Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/13-1.1.1.2-a+b-x-^m-c+d-x-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1917]. This is test number [13].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1917)	0.00 (0)
Mathematica	100.00 (1917)	0.00 (0)
Fricas	83.52 (1601)	16.48 (316)
Maple	81.64 (1565)	18.36 (352)
Maxima	69.27 (1328)	30.73 (589)
Giac	67.87 (1301)	32.13 (616)
Mupad	64.74 (1241)	35.26 (676)
Sympy	63.33 (1214)	36.67 (703)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

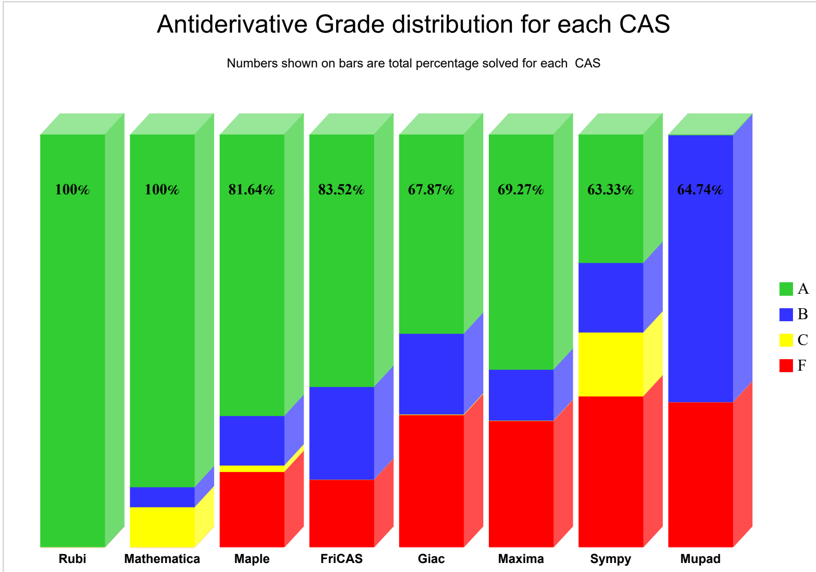
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

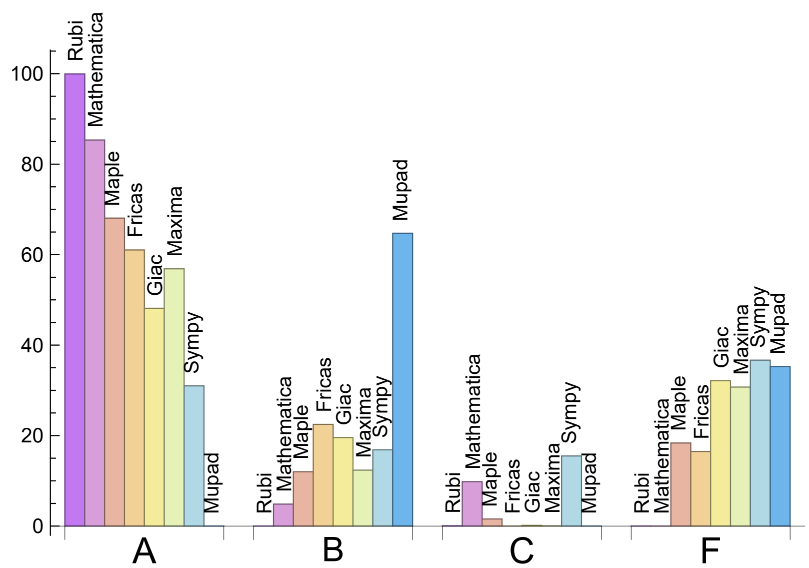
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.95	0.00	0.05	0.00
Mathematica	85.34	4.85	9.81	0.00
Maple	68.08	12.00	1.56	18.36
Fricas	61.03	22.48	0.00	16.48
Maxima	56.86	12.36	0.05	30.73
Giac	48.15	19.56	0.16	32.13
Sympy	30.99	16.85	15.49	36.67
Mupad	N/A	64.74	0.00	35.26

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	352	100.00 %	0.00 %	0.00 %
Fricas	316	100.00 %	0.00 %	0.00 %
Giac	616	68.51 %	2.11 %	29.38 %
Maxima	589	78.10 %	0.00 %	21.90 %
Sympy	703	74.25 %	16.79 %	8.96 %
Mupad	676	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

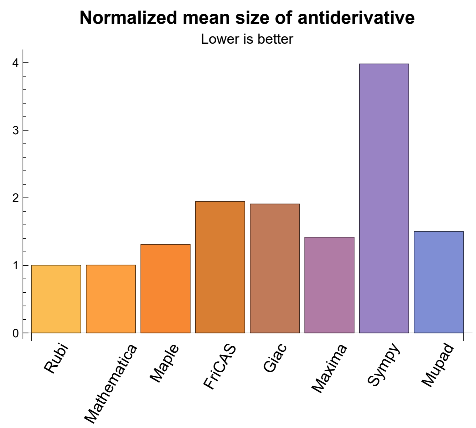
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

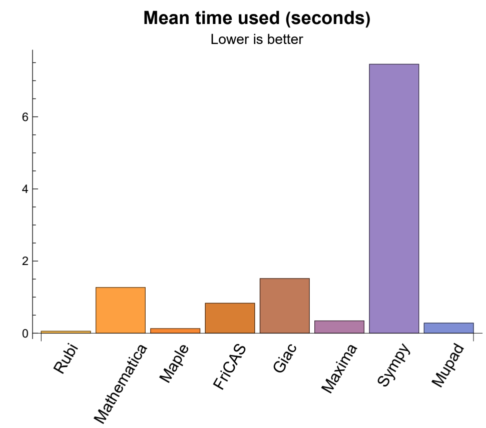
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	108.09	1.00	66.00	1.00
Mathematica	1.27	79.65	1.01	56.00	0.90
Maple	0.13	100.32	1.31	57.00	0.94
Maxima	0.34	107.90	1.42	56.00	0.99
Fricas	0.83	201.07	1.95	77.00	1.34
Sympy	7.46	265.60	3.98	88.00	1.50
Giac	1.52	156.40	1.91	61.00	1.04
Mupad	0.28	122.49	1.50	52.00	0.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {1582, 1583, 1584, 1585, 1586, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1608, 1609, 1610, 1612, 1613, 1614, 1622, 1623, 1624, 1625, 1626, 1627}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
  r"""
  Return the tree size of this expression.
  """
  if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
  expr = SR(expr)
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

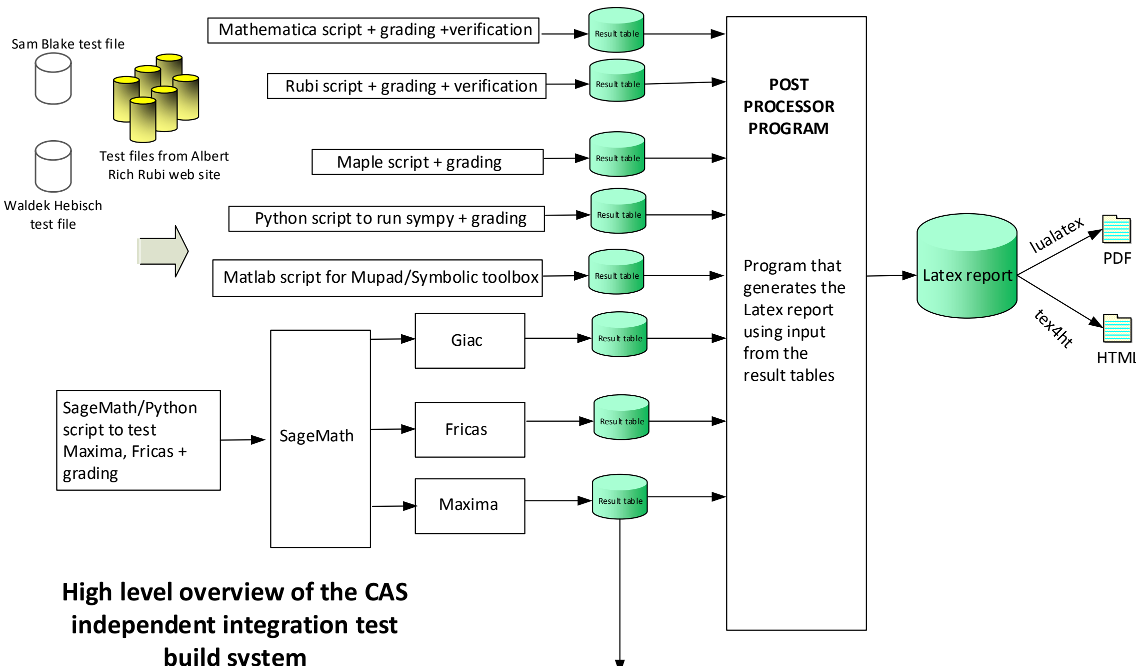
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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B grade: { }

C grade: { 369 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 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1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1800, 1801, 1802, 1803, 1804, 1805, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1826, 1827, 1828, 1829, 1830, 1831, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 1847, 1848, 1849, 1850, 1851, 1852, 1853, 1854, 1855, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 648, 1110, 1154, 1155, 1156, 1157, 1162, 1236, 1246, 1258, 1259, 1268, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1284, 1285, 1288, 1289, 1291, 1292, 1293, 1294, 1295, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1362, 1363, 1365, 1366, 1525, 1527, 1532, 1534, 1537, 1539, 1540, 1543, 1545, 1546, 1548, 1550, 1555, 1628 }

C grade: { 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1189, 1190, 1191, 1192, 1193, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1582, 1583, 1584, 1585, 1586, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1602, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1616, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1680, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1694, 1700, 1701, 1702, 1703, 1704, 1706, 1712, 1713, 1714, 1715, 1716, 1717, 1723, 1724, 1725, 1726, 1727, 1728, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1773, 1780, 1799, 1806, 1825, 1832 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 496, 497, 498, 499, 500, 501, 504, 505, 506, 507, 508, 509, 511, 512, 513, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 580, 581, 582, 583, 587, 588, 589, 590, 591, 592, 594, 595, 596, 599, 600, 601, 605, 606, 607, 608, 609, 610, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 719, 720, 724, 725, 728, 732, 733, 734, 735, 739, 740, 741, 750, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 946, 947, 948, 949, 953, 954, 955, 956, 961, 962, 963, 964, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1102, 1103, 1104, 1106, 1107, 1108, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1131, 1132, 1133, 1134, 1135, 1136, 1140, 1141, 1153, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1167, 1168, 1180, 1181, 1182, 1183, 1186, 1187, 1188, 1196, 1197, 1198, 1214, 1215, 1216, 1226, 1227, 1228, 1229, 1230, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1248, 1249, 1250, 1251, 1252, 1253, 1255, 1256, 1257, 1263, 1264, 1265, 1266, 1267, 1270, 1271, 1272, 1282, 1311, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1469, 1470, 1471, 1472, 1473, 1474, 1481, 1482, 1483, 1484, 1492, 1493, 1494, 1495, 1497, 1498, 1499, 1500, 1501, 1506, 1507, 1508, 1509, 1510, 1511, 1517, 1518, 1519, 1520, 1521, 1530, 1541, 1544, 1578, 1579, 1580, 1581, 1590, 1591, 1592, 1593, 1604, 1605, 1606, 1607, 1618, 1619, 1620, 1621, 1682, 1683, 1684, 1685, 1696, 1697, 1698, 1699, 1708, 1709, 1710, 1711, 1719, 1720, 1721, 1722, 1775, 1776, 1777, 1778, 1782, 1783, 1784, 1785, 1801, 1802, 1803, 1804, 1808, 1809, 1810, 1811, 1827, 1828, 1829, 1830, 1834, 1835, 1836, 1837, 1844, 1848, 1854, 1855, 1860, 1861, 1868, 1869, 1874, 1875, 1881, 1882, 1883, 1884, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 199, 212, 213, 226, 227, 228, 243, 244, 494, 495, 502, 503, 510, 516, 517, 518, 572, 578, 584, 586, 593, 597, 602, 604, 611, 625, 632, 643, 648, 649, 699, 1016, 1017, 1018, 1034, 1066, 1067, 1068, 1069, 1070, 1078, 1079, 1081, 1082, 1083, 1084, 1091, 1098, 1099, 1100, 1101, 1105, 1109, 1110, 1118, 1119, 1129, 1130, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1154, 1155, 1156, 1162, 1166, 1169, 1225, 1236, 1237, 1246, 1247, 1254, 1258, 1259, 1260, 1261, 1262, 1268, 1269, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1343, 1352, 1362, 1363, 1364, 1365, 1366, 1367, 1424, 1434, 1453, 1466, 1467, 1468, 1477, 1478, 1479, 1480, 1488, 1489, 1490, 1491, 1496, 1516, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1542, 1543, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1846, 1847, 1852, 1853, 1859, 1870, 1871, 1876, 1877 }

C grade: { 721, 722, 726, 727, 1171, 1172, 1174, 1175, 1176, 1177, 1178, 1184, 1194, 1195, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1217, 1218, 1219, 1220, 1222, 1223, 1224, 1628 }

F grade: { 369, 579, 585, 598, 603, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 723, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1170, 1173, 1179, 1185, 1189, 1190, 1191, 1192, 1193, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1221, 1231, 1232, 1233, 1234, 1235, 1465, 1475, 1476, 1485, 1486, 1487, 1502, 1503, 1504, 1505, 1512, 1513, 1514, 1515, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1622, 1623, 1624, 1625, 1626, 1627, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1779, 1780, 1781, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1805, 1806, 1807, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1831, 1832, 1833, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 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C grade: { 1027 }

F grade: { 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 754, 755, 760, 761, 762, 763, 769, 770, 771, 778, 779, 808, 809, 810, 811, 817, 818, 819, 824, 825, 826, 827, 856, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 992, 993, 994, 995, 996, 997, 998, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1231, 1232, 1233, 1234, 1235, 1381, 1382, 1383, 1384, 1385, 1386, 1393, 1394, 1395, 1396, 1397, 1398, 1405, 1406, 1407, 1408, 1409, 1410, 1419, 1420, 1421, 1422, 1423, 1430, 1431, 1432, 1433, 1440, 1441, 1442, 1443, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1551, 1554, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 204, 205, 206, 207, 208, 209, 210, 211, 222, 223, 230, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 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2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 224, 225, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 295, 296, 298, 299, 303, 304, 305, 306, 307, 308, 309, 316, 317, 318, 319, 320, 321, 322, 323, 338, 339, 340, 341, 342, 346, 347, 349, 350, 355, 370, 374, 381, 388, 395, 402, 409, 415, 416, 429, 430, 431, 432, 433, 434, 436, 437, 439, 440, 441, 443, 444, 446, 447, 448, 449, 455, 469, 475, 489, 490, 491, 492, 493, 495, 505, 506, 507, 508, 509, 511, 521, 522, 523, 524, 525, 526, 533, 534, 535, 536, 537, 538, 545, 546, 547, 548, 549, 550, 557, 558, 559, 560, 561, 562, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 608, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 625, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 666, 667, 668, 674, 675, 676, 677, 678, 679, 680, 681, 735, 756, 757, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 785, 786, 787, 788, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 812, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 825, 828, 829, 834, 835, 840, 841, 842, 843, 846, 847, 848, 849, 850, 851, 897, 905, 1000, 1001, 1002, 1003, 1008, 1009, 1010, 1011, 1022, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1033, 1035, 1038, 1039, 1040, 1042, 1043, 1044, 1046, 1049, 1050, 1051, 1052, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1104, 1105, 1173, 1191, 1203, 1209, 1221, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1248, 1249, 1251, 1252, 1253, 1255, 1264, 1265, 1266, 1267, 1335, 1336, 1337, 1338, 1339, 1343, 1344, 1345, 1346, 1347, 1348, 1354, 1355, 1356, 1357, 1376, 1377, 1378, 1379, 1380, 1381, 1387, 1388, 1389, 1390, 1392, 1393, 1399, 1400, 1404, 1405, 1418, 1419, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1434, 1435, 1439, 1440, 1451, 1525, 1530, 1532, 1544, 1642, 1855, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917 }

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F grade: { 368, 369, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 590, 591, 592, 593, 608, 609, 610, 611, 629, 630, 631, 632, 649, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 857, 858, 859, 865, 866, 867, 868, 875, 876, 881, 882, 883, 884, 889, 890, 891, 892, 898, 899, 900, 906, 907, 908, 914, 915, 916, 920, 921, 922, 927, 928, 929, 932, 935, 936, 937, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 988, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1231, 1232, 1233, 1234, 1235, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890 }

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C grade: { }

F grade: { 369, 489, 490, 493, 497, 498, 501, 505, 506, 509, 513, 514, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 577, 578, 579, 584, 585, 590, 591, 596, 597, 598, 602, 603, 608, 609, 616, 617, 618, 623, 624, 629, 630, 635, 636, 637, 641, 642, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 756, 757, 758, 761, 762, 764, 765, 766, 767, 768, 771, 772, 773, 774, 775, 776, 780, 788, 796, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 836, 837, 838, 839, 844, 845, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 898, 899, 900, 901, 902, 903, 904, 906, 907, 908, 909, 910, 911, 912, 914, 915, 916, 917, 918, 920, 921, 922, 927, 928, 929, 935, 936, 937, 944, 945, 950, 951, 952, 957, 958, 959, 960, 965, 966, 967, 968, 981, 982, 983, 984, 985, 986, 997, 998, 1063, 1064, 1065, 1066, 1069, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1104, 1106, 1107, 1108, 1116, 1117, 1118, 1119, 1126, 1127, 1128, 1129, 1137, 1138, 1145, 1146, 1153, 1154, 1160, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1231, 1232, 1233, 1234, 1235, 1460, 1461, 1462, 1465, 1471, 1472, 1473, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1492, 1493, 1494, 1502, 1503, 1504, 1505, 1512, 1513, 1514, 1515, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1779, 1780, 1781, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1805, 1806, 1807, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1838, 1839, 1840, 1841, 1842, 1843, 1845, 1849, 1850, 1851, 1856, 1857, 1858, 1862, 1863, 1864, 1865, 1866, 1867, 1872, 1873, 1878, 1879, 1880, 1885, 1886, 1887, 1888, 1889, 1890 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.004	0.000	0.008	0.285	0.734	0.000	1.776	0.041

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.004	0.276	0.998	0.005	1.734	0.005

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00
time (sec)	N/A	0.003	0.000	0.005	0.280	1.137	0.003	1.824	0.006

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.007	0.320	1.009	0.003	2.238	0.004

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	3	3	5	3	3
N.S.	1	1.00	1.00	0.80	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.000	0.000	0.006	0.285	1.113	0.003	2.169	0.008

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00
time (sec)	N/A	0.009	0.000	0.007	0.281	0.829	0.003	1.642	0.002

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00
time (sec)	N/A	0.001	0.000	0.008	0.282	0.871	0.003	2.162	0.002

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.000	0.000	0.008	0.282	1.428	0.003	1.788	0.002

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	18	10	11	11
N.S.	1	1.00	1.00	0.86	0.79	1.29	0.71	0.79	0.79
time (sec)	N/A	0.033	0.000	0.022	0.288	0.920	0.003	1.806	0.003

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71
time (sec)	N/A	0.001	0.000	0.011	0.301	0.690	0.019	1.430	0.118

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71
time (sec)	N/A	0.001	0.000	0.005	0.285	1.880	0.018	1.472	0.022

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71
time (sec)	N/A	0.001	0.000	0.005	0.325	1.041	0.019	1.367	0.010

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	3	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71
time (sec)	N/A	0.000	0.000	0.007	0.291	0.904	0.004	1.155	0.012

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.000	0.000	0.003	0.292	0.666	0.003	1.207	0.002

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	3	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00
time (sec)	N/A	0.000	0.000	0.005	0.360	0.535	0.020	1.316	0.036

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.00	1.00
time (sec)	N/A	0.000	0.000	0.007	0.303	0.997	0.021	1.778	0.035

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.009	0.273	1.059	0.021	1.306	0.014

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.001	0.000	0.007	0.271	1.695	0.020	1.282	0.012

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.025	0.290	1.240	0.021	1.151	0.070

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.020	0.292	1.195	0.022	1.804	0.077

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.016	0.284	0.986	0.021	1.286	0.031

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.017	0.275	0.798	0.021	1.034	0.029

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.000	0.000	0.017	0.286	0.614	0.021	1.799	0.031

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.017	0.335	0.869	0.021	1.308	0.031

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	8	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.89	0.56	0.56
time (sec)	N/A	0.000	0.000	0.020	0.325	0.912	0.021	2.301	0.034

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.001	0.019	0.292	0.930	0.021	1.765	0.073

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.019	0.279	0.936	0.022	2.079	0.066

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.027	0.293	1.055	0.021	1.251	0.065

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.018	0.280	0.972	0.021	1.419	0.065

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56
time (sec)	N/A	0.000	0.000	0.018	0.286	0.871	0.020	1.781	0.040

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	5	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71
time (sec)	N/A	0.000	0.000	0.018	0.306	1.010	0.021	1.710	0.067

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.020	0.286	1.186	0.020	1.872	0.073

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	5	5	8	5	5
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.89	0.56	0.56
time (sec)	N/A	0.000	0.000	0.018	0.288	0.775	0.021	1.924	0.051

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	10	12	11	20
N.S.	1	1.00	1.00	1.09	1.00	0.91	1.09	1.00	1.82
time (sec)	N/A	0.005	0.001	0.007	0.287	0.988	0.005	1.558	0.345

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	17	16	12	17	16	12
N.S.	1	1.00	0.75	1.06	1.00	0.75	1.06	1.00	0.75
time (sec)	N/A	0.017	0.001	0.010	0.293	0.896	0.006	1.602	0.183

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	21	19	22	21
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.83	0.96	0.91
time (sec)	N/A	0.046	0.003	0.105	0.278	1.051	0.012	1.328	0.143

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	104	270	444	93
N.S.	1	1.00	1.00	0.87	0.83	4.52	11.74	19.30	4.04
time (sec)	N/A	0.027	0.022	0.093	0.291	1.815	1.381	1.851	0.183

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	59	153	195	45
N.S.	1	1.00	1.00	0.87	0.83	2.57	6.65	8.48	1.96
time (sec)	N/A	0.008	0.017	0.096	0.277	0.981	0.302	1.271	0.170

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	78	19	19
N.S.	1	1.00	1.00	0.87	0.83	0.83	3.39	0.83	0.83
time (sec)	N/A	0.008	0.013	0.092	0.282	1.058	0.094	1.825	0.075

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	19	31	19	19
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.48	0.90	0.90
time (sec)	N/A	0.008	0.011	0.092	0.271	0.582	0.607	1.206	0.106

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	34	58	19	19
N.S.	1	1.00	1.00	0.95	0.90	1.62	2.76	0.90	0.90
time (sec)	N/A	0.008	0.016	0.084	0.269	0.910	0.526	1.571	0.135

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	68	102	19	19
N.S.	1	1.00	1.00	0.87	0.83	2.96	4.43	0.83	0.83
time (sec)	N/A	0.008	0.015	0.081	0.274	1.135	1.324	1.198	0.179

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.016	0.001	0.011	0.287	0.850	0.006	1.214	0.020

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.001	0.013	0.267	0.551	0.005	1.374	0.019

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.001	0.010	0.269	0.603	0.005	1.178	0.019

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.010	0.292	0.804	0.005	1.456	0.017

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	9	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00
time (sec)	N/A	0.003	0.001	0.012	0.268	0.798	0.018	2.240	0.017

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	7	12	11
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.64	1.09	1.00
time (sec)	N/A	0.006	0.002	0.012	0.274	0.667	0.024	1.479	0.033

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	15	14	11	11	12	11	11
N.S.	1	1.00	0.88	0.82	0.65	0.65	0.71	0.65	0.65
time (sec)	N/A	0.001	0.001	0.010	0.268	0.596	0.028	1.640	0.023

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.004	0.002	0.010	0.284	0.600	0.032	1.444	0.026

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.006	0.001	0.011	0.276	0.935	0.036	1.475	0.027

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.014	0.002	0.071	0.281	1.136	0.007	1.106	0.077

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.008	0.001	0.074	0.272	1.407	0.007	1.409	0.031

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.008	0.001	0.069	0.275	1.580	0.007	1.709	0.030

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43
time (sec)	N/A	0.001	0.001	0.087	0.282	1.645	0.007	1.141	0.029

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	20	20	21	20
N.S.	1	1.00	1.00	0.95	0.91	0.91	0.91	0.95	0.91
time (sec)	N/A	0.006	0.001	0.072	0.274	1.474	0.022	1.303	0.029

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	17	21	20
N.S.	1	1.00	1.00	1.05	1.00	1.20	0.85	1.05	1.00
time (sec)	N/A	0.006	0.001	0.073	0.268	1.066	0.032	1.782	0.066

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	21	26	22	22	23
N.S.	1	1.00	1.00	0.96	0.88	1.08	0.92	0.92	0.96
time (sec)	N/A	0.008	0.003	0.078	0.277	1.523	0.048	1.372	0.044

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	26	25	22	22	24	22	22
N.S.	1	1.00	1.53	1.47	1.29	1.29	1.41	1.29	1.29
time (sec)	N/A	0.001	0.005	0.072	0.274	1.101	0.051	1.124	0.035

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.006	0.003	0.073	0.274	1.443	0.057	1.048	0.035

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.006	0.005	0.084	0.274	1.244	0.063	1.261	0.035

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.010	0.003	0.086	0.271	1.186	0.068	1.402	0.034

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.009	0.005	0.087	0.272	1.106	0.075	1.193	0.036

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.029	0.002	0.074	0.267	1.278	0.008	1.444	0.042

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.014	0.002	0.070	0.272	1.040	0.008	1.490	0.041

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81
time (sec)	N/A	0.013	0.002	0.073	0.269	0.677	0.008	1.461	0.039

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	35	34	34	36	34	34
N.S.	1	1.00	1.33	1.17	1.13	1.13	1.20	1.13	1.13
time (sec)	N/A	0.006	0.001	0.080	0.293	0.787	0.009	1.412	0.040

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	12	31
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.21
time (sec)	N/A	0.001	0.001	0.070	0.266	0.867	0.008	1.307	0.041

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	34	32	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.97	0.91	0.89
time (sec)	N/A	0.008	0.002	0.079	0.268	1.137	0.027	1.760	0.035

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	31	33	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.91	0.97	0.94
time (sec)	N/A	0.010	0.003	0.084	0.270	1.136	0.035	1.301	0.035

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	32	30	37	32	31	32
N.S.	1	1.00	1.00	0.97	0.91	1.12	0.97	0.94	0.97
time (sec)	N/A	0.011	0.004	0.088	0.295	0.970	0.056	1.498	0.030

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	37	36	35	34
N.S.	1	1.00	1.00	0.92	0.92	1.00	0.97	0.95	0.92
time (sec)	N/A	0.012	0.003	0.086	0.288	1.245	0.073	1.382	0.071

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	39	36	33	33	36	33	33
N.S.	1	1.00	2.29	2.12	1.94	1.94	2.12	1.94	1.94
time (sec)	N/A	0.001	0.003	0.073	0.272	1.023	0.078	1.606	0.026

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	35	35	37	35	34
N.S.	1	1.00	1.14	1.00	0.97	0.97	1.03	0.97	0.94
time (sec)	N/A	0.004	0.004	0.073	0.270	0.977	0.086	1.653	0.027

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.010	0.003	0.082	0.273	0.920	0.093	1.533	0.025

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.010	0.003	0.088	0.280	1.112	0.101	1.515	0.026

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	56
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.053	0.002	0.090	0.275	1.094	0.010	1.701	0.025

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83
time (sec)	N/A	0.026	0.002	0.073	0.285	1.013	0.010	1.543	0.023

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83
time (sec)	N/A	0.020	0.002	0.082	0.275	1.026	0.010	1.469	0.024

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	57	56	56	63	56	56
N.S.	1	1.00	1.03	0.89	0.88	0.88	0.98	0.88	0.88
time (sec)	N/A	0.024	0.002	0.086	0.271	1.406	0.011	1.270	0.024

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	67	58	57	57	65	57	57
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.38	1.21	1.21
time (sec)	N/A	0.017	0.002	0.072	0.268	0.865	0.011	1.330	0.024

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	67	58	57	57	65	57	57
N.S.	1	1.00	2.23	1.93	1.90	1.90	2.17	1.90	1.90
time (sec)	N/A	0.006	0.002	0.095	0.266	0.858	0.012	1.464	0.023

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	53	53	60	12	53
N.S.	1	1.00	1.00	0.93	3.79	3.79	4.29	0.86	3.79
time (sec)	N/A	0.001	0.001	0.071	0.274	0.884	0.011	1.395	0.024

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	53	53	60	54	53
N.S.	1	1.00	1.00	0.92	0.90	0.90	1.02	0.92	0.90
time (sec)	N/A	0.013	0.003	0.085	0.286	0.610	0.034	1.625	0.029

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	54	59	56	55	54
N.S.	1	1.00	1.00	0.95	0.93	1.02	0.97	0.95	0.93
time (sec)	N/A	0.015	0.004	0.095	0.271	0.768	0.043	2.207	0.029

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	53	59	60	54	55
N.S.	1	1.00	1.00	0.92	0.88	0.98	1.00	0.90	0.92
time (sec)	N/A	0.017	0.004	0.075	0.280	0.949	0.066	2.018	0.029

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	55	59	60	56	55
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	0.93	0.92
time (sec)	N/A	0.016	0.004	0.098	0.272	0.968	0.085	2.352	0.039

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	54	59	58	55	54
N.S.	1	1.00	1.00	0.95	0.95	1.04	1.02	0.96	0.95
time (sec)	N/A	0.017	0.004	0.095	0.290	0.904	0.112	1.527	0.077

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	56	59	60	57	56
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.98	0.93	0.92
time (sec)	N/A	0.016	0.004	0.073	0.278	0.913	0.137	2.240	0.041

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	65	58	55	55	60	55	55
N.S.	1	1.00	3.82	3.41	3.24	3.24	3.53	3.24	3.24
time (sec)	N/A	0.001	0.003	0.076	0.286	0.538	0.146	2.071	0.039

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	67	58	57	57	61	57	57
N.S.	1	1.00	1.86	1.61	1.58	1.58	1.69	1.58	1.58
time (sec)	N/A	0.004	0.003	0.092	0.289	1.537	0.159	1.784	0.073

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	67	58	57	57	61	57	57
N.S.	1	1.00	1.20	1.04	1.02	1.02	1.09	1.02	1.02
time (sec)	N/A	0.008	0.003	0.080	0.276	1.221	0.171	2.205	0.039

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	57	56
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84
time (sec)	N/A	0.017	0.005	0.089	0.283	1.063	0.181	1.619	0.084

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	61	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.88	0.83	0.83
time (sec)	N/A	0.017	0.003	0.088	0.271	0.754	0.192	2.131	0.082

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	61	57	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.88	0.83	0.83
time (sec)	N/A	0.016	0.003	0.084	0.271	0.961	0.204	2.019	0.042

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	57	56
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84
time (sec)	N/A	0.017	0.003	0.088	0.290	1.035	0.215	1.438	0.040

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	57	56
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84
time (sec)	N/A	0.016	0.003	0.090	0.273	0.897	0.225	2.876	0.038

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	94	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.060	0.002	0.091	0.277	1.013	0.012	2.150	0.153

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	92	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.97	0.83	0.83
time (sec)	N/A	0.034	0.002	0.084	0.306	1.124	0.012	1.751	0.074

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	94	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83
time (sec)	N/A	0.027	0.002	0.089	0.281	0.993	0.012	1.332	0.068

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	92	79	78	78	90	78	78
N.S.	1	1.00	0.96	0.82	0.81	0.81	0.94	0.81	0.81
time (sec)	N/A	0.033	0.002	0.073	0.273	0.960	0.014	1.120	0.060

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	93	80	79	79	92	79	79
N.S.	1	1.00	1.15	0.99	0.98	0.98	1.14	0.98	0.98
time (sec)	N/A	0.026	0.002	0.072	0.282	0.939	0.014	1.193	0.062

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	93	80	79	79	92	79	79
N.S.	1	1.00	1.45	1.25	1.23	1.23	1.44	1.23	1.23
time (sec)	N/A	0.022	0.002	0.094	0.290	1.086	0.014	1.075	0.104

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	93	80	79	79	92	79	31
N.S.	1	1.00	1.98	1.70	1.68	1.68	1.96	1.68	0.66
time (sec)	N/A	0.017	0.002	0.076	0.272	0.972	0.014	0.932	0.121

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	91	80	79	79	90	79	25
N.S.	1	1.00	3.03	2.67	2.63	2.63	3.00	2.63	0.83
time (sec)	N/A	0.005	0.002	0.100	0.297	1.055	0.015	1.073	0.115

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	75	83	12	75
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.36
time (sec)	N/A	0.001	0.001	0.075	0.276	1.156	0.014	1.291	0.060

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	76	75	75	88	76	75
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	0.87	0.86
time (sec)	N/A	0.018	0.003	0.079	0.266	0.993	0.042	1.263	0.072

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	76	81	85	77	76
N.S.	1	1.00	1.00	0.90	0.88	0.94	0.99	0.90	0.88
time (sec)	N/A	0.023	0.003	0.079	0.275	0.786	0.050	1.316	0.055

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	75	81	85	76	77
N.S.	1	1.00	1.00	0.92	0.89	0.96	1.01	0.90	0.92
time (sec)	N/A	0.023	0.004	0.084	0.280	0.883	0.072	1.360	0.051

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	77	81	87	78	77
N.S.	1	1.00	1.00	0.90	0.90	0.94	1.01	0.91	0.90
time (sec)	N/A	0.021	0.004	0.076	0.301	0.807	0.094	2.242	0.051

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	77	77	81	85	78	77
N.S.	1	1.00	1.00	0.90	0.90	0.94	0.99	0.91	0.90
time (sec)	N/A	0.022	0.003	0.077	0.285	0.981	0.116	1.225	0.090

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	77	77	81	83	78	77
N.S.	1	1.00	1.00	0.92	0.92	0.96	0.99	0.93	0.92
time (sec)	N/A	0.021	0.004	0.079	0.283	1.105	0.154	1.337	0.106

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	76	76	81	82	77	81
N.S.	1	1.00	1.00	0.89	0.89	0.95	0.96	0.91	0.95
time (sec)	N/A	0.021	0.004	0.077	0.279	1.065	0.189	1.867	0.110

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	78	78	81	83	79	78
N.S.	1	1.00	1.00	0.88	0.88	0.91	0.93	0.89	0.88
time (sec)	N/A	0.023	0.004	0.091	0.273	0.968	0.217	1.216	0.068

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	87	80	77	77	83	77	77
N.S.	1	1.00	5.12	4.71	4.53	4.53	4.88	4.53	4.53
time (sec)	N/A	0.001	0.003	0.074	0.275	1.286	0.231	0.835	0.067

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	80	79	79	85	79	23
N.S.	1	1.00	2.53	2.22	2.19	2.19	2.36	2.19	0.64
time (sec)	N/A	0.004	0.003	0.073	0.267	0.717	0.246	1.296	0.093

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	93	80	79	79	85	79	79
N.S.	1	1.00	1.66	1.43	1.41	1.41	1.52	1.41	1.41
time (sec)	N/A	0.007	0.003	0.073	0.283	0.761	0.262	1.232	0.109

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	93	80	79	79	85	79	79
N.S.	1	1.00	1.22	1.05	1.04	1.04	1.12	1.04	1.04
time (sec)	N/A	0.013	0.003	0.079	0.301	0.760	0.280	1.725	0.106

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	93	80	79	79	85	79	79
N.S.	1	1.00	0.97	0.83	0.82	0.82	0.89	0.82	0.82
time (sec)	N/A	0.018	0.003	0.075	0.287	0.946	0.294	1.710	0.066

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	80	79	79	85	79	78
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.91	0.85	0.84
time (sec)	N/A	0.022	0.005	0.073	0.270	1.161	0.307	1.396	0.066

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	85	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.89	0.83	0.83
time (sec)	N/A	0.021	0.003	0.077	0.270	1.095	0.324	1.175	0.070

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	80	79	79	85	79	79
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.89	0.83	0.83
time (sec)	N/A	0.022	0.003	0.076	0.279	1.061	0.332	1.745	0.110

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	133	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	0.85	0.85
time (sec)	N/A	0.081	0.003	0.078	0.283	1.256	0.015	1.364	0.150

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	131	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85
time (sec)	N/A	0.042	0.002	0.073	0.277	1.016	0.015	1.659	0.084

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	113	112	112	133	112	112
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	0.85	0.85
time (sec)	N/A	0.040	0.002	0.075	0.280	1.229	0.015	1.620	0.125

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	112	111	111	126	111	111
N.S.	1	1.00	0.85	0.76	0.76	0.76	0.86	0.76	0.76
time (sec)	N/A	0.044	0.003	0.091	0.270	1.544	0.018	1.675	0.087

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	130	113	112	112	131	112	112
N.S.	1	1.00	0.98	0.86	0.85	0.85	0.99	0.85	0.85
time (sec)	N/A	0.039	0.002	0.075	0.275	0.830	0.018	1.129	0.082

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	126	113	112	112	128	112	112
N.S.	1	1.00	1.12	1.01	1.00	1.00	1.14	1.00	1.00
time (sec)	N/A	0.033	0.002	0.075	0.283	0.956	0.018	1.507	0.121

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	132	113	112	112	133	112	112
N.S.	1	1.00	1.35	1.15	1.14	1.14	1.36	1.14	1.14
time (sec)	N/A	0.031	0.003	0.077	0.289	0.913	0.020	2.220	0.123

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	130	113	112	112	131	112	112
N.S.	1	1.00	1.60	1.40	1.38	1.38	1.62	1.38	1.38
time (sec)	N/A	0.030	0.002	0.080	0.278	0.679	0.018	1.211	0.120

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	128	113	112	112	129	112	112
N.S.	1	1.00	2.00	1.77	1.75	1.75	2.02	1.75	1.75
time (sec)	N/A	0.023	0.002	0.076	0.275	0.664	0.018	1.422	0.118

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	126	113	112	112	128	112	31
N.S.	1	1.00	2.68	2.40	2.38	2.38	2.72	2.38	0.66
time (sec)	N/A	0.019	0.002	0.075	0.275	1.371	0.019	1.238	0.070

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	128	113	112	112	129	112	25
N.S.	1	1.00	4.27	3.77	3.73	3.73	4.30	3.73	0.83
time (sec)	N/A	0.005	0.002	0.075	0.277	0.680	0.020	1.527	0.095

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	108	114	12	108
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.71
time (sec)	N/A	0.001	0.001	0.082	0.281	0.646	0.018	1.413	0.112

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	109	108	108	126	109	108
N.S.	1	1.00	1.00	0.89	0.89	0.89	1.03	0.89	0.89
time (sec)	N/A	0.027	0.003	0.118	0.295	1.196	0.055	1.543	0.079

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	109	114	117	110	109
N.S.	1	1.00	1.00	0.96	0.95	0.99	1.02	0.96	0.95
time (sec)	N/A	0.032	0.007	0.078	0.276	1.238	0.065	1.084	0.115

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	108	114	122	109	110
N.S.	1	1.00	1.00	0.92	0.91	0.96	1.03	0.92	0.92
time (sec)	N/A	0.034	0.004	0.096	0.316	1.227	0.087	1.442	0.071

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	108	114	119	109	110
N.S.	1	1.00	1.00	0.96	0.94	0.99	1.03	0.95	0.96
time (sec)	N/A	0.032	0.007	0.077	0.292	0.604	0.109	1.707	0.062

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	121	111	110
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.02	0.93	0.92
time (sec)	N/A	0.033	0.006	0.079	0.272	0.535	0.137	1.158	0.098

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	110	110	114	121	111	110
N.S.	1	1.00	1.00	0.94	0.94	0.97	1.03	0.95	0.94
time (sec)	N/A	0.033	0.007	0.078	0.347	0.480	0.171	1.579	0.099

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	122	111	110
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.03	0.93	0.92
time (sec)	N/A	0.032	0.004	0.077	0.294	0.425	0.204	1.751	0.055

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	110	110	114	119	111	110
N.S.	1	1.00	1.00	0.96	0.96	0.99	1.03	0.97	0.96
time (sec)	N/A	0.034	0.007	0.082	0.272	0.513	0.243	1.091	0.095

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	110	110	114	119	111	110
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.00	0.93	0.92
time (sec)	N/A	0.032	0.004	0.076	0.274	0.485	0.284	1.394	0.068

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	109	114	117	110	114
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	0.96	1.00
time (sec)	N/A	0.033	0.005	0.082	0.273	0.434	0.329	1.795	0.076

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	124	111	111	114	119	112	111
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.96	0.90	0.90
time (sec)	N/A	0.046	0.004	0.077	0.288	0.450	0.374	1.285	0.074

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	114	113	110	110	119	110	110
N.S.	1	1.00	6.71	6.65	6.47	6.47	7.00	6.47	6.47
time (sec)	N/A	0.002	0.007	0.080	0.287	0.526	0.395	1.050	0.134

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	128	113	112	112	121	112	23
N.S.	1	1.00	3.56	3.14	3.11	3.11	3.36	3.11	0.64
time (sec)	N/A	0.006	0.004	0.075	0.281	0.461	0.412	1.239	0.096

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	126	113	112	112	121	112	112
N.S.	1	1.00	2.25	2.02	2.00	2.00	2.16	2.00	2.00
time (sec)	N/A	0.011	0.007	0.075	0.281	0.486	0.440	1.090	0.132

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	128	113	112	112	121	112	112
N.S.	1	1.00	1.68	1.49	1.47	1.47	1.59	1.47	1.47
time (sec)	N/A	0.016	0.006	0.075	0.273	0.553	0.457	1.637	0.094

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	130	113	112	112	121	112	112
N.S.	1	1.00	1.35	1.18	1.17	1.17	1.26	1.17	1.17
time (sec)	N/A	0.031	0.007	0.078	0.270	0.614	0.481	1.626	0.130

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	132	113	112	112	121	112	112
N.S.	1	1.00	1.14	0.97	0.97	0.97	1.04	0.97	0.97
time (sec)	N/A	0.046	0.004	0.085	0.266	0.571	0.499	0.859	0.135

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	126	113	112	112	121	112	112
N.S.	1	1.00	0.93	0.83	0.82	0.82	0.89	0.82	0.82
time (sec)	N/A	0.067	0.007	0.090	0.272	0.664	0.529	1.010	0.134

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	113	112	112	121	112	112
N.S.	1	1.00	1.00	0.87	0.86	0.86	0.93	0.86	0.86
time (sec)	N/A	0.039	0.004	0.076	0.282	1.000	0.544	1.059	0.099

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	126	113	112	112	121	112	111
N.S.	1	1.00	1.00	0.90	0.89	0.89	0.96	0.89	0.88
time (sec)	N/A	0.034	0.006	0.076	0.268	0.819	0.567	1.197	0.137

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	13	13	12	12	13	11
N.S.	1	1.00	0.93	0.87	0.87	0.80	0.80	0.87	0.73
time (sec)	N/A	0.001	0.001	0.009	0.302	0.888	0.005	1.304	0.021

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	18	17	26	22	17	16
N.S.	1	1.00	0.95	0.90	0.85	1.30	1.10	0.85	0.80
time (sec)	N/A	0.004	0.001	0.008	0.263	1.365	0.009	1.440	0.075

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	63	64	63	61	65	62
N.S.	1	1.00	1.00	0.90	0.91	0.90	0.87	0.93	0.89
time (sec)	N/A	0.026	0.004	0.085	0.276	0.875	0.042	1.258	0.080

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89
time (sec)	N/A	0.018	0.004	0.102	0.275	0.796	0.038	1.342	0.100

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91
time (sec)	N/A	0.014	0.003	0.084	0.276	0.906	0.035	0.966	0.038

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.010	0.003	0.077	0.263	0.929	0.031	1.281	0.039

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.006	0.002	0.076	0.275	0.934	0.026	0.797	0.076

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.001	0.078	0.261	1.175	0.008	1.159	0.021

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	20	15
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.002	0.003	0.082	0.268	1.167	0.046	1.600	0.085

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.010	0.004	0.077	0.281	0.974	0.062	1.444	0.051

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.012	0.004	0.082	0.267	1.078	0.073	1.378	0.058

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	48
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86
time (sec)	N/A	0.015	0.004	0.086	0.275	1.335	0.086	0.978	0.105

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	62	65	56	67	60
N.S.	1	1.00	1.00	0.93	0.91	0.96	0.82	0.99	0.88
time (sec)	N/A	0.021	0.004	0.088	0.269	0.892	0.100	0.812	0.065

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	78	82	96	78	103	83
N.S.	1	1.00	0.95	0.96	1.01	1.19	0.96	1.27	1.02
time (sec)	N/A	0.042	0.020	0.082	0.272	1.033	0.089	0.727	0.140

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	68	70	85	71	90	72
N.S.	1	1.00	0.92	0.94	0.97	1.18	0.99	1.25	1.00
time (sec)	N/A	0.031	0.014	0.084	0.268	0.978	0.081	1.116	0.070

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	79	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.36	1.07
time (sec)	N/A	0.025	0.015	0.082	0.276	0.933	0.074	1.206	0.067

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	46	47	62	44	66	50
N.S.	1	1.00	0.93	1.00	1.02	1.35	0.96	1.43	1.09
time (sec)	N/A	0.019	0.010	0.084	0.264	0.657	0.065	1.586	0.079

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	50	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.52	1.09
time (sec)	N/A	0.013	0.010	0.089	0.278	0.786	0.059	0.906	0.081

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	42	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.83	1.00
time (sec)	N/A	0.009	0.005	0.081	0.286	0.676	0.042	1.583	0.036

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00
time (sec)	N/A	0.001	0.004	0.077	0.268	0.596	0.042	1.881	0.029

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	38	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.31	0.90
time (sec)	N/A	0.011	0.010	0.084	0.296	0.564	0.078	1.417	0.122

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	52	45
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.07
time (sec)	N/A	0.015	0.025	0.091	0.266	0.629	0.102	0.991	0.120

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	74	57
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.28	0.98
time (sec)	N/A	0.020	0.037	0.087	0.266	0.569	0.119	1.067	0.110

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	90	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.30	1.00
time (sec)	N/A	0.025	0.037	0.083	0.278	1.015	0.136	0.929	0.080

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	104	79
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.24	0.94
time (sec)	N/A	0.035	0.031	0.091	0.266	1.199	0.151	0.941	0.116

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	89	94	103	129	109	95	91
N.S.	1	1.00	0.90	0.95	1.04	1.30	1.10	0.96	0.92
time (sec)	N/A	0.057	0.019	0.095	0.274	1.250	0.147	1.154	0.234

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	83	91	117	92	83	78
N.S.	1	1.00	0.90	0.97	1.06	1.36	1.07	0.97	0.91
time (sec)	N/A	0.038	0.017	0.085	0.263	1.052	0.137	1.210	0.158

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	72	81	107	85	73	67
N.S.	1	1.00	0.87	0.94	1.05	1.39	1.10	0.95	0.87
time (sec)	N/A	0.032	0.015	0.085	0.271	0.930	0.127	1.601	0.124

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	62	69	95	70	61	54
N.S.	1	1.00	0.86	0.97	1.08	1.48	1.09	0.95	0.84
time (sec)	N/A	0.024	0.014	0.096	0.268	2.062	0.123	0.957	0.078

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	40	49	57	83	58	44	43
N.S.	1	1.00	0.80	0.98	1.14	1.66	1.16	0.88	0.86
time (sec)	N/A	0.019	0.029	0.085	0.275	0.763	0.107	0.696	0.146

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	40	48	61	46	37	46
N.S.	1	1.00	0.80	0.98	1.17	1.49	1.12	0.90	1.12
time (sec)	N/A	0.014	0.009	0.083	0.277	1.019	0.077	0.622	0.093

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	27	32	32	32	18	32
N.S.	1	1.00	1.18	1.59	1.88	1.88	1.88	1.06	1.88
time (sec)	N/A	0.001	0.005	0.085	0.268	1.228	0.069	0.533	0.072

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	24	26	12	26
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86
time (sec)	N/A	0.001	0.002	0.087	0.260	0.906	0.069	0.578	0.068

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	42	51	80	46	43	43
N.S.	1	1.00	0.86	0.98	1.19	1.86	1.07	1.00	1.00
time (sec)	N/A	0.015	0.023	0.095	0.268	1.182	0.124	0.546	0.100

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	56	69	109	66	60	63
N.S.	1	1.00	0.93	0.98	1.21	1.91	1.16	1.05	1.11
time (sec)	N/A	0.020	0.034	0.086	0.283	1.056	0.153	0.799	0.114

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	73	86	130	78	73	79
N.S.	1	1.00	0.89	0.96	1.13	1.71	1.03	0.96	1.04
time (sec)	N/A	0.027	0.037	0.090	0.280	0.912	0.165	1.052	0.119

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	84	97	141	92	86	91
N.S.	1	1.00	0.89	0.94	1.09	1.58	1.03	0.97	1.02
time (sec)	N/A	0.030	0.049	0.092	0.308	0.948	0.180	0.947	0.128

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	94	108	152	102	97	101
N.S.	1	1.00	0.93	0.97	1.11	1.57	1.05	1.00	1.04
time (sec)	N/A	0.037	0.041	0.085	0.287	1.044	0.198	0.947	0.092

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	101	109	125	162	131	106	103
N.S.	1	1.00	0.89	0.96	1.10	1.42	1.15	0.93	0.90
time (sec)	N/A	0.063	0.025	0.085	0.265	0.713	0.217	1.214	0.370

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	90	99	114	151	119	95	90
N.S.	1	1.00	0.86	0.94	1.09	1.44	1.13	0.90	0.86
time (sec)	N/A	0.052	0.022	0.097	0.278	0.688	0.202	1.415	0.221

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	90	87	102	139	107	83	79
N.S.	1	1.00	1.00	0.97	1.13	1.54	1.19	0.92	0.88
time (sec)	N/A	0.044	0.016	0.085	0.275	1.698	0.189	1.454	0.153

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	77	91	129	94	72	66
N.S.	1	1.00	0.84	0.95	1.12	1.59	1.16	0.89	0.81
time (sec)	N/A	0.033	0.020	0.084	0.278	0.720	0.178	0.915	0.125

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	64	79	116	82	55	55
N.S.	1	1.00	0.78	0.98	1.22	1.78	1.26	0.85	0.85
time (sec)	N/A	0.028	0.034	0.084	0.282	0.715	0.158	1.963	0.173

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	44	55	70	94	70	46	45
N.S.	1	1.00	0.76	0.95	1.21	1.62	1.21	0.79	0.78
time (sec)	N/A	0.021	0.012	0.081	0.273	0.849	0.120	1.195	0.070

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	41	54	54	56	29	56
N.S.	1	1.00	1.82	2.41	3.18	3.18	3.29	1.71	3.29
time (sec)	N/A	0.001	0.008	0.090	0.267	0.781	0.102	1.536	0.085

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	43	43	44	18	44
N.S.	1	1.00	0.67	0.90	1.43	1.43	1.47	0.60	1.47
time (sec)	N/A	0.009	0.004	0.079	0.325	0.856	0.097	1.310	0.072

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	35	37	12	37
N.S.	1	1.00	1.00	0.93	0.86	2.50	2.64	0.86	2.64
time (sec)	N/A	0.001	0.002	0.080	0.274	2.219	0.095	0.969	0.077

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	54	73	124	70	54	60
N.S.	1	1.00	0.84	0.95	1.28	2.18	1.23	0.95	1.05
time (sec)	N/A	0.019	0.023	0.085	0.274	0.889	0.166	0.969	0.128

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	69	91	153	90	71	85
N.S.	1	1.00	0.91	0.99	1.30	2.19	1.29	1.01	1.21
time (sec)	N/A	0.027	0.041	0.091	0.273	0.884	0.191	1.234	0.084

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	88	108	174	104	86	101
N.S.	1	1.00	0.85	0.95	1.16	1.87	1.12	0.92	1.09
time (sec)	N/A	0.033	0.039	0.085	0.269	0.757	0.212	1.429	0.136

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	88	99	117	183	114	93	113
N.S.	1	1.00	0.86	0.97	1.15	1.79	1.12	0.91	1.11
time (sec)	N/A	0.039	0.039	0.085	0.289	0.720	0.223	1.239	0.104

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	110	130	196	128	108	123
N.S.	1	1.00	0.86	0.94	1.11	1.68	1.09	0.92	1.05
time (sec)	N/A	0.050	0.047	0.087	0.277	1.123	0.238	1.290	0.173

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	144	180	250	190	128	126
N.S.	1	1.00	0.93	0.96	1.20	1.67	1.27	0.85	0.84
time (sec)	N/A	0.102	0.018	0.091	0.288	0.926	0.414	1.963	1.086

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	128	132	169	239	180	117	115
N.S.	1	1.00	0.92	0.95	1.22	1.72	1.29	0.84	0.83
time (sec)	N/A	0.080	0.021	0.105	0.288	0.936	0.386	1.458	0.553

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	122	157	228	165	105	102
N.S.	1	1.00	0.81	0.95	1.23	1.78	1.29	0.82	0.80
time (sec)	N/A	0.063	0.033	0.087	0.314	0.928	0.359	1.098	0.179

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	109	145	215	153	88	91
N.S.	1	1.00	0.88	0.92	1.23	1.82	1.30	0.75	0.77
time (sec)	N/A	0.052	0.020	0.087	0.289	0.720	0.336	1.140	0.336

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	77	100	136	193	141	79	81
N.S.	1	1.00	0.71	0.92	1.25	1.77	1.29	0.72	0.74
time (sec)	N/A	0.043	0.016	0.083	0.280	0.912	0.267	1.233	0.106

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	64	87	120	120	128	62	72
N.S.	1	1.00	3.76	5.12	7.06	7.06	7.53	3.65	4.24
time (sec)	N/A	0.001	0.008	0.082	0.275	0.626	0.232	1.025	0.122

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	53	72	109	109	116	51	22
N.S.	1	1.00	1.51	2.06	3.11	3.11	3.31	1.46	0.63
time (sec)	N/A	0.004	0.007	0.082	0.300	0.529	0.216	1.451	0.072

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	64	42	57	98	98	104	40	48
N.S.	1	1.23	0.81	1.10	1.88	1.88	2.00	0.77	0.92
time (sec)	N/A	0.022	0.007	0.084	0.293	0.808	0.198	0.974	0.068

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	42	87	87	92	29	31
N.S.	1	1.00	0.66	0.89	1.85	1.85	1.96	0.62	0.66
time (sec)	N/A	0.016	0.006	0.081	0.287	0.650	0.180	1.461	0.080

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	76	76	80	18	18
N.S.	1	1.00	0.67	0.90	2.53	2.53	2.67	0.60	0.60
time (sec)	N/A	0.009	0.005	0.082	0.283	0.805	0.181	1.146	0.100

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	68	73	12	70
N.S.	1	1.00	1.00	0.93	0.86	4.86	5.21	0.86	5.00
time (sec)	N/A	0.001	0.003	0.080	0.282	0.814	0.178	1.499	0.064

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	81	90	139	256	141	87	102
N.S.	1	1.00	0.82	0.91	1.40	2.59	1.42	0.88	1.03
time (sec)	N/A	0.036	0.045	0.090	0.300	1.294	0.289	1.606	0.455

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	108	157	285	162	104	151
N.S.	1	1.00	0.83	0.92	1.34	2.44	1.38	0.89	1.29
time (sec)	N/A	0.055	0.063	0.088	0.290	1.229	0.332	1.775	0.186

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	112	133	174	306	175	119	167
N.S.	1	1.00	0.78	0.92	1.21	2.12	1.22	0.83	1.16
time (sec)	N/A	0.065	0.049	0.086	0.367	1.208	0.346	1.371	0.209

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	144	185	317	187	130	179
N.S.	1	1.00	0.78	0.92	1.18	2.02	1.19	0.83	1.14
time (sec)	N/A	0.077	0.062	0.107	0.497	1.104	0.359	1.857	0.311

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	161	177	234	338	250	149	151
N.S.	1	1.00	0.87	0.95	1.26	1.82	1.34	0.80	0.81
time (sec)	N/A	0.127	0.030	0.086	0.340	0.748	0.653	1.175	0.979

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	167	223	327	236	138	138
N.S.	1	1.00	0.85	0.94	1.26	1.85	1.33	0.78	0.78
time (sec)	N/A	0.102	0.020	0.101	0.368	1.192	0.614	1.468	0.227

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	137	154	211	314	224	121	127
N.S.	1	1.00	0.86	0.97	1.33	1.97	1.41	0.76	0.80
time (sec)	N/A	0.083	0.023	0.097	0.329	0.938	0.572	1.375	0.943

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	111	145	202	292	212	112	117
N.S.	1	1.00	0.72	0.94	1.31	1.90	1.38	0.73	0.76
time (sec)	N/A	0.071	0.022	0.085	0.358	0.942	0.464	1.100	0.187

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	97	131	186	186	199	95	107
N.S.	1	1.00	5.71	7.71	10.94	10.94	11.71	5.59	6.29
time (sec)	N/A	0.001	0.013	0.082	0.324	0.991	0.423	1.573	0.141

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	86	117	175	175	187	84	22
N.S.	1	1.00	2.46	3.34	5.00	5.00	5.34	2.40	0.63
time (sec)	N/A	0.004	0.011	0.083	0.502	0.928	0.391	1.539	0.126

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	75	102	164	164	175	73	85
N.S.	1	1.00	1.44	1.96	3.15	3.15	3.37	1.40	1.63
time (sec)	N/A	0.007	0.011	0.090	0.512	0.727	0.354	1.042	0.140

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	86	153	153	163	62	71
N.S.	1	1.00	0.93	1.25	2.22	2.22	2.36	0.90	1.03
time (sec)	N/A	0.010	0.010	0.083	0.300	0.814	0.329	1.392	0.078

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	53	72	142	142	151	51	61
N.S.	1	1.00	0.65	0.89	1.75	1.75	1.86	0.63	0.75
time (sec)	N/A	0.028	0.010	0.085	0.295	1.001	0.308	1.028	0.077

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	42	57	131	131	139	40	48
N.S.	1	1.00	0.66	0.89	2.05	2.05	2.17	0.62	0.75
time (sec)	N/A	0.022	0.007	0.084	0.539	0.819	0.288	1.292	0.127

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	31	42	120	120	128	29	31
N.S.	1	1.00	0.66	0.89	2.55	2.55	2.72	0.62	0.66
time (sec)	N/A	0.016	0.009	0.095	0.508	0.601	0.279	1.443	0.150

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	27	109	109	116	18	18
N.S.	1	1.00	0.67	0.90	3.63	3.63	3.87	0.60	0.60
time (sec)	N/A	0.010	0.004	0.085	0.475	0.595	0.274	1.025	0.067

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	101	109	12	103
N.S.	1	1.00	1.00	0.93	0.86	7.21	7.79	0.86	7.36
time (sec)	N/A	0.001	0.003	0.089	0.312	1.237	0.269	0.996	0.141

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	127	126	205	388	212	120	145
N.S.	1	1.00	0.90	0.89	1.45	2.75	1.50	0.85	1.03
time (sec)	N/A	0.052	0.077	0.091	0.452	1.431	0.420	2.555	0.762

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	130	147	223	417	233	137	217
N.S.	1	1.00	0.82	0.93	1.41	2.64	1.47	0.87	1.37
time (sec)	N/A	0.089	0.091	0.087	0.332	1.475	0.470	1.575	0.395

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	145	178	240	438	246	152	233
N.S.	1	1.00	0.76	0.93	1.26	2.29	1.29	0.80	1.22
time (sec)	N/A	0.100	0.075	0.094	0.314	1.177	0.491	2.011	0.438

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	156	189	251	449	258	163	245
N.S.	1	1.00	0.79	0.95	1.27	2.27	1.30	0.82	1.24
time (sec)	N/A	0.113	0.086	0.136	0.320	0.685	0.505	1.759	0.586

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	132	132	136	143	133	132
N.S.	1	1.00	1.00	0.94	0.94	0.96	1.01	0.94	0.94
time (sec)	N/A	0.055	0.008	0.083	0.284	0.479	0.360	1.788	0.081

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	121	121	125	131	122	121
N.S.	1	1.00	1.00	0.92	0.92	0.95	0.99	0.92	0.92
time (sec)	N/A	0.046	0.004	0.087	0.302	0.471	0.343	1.704	0.092

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	109	109	114	117	110	114
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	0.96	1.00
time (sec)	N/A	0.038	0.005	0.077	0.289	0.492	0.339	0.778	0.002

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	100	100	103	107	101	100
N.S.	1	1.00	1.00	0.92	0.92	0.94	0.98	0.93	0.92
time (sec)	N/A	0.035	0.004	0.077	0.274	0.474	0.325	1.359	0.082

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	96	91	88	88	95	88	88
N.S.	1	1.00	5.65	5.35	5.18	5.18	5.59	5.18	5.18
time (sec)	N/A	0.001	0.007	0.084	0.265	0.426	0.292	1.734	0.090

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	91	80	79	79	85	79	23
N.S.	1	1.00	2.53	2.22	2.19	2.19	2.36	2.19	0.64
time (sec)	N/A	0.004	0.003	0.074	0.273	0.441	0.254	1.748	0.002

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	80	69	68	68	73	68	68
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.30	1.21	1.21
time (sec)	N/A	0.008	0.006	0.082	0.277	0.404	0.226	1.592	0.098

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	57	57	61	57	56
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84
time (sec)	N/A	0.016	0.006	0.073	0.276	0.446	0.187	1.353	0.002

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	49	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.88	0.82	0.82
time (sec)	N/A	0.013	0.006	0.076	0.274	0.396	0.157	1.200	0.035

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81
time (sec)	N/A	0.010	0.003	0.076	0.289	0.394	0.120	1.163	0.032

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.006	0.005	0.084	0.277	0.714	0.090	2.188	0.036

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76
time (sec)	N/A	0.004	0.002	0.012	0.288	0.757	0.057	1.065	0.029

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	5	7	5	5
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71
time (sec)	N/A	0.000	0.000	0.007	0.272	0.573	0.021	1.510	0.020

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	119	117	120	116	122	114
N.S.	1	1.00	1.00	0.89	0.87	0.90	0.87	0.91	0.85
time (sec)	N/A	0.041	0.005	0.089	0.275	0.523	0.159	1.576	0.127

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	134	135	141	163	139	180	135
N.S.	1	1.00	0.92	0.92	0.97	1.12	0.95	1.23	0.92
time (sec)	N/A	0.063	0.062	0.089	0.273	0.528	0.223	1.173	0.083

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	145	150	163	207	163	152	157
N.S.	1	1.00	0.89	0.92	1.00	1.27	1.00	0.93	0.96
time (sec)	N/A	0.076	0.071	0.089	0.289	0.518	0.271	1.487	0.228

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	15	10
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.88	0.59
time (sec)	N/A	0.002	0.002	0.086	0.282	0.833	0.029	1.132	0.170

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	15	10
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.88	0.59
time (sec)	N/A	0.002	0.002	0.091	0.269	0.724	0.030	1.600	0.143

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	18	21	20	20	18
N.S.	1	1.00	1.00	0.79	0.75	0.88	0.83	0.83	0.75
time (sec)	N/A	0.006	0.002	0.091	0.275	0.940	0.033	1.468	0.054

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	28	26	25	18
N.S.	1	1.00	1.00	0.77	0.74	0.90	0.84	0.81	0.58
time (sec)	N/A	0.007	0.002	0.092	0.287	0.776	0.036	1.268	0.037

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	28	33	31	30	24
N.S.	1	1.00	1.00	0.76	0.74	0.87	0.82	0.79	0.63
time (sec)	N/A	0.008	0.003	0.092	0.274	1.028	0.041	1.439	0.088

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	33	38	36	35	28
N.S.	1	1.00	1.00	0.76	0.73	0.84	0.80	0.78	0.62
time (sec)	N/A	0.009	0.002	0.096	0.273	0.713	0.045	1.216	0.040

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	23	22	32	19	25	20
N.S.	1	1.00	0.93	0.82	0.79	1.14	0.68	0.89	0.71
time (sec)	N/A	0.007	0.010	0.089	0.274	0.896	0.036	1.037	0.055

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	31	48	31	40	34
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.89	1.14	0.97
time (sec)	N/A	0.008	0.010	0.098	0.287	0.573	0.042	0.717	0.089

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	33	38	59	36	51	31
N.S.	1	1.00	0.86	0.79	0.90	1.40	0.86	1.21	0.74
time (sec)	N/A	0.010	0.010	0.099	0.284	0.597	0.047	0.528	0.042

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	43	64	41	60	37
N.S.	1	1.00	0.90	0.78	0.88	1.31	0.84	1.22	0.76
time (sec)	N/A	0.011	0.021	0.104	0.361	1.044	0.050	0.535	0.093

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	43	48	69	46	69	41
N.S.	1	1.00	1.00	0.77	0.86	1.23	0.82	1.23	0.73
time (sec)	N/A	0.014	0.007	0.111	0.322	0.556	0.053	0.474	0.095

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	32	30	50	27	27	29
N.S.	1	1.00	0.74	0.82	0.77	1.28	0.69	0.69	0.74
time (sec)	N/A	0.009	0.015	0.104	0.274	0.495	0.043	0.966	0.125

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	37	41	68	41	37	35
N.S.	1	1.00	0.85	0.80	0.89	1.48	0.89	0.80	0.76
time (sec)	N/A	0.010	0.015	0.122	0.270	0.665	0.052	0.829	0.092

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	42	48	79	46	43	41
N.S.	1	1.00	0.83	0.79	0.91	1.49	0.87	0.81	0.77
time (sec)	N/A	0.011	0.018	0.102	0.276	0.611	0.054	0.791	0.094

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	47	53	84	51	47	47
N.S.	1	1.00	0.82	0.78	0.88	1.40	0.85	0.78	0.78
time (sec)	N/A	0.015	0.014	0.103	0.318	0.486	0.058	0.672	0.046

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	52	58	89	56	52	51
N.S.	1	1.00	0.81	0.78	0.87	1.33	0.84	0.78	0.76
time (sec)	N/A	0.017	0.014	0.104	0.281	0.438	0.062	0.652	0.047

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	9	6	6	7	7	6
N.S.	1	1.00	1.25	1.12	0.75	0.75	0.88	0.88	0.75
time (sec)	N/A	0.001	0.001	0.082	0.299	0.485	0.006	0.639	0.151

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	9	6
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.90	0.60
time (sec)	N/A	0.001	0.001	0.083	0.272	0.418	0.008	0.978	0.077

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	13	12	10	14	13	10
N.S.	1	1.00	1.14	0.93	0.86	0.71	1.00	0.93	0.71
time (sec)	N/A	0.002	0.003	0.091	0.275	0.415	0.009	0.712	0.105

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	20	17	17	16
N.S.	1	1.00	1.00	0.85	0.80	1.00	0.85	0.85	0.80
time (sec)	N/A	0.005	0.004	0.089	0.294	0.432	0.010	0.642	0.108

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	19	19	14
N.S.	1	1.00	1.00	0.86	0.82	0.95	0.86	0.86	0.64
time (sec)	N/A	0.002	0.003	0.087	0.321	0.400	0.009	0.563	0.052

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	23	19	19	16
N.S.	1	1.00	1.00	0.86	0.82	1.05	0.86	0.86	0.73
time (sec)	N/A	0.002	0.003	0.091	0.279	0.447	0.009	0.648	0.057

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	18	24	19	19	16
N.S.	1	1.00	1.00	0.90	0.86	1.14	0.90	0.90	0.76
time (sec)	N/A	0.003	0.005	0.090	0.279	0.387	0.014	0.999	0.152

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	18	24	20	19	14
N.S.	1	1.00	1.10	0.95	0.90	1.20	1.00	0.95	0.70
time (sec)	N/A	0.003	0.007	0.088	0.267	0.419	0.015	0.955	0.175

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	9
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	0.82
time (sec)	N/A	0.002	0.003	0.094	0.273	0.438	0.033	0.688	0.097

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	8	13	9
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.67	1.08	0.75
time (sec)	N/A	0.002	0.002	0.088	0.288	0.494	0.035	0.549	0.036

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	21	14	21	16
N.S.	1	1.00	1.00	1.05	1.00	1.11	0.74	1.11	0.84
time (sec)	N/A	0.006	0.003	0.111	0.286	0.483	0.048	0.543	0.043

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	21	14	19	14
N.S.	1	1.00	1.00	1.00	0.94	1.17	0.78	1.06	0.78
time (sec)	N/A	0.007	0.003	0.110	0.275	0.439	0.052	0.633	0.029

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	15	10	15	20
N.S.	1	1.00	1.00	1.07	1.00	1.07	0.71	1.07	1.43
time (sec)	N/A	0.006	0.004	0.086	0.268	0.407	0.045	1.173	0.036

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	53	1742	116	56
N.S.	1	1.00	0.64	0.69	0.78	0.74	24.19	1.61	0.78
time (sec)	N/A	0.013	0.022	0.102	0.270	0.385	1.191	1.053	0.050

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	93	37
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	1.75	0.70
time (sec)	N/A	0.009	0.006	0.095	0.267	0.380	0.813	0.694	0.046

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	30	202	66	25
N.S.	1	1.00	1.00	0.76	0.76	0.88	5.94	1.94	0.74
time (sec)	N/A	0.006	0.004	0.080	0.272	0.385	0.527	0.949	0.028

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.001	0.003	0.081	0.267	0.407	0.006	0.929	0.021

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	42	73	68	32	27
N.S.	1	1.00	1.00	0.80	1.20	2.09	1.94	0.91	0.77
time (sec)	N/A	0.011	0.006	0.081	0.492	0.428	0.652	1.027	0.094

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	47	93	44	41	31
N.S.	1	1.00	1.00	0.95	1.21	2.38	1.13	1.05	0.79
time (sec)	N/A	0.008	0.013	0.085	0.486	0.402	0.852	1.606	0.053

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	54	88	119	97	66	48
N.S.	1	1.00	0.85	0.83	1.35	1.83	1.49	1.02	0.74
time (sec)	N/A	0.013	0.090	0.095	0.502	0.433	1.951	1.223	0.068

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	66	121	145	122	84	66
N.S.	1	1.00	0.77	0.76	1.39	1.67	1.40	0.97	0.76
time (sec)	N/A	0.017	0.122	0.092	0.475	0.421	4.958	0.772	0.107

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	64	1742	193	56
N.S.	1	1.00	0.64	0.69	0.78	0.89	24.19	2.68	0.78
time (sec)	N/A	0.012	0.027	0.085	0.285	0.382	1.270	0.687	0.048

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	53	733	156	37
N.S.	1	1.00	0.66	0.72	0.77	1.00	13.83	2.94	0.70
time (sec)	N/A	0.009	0.024	0.084	0.280	0.364	0.893	0.720	0.041

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	41	80	119	25
N.S.	1	1.00	0.71	0.76	0.76	1.21	2.35	3.50	0.74
time (sec)	N/A	0.006	0.019	0.090	0.260	0.399	0.124	0.797	0.026

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	58	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	0.75
time (sec)	N/A	0.001	0.008	0.099	0.275	0.412	0.007	0.973	0.016

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	38	52	88	71	44	37
N.S.	1	1.00	0.90	0.78	1.06	1.80	1.45	0.90	0.76
time (sec)	N/A	0.010	0.028	0.101	0.474	0.439	1.054	0.925	0.042

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	48	58	102	92	56	42
N.S.	1	1.00	0.88	0.94	1.14	2.00	1.80	1.10	0.82
time (sec)	N/A	0.010	0.059	0.104	0.485	0.399	1.264	0.953	0.100

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	52	86	124	76	64	46
N.S.	1	1.00	0.85	0.84	1.39	2.00	1.23	1.03	0.74
time (sec)	N/A	0.011	0.102	0.095	0.480	0.429	1.496	1.098	0.057

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	64	119	145	124	84	64
N.S.	1	1.00	0.80	0.76	1.42	1.73	1.48	1.00	0.76
time (sec)	N/A	0.015	0.118	0.095	0.487	0.417	3.528	1.132	0.102

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	75	146	281	56
N.S.	1	1.00	0.64	0.69	0.78	1.04	2.03	3.90	0.78
time (sec)	N/A	0.012	0.028	0.086	0.299	0.455	0.342	2.144	0.050

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	64	124	233	37
N.S.	1	1.00	0.66	0.72	0.77	1.21	2.34	4.40	0.70
time (sec)	N/A	0.009	0.024	0.090	0.273	0.463	0.290	0.930	0.044

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	52	102	182	25
N.S.	1	1.00	0.71	0.76	0.76	1.53	3.00	5.35	0.74
time (sec)	N/A	0.006	0.019	0.102	0.275	0.504	0.232	0.947	0.028

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	39	12	95	12
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	5.94	0.75
time (sec)	N/A	0.001	0.008	0.098	0.287	0.444	0.009	1.105	0.018

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	50	64	114	97	56	52
N.S.	1	1.00	0.86	0.77	0.98	1.75	1.49	0.86	0.80
time (sec)	N/A	0.013	0.048	0.086	0.496	0.494	2.177	0.973	0.053

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	62	71	126	99	74	58
N.S.	1	1.00	0.91	0.94	1.08	1.91	1.50	1.12	0.88
time (sec)	N/A	0.014	0.077	0.099	0.494	0.446	2.045	1.938	0.115

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	63	62	101	133	126	80	64
N.S.	1	1.00	0.81	0.79	1.29	1.71	1.62	1.03	0.82
time (sec)	N/A	0.015	0.104	0.095	0.487	0.426	2.275	0.982	0.053

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	64	115	146	104	79	64
N.S.	1	1.00	0.79	0.79	1.42	1.80	1.28	0.98	0.79
time (sec)	N/A	0.015	0.142	0.095	0.490	0.464	2.589	0.827	0.046

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	76	144	167	155	99	79
N.S.	1	1.00	0.76	0.74	1.40	1.62	1.50	0.96	0.77
time (sec)	N/A	0.020	0.153	0.102	0.487	0.434	7.159	0.695	0.111

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	90	97	116	141	279	781	116
N.S.	1	1.00	0.62	0.66	0.79	0.97	1.91	5.35	0.79
time (sec)	N/A	0.029	0.048	0.087	0.275	0.452	1.397	0.895	0.036

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	85	101	130	257	709	101
N.S.	1	1.00	0.62	0.67	0.80	1.02	2.02	5.58	0.80
time (sec)	N/A	0.024	0.041	0.085	0.277	0.430	1.256	1.690	0.031

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	68	74	86	119	235	637	86
N.S.	1	1.00	0.62	0.67	0.78	1.08	2.14	5.79	0.78
time (sec)	N/A	0.022	0.035	0.089	0.266	0.411	1.136	1.932	0.027

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	57	62	71	108	212	565	71
N.S.	1	1.00	0.63	0.68	0.78	1.19	2.33	6.21	0.78
time (sec)	N/A	0.017	0.030	0.087	0.271	0.425	1.034	1.338	0.025

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	97	190	493	56
N.S.	1	1.00	0.64	0.69	0.78	1.35	2.64	6.85	0.78
time (sec)	N/A	0.013	0.028	0.089	0.280	0.422	0.898	0.865	0.045

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	86	168	421	36
N.S.	1	1.00	0.66	0.72	0.77	1.62	3.17	7.94	0.68
time (sec)	N/A	0.009	0.025	0.098	0.270	0.438	0.806	1.216	0.040

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	74	146	347	25
N.S.	1	1.00	0.71	0.76	0.76	2.18	4.29	10.21	0.74
time (sec)	N/A	0.006	0.022	0.102	0.273	0.478	0.709	1.454	0.030

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	61	12	229	12
N.S.	1	1.00	1.00	0.81	0.75	3.81	0.75	14.31	0.75
time (sec)	N/A	0.001	0.009	0.095	0.276	0.409	0.012	1.568	0.018

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	74	88	158	148	80	76
N.S.	1	1.00	0.80	0.76	0.91	1.63	1.53	0.82	0.78
time (sec)	N/A	0.024	0.044	0.089	0.476	0.466	10.499	1.269	0.037

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	82	85	97	172	150	104	84
N.S.	1	1.00	0.84	0.87	0.99	1.76	1.53	1.06	0.86
time (sec)	N/A	0.024	0.084	0.097	0.488	0.497	10.594	0.840	0.043

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	86	87	131	180	184	112	117
N.S.	1	1.00	0.75	0.76	1.15	1.58	1.61	0.98	1.03
time (sec)	N/A	0.024	0.118	0.105	0.484	0.452	9.956	0.659	0.047

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	89	145	178	184	112	131
N.S.	1	1.00	0.75	0.78	1.27	1.56	1.61	0.98	1.15
time (sec)	N/A	0.026	0.146	0.100	0.489	0.460	9.350	0.688	0.121

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	86	155	177	182	110	94
N.S.	1	1.00	0.74	0.74	1.34	1.53	1.57	0.95	0.81
time (sec)	N/A	0.026	0.161	0.102	0.485	0.560	9.946	1.075	0.062

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	88	169	190	158	109	94
N.S.	1	1.00	0.72	0.74	1.42	1.60	1.33	0.92	0.79
time (sec)	N/A	0.027	0.192	0.125	0.500	0.616	11.056	1.636	0.119

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	100	100	198	211	209	129	109
N.S.	1	1.00	0.71	0.71	1.40	1.50	1.48	0.91	0.77
time (sec)	N/A	0.035	0.213	0.115	0.494	0.569	47.971	1.019	0.133

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	111	112	229	233	0	144	124
N.S.	1	1.00	0.68	0.69	1.40	1.43	0.00	0.88	0.76
time (sec)	N/A	0.045	0.251	0.107	0.503	0.501	0.000	0.969	0.126

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	31	78	148	31	31
N.S.	1	1.00	1.00	0.82	0.79	2.00	3.79	0.79	0.79
time (sec)	N/A	0.007	0.023	0.089	0.502	0.597	0.733	0.808	0.094

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	34	98	117	41	34
N.S.	1	1.00	1.00	0.98	0.81	2.33	2.79	0.98	0.81
time (sec)	N/A	0.007	0.042	0.098	0.504	0.588	0.957	0.866	0.097

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	59	83	124	207	66	54
N.S.	1	1.00	0.85	0.83	1.17	1.75	2.92	0.93	0.76
time (sec)	N/A	0.011	0.086	0.112	0.493	0.513	1.974	1.027	0.103

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	44	43	93	187	43	43
N.S.	1	1.00	0.87	0.80	0.78	1.69	3.40	0.78	0.78
time (sec)	N/A	0.010	0.037	0.100	0.476	0.541	1.222	1.009	0.041

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	54	47	105	197	58	47
N.S.	1	1.00	0.84	0.95	0.82	1.84	3.46	1.02	0.82
time (sec)	N/A	0.010	0.054	0.105	0.492	0.462	1.358	0.727	0.045

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	57	80	129	189	66	52
N.S.	1	1.00	0.82	0.84	1.18	1.90	2.78	0.97	0.76
time (sec)	N/A	0.010	0.090	0.110	0.494	0.457	1.542	0.979	0.097

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	58	57	119	240	57	57
N.S.	1	1.00	0.82	0.79	0.78	1.63	3.29	0.78	0.78
time (sec)	N/A	0.014	0.051	0.092	0.485	0.500	2.269	0.892	0.043

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	69	63	131	245	75	63
N.S.	1	1.00	0.86	0.93	0.85	1.77	3.31	1.01	0.85
time (sec)	N/A	0.014	0.067	0.107	0.499	0.488	2.202	0.912	0.103

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	70	97	139	267	83	69
N.S.	1	1.00	0.78	0.81	1.13	1.62	3.10	0.97	0.80
time (sec)	N/A	0.015	0.098	0.098	0.484	0.471	2.277	1.297	0.095

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	57	61	71	53	3755	61	71
N.S.	1	1.00	0.64	0.69	0.80	0.60	42.19	0.69	0.80
time (sec)	N/A	0.015	0.026	0.086	0.271	0.467	2.368	1.163	0.024

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	49	56	42	1640	49	56
N.S.	1	1.00	0.68	0.72	0.82	0.62	24.12	0.72	0.82
time (sec)	N/A	0.012	0.024	0.092	0.266	0.462	1.239	0.889	0.046

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	41	31	600	37	37
N.S.	1	1.00	0.69	0.73	0.80	0.61	11.76	0.73	0.73
time (sec)	N/A	0.009	0.006	0.088	0.271	0.417	0.821	0.719	0.038

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	26	26	19	162	23	25
N.S.	1	1.00	0.72	0.81	0.81	0.59	5.06	0.72	0.78
time (sec)	N/A	0.005	0.004	0.084	0.304	0.472	0.536	0.678	0.027

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.001	0.002	0.090	0.291	0.413	0.006	1.299	0.018

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.005	0.002	0.082	0.483	0.392	0.434	1.601	0.055

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	40	60	93	44	47	33
N.S.	1	1.00	1.00	0.98	1.46	2.27	1.07	1.15	0.80
time (sec)	N/A	0.008	0.009	0.111	0.479	0.452	1.085	1.147	0.111

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	66	92	123	102	69	51
N.S.	1	1.00	0.82	0.97	1.35	1.81	1.50	1.01	0.75
time (sec)	N/A	0.011	0.079	0.113	0.475	0.449	2.465	1.006	0.061

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	90	121	145	129	84	69
N.S.	1	1.00	0.74	1.00	1.34	1.61	1.43	0.93	0.77
time (sec)	N/A	0.017	0.088	0.095	0.475	0.416	7.658	1.358	0.052

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	57	62	71	63	3606	77	71
N.S.	1	1.00	0.67	0.73	0.84	0.74	42.42	0.91	0.84
time (sec)	N/A	0.016	0.029	0.094	0.276	0.419	2.015	1.367	0.028

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	49	56	51	1538	61	56
N.S.	1	1.00	0.68	0.74	0.85	0.77	23.30	0.92	0.85
time (sec)	N/A	0.013	0.024	0.112	0.260	0.443	1.253	1.201	0.048

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	38	41	40	534	46	35
N.S.	1	1.00	0.69	0.78	0.84	0.82	10.90	0.94	0.71
time (sec)	N/A	0.009	0.022	0.111	0.269	0.420	0.866	1.326	0.040

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	23	26	29	37	29	19
N.S.	1	1.00	0.70	0.77	0.87	0.97	1.23	0.97	0.63
time (sec)	N/A	0.005	0.014	0.114	0.268	0.504	0.262	0.809	0.087

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86
time (sec)	N/A	0.001	0.009	0.115	0.263	0.485	0.008	0.682	0.019

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	45	110	146	37	30
N.S.	1	1.00	1.00	0.82	1.18	2.89	3.84	0.97	0.79
time (sec)	N/A	0.008	0.032	0.132	0.481	0.464	0.779	0.716	0.043

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	54	76	151	73	64	60
N.S.	1	1.00	0.86	0.95	1.33	2.65	1.28	1.12	1.05
time (sec)	N/A	0.011	0.070	0.103	0.473	0.461	1.775	0.737	0.122

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	68	108	189	107	80	90
N.S.	1	1.00	0.77	0.78	1.24	2.17	1.23	0.92	1.03
time (sec)	N/A	0.016	0.102	0.120	0.505	0.432	3.989	1.356	0.059

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	71	74	3456	75	68
N.S.	1	1.00	0.66	0.71	0.82	0.85	39.72	0.86	0.78
time (sec)	N/A	0.016	0.029	0.115	0.279	0.416	2.021	1.884	0.049

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	47	50	56	62	163	59	47
N.S.	1	1.00	0.69	0.74	0.82	0.91	2.40	0.87	0.69
time (sec)	N/A	0.013	0.027	0.098	0.261	0.427	0.376	1.124	0.043

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	41	52	121	39	35
N.S.	1	1.00	0.71	0.73	0.84	1.06	2.47	0.80	0.71
time (sec)	N/A	0.009	0.025	0.101	0.276	0.417	0.402	0.631	0.084

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	26	26	41	80	20	20
N.S.	1	1.00	0.75	0.81	0.81	1.28	2.50	0.62	0.62
time (sec)	N/A	0.006	0.016	0.093	0.265	0.396	0.355	0.661	0.033

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	31	14	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	0.75	0.75
time (sec)	N/A	0.001	0.008	0.092	0.270	0.477	0.011	1.260	0.018

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	43	53	177	697	45	42
N.S.	1	1.00	0.91	0.80	0.98	3.28	12.91	0.83	0.78
time (sec)	N/A	0.011	0.054	0.120	0.493	0.514	1.466	0.839	0.050

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	66	89	221	818	65	73
N.S.	1	1.00	0.85	0.89	1.20	2.99	11.05	0.88	0.99
time (sec)	N/A	0.016	0.084	0.105	0.498	0.497	2.876	0.970	0.109

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	78	81	123	255	464	93	105
N.S.	1	1.00	0.74	0.76	1.16	2.41	4.38	0.88	0.99
time (sec)	N/A	0.022	0.119	0.106	0.481	0.492	10.562	0.905	0.120

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	58	54	19	19
N.S.	1	1.00	1.00	0.80	0.76	2.32	2.16	0.76	0.76
time (sec)	N/A	0.005	0.018	0.095	0.490	0.477	0.554	0.725	0.048

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	44	46	97	121	43	36
N.S.	1	1.00	1.00	1.00	1.05	2.20	2.75	0.98	0.82
time (sec)	N/A	0.007	0.043	0.131	0.494	0.503	1.133	0.838	0.041

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	72	86	128	216	68	57
N.S.	1	1.00	0.81	0.97	1.16	1.73	2.92	0.92	0.77
time (sec)	N/A	0.011	0.072	0.094	0.491	0.647	2.465	0.616	0.045

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	124	437	34	34
N.S.	1	1.00	1.00	0.83	0.81	2.95	10.40	0.81	0.81
time (sec)	N/A	0.007	0.034	0.094	0.476	0.716	0.946	0.859	0.095

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	61	67	164	156	64	52
N.S.	1	1.00	0.82	0.98	1.08	2.65	2.52	1.03	0.84
time (sec)	N/A	0.011	0.064	0.103	0.495	1.537	1.718	1.557	0.063

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	77	104	198	226	81	101
N.S.	1	1.00	0.75	0.81	1.09	2.08	2.38	0.85	1.06
time (sec)	N/A	0.017	0.098	0.103	0.478	1.029	3.982	1.511	0.132

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	49	42	182	1950	42	48
N.S.	1	1.00	0.87	0.82	0.70	3.03	32.50	0.70	0.80
time (sec)	N/A	0.011	0.061	0.097	0.490	0.523	104.594	1.025	0.092

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	67	74	82	226	0	66	70
N.S.	1	1.00	0.83	0.91	1.01	2.79	0.00	0.81	0.86
time (sec)	N/A	0.015	0.087	0.128	0.499	0.504	0.000	1.138	0.118

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	82	90	121	260	1108	97	117
N.S.	1	1.00	0.71	0.78	1.04	2.24	9.55	0.84	1.01
time (sec)	N/A	0.021	0.130	0.117	0.484	0.710	12.864	1.037	0.071

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	14	78	0	11
N.S.	1	1.00	1.00	0.92	0.85	1.08	6.00	0.00	0.85
time (sec)	N/A	0.005	0.180	0.088	0.328	0.791	35.500	0.000	0.413

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	A	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	92	13	0	11	11	73	0	-1
N.S.	1	7.08	1.00	0.00	0.85	0.85	5.62	0.00	-0.08
time (sec)	N/A	0.029	0.013	0.039	0.353	0.988	2.397	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.006	0.003	0.107	0.479	0.604	0.443	0.979	0.002

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	53	1742	117	56
N.S.	1	1.00	0.64	0.69	0.78	0.74	24.19	1.62	0.78
time (sec)	N/A	0.012	0.024	0.099	0.277	0.517	1.238	2.102	0.052

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	92	37
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	1.74	0.70
time (sec)	N/A	0.009	0.022	0.095	0.275	0.450	0.812	1.240	0.038

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	26	30	202	67	25
N.S.	1	1.00	1.00	0.76	0.76	0.88	5.94	1.97	0.74
time (sec)	N/A	0.006	0.017	0.109	0.274	0.469	0.536	1.076	0.028

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.001	0.007	0.107	0.301	0.443	0.006	0.761	0.017

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	113	90	86	91	180	87	107
N.S.	1	1.00	1.24	0.99	0.95	1.00	1.98	0.96	1.18
time (sec)	N/A	0.037	0.076	0.099	0.508	0.471	0.996	1.130	0.121

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	119	95	93	139	643	105	117
N.S.	1	1.00	1.23	0.98	0.96	1.43	6.63	1.08	1.21
time (sec)	N/A	0.025	0.138	0.149	0.499	0.496	1.083	1.052	0.067

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	129	118	139	187	2266	128	196
N.S.	1	1.00	1.02	0.93	1.09	1.47	17.84	1.01	1.54
time (sec)	N/A	0.035	0.202	0.139	0.519	0.474	1.663	1.729	0.230

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	53	1742	117	56
N.S.	1	1.00	0.64	0.69	0.78	0.74	24.19	1.62	0.78
time (sec)	N/A	0.012	0.025	0.107	0.276	0.433	1.266	1.118	0.045

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	42	666	92	37
N.S.	1	1.00	0.66	0.72	0.77	0.79	12.57	1.74	0.70
time (sec)	N/A	0.009	0.022	0.109	0.274	0.490	0.850	1.130	0.041

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	26	26	31	202	68	25
N.S.	1	1.00	1.03	0.76	0.76	0.91	5.94	2.00	0.74
time (sec)	N/A	0.006	0.017	0.092	0.298	0.487	0.563	0.746	0.028

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.001	0.008	0.089	0.264	0.504	0.006	0.779	0.018

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	114	90	85	110	182	86	117
N.S.	1	1.00	1.24	0.98	0.92	1.20	1.98	0.93	1.27
time (sec)	N/A	0.023	0.059	0.091	0.482	0.484	1.068	1.286	0.113

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	120	95	93	252	643	106	127
N.S.	1	1.00	1.28	1.01	0.99	2.68	6.84	1.13	1.35
time (sec)	N/A	0.025	0.136	0.100	0.484	0.748	1.147	2.078	0.114

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	147	118	139	350	2266	129	194
N.S.	1	1.00	1.16	0.93	1.09	2.76	17.84	1.02	1.53
time (sec)	N/A	0.035	0.217	0.119	0.510	0.847	1.719	1.509	0.329

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	64	1844	193	56
N.S.	1	1.00	0.64	0.69	0.78	0.89	25.61	2.68	0.78
time (sec)	N/A	0.012	0.026	0.107	0.272	0.677	1.404	0.847	0.047

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	53	733	157	37
N.S.	1	1.00	0.66	0.72	0.77	1.00	13.83	2.96	0.70
time (sec)	N/A	0.009	0.023	0.098	0.280	0.738	0.943	0.845	0.042

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	41	80	118	25
N.S.	1	1.00	0.71	0.76	0.76	1.21	2.35	3.47	0.74
time (sec)	N/A	0.006	0.018	0.108	0.275	1.101	0.190	0.668	0.027

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	58	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	0.75
time (sec)	N/A	0.001	0.008	0.107	0.282	0.773	0.006	1.423	0.018

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	102	96	98	209	97	123
N.S.	1	1.00	1.16	0.97	0.91	0.93	1.99	0.92	1.17
time (sec)	N/A	0.029	0.068	0.091	0.499	0.838	1.300	1.832	0.060

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	128	106	104	111	719	119	131
N.S.	1	1.00	1.20	0.99	0.97	1.04	6.72	1.11	1.22
time (sec)	N/A	0.029	0.166	0.151	0.488	0.818	1.405	1.261	0.073

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	136	110	136	162	2266	127	174
N.S.	1	1.00	1.10	0.89	1.10	1.31	18.27	1.02	1.40
time (sec)	N/A	0.031	0.222	0.141	0.508	0.606	1.521	1.246	0.122

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	46	50	56	42	1640	49	56
N.S.	1	1.00	0.64	0.69	0.78	0.58	22.78	0.68	0.78
time (sec)	N/A	0.013	0.024	0.109	0.285	0.661	1.199	1.009	0.043

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	38	41	31	600	37	37
N.S.	1	1.00	0.66	0.72	0.77	0.58	11.32	0.70	0.70
time (sec)	N/A	0.009	0.024	0.107	0.294	0.544	0.817	1.724	0.038

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	26	26	20	162	25	25
N.S.	1	1.00	0.71	0.76	0.76	0.59	4.76	0.74	0.74
time (sec)	N/A	0.006	0.015	0.122	0.271	0.747	0.568	1.121	0.029

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.001	0.007	0.108	0.301	0.787	0.006	0.958	0.016

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	95	75	76	213	155	77	99
N.S.	1	1.00	1.20	0.95	0.96	2.70	1.96	0.97	1.25
time (sec)	N/A	0.016	0.049	0.110	0.482	0.895	0.878	1.147	0.086

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	120	104	106	306	831	109	130
N.S.	1	1.00	1.20	1.04	1.06	3.06	8.31	1.09	1.30
time (sec)	N/A	0.023	0.148	0.115	0.496	0.717	1.140	0.987	0.137

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	149	130	142	296	2730	130	182
N.S.	1	1.00	1.15	1.00	1.09	2.28	21.00	1.00	1.40
time (sec)	N/A	0.032	0.138	0.112	0.483	0.924	1.978	0.865	0.226

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	48	58	64	44	4974	57	64
N.S.	1	1.00	0.60	0.72	0.80	0.55	62.18	0.71	0.80
time (sec)	N/A	0.012	0.024	0.108	0.268	0.702	1.319	1.107	0.048

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	44	47	33	1326	43	43
N.S.	1	1.00	0.63	0.75	0.80	0.56	22.47	0.73	0.73
time (sec)	N/A	0.009	0.024	0.108	0.273	0.694	1.102	1.452	0.040

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	26	30	30	22	486	29	29
N.S.	1	1.00	0.68	0.79	0.79	0.58	12.79	0.76	0.76
time (sec)	N/A	0.006	0.015	0.100	0.262	0.725	0.554	0.942	0.029

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	12	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	0.78	0.78
time (sec)	N/A	0.001	0.007	0.106	0.279	0.863	0.006	0.902	0.020

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	100	83	86	285	160	112	117
N.S.	1	1.00	1.22	1.01	1.05	3.48	1.95	1.37	1.43
time (sec)	N/A	0.022	0.065	0.114	0.501	0.719	0.884	1.235	0.095

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	128	113	116	328	838	144	133
N.S.	1	1.00	1.24	1.10	1.13	3.18	8.14	1.40	1.29
time (sec)	N/A	0.023	0.120	0.121	0.511	0.659	1.241	2.446	0.175

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	160	141	159	374	2744	167	216
N.S.	1	1.00	1.18	1.04	1.17	2.75	20.18	1.23	1.59
time (sec)	N/A	0.031	0.117	0.105	0.510	0.721	2.014	2.884	0.220

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	50	56	42	1640	49	56
N.S.	1	1.00	0.66	0.71	0.80	0.60	23.43	0.70	0.80
time (sec)	N/A	0.013	0.026	0.111	0.276	0.681	1.197	1.337	0.045

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	41	31	600	37	37
N.S.	1	1.00	0.69	0.73	0.80	0.61	11.76	0.73	0.73
time (sec)	N/A	0.009	0.023	0.109	0.269	0.953	0.789	1.020	0.041

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	26	26	19	162	23	25
N.S.	1	1.00	0.72	0.81	0.81	0.59	5.06	0.72	0.78
time (sec)	N/A	0.005	0.014	0.108	0.280	1.029	0.516	0.849	0.029

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.001	0.007	0.115	0.278	0.757	0.007	0.784	0.017

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	76	77	115	150	78	95
N.S.	1	1.00	1.16	0.95	0.96	1.44	1.88	0.98	1.19
time (sec)	N/A	0.018	0.060	0.111	0.516	0.599	0.936	0.901	0.166

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	119	104	106	166	830	108	122
N.S.	1	1.00	1.21	1.06	1.08	1.69	8.47	1.10	1.24
time (sec)	N/A	0.023	0.128	0.133	0.482	0.496	1.206	0.817	0.130

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	149	130	142	162	2728	130	175
N.S.	1	1.00	1.15	1.00	1.09	1.25	20.98	1.00	1.35
time (sec)	N/A	0.033	0.128	0.148	0.477	0.503	2.035	0.995	0.131

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	46	49	56	52	1538	62	56
N.S.	1	1.00	0.66	0.70	0.80	0.74	21.97	0.89	0.80
time (sec)	N/A	0.012	0.025	0.123	0.272	0.426	1.268	1.564	0.053

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	38	41	40	534	46	35
N.S.	1	1.00	0.69	0.78	0.84	0.82	10.90	0.94	0.71
time (sec)	N/A	0.009	0.024	0.126	0.280	0.461	0.829	1.411	0.042

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	25	26	29	41	30	20
N.S.	1	1.00	0.72	0.78	0.81	0.91	1.28	0.94	0.62
time (sec)	N/A	0.005	0.014	0.115	0.273	0.509	0.268	1.034	0.030

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86
time (sec)	N/A	0.001	0.009	0.113	0.267	0.962	0.009	0.696	0.021

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	111	95	88	285	184	89	114
N.S.	1	1.00	1.19	1.02	0.95	3.06	1.98	0.96	1.23
time (sec)	N/A	0.023	0.088	0.112	0.487	1.895	1.000	1.051	0.056

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	112	122	407	857	120	173
N.S.	1	1.00	1.09	0.99	1.08	3.60	7.58	1.06	1.53
time (sec)	N/A	0.029	0.210	0.112	0.518	1.265	1.379	0.919	0.068

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	142	126	158	407	2793	140	221
N.S.	1	1.00	0.95	0.85	1.06	2.73	18.74	0.94	1.48
time (sec)	N/A	0.042	0.283	0.131	0.488	1.246	2.986	0.808	0.131

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	86	86	88	138	87	105
N.S.	1	1.00	1.37	1.21	1.21	1.24	1.94	1.23	1.48
time (sec)	N/A	0.024	0.056	0.120	0.478	1.017	0.878	0.559	0.101

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	101	90	90	92	136	91	108
N.S.	1	1.00	1.38	1.23	1.23	1.26	1.86	1.25	1.48
time (sec)	N/A	0.022	0.056	0.117	0.485	1.100	0.891	0.631	0.128

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	96	94	93	134	95	112
N.S.	1	1.00	1.38	1.30	1.27	1.26	1.81	1.28	1.51
time (sec)	N/A	0.022	0.058	0.099	0.494	0.927	0.922	0.604	0.108

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	100	98	97	139	99	115
N.S.	1	1.00	1.39	1.32	1.29	1.28	1.83	1.30	1.51
time (sec)	N/A	0.021	0.073	0.118	0.505	0.872	0.937	0.804	0.072

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	95	87	87	86	134	88	101
N.S.	1	1.00	1.32	1.21	1.21	1.19	1.86	1.22	1.40
time (sec)	N/A	0.017	0.048	0.118	0.479	0.705	0.893	1.121	0.141

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	99	91	91	90	136	92	104
N.S.	1	1.00	1.34	1.23	1.23	1.22	1.84	1.24	1.41
time (sec)	N/A	0.018	0.047	0.115	0.484	0.499	0.900	1.379	0.107

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	95	93	95	134	94	107
N.S.	1	1.00	1.38	1.28	1.26	1.28	1.81	1.27	1.45
time (sec)	N/A	0.017	0.044	0.119	0.483	0.441	0.914	0.880	0.156

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	106	99	97	99	133	98	110
N.S.	1	1.00	1.39	1.30	1.28	1.30	1.75	1.29	1.45
time (sec)	N/A	0.018	0.049	0.117	0.484	0.488	0.947	1.264	0.160

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	30	25	33	87	43	30
N.S.	1	1.00	0.88	1.20	1.00	1.32	3.48	1.72	1.20
time (sec)	N/A	0.007	0.023	0.012	0.266	0.455	0.102	1.664	0.313

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62
time (sec)	N/A	0.003	0.011	0.047	0.268	0.456	0.203	1.121	0.091

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62
time (sec)	N/A	0.003	0.010	0.047	0.287	0.391	0.107	1.198	0.027

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62
time (sec)	N/A	0.003	0.010	0.047	0.305	0.388	0.657	1.394	0.025

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	17	13	12
N.S.	1	1.00	0.84	0.74	0.68	0.63	0.89	0.68	0.63
time (sec)	N/A	0.003	0.009	0.020	0.274	0.456	0.046	0.916	0.025

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	13	14	13	12	15	13	11
N.S.	1	1.00	0.76	0.82	0.76	0.71	0.88	0.76	0.65
time (sec)	N/A	0.003	0.011	0.026	0.277	0.414	0.129	0.553	0.028

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	14	11	11	19	11	13
N.S.	1	1.00	0.79	0.74	0.58	0.58	1.00	0.58	0.68
time (sec)	N/A	0.003	0.013	0.025	0.272	0.435	0.184	0.640	0.027

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	51	43	85	299	117	93
N.S.	1	1.00	0.88	1.19	1.00	1.98	6.95	2.72	2.16
time (sec)	N/A	0.010	0.043	0.087	0.267	0.513	0.173	0.612	0.416

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67
time (sec)	N/A	0.005	0.016	0.087	0.276	0.552	0.283	0.592	0.103

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67
time (sec)	N/A	0.005	0.015	0.088	0.264	0.460	0.159	0.566	0.035

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	1851	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	51.42	0.67	0.67
time (sec)	N/A	0.005	0.014	0.092	0.265	0.422	63.510	0.517	0.042

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	24	32	24	24
N.S.	1	1.00	0.82	0.74	0.71	0.71	0.94	0.71	0.71
time (sec)	N/A	0.005	0.014	0.089	0.286	0.464	0.072	0.622	0.037

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	23	31	24	24
N.S.	1	1.00	0.88	0.78	0.75	0.72	0.97	0.75	0.75
time (sec)	N/A	0.006	0.016	0.095	0.275	0.467	0.150	0.543	0.035

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	23	24	31	23	24
N.S.	1	1.00	0.81	0.78	0.72	0.75	0.97	0.72	0.75
time (sec)	N/A	0.005	0.018	0.089	0.284	0.435	0.218	0.550	0.030

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	61	157	663	224	167
N.S.	1	1.00	0.89	1.18	1.00	2.57	10.87	3.67	2.74
time (sec)	N/A	0.015	0.042	0.089	0.305	0.515	0.242	0.577	0.389

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.008	0.016	0.090	0.279	0.482	0.369	0.563	0.045

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.008	0.016	0.087	0.279	0.549	0.234	0.561	0.046

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	0	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.00	0.69	0.69
time (sec)	N/A	0.008	0.015	0.087	0.384	0.588	0.000	0.503	0.042

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	39	36	35	35	46	35	35
N.S.	1	1.00	0.83	0.77	0.74	0.74	0.98	0.74	0.74
time (sec)	N/A	0.008	0.015	0.110	0.566	1.159	0.103	0.507	0.043

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	39	36	35	34	44	35	35
N.S.	1	1.00	0.87	0.80	0.78	0.76	0.98	0.78	0.78
time (sec)	N/A	0.008	0.018	0.096	0.512	0.689	0.191	0.557	0.047

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	36	34	34	46	34	35
N.S.	1	1.00	0.81	0.77	0.72	0.72	0.98	0.72	0.74
time (sec)	N/A	0.008	0.020	0.090	0.291	0.614	0.218	0.530	0.039

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	54	132	122	59	48
N.S.	1	1.00	0.90	0.79	0.79	1.94	1.79	0.87	0.71
time (sec)	N/A	0.024	0.050	0.105	0.536	0.649	2.838	0.597	0.057

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	42	103	107	45	37
N.S.	1	1.00	0.92	0.81	0.79	1.94	2.02	0.85	0.70
time (sec)	N/A	0.012	0.040	0.101	0.490	0.722	0.690	0.600	0.052

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	85	88	31	28
N.S.	1	1.00	1.00	0.80	0.78	2.12	2.20	0.78	0.70
time (sec)	N/A	0.009	0.023	0.102	0.530	0.511	0.317	0.522	0.041

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	68	73	18	19
N.S.	1	1.00	1.00	0.66	0.62	2.34	2.52	0.62	0.66
time (sec)	N/A	0.006	0.016	0.098	0.502	0.699	0.415	0.574	0.044

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	31	93	85	31	28
N.S.	1	1.00	1.00	0.80	0.78	2.32	2.12	0.78	0.70
time (sec)	N/A	0.009	0.027	0.121	0.486	0.485	0.969	0.570	0.044

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	43	41	118	107	41	38
N.S.	1	1.00	0.91	0.81	0.77	2.23	2.02	0.77	0.72
time (sec)	N/A	0.012	0.040	0.107	0.486	0.510	3.524	0.738	0.102

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	52	144	126	52	49
N.S.	1	1.00	0.90	0.79	0.76	2.12	1.85	0.76	0.72
time (sec)	N/A	0.016	0.050	0.106	0.496	0.608	14.779	0.660	0.110

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	68	59	63	161	389	65	58
N.S.	1	1.00	0.97	0.84	0.90	2.30	5.56	0.93	0.83
time (sec)	N/A	0.016	0.083	0.118	0.497	0.569	14.300	0.611	0.114

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	47	49	134	332	46	46
N.S.	1	1.00	0.95	0.82	0.86	2.35	5.82	0.81	0.81
time (sec)	N/A	0.012	0.072	0.130	0.488	0.562	4.031	0.525	0.124

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	37	115	269	36	34
N.S.	1	1.00	1.00	0.80	0.80	2.50	5.85	0.78	0.74
time (sec)	N/A	0.009	0.055	0.102	0.538	0.496	1.595	0.605	0.040

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	116	277	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.58	6.16	0.78	0.73
time (sec)	N/A	0.009	0.051	0.095	0.569	0.470	2.648	0.580	0.095

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	47	51	147	384	49	48
N.S.	1	1.00	0.96	0.84	0.91	2.62	6.86	0.88	0.86
time (sec)	N/A	0.012	0.072	0.112	0.554	0.491	7.541	0.511	0.124

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	58	64	184	452	58	58
N.S.	1	1.00	0.99	0.84	0.93	2.67	6.55	0.84	0.84
time (sec)	N/A	0.015	0.080	0.114	0.507	0.505	29.408	0.965	0.150

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	68	86	227	762	77	81
N.S.	1	1.00	0.85	0.72	0.91	2.39	8.02	0.81	0.85
time (sec)	N/A	0.022	0.127	0.132	0.516	0.563	96.958	1.376	0.122

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	56	73	200	683	59	69
N.S.	1	1.00	0.85	0.68	0.89	2.44	8.33	0.72	0.84
time (sec)	N/A	0.016	0.114	0.115	0.497	0.735	38.622	1.918	0.143

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	50	61	185	605	47	58
N.S.	1	1.00	0.84	0.71	0.87	2.64	8.64	0.67	0.83
time (sec)	N/A	0.013	0.108	0.099	0.498	0.720	15.885	1.370	0.131

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	52	64	186	627	52	56
N.S.	1	1.00	0.82	0.71	0.88	2.55	8.59	0.71	0.77
time (sec)	N/A	0.013	0.105	0.129	0.498	0.931	7.041	1.215	0.125

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	59	59	60	186	632	47	57
N.S.	1	1.00	0.84	0.84	0.86	2.66	9.03	0.67	0.81
time (sec)	N/A	0.013	0.073	0.108	0.525	0.735	11.408	1.869	0.126

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	56	73	214	779	59	70
N.S.	1	1.00	0.85	0.68	0.89	2.61	9.50	0.72	0.85
time (sec)	N/A	0.015	0.108	0.115	0.513	0.584	30.950	1.013	0.149

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	81	67	86	250	869	71	80
N.S.	1	1.00	0.85	0.71	0.91	2.63	9.15	0.75	0.84
time (sec)	N/A	0.020	0.121	0.134	0.503	0.637	83.768	0.727	0.155

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	70	131	117	61	51
N.S.	1	1.00	0.90	0.79	1.03	1.93	1.72	0.90	0.75
time (sec)	N/A	0.019	0.052	0.124	0.503	0.511	2.919	0.576	0.147

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	43	58	103	102	47	37
N.S.	1	1.00	0.92	0.81	1.09	1.94	1.92	0.89	0.70
time (sec)	N/A	0.012	0.040	0.119	0.494	0.554	0.699	0.552	0.114

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	47	83	83	33	28
N.S.	1	1.00	1.00	0.80	1.18	2.08	2.08	0.82	0.70
time (sec)	N/A	0.009	0.025	0.111	0.495	0.530	0.315	0.572	0.112

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	34	67	68	20	19
N.S.	1	1.00	1.00	0.66	1.17	2.31	2.34	0.69	0.66
time (sec)	N/A	0.007	0.016	0.095	0.483	0.473	0.420	0.610	0.125

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	32	47	91	76	33	28
N.S.	1	1.00	1.00	0.80	1.18	2.28	1.90	0.82	0.70
time (sec)	N/A	0.010	0.027	0.126	0.508	0.451	1.019	0.595	0.056

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	48	43	55	113	99	41	37
N.S.	1	1.00	0.91	0.81	1.04	2.13	1.87	0.77	0.70
time (sec)	N/A	0.012	0.045	0.103	0.497	0.385	3.428	0.514	0.122

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	54	68	143	117	54	48
N.S.	1	1.00	0.90	0.79	1.00	2.10	1.72	0.79	0.71
time (sec)	N/A	0.027	0.048	0.124	0.497	0.411	14.833	0.503	0.129

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	60	81	167	354	69	61
N.S.	1	1.00	1.00	0.86	1.16	2.39	5.06	0.99	0.87
time (sec)	N/A	0.031	0.080	0.116	0.490	0.437	15.262	0.924	0.071

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	48	68	138	301	51	47
N.S.	1	1.00	0.98	0.84	1.19	2.42	5.28	0.89	0.82
time (sec)	N/A	0.026	0.065	0.130	0.504	0.435	3.756	0.595	0.115

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	38	56	123	243	40	35
N.S.	1	1.00	1.04	0.81	1.19	2.62	5.17	0.85	0.74
time (sec)	N/A	0.020	0.058	0.123	0.492	0.417	1.627	0.943	0.112

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	56	122	252	41	34
N.S.	1	1.00	1.00	0.80	1.22	2.65	5.48	0.89	0.74
time (sec)	N/A	0.021	0.052	0.106	0.511	0.418	2.643	1.338	0.055

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	48	69	151	354	52	49
N.S.	1	1.00	0.96	0.84	1.21	2.65	6.21	0.91	0.86
time (sec)	N/A	0.026	0.067	0.130	0.510	0.395	7.121	1.032	0.073

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	59	82	187	416	61	60
N.S.	1	1.00	0.99	0.84	1.17	2.67	5.94	0.87	0.86
time (sec)	N/A	0.033	0.083	0.109	0.557	0.402	30.589	1.370	0.138

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	69	103	227	695	81	83
N.S.	1	1.00	0.85	0.71	1.06	2.34	7.16	0.84	0.86
time (sec)	N/A	0.043	0.122	0.113	0.607	0.389	105.644	0.722	0.141

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	57	90	199	624	63	69
N.S.	1	1.00	0.85	0.68	1.07	2.37	7.43	0.75	0.82
time (sec)	N/A	0.035	0.122	0.141	0.518	0.412	40.129	0.746	0.065

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	51	78	186	552	51	58
N.S.	1	1.00	0.83	0.71	1.08	2.58	7.67	0.71	0.81
time (sec)	N/A	0.073	0.111	0.103	0.563	0.472	17.910	0.681	0.143

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	53	80	183	575	55	57
N.S.	1	1.00	0.80	0.71	1.07	2.44	7.67	0.73	0.76
time (sec)	N/A	0.019	0.094	0.112	0.544	0.420	8.185	0.607	0.137

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	61	77	185	580	51	58
N.S.	1	1.00	0.83	0.85	1.07	2.57	8.06	0.71	0.81
time (sec)	N/A	0.015	0.081	0.115	0.514	0.442	12.329	0.963	0.135

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	57	90	213	716	63	69
N.S.	1	1.00	0.85	0.68	1.07	2.54	8.52	0.75	0.82
time (sec)	N/A	0.016	0.106	0.125	0.516	0.438	35.101	1.195	0.155

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	82	68	103	249	799	73	80
N.S.	1	1.00	0.85	0.70	1.06	2.57	8.24	0.75	0.82
time (sec)	N/A	0.022	0.113	0.133	0.506	0.425	94.621	1.460	0.165

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	88	128	178	162	153	0	-1
N.S.	1	1.00	0.72	1.05	1.46	1.33	1.25	0.00	-0.01
time (sec)	N/A	0.031	0.097	0.115	0.500	0.410	24.060	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	77	106	146	141	122	0	-1
N.S.	1	1.00	0.79	1.08	1.49	1.44	1.24	0.00	-0.01
time (sec)	N/A	0.022	0.085	0.122	0.497	0.469	5.994	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	84	108	114	97	0	52
N.S.	1	1.00	0.85	1.14	1.46	1.54	1.31	0.00	0.70
time (sec)	N/A	0.015	0.057	0.135	0.495	0.461	1.961	0.000	0.150

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	62	70	93	42	0	41
N.S.	1	1.00	1.07	1.41	1.59	2.11	0.95	0.00	0.93
time (sec)	N/A	0.012	0.052	0.120	0.501	0.463	1.067	0.000	0.683

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	47	61	54	89	68	0	-1
N.S.	1	1.00	1.04	1.36	1.20	1.98	1.51	0.00	-0.02
time (sec)	N/A	0.011	0.062	0.106	0.507	0.488	0.822	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	49	15	15	41	33	21
N.S.	1	1.00	1.00	2.33	0.71	0.71	1.95	1.57	1.00
time (sec)	N/A	0.001	0.021	0.119	0.279	0.430	0.792	0.928	0.237

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	39	71	31	34	65	50	32
N.S.	1	1.00	0.89	1.61	0.70	0.77	1.48	1.14	0.73
time (sec)	N/A	0.003	0.071	0.100	0.287	0.582	2.889	0.750	0.255

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	93	46	45	347	66	43
N.S.	1	1.00	0.75	1.37	0.68	0.66	5.10	0.97	0.63
time (sec)	N/A	0.007	0.075	0.113	0.296	0.534	8.651	0.929	0.264

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	135	170	164	323	0	-1
N.S.	1	1.00	0.72	1.06	1.34	1.29	2.54	0.00	-0.01
time (sec)	N/A	0.028	0.145	0.127	0.503	0.542	22.561	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	112	135	142	260	0	-1
N.S.	1	1.00	0.79	1.10	1.32	1.39	2.55	0.00	-0.01
time (sec)	N/A	0.022	0.116	0.128	0.497	0.474	5.565	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	89	95	118	207	0	58
N.S.	1	1.00	0.92	1.16	1.23	1.53	2.69	0.00	0.75
time (sec)	N/A	0.016	0.089	0.110	0.530	0.454	2.302	0.000	0.083

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	53	66	52	94	119	0	43
N.S.	1	1.00	1.15	1.43	1.13	2.04	2.59	0.00	0.93
time (sec)	N/A	0.012	0.061	0.119	0.528	0.487	0.976	0.000	0.593

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	53	66	35	91	148	0	-1
N.S.	1	1.00	1.13	1.40	0.74	1.94	3.15	0.00	-0.02
time (sec)	N/A	0.012	0.065	0.127	0.501	0.548	0.909	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	52	16	23	88	42	21
N.S.	1	1.00	1.00	2.36	0.73	1.05	4.00	1.91	0.95
time (sec)	N/A	0.001	0.066	0.111	0.290	0.458	0.863	1.245	0.243

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	75	33	34	241	61	32
N.S.	1	1.00	0.89	1.63	0.72	0.74	5.24	1.33	0.70
time (sec)	N/A	0.004	0.082	0.123	0.278	0.459	3.155	0.786	0.253

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	52	98	49	46	707	79	43
N.S.	1	1.00	0.73	1.38	0.69	0.65	9.96	1.11	0.61
time (sec)	N/A	0.008	0.094	0.102	0.282	0.499	11.474	0.787	0.269

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	76	121	163	140	117	0	-1
N.S.	1	1.00	0.70	1.12	1.51	1.30	1.08	0.00	-0.01
time (sec)	N/A	0.023	0.094	0.142	0.541	0.480	22.816	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	100	134	121	90	0	-1
N.S.	1	1.00	0.77	1.19	1.60	1.44	1.07	0.00	-0.01
time (sec)	N/A	0.014	0.078	0.104	0.518	0.534	5.140	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	79	98	101	71	0	46
N.S.	1	1.00	0.88	1.23	1.53	1.58	1.11	0.00	0.72
time (sec)	N/A	0.010	0.057	0.112	0.526	0.492	2.093	0.000	0.098

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	46	58	68	86	37	0	40
N.S.	1	1.00	1.15	1.45	1.70	2.15	0.92	0.00	1.00
time (sec)	N/A	0.005	0.045	0.126	0.534	0.546	0.884	0.000	0.617

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	47	49	54	87	48	0	-1
N.S.	1	1.00	1.15	1.20	1.32	2.12	1.17	0.00	-0.02
time (sec)	N/A	0.006	0.054	0.125	0.503	0.494	0.821	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	12	12	37	29	18
N.S.	1	1.00	1.00	1.50	0.67	0.67	2.06	1.61	1.00
time (sec)	N/A	0.001	0.020	0.119	0.285	0.513	0.717	1.073	0.208

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	43	26	25	56	42	26
N.S.	1	1.00	0.82	1.13	0.68	0.66	1.47	1.11	0.68
time (sec)	N/A	0.003	0.056	0.122	0.280	0.536	3.215	0.749	0.215

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	40	59	41	34	270	55	34
N.S.	1	1.00	0.68	1.00	0.69	0.58	4.58	0.93	0.58
time (sec)	N/A	0.005	0.062	0.129	0.287	0.482	8.047	1.141	0.224

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	82	128	147	141	250	0	-1
N.S.	1	1.00	0.73	1.14	1.31	1.26	2.23	0.00	-0.01
time (sec)	N/A	0.020	0.124	0.122	0.525	0.498	22.593	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	72	106	117	125	194	0	-1
N.S.	1	1.00	0.83	1.22	1.34	1.44	2.23	0.00	-0.01
time (sec)	N/A	0.015	0.103	0.123	0.539	0.453	5.373	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	85	81	107	155	0	53
N.S.	1	1.00	0.95	1.31	1.25	1.65	2.38	0.00	0.82
time (sec)	N/A	0.010	0.075	0.125	0.501	0.485	1.823	0.000	0.104

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	52	63	49	89	119	0	42
N.S.	1	1.00	1.27	1.54	1.20	2.17	2.90	0.00	1.02
time (sec)	N/A	0.005	0.057	0.109	0.496	0.445	0.900	0.000	0.562

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	64	35	90	122	0	-1
N.S.	1	1.00	1.26	1.52	0.83	2.14	2.90	0.00	-0.02
time (sec)	N/A	0.007	0.062	0.110	0.524	0.445	0.849	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	29	13	18	83	35	18
N.S.	1	1.00	1.00	1.53	0.68	0.95	4.37	1.84	0.95
time (sec)	N/A	0.001	0.056	0.141	0.287	0.477	0.737	1.419	0.224

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	46	28	25	194	48	26
N.S.	1	1.00	0.78	1.15	0.70	0.62	4.85	1.20	0.65
time (sec)	N/A	0.004	0.065	0.111	0.282	0.502	2.863	1.381	0.219

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	41	63	44	35	556	61	34
N.S.	1	1.00	0.66	1.02	0.71	0.56	8.97	0.98	0.55
time (sec)	N/A	0.006	0.072	0.133	0.282	0.487	10.830	1.289	0.224

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	99	144	212	184	178	0	-1
N.S.	1	1.00	0.69	1.01	1.48	1.29	1.24	0.00	-0.01
time (sec)	N/A	0.035	0.128	0.122	0.499	0.509	64.169	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	122	178	163	153	0	-1
N.S.	1	1.00	0.74	1.03	1.50	1.37	1.29	0.00	-0.01
time (sec)	N/A	0.026	0.092	0.105	0.516	0.459	13.317	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	76	97	144	140	124	0	-1
N.S.	1	1.00	0.80	1.02	1.52	1.47	1.31	0.00	-0.01
time (sec)	N/A	0.022	0.096	0.106	0.504	0.461	4.112	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	78	107	119	75	0	-1
N.S.	1	1.00	0.87	1.10	1.51	1.68	1.06	0.00	-0.01
time (sec)	N/A	0.015	0.079	0.123	0.494	0.437	1.868	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	71	84	109	92	0	-1
N.S.	1	1.00	0.86	1.13	1.33	1.73	1.46	0.00	-0.02
time (sec)	N/A	0.015	0.107	0.102	0.509	0.473	1.490	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	67	67	109	71	0	-1
N.S.	1	1.00	0.86	1.05	1.05	1.70	1.11	0.00	-0.02
time (sec)	N/A	0.014	0.112	0.106	0.527	0.431	1.770	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	104	152	207	185	376	0	-1
N.S.	1	1.00	0.70	1.02	1.39	1.24	2.52	0.00	-0.01
time (sec)	N/A	0.035	0.177	0.109	0.517	0.465	66.963	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	93	129	170	163	323	0	-1
N.S.	1	1.00	0.75	1.04	1.37	1.31	2.60	0.00	-0.01
time (sec)	N/A	0.027	0.138	0.108	0.516	0.483	12.588	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	104	133	141	264	0	-1
N.S.	1	1.00	0.83	1.05	1.34	1.42	2.67	0.00	-0.01
time (sec)	N/A	0.021	0.122	0.107	0.523	0.469	4.019	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	83	93	119	190	0	-1
N.S.	1	1.00	0.92	1.12	1.26	1.61	2.57	0.00	-0.01
time (sec)	N/A	0.015	0.097	0.105	0.523	0.558	1.775	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	74	68	109	197	0	-1
N.S.	1	1.00	0.92	1.12	1.03	1.65	2.98	0.00	-0.02
time (sec)	N/A	0.015	0.108	0.104	0.515	0.634	1.666	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	71	49	115	187	0	-1
N.S.	1	1.00	0.96	1.06	0.73	1.72	2.79	0.00	-0.01
time (sec)	N/A	0.015	0.123	0.105	0.526	0.589	2.292	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	84	135	194	156	136	0	-1
N.S.	1	1.00	0.67	1.07	1.54	1.24	1.08	0.00	-0.01
time (sec)	N/A	0.025	0.115	0.115	0.520	0.565	65.073	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	75	114	163	137	117	0	-1
N.S.	1	1.00	0.71	1.09	1.55	1.30	1.11	0.00	-0.01
time (sec)	N/A	0.018	0.085	0.111	0.525	0.475	12.623	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	65	87	132	124	92	0	-1
N.S.	1	1.00	0.79	1.06	1.61	1.51	1.12	0.00	-0.01
time (sec)	N/A	0.012	0.081	0.122	0.517	0.598	3.739	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	98	105	76	0	-1
N.S.	1	1.00	0.89	1.18	1.61	1.72	1.25	0.00	-0.02
time (sec)	N/A	0.008	0.069	0.116	0.519	0.493	1.777	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	55	81	99	73	0	-1
N.S.	1	1.00	0.88	0.95	1.40	1.71	1.26	0.00	-0.02
time (sec)	N/A	0.009	0.081	0.125	0.531	0.472	1.510	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	55	55	67	108	70	0	-1
N.S.	1	1.00	0.92	0.92	1.12	1.80	1.17	0.00	-0.02
time (sec)	N/A	0.010	0.090	0.129	0.501	0.493	1.677	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	90	143	179	157	289	0	-1
N.S.	1	1.00	0.69	1.09	1.37	1.20	2.21	0.00	-0.01
time (sec)	N/A	0.027	0.148	0.113	0.532	0.455	62.731	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	81	122	147	139	250	0	-1
N.S.	1	1.00	0.74	1.12	1.35	1.28	2.29	0.00	-0.01
time (sec)	N/A	0.019	0.123	0.109	0.533	0.438	12.588	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	94	115	125	197	0	-1
N.S.	1	1.00	0.85	1.12	1.37	1.49	2.35	0.00	-0.01
time (sec)	N/A	0.011	0.099	0.110	0.506	0.531	3.604	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	78	79	107	165	0	-1
N.S.	1	1.00	0.95	1.24	1.25	1.70	2.62	0.00	-0.02
time (sec)	N/A	0.009	0.080	0.112	0.504	0.520	1.684	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	70	63	101	158	0	-1
N.S.	1	1.00	0.97	1.17	1.05	1.68	2.63	0.00	-0.02
time (sec)	N/A	0.008	0.091	0.125	0.516	0.480	1.463	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	70	49	111	184	0	-1
N.S.	1	1.00	1.00	1.13	0.79	1.79	2.97	0.00	-0.02
time (sec)	N/A	0.009	0.099	0.125	0.506	0.457	2.078	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	110	160	244	206	207	0	-1
N.S.	1	1.00	0.67	0.98	1.49	1.26	1.26	0.00	-0.01
time (sec)	N/A	0.041	0.116	0.105	0.512	0.395	191.953	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	99	138	212	185	180	0	-1
N.S.	1	1.00	0.71	0.99	1.51	1.32	1.29	0.00	-0.01
time (sec)	N/A	0.040	0.133	0.119	0.515	0.423	33.307	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	87	113	176	162	155	0	-1
N.S.	1	1.00	0.75	0.97	1.52	1.40	1.34	0.00	-0.01
time (sec)	N/A	0.026	0.117	0.121	0.520	0.427	8.520	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	73	94	141	141	102	0	-1
N.S.	1	1.00	0.79	1.02	1.53	1.53	1.11	0.00	-0.01
time (sec)	N/A	0.018	0.108	0.104	0.510	0.408	3.753	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	84	125	137	126	0	-1
N.S.	1	1.00	0.82	0.94	1.40	1.54	1.42	0.00	-0.01
time (sec)	N/A	0.019	0.120	0.104	0.499	0.549	3.715	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	82	100	138	99	0	-1
N.S.	1	1.00	0.80	0.95	1.16	1.60	1.15	0.00	-0.01
time (sec)	N/A	0.019	0.133	0.109	0.512	0.477	3.398	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	114	169	242	208	435	0	-1
N.S.	1	1.00	0.67	0.99	1.42	1.22	2.54	0.00	-0.01
time (sec)	N/A	0.044	0.189	0.112	0.494	0.498	184.163	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	103	146	207	186	379	0	-1
N.S.	1	1.00	0.71	1.00	1.42	1.27	2.60	0.00	-0.01
time (sec)	N/A	0.034	0.175	0.141	0.533	0.511	33.926	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	121	168	164	326	0	-1
N.S.	1	1.00	0.77	1.00	1.39	1.36	2.69	0.00	-0.01
time (sec)	N/A	0.026	0.153	0.107	0.507	0.477	8.789	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	100	130	142	246	0	-1
N.S.	1	1.00	0.82	1.04	1.35	1.48	2.56	0.00	-0.01
time (sec)	N/A	0.020	0.126	0.104	0.512	0.568	3.906	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	79	88	112	137	267	0	-1
N.S.	1	1.00	0.85	0.95	1.20	1.47	2.87	0.00	-0.01
time (sec)	N/A	0.020	0.134	0.124	0.498	0.525	3.815	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	86	84	139	245	0	-1
N.S.	1	1.00	0.84	0.96	0.93	1.54	2.72	0.00	-0.01
time (sec)	N/A	0.019	0.151	0.120	0.501	0.600	3.578	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	92	147	223	172	158	0	-1
N.S.	1	1.00	0.64	1.02	1.55	1.19	1.10	0.00	-0.01
time (sec)	N/A	0.029	0.111	0.112	0.505	0.605	184.699	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	126	194	155	138	0	-1
N.S.	1	1.00	0.68	1.02	1.58	1.26	1.12	0.00	-0.01
time (sec)	N/A	0.019	0.117	0.110	0.515	0.567	32.701	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	76	99	161	140	119	0	-1
N.S.	1	1.00	0.75	0.97	1.58	1.37	1.17	0.00	-0.01
time (sec)	N/A	0.015	0.099	0.108	0.601	0.931	8.330	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	84	129	123	97	0	-1
N.S.	1	1.00	0.80	1.06	1.63	1.56	1.23	0.00	-0.01
time (sec)	N/A	0.012	0.091	0.112	0.506	0.930	3.712	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	62	63	113	116	94	0	-1
N.S.	1	1.00	0.78	0.80	1.43	1.47	1.19	0.00	-0.01
time (sec)	N/A	0.012	0.095	0.109	0.525	1.129	3.589	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	63	63	96	123	88	0	-1
N.S.	1	1.00	0.78	0.78	1.19	1.52	1.09	0.00	-0.01
time (sec)	N/A	0.012	0.107	0.139	0.501	0.793	3.418	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	98	157	209	173	335	0	-1
N.S.	1	1.00	0.65	1.05	1.39	1.15	2.23	0.00	-0.01
time (sec)	N/A	0.033	0.156	0.133	0.542	1.011	183.344	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	90	135	179	157	292	0	-1
N.S.	1	1.00	0.70	1.05	1.40	1.23	2.28	0.00	-0.01
time (sec)	N/A	0.021	0.143	0.125	0.544	0.709	33.123	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	82	107	145	141	253	0	-1
N.S.	1	1.00	0.77	1.01	1.37	1.33	2.39	0.00	-0.01
time (sec)	N/A	0.016	0.120	0.130	0.504	0.730	8.502	0.000	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	69	91	112	125	207	0	-1
N.S.	1	1.00	0.84	1.11	1.37	1.52	2.52	0.00	-0.01
time (sec)	N/A	0.012	0.111	0.139	0.512	0.654	3.698	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	78	96	117	201	0	-1
N.S.	1	1.00	0.83	0.95	1.17	1.43	2.45	0.00	-0.01
time (sec)	N/A	0.012	0.111	0.120	0.503	0.615	3.582	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	78	79	126	223	0	-1
N.S.	1	1.00	0.83	0.93	0.94	1.50	2.65	0.00	-0.01
time (sec)	N/A	0.012	0.119	0.148	0.514	0.666	3.534	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	109	146	140	128	0	-1
N.S.	1	1.00	0.75	1.08	1.45	1.39	1.27	0.00	-0.01
time (sec)	N/A	0.021	0.094	0.124	0.512	0.473	9.207	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	87	112	119	100	0	-1
N.S.	1	1.00	0.86	1.13	1.45	1.55	1.30	0.00	-0.01
time (sec)	N/A	0.015	0.073	0.104	0.497	0.514	2.820	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	65	73	91	44	0	44
N.S.	1	1.00	1.02	1.35	1.52	1.90	0.92	0.00	0.92
time (sec)	N/A	0.011	0.055	0.107	0.503	0.612	1.130	0.000	0.548

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	48	41	57	22	0	30
N.S.	1	1.00	1.07	1.71	1.46	2.04	0.79	0.00	1.07
time (sec)	N/A	0.009	0.033	0.115	0.488	0.545	0.487	0.000	0.030

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	19	33	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.00	1.74	0.79
time (sec)	N/A	0.001	0.018	0.120	0.273	0.690	0.439	1.614	0.349

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	33	31	23	42	50	25
N.S.	1	1.00	0.61	0.75	0.70	0.52	0.95	1.14	0.57
time (sec)	N/A	0.004	0.059	0.119	0.288	0.754	0.960	1.259	0.342

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	40	55	46	34	287	66	36
N.S.	1	1.00	0.59	0.81	0.68	0.50	4.22	0.97	0.53
time (sec)	N/A	0.008	0.069	0.117	0.274	0.691	3.503	1.200	0.353

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	51	77	61	45	488	82	47
N.S.	1	1.00	0.55	0.84	0.66	0.49	5.30	0.89	0.51
time (sec)	N/A	0.012	0.077	0.125	0.277	1.494	9.873	0.991	0.382

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	76	119	131	175	105	131	-1
N.S.	1	1.00	0.79	1.24	1.36	1.82	1.09	1.36	-0.01
time (sec)	N/A	0.022	0.129	0.129	0.490	0.928	5.461	19.080	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	106	92	145	71	115	-1
N.S.	1	1.00	0.84	1.56	1.35	2.13	1.04	1.69	-0.01
time (sec)	N/A	0.016	0.107	0.128	0.506	0.876	1.913	18.415	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	0	57	119	46	85	-1
N.S.	1	1.00	1.04	0.00	1.19	2.48	0.96	1.77	-0.02
time (sec)	N/A	0.011	0.069	0.028	0.498	0.761	0.810	18.566	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	22	17	45	22
N.S.	1	1.00	1.00	0.84	0.79	1.16	0.89	2.37	1.16
time (sec)	N/A	0.001	0.019	0.117	0.278	0.814	0.441	0.967	0.326

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	33	32	34	41	82	39
N.S.	1	1.00	0.64	0.85	0.82	0.87	1.05	2.10	1.00
time (sec)	N/A	0.003	0.062	0.144	0.284	0.856	0.790	1.631	0.386

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	38	55	50	49	219	98	46
N.S.	1	1.00	0.60	0.87	0.79	0.78	3.48	1.56	0.73
time (sec)	N/A	0.007	0.085	0.128	0.277	0.733	2.181	1.352	0.413

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	49	77	64	58	348	121	58
N.S.	1	1.00	0.56	0.89	0.74	0.67	4.00	1.39	0.67
time (sec)	N/A	0.012	0.094	0.129	0.280	0.613	6.756	1.483	0.426

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	147	109	214	396	197	-1
N.S.	1	1.00	0.79	1.62	1.20	2.35	4.35	2.16	-0.01
time (sec)	N/A	0.020	0.140	0.139	0.496	0.635	4.785	21.512	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	0	69	186	328	165	-1
N.S.	1	1.00	0.87	0.00	1.00	2.70	4.75	2.39	-0.01
time (sec)	N/A	0.015	0.128	0.026	0.495	0.614	2.044	21.860	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	54	15	33	42	86	36
N.S.	1	1.00	1.00	2.57	0.71	1.57	2.00	4.10	1.71
time (sec)	N/A	0.001	0.022	0.119	0.280	0.808	0.695	1.746	0.242

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	32	27	43	92	81	54
N.S.	1	1.00	0.67	0.74	0.63	1.00	2.14	1.88	1.26
time (sec)	N/A	0.003	0.070	0.118	0.289	0.770	0.965	1.668	0.400

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	40	54	46	58	153	159	71
N.S.	1	1.00	0.62	0.84	0.72	0.91	2.39	2.48	1.11
time (sec)	N/A	0.008	0.085	0.138	0.288	0.910	2.124	1.806	0.420

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	49	76	64	71	337	175	88
N.S.	1	1.00	0.58	0.90	0.76	0.85	4.01	2.08	1.05
time (sec)	N/A	0.011	0.092	0.131	0.291	0.860	4.028	1.258	0.471

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	82	116	135	141	270	0	-1
N.S.	1	1.00	0.78	1.10	1.29	1.34	2.57	0.00	-0.01
time (sec)	N/A	0.022	0.120	0.123	0.484	0.898	9.083	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	93	98	119	214	0	-1
N.S.	1	1.00	0.89	1.16	1.22	1.49	2.68	0.00	-0.01
time (sec)	N/A	0.016	0.099	0.127	0.494	0.717	2.810	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	70	56	93	121	0	47
N.S.	1	1.00	1.12	1.40	1.12	1.86	2.42	0.00	0.94
time (sec)	N/A	0.012	0.068	0.123	0.492	0.693	1.229	0.000	0.517

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	51	21	57	54	0	27
N.S.	1	1.00	1.21	1.76	0.72	1.97	1.86	0.00	0.93
time (sec)	N/A	0.010	0.045	0.106	0.501	1.177	0.494	0.000	0.031

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	46	35	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.30	1.75	0.80
time (sec)	N/A	0.001	0.020	0.128	0.290	1.070	0.477	1.344	0.401

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	28	35	32	22	177	54	26
N.S.	1	1.00	0.61	0.76	0.70	0.48	3.85	1.17	0.57
time (sec)	N/A	0.004	0.069	0.118	0.286	0.837	1.046	1.193	0.350

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	81	127	118	181	224	154	-1
N.S.	1	1.00	0.81	1.27	1.18	1.81	2.24	1.54	-0.01
time (sec)	N/A	0.021	0.157	0.128	0.516	1.287	5.682	20.876	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	114	75	152	155	130	-1
N.S.	1	1.00	0.90	1.61	1.06	2.14	2.18	1.83	-0.01
time (sec)	N/A	0.016	0.141	0.130	0.501	1.001	1.932	22.157	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	0	38	128	102	98	-1
N.S.	1	1.00	1.12	0.00	0.76	2.56	2.04	1.96	-0.02
time (sec)	N/A	0.012	0.101	0.031	0.488	0.900	0.932	20.148	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	25	44	53	24
N.S.	1	1.00	1.00	0.85	0.80	1.25	2.20	2.65	1.20
time (sec)	N/A	0.001	0.021	0.116	0.281	0.954	0.462	0.894	0.339

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	26	35	34	38	112	94	42
N.S.	1	1.00	0.63	0.85	0.83	0.93	2.73	2.29	1.02
time (sec)	N/A	0.004	0.078	0.132	0.287	1.019	0.832	1.174	0.400

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	58	52	51	452	112	48
N.S.	1	1.00	0.59	0.88	0.79	0.77	6.85	1.70	0.73
time (sec)	N/A	0.007	0.099	0.136	0.298	1.067	2.434	0.900	0.432

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	160	94	215	971	221	-1
N.S.	1	1.00	0.82	1.68	0.99	2.26	10.22	2.33	-0.01
time (sec)	N/A	0.021	0.172	0.122	0.490	0.705	5.334	20.981	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	52	188	833	194	-1
N.S.	1	1.00	0.92	0.00	0.72	2.61	11.57	2.69	-0.01
time (sec)	N/A	0.015	0.149	0.028	0.489	0.767	2.277	21.704	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	56	16	34	95	102	37
N.S.	1	1.00	1.00	2.55	0.73	1.55	4.32	4.64	1.68
time (sec)	N/A	0.001	0.024	0.107	0.285	0.548	0.735	1.813	0.252

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	34	30	44	197	96	56
N.S.	1	1.00	0.67	0.76	0.67	0.98	4.38	2.13	1.24
time (sec)	N/A	0.004	0.084	0.120	0.281	0.544	1.062	2.378	0.406

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	41	57	50	59	314	189	73
N.S.	1	1.00	0.61	0.85	0.75	0.88	4.69	2.82	1.09
time (sec)	N/A	0.007	0.100	0.155	0.311	0.552	2.237	1.480	0.441

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	80	68	70	688	207	92
N.S.	1	1.00	0.57	0.91	0.77	0.80	7.82	2.35	1.05
time (sec)	N/A	0.012	0.114	0.135	0.279	0.639	5.346	1.720	0.475

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	66	104	134	124	95	0	-1
N.S.	1	1.00	0.75	1.18	1.52	1.41	1.08	0.00	-0.01
time (sec)	N/A	0.014	0.080	0.133	0.496	0.553	8.560	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	83	102	105	75	0	-1
N.S.	1	1.00	0.85	1.24	1.52	1.57	1.12	0.00	-0.01
time (sec)	N/A	0.010	0.069	0.114	0.510	0.480	2.489	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	62	70	87	54	0	43
N.S.	1	1.00	1.14	1.44	1.63	2.02	1.26	0.00	1.00
time (sec)	N/A	0.007	0.049	0.123	0.512	0.539	1.109	0.000	0.585

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	46	41	55	24	0	30
N.S.	1	1.00	1.25	1.92	1.71	2.29	1.00	0.00	1.25
time (sec)	N/A	0.004	0.031	0.121	0.496	0.487	0.444	0.000	0.035

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	29	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	1.81	0.75
time (sec)	N/A	0.001	0.016	0.102	0.275	0.441	0.453	0.975	0.330

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	27	26	17	34	42	17
N.S.	1	1.00	0.61	0.71	0.68	0.45	0.89	1.11	0.45
time (sec)	N/A	0.003	0.049	0.136	0.284	0.397	1.013	1.291	0.316

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	32	43	41	26	224	55	26
N.S.	1	1.00	0.54	0.73	0.69	0.44	3.80	0.93	0.44
time (sec)	N/A	0.005	0.057	0.121	0.274	0.438	3.483	0.959	0.323

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	40	59	56	34	374	68	33
N.S.	1	1.00	0.50	0.74	0.70	0.42	4.68	0.85	0.41
time (sec)	N/A	0.009	0.066	0.128	0.278	0.407	9.809	0.847	0.333

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	65	63	119	152	80	119	-1
N.S.	1	1.00	0.76	0.73	1.38	1.77	0.93	1.38	-0.01
time (sec)	N/A	0.015	0.107	0.134	0.491	0.431	5.023	2.605	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	55	90	134	58	106	-1
N.S.	1	1.00	0.86	0.87	1.43	2.13	0.92	1.68	-0.02
time (sec)	N/A	0.010	0.085	0.126	0.500	0.492	1.734	2.810	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	50	48	57	117	41	82	-1
N.S.	1	1.00	1.14	1.09	1.30	2.66	0.93	1.86	-0.02
time (sec)	N/A	0.007	0.063	0.123	0.504	0.577	0.767	2.754	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	15	44	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.00	2.93	0.73
time (sec)	N/A	0.001	0.017	0.132	0.303	0.549	0.483	1.733	0.307

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	21	27	26	28	34	74	17
N.S.	1	1.00	0.66	0.84	0.81	0.88	1.06	2.31	0.53
time (sec)	N/A	0.003	0.054	0.136	0.286	0.542	0.763	1.163	0.348

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	32	43	41	39	170	86	37
N.S.	1	1.00	0.60	0.81	0.77	0.74	3.21	1.62	0.70
time (sec)	N/A	0.006	0.071	0.130	0.285	0.725	2.203	1.288	0.380

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	39	59	56	47	269	107	46
N.S.	1	1.00	0.53	0.80	0.76	0.64	3.64	1.45	0.62
time (sec)	N/A	0.009	0.077	0.138	0.297	1.254	6.657	1.345	0.426

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	63	105	186	308	182	-1
N.S.	1	1.00	0.77	0.73	1.22	2.16	3.58	2.12	-0.01
time (sec)	N/A	0.014	0.116	0.150	0.493	1.341	3.820	2.875	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	55	69	171	257	154	-1
N.S.	1	1.00	0.89	0.85	1.06	2.63	3.95	2.37	-0.02
time (sec)	N/A	0.010	0.108	0.128	0.503	1.218	1.891	3.359	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	46	12	27	27	82	12
N.S.	1	1.00	1.00	2.56	0.67	1.50	1.50	4.56	0.67
time (sec)	N/A	0.001	0.018	0.103	0.282	0.683	0.739	2.052	0.252

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	26	24	32	75	79	42
N.S.	1	1.00	0.62	0.70	0.65	0.86	2.03	2.14	1.14
time (sec)	N/A	0.002	0.056	0.123	0.272	0.962	1.028	2.128	0.357

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	32	42	40	45	117	145	57
N.S.	1	1.00	0.58	0.76	0.73	0.82	2.13	2.64	1.04
time (sec)	N/A	0.004	0.076	0.131	0.294	0.889	2.083	1.642	0.378

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	40	58	55	55	257	158	71
N.S.	1	1.00	0.56	0.82	0.77	0.77	3.62	2.23	1.00
time (sec)	N/A	0.007	0.077	0.131	0.275	1.081	3.978	1.882	0.418

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	72	111	117	125	204	0	-1
N.S.	1	1.00	0.79	1.22	1.29	1.37	2.24	0.00	-0.01
time (sec)	N/A	0.015	0.103	0.131	0.502	0.863	8.600	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	89	85	107	162	0	-1
N.S.	1	1.00	0.91	1.29	1.23	1.55	2.35	0.00	-0.01
time (sec)	N/A	0.011	0.084	0.110	0.503	1.491	2.477	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	67	52	90	119	0	46
N.S.	1	1.00	1.24	1.49	1.16	2.00	2.64	0.00	1.02
time (sec)	N/A	0.007	0.061	0.127	0.491	1.539	1.058	0.000	0.520

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	50	21	56	56	0	27
N.S.	1	1.00	1.46	2.08	0.88	2.33	2.33	0.00	1.12
time (sec)	N/A	0.005	0.041	0.121	0.487	1.054	0.521	0.000	0.032

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	41	30	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	2.41	1.76	0.76
time (sec)	N/A	0.001	0.017	0.112	0.278	0.988	0.463	2.331	0.310

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	25	29	28	18	139	43	19
N.S.	1	1.00	0.62	0.72	0.70	0.45	3.48	1.08	0.48
time (sec)	N/A	0.003	0.059	0.115	0.274	1.252	1.032	1.715	0.292

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	81	101	155	172	136	-1
N.S.	1	1.00	0.80	0.91	1.13	1.74	1.93	1.53	-0.01
time (sec)	N/A	0.015	0.148	0.138	0.502	1.380	5.080	4.563	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	73	71	138	126	119	-1
N.S.	1	1.00	0.94	1.12	1.09	2.12	1.94	1.83	-0.02
time (sec)	N/A	0.010	0.127	0.138	0.502	0.998	1.716	4.502	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	67	38	122	90	92	-1
N.S.	1	1.00	1.24	1.49	0.84	2.71	2.00	2.04	-0.02
time (sec)	N/A	0.007	0.098	0.105	0.498	1.094	0.849	5.231	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	20	41	50	12
N.S.	1	1.00	1.00	0.81	0.75	1.25	2.56	3.12	0.75
time (sec)	N/A	0.001	0.021	0.110	0.282	1.070	0.455	2.259	0.299

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	28	28	29	90	83	27
N.S.	1	1.00	0.62	0.82	0.82	0.85	2.65	2.44	0.79
time (sec)	N/A	0.002	0.068	0.136	0.285	1.174	0.817	2.202	0.321

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	45	44	40	355	96	38
N.S.	1	1.00	0.59	0.80	0.79	0.71	6.34	1.71	0.68
time (sec)	N/A	0.005	0.086	0.138	0.289	1.006	2.333	1.789	0.362

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	81	86	187	751	200	-1
N.S.	1	1.00	0.81	0.91	0.97	2.10	8.44	2.25	-0.01
time (sec)	N/A	0.015	0.151	0.124	0.483	1.310	4.085	5.746	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	73	50	173	648	178	-1
N.S.	1	1.00	0.96	1.09	0.75	2.58	9.67	2.66	-0.01
time (sec)	N/A	0.011	0.129	0.126	0.496	1.470	1.889	4.900	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	49	13	28	63	95	13
N.S.	1	1.00	1.00	2.58	0.68	1.47	3.32	5.00	0.68
time (sec)	N/A	0.001	0.072	0.127	0.277	0.881	0.779	3.729	0.232

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	24	28	25	33	165	90	45
N.S.	1	1.00	0.62	0.72	0.64	0.85	4.23	2.31	1.15
time (sec)	N/A	0.003	0.073	0.114	0.270	0.730	1.041	1.719	0.357

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	33	45	42	46	245	170	59
N.S.	1	1.00	0.57	0.78	0.72	0.79	4.22	2.93	1.02
time (sec)	N/A	0.005	0.089	0.133	0.269	1.292	2.205	2.464	0.367

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	41	61	58	56	530	183	73
N.S.	1	1.00	0.55	0.81	0.77	0.75	7.07	2.44	0.97
time (sec)	N/A	0.007	0.109	0.117	0.276	1.484	4.977	2.780	0.438

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	35	41	37	27	54	17	31
N.S.	1	1.00	1.30	1.52	1.37	1.00	2.00	0.63	1.15
time (sec)	N/A	0.004	0.060	0.132	0.489	1.015	0.852	2.268	0.570

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	27	14	14	20	6	16
N.S.	1	1.00	4.75	3.38	1.75	1.75	2.50	0.75	2.00
time (sec)	N/A	0.002	0.007	0.125	0.487	1.148	0.415	1.912	0.051

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	48	21	57	42	0	23
N.S.	1	1.00	1.84	2.53	1.11	3.00	2.21	0.00	1.21
time (sec)	N/A	0.003	0.044	0.129	0.491	0.937	0.455	0.000	0.125

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62
time (sec)	N/A	0.003	0.011	0.051	0.301	0.745	0.277	2.021	0.028

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	18	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62
time (sec)	N/A	0.003	0.010	0.053	0.268	0.798	0.204	2.039	0.025

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62
time (sec)	N/A	0.003	0.010	0.053	0.265	0.905	0.106	1.441	0.024

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	16	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62
time (sec)	N/A	0.003	0.010	0.072	0.263	0.854	0.636	1.621	0.025

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	13	13	19	13	13
N.S.	1	1.00	0.81	0.67	0.62	0.62	0.90	0.62	0.62
time (sec)	N/A	0.003	0.010	0.023	0.281	0.684	0.723	1.747	0.024

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	17	13	12
N.S.	1	1.00	0.84	0.74	0.68	0.63	0.89	0.68	0.63
time (sec)	N/A	0.003	0.010	0.026	0.273	0.521	0.457	1.318	0.023

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	12	17	13	13
N.S.	1	1.00	0.89	0.74	0.68	0.63	0.89	0.68	0.68
time (sec)	N/A	0.003	0.012	0.026	0.275	0.829	0.148	1.891	0.027

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	15	14	13	13	17	13	13
N.S.	1	1.00	0.79	0.74	0.68	0.68	0.89	0.68	0.68
time (sec)	N/A	0.003	0.013	0.027	0.270	0.789	0.167	2.173	0.026

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67
time (sec)	N/A	0.005	0.015	0.111	0.280	0.653	0.417	1.629	0.045

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	29	34	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67
time (sec)	N/A	0.005	0.015	0.107	0.280	0.518	0.338	2.004	0.038

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	34	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	0.94	0.67	0.67
time (sec)	N/A	0.005	0.014	0.092	0.269	0.863	0.179	1.579	0.038

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	27	2633	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.75	73.14	0.67	0.67
time (sec)	N/A	0.005	0.013	0.095	0.280	1.321	1.011	1.984	0.035

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	28	25	24	24	1765	24	24
N.S.	1	1.00	0.78	0.69	0.67	0.67	49.03	0.67	0.67
time (sec)	N/A	0.005	0.014	0.105	0.276	1.073	0.963	1.647	0.038

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	24	1741	24	24
N.S.	1	1.00	0.82	0.74	0.71	0.71	51.21	0.71	0.71
time (sec)	N/A	0.005	0.014	0.111	0.263	0.957	0.983	1.716	0.034

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	24	23	1826	24	24
N.S.	1	1.00	0.88	0.78	0.75	0.72	57.06	0.75	0.75
time (sec)	N/A	0.005	0.017	0.097	0.265	0.715	1.065	2.449	0.037

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	28	25	24	23	1957	24	24
N.S.	1	1.00	0.82	0.74	0.71	0.68	57.56	0.71	0.71
time (sec)	N/A	0.006	0.017	0.102	0.269	0.592	0.970	2.616	0.036

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.008	0.017	0.102	0.268	0.606	0.588	1.872	0.043

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	40	49	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69
time (sec)	N/A	0.008	0.016	0.124	0.264	0.764	0.474	1.752	0.044

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	49	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.96	0.69	0.69
time (sec)	N/A	0.008	0.016	0.113	0.287	0.944	0.286	1.503	0.044

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	38	5012	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.75	98.27	0.69	0.69
time (sec)	N/A	0.008	0.016	0.094	0.263	1.377	1.504	2.347	0.048

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	35	35	6246	35	35
N.S.	1	1.00	0.76	0.71	0.69	0.69	122.47	0.69	0.69
time (sec)	N/A	0.008	0.016	0.110	0.298	0.631	1.464	1.619	0.044

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	6667	35	35
N.S.	1	1.00	0.80	0.73	0.71	0.71	136.06	0.71	0.71
time (sec)	N/A	0.007	0.016	0.092	0.371	0.569	1.534	1.573	0.044

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	4004	35	35
N.S.	1	1.00	0.80	0.73	0.71	0.71	81.71	0.71	0.71
time (sec)	N/A	0.008	0.019	0.098	0.327	0.473	1.606	1.000	0.044

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	36	35	35	3964	35	35
N.S.	1	1.00	0.80	0.73	0.71	0.71	80.90	0.71	0.71
time (sec)	N/A	0.008	0.018	0.111	0.347	0.391	1.547	0.978	0.043

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	140	124	130	147	180	138	151
N.S.	1	1.00	1.12	0.99	1.04	1.18	1.44	1.10	1.21
time (sec)	N/A	0.049	0.093	0.122	0.572	0.519	17.686	1.610	0.243

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	140	123	128	116	173	136	126
N.S.	1	1.00	1.14	1.00	1.04	0.94	1.41	1.11	1.02
time (sec)	N/A	0.040	0.077	0.173	0.576	0.496	9.274	1.541	0.068

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	127	112	114	128	162	118	130
N.S.	1	1.00	1.14	1.01	1.03	1.15	1.46	1.06	1.17
time (sec)	N/A	0.026	0.068	0.112	0.483	0.452	3.146	1.180	0.150

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	126	112	115	114	148	119	126
N.S.	1	1.00	1.16	1.03	1.06	1.05	1.36	1.09	1.16
time (sec)	N/A	0.027	0.064	0.146	0.493	0.523	2.094	1.475	0.074

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	103	96	103	313	141	118	120
N.S.	1	1.00	1.03	0.96	1.03	3.13	1.41	1.18	1.20
time (sec)	N/A	0.021	0.055	0.117	0.493	0.509	2.370	1.621	0.113

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	103	95	102	307	141	117	110
N.S.	1	1.00	1.03	0.95	1.02	3.07	1.41	1.17	1.10
time (sec)	N/A	0.021	0.058	0.098	0.482	0.524	3.984	1.657	0.206

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	127	112	111	113	151	125	124
N.S.	1	1.00	1.17	1.03	1.02	1.04	1.39	1.15	1.14
time (sec)	N/A	0.028	0.088	0.118	0.485	0.628	8.902	1.391	0.147

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	126	112	112	147	155	120	138
N.S.	1	1.00	1.14	1.01	1.01	1.32	1.40	1.08	1.24
time (sec)	N/A	0.028	0.086	0.172	0.489	0.559	11.678	2.093	0.072

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	142	124	133	162	595	135	150
N.S.	1	1.00	1.10	0.96	1.03	1.26	4.61	1.05	1.16
time (sec)	N/A	0.034	0.200	0.115	0.491	0.601	89.401	1.082	0.265

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	124	133	147	457	135	142
N.S.	1	1.00	1.14	0.99	1.06	1.18	3.66	1.08	1.14
time (sec)	N/A	0.034	0.197	0.258	0.485	0.806	68.806	0.859	0.152

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	133	118	120	394	527	136	142
N.S.	1	1.00	1.16	1.03	1.04	3.43	4.58	1.18	1.23
time (sec)	N/A	0.027	0.168	0.107	0.479	0.979	39.056	1.003	0.236

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	134	117	120	389	450	136	120
N.S.	1	1.00	1.15	1.00	1.03	3.32	3.85	1.16	1.03
time (sec)	N/A	0.028	0.171	0.102	0.477	1.021	25.536	0.972	0.063

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	133	116	127	396	544	132	144
N.S.	1	1.00	1.15	1.00	1.09	3.41	4.69	1.14	1.24
time (sec)	N/A	0.028	0.158	0.102	0.498	0.710	27.070	1.256	0.358

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	134	117	127	387	434	132	134
N.S.	1	1.00	1.19	1.04	1.12	3.42	3.84	1.17	1.19
time (sec)	N/A	0.029	0.155	0.113	0.494	0.866	39.310	1.761	0.223

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	142	124	132	156	690	145	151
N.S.	1	1.00	1.15	1.00	1.06	1.26	5.56	1.17	1.22
time (sec)	N/A	0.034	0.209	0.145	0.501	1.150	88.308	1.524	0.152

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	142	124	132	189	590	137	166
N.S.	1	1.00	1.11	0.97	1.03	1.48	4.61	1.07	1.30
time (sec)	N/A	0.035	0.224	0.223	0.496	1.642	130.182	1.321	0.165

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	130	143	506	0	146	165
N.S.	1	1.00	1.01	0.93	1.02	3.61	0.00	1.04	1.18
time (sec)	N/A	0.037	0.234	0.119	0.489	0.707	0.000	2.163	0.172

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	130	143	503	0	146	139
N.S.	1	1.00	1.01	0.93	1.02	3.59	0.00	1.04	0.99
time (sec)	N/A	0.035	0.231	0.118	0.510	1.052	0.000	1.792	0.066

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	133	132	153	508	1171	149	172
N.S.	1	1.00	0.93	0.92	1.07	3.55	8.19	1.04	1.20
time (sec)	N/A	0.036	0.220	0.099	0.492	1.199	159.619	1.276	0.265

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	136	132	152	501	899	148	146
N.S.	1	1.00	0.95	0.92	1.06	3.50	6.29	1.03	1.02
time (sec)	N/A	0.036	0.222	0.106	0.490	1.048	129.916	1.156	0.235

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	139	151	510	1175	143	167
N.S.	1	1.00	1.01	0.99	1.08	3.64	8.39	1.02	1.19
time (sec)	N/A	0.034	0.160	0.112	0.489	0.875	140.708	2.093	0.192

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	142	139	151	499	853	143	157
N.S.	1	1.00	1.01	0.99	1.08	3.56	6.09	1.02	1.12
time (sec)	N/A	0.037	0.150	0.126	0.485	1.058	199.838	1.538	0.241

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	153	133	154	211	0	155	174
N.S.	1	1.00	1.01	0.88	1.01	1.39	0.00	1.02	1.14
time (sec)	N/A	0.043	0.268	0.138	0.495	0.794	0.000	1.711	0.087

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	153	133	154	244	0	150	182
N.S.	1	1.00	1.01	0.88	1.01	1.61	0.00	0.99	1.20
time (sec)	N/A	0.043	0.279	0.158	0.490	0.726	0.000	1.725	0.173

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	60	61	82	243	64	46
N.S.	1	1.00	1.00	1.03	1.05	1.41	4.19	1.10	0.79
time (sec)	N/A	0.016	0.062	1.208	0.506	0.584	1.202	1.268	0.068

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	166	1534	187	1277	9996	1925	1274
N.S.	1	1.00	0.89	8.20	1.00	6.83	53.45	10.29	6.81
time (sec)	N/A	0.061	0.194	0.107	0.265	0.621	1.132	1.274	1.370

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	118	156	133	665	4257	992	683
N.S.	1	1.00	0.89	1.17	1.00	5.00	32.01	7.46	5.14
time (sec)	N/A	0.037	0.083	0.105	0.265	0.844	0.644	1.469	0.777

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	72	61	157	663	224	167
N.S.	1	1.00	0.89	1.18	1.00	2.57	10.87	3.67	2.74
time (sec)	N/A	0.012	0.023	0.096	0.276	1.039	0.244	1.087	0.443

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	51	43	85	299	117	93
N.S.	1	1.00	0.88	1.19	1.00	1.98	6.95	2.72	2.16
time (sec)	N/A	0.008	0.024	0.117	0.268	1.165	0.173	1.916	0.370

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	30	25	33	87	43	30
N.S.	1	1.00	0.88	1.20	1.00	1.32	3.48	1.72	1.20
time (sec)	N/A	0.005	0.011	0.010	0.264	0.917	0.104	2.101	0.304

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	61	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.10	0.00	-0.03
time (sec)	N/A	0.004	0.026	0.028	0.000	0.000	0.343	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	262	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	9.03	0.00	-0.03
time (sec)	N/A	0.004	0.021	0.032	0.000	0.000	0.464	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	717	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	24.72	0.00	-0.03
time (sec)	N/A	0.004	0.022	0.036	0.000	0.000	0.632	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	-0.02
time (sec)	N/A	0.009	0.100	0.026	0.000	0.000	5.218	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	-0.02
time (sec)	N/A	0.008	0.088	0.027	0.000	0.000	1.670	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	-0.02
time (sec)	N/A	0.009	0.064	0.025	0.000	0.000	0.749	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.008	0.057	0.028	0.000	0.000	0.703	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.008	0.082	0.029	0.000	0.000	0.887	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	36	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	-0.02
time (sec)	N/A	0.008	0.084	0.026	0.000	0.000	1.680	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	-0.02
time (sec)	N/A	0.009	0.077	0.027	0.000	0.000	1.608	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.009	0.071	0.031	0.000	0.000	1.215	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	36	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.78	0.00	-0.02
time (sec)	N/A	0.010	0.008	0.003	0.000	0.000	0.742	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	31	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.009	0.069	0.028	0.000	0.000	1.917	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	32	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.65	0.00	-0.02
time (sec)	N/A	0.009	0.072	0.027	0.000	0.000	5.019	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	-0.02
time (sec)	N/A	0.010	0.074	0.028	0.000	0.000	13.750	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	37	0	-1
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.19	0.00	-0.03
time (sec)	N/A	0.005	0.038	0.100	0.000	0.000	0.542	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	0	46	0	-1
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.48	0.00	-0.03
time (sec)	N/A	0.003	0.037	0.115	0.000	0.000	0.555	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	43	0	0	36	0	-1
N.S.	1	1.00	1.00	1.19	0.00	0.00	1.00	0.00	-0.03
time (sec)	N/A	0.007	0.040	0.130	0.000	0.000	0.633	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	30	0	0	41	0	-1
N.S.	1	1.00	0.96	0.60	0.00	0.00	0.82	0.00	-0.02
time (sec)	N/A	0.009	0.054	0.098	0.000	0.000	0.608	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	42	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.88	0.00	-0.02
time (sec)	N/A	0.008	0.014	0.027	0.000	0.000	0.699	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	44	0	-1
N.S.	1	1.00	0.94	0.88	0.00	0.00	1.29	0.00	-0.03
time (sec)	N/A	0.004	0.006	0.105	0.000	0.000	0.552	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	30	0	0	53	0	-1
N.S.	1	1.00	0.94	0.88	0.00	0.00	1.56	0.00	-0.03
time (sec)	N/A	0.004	0.006	0.129	0.000	0.000	0.589	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	0	0	42	0	-1
N.S.	1	1.00	1.00	0.90	0.00	0.00	0.86	0.00	-0.02
time (sec)	N/A	0.008	0.007	0.131	0.000	0.000	0.567	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	57	31	0	0	48	0	-1
N.S.	1	1.00	1.54	0.84	0.00	0.00	1.30	0.00	-0.03
time (sec)	N/A	0.004	0.011	0.119	0.000	0.000	0.544	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	0	26	0	-1
N.S.	1	1.00	1.00	0.88	0.00	0.00	1.00	0.00	-0.04
time (sec)	N/A	0.003	0.034	0.110	0.000	0.000	0.480	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	31	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.03	0.00	-0.03
time (sec)	N/A	0.003	0.109	0.032	0.000	0.000	0.557	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	34	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.72	0.00	-0.02
time (sec)	N/A	0.008	0.023	0.040	0.000	0.000	1.773	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	0	0	37	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.71	0.00	-0.02
time (sec)	N/A	0.011	0.007	0.055	0.000	0.000	1.432	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	126	101	143	1318	226	176
N.S.	1	1.00	0.81	1.52	1.22	1.72	15.88	2.72	2.12
time (sec)	N/A	0.023	0.056	0.107	0.282	0.898	0.584	1.730	0.534

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	68	96	597	140	192
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.33	3.20
time (sec)	N/A	0.014	0.038	0.114	0.281	1.240	0.371	2.248	0.557

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	42	53	201	76	94
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	1.95	2.41
time (sec)	N/A	0.009	0.029	0.122	0.271	1.210	0.221	1.031	0.378

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	18	18
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	1.00	1.00
time (sec)	N/A	0.002	0.018	0.112	0.294	1.252	0.006	1.691	0.198

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	83	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.37	0.00	-0.03
time (sec)	N/A	0.005	0.023	0.028	0.000	0.000	0.608	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	354	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	10.11	0.00	-0.03
time (sec)	N/A	0.005	0.022	0.044	0.000	0.000	0.899	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	918	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	24.16	0.00	-0.03
time (sec)	N/A	0.005	0.023	0.030	0.000	0.000	2.018	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	64	77	0	104	0	0	136
N.S.	1	1.00	0.58	0.70	0.00	0.95	0.00	0.00	1.24
time (sec)	N/A	0.027	0.038	0.127	0.000	1.545	0.000	0.000	0.523

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	44	0	64	0	0	80
N.S.	1	1.00	0.61	0.69	0.00	1.00	0.00	0.00	1.25
time (sec)	N/A	0.007	0.031	0.115	0.000	1.449	0.000	0.000	0.447

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	29	0	33	85	0	29
N.S.	1	1.00	0.89	1.04	0.00	1.18	3.04	0.00	1.04
time (sec)	N/A	0.002	0.024	0.116	0.000	1.117	201.000	0.000	0.349

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	0	27	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.69	0.00	-0.03
time (sec)	N/A	0.007	0.025	0.046	0.000	0.000	185.550	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	32	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.71	0.00	-0.02
time (sec)	N/A	0.008	0.021	0.043	0.000	0.000	14.328	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	34	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	-0.02
time (sec)	N/A	0.009	0.024	0.054	0.000	0.000	108.863	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	27	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.60	0.00	-0.02
time (sec)	N/A	0.007	0.086	0.027	0.000	0.000	78.933	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	27	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.60	0.00	-0.02
time (sec)	N/A	0.007	0.055	0.025	0.000	0.000	3.579	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	26	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.60	0.00	-0.02
time (sec)	N/A	0.007	0.062	0.026	0.000	0.000	2.359	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	29	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	-0.02
time (sec)	N/A	0.007	0.097	0.027	0.000	0.000	15.232	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	32	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.71	0.00	-0.02
time (sec)	N/A	0.007	0.073	0.024	0.000	0.000	186.747	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	32	0	0	37	0	-1
N.S.	1	1.00	0.89	0.91	0.00	0.00	1.06	0.00	-0.03
time (sec)	N/A	0.009	0.020	0.132	0.000	0.000	1.137	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	0	0	0	37	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.92	0.00	-0.02
time (sec)	N/A	0.007	0.047	0.060	0.000	0.000	1.141	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	0	0	37	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.71	0.00	-0.02
time (sec)	N/A	0.010	0.024	0.040	0.000	0.000	1.430	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	22	32	197	0	19
N.S.	1	1.00	1.00	1.05	1.16	1.68	10.37	0.00	1.00
time (sec)	N/A	0.002	0.030	0.113	0.273	0.983	208.879	0.000	0.504

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	64	328	0	86
N.S.	1	1.00	0.69	0.71	0.00	1.10	5.66	0.00	1.48
time (sec)	N/A	0.010	0.030	0.121	0.000	0.905	16.275	0.000	0.498

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	41	0	64	328	0	86
N.S.	1	1.00	0.69	0.71	0.00	1.10	5.66	0.00	1.48
time (sec)	N/A	0.009	0.003	0.120	0.000	1.017	16.749	0.000	0.002

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	33	22	29	22	-1
N.S.	1	1.00	0.69	0.60	0.94	0.63	0.83	0.63	-0.03
time (sec)	N/A	0.008	0.004	0.025	0.266	0.943	0.115	2.256	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	31	22	29	22	-1
N.S.	1	1.00	0.69	0.60	0.89	0.63	0.83	0.63	-0.03
time (sec)	N/A	0.007	0.003	0.024	0.330	0.828	0.100	1.091	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	28	22	29	22	-1
N.S.	1	1.00	0.69	0.60	0.80	0.63	0.83	0.63	-0.03
time (sec)	N/A	0.007	0.003	0.022	0.277	0.838	0.080	3.126	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	19	25	20	27	22	20
N.S.	1	1.00	0.67	0.58	0.76	0.61	0.82	0.67	0.61
time (sec)	N/A	0.006	0.003	0.021	0.318	1.628	0.065	1.265	0.542

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	17	0	16	22	17	14
N.S.	1	1.00	0.89	0.63	0.00	0.59	0.81	0.63	0.52
time (sec)	N/A	0.003	0.003	0.020	0.000	0.888	0.067	1.896	0.192

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	20	20	0	19	0	17	-1
N.S.	1	1.00	0.71	0.71	0.00	0.68	0.00	0.61	-0.04
time (sec)	N/A	0.003	0.004	0.024	0.000	1.088	0.000	1.872	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	20	21	0	20	0	20	-1
N.S.	1	1.00	0.62	0.66	0.00	0.62	0.00	0.62	-0.03
time (sec)	N/A	0.005	0.004	0.020	0.000	1.470	0.000	2.703	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	0	18	29	19	28
N.S.	1	1.00	0.85	0.73	0.00	0.69	1.12	0.73	1.08
time (sec)	N/A	0.003	0.004	0.020	0.000	0.980	0.173	2.231	0.144

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	33	24	29	22	-1
N.S.	1	1.00	0.65	0.57	0.89	0.65	0.78	0.59	-0.03
time (sec)	N/A	0.009	0.005	0.028	0.281	0.627	0.227	1.603	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	31	24	29	22	-1
N.S.	1	1.00	0.65	0.57	0.84	0.65	0.78	0.59	-0.03
time (sec)	N/A	0.008	0.005	0.025	0.278	1.166	0.190	1.955	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	24	21	28	24	29	22	-1
N.S.	1	1.00	0.65	0.57	0.76	0.65	0.78	0.59	-0.03
time (sec)	N/A	0.007	0.004	0.023	0.314	0.831	0.160	1.320	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	22	19	25	24	27	22	-1
N.S.	1	1.00	0.59	0.51	0.68	0.65	0.73	0.59	-0.03
time (sec)	N/A	0.007	0.004	0.023	0.282	0.734	0.137	1.090	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	18	22	24	24	22	-1
N.S.	1	1.00	0.68	0.49	0.59	0.65	0.65	0.59	-0.03
time (sec)	N/A	0.006	0.002	0.024	0.281	1.235	0.138	0.956	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	21	0	22	24	22	20
N.S.	1	1.00	0.66	0.60	0.00	0.63	0.69	0.63	0.57
time (sec)	N/A	0.006	0.002	0.021	0.000	0.926	0.145	1.026	0.267

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	21	20	0	18	26	17	14
N.S.	1	1.00	0.72	0.69	0.00	0.62	0.90	0.59	0.48
time (sec)	N/A	0.003	0.002	0.020	0.000	1.286	0.213	2.649	0.222

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	21	20	0	21	0	17	-1
N.S.	1	1.00	0.70	0.67	0.00	0.70	0.00	0.57	-0.03
time (sec)	N/A	0.004	0.004	0.023	0.000	1.048	0.000	1.909	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	33	28	29	28	-1
N.S.	1	1.00	0.59	0.51	0.80	0.68	0.71	0.68	-0.02
time (sec)	N/A	0.012	0.005	0.032	0.289	1.389	0.421	1.720	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	31	28	29	28	-1
N.S.	1	1.00	0.59	0.51	0.76	0.68	0.71	0.68	-0.02
time (sec)	N/A	0.011	0.005	0.031	0.278	0.981	0.358	1.355	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	28	28	29	28	-1
N.S.	1	1.00	0.59	0.51	0.68	0.68	0.71	0.68	-0.02
time (sec)	N/A	0.009	0.005	0.027	0.272	1.331	0.303	2.103	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	22	19	25	28	27	28	-1
N.S.	1	1.00	0.54	0.46	0.61	0.68	0.66	0.68	-0.02
time (sec)	N/A	0.008	0.005	0.026	0.272	1.173	0.263	2.444	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	22	28	24	28	-1
N.S.	1	1.00	0.61	0.44	0.54	0.68	0.59	0.68	-0.02
time (sec)	N/A	0.008	0.002	0.025	0.267	1.269	0.326	1.804	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	21	24	28	24	28	25
N.S.	1	1.00	0.56	0.51	0.59	0.68	0.59	0.68	0.61
time (sec)	N/A	0.008	0.002	0.023	0.277	1.102	0.267	1.020	0.284

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	0	28	27	28	25
N.S.	1	1.00	0.66	0.51	0.00	0.68	0.66	0.68	0.61
time (sec)	N/A	0.007	0.002	0.023	0.000	0.867	0.335	1.757	0.265

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	25	21	0	26	29	28	20
N.S.	1	1.00	0.64	0.54	0.00	0.67	0.74	0.72	0.51
time (sec)	N/A	0.006	0.002	0.021	0.000	1.017	0.339	1.922	0.260

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	33	25	29	22	-1
N.S.	1	1.00	0.69	0.60	0.94	0.71	0.83	0.63	-0.03
time (sec)	N/A	0.006	0.004	0.025	0.272	0.987	0.212	2.666	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	26	23	29	22	23
N.S.	1	1.00	0.69	0.60	0.74	0.66	0.83	0.63	0.66
time (sec)	N/A	0.005	0.003	0.021	0.280	0.856	0.197	2.240	0.251

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	23	20	22	19	27	19	19
N.S.	1	1.00	0.72	0.62	0.69	0.59	0.84	0.59	0.59
time (sec)	N/A	0.003	0.001	0.022	0.284	1.256	0.179	2.484	0.221

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	19	18	20	22	0	23	17
N.S.	1	1.00	0.66	0.62	0.69	0.76	0.00	0.79	0.59
time (sec)	N/A	0.003	0.002	0.030	0.281	1.075	0.000	2.021	0.513

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	18	17	23	0	26	22
N.S.	1	1.00	0.85	0.67	0.63	0.85	0.00	0.96	0.81
time (sec)	N/A	0.004	0.005	0.022	0.269	0.860	0.000	2.690	1.222

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	19	19	21	24	18	25
N.S.	1	1.00	0.88	0.73	0.73	0.81	0.92	0.69	0.96
time (sec)	N/A	0.003	0.005	0.022	0.269	1.378	0.202	2.018	0.157

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	22	21	19	23	29	20	26
N.S.	1	1.00	0.63	0.60	0.54	0.66	0.83	0.57	0.74
time (sec)	N/A	0.005	0.005	0.023	0.269	1.142	0.225	2.690	0.149

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	24	21	19	23	31	20	26
N.S.	1	1.00	0.69	0.60	0.54	0.66	0.89	0.57	0.74
time (sec)	N/A	0.005	0.004	0.025	0.276	0.842	0.254	2.326	0.149

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	20	32	19	27	19	-1
N.S.	1	1.00	0.61	0.53	0.84	0.50	0.71	0.50	-0.03
time (sec)	N/A	0.004	0.003	0.022	0.290	1.187	0.226	2.423	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	21	20	23	22	0	27	30
N.S.	1	1.00	0.60	0.57	0.66	0.63	0.00	0.77	0.86
time (sec)	N/A	0.004	0.003	0.025	0.283	1.095	0.000	1.683	0.320

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	21	21	23	0	30	28
N.S.	1	1.00	0.67	0.64	0.64	0.70	0.00	0.91	0.85
time (sec)	N/A	0.005	0.003	0.024	0.287	0.907	0.000	2.359	0.250

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	22	17	23	21	27	18	25
N.S.	1	1.00	0.76	0.59	0.79	0.72	0.93	0.62	0.86
time (sec)	N/A	0.003	0.003	0.022	0.290	1.155	0.200	3.149	0.149

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	25	18	19	23	26	20	26
N.S.	1	1.00	0.61	0.44	0.46	0.56	0.63	0.49	0.63
time (sec)	N/A	0.005	0.006	0.023	0.270	1.001	0.236	3.530	0.158

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	19	23	26	20	26
N.S.	1	1.00	0.66	0.51	0.46	0.56	0.63	0.49	0.63
time (sec)	N/A	0.005	0.006	0.027	0.293	0.825	0.241	2.630	0.151

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	22	21	19	23	29	20	26
N.S.	1	1.00	0.54	0.51	0.46	0.56	0.71	0.49	0.63
time (sec)	N/A	0.005	0.006	0.026	0.283	1.203	0.280	2.495	0.149

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	19	23	31	20	26
N.S.	1	1.00	0.59	0.51	0.46	0.56	0.76	0.49	0.63
time (sec)	N/A	0.005	0.005	0.028	0.278	1.132	0.331	2.908	0.151

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	22	21	24	23	0	26	-1
N.S.	1	1.00	0.67	0.64	0.73	0.70	0.00	0.79	-0.03
time (sec)	N/A	0.005	0.004	0.021	0.276	1.240	0.000	2.413	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	19	26	21	29	18	25
N.S.	1	1.00	0.83	0.66	0.90	0.72	1.00	0.62	0.86
time (sec)	N/A	0.003	0.005	0.021	0.282	1.214	0.277	1.519	0.153

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	23	23	31	20	26
N.S.	1	1.00	0.59	0.51	0.56	0.56	0.76	0.49	0.63
time (sec)	N/A	0.005	0.003	0.025	0.305	1.144	0.277	1.601	0.151

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	19	23	23	29	20	26
N.S.	1	1.00	0.66	0.46	0.56	0.56	0.71	0.49	0.63
time (sec)	N/A	0.005	0.003	0.024	0.270	0.938	0.277	1.852	0.160

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	18	19	23	26	20	26
N.S.	1	1.00	0.66	0.44	0.46	0.56	0.63	0.49	0.63
time (sec)	N/A	0.005	0.006	0.024	0.278	1.111	0.309	1.827	0.158

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	27	21	19	23	26	20	26
N.S.	1	1.00	0.66	0.51	0.46	0.56	0.63	0.49	0.63
time (sec)	N/A	0.005	0.006	0.029	0.281	1.162	0.387	2.668	0.162

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	22	21	19	23	29	20	26
N.S.	1	1.00	0.54	0.51	0.46	0.56	0.71	0.49	0.63
time (sec)	N/A	0.005	0.006	0.028	0.277	0.859	0.397	2.072	0.159

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	24	21	19	23	31	20	26
N.S.	1	1.00	0.59	0.51	0.46	0.56	0.76	0.49	0.63
time (sec)	N/A	0.005	0.005	0.028	0.284	0.959	0.454	1.395	0.162

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	54	33	49	35	-1
N.S.	1	1.00	0.61	0.56	0.95	0.58	0.86	0.61	-0.02
time (sec)	N/A	0.011	0.004	0.115	0.289	0.695	0.139	1.201	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	52	33	51	35	-1
N.S.	1	1.00	0.61	0.56	0.91	0.58	0.89	0.61	-0.02
time (sec)	N/A	0.011	0.004	0.129	0.286	0.626	0.119	1.612	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	49	33	49	35	-1
N.S.	1	1.00	0.61	0.56	0.86	0.58	0.86	0.61	-0.02
time (sec)	N/A	0.009	0.004	0.126	0.267	0.707	0.113	1.397	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	33	30	44	31	49	35	-1
N.S.	1	1.00	0.60	0.55	0.80	0.56	0.89	0.64	-0.02
time (sec)	N/A	0.009	0.005	0.112	0.309	0.513	0.109	1.054	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	23	0	27	41	29	-1
N.S.	1	1.00	0.96	0.88	0.00	1.04	1.58	1.12	-0.04
time (sec)	N/A	0.003	0.004	0.103	0.000	0.508	0.085	1.159	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	33	0	32	0	32	-1
N.S.	1	1.00	0.67	0.67	0.00	0.65	0.00	0.65	-0.02
time (sec)	N/A	0.007	0.007	0.120	0.000	0.705	0.000	1.177	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	31	32	0	31	0	31	-1
N.S.	1	1.00	0.63	0.65	0.00	0.63	0.00	0.63	-0.02
time (sec)	N/A	0.008	0.008	0.119	0.000	0.634	0.000	1.213	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	36	34	0	33	0	35	-1
N.S.	1	1.00	0.67	0.63	0.00	0.61	0.00	0.65	-0.02
time (sec)	N/A	0.008	0.007	0.102	0.000	0.684	0.000	1.581	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	54	36	49	35	-1
N.S.	1	1.00	0.58	0.53	0.90	0.60	0.82	0.58	-0.02
time (sec)	N/A	0.013	0.006	0.108	0.285	0.647	0.263	1.536	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	52	36	51	35	-1
N.S.	1	1.00	0.58	0.53	0.87	0.60	0.85	0.58	-0.02
time (sec)	N/A	0.012	0.006	0.105	0.277	0.610	0.231	0.851	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	35	32	49	36	49	35	-1
N.S.	1	1.00	0.58	0.53	0.82	0.60	0.82	0.58	-0.02
time (sec)	N/A	0.012	0.006	0.113	0.277	0.471	0.195	1.480	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	33	30	44	36	49	35	-1
N.S.	1	1.00	0.55	0.50	0.73	0.60	0.82	0.58	-0.02
time (sec)	N/A	0.011	0.005	0.125	0.279	0.459	0.168	1.298	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	36	29	40	36	44	35	-1
N.S.	1	1.00	0.60	0.48	0.67	0.60	0.73	0.58	-0.02
time (sec)	N/A	0.010	0.003	0.119	0.270	0.464	0.168	1.242	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	34	32	0	34	44	35	-1
N.S.	1	1.00	0.59	0.55	0.00	0.59	0.76	0.60	-0.02
time (sec)	N/A	0.009	0.003	0.109	0.000	0.428	0.175	1.066	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	23	0	30	41	29	-1
N.S.	1	1.00	0.96	0.85	0.00	1.11	1.52	1.07	-0.04
time (sec)	N/A	0.003	0.004	0.107	0.000	0.460	0.252	1.471	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	34	33	0	35	0	32	-1
N.S.	1	1.00	0.65	0.63	0.00	0.67	0.00	0.62	-0.02
time (sec)	N/A	0.007	0.006	0.105	0.000	0.492	0.000	1.664	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	49	42	49	44	-1
N.S.	1	1.00	0.53	0.48	0.74	0.64	0.74	0.67	-0.02
time (sec)	N/A	0.014	0.006	0.107	0.268	0.423	0.362	1.473	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	30	44	42	49	44	-1
N.S.	1	1.00	0.50	0.45	0.67	0.64	0.74	0.67	-0.02
time (sec)	N/A	0.013	0.006	0.103	0.268	0.421	0.305	1.808	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	36	29	40	42	44	44	-1
N.S.	1	1.00	0.55	0.44	0.61	0.64	0.67	0.67	-0.02
time (sec)	N/A	0.012	0.003	0.109	0.282	0.377	0.313	0.968	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	34	32	40	42	44	44	-1
N.S.	1	1.00	0.52	0.48	0.61	0.64	0.67	0.67	-0.02
time (sec)	N/A	0.011	0.004	0.131	0.267	0.393	0.327	1.246	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	0	42	44	44	-1
N.S.	1	1.00	0.58	0.48	0.00	0.64	0.67	0.67	-0.02
time (sec)	N/A	0.011	0.003	0.111	0.000	0.419	0.384	1.177	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	32	0	40	49	44	-1
N.S.	1	1.00	0.56	0.50	0.00	0.62	0.77	0.69	-0.02
time (sec)	N/A	0.010	0.003	0.121	0.000	0.443	0.384	1.936	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	0	36	46	41	-1
N.S.	1	1.00	0.90	0.79	0.00	1.24	1.59	1.41	-0.03
time (sec)	N/A	0.003	0.004	0.117	0.000	0.422	0.389	1.341	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	35	33	0	41	0	41	-1
N.S.	1	1.00	0.60	0.57	0.00	0.71	0.00	0.71	-0.02
time (sec)	N/A	0.008	0.007	0.122	0.000	0.438	0.000	1.151	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	54	36	49	42	-1
N.S.	1	1.00	0.61	0.56	0.95	0.63	0.86	0.74	-0.02
time (sec)	N/A	0.009	0.005	0.108	0.271	0.437	0.241	1.020	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	47	34	51	42	-1
N.S.	1	1.00	0.61	0.56	0.82	0.60	0.89	0.74	-0.02
time (sec)	N/A	0.010	0.004	0.104	0.268	0.442	0.222	0.869	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	42	30	46	39	-1
N.S.	1	1.00	1.00	0.88	1.75	1.25	1.92	1.62	-0.04
time (sec)	N/A	0.002	0.001	0.121	0.261	0.412	0.204	0.930	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	32	31	35	35	0	43	-1
N.S.	1	1.00	0.62	0.60	0.67	0.67	0.00	0.83	-0.02
time (sec)	N/A	0.007	0.003	0.121	0.279	0.403	0.000	0.989	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	29	35	34	0	42	-1
N.S.	1	1.00	0.72	0.62	0.74	0.72	0.00	0.89	-0.02
time (sec)	N/A	0.008	0.006	0.105	0.273	0.436	0.000	1.075	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	35	34	31	36	0	43	-1
N.S.	1	1.00	0.71	0.69	0.63	0.73	0.00	0.88	-0.02
time (sec)	N/A	0.008	0.006	0.102	0.268	0.392	0.000	1.751	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	33	30	33	32	42	39	33
N.S.	1	1.00	1.27	1.15	1.27	1.23	1.62	1.50	1.27
time (sec)	N/A	0.003	0.008	0.105	0.266	0.406	0.231	1.361	0.183

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	33	34	51	40	42
N.S.	1	1.00	0.61	0.56	0.58	0.60	0.89	0.70	0.74
time (sec)	N/A	0.008	0.006	0.102	0.262	0.465	0.256	1.387	0.187

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	23	52	30	46	39	-1
N.S.	1	1.00	0.96	0.85	1.93	1.11	1.70	1.44	-0.04
time (sec)	N/A	0.003	0.003	0.113	0.271	0.435	0.251	1.471	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	34	33	45	35	0	48	-1
N.S.	1	1.00	0.56	0.54	0.74	0.57	0.00	0.79	-0.02
time (sec)	N/A	0.008	0.005	0.122	0.282	0.471	0.000	0.843	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	32	42	34	0	46	-1
N.S.	1	1.00	0.59	0.57	0.75	0.61	0.00	0.82	-0.02
time (sec)	N/A	0.008	0.005	0.131	0.276	0.466	0.000	0.840	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	34	32	35	36	0	49	-1
N.S.	1	1.00	0.59	0.55	0.60	0.62	0.00	0.84	-0.02
time (sec)	N/A	0.009	0.004	0.106	0.281	0.450	0.000	1.426	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	36	27	37	32	42	39	33
N.S.	1	1.00	1.24	0.93	1.28	1.10	1.45	1.34	1.14
time (sec)	N/A	0.003	0.008	0.102	0.273	0.441	0.224	1.368	0.191

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	33	34	46	40	42
N.S.	1	1.00	0.58	0.48	0.50	0.52	0.70	0.61	0.64
time (sec)	N/A	0.009	0.007	0.116	0.263	0.430	0.247	1.558	0.193

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	32	33	34	46	40	42
N.S.	1	1.00	0.50	0.48	0.50	0.52	0.70	0.61	0.64
time (sec)	N/A	0.010	0.010	0.108	0.269	0.436	0.286	1.420	0.197

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	33	34	51	40	42
N.S.	1	1.00	0.53	0.48	0.50	0.52	0.77	0.61	0.64
time (sec)	N/A	0.010	0.007	0.122	0.266	0.487	0.319	0.881	0.175

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	33	32	45	34	0	42	-1
N.S.	1	1.00	0.59	0.57	0.80	0.61	0.00	0.75	-0.02
time (sec)	N/A	0.009	0.006	0.123	0.284	0.587	0.000	1.029	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	34	38	36	0	43	-1
N.S.	1	1.00	0.62	0.59	0.66	0.62	0.00	0.74	-0.02
time (sec)	N/A	0.009	0.007	0.109	0.279	1.031	0.000	0.820	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	30	44	32	48	39	33
N.S.	1	1.00	1.21	1.03	1.52	1.10	1.66	1.34	1.14
time (sec)	N/A	0.003	0.005	0.101	0.269	1.164	0.278	0.966	0.180

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	30	37	34	51	40	42
N.S.	1	1.00	0.58	0.45	0.56	0.52	0.77	0.61	0.64
time (sec)	N/A	0.009	0.003	0.128	0.269	0.902	0.276	0.814	0.175

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	29	37	34	46	40	42
N.S.	1	1.00	0.58	0.44	0.56	0.52	0.70	0.61	0.64
time (sec)	N/A	0.009	0.009	0.122	0.272	0.614	0.309	1.022	0.176

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	38	32	33	34	46	40	42
N.S.	1	1.00	0.58	0.48	0.50	0.52	0.70	0.61	0.64
time (sec)	N/A	0.009	0.007	0.114	0.295	0.473	0.354	0.987	0.180

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	33	32	33	34	46	40	42
N.S.	1	1.00	0.50	0.48	0.50	0.52	0.70	0.61	0.64
time (sec)	N/A	0.009	0.010	0.114	0.277	0.399	0.381	0.866	0.182

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	35	32	33	34	51	40	42
N.S.	1	1.00	0.53	0.48	0.50	0.52	0.77	0.61	0.64
time (sec)	N/A	0.009	0.007	0.103	0.288	0.423	0.415	0.815	0.180

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	63	128	62	0	81	-1
N.S.	1	1.00	0.62	0.62	1.25	0.61	0.00	0.79	-0.01
time (sec)	N/A	0.024	0.013	0.153	0.286	0.362	0.000	0.614	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	52	52	110	51	0	69	-1
N.S.	1	1.00	0.65	0.65	1.38	0.64	0.00	0.86	-0.01
time (sec)	N/A	0.017	0.011	0.140	0.285	0.362	0.000	0.928	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	40	91	39	0	54	-1
N.S.	1	1.00	0.69	0.69	1.57	0.67	0.00	0.93	-0.02
time (sec)	N/A	0.014	0.009	0.117	0.293	0.388	0.000	1.036	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	29	74	27	0	37	-1
N.S.	1	1.00	0.74	0.76	1.95	0.71	0.00	0.97	-0.03
time (sec)	N/A	0.009	0.006	0.131	0.294	0.380	0.000	2.057	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	21	0	20	0	28	-1
N.S.	1	1.00	0.95	0.95	0.00	0.91	0.00	1.27	-0.05
time (sec)	N/A	0.002	0.004	0.122	0.000	0.451	0.000	1.676	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	26	26	24	64	0	0	-1
N.S.	1	1.00	0.62	0.62	0.57	1.52	0.00	0.00	-0.02
time (sec)	N/A	0.004	0.006	0.130	0.272	0.424	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	32	33	37	31	0	0	-1
N.S.	1	1.00	0.52	0.54	0.61	0.51	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.008	0.132	0.269	0.408	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	53	51	52	44	0	0	-1
N.S.	1	1.00	0.63	0.61	0.62	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.011	0.138	0.260	0.397	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	64	63	124	67	0	81	-1
N.S.	1	1.00	0.60	0.59	1.16	0.63	0.00	0.76	-0.01
time (sec)	N/A	0.024	0.010	0.130	0.289	0.445	0.000	2.296	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	53	52	109	55	0	69	-1
N.S.	1	1.00	0.63	0.62	1.30	0.65	0.00	0.82	-0.01
time (sec)	N/A	0.017	0.009	0.113	0.283	0.458	0.000	2.016	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	40	93	42	0	54	-1
N.S.	1	1.00	0.69	0.66	1.52	0.69	0.00	0.89	-0.02
time (sec)	N/A	0.012	0.004	0.111	0.311	0.434	0.000	1.590	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	30	29	75	29	0	37	-1
N.S.	1	1.00	0.75	0.72	1.88	0.72	0.00	0.92	-0.02
time (sec)	N/A	0.008	0.003	0.129	0.292	0.405	0.000	1.441	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	13	21	0	28	-1
N.S.	1	1.00	0.96	0.91	0.57	0.91	0.00	1.22	-0.04
time (sec)	N/A	0.003	0.003	0.128	0.276	0.420	0.000	1.151	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	26	24	66	0	0	-1
N.S.	1	1.00	0.61	0.59	0.55	1.50	0.00	0.00	-0.02
time (sec)	N/A	0.005	0.006	0.112	0.286	0.452	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	34	33	37	33	0	0	-1
N.S.	1	1.00	0.53	0.52	0.58	0.52	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.008	0.112	0.269	0.484	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	53	51	52	47	0	0	-1
N.S.	1	1.00	0.60	0.58	0.59	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.010	0.138	0.277	0.457	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	65	62	66	59	0	0	-1
N.S.	1	1.00	0.58	0.55	0.59	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.018	0.133	0.292	0.437	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	76	74	146	91	0	116	-1
N.S.	1	1.00	0.54	0.52	1.03	0.64	0.00	0.82	-0.01
time (sec)	N/A	0.032	0.016	0.131	0.286	0.918	0.000	1.404	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	65	63	130	77	0	99	-1
N.S.	1	1.00	0.56	0.54	1.11	0.66	0.00	0.85	-0.01
time (sec)	N/A	0.024	0.005	0.147	0.291	0.742	0.000	1.639	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	54	52	114	63	0	84	-1
N.S.	1	1.00	0.59	0.57	1.24	0.68	0.00	0.91	-0.01
time (sec)	N/A	0.018	0.004	0.132	0.282	0.778	0.000	1.087	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	42	40	97	48	0	66	-1
N.S.	1	1.00	0.63	0.60	1.45	0.72	0.00	0.99	-0.01
time (sec)	N/A	0.013	0.006	0.166	0.278	0.584	0.000	1.825	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	29	77	33	0	46	-1
N.S.	1	1.00	0.68	0.66	1.75	0.75	0.00	1.05	-0.02
time (sec)	N/A	0.008	0.003	0.136	0.286	0.729	0.000	1.455	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	21	13	23	0	34	-1
N.S.	1	1.00	0.88	0.84	0.52	0.92	0.00	1.36	-0.04
time (sec)	N/A	0.003	0.003	0.134	0.285	0.566	0.000	1.157	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	28	26	24	70	0	0	-1
N.S.	1	1.00	0.58	0.54	0.50	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.007	0.140	0.276	0.806	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	34	33	37	37	0	0	-1
N.S.	1	1.00	0.49	0.47	0.53	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.009	0.132	0.298	0.900	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	50	142	54	0	75	-1
N.S.	1	1.00	0.61	0.60	1.71	0.65	0.00	0.90	-0.01
time (sec)	N/A	0.017	0.009	0.117	0.286	0.944	0.000	0.741	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	39	38	100	42	0	66	-1
N.S.	1	1.00	0.64	0.62	1.64	0.69	0.00	1.08	-0.02
time (sec)	N/A	0.012	0.008	0.112	0.291	0.647	0.000	0.632	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	27	27	64	30	0	46	-1
N.S.	1	1.00	0.69	0.69	1.64	0.77	0.00	1.18	-0.03
time (sec)	N/A	0.009	0.006	0.129	0.293	0.488	0.000	0.580	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	46	23	0	32	-1
N.S.	1	1.00	1.00	0.95	2.30	1.15	0.00	1.60	-0.05
time (sec)	N/A	0.002	0.002	0.131	0.276	0.432	0.000	0.498	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	24	35	70	0	0	-1
N.S.	1	1.00	0.66	0.63	0.92	1.84	0.00	0.00	-0.03
time (sec)	N/A	0.004	0.003	0.112	0.284	0.431	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	36	30	37	34	0	0	-1
N.S.	1	1.00	0.67	0.56	0.69	0.63	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.008	0.132	0.289	0.413	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	52	51	55	47	0	0	-1
N.S.	1	1.00	0.68	0.66	0.71	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.009	0.138	0.278	0.497	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	63	62	69	58	0	0	-1
N.S.	1	1.00	0.63	0.62	0.69	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.009	0.135	0.274	0.430	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	53	52	162	54	0	80	-1
N.S.	1	1.00	0.56	0.55	1.71	0.57	0.00	0.84	-0.01
time (sec)	N/A	0.021	0.008	0.133	0.297	0.403	0.000	1.295	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	41	40	140	42	0	72	-1
N.S.	1	1.00	0.59	0.57	2.00	0.60	0.00	1.03	-0.01
time (sec)	N/A	0.014	0.007	0.133	0.315	0.374	0.000	1.480	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	29	29	116	30	0	50	-1
N.S.	1	1.00	0.64	0.64	2.58	0.67	0.00	1.11	-0.02
time (sec)	N/A	0.010	0.005	0.139	0.287	0.365	0.000	1.211	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	21	74	23	0	37	-1
N.S.	1	1.00	0.96	0.91	3.22	1.00	0.00	1.61	-0.04
time (sec)	N/A	0.003	0.003	0.129	0.296	0.515	0.000	0.967	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	27	26	35	70	0	0	-1
N.S.	1	1.00	0.61	0.59	0.80	1.59	0.00	0.00	-0.02
time (sec)	N/A	0.005	0.006	0.131	0.280	0.511	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	35	33	51	34	0	0	-1
N.S.	1	1.00	0.56	0.52	0.81	0.54	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.005	0.114	0.296	0.434	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	51	49	65	47	0	0	-1
N.S.	1	1.00	0.57	0.55	0.73	0.53	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.006	0.112	0.270	0.445	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	66	59	69	58	0	0	-1
N.S.	1	1.00	0.57	0.51	0.60	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.011	0.140	0.288	0.453	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	81	88	135	83	0	96	-1
N.S.	1	1.00	0.76	0.83	1.27	0.78	0.00	0.91	-0.01
time (sec)	N/A	0.029	0.020	0.135	0.292	0.465	0.000	3.007	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	70	76	118	72	0	80	-1
N.S.	1	1.00	0.82	0.89	1.39	0.85	0.00	0.94	-0.01
time (sec)	N/A	0.023	0.015	0.115	0.282	0.484	0.000	1.285	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	62	96	57	0	58	-1
N.S.	1	1.00	0.82	0.95	1.48	0.88	0.00	0.89	-0.02
time (sec)	N/A	0.016	0.014	0.118	0.293	0.435	0.000	0.996	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	41	79	38	0	46	-1
N.S.	1	1.00	0.77	0.87	1.68	0.81	0.00	0.98	-0.02
time (sec)	N/A	0.011	0.009	0.135	0.280	0.486	0.000	0.792	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	23	16	23	32	29	22
N.S.	1	1.00	0.96	0.96	0.67	0.96	1.33	1.21	0.92
time (sec)	N/A	0.003	0.005	0.122	0.274	0.515	0.277	0.859	0.165

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	52	38	42	0	0	-1
N.S.	1	1.00	0.69	0.80	0.58	0.65	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.012	0.115	0.284	0.450	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	74	58	60	0	0	-1
N.S.	1	1.00	0.66	0.85	0.67	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.021	0.113	0.279	0.446	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	82	95	79	77	0	0	-1
N.S.	1	1.00	0.73	0.85	0.71	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.020	0.131	0.279	0.480	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	88	132	91	0	96	-1
N.S.	1	1.00	0.74	0.79	1.19	0.82	0.00	0.86	-0.01
time (sec)	N/A	0.027	0.015	0.155	0.289	0.441	0.000	0.830	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	76	115	79	0	80	-1
N.S.	1	1.00	0.80	0.85	1.29	0.89	0.00	0.90	-0.01
time (sec)	N/A	0.037	0.013	0.135	0.315	0.465	0.000	0.565	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	62	98	63	0	58	-1
N.S.	1	1.00	0.81	0.91	1.44	0.93	0.00	0.85	-0.01
time (sec)	N/A	0.029	0.005	0.121	0.276	0.463	0.000	0.544	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	41	80	43	0	46	-1
N.S.	1	1.00	0.78	0.84	1.63	0.88	0.00	0.94	-0.02
time (sec)	N/A	0.020	0.007	0.130	0.305	0.417	0.000	0.848	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	16	24	37	29	24
N.S.	1	1.00	0.96	0.92	0.64	0.96	1.48	1.16	0.96
time (sec)	N/A	0.006	0.005	0.115	0.284	0.438	0.580	0.807	0.148

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	52	38	47	0	0	-1
N.S.	1	1.00	0.68	0.76	0.56	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.013	0.125	0.375	0.482	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	74	58	65	0	0	-1
N.S.	1	1.00	0.65	0.81	0.64	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.017	0.132	0.293	0.427	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	95	79	82	0	0	-1
N.S.	1	1.00	0.70	0.81	0.68	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.016	0.137	0.289	0.439	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	80	86	168	85	0	107	-1
N.S.	1	1.00	0.75	0.80	1.57	0.79	0.00	1.00	-0.01
time (sec)	N/A	0.080	0.012	0.115	0.310	0.491	0.000	0.633	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	74	129	74	0	97	-1
N.S.	1	1.00	0.80	0.86	1.50	0.86	0.00	1.13	-0.01
time (sec)	N/A	0.024	0.011	0.122	0.299	0.441	0.000	0.657	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	60	88	59	0	72	-1
N.S.	1	1.00	0.81	0.94	1.38	0.92	0.00	1.12	-0.02
time (sec)	N/A	0.015	0.009	0.138	0.307	0.422	0.000	0.576	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	39	68	40	0	53	-1
N.S.	1	1.00	0.81	0.91	1.58	0.93	0.00	1.23	-0.02
time (sec)	N/A	0.010	0.007	0.130	0.298	0.463	0.000	0.584	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	21	25	68	32	25
N.S.	1	1.00	1.00	0.95	0.95	1.14	3.09	1.45	1.14
time (sec)	N/A	0.003	0.003	0.121	0.284	0.402	0.402	0.660	0.156

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	50	61	44	0	0	-1
N.S.	1	1.00	0.75	0.85	1.03	0.75	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.004	0.132	0.291	0.464	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	60	71	57	62	0	0	-1
N.S.	1	1.00	0.77	0.91	0.73	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.016	0.114	0.292	0.480	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	95	76	79	0	0	-1
N.S.	1	1.00	0.79	0.92	0.74	0.77	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.013	0.143	0.290	0.402	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	62	149	63	0	76	-1
N.S.	1	1.00	0.74	0.85	2.04	0.86	0.00	1.04	-0.01
time (sec)	N/A	0.016	0.011	0.130	0.320	0.405	0.000	0.616	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	37	41	108	44	0	59	-1
N.S.	1	1.00	0.76	0.84	2.20	0.90	0.00	1.20	-0.02
time (sec)	N/A	0.011	0.009	0.140	0.290	0.447	0.000	0.690	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	23	47	29	73	36	25
N.S.	1	1.00	0.96	0.92	1.88	1.16	2.92	1.44	1.00
time (sec)	N/A	0.003	0.005	0.137	0.288	0.464	0.550	1.160	0.167

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	46	52	82	48	0	0	-1
N.S.	1	1.00	0.68	0.76	1.21	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.010	0.116	0.288	0.440	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	59	74	79	66	0	0	-1
N.S.	1	1.00	0.66	0.82	0.88	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.008	0.126	0.307	0.507	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	93	98	83	0	0	-1
N.S.	1	1.00	0.68	0.79	0.83	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.006	0.135	0.287	0.593	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	97	136	116	153	0	300	214
N.S.	1	1.00	0.74	1.04	0.89	1.17	0.00	2.29	1.63
time (sec)	N/A	0.026	0.050	0.130	0.294	0.522	0.000	0.666	0.348

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	68	83	80	106	0	200	142
N.S.	1	1.00	0.71	0.86	0.83	1.10	0.00	2.08	1.48
time (sec)	N/A	0.018	0.035	0.112	0.294	0.500	0.000	0.706	0.252

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	46	51	63	0	119	85
N.S.	1	1.00	0.70	0.73	0.81	1.00	0.00	1.89	1.35
time (sec)	N/A	0.013	0.024	0.120	0.327	0.528	0.000	1.049	0.222

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	28	30	0	42	31
N.S.	1	1.00	0.97	0.97	0.93	1.00	0.00	1.40	1.03
time (sec)	N/A	0.005	0.011	0.130	0.291	0.492	0.000	1.682	0.228

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.009	0.033	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.012	0.026	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.013	0.026	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	132	199	157	233	0	426	307
N.S.	1	1.00	0.78	1.18	0.93	1.38	0.00	2.52	1.82
time (sec)	N/A	0.041	0.073	0.132	0.337	0.812	0.000	0.997	0.414

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	136	116	164	0	300	219
N.S.	1	1.00	0.73	1.01	0.86	1.21	0.00	2.22	1.62
time (sec)	N/A	0.029	0.041	0.140	0.280	0.465	0.000	0.728	0.320

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	83	80	113	0	0	146
N.S.	1	1.00	0.71	0.84	0.81	1.14	0.00	0.00	1.47
time (sec)	N/A	0.021	0.024	0.135	0.287	0.522	0.000	0.000	0.261

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	46	51	68	0	119	88
N.S.	1	1.00	0.71	0.71	0.78	1.05	0.00	1.83	1.35
time (sec)	N/A	0.015	0.008	0.114	0.316	0.606	0.000	0.533	0.231

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	28	33	0	42	45
N.S.	1	1.00	0.97	0.94	0.90	1.06	0.00	1.35	1.45
time (sec)	N/A	0.005	0.012	0.124	0.279	0.506	0.000	0.822	0.229

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.009	0.026	0.000	0.000	0.000	0.000	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.010	0.027	0.000	0.000	0.000	0.000	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.010	0.026	0.000	0.000	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	172	280	203	352	0	640	424
N.S.	1	1.00	0.79	1.29	0.94	1.62	0.00	2.95	1.95
time (sec)	N/A	0.056	0.092	0.145	0.299	0.730	0.000	0.917	0.496

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	133	199	157	265	0	0	319
N.S.	1	1.00	0.74	1.11	0.88	1.48	0.00	0.00	1.78
time (sec)	N/A	0.041	0.018	0.137	0.296	0.567	0.000	0.000	0.379

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	99	136	116	186	0	0	229
N.S.	1	1.00	0.69	0.95	0.81	1.30	0.00	0.00	1.60
time (sec)	N/A	0.032	0.015	0.137	0.280	0.744	0.000	0.000	0.317

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	83	80	127	0	0	154
N.S.	1	1.00	0.67	0.79	0.76	1.21	0.00	0.00	1.47
time (sec)	N/A	0.022	0.033	0.118	0.283	0.659	0.000	0.000	0.265

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	46	46	51	76	0	0	94
N.S.	1	1.00	0.67	0.67	0.74	1.10	0.00	0.00	1.36
time (sec)	N/A	0.014	0.008	0.156	0.281	0.715	0.000	0.000	0.239

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	29	28	37	0	0	49
N.S.	1	1.00	0.94	0.88	0.85	1.12	0.00	0.00	1.48
time (sec)	N/A	0.005	0.013	0.118	0.278	0.761	0.000	0.000	0.231

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.010	0.029	0.000	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.010	0.031	0.000	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	96	134	104	158	0	0	186
N.S.	1	1.00	0.78	1.09	0.85	1.28	0.00	0.00	1.51
time (sec)	N/A	0.025	0.031	0.143	0.292	1.289	0.000	0.000	0.369

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	67	81	83	110	0	0	121
N.S.	1	1.00	0.74	0.90	0.92	1.22	0.00	0.00	1.34
time (sec)	N/A	0.018	0.023	0.135	0.287	0.648	0.000	0.000	0.295

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	44	45	66	0	0	71
N.S.	1	1.00	0.73	0.75	0.76	1.12	0.00	0.00	1.20
time (sec)	N/A	0.011	0.016	0.136	0.293	0.915	0.000	0.000	0.277

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	31	33	0	0	36
N.S.	1	1.00	1.00	0.96	1.11	1.18	0.00	0.00	1.29
time (sec)	N/A	0.003	0.006	0.134	0.275	1.110	0.000	0.000	0.220

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.003	0.027	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	48	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.009	0.029	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.009	0.030	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	98	136	104	168	0	0	201
N.S.	1	1.00	0.73	1.01	0.77	1.24	0.00	0.00	1.49
time (sec)	N/A	0.030	0.031	0.137	0.279	0.728	0.000	0.000	0.404

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	83	83	118	0	0	133
N.S.	1	1.00	0.70	0.84	0.84	1.19	0.00	0.00	1.34
time (sec)	N/A	0.023	0.026	0.131	0.280	0.662	0.000	0.000	0.313

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	46	45	72	0	0	80
N.S.	1	1.00	0.69	0.71	0.69	1.11	0.00	0.00	1.23
time (sec)	N/A	0.015	0.018	0.134	0.282	0.788	0.000	0.000	0.289

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	29	31	37	0	0	42
N.S.	1	1.00	0.97	0.94	1.00	1.19	0.00	0.00	1.35
time (sec)	N/A	0.005	0.012	0.138	0.276	0.643	0.000	0.000	0.229

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.008	0.029	0.000	0.000	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.004	0.027	0.000	0.000	0.000	0.000	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.004	0.024	0.000	0.000	0.000	0.000	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.031	0.026	0.000	0.000	0.000	0.000	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	99	136	104	168	0	0	201
N.S.	1	1.00	0.73	1.01	0.77	1.24	0.00	0.00	1.49
time (sec)	N/A	0.031	0.030	0.118	0.287	0.835	0.000	0.000	0.410

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	83	83	118	0	0	133
N.S.	1	1.00	0.71	0.84	0.84	1.19	0.00	0.00	1.34
time (sec)	N/A	0.021	0.023	0.145	0.285	0.711	0.000	0.000	0.355

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	46	45	72	0	0	80
N.S.	1	1.00	0.71	0.71	0.69	1.11	0.00	0.00	1.23
time (sec)	N/A	0.014	0.016	0.137	0.281	0.832	0.000	0.000	0.285

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	31	37	0	0	42
N.S.	1	1.00	1.00	0.94	1.00	1.19	0.00	0.00	1.35
time (sec)	N/A	0.005	0.010	0.128	0.285	1.822	0.000	0.000	0.230

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.008	0.028	0.000	0.000	0.000	0.000	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	47	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.008	0.026	0.000	0.000	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.008	0.027	0.000	0.000	0.000	0.000	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.004	0.024	0.000	0.000	0.000	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	38	40	39	58	0	0	44
N.S.	1	1.00	0.58	0.62	0.60	0.89	0.00	0.00	0.68
time (sec)	N/A	0.024	0.034	0.013	0.283	0.811	0.000	0.000	0.266

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	38	40	39	50	0	0	42
N.S.	1	1.00	0.62	0.66	0.64	0.82	0.00	0.00	0.69
time (sec)	N/A	0.021	0.032	0.011	0.291	0.627	0.000	0.000	0.237

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	40	39	44	0	0	39
N.S.	1	1.00	0.64	0.68	0.66	0.75	0.00	0.00	0.66
time (sec)	N/A	0.020	0.029	0.011	0.281	0.644	0.000	0.000	0.212

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	33	32	32	36	0	0	30
N.S.	1	1.00	0.69	0.67	0.67	0.75	0.00	0.00	0.62
time (sec)	N/A	0.015	0.026	0.012	0.282	2.027	0.000	0.000	0.211

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	38	40	39	53	0	0	48
N.S.	1	1.00	0.58	0.62	0.60	0.82	0.00	0.00	0.74
time (sec)	N/A	0.026	0.034	0.015	0.295	1.488	0.000	0.000	0.259

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	38	40	39	53	0	0	47
N.S.	1	1.00	0.57	0.60	0.58	0.79	0.00	0.00	0.70
time (sec)	N/A	0.026	0.036	0.014	0.277	0.953	0.000	0.000	0.275

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	48	95	64	123	0	0	127
N.S.	1	1.00	0.47	0.92	0.62	1.19	0.00	0.00	1.23
time (sec)	N/A	0.037	0.057	0.105	0.292	0.778	0.000	0.000	0.312

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	48	95	64	105	0	0	121
N.S.	1	1.00	0.49	0.98	0.66	1.08	0.00	0.00	1.25
time (sec)	N/A	0.032	0.053	0.108	0.292	0.652	0.000	0.000	0.279

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	72	95	64	94	0	0	116
N.S.	1	1.00	0.77	1.01	0.68	1.00	0.00	0.00	1.23
time (sec)	N/A	0.029	0.050	0.117	0.289	0.695	0.000	0.000	0.264

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	62	79	57	85	0	0	62
N.S.	1	1.00	0.77	0.98	0.70	1.05	0.00	0.00	0.77
time (sec)	N/A	0.029	0.044	0.131	0.311	0.719	0.000	0.000	0.258

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	62	83	59	92	0	0	66
N.S.	1	1.00	0.67	0.89	0.63	0.99	0.00	0.00	0.71
time (sec)	N/A	0.032	0.049	0.120	0.285	1.661	0.000	0.000	0.319

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	72	95	64	106	0	0	82
N.S.	1	1.00	0.69	0.90	0.61	1.01	0.00	0.00	0.78
time (sec)	N/A	0.038	0.054	0.112	0.284	1.190	0.000	0.000	0.339

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	57	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.032	0.034	0.000	0.000	0.000	0.000	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.029	0.033	0.000	0.000	0.000	0.000	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.028	0.036	0.000	0.000	0.000	0.000	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.027	0.036	0.000	0.000	0.000	0.000	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.031	0.037	0.000	0.000	0.000	0.000	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.032	0.035	0.000	0.000	0.000	0.000	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	32	0	40	0	74	33
N.S.	1	1.00	0.97	0.97	0.00	1.21	0.00	2.24	1.00
time (sec)	N/A	0.007	0.042	0.142	0.000	0.952	0.000	1.598	0.266

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	33	0	40	0	0	34
N.S.	1	1.00	1.06	1.03	0.00	1.25	0.00	0.00	1.06
time (sec)	N/A	0.007	0.047	0.145	0.000	1.044	0.000	0.000	0.235

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	32	0	38	1409	72	33
N.S.	1	1.00	0.97	0.97	0.00	1.15	42.70	2.18	1.00
time (sec)	N/A	0.008	0.037	0.142	0.000	0.872	145.451	0.792	0.217

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	31	0	36	0	0	32
N.S.	1	1.00	0.93	1.03	0.00	1.20	0.00	0.00	1.07
time (sec)	N/A	0.006	0.037	0.129	0.000	1.170	0.000	0.000	0.200

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	27	31	250	0	26
N.S.	1	1.00	1.00	0.96	1.04	1.19	9.62	0.00	1.00
time (sec)	N/A	0.004	0.032	0.125	0.294	1.058	17.617	0.000	0.262

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	38	0	37	0	0	32
N.S.	1	1.00	0.97	1.15	0.00	1.12	0.00	0.00	0.97
time (sec)	N/A	0.008	0.028	0.148	0.000	0.575	0.000	0.000	0.240

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	0	37	0	0	50
N.S.	1	1.00	0.91	0.91	0.00	1.06	0.00	0.00	1.43
time (sec)	N/A	0.008	0.035	0.149	0.000	0.527	0.000	0.000	0.248

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	33	0	37	0	0	51
N.S.	1	1.00	0.97	1.00	0.00	1.12	0.00	0.00	1.55
time (sec)	N/A	0.009	0.034	0.187	0.000	0.897	0.000	0.000	0.251

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	0	49	0	0	50
N.S.	1	1.00	1.00	1.03	0.00	1.29	0.00	0.00	1.32
time (sec)	N/A	0.008	0.051	0.161	0.000	0.647	0.000	0.000	0.338

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	0	57	0	0	39
N.S.	1	1.00	1.00	1.03	0.00	1.46	0.00	0.00	1.00
time (sec)	N/A	0.008	0.013	0.209	0.000	0.616	0.000	0.000	0.265

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.028	0.059	0.000	0.000	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	64	64	0	0	0	0	0	-1
N.S.	1	0.94	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.008	0.049	0.000	0.000	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	31	31	34	31	27
N.S.	1	1.00	1.00	0.94	1.82	1.82	2.00	1.82	1.59
time (sec)	N/A	0.003	0.002	0.134	0.289	0.482	0.033	2.023	0.050

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	18	20	20	20	20	16
N.S.	1	1.00	0.83	0.78	0.87	0.87	0.87	0.87	0.70
time (sec)	N/A	0.004	0.001	0.135	0.273	0.448	0.027	1.306	0.034

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.001	0.000	0.149	0.284	0.386	0.027	0.769	0.010

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	19	14	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.46	1.08	1.00
time (sec)	N/A	0.002	0.002	0.149	0.293	0.464	0.025	1.072	0.048

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	19	19	19	15	15
N.S.	1	1.00	1.00	1.07	1.27	1.27	1.27	1.00	1.00
time (sec)	N/A	0.002	0.003	0.128	0.286	0.414	0.058	1.077	0.037

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	47	47	53	15	49
N.S.	1	1.00	1.00	0.94	2.76	2.76	3.12	0.88	2.88
time (sec)	N/A	0.002	0.004	0.146	0.314	0.437	0.116	1.318	0.153

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	61	61	68	15	63
N.S.	1	1.00	1.00	0.94	3.59	3.59	4.00	0.88	3.71
time (sec)	N/A	0.003	0.004	0.143	0.275	0.489	0.144	1.046	0.064

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	75	75	83	15	77
N.S.	1	1.00	1.00	0.94	4.41	4.41	4.88	0.88	4.53
time (sec)	N/A	0.002	0.005	0.137	0.301	0.441	0.178	2.498	0.054

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	35	35	34	35	27
N.S.	1	1.00	1.00	0.94	2.06	2.06	2.00	2.06	1.59
time (sec)	N/A	0.003	0.002	0.153	0.294	0.583	0.035	1.870	0.156

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	19	18	21	21	20	21	16
N.S.	1	1.00	0.83	0.78	0.91	0.91	0.87	0.91	0.70
time (sec)	N/A	0.004	0.001	0.129	0.286	0.461	0.029	1.891	0.030

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.001	0.000	0.144	0.290	0.523	0.024	1.535	0.009

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	14	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.08	1.00
time (sec)	N/A	0.002	0.002	0.156	0.283	0.553	0.023	2.136	0.142

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	16	16	12	13	13
N.S.	1	1.00	1.00	1.08	1.23	1.23	0.92	1.00	1.00
time (sec)	N/A	0.002	0.003	0.131	0.333	0.553	0.053	1.596	0.040

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	36	36	44	12	38
N.S.	1	1.00	1.00	0.93	2.57	2.57	3.14	0.86	2.71
time (sec)	N/A	0.002	0.004	0.151	0.289	0.673	0.107	2.128	0.047

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	59	59	68	20	61
N.S.	1	1.00	1.00	0.93	3.93	3.93	4.53	1.33	4.07
time (sec)	N/A	0.002	0.005	0.132	0.288	0.798	0.187	1.211	0.050

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	75	75	83	15	77
N.S.	1	1.00	1.00	0.94	4.41	4.41	4.88	0.88	4.53
time (sec)	N/A	0.003	0.005	0.156	0.284	0.706	0.176	1.963	0.173

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	27	649	80	212	141	107
N.S.	1	1.00	1.04	1.12	27.04	3.33	8.83	5.88	4.46
time (sec)	N/A	0.006	0.029	0.168	0.330	0.815	0.673	2.168	0.326

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	114	113	113	124	113	113
N.S.	1	1.00	1.00	6.71	6.65	6.65	7.29	6.65	6.65
time (sec)	N/A	0.003	0.002	0.118	0.290	0.920	0.021	1.537	0.051

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	100	99	99	110	99	99
N.S.	1	1.00	1.00	5.88	5.82	5.82	6.47	5.82	5.82
time (sec)	N/A	0.003	0.002	0.140	0.285	0.614	0.019	0.894	0.041

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	72	71	71	78	71	71
N.S.	1	1.00	1.00	4.80	4.73	4.73	5.20	4.73	4.73
time (sec)	N/A	0.002	0.001	0.128	0.288	0.673	0.016	1.584	0.030

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	48	48	51	48	57
N.S.	1	1.00	1.00	0.94	2.82	2.82	3.00	2.82	3.35
time (sec)	N/A	0.002	0.001	0.150	0.279	0.554	0.024	2.233	0.025

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	37	37	46	18	43
N.S.	1	1.00	1.00	0.94	2.18	2.18	2.71	1.06	2.53
time (sec)	N/A	0.003	0.001	0.149	0.291	0.450	0.026	1.887	0.048

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	26	26	29	26	24
N.S.	1	1.00	1.00	0.94	1.53	1.53	1.71	1.53	1.41
time (sec)	N/A	0.003	0.001	0.151	0.311	0.415	0.026	1.923	0.036

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	15	15	15	15	15	13
N.S.	1	1.00	0.89	0.83	0.83	0.83	0.83	0.83	0.72
time (sec)	N/A	0.003	0.001	0.143	0.293	0.456	0.028	2.561	0.024

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	3	15	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	3.00	1.00
time (sec)	N/A	0.001	0.000	0.153	0.299	0.453	0.026	2.821	0.008

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	17	14	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.08	1.00
time (sec)	N/A	0.003	0.002	0.137	0.286	0.499	0.032	1.328	0.040

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	19	19	17	15	19
N.S.	1	1.00	1.00	1.07	1.27	1.27	1.13	1.00	1.27
time (sec)	N/A	0.003	0.002	0.148	0.288	0.476	0.070	2.795	0.046

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	33	33	36	15	35
N.S.	1	1.00	1.00	0.94	1.94	1.94	2.12	0.88	2.06
time (sec)	N/A	0.003	0.003	0.125	0.286	0.426	0.099	2.140	0.146

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	F	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	6	0	66	11	35
N.S.	1	1.00	1.00	0.82	0.21	0.00	2.36	0.39	1.25
time (sec)	N/A	0.002	0.005	0.158	0.512	0.000	0.545	2.006	0.222

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	45	44	44	44	44	44
N.S.	1	1.00	1.05	1.18	1.16	1.16	1.16	1.16	1.16
time (sec)	N/A	0.008	0.003	0.135	0.280	0.901	0.013	2.586	0.160

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	45	44	44	46	44	44
N.S.	1	1.00	1.11	1.18	1.16	1.16	1.21	1.16	1.16
time (sec)	N/A	0.012	0.002	0.106	0.286	0.663	0.013	2.525	0.048

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	16	18
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.89	1.00
time (sec)	N/A	0.003	0.001	0.037	0.292	0.663	0.007	1.758	0.023

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.000	0.006	0.278	0.695	0.005	2.018	0.017

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	24	23	17	25	23
N.S.	1	1.00	1.00	0.96	1.04	1.00	0.74	1.09	1.00
time (sec)	N/A	0.010	0.004	0.112	0.280	1.062	0.044	2.193	0.046

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	28	31	37	39	29	81	37
N.S.	1	1.00	0.88	0.97	1.16	1.22	0.91	2.53	1.16
time (sec)	N/A	0.013	0.013	0.121	0.287	0.630	0.072	2.070	0.052

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	32	30	30	27	14	13
N.S.	1	1.00	1.00	2.46	2.31	2.31	2.08	1.08	1.00
time (sec)	N/A	0.002	0.006	0.138	0.287	0.599	0.093	1.616	0.148

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	33	54	54	56	23	54
N.S.	1	1.00	0.66	0.87	1.42	1.42	1.47	0.61	1.42
time (sec)	N/A	0.014	0.009	0.112	0.304	0.456	0.128	1.757	0.049

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	24	33	67	67	73	40	67
N.S.	1	1.00	0.63	0.87	1.76	1.76	1.92	1.05	1.76
time (sec)	N/A	0.014	0.008	0.128	0.293	0.421	0.168	1.534	0.167

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	33	84	84	88	25	82
N.S.	1	1.00	0.71	0.87	2.21	2.21	2.32	0.66	2.16
time (sec)	N/A	0.014	0.010	0.136	0.283	0.436	0.235	1.462	0.075

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	73	72	72	78	72	72
N.S.	1	1.00	1.19	1.28	1.26	1.26	1.37	1.26	1.26
time (sec)	N/A	0.022	0.003	0.168	0.276	0.450	0.017	2.086	0.032

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	34	34	36	34	31
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.95	0.89	0.82
time (sec)	N/A	0.012	0.001	0.129	0.290	0.528	0.011	3.110	0.040

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	40	37	36	36	39	36	36
N.S.	1	1.00	1.25	1.16	1.12	1.12	1.22	1.12	1.12
time (sec)	N/A	0.011	0.002	0.125	0.283	0.469	0.012	2.693	0.049

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43
time (sec)	N/A	0.001	0.001	0.125	0.287	0.392	0.008	1.725	0.027

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	31	35	34	31	46	34
N.S.	1	1.00	0.86	0.72	0.81	0.79	0.72	1.07	0.79
time (sec)	N/A	0.010	0.005	0.176	0.284	0.505	0.056	2.065	0.047

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	36	46	57	39	79	46
N.S.	1	1.00	0.85	0.88	1.12	1.39	0.95	1.93	1.12
time (sec)	N/A	0.016	0.021	0.155	0.286	0.546	0.078	1.918	0.149

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	33	48	61	69	54	46	59
N.S.	1	1.00	0.63	0.92	1.17	1.33	1.04	0.88	1.13
time (sec)	N/A	0.019	0.017	0.127	0.282	0.462	0.126	1.929	0.172

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	48	60	60	61	29	58
N.S.	1	1.00	1.11	1.71	2.14	2.14	2.18	1.04	2.07
time (sec)	N/A	0.003	0.013	0.144	0.296	0.409	0.151	2.080	0.046

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	48	78	78	85	64	76
N.S.	1	1.00	0.62	0.86	1.39	1.39	1.52	1.14	1.36
time (sec)	N/A	0.018	0.009	0.149	0.284	0.457	0.187	1.410	0.052

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	38	49	95	95	100	36	91
N.S.	1	1.00	0.67	0.86	1.67	1.67	1.75	0.63	1.60
time (sec)	N/A	0.019	0.015	0.146	0.292	0.418	0.256	1.328	0.187

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	37	49	108	108	117	36	104
N.S.	1	1.00	0.63	0.83	1.83	1.83	1.98	0.61	1.76
time (sec)	N/A	0.020	0.011	0.161	0.290	0.456	0.278	1.137	0.108

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	41	48	52	49	59	48
N.S.	1	1.00	0.69	0.67	0.79	0.85	0.80	0.97	0.79
time (sec)	N/A	0.015	0.005	0.152	0.274	0.477	0.057	1.419	0.049

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	30	34	38	34	45	32
N.S.	1	1.00	0.72	0.70	0.79	0.88	0.79	1.05	0.74
time (sec)	N/A	0.011	0.005	0.149	0.290	0.543	0.047	1.418	0.148

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	15	19	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	0.83	1.06	1.00
time (sec)	N/A	0.007	0.003	0.137	0.282	0.493	0.037	1.013	0.039

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.001	0.129	0.292	0.509	0.007	1.487	0.019

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	35	37	28	22	39	17
N.S.	1	1.00	1.00	2.06	2.18	1.65	1.29	2.29	1.00
time (sec)	N/A	0.006	0.005	0.155	0.283	0.530	0.078	1.681	0.170

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	51	60	60	48	53	42
N.S.	1	1.00	1.26	1.21	1.43	1.43	1.14	1.26	1.00
time (sec)	N/A	0.021	0.012	0.146	0.282	0.627	0.116	2.387	0.071

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	67	82	98	71	69	64
N.S.	1	1.00	1.03	1.06	1.30	1.56	1.13	1.10	1.02
time (sec)	N/A	0.027	0.016	0.184	0.295	0.482	0.161	2.556	0.078

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	45	53	79	51	80	52
N.S.	1	1.00	0.85	0.83	0.98	1.46	0.94	1.48	0.96
time (sec)	N/A	0.023	0.014	0.153	0.275	0.781	0.096	1.507	0.055

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	34	40	61	36	59	39
N.S.	1	1.00	0.85	0.87	1.03	1.56	0.92	1.51	1.00
time (sec)	N/A	0.015	0.013	0.153	0.281	0.514	0.069	1.295	0.168

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	28	28	33	24	54	27
N.S.	1	1.00	0.85	1.04	1.04	1.22	0.89	2.00	1.00
time (sec)	N/A	0.010	0.008	0.145	0.302	0.652	0.064	1.809	0.043

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00
time (sec)	N/A	0.001	0.002	0.128	0.287	0.591	0.047	2.522	0.024

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	50	50	55	51	44	44	37
N.S.	1	1.00	1.22	1.22	1.34	1.24	1.07	1.07	0.90
time (sec)	N/A	0.019	0.013	0.150	0.285	0.581	0.113	1.688	0.177

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	74	66	64	76	49	83	46
N.S.	1	1.00	1.61	1.43	1.39	1.65	1.07	1.80	1.00
time (sec)	N/A	0.011	0.019	0.165	0.337	0.781	0.108	1.178	0.181

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	68	82	108	146	104	81	86
N.S.	1	1.00	0.82	0.99	1.30	1.76	1.25	0.98	1.04
time (sec)	N/A	0.035	0.031	0.178	0.283	0.952	0.218	1.701	0.098

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	73	113	68	62	287	185	-1
N.S.	1	1.00	0.68	1.05	0.63	0.57	2.66	1.71	-0.01
time (sec)	N/A	0.015	0.094	0.164	0.502	1.072	89.423	1.872	0.000

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	99	54	57	252	115	-1
N.S.	1	1.00	0.77	1.12	0.61	0.65	2.86	1.31	-0.01
time (sec)	N/A	0.011	0.079	0.154	0.497	0.634	29.454	2.659	0.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	85	40	52	216	101	-1
N.S.	1	1.00	0.93	1.25	0.59	0.76	3.18	1.49	-0.01
time (sec)	N/A	0.007	0.076	0.163	0.496	0.573	9.191	1.214	0.000

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	52	71	28	47	167	50	-1
N.S.	1	1.00	1.08	1.48	0.58	0.98	3.48	1.04	-0.02
time (sec)	N/A	0.004	0.066	0.159	0.496	0.460	3.382	1.411	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	57	17	38	131	42	37
N.S.	1	1.00	1.32	2.04	0.61	1.36	4.68	1.50	1.32
time (sec)	N/A	0.003	0.004	0.155	0.496	0.501	1.597	1.333	0.205

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	34	42	14	37	99	28	14
N.S.	1	1.00	1.62	2.00	0.67	1.76	4.71	1.33	0.67
time (sec)	N/A	0.003	0.004	0.166	0.511	0.564	0.927	1.295	0.145

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	64	21	48	70	33	-1
N.S.	1	1.00	1.70	2.78	0.91	2.09	3.04	1.43	-0.04
time (sec)	N/A	0.003	0.037	0.186	0.486	1.293	0.788	0.909	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	30	38	33	60	19	34
N.S.	1	1.00	1.00	1.50	1.90	1.65	3.00	0.95	1.70
time (sec)	N/A	0.001	0.053	0.155	0.273	1.085	0.843	0.992	0.270

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	44	64	53	172	22	50
N.S.	1	1.00	0.56	1.07	1.56	1.29	4.20	0.54	1.22
time (sec)	N/A	0.003	0.048	0.149	0.293	0.542	4.005	1.204	0.244

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	58	95	70	566	29	64
N.S.	1	1.00	0.49	0.95	1.56	1.15	9.28	0.48	1.05
time (sec)	N/A	0.006	0.055	0.142	0.278	0.606	14.859	1.084	0.269

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	72	131	85	1561	35	80
N.S.	1	1.00	0.43	0.89	1.62	1.05	19.27	0.43	0.99
time (sec)	N/A	0.009	0.060	0.170	0.284	0.540	47.661	1.034	0.282

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	86	172	100	3648	42	94
N.S.	1	1.00	0.40	0.85	1.70	0.99	36.12	0.42	0.93
time (sec)	N/A	0.013	0.065	0.158	0.327	0.569	134.793	0.920	0.292

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	78	127	66	67	0	237	-1
N.S.	1	1.00	0.72	1.17	0.61	0.61	0.00	2.17	-0.01
time (sec)	N/A	0.013	0.138	0.137	0.568	0.529	0.000	1.001	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	113	52	62	287	185	-1
N.S.	1	1.00	0.82	1.27	0.58	0.70	3.22	2.08	-0.01
time (sec)	N/A	0.009	0.121	0.161	0.501	0.431	109.660	2.038	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	99	40	57	248	91	-1
N.S.	1	1.00	0.99	1.43	0.58	0.83	3.59	1.32	-0.01
time (sec)	N/A	0.006	0.107	0.143	0.629	0.417	34.678	2.304	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	85	29	46	212	101	-1
N.S.	1	1.00	0.94	1.73	0.59	0.94	4.33	2.06	-0.02
time (sec)	N/A	0.005	0.070	0.141	0.550	0.440	11.281	1.059	0.000

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	54	71	28	47	163	66	-1
N.S.	1	1.00	1.12	1.48	0.58	0.98	3.40	1.38	-0.02
time (sec)	N/A	0.004	0.068	0.135	0.500	0.466	4.552	0.985	0.000

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	57	28	40	134	31	-1
N.S.	1	1.00	1.04	1.21	0.60	0.85	2.85	0.66	-0.02
time (sec)	N/A	0.005	0.052	0.159	0.507	0.482	2.234	1.148	0.000

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	72	42	52	99	35	-1
N.S.	1	1.00	1.00	1.76	1.02	1.27	2.41	0.85	-0.02
time (sec)	N/A	0.004	0.105	0.163	0.493	0.528	1.588	1.037	0.000

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	76	66	71	498	38	-1
N.S.	1	1.00	1.12	1.85	1.61	1.73	12.15	0.93	-0.02
time (sec)	N/A	0.004	0.063	0.164	0.506	0.539	2.012	1.125	0.000

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	57	94	52	87	19	50
N.S.	1	1.00	1.00	2.85	4.70	2.60	4.35	0.95	2.50
time (sec)	N/A	0.001	0.045	0.132	0.311	0.477	4.005	1.048	0.252

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	72	131	69	226	22	64
N.S.	1	1.00	0.56	1.76	3.20	1.68	5.51	0.54	1.56
time (sec)	N/A	0.003	0.053	0.138	0.278	0.584	14.460	1.494	0.268

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	86	172	86	675	29	80
N.S.	1	1.00	0.49	1.41	2.82	1.41	11.07	0.48	1.31
time (sec)	N/A	0.006	0.057	0.176	0.281	0.803	46.298	1.368	0.319

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	100	218	101	1751	35	94
N.S.	1	1.00	0.43	1.23	2.69	1.25	21.62	0.43	1.16
time (sec)	N/A	0.010	0.067	0.158	0.270	0.855	133.325	1.068	0.310

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	114	269	116	0	42	110
N.S.	1	1.00	0.40	1.13	2.66	1.15	0.00	0.42	1.09
time (sec)	N/A	0.013	0.074	0.140	0.332	0.794	0.000	1.409	0.328

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	88	155	78	77	0	323	-1
N.S.	1	1.00	0.68	1.19	0.60	0.59	0.00	2.48	-0.01
time (sec)	N/A	0.019	0.186	0.139	0.519	0.909	0.000	1.593	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	141	64	72	0	296	-1
N.S.	1	1.00	0.75	1.28	0.58	0.65	0.00	2.69	-0.01
time (sec)	N/A	0.014	0.157	0.140	0.495	0.765	0.000	2.014	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	127	52	67	0	143	-1
N.S.	1	1.00	0.87	1.41	0.58	0.74	0.00	1.59	-0.01
time (sec)	N/A	0.009	0.134	0.162	0.514	0.838	0.000	2.167	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	113	41	51	284	185	-1
N.S.	1	1.00	0.73	1.61	0.59	0.73	4.06	2.64	-0.01
time (sec)	N/A	0.007	0.086	0.152	0.496	0.724	186.195	1.692	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	99	40	57	245	114	-1
N.S.	1	1.00	0.99	1.43	0.58	0.83	3.55	1.65	-0.01
time (sec)	N/A	0.006	0.097	0.161	0.526	0.747	60.642	1.240	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	85	40	52	212	101	-1
N.S.	1	1.00	0.93	1.25	0.59	0.76	3.12	1.49	-0.01
time (sec)	N/A	0.007	0.069	0.136	0.500	0.818	20.765	1.176	0.000

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	71	42	47	170	39	-1
N.S.	1	1.00	0.84	1.06	0.63	0.70	2.54	0.58	-0.01
time (sec)	N/A	0.008	0.058	0.141	0.509	0.946	9.021	1.443	0.000

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	77	56	58	138	42	-1
N.S.	1	1.00	0.75	1.18	0.86	0.89	2.12	0.65	-0.02
time (sec)	N/A	0.008	0.069	0.184	0.544	0.617	5.569	1.218	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	84	99	75	575	44	-1
N.S.	1	1.00	0.81	1.33	1.57	1.19	9.13	0.70	-0.02
time (sec)	N/A	0.008	0.141	0.164	0.503	0.594	4.527	1.322	0.000

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	84	160	91	1606	44	-1
N.S.	1	1.00	0.81	1.33	2.54	1.44	25.49	0.70	-0.02
time (sec)	N/A	0.005	0.063	0.164	0.545	0.938	7.280	1.242	0.000

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	85	171	66	114	19	64
N.S.	1	1.00	1.00	4.25	8.55	3.30	5.70	0.95	3.20
time (sec)	N/A	0.001	0.049	0.138	0.281	1.116	15.689	1.233	0.279

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	100	218	83	280	22	80
N.S.	1	1.00	0.56	2.44	5.32	2.02	6.83	0.54	1.95
time (sec)	N/A	0.003	0.058	0.144	0.304	1.606	50.924	1.115	0.303

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	114	269	100	784	29	94
N.S.	1	1.00	0.49	1.87	4.41	1.64	12.85	0.48	1.54
time (sec)	N/A	0.006	0.063	0.138	0.282	0.779	148.973	1.321	0.310

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	128	325	115	0	35	110
N.S.	1	1.00	0.43	1.58	4.01	1.42	0.00	0.43	1.36
time (sec)	N/A	0.009	0.066	0.161	0.270	1.346	0.000	1.963	0.315

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	142	386	130	0	42	124
N.S.	1	1.00	0.40	1.41	3.82	1.29	0.00	0.42	1.23
time (sec)	N/A	0.013	0.077	0.160	0.282	0.944	0.000	1.838	0.352

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	45	156	452	145	0	48	140
N.S.	1	1.00	0.37	1.29	3.74	1.20	0.00	0.40	1.16
time (sec)	N/A	0.018	0.082	0.176	0.281	0.880	0.000	2.506	0.370

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	98	42	55	88	42	-1
N.S.	1	1.00	1.00	1.53	0.66	0.86	1.38	0.66	-0.02
time (sec)	N/A	0.009	0.095	0.152	0.488	1.044	16.600	1.640	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	120	42	48	76	34	55
N.S.	1	1.00	1.11	1.94	0.68	0.77	1.23	0.55	0.89
time (sec)	N/A	0.022	0.175	0.155	0.487	1.103	2.992	1.164	0.151

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	63	85	56	52	197	101	-1
N.S.	1	1.00	0.72	0.98	0.64	0.60	2.26	1.16	-0.01
time (sec)	N/A	0.013	0.067	0.138	0.506	1.042	14.704	1.274	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	71	42	47	173	69	-1
N.S.	1	1.00	0.84	1.06	0.63	0.70	2.58	1.03	-0.01
time (sec)	N/A	0.008	0.064	0.162	0.495	0.916	4.112	1.496	0.000

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	57	28	40	138	44	-1
N.S.	1	1.00	1.04	1.21	0.60	0.85	2.94	0.94	-0.02
time (sec)	N/A	0.005	0.056	0.155	0.486	1.255	1.410	1.700	0.000

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	32	41	12	36	99	27	12
N.S.	1	1.00	1.60	2.05	0.60	1.80	4.95	1.35	0.60
time (sec)	N/A	0.003	0.004	0.159	0.496	1.038	0.752	1.663	0.119

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	14	27	2	22	39	13	22
N.S.	1	1.00	7.00	13.50	1.00	11.00	19.50	6.50	11.00
time (sec)	N/A	0.001	0.002	0.136	0.511	0.809	0.450	1.652	0.078

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	23	31	19	13
N.S.	1	1.00	1.00	0.82	0.94	1.35	1.82	1.12	0.76
time (sec)	N/A	0.003	0.015	0.132	0.534	0.877	0.445	0.995	0.283

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	30	38	39	128	22	43
N.S.	1	1.00	0.56	0.73	0.93	0.95	3.12	0.54	1.05
time (sec)	N/A	0.004	0.044	0.134	0.494	0.911	1.164	1.133	0.309

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	44	64	56	303	29	55
N.S.	1	1.00	0.49	0.72	1.05	0.92	4.97	0.48	0.90
time (sec)	N/A	0.006	0.051	0.145	0.494	1.742	4.439	0.922	0.324

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	35	58	95	71	542	35	67
N.S.	1	1.00	0.43	0.72	1.17	0.88	6.69	0.43	0.83
time (sec)	N/A	0.009	0.058	0.135	0.493	1.058	14.417	0.913	0.345

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	40	72	131	86	850	42	80
N.S.	1	1.00	0.40	0.71	1.30	0.85	8.42	0.42	0.79
time (sec)	N/A	0.013	0.060	0.158	0.528	1.375	41.469	0.814	0.363

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	56	84	70	65	206	81	-1
N.S.	1	1.00	0.66	0.99	0.82	0.76	2.42	0.95	-0.01
time (sec)	N/A	0.011	0.088	0.143	0.507	0.984	14.476	1.309	0.000

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	77	56	58	167	73	-1
N.S.	1	1.00	0.75	1.18	0.86	0.89	2.57	1.12	-0.02
time (sec)	N/A	0.008	0.073	0.161	0.499	0.907	4.409	1.300	0.000

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	43	71	41	53	131	70	-1
N.S.	1	1.00	1.05	1.73	1.00	1.29	3.20	1.71	-0.02
time (sec)	N/A	0.005	0.097	0.157	0.502	1.410	1.311	1.596	0.000

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	67	21	50	102	55	-1
N.S.	1	1.00	1.70	2.91	0.91	2.17	4.43	2.39	-0.04
time (sec)	N/A	0.002	0.036	0.136	0.484	1.337	0.730	1.261	0.000

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	16	23	31	43	14
N.S.	1	1.00	1.00	0.83	0.89	1.28	1.72	2.39	0.78
time (sec)	N/A	0.001	0.016	0.138	0.503	1.293	0.460	1.625	0.360

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	13	29	11	22	63	62	14
N.S.	1	1.00	0.72	1.61	0.61	1.22	3.50	3.44	0.78
time (sec)	N/A	0.001	0.029	0.154	0.273	1.072	0.861	1.804	0.305

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	44	40	54	160	67	42
N.S.	1	1.00	0.71	1.05	0.95	1.29	3.81	1.60	1.00
time (sec)	N/A	0.003	0.053	0.136	0.271	0.825	3.016	1.267	0.322

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	33	58	79	59	284	73	55
N.S.	1	1.00	0.53	0.94	1.27	0.95	4.58	1.18	0.89
time (sec)	N/A	0.006	0.058	0.162	0.269	0.940	10.198	1.048	0.337

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	40	72	134	86	425	79	68
N.S.	1	1.00	0.49	0.88	1.63	1.05	5.18	0.96	0.83
time (sec)	N/A	0.009	0.063	0.145	0.278	1.449	30.969	1.190	0.355

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	45	86	201	91	593	85	80
N.S.	1	1.00	0.44	0.84	1.97	0.89	5.81	0.83	0.78
time (sec)	N/A	0.013	0.070	0.139	0.275	1.023	82.947	1.235	0.363

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	61	89	125	85	248	127	-1
N.S.	1	1.00	0.59	0.86	1.21	0.83	2.41	1.23	-0.01
time (sec)	N/A	0.015	0.104	0.167	0.506	0.958	47.551	2.134	0.000

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	56	84	111	81	212	119	-1
N.S.	1	1.00	0.64	0.97	1.28	0.93	2.44	1.37	-0.01
time (sec)	N/A	0.011	0.083	0.161	0.489	1.112	14.609	1.368	0.000

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	79	98	75	162	115	-1
N.S.	1	1.00	0.81	1.25	1.56	1.19	2.57	1.83	-0.02
time (sec)	N/A	0.008	0.130	0.170	0.509	1.019	4.727	2.199	0.000

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	46	73	66	71	128	102	-1
N.S.	1	1.00	1.12	1.78	1.61	1.73	3.12	2.49	-0.02
time (sec)	N/A	0.004	0.057	0.163	0.507	1.538	1.920	1.997	0.000

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	30	38	37	66	89	32
N.S.	1	1.00	1.00	1.50	1.90	1.85	3.30	4.45	1.60
time (sec)	N/A	0.001	0.048	0.163	0.304	1.093	0.859	1.088	0.263

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	23	30	38	38	66	89	33
N.S.	1	1.00	0.56	0.73	0.93	0.93	1.61	2.17	0.80
time (sec)	N/A	0.003	0.041	0.156	0.531	1.592	1.327	1.190	0.310

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	30	43	38	49	167	108	48
N.S.	1	1.00	0.52	0.74	0.66	0.84	2.88	1.86	0.83
time (sec)	N/A	0.006	0.052	0.160	0.293	1.062	2.864	1.567	0.338

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	57	25	35	280	113	41
N.S.	1	1.00	0.56	1.33	0.58	0.81	6.51	2.63	0.95
time (sec)	N/A	0.003	0.044	0.154	0.313	1.181	5.754	1.686	0.365

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	40	72	52	84	425	119	75
N.S.	1	1.00	0.63	1.14	0.83	1.33	6.75	1.89	1.19
time (sec)	N/A	0.006	0.064	0.157	0.346	0.991	18.171	1.317	0.377

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	45	86	91	101	593	125	86
N.S.	1	1.00	0.54	1.04	1.10	1.22	7.14	1.51	1.04
time (sec)	N/A	0.009	0.071	0.164	0.550	1.449	51.379	1.119	0.413

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	50	100	146	114	789	131	99
N.S.	1	1.00	0.49	0.97	1.42	1.11	7.66	1.27	0.96
time (sec)	N/A	0.013	0.075	0.161	0.282	1.826	162.013	0.985	0.423

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	86	198	72	201	0	679	-1
N.S.	1	1.00	0.68	1.57	0.57	1.60	0.00	5.39	-0.01
time (sec)	N/A	0.039	0.249	0.186	0.500	0.720	0.000	1.476	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	150	50	155	0	403	-1
N.S.	1	1.00	0.82	1.56	0.52	1.61	0.00	4.20	-0.01
time (sec)	N/A	0.026	0.174	0.178	0.518	0.696	0.000	1.724	0.000

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	69	102	28	127	0	173	59
N.S.	1	1.00	1.03	1.52	0.42	1.90	0.00	2.58	0.88
time (sec)	N/A	0.022	0.176	0.139	0.529	0.677	0.000	1.285	0.300

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	47	57	8	101	85	49	44
N.S.	1	1.00	1.09	1.33	0.19	2.35	1.98	1.14	1.02
time (sec)	N/A	0.017	0.049	0.137	0.497	1.201	13.229	1.181	0.175

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	47	21	39	82	116	23
N.S.	1	1.00	1.00	1.74	0.78	1.44	3.04	4.30	0.85
time (sec)	N/A	0.002	0.063	0.143	0.286	1.445	2.185	1.005	0.391

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	105	45	57	82	237	62
N.S.	1	1.00	0.69	1.72	0.74	0.93	1.34	3.89	1.02
time (sec)	N/A	0.006	0.076	0.141	0.269	0.967	7.263	1.115	0.415

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	49	163	67	74	85	333	50
N.S.	1	1.00	0.54	1.79	0.74	0.81	0.93	3.66	0.55
time (sec)	N/A	0.012	0.088	0.157	0.306	1.208	35.640	1.540	0.442

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	54	221	89	89	85	437	66
N.S.	1	1.00	0.45	1.83	0.74	0.74	0.70	3.61	0.55
time (sec)	N/A	0.019	0.098	0.137	0.280	1.012	180.069	1.399	0.478

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	103	242	89	232	0	622	-1
N.S.	1	1.00	0.76	1.79	0.66	1.72	0.00	4.61	-0.01
time (sec)	N/A	0.038	0.180	0.161	0.505	1.593	0.000	1.276	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	92	184	63	193	0	354	-1
N.S.	1	1.00	0.90	1.80	0.62	1.89	0.00	3.47	-0.01
time (sec)	N/A	0.027	0.133	0.172	0.499	1.278	0.000	1.491	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	78	126	39	159	0	148	72
N.S.	1	1.00	1.15	1.85	0.57	2.34	0.00	2.18	1.06
time (sec)	N/A	0.019	0.077	0.176	0.516	1.319	0.000	1.063	0.203

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	71	14	108	90	42	53
N.S.	1	1.00	1.26	1.87	0.37	2.84	2.37	1.11	1.39
time (sec)	N/A	0.015	0.048	0.189	0.501	0.963	13.226	1.881	0.177

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	59	25	45	94	103	26
N.S.	1	1.00	0.97	1.97	0.83	1.50	3.13	3.43	0.87
time (sec)	N/A	0.003	0.064	0.149	0.273	0.887	2.348	1.749	0.497

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	46	129	53	72	94	199	80
N.S.	1	1.00	0.69	1.93	0.79	1.07	1.40	2.97	1.19
time (sec)	N/A	0.007	0.077	0.145	0.270	1.744	8.059	1.456	0.584

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	57	202	79	98	97	296	111
N.S.	1	1.00	0.57	2.02	0.79	0.98	0.97	2.96	1.11
time (sec)	N/A	0.014	0.087	0.167	0.270	1.068	39.183	1.793	0.650

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	76	275	105	122	97	393	170
N.S.	1	1.00	0.57	2.07	0.79	0.92	0.73	2.95	1.28
time (sec)	N/A	0.023	0.100	0.151	0.268	1.282	185.105	2.517	0.714

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	193	134	46	65	0	227	-1
N.S.	1	1.00	1.93	1.34	0.46	0.65	0.00	2.27	-0.01
time (sec)	N/A	0.012	1.281	0.175	0.507	0.998	0.000	1.523	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	179	102	34	60	0	125	-1
N.S.	1	1.00	2.42	1.38	0.46	0.81	0.00	1.69	-0.01
time (sec)	N/A	0.007	0.820	0.144	0.489	1.259	0.000	1.663	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	108	70	22	52	187	55	44
N.S.	1	1.00	2.51	1.63	0.51	1.21	4.35	1.28	1.02
time (sec)	N/A	0.004	0.665	0.183	0.482	1.166	2.104	1.301	0.256

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	27	37	9	28	41	21	40
N.S.	1	1.00	2.08	2.85	0.69	2.15	3.15	1.62	3.08
time (sec)	N/A	0.002	0.045	0.161	0.478	1.203	0.998	1.054	0.051

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	59	34	12	26	156	79	24
N.S.	1	1.00	2.11	1.21	0.43	0.93	5.57	2.82	0.86
time (sec)	N/A	0.001	0.691	0.151	0.275	1.267	37.336	2.380	0.461

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	107	66	25	39	0	144	49
N.S.	1	1.00	1.88	1.16	0.44	0.68	0.00	2.53	0.86
time (sec)	N/A	0.004	0.792	0.148	0.273	1.048	0.000	1.566	0.311

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	149	98	37	49	0	205	66
N.S.	1	1.00	1.75	1.15	0.44	0.58	0.00	2.41	0.78
time (sec)	N/A	0.008	1.091	0.138	0.259	1.236	0.000	1.759	0.452

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	89	67	62	199	101	-1
N.S.	1	1.00	0.90	0.98	0.74	0.68	2.19	1.11	-0.01
time (sec)	N/A	0.015	0.090	0.164	0.495	1.465	11.553	1.251	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	72	61	38	52	124	42	41
N.S.	1	1.00	1.41	1.20	0.75	1.02	2.43	0.82	0.80
time (sec)	N/A	0.008	0.062	0.170	0.482	1.714	1.835	2.264	0.207

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	44	31	6	32	26	8	31
N.S.	1	1.00	5.50	3.88	0.75	4.00	3.25	1.00	3.88
time (sec)	N/A	0.003	0.030	0.167	0.471	1.159	0.753	1.481	0.178

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	21	30	30	29	100	53	32
N.S.	1	1.00	0.57	0.81	0.81	0.78	2.70	1.43	0.86
time (sec)	N/A	0.003	0.034	0.161	0.290	1.280	1.218	1.782	0.255

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	43	58	59	49	282	97	69
N.S.	1	1.00	0.54	0.73	0.75	0.62	3.57	1.23	0.87
time (sec)	N/A	0.010	0.041	0.167	0.302	1.096	5.935	1.099	0.370

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	16	30	12	22	71	62	22
N.S.	1	1.00	0.76	1.43	0.57	1.05	3.38	2.95	1.05
time (sec)	N/A	0.001	0.031	0.163	0.342	1.086	0.953	1.346	0.364

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	42	15	29	73	77	26
N.S.	1	1.00	0.79	1.75	0.62	1.21	3.04	3.21	1.08
time (sec)	N/A	0.002	0.043	0.149	0.272	1.644	2.398	1.396	0.463

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	21	30	12	22	92	71	22
N.S.	1	1.00	0.81	1.15	0.46	0.85	3.54	2.73	0.85
time (sec)	N/A	0.001	0.095	0.141	0.280	1.529	8.368	1.250	0.374

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	19	42	15	29	83	79	26
N.S.	1	1.00	0.66	1.45	0.52	1.00	2.86	2.72	0.90
time (sec)	N/A	0.002	0.167	0.179	0.284	0.804	10.812	0.956	0.321

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	76	39	108	88	37	56
N.S.	1	1.00	1.00	1.95	1.00	2.77	2.26	0.95	1.44
time (sec)	N/A	0.017	0.071	0.168	0.277	1.113	13.313	1.710	0.218

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	42	0	0	452	0	0	-1
N.S.	1	1.00	0.17	0.00	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.179	0.100	0.055	0.000	1.064	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	70	104	0	0	0	0	-1
N.S.	1	1.00	0.49	0.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	10.037	0.224	0.000	0.000	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	70	94	0	0	0	0	-1
N.S.	1	1.00	0.66	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.031	0.169	0.000	0.000	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	102	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.44	0.00	-0.01
time (sec)	N/A	0.009	10.022	0.050	0.000	0.000	1.712	0.000	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	94	0	0	0	0	-1
N.S.	1	1.00	0.87	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	10.029	0.169	0.000	0.000	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0	-1
N.S.	1	1.00	0.85	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.012	10.033	0.184	0.000	0.000	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	113	0	0	0	0	-1
N.S.	1	1.00	0.61	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	10.032	0.175	0.000	0.000	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	70	114	0	0	0	0	-1
N.S.	1	1.00	0.47	0.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	10.036	0.201	0.000	0.000	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	111	479	0	194	0	0	-1
N.S.	1	1.00	0.43	1.87	0.00	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.117	0.343	1.432	0.000	0.743	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	77	0	0	221	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.095	0.164	0.050	0.000	0.796	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	0	31	0	0	38
N.S.	1	1.00	1.00	0.94	0.00	0.94	0.00	0.00	1.15
time (sec)	N/A	0.002	0.085	0.159	0.000	0.864	0.000	0.000	0.549

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	44	0	0	46
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	0.69
time (sec)	N/A	0.007	0.088	0.146	0.000	2.391	0.000	0.000	0.666

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	50	0	57	0	0	51
N.S.	1	1.00	0.52	0.50	0.00	0.57	0.00	0.00	0.51
time (sec)	N/A	0.012	0.094	0.162	0.000	1.137	0.000	0.000	0.752

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	57	55	0	68	0	0	57
N.S.	1	1.00	0.43	0.41	0.00	0.51	0.00	0.00	0.43
time (sec)	N/A	0.020	0.100	0.170	0.000	1.032	0.000	0.000	0.794

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	111	465	0	198	0	0	-1
N.S.	1	1.00	0.43	1.82	0.00	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.110	0.443	1.604	0.000	0.839	0.000	0.000	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	77	0	0	221	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.094	0.126	0.053	0.000	0.641	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	0	31	0	0	-1
N.S.	1	1.00	1.00	1.00	0.00	1.00	0.00	0.00	-0.03
time (sec)	N/A	0.003	0.045	0.160	0.000	0.881	0.000	0.000	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	44	0	0	-1
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.006	0.081	0.151	0.000	0.964	0.000	0.000	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	50	0	57	0	0	-1
N.S.	1	1.00	0.52	0.50	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.085	0.159	0.000	0.750	0.000	0.000	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	70	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.022	0.013	0.000	0.000	0.000	0.000	0.000

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.020	0.009	0.000	0.000	0.000	0.000	0.000

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	68	0	0	0	100	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	2.33	0.00	-0.02
time (sec)	N/A	0.005	10.027	0.045	0.000	0.000	2.489	0.000	0.000

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.024	0.020	0.000	0.000	0.000	0.000	0.000

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.032	0.019	0.000	0.000	0.000	0.000	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	121	465	0	235	0	0	-1
N.S.	1	1.00	0.42	1.60	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.123	0.696	1.608	0.000	0.722	0.000	0.000	0.000

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	112	455	0	298	0	0	-1
N.S.	1	1.00	0.42	1.71	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.098	0.208	1.305	0.000	0.702	0.000	0.000	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	31	0	31	0	0	-1
N.S.	1	1.00	1.00	0.94	0.00	0.94	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.057	0.153	0.000	0.743	0.000	0.000	0.000

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	33	0	36	0	0	-1
N.S.	1	1.00	0.69	0.51	0.00	0.55	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.080	0.147	0.000	0.628	0.000	0.000	0.000

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	44	0	56	0	0	-1
N.S.	1	1.00	0.52	0.44	0.00	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.087	0.153	0.000	0.613	0.000	0.000	0.000

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	70	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	10.028	0.016	0.000	0.000	0.000	0.000	0.000

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	70	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.036	0.014	0.000	0.000	0.000	0.000	0.000

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.018	0.015	0.000	0.000	0.000	0.000	0.000

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.010	10.020	0.017	0.000	0.000	0.000	0.000	0.000

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	0	0	95	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	1.17	0.00	-0.01
time (sec)	N/A	0.010	10.029	0.014	0.000	0.000	21.950	0.000	0.000

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	70	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.027	0.019	0.000	0.000	0.000	0.000	0.000

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	70	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	10.026	0.022	0.000	0.000	0.000	0.000	0.000

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	70	96	0	0	0	0	-1
N.S.	1	1.00	0.51	0.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	10.030	0.167	0.000	0.000	0.000	0.000	0.000

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	88	0	0	0	0	-1
N.S.	1	1.00	0.69	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.023	0.167	0.000	0.000	0.000	0.000	0.000

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	94	0	0	0	0	-1
N.S.	1	1.00	0.90	1.21	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.011	10.029	0.173	0.000	0.000	0.000	0.000	0.000

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	68	91	0	0	97	0	-1
N.S.	1	1.00	1.48	1.98	0.00	0.00	2.11	0.00	-0.02
time (sec)	N/A	0.006	10.028	0.199	0.000	0.000	5.654	0.000	0.000

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	107	0	0	0	0	-1
N.S.	1	1.00	0.85	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.011	10.019	0.179	0.000	0.000	0.000	0.000	0.000

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	113	0	0	0	0	-1
N.S.	1	1.00	0.61	0.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	10.025	0.181	0.000	0.000	0.000	0.000	0.000

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	117	480	0	233	0	0	-1
N.S.	1	1.00	0.41	1.67	0.00	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.581	1.569	0.000	0.820	0.000	0.000	0.000

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	109	478	0	296	0	0	-1
N.S.	1	1.00	0.41	1.81	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.094	0.201	1.149	0.000	0.681	0.000	0.000	0.000

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	0	31	0	0	27
N.S.	1	1.00	1.00	1.00	0.00	1.00	0.00	0.00	0.87
time (sec)	N/A	0.002	0.047	0.184	0.000	0.721	0.000	0.000	1.161

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	33	0	36	0	0	40
N.S.	1	1.00	0.67	0.49	0.00	0.54	0.00	0.00	0.60
time (sec)	N/A	0.007	0.098	0.149	0.000	0.791	0.000	0.000	0.597

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	44	0	56	0	0	46
N.S.	1	1.00	0.52	0.44	0.00	0.56	0.00	0.00	0.46
time (sec)	N/A	0.012	0.110	0.150	0.000	0.832	0.000	0.000	0.764

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	70	101	0	0	0	0	-1
N.S.	1	1.00	0.50	0.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	10.032	0.177	0.000	0.000	0.000	0.000	0.000

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	70	107	0	0	0	0	-1
N.S.	1	1.00	0.61	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.033	0.211	0.000	0.000	0.000	0.000	0.000

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	105	0	0	0	0	-1
N.S.	1	1.00	0.85	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.011	10.027	0.196	0.000	0.000	0.000	0.000	0.000

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	68	107	0	0	0	0	-1
N.S.	1	1.00	0.83	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.027	0.201	0.000	0.000	0.000	0.000	0.000

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	70	0	0	0	95	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	1.08	0.00	-0.01
time (sec)	N/A	0.011	10.024	0.067	0.000	0.000	96.847	0.000	0.000

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	70	124	0	0	0	0	-1
N.S.	1	1.00	0.58	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	10.030	0.208	0.000	0.000	0.000	0.000	0.000

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	70	130	0	0	0	0	-1
N.S.	1	1.00	0.45	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	10.028	0.214	0.000	0.000	0.000	0.000	0.000

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	127	490	0	343	0	0	-1
N.S.	1	1.00	0.43	1.65	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.097	0.321	1.345	0.000	0.621	0.000	0.000	0.000

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	50	0	42	0	0	38
N.S.	1	1.00	1.00	1.52	0.00	1.27	0.00	0.00	1.15
time (sec)	N/A	0.002	0.080	0.147	0.000	0.760	0.000	0.000	0.546

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	44	0	0	38
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	0.57
time (sec)	N/A	0.007	0.101	0.167	0.000	1.089	0.000	0.000	0.633

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	44	0	56	0	0	45
N.S.	1	1.00	0.52	0.44	0.00	0.56	0.00	0.00	0.45
time (sec)	N/A	0.013	0.110	0.154	0.000	1.287	0.000	0.000	0.535

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	64	56	0	54	0	0	56
N.S.	1	1.00	0.48	0.42	0.00	0.41	0.00	0.00	0.42
time (sec)	N/A	0.020	0.110	0.165	0.000	0.963	0.000	0.000	0.687

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	103	167	128	819	256	133
N.S.	1	1.00	0.93	1.24	2.01	1.54	9.87	3.08	1.60
time (sec)	N/A	0.020	0.062	0.174	0.316	0.716	0.430	0.781	0.493

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	47	81	58	245	103	66
N.S.	1	1.00	0.81	0.89	1.53	1.09	4.62	1.94	1.25
time (sec)	N/A	0.012	0.047	0.123	0.308	0.737	0.255	1.132	0.319

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.041	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.037	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	53	0	0	0	124	0	-1
N.S.	1	1.00	1.29	0.00	0.00	0.00	3.02	0.00	-0.02
time (sec)	N/A	0.014	0.070	0.082	0.000	0.000	2.086	0.000	0.000

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	72	0	0	0	146	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	2.56	0.00	-0.02
time (sec)	N/A	0.019	0.040	0.063	0.000	0.000	2.813	0.000	0.000

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	0	42	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.10	0.00	-0.05
time (sec)	N/A	0.007	0.055	0.057	0.000	0.000	2.216	0.000	0.000

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	84	97	96	96	100	97	88
N.S.	1	1.00	2.21	2.55	2.53	2.53	2.63	2.55	2.32
time (sec)	N/A	0.015	0.014	0.116	0.267	0.457	0.018	0.683	0.190

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	69	69	73	72	65
N.S.	1	1.00	1.76	1.92	1.82	1.82	1.92	1.89	1.71
time (sec)	N/A	0.019	0.008	0.117	0.283	0.439	0.014	0.584	0.159

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	49	48	48	49	49	47
N.S.	1	1.00	1.21	1.29	1.26	1.26	1.29	1.29	1.24
time (sec)	N/A	0.039	0.006	0.122	0.263	0.487	0.012	0.550	0.047

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.027	0.004	0.011	0.289	0.388	0.006	0.933	0.035

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.003	0.000	0.010	0.307	0.416	0.005	1.129	0.019

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	24	20	26	26
N.S.	1	1.00	1.00	1.04	1.00	0.96	0.80	1.04	1.04
time (sec)	N/A	0.022	0.006	0.124	0.293	0.509	0.061	1.325	0.049

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	33	35	39	27	57	31
N.S.	1	1.00	0.97	1.03	1.09	1.22	0.84	1.78	0.97
time (sec)	N/A	0.022	0.009	0.128	0.268	0.476	0.083	1.033	0.171

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	38	38	39	24	39
N.S.	1	1.00	0.93	1.25	1.36	1.36	1.39	0.86	1.39
time (sec)	N/A	0.007	0.009	0.117	0.272	0.595	0.130	0.777	0.159

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	50	50	53	25	52
N.S.	1	1.00	0.71	0.92	1.32	1.32	1.39	0.66	1.37
time (sec)	N/A	0.017	0.007	0.126	0.309	0.568	0.167	0.633	0.165

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	61	61	65	41	63
N.S.	1	1.00	0.71	0.92	1.61	1.61	1.71	1.08	1.66
time (sec)	N/A	0.015	0.007	0.123	0.271	0.458	0.228	0.560	0.041

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	148	163	156	156	168	170	144
N.S.	1	1.00	2.28	2.51	2.40	2.40	2.58	2.62	2.22
time (sec)	N/A	0.064	0.020	0.133	0.275	0.463	0.023	0.491	0.066

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	124	124	133	130	115
N.S.	1	1.00	1.88	1.92	1.91	1.91	2.05	2.00	1.77
time (sec)	N/A	0.046	0.011	0.132	0.279	0.495	0.019	0.702	0.050

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	79	87	81	81	87	89	74
N.S.	1	1.00	1.22	1.34	1.25	1.25	1.34	1.37	1.14
time (sec)	N/A	0.034	0.007	0.135	0.266	0.390	0.015	0.537	0.166

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	49	48	48	49	49	47
N.S.	1	1.00	1.24	1.29	1.26	1.26	1.29	1.29	1.24
time (sec)	N/A	0.020	0.007	0.115	0.270	0.409	0.011	0.624	0.044

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	20	20	19	12	20
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43
time (sec)	N/A	0.001	0.001	0.120	0.269	0.897	0.007	0.819	0.028

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	56	61	63	44	60	62
N.S.	1	1.00	0.88	1.14	1.24	1.29	0.90	1.22	1.27
time (sec)	N/A	0.014	0.012	0.168	0.278	0.956	0.104	0.754	0.190

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	63	67	92	60	98	71
N.S.	1	1.00	0.92	1.24	1.31	1.80	1.18	1.92	1.39
time (sec)	N/A	0.023	0.029	0.139	0.276	0.960	0.172	0.707	0.200

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	69	79	99	80	68	77
N.S.	1	1.00	0.83	1.17	1.34	1.68	1.36	1.15	1.31
time (sec)	N/A	0.026	0.018	0.143	0.267	0.639	0.270	0.635	0.197

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	70	84	84	88	59	80
N.S.	1	1.00	1.89	2.50	3.00	3.00	3.14	2.11	2.86
time (sec)	N/A	0.003	0.020	0.137	0.273	0.609	0.321	0.609	0.037

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	71	98	98	104	96	39
N.S.	1	1.00	0.86	1.09	1.51	1.51	1.60	1.48	0.60
time (sec)	N/A	0.025	0.014	0.140	0.282	0.439	0.425	1.032	0.193

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	71	109	109	116	61	107
N.S.	1	1.00	0.88	1.09	1.68	1.68	1.78	0.94	1.65
time (sec)	N/A	0.024	0.017	0.152	0.267	0.478	0.515	1.217	0.202

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	71	120	120	128	61	118
N.S.	1	1.00	0.89	1.09	1.85	1.85	1.97	0.94	1.82
time (sec)	N/A	0.024	0.015	0.162	0.276	0.522	0.646	1.181	0.089

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	235	281	277	277	308	303	261
N.S.	1	1.00	2.55	3.05	3.01	3.01	3.35	3.29	2.84
time (sec)	N/A	0.114	0.050	0.133	0.317	0.431	0.031	1.261	0.240

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	229	225	225	243	245	208
N.S.	1	1.00	2.36	2.49	2.45	2.45	2.64	2.66	2.26
time (sec)	N/A	0.083	0.021	0.132	0.277	0.477	0.027	1.279	0.214

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	161	177	167	167	190	188	152
N.S.	1	1.00	1.75	1.92	1.82	1.82	2.07	2.04	1.65
time (sec)	N/A	0.060	0.013	0.138	0.278	0.511	0.023	0.976	0.056

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	122	125	124	124	133	130	115
N.S.	1	1.00	1.88	1.92	1.91	1.91	2.05	2.00	1.77
time (sec)	N/A	0.047	0.010	0.139	0.286	0.636	0.018	0.682	0.046

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	69	69	73	72	65
N.S.	1	1.00	1.76	1.92	1.82	1.82	1.92	1.89	1.71
time (sec)	N/A	0.010	0.006	0.118	0.284	0.509	0.014	0.928	0.032

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	31	31	32	12	31
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.21
time (sec)	N/A	0.001	0.001	0.115	0.284	0.851	0.009	0.826	0.037

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	109	114	116	83	115	118
N.S.	1	1.00	1.01	1.49	1.56	1.59	1.14	1.58	1.62
time (sec)	N/A	0.020	0.022	0.140	0.282	1.018	0.158	0.942	0.201

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	109	118	173	102	167	123
N.S.	1	1.00	0.96	1.45	1.57	2.31	1.36	2.23	1.64
time (sec)	N/A	0.041	0.036	0.173	0.283	1.007	0.267	1.020	0.215

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	114	114	125	188	128	112	130
N.S.	1	1.00	1.46	1.46	1.60	2.41	1.64	1.44	1.67
time (sec)	N/A	0.037	0.033	0.142	0.299	0.905	0.475	0.719	0.821

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	120	142	176	148	118	138
N.S.	1	1.00	0.93	1.40	1.65	2.05	1.72	1.37	1.60
time (sec)	N/A	0.037	0.029	0.143	0.294	0.737	0.650	0.578	0.254

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	91	122	143	143	155	159	135
N.S.	1	1.00	3.25	4.36	5.11	5.11	5.54	5.68	4.82
time (sec)	N/A	0.002	0.022	0.145	0.340	0.530	0.850	0.568	0.072

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	97	121	160	160	172	114	39
N.S.	1	1.00	1.67	2.09	2.76	2.76	2.97	1.97	0.67
time (sec)	N/A	0.008	0.025	0.135	0.297	0.540	1.132	1.112	0.082

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	171	171	184	114	165
N.S.	1	1.00	1.05	1.33	1.86	1.86	2.00	1.24	1.79
time (sec)	N/A	0.037	0.025	0.136	0.282	0.611	1.568	1.015	0.222

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	182	182	196	114	176
N.S.	1	1.00	1.05	1.33	1.98	1.98	2.13	1.24	1.91
time (sec)	N/A	0.036	0.022	0.158	0.279	0.492	1.973	1.292	0.114

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	122	193	193	207	114	187
N.S.	1	1.00	1.05	1.33	2.10	2.10	2.25	1.24	2.03
time (sec)	N/A	0.034	0.025	0.138	0.288	0.502	3.054	0.867	0.232

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	993	1033	1023	1023	1163	1175	997
N.S.	1	1.00	4.96	5.16	5.12	5.12	5.82	5.88	4.98
time (sec)	N/A	0.487	0.095	0.137	0.282	0.492	0.082	0.865	0.554

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	897	925	921	921	1046	1050	892
N.S.	1	1.00	4.48	4.62	4.60	4.60	5.23	5.25	4.46
time (sec)	N/A	0.398	0.074	0.154	0.287	0.523	0.074	0.625	0.357

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	785	817	807	807	935	924	781
N.S.	1	1.00	3.92	4.08	4.04	4.04	4.68	4.62	3.90
time (sec)	N/A	0.324	0.057	0.147	0.295	0.451	0.063	0.637	0.402

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	684	709	706	706	796	798	683
N.S.	1	1.00	3.95	4.10	4.08	4.08	4.60	4.61	3.95
time (sec)	N/A	0.310	0.052	0.135	0.266	0.643	0.057	0.641	0.256

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	574	601	594	594	673	670	570
N.S.	1	1.00	3.99	4.17	4.12	4.12	4.67	4.65	3.96
time (sec)	N/A	0.255	0.043	0.135	0.283	0.691	0.050	0.547	0.214

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	473	493	489	489	549	546	470
N.S.	1	1.00	3.97	4.14	4.11	4.11	4.61	4.59	3.95
time (sec)	N/A	0.194	0.033	0.136	0.279	0.613	0.044	0.543	0.313

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	360	385	376	376	427	420	356
N.S.	1	1.00	3.91	4.18	4.09	4.09	4.64	4.57	3.87
time (sec)	N/A	0.157	0.028	0.135	0.294	0.974	0.038	0.929	0.273

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	261	277	273	273	303	294	249
N.S.	1	1.00	4.02	4.26	4.20	4.20	4.66	4.52	3.83
time (sec)	N/A	0.110	0.020	0.132	0.267	0.990	0.031	0.725	0.107

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	151	169	163	163	178	169	143
N.S.	1	1.00	3.97	4.45	4.29	4.29	4.68	4.45	3.76
time (sec)	N/A	0.011	0.011	0.117	0.266	1.067	0.024	0.625	0.079

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	75	83	12	75
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.36
time (sec)	N/A	0.001	0.001	0.116	0.281	1.045	0.014	0.869	0.057

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	304	491	460	462	408	497	509
N.S.	1	1.00	1.80	2.91	2.72	2.73	2.41	2.94	3.01
time (sec)	N/A	0.050	0.100	0.143	0.280	1.095	0.482	0.751	0.217

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	388	479	467	632	428	567	841
N.S.	1	1.00	2.07	2.56	2.50	3.38	2.29	3.03	4.50
time (sec)	N/A	0.160	0.082	0.169	0.346	1.246	0.914	0.765	0.241

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	389	467	473	703	447	477	690
N.S.	1	1.00	2.10	2.52	2.56	3.80	2.42	2.58	3.73
time (sec)	N/A	0.153	0.089	0.146	0.292	0.977	1.790	0.591	0.266

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	199	459	484	739	474	470	559
N.S.	1	1.00	1.06	2.45	2.59	3.95	2.53	2.51	2.99
time (sec)	N/A	0.149	0.070	0.140	0.302	0.809	22.920	0.735	0.289

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	173	453	494	754	0	660	512
N.S.	1	1.00	0.93	2.42	2.64	4.03	0.00	3.53	2.74
time (sec)	N/A	0.135	0.073	0.145	0.349	0.900	0.000	0.609	0.772

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	389	451	504	732	0	463	508
N.S.	1	1.00	2.15	2.49	2.78	4.04	0.00	2.56	2.81
time (sec)	N/A	0.132	0.105	0.179	0.344	0.571	0.000	0.626	0.339

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	390	456	516	692	0	459	517
N.S.	1	1.00	2.10	2.45	2.77	3.72	0.00	2.47	2.78
time (sec)	N/A	0.119	0.140	0.147	0.325	0.489	0.000	0.799	0.369

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	308	462	534	624	0	466	461
N.S.	1	1.00	1.59	2.38	2.75	3.22	0.00	2.40	2.38
time (sec)	N/A	0.108	0.111	0.147	0.332	0.622	0.000	1.024	0.353

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	353	464	509	509	0	489	571
N.S.	1	1.00	12.61	16.57	18.18	18.18	0.00	17.46	20.39
time (sec)	N/A	0.003	0.081	0.141	0.352	0.480	0.000	1.396	0.169

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	367	464	548	548	0	496	39
N.S.	1	1.00	6.33	8.00	9.45	9.45	0.00	8.55	0.67
time (sec)	N/A	0.007	0.081	0.177	0.308	0.492	0.000	0.921	0.146

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	371	464	559	559	0	496	600
N.S.	1	1.00	4.17	5.21	6.28	6.28	0.00	5.57	6.74
time (sec)	N/A	0.015	0.083	0.143	0.300	0.592	0.000	0.924	0.446

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	369	464	570	570	0	496	548
N.S.	1	1.00	3.08	3.87	4.75	4.75	0.00	4.13	4.57
time (sec)	N/A	0.022	0.079	0.141	0.299	0.479	0.000	1.120	0.518

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	371	464	581	581	0	496	559
N.S.	1	1.00	2.46	3.07	3.85	3.85	0.00	3.28	3.70
time (sec)	N/A	0.033	0.084	0.145	0.308	0.452	0.000	1.097	0.229

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	369	463	592	592	0	496	570
N.S.	1	1.00	1.86	2.34	2.99	2.99	0.00	2.51	2.88
time (sec)	N/A	0.109	0.087	0.171	0.329	0.538	0.000	0.693	0.399

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	371	464	603	603	0	496	581
N.S.	1	1.00	1.86	2.32	3.02	3.02	0.00	2.48	2.90
time (sec)	N/A	0.096	0.088	0.161	0.342	0.489	0.000	0.821	1.238

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	371	464	614	614	0	496	592
N.S.	1	1.00	1.86	2.32	3.07	3.07	0.00	2.48	2.96
time (sec)	N/A	0.094	0.086	0.137	0.320	0.447	0.000	1.062	2.196

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	1817	1891	1877	1877	2088	2186	1847
N.S.	1	1.00	6.61	6.88	6.83	6.83	7.59	7.95	6.72
time (sec)	N/A	1.009	0.187	0.138	0.286	0.492	0.137	1.160	0.984

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	1702	1741	1740	1740	1965	2010	1702
N.S.	1	1.00	6.10	6.24	6.24	6.24	7.04	7.20	6.10
time (sec)	N/A	0.881	0.134	0.136	0.288	0.525	0.128	1.325	1.026

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	1539	1591	1581	1581	1775	1833	1549
N.S.	1	1.00	5.52	5.70	5.67	5.67	6.36	6.57	5.55
time (sec)	N/A	0.768	0.116	0.134	0.282	0.612	0.116	1.157	0.689

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	1397	1441	1437	1437	1598	1656	1404
N.S.	1	1.00	5.59	5.76	5.75	5.75	6.39	6.62	5.62
time (sec)	N/A	0.934	0.120	0.144	0.289	0.550	0.106	0.904	0.788

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	1241	1291	1283	1283	1428	1478	1253
N.S.	1	1.00	5.52	5.74	5.70	5.70	6.35	6.57	5.57
time (sec)	N/A	0.617	0.106	0.154	0.278	0.588	0.098	0.996	0.707

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	1105	1141	1135	1135	1280	1302	1106
N.S.	1	1.00	5.52	5.70	5.68	5.68	6.40	6.51	5.53
time (sec)	N/A	0.540	0.086	0.133	0.301	0.503	0.086	0.809	0.613

Problem 1305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	939	991	977	977	1088	1124	953
N.S.	1	1.00	5.52	5.83	5.75	5.75	6.40	6.61	5.61
time (sec)	N/A	0.467	0.078	0.131	0.288	0.460	0.077	0.691	0.532

Problem 1306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	811	841	835	835	940	948	806
N.S.	1	1.00	5.55	5.76	5.72	5.72	6.44	6.49	5.52
time (sec)	N/A	0.373	0.061	0.135	0.306	0.552	0.068	0.648	0.341

Problem 1307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	660	691	686	686	748	771	664
N.S.	1	1.00	5.55	5.81	5.76	5.76	6.29	6.48	5.58
time (sec)	N/A	0.312	0.050	0.130	0.302	0.600	0.058	0.553	0.426

Problem 1308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	511	541	535	535	586	594	495
N.S.	1	1.00	5.55	5.88	5.82	5.82	6.37	6.46	5.38
time (sec)	N/A	0.248	0.040	0.231	0.285	0.539	0.050	0.547	0.231

Problem 1309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	358	391	384	384	415	417	348
N.S.	1	1.00	5.51	6.02	5.91	5.91	6.38	6.42	5.35
time (sec)	N/A	0.178	0.029	0.141	0.279	0.487	0.041	0.683	0.320

Problem 1310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	220	241	240	240	248	241	208
N.S.	1	1.00	5.79	6.34	6.32	6.32	6.53	6.34	5.47
time (sec)	N/A	0.011	0.017	0.118	0.275	0.473	0.033	0.707	0.129

Problem 1311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	108	114	12	108
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.71
time (sec)	N/A	0.001	0.001	0.113	0.270	0.452	0.019	0.677	0.080

Problem 1312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	591	2929	866	868	799	961	979
N.S.	1	1.00	2.45	12.15	3.59	3.60	3.32	3.99	4.06
time (sec)	N/A	0.070	0.207	0.517	0.296	0.463	0.788	0.562	0.130

Problem 1313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	708	933	874	1124	816	1012	2500
N.S.	1	1.00	2.74	3.62	3.39	4.36	3.16	3.92	9.69
time (sec)	N/A	0.324	0.165	0.151	0.291	0.484	1.422	0.578	0.352

Problem 1314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	708	914	881	1233	843	924	2500
N.S.	1	1.00	2.70	3.49	3.36	4.71	3.22	3.53	9.54
time (sec)	N/A	0.308	0.161	0.158	0.294	0.577	6.266	0.613	0.378

Problem 1315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	427	896	891	1316	0	907	2219
N.S.	1	1.00	1.66	3.47	3.45	5.10	0.00	3.52	8.60
time (sec)	N/A	0.298	0.123	0.168	0.331	0.879	0.000	0.656	0.385

Problem 1316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	359	881	903	1365	0	1168	1494
N.S.	1	1.00	1.37	3.36	3.45	5.21	0.00	4.46	5.70
time (sec)	N/A	0.297	0.129	0.153	0.340	0.933	0.000	0.531	0.381

Problem 1317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	305	870	912	1395	0	883	1141
N.S.	1	1.00	1.17	3.35	3.51	5.37	0.00	3.40	4.39
time (sec)	N/A	0.287	0.138	0.147	0.373	1.493	0.000	0.602	0.396

Problem 1318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	265	862	925	1386	0	878	997
N.S.	1	1.00	1.01	3.29	3.53	5.29	0.00	3.35	3.81
time (sec)	N/A	0.267	0.142	0.150	0.404	1.095	0.000	0.545	0.422

Problem 1319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	239	856	934	1362	0	872	950
N.S.	1	1.00	0.93	3.32	3.62	5.28	0.00	3.38	3.68
time (sec)	N/A	0.250	0.168	0.142	0.406	0.860	0.000	0.830	0.428

Problem 1320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	712	854	945	1296	0	871	946
N.S.	1	1.00	2.76	3.31	3.66	5.02	0.00	3.38	3.67
time (sec)	N/A	0.234	0.206	0.168	0.424	0.700	0.000	1.025	0.263

Problem 1321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	708	859	957	1216	0	867	955
N.S.	1	1.00	2.75	3.34	3.72	4.73	0.00	3.37	3.72
time (sec)	N/A	0.215	0.274	0.171	0.395	0.934	0.000	1.157	0.502

Problem 1322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	591	865	975	1107	0	874	866
N.S.	1	1.00	2.18	3.19	3.60	4.08	0.00	3.23	3.20
time (sec)	N/A	0.190	0.262	0.244	0.336	0.979	0.000	0.901	0.555

Problem 1323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	665	866	920	920	0	951	1066
N.S.	1	1.00	23.75	30.93	32.86	32.86	0.00	33.96	38.07
time (sec)	N/A	0.003	0.187	0.137	0.341	1.189	0.000	0.772	0.458

Problem 1324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	684	867	986	986	0	961	39
N.S.	1	1.00	11.79	14.95	17.00	17.00	0.00	16.57	0.67
time (sec)	N/A	0.007	0.184	0.141	0.340	1.489	0.000	0.582	0.394

Problem 1325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	690	867	997	997	0	961	1098
N.S.	1	1.00	7.75	9.74	11.20	11.20	0.00	10.80	12.34
time (sec)	N/A	0.014	0.191	0.151	0.374	0.904	0.000	1.195	0.475

Problem 1326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	692	867	1008	1008	0	961	1109
N.S.	1	1.00	5.77	7.22	8.40	8.40	0.00	8.01	9.24
time (sec)	N/A	0.021	0.188	0.148	0.359	0.728	0.000	1.279	1.295

Problem 1327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	690	867	1019	1019	0	961	1120
N.S.	1	1.00	4.57	5.74	6.75	6.75	0.00	6.36	7.42
time (sec)	N/A	0.032	0.197	0.145	0.370	0.786	0.000	1.443	2.279

Problem 1328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	694	867	1030	1030	0	961	1131
N.S.	1	1.00	3.81	4.76	5.66	5.66	0.00	5.28	6.21
time (sec)	N/A	0.046	0.187	0.209	0.361	0.817	0.000	0.965	0.584

Problem 1329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	690	867	1041	1041	0	961	1142
N.S.	1	1.00	3.24	4.07	4.89	4.89	0.00	4.51	5.36
time (sec)	N/A	0.062	0.207	0.142	0.368	1.063	0.000	0.765	0.658

Problem 1330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	694	867	1052	1052	0	961	1153
N.S.	1	1.00	2.84	3.55	4.31	4.31	0.00	3.94	4.73
time (sec)	N/A	0.073	0.188	0.163	0.446	0.868	0.000	0.738	12.020

Problem 1331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	692	866	1063	1063	0	961	1164
N.S.	1	1.00	2.53	3.17	3.89	3.89	0.00	3.52	4.26
time (sec)	N/A	0.206	0.191	0.148	0.471	0.562	0.000	0.805	25.721

Problem 1332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	692	867	1074	1074	0	961	1175
N.S.	1	1.00	2.48	3.11	3.85	3.85	0.00	3.44	4.21
time (sec)	N/A	0.185	0.188	0.160	0.439	0.760	0.000	0.837	0.799

Problem 1333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	692	867	1085	1085	0	961	1186
N.S.	1	1.00	2.48	3.11	3.89	3.89	0.00	3.44	4.25
time (sec)	N/A	0.181	0.192	0.181	0.429	0.861	0.000	1.200	1.036

Problem 1334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	167	266	258	259	209	273	280
N.S.	1	1.00	1.37	2.18	2.11	2.12	1.71	2.24	2.30
time (sec)	N/A	0.039	0.046	0.141	0.409	1.253	0.279	1.345	0.074

Problem 1335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	115	189	177	179	136	184	189
N.S.	1	1.00	1.17	1.93	1.81	1.83	1.39	1.88	1.93
time (sec)	N/A	0.028	0.030	0.151	0.344	0.763	0.203	1.244	0.218

Problem 1336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	109	114	115	83	116	118
N.S.	1	1.00	1.00	1.47	1.54	1.55	1.12	1.57	1.59
time (sec)	N/A	0.020	0.021	0.137	0.310	0.643	0.152	0.747	0.065

Problem 1337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	56	60	62	44	60	62
N.S.	1	1.00	0.86	1.12	1.20	1.24	0.88	1.20	1.24
time (sec)	N/A	0.017	0.013	0.177	0.315	0.560	0.103	0.650	0.225

Problem 1338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	27	25
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.04	0.96
time (sec)	N/A	0.014	0.006	0.123	0.384	0.518	0.060	0.614	0.201

Problem 1339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.001	0.123	0.381	0.532	0.008	0.552	0.022

Problem 1340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	26	37	36	26	128	46	25
N.S.	1	1.00	0.72	1.03	1.00	0.72	3.56	1.28	0.69
time (sec)	N/A	0.006	0.010	0.156	0.384	0.503	0.164	0.550	0.258

Problem 1341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	92	93	233	78	46
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.37	0.81
time (sec)	N/A	0.024	0.019	0.164	0.318	0.646	0.397	0.574	0.142

Problem 1342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	202	242	381	165	182
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.01	2.22
time (sec)	N/A	0.033	0.048	0.172	0.306	0.942	0.587	0.528	0.161

Problem 1343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	228	259	264	373	231	339	327
N.S.	1	1.00	1.75	1.99	2.03	2.87	1.78	2.61	2.52
time (sec)	N/A	0.098	0.055	0.175	0.291	1.027	0.494	0.542	0.245

Problem 1344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	165	175	183	267	155	245	203
N.S.	1	1.00	1.59	1.68	1.76	2.57	1.49	2.36	1.95
time (sec)	N/A	0.072	0.038	0.144	0.290	1.272	0.378	0.635	0.073

Problem 1345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	114	108	117	172	102	166	123
N.S.	1	1.00	1.52	1.44	1.56	2.29	1.36	2.21	1.64
time (sec)	N/A	0.044	0.024	0.138	0.291	0.905	0.265	0.621	0.077

Problem 1346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	63	67	92	60	98	71
N.S.	1	1.00	0.92	1.24	1.31	1.80	1.18	1.92	1.39
time (sec)	N/A	0.026	0.025	0.138	0.405	0.711	0.174	0.603	0.236

Problem 1347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	33	34	37	27	57	32
N.S.	1	1.00	1.00	1.06	1.10	1.19	0.87	1.84	1.03
time (sec)	N/A	0.015	0.008	0.121	0.318	0.825	0.080	0.778	0.041

Problem 1348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00
time (sec)	N/A	0.001	0.002	0.126	0.306	1.335	0.043	1.219	0.187

Problem 1349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	58	90	92	233	77	47
N.S.	1	1.00	0.95	1.04	1.61	1.64	4.16	1.38	0.84
time (sec)	N/A	0.022	0.020	0.158	0.293	1.064	0.364	1.240	0.292

Problem 1350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	66	82	208	241	406	153	74
N.S.	1	1.00	0.81	1.01	2.57	2.98	5.01	1.89	0.91
time (sec)	N/A	0.035	0.049	0.161	0.293	0.965	0.602	1.115	0.334

Problem 1351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	98	109	386	494	634	216	330
N.S.	1	1.00	0.90	1.00	3.54	4.53	5.82	1.98	3.03
time (sec)	N/A	0.055	0.052	0.165	0.307	0.715	0.951	1.822	0.396

Problem 1352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	303	351	364	548	340	362	441
N.S.	1	1.00	1.92	2.22	2.30	3.47	2.15	2.29	2.79
time (sec)	N/A	0.141	0.074	0.140	0.329	0.677	1.231	1.411	0.274

Problem 1353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	230	254	271	416	258	264	291
N.S.	1	1.00	1.73	1.91	2.04	3.13	1.94	1.98	2.19
time (sec)	N/A	0.084	0.047	0.140	0.310	0.585	0.935	0.932	0.099

Problem 1354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	167	172	191	291	185	183	196
N.S.	1	1.00	1.62	1.67	1.85	2.83	1.80	1.78	1.90
time (sec)	N/A	0.061	0.037	0.142	0.289	0.821	0.667	0.613	0.099

Problem 1355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	114	114	125	188	128	112	130
N.S.	1	1.00	1.46	1.46	1.60	2.41	1.64	1.44	1.67
time (sec)	N/A	0.039	0.027	0.147	0.302	0.653	0.457	0.692	0.109

Problem 1356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	69	80	100	80	69	77
N.S.	1	1.00	0.81	1.17	1.36	1.69	1.36	1.17	1.31
time (sec)	N/A	0.054	0.016	0.134	0.285	0.794	0.236	0.703	0.228

Problem 1357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	35	38	38	39	24	39
N.S.	1	1.00	0.93	1.25	1.36	1.36	1.39	0.86	1.39
time (sec)	N/A	0.004	0.007	0.121	0.291	0.693	0.126	1.887	0.029

Problem 1358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	24	26	12	26
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86
time (sec)	N/A	0.001	0.002	0.121	0.319	0.583	0.070	1.636	0.024

Problem 1359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	67	81	202	242	381	165	183
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.01	2.23
time (sec)	N/A	0.064	0.037	0.168	0.304	0.485	0.566	1.476	0.299

Problem 1360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	108	386	495	632	217	329
N.S.	1	1.00	0.88	0.98	3.51	4.50	5.75	1.97	2.99
time (sec)	N/A	0.096	0.069	0.174	0.306	0.458	0.934	2.293	0.395

Problem 1361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	128	140	594	760	881	345	542
N.S.	1	1.00	0.90	0.98	4.15	5.31	6.16	2.41	3.79
time (sec)	N/A	0.141	0.075	0.178	0.338	0.465	1.397	1.615	0.526

Problem 1362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	584	705	786	1093	0	723	784
N.S.	1	1.00	2.52	3.04	3.39	4.71	0.00	3.12	3.38
time (sec)	N/A	0.253	0.183	0.145	0.420	0.545	0.000	1.127	0.257

Problem 1363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	474	576	649	852	0	581	649
N.S.	1	1.00	2.27	2.76	3.11	4.08	0.00	2.78	3.11
time (sec)	N/A	0.191	0.134	0.144	0.354	0.479	0.000	1.687	0.430

Problem 1364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	308	462	535	625	0	467	460
N.S.	1	1.00	1.59	2.38	2.76	3.22	0.00	2.41	2.37
time (sec)	N/A	0.148	0.107	0.139	0.333	0.754	0.000	1.662	0.383

Problem 1365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	271	357	398	398	0	369	378
N.S.	1	1.00	9.68	12.75	14.21	14.21	0.00	13.18	13.50
time (sec)	N/A	0.002	0.060	0.136	0.303	0.923	0.000	3.223	0.146

Problem 1366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	205	265	326	326	0	271	39
N.S.	1	1.00	3.53	4.57	5.62	5.62	0.00	4.67	0.67
time (sec)	N/A	0.007	0.039	0.156	0.305	0.945	0.000	1.847	0.277

Problem 1367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	144	186	247	247	267	184	237
N.S.	1	1.00	1.62	2.09	2.78	2.78	3.00	2.07	2.66
time (sec)	N/A	0.014	0.035	0.141	0.311	0.777	111.242	1.331	0.110

Problem 1368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	94	122	182	182	196	114	176
N.S.	1	1.00	1.02	1.33	1.98	1.98	2.13	1.24	1.91
time (sec)	N/A	0.042	0.022	0.139	0.283	0.799	1.945	0.851	0.099

Problem 1369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	55	71	131	131	139	61	129
N.S.	1	1.00	0.85	1.09	2.02	2.02	2.14	0.94	1.98
time (sec)	N/A	0.028	0.017	0.141	0.296	0.663	0.807	1.681	0.086

Problem 1370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	35	94	94	100	25	96
N.S.	1	1.00	0.71	0.92	2.47	2.47	2.63	0.66	2.53
time (sec)	N/A	0.016	0.007	0.125	0.292	0.529	0.513	1.709	0.226

Problem 1371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	79	85	12	81
N.S.	1	1.00	1.00	0.93	0.86	5.64	6.07	0.86	5.79
time (sec)	N/A	0.001	0.003	0.125	0.315	0.448	0.223	1.001	0.223

Problem 1372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	196	192	1418	1589	1776	703	1299
N.S.	1	1.00	0.97	0.95	7.02	7.87	8.79	3.48	6.43
time (sec)	N/A	0.120	0.064	0.198	0.439	0.601	8.336	1.833	0.873

Problem 1373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	213	223	1881	2264	2336	714	1738
N.S.	1	1.00	0.92	0.97	8.14	9.80	10.11	3.09	7.52
time (sec)	N/A	0.190	0.158	0.201	0.555	0.604	26.935	1.194	1.386

Problem 1374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	254	265	2399	3016	0	1029	2224
N.S.	1	1.00	0.92	0.96	8.69	10.93	0.00	3.73	8.06
time (sec)	N/A	0.249	0.137	0.240	0.674	0.682	0.000	1.285	1.911

Problem 1375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	217	121	259	338	314	641	137
N.S.	1	1.00	1.39	0.78	1.66	2.17	2.01	4.11	0.88
time (sec)	N/A	0.040	0.118	0.138	0.290	0.435	2.326	1.843	0.081

Problem 1376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	154	100	181	245	223	470	112
N.S.	1	1.00	1.19	0.78	1.40	1.90	1.73	3.64	0.87
time (sec)	N/A	0.036	0.085	0.163	0.290	0.427	1.872	1.207	0.225

Problem 1377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	78	118	164	146	322	87
N.S.	1	1.00	1.02	0.78	1.18	1.64	1.46	3.22	0.87
time (sec)	N/A	0.023	0.066	0.151	0.302	0.390	1.474	1.184	0.065

Problem 1378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	56	68	99	85	200	68
N.S.	1	1.00	0.86	0.79	0.96	1.39	1.20	2.82	0.96
time (sec)	N/A	0.017	0.039	0.137	0.293	0.644	1.119	1.302	0.240

Problem 1379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	34	33	46	36	100	29
N.S.	1	1.00	0.71	0.81	0.79	1.10	0.86	2.38	0.69
time (sec)	N/A	0.010	0.022	0.123	0.290	0.638	0.876	1.133	0.043

Problem 1380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75
time (sec)	N/A	0.001	0.007	0.119	0.281	0.688	0.007	1.538	0.021

Problem 1381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	61	0	143	61	62	50
N.S.	1	1.00	1.00	0.98	0.00	2.31	0.98	1.00	0.81
time (sec)	N/A	0.038	0.080	0.164	0.000	0.939	1.876	1.313	0.066

Problem 1382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	73	0	232	573	72	61
N.S.	1	1.00	0.99	1.04	0.00	3.31	8.19	1.03	0.87
time (sec)	N/A	0.022	0.186	0.157	0.000	0.728	21.736	1.091	0.242

Problem 1383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	99	106	0	456	1658	126	135
N.S.	1	1.00	0.90	0.96	0.00	4.15	15.07	1.15	1.23
time (sec)	N/A	0.056	0.430	0.144	0.000	0.511	93.909	1.154	0.302

Problem 1384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	131	152	0	785	0	207	207
N.S.	1	1.00	0.90	1.04	0.00	5.38	0.00	1.42	1.42
time (sec)	N/A	0.070	0.644	0.148	0.000	0.630	0.000	2.465	0.374

Problem 1385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	170	217	0	1176	0	311	297
N.S.	1	1.00	0.93	1.19	0.00	6.46	0.00	1.71	1.63
time (sec)	N/A	0.083	1.002	0.145	0.000	0.504	0.000	2.453	0.217

Problem 1386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	224	293	0	1673	0	432	401
N.S.	1	1.00	1.03	1.34	0.00	7.67	0.00	1.98	1.84
time (sec)	N/A	0.104	1.407	0.161	0.000	0.450	0.000	2.113	0.490

Problem 1387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	217	122	259	418	763	1084	137
N.S.	1	1.00	1.37	0.77	1.64	2.65	4.83	6.86	0.87
time (sec)	N/A	0.037	0.126	0.158	0.296	0.485	13.833	1.461	0.244

Problem 1388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	154	100	181	311	559	807	112
N.S.	1	1.00	1.19	0.78	1.40	2.41	4.33	6.26	0.87
time (sec)	N/A	0.031	0.091	0.155	0.295	0.444	10.524	1.095	0.242

Problem 1389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	78	118	216	386	566	87
N.S.	1	1.00	1.02	0.78	1.18	2.16	3.86	5.66	0.87
time (sec)	N/A	0.024	0.067	0.169	0.325	0.476	7.573	1.457	0.249

Problem 1390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	56	68	137	240	360	68
N.S.	1	1.00	0.86	0.79	0.96	1.93	3.38	5.07	0.96
time (sec)	N/A	0.017	0.045	0.145	0.297	0.469	6.442	0.835	0.059

Problem 1391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	34	33	69	146	192	29
N.S.	1	1.00	0.71	0.81	0.79	1.64	3.48	4.57	0.69
time (sec)	N/A	0.010	0.025	0.130	0.299	0.399	0.139	0.765	0.215

Problem 1392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	28	12	58	12
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	0.75
time (sec)	N/A	0.001	0.008	0.135	0.276	0.398	0.009	1.098	0.017

Problem 1393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	99	0	188	82	105	93
N.S.	1	1.00	0.90	1.15	0.00	2.19	0.95	1.22	1.08
time (sec)	N/A	0.033	0.157	0.172	0.000	0.462	7.120	1.199	0.075

Problem 1394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	100	0	210	923	113	109
N.S.	1	1.00	0.98	1.18	0.00	2.47	10.86	1.33	1.28
time (sec)	N/A	0.027	0.249	0.220	0.000	0.755	70.023	2.397	0.105

Problem 1395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	97	0	383	0	108	135
N.S.	1	1.00	0.90	0.97	0.00	3.83	0.00	1.08	1.35
time (sec)	N/A	0.032	0.356	0.158	0.000	0.612	0.000	1.945	0.284

Problem 1396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	126	0	666	0	185	209
N.S.	1	1.00	0.95	0.93	0.00	4.90	0.00	1.36	1.54
time (sec)	N/A	0.039	0.591	0.157	0.000	0.487	0.000	1.669	0.338

Problem 1397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	171	172	0	1043	0	285	296
N.S.	1	1.00	0.99	1.00	0.00	6.06	0.00	1.66	1.72
time (sec)	N/A	0.052	0.952	0.162	0.000	0.542	0.000	2.006	0.371

Problem 1398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	223	237	0	1492	0	410	398
N.S.	1	1.00	1.07	1.14	0.00	7.17	0.00	1.97	1.91
time (sec)	N/A	0.069	1.716	0.158	0.000	0.788	0.000	1.726	0.473

Problem 1399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	217	122	259	497	1292	1599	137
N.S.	1	1.00	1.37	0.77	1.64	3.15	8.18	10.12	0.87
time (sec)	N/A	0.036	0.132	0.150	0.291	0.738	22.827	1.809	0.267

Problem 1400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	154	100	181	377	960	1204	112
N.S.	1	1.00	1.19	0.78	1.40	2.92	7.44	9.33	0.87
time (sec)	N/A	0.028	0.092	0.149	0.315	0.886	16.757	1.790	0.234

Problem 1401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	102	78	118	268	549	857	87
N.S.	1	1.00	1.02	0.78	1.18	2.68	5.49	8.57	0.87
time (sec)	N/A	0.023	0.078	0.145	0.289	0.607	0.415	1.597	0.076

Problem 1402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	61	56	68	174	355	558	68
N.S.	1	1.00	0.86	0.79	0.96	2.45	5.00	7.86	0.96
time (sec)	N/A	0.016	0.051	0.146	0.299	0.625	0.337	0.810	0.067

Problem 1403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	30	34	33	93	194	306	29
N.S.	1	1.00	0.71	0.81	0.79	2.21	4.62	7.29	0.69
time (sec)	N/A	0.010	0.030	0.133	0.283	0.541	0.282	1.930	0.045

Problem 1404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	39	12	95	12
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	5.94	0.75
time (sec)	N/A	0.001	0.009	0.135	0.288	0.438	0.009	1.590	0.021

Problem 1405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	161	0	290	121	171	130
N.S.	1	1.00	0.96	1.44	0.00	2.59	1.08	1.53	1.16
time (sec)	N/A	0.041	0.128	0.174	0.000	0.460	12.999	1.693	0.083

Problem 1406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	116	152	0	330	1312	181	161
N.S.	1	1.00	1.05	1.38	0.00	3.00	11.93	1.65	1.46
time (sec)	N/A	0.037	0.320	0.188	0.000	0.419	124.932	1.952	0.124

Problem 1407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	138	0	344	0	171	199
N.S.	1	1.00	1.00	1.16	0.00	2.89	0.00	1.44	1.67
time (sec)	N/A	0.033	0.451	0.196	0.000	0.450	0.000	1.715	0.160

Problem 1408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	130	0	563	0	161	222
N.S.	1	1.00	0.94	1.03	0.00	4.47	0.00	1.28	1.76
time (sec)	N/A	0.033	0.568	0.165	0.000	0.464	0.000	1.563	0.365

Problem 1409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	172	159	0	894	0	259	309
N.S.	1	1.00	1.06	0.98	0.00	5.52	0.00	1.60	1.91
time (sec)	N/A	0.046	0.966	0.163	0.000	0.417	0.000	2.531	0.412

Problem 1410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	222	205	0	1337	0	380	411
N.S.	1	1.00	1.12	1.04	0.00	6.75	0.00	1.92	2.08
time (sec)	N/A	0.062	1.319	0.164	0.000	0.445	0.000	1.511	0.503

Problem 1411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	30	29	33	105	29	29
N.S.	1	1.00	1.00	0.86	0.83	0.94	3.00	0.83	0.83
time (sec)	N/A	0.006	0.040	0.168	0.502	0.448	0.883	1.636	0.062

Problem 1412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	40	43	46	168	37	45
N.S.	1	1.00	0.77	0.71	0.77	0.82	3.00	0.66	0.80
time (sec)	N/A	0.008	0.053	0.198	0.530	0.417	1.628	1.234	0.041

Problem 1413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	216	121	283	261	728	283	137
N.S.	1	1.00	1.40	0.79	1.84	1.69	4.73	1.84	0.89
time (sec)	N/A	0.035	0.110	0.148	0.292	0.669	38.650	1.641	0.069

Problem 1414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	153	99	204	182	532	204	112
N.S.	1	1.00	1.20	0.78	1.61	1.43	4.19	1.61	0.88
time (sec)	N/A	0.027	0.083	0.145	0.279	1.026	27.185	1.216	0.241

Problem 1415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	76	137	115	366	137	87
N.S.	1	1.00	1.05	0.79	1.43	1.20	3.81	1.43	0.91
time (sec)	N/A	0.021	0.057	0.147	0.282	1.249	17.600	2.255	0.264

Problem 1416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	55	82	64	231	82	68
N.S.	1	1.00	0.87	0.80	1.19	0.93	3.35	1.19	0.99
time (sec)	N/A	0.015	0.038	0.145	0.279	0.954	10.285	1.597	0.067

Problem 1417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	38	39	25	121	39	28
N.S.	1	1.00	0.72	0.95	0.98	0.62	3.02	0.98	0.70
time (sec)	N/A	0.009	0.024	0.131	0.289	0.681	2.266	1.425	0.048

Problem 1418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.001	0.007	0.128	0.284	0.933	0.006	1.632	0.021

Problem 1419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	37	0	119	44	38	38
N.S.	1	1.00	1.00	0.79	0.00	2.53	0.94	0.81	0.81
time (sec)	N/A	0.014	0.050	0.156	0.000	0.735	2.297	1.034	0.273

Problem 1420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	87	0	280	0	87	74
N.S.	1	1.00	0.99	1.14	0.00	3.68	0.00	1.14	0.97
time (sec)	N/A	0.020	0.189	0.160	0.000	0.976	0.000	1.458	0.094

Problem 1421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	96	138	0	549	0	148	142
N.S.	1	1.00	0.84	1.21	0.00	4.82	0.00	1.30	1.25
time (sec)	N/A	0.026	0.277	0.164	0.000	0.993	0.000	1.494	0.332

Problem 1422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	128	187	0	884	0	231	218
N.S.	1	1.00	0.87	1.27	0.00	6.01	0.00	1.57	1.48
time (sec)	N/A	0.037	0.356	0.161	0.000	1.048	0.000	1.787	0.395

Problem 1423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	166	236	0	1325	0	331	307
N.S.	1	1.00	0.92	1.31	0.00	7.36	0.00	1.84	1.71
time (sec)	N/A	0.047	0.569	0.156	0.000	0.766	0.000	2.253	0.455

Problem 1424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	214	324	267	271	243	350	192
N.S.	1	1.00	1.41	2.13	1.76	1.78	1.60	2.30	1.26
time (sec)	N/A	0.036	0.115	0.154	0.283	0.612	18.345	1.125	0.077

Problem 1425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	151	219	189	192	168	240	153
N.S.	1	1.00	1.23	1.78	1.54	1.56	1.37	1.95	1.24
time (sec)	N/A	0.028	0.088	0.155	0.284	0.824	13.061	1.465	0.056

Problem 1426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	136	125	124	109	152	114
N.S.	1	1.00	1.05	1.45	1.33	1.32	1.16	1.62	1.21
time (sec)	N/A	0.023	0.062	0.191	0.276	1.212	8.900	1.148	0.082

Problem 1427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	74	75	73	65	84	67
N.S.	1	1.00	0.88	1.10	1.12	1.09	0.97	1.25	1.00
time (sec)	N/A	0.015	0.044	0.174	0.284	1.179	5.631	1.500	0.264

Problem 1428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	33	37	35	60	34	25
N.S.	1	1.00	0.71	0.87	0.97	0.92	1.58	0.89	0.66
time (sec)	N/A	0.010	0.026	0.165	0.291	0.806	0.275	1.082	0.054

Problem 1429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	20	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86
time (sec)	N/A	0.001	0.010	0.137	0.285	1.090	0.010	1.258	0.024

Problem 1430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	68	0	214	60	69	57
N.S.	1	1.00	1.00	0.99	0.00	3.10	0.87	1.00	0.83
time (sec)	N/A	0.020	0.104	0.187	0.000	1.010	4.515	1.658	0.270

Problem 1431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	90	100	0	423	0	143	123
N.S.	1	1.00	0.91	1.01	0.00	4.27	0.00	1.44	1.24
time (sec)	N/A	0.027	0.304	0.187	0.000	0.744	0.000	1.199	0.186

Problem 1432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	126	122	0	782	0	234	205
N.S.	1	1.00	0.90	0.87	0.00	5.59	0.00	1.67	1.46
time (sec)	N/A	0.035	0.553	0.198	0.000	0.834	0.000	1.568	0.444

Problem 1433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	170	156	0	1204	0	326	294
N.S.	1	1.00	0.98	0.90	0.00	6.96	0.00	1.88	1.70
time (sec)	N/A	0.046	0.705	0.189	0.000	0.596	0.000	1.296	0.541

Problem 1434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	217	294	265	283	196	335	229
N.S.	1	1.00	1.43	1.93	1.74	1.86	1.29	2.20	1.51
time (sec)	N/A	0.036	0.120	0.169	0.288	0.470	24.127	0.880	0.083

Problem 1435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	153	198	187	203	136	229	175
N.S.	1	1.00	1.22	1.58	1.50	1.62	1.09	1.83	1.40
time (sec)	N/A	0.029	0.096	0.164	0.282	0.481	18.514	0.982	0.302

Problem 1436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	122	122	136	461	141	128
N.S.	1	1.00	1.05	1.27	1.27	1.42	4.80	1.47	1.33
time (sec)	N/A	0.023	0.071	0.166	0.284	0.417	0.449	1.583	0.088

Problem 1437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	66	72	85	265	72	68
N.S.	1	1.00	0.93	0.99	1.07	1.27	3.96	1.07	1.01
time (sec)	N/A	0.016	0.052	0.163	0.283	0.435	0.402	2.053	0.072

Problem 1438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	28	46	124	28	29
N.S.	1	1.00	0.72	0.85	0.70	1.15	3.10	0.70	0.72
time (sec)	N/A	0.010	0.027	0.135	0.281	0.452	0.391	2.400	0.246

Problem 1439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	31	14	12	12
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	0.75	0.75
time (sec)	N/A	0.001	0.008	0.135	0.289	0.424	0.011	1.666	0.026

Problem 1440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	85	90	0	398	83	113	100
N.S.	1	1.00	0.91	0.97	0.00	4.28	0.89	1.22	1.08
time (sec)	N/A	0.027	0.186	0.168	0.000	0.427	5.975	2.666	0.333

Problem 1441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	125	121	0	782	0	216	161
N.S.	1	1.00	1.01	0.98	0.00	6.31	0.00	1.74	1.30
time (sec)	N/A	0.035	0.336	0.168	0.000	0.414	0.000	1.344	0.377

Problem 1442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	168	143	0	1226	0	298	243
N.S.	1	1.00	1.01	0.86	0.00	7.34	0.00	1.78	1.46
time (sec)	N/A	0.044	0.698	0.168	0.000	0.408	0.000	1.507	0.285

Problem 1443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	220	177	0	1840	0	432	334
N.S.	1	1.00	1.10	0.88	0.00	9.20	0.00	2.16	1.67
time (sec)	N/A	0.094	0.986	0.174	0.000	0.472	0.000	1.396	0.641

Problem 1444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	95	66	637	17
N.S.	1	1.00	1.14	0.86	0.82	4.32	3.00	28.95	0.77
time (sec)	N/A	0.004	0.018	0.155	0.293	0.379	0.577	1.152	0.050

Problem 1445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	75	66	495	17
N.S.	1	1.00	1.14	0.86	0.82	3.41	3.00	22.50	0.77
time (sec)	N/A	0.003	0.013	0.174	0.282	0.414	0.483	1.184	0.034

Problem 1446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	374	67	88	374	17
N.S.	1	1.00	1.14	0.86	17.00	3.05	4.00	17.00	0.77
time (sec)	N/A	0.003	0.009	0.158	0.288	0.738	0.476	0.667	0.028

Problem 1447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	56	88	266	17
N.S.	1	1.00	1.14	0.86	0.82	2.55	4.00	12.09	0.77
time (sec)	N/A	0.003	0.014	0.160	0.281	0.921	0.489	0.565	0.028

Problem 1448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	45	88	178	17
N.S.	1	1.00	1.14	0.86	0.82	2.05	4.00	8.09	0.77
time (sec)	N/A	0.003	0.010	0.151	0.278	1.295	0.633	0.836	0.028

Problem 1449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	19	18	34	80	106	17
N.S.	1	1.00	1.14	0.86	0.82	1.55	3.64	4.82	0.77
time (sec)	N/A	0.003	0.009	0.178	0.287	1.071	0.677	0.923	0.028

Problem 1450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	26	19	18	23	53	54	17
N.S.	1	1.00	1.18	0.86	0.82	1.05	2.41	2.45	0.77
time (sec)	N/A	0.003	0.009	0.155	0.294	0.898	1.032	0.985	0.029

Problem 1451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	24	19	18	18	29	18	17
N.S.	1	1.00	1.20	0.95	0.90	0.90	1.45	0.90	0.85
time (sec)	N/A	0.003	0.005	0.168	0.278	0.821	1.507	1.177	0.030

Problem 1452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	24	19	18	29	41	18	17
N.S.	1	1.00	1.20	0.95	0.90	1.45	2.05	0.90	0.85
time (sec)	N/A	0.003	0.007	0.167	0.285	0.865	2.313	1.098	0.030

Problem 1453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	21	21	26	22	10
N.S.	1	1.00	1.00	1.57	1.50	1.50	1.86	1.57	0.71
time (sec)	N/A	0.003	0.014	0.164	0.278	0.815	0.294	0.915	0.054

Problem 1454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	19	18	18	61	18	15
N.S.	1	1.00	1.00	0.76	0.72	0.72	2.44	0.72	0.60
time (sec)	N/A	0.006	0.027	0.156	0.526	0.729	0.507	0.943	0.060

Problem 1455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	112	84	86	86	170	87	104
N.S.	1	1.00	1.33	1.00	1.02	1.02	2.02	1.04	1.24
time (sec)	N/A	0.030	0.102	3.086	0.528	0.837	1.235	0.984	0.068

Problem 1456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	20	19	19	112	26	14
N.S.	1	1.00	0.67	0.74	0.70	0.70	4.15	0.96	0.52
time (sec)	N/A	0.004	0.015	0.151	0.274	0.778	0.524	1.229	0.255

Problem 1457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	29	28	24	144	38	21
N.S.	1	1.00	0.61	0.76	0.74	0.63	3.79	1.00	0.55
time (sec)	N/A	0.005	0.016	0.173	0.269	0.712	0.692	1.844	0.046

Problem 1458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	154	161	0	570	0	196	204
N.S.	1	1.00	1.11	1.16	0.00	4.10	0.00	1.41	1.47
time (sec)	N/A	0.082	0.197	0.173	0.000	0.841	0.000	1.525	0.211

Problem 1459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	154	160	0	900	0	207	206
N.S.	1	1.00	1.10	1.14	0.00	6.43	0.00	1.48	1.47
time (sec)	N/A	0.054	0.173	0.166	0.000	0.799	0.000	0.757	0.369

Problem 1460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	166	239	0	702	0	1107	-1
N.S.	1	1.00	0.72	1.04	0.00	3.05	0.00	4.81	-0.00
time (sec)	N/A	0.114	0.448	0.165	0.000	0.702	0.000	0.853	0.000

Problem 1461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	144	206	0	540	0	726	-1
N.S.	1	1.00	0.75	1.07	0.00	2.81	0.00	3.78	-0.01
time (sec)	N/A	0.067	0.365	0.185	0.000	0.575	0.000	0.974	0.000

Problem 1462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	128	173	0	410	0	438	-1
N.S.	1	1.00	0.83	1.12	0.00	2.66	0.00	2.84	-0.01
time (sec)	N/A	0.051	0.265	0.153	0.000	0.927	0.000	1.504	0.000

Problem 1463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	95	140	0	300	0	232	88
N.S.	1	1.00	0.82	1.21	0.00	2.59	0.00	2.00	0.76
time (sec)	N/A	0.038	0.081	0.155	0.000	0.698	0.000	0.883	0.136

Problem 1464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	74	107	0	236	0	93	260
N.S.	1	1.00	1.03	1.49	0.00	3.28	0.00	1.29	3.61
time (sec)	N/A	0.025	0.209	0.163	0.000	0.669	0.000	0.991	4.005

Problem 1465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	0	241	0	131	-1
N.S.	1	1.00	1.00	0.00	0.00	3.65	0.00	1.98	-0.02
time (sec)	N/A	0.023	0.085	0.059	0.000	0.697	0.000	0.812	0.000

Problem 1466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	88	0	65	0	152	27
N.S.	1	1.00	1.00	2.75	0.00	2.03	0.00	4.75	0.84
time (sec)	N/A	0.002	0.034	0.156	0.000	0.740	0.000	1.093	0.717

Problem 1467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	128	0	175	0	447	127
N.S.	1	1.00	0.70	1.94	0.00	2.65	0.00	6.77	1.92
time (sec)	N/A	0.006	0.080	0.156	0.000	0.897	0.000	0.793	0.822

Problem 1468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	168	0	337	0	689	203
N.S.	1	1.00	0.76	1.66	0.00	3.34	0.00	6.82	2.01
time (sec)	N/A	0.013	0.094	0.157	0.000	1.510	0.000	2.006	0.966

Problem 1469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	208	0	532	0	989	292
N.S.	1	1.00	0.68	1.53	0.00	3.91	0.00	7.27	2.15
time (sec)	N/A	0.021	0.139	0.158	0.000	4.564	0.000	1.931	1.184

Problem 1470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	117	248	0	781	0	1345	397
N.S.	1	1.00	0.68	1.45	0.00	4.57	0.00	7.87	2.32
time (sec)	N/A	0.030	0.154	0.156	0.000	7.144	0.000	1.539	1.432

Problem 1471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	166	239	0	702	0	1740	-1
N.S.	1	1.00	0.73	1.05	0.00	3.09	0.00	7.67	-0.00
time (sec)	N/A	0.085	0.517	0.157	0.000	1.240	0.000	2.553	0.000

Problem 1472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	144	206	0	534	0	1071	-1
N.S.	1	1.00	0.76	1.09	0.00	2.83	0.00	5.67	-0.01
time (sec)	N/A	0.063	0.403	0.160	0.000	1.021	0.000	1.489	0.000

Problem 1473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	173	0	410	0	576	-1
N.S.	1	1.00	0.84	1.15	0.00	2.72	0.00	3.81	-0.01
time (sec)	N/A	0.048	0.254	0.158	0.000	0.928	0.000	1.194	0.000

Problem 1474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	140	0	306	0	233	-1
N.S.	1	1.00	0.83	1.24	0.00	2.71	0.00	2.06	-0.01
time (sec)	N/A	0.035	0.175	0.156	0.000	0.945	0.000	1.042	0.000

Problem 1475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	89	0	0	311	0	204	-1
N.S.	1	1.00	0.91	0.00	0.00	3.17	0.00	2.08	-0.01
time (sec)	N/A	0.032	0.435	0.077	0.000	1.291	0.000	1.365	0.000

Problem 1476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	0	0	325	0	455	-1
N.S.	1	1.00	0.88	0.00	0.00	3.53	0.00	4.95	-0.01
time (sec)	N/A	0.029	0.162	0.066	0.000	1.665	0.000	1.368	0.000

Problem 1477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	161	0	104	0	374	27
N.S.	1	1.00	1.00	5.03	0.00	3.25	0.00	11.69	0.84
time (sec)	N/A	0.002	0.046	0.180	0.000	2.153	0.000	1.069	0.796

Problem 1478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	201	0	235	0	1024	178
N.S.	1	1.00	0.70	3.05	0.00	3.56	0.00	15.52	2.70
time (sec)	N/A	0.006	0.099	0.165	0.000	2.070	0.000	0.866	0.926

Problem 1479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	241	0	426	0	1394	268
N.S.	1	1.00	0.72	2.39	0.00	4.22	0.00	13.80	2.65
time (sec)	N/A	0.013	0.126	0.166	0.000	4.070	0.000	0.807	1.113

Problem 1480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	281	0	649	0	1823	376
N.S.	1	1.00	0.70	2.07	0.00	4.77	0.00	13.40	2.76
time (sec)	N/A	0.020	0.155	0.166	0.000	8.285	0.000	1.804	1.333

Problem 1481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	272	0	882	0	3120	-1
N.S.	1	1.00	0.73	1.04	0.00	3.37	0.00	11.91	-0.00
time (sec)	N/A	0.113	0.666	0.165	0.000	1.074	0.000	1.810	0.000

Problem 1482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	169	239	0	702	0	1962	-1
N.S.	1	1.00	0.75	1.07	0.00	3.13	0.00	8.76	-0.00
time (sec)	N/A	0.084	0.528	0.167	0.000	1.020	0.000	1.751	0.000

Problem 1483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	144	206	0	540	0	1083	-1
N.S.	1	1.00	0.77	1.11	0.00	2.90	0.00	5.82	-0.01
time (sec)	N/A	0.065	0.354	0.164	0.000	1.165	0.000	1.217	0.000

Problem 1484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	124	173	0	412	0	446	-1
N.S.	1	1.00	0.84	1.17	0.00	2.78	0.00	3.01	-0.01
time (sec)	N/A	0.048	0.239	0.165	0.000	0.938	0.000	1.376	0.000

Problem 1485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	0	0	439	0	287	-1
N.S.	1	1.00	0.90	0.00	0.00	3.18	0.00	2.08	-0.01
time (sec)	N/A	0.046	0.261	0.057	0.000	1.014	0.000	1.490	0.000

Problem 1486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	121	0	0	475	0	650	-1
N.S.	1	1.00	0.95	0.00	0.00	3.71	0.00	5.08	-0.01
time (sec)	N/A	0.042	0.730	0.056	0.000	1.417	0.000	1.355	0.000

Problem 1487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	463	0	1025	-1
N.S.	1	1.00	0.92	0.00	0.00	3.86	0.00	8.54	-0.01
time (sec)	N/A	0.038	0.136	0.057	0.000	1.926	0.000	1.954	0.000

Problem 1488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	234	0	138	0	706	27
N.S.	1	1.00	1.00	7.31	0.00	4.31	0.00	22.06	0.84
time (sec)	N/A	0.002	0.049	0.158	0.000	1.466	0.000	1.184	0.971

Problem 1489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	274	0	295	0	1826	229
N.S.	1	1.00	0.70	4.15	0.00	4.47	0.00	27.67	3.47
time (sec)	N/A	0.007	0.106	0.158	0.000	5.919	0.000	1.716	1.140

Problem 1490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	314	0	513	0	2316	333
N.S.	1	1.00	0.72	3.11	0.00	5.08	0.00	22.93	3.30
time (sec)	N/A	0.013	0.131	0.159	0.000	10.558	0.000	1.907	1.355

Problem 1491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	354	0	765	0	2868	459
N.S.	1	1.00	0.70	2.60	0.00	5.62	0.00	21.09	3.38
time (sec)	N/A	0.021	0.158	0.166	0.000	16.070	0.000	2.171	1.615

Problem 1492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	164	206	0	542	0	268	-1
N.S.	1	1.00	0.90	1.13	0.00	2.96	0.00	1.46	-0.01
time (sec)	N/A	0.069	0.211	0.158	0.000	1.067	0.000	1.222	0.000

Problem 1493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	125	173	0	412	0	198	-1
N.S.	1	1.00	0.84	1.17	0.00	2.78	0.00	1.34	-0.01
time (sec)	N/A	0.051	0.200	0.155	0.000	1.244	0.000	1.754	0.000

Problem 1494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	140	0	306	0	139	-1
N.S.	1	1.00	0.83	1.24	0.00	2.71	0.00	1.23	-0.01
time (sec)	N/A	0.037	0.180	0.152	0.000	0.820	0.000	1.325	0.000

Problem 1495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	74	107	0	235	0	97	261
N.S.	1	1.00	1.01	1.47	0.00	3.22	0.00	1.33	3.58
time (sec)	N/A	0.025	0.206	0.156	0.000	1.006	0.000	1.323	3.803

Problem 1496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	76	0	178	0	50	45
N.S.	1	1.00	1.00	1.81	0.00	4.24	0.00	1.19	1.07
time (sec)	N/A	0.018	0.062	0.157	0.000	0.764	0.000	1.373	0.288

Problem 1497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	66	26
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	2.20	0.87
time (sec)	N/A	0.002	0.029	0.161	0.000	0.766	0.000	1.279	0.732

Problem 1498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	55	0	118	0	121	71
N.S.	1	1.00	0.70	0.83	0.00	1.79	0.00	1.83	1.08
time (sec)	N/A	0.006	0.080	0.164	0.000	1.112	0.000	1.259	0.894

Problem 1499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	95	0	251	0	227	133
N.S.	1	1.00	0.74	0.94	0.00	2.49	0.00	2.25	1.32
time (sec)	N/A	0.012	0.097	0.162	0.000	0.863	0.000	1.049	1.005

Problem 1500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	135	0	419	0	386	209
N.S.	1	1.00	0.68	0.99	0.00	3.08	0.00	2.84	1.54
time (sec)	N/A	0.020	0.125	0.197	0.000	1.561	0.000	1.221	1.192

Problem 1501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	115	175	0	638	0	596	303
N.S.	1	1.00	0.67	1.02	0.00	3.73	0.00	3.49	1.77
time (sec)	N/A	0.030	0.135	0.155	0.000	3.684	0.000	2.203	1.370

Problem 1502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	166	0	0	603	0	279	-1
N.S.	1	1.00	0.95	0.00	0.00	3.47	0.00	1.60	-0.01
time (sec)	N/A	0.066	0.306	0.057	0.000	1.072	0.000	1.264	0.000

Problem 1503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	0	0	441	0	201	-1
N.S.	1	1.00	0.90	0.00	0.00	3.20	0.00	1.46	-0.01
time (sec)	N/A	0.047	0.252	0.058	0.000	0.668	0.000	1.643	0.000

Problem 1504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	0	0	311	0	137	-1
N.S.	1	1.00	0.89	0.00	0.00	3.17	0.00	1.40	-0.01
time (sec)	N/A	0.032	0.395	0.062	0.000	1.308	0.000	1.017	0.000

Problem 1505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	0	0	241	0	96	-1
N.S.	1	1.00	1.00	0.00	0.00	3.65	0.00	1.45	-0.02
time (sec)	N/A	0.023	0.084	0.056	0.000	0.938	0.000	1.038	0.000

Problem 1506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	47	26
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	1.57	0.87
time (sec)	N/A	0.002	0.028	0.160	0.000	1.259	0.000	1.324	0.741

Problem 1507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	42	65	0	125	0	142	71
N.S.	1	1.00	0.68	1.05	0.00	2.02	0.00	2.29	1.15
time (sec)	N/A	0.007	0.088	0.162	0.000	1.192	0.000	1.846	0.858

Problem 1508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	71	105	0	273	0	368	141
N.S.	1	1.00	0.70	1.04	0.00	2.70	0.00	3.64	1.40
time (sec)	N/A	0.012	0.107	0.162	0.000	0.946	0.000	1.967	1.064

Problem 1509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	145	0	455	0	830	227
N.S.	1	1.00	0.68	1.07	0.00	3.35	0.00	6.10	1.67
time (sec)	N/A	0.022	0.122	0.160	0.000	1.593	0.000	2.797	1.312

Problem 1510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	117	185	0	689	0	1518	337
N.S.	1	1.00	0.68	1.08	0.00	4.03	0.00	8.88	1.97
time (sec)	N/A	0.029	0.151	0.158	0.000	2.725	0.000	2.386	1.500

Problem 1511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	139	225	0	955	0	2438	454
N.S.	1	1.00	0.67	1.09	0.00	4.64	0.00	11.83	2.20
time (sec)	N/A	0.041	0.138	0.161	0.000	5.335	0.000	3.081	1.959

Problem 1512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	165	0	0	879	0	500	-1
N.S.	1	1.00	0.81	0.00	0.00	4.31	0.00	2.45	-0.00
time (sec)	N/A	0.077	0.394	0.057	0.000	1.457	0.000	0.959	0.000

Problem 1513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	166	0	0	657	0	380	-1
N.S.	1	1.00	0.98	0.00	0.00	3.86	0.00	2.24	-0.01
time (sec)	N/A	0.059	0.270	0.055	0.000	1.539	0.000	0.800	0.000

Problem 1514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	123	0	0	475	0	276	-1
N.S.	1	1.00	0.96	0.00	0.00	3.71	0.00	2.16	-0.01
time (sec)	N/A	0.040	0.775	0.054	0.000	0.844	0.000	1.308	0.000

Problem 1515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	0	0	325	0	181	-1
N.S.	1	1.00	0.88	0.00	0.00	3.53	0.00	1.97	-0.01
time (sec)	N/A	0.029	0.167	0.056	0.000	1.226	0.000	1.464	0.000

Problem 1516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	88	0	65	0	51	130
N.S.	1	1.00	1.00	2.75	0.00	2.03	0.00	1.59	4.06
time (sec)	N/A	0.002	0.034	0.155	0.000	1.343	0.000	1.455	0.561

Problem 1517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	55	0	118	0	126	127
N.S.	1	1.00	0.70	0.83	0.00	1.79	0.00	1.91	1.92
time (sec)	N/A	0.006	0.076	0.158	0.000	1.345	0.000	1.248	0.897

Problem 1518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	95	0	273	0	373	132
N.S.	1	1.00	0.74	0.97	0.00	2.79	0.00	3.81	1.35
time (sec)	N/A	0.014	0.118	0.162	0.000	1.191	0.000	1.121	1.030

Problem 1519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	92	135	0	447	0	670	224
N.S.	1	1.00	0.68	1.00	0.00	3.31	0.00	4.96	1.66
time (sec)	N/A	0.020	0.143	0.155	0.000	1.655	0.000	1.879	1.289

Problem 1520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	117	175	0	715	0	1203	346
N.S.	1	1.00	0.68	1.02	0.00	4.16	0.00	6.99	2.01
time (sec)	N/A	0.031	0.157	0.160	0.000	3.270	0.000	2.910	1.531

Problem 1521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	139	215	0	999	0	1964	478
N.S.	1	1.00	0.67	1.04	0.00	4.83	0.00	9.49	2.31
time (sec)	N/A	0.044	0.152	0.178	0.000	6.932	0.000	2.880	1.908

Problem 1522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	26	86	48	31	0	24	50
N.S.	1	1.00	1.37	4.53	2.53	1.63	0.00	1.26	2.63
time (sec)	N/A	0.005	0.040	0.170	0.289	0.751	0.000	2.398	0.308

Problem 1523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	66	33	27	0	23	47
N.S.	1	1.00	1.32	3.47	1.74	1.42	0.00	1.21	2.47
time (sec)	N/A	0.004	0.041	0.169	0.285	0.787	0.000	1.235	0.339

Problem 1524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	66	33	27	0	23	43
N.S.	1	1.00	1.32	3.47	1.74	1.42	0.00	1.21	2.26
time (sec)	N/A	0.004	0.040	0.158	0.296	0.846	0.000	0.920	0.326

Problem 1525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	42	60	32	25	15	21	33
N.S.	1	1.00	2.47	3.53	1.88	1.47	0.88	1.24	1.94
time (sec)	N/A	0.003	0.038	0.142	0.293	0.797	0.573	2.598	0.309

Problem 1526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	64	33	27	0	23	44
N.S.	1	1.00	1.32	3.37	1.74	1.42	0.00	1.21	2.32
time (sec)	N/A	0.004	0.041	0.157	0.282	0.738	0.000	1.252	0.318

Problem 1527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	23	50
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	4.55
time (sec)	N/A	0.002	0.039	0.159	0.274	0.836	13.711	1.517	0.304

Problem 1528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	66	33	27	0	23	46
N.S.	1	1.00	1.32	3.47	1.74	1.42	0.00	1.21	2.42
time (sec)	N/A	0.004	0.039	0.161	0.292	0.634	0.000	1.194	0.292

Problem 1529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	25	66	33	28	0	23	47
N.S.	1	1.00	1.67	4.40	2.20	1.87	0.00	1.53	3.13
time (sec)	N/A	0.003	0.039	0.158	0.287	1.107	0.000	1.177	0.293

Problem 1530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.001	0.002	0.140	0.278	0.903	0.011	1.150	0.257

Problem 1531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	25	66	33	28	0	23	43
N.S.	1	1.00	1.67	4.40	2.20	1.87	0.00	1.53	2.87
time (sec)	N/A	0.003	0.039	0.162	0.309	0.890	0.000	1.533	0.286

Problem 1532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	42	58	32	25	20	21	37
N.S.	1	1.00	2.21	3.05	1.68	1.32	1.05	1.11	1.95
time (sec)	N/A	0.004	0.005	0.143	0.285	0.627	0.594	1.179	0.280

Problem 1533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	65	30	28	0	23	44
N.S.	1	1.00	1.19	3.10	1.43	1.33	0.00	1.10	2.10
time (sec)	N/A	0.005	0.038	0.153	0.271	0.628	0.000	0.875	0.286

Problem 1534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	23	50
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	4.55
time (sec)	N/A	0.002	0.003	0.154	0.286	0.697	13.740	1.123	0.002

Problem 1535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	66	33	28	0	23	50
N.S.	1	1.00	1.19	3.14	1.57	1.33	0.00	1.10	2.38
time (sec)	N/A	0.004	0.039	0.155	0.283	0.831	0.000	1.462	0.283

Problem 1536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	65	21	44	0	18	44
N.S.	1	1.00	1.62	4.06	1.31	2.75	0.00	1.12	2.75
time (sec)	N/A	0.010	0.047	0.161	0.494	1.002	0.000	1.264	0.084

Problem 1537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	39	56	9	31	76	15	44
N.S.	1	1.00	3.55	5.09	0.82	2.82	6.91	1.36	4.00
time (sec)	N/A	0.003	0.037	0.161	0.497	0.858	13.878	1.166	0.078

Problem 1538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	26	66	19	43	0	18	40
N.S.	1	1.00	1.62	4.12	1.19	2.69	0.00	1.12	2.50
time (sec)	N/A	0.010	0.052	0.169	0.495	0.650	0.000	1.094	0.320

Problem 1539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	57	58	18	26	24	17	34
N.S.	1	1.00	5.70	5.80	1.80	2.60	2.40	1.70	3.40
time (sec)	N/A	0.008	0.014	0.148	0.498	0.757	0.566	1.151	0.293

Problem 1540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	59	66	21	44	0	13	41
N.S.	1	1.00	5.36	6.00	1.91	4.00	0.00	1.18	3.73
time (sec)	N/A	0.007	0.017	0.166	0.492	0.964	0.000	1.441	0.303

Problem 1541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	C	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	26	16	0	87	16	47
N.S.	1	1.00	0.97	0.90	0.55	0.00	3.00	0.55	1.62
time (sec)	N/A	0.003	0.007	0.241	0.280	0.000	0.921	0.966	0.069

Problem 1542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	66	21	44	0	23	47
N.S.	1	1.00	1.00	2.54	0.81	1.69	0.00	0.88	1.81
time (sec)	N/A	0.011	0.037	0.160	0.497	0.987	0.000	2.161	0.297

Problem 1543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	59	70	33	30	0	25	49
N.S.	1	1.00	3.69	4.38	2.06	1.88	0.00	1.56	3.06
time (sec)	N/A	0.004	0.044	0.159	0.327	1.012	0.000	0.901	0.312

Problem 1544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	11	11	8	12	11
N.S.	1	1.00	1.00	1.08	0.92	0.92	0.67	1.00	0.92
time (sec)	N/A	0.001	0.001	0.148	0.285	1.391	0.011	0.653	0.034

Problem 1545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	59	70	33	30	0	25	45
N.S.	1	1.00	3.69	4.38	2.06	1.88	0.00	1.56	2.81
time (sec)	N/A	0.004	0.043	0.162	0.275	1.490	0.000	1.066	0.311

Problem 1546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	48	64	32	27	51	23	39
N.S.	1	1.00	2.40	3.20	1.60	1.35	2.55	1.15	1.95
time (sec)	N/A	0.004	0.006	0.142	0.299	0.974	0.600	1.516	0.282

Problem 1547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	27	70	33	30	0	25	46
N.S.	1	1.00	1.23	3.18	1.50	1.36	0.00	1.14	2.09
time (sec)	N/A	0.006	0.053	0.158	0.284	1.180	0.000	1.551	0.285

Problem 1548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	27	61	26	28	78	25	52
N.S.	1	1.00	2.25	5.08	2.17	2.33	6.50	2.08	4.33
time (sec)	N/A	0.002	0.041	0.163	0.268	1.305	14.382	1.411	0.294

Problem 1549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	27	69	30	30	0	25	52
N.S.	1	1.00	1.23	3.14	1.36	1.36	0.00	1.14	2.36
time (sec)	N/A	0.008	0.045	0.171	0.272	1.372	0.000	1.466	0.292

Problem 1550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	57	26	26	75	23	40
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	3.64
time (sec)	N/A	0.003	0.040	0.168	0.284	0.889	13.439	0.967	0.324

Problem 1551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	100	0	175	0	66	66
N.S.	1	1.00	1.19	2.33	0.00	4.07	0.00	1.53	1.53
time (sec)	N/A	0.021	0.058	0.189	0.000	1.178	0.000	0.986	0.495

Problem 1552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	48	41	26	42	23	30
N.S.	1	1.00	1.36	2.18	1.86	1.18	1.91	1.05	1.36
time (sec)	N/A	0.006	0.035	0.162	0.492	1.355	0.474	1.577	0.439

Problem 1553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	57	28	46	56	30	43
N.S.	1	1.00	1.00	2.19	1.08	1.77	2.15	1.15	1.65
time (sec)	N/A	0.011	0.063	0.172	0.499	1.244	0.496	1.276	0.122

Problem 1554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	58	109	0	176	0	69	63
N.S.	1	1.00	1.38	2.60	0.00	4.19	0.00	1.64	1.50
time (sec)	N/A	0.024	0.073	0.183	0.000	1.361	0.000	1.754	0.515

Problem 1555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	38	27	14	14	24	8	16
N.S.	1	1.00	3.80	2.70	1.40	1.40	2.40	0.80	1.60
time (sec)	N/A	0.010	0.032	0.151	0.507	1.157	0.454	1.389	0.290

Problem 1556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	38	31	21	21	42	13	27
N.S.	1	1.00	1.90	1.55	1.05	1.05	2.10	0.65	1.35
time (sec)	N/A	0.007	0.056	0.151	0.491	1.594	0.448	0.999	0.295

Problem 1557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	31	39	11	44	56	21	40
N.S.	1	1.00	1.19	1.50	0.42	1.69	2.15	0.81	1.54
time (sec)	N/A	0.015	0.048	0.167	0.504	1.228	0.489	0.851	0.080

Problem 1558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	84	0	185	0	54	44
N.S.	1	1.00	1.00	1.95	0.00	4.30	0.00	1.26	1.02
time (sec)	N/A	0.042	0.072	0.167	0.000	1.102	0.000	2.093	0.343

Problem 1559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	73	0	0	0	0	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	10.063	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	73	0	0	0	0	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	10.023	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	71	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	10.025	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	71	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	10.025	0.057	0.000	0.000	0.000	0.000	0.000

Problem 1563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	10.033	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	73	0	0	0	0	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	10.031	0.057	0.000	0.000	0.000	0.000	0.000

Problem 1565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.688	10.029	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	804	804	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	10.022	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	762	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	10.024	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	796	796	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	10.031	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	842	842	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	10.041	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	10.040	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	73	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	10.024	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	71	0	0	0	0	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	10.035	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	71	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	10.023	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	73	0	0	0	0	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	10.037	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	220	0	0	717	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	3.27	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.600	0.004	0.000	1.005	0.000	0.000	0.000

Problem 1576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	278	0	0	596	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	3.47	0.00	0.00	-0.01
time (sec)	N/A	0.032	9.453	0.004	0.000	0.770	0.000	0.000	0.000

Problem 1577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	191	0	0	233	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.178	0.068	0.000	0.876	0.000	0.000	0.000

Problem 1578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	65	0	0	92
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	2.88
time (sec)	N/A	0.003	0.035	0.181	0.000	0.623	0.000	0.000	0.713

Problem 1579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	175	0	0	127
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	1.92
time (sec)	N/A	0.007	0.094	0.192	0.000	0.880	0.000	0.000	1.032

Problem 1580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	337	0	0	203
N.S.	1	1.00	0.72	1.04	0.00	3.34	0.00	0.00	2.01
time (sec)	N/A	0.014	0.127	0.167	0.000	0.927	0.000	0.000	1.017

Problem 1581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	533	0	0	293
N.S.	1	1.00	0.70	1.26	0.00	3.92	0.00	0.00	2.15
time (sec)	N/A	0.023	0.141	0.206	0.000	1.050	0.000	0.000	1.152

Problem 1582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	73	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.025	10.048	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	73	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	10.023	0.003	0.000	0.000	0.000	0.000	0.000

Problem 1584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	576	576	71	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	10.032	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	73	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	10.025	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	73	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.571	10.032	0.053	0.000	0.000	0.000	0.000	0.000

Problem 1587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	231	0	0	740	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	3.43	0.00	0.00	-0.00
time (sec)	N/A	0.061	0.582	0.005	0.000	1.758	0.000	0.000	0.000

Problem 1588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	278	0	0	618	0	0	-1
N.S.	1	1.00	1.63	0.00	0.00	3.61	0.00	0.00	-0.01
time (sec)	N/A	0.028	10.017	0.003	0.000	1.350	0.000	0.000	0.000

Problem 1589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	155	0	0	519	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	4.12	0.00	0.00	-0.01
time (sec)	N/A	0.011	0.148	0.064	0.000	1.282	0.000	0.000	0.000

Problem 1590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	42	0	0	-1
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.034	0.184	0.000	0.911	0.000	0.000	0.000

Problem 1591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	-1
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.134	0.164	0.000	1.224	0.000	0.000	0.000

Problem 1592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	251	0	0	-1
N.S.	1	1.00	0.72	1.04	0.00	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.175	0.178	0.000	1.429	0.000	0.000	0.000

Problem 1593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	420	0	0	-1
N.S.	1	1.00	0.70	1.26	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.173	0.214	0.000	1.709	0.000	0.000	0.000

Problem 1594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1365	1365	73	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.916	10.040	0.053	0.000	0.000	0.000	0.000	0.000

Problem 1595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1330	1330	73	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.394	10.028	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1293	1293	73	0	0	0	0	0	-1
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.488	10.030	0.053	0.000	0.000	0.000	0.000	0.000

Problem 1597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1257	1257	73	0	0	0	0	0	-1
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.820	10.037	0.065	0.000	0.000	0.000	0.000	0.000

Problem 1598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1297	1297	71	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.715	10.024	0.057	0.000	0.000	0.000	0.000	0.000

Problem 1599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1335	1335	73	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.413	10.034	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1372	1372	73	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.746	10.032	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	231	0	0	741	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	3.43	0.00	0.00	-0.00
time (sec)	N/A	0.056	0.410	0.006	0.000	1.001	0.000	0.000	0.000

Problem 1602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	73	0	0	619	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	3.66	0.00	0.00	-0.01
time (sec)	N/A	0.027	10.043	0.005	0.000	1.233	0.000	0.000	0.000

Problem 1603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	155	0	0	521	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.146	0.061	0.000	1.213	0.000	0.000	0.000

Problem 1604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	26
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87
time (sec)	N/A	0.003	0.041	0.177	0.000	1.228	0.000	0.000	0.828

Problem 1605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	71
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.08
time (sec)	N/A	0.006	0.097	0.243	0.000	0.823	0.000	0.000	0.977

Problem 1606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	251	0	0	133
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	0.00	1.32
time (sec)	N/A	0.012	0.119	0.171	0.000	1.061	0.000	0.000	1.508

Problem 1607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	419	0	0	209
N.S.	1	1.00	0.70	1.26	0.00	3.08	0.00	0.00	1.54
time (sec)	N/A	0.019	0.154	0.188	0.000	0.814	0.000	0.000	1.273

Problem 1608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	73	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.743	10.041	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	73	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	10.025	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	73	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	10.027	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	71	0	0	0	0	0	-1
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.662	10.021	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	586	586	73	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	10.032	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	73	0	0	0	0	0	-1
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.585	10.027	0.070	0.000	0.000	0.000	0.000	0.000

Problem 1614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	656	656	73	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	10.021	0.066	0.000	0.000	0.000	0.000	0.000

Problem 1615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	267	0	0	423	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.073	0.575	0.009	0.000	0.718	0.000	0.000	0.000

Problem 1616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	73	0	0	306	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.047	10.053	0.007	0.000	0.726	0.000	0.000	0.000

Problem 1617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	191	0	0	233	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.212	0.056	0.000	0.946	0.000	0.000	0.000

Problem 1618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.037	0.197	0.000	1.133	0.000	0.000	0.000

Problem 1619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	53	0	126	0	0	-1
N.S.	1	1.00	0.68	0.80	0.00	1.91	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.104	0.165	0.000	0.833	0.000	0.000	0.000

Problem 1620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	71	105	0	273	0	0	-1
N.S.	1	1.00	0.70	1.04	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.133	0.169	0.000	0.993	0.000	0.000	0.000

Problem 1621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	456	0	0	-1
N.S.	1	1.00	0.70	1.26	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.136	0.168	0.000	1.445	0.000	0.000	0.000

Problem 1622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1355	1355	73	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.726	10.084	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1317	1317	73	0	0	0	0	0	-1
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.405	10.046	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1279	1279	73	0	0	0	0	0	-1
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.058	10.040	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1298	1298	73	0	0	0	0	0	-1
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.088	10.036	0.065	0.000	0.000	0.000	0.000	0.000

Problem 1626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1327	1327	71	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.382	10.030	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1370	1370	73	0	0	0	0	0	-1
N.S.	1	1.00	0.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.699	10.045	0.063	0.000	0.000	0.000	0.000	0.000

Problem 1628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	158	577	0	107	39	0	-1
N.S.	1	1.00	2.05	7.49	0.00	1.39	0.51	0.00	-0.01
time (sec)	N/A	0.010	0.219	0.436	0.000	1.107	1.593	0.000	0.000

Problem 1629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	73	0	0	0	0	0	-1
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	10.068	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	73	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	10.026	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	71	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	10.032	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	71	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	10.052	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	73	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	10.045	0.053	0.000	0.000	0.000	0.000	0.000

Problem 1634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	73	0	0	0	0	0	-1
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	10.055	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	73	0	0	0	0	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	10.051	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	73	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.179	10.046	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	71	0	0	0	0	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	10.020	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	71	0	0	0	0	0	-1
N.S.	1	1.00	0.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	10.043	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	73	0	0	0	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.164	10.053	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	73	0	0	0	0	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	10.041	0.054	0.000	0.000	0.000	0.000	0.000

Problem 1641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	73	0	0	0	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.116	10.073	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	73	0	0	0	218	0	-1
N.S.	1	1.00	0.40	0.00	0.00	0.00	1.20	0.00	-0.01
time (sec)	N/A	0.072	10.041	0.005	0.000	0.000	8.335	0.000	0.000

Problem 1643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	71	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	10.048	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	71	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	10.035	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	73	0	0	0	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	10.069	0.067	0.000	0.000	0.000	0.000	0.000

Problem 1646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	73	0	0	0	0	0	-1
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	10.066	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	73	0	0	0	0	0	-1
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.085	10.080	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	73	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.196	10.057	0.009	0.000	0.000	0.000	0.000	0.000

Problem 1649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	73	0	0	0	0	0	-1
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	10.030	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	73	0	0	0	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	10.023	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	71	0	0	0	0	0	-1
N.S.	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	10.030	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	71	0	0	0	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	10.037	0.067	0.000	0.000	0.000	0.000	0.000

Problem 1653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	73	0	0	0	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	10.073	0.066	0.000	0.000	0.000	0.000	0.000

Problem 1654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	73	0	0	0	0	0	-1
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	10.040	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	10.022	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	10.030	0.063	0.000	0.000	0.000	0.000	0.000

Problem 1657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	71	0	0	0	0	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	10.033	0.084	0.000	0.000	0.000	0.000	0.000

Problem 1658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	73	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	10.055	0.066	0.000	0.000	0.000	0.000	0.000

Problem 1659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	73	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	10.049	0.011	0.000	0.000	0.000	0.000	0.000

Problem 1660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	73	0	0	0	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	10.055	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	73	0	0	0	0	0	-1
N.S.	1	1.00	0.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	10.039	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	71	0	0	0	0	0	-1
N.S.	1	1.00	0.36	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	10.035	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	71	0	0	0	0	0	-1
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	10.037	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	73	0	0	0	0	0	-1
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	10.060	0.065	0.000	0.000	0.000	0.000	0.000

Problem 1665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	73	0	0	0	0	0	-1
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.096	10.073	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	73	0	0	0	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	10.053	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	10.040	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	71	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	10.032	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	71	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	10.041	0.067	0.000	0.000	0.000	0.000	0.000

Problem 1670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	73	0	0	0	0	0	-1
N.S.	1	1.00	0.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	10.056	0.071	0.000	0.000	0.000	0.000	0.000

Problem 1671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	73	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	10.076	0.012	0.000	0.000	0.000	0.000	0.000

Problem 1672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	73	0	0	0	0	0	-1
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	10.064	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	73	0	0	0	0	0	-1
N.S.	1	1.00	0.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	10.065	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	73	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	10.055	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	71	0	0	0	0	0	-1
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	10.040	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	71	0	0	0	0	0	-1
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.178	10.069	0.063	0.000	0.000	0.000	0.000	0.000

Problem 1677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	73	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	10.043	0.087	0.000	0.000	0.000	0.000	0.000

Problem 1678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	174	0	0	2151	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	10.49	0.00	0.00	-0.00
time (sec)	N/A	0.096	0.553	0.006	0.000	0.871	0.000	0.000	0.000

Problem 1679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	0	0	1468	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	8.79	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.441	0.007	0.000	1.396	0.000	0.000	0.000

Problem 1680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	71	0	0	857	0	0	-1
N.S.	1	1.00	0.47	0.00	0.00	5.64	0.00	0.00	-0.01
time (sec)	N/A	0.065	10.054	0.007	0.000	1.504	0.000	0.000	0.000

Problem 1681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	0	0	368	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.297	0.059	0.000	1.198	0.000	0.000	0.000

Problem 1682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	104	0	0	99
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	0.00	3.09
time (sec)	N/A	0.002	0.057	0.173	0.000	0.897	0.000	0.000	0.810

Problem 1683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	235	0	0	178
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	0.00	2.70
time (sec)	N/A	0.006	0.145	0.225	0.000	0.878	0.000	0.000	0.955

Problem 1684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	426	0	0	268
N.S.	1	1.00	0.72	1.04	0.00	4.22	0.00	0.00	2.65
time (sec)	N/A	0.011	0.188	0.218	0.000	0.858	0.000	0.000	1.126

Problem 1685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	649	0	0	376
N.S.	1	1.00	0.70	1.26	0.00	4.77	0.00	0.00	2.76
time (sec)	N/A	0.020	0.194	0.171	0.000	1.057	0.000	0.000	1.360

Problem 1686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	10.066	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	73	0	0	0	0	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	10.048	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	71	0	0	0	0	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	10.049	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	73	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	10.044	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	73	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	10.050	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	73	0	0	0	0	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	10.065	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	10.061	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	0	0	1468	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	8.79	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.402	0.006	0.000	0.785	0.000	0.000	0.000

Problem 1694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	73	0	0	814	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	6.41	0.00	0.00	-0.01
time (sec)	N/A	0.050	10.035	0.003	0.000	0.925	0.000	0.000	0.000

Problem 1695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	0	0	234	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.100	0.065	0.000	0.631	0.000	0.000	0.000

Problem 1696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	42	0	0	-1
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.037	0.189	0.000	0.740	0.000	0.000	0.000

Problem 1697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	-1
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.124	0.218	0.000	0.791	0.000	0.000	0.000

Problem 1698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	252	0	0	-1
N.S.	1	1.00	0.72	1.04	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.154	0.209	0.000	1.695	0.000	0.000	0.000

Problem 1699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	419	0	0	-1
N.S.	1	1.00	0.70	1.26	0.00	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.194	0.210	0.000	3.047	0.000	0.000	0.000

Problem 1700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	73	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	10.050	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	705	705	73	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.443	10.027	0.057	0.000	0.000	0.000	0.000	0.000

Problem 1702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	688	688	73	0	0	0	0	0	-1
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	10.025	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	71	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	10.034	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	760	760	73	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	10.039	0.067	0.000	0.000	0.000	0.000	0.000

Problem 1705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	143	0	0	1457	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	8.72	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.280	0.006	0.000	1.009	0.000	0.000	0.000

Problem 1706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	73	0	0	808	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	6.36	0.00	0.00	-0.01
time (sec)	N/A	0.054	10.032	0.004	0.000	0.953	0.000	0.000	0.000

Problem 1707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	73	0	0	234	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	2.75	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.106	0.067	0.000	1.207	0.000	0.000	0.000

Problem 1708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	26
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87
time (sec)	N/A	0.002	0.042	0.185	0.000	1.161	0.000	0.000	0.706

Problem 1709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	71
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.08
time (sec)	N/A	0.006	0.096	0.187	0.000	1.276	0.000	0.000	0.868

Problem 1710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	105	0	251	0	0	133
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	0.00	1.32
time (sec)	N/A	0.012	0.110	0.204	0.000	0.709	0.000	0.000	1.020

Problem 1711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	171	0	419	0	0	209
N.S.	1	1.00	0.68	1.26	0.00	3.08	0.00	0.00	1.54
time (sec)	N/A	0.020	0.149	0.181	0.000	0.786	0.000	0.000	1.260

Problem 1712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	73	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	10.042	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	73	0	0	0	0	0	-1
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	10.026	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	71	0	0	0	0	0	-1
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.131	10.023	0.072	0.000	0.000	0.000	0.000	0.000

Problem 1715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	73	0	0	0	0	0	-1
N.S.	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.162	10.035	0.069	0.000	0.000	0.000	0.000	0.000

Problem 1716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	73	0	0	0	0	0	-1
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	10.031	0.069	0.000	0.000	0.000	0.000	0.000

Problem 1717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	73	0	0	857	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	5.64	0.00	0.00	-0.01
time (sec)	N/A	0.062	10.059	0.007	0.000	1.003	0.000	0.000	0.000

Problem 1718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	105	0	0	273	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	2.53	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.138	0.056	0.000	0.664	0.000	0.000	0.000

Problem 1719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.042	0.212	0.000	0.732	0.000	0.000	0.000

Problem 1720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	45	53	0	126	0	0	-1
N.S.	1	1.00	0.68	0.80	0.00	1.91	0.00	0.00	-0.02
time (sec)	N/A	0.006	0.098	0.175	0.000	0.576	0.000	0.000	0.000

Problem 1721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	273	0	0	-1
N.S.	1	1.00	0.72	1.04	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.123	0.193	0.000	0.733	0.000	0.000	0.000

Problem 1722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	457	0	0	-1
N.S.	1	1.00	0.70	1.26	0.00	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.136	0.211	0.000	0.832	0.000	0.000	0.000

Problem 1723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	10.063	0.069	0.000	0.000	0.000	0.000	0.000

Problem 1724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	73	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.528	10.041	0.069	0.000	0.000	0.000	0.000	0.000

Problem 1725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	73	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	10.046	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	719	73	0	0	0	0	0	-1
N.S.	1	1.00	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	10.032	0.067	0.000	0.000	0.000	0.000	0.000

Problem 1727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	750	750	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.530	10.045	0.071	0.000	0.000	0.000	0.000	0.000

Problem 1728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	795	795	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	10.034	0.072	0.000	0.000	0.000	0.000	0.000

Problem 1729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	156	0	0	247	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.89	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.214	0.068	0.000	0.787	0.000	0.000	0.000

Problem 1730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	96	0	0	448	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	2.32	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.112	0.059	0.000	1.010	0.000	0.000	0.000

Problem 1731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	10.031	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.033	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.038	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.038	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.033	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	73	0	0	0	0	0	-1
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.379	10.075	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	73	0	0	0	0	0	-1
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	10.029	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	10.026	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	71	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	10.052	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	71	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.170	10.048	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	10.047	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	896	896	73	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.749	10.075	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.554	10.057	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	817	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.468	10.030	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	798	798	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	10.033	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	854	854	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	10.041	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	896	896	73	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.623	10.056	0.054	0.000	0.000	0.000	0.000	0.000

Problem 1748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	890	890	73	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.657	10.030	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	855	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	10.027	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	820	820	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	10.040	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	780	780	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	10.037	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	813	813	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.499	10.044	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	10.054	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	73	0	0	0	0	0	-1
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.232	10.048	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	10.028	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	73	0	0	0	0	0	-1
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	10.030	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	71	0	0	0	0	0	-1
N.S.	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.136	10.033	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	71	0	0	0	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	10.034	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	73	0	0	0	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	10.036	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	880	880	73	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.647	10.062	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	844	844	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	10.037	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	806	806	73	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	10.044	0.057	0.000	0.000	0.000	0.000	0.000

Problem 1763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	817	71	0	0	0	0	0	-1
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.467	10.035	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	844	844	71	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	10.043	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	893	893	73	0	0	0	0	0	-1
N.S.	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.610	10.063	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.072	0.005	0.000	0.000	0.000	0.000	0.000

Problem 1767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.052	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.039	0.003	0.000	0.000	0.000	0.000	0.000

Problem 1769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.036	0.003	0.000	0.000	0.000	0.000	0.000

Problem 1770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.046	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	81	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.048	0.063	0.000	0.000	0.000	0.000	0.000

Problem 1772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	267	0	0	5633	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	13.19	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.868	0.005	0.000	1.139	0.000	0.000	0.000

Problem 1773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	3025	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	8.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	10.033	0.005	0.000	1.090	0.000	0.000	0.000

Problem 1774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	239	0	0	663	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.300	0.054	0.000	1.485	0.000	0.000	0.000

Problem 1775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	65	0	0	130
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	4.06
time (sec)	N/A	0.002	0.037	0.171	0.000	0.661	0.000	0.000	0.565

Problem 1776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	175	0	0	137
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	2.08
time (sec)	N/A	0.006	0.129	0.199	0.000	0.670	0.000	0.000	0.750

Problem 1777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	338	0	0	213
N.S.	1	1.00	0.72	1.04	0.00	3.35	0.00	0.00	2.11
time (sec)	N/A	0.015	0.148	0.171	0.000	1.178	0.000	0.000	0.949

Problem 1778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	533	0	0	302
N.S.	1	1.00	0.70	1.26	0.00	3.92	0.00	0.00	2.22
time (sec)	N/A	0.022	0.172	0.210	0.000	1.023	0.000	0.000	1.147

Problem 1779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	267	0	0	5633	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	13.19	0.00	0.00	-0.00
time (sec)	N/A	0.421	0.890	0.004	0.000	1.101	0.000	0.000	0.000

Problem 1780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	2997	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	7.93	0.00	0.00	-0.00
time (sec)	N/A	0.387	10.050	0.003	0.000	1.121	0.000	0.000	0.000

Problem 1781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	244	0	0	755	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.363	0.388	0.053	0.000	1.305	0.000	0.000	0.000

Problem 1782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	65	0	0	130
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	4.06
time (sec)	N/A	0.002	0.048	0.175	0.000	1.022	0.000	0.000	0.587

Problem 1783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	175	0	0	137
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	2.08
time (sec)	N/A	0.006	0.138	0.177	0.000	1.079	0.000	0.000	0.744

Problem 1784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	338	0	0	214
N.S.	1	1.00	0.72	1.04	0.00	3.35	0.00	0.00	2.12
time (sec)	N/A	0.012	0.171	0.201	0.000	1.288	0.000	0.000	0.944

Problem 1785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	533	0	0	303
N.S.	1	1.00	0.70	1.26	0.00	3.92	0.00	0.00	2.23
time (sec)	N/A	0.021	0.185	0.198	0.000	1.318	0.000	0.000	1.158

Problem 1786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.072	0.063	0.000	0.000	0.000	0.000	0.000

Problem 1787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.043	0.053	0.000	0.000	0.000	0.000	0.000

Problem 1788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.032	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.049	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.071	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.048	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.082	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.054	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.043	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	10.038	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.049	0.007	0.000	0.000	0.000	0.000	0.000

Problem 1797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	81	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.039	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	288	0	0	5633	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	13.29	0.00	0.00	-0.00
time (sec)	N/A	0.393	0.771	0.007	0.000	1.371	0.000	0.000	0.000

Problem 1799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	73	0	0	3084	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	7.65	0.00	0.00	-0.00
time (sec)	N/A	0.371	10.054	0.013	0.000	0.946	0.000	0.000	0.000

Problem 1800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	257	0	0	855	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	2.39	0.00	0.00	-0.00
time (sec)	N/A	0.340	0.497	0.055	0.000	0.990	0.000	0.000	0.000

Problem 1801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	104	0	0	199
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	0.00	6.22
time (sec)	N/A	0.002	0.052	0.171	0.000	0.756	0.000	0.000	0.756

Problem 1802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	235	0	0	189
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	0.00	2.86
time (sec)	N/A	0.006	0.153	0.175	0.000	0.645	0.000	0.000	0.907

Problem 1803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	427	0	0	278
N.S.	1	1.00	0.72	1.04	0.00	4.23	0.00	0.00	2.75
time (sec)	N/A	0.012	0.188	0.201	0.000	1.649	0.000	0.000	1.144

Problem 1804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	649	0	0	385
N.S.	1	1.00	0.70	1.26	0.00	4.77	0.00	0.00	2.83
time (sec)	N/A	0.020	0.241	0.175	0.000	0.890	0.000	0.000	1.429

Problem 1805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	278	0	0	5633	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	13.29	0.00	0.00	-0.00
time (sec)	N/A	0.419	0.796	0.006	0.000	1.318	0.000	0.000	0.000

Problem 1806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	73	0	0	3025	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	8.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	10.029	0.005	0.000	1.322	0.000	0.000	0.000

Problem 1807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	194	0	0	620	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.346	0.190	0.055	0.000	1.341	0.000	0.000	0.000

Problem 1808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	42	0	0	27
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	0.84
time (sec)	N/A	0.002	0.037	0.193	0.000	1.220	0.000	0.000	0.763

Problem 1809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	54	0	118	0	0	127
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.92
time (sec)	N/A	0.007	0.086	0.197	0.000	0.953	0.000	0.000	0.860

Problem 1810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	73	105	0	252	0	0	203
N.S.	1	1.00	0.72	1.04	0.00	2.50	0.00	0.00	2.01
time (sec)	N/A	0.013	0.122	0.205	0.000	1.273	0.000	0.000	1.033

Problem 1811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	95	171	0	420	0	0	292
N.S.	1	1.00	0.70	1.26	0.00	3.09	0.00	0.00	2.15
time (sec)	N/A	0.020	0.142	0.233	0.000	1.023	0.000	0.000	1.200

Problem 1812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	10.042	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.033	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.023	0.051	0.000	0.000	0.000	0.000	0.000

Problem 1815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.032	0.052	0.000	0.000	0.000	0.000	0.000

Problem 1816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.050	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.048	0.060	0.000	0.000	0.000	0.000	0.000

Problem 1818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	71	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.061	0.008	0.000	0.000	0.000	0.000	0.000

Problem 1819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.036	0.006	0.000	0.000	0.000	0.000	0.000

Problem 1820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.024	0.003	0.000	0.000	0.000	0.000	0.000

Problem 1821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.028	0.054	0.000	0.000	0.000	0.000	0.000

Problem 1822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.055	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.037	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	278	0	0	5591	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	13.19	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.550	0.006	0.000	1.137	0.000	0.000	0.000

Problem 1825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	71	0	0	2997	0	0	-1
N.S.	1	1.00	0.19	0.00	0.00	7.93	0.00	0.00	-0.00
time (sec)	N/A	0.333	10.021	0.005	0.000	0.879	0.000	0.000	0.000

Problem 1826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	194	0	0	620	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.184	0.056	0.000	1.120	0.000	0.000	0.000

Problem 1827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03
time (sec)	N/A	0.002	0.042	0.174	0.000	0.590	0.000	0.000	0.000

Problem 1828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	53	0	118	0	0	-1
N.S.	1	1.00	0.70	0.80	0.00	1.79	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.092	0.174	0.000	0.892	0.000	0.000	0.000

Problem 1829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	105	0	252	0	0	-1
N.S.	1	1.00	0.76	1.04	0.00	2.50	0.00	0.00	-0.01
time (sec)	N/A	0.012	0.107	0.200	0.000	0.935	0.000	0.000	0.000

Problem 1830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	93	171	0	420	0	0	-1
N.S.	1	1.00	0.68	1.26	0.00	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.138	0.172	0.000	1.180	0.000	0.000	0.000

Problem 1831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	308	0	0	5690	0	0	-1
N.S.	1	1.00	0.69	0.00	0.00	12.67	0.00	0.00	-0.00
time (sec)	N/A	0.449	0.919	0.009	0.000	1.345	0.000	0.000	0.000

Problem 1832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	71	0	0	3084	0	0	-1
N.S.	1	1.00	0.18	0.00	0.00	7.65	0.00	0.00	-0.00
time (sec)	N/A	0.422	10.063	0.007	0.000	1.469	0.000	0.000	0.000

Problem 1833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	239	0	0	663	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.372	0.284	0.051	0.000	1.150	0.000	0.000	0.000

Problem 1834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	27	0	42	0	0	26
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87
time (sec)	N/A	0.002	0.043	0.174	0.000	0.917	0.000	0.000	0.680

Problem 1835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	53	0	126	0	0	72
N.S.	1	1.00	0.70	0.83	0.00	1.97	0.00	0.00	1.12
time (sec)	N/A	0.007	0.094	0.175	0.000	0.646	0.000	0.000	0.834

Problem 1836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	105	0	273	0	0	132
N.S.	1	1.00	0.74	1.07	0.00	2.79	0.00	0.00	1.35
time (sec)	N/A	0.015	0.123	0.171	0.000	1.557	0.000	0.000	0.960

Problem 1837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	95	171	0	457	0	0	209
N.S.	1	1.00	0.71	1.28	0.00	3.41	0.00	0.00	1.56
time (sec)	N/A	0.022	0.134	0.173	0.000	1.386	0.000	0.000	1.146

Problem 1838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	10.063	0.054	0.000	0.000	0.000	0.000	0.000

Problem 1839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.020	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.029	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.041	0.058	0.000	0.000	0.000	0.000	0.000

Problem 1842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.052	0.062	0.000	0.000	0.000	0.000	0.000

Problem 1843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	10.037	0.056	0.000	0.000	0.000	0.000	0.000

Problem 1844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	106	17	20	23	11
N.S.	1	1.00	1.00	1.09	9.64	1.55	1.82	2.09	1.00
time (sec)	N/A	0.002	0.047	0.150	0.305	0.568	0.091	3.712	0.462

Problem 1845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	74	73	0	0	0	0	0	-1
N.S.	1	1.21	1.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.060	0.092	0.000	0.000	0.000	0.000	0.000

Problem 1846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	94	389	246	497	4058	833	478
N.S.	1	1.00	0.85	3.54	2.24	4.52	36.89	7.57	4.35
time (sec)	N/A	0.038	0.109	0.182	0.324	0.656	1.137	3.074	0.942

Problem 1847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	138	235	1506	385	226
N.S.	1	1.00	0.86	2.04	1.77	3.01	19.31	4.94	2.90
time (sec)	N/A	0.022	0.091	0.191	0.307	0.628	0.582	3.579	0.655

Problem 1848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	49	63	83	377	132	88
N.S.	1	1.00	0.89	1.07	1.37	1.80	8.20	2.87	1.91
time (sec)	N/A	0.014	0.050	0.146	0.264	1.001	0.288	2.257	0.484

Problem 1849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.047	0.055	0.000	0.000	0.000	0.000	0.000

Problem 1850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.008	0.050	0.061	0.000	0.000	0.000	0.000	0.000

Problem 1851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.048	0.087	0.000	0.000	0.000	0.000	0.000

Problem 1852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	386	246	496	4058	833	478
N.S.	1	1.00	0.86	3.48	2.22	4.47	36.56	7.50	4.31
time (sec)	N/A	0.040	0.098	0.178	0.309	0.994	1.061	2.159	0.913

Problem 1853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	159	138	237	1506	385	226
N.S.	1	1.00	0.86	2.04	1.77	3.04	19.31	4.94	2.90
time (sec)	N/A	0.022	0.085	0.188	0.315	0.715	0.552	1.613	0.618

Problem 1854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	46	63	83	377	132	88
N.S.	1	1.00	0.87	0.98	1.34	1.77	8.02	2.81	1.87
time (sec)	N/A	0.013	0.045	0.163	0.288	0.705	0.284	6.127	0.494

Problem 1855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	18	20	20	18	18
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	1.00	1.00
time (sec)	N/A	0.002	0.019	0.150	0.302	1.149	0.007	4.327	0.379

Problem 1856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.044	0.059	0.000	0.000	0.000	0.000	0.000

Problem 1857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	52	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.049	0.064	0.000	0.000	0.000	0.000	0.000

Problem 1858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.047	0.081	0.000	0.000	0.000	0.000	0.000

Problem 1859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	112	322	0	512	0	0	528
N.S.	1	1.00	0.78	2.25	0.00	3.58	0.00	0.00	3.69
time (sec)	N/A	0.047	0.085	0.222	0.000	0.869	0.000	0.000	1.096

Problem 1860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	59	127	0	206	0	0	220
N.S.	1	1.00	0.69	1.48	0.00	2.40	0.00	0.00	2.56
time (sec)	N/A	0.009	0.054	0.189	0.000	0.960	0.000	0.000	0.767

Problem 1861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	45	0	60	0	0	102
N.S.	1	1.00	0.92	1.15	0.00	1.54	0.00	0.00	2.62
time (sec)	N/A	0.003	0.042	0.192	0.000	1.301	0.000	0.000	0.559

Problem 1862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.057	0.084	0.000	0.000	0.000	0.000	0.000

Problem 1863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.042	0.078	0.000	0.000	0.000	0.000	0.000

Problem 1864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	89	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.072	0.085	0.000	0.000	0.000	0.000	0.000

Problem 1865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	92	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.061	0.080	0.000	0.000	0.000	0.000	0.000

Problem 1866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.040	0.073	0.000	0.000	0.000	0.000	0.000

Problem 1867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.048	0.108	0.000	0.000	0.000	0.000	0.000

Problem 1868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	41	0	59	0	0	97
N.S.	1	1.00	1.03	1.11	0.00	1.59	0.00	0.00	2.62
time (sec)	N/A	0.004	0.041	0.189	0.000	0.881	0.000	0.000	0.532

Problem 1869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	60	123	0	207	0	0	214
N.S.	1	1.00	0.75	1.54	0.00	2.59	0.00	0.00	2.68
time (sec)	N/A	0.014	0.053	0.188	0.000	1.008	0.000	0.000	0.736

Problem 1870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	113	318	0	509	0	0	525
N.S.	1	1.00	0.86	2.43	0.00	3.89	0.00	0.00	4.01
time (sec)	N/A	0.034	0.088	0.194	0.000	0.760	0.000	0.000	0.992

Problem 1871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	195	661	0	959	0	0	944
N.S.	1	1.00	1.05	3.55	0.00	5.16	0.00	0.00	5.08
time (sec)	N/A	0.062	0.109	0.211	0.000	0.935	0.000	0.000	1.642

Problem 1872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.006	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.052	0.087	0.000	0.000	0.000	0.000	0.000

Problem 1874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	42	0	58	0	0	98
N.S.	1	1.00	1.00	1.17	0.00	1.61	0.00	0.00	2.72
time (sec)	N/A	0.003	0.042	0.193	0.000	0.835	0.000	0.000	0.556

Problem 1875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	59	124	0	205	0	0	214
N.S.	1	1.00	0.75	1.57	0.00	2.59	0.00	0.00	2.71
time (sec)	N/A	0.011	0.053	0.233	0.000	0.836	0.000	0.000	0.745

Problem 1876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	112	319	0	507	0	0	528
N.S.	1	1.00	0.86	2.45	0.00	3.90	0.00	0.00	4.06
time (sec)	N/A	0.027	0.106	0.244	0.000	0.627	0.000	0.000	1.023

Problem 1877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	195	662	0	954	0	0	945
N.S.	1	1.00	1.05	3.58	0.00	5.16	0.00	0.00	5.11
time (sec)	N/A	0.043	0.120	0.233	0.000	0.905	0.000	0.000	1.609

Problem 1878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.076	0.084	0.000	0.000	0.000	0.000	0.000

Problem 1879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.069	0.076	0.000	0.000	0.000	0.000	0.000

Problem 1880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	0.010	0.004	0.000	0.000	0.000	0.000	0.000

Problem 1881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	57	92	93	233	78	46
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.37	0.81
time (sec)	N/A	0.024	0.019	0.193	0.265	0.923	0.369	1.987	0.439

Problem 1882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	54	57	0	85	0	0	81
N.S.	1	1.00	0.57	0.60	0.00	0.89	0.00	0.00	0.85
time (sec)	N/A	0.026	0.189	0.278	0.000	0.935	0.000	0.000	1.037

Problem 1883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	46	66	0	84	0	0	119
N.S.	1	1.00	0.47	0.68	0.00	0.87	0.00	0.00	1.23
time (sec)	N/A	0.016	0.274	0.204	0.000	1.301	0.000	0.000	2.138

Problem 1884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	46	66	0	84	0	0	142
N.S.	1	1.00	0.47	0.68	0.00	0.87	0.00	0.00	1.46
time (sec)	N/A	0.013	0.132	0.208	0.000	1.062	0.000	0.000	0.850

Problem 1885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	0	0	0	29	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.97	0.00	-0.03
time (sec)	N/A	0.005	0.075	0.059	0.000	0.000	1.174	0.000	0.000

Problem 1886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	0	31	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.89	0.00	-0.03
time (sec)	N/A	0.004	0.064	0.051	0.000	0.000	1.100	0.000	0.000

Problem 1887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	37	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.12	0.00	-0.03
time (sec)	N/A	0.005	0.085	0.051	0.000	0.000	52.967	0.000	0.000

Problem 1888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	42	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.14	0.00	-0.03
time (sec)	N/A	0.004	0.080	0.050	0.000	0.000	53.255	0.000	0.000

Problem 1889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	42	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.89	0.00	-0.02
time (sec)	N/A	0.010	0.032	0.057	0.000	0.000	15.382	0.000	0.000

Problem 1890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	88	0	0	0	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.028	0.092	0.000	0.000	0.000	0.000	0.000

Problem 1891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	22	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.79	0.79
time (sec)	N/A	0.004	0.000	0.013	0.272	1.134	0.007	1.907	0.037

Problem 1892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	8	11	10
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	0.73	0.67
time (sec)	N/A	0.002	0.000	0.013	0.293	1.070	0.005	1.527	0.021

Problem 1893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	5	9	8
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.73
time (sec)	N/A	0.001	0.000	0.012	0.280	0.603	0.005	1.510	0.017

Problem 1894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	7
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	0.78
time (sec)	N/A	0.001	0.000	0.012	0.273	0.681	0.005	1.069	0.029

Problem 1895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	12	10	12	12
N.S.	1	1.00	1.00	1.07	0.86	0.86	0.71	0.86	0.86
time (sec)	N/A	0.002	0.000	0.038	0.271	0.557	0.007	1.553	0.020

Problem 1896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	10
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.91
time (sec)	N/A	0.001	0.000	0.011	0.264	0.570	0.005	2.301	0.019

Problem 1897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	8	11	8
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	0.73	0.53
time (sec)	N/A	0.001	0.000	0.013	0.272	0.845	0.007	1.014	0.021

Problem 1898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	15	14	13
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	0.78	0.72
time (sec)	N/A	0.002	0.000	0.014	0.266	0.673	0.006	0.856	0.024

Problem 1899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	12	16	15
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.60	0.80	0.75
time (sec)	N/A	0.002	0.000	0.007	0.259	0.694	0.006	1.668	0.026

Problem 1900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.001	0.000	0.013	0.278	0.629	0.006	2.441	0.023

Problem 1901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	13	12	13	13
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.92	1.00	1.00
time (sec)	N/A	0.001	0.000	0.012	0.273	0.899	0.007	1.746	0.028

Problem 1902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	27	20	21	20
N.S.	1	1.00	1.00	0.95	0.91	1.23	0.91	0.95	0.91
time (sec)	N/A	0.003	0.006	0.012	0.272	0.960	0.067	1.245	0.037

Problem 1903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	17	15	16	17
N.S.	1	1.00	1.00	0.77	0.73	0.77	0.68	0.73	0.77
time (sec)	N/A	0.002	0.002	0.014	0.280	0.805	0.022	1.204	0.029

Problem 1904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	14	14	11
N.S.	1	1.00	1.00	0.93	0.87	1.13	0.93	0.93	0.73
time (sec)	N/A	0.001	0.002	0.008	0.268	0.861	0.024	1.150	0.029

Problem 1905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	11	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.10	0.70	1.10	1.00
time (sec)	N/A	0.001	0.002	0.012	0.288	0.802	0.026	1.308	0.033

Problem 1906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	10	11	12
N.S.	1	1.00	1.00	0.80	0.73	0.80	0.67	0.73	0.80
time (sec)	N/A	0.001	0.001	0.016	0.283	0.606	0.023	1.554	0.025

Problem 1907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	0.82
time (sec)	N/A	0.001	0.001	0.012	0.275	0.536	0.017	1.400	0.027

Problem 1908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	10	8	11	10
N.S.	1	1.00	1.00	0.92	0.85	0.77	0.62	0.85	0.77
time (sec)	N/A	0.001	0.002	0.010	0.291	0.575	0.021	1.573	0.031

Problem 1909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	12	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.92	0.85
time (sec)	N/A	0.001	0.001	0.010	0.273	0.737	0.018	1.842	0.026

Problem 1910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	11	11	12	11	11
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.71	0.65	0.65
time (sec)	N/A	0.001	0.002	0.039	0.271	0.682	0.008	1.891	0.030

Problem 1911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	8
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62
time (sec)	N/A	0.001	0.001	0.169	0.262	0.910	0.007	1.351	0.024

Problem 1912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	10	9	10	12	9	10
N.S.	1	1.00	0.93	0.67	0.60	0.67	0.80	0.60	0.67
time (sec)	N/A	0.001	0.010	0.023	0.271	0.802	0.009	1.358	0.021

Problem 1913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	18	14	14	15
N.S.	1	1.00	1.00	0.93	0.87	1.20	0.93	0.93	1.00
time (sec)	N/A	0.002	0.016	0.046	0.280	0.573	0.010	1.957	0.291

Problem 1914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	14	12	11	10	14	11	12
N.S.	1	1.00	0.82	0.71	0.65	0.59	0.82	0.65	0.71
time (sec)	N/A	0.001	0.013	0.026	0.260	0.950	0.008	1.656	0.027

Problem 1915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	10	12	11	14	12	11	8
N.S.	1	1.00	0.67	0.80	0.73	0.93	0.80	0.73	0.53
time (sec)	N/A	0.001	0.009	0.044	0.264	0.604	0.012	1.827	0.027

Problem 1916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	16	14	19	16	15
N.S.	1	1.00	1.00	0.71	0.67	0.58	0.79	0.67	0.62
time (sec)	N/A	0.002	0.004	0.025	0.278	0.754	0.009	1.626	0.029

Problem 1917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	15	19	20	16	17
N.S.	1	1.00	1.00	0.70	0.65	0.83	0.87	0.70	0.74
time (sec)	N/A	0.002	0.005	0.045	0.289	1.014	0.010	1.491	0.281

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1884] had the largest ratio of [51]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	1	1.000
2	A	1	1	1.00	1	1.000
3	A	1	1	1.00	1	1.000
4	A	1	1	1.00	1	1.000
5	A	1	1	1.00	3	0.333
6	A	1	1	1.00	1	1.000
7	A	1	1	1.00	1	1.000
8	A	1	1	1.00	3	0.333
9	A	1	1	1.00	13	0.077
10	A	1	1	1.00	3	0.333
11	A	1	1	1.00	3	0.333
12	A	1	1	1.00	3	0.333
13	A	1	1	1.00	1	1.000
14	A	1	1	1.00	1	1.000
15	A	1	1	1.00	3	0.333
16	A	1	1	1.00	3	0.333
17	A	1	1	1.00	3	0.333
18	A	1	1	1.00	3	0.333
19	A	1	1	1.00	3	0.333
20	A	1	1	1.00	5	0.200
21	A	1	1	1.00	5	0.200
22	A	1	1	1.00	5	0.200
23	A	1	1	1.00	5	0.200
24	A	1	1	1.00	5	0.200
25	A	1	1	1.00	5	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	1	1	1.00	5	0.200
27	A	1	1	1.00	5	0.200
28	A	1	1	1.00	5	0.200
29	A	1	1	1.00	5	0.200
30	A	1	1	1.00	5	0.200
31	A	1	1	1.00	5	0.200
32	A	1	1	1.00	5	0.200
33	A	1	1	1.00	5	0.200
34	A	1	1	1.00	3	0.333
35	A	1	1	1.00	5	0.200
36	A	2	2	1.00	17	0.118
37	A	2	2	1.00	13	0.154
38	A	2	2	1.00	13	0.154
39	A	2	2	1.00	13	0.154
40	A	2	2	1.00	13	0.154
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	13	0.154
43	A	2	1	1.00	9	0.111
44	A	2	1	1.00	9	0.111
45	A	2	1	1.00	7	0.143
46	A	1	0	1.00	5	0.000
47	A	2	1	1.00	9	0.111
48	A	2	1	1.00	9	0.111
49	A	1	1	1.00	9	0.111
50	A	2	1	1.00	9	0.111
51	A	2	1	1.00	9	0.111
52	A	2	1	1.00	11	0.091
53	A	2	1	1.00	11	0.091
54	A	2	1	1.00	9	0.111
55	A	1	1	1.00	7	0.143
56	A	2	1	1.00	11	0.091
57	A	2	1	1.00	11	0.091
58	A	2	1	1.00	11	0.091
59	A	1	1	1.00	11	0.091
60	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	1	1.00	11	0.091
62	A	2	1	1.00	11	0.091
63	A	2	1	1.00	11	0.091
64	A	2	1	1.00	11	0.091
65	A	2	1	1.00	11	0.091
66	A	2	1	1.00	11	0.091
67	A	2	1	1.00	9	0.111
68	A	1	1	1.00	7	0.143
69	A	2	1	1.00	11	0.091
70	A	2	1	1.00	11	0.091
71	A	2	1	1.00	11	0.091
72	A	2	1	1.00	11	0.091
73	A	1	1	1.00	11	0.091
74	A	2	2	1.00	11	0.182
75	A	2	1	1.00	11	0.091
76	A	2	1	1.00	11	0.091
77	A	2	1	1.00	11	0.091
78	A	2	1	1.00	11	0.091
79	A	2	1	1.00	11	0.091
80	A	2	1	1.00	11	0.091
81	A	2	1	1.00	11	0.091
82	A	2	1	1.00	9	0.111
83	A	1	1	1.00	7	0.143
84	A	2	1	1.00	11	0.091
85	A	2	1	1.00	11	0.091
86	A	2	1	1.00	11	0.091
87	A	2	1	1.00	11	0.091
88	A	2	1	1.00	11	0.091
89	A	2	1	1.00	11	0.091
90	A	1	1	1.00	11	0.091
91	A	2	2	1.00	11	0.182
92	A	3	2	1.00	11	0.182
93	A	2	1	1.00	11	0.091
94	A	2	1	1.00	11	0.091
95	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	1	1.00	11	0.091
97	A	2	1	1.00	11	0.091
98	A	2	1	1.00	11	0.091
99	A	2	1	1.00	11	0.091
100	A	2	1	1.00	11	0.091
101	A	2	1	1.00	11	0.091
102	A	2	1	1.00	11	0.091
103	A	2	1	1.00	11	0.091
104	A	2	1	1.00	11	0.091
105	A	2	1	1.00	9	0.111
106	A	1	1	1.00	7	0.143
107	A	2	1	1.00	11	0.091
108	A	2	1	1.00	11	0.091
109	A	2	1	1.00	11	0.091
110	A	2	1	1.00	11	0.091
111	A	2	1	1.00	11	0.091
112	A	2	1	1.00	11	0.091
113	A	2	1	1.00	11	0.091
114	A	2	1	1.00	11	0.091
115	A	1	1	1.00	11	0.091
116	A	2	2	1.00	11	0.182
117	A	3	2	1.00	11	0.182
118	A	4	2	1.00	11	0.182
119	A	5	2	1.00	11	0.182
120	A	2	1	1.00	11	0.091
121	A	2	1	1.00	11	0.091
122	A	2	1	1.00	11	0.091
123	A	2	1	1.00	11	0.091
124	A	2	1	1.00	11	0.091
125	A	2	1	1.00	11	0.091
126	A	2	1	1.00	11	0.091
127	A	2	1	1.00	11	0.091
128	A	2	1	1.00	11	0.091
129	A	2	1	1.00	11	0.091
130	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	1	1.00	11	0.091
132	A	2	1	1.00	11	0.091
133	A	2	1	1.00	9	0.111
134	A	1	1	1.00	7	0.143
135	A	2	1	1.00	11	0.091
136	A	2	1	1.00	11	0.091
137	A	2	1	1.00	11	0.091
138	A	2	1	1.00	11	0.091
139	A	2	1	1.00	11	0.091
140	A	2	1	1.00	11	0.091
141	A	2	1	1.00	11	0.091
142	A	2	1	1.00	11	0.091
143	A	2	1	1.00	11	0.091
144	A	2	1	1.00	11	0.091
145	A	2	1	1.00	11	0.091
146	A	1	1	1.00	11	0.091
147	A	2	2	1.00	11	0.182
148	A	3	2	1.00	11	0.182
149	A	4	2	1.00	11	0.182
150	A	5	2	1.00	11	0.182
151	A	6	2	1.00	11	0.182
152	A	7	2	1.00	11	0.182
153	A	2	1	1.00	11	0.091
154	A	2	1	1.00	11	0.091
155	A	1	1	1.00	7	0.143
156	A	1	1	1.00	12	0.083
157	A	2	1	1.00	11	0.091
158	A	2	1	1.00	11	0.091
159	A	2	1	1.00	11	0.091
160	A	2	1	1.00	11	0.091
161	A	2	1	1.00	9	0.111
162	A	1	1	1.00	7	0.143
163	A	3	3	1.00	11	0.273
164	A	2	1	1.00	11	0.091
165	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	1	1.00	11	0.091
167	A	2	1	1.00	11	0.091
168	A	2	1	1.00	11	0.091
169	A	2	1	1.00	11	0.091
170	A	2	1	1.00	11	0.091
171	A	2	1	1.00	11	0.091
172	A	2	1	1.00	11	0.091
173	A	2	1	1.00	9	0.111
174	A	1	1	1.00	7	0.143
175	A	2	1	1.00	11	0.091
176	A	2	1	1.00	11	0.091
177	A	2	1	1.00	11	0.091
178	A	2	1	1.00	11	0.091
179	A	2	1	1.00	11	0.091
180	A	2	1	1.00	11	0.091
181	A	2	1	1.00	11	0.091
182	A	2	1	1.00	11	0.091
183	A	2	1	1.00	11	0.091
184	A	2	1	1.00	11	0.091
185	A	2	1	1.00	11	0.091
186	A	1	1	1.00	9	0.111
187	A	1	1	1.00	7	0.143
188	A	2	1	1.00	11	0.091
189	A	2	1	1.00	11	0.091
190	A	2	1	1.00	11	0.091
191	A	2	1	1.00	11	0.091
192	A	2	1	1.00	11	0.091
193	A	2	1	1.00	11	0.091
194	A	2	1	1.00	11	0.091
195	A	2	1	1.00	11	0.091
196	A	2	1	1.00	11	0.091
197	A	2	1	1.00	11	0.091
198	A	2	1	1.00	11	0.091
199	A	1	1	1.00	11	0.091
200	A	2	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
201	A	1	1	1.00	7	0.143
202	A	2	1	1.00	11	0.091
203	A	2	1	1.00	11	0.091
204	A	2	1	1.00	11	0.091
205	A	2	1	1.00	11	0.091
206	A	2	1	1.00	11	0.091
207	A	2	1	1.00	11	0.091
208	A	2	1	1.00	11	0.091
209	A	2	1	1.00	11	0.091
210	A	2	1	1.00	11	0.091
211	A	2	1	1.00	11	0.091
212	A	1	1	1.00	11	0.091
213	A	2	2	1.00	11	0.182
214	A	2	1	1.23	11	0.091
215	A	2	1	1.00	11	0.091
216	A	2	1	1.00	9	0.111
217	A	1	1	1.00	7	0.143
218	A	2	1	1.00	11	0.091
219	A	2	1	1.00	11	0.091
220	A	2	1	1.00	11	0.091
221	A	2	1	1.00	11	0.091
222	A	2	1	1.00	11	0.091
223	A	2	1	1.00	11	0.091
224	A	2	1	1.00	11	0.091
225	A	2	1	1.00	11	0.091
226	A	1	1	1.00	11	0.091
227	A	2	2	1.00	11	0.182
228	A	3	2	1.00	11	0.182
229	A	4	2	1.00	11	0.182
230	A	2	1	1.00	11	0.091
231	A	2	1	1.00	11	0.091
232	A	2	1	1.00	11	0.091
233	A	2	1	1.00	9	0.111
234	A	1	1	1.00	7	0.143
235	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	1	1.00	11	0.091
237	A	2	1	1.00	11	0.091
238	A	2	1	1.00	11	0.091
239	A	2	1	1.00	11	0.091
240	A	2	1	1.00	11	0.091
241	A	2	1	1.00	11	0.091
242	A	2	1	1.00	11	0.091
243	A	1	1	1.00	11	0.091
244	A	2	2	1.00	11	0.182
245	A	3	2	1.00	11	0.182
246	A	2	1	1.00	11	0.091
247	A	2	1	1.00	11	0.091
248	A	2	1	1.00	11	0.091
249	A	2	1	1.00	11	0.091
250	A	2	1	1.00	9	0.111
251	A	1	1	1.00	3	0.333
252	A	2	1	1.00	11	0.091
253	A	2	1	1.00	11	0.091
254	A	2	1	1.00	11	0.091
255	A	3	3	1.00	11	0.273
256	A	3	3	1.00	11	0.273
257	A	2	1	1.00	11	0.091
258	A	2	1	1.00	11	0.091
259	A	2	1	1.00	11	0.091
260	A	2	1	1.00	11	0.091
261	A	2	1	1.00	11	0.091
262	A	2	1	1.00	11	0.091
263	A	2	1	1.00	11	0.091
264	A	2	1	1.00	11	0.091
265	A	2	1	1.00	11	0.091
266	A	2	1	1.00	11	0.091
267	A	2	1	1.00	11	0.091
268	A	2	1	1.00	11	0.091
269	A	2	1	1.00	11	0.091
270	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	1	1	1.00	7	0.143
272	A	1	1	1.00	7	0.143
273	A	1	1	1.00	11	0.091
274	A	1	1	1.00	13	0.077
275	A	1	1	1.00	15	0.067
276	A	1	1	1.00	15	0.067
277	A	1	1	1.00	15	0.067
278	A	1	1	1.00	15	0.067
279	A	3	3	1.00	11	0.273
280	A	3	3	1.00	11	0.273
281	A	2	1	1.00	11	0.091
282	A	2	1	1.00	11	0.091
283	A	3	1	1.00	17	0.059
284	A	2	1	1.00	13	0.077
285	A	2	1	1.00	13	0.077
286	A	2	1	1.00	11	0.091
287	A	1	1	1.00	9	0.111
288	A	3	3	1.00	13	0.231
289	A	3	3	1.00	13	0.231
290	A	4	4	1.00	13	0.308
291	A	5	4	1.00	13	0.308
292	A	2	1	1.00	13	0.077
293	A	2	1	1.00	13	0.077
294	A	2	1	1.00	11	0.091
295	A	1	1	1.00	9	0.111
296	A	4	3	1.00	13	0.231
297	A	4	4	1.00	13	0.308
298	A	4	3	1.00	13	0.231
299	A	5	4	1.00	13	0.308
300	A	2	1	1.00	13	0.077
301	A	2	1	1.00	13	0.077
302	A	2	1	1.00	11	0.091
303	A	1	1	1.00	9	0.111
304	A	5	3	1.00	13	0.231
305	A	5	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
306	A	5	4	1.00	13	0.308
307	A	5	3	1.00	13	0.231
308	A	6	4	1.00	13	0.308
309	A	2	1	1.00	13	0.077
310	A	2	1	1.00	13	0.077
311	A	2	1	1.00	13	0.077
312	A	2	1	1.00	13	0.077
313	A	2	1	1.00	13	0.077
314	A	2	1	1.00	13	0.077
315	A	2	1	1.00	11	0.091
316	A	1	1	1.00	9	0.111
317	A	7	3	1.00	13	0.231
318	A	7	4	1.00	13	0.308
319	A	7	4	1.00	13	0.308
320	A	7	4	1.00	13	0.308
321	A	7	4	1.00	13	0.308
322	A	7	3	1.00	13	0.231
323	A	8	4	1.00	13	0.308
324	A	9	4	1.00	13	0.308
325	A	3	3	1.00	15	0.200
326	A	3	3	1.00	15	0.200
327	A	4	4	1.00	15	0.267
328	A	4	3	1.00	15	0.200
329	A	4	4	1.00	15	0.267
330	A	4	3	1.00	15	0.200
331	A	5	3	1.00	15	0.200
332	A	5	4	1.00	15	0.267
333	A	5	4	1.00	15	0.267
334	A	2	1	1.00	13	0.077
335	A	2	1	1.00	13	0.077
336	A	2	1	1.00	13	0.077
337	A	2	1	1.00	11	0.091
338	A	1	1	1.00	9	0.111
339	A	2	2	1.00	13	0.154
340	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	4	3	1.00	13	0.231
342	A	5	3	1.00	13	0.231
343	A	2	1	1.00	13	0.077
344	A	2	1	1.00	13	0.077
345	A	2	1	1.00	13	0.077
346	A	2	1	1.00	11	0.091
347	A	1	1	1.00	9	0.111
348	A	3	3	1.00	13	0.231
349	A	4	4	1.00	13	0.308
350	A	5	4	1.00	13	0.308
351	A	2	1	1.00	13	0.077
352	A	2	1	1.00	13	0.077
353	A	2	1	1.00	13	0.077
354	A	2	1	1.00	11	0.091
355	A	1	1	1.00	9	0.111
356	A	4	3	1.00	13	0.231
357	A	5	4	1.00	13	0.308
358	A	6	4	1.00	13	0.308
359	A	2	2	1.00	15	0.133
360	A	3	3	1.00	15	0.200
361	A	4	3	1.00	15	0.200
362	A	3	3	1.00	15	0.200
363	A	4	4	1.00	15	0.267
364	A	5	4	1.00	15	0.267
365	A	4	3	1.00	15	0.200
366	A	5	4	1.00	15	0.267
367	A	6	4	1.00	15	0.267
368	A	2	2	1.00	31	0.065
369	C	5	2	7.08	34	0.059
370	A	3	3	1.00	29	0.103
371	A	2	1	1.00	13	0.077
372	A	2	1	1.00	13	0.077
373	A	2	1	1.00	11	0.091
374	A	1	1	1.00	9	0.111
375	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	5	5	1.00	13	0.385
377	A	6	6	1.00	13	0.462
378	A	2	1	1.00	13	0.077
379	A	2	1	1.00	13	0.077
380	A	2	1	1.00	11	0.091
381	A	1	1	1.00	9	0.111
382	A	5	5	1.00	13	0.385
383	A	5	5	1.00	13	0.385
384	A	6	6	1.00	13	0.462
385	A	2	1	1.00	13	0.077
386	A	2	1	1.00	13	0.077
387	A	2	1	1.00	11	0.091
388	A	1	1	1.00	9	0.111
389	A	6	5	1.00	13	0.385
390	A	6	6	1.00	13	0.462
391	A	6	5	1.00	13	0.385
392	A	2	1	1.00	13	0.077
393	A	2	1	1.00	13	0.077
394	A	2	1	1.00	11	0.091
395	A	1	1	1.00	9	0.111
396	A	4	4	1.00	13	0.308
397	A	5	5	1.00	13	0.385
398	A	6	5	1.00	13	0.385
399	A	2	1	1.00	15	0.067
400	A	2	1	1.00	15	0.067
401	A	2	1	1.00	13	0.077
402	A	1	1	1.00	11	0.091
403	A	4	4	1.00	15	0.267
404	A	5	5	1.00	15	0.333
405	A	6	5	1.00	15	0.333
406	A	2	1	1.00	13	0.077
407	A	2	1	1.00	13	0.077
408	A	2	1	1.00	11	0.091
409	A	1	1	1.00	9	0.111
410	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	5	5	1.00	13	0.385
412	A	6	5	1.00	13	0.385
413	A	2	1	1.00	13	0.077
414	A	2	1	1.00	13	0.077
415	A	2	1	1.00	11	0.091
416	A	1	1	1.00	9	0.111
417	A	5	5	1.00	13	0.385
418	A	6	6	1.00	13	0.462
419	A	7	6	1.00	13	0.462
420	A	4	4	1.00	17	0.235
421	A	4	4	1.00	18	0.222
422	A	4	4	1.00	19	0.210
423	A	4	4	1.00	20	0.200
424	A	4	4	1.00	17	0.235
425	A	4	4	1.00	18	0.222
426	A	4	4	1.00	19	0.210
427	A	4	4	1.00	20	0.200
428	A	2	1	1.00	9	0.111
429	A	2	1	1.00	11	0.091
430	A	2	1	1.00	11	0.091
431	A	2	1	1.00	11	0.091
432	A	2	1	1.00	11	0.091
433	A	2	1	1.00	11	0.091
434	A	2	1	1.00	11	0.091
435	A	2	1	1.00	11	0.091
436	A	2	1	1.00	13	0.077
437	A	2	1	1.00	13	0.077
438	A	2	1	1.00	13	0.077
439	A	2	1	1.00	13	0.077
440	A	2	1	1.00	13	0.077
441	A	2	1	1.00	13	0.077
442	A	2	1	1.00	11	0.091
443	A	2	1	1.00	13	0.077
444	A	2	1	1.00	13	0.077
445	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	2	1	1.00	13	0.077
447	A	2	1	1.00	13	0.077
448	A	2	1	1.00	13	0.077
449	A	5	3	1.00	13	0.231
450	A	4	3	1.00	13	0.231
451	A	3	3	1.00	13	0.231
452	A	2	2	1.00	13	0.154
453	A	3	3	1.00	13	0.231
454	A	4	3	1.00	13	0.231
455	A	5	3	1.00	13	0.231
456	A	5	4	1.00	13	0.308
457	A	4	4	1.00	13	0.308
458	A	3	3	1.00	13	0.231
459	A	3	3	1.00	13	0.231
460	A	4	4	1.00	13	0.308
461	A	5	4	1.00	13	0.308
462	A	6	4	1.00	13	0.308
463	A	5	4	1.00	13	0.308
464	A	4	3	1.00	13	0.231
465	A	4	4	1.00	13	0.308
466	A	4	3	1.00	13	0.231
467	A	5	4	1.00	13	0.308
468	A	6	4	1.00	13	0.308
469	A	5	3	1.00	15	0.200
470	A	4	3	1.00	15	0.200
471	A	3	3	1.00	15	0.200
472	A	2	2	1.00	15	0.133
473	A	3	3	1.00	15	0.200
474	A	4	3	1.00	15	0.200
475	A	5	3	1.00	15	0.200
476	A	5	4	1.00	15	0.267
477	A	4	4	1.00	15	0.267
478	A	3	3	1.00	15	0.200
479	A	3	3	1.00	15	0.200
480	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	5	4	1.00	15	0.267
482	A	6	4	1.00	15	0.267
483	A	5	4	1.00	15	0.267
484	A	4	3	1.00	15	0.200
485	A	4	4	1.00	15	0.267
486	A	4	3	1.00	15	0.200
487	A	5	4	1.00	15	0.267
488	A	6	4	1.00	15	0.267
489	A	7	4	1.00	15	0.267
490	A	6	4	1.00	15	0.267
491	A	5	4	1.00	15	0.267
492	A	4	4	1.00	15	0.267
493	A	4	4	1.00	15	0.267
494	A	1	1	1.00	15	0.067
495	A	2	2	1.00	15	0.133
496	A	3	2	1.00	15	0.133
497	A	7	4	1.00	16	0.250
498	A	6	4	1.00	16	0.250
499	A	5	4	1.00	16	0.250
500	A	4	4	1.00	16	0.250
501	A	4	4	1.00	16	0.250
502	A	1	1	1.00	16	0.062
503	A	2	2	1.00	16	0.125
504	A	3	2	1.00	16	0.125
505	A	6	3	1.00	15	0.200
506	A	5	3	1.00	15	0.200
507	A	4	3	1.00	15	0.200
508	A	3	3	1.00	15	0.200
509	A	3	3	1.00	15	0.200
510	A	1	1	1.00	15	0.067
511	A	2	2	1.00	15	0.133
512	A	3	2	1.00	15	0.133
513	A	6	3	1.00	16	0.188
514	A	5	3	1.00	16	0.188
515	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
516	A	3	3	1.00	16	0.188
517	A	3	3	1.00	16	0.188
518	A	1	1	1.00	16	0.062
519	A	2	2	1.00	16	0.125
520	A	3	2	1.00	16	0.125
521	A	8	4	1.00	15	0.267
522	A	7	4	1.00	15	0.267
523	A	6	4	1.00	15	0.267
524	A	5	4	1.00	15	0.267
525	A	5	5	1.00	15	0.333
526	A	5	4	1.00	15	0.267
527	A	8	4	1.00	16	0.250
528	A	7	4	1.00	16	0.250
529	A	6	4	1.00	16	0.250
530	A	5	4	1.00	16	0.250
531	A	5	5	1.00	16	0.312
532	A	5	4	1.00	16	0.250
533	A	7	3	1.00	15	0.200
534	A	6	3	1.00	15	0.200
535	A	5	3	1.00	15	0.200
536	A	4	3	1.00	15	0.200
537	A	4	4	1.00	15	0.267
538	A	4	3	1.00	15	0.200
539	A	7	3	1.00	16	0.188
540	A	6	3	1.00	16	0.188
541	A	5	3	1.00	16	0.188
542	A	4	3	1.00	16	0.188
543	A	4	4	1.00	16	0.250
544	A	4	3	1.00	16	0.188
545	A	9	4	1.00	15	0.267
546	A	8	4	1.00	15	0.267
547	A	7	4	1.00	15	0.267
548	A	6	4	1.00	15	0.267
549	A	6	5	1.00	15	0.333
550	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	9	4	1.00	16	0.250
552	A	8	4	1.00	16	0.250
553	A	7	4	1.00	16	0.250
554	A	6	4	1.00	16	0.250
555	A	6	5	1.00	16	0.312
556	A	6	5	1.00	16	0.312
557	A	8	3	1.00	15	0.200
558	A	7	3	1.00	15	0.200
559	A	6	3	1.00	15	0.200
560	A	5	3	1.00	15	0.200
561	A	5	4	1.00	15	0.267
562	A	5	4	1.00	15	0.267
563	A	8	3	1.00	16	0.188
564	A	7	3	1.00	16	0.188
565	A	6	3	1.00	16	0.188
566	A	5	3	1.00	16	0.188
567	A	5	4	1.00	16	0.250
568	A	5	4	1.00	16	0.250
569	A	6	4	1.00	15	0.267
570	A	5	4	1.00	15	0.267
571	A	4	4	1.00	15	0.267
572	A	3	3	1.00	15	0.200
573	A	1	1	1.00	15	0.067
574	A	2	2	1.00	15	0.133
575	A	3	2	1.00	15	0.133
576	A	4	2	1.00	15	0.133
577	A	6	5	1.00	15	0.333
578	A	5	5	1.00	15	0.333
579	A	4	4	1.00	15	0.267
580	A	1	1	1.00	15	0.067
581	A	2	2	1.00	15	0.133
582	A	3	2	1.00	15	0.133
583	A	4	2	1.00	15	0.133
584	A	6	5	1.00	15	0.333
585	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	1	1	1.00	15	0.067
587	A	2	2	1.00	15	0.133
588	A	3	2	1.00	15	0.133
589	A	4	2	1.00	15	0.133
590	A	6	4	1.00	16	0.250
591	A	5	4	1.00	16	0.250
592	A	4	4	1.00	16	0.250
593	A	3	3	1.00	16	0.188
594	A	1	1	1.00	16	0.062
595	A	2	2	1.00	16	0.125
596	A	6	5	1.00	16	0.312
597	A	5	5	1.00	16	0.312
598	A	4	4	1.00	16	0.250
599	A	1	1	1.00	16	0.062
600	A	2	2	1.00	16	0.125
601	A	3	2	1.00	16	0.125
602	A	6	5	1.00	16	0.312
603	A	5	4	1.00	16	0.250
604	A	1	1	1.00	16	0.062
605	A	2	2	1.00	16	0.125
606	A	3	2	1.00	16	0.125
607	A	4	2	1.00	16	0.125
608	A	5	3	1.00	15	0.200
609	A	4	3	1.00	15	0.200
610	A	3	3	1.00	15	0.200
611	A	2	2	1.00	15	0.133
612	A	1	1	1.00	15	0.067
613	A	2	2	1.00	15	0.133
614	A	3	2	1.00	15	0.133
615	A	4	2	1.00	15	0.133
616	A	5	4	1.00	15	0.267
617	A	4	4	1.00	15	0.267
618	A	3	3	1.00	15	0.200
619	A	1	1	1.00	15	0.067
620	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	3	2	1.00	15	0.133
622	A	4	2	1.00	15	0.133
623	A	5	4	1.00	15	0.267
624	A	4	3	1.00	15	0.200
625	A	1	1	1.00	15	0.067
626	A	2	2	1.00	15	0.133
627	A	3	2	1.00	15	0.133
628	A	4	2	1.00	15	0.133
629	A	5	3	1.00	16	0.188
630	A	4	3	1.00	16	0.188
631	A	3	3	1.00	16	0.188
632	A	2	2	1.00	16	0.125
633	A	1	1	1.00	16	0.062
634	A	2	2	1.00	16	0.125
635	A	5	4	1.00	16	0.250
636	A	4	4	1.00	16	0.250
637	A	3	3	1.00	16	0.188
638	A	1	1	1.00	16	0.062
639	A	2	2	1.00	16	0.125
640	A	3	2	1.00	16	0.125
641	A	5	4	1.00	16	0.250
642	A	4	3	1.00	16	0.188
643	A	1	1	1.00	16	0.062
644	A	2	2	1.00	16	0.125
645	A	3	2	1.00	16	0.125
646	A	4	2	1.00	16	0.125
647	A	4	4	1.00	15	0.267
648	A	3	3	1.00	15	0.200
649	A	2	2	1.00	16	0.125
650	A	2	1	1.00	11	0.091
651	A	2	1	1.00	11	0.091
652	A	2	1	1.00	11	0.091
653	A	2	1	1.00	11	0.091
654	A	2	1	1.00	11	0.091
655	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	2	1	1.00	11	0.091
657	A	2	1	1.00	11	0.091
658	A	2	1	1.00	13	0.077
659	A	2	1	1.00	13	0.077
660	A	2	1	1.00	13	0.077
661	A	2	1	1.00	13	0.077
662	A	2	1	1.00	13	0.077
663	A	2	1	1.00	13	0.077
664	A	2	1	1.00	13	0.077
665	A	2	1	1.00	13	0.077
666	A	2	1	1.00	13	0.077
667	A	2	1	1.00	13	0.077
668	A	2	1	1.00	13	0.077
669	A	2	1	1.00	13	0.077
670	A	2	1	1.00	13	0.077
671	A	2	1	1.00	13	0.077
672	A	2	1	1.00	13	0.077
673	A	2	1	1.00	13	0.077
674	A	6	5	1.00	13	0.385
675	A	6	5	1.00	13	0.385
676	A	5	5	1.00	13	0.385
677	A	5	5	1.00	13	0.385
678	A	4	4	1.00	13	0.308
679	A	4	4	1.00	13	0.308
680	A	5	5	1.00	13	0.385
681	A	5	5	1.00	13	0.385
682	A	6	6	1.00	13	0.462
683	A	6	6	1.00	13	0.462
684	A	5	5	1.00	13	0.385
685	A	5	5	1.00	13	0.385
686	A	5	5	1.00	13	0.385
687	A	5	5	1.00	13	0.385
688	A	6	6	1.00	13	0.462
689	A	6	6	1.00	13	0.462
690	A	6	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	6	5	1.00	13	0.385
692	A	6	6	1.00	13	0.462
693	A	6	6	1.00	13	0.462
694	A	6	5	1.00	13	0.385
695	A	6	5	1.00	13	0.385
696	A	7	6	1.00	13	0.462
697	A	7	6	1.00	13	0.462
698	A	5	5	1.00	15	0.333
699	A	2	1	1.00	11	0.091
700	A	2	1	1.00	11	0.091
701	A	2	1	1.00	11	0.091
702	A	2	1	1.00	11	0.091
703	A	2	1	1.00	9	0.111
704	A	1	1	1.00	11	0.091
705	A	1	1	1.00	11	0.091
706	A	1	1	1.00	11	0.091
707	A	2	2	1.00	13	0.154
708	A	2	2	1.00	13	0.154
709	A	2	2	1.00	13	0.154
710	A	2	2	1.00	13	0.154
711	A	2	2	1.00	13	0.154
712	A	2	2	1.00	13	0.154
713	A	2	2	1.00	15	0.133
714	A	2	2	1.00	15	0.133
715	A	2	2	1.00	13	0.154
716	A	2	2	1.00	15	0.133
717	A	2	2	1.00	15	0.133
718	A	2	2	1.00	15	0.133
719	A	1	1	1.00	13	0.077
720	A	1	1	1.00	13	0.077
721	A	1	1	1.00	13	0.077
722	A	3	3	1.00	13	0.231
723	A	2	2	1.00	15	0.133
724	A	1	1	1.00	15	0.067
725	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
726	A	3	3	1.00	15	0.200
727	A	1	1	1.00	15	0.067
728	A	1	1	1.00	13	0.077
729	A	1	1	1.00	14	0.071
730	A	2	2	1.00	11	0.182
731	A	2	2	1.00	13	0.154
732	A	2	1	1.00	11	0.091
733	A	2	1	1.00	11	0.091
734	A	2	1	1.00	9	0.111
735	A	1	1	1.00	7	0.143
736	A	1	1	1.00	11	0.091
737	A	1	1	1.00	11	0.091
738	A	1	1	1.00	11	0.091
739	A	3	2	1.00	15	0.133
740	A	2	2	1.00	15	0.133
741	A	1	1	1.00	15	0.067
742	A	2	2	1.00	15	0.133
743	A	2	2	1.00	13	0.154
744	A	2	2	1.00	15	0.133
745	A	2	2	1.00	13	0.154
746	A	2	2	1.00	13	0.154
747	A	2	2	1.00	13	0.154
748	A	2	2	1.00	13	0.154
749	A	2	2	1.00	13	0.154
750	A	1	1	1.00	13	0.077
751	A	1	1	1.00	15	0.067
752	A	2	2	1.00	13	0.154
753	A	1	1	1.00	17	0.059
754	A	2	2	1.00	15	0.133
755	A	2	2	1.00	19	0.105
756	A	3	2	1.00	18	0.111
757	A	3	2	1.00	18	0.111
758	A	3	2	1.00	16	0.125
759	A	3	2	1.00	15	0.133
760	A	2	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	3	2	1.00	18	0.111
762	A	3	2	1.00	18	0.111
763	A	2	2	1.00	18	0.111
764	A	3	2	1.00	18	0.111
765	A	3	2	1.00	18	0.111
766	A	3	2	1.00	16	0.125
767	A	3	2	1.00	15	0.133
768	A	3	2	1.00	18	0.111
769	A	3	2	1.00	18	0.111
770	A	2	1	1.00	18	0.056
771	A	3	2	1.00	18	0.111
772	A	3	2	1.00	18	0.111
773	A	3	2	1.00	18	0.111
774	A	3	2	1.00	16	0.125
775	A	3	2	1.00	15	0.133
776	A	3	2	1.00	18	0.111
777	A	3	2	1.00	18	0.111
778	A	3	2	1.00	18	0.111
779	A	3	2	1.00	18	0.111
780	A	3	2	1.00	18	0.111
781	A	3	2	1.00	18	0.111
782	A	2	1	1.00	16	0.062
783	A	3	2	1.00	15	0.133
784	A	3	2	1.00	18	0.111
785	A	2	2	1.00	18	0.111
786	A	3	2	1.00	18	0.111
787	A	3	2	1.00	18	0.111
788	A	2	1	1.00	18	0.056
789	A	3	2	1.00	18	0.111
790	A	3	2	1.00	16	0.125
791	A	2	2	1.00	15	0.133
792	A	3	2	1.00	18	0.111
793	A	3	2	1.00	18	0.111
794	A	3	2	1.00	18	0.111
795	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	3	2	1.00	18	0.111
797	A	2	2	1.00	18	0.111
798	A	3	2	1.00	16	0.125
799	A	3	2	1.00	15	0.133
800	A	3	2	1.00	18	0.111
801	A	3	2	1.00	18	0.111
802	A	3	2	1.00	18	0.111
803	A	3	2	1.00	18	0.111
804	A	3	2	1.00	20	0.100
805	A	3	2	1.00	20	0.100
806	A	3	2	1.00	18	0.111
807	A	3	2	1.00	17	0.118
808	A	2	2	1.00	20	0.100
809	A	3	2	1.00	20	0.100
810	A	3	2	1.00	20	0.100
811	A	3	2	1.00	20	0.100
812	A	3	2	1.00	20	0.100
813	A	3	2	1.00	20	0.100
814	A	3	2	1.00	18	0.111
815	A	3	2	1.00	17	0.118
816	A	3	2	1.00	20	0.100
817	A	3	2	1.00	20	0.100
818	A	2	2	1.00	20	0.100
819	A	3	2	1.00	20	0.100
820	A	3	2	1.00	18	0.111
821	A	3	2	1.00	17	0.118
822	A	3	2	1.00	20	0.100
823	A	3	2	1.00	20	0.100
824	A	3	2	1.00	20	0.100
825	A	3	2	1.00	20	0.100
826	A	2	2	1.00	20	0.100
827	A	3	2	1.00	20	0.100
828	A	3	2	1.00	20	0.100
829	A	3	2	1.00	20	0.100
830	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	3	2	1.00	17	0.118
832	A	3	2	1.00	20	0.100
833	A	3	2	1.00	20	0.100
834	A	2	2	1.00	20	0.100
835	A	3	2	1.00	20	0.100
836	A	2	2	1.00	20	0.100
837	A	3	2	1.00	20	0.100
838	A	3	2	1.00	18	0.111
839	A	3	2	1.00	17	0.118
840	A	2	2	1.00	20	0.100
841	A	3	2	1.00	20	0.100
842	A	3	2	1.00	20	0.100
843	A	3	2	1.00	20	0.100
844	A	3	2	1.00	20	0.100
845	A	3	2	1.00	20	0.100
846	A	2	2	1.00	18	0.111
847	A	3	2	1.00	17	0.118
848	A	3	2	1.00	20	0.100
849	A	3	2	1.00	20	0.100
850	A	3	2	1.00	20	0.100
851	A	3	2	1.00	20	0.100
852	A	3	2	1.00	20	0.100
853	A	3	2	1.00	20	0.100
854	A	3	2	1.00	18	0.111
855	A	3	2	1.00	17	0.118
856	A	2	2	1.00	20	0.100
857	A	4	4	1.00	20	0.200
858	A	3	2	1.00	20	0.100
859	A	3	2	1.00	20	0.100
860	A	3	2	1.00	18	0.111
861	A	3	2	1.00	17	0.118
862	A	3	2	1.00	20	0.100
863	A	3	2	1.00	20	0.100
864	A	2	2	1.00	20	0.100
865	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	3	2	1.00	20	0.100
867	A	3	2	1.00	20	0.100
868	A	3	2	1.00	20	0.100
869	A	3	2	1.00	17	0.118
870	A	3	2	1.00	20	0.100
871	A	3	2	1.00	20	0.100
872	A	3	2	1.00	20	0.100
873	A	3	2	1.00	20	0.100
874	A	2	2	1.00	20	0.100
875	A	4	4	1.00	20	0.200
876	A	3	2	1.00	20	0.100
877	A	3	2	1.00	20	0.100
878	A	3	2	1.00	20	0.100
879	A	3	2	1.00	20	0.100
880	A	2	2	1.00	18	0.111
881	A	4	4	1.00	17	0.235
882	A	3	2	1.00	20	0.100
883	A	3	2	1.00	20	0.100
884	A	3	2	1.00	20	0.100
885	A	3	2	1.00	20	0.100
886	A	3	2	1.00	20	0.100
887	A	3	2	1.00	20	0.100
888	A	2	2	1.00	20	0.100
889	A	4	4	1.00	20	0.200
890	A	3	2	1.00	18	0.111
891	A	3	2	1.00	17	0.118
892	A	3	2	1.00	20	0.100
893	A	3	2	1.00	20	0.100
894	A	3	2	1.00	20	0.100
895	A	3	2	1.00	18	0.111
896	A	3	2	1.00	17	0.118
897	A	2	2	1.00	20	0.100
898	A	3	2	1.00	20	0.100
899	A	3	2	1.00	20	0.100
900	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
901	A	3	2	1.00	18	0.111
902	A	3	2	1.00	17	0.118
903	A	3	2	1.00	20	0.100
904	A	3	2	1.00	20	0.100
905	A	2	2	1.00	20	0.100
906	A	3	2	1.00	20	0.100
907	A	3	2	1.00	20	0.100
908	A	3	2	1.00	20	0.100
909	A	3	2	1.00	20	0.100
910	A	3	2	1.00	20	0.100
911	A	3	2	1.00	20	0.100
912	A	3	2	1.00	20	0.100
913	A	2	2	1.00	18	0.111
914	A	3	2	1.00	17	0.118
915	A	3	2	1.00	20	0.100
916	A	3	2	1.00	20	0.100
917	A	3	2	1.00	20	0.100
918	A	3	2	1.00	20	0.100
919	A	2	2	1.00	20	0.100
920	A	3	2	1.00	20	0.100
921	A	3	2	1.00	18	0.111
922	A	3	2	1.00	17	0.118
923	A	3	2	1.00	20	0.100
924	A	3	2	1.00	18	0.111
925	A	3	2	1.00	17	0.118
926	A	2	2	1.00	20	0.100
927	A	2	2	1.00	20	0.100
928	A	2	2	1.00	20	0.100
929	A	2	2	1.00	20	0.100
930	A	3	2	1.00	18	0.111
931	A	3	2	1.00	17	0.118
932	A	3	2	1.00	20	0.100
933	A	3	2	1.00	20	0.100
934	A	2	2	1.00	20	0.100
935	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	2	2	1.00	20	0.100
937	A	2	2	1.00	20	0.100
938	A	3	2	1.00	17	0.118
939	A	3	2	1.00	20	0.100
940	A	3	2	1.00	20	0.100
941	A	3	2	1.00	20	0.100
942	A	3	2	1.00	20	0.100
943	A	2	2	1.00	20	0.100
944	A	2	2	1.00	20	0.100
945	A	2	2	1.00	20	0.100
946	A	3	2	1.00	20	0.100
947	A	3	2	1.00	20	0.100
948	A	3	2	1.00	20	0.100
949	A	2	2	1.00	18	0.111
950	A	2	2	1.00	17	0.118
951	A	2	2	1.00	20	0.100
952	A	2	2	1.00	20	0.100
953	A	3	2	1.00	20	0.100
954	A	3	2	1.00	20	0.100
955	A	3	2	1.00	20	0.100
956	A	2	2	1.00	20	0.100
957	A	2	2	1.00	20	0.100
958	A	2	2	1.00	18	0.111
959	A	2	2	1.00	17	0.118
960	A	2	2	1.00	20	0.100
961	A	3	2	1.00	20	0.100
962	A	3	2	1.00	20	0.100
963	A	3	2	1.00	20	0.100
964	A	2	2	1.00	20	0.100
965	A	2	2	1.00	20	0.100
966	A	2	2	1.00	20	0.100
967	A	2	2	1.00	20	0.100
968	A	2	2	1.00	18	0.111
969	A	4	3	1.00	20	0.150
970	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	4	3	1.00	20	0.150
972	A	4	3	1.00	20	0.150
973	A	4	3	1.00	20	0.150
974	A	4	3	1.00	20	0.150
975	A	4	3	1.00	22	0.136
976	A	4	3	1.00	22	0.136
977	A	4	3	1.00	22	0.136
978	A	4	3	1.00	22	0.136
979	A	4	3	1.00	22	0.136
980	A	4	3	1.00	22	0.136
981	A	4	4	1.00	22	0.182
982	A	4	4	1.00	22	0.182
983	A	4	4	1.00	22	0.182
984	A	4	4	1.00	22	0.182
985	A	4	4	1.00	22	0.182
986	A	4	4	1.00	22	0.182
987	A	2	2	1.00	22	0.091
988	A	2	2	1.00	22	0.091
989	A	2	2	1.00	20	0.100
990	A	2	2	1.00	19	0.105
991	A	2	2	1.00	22	0.091
992	A	2	2	1.00	20	0.100
993	A	2	2	1.00	22	0.091
994	A	2	2	1.00	22	0.091
995	A	2	2	1.00	25	0.080
996	A	3	3	1.00	27	0.111
997	A	3	3	1.00	18	0.167
998	A	4	4	0.94	20	0.200
999	A	2	2	1.00	20	0.100
1000	A	2	1	1.00	20	0.050
1001	A	2	2	1.00	20	0.100
1002	A	2	2	1.00	20	0.100
1003	A	2	2	1.00	18	0.111
1004	A	2	2	1.00	20	0.100
1005	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	2	2	1.00	20	0.100
1007	A	2	2	1.00	20	0.100
1008	A	2	1	1.00	20	0.050
1009	A	2	2	1.00	20	0.100
1010	A	2	2	1.00	20	0.100
1011	A	2	2	1.00	18	0.111
1012	A	2	2	1.00	20	0.100
1013	A	2	2	1.00	20	0.100
1014	A	2	2	1.00	20	0.100
1015	A	2	2	1.00	18	0.111
1016	A	2	2	1.00	18	0.111
1017	A	2	2	1.00	18	0.111
1018	A	2	2	1.00	16	0.125
1019	A	2	2	1.00	18	0.111
1020	A	2	2	1.00	18	0.111
1021	A	2	2	1.00	18	0.111
1022	A	2	1	1.00	18	0.056
1023	A	2	2	1.00	18	0.111
1024	A	2	2	1.00	18	0.111
1025	A	2	2	1.00	18	0.111
1026	A	2	2	1.00	18	0.111
1027	A	2	2	1.00	19	0.105
1028	A	2	1	1.00	17	0.059
1029	A	2	1	1.00	17	0.059
1030	A	2	1	1.00	15	0.067
1031	A	1	0	1.00	5	0.000
1032	A	2	1	1.00	17	0.059
1033	A	2	1	1.00	17	0.059
1034	A	1	1	1.00	17	0.059
1035	A	2	1	1.00	17	0.059
1036	A	2	1	1.00	17	0.059
1037	A	2	1	1.00	17	0.059
1038	A	2	1	1.00	19	0.053
1039	A	3	2	1.00	19	0.105
1040	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1041	A	1	1	1.00	7	0.143
1042	A	2	1	1.00	19	0.053
1043	A	2	1	1.00	19	0.053
1044	A	2	1	1.00	19	0.053
1045	A	1	1	1.00	19	0.053
1046	A	2	1	1.00	19	0.053
1047	A	2	1	1.00	19	0.053
1048	A	2	1	1.00	19	0.053
1049	A	2	1	1.00	19	0.053
1050	A	2	1	1.00	19	0.053
1051	A	2	1	1.00	17	0.059
1052	A	1	1	1.00	7	0.143
1053	A	2	2	1.00	19	0.105
1054	A	3	2	1.00	19	0.105
1055	A	3	2	1.00	19	0.105
1056	A	2	1	1.00	19	0.053
1057	A	2	1	1.00	19	0.053
1058	A	2	1	1.00	17	0.059
1059	A	1	1	1.00	7	0.143
1060	A	3	2	1.00	19	0.105
1061	A	3	3	1.00	19	0.158
1062	A	3	2	1.00	19	0.105
1063	A	7	4	1.00	17	0.235
1064	A	6	4	1.00	17	0.235
1065	A	5	4	1.00	17	0.235
1066	A	4	4	1.00	17	0.235
1067	A	3	3	1.00	17	0.176
1068	A	3	3	1.00	17	0.176
1069	A	3	3	1.00	17	0.176
1070	A	1	1	1.00	17	0.059
1071	A	2	2	1.00	17	0.118
1072	A	3	2	1.00	17	0.118
1073	A	4	2	1.00	17	0.118
1074	A	5	2	1.00	17	0.118
1075	A	7	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1076	A	6	4	1.00	17	0.235
1077	A	5	4	1.00	17	0.235
1078	A	4	3	1.00	17	0.176
1079	A	4	4	1.00	17	0.235
1080	A	4	3	1.00	17	0.176
1081	A	4	4	1.00	17	0.235
1082	A	4	3	1.00	17	0.176
1083	A	1	1	1.00	17	0.059
1084	A	2	2	1.00	17	0.118
1085	A	3	2	1.00	17	0.118
1086	A	4	2	1.00	17	0.118
1087	A	5	2	1.00	17	0.118
1088	A	8	4	1.00	17	0.235
1089	A	7	4	1.00	17	0.235
1090	A	6	4	1.00	17	0.235
1091	A	5	3	1.00	17	0.176
1092	A	5	4	1.00	17	0.235
1093	A	5	4	1.00	17	0.235
1094	A	5	3	1.00	17	0.176
1095	A	5	4	1.00	17	0.235
1096	A	5	4	1.00	17	0.235
1097	A	5	3	1.00	17	0.176
1098	A	1	1	1.00	17	0.059
1099	A	2	2	1.00	17	0.118
1100	A	3	2	1.00	17	0.118
1101	A	4	2	1.00	17	0.118
1102	A	5	2	1.00	17	0.118
1103	A	6	2	1.00	17	0.118
1104	A	4	3	1.00	20	0.150
1105	A	3	3	1.00	28	0.107
1106	A	6	3	1.00	17	0.176
1107	A	5	3	1.00	17	0.176
1108	A	4	3	1.00	17	0.176
1109	A	3	3	1.00	17	0.176
1110	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1111	A	1	1	1.00	17	0.059
1112	A	2	2	1.00	17	0.118
1113	A	3	2	1.00	17	0.118
1114	A	4	2	1.00	17	0.118
1115	A	5	2	1.00	17	0.118
1116	A	6	4	1.00	17	0.235
1117	A	5	4	1.00	17	0.235
1118	A	4	4	1.00	17	0.235
1119	A	3	3	1.00	17	0.176
1120	A	1	1	1.00	17	0.059
1121	A	1	1	1.00	17	0.059
1122	A	2	2	1.00	17	0.118
1123	A	3	2	1.00	17	0.118
1124	A	4	2	1.00	17	0.118
1125	A	5	2	1.00	17	0.118
1126	A	7	4	1.00	17	0.235
1127	A	6	4	1.00	17	0.235
1128	A	5	4	1.00	17	0.235
1129	A	4	3	1.00	17	0.176
1130	A	1	1	1.00	17	0.059
1131	A	2	2	1.00	17	0.118
1132	A	3	2	1.00	17	0.118
1133	A	2	2	1.00	17	0.118
1134	A	3	3	1.00	17	0.176
1135	A	4	3	1.00	17	0.176
1136	A	5	3	1.00	17	0.176
1137	A	6	4	1.00	20	0.200
1138	A	5	4	1.00	20	0.200
1139	A	4	4	1.00	20	0.200
1140	A	3	3	1.00	20	0.150
1141	A	1	1	1.00	20	0.050
1142	A	2	2	1.00	20	0.100
1143	A	3	2	1.00	20	0.100
1144	A	4	2	1.00	20	0.100
1145	A	6	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1146	A	5	4	1.00	23	0.174
1147	A	4	4	1.00	23	0.174
1148	A	3	3	1.00	23	0.130
1149	A	1	1	1.00	23	0.043
1150	A	2	2	1.00	23	0.087
1151	A	3	2	1.00	23	0.087
1152	A	4	2	1.00	23	0.087
1153	A	5	3	1.00	19	0.158
1154	A	4	3	1.00	19	0.158
1155	A	3	3	1.00	19	0.158
1156	A	2	2	1.00	19	0.105
1157	A	1	1	1.00	19	0.053
1158	A	2	2	1.00	19	0.105
1159	A	3	2	1.00	19	0.105
1160	A	7	4	1.00	17	0.235
1161	A	5	4	1.00	17	0.235
1162	A	3	3	1.00	17	0.176
1163	A	2	2	1.00	17	0.118
1164	A	4	2	1.00	17	0.118
1165	A	1	1	1.00	17	0.059
1166	A	1	1	1.00	20	0.050
1167	A	1	1	1.00	17	0.059
1168	A	1	1	1.00	20	0.050
1169	A	3	3	1.00	23	0.130
1170	A	11	8	1.00	20	0.400
1171	A	6	5	1.00	25	0.200
1172	A	5	5	1.00	25	0.200
1173	A	4	4	1.00	25	0.160
1174	A	4	4	1.00	25	0.160
1175	A	4	4	1.00	25	0.160
1176	A	5	5	1.00	25	0.200
1177	A	6	5	1.00	25	0.200
1178	A	12	9	1.00	25	0.360
1179	A	11	8	1.00	25	0.320
1180	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1181	A	2	2	1.00	25	0.080
1182	A	3	2	1.00	25	0.080
1183	A	4	2	1.00	25	0.080
1184	A	12	9	1.00	25	0.360
1185	A	11	8	1.00	25	0.320
1186	A	1	1	1.00	25	0.040
1187	A	2	2	1.00	25	0.080
1188	A	3	2	1.00	25	0.080
1189	A	5	4	1.00	25	0.160
1190	A	4	4	1.00	25	0.160
1191	A	3	3	1.00	25	0.120
1192	A	4	4	1.00	25	0.160
1193	A	5	4	1.00	25	0.160
1194	A	13	10	1.00	25	0.400
1195	A	12	9	1.00	25	0.360
1196	A	1	1	1.00	25	0.040
1197	A	2	2	1.00	25	0.080
1198	A	3	2	1.00	25	0.080
1199	A	6	5	1.00	25	0.200
1200	A	5	5	1.00	25	0.200
1201	A	4	4	1.00	25	0.160
1202	A	4	4	1.00	25	0.160
1203	A	4	4	1.00	25	0.160
1204	A	5	5	1.00	25	0.200
1205	A	6	5	1.00	25	0.200
1206	A	6	6	1.00	25	0.240
1207	A	5	5	1.00	25	0.200
1208	A	4	4	1.00	25	0.160
1209	A	3	3	1.00	25	0.120
1210	A	4	4	1.00	25	0.160
1211	A	5	4	1.00	25	0.160
1212	A	13	10	1.00	25	0.400
1213	A	12	9	1.00	25	0.360
1214	A	1	1	1.00	25	0.040
1215	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1216	A	3	2	1.00	25	0.080
1217	A	6	5	1.00	25	0.200
1218	A	5	5	1.00	25	0.200
1219	A	4	4	1.00	25	0.160
1220	A	5	5	1.00	25	0.200
1221	A	4	4	1.00	25	0.160
1222	A	5	5	1.00	25	0.200
1223	A	6	5	1.00	25	0.200
1224	A	13	9	1.00	25	0.360
1225	A	1	1	1.00	25	0.040
1226	A	2	2	1.00	25	0.080
1227	A	3	2	1.00	25	0.080
1228	A	4	2	1.00	25	0.080
1229	A	2	1	1.00	19	0.053
1230	A	2	1	1.00	17	0.059
1231	A	1	1	1.00	19	0.053
1232	A	1	1	1.00	19	0.053
1233	A	3	3	1.00	16	0.188
1234	A	3	3	1.00	19	0.158
1235	A	2	2	1.00	15	0.133
1236	A	2	1	1.00	13	0.077
1237	A	2	1	1.00	13	0.077
1238	A	2	1	1.00	13	0.077
1239	A	2	1	1.00	11	0.091
1240	A	1	0	1.00	5	0.000
1241	A	2	1	1.00	13	0.077
1242	A	2	1	1.00	13	0.077
1243	A	1	1	1.00	13	0.077
1244	A	2	1	1.00	13	0.077
1245	A	2	1	1.00	13	0.077
1246	A	2	1	1.00	15	0.067
1247	A	2	1	1.00	15	0.067
1248	A	2	1	1.00	15	0.067
1249	A	2	1	1.00	13	0.077
1250	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1251	A	2	1	1.00	15	0.067
1252	A	2	1	1.00	15	0.067
1253	A	2	1	1.00	15	0.067
1254	A	1	1	1.00	15	0.067
1255	A	2	1	1.00	15	0.067
1256	A	2	1	1.00	15	0.067
1257	A	2	1	1.00	15	0.067
1258	A	2	1	1.00	15	0.067
1259	A	2	1	1.00	15	0.067
1260	A	2	1	1.00	15	0.067
1261	A	2	1	1.00	15	0.067
1262	A	2	1	1.00	13	0.077
1263	A	1	1	1.00	7	0.143
1264	A	2	1	1.00	15	0.067
1265	A	2	1	1.00	15	0.067
1266	A	2	1	1.00	15	0.067
1267	A	2	1	1.00	15	0.067
1268	A	1	1	1.00	15	0.067
1269	A	2	2	1.00	15	0.133
1270	A	2	1	1.00	15	0.067
1271	A	2	1	1.00	15	0.067
1272	A	2	1	1.00	15	0.067
1273	A	2	1	1.00	15	0.067
1274	A	2	1	1.00	15	0.067
1275	A	2	1	1.00	15	0.067
1276	A	2	1	1.00	15	0.067
1277	A	2	1	1.00	15	0.067
1278	A	2	1	1.00	15	0.067
1279	A	2	1	1.00	15	0.067
1280	A	2	1	1.00	15	0.067
1281	A	2	1	1.00	13	0.077
1282	A	1	1	1.00	7	0.143
1283	A	2	1	1.00	15	0.067
1284	A	2	1	1.00	15	0.067
1285	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1286	A	2	1	1.00	15	0.067
1287	A	2	1	1.00	15	0.067
1288	A	2	1	1.00	15	0.067
1289	A	2	1	1.00	15	0.067
1290	A	2	1	1.00	15	0.067
1291	A	1	1	1.00	15	0.067
1292	A	2	2	1.00	15	0.133
1293	A	3	2	1.00	15	0.133
1294	A	4	2	1.00	15	0.133
1295	A	5	2	1.00	15	0.133
1296	A	2	1	1.00	15	0.067
1297	A	2	1	1.00	15	0.067
1298	A	2	1	1.00	15	0.067
1299	A	2	1	1.00	15	0.067
1300	A	2	1	1.00	15	0.067
1301	A	2	1	1.00	15	0.067
1302	A	2	1	1.00	15	0.067
1303	A	2	1	1.00	15	0.067
1304	A	2	1	1.00	15	0.067
1305	A	2	1	1.00	15	0.067
1306	A	2	1	1.00	15	0.067
1307	A	2	1	1.00	15	0.067
1308	A	2	1	1.00	15	0.067
1309	A	2	1	1.00	15	0.067
1310	A	2	1	1.00	13	0.077
1311	A	1	1	1.00	7	0.143
1312	A	2	1	1.00	15	0.067
1313	A	2	1	1.00	15	0.067
1314	A	2	1	1.00	15	0.067
1315	A	2	1	1.00	15	0.067
1316	A	2	1	1.00	15	0.067
1317	A	2	1	1.00	15	0.067
1318	A	2	1	1.00	15	0.067
1319	A	2	1	1.00	15	0.067
1320	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1321	A	2	1	1.00	15	0.067
1322	A	2	1	1.00	15	0.067
1323	A	1	1	1.00	15	0.067
1324	A	2	2	1.00	15	0.133
1325	A	3	2	1.00	15	0.133
1326	A	4	2	1.00	15	0.133
1327	A	5	2	1.00	15	0.133
1328	A	6	2	1.00	15	0.133
1329	A	7	2	1.00	15	0.133
1330	A	8	2	1.00	15	0.133
1331	A	2	1	1.00	15	0.067
1332	A	2	1	1.00	15	0.067
1333	A	2	1	1.00	15	0.067
1334	A	2	1	1.00	15	0.067
1335	A	2	1	1.00	15	0.067
1336	A	2	1	1.00	15	0.067
1337	A	2	1	1.00	15	0.067
1338	A	2	1	1.00	13	0.077
1339	A	1	1	1.00	7	0.143
1340	A	3	2	1.00	15	0.133
1341	A	2	1	1.00	15	0.067
1342	A	2	1	1.00	15	0.067
1343	A	2	1	1.00	15	0.067
1344	A	2	1	1.00	15	0.067
1345	A	2	1	1.00	15	0.067
1346	A	2	1	1.00	15	0.067
1347	A	2	1	1.00	13	0.077
1348	A	1	1	1.00	7	0.143
1349	A	2	1	1.00	15	0.067
1350	A	2	1	1.00	15	0.067
1351	A	2	1	1.00	15	0.067
1352	A	2	1	1.00	15	0.067
1353	A	2	1	1.00	15	0.067
1354	A	2	1	1.00	15	0.067
1355	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1356	A	2	1	1.00	15	0.067
1357	A	1	1	1.00	13	0.077
1358	A	1	1	1.00	7	0.143
1359	A	2	1	1.00	15	0.067
1360	A	2	1	1.00	15	0.067
1361	A	2	1	1.00	15	0.067
1362	A	2	1	1.00	15	0.067
1363	A	2	1	1.00	15	0.067
1364	A	2	1	1.00	15	0.067
1365	A	1	1	1.00	15	0.067
1366	A	2	2	1.00	15	0.133
1367	A	3	2	1.00	15	0.133
1368	A	2	1	1.00	15	0.067
1369	A	2	1	1.00	15	0.067
1370	A	2	1	1.00	13	0.077
1371	A	1	1	1.00	7	0.143
1372	A	2	1	1.00	15	0.067
1373	A	2	1	1.00	15	0.067
1374	A	2	1	1.00	15	0.067
1375	A	2	1	1.00	17	0.059
1376	A	2	1	1.00	17	0.059
1377	A	2	1	1.00	17	0.059
1378	A	2	1	1.00	17	0.059
1379	A	2	1	1.00	15	0.067
1380	A	1	1	1.00	9	0.111
1381	A	3	3	1.00	17	0.176
1382	A	3	3	1.00	17	0.176
1383	A	4	4	1.00	17	0.235
1384	A	5	4	1.00	17	0.235
1385	A	6	4	1.00	17	0.235
1386	A	7	4	1.00	17	0.235
1387	A	2	1	1.00	17	0.059
1388	A	2	1	1.00	17	0.059
1389	A	2	1	1.00	17	0.059
1390	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1391	A	2	1	1.00	15	0.067
1392	A	1	1	1.00	9	0.111
1393	A	4	3	1.00	17	0.176
1394	A	4	4	1.00	17	0.235
1395	A	4	3	1.00	17	0.176
1396	A	5	4	1.00	17	0.235
1397	A	6	4	1.00	17	0.235
1398	A	7	4	1.00	17	0.235
1399	A	2	1	1.00	17	0.059
1400	A	2	1	1.00	17	0.059
1401	A	2	1	1.00	17	0.059
1402	A	2	1	1.00	17	0.059
1403	A	2	1	1.00	15	0.067
1404	A	1	1	1.00	9	0.111
1405	A	5	3	1.00	17	0.176
1406	A	5	4	1.00	17	0.235
1407	A	5	4	1.00	17	0.235
1408	A	5	3	1.00	17	0.176
1409	A	6	4	1.00	17	0.235
1410	A	7	4	1.00	17	0.235
1411	A	3	3	1.00	13	0.231
1412	A	4	4	1.00	13	0.308
1413	A	2	1	1.00	17	0.059
1414	A	2	1	1.00	17	0.059
1415	A	2	1	1.00	17	0.059
1416	A	2	1	1.00	17	0.059
1417	A	2	1	1.00	15	0.067
1418	A	1	1	1.00	9	0.111
1419	A	2	2	1.00	17	0.118
1420	A	3	3	1.00	17	0.176
1421	A	4	3	1.00	17	0.176
1422	A	5	3	1.00	17	0.176
1423	A	6	3	1.00	17	0.176
1424	A	2	1	1.00	17	0.059
1425	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1426	A	2	1	1.00	17	0.059
1427	A	2	1	1.00	17	0.059
1428	A	2	1	1.00	15	0.067
1429	A	1	1	1.00	9	0.111
1430	A	3	3	1.00	17	0.176
1431	A	4	4	1.00	17	0.235
1432	A	5	4	1.00	17	0.235
1433	A	6	4	1.00	17	0.235
1434	A	2	1	1.00	17	0.059
1435	A	2	1	1.00	17	0.059
1436	A	2	1	1.00	17	0.059
1437	A	2	1	1.00	17	0.059
1438	A	2	1	1.00	15	0.067
1439	A	1	1	1.00	9	0.111
1440	A	4	3	1.00	17	0.176
1441	A	5	4	1.00	17	0.235
1442	A	6	4	1.00	17	0.235
1443	A	7	4	1.00	17	0.235
1444	A	2	2	1.00	20	0.100
1445	A	2	2	1.00	20	0.100
1446	A	2	2	1.00	20	0.100
1447	A	2	2	1.00	20	0.100
1448	A	2	2	1.00	20	0.100
1449	A	2	2	1.00	20	0.100
1450	A	2	2	1.00	20	0.100
1451	A	2	2	1.00	20	0.100
1452	A	2	2	1.00	20	0.100
1453	A	2	2	1.00	13	0.154
1454	A	2	2	1.00	17	0.118
1455	A	5	5	1.00	15	0.333
1456	A	2	1	1.00	13	0.077
1457	A	2	1	1.00	15	0.067
1458	A	4	4	1.00	17	0.235
1459	A	4	4	1.00	17	0.235
1460	A	8	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1461	A	7	4	1.00	19	0.210
1462	A	6	4	1.00	19	0.210
1463	A	5	4	1.00	19	0.210
1464	A	4	4	1.00	19	0.210
1465	A	4	4	1.00	19	0.210
1466	A	1	1	1.00	19	0.053
1467	A	2	2	1.00	19	0.105
1468	A	3	2	1.00	19	0.105
1469	A	4	2	1.00	19	0.105
1470	A	5	2	1.00	19	0.105
1471	A	8	4	1.00	19	0.210
1472	A	7	4	1.00	19	0.210
1473	A	6	4	1.00	19	0.210
1474	A	5	4	1.00	19	0.210
1475	A	5	5	1.00	19	0.263
1476	A	5	4	1.00	19	0.210
1477	A	1	1	1.00	19	0.053
1478	A	2	2	1.00	19	0.105
1479	A	3	2	1.00	19	0.105
1480	A	4	2	1.00	19	0.105
1481	A	9	4	1.00	19	0.210
1482	A	8	4	1.00	19	0.210
1483	A	7	4	1.00	19	0.210
1484	A	6	4	1.00	19	0.210
1485	A	6	5	1.00	19	0.263
1486	A	6	5	1.00	19	0.263
1487	A	6	4	1.00	19	0.210
1488	A	1	1	1.00	19	0.053
1489	A	2	2	1.00	19	0.105
1490	A	3	2	1.00	19	0.105
1491	A	4	2	1.00	19	0.105
1492	A	7	4	1.00	19	0.210
1493	A	6	4	1.00	19	0.210
1494	A	5	4	1.00	19	0.210
1495	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1496	A	3	3	1.00	19	0.158
1497	A	1	1	1.00	19	0.053
1498	A	2	2	1.00	19	0.105
1499	A	3	2	1.00	19	0.105
1500	A	4	2	1.00	19	0.105
1501	A	5	2	1.00	19	0.105
1502	A	7	5	1.00	19	0.263
1503	A	6	5	1.00	19	0.263
1504	A	5	5	1.00	19	0.263
1505	A	4	4	1.00	19	0.210
1506	A	1	1	1.00	19	0.053
1507	A	2	2	1.00	19	0.105
1508	A	3	2	1.00	19	0.105
1509	A	4	2	1.00	19	0.105
1510	A	5	2	1.00	19	0.105
1511	A	6	2	1.00	19	0.105
1512	A	8	5	1.00	19	0.263
1513	A	7	5	1.00	19	0.263
1514	A	6	5	1.00	19	0.263
1515	A	5	4	1.00	19	0.210
1516	A	1	1	1.00	19	0.053
1517	A	2	2	1.00	19	0.105
1518	A	3	2	1.00	19	0.105
1519	A	4	2	1.00	19	0.105
1520	A	5	2	1.00	19	0.105
1521	A	6	2	1.00	19	0.105
1522	A	2	2	1.00	20	0.100
1523	A	2	2	1.00	19	0.105
1524	A	2	2	1.00	19	0.105
1525	A	2	2	1.00	17	0.118
1526	A	2	2	1.00	19	0.105
1527	A	1	1	1.00	19	0.053
1528	A	2	2	1.00	19	0.105
1529	A	2	2	1.00	19	0.105
1530	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1531	A	2	2	1.00	19	0.105
1532	A	2	2	1.00	17	0.118
1533	A	2	2	1.00	19	0.105
1534	A	1	1	1.00	19	0.053
1535	A	2	2	1.00	19	0.105
1536	A	3	3	1.00	20	0.150
1537	A	2	2	1.00	20	0.100
1538	A	3	3	1.00	20	0.150
1539	A	3	3	1.00	18	0.167
1540	A	3	3	1.00	20	0.150
1541	A	2	2	1.00	20	0.100
1542	A	3	3	1.00	20	0.150
1543	A	2	2	1.00	21	0.095
1544	A	1	1	1.00	8	0.125
1545	A	2	2	1.00	21	0.095
1546	A	2	2	1.00	19	0.105
1547	A	2	2	1.00	21	0.095
1548	A	1	1	1.00	21	0.048
1549	A	2	2	1.00	21	0.095
1550	A	1	1	1.00	19	0.053
1551	A	2	2	1.00	29	0.069
1552	A	2	2	1.00	15	0.133
1553	A	2	2	1.00	19	0.105
1554	A	2	2	1.00	29	0.069
1555	A	3	3	1.00	15	0.200
1556	A	2	2	1.00	15	0.133
1557	A	2	2	1.00	19	0.105
1558	A	3	3	1.00	20	0.150
1559	A	5	3	1.00	19	0.158
1560	A	4	3	1.00	19	0.158
1561	A	3	3	1.00	19	0.158
1562	A	3	3	1.00	19	0.158
1563	A	4	4	1.00	19	0.210
1564	A	5	4	1.00	19	0.210
1565	A	6	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1566	A	5	5	1.00	19	0.263
1567	A	4	4	1.00	19	0.210
1568	A	5	5	1.00	19	0.263
1569	A	6	5	1.00	19	0.263
1570	A	4	3	1.00	19	0.158
1571	A	3	3	1.00	19	0.158
1572	A	2	2	1.00	19	0.105
1573	A	3	3	1.00	19	0.158
1574	A	4	3	1.00	19	0.158
1575	A	3	2	1.00	19	0.105
1576	A	2	2	1.00	19	0.105
1577	A	2	2	1.00	19	0.105
1578	A	1	1	1.00	19	0.053
1579	A	2	2	1.00	19	0.105
1580	A	3	2	1.00	19	0.105
1581	A	4	2	1.00	19	0.105
1582	A	6	4	1.00	19	0.210
1583	A	5	4	1.00	19	0.210
1584	A	4	4	1.00	19	0.210
1585	A	4	4	1.00	19	0.210
1586	A	5	5	1.00	19	0.263
1587	A	3	2	1.00	19	0.105
1588	A	2	2	1.00	19	0.105
1589	A	1	1	1.00	19	0.053
1590	A	1	1	1.00	19	0.053
1591	A	2	2	1.00	19	0.105
1592	A	3	2	1.00	19	0.105
1593	A	4	2	1.00	19	0.105
1594	A	8	6	1.00	19	0.316
1595	A	7	6	1.00	19	0.316
1596	A	6	6	1.00	19	0.316
1597	A	5	5	1.00	19	0.263
1598	A	6	6	1.00	19	0.316
1599	A	7	6	1.00	19	0.316
1600	A	8	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1601	A	3	2	1.00	19	0.105
1602	A	2	2	1.00	19	0.105
1603	A	1	1	1.00	19	0.053
1604	A	1	1	1.00	19	0.053
1605	A	2	2	1.00	19	0.105
1606	A	3	2	1.00	19	0.105
1607	A	4	2	1.00	19	0.105
1608	A	6	4	1.00	19	0.210
1609	A	5	4	1.00	19	0.210
1610	A	4	4	1.00	19	0.210
1611	A	3	3	1.00	19	0.158
1612	A	4	4	1.00	19	0.210
1613	A	5	4	1.00	19	0.210
1614	A	6	4	1.00	19	0.210
1615	A	4	3	1.00	19	0.158
1616	A	3	3	1.00	19	0.158
1617	A	2	2	1.00	19	0.105
1618	A	1	1	1.00	19	0.053
1619	A	2	2	1.00	19	0.105
1620	A	3	2	1.00	19	0.105
1621	A	4	2	1.00	19	0.105
1622	A	8	7	1.00	19	0.368
1623	A	7	7	1.00	19	0.368
1624	A	6	6	1.00	19	0.316
1625	A	6	6	1.00	19	0.316
1626	A	7	6	1.00	19	0.316
1627	A	8	6	1.00	19	0.316
1628	A	2	2	1.00	15	0.133
1629	A	6	4	1.00	19	0.210
1630	A	5	4	1.00	19	0.210
1631	A	4	4	1.00	19	0.210
1632	A	4	4	1.00	19	0.210
1633	A	5	5	1.00	19	0.263
1634	A	6	5	1.00	19	0.263
1635	A	10	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1636	A	9	8	1.00	19	0.421
1637	A	8	8	1.00	19	0.421
1638	A	8	8	1.00	19	0.421
1639	A	9	9	1.00	19	0.474
1640	A	10	9	1.00	19	0.474
1641	A	7	4	1.00	19	0.210
1642	A	6	4	1.00	19	0.210
1643	A	5	4	1.00	19	0.210
1644	A	5	5	1.00	19	0.263
1645	A	5	4	1.00	19	0.210
1646	A	6	5	1.00	19	0.263
1647	A	7	5	1.00	19	0.263
1648	A	10	8	1.00	19	0.421
1649	A	9	8	1.00	19	0.421
1650	A	8	8	1.00	19	0.421
1651	A	7	7	1.00	19	0.368
1652	A	8	8	1.00	19	0.421
1653	A	9	8	1.00	19	0.421
1654	A	5	4	1.00	19	0.210
1655	A	4	4	1.00	19	0.210
1656	A	3	3	1.00	19	0.158
1657	A	4	4	1.00	19	0.210
1658	A	5	4	1.00	19	0.210
1659	A	10	9	1.00	19	0.474
1660	A	9	9	1.00	19	0.474
1661	A	8	8	1.00	19	0.421
1662	A	8	8	1.00	19	0.421
1663	A	9	8	1.00	19	0.421
1664	A	10	8	1.00	19	0.421
1665	A	7	5	1.00	19	0.263
1666	A	5	5	1.00	19	0.263
1667	A	4	4	1.00	19	0.210
1668	A	4	4	1.00	19	0.210
1669	A	5	4	1.00	19	0.210
1670	A	6	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1671	A	11	9	1.00	19	0.474
1672	A	10	9	1.00	19	0.474
1673	A	9	8	1.00	19	0.421
1674	A	9	9	1.00	19	0.474
1675	A	9	8	1.00	19	0.421
1676	A	10	8	1.00	19	0.421
1677	A	11	8	1.00	19	0.421
1678	A	8	6	1.00	19	0.316
1679	A	7	6	1.00	19	0.316
1680	A	7	7	1.00	19	0.368
1681	A	7	6	1.00	19	0.316
1682	A	1	1	1.00	19	0.053
1683	A	2	2	1.00	19	0.105
1684	A	3	2	1.00	19	0.105
1685	A	4	2	1.00	19	0.105
1686	A	7	4	1.00	19	0.210
1687	A	6	4	1.00	19	0.210
1688	A	5	4	1.00	19	0.210
1689	A	5	5	1.00	19	0.263
1690	A	5	4	1.00	19	0.210
1691	A	6	5	1.00	19	0.263
1692	A	7	5	1.00	19	0.263
1693	A	7	6	1.00	19	0.316
1694	A	6	6	1.00	19	0.316
1695	A	5	5	1.00	19	0.263
1696	A	1	1	1.00	19	0.053
1697	A	2	2	1.00	19	0.105
1698	A	3	2	1.00	19	0.105
1699	A	4	2	1.00	19	0.105
1700	A	7	6	1.00	19	0.316
1701	A	6	6	1.00	19	0.316
1702	A	5	5	1.00	19	0.263
1703	A	6	6	1.00	19	0.316
1704	A	7	6	1.00	19	0.316
1705	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1706	A	6	6	1.00	19	0.316
1707	A	5	5	1.00	19	0.263
1708	A	1	1	1.00	19	0.053
1709	A	2	2	1.00	19	0.105
1710	A	3	2	1.00	19	0.105
1711	A	4	2	1.00	19	0.105
1712	A	5	4	1.00	19	0.210
1713	A	4	4	1.00	19	0.210
1714	A	3	3	1.00	19	0.158
1715	A	4	4	1.00	19	0.210
1716	A	5	4	1.00	19	0.210
1717	A	7	7	1.00	19	0.368
1718	A	6	6	1.00	19	0.316
1719	A	1	1	1.00	19	0.053
1720	A	2	2	1.00	19	0.105
1721	A	3	2	1.00	19	0.105
1722	A	4	2	1.00	19	0.105
1723	A	8	7	1.00	19	0.368
1724	A	7	7	1.00	19	0.368
1725	A	6	6	1.00	19	0.316
1726	A	6	6	1.00	19	0.316
1727	A	7	6	1.00	19	0.316
1728	A	8	6	1.00	19	0.316
1729	A	11	8	1.00	20	0.400
1730	A	11	8	1.00	20	0.400
1731	A	2	2	1.00	19	0.105
1732	A	2	2	1.00	19	0.105
1733	A	2	2	1.00	19	0.105
1734	A	2	2	1.00	19	0.105
1735	A	2	2	1.00	19	0.105
1736	A	6	3	1.00	19	0.158
1737	A	5	3	1.00	19	0.158
1738	A	4	3	1.00	19	0.158
1739	A	3	3	1.00	19	0.158
1740	A	3	3	1.00	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1741	A	4	4	1.00	19	0.210
1742	A	7	5	1.00	19	0.263
1743	A	6	5	1.00	19	0.263
1744	A	5	5	1.00	19	0.263
1745	A	5	5	1.00	19	0.263
1746	A	6	6	1.00	19	0.316
1747	A	7	6	1.00	19	0.316
1748	A	7	5	1.00	19	0.263
1749	A	6	5	1.00	19	0.263
1750	A	5	5	1.00	19	0.263
1751	A	4	4	1.00	19	0.210
1752	A	5	5	1.00	19	0.263
1753	A	6	5	1.00	19	0.263
1754	A	5	3	1.00	19	0.158
1755	A	4	3	1.00	19	0.158
1756	A	3	3	1.00	19	0.158
1757	A	2	2	1.00	19	0.105
1758	A	3	3	1.00	19	0.158
1759	A	4	3	1.00	19	0.158
1760	A	7	6	1.00	19	0.316
1761	A	6	6	1.00	19	0.316
1762	A	5	5	1.00	19	0.263
1763	A	5	5	1.00	19	0.263
1764	A	6	5	1.00	19	0.263
1765	A	7	5	1.00	19	0.263
1766	A	2	2	1.00	19	0.105
1767	A	2	2	1.00	19	0.105
1768	A	2	2	1.00	19	0.105
1769	A	2	2	1.00	19	0.105
1770	A	2	2	1.00	19	0.105
1771	A	2	2	1.00	19	0.105
1772	A	14	9	1.00	19	0.474
1773	A	13	9	1.00	19	0.474
1774	A	13	9	1.00	19	0.474
1775	A	1	1	1.00	19	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1776	A	2	2	1.00	19	0.105
1777	A	3	2	1.00	19	0.105
1778	A	4	2	1.00	19	0.105
1779	A	14	9	1.00	19	0.474
1780	A	13	9	1.00	19	0.474
1781	A	13	9	1.00	19	0.474
1782	A	1	1	1.00	19	0.053
1783	A	2	2	1.00	19	0.105
1784	A	3	2	1.00	19	0.105
1785	A	4	2	1.00	19	0.105
1786	A	2	2	1.00	19	0.105
1787	A	2	2	1.00	19	0.105
1788	A	2	2	1.00	19	0.105
1789	A	2	2	1.00	19	0.105
1790	A	2	2	1.00	19	0.105
1791	A	2	2	1.00	19	0.105
1792	A	2	2	1.00	19	0.105
1793	A	2	2	1.00	19	0.105
1794	A	2	2	1.00	19	0.105
1795	A	2	2	1.00	19	0.105
1796	A	2	2	1.00	19	0.105
1797	A	2	2	1.00	19	0.105
1798	A	14	9	1.00	19	0.474
1799	A	14	10	1.00	19	0.526
1800	A	14	9	1.00	19	0.474
1801	A	1	1	1.00	19	0.053
1802	A	2	2	1.00	19	0.105
1803	A	3	2	1.00	19	0.105
1804	A	4	2	1.00	19	0.105
1805	A	14	9	1.00	19	0.474
1806	A	13	9	1.00	19	0.474
1807	A	12	8	1.00	19	0.421
1808	A	1	1	1.00	19	0.053
1809	A	2	2	1.00	19	0.105
1810	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1811	A	4	2	1.00	19	0.105
1812	A	2	2	1.00	19	0.105
1813	A	2	2	1.00	19	0.105
1814	A	2	2	1.00	19	0.105
1815	A	2	2	1.00	19	0.105
1816	A	2	2	1.00	19	0.105
1817	A	2	2	1.00	19	0.105
1818	A	2	2	1.00	19	0.105
1819	A	2	2	1.00	19	0.105
1820	A	2	2	1.00	19	0.105
1821	A	2	2	1.00	19	0.105
1822	A	2	2	1.00	19	0.105
1823	A	2	2	1.00	19	0.105
1824	A	14	9	1.00	19	0.474
1825	A	13	9	1.00	19	0.474
1826	A	12	8	1.00	19	0.421
1827	A	1	1	1.00	19	0.053
1828	A	2	2	1.00	19	0.105
1829	A	3	2	1.00	19	0.105
1830	A	4	2	1.00	19	0.105
1831	A	15	10	1.00	19	0.526
1832	A	14	10	1.00	19	0.526
1833	A	13	9	1.00	19	0.474
1834	A	1	1	1.00	19	0.053
1835	A	2	2	1.00	19	0.105
1836	A	3	2	1.00	19	0.105
1837	A	4	2	1.00	19	0.105
1838	A	2	2	1.00	19	0.105
1839	A	2	2	1.00	19	0.105
1840	A	2	2	1.00	19	0.105
1841	A	2	2	1.00	19	0.105
1842	A	2	2	1.00	19	0.105
1843	A	2	2	1.00	19	0.105
1844	A	1	1	1.00	16	0.062
1845	A	2	2	1.21	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1846	A	2	1	1.00	15	0.067
1847	A	2	1	1.00	15	0.067
1848	A	2	1	1.00	13	0.077
1849	A	1	1	1.00	15	0.067
1850	A	1	1	1.00	15	0.067
1851	A	1	1	1.00	15	0.067
1852	A	2	1	1.00	15	0.067
1853	A	2	1	1.00	15	0.067
1854	A	2	1	1.00	13	0.077
1855	A	1	1	1.00	7	0.143
1856	A	1	1	1.00	15	0.067
1857	A	1	1	1.00	15	0.067
1858	A	1	1	1.00	15	0.067
1859	A	3	2	1.00	19	0.105
1860	A	2	2	1.00	19	0.105
1861	A	1	1	1.00	19	0.053
1862	A	2	2	1.00	19	0.105
1863	A	2	2	1.00	17	0.118
1864	A	2	2	1.00	19	0.105
1865	A	2	2	1.00	19	0.105
1866	A	2	2	1.00	17	0.118
1867	A	2	2	1.00	19	0.105
1868	A	1	1	1.00	19	0.053
1869	A	2	2	1.00	19	0.105
1870	A	3	2	1.00	19	0.105
1871	A	4	2	1.00	19	0.105
1872	A	2	2	1.00	17	0.118
1873	A	2	2	1.00	19	0.105
1874	A	1	1	1.00	19	0.053
1875	A	2	2	1.00	19	0.105
1876	A	3	2	1.00	19	0.105
1877	A	4	2	1.00	19	0.105
1878	A	2	2	1.00	21	0.095
1879	A	2	2	1.00	21	0.095
1880	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1881	A	3	2	1.00	24	0.083
1882	A	2	2	1.00	28	0.071
1883	A	2	2	1.00	44	0.045
1884	A	2	2	1.00	51	0.039
1885	A	1	1	1.00	15	0.067
1886	A	1	1	1.00	15	0.067
1887	A	1	1	1.00	15	0.067
1888	A	1	1	1.00	15	0.067
1889	A	1	1	1.00	17	0.059
1890	A	2	2	1.00	27	0.074
1891	A	1	0	1.00	15	0.000
1892	A	1	0	1.00	9	0.000
1893	A	1	0	1.00	5	0.000
1894	A	1	0	1.00	5	0.000
1895	A	1	0	1.00	9	0.000
1896	A	1	0	1.00	9	0.000
1897	A	1	0	1.00	15	0.000
1898	A	1	0	1.00	10	0.000
1899	A	1	0	1.00	10	0.000
1900	A	1	0	1.00	12	0.000
1901	A	1	0	1.00	15	0.000
1902	A	1	0	1.00	17	0.000
1903	A	1	0	1.00	8	0.000
1904	A	1	0	1.00	10	0.000
1905	A	1	0	1.00	11	0.000
1906	A	1	0	1.00	11	0.000
1907	A	1	0	1.00	6	0.000
1908	A	1	0	1.00	11	0.000
1909	A	1	0	1.00	10	0.000
1910	A	1	0	1.00	11	0.000
1911	A	1	0	1.00	7	0.000
1912	A	1	0	1.00	17	0.000
1913	A	1	0	1.00	18	0.000
1914	A	1	0	1.00	11	0.000
1915	A	1	0	1.00	15	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1916	A	1	0	1.00	18	0.000
1917	A	1	0	1.00	20	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int 0\,dx$

Optimal. Leaf size=1

$$0$$

[Out] 0

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {8}

$$0$$

Antiderivative was successfully verified.

[In] Int[0,x]

[Out] 0

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 0\,dx = 0$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$0$$

Antiderivative was successfully verified.

[In] Integrate[0,x]

[Out] 0

Maple [A]
time = 0.01, size = 2, normalized size = 2.00

method	result	size

default	0	2
norman	0	2
meijerg	0	2
risch	0	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x,method=_RETURNVERBOSE)
```

```
[Out] 0
```

Maxima [A]
time = 0.28, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="maxima")
```

```
[Out] 0
```

Fricas [A]
time = 0.73, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x)
```

```
[Out] 0
```

Giac [A]
time = 1.78, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="giac")
```

```
[Out] 0
```

Mupad [B]
time = 0.04, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
```

```
[Out] 0
```

3.2 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$x$$

Antiderivative was successfully verified.

[In] Int[1,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[1,x]

[Out] x

Maple [A]
time = 0.00, size = 2, normalized size = 2.00

method	result	size

default	x	2
norman	x	2
risch	x	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.28, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 1.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Giac [A]
time = 1.73, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 0.01, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

3.3 $\int 5\,dx$

Optimal. Leaf size=3

$$5x$$

[Out] 5*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$5x$$

Antiderivative was successfully verified.

[In] Int[5,x]

[Out] 5*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 5\,dx = 5x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$5x$$

Antiderivative was successfully verified.

[In] Integrate[5,x]

[Out] 5*x

Maple [A]
time = 0.00, size = 4, normalized size = 1.33

method	result	size

default	$5x$	4
norman	$5x$	4
risch	$5x$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x,method=_RETURNVERBOSE)
```

```
[Out] 5*x
```

Maxima [A]
time = 0.28, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="maxima")
```

```
[Out] 5*x
```

Fricas [A]
time = 1.14, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="fricas")
```

```
[Out] 5*x
```

Sympy [A]
time = 0.00, size = 2, normalized size = 0.67

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x)
```

```
[Out] 5*x
```

Giac [A]
time = 1.82, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="giac")
```

```
[Out] 5*x
```

Mupad [B]
time = 0.01, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x)
```

```
[Out] 5*x
```

3.4 $\int -2\,dx$

Optimal. Leaf size=3

$$-2x$$

[Out] -2*x

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$-2x$$

Antiderivative was successfully verified.

[In] Int[-2,x]

[Out] -2*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -2\,dx = -2x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Antiderivative was successfully verified.

[In] Integrate[-2,x]

[Out] -2*x

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size

default	$-2x$	4
norman	$-2x$	4
risch	$-2x$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*x
```

Maxima [A]
time = 0.32, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="maxima")
```

```
[Out] -2*x
```

Fricas [A]
time = 1.01, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="fricas")
```

```
[Out] -2*x
```

Sympy [A]
time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x)
```

```
[Out] -2*x
```

Giac [A]
time = 2.24, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="giac")
```

```
[Out] -2*x
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
```

```
[Out] -2*x
```

3.5 $\int -\frac{3}{2}\,dx$

Optimal. Leaf size=5

$$-\frac{3x}{2}$$

```
[Out] -3/2*x
```

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {8}

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-3/2,x]
```

```
[Out] (-3*x)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -\frac{3}{2}\,dx = -\frac{3x}{2}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/2,x]
```

```
[Out] (-3*x)/2
```

Maple [A]
time = 0.01, size = 4, normalized size = 0.80

method	result	size
default	$-\frac{3x}{2}$	4
norman	$-\frac{3x}{2}$	4
risch	$-\frac{3x}{2}$	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x
```

Maxima [A]
time = 0.28, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="maxima")
```

```
[Out] -3/2*x
```

Fricas [A]
time = 1.11, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="fricas")
```

```
[Out] -3/2*x
```

Sympy [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x)
```

```
[Out] -3*x/2
```

Giac [A]
time = 2.17, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="giac")
```

```
[Out] -3/2*x
```

Mupad [B]
time = 0.01, size = 3, normalized size = 0.60

$$-\frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
```

```
[Out] -(3*x)/2
```

3.6 $\int \pi \, dx$

Optimal. Leaf size=3

$$\pi x$$

[Out] Pi*x

Rubi [A]
time = 0.01, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$\pi x$$

Antiderivative was successfully verified.

[In] Int[Pi,x]

[Out] Pi*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \pi \, dx = \pi x$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$\pi x$$

Antiderivative was successfully verified.

[In] Integrate[Pi,x]

[Out] Pi*x

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size

default	πx	4
norman	πx	4
risch	πx	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x
```

Maxima [A]
time = 0.28, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="maxima")
```

```
[Out] pi*x
```

Fricas [A]
time = 0.83, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="fricas")
```

```
[Out] pi*x
```

Sympy [A]
time = 0.00, size = 2, normalized size = 0.67

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x)
```

```
[Out] pi*x
```

Giac [A]
time = 1.64, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="giac")
```

```
[Out] pi*x
```

Mupad [B]
time = 0.00, size = 3, normalized size = 1.00

$$\Pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x)
```

```
[Out] Pi*x
```

3.7 $\int a\,dx$

Optimal. Leaf size=3

$$ax$$

```
[Out] a*x
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = $\{8\}$

$$ax$$

Antiderivative was successfully verified.

```
[In] Int[a,x]
```

```
[Out] a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int a\,dx = ax$$

Mathematica [A]
time = 0.00, size = 3, normalized size = 1.00

$$ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a,x]
```

```
[Out] a*x
```

Maple [A]
time = 0.01, size = 4, normalized size = 1.33

method	result	size

default	ax	4
norman	ax	4
risch	ax	4

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x,method=_RETURNVERBOSE)
```

```
[Out] a*x
```

Maxima [A]
time = 0.28, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="maxima")
```

```
[Out] a*x
```

Fricas [A]
time = 0.87, size = 3, normalized size = 1.00

$$xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="fricas")
```

```
[Out] x*a
```

Sympy [A]
time = 0.00, size = 2, normalized size = 0.67

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x)
```

```
[Out] a*x
```

Giac [A]
time = 2.16, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="giac")
```

```
[Out] a*x
```

Mupad [**B**]
time = 0.00, size = 3, normalized size = 1.00

$$a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x)
```

```
[Out] a*x
```

3.8 $\int 3a\,dx$

Optimal. Leaf size=4

$$3ax$$

```
[Out] 3*a*x
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{8\}$

$$3ax$$

Antiderivative was successfully verified.

```
[In] Int[3*a,x]
[Out] 3*a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 3a\,dx = 3ax$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$3ax$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a,x]
[Out] 3*a*x
```

Maple [A]
time = 0.01, size = 5, normalized size = 1.25

method	result	size

default	$3ax$	5
norman	$3ax$	5
risch	$3ax$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a*x
```

Maxima [A]
time = 0.28, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="maxima")
```

```
[Out] 3*a*x
```

Fricas [A]
time = 1.43, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="fricas")
```

```
[Out] 3*a*x
```

Sympy [A]
time = 0.00, size = 3, normalized size = 0.75

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x)
```

```
[Out] 3*a*x
```

Giac [A]
time = 1.79, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="giac")
```

```
[Out] 3*a*x
```

Mupad [B]
time = 0.00, size = 4, normalized size = 1.00

$$3\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a,x)
```

```
[Out] 3*a*x
```

3.9 $\int \frac{\pi}{\sqrt{16-e^2}}\,dx$

Optimal. Leaf size=14

$$\frac{\pi x}{\sqrt{16-e^2}}$$

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {8}

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Pi/Sqrt[16 - E^2],x]
```

```
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\pi}{\sqrt{16-e^2}}\,dx = \frac{\pi x}{\sqrt{16-e^2}}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi/Sqrt[16 - E^2],x]
```

```
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.86

method	result	size
default	$\frac{\pi x}{\sqrt{16-e^{2}}}$	12
norman	$-\frac{\pi\sqrt{16-e^{2}}\, x}{-16+e^{2}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16-exp(2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

Maxima [**A**]
time = 0.29, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^{2}+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="maxima")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

Fricas [**A**]
time = 0.92, size = 18, normalized size = 1.29

$$-\frac{\pi x\sqrt{-e^{2}+16}}{e^{2}-16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="fricas")
```

```
[Out] -pi*x*sqrt(-e^2 + 16)/(e^2 - 16)
```

Sympy [**A**]
time = 0.00, size = 10, normalized size = 0.71

$$\frac{\pi x}{\sqrt{16-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))**(1/2),x)
```

```
[Out] pi*x/sqrt(16 - exp(2))
```

Giac [**A**]
time = 1.81, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^{2}+16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="giac")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

Mupad [B]
time = 0.00, size = 11, normalized size = 0.79

$$\frac{\Pi\,x}{\sqrt{16-\mathrm{e}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16 - exp(2))^(1/2),x)
```

```
[Out] (Pi*x)/(16 - exp(2))^(1/2)
```

3.10 $\int x^{100} \, dx$

Optimal. Leaf size=7

$$\frac{x^{101}}{101}$$

[Out] 1/101*x^101

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

[In] Int[x^100,x]

[Out] x^101/101

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{100} \, dx = \frac{x^{101}}{101}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

[In] Integrate[x^100,x]

[Out] x^101/101

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^{101}}{101}$	6
default	$\frac{x^{101}}{101}$	6
risch	$\frac{x^{101}}{101}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x,method=_RETURNVERBOSE)
```

```
[Out] 1/101*x^101
```

Maxima [A]
time = 0.30, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="maxima")
```

```
[Out] 1/101*x^101
```

Fricas [A]
time = 0.69, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="fricas")
```

```
[Out] 1/101*x^101
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**100,x)
```

```
[Out] x**101/101
```

Giac [A]
time = 1.43, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="giac")
```

```
[Out] 1/101*x^101
```

Mupad [B]
time = 0.12, size = 5, normalized size = 0.71

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] x^101/101
```

3.11 $\int x^3 \, dx$

Optimal. Leaf size=7

$$\frac{x^4}{4}$$

```
[Out] 1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3,x]
```

```
[Out] x^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \, dx = \frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3,x]
```

```
[Out] x^4/4
```

Maple [A]
time = 0.00, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^4}{4}$	6
default	$\frac{x^4}{4}$	6
norman	$\frac{x^4}{4}$	6
risch	$\frac{x^4}{4}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4
```

Fricas [A]
time = 1.88, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3,x)
```

```
[Out] x**4/4
```

Giac [**A**]
time = 1.47, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4
```

Mupad [**B**]
time = 0.02, size = 5, normalized size = 0.71

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
```

```
[Out] x^4/4
```

3.12 $\int x^2 \, dx$

Optimal. Leaf size=7

$$\frac{x^3}{3}$$

[Out] 1/3*x^3

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2,x]

[Out] x^3/3

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2 \, dx = \frac{x^3}{3}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2,x]

[Out] x^3/3

Maple [A]
time = 0.00, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^3}{3}$	6
default	$\frac{x^3}{3}$	6
norman	$\frac{x^3}{3}$	6
risch	$\frac{x^3}{3}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3
```

Maxima [A]
time = 0.32, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3
```

Fricas [A]
time = 1.04, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2,x)
```

```
[Out] x**3/3
```

Giac [A]
time = 1.37, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
```

```
[Out] x^3/3
```

3.13 $\int x\,dx$

Optimal. Leaf size=7

$$\frac{x^2}{2}$$

[Out] 1/2*x^2

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {30}

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[x,x]

[Out] x^2/2

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\,dx = \frac{x^2}{2}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x,x]

[Out] x^2/2

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$\frac{x^2}{2}$	6
default	$\frac{x^2}{2}$	6
norman	$\frac{x^2}{2}$	6
risch	$\frac{x^2}{2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2
```

Fricas [A]
time = 0.90, size = 5, normalized size = 0.71

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2
```

Sympy [A]
time = 0.00, size = 3, normalized size = 0.43

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x)
```

```
[Out] x**2/2
```

Giac [A]
time = 1.16, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="giac")
```

```
[Out] 1/2*x^2
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x)
```

```
[Out] x^2/2
```

3.14 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$x$$

Antiderivative was successfully verified.

[In] Int[1,x]

[Out] x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[1,x]

[Out] x

Maple [A]
time = 0.00, size = 2, normalized size = 2.00

method	result	size

default	x	2
norman	x	2
risch	x	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 0.67, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

Giac [A]
time = 1.21, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

3.15 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{29\}$

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A]
time = 0.00, size = 3, normalized size = 1.50

method	result	size

default	$\ln(x)$	3
norman	$\ln(x)$	3
risch	$\ln(x)$	3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)
```

Maxima [A]
time = 0.36, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A]
time = 0.53, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A]
time = 0.02, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A]
time = 1.32, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

Mupad [B]
time = 0.04, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

3.16 $\int \frac{1}{x^2}\,dx$

Optimal. Leaf size=5

$$-\frac{1}{x}$$

[Out] -1/x

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[x^(-2),x]

[Out] -x^(-1)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2),x]

[Out] -x^(-1)

Maple [A]
time = 0.01, size = 6, normalized size = 1.20

method	result	size
gosper	$-\frac{1}{x}$	6
default	$-\frac{1}{x}$	6
norman	$-\frac{1}{x}$	6
risch	$-\frac{1}{x}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x
```

Maxima [A]
time = 0.30, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="maxima")
```

```
[Out] -1/x
```

Fricas [A]
time = 1.00, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="fricas")
```

```
[Out] -1/x
```

Sympy [A]
time = 0.02, size = 3, normalized size = 0.60

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2,x)
```

```
[Out] -1/x
```

Giac [A]
time = 1.78, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="giac")
[Out] -1/x
```

Mupad [B]
time = 0.03, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x)
[Out] -1/x
```

3.17 $\int \frac{1}{x^3} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3),x]
```

```
[Out] -1/2*1/x^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3} \, dx = -\frac{1}{2x^2}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3),x]
```

```
[Out] -1/2*1/x^2
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{2x^2}$	6
default	$-\frac{1}{2x^2}$	6
norman	$-\frac{1}{2x^2}$	6
risch	$-\frac{1}{2x^2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2
```

Fricas [A]
time = 1.06, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="fricas")
```

```
[Out] -1/2/x^2
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3,x)
```

```
[Out] -1/(2*x**2)
```

Giac [**A**]
time = 1.31, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="giac")
```

```
[Out] -1/2/x^2
```

Mupad [**B**]
time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
```

```
[Out] -1/(2*x^2)
```

3.18 $\int \frac{1}{x^4} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{3x^3}$$

```
[Out] -1/3/x^3
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4),x]
```

```
[Out] -1/3*1/x^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4} \, dx = -\frac{1}{3x^3}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4),x]
```

```
[Out] -1/3*1/x^3
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{3x^3}$	6
default	$-\frac{1}{3x^3}$	6
norman	$-\frac{1}{3x^3}$	6
risch	$-\frac{1}{3x^3}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="maxima")
```

```
[Out] -1/3/x^3
```

Fricas [A]
time = 1.69, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="fricas")
```

```
[Out] -1/3/x^3
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4,x)
```

```
[Out] -1/(3*x**3)
```

Giac [A]
time = 1.28, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="giac")
```

```
[Out] -1/3/x^3
```

Mupad [B]
time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
```

```
[Out] -1/(3*x^3)
```

3.19 $\int \frac{1}{x^{100}} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{99x^{99}}$$

```
[Out] -1/99/x^99
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = $\{30\}$

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-100),x]
[Out] -1/99*1/x^99
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{100}} \, dx = -\frac{1}{99x^{99}}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-100),x]
[Out] -1/99*1/x^99
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{99x^{99}}$	6
default	$-\frac{1}{99x^{99}}$	6
risch	$-\frac{1}{99x^{99}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x,method=_RETURNVERBOSE)
```

```
[Out] -1/99/x^99
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="maxima")
```

```
[Out] -1/99/x^99
```

Fricas [A]
time = 1.24, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="fricas")
```

```
[Out] -1/99/x^99
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**100,x)
```

```
[Out] -1/(99*x**99)
```

Giac [A]
time = 1.15, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="giac")
```

```
[Out] -1/99/x^99
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/(99*x^99)
```

3.20 $\int x^{5/2}\, dx$

Optimal. Leaf size=9

$$\frac{2x^{7/2}}{7}$$

[Out] 2/7*x^(7/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30}

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2),x]

[Out] (2*x^(7/2))/7

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\, dx = \frac{2x^{7/2}}{7}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/2),x]

[Out] (2*x^(7/2))/7

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}}{7}$	6
derivativedivides	$\frac{2x^{\frac{7}{2}}}{7}$	6
default	$\frac{2x^{\frac{7}{2}}}{7}$	6
trager	$\frac{2x^{\frac{7}{2}}}{7}$	6
risch	$\frac{2x^{\frac{7}{2}}}{7}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^(7/2)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2)
```

Fricas [A]
time = 1.20, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*x^(7/2)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2),x)
```

```
[Out] 2*x**(7/2)/7
```

Giac [A]
time = 1.80, size = 5, normalized size = 0.56

$$\frac{2}{7} x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2)
```

Mupad [B]
time = 0.08, size = 5, normalized size = 0.56

$$\frac{2 x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x)
```

```
[Out] (2*x^(7/2))/7
```

3.21 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

[Out] 2/5*x^(5/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2),x]

[Out] (2*x^(5/2))/5

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2),x]

[Out] (2*x^(5/2))/5

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}}{5}$	6
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}$	6
default	$\frac{2x^{\frac{5}{2}}}{5}$	6
trager	$\frac{2x^{\frac{5}{2}}}{5}$	6
risch	$\frac{2x^{\frac{5}{2}}}{5}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

Fricas [A]
time = 0.99, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5
```

Giac [A]
time = 1.29, size = 5, normalized size = 0.56

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] (2*x^(5/2))/5
```

3.22 $\int \sqrt{x}\, dx$

Optimal. Leaf size=9

$$\frac{2x^{3/2}}{3}$$

```
[Out] 2/3*x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x],x]
[Out] (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\, dx = \frac{2x^{3/2}}{3}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x],x]
[Out] (2*x^(3/2))/3
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3}$	6
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}$	6
default	$\frac{2x^{\frac{3}{2}}}{3}$	6
trager	$\frac{2x^{\frac{3}{2}}}{3}$	6
risch	$\frac{2x^{\frac{3}{2}}}{3}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="maxima")
[Out] 2/3*x^(3/2)
```

Fricas [A]
time = 0.80, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="fricas")
[Out] 2/3*x^(3/2)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3
```

Giac [A]
time = 1.03, size = 5, normalized size = 0.56

$$\frac{2}{3} x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.56

$$\frac{2 x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
```

```
[Out] (2*x^(3/2))/3
```

3.23 $\int \frac{1}{\sqrt{x}}\,dx$

Optimal. Leaf size=7

$$2\sqrt{x}$$

```
[Out] 2*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$2\sqrt{x}$	6
derivativedivides	$2\sqrt{x}$	6
default	$2\sqrt{x}$	6
trager	$2\sqrt{x}$	6
risch	$2\sqrt{x}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)
```

Fricas [A]
time = 0.61, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2),x)
```

```
[Out] 2*sqrt(x)
```

Giac [A]
time = 1.80, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

3.24 $\int \frac{1}{x^{3/2}} \, dx$

Optimal. Leaf size=7

$$-\frac{2}{\sqrt{x}}$$

```
[Out] -2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}} \, dx = -\frac{2}{\sqrt{x}}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{2}{\sqrt{x}}$	6
derivativedivides	$-\frac{2}{\sqrt{x}}$	6
default	$-\frac{2}{\sqrt{x}}$	6
trager	$-\frac{2}{\sqrt{x}}$	6
risch	$-\frac{2}{\sqrt{x}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/x^(1/2)
```

Maxima [A]
time = 0.34, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="maxima")
```

```
[Out] -2/sqrt(x)
```

Fricas [A]
time = 0.87, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2/sqrt(x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2),x)
```

```
[Out] -2/sqrt(x)
```

Giac [**A**]
time = 1.31, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="giac")
```

```
[Out] -2/sqrt(x)
```

Mupad [**B**]
time = 0.03, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
```

```
[Out] -2/x^(1/2)
```

3.25 $\int \frac{1}{x^{5/2}} \, dx$

Optimal. Leaf size=9

$$-\frac{2}{3x^{3/2}}$$

```
[Out] -2/3/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/2),x]
```

```
[Out] -2/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}} \, dx = -\frac{2}{3x^{3/2}}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/2),x]
```

```
[Out] -2/(3*x^(3/2))
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$-\frac{2}{3x^{\frac{3}{2}}}$	6
derivativedivides	$-\frac{2}{3x^{\frac{3}{2}}}$	6
default	$-\frac{2}{3x^{\frac{3}{2}}}$	6
trager	$-\frac{2}{3x^{\frac{3}{2}}}$	6
risch	$-\frac{2}{3x^{\frac{3}{2}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/x^(3/2)
```

Maxima [A]
time = 0.33, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/x^(3/2)
```

Fricas [A]
time = 0.91, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3/x^(3/2)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.89

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2),x)
```

```
[Out] -2/(3*x**(3/2))
```

Giac [A]
time = 2.30, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/x^(3/2)
```

Mupad [B]
time = 0.03, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x)
```

```
[Out] -2/(3*x^(3/2))
```

3.26 $\int x^{5/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{8/3}}{8}$$

[Out] 3/8*x^(8/3)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30}

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

[In] Int[x^(5/3),x]

[Out] (3*x^(8/3))/8

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/3}\,dx = \frac{3x^{8/3}}{8}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/3),x]

[Out] (3*x^(8/3))/8

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}}{8}$	6
derivativedivides	$\frac{3x^{\frac{8}{3}}}{8}$	6
default	$\frac{3x^{\frac{8}{3}}}{8}$	6
trager	$\frac{3x^{\frac{8}{3}}}{8}$	6
risch	$\frac{3x^{\frac{8}{3}}}{8}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^(8/3)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{8}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/8*x^(8/3)
```

Fricas [A]
time = 0.93, size = 5, normalized size = 0.56

$$\frac{3}{8}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/8*x^(8/3)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3),x)
```

```
[Out] 3*x**(8/3)/8
```

Giac [A]
time = 1.76, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="giac")
```

```
[Out] 3/8*x^(8/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
```

```
[Out] (3*x^(8/3))/8
```

3.27 $\int x^{4/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{7/3}}{7}$$

```
[Out] 3/7*x^(7/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30}

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3),x]
```

```
[Out] (3*x^(7/3))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{4/3}\,dx = \frac{3x^{7/3}}{7}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3),x]
```

```
[Out] (3*x^(7/3))/7
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}}{7}$	6
derivativedivides	$\frac{3x^{\frac{7}{3}}}{7}$	6
default	$\frac{3x^{\frac{7}{3}}}{7}$	6
trager	$\frac{3x^{\frac{7}{3}}}{7}$	6
risch	$\frac{3x^{\frac{7}{3}}}{7}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/7*x^(7/3)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.56

$$\frac{3}{7}\,x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="maxima")
[Out] 3/7*x^(7/3)
```

Fricas [A]
time = 0.94, size = 5, normalized size = 0.56

$$\frac{3}{7}\,x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="fricas")
[Out] 3/7*x^(7/3)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3),x)
```

```
[Out] 3*x**(7/3)/7
```

Giac [A]
time = 2.08, size = 5, normalized size = 0.56

$$\frac{3}{7}\,x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="giac")
```

```
[Out] 3/7*x^(7/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x)
```

```
[Out] (3*x^(7/3))/7
```

3.28 $\int x^{2/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{5/3}}{5}$$

```
[Out] 3/5*x^(5/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{2/3}\,dx = \frac{3x^{5/3}}{5}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3),x]
```

```
[Out] (3*x^(5/3))/5
```

Maple [A]
time = 0.03, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}}{5}$	6
derivativedivides	$\frac{3x^{\frac{5}{3}}}{5}$	6
default	$\frac{3x^{\frac{5}{3}}}{5}$	6
trager	$\frac{3x^{\frac{5}{3}}}{5}$	6
risch	$\frac{3x^{\frac{5}{3}}}{5}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*x^(5/3)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*x^(5/3)
```

Fricas [A]
time = 1.06, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*x^(5/3)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3),x)
```

```
[Out] 3*x**(5/3)/5
```

Giac [A]
time = 1.25, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*x^(5/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
```

```
[Out] (3*x^(5/3))/5
```

3.29 $\int \sqrt[3]{x}\, dx$

Optimal. Leaf size=9

$$\frac{3x^{4/3}}{4}$$

[Out] 3/4*x^(4/3)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

[In] Int[x^(1/3),x]

[Out] (3*x^(4/3))/4

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x}\, dx = \frac{3x^{4/3}}{4}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1/3),x]

[Out] (3*x^(4/3))/4

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}}{4}$	6
derivativedivides	$\frac{3x^{\frac{4}{3}}}{4}$	6
default	$\frac{3x^{\frac{4}{3}}}{4}$	6
trager	$\frac{3x^{\frac{4}{3}}}{4}$	6
risch	$\frac{3x^{\frac{4}{3}}}{4}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^(4/3)
```

Maxima [A]
time = 0.28, size = 5, normalized size = 0.56

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)
```

Fricas [A]
time = 0.97, size = 5, normalized size = 0.56

$$\frac{3}{4}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*x^(4/3)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3),x)
```

```
[Out] 3*x**(4/3)/4
```

Giac [A]
time = 1.42, size = 5, normalized size = 0.56

$$\frac{3}{4}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)
```

Mupad [B]
time = 0.06, size = 5, normalized size = 0.56

$$\frac{3\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
```

```
[Out] (3*x^(4/3))/4
```

3.30 $\int \frac{1}{\sqrt[3]{x}} \, dx$

Optimal. Leaf size=9

$$\frac{3x^{2/3}}{2}$$

```
[Out] 3/2*x^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {30}

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}} \, dx = \frac{3x^{2/3}}{2}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{2}{3}}}{2}$	6
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2}$	6
default	$\frac{3x^{\frac{2}{3}}}{2}$	6
trager	$\frac{3x^{\frac{2}{3}}}{2}$	6
risch	$\frac{3x^{\frac{2}{3}}}{2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.56

$$\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="maxima")
[Out] 3/2*x^(2/3)
```

Fricas [A]
time = 0.87, size = 5, normalized size = 0.56

$$\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="fricas")
[Out] 3/2*x^(2/3)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3),x)
```

```
[Out] 3*x**(2/3)/2
```

Giac [A]
time = 1.78, size = 5, normalized size = 0.56

$$\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3)
```

Mupad [B]
time = 0.04, size = 5, normalized size = 0.56

$$\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
```

```
[Out] (3*x^(2/3))/2
```

3.31 $\int \frac{1}{x^{2/3}} \, dx$

Optimal. Leaf size=7

$$3\sqrt[3]{x}$$

[Out] 3*x^(1/3)

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{2/3}} \, dx = 3\sqrt[3]{x}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size

gosper	$3x^{\frac{1}{3}}$	6
derivativedivides	$3x^{\frac{1}{3}}$	6
default	$3x^{\frac{1}{3}}$	6
trager	$3x^{\frac{1}{3}}$	6
risch	$3x^{\frac{1}{3}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)
```

Maxima [A]
time = 0.31, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3)
```

Fricas [A]
time = 1.01, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.71

$$3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3),x)
```

```
[Out] 3*x**(1/3)
```

Giac [A]
time = 1.71, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="giac")
```

```
[Out] 3*x^(1/3)
```

Mupad [B]
time = 0.07, size = 5, normalized size = 0.71

$$3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

3.32 $\int \frac{1}{x^{4/3}}\,dx$

Optimal. Leaf size=7

$$-\frac{3}{\sqrt[3]{x}}$$

```
[Out] -3/x^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4/3),x]
```

```
[Out] -3/x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{4/3}}\,dx = -\frac{3}{\sqrt[3]{x}}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4/3),x]
```

```
[Out] -3/x^(1/3)
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{3}{x^{\frac{1}{3}}}$	6
derivativedivides	$-\frac{3}{x^{\frac{1}{3}}}$	6
default	$-\frac{3}{x^{\frac{1}{3}}}$	6
trager	$-\frac{3}{x^{\frac{1}{3}}}$	6
risch	$-\frac{3}{x^{\frac{1}{3}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/x^(1/3)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="maxima")
```

```
[Out] -3/x^(1/3)
```

Fricas [A]
time = 1.19, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="fricas")
```

```
[Out] -3/x^(1/3)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3),x)
```

[Out] -3/x**(1/3)

Giac [A]
time = 1.87, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(4/3),x, algorithm="giac")

[Out] -3/x^(1/3)

Mupad [B]
time = 0.07, size = 5, normalized size = 0.71

$$-\frac{3}{x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(4/3),x)

[Out] -3/x^(1/3)

3.33 $\int \frac{1}{x^{5/3}} \, dx$

Optimal. Leaf size=9

$$-\frac{3}{2x^{2/3}}$$

```
[Out] -3/2/x^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{30\}$

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/3}} \, dx = -\frac{3}{2x^{2/3}}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

Maple [A]
time = 0.02, size = 6, normalized size = 0.67

method	result	size
gosper	$-\frac{3}{2x^{\frac{2}{3}}}$	6
derivativedivides	$-\frac{3}{2x^{\frac{2}{3}}}$	6
default	$-\frac{3}{2x^{\frac{2}{3}}}$	6
trager	$-\frac{3}{2x^{\frac{2}{3}}}$	6
risch	$-\frac{3}{2x^{\frac{2}{3}}}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/x^(2/3)
```

Maxima [A]
time = 0.29, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="maxima")
```

```
[Out] -3/2/x^(2/3)
```

Fricas [A]
time = 0.78, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="fricas")
```

```
[Out] -3/2/x^(2/3)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.89

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3),x)
```

```
[Out] -3/(2*x**(2/3))
```

Giac [A]
time = 1.92, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="giac")
```

```
[Out] -3/2/x^(2/3)
```

Mupad [B]
time = 0.05, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
```

```
[Out] -3/(2*x^(2/3))
```

3.34 $\int x^n \, dx$

Optimal. Leaf size=11

$$\frac{x^{1+n}}{1+n}$$

[Out] x^(1+n)/(1+n)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n \, dx = \frac{x^{1+n}}{1+n}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^{1+n}}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Maple [A]
time = 0.01, size = 12, normalized size = 1.09

method	result	size
risch	$\frac{x\,x^n}{1+n}$	11
gosper	$\frac{x^{1+n}}{1+n}$	12
default	$\frac{x^{1+n}}{1+n}$	12
norman	$\frac{x\,\mathrm{e}^{n\ln(x)}}{1+n}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x,method=_RETURNVERBOSE)
[Out] x^(1+n)/(1+n)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")
[Out] x^(n + 1)/(n + 1)
```

Fricas [A]
time = 0.99, size = 10, normalized size = 0.91

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
[Out] x*x^n/(n + 1)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
```

```
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Giac [A]
time = 1.56, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="giac")
```

```
[Out] x^(n + 1)/(n + 1)
```

Mupad [B]
time = 0.35, size = 20, normalized size = 1.82

$$\begin{cases} \ln(x) & \text{if } n=-1 \\ \frac{x^{n+1}}{n+1} & \text{if } n\neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
```

```
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

3.35 $\int (bx)^n \, dx$

Optimal. Leaf size=16

$$\frac{(bx)^{1+n}}{b(1+n)}$$

```
[Out] (b*x)^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{32\}$

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^n,x]
```

```
[Out] (b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^n \, dx = \frac{(bx)^{1+n}}{b(1+n)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.75

$$\frac{x(bx)^n}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^n,x]
```

```
[Out] (x*(b*x)^n)/(1 + n)
```

Maple [A]
time = 0.01, size = 17, normalized size = 1.06

method	result	size
gosper	$\frac{x(bx)^n}{1+n}$	13
risch	$\frac{x(bx)^n}{1+n}$	13
norman	$\frac{x\,e^{n\ln(bx)}}{1+n}$	15
default	$\frac{(bx)^{1+n}}{b(1+n)}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x)^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="maxima")
```

```
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 0.90, size = 12, normalized size = 0.75

$$\frac{(bx)^n\,x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="fricas")
```

```
[Out] (b*x)^n*x/(n + 1)
```

Sympy [A]
time = 0.01, size = 17, normalized size = 1.06

$$\frac{\begin{cases} \frac{(bx)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log{(bx)} & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**n,x)
```

```
[Out] Piecewise(((b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(b*x), True))/b
```

Giac [A]
time = 1.60, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="giac")
```

```
[Out] (b*x)^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 0.18, size = 12, normalized size = 0.75

$$\frac{x\,(b\,x)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^n,x)
```

```
[Out] (x*(b*x)^n)/(n + 1)
```

3.36 $\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

```
[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e
```

Rubi [A]
time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {33, 31}

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Maple [A]
time = 0.10, size = 22, normalized size = 0.96

method	result	size
default	$\frac{\ln\left(ce+dex+\sqrt{-a}\right)}{de}$	22
norman	$\frac{\ln\left(ce+dex+\sqrt{-a}\right)}{de}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(d*x+c)+(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.91

$$\frac{e^{(-1)}\log\left((dx+c)e+\sqrt{-a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] e^(-1)*log((d*x + c)*e + sqrt(-a))/d
```

Fricas [A]
time = 1.05, size = 21, normalized size = 0.91

$$\frac{e^{(-1)}\log\left((dx+c)e+\sqrt{-a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] e^(-1)*log((d*x + c)*e + sqrt(-a))/d
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.83

$$\frac{\log\left(ce+dex+\sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)**(1/2)),x)
```

```
[Out] log(c*e + d*e*x + sqrt(-a))/(d*e)
```

Giac [A]
time = 1.33, size = 22, normalized size = 0.96

$$\frac{e^{(-1)}\log\left(\left|(dx+c)e+\sqrt{-a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] e^(-1)*log(abs((d*x + c)*e + sqrt(-a)))/d
```

Mupad [B]
time = 0.14, size = 21, normalized size = 0.91

$$\frac{\ln\left(\sqrt{-a}+c\,e+d\,e\,x\right)}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-a)^(1/2) + e*(c + d*x)),x)
```

```
[Out] log((-a)^(1/2) + c*e + d*e*x)/(d*e)
```

3.37 $\int (c+d(a+bx))^{5/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(c+d(a+bx))^{7/2}}{7bd}$$

```
[Out] 2/7*(c+d*(b*x+a))^(7/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{5/2}\,dx,x,a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$\frac{2(c+ad+bdx)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(7/2))/(7*b*d)
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.87

method	result	si
gosper	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7bd}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7bd}$	20
default	$\frac{2(bdx+ad+c)^{\frac{7}{2}}}{7bd}$	20
trager	$\frac{2(b^{3}d^{3}x^{3}+3a\,b^{2}d^{3}x^{2}+3a^{2}b\,d^{3}x+3b^{2}c\,d^{2}x^{2}+a^{3}d^{3}+6abc\,d^{2}x+3a^{2}c\,d^{2}+3b\,c^{2}dx+3a\,c^{2}d+c^{3})\sqrt{bdx+ad+c}}{7bd}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(b*d*x+a*d+c)^(7/2)/b/d
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*((b*x + a)*d + c)^(7/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(19) = 38.
time = 1.82, size = 104, normalized size = 4.52

$$\frac{2\left(b^{3}d^{3}x^{3}+a^{3}d^{3}+3\,a^{2}cd^{2}+3\,ac^{2}d+c^{3}+3\,(ab^{2}d^{3}+b^{2}cd^{2})x^{2}+3\,(a^{2}bd^{3}+2\,abcd^{2}+bc^{2}d)x\right)\sqrt{bdx+ad+c}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*d^3*x^3 + a^3*d^3 + 3*a^2*c*d^2 + 3*a*c^2*d + c^3 + 3*(a*b^2*d^3 +
 b^2*c*d^2)*x^2 + 3*(a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d)*x)*sqrt(b*d*x + a*d
 + c)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(17) = 34$.
time = 1.38, size = 270, normalized size = 11.74

$$\begin{cases} c^{\frac{5}{2}}x & \text{for } b = 0 \wedge d = 0 \\ x(ad + c)^{\frac{5}{2}} & \text{for } b = 0 \\ c^{\frac{5}{2}}x & \text{for } d = 0 \\ \frac{2a^3d^2\sqrt{ad + bdx + c}}{7b} + \frac{6a^2d^2x\sqrt{ad + bdx + c}}{7} + \frac{6a^2cd\sqrt{ad + bdx + c}}{7b} + \frac{6abd^2x^2\sqrt{ad + bdx + c}}{7} + \frac{12acdx\sqrt{ad + bdx + c}}{7} + \frac{6ac^2\sqrt{ad + bdx + c}}{7b} + \frac{2b^2d^2x^3\sqrt{ad + bdx + c}}{7} + \frac{6bcdx^2\sqrt{ad + bdx + c}}{7} + \frac{6c^2x\sqrt{ad + bdx + c}}{7} + \frac{2c^3\sqrt{ad + bdx + c}}{7bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((c**(5/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(5/2), Eq(b, 0))
, (c**(5/2)*x, Eq(d, 0)), (2*a**3*d**2*sqrt(a*d + b*d*x + c)/(7*b) + 6*a**2
*d**2*x*sqrt(a*d + b*d*x + c)/7 + 6*a**2*c*d*sqrt(a*d + b*d*x + c)/(7*b) +
6*a*b*d**2*x**2*sqrt(a*d + b*d*x + c)/7 + 12*a*c*d*x*sqrt(a*d + b*d*x + c)/
7 + 6*a*c**2*sqrt(a*d + b*d*x + c)/(7*b) + 2*b**2*d**2*x**3*sqrt(a*d + b*d*
x + c)/7 + 6*b*c*d*x**2*sqrt(a*d + b*d*x + c)/7 + 6*c**2*x*sqrt(a*d + b*d*x
 + c)/7 + 2*c**3*sqrt(a*d + b*d*x + c)/(7*b*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(19) = 38$.
time = 1.85, size = 444, normalized size = 19.30

$$\frac{2\left(35\,(bdx + ad + c)^{\frac{3}{2}}a^2d^2 - 35\left(3\sqrt{bdx + ad + c}\,ad - (bdx + ad + c)^{\frac{3}{2}} + 3\sqrt{bdx + ad + c}\,c\right)a^2d^2 - 21\,(bdx + ad + c)^{\frac{5}{2}}ad + 70\,(bdx + ad + c)^{\frac{3}{2}}acd - 70\left(3\sqrt{bdx + ad + c}\,ad - (bdx + ad + c)^{\frac{3}{2}} + 3\sqrt{bdx + ad + c}\,c\right)acd + 5\,(bdx + ad + c)^{\frac{7}{2}} - 21\,(bdx + ad + c)^{\frac{5}{2}}c + 35\,(bdx + ad + c)^{\frac{3}{2}}c^2 - 35\left(3\sqrt{bdx + ad + c}\,ad - (bdx + ad + c)^{\frac{3}{2}} + 3\sqrt{bdx + ad + c}\,c\right)c^2 + 7\left(15\sqrt{bdx + ad + c}\,a^2d^2 - 10\,(bdx + ad + c)^{\frac{3}{2}}ad + 30\sqrt{bdx + ad + c}\,acd + 3\,(bdx + ad + c)^{\frac{5}{2}} - 10\,(bdx + ad + c)^{\frac{3}{2}}c + 15\sqrt{bdx + ad + c}\,c^2\right)ad + 7\left(15\sqrt{bdx + ad + c}\,a^2d^2 - 10\,(bdx + ad + c)^{\frac{3}{2}}ad + 30\sqrt{bdx + ad + c}\,acd + 3\,(bdx + ad + c)^{\frac{5}{2}} - 10\,(bdx + ad + c)^{\frac{3}{2}}c + 15\sqrt{bdx + ad + c}\,c^2\right)c\right)}{35\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*(b*d*x + a*d + c)^(3/2)*a^2*d^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d
- (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a^2*d^2 - 21*(b*d*x
+ a*d + c)^(5/2)*a*d + 70*(b*d*x + a*d + c)^(3/2)*a*c*d - 70*(3*sqrt(b*d*x
+ a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*c*d
 + 5*(b*d*x + a*d + c)^(7/2) - 21*(b*d*x + a*d + c)^(5/2)*c + 35*(b*d*x + a
*d + c)^(3/2)*c^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/
2) + 3*sqrt(b*d*x + a*d + c)*c)*c^2 + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 -
 10*(b*d*x + a*d + c)^(3/2)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x
 + a*d + c)^(5/2) - 10*(b*d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)
*c^2)*a*d + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2
)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x + a*d + c)^(5/2) - 10*(b*
d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)*c^2)*c)/(b*d)
```

Mupad [B]
time = 0.18, size = 93, normalized size = 4.04

$$\frac{6\,x\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^2}{7}+\frac{2\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^3}{7\,b\,d}+\frac{2\,b^2\,d^2\,x^3\,\sqrt{c+d\,\left(a+b\,x\right)}}{7}+\frac{6\,b\,d\,x^2\,\sqrt{c+d\,\left(a+b\,x\right)}\,\left(c+a\,d\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(5/2),x)
```

```
[Out] (6*x*(c + d*(a + b*x))^(1/2)*(c + a*d)^2)/7 + (2*(c + d*(a + b*x))^(1/2)*(c
 + a*d)^3)/(7*b*d) + (2*b^2*d^2*x^3*(c + d*(a + b*x))^(1/2))/7 + (6*b*d*x^2
*(c + d*(a + b*x))^(1/2)*(c + a*d))/7
```

3.38 $\int (c+d(a+bx))^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(c+d(a+bx))^{5/2}}{5bd}$$

```
[Out] 2/5*(c+d*(b*x+a))^(5/2)/b/d
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{3/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$\frac{2(c+ad+bdx)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(5/2))/(5*b*d)
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.87

method	result	size
gosper	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5bd}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5bd}$	20
default	$\frac{2(bdx+ad+c)^{\frac{5}{2}}}{5bd}$	20
trager	$\frac{2\left(b^{2}d^{2}x^{2}+2ab\,d^{2}x+a^{2}d^{2}+2bcdx+2acd+c^{2}\right)\sqrt{bdx+ad+c}}{5bd}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*d*x+a*d+c)^(5/2)/b/d
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*((b*x + a)*d + c)^(5/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(19) = 38$.
time = 0.98, size = 59, normalized size = 2.57

$$\frac{2\left(b^{2}d^{2}x^{2}+a^{2}d^{2}+2\,acd+c^{2}+2\,(abd^{2}+bcd)x\right)\sqrt{bdx+ad+c}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*d^2*x^2 + a^2*d^2 + 2*a*c*d + c^2 + 2*(a*b*d^2 + b*c*d)*x)*sqrt(b*
d*x + a*d + c)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(17) = 34$.
time = 0.30, size = 153, normalized size = 6.65

$$\begin{cases} c^{\frac{3}{2}}x & \text{for } d = 0 \wedge (b = 0 \vee d = 0) \\ x(ad+c)^{\frac{3}{2}} & \text{for } b = 0 \\ \frac{2a^2d\sqrt{ad+bdx+c}}{5b} + \frac{4adx\sqrt{ad+bdx+c}}{5} + \frac{4ac\sqrt{ad+bdx+c}}{5b} + \frac{2bdx^2\sqrt{ad+bdx+c}}{5} + \frac{4cx\sqrt{ad+bdx+c}}{5} + \frac{2c^2\sqrt{ad+bdx+c}}{5bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x, Eq(d, 0) & (Eq(b, 0) | Eq(d, 0))), (x*(a*d + c)**(3/
2), Eq(b, 0)), (2*a**2*d*sqrt(a*d + b*d*x + c)/(5*b) + 4*a*d*x*sqrt(a*d + b
*d*x + c)/5 + 4*a*c*sqrt(a*d + b*d*x + c)/(5*b) + 2*b*d*x**2*sqrt(a*d + b*d
*x + c)/5 + 4*c*x*sqrt(a*d + b*d*x + c)/5 + 2*c**2*sqrt(a*d + b*d*x + c)/(5
*b*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(19) = 38$.
time = 1.27, size = 195, normalized size = 8.48

$$\frac{2\left(30\sqrt{bdx+ad+c}\,a^2d^2 - 10\,(bdx+ad+c)^{\frac{3}{2}}ad + 60\sqrt{bdx+ad+c}\,acd - 10\left(3\sqrt{bdx+ad+c}\,ad - (bdx+ad+c)^{\frac{3}{2}} + 3\sqrt{bdx+ad+c}\,c\right)ad + 3\,(bdx+ad+c)^{\frac{5}{2}} - 10\,(bdx+ad+c)^{\frac{3}{2}}c + 30\sqrt{bdx+ad+c}\,c^2 - 10\left(3\sqrt{bdx+ad+c}\,ad - (bdx+ad+c)^{\frac{3}{2}} + 3\sqrt{bdx+ad+c}\,c\right)c\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(30*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2)*a*d + 6
0*sqrt(b*d*x + a*d + c)*a*c*d - 10*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x +
a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*d + 3*(b*d*x + a*d + c)^(5/2)
 - 10*(b*d*x + a*d + c)^(3/2)*c + 30*sqrt(b*d*x + a*d + c)*c^2 - 10*(3*sqrt
(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c
)*c)/(b*d)
```

Mupad [B]
time = 0.17, size = 45, normalized size = 1.96

$$\sqrt{c+d\,\left(a+b\,x\right)}\,\left(x\,\left(\frac{4\,c}{5}+\frac{4\,a\,d}{5}\right)+\frac{2\,{\left(c+a\,d\right)}^2}{5\,b\,d}+\frac{2\,b\,d\,x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(3/2),x)
```

```
[Out] (c + d*(a + b*x))^(1/2)*(x*((4*c)/5 + (4*a*d)/5) + (2*(c + a*d)^2)/(5*b*d)
+ (2*b*d*x^2)/5)
```

3.39 $\int \sqrt{c+d(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{2(c+d(a+bx))^{3/2}}{3bd}$$

[Out] 2/3*(c+d*(b*x+a))^(3/2)/b/d

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d*(a + b*x)],x]

[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+d(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{c+dx}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(c+ad+bdx)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*(a + b*x)],x]
[Out] (2*(c + a*d + b*d*x)^(3/2))/(3*b*d)
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.87

method	result	size
gosper	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20
derivativedivides	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20
default	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20
trager	$\frac{2(bdx+ad+c)^{\frac{3}{2}}}{3bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*d*x+a*d+c)^(3/2)/b/d
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.83

$$\frac{2\left((bx+a)d+c\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2/3*((b*x + a)*d + c)^(3/2)/(b*d)
```

Fricas [A]
time = 1.06, size = 19, normalized size = 0.83

$$\frac{2\left(bdx+ad+c\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.
time = 0.09, size = 78, normalized size = 3.39

$$\begin{cases} \sqrt{c}\, x & \text{for}\ d = 0 \wedge (b = 0 \vee d = 0) \\ x\sqrt{ad + c} & \text{for}\ b = 0 \\ \frac{2a\sqrt{ad + bdx + c}}{3b} + \frac{2x\sqrt{ad + bdx + c}}{3} + \frac{2c\sqrt{ad + bdx + c}}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((sqrt(c)*x, Eq(d, 0) & (Eq(b, 0) | Eq(d, 0))), (x*sqrt(a*d + c),
Eq(b, 0)), (2*a*sqrt(a*d + b*d*x + c)/(3*b) + 2*x*sqrt(a*d + b*d*x + c)/3 +
 2*c*sqrt(a*d + b*d*x + c)/(3*b*d), True))
```

Giac [A]
time = 1.83, size = 19, normalized size = 0.83

$$\frac{2\,(bdx + ad + c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

Mupad [B]
time = 0.08, size = 19, normalized size = 0.83

$$\frac{2\,(c + d\,(a + b\,x))^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(1/2),x)
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

3.40 $\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{c+d(a+bx)}}{bd}$$

[Out] 2*(c+d*(b*x+a))^(1/2)/b/d

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[c + d*(a + b*x)],x]

[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}}\,dx, x, a+bx\right)}{b}\\ &= \frac{2\sqrt{c+d(a+bx)}}{bd}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{c+ad+bdx}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*(a + b*x)],x]
[Out] (2*Sqrt[c + a*d + b*d*x])/(b*d)
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.95

method	result	size
gosper	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20
derivativedivides	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20
default	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20
trager	$\frac{2\sqrt{bdx+ad+c}}{bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*d*x+a*d+c)^(1/2)/b/d
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.90

$$\frac{2\sqrt{(bx+a)d+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt((b*x + a)*d + c)/(b*d)
```

Fricas [A]
time = 0.58, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.61, size = 31, normalized size = 1.48

$$\begin{cases} \frac{x}{\sqrt{ad+c}} & \text{for } b = 0 \\ \frac{x}{\sqrt{c}} & \text{for } d = 0 \\ \frac{2\sqrt{c+d(a+bx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((x/sqrt(a*d + c), Eq(b, 0)), (x/sqrt(c), Eq(d, 0)), (2*sqrt(c + d
*(a + b*x))/(b*d), True))
```

Giac [A]
time = 1.21, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx+ad+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

Mupad [B]
time = 0.11, size = 19, normalized size = 0.90

$$\frac{2\sqrt{c+d(a+bx)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(1/2),x)
```

```
[Out] (2*(c + d*(a + b*x))^(1/2))/(b*d)
```

3.41 $\int \frac{1}{(c+d(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=21

$$-\frac{2}{bd\sqrt{c+d(a+bx)}}$$

```
[Out] -2/b/d/(c+d*(b*x+a))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{3/2}} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{3/2}} \, dx, x, a+bx\right)}{b}\\ &= -\frac{2}{bd\sqrt{c+d(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{c+ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + a*d + b*d*x])
```

Maple [A]
time = 0.08, size = 20, normalized size = 0.95

method	result	size
gosper	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20
derivativedivides	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20
default	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20
trager	$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(b*d*x+a*d+c)^(1/2)/b/d
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{(bx+a)d+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt((b*x + a)*d + c)*b*d)
```

Fricas [A]
time = 0.91, size = 34, normalized size = 1.62

$$-\frac{2\sqrt{bdx+ad+c}}{b^2d^2x+abd^2+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*d*x + a*d + c)/(b^2*d^2*x + a*b*d^2 + b*c*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.
time = 0.53, size = 58, normalized size = 2.76

$$\begin{cases} \frac{x}{c^{\frac{3}{2}}} & \text{for } b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{\frac{3}{2}}} & \text{for } b = 0 \\ \frac{x}{c^{\frac{3}{2}}} & \text{for } d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{abd^2+b^2d^2x+bcd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((x/c**(3/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(3/2), Eq(b, 0))
, (x/c**(3/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(a*b*d**2 + b**2*d**2*x
 + b*c*d), True))
```

Giac [A]
time = 1.57, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*d*x + a*d + c)*b*d)
```

Mupad [B]
time = 0.13, size = 19, normalized size = 0.90

$$-\frac{2}{b\,d\,\sqrt{c+d\,(a+b\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(3/2),x)
```

```
[Out] -2/(b*d*(c + d*(a + b*x))^(1/2))
```

3.42 $\int \frac{1}{(c+d(a+bx))^{5/2}} dx$

Optimal. Leaf size=23

$$-\frac{2}{3bd(c+d(a+bx))^{3/2}}$$

[Out] -2/3/b/d/(c+d*(b*x+a))^(3/2)

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*(a + b*x))^(-5/2),x]

[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{5/2}} dx, x, a+bx\right)}{b}\\ &= -\frac{2}{3bd(c+d(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2}{3bd(c+ad+bdx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + a*d + b*d*x)^(3/2))
```

Maple [A]
time = 0.08, size = 20, normalized size = 0.87

method	result	size
gosper	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}} bd}$	20
derivativedivides	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}} bd}$	20
default	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}} bd}$	20
trager	$-\frac{2}{3(bdx+ad+c)^{\frac{3}{2}} bd}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(b*d*x+a*d+c)^(3/2)/b/d
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.83

$$-\frac{2}{3\left((bx+a)d+c\right)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/(((b*x + a)*d + c)^(3/2)*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(19) = 38.
time = 1.14, size = 68, normalized size = 2.96

$$-\frac{2\sqrt{bdx+ad+c}}{3\left(b^3d^3x^2+a^2bd^3+2\,abcd^2+bc^2d+2\left(ab^2d^3+b^2cd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*d*x + a*d + c)/(b^3*d^3*x^2 + a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d
 + 2*(a*b^2*d^3 + b^2*c*d^2)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(19) = 38$.
time = 1.32, size = 102, normalized size = 4.43

$$\begin{cases} \frac{x}{c^{\frac{5}{2}}} & \text{for } b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{\frac{5}{2}}} & \text{for } b = 0 \\ \frac{x}{c^{\frac{5}{2}}} & \text{for } d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((x/c**(5/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(5/2), Eq(b, 0))
, (x/c**(5/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(3*a**2*b*d**3 + 6*a*b*
*2*d**3*x + 6*a*b*c*d**2 + 3*b**3*d**3*x**2 + 6*b**2*c*d**2*x + 3*b*c**2*d)
, True))
```

Giac [A]
time = 1.20, size = 19, normalized size = 0.83

$$-\frac{2}{3\,(bdx+ad+c)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*d*x + a*d + c)^(3/2)*b*d)
```

Mupad [B]
time = 0.18, size = 19, normalized size = 0.83

$$-\frac{2}{3\,b\,d\,\left(c+d\,\left(a+b\,x\right)\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(5/2),x)
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

3.43 $\int x^3(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = $\{45\}$

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)\,dx&=\int\left(ax^3+bx^4\right)\,dx\\&=\frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A]
time = 0.85, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

Giac [**A**]
time = 1.21, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [**B**]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,\left(5\,a+4\,b\,x\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

3.44 $\int x^2(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[Out] 1/3*x^3*a+1/4*b*x^4
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = $\{45\}$

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)\,dx&=\int \left(ax^2+bx^3\right)\,dx\\&=\frac{ax^3}{3}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/3*a*x^3+1/4*b*x^4
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="maxima")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A]
time = 0.55, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="fricas")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a),x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Giac [A]
time = 1.37, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,(4\,a+3\,b\,x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x),x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

3.45 $\int x(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{45\}$

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x),x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)\,dx &= \int \left(ax+bx^2\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x),x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A]
time = 0.60, size = 13, normalized size = 0.76

$$\frac{1}{3}x^3b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b + 1/2*x^2*a
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

Giac [A]
time = 1.18, size = 13, normalized size = 0.76

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\,(3\,a+2\,b\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x),x)
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

3.46 $\int (a + bx)\,dx$

Optimal. Leaf size=12

$$ax + \frac{bx^2}{2}$$

[Out] a*x+1/2*b*x^2

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[a + b*x,x]

[Out] a*x + (b*x^2)/2

Rubi steps

$$\int (a + bx)\,dx = ax + \frac{bx^2}{2}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*x,x]

[Out] a*x + (b*x^2)/2

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{1}{2}x^2b + ax$	11
default	$\frac{1}{2}x^2b + ax$	11

norman	$\frac{1}{2}x^2b + ax$	11
risch	$\frac{1}{2}x^2b + ax$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*b+a*x
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.80, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A]
time = 0.00, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A]
time = 1.46, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

3.47 $\int \frac{a+bx}{x}\,dx$

Optimal. Leaf size=8

$$bx + a\log(x)$$

[Out] b*x+a*ln(x)

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$a\log(x) + bx$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x,x]

[Out] b*x + a*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x}\,dx &= \int \left(b+\frac{a}{x}\right)\,dx\\ &= bx + a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x,x]

[Out] b*x + a*Log[x]

Maple [A]
time = 0.01, size = 9, normalized size = 1.12

method	result	size
default	$bx + a\ln(x)$	9
norman	$bx + a\ln(x)$	9
risch	$bx + a\ln(x)$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] b*x+a*ln(x)
```

Maxima [A]
time = 0.27, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="maxima")
```

```
[Out] b*x + a*log(x)
```

Fricas [A]
time = 0.80, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.88

$$a\log(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x)
```

```
[Out] a*log(x) + b*x
```

Giac [A]
time = 2.24, size = 9, normalized size = 1.12

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="giac")
```

```
[Out] b*x + a*log(abs(x))
```

Mupad [B]
time = 0.02, size = 8, normalized size = 1.00

$$b\,x+a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x,x)
```

```
[Out] b*x + a*log(x)
```

3.48 $\int \frac{a+bx}{x^2} \, dx$

Optimal. Leaf size=11

$$-\frac{a}{x}+b\log(x)$$

[Out] -a/x+b*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$b\log(x)-\frac{a}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^2,x]

[Out] -(a/x) + b*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2} \, dx &= \int \left(\frac{a}{x^2}+\frac{b}{x}\right) dx\\ &= -\frac{a}{x}+b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{a}{x}+b\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^2,x]

[Out] -(a/x) + b*Log[x]

Maple [A]
time = 0.01, size = 12, normalized size = 1.09

method	result	size
default	$-\frac{a}{x}+b\ln\left(x\right)$	12
norman	$-\frac{a}{x}+b\ln\left(x\right)$	12
risch	$-\frac{a}{x}+b\ln\left(x\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*ln(x)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$b\log\left(x\right)-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="maxima")
[Out] b*log(x) - a/x
```

Fricas [A]
time = 0.67, size = 13, normalized size = 1.18

$$\frac{bx\log\left(x\right)-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="fricas")
[Out] (b*x*log(x) - a)/x
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.64

$$-\frac{a}{x}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2,x)
[Out] -a/x + b*log(x)
```

Giac [A]
time = 1.48, size = 12, normalized size = 1.09

$$b\log\left(|x|\right) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] b*log(abs(x)) - a/x
```

Mupad [B]
time = 0.03, size = 11, normalized size = 1.00

$$b\,\ln\left(x\right) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^2,x)
```

```
[Out] b*log(x) - a/x
```

3.49 $\int \frac{a+bx}{x^3}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^2}{2ax^2}$$

[Out] -1/2*(b*x+a)^2/a/x^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {37}

$$-\frac{(a+bx)^2}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^3,x]

[Out] -1/2*(a + b*x)^2/(a*x^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{x^3}\,dx = -\frac{(a+bx)^2}{2ax^2}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 0.88

$$-\frac{a}{2x^2}-\frac{b}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^3,x]

[Out] -1/2*a/x^2 - b/x

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2bx+a}{2x^2}$	12
norman	$\frac{-bx-\frac{a}{2}}{x^2}$	13
risch	$\frac{-bx-\frac{a}{2}}{x^2}$	13
default	$-\frac{b}{x}-\frac{a}{2x^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/x-1/2/x^2*a
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Fricas [A]
time = 0.60, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{-a-2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3,x)
```

```
[Out] (-a - 2*b*x)/(2*x**2)
```

Giac [A]
time = 1.64, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/x^2
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.65

$$-\frac{a+2\,b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^3,x)
```

```
[Out] -(a + 2*b*x)/(2*x^2)
```

3.50 $\int \frac{a+bx}{x^4} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

```
[Out] -1/3*a/x^3-1/2*b/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^4} \, dx &= \int \left(\frac{a}{x^4}+\frac{b}{x^3}\right) \, dx \\ &= -\frac{a}{3x^3}-\frac{b}{2x^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{3x^3}-\frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^3}$	13
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^3}$	13
gosper	$-\frac{3bx+2a}{6x^3}$	14
default	$-\frac{a}{3x^3}-\frac{b}{2x^2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/2/x^2*b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Fricas [A]
time = 0.60, size = 13, normalized size = 0.76

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.82

$$\frac{-2a-3bx}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4,x)
```

```
[Out] (-2*a - 3*b*x)/(6*x**3)
```

Giac [A]
time = 1.44, size = 13, normalized size = 0.76

$$-\frac{3\,bx+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + 2*a)/x^3
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,a+3\,b\,x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^4,x)
```

```
[Out] -(2*a + 3*b*x)/(6*x^3)
```

3.51 $\int \frac{a+bx}{x^5} dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

[Out] -1/4*a/x^4-1/3*b/x^3

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/x^5,x]

[Out] -1/4*a/x^4 - b/(3*x^3)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^5}\,dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx\\ &= -\frac{a}{4x^4}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/x^5,x]

```
[Out] -1/4*a/x^4 - b/(3*x^3)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^4}$	13
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^4}$	13
gosper	$-\frac{4bx+3a}{12x^4}$	14
default	$-\frac{a}{4x^4}-\frac{b}{3x^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4-1/3*b/x^3
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="maxima")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Fricas [A]
time = 0.93, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="fricas")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.82

$$\frac{-3a-4bx}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**5,x)
```

```
[Out] (-3*a - 4*b*x)/(12*x**4)
```

Giac [A]
time = 1.48, size = 13, normalized size = 0.76

$$-\frac{4\,bx + 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(4*b*x + 3*a)/x^4
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{3\,a + 4\,b\,x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^5,x)
```

```
[Out] -(3*a + 4*b*x)/(12*x^4)
```

3.52 $\int x^3(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^2\,dx &= \int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
default	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
norman	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Fricas [A]
time = 1.14, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4}+\frac{2abx^5}{5}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Giac [A]
time = 1.11, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B]
time = 0.08, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

3.53 $\int x^2(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^2\,dx &= \int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
default	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
norman	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Fricas [A]
time = 1.41, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Giac [A]
time = 1.41, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

3.54 $\int x(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

```
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {45}

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^2\,dx &= \int \left(a^2x+2abx^2+b^2x^3\right)\,dx\\ &= \frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
gosper	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
default	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
norman	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25
risch	$\frac{1}{2}a^2x^2+\frac{2}{3}ab\,x^3+\frac{1}{4}b^2x^4$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Fricas [A]
time = 1.58, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.87

$$\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2,x)
```

```
[Out] a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

Giac [A]
time = 1.71, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4+\frac{2}{3}\,abx^3+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2}+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^4)/4 + (2*a*b*x^3)/3
```

3.55 $\int (a+bx)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

[Out] 1/3*(b*x+a)^3/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2\,dx = \frac{(a+bx)^3}{3b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
gosper	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
norman	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
risch	$\frac{b^2x^3}{3} + ab\,x^2 + a^2x + \frac{a^3}{3b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/b
```

Maxima [A]
time = 0.28, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A]
time = 1.65, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.01, size = 19, normalized size = 1.36

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A]
time = 1.14, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3/b
```

Mupad [B]
time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
```

```
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

3.56 $\int \frac{(a+bx)^2}{x} \, dx$

Optimal. Leaf size=22

$$2abx + \frac{b^2x^2}{2} + a^2\log(x)$$

[Out] 2*a*b*x+1/2*b^2*x^2+a^2*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/x,x]

[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x} \, dx &= \int \left(2ab + \frac{a^2}{x} + b^2x\right) dx \\ &= 2abx + \frac{b^2x^2}{2} + a^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$2abx + \frac{b^2x^2}{2} + a^2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2/x,x]

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Maple [A]
time = 0.07, size = 21, normalized size = 0.95

method	result	size
default	$2abx+\frac{x^2b^2}{2}+a^2\ln\left(x\right)$	21
norman	$2abx+\frac{x^2b^2}{2}+a^2\ln\left(x\right)$	21
risch	$2abx+\frac{x^2b^2}{2}+a^2\ln\left(x\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x,x,method=_RETURNVERBOSE)
[Out] 2*a*b*x+1/2*x^2*b^2+a^2*ln(x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2+2\,abx+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Fricas [A]
time = 1.47, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2+2\,abx+a^2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

Sympy [A]
time = 0.02, size = 20, normalized size = 0.91

$$a^2\log\left(x\right)+2abx+\frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x,x)
```

```
[Out] a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

Giac [A]
time = 1.30, size = 21, normalized size = 0.95

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.91

$$a^2\,\ln\left(x\right)+\frac{b^2\,x^2}{2}+2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x,x)
```

```
[Out] a^2*log(x) + (b^2*x^2)/2 + 2*a*b*x
```

3.57 $\int \frac{(a+bx)^2}{x^2} dx$

Optimal. Leaf size=20

$$-\frac{a^2}{x}+b^2x+2ab\log(x)$$

```
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2}\,dx &= \int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx\\ &= -\frac{a^2}{x}+b^2x+2ab\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-\frac{a^2}{x}+b^2x+2ab\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Maple [A]
time = 0.07, size = 21, normalized size = 1.05

method	result	size
default	$-\frac{a^2}{x} + b^2 x + 2ab\ln(x)$	21
risch	$-\frac{a^2}{x} + b^2 x + 2ab\ln(x)$	21
norman	$\frac{x^2 b^2 - a^2}{x} + 2ab\ln(x)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.00

$$b^2 x + 2\,ab\log(x) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="maxima")
[Out] b^2*x + 2*a*b*log(x) - a^2/x
```

Fricas [A]
time = 1.07, size = 24, normalized size = 1.20

$$\frac{b^2 x^2 + 2\,abx\log(x) - a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.85

$$-\frac{a^2}{x} + 2ab\log(x) + b^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*log(x) + b**2*x
```

Giac [A]
time = 1.78, size = 21, normalized size = 1.05

$$b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

Mupad [B]
time = 0.07, size = 20, normalized size = 1.00

$$b^2\,x - \frac{a^2}{x} + 2\,a\,b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^2,x)
```

```
[Out] b^2*x - a^2/x + 2*a*b*log(x)
```

3.58 $\int \frac{(a+bx)^2}{x^3} \, dx$

Optimal. Leaf size=24

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

```
[Out] -1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3}\,dx &= \int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx\\ &= -\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)\end{aligned}$$

Mathematica [A]

time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

Maple [A]
time = 0.08, size = 23, normalized size = 0.96

method	result	size
default	$-\frac{a^2}{2x^2}-\frac{2ab}{x}+b^2\ln\left(x\right)$	23
norman	$\frac{-\frac{1}{2}a^2-2abx}{x^2}+b^2\ln\left(x\right)$	23
risch	$\frac{-\frac{1}{2}a^2-2abx}{x^2}+b^2\ln\left(x\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.88

$$b^2\log\left(x\right)-\frac{4\,abx+a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="maxima")
[Out] b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

Fricas [A]
time = 1.52, size = 26, normalized size = 1.08

$$\frac{2\,b^2x^2\log\left(x\right)-4\,abx-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

Sympy [A]
time = 0.05, size = 22, normalized size = 0.92

$$b^2\log\left(x\right)+\frac{-a^2-4abx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3,x)
```

```
[Out] b**2*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

Giac [A]
time = 1.37, size = 22, normalized size = 0.92

$$b^2 \log (|x|) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="giac")
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.96

$$b^2 \ln (x) - \frac{\frac{a^2}{2} + 2\,b\,x\,a}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^3,x)
[Out] b^2*log(x) - (a^2/2 + 2*a*b*x)/x^2
```

3.59 $\int \frac{(a+bx)^2}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^3}{3ax^3}$$

```
[Out] -1/3*(b*x+a)^3/x^3/a
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$-\frac{(a+bx)^3}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^4,x]
```

```
[Out] -1/3*(a + b*x)^3/(a*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4} dx = -\frac{(a+bx)^3}{3ax^3}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.53

$$-\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (a*b)/x^2 - b^2/x
```

Maple [A]
time = 0.07, size = 25, normalized size = 1.47

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3x^3}$	23
norman	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{x^3}$	24
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{x^3}$	24
default	$-\frac{b^2}{x}-\frac{a^2}{3x^3}-\frac{ab}{x^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -b^2/x-1/3*a^2/x^3-a*b/x^2
```

Maxima [A]
time = 0.27, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Fricas [A]
time = 1.10, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Sympy [A]
time = 0.05, size = 24, normalized size = 1.41

$$\frac{-a^2-3abx-3b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

Giac [A]
time = 1.12, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

Mupad [B]
time = 0.04, size = 22, normalized size = 1.29

$$-\frac{\frac{a^2}{3}+a\,b\,x+b^2\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^4,x)
```

```
[Out] -(a^2/3 + b^2*x^2 + a*b*x)/x^3
```

3.60 $\int \frac{(a+bx)^2}{x^5}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

```
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^5}\,dx &= \int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx\\ &= -\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Maple [A]
time = 0.07, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^4}$	24
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^4}$	24
gosper	$-\frac{6x^2b^2+8abx+3a^2}{12x^4}$	25
default	$-\frac{a^2}{4x^4}-\frac{2ab}{3x^3}-\frac{b^2}{2x^2}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="maxima")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Fricas [A]
time = 1.44, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.87

$$\frac{-3a^2-8abx-6b^2x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**5,x)
```

```
[Out] (-3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

Giac [A]
time = 1.05, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2+8\,abx+3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{4}+\frac{2\,a\,b\,x}{3}+\frac{b^2\,x^2}{2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^5,x)
```

```
[Out] -(a^2/4 + (b^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

3.61 $\int \frac{(a+bx)^2}{x^6} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

```
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^6}\,dx &= \int \left(\frac{a^2}{x^6} + \frac{2ab}{x^5} + \frac{b^2}{x^4}\right) dx \\ &= -\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{5x^5} - \frac{ab}{2x^4} - \frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{3}x^2b^2-\frac{1}{2}abx-\frac{1}{5}a^2}{x^5}$	24
risch	$\frac{-\frac{1}{3}x^2b^2-\frac{1}{2}abx-\frac{1}{5}a^2}{x^5}$	24
gosper	$-\frac{10x^2b^2+15abx+6a^2}{30x^5}$	25
default	$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Fricas [A]
time = 1.24, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.87

$$\frac{-6a^2-15abx-10b^2x^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**6,x)
```

```
[Out] (-6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

Giac [A]
time = 1.26, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{5}+\frac{a\,b\,x}{2}+\frac{b^2\,x^2}{3}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^6,x)
```

```
[Out] -(a^2/5 + (b^2*x^2)/3 + (a*b*x)/2)/x^5
```

3.62 $\int \frac{(a+bx)^2}{x^7} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

```
[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

Rubi [A]

time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^7,x]
```

```
[Out] -1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^7,x]
```

[Out] -1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)

Maple [A]
time = 0.09, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{4}x^{2}b^{2}-\frac{2}{5}abx-\frac{1}{6}a^{2}}{x^{6}}$	24
risch	$\frac{-\frac{1}{4}x^{2}b^{2}-\frac{2}{5}abx-\frac{1}{6}a^{2}}{x^{6}}$	24
gosper	$-\frac{15x^{2}b^{2}+24abx+10a^{2}}{60x^{6}}$	25
default	$-\frac{a^{2}}{6x^{6}}-\frac{2ab}{5x^{5}}-\frac{b^{2}}{4x^{4}}$	25

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2/x^7,x,method=_RETURNVERBOSE)

[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$-\frac{15\,b^{2}x^{2}+24\,abx+10\,a^{2}}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/x^7,x, algorithm="maxima")

[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6

Fricas [A]
time = 1.19, size = 24, normalized size = 0.80

$$-\frac{15\,b^{2}x^{2}+24\,abx+10\,a^{2}}{60\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/x^7,x, algorithm="fricas")

[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6

Sympy [A]
time = 0.07, size = 26, normalized size = 0.87

$$\frac{-10a^{2}-24abx-15b^{2}x^{2}}{60x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**7,x)
```

```
[Out] (-10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

Giac [A]
time = 1.40, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{6}+\frac{2\,a\,b\,x}{5}+\frac{b^2\,x^2}{4}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^7,x)
```

```
[Out] -(a^2/6 + (b^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

3.63 $\int \frac{(a+bx)^2}{x^8} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

```
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^8}\,dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx \\ &= -\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{5}x^2b^2-\frac{1}{3}abx-\frac{1}{7}a^2}{x^7}$	24
risch	$\frac{-\frac{1}{5}x^2b^2-\frac{1}{3}abx-\frac{1}{7}a^2}{x^7}$	24
gosper	$-\frac{21x^2b^2+35abx+15a^2}{105x^7}$	25
default	$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Fricas [A]
time = 1.11, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Sympy [A]
time = 0.07, size = 26, normalized size = 0.87

$$\frac{-15a^2-35abx-21b^2x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**8,x)
```

```
[Out] (-15*a**2 - 35*a*b*x - 21*b**2*x**2)/(105*x**7)
```

Giac [A]
time = 1.19, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{7}+\frac{a\,b\,x}{3}+\frac{b^2\,x^2}{5}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^8,x)
```

```
[Out] -(a^2/7 + (b^2*x^2)/5 + (a*b*x)/3)/x^7
```

3.64 $\int x^4(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

```
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^3\,dx&=\int\left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)\,dx\\&=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
default	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
norman	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36
risch	$\frac{1}{5}a^3x^5+\frac{1}{2}a^2b\,x^6+\frac{3}{7}a\,b^2x^7+\frac{1}{8}b^3x^8$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Fricas [A]
time = 1.28, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8+\frac{3}{7}\,ab^2x^7+\frac{1}{2}\,a^2bx^6+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5}+\frac{a^2bx^6}{2}+\frac{3ab^2x^7}{7}+\frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**3,x)
```

```
[Out] a**3*x**5/5 + a**2*b*x**6/2 + 3*a*b**2*x**7/7 + b**3*x**8/8
```

Giac [A]
time = 1.44, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8 + \frac{3}{7}\,ab^2x^7 + \frac{1}{2}\,a^2bx^6 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5} + \frac{a^2\,b\,x^6}{2} + \frac{3\,a\,b^2\,x^7}{7} + \frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^8)/8 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7
```

3.65 $\int x^3(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

```
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^3\,dx&=\int\left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)\,dx\\&=\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^3,x]
```

```
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
default	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
norman	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36
risch	$\frac{1}{4}a^3x^4+\frac{3}{5}a^2b\,x^5+\frac{1}{2}a\,b^2x^6+\frac{1}{7}b^3x^7$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Fricas [A]
time = 1.04, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7+\frac{1}{2}\,ab^2x^6+\frac{3}{5}\,a^2bx^5+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Sympy [A]
time = 0.01, size = 37, normalized size = 0.86

$$\frac{a^3x^4}{4}+\frac{3a^2bx^5}{5}+\frac{ab^2x^6}{2}+\frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**3,x)
```

```
[Out] a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7
```

Giac [A]
time = 1.49, size = 35, normalized size = 0.81

$$\frac{1}{7}b^3x^7 + \frac{1}{2}ab^2x^6 + \frac{3}{5}a^2bx^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{ab^2x^6}{2} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^7)/7 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2
```

3.66 $\int x^2(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

```
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^3\,dx &= \int \left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)\,dx\\ &= \frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Maple [A]
time = 0.07, size = 36, normalized size = 0.84

method	result	size
gosper	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
default	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
norman	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36
risch	$\frac{1}{3}a^3x^3+\frac{3}{4}a^2b\,x^4+\frac{3}{5}a\,b^2x^5+\frac{1}{6}b^3x^6$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Fricas [A]
time = 0.68, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Sympy [A]
time = 0.01, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3}+\frac{3a^2bx^4}{4}+\frac{3ab^2x^5}{5}+\frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6
```

Giac [A]
time = 1.46, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6+\frac{3}{5}\,ab^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3}+\frac{3\,a^2\,b\,x^4}{4}+\frac{3\,a\,b^2\,x^5}{5}+\frac{b^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^6)/6 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5
```

3.67 $\int x(a+bx)^3\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}$$

```
[Out] -1/4*a*(b*x+a)^4/b^2+1/5*(b*x+a)^5/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = $\{45\}$

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^3,x]
```

```
[Out] -1/4*(a*(a + b*x)^4)/b^2 + (a + b*x)^5/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^3\,dx&=\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx\\&=-\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.33

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}ab^2x^4+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^3,x]
```

```
[Out] (a^3*x^2)/2 + a^2*b*x^3 + (3*a*b^2*x^4)/4 + (b^3*x^5)/5
```

Maple [A]
time = 0.08, size = 35, normalized size = 1.17

method	result	size
gosper	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
default	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
norman	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35
risch	$\frac{1}{5}b^3x^5+\frac{3}{4}a\,b^2x^4+a^2b\,x^3+\frac{1}{2}a^3x^2$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/5*b^3*x^5+3/4*a*b^2*x^4+a^2*b*x^3+1/2*a^3*x^2
```

Maxima [A]
time = 0.29, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="maxima")
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Fricas [A]
time = 0.79, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5+\frac{3}{4}\,ab^2x^4+a^2bx^3+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="fricas")
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Sympy [A]
time = 0.01, size = 36, normalized size = 1.20

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3ab^2x^4}{4}+\frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**3,x)
```

```
[Out] a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5
```

Giac [**A**]
time = 1.41, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

Mupad [**B**]
time = 0.04, size = 34, normalized size = 1.13

$$\frac{a^3\,x^2}{2} + a^2\,b\,x^3 + \frac{3\,a\,b^2\,x^4}{4} + \frac{b^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^5)/5 + a^2*b*x^3 + (3*a*b^2*x^4)/4
```

3.68 $\int (a+bx)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^4}{4b}$$

```
[Out] 1/4*(b*x+a)^4/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3,x]
```

```
[Out] (a + b*x)^4/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3 \, dx = \frac{(a+bx)^4}{4b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3,x]
```

```
[Out] (a + b*x)^4/(4*b)
```

Maple [A]
time = 0.07, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^4}{4b}$	13
gosper	$\frac{1}{4}b^3x^4 + a\,b^2x^3 + \frac{3}{2}a^2b\,x^2 + a^3x$	32
norman	$\frac{1}{4}b^3x^4 + a\,b^2x^3 + \frac{3}{2}a^2b\,x^2 + a^3x$	32
risch	$\frac{b^3x^4}{4} + a\,b^2x^3 + \frac{3a^2b\,x^2}{2} + a^3x + \frac{a^4}{4b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.27, size = 31, normalized size = 2.21

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.87, size = 31, normalized size = 2.21

$$\frac{1}{4}b^3x^4 + ab^2x^3 + \frac{3}{2}a^2bx^2 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.
time = 0.01, size = 32, normalized size = 2.29

$$a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

Giac [A]
time = 1.31, size = 12, normalized size = 0.86

$$\frac{(bx+a)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*x + a)^4/b
```

Mupad [B]
time = 0.04, size = 31, normalized size = 2.21

$$a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3,x)
```

```
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

3.69 $\int \frac{(a+bx)^3}{x} \, dx$

Optimal. Leaf size=35

$$3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}+a^3\log(x)$$

```
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$a^3\log(x)+3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x}\,dx &= \int \left(3a^2b+\frac{a^3}{x}+3ab^2x+b^3x^2\right)dx\\ &= 3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}+a^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$3a^2bx+\frac{3}{2}ab^2x^2+\frac{b^3x^3}{3}+a^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Maple [A]
time = 0.08, size = 32, normalized size = 0.91

method	result	size
default	$3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln\left(x\right)$	32
norman	$3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln\left(x\right)$	32
risch	$3a^2bx + \frac{3a\,b^2x^2}{2} + \frac{b^3x^3}{3} + a^3\ln\left(x\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.89

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Fricas [A]
time = 1.14, size = 31, normalized size = 0.89

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

Sympy [A]
time = 0.03, size = 34, normalized size = 0.97

$$a^3\log\left(x\right) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3
```

Giac [A]
time = 1.76, size = 32, normalized size = 0.91

$$\frac{1}{3}\,b^3x^3 + \frac{3}{2}\,ab^2x^2 + 3\,a^2bx + a^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

Mupad [B]
time = 0.03, size = 31, normalized size = 0.89

$$a^3\,\ln\left(x\right) + \frac{b^3\,x^3}{3} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x,x)
```

```
[Out] a^3*log(x) + (b^3*x^3)/3 + (3*a*b^2*x^2)/2 + 3*a^2*b*x
```

3.70 $\int \frac{(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)$$

```
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^2}\,dx &= \int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)dx\\ &= -\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Maple [A]
time = 0.08, size = 33, normalized size = 0.97

method	result	size
default	$-\frac{a^3}{x}+3a\,b^2x+\frac{b^3x^2}{2}+3a^2b\ln\left(x\right)$	33
risch	$-\frac{a^3}{x}+3a\,b^2x+\frac{b^3x^2}{2}+3a^2b\ln\left(x\right)$	33
norman	$\frac{-a^3+\frac{1}{2}b^3x^3+3a\,b^2x^2}{x}+3a^2b\ln\left(x\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^2,x,method=_RETURNVERBOSE)
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

Maxima [A]
time = 0.27, size = 32, normalized size = 0.94

$$\frac{1}{2}\,b^3x^2+3\,ab^2x+3\,a^2b\log\left(x\right)-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(x) - a^3/x
```

Fricas [A]
time = 1.14, size = 36, normalized size = 1.06

$$\frac{b^3x^3+6\,ab^2x^2+6\,a^2bx\log\left(x\right)-2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="fricas")
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

Sympy [A]
time = 0.03, size = 31, normalized size = 0.91

$$-\frac{a^3}{x}+3a^2b\log\left(x\right)+3ab^2x+\frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2
```

Giac [A]
time = 1.30, size = 33, normalized size = 0.97

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log\left(|x|\right) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.94

$$\frac{b^3\,x^2}{2} - \frac{a^3}{x} + 3\,a^2\,b\,\ln\left(x\right) + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^2,x)
```

```
[Out] (b^3*x^2)/2 - a^3/x + 3*a^2*b*log(x) + 3*a*b^2*x
```

3.71 $\int \frac{(a+bx)^3}{x^3} \, dx$

Optimal. Leaf size=33

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)$$

```
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^3}\,dx &= \int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)dx\\ &= -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Maple [A]
time = 0.09, size = 32, normalized size = 0.97

method	result	size
default	$-\frac{a^{3}}{2x^{2}}-\frac{3a^{2}b}{x}+b^{3}x+3a\,b^{2}\ln\left(x\right)$	32
risch	$b^{3}x+\frac{-3a^{2}bx-\frac{1}{2}a^{3}}{x^{2}}+3a\,b^{2}\ln\left(x\right)$	32
norman	$\frac{b^{3}x^{3}-\frac{1}{2}a^{3}-3a^{2}bx}{x^{2}}+3a\,b^{2}\ln\left(x\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.91

$$b^{3}x+3\,ab^{2}\log\left(x\right)-\frac{6\,a^{2}bx+a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="maxima")
[Out] b^3*x + 3*a*b^2*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Fricas [A]
time = 0.97, size = 37, normalized size = 1.12

$$\frac{2\,b^{3}x^{3}+6\,ab^{2}x^{2}\log\left(x\right)-6\,a^{2}bx-a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

Sympy [A]
time = 0.06, size = 32, normalized size = 0.97

$$3ab^{2}\log\left(x\right)+b^{3}x+\frac{-a^{3}-6a^{2}bx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**3,x)
```

```
[Out] 3*a*b**2*log(x) + b**3*x + (-a**3 - 6*a**2*b*x)/(2*x**2)
```

Giac [**A**]
time = 1.50, size = 31, normalized size = 0.94

$$b^3x + 3\,ab^2\log\left(|x|\right) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="giac")
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

Mupad [**B**]
time = 0.03, size = 32, normalized size = 0.97

$$b^3\,x - \frac{\frac{a^3}{2} + 3\,b\,x\,a^2}{x^2} + 3\,a\,b^2\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^3,x)
[Out] b^3*x - (a^3/2 + 3*a^2*b*x)/x^2 + 3*a*b^2*log(x)
```

3.72 $\int \frac{(a+bx)^3}{x^4}\,dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

```
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^4}\,dx &= \int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Maple [A]
time = 0.09, size = 34, normalized size = 0.92

method	result	size
default	$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3a\,b^2}{x}+b^3\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{3}a^3-3a\,b^2x^2-\frac{3}{2}a^2bx}{x^3}+b^3\ln\left(x\right)$	34
risch	$\frac{-\frac{1}{3}a^3-3a\,b^2x^2-\frac{3}{2}a^2bx}{x^3}+b^3\ln\left(x\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.92

$$b^3\log\left(x\right)-\frac{18\,ab^2x^2+9\,a^2bx+2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="maxima")
[Out] b^3*log(x) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Fricas [A]
time = 1.24, size = 37, normalized size = 1.00

$$\frac{6\,b^3x^3\log\left(x\right)-18\,ab^2x^2-9\,a^2bx-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="fricas")
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

Sympy [A]
time = 0.07, size = 36, normalized size = 0.97

$$b^3\log\left(x\right)+\frac{-2a^3-9a^2bx-18ab^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**4,x)
```

```
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

Giac [A]
time = 1.38, size = 35, normalized size = 0.95

$$b^3 \log(|x|) - \frac{18\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

Mupad [B]
time = 0.07, size = 34, normalized size = 0.92

$$b^3\,\ln(x) - \frac{\frac{a^3}{3} + \frac{3\,a^2\,b\,x}{2} + 3\,a\,b^2\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^4,x)
```

```
[Out] b^3*log(x) - (a^3/3 + 3*a*b^2*x^2 + (3*a^2*b*x)/2)/x^3
```

3.73 $\int \frac{(a+bx)^3}{x^5} \, dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^4}{4ax^4}$$

```
[Out] -1/4*(b*x+a)^4/a/x^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$-\frac{(a+bx)^4}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^5,x]
```

```
[Out] -1/4*(a + b*x)^4/(a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^3}{x^5} \, dx = -\frac{(a+bx)^4}{4ax^4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.
time = 0.00, size = 39, normalized size = 2.29

$$-\frac{a^3}{4x^4} - \frac{a^2 b}{x^3} - \frac{3ab^2}{2x^2} - \frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (a^2*b)/x^3 - (3*a*b^2)/(2*x^2) - b^3/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.07, size = 36, normalized size = 2.12

method	result	size
gosper	$-\frac{4b^3x^3+6a\,b^2x^2+4a^2bx+a^3}{4x^4}$	34
norman	$\frac{-b^3x^3-\frac{3}{2}a\,b^2x^2-a^2bx-\frac{1}{4}a^3}{x^4}$	35
risch	$\frac{-b^3x^3-\frac{3}{2}a\,b^2x^2-a^2bx-\frac{1}{4}a^3}{x^4}$	35
default	$-\frac{b^3}{x}-\frac{a^2b}{x^3}-\frac{a^3}{4x^4}-\frac{3a\,b^2}{2x^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/x-a^2*b/x^3-1/4*a^3/x^4-3/2*a*b^2/x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.27, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 1.02, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.08, size = 36, normalized size = 2.12

$$\frac{-a^3-4a^2bx-6ab^2x^2-4b^3x^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**5,x)
```

```
[Out] (-a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 1.61, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

Mupad [B]
time = 0.03, size = 33, normalized size = 1.94

$$-\frac{\frac{a^3}{4} + a^2\,b\,x + \frac{3\,a\,b^2\,x^2}{2} + b^3\,x^3}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^5,x)
```

```
[Out] -(a^3/4 + b^3*x^3 + (3*a*b^2*x^2)/2 + a^2*b*x)/x^4
```

3.74 $\int \frac{(a+bx)^3}{x^6}\,dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^4}{5ax^5}+\frac{b(a+bx)^4}{20a^2x^4}$$

```
[Out] -1/5*(b*x+a)^4/a/x^5+1/20*b*(b*x+a)^4/a^2/x^4
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{b(a+bx)^4}{20a^2x^4}-\frac{(a+bx)^4}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^6,x]
```

```
[Out] -1/5*(a + b*x)^4/(a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^6}\,dx &= -\frac{(a+bx)^4}{5ax^5}-\frac{b\int \frac{(a+bx)^3}{x^5}\,dx}{5a}\\ &= -\frac{(a+bx)^4}{5ax^5}+\frac{b(a+bx)^4}{20a^2x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.14

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{4x^4}-\frac{ab^2}{x^3}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^6,x]
[Out] -1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) - (a*b^2)/x^3 - b^3/(2*x^2)
```

Maple [A]
time = 0.07, size = 36, normalized size = 1.00

method	result	size
norman	$\frac{-\frac{1}{2}b^3x^3-a\,b^2x^2-\frac{3}{4}a^2bx-\frac{1}{5}a^3}{x^5}$	35
risch	$\frac{-\frac{1}{2}b^3x^3-a\,b^2x^2-\frac{3}{4}a^2bx-\frac{1}{5}a^3}{x^5}$	35
gosper	$-\frac{10b^3x^3+20a\,b^2x^2+15a^2bx+4a^3}{20x^5}$	36
default	$-\frac{a\,b^2}{x^3}-\frac{3a^2b}{4x^4}-\frac{b^3}{2x^2}-\frac{a^3}{5x^5}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^6,x,method=_RETURNVERBOSE)
[Out] -a*b^2/x^3-3/4*a^2*b/x^4-1/2*b^3/x^2-1/5*a^3/x^5
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="maxima")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Fricas [A]
time = 0.98, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3+20\,ab^2x^2+15\,a^2bx+4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Sympy [A]
time = 0.09, size = 37, normalized size = 1.03

$$\frac{-4a^3 - 15a^2bx - 20ab^2x^2 - 10b^3x^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**6,x)
[Out] (-4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

Giac [A]
time = 1.65, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="giac")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.94

$$-\frac{\frac{a^3}{5} + \frac{3\,a^2\,b\,x}{4} + a\,b^2\,x^2 + \frac{b^3\,x^3}{2}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^6,x)
[Out] -(a^3/5 + (b^3*x^3)/2 + a*b^2*x^2 + (3*a^2*b*x)/4)/x^5
```

3.75 $\int \frac{(a+bx)^3}{x^7} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

```
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^7}\,dx &= \int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{3}b^3x^3-\frac{3}{4}a\,b^2x^2-\frac{3}{5}a^2bx-\frac{1}{6}a^3}{x^6}$	35
risch	$\frac{-\frac{1}{3}b^3x^3-\frac{3}{4}a\,b^2x^2-\frac{3}{5}a^2bx-\frac{1}{6}a^3}{x^6}$	35
gosper	$-\frac{20b^3x^3+45a\,b^2x^2+36a^2bx+10a^3}{60x^6}$	36
default	$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3a\,b^2}{4x^4}-\frac{b^3}{3x^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="maxima")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Fricas [A]
time = 0.92, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="fricas")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Sympy [A]
time = 0.09, size = 37, normalized size = 0.86

$$\frac{-10a^3-36a^2bx-45ab^2x^2-20b^3x^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**7,x)
```

```
[Out] (-10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

Giac [A]
time = 1.53, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{6}+\frac{3\,a^2\,b\,x}{5}+\frac{3\,a\,b^2\,x^2}{4}+\frac{b^3\,x^3}{3}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^7,x)
```

```
[Out] -(a^3/6 + (b^3*x^3)/3 + (3*a*b^2*x^2)/4 + (3*a^2*b*x)/5)/x^6
```

3.76 $\int \frac{(a+bx)^3}{x^8} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

```
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^8} dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right) dx\\ &= -\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{4}b^3x^3-\frac{3}{5}a\,b^2x^2-\frac{1}{2}a^2bx-\frac{1}{7}a^3}{x^7}$	35
risch	$\frac{-\frac{1}{4}b^3x^3-\frac{3}{5}a\,b^2x^2-\frac{1}{2}a^2bx-\frac{1}{7}a^3}{x^7}$	35
gosper	$-\frac{35b^3x^3+84a\,b^2x^2+70a^2bx+20a^3}{140x^7}$	36
default	$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3a\,b^2}{5x^5}-\frac{b^3}{4x^4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^8,x,method=_RETURNVERBOSE)
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="maxima")
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Fricas [A]
time = 1.11, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="fricas")
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Sympy [A]
time = 0.10, size = 37, normalized size = 0.86

$$\frac{-20a^3-70a^2bx-84ab^2x^2-35b^3x^3}{140x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**8,x)
```

```
[Out] (-20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

Giac [A]
time = 1.52, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{7}+\frac{a^2\,b\,x}{2}+\frac{3\,a\,b^2\,x^2}{5}+\frac{b^3\,x^3}{4}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^8,x)
```

```
[Out] -(a^3/7 + (b^3*x^3)/4 + (3*a*b^2*x^2)/5 + (a^2*b*x)/2)/x^7
```

3.77 $\int x^6(a+bx)^5\,dx$

Optimal. Leaf size=66

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^5\,dx&=\int\left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx\\&=\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.86

method	result	size
gosper	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
default	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
norman	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57
risch	$\frac{1}{7}a^5x^7+\frac{5}{8}a^4b\,x^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}a\,b^4x^{11}+\frac{1}{12}b^5x^{12}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Fricas [A]
time = 1.09, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Sympy [A]
time = 0.01, size = 63, normalized size = 0.95

$$\frac{a^5x^7}{7}+\frac{5a^4bx^8}{8}+\frac{10a^3b^2x^9}{9}+a^2b^3x^{10}+\frac{5ab^4x^{11}}{11}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**9/9 + a**2*b**3*x**10 + 5*a
*b**4*x**11/11 + b**5*x**12/12
```

Giac [A]
time = 1.70, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

Mupad [B]
time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^8}{8}+\frac{10\,a^3\,b^2\,x^9}{9}+a^2\,b^3\,x^{10}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^12)/12 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^11)/11 + (10*a^3
*b^2*x^9)/9 + a^2*b^3*x^10
```

3.78 $\int x^5(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^5\,dx &= \int \left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)\,dx\\ &= \frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Maple [A]
time = 0.07, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
default	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
norman	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58
risch	$\frac{1}{6}a^5x^6+\frac{5}{7}a^4b\,x^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}a\,b^4x^{10}+\frac{1}{11}b^5x^{11}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Fricas [A]
time = 1.01, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.94

$$\frac{a^5x^6}{6}+\frac{5a^4bx^7}{7}+\frac{5a^3b^2x^8}{4}+\frac{10a^2b^3x^9}{9}+\frac{ab^4x^{10}}{2}+\frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**7/7 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**9/9 +
a*b**4*x**10/2 + b**5*x**11/11
```

Giac [A]
time = 1.54, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11}+\frac{1}{2}\,ab^4x^{10}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{4}\,a^3b^2x^8+\frac{5}{7}\,a^4bx^7+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^6}{6}+\frac{5\,a^4\,b\,x^7}{7}+\frac{5\,a^3\,b^2\,x^8}{4}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{a\,b^4\,x^{10}}{2}+\frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^11)/11 + (5*a^4*b*x^7)/7 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^8)/4 + (10*a^2*b^3*x^9)/9
```

3.79 $\int x^4(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^5\,dx&=\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)\,dx\\&=\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.84

method	result	size
gosper	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
default	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
norman	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58
risch	$\frac{1}{5}a^5x^5+\frac{5}{6}a^4b\,x^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}a\,b^4x^9+\frac{1}{10}b^5x^{10}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Fricas [A]
time = 1.03, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Sympy [A]
time = 0.01, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^6}{6}+\frac{10a^3b^2x^7}{7}+\frac{5a^2b^3x^8}{4}+\frac{5ab^4x^9}{9}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**7/7 + 5*a**2*b**3*x**8/4 +
5*a*b**4*x**9/9 + b**5*x**10/10
```

Giac [A]
time = 1.47, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

Mupad [B]
time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5}+\frac{5\,a^4\,b\,x^6}{6}+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{5\,a^2\,b^3\,x^8}{4}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^10)/10 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^9)/9 + (10*a^3*b
^2*x^7)/7 + (5*a^2*b^3*x^8)/4
```

3.80 $\int x^3(a+bx)^5\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}$$

```
[Out] -1/6*a^3*(b*x+a)^6/b^4+3/7*a^2*(b*x+a)^7/b^4-3/8*a*(b*x+a)^8/b^4+1/9*(b*x+a
)^9/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^5,x]
```

```
[Out] -1/6*(a^3*(a + b*x)^6)/b^4 + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^8
)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^5\,dx &= \int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.03

$$\frac{a^5x^4}{4}+a^4bx^5+\frac{5}{3}a^3b^2x^6+\frac{10}{7}a^2b^3x^7+\frac{5}{8}ab^4x^8+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^5,x]
```

```
[Out] (a^5*x^4)/4 + a^4*b*x^5 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^7)/7 + (5*a*b^4
*x^8)/8 + (b^5*x^9)/9
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.89

method	result	size
gosper	$\frac{1}{9}b^{5}x^{9}+\frac{5}{8}a\,b^{4}x^{8}+\frac{10}{7}a^{2}b^{3}x^{7}+\frac{5}{3}a^{3}b^{2}x^{6}+a^{4}b\,x^{5}+\frac{1}{4}a^{5}x^{4}$	57
default	$\frac{1}{9}b^{5}x^{9}+\frac{5}{8}a\,b^{4}x^{8}+\frac{10}{7}a^{2}b^{3}x^{7}+\frac{5}{3}a^{3}b^{2}x^{6}+a^{4}b\,x^{5}+\frac{1}{4}a^{5}x^{4}$	57
norman	$\frac{1}{9}b^{5}x^{9}+\frac{5}{8}a\,b^{4}x^{8}+\frac{10}{7}a^{2}b^{3}x^{7}+\frac{5}{3}a^{3}b^{2}x^{6}+a^{4}b\,x^{5}+\frac{1}{4}a^{5}x^{4}$	57
risch	$\frac{1}{9}b^{5}x^{9}+\frac{5}{8}a\,b^{4}x^{8}+\frac{10}{7}a^{2}b^{3}x^{7}+\frac{5}{3}a^{3}b^{2}x^{6}+a^{4}b\,x^{5}+\frac{1}{4}a^{5}x^{4}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*x^9+5/8*a*b^4*x^8+10/7*a^2*b^3*x^7+5/3*a^3*b^2*x^6+a^4*b*x^5+1/4*a^
5*x^4
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^{5}x^{9}+\frac{5}{8}\,ab^{4}x^{8}+\frac{10}{7}\,a^{2}b^{3}x^{7}+\frac{5}{3}\,a^{3}b^{2}x^{6}+a^{4}bx^{5}+\frac{1}{4}\,a^{5}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Fricas [A]
time = 1.41, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^{5}x^{9}+\frac{5}{8}\,ab^{4}x^{8}+\frac{10}{7}\,a^{2}b^{3}x^{7}+\frac{5}{3}\,a^{3}b^{2}x^{6}+a^{4}bx^{5}+\frac{1}{4}\,a^{5}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Sympy [A]
time = 0.01, size = 63, normalized size = 0.98

$$\frac{a^{5}x^{4}}{4}+a^{4}bx^{5}+\frac{5a^{3}b^{2}x^{6}}{3}+\frac{10a^{2}b^{3}x^{7}}{7}+\frac{5ab^{4}x^{8}}{8}+\frac{b^{5}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**5,x)
```

```
[Out] a**5*x**4/4 + a**4*b*x**5 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**7/7 + 5*a*
b**4*x**8/8 + b**5*x**9/9
```

Giac [A]
time = 1.27, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9+\frac{5}{8}\,ab^4x^8+\frac{10}{7}\,a^2b^3x^7+\frac{5}{3}\,a^3b^2x^6+a^4bx^5+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

Mupad [B]
time = 0.02, size = 56, normalized size = 0.88

$$\frac{a^5\,x^4}{4}+a^4\,b\,x^5+\frac{5\,a^3\,b^2\,x^6}{3}+\frac{10\,a^2\,b^3\,x^7}{7}+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^9)/9 + a^4*b*x^5 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x^6)/3
 + (10*a^2*b^3*x^7)/7
```

3.81 $\int x^2(a+bx)^5\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}$$

```
[Out] 1/6*a^2*(b*x+a)^6/b^3-2/7*a*(b*x+a)^7/b^3+1/8*(b*x+a)^8/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^5,x]
```

```
[Out] (a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^5\,dx&=\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx\\&=\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.43

$$\frac{a^5x^3}{3}+\frac{5}{4}a^4bx^4+2a^3b^2x^5+\frac{5}{3}a^2b^3x^6+\frac{5}{7}ab^4x^7+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^5,x]
```

```
[Out] (a^5*x^3)/3 + (5*a^4*b*x^4)/4 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^6)/3 + (5*a*b^
4*x^7)/7 + (b^5*x^8)/8
```

Maple [A]
time = 0.07, size = 58, normalized size = 1.23

method	result	size
gosper	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
default	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
norman	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58
risch	$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^5*x^8+5/7*a*b^4*x^7+5/3*a^2*b^3*x^6+2*a^3*b^2*x^5+5/4*a^4*b*x^4+1/3*a
^5*x^3
```

Maxima [A]
time = 0.27, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Fricas [A]
time = 0.87, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Sympy [A]
time = 0.01, size = 65, normalized size = 1.38

$$\frac{a^5x^3}{3}+\frac{5a^4bx^4}{4}+2a^3b^2x^5+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^7}{7}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**5,x)
```

```
[Out] a**5*x**3/3 + 5*a**4*b*x**4/4 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**6/3 + 5*a
*b**4*x**7/7 + b**5*x**8/8
```

Giac [A]
time = 1.33, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

Mupad [B]
time = 0.02, size = 57, normalized size = 1.21

$$\frac{a^5\,x^3}{3}+\frac{5\,a^4\,b\,x^4}{4}+2\,a^3\,b^2\,x^5+\frac{5\,a^2\,b^3\,x^6}{3}+\frac{5\,a\,b^4\,x^7}{7}+\frac{b^5\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^8)/8 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^7)/7 + 2*a^3*b^2*x
^5 + (5*a^2*b^3*x^6)/3
```

3.82 $\int x(a+bx)^5 \, dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}$$

```
[Out] -1/6*a*(b*x+a)^6/b^2+1/7*(b*x+a)^7/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^5,x]
```

```
[Out] -1/6*(a*(a + b*x)^6)/b^2 + (a + b*x)^7/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^5 \, dx &= \int \left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right) dx\\ &= -\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.
time = 0.00, size = 67, normalized size = 2.23

$$\frac{a^5x^2}{2}+\frac{5}{3}a^4bx^3+\frac{5}{2}a^3b^2x^4+2a^2b^3x^5+\frac{5}{6}ab^4x^6+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^5,x]
```

```
[Out] (a^5*x^2)/2 + (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^4)/2 + 2*a^2*b^3*x^5 + (5*a*b^
4*x^6)/6 + (b^5*x^7)/7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.10, size = 58, normalized size = 1.93

method	result	size
gosper	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58
default	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58
norman	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58
risch	$\frac{1}{7}b^5x^7 + \frac{5}{6}a\,b^4x^6 + 2a^2b^3x^5 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^4b\,x^3 + \frac{1}{2}a^5x^2$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^5*x^7+5/6*a*b^4*x^6+2*a^2*b^3*x^5+5/2*a^3*b^2*x^4+5/3*a^4*b*x^3+1/2*a
^5*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.27, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.86, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(24) = 48$.
time = 0.01, size = 65, normalized size = 2.17

$$\frac{a^5x^2}{2} + \frac{5a^4bx^3}{3} + \frac{5a^3b^2x^4}{2} + 2a^2b^3x^5 + \frac{5ab^4x^6}{6} + \frac{b^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**4/2 + 2*a**2*b**3*x**5 + 5*a
*b**4*x**6/6 + b**5*x**7/7
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 1.46, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^5x^7 + \frac{5}{6}\,ab^4x^6 + 2\,a^2b^3x^5 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{3}\,a^4bx^3 + \frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

Mupad [B]
time = 0.02, size = 57, normalized size = 1.90

$$\frac{a^5\,x^2}{2} + \frac{5\,a^4\,b\,x^3}{3} + \frac{5\,a^3\,b^2\,x^4}{2} + 2\,a^2\,b^3\,x^5 + \frac{5\,a\,b^4\,x^6}{6} + \frac{b^5\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^7)/7 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^6)/6 + (5*a^3*b^2*
x^4)/2 + 2*a^2*b^3*x^5
```

3.83 $\int (a+bx)^5\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

```
[Out] 1/6*(b*x+a)^6/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^5\,dx = \frac{(a+bx)^6}{6b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Maple [A]
time = 0.07, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^6}{6b}$	13
gosper	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4b\,x^2+a^5x$	54
norman	$\frac{1}{6}b^5x^6+a\,b^4x^5+\frac{5}{2}a^2b^3x^4+\frac{10}{3}a^3b^2x^3+\frac{5}{2}a^4b\,x^2+a^5x$	54
risch	$\frac{b^5x^6}{6}+a\,b^4x^5+\frac{5a^2b^3x^4}{2}+\frac{10a^3b^2x^3}{3}+\frac{5a^4b\,x^2}{2}+a^5x+\frac{a^6}{6b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x+a)^6/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.
time = 0.27, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(12) = 24$.
time = 0.88, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6+ab^4x^5+\frac{5}{2}\,a^2b^3x^4+\frac{10}{3}\,a^3b^2x^3+\frac{5}{2}\,a^4bx^2+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^
2 + a^5*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(8) = 16$.
time = 0.01, size = 60, normalized size = 4.29

$$a^5x+\frac{5a^4bx^2}{2}+\frac{10a^3b^2x^3}{3}+\frac{5a^2b^3x^4}{2}+ab^4x^5+\frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**
4*x**5 + b**5*x**6/6
```

Giac [A]
time = 1.40, size = 12, normalized size = 0.86

$$\frac{(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(b*x + a)^6/b
```

Mupad [B]
time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x+\frac{5\,a^4\,b\,x^2}{2}+\frac{10\,a^3\,b^2\,x^3}{3}+\frac{5\,a^2\,b^3\,x^4}{2}+a\,b^4\,x^5+\frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5
*a^2*b^3*x^4)/2
```

3.84 $\int \frac{(a+bx)^5}{x}\,dx$

Optimal. Leaf size=59

$$5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}+a^5\log(x)$$

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x}\,dx &= \int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx\\ &= 5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}+a^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 59, normalized size = 1.00

$$5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}+a^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x,x]
```

```
[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.92

method	result	size
default	$5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$	54
norman	$5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$	54
risch	$5a^4bx+5a^3b^2x^2+\frac{10a^2b^3x^3}{3}+\frac{5a\,b^4x^4}{4}+\frac{b^5x^5}{5}+a^5\ln\left(x\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

Maxima [A]
time = 0.29, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Fricas [A]
time = 0.61, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5+\frac{5}{4}\,ab^4x^4+\frac{10}{3}\,a^2b^3x^3+5\,a^3b^2x^2+5\,a^4bx+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

Sympy [A]
time = 0.03, size = 60, normalized size = 1.02

$$a^5 \log(x) + 5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5ab^4x^4}{4} + \frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x + 5*a**3*b**2*x**2 + 10*a**2*b**3*x**3/3 + 5*a*b**
4*x**4/4 + b**5*x**5/5
```

Giac [A]
time = 1.63, size = 54, normalized size = 0.92

$$\frac{1}{5}b^5x^5 + \frac{5}{4}ab^4x^4 + \frac{10}{3}a^2b^3x^3 + 5a^3b^2x^2 + 5a^4bx + a^5\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(abs(x))
```

Mupad [B]
time = 0.03, size = 53, normalized size = 0.90

$$a^5 \ln(x) + \frac{b^5x^5}{5} + \frac{5ab^4x^4}{4} + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + 5a^4bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^5)/5 + (5*a*b^4*x^4)/4 + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^
3)/3 + 5*a^4*b*x
```

3.85 $\int \frac{(a+bx)^5}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)$$

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5}{x}+5a^4b\log(x)+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^2}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx\\ &= -\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 1.00

$$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.95

method	result	size
default	$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5a\,b^4x^3}{3}+\frac{b^5x^4}{4}+5a^4b\ln\left(x\right)$	55
risch	$-\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5a\,b^4x^3}{3}+\frac{b^5x^4}{4}+5a^4b\ln\left(x\right)$	55
norman	$\frac{-a^5+\frac{1}{4}b^5x^5+\frac{5}{3}a\,b^4x^4+5a^2b^3x^3+10a^3b^2x^2}{x}+5a^4b\ln\left(x\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.93

$$\frac{1}{4}\,b^5x^4+\frac{5}{3}\,ab^4x^3+5\,a^2b^3x^2+10\,a^3b^2x+5\,a^4b\log\left(x\right)-\frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(x)
 - a^5/x
```

Fricas [A]
time = 0.77, size = 59, normalized size = 1.02

$$\frac{3\,b^5x^5+20\,ab^4x^4+60\,a^2b^3x^3+120\,a^3b^2x^2+60\,a^4bx\log\left(x\right)-12\,a^5}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4*
b*x*log(x) - 12*a^5)/x
```

Sympy [A]
time = 0.04, size = 56, normalized size = 0.97

$$-\frac{a^5}{x}+5a^4b\log\left(x\right)+10a^3b^2x+5a^2b^3x^2+\frac{5ab^4x^3}{3}+\frac{b^5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*log(x) + 10*a**3*b**2*x + 5*a**2*b**3*x**2 + 5*a*b**4*x*
*3/3 + b**5*x**4/4
```

Giac [A]
time = 2.21, size = 55, normalized size = 0.95

$$\frac{1}{4}\,b^5x^4 + \frac{5}{3}\,ab^4x^3 + 5\,a^2b^3x^2 + 10\,a^3b^2x + 5\,a^4b\log\left(|x|\right) - \frac{a^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(ab
s(x)) - a^5/x
```

Mupad [B]
time = 0.03, size = 54, normalized size = 0.93

$$\frac{b^5\,x^4}{4} - \frac{a^5}{x} + 10\,a^3\,b^2\,x + \frac{5\,a\,b^4\,x^3}{3} + 5\,a^4\,b\,\ln\left(x\right) + 5\,a^2\,b^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^2,x)
```

```
[Out] (b^5*x^4)/4 - a^5/x + 10*a^3*b^2*x + (5*a*b^4*x^3)/3 + 5*a^4*b*log(x) + 5*a
^2*b^3*x^2
```

3.86 $\int \frac{(a+bx)^5}{x^3} dx$

Optimal. Leaf size=60

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)$$

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^3}\,dx &= \int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx\\ &= -\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Maple [A]
time = 0.08, size = 55, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5a\,b^4x^2}{2}+\frac{b^5x^3}{3}+10a^3b^2\ln\left(x\right)$	55
risch	$\frac{b^5x^3}{3}+\frac{5a\,b^4x^2}{2}+10a^2b^3x+\frac{-5a^4bx-\frac{1}{2}a^5}{x^2}+10a^3b^2\ln\left(x\right)$	55
norman	$\frac{-\frac{1}{2}a^5+\frac{1}{3}b^5x^5+\frac{5}{2}a\,b^4x^4+10a^2b^3x^3-5a^4bx}{x^2}+10a^3b^2\ln\left(x\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.88

$$\frac{1}{3}\,b^5x^3+\frac{5}{2}\,ab^4x^2+10\,a^2b^3x+10\,a^3b^2\log\left(x\right)-\frac{10\,a^4bx+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(x) - 1/2*(10*a^
4*b*x + a^5)/x^2
```

Fricas [A]
time = 0.95, size = 59, normalized size = 0.98

$$\frac{2\,b^5x^5+15\,ab^4x^4+60\,a^2b^3x^3+60\,a^3b^2x^2\log\left(x\right)-30\,a^4bx-3\,a^5}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 30
*a^4*b*x - 3*a^5)/x^2
```

Sympy [A]
time = 0.07, size = 60, normalized size = 1.00

$$10a^3b^2\log(x) + 10a^2b^3x + \frac{5ab^4x^2}{2} + \frac{b^5x^3}{3} + \frac{-a^5 - 10a^4bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**3,x)
```

```
[Out] 10*a**3*b**2*log(x) + 10*a**2*b**3*x + 5*a*b**4*x**2/2 + b**5*x**3/3 + (-a*
*5 - 10*a**4*b*x)/(2*x**2)
```

Giac [A]
time = 2.02, size = 54, normalized size = 0.90

$$\frac{1}{3}\,b^5x^3 + \frac{5}{2}\,ab^4x^2 + 10\,a^2b^3x + 10\,a^3b^2\log(|x|) - \frac{10\,a^4bx + a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(abs(x)) - 1/2*(
10*a^4*b*x + a^5)/x^2
```

Mupad [B]
time = 0.03, size = 55, normalized size = 0.92

$$\frac{b^5\,x^3}{3} - \frac{\frac{a^5}{2} + 5\,b\,x\,a^4}{x^2} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^2}{2} + 10\,a^3\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^3,x)
```

```
[Out] (b^5*x^3)/3 - (a^5/2 + 5*a^4*b*x)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + 10
*a^3*b^2*log(x)
```

3.87 $\int \frac{(a+bx)^5}{x^4} dx$

Optimal. Leaf size=60

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)$$

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^4}\,dx &= \int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.92

method	result	size
default	$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5a\,b^4x+\frac{b^5x^2}{2}+10a^2b^3\ln\left(x\right)$	55
risch	$\frac{b^5x^2}{2}+5a\,b^4x+\frac{-10a^3b^2x^2-\frac{5}{2}a^4bx-\frac{1}{3}a^5}{x^3}+10a^2b^3\ln\left(x\right)$	55
norman	$\frac{-\frac{1}{3}a^5+\frac{1}{2}b^5x^5+5a\,b^4x^4-10a^3b^2x^2-\frac{5}{2}a^4bx}{x^3}+10a^2b^3\ln\left(x\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.92

$$\frac{1}{2}\,b^5x^2+5\,ab^4x+10\,a^2b^3\log\left(x\right)-\frac{60\,a^3b^2x^2+15\,a^4bx+2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(x) - 1/6*(60*a^3*b^2*x^2 + 15*a^4*
b*x + 2*a^5)/x^3
```

Fricas [A]
time = 0.97, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^5+30\,ab^4x^4+60\,a^2b^3x^3\log\left(x\right)-60\,a^3b^2x^2-15\,a^4bx-2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 15
*a^4*b*x - 2*a^5)/x^3
```

Sympy [A]
time = 0.09, size = 60, normalized size = 1.00

$$10a^2b^3\log(x) + 5ab^4x + \frac{b^5x^2}{2} + \frac{-2a^5 - 15a^4bx - 60a^3b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**4,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x + b**5*x**2/2 + (-2*a**5 - 15*a**4*b*x - 6
0*a**3*b**2*x**2)/(6*x**3)
```

Giac [A]
time = 2.35, size = 56, normalized size = 0.93

$$\frac{1}{2}b^5x^2 + 5ab^4x + 10a^2b^3\log(|x|) - \frac{60a^3b^2x^2 + 15a^4bx + 2a^5}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(abs(x)) - 1/6*(60*a^3*b^2*x^2 + 15
*a^4*b*x + 2*a^5)/x^3
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.92

$$\frac{b^5x^2}{2} - \frac{\frac{a^5}{3} + \frac{5a^4bx}{2} + 10a^3b^2x^2}{x^3} + 10a^2b^3\ln(x) + 5ab^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^4,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/3 + 10*a^3*b^2*x^2 + (5*a^4*b*x)/2)/x^3 + 10*a^2*b^3*log
(x) + 5*a*b^4*x
```

3.88 $\int \frac{(a+bx)^5}{x^5} dx$

Optimal. Leaf size=57

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)$$

```
[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^5}\,dx &= \int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right) dx\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.95

method	result	size
default	$-\frac{a^{5}}{4x^{4}}-\frac{5a^{4}b}{3x^{3}}-\frac{5a^{3}b^{2}}{x^{2}}-\frac{10a^{2}b^{3}}{x}+b^{5}x+5a\,b^{4}\ln\left(x\right)$	54
risch	$b^{5}x+\frac{-10a^{2}b^{3}x^{3}-5a^{3}b^{2}x^{2}-\frac{5}{3}a^{4}bx-\frac{1}{4}a^{5}}{x^{4}}+5a\,b^{4}\ln\left(x\right)$	54
norman	$\frac{b^{5}x^{5}-\frac{1}{4}a^{5}-10a^{2}b^{3}x^{3}-5a^{3}b^{2}x^{2}-\frac{5}{3}a^{4}bx}{x^{4}}+5a\,b^{4}\ln\left(x\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.95

$$b^{5}x+5\,ab^{4}\log\left(x\right)-\frac{120\,a^{2}b^{3}x^{3}+60\,a^{3}b^{2}x^{2}+20\,a^{4}bx+3\,a^{5}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] b^5*x + 5*a*b^4*log(x) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a^4*b*
x + 3*a^5)/x^4
```

Fricas [A]
time = 0.90, size = 59, normalized size = 1.04

$$\frac{12\,b^{5}x^{5}+60\,ab^{4}x^{4}\log\left(x\right)-120\,a^{2}b^{3}x^{3}-60\,a^{3}b^{2}x^{2}-20\,a^{4}bx-3\,a^{5}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2 -
 20*a^4*b*x - 3*a^5)/x^4
```

Sympy [A]
time = 0.11, size = 58, normalized size = 1.02

$$5ab^{4}\log\left(x\right)+b^{5}x+\frac{-3a^{5}-20a^{4}bx-60a^{3}b^{2}x^{2}-120a^{2}b^{3}x^{3}}{12x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**5,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x + (-3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120
*a**2*b**3*x**3)/(12*x**4)
```

Giac [A]
time = 1.53, size = 55, normalized size = 0.96

$$b^5x + 5\,ab^4\log\left(|x|\right) - \frac{120\,a^2b^3x^3 + 60\,a^3b^2x^2 + 20\,a^4bx + 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="giac")
```

```
[Out] b^5*x + 5*a*b^4*log(abs(x)) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a
^4*b*x + 3*a^5)/x^4
```

Mupad [B]
time = 0.08, size = 54, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{4} + \frac{5\,a^4\,b\,x}{3} + 5\,a^3\,b^2\,x^2 + 10\,a^2\,b^3\,x^3}{x^4} + 5\,a\,b^4\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^5,x)
```

```
[Out] b^5*x - (a^5/4 + 5*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + (5*a^4*b*x)/3)/x^4 + 5*a*
b^4*log(x)
```

3.89 $\int \frac{(a+bx)^5}{x^6} dx$

Optimal. Leaf size=61

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^6}\,dx &= \int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Maple [A]
time = 0.07, size = 56, normalized size = 0.92

method	result	size
default	$-\frac{a^{5}}{5x^{5}}-\frac{5a^{4}b}{4x^{4}}-\frac{10a^{3}b^{2}}{3x^{3}}-\frac{5a^{2}b^{3}}{x^{2}}-\frac{5a\,b^{4}}{x}+b^{5}\ln\left(x\right)$	56
norman	$\frac{-\frac{1}{5}a^{5}-5a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}-\frac{10}{3}a^{3}b^{2}x^{2}-\frac{5}{4}a^{4}bx}{x^{5}}+b^{5}\ln\left(x\right)$	56
risch	$\frac{-\frac{1}{5}a^{5}-5a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}-\frac{10}{3}a^{3}b^{2}x^{2}-\frac{5}{4}a^{4}bx}{x^{5}}+b^{5}\ln\left(x\right)$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.92

$$b^{5}\log\left(x\right)-\frac{300\,ab^{4}x^{4}+300\,a^{2}b^{3}x^{3}+200\,a^{3}b^{2}x^{2}+75\,a^{4}bx+12\,a^{5}}{60\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 + 75*a
^4*b*x + 12*a^5)/x^5
```

Fricas [A]
time = 0.91, size = 59, normalized size = 0.97

$$\frac{60\,b^{5}x^{5}\log\left(x\right)-300\,ab^{4}x^{4}-300\,a^{2}b^{3}x^{3}-200\,a^{3}b^{2}x^{2}-75\,a^{4}bx-12\,a^{5}}{60\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^2
 - 75*a^4*b*x - 12*a^5)/x^5
```

Sympy [A]
time = 0.14, size = 60, normalized size = 0.98

$$b^5\log(x)+\frac{-12a^5-75a^4bx-200a^3b^2x^2-300a^2b^3x^3-300ab^4x^4}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**6,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a**2*b**3*
x**3 - 300*a*b**4*x**4)/(60*x**5)
```

Giac [A]
time = 2.24, size = 57, normalized size = 0.93

$$b^5\log(|x|)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x)) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 +
 75*a^4*b*x + 12*a^5)/x^5
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.92

$$b^5\,\ln(x)-\frac{\frac{a^5}{5}+\frac{5\,a^4\,b\,x}{4}+\frac{10\,a^3\,b^2\,x^2}{3}+5\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^6,x)
```

```
[Out] b^5*log(x) - (a^5/5 + 5*a*b^4*x^4 + (10*a^3*b^2*x^2)/3 + 5*a^2*b^3*x^3 + (5
*a^4*b*x)/4)/x^5
```

3.90 $\int \frac{(a+bx)^5}{x^7} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^6}{6ax^6}$$

[Out] -1/6*(b*x+a)^6/a/x^6

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^6}{6ax^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/x^7,x]

[Out] -1/6*(a + b*x)^6/(a*x^6)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^7} dx = -\frac{(a+bx)^6}{6ax^6}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(17) = 34.
time = 0.00, size = 65, normalized size = 3.82

$$-\frac{a^5}{6x^6} - \frac{a^4b}{x^5} - \frac{5a^3b^2}{2x^4} - \frac{10a^2b^3}{3x^3} - \frac{5ab^4}{2x^2} - \frac{b^5}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/x^7,x]

```
[Out] -1/6*a^5/x^6 - (a^4*b)/x^5 - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/(3*x^3) - (
5*a*b^4)/(2*x^2) - b^5/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(15) = 30$.
time = 0.08, size = 58, normalized size = 3.41

method	result	size
gosper	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6x^6}$	56
norman	$\frac{-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5}{x^6}$	57
risch	$\frac{-b^5x^5-\frac{5}{2}a\,b^4x^4-\frac{10}{3}a^2b^3x^3-\frac{5}{2}a^3b^2x^2-a^4bx-\frac{1}{6}a^5}{x^6}$	57
default	$-\frac{b^5}{x}-\frac{10a^2b^3}{3x^3}-\frac{5a^3b^2}{2x^4}-\frac{5a\,b^4}{2x^2}-\frac{a^4b}{x^5}-\frac{a^5}{6x^6}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -b^5/x-10/3*a^2*b^3/x^3-5/2*a^3*b^2/x^4-5/2*a*b^4/x^2-a^4*b/x^5-1/6*a^5/x^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.
time = 0.29, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.
time = 0.54, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.
time = 0.15, size = 60, normalized size = 3.53

$$\frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*x**6)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(15) = 30$.
time = 2.07, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

Mupad [**B**]
time = 0.04, size = 55, normalized size = 3.24

$$-\frac{\frac{a^5}{6} + a^4\,b\,x + \frac{5\,a^3\,b^2\,x^2}{2} + \frac{10\,a^2\,b^3\,x^3}{3} + \frac{5\,a\,b^4\,x^4}{2} + b^5\,x^5}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^7,x)
```

```
[Out] -(a^5/6 + b^5*x^5 + (5*a*b^4*x^4)/2 + (5*a^3*b^2*x^2)/2 + (10*a^2*b^3*x^3)/
3 + a^4*b*x)/x^6
```

3.91 $\int \frac{(a+bx)^5}{x^8} \, dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^6}{7ax^7}+\frac{b(a+bx)^6}{42a^2x^6}$$

```
[Out] -1/7*(b*x+a)^6/a/x^7+1/42*b*(b*x+a)^6/a^2/x^6
```

Rubi [A]

time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b(a+bx)^6}{42a^2x^6}-\frac{(a+bx)^6}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^8,x]
```

```
[Out] -1/7*(a + b*x)^6/(a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^8} \, dx &= -\frac{(a+bx)^6}{7ax^7}-\frac{b\int \frac{(a+bx)^5}{x^7} \, dx}{7a}\\ &= -\frac{(a+bx)^6}{7ax^7}+\frac{b(a+bx)^6}{42a^2x^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.86

$$-\frac{a^5}{7x^7}-\frac{5a^4b}{6x^6}-\frac{2a^3b^2}{x^5}-\frac{5a^2b^3}{2x^4}-\frac{5ab^4}{3x^3}-\frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (5*a^4*b)/(6*x^6) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/(2*x^4) -
(5*a*b^4)/(3*x^3) - b^5/(2*x^2)
```

Maple [A]
time = 0.09, size = 58, normalized size = 1.61

method	result	size
norman	$\frac{-\frac{1}{2}b^5x^5-\frac{5}{3}a\,b^4x^4-\frac{5}{2}a^2b^3x^3-2a^3b^2x^2-\frac{5}{6}a^4bx-\frac{1}{7}a^5}{x^7}$	57
risch	$\frac{-\frac{1}{2}b^5x^5-\frac{5}{3}a\,b^4x^4-\frac{5}{2}a^2b^3x^3-2a^3b^2x^2-\frac{5}{6}a^4bx-\frac{1}{7}a^5}{x^7}$	57
gosper	$-\frac{21b^5x^5+70a\,b^4x^4+105a^2b^3x^3+84a^3b^2x^2+35a^4bx+6a^5}{42x^7}$	58
default	$-\frac{5a\,b^4}{3x^3}-\frac{5a^2b^3}{2x^4}-\frac{b^5}{2x^2}-\frac{2a^3b^2}{x^5}-\frac{5a^4b}{6x^6}-\frac{a^5}{7x^7}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3*a*b^4/x^3-5/2*a^2*b^3/x^4-1/2*b^5/x^2-2*a^3*b^2/x^5-5/6*a^4*b/x^6-1/7*
a^5/x^7
```

Maxima [A]
time = 0.29, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Fricas [A]
time = 1.54, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5+70\,ab^4x^4+105\,a^2b^3x^3+84\,a^3b^2x^2+35\,a^4bx+6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(29) = 58$.
time = 0.16, size = 61, normalized size = 1.69

$$\frac{-6a^5 - 35a^4bx - 84a^3b^2x^2 - 105a^2b^3x^3 - 70ab^4x^4 - 21b^5x^5}{42x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**8,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b**4
*x**4 - 21*b**5*x**5)/(42*x**7)
```

Giac [A]
time = 1.78, size = 57, normalized size = 1.58

$$-\frac{21\,b^5x^5 + 70\,ab^4x^4 + 105\,a^2b^3x^3 + 84\,a^3b^2x^2 + 35\,a^4bx + 6\,a^5}{42\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

Mupad [B]
time = 0.07, size = 57, normalized size = 1.58

$$-\frac{\frac{a^5}{7} + \frac{5\,a^4\,b\,x}{6} + 2\,a^3\,b^2\,x^2 + \frac{5\,a^2\,b^3\,x^3}{2} + \frac{5\,a\,b^4\,x^4}{3} + \frac{b^5\,x^5}{2}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^8,x)
```

```
[Out] -(a^5/7 + (b^5*x^5)/2 + (5*a*b^4*x^4)/3 + 2*a^3*b^2*x^2 + (5*a^2*b^3*x^3)/2
 + (5*a^4*b*x)/6)/x^7
```

3.92 $\int \frac{(a+bx)^5}{x^9} \, dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^6}{8ax^8}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{b^2(a+bx)^6}{168a^3x^6}$$

```
[Out] -1/8*(b*x+a)^6/a/x^8+1/28*b*(b*x+a)^6/a^2/x^7-1/168*b^2*(b*x+a)^6/a^3/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^9,x]
```

```
[Out] -1/8*(a + b*x)^6/(a*x^8) + (b*(a + b*x)^6)/(28*a^2*x^7) - (b^2*(a + b*x)^6)
/(168*a^3*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^9}\,dx = -\frac{(a+bx)^6}{8ax^8} - \frac{b\int \frac{(a+bx)^5}{x^8}\,dx}{4a}$$
$$= -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} + \frac{b^2\int \frac{(a+bx)^5}{x^7}\,dx}{28a^2}$$
$$= -\frac{(a+bx)^6}{8ax^8} + \frac{b(a+bx)^6}{28a^2x^7} - \frac{b^2(a+bx)^6}{168a^3x^6}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.20

$$-\frac{a^5}{8x^8} - \frac{5a^4b}{7x^7} - \frac{5a^3b^2}{3x^6} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{4x^4} - \frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(3*x^6) - (2*a^2*b^3)/x^5 -
(5*a*b^4)/(4*x^4) - b^5/(3*x^3)
```

Maple [A]
time = 0.08, size = 58, normalized size = 1.04

method	result	size
norman	$\frac{-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5}{x^8}$	57
risch	$\frac{-\frac{1}{3}b^5x^5-\frac{5}{4}a\,b^4x^4-2a^2b^3x^3-\frac{5}{3}a^3b^2x^2-\frac{5}{7}a^4bx-\frac{1}{8}a^5}{x^8}$	57
gosper	$-\frac{56b^5x^5+210a\,b^4x^4+336a^2b^3x^3+280a^3b^2x^2+120a^4bx+21a^5}{168x^8}$	58
default	$-\frac{b^5}{3x^3}-\frac{5a\,b^4}{4x^4}-\frac{a^5}{8x^8}-\frac{2a^2b^3}{x^5}-\frac{5a^3b^2}{3x^6}-\frac{5a^4b}{7x^7}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^5/x^3-5/4*a*b^4/x^4-1/8*a^5/x^8-2*a^2*b^3/x^5-5/3*a^3*b^2/x^6-5/7*a^
4*b/x^7
```

Maxima [A]
time = 0.28, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5 + 210\,ab^4x^4 + 336\,a^2b^3x^3 + 280\,a^3b^2x^2 + 120\,a^4bx + 21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Fricas [A]
time = 1.22, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Sympy [A]
time = 0.17, size = 61, normalized size = 1.09

$$\frac{-21a^5-120a^4bx-280a^3b^2x^2-336a^2b^3x^3-210ab^4x^4-56b^5x^5}{168x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**9,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210*a*
b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

Giac [A]
time = 2.21, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="giac")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

Mupad [B]
time = 0.04, size = 57, normalized size = 1.02

$$-\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x}{7}+\frac{5\,a^3\,b^2\,x^2}{3}+2\,a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{4}+\frac{b^5\,x^5}{3}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^9,x)
```

```
[Out] -(a^5/8 + (b^5*x^5)/3 + (5*a*b^4*x^4)/4 + (5*a^3*b^2*x^2)/3 + 2*a^2*b^3*x^3
 + (5*a^4*b*x)/7)/x^8
```

3.93 $\int \frac{(a+bx)^5}{x^{10}} dx$

Optimal. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
risch	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
gosper	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{a\,b^4}{x^5}-\frac{b^5}{4x^4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Fricas [A]
time = 1.06, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [A]
time = 0.18, size = 61, normalized size = 0.91

$$\frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

Giac [A]
time = 1.62, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Mupad [B]
time = 0.08, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x}{8} + \frac{10\,a^3\,b^2\,x^2}{7} + \frac{5\,a^2\,b^3\,x^3}{3} + a\,b^4\,x^4 + \frac{b^5\,x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

3.94 $\int \frac{(a+bx)^5}{x^{11}} \, dx$

Optimal. Leaf size=69

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{11}}\,dx &= \int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right)dx\\ &= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5}{x^{10}}$	57
risch	$\frac{-\frac{1}{5}b^5x^5-\frac{5}{6}a\,b^4x^4-\frac{10}{7}a^2b^3x^3-\frac{5}{4}a^3b^2x^2-\frac{5}{9}a^4bx-\frac{1}{10}a^5}{x^{10}}$	57
gosper	$-\frac{252b^5x^5+1050a\,b^4x^4+1800a^2b^3x^3+1575a^3b^2x^2+700a^4bx+126a^5}{1260x^{10}}$	58
default	$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a\,b^4}{6x^6}-\frac{b^5}{5x^5}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Fricas [A]
time = 0.75, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5+1050\,ab^4x^4+1800\,a^2b^3x^3+1575\,a^3b^2x^2+700\,a^4bx+126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Sympy [A]
time = 0.19, size = 61, normalized size = 0.88

$$\frac{-126a^5 - 700a^4bx - 1575a^3b^2x^2 - 1800a^2b^3x^3 - 1050ab^4x^4 - 252b^5x^5}{1260x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**11,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 - 105
0*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

Giac [A]
time = 2.13, size = 57, normalized size = 0.83

$$-\frac{252\,b^5x^5 + 1050\,ab^4x^4 + 1800\,a^2b^3x^3 + 1575\,a^3b^2x^2 + 700\,a^4bx + 126\,a^5}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

Mupad [B]
time = 0.08, size = 57, normalized size = 0.83

$$-\frac{\frac{a^5}{10} + \frac{5\,a^4\,b\,x}{9} + \frac{5\,a^3\,b^2\,x^2}{4} + \frac{10\,a^2\,b^3\,x^3}{7} + \frac{5\,a\,b^4\,x^4}{6} + \frac{b^5\,x^5}{5}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^11,x)
```

```
[Out] -(a^5/10 + (b^5*x^5)/5 + (5*a*b^4*x^4)/6 + (5*a^3*b^2*x^2)/4 + (10*a^2*b^3*
x^3)/7 + (5*a^4*b*x)/9)/x^10
```

3.95 $\int \frac{(a+bx)^5}{x^{12}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{12}}\,dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{11}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^9}+\frac{5ab^4}{x^8}+\frac{b^5}{x^7}\right)dx\\ &= -\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Maple [A]
time = 0.08, size = 58, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5}{x^{11}}$	57
risch	$\frac{-\frac{1}{6}b^5x^5-\frac{5}{7}a\,b^4x^4-\frac{5}{4}a^2b^3x^3-\frac{10}{9}a^3b^2x^2-\frac{1}{2}a^4bx-\frac{1}{11}a^5}{x^{11}}$	57
gosper	$-\frac{462b^5x^5+1980a\,b^4x^4+3465a^2b^3x^3+3080a^3b^2x^2+1386a^4bx+252a^5}{2772x^{11}}$	58
default	$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5a\,b^4}{7x^7}-\frac{b^5}{6x^6}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Fricas [A]
time = 0.96, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5+1980\,ab^4x^4+3465\,a^2b^3x^3+3080\,a^3b^2x^2+1386\,a^4bx+252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Sympy [A]
time = 0.20, size = 61, normalized size = 0.88

$$\frac{-252a^5 - 1386a^4bx - 3080a^3b^2x^2 - 3465a^2b^3x^3 - 1980ab^4x^4 - 462b^5x^5}{2772x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**12,x)
```

```
[Out] (-252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 - 19
80*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

Giac [A]
time = 2.02, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5 + 1980\,ab^4x^4 + 3465\,a^2b^3x^3 + 3080\,a^3b^2x^2 + 1386\,a^4bx + 252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

Mupad [B]
time = 0.04, size = 57, normalized size = 0.83

$$-\frac{\frac{a^5}{11} + \frac{a^4\,b\,x}{2} + \frac{10\,a^3\,b^2\,x^2}{9} + \frac{5\,a^2\,b^3\,x^3}{4} + \frac{5\,a\,b^4\,x^4}{7} + \frac{b^5\,x^5}{6}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^12,x)
```

```
[Out] -(a^5/11 + (b^5*x^5)/6 + (5*a*b^4*x^4)/7 + (10*a^3*b^2*x^2)/9 + (5*a^2*b^3*
x^3)/4 + (a^4*b*x)/2)/x^11
```

3.96 $\int \frac{(a+bx)^5}{x^{13}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{13}}\,dx &= \int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{7}b^5x^5-\frac{5}{8}a\,b^4x^4-\frac{10}{9}a^2b^3x^3-a^3b^2x^2-\frac{5}{11}a^4bx-\frac{1}{12}a^5}{x^{12}}$	57
risch	$\frac{-\frac{1}{7}b^5x^5-\frac{5}{8}a\,b^4x^4-\frac{10}{9}a^2b^3x^3-a^3b^2x^2-\frac{5}{11}a^4bx-\frac{1}{12}a^5}{x^{12}}$	57
gosper	$-\frac{792b^5x^5+3465a\,b^4x^4+6160a^2b^3x^3+5544a^3b^2x^2+2520a^4bx+462a^5}{5544x^{12}}$	58
default	$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a\,b^4}{8x^8}-\frac{b^5}{7x^7}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Fricas [A]
time = 1.03, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5+3465\,ab^4x^4+6160\,a^2b^3x^3+5544\,a^3b^2x^2+2520\,a^4bx+462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Sympy [A]
time = 0.21, size = 61, normalized size = 0.91

$$\frac{-462a^5 - 2520a^4bx - 5544a^3b^2x^2 - 6160a^2b^3x^3 - 3465ab^4x^4 - 792b^5x^5}{5544x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**13,x)
```

```
[Out] (-462*a**5 - 2520*a**4*b*x - 5544*a**3*b**2*x**2 - 6160*a**2*b**3*x**3 - 34
65*a*b**4*x**4 - 792*b**5*x**5)/(5544*x**12)
```

Giac [A]
time = 1.44, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{12} + \frac{5\,a^4\,b\,x}{11} + a^3\,b^2\,x^2 + \frac{10\,a^2\,b^3\,x^3}{9} + \frac{5\,a\,b^4\,x^4}{8} + \frac{b^5\,x^5}{7}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^5)/7 + (5*a*b^4*x^4)/8 + a^3*b^2*x^2 + (10*a^2*b^3*x^3)/9
 + (5*a^4*b*x)/11)/x^12
```

3.97 $\int \frac{(a+bx)^5}{x^{14}} dx$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{14}} dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{13}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^9}\right) dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{8}b^5x^5-\frac{5}{9}a\,b^4x^4-a^2b^3x^3-\frac{10}{11}a^3b^2x^2-\frac{5}{12}a^4bx-\frac{1}{13}a^5}{x^{13}}$	57
risch	$\frac{-\frac{1}{8}b^5x^5-\frac{5}{9}a\,b^4x^4-a^2b^3x^3-\frac{10}{11}a^3b^2x^2-\frac{5}{12}a^4bx-\frac{1}{13}a^5}{x^{13}}$	57
gosper	$-\frac{1287b^5x^5+5720a\,b^4x^4+10296a^2b^3x^3+9360a^3b^2x^2+4290a^4bx+792a^5}{10296x^{13}}$	58
default	$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a\,b^4}{9x^9}-\frac{b^5}{8x^8}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Fricas [A]
time = 0.90, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5+5720\,ab^4x^4+10296\,a^2b^3x^3+9360\,a^3b^2x^2+4290\,a^4bx+792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Sympy [A]
time = 0.22, size = 61, normalized size = 0.91

$$\frac{-792a^5 - 4290a^4bx - 9360a^3b^2x^2 - 10296a^2b^3x^3 - 5720ab^4x^4 - 1287b^5x^5}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**14,x)
```

```
[Out] (-792*a**5 - 4290*a**4*b*x - 9360*a**3*b**2*x**2 - 10296*a**2*b**3*x**3 - 5
720*a*b**4*x**4 - 1287*b**5*x**5)/(10296*x**13)
```

Giac [A]
time = 2.88, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5 + 5720\,ab^4x^4 + 10296\,a^2b^3x^3 + 9360\,a^3b^2x^2 + 4290\,a^4bx + 792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{13} + \frac{5\,a^4\,b\,x}{12} + \frac{10\,a^3\,b^2\,x^2}{11} + a^2\,b^3\,x^3 + \frac{5\,a\,b^4\,x^4}{9} + \frac{b^5\,x^5}{8}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^5)/8 + (5*a*b^4*x^4)/9 + (10*a^3*b^2*x^2)/11 + a^2*b^3*x^
3 + (5*a^4*b*x)/12)/x^13
```

3.98 $\int x^8(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^7\,dx&=\int\left(a^7x^8+7a^6bx^9+21a^5b^2x^{10}+35a^4b^3x^{11}+35a^3b^4x^{12}+21a^2b^5x^{13}+7ab^6x^{14}+b^7x^{15}\right)\\&=\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Maple [A]
time = 0.09, size = 80, normalized size = 0.84

method	result	size
gosper	$\frac{1}{9}a^{7}x^{9}+\frac{7}{10}a^{6}b\,x^{10}+\frac{21}{11}a^{5}b^{2}x^{11}+\frac{35}{12}a^{4}b^{3}x^{12}+\frac{35}{13}a^{3}b^{4}x^{13}+\frac{3}{2}a^{2}b^{5}x^{14}+\frac{7}{15}a\,b^{6}x^{15}+\frac{1}{16}b^{7}x^{16}$	80
default	$\frac{1}{9}a^{7}x^{9}+\frac{7}{10}a^{6}b\,x^{10}+\frac{21}{11}a^{5}b^{2}x^{11}+\frac{35}{12}a^{4}b^{3}x^{12}+\frac{35}{13}a^{3}b^{4}x^{13}+\frac{3}{2}a^{2}b^{5}x^{14}+\frac{7}{15}a\,b^{6}x^{15}+\frac{1}{16}b^{7}x^{16}$	80
norman	$\frac{1}{9}a^{7}x^{9}+\frac{7}{10}a^{6}b\,x^{10}+\frac{21}{11}a^{5}b^{2}x^{11}+\frac{35}{12}a^{4}b^{3}x^{12}+\frac{35}{13}a^{3}b^{4}x^{13}+\frac{3}{2}a^{2}b^{5}x^{14}+\frac{7}{15}a\,b^{6}x^{15}+\frac{1}{16}b^{7}x^{16}$	80
risch	$\frac{1}{9}a^{7}x^{9}+\frac{7}{10}a^{6}b\,x^{10}+\frac{21}{11}a^{5}b^{2}x^{11}+\frac{35}{12}a^{4}b^{3}x^{12}+\frac{35}{13}a^{3}b^{4}x^{13}+\frac{3}{2}a^{2}b^{5}x^{14}+\frac{7}{15}a\,b^{6}x^{15}+\frac{1}{16}b^{7}x^{16}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^{7}x^{16}+\frac{7}{15}\,ab^{6}x^{15}+\frac{3}{2}\,a^{2}b^{5}x^{14}+\frac{35}{13}\,a^{3}b^{4}x^{13}+\frac{35}{12}\,a^{4}b^{3}x^{12}+\frac{21}{11}\,a^{5}b^{2}x^{11}+\frac{7}{10}\,a^{6}bx^{10}+\frac{1}{9}\,a^{7}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Fricas [A]
time = 1.01, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^{7}x^{16}+\frac{7}{15}\,ab^{6}x^{15}+\frac{3}{2}\,a^{2}b^{5}x^{14}+\frac{35}{13}\,a^{3}b^{4}x^{13}+\frac{35}{12}\,a^{4}b^{3}x^{12}+\frac{21}{11}\,a^{5}b^{2}x^{11}+\frac{7}{10}\,a^{6}bx^{10}+\frac{1}{9}\,a^{7}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Sympy [A]
time = 0.01, size = 94, normalized size = 0.99

$$\frac{a^7x^9}{9}+\frac{7a^6bx^{10}}{10}+\frac{21a^5b^2x^{11}}{11}+\frac{35a^4b^3x^{12}}{12}+\frac{35a^3b^4x^{13}}{13}+\frac{3a^2b^5x^{14}}{2}+\frac{7ab^6x^{15}}{15}+\frac{b^7x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**7,x)
```

```
[Out] a**7*x**9/9 + 7*a**6*b*x**10/10 + 21*a**5*b**2*x**11/11 + 35*a**4*b**3*x**1
2/12 + 35*a**3*b**4*x**13/13 + 3*a**2*b**5*x**14/2 + 7*a*b**6*x**15/15 + b*
*7*x**16/16
```

Giac [A]
time = 2.15, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16}+\frac{7}{15}\,ab^6x^{15}+\frac{3}{2}\,a^2b^5x^{14}+\frac{35}{13}\,a^3b^4x^{13}+\frac{35}{12}\,a^4b^3x^{12}+\frac{21}{11}\,a^5b^2x^{11}+\frac{7}{10}\,a^6bx^{10}+\frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

Mupad [B]
time = 0.15, size = 79, normalized size = 0.83

$$\frac{a^7\,x^9}{9}+\frac{7\,a^6\,b\,x^{10}}{10}+\frac{21\,a^5\,b^2\,x^{11}}{11}+\frac{35\,a^4\,b^3\,x^{12}}{12}+\frac{35\,a^3\,b^4\,x^{13}}{13}+\frac{3\,a^2\,b^5\,x^{14}}{2}+\frac{7\,a\,b^6\,x^{15}}{15}+\frac{b^7\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^7,x)
```

```
[Out] (a^7*x^9)/9 + (b^7*x^16)/16 + (7*a^6*b*x^10)/10 + (7*a*b^6*x^15)/15 + (21*a
^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*
x^14)/2
```

3.99 $\int x^7(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^7\,dx&=\int\left(a^7x^7+7a^6bx^8+21a^5b^2x^9+35a^4b^3x^{10}+35a^3b^4x^{11}+21a^2b^5x^{12}+7ab^6x^{13}+b^7x^{14}\right)\\&=\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.84

method	result	size
gosper	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80
default	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80
norman	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80
risch	$\frac{1}{8}a^7x^8+\frac{7}{9}a^6b\,x^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}a\,b^6x^{14}+\frac{1}{15}b^7x^{15}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

Maxima [A]
time = 0.31, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Fricas [A]
time = 1.12, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Sympy [A]
time = 0.01, size = 92, normalized size = 0.97

$$\frac{a^7x^8}{8} + \frac{7a^6bx^9}{9} + \frac{21a^5b^2x^{10}}{10} + \frac{35a^4b^3x^{11}}{11} + \frac{35a^3b^4x^{12}}{12} + \frac{21a^2b^5x^{13}}{13} + \frac{ab^6x^{14}}{2} + \frac{b^7x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**7,x)
```

```
[Out] a**7*x**8/8 + 7*a**6*b*x**9/9 + 21*a**5*b**2*x**10/10 + 35*a**4*b**3*x**11/
11 + 35*a**3*b**4*x**12/12 + 21*a**2*b**5*x**13/13 + a*b**6*x**14/2 + b**7*
x**15/15
```

Giac [A]
time = 1.75, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15} + \frac{1}{2}\,ab^6x^{14} + \frac{21}{13}\,a^2b^5x^{13} + \frac{35}{12}\,a^3b^4x^{12} + \frac{35}{11}\,a^4b^3x^{11} + \frac{21}{10}\,a^5b^2x^{10} + \frac{7}{9}\,a^6bx^9 + \frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^8}{8} + \frac{7\,a^6\,b\,x^9}{9} + \frac{21\,a^5\,b^2\,x^{10}}{10} + \frac{35\,a^4\,b^3\,x^{11}}{11} + \frac{35\,a^3\,b^4\,x^{12}}{12} + \frac{21\,a^2\,b^5\,x^{13}}{13} + \frac{a\,b^6\,x^{14}}{2} + \frac{b^7\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^7,x)
```

```
[Out] (a^7*x^8)/8 + (b^7*x^15)/15 + (7*a^6*b*x^9)/9 + (a*b^6*x^14)/2 + (21*a^5*b^
2*x^10)/10 + (35*a^4*b^3*x^11)/11 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13
)/13
```

3.100 $\int x^6(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^7\,dx &= \int \left(a^7x^6+7a^6bx^7+21a^5b^2x^8+35a^4b^3x^9+35a^3b^4x^{10}+21a^2b^5x^{11}+7ab^6x^{12}+b^7x^{13}\right)\,dx\\ &= \frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Maple [A]
time = 0.09, size = 80, normalized size = 0.84

method	result	size
gosper	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
default	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
norman	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80
risch	$\frac{1}{7}a^7x^7+\frac{7}{8}a^6b\,x^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}a\,b^6x^{13}+\frac{1}{14}b^7x^{14}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Fricas [A]
time = 0.99, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Sympy [A]
time = 0.01, size = 94, normalized size = 0.99

$$\frac{a^7x^7}{7}+\frac{7a^6bx^8}{8}+\frac{7a^5b^2x^9}{3}+\frac{7a^4b^3x^{10}}{2}+\frac{35a^3b^4x^{11}}{11}+\frac{7a^2b^5x^{12}}{4}+\frac{7ab^6x^{13}}{13}+\frac{b^7x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**7,x)
```

```
[Out] a**7*x**7/7 + 7*a**6*b*x**8/8 + 7*a**5*b**2*x**9/3 + 7*a**4*b**3*x**10/2 +
35*a**3*b**4*x**11/11 + 7*a**2*b**5*x**12/4 + 7*a*b**6*x**13/13 + b**7*x**1
4/14
```

Giac [A]
time = 1.33, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^7x^{14}+\frac{7}{13}\,ab^6x^{13}+\frac{7}{4}\,a^2b^5x^{12}+\frac{35}{11}\,a^3b^4x^{11}+\frac{7}{2}\,a^4b^3x^{10}+\frac{7}{3}\,a^5b^2x^9+\frac{7}{8}\,a^6bx^8+\frac{1}{7}\,a^7x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^7}{7}+\frac{7\,a^6\,b\,x^8}{8}+\frac{7\,a^5\,b^2\,x^9}{3}+\frac{7\,a^4\,b^3\,x^{10}}{2}+\frac{35\,a^3\,b^4\,x^{11}}{11}+\frac{7\,a^2\,b^5\,x^{12}}{4}+\frac{7\,a\,b^6\,x^{13}}{13}+\frac{b^7\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^7,x)
```

```
[Out] (a^7*x^7)/7 + (b^7*x^14)/14 + (7*a^6*b*x^8)/8 + (7*a*b^6*x^13)/13 + (7*a^5*
b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4
```

3.101 $\int x^5(a+bx)^7\,dx$

Optimal. Leaf size=96

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+bx)^{13}}{13b^6}$$

```
[Out] -1/8*a^5*(b*x+a)^8/b^6+5/9*a^4*(b*x+a)^9/b^6-a^3*(b*x+a)^10/b^6+10/11*a^2*(
b*x+a)^11/b^6-5/12*a*(b*x+a)^12/b^6+1/13*(b*x+a)^13/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^7,x]
```

```
[Out] -1/8*(a^5*(a + b*x)^8)/b^6 + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^1
0)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (a
+ b*x)^13/(13*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^7\,dx &= \int \left(-\frac{a^5(a+bx)^7}{b^5}+\frac{5a^4(a+bx)^8}{b^5}-\frac{10a^3(a+bx)^9}{b^5}+\frac{10a^2(a+bx)^{10}}{b^5}-\frac{5a(a+bx)^{11}}{b^5}\right. \\ &= -\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+}{1}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 92, normalized size = 0.96

$$\frac{a^7x^6}{6}+a^6bx^7+\frac{21}{8}a^5b^2x^8+\frac{35}{9}a^4b^3x^9+\frac{7}{2}a^3b^4x^{10}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{12}ab^6x^{12}+\frac{b^7x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^7,x]
```

```
[Out] (a^7*x^6)/6 + a^6*b*x^7 + (21*a^5*b^2*x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*
b^4*x^10)/2 + (21*a^2*b^5*x^11)/11 + (7*a*b^6*x^12)/12 + (b^7*x^13)/13
```

Maple [A]
time = 0.07, size = 79, normalized size = 0.82

method	result	size
gosper	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
default	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
norman	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79
risch	$\frac{1}{13}b^7x^{13}+\frac{7}{12}a\,b^6x^{12}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{2}a^3b^4x^{10}+\frac{35}{9}a^4b^3x^9+\frac{21}{8}a^5b^2x^8+a^6b\,x^7+\frac{1}{6}a^7x^6$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^7*x^13+7/12*a*b^6*x^12+21/11*a^2*b^5*x^11+7/2*a^3*b^4*x^10+35/9*a^4*
b^3*x^9+21/8*a^5*b^2*x^8+a^6*b*x^7+1/6*a^7*x^6
```

Maxima [A]
time = 0.27, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^7x^{13}+\frac{7}{12}\,ab^6x^{12}+\frac{21}{11}\,a^2b^5x^{11}+\frac{7}{2}\,a^3b^4x^{10}+\frac{35}{9}\,a^4b^3x^9+\frac{21}{8}\,a^5b^2x^8+a^6bx^7+\frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Fricas [A]
time = 0.96, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^7x^{13}+\frac{7}{12}\,ab^6x^{12}+\frac{21}{11}\,a^2b^5x^{11}+\frac{7}{2}\,a^3b^4x^{10}+\frac{35}{9}\,a^4b^3x^9+\frac{21}{8}\,a^5b^2x^8+a^6bx^7+\frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Sympy [A]
time = 0.01, size = 90, normalized size = 0.94

$$\frac{a^7x^6}{6} + a^6bx^7 + \frac{21a^5b^2x^8}{8} + \frac{35a^4b^3x^9}{9} + \frac{7a^3b^4x^{10}}{2} + \frac{21a^2b^5x^{11}}{11} + \frac{7ab^6x^{12}}{12} + \frac{b^7x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**7,x)
```

```
[Out] a**7*x**6/6 + a**6*b*x**7 + 21*a**5*b**2*x**8/8 + 35*a**4*b**3*x**9/9 + 7*a
**3*b**4*x**10/2 + 21*a**2*b**5*x**11/11 + 7*a*b**6*x**12/12 + b**7*x**13/1
3
```

Giac [A]
time = 1.12, size = 78, normalized size = 0.81

$$\frac{1}{13}\,b^7x^{13} + \frac{7}{12}\,ab^6x^{12} + \frac{21}{11}\,a^2b^5x^{11} + \frac{7}{2}\,a^3b^4x^{10} + \frac{35}{9}\,a^4b^3x^9 + \frac{21}{8}\,a^5b^2x^8 + a^6bx^7 + \frac{1}{6}\,a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

Mupad [B]
time = 0.06, size = 78, normalized size = 0.81

$$\frac{a^7\,x^6}{6} + a^6\,b\,x^7 + \frac{21\,a^5\,b^2\,x^8}{8} + \frac{35\,a^4\,b^3\,x^9}{9} + \frac{7\,a^3\,b^4\,x^{10}}{2} + \frac{21\,a^2\,b^5\,x^{11}}{11} + \frac{7\,a\,b^6\,x^{12}}{12} + \frac{b^7\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^7,x)
```

```
[Out] (a^7*x^6)/6 + (b^7*x^13)/13 + a^6*b*x^7 + (7*a*b^6*x^12)/12 + (21*a^5*b^2*x
^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*b^4*x^10)/2 + (21*a^2*b^5*x^11)/11
```

3.102 $\int x^4(a+bx)^7\,dx$

Optimal. Leaf size=81

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}$$

```
[Out] 1/8*a^4*(b*x+a)^8/b^5-4/9*a^3*(b*x+a)^9/b^5+3/5*a^2*(b*x+a)^10/b^5-4/11*a*(
b*x+a)^11/b^5+1/12*(b*x+a)^12/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^7,x]
```

```
[Out] (a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)^
10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^7\,dx &= \int \left(\frac{a^4(a+bx)^7}{b^4}-\frac{4a^3(a+bx)^8}{b^4}+\frac{6a^2(a+bx)^9}{b^4}-\frac{4a(a+bx)^{10}}{b^4}+\frac{(a+bx)^{11}}{b^4}\right)dx\\ &= \frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.15

$$\frac{a^7x^5}{5}+\frac{7}{6}a^6bx^6+3a^5b^2x^7+\frac{35}{8}a^4b^3x^8+\frac{35}{9}a^3b^4x^9+\frac{21}{10}a^2b^5x^{10}+\frac{7}{11}ab^6x^{11}+\frac{b^7x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^7,x]
```

```
[Out] (a^7*x^5)/5 + (7*a^6*b*x^6)/6 + 3*a^5*b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^
3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10 + (7*a*b^6*x^11)/11 + (b^7*x^12)/12
```

Maple [A]
time = 0.07, size = 80, normalized size = 0.99

method	result	size
gosper	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
default	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
norman	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80
risch	$\frac{1}{12}b^7x^{12}+\frac{7}{11}a\,b^6x^{11}+\frac{21}{10}a^2b^5x^{10}+\frac{35}{9}a^3b^4x^9+\frac{35}{8}a^4b^3x^8+3a^5b^2x^7+\frac{7}{6}a^6b\,x^6+\frac{1}{5}a^7x^5$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^7*x^12+7/11*a*b^6*x^11+21/10*a^2*b^5*x^10+35/9*a^3*b^4*x^9+35/8*a^4*
b^3*x^8+3*a^5*b^2*x^7+7/6*a^6*b*x^6+1/5*a^7*x^5
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Fricas [A]
time = 0.94, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Sympy [A]
time = 0.01, size = 92, normalized size = 1.14

$$\frac{a^7x^5}{5}+\frac{7a^6bx^6}{6}+3a^5b^2x^7+\frac{35a^4b^3x^8}{8}+\frac{35a^3b^4x^9}{9}+\frac{21a^2b^5x^{10}}{10}+\frac{7ab^6x^{11}}{11}+\frac{b^7x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**7,x)
```

```
[Out] a**7*x**5/5 + 7*a**6*b*x**6/6 + 3*a**5*b**2*x**7 + 35*a**4*b**3*x**8/8 + 35
*a**3*b**4*x**9/9 + 21*a**2*b**5*x**10/10 + 7*a*b**6*x**11/11 + b**7*x**12/
12
```

Giac [A]
time = 1.19, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12}+\frac{7}{11}\,ab^6x^{11}+\frac{21}{10}\,a^2b^5x^{10}+\frac{35}{9}\,a^3b^4x^9+\frac{35}{8}\,a^4b^3x^8+3\,a^5b^2x^7+\frac{7}{6}\,a^6bx^6+\frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

Mupad [B]
time = 0.06, size = 79, normalized size = 0.98

$$\frac{a^7\,x^5}{5}+\frac{7\,a^6\,b\,x^6}{6}+3\,a^5\,b^2\,x^7+\frac{35\,a^4\,b^3\,x^8}{8}+\frac{35\,a^3\,b^4\,x^9}{9}+\frac{21\,a^2\,b^5\,x^{10}}{10}+\frac{7\,a\,b^6\,x^{11}}{11}+\frac{b^7\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^7,x)
```

```
[Out] (a^7*x^5)/5 + (b^7*x^12)/12 + (7*a^6*b*x^6)/6 + (7*a*b^6*x^11)/11 + 3*a^5*b
^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10
```

3.103 $\int x^3(a+bx)^7\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}$$

```
[Out] -1/8*a^3*(b*x+a)^8/b^4+1/3*a^2*(b*x+a)^9/b^4-3/10*a*(b*x+a)^10/b^4+1/11*(b*
x+a)^11/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^7,x]
```

```
[Out] -1/8*(a^3*(a + b*x)^8)/b^4 + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10)
/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^7\,dx&=\int\left(-\frac{a^3(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^8}{b^3}-\frac{3a(a+bx)^9}{b^3}+\frac{(a+bx)^{10}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.45

$$\frac{a^7x^4}{4}+\frac{7}{5}a^6bx^5+\frac{7}{2}a^5b^2x^6+5a^4b^3x^7+\frac{35}{8}a^3b^4x^8+\frac{7}{3}a^2b^5x^9+\frac{7}{10}ab^6x^{10}+\frac{b^7x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^7,x]
```

```
[Out] (a^7*x^4)/4 + (7*a^6*b*x^5)/5 + (7*a^5*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3
*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3 + (7*a*b^6*x^10)/10 + (b^7*x^11)/11
```

Maple [A]
time = 0.09, size = 80, normalized size = 1.25

method	result	size
gosper	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
default	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
norman	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80
risch	$\frac{1}{11}b^7x^{11}+\frac{7}{10}a\,b^6x^{10}+\frac{7}{3}a^2b^5x^9+\frac{35}{8}a^3b^4x^8+5a^4b^3x^7+\frac{7}{2}a^5b^2x^6+\frac{7}{5}a^6b\,x^5+\frac{1}{4}a^7x^4$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^7*x^11+7/10*a*b^6*x^10+7/3*a^2*b^5*x^9+35/8*a^3*b^4*x^8+5*a^4*b^3*x^
7+7/2*a^5*b^2*x^6+7/5*a^6*b*x^5+1/4*a^7*x^4
```

Maxima [A]
time = 0.29, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Fricas [A]
time = 1.09, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Sympy [A]
time = 0.01, size = 92, normalized size = 1.44

$$\frac{a^7x^4}{4}+\frac{7a^6bx^5}{5}+\frac{7a^5b^2x^6}{2}+5a^4b^3x^7+\frac{35a^3b^4x^8}{8}+\frac{7a^2b^5x^9}{3}+\frac{7ab^6x^{10}}{10}+\frac{b^7x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**7,x)
```

```
[Out] a**7*x**4/4 + 7*a**6*b*x**5/5 + 7*a**5*b**2*x**6/2 + 5*a**4*b**3*x**7 + 35*
a**3*b**4*x**8/8 + 7*a**2*b**5*x**9/3 + 7*a*b**6*x**10/10 + b**7*x**11/11
```

Giac [A]
time = 1.08, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11}+\frac{7}{10}\,ab^6x^{10}+\frac{7}{3}\,a^2b^5x^9+\frac{35}{8}\,a^3b^4x^8+5\,a^4b^3x^7+\frac{7}{2}\,a^5b^2x^6+\frac{7}{5}\,a^6bx^5+\frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

Mupad [B]
time = 0.10, size = 79, normalized size = 1.23

$$\frac{a^7\,x^4}{4}+\frac{7\,a^6\,b\,x^5}{5}+\frac{7\,a^5\,b^2\,x^6}{2}+5\,a^4\,b^3\,x^7+\frac{35\,a^3\,b^4\,x^8}{8}+\frac{7\,a^2\,b^5\,x^9}{3}+\frac{7\,a\,b^6\,x^{10}}{10}+\frac{b^7\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^7,x)
```

```
[Out] (a^7*x^4)/4 + (b^7*x^11)/11 + (7*a^6*b*x^5)/5 + (7*a*b^6*x^10)/10 + (7*a^5*
b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3
```

3.104 $\int x^2(a+bx)^7\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}$$

```
[Out] 1/8*a^2*(b*x+a)^8/b^3-2/9*a*(b*x+a)^9/b^3+1/10*(b*x+a)^10/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^7,x]
```

```
[Out] (a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b^
3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^7\,dx &= \int\left(\frac{a^2(a+bx)^7}{b^2}-\frac{2a(a+bx)^8}{b^2}+\frac{(a+bx)^9}{b^2}\right)dx\\ &= \frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.98

$$\frac{a^7x^3}{3}+\frac{7}{4}a^6bx^4+\frac{21}{5}a^5b^2x^5+\frac{35}{6}a^4b^3x^6+5a^3b^4x^7+\frac{21}{8}a^2b^5x^8+\frac{7}{9}ab^6x^9+\frac{b^7x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^7,x]
```

```
[Out] (a^7*x^3)/3 + (7*a^6*b*x^4)/4 + (21*a^5*b^2*x^5)/5 + (35*a^4*b^3*x^6)/6 + 5
*a^3*b^4*x^7 + (21*a^2*b^5*x^8)/8 + (7*a*b^6*x^9)/9 + (b^7*x^10)/10
```

Maple [A]
time = 0.08, size = 80, normalized size = 1.70

method	result	size
gosper	$\frac{1}{10}b^{7}x^{10}+\frac{7}{9}a\,b^{6}x^{9}+\frac{21}{8}a^{2}b^{5}x^{8}+5a^{3}b^{4}x^{7}+\frac{35}{6}a^{4}b^{3}x^{6}+\frac{21}{5}a^{5}b^{2}x^{5}+\frac{7}{4}a^{6}b\,x^{4}+\frac{1}{3}a^{7}x^{3}$	80
default	$\frac{1}{10}b^{7}x^{10}+\frac{7}{9}a\,b^{6}x^{9}+\frac{21}{8}a^{2}b^{5}x^{8}+5a^{3}b^{4}x^{7}+\frac{35}{6}a^{4}b^{3}x^{6}+\frac{21}{5}a^{5}b^{2}x^{5}+\frac{7}{4}a^{6}b\,x^{4}+\frac{1}{3}a^{7}x^{3}$	80
norman	$\frac{1}{10}b^{7}x^{10}+\frac{7}{9}a\,b^{6}x^{9}+\frac{21}{8}a^{2}b^{5}x^{8}+5a^{3}b^{4}x^{7}+\frac{35}{6}a^{4}b^{3}x^{6}+\frac{21}{5}a^{5}b^{2}x^{5}+\frac{7}{4}a^{6}b\,x^{4}+\frac{1}{3}a^{7}x^{3}$	80
risch	$\frac{1}{10}b^{7}x^{10}+\frac{7}{9}a\,b^{6}x^{9}+\frac{21}{8}a^{2}b^{5}x^{8}+5a^{3}b^{4}x^{7}+\frac{35}{6}a^{4}b^{3}x^{6}+\frac{21}{5}a^{5}b^{2}x^{5}+\frac{7}{4}a^{6}b\,x^{4}+\frac{1}{3}a^{7}x^{3}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^7*x^10+7/9*a*b^6*x^9+21/8*a^2*b^5*x^8+5*a^3*b^4*x^7+35/6*a^4*b^3*x^6
+21/5*a^5*b^2*x^5+7/4*a^6*b*x^4+1/3*a^7*x^3
```

Maxima [A]
time = 0.27, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^{7}x^{10}+\frac{7}{9}\,ab^{6}x^{9}+\frac{21}{8}\,a^{2}b^{5}x^{8}+5\,a^{3}b^{4}x^{7}+\frac{35}{6}\,a^{4}b^{3}x^{6}+\frac{21}{5}\,a^{5}b^{2}x^{5}+\frac{7}{4}\,a^{6}bx^{4}+\frac{1}{3}\,a^{7}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Fricas [A]
time = 0.97, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^{7}x^{10}+\frac{7}{9}\,ab^{6}x^{9}+\frac{21}{8}\,a^{2}b^{5}x^{8}+5\,a^{3}b^{4}x^{7}+\frac{35}{6}\,a^{4}b^{3}x^{6}+\frac{21}{5}\,a^{5}b^{2}x^{5}+\frac{7}{4}\,a^{6}bx^{4}+\frac{1}{3}\,a^{7}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 0.01, size = 92, normalized size = 1.96

$$\frac{a^{7}x^{3}}{3}+\frac{7a^{6}bx^{4}}{4}+\frac{21a^{5}b^{2}x^{5}}{5}+\frac{35a^{4}b^{3}x^{6}}{6}+5a^{3}b^{4}x^{7}+\frac{21a^{2}b^{5}x^{8}}{8}+\frac{7ab^{6}x^{9}}{9}+\frac{b^{7}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**7,x)
```

```
[Out] a**7*x**3/3 + 7*a**6*b*x**4/4 + 21*a**5*b**2*x**5/5 + 35*a**4*b**3*x**6/6 +
 5*a**3*b**4*x**7 + 21*a**2*b**5*x**8/8 + 7*a*b**6*x**9/9 + b**7*x**10/10
```

Giac [A]
time = 0.93, size = 79, normalized size = 1.68

$$\frac{1}{10}b^7x^{10} + \frac{7}{9}ab^6x^9 + \frac{21}{8}a^2b^5x^8 + 5a^3b^4x^7 + \frac{35}{6}a^4b^3x^6 + \frac{21}{5}a^5b^2x^5 + \frac{7}{4}a^6bx^4 + \frac{1}{3}a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

Mupad [B]
time = 0.12, size = 31, normalized size = 0.66

$$\frac{(a+b\,x)^8\,(8\,a^2-64\,a\,b\,x+288\,b^2\,x^2)}{2880\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^8*(8*a^2 + 288*b^2*x^2 - 64*a*b*x))/(2880*b^3)
```

3.105 $\int x(a+bx)^7\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}$$

```
[Out] -1/8*a*(b*x+a)^8/b^2+1/9*(b*x+a)^9/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^7,x]
```

```
[Out] -1/8*(a*(a + b*x)^8)/b^2 + (a + b*x)^9/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^7\,dx &= \int\left(-\frac{a(a+bx)^7}{b}+\frac{(a+bx)^8}{b}\right)dx\\ &= -\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(30) = 60.
time = 0.00, size = 91, normalized size = 3.03

$$\frac{a^7x^2}{2}+\frac{7}{3}a^6bx^3+\frac{21}{4}a^5b^2x^4+7a^4b^3x^5+\frac{35}{6}a^3b^4x^6+3a^2b^5x^7+\frac{7}{8}ab^6x^8+\frac{b^7x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^7,x]
```

[Out] (a^7*x^2)/2 + (7*a^6*b*x^3)/3 + (21*a^5*b^2*x^4)/4 + 7*a^4*b^3*x^5 + (35*a^3*b^4*x^6)/6 + 3*a^2*b^5*x^7 + (7*a*b^6*x^8)/8 + (b^7*x^9)/9

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.
time = 0.10, size = 80, normalized size = 2.67

method	result	size
gosper	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80
default	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80
norman	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80
risch	$\frac{1}{9}b^7x^9 + \frac{7}{8}a\,b^6x^8 + 3a^2b^5x^7 + \frac{35}{6}a^3b^4x^6 + 7a^4b^3x^5 + \frac{21}{4}a^5b^2x^4 + \frac{7}{3}a^6b\,x^3 + \frac{1}{2}a^7x^2$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x+a)^7,x,method=_RETURNVERBOSE)

[Out] 1/9*b^7*x^9+7/8*a*b^6*x^8+3*a^2*b^5*x^7+35/6*a^3*b^4*x^6+7*a^4*b^3*x^5+21/4*a^5*b^2*x^4+7/3*a^6*b*x^3+1/2*a^7*x^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.
time = 0.30, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^7,x, algorithm="maxima")

[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.
time = 1.06, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^7,x, algorithm="fricas")

[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(24) = 48$.
time = 0.01, size = 90, normalized size = 3.00

$$\frac{a^7x^2}{2} + \frac{7a^6bx^3}{3} + \frac{21a^5b^2x^4}{4} + 7a^4b^3x^5 + \frac{35a^3b^4x^6}{6} + 3a^2b^5x^7 + \frac{7ab^6x^8}{8} + \frac{b^7x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**7,x)
```

```
[Out] a**7*x**2/2 + 7*a**6*b*x**3/3 + 21*a**5*b**2*x**4/4 + 7*a**4*b**3*x**5 + 35
*a**3*b**4*x**6/6 + 3*a**2*b**5*x**7 + 7*a*b**6*x**8/8 + b**7*x**9/9
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(26) = 52$.
time = 1.07, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9 + \frac{7}{8}\,ab^6x^8 + 3\,a^2b^5x^7 + \frac{35}{6}\,a^3b^4x^6 + 7\,a^4b^3x^5 + \frac{21}{4}\,a^5b^2x^4 + \frac{7}{3}\,a^6bx^3 + \frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

Mupad [B]
time = 0.12, size = 25, normalized size = 0.83

$$-\frac{2\left(\frac{a\,(a+b\,x)^8}{16}-\frac{(a+b\,x)^9}{18}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^7,x)
```

```
[Out] -(2*((a*(a + b*x)^8)/16 - (a + b*x)^9/18))/b^2
```

3.106 $\int (a+bx)^7\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^8}{8b}$$

```
[Out] 1/8*(b*x+a)^8/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^7\,dx = \frac{(a+bx)^8}{8b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^8}{8b}$	13
gosper	$\frac{1}{8}b^7x^8+a\,b^6x^7+\frac{7}{2}a^2b^5x^6+7a^3b^4x^5+\frac{35}{4}a^4b^3x^4+7a^5b^2x^3+\frac{7}{2}a^6b\,x^2+a^7x$	76
norman	$\frac{1}{8}b^7x^8+a\,b^6x^7+\frac{7}{2}a^2b^5x^6+7a^3b^4x^5+\frac{35}{4}a^4b^3x^4+7a^5b^2x^3+\frac{7}{2}a^6b\,x^2+a^7x$	76
risch	$\frac{b^7x^8}{8}+a\,b^6x^7+\frac{7a^2b^5x^6}{2}+7a^3b^4x^5+\frac{35a^4b^3x^4}{4}+7a^5b^2x^3+\frac{7a^6b\,x^2}{2}+a^7x+\frac{a^8}{8b}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(b*x+a)^8/b
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x + a)^8/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(12) = 24$.
time = 1.16, size = 75, normalized size = 5.36

$$\frac{1}{8}\,b^7x^8+ab^6x^7+\frac{7}{2}\,a^2b^5x^6+7\,a^3b^4x^5+\frac{35}{4}\,a^4b^3x^4+7\,a^5b^2x^3+\frac{7}{2}\,a^6bx^2+a^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*b^7*x^8 + a*b^6*x^7 + 7/2*a^2*b^5*x^6 + 7*a^3*b^4*x^5 + 35/4*a^4*b^3*x^
4 + 7*a^5*b^2*x^3 + 7/2*a^6*b*x^2 + a^7*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.
time = 0.01, size = 83, normalized size = 5.93

$$a^7x+\frac{7a^6bx^2}{2}+7a^5b^2x^3+\frac{35a^4b^3x^4}{4}+7a^3b^4x^5+\frac{7a^2b^5x^6}{2}+ab^6x^7+\frac{b^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7,x)
```

```
[Out] a**7*x + 7*a**6*b*x**2/2 + 7*a**5*b**2*x**3 + 35*a**4*b**3*x**4/4 + 7*a**3*
b**4*x**5 + 7*a**2*b**5*x**6/2 + a*b**6*x**7 + b**7*x**8/8
```

Giac [A]
time = 1.29, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(b*x + a)^8/b
```

Mupad [B]
time = 0.06, size = 75, normalized size = 5.36

$$a^7\,x+\frac{7\,a^6\,b\,x^2}{2}+7\,a^5\,b^2\,x^3+\frac{35\,a^4\,b^3\,x^4}{4}+7\,a^3\,b^4\,x^5+\frac{7\,a^2\,b^5\,x^6}{2}+a\,b^6\,x^7+\frac{b^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7,x)
```

```
[Out] a^7*x + (b^7*x^8)/8 + (7*a^6*b*x^2)/2 + a*b^6*x^7 + 7*a^5*b^2*x^3 + (35*a^4
*b^3*x^4)/4 + 7*a^3*b^4*x^5 + (7*a^2*b^5*x^6)/2
```

3.107 $\int \frac{(a+bx)^7}{x}\,dx$

Optimal. Leaf size=87

$$7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)$$

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$a^7\log(x)+7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x}\,dx &= \int \left(7a^6b+\frac{a^7}{x}+21a^5b^2x+35a^4b^3x^2+35a^3b^4x^3+21a^2b^5x^4+7ab^6x^5+b^7x^6\right)dx\\ &= 7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 87, normalized size = 1.00

$$7a^6bx+\frac{21}{2}a^5b^2x^2+\frac{35}{3}a^4b^3x^3+\frac{35}{4}a^3b^4x^4+\frac{21}{5}a^2b^5x^5+\frac{7}{6}ab^6x^6+\frac{b^7x^7}{7}+a^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.87

method	result	size
default	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76
norman	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76
risch	$7a^6bx+\frac{21a^5b^2x^2}{2}+\frac{35a^4b^3x^3}{3}+\frac{35a^3b^4x^4}{4}+\frac{21a^2b^5x^5}{5}+\frac{7a\,b^6x^6}{6}+\frac{b^7x^7}{7}+a^7\ln\left(x\right)$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x,x,method=_RETURNVERBOSE)
```

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

Maxima [A]
time = 0.27, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^7x^7+\frac{7}{6}\,ab^6x^6+\frac{21}{5}\,a^2b^5x^5+\frac{35}{4}\,a^3b^4x^4+\frac{35}{3}\,a^4b^3x^3+\frac{21}{2}\,a^5b^2x^2+7\,a^6bx+a^7\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="maxima")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Fricas [A]
time = 0.99, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^7x^7+\frac{7}{6}\,ab^6x^6+\frac{21}{5}\,a^2b^5x^5+\frac{35}{4}\,a^3b^4x^4+\frac{35}{3}\,a^4b^3x^3+\frac{21}{2}\,a^5b^2x^2+7\,a^6bx+a^7\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="fricas")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

Sympy [A]
time = 0.04, size = 88, normalized size = 1.01

$$a^{7}\log\left(x\right)+7a^{6}bx+\frac{21a^{5}b^{2}x^{2}}{2}+\frac{35a^{4}b^{3}x^{3}}{3}+\frac{35a^{3}b^{4}x^{4}}{4}+\frac{21a^{2}b^{5}x^{5}}{5}+\frac{7ab^{6}x^{6}}{6}+\frac{b^{7}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x,x)
```

```
[Out] a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**3/3 + 35*a
**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x**7/7
```

Giac [A]
time = 1.26, size = 76, normalized size = 0.87

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="giac")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(abs(x))
```

Mupad [B]
time = 0.07, size = 75, normalized size = 0.86

$$a^{7}\,\ln\left(x\right)+\frac{b^{7}\,x^{7}}{7}+\frac{7\,a\,b^{6}\,x^{6}}{6}+\frac{21\,a^{5}\,b^{2}\,x^{2}}{2}+\frac{35\,a^{4}\,b^{3}\,x^{3}}{3}+\frac{35\,a^{3}\,b^{4}\,x^{4}}{4}+\frac{21\,a^{2}\,b^{5}\,x^{5}}{5}+7\,a^{6}\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x,x)
```

```
[Out] a^7*log(x) + (b^7*x^7)/7 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/2 + (35*a^4*b
^3*x^3)/3 + (35*a^3*b^4*x^4)/4 + (21*a^2*b^5*x^5)/5 + 7*a^6*b*x
```

3.108 $\int \frac{(a+bx)^7}{x^2} \, dx$

Optimal. Leaf size=86

$$-\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)$$

```
[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*
a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^7}{x}+7a^6b\log(x)+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^2}\,dx &= \int \left(21a^5b^2+\frac{a^7}{x^2}+\frac{7a^6b}{x}+35a^4b^3x+35a^3b^4x^2+21a^2b^5x^3+7ab^6x^4+b^7x^5\right)dx\\ &= -\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{x}+21a^5b^2x+\frac{35}{2}a^4b^3x^2+\frac{35}{3}a^3b^4x^3+\frac{21}{4}a^2b^5x^4+\frac{7}{5}ab^6x^5+\frac{b^7x^6}{6}+7a^6b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.90

method	result	size
default	$-\frac{a^7}{x}+21a^5b^2x+\frac{35a^4b^3x^2}{2}+\frac{35a^3b^4x^3}{3}+\frac{21a^2b^5x^4}{4}+\frac{7a\,b^6x^5}{5}+\frac{b^7x^6}{6}+7a^6b\ln\left(x\right)$	77
risch	$-\frac{a^7}{x}+21a^5b^2x+\frac{35a^4b^3x^2}{2}+\frac{35a^3b^4x^3}{3}+\frac{21a^2b^5x^4}{4}+\frac{7a\,b^6x^5}{5}+\frac{b^7x^6}{6}+7a^6b\ln\left(x\right)$	77
norman	$\frac{-a^7+\frac{1}{6}b^7x^7+\frac{7}{5}a\,b^6x^6+\frac{21}{4}a^2b^5x^5+\frac{35}{3}a^3b^4x^4+\frac{35}{2}a^4b^3x^3+21a^5b^2x^2}{x}+7a^6b\ln\left(x\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*
a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.88

$$\frac{1}{6}\,b^7x^6+\frac{7}{5}\,ab^6x^5+\frac{21}{4}\,a^2b^5x^4+\frac{35}{3}\,a^3b^4x^3+\frac{35}{2}\,a^4b^3x^2+21\,a^5b^2x+7\,a^6b\log\left(x\right)-\frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(x) - a^7/x
```

Fricas [A]
time = 0.79, size = 81, normalized size = 0.94

$$\frac{10\,b^7x^7+84\,ab^6x^6+315\,a^2b^5x^5+700\,a^3b^4x^4+1050\,a^4b^3x^3+1260\,a^5b^2x^2+420\,a^6bx\log\left(x\right)-60\,a^7}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*x^7 + 84*a*b^6*x^6 + 315*a^2*b^5*x^5 + 700*a^3*b^4*x^4 + 1050*
a^4*b^3*x^3 + 1260*a^5*b^2*x^2 + 420*a^6*b*x*log(x) - 60*a^7)/x
```

Sympy [A]
time = 0.05, size = 85, normalized size = 0.99

$$-\frac{a^7}{x} + 7a^6b\log(x) + 21a^5b^2x + \frac{35a^4b^3x^2}{2} + \frac{35a^3b^4x^3}{3} + \frac{21a^2b^5x^4}{4} + \frac{7ab^6x^5}{5} + \frac{b^7x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**2,x)
```

```
[Out] -a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*a**3*
b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6
```

Giac [A]
time = 1.32, size = 77, normalized size = 0.90

$$\frac{1}{6}\,b^7x^6 + \frac{7}{5}\,ab^6x^5 + \frac{21}{4}\,a^2b^5x^4 + \frac{35}{3}\,a^3b^4x^3 + \frac{35}{2}\,a^4b^3x^2 + 21\,a^5b^2x + 7\,a^6b\log(|x|) - \frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="giac")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(abs(x)) - a^7/x
```

Mupad [B]
time = 0.05, size = 76, normalized size = 0.88

$$\frac{b^7\,x^6}{6} - \frac{a^7}{x} + 21\,a^5\,b^2\,x + \frac{7\,a\,b^6\,x^5}{5} + 7\,a^6\,b\,\ln(x) + \frac{35\,a^4\,b^3\,x^2}{2} + \frac{35\,a^3\,b^4\,x^3}{3} + \frac{21\,a^2\,b^5\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^2,x)
```

```
[Out] (b^7*x^6)/6 - a^7/x + 21*a^5*b^2*x + (7*a*b^6*x^5)/5 + 7*a^6*b*log(x) + (35
*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2*b^5*x^4)/4
```

3.109 $\int \frac{(a+bx)^7}{x^3} dx$

Optimal. Leaf size=84

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)$$

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^3,x]
```

```
[Out] -1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^3}\,dx &= \int \left(35a^4b^3+\frac{a^7}{x^3}+\frac{7a^6b}{x^2}+\frac{21a^5b^2}{x}+35a^3b^4x+21a^2b^5x^2+7ab^6x^3+b^7x^4\right)dx\\ &= -\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^3,x]
```

```
[Out] -1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.92

method	result	size
default	$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35a^3b^4x^2}{2}+7a^2b^5x^3+\frac{7a\,b^6x^4}{4}+\frac{b^7x^5}{5}+21a^5b^2\ln(x)$	77
risch	$\frac{b^7x^5}{5}+\frac{7a\,b^6x^4}{4}+7a^2b^5x^3+\frac{35a^3b^4x^2}{2}+35a^4b^3x+\frac{-7a^6bx-\frac{1}{2}a^7}{x^2}+21a^5b^2\ln(x)$	77
norman	$\frac{-\frac{1}{2}a^7+\frac{1}{5}b^7x^7+\frac{7}{4}a\,b^6x^6+7a^2b^5x^5+\frac{35}{2}a^3b^4x^4+35a^4b^3x^3-7a^6bx}{x^2}+21a^5b^2\ln(x)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.89

$$\frac{1}{5}\,b^7x^5+\frac{7}{4}\,ab^6x^4+7\,a^2b^5x^3+\frac{35}{2}\,a^3b^4x^2+35\,a^4b^3x+21\,a^5b^2\log(x)-\frac{14\,a^6bx+a^7}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(x) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Fricas [A]
time = 0.88, size = 81, normalized size = 0.96

$$\frac{4\,b^7x^7+35\,ab^6x^6+140\,a^2b^5x^5+350\,a^3b^4x^4+700\,a^4b^3x^3+420\,a^5b^2x^2\log(x)-140\,a^6bx-10\,a^7}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 + 35*a*b^6*x^6 + 140*a^2*b^5*x^5 + 350*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 420*a^5*b^2*x^2*log(x) - 140*a^6*b*x - 10*a^7)/x^2
```

Sympy [A]
time = 0.07, size = 85, normalized size = 1.01

$$21a^5b^2\log(x) + 35a^4b^3x + \frac{35a^3b^4x^2}{2} + 7a^2b^5x^3 + \frac{7ab^6x^4}{4} + \frac{b^7x^5}{5} + \frac{-a^7 - 14a^6bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**3,x)
```

```
[Out] 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a**3*b**4*x**2/2 + 7*a**2*b**5*x*
*3 + 7*a*b**6*x**4/4 + b**7*x**5/5 + (-a**7 - 14*a**6*b*x)/(2*x**2)
```

Giac [A]
time = 1.36, size = 76, normalized size = 0.90

$$\frac{1}{5}\,b^7x^5 + \frac{7}{4}\,ab^6x^4 + 7\,a^2b^5x^3 + \frac{35}{2}\,a^3b^4x^2 + 35\,a^4b^3x + 21\,a^5b^2\log(|x|) - \frac{14\,a^6bx + a^7}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="giac")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(abs(x)) - 1/2*(14*a^6*b*x + a^7)/x^2
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.92

$$\frac{b^7\,x^5}{5} - \frac{\frac{a^7}{2} + 7\,b\,x\,a^6}{x^2} + 35\,a^4\,b^3\,x + \frac{7\,a\,b^6\,x^4}{4} + \frac{35\,a^3\,b^4\,x^2}{2} + 7\,a^2\,b^5\,x^3 + 21\,a^5\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^3,x)
```

```
[Out] (b^7*x^5)/5 - (a^7/2 + 7*a^6*b*x)/x^2 + 35*a^4*b^3*x + (7*a*b^6*x^4)/4 + (3
5*a^3*b^4*x^2)/2 + 7*a^2*b^5*x^3 + 21*a^5*b^2*log(x)
```

3.110 $\int \frac{(a+bx)^7}{x^4} \, dx$

Optimal. Leaf size=86

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)$$

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^4,x]
```

```
[Out] -1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^4}\,dx &= \int \left(35a^3b^4+\frac{a^7}{x^4}+\frac{7a^6b}{x^3}+\frac{21a^5b^2}{x^2}+\frac{35a^4b^3}{x}+21a^2b^5x+7ab^6x^2+b^7x^3\right)dx\\ &= -\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^4,x]
```

```
[Out] -1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.90

method	result	size
default	$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21a^2b^5x^2}{2}+\frac{7a\,b^6x^3}{3}+\frac{b^7x^4}{4}+35a^4b^3\ln\left(x\right)$	77
risch	$\frac{b^7x^4}{4}+\frac{7a\,b^6x^3}{3}+\frac{21a^2b^5x^2}{2}+35a^3b^4x+\frac{-21a^5b^2x^2-\frac{7}{2}a^6bx-\frac{1}{3}a^7}{x^3}+35a^4b^3\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{3}a^7+\frac{1}{4}b^7x^7+\frac{7}{3}a\,b^6x^6+\frac{21}{2}a^2b^5x^5+35a^3b^4x^4-21a^5b^2x^2-\frac{7}{2}a^6bx}{x^3}+35a^4b^3\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

Maxima [A]
time = 0.30, size = 77, normalized size = 0.90

$$\frac{1}{4}\,b^7x^4+\frac{7}{3}\,ab^6x^3+\frac{21}{2}\,a^2b^5x^2+35\,a^3b^4x+35\,a^4b^3\log\left(x\right)-\frac{126\,a^5b^2x^2+21\,a^6bx+2\,a^7}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(x) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Fricas [A]
time = 0.81, size = 81, normalized size = 0.94

$$\frac{3\,b^7x^7+28\,ab^6x^6+126\,a^2b^5x^5+420\,a^3b^4x^4+420\,a^4b^3x^3\log\left(x\right)-252\,a^5b^2x^2-42\,a^6bx-4\,a^7}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 + 28*a*b^6*x^6 + 126*a^2*b^5*x^5 + 420*a^3*b^4*x^4 + 420*a^
4*b^3*x^3*log(x) - 252*a^5*b^2*x^2 - 42*a^6*b*x - 4*a^7)/x^3
```

Sympy [A]
time = 0.09, size = 87, normalized size = 1.01

$$35a^4b^3\log{(x)} + 35a^3b^4x + \frac{21a^2b^5x^2}{2} + \frac{7ab^6x^3}{3} + \frac{b^7x^4}{4} + \frac{-2a^7 - 21a^6bx - 126a^5b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**4,x)
```

```
[Out] 35*a**4*b**3*log(x) + 35*a**3*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/
3 + b**7*x**4/4 + (-2*a**7 - 21*a**6*b*x - 126*a**5*b**2*x**2)/(6*x**3)
```

Giac [A]
time = 2.24, size = 78, normalized size = 0.91

$$\frac{1}{4}\,b^7x^4 + \frac{7}{3}\,ab^6x^3 + \frac{21}{2}\,a^2b^5x^2 + 35\,a^3b^4x + 35\,a^4b^3\log{(|x|)} - \frac{126\,a^5b^2x^2 + 21\,a^6bx + 2\,a^7}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="giac")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(abs(x)) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

Mupad [B]
time = 0.05, size = 77, normalized size = 0.90

$$\frac{b^7\,x^4}{4} - \frac{\frac{a^7}{3} + \frac{7\,a^6\,b\,x}{2} + 21\,a^5\,b^2\,x^2}{x^3} + 35\,a^3\,b^4\,x + \frac{7\,a\,b^6\,x^3}{3} + \frac{21\,a^2\,b^5\,x^2}{2} + 35\,a^4\,b^3\,\ln{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^4,x)
```

```
[Out] (b^7*x^4)/4 - (a^7/3 + 21*a^5*b^2*x^2 + (7*a^6*b*x)/2)/x^3 + 35*a^3*b^4*x +
 (7*a*b^6*x^3)/3 + (21*a^2*b^5*x^2)/2 + 35*a^4*b^3*log(x)
```

3.111 $\int \frac{(a+bx)^7}{x^5} dx$

Optimal. Leaf size=86

$$-\frac{a^7}{4x^4} - \frac{7a^6b}{3x^3} - \frac{21a^5b^2}{2x^2} - \frac{35a^4b^3}{x} + 21a^2b^5x + \frac{7}{2}ab^6x^2 + \frac{b^7x^3}{3} + 35a^3b^4\log(x)$$

```
[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a
*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{4x^4} - \frac{7a^6b}{3x^3} - \frac{21a^5b^2}{2x^2} - \frac{35a^4b^3}{x} + 35a^3b^4\log(x) + 21a^2b^5x + \frac{7}{2}ab^6x^2 + \frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^5,x]
```

```
[Out] -1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{x^5}\,dx &= \int \left(21a^2b^5 + \frac{a^7}{x^5} + \frac{7a^6b}{x^4} + \frac{21a^5b^2}{x^3} + \frac{35a^4b^3}{x^2} + \frac{35a^3b^4}{x} + 7ab^6x + b^7x^2\right) dx \\
&= -\frac{a^7}{4x^4} - \frac{7a^6b}{3x^3} - \frac{21a^5b^2}{2x^2} - \frac{35a^4b^3}{x} + 21a^2b^5x + \frac{7}{2}ab^6x^2 + \frac{b^7x^3}{3} + 35a^3b^4\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{4x^4} - \frac{7a^6b}{3x^3} - \frac{21a^5b^2}{2x^2} - \frac{35a^4b^3}{x} + 21a^2b^5x + \frac{7}{2}ab^6x^2 + \frac{b^7x^3}{3} + 35a^3b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^5,x]
```

```
[Out] -1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.90

method	result	size
default	$-\frac{a^{7}}{4x^{4}}-\frac{7a^{6}b}{3x^{3}}-\frac{21a^{5}b^{2}}{2x^{2}}-\frac{35a^{4}b^{3}}{x}+21a^{2}b^{5}x+\frac{7a\,b^{6}x^{2}}{2}+\frac{b^{7}x^{3}}{3}+35a^{3}b^{4}\ln\left(x\right)$	77
risch	$\frac{b^{7}x^{3}}{3}+\frac{7a\,b^{6}x^{2}}{2}+21a^{2}b^{5}x+\frac{-35a^{4}b^{3}x^{3}-\frac{21}{2}a^{5}b^{2}x^{2}-\frac{7}{3}a^{6}bx-\frac{1}{4}a^{7}}{x^{4}}+35a^{3}b^{4}\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{4}a^{7}+\frac{1}{3}b^{7}x^{7}+\frac{7}{2}a\,b^{6}x^{6}+21a^{2}b^{5}x^{5}-35a^{4}b^{3}x^{3}-\frac{21}{2}a^{5}b^{2}x^{2}-\frac{7}{3}a^{6}bx}{x^{4}}+35a^{3}b^{4}\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a
*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.90

$$\frac{1}{3}\,b^{7}x^{3}+\frac{7}{2}\,ab^{6}x^{2}+21\,a^{2}b^{5}x+35\,a^{3}b^{4}\log\left(x\right)-\frac{420\,a^{4}b^{3}x^{3}+126\,a^{5}b^{2}x^{2}+28\,a^{6}bx+3\,a^{7}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(x) - 1/12*(420*
a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Fricas [A]
time = 0.98, size = 81, normalized size = 0.94

$$\frac{4\,b^{7}x^{7}+42\,ab^{6}x^{6}+252\,a^{2}b^{5}x^{5}+420\,a^{3}b^{4}x^{4}\log\left(x\right)-420\,a^{4}b^{3}x^{3}-126\,a^{5}b^{2}x^{2}-28\,a^{6}bx-3\,a^{7}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*x^7 + 42*a*b^6*x^6 + 252*a^2*b^5*x^5 + 420*a^3*b^4*x^4*log(x) -
 420*a^4*b^3*x^3 - 126*a^5*b^2*x^2 - 28*a^6*b*x - 3*a^7)/x^4
```

Sympy [A]
time = 0.12, size = 85, normalized size = 0.99

$$35a^3b^4\log{(x)} + 21a^2b^5x + \frac{7ab^6x^2}{2} + \frac{b^7x^3}{3} + \frac{-3a^7 - 28a^6bx - 126a^5b^2x^2 - 420a^4b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**5,x)
```

```
[Out] 35*a**3*b**4*log(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3 + (-3*
a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 420*a**4*b**3*x**3)/(12*x**4)
```

Giac [A]
time = 1.22, size = 78, normalized size = 0.91

$$\frac{1}{3}\,b^7x^3 + \frac{7}{2}\,ab^6x^2 + 21\,a^2b^5x + 35\,a^3b^4\log\left(|x|\right) - \frac{420\,a^4b^3x^3 + 126\,a^5b^2x^2 + 28\,a^6bx + 3\,a^7}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="giac")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(abs(x)) - 1/12*
(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

Mupad [B]
time = 0.09, size = 77, normalized size = 0.90

$$\frac{b^7\,x^3}{3} - \frac{\frac{a^7}{4} + \frac{7\,a^6\,b\,x}{3} + \frac{21\,a^5\,b^2\,x^2}{2} + 35\,a^4\,b^3\,x^3}{x^4} + 21\,a^2\,b^5\,x + \frac{7\,a\,b^6\,x^2}{2} + 35\,a^3\,b^4\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^5,x)
```

```
[Out] (b^7*x^3)/3 - (a^7/4 + (21*a^5*b^2*x^2)/2 + 35*a^4*b^3*x^3 + (7*a^6*b*x)/3)
/x^4 + 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + 35*a^3*b^4*log(x)
```

3.112 $\int \frac{(a+bx)^7}{x^6}\,dx$

Optimal. Leaf size=84

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)$$

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^6,x]
```

```
[Out] -1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^6}\,dx &= \int \left(7ab^6+\frac{a^7}{x^6}+\frac{7a^6b}{x^5}+\frac{21a^5b^2}{x^4}+\frac{35a^4b^3}{x^3}+\frac{35a^3b^4}{x^2}+\frac{21a^2b^5}{x}+b^7x\right)dx\\ &= -\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^6,x]
```

```
[Out] -1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.92

method	result	size
default	$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7a\,b^6x+\frac{b^7x^2}{2}+21a^2b^5\ln\left(x\right)$	77
risch	$\frac{b^7x^2}{2}+7a\,b^6x+\frac{-35a^3b^4x^4-\frac{35}{2}a^4b^3x^3-7a^5b^2x^2-\frac{7}{4}a^6bx-\frac{1}{5}a^7}{x^5}+21a^2b^5\ln\left(x\right)$	77
norman	$\frac{-\frac{1}{5}a^7+\frac{1}{2}b^7x^7+7a\,b^6x^6-35a^3b^4x^4-\frac{35}{2}a^4b^3x^3-7a^5b^2x^2-\frac{7}{4}a^6bx}{x^5}+21a^2b^5\ln\left(x\right)$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.92

$$\frac{1}{2}\,b^7x^2+7\,ab^6x+21\,a^2b^5\log\left(x\right)-\frac{700\,a^3b^4x^4+350\,a^4b^3x^3+140\,a^5b^2x^2+35\,a^6bx+4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(x) - 1/20*(700*a^3*b^4*x^4 + 350*a
^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Fricas [A]
time = 1.10, size = 81, normalized size = 0.96

$$\frac{10\,b^7x^7+140\,ab^6x^6+420\,a^2b^5x^5\log\left(x\right)-700\,a^3b^4x^4-350\,a^4b^3x^3-140\,a^5b^2x^2-35\,a^6bx-4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*x^7 + 140*a*b^6*x^6 + 420*a^2*b^5*x^5*log(x) - 700*a^3*b^4*x^4
 - 350*a^4*b^3*x^3 - 140*a^5*b^2*x^2 - 35*a^6*b*x - 4*a^7)/x^5
```

Sympy [A]
time = 0.15, size = 83, normalized size = 0.99

$$21a^2b^5\log{(x)} + 7ab^6x + \frac{b^7x^2}{2} + \frac{-4a^7 - 35a^6bx - 140a^5b^2x^2 - 350a^4b^3x^3 - 700a^3b^4x^4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**6,x)
```

```
[Out] 21*a**2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2 + (-4*a**7 - 35*a**6*b*x - 1
40*a**5*b**2*x**2 - 350*a**4*b**3*x**3 - 700*a**3*b**4*x**4)/(20*x**5)
```

Giac [A]
time = 1.34, size = 78, normalized size = 0.93

$$\frac{1}{2}\,b^7x^2 + 7\,ab^6x + 21\,a^2b^5\log\left(|x|\right) - \frac{700\,a^3b^4x^4 + 350\,a^4b^3x^3 + 140\,a^5b^2x^2 + 35\,a^6bx + 4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="giac")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(abs(x)) - 1/20*(700*a^3*b^4*x^4 +
350*a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

Mupad [B]
time = 0.11, size = 77, normalized size = 0.92

$$\frac{b^7\,x^2}{2} - \frac{\frac{a^7}{5} + \frac{7\,a^6\,b\,x}{4} + 7\,a^5\,b^2\,x^2 + \frac{35\,a^4\,b^3\,x^3}{2} + 35\,a^3\,b^4\,x^4}{x^5} + 21\,a^2\,b^5\,\ln\left(x\right) + 7\,a\,b^6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^6,x)
```

```
[Out] (b^7*x^2)/2 - (a^7/5 + 7*a^5*b^2*x^2 + (35*a^4*b^3*x^3)/2 + 35*a^3*b^4*x^4
+ (7*a^6*b*x)/4)/x^5 + 21*a^2*b^5*log(x) + 7*a*b^6*x
```

3.113 $\int \frac{(a+bx)^7}{x^7} dx$

Optimal. Leaf size=85

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)$$

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^7,x]
```

```
[Out] -1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^7}\,dx &= \int \left(b^7+\frac{a^7}{x^7}+\frac{7a^6b}{x^6}+\frac{21a^5b^2}{x^5}+\frac{35a^4b^3}{x^4}+\frac{35a^3b^4}{x^3}+\frac{21a^2b^5}{x^2}+\frac{7ab^6}{x}\right)dx\\ &= -\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 85, normalized size = 1.00

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^7,x]
```

```
[Out] -1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Maple [A]
time = 0.08, size = 76, normalized size = 0.89

method	result	size
default	$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7a\,b^6\ln\left(x\right)$	76
risch	$b^7x+\frac{-21a^2b^5x^5-\frac{35}{2}a^3b^4x^4-\frac{35}{3}a^4b^3x^3-\frac{21}{4}a^5b^2x^2-\frac{7}{5}a^6bx-\frac{1}{6}a^7}{x^6}+7a\,b^6\ln\left(x\right)$	76
norman	$\frac{b^7x^7-\frac{1}{6}a^7-21a^2b^5x^5-\frac{35}{2}a^3b^4x^4-\frac{35}{3}a^4b^3x^3-\frac{21}{4}a^5b^2x^2-\frac{7}{5}a^6bx}{x^6}+7a\,b^6\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.89

$$b^7x+7\,ab^6\log\left(x\right)-\frac{1260\,a^2b^5x^5+1050\,a^3b^4x^4+700\,a^4b^3x^3+315\,a^5b^2x^2+84\,a^6bx+10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="maxima")
```

```
[Out] b^7*x + 7*a*b^6*log(x) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Fricas [A]
time = 1.07, size = 81, normalized size = 0.95

$$\frac{60\,b^7x^7+420\,ab^6x^6\log\left(x\right)-1260\,a^2b^5x^5-1050\,a^3b^4x^4-700\,a^4b^3x^3-315\,a^5b^2x^2-84\,a^6bx-10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 420*a*b^6*x^6*log(x) - 1260*a^2*b^5*x^5 - 1050*a^3*b^4*x
^4 - 700*a^4*b^3*x^3 - 315*a^5*b^2*x^2 - 84*a^6*b*x - 10*a^7)/x^6
```

Sympy [A]
time = 0.19, size = 82, normalized size = 0.96

$$7ab^6\log(x) + b^7x + \frac{-10a^7 - 84a^6bx - 315a^5b^2x^2 - 700a^4b^3x^3 - 1050a^3b^4x^4 - 1260a^2b^5x^5}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**7,x)
```

```
[Out] 7*a*b**6*log(x) + b**7*x + (-10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 - 7
00*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*x**6)
```

Giac [A]
time = 1.87, size = 77, normalized size = 0.91

$$b^7x + 7\,ab^6\log(|x|) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="giac")
```

```
[Out] b^7*x + 7*a*b^6*log(abs(x)) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 7
00*a^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

Mupad [B]
time = 0.11, size = 81, normalized size = 0.95

$$-\frac{10\,a^7 - 60\,b^7\,x^7 + 315\,a^5\,b^2\,x^2 + 700\,a^4\,b^3\,x^3 + 1050\,a^3\,b^4\,x^4 + 1260\,a^2\,b^5\,x^5 + 84\,a^6\,b\,x - 420\,a\,b^6\,x^6\,\ln(x)}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^7,x)
```

```
[Out] -(10*a^7 - 60*b^7*x^7 + 315*a^5*b^2*x^2 + 700*a^4*b^3*x^3 + 1050*a^3*b^4*x^
4 + 1260*a^2*b^5*x^5 + 84*a^6*b*x - 420*a*b^6*x^6*log(x))/(60*x^6)
```

3.114 $\int \frac{(a+bx)^7}{x^8} \, dx$

Optimal. Leaf size=89

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^8,x]
```

```
[Out] -1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^8}\,dx &= \int \left(\frac{a^7}{x^8}+\frac{7a^6b}{x^7}+\frac{21a^5b^2}{x^6}+\frac{35a^4b^3}{x^5}+\frac{35a^3b^4}{x^4}+\frac{21a^2b^5}{x^3}+\frac{7ab^6}{x^2}+\frac{b^7}{x}\right)dx\\ &= -\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 89, normalized size = 1.00

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^8,x]
```

```
[Out] -1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Maple [A]
time = 0.09, size = 78, normalized size = 0.88

method	result	size
default	$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a\,b^6}{x}+b^7\ln\left(x\right)$	78
norman	$\frac{-\frac{1}{7}a^7-7a\,b^6x^6-\frac{21}{2}a^2b^5x^5-\frac{35}{3}a^3b^4x^4-\frac{35}{4}a^4b^3x^3-\frac{21}{5}a^5b^2x^2-\frac{7}{6}a^6bx}{x^7}+b^7\ln\left(x\right)$	78
risch	$\frac{-\frac{1}{7}a^7-7a\,b^6x^6-\frac{21}{2}a^2b^5x^5-\frac{35}{3}a^3b^4x^4-\frac{35}{4}a^4b^3x^3-\frac{21}{5}a^5b^2x^2-\frac{7}{6}a^6bx}{x^7}+b^7\ln\left(x\right)$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

Maxima [A]
time = 0.27, size = 78, normalized size = 0.88

$$b^7\log\left(x\right)-\frac{2940\,ab^6x^6+4410\,a^2b^5x^5+4900\,a^3b^4x^4+3675\,a^4b^3x^3+1764\,a^5b^2x^2+490\,a^6bx+60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="maxima")
```

```
[Out] b^7*log(x) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x^4 +
3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Fricas [A]
time = 0.97, size = 81, normalized size = 0.91

$$\frac{420\,b^7x^7\log\left(x\right)-2940\,ab^6x^6-4410\,a^2b^5x^5-4900\,a^3b^4x^4-3675\,a^4b^3x^3-1764\,a^5b^2x^2-490\,a^6bx-60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="fricas")
```

```
[Out] 1/420*(420*b^7*x^7*log(x) - 2940*a*b^6*x^6 - 4410*a^2*b^5*x^5 - 4900*a^3*b^
4*x^4 - 3675*a^4*b^3*x^3 - 1764*a^5*b^2*x^2 - 490*a^6*b*x - 60*a^7)/x^7
```

Sympy [A]
time = 0.22, size = 83, normalized size = 0.93

$$b^7 \log(x) + \frac{-60a^7 - 490a^6bx - 1764a^5b^2x^2 - 3675a^4b^3x^3 - 4900a^3b^4x^4 - 4410a^2b^5x^5 - 2940ab^6x^6}{420x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**8,x)
```

```
[Out] b**7*log(x) + (-60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 3675*a**4*b*
*3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6*x**6)/(42
0*x**7)
```

Giac [A]
time = 1.22, size = 79, normalized size = 0.89

$$b^7 \log(|x|) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(x)) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x
^4 + 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

Mupad [B]
time = 0.07, size = 78, normalized size = 0.88

$$b^7\,\ln(x) - \frac{\frac{a^7}{7} + \frac{7\,a^6\,b\,x}{6} + \frac{21\,a^5\,b^2\,x^2}{5} + \frac{35\,a^4\,b^3\,x^3}{4} + \frac{35\,a^3\,b^4\,x^4}{3} + \frac{21\,a^2\,b^5\,x^5}{2} + 7\,a\,b^6\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^8,x)
```

```
[Out] b^7*log(x) - (a^7/7 + 7*a*b^6*x^6 + (21*a^5*b^2*x^2)/5 + (35*a^4*b^3*x^3)/4
 + (35*a^3*b^4*x^4)/3 + (21*a^2*b^5*x^5)/2 + (7*a^6*b*x)/6)/x^7
```

3.115 $\int \frac{(a+bx)^7}{x^9} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^8}{8ax^8}$$

[Out] -1/8*(b*x+a)^8/a/x^8

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^8}{8ax^8}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^7/x^9,x]

[Out] -1/8*(a + b*x)^8/(a*x^8)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^9} dx = -\frac{(a+bx)^8}{8ax^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(17) = 34.
time = 0.00, size = 87, normalized size = 5.12

$$-\frac{a^7}{8x^8} - \frac{a^6b}{x^7} - \frac{7a^5b^2}{2x^6} - \frac{7a^4b^3}{x^5} - \frac{35a^3b^4}{4x^4} - \frac{7a^2b^5}{x^3} - \frac{7ab^6}{2x^2} - \frac{b^7}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^7/x^9,x]

[Out] -1/8*a^7/x^8 - (a^6*b)/x^7 - (7*a^5*b^2)/(2*x^6) - (7*a^4*b^3)/x^5 - (35*a^3*b^4)/(4*x^4) - (7*a^2*b^5)/x^3 - (7*a*b^6)/(2*x^2) - b^7/x

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(15) = 30$.
time = 0.07, size = 80, normalized size = 4.71

method	result	size
gosper	$-\frac{8b^7x^7+28a\,b^6x^6+56a^2b^5x^5+70a^3b^4x^4+56a^4b^3x^3+28a^5b^2x^2+8a^6bx+a^7}{8x^8}$	78
norman	$\frac{-b^7x^7-\frac{7}{2}a\,b^6x^6-7a^2b^5x^5-\frac{35}{4}a^3b^4x^4-7a^4b^3x^3-\frac{7}{2}a^5b^2x^2-a^6bx-\frac{1}{8}a^7}{x^8}$	79
risch	$\frac{-b^7x^7-\frac{7}{2}a\,b^6x^6-7a^2b^5x^5-\frac{35}{4}a^3b^4x^4-7a^4b^3x^3-\frac{7}{2}a^5b^2x^2-a^6bx-\frac{1}{8}a^7}{x^8}$	79
default	$-\frac{b^7}{x}-\frac{7a^2b^5}{x^3}-\frac{35a^3b^4}{4x^4}-\frac{7a\,b^6}{2x^2}-\frac{a^7}{8x^8}-\frac{7a^4b^3}{x^5}-\frac{7a^5b^2}{2x^6}-\frac{a^6b}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^7/x^9,x,method=_RETURNVERBOSE)

[Out] -b^7/x-7*a^2*b^5/x^3-35/4*a^3*b^4/x^4-7/2*a*b^6/x^2-1/8*a^7/x^8-7*a^4*b^3/x^5-7/2*a^5*b^2/x^6-a^6*b/x^7

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.
time = 0.27, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^9,x, algorithm="maxima")

[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.
time = 1.29, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^9,x, algorithm="fricas")

[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(14) = 28$.
time = 0.23, size = 83, normalized size = 4.88

$$\frac{-a^7 - 8a^6bx - 28a^5b^2x^2 - 56a^4b^3x^3 - 70a^3b^4x^4 - 56a^2b^5x^5 - 28ab^6x^6 - 8b^7x^7}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**9,x)
```

```
[Out] (-a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b**4*
x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.
time = 0.84, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

Mupad [B]
time = 0.07, size = 77, normalized size = 4.53

$$-\frac{\frac{a^7}{8} + a^6\,b\,x + \frac{7\,a^5\,b^2\,x^2}{2} + 7\,a^4\,b^3\,x^3 + \frac{35\,a^3\,b^4\,x^4}{4} + 7\,a^2\,b^5\,x^5 + \frac{7\,a\,b^6\,x^6}{2} + b^7\,x^7}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^9,x)
```

```
[Out] -(a^7/8 + b^7*x^7 + (7*a*b^6*x^6)/2 + (7*a^5*b^2*x^2)/2 + 7*a^4*b^3*x^3 + (
35*a^3*b^4*x^4)/4 + 7*a^2*b^5*x^5 + a^6*b*x)/x^8
```

3.116 $\int \frac{(a+bx)^7}{x^{10}} \, dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}$$

```
[Out] -1/9*(b*x+a)^8/a/x^9+1/72*b*(b*x+a)^8/a^2/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}} \, dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9} \, dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(36) = 72.

time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.

time = 0.07, size = 80, normalized size = 2.22

method	result	size
norman	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
risch	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
gosper	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80
default	$-\frac{7a\,b^6}{3x^3}-\frac{21a^2b^5}{4x^4}-\frac{a^7}{9x^9}-\frac{b^7}{2x^2}-\frac{7a^6b}{8x^8}-\frac{7a^3b^4}{x^5}-\frac{35a^4b^3}{6x^6}-\frac{3a^5b^2}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*a*b^6/x^3-21/4*a^2*b^5/x^4-1/9*a^7/x^9-1/2*b^7/x^2-7/8*a^6*b/x^8-7*a^3
*b^4/x^5-35/6*a^4*b^3/x^6-3*a^5*b^2/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.

time = 0.27, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.

time = 0.72, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.
time = 0.25, size = 85, normalized size = 2.36

$$\frac{-8a^{7}-63a^{6}bx-216a^{5}b^{2}x^{2}-420a^{4}b^{3}x^{3}-504a^{3}b^{4}x^{4}-378a^{2}b^{5}x^{5}-168ab^{6}x^{6}-36b^{7}x^{7}}{72x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 1.30, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Mupad [B]
time = 0.09, size = 23, normalized size = 0.64

$$-\frac{\left(8\,a-b\,x\right)\,{\left(a+b\,x\right)}^{8}}{72\,a^{2}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

3.117 $\int \frac{(a+bx)^7}{x^{11}} \, dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^8}{10ax^{10}}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{b^2(a+bx)^8}{360a^3x^8}$$

```
[Out] -1/10*(b*x+a)^8/a/x^10+1/45*b*(b*x+a)^8/a^2/x^9-1/360*b^2*(b*x+a)^8/a^3/x^8
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^11,x]
```

```
[Out] -1/10*(a + b*x)^8/(a*x^10) + (b*(a + b*x)^8)/(45*a^2*x^9) - (b^2*(a + b*x)^
8)/(360*a^3*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{11}}\,dx = -\frac{(a+bx)^8}{10ax^{10}} - \frac{b\int \frac{(a+bx)^7}{x^{10}}\,dx}{5a}$$
$$= -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} + \frac{b^2\int \frac{(a+bx)^7}{x^9}\,dx}{45a^2}$$
$$= -\frac{(a+bx)^8}{10ax^{10}} + \frac{b(a+bx)^8}{45a^2x^9} - \frac{b^2(a+bx)^8}{360a^3x^8}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.66

$$-\frac{a^7}{10x^{10}} - \frac{7a^6b}{9x^9} - \frac{21a^5b^2}{8x^8} - \frac{5a^4b^3}{x^7} - \frac{35a^3b^4}{6x^6} - \frac{21a^2b^5}{5x^5} - \frac{7ab^6}{4x^4} - \frac{b^7}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^11,x]
```

```
[Out] -1/10*a^7/x^10 - (7*a^6*b)/(9*x^9) - (21*a^5*b^2)/(8*x^8) - (5*a^4*b^3)/x^7
 - (35*a^3*b^4)/(6*x^6) - (21*a^2*b^5)/(5*x^5) - (7*a*b^6)/(4*x^4) - b^7/(3
*x^3)
```

Maple [A]
time = 0.07, size = 80, normalized size = 1.43

method	result	size
norman	$\frac{-\frac{1}{3}b^7x^7-\frac{7}{4}a\,b^6x^6-\frac{21}{5}a^2b^5x^5-\frac{35}{6}a^3b^4x^4-5a^4b^3x^3-\frac{21}{8}a^5b^2x^2-\frac{7}{9}a^6bx-\frac{1}{10}a^7}{x^{10}}$	79
risch	$\frac{-\frac{1}{3}b^7x^7-\frac{7}{4}a\,b^6x^6-\frac{21}{5}a^2b^5x^5-\frac{35}{6}a^3b^4x^4-5a^4b^3x^3-\frac{21}{8}a^5b^2x^2-\frac{7}{9}a^6bx-\frac{1}{10}a^7}{x^{10}}$	79
gosper	$-\frac{120b^7x^7+630a\,b^6x^6+1512a^2b^5x^5+2100a^3b^4x^4+1800a^4b^3x^3+945a^5b^2x^2+280a^6bx+36a^7}{360x^{10}}$	80
default	$-\frac{a^7}{10x^{10}}-\frac{b^7}{3x^3}-\frac{7a\,b^6}{4x^4}-\frac{7a^6b}{9x^9}-\frac{21a^5b^2}{8x^8}-\frac{21a^2b^5}{5x^5}-\frac{35a^3b^4}{6x^6}-\frac{5a^4b^3}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*a^7/x^10-1/3*b^7/x^3-7/4*a*b^6/x^4-7/9*a^6*b/x^9-21/8*a^5*b^2/x^8-21/
5*a^2*b^5/x^5-35/6*a^3*b^4/x^6-5*a^4*b^3/x^7
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7 + 630\,ab^6x^6 + 1512\,a^2b^5x^5 + 2100\,a^3b^4x^4 + 1800\,a^4b^3x^3 + 945\,a^5b^2x^2 + 280\,a^6bx + 36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Fricas [A]
time = 0.76, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Sympy [A]
time = 0.26, size = 85, normalized size = 1.52

$$\frac{-36a^7-280a^6bx-945a^5b^2x^2-1800a^4b^3x^3-2100a^3b^4x^4-1512a^2b^5x^5-630ab^6x^6-120b^7x^7}{360x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**11,x)
```

```
[Out] (-36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 2100*
a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)/(36
0*x**10)
```

Giac [A]
time = 1.23, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7+630\,ab^6x^6+1512\,a^2b^5x^5+2100\,a^3b^4x^4+1800\,a^4b^3x^3+945\,a^5b^2x^2+280\,a^6bx+36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

Mupad [B]
time = 0.11, size = 79, normalized size = 1.41

$$-\frac{\frac{a^7}{10}+\frac{7\,a^6\,b\,x}{9}+\frac{21\,a^5\,b^2\,x^2}{8}+5\,a^4\,b^3\,x^3+\frac{35\,a^3\,b^4\,x^4}{6}+\frac{21\,a^2\,b^5\,x^5}{5}+\frac{7\,a\,b^6\,x^6}{4}+\frac{b^7\,x^7}{3}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^11,x)
```

```
[Out] -(a^7/10 + (b^7*x^7)/3 + (7*a*b^6*x^6)/4 + (21*a^5*b^2*x^2)/8 + 5*a^4*b^3*x
^3 + (35*a^3*b^4*x^4)/6 + (21*a^2*b^5*x^5)/5 + (7*a^6*b*x)/9)/x^10
```

3.118 $\int \frac{(a+bx)^7}{x^{12}}\,dx$

Optimal. Leaf size=76

$$-\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{b^3(a+bx)^8}{1320a^4x^8}$$

```
[Out] -1/11*(b*x+a)^8/a/x^11+3/110*b*(b*x+a)^8/a^2/x^10-1/165*b^2*(b*x+a)^8/a^3/x
^9+1/1320*b^3*(b*x+a)^8/a^4/x^8
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b^3(a+bx)^8}{1320a^4x^8}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{(a+bx)^8}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^12,x]
```

```
[Out] -1/11*(a + b*x)^8/(a*x^11) + (3*b*(a + b*x)^8)/(110*a^2*x^10) - (b^2*(a + b
*x)^8)/(165*a^3*x^9) + (b^3*(a + b*x)^8)/(1320*a^4*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^{12}}\,dx = -\frac{(a+bx)^8}{11ax^{11}} - \frac{(3b)\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} + \frac{(3b^2)\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^2}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} - \frac{b^3\int \frac{(a+bx)^7}{x^9}\,dx}{165a^3}$$
$$= -\frac{(a+bx)^8}{11ax^{11}} + \frac{3b(a+bx)^8}{110a^2x^{10}} - \frac{b^2(a+bx)^8}{165a^3x^9} + \frac{b^3(a+bx)^8}{1320a^4x^8}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.22

$$-\frac{a^7}{11x^{11}} - \frac{7a^6b}{10x^{10}} - \frac{7a^5b^2}{3x^9} - \frac{35a^4b^3}{8x^8} - \frac{5a^3b^4}{x^7} - \frac{7a^2b^5}{2x^6} - \frac{7ab^6}{5x^5} - \frac{b^7}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^12,x]
```

```
[Out] -1/11*a^7/x^11 - (7*a^6*b)/(10*x^10) - (7*a^5*b^2)/(3*x^9) - (35*a^4*b^3)/(
8*x^8) - (5*a^3*b^4)/x^7 - (7*a^2*b^5)/(2*x^6) - (7*a*b^6)/(5*x^5) - b^7/(4
*x^4)
```

Maple [A]
time = 0.08, size = 80, normalized size = 1.05

method	result	size
norman	$\frac{-\frac{1}{4}b^7x^7-\frac{7}{5}a\,b^6x^6-\frac{7}{2}a^2b^5x^5-5a^3b^4x^4-\frac{35}{8}a^4b^3x^3-\frac{7}{3}a^5b^2x^2-\frac{7}{10}a^6bx-\frac{1}{11}a^7}{x^{11}}$	79
risch	$\frac{-\frac{1}{4}b^7x^7-\frac{7}{5}a\,b^6x^6-\frac{7}{2}a^2b^5x^5-5a^3b^4x^4-\frac{35}{8}a^4b^3x^3-\frac{7}{3}a^5b^2x^2-\frac{7}{10}a^6bx-\frac{1}{11}a^7}{x^{11}}$	79
gosper	$-\frac{330b^7x^7+1848a\,b^6x^6+4620a^2b^5x^5+6600a^3b^4x^4+5775a^4b^3x^3+3080a^5b^2x^2+924a^6bx+120a^7}{1320x^{11}}$	80
default	$-\frac{7a^6b}{10x^{10}}-\frac{a^7}{11x^{11}}-\frac{b^7}{4x^4}-\frac{7a^5b^2}{3x^9}-\frac{35a^4b^3}{8x^8}-\frac{7a\,b^6}{5x^5}-\frac{7a^2b^5}{2x^6}-\frac{5a^3b^4}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -7/10*a^6*b/x^10-1/11*a^7/x^11-1/4*b^7/x^4-7/3*a^5*b^2/x^9-35/8*a^4*b^3/x^8
-7/5*a*b^6/x^5-7/2*a^2*b^5/x^6-5*a^3*b^4/x^7
```

Maxima [A]
time = 0.30, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7 + 1848\,ab^6x^6 + 4620\,a^2b^5x^5 + 6600\,a^3b^4x^4 + 5775\,a^4b^3x^3 + 3080\,a^5b^2x^2 + 924\,a^6bx + 120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Fricas [A]
time = 0.76, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Sympy [A]
time = 0.28, size = 85, normalized size = 1.12

$$\frac{-120a^7-924a^6bx-3080a^5b^2x^2-5775a^4b^3x^3-6600a^3b^4x^4-4620a^2b^5x^5-1848ab^6x^6-330b^7x^7}{1320x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**12,x)
```

```
[Out] (-120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 - 660
0*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x**7)/
(1320*x**11)
```

Giac [A]
time = 1.73, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

Mupad [B]
time = 0.11, size = 79, normalized size = 1.04

$$-\frac{\frac{a^7}{11}+\frac{7\,a^6\,b\,x}{10}+\frac{7\,a^5\,b^2\,x^2}{3}+\frac{35\,a^4\,b^3\,x^3}{8}+5\,a^3\,b^4\,x^4+\frac{7\,a^2\,b^5\,x^5}{2}+\frac{7\,a\,b^6\,x^6}{5}+\frac{b^7\,x^7}{4}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^12,x)
```

```
[Out] -(a^7/11 + (b^7*x^7)/4 + (7*a*b^6*x^6)/5 + (7*a^5*b^2*x^2)/3 + (35*a^4*b^3*
x^3)/8 + 5*a^3*b^4*x^4 + (7*a^2*b^5*x^5)/2 + (7*a^6*b*x)/10)/x^11
```

3.119 $\int \frac{(a+bx)^7}{x^{13}} \, dx$

Optimal. Leaf size=96

$$-\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^4(a+bx)^8}{3960a^5x^8}$$

```
[Out] -1/12*(b*x+a)^8/a/x^12+1/33*b*(b*x+a)^8/a^2/x^11-1/110*b^2*(b*x+a)^8/a^3/x^
10+1/495*b^3*(b*x+a)^8/a^4/x^9-1/3960*b^4*(b*x+a)^8/a^5/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^13,x]
```

```
[Out] -1/12*(a + b*x)^8/(a*x^12) + (b*(a + b*x)^8)/(33*a^2*x^11) - (b^2*(a + b*x)
^8)/(110*a^3*x^10) + (b^3*(a + b*x)^8)/(495*a^4*x^9) - (b^4*(a + b*x)^8)/(3
960*a^5*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{13}}\,dx &= -\frac{(a+bx)^8}{12ax^{12}} - \frac{b\int \frac{(a+bx)^7}{x^{12}}\,dx}{3a} \\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} + \frac{b^2\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a^2} \\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} - \frac{b^3\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^3} \\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} + \frac{b^4\int \frac{(a+bx)^7}{x^9}\,dx}{495a^4} \\ &= -\frac{(a+bx)^8}{12ax^{12}} + \frac{b(a+bx)^8}{33a^2x^{11}} - \frac{b^2(a+bx)^8}{110a^3x^{10}} + \frac{b^3(a+bx)^8}{495a^4x^9} - \frac{b^4(a+bx)^8}{3960a^5x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 0.97

$$-\frac{a^7}{12x^{12}} - \frac{7a^6b}{11x^{11}} - \frac{21a^5b^2}{10x^{10}} - \frac{35a^4b^3}{9x^9} - \frac{35a^3b^4}{8x^8} - \frac{3a^2b^5}{x^7} - \frac{7ab^6}{6x^6} - \frac{b^7}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^13,x]
```

```
[Out] -1/12*a^7/x^12 - (7*a^6*b)/(11*x^11) - (21*a^5*b^2)/(10*x^10) - (35*a^4*b^3
)/(9*x^9) - (35*a^3*b^4)/(8*x^8) - (3*a^2*b^5)/x^7 - (7*a*b^6)/(6*x^6) - b^
7/(5*x^5)
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{5}b^7x^7-\frac{7}{6}a\,b^6x^6-3a^2b^5x^5-\frac{35}{8}a^3b^4x^4-\frac{35}{9}a^4b^3x^3-\frac{21}{10}a^5b^2x^2-\frac{7}{11}a^6bx-\frac{1}{12}a^7}{x^{12}}$	79
risch	$\frac{-\frac{1}{5}b^7x^7-\frac{7}{6}a\,b^6x^6-3a^2b^5x^5-\frac{35}{8}a^3b^4x^4-\frac{35}{9}a^4b^3x^3-\frac{21}{10}a^5b^2x^2-\frac{7}{11}a^6bx-\frac{1}{12}a^7}{x^{12}}$	79
gosper	$-\frac{792b^7x^7+4620a\,b^6x^6+11880a^2b^5x^5+17325a^3b^4x^4+15400a^4b^3x^3+8316a^5b^2x^2+2520a^6bx+330a^7}{3960x^{12}}$	80
default	$-\frac{21a^5b^2}{10x^{10}}-\frac{7a^6b}{11x^{11}}-\frac{a^7}{12x^{12}}-\frac{35a^4b^3}{9x^9}-\frac{35a^3b^4}{8x^8}-\frac{b^7}{5x^5}-\frac{7a\,b^6}{6x^6}-\frac{3a^2b^5}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -21/10*a^5*b^2/x^10-7/11*a^6*b/x^11-1/12*a^7/x^12-35/9*a^4*b^3/x^9-35/8*a^3
*b^4/x^8-1/5*b^7/x^5-7/6*a*b^6/x^6-3*a^2*b^5/x^7
```

Maxima [A]

time = 0.29, size = 79, normalized size = 0.82

$$-\frac{792\,b^{7}x^{7}+4620\,ab^{6}x^{6}+11880\,a^{2}b^{5}x^{5}+17325\,a^{3}b^{4}x^{4}+15400\,a^{4}b^{3}x^{3}+8316\,a^{5}b^{2}x^{2}+2520\,a^{6}bx+330\,a^{7}}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Fricas [A]

time = 0.95, size = 79, normalized size = 0.82

$$-\frac{792\,b^{7}x^{7}+4620\,ab^{6}x^{6}+11880\,a^{2}b^{5}x^{5}+17325\,a^{3}b^{4}x^{4}+15400\,a^{4}b^{3}x^{3}+8316\,a^{5}b^{2}x^{2}+2520\,a^{6}bx+330\,a^{7}}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Sympy [A]

time = 0.29, size = 85, normalized size = 0.89

$$\frac{-330a^{7}-2520a^{6}bx-8316a^{5}b^{2}x^{2}-15400a^{4}b^{3}x^{3}-17325a^{3}b^{4}x^{4}-11880a^{2}b^{5}x^{5}-4620ab^{6}x^{6}-792b^{7}x^{7}}{3960x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**13,x)
```

```
[Out] (-330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3 - 1
7325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**7*x*
*7)/(3960*x**12)
```

Giac [A]

time = 1.71, size = 79, normalized size = 0.82

$$-\frac{792\,b^{7}x^{7}+4620\,ab^{6}x^{6}+11880\,a^{2}b^{5}x^{5}+17325\,a^{3}b^{4}x^{4}+15400\,a^{4}b^{3}x^{3}+8316\,a^{5}b^{2}x^{2}+2520\,a^{6}bx+330\,a^{7}}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.82

$$-\frac{\frac{a^7}{12}+\frac{7\,a^6\,b\,x}{11}+\frac{21\,a^5\,b^2\,x^2}{10}+\frac{35\,a^4\,b^3\,x^3}{9}+\frac{35\,a^3\,b^4\,x^4}{8}+3\,a^2\,b^5\,x^5+\frac{7\,a\,b^6\,x^6}{6}+\frac{b^7\,x^7}{5}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^13,x)
```

```
[Out] -(a^7/12 + (b^7*x^7)/5 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/10 + (35*a^4*b^
3*x^3)/9 + (35*a^3*b^4*x^4)/8 + 3*a^2*b^5*x^5 + (7*a^6*b*x)/11)/x^12
```

3.120 $\int \frac{(a+bx)^7}{x^{14}} dx$

Optimal. Leaf size=93

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

```
[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3
*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^14,x]
```

```
[Out] -1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{14}}\,dx &= \int \left(\frac{a^7}{x^{14}}+\frac{7a^6b}{x^{13}}+\frac{21a^5b^2}{x^{12}}+\frac{35a^4b^3}{x^{11}}+\frac{35a^3b^4}{x^{10}}+\frac{21a^2b^5}{x^9}+\frac{7ab^6}{x^8}+\frac{b^7}{x^7}\right)dx\\ &= -\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 93, normalized size = 1.00

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^7/x^14,x]

[Out] -1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7/(6*x^6)

Maple [A]
time = 0.07, size = 80, normalized size = 0.86

method	result	size
norman	$\frac{-\frac{1}{6}b^7x^7-a\,b^6x^6-\frac{21}{8}a^2b^5x^5-\frac{35}{9}a^3b^4x^4-\frac{7}{2}a^4b^3x^3-\frac{21}{11}a^5b^2x^2-\frac{7}{12}a^6bx-\frac{1}{13}a^7}{x^{13}}$	79
risch	$\frac{-\frac{1}{6}b^7x^7-a\,b^6x^6-\frac{21}{8}a^2b^5x^5-\frac{35}{9}a^3b^4x^4-\frac{7}{2}a^4b^3x^3-\frac{21}{11}a^5b^2x^2-\frac{7}{12}a^6bx-\frac{1}{13}a^7}{x^{13}}$	79
gosper	$-\frac{1716b^7x^7+10296a\,b^6x^6+27027a^2b^5x^5+40040a^3b^4x^4+36036a^4b^3x^3+19656a^5b^2x^2+6006a^6bx+792a^7}{10296x^{13}}$	80
default	$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{a\,b^6}{x^7}-\frac{b^7}{6x^6}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^7/x^14,x,method=_RETURNVERBOSE)

[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6

Maxima [A]
time = 0.27, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7+10296\,ab^6x^6+27027\,a^2b^5x^5+40040\,a^3b^4x^4+36036\,a^4b^3x^3+19656\,a^5b^2x^2+6006\,a^6bx+792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^14,x, algorithm="maxima")

[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^13

Fricas [A]
time = 1.16, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7+10296\,ab^6x^6+27027\,a^2b^5x^5+40040\,a^3b^4x^4+36036\,a^4b^3x^3+19656\,a^5b^2x^2+6006\,a^6bx+792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^7/x^14,x, algorithm="fricas")

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Sympy [A]
time = 0.31, size = 85, normalized size = 0.91

$$\frac{-792a^7 - 6006a^6bx - 19656a^5b^2x^2 - 36036a^4b^3x^3 - 40040a^3b^4x^4 - 27027a^2b^5x^5 - 10296ab^6x^6 - 1716b^7x^7}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**14,x)
```

```
[Out] (-792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3 -
40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*b**7
*x**7)/(10296*x**13)
```

Giac [A]
time = 1.40, size = 79, normalized size = 0.85

$$-\frac{1716\,b^7x^7 + 10296\,ab^6x^6 + 27027\,a^2b^5x^5 + 40040\,a^3b^4x^4 + 36036\,a^4b^3x^3 + 19656\,a^5b^2x^2 + 6006\,a^6bx + 792\,a^7}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

Mupad [B]
time = 0.07, size = 78, normalized size = 0.84

$$-\frac{\frac{a^7}{13} + \frac{7\,a^6\,b\,x}{12} + \frac{21\,a^5\,b^2\,x^2}{11} + \frac{7\,a^4\,b^3\,x^3}{2} + \frac{35\,a^3\,b^4\,x^4}{9} + \frac{21\,a^2\,b^5\,x^5}{8} + a\,b^6\,x^6 + \frac{b^7\,x^7}{6}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^14,x)
```

```
[Out] -(a^7/13 + (b^7*x^7)/6 + a*b^6*x^6 + (21*a^5*b^2*x^2)/11 + (7*a^4*b^3*x^3)/
2 + (35*a^3*b^4*x^4)/9 + (21*a^2*b^5*x^5)/8 + (7*a^6*b*x)/12)/x^13
```

3.121 $\int \frac{(a+bx)^7}{x^{15}}\,dx$

Optimal. Leaf size=95

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^15,x]
```

```
[Out] -1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{15}}\,dx &= \int \left(\frac{a^7}{x^{15}}+\frac{7a^6b}{x^{14}}+\frac{21a^5b^2}{x^{13}}+\frac{35a^4b^3}{x^{12}}+\frac{35a^3b^4}{x^{11}}+\frac{21a^2b^5}{x^{10}}+\frac{7ab^6}{x^9}+\frac{b^7}{x^8}\right)dx\\ &= -\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^15,x]
```

```
[Out] -1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{14}a^7-\frac{7}{13}a^6bx-\frac{7}{4}a^5b^2x^2-\frac{35}{11}a^4b^3x^3-\frac{7}{2}a^3b^4x^4-\frac{7}{3}a^2b^5x^5-\frac{7}{8}a\,b^6x^6-\frac{1}{7}b^7x^7}{x^{14}}$	79
risch	$\frac{-\frac{1}{14}a^7-\frac{7}{13}a^6bx-\frac{7}{4}a^5b^2x^2-\frac{35}{11}a^4b^3x^3-\frac{7}{2}a^3b^4x^4-\frac{7}{3}a^2b^5x^5-\frac{7}{8}a\,b^6x^6-\frac{1}{7}b^7x^7}{x^{14}}$	79
gosper	$-\frac{3432b^7x^7+21021a\,b^6x^6+56056a^2b^5x^5+84084a^3b^4x^4+76440a^4b^3x^3+42042a^5b^2x^2+12936a^6bx+1716a^7}{24024x^{14}}$	80
default	$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a\,b^6}{8x^8}-\frac{b^7}{7x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.83

$$-\frac{3432\,b^7x^7+21021\,ab^6x^6+56056\,a^2b^5x^5+84084\,a^3b^4x^4+76440\,a^4b^3x^3+42042\,a^5b^2x^2+12936\,a^6bx+1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Fricas [A]
time = 1.10, size = 79, normalized size = 0.83

$$-\frac{3432\,b^7x^7+21021\,ab^6x^6+56056\,a^2b^5x^5+84084\,a^3b^4x^4+76440\,a^4b^3x^3+42042\,a^5b^2x^2+12936\,a^6bx+1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Sympy [A]
time = 0.32, size = 85, normalized size = 0.89

$$\frac{-1716a^7 - 12936a^6bx - 42042a^5b^2x^2 - 76440a^4b^3x^3 - 84084a^3b^4x^4 - 56056a^2b^5x^5 - 21021ab^6x^6 - 3432b^7x^7}{24024x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**15,x)
```

```
[Out] (-1716*a**7 - 12936*a**6*b*x - 42042*a**5*b**2*x**2 - 76440*a**4*b**3*x**3
- 84084*a**3*b**4*x**4 - 56056*a**2*b**5*x**5 - 21021*a*b**6*x**6 - 3432*b*
*7*x**7)/(24024*x**14)
```

Giac [A]
time = 1.17, size = 79, normalized size = 0.83

$$-\frac{3432\,b^7x^7 + 21021\,ab^6x^6 + 56056\,a^2b^5x^5 + 84084\,a^3b^4x^4 + 76440\,a^4b^3x^3 + 42042\,a^5b^2x^2 + 12936\,a^6bx + 1716\,a^7}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

Mupad [B]
time = 0.07, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{14} + \frac{7\,a^6\,b\,x}{13} + \frac{7\,a^5\,b^2\,x^2}{4} + \frac{35\,a^4\,b^3\,x^3}{11} + \frac{7\,a^3\,b^4\,x^4}{2} + \frac{7\,a^2\,b^5\,x^5}{3} + \frac{7\,a\,b^6\,x^6}{8} + \frac{b^7\,x^7}{7}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^15,x)
```

```
[Out] -(a^7/14 + (b^7*x^7)/7 + (7*a*b^6*x^6)/8 + (7*a^5*b^2*x^2)/4 + (35*a^4*b^3*
x^3)/11 + (7*a^3*b^4*x^4)/2 + (7*a^2*b^5*x^5)/3 + (7*a^6*b*x)/13)/x^14
```

3.122 $\int \frac{(a+bx)^7}{x^{16}} dx$

Optimal. Leaf size=95

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^16,x]
```

```
[Out] -1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{16}} dx &= \int \left(\frac{a^7}{x^{16}}+\frac{7a^6b}{x^{15}}+\frac{21a^5b^2}{x^{14}}+\frac{35a^4b^3}{x^{13}}+\frac{35a^3b^4}{x^{12}}+\frac{21a^2b^5}{x^{11}}+\frac{7ab^6}{x^{10}}+\frac{b^7}{x^9}\right) dx\\ &= -\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^16,x]
```

```
[Out] -1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Maple [A]
time = 0.08, size = 80, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{15}a^7-\frac{1}{2}a^6bx-\frac{21}{13}a^5b^2x^2-\frac{35}{12}a^4b^3x^3-\frac{35}{11}a^3b^4x^4-\frac{21}{10}a^2b^5x^5-\frac{7}{9}a\,b^6x^6-\frac{1}{8}b^7x^7}{x^{15}}$	79
risch	$\frac{-\frac{1}{15}a^7-\frac{1}{2}a^6bx-\frac{21}{13}a^5b^2x^2-\frac{35}{12}a^4b^3x^3-\frac{35}{11}a^3b^4x^4-\frac{21}{10}a^2b^5x^5-\frac{7}{9}a\,b^6x^6-\frac{1}{8}b^7x^7}{x^{15}}$	79
gosper	$-\frac{6435b^7x^7+40040a\,b^6x^6+108108a^2b^5x^5+163800a^3b^4x^4+150150a^4b^3x^3+83160a^5b^2x^2+25740a^6bx+3432a^7}{51480x^{15}}$	80
default	$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7a\,b^6}{9x^9}-\frac{b^7}{8x^8}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Fricas [A]
time = 1.06, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Sympy [A]
time = 0.33, size = 85, normalized size = 0.89

$$\frac{-3432a^7 - 25740a^6bx - 83160a^5b^2x^2 - 150150a^4b^3x^3 - 163800a^3b^4x^4 - 108108a^2b^5x^5 - 40040ab^6x^6 - 6435b^7x^7}{51480x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**16,x)
```

```
[Out] (-3432*a**7 - 25740*a**6*b*x - 83160*a**5*b**2*x**2 - 150150*a**4*b**3*x**3
 - 163800*a**3*b**4*x**4 - 108108*a**2*b**5*x**5 - 40040*a*b**6*x**6 - 6435
*b**7*x**7)/(51480*x**15)
```

Giac [A]
time = 1.74, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7 + 40040\,ab^6x^6 + 108108\,a^2b^5x^5 + 163800\,a^3b^4x^4 + 150150\,a^4b^3x^3 + 83160\,a^5b^2x^2 + 25740\,a^6bx + 3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

Mupad [B]
time = 0.11, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{15} + \frac{a^6\,b\,x}{2} + \frac{21\,a^5\,b^2\,x^2}{13} + \frac{35\,a^4\,b^3\,x^3}{12} + \frac{35\,a^3\,b^4\,x^4}{11} + \frac{21\,a^2\,b^5\,x^5}{10} + \frac{7\,a\,b^6\,x^6}{9} + \frac{b^7\,x^7}{8}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^16,x)
```

```
[Out] -(a^7/15 + (b^7*x^7)/8 + (7*a*b^6*x^6)/9 + (21*a^5*b^2*x^2)/13 + (35*a^4*b^
3*x^3)/12 + (35*a^3*b^4*x^4)/11 + (21*a^2*b^5*x^5)/10 + (a^6*b*x)/2)/x^15
```

3.123 $\int x^{11}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab$$

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Rubi [A]
time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{11}(a+bx)^{10}\,dx &= \int \left(a^{10}x^{11}+10a^9bx^{12}+45a^8b^2x^{13}+120a^7b^3x^{14}+210a^6b^4x^{15}+252a^5b^5x^{16}+210a^4b^6x\right.\\ &= \frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$
default	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$
norman	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$
risch	$\frac{1}{12}a^{10}x^{12}+\frac{10}{13}a^{9}b\,x^{13}+\frac{45}{14}a^{8}b^{2}x^{14}+8a^{7}b^{3}x^{15}+\frac{105}{8}a^{6}b^{4}x^{16}+\frac{252}{17}a^{5}b^{5}x^{17}+\frac{35}{3}a^{4}b^{6}x^{18}+\frac{120}{19}a^{3}b^{7}x^{19}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

Maxima [A]
time = 0.28, size = 112, normalized size = 0.85

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{14}+\frac{10}{13}\,a^{9}bx^{13}+\frac{1}{12}\,a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Fricas [A]
time = 1.26, size = 112, normalized size = 0.85

$$\frac{1}{22}\,b^{10}x^{22}+\frac{10}{21}\,ab^{9}x^{21}+\frac{9}{4}\,a^{2}b^{8}x^{20}+\frac{120}{19}\,a^{3}b^{7}x^{19}+\frac{35}{3}\,a^{4}b^{6}x^{18}+\frac{252}{17}\,a^{5}b^{5}x^{17}+\frac{105}{8}\,a^{6}b^{4}x^{16}+8\,a^{7}b^{3}x^{15}+\frac{45}{14}\,a^{8}b^{2}x^{14}+\frac{10}{13}\,a^{9}bx^{13}+\frac{1}{12}\,a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Sympy [A]
time = 0.01, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{12}}{12} + \frac{10a^9bx^{13}}{13} + \frac{45a^8b^2x^{14}}{14} + 8a^7b^3x^{15} + \frac{105a^6b^4x^{16}}{8} + \frac{252a^5b^5x^{17}}{17} + \frac{35a^4b^6x^{18}}{3} + \frac{120a^3b^7x^{19}}{19} + \frac{9a^2b^8x^{20}}{4} + \frac{10ab^9x^{21}}{21} + \frac{b^{10}x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x+a)**10,x)
```

```
[Out] a**10*x**12/12 + 10*a**9*b*x**13/13 + 45*a**8*b**2*x**14/14 + 8*a**7*b**3*x
**15 + 105*a**6*b**4*x**16/8 + 252*a**5*b**5*x**17/17 + 35*a**4*b**6*x**18/
3 + 120*a**3*b**7*x**19/19 + 9*a**2*b**8*x**20/4 + 10*a*b**9*x**21/21 + b**
10*x**22/22
```

Giac [A]
time = 1.36, size = 112, normalized size = 0.85

$$\frac{1}{22}\,b^{10}x^{22} + \frac{10}{21}\,ab^9x^{21} + \frac{9}{4}\,a^2b^8x^{20} + \frac{120}{19}\,a^3b^7x^{19} + \frac{35}{3}\,a^4b^6x^{18} + \frac{252}{17}\,a^5b^5x^{17} + \frac{105}{8}\,a^6b^4x^{16} + 8\,a^7b^3x^{15} + \frac{45}{14}\,a^8b^2x^{14} + \frac{10}{13}\,a^9bx^{13} + \frac{1}{12}\,a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

Mupad [B]
time = 0.15, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{12}}{12} + \frac{10\,a^9\,b\,x^{13}}{13} + \frac{45\,a^8\,b^2\,x^{14}}{14} + 8\,a^7\,b^3\,x^{15} + \frac{105\,a^6\,b^4\,x^{16}}{8} + \frac{252\,a^5\,b^5\,x^{17}}{17} + \frac{35\,a^4\,b^6\,x^{18}}{3} + \frac{120\,a^3\,b^7\,x^{19}}{19} + \frac{9\,a^2\,b^8\,x^{20}}{4} + \frac{10\,a\,b^9\,x^{21}}{21} + \frac{b^{10}\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x)^10,x)
```

```
[Out] (a^10*x^12)/12 + (b^10*x^22)/22 + (10*a^9*b*x^13)/13 + (10*a*b^9*x^21)/21 +
 (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^
5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)
/4
```

3.124 $\int x^{10}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9$$

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{10}(a+bx)^{10}\,dx &= \int \left(a^{10}x^{10}+10a^9bx^{11}+45a^8b^2x^{12}+120a^7b^3x^{13}+210a^6b^4x^{14}+252a^5b^5x^{15}+210a^4b\right.\\ &= \frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Maple [A]
time = 0.07, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}b^{7}x^{18}+$
default	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}b^{7}x^{18}+$
norman	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}b^{7}x^{18}+$
risch	$\frac{1}{11}a^{10}x^{11}+\frac{5}{6}a^{9}b\,x^{12}+\frac{45}{13}a^{8}b^{2}x^{13}+\frac{60}{7}a^{7}b^{3}x^{14}+14a^{6}b^{4}x^{15}+\frac{63}{4}a^{5}b^{5}x^{16}+\frac{210}{17}a^{4}b^{6}x^{17}+\frac{20}{3}a^{3}b^{7}x^{18}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

Maxima [A]
time = 0.28, size = 112, normalized size = 0.85

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}+\frac{5}{6}\,a^{9}bx^{12}+\frac{1}{11}\,a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Fricas [A]
time = 1.02, size = 112, normalized size = 0.85

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^{9}x^{20}+\frac{45}{19}\,a^{2}b^{8}x^{19}+\frac{20}{3}\,a^{3}b^{7}x^{18}+\frac{210}{17}\,a^{4}b^{6}x^{17}+\frac{63}{4}\,a^{5}b^{5}x^{16}+14\,a^{6}b^{4}x^{15}+\frac{60}{7}\,a^{7}b^{3}x^{14}+\frac{45}{13}\,a^{8}b^{2}x^{13}+\frac{5}{6}\,a^{9}bx^{12}+\frac{1}{11}\,a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Sympy [A]
time = 0.01, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{11}}{11}+\frac{5a^9bx^{12}}{6}+\frac{45a^8b^2x^{13}}{13}+\frac{60a^7b^3x^{14}}{7}+14a^6b^4x^{15}+\frac{63a^5b^5x^{16}}{4}+\frac{210a^4b^6x^{17}}{17}+\frac{20a^3b^7x^{18}}{3}+\frac{45a^2b^8x^{19}}{19}+\frac{ab^9x^{20}}{2}+\frac{b^{10}x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b*x+a)**10,x)
```

```
[Out] a**10*x**11/11 + 5*a**9*b*x**12/6 + 45*a**8*b**2*x**13/13 + 60*a**7*b**3*x*
*14/7 + 14*a**6*b**4*x**15 + 63*a**5*b**5*x**16/4 + 210*a**4*b**6*x**17/17
+ 20*a**3*b**7*x**18/3 + 45*a**2*b**8*x**19/19 + a*b**9*x**20/2 + b**10*x**
21/21
```

Giac [A]
time = 1.66, size = 112, normalized size = 0.85

$$\frac{1}{21}\,b^{10}x^{21}+\frac{1}{2}\,ab^9x^{20}+\frac{45}{19}\,a^2b^8x^{19}+\frac{20}{3}\,a^3b^7x^{18}+\frac{210}{17}\,a^4b^6x^{17}+\frac{63}{4}\,a^5b^5x^{16}+14\,a^6b^4x^{15}+\frac{60}{7}\,a^7b^3x^{14}+\frac{45}{13}\,a^8b^2x^{13}+\frac{5}{6}\,a^9bx^{12}+\frac{1}{11}\,a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

Mupad [B]
time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{11}}{11}+\frac{5\,a^9\,b\,x^{12}}{6}+\frac{45\,a^8\,b^2\,x^{13}}{13}+\frac{60\,a^7\,b^3\,x^{14}}{7}+14\,a^6\,b^4\,x^{15}+\frac{63\,a^5\,b^5\,x^{16}}{4}+\frac{210\,a^4\,b^6\,x^{17}}{17}+\frac{20\,a^3\,b^7\,x^{18}}{3}+\frac{45\,a^2\,b^8\,x^{19}}{19}+\frac{a\,b^9\,x^{20}}{2}+\frac{b^{10}\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b*x)^10,x)
```

```
[Out] (a^10*x^11)/11 + (b^10*x^21)/21 + (5*a^9*b*x^12)/6 + (a*b^9*x^20)/2 + (45*a
^8*b^2*x^13)/13 + (60*a^7*b^3*x^14)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)
/4 + (210*a^4*b^6*x^17)/17 + (20*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19
```

3.125 $\int x^9(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}a$$

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^9(a+bx)^{10}\,dx &= \int \left(a^{10}x^9+10a^9bx^{10}+45a^8b^2x^{11}+120a^7b^3x^{12}+210a^6b^4x^{13}+252a^5b^5x^{14}+210a^4b^6x^1\right.\\ &= \frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$
default	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$
norman	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$
risch	$\frac{1}{10}a^{10}x^{10}+\frac{10}{11}a^9b\,x^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

Maxima [A]
time = 0.28, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20}+\frac{10}{19}\,ab^9x^{19}+\frac{5}{2}\,a^2b^8x^{18}+\frac{120}{17}\,a^3b^7x^{17}+\frac{105}{8}\,a^4b^6x^{16}+\frac{84}{5}\,a^5b^5x^{15}+15\,a^6b^4x^{14}+\frac{120}{13}\,a^7b^3x^{13}+\frac{15}{4}\,a^8b^2x^{12}+\frac{10}{11}\,a^9bx^{11}+\frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Fricas [A]
time = 1.23, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20}+\frac{10}{19}\,ab^9x^{19}+\frac{5}{2}\,a^2b^8x^{18}+\frac{120}{17}\,a^3b^7x^{17}+\frac{105}{8}\,a^4b^6x^{16}+\frac{84}{5}\,a^5b^5x^{15}+15\,a^6b^4x^{14}+\frac{120}{13}\,a^7b^3x^{13}+\frac{15}{4}\,a^8b^2x^{12}+\frac{10}{11}\,a^9bx^{11}+\frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Sympy [A]
time = 0.01, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{10}}{10}+\frac{10a^9bx^{11}}{11}+\frac{15a^8b^2x^{12}}{4}+\frac{120a^7b^3x^{13}}{13}+15a^6b^4x^{14}+\frac{84a^5b^5x^{15}}{5}+\frac{105a^4b^6x^{16}}{8}+\frac{120a^3b^7x^{17}}{17}+\frac{5a^2b^8x^{18}}{2}+\frac{10ab^9x^{19}}{19}+\frac{b^{10}x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x+a)**10,x)
```

```
[Out] a**10*x**10/10 + 10*a**9*b*x**11/11 + 15*a**8*b**2*x**12/4 + 120*a**7*b**3*
x**13/13 + 15*a**6*b**4*x**14 + 84*a**5*b**5*x**15/5 + 105*a**4*b**6*x**16/
8 + 120*a**3*b**7*x**17/17 + 5*a**2*b**8*x**18/2 + 10*a*b**9*x**19/19 + b**
10*x**20/20
```

Giac [A]
time = 1.62, size = 112, normalized size = 0.85

$$\frac{1}{20}b^{10}x^{20}+\frac{10}{19}ab^9x^{19}+\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{1}{10}a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

Mupad [B]
time = 0.12, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{10}}{10}+\frac{10\,a^9\,b\,x^{11}}{11}+\frac{15\,a^8\,b^2\,x^{12}}{4}+\frac{120\,a^7\,b^3\,x^{13}}{13}+15\,a^6\,b^4\,x^{14}+\frac{84\,a^5\,b^5\,x^{15}}{5}+\frac{105\,a^4\,b^6\,x^{16}}{8}+\frac{120\,a^3\,b^7\,x^{17}}{17}+\frac{5\,a^2\,b^8\,x^{18}}{2}+\frac{10\,a\,b^9\,x^{19}}{19}+\frac{b^{10}\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x)^10,x)
```

```
[Out] (a^10*x^10)/10 + (b^10*x^20)/20 + (10*a^9*b*x^11)/11 + (10*a*b^9*x^19)/19 +
 (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^
5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)
/2
```

3.126 $\int x^8(a+bx)^{10}\,dx$

Optimal. Leaf size=147

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}$$

```
[Out] 1/11*a^8*(b*x+a)^11/b^9-2/3*a^7*(b*x+a)^12/b^9+28/13*a^6*(b*x+a)^13/b^9-4*a
^5*(b*x+a)^14/b^9+14/3*a^4*(b*x+a)^15/b^9-7/2*a^3*(b*x+a)^16/b^9+28/17*a^2*
(b*x+a)^17/b^9-4/9*a*(b*x+a)^18/b^9+1/19*(b*x+a)^19/b^9
```

Rubi [A]
time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}+\frac{(a+bx)^{19}}{19b^9}-\frac{4a(a+bx)^{18}}{9b^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^10,x]
```

```
[Out] (a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a + b
*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^9)
 - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*(a
+ b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^{10}\,dx &= \int\left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+bx)}{b^8}\right.\\ &= \frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\end{aligned}$$

Mathematica [A]
time = 0.00, size = 125, normalized size = 0.85

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}a^2b^8x^{17}+\frac{5}{9}ab^9x^{18}+\frac{b^{10}x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^10,x]
```

```
[Out] (a^10*x^9)/9 + a^9*b*x^10 + (45*a^8*b^2*x^11)/11 + 10*a^7*b^3*x^12 + (210*a
^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 +
 (45*a^2*b^8*x^17)/17 + (5*a*b^9*x^18)/9 + (b^10*x^19)/19
```

Maple [A]
time = 0.09, size = 112, normalized size = 0.76

method	result
gosper	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}$
default	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}$
norman	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}$
risch	$\frac{1}{9}a^{10}x^9+a^9b\,x^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^10*x^9+a^9*b*x^10+45/11*a^8*b^2*x^11+10*a^7*b^3*x^12+210/13*a^6*b^4*x
^13+18*a^5*b^5*x^14+14*a^4*b^6*x^15+15/2*a^3*b^7*x^16+45/17*a^2*b^8*x^17+5/
9*a*b^9*x^18+1/19*b^10*x^19
```

Maxima [A]
time = 0.27, size = 111, normalized size = 0.76

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^9x^{18}+\frac{45}{17}\,a^2b^8x^{17}+\frac{15}{2}\,a^3b^7x^{16}+14\,a^4b^6x^{15}+18\,a^5b^5x^{14}+\frac{210}{13}\,a^6b^4x^{13}+10\,a^7b^3x^{12}+\frac{45}{11}\,a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}\,a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Fricas [A]
time = 1.54, size = 111, normalized size = 0.76

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^9x^{18}+\frac{45}{17}\,a^2b^8x^{17}+\frac{15}{2}\,a^3b^7x^{16}+14\,a^4b^6x^{15}+18\,a^5b^5x^{14}+\frac{210}{13}\,a^6b^4x^{13}+10\,a^7b^3x^{12}+\frac{45}{11}\,a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}\,a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Sympy [A]
time = 0.02, size = 126, normalized size = 0.86

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45a^8b^2x^{11}}{11}+10a^7b^3x^{12}+\frac{210a^6b^4x^{13}}{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15a^3b^7x^{16}}{2}+\frac{45a^2b^8x^{17}}{17}+\frac{5ab^9x^{18}}{9}+\frac{b^{10}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**10,x)
```

```
[Out] a**10*x**9/9 + a**9*b*x**10 + 45*a**8*b**2*x**11/11 + 10*a**7*b**3*x**12 +
210*a**6*b**4*x**13/13 + 18*a**5*b**5*x**14 + 14*a**4*b**6*x**15 + 15*a**3*
b**7*x**16/2 + 45*a**2*b**8*x**17/17 + 5*a*b**9*x**18/9 + b**10*x**19/19
```

Giac [A]
time = 1.67, size = 111, normalized size = 0.76

$$\frac{1}{19}\,b^{10}x^{19}+\frac{5}{9}\,ab^9x^{18}+\frac{45}{17}\,a^2b^8x^{17}+\frac{15}{2}\,a^3b^7x^{16}+14\,a^4b^6x^{15}+18\,a^5b^5x^{14}+\frac{210}{13}\,a^6b^4x^{13}+10\,a^7b^3x^{12}+\frac{45}{11}\,a^8b^2x^{11}+a^9bx^{10}+\frac{1}{9}\,a^{10}x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

Mupad [B]
time = 0.09, size = 111, normalized size = 0.76

$$\frac{a^{10}\,x^9}{9}+a^9\,b\,x^{10}+\frac{45\,a^8\,b^2\,x^{11}}{11}+10\,a^7\,b^3\,x^{12}+\frac{210\,a^6\,b^4\,x^{13}}{13}+18\,a^5\,b^5\,x^{14}+14\,a^4\,b^6\,x^{15}+\frac{15\,a^3\,b^7\,x^{16}}{2}+\frac{45\,a^2\,b^8\,x^{17}}{17}+\frac{5\,a\,b^9\,x^{18}}{9}+\frac{b^{10}\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^10,x)
```

```
[Out] (a^10*x^9)/9 + (b^10*x^19)/19 + a^9*b*x^10 + (5*a*b^9*x^18)/9 + (45*a^8*b^2
*x^11)/11 + 10*a^7*b^3*x^12 + (210*a^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*
a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 + (45*a^2*b^8*x^17)/17
```

3.127 $\int x^7(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a(a+bx)}{17b^8}$$

```
[Out] -1/11*a^7*(b*x+a)^11/b^8+7/12*a^6*(b*x+a)^12/b^8-21/13*a^5*(b*x+a)^13/b^8+5
/2*a^4*(b*x+a)^14/b^8-7/3*a^3*(b*x+a)^15/b^8+21/16*a^2*(b*x+a)^16/b^8-7/17*
a*(b*x+a)^17/b^8+1/18*(b*x+a)^18/b^8
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^10,x]
```

```
[Out] -1/11*(a^7*(a + b*x)^11)/b^8 + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a +
 b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/(3
*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (a +
 b*x)^18/(18*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^{10}\,dx &= \int \left(-\frac{a^7(a+bx)^{10}}{b^7}+\frac{7a^6(a+bx)^{11}}{b^7}-\frac{21a^5(a+bx)^{12}}{b^7}+\frac{35a^4(a+bx)^{13}}{b^7}-\frac{35a^3(a+bx}{b^7}\right.\\ &= -\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 130, normalized size = 0.98

$$\frac{a^{10}x^8}{8}+\frac{10}{9}a^9bx^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}a^2b^8x^{16}+\frac{10}{17}ab^9x^{17}+\frac{b^{10}x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^10,x]
```

```
[Out] (a^10*x^8)/8 + (10*a^9*b*x^9)/9 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/1
1 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b
^7*x^15 + (45*a^2*b^8*x^16)/16 + (10*a*b^9*x^17)/17 + (b^10*x^18)/18
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.86

method	result
gosper	$\frac{1}{8}a^{10}x^8 + \frac{10}{9}a^9b\,x^9 + \frac{9}{2}a^8b^2x^{10} + \frac{120}{11}a^7b^3x^{11} + \frac{35}{2}a^6b^4x^{12} + \frac{252}{13}a^5b^5x^{13} + 15a^4b^6x^{14} + 8a^3b^7x^{15} +$
default	$\frac{1}{8}a^{10}x^8 + \frac{10}{9}a^9b\,x^9 + \frac{9}{2}a^8b^2x^{10} + \frac{120}{11}a^7b^3x^{11} + \frac{35}{2}a^6b^4x^{12} + \frac{252}{13}a^5b^5x^{13} + 15a^4b^6x^{14} + 8a^3b^7x^{15} +$
norman	$\frac{1}{8}a^{10}x^8 + \frac{10}{9}a^9b\,x^9 + \frac{9}{2}a^8b^2x^{10} + \frac{120}{11}a^7b^3x^{11} + \frac{35}{2}a^6b^4x^{12} + \frac{252}{13}a^5b^5x^{13} + 15a^4b^6x^{14} + 8a^3b^7x^{15} +$
risch	$\frac{1}{8}a^{10}x^8 + \frac{10}{9}a^9b\,x^9 + \frac{9}{2}a^8b^2x^{10} + \frac{120}{11}a^7b^3x^{11} + \frac{35}{2}a^6b^4x^{12} + \frac{252}{13}a^5b^5x^{13} + 15a^4b^6x^{14} + 8a^3b^7x^{15} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a^10*x^8+10/9*a^9*b*x^9+9/2*a^8*b^2*x^10+120/11*a^7*b^3*x^11+35/2*a^6*b
^4*x^12+252/13*a^5*b^5*x^13+15*a^4*b^6*x^14+8*a^3*b^7*x^15+45/16*a^2*b^8*x^
16+10/17*a*b^9*x^17+1/18*b^10*x^18
```

Maxima [A]
time = 0.28, size = 112, normalized size = 0.85

$$\frac{1}{18}\,b^{10}x^{18} + \frac{10}{17}\,ab^9x^{17} + \frac{45}{16}\,a^2b^8x^{16} + 8\,a^3b^7x^{15} + 15\,a^4b^6x^{14} + \frac{252}{13}\,a^5b^5x^{13} + \frac{35}{2}\,a^6b^4x^{12} + \frac{120}{11}\,a^7b^3x^{11} + \frac{9}{2}\,a^8b^2x^{10} + \frac{10}{9}\,a^9bx^9 + \frac{1}{8}\,a^{10}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

Fricas [A]
time = 0.83, size = 112, normalized size = 0.85

$$\frac{1}{18}\,b^{10}x^{18} + \frac{10}{17}\,ab^9x^{17} + \frac{45}{16}\,a^2b^8x^{16} + 8\,a^3b^7x^{15} + 15\,a^4b^6x^{14} + \frac{252}{13}\,a^5b^5x^{13} + \frac{35}{2}\,a^6b^4x^{12} + \frac{120}{11}\,a^7b^3x^{11} + \frac{9}{2}\,a^8b^2x^{10} + \frac{10}{9}\,a^9bx^9 + \frac{1}{8}\,a^{10}x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="fricas")
```

[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 15*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8

Sympy [A]
time = 0.02, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{8}}{8}+\frac{10a^{9}bx^{9}}{9}+\frac{9a^{8}b^{2}x^{10}}{2}+\frac{120a^{7}b^{3}x^{11}}{11}+\frac{35a^{6}b^{4}x^{12}}{2}+\frac{252a^{5}b^{5}x^{13}}{13}+15a^{4}b^{6}x^{14}+8a^{3}b^{7}x^{15}+\frac{45a^{2}b^{8}x^{16}}{16}+\frac{10ab^{9}x^{17}}{17}+\frac{b^{10}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(b*x+a)**10,x)

[Out] a**10*x**8/8 + 10*a**9*b*x**9/9 + 9*a**8*b**2*x**10/2 + 120*a**7*b**3*x**11/11 + 35*a**6*b**4*x**12/2 + 252*a**5*b**5*x**13/13 + 15*a**4*b**6*x**14 + 8*a**3*b**7*x**15 + 45*a**2*b**8*x**16/16 + 10*a*b**9*x**17/17 + b**10*x**18/18

Giac [A]
time = 1.13, size = 112, normalized size = 0.85

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}ab^{9}x^{17}+\frac{45}{16}a^{2}b^{8}x^{16}+8a^{3}b^{7}x^{15}+15a^{4}b^{6}x^{14}+\frac{252}{13}a^{5}b^{5}x^{13}+\frac{35}{2}a^{6}b^{4}x^{12}+\frac{120}{11}a^{7}b^{3}x^{11}+\frac{9}{2}a^{8}b^{2}x^{10}+\frac{10}{9}a^{9}bx^{9}+\frac{1}{8}a^{10}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^10,x, algorithm="giac")

[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 15*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8

Mupad [B]
time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{8}}{8}+\frac{10\,a^{9}\,b\,x^{9}}{9}+\frac{9\,a^{8}\,b^{2}\,x^{10}}{2}+\frac{120\,a^{7}\,b^{3}\,x^{11}}{11}+\frac{35\,a^{6}\,b^{4}\,x^{12}}{2}+\frac{252\,a^{5}\,b^{5}\,x^{13}}{13}+15\,a^{4}\,b^{6}\,x^{14}+8\,a^{3}\,b^{7}\,x^{15}+\frac{45\,a^{2}\,b^{8}\,x^{16}}{16}+\frac{10\,a\,b^{9}\,x^{17}}{17}+\frac{b^{10}\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(a + b*x)^10,x)

[Out] (a^10*x^8)/8 + (b^10*x^18)/18 + (10*a^9*b*x^9)/9 + (10*a*b^9*x^17)/17 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/11 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b^7*x^15 + (45*a^2*b^8*x^16)/16

3.128 $\int x^6(a+bx)^{10}\,dx$

Optimal. Leaf size=112

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a(a+bx)^{16}}{8b^7}+\frac{(a+bx)^{17}}{17b^7}$$

```
[Out] 1/11*a^6*(b*x+a)^11/b^7-1/2*a^5*(b*x+a)^12/b^7+15/13*a^4*(b*x+a)^13/b^7-10/
7*a^3*(b*x+a)^14/b^7+a^2*(b*x+a)^15/b^7-3/8*a*(b*x+a)^16/b^7+1/17*(b*x+a)^1
7/b^7
```

Rubi [A]
time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^10,x]
```

```
[Out] (a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*x
)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 - (
3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^6(a+bx)^{10}\,dx=\int\left(\frac{a^6(a+bx)^{10}}{b^6}-\frac{6a^5(a+bx)^{11}}{b^6}+\frac{15a^4(a+bx)^{12}}{b^6}-\frac{20a^3(a+bx)^{13}}{b^6}+\frac{15a^2(a+bx)}{b^6}\right.$$

$$=\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a}{}$$

Mathematica [A]
time = 0.00, size = 126, normalized size = 1.12

$$\frac{a^{10}x^7}{7}+\frac{5}{4}a^9bx^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2b^8x^{15}+\frac{5}{8}ab^9x^{16}+\frac{b^{10}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^10,x]
```

```
[Out] (a^10*x^7)/7 + (5*a^9*b*x^8)/4 + 5*a^8*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6
*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)
/7 + 3*a^2*b^8*x^15 + (5*a*b^9*x^16)/8 + (b^10*x^17)/17
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.01

method	result
gosper	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a$
default	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a$
norman	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a$
risch	$\frac{1}{7}a^{10}x^7+\frac{5}{4}a^9b\,x^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^10*x^7+5/4*a^9*b*x^8+5*a^8*b^2*x^9+12*a^7*b^3*x^10+210/11*a^6*b^4*x^1
1+21*a^5*b^5*x^12+210/13*a^4*b^6*x^13+60/7*a^3*b^7*x^14+3*a^2*b^8*x^15+5/8*
a*b^9*x^16+1/17*b^10*x^17
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.00

$$\frac{1}{17}\,b^{10}x^{17}+\frac{5}{8}\,ab^9x^{16}+3\,a^2b^8x^{15}+\frac{60}{7}\,a^3b^7x^{14}+\frac{210}{13}\,a^4b^6x^{13}+21\,a^5b^5x^{12}+\frac{210}{11}\,a^6b^4x^{11}+12\,a^7b^3x^{10}+5\,a^8b^2x^9+\frac{5}{4}\,a^9bx^8+\frac{1}{7}\,a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Fricas [A]
time = 0.96, size = 112, normalized size = 1.00

$$\frac{1}{17}\,b^{10}x^{17}+\frac{5}{8}\,ab^9x^{16}+3\,a^2b^8x^{15}+\frac{60}{7}\,a^3b^7x^{14}+\frac{210}{13}\,a^4b^6x^{13}+21\,a^5b^5x^{12}+\frac{210}{11}\,a^6b^4x^{11}+12\,a^7b^3x^{10}+5\,a^8b^2x^9+\frac{5}{4}\,a^9bx^8+\frac{1}{7}\,a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Sympy [A]
time = 0.02, size = 128, normalized size = 1.14

$$\frac{a^{10}x^7}{7}+\frac{5a^9bx^8}{4}+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210a^6b^4x^{11}}{11}+21a^5b^5x^{12}+\frac{210a^4b^6x^{13}}{13}+\frac{60a^3b^7x^{14}}{7}+3a^2b^8x^{15}+\frac{5ab^9x^{16}}{8}+\frac{b^{10}x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**10,x)
```

```
[Out] a**10*x**7/7 + 5*a**9*b*x**8/4 + 5*a**8*b**2*x**9 + 12*a**7*b**3*x**10 + 21
0*a**6*b**4*x**11/11 + 21*a**5*b**5*x**12 + 210*a**4*b**6*x**13/13 + 60*a**
3*b**7*x**14/7 + 3*a**2*b**8*x**15 + 5*a*b**9*x**16/8 + b**10*x**17/17
```

Giac [A]
time = 1.51, size = 112, normalized size = 1.00

$$\frac{1}{17}b^{10}x^{17}+\frac{5}{8}ab^9x^{16}+3a^2b^8x^{15}+\frac{60}{7}a^3b^7x^{14}+\frac{210}{13}a^4b^6x^{13}+21a^5b^5x^{12}+\frac{210}{11}a^6b^4x^{11}+12a^7b^3x^{10}+5a^8b^2x^9+\frac{5}{4}a^9bx^8+\frac{1}{7}a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.00

$$\frac{a^{10}\,x^7}{7}+\frac{5\,a^9\,b\,x^8}{4}+5\,a^8\,b^2\,x^9+12\,a^7\,b^3\,x^{10}+\frac{210\,a^6\,b^4\,x^{11}}{11}+21\,a^5\,b^5\,x^{12}+\frac{210\,a^4\,b^6\,x^{13}}{13}+\frac{60\,a^3\,b^7\,x^{14}}{7}+3\,a^2\,b^8\,x^{15}+\frac{5\,a\,b^9\,x^{16}}{8}+\frac{b^{10}\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^10,x)
```

```
[Out] (a^10*x^7)/7 + (b^10*x^17)/17 + (5*a^9*b*x^8)/4 + (5*a*b^9*x^16)/8 + 5*a^8*
b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*
a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)/7 + 3*a^2*b^8*x^15
```

3.129 $\int x^5(a+bx)^{10}\,dx$

Optimal. Leaf size=98

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\frac{(a+bx)^{16}}{16b^6}$$

```
[Out] -1/11*a^5*(b*x+a)^11/b^6+5/12*a^4*(b*x+a)^12/b^6-10/13*a^3*(b*x+a)^13/b^6+5
/7*a^2*(b*x+a)^14/b^6-1/3*a*(b*x+a)^15/b^6+1/16*(b*x+a)^16/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^10,x]
```

```
[Out] -1/11*(a^5*(a + b*x)^11)/b^6 + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a +
 b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b^6
) + (a + b*x)^16/(16*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{10}\,dx &= \int \left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a+bx)^{14}}{b^5}\right.\\ &= -\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.35

$$\frac{a^{10}x^6}{6}+\frac{10}{7}a^9bx^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+\frac{45}{14}a^2b^8x^{14}+\frac{2}{3}ab^9x^{15}+\frac{b^{10}x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^10,x]
```

```
[Out] (a^10*x^6)/6 + (10*a^9*b*x^7)/7 + (45*a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 +
 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/11 + (35*a^4*b^6*x^12)/2 + (120*a^3*b
^7*x^13)/13 + (45*a^2*b^8*x^14)/14 + (2*a*b^9*x^15)/3 + (b^10*x^16)/16
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.15

method	result
gosper	$\frac{1}{6}a^{10}x^{6}+\frac{10}{7}a^{9}b\,x^{7}+\frac{45}{8}a^{8}b^{2}x^{8}+\frac{40}{3}a^{7}b^{3}x^{9}+21a^{6}b^{4}x^{10}+\frac{252}{11}a^{5}b^{5}x^{11}+\frac{35}{2}a^{4}b^{6}x^{12}+\frac{120}{13}a^{3}b^{7}x^{13}+$
default	$\frac{1}{6}a^{10}x^{6}+\frac{10}{7}a^{9}b\,x^{7}+\frac{45}{8}a^{8}b^{2}x^{8}+\frac{40}{3}a^{7}b^{3}x^{9}+21a^{6}b^{4}x^{10}+\frac{252}{11}a^{5}b^{5}x^{11}+\frac{35}{2}a^{4}b^{6}x^{12}+\frac{120}{13}a^{3}b^{7}x^{13}+$
norman	$\frac{1}{6}a^{10}x^{6}+\frac{10}{7}a^{9}b\,x^{7}+\frac{45}{8}a^{8}b^{2}x^{8}+\frac{40}{3}a^{7}b^{3}x^{9}+21a^{6}b^{4}x^{10}+\frac{252}{11}a^{5}b^{5}x^{11}+\frac{35}{2}a^{4}b^{6}x^{12}+\frac{120}{13}a^{3}b^{7}x^{13}+$
risch	$\frac{1}{6}a^{10}x^{6}+\frac{10}{7}a^{9}b\,x^{7}+\frac{45}{8}a^{8}b^{2}x^{8}+\frac{40}{3}a^{7}b^{3}x^{9}+21a^{6}b^{4}x^{10}+\frac{252}{11}a^{5}b^{5}x^{11}+\frac{35}{2}a^{4}b^{6}x^{12}+\frac{120}{13}a^{3}b^{7}x^{13}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^10*x^6+10/7*a^9*b*x^7+45/8*a^8*b^2*x^8+40/3*a^7*b^3*x^9+21*a^6*b^4*x^
10+252/11*a^5*b^5*x^11+35/2*a^4*b^6*x^12+120/13*a^3*b^7*x^13+45/14*a^2*b^8*
x^14+2/3*a*b^9*x^15+1/16*b^10*x^16
```

Maxima [A]
time = 0.29, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^{9}x^{15}+\frac{45}{14}\,a^{2}b^{8}x^{14}+\frac{120}{13}\,a^{3}b^{7}x^{13}+\frac{35}{2}\,a^{4}b^{6}x^{12}+\frac{252}{11}\,a^{5}b^{5}x^{11}+21\,a^{6}b^{4}x^{10}+\frac{40}{3}\,a^{7}b^{3}x^{9}+\frac{45}{8}\,a^{8}b^{2}x^{8}+\frac{10}{7}\,a^{9}bx^{7}+\frac{1}{6}\,a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Fricas [A]
time = 0.91, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^{9}x^{15}+\frac{45}{14}\,a^{2}b^{8}x^{14}+\frac{120}{13}\,a^{3}b^{7}x^{13}+\frac{35}{2}\,a^{4}b^{6}x^{12}+\frac{252}{11}\,a^{5}b^{5}x^{11}+21\,a^{6}b^{4}x^{10}+\frac{40}{3}\,a^{7}b^{3}x^{9}+\frac{45}{8}\,a^{8}b^{2}x^{8}+\frac{10}{7}\,a^{9}bx^{7}+\frac{1}{6}\,a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Sympy [A]
time = 0.02, size = 133, normalized size = 1.36

$$\frac{a^{10}x^6}{6}+\frac{10a^9bx^7}{7}+\frac{45a^8b^2x^8}{8}+\frac{40a^7b^3x^9}{3}+21a^6b^4x^{10}+\frac{252a^5b^5x^{11}}{11}+\frac{35a^4b^6x^{12}}{2}+\frac{120a^3b^7x^{13}}{13}+\frac{45a^2b^8x^{14}}{14}+\frac{2ab^9x^{15}}{3}+\frac{b^{10}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**10,x)
```

```
[Out] a**10*x**6/6 + 10*a**9*b*x**7/7 + 45*a**8*b**2*x**8/8 + 40*a**7*b**3*x**9/3
 + 21*a**6*b**4*x**10 + 252*a**5*b**5*x**11/11 + 35*a**4*b**6*x**12/2 + 120
*a**3*b**7*x**13/13 + 45*a**2*b**8*x**14/14 + 2*a*b**9*x**15/3 + b**10*x**1
6/16
```

Giac [A]
time = 2.22, size = 112, normalized size = 1.14

$$\frac{1}{16}\,b^{10}x^{16}+\frac{2}{3}\,ab^9x^{15}+\frac{45}{14}\,a^2b^8x^{14}+\frac{120}{13}\,a^3b^7x^{13}+\frac{35}{2}\,a^4b^6x^{12}+\frac{252}{11}\,a^5b^5x^{11}+21\,a^6b^4x^{10}+\frac{40}{3}\,a^7b^3x^9+\frac{45}{8}\,a^8b^2x^8+\frac{10}{7}\,a^9bx^7+\frac{1}{6}\,a^{10}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.14

$$\frac{a^{10}\,x^6}{6}+\frac{10\,a^9\,b\,x^7}{7}+\frac{45\,a^8\,b^2\,x^8}{8}+\frac{40\,a^7\,b^3\,x^9}{3}+21\,a^6\,b^4\,x^{10}+\frac{252\,a^5\,b^5\,x^{11}}{11}+\frac{35\,a^4\,b^6\,x^{12}}{2}+\frac{120\,a^3\,b^7\,x^{13}}{13}+\frac{45\,a^2\,b^8\,x^{14}}{14}+\frac{2\,a\,b^9\,x^{15}}{3}+\frac{b^{10}\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^10,x)
```

```
[Out] (a^10*x^6)/6 + (b^10*x^16)/16 + (10*a^9*b*x^7)/7 + (2*a*b^9*x^15)/3 + (45*a
^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 + 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/1
1 + (35*a^4*b^6*x^12)/2 + (120*a^3*b^7*x^13)/13 + (45*a^2*b^8*x^14)/14
```

3.130 $\int x^4(a+bx)^{10}\,dx$

Optimal. Leaf size=81

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}$$

```
[Out] 1/11*a^4*(b*x+a)^11/b^5-1/3*a^3*(b*x+a)^12/b^5+6/13*a^2*(b*x+a)^13/b^5-2/7*
a*(b*x+a)^14/b^5+1/15*(b*x+a)^15/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^10,x]
```

```
[Out] (a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x)
^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{10}\,dx &= \int\left(\frac{a^4(a+bx)^{10}}{b^4}-\frac{4a^3(a+bx)^{11}}{b^4}+\frac{6a^2(a+bx)^{12}}{b^4}-\frac{4a(a+bx)^{13}}{b^4}+\frac{(a+bx)^{14}}{b^4}\right)dx\\ &= \frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 130, normalized size = 1.60

$$\frac{a^{10}x^5}{5}+\frac{5}{3}a^9bx^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a^2b^8x^{13}+\frac{5}{7}ab^9x^{14}+\frac{b^{10}x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^10,x]
```

```
[Out] (a^10*x^5)/5 + (5*a^9*b*x^6)/3 + (45*a^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*
a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*
x^12 + (45*a^2*b^8*x^13)/13 + (5*a*b^9*x^14)/7 + (b^10*x^15)/15
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.40

method	result
gosper	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a$
default	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a$
norman	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a$
risch	$\frac{1}{5}a^{10}x^5+\frac{5}{3}a^9b\,x^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^10*x^5+5/3*a^9*b*x^6+45/7*a^8*b^2*x^7+15*a^7*b^3*x^8+70/3*a^6*b^4*x^9
+126/5*a^5*b^5*x^10+210/11*a^4*b^6*x^11+10*a^3*b^7*x^12+45/13*a^2*b^8*x^13+
5/7*a*b^9*x^14+1/15*b^10*x^15
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15}+\frac{5}{7}\,ab^9x^{14}+\frac{45}{13}\,a^2b^8x^{13}+10\,a^3b^7x^{12}+\frac{210}{11}\,a^4b^6x^{11}+\frac{126}{5}\,a^5b^5x^{10}+\frac{70}{3}\,a^6b^4x^9+15\,a^7b^3x^8+\frac{45}{7}\,a^8b^2x^7+\frac{5}{3}\,a^9bx^6+\frac{1}{5}\,a^{10}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Fricas [A]
time = 0.68, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15}+\frac{5}{7}\,ab^9x^{14}+\frac{45}{13}\,a^2b^8x^{13}+10\,a^3b^7x^{12}+\frac{210}{11}\,a^4b^6x^{11}+\frac{126}{5}\,a^5b^5x^{10}+\frac{70}{3}\,a^6b^4x^9+15\,a^7b^3x^8+\frac{45}{7}\,a^8b^2x^7+\frac{5}{3}\,a^9bx^6+\frac{1}{5}\,a^{10}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Sympy [A]
time = 0.02, size = 131, normalized size = 1.62

$$\frac{a^{10}x^5}{5} + \frac{5a^9bx^6}{3} + \frac{45a^8b^2x^7}{7} + 15a^7b^3x^8 + \frac{70a^6b^4x^9}{3} + \frac{126a^5b^5x^{10}}{5} + \frac{210a^4b^6x^{11}}{11} + 10a^3b^7x^{12} + \frac{45a^2b^8x^{13}}{13} + \frac{5ab^9x^{14}}{7} + \frac{b^{10}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**10,x)
```

```
[Out] a**10*x**5/5 + 5*a**9*b*x**6/3 + 45*a**8*b**2*x**7/7 + 15*a**7*b**3*x**8 +
70*a**6*b**4*x**9/3 + 126*a**5*b**5*x**10/5 + 210*a**4*b**6*x**11/11 + 10*a
**3*b**7*x**12 + 45*a**2*b**8*x**13/13 + 5*a*b**9*x**14/7 + b**10*x**15/15
```

Giac [A]
time = 1.21, size = 112, normalized size = 1.38

$$\frac{1}{15}\,b^{10}x^{15} + \frac{5}{7}\,ab^9x^{14} + \frac{45}{13}\,a^2b^8x^{13} + 10\,a^3b^7x^{12} + \frac{210}{11}\,a^4b^6x^{11} + \frac{126}{5}\,a^5b^5x^{10} + \frac{70}{3}\,a^6b^4x^9 + 15\,a^7b^3x^8 + \frac{45}{7}\,a^8b^2x^7 + \frac{5}{3}\,a^9bx^6 + \frac{1}{5}\,a^{10}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

Mupad [B]
time = 0.12, size = 112, normalized size = 1.38

$$\frac{a^{10}\,x^5}{5} + \frac{5\,a^9\,b\,x^6}{3} + \frac{45\,a^8\,b^2\,x^7}{7} + 15\,a^7\,b^3\,x^8 + \frac{70\,a^6\,b^4\,x^9}{3} + \frac{126\,a^5\,b^5\,x^{10}}{5} + \frac{210\,a^4\,b^6\,x^{11}}{11} + 10\,a^3\,b^7\,x^{12} + \frac{45\,a^2\,b^8\,x^{13}}{13} + \frac{5\,a\,b^9\,x^{14}}{7} + \frac{b^{10}\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^10,x)
```

```
[Out] (a^10*x^5)/5 + (b^10*x^15)/15 + (5*a^9*b*x^6)/3 + (5*a*b^9*x^14)/7 + (45*a^
8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 +
 (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*x^12 + (45*a^2*b^8*x^13)/13
```

3.131 $\int x^3(a+bx)^{10}\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}$$

```
[Out] -1/11*a^3*(b*x+a)^11/b^4+1/4*a^2*(b*x+a)^12/b^4-3/13*a*(b*x+a)^13/b^4+1/14*
(b*x+a)^14/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^10,x]
```

```
[Out] -1/11*(a^3*(a + b*x)^11)/b^4 + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)^
13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{10}\,dx&=\int\left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}+\frac{(a+bx)^{13}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 128, normalized size = 2.00

$$\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2b^8x^{12}+\frac{10}{13}ab^9x^{13}+\frac{b^{10}x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^10,x]
```

```
[Out] (a^10*x^4)/4 + 2*a^9*b*x^5 + (15*a^8*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (10
5*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11
 + (15*a^2*b^8*x^12)/4 + (10*a*b^9*x^13)/13 + (b^10*x^14)/14
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.77

method	result
gosper	$\frac{1}{4}a^{10}x^4 + 2a^9b\,x^5 + \frac{15}{2}a^8b^2x^6 + \frac{120}{7}a^7b^3x^7 + \frac{105}{4}a^6b^4x^8 + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120}{11}a^3b^7x^{11} + \frac{15}{4}$
default	$\frac{1}{4}a^{10}x^4 + 2a^9b\,x^5 + \frac{15}{2}a^8b^2x^6 + \frac{120}{7}a^7b^3x^7 + \frac{105}{4}a^6b^4x^8 + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120}{11}a^3b^7x^{11} + \frac{15}{4}$
norman	$\frac{1}{4}a^{10}x^4 + 2a^9b\,x^5 + \frac{15}{2}a^8b^2x^6 + \frac{120}{7}a^7b^3x^7 + \frac{105}{4}a^6b^4x^8 + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120}{11}a^3b^7x^{11} + \frac{15}{4}$
risch	$\frac{1}{4}a^{10}x^4 + 2a^9b\,x^5 + \frac{15}{2}a^8b^2x^6 + \frac{120}{7}a^7b^3x^7 + \frac{105}{4}a^6b^4x^8 + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120}{11}a^3b^7x^{11} + \frac{15}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^10*x^4+2*a^9*b*x^5+15/2*a^8*b^2*x^6+120/7*a^7*b^3*x^7+105/4*a^6*b^4*x
^8+28*a^5*b^5*x^9+21*a^4*b^6*x^10+120/11*a^3*b^7*x^11+15/4*a^2*b^8*x^12+10/
13*a*b^9*x^13+1/14*b^10*x^14
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.75

$$\frac{1}{14}\,b^{10}x^{14} + \frac{10}{13}\,ab^9x^{13} + \frac{15}{4}\,a^2b^8x^{12} + \frac{120}{11}\,a^3b^7x^{11} + 21\,a^4b^6x^{10} + 28\,a^5b^5x^9 + \frac{105}{4}\,a^6b^4x^8 + \frac{120}{7}\,a^7b^3x^7 + \frac{15}{2}\,a^8b^2x^6 + 2\,a^9bx^5 + \frac{1}{4}\,a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Fricas [A]
time = 0.66, size = 112, normalized size = 1.75

$$\frac{1}{14}\,b^{10}x^{14} + \frac{10}{13}\,ab^9x^{13} + \frac{15}{4}\,a^2b^8x^{12} + \frac{120}{11}\,a^3b^7x^{11} + 21\,a^4b^6x^{10} + 28\,a^5b^5x^9 + \frac{105}{4}\,a^6b^4x^8 + \frac{120}{7}\,a^7b^3x^7 + \frac{15}{2}\,a^8b^2x^6 + 2\,a^9bx^5 + \frac{1}{4}\,a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.

time = 0.02, size = 129, normalized size = 2.02

$$\frac{a^{10}x^4}{4} + 2a^9bx^5 + \frac{15a^8b^2x^6}{2} + \frac{120a^7b^3x^7}{7} + \frac{105a^6b^4x^8}{4} + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120a^3b^7x^{11}}{11} + \frac{15a^2b^8x^{12}}{4} + \frac{10ab^9x^{13}}{13} + \frac{b^{10}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**10,x)
```

```
[Out] a**10*x**4/4 + 2*a**9*b*x**5 + 15*a**8*b**2*x**6/2 + 120*a**7*b**3*x**7/7 +
 105*a**6*b**4*x**8/4 + 28*a**5*b**5*x**9 + 21*a**4*b**6*x**10 + 120*a**3*b
**7*x**11/11 + 15*a**2*b**8*x**12/4 + 10*a*b**9*x**13/13 + b**10*x**14/14
```

Giac [A]

time = 1.42, size = 112, normalized size = 1.75

$$\frac{1}{14}\,b^{10}x^{14} + \frac{10}{13}\,ab^9x^{13} + \frac{15}{4}\,a^2b^8x^{12} + \frac{120}{11}\,a^3b^7x^{11} + 21\,a^4b^6x^{10} + 28\,a^5b^5x^9 + \frac{105}{4}\,a^6b^4x^8 + \frac{120}{7}\,a^7b^3x^7 + \frac{15}{2}\,a^8b^2x^6 + 2\,a^9bx^5 + \frac{1}{4}\,a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

Mupad [B]

time = 0.12, size = 112, normalized size = 1.75

$$\frac{a^{10}\,x^4}{4} + 2\,a^9\,b\,x^5 + \frac{15\,a^8\,b^2\,x^6}{2} + \frac{120\,a^7\,b^3\,x^7}{7} + \frac{105\,a^6\,b^4\,x^8}{4} + 28\,a^5\,b^5\,x^9 + 21\,a^4\,b^6\,x^{10} + \frac{120\,a^3\,b^7\,x^{11}}{11} + \frac{15\,a^2\,b^8\,x^{12}}{4} + \frac{10\,a\,b^9\,x^{13}}{13} + \frac{b^{10}\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^10,x)
```

```
[Out] (a^10*x^4)/4 + (b^10*x^14)/14 + 2*a^9*b*x^5 + (10*a*b^9*x^13)/13 + (15*a^8*
b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (105*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 2
1*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11 + (15*a^2*b^8*x^12)/4
```

3.132 $\int x^2(a+bx)^{10}\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}$$

```
[Out] 1/11*a^2*(b*x+a)^11/b^3-1/6*a*(b*x+a)^12/b^3+1/13*(b*x+a)^13/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^10,x]
```

```
[Out] (a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*b
^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{10}\,dx&=\int\left(\frac{a^2(a+bx)^{10}}{b^2}-\frac{2a(a+bx)^{11}}{b^2}+\frac{(a+bx)^{12}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(47) = 94.
time = 0.00, size = 126, normalized size = 2.68

$$\frac{a^{10}x^3}{3}+\frac{5}{2}a^9bx^4+9a^8b^2x^5+20a^7b^3x^6+30a^6b^4x^7+\frac{63}{2}a^5b^5x^8+\frac{70}{3}a^4b^6x^9+12a^3b^7x^{10}+\frac{45}{11}a^2b^8x^{11}+\frac{5}{6}ab^9x^{12}+\frac{b^{10}x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^10,x]
```

```
[Out] (a^10*x^3)/3 + (5*a^9*b*x^4)/2 + 9*a^8*b^2*x^5 + 20*a^7*b^3*x^6 + 30*a^6*b^
4*x^7 + (63*a^5*b^5*x^8)/2 + (70*a^4*b^6*x^9)/3 + 12*a^3*b^7*x^10 + (45*a^2
*b^8*x^11)/11 + (5*a*b^9*x^12)/6 + (b^10*x^13)/13
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.08, size = 113, normalized size = 2.40

method	result
gosper	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8$
default	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8$
norman	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8$
risch	$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^10*x^13+5/6*a*b^9*x^12+45/11*a^2*b^8*x^11+12*a^3*b^7*x^10+70/3*a^4*b
^6*x^9+63/2*a^5*b^5*x^8+30*a^6*b^4*x^7+20*a^7*b^3*x^6+9*a^8*b^2*x^5+5/2*a^9
*b*x^4+1/3*a^10*x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 0.28, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 1.37, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.
time = 0.02, size = 128, normalized size = 2.72

$$\frac{a^{10}x^3}{3} + \frac{5a^9bx^4}{2} + 9a^8b^2x^5 + 20a^7b^3x^6 + 30a^6b^4x^7 + \frac{63a^5b^5x^8}{2} + \frac{70a^4b^6x^9}{3} + 12a^3b^7x^{10} + \frac{45a^2b^8x^{11}}{11} + \frac{5ab^9x^{12}}{6} + \frac{b^{10}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**10,x)
```

```
[Out] a**10*x**3/3 + 5*a**9*b*x**4/2 + 9*a**8*b**2*x**5 + 20*a**7*b**3*x**6 + 30*
a**6*b**4*x**7 + 63*a**5*b**5*x**8/2 + 70*a**4*b**6*x**9/3 + 12*a**3*b**7*x
**10 + 45*a**2*b**8*x**11/11 + 5*a*b**9*x**12/6 + b**10*x**13/13
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(41) = 82$.
time = 1.24, size = 112, normalized size = 2.38

$$\frac{1}{13}\,b^{10}x^{13} + \frac{5}{6}\,ab^9x^{12} + \frac{45}{11}\,a^2b^8x^{11} + 12\,a^3b^7x^{10} + \frac{70}{3}\,a^4b^6x^9 + \frac{63}{2}\,a^5b^5x^8 + 30\,a^6b^4x^7 + 20\,a^7b^3x^6 + 9\,a^8b^2x^5 + \frac{5}{2}\,a^9bx^4 + \frac{1}{3}\,a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

Mupad [B]
time = 0.07, size = 31, normalized size = 0.66

$$\frac{(a+b\,x)^{11}\,(8\,a^2-88\,a\,b\,x+528\,b^2\,x^2)}{6864\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^11*(8*a^2 + 528*b^2*x^2 - 88*a*b*x))/(6864*b^3)
```

3.133 $\int x(a+bx)^{10}\,dx$

Optimal. Leaf size=30

$$-\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}$$

[Out] -1/11*a*(b*x+a)^11/b^2+1/12*(b*x+a)^12/b^2

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x)^10,x]

[Out] -1/11*(a*(a + b*x)^11)/b^2 + (a + b*x)^12/(12*b^2)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{10}\,dx &= \int\left(-\frac{a(a+bx)^{10}}{b}+\frac{(a+bx)^{11}}{b}\right)dx\\ &= -\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(30) = 60.
time = 0.00, size = 128, normalized size = 4.27

$$\frac{a^{10}x^2}{2}+\frac{10}{3}a^9bx^3+\frac{45}{4}a^8b^2x^4+24a^7b^3x^5+35a^6b^4x^6+36a^5b^5x^7+\frac{105}{4}a^4b^6x^8+\frac{40}{3}a^3b^7x^9+\frac{9}{2}a^2b^8x^{10}+\frac{10}{11}ab^9x^{11}+\frac{b^{10}x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x)^10,x]

```
[Out] (a^10*x^2)/2 + (10*a^9*b*x^3)/3 + (45*a^8*b^2*x^4)/4 + 24*a^7*b^3*x^5 + 35*
a^6*b^4*x^6 + 36*a^5*b^5*x^7 + (105*a^4*b^6*x^8)/4 + (40*a^3*b^7*x^9)/3 + (
9*a^2*b^8*x^10)/2 + (10*a*b^9*x^11)/11 + (b^10*x^12)/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.
time = 0.08, size = 113, normalized size = 3.77

method	result
gosper	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x^5 + \frac{45}{4}$
default	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x^5 + \frac{45}{4}$
norman	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x^5 + \frac{45}{4}$
risch	$\frac{1}{12}b^{10}x^{12} + \frac{10}{11}a\,b^9x^{11} + \frac{9}{2}a^2b^8x^{10} + \frac{40}{3}a^3b^7x^9 + \frac{105}{4}a^4b^6x^8 + 36a^5b^5x^7 + 35a^6b^4x^6 + 24a^7b^3x^5 + \frac{45}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^10*x^12+10/11*a*b^9*x^11+9/2*a^2*b^8*x^10+40/3*a^3*b^7*x^9+105/4*a^4
*b^6*x^8+36*a^5*b^5*x^7+35*a^6*b^4*x^6+24*a^7*b^3*x^5+45/4*a^8*b^2*x^4+10/3
*a^9*b*x^3+1/2*a^10*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.
time = 0.28, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^9x^{11} + \frac{9}{2}\,a^2b^8x^{10} + \frac{40}{3}\,a^3b^7x^9 + \frac{105}{4}\,a^4b^6x^8 + 36\,a^5b^5x^7 + 35\,a^6b^4x^6 + 24\,a^7b^3x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{10}{3}\,a^9bx^3 + \frac{1}{2}\,a^{10}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(26) = 52$.
time = 0.68, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^9x^{11} + \frac{9}{2}\,a^2b^8x^{10} + \frac{40}{3}\,a^3b^7x^9 + \frac{105}{4}\,a^4b^6x^8 + 36\,a^5b^5x^7 + 35\,a^6b^4x^6 + 24\,a^7b^3x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{10}{3}\,a^9bx^3 + \frac{1}{2}\,a^{10}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(24) = 48.
time = 0.02, size = 129, normalized size = 4.30

$$\frac{a^{10}x^2}{2} + \frac{10a^9bx^3}{3} + \frac{45a^8b^2x^4}{4} + 24a^7b^3x^5 + 35a^6b^4x^6 + 36a^5b^5x^7 + \frac{105a^4b^6x^8}{4} + \frac{40a^3b^7x^9}{3} + \frac{9a^2b^8x^{10}}{2} + \frac{10ab^9x^{11}}{11} + \frac{b^{10}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**10,x)
```

```
[Out] a**10*x**2/2 + 10*a**9*b*x**3/3 + 45*a**8*b**2*x**4/4 + 24*a**7*b**3*x**5 +
 35*a**6*b**4*x**6 + 36*a**5*b**5*x**7 + 105*a**4*b**6*x**8/4 + 40*a**3*b**
7*x**9/3 + 9*a**2*b**8*x**10/2 + 10*a*b**9*x**11/11 + b**10*x**12/12
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(26) = 52.
time = 1.53, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12} + \frac{10}{11}\,ab^9x^{11} + \frac{9}{2}\,a^2b^8x^{10} + \frac{40}{3}\,a^3b^7x^9 + \frac{105}{4}\,a^4b^6x^8 + 36\,a^5b^5x^7 + 35\,a^6b^4x^6 + 24\,a^7b^3x^5 + \frac{45}{4}\,a^8b^2x^4 + \frac{10}{3}\,a^9bx^3 + \frac{1}{2}\,a^{10}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

Mupad [B]
time = 0.09, size = 25, normalized size = 0.83

$$-\frac{2\left(\frac{a\,(a+b\,x)^{11}}{22}-\frac{(a+b\,x)^{12}}{24}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^10,x)
```

```
[Out] -(2*((a*(a + b*x)^11)/22 - (a + b*x)^12/24))/b^2
```

3.134 $\int (a+bx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

[Out] 1/11*(b*x+a)^11/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^10,x]

[Out] (a + b*x)^11/(11*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{10}\,dx = \frac{(a+bx)^{11}}{11b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^10,x]

[Out] (a + b*x)^11/(11*b)

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result
default	$\frac{(bx+a)^{11}}{11b}$
gosper	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a^8b^2x$
norman	$\frac{1}{11}b^{10}x^{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a^8b^2x$
risch	$\frac{b^{10}x^{11}}{11}+a\,b^9x^{10}+5a^2b^8x^9+15a^3b^7x^8+30a^4b^6x^7+42a^5b^5x^6+42a^6b^4x^5+30a^7b^3x^4+15a^8b^2x^3+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*(b*x+a)^11/b
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(b*x + a)^11/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.
time = 0.65, size = 108, normalized size = 7.71

$$\frac{1}{11}\,b^{10}x^{11}+ab^9x^{10}+5\,a^2b^8x^9+15\,a^3b^7x^8+30\,a^4b^6x^7+42\,a^5b^5x^6+42\,a^6b^4x^5+30\,a^7b^3x^4+15\,a^8b^2x^3+5\,a^9bx^2+a^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/11*b^10*x^11 + a*b^9*x^10 + 5*a^2*b^8*x^9 + 15*a^3*b^7*x^8 + 30*a^4*b^6*x
^7 + 42*a^5*b^5*x^6 + 42*a^6*b^4*x^5 + 30*a^7*b^3*x^4 + 15*a^8*b^2*x^3 + 5*
a^9*b*x^2 + a^10*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(8) = 16.
time = 0.02, size = 114, normalized size = 8.14

$$a^{10}x+5a^9bx^2+15a^8b^2x^3+30a^7b^3x^4+42a^6b^4x^5+42a^5b^5x^6+30a^4b^6x^7+15a^3b^7x^8+5a^2b^8x^9+ab^9x^{10}+\frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10,x)
```

```
[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b
**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a*
*2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11
```

Giac [A]
time = 1.41, size = 12, normalized size = 0.86

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/11*(b*x + a)^11/b
```

Mupad [B]
time = 0.11, size = 108, normalized size = 7.71

$$a^{10}\,x+5\,a^9\,b\,x^2+15\,a^8\,b^2\,x^3+30\,a^7\,b^3\,x^4+42\,a^6\,b^4\,x^5+42\,a^5\,b^5\,x^6+30\,a^4\,b^6\,x^7+15\,a^3\,b^7\,x^8+5\,a^2\,b^8\,x^9+a\,b^9\,x^{10}+\frac{b^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10,x)
```

```
[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^
7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x
^8 + 5*a^2*b^8*x^9
```

3.135 $\int \frac{(a+bx)^{10}}{x} \, dx$

Optimal. Leaf size=122

$$10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}+a$$

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x}\,dx = \int \left(10a^9b+\frac{a^{10}}{x}+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5+120a^3b^7\right.$$

$$= 10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a$$

Mathematica [A]
time = 0.00, size = 122, normalized size = 1.00

$$10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}+a^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Maple [A]
time = 0.12, size = 109, normalized size = 0.89

method	result
default	$10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10a\,b^9x^9}{9}+$
norman	$10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10a\,b^9x^9}{9}+$
risch	$10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10a\,b^9x^9}{9}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x,x,method=_RETURNVERBOSE)
```

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^9x^9+\frac{45}{8}\,a^2b^8x^8+\frac{120}{7}\,a^3b^7x^7+35\,a^4b^6x^6+\frac{252}{5}\,a^5b^5x^5+\frac{105}{2}\,a^6b^4x^4+40\,a^7b^3x^3+\frac{45}{2}\,a^8b^2x^2+10\,a^9bx+a^{10}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Fricas [A]
time = 1.20, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10}+\frac{10}{9}\,ab^9x^9+\frac{45}{8}\,a^2b^8x^8+\frac{120}{7}\,a^3b^7x^7+35\,a^4b^6x^6+\frac{252}{5}\,a^5b^5x^5+\frac{105}{2}\,a^6b^4x^4+40\,a^7b^3x^3+\frac{45}{2}\,a^8b^2x^2+10\,a^9bx+a^{10}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

Sympy [A]
time = 0.06, size = 126, normalized size = 1.03

$$a^{10}\log(x)+10a^9bx+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+\frac{10ab^9x^9}{9}+\frac{b^{10}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x,x)
```

```
[Out] a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**3 + 105*
a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a**3*b**7
*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10
```

Giac [A]
time = 1.54, size = 109, normalized size = 0.89

$$\frac{1}{10}b^{10}x^{10}+\frac{10}{9}ab^9x^9+\frac{45}{8}a^2b^8x^8+\frac{120}{7}a^3b^7x^7+35a^4b^6x^6+\frac{252}{5}a^5b^5x^5+\frac{105}{2}a^6b^4x^4+40a^7b^3x^3+\frac{45}{2}a^8b^2x^2+10a^9bx+a^{10}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="giac")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(abs(x))
```

Mupad [B]
time = 0.08, size = 108, normalized size = 0.89

$$a^{10}\ln(x)+\frac{b^{10}x^{10}}{10}+\frac{10ab^9x^9}{9}+\frac{45a^8b^2x^2}{2}+40a^7b^3x^3+\frac{105a^6b^4x^4}{2}+\frac{252a^5b^5x^5}{5}+35a^4b^6x^6+\frac{120a^3b^7x^7}{7}+\frac{45a^2b^8x^8}{8}+10a^9bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x,x)
```

```
[Out] a^10*log(x) + (b^10*x^10)/10 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/2 + 40*a
^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (
120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)/8 + 10*a^9*b*x
```

3.136 $\int \frac{(a+bx)^{10}}{x^2} dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}+10a^9b\log($$

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^2}\,dx &= \int \left(45a^8b^2+\frac{a^{10}}{x^2}+\frac{10a^9b}{x}+120a^7b^3x+210a^6b^4x^2+252a^5b^5x^3+210a^4b^6x^4+120a^3b^7\right.\\ &= -\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}+10a^9b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.96

method	result
default	$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5a\,b^9x^8}{4}+b$
risch	$-\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5a\,b^9x^8}{4}+b$
norman	$\frac{-a^{10}+\frac{1}{9}b^{10}x^{10}+\frac{5}{4}a\,b^9x^9+\frac{45}{7}a^2b^8x^8+20a^3b^7x^7+42a^4b^6x^6+63a^5b^5x^5+70a^6b^4x^4+60a^7b^3x^3+45a^8b^2x^2}{x}+10a^9b\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

Maxima [A]
time = 0.28, size = 109, normalized size = 0.95

$$\frac{1}{9}b^{10}x^9+\frac{5}{4}ab^9x^8+\frac{45}{7}a^2b^8x^7+20\,a^3b^7x^6+42\,a^4b^6x^5+63\,a^5b^5x^4+70\,a^6b^4x^3+60\,a^7b^3x^2+45\,a^8b^2x+10\,a^9b\log(x)-\frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(x) - a^10/x
```

Fricas [A]
time = 1.24, size = 114, normalized size = 0.99

$$\frac{28\,b^{10}x^{10}+315\,ab^9x^9+1620\,a^2b^8x^8+5040\,a^3b^7x^7+10584\,a^4b^6x^6+15876\,a^5b^5x^5+17640\,a^6b^4x^4+15120\,a^7b^3x^3+11340\,a^8b^2x^2+2520\,a^9bx\log(x)-252\,a^{10}}{252\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*x^10 + 315*a*b^9*x^9 + 1620*a^2*b^8*x^8 + 5040*a^3*b^7*x^7 +
 10584*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 17640*a^6*b^4*x^4 + 15120*a^7*b^3*
x^3 + 11340*a^8*b^2*x^2 + 2520*a^9*b*x*log(x) - 252*a^10)/x
```

Sympy [A]
time = 0.06, size = 117, normalized size = 1.02

$$-\frac{a^{10}}{x}+10a^9b\log\left(x\right)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45a^2b^8x^7}{7}+\frac{5ab^9x^8}{4}+\frac{b^{10}x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**2,x)
```

```
[Out] -a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b**3*x**2 + 70*a**6*
b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a**3*b**7*x**6 + 45*
a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9
```

Giac [A]
time = 1.08, size = 110, normalized size = 0.96

$$\frac{1}{9}\,b^{10}x^9+\frac{5}{4}\,ab^9x^8+\frac{45}{7}\,a^2b^8x^7+20\,a^3b^7x^6+42\,a^4b^6x^5+63\,a^5b^5x^4+70\,a^6b^4x^3+60\,a^7b^3x^2+45\,a^8b^2x+10\,a^9b\log\left(|x|\right)-\frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(abs(x)) - a^10/x
```

Mupad [B]
time = 0.12, size = 109, normalized size = 0.95

$$\frac{b^{10}\,x^9}{9}-\frac{a^{10}}{x}+45\,a^8\,b^2\,x+\frac{5\,a\,b^9\,x^8}{4}+10\,a^9\,b\,\ln\left(x\right)+60\,a^7\,b^3\,x^2+70\,a^6\,b^4\,x^3+63\,a^5\,b^5\,x^4+42\,a^4\,b^6\,x^5+20\,a^3\,b^7\,x^6+\frac{45\,a^2\,b^8\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^2,x)
```

```
[Out] (b^10*x^9)/9 - a^10/x + 45*a^8*b^2*x + (5*a*b^9*x^8)/4 + 10*a^9*b*log(x) +
60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4 + 42*a^4*b^6*x^5 + 20*a^3*
b^7*x^6 + (45*a^2*b^8*x^7)/7
```

3.137 $\int \frac{(a+bx)^{10}}{x^3}\,dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}+45a^8b^2\,\mathrm{lo}$$

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^3}\,dx = \int \left(120a^7b^3+\frac{a^{10}}{x^3}+\frac{10a^9b}{x^2}+\frac{45a^8b^2}{x}+210a^6b^4x+252a^5b^5x^2+210a^4b^6x^3+120a^3b^7x^4+\right.$$
$$= -\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6$$

Mathematica [A]
time = 0.00, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}+45a^8b^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Maple [A]
time = 0.10, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105a^4b^6x^4}{2}+24a^3b^7x^5+\frac{15a^2b^8x^6}{2}+\frac{10a\,b^9x^7}{7}+b$
risch	$\frac{b^{10}x^8}{8}+\frac{10a\,b^9x^7}{7}+\frac{15a^2b^8x^6}{2}+24a^3b^7x^5+\frac{105a^4b^6x^4}{2}+84a^5b^5x^3+105a^6b^4x^2+120a^7b^3x+\frac{-10a^9bx-\frac{1}{2}}{x^2}$
norman	$\frac{-\frac{1}{2}a^{10}+\frac{1}{8}b^{10}x^{10}+\frac{10}{7}a\,b^9x^9+\frac{15}{2}a^2b^8x^8+24a^3b^7x^7+\frac{105}{2}a^4b^6x^6+84a^5b^5x^5+105a^6b^4x^4+120a^7b^3x^3-10a^9bx}{x^2}+45a^8b^2\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

Maxima [A]
time = 0.32, size = 108, normalized size = 0.91

$$\frac{1}{8}b^{10}x^8+\frac{10}{7}ab^9x^7+\frac{15}{2}a^2b^8x^6+24\,a^3b^7x^5+\frac{105}{2}a^4b^6x^4+84\,a^5b^5x^3+105\,a^6b^4x^2+120\,a^7b^3x+45\,a^8b^2\log(x)-\frac{20\,a^9bx+a^{10}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(x) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Fricas [A]
time = 1.23, size = 114, normalized size = 0.96

$$\frac{7\,b^{10}x^{10}+80\,ab^9x^9+420\,a^2b^8x^8+1344\,a^3b^7x^7+2940\,a^4b^6x^6+4704\,a^5b^5x^5+5880\,a^6b^4x^4+6720\,a^7b^3x^3+2520\,a^8b^2x^2\log(x)-560\,a^9bx-28\,a^{10}}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*x^10 + 80*a*b^9*x^9 + 420*a^2*b^8*x^8 + 1344*a^3*b^7*x^7 + 294
0*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 5880*a^6*b^4*x^4 + 6720*a^7*b^3*x^3 + 25
20*a^8*b^2*x^2*log(x) - 560*a^9*b*x - 28*a^10)/x^2
```

Sympy [A]
time = 0.09, size = 122, normalized size = 1.03

$$45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105a^4b^6x^4}{2}+24a^3b^7x^5+\frac{15a^2b^8x^6}{2}+\frac{10ab^9x^7}{7}+\frac{b^{10}x^8}{8}+\frac{-a^{10}-20a^9bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**3,x)
```

```
[Out] 45*a**8*b**2*log(x) + 120*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x
**3 + 105*a**4*b**6*x**4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a
*b**9*x**7/7 + b**10*x**8/8 + (-a**10 - 20*a**9*b*x)/(2*x**2)
```

Giac [A]
time = 1.44, size = 109, normalized size = 0.92

$$\frac{1}{8}b^{10}x^8+\frac{10}{7}ab^9x^7+\frac{15}{2}a^2b^8x^6+24a^3b^7x^5+\frac{105}{2}a^4b^6x^4+84a^5b^5x^3+105a^6b^4x^2+120a^7b^3x+45a^8b^2\log(|x|)-\frac{20a^9bx+a^{10}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(abs(x)) - 1/2*(20*a^9*b*x + a^10)/x^2
```

Mupad [B]
time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}x^8}{8}-\frac{\frac{a^{10}}{2}+10bxa^9}{x^2}+120a^7b^3x+\frac{10ab^9x^7}{7}+105a^6b^4x^2+84a^5b^5x^3+\frac{105a^4b^6x^4}{2}+24a^3b^7x^5+\frac{15a^2b^8x^6}{2}+45a^8b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^3,x)
```

```
[Out] (b^10*x^8)/8 - (a^10/2 + 10*a^9*b*x)/x^2 + 120*a^7*b^3*x + (10*a*b^9*x^7)/7
 + 105*a^6*b^4*x^2 + 84*a^5*b^5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5
+ (15*a^2*b^8*x^6)/2 + 45*a^8*b^2*log(x)
```

3.138 $\int \frac{(a+bx)^{10}}{x^4}\,dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}+120a^7b^3\log(x)$$

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^4}\,dx &= \int \left(210a^6b^4+\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+252a^5b^5x+210a^4b^6x^2+120a^3b^7x^3+\right.\\ &= -\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}a\end{aligned}$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}+120a^7b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.96

method	result
default	$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5ab^9x^6}{3}+\frac{b^{10}x^7}{7}+$
risch	$\frac{b^{10}x^7}{7}+\frac{5ab^9x^6}{3}+9a^2b^8x^5+30a^3b^7x^4+70a^4b^6x^3+126a^5b^5x^2+210a^6b^4x+\frac{-45a^8b^2x^2-5a^9bx-\frac{1}{3}a^{10}}{x^3}+$
norman	$\frac{-\frac{1}{3}a^{10}+\frac{1}{7}b^{10}x^{10}+\frac{5}{3}ab^9x^9+9a^2b^8x^8+30a^3b^7x^7+70a^4b^6x^6+126a^5b^5x^5+210a^6b^4x^4-45a^8b^2x^2-5a^9bx}{x^3}+120a^7b^3\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.94

$$\frac{1}{7}b^{10}x^7+\frac{5}{3}ab^9x^6+9a^2b^8x^5+30a^3b^7x^4+70a^4b^6x^3+126a^5b^5x^2+210a^6b^4x+120a^7b^3\log(x)-\frac{135a^8b^2x^2+15a^9bx+a^{10}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(x) - 1/3*(135*a^8*b
^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Fricas [A]
time = 0.60, size = 114, normalized size = 0.99

$$\frac{3b^{10}x^{10}+35ab^9x^9+189a^2b^8x^8+630a^3b^7x^7+1470a^4b^6x^6+2646a^5b^5x^5+4410a^6b^4x^4+2520a^7b^3x^3\log(x)-945a^8b^2x^2-105a^9bx-7a^{10}}{21x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*x^10 + 35*a*b^9*x^9 + 189*a^2*b^8*x^8 + 630*a^3*b^7*x^7 + 1470
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 4410*a^6*b^4*x^4 + 2520*a^7*b^3*x^3*log(x
) - 945*a^8*b^2*x^2 - 105*a^9*b*x - 7*a^10)/x^3
```

Sympy [A]
time = 0.11, size = 119, normalized size = 1.03

$$120a^7b^3\log(x) + 210a^6b^4x + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + \frac{5ab^9x^6}{3} + \frac{b^{10}x^7}{7} + \frac{-a^{10} - 15a^9bx - 135a^8b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**4,x)
```

```
[Out] 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b**6*
x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x**7/
7 + (-a**10 - 15*a**9*b*x - 135*a**8*b**2*x**2)/(3*x**3)
```

Giac [A]
time = 1.71, size = 109, normalized size = 0.95

$$\frac{1}{7}b^{10}x^7 + \frac{5}{3}ab^9x^6 + 9a^2b^8x^5 + 30a^3b^7x^4 + 70a^4b^6x^3 + 126a^5b^5x^2 + 210a^6b^4x + 120a^7b^3\log(|x|) - \frac{135a^8b^2x^2 + 15a^9bx + a^{10}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(abs(x)) - 1/3*(135*
a^8*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

Mupad [B]
time = 0.06, size = 110, normalized size = 0.96

$$\frac{b^{10}x^7}{7} - \frac{\frac{a^{10}}{3} + 5a^9bx + 45a^8b^2x^2}{x^3} + 210a^6b^4x + \frac{5ab^9x^6}{3} + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + 120a^7b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^4,x)
```

```
[Out] (b^10*x^7)/7 - (a^10/3 + 45*a^8*b^2*x^2 + 5*a^9*b*x)/x^3 + 210*a^6*b^4*x +
(5*a*b^9*x^6)/3 + 126*a^5*b^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2
*b^8*x^5 + 120*a^7*b^3*log(x)
```

3.139 $\int \frac{(a+bx)^{10}}{x^5} dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}+210a^6b^4\log(x)$$

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^5}\,dx &= \int \left(252a^5b^5+\frac{a^{10}}{x^5}+\frac{10a^9b}{x^4}+\frac{45a^8b^2}{x^3}+\frac{120a^7b^3}{x^2}+\frac{210a^6b^4}{x}+210a^4b^6x+120a^3b^7x^2+45\right.\\ &= -\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2a\end{aligned}$$

Mathematica [A]
time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}+210a^6b^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45a^2b^8x^4}{4}+2a\,b^9x^5+\frac{b^{10}x^6}{6}$ -
risch	$\frac{b^{10}x^6}{6}+2a\,b^9x^5+\frac{45a^2b^8x^4}{4}+40a^3b^7x^3+105a^4b^6x^2+252a^5b^5x+\frac{-120a^7b^3x^3-\frac{45}{2}a^8b^2x^2-\frac{10}{3}a^9bx-\frac{1}{4}a^{10}}{x^4}$ +
norman	$\frac{-\frac{1}{4}a^{10}+\frac{1}{6}b^{10}x^{10}+2a\,b^9x^9+\frac{45}{4}a^2b^8x^8+40a^3b^7x^7+105a^4b^6x^6+252a^5b^5x^5-120a^7b^3x^3-\frac{45}{2}a^8b^2x^2-\frac{10}{3}a^9bx}{x^4}+210a^6b^4\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

Maxima [A]
time = 0.27, size = 110, normalized size = 0.92

$$\frac{1}{6}\,b^{10}x^6+2\,ab^9x^5+\frac{45}{4}\,a^2b^8x^4+40\,a^3b^7x^3+105\,a^4b^6x^2+252\,a^5b^5x+210\,a^6b^4\log\left(x\right)-\frac{1440\,a^7b^3x^3+270\,a^8b^2x^2+40\,a^9bx+3\,a^{10}}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(x) - 1/12*(1440*a^7*b^3*x^3 + 270*a
^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Fricas [A]
time = 0.54, size = 114, normalized size = 0.96

$$\frac{2\,b^{10}x^{10}+24\,ab^9x^9+135\,a^2b^8x^8+480\,a^3b^7x^7+1260\,a^4b^6x^6+3024\,a^5b^5x^5+2520\,a^6b^4x^4\log\left(x\right)-1440\,a^7b^3x^3-270\,a^8b^2x^2-40\,a^9bx-3\,a^{10}}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*x^10 + 24*a*b^9*x^9 + 135*a^2*b^8*x^8 + 480*a^3*b^7*x^7 + 1260
*a^4*b^6*x^6 + 3024*a^5*b^5*x^5 + 2520*a^6*b^4*x^4*log(x) - 1440*a^7*b^3*x^
3 - 270*a^8*b^2*x^2 - 40*a^9*b*x - 3*a^10)/x^4
```

Sympy [A]
time = 0.14, size = 121, normalized size = 1.02

$$210a^6b^4\log(x) + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45a^2b^8x^4}{4} + 2ab^9x^5 + \frac{b^{10}x^6}{6} + \frac{-3a^{10} - 40a^9bx - 270a^8b^2x^2 - 1440a^7b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**5,x)
```

```
[Out] 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105*a**4*b**6*x**2 + 40*a**3*b**7*
x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 + b**10*x**6/6 + (-3*a**10 - 40*
a**9*b*x - 270*a**8*b**2*x**2 - 1440*a**7*b**3*x**3)/(12*x**4)
```

Giac [A]
time = 1.16, size = 111, normalized size = 0.93

$$\frac{1}{6}b^{10}x^6 + 2ab^9x^5 + \frac{45}{4}a^2b^8x^4 + 40a^3b^7x^3 + 105a^4b^6x^2 + 252a^5b^5x + 210a^6b^4\log(|x|) - \frac{1440a^7b^3x^3 + 270a^8b^2x^2 + 40a^9bx + 3a^{10}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(abs(x)) - 1/12*(1440*a^7*b^3*x^3 +
270*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

Mupad [B]
time = 0.10, size = 110, normalized size = 0.92

$$\frac{b^{10}x^6}{6} - \frac{\frac{a^{10}}{4} + \frac{10a^9bx}{3} + \frac{45a^8b^2x^2}{2} + 120a^7b^3x^3}{x^4} + 252a^5b^5x + 2ab^9x^5 + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45a^2b^8x^4}{4} + 210a^6b^4\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^5,x)
```

```
[Out] (b^10*x^6)/6 - (a^10/4 + (45*a^8*b^2*x^2)/2 + 120*a^7*b^3*x^3 + (10*a^9*b*x
)/3)/x^4 + 252*a^5*b^5*x + 2*a*b^9*x^5 + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 +
 (45*a^2*b^8*x^4)/4 + 210*a^6*b^4*log(x)
```

3.140 $\int \frac{(a+bx)^{10}}{x^6} dx$

Optimal. Leaf size=117

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}+252a^5b^5\log(x)$$

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^6}\,dx &= \int \left(210a^4b^6+\frac{a^{10}}{x^6}+\frac{10a^9b}{x^5}+\frac{45a^8b^2}{x^4}+\frac{120a^7b^3}{x^3}+\frac{210a^6b^4}{x^2}+\frac{252a^5b^5}{x}+120a^3b^7x+45\right.\\ &= -\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 117, normalized size = 1.00

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}+252a^5b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.94

method	result
default	$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5a\,b^9x^4}{2}+\frac{b^{10}x^5}{5}+252a$
risch	$\frac{b^{10}x^5}{5}+\frac{5a\,b^9x^4}{2}+15a^2b^8x^3+60a^3b^7x^2+210a^4b^6x+\frac{-210a^6b^4x^4-60a^7b^3x^3-15a^8b^2x^2-\frac{5}{2}a^9bx-\frac{1}{5}a^{10}}{x^5}+252a^5$
norman	$\frac{-\frac{1}{5}a^{10}+\frac{1}{5}b^{10}x^{10}+\frac{5}{2}a\,b^9x^9+15a^2b^8x^8+60a^3b^7x^7+210a^4b^6x^6-210a^6b^4x^4-60a^7b^3x^3-15a^8b^2x^2-\frac{5}{2}a^9bx}{x^5}+252a^5b^5\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

Maxima [A]
time = 0.35, size = 110, normalized size = 0.94

$$\frac{1}{5}b^{10}x^5+\frac{5}{2}ab^9x^4+15\,a^2b^8x^3+60\,a^3b^7x^2+210\,a^4b^6x+252\,a^5b^5\log(x)-\frac{2100\,a^6b^4x^4+600\,a^7b^3x^3+150\,a^8b^2x^2+25\,a^9bx+2\,a^{10}}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(x) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 + 150*a
^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Fricas [A]
time = 0.48, size = 114, normalized size = 0.97

$$\frac{2\,b^{10}x^{10}+25\,ab^9x^9+150\,a^2b^8x^8+600\,a^3b^7x^7+2100\,a^4b^6x^6+2520\,a^5b^5x^5\log(x)-2100\,a^6b^4x^4-600\,a^7b^3x^3-150\,a^8b^2x^2-25\,a^9bx-2\,a^{10}}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*x^10 + 25*a*b^9*x^9 + 150*a^2*b^8*x^8 + 600*a^3*b^7*x^7 + 2100
*a^4*b^6*x^6 + 2520*a^5*b^5*x^5*log(x) - 2100*a^6*b^4*x^4 - 600*a^7*b^3*x^3
 - 150*a^8*b^2*x^2 - 25*a^9*b*x - 2*a^10)/x^5
```

Sympy [A]
time = 0.17, size = 121, normalized size = 1.03

$$252a^5b^5\log(x) + 210a^4b^6x + 60a^3b^7x^2 + 15a^2b^8x^3 + \frac{5ab^9x^4}{2} + \frac{b^{10}x^5}{5} + \frac{-2a^{10} - 25a^9bx - 150a^8b^2x^2 - 600a^7b^3x^3 - 2100a^6b^4x^4}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**6,x)
```

```
[Out] 252*a**5*b**5*log(x) + 210*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x
**3 + 5*a*b**9*x**4/2 + b**10*x**5/5 + (-2*a**10 - 25*a**9*b*x - 150*a**8*b
**2*x**2 - 600*a**7*b**3*x**3 - 2100*a**6*b**4*x**4)/(10*x**5)
```

Giac [A]
time = 1.58, size = 111, normalized size = 0.95

$$\frac{1}{5}b^{10}x^5 + \frac{5}{2}ab^9x^4 + 15a^2b^8x^3 + 60a^3b^7x^2 + 210a^4b^6x + 252a^5b^5\log(|x|) - \frac{2100a^6b^4x^4 + 600a^7b^3x^3 + 150a^8b^2x^2 + 25a^9bx + 2a^{10}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(abs(x)) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 +
150*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

Mupad [B]
time = 0.10, size = 110, normalized size = 0.94

$$\frac{b^{10}x^5}{5} - \frac{\frac{a^{10}}{5} + \frac{5a^9bx}{2} + 15a^8b^2x^2 + 60a^7b^3x^3 + 210a^6b^4x^4}{x^5} + 210a^4b^6x + \frac{5ab^9x^4}{2} + 60a^3b^7x^2 + 15a^2b^8x^3 + 252a^5b^5\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^6,x)
```

```
[Out] (b^10*x^5)/5 - (a^10/5 + 15*a^8*b^2*x^2 + 60*a^7*b^3*x^3 + 210*a^6*b^4*x^4
+ (5*a^9*b*x)/2)/x^5 + 210*a^4*b^6*x + (5*a*b^9*x^4)/2 + 60*a^3*b^7*x^2 + 1
5*a^2*b^8*x^3 + 252*a^5*b^5*log(x)
```

3.141 $\int \frac{(a+bx)^{10}}{x^7} dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}+210a^4b^6\log(x)$$

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^7}\,dx &= \int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}+45a^2\right.\\ &= -\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9\end{aligned}$$

Mathematica [A]
time = 0.00, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}+210a^4b^6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45a^2b^8x^2}{2}+\frac{10a\,b^9x^3}{3}+\frac{b^{10}x^4}{4}+210a^4b$
risch	$\frac{b^{10}x^4}{4}+\frac{10a\,b^9x^3}{3}+\frac{45a^2b^8x^2}{2}+120a^3b^7x+\frac{-252a^5b^5x^5-105a^6b^4x^4-40a^7b^3x^3-\frac{45}{4}a^8b^2x^2-2a^9bx-\frac{1}{6}a^{10}}{x^6}+210a^4b^6$
norman	$\frac{-\frac{1}{6}a^{10}+\frac{1}{4}b^{10}x^{10}+\frac{10}{3}a\,b^9x^9+\frac{45}{2}a^2b^8x^8+120a^3b^7x^7-252a^5b^5x^5-105a^6b^4x^4-40a^7b^3x^3-\frac{45}{4}a^8b^2x^2-2a^9bx}{x^6}+210a^4b^6\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

Maxima [A]
time = 0.29, size = 110, normalized size = 0.92

$$\frac{1}{4}\,b^{10}x^4+\frac{10}{3}\,ab^9x^3+\frac{45}{2}\,a^2b^8x^2+120\,a^3b^7x+210\,a^4b^6\log\left(x\right)-\frac{3024\,a^5b^5x^5+1260\,a^6b^4x^4+480\,a^7b^3x^3+135\,a^8b^2x^2+24\,a^9bx+2\,a^{10}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(x) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x^3 +
135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Fricas [A]
time = 0.43, size = 114, normalized size = 0.96

$$\frac{3\,b^{10}x^{10}+40\,ab^9x^9+270\,a^2b^8x^8+1440\,a^3b^7x^7+2520\,a^4b^6x^6\log\left(x\right)-3024\,a^5b^5x^5-1260\,a^6b^4x^4-480\,a^7b^3x^3-135\,a^8b^2x^2-24\,a^9bx-2\,a^{10}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 + 40*a*b^9*x^9 + 270*a^2*b^8*x^8 + 1440*a^3*b^7*x^7 + 252
0*a^4*b^6*x^6*log(x) - 3024*a^5*b^5*x^5 - 1260*a^6*b^4*x^4 - 480*a^7*b^3*x^
3 - 135*a^8*b^2*x^2 - 24*a^9*b*x - 2*a^10)/x^6
```

Sympy [A]
time = 0.20, size = 122, normalized size = 1.03

$$210a^4b^6\log(x) + 120a^3b^7x + \frac{45a^2b^8x^2}{2} + \frac{10ab^9x^3}{3} + \frac{b^{10}x^4}{4} + \frac{-2a^{10} - 24a^9bx - 135a^8b^2x^2 - 480a^7b^3x^3 - 1260a^6b^4x^4 - 3024a^5b^5x^5}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**7,x)
```

```
[Out] 210*a**4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x**
*3/3 + b**10*x**4/4 + (-2*a**10 - 24*a**9*b*x - 135*a**8*b**2*x**2 - 480*a*
*7*b**3*x**3 - 1260*a**6*b**4*x**4 - 3024*a**5*b**5*x**5)/(12*x**6)
```

Giac [A]
time = 1.75, size = 111, normalized size = 0.93

$$\frac{1}{4}b^{10}x^4 + \frac{10}{3}ab^9x^3 + \frac{45}{2}a^2b^8x^2 + 120a^3b^7x + 210a^4b^6\log(|x|) - \frac{3024a^5b^5x^5 + 1260a^6b^4x^4 + 480a^7b^3x^3 + 135a^8b^2x^2 + 24a^9bx + 2a^{10}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(abs(x)) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x
^3 + 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

Mupad [B]
time = 0.05, size = 110, normalized size = 0.92

$$\frac{b^{10}x^4}{4} - \frac{\frac{a^{10}}{6} + 2a^9bx + \frac{45a^8b^2x^2}{4} + 40a^7b^3x^3 + 105a^6b^4x^4 + 252a^5b^5x^5}{x^6} + 120a^3b^7x + \frac{10ab^9x^3}{3} + \frac{45a^2b^8x^2}{2} + 210a^4b^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^7,x)
```

```
[Out] (b^10*x^4)/4 - (a^10/6 + (45*a^8*b^2*x^2)/4 + 40*a^7*b^3*x^3 + 105*a^6*b^4*
x^4 + 252*a^5*b^5*x^5 + 2*a^9*b*x)/x^6 + 120*a^3*b^7*x + (10*a*b^9*x^3)/3 +
 (45*a^2*b^8*x^2)/2 + 210*a^4*b^6*log(x)
```

3.142 $\int \frac{(a+bx)^{10}}{x^8}\,dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\log(x)$$

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^8}\,dx &= \int \left(45a^2b^8+\frac{a^{10}}{x^8}+\frac{10a^9b}{x^7}+\frac{45a^8b^2}{x^6}+\frac{120a^7b^3}{x^5}+\frac{210a^6b^4}{x^4}+\frac{252a^5b^5}{x^3}+\frac{210a^4b^6}{x^2}+\frac{120a}{x}\right.\\ &= -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.96

method	result
default	$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5a\,b^9x^2+\frac{b^{10}x^3}{3}+120a^3b^7\ln($
risch	$\frac{b^{10}x^3}{3}+5a\,b^9x^2+45a^2b^8x+\frac{-210a^4b^6x^6-126a^5b^5x^5-70a^6b^4x^4-30a^7b^3x^3-9a^8b^2x^2-\frac{5}{3}a^9bx-\frac{1}{7}a^{10}}{x^7}+120a^3b^7\ln(x$
norman	$\frac{-\frac{1}{7}a^{10}+\frac{1}{3}b^{10}x^{10}+5a\,b^9x^9+45a^2b^8x^8-210a^4b^6x^6-126a^5b^5x^5-70a^6b^4x^4-30a^7b^3x^3-9a^8b^2x^2-\frac{5}{3}a^9bx}{x^7}+120a^3b^7\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

Maxima [A]
time = 0.27, size = 110, normalized size = 0.96

$$\frac{1}{3}b^{10}x^3+5\,ab^9x^2+45\,a^2b^8x+120\,a^3b^7\log(x)-\frac{4410\,a^4b^6x^6+2646\,a^5b^5x^5+1470\,a^6b^4x^4+630\,a^7b^3x^3+189\,a^8b^2x^2+35\,a^9bx+3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(x) - 1/21*(4410
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 + 189*
a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Fricas [A]
time = 0.51, size = 114, normalized size = 0.99

$$\frac{7\,b^{10}x^{10}+105\,ab^9x^9+945\,a^2b^8x^8+2520\,a^3b^7x^7\log(x)-4410\,a^4b^6x^6-2646\,a^5b^5x^5-1470\,a^6b^4x^4-630\,a^7b^3x^3-189\,a^8b^2x^2-35\,a^9bx-3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*x^10 + 105*a*b^9*x^9 + 945*a^2*b^8*x^8 + 2520*a^3*b^7*x^7*log(
x) - 4410*a^4*b^6*x^6 - 2646*a^5*b^5*x^5 - 1470*a^6*b^4*x^4 - 630*a^7*b^3*x
^3 - 189*a^8*b^2*x^2 - 35*a^9*b*x - 3*a^10)/x^7
```

Sympy [A]
time = 0.24, size = 119, normalized size = 1.03

$$120a^3b^7\log(x) + 45a^2b^8x + 5ab^9x^2 + \frac{b^{10}x^3}{3} + \frac{-3a^{10} - 35a^9bx - 189a^8b^2x^2 - 630a^7b^3x^3 - 1470a^6b^4x^4 - 2646a^5b^5x^5 - 4410a^4b^6x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**8,x)
```

```
[Out] 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 + b**10*x**3/3 + (-3*
a**10 - 35*a**9*b*x - 189*a**8*b**2*x**2 - 630*a**7*b**3*x**3 - 1470*a**6*b
**4*x**4 - 2646*a**5*b**5*x**5 - 4410*a**4*b**6*x**6)/(21*x**7)
```

Giac [A]
time = 1.09, size = 111, normalized size = 0.97

$$\frac{1}{3}b^{10}x^3 + 5ab^9x^2 + 45a^2b^8x + 120a^3b^7\log(|x|) - \frac{4410a^4b^6x^6 + 2646a^5b^5x^5 + 1470a^6b^4x^4 + 630a^7b^3x^3 + 189a^8b^2x^2 + 35a^9bx + 3a^{10}}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(abs(x)) - 1/21*
(4410*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 +
 189*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

Mupad [B]
time = 0.10, size = 110, normalized size = 0.96

$$\frac{b^{10}x^3}{3} - \frac{\frac{a^{10}}{7} + \frac{5a^9bx}{3} + 9a^8b^2x^2 + 30a^7b^3x^3 + 70a^6b^4x^4 + 126a^5b^5x^5 + 210a^4b^6x^6}{x^7} + 45a^2b^8x + 5ab^9x^2 + 120a^3b^7\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^8,x)
```

```
[Out] (b^10*x^3)/3 - (a^10/7 + 9*a^8*b^2*x^2 + 30*a^7*b^3*x^3 + 70*a^6*b^4*x^4 +
126*a^5*b^5*x^5 + 210*a^4*b^6*x^6 + (5*a^9*b*x)/3)/x^7 + 45*a^2*b^8*x + 5*a
*b^9*x^2 + 120*a^3*b^7*log(x)
```

3.143 $\int \frac{(a+bx)^{10}}{x^9}\,dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\frac{b^{10}x^2}{2}+45a^2b^8\log(x)$$

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^9}\,dx &= \int \left(10ab^9+\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b^6}{x^3}+\frac{120a^3b\right.\\ &= -\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\frac{b^{10}x^2}{2}+45a^2b^8\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Maple [A]
time = 0.08, size = 110, normalized size = 0.92

method	result
default	$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10a\,b^9x+\frac{b^{10}x^2}{2}+45a^2b^8\ln$
risch	$\frac{b^{10}x^2}{2}+10a\,b^9x+\frac{-120a^3b^7x^7-105a^4b^6x^6-84a^5b^5x^5-\frac{105}{2}a^6b^4x^4-24a^7b^3x^3-\frac{15}{2}a^8b^2x^2-\frac{10}{7}a^9bx-\frac{1}{8}a^{10}}{x^8}+45a^2b^8\ln(x$
norman	$\frac{-\frac{1}{8}a^{10}+\frac{1}{2}b^{10}x^{10}+10a\,b^9x^9-120a^3b^7x^7-105a^4b^6x^6-84a^5b^5x^5-\frac{105}{2}a^6b^4x^4-24a^7b^3x^3-\frac{15}{2}a^8b^2x^2-\frac{10}{7}a^9bx}{x^8}+45a^2b^8\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

Maxima [A]
time = 0.27, size = 110, normalized size = 0.92

$$\frac{1}{2}\,b^{10}x^2+10\,ab^9x+45\,a^2b^8\log(x)-\frac{6720\,a^3b^7x^7+5880\,a^4b^6x^6+4704\,a^5b^5x^5+2940\,a^6b^4x^4+1344\,a^7b^3x^3+420\,a^8b^2x^2+80\,a^9bx+7\,a^{10}}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(x) - 1/56*(6720*a^3*b^7*x^7 + 58
80*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^3 + 4
20*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Fricas [A]
time = 0.49, size = 114, normalized size = 0.96

$$\frac{28\,b^{10}x^{10}+560\,ab^9x^9+2520\,a^2b^8x^8\log(x)-6720\,a^3b^7x^7-5880\,a^4b^6x^6-4704\,a^5b^5x^5-2940\,a^6b^4x^4-1344\,a^7b^3x^3-420\,a^8b^2x^2-80\,a^9bx-7\,a^{10}}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*x^10 + 560*a*b^9*x^9 + 2520*a^2*b^8*x^8*log(x) - 6720*a^3*b^7
*x^7 - 5880*a^4*b^6*x^6 - 4704*a^5*b^5*x^5 - 2940*a^6*b^4*x^4 - 1344*a^7*b^
3*x^3 - 420*a^8*b^2*x^2 - 80*a^9*b*x - 7*a^10)/x^8
```

Sympy [A]
time = 0.28, size = 119, normalized size = 1.00

$$45a^2b^8\log(x) + 10ab^9x + \frac{b^{10}x^2}{2} + \frac{-7a^{10} - 80a^9bx - 420a^8b^2x^2 - 1344a^7b^3x^3 - 2940a^6b^4x^4 - 4704a^5b^5x^5 - 5880a^4b^6x^6 - 6720a^3b^7x^7}{56x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**9,x)
```

```
[Out] 45*a**2*b**8*log(x) + 10*a*b**9*x + b**10*x**2/2 + (-7*a**10 - 80*a**9*b*x
- 420*a**8*b**2*x**2 - 1344*a**7*b**3*x**3 - 2940*a**6*b**4*x**4 - 4704*a**
5*b**5*x**5 - 5880*a**4*b**6*x**6 - 6720*a**3*b**7*x**7)/(56*x**8)
```

Giac [A]
time = 1.39, size = 111, normalized size = 0.93

$$\frac{1}{2}b^{10}x^2 + 10\,ab^9x + 45\,a^2b^8\log(|x|) - \frac{6720\,a^3b^7x^7 + 5880\,a^4b^6x^6 + 4704\,a^5b^5x^5 + 2940\,a^6b^4x^4 + 1344\,a^7b^3x^3 + 420\,a^8b^2x^2 + 80\,a^9bx + 7\,a^{10}}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(abs(x)) - 1/56*(6720*a^3*b^7*x^7
 + 5880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^
3 + 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

Mupad [B]
time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^2}{2} - \frac{\frac{a^{10}}{8} + \frac{10\,a^9\,b\,x}{7} + \frac{15\,a^8\,b^2\,x^2}{2} + 24\,a^7\,b^3\,x^3 + \frac{105\,a^6\,b^4\,x^4}{2} + 84\,a^5\,b^5\,x^5 + 105\,a^4\,b^6\,x^6 + 120\,a^3\,b^7\,x^7}{x^8} + 45\,a^2\,b^8\,\ln(x) + 10\,a\,b^9\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^9,x)
```

```
[Out] (b^10*x^2)/2 - (a^10/8 + (15*a^8*b^2*x^2)/2 + 24*a^7*b^3*x^3 + (105*a^6*b^4
*x^4)/2 + 84*a^5*b^5*x^5 + 105*a^4*b^6*x^6 + 120*a^3*b^7*x^7 + (10*a^9*b*x)
/7)/x^8 + 45*a^2*b^8*log(x) + 10*a*b^9*x
```

3.144 $\int \frac{(a+bx)^{10}}{x^{10}} \, dx$

Optimal. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{10}}\,dx &= \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^1\end{aligned}$$

Mathematica [A]
time = 0.00, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.96

method	result	si
default	$-\frac{a^{10}}{9x^{9}}-\frac{5a^{9}b}{4x^{8}}-\frac{45a^{8}b^{2}}{7x^{7}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{42a^{6}b^{4}}{x^{5}}-\frac{63a^{5}b^{5}}{x^{4}}-\frac{70a^{4}b^{6}}{x^{3}}-\frac{60a^{3}b^{7}}{x^{2}}-\frac{45a^{2}b^{8}}{x}+b^{10}x+10a\,b^{9}\ln\left(x\right)$	10
risch	$b^{10}x+\frac{-45a^{2}b^{8}x^{8}-60a^{3}b^{7}x^{7}-70a^{4}b^{6}x^{6}-63a^{5}b^{5}x^{5}-42a^{6}b^{4}x^{4}-20a^{7}b^{3}x^{3}-\frac{45}{7}a^{8}b^{2}x^{2}-\frac{5}{4}a^{9}bx-\frac{1}{9}a^{10}}{x^{9}}+10a\,b^{9}\ln\left(x\right)$	10
norman	$\frac{b^{10}x^{10}-\frac{1}{9}a^{10}-45a^{2}b^{8}x^{8}-60a^{3}b^{7}x^{7}-70a^{4}b^{6}x^{6}-63a^{5}b^{5}x^{5}-42a^{6}b^{4}x^{4}-20a^{7}b^{3}x^{3}-\frac{45}{7}a^{8}b^{2}x^{2}-\frac{5}{4}a^{9}bx}{x^{9}}+10a\,b^{9}\ln\left(x\right)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Maxima [A]
time = 0.27, size = 109, normalized size = 0.96

$$b^{10}x+10\,ab^{9}\log\left(x\right)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+1620\,a^{8}b^{2}x^{2}+315\,a^{9}bx+28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Fricas [A]
time = 0.43, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log\left(x\right)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A]
time = 0.33, size = 117, normalized size = 1.03

$$10ab^9\log(x) + b^{10}x + \frac{-28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 17640a^4b^6x^6 - 15120a^3b^7x^7 - 11340a^2b^8x^8}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

Giac [A]
time = 1.80, size = 110, normalized size = 0.96

$$b^{10}x + 10\,ab^9\log(|x|) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b^2x^2 + 315\,a^9bx + 28\,a^{10}}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Mupad [B]
time = 0.08, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^8\,b^2\,x^2}{7} + 20\,a^7\,b^3\,x^3 + 42\,a^6\,b^4\,x^4 + 63\,a^5\,b^5\,x^5 + 70\,a^4\,b^6\,x^6 + 60\,a^3\,b^7\,x^7 + 45\,a^2\,b^8\,x^8 + \frac{5\,a^9\,b\,x}{4} - 10\,a\,b^9\,x^9\,\ln(x)}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

3.145 $\int \frac{(a+bx)^{10}}{x^{11}}\,dx$

Optimal. Leaf size=124

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

```
[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^
4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-1
0*a*b^9/x+b^10*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^11,x]
```

```
[Out] -1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{11}}\,dx &= \int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b^7}{x^4}+\frac{45a\right.\\ &= -\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 124, normalized size = 1.00

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^10/x^11,x]

[Out] -1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4) - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]

Maple [A]
time = 0.08, size = 111, normalized size = 0.90

method	result
default	$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a\,b^9}{x}+b^{10}\ln\left(x\right)$
norman	$\frac{-\frac{1}{10}a^{10}-10a\,b^9x^9-\frac{45}{2}a^2b^8x^8-40a^3b^7x^7-\frac{105}{2}a^4b^6x^6-\frac{252}{5}a^5b^5x^5-35a^6b^4x^4-\frac{120}{7}a^7b^3x^3-\frac{45}{8}a^8b^2x^2-\frac{10}{9}a^9bx}{x^{10}}+b^{10}\ln\left(x\right)$
risch	$\frac{-\frac{1}{10}a^{10}-10a\,b^9x^9-\frac{45}{2}a^2b^8x^8-40a^3b^7x^7-\frac{105}{2}a^4b^6x^6-\frac{252}{5}a^5b^5x^5-35a^6b^4x^4-\frac{120}{7}a^7b^3x^3-\frac{45}{8}a^8b^2x^2-\frac{10}{9}a^9bx}{x^{10}}+b^{10}\ln\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^10/x^11,x,method=_RETURNVERBOSE)

[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-10*a*b^9/x+b^10*ln(x)

Maxima [A]
time = 0.29, size = 111, normalized size = 0.90

$$b^{10}\log\left(x\right)-\frac{25200\,ab^9x^9+56700\,a^2b^8x^8+100800\,a^3b^7x^7+132300\,a^4b^6x^6+127008\,a^5b^5x^5+88200\,a^6b^4x^4+43200\,a^7b^3x^3+14175\,a^8b^2x^2+2800\,a^9bx+252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10/x^11,x, algorithm="maxima")

[Out] b^10*log(x) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10

Fricas [A]
time = 0.45, size = 114, normalized size = 0.92

$$\frac{2520\,b^{10}x^{10}\log\left(x\right)-25200\,ab^9x^9-56700\,a^2b^8x^8-100800\,a^3b^7x^7-132300\,a^4b^6x^6-127008\,a^5b^5x^5-88200\,a^6b^4x^4-43200\,a^7b^3x^3-14175\,a^8b^2x^2-2800\,a^9bx-252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10/x^11,x, algorithm="fricas")

[Out] 1/2520*(2520*b^10*x^10*log(x) - 25200*a*b^9*x^9 - 56700*a^2*b^8*x^8 - 100800*a^3*b^7*x^7 - 132300*a^4*b^6*x^6 - 127008*a^5*b^5*x^5 - 88200*a^6*b^4*x^4 - 43200*a^7*b^3*x^3 - 14175*a^8*b^2*x^2 - 2800*a^9*b*x - 252*a^10)/x^10

Sympy [A]
time = 0.37, size = 119, normalized size = 0.96

$$b^{10}\log(x)+\frac{-252a^{10}-2800a^{9}bx-14175a^{8}b^{2}x^{2}-43200a^{7}b^{3}x^{3}-88200a^{6}b^{4}x^{4}-127008a^{5}b^{5}x^{5}-132300a^{4}b^{6}x^{6}-100800a^{3}b^{7}x^{7}-56700a^{2}b^{8}x^{8}-25200ab^{9}x^{9}}{2520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**11,x)
```

```
[Out] b**10*log(x) + (-252*a**10 - 2800*a**9*b*x - 14175*a**8*b**2*x**2 - 43200*a
**7*b**3*x**3 - 88200*a**6*b**4*x**4 - 127008*a**5*b**5*x**5 - 132300*a**4*
b**6*x**6 - 100800*a**3*b**7*x**7 - 56700*a**2*b**8*x**8 - 25200*a*b**9*x**
9)/(2520*x**10)
```

Giac [A]
time = 1.28, size = 112, normalized size = 0.90

$$b^{10}\log(|x|)-\frac{25200\,ab^{9}x^{9}+56700\,a^{2}b^{8}x^{8}+100800\,a^{3}b^{7}x^{7}+132300\,a^{4}b^{6}x^{6}+127008\,a^{5}b^{5}x^{5}+88200\,a^{6}b^{4}x^{4}+43200\,a^{7}b^{3}x^{3}+14175\,a^{8}b^{2}x^{2}+2800\,a^{9}bx+252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="giac")
```

```
[Out] b^10*log(abs(x)) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3
*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43
200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

Mupad [B]
time = 0.07, size = 111, normalized size = 0.90

$$b^{10}\,\ln(x)-\frac{\frac{a^{10}}{10}+\frac{10\,a^9\,b\,x}{9}+\frac{45\,a^8\,b^2\,x^2}{8}+\frac{120\,a^7\,b^3\,x^3}{7}+35\,a^6\,b^4\,x^4+\frac{252\,a^5\,b^5\,x^5}{5}+\frac{105\,a^4\,b^6\,x^6}{2}+40\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{2}+10\,a\,b^9\,x^9}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^11,x)
```

```
[Out] b^10*log(x) - (a^10/10 + 10*a*b^9*x^9 + (45*a^8*b^2*x^2)/8 + (120*a^7*b^3*x
^3)/7 + 35*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/5 + (105*a^4*b^6*x^6)/2 + 40*a^3
*b^7*x^7 + (45*a^2*b^8*x^8)/2 + (10*a^9*b*x)/9)/x^10
```

3.146 $\int \frac{(a+bx)^{10}}{x^{12}}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

```
[Out] -1/11*(b*x+a)^11/a/x^11
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^12,x]
```

```
[Out] -1/11*(a + b*x)^11/(a*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{12}}\,dx = -\frac{(a+bx)^{11}}{11ax^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(17) = 34.
time = 0.01, size = 114, normalized size = 6.71

$$-\frac{a^{10}}{11x^{11}}-\frac{a^9b}{x^{10}}-\frac{5a^8b^2}{x^9}-\frac{15a^7b^3}{x^8}-\frac{30a^6b^4}{x^7}-\frac{42a^5b^5}{x^6}-\frac{42a^4b^6}{x^5}-\frac{30a^3b^7}{x^4}-\frac{15a^2b^8}{x^3}-\frac{5ab^9}{x^2}-\frac{b^{10}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^12,x]
```

```
[Out] -1/11*a^10/x^11 - (a^9*b)/x^10 - (5*a^8*b^2)/x^9 - (15*a^7*b^3)/x^8 - (30*a
^6*b^4)/x^7 - (42*a^5*b^5)/x^6 - (42*a^4*b^6)/x^5 - (30*a^3*b^7)/x^4 - (15*
a^2*b^8)/x^3 - (5*a*b^9)/x^2 - b^10/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(15) = 30$.
time = 0.08, size = 113, normalized size = 6.65

method	result	size
gosper	$-\frac{11b^{10}x^{10}+55a\,b^{9}x^{9}+165a^{2}b^{8}x^{8}+330a^{3}b^{7}x^{7}+462a^{4}b^{6}x^{6}+462a^{5}b^{5}x^{5}+330a^{6}b^{4}x^{4}+165a^{7}b^{3}x^{3}+55a^{8}b^{2}x^{2}+11a^{9}bx+a^{10}}{11x^{11}}$	111
norman	$\frac{-b^{10}x^{10}-5a\,b^{9}x^{9}-15a^{2}b^{8}x^{8}-30a^{3}b^{7}x^{7}-42a^{4}b^{6}x^{6}-42a^{5}b^{5}x^{5}-30a^{6}b^{4}x^{4}-15a^{7}b^{3}x^{3}-5a^{8}b^{2}x^{2}-a^{9}bx-\frac{1}{11}a^{10}}{x^{11}}$	112
risch	$\frac{-b^{10}x^{10}-5a\,b^{9}x^{9}-15a^{2}b^{8}x^{8}-30a^{3}b^{7}x^{7}-42a^{4}b^{6}x^{6}-42a^{5}b^{5}x^{5}-30a^{6}b^{4}x^{4}-15a^{7}b^{3}x^{3}-5a^{8}b^{2}x^{2}-a^{9}bx-\frac{1}{11}a^{10}}{x^{11}}$	112
default	$-\frac{a^{9}b}{x^{10}}-\frac{a^{10}}{11x^{11}}-\frac{b^{10}}{x}-\frac{15a^{2}b^{8}}{x^{3}}-\frac{30a^{3}b^{7}}{x^{4}}-\frac{5a^{8}b^{2}}{x^{9}}-\frac{5a\,b^{9}}{x^{2}}-\frac{15a^{7}b^{3}}{x^{8}}-\frac{42a^{4}b^{6}}{x^{5}}-\frac{42a^{5}b^{5}}{x^{6}}-\frac{30a^{6}b^{4}}{x^{7}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -a^9*b/x^10-1/11*a^10/x^11-b^10/x-15*a^2*b^8/x^3-30*a^3*b^7/x^4-5*a^8*b^2/x
^9-5*a*b^9/x^2-15*a^7*b^3/x^8-42*a^4*b^6/x^5-42*a^5*b^5/x^6-30*a^6*b^4/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.
time = 0.29, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.
time = 0.53, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(14) = 28$.

time = 0.40, size = 119, normalized size = 7.00

$$\frac{-a^{10} - 11a^9bx - 55a^8b^2x^2 - 165a^7b^3x^3 - 330a^6b^4x^4 - 462a^5b^5x^5 - 462a^4b^6x^6 - 330a^3b^7x^7 - 165a^2b^8x^8 - 55ab^9x^9 - 11b^{10}x^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**12,x)
```

```
[Out] (-a**10 - 11*a**9*b*x - 55*a**8*b**2*x**2 - 165*a**7*b**3*x**3 - 330*a**6*b
**4*x**4 - 462*a**5*b**5*x**5 - 462*a**4*b**6*x**6 - 330*a**3*b**7*x**7 - 1
65*a**2*b**8*x**8 - 55*a*b**9*x**9 - 11*b**10*x**10)/(11*x**11)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(15) = 30$.

time = 1.05, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10} + 55\,ab^9x^9 + 165\,a^2b^8x^8 + 330\,a^3b^7x^7 + 462\,a^4b^6x^6 + 462\,a^5b^5x^5 + 330\,a^6b^4x^4 + 165\,a^7b^3x^3 + 55\,a^8b^2x^2 + 11\,a^9bx + a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

Mupad [B]

time = 0.13, size = 110, normalized size = 6.47

$$-\frac{\frac{a^{10}}{11} + a^9\,b\,x + 5\,a^8\,b^2\,x^2 + 15\,a^7\,b^3\,x^3 + 30\,a^6\,b^4\,x^4 + 42\,a^5\,b^5\,x^5 + 42\,a^4\,b^6\,x^6 + 30\,a^3\,b^7\,x^7 + 15\,a^2\,b^8\,x^8 + 5\,a\,b^9\,x^9 + b^{10}\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^12,x)
```

```
[Out] -(a^10/11 + b^10*x^10 + 5*a*b^9*x^9 + 5*a^8*b^2*x^2 + 15*a^7*b^3*x^3 + 30*a
^6*b^4*x^4 + 42*a^5*b^5*x^5 + 42*a^4*b^6*x^6 + 30*a^3*b^7*x^7 + 15*a^2*b^8*
x^8 + a^9*b*x)/x^11
```

3.147 $\int \frac{(a+bx)^{10}}{x^{13}} dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^{11}}{12ax^{12}}+\frac{b(a+bx)^{11}}{132a^2x^{11}}$$

```
[Out] -1/12*(b*x+a)^11/a/x^12+1/132*b*(b*x+a)^11/a^2/x^11
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^13,x]
```

```
[Out] -1/12*(a + b*x)^11/(a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{13}}\,dx &= -\frac{(a+bx)^{11}}{12ax^{12}}-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{12a}\\ &= -\frac{(a+bx)^{11}}{12ax^{12}}+\frac{b(a+bx)^{11}}{132a^2x^{11}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(36) = 72.
time = 0.00, size = 128, normalized size = 3.56

$$-\frac{a^{10}}{12x^{12}}-\frac{10a^9b}{11x^{11}}-\frac{9a^8b^2}{2x^{10}}-\frac{40a^7b^3}{3x^9}-\frac{105a^6b^4}{4x^8}-\frac{36a^5b^5}{x^7}-\frac{35a^4b^6}{x^6}-\frac{24a^3b^7}{x^5}-\frac{45a^2b^8}{4x^4}-\frac{10ab^9}{3x^3}-\frac{b^{10}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^13,x]
```

```
[Out] -1/12*a^10/x^12 - (10*a^9*b)/(11*x^11) - (9*a^8*b^2)/(2*x^10) - (40*a^7*b^3
)/(3*x^9) - (105*a^6*b^4)/(4*x^8) - (36*a^5*b^5)/x^7 - (35*a^4*b^6)/x^6 - (
24*a^3*b^7)/x^5 - (45*a^2*b^8)/(4*x^4) - (10*a*b^9)/(3*x^3) - b^10/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(32) = 64.
time = 0.08, size = 113, normalized size = 3.14

method	result
norman	$\frac{-\frac{1}{2}b^{10}x^{10}-\frac{10}{3}a\,b^9x^9-\frac{45}{4}a^2b^8x^8-24a^3b^7x^7-35a^4b^6x^6-36a^5b^5x^5-\frac{105}{4}a^6b^4x^4-\frac{40}{3}a^7b^3x^3-\frac{9}{2}a^8b^2x^2-\frac{10}{11}a^9bx-\frac{1}{12}a^{10}}{x^{12}}$
risch	$\frac{-\frac{1}{2}b^{10}x^{10}-\frac{10}{3}a\,b^9x^9-\frac{45}{4}a^2b^8x^8-24a^3b^7x^7-35a^4b^6x^6-36a^5b^5x^5-\frac{105}{4}a^6b^4x^4-\frac{40}{3}a^7b^3x^3-\frac{9}{2}a^8b^2x^2-\frac{10}{11}a^9bx-\frac{1}{12}a^{10}}{x^{12}}$
gosper	$-\frac{66b^{10}x^{10}+440a\,b^9x^9+1485a^2b^8x^8+3168a^3b^7x^7+4620a^4b^6x^6+4752a^5b^5x^5+3465a^6b^4x^4+1760a^7b^3x^3+594a^8b^2x^2+120a^9bx+11}{132x^{12}}$
default	$-\frac{9a^8b^2}{2x^{10}}-\frac{10a^9b}{11x^{11}}-\frac{10a\,b^9}{3x^3}-\frac{45a^2b^8}{4x^4}-\frac{a^{10}}{12x^{12}}-\frac{40a^7b^3}{3x^9}-\frac{b^{10}}{2x^2}-\frac{105a^6b^4}{4x^8}-\frac{24a^3b^7}{x^5}-\frac{35a^4b^6}{x^6}-\frac{36a^5b^5}{x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -9/2*a^8*b^2/x^10-10/11*a^9*b/x^11-10/3*a*b^9/x^3-45/4*a^2*b^8/x^4-1/12*a^1
0/x^12-40/3*a^7*b^3/x^9-1/2*b^10/x^2-105/4*a^6*b^4/x^8-24*a^3*b^7/x^5-35*a^
4*b^6/x^6-36*a^5*b^5/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(32) = 64.
time = 0.28, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^9x^9+1485\,a^2b^8x^8+3168\,a^3b^7x^7+4620\,a^4b^6x^6+4752\,a^5b^5x^5+3465\,a^6b^4x^4+1760\,a^7b^3x^3+594\,a^8b^2x^2+120\,a^9bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(32) = 64$.

time = 0.46, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^{9}x^{9}+1485\,a^{2}b^{8}x^{8}+3168\,a^{3}b^{7}x^{7}+4620\,a^{4}b^{6}x^{6}+4752\,a^{5}b^{5}x^{5}+3465\,a^{6}b^{4}x^{4}+1760\,a^{7}b^{3}x^{3}+594\,a^{8}b^{2}x^{2}+120\,a^{9}bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="fricas")
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(29) = 58$.

time = 0.41, size = 121, normalized size = 3.36

$$\frac{-11a^{10}-120a^{9}bx-594a^{8}b^{2}x^{2}-1760a^{7}b^{3}x^{3}-3465a^{6}b^{4}x^{4}-4752a^{5}b^{5}x^{5}-4620a^{4}b^{6}x^{6}-3168a^{3}b^{7}x^{7}-1485a^{2}b^{8}x^{8}-440ab^{9}x^{9}-66b^{10}x^{10}}{132x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**13,x)
[Out] (-11*a**10 - 120*a**9*b*x - 594*a**8*b**2*x**2 - 1760*a**7*b**3*x**3 - 3465
*a**6*b**4*x**4 - 4752*a**5*b**5*x**5 - 4620*a**4*b**6*x**6 - 3168*a**3*b**
7*x**7 - 1485*a**2*b**8*x**8 - 440*a*b**9*x**9 - 66*b**10*x**10)/(132*x**12
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(32) = 64$.

time = 1.24, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^{9}x^{9}+1485\,a^{2}b^{8}x^{8}+3168\,a^{3}b^{7}x^{7}+4620\,a^{4}b^{6}x^{6}+4752\,a^{5}b^{5}x^{5}+3465\,a^{6}b^{4}x^{4}+1760\,a^{7}b^{3}x^{3}+594\,a^{8}b^{2}x^{2}+120\,a^{9}bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="giac")
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

Mupad [B]

time = 0.10, size = 23, normalized size = 0.64

$$-\frac{(11\,a-b\,x)\,{(a+b\,x)}^{11}}{132\,a^2\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^13,x)
[Out] -((11*a - b*x)*(a + b*x)^11)/(132*a^2*x^12)
```

3.148 $\int \frac{(a+bx)^{10}}{x^{14}}\,dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^{11}}{13ax^{13}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}$$

```
[Out] -1/13*(b*x+a)^11/a/x^13+1/78*b*(b*x+a)^11/a^2/x^12-1/858*b^2*(b*x+a)^11/a^3
/x^11
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^14,x]
```

```
[Out] -1/13*(a + b*x)^11/(a*x^13) + (b*(a + b*x)^11)/(78*a^2*x^12) - (b^2*(a + b*
x)^11)/(858*a^3*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx = -\frac{(a+bx)^{11}}{13ax^{13}} - \frac{(2b)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{13a}$$
$$= -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{78a^2}$$
$$= -\frac{(a+bx)^{11}}{13ax^{13}} + \frac{b(a+bx)^{11}}{78a^2x^{12}} - \frac{b^2(a+bx)^{11}}{858a^3x^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(56) = 112$.

time = 0.01, size = 126, normalized size = 2.25

$$-\frac{a^{10}}{13x^{13}} - \frac{5a^9b}{6x^{12}} - \frac{45a^8b^2}{11x^{11}} - \frac{12a^7b^3}{x^{10}} - \frac{70a^6b^4}{3x^9} - \frac{63a^5b^5}{2x^8} - \frac{30a^4b^6}{x^7} - \frac{20a^3b^7}{x^6} - \frac{9a^2b^8}{x^5} - \frac{5ab^9}{2x^4} - \frac{b^{10}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^14,x]
```

```
[Out] -1/13*a^10/x^13 - (5*a^9*b)/(6*x^12) - (45*a^8*b^2)/(11*x^11) - (12*a^7*b^3
)/x^10 - (70*a^6*b^4)/(3*x^9) - (63*a^5*b^5)/(2*x^8) - (30*a^4*b^6)/x^7 - (
20*a^3*b^7)/x^6 - (9*a^2*b^8)/x^5 - (5*a*b^9)/(2*x^4) - b^10/(3*x^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.

time = 0.08, size = 113, normalized size = 2.02

method	result
norman	$\frac{-\frac{1}{3}b^{10}x^{10}-\frac{5}{2}a\,b^9x^9-9a^2b^8x^8-20a^3b^7x^7-30a^4b^6x^6-\frac{63}{2}a^5b^5x^5-\frac{70}{3}a^6b^4x^4-12a^7b^3x^3-\frac{45}{11}a^8b^2x^2-\frac{5}{6}a^9bx-\frac{1}{13}a^{10}}{x^{13}}$
risch	$\frac{-\frac{1}{3}b^{10}x^{10}-\frac{5}{2}a\,b^9x^9-9a^2b^8x^8-20a^3b^7x^7-30a^4b^6x^6-\frac{63}{2}a^5b^5x^5-\frac{70}{3}a^6b^4x^4-12a^7b^3x^3-\frac{45}{11}a^8b^2x^2-\frac{5}{6}a^9bx-\frac{1}{13}a^{10}}{x^{13}}$
gosper	$-\frac{286b^{10}x^{10}+2145a\,b^9x^9+7722a^2b^8x^8+17160a^3b^7x^7+25740a^4b^6x^6+27027a^5b^5x^5+20020a^6b^4x^4+10296a^7b^3x^3+3510a^8b^2x^2+715a^9}{858x^{13}}$
default	$-\frac{12a^7b^3}{x^{10}}-\frac{45a^8b^2}{11x^{11}}-\frac{b^{10}}{3x^3}-\frac{5a\,b^9}{2x^4}-\frac{5a^9b}{6x^{12}}-\frac{70a^6b^4}{3x^9}-\frac{63a^5b^5}{2x^8}-\frac{9a^2b^8}{x^5}-\frac{a^{10}}{13x^{13}}-\frac{20a^3b^7}{x^6}-\frac{30a^4b^6}{x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -12*a^7*b^3/x^10-45/11*a^8*b^2/x^11-1/3*b^10/x^3-5/2*a*b^9/x^4-5/6*a^9*b/x^
12-70/3*a^6*b^4/x^9-63/2*a^5*b^5/x^8-9*a^2*b^8/x^5-1/13*a^10/x^13-20*a^3*b^
7/x^6-30*a^4*b^6/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(50) = 100$.

time = 0.28, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(50) = 100.
time = 0.49, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(48) = 96.
time = 0.44, size = 121, normalized size = 2.16

$$\frac{-66a^{10}-715a^{9}bx-3510a^{8}b^{2}x^{2}-10296a^{7}b^{3}x^{3}-20020a^{6}b^{4}x^{4}-27027a^{5}b^{5}x^{5}-25740a^{4}b^{6}x^{6}-17160a^{3}b^{7}x^{7}-7722a^{2}b^{8}x^{8}-2145ab^{9}x^{9}-286b^{10}x^{10}}{858x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**14,x)
```

```
[Out] (-66*a**10 - 715*a**9*b*x - 3510*a**8*b**2*x**2 - 10296*a**7*b**3*x**3 - 20
020*a**6*b**4*x**4 - 27027*a**5*b**5*x**5 - 25740*a**4*b**6*x**6 - 17160*a*
*3*b**7*x**7 - 7722*a**2*b**8*x**8 - 2145*a*b**9*x**9 - 286*b**10*x**10)/(8
58*x**13)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(50) = 100.
time = 1.09, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="giac")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

Mupad [B]
time = 0.13, size = 112, normalized size = 2.00

$$-\frac{\frac{a^{10}}{13}+\frac{5\,a^9\,b\,x}{6}+\frac{45\,a^8\,b^2\,x^2}{11}+12\,a^7\,b^3\,x^3+\frac{70\,a^6\,b^4\,x^4}{3}+\frac{63\,a^5\,b^5\,x^5}{2}+30\,a^4\,b^6\,x^6+20\,a^3\,b^7\,x^7+9\,a^2\,b^8\,x^8+\frac{5\,a\,b^9\,x^9}{2}+\frac{b^{10}\,x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^14,x)
```

```
[Out] -(a^10/13 + (b^10*x^10)/3 + (5*a*b^9*x^9)/2 + (45*a^8*b^2*x^2)/11 + 12*a^7*
b^3*x^3 + (70*a^6*b^4*x^4)/3 + (63*a^5*b^5*x^5)/2 + 30*a^4*b^6*x^6 + 20*a^3
*b^7*x^7 + 9*a^2*b^8*x^8 + (5*a^9*b*x)/6)/x^13
```

3.149 $\int \frac{(a+bx)^{10}}{x^{15}}\,dx$

Optimal. Leaf size=76

$$-\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}$$

```
[Out] -1/14*(b*x+a)^11/a/x^14+3/182*b*(b*x+a)^11/a^2/x^13-1/364*b^2*(b*x+a)^11/a^
3/x^12+1/4004*b^3*(b*x+a)^11/a^4/x^11
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^15,x]
```

```
[Out] -1/14*(a + b*x)^11/(a*x^14) + (3*b*(a + b*x)^11)/(182*a^2*x^13) - (b^2*(a +
 b*x)^11)/(364*a^3*x^12) + (b^3*(a + b*x)^11)/(4004*a^4*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx = -\frac{(a+bx)^{11}}{14ax^{14}} - \frac{(3b)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{14a}$$
$$= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} + \frac{(3b^2)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{91a^2}$$
$$= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{364a^3}$$
$$= -\frac{(a+bx)^{11}}{14ax^{14}} + \frac{3b(a+bx)^{11}}{182a^2x^{13}} - \frac{b^2(a+bx)^{11}}{364a^3x^{12}} + \frac{b^3(a+bx)^{11}}{4004a^4x^{11}}$$

Mathematica [A]
time = 0.01, size = 128, normalized size = 1.68

$$-\frac{a^{10}}{14x^{14}} - \frac{10a^9b}{13x^{13}} - \frac{15a^8b^2}{4x^{12}} - \frac{120a^7b^3}{11x^{11}} - \frac{21a^6b^4}{x^{10}} - \frac{28a^5b^5}{x^9} - \frac{105a^4b^6}{4x^8} - \frac{120a^3b^7}{7x^7} - \frac{15a^2b^8}{2x^6} - \frac{2ab^9}{x^5} - \frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^15,x]
```

```
[Out] -1/14*a^10/x^14 - (10*a^9*b)/(13*x^13) - (15*a^8*b^2)/(4*x^12) - (120*a^7*b
^3)/(11*x^11) - (21*a^6*b^4)/x^10 - (28*a^5*b^5)/x^9 - (105*a^4*b^6)/(4*x^8
) - (120*a^3*b^7)/(7*x^7) - (15*a^2*b^8)/(2*x^6) - (2*a*b^9)/x^5 - b^10/(4*
x^4)
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.49

method	result
norman	$\frac{-\frac{1}{14}a^{10}-\frac{10}{13}a^9bx-\frac{15}{4}a^8b^2x^2-\frac{120}{11}a^7b^3x^3-21a^6b^4x^4-28a^5b^5x^5-\frac{105}{4}a^4b^6x^6-\frac{120}{7}a^3b^7x^7-\frac{15}{2}a^2b^8x^8-2a\,b^9x^9-\frac{1}{4}b^{10}x^{10}}{x^{14}}$
risch	$\frac{-\frac{1}{14}a^{10}-\frac{10}{13}a^9bx-\frac{15}{4}a^8b^2x^2-\frac{120}{11}a^7b^3x^3-21a^6b^4x^4-28a^5b^5x^5-\frac{105}{4}a^4b^6x^6-\frac{120}{7}a^3b^7x^7-\frac{15}{2}a^2b^8x^8-2a\,b^9x^9-\frac{1}{4}b^{10}x^{10}}{x^{14}}$
gosper	$-\frac{1001b^{10}x^{10}+8008a\,b^9x^9+30030a^2b^8x^8+68640a^3b^7x^7+105105a^4b^6x^6+112112a^5b^5x^5+84084a^6b^4x^4+43680a^7b^3x^3+15015a^8b^2x^2+}{4004x^{14}}$
default	$-\frac{21a^6b^4}{x^{10}}-\frac{120a^7b^3}{11x^{11}}-\frac{a^{10}}{14x^{14}}-\frac{b^{10}}{4x^4}-\frac{15a^8b^2}{4x^{12}}-\frac{28a^5b^5}{x^9}-\frac{105a^4b^6}{4x^8}-\frac{2a\,b^9}{x^5}-\frac{10a^9b}{13x^{13}}-\frac{15a^2b^8}{2x^6}-\frac{120a^3b^7}{7x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -21*a^6*b^4/x^10-120/11*a^7*b^3/x^11-1/14*a^10/x^14-1/4*b^10/x^4-15/4*a^8*b
^2/x^12-28*a^5*b^5/x^9-105/4*a^4*b^6/x^8-2*a*b^9/x^5-10/13*a^9*b/x^13-15/2*
a^2*b^8/x^6-120/7*a^3*b^7/x^7
```

Maxima [A]
time = 0.27, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Fricas [A]
time = 0.55, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Sympy [A]
time = 0.46, size = 121, normalized size = 1.59

$$\frac{-286a^{10}-3080a^{9}bx-15015a^{8}b^{2}x^{2}-43680a^{7}b^{3}x^{3}-84084a^{6}b^{4}x^{4}-112112a^{5}b^{5}x^{5}-105105a^{4}b^{6}x^{6}-68640a^{3}b^{7}x^{7}-30030a^{2}b^{8}x^{8}-8008ab^{9}x^{9}-1001b^{10}x^{10}}{4004x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**15,x)
```

```
[Out] (-286*a**10 - 3080*a**9*b*x - 15015*a**8*b**2*x**2 - 43680*a**7*b**3*x**3 -
 84084*a**6*b**4*x**4 - 112112*a**5*b**5*x**5 - 105105*a**4*b**6*x**6 - 686
40*a**3*b**7*x**7 - 30030*a**2*b**8*x**8 - 8008*a*b**9*x**9 - 1001*b**10*x*
*10)/(4004*x**14)
```

Giac [A]
time = 1.64, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

Mupad [B]
time = 0.09, size = 112, normalized size = 1.47

$$-\frac{\frac{a^{10}}{14}+\frac{10\,a^9\,b\,x}{13}+\frac{15\,a^8\,b^2\,x^2}{4}+\frac{120\,a^7\,b^3\,x^3}{11}+21\,a^6\,b^4\,x^4+28\,a^5\,b^5\,x^5+\frac{105\,a^4\,b^6\,x^6}{4}+\frac{120\,a^3\,b^7\,x^7}{7}+\frac{15\,a^2\,b^8\,x^8}{2}+2\,a\,b^9\,x^9+\frac{b^{10}\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^15,x)
```

```
[Out] -(a^10/14 + (b^10*x^10)/4 + 2*a*b^9*x^9 + (15*a^8*b^2*x^2)/4 + (120*a^7*b^3
*x^3)/11 + 21*a^6*b^4*x^4 + 28*a^5*b^5*x^5 + (105*a^4*b^6*x^6)/4 + (120*a^3
*b^7*x^7)/7 + (15*a^2*b^8*x^8)/2 + (10*a^9*b*x)/13)/x^14
```

3.150 $\int \frac{(a+bx)^{10}}{x^{16}} dx$

Optimal. Leaf size=96

$$-\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}$$

```
[Out] -1/15*(b*x+a)^11/a/x^15+2/105*b*(b*x+a)^11/a^2/x^14-2/455*b^2*(b*x+a)^11/a^
3/x^13+1/1365*b^3*(b*x+a)^11/a^4/x^12-1/15015*b^4*(b*x+a)^11/a^5/x^11
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^16,x]
```

```
[Out] -1/15*(a + b*x)^11/(a*x^15) + (2*b*(a + b*x)^11)/(105*a^2*x^14) - (2*b^2*(a
 + b*x)^11)/(455*a^3*x^13) + (b^3*(a + b*x)^11)/(1365*a^4*x^12) - (b^4*(a +
 b*x)^11)/(15015*a^5*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{16}}\,dx &= -\frac{(a+bx)^{11}}{15ax^{15}} - \frac{(4b)\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{15a} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} + \frac{(2b^2)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{35a^2} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} - \frac{(4b^3)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{455a^3} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{1365a^4} \\
&= -\frac{(a+bx)^{11}}{15ax^{15}} + \frac{2b(a+bx)^{11}}{105a^2x^{14}} - \frac{2b^2(a+bx)^{11}}{455a^3x^{13}} + \frac{b^3(a+bx)^{11}}{1365a^4x^{12}} - \frac{b^4(a+bx)^{11}}{15015a^5x^{11}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 130, normalized size = 1.35

$$
-\frac{a^{10}}{15x^{15}} - \frac{5a^9b}{7x^{14}} - \frac{45a^8b^2}{13x^{13}} - \frac{10a^7b^3}{x^{12}} - \frac{210a^6b^4}{11x^{11}} - \frac{126a^5b^5}{5x^{10}} - \frac{70a^4b^6}{3x^9} - \frac{15a^3b^7}{x^8} - \frac{45a^2b^8}{7x^7} - \frac{5ab^9}{3x^6} - \frac{b^{10}}{5x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^16,x]
```

```
[Out] -1/15*a^10/x^15 - (5*a^9*b)/(7*x^14) - (45*a^8*b^2)/(13*x^13) - (10*a^7*b^3
)/x^12 - (210*a^6*b^4)/(11*x^11) - (126*a^5*b^5)/(5*x^10) - (70*a^4*b^6)/(3
*x^9) - (15*a^3*b^7)/x^8 - (45*a^2*b^8)/(7*x^7) - (5*a*b^9)/(3*x^6) - b^10/
(5*x^5)
```

Maple [A]
time = 0.08, size = 113, normalized size = 1.18

method	result
norman	$\frac{-\frac{1}{15}a^{10}-\frac{5}{7}a^9bx-\frac{45}{13}a^8b^2x^2-10a^7b^3x^3-\frac{210}{11}a^6b^4x^4-\frac{126}{5}a^5b^5x^5-\frac{70}{3}a^4b^6x^6-15a^3b^7x^7-\frac{45}{7}a^2b^8x^8-\frac{5}{3}a\,b^9x^9-\frac{1}{5}b^{10}x^{10}}{x^{15}}$
risch	$\frac{-\frac{1}{15}a^{10}-\frac{5}{7}a^9bx-\frac{45}{13}a^8b^2x^2-10a^7b^3x^3-\frac{210}{11}a^6b^4x^4-\frac{126}{5}a^5b^5x^5-\frac{70}{3}a^4b^6x^6-15a^3b^7x^7-\frac{45}{7}a^2b^8x^8-\frac{5}{3}a\,b^9x^9-\frac{1}{5}b^{10}x^{10}}{x^{15}}$
gosper	$-\frac{3003b^{10}x^{10}+25025a\,b^9x^9+96525a^2b^8x^8+225225a^3b^7x^7+350350a^4b^6x^6+378378a^5b^5x^5+286650a^6b^4x^4+150150a^7b^3x^3+51975a^8b}{15015x^{15}}$
default	$-\frac{126a^5b^5}{5x^{10}}-\frac{210a^6b^4}{11x^{11}}-\frac{5a^9b}{7x^{14}}-\frac{10a^7b^3}{x^{12}}-\frac{70a^4b^6}{3x^9}-\frac{15a^3b^7}{x^8}-\frac{b^{10}}{5x^5}-\frac{45a^8b^2}{13x^{13}}-\frac{a^{10}}{15x^{15}}-\frac{5a\,b^9}{3x^6}-\frac{45a^2b^8}{7x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -126/5*a^5*b^5/x^10-210/11*a^6*b^4/x^11-5/7*a^9*b/x^14-10*a^7*b^3/x^12-70/3
*a^4*b^6/x^9-15*a^3*b^7/x^8-1/5*b^10/x^5-45/13*a^8*b^2/x^13-1/15*a^10/x^15-
5/3*a*b^9/x^6-45/7*a^2*b^8/x^7
```

Maxima [A]
time = 0.27, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Fricas [A]
time = 0.61, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Sympy [A]
time = 0.48, size = 121, normalized size = 1.26

$$\frac{-1001a^{10}-10725a^{9}bx-51975a^{8}b^{2}x^{2}-150150a^{7}b^{3}x^{3}-286650a^{6}b^{4}x^{4}-378378a^{5}b^{5}x^{5}-350350a^{4}b^{6}x^{6}-225225a^{3}b^{7}x^{7}-96525a^{2}b^{8}x^{8}-25025ab^{9}x^{9}-3003b^{10}x^{10}}{15015x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**16,x)
```

```
[Out] (-1001*a**10 - 10725*a**9*b*x - 51975*a**8*b**2*x**2 - 150150*a**7*b**3*x**
3 - 286650*a**6*b**4*x**4 - 378378*a**5*b**5*x**5 - 350350*a**4*b**6*x**6 -
 225225*a**3*b**7*x**7 - 96525*a**2*b**8*x**8 - 25025*a*b**9*x**9 - 3003*b*
*10*x**10)/(15015*x**15)
```

Giac [A]
time = 1.63, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

Mupad [B]
time = 0.13, size = 112, normalized size = 1.17

$$-\frac{\frac{a^{10}}{15}+\frac{5\,a^9\,b\,x}{7}+\frac{45\,a^8\,b^2\,x^2}{13}+10\,a^7\,b^3\,x^3+\frac{210\,a^6\,b^4\,x^4}{11}+\frac{126\,a^5\,b^5\,x^5}{5}+\frac{70\,a^4\,b^6\,x^6}{3}+15\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{7}+\frac{5\,a\,b^9\,x^9}{3}+\frac{b^{10}\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^16,x)
```

```
[Out] -(a^10/15 + (b^10*x^10)/5 + (5*a*b^9*x^9)/3 + (45*a^8*b^2*x^2)/13 + 10*a^7*
b^3*x^3 + (210*a^6*b^4*x^4)/11 + (126*a^5*b^5*x^5)/5 + (70*a^4*b^6*x^6)/3 +
 15*a^3*b^7*x^7 + (45*a^2*b^8*x^8)/7 + (5*a^9*b*x)/7)/x^15
```

3.151 $\int \frac{(a+bx)^{10}}{x^{17}} dx$

Optimal. Leaf size=116

$$-\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}$$

```
[Out] -1/16*(b*x+a)^11/a/x^16+1/48*b*(b*x+a)^11/a^2/x^15-1/168*b^2*(b*x+a)^11/a^3
/x^14+1/728*b^3*(b*x+a)^11/a^4/x^13-1/4368*b^4*(b*x+a)^11/a^5/x^12+1/48048*
b^5*(b*x+a)^11/a^6/x^11
```

Rubi [A]
time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^17,x]
```

```
[Out] -1/16*(a + b*x)^11/(a*x^16) + (b*(a + b*x)^11)/(48*a^2*x^15) - (b^2*(a + b*
x)^11)/(168*a^3*x^14) + (b^3*(a + b*x)^11)/(728*a^4*x^13) - (b^4*(a + b*x)^
11)/(4368*a^5*x^12) + (b^5*(a + b*x)^11)/(48048*a^6*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{17}}\,dx &= -\frac{(a+bx)^{11}}{16ax^{16}} - \frac{(5b)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{16a} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} + \frac{b^2\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{12a^2} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{56a^3} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} + \frac{b^4\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{364a^4} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} - \frac{b^5\int \frac{(a+bx)^{10}}{x^{12}}\,d}{4368a^5} \\
&= -\frac{(a+bx)^{11}}{16ax^{16}} + \frac{b(a+bx)^{11}}{48a^2x^{15}} - \frac{b^2(a+bx)^{11}}{168a^3x^{14}} + \frac{b^3(a+bx)^{11}}{728a^4x^{13}} - \frac{b^4(a+bx)^{11}}{4368a^5x^{12}} + \frac{b^5(a+bx)^{11}}{48048a^6x^{11}}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 132, normalized size = 1.14

$$
-\frac{a^{10}}{16x^{16}} - \frac{2a^9b}{3x^{15}} - \frac{45a^8b^2}{14x^{14}} - \frac{120a^7b^3}{13x^{13}} - \frac{35a^6b^4}{2x^{12}} - \frac{252a^5b^5}{11x^{11}} - \frac{21a^4b^6}{x^{10}} - \frac{40a^3b^7}{3x^9} - \frac{45a^2b^8}{8x^8} - \frac{10ab^9}{7x^7} - \frac{b^{10}}{6x^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^17,x]
```

```
[Out] -1/16*a^10/x^16 - (2*a^9*b)/(3*x^15) - (45*a^8*b^2)/(14*x^14) - (120*a^7*b^
3)/(13*x^13) - (35*a^6*b^4)/(2*x^12) - (252*a^5*b^5)/(11*x^11) - (21*a^4*b^
6)/x^10 - (40*a^3*b^7)/(3*x^9) - (45*a^2*b^8)/(8*x^8) - (10*a*b^9)/(7*x^7)
- b^10/(6*x^6)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.97

method	result
norman	$\frac{-\frac{1}{16}a^{10}-\frac{2}{3}a^9bx-\frac{45}{14}a^8b^2x^2-\frac{120}{13}a^7b^3x^3-\frac{35}{2}a^6b^4x^4-\frac{252}{11}a^5b^5x^5-21a^4b^6x^6-\frac{40}{3}a^3b^7x^7-\frac{45}{8}a^2b^8x^8-\frac{10}{7}ab^9x^9-\frac{1}{6}b^{10}x^{10}}{x^{16}}$
risch	$\frac{-\frac{1}{16}a^{10}-\frac{2}{3}a^9bx-\frac{45}{14}a^8b^2x^2-\frac{120}{13}a^7b^3x^3-\frac{35}{2}a^6b^4x^4-\frac{252}{11}a^5b^5x^5-21a^4b^6x^6-\frac{40}{3}a^3b^7x^7-\frac{45}{8}a^2b^8x^8-\frac{10}{7}ab^9x^9-\frac{1}{6}b^{10}x^{10}}{x^{16}}$
gosper	$-\frac{8008b^{10}x^{10}+68640ab^9x^9+270270a^2b^8x^8+640640a^3b^7x^7+1009008a^4b^6x^6+1100736a^5b^5x^5+840840a^6b^4x^4+443520a^7b^3x^3+15444}{48048x^{16}}$
default	$-\frac{21a^4b^6}{x^{10}}-\frac{252a^5b^5}{11x^{11}}-\frac{45a^8b^2}{14x^{14}}-\frac{35a^6b^4}{2x^{12}}-\frac{40a^3b^7}{3x^9}-\frac{45a^2b^8}{8x^8}-\frac{120a^7b^3}{13x^{13}}-\frac{2a^9b}{3x^{15}}-\frac{b^{10}}{6x^6}-\frac{10ab^9}{7x^7}-\frac{a^{10}}{16x^{16}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -21*a^4*b^6/x^10-252/11*a^5*b^5/x^11-45/14*a^8*b^2/x^14-35/2*a^6*b^4/x^12-4
0/3*a^3*b^7/x^9-45/8*a^2*b^8/x^8-120/13*a^7*b^3/x^13-2/3*a^9*b/x^15-1/6*b^1
0/x^6-10/7*a*b^9/x^7-1/16*a^10/x^16
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^{9}x^{9}+270270\,a^{2}b^{8}x^{8}+640640\,a^{3}b^{7}x^{7}+1009008\,a^{4}b^{6}x^{6}+1100736\,a^{5}b^{5}x^{5}+840840\,a^{6}b^{4}x^{4}+443520\,a^{7}b^{3}x^{3}+154440\,a^{8}b^{2}x^{2}+32032\,a^{9}bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Fricas [A]
time = 0.57, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^{9}x^{9}+270270\,a^{2}b^{8}x^{8}+640640\,a^{3}b^{7}x^{7}+1009008\,a^{4}b^{6}x^{6}+1100736\,a^{5}b^{5}x^{5}+840840\,a^{6}b^{4}x^{4}+443520\,a^{7}b^{3}x^{3}+154440\,a^{8}b^{2}x^{2}+32032\,a^{9}bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Sympy [A]
time = 0.50, size = 121, normalized size = 1.04

$$\frac{-3003a^{10}-32032a^{9}bx-154440a^{8}b^{2}x^{2}-443520a^{7}b^{3}x^{3}-840840a^{6}b^{4}x^{4}-1100736a^{5}b^{5}x^{5}-1009008a^{4}b^{6}x^{6}-640640a^{3}b^{7}x^{7}-270270a^{2}b^{8}x^{8}-68640ab^{9}x^{9}-8008b^{10}x^{10}}{48048x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**17,x)
```

```
[Out] (-3003*a**10 - 32032*a**9*b*x - 154440*a**8*b**2*x**2 - 443520*a**7*b**3*x*
*3 - 840840*a**6*b**4*x**4 - 1100736*a**5*b**5*x**5 - 1009008*a**4*b**6*x**
6 - 640640*a**3*b**7*x**7 - 270270*a**2*b**8*x**8 - 68640*a*b**9*x**9 - 800
8*b**10*x**10)/(48048*x**16)
```

Giac [A]
time = 0.86, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^{9}x^{9}+270270\,a^{2}b^{8}x^{8}+640640\,a^{3}b^{7}x^{7}+1009008\,a^{4}b^{6}x^{6}+1100736\,a^{5}b^{5}x^{5}+840840\,a^{6}b^{4}x^{4}+443520\,a^{7}b^{3}x^{3}+154440\,a^{8}b^{2}x^{2}+32032\,a^{9}bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

Mupad [B]
time = 0.13, size = 112, normalized size = 0.97

$$-\frac{\frac{a^{10}}{16}+\frac{2\,a^9\,b\,x}{3}+\frac{45\,a^8\,b^2\,x^2}{14}+\frac{120\,a^7\,b^3\,x^3}{13}+\frac{35\,a^6\,b^4\,x^4}{2}+\frac{252\,a^5\,b^5\,x^5}{11}+21\,a^4\,b^6\,x^6+\frac{40\,a^3\,b^7\,x^7}{3}+\frac{45\,a^2\,b^8\,x^8}{8}+\frac{10\,a\,b^9\,x^9}{7}+\frac{b^{10}\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^17,x)
```

```
[Out] -(a^10/16 + (b^10*x^10)/6 + (10*a*b^9*x^9)/7 + (45*a^8*b^2*x^2)/14 + (120*a
^7*b^3*x^3)/13 + (35*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/11 + 21*a^4*b^6*x^6
 + (40*a^3*b^7*x^7)/3 + (45*a^2*b^8*x^8)/8 + (2*a^9*b*x)/3)/x^16
```

3.152 $\int \frac{(a+bx)^{10}}{x^{18}}\,dx$

Optimal. Leaf size=136

$$-\frac{(a+bx)^{11}}{17ax^{17}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}$$

```
[Out] -1/17*(b*x+a)^11/a/x^17+3/136*b*(b*x+a)^11/a^2/x^16-1/136*b^2*(b*x+a)^11/a^
3/x^15+1/476*b^3*(b*x+a)^11/a^4/x^14-3/6188*b^4*(b*x+a)^11/a^5/x^13+1/12376
*b^5*(b*x+a)^11/a^6/x^12-1/136136*b^6*(b*x+a)^11/a^7/x^11
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^18,x]
```

```
[Out] -1/17*(a + b*x)^11/(a*x^17) + (3*b*(a + b*x)^11)/(136*a^2*x^16) - (b^2*(a +
 b*x)^11)/(136*a^3*x^15) + (b^3*(a + b*x)^11)/(476*a^4*x^14) - (3*b^4*(a +
b*x)^11)/(6188*a^5*x^13) + (b^5*(a + b*x)^11)/(12376*a^6*x^12) - (b^6*(a +
b*x)^11)/(136136*a^7*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{(a+bx)^{11}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx)^{10}}{x^{17}}\,dx}{17a}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} + \frac{(15b^2)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{136a^2}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{34a^3}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} + \frac{(3b^4)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{476a^4}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} - \frac{(3b^5)\int \frac{(a+}{x}}{3094}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)}{12376a^6x^1}$$

$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)}{12376a^6x^1}$$

Mathematica [A]
time = 0.01, size = 126, normalized size = 0.93

$$-\frac{a^{10}}{17x^{17}} - \frac{5a^9b}{8x^{16}} - \frac{3a^8b^2}{x^{15}} - \frac{60a^7b^3}{7x^{14}} - \frac{210a^6b^4}{13x^{13}} - \frac{21a^5b^5}{x^{12}} - \frac{210a^4b^6}{11x^{11}} - \frac{12a^3b^7}{x^{10}} - \frac{5a^2b^8}{x^9} - \frac{5ab^9}{4x^8} - \frac{b^{10}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^18,x]
```

```
[Out] -1/17*a^10/x^17 - (5*a^9*b)/(8*x^16) - (3*a^8*b^2)/x^15 - (60*a^7*b^3)/(7*x
^14) - (210*a^6*b^4)/(13*x^13) - (21*a^5*b^5)/x^12 - (210*a^4*b^6)/(11*x^11
) - (12*a^3*b^7)/x^10 - (5*a^2*b^8)/x^9 - (5*a*b^9)/(4*x^8) - b^10/(7*x^7)
```

Maple [A]
time = 0.09, size = 113, normalized size = 0.83

method	result
norman	$\frac{-\frac{1}{17}a^{10}-\frac{5}{8}a^9bx-3a^8b^2x^2-\frac{60}{7}a^7b^3x^3-\frac{210}{13}a^6b^4x^4-21a^5b^5x^5-\frac{210}{11}a^4b^6x^6-12a^3b^7x^7-5a^2b^8x^8-\frac{5}{4}a\,b^9x^9-\frac{1}{7}b^{10}x^{10}}{x^{17}}$
risch	$\frac{-\frac{1}{17}a^{10}-\frac{5}{8}a^9bx-3a^8b^2x^2-\frac{60}{7}a^7b^3x^3-\frac{210}{13}a^6b^4x^4-21a^5b^5x^5-\frac{210}{11}a^4b^6x^6-12a^3b^7x^7-5a^2b^8x^8-\frac{5}{4}a\,b^9x^9-\frac{1}{7}b^{10}x^{10}}{x^{17}}$
gosper	$-\frac{19448b^{10}x^{10}+170170a\,b^9x^9+680680a^2b^8x^8+1633632a^3b^7x^7+2598960a^4b^6x^6+2858856a^5b^5x^5+2199120a^6b^4x^4+1166880a^7b^3x^3+}{136136x^{17}}$
default	$-\frac{12a^3b^7}{x^{10}}-\frac{210a^4b^6}{11x^{11}}-\frac{60a^7b^3}{7x^{14}}-\frac{a^{10}}{17x^{17}}-\frac{21a^5b^5}{x^{12}}-\frac{5a^2b^8}{x^9}-\frac{5a\,b^9}{4x^8}-\frac{210a^6b^4}{13x^{13}}-\frac{3a^8b^2}{x^{15}}-\frac{b^{10}}{7x^7}-\frac{5a^9b}{8x^{16}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -12*a^3*b^7/x^10-210/11*a^4*b^6/x^11-60/7*a^7*b^3/x^14-1/17*a^10/x^17-21*a^
5*b^5/x^12-5*a^2*b^8/x^9-5/4*a*b^9/x^8-210/13*a^6*b^4/x^13-3*a^8*b^2/x^15-1
/7*b^10/x^7-5/8*a^9*b/x^16
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^9x^9+680680\,a^2b^8x^8+1633632\,a^3b^7x^7+2598960\,a^4b^6x^6+2858856\,a^5b^5x^5+2199120\,a^6b^4x^4+1166880\,a^7b^3x^3+408408\,a^8b^2x^2+85085\,a^9bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Fricas [A]
time = 0.66, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^9x^9+680680\,a^2b^8x^8+1633632\,a^3b^7x^7+2598960\,a^4b^6x^6+2858856\,a^5b^5x^5+2199120\,a^6b^4x^4+1166880\,a^7b^3x^3+408408\,a^8b^2x^2+85085\,a^9bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Sympy [A]
time = 0.53, size = 121, normalized size = 0.89

$$\frac{-8008a^{10}-85085a^9bx-408408a^8b^2x^2-1166880a^7b^3x^3-2199120a^6b^4x^4-2858856a^5b^5x^5-2598960a^4b^6x^6-1633632a^3b^7x^7-680680a^2b^8x^8-170170ab^9x^9-19448b^{10}x^{10}}{136136x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**18,x)
```

```
[Out] (-8008*a**10 - 85085*a**9*b*x - 408408*a**8*b**2*x**2 - 1166880*a**7*b**3*x
**3 - 2199120*a**6*b**4*x**4 - 2858856*a**5*b**5*x**5 - 2598960*a**4*b**6*x
**6 - 1633632*a**3*b**7*x**7 - 680680*a**2*b**8*x**8 - 170170*a*b**9*x**9 -
 19448*b**10*x**10)/(136136*x**17)
```

Giac [A]
time = 1.01, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^9x^9+680680\,a^2b^8x^8+1633632\,a^3b^7x^7+2598960\,a^4b^6x^6+2858856\,a^5b^5x^5+2199120\,a^6b^4x^4+1166880\,a^7b^3x^3+408408\,a^8b^2x^2+85085\,a^9bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

Mupad [B]
time = 0.13, size = 112, normalized size = 0.82

$$-\frac{\frac{a^{10}}{17}+\frac{5\,a^9\,b\,x}{8}+3\,a^8\,b^2\,x^2+\frac{60\,a^7\,b^3\,x^3}{7}+\frac{210\,a^6\,b^4\,x^4}{13}+21\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{11}+12\,a^3\,b^7\,x^7+5\,a^2\,b^8\,x^8+\frac{5\,a\,b^9\,x^9}{4}+\frac{b^{10}\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^18,x)
```

```
[Out] -(a^10/17 + (b^10*x^10)/7 + (5*a*b^9*x^9)/4 + 3*a^8*b^2*x^2 + (60*a^7*b^3*x
^3)/7 + (210*a^6*b^4*x^4)/13 + 21*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/11 + 12*a
^3*b^7*x^7 + 5*a^2*b^8*x^8 + (5*a^9*b*x)/8)/x^17
```

3.153 $\int \frac{(a+bx)^{10}}{x^{19}} \, dx$

Optimal. Leaf size=130

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

Rubi [A]
time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^19,x]
```

```
[Out] -1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{19}} \, dx &= \int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b^7}{x^{12}}+45\right.\\ &= -\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\end{aligned}$$

Mathematica [A]
time = 0.00, size = 130, normalized size = 1.00

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^19,x]
```

```
[Out] -1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.87

method	result
norman	$\frac{-\frac{1}{18}a^{10}-\frac{10}{17}a^{9}bx-\frac{45}{16}a^{8}b^{2}x^{2}-8a^{7}b^{3}x^{3}-15a^{6}b^{4}x^{4}-\frac{252}{13}a^{5}b^{5}x^{5}-\frac{35}{2}a^{4}b^{6}x^{6}-\frac{120}{11}a^{3}b^{7}x^{7}-\frac{9}{2}a^{2}b^{8}x^{8}-\frac{10}{9}a\,b^{9}x^{9}-\frac{1}{8}b^{10}x^{10}}{x^{18}}$
risch	$\frac{-\frac{1}{18}a^{10}-\frac{10}{17}a^{9}bx-\frac{45}{16}a^{8}b^{2}x^{2}-8a^{7}b^{3}x^{3}-15a^{6}b^{4}x^{4}-\frac{252}{13}a^{5}b^{5}x^{5}-\frac{35}{2}a^{4}b^{6}x^{6}-\frac{120}{11}a^{3}b^{7}x^{7}-\frac{9}{2}a^{2}b^{8}x^{8}-\frac{10}{9}a\,b^{9}x^{9}-\frac{1}{8}b^{10}x^{10}}{x^{18}}$
gosper	$-\frac{43758b^{10}x^{10}+388960a\,b^{9}x^{9}+1575288a^{2}b^{8}x^{8}+3818880a^{3}b^{7}x^{7}+6126120a^{4}b^{6}x^{6}+6785856a^{5}b^{5}x^{5}+5250960a^{6}b^{4}x^{4}+2800512a^{7}b^{3}x^{3}-}{350064x^{18}}$
default	$-\frac{a^{10}}{18x^{18}}-\frac{10a^{9}b}{17x^{17}}-\frac{45a^{8}b^{2}}{16x^{16}}-\frac{8a^{7}b^{3}}{x^{15}}-\frac{15a^{6}b^{4}}{x^{14}}-\frac{252a^{5}b^{5}}{13x^{13}}-\frac{35a^{4}b^{6}}{2x^{12}}-\frac{120a^{3}b^{7}}{11x^{11}}-\frac{9a^{2}b^{8}}{2x^{10}}-\frac{10a\,b^{9}}{9x^{9}}-\frac{b^{10}}{8x^{8}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

Maxima [A]
time = 0.28, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

Fricas [A]
time = 1.00, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

Sympy [A]
time = 0.54, size = 121, normalized size = 0.93

$$\frac{-19448a^{10} - 205920a^{9}bx - 984555a^{8}b^{2}x^{2} - 2800512a^{7}b^{3}x^{3} - 5250960a^{6}b^{4}x^{4} - 6785856a^{5}b^{5}x^{5} - 6126120a^{4}b^{6}x^{6} - 3818880a^{3}b^{7}x^{7} - 1575288a^{2}b^{8}x^{8} - 388960ab^{9}x^{9} - 43758b^{10}x^{10}}{350064x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**19,x)
```

```
[Out] (-19448*a**10 - 205920*a**9*b*x - 984555*a**8*b**2*x**2 - 2800512*a**7*b**3
*x**3 - 5250960*a**6*b**4*x**4 - 6785856*a**5*b**5*x**5 - 6126120*a**4*b**6
*x**6 - 3818880*a**3*b**7*x**7 - 1575288*a**2*b**8*x**8 - 388960*a*b**9*x**
9 - 43758*b**10*x**10)/(350064*x**18)
```

Giac [A]
time = 1.06, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10} + 388960\,ab^{9}x^{9} + 1575288\,a^{2}b^{8}x^{8} + 3818880\,a^{3}b^{7}x^{7} + 6126120\,a^{4}b^{6}x^{6} + 6785856\,a^{5}b^{5}x^{5} + 5250960\,a^{6}b^{4}x^{4} + 2800512\,a^{7}b^{3}x^{3} + 984555\,a^{8}b^{2}x^{2} + 205920\,a^{9}bx + 19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

Mupad [B]
time = 0.10, size = 112, normalized size = 0.86

$$-\frac{\frac{a^{10}}{18} + \frac{10\,a^9\,b\,x}{17} + \frac{45\,a^8\,b^2\,x^2}{16} + 8\,a^7\,b^3\,x^3 + 15\,a^6\,b^4\,x^4 + \frac{252\,a^5\,b^5\,x^5}{13} + \frac{35\,a^4\,b^6\,x^6}{2} + \frac{120\,a^3\,b^7\,x^7}{11} + \frac{9\,a^2\,b^8\,x^8}{2} + \frac{10\,a\,b^9\,x^9}{9} + \frac{b^{10}\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^19,x)
```

```
[Out] -(a^10/18 + (b^10*x^10)/8 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/16 + 8*a^7*
b^3*x^3 + 15*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/13 + (35*a^4*b^6*x^6)/2 + (120
*a^3*b^7*x^7)/11 + (9*a^2*b^8*x^8)/2 + (10*a^9*b*x)/17)/x^18
```

3.154 $\int \frac{(a+bx)^{10}}{x^{20}} dx$

Optimal. Leaf size=126

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^20,x]
```

```
[Out] -1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{20}}\,dx = \int \left(\frac{a^{10}}{x^{20}}+\frac{10a^9b}{x^{19}}+\frac{45a^8b^2}{x^{18}}+\frac{120a^7b^3}{x^{17}}+\frac{210a^6b^4}{x^{16}}+\frac{252a^5b^5}{x^{15}}+\frac{210a^4b^6}{x^{14}}+\frac{120a^3b^7}{x^{13}}+\frac{45a}{x^1}\right.$$
$$= -\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-$$

Mathematica [A]
time = 0.01, size = 126, normalized size = 1.00

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^20,x]
```

```
[Out] -1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Maple [A]
time = 0.08, size = 113, normalized size = 0.90

method	result
norman	$\frac{-\frac{1}{19}a^{10}-\frac{5}{9}a^{9}bx-\frac{45}{17}a^{8}b^{2}x^{2}-\frac{15}{2}a^{7}b^{3}x^{3}-14a^{6}b^{4}x^{4}-18a^{5}b^{5}x^{5}-\frac{210}{13}a^{4}b^{6}x^{6}-10a^{3}b^{7}x^{7}-\frac{45}{11}a^{2}b^{8}x^{8}-a\,b^{9}x^{9}-\frac{1}{9}b^{10}x^{10}}{x^{19}}$
risch	$\frac{-\frac{1}{19}a^{10}-\frac{5}{9}a^{9}bx-\frac{45}{17}a^{8}b^{2}x^{2}-\frac{15}{2}a^{7}b^{3}x^{3}-14a^{6}b^{4}x^{4}-18a^{5}b^{5}x^{5}-\frac{210}{13}a^{4}b^{6}x^{6}-10a^{3}b^{7}x^{7}-\frac{45}{11}a^{2}b^{8}x^{8}-a\,b^{9}x^{9}-\frac{1}{9}b^{10}x^{10}}{x^{19}}$
gosper	$-\frac{92378b^{10}x^{10}+831402a\,b^{9}x^{9}+3401190a^{2}b^{8}x^{8}+8314020a^{3}b^{7}x^{7}+13430340a^{4}b^{6}x^{6}+14965236a^{5}b^{5}x^{5}+11639628a^{6}b^{4}x^{4}+6235515a^{7}}{831402x^{19}}$
default	$-\frac{a^{10}}{19x^{19}}-\frac{5a^{9}b}{9x^{18}}-\frac{45a^{8}b^{2}}{17x^{17}}-\frac{15a^{7}b^{3}}{2x^{16}}-\frac{14a^{6}b^{4}}{x^{15}}-\frac{18a^{5}b^{5}}{x^{14}}-\frac{210a^{4}b^{6}}{13x^{13}}-\frac{10a^{3}b^{7}}{x^{12}}-\frac{45a^{2}b^{8}}{11x^{11}}-\frac{a\,b^{9}}{x^{10}}-\frac{b^{10}}{9x^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^20,x,method=_RETURNVERBOSE)
```

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+11639628\,a^{6}b^{4}x^{4}+6235515\,a^{7}b^{3}x^{3}+2200770\,a^{8}b^{2}x^{2}+461890\,a^{9}bx+43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Fricas [A]
time = 0.82, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10}+831402\,ab^{9}x^{9}+3401190\,a^{2}b^{8}x^{8}+8314020\,a^{3}b^{7}x^{7}+13430340\,a^{4}b^{6}x^{6}+14965236\,a^{5}b^{5}x^{5}+11639628\,a^{6}b^{4}x^{4}+6235515\,a^{7}b^{3}x^{3}+2200770\,a^{8}b^{2}x^{2}+461890\,a^{9}bx+43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Sympy [A]
time = 0.57, size = 121, normalized size = 0.96

$$\frac{-43758a^{10} - 461890a^9bx - 2200770a^8b^2x^2 - 6235515a^7b^3x^3 - 11639628a^6b^4x^4 - 14965236a^5b^5x^5 - 13430340a^4b^6x^6 - 8314020a^3b^7x^7 - 3401190a^2b^8x^8 - 831402ab^9x^9 - 92378b^{10}x^{10}}{831402x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**20,x)
```

```
[Out] (-43758*a**10 - 461890*a**9*b*x - 2200770*a**8*b**2*x**2 - 6235515*a**7*b**
3*x**3 - 11639628*a**6*b**4*x**4 - 14965236*a**5*b**5*x**5 - 13430340*a**4*
b**6*x**6 - 8314020*a**3*b**7*x**7 - 3401190*a**2*b**8*x**8 - 831402*a*b**9
*x**9 - 92378*b**10*x**10)/(831402*x**19)
```

Giac [A]
time = 1.20, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^9x^9 + 3401190\,a^2b^8x^8 + 8314020\,a^3b^7x^7 + 13430340\,a^4b^6x^6 + 14965236\,a^5b^5x^5 + 11639628\,a^6b^4x^4 + 6235515\,a^7b^3x^3 + 2200770\,a^8b^2x^2 + 461890\,a^9bx + 43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

Mupad [B]
time = 0.14, size = 111, normalized size = 0.88

$$-\frac{\frac{a^{10}}{19} + \frac{5\,a^9\,b\,x}{9} + \frac{45\,a^8\,b^2\,x^2}{17} + \frac{15\,a^7\,b^3\,x^3}{2} + 14\,a^6\,b^4\,x^4 + 18\,a^5\,b^5\,x^5 + \frac{210\,a^4\,b^6\,x^6}{13} + 10\,a^3\,b^7\,x^7 + \frac{45\,a^2\,b^8\,x^8}{11} + a\,b^9\,x^9 + \frac{b^{10}\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^20,x)
```

```
[Out] -(a^10/19 + (b^10*x^10)/9 + a*b^9*x^9 + (45*a^8*b^2*x^2)/17 + (15*a^7*b^3*x
^3)/2 + 14*a^6*b^4*x^4 + 18*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/13 + 10*a^3*b^7
*x^7 + (45*a^2*b^8*x^8)/11 + (5*a^9*b*x)/9)/x^19
```

3.155 $\int c(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^2}{2b}$$

```
[Out] 1/2*c*(b*x+a)^2/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{9\}$

$$\frac{c(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[c*(a + b*x),x]
[Out] (c*(a + b*x)^2)/(2*b)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[a*((b + c*x)^2/(2*c)), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int c(a+bx)\,dx = \frac{c(a+bx)^2}{2b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 0.93

$$c\left(ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[c*(a + b*x),x]
[Out] c*(a*x + (b*x^2)/2)
```

Maple [A]
time = 0.01, size = 13, normalized size = 0.87

method	result	size
gosper	$\frac{x(bx+2a)c}{2}$	12
default	$\left(\frac{1}{2}x^2b+ax\right)c$	13
norman	$acx+\frac{1}{2}bc\,x^2$	13
risch	$acx+\frac{1}{2}bc\,x^2$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(b*x+a),x,method=_RETURNVERBOSE)
[Out] (1/2*x^2*b+a*x)*c
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^2+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

Fricas [A]
time = 0.89, size = 12, normalized size = 0.80

$$\frac{1}{2}x^2cb+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="fricas")
[Out] 1/2*x^2*c*b + x*c*a
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.80

$$acx+\frac{bcx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x)
[Out] a*c*x + b*c*x**2/2
```

Giac [A]
time = 1.30, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^2+2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.73

$$\frac{c\,x\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(a + b*x),x)
```

```
[Out] (c*x*(2*a + b*x))/2
```

3.156 $\int \frac{(c+d)(a+bx)}{e}\,dx$

Optimal. Leaf size=20

$$\frac{(c+d)(a+bx)^2}{2be}$$

```
[Out] 1/2*(c+d)*(b*x+a)^2/b/e
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {9}

$$\frac{(c+d)(a+bx)^2}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d)*(a + b*x))/e,x]
```

```
[Out] ((c + d)*(a + b*x)^2)/(2*b*e)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[a*((b + c*x)^2/(2*c)), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(c+d)(a+bx)}{e}\,dx = \frac{(c+d)(a+bx)^2}{2be}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.95

$$\frac{(c+d)\left(ax+\frac{bx^2}{2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d)*(a + b*x))/e,x]
```

```
[Out] ((c + d)*(a*x + (b*x^2)/2))/e
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.90

method	result	size
gosper	$\frac{x(bx+2a)(c+d)}{2e}$	17
default	$\frac{(\frac{1}{2}x^2b+ax)(c+d)}{e}$	18
norman	$\frac{a(c+d)x}{e}+\frac{(c+d)b\,x^2}{2e}$	23
risch	$\frac{axc}{e}+\frac{axd}{e}+\frac{b\,x^2c}{2e}+\frac{b\,x^2d}{2e}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d)*(b*x+a)/e,x,method=_RETURNVERBOSE)
[Out] (1/2*x^2*b+a*x)*(c+d)/e
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.85

$$\frac{1}{2}\left(bx^2+2\,ax\right)(c+d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

Fricas [A]
time = 1.37, size = 26, normalized size = 1.30

$$\frac{1}{2}\left((bc+bd)x^2+2\,(ac+ad)x\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="fricas")
[Out] 1/2*((b*c + b*d)*x^2 + 2*(a*c + a*d)*x)*e^(-1)
```

Sympy [A]
time = 0.01, size = 22, normalized size = 1.10

$$\frac{x^2(bc+bd)}{2e}+\frac{x(ac+ad)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x)
[Out] x**2*(b*c + b*d)/(2*e) + x*(a*c + a*d)/e
```

Giac [A]
time = 1.44, size = 17, normalized size = 0.85

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)(c+d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

Mupad [B]
time = 0.07, size = 16, normalized size = 0.80

$$\frac{x\,(c+d)\,\left(2\,a+b\,x\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d)*(a + b*x))/e,x)
```

```
[Out] (x*(c + d)*(2*a + b*x))/(2*e)
```

3.157 $\int \frac{x^5}{a+bx}\,dx$

Optimal. Leaf size=70

$$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}$$

```
[Out] a^4*x/b^5-1/2*a^3*x^2/b^4+1/3*a^2*x^3/b^3-1/4*a*x^4/b^2+1/5*x^5/b-a^5*ln(b*
x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx}\,dx&=\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx\\&=\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 70, normalized size = 1.00

$$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Maple [A]
time = 0.08, size = 63, normalized size = 0.90

method	result	size
default	$\frac{\frac{1}{5}b^4x^5-\frac{1}{4}a\,b^3x^4+\frac{1}{3}a^2b^2x^3-\frac{1}{2}a^3b\,x^2+a^4x}{b^5}-\frac{a^5\ln(bx+a)}{b^6}$	63
norman	$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{a\,x^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\ln(bx+a)}{b^6}$	63
risch	$\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{a\,x^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\ln(bx+a)}{b^6}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*b^4*x^5-1/4*a*b^3*x^4+1/3*a^2*b^2*x^3-1/2*a^3*b*x^2+a^4*x)-a^5*l
n(b*x+a)/b^6
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.91

$$-\frac{a^5\log(bx+a)}{b^6}+\frac{12\,b^4x^5-15\,ab^3x^4+20\,a^2b^2x^3-30\,a^3bx^2+60\,a^4x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -a^5*log(b*x + a)/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x^3 -
30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Fricas [A]
time = 0.87, size = 63, normalized size = 0.90

$$\frac{12\,b^5x^5-15\,ab^4x^4+20\,a^2b^3x^3-30\,a^3b^2x^2+60\,a^4bx-60\,a^5\log(bx+a)}{60\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*x^5 - 15*a*b^4*x^4 + 20*a^2*b^3*x^3 - 30*a^3*b^2*x^2 + 60*a^4*
b*x - 60*a^5*log(b*x + a))/b^6
```

Sympy [A]
time = 0.04, size = 61, normalized size = 0.87

$$-\frac{a^5\log\left(a+bx\right)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a),x)
```

```
[Out] -a**5*log(a + b*x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b
**3) - a*x**4/(4*b**2) + x**5/(5*b)
```

Giac [A]
time = 1.26, size = 65, normalized size = 0.93

$$-\frac{a^5\log\left(|bx+a|\right)}{b^6}+\frac{12\,b^4x^5-15\,ab^3x^4+20\,a^2b^2x^3-30\,a^3bx^2+60\,a^4x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -a^5*log(abs(b*x + a))/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x
^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

Mupad [B]
time = 0.08, size = 62, normalized size = 0.89

$$\frac{x^5}{5\,b}-\frac{a^5\,\ln\left(a+b\,x\right)}{b^6}-\frac{a\,x^4}{4\,b^2}+\frac{a^4\,x}{b^5}+\frac{a^2\,x^3}{3\,b^3}-\frac{a^3\,x^2}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x),x)
```

```
[Out] x^5/(5*b) - (a^5*log(a + b*x))/b^6 - (a*x^4)/(4*b^2) + (a^4*x)/b^5 + (a^2*x
^3)/(3*b^3) - (a^3*x^2)/(2*b^4)
```

3.158 $\int \frac{x^4}{a+bx} \, dx$

Optimal. Leaf size=57

$$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{a+bx}\,dx &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.00

$$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{1}{3}a\,b^2x^3-\frac{1}{2}a^2b\,x^2+a^3x}{b^4}+\frac{a^4\ln(bx+a)}{b^5}$	52
norman	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52
risch	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.91

$$\frac{a^4\log\left(bx+a\right)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

Fricas [A]
time = 0.80, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

Sympy [A]
time = 0.04, size = 49, normalized size = 0.86

$$\frac{a^4\log\left(a+bx\right)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

Giac [A]
time = 1.34, size = 53, normalized size = 0.93

$$\frac{a^4\log\left(|bx+a|\right)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

Mupad [B]
time = 0.10, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b}+\frac{a^4\,\ln\left(a+b\,x\right)}{b^5}-\frac{a\,x^3}{3\,b^2}-\frac{a^3\,x}{b^4}+\frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

3.159 $\int \frac{x^3}{a+bx}\,dx$

Optimal. Leaf size=44

$$\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}$$

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{a+bx}\,dx &= \int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 1.00

$$\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.93

method	result	size
default	$\frac{\frac{1}{3} b^{2} x^{3}-\frac{1}{2} a b \,x^{2}+a^{2} x}{b^{3}}-\frac{a^{3} \ln \left(b x +a \right)}{b^{4}}$	41
norman	$\frac{a^{2} x}{b^{3}}-\frac{a \,x^{2}}{2 b^{2}}+\frac{x^{3}}{3 b}-\frac{a^{3} \ln \left(b x +a \right)}{b^{4}}$	41
risch	$\frac{a^{2} x}{b^{3}}-\frac{a \,x^{2}}{2 b^{2}}+\frac{x^{3}}{3 b}-\frac{a^{3} \ln \left(b x +a \right)}{b^{4}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.95

$$-\frac{a^{3} \log \left(b x+a\right)}{b^{4}}+\frac{2 \, b^{2} x^{3}-3 \, a b x^{2}+6 \, a^{2} x}{6 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Fricas [A]
time = 0.91, size = 41, normalized size = 0.93

$$\frac{2 \, b^{3} x^{3}-3 \, a b^{2} x^{2}+6 \, a^{2} b x-6 \, a^{3} \log \left(b x+a\right)}{6 \, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A]
time = 0.03, size = 37, normalized size = 0.84

$$-\frac{a^{3} \log{\left(a+b x \right)}}{b^{4}}+\frac{a^{2} x}{b^{3}}-\frac{a x^{2}}{2 b^{2}}+\frac{x^{3}}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Giac [**A**]
time = 0.97, size = 43, normalized size = 0.98

$$-\frac{a^3 \log\left(|bx + a|\right)}{b^4} + \frac{2\, b^2 x^3 - 3\, abx^2 + 6\, a^2 x}{6\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="giac")
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [**B**]
time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\, b} - \frac{a^3\, \ln\left(a + b\, x\right)}{b^4} - \frac{a\, x^2}{2\, b^2} + \frac{a^2\, x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x),x)
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

3.160 $\int \frac{x^2}{a+bx}\,dx$

Optimal. Leaf size=31

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a+bx}\,dx &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}x^2b+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
norman	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*x^2*b+a*x)+a^2*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.26, size = 29, normalized size = 0.94

$$\frac{a^2\log(bx+a)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A]
time = 0.93, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.84

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [A]
time = 1.28, size = 30, normalized size = 0.97

$$\frac{a^2 \log(|bx+a|)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B]
time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln(a+b\,x) + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.161 $\int \frac{x}{a+bx}\,dx$

Optimal. Leaf size=18

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx}\,dx &= \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx\\ &= \frac{x}{b}-\frac{a\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A]
time = 0.08, size = 19, normalized size = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.28, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A]
time = 0.93, size = 17, normalized size = 0.94

$$\frac{bx-a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A]
time = 0.03, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a+bx\right)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

Giac [A]
time = 0.80, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a\log\left(|bx+a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B]
time = 0.08, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a+b\,x\right)-b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x),x)
```

```
[Out] -(a*log(a + b*x) - b*x)/b^2
```

3.162 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

[Out] ln(b*x+a)/b

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Maple [A]
time = 0.08, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A]
time = 0.26, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A]
time = 1.18, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

Giac [**A**]
time = 1.16, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [**B**]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a + b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

3.163 $\int \frac{1}{x(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx)}\,dx &= \frac{\int \frac{1}{x}\,dx}{a} - \frac{b\int \frac{1}{a+bx}\,dx}{a}\\
&= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A]
time = 0.08, size = 19, normalized size = 1.06

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
risch	$\frac{\ln(-x)}{a}-\frac{\ln(bx+a)}{a}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A]
time = 0.27, size = 18, normalized size = 1.00

$$-\frac{\log\left(bx+a\right)}{a}+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A]
time = 1.17, size = 16, normalized size = 0.89

$$-\frac{\log\left(bx+a\right)-\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [**A**]
time = 0.05, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [**A**]
time = 1.60, size = 20, normalized size = 1.11

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [**B**]
time = 0.09, size = 15, normalized size = 0.83

$$-\frac{2\operatorname{atanh}\left(\frac{2bx}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)),x)
```

```
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

3.164 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A]
time = 0.08, size = 29, normalized size = 1.04

method	result	size
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A]
time = 0.28, size = 28, normalized size = 1.00

$$\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A]
time = 0.97, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A]
time = 0.06, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A]
time = 1.44, size = 30, normalized size = 1.07

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)),x)
```

```
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.165 $\int \frac{1}{x^3(a+bx)}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{46\}$

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)}\,dx &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.00

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A]
time = 0.08, size = 41, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b}{a^{2}x}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	41
norman	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	41
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln(-x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.95

$$-\frac{b^{2}\log\left(bx+a\right)}{a^{3}}+\frac{b^{2}\log\left(x\right)}{a^{3}}+\frac{2\,bx-a}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Fricas [A]
time = 1.08, size = 41, normalized size = 0.98

$$-\frac{2\,b^{2}x^{2}\log\left(bx+a\right)-2\,b^{2}x^{2}\log\left(x\right)-2\,abx+a^{2}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A]
time = 0.07, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^{2}x^{2}}+\frac{b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Giac [A]
time = 1.38, size = 45, normalized size = 1.07

$$-\frac{b^2 \log(|bx+a|)}{a^3} + \frac{b^2 \log(|x|)}{a^3} + \frac{2\,abx - a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B]
time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2} - a\,b\,x}{a^3\,x^2} - \frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.166 $\int \frac{1}{x^4(a+bx)}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

```
[Out] -1/3/x^3/a+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)}\,dx &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.00

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Maple [A]
time = 0.09, size = 53, normalized size = 0.95

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b}{2a^{2}x^{2}}-\frac{b^{2}}{a^{3}x}-\frac{b^{3}\ln(x)}{a^{4}}+\frac{b^{3}\ln(bx+a)}{a^{4}}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}+\frac{b^{3}\ln(bx+a)}{a^{4}}-\frac{b^{3}\ln(x)}{a^{4}}$	53
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^{2}}-\frac{b^{2}x^{2}}{a^{3}}}{x^{3}}-\frac{b^{3}\ln(x)}{a^{4}}+\frac{b^{3}\ln(-bx-a)}{a^{4}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.91

$$\frac{b^{3}\log\left(bx+a\right)}{a^{4}}-\frac{b^{3}\log\left(x\right)}{a^{4}}-\frac{6\,b^{2}x^{2}-3\,abx+2\,a^{2}}{6\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

Fricas [A]
time = 1.33, size = 54, normalized size = 0.96

$$\frac{6\,b^{3}x^{3}\log\left(bx+a\right)-6\,b^{3}x^{3}\log\left(x\right)-6\,ab^{2}x^{2}+3\,a^{2}bx-2\,a^{3}}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

Sympy [A]
time = 0.09, size = 44, normalized size = 0.79

$$\frac{-2a^{2}+3abx-6b^{2}x^{2}}{6a^{3}x^{3}}+\frac{b^{3}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

Giac [A]
time = 0.98, size = 56, normalized size = 1.00

$$\frac{b^3\log\left(|bx+a|\right)}{a^4}-\frac{b^3\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

Mupad [B]
time = 0.10, size = 48, normalized size = 0.86

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

3.167 $\int \frac{1}{x^5(a+bx)} dx$

Optimal. Leaf size=68

$$-\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}$$

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}+\frac{b^3}{a^4x}-\frac{b^2}{2a^3x^2}+\frac{b}{3a^2x^3}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x
])/a^5 - (b^4*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)}\,dx &= \int \left(\frac{1}{ax^5}-\frac{b}{a^2x^4}+\frac{b^2}{a^3x^3}-\frac{b^3}{a^4x^2}+\frac{b^4}{a^5x}-\frac{b^5}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.00

$$-\frac{1}{4ax^4}+\frac{b}{3a^2x^3}-\frac{b^2}{2a^3x^2}+\frac{b^3}{a^4x}+\frac{b^4\log(x)}{a^5}-\frac{b^4\log(a+bx)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x
])/a^5 - (b^4*Log[a + b*x])/a^5
```

Maple [A]
time = 0.09, size = 63, normalized size = 0.93

method	result	size
default	$-\frac{1}{4a\,x^{4}}+\frac{b}{3a^{2}x^{3}}-\frac{b^{2}}{2a^{3}x^{2}}+\frac{b^{3}}{a^{4}x}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}$	63
norman	$\frac{\frac{b^{3}x^{3}}{a^{4}}-\frac{1}{4a}+\frac{bx}{3a^{2}}-\frac{b^{2}x^{2}}{2a^{3}}}{x^{4}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}$	63
risch	$\frac{\frac{b^{3}x^{3}}{a^{4}}-\frac{1}{4a}+\frac{bx}{3a^{2}}-\frac{b^{2}x^{2}}{2a^{3}}}{x^{4}}-\frac{b^{4}\ln\left(bx+a\right)}{a^{5}}+\frac{b^{4}\ln\left(-x\right)}{a^{5}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

Maxima [A]
time = 0.27, size = 62, normalized size = 0.91

$$-\frac{b^{4}\log\left(bx+a\right)}{a^{5}}+\frac{b^{4}\log\left(x\right)}{a^{5}}+\frac{12\,b^{3}x^{3}-6\,ab^{2}x^{2}+4\,a^{2}bx-3\,a^{3}}{12\,a^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^4*log(b*x + a)/a^5 + b^4*log(x)/a^5 + 1/12*(12*b^3*x^3 - 6*a*b^2*x^2 + 4
*a^2*b*x - 3*a^3)/(a^4*x^4)
```

Fricas [A]
time = 0.89, size = 65, normalized size = 0.96

$$-\frac{12\,b^{4}x^{4}\log\left(bx+a\right)-12\,b^{4}x^{4}\log\left(x\right)-12\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-4\,a^{3}bx+3\,a^{4}}{12\,a^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^4*x^4*log(b*x + a) - 12*b^4*x^4*log(x) - 12*a*b^3*x^3 + 6*a^2*b
^2*x^2 - 4*a^3*b*x + 3*a^4)/(a^5*x^4)
```

Sympy [A]
time = 0.10, size = 56, normalized size = 0.82

$$\frac{-3a^3+4a^2bx-6ab^2x^2+12b^3x^3}{12a^4x^4}+\frac{b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x - 6*a*b**2*x**2 + 12*b**3*x**3)/(12*a**4*x**4) + b**4
*(log(x) - log(a/b + x))/a**5
```

Giac [A]
time = 0.81, size = 67, normalized size = 0.99

$$-\frac{b^4\log\left(|bx+a|\right)}{a^5}+\frac{b^4\log\left(|x|\right)}{a^5}+\frac{12\,ab^3x^3-6\,a^2b^2x^2+4\,a^3bx-3\,a^4}{12\,a^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(b*x + a))/a^5 + b^4*log(abs(x))/a^5 + 1/12*(12*a*b^3*x^3 - 6*a
^2*b^2*x^2 + 4*a^3*b*x - 3*a^4)/(a^5*x^4)
```

Mupad [B]
time = 0.07, size = 60, normalized size = 0.88

$$-\frac{\frac{a^4}{4}-\frac{a^3\,b\,x}{3}+\frac{a^2\,b^2\,x^2}{2}-a\,b^3\,x^3}{a^5\,x^4}-\frac{2\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)),x)
```

```
[Out] - (a^4/4 - a*b^3*x^3 + (a^2*b^2*x^2)/2 - (a^3*b*x)/3)/(a^5*x^4) - (2*b^4*at
anh((2*b*x)/a + 1))/a^5
```

3.168 $\int \frac{x^6}{(a+bx)^2} dx$

Optimal. Leaf size=81

$$\frac{5a^4x}{b^6} - \frac{2a^3x^2}{b^5} + \frac{a^2x^3}{b^4} - \frac{ax^4}{2b^3} + \frac{x^5}{5b^2} - \frac{a^6}{b^7(a+bx)} - \frac{6a^5\log(a+bx)}{b^7}$$

```
[Out] 5*a^4*x/b^6-2*a^3*x^2/b^5+a^2*x^3/b^4-1/2*a*x^4/b^3+1/5*x^5/b^2-a^6/b^7/(b*
x+a)-6*a^5*ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^6}{b^7(a+bx)} - \frac{6a^5\log(a+bx)}{b^7} + \frac{5a^4x}{b^6} - \frac{2a^3x^2}{b^5} + \frac{a^2x^3}{b^4} - \frac{ax^4}{2b^3} + \frac{x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^2,x]
```

```
[Out] (5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5*
b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(a+bx)^2} dx &= \int \left(\frac{5a^4}{b^6} - \frac{4a^3x}{b^5} + \frac{3a^2x^2}{b^4} - \frac{2ax^3}{b^3} + \frac{x^4}{b^2} + \frac{a^6}{b^6(a+bx)^2} - \frac{6a^5}{b^6(a+bx)}\right) dx \\
&= \frac{5a^4x}{b^6} - \frac{2a^3x^2}{b^5} + \frac{a^2x^3}{b^4} - \frac{ax^4}{2b^3} + \frac{x^5}{5b^2} - \frac{a^6}{b^7(a+bx)} - \frac{6a^5\log(a+bx)}{b^7}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.95

$$\frac{50a^4bx - 20a^3b^2x^2 + 10a^2b^3x^3 - 5ab^4x^4 + 2b^5x^5 - \frac{10a^6}{a+bx} - 60a^5\log(a+bx)}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^2,x]
```

```
[Out] (50*a^4*b*x - 20*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 5*a*b^4*x^4 + 2*b^5*x^5 - (
10*a^6)/(a + b*x) - 60*a^5*Log[a + b*x])/(10*b^7)
```

Maple [A]
time = 0.08, size = 78, normalized size = 0.96

method	result	size
default	$\frac{\frac{1}{5}b^4x^5-\frac{1}{2}a\,b^3x^4+a^2b^2x^3-2a^3b\,x^2+5a^4x}{b^6}-\frac{a^6}{b^7(bx+a)}-\frac{6a^5\ln(bx+a)}{b^7}$	78
risch	$\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a\,x^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(bx+a)}-\frac{6a^5\ln(bx+a)}{b^7}$	78
norman	$\frac{\frac{x^6}{5b}-\frac{3a\,x^5}{10b^2}-\frac{6a^6}{b^7}-\frac{a^3x^3}{b^4}+\frac{3a^4x^2}{b^5}+\frac{a^2x^4}{2b^3}}{bx+a}-\frac{6a^5\ln(bx+a)}{b^7}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/5*b^4*x^5-1/2*a*b^3*x^4+a^2*b^2*x^3-2*a^3*b*x^2+5*a^4*x)-a^6/b^7/(
b*x+a)-6*a^5*ln(b*x+a)/b^7
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.01

$$-\frac{a^6}{b^8x+ab^7}-\frac{6\,a^5\log\left(bx+a\right)}{b^7}+\frac{2\,b^4x^5-5\,ab^3x^4+10\,a^2b^2x^3-20\,a^3bx^2+50\,a^4x}{10\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^6/(b^8*x + a*b^7) - 6*a^5*log(b*x + a)/b^7 + 1/10*(2*b^4*x^5 - 5*a*b^3*x
^4 + 10*a^2*b^2*x^3 - 20*a^3*b*x^2 + 50*a^4*x)/b^6
```

Fricas [A]
time = 1.03, size = 96, normalized size = 1.19

$$\frac{2\,b^6x^6-3\,ab^5x^5+5\,a^2b^4x^4-10\,a^3b^3x^3+30\,a^4b^2x^2+50\,a^5bx-10\,a^6-60\,(a^5bx+a^6)\log\left(bx+a\right)}{10\,(b^8x+ab^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^6*x^6 - 3*a*b^5*x^5 + 5*a^2*b^4*x^4 - 10*a^3*b^3*x^3 + 30*a^4*b^2
*x^2 + 50*a^5*b*x - 10*a^6 - 60*(a^5*b*x + a^6)*log(b*x + a))/(b^8*x + a*b^
7)
```

Sympy [A]
time = 0.09, size = 78, normalized size = 0.96

$$-\frac{a^6}{ab^7+b^8x}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**2,x)
```

```
[Out] -a**6/(a*b**7 + b**8*x) - 6*a**5*log(a + b*x)/b**7 + 5*a**4*x/b**6 - 2*a**3
*x**2/b**5 + a**2*x**3/b**4 - a*x**4/(2*b**3) + x**5/(5*b**2)
```

Giac [A]
time = 0.73, size = 103, normalized size = 1.27

$$-\frac{(bx+a)^5\left(\frac{15\,a}{bx+a}-\frac{50\,a^2}{(bx+a)^2}+\frac{100\,a^3}{(bx+a)^3}-\frac{150\,a^4}{(bx+a)^4}-2\right)}{10\,b^7}+\frac{6\,a^5\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^7}-\frac{a^6}{(bx+a)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/10*(b*x + a)^5*(15*a/(b*x + a) - 50*a^2/(b*x + a)^2 + 100*a^3/(b*x + a)^
3 - 150*a^4/(b*x + a)^4 - 2)/b^7 + 6*a^5*log(abs(b*x + a)/((b*x + a)^2*abs(
b)))/b^7 - a^6/((b*x + a)*b^7)
```

Mupad [B]
time = 0.14, size = 83, normalized size = 1.02

$$\frac{x^5}{5\,b^2}-\frac{6\,a^5\,\ln(a+b\,x)}{b^7}-\frac{a\,x^4}{2\,b^3}+\frac{5\,a^4\,x}{b^6}+\frac{a^2\,x^3}{b^4}-\frac{2\,a^3\,x^2}{b^5}-\frac{a^6}{b\,(x\,b^7+a\,b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^2,x)
```

```
[Out] x^5/(5*b^2) - (6*a^5*log(a + b*x))/b^7 - (a*x^4)/(2*b^3) + (5*a^4*x)/b^6 +
(a^2*x^3)/b^4 - (2*a^3*x^2)/b^5 - a^6/(b*(a*b^6 + b^7*x))
```

3.169 $\int \frac{x^5}{(a+bx)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}$$

```
[Out] -4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a^
4*ln(b*x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^2,x]
```

```
[Out] (-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a^
5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^2}\,dx&=\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx\\&=-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 0.92

$$\frac{-48a^3bx+18a^2b^2x^2-8ab^3x^3+3b^4x^4+\frac{12a^5}{a+bx}+60a^4\log(a+bx)}{12b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b*x)^2,x]

[Out] (-48*a^3*b*x + 18*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4 + (12*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(12*b^6)

Maple [A]
time = 0.08, size = 68, normalized size = 0.94

method	result	size
risch	$-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2a\,x^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(bx+a)}+\frac{5a^4\ln(bx+a)}{b^6}$	67
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{2}{3}a\,b^2x^3-\frac{3}{2}a^2b\,x^2+4a^3x}{b^5}+\frac{a^5}{b^6(bx+a)}+\frac{5a^4\ln(bx+a)}{b^6}$	68
norman	$\frac{\frac{5a^5}{b^6}+\frac{x^5}{4b}-\frac{5a\,x^4}{12b^2}+\frac{5a^2x^3}{6b^3}-\frac{5a^3x^2}{2b^4}}{bx+a}+\frac{5a^4\ln(bx+a)}{b^6}$	72

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/b^5*(-1/4*b^3*x^4+2/3*a*b^2*x^3-3/2*a^2*b*x^2+4*a^3*x)+a^5/b^6/(b*x+a)+5*a^4*ln(b*x+a)/b^6

Maxima [A]
time = 0.27, size = 70, normalized size = 0.97

$$\frac{a^5}{b^7x+ab^6}+\frac{5\,a^4\log\left(bx+a\right)}{b^6}+\frac{3\,b^3x^4-8\,ab^2x^3+18\,a^2bx^2-48\,a^3x}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x+a)^2,x, algorithm="maxima")

[Out] a^5/(b^7*x + a*b^6) + 5*a^4*log(b*x + a)/b^6 + 1/12*(3*b^3*x^4 - 8*a*b^2*x^3 + 18*a^2*b*x^2 - 48*a^3*x)/b^5

Fricas [A]
time = 0.98, size = 85, normalized size = 1.18

$$\frac{3\,b^5x^5-5\,ab^4x^4+10\,a^2b^3x^3-30\,a^3b^2x^2-48\,a^4bx+12\,a^5+60\,(a^4bx+a^5)\log\left(bx+a\right)}{12\,(b^7x+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(b*x+a)^2,x, algorithm="fricas")

[Out] 1/12*(3*b^5*x^5 - 5*a*b^4*x^4 + 10*a^2*b^3*x^3 - 30*a^3*b^2*x^2 - 48*a^4*b*x + 12*a^5 + 60*(a^4*b*x + a^5)*log(b*x + a))/(b^7*x + a*b^6)

Sympy [A]
time = 0.08, size = 71, normalized size = 0.99

$$\frac{a^5}{ab^6 + b^7x} + \frac{5a^4 \log(a + bx)}{b^6} - \frac{4a^3x}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{2ax^3}{3b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2,x)
```

```
[Out] a**5/(a*b**6 + b**7*x) + 5*a**4*log(a + b*x)/b**6 - 4*a**3*x/b**5 + 3*a**2*
x**2/(2*b**4) - 2*a*x**3/(3*b**3) + x**4/(4*b**2)
```

Giac [A]
time = 1.12, size = 90, normalized size = 1.25

$$-\frac{(bx + a)^4 \left(\frac{20\,a}{bx+a} - \frac{60\,a^2}{(bx+a)^2} + \frac{120\,a^3}{(bx+a)^3} - 3\right)}{12\,b^6} - \frac{5\,a^4 \log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^6} + \frac{a^5}{(bx + a)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/12*(b*x + a)^4*(20*a/(b*x + a) - 60*a^2/(b*x + a)^2 + 120*a^3/(b*x + a)^
3 - 3)/b^6 - 5*a^4*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^6 + a^5/((b*x +
 a)*b^6)
```

Mupad [B]
time = 0.07, size = 72, normalized size = 1.00

$$\frac{x^4}{4\,b^2} + \frac{5\,a^4\,\ln(a + b\,x)}{b^6} - \frac{2\,a\,x^3}{3\,b^3} - \frac{4\,a^3\,x}{b^5} + \frac{3\,a^2\,x^2}{2\,b^4} + \frac{a^5}{b\,\left(x\,b^6 + a\,b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^2,x)
```

```
[Out] x^4/(4*b^2) + (5*a^4*log(a + b*x))/b^6 - (2*a*x^3)/(3*b^3) - (4*a^3*x)/b^5
+ (3*a^2*x^2)/(2*b^4) + a^5/(b*(a*b^5 + b^6*x))
```

3.170 $\int \frac{x^4}{(a+bx)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}$$

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^2}\,dx &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.93

$$\frac{9a^2bx-3ab^2x^2+b^3x^3-\frac{3a^4}{a+bx}-12a^3\log(a+bx)}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^4}{3b}-\frac{2a\,x^3}{3b^2}-\frac{4a^4}{b^5}+\frac{2a^2x^2}{b^3}}{bx+a}-\frac{4a^3\ln(bx+a)}{b^5}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 1.02

$$-\frac{a^4}{b^6x+ab^5}-\frac{4\,a^3\log\left(bx+a\right)}{b^5}+\frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

Fricas [A]
time = 0.93, size = 73, normalized size = 1.26

$$\frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log\left(bx+a\right)}{3\,(b^6x+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A]
time = 0.07, size = 54, normalized size = 0.93

$$-\frac{a^{4}}{ab^{5}+b^{6}x}-\frac{4a^{3}\log\left(a+bx\right)}{b^{5}}+\frac{3a^{2}x}{b^{4}}-\frac{ax^{2}}{b^{3}}+\frac{x^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

Giac [A]
time = 1.21, size = 79, normalized size = 1.36

$$-\frac{(bx+a)^{3}\left(\frac{6\,a}{bx+a}-\frac{18\,a^{2}}{(bx+a)^{2}}-1\right)}{3\,b^{5}}+\frac{4\,a^{3}\log\left(\frac{|bx+a|}{(bx+a)^{2}|b|}\right)}{b^{5}}-\frac{a^{4}}{(bx+a)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/b^5 + 4*a^3*log(a
bs(b*x + a)/((b*x + a)^2*abs(b)))/b^5 - a^4/((b*x + a)*b^5)
```

Mupad [B]
time = 0.07, size = 62, normalized size = 1.07

$$\frac{x^{3}}{3\,b^{2}}-\frac{4\,a^{3}\,\ln\left(a+b\,x\right)}{b^{5}}-\frac{a\,x^{2}}{b^{3}}+\frac{3\,a^{2}\,x}{b^{4}}-\frac{a^{4}}{b\,\left(x\,b^{5}+a\,b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

3.171 $\int \frac{x^3}{(a+bx)^2}\,dx$

Optimal. Leaf size=46

$$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}$$

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int\frac{x^3}{(a+bx)^2}\,dx&=\int\left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx\\&=-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.93

$$\frac{-4abx+b^2x^2+\frac{2a^3}{a+bx}+6a^2\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Maple [A]
time = 0.08, size = 46, normalized size = 1.00

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}x^2b+2ax}{b^3}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	46
norman	$\frac{\frac{3a^3}{b^4}+\frac{x^3}{2b}-\frac{3a\,x^2}{2b^2}}{bx+a}+\frac{3a^2\ln(bx+a)}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/2*x^2*b+2*a*x)+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.26, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log\left(bx+a\right)}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Fricas [A]
time = 0.66, size = 62, normalized size = 1.35

$$\frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log\left(bx+a\right)}{2\,(b^5x+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [A]
time = 0.06, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4+b^5x}+\frac{3a^2\log\left(a+bx\right)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

Giac [A]
time = 1.59, size = 66, normalized size = 1.43

$$-\frac{(bx+a)^2\left(\frac{6a}{bx+a}-1\right)}{2b^4}-\frac{3a^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4}+\frac{a^3}{(bx+a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x + a)^2*(6*a/(b*x + a) - 1)/b^4 - 3*a^2*log(abs(b*x + a)/((b*x + a
)^2*abs(b)))/b^4 + a^3/((b*x + a)*b^4)
```

Mupad [B]
time = 0.08, size = 50, normalized size = 1.09

$$\frac{x^2}{2b^2}+\frac{3a^2\ln(a+bx)}{b^4}+\frac{a^3}{b\left(xb^4+ab^3\right)}-\frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

3.172 $\int \frac{x^2}{(a+bx)^2} \, dx$

Optimal. Leaf size=33

$$\frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}$$

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^2} \, dx &= \int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a+bx)^2} - \frac{2a}{b^2(a+bx)}\right) dx \\ &= \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.88

$$\frac{bx - \frac{a^2}{a+bx} - 2a\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^2,x]
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Maple [A]
time = 0.09, size = 34, normalized size = 1.03

method	result	size
default	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
risch	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
norman	$\frac{\frac{x^2}{b}-\frac{2a^2}{b^3}}{bx+a}-\frac{2a\ln(bx+a)}{b^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.28, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x+ab^3}+\frac{x}{b^2}-\frac{2\,a\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Fricas [A]
time = 0.79, size = 47, normalized size = 1.42

$$\frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="fricas")
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3+b^4x}-\frac{2a\log\left(a+bx\right)}{b^3}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2,x)
```

```
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Giac [A]
time = 0.91, size = 50, normalized size = 1.52

$$\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3}+\frac{bx+a}{b^3}-\frac{a^2}{(bx+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/((b*x
+ a)*b^3)
```

Mupad [B]
time = 0.08, size = 36, normalized size = 1.09

$$\frac{x}{b^2}-\frac{a^2}{x\,b^4+a\,b^3}-\frac{2\,a\,\ln\left(a+b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^2,x)
```

```
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

3.173 $\int \frac{x}{(a+bx)^2} dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^2} dx &= \int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx \\ &= \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx} + \log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

Maple [A]
time = 0.08, size = 24, normalized size = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.13

$$\frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Fricas [A]
time = 0.68, size = 28, normalized size = 1.22

$$\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.87

$$\frac{a}{ab^2+b^3x}+\frac{\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2,x)
```

```
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Giac [A]
time = 1.58, size = 42, normalized size = 1.83

$$-\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b
```

Mupad [B]
time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b^2}+\frac{a}{b^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^2,x)
```

```
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

3.174 $\int \frac{1}{(a+bx)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2}\,dx = -\frac{1}{b(a+bx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A]
time = 0.08, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A]
time = 0.60, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A]
time = 1.88, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

3.175 $\int \frac{1}{x(a+bx)^2} dx$

Optimal. Leaf size=29

$$\frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}$$

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^2),x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}+\log(x)-\log(a+bx)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^2),x]
```

```
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Maple [A]
time = 0.08, size = 30, normalized size = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}+\frac{\ln(-x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Maxima [A]
time = 0.30, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="maxima")
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Fricas [A]
time = 0.56, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.76

$$\frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2,x)
```

```
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Giac [A]
time = 1.42, size = 38, normalized size = 1.31

$$b\left(\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2b}+\frac{1}{(bx+a)ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="giac")
[Out] b*(log(abs(-a/(b*x + a) + 1))/(a^2*b) + 1/((b*x + a)*a*b))
```

Mupad [B]
time = 0.12, size = 26, normalized size = 0.90

$$\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^2),x)
[Out] 1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2
```

3.176 $\int \frac{1}{x^2(a+bx)^2} \, dx$

Optimal. Leaf size=42

$$-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A]
time = 0.09, size = 43, normalized size = 1.02

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b}{a^{2} \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	43
risch	$\frac{-\frac{2 b x}{a^{2}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(-b x -a \right)}{a^{3}}$	49
norman	$\frac{\frac{2 b^{2} x^{2}}{a^{3}}-\frac{1}{a}}{x \left(b x +a \right)}-\frac{2 b \ln \left(x \right)}{a^{3}}+\frac{2 b \ln \left(b x +a \right)}{a^{3}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A]
time = 0.27, size = 45, normalized size = 1.07

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A]
time = 0.63, size = 63, normalized size = 1.50

$$-\frac{2\,abx+a^2-2\,(b^2x^2+abx)\log\left(bx+a\right)+2\,(b^2x^2+abx)\log\left(x\right)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A]
time = 0.10, size = 37, normalized size = 0.88

$$\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Giac [A]
time = 0.99, size = 52, normalized size = 1.24

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

Mupad [B]
time = 0.12, size = 45, normalized size = 1.07

$$\frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3}-\frac{1}{a\,x\,\left(a+b\,x\right)}-\frac{2\,b}{a^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

3.177 $\int \frac{1}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}$$

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{46\}$

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^2}\,dx&=\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\&=-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.91

$$\frac{a\left(-\frac{a}{x^2}+\frac{4b}{x}+\frac{2b^2}{a+bx}\right)+6b^2\log(x)-6b^2\log(a+bx)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^2),x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a + b*x])/(2*a^4)
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.98

method	result	size
default	$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	57
norman	$\frac{-\frac{3b^3x^3}{a^4}-\frac{1}{2a}+\frac{3bx}{2a^2}}{x^2(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	61
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)}-\frac{3b^2\ln(bx+a)}{a^4}+\frac{3b^2\ln(-x)}{a^4}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Maxima [A]
time = 0.27, size = 64, normalized size = 1.10

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\,(a^3bx^3+a^4x^2)}-\frac{3\,b^2\log(bx+a)}{a^4}+\frac{3\,b^2\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/a^4 + 3*b^2*log(x)/a^4
```

Fricas [A]
time = 0.57, size = 86, normalized size = 1.48

$$\frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,(b^3x^3+ab^2x^2)\log(bx+a)+6\,(b^3x^3+ab^2x^2)\log(x)}{2\,(a^4bx^3+a^5x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) + 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A]
time = 0.12, size = 54, normalized size = 0.93

$$\frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3}+\frac{3b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

Giac [A]
time = 1.07, size = 74, normalized size = 1.28

$$\frac{3\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4}+\frac{b^2}{(bx+a)a^3}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{2\,a^4\left(\frac{a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(-a/(b*x + a) + 1))/a^4 + b^2/((b*x + a)*a^3) - 1/2*(6*a*b^2/(
b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2)
```

Mupad [B]
time = 0.11, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2}-\frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^2),x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

3.178 $\int \frac{1}{x^4(a+bx)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4
*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.96

$$-\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)}+12b^3\log(x)-12b^3\log(a+bx)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Maple [A]
time = 0.08, size = 68, normalized size = 0.99

method	result	size
default	$-\frac{1}{3a^{2}x^{3}}+\frac{b}{a^{3}x^{2}}-\frac{3b^{2}}{a^{4}x}-\frac{b^{3}}{a^{4}(bx+a)}-\frac{4b^{3}\ln(x)}{a^{5}}+\frac{4b^{3}\ln(bx+a)}{a^{5}}$	68
norman	$\frac{\frac{4b^{4}x^{4}}{a^{5}}-\frac{1}{3a}+\frac{2bx}{3a^{2}}-\frac{2b^{2}x^{2}}{a^{3}}}{x^{3}(bx+a)}-\frac{4b^{3}\ln(x)}{a^{5}}+\frac{4b^{3}\ln(bx+a)}{a^{5}}$	72
risch	$\frac{-\frac{4b^{3}x^{3}}{a^{4}}-\frac{2b^{2}x^{2}}{a^{3}}+\frac{2bx}{3a^{2}}-\frac{1}{3a}}{x^{3}(bx+a)}-\frac{4b^{3}\ln(x)}{a^{5}}+\frac{4b^{3}\ln(-bx-a)}{a^{5}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Maxima [A]
time = 0.28, size = 73, normalized size = 1.06

$$-\frac{12\,b^{3}x^{3}+6\,ab^{2}x^{2}-2\,a^{2}bx+a^{3}}{3\,(a^{4}bx^{4}+a^{5}x^{3})}+\frac{4\,b^{3}\log(bx+a)}{a^{5}}-\frac{4\,b^{3}\log(x)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Fricas [A]
time = 1.02, size = 95, normalized size = 1.38

$$-\frac{12\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-2\,a^{3}bx+a^{4}-12\,(b^{4}x^{4}+ab^{3}x^{3})\log(bx+a)+12\,(b^{4}x^{4}+ab^{3}x^{3})\log(x)}{3\,(a^{5}bx^{4}+a^{6}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [A]
time = 0.14, size = 66, normalized size = 0.96

$$\frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Giac [A]
time = 0.93, size = 90, normalized size = 1.30

$$-\frac{4\,b^3\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^5}-\frac{b^3}{(bx+a)a^4}-\frac{\frac{30\,ab^3}{bx+a}-\frac{18\,a^2b^3}{(bx+a)^2}-13\,b^3}{3\,a^5\left(\frac{a}{bx+a}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*b^3*log(abs(-a/(b*x + a) + 1))/a^5 - b^3/((b*x + a)*a^4) - 1/3*(30*a*b^3
/(b*x + a) - 18*a^2*b^3/(b*x + a)^2 - 13*b^3)/(a^5*(a/(b*x + a) - 1)^3)
```

Mupad [B]
time = 0.08, size = 69, normalized size = 1.00

$$\frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^2),x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

3.179 $\int \frac{1}{x^5(a+bx)^2} dx$

Optimal. Leaf size=84

$$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}$$

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{46\}$

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^2),x]
```

```
[Out] -1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x
) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}+60b^4\log(x)-60b^4\log(a+bx)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Maple [A]
time = 0.09, size = 79, normalized size = 0.94

method	result	size
default	$-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	79
norman	$\frac{-\frac{5b^5x^5}{a^6}-\frac{1}{4a}+\frac{5bx}{12a^2}-\frac{5b^2x^2}{6a^3}+\frac{5b^3x^3}{2a^4}}{x^4(bx+a)}+\frac{5b^4\ln(x)}{a^6}-\frac{5b^4\ln(bx+a)}{a^6}$	83
risch	$\frac{\frac{5b^4x^4}{a^5}+\frac{5b^3x^3}{2a^4}-\frac{5b^2x^2}{6a^3}+\frac{5bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)}-\frac{5b^4\ln(bx+a)}{a^6}+\frac{5b^4\ln(-x)}{a^6}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.27, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\,(a^5bx^5+a^6x^4)}-\frac{5\,b^4\log{(bx+a)}}{a^6}+\frac{5\,b^4\log{(x)}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Fricas [A]
time = 1.20, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5-60\,(b^5x^5+ab^4x^4)\log{(bx+a)}+60\,(b^5x^5+ab^4x^4)\log{(x)}}{12\,(a^6bx^5+a^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

Sympy [A]
time = 0.15, size = 80, normalized size = 0.95

$$\frac{-3a^4 + 5a^3bx - 10a^2b^2x^2 + 30ab^3x^3 + 60b^4x^4}{12a^6x^4 + 12a^5bx^5} + \frac{5b^4\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Giac [A]
time = 0.94, size = 104, normalized size = 1.24

$$\frac{5\,b^4\log\left(\left|-\frac{a}{bx+a} + 1\right|\right)}{a^6} + \frac{b^4}{(bx+a)a^5} - \frac{\frac{260\,ab^4}{bx+a} - \frac{300\,a^2b^4}{(bx+a)^2} + \frac{120\,a^3b^4}{(bx+a)^3} - 77\,b^4}{12\,a^6\left(\frac{a}{bx+a} - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^4*log(abs(-a/(b*x + a) + 1))/a^6 + b^4/((b*x + a)*a^5) - 1/12*(260*a*b^
4/(b*x + a) - 300*a^2*b^4/(b*x + a)^2 + 120*a^3*b^4/(b*x + a)^3 - 77*b^4)/(
a^6*(a/(b*x + a) - 1)^4)
```

Mupad [B]
time = 0.12, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4} - \frac{5\,b^2\,x^2}{6\,a^3} - \frac{1}{4\,a} + \frac{5\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{12\,a^2}}{b\,x^5 + a\,x^4} - \frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

3.180 $\int \frac{x^7}{(a+bx)^3}\,dx$

Optimal. Leaf size=99

$$\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}$$

```
[Out] 15*a^4*x/b^7-5*a^3*x^2/b^6+2*a^2*x^3/b^5-3/4*a*x^4/b^4+1/5*x^5/b^3+1/2*a^7/
b^8/(b*x+a)^2-7*a^6/b^8/(b*x+a)-21*a^5*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.06, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^3,x]
```

```
[Out] (15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x^
5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*Log
[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^3}\,dx &= \int \left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right)dx\\ &= \frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 89, normalized size = 0.90

$$\frac{300a^4bx-100a^3b^2x^2+40a^2b^3x^3-15ab^4x^4+4b^5x^5+\frac{10a^7}{(a+bx)^2}-\frac{140a^6}{a+bx}-420a^5\log(a+bx)}{20b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^3,x]
```

```
[Out] (300*a^4*b*x - 100*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 15*a*b^4*x^4 + 4*b^5*x^5
+ (10*a^7)/(a + b*x)^2 - (140*a^6)/(a + b*x) - 420*a^5*Log[a + b*x])/(20*b^
8)
```

Maple [A]
time = 0.10, size = 94, normalized size = 0.95

method	result	size
risch	$\frac{x^5}{5b^3}-\frac{3a\,x^4}{4b^4}+\frac{2a^2x^3}{b^5}-\frac{5a^3x^2}{b^6}+\frac{15a^4x}{b^7}+\frac{-7a^6x-\frac{13a^7}{2b}}{b^7(bx+a)^2}-\frac{21a^5\ln(bx+a)}{b^8}$	90
norman	$\frac{\frac{x^7}{5b}-\frac{7a\,x^6}{20b^2}-\frac{63a^7}{2b^8}-\frac{7a^3x^4}{4b^4}+\frac{7a^4x^3}{b^5}+\frac{7a^2x^5}{10b^3}-\frac{42a^6x}{b^7}}{(bx+a)^2}-\frac{21a^5\ln(bx+a)}{b^8}$	92
default	$\frac{\frac{1}{5}b^4x^5-\frac{3}{4}a\,b^3x^4+2a^2b^2x^3-5a^3b\,x^2+15a^4x}{b^7}-\frac{7a^6}{b^8(bx+a)}+\frac{a^7}{2b^8(bx+a)^2}-\frac{21a^5\ln(bx+a)}{b^8}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/5*b^4*x^5-3/4*a*b^3*x^4+2*a^2*b^2*x^3-5*a^3*b*x^2+15*a^4*x)-7*a^6/
b^8/(b*x+a)+1/2*a^7/b^8/(b*x+a)^2-21*a^5*ln(b*x+a)/b^8
```

Maxima [A]
time = 0.27, size = 103, normalized size = 1.04

$$-\frac{14\,a^6bx+13\,a^7}{2\,(b^{10}x^2+2\,ab^9x+a^2b^8)}-\frac{21\,a^5\log(bx+a)}{b^8}+\frac{4\,b^4x^5-15\,ab^3x^4+40\,a^2b^2x^3-100\,a^3bx^2+300\,a^4x}{20\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(14*a^6*b*x + 13*a^7)/(b^10*x^2 + 2*a*b^9*x + a^2*b^8) - 21*a^5*log(b*
x + a)/b^8 + 1/20*(4*b^4*x^5 - 15*a*b^3*x^4 + 40*a^2*b^2*x^3 - 100*a^3*b*x^
2 + 300*a^4*x)/b^7
```

Fricas [A]
time = 1.25, size = 129, normalized size = 1.30

$$\frac{4\,b^7x^7-7\,ab^6x^6+14\,a^2b^5x^5-35\,a^3b^4x^4+140\,a^4b^3x^3+500\,a^5b^2x^2+160\,a^6bx-130\,a^7-420\,(a^5b^2x^2+2\,a^6bx+a^7)\log(bx+a)}{20\,(b^{10}x^2+2\,ab^9x+a^2b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 - 7*a*b^6*x^6 + 14*a^2*b^5*x^5 - 35*a^3*b^4*x^4 + 140*a^4*b
^3*x^3 + 500*a^5*b^2*x^2 + 160*a^6*b*x - 130*a^7 - 420*(a^5*b^2*x^2 + 2*a^6
*b*x + a^7)*log(b*x + a))/(b^10*x^2 + 2*a*b^9*x + a^2*b^8)
```

Sympy [A]
time = 0.15, size = 109, normalized size = 1.10

$$-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{-13a^7-14a^6bx}{2a^2b^8+4ab^9x+2b^{10}x^2}+\frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x)/b**8 + 15*a**4*x/b**7 - 5*a**3*x**2/b**6 + 2*a**2*x**
3/b**5 - 3*a*x**4/(4*b**4) + (-13*a**7 - 14*a**6*b*x)/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + x**5/(5*b**3)
```

Giac [A]
time = 1.15, size = 95, normalized size = 0.96

$$-\frac{21\,a^5\log(|bx+a|)}{b^8}-\frac{14\,a^6bx+13\,a^7}{2\,(bx+a)^2b^8}+\frac{4\,b^{12}x^5-15\,ab^{11}x^4+40\,a^2b^{10}x^3-100\,a^3b^9x^2+300\,a^4b^8x}{20\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -21*a^5*log(abs(b*x + a))/b^8 - 1/2*(14*a^6*b*x + 13*a^7)/((b*x + a)^2*b^8)
 + 1/20*(4*b^12*x^5 - 15*a*b^11*x^4 + 40*a^2*b^10*x^3 - 100*a^3*b^9*x^2 + 3
00*a^4*b^8*x)/b^15
```

Mupad [B]
time = 0.23, size = 91, normalized size = 0.92

$$-\frac{\frac{7\,a\,(a+b\,x)^4}{4}-\frac{(a+b\,x)^5}{5}-7\,a^2\,(a+b\,x)^3+\frac{35\,a^3\,(a+b\,x)^2}{2}+\frac{7\,a^6}{a+b\,x}-\frac{a^7}{2\,(a+b\,x)^2}+21\,a^5\,\ln(a+b\,x)-35\,a^4\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^3,x)
```

```
[Out] -((7*a*(a + b*x)^4)/4 - (a + b*x)^5/5 - 7*a^2*(a + b*x)^3 + (35*a^3*(a + b*
x)^2)/2 + (7*a^6)/(a + b*x) - a^7/(2*(a + b*x)^2) + 21*a^5*log(a + b*x) - 3
5*a^4*b*x)/b^8
```

3.181 $\int \frac{x^6}{(a+bx)^3} \, dx$

Optimal. Leaf size=86

$$-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}$$

```
[Out] -10*a^3*x/b^6+3*a^2*x^2/b^5-a*x^3/b^4+1/4*x^4/b^3-1/2*a^6/b^7/(b*x+a)^2+6*a
^5/b^7/(b*x+a)+15*a^4*ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^3,x]
```

```
[Out] (-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7*
(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^3}\,dx &= \int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx\\ &= -\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.90

$$\frac{-40a^3bx+12a^2b^2x^2-4ab^3x^3+b^4x^4-\frac{2a^6}{(a+bx)^2}+\frac{24a^5}{a+bx}+60a^4\log(a+bx)}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^3,x]
```

```
[Out] (-40*a^3*b*x + 12*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 - (2*a^6)/(a + b*x)^2
 + (24*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(4*b^7)
```

Maple [A]
time = 0.08, size = 83, normalized size = 0.97

method	result	size
risch	$\frac{x^4}{4b^3}-\frac{a\,x^3}{b^4}+\frac{3a^2x^2}{b^5}-\frac{10a^3x}{b^6}+\frac{6a^5x+\frac{11a^6}{2b}}{b^6(bx+a)^2}+\frac{15a^4\ln(bx+a)}{b^7}$	79
norman	$\frac{\frac{x^6}{4b}-\frac{a\,x^5}{2b^2}+\frac{45a^6}{2b^7}-\frac{5a^3x^3}{b^4}+\frac{5a^2x^4}{4b^3}+\frac{30a^5x}{b^6}}{(bx+a)^2}+\frac{15a^4\ln(bx+a)}{b^7}$	81
default	$-\frac{-\frac{1}{4}b^3x^4+a\,b^2x^3-3a^2b\,x^2+10a^3x}{b^6}+\frac{6a^5}{b^7(bx+a)}-\frac{a^6}{2b^7(bx+a)^2}+\frac{15a^4\ln(bx+a)}{b^7}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/4*b^3*x^4+a*b^2*x^3-3*a^2*b*x^2+10*a^3*x)+6*a^5/b^7/(b*x+a)-1/2*
a^6/b^7/(b*x+a)^2+15*a^4*ln(b*x+a)/b^7
```

Maxima [A]
time = 0.26, size = 91, normalized size = 1.06

$$\frac{12\,a^5bx+11\,a^6}{2\,(b^9x^2+2\,ab^8x+a^2b^7)}+\frac{15\,a^4\log(bx+a)}{b^7}+\frac{b^3x^4-4\,ab^2x^3+12\,a^2bx^2-40\,a^3x}{4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^5*b*x + 11*a^6)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7) + 15*a^4*log(b*x
+ a)/b^7 + 1/4*(b^3*x^4 - 4*a*b^2*x^3 + 12*a^2*b*x^2 - 40*a^3*x)/b^6
```

Fricas [A]
time = 1.05, size = 117, normalized size = 1.36

$$\frac{b^6x^6-2\,ab^5x^5+5\,a^2b^4x^4-20\,a^3b^3x^3-68\,a^4b^2x^2-16\,a^5bx+22\,a^6+60\,(a^4b^2x^2+2\,a^5bx+a^6)\log(bx+a)}{4\,(b^9x^2+2\,ab^8x+a^2b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*x^6 - 2*a*b^5*x^5 + 5*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 68*a^4*b^2*x^
2 - 16*a^5*b*x + 22*a^6 + 60*(a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a))/
(b^9*x^2 + 2*a*b^8*x + a^2*b^7)
```

Sympy [A]
time = 0.14, size = 92, normalized size = 1.07

$$\frac{15a^4\log{(a+bx)}}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{11a^6+12a^5bx}{2a^2b^7+4ab^8x+2b^9x^2}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x)/b**7 - 10*a**3*x/b**6 + 3*a**2*x**2/b**5 - a*x**3/b**4
 + (11*a**6 + 12*a**5*b*x)/(2*a**2*b**7 + 4*a*b**8*x + 2*b**9*x**2) + x**4/
(4*b**3)
```

Giac [A]
time = 1.21, size = 83, normalized size = 0.97

$$\frac{15\,a^4\log{(|bx+a|)}}{b^7}+\frac{12\,a^5bx+11\,a^6}{2\,(bx+a)^2b^7}+\frac{b^9x^4-4\,ab^8x^3+12\,a^2b^7x^2-40\,a^3b^6x}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 15*a^4*log(abs(b*x + a))/b^7 + 1/2*(12*a^5*b*x + 11*a^6)/((b*x + a)^2*b^7)
+ 1/4*(b^9*x^4 - 4*a*b^8*x^3 + 12*a^2*b^7*x^2 - 40*a^3*b^6*x)/b^12
```

Mupad [B]
time = 0.16, size = 78, normalized size = 0.91

$$\frac{\frac{(a+b\,x)^4}{4}-2\,a\,(a+b\,x)^3+\frac{15\,a^2\,(a+b\,x)^2}{2}+\frac{6\,a^5}{a+b\,x}-\frac{a^6}{2\,(a+b\,x)^2}+15\,a^4\,\ln{(a+b\,x)}-20\,a^3\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^4/4 - 2*a*(a + b*x)^3 + (15*a^2*(a + b*x)^2)/2 + (6*a^5)/(a + b*
x) - a^6/(2*(a + b*x)^2) + 15*a^4*log(a + b*x) - 20*a^3*b*x)/b^7
```

3.182 $\int \frac{x^5}{(a+bx)^3} \, dx$

Optimal. Leaf size=77

$$\frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3} + \frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} - \frac{10a^3\log(a+bx)}{b^6}$$

```
[Out] 6*a^2*x/b^5-3/2*a*x^2/b^4+1/3*x^3/b^3+1/2*a^5/b^6/(b*x+a)^2-5*a^4/b^6/(b*x+
a)-10*a^3*ln(b*x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} - \frac{10a^3\log(a+bx)}{b^6} + \frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^3,x]
```

```
[Out] (6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2) -
 (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^3} \, dx &= \int \left(\frac{6a^2}{b^5} - \frac{3ax}{b^4} + \frac{x^2}{b^3} - \frac{a^5}{b^5(a+bx)^3} + \frac{5a^4}{b^5(a+bx)^2} - \frac{10a^3}{b^5(a+bx)}\right) dx \\ &= \frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3} + \frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} - \frac{10a^3\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 0.87

$$\frac{36a^2bx - 9ab^2x^2 + 2b^3x^3 + \frac{3a^5}{(a+bx)^2} - \frac{30a^4}{a+bx} - 60a^3\log(a+bx)}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^3,x]
```

```
[Out] (36*a^2*b*x - 9*a*b^2*x^2 + 2*b^3*x^3 + (3*a^5)/(a + b*x)^2 - (30*a^4)/(a +
 b*x) - 60*a^3*Log[a + b*x])/(6*b^6)
```

Maple [A]
time = 0.08, size = 72, normalized size = 0.94

method	result	size
risch	$\frac{x^3}{3b^3} - \frac{3a\,x^2}{2b^4} + \frac{6a^2x}{b^5} + \frac{-5a^4x - \frac{9a^5}{2b}}{b^5(bx+a)^2} - \frac{10a^3\ln(bx+a)}{b^6}$	68
norman	$\frac{\frac{x^5}{3b} - \frac{5a\,x^4}{6b^2} + \frac{10a^2x^3}{3b^3} - \frac{15a^5}{b^6} - \frac{20a^4x}{b^5}}{(bx+a)^2} - \frac{10a^3\ln(bx+a)}{b^6}$	70
default	$\frac{\frac{1}{3}b^2x^3 - \frac{3}{2}ab\,x^2 + 6a^2x}{b^5} - \frac{5a^4}{b^6(bx+a)} + \frac{a^5}{2b^6(bx+a)^2} - \frac{10a^3\ln(bx+a)}{b^6}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/3*b^2*x^3-3/2*a*b*x^2+6*a^2*x)-5*a^4/b^6/(b*x+a)+1/2*a^5/b^6/(b*x+
a)^2-10*a^3*ln(b*x+a)/b^6
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.05

$$-\frac{10\,a^4bx + 9\,a^5}{2\,(b^8x^2 + 2\,ab^7x + a^2b^6)} - \frac{10\,a^3\log(bx+a)}{b^6} + \frac{2\,b^2x^3 - 9\,abx^2 + 36\,a^2x}{6\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*a^4*b*x + 9*a^5)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) - 10*a^3*log(b*x
+ a)/b^6 + 1/6*(2*b^2*x^3 - 9*a*b*x^2 + 36*a^2*x)/b^5
```

Fricas [A]
time = 0.93, size = 107, normalized size = 1.39

$$\frac{2\,b^5x^5 - 5\,ab^4x^4 + 20\,a^2b^3x^3 + 63\,a^3b^2x^2 + 6\,a^4bx - 27\,a^5 - 60\,(a^3b^2x^2 + 2\,a^4bx + a^5)\log(bx+a)}{6\,(b^8x^2 + 2\,ab^7x + a^2b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 - 5*a*b^4*x^4 + 20*a^2*b^3*x^3 + 63*a^3*b^2*x^2 + 6*a^4*b*x
- 27*a^5 - 60*(a^3*b^2*x^2 + 2*a^4*b*x + a^5)*log(b*x + a))/(b^8*x^2 + 2*a*
b^7*x + a^2*b^6)
```

Sympy [A]
time = 0.13, size = 85, normalized size = 1.10

$$-\frac{10a^3\log(a+bx)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{-9a^5-10a^4bx}{2a^2b^6+4ab^7x+2b^8x^2}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**3,x)
```

```
[Out] -10*a**3*log(a + b*x)/b**6 + 6*a**2*x/b**5 - 3*a*x**2/(2*b**4) + (-9*a**5 -
 10*a**4*b*x)/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) + x**3/(3*b**3)
```

Giac [A]
time = 1.60, size = 73, normalized size = 0.95

$$-\frac{10\,a^3\log(|bx+a|)}{b^6}-\frac{10\,a^4bx+9\,a^5}{2\,(bx+a)^2b^6}+\frac{2\,b^6x^3-9\,ab^5x^2+36\,a^2b^4x}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -10*a^3*log(abs(b*x + a))/b^6 - 1/2*(10*a^4*b*x + 9*a^5)/((b*x + a)^2*b^6)
+ 1/6*(2*b^6*x^3 - 9*a*b^5*x^2 + 36*a^2*b^4*x)/b^9
```

Mupad [B]
time = 0.12, size = 67, normalized size = 0.87

$$-\frac{\frac{5\,a\,(a+b\,x)^2}{2}-\frac{(a+b\,x)^3}{3}+\frac{5\,a^4}{a+b\,x}-\frac{a^5}{2\,(a+b\,x)^2}+10\,a^3\,\ln(a+b\,x)-10\,a^2\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^3,x)
```

```
[Out] -((5*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (5*a^4)/(a + b*x) - a^5/(2*(a + b*x
)^2) + 10*a^3*log(a + b*x) - 10*a^2*b*x)/b^6
```

3.183 $\int \frac{x^4}{(a+bx)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}$$

```
[Out] -3*a*x/b^4+1/2*x^2/b^3-1/2*a^4/b^5/(b*x+a)^2+4*a^3/b^5/(b*x+a)+6*a^2*ln(b*x
+a)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^3,x]
```

```
[Out] (-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b*
x)) + (6*a^2*Log[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^3}\,dx &= \int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right)dx\\ &= -\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 0.86

$$\frac{-6abx+b^2x^2-\frac{a^4}{(a+bx)^2}+\frac{8a^3}{a+bx}+12a^2\log(a+bx)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^3,x]
```

```
[Out] (-6*a*b*x + b^2*x^2 - a^4/(a + b*x)^2 + (8*a^3)/(a + b*x) + 12*a^2*Log[a + b*x])/(2*b^5)
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.97

method	result	size
risch	$\frac{x^2}{2b^3}-\frac{3ax}{b^4}+\frac{4a^3x+\frac{7a^4}{2b}}{b^4(bx+a)^2}+\frac{6a^2\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{9a^4}{b^5}+\frac{x^4}{2b}-\frac{2a\,x^3}{b^2}+\frac{12a^3x}{b^4}}{(bx+a)^2}+\frac{6a^2\ln(bx+a)}{b^5}$	59
default	$-\frac{-\frac{1}{2}x^2b+3ax}{b^4}+\frac{4a^3}{b^5(bx+a)}-\frac{a^4}{2b^5(bx+a)^2}+\frac{6a^2\ln(bx+a)}{b^5}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/2*x^2*b+3*a*x)+4*a^3/b^5/(b*x+a)-1/2*a^4/b^5/(b*x+a)^2+6*a^2*ln(b*x+a)/b^5
```

Maxima [A]
time = 0.27, size = 69, normalized size = 1.08

$$\frac{8\,a^3bx+7\,a^4}{2\,(b^7x^2+2\,ab^6x+a^2b^5)}+\frac{6\,a^2\log{(bx+a)}}{b^5}+\frac{bx^2-6\,ax}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*b*x + 7*a^4)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a)/b^5 + 1/2*(b*x^2 - 6*a*x)/b^4
```

Fricas [A]
time = 2.06, size = 95, normalized size = 1.48

$$\frac{b^4x^4-4\,ab^3x^3-11\,a^2b^2x^2+2\,a^3bx+7\,a^4+12\,(a^2b^2x^2+2\,a^3bx+a^4)\log{(bx+a)}}{2\,(b^7x^2+2\,ab^6x+a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A]
time = 0.12, size = 70, normalized size = 1.09

$$\frac{6a^2\log\left(a+bx\right)}{b^5}-\frac{3ax}{b^4}+\frac{7a^4+8a^3bx}{2a^2b^5+4ab^6x+2b^7x^2}+\frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**3,x)
```

```
[Out] 6*a**2*log(a + b*x)/b**5 - 3*a*x/b**4 + (7*a**4 + 8*a**3*b*x)/(2*a**2*b**5
+ 4*a*b**6*x + 2*b**7*x**2) + x**2/(2*b**3)
```

Giac [A]
time = 0.96, size = 61, normalized size = 0.95

$$\frac{6\,a^2\log\left(|bx+a|\right)}{b^5}+\frac{b^3x^2-6\,ab^2x}{2\,b^6}+\frac{8\,a^3bx+7\,a^4}{2\,(bx+a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

Mupad [B]
time = 0.08, size = 54, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{4\,a^3}{a+b\,x}-\frac{a^4}{2\,(a+b\,x)^2}+6\,a^2\,\ln\left(a+b\,x\right)-4\,a\,b\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^2/2 + (4*a^3)/(a + b*x) - a^4/(2*(a + b*x)^2) + 6*a^2*log(a + b*
x) - 4*a*b*x)/b^5
```

3.184 $\int \frac{x^3}{(a+bx)^3} \, dx$

Optimal. Leaf size=50

$$\frac{x}{b^3}+\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}$$

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^3,x]
```

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^3}\,dx &= \int \left(\frac{1}{b^3}-\frac{a^3}{b^3(a+bx)^3}+\frac{3a^2}{b^3(a+bx)^2}-\frac{3a}{b^3(a+bx)}\right)dx\\ &= \frac{x}{b^3}+\frac{a^3}{2b^4(a+bx)^2}-\frac{3a^2}{b^4(a+bx)}-\frac{3a\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.80

$$-\frac{-2bx+\frac{a^2(5a+6bx)}{(a+bx)^2}+6a\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^3,x]
```

```
[Out] -1/2*(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/b^4
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.98

method	result	size
risch	$\frac{x}{b^3}+\frac{-3a^2x-\frac{5a^3}{2b}}{b^3(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	45
norman	$\frac{\frac{x^3}{b}-\frac{9a^3}{2b^4}-\frac{6a^2x}{b^3}}{(bx+a)^2}-\frac{3a\ln(bx+a)}{b^4}$	47
default	$\frac{x}{b^3}+\frac{a^3}{2b^4(bx+a)^2}-\frac{3a^2}{b^4(bx+a)}-\frac{3a\ln(bx+a)}{b^4}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

Maxima [A]
time = 0.27, size = 57, normalized size = 1.14

$$-\frac{6\,a^2bx+5\,a^3}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}+\frac{x}{b^3}-\frac{3\,a\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log(
b*x + a)/b^4
```

Fricas [A]
time = 0.76, size = 83, normalized size = 1.66

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\,(ab^2x^2+2\,a^2bx+a^3)\log(bx+a)}{2\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A]
time = 0.11, size = 58, normalized size = 1.16

$$-\frac{3a\log{(a+bx)}}{b^4}+\frac{-5a^3-6a^2bx}{2a^2b^4+4ab^5x+2b^6x^2}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**3,x)
```

```
[Out] -3*a*log(a + b*x)/b**4 + (-5*a**3 - 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + x/b**3
```

Giac [A]
time = 0.70, size = 44, normalized size = 0.88

$$\frac{x}{b^3}-\frac{3\,a\log{(|bx+a|)}}{b^4}-\frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

Mupad [B]
time = 0.15, size = 43, normalized size = 0.86

$$-\frac{3\,a\,\ln{(a+b\,x)}-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,(a+b\,x)^2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^3,x)
```

```
[Out] -(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2))/b^4
```

3.185 $\int \frac{x^2}{(a+bx)^3} \, dx$

Optimal. Leaf size=41

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^3,x]
```

```
[Out] -1/2*a^2/(b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^3} \, dx &= \int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right) \, dx\\ &= -\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.80

$$\frac{\frac{a(3a+4bx)}{(a+bx)^2}+2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

Maple [A]
time = 0.08, size = 40, normalized size = 0.98

method	result	size
norman	$\frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
risch	$\frac{\frac{3a^2}{2b^3}+\frac{2ax}{b^2}}{(bx+a)^2}+\frac{\ln(bx+a)}{b^3}$	36
default	$-\frac{a^2}{2b^3(bx+a)^2}+\frac{2a}{b^3(bx+a)}+\frac{\ln(bx+a)}{b^3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.17

$$\frac{4\,abx+3\,a^2}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}+\frac{\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

Fricas [A]
time = 1.02, size = 61, normalized size = 1.49

$$\frac{4\,abx+3\,a^2+2\,(b^2x^2+2\,abx+a^2)\log(bx+a)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

Sympy [A]
time = 0.08, size = 46, normalized size = 1.12

$$\frac{3a^2+4abx}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{\log(a+bx)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)/
b**3
```

Giac [A]
time = 0.62, size = 37, normalized size = 0.90

$$\frac{\log\left(|bx+a|\right)}{b^3}+\frac{4\,ax+\frac{3\,a^2}{b}}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.09, size = 46, normalized size = 1.12

$$\frac{\ln\left(a+b\,x\right)}{b^3}+\frac{\frac{3\,a^2}{2\,b^3}+\frac{2\,a\,x}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^3,x)
```

```
[Out] log(a + b*x)/b^3 + ((3*a^2)/(2*b^3) + (2*a*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*x
)
```

3.186 $\int \frac{x}{(a+bx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{x^2}{2a(a+bx)^2}$$

```
[Out] 1/2*x^2/a/(b*x+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{37\}$

$$\frac{x^2}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^3,x]
```

```
[Out] x^2/(2*a*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{(a+bx)^3}\,dx = \frac{x^2}{2a(a+bx)^2}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.18

$$-\frac{a+2bx}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^3,x]
```

```
[Out] -1/2*(a + 2*b*x)/(b^2*(a + b*x)^2)
```

Maple [A]
time = 0.08, size = 27, normalized size = 1.59

method	result	size
gosper	$-\frac{2bx+a}{2(bx+a)^{2}b^{2}}$	19
norman	$\frac{-\frac{x}{b}-\frac{a}{2b^{2}}}{(bx+a)^{2}}$	22
risch	$\frac{-\frac{x}{b}-\frac{a}{2b^{2}}}{(bx+a)^{2}}$	22
default	$-\frac{1}{b^{2}(bx+a)}+\frac{a}{2b^{2}(bx+a)^{2}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/(b*x+a)+1/2*a/b^2/(b*x+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.
time = 0.27, size = 32, normalized size = 1.88

$$-\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.
time = 1.23, size = 32, normalized size = 1.88

$$-\frac{2\,bx+a}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.
time = 0.07, size = 32, normalized size = 1.88

$$\frac{-a-2bx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**3,x)
```

```
[Out] (-a - 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Giac [A]
time = 0.53, size = 18, normalized size = 1.06

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.07, size = 32, normalized size = 1.88

$$-\frac{\frac{a}{2\,b^2}+\frac{x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^3,x)
```

```
[Out] -(a/(2*b^2) + x/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.187 $\int \frac{1}{(a+bx)^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

```
[Out] -1/2/b/(b*x+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3} \, dx = -\frac{1}{2b(a+bx)^2}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{2b(bx+a)^2}$	13
default	$-\frac{1}{2b(bx+a)^2}$	13
norman	$-\frac{1}{2b(bx+a)^2}$	13
risch	$-\frac{1}{2b(bx+a)^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/(b*x+a)^2
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(bx+a\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*x + a)^2*b)
```

Fricas [A]
time = 0.91, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^{3}x^{2}+2\,ab^{2}x+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2a^{2}b+4ab^{2}x+2b^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3,x)
```

```
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

Giac [A]
time = 0.58, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(bx+a\right)^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b)
```

Mupad [B]
time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^3,x)
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

3.188 $\int \frac{1}{x(a+bx)^3}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}$$

```
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{46\}$

$$-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a^2(a+bx)}+\frac{1}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^3),x]
```

```
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx\\ &= \frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.86

$$\frac{\frac{a(3a+2bx)}{(a+bx)^2}+2\log(x)-2\log(a+bx)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.98

method	result	size
risch	$\frac{\frac{bx}{a^2}+\frac{3}{2a}}{(bx+a)^2}-\frac{\ln(bx+a)}{a^3}+\frac{\ln(-x)}{a^3}$	41
default	$\frac{1}{2a(bx+a)^2}+\frac{1}{a^2(bx+a)}+\frac{\ln(x)}{a^3}-\frac{\ln(bx+a)}{a^3}$	42
norman	$\frac{-\frac{2bx}{a^2}-\frac{3b^2x^2}{2a^3}}{(bx+a)^2}+\frac{\ln(x)}{a^3}-\frac{\ln(bx+a)}{a^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

Maxima [A]
time = 0.27, size = 51, normalized size = 1.19

$$\frac{2\,bx+3\,a}{2\,(a^2b^2x^2+2\,a^3bx+a^4)}-\frac{\log{(bx+a)}}{a^3}+\frac{\log{(x)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

Fricas [A]
time = 1.18, size = 80, normalized size = 1.86

$$\frac{2\,abx+3\,a^2-2\,(b^2x^2+2\,abx+a^2)\log{(bx+a)}+2\,(b^2x^2+2\,abx+a^2)\log{(x)}}{2\,(a^3b^2x^2+2\,a^4bx+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

Sympy [A]
time = 0.12, size = 46, normalized size = 1.07

$$\frac{3a+2bx}{2a^4+4a^3bx+2a^2b^2x^2}+\frac{\log{(x)}-\log{\left(\frac{a}{b}+x\right)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**3,x)
```

```
[Out] (3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
+ x))/a**3
```

Giac [A]
time = 0.55, size = 43, normalized size = 1.00

$$-\frac{\log\left(|bx+a|\right)}{a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{2\,abx+3\,a^2}{2\,(bx+a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

Mupad [B]
time = 0.10, size = 43, normalized size = 1.00

$$\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,{\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^3),x)
```

```
[Out] (1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2)
```

3.189 $\int \frac{1}{x^2(a+bx)^3} dx$

Optimal. Leaf size=57

$$-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}$$

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{2b}{a^3(a+bx)}-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.93

$$-\frac{\frac{a(2a^2+9abx+6b^2x^2)}{x(a+bx)^2}+6b\log(x)-6b\log(a+bx)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^3),x]
```

```
[Out] -1/2*((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b*
Log[a + b*x])/a^4
```

Maple [A]
time = 0.09, size = 56, normalized size = 0.98

method	result	size
default	$-\frac{1}{a^{3} x}-\frac{b}{2 a^{2} \left(b x +a \right)^{2}}-\frac{2 b}{a^{3} \left(b x +a \right)}-\frac{3 b \ln \left(x \right)}{a^{4}}+\frac{3 b \ln \left(b x +a \right)}{a^{4}}$	56
risch	$\frac{-\frac{3 b^{2} x^{2}}{a^{3}}-\frac{9 b x}{2 a^{2}}-\frac{1}{a}}{x \left(b x +a \right)^{2}}-\frac{3 b \ln \left(x \right)}{a^{4}}+\frac{3 b \ln \left(-b x -a \right)}{a^{4}}$	60
norman	$\frac{-\frac{1}{a}+\frac{6 b^{2} x^{2}}{a^{3}}+\frac{9 b^{3} x^{3}}{2 a^{4}}}{x \left(b x +a \right)^{2}}-\frac{3 b \ln \left(x \right)}{a^{4}}+\frac{3 b \ln \left(b x +a \right)}{a^{4}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

Maxima [A]
time = 0.28, size = 69, normalized size = 1.21

$$-\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log\left(x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

Fricas [A]
time = 1.06, size = 109, normalized size = 1.91

$$-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log\left(bx+a\right)+6\,(b^3x^3+2\,ab^2x^2+a^2bx)\log\left(x\right)}{2\,(a^4b^2x^3+2\,a^5bx^2+a^6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

Sympy [A]
time = 0.15, size = 66, normalized size = 1.16

$$\frac{-2a^2 - 9abx - 6b^2x^2}{2a^5x + 4a^4bx^2 + 2a^3b^2x^3} + \frac{3b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x - 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*x
**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

Giac [A]
time = 0.80, size = 60, normalized size = 1.05

$$\frac{3\,b\log\left(|bx+a|\right)}{a^4} - \frac{3\,b\log\left(|x|\right)}{a^4} - \frac{6\,ab^2x^2 + 9\,a^2bx + 2\,a^3}{2\,(bx+a)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

Mupad [B]
time = 0.11, size = 63, normalized size = 1.11

$$\frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4} - \frac{\frac{1}{a} + \frac{3\,b^2\,x^2}{a^3} + \frac{9\,b\,x}{2\,a^2}}{a^2\,x + 2\,a\,b\,x^2 + b^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^3),x)
```

```
[Out] (6*b*atanh((2*b*x)/a + 1))/a^4 - (1/a + (3*b^2*x^2)/a^3 + (9*b*x)/(2*a^2))/
(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

3.190 $\int \frac{1}{x^3(a+bx)^3} dx$

Optimal. Leaf size=76

$$-\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}$$

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b^2}{a^4(a+bx)}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/2*1/(a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(
a + b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.89

$$\frac{\frac{a(-a^3+4a^2bx+18ab^2x^2+12b^3x^3)}{x^2(a+bx)^2}+12b^2\log(x)-12b^2\log(a+bx)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12*
b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

Maple [A]
time = 0.09, size = 73, normalized size = 0.96

method	result	size
norman	$\frac{-\frac{9b^4x^4}{a^5}-\frac{1}{2a}+\frac{2bx}{a^2}-\frac{12b^3x^3}{a^4}}{x^2(bx+a)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$	72
default	$-\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(bx+a)^2}+\frac{3b^2}{a^4(bx+a)}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$	73
risch	$\frac{\frac{6b^3x^3}{a^4}+\frac{9b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^2}+\frac{6b^2\ln(-x)}{a^5}-\frac{6b^2\ln(bx+a)}{a^5}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.13

$$\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}-\frac{6\,b^2\log\left(bx+a\right)}{a^5}+\frac{6\,b^2\log\left(x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

Fricas [A]
time = 0.91, size = 130, normalized size = 1.71

$$\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\log\left(bx+a\right)+12\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\log\left(x\right)}{2\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [A]
time = 0.16, size = 78, normalized size = 1.03

$$\frac{-a^3 + 4a^2bx + 18ab^2x^2 + 12b^3x^3}{2a^6x^2 + 4a^5bx^3 + 2a^4b^2x^4} + \frac{6b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5*
b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

Giac [A]
time = 1.05, size = 73, normalized size = 0.96

$$-\frac{6\,b^2\log\left(|bx + a|\right)}{a^5} + \frac{6\,b^2\log\left(|x|\right)}{a^5} + \frac{12\,b^3x^3 + 18\,ab^2x^2 + 4\,a^2bx - a^3}{2\,(bx^2 + ax)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

Mupad [B]
time = 0.12, size = 79, normalized size = 1.04

$$\frac{\frac{9\,b^2\,x^2}{a^3} - \frac{1}{2\,a} + \frac{6\,b^3\,x^3}{a^4} + \frac{2\,b\,x}{a^2}}{a^2\,x^2 + 2\,a\,b\,x^3 + b^2\,x^4} - \frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^3),x)
```

```
[Out] ((9*b^2*x^2)/a^3 - 1/(2*a) + (6*b^3*x^3)/a^4 + (2*b*x)/a^2)/(a^2*x^2 + b^2*
x^4 + 2*a*b*x^3) - (12*b^2*atanh((2*b*x)/a + 1))/a^5
```

3.191 $\int \frac{1}{x^4(a+bx)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}$$

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}-\frac{4b^3}{a^5(a+bx)}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/3*1/(a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b*
x)^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b*x
])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 0.89

$$-\frac{\frac{a(2a^4-5a^3bx+20a^2b^2x^2+90ab^3x^3+60b^4x^4)}{x^3(a+bx)^2}+60b^3\log(x)-60b^3\log(a+bx)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/6*((a*(2*a^4 - 5*a^3*b*x + 20*a^2*b^2*x^2 + 90*a*b^3*x^3 + 60*b^4*x^4))/
(x^3*(a + b*x)^2) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a^6
```

Maple [A]
time = 0.09, size = 84, normalized size = 0.94

method	result	size
norman	$\frac{\frac{15b^5x^5}{a^6}-\frac{1}{3a}+\frac{5bx}{6a^2}-\frac{10b^2x^2}{3a^3}+\frac{20b^4x^4}{a^5}}{x^3(bx+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(bx+a)}{a^6}$	83
default	$-\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(bx+a)^2}-\frac{4b^3}{a^5(bx+a)}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(bx+a)}{a^6}$	84
risch	$\frac{-\frac{10b^4x^4}{a^5}-\frac{15b^3x^3}{a^4}-\frac{10b^2x^2}{3a^3}+\frac{5bx}{6a^2}-\frac{1}{3a}}{x^3(bx+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln(-bx-a)}{a^6}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.31, size = 97, normalized size = 1.09

$$-\frac{60\,b^4x^4+90\,ab^3x^3+20\,a^2b^2x^2-5\,a^3bx+2\,a^4}{6\,(a^5b^2x^5+2\,a^6bx^4+a^7x^3)}+\frac{10\,b^3\log\left(bx+a\right)}{a^6}-\frac{10\,b^3\log\left(x\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(60*b^4*x^4 + 90*a*b^3*x^3 + 20*a^2*b^2*x^2 - 5*a^3*b*x + 2*a^4)/(a^5*
b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3) + 10*b^3*log(b*x + a)/a^6 - 10*b^3*log(x)/
a^6
```

Fricas [A]
time = 0.95, size = 141, normalized size = 1.58

$$-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5-60\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\log\left(bx+a\right)+60\,(b^5x^5+2\,ab^4x^4+a^2b^3x^3)\log\left(x\right)}{6\,(a^6b^2x^5+2\,a^7bx^4+a^8x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*a*b^4*x^4 + 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5 -
60*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*log(b*x + a) + 60*(b^5*x^5 + 2*a*b
^4*x^4 + a^2*b^3*x^3)*log(x))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

Sympy [A]
time = 0.18, size = 92, normalized size = 1.03

$$\frac{-2a^4+5a^3bx-20a^2b^2x^2-90ab^3x^3-60b^4x^4}{6a^7x^3+12a^6bx^4+6a^5b^2x^5}+\frac{10b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x - 20*a**2*b**2*x**2 - 90*a*b**3*x**3 - 60*b**4*x**4)/
(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 10*b**3*(-log(x) + log(
a/b + x))/a**6
```

Giac [A]
time = 0.95, size = 86, normalized size = 0.97

$$\frac{10\,b^3\log(|bx+a|)}{a^6}-\frac{10\,b^3\log(|x|)}{a^6}-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5}{6\,(bx+a)^2a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 10*b^3*log(abs(b*x + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/6*(60*a*b^4*x^4 +
 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5)/((b*x + a)^2*a^6*x^3)
```

Mupad [B]
time = 0.13, size = 91, normalized size = 1.02

$$\frac{20\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}-\frac{\frac{1}{3\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{15\,b^3\,x^3}{a^4}+\frac{10\,b^4\,x^4}{a^5}-\frac{5\,b\,x}{6\,a^2}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^3),x)
```

```
[Out] (20*b^3*atanh((2*b*x)/a + 1))/a^6 - (1/(3*a) + (10*b^2*x^2)/(3*a^3) + (15*b
^3*x^3)/a^4 + (10*b^4*x^4)/a^5 - (5*b*x)/(6*a^2))/(a^2*x^3 + b^2*x^5 + 2*a*
b*x^4)
```

3.192 $\int \frac{1}{x^5(a+bx)^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}$$

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{5b^4}{a^6(a+bx)}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}-\frac{3b^2}{a^5x^2}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^3),x]
```

```
[Out] -1/4*1/(a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^4
/(2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (15*
b^4*Log[a + b*x])/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx)}\right) \\ &= -\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\end{aligned}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.93

$$\frac{\frac{a\left(-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5\right)}{x^4(a+bx)^2}+60b^4\log(x)-60b^4\log(a+bx)}{4a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x - 5*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 90*a*b^4*x^4 + 60*
b^5*x^5))/(x^4*(a + b*x)^2) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(4*a^7)
```

Maple [A]
time = 0.08, size = 94, normalized size = 0.97

method	result	size
default	$-\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(bx+a)^2}+\frac{5b^4}{a^6(bx+a)}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$	94
norman	$\frac{-\frac{1}{4a}+\frac{bx}{2a^2}-\frac{5b^2x^2}{4a^3}+\frac{5b^3x^3}{a^4}-\frac{30b^5x^5}{a^6}-\frac{45b^6x^6}{2a^7}}{x^4(bx+a)^2}+\frac{15b^4\ln(x)}{a^7}-\frac{15b^4\ln(bx+a)}{a^7}$	94
risch	$\frac{\frac{15b^5x^5}{a^6}+\frac{45b^4x^4}{2a^5}+\frac{5b^3x^3}{a^4}-\frac{5b^2x^2}{4a^3}+\frac{bx}{2a^2}-\frac{1}{4a}}{x^4(bx+a)^2}-\frac{15b^4\ln(bx+a)}{a^7}+\frac{15b^4\ln(-x)}{a^7}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

Maxima [A]
time = 0.29, size = 108, normalized size = 1.11

$$\frac{60\,b^5x^5+90\,ab^4x^4+20\,a^2b^3x^3-5\,a^3b^2x^2+2\,a^4bx-a^5}{4\,(a^6b^2x^6+2\,a^7bx^5+a^8x^4)}-\frac{15\,b^4\log\left(bx+a\right)}{a^7}+\frac{15\,b^4\log\left(x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(60*b^5*x^5 + 90*a*b^4*x^4 + 20*a^2*b^3*x^3 - 5*a^3*b^2*x^2 + 2*a^4*b*x
 - a^5)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4) - 15*b^4*log(b*x + a)/a^7 + 1
5*b^4*log(x)/a^7
```

Fricas [A]
time = 1.04, size = 152, normalized size = 1.57

$$\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6-60\,(b^6x^6+2\,ab^5x^5+a^2b^4x^4)\log\left(bx+a\right)+60\,(b^6x^6+2\,ab^5x^5+a^2b^4x^4)\log\left(x\right)}{4\,(a^7b^2x^6+2\,a^8bx^5+a^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5
*b*x - a^6 - 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(b*x + a) + 60*(b^
```

```
6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(x))/(a^7*b^2*x^6 + 2*a^8*b*x^5 + a^9
*x^4)
```

Sympy [A]
time = 0.20, size = 102, normalized size = 1.05

$$\frac{-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5}{4a^8x^4+8a^7bx^5+4a^6b^2x^6}+\frac{15b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x - 5*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 90*a*b**4*x**4
 + 60*b**5*x**5)/(4*a**8*x**4 + 8*a**7*b*x**5 + 4*a**6*b**2*x**6) + 15*b**4
*(log(x) - log(a/b + x))/a**7
```

Giac [A]
time = 0.95, size = 97, normalized size = 1.00

$$-\frac{15\,b^4\log\left(|bx+a|\right)}{a^7}+\frac{15\,b^4\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+90\,a^2b^4x^4+20\,a^3b^3x^3-5\,a^4b^2x^2+2\,a^5bx-a^6}{4\,(bx+a)^2a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15*b^4*log(abs(b*x + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/4*(60*a*b^5*x^5
+ 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6)/((b*x
+ a)^2*a^7*x^4)
```

Mupad [B]
time = 0.09, size = 101, normalized size = 1.04

$$\frac{\frac{5\,b^3\,x^3}{a^4}-\frac{5\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{45\,b^4\,x^4}{2\,a^5}+\frac{15\,b^5\,x^5}{a^6}+\frac{b\,x}{2\,a^2}}{a^2\,x^4+2\,a\,b\,x^5+b^2\,x^6}-\frac{30\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^3),x)
```

```
[Out] ((5*b^3*x^3)/a^4 - (5*b^2*x^2)/(4*a^3) - 1/(4*a) + (45*b^4*x^4)/(2*a^5) + (
15*b^5*x^5)/a^6 + (b*x)/(2*a^2))/(a^2*x^4 + b^2*x^6 + 2*a*b*x^5) - (30*b^4*
atanh((2*b*x)/a + 1))/a^7
```

3.193 $\int \frac{x^8}{(a+bx)^4} \, dx$

Optimal. Leaf size=114

$$\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}$$

```
[Out] 35*a^4*x/b^8-10*a^3*x^2/b^7+10/3*a^2*x^3/b^6-a*x^4/b^5+1/5*x^5/b^4-1/3*a^8/
b^9/(b*x+a)^3+4*a^7/b^9/(b*x+a)^2-28*a^6/b^9/(b*x+a)-56*a^5*ln(b*x+a)/b^9
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^4,x]
```

```
[Out] (35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x^
5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6)/
(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(a+bx)^4}\,dx &= \int \left(\frac{35a^4}{b^8}-\frac{20a^3x}{b^7}+\frac{10a^2x^2}{b^6}-\frac{4ax^3}{b^5}+\frac{x^4}{b^4}+\frac{a^8}{b^8(a+bx)^4}-\frac{8a^7}{b^8(a+bx)^3}+\frac{28a^6}{b^8(a+bx)^2}-\right.\\ &= \frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-5\end{aligned}$$

Mathematica [A]
time = 0.03, size = 101, normalized size = 0.89

$$\frac{525a^4bx-150a^3b^2x^2+50a^2b^3x^3-15ab^4x^4+3b^5x^5-\frac{5a^8}{(a+bx)^3}+\frac{60a^7}{(a+bx)^2}-\frac{420a^6}{a+bx}-840a^5\log(a+bx)}{15b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^4,x]
```

```
[Out] (525*a^4*b*x - 150*a^3*b^2*x^2 + 50*a^2*b^3*x^3 - 15*a*b^4*x^4 + 3*b^5*x^5
- (5*a^8)/(a + b*x)^3 + (60*a^7)/(a + b*x)^2 - (420*a^6)/(a + b*x) - 840*a^
5*Log[a + b*x])/(15*b^9)
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.96

method	result	size
risch	$\frac{x^5}{5b^4}-\frac{a\,x^4}{b^5}+\frac{10a^2x^3}{3b^6}-\frac{10a^3x^2}{b^7}+\frac{35a^4x}{b^8}+\frac{-28a^6b\,x^2-52a^7x-\frac{73a^8}{3b}}{b^8(bx+a)^3}-\frac{56a^5\ln(bx+a)}{b^9}$	99
norman	$\frac{\frac{x^8}{5b}-\frac{2a\,x^7}{5b^2}-\frac{14a^3x^5}{5b^4}-\frac{308a^8}{3b^9}+\frac{14a^4x^4}{b^5}+\frac{14a^2x^6}{15b^3}-\frac{168a^6x^2}{b^7}-\frac{252a^7x}{b^8}}{(bx+a)^3}-\frac{56a^5\ln(bx+a)}{b^9}$	103
default	$\frac{\frac{1}{5}b^4x^5-a\,b^3x^4+\frac{10}{3}a^2b^2x^3-10a^3b\,x^2+35a^4x}{b^8}-\frac{28a^6}{b^9(bx+a)}+\frac{4a^7}{b^9(bx+a)^2}-\frac{56a^5\ln(bx+a)}{b^9}-\frac{a^8}{3b^9(bx+a)^3}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^8*(1/5*b^4*x^5-a*b^3*x^4+10/3*a^2*b^2*x^3-10*a^3*b*x^2+35*a^4*x)-28*a^6
/b^9/(b*x+a)+4*a^7/b^9/(b*x+a)^2-56*a^5*ln(b*x+a)/b^9-1/3*a^8/b^9/(b*x+a)^3
```

Maxima [A]
time = 0.26, size = 125, normalized size = 1.10

$$-\frac{84\,a^6b^2x^2+156\,a^7bx+73\,a^8}{3\,(b^{12}x^3+3\,ab^{11}x^2+3\,a^2b^{10}x+a^3b^9)}-\frac{56\,a^5\log(bx+a)}{b^9}+\frac{3\,b^4x^5-15\,ab^3x^4+50\,a^2b^2x^3-150\,a^3bx^2+525\,a^4x}{15\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)/(b^12*x^3 + 3*a*b^11*x^2 + 3*a
^2*b^10*x + a^3*b^9) - 56*a^5*log(b*x + a)/b^9 + 1/15*(3*b^4*x^5 - 15*a*b^3
*x^4 + 50*a^2*b^2*x^3 - 150*a^3*b*x^2 + 525*a^4*x)/b^8
```

Fricas [A]
time = 0.71, size = 162, normalized size = 1.42

$$\frac{3\,b^8x^8-6\,ab^7x^7+14\,a^2b^6x^6-42\,a^3b^5x^5+210\,a^4b^4x^4+1175\,a^5b^3x^3+1005\,a^6b^2x^2-255\,a^7bx-365\,a^8-840\,(a^5b^3x^3+3\,a^6b^2x^2+3\,a^7bx+a^8)\log(bx+a)}{15\,(b^{12}x^3+3\,ab^{11}x^2+3\,a^2b^{10}x+a^3b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^8*x^8 - 6*a*b^7*x^7 + 14*a^2*b^6*x^6 - 42*a^3*b^5*x^5 + 210*a^4*b
^4*x^4 + 1175*a^5*b^3*x^3 + 1005*a^6*b^2*x^2 - 255*a^7*b*x - 365*a^8 - 840*
```

```
(a^5*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8)*log(b*x + a))/(b^12*x^3 + 3
*a*b^11*x^2 + 3*a^2*b^10*x + a^3*b^9)
```

Sympy [A]
time = 0.22, size = 131, normalized size = 1.15

$$-\frac{56a^5\log\left(a+bx\right)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{-73a^8-156a^7bx-84a^6b^2x^2}{3a^3b^9+9a^2b^{10}x+9ab^{11}x^2+3b^{12}x^3}+\frac{x^5}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**4,x)
```

```
[Out] -56*a**5*log(a + b*x)/b**9 + 35*a**4*x/b**8 - 10*a**3*x**2/b**7 + 10*a**2*x
**3/(3*b**6) - a*x**4/b**5 + (-73*a**8 - 156*a**7*b*x - 84*a**6*b**2*x**2)/
(3*a**3*b**9 + 9*a**2*b**10*x + 9*a*b**11*x**2 + 3*b**12*x**3) + x**5/(5*b*
*4)
```

Giac [A]
time = 1.21, size = 106, normalized size = 0.93

$$-\frac{56\,a^5\log\left(|bx+a|\right)}{b^9}-\frac{84\,a^6b^2x^2+156\,a^7bx+73\,a^8}{3\,(bx+a)^3b^9}+\frac{3\,b^{16}x^5-15\,ab^{15}x^4+50\,a^2b^{14}x^3-150\,a^3b^{13}x^2+525\,a^4b^{12}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -56*a^5*log(abs(b*x + a))/b^9 - 1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)
/((b*x + a)^3*b^9) + 1/15*(3*b^16*x^5 - 15*a*b^15*x^4 + 50*a^2*b^14*x^3 - 1
50*a^3*b^13*x^2 + 525*a^4*b^12*x)/b^20
```

Mupad [B]
time = 0.37, size = 103, normalized size = 0.90

$$-\frac{2\,a\,(a+b\,x)^4-\frac{(a+b\,x)^5}{5}-\frac{28\,a^2\,(a+b\,x)^3}{3}+28\,a^3\,(a+b\,x)^2+\frac{28\,a^6}{a+b\,x}-\frac{4\,a^7}{(a+b\,x)^2}+\frac{a^8}{3\,(a+b\,x)^3}+56\,a^5\,\ln\left(a+b\,x\right)-70\,a^4\,b\,x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^4,x)
```

```
[Out] -(2*a*(a + b*x)^4 - (a + b*x)^5/5 - (28*a^2*(a + b*x)^3)/3 + 28*a^3*(a + b*
x)^2 + (28*a^6)/(a + b*x) - (4*a^7)/(a + b*x)^2 + a^8/(3*(a + b*x)^3) + 56*
a^5*log(a + b*x) - 70*a^4*b*x)/b^9
```

3.194 $\int \frac{x^7}{(a+bx)^4}\,dx$

Optimal. Leaf size=105

$$-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}$$

```
[Out] -20*a^3*x/b^7+5*a^2*x^2/b^6-4/3*a*x^3/b^5+1/4*x^4/b^4+1/3*a^7/b^8/(b*x+a)^3
-7/2*a^6/b^8/(b*x+a)^2+21*a^5/b^8/(b*x+a)+35*a^4*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^4,x]
```

```
[Out] (-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/(
3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x))
 + (35*a^4*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^4}\,dx &= \int \left(-\frac{20a^3}{b^7}+\frac{10a^2x}{b^6}-\frac{4ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^7}{b^7(a+bx)^4}+\frac{7a^6}{b^7(a+bx)^3}-\frac{21a^5}{b^7(a+bx)^2}+\frac{35a^4}{b^7(a+bx)}\right)dx\\ &= -\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 90, normalized size = 0.86

$$\frac{-240a^3bx+60a^2b^2x^2-16ab^3x^3+3b^4x^4+\frac{4a^7}{(a+bx)^3}-\frac{42a^6}{(a+bx)^2}+\frac{252a^5}{a+bx}+420a^4\log(a+bx)}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^4,x]
```

```
[Out] (-240*a^3*b*x + 60*a^2*b^2*x^2 - 16*a*b^3*x^3 + 3*b^4*x^4 + (4*a^7)/(a + b*
x)^3 - (42*a^6)/(a + b*x)^2 + (252*a^5)/(a + b*x) + 420*a^4*Log[a + b*x])/(
12*b^8)
```

Maple [A]
time = 0.10, size = 99, normalized size = 0.94

method	result	size
risch	$\frac{x^4}{4b^4}-\frac{4a\,x^3}{3b^5}+\frac{5a^2x^2}{b^6}-\frac{20a^3x}{b^7}+\frac{21a^5b\,x^2+\frac{77a^6x}{2}+\frac{107a^7}{6b}}{b^7(bx+a)^3}+\frac{35a^4\ln(bx+a)}{b^8}$	88
norman	$\frac{\frac{x^7}{4b}-\frac{7a\,x^6}{12b^2}-\frac{35a^3x^4}{4b^4}+\frac{385a^7}{6b^8}+\frac{7a^2x^5}{4b^3}+\frac{105a^5x^2}{b^6}+\frac{315a^6x}{2b^7}}{(bx+a)^3}+\frac{35a^4\ln(bx+a)}{b^8}$	92
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{4}{3}a\,b^2x^3-5a^2b\,x^2+20a^3x}{b^7}+\frac{21a^5}{b^8(bx+a)}-\frac{7a^6}{2b^8(bx+a)^2}+\frac{35a^4\ln(bx+a)}{b^8}+\frac{a^7}{3b^8(bx+a)^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^7*(-1/4*b^3*x^4+4/3*a*b^2*x^3-5*a^2*b*x^2+20*a^3*x)+21*a^5/b^8/(b*x+a)
-7/2*a^6/b^8/(b*x+a)^2+35*a^4*ln(b*x+a)/b^8+1/3*a^7/b^8/(b*x+a)^3
```

Maxima [A]
time = 0.28, size = 114, normalized size = 1.09

$$\frac{126\,a^5b^2x^2+231\,a^6bx+107\,a^7}{6\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}+\frac{35\,a^4\log\left(bx+a\right)}{b^8}+\frac{3\,b^3x^4-16\,ab^2x^3+60\,a^2bx^2-240\,a^3x}{12\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7)/(b^11*x^3 + 3*a*b^10*x^2 + 3*
a^2*b^9*x + a^3*b^8) + 35*a^4*log(b*x + a)/b^8 + 1/12*(3*b^3*x^4 - 16*a*b^2
*x^3 + 60*a^2*b*x^2 - 240*a^3*x)/b^7
```

Fricas [A]
time = 0.69, size = 151, normalized size = 1.44

$$\frac{3\,b^7x^7-7\,ab^6x^6+21\,a^2b^5x^5-105\,a^3b^4x^4-556\,a^4b^3x^3-408\,a^5b^2x^2+222\,a^6bx+214\,a^7+420\,(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7)\log\left(bx+a\right)}{12\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 - 7*a*b^6*x^6 + 21*a^2*b^5*x^5 - 105*a^3*b^4*x^4 - 556*a^4*
b^3*x^3 - 408*a^5*b^2*x^2 + 222*a^6*b*x + 214*a^7 + 420*(a^4*b^3*x^3 + 3*a^
5*b^2*x^2 + 3*a^6*b*x + a^7)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2
*b^9*x + a^3*b^8)
```

Sympy [A]
time = 0.20, size = 119, normalized size = 1.13

$$\frac{35a^4\log{(a+bx)}}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{107a^7+231a^6bx+126a^5b^2x^2}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3}+\frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**4,x)
```

```
[Out] 35*a**4*log(a + b*x)/b**8 - 20*a**3*x/b**7 + 5*a**2*x**2/b**6 - 4*a*x**3/(3
*b**5) + (107*a**7 + 231*a**6*b*x + 126*a**5*b**2*x**2)/(6*a**3*b**8 + 18*a
**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*x**3) + x**4/(4*b**4)
```

Giac [A]
time = 1.42, size = 95, normalized size = 0.90

$$\frac{35\,a^4\log{(|bx+a|)}}{b^8}+\frac{126\,a^5b^2x^2+231\,a^6bx+107\,a^7}{6\,(bx+a)^3b^8}+\frac{3\,b^{12}x^4-16\,ab^{11}x^3+60\,a^2b^{10}x^2-240\,a^3b^9x}{12\,b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*a^4*log(abs(b*x + a))/b^8 + 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7
)/((b*x + a)^3*b^8) + 1/12*(3*b^12*x^4 - 16*a*b^11*x^3 + 60*a^2*b^10*x^2 -
240*a^3*b^9*x)/b^16
```

Mupad [B]
time = 0.22, size = 90, normalized size = 0.86

$$\frac{\frac{(a+b\,x)^4}{4}-\frac{7\,a\,(a+b\,x)^3}{3}+\frac{21\,a^2\,(a+b\,x)^2}{2}+\frac{21\,a^5}{a+b\,x}-\frac{7\,a^6}{2\,(a+b\,x)^2}+\frac{a^7}{3\,(a+b\,x)^3}+35\,a^4\,\ln{(a+b\,x)}-35\,a^3\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^4/4 - (7*a*(a + b*x)^3)/3 + (21*a^2*(a + b*x)^2)/2 + (21*a^5)/(a
 + b*x) - (7*a^6)/(2*(a + b*x)^2) + a^7/(3*(a + b*x)^3) + 35*a^4*log(a + b*
x) - 35*a^3*b*x)/b^8
```

3.195 $\int \frac{x^6}{(a+bx)^4} dx$

Optimal. Leaf size=90

$$\frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{x^3}{3b^4} - \frac{a^6}{3b^7(a+bx)^3} + \frac{3a^5}{b^7(a+bx)^2} - \frac{15a^4}{b^7(a+bx)} - \frac{20a^3\log(a+bx)}{b^7}$$

```
[Out] 10*a^2*x/b^6-2*a*x^2/b^5+1/3*x^3/b^4-1/3*a^6/b^7/(b*x+a)^3+3*a^5/b^7/(b*x+a
)^2-15*a^4/b^7/(b*x+a)-20*a^3*ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^6}{3b^7(a+bx)^3} + \frac{3a^5}{b^7(a+bx)^2} - \frac{15a^4}{b^7(a+bx)} - \frac{20a^3\log(a+bx)}{b^7} + \frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^4}\,dx &= \int \left(\frac{10a^2}{b^6} - \frac{4ax}{b^5} + \frac{x^2}{b^4} + \frac{a^6}{b^6(a+bx)^4} - \frac{6a^5}{b^6(a+bx)^3} + \frac{15a^4}{b^6(a+bx)^2} - \frac{20a^3}{b^6(a+bx)}\right) dx\\ &= \frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{x^3}{3b^4} - \frac{a^6}{3b^7(a+bx)^3} + \frac{3a^5}{b^7(a+bx)^2} - \frac{15a^4}{b^7(a+bx)} - \frac{20a^3\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 90, normalized size = 1.00

$$\frac{10a^2x}{b^6} - \frac{2ax^2}{b^5} + \frac{x^3}{3b^4} - \frac{a^6}{3b^7(a+bx)^3} + \frac{3a^5}{b^7(a+bx)^2} - \frac{15a^4}{b^7(a+bx)} - \frac{20a^3\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Maple [A]
time = 0.08, size = 87, normalized size = 0.97

method	result	size
risch	$\frac{x^3}{3b^4}-\frac{2a\,x^2}{b^5}+\frac{10a^2x}{b^6}+\frac{-15a^4b\,x^2-27a^5x-\frac{37a^6}{3b}}{b^6(bx+a)^3}-\frac{20a^3\ln(bx+a)}{b^7}$	77
norman	$\frac{\frac{x^6}{3b}-\frac{a\,x^5}{b^2}-\frac{110a^6}{3b^7}+\frac{5a^2x^4}{b^3}-\frac{60a^4x^2}{b^5}-\frac{90a^5x}{b^6}}{(bx+a)^3}-\frac{20a^3\ln(bx+a)}{b^7}$	81
default	$\frac{\frac{1}{3}b^2x^3-2ab\,x^2+10a^2x}{b^6}-\frac{15a^4}{b^7(bx+a)}+\frac{3a^5}{b^7(bx+a)^2}-\frac{20a^3\ln(bx+a)}{b^7}-\frac{a^6}{3b^7(bx+a)^3}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/3*b^2*x^3-2*a*b*x^2+10*a^2*x)-15*a^4/b^7/(b*x+a)+3*a^5/b^7/(b*x+a)
^2-20*a^3*ln(b*x+a)/b^7-1/3*a^6/b^7/(b*x+a)^3
```

Maxima [A]
time = 0.28, size = 102, normalized size = 1.13

$$-\frac{45\,a^4b^2x^2+81\,a^5bx+37\,a^6}{3\,(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7)}-\frac{20\,a^3\log\left(bx+a\right)}{b^7}+\frac{b^2x^3-6\,abx^2+30\,a^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2
*b^8*x + a^3*b^7) - 20*a^3*log(b*x + a)/b^7 + 1/3*(b^2*x^3 - 6*a*b*x^2 + 30
*a^2*x)/b^6
```

Fricas [A]
time = 1.70, size = 139, normalized size = 1.54

$$\frac{b^6x^6-3\,ab^5x^5+15\,a^2b^4x^4+73\,a^3b^3x^3+39\,a^4b^2x^2-51\,a^5bx-37\,a^6-60\,(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6)\log\left(bx+a\right)}{3\,(b^{10}x^3+3\,ab^9x^2+3\,a^2b^8x+a^3b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*x^6 - 3*a*b^5*x^5 + 15*a^2*b^4*x^4 + 73*a^3*b^3*x^3 + 39*a^4*b^2*x
^2 - 51*a^5*b*x - 37*a^6 - 60*(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*x + a^
6)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

Sympy [A]
time = 0.19, size = 107, normalized size = 1.19

$$-\frac{20a^3\log{(a+bx)}}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{-37a^6-81a^5bx-45a^4b^2x^2}{3a^3b^7+9a^2b^8x+9ab^9x^2+3b^{10}x^3}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**4,x)
```

```
[Out] -20*a**3*log(a + b*x)/b**7 + 10*a**2*x/b**6 - 2*a*x**2/b**5 + (-37*a**6 - 8
1*a**5*b*x - 45*a**4*b**2*x**2)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3) + x**3/(3*b**4)
```

Giac [A]
time = 1.45, size = 83, normalized size = 0.92

$$-\frac{20\,a^3\log\left(|bx+a|\right)}{b^7}-\frac{45\,a^4b^2x^2+81\,a^5bx+37\,a^6}{3\,(bx+a)^3b^7}+\frac{b^8x^3-6\,ab^7x^2+30\,a^2b^6x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -20*a^3*log(abs(b*x + a))/b^7 - 1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/
((b*x + a)^3*b^7) + 1/3*(b^8*x^3 - 6*a*b^7*x^2 + 30*a^2*b^6*x)/b^12
```

Mupad [B]
time = 0.15, size = 79, normalized size = 0.88

$$-\frac{3\,a\,{(a+b\,x)}^2-\frac{{(a+b\,x)}^3}{3}+\frac{15\,a^4}{a+b\,x}-\frac{3\,a^5}{{(a+b\,x)}^2}+\frac{a^6}{3\,{(a+b\,x)}^3}+20\,a^3\,\ln\left(a+b\,x\right)-15\,a^2\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^4,x)
```

```
[Out] -(3*a*(a + b*x)^2 - (a + b*x)^3/3 + (15*a^4)/(a + b*x) - (3*a^5)/(a + b*x)^
2 + a^6/(3*(a + b*x)^3) + 20*a^3*log(a + b*x) - 15*a^2*b*x)/b^7
```

3.196 $\int \frac{x^5}{(a+bx)^4} \, dx$

Optimal. Leaf size=81

$$-\frac{4ax}{b^5}+\frac{x^2}{2b^4}+\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}$$

```
[Out] -4*a*x/b^5+1/2*x^2/b^4+1/3*a^5/b^6/(b*x+a)^3-5/2*a^4/b^6/(b*x+a)^2+10*a^3/b
^6/(b*x+a)+10*a^2*ln(b*x+a)/b^6
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^4,x]
```

```
[Out] (-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a +
b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^4}\,dx &= \int \left(-\frac{4a}{b^5}+\frac{x}{b^4}-\frac{a^5}{b^5(a+bx)^4}+\frac{5a^4}{b^5(a+bx)^3}-\frac{10a^3}{b^5(a+bx)^2}+\frac{10a^2}{b^5(a+bx)}\right)dx\\ &= -\frac{4ax}{b^5}+\frac{x^2}{2b^4}+\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 0.84

$$\frac{-24abx+3b^2x^2+\frac{2a^5}{(a+bx)^3}-\frac{15a^4}{(a+bx)^2}+\frac{60a^3}{a+bx}+60a^2\log(a+bx)}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^4,x]
```

```
[Out] (-24*a*b*x + 3*b^2*x^2 + (2*a^5)/(a + b*x)^3 - (15*a^4)/(a + b*x)^2 + (60*a
^3)/(a + b*x) + 60*a^2*Log[a + b*x])/(6*b^6)
```

Maple [A]
time = 0.08, size = 77, normalized size = 0.95

method	result	size
risch	$\frac{x^2}{2b^4}-\frac{4ax}{b^5}+\frac{10a^3b\,x^2+\frac{35a^4x}{2}+\frac{47a^5}{6b}}{b^5(bx+a)^3}+\frac{10a^2\ln(bx+a)}{b^6}$	66
norman	$\frac{\frac{x^5}{2b}-\frac{5a\,x^4}{2b^2}+\frac{55a^5}{3b^6}+\frac{30a^3x^2}{b^4}+\frac{45a^4x}{b^5}}{(bx+a)^3}+\frac{10a^2\ln(bx+a)}{b^6}$	70
default	$-\frac{-\frac{1}{2}x^2b+4ax}{b^5}+\frac{10a^3}{b^6(bx+a)}-\frac{5a^4}{2b^6(bx+a)^2}+\frac{10a^2\ln(bx+a)}{b^6}+\frac{a^5}{3b^6(bx+a)^3}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/2*x^2*b+4*a*x)+10*a^3/b^6/(b*x+a)-5/2*a^4/b^6/(b*x+a)^2+10*a^2*l
n(b*x+a)/b^6+1/3*a^5/b^6/(b*x+a)^3
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.12

$$\frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\,(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6)}+\frac{10\,a^2\log(bx+a)}{b^6}+\frac{bx^2-8\,ax}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*a^3*b^2*x^2 + 105*a^4*b*x + 47*a^5)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*
b^7*x + a^3*b^6) + 10*a^2*log(b*x + a)/b^6 + 1/2*(b*x^2 - 8*a*x)/b^5
```

Fricas [A]
time = 0.72, size = 129, normalized size = 1.59

$$\frac{3\,b^5x^5-15\,ab^4x^4-63\,a^2b^3x^3-9\,a^3b^2x^2+81\,a^4bx+47\,a^5+60\,(a^2b^3x^3+3\,a^3b^2x^2+3\,a^4bx+a^5)\log(bx+a)}{6\,(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 - 15*a*b^4*x^4 - 63*a^2*b^3*x^3 - 9*a^3*b^2*x^2 + 81*a^4*b*x
 + 47*a^5 + 60*(a^2*b^3*x^3 + 3*a^3*b^2*x^2 + 3*a^4*b*x + a^5)*log(b*x + a)
)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

Sympy [A]
time = 0.18, size = 94, normalized size = 1.16

$$\frac{10a^2\log{(a+bx)}}{b^6}-\frac{4ax}{b^5}+\frac{47a^5+105a^4bx+60a^3b^2x^2}{6a^3b^6+18a^2b^7x+18ab^8x^2+6b^9x^3}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**4,x)
```

```
[Out] 10*a**2*log(a + b*x)/b**6 - 4*a*x/b**5 + (47*a**5 + 105*a**4*b*x + 60*a**3*
b**2*x**2)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
x**2/(2*b**4)
```

Giac [A]
time = 0.92, size = 72, normalized size = 0.89

$$\frac{10\,a^2\log{(|bx+a|)}}{b^6}+\frac{b^4x^2-8\,ab^3x}{2\,b^8}+\frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\,(bx+a)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 10*a^2*log(abs(b*x + a))/b^6 + 1/2*(b^4*x^2 - 8*a*b^3*x)/b^8 + 1/6*(60*a^3*
b^2*x^2 + 105*a^4*b*x + 47*a^5)/((b*x + a)^3*b^6)
```

Mupad [B]
time = 0.12, size = 66, normalized size = 0.81

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{10\,a^3}{a+b\,x}-\frac{5\,a^4}{2\,(a+b\,x)^2}+\frac{a^5}{3\,(a+b\,x)^3}+10\,a^2\,\ln{(a+b\,x)}-5\,a\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^2/2 + (10*a^3)/(a + b*x) - (5*a^4)/(2*(a + b*x)^2) + a^5/(3*(a +
 b*x)^3) + 10*a^2*log(a + b*x) - 5*a*b*x)/b^6
```

3.197 $\int \frac{x^4}{(a+bx)^4} \, dx$

Optimal. Leaf size=65

$$\frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}$$

```
[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*
x+a)/b^5
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^4,x]
```

```
[Out] x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5*
(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^4}\,dx &= \int \left(\frac{1}{b^4}+\frac{a^4}{b^4(a+bx)^4}-\frac{4a^3}{b^4(a+bx)^3}+\frac{6a^2}{b^4(a+bx)^2}-\frac{4a}{b^4(a+bx)}\right)dx\\ &= \frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.78

$$-\frac{-3bx+\frac{a^2(13a^2+30abx+18b^2x^2)}{(a+bx)^3}+12a\log(a+bx)}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^4,x]
```

```
[Out] -1/3*(-3*b*x + (a^2*(13*a^2 + 30*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 12*a*Lo
g[a + b*x])/b^5
```

Maple [A]
time = 0.08, size = 64, normalized size = 0.98

method	result	size
risch	$\frac{x}{b^4}+\frac{-6a^2b\,x^2-10a^3x-\frac{13a^4}{3b}}{b^4(bx+a)^3}-\frac{4a\ln(bx+a)}{b^5}$	54
norman	$\frac{\frac{x^4}{b}-\frac{22a^4}{3b^5}-\frac{12a^2x^2}{b^3}-\frac{18a^3x}{b^4}}{(bx+a)^3}-\frac{4a\ln(bx+a)}{b^5}$	58
default	$\frac{x}{b^4}-\frac{a^4}{3b^5(bx+a)^3}+\frac{2a^3}{b^5(bx+a)^2}-\frac{6a^2}{b^5(bx+a)}-\frac{4a\ln(bx+a)}{b^5}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*
x+a)/b^5
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.22

$$-\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)}+\frac{x}{b^4}-\frac{4\,a\log{(bx+a)}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a^4)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*
b^6*x + a^3*b^5) + x/b^4 - 4*a*log(b*x + a)/b^5
```

Fricas [A]
time = 0.71, size = 116, normalized size = 1.78

$$\frac{3\,b^4x^4+9\,ab^3x^3-9\,a^2b^2x^2-27\,a^3bx-13\,a^4-12\,(ab^3x^3+3\,a^2b^2x^2+3\,a^3bx+a^4)\log{(bx+a)}}{3\,(b^8x^3+3\,ab^7x^2+3\,a^2b^6x+a^3b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 9*a*b^3*x^3 - 9*a^2*b^2*x^2 - 27*a^3*b*x - 13*a^4 - 12*(a*
b^3*x^3 + 3*a^2*b^2*x^2 + 3*a^3*b*x + a^4)*log(b*x + a))/(b^8*x^3 + 3*a*b^7
*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [A]
time = 0.16, size = 82, normalized size = 1.26

$$-\frac{4a\log{(a+bx)}}{b^5}+\frac{-13a^4-30a^3bx-18a^2b^2x^2}{3a^3b^5+9a^2b^6x+9ab^7x^2+3b^8x^3}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**4,x)
```

```
[Out] -4*a*log(a + b*x)/b**5 + (-13*a**4 - 30*a**3*b*x - 18*a**2*b**2*x**2)/(3*a*
*3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + x/b**4
```

Giac [A]
time = 1.96, size = 55, normalized size = 0.85

$$\frac{x}{b^4}-\frac{4\,a\log{(|bx+a|)}}{b^5}-\frac{18\,a^2b^2x^2+30\,a^3bx+13\,a^4}{3\,(bx+a)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] x/b^4 - 4*a*log(abs(b*x + a))/b^5 - 1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a
^4)/((b*x + a)^3*b^5)
```

Mupad [B]
time = 0.17, size = 55, normalized size = 0.85

$$-\frac{4\,a\,\ln{(a+b\,x)}-b\,x+\frac{6\,a^2}{a+b\,x}-\frac{2\,a^3}{(a+b\,x)^2}+\frac{a^4}{3\,(a+b\,x)^3}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^4,x)
```

```
[Out] -(4*a*log(a + b*x) - b*x + (6*a^2)/(a + b*x) - (2*a^3)/(a + b*x)^2 + a^4/(3
*(a + b*x)^3))/b^5
```

3.198 $\int \frac{x^3}{(a+bx)^4} dx$

Optimal. Leaf size=58

$$\frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

```
[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^4,x]
```

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx)^4} dx &= \int \left(-\frac{a^3}{b^3(a+bx)^4} + \frac{3a^2}{b^3(a+bx)^3} - \frac{3a}{b^3(a+bx)^2} + \frac{1}{b^3(a+bx)}\right) dx \\ &= \frac{a^3}{3b^4(a+bx)^3} - \frac{3a^2}{2b^4(a+bx)^2} + \frac{3a}{b^4(a+bx)} + \frac{\log(a+bx)}{b^4} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.76

$$\frac{\frac{a(11a^2+27abx+18b^2x^2)}{(a+bx)^3} + 6\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^4,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4)
```

Maple [A]
time = 0.08, size = 55, normalized size = 0.95

method	result	size
norman	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
risch	$\frac{\frac{11a^3}{6b^4}+\frac{3a\,x^2}{b^2}+\frac{9a^2x}{2b^3}}{(bx+a)^3}+\frac{\ln(bx+a)}{b^4}$	47
default	$\frac{a^3}{3b^4(bx+a)^3}-\frac{3a^2}{2b^4(bx+a)^2}+\frac{3a}{b^4(bx+a)}+\frac{\ln(bx+a)}{b^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

Maxima [A]
time = 0.27, size = 70, normalized size = 1.21

$$\frac{18\,ab^2x^2+27\,a^2bx+11\,a^3}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}+\frac{\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5
*x + a^3*b^4) + log(b*x + a)/b^4
```

Fricas [A]
time = 0.85, size = 94, normalized size = 1.62

$$\frac{18\,ab^2x^2+27\,a^2bx+11\,a^3+6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log\left(bx+a\right)}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A]
time = 0.12, size = 70, normalized size = 1.21

$$\frac{11a^3 + 27a^2bx + 18ab^2x^2}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{\log(a + bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

Giac [A]
time = 1.20, size = 46, normalized size = 0.79

$$\frac{\log(|bx + a|)}{b^4} + \frac{18\,abx^2 + 27\,a^2x + \frac{11\,a^3}{b}}{6\,(bx + a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^3
*b^3)
```

Mupad [B]
time = 0.07, size = 45, normalized size = 0.78

$$\frac{\ln(a + b\,x) + \frac{3\,a}{a+b\,x} - \frac{3\,a^2}{2\,(a+b\,x)^2} + \frac{a^3}{3\,(a+b\,x)^3}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^4,x)
```

```
[Out] (log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a + b*x
)^3))/b^4
```

3.199 $\int \frac{x^2}{(a+bx)^4} dx$

Optimal. Leaf size=17

$$\frac{x^3}{3a(a+bx)^3}$$

```
[Out] 1/3*x^3/a/(b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$\frac{x^3}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^4,x]
```

```
[Out] x^3/(3*a*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx)^4} dx = \frac{x^3}{3a(a+bx)^3}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.82

$$-\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.
time = 0.09, size = 41, normalized size = 2.41

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3(bx+a)^3b^3}$	30
norman	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
risch	$\frac{-\frac{x^2}{b}-\frac{ax}{b^2}-\frac{a^2}{3b^3}}{(bx+a)^3}$	33
default	$-\frac{1}{b^3(bx+a)}+\frac{a}{b^3(bx+a)^2}-\frac{a^2}{3b^3(bx+a)^3}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3/(b*x+a)+a/b^3/(b*x+a)^2-1/3/b^3*a^2/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.
time = 0.27, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(15) = 30$.
time = 0.78, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(12) = 24$.
time = 0.10, size = 56, normalized size = 3.29

$$\frac{-a^2 - 3abx - 3b^2x^2}{3a^3b^3 + 9a^2b^4x + 9ab^5x^2 + 3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**
2 + 3*b**6*x**3)
```

Giac [A]
time = 1.54, size = 29, normalized size = 1.71

$$-\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

Mupad [B]
time = 0.09, size = 56, normalized size = 3.29

$$-\frac{a^2 + 3\,a\,b\,x + 3\,b^2\,x^2}{3\,a^3\,b^3 + 9\,a^2\,b^4\,x + 9\,a\,b^5\,x^2 + 3\,b^6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^4,x)
```

```
[Out] -(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5
*x^2)
```

3.200 $\int \frac{x}{(a+bx)^4}\,dx$

Optimal. Leaf size=30

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

```
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {45}

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^4,x]
```

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^4}\,dx &= \int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right)dx\\ &= \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.67

$$-\frac{a+3bx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

Maple [A]
time = 0.08, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{3bx+a}{6(bx+a)^{3}b^{2}}$	19
norman	$\frac{-\frac{x}{2b}-\frac{a}{6b^{2}}}{(bx+a)^{3}}$	22
risch	$\frac{-\frac{x}{2b}-\frac{a}{6b^{2}}}{(bx+a)^{3}}$	22
default	$\frac{a}{3b^{2}(bx+a)^{3}}-\frac{1}{2b^{2}(bx+a)^{2}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

Maxima [A]
time = 0.33, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,(b^{5}x^{3}+3\,ab^{4}x^{2}+3\,a^{2}b^{3}x+a^{3}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="maxima")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Fricas [A]
time = 0.86, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,(b^{5}x^{3}+3\,ab^{4}x^{2}+3\,a^{2}b^{3}x+a^{3}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

Sympy [A]
time = 0.10, size = 44, normalized size = 1.47

$$\frac{-a-3bx}{6a^{3}b^{2}+18a^{2}b^{3}x+18ab^{4}x^{2}+6b^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**4,x)
```

```
[Out] (-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

Giac [A]
time = 1.31, size = 18, normalized size = 0.60

$$-\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

Mupad [B]
time = 0.07, size = 44, normalized size = 1.47

$$-\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^4,x)
```

```
[Out] -(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.201 $\int \frac{1}{(a+bx)^4} dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(a+bx)^3}$$

```
[Out] -1/3/b/(b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4),x]
```

```
[Out] -1/3*1/(b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4} dx = -\frac{1}{3b(a+bx)^3}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4),x]
```

```
[Out] -1/3*1/(b*(a + b*x)^3)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{3b(bx+a)^3}$	13
default	$-\frac{1}{3b(bx+a)^3}$	13
norman	$-\frac{1}{3b(bx+a)^3}$	13
risch	$-\frac{1}{3b(bx+a)^3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/(b*x+a)^3
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(bx+a)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.
time = 2.22, size = 35, normalized size = 2.50

$$-\frac{1}{3\,(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.
time = 0.10, size = 37, normalized size = 2.64

$$-\frac{1}{3a^3b + 9a^2b^2x + 9ab^3x^2 + 3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4,x)
```

```
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

Giac [A]
time = 0.97, size = 12, normalized size = 0.86

$$-\frac{1}{3(bx+a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/((b*x + a)^3*b)
```

Mupad [B]
time = 0.08, size = 37, normalized size = 2.64

$$-\frac{1}{3\,a^3\,b+9\,a^2\,b^2\,x+9\,a\,b^3\,x^2+3\,b^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^4,x)
```

```
[Out] -1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

3.202 $\int \frac{1}{x(a+bx)^4} \, dx$

Optimal. Leaf size=57

$$\frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}$$

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^4),x]
```

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^4} \, dx &= \int \left(\frac{1}{a^4x}-\frac{b}{a(a+bx)^4}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^4(a+bx)}\right) dx\\ &= \frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.84

$$\frac{\frac{a(11a^2+15abx+6b^2x^2)}{(a+bx)^3}+6\log(x)-6\log(a+bx)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^4),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*x
])/(6*a^4)
```

Maple [A]
time = 0.08, size = 54, normalized size = 0.95

method	result	size
risch	$\frac{\frac{b^2x^2}{a^3}+\frac{5bx}{2a^2}+\frac{11}{6a}}{(bx+a)^3}+\frac{\ln(-x)}{a^4}-\frac{\ln(bx+a)}{a^4}$	52
default	$\frac{1}{3a(bx+a)^3}+\frac{1}{2a^2(bx+a)^2}+\frac{1}{a^3(bx+a)}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$	54
norman	$\frac{-\frac{3bx}{a^2}-\frac{9b^2x^2}{2a^3}-\frac{11b^3x^3}{6a^4}}{(bx+a)^3}+\frac{\ln(x)}{a^4}-\frac{\ln(bx+a)}{a^4}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

Maxima [A]
time = 0.27, size = 73, normalized size = 1.28

$$\frac{6\,b^2x^2+15\,abx+11\,a^2}{6\,(a^3b^3x^3+3\,a^4b^2x^2+3\,a^5bx+a^6)}-\frac{\log(bx+a)}{a^4}+\frac{\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*
x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.
time = 0.89, size = 124, normalized size = 2.18

$$\frac{6\,ab^2x^2+15\,a^2bx+11\,a^3-6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log(bx+a)+6\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\log(x)}{6\,(a^4b^3x^3+3\,a^5b^2x^2+3\,a^6bx+a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(x)
)/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

Sympy [A]
time = 0.17, size = 70, normalized size = 1.23

$$\frac{11a^2+15abx+6b^2x^2}{6a^6+18a^5bx+18a^4b^2x^2+6a^3b^3x^3}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**4,x)
```

```
[Out] (11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x**
2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

Giac [A]
time = 0.97, size = 54, normalized size = 0.95

$$-\frac{\log(|bx+a|)}{a^4}+\frac{\log(|x|)}{a^4}+\frac{6\,ab^2x^2+15\,a^2bx+11\,a^3}{6\,(bx+a)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
11*a^3)/((b*x + a)^3*a^4)
```

Mupad [B]
time = 0.13, size = 60, normalized size = 1.05

$$\frac{\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,(a+b\,x)^2}}{a}+\frac{1}{3\,a\,(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^4),x)
```

```
[Out] ((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(3
*a*(a + b*x)^3)
```

3.203 $\int \frac{1}{x^2(a+bx)^4}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}$$

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^2}-\frac{4b}{a^5x}+\frac{b^2}{a^2(a+bx)^4}+\frac{2b^2}{a^3(a+bx)^3}+\frac{3b^2}{a^4(a+bx)^2}+\frac{4b^2}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.91

$$-\frac{\frac{a(3a^3+22a^2bx+30ab^2x^2+12b^3x^3)}{x(a+bx)^3}+12b\log(x)-12b\log(a+bx)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
+ 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

Maple [A]
time = 0.09, size = 69, normalized size = 0.99

method	result	size
default	$-\frac{1}{a^{4} x}-\frac{b}{3 a^{2} \left(b x +a \right)^{3}}-\frac{b}{a^{3} \left(b x +a \right)^{2}}-\frac{3 b}{a^{4} \left(b x +a \right)}-\frac{4 b \ln \left(x \right)}{a^{5}}+\frac{4 b \ln \left(b x +a \right)}{a^{5}}$	69
risch	$\frac{-\frac{4 b^{3} x^{3}}{a^{4}}-\frac{10 b^{2} x^{2}}{a^{3}}-\frac{22 b x}{3 a^{2}}-\frac{1}{a}}{x \left(b x +a \right)^{3}}+\frac{4 b \ln \left(-b x -a \right)}{a^{5}}-\frac{4 b \ln \left(x \right)}{a^{5}}$	71
norman	$\frac{-\frac{1}{a}+\frac{12 b^{2} x^{2}}{a^{3}}+\frac{18 b^{3} x^{3}}{a^{4}}+\frac{22 b^{4} x^{4}}{3 a^{5}}}{x \left(b x +a \right)^{3}}-\frac{4 b \ln \left(x \right)}{a^{5}}+\frac{4 b \ln \left(b x +a \right)}{a^{5}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.30

$$-\frac{12 \, b^{3} x^{3} + 30 \, a b^{2} x^{2} + 22 \, a^{2} b x + 3 \, a^{3}}{3 \, {\left(a^{4} b^{3} x^{4} + 3 \, a^{5} b^{2} x^{3} + 3 \, a^{6} b x^{2} + a^{7} x\right)}} + \frac{4 \, b \log\left(b x + a\right)}{a^{5}} - \frac{4 \, b \log\left(x\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5*
b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(68) = 136.
time = 0.88, size = 153, normalized size = 2.19

$$-\frac{12 \, a b^{3} x^{3} + 30 \, a^{2} b^{2} x^{2} + 22 \, a^{3} b x + 3 \, a^{4} - 12 \, {\left(b^{4} x^{4} + 3 \, a b^{3} x^{3} + 3 \, a^{2} b^{2} x^{2} + a^{3} b x\right)} \log\left(b x + a\right) + 12 \, {\left(b^{4} x^{4} + 3 \, a b^{3} x^{3} + 3 \, a^{2} b^{2} x^{2} + a^{3} b x\right)} \log\left(x\right)}{3 \, {\left(a^{5} b^{3} x^{4} + 3 \, a^{6} b^{2} x^{3} + 3 \, a^{7} b x^{2} + a^{8} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3*
a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3*x
```

```
^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*
b*x^2 + a^8*x)
```

Sympy [A]
time = 0.19, size = 90, normalized size = 1.29

$$\frac{-3a^3 - 22a^2bx - 30ab^2x^2 - 12b^3x^3}{3a^7x + 9a^6bx^2 + 9a^5b^2x^3 + 3a^4b^3x^4} + \frac{4b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**4,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6*
b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b + x)
)/a**5
```

Giac [A]
time = 1.23, size = 71, normalized size = 1.01

$$\frac{4\,b\log\left(|bx+a|\right)}{a^5} - \frac{4\,b\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x^3 + 30\,a^2b^2x^2 + 22\,a^3bx + 3\,a^4}{3\,(bx+a)^3a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a^
2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

Mupad [B]
time = 0.08, size = 85, normalized size = 1.21

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{a}+\frac{10\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}+\frac{22\,b\,x}{3\,a^2}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^4),x)
```

```
[Out] (8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
+ (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

3.204 $\int \frac{1}{x^3(a+bx)^4} \, dx$

Optimal. Leaf size=93

$$-\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(a+bx)^3}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{6b^2}{a^5(a+bx)}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}$$

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}+\frac{6b^2}{a^5(a+bx)}+\frac{4b}{a^5x}+\frac{3b^2}{2a^4(a+bx)^2}-\frac{1}{2a^4x^2}+\frac{b^2}{3a^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^4),x]
```

```
[Out] -1/2*1/(a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4
*(a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log
[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^3}-\frac{4b}{a^5x^2}+\frac{10b^2}{a^6x}-\frac{b^3}{a^3(a+bx)^4}-\frac{3b^3}{a^4(a+bx)^3}-\frac{6b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}\right)\,dx\\ &= -\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(a+bx)^3}+\frac{3b^2}{2a^4(a+bx)^2}+\frac{6b^2}{a^5(a+bx)}+\frac{10b^2\log(x)}{a^6}-\frac{10b^2\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.85

$$\frac{\frac{a(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4)}{x^2(a+bx)^3}+60b^2\log(x)-60b^2\log(a+bx)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/(
x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

Maple [A]
time = 0.08, size = 88, normalized size = 0.95

method	result	size
norman	$\frac{-\frac{1}{2a}+\frac{5bx}{2a^2}-\frac{30b^3x^3}{a^4}-\frac{45b^4x^4}{a^5}-\frac{55b^5x^5}{3a^6}}{x^2(bx+a)^3}+\frac{10b^2\ln(x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$	83
risch	$\frac{\frac{10b^4x^4}{a^5}+\frac{25b^3x^3}{a^4}+\frac{55b^2x^2}{3a^3}+\frac{5bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)^3}+\frac{10b^2\ln(-x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$	85
default	$-\frac{1}{2a^4x^2}+\frac{4b}{a^5x}+\frac{b^2}{3a^3(bx+a)^3}+\frac{3b^2}{2a^4(bx+a)^2}+\frac{6b^2}{a^5(bx+a)}+\frac{10b^2\ln(x)}{a^6}-\frac{10b^2\ln(bx+a)}{a^6}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

Maxima [A]
time = 0.27, size = 108, normalized size = 1.16

$$\frac{60\,b^4x^4+150\,ab^3x^3+110\,a^2b^2x^2+15\,a^3bx-3\,a^4}{6\,(a^5b^3x^5+3\,a^6b^2x^4+3\,a^7bx^3+a^8x^2)}-\frac{10\,b^2\log{(bx+a)}}{a^6}+\frac{10\,b^2\log{(x)}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a^
5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/a^
6 + 10*b^2*log(x)/a^6
```

Fricas [A]
time = 0.76, size = 174, normalized size = 1.87

$$\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5-60\,(b^5x^5+3\,ab^4x^4+3\,a^2b^3x^3+a^3b^2x^2)\log{(bx+a)}+60\,(b^5x^5+3\,ab^4x^4+3\,a^2b^3x^3+a^3b^2x^2)\log{(x)}}{6\,(a^6b^3x^5+3\,a^7b^2x^4+3\,a^8bx^3+a^9x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
- 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) + 6
```

```
0*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3*x^
5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

Sympy [A]
time = 0.21, size = 104, normalized size = 1.12

$$\frac{-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4}{6a^8x^2+18a^7bx^3+18a^6b^2x^4+6a^5b^3x^5}+\frac{10b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**4,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x**
4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5) +
10*b**2*(log(x) - log(a/b + x))/a**6
```

Giac [A]
time = 1.43, size = 86, normalized size = 0.92

$$-\frac{10\,b^2\log\left(|bx+a|\right)}{a^6}+\frac{10\,b^2\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^4+150\,a^2b^3x^3+110\,a^3b^2x^2+15\,a^4bx-3\,a^5}{6\,(bx+a)^3a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
+ 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^6*
x^2)
```

Mupad [B]
time = 0.14, size = 101, normalized size = 1.09

$$\frac{\frac{55\,b^2\,x^2}{3\,a^3}-\frac{1}{2\,a}+\frac{25\,b^3\,x^3}{a^4}+\frac{10\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{2\,a^2}}{a^3\,x^2+3\,a^2\,b\,x^3+3\,a\,b^2\,x^4+b^3\,x^5}-\frac{20\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^4),x)
```

```
[Out] ((55*b^2*x^2)/(3*a^3) - 1/(2*a) + (25*b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 + (5*
b*x)/(2*a^2))/(a^3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (20*b^2*ata
nh((2*b*x)/a + 1))/a^6
```

3.205 $\int \frac{1}{x^4(a+bx)^4}\,dx$

Optimal. Leaf size=102

$$-\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}$$

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Rubi [A]
time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}-\frac{10b^3}{a^6(a+bx)}-\frac{10b^2}{a^6x}-\frac{2b^3}{a^5(a+bx)^2}+\frac{2b}{a^5x^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/3*1/(a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*x
)^3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log[x
])/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^4}-\frac{4b}{a^5x^3}+\frac{10b^2}{a^6x^2}-\frac{20b^3}{a^7x}+\frac{b^4}{a^4(a+bx)^4}+\frac{4b^4}{a^5(a+bx)^3}+\frac{10b^4}{a^6(a+bx)^2}+\frac{20\cdots}{a^7(a\cdots}\right.\\ &= -\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\cdots\end{aligned}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.86

$$-\frac{\frac{a(a^5-3a^4bx+15a^3b^2x^2+110a^2b^3x^3+150ab^4x^4+60b^5x^5)}{x^3(a+bx)^3}+60b^3\log(x)-60b^3\log(a+bx)}{3a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(a^5 - 3*a^4*b*x + 15*a^3*b^2*x^2 + 110*a^2*b^3*x^3 + 150*a*b^4*x^
4 + 60*b^5*x^5))/(x^3*(a + b*x)^3) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a
^7
```

Maple [A]
time = 0.08, size = 99, normalized size = 0.97

method	result	size
norman	$\frac{\frac{bx}{a^2}-\frac{1}{3a}-\frac{5b^2x^2}{a^3}+\frac{60b^4x^4}{a^5}+\frac{90b^5x^5}{a^6}+\frac{110b^6x^6}{3a^7}}{x^3(bx+a)^3}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln(bx+a)}{a^7}$	93
risch	$\frac{-\frac{20b^5x^5}{a^6}-\frac{50b^4x^4}{a^5}-\frac{110b^3x^3}{3a^4}-\frac{5b^2x^2}{a^3}+\frac{bx}{a^2}-\frac{1}{3a}}{x^3(bx+a)^3}+\frac{20b^3\ln(-bx-a)}{a^7}-\frac{20b^3\ln(x)}{a^7}$	96
default	$-\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(bx+a)^3}-\frac{2b^3}{a^5(bx+a)^2}-\frac{10b^3}{a^6(bx+a)}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln(bx+a)}{a^7}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

Maxima [A]
time = 0.29, size = 117, normalized size = 1.15

$$-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\,(a^6b^3x^6+3\,a^7b^2x^5+3\,a^8bx^4+a^9x^3)}+\frac{20\,b^3\log(bx+a)}{a^7}-\frac{20\,b^3\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(60*b^5*x^5 + 150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4
*b*x + a^5)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*x^3) + 20*b^3*
log(b*x + a)/a^7 - 20*b^3*log(x)/a^7
```

Fricas [A]
time = 0.72, size = 183, normalized size = 1.79

$$-\frac{60\,ab^5x^5+150\,a^2b^4x^4+110\,a^3b^3x^3+15\,a^4b^2x^2-3\,a^5bx+a^6-60\,(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3)\log(bx+a)+60\,(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3)\log(x)}{3\,(a^7b^3x^6+3\,a^8b^2x^5+3\,a^9bx^4+a^{10}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(60*a*b^5*x^5 + 150*a^2*b^4*x^4 + 110*a^3*b^3*x^3 + 15*a^4*b^2*x^2 - 3
*a^5*b*x + a^6 - 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*l
og(b*x + a) + 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*log(
x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

Sympy [A]
time = 0.22, size = 114, normalized size = 1.12

$$\frac{-a^5 + 3a^4bx - 15a^3b^2x^2 - 110a^2b^3x^3 - 150ab^4x^4 - 60b^5x^5}{3a^9x^3 + 9a^8bx^4 + 9a^7b^2x^5 + 3a^6b^3x^6} + \frac{20b^3\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**4,x)
```

```
[Out] (-a**5 + 3*a**4*b*x - 15*a**3*b**2*x**2 - 110*a**2*b**3*x**3 - 150*a*b**4*x
**4 - 60*b**5*x**5)/(3*a**9*x**3 + 9*a**8*b*x**4 + 9*a**7*b**2*x**5 + 3*a**
6*b**3*x**6) + 20*b**3*(-log(x) + log(a/b + x))/a**7
```

Giac [A]
time = 1.24, size = 93, normalized size = 0.91

$$\frac{20\,b^3\log\left(|bx+a|\right)}{a^7} - \frac{20\,b^3\log\left(|x|\right)}{a^7} - \frac{60\,b^5x^5 + 150\,ab^4x^4 + 110\,a^2b^3x^3 + 15\,a^3b^2x^2 - 3\,a^4bx + a^5}{3\left(bx^2+ax\right)^3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 20*b^3*log(abs(b*x + a))/a^7 - 20*b^3*log(abs(x))/a^7 - 1/3*(60*b^5*x^5 + 1
50*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4*b*x + a^5)/((b*x^2
+ a*x)^3*a^6)
```

Mupad [B]
time = 0.10, size = 113, normalized size = 1.11

$$\frac{40\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7} - \frac{\frac{1}{3\,a} + \frac{5\,b^2\,x^2}{a^3} + \frac{110\,b^3\,x^3}{3\,a^4} + \frac{50\,b^4\,x^4}{a^5} + \frac{20\,b^5\,x^5}{a^6} - \frac{b\,x}{a^2}}{a^3\,x^3 + 3\,a^2\,b\,x^4 + 3\,a\,b^2\,x^5 + b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^4),x)
```

```
[Out] (40*b^3*atanh((2*b*x)/a + 1))/a^7 - (1/(3*a) + (5*b^2*x^2)/a^3 + (110*b^3*x
^3)/(3*a^4) + (50*b^4*x^4)/a^5 + (20*b^5*x^5)/a^6 - (b*x)/a^2)/(a^3*x^3 + b
^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^5)
```

3.206 $\int \frac{1}{x^5(a+bx)^4}\,dx$

Optimal. Leaf size=117

$$-\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}$$

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

Rubi [A]

time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{15b^4}{a^7(a+bx)}+\frac{20b^3}{a^7x}+\frac{5b^4}{2a^6(a+bx)^2}-\frac{5b^2}{a^6x^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^4),x]
```

```
[Out] -1/4*1/(a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x)
 + b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*(a
 + b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^5}-\frac{4b}{a^5x^4}+\frac{10b^2}{a^6x^3}-\frac{20b^3}{a^7x^2}+\frac{35b^4}{a^8x}-\frac{b^5}{a^5(a+bx)^4}-\frac{5b^5}{a^6(a+bx)^3}-\frac{15b^5}{a^7(a+bx)^2}-\right.\\ &= -\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4\log}{a^8}\end{aligned}$$

Mathematica [A]

time = 0.05, size = 101, normalized size = 0.86

$$\frac{\frac{a(-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6)}{x^4(a+bx)^3}+420b^4\log(x)-420b^4\log(a+bx)}{12a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^6 + 7*a^5*b*x - 21*a^4*b^2*x^2 + 105*a^3*b^3*x^3 + 770*a^2*b^4*x^
4 + 1050*a*b^5*x^5 + 420*b^6*x^6))/(x^4*(a + b*x)^3) + 420*b^4*Log[x] - 420
*b^4*Log[a + b*x])/(12*a^8)
```

Maple [A]
time = 0.09, size = 110, normalized size = 0.94

method	result	size
norman	$\frac{-\frac{1}{4a}+\frac{7bx}{12a^2}-\frac{7b^2x^2}{4a^3}+\frac{35b^3x^3}{4a^4}-\frac{105b^5x^5}{a^6}-\frac{315b^6x^6}{2a^7}-\frac{385b^7x^7}{6a^8}}{x^4(bx+a)^3}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$	105
risch	$\frac{\frac{35b^6x^6}{a^7}+\frac{175b^5x^5}{2a^6}+\frac{385b^4x^4}{6a^5}+\frac{35b^3x^3}{4a^4}-\frac{7b^2x^2}{4a^3}+\frac{7bx}{12a^2}-\frac{1}{4a}}{x^4(bx+a)^3}+\frac{35b^4\ln(-x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$	107
default	$-\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(bx+a)^3}+\frac{5b^4}{2a^6(bx+a)^2}+\frac{15b^4}{a^7(bx+a)}+\frac{35b^4\ln(x)}{a^8}-\frac{35b^4\ln(bx+a)}{a^8}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

Maxima [A]
time = 0.28, size = 130, normalized size = 1.11

$$\frac{420\,b^6x^6+1050\,ab^5x^5+770\,a^2b^4x^4+105\,a^3b^3x^3-21\,a^4b^2x^2+7\,a^5bx-3\,a^6}{12\,(a^7b^3x^7+3\,a^8b^2x^6+3\,a^9bx^5+a^{10}x^4)}-\frac{35\,b^4\log(bx+a)}{a^8}+\frac{35\,b^4\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(420*b^6*x^6 + 1050*a*b^5*x^5 + 770*a^2*b^4*x^4 + 105*a^3*b^3*x^3 - 21
*a^4*b^2*x^2 + 7*a^5*b*x - 3*a^6)/(a^7*b^3*x^7 + 3*a^8*b^2*x^6 + 3*a^9*b*x^
5 + a^10*x^4) - 35*b^4*log(b*x + a)/a^8 + 35*b^4*log(x)/a^8
```

Fricas [A]
time = 1.12, size = 196, normalized size = 1.68

$$\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7-420\,(b^7x^7+3\,ab^6x^6+3\,a^2b^5x^5+a^3b^4x^4)\log(bx+a)+420\,(b^7x^7+3\,ab^6x^6+3\,a^2b^5x^5+a^3b^4x^4)\log(x)}{12\,(a^8b^3x^7+3\,a^9b^2x^6+3\,a^{10}bx^5+a^{11}x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(420*a*b^6*x^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3
- 21*a^5*b^2*x^2 + 7*a^6*b*x - 3*a^7 - 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b
^5*x^5 + a^3*b^4*x^4)*log(b*x + a) + 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b^5
*x^5 + a^3*b^4*x^4)*log(x))/(a^8*b^3*x^7 + 3*a^9*b^2*x^6 + 3*a^10*b*x^5 + a
^11*x^4)
```

Sympy [A]
time = 0.24, size = 128, normalized size = 1.09

$$\frac{-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6}{12a^{10}x^4+36a^9bx^5+36a^8b^2x^6+12a^7b^3x^7}+\frac{35b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**4,x)
```

```
[Out] (-3*a**6 + 7*a**5*b*x - 21*a**4*b**2*x**2 + 105*a**3*b**3*x**3 + 770*a**2*b
**4*x**4 + 1050*a*b**5*x**5 + 420*b**6*x**6)/(12*a**10*x**4 + 36*a**9*b*x**
5 + 36*a**8*b**2*x**6 + 12*a**7*b**3*x**7) + 35*b**4*(log(x) - log(a/b + x)
)/a**8
```

Giac [A]
time = 1.29, size = 108, normalized size = 0.92

$$-\frac{35\,b^4\log\left(|bx+a|\right)}{a^8}+\frac{35\,b^4\log\left(|x|\right)}{a^8}+\frac{420\,ab^6x^6+1050\,a^2b^5x^5+770\,a^3b^4x^4+105\,a^4b^3x^3-21\,a^5b^2x^2+7\,a^6bx-3\,a^7}{12\,(bx+a)^3a^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -35*b^4*log(abs(b*x + a))/a^8 + 35*b^4*log(abs(x))/a^8 + 1/12*(420*a*b^6*x^
6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3 - 21*a^5*b^2*x^2 +
 7*a^6*b*x - 3*a^7)/((b*x + a)^3*a^8*x^4)
```

Mupad [B]
time = 0.17, size = 123, normalized size = 1.05

$$\frac{\frac{35\,b^3\,x^3}{4\,a^4}-\frac{7\,b^2\,x^2}{4\,a^3}-\frac{1}{4\,a}+\frac{385\,b^4\,x^4}{6\,a^5}+\frac{175\,b^5\,x^5}{2\,a^6}+\frac{35\,b^6\,x^6}{a^7}+\frac{7\,b\,x}{12\,a^2}}{a^3\,x^4+3\,a^2\,b\,x^5+3\,a\,b^2\,x^6+b^3\,x^7}-\frac{70\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^4),x)
```

```
[Out] ((35*b^3*x^3)/(4*a^4) - (7*b^2*x^2)/(4*a^3) - 1/(4*a) + (385*b^4*x^4)/(6*a^
5) + (175*b^5*x^5)/(2*a^6) + (35*b^6*x^6)/a^7 + (7*b*x)/(12*a^2))/(a^3*x^4
+ b^3*x^7 + 3*a^2*b*x^5 + 3*a*b^2*x^6) - (70*b^4*atanh((2*b*x)/a + 1))/a^8
```

3.207 $\int \frac{x^{10}}{(a+bx)^7}\,dx$

Optimal. Leaf size=150

$$-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\dots$$

```
[Out] -84*a^3*x/b^10+14*a^2*x^2/b^9-7/3*a*x^3/b^8+1/4*x^4/b^7-1/6*a^10/b^11/(b*x+
a)^6+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4+40*a^7/b^11/(b*x+a)^3-105
*a^6/b^11/(b*x+a)^2+252*a^5/b^11/(b*x+a)+210*a^4*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.10, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\frac{252a^5}{b^{11}(a+bx)}+\frac{210a^4\log(a+bx)}{b^{11}}-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^7,x]
```

```
[Out] (-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^1
0/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a +
 b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) + (25
2*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx)^7}\,dx=\int \left(-\frac{84a^3}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{b^8}+\frac{x^3}{b^7}+\frac{a^{10}}{b^{10}(a+bx)^7}-\frac{10a^9}{b^{10}(a+bx)^6}+\frac{45a^8}{b^{10}(a+bx)^5}-\dots\right.$$

$$=-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\dots$$

Mathematica [A]
time = 0.02, size = 139, normalized size = 0.93

$$\frac{2131a^{10}+10266a^9bx+18105a^8b^2x^2+11540a^7b^3x^3-3945a^6b^4x^4-9138a^5b^5x^5-4043a^4b^6x^6-360a^3b^7x^7+45a^2b^8x^8-10ab^9x^9+3b^{10}x^{10}+2520a^4(a+bx)^6\log(a+bx)}{12b^{11}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^7,x]
```

```
[Out] (2131*a^10 + 10266*a^9*b*x + 18105*a^8*b^2*x^2 + 11540*a^7*b^3*x^3 - 3945*a
^6*b^4*x^4 - 9138*a^5*b^5*x^5 - 4043*a^4*b^6*x^6 - 360*a^3*b^7*x^7 + 45*a^2
*b^8*x^8 - 10*a*b^9*x^9 + 3*b^10*x^10 + 2520*a^4*(a + b*x)^6*Log[a + b*x])/
(12*b^11*(a + b*x)^6)
```

Maple [A]
time = 0.09, size = 144, normalized size = 0.96

method	result
risch	$\frac{x^4}{4b^7} - \frac{7a\,x^3}{3b^8} + \frac{14a^2x^2}{b^9} - \frac{84a^3x}{b^{10}} + \frac{252a^5b^4x^5+1155a^6b^3x^4+2140a^7b^2x^3+\frac{7995a^8b\,x^2}{4}+\frac{1879a^9x}{2}+\frac{2131a^{10}}{12b}}{b^{10}(bx+a)^6} + \frac{210a^4\ln(bx+a)}{b^{11}}$
norman	$\frac{\frac{x^{10}}{4b}-\frac{5a\,x^9}{6b^2}-\frac{30a^3x^7}{b^4}+\frac{1029a^{10}}{2b^{11}}+\frac{15a^2x^8}{4b^3}+\frac{1260a^5x^5}{b^6}+\frac{4725a^6x^4}{b^7}+\frac{7700a^7x^3}{b^8}+\frac{13125a^8x^2}{2b^9}+\frac{2877a^9x}{b^{10}}}{(bx+a)^6} + \frac{210a^4\ln(bx+a)}{b^{11}}$
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{7}{3}a\,b^2x^3-14a^2b\,x^2+84a^3x}{b^{10}} + \frac{252a^5}{b^{11}(bx+a)} + \frac{2a^9}{b^{11}(bx+a)^5} - \frac{45a^8}{4b^{11}(bx+a)^4} - \frac{105a^6}{b^{11}(bx+a)^2} - \frac{a^{10}}{6b^{11}(bx+a)^6} + \frac{210a^4\ln}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^10*(-1/4*b^3*x^4+7/3*a*b^2*x^3-14*a^2*b*x^2+84*a^3*x)+252*a^5/b^11/(b*
x+a)+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4-105*a^6/b^11/(b*x+a)^2-1/
6*a^10/b^11/(b*x+a)^6+210*a^4*ln(b*x+a)/b^11+40*a^7/b^11/(b*x+a)^3
```

Maxima [A]
time = 0.29, size = 180, normalized size = 1.20

$$\frac{3024\,a^5b^5x^5 + 13860\,a^6b^4x^4 + 25680\,a^7b^3x^3 + 23985\,a^8b^2x^2 + 11274\,a^9bx + 2131\,a^{10}}{12\,(b^{17}x^6 + 6\,ab^{16}x^5 + 15\,a^2b^{15}x^4 + 20\,a^3b^{14}x^3 + 15\,a^4b^{13}x^2 + 6\,a^5b^{12}x + a^6b^{11})} + \frac{210\,a^4\log(bx+a)}{b^{11}} + \frac{3\,b^3x^4 - 28\,ab^2x^3 + 168\,a^2bx^2 - 1008\,a^3x}{12\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4 + 25680*a^7*b^3*x^3 + 23985*a^8*
b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15
*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 210*a
^4*log(b*x + a)/b^11 + 1/12*(3*b^3*x^4 - 28*a*b^2*x^3 + 168*a^2*b*x^2 - 100
8*a^3*x)/b^10
```

Fricas [A]
time = 0.93, size = 250, normalized size = 1.67

$$\frac{3\,b^{10}x^{10} - 10\,ab^9x^9 + 45\,a^2b^8x^8 - 360\,a^3b^7x^7 - 4043\,a^4b^6x^6 - 9138\,a^5b^5x^5 - 3945\,a^6b^4x^4 + 11540\,a^7b^3x^3 + 18105\,a^8b^2x^2 + 10266\,a^9bx + 2131\,a^{10} + 2520\,(a^4b^6x^6 + 6\,a^5b^5x^5 + 15\,a^6b^4x^4 + 20\,a^7b^3x^3 + 15\,a^8b^2x^2 + 6\,a^9bx + a^{10})\log(bx+a)}{12\,(b^{17}x^6 + 6\,ab^{16}x^5 + 15\,a^2b^{15}x^4 + 20\,a^3b^{14}x^3 + 15\,a^4b^{13}x^2 + 6\,a^5b^{12}x + a^6b^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 - 10*a*b^9*x^9 + 45*a^2*b^8*x^8 - 360*a^3*b^7*x^7 - 4043*
a^4*b^6*x^6 - 9138*a^5*b^5*x^5 - 3945*a^6*b^4*x^4 + 11540*a^7*b^3*x^3 + 181
05*a^8*b^2*x^2 + 10266*a^9*b*x + 2131*a^10 + 2520*(a^4*b^6*x^6 + 6*a^5*b^5*
x^5 + 15*a^6*b^4*x^4 + 20*a^7*b^3*x^3 + 15*a^8*b^2*x^2 + 6*a^9*b*x + a^10)*
log(b*x + a))/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3
+ 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11)
```

Sympy [A]
time = 0.41, size = 190, normalized size = 1.27

$$\frac{210a^4\log{(a+bx)}}{b^{11}} - \frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{2131a^{10} + 11274a^9bx + 23985a^8b^2x^2 + 25680a^7b^3x^3 + 13860a^6b^4x^4 + 3024a^5b^5x^5}{12a^6b^{11} + 72a^5b^{12}x + 180a^4b^{13}x^2 + 240a^3b^{14}x^3 + 180a^2b^{15}x^4 + 72ab^{16}x^5 + 12b^{17}x^6} + \frac{x^4}{4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**7,x)
```

```
[Out] 210*a**4*log(a + b*x)/b**11 - 84*a**3*x/b**10 + 14*a**2*x**2/b**9 - 7*a*x**
3/(3*b**8) + (2131*a**10 + 11274*a**9*b*x + 23985*a**8*b**2*x**2 + 25680*a*
*7*b**3*x**3 + 13860*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*a**6*b**11 +
 72*a**5*b**12*x + 180*a**4*b**13*x**2 + 240*a**3*b**14*x**3 + 180*a**2*b**
15*x**4 + 72*a*b**16*x**5 + 12*b**17*x**6) + x**4/(4*b**7)
```

Giac [A]
time = 1.96, size = 128, normalized size = 0.85

$$\frac{210\,a^4\log{(|bx+a|)}}{b^{11}} + \frac{3024\,a^5b^5x^5 + 13860\,a^6b^4x^4 + 25680\,a^7b^3x^3 + 23985\,a^8b^2x^2 + 11274\,a^9bx + 2131\,a^{10}}{12\,(bx+a)^6b^{11}} + \frac{3\,b^{21}x^4 - 28\,ab^{20}x^3 + 168\,a^2b^{19}x^2 - 1008\,a^3b^{18}x}{12\,b^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*a^4*log(abs(b*x + a))/b^11 + 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4
 + 25680*a^7*b^3*x^3 + 23985*a^8*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/((b*x
 + a)^6*b^11) + 1/12*(3*b^21*x^4 - 28*a*b^20*x^3 + 168*a^2*b^19*x^2 - 1008*
a^3*b^18*x)/b^28
```

Mupad [B]
time = 1.09, size = 126, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^4}{4} - \frac{10\,a\,(a+b\,x)^3}{3} + \frac{45\,a^2\,(a+b\,x)^2}{2} + \frac{252\,a^5}{a+b\,x} - \frac{105\,a^6}{(a+b\,x)^2} + \frac{40\,a^7}{(a+b\,x)^3} - \frac{45\,a^8}{4\,(a+b\,x)^4} + \frac{2\,a^9}{(a+b\,x)^5} - \frac{a^{10}}{6\,(a+b\,x)^6} + 210\,a^4\,\ln{(a+b\,x)} - 120\,a^3\,b\,x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^4/4 - (10*a*(a + b*x)^3)/3 + (45*a^2*(a + b*x)^2)/2 + (252*a^5)/
(a + b*x) - (105*a^6)/(a + b*x)^2 + (40*a^7)/(a + b*x)^3 - (45*a^8)/(4*(a +
 b*x)^4) + (2*a^9)/(a + b*x)^5 - a^10/(6*(a + b*x)^6) + 210*a^4*log(a + b*x
) - 120*a^3*b*x)/b^11
```

3.208 $\int \frac{x^9}{(a+bx)^7}\,dx$

Optimal. Leaf size=139

$$\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}-$$

```
[Out] 28*a^2*x/b^9-7/2*a*x^2/b^8+1/3*x^3/b^7+1/6*a^9/b^10/(b*x+a)^6-9/5*a^8/b^10/
(b*x+a)^5+9*a^7/b^10/(b*x+a)^4-28*a^6/b^10/(b*x+a)^3+63*a^5/b^10/(b*x+a)^2-
126*a^4/b^10/(b*x+a)-84*a^3*ln(b*x+a)/b^10
```

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^7,x]
```

```
[Out] (28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6)
 - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(b^
10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x))
- (84*a^3*Log[a + b*x])/b^10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^7}\,dx &= \int \left(\frac{28a^2}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{b^7}-\frac{a^9}{b^9(a+bx)^7}+\frac{9a^8}{b^9(a+bx)^6}-\frac{36a^7}{b^9(a+bx)^5}+\frac{84a^6}{b^9(a+bx)^4}-\frac{126}{b^9(a+}\right.\\ &= \frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{}{b^1}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 128, normalized size = 0.92

$$-\frac{2509a^9+12534a^8bx+23775a^7b^2x^2+19100a^6b^3x^3+1725a^5b^4x^4-6870a^4b^5x^5-3665a^3b^6x^6-360a^2b^7x^7+45ab^8x^8-10b^9x^9+2520a^3(a+bx)^6\log(a+bx)}{30b^{10}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^7,x]
```

```
[Out] -1/30*(2509*a^9 + 12534*a^8*b*x + 23775*a^7*b^2*x^2 + 19100*a^6*b^3*x^3 + 1
725*a^5*b^4*x^4 - 6870*a^4*b^5*x^5 - 3665*a^3*b^6*x^6 - 360*a^2*b^7*x^7 + 4
5*a*b^8*x^8 - 10*b^9*x^9 + 2520*a^3*(a + b*x)^6*Log[a + b*x])/(b^10*(a + b*
x)^6)
```

Maple [A]
time = 0.10, size = 132, normalized size = 0.95

method	result
risch	$\frac{x^3}{3b^7} - \frac{7a\,x^2}{2b^8} + \frac{28a^2x}{b^9} + \frac{-126a^4b^4x^5 - 567a^5b^3x^4 - 1036a^6b^2x^3 - 957a^7b\,x^2 - \frac{2229a^8x}{5} - \frac{2509a^9}{30b}}{b^9(bx+a)^6} - \frac{84a^3\ln(bx+a)}{b^{10}}$
norman	$\frac{\frac{x^9}{3b} - \frac{3a\,x^8}{2b^2} - \frac{1029a^9}{5b^{10}} + \frac{12a^2x^7}{b^3} - \frac{504a^4x^5}{b^5} - \frac{1890a^5x^4}{b^6} - \frac{3080a^6x^3}{b^7} - \frac{2625a^7x^2}{b^8} - \frac{5754a^8x}{5b^9}}{(bx+a)^6} - \frac{84a^3\ln(bx+a)}{b^{10}}$
default	$\frac{\frac{1}{3}b^2x^3 - \frac{7}{2}ab\,x^2 + 28a^2x}{b^9} - \frac{126a^4}{b^{10}(bx+a)} + \frac{9a^7}{b^{10}(bx+a)^4} + \frac{63a^5}{b^{10}(bx+a)^2} + \frac{a^9}{6b^{10}(bx+a)^6} - \frac{84a^3\ln(bx+a)}{b^{10}} - \frac{9a^8}{5b^{10}(bx+a)^5} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^9*(1/3*b^2*x^3-7/2*a*b*x^2+28*a^2*x)-126*a^4/b^10/(b*x+a)+9*a^7/b^10/(b
*x+a)^4+63*a^5/b^10/(b*x+a)^2+1/6*a^9/b^10/(b*x+a)^6-84*a^3*ln(b*x+a)/b^10-
9/5*a^8/b^10/(b*x+a)^5-28*a^6/b^10/(b*x+a)^3
```

Maxima [A]
time = 0.29, size = 169, normalized size = 1.22

$$-\frac{3780\,a^4b^5x^5 + 17010\,a^5b^4x^4 + 31080\,a^6b^3x^3 + 28710\,a^7b^2x^2 + 13374\,a^8bx + 2509\,a^9}{30\,(b^{16}x^6 + 6\,ab^{15}x^5 + 15\,a^2b^{14}x^4 + 20\,a^3b^{13}x^3 + 15\,a^4b^{12}x^2 + 6\,a^5b^{11}x + a^6b^{10})} - \frac{84\,a^3\log(bx+a)}{b^{10}} + \frac{2\,b^2x^3 - 21\,abx^2 + 168\,a^2x}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4 + 31080*a^6*b^3*x^3 + 28710*a^7
*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/(b^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^14
*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 + 6*a^5*b^11*x + a^6*b^10) - 84*a^
3*log(b*x + a)/b^10 + 1/6*(2*b^2*x^3 - 21*a*b*x^2 + 168*a^2*x)/b^9
```

Fricas [A]
time = 0.94, size = 239, normalized size = 1.72

$$\frac{10\,b^9x^9 - 45\,ab^8x^8 + 360\,a^2b^7x^7 + 3665\,a^3b^6x^6 + 6870\,a^4b^5x^5 - 1725\,a^5b^4x^4 - 19100\,a^6b^3x^3 - 23775\,a^7b^2x^2 - 12534\,a^8bx - 2509\,a^9 - 2520\,(a^3b^6x^6 + 6\,a^4b^5x^5 + 15\,a^5b^4x^4 + 20\,a^6b^3x^3 + 15\,a^7b^2x^2 + 6\,a^8bx + a^9)\log(bx+a)}{30\,(b^{16}x^6 + 6\,ab^{15}x^5 + 15\,a^2b^{14}x^4 + 20\,a^3b^{13}x^3 + 15\,a^4b^{12}x^2 + 6\,a^5b^{11}x + a^6b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^9*x^9 - 45*a*b^8*x^8 + 360*a^2*b^7*x^7 + 3665*a^3*b^6*x^6 + 6870
*a^4*b^5*x^5 - 1725*a^5*b^4*x^4 - 19100*a^6*b^3*x^3 - 23775*a^7*b^2*x^2 - 1
2534*a^8*b*x - 2509*a^9 - 2520*(a^3*b^6*x^6 + 6*a^4*b^5*x^5 + 15*a^5*b^4*x^
4 + 20*a^6*b^3*x^3 + 15*a^7*b^2*x^2 + 6*a^8*b*x + a^9)*log(b*x + a))/(b^16*
x^6 + 6*a*b^15*x^5 + 15*a^2*b^14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 +
6*a^5*b^11*x + a^6*b^10)
```

Sympy [A]
time = 0.39, size = 180, normalized size = 1.29

$$-\frac{84a^3\log{(a+bx)}}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{-2509a^9-13374a^8bx-28710a^7b^2x^2-31080a^6b^3x^3-17010a^5b^4x^4-3780a^4b^5x^5}{30a^6b^{10}+180a^5b^{11}x+450a^4b^{12}x^2+600a^3b^{13}x^3+450a^2b^{14}x^4+180ab^{15}x^5+30b^{16}x^6}+\frac{x^3}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**7,x)
```

```
[Out] -84*a**3*log(a + b*x)/b**10 + 28*a**2*x/b**9 - 7*a*x**2/(2*b**8) + (-2509*a
**9 - 13374*a**8*b*x - 28710*a**7*b**2*x**2 - 31080*a**6*b**3*x**3 - 17010*
a**5*b**4*x**4 - 3780*a**4*b**5*x**5)/(30*a**6*b**10 + 180*a**5*b**11*x + 4
50*a**4*b**12*x**2 + 600*a**3*b**13*x**3 + 450*a**2*b**14*x**4 + 180*a*b**1
5*x**5 + 30*b**16*x**6) + x**3/(3*b**7)
```

Giac [A]
time = 1.46, size = 117, normalized size = 0.84

$$-\frac{84\,a^3\log\left(|bx+a|\right)}{b^{10}}-\frac{3780\,a^4b^5x^5+17010\,a^5b^4x^4+31080\,a^6b^3x^3+28710\,a^7b^2x^2+13374\,a^8bx+2509\,a^9}{30\,(bx+a)^6b^{10}}+\frac{2\,b^{14}x^3-21\,ab^{13}x^2+168\,a^2b^{12}x}{6\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -84*a^3*log(abs(b*x + a))/b^10 - 1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4
 + 31080*a^6*b^3*x^3 + 28710*a^7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/((b*x
+ a)^6*b^10) + 1/6*(2*b^14*x^3 - 21*a*b^13*x^2 + 168*a^2*b^12*x)/b^21
```

Mupad [B]
time = 0.55, size = 115, normalized size = 0.83

$$-\frac{\frac{9\,a\,(a+b\,x)^2}{2}-\frac{(a+b\,x)^3}{3}+\frac{126\,a^4}{a+b\,x}-\frac{63\,a^5}{(a+b\,x)^2}+\frac{28\,a^6}{(a+b\,x)^3}-\frac{9\,a^7}{(a+b\,x)^4}+\frac{9\,a^8}{5\,(a+b\,x)^5}-\frac{a^9}{6\,(a+b\,x)^6}+84\,a^3\,\ln\left(a+b\,x\right)-36\,a^2\,b\,x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^7,x)
```

```
[Out] -((9*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (126*a^4)/(a + b*x) - (63*a^5)/(a +
 b*x)^2 + (28*a^6)/(a + b*x)^3 - (9*a^7)/(a + b*x)^4 + (9*a^8)/(5*(a + b*x)
^5) - a^9/(6*(a + b*x)^6) + 84*a^3*log(a + b*x) - 36*a^2*b*x)/b^10
```

3.209 $\int \frac{x^8}{(a+bx)^7}\,dx$

Optimal. Leaf size=128

$$-\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a}{b^9}$$

```
[Out] -7*a*x/b^8+1/2*x^2/b^7-1/6*a^8/b^9/(b*x+a)^6+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4+56/3*a^5/b^9/(b*x+a)^3-35*a^4/b^9/(b*x+a)^2+56*a^3/b^9/(b*x+a)+
28*a^2*ln(b*x+a)/b^9
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^7,x]
```

```
[Out] (-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*a^
4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/b^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^7}\,dx = \int \left(-\frac{7a}{b^8}+\frac{x}{b^7}+\frac{a^8}{b^8(a+bx)^7}-\frac{8a^7}{b^8(a+bx)^6}+\frac{28a^6}{b^8(a+bx)^5}-\frac{56a^5}{b^8(a+bx)^4}+\frac{70a^4}{b^8(a+bx)^3}\right.$$
$$= -\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}$$

Mathematica [A]
time = 0.03, size = 104, normalized size = 0.81

$$\frac{-210abx+15b^2x^2-\frac{5a^8}{(a+bx)^6}+\frac{48a^7}{(a+bx)^5}-\frac{210a^6}{(a+bx)^4}+\frac{560a^5}{(a+bx)^3}-\frac{1050a^4}{(a+bx)^2}+\frac{1680a^3}{a+bx}+840a^2\log(a+bx)}{30b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^7,x]
```

```
[Out] (-210*a*b*x + 15*b^2*x^2 - (5*a^8)/(a + b*x)^6 + (48*a^7)/(a + b*x)^5 - (21
0*a^6)/(a + b*x)^4 + (560*a^5)/(a + b*x)^3 - (1050*a^4)/(a + b*x)^2 + (1680
*a^3)/(a + b*x) + 840*a^2*Log[a + b*x])/(30*b^9)
```

Maple [A]
time = 0.09, size = 122, normalized size = 0.95

method	result	si
risch	$\frac{x^2}{2b^7}-\frac{7ax}{b^8}+\frac{56a^3b^4x^5+245a^4b^3x^4+\frac{1316a^5b^2x^3}{3}+399a^6b\,x^2+\frac{918a^7x}{5}+\frac{341a^8}{10b}}{b^8(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}$	9
norman	$\frac{\frac{x^8}{2b}-\frac{4a\,x^7}{b^2}+\frac{343a^8}{5b^9}+\frac{168a^3x^5}{b^4}+\frac{630a^4x^4}{b^5}+\frac{3080a^5x^3}{3b^6}+\frac{875a^6x^2}{b^7}+\frac{1918a^7x}{5b^8}}{(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}$	1
default	$-\frac{-\frac{1}{2}x^2b+7ax}{b^8}+\frac{56a^3}{b^9(bx+a)}+\frac{8a^7}{5b^9(bx+a)^5}-\frac{7a^6}{b^9(bx+a)^4}-\frac{35a^4}{b^9(bx+a)^2}-\frac{a^8}{6b^9(bx+a)^6}+\frac{28a^2\ln(bx+a)}{b^9}+\frac{56a^5}{3b^9(bx+a)^3}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^8*(-1/2*x^2*b+7*a*x)+56*a^3/b^9/(b*x+a)+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4-35*a^4/b^9/(b*x+a)^2-1/6*a^8/b^9/(b*x+a)^6+28*a^2*ln(b*x+a)/b^9
+56/3*a^5/b^9/(b*x+a)^3
```

Maxima [A]
time = 0.31, size = 157, normalized size = 1.23

$$\frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\,(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^{11}x^2+6\,a^5b^{10}x+a^6b^9)}+\frac{28\,a^2\log(bx+a)}{b^9}+\frac{bx^2-14\,ax}{2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(1680*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b
^2*x^2 + 5508*a^7*b*x + 1023*a^8)/(b^15*x^6 + 6*a*b^14*x^5 + 15*a^2*b^13*x^
4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*b^9) + 28*a^2*lo
g(b*x + a)/b^9 + 1/2*(b*x^2 - 14*a*x)/b^8
```

Fricas [A]
time = 0.93, size = 228, normalized size = 1.78

$$\frac{15\,b^8x^8-120\,ab^7x^7-1035\,a^2b^6x^6-1170\,a^3b^5x^5+3375\,a^4b^4x^4+10100\,a^5b^3x^3+10725\,a^6b^2x^2+5298\,a^7bx+1023\,a^8+840\,(a^2b^6x^6+6\,a^3b^5x^5+15\,a^4b^4x^4+20\,a^5b^3x^3+15\,a^6b^2x^2+6\,a^7bx+a^8)\log(bx+a)}{30\,(b^{15}x^6+6\,ab^{14}x^5+15\,a^2b^{13}x^4+20\,a^3b^{12}x^3+15\,a^4b^{11}x^2+6\,a^5b^{10}x+a^6b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(15*b^8*x^8 - 120*a*b^7*x^7 - 1035*a^2*b^6*x^6 - 1170*a^3*b^5*x^5 + 33
75*a^4*b^4*x^4 + 10100*a^5*b^3*x^3 + 10725*a^6*b^2*x^2 + 5298*a^7*b*x + 102
3*a^8 + 840*(a^2*b^6*x^6 + 6*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 20*a^5*b^3*x^3
+ 15*a^6*b^2*x^2 + 6*a^7*b*x + a^8)*log(b*x + a))/(b^15*x^6 + 6*a*b^14*x^5
+ 15*a^2*b^13*x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*
b^9)
```

Sympy [A]
time = 0.36, size = 165, normalized size = 1.29

$$\frac{28a^2\log{(a+bx)}}{b^9}-\frac{7ax}{b^8}+\frac{1023a^8+5508a^7bx+11970a^6b^2x^2+13160a^5b^3x^3+7350a^4b^4x^4+1680a^3b^5x^5}{30a^6b^9+180a^5b^{10}x+450a^4b^{11}x^2+600a^3b^{12}x^3+450a^2b^{13}x^4+180ab^{14}x^5+30b^{15}x^6}+\frac{x^2}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**7,x)
```

```
[Out] 28*a**2*log(a + b*x)/b**9 - 7*a*x/b**8 + (1023*a**8 + 5508*a**7*b*x + 11970
*a**6*b**2*x**2 + 13160*a**5*b**3*x**3 + 7350*a**4*b**4*x**4 + 1680*a**3*b*
*5*x**5)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*
b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + x**2
/(2*b**7)
```

Giac [A]
time = 1.10, size = 105, normalized size = 0.82

$$\frac{28\,a^2\log{(|bx+a|)}}{b^9}+\frac{b^7x^2-14\,ab^6x}{2\,b^{14}}+\frac{1680\,a^3b^5x^5+7350\,a^4b^4x^4+13160\,a^5b^3x^3+11970\,a^6b^2x^2+5508\,a^7bx+1023\,a^8}{30\,(bx+a)^6b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 28*a^2*log(abs(b*x + a))/b^9 + 1/2*(b^7*x^2 - 14*a*b^6*x)/b^14 + 1/30*(1680
*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b^2*x^2 + 5
508*a^7*b*x + 1023*a^8)/((b*x + a)^6*b^9)
```

Mupad [B]
time = 0.18, size = 102, normalized size = 0.80

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{56\,a^3}{a+b\,x}-\frac{35\,a^4}{(a+b\,x)^2}+\frac{56\,a^5}{3\,(a+b\,x)^3}-\frac{7\,a^6}{(a+b\,x)^4}+\frac{8\,a^7}{5\,(a+b\,x)^5}-\frac{a^8}{6\,(a+b\,x)^6}+28\,a^2\,\ln{(a+b\,x)}-8\,a\,b\,x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^2/2 + (56*a^3)/(a + b*x) - (35*a^4)/(a + b*x)^2 + (56*a^5)/(3*(a
 + b*x)^3) - (7*a^6)/(a + b*x)^4 + (8*a^7)/(5*(a + b*x)^5) - a^8/(6*(a + b*
x)^6) + 28*a^2*log(a + b*x) - 8*a*b*x)/b^9
```

3.210 $\int \frac{x^7}{(a+bx)^7}\,dx$

Optimal. Leaf size=118

$$\frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}$$

```
[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35
/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a
)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^7,x]
```

```
[Out] x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(4
*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b*x
)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^7}\,dx &= \int \left(\frac{1}{b^7}-\frac{a^7}{b^7(a+bx)^7}+\frac{7a^6}{b^7(a+bx)^6}-\frac{21a^5}{b^7(a+bx)^5}+\frac{35a^4}{b^7(a+bx)^4}-\frac{35a^3}{b^7(a+bx)^3}+\frac{21a}{b^7(a+\right.\\ &= \frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21}{b^8(a-}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 0.88

$$-\frac{669a^7+3594a^6bx+7725a^5b^2x^2+8200a^4b^3x^3+4050a^3b^4x^4+360a^2b^5x^5-360ab^6x^6-60b^7x^7+420a(a+bx)^6\log(a+bx)}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7 + 3594*a^6*b*x + 7725*a^5*b^2*x^2 + 8200*a^4*b^3*x^3 + 4050*
a^3*b^4*x^4 + 360*a^2*b^5*x^5 - 360*a*b^6*x^6 - 60*b^7*x^7 + 420*a*(a + b*x
)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

Maple [A]
time = 0.09, size = 109, normalized size = 0.92

method	result	size
risch	$\frac{x}{b^{7}}+\frac{-21 a^{2} b^{4} x^{5}-\frac{175 a^{3} b^{3} x^{4}}{2}-\frac{455 a^{4} b^{2} x^{3}}{3}-\frac{539 a^{5} b\,x^{2}}{4}-\frac{609 a^{6} x}{10}-\frac{223 a^{7}}{20 b}}{b^{7} \left(b x +a \right)^{6}}-\frac{7 a \ln \left(b x +a \right)}{b^{8}}$	87
norman	$\frac{\frac{x^{7}}{b}-\frac{343 a^{7}}{20 b^{8}}-\frac{42 a^{2} x^{5}}{b^{3}}-\frac{315 a^{3} x^{4}}{2 b^{4}}-\frac{770 a^{4} x^{3}}{3 b^{5}}-\frac{875 a^{5} x^{2}}{4 b^{6}}-\frac{959 a^{6} x}{10 b^{7}}}{\left(b x +a \right)^{6}}-\frac{7 a \ln \left(b x +a \right)}{b^{8}}$	91
default	$\frac{x}{b^{7}}+\frac{a^{7}}{6 b^{8} \left(b x +a \right)^{6}}-\frac{7 a^{6}}{5 b^{8} \left(b x +a \right)^{5}}+\frac{21 a^{5}}{4 b^{8} \left(b x +a \right)^{4}}-\frac{35 a^{4}}{3 b^{8} \left(b x +a \right)^{3}}+\frac{35 a^{3}}{2 b^{8} \left(b x +a \right)^{2}}-\frac{21 a^{2}}{b^{8} \left(b x +a \right)}-\frac{7 a \ln \left(b x +a \right)}{b^{8}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35
/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a
)/b^8
```

Maxima [A]
time = 0.29, size = 145, normalized size = 1.23

$$-\frac{1260\,a^{2}b^{5}x^{5}+5250\,a^{3}b^{4}x^{4}+9100\,a^{4}b^{3}x^{3}+8085\,a^{5}b^{2}x^{2}+3654\,a^{6}bx+669\,a^{7}}{60\,\left(b^{14}x^{6}+6\,ab^{13}x^{5}+15\,a^{2}b^{12}x^{4}+20\,a^{3}b^{11}x^{3}+15\,a^{4}b^{10}x^{2}+6\,a^{5}b^{9}x+a^{6}b^{8}\right)}+\frac{x}{b^{7}}-\frac{7\,a\log\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^
2*x^2 + 3654*a^6*b*x + 669*a^7)/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4
+ 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8) + x/b^7 - 7*a*
log(b*x + a)/b^8
```

Fricas [A]
time = 0.72, size = 215, normalized size = 1.82

$$\frac{60\,b^{7}x^{7}+360\,ab^{6}x^{6}-360\,a^{2}b^{5}x^{5}-4050\,a^{3}b^{4}x^{4}-8200\,a^{4}b^{3}x^{3}-7725\,a^{5}b^{2}x^{2}-3594\,a^{6}bx-669\,a^{7}-420\,\left(ab^{6}x^{6}+6\,a^{2}b^{5}x^{5}+15\,a^{3}b^{4}x^{4}+20\,a^{4}b^{3}x^{3}+15\,a^{5}b^{2}x^{2}+6\,a^{6}bx+a^{7}\right)\log\left(bx+a\right)}{60\,\left(b^{14}x^{6}+6\,ab^{13}x^{5}+15\,a^{2}b^{12}x^{4}+20\,a^{3}b^{11}x^{3}+15\,a^{4}b^{10}x^{2}+6\,a^{5}b^{9}x+a^{6}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 360*a*b^6*x^6 - 360*a^2*b^5*x^5 - 4050*a^3*b^4*x^4 - 820
0*a^4*b^3*x^3 - 7725*a^5*b^2*x^2 - 3594*a^6*b*x - 669*a^7 - 420*(a*b^6*x^6
+ 6*a^2*b^5*x^5 + 15*a^3*b^4*x^4 + 20*a^4*b^3*x^3 + 15*a^5*b^2*x^2 + 6*a^6*
b*x + a^7)*log(b*x + a))/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^
3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8)
```

Sympy [A]
time = 0.34, size = 153, normalized size = 1.30

$$-\frac{7a\log(a+bx)}{b^8}+\frac{-669a^7-3654a^6bx-8085a^5b^2x^2-9100a^4b^3x^3-5250a^3b^4x^4-1260a^2b^5x^5}{60a^6b^8+360a^5b^9x+900a^4b^{10}x^2+1200a^3b^{11}x^3+900a^2b^{12}x^4+360ab^{13}x^5+60b^{14}x^6}+\frac{x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**7,x)
```

```
[Out] -7*a*log(a + b*x)/b**8 + (-669*a**7 - 3654*a**6*b*x - 8085*a**5*b**2*x**2 -
 9100*a**4*b**3*x**3 - 5250*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*a**6*
b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a
**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) + x/b**7
```

Giac [A]
time = 1.14, size = 88, normalized size = 0.75

$$\frac{x}{b^7}-\frac{7\,a\log(|bx+a|)}{b^8}-\frac{1260\,a^2b^5x^5+5250\,a^3b^4x^4+9100\,a^4b^3x^3+8085\,a^5b^2x^2+3654\,a^6bx+669\,a^7}{60\,(bx+a)^6b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] x/b^7 - 7*a*log(abs(b*x + a))/b^8 - 1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x
^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^2*x^2 + 3654*a^6*b*x + 669*a^7)/((b*x +
a)^6*b^8)
```

Mupad [B]
time = 0.34, size = 91, normalized size = 0.77

$$-\frac{7\,a\,\ln(a+b\,x)-b\,x+\frac{21\,a^2}{a+b\,x}-\frac{35\,a^3}{2\,(a+b\,x)^2}+\frac{35\,a^4}{3\,(a+b\,x)^3}-\frac{21\,a^5}{4\,(a+b\,x)^4}+\frac{7\,a^6}{5\,(a+b\,x)^5}-\frac{a^7}{6\,(a+b\,x)^6}}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^7,x)
```

```
[Out] -(7*a*log(a + b*x) - b*x + (21*a^2)/(a + b*x) - (35*a^3)/(2*(a + b*x)^2) +
(35*a^4)/(3*(a + b*x)^3) - (21*a^5)/(4*(a + b*x)^4) + (7*a^6)/(5*(a + b*x)^
5) - a^7/(6*(a + b*x)^6))/b^8
```

3.211 $\int \frac{x^6}{(a+bx)^7} dx$

Optimal. Leaf size=109

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^7,x]
```

```
[Out] -1/6*a^6/(b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7*
(a + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2)
+ (6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6}{(a+bx)^7}\,dx = \int \left(\frac{a^6}{b^6(a+bx)^7}-\frac{6a^5}{b^6(a+bx)^6}+\frac{15a^4}{b^6(a+bx)^5}-\frac{20a^3}{b^6(a+bx)^4}+\frac{15a^2}{b^6(a+bx)^3}-\frac{6a}{b^6(a+bx)^2}\right.$$
$$= -\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+b}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.71

$$\frac{\frac{a(147a^5+822a^4bx+1875a^3b^2x^2+2200a^2b^3x^3+1350ab^4x^4+360b^5x^5)}{(a+bx)^6}+60\log(a+bx)}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^7,x]
```

```
[Out] ((a*(147*a^5 + 822*a^4*b*x + 1875*a^3*b^2*x^2 + 2200*a^2*b^3*x^3 + 1350*a*b
^4*x^4 + 360*b^5*x^5))/(a + b*x)^6 + 60*Log[a + b*x])/(60*b^7)
```

Maple [A]
time = 0.08, size = 100, normalized size = 0.92

method	result	size
norman	$\frac{\frac{49a^6}{20b^7}+\frac{6a\,x^5}{b^2}+\frac{45a^2x^4}{2b^3}+\frac{125a^4x^2}{4b^5}+\frac{137a^5x}{10b^6}+\frac{110a^3x^3}{3b^4}}{(bx+a)^6}+\frac{\ln(bx+a)}{b^7}$	80
risch	$\frac{\frac{49a^6}{20b^7}+\frac{6a\,x^5}{b^2}+\frac{45a^2x^4}{2b^3}+\frac{125a^4x^2}{4b^5}+\frac{137a^5x}{10b^6}+\frac{110a^3x^3}{3b^4}}{(bx+a)^6}+\frac{\ln(bx+a)}{b^7}$	80
default	$-\frac{a^6}{6b^7(bx+a)^6}+\frac{6a^5}{5b^7(bx+a)^5}-\frac{15a^4}{4b^7(bx+a)^4}+\frac{20a^3}{3b^7(bx+a)^3}-\frac{15a^2}{2b^7(bx+a)^2}+\frac{6a}{b^7(bx+a)}+\frac{\ln(bx+a)}{b^7}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

Maxima [A]
time = 0.28, size = 136, normalized size = 1.25

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6}{60\,(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7)}+\frac{\log(bx+a)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6)/(b^13*x^6 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*
a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b^8*x + a^6*b^7) + log(b*x + a)/b^7
```

Fricas [A]
time = 0.91, size = 193, normalized size = 1.77

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6+60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\log(bx+a)}{60\,(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6 + 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20
```

```
*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a))/(b^13*x^6 +
6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b
^8*x + a^6*b^7)
```

Sympy [A]
time = 0.27, size = 141, normalized size = 1.29

$$\frac{147a^6 + 822a^5bx + 1875a^4b^2x^2 + 2200a^3b^3x^3 + 1350a^2b^4x^4 + 360ab^5x^5}{60a^6b^7 + 360a^5b^8x + 900a^4b^9x^2 + 1200a^3b^{10}x^3 + 900a^2b^{11}x^4 + 360ab^{12}x^5 + 60b^{13}x^6} + \frac{\log{(a + bx)}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**7,x)
```

```
[Out] (147*a**6 + 822*a**5*b*x + 1875*a**4*b**2*x**2 + 2200*a**3*b**3*x**3 + 1350
*a**2*b**4*x**4 + 360*a*b**5*x**5)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + log(a + b*x)/b**7
```

Giac [A]
time = 1.23, size = 79, normalized size = 0.72

$$\frac{\log\left(|bx + a|\right)}{b^7} + \frac{360\,ab^4x^5 + 1350\,a^2b^3x^4 + 2200\,a^3b^2x^3 + 1875\,a^4bx^2 + 822\,a^5x + \frac{147\,a^6}{b}}{60\,(bx + a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^7 + 1/60*(360*a*b^4*x^5 + 1350*a^2*b^3*x^4 + 2200*a^3*b
^2*x^3 + 1875*a^4*b*x^2 + 822*a^5*x + 147*a^6/b)/((b*x + a)^6*b^6)
```

Mupad [B]
time = 0.11, size = 81, normalized size = 0.74

$$\frac{\ln\left(a + b\,x\right) + \frac{6\,a}{a+b\,x} - \frac{15\,a^2}{2\,(a+b\,x)^2} + \frac{20\,a^3}{3\,(a+b\,x)^3} - \frac{15\,a^4}{4\,(a+b\,x)^4} + \frac{6\,a^5}{5\,(a+b\,x)^5} - \frac{a^6}{6\,(a+b\,x)^6}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^7,x)
```

```
[Out] (log(a + b*x) + (6*a)/(a + b*x) - (15*a^2)/(2*(a + b*x)^2) + (20*a^3)/(3*(a
 + b*x)^3) - (15*a^4)/(4*(a + b*x)^4) + (6*a^5)/(5*(a + b*x)^5) - a^6/(6*(a
 + b*x)^6))/b^7
```

3.212 $\int \frac{x^5}{(a+bx)^7}\,dx$

Optimal. Leaf size=17

$$\frac{x^6}{6a(a+bx)^6}$$

[Out] 1/6*x^6/a/(b*x+a)^6

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{37\}$

$$\frac{x^6}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b*x)^7,x]

[Out] x^6/(6*a*(a + b*x)^6)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^7}\,dx = \frac{x^6}{6a(a+bx)^6}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(17) = 34.
time = 0.01, size = 64, normalized size = 3.76

$$-\frac{a^5+6a^4bx+15a^3b^2x^2+20a^2b^3x^3+15ab^4x^4+6b^5x^5}{6b^6(a+bx)^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a + b*x)^7,x]

```
[Out] -1/6*(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15*a*b^4*x^4 + 6*
b^5*x^5)/(b^6*(a + b*x)^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(15) = 30$.
time = 0.08, size = 87, normalized size = 5.12

method	result	size
gosper	$-\frac{6b^5x^5+15a\,b^4x^4+20a^2b^3x^3+15a^3b^2x^2+6a^4bx+a^5}{6(bx+a)^6b^6}$	63
norman	$\frac{-\frac{x^5}{b}-\frac{5a\,x^4}{2b^2}-\frac{10a^2x^3}{3b^3}-\frac{5a^3x^2}{2b^4}-\frac{a^4x}{b^5}-\frac{a^5}{6b^6}}{(bx+a)^6}$	66
risch	$\frac{-\frac{x^5}{b}-\frac{5a\,x^4}{2b^2}-\frac{10a^2x^3}{3b^3}-\frac{5a^3x^2}{2b^4}-\frac{a^4x}{b^5}-\frac{a^5}{6b^6}}{(bx+a)^6}$	66
default	$-\frac{1}{b^6(bx+a)}+\frac{5a^3}{2b^6(bx+a)^4}+\frac{5a}{2b^6(bx+a)^2}+\frac{a^5}{6b^6(bx+a)^6}-\frac{10a^2}{3b^6(bx+a)^3}-\frac{a^4}{b^6(bx+a)^5}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6/(b*x+a)+5/2*a^3/b^6/(b*x+a)^4+5/2*a/b^6/(b*x+a)^2+1/6*a^5/b^6/(b*x+a
)^6-10/3/b^6*a^2/(b*x+a)^3-a^4/b^6/(b*x+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.
time = 0.28, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,(b^{12}x^6+6\,ab^{11}x^5+15\,a^2b^{10}x^4+20\,a^3b^9x^3+15\,a^4b^8x^2+6\,a^5b^7x+a^6b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.
time = 0.63, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,(b^{12}x^6+6\,ab^{11}x^5+15\,a^2b^{10}x^4+20\,a^3b^9x^3+15\,a^4b^8x^2+6\,a^5b^7x+a^6b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(12) = 24$.
time = 0.23, size = 128, normalized size = 7.53

$$\frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6a^6b^6 + 36a^5b^7x + 90a^4b^8x^2 + 120a^3b^9x^3 + 90a^2b^{10}x^4 + 36ab^{11}x^5 + 6b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*a**6*b**6 + 36*a**5*b**7*x + 90*a**4*b**8*x**2 + 120*a*
*3*b**9*x**3 + 90*a**2*b**10*x**4 + 36*a*b**11*x**5 + 6*b**12*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(15) = 30$.
time = 1.03, size = 62, normalized size = 3.65

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/((b*x + a)^6*b^6)
```

Mupad [B]
time = 0.12, size = 72, normalized size = 4.24

$$\frac{\frac{5\,a}{2\,(a+b\,x)^2} - \frac{1}{a+b\,x} - \frac{10\,a^2}{3\,(a+b\,x)^3} + \frac{5\,a^3}{2\,(a+b\,x)^4} - \frac{a^4}{(a+b\,x)^5} + \frac{a^5}{6\,(a+b\,x)^6}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^7,x)
```

```
[Out] ((5*a)/(2*(a + b*x)^2) - 1/(a + b*x) - (10*a^2)/(3*(a + b*x)^3) + (5*a^3)/(
2*(a + b*x)^4) - a^4/(a + b*x)^5 + a^5/(6*(a + b*x)^6))/b^6
```

3.213 $\int \frac{x^4}{(a+bx)^7} dx$

Optimal. Leaf size=35

$$\frac{x^5}{6a(a+bx)^6} + \frac{x^5}{30a^2(a+bx)^5}$$

```
[Out] 1/6*x^5/a/(b*x+a)^6+1/30*x^5/a^2/(b*x+a)^5
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{x^5}{30a^2(a+bx)^5} + \frac{x^5}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^7,x]
```

```
[Out] x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^7} dx &= \frac{x^5}{6a(a+bx)^6} + \frac{\int \frac{x^4}{(a+bx)^6} dx}{6a} \\ &= \frac{x^5}{6a(a+bx)^6} + \frac{x^5}{30a^2(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.51

$$-\frac{a^4+6a^3bx+15a^2b^2x^2+20ab^3x^3+15b^4x^4}{30b^5(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^7,x]
```

```
[Out] -1/30*(a^4 + 6*a^3*b*x + 15*a^2*b^2*x^2 + 20*a*b^3*x^3 + 15*b^4*x^4)/(b^5*(
a + b*x)^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 0.08, size = 72, normalized size = 2.06

method	result	size
gosper	$-\frac{15b^4x^4+20a\,b^3x^3+15a^2b^2x^2+6a^3bx+a^4}{30(bx+a)^6b^5}$	52
norman	$\frac{-\frac{x^4}{2b}-\frac{2a\,x^3}{3b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3x}{5b^4}-\frac{a^4}{30b^5}}{(bx+a)^6}$	55
risch	$\frac{-\frac{x^4}{2b}-\frac{2a\,x^3}{3b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3x}{5b^4}-\frac{a^4}{30b^5}}{(bx+a)^6}$	55
default	$-\frac{3a^2}{2b^5(bx+a)^4}-\frac{1}{2b^5(bx+a)^2}-\frac{a^4}{6b^5(bx+a)^6}+\frac{4a^3}{5b^5(bx+a)^5}+\frac{4a}{3b^5(bx+a)^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/b^5*a^2/(b*x+a)^4-1/2/b^5/(b*x+a)^2-1/6*a^4/b^5/(b*x+a)^6+4/5*a^3/b^5/
(b*x+a)^5+4/3*a/b^5/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.
time = 0.30, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\,(b^{11}x^6+6\,ab^{10}x^5+15\,a^2b^9x^4+20\,a^3b^8x^3+15\,a^4b^7x^2+6\,a^5b^6x+a^6b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.
time = 0.53, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(b^{11}x^6 + 6\,ab^{10}x^5 + 15\,a^2b^9x^4 + 20\,a^3b^8x^3 + 15\,a^4b^7x^2 + 6\,a^5b^6x + a^6b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(27) = 54$.
time = 0.22, size = 116, normalized size = 3.31

$$\frac{-a^4 - 6a^3bx - 15a^2b^2x^2 - 20ab^3x^3 - 15b^4x^4}{30a^6b^5 + 180a^5b^6x + 450a^4b^7x^2 + 600a^3b^8x^3 + 450a^2b^9x^4 + 180ab^{10}x^5 + 30b^{11}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**7,x)
```

```
[Out] (-a**4 - 6*a**3*b*x - 15*a**2*b**2*x**2 - 20*a*b**3*x**3 - 15*b**4*x**4)/(3
0*a**6*b**5 + 180*a**5*b**6*x + 450*a**4*b**7*x**2 + 600*a**3*b**8*x**3 + 4
50*a**2*b**9*x**4 + 180*a*b**10*x**5 + 30*b**11*x**6)
```

Giac [A]
time = 1.45, size = 51, normalized size = 1.46

$$-\frac{15\,b^4x^4 + 20\,ab^3x^3 + 15\,a^2b^2x^2 + 6\,a^3bx + a^4}{30\,(bx + a)^6b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/((b*x
+ a)^6*b^5)
```

Mupad [B]
time = 0.07, size = 22, normalized size = 0.63

$$\frac{x^5\,(6\,a + b\,x)}{30\,a^2\,(a + b\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^7,x)
```

```
[Out] (x^5*(6*a + b*x))/(30*a^2*(a + b*x)^6)
```

3.214 $\int \frac{x^3}{(a+bx)^7}\,dx$

Optimal. Leaf size=52

$$\frac{x^4}{6a(a+bx)^6}+\frac{x^4}{15a^2(a+bx)^5}+\frac{x^4}{60a^3(a+bx)^4}$$

```
[Out] 1/6*x^4/a/(b*x+a)^6+1/15*x^4/a^2/(b*x+a)^5+1/60*x^4/a^3/(b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^7,x]
```

```
[Out] a^3/(6*b^4*(a + b*x)^6) - (3*a^2)/(5*b^4*(a + b*x)^5) + (3*a)/(4*b^4*(a + b
*x)^4) - 1/(3*b^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^7}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^7}+\frac{3a^2}{b^3(a+bx)^6}-\frac{3a}{b^3(a+bx)^5}+\frac{1}{b^3(a+bx)^4}\right)dx\\ &= \frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.81

$$-\frac{a^3+6a^2bx+15ab^2x^2+20b^3x^3}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3 + 6*a^2*b*x + 15*a*b^2*x^2 + 20*b^3*x^3)/(b^4*(a + b*x)^6)
```

Maple [A]
time = 0.08, size = 57, normalized size = 1.10

method	result	size
gosper	$-\frac{20 b^{3} x^{3}+15 a\,b^{2} x^{2}+6 a^{2} b x +a^{3}}{60 \left(b x +a \right)^{6} b^{4}}$	41
norman	$\frac{-\frac{x^{3}}{3 b}-\frac{a\,x^{2}}{4 b^{2}}-\frac{a^{2} x}{10 b^{3}}-\frac{a^{3}}{60 b^{4}}}{\left(b x +a \right)^{6}}$	44
risch	$\frac{-\frac{x^{3}}{3 b}-\frac{a\,x^{2}}{4 b^{2}}-\frac{a^{2} x}{10 b^{3}}-\frac{a^{3}}{60 b^{4}}}{\left(b x +a \right)^{6}}$	44
default	$\frac{a^{3}}{6 b^{4} \left(b x +a \right)^{6}}-\frac{3 a^{2}}{5 b^{4} \left(b x +a \right)^{5}}+\frac{3 a}{4 b^{4} \left(b x +a \right)^{4}}-\frac{1}{3 b^{4} \left(b x +a \right)^{3}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^3/b^4/(b*x+a)^6-3/5*a^2/b^4/(b*x+a)^5+3/4*a/b^4/(b*x+a)^4-1/3/b^4/(b*
x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.
time = 0.29, size = 98, normalized size = 1.88

$$-\frac{20 \, b^{3} x^{3} + 15 \, a b^{2} x^{2} + 6 \, a^{2} b x + a^{3}}{60 \, {\left(b^{10} x^{6} + 6 \, a b^{9} x^{5} + 15 \, a^{2} b^{8} x^{4} + 20 \, a^{3} b^{7} x^{3} + 15 \, a^{4} b^{6} x^{2} + 6 \, a^{5} b^{5} x + a^{6} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(46) = 92$.
time = 0.81, size = 98, normalized size = 1.88

$$-\frac{20 \, b^{3} x^{3} + 15 \, a b^{2} x^{2} + 6 \, a^{2} b x + a^{3}}{60 \, {\left(b^{10} x^{6} + 6 \, a b^{9} x^{5} + 15 \, a^{2} b^{8} x^{4} + 20 \, a^{3} b^{7} x^{3} + 15 \, a^{4} b^{6} x^{2} + 6 \, a^{5} b^{5} x + a^{6} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.
time = 0.20, size = 104, normalized size = 2.00

$$\frac{-a^3 - 6a^2bx - 15ab^2x^2 - 20b^3x^3}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**7,x)
```

```
[Out] (-a**3 - 6*a**2*b*x - 15*a*b**2*x**2 - 20*b**3*x**3)/(60*a**6*b**4 + 360*a*
*5*b**5*x + 900*a**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 +
 360*a*b**9*x**5 + 60*b**10*x**6)
```

Giac [A]
time = 0.97, size = 40, normalized size = 0.77

$$-\frac{20\, b^3x^3 + 15\, ab^2x^2 + 6\, a^2bx + a^3}{60\, (bx + a)^6 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/((b*x + a)^6*b^4)
```

Mupad [B]
time = 0.07, size = 48, normalized size = 0.92

$$\frac{\frac{3\,a}{4\,{\left(a+b\,x\right)}^4}-\frac{1}{3\,{\left(a+b\,x\right)}^3}-\frac{3\,a^2}{5\,{\left(a+b\,x\right)}^5}+\frac{a^3}{6\,{\left(a+b\,x\right)}^6}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^7,x)
```

```
[Out] ((3*a)/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5) + a^3/
(6*(a + b*x)^6))/b^4
```

3.215 $\int \frac{x^2}{(a+bx)^7} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^7,x]
```

```
[Out] -1/6*a^2/(b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x)
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^7}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^7}-\frac{2a}{b^2(a+bx)^6}+\frac{1}{b^2(a+bx)^5}\right)dx\\ &= -\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+6abx+15b^2x^2}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2 + 6*a*b*x + 15*b^2*x^2)/(b^3*(a + b*x)^6)
```

Maple [A]
time = 0.08, size = 42, normalized size = 0.89

method	result	size
gosper	$-\frac{15x^2b^2+6abx+a^2}{60(bx+a)^6b^3}$	30
norman	$\frac{-\frac{x^2}{4b}-\frac{ax}{10b^2}-\frac{a^2}{60b^3}}{(bx+a)^6}$	33
risch	$\frac{-\frac{x^2}{4b}-\frac{ax}{10b^2}-\frac{a^2}{60b^3}}{(bx+a)^6}$	33
default	$-\frac{a^2}{6b^3(bx+a)^6}+\frac{2a}{5b^3(bx+a)^5}-\frac{1}{4b^3(bx+a)^4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.
time = 0.29, size = 87, normalized size = 1.85

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(41) = 82$.
time = 0.65, size = 87, normalized size = 1.85

$$-\frac{15\,b^2x^2+6\,abx+a^2}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(42) = 84$.
time = 0.18, size = 92, normalized size = 1.96

$$\frac{-a^2 - 6abx - 15b^2x^2}{60a^6b^3 + 360a^5b^4x + 900a^4b^5x^2 + 1200a^3b^6x^3 + 900a^2b^7x^4 + 360ab^8x^5 + 60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**7,x)
```

```
[Out] (-a**2 - 6*a*b*x - 15*b**2*x**2)/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4
*b**5*x**2 + 1200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 6
0*b**9*x**6)
```

Giac [A]
time = 1.46, size = 29, normalized size = 0.62

$$-\frac{15\,b^2x^2 + 6\,abx + a^2}{60\,(bx + a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/((b*x + a)^6*b^3)
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.66

$$-\frac{8\,a^2 + 48\,a\,b\,x + 120\,b^2\,x^2}{480\,b^3\,(a + b\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^7,x)
```

```
[Out] -(8*a^2 + 120*b^2*x^2 + 48*a*b*x)/(480*b^3*(a + b*x)^6)
```

3.216 $\int \frac{x}{(a+bx)^7} \, dx$

Optimal. Leaf size=30

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

```
[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{45\}$

$$\frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^7,x]
```

```
[Out] a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(a+bx)^7} \, dx &= \int \left(-\frac{a}{b(a+bx)^7} + \frac{1}{b(a+bx)^6}\right) dx \\ &= \frac{a}{6b^2(a+bx)^6} - \frac{1}{5b^2(a+bx)^5} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.67

$$-\frac{a+6bx}{30b^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^7,x]
```

```
[Out] -1/30*(a + 6*b*x)/(b^2*(a + b*x)^6)
```

Maple [A]
time = 0.08, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{6bx+a}{30(bx+a)^6 b^2}$	19
norman	$\frac{-\frac{x}{5b}-\frac{a}{30b^2}}{(bx+a)^6}$	22
risch	$\frac{-\frac{x}{5b}-\frac{a}{30b^2}}{(bx+a)^6}$	22
default	$\frac{a}{6b^2(bx+a)^6}-\frac{1}{5b^2(bx+a)^5}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.28, size = 76, normalized size = 2.53

$$-\frac{6\,bx+a}{30\,(b^8x^6+6\,ab^7x^5+15\,a^2b^6x^4+20\,a^3b^5x^3+15\,a^4b^4x^2+6\,a^5b^3x+a^6b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(26) = 52$.
time = 0.81, size = 76, normalized size = 2.53

$$-\frac{6\,bx+a}{30\,(b^8x^6+6\,ab^7x^5+15\,a^2b^6x^4+20\,a^3b^5x^3+15\,a^4b^4x^2+6\,a^5b^3x+a^6b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(26) = 52$.
time = 0.18, size = 80, normalized size = 2.67

$$\frac{-a-6bx}{30a^6b^2+180a^5b^3x+450a^4b^4x^2+600a^3b^5x^3+450a^2b^6x^4+180ab^7x^5+30b^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**7,x)
```

```
[Out] (-a - 6*b*x)/(30*a**6*b**2 + 180*a**5*b**3*x + 450*a**4*b**4*x**2 + 600*a**
3*b**5*x**3 + 450*a**2*b**6*x**4 + 180*a*b**7*x**5 + 30*b**8*x**6)
```

Giac [A]
time = 1.15, size = 18, normalized size = 0.60

$$-\frac{6\,bx+a}{30\,(bx+a)^6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(6*b*x + a)/((b*x + a)^6*b^2)
```

Mupad [B]
time = 0.10, size = 18, normalized size = 0.60

$$-\frac{a+6\,b\,x}{30\,b^2\,(a+b\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^7,x)
```

```
[Out] -(a + 6*b*x)/(30*b^2*(a + b*x)^6)
```

3.217 $\int \frac{1}{(a+bx)^7} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{6b(a+bx)^6}$$

```
[Out] -1/6/b/(b*x+a)^6
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-7),x]
```

```
[Out] -1/6*1/(b*(a + b*x)^6)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^7} \, dx = -\frac{1}{6b(a+bx)^6}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-7),x]
```

```
[Out] -1/6*1/(b*(a + b*x)^6)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{6b(bx+a)^6}$	13
default	$-\frac{1}{6b(bx+a)^6}$	13
norman	$-\frac{1}{6b(bx+a)^6}$	13
risch	$-\frac{1}{6b(bx+a)^6}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b/(b*x+a)^6
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$-\frac{1}{6\,(bx+a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(12) = 24$.
time = 0.81, size = 68, normalized size = 4.86

$$-\frac{1}{6\,(b^7x^6 + 6\,ab^6x^5 + 15\,a^2b^5x^4 + 20\,a^3b^4x^3 + 15\,a^4b^3x^2 + 6\,a^5b^2x + a^6b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b^4*x^3 + 15*a^4*b^3*
x^2 + 6*a^5*b^2*x + a^6*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(12) = 24$.
time = 0.18, size = 73, normalized size = 5.21

$$-\frac{1}{6a^6b + 36a^5b^2x + 90a^4b^3x^2 + 120a^3b^4x^3 + 90a^2b^5x^4 + 36ab^6x^5 + 6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**7,x)
```

```
[Out] -1/(6*a**6*b + 36*a**5*b**2*x + 90*a**4*b**3*x**2 + 120*a**3*b**4*x**3 + 90
*a**2*b**5*x**4 + 36*a*b**6*x**5 + 6*b**7*x**6)
```

Giac [A]
time = 1.50, size = 12, normalized size = 0.86

$$-\frac{1}{6\,(bx+a)^6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

Mupad [B]
time = 0.06, size = 70, normalized size = 5.00

$$-\frac{1}{6\,a^6\,b+36\,a^5\,b^2\,x+90\,a^4\,b^3\,x^2+120\,a^3\,b^4\,x^3+90\,a^2\,b^5\,x^4+36\,a\,b^6\,x^5+6\,b^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^7,x)
```

```
[Out] -1/(6*a^6*b + 6*b^7*x^6 + 36*a^5*b^2*x + 36*a*b^6*x^5 + 90*a^4*b^3*x^2 + 12
0*a^3*b^4*x^3 + 90*a^2*b^5*x^4)
```

3.218 $\int \frac{1}{x(a+bx)^7}\,dx$

Optimal. Leaf size=99

$$\frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx)}+\frac{\log(x)}{a^7}-\frac{\log(a+bx)}{a^7}$$

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^7),x]
```

```
[Out] 1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3*
a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7 -
 Log[a + b*x]/a^7
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x}-\frac{b}{a(a+bx)^7}-\frac{b}{a^2(a+bx)^6}-\frac{b}{a^3(a+bx)^5}-\frac{b}{a^4(a+bx)^4}-\frac{b}{a^5(a+bx)^3}-\frac{}{a^6(}\right.\\ &= \frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 0.82

$$\frac{\frac{a\left(147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5\right)}{(a+bx)^6}+60\log(x)-60\log(a+bx)}{60a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^7),x]
```

```
[Out] ((a*(147*a^5 + 522*a^4*b*x + 855*a^3*b^2*x^2 + 740*a^2*b^3*x^3 + 330*a*b^4*
x^4 + 60*b^5*x^5))/(a + b*x)^6 + 60*Log[x] - 60*Log[a + b*x])/(60*a^7)
```

Maple [A]
time = 0.09, size = 90, normalized size = 0.91

method	result	size
risch	$\frac{\frac{b^5x^5}{a^6}+\frac{11b^4x^4}{2a^5}+\frac{37b^3x^3}{3a^4}+\frac{57b^2x^2}{4a^3}+\frac{87bx}{10a^2}+\frac{49}{20a}}{(bx+a)^6}-\frac{\ln(bx+a)}{a^7}+\frac{\ln(-x)}{a^7}$	85
default	$\frac{1}{6a(bx+a)^6}+\frac{1}{5a^2(bx+a)^5}+\frac{1}{4a^3(bx+a)^4}+\frac{1}{3a^4(bx+a)^3}+\frac{1}{2a^5(bx+a)^2}+\frac{1}{a^6(bx+a)}+\frac{\ln(x)}{a^7}-\frac{\ln(bx+a)}{a^7}$	90
norman	$\frac{-\frac{6bx}{a^2}-\frac{45b^2x^2}{2a^3}-\frac{110b^3x^3}{3a^4}-\frac{125b^4x^4}{4a^5}-\frac{137b^5x^5}{10a^6}-\frac{49b^6x^6}{20a^7}}{(bx+a)^6}+\frac{\ln(x)}{a^7}-\frac{\ln(bx+a)}{a^7}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

Maxima [A]
time = 0.30, size = 139, normalized size = 1.40

$$\frac{60\,b^5x^5+330\,ab^4x^4+740\,a^2b^3x^3+855\,a^3b^2x^2+522\,a^4bx+147\,a^5}{60\,(a^6b^6x^6+6\,a^7b^5x^5+15\,a^8b^4x^4+20\,a^9b^3x^3+15\,a^{10}b^2x^2+6\,a^{11}bx+a^{12})}-\frac{\log(bx+a)}{a^7}+\frac{\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(60*b^5*x^5 + 330*a*b^4*x^4 + 740*a^2*b^3*x^3 + 855*a^3*b^2*x^2 + 522*
a^4*b*x + 147*a^5)/(a^6*b^6*x^6 + 6*a^7*b^5*x^5 + 15*a^8*b^4*x^4 + 20*a^9*b
^3*x^3 + 15*a^10*b^2*x^2 + 6*a^11*b*x + a^12) - log(b*x + a)/a^7 + log(x)/a
^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(89) = 178$.
time = 1.29, size = 256, normalized size = 2.59

$$\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6-60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\log(bx+a)+60\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\log(x)}{60\,(a^7b^6x^6+6\,a^8b^5x^5+15\,a^9b^4x^4+20\,a^{10}b^3x^3+15\,a^{11}b^2x^2+6\,a^{12}bx+a^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 +
522*a^5*b*x + 147*a^6 - 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3
*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a) + 60*(b^6*x^6 + 6
*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x +
 a^6)*log(x))/(a^7*b^6*x^6 + 6*a^8*b^5*x^5 + 15*a^9*b^4*x^4 + 20*a^10*b^3*x
^3 + 15*a^11*b^2*x^2 + 6*a^12*b*x + a^13)
```

Sympy [A]
time = 0.29, size = 141, normalized size = 1.42

$$\frac{147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5}{60a^{12}+360a^{11}bx+900a^{10}b^2x^2+1200a^9b^3x^3+900a^8b^4x^4+360a^7b^5x^5+60a^6b^6x^6}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**7,x)
```

```
[Out] (147*a**5 + 522*a**4*b*x + 855*a**3*b**2*x**2 + 740*a**2*b**3*x**3 + 330*a*
b**4*x**4 + 60*b**5*x**5)/(60*a**12 + 360*a**11*b*x + 900*a**10*b**2*x**2 +
 1200*a**9*b**3*x**3 + 900*a**8*b**4*x**4 + 360*a**7*b**5*x**5 + 60*a**6*b*
*6*x**6) + (log(x) - log(a/b + x))/a**7
```

Giac [A]
time = 1.61, size = 87, normalized size = 0.88

$$-\frac{\log(|bx+a|)}{a^7}+\frac{\log(|x|)}{a^7}+\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6}{60\,(bx+a)^6a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^7 + log(abs(x))/a^7 + 1/60*(60*a*b^5*x^5 + 330*a^2*b^4
*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 + 522*a^5*b*x + 147*a^6)/((b*x + a
)^6*a^7)
```

Mupad [B]
time = 0.45, size = 102, normalized size = 1.03

$$-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{15\,b^2\,x^2}{2\,(a+b\,x)^2}+\frac{20\,b^3\,x^3}{3\,(a+b\,x)^3}-\frac{15\,b^4\,x^4}{4\,(a+b\,x)^4}+\frac{6\,b^5\,x^5}{5\,(a+b\,x)^5}-\frac{b^6\,x^6}{6\,(a+b\,x)^6}+\frac{6\,b\,x}{a+b\,x}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^7),x)
```

```
[Out] -(log((a + b*x)/x) - (15*b^2*x^2)/(2*(a + b*x)^2) + (20*b^3*x^3)/(3*(a + b*
x)^3) - (15*b^4*x^4)/(4*(a + b*x)^4) + (6*b^5*x^5)/(5*(a + b*x)^5) - (b^6*x
^6)/(6*(a + b*x)^6) + (6*b*x)/(a + b*x))/a^7
```

3.219 $\int \frac{1}{x^2(a+bx)^7} dx$

Optimal. Leaf size=117

$$-\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-\frac{6b}{a^7(a+bx)}-\frac{7b\log(x)}{a^8}+$$

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

Rubi [A]
time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}-\frac{6b}{a^7(a+bx)}-\frac{1}{a^7x}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(4
*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2) -
 (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x^2}-\frac{7b}{a^8x}+\frac{b^2}{a^2(a+bx)^7}+\frac{2b^2}{a^3(a+bx)^6}+\frac{3b^2}{a^4(a+bx)^5}+\frac{4b^2}{a^5(a+bx)^4}+\frac{5b^2}{a^6(a+\right.\\ &= -\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.83

$$-\frac{\frac{a(60a^6+1029a^5bx+3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+2310ab^5x^5+420b^6x^6)}{x(a+bx)^6}+420b\log(x)-420b\log(a+bx)}{60a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -1/60*((a*(60*a^6 + 1029*a^5*b*x + 3654*a^4*b^2*x^2 + 5985*a^3*b^3*x^3 + 51
80*a^2*b^4*x^4 + 2310*a*b^5*x^5 + 420*b^6*x^6))/(x*(a + b*x)^6) + 420*b*Log
[x] - 420*b*Log[a + b*x])/a^8
```

Maple [A]
time = 0.09, size = 108, normalized size = 0.92

method	result
risch	$\frac{-\frac{7b^6x^6}{a^7}-\frac{77b^5x^5}{2a^6}-\frac{259b^4x^4}{3a^5}-\frac{399b^3x^3}{4a^4}-\frac{609b^2x^2}{10a^3}-\frac{343bx}{20a^2}-\frac{1}{a}}{x(bx+a)^6}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(-bx-a)}{a^8}$
norman	$\frac{-\frac{1}{a}+\frac{42b^2x^2}{a^3}+\frac{315b^3x^3}{2a^4}+\frac{770b^4x^4}{3a^5}+\frac{875b^5x^5}{4a^6}+\frac{959b^6x^6}{10a^7}+\frac{343b^7x^7}{20a^8}}{x(bx+a)^6}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(bx+a)}{a^8}$
default	$-\frac{1}{a^7x}-\frac{b}{6a^2(bx+a)^6}-\frac{2b}{5a^3(bx+a)^5}-\frac{3b}{4a^4(bx+a)^4}-\frac{4b}{3a^5(bx+a)^3}-\frac{5b}{2a^6(bx+a)^2}-\frac{6b}{a^7(bx+a)}-\frac{7b\ln(x)}{a^8}+\frac{7b\ln(bx+a)}{a^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

Maxima [A]
time = 0.29, size = 157, normalized size = 1.34

$$-\frac{420\,b^6x^6+2310\,ab^5x^5+5180\,a^2b^4x^4+5985\,a^3b^3x^3+3654\,a^4b^2x^2+1029\,a^5bx+60\,a^6}{60\,(a^7b^6x^7+6\,a^8b^5x^6+15\,a^9b^4x^5+20\,a^{10}b^3x^4+15\,a^{11}b^2x^3+6\,a^{12}bx^2+a^{13}x)}+\frac{7\,b\log(bx+a)}{a^8}-\frac{7\,b\log(x)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(420*b^6*x^6 + 2310*a*b^5*x^5 + 5180*a^2*b^4*x^4 + 5985*a^3*b^3*x^3 +
 3654*a^4*b^2*x^2 + 1029*a^5*b*x + 60*a^6)/(a^7*b^6*x^7 + 6*a^8*b^5*x^6 + 1
5*a^9*b^4*x^5 + 20*a^10*b^3*x^4 + 15*a^11*b^2*x^3 + 6*a^12*b*x^2 + a^13*x)
+ 7*b*log(b*x + a)/a^8 - 7*b*log(x)/a^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(107) = 214$.
time = 1.23, size = 285, normalized size = 2.44

$$-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7-420\,(b^7x^7+6\,ab^6x^6+15\,a^2b^5x^5+20\,a^3b^4x^4+15\,a^4b^3x^3+6\,a^5b^2x^2+a^6bx)\log(bx+a)+420\,(b^7x^7+6\,ab^6x^6+15\,a^2b^5x^5+20\,a^3b^4x^4+15\,a^4b^3x^3+6\,a^5b^2x^2+a^6bx)\log(x)}{60\,(a^8b^6x^7+6\,a^9b^5x^6+15\,a^{10}b^4x^5+20\,a^{11}b^3x^4+15\,a^{12}b^2x^3+6\,a^{13}bx^2+a^{14}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(420*a*b^6*x^6 + 2310*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x
^3 + 3654*a^5*b^2*x^2 + 1029*a^6*b*x + 60*a^7 - 420*(b^7*x^7 + 6*a*b^6*x^6
+ 15*a^2*b^5*x^5 + 20*a^3*b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*
x)*log(b*x + a) + 420*(b^7*x^7 + 6*a*b^6*x^6 + 15*a^2*b^5*x^5 + 20*a^3*b^4*
x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*x)*log(x))/(a^8*b^6*x^7 + 6*a^
9*b^5*x^6 + 15*a^10*b^4*x^5 + 20*a^11*b^3*x^4 + 15*a^12*b^2*x^3 + 6*a^13*b*
x^2 + a^14*x)
```

Sympy [A]
time = 0.33, size = 162, normalized size = 1.38

$$\frac{-60a^6-1029a^5bx-3654a^4b^2x^2-5985a^3b^3x^3-5180a^2b^4x^4-2310ab^5x^5-420b^6x^6}{60a^{13}x+360a^{12}bx^2+900a^{11}b^2x^3+1200a^{10}b^3x^4+900a^9b^4x^5+360a^8b^5x^6+60a^7b^6x^7}+\frac{7b\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**7,x)
```

```
[Out] (-60*a**6 - 1029*a**5*b*x - 3654*a**4*b**2*x**2 - 5985*a**3*b**3*x**3 - 518
0*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 420*b**6*x**6)/(60*a**13*x + 360*a**1
2*b*x**2 + 900*a**11*b**2*x**3 + 1200*a**10*b**3*x**4 + 900*a**9*b**4*x**5
+ 360*a**8*b**5*x**6 + 60*a**7*b**6*x**7) + 7*b*(-log(x) + log(a/b + x))/a*
*8
```

Giac [A]
time = 1.77, size = 104, normalized size = 0.89

$$\frac{7\,b\log\left(|bx+a|\right)}{a^8}-\frac{7\,b\log\left(|x|\right)}{a^8}-\frac{420\,ab^6x^6+2310\,a^2b^5x^5+5180\,a^3b^4x^4+5985\,a^4b^3x^3+3654\,a^5b^2x^2+1029\,a^6bx+60\,a^7}{60\,(bx+a)^6a^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 7*b*log(abs(b*x + a))/a^8 - 7*b*log(abs(x))/a^8 - 1/60*(420*a*b^6*x^6 + 231
0*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x^3 + 3654*a^5*b^2*x^2 + 10
29*a^6*b*x + 60*a^7)/((b*x + a)^6*a^8*x)
```

Mupad [B]
time = 0.19, size = 151, normalized size = 1.29

$$\frac{14\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8}-\frac{\frac{1}{a}+\frac{609\,b^2\,x^2}{10\,a^3}+\frac{399\,b^3\,x^3}{4\,a^4}+\frac{259\,b^4\,x^4}{3\,a^5}+\frac{77\,b^5\,x^5}{2\,a^6}+\frac{7\,b^6\,x^6}{a^7}+\frac{343\,b\,x}{20\,a^2}}{a^6\,x+6\,a^5\,b\,x^2+15\,a^4\,b^2\,x^3+20\,a^3\,b^3\,x^4+15\,a^2\,b^4\,x^5+6\,a\,b^5\,x^6+b^6\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^7),x)
```

```
[Out] (14*b*atanh((2*b*x)/a + 1))/a^8 - (1/a + (609*b^2*x^2)/(10*a^3) + (399*b^3*
x^3)/(4*a^4) + (259*b^4*x^4)/(3*a^5) + (77*b^5*x^5)/(2*a^6) + (7*b^6*x^6)/a
^7 + (343*b*x)/(20*a^2))/(a^6*x + b^6*x^7 + 6*a^5*b*x^2 + 6*a*b^5*x^6 + 15*
a^4*b^2*x^3 + 20*a^3*b^3*x^4 + 15*a^2*b^4*x^5)
```

3.220 $\int \frac{1}{x^3(a+bx)^7}\,dx$

Optimal. Leaf size=144

$$-\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15b^2}{2a^7(a+bx)^2}+\frac{21b^2}{a^8(a+bx)}+\frac{28b^2 \mathrm{l}}{a}$$

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{21b^2}{a^8(a+bx)}+\frac{7b}{a^8x}+\frac{15b^2}{2a^7(a+bx)^2}-\frac{1}{2a^7x^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^7),x]
```

```
[Out] -1/2*1/(a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^4
*(a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3)
+ (15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[x])
/a^9 - (28*b^2*Log[a + b*x])/a^9
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^7}\,dx = \int \left(\frac{1}{a^7x^3}-\frac{7b}{a^8x^2}+\frac{28b^2}{a^9x}-\frac{b^3}{a^3(a+bx)^7}-\frac{3b^3}{a^4(a+bx)^6}-\frac{6b^3}{a^5(a+bx)^5}-\frac{10b^3}{a^6(a+bx)^4}-\right.$$
$$= -\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15}{2a^7(a}$$

Mathematica [A]
time = 0.05, size = 112, normalized size = 0.78

$$\frac{\frac{a(-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7)}{x^2(a+bx)^6}+840b^2\log(x)-840b^2\log(a+bx)}{30a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^7),x]
```

```
[Out] ((a*(-15*a^7 + 120*a^6*b*x + 2058*a^5*b^2*x^2 + 7308*a^4*b^3*x^3 + 11970*a^
3*b^4*x^4 + 10360*a^2*b^5*x^5 + 4620*a*b^6*x^6 + 840*b^7*x^7))/(x^2*(a + b*
x)^6) + 840*b^2*Log[x] - 840*b^2*Log[a + b*x])/(30*a^9)
```

Maple [A]
time = 0.09, size = 133, normalized size = 0.92

method	result
norman	$\frac{-\frac{1}{2a}+\frac{4bx}{a^2}-\frac{168b^3x^3}{a^4}-\frac{630b^4x^4}{a^5}-\frac{3080b^5x^5}{3a^6}-\frac{875b^6x^6}{a^7}-\frac{1918b^7x^7}{5a^8}-\frac{343b^8x^8}{5a^9}}{x^2(bx+a)^6}+\frac{28b^2\ln(x)}{a^9}-\frac{28b^2\ln(bx+a)}{a^9}$
risch	$\frac{\frac{28b^7x^7}{a^8}+\frac{154b^6x^6}{a^7}+\frac{1036b^5x^5}{3a^6}+\frac{399b^4x^4}{a^5}+\frac{1218b^3x^3}{5a^4}+\frac{343b^2x^2}{5a^3}+\frac{4bx}{a^2}-\frac{1}{2a}}{x^2(bx+a)^6}-\frac{28b^2\ln(bx+a)}{a^9}+\frac{28b^2\ln(-x)}{a^9}$
default	$-\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(bx+a)^6}+\frac{3b^2}{5a^4(bx+a)^5}+\frac{3b^2}{2a^5(bx+a)^4}+\frac{10b^2}{3a^6(bx+a)^3}+\frac{15b^2}{2a^7(bx+a)^2}+\frac{21b^2}{a^8(bx+a)}+\frac{28b^2\ln(x)}{a^9}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

Maxima [A]
time = 0.37, size = 174, normalized size = 1.21

$$\frac{840\,b^7x^7+4620\,ab^6x^6+10360\,a^2b^5x^5+11970\,a^3b^4x^4+7308\,a^4b^3x^3+2058\,a^5b^2x^2+120\,a^6bx-15\,a^7}{30\,(a^8b^6x^8+6\,a^9b^5x^7+15\,a^{10}b^4x^6+20\,a^{11}b^3x^5+15\,a^{12}b^2x^4+6\,a^{13}bx^3+a^{14}x^2)}-\frac{28\,b^2\log(bx+a)}{a^9}+\frac{28\,b^2\log(x)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(840*b^7*x^7 + 4620*a*b^6*x^6 + 10360*a^2*b^5*x^5 + 11970*a^3*b^4*x^4
+ 7308*a^4*b^3*x^3 + 2058*a^5*b^2*x^2 + 120*a^6*b*x - 15*a^7)/(a^8*b^6*x^8
+ 6*a^9*b^5*x^7 + 15*a^10*b^4*x^6 + 20*a^11*b^3*x^5 + 15*a^12*b^2*x^4 + 6*a
^13*b*x^3 + a^14*x^2) - 28*b^2*log(b*x + a)/a^9 + 28*b^2*log(x)/a^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(132) = 264.
time = 1.21, size = 306, normalized size = 2.12

$$\frac{840\,ab^7x^7+4620\,a^2b^6x^6+10360\,a^3b^5x^5+11970\,a^4b^4x^4+7308\,a^5b^3x^3+2058\,a^6b^2x^2+120\,a^7bx-15\,a^8-840\,(b^8x^8+6\,ab^7x^7+15\,a^2b^6x^6+20\,a^3b^5x^5+15\,a^4b^4x^4+6\,a^5b^3x^3+a^6b^2x^2)\log(bx+a)+840\,(b^8x^8+6\,ab^7x^7+15\,a^2b^6x^6+20\,a^3b^5x^5+15\,a^4b^4x^4+6\,a^5b^3x^3+a^6b^2x^2)\log(x)}{30\,(a^9b^6x^8+6\,a^{10}b^5x^7+15\,a^{11}b^4x^6+20\,a^{12}b^3x^5+15\,a^{13}b^2x^4+6\,a^{14}bx^3+a^{15}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="fricas")

[Out] 1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8 - 840*(b^8*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*x^2)*log(b*x + a) + 840*(b^8*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*x^2)*log(x))/(a^9*b^6*x^8 + 6*a^10*b^5*x^7 + 15*a^11*b^4*x^6 + 20*a^12*b^3*x^5 + 15*a^13*b^2*x^4 + 6*a^14*b*x^3 + a^15*x^2)

Sympy [A]
time = 0.35, size = 175, normalized size = 1.22

$$\frac{-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7}{30a^{14}x^2+180a^{13}bx^3+450a^{12}b^2x^4+600a^{11}b^3x^5+450a^{10}b^4x^6+180a^9b^5x^7+30a^8b^6x^8}+\frac{28b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(b*x+a)**7,x)

[Out] (-15*a**7 + 120*a**6*b*x + 2058*a**5*b**2*x**2 + 7308*a**4*b**3*x**3 + 11970*a**3*b**4*x**4 + 10360*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 840*b**7*x**7)/(30*a**14*x**2 + 180*a**13*b*x**3 + 450*a**12*b**2*x**4 + 600*a**11*b**3*x**5 + 450*a**10*b**4*x**6 + 180*a**9*b**5*x**7 + 30*a**8*b**6*x**8) + 28*b**2*(log(x) - log(a/b + x))/a**9

Giac [A]
time = 1.37, size = 119, normalized size = 0.83

$$-\frac{28\,b^2\log\left(|bx+a|\right)}{a^9}+\frac{28\,b^2\log\left(|x|\right)}{a^9}+\frac{840\,ab^7x^7+4620\,a^2b^6x^6+10360\,a^3b^5x^5+11970\,a^4b^4x^4+7308\,a^5b^3x^3+2058\,a^6b^2x^2+120\,a^7bx-15\,a^8}{30\,(bx+a)^6a^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="giac")

[Out] -28*b^2*log(abs(b*x + a))/a^9 + 28*b^2*log(abs(x))/a^9 + 1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8)/((b*x + a)^6*a^9*x^2)

Mupad [B]
time = 0.21, size = 167, normalized size = 1.16

$$\frac{\frac{343\,b^2\,x^2}{5\,a^3}-\frac{1}{2\,a}+\frac{1218\,b^3\,x^3}{5\,a^4}+\frac{399\,b^4\,x^4}{a^5}+\frac{1036\,b^5\,x^5}{3\,a^6}+\frac{154\,b^6\,x^6}{a^7}+\frac{28\,b^7\,x^7}{a^8}+\frac{4\,b\,x}{a^2}}{a^6\,x^2+6\,a^5\,b\,x^3+15\,a^4\,b^2\,x^4+20\,a^3\,b^3\,x^5+15\,a^2\,b^4\,x^6+6\,a\,b^5\,x^7+b^6\,x^8}-\frac{56\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*x)^7),x)

[Out] ((343*b^2*x^2)/(5*a^3) - 1/(2*a) + (1218*b^3*x^3)/(5*a^4) + (399*b^4*x^4)/a^5 + (1036*b^5*x^5)/(3*a^6) + (154*b^6*x^6)/a^7 + (28*b^7*x^7)/a^8 + (4*b*x)/a^2)/(a^6*x^2 + b^6*x^8 + 6*a^5*b*x^3 + 6*a*b^5*x^7 + 15*a^4*b^2*x^4 + 20*a^3*b^3*x^5 + 15*a^2*b^4*x^6) - (56*b^2*atanh((2*b*x)/a + 1))/a^9

3.221 $\int \frac{1}{x^4(a+bx)^7} \, dx$

Optimal. Leaf size=157

$$-\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{35b^3}{2a^8(a+bx)^2}-\frac{56b^3}{a^9(a+bx)}$$

```
[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(
b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^
2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Rubi [A]
time = 0.08, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}-\frac{56b^3}{a^9(a+bx)}-\frac{28b^2}{a^9x}-\frac{35b^3}{2a^8(a+bx)^2}+\frac{7b}{2a^8x^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{1}{3a^7x^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/3*1/(a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a + b
*x)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*b^
3)/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a +
b*x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^7} \, dx &= \int \left(\frac{1}{a^7x^4}-\frac{7b}{a^8x^3}+\frac{28b^2}{a^9x^2}-\frac{84b^3}{a^{10}x}+\frac{b^4}{a^4(a+bx)^7}+\frac{4b^4}{a^5(a+bx)^6}+\frac{10b^4}{a^6(a+bx)^5}+\frac{20}{a^7(a+\right.\\ &= -\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 123, normalized size = 0.78

$$-\frac{\frac{a(10a^8-45a^7bx+360a^6b^2x^2+6174a^5b^3x^3+21924a^4b^4x^4+35910a^3b^5x^5+31080a^2b^6x^6+13860ab^7x^7+2520b^8x^8)}{x^3(a+bx)^6}+2520b^3\log(x)-2520b^3\log(a+bx)}{30a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/30*((a*(10*a^8 - 45*a^7*b*x + 360*a^6*b^2*x^2 + 6174*a^5*b^3*x^3 + 21924
*a^4*b^4*x^4 + 35910*a^3*b^5*x^5 + 31080*a^2*b^6*x^6 + 13860*a*b^7*x^7 + 25
20*b^8*x^8))/(x^3*(a + b*x)^6) + 2520*b^3*Log[x] - 2520*b^3*Log[a + b*x])/a
^10
```

Maple [A]
time = 0.11, size = 144, normalized size = 0.92

method	result
norman	$\frac{-\frac{1}{3a}+\frac{3bx}{2a^2}-\frac{12b^2x^2}{a^3}+\frac{504b^4x^4}{a^5}+\frac{1890b^5x^5}{a^6}+\frac{3080b^6x^6}{a^7}+\frac{2625b^7x^7}{a^8}+\frac{5754b^8x^8}{5a^9}+\frac{1029b^9x^9}{5a^{10}}}{x^3(bx+a)^6}-\frac{84b^3\ln(x)}{a^{10}}+\frac{84b^3\ln(bx+a)}{a^{10}}$
risch	$\frac{-\frac{84b^8x^8}{a^9}-\frac{462b^7x^7}{a^8}-\frac{1036b^6x^6}{a^7}-\frac{1197b^5x^5}{a^6}-\frac{3654b^4x^4}{5a^5}-\frac{1029b^3x^3}{5a^4}-\frac{12b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{3a}}{x^3(bx+a)^6}+\frac{84b^3\ln(-bx-a)}{a^{10}}-\frac{84b^3\ln(x)}{a^{10}}$
default	$-\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(bx+a)^6}-\frac{4b^3}{5a^5(bx+a)^5}-\frac{5b^3}{2a^6(bx+a)^4}-\frac{20b^3}{3a^7(bx+a)^3}-\frac{35b^3}{2a^8(bx+a)^2}-\frac{56b^3}{a^9(bx+a)}-84$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(
b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^
2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10
```

Maxima [A]
time = 0.50, size = 185, normalized size = 1.18

$$-\frac{2520\,b^8x^8+13860\,ab^7x^7+31080\,a^2b^6x^6+35910\,a^3b^5x^5+21924\,a^4b^4x^4+6174\,a^5b^3x^3+360\,a^6b^2x^2-45\,a^7bx+10\,a^8}{30\,(a^9b^6x^9+6\,a^{10}b^5x^8+15\,a^{11}b^4x^7+20\,a^{12}b^3x^6+15\,a^{13}b^2x^5+6\,a^{14}bx^4+a^{15}x^3)}+\frac{84\,b^3\log(bx+a)}{a^{10}}-\frac{84\,b^3\log(x)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(2520*b^8*x^8 + 13860*a*b^7*x^7 + 31080*a^2*b^6*x^6 + 35910*a^3*b^5*x
^5 + 21924*a^4*b^4*x^4 + 6174*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 45*a^7*b*x +
10*a^8)/(a^9*b^6*x^9 + 6*a^10*b^5*x^8 + 15*a^11*b^4*x^7 + 20*a^12*b^3*x^6 +
 15*a^13*b^2*x^5 + 6*a^14*b*x^4 + a^15*x^3) + 84*b^3*log(b*x + a)/a^10 - 84
*b^3*log(x)/a^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(143) = 286.
time = 1.10, size = 317, normalized size = 2.02

$$-\frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2-45\,a^8bx+10\,a^9-2520\,(b^9x^9+6\,ab^8x^8+15\,a^2b^7x^7+20\,a^3b^6x^6+15\,a^4b^5x^5+6\,a^5b^4x^4+a^6b^3x^3)\log(bx+a)+2520\,(b^9x^9+6\,ab^8x^8+15\,a^2b^7x^7+20\,a^3b^6x^6+15\,a^4b^5x^5+6\,a^5b^4x^4+a^6b^3x^3)\log(x)}{30\,(a^{10}b^6x^9+6\,a^{11}b^5x^8+15\,a^{12}b^4x^7+20\,a^{13}b^3x^6+15\,a^{14}b^2x^5+6\,a^{15}bx^4+a^{16}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b
^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*
x + 10*a^9 - 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6
+ 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(b*x + a) + 2520*(b^9*x^
9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6*a^5*
b^4*x^4 + a^6*b^3*x^3)*log(x))/(a^10*b^6*x^9 + 6*a^11*b^5*x^8 + 15*a^12*b^4
*x^7 + 20*a^13*b^3*x^6 + 15*a^14*b^2*x^5 + 6*a^15*b*x^4 + a^16*x^3)
```

Sympy [A]
time = 0.36, size = 187, normalized size = 1.19

$$\frac{-10a^8+45a^7bx-360a^6b^2x^2-6174a^5b^3x^3-21924a^4b^4x^4-35910a^3b^5x^5-31080a^2b^6x^6-13860ab^7x^7-2520b^8x^8}{30a^{15}x^3+180a^{14}bx^4+450a^{13}b^2x^5+600a^{12}b^3x^6+450a^{11}b^4x^7+180a^{10}b^5x^8+30a^9b^6x^9}+\frac{84b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**7,x)
```

```
[Out] (-10*a**8 + 45*a**7*b*x - 360*a**6*b**2*x**2 - 6174*a**5*b**3*x**3 - 21924*
a**4*b**4*x**4 - 35910*a**3*b**5*x**5 - 31080*a**2*b**6*x**6 - 13860*a*b**7
*x**7 - 2520*b**8*x**8)/(30*a**15*x**3 + 180*a**14*b*x**4 + 450*a**13*b**2*
x**5 + 600*a**12*b**3*x**6 + 450*a**11*b**4*x**7 + 180*a**10*b**5*x**8 + 30
*a**9*b**6*x**9) + 84*b**3*(-log(x) + log(a/b + x))/a**10
```

Giac [A]
time = 1.86, size = 130, normalized size = 0.83

$$\frac{84\,b^3\log\left(|bx+a|\right)}{a^{10}}-\frac{84\,b^3\log\left(|x|\right)}{a^{10}}-\frac{2520\,ab^8x^8+13860\,a^2b^7x^7+31080\,a^3b^6x^6+35910\,a^4b^5x^5+21924\,a^5b^4x^4+6174\,a^6b^3x^3+360\,a^7b^2x^2-45\,a^8bx+10\,a^9}{30\,(bx+a)^6a^{10}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 84*b^3*log(abs(b*x + a))/a^10 - 84*b^3*log(abs(x))/a^10 - 1/30*(2520*a*b^8*
x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a^5
*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9)/((b*x
+ a)^6*a^10*x^3)
```

Mupad [B]
time = 0.31, size = 179, normalized size = 1.14

$$\frac{168\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{10}}-\frac{\frac{1}{3\,a}+\frac{12\,b^2\,x^2}{a^3}+\frac{1029\,b^3\,x^3}{5\,a^4}+\frac{3654\,b^4\,x^4}{5\,a^5}+\frac{1197\,b^5\,x^5}{a^6}+\frac{1036\,b^6\,x^6}{a^7}+\frac{462\,b^7\,x^7}{a^8}+\frac{84\,b^8\,x^8}{a^9}-\frac{3\,b\,x}{2\,a^2}}{a^6\,x^3+6\,a^5\,b\,x^4+15\,a^4\,b^2\,x^5+20\,a^3\,b^3\,x^6+15\,a^2\,b^4\,x^7+6\,a\,b^5\,x^8+b^6\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^7),x)
```

```
[Out] (168*b^3*atanh((2*b*x)/a + 1))/a^10 - (1/(3*a) + (12*b^2*x^2)/a^3 + (1029*b
^3*x^3)/(5*a^4) + (3654*b^4*x^4)/(5*a^5) + (1197*b^5*x^5)/a^6 + (1036*b^6*x
^6)/a^7 + (462*b^7*x^7)/a^8 + (84*b^8*x^8)/a^9 - (3*b*x)/(2*a^2))/(a^6*x^3
+ b^6*x^9 + 6*a^5*b*x^4 + 6*a*b^5*x^8 + 15*a^4*b^2*x^5 + 20*a^3*b^3*x^6 + 1
5*a^2*b^4*x^7)
```

3.222 $\int \frac{x^{12}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=186

$$\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+b}$$

```
[Out] 55*a^2*x/b^12-5*a*x^2/b^11+1/3*x^3/b^10-1/9*a^12/b^13/(b*x+a)^9+3/2*a^11/b^
13/(b*x+a)^8-66/7*a^10/b^13/(b*x+a)^7+110/3*a^9/b^13/(b*x+a)^6-99*a^8/b^13/
(b*x+a)^5+198*a^7/b^13/(b*x+a)^4-308*a^6/b^13/(b*x+a)^3+396*a^5/b^13/(b*x+a
)^2-495*a^4/b^13/(b*x+a)-220*a^3*ln(b*x+a)/b^13
```

Rubi [A]

time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{396a^5}{b^{13}(a+bx)^2}-\frac{495a^4}{b^{13}(a+bx)}-\frac{220a^3\log(a+bx)}{b^{13}}+\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x)^10,x]
```

```
[Out] (55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9)
 + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*a^
9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*(a
+ b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2) - (
495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{12}}{(a+bx)^{10}}\,dx &= \int \left(\frac{55a^2}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{b^{10}}+\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220a^9}{b^{12}(a+bx)^7}\right.\\ &= \frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 161, normalized size = 0.87

$$-\frac{35201a^{12}+289089a^{11}bx+1031616a^{10}b^2x^2+2074464a^9b^3x^3+2529576a^8b^4x^4+1831032a^7b^5x^5+638568a^6b^6x^6-58968a^5b^7x^7-139482a^4b^8x^8-43218a^3b^9x^9-2772a^2b^{10}x^{10}+252ab^{11}x^{11}-42b^{12}x^{12}+27720a^3(a+bx)^9\log(a+bx)}{126b^{13}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x)^10,x]
```

```
[Out] -1/126*(35201*a^12 + 289089*a^11*b*x + 1031616*a^10*b^2*x^2 + 2074464*a^9*b
^3*x^3 + 2529576*a^8*b^4*x^4 + 1831032*a^7*b^5*x^5 + 638568*a^6*b^6*x^6 - 5
8968*a^5*b^7*x^7 - 139482*a^4*b^8*x^8 - 43218*a^3*b^9*x^9 - 2772*a^2*b^10*x
^10 + 252*a*b^11*x^11 - 42*b^12*x^12 + 27720*a^3*(a + b*x)^9*Log[a + b*x])/
(b^13*(a + b*x)^9)
```

Maple [A]
time = 0.09, size = 177, normalized size = 0.95

method	result
risch	$\frac{x^3}{3b^{10}}-\frac{5a\,x^2}{b^{11}}+\frac{55a^2x}{b^{12}}+\frac{-495a^4b^7x^8-3564a^5b^6x^7-11396a^6b^5x^6-21054a^7b^4x^5-24519a^8b^3x^4-\frac{55198a^9b^2x^3}{3}-\frac{60742a^{10}b\,x^2}{7}-328}{b^{12}(bx+a)^9}$
norman	$\frac{\frac{x^{12}}{3b}-\frac{2a\,x^{11}}{b^2}-\frac{78419a^{12}}{126b^{13}}+\frac{22a^2x^{10}}{b^3}-\frac{1980a^4x^8}{b^5}-\frac{11880a^5x^7}{b^6}-\frac{33880a^6x^6}{b^7}-\frac{57750a^7x^5}{b^8}-\frac{63294a^8x^4}{b^9}-\frac{45276a^9x^3}{b^{10}}-\frac{143748a^{10}x^2}{7b^{11}}-\frac{75339a^{11}x}{14b^{12}}}{(bx+a)^9}-$
default	$\frac{\frac{1}{3}b^2x^3-5ab\,x^2+55a^2x}{b^{12}}-\frac{495a^4}{b^{13}(bx+a)}+\frac{198a^7}{b^{13}(bx+a)^4}-\frac{66a^{10}}{7b^{13}(bx+a)^7}-\frac{a^{12}}{9b^{13}(bx+a)^9}+\frac{396a^5}{b^{13}(bx+a)^2}+\frac{110a^9}{3b^{13}(bx+a)^6}+\frac{3a}{2b^{13}(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^12*(1/3*b^2*x^3-5*a*b*x^2+55*a^2*x)-495*a^4/b^13/(b*x+a)+198*a^7/b^13/(
b*x+a)^4-66/7*a^10/b^13/(b*x+a)^7-1/9*a^12/b^13/(b*x+a)^9+396*a^5/b^13/(b*x
+a)^2+110/3*a^9/b^13/(b*x+a)^6+3/2*a^11/b^13/(b*x+a)^8-99*a^8/b^13/(b*x+a)^
5-220*a^3*ln(b*x+a)/b^13-308*a^6/b^13/(b*x+a)^3
```

Maxima [A]
time = 0.34, size = 234, normalized size = 1.26

$$-\frac{62370\,a^4b^8x^8+449064\,a^5b^7x^7+1435896\,a^6b^6x^6+2652804\,a^7b^5x^5+3089394\,a^8b^4x^4+2318316\,a^9b^3x^3+1093356\,a^{10}b^2x^2+296019\,a^{11}bx+35201\,a^{12}}{126\,(b^{22}x^9+9\,ab^{21}x^8+36\,a^2b^{20}x^7+84\,a^3b^{19}x^6+126\,a^4b^{18}x^5+126\,a^5b^{17}x^4+84\,a^6b^{16}x^3+36\,a^7b^{15}x^2+9\,a^8b^{14}x+a^9b^{13})}-\frac{220\,a^3\log(bx+a)}{b^{13}}+\frac{b^2x^3-15\,abx^2+165\,a^2x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 2652
804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^10*
b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^2*b
^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^6*b^
```

```
16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13) - 220*a^3*log(b*x + a)/
b^13 + 1/3*(b^2*x^3 - 15*a*b*x^2 + 165*a^2*x)/b^12
```

Fricas [A]
time = 0.75, size = 338, normalized size = 1.82

$$\frac{42 b^{12}x^{12} - 252 ab^{11}x^{11} + 2772 a^2b^{10}x^{10} + 43218 a^3b^9x^9 + 139482 a^4b^8x^8 + 58968 a^5b^7x^7 - 638568 a^6b^6x^6 - 1831032 a^7b^5x^5 - 2529576 a^8b^4x^4 - 2074464 a^9b^3x^3 - 1031616 a^{10}b^2x^2 - 289089 a^{11}bx - 35201 a^{12} - 27720 (a^3b^9x^9 + 9 a^4b^8x^8 + 36 a^5b^7x^7 + 84 a^6b^6x^6 + 126 a^7b^5x^5 + 126 a^8b^4x^4 + 84 a^9b^3x^3 + 36 a^{10}b^2x^2 + 9 a^{11}bx + a^{12}) \log (bx + a)}{126 (b^{22}x^9 + 9 ab^{21}x^8 + 36 a^2b^{20}x^7 + 84 a^3b^{19}x^6 + 126 a^4b^{18}x^5 + 126 a^5b^{17}x^4 + 84 a^6b^{16}x^3 + 36 a^7b^{15}x^2 + 9 a^8b^{14}x + a^9b^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 - 252*a*b^11*x^11 + 2772*a^2*b^10*x^10 + 43218*a^3*b^9*
x^9 + 139482*a^4*b^8*x^8 + 58968*a^5*b^7*x^7 - 638568*a^6*b^6*x^6 - 1831032
*a^7*b^5*x^5 - 2529576*a^8*b^4*x^4 - 2074464*a^9*b^3*x^3 - 1031616*a^10*b^2
*x^2 - 289089*a^11*b*x - 35201*a^12 - 27720*(a^3*b^9*x^9 + 9*a^4*b^8*x^8 +
36*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 126*a^7*b^5*x^5 + 126*a^8*b^4*x^4 + 84*a^
9*b^3*x^3 + 36*a^10*b^2*x^2 + 9*a^11*b*x + a^12)*log(b*x + a))/(b^22*x^9 +
9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a
^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13)
```

Sympy [A]
time = 0.65, size = 250, normalized size = 1.34

$$-\frac{220a^3 \log (a + bx)}{b^{13}} + \frac{55a^2x}{b^{12}} - \frac{5ax^2}{b^{11}} + \frac{-35201a^{12} - 296019a^{11}bx - 1093356a^{10}b^2x^2 - 2318316a^9b^3x^3 - 3089394a^8b^4x^4 - 2652804a^7b^5x^5 - 1435896a^6b^6x^6 - 449064a^5b^7x^7 - 62370a^4b^8x^8}{126a^9b^{13} + 1134a^8b^{14}x + 4536a^7b^{15}x^2 + 10584a^6b^{16}x^3 + 15876a^5b^{17}x^4 + 15876a^4b^{18}x^5 + 10584a^3b^{19}x^6 + 4536a^2b^{20}x^7 + 1134ab^{21}x^8 + 126b^{22}x^9} + \frac{x^3}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x+a)**10,x)
```

```
[Out] -220*a**3*log(a + b*x)/b**13 + 55*a**2*x/b**12 - 5*a*x**2/b**11 + (-35201*a
**12 - 296019*a**11*b*x - 1093356*a**10*b**2*x**2 - 2318316*a**9*b**3*x**3
- 3089394*a**8*b**4*x**4 - 2652804*a**7*b**5*x**5 - 1435896*a**6*b**6*x**6
- 449064*a**5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*a**9*b**13 + 1134*a**8
*b**14*x + 4536*a**7*b**15*x**2 + 10584*a**6*b**16*x**3 + 15876*a**5*b**17*
x**4 + 15876*a**4*b**18*x**5 + 10584*a**3*b**19*x**6 + 4536*a**2*b**20*x**7
 + 1134*a*b**21*x**8 + 126*b**22*x**9) + x**3/(3*b**10)
```

Giac [A]
time = 1.18, size = 149, normalized size = 0.80

$$-\frac{220 a^3 \log (|bx + a|)}{b^{13}} - \frac{62370 a^4b^8x^8 + 449064 a^5b^7x^7 + 1435896 a^6b^6x^6 + 2652804 a^7b^5x^5 + 3089394 a^8b^4x^4 + 2318316 a^9b^3x^3 + 1093356 a^{10}b^2x^2 + 296019 a^{11}bx + 35201 a^{12}}{126 (bx + a)^9 b^{13}} + \frac{b^{20}x^3 - 15 ab^{19}x^2 + 165 a^2b^{18}x}{3 b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -220*a^3*log(abs(b*x + a))/b^13 - 1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7
*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 23
```

```
18316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/((
b*x + a)^9*b^13) + 1/3*(b^20*x^3 - 15*a*b^19*x^2 + 165*a^2*b^18*x)/b^30
```

Mupad [B]
time = 0.98, size = 151, normalized size = 0.81

$$-\frac{6\,a\,{\left(a+b\,x\right)}^2-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{495\,a^4}{a+b\,x}-\frac{396\,a^5}{{\left(a+b\,x\right)}^2}+\frac{308\,a^6}{{\left(a+b\,x\right)}^3}-\frac{198\,a^7}{{\left(a+b\,x\right)}^4}+\frac{99\,a^8}{{\left(a+b\,x\right)}^5}-\frac{110\,a^9}{3\,{\left(a+b\,x\right)}^6}+\frac{66\,a^{10}}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^{11}}{2\,{\left(a+b\,x\right)}^8}+\frac{a^{12}}{9\,{\left(a+b\,x\right)}^9}+220\,a^3\,\ln\left(a+b\,x\right)-66\,a^2\,b\,x}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x)^10,x)
```

```
[Out] -(6*a*(a + b*x)^2 - (a + b*x)^3/3 + (495*a^4)/(a + b*x) - (396*a^5)/(a + b*
x)^2 + (308*a^6)/(a + b*x)^3 - (198*a^7)/(a + b*x)^4 + (99*a^8)/(a + b*x)^5
 - (110*a^9)/(3*(a + b*x)^6) + (66*a^10)/(7*(a + b*x)^7) - (3*a^11)/(2*(a +
 b*x)^8) + a^12/(9*(a + b*x)^9) + 220*a^3*log(a + b*x) - 66*a^2*b*x)/b^13
```

3.223 $\int \frac{x^{11}}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=177

$$-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+$$

```
[Out] -10*a*x/b^11+1/2*x^2/b^10+1/9*a^11/b^12/(b*x+a)^9-11/8*a^10/b^12/(b*x+a)^8+
55/7*a^9/b^12/(b*x+a)^7-55/2*a^8/b^12/(b*x+a)^6+66*a^7/b^12/(b*x+a)^5-231/2
*a^6/b^12/(b*x+a)^4+154*a^5/b^12/(b*x+a)^3-165*a^4/b^12/(b*x+a)^2+165*a^3/b
^12/(b*x+a)+55*a^2*ln(b*x+a)/b^12
```

Rubi [A]
time = 0.10, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165a^4}{b^{12}(a+bx)^2}+\frac{165a^3}{b^{12}(a+bx)}+\frac{55a^2\log(a+bx)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x)^10,x]
```

```
[Out] (-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b^
12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b*x
)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (154*
a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^12*(a
 + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx)^{10}} \, dx = \int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\frac{33}{b^{11}(a}\right.$$

$$= -\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{}{b^{12}(}$$

Mathematica [A]
time = 0.02, size = 150, normalized size = 0.85

$$\frac{42131a^{11}+351459a^{10}bx+1281096a^9b^2x^2+2656584a^8b^3x^3+3402756a^7b^4x^4+2704212a^6b^5x^5+1220688a^5b^6x^6+190512a^4b^7x^7-77112a^3b^8x^8-36288a^2b^9x^9-2772ab^{10}x^{10}+252b^{11}x^{11}+27720a^2(a+bx)^9\log(a+bx)}{504b^{12}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x + 1281096*a^9*b^2*x^2 + 2656584*a^8*b^3*x^3 +
 3402756*a^7*b^4*x^4 + 2704212*a^6*b^5*x^5 + 1220688*a^5*b^6*x^6 + 190512*a
^4*b^7*x^7 - 77112*a^3*b^8*x^8 - 36288*a^2*b^9*x^9 - 2772*a*b^10*x^10 + 252
*b^11*x^11 + 27720*a^2*(a + b*x)^9*Log[a + b*x])/(504*b^12*(a + b*x)^9)
```

Maple [A]
time = 0.10, size = 167, normalized size = 0.94

method	result
risch	$\frac{x^2}{2b^{10}}-\frac{10ax}{b^{11}}+\frac{165a^3b^7x^8+1155a^4b^6x^7+3619a^5b^5x^6+\frac{13167a^6b^4x^5}{2}+\frac{15147a^7b^3x^4}{2}+\frac{11253a^8b^2x^3}{2}+\frac{36839a^9b\,x^2}{14}+\frac{39611a^{10}x}{56}+\frac{42131a^{1}}{504b}}{b^{11}(bx+a)^9}$
norman	$\frac{\frac{x^{11}}{2b}-\frac{11a\,x^{10}}{2b^2}+\frac{78419a^{11}}{504b^{12}}+\frac{495a^3x^8}{b^4}+\frac{2970a^4x^7}{b^5}+\frac{8470a^5x^6}{b^6}+\frac{28875a^6x^5}{2b^7}+\frac{31647a^7x^4}{2b^8}+\frac{11319a^8x^3}{b^9}+\frac{35937a^9x^2}{7b^{10}}+\frac{75339a^{10}x}{56b^{11}}}{(bx+a)^9}+\frac{55a^2\ln(bx+}{b^{12}}$
default	$-\frac{-\frac{1}{2}x^2b+10ax}{b^{11}}+\frac{165a^3}{b^{12}(bx+a)}-\frac{231a^6}{2b^{12}(bx+a)^4}+\frac{a^{11}}{9b^{12}(bx+a)^9}+\frac{66a^7}{b^{12}(bx+a)^5}-\frac{165a^4}{b^{12}(bx+a)^2}-\frac{55a^8}{2b^{12}(bx+a)^6}+\frac{55a^2\ln(bx+a}{b^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^11*(-1/2*x^2*b+10*a*x)+165*a^3/b^12/(b*x+a)-231/2*a^6/b^12/(b*x+a)^4+1
/9*a^11/b^12/(b*x+a)^9+66*a^7/b^12/(b*x+a)^5-165*a^4/b^12/(b*x+a)^2-55/2*a^
8/b^12/(b*x+a)^6+55*a^2*ln(b*x+a)/b^12-11/8*a^10/b^12/(b*x+a)^8+154*a^5/b^1
2/(b*x+a)^3+55/7*a^9/b^12/(b*x+a)^7
```

Maxima [A]
time = 0.37, size = 223, normalized size = 1.26

$$\frac{83160\,a^3b^8x^8+582120\,a^4b^7x^7+1823976\,a^5b^6x^6+3318084\,a^6b^5x^5+3817044\,a^7b^4x^4+2835756\,a^8b^3x^3+1326204\,a^9b^2x^2+356499\,a^{10}bx+42131\,a^{11}}{504\,(b^{21}x^9+9\,ab^{20}x^8+36\,a^2b^{19}x^7+84\,a^3b^{18}x^6+126\,a^4b^{17}x^5+126\,a^5b^{16}x^4+84\,a^6b^{15}x^3+36\,a^7b^{14}x^2+9\,a^8b^{13}x+a^9b^{12})}+\frac{55\,a^2\log(bx+a)}{b^{12}}+\frac{bx^2-20\,ax}{2\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 33180
84*a^6*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^
2*x^2 + 356499*a^10*b*x + 42131*a^11)/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^2*b^1
9*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6*b^15
*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12) + 55*a^2*log(b*x + a)/b^1
2 + 1/2*(b*x^2 - 20*a*x)/b^11
```

Fricas [A]
time = 1.19, size = 327, normalized size = 1.85

$$\frac{252\,b^{11}x^{11} - 2772\,ab^{10}x^{10} - 36288\,a^{2}b^{9}x^{9} - 77112\,a^{3}b^{8}x^{8} + 190512\,a^{4}b^{7}x^{7} + 1220688\,a^{5}b^{6}x^{6} + 2704212\,a^{6}b^{5}x^{5} + 3402756\,a^{7}b^{4}x^{4} + 2656584\,a^{8}b^{3}x^{3} + 1281096\,a^{9}b^{2}x^{2} + 351459\,a^{10}bx + 42131\,a^{11} + 27720\,(a^{2}b^{9}x^{9} + 9\,a^{3}b^{8}x^{8} + 36\,a^{4}b^{7}x^{7} + 84\,a^{5}b^{6}x^{6} + 126\,a^{6}b^{5}x^{5} + 126\,a^{7}b^{4}x^{4} + 84\,a^{8}b^{3}x^{3} + 36\,a^{9}b^{2}x^{2} + 9\,a^{10}bx + a^{11})\log(bx+a)}{504\,(b^{21}x^{9} + 9\,ab^{20}x^{8} + 36\,a^{2}b^{19}x^{7} + 84\,a^{3}b^{18}x^{6} + 126\,a^{4}b^{17}x^{5} + 126\,a^{5}b^{16}x^{4} + 84\,a^{6}b^{15}x^{3} + 36\,a^{7}b^{14}x^{2} + 9\,a^{8}b^{13}x + a^{9}b^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 - 2772*a*b^10*x^10 - 36288*a^2*b^9*x^9 - 77112*a^3*b^8
*x^8 + 190512*a^4*b^7*x^7 + 1220688*a^5*b^6*x^6 + 2704212*a^6*b^5*x^5 + 340
2756*a^7*b^4*x^4 + 2656584*a^8*b^3*x^3 + 1281096*a^9*b^2*x^2 + 351459*a^10*
b*x + 42131*a^11 + 27720*(a^2*b^9*x^9 + 9*a^3*b^8*x^8 + 36*a^4*b^7*x^7 + 84
*a^5*b^6*x^6 + 126*a^6*b^5*x^5 + 126*a^7*b^4*x^4 + 84*a^8*b^3*x^3 + 36*a^9*
b^2*x^2 + 9*a^10*b*x + a^11)*log(b*x + a))/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^
2*b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6
*b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12)
```

Sympy [A]
time = 0.61, size = 236, normalized size = 1.33

$$\frac{55a^2\log(a+bx)}{b^{12}} - \frac{10ax}{b^{11}} + \frac{42131a^{11} + 356499a^{10}bx + 1326204a^{9}b^{2}x^{2} + 2835756a^{8}b^{3}x^{3} + 3817044a^{7}b^{4}x^{4} + 3318084a^{6}b^{5}x^{5} + 1823976a^{5}b^{6}x^{6} + 582120a^{4}b^{7}x^{7} + 83160a^{3}b^{8}x^{8}}{504a^{9}b^{12} + 4536a^{8}b^{13}x + 18144a^{7}b^{14}x^{2} + 42336a^{6}b^{15}x^{3} + 63504a^{5}b^{16}x^{4} + 63504a^{4}b^{17}x^{5} + 42336a^{3}b^{18}x^{6} + 18144a^{2}b^{19}x^{7} + 4536ab^{20}x^{8} + 504b^{21}x^{9}} + \frac{x^2}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x)/b**12 - 10*a*x/b**11 + (42131*a**11 + 356499*a**10*b*x
 + 1326204*a**9*b**2*x**2 + 2835756*a**8*b**3*x**3 + 3817044*a**7*b**4*x**4
 + 3318084*a**6*b**5*x**5 + 1823976*a**5*b**6*x**6 + 582120*a**4*b**7*x**7
+ 83160*a**3*b**8*x**8)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b*
*14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17
*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 +
 504*b**21*x**9) + x**2/(2*b**10)
```

Giac [A]
time = 1.47, size = 138, normalized size = 0.78

$$\frac{55\,a^2\log(|bx+a|)}{b^{12}} + \frac{b^{10}x^2 - 20\,ab^9x}{2\,b^{20}} + \frac{83160\,a^{3}b^{8}x^{8} + 582120\,a^{4}b^{7}x^{7} + 1823976\,a^{5}b^{6}x^{6} + 3318084\,a^{6}b^{5}x^{5} + 3817044\,a^{7}b^{4}x^{4} + 2835756\,a^{8}b^{3}x^{3} + 1326204\,a^{9}b^{2}x^{2} + 356499\,a^{10}bx + 42131\,a^{11}}{504\,(bx+a)^{9}b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 55*a^2*log(abs(b*x + a))/b^12 + 1/2*(b^10*x^2 - 20*a*b^9*x)/b^20 + 1/504*(8
3160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318084*a^6*b
^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^2*x^2 +
356499*a^10*b*x + 42131*a^11)/((b*x + a)^9*b^12)
```

Mupad [B]
time = 0.23, size = 138, normalized size = 0.78

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{165\,a^3}{a+b\,x}-\frac{165\,a^4}{(a+b\,x)^2}+\frac{154\,a^5}{(a+b\,x)^3}-\frac{231\,a^6}{2\,(a+b\,x)^4}+\frac{66\,a^7}{(a+b\,x)^5}-\frac{55\,a^8}{2\,(a+b\,x)^6}+\frac{55\,a^9}{7\,(a+b\,x)^7}-\frac{11\,a^{10}}{8\,(a+b\,x)^8}+\frac{a^{11}}{9\,(a+b\,x)^9}+55\,a^2\,\ln\left(a+b\,x\right)-11\,a\,b\,x}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^2/2 + (165*a^3)/(a + b*x) - (165*a^4)/(a + b*x)^2 + (154*a^5)/(a
 + b*x)^3 - (231*a^6)/(2*(a + b*x)^4) + (66*a^7)/(a + b*x)^5 - (55*a^8)/(2*
(a + b*x)^6) + (55*a^9)/(7*(a + b*x)^7) - (11*a^10)/(8*(a + b*x)^8) + a^11/
(9*(a + b*x)^9) + 55*a^2*log(a + b*x) - 11*a*b*x)/b^12
```

3.224 $\int \frac{x^{10}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=159

$$\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}$$

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

Rubi [A]
time = 0.08, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{60a^3}{b^{11}(a+bx)^2}-\frac{45a^2}{b^{11}(a+bx)}-\frac{10a\log(a+bx)}{b^{11}}+\frac{x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^10,x]
```

```
[Out] x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^8
)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a +
b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (60*a
^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x])/b^
11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx = \int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\right.$$
$$=\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+$$

Mathematica [A]
time = 0.02, size = 137, normalized size = 0.86

$$-\frac{4861a^{10}+41229a^9bx+153576a^8b^2x^2+328104a^7b^3x^3+439236a^6b^4x^4+375732a^5b^5x^5+197568a^4b^6x^6+54432a^3b^7x^7+2268a^2b^8x^8-2268ab^9x^9-252b^{10}x^{10}+2520a(a+bx)^9\log(a+bx)}{252b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10 + 41229*a^9*b*x + 153576*a^8*b^2*x^2 + 328104*a^7*b^3*x^3
 + 439236*a^6*b^4*x^4 + 375732*a^5*b^5*x^5 + 197568*a^4*b^6*x^6 + 54432*a^3
*b^7*x^7 + 2268*a^2*b^8*x^8 - 2268*a*b^9*x^9 - 252*b^10*x^10 + 2520*a*(a +
b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

Maple [A]
time = 0.10, size = 154, normalized size = 0.97

method	result
risch	$\frac{x}{b^{10}}+\frac{-45a^2b^7x^8-300a^3b^6x^7-910a^4b^5x^6-1617a^5b^4x^5-1827a^6b^3x^4-1338a^7b^2x^3-\frac{4329a^8b\,x^2}{7}-\frac{4609a^9x}{28}-\frac{4861a^{10}}{252b}}{b^{10}(bx+a)^9}-\frac{10a\ln(bx+a}{b^{11}}$
norman	$\frac{\frac{x^{10}}{b}-\frac{7129a^{10}}{252b^{11}}-\frac{90a^2x^8}{b^3}-\frac{540a^3x^7}{b^4}-\frac{1540a^4x^6}{b^5}-\frac{2625a^5x^5}{b^6}-\frac{2877a^6x^4}{b^7}-\frac{2058a^7x^3}{b^8}-\frac{6534a^8x^2}{7b^9}-\frac{6849a^9x}{28b^{10}}}{(bx+a)^9}-\frac{10a\ln(bx+a)}{b^{11}}$
default	$\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(bx+a)^9}+\frac{5a^9}{4b^{11}(bx+a)^8}-\frac{45a^8}{7b^{11}(bx+a)^7}+\frac{20a^7}{b^{11}(bx+a)^6}-\frac{42a^6}{b^{11}(bx+a)^5}+\frac{63a^5}{b^{11}(bx+a)^4}-\frac{70a^4}{b^{11}(bx+a)^3}+\frac{60a^3}{b^{11}(bx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

Maxima [A]
time = 0.33, size = 211, normalized size = 1.33

$$-\frac{11340\,a^2b^8x^8+75600\,a^3b^7x^7+229320\,a^4b^6x^6+407484\,a^5b^5x^5+460404\,a^6b^4x^4+337176\,a^7b^3x^3+155844\,a^8b^2x^2+41481\,a^9bx+4861\,a^{10}}{252\,(b^{20}x^9+9\,ab^{19}x^8+36\,a^2b^{18}x^7+84\,a^3b^{17}x^6+126\,a^4b^{16}x^5+126\,a^5b^{15}x^4+84\,a^6b^{14}x^3+36\,a^7b^{13}x^2+9\,a^8b^{12}x+a^9b^{11})}+\frac{x}{b^{10}}-\frac{10\,a\log(bx+a)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(11340*a^2*b^8*x^8 + 75600*a^3*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484
*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2
 + 41481*a^9*b*x + 4861*a^10)/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 +
84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 3
6*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + x/b^10 - 10*a*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(153) = 306$.
time = 0.94, size = 314, normalized size = 1.97

$$\frac{252\,b^{10}x^{10}+2268\,ab^{9}x^{9}-2268\,a^{2}b^{8}x^{8}-54432\,a^{3}b^{7}x^{7}-197568\,a^{4}b^{6}x^{6}-375732\,a^{5}b^{5}x^{5}-439236\,a^{6}b^{4}x^{4}-328104\,a^{7}b^{3}x^{3}-153576\,a^{8}b^{2}x^{2}-41229\,a^{9}bx-4861\,a^{10}-2520\,(ab^{9}x^{9}+9\,a^{2}b^{8}x^{8}+36\,a^{3}b^{7}x^{7}+84\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+126\,a^{6}b^{4}x^{4}+84\,a^{7}b^{3}x^{3}+36\,a^{8}b^{2}x^{2}+9\,a^{9}bx+a^{10})\log(bx+a)}{252\,(b^{20}x^{9}+9\,ab^{19}x^{8}+36\,a^{2}b^{18}x^{7}+84\,a^{3}b^{17}x^{6}+126\,a^{4}b^{16}x^{5}+126\,a^{5}b^{15}x^{4}+84\,a^{6}b^{14}x^{3}+36\,a^{7}b^{13}x^{2}+9\,a^{8}b^{12}x+a^{9}b^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2268*a*b^9*x^9 - 2268*a^2*b^8*x^8 - 54432*a^3*b^7*x^
7 - 197568*a^4*b^6*x^6 - 375732*a^5*b^5*x^5 - 439236*a^6*b^4*x^4 - 328104*a
^7*b^3*x^3 - 153576*a^8*b^2*x^2 - 41229*a^9*b*x - 4861*a^10 - 2520*(a*b^9*x
^9 + 9*a^2*b^8*x^8 + 36*a^3*b^7*x^7 + 84*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 12
6*a^6*b^4*x^4 + 84*a^7*b^3*x^3 + 36*a^8*b^2*x^2 + 9*a^9*b*x + a^10)*log(b*x
 + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a
^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*
b^12*x + a^9*b^11)
```

Sympy [A]
time = 0.57, size = 224, normalized size = 1.41

$$-\frac{10a\log(a+bx)}{b^{11}}+\frac{-4861a^{10}-41481a^{9}bx-155844a^{8}b^{2}x^{2}-337176a^{7}b^{3}x^{3}-460404a^{6}b^{4}x^{4}-407484a^{5}b^{5}x^{5}-229320a^{4}b^{6}x^{6}-75600a^{3}b^{7}x^{7}-11340a^{2}b^{8}x^{8}}{252a^{9}b^{11}+2268a^{8}b^{12}x+9072a^{7}b^{13}x^{2}+21168a^{6}b^{14}x^{3}+31752a^{5}b^{15}x^{4}+31752a^{4}b^{16}x^{5}+21168a^{3}b^{17}x^{6}+9072a^{2}b^{18}x^{7}+2268ab^{19}x^{8}+252b^{20}x^{9}}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**10,x)
```

```
[Out] -10*a*log(a + b*x)/b**11 + (-4861*a**10 - 41481*a**9*b*x - 155844*a**8*b**2
*x**2 - 337176*a**7*b**3*x**3 - 460404*a**6*b**4*x**4 - 407484*a**5*b**5*x*
*5 - 229320*a**4*b**6*x**6 - 75600*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(
252*a**9*b**11 + 2268*a**8*b**12*x + 9072*a**7*b**13*x**2 + 21168*a**6*b**1
4*x**3 + 31752*a**5*b**15*x**4 + 31752*a**4*b**16*x**5 + 21168*a**3*b**17*x
**6 + 9072*a**2*b**18*x**7 + 2268*a*b**19*x**8 + 252*b**20*x**9) + x/b**10
```

Giac [A]
time = 1.37, size = 121, normalized size = 0.76

$$\frac{x}{b^{10}}-\frac{10\,a\log(|bx+a|)}{b^{11}}-\frac{11340\,a^{2}b^{8}x^{8}+75600\,a^{3}b^{7}x^{7}+229320\,a^{4}b^{6}x^{6}+407484\,a^{5}b^{5}x^{5}+460404\,a^{6}b^{4}x^{4}+337176\,a^{7}b^{3}x^{3}+155844\,a^{8}b^{2}x^{2}+41481\,a^{9}bx+4861\,a^{10}}{252\,(bx+a)^{9}b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] x/b^10 - 10*a*log(abs(b*x + a))/b^11 - 1/252*(11340*a^2*b^8*x^8 + 75600*a^3
*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 3
37176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2 + 41481*a^9*b*x + 4861*a^10)/((b*x +
 a)^9*b^11)
```

Mupad [B]
time = 0.94, size = 127, normalized size = 0.80

$$-\frac{10\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{45\,a^2}{a+b\,x}-\frac{60\,a^3}{{\left(a+b\,x\right)}^2}+\frac{70\,a^4}{{\left(a+b\,x\right)}^3}-\frac{63\,a^5}{{\left(a+b\,x\right)}^4}+\frac{42\,a^6}{{\left(a+b\,x\right)}^5}-\frac{20\,a^7}{{\left(a+b\,x\right)}^6}+\frac{45\,a^8}{7\,{\left(a+b\,x\right)}^7}-\frac{5\,a^9}{4\,{\left(a+b\,x\right)}^8}+\frac{a^{10}}{9\,{\left(a+b\,x\right)}^9}}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^10,x)
```

```
[Out] -(10*a*log(a + b*x) - b*x + (45*a^2)/(a + b*x) - (60*a^3)/(a + b*x)^2 + (70
*a^4)/(a + b*x)^3 - (63*a^5)/(a + b*x)^4 + (42*a^6)/(a + b*x)^5 - (20*a^7)/
(a + b*x)^6 + (45*a^8)/(7*(a + b*x)^7) - (5*a^9)/(4*(a + b*x)^8) + a^10/(9*
(a + b*x)^9))/b^11
```

3.225 $\int \frac{x^9}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=154

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-$$

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

Rubi [A]
time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a^2}{b^{10}(a+bx)^2}+\frac{9a}{b^{10}(a+bx)}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^10,x]
```

```
[Out] a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10*
(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^5)
 - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2)/(
b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^9}{(a+bx)^{10}}\,dx = \int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a}{b^9(a+}\right.$$

$$=\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{6}{2b^{10}(}$$

Mathematica [A]
time = 0.02, size = 111, normalized size = 0.72

$$\frac{a(7129a^8+61641a^7bx+235224a^6b^2x^2+518616a^5b^3x^3+725004a^4b^4x^4+661500a^3b^5x^5+388080a^2b^6x^6+136080ab^7x^7+22680b^8x^8)}{2520b^{10}(a+bx)^9}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^10,x]
```

```
[Out] (a*(7129*a^8 + 61641*a^7*b*x + 235224*a^6*b^2*x^2 + 518616*a^5*b^3*x^3 + 72
5004*a^4*b^4*x^4 + 661500*a^3*b^5*x^5 + 388080*a^2*b^6*x^6 + 136080*a*b^7*x
^7 + 22680*b^8*x^8))/(2520*b^10*(a + b*x)^9) + Log[a + b*x]/b^10
```

Maple [A]
time = 0.08, size = 145, normalized size = 0.94

method	result
norman	$\frac{\frac{7129a^9}{2520b^{10}}+\frac{9a\,x^8}{b^2}+\frac{54a^2x^7}{b^3}+\frac{154a^3x^6}{b^4}+\frac{525a^4x^5}{2b^5}+\frac{2877a^5x^4}{10b^6}+\frac{1029a^6x^3}{5b^7}+\frac{3267a^7x^2}{35b^8}+\frac{6849a^8x}{280b^9}}{(bx+a)^9}+\frac{\ln(bx+a)}{b^{10}}$
risch	$\frac{\frac{7129a^9}{2520b^{10}}+\frac{9a\,x^8}{b^2}+\frac{54a^2x^7}{b^3}+\frac{154a^3x^6}{b^4}+\frac{525a^4x^5}{2b^5}+\frac{2877a^5x^4}{10b^6}+\frac{1029a^6x^3}{5b^7}+\frac{3267a^7x^2}{35b^8}+\frac{6849a^8x}{280b^9}}{(bx+a)^9}+\frac{\ln(bx+a)}{b^{10}}$
default	$\frac{a^9}{9b^{10}(bx+a)^9}-\frac{9a^8}{8b^{10}(bx+a)^8}+\frac{36a^7}{7b^{10}(bx+a)^7}-\frac{14a^6}{b^{10}(bx+a)^6}+\frac{126a^5}{5b^{10}(bx+a)^5}-\frac{63a^4}{2b^{10}(bx+a)^4}+\frac{28a^3}{b^{10}(bx+a)^3}-\frac{18a^2}{b^{10}(bx+a)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

Maxima [A]
time = 0.36, size = 202, normalized size = 1.31

$$\frac{22680\,ab^8x^8+136080\,a^2b^7x^7+388080\,a^3b^6x^6+661500\,a^4b^5x^5+725004\,a^5b^4x^4+518616\,a^6b^3x^3+235224\,a^7b^2x^2+61641\,a^8bx+7129\,a^9}{2520\,(b^{19}x^9+9\,ab^{18}x^8+36\,a^2b^{17}x^7+84\,a^3b^{16}x^6+126\,a^4b^{15}x^5+126\,a^5b^{14}x^4+84\,a^6b^{13}x^3+36\,a^7b^{12}x^2+9\,a^8b^{11}x+a^9b^{10})}+\frac{\log(bx+a)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9)/(b^19*x^9 + 9*a*b^18*x^8 + 36*a^2*b^17*x^7 + 84
*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^6*b^13*x^3 + 36*
a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10) + log(b*x + a)/b^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(144) = 288$.
time = 0.94, size = 292, normalized size = 1.90

$$\frac{22680\,ab^8x^8+136080\,a^2b^7x^7+388080\,a^3b^6x^6+661500\,a^4b^5x^5+725004\,a^5b^4x^4+518616\,a^6b^3x^3+235224\,a^7b^2x^2+61641\,a^8bx+7129\,a^9+2520\,(b^9x^9+9\,ab^8x^8+36\,a^2b^7x^7+84\,a^3b^6x^6+126\,a^4b^5x^5+126\,a^5b^4x^4+84\,a^6b^3x^3+36\,a^7b^2x^2+9\,a^8bx+a^9)\log(bx+a)}{2520\,(b^{19}x^9+9\,ab^{18}x^8+36\,a^2b^{17}x^7+84\,a^3b^{16}x^6+126\,a^4b^{15}x^5+126\,a^5b^{14}x^4+84\,a^6b^{13}x^3+36\,a^7b^{12}x^2+9\,a^8b^{11}x+a^9b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9 + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 +
 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a
^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a))/(b^19*x^9 + 9*a*b^18*x^8 + 36*a
^2*b^17*x^7 + 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^
6*b^13*x^3 + 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10)
```

Sympy [A]
time = 0.46, size = 212, normalized size = 1.38

$$\frac{7129a^9 + 61641a^8bx + 235224a^7b^2x^2 + 518616a^6b^3x^3 + 725004a^5b^4x^4 + 661500a^4b^5x^5 + 388080a^3b^6x^6 + 136080a^2b^7x^7 + 22680ab^8x^8}{2520a^9b^{10} + 22680a^8b^{11}x + 90720a^7b^{12}x^2 + 211680a^6b^{13}x^3 + 317520a^5b^{14}x^4 + 317520a^4b^{15}x^5 + 211680a^3b^{16}x^6 + 90720a^2b^{17}x^7 + 22680ab^{18}x^8 + 2520b^{19}x^9} + \frac{\log(a + bx)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x + 235224*a**7*b**2*x**2 + 518616*a**6*b**3*x**3
 + 725004*a**5*b**4*x**4 + 661500*a**4*b**5*x**5 + 388080*a**3*b**6*x**6 +
136080*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*a**9*b**10 + 22680*a**8*b*
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + log(a + b*x)/b**10
```

Giac [A]
time = 1.10, size = 112, normalized size = 0.73

$$\frac{\log(|bx + a|)}{b^{10}} + \frac{22680\,ab^7x^8 + 136080\,a^2b^6x^7 + 388080\,a^3b^5x^6 + 661500\,a^4b^4x^5 + 725004\,a^5b^3x^4 + 518616\,a^6b^2x^3 + 235224\,a^7bx^2 + 61641\,a^8x + \frac{7129\,a^9}{b}}{2520\,(bx + a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^10 + 1/2520*(22680*a*b^7*x^8 + 136080*a^2*b^6*x^7 + 388
080*a^3*b^5*x^6 + 661500*a^4*b^4*x^5 + 725004*a^5*b^3*x^4 + 518616*a^6*b^2*
x^3 + 235224*a^7*b*x^2 + 61641*a^8*x + 7129*a^9/b)/((b*x + a)^9*b^9)
```

Mupad [B]
time = 0.19, size = 117, normalized size = 0.76

$$\frac{\ln(a + b\,x) + \frac{9\,a}{a+b\,x} - \frac{18\,a^2}{(a+b\,x)^2} + \frac{28\,a^3}{(a+b\,x)^3} - \frac{63\,a^4}{2\,(a+b\,x)^4} + \frac{126\,a^5}{5\,(a+b\,x)^5} - \frac{14\,a^6}{(a+b\,x)^6} + \frac{36\,a^7}{7\,(a+b\,x)^7} - \frac{9\,a^8}{8\,(a+b\,x)^8} + \frac{a^9}{9\,(a+b\,x)^9}}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^10,x)
```

```
[Out] (log(a + b*x) + (9*a)/(a + b*x) - (18*a^2)/(a + b*x)^2 + (28*a^3)/(a + b*x)
^3 - (63*a^4)/(2*(a + b*x)^4) + (126*a^5)/(5*(a + b*x)^5) - (14*a^6)/(a + b
*x)^6 + (36*a^7)/(7*(a + b*x)^7) - (9*a^8)/(8*(a + b*x)^8) + a^9/(9*(a + b*
x)^9))/b^10
```

3.226 $\int \frac{x^8}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=17

$$\frac{x^9}{9a(a+bx)^9}$$

```
[Out] 1/9*x^9/a/(b*x+a)^9
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$\frac{x^9}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^10,x]
```

```
[Out] x^9/(9*a*(a + b*x)^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^{10}} \, dx = \frac{x^9}{9a(a+bx)^9}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 97 vs. 2(17) = 34.
time = 0.01, size = 97, normalized size = 5.71

$$-\frac{a^8+9a^7bx+36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+36ab^7x^7+9b^8x^8}{9b^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^10,x]
```

```
[Out] -1/9*(a^8 + 9*a^7*b*x + 36*a^6*b^2*x^2 + 84*a^5*b^3*x^3 + 126*a^4*b^4*x^4 +
 126*a^3*b^5*x^5 + 84*a^2*b^6*x^6 + 36*a*b^7*x^7 + 9*b^8*x^8)/(b^9*(a + b*x
)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(15) = 30$.
time = 0.08, size = 131, normalized size = 7.71

method	result
gosper	$-\frac{9x^8b^8+36a\,x^7b^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5x^3b^3+36a^6x^2b^2+9a^7xb+a^8}{9(bx+a)^9b^9}$
norman	$\frac{-\frac{x^8}{b}-\frac{4a\,x^7}{b^2}-\frac{28a^2x^6}{3b^3}-\frac{14a^3x^5}{b^4}-\frac{14a^4x^4}{b^5}-\frac{28a^5x^3}{3b^6}-\frac{4a^6x^2}{b^7}-\frac{a^7x}{b^8}-\frac{a^8}{9b^9}}{(bx+a)^9}$
risch	$\frac{-\frac{x^8}{b}-\frac{4a\,x^7}{b^2}-\frac{28a^2x^6}{3b^3}-\frac{14a^3x^5}{b^4}-\frac{14a^4x^4}{b^5}-\frac{28a^5x^3}{3b^6}-\frac{4a^6x^2}{b^7}-\frac{a^7x}{b^8}-\frac{a^8}{9b^9}}{(bx+a)^9}$
default	$-\frac{1}{b^9(bx+a)}-\frac{14a^4}{b^9(bx+a)^5}+\frac{14a^3}{b^9(bx+a)^4}-\frac{a^8}{9b^9(bx+a)^9}+\frac{4a}{b^9(bx+a)^2}+\frac{28a^5}{3b^9(bx+a)^6}-\frac{4a^6}{b^9(bx+a)^7}+\frac{a^7}{b^9(bx+a)^8}-\frac{28}{3b^9(bx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^9/(b*x+a)-14*a^4/b^9/(b*x+a)^5+14*a^3/b^9/(b*x+a)^4-1/9*a^8/b^9/(b*x+a
)^9+4/b^9*a/(b*x+a)^2+28/3*a^5/b^9/(b*x+a)^6-4*a^6/b^9/(b*x+a)^7+a^7/b^9/(b
*x+a)^8-28/3/b^9*a^2/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(15) = 30$.
time = 0.32, size = 186, normalized size = 10.94

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,(b^{18}x^9+9\,ab^{17}x^8+36\,a^2b^{16}x^7+84\,a^3b^{15}x^6+126\,a^4b^{14}x^5+126\,a^5b^{13}x^4+84\,a^6b^{12}x^3+36\,a^7b^{11}x^2+9\,a^8b^{10}x+a^9b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(15) = 30$.
time = 0.99, size = 186, normalized size = 10.94

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,(b^{18}x^9+9\,ab^{17}x^8+36\,a^2b^{16}x^7+84\,a^3b^{15}x^6+126\,a^4b^{14}x^5+126\,a^5b^{13}x^4+84\,a^6b^{12}x^3+36\,a^7b^{11}x^2+9\,a^8b^{10}x+a^9b^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(12) = 24$.
time = 0.42, size = 199, normalized size = 11.71

$$\frac{-a^8 - 9a^7bx - 36a^6b^2x^2 - 84a^5b^3x^3 - 126a^4b^4x^4 - 126a^3b^5x^5 - 84a^2b^6x^6 - 36ab^7x^7 - 9b^8x^8}{9a^9b^9 + 81a^8b^{10}x + 324a^7b^{11}x^2 + 756a^6b^{12}x^3 + 1134a^5b^{13}x^4 + 1134a^4b^{14}x^5 + 756a^3b^{15}x^6 + 324a^2b^{16}x^7 + 81ab^{17}x^8 + 9b^{18}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*a**9*b**9 + 81*a**8*b**10*x + 324*a**7*b**11*x**2 + 756*a**6*b**12*x
**3 + 1134*a**5*b**13*x**4 + 1134*a**4*b**14*x**5 + 756*a**3*b**15*x**6 + 3
24*a**2*b**16*x**7 + 81*a*b**17*x**8 + 9*b**18*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(15) = 30$.
time = 1.57, size = 95, normalized size = 5.59

$$-\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx + a^8}{9\,(bx+a)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/((b*x + a)^9*
b^9)
```

Mupad [B]
time = 0.14, size = 107, normalized size = 6.29

$$-\frac{\frac{1}{a+b\,x} - \frac{4\,a}{(a+b\,x)^2} + \frac{28\,a^2}{3\,(a+b\,x)^3} - \frac{14\,a^3}{(a+b\,x)^4} + \frac{14\,a^4}{(a+b\,x)^5} - \frac{28\,a^5}{3\,(a+b\,x)^6} + \frac{4\,a^6}{(a+b\,x)^7} - \frac{a^7}{(a+b\,x)^8} + \frac{a^8}{9\,(a+b\,x)^9}}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^10,x)
```

```
[Out] -(1/(a + b*x) - (4*a)/(a + b*x)^2 + (28*a^2)/(3*(a + b*x)^3) - (14*a^3)/(a
+ b*x)^4 + (14*a^4)/(a + b*x)^5 - (28*a^5)/(3*(a + b*x)^6) + (4*a^6)/(a + b
*x)^7 - a^7/(a + b*x)^8 + a^8/(9*(a + b*x)^9))/b^9
```

3.227 $\int \frac{x^7}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=35

$$\frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}$$

```
[Out] 1/9*x^8/a/(b*x+a)^9+1/72*x^8/a^2/(b*x+a)^8
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^10,x]
```

```
[Out] x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^{10}}\,dx &= \frac{x^8}{9a(a+bx)^9}+\frac{\int \frac{x^7}{(a+bx)^9}\,dx}{9a}\\ &= \frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 86 vs. 2(35) = 70.
time = 0.01, size = 86, normalized size = 2.46

$$-\frac{a^7+9a^6bx+36a^5b^2x^2+84a^4b^3x^3+126a^3b^4x^4+126a^2b^5x^5+84ab^6x^6+36b^7x^7}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7 + 9*a^6*b*x + 36*a^5*b^2*x^2 + 84*a^4*b^3*x^3 + 126*a^3*b^4*x^4
+ 126*a^2*b^5*x^5 + 84*a*b^6*x^6 + 36*b^7*x^7)/(b^8*(a + b*x)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(31) = 62.
time = 0.08, size = 117, normalized size = 3.34

method	result	siz
gosper	$-\frac{36b^7x^7+84a\,b^6x^6+126a^2b^5x^5+126a^3b^4x^4+84a^4b^3x^3+36a^5b^2x^2+9a^6bx+a^7}{72(bx+a)^9b^8}$	85
norman	$\frac{-\frac{x^7}{2b}-\frac{7a\,x^6}{6b^2}-\frac{7a^2x^5}{4b^3}-\frac{7a^3x^4}{4b^4}-\frac{7a^4x^3}{6b^5}-\frac{a^5x^2}{2b^6}-\frac{a^6x}{8b^7}-\frac{a^7}{72b^8}}{(bx+a)^9}$	88
risch	$\frac{-\frac{x^7}{2b}-\frac{7a\,x^6}{6b^2}-\frac{7a^2x^5}{4b^3}-\frac{7a^3x^4}{4b^4}-\frac{7a^4x^3}{6b^5}-\frac{a^5x^2}{2b^6}-\frac{a^6x}{8b^7}-\frac{a^7}{72b^8}}{(bx+a)^9}$	88
default	$-\frac{21a^2}{4b^8(bx+a)^4}+\frac{3a^5}{b^8(bx+a)^7}+\frac{a^7}{9b^8(bx+a)^9}-\frac{7a^6}{8b^8(bx+a)^8}-\frac{1}{2b^8(bx+a)^2}-\frac{35a^4}{6b^8(bx+a)^6}+\frac{7a^3}{b^8(bx+a)^5}+\frac{7a}{3b^8(bx+a)^3}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -21/4*a^2/b^8/(b*x+a)^4+3*a^5/b^8/(b*x+a)^7+1/9*a^7/b^8/(b*x+a)^9-7/8*a^6/b
^8/(b*x+a)^8-1/2/b^8/(b*x+a)^2-35/6*a^4/b^8/(b*x+a)^6+7*a^3/b^8/(b*x+a)^5+7
/3/b^8*a/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(31) = 62.
time = 0.50, size = 175, normalized size = 5.00

$$-\frac{36\,b^7x^7+84\,ab^6x^6+126\,a^2b^5x^5+126\,a^3b^4x^4+84\,a^4b^3x^3+36\,a^5b^2x^2+9\,a^6bx+a^7}{72\,(b^{17}x^9+9\,ab^{16}x^8+36\,a^2b^{15}x^7+84\,a^3b^{14}x^6+126\,a^4b^{13}x^5+126\,a^5b^{12}x^4+84\,a^6b^{11}x^3+36\,a^7b^{10}x^2+9\,a^8b^9x+a^9b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
```

```
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(31) = 62$.
time = 0.93, size = 175, normalized size = 5.00

$$-\frac{36\,b^{7}x^{7}+84\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+126\,a^{3}b^{4}x^{4}+84\,a^{4}b^{3}x^{3}+36\,a^{5}b^{2}x^{2}+9\,a^{6}bx+a^{7}}{72\,(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a^{9}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.
time = 0.39, size = 187, normalized size = 5.34

$$\frac{-a^{7}-9a^{6}bx-36a^{5}b^{2}x^{2}-84a^{4}b^{3}x^{3}-126a^{3}b^{4}x^{4}-126a^{2}b^{5}x^{5}-84ab^{6}x^{6}-36b^{7}x^{7}}{72a^{9}b^{8}+648a^{8}b^{9}x+2592a^{7}b^{10}x^{2}+6048a^{6}b^{11}x^{3}+9072a^{5}b^{12}x^{4}+9072a^{4}b^{13}x^{5}+6048a^{3}b^{14}x^{6}+2592a^{2}b^{15}x^{7}+648ab^{16}x^{8}+72b^{17}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x - 36*a**5*b**2*x**2 - 84*a**4*b**3*x**3 - 126*a**3*b**4
*x**4 - 126*a**2*b**5*x**5 - 84*a*b**6*x**6 - 36*b**7*x**7)/(72*a**9*b**8 +
 648*a**8*b**9*x + 2592*a**7*b**10*x**2 + 6048*a**6*b**11*x**3 + 9072*a**5*
b**12*x**4 + 9072*a**4*b**13*x**5 + 6048*a**3*b**14*x**6 + 2592*a**2*b**15*
x**7 + 648*a*b**16*x**8 + 72*b**17*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(31) = 62$.
time = 1.54, size = 84, normalized size = 2.40

$$-\frac{36\,b^{7}x^{7}+84\,ab^{6}x^{6}+126\,a^{2}b^{5}x^{5}+126\,a^{3}b^{4}x^{4}+84\,a^{4}b^{3}x^{3}+36\,a^{5}b^{2}x^{2}+9\,a^{6}bx+a^{7}}{72\,(bx+a)^{9}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/((b*x + a)^9*b^8)
```

Mupad [B]
time = 0.13, size = 22, normalized size = 0.63

$$\frac{x^{8}\,(9\,a+b\,x)}{72\,a^{2}\,(a+b\,x)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^10,x)
[Out] (x^8*(9*a + b*x))/(72*a^2*(a + b*x)^9)
```

3.228 $\int \frac{x^6}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=52

$$\frac{x^7}{9a(a+bx)^9}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{252a^3(a+bx)^7}$$

```
[Out] 1/9*x^7/a/(b*x+a)^9+1/36*x^7/a^2/(b*x+a)^8+1/252*x^7/a^3/(b*x+a)^7
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{x^7}{252a^3(a+bx)^7}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^10,x]
```

```
[Out] x^7/(9*a*(a + b*x)^9) + x^7/(36*a^2*(a + b*x)^8) + x^7/(252*a^3*(a + b*x)^7
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^{10}}\,dx &= \frac{x^7}{9a(a+bx)^9}+\frac{2\int \frac{x^6}{(a+bx)^9}\,dx}{9a}\\ &= \frac{x^7}{9a(a+bx)^9}+\frac{x^7}{36a^2(a+bx)^8}+\frac{\int \frac{x^6}{(a+bx)^8}\,dx}{36a^2}\\ &= \frac{x^7}{9a(a+bx)^9}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{252a^3(a+bx)^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 75, normalized size = 1.44

$$-\frac{a^6+9a^5bx+36a^4b^2x^2+84a^3b^3x^3+126a^2b^4x^4+126ab^5x^5+84b^6x^6}{252b^7(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^6 + 9*a^5*b*x + 36*a^4*b^2*x^2 + 84*a^3*b^3*x^3 + 126*a^2*b^4*x^4
 + 126*a*b^5*x^5 + 84*b^6*x^6)/(b^7*(a + b*x)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(46) = 92$.
time = 0.09, size = 102, normalized size = 1.96

method	result	size
gosper	$-\frac{84x^6b^6+126a\,x^5b^5+126a^2x^4b^4+84a^3x^3b^3+36a^4x^2b^2+9a^5xb+a^6}{252(bx+a)^9b^7}$	74
norman	$\frac{-\frac{x^6}{3b}-\frac{a\,x^5}{2b^2}-\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{3b^4}-\frac{a^4x^2}{7b^5}-\frac{a^5x}{28b^6}-\frac{a^6}{252b^7}}{(bx+a)^9}$	77
risch	$\frac{-\frac{x^6}{3b}-\frac{a\,x^5}{2b^2}-\frac{a^2x^4}{2b^3}-\frac{a^3x^3}{3b^4}-\frac{a^4x^2}{7b^5}-\frac{a^5x}{28b^6}-\frac{a^6}{252b^7}}{(bx+a)^9}$	77
default	$\frac{3a^5}{4b^7(bx+a)^8}+\frac{3a}{2b^7(bx+a)^4}-\frac{3a^2}{b^7(bx+a)^5}-\frac{a^6}{9b^7(bx+a)^9}+\frac{10a^3}{3b^7(bx+a)^6}-\frac{15a^4}{7b^7(bx+a)^7}-\frac{1}{3b^7(bx+a)^3}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*a^5/b^7/(b*x+a)^8+3/2*a/b^7/(b*x+a)^4-3/b^7*a^2/(b*x+a)^5-1/9*a^6/b^7/(
b*x+a)^9+10/3*a^3/b^7/(b*x+a)^6-15/7*a^4/b^7/(b*x+a)^7-1/3/b^7/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(46) = 92$.
time = 0.51, size = 164, normalized size = 3.15

$$-\frac{84\,b^6x^6+126\,ab^5x^5+126\,a^2b^4x^4+84\,a^3b^3x^3+36\,a^4b^2x^2+9\,a^5bx+a^6}{252\,(b^{16}x^9+9\,ab^{15}x^8+36\,a^2b^{14}x^7+84\,a^3b^{13}x^6+126\,a^4b^{12}x^5+126\,a^5b^{11}x^4+84\,a^6b^{10}x^3+36\,a^7b^9x^2+9\,a^8b^8x+a^9b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(46) = 92$.
time = 0.73, size = 164, normalized size = 3.15

$$-\frac{84\,b^{6}x^{6}+126\,ab^{5}x^{5}+126\,a^{2}b^{4}x^{4}+84\,a^{3}b^{3}x^{3}+36\,a^{4}b^{2}x^{2}+9\,a^{5}bx+a^{6}}{252\,(b^{16}x^{9}+9\,ab^{15}x^{8}+36\,a^{2}b^{14}x^{7}+84\,a^{3}b^{13}x^{6}+126\,a^{4}b^{12}x^{5}+126\,a^{5}b^{11}x^{4}+84\,a^{6}b^{10}x^{3}+36\,a^{7}b^{9}x^{2}+9\,a^{8}b^{8}x+a^{9}b^{7})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(42) = 84$.
time = 0.35, size = 175, normalized size = 3.37

$$\frac{-a^{6}-9a^{5}bx-36a^{4}b^{2}x^{2}-84a^{3}b^{3}x^{3}-126a^{2}b^{4}x^{4}-126ab^{5}x^{5}-84b^{6}x^{6}}{252a^{9}b^{7}+2268a^{8}b^{8}x+9072a^{7}b^{9}x^{2}+21168a^{6}b^{10}x^{3}+31752a^{5}b^{11}x^{4}+31752a^{4}b^{12}x^{5}+21168a^{3}b^{13}x^{6}+9072a^{2}b^{14}x^{7}+2268ab^{15}x^{8}+252b^{16}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x - 36*a**4*b**2*x**2 - 84*a**3*b**3*x**3 - 126*a**2*b**4
*x**4 - 126*a*b**5*x**5 - 84*b**6*x**6)/(252*a**9*b**7 + 2268*a**8*b**8*x +
 9072*a**7*b**9*x**2 + 21168*a**6*b**10*x**3 + 31752*a**5*b**11*x**4 + 3175
2*a**4*b**12*x**5 + 21168*a**3*b**13*x**6 + 9072*a**2*b**14*x**7 + 2268*a*b
**15*x**8 + 252*b**16*x**9)
```

Giac [A]
time = 1.04, size = 73, normalized size = 1.40

$$-\frac{84\,b^{6}x^{6}+126\,ab^{5}x^{5}+126\,a^{2}b^{4}x^{4}+84\,a^{3}b^{3}x^{3}+36\,a^{4}b^{2}x^{2}+9\,a^{5}bx+a^{6}}{252\,(bx+a)^{9}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="giac")
```

[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*a^4*b^2*x^2 + 9*a^5*b*x + a^6)/((b*x + a)^9*b^7)

Mupad [B]
time = 0.14, size = 85, normalized size = 1.63

$$-\frac{\frac{1}{3\,{\left(a+b\,x\right)}^3}-\frac{3\,a}{2\,{\left(a+b\,x\right)}^4}+\frac{3\,a^2}{{\left(a+b\,x\right)}^5}-\frac{10\,a^3}{3\,{\left(a+b\,x\right)}^6}+\frac{15\,a^4}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^5}{4\,{\left(a+b\,x\right)}^8}+\frac{a^6}{9\,{\left(a+b\,x\right)}^9}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a + b*x)^10,x)

[Out] -(1/(3*(a + b*x)^3) - (3*a)/(2*(a + b*x)^4) + (3*a^2)/(a + b*x)^5 - (10*a^3)/(3*(a + b*x)^6) + (15*a^4)/(7*(a + b*x)^7) - (3*a^5)/(4*(a + b*x)^8) + a^6/(9*(a + b*x)^9))/b^7

3.229 $\int \frac{x^5}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=69

$$\frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{504a^4(a+bx)^6}$$

```
[Out] 1/9*x^6/a/(b*x+a)^9+1/24*x^6/a^2/(b*x+a)^8+1/84*x^6/a^3/(b*x+a)^7+1/504*x^6
/a^4/(b*x+a)^6
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^10,x]
```

```
[Out] x^6/(9*a*(a + b*x)^9) + x^6/(24*a^2*(a + b*x)^8) + x^6/(84*a^3*(a + b*x)^7)
 + x^6/(504*a^4*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^{10}}\,dx = \frac{x^6}{9a(a+bx)^9} + \frac{\int \frac{x^5}{(a+bx)^9}\,dx}{3a}$$
$$= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{\int \frac{x^5}{(a+bx)^8}\,dx}{12a^2}$$
$$= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{\int \frac{x^5}{(a+bx)^7}\,dx}{84a^3}$$
$$= \frac{x^6}{9a(a+bx)^9} + \frac{x^6}{24a^2(a+bx)^8} + \frac{x^6}{84a^3(a+bx)^7} + \frac{x^6}{504a^4(a+bx)^6}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.93

$$-\frac{a^5 + 9a^4bx + 36a^3b^2x^2 + 84a^2b^3x^3 + 126ab^4x^4 + 126b^5x^5}{504b^6(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^5 + 9*a^4*b*x + 36*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 126*a*b^4*x^4 +
 126*b^5*x^5)/(b^6*(a + b*x)^9)
```

Maple [A]
time = 0.08, size = 86, normalized size = 1.25

method	result	size
gosper	$-\frac{126b^5x^5+126a\,b^4x^4+84a^2b^3x^3+36a^3b^2x^2+9a^4bx+a^5}{504(bx+a)^9b^6}$	63
norman	$\frac{-\frac{x^5}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^3}{6b^3}-\frac{a^3x^2}{14b^4}-\frac{a^4x}{56b^5}-\frac{a^5}{504b^6}}{(bx+a)^9}$	66
risch	$\frac{-\frac{x^5}{4b}-\frac{a\,x^4}{4b^2}-\frac{a^2x^3}{6b^3}-\frac{a^3x^2}{14b^4}-\frac{a^4x}{56b^5}-\frac{a^5}{504b^6}}{(bx+a)^9}$	66
default	$\frac{a}{b^6(bx+a)^5}-\frac{1}{4b^6(bx+a)^4}+\frac{a^5}{9b^6(bx+a)^9}-\frac{5a^2}{3b^6(bx+a)^6}+\frac{10a^3}{7b^6(bx+a)^7}-\frac{5a^4}{8b^6(bx+a)^8}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] a/b^6/(b*x+a)^5-1/4/b^6/(b*x+a)^4+1/9*a^5/b^6/(b*x+a)^9-5/3/b^6*a^2/(b*x+a)
^6+10/7*a^3/b^6/(b*x+a)^7-5/8*a^4/b^6/(b*x+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.
time = 0.30, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+9\,a^8b^7x+a^9b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(61) = 122$.
time = 0.81, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+9\,a^8b^7x+a^9b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.
time = 0.33, size = 163, normalized size = 2.36

$$\frac{-a^5-9a^4bx-36a^3b^2x^2-84a^2b^3x^3-126ab^4x^4-126b^5x^5}{504a^9b^6+4536a^8b^7x+18144a^7b^8x^2+42336a^6b^9x^3+63504a^5b^{10}x^4+63504a^4b^{11}x^5+42336a^3b^{12}x^6+18144a^2b^{13}x^7+4536ab^{14}x^8+504b^{15}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**10,x)
```

```
[Out] (-a**5 - 9*a**4*b*x - 36*a**3*b**2*x**2 - 84*a**2*b**3*x**3 - 126*a*b**4*x*
*4 - 126*b**5*x**5)/(504*a**9*b**6 + 4536*a**8*b**7*x + 18144*a**7*b**8*x**
2 + 42336*a**6*b**9*x**3 + 63504*a**5*b**10*x**4 + 63504*a**4*b**11*x**5 +
42336*a**3*b**12*x**6 + 18144*a**2*b**13*x**7 + 4536*a*b**14*x**8 + 504*b**
15*x**9)
```

Giac [A]
time = 1.39, size = 62, normalized size = 0.90

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(bx+a)^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/((b*x + a)^9*b^6)
```

Mupad [B]
time = 0.08, size = 71, normalized size = 1.03

$$\frac{\frac{a}{(a+b\,x)^5}-\frac{1}{4\,(a+b\,x)^4}-\frac{5\,a^2}{3\,(a+b\,x)^6}+\frac{10\,a^3}{7\,(a+b\,x)^7}-\frac{5\,a^4}{8\,(a+b\,x)^8}+\frac{a^5}{9\,(a+b\,x)^9}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^10,x)
```

```
[Out] (a/(a + b*x)^5 - 1/(4*(a + b*x)^4) - (5*a^2)/(3*(a + b*x)^6) + (10*a^3)/(7*
(a + b*x)^7) - (5*a^4)/(8*(a + b*x)^8) + a^5/(9*(a + b*x)^9))/b^6
```

3.230 $\int \frac{x^4}{(a+bx)^{10}}\, dx$

Optimal. Leaf size=81

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^10,x]
```

```
[Out] -1/9*a^4/(b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a +
b*x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{10}}\, dx &= \int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right) dx\\ &= -\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.65

$$-\frac{a^4+9a^3bx+36a^2b^2x^2+84ab^3x^3+126b^4x^4}{630b^5(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^10,x]
```

```
[Out] -1/630*(a^4 + 9*a^3*b*x + 36*a^2*b^2*x^2 + 84*a*b^3*x^3 + 126*b^4*x^4)/(b^5
*(a + b*x)^9)
```

Maple [A]
time = 0.08, size = 72, normalized size = 0.89

method	result	size
gosper	$-\frac{126 b^{4} x^{4}+84 a\,b^{3} x^{3}+36 a^{2} b^{2} x^{2}+9 a^{3} b x +a^{4}}{630 \left(b x +a \right)^{9} b^{5}}$	52
norman	$\frac{-\frac{x^{4}}{5 b}-\frac{2 a\,x^{3}}{15 b^{2}}-\frac{2 a^{2} x^{2}}{35 b^{3}}-\frac{a^{3} x}{70 b^{4}}-\frac{a^{4}}{630 b^{5}}}{\left(b x +a \right)^{9}}$	55
risch	$\frac{-\frac{x^{4}}{5 b}-\frac{2 a\,x^{3}}{15 b^{2}}-\frac{2 a^{2} x^{2}}{35 b^{3}}-\frac{a^{3} x}{70 b^{4}}-\frac{a^{4}}{630 b^{5}}}{\left(b x +a \right)^{9}}$	55
default	$-\frac{a^{4}}{9 b^{5} \left(b x +a \right)^{9}}+\frac{a^{3}}{2 b^{5} \left(b x +a \right)^{8}}-\frac{6 a^{2}}{7 b^{5} \left(b x +a \right)^{7}}+\frac{2 a}{3 b^{5} \left(b x +a \right)^{6}}-\frac{1}{5 b^{5} \left(b x +a \right)^{5}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

Maxima [A]
time = 0.30, size = 142, normalized size = 1.75

$$-\frac{126 \, b^{4} x^{4} + 84 \, a b^{3} x^{3} + 36 \, a^{2} b^{2} x^{2} + 9 \, a^{3} b x + a^{4}}{630 \, {\left(b^{14} x^{9} + 9 \, a b^{13} x^{8} + 36 \, a^{2} b^{12} x^{7} + 84 \, a^{3} b^{11} x^{6} + 126 \, a^{4} b^{10} x^{5} + 126 \, a^{5} b^{9} x^{4} + 84 \, a^{6} b^{8} x^{3} + 36 \, a^{7} b^{7} x^{2} + 9 \, a^{8} b^{6} x + a^{9} b^{5}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

Fricas [A]
time = 1.00, size = 142, normalized size = 1.75

$$-\frac{126 \, b^{4} x^{4} + 84 \, a b^{3} x^{3} + 36 \, a^{2} b^{2} x^{2} + 9 \, a^{3} b x + a^{4}}{630 \, {\left(b^{14} x^{9} + 9 \, a b^{13} x^{8} + 36 \, a^{2} b^{12} x^{7} + 84 \, a^{3} b^{11} x^{6} + 126 \, a^{4} b^{10} x^{5} + 126 \, a^{5} b^{9} x^{4} + 84 \, a^{6} b^{8} x^{3} + 36 \, a^{7} b^{7} x^{2} + 9 \, a^{8} b^{6} x + a^{9} b^{5}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(75) = 150$.
time = 0.31, size = 151, normalized size = 1.86

$$\frac{-a^4 - 9a^3bx - 36a^2b^2x^2 - 84ab^3x^3 - 126b^4x^4}{630a^9b^5 + 5670a^8b^6x + 22680a^7b^7x^2 + 52920a^6b^8x^3 + 79380a^5b^9x^4 + 79380a^4b^{10}x^5 + 52920a^3b^{11}x^6 + 22680a^2b^{12}x^7 + 5670ab^{13}x^8 + 630b^{14}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x - 36*a**2*b**2*x**2 - 84*a*b**3*x**3 - 126*b**4*x**4)/(
630*a**9*b**5 + 5670*a**8*b**6*x + 22680*a**7*b**7*x**2 + 52920*a**6*b**8*x
**3 + 79380*a**5*b**9*x**4 + 79380*a**4*b**10*x**5 + 52920*a**3*b**11*x**6
+ 22680*a**2*b**12*x**7 + 5670*a*b**13*x**8 + 630*b**14*x**9)
```

Giac [A]
time = 1.03, size = 51, normalized size = 0.63

$$-\frac{126\,b^4x^4 + 84\,ab^3x^3 + 36\,a^2b^2x^2 + 9\,a^3bx + a^4}{630\,(bx+a)^9b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/((b*
x + a)^9*b^5)
```

Mupad [B]
time = 0.08, size = 61, normalized size = 0.75

$$-\frac{\frac{1}{5\,(a+b\,x)^5} - \frac{2\,a}{3\,(a+b\,x)^6} + \frac{6\,a^2}{7\,(a+b\,x)^7} - \frac{a^3}{2\,(a+b\,x)^8} + \frac{a^4}{9\,(a+b\,x)^9}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^10,x)
```

```
[Out] -(1/(5*(a + b*x)^5) - (2*a)/(3*(a + b*x)^6) + (6*a^2)/(7*(a + b*x)^7) - a^3
/(2*(a + b*x)^8) + a^4/(9*(a + b*x)^9))/b^5
```

3.231 $\int \frac{x^3}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=64

$$\frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}$$

```
[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*
x+a)^6
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^10,x]
```

```
[Out] a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a + b
*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{10}} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{10}} + \frac{3a^2}{b^3(a+bx)^9} - \frac{3a}{b^3(a+bx)^8} + \frac{1}{b^3(a+bx)^7}\right) dx \\ &= \frac{a^3}{9b^4(a+bx)^9} - \frac{3a^2}{8b^4(a+bx)^8} + \frac{3a}{7b^4(a+bx)^7} - \frac{1}{6b^4(a+bx)^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.66

$$-\frac{a^3 + 9a^2bx + 36ab^2x^2 + 84b^3x^3}{504b^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^3 + 9*a^2*b*x + 36*a*b^2*x^2 + 84*b^3*x^3)/(b^4*(a + b*x)^9)
```

Maple [A]
time = 0.08, size = 57, normalized size = 0.89

method	result	size
gosper	$-\frac{84b^{3}x^{3}+36a\,b^{2}x^{2}+9a^{2}bx+a^{3}}{504\left(bx+a\right)^{9}b^{4}}$	41
norman	$\frac{-\frac{x^{3}}{6b}-\frac{a\,x^{2}}{14b^{2}}-\frac{a^{2}x}{56b^{3}}-\frac{a^{3}}{504b^{4}}}{\left(bx+a\right)^{9}}$	44
risch	$\frac{-\frac{x^{3}}{6b}-\frac{a\,x^{2}}{14b^{2}}-\frac{a^{2}x}{56b^{3}}-\frac{a^{3}}{504b^{4}}}{\left(bx+a\right)^{9}}$	44
default	$\frac{a^{3}}{9b^{4}\left(bx+a\right)^{9}}-\frac{3a^{2}}{8b^{4}\left(bx+a\right)^{8}}+\frac{3a}{7b^{4}\left(bx+a\right)^{7}}-\frac{1}{6b^{4}\left(bx+a\right)^{6}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*
x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(56) = 112.
time = 0.54, size = 131, normalized size = 2.05

$$-\frac{84\,b^{3}x^{3}+36\,ab^{2}x^{2}+9\,a^{2}bx+a^{3}}{504\,\left(b^{13}x^{9}+9\,ab^{12}x^{8}+36\,a^{2}b^{11}x^{7}+84\,a^{3}b^{10}x^{6}+126\,a^{4}b^{9}x^{5}+126\,a^{5}b^{8}x^{4}+84\,a^{6}b^{7}x^{3}+36\,a^{7}b^{6}x^{2}+9\,a^{8}b^{5}x+a^{9}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(56) = 112.
time = 0.82, size = 131, normalized size = 2.05

$$-\frac{84\,b^{3}x^{3}+36\,ab^{2}x^{2}+9\,a^{2}bx+a^{3}}{504\,\left(b^{13}x^{9}+9\,ab^{12}x^{8}+36\,a^{2}b^{11}x^{7}+84\,a^{3}b^{10}x^{6}+126\,a^{4}b^{9}x^{5}+126\,a^{5}b^{8}x^{4}+84\,a^{6}b^{7}x^{3}+36\,a^{7}b^{6}x^{2}+9\,a^{8}b^{5}x+a^{9}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(60) = 120$.
time = 0.29, size = 139, normalized size = 2.17

$$\frac{-a^3 - 9a^2bx - 36ab^2x^2 - 84b^3x^3}{504a^9b^4 + 4536a^8b^5x + 18144a^7b^6x^2 + 42336a^6b^7x^3 + 63504a^5b^8x^4 + 63504a^4b^9x^5 + 42336a^3b^{10}x^6 + 18144a^2b^{11}x^7 + 4536ab^{12}x^8 + 504b^{13}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**10,x)
```

```
[Out] (-a**3 - 9*a**2*b*x - 36*a*b**2*x**2 - 84*b**3*x**3)/(504*a**9*b**4 + 4536*
a**8*b**5*x + 18144*a**7*b**6*x**2 + 42336*a**6*b**7*x**3 + 63504*a**5*b**8
*x**4 + 63504*a**4*b**9*x**5 + 42336*a**3*b**10*x**6 + 18144*a**2*b**11*x**
7 + 4536*a*b**12*x**8 + 504*b**13*x**9)
```

Giac [A]
time = 1.29, size = 40, normalized size = 0.62

$$-\frac{84\,b^3x^3 + 36\,ab^2x^2 + 9\,a^2bx + a^3}{504\,(bx+a)^9b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/((b*x + a)^9*b^4)
```

Mupad [B]
time = 0.13, size = 48, normalized size = 0.75

$$\frac{\frac{3\,a}{7\,(a+b\,x)^7} - \frac{1}{6\,(a+b\,x)^6} - \frac{3\,a^2}{8\,(a+b\,x)^8} + \frac{a^3}{9\,(a+b\,x)^9}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^10,x)
```

```
[Out] ((3*a)/(7*(a + b*x)^7) - 1/(6*(a + b*x)^6) - (3*a^2)/(8*(a + b*x)^8) + a^3/
(9*(a + b*x)^9))/b^4
```

3.232 $\int \frac{x^2}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

```
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^10,x]
```

```
[Out] -1/9*a^2/(b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{10}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right)dx\\ &= -\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+9abx+36b^2x^2}{252b^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^10,x]
[Out] -1/252*(a^2 + 9*a*b*x + 36*b^2*x^2)/(b^3*(a + b*x)^9)
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.89

method	result	size
gosper	$-\frac{36x^2b^2+9abx+a^2}{252(bx+a)^9 b^3}$	30
norman	$\frac{-\frac{x^2}{7b}-\frac{ax}{28b^2}-\frac{a^2}{252b^3}}{(bx+a)^9}$	33
risch	$\frac{-\frac{x^2}{7b}-\frac{ax}{28b^2}-\frac{a^2}{252b^3}}{(bx+a)^9}$	33
default	$-\frac{a^2}{9b^3(bx+a)^9}+\frac{a}{4b^3(bx+a)^8}-\frac{1}{7b^3(bx+a)^7}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(41) = 82.
time = 0.51, size = 120, normalized size = 2.55

$$-\frac{36\,b^2x^2+9\,abx+a^2}{252\,(b^{12}x^9+9\,ab^{11}x^8+36\,a^2b^{10}x^7+84\,a^3b^9x^6+126\,a^4b^8x^5+126\,a^5b^7x^4+84\,a^6b^6x^3+36\,a^7b^5x^2+9\,a^8b^4x+a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="maxima")
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(41) = 82.
time = 0.60, size = 120, normalized size = 2.55

$$-\frac{36\,b^2x^2+9\,abx+a^2}{252\,(b^{12}x^9+9\,ab^{11}x^8+36\,a^2b^{10}x^7+84\,a^3b^9x^6+126\,a^4b^8x^5+126\,a^5b^7x^4+84\,a^6b^6x^3+36\,a^7b^5x^2+9\,a^8b^4x+a^9b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="fricas")
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(41) = 82$.
time = 0.28, size = 128, normalized size = 2.72

$$\frac{-a^2 - 9abx - 36b^2x^2}{252a^9b^3 + 2268a^8b^4x + 9072a^7b^5x^2 + 21168a^6b^6x^3 + 31752a^5b^7x^4 + 31752a^4b^8x^5 + 21168a^3b^9x^6 + 9072a^2b^{10}x^7 + 2268ab^{11}x^8 + 252b^{12}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x - 36*b**2*x**2)/(252*a**9*b**3 + 2268*a**8*b**4*x + 9072*a
**7*b**5*x**2 + 21168*a**6*b**6*x**3 + 31752*a**5*b**7*x**4 + 31752*a**4*b*
*8*x**5 + 21168*a**3*b**9*x**6 + 9072*a**2*b**10*x**7 + 2268*a*b**11*x**8 +
 252*b**12*x**9)
```

Giac [A]
time = 1.44, size = 29, normalized size = 0.62

$$-\frac{36\,b^2x^2 + 9\,abx + a^2}{252\,(bx + a)^9b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/((b*x + a)^9*b^3)
```

Mupad [B]
time = 0.15, size = 31, normalized size = 0.66

$$-\frac{8\,a^2 + 72\,a\,b\,x + 288\,b^2\,x^2}{2016\,b^3\,(a + b\,x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^10,x)
```

```
[Out] -(8*a^2 + 288*b^2*x^2 + 72*a*b*x)/(2016*b^3*(a + b*x)^9)
```

3.233 $\int \frac{x}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=30

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^10,x]
```

```
[Out] a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{10}}+\frac{1}{b(a+bx)^9}\right)dx\\ &= \frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.67

$$-\frac{a+9bx}{72b^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^10,x]
```

```
[Out] -1/72*(a + 9*b*x)/(b^2*(a + b*x)^9)
```

Maple [A]
time = 0.08, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{9bx+a}{72(bx+a)^{9}b^{2}}$	19
norman	$\frac{-\frac{x}{8b}-\frac{a}{72b^{2}}}{(bx+a)^{9}}$	22
risch	$\frac{-\frac{x}{8b}-\frac{a}{72b^{2}}}{(bx+a)^{9}}$	22
default	$\frac{a}{9b^{2}(bx+a)^{9}}-\frac{1}{8b^{2}(bx+a)^{8}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.
time = 0.47, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}b^{3}x+a^{9}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(26) = 52$.
time = 0.60, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,(b^{11}x^{9}+9\,ab^{10}x^{8}+36\,a^{2}b^{9}x^{7}+84\,a^{3}b^{8}x^{6}+126\,a^{4}b^{7}x^{5}+126\,a^{5}b^{6}x^{4}+84\,a^{6}b^{5}x^{3}+36\,a^{7}b^{4}x^{2}+9\,a^{8}b^{3}x+a^{9}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(26) = 52$.
time = 0.27, size = 116, normalized size = 3.87

$$\frac{-a - 9bx}{72a^9b^2 + 648a^8b^3x + 2592a^7b^4x^2 + 6048a^6b^5x^3 + 9072a^5b^6x^4 + 9072a^4b^7x^5 + 6048a^3b^8x^6 + 2592a^2b^9x^7 + 648ab^{10}x^8 + 72b^{11}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**10,x)
```

```
[Out] (-a - 9*b*x)/(72*a**9*b**2 + 648*a**8*b**3*x + 2592*a**7*b**4*x**2 + 6048*a
**6*b**5*x**3 + 9072*a**5*b**6*x**4 + 9072*a**4*b**7*x**5 + 6048*a**3*b**8*
x**6 + 2592*a**2*b**9*x**7 + 648*a*b**10*x**8 + 72*b**11*x**9)
```

Giac [A]
time = 1.03, size = 18, normalized size = 0.60

$$-\frac{9\,bx + a}{72\,(bx + a)^9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(9*b*x + a)/((b*x + a)^9*b^2)
```

Mupad [B]
time = 0.07, size = 18, normalized size = 0.60

$$-\frac{a + 9\,b\,x}{72\,b^2\,(a + b\,x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^10,x)
```

```
[Out] -(a + 9*b*x)/(72*b^2*(a + b*x)^9)
```

3.234 $\int \frac{1}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

```
[Out] -1/9/b/(b*x+a)^9
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-10),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9b(a+bx)^9}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-10),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{9b(bx+a)^9}$	13
default	$-\frac{1}{9b(bx+a)^9}$	13
norman	$-\frac{1}{9b(bx+a)^9}$	13
risch	$-\frac{1}{9b(bx+a)^9}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/b/(b*x+a)^9
```

Maxima [A]
time = 0.31, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(12) = 24$.
time = 1.24, size = 101, normalized size = 7.21

$$-\frac{1}{9\,(b^{10}x^9+9\,ab^9x^8+36\,a^2b^8x^7+84\,a^3b^7x^6+126\,a^4b^6x^5+126\,a^5b^5x^4+84\,a^6b^4x^3+36\,a^7b^3x^2+9\,a^8b^2x+a^9b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(12) = 24$.
time = 0.27, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**10,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

Giac [A]
time = 1.00, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^{9}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

Mupad [B]
time = 0.14, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x^7+81\,a\,b^9\,x^8+9\,b^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^10,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

3.235 $\int \frac{1}{x(a+bx)^{10}}\,dx$

Optimal. Leaf size=141

$$\frac{1}{9a(a+bx)^9}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{2a^8(\cdots}$$

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

Rubi [A]

time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^10),x]
```

```
[Out] 1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6*
a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a^7
*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10 - L
og[a + b*x]/a^10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x}-\frac{b}{a(a+bx)^{10}}-\frac{b}{a^2(a+bx)^9}-\frac{b}{a^3(a+bx)^8}-\frac{b}{a^4(a+bx)^7}-\frac{b}{a^5(a+bx)^6}-\cdots\right.\\ &= \frac{1}{9a(a+bx)^9}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{4a^6(a+\cdots}\end{aligned}$$

Mathematica [A]

time = 0.08, size = 127, normalized size = 0.90

$$\frac{280a^8+315a^7(a+bx)+360a^6(a+bx)^2+420a^5(a+bx)^3+504a^4(a+bx)^4+630a^3(a+bx)^5+840a^2(a+bx)^6+1260a(a+bx)^7+2520(a+bx)^8}{2520a^9(a+bx)^9}+\frac{\log(x)}{a^{10}}-\frac{\log(a+bx)}{a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^10),x]
```

```
[Out] (280*a^8 + 315*a^7*(a + b*x) + 360*a^6*(a + b*x)^2 + 420*a^5*(a + b*x)^3 +
504*a^4*(a + b*x)^4 + 630*a^3*(a + b*x)^5 + 840*a^2*(a + b*x)^6 + 1260*a*(a
 + b*x)^7 + 2520*(a + b*x)^8)/(2520*a^9*(a + b*x)^9) + Log[x]/a^10 - Log[a
+ b*x]/a^10
```

Maple [A]
time = 0.09, size = 126, normalized size = 0.89

method	result
risch	$\frac{\frac{b^8x^8}{a^9}+\frac{17b^7x^7}{2a^8}+\frac{191b^6x^6}{6a^7}+\frac{275b^5x^5}{4a^6}+\frac{1879b^4x^4}{20a^5}+\frac{2509b^3x^3}{30a^4}+\frac{3349b^2x^2}{70a^3}+\frac{4609bx}{280a^2}+\frac{7129}{2520a}}{(bx+a)^9}-\frac{\ln(bx+a)}{a^{10}}+\frac{\ln(-x)}{a^{10}}$
norman	$\frac{-\frac{9bx}{a^2}-\frac{54b^2x^2}{a^3}-\frac{154b^3x^3}{a^4}-\frac{525b^4x^4}{2a^5}-\frac{2877b^5x^5}{10a^6}-\frac{1029b^6x^6}{5a^7}-\frac{3267b^7x^7}{35a^8}-\frac{6849b^8x^8}{280a^9}-\frac{7129b^9x^9}{2520a^{10}}}{(bx+a)^9}+\frac{\ln(x)}{a^{10}}-\frac{\ln(bx+a)}{a^{10}}$
default	$\frac{1}{9a(bx+a)^9}+\frac{1}{8a^2(bx+a)^8}+\frac{1}{7a^3(bx+a)^7}+\frac{1}{6a^4(bx+a)^6}+\frac{1}{5a^5(bx+a)^5}+\frac{1}{4a^6(bx+a)^4}+\frac{1}{3a^7(bx+a)^3}+\frac{1}{2a^8(bx+a)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

Maxima [A]
time = 0.45, size = 205, normalized size = 1.45

$$\frac{2520\,b^8x^8+21420\,ab^7x^7+80220\,a^2b^6x^6+173250\,a^3b^5x^5+236754\,a^4b^4x^4+210756\,a^5b^3x^3+120564\,a^6b^2x^2+41481\,a^7bx+7129\,a^8}{2520\,(a^9b^9x^9+9\,a^{10}b^8x^8+36\,a^{11}b^7x^7+84\,a^{12}b^6x^6+126\,a^{13}b^5x^5+126\,a^{14}b^4x^4+84\,a^{15}b^3x^3+36\,a^{16}b^2x^2+9\,a^{17}bx+a^{18})}-\frac{\log(bx+a)}{a^{10}}+\frac{\log(x)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(2520*b^8*x^8 + 21420*a*b^7*x^7 + 80220*a^2*b^6*x^6 + 173250*a^3*b^5
*x^5 + 236754*a^4*b^4*x^4 + 210756*a^5*b^3*x^3 + 120564*a^6*b^2*x^2 + 41481
*a^7*b*x + 7129*a^8)/(a^9*b^9*x^9 + 9*a^10*b^8*x^8 + 36*a^11*b^7*x^7 + 84*a
^12*b^6*x^6 + 126*a^13*b^5*x^5 + 126*a^14*b^4*x^4 + 84*a^15*b^3*x^3 + 36*a^
16*b^2*x^2 + 9*a^17*b*x + a^18) - log(b*x + a)/a^10 + log(x)/a^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(125) = 250.
time = 1.43, size = 388, normalized size = 2.75

$$\frac{2520\,ab^8x^8+21420\,a^2b^7x^7+80220\,a^3b^6x^6+173250\,a^4b^5x^5+236754\,a^5b^4x^4+210756\,a^6b^3x^3+120564\,a^7b^2x^2+41481\,a^8bx+7129\,a^9-2520\,(b^9x^9+9\,ab^8x^8+36\,a^2b^7x^7+84\,a^3b^6x^6+126\,a^4b^5x^5+126\,a^5b^4x^4+84\,a^6b^3x^3+36\,a^7b^2x^2+9\,a^8bx+a^9)\log(bx+a)+2520\,(b^9x^9+9\,ab^8x^8+36\,a^2b^7x^7+84\,a^3b^6x^6+126\,a^4b^5x^5+126\,a^5b^4x^4+84\,a^6b^3x^3+36\,a^7b^2x^2+9\,a^8bx+a^9)\log(x)}{2520\,(a^{10}b^9x^9+9\,a^{11}b^8x^8+36\,a^{12}b^7x^7+84\,a^{13}b^6x^6+126\,a^{14}b^5x^5+126\,a^{15}b^4x^4+84\,a^{16}b^3x^3+36\,a^{17}b^2x^2+9\,a^{18}bx+a^{19})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*a*b^8*x^8 + 21420*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4
*b^5*x^5 + 236754*a^5*b^4*x^4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 4
1481*a^8*b*x + 7129*a^9 - 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 + 84
*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a^7*
b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a) + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*
a^2*b^7*x^7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b
^3*x^3 + 36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(x))/(a^10*b^9*x^9 + 9*a^11*b
^8*x^8 + 36*a^12*b^7*x^7 + 84*a^13*b^6*x^6 + 126*a^14*b^5*x^5 + 126*a^15*b^
4*x^4 + 84*a^16*b^3*x^3 + 36*a^17*b^2*x^2 + 9*a^18*b*x + a^19)
```

Sympy [A]
time = 0.42, size = 212, normalized size = 1.50

$$\frac{7129a^8 + 41481a^7bx + 120564a^6b^2x^2 + 210756a^5b^3x^3 + 236754a^4b^4x^4 + 173250a^3b^5x^5 + 80220a^2b^6x^6 + 21420ab^7x^7 + 2520b^8x^8}{2520a^{18} + 22680a^{17}bx + 90720a^{16}b^2x^2 + 211680a^{15}b^3x^3 + 317520a^{14}b^4x^4 + 317520a^{13}b^5x^5 + 211680a^{12}b^6x^6 + 90720a^{11}b^7x^7 + 22680a^{10}b^8x^8 + 2520a^9b^9x^9} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x + 120564*a**6*b**2*x**2 + 210756*a**5*b**3*x**3
 + 236754*a**4*b**4*x**4 + 173250*a**3*b**5*x**5 + 80220*a**2*b**6*x**6 + 2
1420*a*b**7*x**7 + 2520*b**8*x**8)/(2520*a**18 + 22680*a**17*b*x + 90720*a*
*16*b**2*x**2 + 211680*a**15*b**3*x**3 + 317520*a**14*b**4*x**4 + 317520*a*
*13*b**5*x**5 + 211680*a**12*b**6*x**6 + 90720*a**11*b**7*x**7 + 22680*a**1
0*b**8*x**8 + 2520*a**9*b**9*x**9) + (log(x) - log(a/b + x))/a**10
```

Giac [A]
time = 2.55, size = 120, normalized size = 0.85

$$-\frac{\log(|bx+a|)}{a^{10}} + \frac{\log(|x|)}{a^{10}} + \frac{2520\,ab^8x^8 + 21420\,a^2b^7x^7 + 80220\,a^3b^6x^6 + 173250\,a^4b^5x^5 + 236754\,a^5b^4x^4 + 210756\,a^6b^3x^3 + 120564\,a^7b^2x^2 + 41481\,a^8bx + 7129\,a^9}{2520\,(bx+a)^9a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^10 + log(abs(x))/a^10 + 1/2520*(2520*a*b^8*x^8 + 21420
*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4*b^5*x^5 + 236754*a^5*b^4*x^4
+ 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 41481*a^8*b*x + 7129*a^9)/((b*x
 + a)^9*a^10)
```

Mupad [B]
time = 0.76, size = 145, normalized size = 1.03

$$\frac{1}{9\,a\,{\left(a+b\,x\right)}^9}-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{14\,b^2\,x^2}{{\left(a+b\,x\right)}^2}+\frac{56\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{35\,b^4\,x^4}{2\,{\left(a+b\,x\right)}^4}+\frac{56\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,b^6\,x^6}{3\,{\left(a+b\,x\right)}^6}+\frac{8\,b^7\,x^7}{7\,{\left(a+b\,x\right)}^7}-\frac{b^8\,x^8}{8\,{\left(a+b\,x\right)}^8}+\frac{8\,b\,x}{a+b\,x}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^10),x)
```

```
[Out] 1/(9*a*(a + b*x)^9) - (log((a + b*x)/x) - (14*b^2*x^2)/(a + b*x)^2 + (56*b^
3*x^3)/(3*(a + b*x)^3) - (35*b^4*x^4)/(2*(a + b*x)^4) + (56*b^5*x^5)/(5*(a
+ b*x)^5) - (14*b^6*x^6)/(3*(a + b*x)^6) + (8*b^7*x^7)/(7*(a + b*x)^7) - (b
^8*x^8)/(8*(a + b*x)^8) + (8*b*x)/(a + b*x))/a^10
```

3.236 $\int \frac{1}{x^2(a+bx)^{10}}\,dx$

Optimal. Leaf size=158

$$-\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}-\frac{3b}{2a^7(a+bx)^4}-\frac{7b}{3a^8(a+bx)}$$

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

Rubi [A]

time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{9b}{a^{10}(a+bx)}-\frac{1}{a^{10}x}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}-\frac{b}{9a^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a^
4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/(2
*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2) - (
9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\frac{5}{a^6(a\right.\\ &= -\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}\end{aligned}$$

Mathematica [A]

time = 0.09, size = 130, normalized size = 0.82

$$-\frac{\frac{a\left(252a^9+7129a^8bx+41481a^7b^2x^2+120564a^6b^3x^3+210756a^5b^4x^4+236754a^4b^5x^5+173250a^3b^6x^6+80220a^2b^7x^7+21420ab^8x^8+2520b^9x^9\right)}{x(a+bx)^9}+2520b\log(x)-2520b\log(a+bx)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -1/252*((a*(252*a^9 + 7129*a^8*b*x + 41481*a^7*b^2*x^2 + 120564*a^6*b^3*x^3
 + 210756*a^5*b^4*x^4 + 236754*a^4*b^5*x^5 + 173250*a^3*b^6*x^6 + 80220*a^2
*b^7*x^7 + 21420*a*b^8*x^8 + 2520*b^9*x^9))/(x*(a + b*x)^9) + 2520*b*Log[x]
 - 2520*b*Log[a + b*x])/a^11
```

Maple [A]
time = 0.09, size = 147, normalized size = 0.93

method	result
risch	$\frac{-\frac{10b^9x^9}{a^{10}}-\frac{85b^8x^8}{a^9}-\frac{955b^7x^7}{3a^8}-\frac{1375b^6x^6}{2a^7}-\frac{1879b^5x^5}{2a^6}-\frac{2509b^4x^4}{3a^5}-\frac{3349b^3x^3}{7a^4}-\frac{4609b^2x^2}{28a^3}-\frac{7129bx}{252a^2}-\frac{1}{a}}{x(bx+a)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{10b\ln(-bx-a)}{a^{11}}$
norman	$\frac{-\frac{1}{a}+\frac{90b^2x^2}{a^3}+\frac{540b^3x^3}{a^4}+\frac{1540b^4x^4}{a^5}+\frac{2625b^5x^5}{a^6}+\frac{2877b^6x^6}{a^7}+\frac{2058b^7x^7}{a^8}+\frac{6534b^8x^8}{7a^9}+\frac{6849b^9x^9}{28a^{10}}+\frac{7129b^{10}x^{10}}{252a^{11}}}{x(bx+a)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{10b\ln(bx+a}{a^{11}}$
default	$-\frac{1}{a^{10}x}-\frac{b}{9a^2(bx+a)^9}-\frac{b}{4a^3(bx+a)^8}-\frac{3b}{7a^4(bx+a)^7}-\frac{2b}{3a^5(bx+a)^6}-\frac{b}{a^6(bx+a)^5}-\frac{3b}{2a^7(bx+a)^4}-\frac{7b}{3a^8(bx+a)^3}-\frac{}{a^9(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

Maxima [A]
time = 0.33, size = 223, normalized size = 1.41

$$-\frac{2520\,b^9x^9+21420\,ab^8x^8+80220\,a^2b^7x^7+173250\,a^3b^6x^6+236754\,a^4b^5x^5+210756\,a^5b^4x^4+120564\,a^6b^3x^3+41481\,a^7b^2x^2+7129\,a^8bx+252\,a^9}{252\,(a^{10}b^9x^{10}+9\,a^{11}b^8x^9+36\,a^{12}b^7x^8+84\,a^{13}b^6x^7+126\,a^{14}b^5x^6+126\,a^{15}b^4x^5+84\,a^{16}b^3x^4+36\,a^{17}b^2x^3+9\,a^{18}bx^2+a^{19}x)}+\frac{10\,b\log(bx+a)}{a^{11}}-\frac{10\,b\log(x)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 21420*a*b^8*x^8 + 80220*a^2*b^7*x^7 + 173250*a^3*b^6
*x^6 + 236754*a^4*b^5*x^5 + 210756*a^5*b^4*x^4 + 120564*a^6*b^3*x^3 + 41481
*a^7*b^2*x^2 + 7129*a^8*b*x + 252*a^9)/(a^10*b^9*x^10 + 9*a^11*b^8*x^9 + 36
*a^12*b^7*x^8 + 84*a^13*b^6*x^7 + 126*a^14*b^5*x^6 + 126*a^15*b^4*x^5 + 84*
a^16*b^3*x^4 + 36*a^17*b^2*x^3 + 9*a^18*b*x^2 + a^19*x) + 10*b*log(b*x + a)
/a^11 - 10*b*log(x)/a^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(146) = 292$.
time = 1.47, size = 417, normalized size = 2.64

$$-\frac{2520\,ab^9x^9+21420\,a^2b^8x^8+80220\,a^3b^7x^7+173250\,a^4b^6x^6+236754\,a^5b^5x^5+210756\,a^6b^4x^4+120564\,a^7b^3x^3+41481\,a^8b^2x^2+7129\,a^9bx+252\,a^{10}-2520\,(b^{10}x^{10}+9\,ab^9x^9+36\,a^2b^8x^8+84\,a^3b^7x^7+126\,a^4b^6x^6+126\,a^5b^5x^5+84\,a^6b^4x^4+36\,a^7b^3x^3+9\,a^8b^2x^2+a^9bx)\log(bx+a)+2520\,(b^{10}x^{10}+9\,ab^9x^9+36\,a^2b^8x^8+84\,a^3b^7x^7+126\,a^4b^6x^6+126\,a^5b^5x^5+84\,a^6b^4x^4+36\,a^7b^3x^3+9\,a^8b^2x^2+a^9bx)\log(x)}{252\,(a^{11}b^9x^{10}+9\,a^{12}b^8x^9+36\,a^{13}b^7x^8+84\,a^{14}b^6x^7+126\,a^{15}b^5x^6+126\,a^{16}b^4x^5+84\,a^{17}b^3x^4+36\,a^{18}b^2x^3+9\,a^{19}bx^2+a^{20}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4
*b^6*x^6 + 236754*a^5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 4
1481*a^8*b^2*x^2 + 7129*a^9*b*x + 252*a^10 - 2520*(b^10*x^10 + 9*a*b^9*x^9
+ 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 84*
a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^9*b*x)*log(b*x + a) + 2520
*(b^10*x^10 + 9*a*b^9*x^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x
^6 + 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^
9*b*x)*log(x))/(a^11*b^9*x^10 + 9*a^12*b^8*x^9 + 36*a^13*b^7*x^8 + 84*a^14*
b^6*x^7 + 126*a^15*b^5*x^6 + 126*a^16*b^4*x^5 + 84*a^17*b^3*x^4 + 36*a^18*b
^2*x^3 + 9*a^19*b*x^2 + a^20*x)
```

Sympy [A]
time = 0.47, size = 233, normalized size = 1.47

$$\frac{-252a^9-7129a^8bx-41481a^7b^2x^2-120564a^6b^3x^3-210756a^5b^4x^4-236754a^4b^5x^5-173250a^3b^6x^6-80220a^2b^7x^7-21420ab^8x^8-2520b^9x^9}{252a^{19}x+2268a^{18}bx^2+9072a^{17}b^2x^3+21168a^{16}b^3x^4+31752a^{15}b^4x^5+31752a^{14}b^5x^6+21168a^{13}b^6x^7+9072a^{12}b^7x^8+2268a^{11}b^8x^9+252a^{10}b^9x^{10}}+\frac{10b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x - 41481*a**7*b**2*x**2 - 120564*a**6*b**3*x**3 -
 210756*a**5*b**4*x**4 - 236754*a**4*b**5*x**5 - 173250*a**3*b**6*x**6 - 80
220*a**2*b**7*x**7 - 21420*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**19*x + 226
8*a**18*b*x**2 + 9072*a**17*b**2*x**3 + 21168*a**16*b**3*x**4 + 31752*a**15
*b**4*x**5 + 31752*a**14*b**5*x**6 + 21168*a**13*b**6*x**7 + 9072*a**12*b**
7*x**8 + 2268*a**11*b**8*x**9 + 252*a**10*b**9*x**10) + 10*b*(-log(x) + log
(a/b + x))/a**11
```

Giac [A]
time = 1.58, size = 137, normalized size = 0.87

$$\frac{10\,b\log\left(|bx+a|\right)}{a^{11}}-\frac{10\,b\log\left(|x|\right)}{a^{11}}-\frac{2520\,ab^9x^9+21420\,a^2b^8x^8+80220\,a^3b^7x^7+173250\,a^4b^6x^6+236754\,a^5b^5x^5+210756\,a^6b^4x^4+120564\,a^7b^3x^3+41481\,a^8b^2x^2+7129\,a^9bx+252\,a^{10}}{252\,(bx+a)^9a^{11}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 10*b*log(abs(b*x + a))/a^11 - 10*b*log(abs(x))/a^11 - 1/252*(2520*a*b^9*x^9
 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4*b^6*x^6 + 236754*a^5*
b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 41481*a^8*b^2*x^2 + 712
9*a^9*b*x + 252*a^10)/((b*x + a)^9*a^11*x)
```

Mupad [B]
time = 0.39, size = 217, normalized size = 1.37

$$\frac{20\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{a}+\frac{4609\,b^2\,x^2}{28\,a^3}+\frac{3349\,b^3\,x^3}{7\,a^4}+\frac{2509\,b^4\,x^4}{3\,a^5}+\frac{1879\,b^5\,x^5}{2\,a^6}+\frac{1375\,b^6\,x^6}{2\,a^7}+\frac{955\,b^7\,x^7}{3\,a^8}+\frac{85\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}+\frac{7129\,b\,x}{252\,a^2}}{a^9\,x+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^3+84\,a^6\,b^3\,x^4+126\,a^5\,b^4\,x^5+126\,a^4\,b^5\,x^6+84\,a^3\,b^6\,x^7+36\,a^2\,b^7\,x^8+9\,a\,b^8\,x^9+b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^10),x)
```

```
[Out] (20*b*atanh((2*b*x)/a + 1))/a^11 - (1/a + (4609*b^2*x^2)/(28*a^3) + (3349*b
^3*x^3)/(7*a^4) + (2509*b^4*x^4)/(3*a^5) + (1879*b^5*x^5)/(2*a^6) + (1375*b
^6*x^6)/(2*a^7) + (955*b^7*x^7)/(3*a^8) + (85*b^8*x^8)/a^9 + (10*b^9*x^9)/a
^10 + (7129*b*x)/(252*a^2))/(a^9*x + b^9*x^10 + 9*a^8*b*x^2 + 9*a*b^8*x^9 +
 36*a^7*b^2*x^3 + 84*a^6*b^3*x^4 + 126*a^5*b^4*x^5 + 126*a^4*b^5*x^6 + 84*a
^3*b^6*x^7 + 36*a^2*b^7*x^8)
```

3.237 $\int \frac{1}{x^3(a+bx)^{10}}\,dx$

Optimal. Leaf size=191

$$-\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(a+bx)^9}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{3b^2}{a^7(a+bx)^5}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{}{3a}$$

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Rubi [A]
time = 0.10, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$\frac{55b^2\log(x)}{a^{12}}-\frac{55b^2\log(a+bx)}{a^{12}}+\frac{45b^2}{a^{11}(a+bx)}+\frac{10b}{a^{11}x}+\frac{18b^2}{a^{10}(a+bx)^2}-\frac{1}{2a^{10}x^2}+\frac{28b^2}{3a^9(a+bx)^3}+\frac{21b^2}{4a^8(a+bx)^4}+\frac{3b^2}{a^7(a+bx)^5}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{b^2}{9a^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^10),x]
```

```
[Out] -1/2*1/(a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8*
a^4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)^6
) + (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/(3*
a^9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*x))
+ (55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^3}-\frac{10b}{a^{11}x^2}+\frac{55b^2}{a^{12}x}-\frac{b^3}{a^3(a+bx)^{10}}-\frac{3b^3}{a^4(a+bx)^9}-\frac{6b^3}{a^5(a+bx)^8}-\frac{10b^3}{a^6(a+bx)^7}\right.\\ &= -\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(a+bx)^9}+\frac{3b^2}{8a^4(a+bx)^8}+\frac{6b^2}{7a^5(a+bx)^7}+\frac{5b^2}{3a^6(a+bx)^6}+\frac{}{a^7(}\end{aligned}$$

Mathematica [A]

time = 0.08, size = 145, normalized size = 0.76

$$\frac{\frac{a\left(-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10}\right)}{x^2(a+bx)^9}+27720b^2\log(x)-27720b^2\log(a+bx)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x + 78419*a^8*b^2*x^2 + 456291*a^7*b^3*x^3 + 13
26204*a^6*b^4*x^4 + 2318316*a^5*b^5*x^5 + 2604294*a^4*b^6*x^6 + 1905750*a^3
*b^7*x^7 + 882420*a^2*b^8*x^8 + 235620*a*b^9*x^9 + 27720*b^10*x^10))/(x^2*(
a + b*x)^9) + 27720*b^2*Log[x] - 27720*b^2*Log[a + b*x])/(504*a^12)
```

Maple [A]

time = 0.09, size = 178, normalized size = 0.93

method	result
norman	$\frac{-\frac{1}{2a}+\frac{11bx}{2a^2}-\frac{495b^3x^3}{a^4}-\frac{2970b^4x^4}{a^5}-\frac{8470b^5x^5}{a^6}-\frac{28875b^6x^6}{2a^7}-\frac{31647b^7x^7}{2a^8}-\frac{11319b^8x^8}{a^9}-\frac{35937b^9x^9}{7a^{10}}-\frac{75339b^{10}x^{10}}{56a^{11}}-\frac{78419b^{11}x^{11}}{504a^{12}}}{x^2(bx+a)^9}+\frac{55b^2\ln(}{a^{12}}$
risch	$\frac{\frac{55b^{10}x^{10}}{a^{11}}+\frac{935b^9x^9}{2a^{10}}+\frac{10505b^8x^8}{6a^9}+\frac{15125b^7x^7}{4a^8}+\frac{20669b^6x^6}{4a^7}+\frac{27599b^5x^5}{6a^6}+\frac{36839b^4x^4}{14a^5}+\frac{50699b^3x^3}{56a^4}+\frac{78419b^2x^2}{504a^3}+\frac{11bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)^9}-\frac{55b^2\ln(bx+a}{a^{12}}$
default	$-\frac{1}{2a^{10}x^2}+\frac{10b}{a^{11}x}+\frac{b^2}{9a^3(bx+a)^9}+\frac{3b^2}{8a^4(bx+a)^8}+\frac{6b^2}{7a^5(bx+a)^7}+\frac{5b^2}{3a^6(bx+a)^6}+\frac{3b^2}{a^7(bx+a)^5}+\frac{21b^2}{4a^8(bx+a)^4}+\frac{28b^2}{3a^9(bx+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

Maxima [A]

time = 0.31, size = 240, normalized size = 1.26

$$\frac{27720\,b^{10}x^{10}+235620\,ab^9x^9+882420\,a^2b^8x^8+1905750\,a^3b^7x^7+2604294\,a^4b^6x^6+2318316\,a^5b^5x^5+1326204\,a^6b^4x^4+456291\,a^7b^3x^3+78419\,a^8b^2x^2+2772\,a^9bx-252\,a^{10}}{504\left(a^{11}b^9x^{11}+9\,a^{12}b^8x^{10}+36\,a^{13}b^7x^9+84\,a^{14}b^6x^8+126\,a^{15}b^5x^7+126\,a^{16}b^4x^6+84\,a^{17}b^3x^5+36\,a^{18}b^2x^4+9\,a^{19}bx^3+a^{20}x^2\right)}-\frac{55\,b^2\log\left(bx+a\right)}{a^{12}}+\frac{55\,b^2\log\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(27720*b^10*x^10 + 235620*a*b^9*x^9 + 882420*a^2*b^8*x^8 + 1905750*a^
3*b^7*x^7 + 2604294*a^4*b^6*x^6 + 2318316*a^5*b^5*x^5 + 1326204*a^6*b^4*x^4
 + 456291*a^7*b^3*x^3 + 78419*a^8*b^2*x^2 + 2772*a^9*b*x - 252*a^10)/(a^11*
b^9*x^11 + 9*a^12*b^8*x^10 + 36*a^13*b^7*x^9 + 84*a^14*b^6*x^8 + 126*a^15*b
```

```
^5*x^7 + 126*a^16*b^4*x^6 + 84*a^17*b^3*x^5 + 36*a^18*b^2*x^4 + 9*a^19*b*x^
3 + a^20*x^2) - 55*b^2*log(b*x + a)/a^12 + 55*b^2*log(x)/a^12
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(177) = 354$.
time = 1.18, size = 438, normalized size = 2.29

$$\frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 2604294\,a^5b^6x^6 + 2318316\,a^6b^5x^5 + 1326204\,a^7b^4x^4 + 456291\,a^8b^3x^3 + 78419\,a^9b^2x^2 + 2772\,a^{10}bx - 252\,a^{11} - 27720\,(b^{11}x^{11} + 9\,ab^{10}x^{10} + 36\,a^2b^9x^9 + 84\,a^3b^8x^8 + 126\,a^4b^7x^7 + 126\,a^5b^6x^6 + 84\,a^6b^5x^5 + 36\,a^7b^4x^4 + 9\,a^8b^3x^3 + a^9b^2x^2)\log(bx+a) + 27720\,(b^{11}x^{11} + 9\,ab^{10}x^{10} + 36\,a^2b^9x^9 + 84\,a^3b^8x^8 + 126\,a^4b^7x^7 + 126\,a^5b^6x^6 + 84\,a^6b^5x^5 + 36\,a^7b^4x^4 + 9\,a^8b^3x^3 + a^9b^2x^2)\log(x)}{504\,(a^{12}b^9x^{11} + 9\,a^{13}b^8x^{10} + 36\,a^{14}b^7x^9 + 84\,a^{15}b^6x^8 + 126\,a^{16}b^5x^7 + 126\,a^{17}b^4x^6 + 84\,a^{18}b^3x^5 + 36\,a^{19}b^2x^4 + 9\,a^{20}bx^3 + a^{21}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(27720*a*b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 190575
0*a^4*b^7*x^7 + 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4
*x^4 + 456291*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11 -
27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^
4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x
^3 + a^9*b^2*x^2)*log(b*x + a) + 27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*
b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x
^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x^3 + a^9*b^2*x^2)*log(x))/(a^12*b^9*x^11 +
 9*a^13*b^8*x^10 + 36*a^14*b^7*x^9 + 84*a^15*b^6*x^8 + 126*a^16*b^5*x^7 + 1
26*a^17*b^4*x^6 + 84*a^18*b^3*x^5 + 36*a^19*b^2*x^4 + 9*a^20*b*x^3 + a^21*x
^2)
```

Sympy [A]
time = 0.49, size = 246, normalized size = 1.29

$$\frac{-252a^{10} + 2772a^9bx + 78419a^8b^2x^2 + 456291a^7b^3x^3 + 1326204a^6b^4x^4 + 2318316a^5b^5x^5 + 2604294a^4b^6x^6 + 1905750a^3b^7x^7 + 882420a^2b^8x^8 + 235620ab^9x^9 + 27720b^{10}x^{10}}{504a^{20}x^2 + 4536a^{19}bx^3 + 18144a^{18}b^2x^4 + 42336a^{17}b^3x^5 + 63504a^{16}b^4x^6 + 63504a^{15}b^5x^7 + 42336a^{14}b^6x^8 + 18144a^{13}b^7x^9 + 4536a^{12}b^8x^{10} + 504a^{11}b^9x^{11}} + \frac{55b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x + 78419*a**8*b**2*x**2 + 456291*a**7*b**3*x**3
+ 1326204*a**6*b**4*x**4 + 2318316*a**5*b**5*x**5 + 2604294*a**4*b**6*x**6
+ 1905750*a**3*b**7*x**7 + 882420*a**2*b**8*x**8 + 235620*a*b**9*x**9 + 277
20*b**10*x**10)/(504*a**20*x**2 + 4536*a**19*b*x**3 + 18144*a**18*b**2*x**4
 + 42336*a**17*b**3*x**5 + 63504*a**16*b**4*x**6 + 63504*a**15*b**5*x**7 +
42336*a**14*b**6*x**8 + 18144*a**13*b**7*x**9 + 4536*a**12*b**8*x**10 + 504
*a**11*b**9*x**11) + 55*b**2*(log(x) - log(a/b + x))/a**12
```

Giac [A]
time = 2.01, size = 152, normalized size = 0.80

$$-\frac{55\,b^2\log(|bx+a|)}{a^{12}} + \frac{55\,b^2\log(|x|)}{a^{12}} + \frac{27720\,ab^{10}x^{10} + 235620\,a^2b^9x^9 + 882420\,a^3b^8x^8 + 1905750\,a^4b^7x^7 + 2604294\,a^5b^6x^6 + 2318316\,a^6b^5x^5 + 1326204\,a^7b^4x^4 + 456291\,a^8b^3x^3 + 78419\,a^9b^2x^2 + 2772\,a^{10}bx - 252\,a^{11}}{504\,(bx+a)^9a^{12}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -55*b^2*log(abs(b*x + a))/a^12 + 55*b^2*log(abs(x))/a^12 + 1/504*(27720*a*b
^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 1905750*a^4*b^7*x^7 +
2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4*x^4 + 456291*a^
8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11)/((b*x + a)^9*a^12
*x^2)
```

Mupad [B]
time = 0.44, size = 233, normalized size = 1.22

$$\frac{\frac{78419\,b^2\,x^2}{504\,a^3}-\frac{1}{2\,a}+\frac{50699\,b^3\,x^3}{56\,a^4}+\frac{36839\,b^4\,x^4}{14\,a^5}+\frac{27599\,b^5\,x^5}{6\,a^6}+\frac{20669\,b^6\,x^6}{4\,a^7}+\frac{15125\,b^7\,x^7}{4\,a^8}+\frac{10505\,b^8\,x^8}{6\,a^9}+\frac{935\,b^9\,x^9}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}+\frac{11\,b\,x}{2\,a^2}}{a^9\,x^2+9\,a^8\,b\,x^3+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^5+126\,a^5\,b^4\,x^6+126\,a^4\,b^5\,x^7+84\,a^3\,b^6\,x^8+36\,a^2\,b^7\,x^9+9\,a\,b^8\,x^{10}+b^9\,x^{11}}-\frac{110\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^10),x)
```

```
[Out] ((78419*b^2*x^2)/(504*a^3) - 1/(2*a) + (50699*b^3*x^3)/(56*a^4) + (36839*b^
4*x^4)/(14*a^5) + (27599*b^5*x^5)/(6*a^6) + (20669*b^6*x^6)/(4*a^7) + (1512
5*b^7*x^7)/(4*a^8) + (10505*b^8*x^8)/(6*a^9) + (935*b^9*x^9)/(2*a^10) + (55
*b^10*x^10)/a^11 + (11*b*x)/(2*a^2))/(a^9*x^2 + b^9*x^11 + 9*a^8*b*x^3 + 9*
a*b^8*x^10 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^5 + 126*a^5*b^4*x^6 + 126*a^4*b^
5*x^7 + 84*a^3*b^6*x^8 + 36*a^2*b^7*x^9) - (110*b^2*atanh((2*b*x)/a + 1))/a
^12
```

3.238 $\int \frac{1}{x^4(a+bx)^{10}}\,dx$

Optimal. Leaf size=198

$$-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{7b^3}{a^8(a+bx)^5}-\frac{14b^3}{a^9(a+bx}$$

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Rubi [A]

time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}-\frac{165b^3}{a^{12}(a+bx)}-\frac{55b^2}{a^{12}x}-\frac{60b^3}{a^{11}(a+bx)^2}+\frac{5b}{a^{11}x^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{1}{3a^{10}x^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-\frac{b^3}{9a^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/3*1/(a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a +
b*x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^3)
/(3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*x)^
4) - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b^3)/
(a^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^4}-\frac{10b}{a^{11}x^3}+\frac{55b^2}{a^{12}x^2}-\frac{220b^3}{a^{13}x}+\frac{b^4}{a^4(a+bx)^{10}}+\frac{4b^4}{a^5(a+bx)^9}+\frac{10b^4}{a^6(a+bx)^8}+\frac{}{a^7}\right.\\ &= -\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx}\end{aligned}$$

Mathematica [A]

time = 0.09, size = 156, normalized size = 0.79

$$-\frac{\frac{a(42a^{11}-252a^{10}bx+2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7+1905750a^3b^8x^8+882420a^2b^9x^9+235620ab^{10}x^{10}+27720b^{11}x^{11})}{x^3(a+bx)^9}+27720b^3\log(x)-27720b^3\log(a+bx)}{126a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/126*((a*(42*a^11 - 252*a^10*b*x + 2772*a^9*b^2*x^2 + 78419*a^8*b^3*x^3 +
 456291*a^7*b^4*x^4 + 1326204*a^6*b^5*x^5 + 2318316*a^5*b^6*x^6 + 2604294*a
^4*b^7*x^7 + 1905750*a^3*b^8*x^8 + 882420*a^2*b^9*x^9 + 235620*a*b^10*x^10
+ 27720*b^11*x^11))/(x^3*(a + b*x)^9) + 27720*b^3*Log[x] - 27720*b^3*Log[a
+ b*x])/a^13
```

Maple [A]
time = 0.14, size = 189, normalized size = 0.95

method	result
norman	$\frac{-\frac{1}{3a}+\frac{2bx}{a^2}-\frac{22b^2x^2}{a^3}+\frac{1980b^4x^4}{a^5}+\frac{11880b^5x^5}{a^6}+\frac{33880b^6x^6}{a^7}+\frac{57750b^7x^7}{a^8}+\frac{63294b^8x^8}{a^9}+\frac{45276b^9x^9}{a^{10}}+\frac{143748b^{10}x^{10}}{7a^{11}}+\frac{75339b^{11}x^{11}}{14a^{12}}+\frac{78419b^{12}x^{12}}{126a^{13}}}{x^3(bx+a)^9}$
risch	$\frac{-\frac{220b^{11}x^{11}}{a^{12}}-\frac{1870b^{10}x^{10}}{a^{11}}-\frac{21010b^9x^9}{3a^{10}}-\frac{15125b^8x^8}{a^9}-\frac{20669b^7x^7}{a^8}-\frac{55198b^6x^6}{3a^7}-\frac{73678b^5x^5}{7a^6}-\frac{50699b^4x^4}{14a^5}-\frac{78419b^3x^3}{126a^4}-\frac{22b^2x^2}{a^3}+\frac{2bx}{a^2}-\frac{1}{3a}}{x^3(bx+a)^9}-$
default	$-\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(bx+a)^9}-\frac{b^3}{2a^5(bx+a)^8}-\frac{10b^3}{7a^6(bx+a)^7}-\frac{10b^3}{3a^7(bx+a)^6}-\frac{7b^3}{a^8(bx+a)^5}-\frac{14b^3}{a^9(bx+a)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

Maxima [A]
time = 0.32, size = 251, normalized size = 1.27

$$-\frac{27720\,b^{11}x^{11}+235620\,ab^{10}x^{10}+882420\,a^2b^9x^9+1905750\,a^3b^8x^8+2604294\,a^4b^7x^7+2318316\,a^5b^6x^6+1326204\,a^6b^5x^5+456291\,a^7b^4x^4+78419\,a^8b^3x^3+2772\,a^9b^2x^2-252\,a^{10}bx+42\,a^{11}}{126\,(a^{12}b^9x^{12}+9\,a^{13}b^8x^{11}+36\,a^{14}b^7x^{10}+84\,a^{15}b^6x^9+126\,a^{16}b^5x^8+126\,a^{17}b^4x^7+84\,a^{18}b^3x^6+36\,a^{19}b^2x^5+9\,a^{20}bx^4+a^{21}x^3)}+\frac{220\,b^3\log(bx+a)}{a^{13}}-\frac{220\,b^3\log(x)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(27720*b^11*x^11 + 235620*a*b^10*x^10 + 882420*a^2*b^9*x^9 + 1905750
*a^3*b^8*x^8 + 2604294*a^4*b^7*x^7 + 2318316*a^5*b^6*x^6 + 1326204*a^6*b^5*
x^5 + 456291*a^7*b^4*x^4 + 78419*a^8*b^3*x^3 + 2772*a^9*b^2*x^2 - 252*a^10*
b*x + 42*a^11)/(a^12*b^9*x^12 + 9*a^13*b^8*x^11 + 36*a^14*b^7*x^10 + 84*a^1
5*b^6*x^9 + 126*a^16*b^5*x^8 + 126*a^17*b^4*x^7 + 84*a^18*b^3*x^6 + 36*a^19
*b^2*x^5 + 9*a^20*b*x^4 + a^21*x^3) + 220*b^3*log(b*x + a)/a^13 - 220*b^3*l
og(x)/a^13
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(188) = 376$.

time = 0.69, size = 449, normalized size = 2.27

$$-\frac{27720\,ab^{11}x^{11}+235620\,a^{2}b^{10}x^{10}+882420\,a^{3}b^{9}x^{9}+1905750\,a^{4}b^{8}x^{8}+2604294\,a^{5}b^{7}x^{7}+2318316\,a^{6}b^{6}x^{6}+1326204\,a^{7}b^{5}x^{5}+456291\,a^{8}b^{4}x^{4}+78419\,a^{9}b^{3}x^{3}+2772\,a^{10}b^{2}x^{2}-252\,a^{11}bx+42\,a^{12}-27720\,(b^{12}x^{12}+9\,ab^{11}x^{11}+36\,a^{2}b^{10}x^{10}+84\,a^{3}b^{9}x^{9}+126\,a^{4}b^{8}x^{8}+126\,a^{5}b^{7}x^{7}+84\,a^{6}b^{6}x^{6}+36\,a^{7}b^{5}x^{5}+9\,a^{8}b^{4}x^{4}+a^{9}b^{3}x^{3})\log(bx+a)+27720\,(b^{12}x^{12}+9\,ab^{11}x^{11}+36\,a^{2}b^{10}x^{10}+84\,a^{3}b^{9}x^{9}+126\,a^{4}b^{8}x^{8}+126\,a^{5}b^{7}x^{7}+84\,a^{6}b^{6}x^{6}+36\,a^{7}b^{5}x^{5}+9\,a^{8}b^{4}x^{4}+a^{9}b^{3}x^{3})\log(x)}{126\,(a^{13}b^{9}x^{12}+9\,a^{14}b^{8}x^{11}+36\,a^{15}b^{7}x^{10}+84\,a^{16}b^{6}x^{9}+126\,a^{17}b^{5}x^{8}+126\,a^{18}b^{4}x^{7}+84\,a^{19}b^{3}x^{6}+36\,a^{20}b^{2}x^{5}+9\,a^{21}bx^{4}+a^{22}x^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/126*(27720*a*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 190
5750*a^4*b^8*x^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*
b^5*x^5 + 456291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*
a^11*b*x + 42*a^12 - 27720*(b^12*x^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^10 +
84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^
7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)*log(b*x + a) + 27720*(b^12*x^12 +
9*a*b^11*x^11 + 36*a^2*b^10*x^10 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a
^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)
*log(x))/(a^13*b^9*x^12 + 9*a^14*b^8*x^11 + 36*a^15*b^7*x^10 + 84*a^16*b^6*
x^9 + 126*a^17*b^5*x^8 + 126*a^18*b^4*x^7 + 84*a^19*b^3*x^6 + 36*a^20*b^2*x
^5 + 9*a^21*b*x^4 + a^22*x^3)
```

Sympy [A]
time = 0.51, size = 258, normalized size = 1.30

$$\frac{-42a^{11}+252a^{10}bx-2772a^{9}b^{2}x^{2}-78419a^{8}b^{3}x^{3}-456291a^{7}b^{4}x^{4}-1326204a^{6}b^{5}x^{5}-2318316a^{5}b^{6}x^{6}-2604294a^{4}b^{7}x^{7}-1905750a^{3}b^{8}x^{8}-882420a^{2}b^{9}x^{9}-235620ab^{10}x^{10}-27720b^{11}x^{11}}{126a^{21}x^{3}+1134a^{20}bx^{4}+4536a^{19}b^{2}x^{5}+10584a^{18}b^{3}x^{6}+15876a^{17}b^{4}x^{7}+15876a^{16}b^{5}x^{8}+10584a^{15}b^{6}x^{9}+4536a^{14}b^{7}x^{10}+1134a^{13}b^{8}x^{11}+126a^{12}b^{9}x^{12}}+\frac{220b^{3}\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x - 2772*a**9*b**2*x**2 - 78419*a**8*b**3*x**3 - 4
56291*a**7*b**4*x**4 - 1326204*a**6*b**5*x**5 - 2318316*a**5*b**6*x**6 - 26
04294*a**4*b**7*x**7 - 1905750*a**3*b**8*x**8 - 882420*a**2*b**9*x**9 - 235
620*a*b**10*x**10 - 27720*b**11*x**11)/(126*a**21*x**3 + 1134*a**20*b*x**4
+ 4536*a**19*b**2*x**5 + 10584*a**18*b**3*x**6 + 15876*a**17*b**4*x**7 + 15
876*a**16*b**5*x**8 + 10584*a**15*b**6*x**9 + 4536*a**14*b**7*x**10 + 1134*
a**13*b**8*x**11 + 126*a**12*b**9*x**12) + 220*b**3*(-log(x) + log(a/b + x)
)/a**13
```

Giac [A]
time = 1.76, size = 163, normalized size = 0.82

$$\frac{220\,b^{3}\log(|bx+a|)}{a^{13}}-\frac{220\,b^{3}\log(|x|)}{a^{13}}-\frac{27720\,ab^{11}x^{11}+235620\,a^{2}b^{10}x^{10}+882420\,a^{3}b^{9}x^{9}+1905750\,a^{4}b^{8}x^{8}+2604294\,a^{5}b^{7}x^{7}+2318316\,a^{6}b^{6}x^{6}+1326204\,a^{7}b^{5}x^{5}+456291\,a^{8}b^{4}x^{4}+78419\,a^{9}b^{3}x^{3}+2772\,a^{10}b^{2}x^{2}-252\,a^{11}bx+42\,a^{12}}{126\,(bx+a)^{9}a^{13}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 220*b^3*log(abs(b*x + a))/a^13 - 220*b^3*log(abs(x))/a^13 - 1/126*(27720*a*
b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 1905750*a^4*b^8*x^8
```

```
 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*b^5*x^5 + 456291
*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*a^11*b*x + 42*a^
12)/((b*x + a)^9*a^13*x^3)
```

Mupad [B]
time = 0.59, size = 245, normalized size = 1.24

$$\frac{440\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{13}}-\frac{\frac{1}{3\,a}+\frac{22\,b^2\,x^2}{a^3}+\frac{78419\,b^3\,x^3}{126\,a^4}+\frac{50699\,b^4\,x^4}{14\,a^5}+\frac{73678\,b^5\,x^5}{7\,a^6}+\frac{55198\,b^6\,x^6}{3\,a^7}+\frac{20669\,b^7\,x^7}{a^8}+\frac{15125\,b^8\,x^8}{a^9}+\frac{21010\,b^9\,x^9}{3\,a^{10}}+\frac{1870\,b^{10}\,x^{10}}{a^{11}}+\frac{220\,b^{11}\,x^{11}}{a^{12}}-\frac{2\,b\,x}{a^2}}{a^9\,x^3+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^5+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^7+126\,a^4\,b^5\,x^8+84\,a^3\,b^6\,x^9+36\,a^2\,b^7\,x^{10}+9\,a\,b^8\,x^{11}+b^9\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^10),x)
```

```
[Out] (440*b^3*atanh((2*b*x)/a + 1))/a^13 - (1/(3*a) + (22*b^2*x^2)/a^3 + (78419*
b^3*x^3)/(126*a^4) + (50699*b^4*x^4)/(14*a^5) + (73678*b^5*x^5)/(7*a^6) + (
55198*b^6*x^6)/(3*a^7) + (20669*b^7*x^7)/a^8 + (15125*b^8*x^8)/a^9 + (21010
*b^9*x^9)/(3*a^10) + (1870*b^10*x^10)/a^11 + (220*b^11*x^11)/a^12 - (2*b*x)
/a^2)/(a^9*x^3 + b^9*x^12 + 9*a^8*b*x^4 + 9*a*b^8*x^11 + 36*a^7*b^2*x^5 + 8
4*a^6*b^3*x^6 + 126*a^5*b^4*x^7 + 126*a^4*b^5*x^8 + 84*a^3*b^6*x^9 + 36*a^2
*b^7*x^10)
```

3.239 $\int \frac{(a+bx)^{12}}{x^{10}} \, dx$

Optimal. Leaf size=141

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12/x^10,x]
```

```
[Out] -1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{12}}{x^{10}}\,dx &= \int \left(66a^2b^{10}+\frac{a^{12}}{x^{10}}+\frac{12a^{11}b}{x^9}+\frac{66a^{10}b^2}{x^8}+\frac{220a^9b^3}{x^7}+\frac{495a^8b^4}{x^6}+\frac{792a^7b^5}{x^5}+\frac{924a^6b^6}{x^4}+\frac{792\ldots}{\ldots}\right.\\ &= -\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b\ldots}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 141, normalized size = 1.00

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}+220a^3b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12/x^10,x]
```

```
[Out] -1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Maple [A]
time = 0.08, size = 132, normalized size = 0.94

method	result
default	$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+6a\,b^1$
risch	$\frac{b^{12}x^3}{3}+6a\,b^{11}x^2+66a^2b^{10}x+\frac{-495a^4b^8x^8-396a^5b^7x^7-308a^6b^6x^6-198a^7b^5x^5-99a^8b^4x^4-\frac{110}{3}a^9b^3x^3-\frac{66}{7}a^{10}b^2x^2-\frac{3}{2}a}{x^9}$
norman	$\frac{-\frac{1}{9}a^{12}+\frac{1}{3}b^{12}x^{12}+6a\,b^{11}x^{11}+66a^2b^{10}x^{10}-495a^4b^8x^8-396a^5b^7x^7-308a^6b^6x^6-198a^7b^5x^5-99a^8b^4x^4-\frac{110}{3}a^9b^3x^3-\frac{66}{7}a^{10}b^2x^2-\frac{3}{2}a}{x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

Maxima [A]
time = 0.28, size = 132, normalized size = 0.94

$$\frac{1}{3}\,b^{12}x^3+6\,ab^{11}x^2+66\,a^2b^{10}x+220\,a^3b^9\log\left(x\right)-\frac{62370\,a^4b^8x^8+49896\,a^5b^7x^7+38808\,a^6b^6x^6+24948\,a^7b^5x^5+12474\,a^8b^4x^4+4620\,a^9b^3x^3+1188\,a^{10}b^2x^2+189\,a^{11}bx+14\,a^{12}}{126\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(x) - 1/126*(6
2370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b^5*x^
5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^11*b*x
 + 14*a^12)/x^9
```

Fricas [A]
time = 0.48, size = 136, normalized size = 0.96

$$\frac{42\,b^{12}x^{12}+756\,ab^{11}x^{11}+8316\,a^2b^{10}x^{10}+27720\,a^3b^9x^9\log\left(x\right)-62370\,a^4b^8x^8-49896\,a^5b^7x^7-38808\,a^6b^6x^6-24948\,a^7b^5x^5-12474\,a^8b^4x^4-4620\,a^9b^3x^3-1188\,a^{10}b^2x^2-189\,a^{11}bx-14\,a^{12}}{126\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 + 756*a*b^11*x^11 + 8316*a^2*b^10*x^10 + 27720*a^3*b^9*
x^9*log(x) - 62370*a^4*b^8*x^8 - 49896*a^5*b^7*x^7 - 38808*a^6*b^6*x^6 - 24
948*a^7*b^5*x^5 - 12474*a^8*b^4*x^4 - 4620*a^9*b^3*x^3 - 1188*a^10*b^2*x^2
- 189*a^11*b*x - 14*a^12)/x^9
```

Sympy [A]
time = 0.36, size = 143, normalized size = 1.01

$$220a^3b^9\log(x) + 66a^2b^{10}x + 6ab^{11}x^2 + \frac{b^{12}x^3}{3} + \frac{-14a^{12} - 189a^{11}bx - 1188a^{10}b^2x^2 - 4620a^9b^3x^3 - 12474a^8b^4x^4 - 24948a^7b^5x^5 - 38808a^6b^6x^6 - 49896a^5b^7x^7 - 62370a^4b^8x^8}{126x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12/x**10,x)
```

```
[Out] 220*a**3*b**9*log(x) + 66*a**2*b**10*x + 6*a*b**11*x**2 + b**12*x**3/3 + (-
14*a**12 - 189*a**11*b*x - 1188*a**10*b**2*x**2 - 4620*a**9*b**3*x**3 - 124
74*a**8*b**4*x**4 - 24948*a**7*b**5*x**5 - 38808*a**6*b**6*x**6 - 49896*a**
5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*x**9)
```

Giac [A]
time = 1.79, size = 133, normalized size = 0.94

$$\frac{1}{3}b^{12}x^3 + 6ab^{11}x^2 + 66a^2b^{10}x + 220a^3b^9\log(|x|) - \frac{62370a^4b^8x^8 + 49896a^5b^7x^7 + 38808a^6b^6x^6 + 24948a^7b^5x^5 + 12474a^8b^4x^4 + 4620a^9b^3x^3 + 1188a^{10}b^2x^2 + 189a^{11}bx + 14a^{12}}{126x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(abs(x)) - 1/1
26*(62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b
^5*x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^1
1*b*x + 14*a^12)/x^9
```

Mupad [B]
time = 0.08, size = 132, normalized size = 0.94

$$\frac{b^{12}x^3}{3} - \frac{\frac{a^{12}}{9} + \frac{3a^{11}bx}{2} + \frac{66a^{10}b^2x^2}{7} + \frac{110a^9b^3x^3}{3} + 99a^8b^4x^4 + 198a^7b^5x^5 + 308a^6b^6x^6 + 396a^5b^7x^7 + 495a^4b^8x^8}{x^9} + 66a^2b^{10}x + 6ab^{11}x^2 + 220a^3b^9\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12/x^10,x)
```

```
[Out] (b^12*x^3)/3 - (a^12/9 + (66*a^10*b^2*x^2)/7 + (110*a^9*b^3*x^3)/3 + 99*a^8
*b^4*x^4 + 198*a^7*b^5*x^5 + 308*a^6*b^6*x^6 + 396*a^5*b^7*x^7 + 495*a^4*b^
8*x^8 + (3*a^11*b*x)/2)/x^9 + 66*a^2*b^10*x + 6*a*b^11*x^2 + 220*a^3*b^9*lo
g(x)
```

3.240 $\int \frac{(a+bx)^{11}}{x^{10}}\,dx$

Optimal. Leaf size=132

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11ab^{10}x+\frac{b^{11}x^2}{2}+55a^2b^9\ \mathrm{l}$$

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Rubi [A]

time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11/x^10,x]
```

```
[Out] -1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{11}}{x^{10}}\,dx &= \int \left(11ab^{10}+\frac{a^{11}}{x^{10}}+\frac{11a^{10}b}{x^9}+\frac{55a^9b^2}{x^8}+\frac{165a^8b^3}{x^7}+\frac{330a^7b^4}{x^6}+\frac{462a^6b^5}{x^5}+\frac{462a^5b^6}{x^4}+330\right.\\ &= -\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3}{x}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 132, normalized size = 1.00

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+11ab^{10}x+\frac{b^{11}x^2}{2}+55a^2b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11/x^10,x]
```

```
[Out] -1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Maple [A]
time = 0.09, size = 121, normalized size = 0.92

method	result
default	$-\frac{a^{11}}{9x^{9}}-\frac{11a^{10}b}{8x^{8}}-\frac{55a^{9}b^{2}}{7x^{7}}-\frac{55a^{8}b^{3}}{2x^{6}}-\frac{66a^{7}b^{4}}{x^{5}}-\frac{231a^{6}b^{5}}{2x^{4}}-\frac{154a^{5}b^{6}}{x^{3}}-\frac{165a^{4}b^{7}}{x^{2}}-\frac{165a^{3}b^{8}}{x}+11a\,b^{10}x+\frac{b^{11}x^{2}}{2}+$
risch	$\frac{b^{11}x^{2}}{2}+11a\,b^{10}x+\frac{-165a^{3}b^{8}x^{8}-165a^{4}b^{7}x^{7}-154a^{5}b^{6}x^{6}-\frac{231}{2}a^{6}b^{5}x^{5}-66a^{7}b^{4}x^{4}-\frac{55}{2}a^{8}b^{3}x^{3}-\frac{55}{7}b^{2}a^{9}x^{2}-\frac{11}{8}a^{10}bx-\frac{1}{9}a^{11}}{x^{9}}+55$
norman	$\frac{-\frac{1}{9}a^{11}+\frac{1}{2}b^{11}x^{11}+11a\,b^{10}x^{10}-165a^{3}b^{8}x^{8}-165a^{4}b^{7}x^{7}-154a^{5}b^{6}x^{6}-\frac{231}{2}a^{6}b^{5}x^{5}-66a^{7}b^{4}x^{4}-\frac{55}{2}a^{8}b^{3}x^{3}-\frac{11}{8}a^{10}bx-\frac{55}{7}b^{2}a^{9}x^{2}}{x^{9}}+55a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

Maxima [A]
time = 0.30, size = 121, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^{2}+11\,ab^{10}x+55\,a^{2}b^{9}\log\left(x\right)-\frac{83160\,a^{3}b^{8}x^{8}+83160\,a^{4}b^{7}x^{7}+77616\,a^{5}b^{6}x^{6}+58212\,a^{6}b^{5}x^{5}+33264\,a^{7}b^{4}x^{4}+13860\,a^{8}b^{3}x^{3}+3960\,a^{9}b^{2}x^{2}+693\,a^{10}bx+56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="maxima")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(x) - 1/504*(83160*a^3*b^8*x^8 +
 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7*b^4*
x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x^9
```

Fricas [A]
time = 0.47, size = 125, normalized size = 0.95

$$\frac{252\,b^{11}x^{11}+5544\,ab^{10}x^{10}+27720\,a^{2}b^{9}x^{9}\log\left(x\right)-83160\,a^{3}b^{8}x^{8}-83160\,a^{4}b^{7}x^{7}-77616\,a^{5}b^{6}x^{6}-58212\,a^{6}b^{5}x^{5}-33264\,a^{7}b^{4}x^{4}-13860\,a^{8}b^{3}x^{3}-3960\,a^{9}b^{2}x^{2}-693\,a^{10}bx-56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 + 5544*a*b^10*x^10 + 27720*a^2*b^9*x^9*log(x) - 83160*
a^3*b^8*x^8 - 83160*a^4*b^7*x^7 - 77616*a^5*b^6*x^6 - 58212*a^6*b^5*x^5 - 3
3264*a^7*b^4*x^4 - 13860*a^8*b^3*x^3 - 3960*a^9*b^2*x^2 - 693*a^10*b*x - 56
*a^11)/x^9
```

Sympy [A]
time = 0.34, size = 131, normalized size = 0.99

$$55a^2b^9\log{(x)} + 11ab^{10}x + \frac{b^{11}x^2}{2} + \frac{-56a^{11} - 693a^{10}bx - 3960a^9b^2x^2 - 13860a^8b^3x^3 - 33264a^7b^4x^4 - 58212a^6b^5x^5 - 77616a^5b^6x^6 - 83160a^4b^7x^7 - 83160a^3b^8x^8}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11/x**10,x)
```

```
[Out] 55*a**2*b**9*log(x) + 11*a*b**10*x + b**11*x**2/2 + (-56*a**11 - 693*a**10*
b*x - 3960*a**9*b**2*x**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 - 5
8212*a**6*b**5*x**5 - 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 83160*a
**3*b**8*x**8)/(504*x**9)
```

Giac [A]
time = 1.70, size = 122, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^2 + 11\,ab^{10}x + 55\,a^2b^9\log\left(|x|\right) - \frac{83160\,a^3b^8x^8 + 83160\,a^4b^7x^7 + 77616\,a^5b^6x^6 + 58212\,a^6b^5x^5 + 33264\,a^7b^4x^4 + 13860\,a^8b^3x^3 + 3960\,a^9b^2x^2 + 693\,a^{10}bx + 56\,a^{11}}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="giac")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(abs(x)) - 1/504*(83160*a^3*b^8*
x^8 + 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7
*b^4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x
^9
```

Mupad [B]
time = 0.09, size = 121, normalized size = 0.92

$$\frac{b^{11}\,x^2}{2} - \frac{\frac{a^{11}}{9} + \frac{11\,a^{10}\,b\,x}{8} + \frac{55\,a^9\,b^2\,x^2}{7} + \frac{55\,a^8\,b^3\,x^3}{2} + 66\,a^7\,b^4\,x^4 + \frac{231\,a^6\,b^5\,x^5}{2} + 154\,a^5\,b^6\,x^6 + 165\,a^4\,b^7\,x^7 + 165\,a^3\,b^8\,x^8}{x^9} + 55\,a^2\,b^9\,\ln\left(x\right) + 11\,a\,b^{10}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11/x^10,x)
```

```
[Out] (b^11*x^2)/2 - (a^11/9 + (55*a^9*b^2*x^2)/7 + (55*a^8*b^3*x^3)/2 + 66*a^7*b
^4*x^4 + (231*a^6*b^5*x^5)/2 + 154*a^5*b^6*x^6 + 165*a^4*b^7*x^7 + 165*a^3*
b^8*x^8 + (11*a^10*b*x)/8)/x^9 + 55*a^2*b^9*log(x) + 11*a*b^10*x
```

3.241 $\int \frac{(a+bx)^{10}}{x^{10}} \, dx$

Optimal. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{10}}\,dx &= \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+10ab^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Maple [A]
time = 0.08, size = 109, normalized size = 0.96

method	result	s
default	$-\frac{a^{10}}{9x^{9}}-\frac{5a^{9}b}{4x^{8}}-\frac{45a^{8}b^{2}}{7x^{7}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{42a^{6}b^{4}}{x^{5}}-\frac{63a^{5}b^{5}}{x^{4}}-\frac{70a^{4}b^{6}}{x^{3}}-\frac{60a^{3}b^{7}}{x^{2}}-\frac{45a^{2}b^{8}}{x}+b^{10}x+10a\,b^{9}\ln\left(x\right)$	1
risch	$b^{10}x+\frac{-45a^{2}b^{8}x^{8}-60a^{3}b^{7}x^{7}-70a^{4}b^{6}x^{6}-63a^{5}b^{5}x^{5}-42a^{6}b^{4}x^{4}-20a^{7}b^{3}x^{3}-\frac{45}{7}a^{8}b^{2}x^{2}-\frac{5}{4}a^{9}bx-\frac{1}{9}a^{10}}{x^{9}}+10a\,b^{9}\ln\left(x\right)$	1
norman	$\frac{b^{10}x^{10}-\frac{1}{9}a^{10}-45a^{2}b^{8}x^{8}-60a^{3}b^{7}x^{7}-70a^{4}b^{6}x^{6}-63a^{5}b^{5}x^{5}-42a^{6}b^{4}x^{4}-20a^{7}b^{3}x^{3}-\frac{45}{7}a^{8}b^{2}x^{2}-\frac{5}{4}a^{9}bx}{x^{9}}+10a\,b^{9}\ln\left(x\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

Maxima [A]
time = 0.29, size = 109, normalized size = 0.96

$$b^{10}x+10\,ab^{9}\log\left(x\right)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+1620\,a^{8}b^{2}x^{2}+315\,a^{9}bx+28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Fricas [A]
time = 0.49, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log\left(x\right)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

Sympy [A]
time = 0.34, size = 117, normalized size = 1.03

$$10ab^9\log(x) + b^{10}x + \frac{-28a^{10} - 315a^9bx - 1620a^8b^2x^2 - 5040a^7b^3x^3 - 10584a^6b^4x^4 - 15876a^5b^5x^5 - 17640a^4b^6x^6 - 15120a^3b^7x^7 - 11340a^2b^8x^8}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

Giac [A]
time = 0.78, size = 110, normalized size = 0.96

$$b^{10}x + 10\,ab^9\log(|x|) - \frac{11340\,a^2b^8x^8 + 15120\,a^3b^7x^7 + 17640\,a^4b^6x^6 + 15876\,a^5b^5x^5 + 10584\,a^6b^4x^4 + 5040\,a^7b^3x^3 + 1620\,a^8b^2x^2 + 315\,a^9bx + 28\,a^{10}}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

Mupad [B]
time = 0.00, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^8\,b^2\,x^2}{7} + 20\,a^7\,b^3\,x^3 + 42\,a^6\,b^4\,x^4 + 63\,a^5\,b^5\,x^5 + 70\,a^4\,b^6\,x^6 + 60\,a^3\,b^7\,x^7 + 45\,a^2\,b^8\,x^8 + \frac{5\,a^9\,b\,x}{4} - 10\,a\,b^9\,x^9\,\ln(x)}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

3.242 $\int \frac{(a+bx)^9}{x^{10}}\,dx$

Optimal. Leaf size=109

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/x^10,x]
```

```
[Out] -1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^9}{x^{10}}\,dx &= \int \left(\frac{a^9}{x^{10}}+\frac{9a^8b}{x^9}+\frac{36a^7b^2}{x^8}+\frac{84a^6b^3}{x^7}+\frac{126a^5b^4}{x^6}+\frac{126a^4b^5}{x^5}+\frac{84a^3b^6}{x^4}+\frac{36a^2b^7}{x^3}+\frac{9ab^8}{x^2}+\right.\\ &= -\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\,\mathrm{lo}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 109, normalized size = 1.00

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/x^10,x]
```

```
[Out] -1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Maple [A]
time = 0.08, size = 100, normalized size = 0.92

method	result	size
default	$-\frac{a^{9}}{9x^{9}}-\frac{9a^{8}b}{8x^{8}}-\frac{36a^{7}b^{2}}{7x^{7}}-\frac{14a^{6}b^{3}}{x^{6}}-\frac{126a^{5}b^{4}}{5x^{5}}-\frac{63a^{4}b^{5}}{2x^{4}}-\frac{28a^{3}b^{6}}{x^{3}}-\frac{18a^{2}b^{7}}{x^{2}}-\frac{9a\,b^{8}}{x}+b^{9}\ln\left(x\right)$	100
norman	$\frac{-\frac{1}{9}a^{9}-9a\,b^{8}x^{8}-18a^{2}b^{7}x^{7}-28a^{3}b^{6}x^{6}-\frac{63}{2}a^{4}b^{5}x^{5}-\frac{126}{5}a^{5}b^{4}x^{4}-14a^{6}b^{3}x^{3}-\frac{36}{7}a^{7}b^{2}x^{2}-\frac{9}{8}a^{8}bx}{x^{9}}+b^{9}\ln\left(x\right)$	100
risch	$\frac{-\frac{1}{9}a^{9}-9a\,b^{8}x^{8}-18a^{2}b^{7}x^{7}-28a^{3}b^{6}x^{6}-\frac{63}{2}a^{4}b^{5}x^{5}-\frac{126}{5}a^{5}b^{4}x^{4}-14a^{6}b^{3}x^{3}-\frac{36}{7}a^{7}b^{2}x^{2}-\frac{9}{8}a^{8}bx}{x^{9}}+b^{9}\ln\left(x\right)$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

Maxima [A]
time = 0.27, size = 100, normalized size = 0.92

$$b^{9}\log\left(x\right)-\frac{22680\,ab^{8}x^{8}+45360\,a^{2}b^{7}x^{7}+70560\,a^{3}b^{6}x^{6}+79380\,a^{4}b^{5}x^{5}+63504\,a^{5}b^{4}x^{4}+35280\,a^{6}b^{3}x^{3}+12960\,a^{7}b^{2}x^{2}+2835\,a^{8}bx+280\,a^{9}}{2520\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="maxima")
```

```
[Out] b^9*log(x) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b^6*x^
6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*a^7*b
^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Fricas [A]
time = 0.47, size = 103, normalized size = 0.94

$$\frac{2520\,b^{9}x^{9}\log\left(x\right)-22680\,ab^{8}x^{8}-45360\,a^{2}b^{7}x^{7}-70560\,a^{3}b^{6}x^{6}-79380\,a^{4}b^{5}x^{5}-63504\,a^{5}b^{4}x^{4}-35280\,a^{6}b^{3}x^{3}-12960\,a^{7}b^{2}x^{2}-2835\,a^{8}bx-280\,a^{9}}{2520\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(x) - 22680*a*b^8*x^8 - 45360*a^2*b^7*x^7 - 70560*a
^3*b^6*x^6 - 79380*a^4*b^5*x^5 - 63504*a^5*b^4*x^4 - 35280*a^6*b^3*x^3 - 12
960*a^7*b^2*x^2 - 2835*a^8*b*x - 280*a^9)/x^9
```

Sympy [A]
time = 0.32, size = 107, normalized size = 0.98

$$b^9\log(x)+\frac{-280a^9-2835a^8bx-12960a^7b^2x^2-35280a^6b^3x^3-63504a^5b^4x^4-79380a^4b^5x^5-70560a^3b^6x^6-45360a^2b^7x^7-22680ab^8x^8}{2520x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/x**10,x)
```

```
[Out] b**9*log(x) + (-280*a**9 - 2835*a**8*b*x - 12960*a**7*b**2*x**2 - 35280*a**
6*b**3*x**3 - 63504*a**5*b**4*x**4 - 79380*a**4*b**5*x**5 - 70560*a**3*b**6
*x**6 - 45360*a**2*b**7*x**7 - 22680*a*b**8*x**8)/(2520*x**9)
```

Giac [A]
time = 1.36, size = 101, normalized size = 0.93

$$b^9\log(|x|)-\frac{22680\,ab^8x^8+45360\,a^2b^7x^7+70560\,a^3b^6x^6+79380\,a^4b^5x^5+63504\,a^5b^4x^4+35280\,a^6b^3x^3+12960\,a^7b^2x^2+2835\,a^8bx+280\,a^9}{2520\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="giac")
```

```
[Out] b^9*log(abs(x)) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b
^6*x^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*
a^7*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

Mupad [B]
time = 0.08, size = 100, normalized size = 0.92

$$b^9\,\ln(x)-\frac{\frac{a^9}{9}+\frac{9\,a^8\,b\,x}{8}+\frac{36\,a^7\,b^2\,x^2}{7}+14\,a^6\,b^3\,x^3+\frac{126\,a^5\,b^4\,x^4}{5}+\frac{63\,a^4\,b^5\,x^5}{2}+28\,a^3\,b^6\,x^6+18\,a^2\,b^7\,x^7+9\,a\,b^8\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/x^10,x)
```

```
[Out] b^9*log(x) - (a^9/9 + 9*a*b^8*x^8 + (36*a^7*b^2*x^2)/7 + 14*a^6*b^3*x^3 + (
126*a^5*b^4*x^4)/5 + (63*a^4*b^5*x^5)/2 + 28*a^3*b^6*x^6 + 18*a^2*b^7*x^7 +
 (9*a^8*b*x)/8)/x^9
```

3.243 $\int \frac{(a+bx)^8}{x^{10}} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^9}{9ax^9}$$

[Out] -1/9*(b*x+a)^9/a/x^9

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {37}

$$-\frac{(a+bx)^9}{9ax^9}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^8/x^10,x]

[Out] -1/9*(a + b*x)^9/(a*x^9)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^8}{x^{10}} dx = -\frac{(a+bx)^9}{9ax^9}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(17) = 34.
time = 0.01, size = 96, normalized size = 5.65

$$-\frac{a^8}{9x^9} - \frac{a^7b}{x^8} - \frac{4a^6b^2}{x^7} - \frac{28a^5b^3}{3x^6} - \frac{14a^4b^4}{x^5} - \frac{14a^3b^5}{x^4} - \frac{28a^2b^6}{3x^3} - \frac{4ab^7}{x^2} - \frac{b^8}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^8/x^10,x]

```
[Out] -1/9*a^8/x^9 - (a^7*b)/x^8 - (4*a^6*b^2)/x^7 - (28*a^5*b^3)/(3*x^6) - (14*a
^4*b^4)/x^5 - (14*a^3*b^5)/x^4 - (28*a^2*b^6)/(3*x^3) - (4*a*b^7)/x^2 - b^8
/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(15) = 30$.
time = 0.08, size = 91, normalized size = 5.35

method	result	size
gosper	$-\frac{9b^8x^8+36a\,b^7x^7+84a^2x^6b^6+126a^3x^5b^5+126a^4x^4b^4+84a^5x^3b^3+36a^6x^2b^2+9a^7xb+a^8}{9x^9}$	89
norman	$\frac{-b^8x^8-4a\,b^7x^7-\frac{28}{3}a^2x^6b^6-14a^3x^5b^5-14a^4x^4b^4-\frac{28}{3}a^5x^3b^3-4a^6x^2b^2-a^7xb-\frac{1}{9}a^8}{x^9}$	90
risch	$\frac{-b^8x^8-4a\,b^7x^7-\frac{28}{3}a^2x^6b^6-14a^3x^5b^5-14a^4x^4b^4-\frac{28}{3}a^5x^3b^3-4a^6x^2b^2-a^7xb-\frac{1}{9}a^8}{x^9}$	90
default	$-\frac{b^8}{x}-\frac{28b^6a^2}{3x^3}-\frac{14a^3b^5}{x^4}-\frac{a^8}{9x^9}-\frac{4a\,b^7}{x^2}-\frac{a^7b}{x^8}-\frac{14a^4b^4}{x^5}-\frac{28a^5b^3}{3x^6}-\frac{4b^2a^6}{x^7}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -b^8/x-28/3*b^6*a^2/x^3-14*a^3*b^5/x^4-1/9*a^8/x^9-4*a*b^7/x^2-a^7*b/x^8-14
*a^4*b^4/x^5-28/3*a^5*b^3/x^6-4*b^2*a^6/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.
time = 0.27, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.
time = 0.43, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(14) = 28$.
time = 0.29, size = 95, normalized size = 5.59

$$\frac{-a^8 - 9a^7bx - 36a^6b^2x^2 - 84a^5b^3x^3 - 126a^4b^4x^4 - 126a^3b^5x^5 - 84a^2b^6x^6 - 36ab^7x^7 - 9b^8x^8}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/x**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(15) = 30$.
time = 1.73, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8 + 36\,ab^7x^7 + 84\,a^2b^6x^6 + 126\,a^3b^5x^5 + 126\,a^4b^4x^4 + 84\,a^5b^3x^3 + 36\,a^6b^2x^2 + 9\,a^7bx + a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

Mupad [B]
time = 0.09, size = 88, normalized size = 5.18

$$-\frac{\frac{a^8}{9} + a^7\,b\,x + 4\,a^6\,b^2\,x^2 + \frac{28\,a^5\,b^3\,x^3}{3} + 14\,a^4\,b^4\,x^4 + 14\,a^3\,b^5\,x^5 + \frac{28\,a^2\,b^6\,x^6}{3} + 4\,a\,b^7\,x^7 + b^8\,x^8}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/x^10,x)
```

```
[Out] -(a^8/9 + b^8*x^8 + 4*a*b^7*x^7 + 4*a^6*b^2*x^2 + (28*a^5*b^3*x^3)/3 + 14*a
^4*b^4*x^4 + 14*a^3*b^5*x^5 + (28*a^2*b^6*x^6)/3 + a^7*b*x)/x^9
```

3.244 $\int \frac{(a+bx)^7}{x^{10}} \, dx$

Optimal. Leaf size=36

$$-\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}$$

```
[Out] -1/9*(b*x+a)^8/a/x^9+1/72*b*(b*x+a)^8/a^2/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {47, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*(a + b*x)^8/(a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}} \, dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9} \, dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. $2(36) = 72$.
time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 0.07, size = 80, normalized size = 2.22

method	result	size
norman	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
risch	$\frac{-\frac{1}{2}b^7x^7-\frac{7}{3}a\,b^6x^6-\frac{21}{4}a^2b^5x^5-7a^3b^4x^4-\frac{35}{6}a^4b^3x^3-3a^5b^2x^2-\frac{7}{8}a^6bx-\frac{1}{9}a^7}{x^9}$	79
gosper	$-\frac{36b^7x^7+168a\,b^6x^6+378a^2b^5x^5+504a^3b^4x^4+420a^4b^3x^3+216a^5b^2x^2+63a^6bx+8a^7}{72x^9}$	80
default	$-\frac{7a\,b^6}{3x^3}-\frac{21a^2b^5}{4x^4}-\frac{a^7}{9x^9}-\frac{b^7}{2x^2}-\frac{7a^6b}{8x^8}-\frac{7a^3b^4}{x^5}-\frac{35a^4b^3}{6x^6}-\frac{3a^5b^2}{x^7}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -7/3*a*b^6/x^3-21/4*a^2*b^5/x^4-1/9*a^7/x^9-1/2*b^7/x^2-7/8*a^6*b/x^8-7*a^3
*b^4/x^5-35/6*a^4*b^3/x^6-3*a^5*b^2/x^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 0.27, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.

time = 0.44, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.
time = 0.25, size = 85, normalized size = 2.36

$$\frac{-8a^7-63a^6bx-216a^5b^2x^2-420a^4b^3x^3-504a^3b^4x^4-378a^2b^5x^5-168ab^6x^6-36b^7x^7}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 1.75, size = 79, normalized size = 2.19

$$-\frac{36\,b^7x^7+168\,ab^6x^6+378\,a^2b^5x^5+504\,a^3b^4x^4+420\,a^4b^3x^3+216\,a^5b^2x^2+63\,a^6bx+8\,a^7}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

Mupad [B]
time = 0.00, size = 23, normalized size = 0.64

$$-\frac{(8\,a-b\,x)\,\left(a+b\,x\right)^8}{72\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

3.245 $\int \frac{(a+bx)^6}{x^{10}} \, dx$

Optimal. Leaf size=56

$$-\frac{(a+bx)^7}{9ax^9}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{b^2(a+bx)^7}{252a^3x^7}$$

```
[Out] -1/9*(b*x+a)^7/a/x^9+1/36*b*(b*x+a)^7/a^2/x^8-1/252*b^2*(b*x+a)^7/a^3/x^7
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {47, 37}

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/x^10,x]
```

```
[Out] -1/9*(a + b*x)^7/(a*x^9) + (b*(a + b*x)^7)/(36*a^2*x^8) - (b^2*(a + b*x)^7)
/(252*a^3*x^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^6}{x^{10}}\,dx = -\frac{(a+bx)^7}{9ax^9} - \frac{(2b)\int \frac{(a+bx)^6}{x^9}\,dx}{9a}$$
$$= -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} + \frac{b^2\int \frac{(a+bx)^6}{x^8}\,dx}{36a^2}$$
$$= -\frac{(a+bx)^7}{9ax^9} + \frac{b(a+bx)^7}{36a^2x^8} - \frac{b^2(a+bx)^7}{252a^3x^7}$$

Mathematica [A]
time = 0.01, size = 80, normalized size = 1.43

$$-\frac{a^6}{9x^9} - \frac{3a^5b}{4x^8} - \frac{15a^4b^2}{7x^7} - \frac{10a^3b^3}{3x^6} - \frac{3a^2b^4}{x^5} - \frac{3ab^5}{2x^4} - \frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (3*a^5*b)/(4*x^8) - (15*a^4*b^2)/(7*x^7) - (10*a^3*b^3)/(3*x
^6) - (3*a^2*b^4)/x^5 - (3*a*b^5)/(2*x^4) - b^6/(3*x^3)
```

Maple [A]
time = 0.08, size = 69, normalized size = 1.23

method	result	size
norman	$\frac{-\frac{1}{3}x^6b^6-\frac{3}{2}a\,x^5b^5-3a^2x^4b^4-\frac{10}{3}a^3b^3x^3-\frac{15}{7}a^4x^2b^2-\frac{3}{4}a^5xb-\frac{1}{9}a^6}{x^9}$	68
risch	$\frac{-\frac{1}{3}x^6b^6-\frac{3}{2}a\,x^5b^5-3a^2x^4b^4-\frac{10}{3}a^3b^3x^3-\frac{15}{7}a^4x^2b^2-\frac{3}{4}a^5xb-\frac{1}{9}a^6}{x^9}$	68
gosper	$-\frac{84x^6b^6+378a\,x^5b^5+756a^2x^4b^4+840a^3b^3x^3+540a^4x^2b^2+189a^5xb+28a^6}{252x^9}$	69
default	$-\frac{b^6}{3x^3}-\frac{3a\,b^5}{2x^4}-\frac{a^6}{9x^9}-\frac{3a^5b}{4x^8}-\frac{3a^2b^4}{x^5}-\frac{10a^3b^3}{3x^6}-\frac{15a^4b^2}{7x^7}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^6/x^3-3/2*a*b^5/x^4-1/9*a^6/x^9-3/4*a^5*b/x^8-3*a^2*b^4/x^5-10/3*a^3
*b^3/x^6-15/7*a^4*b^2/x^7
```

Maxima [A]
time = 0.28, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6 + 378\,ab^5x^5 + 756\,a^2b^4x^4 + 840\,a^3b^3x^3 + 540\,a^4b^2x^2 + 189\,a^5bx + 28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Fricas [A]
time = 0.40, size = 68, normalized size = 1.21

$$-\frac{84\,b^{6}x^{6}+378\,ab^{5}x^{5}+756\,a^{2}b^{4}x^{4}+840\,a^{3}b^{3}x^{3}+540\,a^{4}b^{2}x^{2}+189\,a^{5}bx+28\,a^{6}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Sympy [A]
time = 0.23, size = 73, normalized size = 1.30

$$\frac{-28a^{6}-189a^{5}bx-540a^{4}b^{2}x^{2}-840a^{3}b^{3}x^{3}-756a^{2}b^{4}x^{4}-378ab^{5}x^{5}-84b^{6}x^{6}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/x**10,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756*a*
*2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

Giac [A]
time = 1.59, size = 68, normalized size = 1.21

$$-\frac{84\,b^{6}x^{6}+378\,ab^{5}x^{5}+756\,a^{2}b^{4}x^{4}+840\,a^{3}b^{3}x^{3}+540\,a^{4}b^{2}x^{2}+189\,a^{5}bx+28\,a^{6}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

Mupad [B]
time = 0.10, size = 68, normalized size = 1.21

$$-\frac{\frac{a^{6}}{9}+\frac{3\,a^{5}\,b\,x}{4}+\frac{15\,a^{4}\,b^{2}\,x^{2}}{7}+\frac{10\,a^{3}\,b^{3}\,x^{3}}{3}+3\,a^{2}\,b^{4}\,x^{4}+\frac{3\,a\,b^{5}\,x^{5}}{2}+\frac{b^{6}\,x^{6}}{3}}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/x^10,x)
```

```
[Out] -(a^6/9 + (b^6*x^6)/3 + (3*a*b^5*x^5)/2 + (15*a^4*b^2*x^2)/7 + (10*a^3*b^3*
x^3)/3 + 3*a^2*b^4*x^4 + (3*a^5*b*x)/4)/x^9
```

3.246 $\int \frac{(a+bx)^5}{x^{10}} \, dx$

Optimal. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{x^{10}} \, dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right) dx \\
&= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Maple [A]
time = 0.07, size = 58, normalized size = 0.87

method	result	size
norman	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
risch	$\frac{-\frac{1}{4}b^5x^5-a\,b^4x^4-\frac{5}{3}a^2b^3x^3-\frac{10}{7}a^3b^2x^2-\frac{5}{8}a^4bx-\frac{1}{9}a^5}{x^9}$	57
gosper	$-\frac{126b^5x^5+504a\,b^4x^4+840a^2b^3x^3+720a^3b^2x^2+315a^4bx+56a^5}{504x^9}$	58
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{a\,b^4}{x^5}-\frac{b^5}{4x^4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Fricas [A]
time = 0.45, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Sympy [A]
time = 0.19, size = 61, normalized size = 0.91

$$\frac{-56a^5 - 315a^4bx - 720a^3b^2x^2 - 840a^2b^3x^3 - 504ab^4x^4 - 126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

Giac [A]
time = 1.35, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5 + 504\,ab^4x^4 + 840\,a^2b^3x^3 + 720\,a^3b^2x^2 + 315\,a^4bx + 56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

Mupad [B]
time = 0.00, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x}{8} + \frac{10\,a^3\,b^2\,x^2}{7} + \frac{5\,a^2\,b^3\,x^3}{3} + a\,b^4\,x^4 + \frac{b^5\,x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

3.247 $\int \frac{(a+bx)^4}{x^{10}} dx$

Optimal. Leaf size=56

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

```
[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) - b^4/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^4}{x^{10}} dx &= \int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^9}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^7}+\frac{b^4}{x^6}\right) dx \\ &= -\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Maple [A]
time = 0.08, size = 47, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{5}b^4x^4-\frac{2}{3}a\,b^3x^3-\frac{6}{7}a^2b^2x^2-\frac{1}{2}a^3bx-\frac{1}{9}a^4}{x^9}$	46
risch	$\frac{-\frac{1}{5}b^4x^4-\frac{2}{3}a\,b^3x^3-\frac{6}{7}a^2b^2x^2-\frac{1}{2}a^3bx-\frac{1}{9}a^4}{x^9}$	46
gosper	$-\frac{126b^4x^4+420a\,b^3x^3+540a^2b^2x^2+315a^3bx+70a^4}{630x^9}$	47
default	$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2a\,b^3}{3x^6}-\frac{b^4}{5x^5}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Fricas [A]
time = 0.40, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Sympy [A]
time = 0.16, size = 49, normalized size = 0.88

$$\frac{-70a^4-315a^3bx-540a^2b^2x^2-420ab^3x^3-126b^4x^4}{630x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/x**10,x)
```

```
[Out] (-70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b**4*
x**4)/(630*x**9)
```

Giac [A]
time = 1.20, size = 46, normalized size = 0.82

$$-\frac{126 b^4x^4 + 420 ab^3x^3 + 540 a^2b^2x^2 + 315 a^3bx + 70 a^4}{630 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

Mupad [B]
time = 0.03, size = 46, normalized size = 0.82

$$-\frac{\frac{a^4}{9} + \frac{a^3 b x}{2} + \frac{6 a^2 b^2 x^2}{7} + \frac{2 a b^3 x^3}{3} + \frac{b^4 x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/x^10,x)
```

```
[Out] -(a^4/9 + (b^4*x^4)/5 + (2*a*b^3*x^3)/3 + (6*a^2*b^2*x^2)/7 + (a^3*b*x)/2)/
x^9
```

3.248 $\int \frac{(a+bx)^3}{x^{10}} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

```
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^3}{x^{10}} dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right) dx \\
&= -\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^10,x]
```

```
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.84

method	result	size
norman	$\frac{-\frac{1}{6}b^3x^3-\frac{3}{7}a\,b^2x^2-\frac{3}{8}a^2bx-\frac{1}{9}a^3}{x^9}$	35
risch	$\frac{-\frac{1}{6}b^3x^3-\frac{3}{7}a\,b^2x^2-\frac{3}{8}a^2bx-\frac{1}{9}a^3}{x^9}$	35
gosper	$-\frac{84b^3x^3+216a\,b^2x^2+189a^2bx+56a^3}{504x^9}$	36
default	$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3a\,b^2}{7x^7}-\frac{b^3}{6x^6}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="maxima")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Fricas [A]
time = 0.39, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="fricas")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Sympy [A]
time = 0.12, size = 37, normalized size = 0.86

$$\frac{-56a^3-189a^2bx-216ab^2x^2-84b^3x^3}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**10,x)
```

```
[Out] (-56*a**3 - 189*a**2*b*x - 216*a*b**2*x**2 - 84*b**3*x**3)/(504*x**9)
```

Giac [A]
time = 1.16, size = 35, normalized size = 0.81

$$-\frac{84 b^3x^3 + 216 ab^2x^2 + 189 a^2bx + 56 a^3}{504 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

Mupad [B]
time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{9} + \frac{3 a^2 b x}{8} + \frac{3 a b^2 x^2}{7} + \frac{b^3 x^3}{6}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^10,x)
```

```
[Out] -(a^3/9 + (b^3*x^3)/6 + (3*a*b^2*x^2)/7 + (3*a^2*b*x)/8)/x^9
```

3.249 $\int \frac{(a+bx)^2}{x^{10}} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/x^10,x]

[Out] -1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2/x^10,x]

```
[Out] -1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Maple [A]
time = 0.08, size = 25, normalized size = 0.83

method	result	size
norman	$\frac{-\frac{1}{7}x^2b^2-\frac{1}{4}abx-\frac{1}{9}a^2}{x^9}$	24
risch	$\frac{-\frac{1}{7}x^2b^2-\frac{1}{4}abx-\frac{1}{9}a^2}{x^9}$	24
gosper	$-\frac{36x^2b^2+63abx+28a^2}{252x^9}$	25
default	$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Fricas [A]
time = 0.71, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Sympy [A]
time = 0.09, size = 26, normalized size = 0.87

$$\frac{-28a^2-63abx-36b^2x^2}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**10,x)
[Out] (-28*a**2 - 63*a*b*x - 36*b**2*x**2)/(252*x**9)
```

Giac [A]
time = 2.19, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="giac")
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{9}+\frac{a\,b\,x}{4}+\frac{b^2\,x^2}{7}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^10,x)
[Out] -(a^2/9 + (b^2*x^2)/7 + (a*b*x)/4)/x^9
```

3.250 $\int \frac{a+bx}{x^{10}} dx$

Optimal. Leaf size=17

$$-\frac{a}{9x^9} - \frac{b}{8x^8}$$

```
[Out] -1/9*a/x^9-1/8*b/x^8
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$-\frac{a}{9x^9} - \frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^10,x]
```

```
[Out] -1/9*a/x^9 - b/(8*x^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{x^{10}}\,dx &= \int \left(\frac{a}{x^{10}}+\frac{b}{x^9}\right)\,dx \\ &= -\frac{a}{9x^9} - \frac{b}{8x^8} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{9x^9} - \frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^10,x]
```

```
[Out] -1/9*a/x^9 - b/(8*x^8)
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.82

method	result	size
norman	$\frac{-\frac{bx}{8}-\frac{a}{9}}{x^9}$	13
risch	$\frac{-\frac{bx}{8}-\frac{a}{9}}{x^9}$	13
gosper	$-\frac{9bx+8a}{72x^9}$	14
default	$-\frac{a}{9x^9}-\frac{b}{8x^8}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a/x^9-1/8*b/x^8
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="maxima")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Fricas [A]
time = 0.76, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="fricas")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Sympy [A]
time = 0.06, size = 14, normalized size = 0.82

$$\frac{-8a-9bx}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**10,x)
```

```
[Out] (-8*a - 9*b*x)/(72*x**9)
```

Giac [A]
time = 1.06, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(9*b*x + 8*a)/x^9
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.76

$$-\frac{8\,a+9\,b\,x}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^10,x)
```

```
[Out] -(8*a + 9*b*x)/(72*x^9)
```

3.251 $\int \frac{1}{x^{10}} dx$

Optimal. Leaf size=7

$$-\frac{1}{9x^9}$$

```
[Out] -1/9/x^9
```

Rubi [A]
time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {30}

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-10),x]
[Out] -1/9*1/x^9
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}} dx = -\frac{1}{9x^9}$$

Mathematica [A]
time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-10),x]
[Out] -1/9*1/x^9
```

Maple [A]
time = 0.01, size = 6, normalized size = 0.86

method	result	size
gosper	$-\frac{1}{9x^9}$	6
default	$-\frac{1}{9x^9}$	6
norman	$-\frac{1}{9x^9}$	6
risch	$-\frac{1}{9x^9}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9/x^9
```

Maxima [A]
time = 0.27, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="maxima")
[Out] -1/9/x^9
```

Fricas [A]
time = 0.57, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="fricas")
[Out] -1/9/x^9
```

Sympy [A]
time = 0.02, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10,x)
[Out] -1/(9*x**9)
```

Giac [A]
time = 1.51, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="giac")
```

```
[Out] -1/9/x^9
```

Mupad [B]
time = 0.02, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/(9*x^9)
```

3.252 $\int \frac{1}{x^{10}(a+bx)}\,dx$

Optimal. Leaf size=134

$$-\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}$$

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

Rubi [A]
time = 0.04, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5
*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*
x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)}\,dx &= \int \left(\frac{1}{ax^{10}}-\frac{b}{a^2x^9}+\frac{b^2}{a^3x^8}-\frac{b^3}{a^4x^7}+\frac{b^4}{a^5x^6}-\frac{b^5}{a^6x^5}+\frac{b^6}{a^7x^4}-\frac{b^7}{a^8x^3}+\frac{b^8}{a^9x^2}-\frac{b^9}{a^{10}x}+\frac{}{a^{10}}\right. \\ &= -\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(}{a^{10}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 134, normalized size = 1.00

$$-\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5
*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*
x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Maple [A]
time = 0.09, size = 119, normalized size = 0.89

method	result	size
default	$-\frac{1}{9a\,x^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(bx+a)}{a^{10}}$	119
norman	$\frac{-\frac{1}{9a}+\frac{bx}{8a^2}-\frac{b^2x^2}{7a^3}+\frac{b^3x^3}{6a^4}-\frac{b^4x^4}{5a^5}+\frac{b^5x^5}{4a^6}-\frac{b^6x^6}{3a^7}+\frac{b^7x^7}{2a^8}-\frac{b^8x^8}{a^9}}{x^9}+\frac{b^9\ln(bx+a)}{a^{10}}-\frac{b^9\ln(x)}{a^{10}}$	119
risch	$\frac{-\frac{1}{9a}+\frac{bx}{8a^2}-\frac{b^2x^2}{7a^3}+\frac{b^3x^3}{6a^4}-\frac{b^4x^4}{5a^5}+\frac{b^5x^5}{4a^6}-\frac{b^6x^6}{3a^7}+\frac{b^7x^7}{2a^8}-\frac{b^8x^8}{a^9}}{x^9}-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(-bx-a)}{a^{10}}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

Maxima [A]
time = 0.27, size = 117, normalized size = 0.87

$$\frac{b^9\log(bx+a)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}-\frac{2520\,b^8x^8-1260\,ab^7x^7+840\,a^2b^6x^6-630\,a^3b^5x^5+504\,a^4b^4x^4-420\,a^5b^3x^3+360\,a^6b^2x^2-315\,a^7bx+280\,a^8}{2520\,a^9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^9*log(b*x + a)/a^10 - b^9*log(x)/a^10 - 1/2520*(2520*b^8*x^8 - 1260*a*b^7
*x^7 + 840*a^2*b^6*x^6 - 630*a^3*b^5*x^5 + 504*a^4*b^4*x^4 - 420*a^5*b^3*x^
3 + 360*a^6*b^2*x^2 - 315*a^7*b*x + 280*a^8)/(a^9*x^9)
```

Fricas [A]
time = 0.52, size = 120, normalized size = 0.90

$$\frac{2520\,b^9x^9\log(bx+a)-2520\,b^9x^9\log(x)-2520\,ab^8x^8+1260\,a^2b^7x^7-840\,a^3b^6x^6+630\,a^4b^5x^5-504\,a^5b^4x^4+420\,a^6b^3x^3-360\,a^7b^2x^2+315\,a^8bx-280\,a^9}{2520\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(b*x + a) - 2520*b^9*x^9*log(x) - 2520*a*b^8*x^8 +
1260*a^2*b^7*x^7 - 840*a^3*b^6*x^6 + 630*a^4*b^5*x^5 - 504*a^5*b^4*x^4 + 42
0*a^6*b^3*x^3 - 360*a^7*b^2*x^2 + 315*a^8*b*x - 280*a^9)/(a^10*x^9)
```

Sympy [A]
time = 0.16, size = 116, normalized size = 0.87

$$\frac{-280a^8+315a^7bx-360a^6b^2x^2+420a^5b^3x^3-504a^4b^4x^4+630a^3b^5x^5-840a^2b^6x^6+1260ab^7x^7-2520b^8x^8}{2520a^9x^9}+\frac{b^9\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a),x)
```

```
[Out] (-280*a**8 + 315*a**7*b*x - 360*a**6*b**2*x**2 + 420*a**5*b**3*x**3 - 504*a
**4*b**4*x**4 + 630*a**3*b**5*x**5 - 840*a**2*b**6*x**6 + 1260*a*b**7*x**7
- 2520*b**8*x**8)/(2520*a**9*x**9) + b**9*(-log(x) + log(a/b + x))/a**10
```

Giac [A]
time = 1.58, size = 122, normalized size = 0.91

$$\frac{b^9\log\left(|bx+a|\right)}{a^{10}}-\frac{b^9\log\left(|x|\right)}{a^{10}}-\frac{2520\,ab^8x^8-1260\,a^2b^7x^7+840\,a^3b^6x^6-630\,a^4b^5x^5+504\,a^5b^4x^4-420\,a^6b^3x^3+360\,a^7b^2x^2-315\,a^8bx+280\,a^9}{2520\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="giac")
```

```
[Out] b^9*log(abs(b*x + a))/a^10 - b^9*log(abs(x))/a^10 - 1/2520*(2520*a*b^8*x^8
- 1260*a^2*b^7*x^7 + 840*a^3*b^6*x^6 - 630*a^4*b^5*x^5 + 504*a^5*b^4*x^4 -
420*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 315*a^8*b*x + 280*a^9)/(a^10*x^9)
```

Mupad [B]
time = 0.13, size = 114, normalized size = 0.85

$$-\frac{280\,a^9+2520\,a\,b^8\,x^8-5040\,b^9\,x^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)+360\,a^7\,b^2\,x^2-420\,a^6\,b^3\,x^3+504\,a^5\,b^4\,x^4-630\,a^4\,b^5\,x^5+840\,a^3\,b^6\,x^6-1260\,a^2\,b^7\,x^7-315\,a^8\,b\,x}{2520\,a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)),x)
```

```
[Out] -(280*a^9 + 2520*a*b^8*x^8 - 5040*b^9*x^9*atanh((2*b*x)/a + 1) + 360*a^7*b^
2*x^2 - 420*a^6*b^3*x^3 + 504*a^5*b^4*x^4 - 630*a^4*b^5*x^5 + 840*a^3*b^6*x
^6 - 1260*a^2*b^7*x^7 - 315*a^8*b*x)/(2520*a^10*x^9)
```

3.253 $\int \frac{1}{x^{10}(a+bx)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+bx)}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+}{a^{11}}$$

```
[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*
b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*
b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Rubi [A]

time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}+\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/9*1/(a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^6
) - b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(a^
9*x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11 + (
10*b^9*Log[a + b*x])/a^11
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^{10}}-\frac{2b}{a^3x^9}+\frac{3b^2}{a^4x^8}-\frac{4b^3}{a^5x^7}+\frac{5b^4}{a^6x^6}-\frac{6b^5}{a^7x^5}+\frac{7b^6}{a^8x^4}-\frac{8b^7}{a^9x^3}+\frac{9b^8}{a^{10}x^2}-\frac{10b^9}{a^{11}x}+\ldots\right.\\ &= -\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+\ldots}\end{aligned}$$

Mathematica [A]

time = 0.06, size = 134, normalized size = 0.92

$$-\frac{\frac{a(28a^9-35a^8bx+45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7+1260ab^8x^8+2520b^9x^9)}{x^9(a+bx)}+2520b^9\log(x)-2520b^9\log(a+bx)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/252*((a*(28*a^9 - 35*a^8*b*x + 45*a^7*b^2*x^2 - 60*a^6*b^3*x^3 + 84*a^5*
b^4*x^4 - 126*a^4*b^5*x^5 + 210*a^3*b^6*x^6 - 420*a^2*b^7*x^7 + 1260*a*b^8*
x^8 + 2520*b^9*x^9))/(x^9*(a + b*x)) + 2520*b^9*Log[x] - 2520*b^9*Log[a + b
*x])/a^11
```

Maple [A]
time = 0.09, size = 135, normalized size = 0.92

method	result
default	$-\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(bx+a)}-\frac{10b^9\ln(x)}{a^{11}}+10b^9\ln$
norman	$\frac{\frac{10b^{10}x^{10}}{a^{11}}-\frac{1}{9a}+\frac{5bx}{36a^2}-\frac{5b^2x^2}{28a^3}+\frac{5b^3x^3}{21a^4}-\frac{b^4x^4}{3a^5}+\frac{b^5x^5}{2a^6}-\frac{5b^6x^6}{6a^7}+\frac{5b^7x^7}{3a^8}-\frac{5b^8x^8}{a^9}}{x^9(bx+a)}-\frac{10b^9\ln(x)}{a^{11}}+\frac{10b^9\ln(bx+a)}{a^{11}}$
risch	$\frac{-\frac{10b^9x^9}{a^{10}}-\frac{5b^8x^8}{a^9}+\frac{5b^7x^7}{3a^8}-\frac{5b^6x^6}{6a^7}+\frac{b^5x^5}{2a^6}-\frac{b^4x^4}{3a^5}+\frac{5b^3x^3}{21a^4}-\frac{5b^2x^2}{28a^3}+\frac{5bx}{36a^2}-\frac{1}{9a}}{x^9(bx+a)}+\frac{10b^9\ln(-bx-a)}{a^{11}}-\frac{10b^9\ln(x)}{a^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*
b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*
b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11
```

Maxima [A]
time = 0.27, size = 141, normalized size = 0.97

$$-\frac{2520\,b^9x^9+1260\,ab^8x^8-420\,a^2b^7x^7+210\,a^3b^6x^6-126\,a^4b^5x^5+84\,a^5b^4x^4-60\,a^6b^3x^3+45\,a^7b^2x^2-35\,a^8bx+28\,a^9}{252\,(a^{10}bx^{10}+a^{11}x^9)}+\frac{10\,b^9\log(bx+a)}{a^{11}}-\frac{10\,b^9\log(x)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 1260*a*b^8*x^8 - 420*a^2*b^7*x^7 + 210*a^3*b^6*x^6 -
 126*a^4*b^5*x^5 + 84*a^5*b^4*x^4 - 60*a^6*b^3*x^3 + 45*a^7*b^2*x^2 - 35*a^
8*b*x + 28*a^9)/(a^10*b*x^10 + a^11*x^9) + 10*b^9*log(b*x + a)/a^11 - 10*b^
9*log(x)/a^11
```

Fricas [A]
time = 0.53, size = 163, normalized size = 1.12

$$-\frac{2520\,ab^9x^9+1260\,a^2b^8x^8-420\,a^3b^7x^7+210\,a^4b^6x^6-126\,a^5b^5x^5+84\,a^6b^4x^4-60\,a^7b^3x^3+45\,a^8b^2x^2-35\,a^9bx+28\,a^{10}-2520\,(b^{10}x^{10}+ab^9x^9)\log(bx+a)+2520\,(b^{10}x^{10}+ab^9x^9)\log(x)}{252\,(a^{11}bx^{10}+a^{12}x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="fricas")

[Out] -1/252*(2520*a*b^9*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*x^6 - 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 - 35*a^9*b*x + 28*a^10 - 2520*(b^10*x^10 + a*b^9*x^9)*log(b*x + a) + 2520*(b^10*x^10 + a*b^9*x^9)*log(x))/(a^11*b*x^10 + a^12*x^9)

Sympy [A]
time = 0.22, size = 139, normalized size = 0.95

$$\frac{-28a^9+35a^8bx-45a^7b^2x^2+60a^6b^3x^3-84a^5b^4x^4+126a^4b^5x^5-210a^3b^6x^6+420a^2b^7x^7-1260ab^8x^8-2520b^9x^9}{252a^{11}x^9+252a^{10}bx^{10}}+\frac{10b^9\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**10/(b*x+a)**2,x)

[Out] (-28*a**9 + 35*a**8*b*x - 45*a**7*b**2*x**2 + 60*a**6*b**3*x**3 - 84*a**5*b**4*x**4 + 126*a**4*b**5*x**5 - 210*a**3*b**6*x**6 + 420*a**2*b**7*x**7 - 1260*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**11*x**9 + 252*a**10*b*x**10) + 10*b**9*(-log(x) + log(a/b + x))/a**11

Giac [A]
time = 1.17, size = 180, normalized size = 1.23

$$-\frac{10\,b^9\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{11}}-\frac{b^9}{(bx+a)a^{10}}-\frac{\frac{41481\,ab^9}{bx+a}-\frac{155844\,a^2b^9}{(bx+a)^2}+\frac{337176\,a^3b^9}{(bx+a)^3}-\frac{460404\,a^4b^9}{(bx+a)^4}+\frac{407484\,a^5b^9}{(bx+a)^5}-\frac{229320\,a^6b^9}{(bx+a)^6}+\frac{75600\,a^7b^9}{(bx+a)^7}-\frac{11340\,a^8b^9}{(bx+a)^8}-4861\,b^9}{252\,a^{11}\left(\frac{a}{bx+a}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="giac")

[Out] -10*b^9*log(abs(-a/(b*x + a) + 1))/a^11 - b^9/((b*x + a)*a^10) - 1/252*(41481*a*b^9/(b*x + a) - 155844*a^2*b^9/(b*x + a)^2 + 337176*a^3*b^9/(b*x + a)^3 - 460404*a^4*b^9/(b*x + a)^4 + 407484*a^5*b^9/(b*x + a)^5 - 229320*a^6*b^9/(b*x + a)^6 + 75600*a^7*b^9/(b*x + a)^7 - 11340*a^8*b^9/(b*x + a)^8 - 4861*b^9)/(a^11*(a/(b*x + a) - 1)^9)

Mupad [B]
time = 0.08, size = 135, normalized size = 0.92

$$\frac{20\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{9\,a}+\frac{5\,b^2\,x^2}{28\,a^3}-\frac{5\,b^3\,x^3}{21\,a^4}+\frac{b^4\,x^4}{3\,a^5}-\frac{b^5\,x^5}{2\,a^6}+\frac{5\,b^6\,x^6}{6\,a^7}-\frac{5\,b^7\,x^7}{3\,a^8}+\frac{5\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}-\frac{5\,b\,x}{36\,a^2}}{b\,x^{10}+a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^10*(a + b*x)^2),x)

[Out] (20*b^9*atanh((2*b*x)/a + 1))/a^11 - (1/(9*a) + (5*b^2*x^2)/(28*a^3) - (5*b^3*x^3)/(21*a^4) + (b^4*x^4)/(3*a^5) - (b^5*x^5)/(2*a^6) + (5*b^6*x^6)/(6*a^7) - (5*b^7*x^7)/(3*a^8) + (5*b^8*x^8)/a^9 + (10*b^9*x^9)/a^10 - (5*b*x)/(36*a^2))/(a*x^9 + b*x^10)

3.254 $\int \frac{1}{x^{10}(a+bx)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{10b^9}{a^{11}(a+bx)}-\frac{55b^9\,\mathrm{lo}}{a^{12}}$$

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

Rubi [A]
time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx)}{a^{12}}-\frac{10b^9}{a^{11}(a+bx)}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}+\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}+\frac{3b}{8a^4x^8}-\frac{1}{9a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/9*1/(a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^6
*x^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) + (
18*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*b^9
)/(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx = \int \left(\frac{1}{a^3x^{10}}-\frac{3b}{a^4x^9}+\frac{6b^2}{a^5x^8}-\frac{10b^3}{a^6x^7}+\frac{15b^4}{a^7x^6}-\frac{21b^5}{a^8x^5}+\frac{28b^6}{a^9x^4}-\frac{36b^7}{a^{10}x^3}+\frac{45b^8}{a^{11}x^2}-\frac{55b^9}{a^{12}x}+\right.$$
$$= -\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{}{2a^{10}}$$

Mathematica [A]
time = 0.07, size = 145, normalized size = 0.89

$$-\frac{\frac{a(56a^{10}-77a^9bx+110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8+41580ab^9x^9+27720b^{10}x^{10})}{x^9(a+bx)^2}+27720b^9\log(x)-27720b^9\log(a+bx)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/504*((a*(56*a^10 - 77*a^9*b*x + 110*a^8*b^2*x^2 - 165*a^7*b^3*x^3 + 264*
a^6*b^4*x^4 - 462*a^5*b^5*x^5 + 924*a^4*b^6*x^6 - 2310*a^3*b^7*x^7 + 9240*a
^2*b^8*x^8 + 41580*a*b^9*x^9 + 27720*b^10*x^10))/(x^9*(a + b*x)^2) + 27720*
b^9*Log[x] - 27720*b^9*Log[a + b*x])/a^12
```

Maple [A]
time = 0.09, size = 150, normalized size = 0.92

method	result
norman	$\frac{-\frac{1}{9a}+\frac{11bx}{72a^2}-\frac{55b^2x^2}{252a^3}+\frac{55b^3x^3}{168a^4}-\frac{11b^4x^4}{21a^5}+\frac{11b^5x^5}{12a^6}-\frac{11b^6x^6}{6a^7}+\frac{55b^7x^7}{12a^8}-\frac{55b^8x^8}{3a^9}+\frac{110b^{10}x^{10}}{a^{11}}+\frac{165b^{11}x^{11}}{2a^{12}}}{x^9(bx+a)^2}-\frac{55b^9\ln(x)}{a^{12}}+\frac{55b^9\ln(bx+a)}{a^{12}}$
default	$-\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(bx+a)^2}-\frac{10b^9}{a^{11}(bx+a)}-55b^9$
risch	$\frac{-\frac{55b^{10}x^{10}}{a^{11}}-\frac{165b^9x^9}{2a^{10}}-\frac{55b^8x^8}{3a^9}+\frac{55b^7x^7}{12a^8}-\frac{11b^6x^6}{6a^7}+\frac{11b^5x^5}{12a^6}-\frac{11b^4x^4}{21a^5}+\frac{55b^3x^3}{168a^4}-\frac{55b^2x^2}{252a^3}+\frac{11bx}{72a^2}-\frac{1}{9a}}{x^9(bx+a)^2}+\frac{55b^9\ln(-bx-a)}{a^{12}}-\frac{55b^9\ln(x)}{a^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

Maxima [A]
time = 0.29, size = 163, normalized size = 1.00

$$-\frac{27720\,b^{10}x^{10}+41580\,ab^9x^9+9240\,a^2b^8x^8-2310\,a^3b^7x^7+924\,a^4b^6x^6-462\,a^5b^5x^5+264\,a^6b^4x^4-165\,a^7b^3x^3+110\,a^8b^2x^2-77\,a^9bx+56\,a^{10}}{504\,(a^{11}b^2x^{11}+2\,a^{12}bx^{10}+a^{13}x^9)}+\frac{55\,b^9\log(bx+a)}{a^{12}}-\frac{55\,b^9\log(x)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/504*(27720*b^10*x^10 + 41580*a*b^9*x^9 + 9240*a^2*b^8*x^8 - 2310*a^3*b^7
*x^7 + 924*a^4*b^6*x^6 - 462*a^5*b^5*x^5 + 264*a^6*b^4*x^4 - 165*a^7*b^3*x^
3 + 110*a^8*b^2*x^2 - 77*a^9*b*x + 56*a^10)/(a^11*b^2*x^11 + 2*a^12*b*x^10
+ a^13*x^9) + 55*b^9*log(b*x + a)/a^12 - 55*b^9*log(x)/a^12
```

Fricas [A]
time = 0.52, size = 207, normalized size = 1.27

$$-\frac{27720\,ab^{10}x^{10}+41580\,a^2b^9x^9+9240\,a^3b^8x^8-2310\,a^4b^7x^7+924\,a^5b^6x^6-462\,a^6b^5x^5+264\,a^7b^4x^4-165\,a^8b^3x^3+110\,a^9b^2x^2-77\,a^{10}bx+56\,a^{11}-27720\,(b^{11}x^{11}+2\,ab^{10}x^{10}+a^2b^9x^9)\log(bx+a)+27720\,(b^{11}x^{11}+2\,ab^{10}x^{10}+a^2b^9x^9)\log(x)}{504\,(a^{12}b^2x^{11}+2\,a^{13}bx^{10}+a^{14}x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/504*(27720*a*b^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4
*b^7*x^7 + 924*a^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^
3*x^3 + 110*a^9*b^2*x^2 - 77*a^10*b*x + 56*a^11 - 27720*(b^11*x^11 + 2*a*b^
10*x^10 + a^2*b^9*x^9)*log(b*x + a) + 27720*(b^11*x^11 + 2*a*b^10*x^10 + a^
2*b^9*x^9)*log(x))/(a^12*b^2*x^11 + 2*a^13*b*x^10 + a^14*x^9)
```

Sympy [A]
time = 0.27, size = 163, normalized size = 1.00

$$\frac{-56a^{10} + 77a^9bx - 110a^8b^2x^2 + 165a^7b^3x^3 - 264a^6b^4x^4 + 462a^5b^5x^5 - 924a^4b^6x^6 + 2310a^3b^7x^7 - 9240a^2b^8x^8 - 41580ab^9x^9 - 27720b^{10}x^{10}}{504a^{13}x^9 + 1008a^{12}bx^{10} + 504a^{11}b^2x^{11}} + \frac{55b^9\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**3,x)
```

```
[Out] (-56*a**10 + 77*a**9*b*x - 110*a**8*b**2*x**2 + 165*a**7*b**3*x**3 - 264*a*
*6*b**4*x**4 + 462*a**5*b**5*x**5 - 924*a**4*b**6*x**6 + 2310*a**3*b**7*x**
7 - 9240*a**2*b**8*x**8 - 41580*a*b**9*x**9 - 27720*b**10*x**10)/(504*a**13
*x**9 + 1008*a**12*b*x**10 + 504*a**11*b**2*x**11) + 55*b**9*(-log(x) + log
(a/b + x))/a**12
```

Giac [A]
time = 1.49, size = 152, normalized size = 0.93

$$\frac{55\,b^9\log\left(|bx+a|\right)}{a^{12}} - \frac{55\,b^9\log\left(|x|\right)}{a^{12}} - \frac{27720\,ab^{10}x^{10} + 41580\,a^2b^9x^9 + 9240\,a^3b^8x^8 - 2310\,a^4b^7x^7 + 924\,a^5b^6x^6 - 462\,a^6b^5x^5 + 264\,a^7b^4x^4 - 165\,a^8b^3x^3 + 110\,a^9b^2x^2 - 77\,a^{10}bx + 56\,a^{11}}{504\,(bx+a)^2a^{12}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 55*b^9*log(abs(b*x + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/504*(27720*a*b^
10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4*b^7*x^7 + 924*a^5
*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^3*x^3 + 110*a^9*b^
2*x^2 - 77*a^10*b*x + 56*a^11)/((b*x + a)^2*a^12*x^9)
```

Mupad [B]
time = 0.23, size = 157, normalized size = 0.96

$$\frac{110\,b^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}} - \frac{\frac{1}{9\,a} + \frac{55\,b^2\,x^2}{252\,a^3} - \frac{55\,b^3\,x^3}{168\,a^4} + \frac{11\,b^4\,x^4}{21\,a^5} - \frac{11\,b^5\,x^5}{12\,a^6} + \frac{11\,b^6\,x^6}{6\,a^7} - \frac{55\,b^7\,x^7}{12\,a^8} + \frac{55\,b^8\,x^8}{3\,a^9} + \frac{165\,b^9\,x^9}{2\,a^{10}} + \frac{55\,b^{10}\,x^{10}}{a^{11}} - \frac{11\,b\,x}{72\,a^2}}{a^2\,x^9 + 2\,a\,b\,x^{10} + b^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^3),x)
```

```
[Out] (110*b^9*atanh((2*b*x)/a + 1))/a^12 - (1/(9*a) + (55*b^2*x^2)/(252*a^3) - (
55*b^3*x^3)/(168*a^4) + (11*b^4*x^4)/(21*a^5) - (11*b^5*x^5)/(12*a^6) + (11
*b^6*x^6)/(6*a^7) - (55*b^7*x^7)/(12*a^8) + (55*b^8*x^8)/(3*a^9) + (165*b^9
*x^9)/(2*a^10) + (55*b^10*x^10)/a^11 - (11*b*x)/(72*a^2))/(a^2*x^9 + b^2*x^
11 + 2*a*b*x^10)
```

3.255 $\int \frac{1}{x(2+3x)}\,dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)$$

[Out] 1/2*ln(x)-1/2*ln(2+3*x)

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(2 + 3*x)),x]

[Out] Log[x]/2 - Log[2 + 3*x]/2

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(2+3x)}\,dx &= \frac{1}{2}\int \frac{1}{x}\,dx - \frac{3}{2}\int \frac{1}{2+3x}\,dx\\ &= \frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{2} - \frac{1}{2}\log(2 + 3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\ln(x)}{2} - \frac{\ln(2+3x)}{2}$	14
norman	$\frac{\ln(x)}{2} - \frac{\ln(2+3x)}{2}$	14
risch	$\frac{\ln(x)}{2} - \frac{\ln(2+3x)}{2}$	14
meijerg	$\frac{\ln(x)}{2} - \frac{\ln(2)}{2} + \frac{\ln(3)}{2} - \frac{\ln(1+\frac{3x}{2})}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$-\frac{1}{2}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Fricas [A]
time = 0.83, size = 13, normalized size = 0.76

$$-\frac{1}{2}\log(3x+2) + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{\log(x)}{2} - \frac{\log\left(x + \frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x)
```

```
[Out] log(x)/2 - log(x + 2/3)/2
```

Giac [A]
time = 1.13, size = 15, normalized size = 0.88

$$-\frac{1}{2}\log\left(|3x+2|\right) + \frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B]
time = 0.17, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{2}{x}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + 2)),x)
```

```
[Out] -log(2/x + 3)/2
```

3.256 $\int \frac{1}{x(4+6x)} \, dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)$$

```
[Out] 1/4*ln(x)-1/4*ln(2+3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)} \, dx &= \frac{1}{4}\int \frac{1}{x} \, dx - \frac{3}{2}\int \frac{1}{4+6x} \, dx \\ &= \frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{4}-\frac{1}{4}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.82

method	result	size
default	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
norman	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
risch	$\frac{\ln(x)}{4}-\frac{\ln(2+3x)}{4}$	14
meijerg	$\frac{\ln(x)}{4}-\frac{\ln(2)}{4}+\frac{\ln(3)}{4}-\frac{\ln(1+\frac{3x}{2})}{4}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(x)-1/4*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.76

$$-\frac{1}{4}\log\left(3x+2\right)+\frac{1}{4}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

Fricas [A]
time = 0.72, size = 13, normalized size = 0.76

$$-\frac{1}{4}\log\left(3x+2\right)+\frac{1}{4}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="fricas")
```

[Out] -1/4*log(3*x + 2) + 1/4*log(x)

Sympy [**A**]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{\log(x)}{4} - \frac{\log\left(x + \frac{2}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x)
```

```
[Out] log(x)/4 - log(x + 2/3)/4
```

Giac [**A**]
time = 1.60, size = 15, normalized size = 0.88

$$-\frac{1}{4}\log\left(|3x + 2|\right) + \frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(3*x + 2)) + 1/4*log(abs(x))
```

Mupad [**B**]
time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{4}{x} + 6\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)),x)
```

```
[Out] -log(4/x + 6)/4
```

3.257 $\int \frac{1}{x^2(4+6x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)$$

```
[Out] -1/4/x-3/8*ln(x)+3/8*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)),x]
```

```
[Out] -1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)}\,dx &= \int \left(\frac{1}{4x^2}-\frac{3}{8x}+\frac{9}{8(2+3x)}\right)dx\\ &= -\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)),x]
```

```
[Out] -1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.79

method	result	size
default	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
norman	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
risch	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2+3x)}{8}$	19
meijerg	$-\frac{1}{4x}-\frac{3\ln(x)}{8}+\frac{3\ln(2)}{8}-\frac{3\ln(3)}{8}+\frac{3\ln(1+\frac{3x}{2})}{8}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x-3/8*ln(x)+3/8*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.75

$$-\frac{1}{4\,x}+\frac{3}{8}\,\log\left(3\,x+2\right)-\frac{3}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4/x + 3/8*log(3*x + 2) - 3/8*log(x)
```

Fricas [A]
time = 0.94, size = 21, normalized size = 0.88

$$\frac{3\,x\log\left(3\,x+2\right)-3\,x\log\left(x\right)-2}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*x*log(3*x + 2) - 3*x*log(x) - 2)/x
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.83

$$-\frac{3\log\left(x\right)}{8}+\frac{3\log\left(x+\frac{2}{3}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x),x)
[Out] -3*log(x)/8 + 3*log(x + 2/3)/8 - 1/(4*x)
```

Giac [A]
time = 1.47, size = 20, normalized size = 0.83

$$-\frac{1}{4x}+\frac{3}{8}\log\left(|3x+2|\right)-\frac{3}{8}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="giac")
[Out] -1/4/x + 3/8*log(abs(3*x + 2)) - 3/8*log(abs(x))
```

Mupad [B]
time = 0.05, size = 18, normalized size = 0.75

$$-\frac{3\ln\left(\frac{x}{6x+4}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)),x)
[Out] - (3*log(x/(6*x + 4)))/8 - 1/(4*x)
```

3.258 $\int \frac{1}{x^3(4+6x)} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)$$

```
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)),x]
```

```
[Out] -1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)}\,dx &= \int \left(\frac{1}{4x^3}-\frac{3}{8x^2}+\frac{9}{16x}-\frac{27}{16(2+3x)}\right)dx\\ &= -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)),x]
```

```
[Out] -1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Maple [A]
time = 0.09, size = 24, normalized size = 0.77

method	result	size
norman	$\frac{-\frac{1}{8}+\frac{3x}{8}}{x^2}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	23
risch	$\frac{-\frac{1}{8}+\frac{3x}{8}}{x^2}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	23
default	$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\ln(x)}{16}-\frac{9\ln(2+3x)}{16}$	24
meijerg	$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\ln(x)}{16}-\frac{9\ln(2)}{16}+\frac{9\ln(3)}{16}-\frac{9\ln\left(1+\frac{3x}{2}\right)}{16}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(2+3*x)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.74

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log\left(3\,x+2\right)+\frac{9}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(3*x + 2) + 9/16*log(x)
```

Fricas [A]
time = 0.78, size = 28, normalized size = 0.90

$$-\frac{9\,x^2\log\left(3\,x+2\right)-9\,x^2\log\left(x\right)-6\,x+2}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/16*(9*x^2*log(3*x + 2) - 9*x^2*log(x) - 6*x + 2)/x^2
```

Sympy [A]
time = 0.04, size = 26, normalized size = 0.84

$$\frac{9\log\left(x\right)}{16}-\frac{9\log\left(x+\frac{2}{3}\right)}{16}+\frac{3x-1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x),x)
```

```
[Out] 9*log(x)/16 - 9*log(x + 2/3)/16 + (3*x - 1)/(8*x**2)
```

Giac [A]
time = 1.27, size = 25, normalized size = 0.81

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log\left(|3\,x+2|\right)+\frac{9}{16}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="giac")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(abs(3*x + 2)) + 9/16*log(abs(x))
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.58

$$\frac{\frac{3\,x}{8}-\frac{1}{8}}{x^2}-\frac{9\,\mathrm{atanh}(3\,x+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)),x)
```

```
[Out] ((3*x)/8 - 1/8)/x^2 - (9*atanh(3*x + 1))/8
```

3.259 $\int \frac{1}{x^4(4+6x)}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

```
[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)),x]
[Out] -1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)}\,dx &= \int \left(\frac{1}{4x^4}-\frac{3}{8x^3}+\frac{9}{16x^2}-\frac{27}{32x}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.00

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)),x]
```

[Out] -1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32

Maple [A]
time = 0.09, size = 29, normalized size = 0.76

method	result	size
norman	$\frac{-\frac{1}{12}+\frac{3}{16}x-\frac{9}{16}x^2}{x^3}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	28
risch	$\frac{-\frac{1}{12}+\frac{3}{16}x-\frac{9}{16}x^2}{x^3}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	28
default	$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	29
meijerg	$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\ln(x)}{32}+\frac{27\ln(2)}{32}-\frac{27\ln(3)}{32}+\frac{27\ln\left(1+\frac{3x}{2}\right)}{32}$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(4+6*x),x,method=_RETURNVERBOSE)

[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(2+3*x)

Maxima [A]
time = 0.27, size = 28, normalized size = 0.74

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(4+6*x),x, algorithm="maxima")

[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(3*x + 2) - 27/32*log(x)

Fricas [A]
time = 1.03, size = 33, normalized size = 0.87

$$\frac{81\,x^3\log\left(3\,x+2\right)-81\,x^3\log\left(x\right)-54\,x^2+18\,x-8}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(4+6*x),x, algorithm="fricas")

[Out] 1/96*(81*x^3*log(3*x + 2) - 81*x^3*log(x) - 54*x^2 + 18*x - 8)/x^3

Sympy [A]
time = 0.04, size = 31, normalized size = 0.82

$$-\frac{27\log\left(x\right)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-27x^2+9x-4}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x),x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-27*x**2 + 9*x - 4)/(48*x**3)
```

Giac [A]
time = 1.44, size = 30, normalized size = 0.79

$$-\frac{27\,x^2-9\,x+4}{48\,x^3}+\frac{27}{32}\,\log\left(|3\,x+2|\right)-\frac{27}{32}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(abs(3*x + 2)) - 27/32*log(abs(x))
```

Mupad [B]
time = 0.09, size = 24, normalized size = 0.63

$$\frac{27\,\mathrm{atanh}(3\,x+1)}{16}-\frac{\frac{9\,x^2}{16}-\frac{3\,x}{16}+\frac{1}{12}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((9*x^2)/16 - (3*x)/16 + 1/12)/x^3
```

3.260 $\int \frac{1}{x^5(4+6x)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)$$

```
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)}\,dx &= \int \left(\frac{1}{4x^5}-\frac{3}{8x^4}+\frac{9}{16x^3}-\frac{27}{32x^2}+\frac{81}{64x}-\frac{243}{64(2+3x)}\right)dx\\ &= -\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.00

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)),x]
[Out] -1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.76

method	result	size
norman	$\frac{-\frac{1}{16}+\frac{1}{8}x-\frac{9}{32}x^2+\frac{27}{32}x^3}{x^4}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	33
risch	$\frac{-\frac{1}{16}+\frac{1}{8}x-\frac{9}{32}x^2+\frac{27}{32}x^3}{x^4}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	33
default	$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}-\frac{81\ln(2+3x)}{64}$	34
meijerg	$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\ln(x)}{64}-\frac{81\ln(2)}{64}+\frac{81\ln(3)}{64}-\frac{81\ln(1+\frac{3x}{2})}{64}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x),x,method=_RETURNVERBOSE)
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.73

$$\frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4}-\frac{81}{64}\,\log\left(3\,x+2\right)+\frac{81}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="maxima")
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(3*x + 2) + 81/64*log(x)
```

Fricas [A]
time = 0.71, size = 38, normalized size = 0.84

$$-\frac{81\,x^4\log\left(3\,x+2\right)-81\,x^4\log\left(x\right)-54\,x^3+18\,x^2-8\,x+4}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="fricas")
[Out] -1/64*(81*x^4*log(3*x + 2) - 81*x^4*log(x) - 54*x^3 + 18*x^2 - 8*x + 4)/x^4
```

Sympy [A]
time = 0.05, size = 36, normalized size = 0.80

$$\frac{81\log\left(x\right)}{64}-\frac{81\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^3-9x^2+4x-2}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x),x)
```

```
[Out] 81*log(x)/64 - 81*log(x + 2/3)/64 + (27*x**3 - 9*x**2 + 4*x - 2)/(32*x**4)
```

Giac [A]
time = 1.22, size = 35, normalized size = 0.78

$$\frac{27\,x^3-9\,x^2+4\,x-2}{32\,x^4}-\frac{81}{64}\,\log\left(|3\,x+2|\right)+\frac{81}{64}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="giac")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(abs(3*x + 2)) + 81/64*log(a
bs(x))
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.62

$$\frac{\frac{27\,x^3}{32}-\frac{9\,x^2}{32}+\frac{x}{8}-\frac{1}{16}}{x^4}-\frac{81\,\mathrm{atanh}(3\,x+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)),x)
```

```
[Out] (x/8 - (9*x^2)/32 + (27*x^3)/32 - 1/16)/x^4 - (81*atanh(3*x + 1))/32
```

3.261 $\int \frac{1}{x(4+6x)^2}\,dx$

Optimal. Leaf size=28

$$\frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)$$

[Out] 1/8/(2+3*x)+1/16*ln(x)-1/16*ln(2+3*x)

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{46\}$

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(4 + 6*x)^2),x]

[Out] 1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^2}\,dx &= \int \left(\frac{1}{16x}-\frac{3}{8(2+3x)^2}-\frac{3}{16(2+3x)}\right)dx\\ &= \frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{16}\left(\frac{2}{2+3x}+\log(-6x)-\log(4+6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^2),x]
[Out] (2/(2 + 3*x) + Log[-6*x] - Log[4 + 6*x])/16
```

Maple [A]
time = 0.09, size = 23, normalized size = 0.82

method	result	size
risch	$\frac{1}{16+24x}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	21
default	$\frac{1}{16+24x}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	23
norman	$-\frac{3x}{16(2+3x)}+\frac{\ln(x)}{16}-\frac{\ln(2+3x)}{16}$	24
meijerg	$\frac{1}{16}+\frac{\ln(x)}{16}-\frac{\ln(2)}{16}+\frac{\ln(3)}{16}-\frac{3x}{16(2+3x)}-\frac{\ln\left(1+\frac{3x}{2}\right)}{16}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/8/(2+3*x)+1/16*ln(x)-1/16*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.79

$$\frac{1}{8\,(3\,x+2)}-\frac{1}{16}\,\log\left(3\,x+2\right)+\frac{1}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="maxima")
[Out] 1/8/(3*x + 2) - 1/16*log(3*x + 2) + 1/16*log(x)
```

Fricas [A]
time = 0.90, size = 32, normalized size = 1.14

$$-\frac{(3\,x+2)\log\left(3\,x+2\right)-(3\,x+2)\log\left(x\right)-2}{16\,(3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="fricas")
[Out] -1/16*((3*x + 2)*log(3*x + 2) - (3*x + 2)*log(x) - 2)/(3*x + 2)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.68

$$\frac{\log\left(x\right)}{16}-\frac{\log\left(x+\frac{2}{3}\right)}{16}+\frac{1}{24x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**2,x)
```

```
[Out] log(x)/16 - log(x + 2/3)/16 + 1/(24*x + 16)
```

Giac [A]
time = 1.04, size = 25, normalized size = 0.89

$$\frac{1}{8\,(3\,x+2)}+\frac{1}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 1/8/(3*x + 2) + 1/16*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B]
time = 0.06, size = 20, normalized size = 0.71

$$\frac{1}{8\,\left(3\,x+2\right)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^2),x)
```

```
[Out] 1/(8*(3*x + 2)) - log((6*x + 4)/x)/16
```

3.262 $\int \frac{1}{x^2(4+6x)^2} dx$

Optimal. Leaf size=35

$$-\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)$$

```
[Out] -1/16/x-3/16/(2+3*x)-3/16*ln(x)+3/16*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] -1/16*1/x - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^2}-\frac{3}{16x}+\frac{9}{16(2+3x)^2}+\frac{9}{16(2+3x)}\right)dx\\ &= -\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{1}{x}-\frac{3}{2+3x}-3\log(x)+3\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^2),x]
[Out] (-x^(-1) - 3/(2 + 3*x) - 3*Log[x] + 3*Log[2 + 3*x])/16
```

Maple [A]
time = 0.10, size = 28, normalized size = 0.80

method	result	size
default	$-\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	28
risch	$\frac{-\frac{3x}{8}-\frac{1}{8}}{x(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	31
norman	$\frac{-\frac{1}{8}+\frac{9x^2}{16}}{x(2+3x)}-\frac{3\ln(x)}{16}+\frac{3\ln(2+3x)}{16}$	32
meijerg	$-\frac{1}{16x}-\frac{3}{32}-\frac{3\ln(x)}{16}+\frac{3\ln(2)}{16}-\frac{3\ln(3)}{16}+\frac{27x}{64(\frac{9x}{2}+3)}+\frac{3\ln(1+\frac{3x}{2})}{16}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/16/x-3/16/(2+3*x)-3/16*ln(x)+3/16*ln(2+3*x)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.89

$$-\frac{3\,x+1}{8\,(3\,x^2+2\,x)}+\frac{3}{16}\,\log\left(3\,x+2\right)-\frac{3}{16}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="maxima")
[Out] -1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(3*x + 2) - 3/16*log(x)
```

Fricas [A]
time = 0.57, size = 48, normalized size = 1.37

$$\frac{3\,(3\,x^2+2\,x)\log\left(3\,x+2\right)-3\,(3\,x^2+2\,x)\log\left(x\right)-6\,x-2}{16\,(3\,x^2+2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="fricas")
[Out] 1/16*(3*(3*x^2 + 2*x)*log(3*x + 2) - 3*(3*x^2 + 2*x)*log(x) - 6*x - 2)/(3*x
^2 + 2*x)
```

Sympy [A]
time = 0.04, size = 31, normalized size = 0.89

$$\frac{-3x-1}{24x^2+16x}-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+\frac{2}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**2,x)
[Out] (-3*x - 1)/(24*x**2 + 16*x) - 3*log(x)/16 + 3*log(x + 2/3)/16
```

Giac [A]
time = 0.72, size = 40, normalized size = 1.14

$$-\frac{3}{16\,(3\,x+2)}+\frac{3}{32\,\left(\frac{2}{3\,x+2}-1\right)}-\frac{3}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="giac")
[Out] -3/16/(3*x + 2) + 3/32/(2/(3*x + 2) - 1) - 3/16*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B]
time = 0.09, size = 34, normalized size = 0.97

$$\frac{3\,\ln\left(\frac{6\,x+4}{x}\right)}{16}-\frac{3}{4\,\left(6\,x+4\right)}-\frac{1}{4\,x\,\left(6\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^2),x)
[Out] (3*log((6*x + 4)/x))/16 - 3/(4*(6*x + 4)) - 1/(4*x*(6*x + 4))
```

3.263 $\int \frac{1}{x^3(4+6x)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)$$

```
[Out] -1/32/x^2+3/16/x+9/32/(2+3*x)+27/64*ln(x)-27/64*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] -1/32*1/x^2 + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3*
x])/64
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^3}-\frac{3}{16x^2}+\frac{27}{64x}-\frac{27}{32(2+3x)^2}-\frac{81}{64(2+3x)}\right)dx\\ &= -\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.86

$$\frac{1}{64}\left(-\frac{2}{x^2}+\frac{12}{x}+\frac{18}{2+3x}+27\log(x)-27\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^2),x]
```

```
[Out] (-2/x^2 + 12/x + 18/(2 + 3*x) + 27*Log[x] - 27*Log[2 + 3*x])/64
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.79

method	result	size
default	$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	33
norman	$\frac{-\frac{1}{16}-\frac{81}{64}x^3+\frac{9}{32}x}{x^2(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	35
risch	$\frac{\frac{27}{32}x^2+\frac{9}{32}x-\frac{1}{16}}{x^2(2+3x)}+\frac{27\ln(x)}{64}-\frac{27\ln(2+3x)}{64}$	36
meijerg	$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{64}+\frac{27\ln(x)}{64}-\frac{27\ln(2)}{64}+\frac{27\ln(3)}{64}-\frac{27x}{32(4+6x)}-\frac{27\ln\left(1+\frac{3x}{2}\right)}{64}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/x^2+3/16/x+9/32/(2+3*x)+27/64*ln(x)-27/64*ln(2+3*x)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.90

$$\frac{27\,x^2+9\,x-2}{32\,(3\,x^3+2\,x^2)}-\frac{27}{64}\,\log\left(3\,x+2\right)+\frac{27}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^2 + 9*x - 2)/(3*x^3 + 2*x^2) - 27/64*log(3*x + 2) + 27/64*log(x)
```

Fricas [A]
time = 0.60, size = 59, normalized size = 1.40

$$\frac{54\,x^2-27\,(3\,x^3+2\,x^2)\log\left(3\,x+2\right)+27\,(3\,x^3+2\,x^2)\log\left(x\right)+18\,x-4}{64\,(3\,x^3+2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(54*x^2 - 27*(3*x^3 + 2*x^2)*log(3*x + 2) + 27*(3*x^3 + 2*x^2)*log(x)
+ 18*x - 4)/(3*x^3 + 2*x^2)
```

Sympy [A]
time = 0.05, size = 36, normalized size = 0.86

$$\frac{27\log\left(x\right)}{64}-\frac{27\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^2+9x-2}{96x^3+64x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**2,x)
```

```
[Out] 27*log(x)/64 - 27*log(x + 2/3)/64 + (27*x**2 + 9*x - 2)/(96*x**3 + 64*x**2)
```

Giac [A]
time = 0.53, size = 51, normalized size = 1.21

$$\frac{9}{32\,(3\,x+2)}-\frac{9\,\left(\frac{12}{3\,x+2}-5\right)}{128\,\left(\frac{2}{3\,x+2}-1\right)^2}+\frac{27}{64}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 9/32/(3*x + 2) - 9/128*(12/(3*x + 2) - 5)/(2/(3*x + 2) - 1)^2 + 27/64*log(a
bs(-2/(3*x + 2) + 1))
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.74

$$\frac{\frac{9\,x^2}{32}+\frac{3\,x}{32}-\frac{1}{48}}{x^3+\frac{2\,x^2}{3}}-\frac{27\,\mathrm{atanh}(3\,x+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^2),x)
```

```
[Out] ((3*x)/32 + (9*x^2)/32 - 1/48)/((2*x^2)/3 + x^3) - (27*atanh(3*x + 1))/32
```

3.264 $\int \frac{1}{x^4(4+6x)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)$$

```
[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(2+3*x)-27/32*ln(x)+27/32*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] -1/48*1/x^3 + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32 +
 (27*Log[2 + 3*x])/32
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^2} \, dx &= \int \left(\frac{1}{16x^4}-\frac{3}{16x^3}+\frac{27}{64x^2}-\frac{27}{32x}+\frac{81}{64(2+3x)^2}+\frac{81}{32(2+3x)}\right) dx \\ &= -\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.90

$$\frac{1}{192}\left(-\frac{4(2-6x+27x^2+81x^3)}{x^3(2+3x)}-162\log(x)+162\log(2+3x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(4 + 6*x)^2),x]

[Out] ((-4*(2 - 6*x + 27*x^2 + 81*x^3))/(x^3*(2 + 3*x)) - 162*Log[x] + 162*Log[2 + 3*x])/192

Maple [A]
time = 0.10, size = 38, normalized size = 0.78

method	result	size
default	$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	38
norman	$\frac{-\frac{1}{24}+\frac{81}{32}x^4+\frac{1}{8}x-\frac{9}{16}x^2}{x^3(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	40
risch	$\frac{-\frac{27}{16}x^3-\frac{9}{16}x^2+\frac{1}{8}x-\frac{1}{24}}{x^3(2+3x)}-\frac{27\ln(x)}{32}+\frac{27\ln(2+3x)}{32}$	41
meijerg	$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{128}-\frac{27\ln(x)}{32}+\frac{27\ln(2)}{32}-\frac{27\ln(3)}{32}+\frac{405x}{256(5+\frac{15x}{2})}+\frac{27\ln(1+\frac{3x}{2})}{32}$	48

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(4+6*x)^2,x,method=_RETURNVERBOSE)

[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(2+3*x)-27/32*ln(x)+27/32*ln(2+3*x)

Maxima [A]
time = 0.36, size = 43, normalized size = 0.88

$$-\frac{81\,x^3+27\,x^2-6\,x+2}{48\,(3\,x^4+2\,x^3)}+\frac{27}{32}\,\log\left(3\,x+2\right)-\frac{27}{32}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="maxima")

[Out] -1/48*(81*x^3 + 27*x^2 - 6*x + 2)/(3*x^4 + 2*x^3) + 27/32*log(3*x + 2) - 27/32*log(x)

Fricas [A]
time = 1.04, size = 64, normalized size = 1.31

$$-\frac{162\,x^3+54\,x^2-81\,(3\,x^4+2\,x^3)\log\left(3\,x+2\right)+81\,(3\,x^4+2\,x^3)\log\left(x\right)-12\,x+4}{96\,(3\,x^4+2\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="fricas")

[Out] -1/96*(162*x^3 + 54*x^2 - 81*(3*x^4 + 2*x^3)*log(3*x + 2) + 81*(3*x^4 + 2*x^3)*log(x) - 12*x + 4)/(3*x^4 + 2*x^3)

Sympy [A]
time = 0.05, size = 41, normalized size = 0.84

$$-\frac{27\log\left(x\right)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-81x^{3}-27x^{2}+6x-2}{144x^{4}+96x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**2,x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-81*x**3 - 27*x**2 + 6*x - 2)/(144*x*
*4 + 96*x**3)
```

Giac [A]
time = 0.54, size = 60, normalized size = 1.22

$$-\frac{27}{64\,(3\,x+2)}-\frac{9\left(\frac{60}{3\,x+2}-\frac{72}{(3\,x+2)^{2}}-13\right)}{128\left(\frac{2}{3\,x+2}-1\right)^{3}}-\frac{27}{32}\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] -27/64/(3*x + 2) - 9/128*(60/(3*x + 2) - 72/(3*x + 2)^2 - 13)/(2/(3*x + 2)
- 1)^3 - 27/32*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B]
time = 0.09, size = 37, normalized size = 0.76

$$\frac{27\,\mathrm{atanh}(3\,x+1)}{16}-\frac{\frac{9\,x^{3}}{16}+\frac{3\,x^{2}}{16}-\frac{x}{24}+\frac{1}{72}}{x^{4}+\frac{2\,x^{3}}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^2),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((3*x^2)/16 - x/24 + (9*x^3)/16 + 1/72)/((2*x^3)/3
 + x^4)
```

3.265 $\int \frac{1}{x^5(4+6x)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)$$

```
[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(2+3*x)+405/256*ln(x)-405/256*
ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^5}-\frac{3}{16x^4}+\frac{27}{64x^3}-\frac{27}{32x^2}+\frac{405}{256x}-\frac{243}{128(2+3x)^2}-\frac{1215}{256(2+3x)}\right)dx\\ &= -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Maple [A]
time = 0.11, size = 43, normalized size = 0.77

method	result	size
default	$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	43
norman	$\frac{-\frac{1}{32}-\frac{1215}{256}x^5+\frac{5}{64}x-\frac{15}{64}x^2+\frac{135}{128}x^3}{x^4(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	45
risch	$\frac{\frac{405}{128}x^4+\frac{135}{128}x^3-\frac{15}{64}x^2+\frac{5}{64}x-\frac{1}{32}}{x^4(2+3x)}+\frac{405\ln(x)}{256}-\frac{405\ln(2+3x)}{256}$	46
meijerg	$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{256}+\frac{405\ln(x)}{256}-\frac{405\ln(2)}{256}+\frac{405\ln(3)}{256}-\frac{729x}{256(9x+6)}-\frac{405\ln(1+\frac{3x}{2})}{256}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(2+3*x)+405/256*ln(x)-405/256*
ln(2+3*x)
```

Maxima [A]
time = 0.32, size = 48, normalized size = 0.86

$$\frac{405\,x^4+135\,x^3-30\,x^2+10\,x-4}{128\,(3\,x^5+2\,x^4)}-\frac{405}{256}\,\log\left(3\,x+2\right)+\frac{405}{256}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/(3*x^5 + 2*x^4) - 405/256*log
(3*x + 2) + 405/256*log(x)
```

Fricas [A]
time = 0.56, size = 69, normalized size = 1.23

$$\frac{810\,x^4+270\,x^3-60\,x^2-405\,(3\,x^5+2\,x^4)\log\left(3\,x+2\right)+405\,(3\,x^5+2\,x^4)\log\left(x\right)+20\,x-8}{256\,(3\,x^5+2\,x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/256*(810*x^4 + 270*x^3 - 60*x^2 - 405*(3*x^5 + 2*x^4)*log(3*x + 2) + 405*
(3*x^5 + 2*x^4)*log(x) + 20*x - 8)/(3*x^5 + 2*x^4)
```

Sympy [A]
time = 0.05, size = 46, normalized size = 0.82

$$\frac{405\log(x)}{256} - \frac{405\log\left(x+\frac{2}{3}\right)}{256} + \frac{405x^4+135x^3-30x^2+10x-4}{384x^5+256x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**2,x)
```

```
[Out] 405*log(x)/256 - 405*log(x + 2/3)/256 + (405*x**4 + 135*x**3 - 30*x**2 + 10
*x - 4)/(384*x**5 + 256*x**4)
```

Giac [A]
time = 0.47, size = 69, normalized size = 1.23

$$\frac{81}{128\,(3\,x+2)} - \frac{27\left(\frac{520}{3\,x+2} - \frac{1200}{(3\,x+2)^2} + \frac{960}{(3\,x+2)^3} - 77\right)}{1024\left(\frac{2}{3\,x+2}-1\right)^4} + \frac{405}{256}\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 81/128/(3*x + 2) - 27/1024*(520/(3*x + 2) - 1200/(3*x + 2)^2 + 960/(3*x + 2
)^3 - 77)/(2/(3*x + 2) - 1)^4 + 405/256*log(abs(-2/(3*x + 2) + 1))
```

Mupad [B]
time = 0.09, size = 41, normalized size = 0.73

$$\frac{\frac{135\,x^4}{128} + \frac{45\,x^3}{128} - \frac{5\,x^2}{64} + \frac{5\,x}{192} - \frac{1}{96}}{x^5 + \frac{2\,x^4}{3}} - \frac{405\,\mathrm{atanh}(3\,x+1)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^2),x)
```

```
[Out] ((5*x)/192 - (5*x^2)/64 + (45*x^3)/128 + (135*x^4)/128 - 1/96)/((2*x^4)/3 +
 x^5) - (405*atanh(3*x + 1))/128
```

3.266 $\int \frac{1}{x(4+6x)^3}\,dx$

Optimal. Leaf size=39

$$\frac{1}{32(2+3x)^2}+\frac{1}{32(2+3x)}+\frac{\log(x)}{64}-\frac{1}{64}\log(2+3x)$$

```
[Out] 1/32/(2+3*x)^2+1/32/(2+3*x)+1/64*ln(x)-1/64*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{46\}$

$$\frac{1}{32(3x+2)}+\frac{1}{32(3x+2)^2}+\frac{\log(x)}{64}-\frac{1}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^3),x]
```

```
[Out] 1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^3}\,dx &= \int \left(\frac{1}{64x}-\frac{3}{16(2+3x)^3}-\frac{3}{32(2+3x)^2}-\frac{3}{64(2+3x)}\right)dx\\ &= \frac{1}{32(2+3x)^2}+\frac{1}{32(2+3x)}+\frac{\log(x)}{64}-\frac{1}{64}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.74

$$\frac{1}{64}\left(\frac{6(1+x)}{(2+3x)^2}+\log(-6x)-\log(4+6x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(4 + 6*x)^3),x]

[Out] ((6*(1 + x))/(2 + 3*x)^2 + Log[-6*x] - Log[4 + 6*x])/64

Maple [A]
time = 0.10, size = 32, normalized size = 0.82

method	result	size
risch	$\frac{\frac{3x}{32}+\frac{3}{32}}{(2+3x)^2}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	28
norman	$\frac{-\frac{3}{16}x-\frac{27}{128}x^2}{(2+3x)^2}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	31
default	$\frac{1}{32(2+3x)^2}+\frac{1}{64+96x}+\frac{\ln(x)}{64}-\frac{\ln(2+3x)}{64}$	32
meijerg	$\frac{3}{128}+\frac{\ln(x)}{64}-\frac{\ln(2)}{64}+\frac{\ln(3)}{64}-\frac{3x\left(\frac{9x}{2}+4\right)}{256\left(1+\frac{3x}{2}\right)^2}-\frac{\ln\left(1+\frac{3x}{2}\right)}{64}$	38

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(4+6*x)^3,x,method=_RETURNVERBOSE)

[Out] 1/32/(2+3*x)^2+1/32/(2+3*x)+1/64*ln(x)-1/64*ln(2+3*x)

Maxima [A]
time = 0.27, size = 30, normalized size = 0.77

$$\frac{3\,(x+1)}{32\,(9\,x^2+12\,x+4)}-\frac{1}{64}\,\log\left(3\,x+2\right)+\frac{1}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(4+6*x)^3,x, algorithm="maxima")

[Out] 3/32*(x + 1)/(9*x^2 + 12*x + 4) - 1/64*log(3*x + 2) + 1/64*log(x)

Fricas [A]
time = 0.50, size = 50, normalized size = 1.28

$$-\frac{(9\,x^2+12\,x+4)\log\left(3\,x+2\right)-(9\,x^2+12\,x+4)\log\left(x\right)-6\,x-6}{64\,(9\,x^2+12\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(4+6*x)^3,x, algorithm="fricas")

[Out] -1/64*((9*x^2 + 12*x + 4)*log(3*x + 2) - (9*x^2 + 12*x + 4)*log(x) - 6*x - 6)/(9*x^2 + 12*x + 4)

Sympy [A]
time = 0.04, size = 27, normalized size = 0.69

$$\frac{3x+3}{288x^2+384x+128}+\frac{\log\left(x\right)}{64}-\frac{\log\left(x+\frac{2}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**3,x)
```

```
[Out] (3*x + 3)/(288*x**2 + 384*x + 128) + log(x)/64 - log(x + 2/3)/64
```

Giac [A]
time = 0.97, size = 27, normalized size = 0.69

$$\frac{3\,(x+1)}{32\,(3\,x+2)^2}-\frac{1}{64}\,\log\left(|3\,x+2|\right)+\frac{1}{64}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 3/32*(x + 1)/(3*x + 2)^2 - 1/64*log(abs(3*x + 2)) + 1/64*log(abs(x))
```

Mupad [B]
time = 0.13, size = 29, normalized size = 0.74

$$\frac{1}{32\,(3\,x+2)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{64}+\frac{1}{8\,(6\,x+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^3),x)
```

```
[Out] 1/(32*(3*x + 2)) - log((6*x + 4)/x)/64 + 1/(8*(6*x + 4)^2)
```

3.267 $\int \frac{1}{x^2(4+6x)^3}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\log(x)}{128}+\frac{9}{128}\log(2+3x)$$

```
[Out] -1/64/x-3/64/(2+3*x)^2-3/32/(2+3*x)-9/128*ln(x)+9/128*ln(2+3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] -1/64*1/x - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Log
[2 + 3*x])/128
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^2}-\frac{9}{128x}+\frac{9}{32(2+3x)^3}+\frac{9}{32(2+3x)^2}+\frac{27}{128(2+3x)}\right)dx\\ &= -\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\log(x)}{128}+\frac{9}{128}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.85

$$\frac{1}{128}\left(-\frac{2(4+27x+27x^2)}{x(2+3x)^2}-9\log(x)+9\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^3),x]
[Out] ((-2*(4 + 27*x + 27*x^2))/(x*(2 + 3*x)^2) - 9*Log[x] + 9*Log[2 + 3*x])/128
```

Maple [A]
time = 0.12, size = 37, normalized size = 0.80

method	result	size
risch	$\frac{-\frac{27}{64}x^2-\frac{27}{64}x-\frac{1}{16}}{x(2+3x)^2}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	36
default	$-\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	37
norman	$\frac{-\frac{1}{16}+\frac{27}{32}x^2+\frac{243}{256}x^3}{x(2+3x)^2}-\frac{9\ln(x)}{128}+\frac{9\ln(2+3x)}{128}$	37
meijerg	$-\frac{1}{64x}-\frac{15}{256}-\frac{9\ln(x)}{128}+\frac{9\ln(2)}{128}-\frac{9\ln(3)}{128}+\frac{9x\left(\frac{15x}{2}+6\right)}{512\left(1+\frac{3x}{2}\right)^2}+\frac{9\ln\left(1+\frac{3x}{2}\right)}{128}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/64/x-3/64/(2+3*x)^2-3/32/(2+3*x)-9/128*ln(x)+9/128*ln(2+3*x)
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.89

$$-\frac{27\,x^2+27\,x+4}{64\,(9\,x^3+12\,x^2+4\,x)}+\frac{9}{128}\,\log\left(3\,x+2\right)-\frac{9}{128}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="maxima")
[Out] -1/64*(27*x^2 + 27*x + 4)/(9*x^3 + 12*x^2 + 4*x) + 9/128*log(3*x + 2) - 9/1
28*log(x)
```

Fricas [A]
time = 0.67, size = 68, normalized size = 1.48

$$-\frac{54\,x^2-9\,(9\,x^3+12\,x^2+4\,x)\log\left(3\,x+2\right)+9\,(9\,x^3+12\,x^2+4\,x)\log\left(x\right)+54\,x+8}{128\,(9\,x^3+12\,x^2+4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="fricas")
[Out] -1/128*(54*x^2 - 9*(9*x^3 + 12*x^2 + 4*x)*log(3*x + 2) + 9*(9*x^3 + 12*x^2
+ 4*x)*log(x) + 54*x + 8)/(9*x^3 + 12*x^2 + 4*x)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 0.89

$$\frac{-27x^2-27x-4}{576x^3+768x^2+256x}-\frac{9\log{(x)}}{128}+\frac{9\log{\left(x+\frac{2}{3}\right)}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**3,x)
```

```
[Out] (-27*x**2 - 27*x - 4)/(576*x**3 + 768*x**2 + 256*x) - 9*log(x)/128 + 9*log(
x + 2/3)/128
```

Giac [A]
time = 0.83, size = 37, normalized size = 0.80

$$-\frac{27\,x^2+27\,x+4}{64\,(3\,x+2)^2x}+\frac{9}{128}\,\log\left(|3\,x+2|\right)-\frac{9}{128}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/((3*x + 2)^2*x) + 9/128*log(abs(3*x + 2)) - 9/128
*log(abs(x))
```

Mupad [B]
time = 0.09, size = 35, normalized size = 0.76

$$\frac{9\,\mathrm{atanh}(3\,x+1)}{64}-\frac{\frac{3\,x^2}{64}+\frac{3\,x}{64}+\frac{1}{144}}{x^3+\frac{4\,x^2}{3}+\frac{4\,x}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^3),x)
```

```
[Out] (9*atanh(3*x + 1))/64 - ((3*x)/64 + (3*x^2)/64 + 1/144)/((4*x)/9 + (4*x^2)/
3 + x^3)
```

3.268 $\int \frac{1}{x^3(4+6x)^3}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)$$

```
[Out] -1/128/x^2+9/128/x+9/128/(2+3*x)^2+27/128/(2+3*x)+27/128*ln(x)-27/128*ln(2+
3*x)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] -1/128*1/x^2 + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*L
og[x])/128 - (27*Log[2 + 3*x])/128
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^3}-\frac{9}{128x^2}+\frac{27}{128x}-\frac{27}{64(2+3x)^3}-\frac{81}{128(2+3x)^2}-\frac{81}{128(2+3x)}\right)dx\\ &= -\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.83

$$\frac{1}{128}\left(\frac{2(-2+12x+81x^2+81x^3)}{x^2(2+3x)^2}+27\log(x)-27\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-2 + 12*x + 81*x^2 + 81*x^3))/(x^2*(2 + 3*x)^2) + 27*Log[x] - 27*Log[2
 + 3*x])/128
```

Maple [A]
time = 0.10, size = 42, normalized size = 0.79

method	result	size
norman	$\frac{-\frac{1}{32}-\frac{81}{32}x^3-\frac{729}{256}x^4+\frac{3}{16}x}{x^2(2+3x)^2}+\frac{27\ln(x)}{128}-\frac{27\ln(2+3x)}{128}$	40
risch	$\frac{\frac{81}{64}x^3+\frac{81}{64}x^2+\frac{3}{16}x-\frac{1}{32}}{x^2(2+3x)^2}+\frac{27\ln(x)}{128}-\frac{27\ln(2+3x)}{128}$	41
default	$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\ln(x)}{128}-\frac{27\ln(2+3x)}{128}$	42
meijerg	$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{63}{512}+\frac{27\ln(x)}{128}-\frac{27\ln(2)}{128}+\frac{27\ln(3)}{128}-\frac{27x\left(\frac{21x}{2}+8\right)}{1024\left(1+\frac{3x}{2}\right)^2}-\frac{27\ln\left(1+\frac{3x}{2}\right)}{128}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/x^2+9/128/x+9/128/(2+3*x)^2+27/128/(2+3*x)+27/128*ln(x)-27/128*ln(2+
3*x)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.91

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\,(9\,x^4+12\,x^3+4\,x^2)}-\frac{27}{128}\,\log\left(3\,x+2\right)+\frac{27}{128}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(9*x^4 + 12*x^3 + 4*x^2) - 27/128*log(3*x
 + 2) + 27/128*log(x)
```

Fricas [A]
time = 0.61, size = 79, normalized size = 1.49

$$\frac{162\,x^3+162\,x^2-27\,(9\,x^4+12\,x^3+4\,x^2)\log\left(3\,x+2\right)+27\,(9\,x^4+12\,x^3+4\,x^2)\log\left(x\right)+24\,x-4}{128\,(9\,x^4+12\,x^3+4\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(162*x^3 + 162*x^2 - 27*(9*x^4 + 12*x^3 + 4*x^2)*log(3*x + 2) + 27*(9
*x^4 + 12*x^3 + 4*x^2)*log(x) + 24*x - 4)/(9*x^4 + 12*x^3 + 4*x^2)
```

Sympy [A]
time = 0.05, size = 46, normalized size = 0.87

$$\frac{27\log(x)}{128} - \frac{27\log\left(x + \frac{2}{3}\right)}{128} + \frac{81x^3 + 81x^2 + 12x - 2}{576x^4 + 768x^3 + 256x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**3,x)
```

```
[Out] 27*log(x)/128 - 27*log(x + 2/3)/128 + (81*x**3 + 81*x**2 + 12*x - 2)/(576*x
**4 + 768*x**3 + 256*x**2)
```

Giac [A]
time = 0.79, size = 43, normalized size = 0.81

$$\frac{81\,x^3 + 81\,x^2 + 12\,x - 2}{64\,(3\,x^2 + 2\,x)^2} - \frac{27}{128}\,\log(|3\,x + 2|) + \frac{27}{128}\,\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(3*x^2 + 2*x)^2 - 27/128*log(abs(3*x + 2)
) + 27/128*log(abs(x))
```

Mupad [B]
time = 0.09, size = 41, normalized size = 0.77

$$\frac{\frac{9\,x^3}{64} + \frac{9\,x^2}{64} + \frac{x}{48} - \frac{1}{288}}{x^4 + \frac{4\,x^3}{3} + \frac{4\,x^2}{9}} - \frac{27\,\mathrm{atanh}(3\,x + 1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^3),x)
```

```
[Out] (x/48 + (9*x^2)/64 + (9*x^3)/64 - 1/288)/((4*x^2)/9 + (4*x^3)/3 + x^4) - (2
7*atanh(3*x + 1))/64
```

3.269 $\int \frac{1}{x^4(4+6x)^3}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3x)$$

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(2+3*x)^2-27/64/(2+3*x)-135/256*ln(x)+
135/256*ln(2+3*x)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {46}

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] -1/192*1/x^3 + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
+ 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^4}-\frac{9}{128x^3}+\frac{27}{128x^2}-\frac{135}{256x}+\frac{81}{128(2+3x)^3}+\frac{81}{64(2+3x)^2}+\frac{405}{256(2+3x)}\right)dx\\ &= -\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.82

$$\frac{1}{768}\left(-\frac{2(8-30x+180x^2+1215x^3+1215x^4)}{x^3(2+3x)^2}-405\log(x)+405\log(2+3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(8 - 30*x + 180*x^2 + 1215*x^3 + 1215*x^4))/(x^3*(2 + 3*x)^2) - 405*Lo
g[x] + 405*Log[2 + 3*x])/768
```

Maple [A]
time = 0.10, size = 47, normalized size = 0.78

method	result	size
norman	$\frac{-\frac{1}{48}+\frac{405}{64}x^4+\frac{3645}{512}x^5+\frac{5}{64}x-\frac{15}{32}x^2}{x^3(2+3x)^2}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$	45
risch	$\frac{-\frac{405}{128}x^4-\frac{405}{128}x^3-\frac{15}{32}x^2+\frac{5}{64}x-\frac{1}{48}}{x^3(2+3x)^2}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$	46
default	$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\ln(x)}{256}+\frac{135\ln(2+3x)}{256}$	47
meijerg	$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{243}{1024}-\frac{135\ln(x)}{256}+\frac{135\ln(2)}{256}-\frac{135\ln(3)}{256}+\frac{81x\left(\frac{27x}{2}+10\right)}{2048\left(1+\frac{3x}{2}\right)^2}+\frac{135\ln\left(1+\frac{3x}{2}\right)}{256}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(2+3*x)^2-27/64/(2+3*x)-135/256*ln(x)+
135/256*ln(2+3*x)
```

Maxima [A]
time = 0.32, size = 53, normalized size = 0.88

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(9\,x^5+12\,x^4+4\,x^3)}+\frac{135}{256}\,\log\left(3\,x+2\right)-\frac{135}{256}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/(9*x^5 + 12*x^4 + 4*x^3)
+ 135/256*log(3*x + 2) - 135/256*log(x)
```

Fricas [A]
time = 0.49, size = 84, normalized size = 1.40

$$-\frac{2430\,x^4+2430\,x^3+360\,x^2-405\,(9\,x^5+12\,x^4+4\,x^3)\log\left(3\,x+2\right)+405\,(9\,x^5+12\,x^4+4\,x^3)\log\left(x\right)-60\,x+16}{768\,(9\,x^5+12\,x^4+4\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/768*(2430*x^4 + 2430*x^3 + 360*x^2 - 405*(9*x^5 + 12*x^4 + 4*x^3)*log(3*
x + 2) + 405*(9*x^5 + 12*x^4 + 4*x^3)*log(x) - 60*x + 16)/(9*x^5 + 12*x^4 +
 4*x^3)
```

Sympy [A]
time = 0.06, size = 51, normalized size = 0.85

$$-\frac{135\log{(x)}}{256}+\frac{135\log{\left(x+\frac{2}{3}\right)}}{256}+\frac{-1215x^4-1215x^3-180x^2+30x-8}{3456x^5+4608x^4+1536x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**3,x)
```

```
[Out] -135*log(x)/256 + 135*log(x + 2/3)/256 + (-1215*x**4 - 1215*x**3 - 180*x**2
 + 30*x - 8)/(3456*x**5 + 4608*x**4 + 1536*x**3)
```

Giac [A]
time = 0.67, size = 47, normalized size = 0.78

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(3\,x+2)^2x^3}+\frac{135}{256}\,\log\left(|3\,x+2|\right)-\frac{135}{256}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/((3*x + 2)^2*x^3) + 135/2
56*log(abs(3*x + 2)) - 135/256*log(abs(x))
```

Mupad [B]
time = 0.05, size = 47, normalized size = 0.78

$$\frac{135\,\mathrm{atanh}(3\,x+1)}{128}-\frac{\frac{45\,x^4}{128}+\frac{45\,x^3}{128}+\frac{5\,x^2}{96}-\frac{5\,x}{576}+\frac{1}{432}}{x^5+\frac{4\,x^4}{3}+\frac{4\,x^3}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^3),x)
```

```
[Out] (135*atanh(3*x + 1))/128 - ((5*x^2)/96 - (5*x)/576 + (45*x^3)/128 + (45*x^4
)/128 + 1/432)/((4*x^3)/9 + (4*x^4)/3 + x^5)
```

3.270 $\int \frac{1}{x^5(4+6x)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{1215\log(2+3x)}{1024}$$

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(2+3*x)^2+405/512/(2+3*x)+
1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] -1/256*1/x^4 + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x)
^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^5}-\frac{9}{128x^4}+\frac{27}{128x^3}-\frac{135}{256x^2}+\frac{1215}{1024x}-\frac{243}{256(2+3x)^3}-\frac{1215}{512(2+3x)^2}-\frac{}{102}\right.\\ &= -\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{12}{}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.81

$$\frac{\frac{2(-8+24x-90x^2+540x^3+3645x^4+3645x^5)}{x^4(2+3x)^2}+1215\log(x)-1215\log(2+3x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-8 + 24*x - 90*x^2 + 540*x^3 + 3645*x^4 + 3645*x^5))/(x^4*(2 + 3*x)^2)
 + 1215*Log[x] - 1215*Log[2 + 3*x])/1024
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.78

method	result
norman	$\frac{-\frac{1}{64}-\frac{3645}{256}x^{5}-\frac{32805}{2048}x^{6}+\frac{3}{64}x-\frac{45}{256}x^{2}+\frac{135}{128}x^{3}}{x^{4}(2+3x)^{2}}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
risch	$\frac{\frac{3645}{512}x^{5}+\frac{3645}{512}x^{4}+\frac{135}{128}x^{3}-\frac{45}{256}x^{2}+\frac{3}{64}x-\frac{1}{64}}{x^{4}(2+3x)^{2}}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
default	$-\frac{1}{256x^{4}}+\frac{3}{128x^{3}}-\frac{27}{256x^{2}}+\frac{135}{256x}+\frac{81}{512(2+3x)^{2}}+\frac{405}{512(2+3x)}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2+3x)}{1024}$
meijerg	$-\frac{1}{256x^{4}}+\frac{3}{128x^{3}}-\frac{27}{256x^{2}}+\frac{135}{256x}+\frac{891}{2048}+\frac{1215\ln(x)}{1024}-\frac{1215\ln(2)}{1024}+\frac{1215\ln(3)}{1024}-\frac{243x\left(\frac{33x}{2}+12\right)}{4096\left(1+\frac{3x}{2}\right)^{2}}-\frac{1215\ln\left(1+\frac{3x}{2}\right)}{1024}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(2+3*x)^2+405/512/(2+3*x)+
1215/1024*ln(x)-1215/1024*ln(2+3*x)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.87

$$\frac{3645\,x^{5}+3645\,x^{4}+540\,x^{3}-90\,x^{2}+24\,x-8}{512\,(9\,x^{6}+12\,x^{5}+4\,x^{4})}-\frac{1215}{1024}\,\log\left(3\,x+2\right)+\frac{1215}{1024}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/(9*x^6 + 12*x^5 +
 4*x^4) - 1215/1024*log(3*x + 2) + 1215/1024*log(x)
```

Fricas [A]
time = 0.44, size = 89, normalized size = 1.33

$$\frac{7290\,x^{5}+7290\,x^{4}+1080\,x^{3}-180\,x^{2}-1215\,(9\,x^{6}+12\,x^{5}+4\,x^{4})\log\left(3\,x+2\right)+1215\,(9\,x^{6}+12\,x^{5}+4\,x^{4})\log\left(x\right)+48\,x-16}{1024\,(9\,x^{6}+12\,x^{5}+4\,x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(7290*x^5 + 7290*x^4 + 1080*x^3 - 180*x^2 - 1215*(9*x^6 + 12*x^5 + 4
*x^4)*log(3*x + 2) + 1215*(9*x^6 + 12*x^5 + 4*x^4)*log(x) + 48*x - 16)/(9*x
^6 + 12*x^5 + 4*x^4)
```

Sympy [A]
time = 0.06, size = 56, normalized size = 0.84

$$\frac{1215\log(x)}{1024} - \frac{1215\log\left(x+\frac{2}{3}\right)}{1024} + \frac{3645x^5 + 3645x^4 + 540x^3 - 90x^2 + 24x - 8}{4608x^6 + 6144x^5 + 2048x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**3,x)
```

```
[Out] 1215*log(x)/1024 - 1215*log(x + 2/3)/1024 + (3645*x**5 + 3645*x**4 + 540*x*
*3 - 90*x**2 + 24*x - 8)/(4608*x**6 + 6144*x**5 + 2048*x**4)
```

Giac [A]
time = 0.65, size = 52, normalized size = 0.78

$$\frac{3645\,x^5 + 3645\,x^4 + 540\,x^3 - 90\,x^2 + 24\,x - 8}{512\,(3\,x+2)^2x^4} - \frac{1215}{1024}\log\left(|3\,x+2|\right) + \frac{1215}{1024}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/((3*x + 2)^2*x^4)
 - 1215/1024*log(abs(3*x + 2)) + 1215/1024*log(abs(x))
```

Mupad [B]
time = 0.05, size = 51, normalized size = 0.76

$$\frac{\frac{405\,x^5}{512} + \frac{405\,x^4}{512} + \frac{15\,x^3}{128} - \frac{5\,x^2}{256} + \frac{x}{192} - \frac{1}{576}}{x^6 + \frac{4\,x^5}{3} + \frac{4\,x^4}{9}} - \frac{1215\,\mathrm{atanh}(3\,x+1)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^3),x)
```

```
[Out] (x/192 - (5*x^2)/256 + (15*x^3)/128 + (405*x^4)/512 + (405*x^5)/512 - 1/576
)/((4*x^4)/9 + (4*x^5)/3 + x^6) - (1215*atanh(3*x + 1))/512
```

3.271 $\int \frac{1}{2+2x} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\log(1+x)$$

```
[Out] 1/2*ln(1+x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{31\}$

$$\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)^(-1),x]
```

```
[Out] Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+2x} \, dx = \frac{1}{2}\log(1+x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.25

$$\frac{1}{2}\log(2+2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)^(-1),x]
```

```
[Out] Log[2 + 2*x]/2
```

Maple [A]
time = 0.08, size = 9, normalized size = 1.12

method	result	size
meijerg	$\frac{\ln(1+x)}{2}$	7
risch	$\frac{\ln(1+x)}{2}$	7
default	$\frac{\ln(2+2x)}{2}$	9
norman	$\frac{\ln(2+2x)}{2}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+2*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(2+2*x)
```

Maxima [A]
time = 0.30, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1)
```

Fricas [A]
time = 0.49, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.88

$$\frac{\log\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x)
```

```
[Out] log(2*x + 2)/2
```

Giac [A]
time = 0.64, size = 7, normalized size = 0.88

$$\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1))
```

Mupad [B]
time = 0.15, size = 6, normalized size = 0.75

$$\frac{\ln\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 2),x)
```

```
[Out] log(x + 1)/2
```

3.272 $\int \frac{1}{4-6x} \, dx$

Optimal. Leaf size=10

$$-\frac{1}{6} \log(2-3x)$$

```
[Out] -1/6*ln(2-3*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$-\frac{1}{6} \log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*x)^(-1),x]
```

```
[Out] -1/6*Log[2 - 3*x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{4-6x} \, dx = -\frac{1}{6} \log(2-3x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{6} \log(4-6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*x)^(-1),x]
```

```
[Out] -1/6*Log[4 - 6*x]
```

Maple [A]
time = 0.08, size = 9, normalized size = 0.90

method	result	size
default	$-\frac{\ln(4-6x)}{6}$	9
norman	$-\frac{\ln(6x-4)}{6}$	9
meijerg	$-\frac{\ln\left(1-\frac{3x}{2}\right)}{6}$	9
risch	$-\frac{\ln(-2+3x)}{6}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(4-6*x)
```

Maxima [A]
time = 0.27, size = 8, normalized size = 0.80

$$-\frac{1}{6}\log\left(3x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="maxima")
```

```
[Out] -1/6*log(3*x - 2)
```

Fricas [A]
time = 0.42, size = 8, normalized size = 0.80

$$-\frac{1}{6}\log\left(3x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(3*x - 2)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.80

$$-\frac{\log\left(6x-4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x)
```

```
[Out] -log(6*x - 4)/6
```

Giac [A]
time = 0.98, size = 9, normalized size = 0.90

$$-\frac{1}{6}\log\left(|3x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(3*x - 2))
```

Mupad [B]
time = 0.08, size = 6, normalized size = 0.60

$$-\frac{\ln\left(x-\frac{2}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(6*x - 4),x)
```

```
[Out] -log(x - 2/3)/6
```

3.273 $\int \frac{1}{a+\sqrt{a}\,x}\,dx$

Optimal. Leaf size=14

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

```
[Out] ln(x+a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{31\}$

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] Log[Sqrt[a] + x]/Sqrt[a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{a}\,x}\,dx = \frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.14

$$\frac{\log\left(a+\sqrt{a}\,x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[a]*x]/Sqrt[a]
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
default	$\frac{\ln\left(a+x\sqrt{a}\right)}{\sqrt{a}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*a^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a+x*a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{\log\left(\sqrt{a}\,x+a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

Fricas [A]
time = 0.42, size = 10, normalized size = 0.71

$$\frac{\log\left(x+\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="fricas")
```

```
[Out] log(x + sqrt(a))/sqrt(a)
```

Sympy [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{\log\left(\sqrt{a}\,x+a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a**(1/2)),x)
```

```
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

Giac [A]
time = 0.71, size = 13, normalized size = 0.93

$$\frac{\log\left(\left|\sqrt{a}\,x+a\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(a)*x + a))/sqrt(a)
```

Mupad [B]
time = 0.11, size = 10, normalized size = 0.71

$$\frac{\ln\left(x+\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a^(1/2)*x),x)
[Out] log(x + a^(1/2))/a^(1/2)
```

3.274 $\int \frac{1}{a+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

```
[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {31}

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{-a}\,x}\,dx=\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A]
time = 0.09, size = 17, normalized size = 0.85

method	result	size
default	$\frac{\ln\left(a+x\sqrt{-a}\right)}{\sqrt{-a}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.80

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(-a)*x + a)/sqrt(-a)
```

Fricas [A]
time = 0.43, size = 20, normalized size = 1.00

$$-\frac{\sqrt{-a}\,\log\left(x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(x - sqrt(-a))/a
```

Sympy [A]
time = 0.01, size = 17, normalized size = 0.85

$$\frac{\log\left(a+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)**(1/2)),x)
[Out] log(a + x*sqrt(-a))/sqrt(-a)
```

Giac [A]
time = 0.64, size = 17, normalized size = 0.85

$$\frac{\log\left(\left|\sqrt{-a}\,x+a\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + a))/sqrt(-a)
```

Mupad [B]
time = 0.11, size = 16, normalized size = 0.80

$$\frac{\ln\left(x-\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - (-a)^(1/2))/(-a)^(1/2)
```

3.275 $\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=22

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

```
[Out] ln(a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{31\}$

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx=\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.86

method	result	size
default	$\frac{\ln\left(a^{2}+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.32, size = 18, normalized size = 0.82

$$\frac{\log\left(a^{2}+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(a^2 + sqrt(-a)*x)/sqrt(-a)
```

Fricas [A]
time = 0.40, size = 21, normalized size = 0.95

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(-sqrt(-a)*a + x)/a
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.86

$$\frac{\log\left(a^{2}+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x*(-a)**(1/2)),x)
[Out] log(a**2 + x*sqrt(-a))/sqrt(-a)
```

Giac [A]
time = 0.56, size = 19, normalized size = 0.86

$$\frac{\log\left(\left|a^2+\sqrt{-a}\,x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a^2 + sqrt(-a)*x))/sqrt(-a)
```

Mupad [B]
time = 0.05, size = 14, normalized size = 0.64

$$\frac{\ln\left(x+(-a)^{3/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + (-a)^(3/2))/(-a)^(1/2)
```

3.276 $\int \frac{1}{a^3+\sqrt{-a}\, x} \, dx$

Optimal. Leaf size=22

$$\frac{\log \left(a^3+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

```
[Out] ln(a^3+x*(-a)^(1/2))/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{31\}$

$$\frac{\log \left(a^3+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^3+\sqrt{-a}\, x} \, dx=\frac{\log \left(a^3+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log \left(a^3+\sqrt{-a}\, x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.86

method	result	size
default	$\frac{\ln\left(a^3+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(a^3+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(a^3 + sqrt(-a)*x)/sqrt(-a)
```

Fricas [A]
time = 0.45, size = 23, normalized size = 1.05

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a^2+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(-sqrt(-a)*a^2 + x)/a
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.86

$$\frac{\log\left(a^3+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x*(-a)**(1/2)),x)
[Out] log(a**3 + x*sqrt(-a))/sqrt(-a)
```

Giac [A]
time = 0.65, size = 19, normalized size = 0.86

$$\frac{\log\left(\left|a^3+\sqrt{-a}\,x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="giac")
[Out] log(abs(a^3 + sqrt(-a)*x))/sqrt(-a)
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.73

$$\frac{\ln\left(x-(-a)^{5/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + (-a)^(1/2)*x),x)
[Out] log(x - (-a)^(5/2))/(-a)^(1/2)
```

3.277 $\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

```
[Out] ln(1-(-a)^(3/2)*x)/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {31}

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 - (-a)^(3/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx = \frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 1.00

$$\frac{\log\left(1+\sqrt{-a}\,ax\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + Sqrt[-a]*a*x]/Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.90

method	result	size
default	$\frac{\ln\left(\frac{1}{a}+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1/a+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.86

$$\frac{\log\left(\sqrt{-a}\,x+\frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a)/sqrt(-a)
```

Fricas [A]
time = 0.39, size = 24, normalized size = 1.14

$$-\frac{\sqrt{-a}\,\log\left(a^2x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^2*x - sqrt(-a))/a
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.90

$$\frac{\log\left(ax\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)**(1/2)),x)
```

```
[Out] log(a*x*sqrt(-a) + 1)/sqrt(-a)
```

Giac [**A**]
time = 1.00, size = 19, normalized size = 0.90

$$\frac{\log\left(\left|\sqrt{-a}\,x+\frac{1}{a}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a))/sqrt(-a)
```

Mupad [**B**]
time = 0.15, size = 16, normalized size = 0.76

$$\frac{\ln\left(x-\frac{1}{(-a)^{3/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - 1/(-a)^(3/2))/(-a)^(1/2)
```

3.278 $\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

```
[Out] ln(1+(-a)^(5/2)*x)/(-a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{31\}$

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-2) + Sqrt[-a]*x)^(-1),x]
[Out] Log[1 + (-a)^(5/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx = \frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.10

$$\frac{\log\left(\frac{1}{a^2}+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-2) + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^(-2) + Sqrt[-a]*x]/Sqrt[-a]
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.95

method	result	size
default	$\frac{\ln\left(\frac{1}{a^2}+x\sqrt{-a}\right)}{\sqrt{-a}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2+x*(-a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] ln(1/a^2+x*(-a)^(1/2))/(-a)^(1/2)
```

Maxima [A]
time = 0.27, size = 18, normalized size = 0.90

$$\frac{\log\left(\sqrt{-a}\,x+\frac{1}{a^2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(-a)*x + 1/a^2)/sqrt(-a)
```

Fricas [A]
time = 0.42, size = 24, normalized size = 1.20

$$-\frac{\sqrt{-a}\,\log\left(a^3x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(a^3*x - sqrt(-a))/a
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{\log\left(a^2x\sqrt{-a}+1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a**2+x*(-a)**(1/2)),x)
[Out] log(a**2*x*sqrt(-a) + 1)/sqrt(-a)
```

Giac [A]
time = 0.95, size = 19, normalized size = 0.95

$$\frac{\log\left(\left|\sqrt{-a}\,x+\frac{1}{a^2}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a^2))/sqrt(-a)
```

Mupad [B]
time = 0.18, size = 14, normalized size = 0.70

$$\frac{\ln\left(x+\frac{1}{(-a)^{5/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + 1/(-a)^(5/2))/(-a)^(1/2)
```

3.279 $\int \frac{1}{x(1+bx)} dx$

Optimal. Leaf size=11

$$\log(x) - \log(1 + bx)$$

[Out] ln(x)-ln(b*x+1)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\log(x) - \log(bx + 1)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + b*x)),x]

[Out] Log[x] - Log[1 + b*x]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+bx)} dx &= -\left(b \int \frac{1}{1+bx} dx\right) + \int \frac{1}{x} dx \\ &= \log(x) - \log(1+bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\log(x) - \log(1 + bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x)),x]
```

```
[Out] Log[x] - Log[1 + b*x]
```

Maple [A]
time = 0.09, size = 12, normalized size = 1.09

method	result	size
default	$\ln(x)-\ln(bx+1)$	12
norman	$\ln(x)-\ln(bx+1)$	12
meijerg	$\ln(x)+\ln(b)-\ln(bx+1)$	14
risch	$\ln(-x)-\ln(bx+1)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-ln(b*x+1)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$-\log(bx+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="maxima")
```

```
[Out] -log(b*x + 1) + log(x)
```

Fricas [A]
time = 0.44, size = 11, normalized size = 1.00

$$-\log(bx+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="fricas")
```

```
[Out] -log(b*x + 1) + log(x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$\log(x)-\log\left(x+\frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x)
```

```
[Out] log(x) - log(x + 1/b)
```

Giac [**A**]
time = 0.69, size = 13, normalized size = 1.18

$$-\log\left(|bx+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + 1)) + log(abs(x))
```

Mupad [**B**]
time = 0.10, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}(2\,b\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + 1)),x)
```

```
[Out] -2*atanh(2*b*x + 1)
```

3.280 $\int \frac{1}{x(-1+bx)}\,dx$

Optimal. Leaf size=12

$$-\log(x)+\log(1-bx)$$

```
[Out] -ln(x)+ln(-b*x+1)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(-1+bx)}\,dx &= b\int \frac{1}{-1+bx}\,dx-\int \frac{1}{x}\,dx\\ &= -\log(x)+\log(1-bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\log(x)+\log(1-bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

Maple [A]
time = 0.09, size = 12, normalized size = 1.00

method	result	size
default	$-\ln\left(x\right)+\ln\left(bx-1\right)$	12
norman	$-\ln\left(x\right)+\ln\left(bx-1\right)$	12
risch	$-\ln\left(x\right)+\ln\left(-bx+1\right)$	13
meijerg	$-\ln\left(x\right)-\ln\left(-b\right)+\ln\left(-bx+1\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x)+ln(b*x-1)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.92

$$\log\left(bx-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="maxima")
```

```
[Out] log(b*x - 1) - log(x)
```

Fricas [A]
time = 0.49, size = 11, normalized size = 0.92

$$\log\left(bx-1\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="fricas")
```

```
[Out] log(b*x - 1) - log(x)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.67

$$-\log\left(x\right)+\log\left(x-\frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x)
```

```
[Out] -log(x) + log(x - 1/b)
```

Giac [**A**]
time = 0.55, size = 13, normalized size = 1.08

$$\log(|bx - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="giac")
```

```
[Out] log(abs(b*x - 1)) - log(abs(x))
```

Mupad [**B**]
time = 0.04, size = 9, normalized size = 0.75

$$-2\,\mathrm{atanh}(2\,b\,x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - 1)),x)
```

```
[Out] -2*atanh(2*b*x - 1)
```

3.281 $\int \frac{1}{x^2(1+bx)} dx$

Optimal. Leaf size=19

$$-\frac{1}{x} - b\log(x) + b\log(1 + bx)$$

[Out] -1/x-b*ln(x)+b*ln(b*x+1)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{46\}$

$$-b\log(x) + b\log(bx + 1) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 + b*x)),x]

[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(1 + bx)} dx &= \int \left(\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{1 + bx}\right) dx \\ &= -\frac{1}{x} - b\log(x) + b\log(1 + bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$-\frac{1}{x} - b\log(x) + b\log(1 + bx)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(1 + b*x)),x]

```
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Maple [A]
time = 0.11, size = 20, normalized size = 1.05

method	result	size
default	$-\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx+1\right)$	20
norman	$-\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx+1\right)$	20
risch	$-\frac{1}{x}+b\ln\left(-bx-1\right)-b\ln\left(x\right)$	21
meijerg	$b\left(-\frac{1}{xb}-\ln\left(x\right)-\ln\left(b\right)+\ln\left(bx+1\right)\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+1),x,method=_RETURNVERBOSE)
[Out] -1/x-b*ln(x)+b*ln(b*x+1)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 1.00

$$b\log\left(bx+1\right)-b\log\left(x\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="maxima")
[Out] b*log(b*x + 1) - b*log(x) - 1/x
```

Fricas [A]
time = 0.48, size = 21, normalized size = 1.11

$$\frac{bx\log\left(bx+1\right)-bx\log\left(x\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="fricas")
[Out] (b*x*log(b*x + 1) - b*x*log(x) - 1)/x
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.74

$$b\left(-\log\left(x\right)+\log\left(x+\frac{1}{b}\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+1),x)
```

```
[Out] b*(-log(x) + log(x + 1/b)) - 1/x
```

Giac [**A**]
time = 0.54, size = 21, normalized size = 1.11

$$b\log\left(|bx+1|\right) - b\log\left(|x|\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + 1)) - b*log(abs(x)) - 1/x
```

Mupad [**B**]
time = 0.04, size = 16, normalized size = 0.84

$$2\,b\,\mathrm{atanh}(2\,b\,x+1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)),x)
```

```
[Out] 2*b*atanh(2*b*x + 1) - 1/x
```

3.282 $\int \frac{1}{x^2(-1+bx)} dx$

Optimal. Leaf size=18

$$\frac{1}{x} - b\log(x) + b\log(1 - bx)$$

```
[Out] 1/x-b*ln(x)+b*ln(-b*x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {46}

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(-1+bx)} dx &= \int \left(-\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{-1+bx}\right) dx \\ &= \frac{1}{x} - b\log(x) + b\log(1 - bx) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{x} - b\log(x) + b\log(1 - bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-1 + b*x)),x]
```

```
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Maple [A]
time = 0.11, size = 18, normalized size = 1.00

method	result	size
default	$\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx-1\right)$	18
norman	$\frac{1}{x}-b\ln\left(x\right)+b\ln\left(bx-1\right)$	18
risch	$\frac{1}{x}-b\ln\left(x\right)+b\ln\left(-bx+1\right)$	19
meijerg	$b\left(\frac{1}{xb}-\ln\left(x\right)-\ln\left(-b\right)+\ln\left(-bx+1\right)\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x-b*ln(x)+b*ln(b*x-1)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.94

$$b\log\left(bx-1\right)-b\log\left(x\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="maxima")
```

```
[Out] b*log(b*x - 1) - b*log(x) + 1/x
```

Fricas [A]
time = 0.44, size = 21, normalized size = 1.17

$$\frac{bx\log\left(bx-1\right)-bx\log\left(x\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x - 1) - b*x*log(x) + 1)/x
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.78

$$b\left(-\log\left(x\right)+\log\left(x-\frac{1}{b}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-1),x)
[Out] b*(-log(x) + log(x - 1/b)) + 1/x
```

Giac [A]
time = 0.63, size = 19, normalized size = 1.06

$$b\log\left(|bx-1|\right)-b\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="giac")
[Out] b*log(abs(b*x - 1)) - b*log(abs(x)) + 1/x
```

Mupad [B]
time = 0.03, size = 14, normalized size = 0.78

$$\frac{1}{x}-2\,b\,\mathrm{atanh}(2\,b\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - 1)),x)
[Out] 1/x - 2*b*atanh(2*b*x - 1)
```

3.283 $\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx$

Optimal. Leaf size=14

$$-\frac{1}{x}+b\log(1+bx)$$

[Out] -1/x+b*ln(b*x+1)

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {46}

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[b/x + 1/(x^2*(1 + b*x)),x]

[Out] -x^(-1) + b*Log[1 + b*x]

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx &= b\log(x)+\int \frac{1}{x^2(1+bx)}\,dx\\ &= b\log(x)+\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right) dx\\ &= -\frac{1}{x}+b\log(1+bx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{x}+b\log(1+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Maple [A]
time = 0.09, size = 15, normalized size = 1.07

method	result	size
default	$-\frac{1}{x}+b\ln\left(bx+1\right)$	15
norman	$-\frac{1}{x}+b\ln\left(bx+1\right)$	15
risch	$-\frac{1}{x}+b\ln\left(-bx-1\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/x+1/x^2/(b*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+b*ln(b*x+1)
```

Maxima [A]
time = 0.27, size = 14, normalized size = 1.00

$$b\log\left(bx+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="maxima")
```

```
[Out] b*log(b*x + 1) - 1/x
```

Fricas [A]
time = 0.41, size = 15, normalized size = 1.07

$$\frac{bx\log\left(bx+1\right)-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + 1) - 1)/x
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.71

$$b\log\left(bx+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x**2/(b*x+1),x)
```

```
[Out] b*log(b*x + 1) - 1/x
```

Giac [A]
time = 1.17, size = 15, normalized size = 1.07

$$b \log (|bx + 1|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="giac")
[Out] b*log(abs(b*x + 1)) - 1/x
```

Mupad [B]
time = 0.04, size = 20, normalized size = 1.43

$$b\,\ln\left(x\right)+2\,b\,\mathrm{atanh}(2\,b\,x+1)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)) + b/x,x)
[Out] b*log(x) + 2*b*atanh(2*b*x + 1) - 1/x
```

3.284 $\int x^3 \sqrt{a+bx}\, dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}$$

```
[Out] -2/3*a^3*(b*x+a)^(3/2)/b^4+6/5*a^2*(b*x+a)^(5/2)/b^4-6/7*a*(b*x+a)^(7/2)/b^
4+2/9*(b*x+a)^(9/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*(
a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx}\,dx &= \int \left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx\\ &= -\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(315
*b^4)
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)}{315b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{6a(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{6a(bx+a)^{\frac{7}{2}}}{7}+\frac{6a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}}{b^4}$	50
trager	$-\frac{2\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)\sqrt{bx+a}}{315b^4}$	54
risch	$-\frac{2\left(-35b^4x^4-5a\,b^3x^3+6a^2b^2x^2-8a^3bx+16a^4\right)\sqrt{bx+a}}{315b^4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b*x+a)^(5/2)-1/3*a^3*
(b*x+a)^(3/2))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^4}-\frac{6\,(bx+a)^{\frac{7}{2}}a}{7\,b^4}+\frac{6\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^4}-\frac{2\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^4 - 6/7*(b*x + a)^(7/2)*a/b^4 + 6/5*(b*x + a)^(5/2)*a
^2/b^4 - 2/3*(b*x + a)^(3/2)*a^3/b^4
```

Fricas [A]
time = 0.38, size = 53, normalized size = 0.74

$$\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.
time = 1.19, size = 1742, normalized size = 24.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(49/2)*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a*
*18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b*
*9*x**5 + 315*a**14*b**10*x**6) + 32*a**(49/2)/(315*a**20*b**4 + 1890*a**19
*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**
4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 176*a**(47/2)*b*x*sqrt(1
 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300
*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*
b**10*x**6) + 192*a**(47/2)*b*x/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) - 396*a**(45/2)*b**2*x**2*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 480*a**(45/2)*b**2*x**2/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**
18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**
9*x**5 + 315*a**14*b**10*x**6) - 462*a**(43/2)*b**3*x**3*sqrt(1 + b*x/a)/(3
15*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*
x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
 + 640*a**(43/2)*b**3*x**3/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*
b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x
**5 + 315*a**14*b**10*x**6) - 210*a**(41/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*
a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**
3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 4
80*a**(41/2)*b**4*x**4/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**
6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5
 + 315*a**14*b**10*x**6) + 378*a**(39/2)*b**5*x**5*sqrt(1 + b*x/a)/(315*a**
20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 +
 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 192*
a**(39/2)*b**5*x**5/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x
**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 +
315*a**14*b**10*x**6) + 1134*a**(37/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**20
*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4
725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 32*a**
```

```
(37/2)*b**6*x**6/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2
 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315
*a**14*b**10*x**6) + 1494*a**(35/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**20*b*
*4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725
*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 1098*a**(
33/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) + 430*a**(31/2)*b**9*x**9*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 70*a**(29/2)*b**10*x**10*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*
b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4
 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

time = 1.05, size = 116, normalized size = 1.61

$$\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a}{b^{3}}+\frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}}{b^{3}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
 - 35*sqrt(b*x + a)*a^3)*a/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*
a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a
^4)/b^3)/b
```

Mupad [**B**]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^4) - (2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a +
 b*x)^(5/2))/(5*b^4) - (6*a*(a + b*x)^(7/2))/(7*b^4)
```

3.285 $\int x^2 \sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3-4/5*a*(b*x+a)^(5/2)/b^3+2/7*(b*x+a)^(7/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx&=\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}\left(15x^{2}b^{2}-12abx+8a^{2}\right)}{105b^{3}}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	38
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^{2}(bx+a)^{\frac{3}{2}}}{3}}{b^{3}}$	38
trager	$\frac{2\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)\sqrt{bx+a}}{105b^{3}}$	43
risch	$\frac{2\left(15b^{3}x^{3}+3a\,b^{2}x^{2}-4a^{2}bx+8a^{3}\right)\sqrt{bx+a}}{105b^{3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{3}}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^{3}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.79

$$\frac{2\,(15\,b^{3}x^{3}+3\,ab^{2}x^{2}-4\,a^{2}bx+8\,a^{3})\sqrt{bx+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.
time = 0.81, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(41) = 82$.
time = 0.69, size = 93, normalized size = 1.75

$$\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.70

$$\frac{30\,{\left(a+b\,x\right)}^{7/2}-84\,a\,{\left(a+b\,x\right)}^{5/2}+70\,a^2\,{\left(a+b\,x\right)}^{3/2}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/2),x)
```

```
[Out] (30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b
^3)
```

3.286 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2+2/5*(b*x+a)^(5/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx&=\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$\frac{2\sqrt{a+bx}\left(-2a^2+abx+3b^2x^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2)
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2a(bx+a)^{\frac{3}{2}}}{3}}{b^2}$	26
trager	$-\frac{2(-3x^2b^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32
risch	$-\frac{2(-3x^2b^2-abx+2a^2)\sqrt{bx+a}}{15b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Fricas [A]
time = 0.39, size = 30, normalized size = 0.88

$$\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.
time = 0.53, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 0.95, size = 66, normalized size = 1.94

$$\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a}{b}+\frac{3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10\,a\,(a+b\,x)^{3/2}-6\,(a+b\,x)^{5/2}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/2),x)
```

```
[Out] -(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

3.287 $\int \sqrt{a+bx}\, dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\, dx = \frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Maple [A]
time = 0.08, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
trager	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
risch	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x+a)^(3/2)/b
```

Maxima [**A**]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [**A**]
time = 0.41, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [**A**]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

Giac [A]
time = 0.93, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(b*x + a)^(3/2)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,(a+b\,x)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2),x)
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

3.288 $\int \frac{\sqrt{a+bx}}{x} \, dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$2\sqrt{a+bx} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]/x,x]

[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x}\,dx &= 2\sqrt{a+bx}+a\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2\sqrt{a+bx}+\frac{(2a)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.80

method	result	size
derivativedivides	$-2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28
default	$-2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx+a}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.49, size = 42, normalized size = 1.20

$$\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

Fricas [A]
time = 0.43, size = 73, normalized size = 2.09

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a},2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.
time = 0.65, size = 68, normalized size = 1.94

$$-2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 1.03, size = 32, normalized size = 0.91

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

Mupad [B]
time = 0.09, size = 27, normalized size = 0.77

$$2\sqrt{a+b\,x}-2\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x,x)
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

3.289 $\int \frac{\sqrt{a+bx}}{x^2} \, dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {43, 65, 214}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= -\frac{\sqrt{a+bx}}{x} + \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)$$
$$= -\frac{\sqrt{a+bx}}{x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.08, size = 37, normalized size = 0.95

method	result	size
risch	$-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx+a}}{x}$	32
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2bx} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37
default	$2b\left(-\frac{\sqrt{bx+a}}{2bx} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.49, size = 47, normalized size = 1.21

$$\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

Fricas [A]
time = 0.40, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2ax},\ \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A]
time = 0.85, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

Giac [A]
time = 1.61, size = 41, normalized size = 1.05

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+b\,x}}{x}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^2,x)
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.290 $\int \frac{\sqrt{a+bx}}{x^3} \, dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x+a)^(1/2)/x^2-1/4*b*
(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*x]/x^2 - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^3}\,dx &= -\frac{\sqrt{a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a}\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a}\\
&= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 0.85

$$-\frac{\sqrt{a+bx}\,(2a+bx)}{4ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b*x]*(2*a + b*x))/(a*x^2) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*a^(3/2))
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.83

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(bx+2a)}{4x^2a}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	44

derivativedivides	$2b^2\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54
default	$2b^2\left(-\frac{\frac{(bx+a)^{\frac{3}{2}}}{8a}+\frac{\sqrt{bx+a}}{8}}{b^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-(1/8/a*(b*x+a)^(3/2)+1/8*(b*x+a)^(1/2))/b^2/x^2+1/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.50, size = 88, normalized size = 1.35

$$-\frac{b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{3}{2}}b^2+\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) -
 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a - 2*(b*x +
a)*a^2 + a^3)
```

Fricas [A]
time = 0.43, size = 119, normalized size = 1.83

$$\left[\frac{\sqrt{a}\,b^2x^2\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(abx+2\,a^2)\sqrt{bx+a}}{8\,a^2x^2},-\frac{\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(abx+2\,a^2)\sqrt{bx+a}}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

Sympy [A]
time = 1.95, size = 97, normalized size = 1.49

$$-\frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(b
*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)/(
sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

Giac [A]
time = 1.22, size = 66, normalized size = 1.02

$$-\frac{\frac{b^{3}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{(bx+a)^{\frac{3}{2}}b^{3}+\sqrt{bx+a}\,ab^{3}}{ab^{2}x^{2}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.07, size = 48, normalized size = 0.74

$$\frac{b^{2}\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{(a+b\,x)^{3/2}}{4\,a\,x^{2}}-\frac{\sqrt{a+b\,x}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^3,x)
```

```
[Out] (b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x)^(3/2)/(4*a*x^2
) - (a + b*x)^(1/2)/(4*x^2)
```

3.291 $\int \frac{\sqrt{a+bx}}{x^4}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)-1/3*(b*x+a)^(1/2)/x^3-1/12*
b*(b*x+a)^(1/2)/a/x^2+1/8*b^2*(b*x+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 214}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x]/x^3 - (b*Sqrt[a + b*x])/(12*a*x^2) + (b^2*Sqrt[a + b*x])
/(8*a^2*x) - (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^4}\,dx &= -\frac{\sqrt{a+bx}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}\,dx \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^2} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a^2} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 0.77

$$-\frac{\sqrt{a+bx}\,(8a^2+2abx-3b^2x^2)}{24a^2x^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2*x^2))/(a^2*x^3) - (b^3*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Maple [A]
time = 0.09, size = 66, normalized size = 0.76

method	result	size

risch	$-\frac{\sqrt{bx+a}\,(-3x^2b^2+2abx+8a^2)}{24x^3a^2}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$	56
derivativedivides	$2b^3\left(-\frac{-\frac{(bx+a)^{\frac{5}{2}}}{16a^2}+\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{\sqrt{bx+a}}{16}}{b^3x^3}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	66
default	$2b^3\left(-\frac{-\frac{(bx+a)^{\frac{5}{2}}}{16a^2}+\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{\sqrt{bx+a}}{16}}{b^3x^3}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-(-1/16/a^2*(b*x+a)^(5/2)+1/6/a*(b*x+a)^(3/2)+1/16*(b*x+a)^(1/2))/b^
3/x^3-1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2))
```

Maxima [A]
time = 0.48, size = 121, normalized size = 1.39

$$\frac{b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}b^3-8\,(bx+a)^{\frac{3}{2}}ab^3-3\,\sqrt{bx+a}\,a^2b^3}{24\,\left((bx+a)^3a^2-3\,(bx+a)^2a^3+3\,(bx+a)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
 1/24*(3*(b*x + a)^(5/2)*b^3 - 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a^
2*b^3)/((b*x + a)^3*a^2 - 3*(b*x + a)^2*a^3 + 3*(b*x + a)*a^4 - a^5)
```

Fricas [A]
time = 0.42, size = 145, normalized size = 1.67

$$\left[\frac{3\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(3\,ab^2x^2-2\,a^2bx-8\,a^3)\sqrt{bx+a}}{48\,a^3x^3},\frac{3\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,ab^2x^2-2\,a^2bx-8\,a^3)\sqrt{bx+a}}{24\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
```

```
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

Sympy [A]
time = 4.96, size = 122, normalized size = 1.40

$$-\frac{a}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{5}{2}}}{8a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**4,x)
```

```
[Out] -a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*sqrt(b)/(12*x**(5/2)*sqrt(a/(
b*x) + 1)) + b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + b**(5/2)/(8*a**2*
sqrt(x)*sqrt(a/(b*x) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a**(5
/2))
```

Giac [A]
time = 0.77, size = 84, normalized size = 0.97

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4-8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}\,a^2b^4}{a^2b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
/b
```

Mupad [B]
time = 0.11, size = 66, normalized size = 0.76

$$\frac{(a+b\,x)^{5/2}}{8\,a^2\,x^3}-\frac{(a+b\,x)^{3/2}}{3\,a\,x^3}-\frac{\sqrt{a+b\,x}}{8\,x^3}+\frac{b^3\operatorname{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^4,x)
```

```
[Out] (a + b*x)^(5/2)/(8*a^2*x^3) - (a + b*x)^(3/2)/(3*a*x^3) - (a + b*x)^(1/2)/(
8*x^3) + (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(5/2))
```

3.292 $\int x^3(a+bx)^{3/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}$$

```
[Out] -2/5*a^3*(b*x+a)^(5/2)/b^4+6/7*a^2*(b*x+a)^(7/2)/b^4-2/3*a*(b*x+a)^(9/2)/b^
4+2/11*(b*x+a)^(11/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*(
a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{3/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{5/2}\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(11
55*b^4)
```

Maple [A]
time = 0.08, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)}{1155b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{2a(bx+a)^{\frac{9}{2}}}{3}+\frac{6a^2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^3(bx+a)^{\frac{5}{2}}}{5}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{2a(bx+a)^{\frac{9}{2}}}{3}+\frac{6a^2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a^3(bx+a)^{\frac{5}{2}}}{5}}{b^4}$	50
trager	$-\frac{2\left(-105b^5x^5-140a\,b^4x^4-5a^2b^3x^3+6a^3b^2x^2-8a^4bx+16a^5\right)\sqrt{bx+a}}{1155b^4}$	65
risch	$-\frac{2\left(-105b^5x^5-140a\,b^4x^4-5a^2b^3x^3+6a^3b^2x^2-8a^4bx+16a^5\right)\sqrt{bx+a}}{1155b^4}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/11*(b*x+a)^(11/2)-1/3*a*(b*x+a)^(9/2)+3/7*a^2*(b*x+a)^(7/2)-1/5*a^
3*(b*x+a)^(5/2))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^4}-\frac{2\,(bx+a)^{\frac{9}{2}}a}{3\,b^4}+\frac{6\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^4}-\frac{2\,(bx+a)^{\frac{5}{2}}a^3}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^4 - 2/3*(b*x + a)^(9/2)*a/b^4 + 6/7*(b*x + a)^(7/2)
*a^2/b^4 - 2/5*(b*x + a)^(5/2)*a^3/b^4
```

Fricas [A]
time = 0.38, size = 64, normalized size = 0.89

$$\frac{2\,(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.
time = 1.27, size = 1742, normalized size = 24.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(3/2),x)
```

```
[Out] -32*a**(51/2)*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*
a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**1
5*b**9*x**5 + 1155*a**14*b**10*x**6) + 32*a**(51/2)/(1155*a**20*b**4 + 6930
*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16
*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 176*a**(49/2)*
b*x*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6
*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**
5 + 1155*a**14*b**10*x**6) + 192*a**(49/2)*b*x/(1155*a**20*b**4 + 6930*a**1
9*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8
*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 396*a**(47/2)*b**2*
x**2*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**
6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x*
*5 + 1155*a**14*b**10*x**6) + 480*a**(47/2)*b**2*x**2/(1155*a**20*b**4 + 69
30*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**
16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 462*a**(45/2
)*b**3*x**3*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a*
*18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*
b**9*x**5 + 1155*a**14*b**10*x**6) + 640*a**(45/2)*b**3*x**3/(1155*a**20*b*
*4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17
325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 480*a
**(43/2)*b**4*x**4/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*
x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5
 + 1155*a**14*b**10*x**6) + 1848*a**(41/2)*b**5*x**5*sqrt(1 + b*x/a)/(1155*
a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x
**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
 + 192*a**(41/2)*b**5*x**5/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**
18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b
**9*x**5 + 1155*a**14*b**10*x**6) + 5544*a**(39/2)*b**6*x**6*sqrt(1 + b*x/a
)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**1
7*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**
10*x**6) + 32*a**(39/2)*b**6*x**6/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
```

```
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 8844*a**(37/2)*b**7*x**7*sqrt(1
+ b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 231
00*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a*
*14*b**10*x**6) + 8448*a**(35/2)*b**8*x**8*sqrt(1 + b*x/a)/(1155*a**20*b**4
 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1732
5*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 4840*a*
*(33/2)*b**9*x**9*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 1540*a**(31/2)*b**10*x**10*sqrt(
1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 2
3100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*
a**14*b**10*x**6) + 210*a**(29/2)*b**11*x**11*sqrt(1 + b*x/a)/(1155*a**20*b
**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1
7325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(56) = 112.
time = 0.69, size = 193, normalized size = 2.68

$$\frac{2\left(\frac{99\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^2}{b^3}+\frac{22\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a}{b^3}+\frac{5\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)}{b^3}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a
^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)
^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x
 + a)*a^4)*a/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*
x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 69
3*sqrt(b*x + a)*a^5)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b^4) - (2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a
 + b*x)^(7/2))/(7*b^4) - (2*a*(a + b*x)^(9/2))/(3*b^4)
```

3.293 $\int x^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}$$

```
[Out] 2/5*a^2*(b*x+a)^(5/2)/b^3-4/7*a*(b*x+a)^(7/2)/b^3+2/9*(b*x+a)^(9/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + b
*x)^(9/2))/(9*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{3/2}\,dx&=\int\left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{5/2}\,(8a^2-20abx+35b^2x^2)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3)
```

Maple [A]
time = 0.08, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}(35x^2b^2-20abx+8a^2)}{315b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{4a(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{4a(bx+a)^{\frac{7}{2}}}{7}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}}{b^3}$	38
trager	$\frac{2(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{bx+a}}{315b^3}$	54
risch	$\frac{2(35b^4x^4+50a\,b^3x^3+3a^2b^2x^2-4a^3bx+8a^4)\sqrt{bx+a}}{315b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/9*(b*x+a)^(9/2)-2/7*a*(b*x+a)^(7/2)+1/5*a^2*(b*x+a)^(5/2))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^3}-\frac{4\,(bx+a)^{\frac{7}{2}}a}{7\,b^3}+\frac{2\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^3 - 4/7*(b*x + a)^(7/2)*a/b^3 + 2/5*(b*x + a)^(5/2)*a
^2/b^3
```

Fricas [A]
time = 0.36, size = 53, normalized size = 1.00

$$\frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(49) = 98$.

time = 0.89, size = 733, normalized size = 13.83

$$\frac{16a^{\frac{25}{2}}\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{25}{2}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{40a^{\frac{23}{2}}bx\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{23}{2}}bx}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{30a^{\frac{21}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{21}{2}}b^2x^2}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{110a^{\frac{19}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{19}{2}}b^3x^3}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{380a^{\frac{17}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{516a^{\frac{15}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{310a^{\frac{13}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{70a^{\frac{11}{2}}b^7x^7\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(3/2),x)
```

```
[Out] 16*a**(25/2)*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b*
*5*x**2 + 315*a**5*b**6*x**3) - 16*a**(25/2)/(315*a**8*b**3 + 945*a**7*b**4
*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 40*a**(23/2)*b*x*sqrt(1 + b
*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6
*x**3) - 48*a**(23/2)*b*x/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*
x**2 + 315*a**5*b**6*x**3) + 30*a**(21/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a*
*8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 48*a
**(21/2)*b**2*x**2/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 +
315*a**5*b**6*x**3) + 110*a**(19/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**8*b**
3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 16*a**(19/
2)*b**3*x**3/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a*
*5*b**6*x**3) + 380*a**(17/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*a**8*b**3 + 94
5*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 516*a**(15/2)*b*
*5*x**5*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x*
*2 + 315*a**5*b**6*x**3) + 310*a**(13/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**
8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 70*a*
*(11/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a*
*6*b**5*x**2 + 315*a**5*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(41) = 82$.

time = 0.72, size = 156, normalized size = 2.94

$$\frac{2\left(\frac{21\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^2}{b^2}+\frac{18\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a}{b^2}+\frac{35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4}{b^2}\right)}{315b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2
)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(
7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x +
 a)*a^4)/b^2)/b
```

Mupad [B]

time = 0.04, size = 37, normalized size = 0.70

$$\frac{70\,(a+b\,x)^{9/2}-180\,a\,(a+b\,x)^{7/2}+126\,a^2\,(a+b\,x)^{5/2}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(3/2),x)
```

```
[Out] (70*(a + b*x)^(9/2) - 180*a*(a + b*x)^(7/2) + 126*a^2*(a + b*x)^(5/2))/(315
*b^3)
```

3.294 $\int x(a+bx)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}$$

```
[Out] -2/5*a*(b*x+a)^(5/2)/b^2+2/7*(b*x+a)^(7/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{3/2}\,dx&=\int\left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{5/2}(-2a+5bx)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-2*a + 5*b*x))/(35*b^2)
```

Maple [A]
time = 0.09, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}}{b^2}$	26
trager	$-\frac{2(-5b^3x^3-8a\,b^2x^2-a^2bx+2a^3)\sqrt{bx+a}}{35b^2}$	43
risch	$-\frac{2(-5b^3x^3-8a\,b^2x^2-a^2bx+2a^3)\sqrt{bx+a}}{35b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/7*(b*x+a)^(7/2)-1/5*a*(b*x+a)^(5/2))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^2}-\frac{2\,(bx+a)^{\frac{5}{2}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="maxima")
[Out] 2/7*(b*x + a)^(7/2)/b^2 - 2/5*(b*x + a)^(5/2)*a/b^2
```

Fricas [A]
time = 0.40, size = 41, normalized size = 1.21

$$\frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

time = 0.12, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{4a^3\sqrt{a+bx}}{35b^2} + \frac{2a^2x\sqrt{a+bx}}{35b} + \frac{16ax^2\sqrt{a+bx}}{35} + \frac{2bx^3\sqrt{a+bx}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**3*sqrt(a + b*x)/(35*b**2) + 2*a**2*x*sqrt(a + b*x)/(35*b)
+ 16*a*x**2*sqrt(a + b*x)/35 + 2*b*x**3*sqrt(a + b*x)/7, Ne(b, 0)), (a**(3/
2)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(26) = 52.
time = 0.80, size = 119, normalized size = 3.50

$$\frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^2}{b}+\frac{14\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a}{b}+\frac{3\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a/b + 3*(5*(b*x + a)^(7/2
) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b
)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{14\,a\,{\left(a+b\,x\right)}^{5/2}-10\,{\left(a+b\,x\right)}^{7/2}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(3/2),x)
```

```
[Out] -(14*a*(a + b*x)^(5/2) - 10*(a + b*x)^(7/2))/(35*b^2)
```

3.295 $\int (a+bx)^{3/2} dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{5/2}}{5b}$$

[Out] 2/5*(b*x+a)^(5/2)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2),x]

[Out] (2*(a + b*x)^(5/2))/(5*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{3/2} dx = \frac{2(a+bx)^{5/2}}{5b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(3/2),x]

[Out] (2*(a + b*x)^(5/2))/(5*b)

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
default	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
trager	$\frac{2(x^2b^2+2abx+a^2)\sqrt{bx+a}}{5b}$	29
risch	$\frac{2(x^2b^2+2abx+a^2)\sqrt{bx+a}}{5b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*x+a)^(5/2)/b
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.41, size = 28, normalized size = 1.75

$$\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2),x)
```

```
[Out] 2*(a + b*x)**(5/2)/(5*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(12) = 24.
time = 0.97, size = 58, normalized size = 3.62

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+30\,\sqrt{bx+a}\,a^2+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 30*sqrt(b*x + a)*a^2 + 10*
((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

3.296 $\int \frac{(a+bx)^{3/2}}{x}\,dx$

Optimal. Leaf size=49

$$2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {52, 65, 214}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x,x]
```

```
[Out] 2*a*Sqrt[a + b*x] + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(a+bx)^{3/2}+a\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+a^2\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\
&= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 44, normalized size = 0.90

$$\frac{2}{3}\sqrt{a+bx}\,(4a+bx)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x,x]
[Out] (2*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [**A**]
time = 0.10, size = 38, normalized size = 0.78

method	result	size
derivativedivides	$\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a\sqrt{bx+a}$	38
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a\sqrt{bx+a}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

Maxima [**A**]
time = 0.47, size = 52, normalized size = 1.06

$$a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(b*x
 + a)^(3/2) + 2*sqrt(b*x + a)*a
```

Fricas [A]
time = 0.44, size = 88, normalized size = 1.80

$$\left[a^{\frac{3}{2}} \log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a} + 2a}{x}\right) + \frac{2}{3}(bx+4a)\sqrt{bx+a}, 2\sqrt{-a}\, a \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \frac{2}{3}(bx+4a)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

Sympy [A]
time = 1.05, size = 71, normalized size = 1.45

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{3} + a^{\frac{3}{2}}\log\left(\frac{bx}{a}\right) - 2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right) + \frac{2\sqrt{a}\,bx\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x/a)/3 + a**(3/2)*log(b*x/a) - 2*a**(3/2)*log(sqrt(1
+ b*x/a) + 1) + 2*sqrt(a)*b*x*sqrt(1 + b*x/a)/3
```

Giac [A]
time = 0.93, size = 44, normalized size = 0.90

$$\frac{2\,a^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2}{3}(bx+a)^{\frac{3}{2}} + 2\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sqr
t(b*x + a)*a
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.76

$$2\,a\,\sqrt{a+b\,x}+\frac{2\,{\left(a+b\,x\right)}^{3/2}}{3}-2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x,x)
[Out] 2*a*(a + b*x)^(1/2) + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*atanh((a + b*x)^(1/
2)/a^(1/2))
```

3.297 $\int \frac{(a+bx)^{3/2}}{x^2} dx$

Optimal. Leaf size=51

$$3b\sqrt{a+bx} - \frac{(a+bx)^{3/2}}{x} - 3\sqrt{a}\, b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -(b*x+a)^(3/2)/x-3*b*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+3*b*(b*x+a)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-\frac{(a+bx)^{3/2}}{x} + 3b\sqrt{a+bx} - 3\sqrt{a}\, b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[a + b*x] - (a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x^2}\,dx &= -\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+(3a)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.88

$$-\frac{(a-2bx)\sqrt{a+bx}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] -(((a - 2*b*x)*Sqrt[a + b*x])/x) - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]
```

Maple [A]
time = 0.10, size = 48, normalized size = 0.94

method	result	size
risch	$-\frac{a\sqrt{bx+a}}{x}+\frac{b\left(4\sqrt{bx+a}-6\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\right)}{2}$	45

derivativedivides	$2b\left(\sqrt{bx+a}-a\left(\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	48
default	$2b\left(\sqrt{bx+a}-a\left(\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*((b*x+a)^(1/2)-a*(1/2*(b*x+a)^(1/2)/b/x+3/2*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 58, normalized size = 1.14

$$\frac{3}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b-\frac{\sqrt{bx+a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(a)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*
sqrt(b*x + a)*b - sqrt(b*x + a)*a/x
```

Fricas [A]
time = 0.40, size = 102, normalized size = 2.00

$$\left[\frac{3\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2(2bx-a)\sqrt{bx+a}}{2x},\frac{3\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2bx-a)\sqrt{bx+a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.
time = 1.26, size = 92, normalized size = 1.80

$$-3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{a\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**2,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt
(a/(b*x) + 1)) + a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)
/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 0.95, size = 56, normalized size = 1.10

$$\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,b^2-\frac{\sqrt{bx+a}\,ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*b^2 - sq
rt(b*x + a)*a*b/x)/b
```

Mupad [B]
time = 0.10, size = 42, normalized size = 0.82

$$2\,b\,\sqrt{a+b\,x}\,-3\,\sqrt{a}\,\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)-\frac{a\,\sqrt{a+b\,x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^2,x)
```

```
[Out] 2*b*(a + b*x)^(1/2) - 3*a^(1/2)*b*atanh((a + b*x)^(1/2)/a^(1/2)) - (a*(a +
b*x)^(1/2))/x
```

3.298 $\int \frac{(a+bx)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=62

$$-\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/2*(b*x+a)^(3/2)/x^2-3/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)-3/4*b
*(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b\sqrt{a+bx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[a + b*x])/(4*x) - (a + b*x)^(3/2)/(2*x^2) - (3*b^2*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^3}\,dx &= -\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\
&= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 53, normalized size = 0.85

$$
-\frac{\sqrt{a+bx}\,(2a+5bx)}{4x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b*x]*(2*a + 5*b*x))/x^2 - (3*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*Sqrt[a])
```

Maple [A]
time = 0.10, size = 52, normalized size = 0.84

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(5bx+2a)}{4x^2}-\frac{3b^2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4\sqrt{a}}$	42
derivativedivides	$2b^2\left(-\frac{\frac{5(bx+a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx+a}}{8}}{b^2x^2}-\frac{3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	52
default	$2b^2\left(-\frac{\frac{5(bx+a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx+a}}{8}}{b^2x^2}-\frac{3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-(5/8*(b*x+a)^(3/2)-3/8*a*(b*x+a)^(1/2))/b^2/x^2-3/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.48, size = 86, normalized size = 1.39

$$\frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^2-3\,\sqrt{bx+a}\,ab^2}{4\,\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
1/4*(5*(b*x + a)^(3/2)*b^2 - 3*sqrt(b*x + a)*a*b^2)/((b*x + a)^2 - 2*(b*x +
 a)*a + a^2)
```

Fricas [A]
time = 0.43, size = 124, normalized size = 2.00

$$\left[\frac{3\,\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(5\,abx+2\,a^2)\sqrt{bx+a}}{8\,ax^2},\frac{3\,\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(5\,abx+2\,a^2)\sqrt{bx+a}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

Sympy [A]
time = 1.50, size = 76, normalized size = 1.23

$$-\frac{a\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(a/(b*x) + 1)/(4
*sqrt(x)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

Giac [A]
time = 1.10, size = 64, normalized size = 1.03

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3-3\,\sqrt{bx+a}\,ab^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3
 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.06, size = 46, normalized size = 0.74

$$\frac{3\,a\,\sqrt{a+b\,x}}{4\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^3,x)
```

```
[Out] (3*a*(a + b*x)^(1/2))/(4*x^2) - (3*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a
^(1/2)) - (5*(a + b*x)^(3/2))/(4*x^2)
```

3.299 $\int \frac{(a+bx)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

```
[Out] -1/3*(b*x+a)^(3/2)/x^3+1/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-1/4*b
*(b*x+a)^(1/2)/x^2-1/8*b^2*(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b\sqrt{a+bx}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] -1/4*(b*Sqrt[a + b*x])/x^2 - (b^2*Sqrt[a + b*x])/(8*a*x) - (a + b*x)^(3/2)/
(3*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^4}\,dx &= -\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}\,dx \\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{1}{8}b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx \\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a} \\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}-\frac{b^2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a} \\
&= -\frac{b\sqrt{a+bx}}{4x^2}-\frac{b^2\sqrt{a+bx}}{8ax}-\frac{(a+bx)^{3/2}}{3x^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 67, normalized size = 0.80

$$
-\frac{\sqrt{a+bx}\,(8a^2+14abx+3b^2x^2)}{24ax^3}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^2))/(a*x^3) + (b^3*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.76

method	result	size

risch	$-\frac{\sqrt{bx+a}\,(3x^2b^2+14abx+8a^2)}{24x^3a}+\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}$	56
derivativedivides	$2b^3\left(-\frac{\frac{(bx+a)^{\frac{5}{2}}}{16a}+\frac{(bx+a)^{\frac{3}{2}}}{6}-\frac{a\sqrt{bx+a}}{16}}{b^3x^3}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	64
default	$2b^3\left(-\frac{\frac{(bx+a)^{\frac{5}{2}}}{16a}+\frac{(bx+a)^{\frac{3}{2}}}{6}-\frac{a\sqrt{bx+a}}{16}}{b^3x^3}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-(1/16/a*(b*x+a)^(5/2)+1/6*(b*x+a)^(3/2)-1/16*a*(b*x+a)^(1/2))/b^3/x
^3+1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.49, size = 119, normalized size = 1.42

$$-\frac{b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}b^3+8\,(bx+a)^{\frac{3}{2}}ab^3-3\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3a-3\,(bx+a)^2a^2+3\,(bx+a)a^3-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
 - 1/24*(3*(b*x + a)^(5/2)*b^3 + 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a
^2*b^3)/((b*x + a)^3*a - 3*(b*x + a)^2*a^2 + 3*(b*x + a)*a^3 - a^4)
```

Fricas [A]
time = 0.42, size = 145, normalized size = 1.73

$$\left[\frac{3\,\sqrt{a}\,b^3x^3\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,a^2x^3},-\frac{3\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
```

```
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

Sympy [A]
time = 3.53, size = 124, normalized size = 1.48

$$-\frac{a^2}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{17b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**4,x)
```

```
[Out] -a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*a*sqrt(b)/(12*x**(5/2)*sq
rt(a/(b*x) + 1)) - 17*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - b**(5/2)/(
8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a
**(3/2))
```

Giac [A]
time = 1.13, size = 84, normalized size = 1.00

$$-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4+8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}\,a^2b^4}{ab^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x + a)^(5/
2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a*b^3*x^3))/b
```

Mupad [B]
time = 0.10, size = 64, normalized size = 0.76

$$\frac{a\,\sqrt{a+b\,x}}{8\,x^3}-\frac{{\left(a+b\,x\right)}^{5/2}}{8\,a\,x^3}-\frac{{\left(a+b\,x\right)}^{3/2}}{3\,x^3}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{8\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^4,x)
```

```
[Out] (a*(a + b*x)^(1/2))/(8*x^3) - (a + b*x)^(5/2)/(8*a*x^3) - (b^3*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(3/2)) - (a + b*x)^(3/2)/(3*x^3)
```

3.300 $\int x^3(a+bx)^{5/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}$$

```
[Out] -2/7*a^3*(b*x+a)^(7/2)/b^4+2/3*a^2*(b*x+a)^(9/2)/b^4-6/11*a*(b*x+a)^(11/2)/
b^4+2/13*(b*x+a)^(13/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*(
a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{5/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{7/2}\left(-16a^3+56a^2bx-126ab^2x^2+231b^3x^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(3
003*b^4)
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-231b^3x^3+126a\,b^2x^2-56a^2bx+16a^3\right)}{3003b^4}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{6a(bx+a)^{\frac{11}{2}}}{11}+\frac{2a^2(bx+a)^{\frac{9}{2}}}{3}-\frac{2a^3(bx+a)^{\frac{7}{2}}}{7}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{6a(bx+a)^{\frac{11}{2}}}{11}+\frac{2a^2(bx+a)^{\frac{9}{2}}}{3}-\frac{2a^3(bx+a)^{\frac{7}{2}}}{7}}{b^4}$	50
trager	$-\frac{2\left(-231x^6b^6-567a\,x^5b^5-371a^2x^4b^4-5a^3b^3x^3+6a^4x^2b^2-8a^5xb+16a^6\right)\sqrt{bx+a}}{3003b^4}$	76
risch	$-\frac{2\left(-231x^6b^6-567a\,x^5b^5-371a^2x^4b^4-5a^3b^3x^3+6a^4x^2b^2-8a^5xb+16a^6\right)\sqrt{bx+a}}{3003b^4}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/13*(b*x+a)^(13/2)-3/11*a*(b*x+a)^(11/2)+1/3*a^2*(b*x+a)^(9/2)-1/7*
a^3*(b*x+a)^(7/2))
```

Maxima [A]
time = 0.30, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^4}-\frac{6\,(bx+a)^{\frac{11}{2}}a}{11\,b^4}+\frac{2\,(bx+a)^{\frac{9}{2}}a^2}{3\,b^4}-\frac{2\,(bx+a)^{\frac{7}{2}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^4 - 6/11*(b*x + a)^(11/2)*a/b^4 + 2/3*(b*x + a)^(9/
2)*a^2/b^4 - 2/7*(b*x + a)^(7/2)*a^3/b^4
```

Fricas [A]
time = 0.45, size = 75, normalized size = 1.04

$$\frac{2\,(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6)\sqrt{bx+a}}{3003\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(68) = 136$.

time = 0.34, size = 146, normalized size = 2.03

$$\begin{cases} -\frac{32a^6\sqrt{a+bx}}{3003b^4} + \frac{16a^5x\sqrt{a+bx}}{3003b^3} - \frac{4a^4x^2\sqrt{a+bx}}{1001b^2} + \frac{10a^3x^3\sqrt{a+bx}}{3003b} + \frac{106a^2x^4\sqrt{a+bx}}{429} + \frac{54abx^5\sqrt{a+bx}}{143} + \frac{2b^2x^6\sqrt{a+bx}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**6*sqrt(a + b*x)/(3003*b**4) + 16*a**5*x*sqrt(a + b*x)/(30
03*b**3) - 4*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*a**3*x**3*sqrt(a + b*
x)/(3003*b) + 106*a**2*x**4*sqrt(a + b*x)/429 + 54*a*b*x**5*sqrt(a + b*x)/1
43 + 2*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(56) = 112$.

time = 2.14, size = 281, normalized size = 3.90

$$\frac{2\left(\frac{429\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\,a^3\right)a^3}{b^3} + \frac{143\left(35(bx+a)^{\frac{9}{2}} - 180(bx+a)^{\frac{7}{2}}a + 378(bx+a)^{\frac{5}{2}}a^2 - 420(bx+a)^{\frac{3}{2}}a^3 + 315\sqrt{bx+a}\,a^4\right)a^2}{b^3} + \frac{65\left(63(bx+a)^{\frac{11}{2}} - 385(bx+a)^{\frac{9}{2}}a + 990(bx+a)^{\frac{7}{2}}a^2 - 1386(bx+a)^{\frac{5}{2}}a^3 + 1155(bx+a)^{\frac{3}{2}}a^4 - 693\sqrt{bx+a}\,a^5\right)a}{b^3} + \frac{5\left(231(bx+a)^{\frac{13}{2}} - 1638(bx+a)^{\frac{11}{2}}a + 5005(bx+a)^{\frac{9}{2}}a^2 - 8580(bx+a)^{\frac{7}{2}}a^3 + 9009(bx+a)^{\frac{5}{2}}a^4 - 6006(bx+a)^{\frac{3}{2}}a^5 + 3003\sqrt{bx+a}\,a^6\right)}{b^3}\right)}{15015\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)
*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*a^2/b^3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)
^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x
 + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

Mupad [B]

time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(13/2))/(13*b^4) - (2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a
 + b*x)^(9/2))/(3*b^4) - (6*a*(a + b*x)^(11/2))/(11*b^4)
```

3.301 $\int x^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}$$

```
[Out] 2/7*a^2*(b*x+a)^(7/2)/b^3-4/9*a*(b*x+a)^(9/2)/b^3+2/11*(b*x+a)^(11/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a + b
*x)^(11/2))/(11*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{5/2}\,dx&=\int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{7/2}\left(8a^2-28abx+63b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(5/2),x]
[Out] (2*(a + b*x)^(7/2)*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3)
```

Maple [A]
time = 0.09, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}\left(63x^2b^2-28abx+8a^2\right)}{693b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{4a(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^2(bx+a)^{\frac{7}{2}}}{7}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{11}{2}}}{11}-\frac{4a(bx+a)^{\frac{9}{2}}}{9}+\frac{2a^2(bx+a)^{\frac{7}{2}}}{7}}{b^3}$	38
trager	$\frac{2\left(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5\right)\sqrt{bx+a}}{693b^3}$	65
risch	$\frac{2\left(63b^5x^5+161a\,b^4x^4+113a^2b^3x^3+3a^3b^2x^2-4a^4bx+8a^5\right)\sqrt{bx+a}}{693b^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(1/11*(b*x+a)^(11/2)-2/9*a*(b*x+a)^(9/2)+1/7*a^2*(b*x+a)^(7/2))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^3}-\frac{4\,(bx+a)^{\frac{9}{2}}a}{9\,b^3}+\frac{2\,(bx+a)^{\frac{7}{2}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="maxima")
[Out] 2/11*(b*x + a)^(11/2)/b^3 - 4/9*(b*x + a)^(9/2)*a/b^3 + 2/7*(b*x + a)^(7/2)
*a^2/b^3
```

Fricas [A]
time = 0.46, size = 64, normalized size = 1.21

$$\frac{2\,(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5)\sqrt{bx+a}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="fricas")
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(49) = 98$.
time = 0.29, size = 124, normalized size = 2.34

$$\begin{cases} \frac{16a^5\sqrt{a+bx}}{693b^3} - \frac{8a^4x\sqrt{a+bx}}{693b^2} + \frac{2a^3x^2\sqrt{a+bx}}{231b} + \frac{226a^2x^3\sqrt{a+bx}}{693} + \frac{46abx^4\sqrt{a+bx}}{99} + \frac{2b^2x^5\sqrt{a+bx}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**5*sqrt(a + b*x)/(693*b**3) - 8*a**4*x*sqrt(a + b*x)/(693*b
**2) + 2*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*a**2*x**3*sqrt(a + b*x)/693
+ 46*a*b*x**4*sqrt(a + b*x)/99 + 2*b**2*x**5*sqrt(a + b*x)/11, Ne(b, 0)), (
a**(5/2)*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(41) = 82$.
time = 0.93, size = 233, normalized size = 4.40

$$\frac{2\left(\frac{231\left(3(bx+a)^{\frac{5}{2}} - 10(bx+a)^{\frac{3}{2}}a + 15\sqrt{bx+a}\,a^2\right)a^3}{b^2} + \frac{297\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\,a^3\right)a^2}{b^2} + \frac{33\left(35(bx+a)^{\frac{9}{2}} - 180(bx+a)^{\frac{7}{2}}a + 378(bx+a)^{\frac{5}{2}}a^2 - 420(bx+a)^{\frac{3}{2}}a^3 + 315\sqrt{bx+a}\,a^4\right)a}{b^2} + \frac{5\left(63(bx+a)^{\frac{11}{2}} - 385(bx+a)^{\frac{9}{2}}a + 990(bx+a)^{\frac{7}{2}}a^2 - 1386(bx+a)^{\frac{5}{2}}a^3 + 1155(bx+a)^{\frac{3}{2}}a^4 - 693\sqrt{bx+a}\,a^5\right)}{b^2}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*a^3/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*(35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a/b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{126\,{\left(a+b\,x\right)}^{11/2}-308\,a\,{\left(a+b\,x\right)}^{9/2}+198\,a^2\,{\left(a+b\,x\right)}^{7/2}}{693\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(5/2),x)
```

```
[Out] (126*(a + b*x)^(11/2) - 308*a*(a + b*x)^(9/2) + 198*a^2*(a + b*x)^(7/2))/(6
93*b^3)
```

3.302 $\int x(a+bx)^{5/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}$$

```
[Out] -2/7*a*(b*x+a)^(7/2)/b^2+2/9*(b*x+a)^(9/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{5/2}\,dx&=\int\left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}(-2a+7bx)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-2*a + 7*b*x))/(63*b^2)
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bx+2a)}{63b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(bx+a)^{\frac{7}{2}}}{7}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{2a(bx+a)^{\frac{7}{2}}}{7}}{b^2}$	26
trager	$-\frac{2(-7b^4x^4-19a\,b^3x^3-15a^2b^2x^2-a^3bx+2a^4)\sqrt{bx+a}}{63b^2}$	54
risch	$-\frac{2(-7b^4x^4-19a\,b^3x^3-15a^2b^2x^2-a^3bx+2a^4)\sqrt{bx+a}}{63b^2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/9*(b*x+a)^(9/2)-1/7*a*(b*x+a)^(7/2))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^2}-\frac{2\,(bx+a)^{\frac{7}{2}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="maxima")
[Out] 2/9*(b*x + a)^(9/2)/b^2 - 2/7*(b*x + a)^(7/2)*a/b^2
```

Fricas [A]
time = 0.50, size = 52, normalized size = 1.53

$$\frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="fricas")
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.
time = 0.23, size = 102, normalized size = 3.00

$$\begin{cases} -\frac{4a^4\sqrt{a+bx}}{63b^2} + \frac{2a^3x\sqrt{a+bx}}{63b} + \frac{10a^2x^2\sqrt{a+bx}}{21} + \frac{38abx^3\sqrt{a+bx}}{63} + \frac{2b^2x^4\sqrt{a+bx}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a**4*sqrt(a + b*x)/(63*b**2) + 2*a**3*x*sqrt(a + b*x)/(63*b)
+ 10*a**2*x**2*sqrt(a + b*x)/21 + 38*a*b*x**3*sqrt(a + b*x)/63 + 2*b**2*x**
4*sqrt(a + b*x)/9, Ne(b, 0)), (a**(5/2)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(26) = 52$.
time = 0.95, size = 182, normalized size = 5.35

$$\frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^3}{b}+\frac{63\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^2}{b}+\frac{27\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a}{b}+\frac{35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4}{b}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2/b + 27*(5*(b*x + a)^
(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^
3)*a/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{18\,a\,{\left(a+b\,x\right)}^{7/2}-14\,{\left(a+b\,x\right)}^{9/2}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^(7/2) - 14*(a + b*x)^(9/2))/(63*b^2)
```

3.303 $\int (a+bx)^{5/2} dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{7/2}}{7b}$$

[Out] 2/7*(b*x+a)^(7/2)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(5/2),x]

[Out] (2*(a + b*x)^(7/2))/(7*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{5/2} dx = \frac{2(a+bx)^{7/2}}{7b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(5/2),x]

[Out] (2*(a + b*x)^(7/2))/(7*b)

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
default	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$	13
trager	$\frac{2(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)\sqrt{bx+a}}{7b}$	40
risch	$\frac{2(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)\sqrt{bx+a}}{7b}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(b*x+a)^(7/2)/b
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.
time = 0.44, size = 39, normalized size = 2.44

$$\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2),x)
```

```
[Out] 2*(a + b*x)**(7/2)/(7*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(12) = 24.
time = 1.10, size = 95, normalized size = 5.94

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2+35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2+7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 3
5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2 + 7*(3*(b*x + a)^(5/2) - 10*(b*
x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

3.304 $\int \frac{(a+bx)^{5/2}}{x}\,dx$

Optimal. Leaf size=65

$$2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[a + b*x] + (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 - 2*a
^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(a+bx)^{5/2}+a\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= \frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+a^2\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+a^3\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+\frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.86

$$\frac{2}{15}\sqrt{a+bx}\,\left(23a^2+11abx+3b^2x^2\right)-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(23*a^2 + 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.77

method	result	size
derivativedivides	$\frac{2a(bx+a)^{\frac{3}{2}}}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^2\sqrt{bx+a}$	50
default	$\frac{2a(bx+a)^{\frac{3}{2}}}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^2\sqrt{bx+a}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.50, size = 64, normalized size = 0.98

$$a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a+2\,\sqrt{bx+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] a^(5/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/5*(b*x
 + a)^(5/2) + 2/3*(b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

Fricas [A]
time = 0.49, size = 114, normalized size = 1.75

$$\left[a^{\frac{5}{2}}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+\frac{2}{15}\,\left(3\,b^2x^2+11\,abx+23\,a^2\right)\sqrt{bx+a}\,,2\,\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\frac{2}{15}\,\left(3\,b^2x^2+11\,abx+23\,a^2\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

Sympy [A]
time = 2.18, size = 97, normalized size = 1.49

$$\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{15}+a^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{22a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}\,b^2x^2\sqrt{1+\frac{bx}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x,x)
```

```
[Out] 46*a**(5/2)*sqrt(1 + b*x/a)/15 + a**(5/2)*log(b*x/a) - 2*a**(5/2)*log(sqrt(
1 + b*x/a) + 1) + 22*a**(3/2)*b*x*sqrt(1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*
sqrt(1 + b*x/a)/5
```

Giac [A]
time = 0.97, size = 56, normalized size = 0.86

$$\frac{2\,a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a+2\,\sqrt{bx+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/5*(b*x + a)^(5/2) + 2/3*(
b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.80

$$\frac{2\,a\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5}+2\,a^2\,\sqrt{a+b\,x}+a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 + 2*a^2*(a + b*x)^(1/2) + a
^(5/2)*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

3.305 $\int \frac{(a+bx)^{5/2}}{x^2} dx$

Optimal. Leaf size=66

$$5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 5/3*b*(b*x+a)^(3/2)-(b*x+a)^(5/2)/x-5*a^(3/2)*b*arctanh((b*x+a)^(1/2)/a^(1/
2))+5*a*b*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] 5*a*b*Sqrt[a + b*x] + (5*b*(a + b*x)^(3/2))/3 - (a + b*x)^(5/2)/x - 5*a^(3/
2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/2}}{x^2}\,dx &= -\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(a+bx)^{3/2}}{x}\,dx\\ &= \frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5ab)\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5a^2b)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+(5a^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\ &= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.91

$$\frac{\sqrt{a+bx}\,(-3a^2+14abx+2b^2x^2)}{3x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a^2 + 14*a*b*x + 2*b^2*x^2))/(3*x) - 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.94

method	result	size
risch	$-\frac{a^2\sqrt{bx+a}}{x}+\frac{b\left(\frac{4(bx+a)^{\frac{3}{2}}}{3}+8a\sqrt{bx+a}-10a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{2}$	57

derivativedivides	$2b\left(\frac{(bx+a)^{\frac{3}{2}}}{3}+2a\sqrt{bx+a}-a^{2}\left(\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	62
default	$2b\left(\frac{(bx+a)^{\frac{3}{2}}}{3}+2a\sqrt{bx+a}-a^{2}\left(\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/3*(b*x+a)^(3/2)+2*a*(b*x+a)^(1/2)-a^2*(1/2*(b*x+a)^(1/2)/b/x+5/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 71, normalized size = 1.08

$$\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b+4\,\sqrt{bx+a}\,ab-\frac{\sqrt{bx+a}\,a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/2*a^(3/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
3*(b*x + a)^(3/2)*b + 4*sqrt(b*x + a)*a*b - sqrt(b*x + a)*a^2/x
```

Fricas [A]
time = 0.45, size = 126, normalized size = 1.91

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(2\,b^{2}x^{2}+14\,abx-3\,a^{2})\sqrt{bx+a}}{6\,x},\ \frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(2\,b^{2}x^{2}+14\,abx-3\,a^{2})\sqrt{bx+a}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

Sympy [A]
time = 2.05, size = 99, normalized size = 1.50

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}-5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,b^{2}x\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x/a)/x + 14*a**(3/2)*b*sqrt(1 + b*x/a)/3 + 5*a**(3/2)*
b*log(b*x/a)/2 - 5*a**(3/2)*b*log(sqrt(1 + b*x/a) + 1) + 2*sqrt(a)*b**2*x*s
qrt(1 + b*x/a)/3
```

Giac [A]
time = 1.94, size = 74, normalized size = 1.12

$$\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,(bx+a)^{\frac{3}{2}}b^2+12\,\sqrt{bx+a}\,ab^2-\frac{3\,\sqrt{bx+a}\,a^2b}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*(b*x + a)^(3/2)
*b^2 + 12*sqrt(b*x + a)*a*b^2 - 3*sqrt(b*x + a)*a^2*b/x)/b
```

Mupad [B]
time = 0.11, size = 58, normalized size = 0.88

$$\frac{2\,b\,{\left(a+b\,x\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{a+b\,x}}{x}+4\,a\,b\,\sqrt{a+b\,x}+a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(3/2))/3 - (a^2*(a + b*x)^(1/2))/x + a^(3/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*5i + 4*a*b*(a + b*x)^(1/2)
```

3.306 $\int \frac{(a+bx)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=78

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -5/4*b*(b*x+a)^(3/2)/x-1/2*(b*x+a)^(5/2)/x^2-15/4*b^2*arctanh((b*x+a)^(1/2)
/a^(1/2))*a^(1/2)+15/4*b^2*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$\frac{15}{4}b^2\sqrt{a+bx}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{2x^2}-\frac{5b(a+bx)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x])/4 - (5*b*(a + b*x)^(3/2))/(4*x) - (a + b*x)^(5/2)/(2
*x^2) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/2}}{x^3}\,dx &= -\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(a+bx)^{3/2}}{x^2}\,dx\\ &= -\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(15ab)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.\\ &= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.81

$$\frac{1}{4}\left(\frac{\sqrt{a+bx}\,\left(-2a^2-9abx+8b^2x^2\right)}{x^2}-15\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] ((Sqrt[a + b*x]*(-2*a^2 - 9*a*b*x + 8*b^2*x^2))/x^2 - 15*Sqrt[a]*b^2*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/4
```

Maple [A]
time = 0.10, size = 62, normalized size = 0.79

method	result	size
risch	$-\frac{a\sqrt{bx+a}\,(9bx+2a)}{4x^2}+\frac{b^2\left(16\sqrt{bx+a}-30\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\right)}{8}$	55

derivativedivides	$2b^2\left(\sqrt{bx+a}-a\left(\frac{\frac{9(bx+a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	62
default	$2b^2\left(\sqrt{bx+a}-a\left(\frac{\frac{9(bx+a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 2*b^2*((b*x+a)^(1/2)-a*((9/8*(b*x+a)^(3/2)-7/8*a*(b*x+a)^(1/2))/b^2/x^2+15/
8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 101, normalized size = 1.29

$$\frac{15}{8}\sqrt{a}\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^2-\frac{9\,(bx+a)^{\frac{3}{2}}ab^2-7\sqrt{bx+a}\,a^2b^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="maxima")
[Out] 15/8*sqrt(a)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2*sqrt(b*x + a)*b^2 - 1/4*(9*(b*x + a)^(3/2)*a*b^2 - 7*sqrt(b*x + a)*a^2*b
^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

Fricas [A]
time = 0.43, size = 133, normalized size = 1.71

$$\left[\frac{15\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(8\,b^2x^2-9\,abx-2\,a^2)\sqrt{bx+a}}{8\,x^2},\frac{15\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(8\,b^2x^2-9\,abx-2\,a^2)\sqrt{bx+a}}{4\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="fricas")
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

Sympy [A]
time = 2.28, size = 126, normalized size = 1.62

$$-\frac{15\sqrt{a}\,b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4}-\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**3,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**3/(2*sqrt(b)*x**(5
/2)*sqrt(a/(b*x) + 1)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - a
*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(a/(b*x) +
 1)
```

Giac [A]
time = 0.98, size = 80, normalized size = 1.03

$$\frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,\sqrt{bx+a}\,b^3-\frac{9\,(bx+a)^{\frac{3}{2}}ab^3-7\,\sqrt{bx+a}\,a^2b^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x + a)*b^3
 - (9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x + a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.05, size = 64, normalized size = 0.82

$$2\,b^2\,\sqrt{a+b\,x}+\frac{7\,a^2\,\sqrt{a+b\,x}}{4\,x^2}-\frac{9\,a\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}+\frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,15{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(a + b*x)^(1/2) + (7*a^2*(a + b*x)^(1/2))/(4*x^2) + (a^(1/2)*b^2*atan
(((a + b*x)^(1/2)*1i)/a^(1/2))*15i)/4 - (9*a*(a + b*x)^(3/2))/(4*x^2)
```

3.307 $\int \frac{(a+bx)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

```
[Out] -5/12*b*(b*x+a)^(3/2)/x^2-1/3*(b*x+a)^(5/2)/x^3-5/8*b^3*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(1/2)-5/8*b^2*(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {43, 65, 214}

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b(a+bx)^{3/2}}{12x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(8*x) - (5*b*(a + b*x)^(3/2))/(12*x^2) - (a + b*x)^(
5/2)/(3*x^3) - (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^4}\,dx &= -\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{6}(5b)\int \frac{(a+bx)^{3/2}}{x^3}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{16}\left(5b^3\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}+\frac{1}{8}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+\right.\\
&= -\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 64, normalized size = 0.79

$$
-\frac{\sqrt{a+bx}\left(8a^2+26abx+33b^2x^2\right)}{24x^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 + 26*a*b*x + 33*b^2*x^2))/x^3 - (5*b^3*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.79

method	result	size
risch	$-\frac{\sqrt{bx+a}\,\left(33x^2b^2+26abx+8a^2\right)}{24x^3}-\frac{5b^3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}$	53
derivativedivides	$2b^3\left(-\frac{\frac{11(bx+a)^{\frac{5}{2}}}{16}-\frac{5a(bx+a)^{\frac{3}{2}}}{6}+\frac{5a^2\sqrt{bx+a}}{16}}{b^3x^3}-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}\right)$	64

default	$2b^3\left(-\frac{\frac{11(bx+a)^{\frac{5}{2}}}{16}-\frac{5a(bx+a)^{\frac{3}{2}}}{6}+\frac{5a^2\sqrt{bx+a}}{16}}{b^3x^3}-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-(11/16*(b*x+a)^(5/2)-5/6*a*(b*x+a)^(3/2)+5/16*a^2*(b*x+a)^(1/2))/b^
3/x^3-5/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.49, size = 115, normalized size = 1.42

$$\frac{5\,b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,\sqrt{a}}-\frac{33\,(bx+a)^{\frac{5}{2}}b^3-40\,(bx+a)^{\frac{3}{2}}ab^3+15\,\sqrt{bx+a}\,a^2b^3}{24\,\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
 1/24*(33*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 15*sqrt(b*x + a)
*a^2*b^3)/((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

Fricas [A]
time = 0.46, size = 146, normalized size = 1.80

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{48\,ax^3},\frac{15\,\sqrt{-a}\,b^3x^3\,\mathrm{arctan}\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(33\,ab^2x^2+26\,a^2bx+8\,a^3)\sqrt{bx+a}}{24\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

Sympy [A]
time = 2.59, size = 104, normalized size = 1.28

$$-\frac{a^2\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{12x^{\frac{3}{2}}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{8\sqrt{x}}-\frac{5b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x**(5/2)) - 13*a*b**(3/2)*sqrt(a/(b*x) +
 1)/(12*x**(3/2)) - 11*b**(5/2)*sqrt(a/(b*x) + 1)/(8*sqrt(x)) - 5*b**3*asin
h(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a))
```

Giac [A]
time = 0.83, size = 79, normalized size = 0.98

$$\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{33\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+15\,\sqrt{bx+a}\,a^2b^4}{b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (33*(b*x + a)^(5/2)*
b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a^2*b^4)/(b^3*x^3))/b
```

Mupad [B]
time = 0.05, size = 64, normalized size = 0.79

$$\frac{5\,a\,{\left(a+b\,x\right)}^{3/2}}{3\,x^3}-\frac{5\,a^2\,\sqrt{a+b\,x}}{8\,x^3}-\frac{11\,{\left(a+b\,x\right)}^{5/2}}{8\,x^3}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^4,x)
```

```
[Out] (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(1/2)) - (5*a^2*(a + b*x)^
(1/2))/(8*x^3) - (11*(a + b*x)^(5/2))/(8*x^3) + (5*a*(a + b*x)^(3/2))/(3*x^
3)
```

3.308 $\int \frac{(a+bx)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=103

$$-\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

```
[Out] -5/24*b*(b*x+a)^(3/2)/x^3-1/4*(b*x+a)^(5/2)/x^4+5/64*b^4*arctanh((b*x+a)^(1
/2)/a^(1/2))/a^(3/2)-5/32*b^2*(b*x+a)^(1/2)/x^2-5/64*b^3*(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{5b(a+bx)^{3/2}}{24x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(32*x^2) - (5*b^3*Sqrt[a + b*x])/(64*a*x) - (5*b*(a
+ b*x)^(3/2))/(24*x^3) - (a + b*x)^(5/2)/(4*x^4) + (5*b^4*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^5}\,dx &= -\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{8}(5b)\int \frac{(a+bx)^{3/2}}{x^4}\,dx\\
&= -\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^3}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{64}\left(5b^3\right)\int \frac{1}{x^2\sqrt{a+bx}}\,dx\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^4\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{128a}\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^3\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{64a}\\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 78, normalized size = 0.76

$$-\frac{\sqrt{a+bx}\,\left(48a^3+136a^2bx+118ab^2x^2+15b^3x^3\right)}{192ax^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] -1/192*(Sqrt[a + b*x]*(48*a^3 + 136*a^2*b*x + 118*a*b^2*x^2 + 15*b^3*x^3))/
(a*x^4) + (5*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(3/2))
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.74

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(15b^3x^3+118a\,b^2x^2+136a^2bx+48a^3)}{192x^4a}+\frac{5b^4\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64a^{\frac{3}{2}}}$	67
derivativedivides	$2b^4\left(-\frac{\frac{5(bx+a)^{\frac{7}{2}}}{128a}+\frac{73(bx+a)^{\frac{5}{2}}}{384}-\frac{55a(bx+a)^{\frac{3}{2}}}{384}+\frac{5a^2\sqrt{bx+a}}{128}}{b^4x^4}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}\right)$	76
default	$2b^4\left(-\frac{\frac{5(bx+a)^{\frac{7}{2}}}{128a}+\frac{73(bx+a)^{\frac{5}{2}}}{384}-\frac{55a(bx+a)^{\frac{3}{2}}}{384}+\frac{5a^2\sqrt{bx+a}}{128}}{b^4x^4}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}\right)$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4*(-(5/128/a*(b*x+a)^(7/2)+73/384*(b*x+a)^(5/2)-55/384*a*(b*x+a)^(3/2)+
5/128*a^2*(b*x+a)^(1/2))/b^4/x^4+5/128*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/
2))
```

Maxima [A]

time = 0.49, size = 144, normalized size = 1.40

$$-\frac{5\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}}-\frac{15\,(bx+a)^{\frac{7}{2}}b^4+73\,(bx+a)^{\frac{5}{2}}ab^4-55\,(bx+a)^{\frac{3}{2}}a^2b^4+15\,\sqrt{bx+a}\,a^3b^4}{192\,\left((bx+a)^4a-4\,(bx+a)^3a^2+6\,(bx+a)^2a^3-4\,(bx+a)a^4+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -5/128*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
 - 1/192*(15*(b*x + a)^(7/2)*b^4 + 73*(b*x + a)^(5/2)*a*b^4 - 55*(b*x + a)^
(3/2)*a^2*b^4 + 15*sqrt(b*x + a)*a^3*b^4)/((b*x + a)^4*a - 4*(b*x + a)^3*a^
2 + 6*(b*x + a)^2*a^3 - 4*(b*x + a)*a^4 + a^5)
```

Fricas [A]

time = 0.43, size = 167, normalized size = 1.62

$$\left[\frac{15\,\sqrt{a}\,b^4x^4\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{384\,a^2x^4},-\frac{15\,\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^3x^3+118\,a^2b^2x^2+136\,a^3bx+48\,a^4)\sqrt{bx+a}}{192\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

Sympy [A]
time = 7.16, size = 155, normalized size = 1.50

$$-\frac{a^3}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{127ab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{133b^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{7}{2}}}{64a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**5,x)
```

```
[Out] -a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*a**2*sqrt(b)/(24*x**(7/2)
*sqrt(a/(b*x) + 1)) - 127*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)) - 133*
b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(7/2)/(64*a*sqrt(x)*sqrt(a
/(b*x) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(3/2))
```

Giac [A]
time = 0.70, size = 99, normalized size = 0.96

$$-\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx+a}\,a^3b^5}{ab^4x^4}}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x + a)^
(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)^(3/2)*a^2*b^5 + 15*sqrt
(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

Mupad [B]
time = 0.11, size = 79, normalized size = 0.77

$$\frac{55\,a\,{(a+b\,x)}^{3/2}}{192\,x^4}-\frac{5\,a^2\,\sqrt{a+b\,x}}{64\,x^4}-\frac{5\,{(a+b\,x)}^{7/2}}{64\,a\,x^4}-\frac{73\,{(a+b\,x)}^{5/2}}{192\,x^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{64\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^5,x)
```

```
[Out] (55*a*(a + b*x)^(3/2))/(192*x^4) - (5*a^2*(a + b*x)^(1/2))/(64*x^4) - (5*(a
 + b*x)^(7/2))/(64*a*x^4) - (b^4*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(64
*a^(3/2)) - (73*(a + b*x)^(5/2))/(192*x^4)
```

3.309 $\int x^7(a+bx)^{9/2}\,dx$

Optimal. Leaf size=146

$$-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{2}}{b^8}$$

```
[Out] -2/11*a^7*(b*x+a)^(11/2)/b^8+14/13*a^6*(b*x+a)^(13/2)/b^8-14/5*a^5*(b*x+a)^
(15/2)/b^8+70/17*a^4*(b*x+a)^(17/2)/b^8-70/19*a^3*(b*x+a)^(19/2)/b^8+2*a^2*
(b*x+a)^(21/2)/b^8-14/23*a*(b*x+a)^(23/2)/b^8+2/25*(b*x+a)^(25/2)/b^8
```

Rubi [A]
time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{21/2}}{b^8}+\frac{2(a+bx)^{25/2}}{25b^8}-\frac{14a(a+bx)^{23/2}}{23b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) - (
14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (70
*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(a +
 b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^7(a+bx)^{9/2}\,dx = \int \left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-35a\right.$$

$$=-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-70a^3($$

Mathematica [A]
time = 0.05, size = 90, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-2048a^7+11264a^6bx-36608a^5b^2x^2+91520a^4b^3x^3-194480a^3b^4x^4+369512a^2b^5x^5-646646ab^6x^6+1062347b^7x^7\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2048*a^7 + 11264*a^6*b*x - 36608*a^5*b^2*x^2 + 91520*
a^4*b^3*x^3 - 194480*a^3*b^4*x^4 + 369512*a^2*b^5*x^5 - 646646*a*b^6*x^6 +
1062347*b^7*x^7))/(26558675*b^8)
```

Maple [A]
time = 0.09, size = 97, normalized size = 0.66

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}(-1062347b^7x^7+646646a\,b^6x^6-369512a^2b^5x^5+194480a^3b^4x^4-91520a^4b^3x^3+36608a^5b^2x^2-11264a^6bx+}{26558675b^8}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{25}{2}}}{25}-\frac{14a(bx+a)^{\frac{23}{2}}}{23}+2a^2(bx+a)^{\frac{21}{2}}-\frac{70a^3(bx+a)^{\frac{19}{2}}}{19}+\frac{70a^4(bx+a)^{\frac{17}{2}}}{17}-\frac{14a^5(bx+a)^{\frac{15}{2}}}{5}+\frac{14a^6(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^7(bx+a)^{\frac{11}{2}}}{11}}{b^8}$
default	$\frac{\frac{2(bx+a)^{\frac{25}{2}}}{25}-\frac{14a(bx+a)^{\frac{23}{2}}}{23}+2a^2(bx+a)^{\frac{21}{2}}-\frac{70a^3(bx+a)^{\frac{19}{2}}}{19}+\frac{70a^4(bx+a)^{\frac{17}{2}}}{17}-\frac{14a^5(bx+a)^{\frac{15}{2}}}{5}+\frac{14a^6(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^7(bx+a)^{\frac{11}{2}}}{11}}{b^8}$
trager	$-\frac{2(-1062347b^{12}x^{12}-4665089a\,b^{11}x^{11}-7759752a^2b^{10}x^{10}-5810090a^3b^9x^9-1659515a^4b^8x^8-429a^5b^7x^7+462a^6b^6x^6-5}{26558675b^8}$
risch	$-\frac{2(-1062347b^{12}x^{12}-4665089a\,b^{11}x^{11}-7759752a^2b^{10}x^{10}-5810090a^3b^9x^9-1659515a^4b^8x^8-429a^5b^7x^7+462a^6b^6x^6-5}{26558675b^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^8*(1/25*(b*x+a)^(25/2)-7/23*a*(b*x+a)^(23/2)+a^2*(b*x+a)^(21/2)-35/19*a
^3*(b*x+a)^(19/2)+35/17*a^4*(b*x+a)^(17/2)-7/5*a^5*(b*x+a)^(15/2)+7/13*a^6*
(b*x+a)^(13/2)-1/11*a^7*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 116, normalized size = 0.79

$$\frac{2\,(bx+a)^{\frac{25}{2}}}{25\,b^8}-\frac{14\,(bx+a)^{\frac{23}{2}}\,a}{23\,b^8}+\frac{2\,(bx+a)^{\frac{21}{2}}\,a^2}{b^8}-\frac{70\,(bx+a)^{\frac{19}{2}}\,a^3}{19\,b^8}+\frac{70\,(bx+a)^{\frac{17}{2}}\,a^4}{17\,b^8}-\frac{14\,(bx+a)^{\frac{15}{2}}\,a^5}{5\,b^8}+\frac{14\,(bx+a)^{\frac{13}{2}}\,a^6}{13\,b^8}-\frac{2\,(bx+a)^{\frac{11}{2}}\,a^7}{11\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/25*(b*x + a)^(25/2)/b^8 - 14/23*(b*x + a)^(23/2)*a/b^8 + 2*(b*x + a)^(21/
2)*a^2/b^8 - 70/19*(b*x + a)^(19/2)*a^3/b^8 + 70/17*(b*x + a)^(17/2)*a^4/b^
8 - 14/5*(b*x + a)^(15/2)*a^5/b^8 + 14/13*(b*x + a)^(13/2)*a^6/b^8 - 2/11*(
b*x + a)^(11/2)*a^7/b^8
```

Fricas [A]
time = 0.45, size = 141, normalized size = 0.97

$$\frac{2\,(1062347\,b^{12}x^{12}+4665089\,ab^{11}x^{11}+7759752\,a^2b^{10}x^{10}+5810090\,a^3b^9x^9+1659515\,a^4b^8x^8+429\,a^5b^7x^7-462\,a^6b^6x^6+504\,a^7b^5x^5-560\,a^8b^4x^4+640\,a^9b^3x^3-768\,a^{10}b^2x^2+1024\,a^{11}bx-2048\,a^{12})\sqrt{bx+a}}{26558675\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/26558675*(1062347*b^12*x^12 + 4665089*a*b^11*x^11 + 7759752*a^2*b^10*x^10
 + 5810090*a^3*b^9*x^9 + 1659515*a^4*b^8*x^8 + 429*a^5*b^7*x^7 - 462*a^6*b^
6*x^6 + 504*a^7*b^5*x^5 - 560*a^8*b^4*x^4 + 640*a^9*b^3*x^3 - 768*a^10*b^2*
x^2 + 1024*a^11*b*x - 2048*a^12)*sqrt(b*x + a)/b^8
```

Sympy [**A**]
time = 1.40, size = 279, normalized size = 1.91

$$\begin{cases} -\frac{4096a^{12}\sqrt{a+bx}}{26558675b^8} + \frac{2048a^{11}x\sqrt{a+bx}}{26558675b^7} - \frac{1536a^{10}x^2\sqrt{a+bx}}{26558675b^6} + \frac{256a^9x^3\sqrt{a+bx}}{5311735b^5} - \frac{224a^8x^4\sqrt{a+bx}}{5311735b^4} + \frac{1008a^7x^5\sqrt{a+bx}}{26558675b^3} - \frac{84a^6x^6\sqrt{a+bx}}{2414425b^2} + \frac{6a^5x^7\sqrt{a+bx}}{185725b} + \frac{4642a^4x^8\sqrt{a+bx}}{37145} + \frac{956a^3bx^9\sqrt{a+bx}}{2185} + \frac{336a^2b^2x^{10}\sqrt{a+bx}}{575} + \frac{202ab^3x^{11}\sqrt{a+bx}}{575} + \frac{2b^4x^{12}\sqrt{a+bx}}{25} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4096*a**12*sqrt(a + b*x)/(26558675*b**8) + 2048*a**11*x*sqrt(a
+ b*x)/(26558675*b**7) - 1536*a**10*x**2*sqrt(a + b*x)/(26558675*b**6) + 25
6*a**9*x**3*sqrt(a + b*x)/(5311735*b**5) - 224*a**8*x**4*sqrt(a + b*x)/(531
1735*b**4) + 1008*a**7*x**5*sqrt(a + b*x)/(26558675*b**3) - 84*a**6*x**6*sq
rt(a + b*x)/(2414425*b**2) + 6*a**5*x**7*sqrt(a + b*x)/(185725*b) + 4642*a*
*4*x**8*sqrt(a + b*x)/37145 + 956*a**3*b*x**9*sqrt(a + b*x)/2185 + 336*a**2
*b**2*x**10*sqrt(a + b*x)/575 + 202*a*b**3*x**11*sqrt(a + b*x)/575 + 2*b**4
*x**12*sqrt(a + b*x)/25, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(116) = 232.
time = 0.90, size = 781, normalized size = 5.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/1673196525*(260015*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a^5/b^7 + 76475*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^4/b^7 + 72450*(12
155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*
a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230
*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2
)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a^3/b^7 + 17
```

```
250*(46189*(b*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)
^(17/2)*a^2 - 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4
- 22221108*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(
b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*
a^9 + 969969*sqrt(b*x + a)*a^10)*a^2/b^7 + 4125*(88179*(b*x + a)^(23/2) - 1
062347*(b*x + a)^(21/2)*a + 5870865*(b*x + a)^(19/2)*a^2 - 19684665*(b*x +
a)^(17/2)*a^3 + 44618574*(b*x + a)^(15/2)*a^4 - 72076158*(b*x + a)^(13/2)*a
^5 + 85180914*(b*x + a)^(11/2)*a^6 - 74364290*(b*x + a)^(9/2)*a^7 + 4780561
5*(b*x + a)^(7/2)*a^8 - 22309287*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3
/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)*a/b^7 + 99*(676039*(b*x + a)^(25/2)
- 8817900*(b*x + a)^(23/2)*a + 53117350*(b*x + a)^(21/2)*a^2 - 195695500*(b
*x + a)^(19/2)*a^3 + 492116625*(b*x + a)^(17/2)*a^4 - 892371480*(b*x + a)^(
15/2)*a^5 + 1201269300*(b*x + a)^(13/2)*a^6 - 1216870200*(b*x + a)^(11/2)*a
^7 + 929553625*(b*x + a)^(9/2)*a^8 - 531173500*(b*x + a)^(7/2)*a^9 + 223092
870*(b*x + a)^(5/2)*a^10 - 67603900*(b*x + a)^(3/2)*a^11 + 16900975*sqrt(b*
x + a)*a^12)/b^7)/b
```

Mupad [B]
time = 0.04, size = 116, normalized size = 0.79

$$\frac{2\,{\left(a+b\,x\right)}^{25/2}}{25\,b^8}-\frac{2\,a^7\,{\left(a+b\,x\right)}^{11/2}}{11\,b^8}+\frac{14\,a^6\,{\left(a+b\,x\right)}^{13/2}}{13\,b^8}-\frac{14\,a^5\,{\left(a+b\,x\right)}^{15/2}}{5\,b^8}+\frac{70\,a^4\,{\left(a+b\,x\right)}^{17/2}}{17\,b^8}-\frac{70\,a^3\,{\left(a+b\,x\right)}^{19/2}}{19\,b^8}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{21/2}}{b^8}-\frac{14\,a\,{\left(a+b\,x\right)}^{23/2}}{23\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(25/2))/(25*b^8) - (2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6
*(a + b*x)^(13/2))/(13*b^8) - (14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(
a + b*x)^(17/2))/(17*b^8) - (70*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a
+ b*x)^(21/2))/b^8 - (14*a*(a + b*x)^(23/2))/(23*b^8)
```

3.310 $\int x^6(a+bx)^{9/2}\,dx$

Optimal. Leaf size=127

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}+$$

```
[Out] 2/11*a^6*(b*x+a)^(11/2)/b^7-12/13*a^5*(b*x+a)^(13/2)/b^7+2*a^4*(b*x+a)^(15/
2)/b^7-40/17*a^3*(b*x+a)^(17/2)/b^7+30/19*a^2*(b*x+a)^(19/2)/b^7-4/7*a*(b*x
+a)^(21/2)/b^7+2/23*(b*x+a)^(23/2)/b^7
```

Rubi [A]
time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (2
*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2*(
a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x)^(
23/2))/(23*b^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^6(a+bx)^{9/2}\,dx=\int\left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+15a^2(\right.$$

$$=\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a-}{19}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(1024a^6-5632a^5bx+18304a^4b^2x^2-45760a^3b^3x^3+97240a^2b^4x^4-184756ab^5x^5+323323b^6x^6\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(1024*a^6 - 5632*a^5*b*x + 18304*a^4*b^2*x^2 - 45760*a^
3*b^3*x^3 + 97240*a^2*b^4*x^4 - 184756*a*b^5*x^5 + 323323*b^6*x^6))/(743642
9*b^7)
```

Maple [A]
time = 0.08, size = 85, normalized size = 0.67

method	result
gosper	$\frac{2(bx+a)^{\frac{11}{2}}\left(323323x^6b^6-184756a\,x^5b^5+97240a^2x^4b^4-45760a^3b^3x^3+18304a^4x^2b^2-5632a^5xb+1024a^6\right)}{7436429b^7}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{23}{2}}}{23}-\frac{4a(bx+a)^{\frac{21}{2}}}{7}+\frac{30a^2(bx+a)^{\frac{19}{2}}}{19}-\frac{40a^3(bx+a)^{\frac{17}{2}}}{17}+2a^4(bx+a)^{\frac{15}{2}}-\frac{12a^5(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^6(bx+a)^{\frac{11}{2}}}{11}}{b^7}$
default	$\frac{\frac{2(bx+a)^{\frac{23}{2}}}{23}-\frac{4a(bx+a)^{\frac{21}{2}}}{7}+\frac{30a^2(bx+a)^{\frac{19}{2}}}{19}-\frac{40a^3(bx+a)^{\frac{17}{2}}}{17}+2a^4(bx+a)^{\frac{15}{2}}-\frac{12a^5(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^6(bx+a)^{\frac{11}{2}}}{11}}{b^7}$
trager	$\frac{2\left(323323b^{11}x^{11}+1431859a\,b^{10}x^{10}+2406690a^2b^9x^9+1826110a^3b^8x^8+530959a^4b^7x^7+231a^5b^6x^6-252a^6b^5x^5+280a^7b^4x^4\right.}{7436429b^7}$
risch	$\frac{2\left(323323b^{11}x^{11}+1431859a\,b^{10}x^{10}+2406690a^2b^9x^9+1826110a^3b^8x^8+530959a^4b^7x^7+231a^5b^6x^6-252a^6b^5x^5+280a^7b^4x^4\right.}{7436429b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^7*(1/23*(b*x+a)^(23/2)-2/7*a*(b*x+a)^(21/2)+15/19*a^2*(b*x+a)^(19/2)-20
/17*a^3*(b*x+a)^(17/2)+a^4*(b*x+a)^(15/2)-6/13*a^5*(b*x+a)^(13/2)+1/11*a^6*
(b*x+a)^(11/2))
```

Maxima [A]
time = 0.28, size = 101, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{23}{2}}}{23\,b^7}-\frac{4\,(bx+a)^{\frac{21}{2}}a}{7\,b^7}+\frac{30\,(bx+a)^{\frac{19}{2}}a^2}{19\,b^7}-\frac{40\,(bx+a)^{\frac{17}{2}}a^3}{17\,b^7}+\frac{2\,(bx+a)^{\frac{15}{2}}a^4}{b^7}-\frac{12\,(bx+a)^{\frac{13}{2}}a^5}{13\,b^7}+\frac{2\,(bx+a)^{\frac{11}{2}}a^6}{11\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/23*(b*x + a)^(23/2)/b^7 - 4/7*(b*x + a)^(21/2)*a/b^7 + 30/19*(b*x + a)^(1
9/2)*a^2/b^7 - 40/17*(b*x + a)^(17/2)*a^3/b^7 + 2*(b*x + a)^(15/2)*a^4/b^7
- 12/13*(b*x + a)^(13/2)*a^5/b^7 + 2/11*(b*x + a)^(11/2)*a^6/b^7
```

Fricas [A]
time = 0.43, size = 130, normalized size = 1.02

$$\frac{2\,(323323\,b^{11}x^{11}+1431859\,ab^{10}x^{10}+2406690\,a^2b^9x^9+1826110\,a^3b^8x^8+530959\,a^4b^7x^7+231\,a^5b^6x^6-252\,a^6b^5x^5+280\,a^7b^4x^4-320\,a^8b^3x^3+384\,a^9b^2x^2-512\,a^{10}bx+1024\,a^{11})\sqrt{bx+a}}{7436429\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="fricas")

[Out] 2/7436429*(323323*b^11*x^11 + 1431859*a*b^10*x^10 + 2406690*a^2*b^9*x^9 + 1826110*a^3*b^8*x^8 + 530959*a^4*b^7*x^7 + 231*a^5*b^6*x^6 - 252*a^6*b^5*x^5 + 280*a^7*b^4*x^4 - 320*a^8*b^3*x^3 + 384*a^9*b^2*x^2 - 512*a^10*b*x + 1024*a^11)*sqrt(b*x + a)/b^7

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(122) = 244$.
time = 1.26, size = 257, normalized size = 2.02

$$\begin{cases} \frac{2048a^{11}\sqrt{a+bx}}{7436429b^7} - \frac{1024a^{10}x\sqrt{a+bx}}{7436429b^6} + \frac{768a^9x^2\sqrt{a+bx}}{7436429b^5} - \frac{640a^8x^3\sqrt{a+bx}}{7436429b^4} + \frac{80a^7x^4\sqrt{a+bx}}{1062347b^3} - \frac{72a^6x^5\sqrt{a+bx}}{1062347b^2} + \frac{6a^5x^6\sqrt{a+bx}}{96577b} + \frac{7426a^4x^7\sqrt{a+bx}}{52003} + \frac{25540a^3bx^8\sqrt{a+bx}}{52003} + \frac{1980a^2b^2x^9\sqrt{a+bx}}{3059} + \frac{62ab^3x^{10}\sqrt{a+bx}}{161} + \frac{2b^4x^{11}\sqrt{a+bx}}{23} & \text{for}\ b \neq 0 \\ \frac{a^{\frac{9}{2}}x^7}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(b*x+a)**(9/2),x)

[Out] Piecewise((2048*a**11*sqrt(a + b*x)/(7436429*b**7) - 1024*a**10*x*sqrt(a + b*x)/(7436429*b**6) + 768*a**9*x**2*sqrt(a + b*x)/(7436429*b**5) - 640*a**8*x**3*sqrt(a + b*x)/(7436429*b**4) + 80*a**7*x**4*sqrt(a + b*x)/(1062347*b**3) - 72*a**6*x**5*sqrt(a + b*x)/(1062347*b**2) + 6*a**5*x**6*sqrt(a + b*x)/(96577*b) + 7426*a**4*x**7*sqrt(a + b*x)/52003 + 25540*a**3*b*x**8*sqrt(a + b*x)/52003 + 1980*a**2*b**2*x**9*sqrt(a + b*x)/3059 + 62*a*b**3*x**10*sqrt(a + b*x)/161 + 2*b**4*x**11*sqrt(a + b*x)/23, Ne(b, 0)), (a**(9/2)*x**7/7, True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(101) = 202$.
time = 1.69, size = 709, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="giac")

[Out] 2/66927861*(22287*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^5/b^6 + 52003*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^4/b^6 + 6118*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^3/b^6 + 2898*(12155*(b*x + a)^(19/2) - 122265*(b

```
*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a
^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(
b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a
^8 - 230945*sqrt(b*x + a)*a^9)*a^2/b^6 + 345*(46189*(b*x + a)^(21/2) - 5105
10*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x + a)^(1
5/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a^5 +
22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*(b*x
+ a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^10)*a
/b^6 + 33*(88179*(b*x + a)^(23/2) - 1062347*(b*x + a)^(21/2)*a + 5870865*(b
*x + a)^(19/2)*a^2 - 19684665*(b*x + a)^(17/2)*a^3 + 44618574*(b*x + a)^(15
/2)*a^4 - 72076158*(b*x + a)^(13/2)*a^5 + 85180914*(b*x + a)^(11/2)*a^6 - 7
4364290*(b*x + a)^(9/2)*a^7 + 47805615*(b*x + a)^(7/2)*a^8 - 22309287*(b*x
+ a)^(5/2)*a^9 + 7436429*(b*x + a)^(3/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)
/b^6)/b
```

Mupad [B]
time = 0.03, size = 101, normalized size = 0.80

$$\frac{2\,{\left(a+b\,x\right)}^{23/2}}{23\,b^7}+\frac{2\,a^6\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7}-\frac{12\,a^5\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{15/2}}{b^7}-\frac{40\,a^3\,{\left(a+b\,x\right)}^{17/2}}{17\,b^7}+\frac{30\,a^2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^7}-\frac{4\,a\,{\left(a+b\,x\right)}^{21/2}}{7\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(23/2))/(23*b^7) + (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5
*(a + b*x)^(13/2))/(13*b^7) + (2*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b
*x)^(17/2))/(17*b^7) + (30*a^2*(a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^
(21/2))/(7*b^7)
```

3.311 $\int x^5(a+bx)^{9/2}\,dx$

Optimal. Leaf size=110

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}+\frac{2(a+bx)^{21/2}}{21b^6}$$

```
[Out] -2/11*a^5*(b*x+a)^(11/2)/b^6+10/13*a^4*(b*x+a)^(13/2)/b^6-4/3*a^3*(b*x+a)^(
15/2)/b^6+20/17*a^2*(b*x+a)^(17/2)/b^6-10/19*a*(b*x+a)^(19/2)/b^6+2/21*(b*x
+a)^(21/2)/b^6
```

Rubi [A]
time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (
4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*
a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{9/2}\,dx &= \int \left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\frac{5a(\right.\\ &= -\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a}{1}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-256a^5+1408a^4bx-4576a^3b^2x^2+11440a^2b^3x^3-24310ab^4x^4+46189b^5x^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-256*a^5 + 1408*a^4*b*x - 4576*a^3*b^2*x^2 + 11440*a^2
*b^3*x^3 - 24310*a*b^4*x^4 + 46189*b^5*x^5))/(969969*b^6)
```

Maple [A]
time = 0.09, size = 74, normalized size = 0.67

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-46189b^5x^5+24310a\,b^4x^4-11440a^2b^3x^3+4576a^3b^2x^2-1408a^4bx+256a^5\right)}{969969b^6}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{21}{2}}}{21}-\frac{10a(bx+a)^{\frac{19}{2}}}{19}+\frac{20a^2(bx+a)^{\frac{17}{2}}}{17}-\frac{4a^3(bx+a)^{\frac{15}{2}}}{3}+\frac{10a^4(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^5(bx+a)^{\frac{11}{2}}}{11}}{b^6}$
default	$\frac{\frac{2(bx+a)^{\frac{21}{2}}}{21}-\frac{10a(bx+a)^{\frac{19}{2}}}{19}+\frac{20a^2(bx+a)^{\frac{17}{2}}}{17}-\frac{4a^3(bx+a)^{\frac{15}{2}}}{3}+\frac{10a^4(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^5(bx+a)^{\frac{11}{2}}}{11}}{b^6}$
trager	$-\frac{2(-46189b^{10}x^{10}-206635a\,b^9x^9-351780a^2b^8x^8-271414a^3b^7x^7-80773a^4b^6x^6-63a^5b^5x^5+70a^6b^4x^4-80a^7b^3x^3+96a^8}{969969b^6}$
risch	$-\frac{2(-46189b^{10}x^{10}-206635a\,b^9x^9-351780a^2b^8x^8-271414a^3b^7x^7-80773a^4b^6x^6-63a^5b^5x^5+70a^6b^4x^4-80a^7b^3x^3+96a^8}{969969b^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^6*(1/21*(b*x+a)^(21/2)-5/19*a*(b*x+a)^(19/2)+10/17*a^2*(b*x+a)^(17/2)-2
/3*a^3*(b*x+a)^(15/2)+5/13*a^4*(b*x+a)^(13/2)-1/11*a^5*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 86, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{21}{2}}}{21\,b^6}-\frac{10\,(bx+a)^{\frac{19}{2}}a}{19\,b^6}+\frac{20\,(bx+a)^{\frac{17}{2}}a^2}{17\,b^6}-\frac{4\,(bx+a)^{\frac{15}{2}}a^3}{3\,b^6}+\frac{10\,(bx+a)^{\frac{13}{2}}a^4}{13\,b^6}-\frac{2\,(bx+a)^{\frac{11}{2}}a^5}{11\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x + a)^(21/2)/b^6 - 10/19*(b*x + a)^(19/2)*a/b^6 + 20/17*(b*x + a)^
(17/2)*a^2/b^6 - 4/3*(b*x + a)^(15/2)*a^3/b^6 + 10/13*(b*x + a)^(13/2)*a^4/
b^6 - 2/11*(b*x + a)^(11/2)*a^5/b^6
```

Fricas [A]
time = 0.41, size = 119, normalized size = 1.08

$$\frac{2\,(46189\,b^{10}x^{10}+206635\,ab^9x^9+351780\,a^2b^8x^8+271414\,a^3b^7x^7+80773\,a^4b^6x^6+63\,a^5b^5x^5-70\,a^6b^4x^4+80\,a^7b^3x^3-96\,a^8b^2x^2+128\,a^9bx-256\,a^{10})\sqrt{bx+a}}{969969\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(46189*b^10*x^10 + 206635*a*b^9*x^9 + 351780*a^2*b^8*x^8 + 271414*
a^3*b^7*x^7 + 80773*a^4*b^6*x^6 + 63*a^5*b^5*x^5 - 70*a^6*b^4*x^4 + 80*a^7*
b^3*x^3 - 96*a^8*b^2*x^2 + 128*a^9*b*x - 256*a^10)*sqrt(b*x + a)/b^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(105) = 210$.
time = 1.14, size = 235, normalized size = 2.14

$$\begin{cases} -\frac{512a^{10}\sqrt{a+bx}}{969969b^6} + \frac{256a^9x\sqrt{a+bx}}{969969b^5} - \frac{64a^8x^2\sqrt{a+bx}}{323323b^4} + \frac{160a^7x^3\sqrt{a+bx}}{969969b^3} - \frac{20a^6x^4\sqrt{a+bx}}{138567b^2} + \frac{6a^5x^5\sqrt{a+bx}}{46189b} + \frac{2098a^4x^6\sqrt{a+bx}}{12597} + \frac{3796a^3bx^7\sqrt{a+bx}}{6783} + \frac{1640a^2b^2x^8\sqrt{a+bx}}{2261} + \frac{170ab^3x^9\sqrt{a+bx}}{399} + \frac{2b^4x^{10}\sqrt{a+bx}}{21} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-512*a**10*sqrt(a + b*x)/(969969*b**6) + 256*a**9*x*sqrt(a + b*x
)/(969969*b**5) - 64*a**8*x**2*sqrt(a + b*x)/(323323*b**4) + 160*a**7*x**3*
sqrt(a + b*x)/(969969*b**3) - 20*a**6*x**4*sqrt(a + b*x)/(138567*b**2) + 6*
a**5*x**5*sqrt(a + b*x)/(46189*b) + 2098*a**4*x**6*sqrt(a + b*x)/12597 + 37
96*a**3*b*x**7*sqrt(a + b*x)/6783 + 1640*a**2*b**2*x**8*sqrt(a + b*x)/2261
+ 170*a*b**3*x**9*sqrt(a + b*x)/399 + 2*b**4*x**10*sqrt(a + b*x)/21, Ne(b,
0)), (a**(9/2)*x**6/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(86) = 172$.
time = 1.93, size = 637, normalized size = 5.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/2909907*(4199*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)*a^5/b^5 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2
)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^
(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^4/b^5 + 45
22*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)
*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^3/b^5
 + 266*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)
^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 87
5160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3
/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2/b^5 + 63*(12155*(b*x + a)^(19/2) -
122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)
^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 +
```

```
2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a
)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a/b^5 + 3*(46189*(b*x + a)^(21/2) -
 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x +
a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a
^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*
(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^
10)/b^5)/b
```

Mupad [B]
time = 0.03, size = 86, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{21/2}}{21\,b^6}-\frac{2\,a^5\,{\left(a+b\,x\right)}^{11/2}}{11\,b^6}+\frac{10\,a^4\,{\left(a+b\,x\right)}^{13/2}}{13\,b^6}-\frac{4\,a^3\,{\left(a+b\,x\right)}^{15/2}}{3\,b^6}+\frac{20\,a^2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^6}-\frac{10\,a\,{\left(a+b\,x\right)}^{19/2}}{19\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(21/2))/(21*b^6) - (2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4
*(a + b*x)^(13/2))/(13*b^6) - (4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a
 + b*x)^(17/2))/(17*b^6) - (10*a*(a + b*x)^(19/2))/(19*b^6)
```

3.312 $\int x^4(a+bx)^{9/2}\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19/2}}{19b^5}$$

```
[Out] 2/11*a^4*(b*x+a)^(11/2)/b^5-8/13*a^3*(b*x+a)^(13/2)/b^5+4/5*a^2*(b*x+a)^(15
/2)/b^5-8/17*a*(b*x+a)^(17/2)/b^5+2/19*(b*x+a)^(19/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4*
a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a + b
*x)^(19/2))/(19*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{9/2}\,dx &= \int \left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a+bx)^{1}}{b^4}\right.\\ &= \frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19}}{19b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.63

$$\frac{2(a+bx)^{11/2}\left(128a^4-704a^3bx+2288a^2b^2x^2-5720ab^3x^3+12155b^4x^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(128*a^4 - 704*a^3*b*x + 2288*a^2*b^2*x^2 - 5720*a*b^3*
x^3 + 12155*b^4*x^4))/(230945*b^5)
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.68

method	result
gosper	$\frac{2(bx+a)^{\frac{11}{2}}\left(12155b^4x^4-5720a\,b^3x^3+2288a^2b^2x^2-704a^3bx+128a^4\right)}{230945b^5}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{19}{2}}}{19}-\frac{8a(bx+a)^{\frac{17}{2}}}{17}+\frac{4a^2(bx+a)^{\frac{15}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^4(bx+a)^{\frac{11}{2}}}{11}}{b^5}$
default	$\frac{\frac{2(bx+a)^{\frac{19}{2}}}{19}-\frac{8a(bx+a)^{\frac{17}{2}}}{17}+\frac{4a^2(bx+a)^{\frac{15}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^4(bx+a)^{\frac{11}{2}}}{11}}{b^5}$
trager	$\frac{2(12155b^9x^9+55055a\,b^8x^8+95238a^2b^7x^7+75086a^3b^6x^6+23063a^4b^5x^5+35a^5b^4x^4-40a^6b^3x^3+48a^7b^2x^2-64a^8bx+128a^9}{230945b^5}$
risch	$\frac{2(12155b^9x^9+55055a\,b^8x^8+95238a^2b^7x^7+75086a^3b^6x^6+23063a^4b^5x^5+35a^5b^4x^4-40a^6b^3x^3+48a^7b^2x^2-64a^8bx+128a^9}{230945b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/19*(b*x+a)^(19/2)-4/17*a*(b*x+a)^(17/2)+2/5*a^2*(b*x+a)^(15/2)-4/1
3*a^3*(b*x+a)^(13/2)+1/11*a^4*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{19}{2}}}{19\,b^5}-\frac{8\,(bx+a)^{\frac{17}{2}}a}{17\,b^5}+\frac{4\,(bx+a)^{\frac{15}{2}}a^2}{5\,b^5}-\frac{8\,(bx+a)^{\frac{13}{2}}a^3}{13\,b^5}+\frac{2\,(bx+a)^{\frac{11}{2}}a^4}{11\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/19*(b*x + a)^(19/2)/b^5 - 8/17*(b*x + a)^(17/2)*a/b^5 + 4/5*(b*x + a)^(15
/2)*a^2/b^5 - 8/13*(b*x + a)^(13/2)*a^3/b^5 + 2/11*(b*x + a)^(11/2)*a^4/b^5
```

Fricas [A]
time = 0.43, size = 108, normalized size = 1.19

$$\frac{2\,(12155\,b^9x^9+55055\,ab^8x^8+95238\,a^2b^7x^7+75086\,a^3b^6x^6+23063\,a^4b^5x^5+35\,a^5b^4x^4-40\,a^6b^3x^3+48\,a^7b^2x^2-64\,a^8bx+128\,a^9)\sqrt{bx+a}}{230945\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/230945*(12155*b^9*x^9 + 55055*a*b^8*x^8 + 95238*a^2*b^7*x^7 + 75086*a^3*b
^6*x^6 + 23063*a^4*b^5*x^5 + 35*a^5*b^4*x^4 - 40*a^6*b^3*x^3 + 48*a^7*b^2*x
^2 - 64*a^8*b*x + 128*a^9)*sqrt(b*x + a)/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.
time = 1.03, size = 212, normalized size = 2.33

$$\begin{cases} \frac{256a^9\sqrt{a+bx}}{230945b^5} - \frac{128a^8x\sqrt{a+bx}}{230945b^4} + \frac{96a^7x^2\sqrt{a+bx}}{230945b^3} - \frac{16a^6x^3\sqrt{a+bx}}{46189b^2} + \frac{14a^5x^4\sqrt{a+bx}}{46189b} + \frac{46126a^4x^5\sqrt{a+bx}}{230945} + \frac{13652a^3bx^6\sqrt{a+bx}}{20995} + \frac{1332a^2b^2x^7\sqrt{a+bx}}{1615} + \frac{154ab^3x^8\sqrt{a+bx}}{323} + \frac{2b^4x^9\sqrt{a+bx}}{19} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((256*a**9*sqrt(a + b*x)/(230945*b**5) - 128*a**8*x*sqrt(a + b*x)/
(230945*b**4) + 96*a**7*x**2*sqrt(a + b*x)/(230945*b**3) - 16*a**6*x**3*sqr
t(a + b*x)/(46189*b**2) + 14*a**5*x**4*sqrt(a + b*x)/(46189*b) + 46126*a**4
*x**5*sqrt(a + b*x)/230945 + 13652*a**3*b*x**6*sqrt(a + b*x)/20995 + 1332*a
**2*b**2*x**7*sqrt(a + b*x)/1615 + 154*a*b**3*x**8*sqrt(a + b*x)/323 + 2*b*
*4*x**9*sqrt(a + b*x)/19, Ne(b, 0)), (a**(9/2)*x**5/5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(71) = 142$.
time = 1.34, size = 565, normalized size = 6.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/14549535*(46189*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^5/b^4 + 1
04975*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)
*a^5)*a^4/b^4 + 48450*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 500
5*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3/b^4 + 22610*(429*
(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 2
5025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2
)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^2/b^4 + 665*(
6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*
a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*
x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7
+ 109395*sqrt(b*x + a)*a^8)*a/b^4 + 63*(12155*(b*x + a)^(19/2) - 122265*(b*
x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^
```

```
3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b
*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^
8 - 230945*sqrt(b*x + a)*a^9)/b^4)/b
```

Mupad [B]
time = 0.02, size = 71, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^5}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{11/2}}{11\,b^5}-\frac{8\,a^3\,{\left(a+b\,x\right)}^{13/2}}{13\,b^5}+\frac{4\,a^2\,{\left(a+b\,x\right)}^{15/2}}{5\,b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{17/2}}{17\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(19/2))/(19*b^5) + (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*
(a + b*x)^(13/2))/(13*b^5) + (4*a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b
*x)^(17/2))/(17*b^5)
```

3.313 $\int x^3(a+bx)^{9/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}$$

```
[Out] -2/11*a^3*(b*x+a)^(11/2)/b^4+6/13*a^2*(b*x+a)^(13/2)/b^4-2/5*a*(b*x+a)^(15/
2)/b^4+2/17*(b*x+a)^(17/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (2
*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{9/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{11/2}\left(-16a^3+88a^2bx-286ab^2x^2+715b^3x^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-16*a^3 + 88*a^2*b*x - 286*a*b^2*x^2 + 715*b^3*x^3))/(
12155*b^4)
```

Maple [A]
time = 0.09, size = 50, normalized size = 0.69

method	result
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}\left(-715b^3x^3+286a\,b^2x^2-88a^2bx+16a^3\right)}{12155b^4}$
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{17}{2}}}{17}-\frac{2a(bx+a)^{\frac{15}{2}}}{5}+\frac{6a^2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^3(bx+a)^{\frac{11}{2}}}{11}}{b^4}$
default	$\frac{\frac{2(bx+a)^{\frac{17}{2}}}{17}-\frac{2a(bx+a)^{\frac{15}{2}}}{5}+\frac{6a^2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a^3(bx+a)^{\frac{11}{2}}}{11}}{b^4}$
trager	$-\frac{2\left(-715b^8x^8-3289a\,b^7x^7-5808a^2x^6b^6-4714a^3x^5b^5-1515a^4x^4b^4-5a^5x^3b^3+6a^6x^2b^2-8a^7xb+16a^8\right)\sqrt{bx+a}}{12155b^4}$
risch	$-\frac{2\left(-715b^8x^8-3289a\,b^7x^7-5808a^2x^6b^6-4714a^3x^5b^5-1515a^4x^4b^4-5a^5x^3b^3+6a^6x^2b^2-8a^7xb+16a^8\right)\sqrt{bx+a}}{12155b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/17*(b*x+a)^(17/2)-1/5*a*(b*x+a)^(15/2)+3/13*a^2*(b*x+a)^(13/2)-1/1
1*a^3*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{17}{2}}}{17\,b^4}-\frac{2\,(bx+a)^{\frac{15}{2}}a}{5\,b^4}+\frac{6\,(bx+a)^{\frac{13}{2}}a^2}{13\,b^4}-\frac{2\,(bx+a)^{\frac{11}{2}}a^3}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/17*(b*x + a)^(17/2)/b^4 - 2/5*(b*x + a)^(15/2)*a/b^4 + 6/13*(b*x + a)^(13
/2)*a^2/b^4 - 2/11*(b*x + a)^(11/2)*a^3/b^4
```

Fricas [A]
time = 0.42, size = 97, normalized size = 1.35

$$\frac{2\,(715\,b^8x^8+3289\,ab^7x^7+5808\,a^2b^6x^6+4714\,a^3b^5x^5+1515\,a^4b^4x^4+5\,a^5b^3x^3-6\,a^6b^2x^2+8\,a^7bx-16\,a^8)\sqrt{bx+a}}{12155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/12155*(715*b^8*x^8 + 3289*a*b^7*x^7 + 5808*a^2*b^6*x^6 + 4714*a^3*b^5*x^5
 + 1515*a^4*b^4*x^4 + 5*a^5*b^3*x^3 - 6*a^6*b^2*x^2 + 8*a^7*b*x - 16*a^8)*s
qrt(b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(68) = 136$.
time = 0.90, size = 190, normalized size = 2.64

$$\begin{cases} -\frac{32a^8\sqrt{a+bx}}{12155b^4} + \frac{16a^7x\sqrt{a+bx}}{12155b^3} - \frac{12a^6x^2\sqrt{a+bx}}{12155b^2} + \frac{2a^5x^3\sqrt{a+bx}}{2431b} + \frac{606a^4x^4\sqrt{a+bx}}{2431} + \frac{9428a^3bx^5\sqrt{a+bx}}{12155} + \frac{1056a^2b^2x^6\sqrt{a+bx}}{1105} + \frac{46ab^3x^7\sqrt{a+bx}}{85} + \frac{2b^4x^8\sqrt{a+bx}}{17} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-32*a**8*sqrt(a + b*x)/(12155*b**4) + 16*a**7*x*sqrt(a + b*x)/(1
2155*b**3) - 12*a**6*x**2*sqrt(a + b*x)/(12155*b**2) + 2*a**5*x**3*sqrt(a +
 b*x)/(2431*b) + 606*a**4*x**4*sqrt(a + b*x)/2431 + 9428*a**3*b*x**5*sqrt(a
 + b*x)/12155 + 1056*a**2*b**2*x**6*sqrt(a + b*x)/1105 + 46*a*b**3*x**7*sqr
t(a + b*x)/85 + 2*b**4*x**8*sqrt(a + b*x)/17, Ne(b, 0)), (a**(9/2)*x**4/4,
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(56) = 112$.
time = 0.86, size = 493, normalized size = 6.85

$$\frac{2\left(\frac{21879\left(5(bx+a)^{\frac{7}{2}} - 21(bx+a)^{\frac{5}{2}}a + 35(bx+a)^{\frac{3}{2}}a^2 - 35\sqrt{bx+a}\,a^3\right)a^5}{b^3} + \frac{12155\left(35(bx+a)^{\frac{9}{2}} - 180(bx+a)^{\frac{7}{2}}a + 378(bx+a)^{\frac{5}{2}}a^2 - 420(bx+a)^{\frac{3}{2}}a^3 + 315\sqrt{bx+a}\,a^4\right)a^4}{b^3} + \frac{11050\left(63(bx+a)^{\frac{11}{2}} - 385(bx+a)^{\frac{9}{2}}a + 990(bx+a)^{\frac{7}{2}}a^2 - 1386(bx+a)^{\frac{5}{2}}a^3 + 1155(bx+a)^{\frac{3}{2}}a^4 - 693\sqrt{bx+a}\,a^5\right)a^3}{b^3} + \frac{2550\left(231(bx+a)^{\frac{13}{2}} - 1638(bx+a)^{\frac{11}{2}}a + 5005(bx+a)^{\frac{9}{2}}a^2 - 8580(bx+a)^{\frac{7}{2}}a^3 + 9009(bx+a)^{\frac{5}{2}}a^4 - 6006(bx+a)^{\frac{3}{2}}a^5 + 3003\sqrt{bx+a}\,a^6\right)a^2}{b^3} + \frac{595\left(429(bx+a)^{\frac{15}{2}} - 3465(bx+a)^{\frac{13}{2}}a + 12285(bx+a)^{\frac{11}{2}}a^2 - 25025(bx+a)^{\frac{9}{2}}a^3 + 32175(bx+a)^{\frac{7}{2}}a^4 - 27027(bx+a)^{\frac{5}{2}}a^5 + 15015(bx+a)^{\frac{3}{2}}a^6 - 6435\sqrt{bx+a}\,a^7\right)a}{b^3} + \frac{7\left(6435(bx+a)^{\frac{17}{2}} - 58344(bx+a)^{\frac{15}{2}}a + 235620(bx+a)^{\frac{13}{2}}a^2 - 556920(bx+a)^{\frac{11}{2}}a^3 + 850850(bx+a)^{\frac{9}{2}}a^4 - 875160(bx+a)^{\frac{7}{2}}a^5 + 612612(bx+a)^{\frac{5}{2}}a^6 - 291720(bx+a)^{\frac{3}{2}}a^7 + 109395\sqrt{bx+a}\,a^8\right)}{b^3}\right)}{765765\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/765765*(21879*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^5/b^3 + 12155*(35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^4/b^3 + 11050*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/
2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^
(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3/b^3 + 2550*(231*(b*x + a)^(13/2) - 1
638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^
3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a
)*a^6)*a^2/b^3 + 595*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a/b^3 + 7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 23
5620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^
(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 29172
0*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)/b^3)/b
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{17/2}}{17\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}+\frac{6\,a^2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a\,{\left(a+b\,x\right)}^{15/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(17/2))/(17*b^4) - (2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*
(a + b*x)^(13/2))/(13*b^4) - (2*a*(a + b*x)^(15/2))/(5*b^4)
```

3.314 $\int x^2(a+bx)^{9/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}$$

```
[Out] 2/11*a^2*(b*x+a)^(11/2)/b^3-4/13*a*(b*x+a)^(13/2)/b^3+2/15*(b*x+a)^(15/2)/b
^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(a
 + b*x)^(15/2))/(15*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{9/2}\,dx&=\int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{11/2}\left(8a^2-44abx+143b^2x^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x)^(9/2),x]

[Out] (2*(a + b*x)^(11/2)*(8*a^2 - 44*a*b*x + 143*b^2*x^2))/(2145*b^3)

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{2(bx+a)^{\frac{11}{2}}\left(143x^2b^2-44abx+8a^2\right)}{2145b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{15}{2}}}{15}-\frac{4a(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^2(bx+a)^{\frac{11}{2}}}{11}}{b^3}$	38
default	$\frac{\frac{2(bx+a)^{\frac{15}{2}}}{15}-\frac{4a(bx+a)^{\frac{13}{2}}}{13}+\frac{2a^2(bx+a)^{\frac{11}{2}}}{11}}{b^3}$	38
trager	$\frac{2\left(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+8a^7\right)\sqrt{bx+a}}{2145b^3}$	87
risch	$\frac{2\left(143b^7x^7+671a\,b^6x^6+1218a^2b^5x^5+1030a^3b^4x^4+355a^4b^3x^3+3a^5b^2x^2-4a^6bx+8a^7\right)\sqrt{bx+a}}{2145b^3}$	87

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] 2/b^3*(1/15*(b*x+a)^(15/2)-2/13*a*(b*x+a)^(13/2)+1/11*a^2*(b*x+a)^(11/2))

Maxima [A]
time = 0.27, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{15}{2}}}{15\,b^3}-\frac{4\,(bx+a)^{\frac{13}{2}}a}{13\,b^3}+\frac{2\,(bx+a)^{\frac{11}{2}}a^2}{11\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="maxima")

[Out] 2/15*(b*x + a)^(15/2)/b^3 - 4/13*(b*x + a)^(13/2)*a/b^3 + 2/11*(b*x + a)^(1
1/2)*a^2/b^3

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(41) = 82$.
time = 0.44, size = 86, normalized size = 1.62

$$\frac{2\,(143\,b^7x^7+671\,ab^6x^6+1218\,a^2b^5x^5+1030\,a^3b^4x^4+355\,a^4b^3x^3+3\,a^5b^2x^2-4\,a^6bx+8\,a^7)\sqrt{bx+a}}{2145\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="fricas")

```
[Out] 2/2145*(143*b^7*x^7 + 671*a*b^6*x^6 + 1218*a^2*b^5*x^5 + 1030*a^3*b^4*x^4 +
 355*a^4*b^3*x^3 + 3*a^5*b^2*x^2 - 4*a^6*b*x + 8*a^7)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(49) = 98$.
time = 0.81, size = 168, normalized size = 3.17

$$\begin{cases} \frac{16a^7\sqrt{a+bx}}{2145b^3} - \frac{8a^6x\sqrt{a+bx}}{2145b^2} + \frac{2a^5x^2\sqrt{a+bx}}{715b} + \frac{142a^4x^3\sqrt{a+bx}}{429} + \frac{412a^3bx^4\sqrt{a+bx}}{429} + \frac{812a^2b^2x^5\sqrt{a+bx}}{715} + \frac{122ab^3x^6\sqrt{a+bx}}{195} + \frac{2b^4x^7\sqrt{a+bx}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((16*a**7*sqrt(a + b*x)/(2145*b**3) - 8*a**6*x*sqrt(a + b*x)/(2145
*b**2) + 2*a**5*x**2*sqrt(a + b*x)/(715*b) + 142*a**4*x**3*sqrt(a + b*x)/42
9 + 412*a**3*b*x**4*sqrt(a + b*x)/429 + 812*a**2*b**2*x**5*sqrt(a + b*x)/71
5 + 122*a*b**3*x**6*sqrt(a + b*x)/195 + 2*b**4*x**7*sqrt(a + b*x)/15, Ne(b,
 0)), (a**(9/2)*x**3/3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(41) = 82$.
time = 1.22, size = 421, normalized size = 7.94

$$\frac{2\left(\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^5}{b^2}+\frac{6435\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^4}{b^2}+\frac{1430\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^3}{b^2}+\frac{650\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a^2}{b^2}+\frac{75\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)a}{b^2}+\frac{7\left(429(bx+a)^{\frac{15}{2}}-3465(bx+a)^{\frac{13}{2}}a+12285(bx+a)^{\frac{11}{2}}a^2-25025(bx+a)^{\frac{9}{2}}a^3+32175(bx+a)^{\frac{7}{2}}a^4-27027(bx+a)^{\frac{5}{2}}a^5+15015(bx+a)^{\frac{3}{2}}a^6-6435\sqrt{bx+a}\,a^7\right)}{b^2}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*
a^2)*a^5/b^2 + 6435*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^4/b^2 + 1430*(35*(b*x + a)^(9/2) - 18
0*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 3
15*sqrt(b*x + a)*a^4)*a^3/b^2 + 650*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9
/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)
^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2/b^2 + 75*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*a/b^2 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*
x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 -
 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)
*a^7)/b^2)/b
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.68

$$\frac{\frac{2\,{\left(a+b\,x\right)}^{15/2}}{15}-\frac{4\,a\,{\left(a+b\,x\right)}^{13/2}}{13}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{11/2}}{11}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(9/2),x)
```

```
[Out] ((2*(a + b*x)^(15/2))/15 - (4*a*(a + b*x)^(13/2))/13 + (2*a^2*(a + b*x)^(11
/2))/11)/b^3
```

3.315 $\int x(a+bx)^{9/2}\,dx$

Optimal. Leaf size=34

$$-\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}$$

```
[Out] -2/11*a*(b*x+a)^(11/2)/b^2+2/13*(b*x+a)^(13/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(9/2),x]
[Out] (-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{9/2}\,dx&=\int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}(-2a+11bx)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2*a + 11*b*x))/(143*b^2)
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{11}{2}}(-11bx+2a)}{143b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a(bx+a)^{\frac{11}{2}}}{11}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{13}{2}}}{13}-\frac{2a(bx+a)^{\frac{11}{2}}}{11}}{b^2}$	26
trager	$-\frac{2(-11x^6b^6-53a\,x^5b^5-100a^2x^4b^4-90a^3b^3x^3-35a^4x^2b^2-a^5xb+2a^6)\sqrt{bx+a}}{143b^2}$	76
risch	$-\frac{2(-11x^6b^6-53a\,x^5b^5-100a^2x^4b^4-90a^3b^3x^3-35a^4x^2b^2-a^5xb+2a^6)\sqrt{bx+a}}{143b^2}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/13*(b*x+a)^(13/2)-1/11*a*(b*x+a)^(11/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^2}-\frac{2\,(bx+a)^{\frac{11}{2}}a}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^2 - 2/11*(b*x + a)^(11/2)*a/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.
time = 0.48, size = 74, normalized size = 2.18

$$\frac{2\,(11\,b^6x^6+53\,ab^5x^5+100\,a^2b^4x^4+90\,a^3b^3x^3+35\,a^4b^2x^2+a^5bx-2\,a^6)\sqrt{bx+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*b^6*x^6 + 53*a*b^5*x^5 + 100*a^2*b^4*x^4 + 90*a^3*b^3*x^3 + 35*a^
4*b^2*x^2 + a^5*b*x - 2*a^6)*sqrt(b*x + a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(31) = 62$.

time = 0.71, size = 146, normalized size = 4.29

$$\begin{cases} -\frac{4a^6\sqrt{a+bx}}{143b^2} + \frac{2a^5x\sqrt{a+bx}}{143b} + \frac{70a^4x^2\sqrt{a+bx}}{143} + \frac{180a^3bx^3\sqrt{a+bx}}{143} + \frac{200a^2b^2x^4\sqrt{a+bx}}{143} + \frac{106ab^3x^5\sqrt{a+bx}}{143} + \frac{2b^4x^6\sqrt{a+bx}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4*a**6*sqrt(a + b*x)/(143*b**2) + 2*a**5*x*sqrt(a + b*x)/(143*b
) + 70*a**4*x**2*sqrt(a + b*x)/143 + 180*a**3*b*x**3*sqrt(a + b*x)/143 + 20
0*a**2*b**2*x**4*sqrt(a + b*x)/143 + 106*a*b**3*x**5*sqrt(a + b*x)/143 + 2*
b**4*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(9/2)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(26) = 52$.

time = 1.45, size = 347, normalized size = 10.21

$$\frac{2\left(\frac{3003\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^5}{b}+\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^4}{b}+\frac{2574\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^3}{b}+\frac{286\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^2}{b}+\frac{65\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a}{b}+\frac{3\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)}{b}\right)}{9009\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/9009*(3003*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^5/b + 3003*(3*(b*x + a
)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^4/b + 2574*(5*(b*x
 + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x +
 a)*a^3)*a^3/b + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x
 + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b +
65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 -
 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^
5)*a/b + 3*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)
^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*
x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b)/b
```

Mupad [B]

time = 0.03, size = 25, normalized size = 0.74

$$-\frac{26\,a\,(a+b\,x)^{11/2}-22\,(a+b\,x)^{13/2}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(9/2),x)
```

```
[Out] -(26*a*(a + b*x)^(11/2) - 22*(a + b*x)^(13/2))/(143*b^2)
```

3.316 $\int (a+bx)^{9/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{11/2}}{11b}$$

```
[Out] 2/11*(b*x+a)^(11/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{9/2}\,dx = \frac{2(a+bx)^{11/2}}{11b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
derivativedivides	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
default	$\frac{2(bx+a)^{\frac{11}{2}}}{11b}$	13
trager	$\frac{2(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)\sqrt{bx+a}}{11b}$	62
risch	$\frac{2(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)\sqrt{bx+a}}{11b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/11*(b*x+a)^(11/2)/b
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="maxima")
[Out] 2/11*(b*x + a)^(11/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.
time = 0.41, size = 61, normalized size = 3.81

$$\frac{2\,(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5)\sqrt{bx+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="fricas")
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2),x)
```

```
[Out] 2*(a + b*x)**(11/2)/(11*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(12) = 24.
time = 1.57, size = 229, normalized size = 14.31

$$\frac{2\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4+1155\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^4+462\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^3+198\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2+11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/693*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 + 1155*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a^4 + 462*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)
*a + 15*sqrt(b*x + a)*a^2)*a^3 + 198*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2 + 11*(35*(b*x + a)
^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3
/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

3.317 $\int \frac{(a+bx)^{9/2}}{x}\,dx$

Optimal. Leaf size=97

$$2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 214}

$$-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x,x]
```

```
[Out] 2*a^4*Sqrt[a + b*x] + (2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5
 + (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 - 2*a^(9/2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \frac{2}{9}(a+bx)^{9/2} + a\int \frac{(a+bx)^{7/2}}{x}\,dx$$
$$= \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^2\int \frac{(a+bx)^{5/2}}{x}\,dx$$
$$= \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^3\int \frac{(a+bx)^{3/2}}{x}\,dx$$
$$= \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^4\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^5\int$$
$$= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + \frac{(2a^5)}{}$$
$$= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} - 2a^{9/2}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.80

$$\frac{2}{315}\sqrt{a+bx}\left(563a^4+506a^3bx+408a^2b^2x^2+185ab^3x^3+35b^4x^4\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(563*a^4 + 506*a^3*b*x + 408*a^2*b^2*x^2 + 185*a*b^3*x^3 +
 35*b^4*x^4))/315 - 2*a^(9/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.09, size = 74, normalized size = 0.76

method	result
derivativedivides	$\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}+\frac{2a(bx+a)^{\frac{7}{2}}}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}-2a^{\frac{9}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^4\sqrt{bx+a}$
default	$\frac{2a^3(bx+a)^{\frac{3}{2}}}{3}+\frac{2a^2(bx+a)^{\frac{5}{2}}}{5}+\frac{2a(bx+a)^{\frac{7}{2}}}{7}+\frac{2(bx+a)^{\frac{9}{2}}}{9}-2a^{\frac{9}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2a^4\sqrt{bx+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

Maxima [A]
time = 0.48, size = 88, normalized size = 0.91

$$a^{\frac{9}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{9}(bx+a)^{\frac{9}{2}}+\frac{2}{7}(bx+a)^{\frac{7}{2}}a+\frac{2}{5}(bx+a)^{\frac{5}{2}}a^2+\frac{2}{3}(bx+a)^{\frac{3}{2}}a^3+2\sqrt{bx+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] a^(9/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/9*(b*x
 + a)^(9/2) + 2/7*(b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x +
a)^(3/2)*a^3 + 2*sqrt(b*x + a)*a^4
```

Fricas [A]
time = 0.47, size = 158, normalized size = 1.63

$$\left[a^{\frac{9}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a},\,2\sqrt{-a}\,a^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(9/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/315*(35*b^4*x^4 +
 185*a*b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a), 2*
sqrt(-a)*a^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/315*(35*b^4*x^4 + 185*a*b
^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a)]
```

Sympy [A]
time = 10.50, size = 148, normalized size = 1.53

$$\frac{1126a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{315}+a^{\frac{9}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{1012a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{315}+\frac{272a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105}+\frac{74a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{63}+\frac{2\sqrt{a}\,b^4x^4\sqrt{1+\frac{bx}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x,x)
```

```
[Out] 1126*a**(9/2)*sqrt(1 + b*x/a)/315 + a**(9/2)*log(b*x/a) - 2*a**(9/2)*log(sq
rt(1 + b*x/a) + 1) + 1012*a**(7/2)*b*x*sqrt(1 + b*x/a)/315 + 272*a**(5/2)*b
**2*x**2*sqrt(1 + b*x/a)/105 + 74*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/63 + 2
*sqrt(a)*b**4*x**4*sqrt(1 + b*x/a)/9
```

Giac [A]
time = 1.27, size = 80, normalized size = 0.82

$$\frac{2a^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{9}(bx+a)^{\frac{9}{2}}+\frac{2}{7}(bx+a)^{\frac{7}{2}}a+\frac{2}{5}(bx+a)^{\frac{5}{2}}a^2+\frac{2}{3}(bx+a)^{\frac{3}{2}}a^3+2\sqrt{bx+a}\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x + a)^(9/2) + 2/7*(
b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x + a)^(3/2)*a^3 + 2*sq
rt(b*x + a)*a^4
```

Mupad [B]
time = 0.04, size = 76, normalized size = 0.78

$$\frac{2\,a\,{\left(a+b\,x\right)}^{7/2}}{7}+\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9}+2\,a^4\,\sqrt{a+b\,x}+\frac{2\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5}+a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 + 2*a^4*(a + b*x)^(1/2) + (
2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5 + a^(9/2)*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*2i
```

3.318 $\int \frac{(a+bx)^{9/2}}{x^2} dx$

Optimal. Leaf size=98

$$9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 3*a^2*b*(b*x+a)^(3/2)+9/5*a*b*(b*x+a)^(5/2)+9/7*b*(b*x+a)^(7/2)-(b*x+a)^(9/
2)/x-9*a^(7/2)*b*arctanh((b*x+a)^(1/2)/a^(1/2))+9*a^3*b*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] 9*a^3*b*Sqrt[a + b*x] + 3*a^2*b*(a + b*x)^(3/2) + (9*a*b*(a + b*x)^(5/2))/5
 + (9*b*(a + b*x)^(7/2))/7 - (a + b*x)^(9/2)/x - 9*a^(7/2)*b*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^2}\,dx &= -\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9b)\int \frac{(a+bx)^{7/2}}{x}\,dx\\ &= \frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9ab)\int \frac{(a+bx)^{5/2}}{x}\,dx\\ &= \frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9a^2b)\int \frac{(a+bx)^{3/2}}{x}\,dx\\ &= 3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9a^3b)\int \frac{\sqrt{a+bx}}{x}\\ &= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+\frac{1}{2}(9a\\ &= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}+(9a^4\\ &= 9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}+\frac{9}{5}ab(a+bx)^{5/2}+\frac{9}{7}b(a+bx)^{7/2}-\frac{(a+bx)^{9/2}}{x}-9a^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 82, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,(-35a^4+388a^3bx+156a^2b^2x^2+58ab^3x^3+10b^4x^4)}{35x}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-35*a^4 + 388*a^3*b*x + 156*a^2*b^2*x^2 + 58*a*b^3*x^3 + 10
*b^4*x^4))/(35*x) - 9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.87

method	result
risch	$-\frac{a^4\sqrt{bx+a}}{x}+\frac{b\left(\frac{4(bx+a)^{\frac{7}{2}}}{7}+\frac{8a(bx+a)^{\frac{5}{2}}}{5}+4a^2(bx+a)^{\frac{3}{2}}+16a^3\sqrt{bx+a}-18a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{2}$
derivativedivides	$2b\left(\frac{(bx+a)^{\frac{7}{2}}}{7}+\frac{2a(bx+a)^{\frac{5}{2}}}{5}+a^2(bx+a)^{\frac{3}{2}}+4a^3\sqrt{bx+a}-a^4\left(\frac{\sqrt{bx+a}}{2bx}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{2\sqrt{a}}\right.\right.$
default	$2b\left(\frac{(bx+a)^{\frac{7}{2}}}{7}+\frac{2a(bx+a)^{\frac{5}{2}}}{5}+a^2(bx+a)^{\frac{3}{2}}+4a^3\sqrt{bx+a}-a^4\left(\frac{\sqrt{bx+a}}{2bx}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{2\sqrt{a}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/7*(b*x+a)^(7/2)+2/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)+4*a^3*(b*x+a)^
(1/2)-a^4*(1/2*(b*x+a)^(1/2)/b/x+9/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
))
```

Maxima [A]
time = 0.49, size = 97, normalized size = 0.99

$$\frac{9}{2}\,a^{\frac{7}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{7}\,(bx+a)^{\frac{7}{2}}b+\frac{4}{5}\,(bx+a)^{\frac{5}{2}}ab+2\,(bx+a)^{\frac{3}{2}}a^2b+8\,\sqrt{bx+a}\,a^3b-\frac{\sqrt{bx+a}\,a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/2*a^(7/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
7*(b*x + a)^(7/2)*b + 4/5*(b*x + a)^(5/2)*a*b + 2*(b*x + a)^(3/2)*a^2*b + 8
*sqrt(b*x + a)*a^3*b - sqrt(b*x + a)*a^4/x
```

Fricas [A]
time = 0.50, size = 172, normalized size = 1.76

$$\left[\frac{315\,a^{\frac{7}{2}}bx\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4)\sqrt{bx+a}}{70\,x},\ \frac{315\,\sqrt{-a}\,a^3bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4)\sqrt{bx+a}}{35\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/70*(315*a^(7/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(10
*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x
+ a))/x, 1/35*(315*sqrt(-a)*a^3*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (10*
```

```
b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x +
 a))/x]
```

Sympy [A]
time = 10.59, size = 150, normalized size = 1.53

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{388a^{\frac{7}{2}}b\sqrt{1+\frac{bx}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx}{a}\right)}{2}-9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{156a^{\frac{5}{2}}b^2x\sqrt{1+\frac{bx}{a}}}{35}+\frac{58a^{\frac{3}{2}}b^3x^2\sqrt{1+\frac{bx}{a}}}{35}+\frac{2\sqrt{a}\,b^4x^3\sqrt{1+\frac{bx}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x/a)/x + 388*a**(7/2)*b*sqrt(1 + b*x/a)/35 + 9*a**(7/2
)*b*log(b*x/a)/2 - 9*a**(7/2)*b*log(sqrt(1 + b*x/a) + 1) + 156*a**(5/2)*b**
2*x**sqrt(1 + b*x/a)/35 + 58*a**(3/2)*b**3*x**2*sqrt(1 + b*x/a)/35 + 2*sqrt(
a)*b**4*x**3*sqrt(1 + b*x/a)/7
```

Giac [A]
time = 0.84, size = 104, normalized size = 1.06

$$\frac{\frac{315\,a^4b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,(bx+a)^{\frac{7}{2}}b^2+28\,(bx+a)^{\frac{5}{2}}ab^2+70\,(bx+a)^{\frac{3}{2}}a^2b^2+280\,\sqrt{bx+a}\,a^3b^2-\frac{35\,\sqrt{bx+a}\,a^4b}{x}}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/35*(315*a^4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 10*(b*x + a)^(7
/2)*b^2 + 28*(b*x + a)^(5/2)*a*b^2 + 70*(b*x + a)^(3/2)*a^2*b^2 + 280*sqrt(
b*x + a)*a^3*b^2 - 35*sqrt(b*x + a)*a^4*b/x)/b
```

Mupad [B]
time = 0.04, size = 84, normalized size = 0.86

$$\frac{2\,b\,(a+b\,x)^{7/2}}{7}-\frac{a^4\,\sqrt{a+b\,x}}{x}+\frac{4\,a\,b\,(a+b\,x)^{5/2}}{5}+8\,a^3\,b\,\sqrt{a+b\,x}+2\,a^2\,b\,(a+b\,x)^{3/2}+a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(7/2))/7 - (a^4*(a + b*x)^(1/2))/x + a^(7/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*9i + (4*a*b*(a + b*x)^(5/2))/5 + 8*a^3*b*(a + b*x)^(1
/2) + 2*a^2*b*(a + b*x)^(3/2)
```

3.319 $\int \frac{(a+bx)^{9/2}}{x^3} dx$

Optimal. Leaf size=114

$$\frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.$$

```
[Out] 21/4*a*b^2*(b*x+a)^(3/2)+63/20*b^2*(b*x+a)^(5/2)-9/4*b*(b*x+a)^(7/2)/x-1/2*
(b*x+a)^(9/2)/x^2-63/4*a^(5/2)*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))+63/4*a^2*
b^2*(b*x+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x])/4 + (21*a*b^2*(a + b*x)^(3/2))/4 + (63*b^2*(a +
b*x)^(5/2))/20 - (9*b*(a + b*x)^(7/2))/(4*x) - (a + b*x)^(9/2)/(2*x^2) - (6
3*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^3}\,dx &= -\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{4}(9b)\int \frac{(a+bx)^{7/2}}{x^2}\,dx\\ &= -\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x}\,dx\\ &= \frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63ab^2\right)\int \frac{(a+bx)^{3/2}}{x}\,dx\\ &= \frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\frac{1}{8}\left(63a^2b^2\right)\int \frac{\sqrt{a+}}{x}\\ &= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\\ &= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}+\\ &= \frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{21}{4}ab^2(a+bx)^{3/2}+\frac{63}{20}b^2(a+bx)^{5/2}-\frac{9b(a+bx)^{7/2}}{4x}-\frac{(a+bx)^{9/2}}{2x^2}-\end{aligned}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.75

$$\frac{\sqrt{a+bx}\,\left(-10a^4-85a^3bx+288a^2b^2x^2+56ab^3x^3+8b^4x^4\right)}{20x^2}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x]*(-10*a^4 - 85*a^3*b*x + 288*a^2*b^2*x^2 + 56*a*b^3*x^3 + 8*b
^4*x^4))/(20*x^2) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Maple [A]
time = 0.10, size = 87, normalized size = 0.76

method	result
risch	$-\frac{a^3\sqrt{bx+a}\,(17bx+2a)}{4x^2}+\frac{b^2\left(\frac{16(bx+a)^{\frac{5}{2}}}{5}+16a(bx+a)^{\frac{3}{2}}+96a^2\sqrt{bx+a}-126a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{8}$
derivativedivides	$2b^2\left(\frac{(bx+a)^{\frac{5}{2}}}{5}+a(bx+a)^{\frac{3}{2}}+6a^2\sqrt{bx+a}-a^3\left(\frac{\frac{17(bx+a)^{\frac{3}{2}}}{8}-\frac{15a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{63\operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.}{8\sqrt{a}}\right.\right.$
default	$2b^2\left(\frac{(bx+a)^{\frac{5}{2}}}{5}+a(bx+a)^{\frac{3}{2}}+6a^2\sqrt{bx+a}-a^3\left(\frac{\frac{17(bx+a)^{\frac{3}{2}}}{8}-\frac{15a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{63\operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.}{8\sqrt{a}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/5*(b*x+a)^(5/2)+a*(b*x+a)^(3/2)+6*a^2*(b*x+a)^(1/2)-a^3*((17/8*(b*
x+a)^(3/2)-15/8*a*(b*x+a)^(1/2))/b^2/x^2+63/8*arctanh((b*x+a)^(1/2)/a^(1/2)
)/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 131, normalized size = 1.15

$$\frac{63}{8}\,a^{\frac{5}{2}}b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}b^2+2\,(bx+a)^{\frac{3}{2}}ab^2+12\,\sqrt{bx+a}\,a^2b^2-\frac{17\,(bx+a)^{\frac{3}{2}}a^3b^2-15\,\sqrt{bx+a}\,a^4b^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] 63/8*a^(5/2)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2/5*(b*x + a)^(5/2)*b^2 + 2*(b*x + a)^(3/2)*a*b^2 + 12*sqrt(b*x + a)*a^2*b
^2 - 1/4*(17*(b*x + a)^(3/2)*a^3*b^2 - 15*sqrt(b*x + a)*a^4*b^2)/((b*x + a)
^2 - 2*(b*x + a)*a + a^2)
```

Fricas [A]
time = 0.45, size = 180, normalized size = 1.58

$$\left[\frac{315\,a^{\frac{5}{2}}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4)\sqrt{bx+a}}{40\,x^2},\ \frac{315\,\sqrt{-a}\,a^2b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4)\sqrt{bx+a}}{20\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*a^(5/2)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt(b*
```

```
x + a))/x^2, 1/20*(315*sqrt(-a)*a^2*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt
(b*x + a))/x^2]
```

Sympy [A]
time = 9.96, size = 184, normalized size = 1.61

$$-\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{a^5}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{19a^4\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{203a^3b^{\frac{3}{2}}}{20\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{86a^2b^{\frac{5}{2}}\sqrt{x}}{5\sqrt{\frac{a}{bx}+1}}+\frac{16ab^{\frac{7}{2}}x^{\frac{3}{2}}}{5\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{5}{2}}}{5\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**3,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**5/(2*sqrt(b)*x**(
5/2)*sqrt(a/(b*x) + 1)) - 19*a**4*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) +
203*a**3*b**(3/2)/(20*sqrt(x)*sqrt(a/(b*x) + 1)) + 86*a**2*b**(5/2)*sqrt(x)
/(5*sqrt(a/(b*x) + 1)) + 16*a*b**(7/2)*x**(3/2)/(5*sqrt(a/(b*x) + 1)) + 2*b
**(9/2)*x**(5/2)/(5*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.66, size = 112, normalized size = 0.98

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\,(bx+a)^{\frac{5}{2}}b^3+40\,(bx+a)^{\frac{3}{2}}ab^3+240\,\sqrt{bx+a}\,a^2b^3-\frac{5\left(17\,(bx+a)^{\frac{3}{2}}a^3b^3-15\,\sqrt{bx+a}\,a^4b^3\right)}{b^2x^2}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*(315*a^3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*(b*x + a)^(5/
2)*b^3 + 40*(b*x + a)^(3/2)*a*b^3 + 240*sqrt(b*x + a)*a^2*b^3 - 5*(17*(b*x
+ a)^(3/2)*a^3*b^3 - 15*sqrt(b*x + a)*a^4*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.05, size = 117, normalized size = 1.03

$$\frac{2\,b^2\,{(a+b\,x)}^{5/2}}{5}+\frac{\frac{15\,a^4\,b^2\,\sqrt{a+b\,x}}{4}-\frac{17\,a^3\,b^2\,{(a+b\,x)}^{3/2}}{4}}{{(a+b\,x)}^2-2\,a\,(a+b\,x)+a^2}+12\,a^2\,b^2\,\sqrt{a+b\,x}+2\,a\,b^2\,{(a+b\,x)}^{3/2}+\frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^3,x)
```

```
[Out] (2*b^2*(a + b*x)^(5/2))/5 + ((15*a^4*b^2*(a + b*x)^(1/2))/4 - (17*a^3*b^2*(
a + b*x)^(3/2))/4)/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + 12*a^2*b^2*(a + b*
x)^(1/2) + (a^(5/2)*b^2*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/4 + 2*a*b^2
*(a + b*x)^(3/2)
```

3.320 $\int \frac{(a+bx)^{9/2}}{x^4}\,dx$

Optimal. Leaf size=114

$$\frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 35/8*b^3*(b*x+a)^(3/2)-21/8*b^2*(b*x+a)^(5/2)/x-3/4*b*(b*x+a)^(7/2)/x^2-1/3
*(b*x+a)^(9/2)/x^3-105/8*a^(3/2)*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))+105/8*a
*b^3*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x])/8 + (35*b^3*(a + b*x)^(3/2))/8 - (21*b^2*(a + b*x
)^(5/2))/(8*x) - (3*b*(a + b*x)^(7/2))/(4*x^2) - (a + b*x)^(9/2)/(3*x^3) -
(105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^4}\,dx &= -\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{2}(3b)\int \frac{(a+bx)^{7/2}}{x^3}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{8}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^2}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+\frac{1}{16}\left(105ab^3\right)\int \frac{\sqrt{a-}}{x} \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+ \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}+ \\
&= \frac{105}{8}ab^3\sqrt{a+bx}+\frac{35}{8}b^3(a+bx)^{3/2}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{3b(a+bx)^{7/2}}{4x^2}-\frac{(a+bx)^{9/2}}{3x^3}-
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 85, normalized size = 0.75

$$\frac{1}{24}\left(\frac{\sqrt{a+bx}\left(-8a^4-50a^3bx-165a^2b^2x^2+208ab^3x^3+16b^4x^4\right)}{x^3}-315a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] ((Sqrt[a + b*x]*(-8*a^4 - 50*a^3*b*x - 165*a^2*b^2*x^2 + 208*a*b^3*x^3 + 16
*b^4*x^4))/x^3 - 315*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/24
```

Maple [A]
time = 0.10, size = 89, normalized size = 0.78

method	result
risch	$-\frac{a^2\sqrt{bx+a}\,(165x^2b^2+50abx+8a^2)}{24x^3}+\frac{b^3\left(\frac{32(bx+a)^{\frac{3}{2}}}{3}+128a\sqrt{bx+a}-210a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{16}$
derivativedivides	$2b^3\left(\frac{(bx+a)^{\frac{3}{2}}}{3}+4a\sqrt{bx+a}-a^2\left(-\frac{-\frac{55(bx+a)^{\frac{5}{2}}}{16}+\frac{35a(bx+a)^{\frac{3}{2}}}{6}-\frac{41a^2\sqrt{bx+a}}{16}}{b^3x^3}+\frac{105\operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.}{16\sqrt{a}}\right.\right.$
default	$2b^3\left(\frac{(bx+a)^{\frac{3}{2}}}{3}+4a\sqrt{bx+a}-a^2\left(-\frac{-\frac{55(bx+a)^{\frac{5}{2}}}{16}+\frac{35a(bx+a)^{\frac{3}{2}}}{6}-\frac{41a^2\sqrt{bx+a}}{16}}{b^3x^3}+\frac{105\operatorname{arctanh}\left(\frac{\sqrt{b}}{}\right.}{16\sqrt{a}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(1/3*(b*x+a)^(3/2)+4*a*(b*x+a)^(1/2)-a^2*(-(-55/16*(b*x+a)^(5/2)+35/6
*a*(b*x+a)^(3/2)-41/16*a^2*(b*x+a)^(1/2))/b^3/x^3+105/16*arctanh((b*x+a)^(1
/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 145, normalized size = 1.27

$$\frac{105}{16}\,a^{\frac{3}{2}}b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b^3+8\,\sqrt{bx+a}\,ab^3-\frac{165\,(bx+a)^{\frac{5}{2}}a^2b^3-280\,(bx+a)^{\frac{3}{2}}a^3b^3+123\,\sqrt{bx+a}\,a^4b^3}{24\,\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 105/16*a^(3/2)*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
 + 2/3*(b*x + a)^(3/2)*b^3 + 8*sqrt(b*x + a)*a*b^3 - 1/24*(165*(b*x + a)^(5
/2)*a^2*b^3 - 280*(b*x + a)^(3/2)*a^3*b^3 + 123*sqrt(b*x + a)*a^4*b^3)/((b*
x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

Fricas [A]
time = 0.46, size = 178, normalized size = 1.56

$$\left[\frac{315\,a^{\frac{3}{2}}b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-8\,a^4)\sqrt{bx+a}}{48\,x^3},\ \frac{315\,\sqrt{-a}\,ab^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-8\,a^4)\sqrt{bx+a}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(315*a^(3/2)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt(b
```

```
*x + a))/x^3, 1/24*(315*sqrt(-a)*a*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a)
 + (16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt
(b*x + a))/x^3]
```

Sympy [A]
time = 9.35, size = 184, normalized size = 1.61

$$-\frac{105a^{\frac{3}{2}}b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8}-\frac{a^5}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{29a^4\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{215a^3b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{43a^2b^{\frac{5}{2}}}{24\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{28ab^{\frac{7}{2}}\sqrt{x}}{3\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**4,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/8 - a**5/(3*sqrt(b)*x**
(7/2)*sqrt(a/(b*x) + 1)) - 29*a**4*sqrt(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1))
- 215*a**3*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) + 43*a**2*b**(5/2)/(24*
sqrt(x)*sqrt(a/(b*x) + 1)) + 28*a*b**(7/2)*sqrt(x)/(3*sqrt(a/(b*x) + 1)) +
2*b**(9/2)*x**(3/2)/(3*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.69, size = 112, normalized size = 0.98

$$\frac{\frac{315\,a^2b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\,(bx+a)^{\frac{3}{2}}b^4+192\,\sqrt{bx+a}\,ab^4-\frac{165\,(bx+a)^{\frac{5}{2}}a^2b^4-280\,(bx+a)^{\frac{3}{2}}a^3b^4+123\,\sqrt{bx+a}\,a^4b^4}{b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(315*a^2*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 16*(b*x + a)^(3
/2)*b^4 + 192*sqrt(b*x + a)*a*b^4 - (165*(b*x + a)^(5/2)*a^2*b^4 - 280*(b*x
 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x + a)*a^4*b^4)/(b^3*x^3))/b
```

Mupad [B]
time = 0.12, size = 131, normalized size = 1.15

$$\frac{2\,b^3\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{\frac{41\,a^4\,b^3\,\sqrt{a+b\,x}}{8}-\frac{35\,a^3\,b^3\,{\left(a+b\,x\right)}^{3/2}}{3}+\frac{55\,a^2\,b^3\,{\left(a+b\,x\right)}^{5/2}}{8}}{3\,a\,{\left(a+b\,x\right)}^2-3\,a^2\,\left(a+b\,x\right)-{\left(a+b\,x\right)}^3+a^3}+8\,a\,b^3\,\sqrt{a+b\,x}+\frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,105{}\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^4,x)
```

```
[Out] (2*b^3*(a + b*x)^(3/2))/3 + ((41*a^4*b^3*(a + b*x)^(1/2))/8 - (35*a^3*b^3*(
a + b*x)^(3/2))/3 + (55*a^2*b^3*(a + b*x)^(5/2))/8)/(3*a*(a + b*x)^2 - 3*a^
2*(a + b*x) - (a + b*x)^3 + a^3) + (a^(3/2)*b^3*atan(((a + b*x)^(1/2)*1i)/a
^(1/2))*105i)/8 + 8*a*b^3*(a + b*x)^(1/2)
```

3.321 $\int \frac{(a+bx)^{9/2}}{x^5} dx$

Optimal. Leaf size=116

$$\frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -105/64*b^3*(b*x+a)^(3/2)/x-21/32*b^2*(b*x+a)^(5/2)/x^2-3/8*b*(b*x+a)^(7/2)
/x^3-1/4*(b*x+a)^(9/2)/x^4-315/64*b^4*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2
)+315/64*b^4*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 214}

$$\frac{315}{64}b^4\sqrt{a+bx}-\frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{(a+bx)^{9/2}}{4x^4}-\frac{3b(a+bx)^{7/2}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x])/64 - (105*b^3*(a + b*x)^(3/2))/(64*x) - (21*b^2*(a
+ b*x)^(5/2))/(32*x^2) - (3*b*(a + b*x)^(7/2))/(8*x^3) - (a + b*x)^(9/2)/(4
*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^5}\,dx &= -\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{8}(9b)\int \frac{(a+bx)^{7/2}}{x^4}\,dx\\ &= -\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{16}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^3}\,dx\\ &= -\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{64}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x^2}\,dx\\ &= -\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\frac{1}{128}\left(315b^4\right)\int \frac{\sqrt{}}{}\\ &= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\\ &= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}+\\ &= \frac{315}{64}b^4\sqrt{a+bx}-\frac{105b^3(a+bx)^{3/2}}{64x}-\frac{21b^2(a+bx)^{5/2}}{32x^2}-\frac{3b(a+bx)^{7/2}}{8x^3}-\frac{(a+bx)^{9/2}}{4x^4}-\end{aligned}$$

Mathematica [A]
time = 0.16, size = 86, normalized size = 0.74

$$\frac{1}{64}\left(-\frac{\sqrt{a+bx}\,\left(16a^4+88a^3bx+210a^2b^2x^2+325ab^3x^3-128b^4x^4\right)}{x^4}-315\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^5,x]
[Out] (-((Sqrt[a + b*x]*(16*a^4 + 88*a^3*b*x + 210*a^2*b^2*x^2 + 325*a*b^3*x^3 -
128*b^4*x^4))/x^4) - 315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.74

method	result
risch	$-\frac{a\sqrt{bx+a}\,(325b^3x^3+210a\,b^2x^2+88a^2bx+16a^3)}{64x^4}+\frac{b^4\left(256\sqrt{bx+a}-630\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a}\right)}{128}$
derivativedivides	$2b^4\left(\sqrt{bx+a}-a\left(\frac{\frac{325(bx+a)^{\frac{7}{2}}}{128}-\frac{765a(bx+a)^{\frac{5}{2}}}{128}+\frac{643a^2(bx+a)^{\frac{3}{2}}}{128}-\frac{187a^3\sqrt{bx+a}}{128}}{b^4x^4}+\frac{315\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}\right)\right)$
default	$2b^4\left(\sqrt{bx+a}-a\left(\frac{\frac{325(bx+a)^{\frac{7}{2}}}{128}-\frac{765a(bx+a)^{\frac{5}{2}}}{128}+\frac{643a^2(bx+a)^{\frac{3}{2}}}{128}-\frac{187a^3\sqrt{bx+a}}{128}}{b^4x^4}+\frac{315\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4*((b*x+a)^(1/2)-a*((325/128*(b*x+a)^(7/2)-765/128*a*(b*x+a)^(5/2)+643/
128*a^2*(b*x+a)^(3/2)-187/128*a^3*(b*x+a)^(1/2))/b^4/x^4+315/128*arctanh((b
*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 155, normalized size = 1.34

$$\frac{315}{128}\sqrt{a}\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^4-\frac{325\,(bx+a)^{\frac{7}{2}}ab^4-765\,(bx+a)^{\frac{5}{2}}a^2b^4+643\,(bx+a)^{\frac{3}{2}}a^3b^4-187\sqrt{bx+a}\,a^4b^4}{64\left((bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2-4\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))
) + 2*sqrt(b*x + a)*b^4 - 1/64*(325*(b*x + a)^(7/2)*a*b^4 - 765*(b*x + a)^(
5/2)*a^2*b^4 + 643*(b*x + a)^(3/2)*a^3*b^4 - 187*sqrt(b*x + a)*a^4*b^4)/((b
*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2 - 4*(b*x + a)*a^3 + a^4)
```

Fricas [A]
time = 0.56, size = 177, normalized size = 1.53

$$\left[\frac{315\sqrt{a}\,b^4x^4\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4)\sqrt{bx+a}}{128\,x^4},\ \frac{315\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(128\,b^4x^4-325\,ab^3x^3-210\,a^2b^2x^2-88\,a^3bx-16\,a^4)\sqrt{bx+a}}{64\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(315*sqrt(a)*b^4*x^4*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*sqr
```

```
t(b*x + a))/x^4, 1/64*(315*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*s
qrt(b*x + a))/x^4]
```

Sympy [A]
time = 9.95, size = 182, normalized size = 1.57

$$-\frac{315\sqrt{a}\,b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64}-\frac{a^5}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{13a^4\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{149a^3b^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{535a^2b^{\frac{5}{2}}}{64x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{197ab^{\frac{7}{2}}}{64\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**5,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/64 - a**5/(4*sqrt(b)*x**
(9/2)*sqrt(a/(b*x) + 1)) - 13*a**4*sqrt(b)/(8*x**(7/2)*sqrt(a/(b*x) + 1)) -
 149*a**3*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) - 535*a**2*b**(5/2)/(64*
x**(3/2)*sqrt(a/(b*x) + 1)) - 197*a*b**(7/2)/(64*sqrt(x)*sqrt(a/(b*x) + 1))
 + 2*b**(9/2)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A]
time = 1.08, size = 110, normalized size = 0.95

$$\frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\sqrt{bx+a}\,b^5-\frac{325\,(bx+a)^{\frac{7}{2}}ab^5-765\,(bx+a)^{\frac{5}{2}}a^2b^5+643\,(bx+a)^{\frac{3}{2}}a^3b^5-187\sqrt{bx+a}\,a^4b^5}{b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(315*a*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x + a)
*b^5 - (325*(b*x + a)^(7/2)*a*b^5 - 765*(b*x + a)^(5/2)*a^2*b^5 + 643*(b*x
+ a)^(3/2)*a^3*b^5 - 187*sqrt(b*x + a)*a^4*b^5)/(b^4*x^4))/b
```

Mupad [B]
time = 0.06, size = 94, normalized size = 0.81

$$2\,b^4\sqrt{a+b\,x}+\frac{187\,a^4\sqrt{a+b\,x}}{64\,x^4}-\frac{643\,a^3\,(a+b\,x)^{3/2}}{64\,x^4}+\frac{765\,a^2\,(a+b\,x)^{5/2}}{64\,x^4}-\frac{325\,a\,(a+b\,x)^{7/2}}{64\,x^4}+\frac{\sqrt{a}\,b^4\operatorname{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)315\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*(a + b*x)^(1/2) + (187*a^4*(a + b*x)^(1/2))/(64*x^4) - (643*a^3*(a +
b*x)^(3/2))/(64*x^4) + (765*a^2*(a + b*x)^(5/2))/(64*x^4) + (a^(1/2)*b^4*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*315i)/64 - (325*a*(a + b*x)^(7/2))/(64*x^4
)
```

3.322 $\int \frac{(a+bx)^{9/2}}{x^6}\,dx$

Optimal. Leaf size=119

$$-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}$$

```
[Out] -21/64*b^3*(b*x+a)^(3/2)/x^2-21/80*b^2*(b*x+a)^(5/2)/x^3-9/40*b*(b*x+a)^(7/
2)/x^4-1/5*(b*x+a)^(9/2)/x^5-63/128*b^5*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1
/2)-63/128*b^4*(b*x+a)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 214}

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{9b(a+bx)^{7/2}}{40x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x])/(128*x) - (21*b^3*(a + b*x)^(3/2))/(64*x^2) - (21*b
^2*(a + b*x)^(5/2))/(80*x^3) - (9*b*(a + b*x)^(7/2))/(40*x^4) - (a + b*x)^(
9/2)/(5*x^5) - (63*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = -\frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{10}(9b)\int \frac{(a+bx)^{7/2}}{x^5}\,dx$$

$$= -\frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{80}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x^4}\,dx$$

$$= -\frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{32}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$

$$= -\frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{128}\left(63b^4\right)\int \frac{\sqrt{a \cdots}}{x \cdots}$$

$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \cdots$$

$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \cdots$$

$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} - \cdots$$

Mathematica [A]
time = 0.19, size = 86, normalized size = 0.72

$$\frac{1}{640}\left(-\frac{\sqrt{a+bx}\,\left(128a^4+656a^3bx+1368a^2b^2x^2+1490ab^3x^3+965b^4x^4\right)}{x^5} - \frac{315b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-((Sqrt[a + b*x]*(128*a^4 + 656*a^3*b*x + 1368*a^2*b^2*x^2 + 1490*a*b^3*x^
3 + 965*b^4*x^4))/x^5) - (315*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/
640
```

Maple [A]
time = 0.12, size = 88, normalized size = 0.74

method	result
risch	$-\frac{\sqrt{bx+a}\,\left(965b^4x^4+1490a\,b^3x^3+1368a^2b^2x^2+656a^3bx+128a^4\right)}{640x^5} - \frac{63b^5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}$

derivativedivides	$2b^5\left(-\frac{\frac{193(bx+a)^{\frac{9}{2}}}{256}-\frac{237a(bx+a)^{\frac{7}{2}}}{128}+\frac{21a^2(bx+a)^{\frac{5}{2}}}{10}-\frac{147a^3(bx+a)^{\frac{3}{2}}}{128}+\frac{63a^4\sqrt{bx+a}}{256}}{b^5x^5}-\frac{63\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}\right.$
default	$2b^5\left(-\frac{\frac{193(bx+a)^{\frac{9}{2}}}{256}-\frac{237a(bx+a)^{\frac{7}{2}}}{128}+\frac{21a^2(bx+a)^{\frac{5}{2}}}{10}-\frac{147a^3(bx+a)^{\frac{3}{2}}}{128}+\frac{63a^4\sqrt{bx+a}}{256}}{b^5x^5}-\frac{63\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^5*(-(193/256*(b*x+a)^(9/2)-237/128*a*(b*x+a)^(7/2)+21/10*a^2*(b*x+a)^(5
/2)-147/128*a^3*(b*x+a)^(3/2)+63/256*a^4*(b*x+a)^(1/2))/b^5/x^5-63/256*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.50, size = 169, normalized size = 1.42

$$\frac{63\,b^5\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{256\,\sqrt{a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^5-2370\,(bx+a)^{\frac{7}{2}}ab^5+2688\,(bx+a)^{\frac{5}{2}}a^2b^5-1470\,(bx+a)^{\frac{3}{2}}a^3b^5+315\,\sqrt{bx+a}\,a^4b^5}{640\,\left((bx+a)^5-5\,(bx+a)^4a+10\,(bx+a)^3a^2-10\,(bx+a)^2a^3+5\,(bx+a)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 63/256*b^5*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
 - 1/640*(965*(b*x + a)^(9/2)*b^5 - 2370*(b*x + a)^(7/2)*a*b^5 + 2688*(b*x
+ a)^(5/2)*a^2*b^5 - 1470*(b*x + a)^(3/2)*a^3*b^5 + 315*sqrt(b*x + a)*a^4*b
^5)/((b*x + a)^5 - 5*(b*x + a)^4*a + 10*(b*x + a)^3*a^2 - 10*(b*x + a)^2*a^
3 + 5*(b*x + a)*a^4 - a^5)
```

Fricas [A]
time = 0.62, size = 190, normalized size = 1.60

$$\left[\frac{315\,\sqrt{a}\,b^5x^5\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(965\,ab^4x^4+1490\,a^2b^3x^3+1368\,a^3b^2x^2+656\,a^4bx+128\,a^5)\sqrt{bx+a}}{1280\,ax^5},\ \frac{315\,\sqrt{-a}\,b^5x^5\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-(965\,ab^4x^4+1490\,a^2b^3x^3+1368\,a^3b^2x^2+656\,a^4bx+128\,a^5)\sqrt{bx+a}}{640\,ax^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(315*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
 2*(965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 656*a^4*b*x + 128
*a^5)*sqrt(b*x + a))/(a*x^5), 1/640*(315*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x +
 a)*sqrt(-a)/a) - (965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 65
6*a^4*b*x + 128*a^5)*sqrt(b*x + a))/(a*x^5)]
```

Sympy [A]
time = 11.06, size = 158, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^{\frac{9}{2}}}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{40x^{\frac{7}{2}}}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{80x^{\frac{5}{2}}}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{64x^{\frac{3}{2}}}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{128\sqrt{x}}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{128\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**6,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**(9/2)) - 41*a**3*b**(3/2)*sqrt(a/(b*x
) + 1)/(40*x**(7/2)) - 171*a**2*b**(5/2)*sqrt(a/(b*x) + 1)/(80*x**(5/2)) -
149*a*b**(7/2)*sqrt(a/(b*x) + 1)/(64*x**(3/2)) - 193*b**(9/2)*sqrt(a/(b*x)
+ 1)/(128*sqrt(x)) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*sqrt(a))
```

Giac [A]
time = 1.64, size = 109, normalized size = 0.92

$$\frac{\frac{315\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^6-2370\,(bx+a)^{\frac{7}{2}}ab^6+2688\,(bx+a)^{\frac{5}{2}}a^2b^6-1470\,(bx+a)^{\frac{3}{2}}a^3b^6+315\,\sqrt{bx+a}\,a^4b^6}{b^5x^5}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/640*(315*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (965*(b*x + a)^(9/
2)*b^6 - 2370*(b*x + a)^(7/2)*a*b^6 + 2688*(b*x + a)^(5/2)*a^2*b^6 - 1470*(
b*x + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x + a)*a^4*b^6)/(b^5*x^5))/b
```

Mupad [B]
time = 0.12, size = 94, normalized size = 0.79

$$\frac{147\,a^3\,(a+b\,x)^{3/2}}{64\,x^5}-\frac{63\,a^4\,\sqrt{a+b\,x}}{128\,x^5}-\frac{193\,(a+b\,x)^{9/2}}{128\,x^5}-\frac{21\,a^2\,(a+b\,x)^{5/2}}{5\,x^5}+\frac{237\,a\,(a+b\,x)^{7/2}}{64\,x^5}+\frac{b^5\operatorname{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{128\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^6,x)
```

```
[Out] (147*a^3*(a + b*x)^(3/2))/(64*x^5) - (63*a^4*(a + b*x)^(1/2))/(128*x^5) - (
193*(a + b*x)^(9/2))/(128*x^5) - (21*a^2*(a + b*x)^(5/2))/(5*x^5) + (b^5*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/(128*a^(1/2)) + (237*a*(a + b*x)^(7/2
))/(64*x^5)
```

3.323 $\int \frac{(a+bx)^{9/2}}{x^7} dx$

Optimal. Leaf size=141

$$-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{21b^6\tanh^{-1}}{5}$$

```
[Out] -7/64*b^3*(b*x+a)^(3/2)/x^3-21/160*b^2*(b*x+a)^(5/2)/x^4-3/20*b*(b*x+a)^(7/
2)/x^5-1/6*(b*x+a)^(9/2)/x^6+21/512*b^6*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3
/2)-21/256*b^4*(b*x+a)^(1/2)/x^2-21/512*b^5*(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{(a+bx)^{9/2}}{6x^6}-\frac{3b(a+bx)^{7/2}}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x])/(256*x^2) - (21*b^5*Sqrt[a + b*x])/(512*a*x) - (7*b
^3*(a + b*x)^(3/2))/(64*x^3) - (21*b^2*(a + b*x)^(5/2))/(160*x^4) - (3*b*(a
 + b*x)^(7/2))/(20*x^5) - (a + b*x)^(9/2)/(6*x^6) + (21*b^6*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^7}\,dx &= -\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{4}(3b)\int \frac{(a+bx)^{7/2}}{x^6}\,dx\\
&= -\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{40}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^5}\,dx\\
&= -\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{64}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^4}\,dx\\
&= -\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1}{128}\left(21b^4\right)\int \frac{\sqrt{a+
}{x^3
\\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}+\frac{1
}{51
\\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-\\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{3b(a+bx)^{7/2}}{20x^5}-
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 100, normalized size = 0.71

$$-\frac{\sqrt{a+bx}\,\left(1280a^5+6272a^4bx+12144a^3b^2x^2+11432a^2b^3x^3+4910ab^4x^4+315b^5x^5\right)}{7680ax^6}+\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] -1/7680*(Sqrt[a + b*x]*(1280*a^5 + 6272*a^4*b*x + 12144*a^3*b^2*x^2 + 11432
*a^2*b^3*x^3 + 4910*a*b^4*x^4 + 315*b^5*x^5))/(a*x^6) + (21*b^6*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/(512*a^(3/2))
```

Maple [A]
time = 0.12, size = 100, normalized size = 0.71

method	result
risch	$-\frac{\sqrt{bx+a}\,(315b^5x^5+4910a\,b^4x^4+11432a^2b^3x^3+12144a^3b^2x^2+6272a^4bx+1280a^5)}{7680x^6a}+\frac{21b^6\,\mathrm{arctanh}\left(\frac{\sqrt{bx+}}{\sqrt{a}}\right.}{512a^{\frac{3}{2}}}$
derivativedivides	$2b^6\left(-\frac{\frac{21(bx+a)^{\frac{11}{2}}}{1024a}+\frac{667(bx+a)^{\frac{9}{2}}}{3072}-\frac{843a(bx+a)^{\frac{7}{2}}}{2560}+\frac{693a^2(bx+a)^{\frac{5}{2}}}{2560}-\frac{119a^3(bx+a)^{\frac{3}{2}}}{1024}+\frac{21a^4\sqrt{bx+a}}{1024}}{b^6x^6}+\frac{21\,\mathrm{arctanh}\left(\right.}{10}\right.$
default	$2b^6\left(-\frac{\frac{21(bx+a)^{\frac{11}{2}}}{1024a}+\frac{667(bx+a)^{\frac{9}{2}}}{3072}-\frac{843a(bx+a)^{\frac{7}{2}}}{2560}+\frac{693a^2(bx+a)^{\frac{5}{2}}}{2560}-\frac{119a^3(bx+a)^{\frac{3}{2}}}{1024}+\frac{21a^4\sqrt{bx+a}}{1024}}{b^6x^6}+\frac{21\,\mathrm{arctanh}\left(\right.}{10}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^6*(-(21/1024/a*(b*x+a)^(11/2)+667/3072*(b*x+a)^(9/2)-843/2560*a*(b*x+a)
^(7/2)+693/2560*a^2*(b*x+a)^(5/2)-119/1024*a^3*(b*x+a)^(3/2)+21/1024*a^4*(b
*x+a)^(1/2))/b^6/x^6+21/1024*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.49, size = 198, normalized size = 1.40

$$-\frac{21\,b^6\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{1024\,a^{\frac{3}{2}}}-\frac{315\,(bx+a)^{\frac{11}{2}}b^6+3335\,(bx+a)^{\frac{9}{2}}ab^6-5058\,(bx+a)^{\frac{7}{2}}a^2b^6+4158\,(bx+a)^{\frac{5}{2}}a^3b^6-1785\,(bx+a)^{\frac{3}{2}}a^4b^6+315\,\sqrt{bx+a}\,a^5b^6}{7680\,\left((bx+a)^6a-6\,(bx+a)^5a^2+15\,(bx+a)^4a^3-20\,(bx+a)^3a^4+15\,(bx+a)^2a^5-6\,(bx+a)a^6+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -21/1024*b^6*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/
2) - 1/7680*(315*(b*x + a)^(11/2)*b^6 + 3335*(b*x + a)^(9/2)*a*b^6 - 5058*(
b*x + a)^(7/2)*a^2*b^6 + 4158*(b*x + a)^(5/2)*a^3*b^6 - 1785*(b*x + a)^(3/2
)*a^4*b^6 + 315*sqrt(b*x + a)*a^5*b^6)/((b*x + a)^6*a - 6*(b*x + a)^5*a^2 +
 15*(b*x + a)^4*a^3 - 20*(b*x + a)^3*a^4 + 15*(b*x + a)^2*a^5 - 6*(b*x + a)
*a^6 + a^7)
```

Fricas [A]
time = 0.57, size = 211, normalized size = 1.50

$$\left[\frac{315\,\sqrt{a}\,b^6x^6\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+1280\,a^6)\sqrt{bx+a}}{15360\,a^2x^6},\,-\frac{315\,\sqrt{-a}\,b^6x^6\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(315\,ab^5x^5+4910\,a^2b^4x^4+11432\,a^3b^3x^3+12144\,a^4b^2x^2+6272\,a^5bx+1280\,a^6)\sqrt{bx+a}}{7680\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/15360*(315*sqrt(a)*b^6*x^6*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
- 2*(315*a*b^5*x^5 + 4910*a^2*b^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x
^2 + 6272*a^5*b*x + 1280*a^6)*sqrt(b*x + a))/(a^2*x^6), -1/7680*(315*sqrt(-
a)*b^6*x^6*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^5*x^5 + 4910*a^2*b^4
*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x^2 + 6272*a^5*b*x + 1280*a^6)*sqr
t(b*x + a))/(a^2*x^6)]
```

Sympy [A]
time = 47.97, size = 209, normalized size = 1.48

$$-\frac{a^5}{6\sqrt{b}\,x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{59a^4\sqrt{b}}{60x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{480x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{960x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{8171ab^{\frac{7}{2}}}{3840x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1045b^{\frac{9}{2}}}{1536x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{21b^{\frac{11}{2}}}{512a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{21b^6\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{512a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**7,x)
```

```
[Out] -a**5/(6*sqrt(b)*x**(13/2)*sqrt(a/(b*x) + 1)) - 59*a**4*sqrt(b)/(60*x**(11/
2)*sqrt(a/(b*x) + 1)) - 1151*a**3*b**(3/2)/(480*x**(9/2)*sqrt(a/(b*x) + 1))
 - 2947*a**2*b**(5/2)/(960*x**(7/2)*sqrt(a/(b*x) + 1)) - 8171*a*b**(7/2)/(3
840*x**(5/2)*sqrt(a/(b*x) + 1)) - 1045*b**(9/2)/(1536*x**(3/2)*sqrt(a/(b*x)
 + 1)) - 21*b**(11/2)/(512*a*sqrt(x)*sqrt(a/(b*x) + 1)) + 21*b**6*asinh(sqr
t(a)/(sqrt(b)*sqrt(x)))/(512*a**(3/2))
```

Giac [A]
time = 1.02, size = 129, normalized size = 0.91

$$-\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{315\,(bx+a)^{\frac{11}{2}}b^7+3335\,(bx+a)^{\frac{9}{2}}ab^7-5058\,(bx+a)^{\frac{7}{2}}a^2b^7+4158\,(bx+a)^{\frac{5}{2}}a^3b^7-1785\,(bx+a)^{\frac{3}{2}}a^4b^7+315\,\sqrt{bx+a}\,a^5b^7}{ab^6x^6}}{7680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/7680*(315*b^7*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x +
a)^(11/2)*b^7 + 3335*(b*x + a)^(9/2)*a*b^7 - 5058*(b*x + a)^(7/2)*a^2*b^7 +
 4158*(b*x + a)^(5/2)*a^3*b^7 - 1785*(b*x + a)^(3/2)*a^4*b^7 + 315*sqrt(b*x
 + a)*a^5*b^7)/(a*b^6*x^6))/b
```

Mupad [B]
time = 0.13, size = 109, normalized size = 0.77

$$\frac{119\,a^3\,(a+b\,x)^{3/2}}{512\,x^6}-\frac{21\,a^4\,\sqrt{a+b\,x}}{512\,x^6}-\frac{667\,(a+b\,x)^{9/2}}{1536\,x^6}-\frac{693\,a^2\,(a+b\,x)^{5/2}}{1280\,x^6}-\frac{21\,(a+b\,x)^{11/2}}{512\,a\,x^6}+\frac{843\,a\,(a+b\,x)^{7/2}}{1280\,x^6}-\frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,21\mathrm{i}}{512\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^7,x)
```

```
[Out] (119*a^3*(a + b*x)^(3/2))/(512*x^6) - (21*a^4*(a + b*x)^(1/2))/(512*x^6) -
(667*(a + b*x)^(9/2))/(1536*x^6) - (693*a^2*(a + b*x)^(5/2))/(1280*x^6) - (
21*(a + b*x)^(11/2))/(512*a*x^6) - (b^6*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*
21i)/(512*a^(3/2)) + (843*a*(a + b*x)^(7/2))/(1280*x^6)
```

3.324 $\int \frac{(a+bx)^{9/2}}{x^8} \, dx$

Optimal. Leaf size=163

$$-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}-$$

```
[Out] -3/64*b^3*(b*x+a)^(3/2)/x^4-3/40*b^2*(b*x+a)^(5/2)/x^5-3/28*b*(b*x+a)^(7/2)
/x^6-1/7*(b*x+a)^(9/2)/x^7-9/1024*b^7*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2
)-3/128*b^4*(b*x+a)^(1/2)/x^3-3/512*b^5*(b*x+a)^(1/2)/a/x^2+9/1024*b^6*(b*x
+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 44, 65, 214}

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{(a+bx)^{9/2}}{7x^7}-\frac{3b(a+bx)^{7/2}}{28x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x])/(128*x^3) - (3*b^5*Sqrt[a + b*x])/(512*a*x^2) + (9*b
^6*Sqrt[a + b*x])/(1024*a^2*x) - (3*b^3*(a + b*x)^(3/2))/(64*x^4) - (3*b^2*
(a + b*x)^(5/2))/(40*x^5) - (3*b*(a + b*x)^(7/2))/(28*x^6) - (a + b*x)^(9/2
)/(7*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{9/2}}{x^8}\,dx &= -\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{14}(9b)\int \frac{(a+bx)^{7/2}}{x^7}\,dx\\ &= -\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{8}\left(3b^2\right)\int \frac{(a+bx)^{5/2}}{x^6}\,dx\\ &= -\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{16}\left(3b^3\right)\int \frac{(a+bx)^{3/2}}{x^5}\,dx\\ &= -\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{128}\left(9b^4\right)\int \frac{\sqrt{a+\cdots}}{x^4}\\ &= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}+\frac{1}{25\cdots}\\ &= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\cdots\\ &= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\cdots\\ &= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\cdots\\ &= -\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^5\sqrt{a+bx}}{512ax^2}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\cdots\end{aligned}$$

Mathematica [A]
time = 0.25, size = 111, normalized size = 0.68

$$-\frac{\sqrt{a+bx}\left(5120a^6+24320a^5bx+44928a^4b^2x^2+39056a^3b^3x^3+14168a^2b^4x^4+210ab^5x^5-315b^6x^6\right)}{35840a^2x^7}-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] -1/35840*(Sqrt[a + b*x]*(5120*a^6 + 24320*a^5*b*x + 44928*a^4*b^2*x^2 + 390
56*a^3*b^3*x^3 + 14168*a^2*b^4*x^4 + 210*a*b^5*x^5 - 315*b^6*x^6))/(a^2*x^7
) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Maple [A]

time = 0.11, size = 112, normalized size = 0.69

method	result
risch	$-\frac{\sqrt{bx+a}\,\left(-315x^{6}b^{6}+210a\,x^{5}b^{5}+14168a^{2}x^{4}b^{4}+39056a^{3}b^{3}x^{3}+44928a^{4}x^{2}b^{2}+24320a^{5}xb+5120a^{6}\right)}{35840x^{7}a^{2}}-\frac{9b^{7}\,\mathrm{arctanh}}{}$
derivativedivides	$2b^{7}\left(-\frac{-\frac{9(bx+a)^{\frac{13}{2}}}{2048a^{2}}+\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{1199(bx+a)^{\frac{9}{2}}}{10240}-\frac{9a(bx+a)^{\frac{7}{2}}}{70}+\frac{849a^{2}(bx+a)^{\frac{5}{2}}}{10240}-\frac{15a^{3}(bx+a)^{\frac{3}{2}}}{512}+\frac{9a^{4}\sqrt{bx+a}}{2048}}{b^{7}x^{7}}-\frac{9\,a}{}\right.$
default	$2b^{7}\left(-\frac{-\frac{9(bx+a)^{\frac{13}{2}}}{2048a^{2}}+\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{1199(bx+a)^{\frac{9}{2}}}{10240}-\frac{9a(bx+a)^{\frac{7}{2}}}{70}+\frac{849a^{2}(bx+a)^{\frac{5}{2}}}{10240}-\frac{15a^{3}(bx+a)^{\frac{3}{2}}}{512}+\frac{9a^{4}\sqrt{bx+a}}{2048}}{b^{7}x^{7}}-\frac{9\,a}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^7*(-(-9/2048/a^2*(b*x+a)^(13/2)+15/512/a*(b*x+a)^(11/2)+1199/10240*(b*x
+a)^(9/2)-9/70*a*(b*x+a)^(7/2)+849/10240*a^2*(b*x+a)^(5/2)-15/512*a^3*(b*x+
a)^(3/2)+9/2048*a^4*(b*x+a)^(1/2))/b^7/x^7-9/2048*arctanh((b*x+a)^(1/2)/a^(
1/2))/a^(5/2))
```

Maxima [A]

time = 0.50, size = 229, normalized size = 1.40

$$\frac{9\,b^{7}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2048\,a^{\frac{5}{2}}}+\frac{315\,(bx+a)^{\frac{13}{2}}b^{7}-2100\,(bx+a)^{\frac{11}{2}}ab^{7}-8393\,(bx+a)^{\frac{9}{2}}a^{2}b^{7}+9216\,(bx+a)^{\frac{7}{2}}a^{3}b^{7}-5943\,(bx+a)^{\frac{5}{2}}a^{4}b^{7}+2100\,(bx+a)^{\frac{3}{2}}a^{5}b^{7}-315\,\sqrt{bx+a}\,a^{6}b^{7}}{35840\,\left((bx+a)^{7}a^{2}-7\,(bx+a)^{6}a^{3}+21\,(bx+a)^{5}a^{4}-35\,(bx+a)^{4}a^{5}+35\,(bx+a)^{3}a^{6}-21\,(bx+a)^{2}a^{7}+7\,(bx+a)a^{8}-a^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2048*b^7*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
 + 1/35840*(315*(b*x + a)^(13/2)*b^7 - 2100*(b*x + a)^(11/2)*a*b^7 - 8393*(
b*x + a)^(9/2)*a^2*b^7 + 9216*(b*x + a)^(7/2)*a^3*b^7 - 5943*(b*x + a)^(5/2
)*a^4*b^7 + 2100*(b*x + a)^(3/2)*a^5*b^7 - 315*sqrt(b*x + a)*a^6*b^7)/((b*x
 + a)^7*a^2 - 7*(b*x + a)^6*a^3 + 21*(b*x + a)^5*a^4 - 35*(b*x + a)^4*a^5 +
 35*(b*x + a)^3*a^6 - 21*(b*x + a)^2*a^7 + 7*(b*x + a)*a^8 - a^9)
```

Fricas [A]
time = 0.50, size = 233, normalized size = 1.43

$$\left[\frac{315\,\sqrt{a}\,b^{7}x^{7}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(315\,ab^{6}x^{6}-210\,a^{2}b^{5}x^{5}-14168\,a^{3}b^{4}x^{4}-39056\,a^{4}b^{3}x^{3}-44928\,a^{5}b^{2}x^{2}-24320\,a^{6}bx-5120\,a^{7})\sqrt{bx+a}}{71680\,a^{3}x^{7}},\ \frac{315\,\sqrt{-a}\,b^{7}x^{7}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(315\,ab^{6}x^{6}-210\,a^{2}b^{5}x^{5}-14168\,a^{3}b^{4}x^{4}-39056\,a^{4}b^{3}x^{3}-44928\,a^{5}b^{2}x^{2}-24320\,a^{6}bx-5120\,a^{7})\sqrt{bx+a}}{35840\,a^{3}x^{7}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/71680*(315*sqrt(a)*b^7*x^7*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(315*a*b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^
3 - 44928*a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7),
 1/35840*(315*sqrt(-a)*b^7*x^7*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^
6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^3 - 44928*a^5
*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.97, size = 144, normalized size = 0.88

$$\frac{\frac{315\,b^{8}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{315\,(bx+a)^{\frac{13}{2}}b^{8}-2100\,(bx+a)^{\frac{11}{2}}ab^{8}-8393\,(bx+a)^{\frac{9}{2}}a^{2}b^{8}+9216\,(bx+a)^{\frac{7}{2}}a^{3}b^{8}-5943\,(bx+a)^{\frac{5}{2}}a^{4}b^{8}+2100\,(bx+a)^{\frac{3}{2}}a^{5}b^{8}-315\,\sqrt{bx+a}\,a^{6}b^{8}}{a^{2}b^{7}x^{7}}}{35840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/35840*(315*b^8*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*x
+ a)^(13/2)*b^8 - 2100*(b*x + a)^(11/2)*a*b^8 - 8393*(b*x + a)^(9/2)*a^2*b^
8 + 9216*(b*x + a)^(7/2)*a^3*b^8 - 5943*(b*x + a)^(5/2)*a^4*b^8 + 2100*(b*x
 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x + a)*a^6*b^8)/(a^2*b^7*x^7))/b
```

Mupad [B]
time = 0.13, size = 124, normalized size = 0.76

$$\frac{15\,a^{3}\,{\left(a+b\,x\right)}^{3/2}}{256\,x^{7}}-\frac{9\,a^{4}\,\sqrt{a+b\,x}}{1024\,x^{7}}-\frac{1199\,{\left(a+b\,x\right)}^{9/2}}{5120\,x^{7}}-\frac{849\,a^{2}\,{\left(a+b\,x\right)}^{5/2}}{5120\,x^{7}}-\frac{15\,{\left(a+b\,x\right)}^{11/2}}{256\,a\,x^{7}}+\frac{9\,{\left(a+b\,x\right)}^{13/2}}{1024\,a^{2}\,x^{7}}+\frac{9\,a\,{\left(a+b\,x\right)}^{7/2}}{35\,x^{7}}+\frac{b^{7}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,9{}\mathrm{i}}{1024\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^8,x)
```

```
[Out] (15*a^3*(a + b*x)^(3/2))/(256*x^7) - (9*a^4*(a + b*x)^(1/2))/(1024*x^7) - (
1199*(a + b*x)^(9/2))/(5120*x^7) - (849*a^2*(a + b*x)^(5/2))/(5120*x^7) - (
15*(a + b*x)^(11/2))/(256*a*x^7) + (9*(a + b*x)^(13/2))/(1024*a^2*x^7) + (b
^7*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*9i)/(1024*a^(5/2)) + (9*a*(a + b*x)^(
7/2))/(35*x^7)
```

3.325 $\int \frac{\sqrt{-a+bx}}{x}\,dx$

Optimal. Leaf size=39

$$2\sqrt{-a+bx}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 65, 211}

$$2\sqrt{bx-a}-2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x}\,dx = 2\sqrt{-a+bx} - a\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= 2\sqrt{-a+bx} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}$$
$$= 2\sqrt{-a+bx} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$2\sqrt{-a+bx} - 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x,x]
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.09, size = 32, normalized size = 0.82

method	result	size
derivativedivides	$-2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx-a}$	32
default	$-2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2\sqrt{bx-a}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

Maxima [A]
time = 0.50, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="maxima")
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

Fricas [A]
time = 0.60, size = 78, normalized size = 2.00

$$\left[\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}, -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="fricas")
[Out] [sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(b*x - a),
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.73, size = 148, normalized size = 3.79

$$\begin{cases} -2i\sqrt{a}\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2ia}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2i\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ 2\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x,x)
[Out] Piecewise((-2*I*sqrt(a)*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*I*a/(sqrt(b)*s
qrt(x)*sqrt(a/(b*x) - 1)) - 2*I*sqrt(b)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b
*x)) > 1), (2*sqrt(a)*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - 2*a/(sqrt(b)*sqrt(x
)*sqrt(-a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Giac [A]
time = 0.81, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="giac")
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

Mupad [B]
time = 0.09, size = 31, normalized size = 0.79

$$2\sqrt{b\,x-a}-2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x,x)
[Out] 2*(b*x - a)^(1/2) - 2*a^(1/2)*atan((b*x - a)^(1/2)/a^(1/2))
```

3.326 $\int \frac{\sqrt{-a+bx}}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{-a+bx}}{x}+\frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-(b*x-a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {43, 65, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x^2}\,dx = -\frac{\sqrt{-a+bx}}{x} + \frac{1}{2}b\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= -\frac{\sqrt{-a+bx}}{x} + \text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)$$
$$= -\frac{\sqrt{-a+bx}}{x} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.00

$$-\frac{\sqrt{-a+bx}}{x} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^2,x]
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.10, size = 41, normalized size = 0.98

method	result	size
risch	$\frac{-bx+a}{x\sqrt{bx-a}} + \frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	40
derivativedivides	$2b\left(-\frac{\sqrt{bx-a}}{2bx} + \frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	41
default	$2b\left(-\frac{\sqrt{bx-a}}{2bx} + \frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x-a)^(1/2)/b/x+1/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.50, size = 34, normalized size = 0.81

$$\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] b*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)/x
```

Fricas [A]
time = 0.59, size = 98, normalized size = 2.33

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}\,a}{2\,ax},\frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(
b*x - a)*a)/(a*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x -
a)*a)/(a*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 117, normalized size = 2.79

$$\begin{cases}-\frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{\sqrt{x}}+\frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right|>1\\ \frac{a}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**2,x)
```

```
[Out] Piecewise((-I*sqrt(b)*sqrt(a/(b*x) - 1)/sqrt(x) + I*b*acosh(sqrt(a)/(sqrt(b
)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1), (a/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x)
+ 1)) - sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(b)*sqrt
(x)))/sqrt(a), True))
```

Giac [**A**]
time = 0.87, size = 41, normalized size = 0.98

$$\frac{\frac{b^{2} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="giac")
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)*b/x)/b
```

Mupad [**B**]
time = 0.10, size = 34, normalized size = 0.81

$$\frac{b \operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{b\,x-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^2,x)
[Out] (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2) - (b*x - a)^(1/2)/x
```

3.327 $\int \frac{\sqrt{-a+bx}}{x^3} \, dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-1/2*(b*x-a)^(1/2)/x^2+1/4*b*(
b*x-a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 44, 65, 211}

$$\frac{b^2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{bx-a}}{2x^2}+\frac{b\sqrt{bx-a}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -1/2*Sqrt[-a + b*x]/x^2 + (b*Sqrt[-a + b*x])/(4*a*x) + (b^2*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-a+bx}}{x^3}\,dx &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{-a+bx}}\,dx\\ &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a}\\ &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a}\\ &= -\frac{\sqrt{-a+bx}}{2x^2}+\frac{b\sqrt{-a+bx}}{4ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.85

$$-\frac{(2a-bx)\sqrt{-a+bx}}{4ax^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -1/4*((2*a - b*x)*Sqrt[-a + b*x])/(a*x^2) + (b^2*ArcTan[Sqrt[-a + b*x]/Sqrt
[a]])/(4*a^(3/2))
```

Maple [A]
time = 0.11, size = 59, normalized size = 0.83

method	result	size
risch	$\frac{(-bx+a)(-bx+2a)}{4x^2\sqrt{bx-a}\;a}+\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}}$	55

derivativedivides	$2b^2\left(\frac{\frac{(bx-a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx-a}}{8}}{b^2x^2}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	59
default	$2b^2\left(\frac{\frac{(bx-a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx-a}}{8}}{b^2x^2}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((1/8/a*(b*x-a)^(3/2)-1/8*(b*x-a)^(1/2))/b^2/x^2+1/8*arctan((b*x-a)^(
1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.49, size = 83, normalized size = 1.17

$$\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}}+\frac{(bx-a)^{\frac{3}{2}}b^2-\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2a+2\,(bx-a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + 1/4*((b*x - a)^(3/2)*b^2 -
sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a + 2*(b*x - a)*a^2 + a^3)
```

Fricas [A]
time = 0.51, size = 124, normalized size = 1.75

$$\left[-\frac{\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(abx-2\,a^2)\sqrt{bx-a}}{8\,a^2x^2},\frac{\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(abx-2\,a^2)\sqrt{bx-a}}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(
a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2), 1/4*(sqrt(a)*b^2*x^2*arctan(sqrt(b
*x - a)/sqrt(a)) + (a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2)]
```

Sympy [C] Result contains complex when optimal does not.

time = 1.97, size = 207, normalized size = 2.92

$$\begin{cases} -\frac{ia}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{3i\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{ib^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}-1}} + \frac{ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**3,x)
```

```
[Out] Piecewise((-I*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(4*x**
(3/2)*sqrt(a/(b*x) - 1)) - I*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) - 1)) + I*b
**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), Abs(a/(b*x)) > 1), (a/(2
*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x)
 + 1)) + b**(3/2)/(4*a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b**2*asin(sqrt(a)/(sqr
t(b)*sqrt(x)))/(4*a**(3/2)), True))
```

Giac [A]
time = 1.03, size = 66, normalized size = 0.93

$$\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{(bx-a)^{\frac{3}{2}}b^3 - \sqrt{bx-a}\,ab^3}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + ((b*x - a)^(3/2)*b^3 - sqr
t(b*x - a)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.10, size = 54, normalized size = 0.76

$$\frac{b^2\operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{3/2}} - \frac{\sqrt{b\,x-a}}{4\,x^2} + \frac{(b\,x-a)^{3/2}}{4\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^3,x)
```

```
[Out] (b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (b*x - a)^(1/2)/(4*x^2) +
 (b*x - a)^(3/2)/(4*a*x^2)
```

3.328 $\int \frac{(-a+bx)^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$-2a\sqrt{-a+bx}+\frac{2}{3}(-a+bx)^{3/2}+2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 65, 211}

$$2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x,x]
```

```
[Out] -2*a*Sqrt[-a + b*x] + (2*(-a + b*x)^(3/2))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(-a+bx)^{3/2} - a\int \frac{\sqrt{-a+bx}}{x}\,dx\\ &= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + a^2\int \frac{1}{x\sqrt{-a+bx}}\,dx\\ &= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + \frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}\\ &= -2a\sqrt{-a+bx} + \frac{2}{3}(-a+bx)^{3/2} + 2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.87

$$\frac{2}{3}(-4a+bx)\sqrt{-a+bx} + 2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x,x]
[Out] (2*(-4*a + b*x)*Sqrt[-a + b*x])/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
]
```

Maple [A]
time = 0.10, size = 44, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2(bx-a)^{\frac{3}{2}}}{3}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}$	44
default	$\frac{2(bx-a)^{\frac{3}{2}}}{3}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

Maxima [A]
time = 0.48, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

Fricas [A]
time = 0.54, size = 93, normalized size = 1.69

$$\left[\sqrt{-a}\,a\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}\,-2\,a}{x}\right)+\frac{2}{3}\,\sqrt{bx-a}\,(bx-4\,a),2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,\sqrt{bx-a}\,(bx-4\,a)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/3*sqrt(b*x -
a)*(b*x - 4*a), 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*sqrt(b*x - a)
*(b*x - 4*a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.22, size = 187, normalized size = 3.40

$$\begin{cases} -\frac{8a^{\frac{3}{2}}\sqrt{-1+\frac{bx}{a}}}{3}-ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)+2ia^{\frac{3}{2}}\log\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-2a^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\sqrt{a}\,bx\sqrt{-1+\frac{bx}{a}}}{3} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{8ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3}-ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)+2ia^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}\,bx\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x,x)
```

```
[Out] Piecewise((-8*a**(3/2)*sqrt(-1 + b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**
(3/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) - 2*a**(3/2)*asin(sqrt(a)/(sqrt(b)*sqrt(
x))) + 2*sqrt(a)*b*x*sqrt(-1 + b*x/a)/3, Abs(b*x/a) > 1), (-8*I*a**(3/2)*sq
rt(1 - b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**(3/2)*log(sqrt(1 - b*x/a)
+ 1) + 2*I*sqrt(a)*b*x*sqrt(1 - b*x/a)/3, True))
```

Giac [A]
time = 1.01, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="giac")
```

[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x - a)*a

Mupad [B]
time = 0.04, size = 43, normalized size = 0.78

$$2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-2\,a\,\sqrt{b\,x-a}+\frac{2\,{\left(b\,x-a\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x - a)^(3/2)/x,x)

[Out] 2*a^(3/2)*atan((b*x - a)^(1/2)/a^(1/2)) - 2*a*(b*x - a)^(1/2) + (2*(b*x - a)^(3/2))/3

3.329 $\int \frac{(-a+bx)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=57

$$3b\sqrt{-a+bx} - \frac{(-a+bx)^{3/2}}{x} - 3\sqrt{a}\, b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -(b*x-a)^(3/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+3*b*(b*x-a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 211}

$$-3\sqrt{a}\, b\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - \frac{(bx-a)^{3/2}}{x} + 3b\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[-a + b*x] - (-a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^2}\,dx &= -\frac{(-a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-(3a)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= 3b\sqrt{-a+bx}-\frac{(-a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 48, normalized size = 0.84

$$\frac{\sqrt{-a+bx}\,(a+2bx)}{x}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^2,x]
[Out] (Sqrt[-a + b*x]*(a + 2*b*x))/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Maple [**A**]
time = 0.10, size = 54, normalized size = 0.95

method	result	size
derivativedivides	$2b\left(\sqrt{bx-a}-a\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	54

default	$2b\left(\sqrt{bx-a}-a\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	54
risch	$-\frac{a(-bx+a)}{x\sqrt{bx-a}}-3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}+2b\sqrt{bx-a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*((b*x-a)^(1/2)-a*(-1/2*(b*x-a)^(1/2)/b/x+3/2*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(1/2)))
```

Maxima [A]
time = 0.49, size = 47, normalized size = 0.82

$$-3\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\,b+\frac{\sqrt{bx-a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3*sqrt(a)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b + sqrt(b*x -
 a)*a/x
```

Fricas [A]
time = 0.46, size = 105, normalized size = 1.84

$$\left[\frac{3\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2(2bx+a)\sqrt{bx-a}}{2x},-\frac{3\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(2bx+a)\sqrt{bx-a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(2*b
*x + a)*sqrt(b*x - a))/x, -(3*sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - (
2*b*x + a)*sqrt(b*x - a))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.36, size = 197, normalized size = 3.46

$$\begin{cases} -3i\sqrt{a}\,b\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{ia^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{ia\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ 3\sqrt{a}\,b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{a\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**2,x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + I*a**2/(sqrt(b
)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) - 1)) - 2
*I*b**(3/2)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (3*sqrt(a)*b*asin
(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) -
a*sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)/sqrt(-a/(b*x) +
 1), True))
```

Giac [A]
time = 0.73, size = 58, normalized size = 1.02

$$-\frac{3\sqrt{a}\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\sqrt{bx-a}\,b^2-\frac{\sqrt{bx-a}\,ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(3*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) - 2*sqrt(b*x - a)*b^2 - sqrt(
b*x - a)*a*b/x)/b
```

Mupad [B]
time = 0.04, size = 47, normalized size = 0.82

$$2\,b\,\sqrt{b\,x-a}+\frac{a\,\sqrt{b\,x-a}}{x}-3\,\sqrt{a}\,\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^2,x)
```

```
[Out] 2*b*(b*x - a)^(1/2) + (a*(b*x - a)^(1/2))/x - 3*a^(1/2)*b*atan((b*x - a)^(1
/2)/a^(1/2))
```

3.330 $\int \frac{(-a+bx)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=68

$$-\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

```
[Out] -1/2*(b*x-a)^(3/2)/x^2+3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-3/4*b*
(b*x-a)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {43, 65, 211}

$$\frac{3b^2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(bx-a)^{3/2}}{2x^2}-\frac{3b\sqrt{bx-a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[-a + b*x])/(4*x) - (-a + b*x)^(3/2)/(2*x^2) + (3*b^2*ArcTan[Sqrt
[-a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^3}\,dx &= -\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{-a+bx}}{x^2}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 56, normalized size = 0.82

$$\frac{1}{4}\left(\frac{(2a-5bx)\sqrt{-a+bx}}{x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^3,x]
[Out] (((2*a - 5*b*x)*Sqrt[-a + b*x])/x^2 + (3*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
)/Sqrt[a])/4
```

Maple [**A**]
time = 0.11, size = 57, normalized size = 0.84

method	result	size
risch	$-\frac{(-bx+a)(-5bx+2a)}{4x^2\sqrt{bx-a}}+\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}$	52
derivativedivides	$2b^2\left(\frac{-\frac{5(bx-a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	57

default	$2b^2\left(\frac{-\frac{5(bx-a)^{\frac{3}{2}}}{8}-\frac{3a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((-5/8*(b*x-a)^(3/2)-3/8*a*(b*x-a)^(1/2))/b^2/x^2+3/8*arctan((b*x-a)^
(1/2)/a^(1/2))/a^(1/2))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.18

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^2+3\,\sqrt{bx-a}\,ab^2}{4\,\left((bx-a)^2+2\,(bx-a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - 1/4*(5*(b*x - a)^(3/2)*b^2
+ 3*sqrt(b*x - a)*a*b^2)/((b*x - a)^2 + 2*(b*x - a)*a + a^2)
```

Fricas [A]
time = 0.46, size = 129, normalized size = 1.90

$$\left[-\frac{3\,\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,(5\,abx-2\,a^2)\sqrt{bx-a}}{8\,ax^2},\frac{3\,\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(5\,abx-2\,a^2)\sqrt{bx-a}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(sq
rt(b*x - a)/sqrt(a)) - (5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.54, size = 189, normalized size = 2.78

$$\begin{cases} \frac{ia\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{2x^{\frac{3}{2}}}-\frac{5ib^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{4\sqrt{x}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\sqrt{a}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{a^2}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{7a\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{5b^{\frac{3}{2}}}{4\sqrt{x}\,\sqrt{-\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**3,x)
```

```
[Out] Piecewise((I*a*sqrt(b)*sqrt(a/(b*x) - 1)/(2*x**(3/2)) - 5*I*b**(3/2)*sqrt(a
/(b*x) - 1)/(4*sqrt(x)) + 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt
(a)), Abs(a/(b*x)) > 1), (-a**2/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 7
*a*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) - 5*b**(3/2)/(4*sqrt(x)*sqrt(-a/
(b*x) + 1)) - 3*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), True))
```

Giac [A]
time = 0.98, size = 66, normalized size = 0.97

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^3+3\,\sqrt{bx-a}\,ab^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - (5*(b*x - a)^(3/2)*b^3 +
 3*sqrt(b*x - a)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.10, size = 52, normalized size = 0.76

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,{\left(b\,x-a\right)}^{3/2}}{4\,x^2}-\frac{3\,a\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^3,x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(1/2)) - (5*(b*x - a)^(3/2))/(4*
x^2) - (3*a*(b*x - a)^(1/2))/(4*x^2)
```

3.331 $\int \frac{(-a+bx)^{5/2}}{x} \, dx$

Optimal. Leaf size=73

$$2a^2\sqrt{-a+bx} - \frac{2}{3}a(-a+bx)^{3/2} + \frac{2}{5}(-a+bx)^{5/2} - 2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 65, 211}

$$-2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2a^2\sqrt{bx-a} - \frac{2}{3}a(bx-a)^{3/2} + \frac{2}{5}(bx-a)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[-a + b*x] - (2*a*(-a + b*x)^(3/2))/3 + (2*(-a + b*x)^(5/2))/5 -
2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(-a+bx)^{5/2}-a\int \frac{(-a+bx)^{3/2}}{x}\,dx\\ &= -\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}+a^2\int \frac{\sqrt{-a+bx}}{x}\,dx\\ &= 2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-a^3\int \frac{1}{x\sqrt{-a+bx}}\,dx\\ &= 2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-\frac{(2a^3)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a\right.}}{b}\\ &= 2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.82

$$\frac{2}{15}\sqrt{-a+bx}\left(23a^2-11abx+3b^2x^2\right)-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*x]*(23*a^2 - 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.09, size = 58, normalized size = 0.79

method	result	size
derivativedivides	$-\frac{2a(bx-a)^{\frac{3}{2}}}{3}+\frac{2(bx-a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^2\sqrt{bx-a}$	58
default	$-\frac{2a(bx-a)^{\frac{3}{2}}}{3}+\frac{2(bx-a)^{\frac{5}{2}}}{5}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^2\sqrt{bx-a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

Maxima [A]
time = 0.49, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

Fricas [A]
time = 0.50, size = 119, normalized size = 1.63

$$\left[\sqrt{-a}\,a^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}\,-2\,a}{x}\right)+\frac{2}{15}\,(3\,b^2x^2-11\,abx+23\,a^2)\sqrt{bx-a}\,,-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{15}\,(3\,b^2x^2-11\,abx+23\,a^2)\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/15*(3*b^2*x
^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a), -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt
(a)) + 2/15*(3*b^2*x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.27, size = 240, normalized size = 3.29

$$\begin{cases}\frac{46a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+2a^{\frac{5}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{22a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}\,b^2x^2\sqrt{-1+\frac{bx}{a}}}{5} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ \frac{46ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)-\frac{22ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15}+\frac{2i\sqrt{a}\,b^2x^2\sqrt{1-\frac{bx}{a}}}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x,x)
```

```
[Out] Piecewise((46*a**(5/2)*sqrt(-1 + b*x/a)/15 + I*a**(5/2)*log(b*x/a) - 2*I*a*
*(5/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) + 2*a**(5/2)*asin(sqrt(a)/(sqrt(b)*sqrt
(x))) - 22*a**(3/2)*b*x*sqrt(-1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*sqrt(-1 +
 b*x/a)/5, Abs(b*x/a) > 1), (46*I*a**(5/2)*sqrt(1 - b*x/a)/15 + I*a**(5/2)*
log(b*x/a) - 2*I*a**(5/2)*log(sqrt(1 - b*x/a) + 1) - 22*I*a**(3/2)*b*x*sqrt
(1 - b*x/a)/15 + 2*I*sqrt(a)*b**2*x**2*sqrt(1 - b*x/a)/5, True))
```

Giac [A]
time = 0.89, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

Mupad [B]
time = 0.04, size = 57, normalized size = 0.78

$$\frac{2\,{\left(b\,x-a\right)}^{5/2}}{5}-\frac{2\,a\,{\left(b\,x-a\right)}^{3/2}}{3}-2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)+2\,a^2\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x,x)
```

```
[Out] (2*(b*x - a)^(5/2))/5 - (2*a*(b*x - a)^(3/2))/3 - 2*a^(5/2)*atan((b*x - a)^
(1/2)/a^(1/2)) + 2*a^2*(b*x - a)^(1/2)
```

3.332 $\int \frac{(-a+bx)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=74

$$-5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] 5/3*b*(b*x-a)^(3/2)-(b*x-a)^(5/2)/x+5*a^(3/2)*b*arctan((b*x-a)^(1/2)/a^(1/2
))-5*a*b*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {43, 52, 65, 211}

$$5a^{3/2}b\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -5*a*b*Sqrt[-a + b*x] + (5*b*(-a + b*x)^(3/2))/3 - (-a + b*x)^(5/2)/x + 5*a
^(3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{5/2}}{x^2}\,dx &= -\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(-a+bx)^{3/2}}{x}\,dx\\ &= \frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}-\frac{1}{2}(5ab)\int \frac{\sqrt{-a+bx}}{x}\,dx\\ &= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\ &= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{\right.\\ &= -5ab\sqrt{-a+bx}+\frac{5}{3}b(-a+bx)^{3/2}-\frac{(-a+bx)^{5/2}}{x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 0.86

$$-\frac{\sqrt{-a+bx}\,\left(3a^2+14abx-2b^2x^2\right)}{3x}+5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -1/3*(Sqrt[-a + b*x]*(3*a^2 + 14*a*b*x - 2*b^2*x^2))/x + 5*a^(3/2)*b*ArcTan
[Sqrt[-a + b*x]/Sqrt[a]]
```

Maple [A]
time = 0.11, size = 69, normalized size = 0.93

method	result	size

derivativedivides	$2b\left(\frac{(bx-a)^{\frac{3}{2}}}{3}-2a\sqrt{bx-a}+a^2\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	69
default	$2b\left(\frac{(bx-a)^{\frac{3}{2}}}{3}-2a\sqrt{bx-a}+a^2\left(-\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)\right)$	69
risch	$\frac{a^2(-bx+a)}{x\sqrt{bx-a}}+\frac{2b(bx-a)^{\frac{3}{2}}}{3}-4ab\sqrt{bx-a}+5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/3*(b*x-a)^(3/2)-2*a*(b*x-a)^(1/2)+a^2*(-1/2*(b*x-a)^(1/2)/b/x+5/2*ar
ctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 63, normalized size = 0.85

$$5\,a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}b-4\,\sqrt{bx-a}\,ab-\frac{\sqrt{bx-a}\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5*a^(3/2)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2)*b - 4*sqrt(
b*x - a)*a*b - sqrt(b*x - a)*a^2/x
```

Fricas [A]
time = 0.49, size = 131, normalized size = 1.77

$$\left[\frac{15\,\sqrt{-a}\,abx\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,(2\,b^2x^2-14\,abx-3\,a^2)\sqrt{bx-a}}{6\,x},\frac{15\,a^{\frac{3}{2}}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(2\,b^2x^2-14\,abx-3\,a^2)\sqrt{bx-a}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*sqrt(-a)*a*b*x*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(
2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x, 1/3*(15*a^(3/2)*b*x*arctan(
sqrt(b*x - a)/sqrt(a)) + (2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.20, size = 245, normalized size = 3.31

$$\begin{cases} -\frac{a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{x} - \frac{14a^{\frac{3}{2}}b\sqrt{-1+\frac{bx}{a}}}{3} - \frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2} + 5ia^{\frac{3}{2}}b\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - 5a^{\frac{3}{2}}b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right) + \frac{2\sqrt{a}\,b^2x\sqrt{-1+\frac{bx}{a}}}{3} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x} - \frac{14ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3} - \frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2} + 5ia^{\frac{3}{2}}b\log\left(\sqrt{1-\frac{bx}{a}}+1\right) + \frac{2i\sqrt{a}\,b^2x\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**2,x)
```

```
[Out] Piecewise((-a**(5/2)*sqrt(-1 + b*x/a)/x - 14*a**(3/2)*b*sqrt(-1 + b*x/a)/3
- 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqrt(b)*sqrt(x)/sqrt(a))
 - 5*a**(3/2)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*sqrt(a)*b**2*x*sqrt(-1
+ b*x/a)/3, Abs(b*x/a) > 1), (-I*a**(5/2)*sqrt(1 - b*x/a)/x - 14*I*a**(3/2)
*b*sqrt(1 - b*x/a)/3 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqr
t(1 - b*x/a) + 1) + 2*I*sqrt(a)*b**2*x*sqrt(1 - b*x/a)/3, True))
```

Giac [A]
time = 0.91, size = 75, normalized size = 1.01

$$\frac{15\,a^{\frac{3}{2}}b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + 2\,(bx-a)^{\frac{3}{2}}b^2 - 12\,\sqrt{bx-a}\,ab^2 - \frac{3\,\sqrt{bx-a}\,a^2b}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^(3/2)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*(b*x - a)^(3/2)*b^2 -
 12*sqrt(b*x - a)*a*b^2 - 3*sqrt(b*x - a)*a^2*b/x)/b
```

Mupad [B]
time = 0.10, size = 63, normalized size = 0.85

$$\frac{2\,b\,(b\,x-a)^{3/2}}{3} - \frac{a^2\,\sqrt{b\,x-a}}{x} + 5\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right) - 4\,a\,b\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^2,x)
```

```
[Out] (2*b*(b*x - a)^(3/2))/3 - (a^2*(b*x - a)^(1/2))/x + 5*a^(3/2)*b*atan((b*x -
 a)^(1/2)/a^(1/2)) - 4*a*b*(b*x - a)^(1/2)
```

3.333 $\int \frac{(-a+bx)^{5/2}}{x^3} dx$

Optimal. Leaf size=86

$$\frac{15}{4}b^2\sqrt{-a+bx} - \frac{5b(-a+bx)^{3/2}}{4x} - \frac{(-a+bx)^{5/2}}{2x^2} - \frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

```
[Out] -5/4*b*(b*x-a)^(3/2)/x-1/2*(b*x-a)^(5/2)/x^2-15/4*b^2*arctan((b*x-a)^(1/2)/
a^(1/2))*a^(1/2)+15/4*b^2*(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 211}

$$-\frac{15}{4}\sqrt{a}\,b^2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{15}{4}b^2\sqrt{bx-a} - \frac{(bx-a)^{5/2}}{2x^2} - \frac{5b(bx-a)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[-a + b*x])/4 - (5*b*(-a + b*x)^(3/2))/(4*x) - (-a + b*x)^(5/2)
/(2*x^2) - (15*Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x^3}\,dx &= -\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(-a+bx)^{3/2}}{x^2}\,dx\\
&= -\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{1}{4}(15ab)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.\\
&= \frac{15}{4}b^2\sqrt{-a+bx}-\frac{5b(-a+bx)^{3/2}}{4x}-\frac{(-a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 0.78

$$
\frac{1}{4}\left(\frac{\sqrt{-a+bx}\,\left(-2a^2+9abx+8b^2x^2\right)}{x^2}-15\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] ((Sqrt[-a + b*x]*(-2*a^2 + 9*a*b*x + 8*b^2*x^2))/x^2 - 15*Sqrt[a]*b^2*ArcTa
n[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Maple [A]
time = 0.10, size = 70, normalized size = 0.81

method	result	size
risch	$\frac{a(-bx+a)(-9bx+2a)}{4x^2\sqrt{bx-a}}-\frac{15b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\sqrt{a}}{4}+2b^2\sqrt{bx-a}$	67

derivativedivides	$2b^2\left(\sqrt{bx-a}-a\left(\frac{-\frac{9(bx-a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	70
default	$2b^2\left(\sqrt{bx-a}-a\left(\frac{-\frac{9(bx-a)^{\frac{3}{2}}}{8}-\frac{7a\sqrt{bx-a}}{8}}{b^2x^2}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}\right)\right)$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*((b*x-a)^(1/2)-a*((-9/8*(b*x-a)^(3/2)-7/8*a*(b*x-a)^(1/2))/b^2/x^2+15
/8*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.48, size = 97, normalized size = 1.13

$$-\frac{15}{4}\sqrt{a}\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\,b^2+\frac{9(bx-a)^{\frac{3}{2}}ab^2+7\sqrt{bx-a}\,a^2b^2}{4\left((bx-a)^2+2(bx-a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -15/4*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b^2 + 1/4
*(9*(b*x - a)^(3/2)*a*b^2 + 7*sqrt(b*x - a)*a^2*b^2)/((b*x - a)^2 + 2*(b*x
- a)*a + a^2)
```

Fricas [A]
time = 0.47, size = 139, normalized size = 1.62

$$\left[\frac{15\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(8b^2x^2+9abx-2a^2\right)\sqrt{bx-a}}{8x^2},\,-\frac{15\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(8b^2x^2+9abx-2a^2\right)\sqrt{bx-a}}{4x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a))/x^2, -1/4*(15*sqrt(a)*b^2*x^2
*arctan(sqrt(b*x - a)/sqrt(a)) - (8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a
))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.28, size = 267, normalized size = 3.10

$$\begin{cases} -\frac{15i\sqrt{a}\,b^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} - \frac{ia^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{11ia^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{iab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{2ib^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{15\sqrt{a}\,b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4} + \frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**3,x)
```

```
[Out] Piecewise((-15*I*sqrt(a)*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - I*a**3/(
2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 11*I*a**2*sqrt(b)/(4*x**(3/2)*sqrt(
a/(b*x) - 1)) - I*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*b**(5/2)*s
qrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (15*sqrt(a)*b**2*asin(sqrt(a)/
(sqrt(b)*sqrt(x)))/4 + a**3/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 11*a*
*2*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) + a*b**(3/2)/(4*sqrt(x)*sqrt(-a/
(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

Giac [A]
time = 1.30, size = 83, normalized size = 0.97

$$-\frac{15\sqrt{a}\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) - 8\sqrt{bx-a}\,b^3 - \frac{9\,(bx-a)^{\frac{3}{2}}ab^3+7\sqrt{bx-a}\,a^2b^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^3*arctan(sqrt(b*x - a)/sqrt(a)) - 8*sqrt(b*x - a)*b^3 -
(9*(b*x - a)^(3/2)*a*b^3 + 7*sqrt(b*x - a)*a^2*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.09, size = 69, normalized size = 0.80

$$2\,b^2\,\sqrt{b\,x-a}-\frac{15\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4}+\frac{9\,a\,{\left(b\,x-a\right)}^{3/2}}{4\,x^2}+\frac{7\,a^2\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(b*x - a)^(1/2) - (15*a^(1/2)*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/4 +
(9*a*(b*x - a)^(3/2))/(4*x^2) + (7*a^2*(b*x - a)^(1/2))/(4*x^2)
```

3.334 $\int \frac{x^4}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}+\frac{2(a+bx)^{9/2}}{9b^5}$$

```
[Out] -8/3*a^3*(b*x+a)^(3/2)/b^5+12/5*a^2*(b*x+a)^(5/2)/b^5-8/7*a*(b*x+a)^(7/2)/b
^5+2/9*(b*x+a)^(9/2)/b^5+2*a^4*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a +
b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))/(
9*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^4}{b^4\sqrt{a+bx}}-\frac{4a^3\sqrt{a+bx}}{b^4}+\frac{6a^2(a+bx)^{3/2}}{b^4}-\frac{4a(a+bx)^{5/2}}{b^4}+\frac{(a+bx)^{7/2}}{b^4}\right)dx\\ &= \frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}+\frac{2(a+bx)^{9/2}}{9b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\,(128a^4-64a^3bx+48a^2b^2x^2-40ab^3x^3+35b^4x^4)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4 - 64*a^3*b*x + 48*a^2*b^2*x^2 - 40*a*b^3*x^3 + 35
*b^4*x^4))/(315*b^5)
```

Maple [A]
time = 0.09, size = 61, normalized size = 0.69

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
trager	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
risch	$\frac{2\sqrt{bx+a}\,(35b^4x^4-40a\,b^3x^3+48a^2b^2x^2-64a^3bx+128a^4)}{315b^5}$	54
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{8a(bx+a)^{\frac{7}{2}}}{7}+\frac{12a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{3}{2}}}{3}+2a^4\sqrt{bx+a}}{b^5}$	61
default	$\frac{\frac{2(bx+a)^{\frac{9}{2}}}{9}-\frac{8a(bx+a)^{\frac{7}{2}}}{7}+\frac{12a^2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a^3(bx+a)^{\frac{3}{2}}}{3}+2a^4\sqrt{bx+a}}{b^5}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/9*(b*x+a)^(9/2)-4/7*a*(b*x+a)^(7/2)+6/5*a^2*(b*x+a)^(5/2)-4/3*a^3*
(b*x+a)^(3/2)+a^4*(b*x+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^5}-\frac{8\,(bx+a)^{\frac{7}{2}}a}{7\,b^5}+\frac{12\,(bx+a)^{\frac{5}{2}}a^2}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a^3}{3\,b^5}+\frac{2\,\sqrt{bx+a}\,a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^5 - 8/7*(b*x + a)^(7/2)*a/b^5 + 12/5*(b*x + a)^(5/2)*
a^2/b^5 - 8/3*(b*x + a)^(3/2)*a^3/b^5 + 2*sqrt(b*x + a)*a^4/b^5
```

Fricas [A]
time = 0.47, size = 53, normalized size = 0.60

$$\frac{2\,(35\,b^4x^4-40\,ab^3x^3+48\,a^2b^2x^2-64\,a^3bx+128\,a^4)\sqrt{bx+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3755 vs. $2(85) = 170$.
time = 2.37, size = 3755, normalized size = 42.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(1/2),x)
```

```
[Out] 256*a**(89/2)*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a
**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**3
5*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**3
2*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(89/
2)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10) + 2432*a**(87/2)*b*x*sqrt(1 + b*x/a)/(31
5*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8
*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*
x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x
**9 + 315*a**30*b**15*x**10) - 2560*a**(87/2)*b*x/(315*a**40*b**5 + 3150*a*
*39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b*
*9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**
12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*
x**10) + 10336*a**(85/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a
**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b
**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b*
*12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15
*x**10) - 11520*a**(85/2)*b**2*x**2/(315*a**40*b**5 + 3150*a**39*b**6*x + 1
4175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 7938
0*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 1417
5*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 25840
*a**(83/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x +
14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 793
80*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 141
75*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 3072
0*a**(83/2)*b**3*x**3/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**
7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*
x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*
x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 41990*a**(81/2)*b**
4*x**4*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b*
```

```
*7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10
*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13
*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 53760*a**(81/2)*b*
*4*x**4/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800
*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a
**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a*
*31*b**14*x**9 + 315*a**30*b**15*x**10) + 46252*a**(79/2)*b**5*x**5*sqrt(1
+ b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 3780
0*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*
a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a
**31*b**14*x**9 + 315*a**30*b**15*x**10) - 64512*a**(79/2)*b**5*x**5/(315*a
**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x*
*3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**
6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9
 + 315*a**30*b**15*x**10) + 35214*a**(77/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*
a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x
**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x*
*6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**
9 + 315*a**30*b**15*x**10) - 53760*a**(77/2)*b**6*x**6/(315*a**40*b**5 + 31
50*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**
36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**3
3*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b
**15*x**10) + 19632*a**(75/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3
150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a*
*36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**
33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*
b**15*x**10) - 30720*a**(75/2)*b**7*x**7/(315*a**40*b**5 + 3150*a**39*b**6*
x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 +
 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 +
 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) +
10860*a**(73/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6
*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4
+ 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7
+ 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) -
 11520*a**(73/2)*b**8*x**8/(315*a**40*b**5 + 31...
```

Giac [A]
time = 1.16, size = 61, normalized size = 0.69

$$\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)}{315\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

Mupad [B]
time = 0.02, size = 71, normalized size = 0.80

$$\frac{2\,{\left(a+b\,x\right)}^{9/2}}{9\,b^5}+\frac{2\,a^4\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a^3\,{\left(a+b\,x\right)}^{3/2}}{3\,b^5}+\frac{12\,a^2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^5) + (2*a^4*(a + b*x)^(1/2))/b^5 - (8*a^3*(a + b*x
)^(3/2))/(3*b^5) + (12*a^2*(a + b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))
/(7*b^5)
```

3.335 $\int \frac{x^3}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}$$

```
[Out] 2*a^2*(b*x+a)^(3/2)/b^4-6/5*a*(b*x+a)^(5/2)/b^4+2/7*(b*x+a)^(7/2)/b^4-2*a^3
*(b*x+a)^(1/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^(
5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right)dx\\ &= -\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.68

$$\frac{2\sqrt{a+bx}\,(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4)
```

Maple [A]
time = 0.09, size = 49, normalized size = 0.72

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$	43
trager	$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$	43
risch	$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$	43
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{6a(bx+a)^{\frac{5}{2}}}{5}+2a^{2}(bx+a)^{\frac{3}{2}}-2a^{3}\sqrt{bx+a}}{b^{4}}$	49
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{6a(bx+a)^{\frac{5}{2}}}{5}+2a^{2}(bx+a)^{\frac{3}{2}}-2a^{3}\sqrt{bx+a}}{b^{4}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/7*(b*x+a)^(7/2)-3/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)-a^3*(b*x+a)^
(1/2))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{4}}-\frac{6\,(bx+a)^{\frac{5}{2}}a}{5\,b^{4}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{b^{4}}-\frac{2\,\sqrt{bx+a}\,a^{3}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^4 - 6/5*(b*x + a)^(5/2)*a/b^4 + 2*(b*x + a)^(3/2)*a^2
/b^4 - 2*sqrt(b*x + a)*a^3/b^4
```

Fricas [A]
time = 0.46, size = 42, normalized size = 0.62

$$\frac{2\,(5\,b^{3}x^{3}-6\,ab^{2}x^{2}+8\,a^{2}bx-16\,a^{3})\sqrt{bx+a}}{35\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(65) = 130$.
time = 1.24, size = 1640, normalized size = 24.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(47/2)*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18
*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**
5 + 35*a**14*b**10*x**6) + 32*a**(47/2)/(35*a**20*b**4 + 210*a**19*b**5*x +
 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**1
5*b**9*x**5 + 35*a**14*b**10*x**6) - 176*a**(45/2)*b*x*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(45/2)*b*x/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a*
*17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*
x**6) - 396*a**(43/2)*b**2*x**2*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*
b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 +
210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 480*a**(43/2)*b**2*x**2/(35*a*
*20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 5
25*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 462*a**(4
1/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6) + 640*a**(41/2)*b**3*x**3/(35*a**20*b**4 + 210*a*
*19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**
4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 280*a**(39/2)*b**4*x**4*sq
rt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700
*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**
10*x**6) + 480*a**(39/2)*b**4*x**4/(35*a**20*b**4 + 210*a**19*b**5*x + 525*
a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**
9*x**5 + 35*a**14*b**10*x**6) - 42*a**(37/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(37/2)*b**5*x**5/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 +
700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*
b**10*x**6) + 84*a**(35/2)*b**6*x**6*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a
**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x*
*4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 32*a**(35/2)*b**6*x**6/(3
5*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3
 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 94*a*
*(33/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a
```

```
**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9
*x**5 + 35*a**14*b**10*x**6) + 48*a**(31/2)*b**8*x**8*sqrt(1 + b*x/a)/(35*a
**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 10*a**(2
9/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6)
```

Giac [A]
time = 0.89, size = 49, normalized size = 0.72

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.82

$$\frac{2\,(a+b\,x)^{7/2}}{7\,b^4}-\frac{2\,a^3\,\sqrt{a+b\,x}}{b^4}+\frac{2\,a^2\,(a+b\,x)^{3/2}}{b^4}-\frac{6\,a\,(a+b\,x)^{5/2}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^4) - (2*a^3*(a + b*x)^(1/2))/b^4 + (2*a^2*(a + b*x
)^(3/2))/b^4 - (6*a*(a + b*x)^(5/2))/(5*b^4)
```

3.336 $\int \frac{x^2}{\sqrt{a+bx}} dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3}$$

```
[Out] -4/3*a*(b*x+a)^(3/2)/b^3+2/5*(b*x+a)^(5/2)/b^3+2*a^2*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2a^2\sqrt{a+bx}}{b^3} + \frac{2(a+bx)^{5/2}}{5b^3} - \frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx}} dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}} - \frac{2a\sqrt{a+bx}}{b^2} + \frac{(a+bx)^{3/2}}{b^2}\right) dx \\ &= \frac{2a^2\sqrt{a+bx}}{b^3} - \frac{4a(a+bx)^{3/2}}{3b^3} + \frac{2(a+bx)^{5/2}}{5b^3} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\,(8a^2 - 4abx + 3b^2x^2)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Maple [A]
time = 0.09, size = 37, normalized size = 0.73

method	result	size
gosper	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4abx+8a^2)}{15b^3}$	32
trager	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4abx+8a^2)}{15b^3}$	32
risch	$\frac{2\sqrt{bx+a}\,(3x^2b^2-4abx+8a^2)}{15b^3}$	32
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{4a(bx+a)^{\frac{3}{2}}}{3}+2a^2\sqrt{bx+a}}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(1/5*(b*x+a)^(5/2)-2/3*a*(b*x+a)^(3/2)+a^2*(b*x+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}\,a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

Fricas [A]
time = 0.42, size = 31, normalized size = 0.61

$$\frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(48) = 96$.
time = 0.82, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Giac [A]
time = 0.72, size = 37, normalized size = 0.73

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,{\left(a+b\,x\right)}^{5/2}-20\,a\,{\left(a+b\,x\right)}^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/2),x)
```

```
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

3.337 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$-\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b^2-2*a*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\ &= -\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.72

$$\frac{2(-2a+bx)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
risch	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-2a\sqrt{bx+a}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(1/3*(b*x+a)^(3/2)-a*(b*x+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 26, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2}-\frac{2\,\sqrt{bx+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Fricas [A]
time = 0.47, size = 19, normalized size = 0.59

$$\frac{2\,\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.

time = 0.54, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x}+\frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Giac [A]

time = 0.68, size = 23, normalized size = 0.72

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

Mupad [B]

time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6\,a\,\sqrt{a+b\,x}-2\,{(a+b\,x)}^{3/2}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/2),x)
```

```
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

3.338 $\int \frac{1}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

```
[Out] 2*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{2\sqrt{bx+a}}{b}$	13
derivativedivides	$\frac{2\sqrt{bx+a}}{b}$	13
default	$\frac{2\sqrt{bx+a}}{b}$	13
trager	$\frac{2\sqrt{bx+a}}{b}$	13
risch	$\frac{2\sqrt{bx+a}}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A]
time = 0.41, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

Giac [A]
time = 1.30, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{a+b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2))/b
```

3.339 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.08, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.48, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.39, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A]
time = 0.43, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A]
time = 1.60, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.74

$$-\frac{2\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.340 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{a+bx}}{ax}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)-(b*x+a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 214}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{ax} - \frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}$$

$$= -\frac{\sqrt{a+bx}}{ax} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a}$$

$$= -\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{ax} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.11, size = 40, normalized size = 0.98

method	result	size
risch	$\frac{b\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{\sqrt{bx+a}}{ax}$	34
derivativedivides	$2b\left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40
default	$2b\left(-\frac{\sqrt{bx+a}}{2abx} + \frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.48, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a}\,b}{(bx+a)a-a^2}-\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

Fricas [A]
time = 0.45, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,a}{2\,a^2x},-\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A]
time = 1.08, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

Giac [A]
time = 1.15, size = 47, normalized size = 1.15

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\sqrt{bx+a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

Mupad [B]
time = 0.11, size = 33, normalized size = 0.80

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\,x}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

3.341 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

```
[Out] -3/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)-1/2*(b*x+a)^(1/2)/a/x^2+3/4
*b*(b*x+a)^(1/2)/a^2/x
```

Rubi [A]

time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 214}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] -1/2*Sqrt[a + b*x]/(a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{2ax^2}-\frac{(3b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a}\\
&= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}+\frac{(3b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^2}\\
&= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}+\frac{(3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a^2}\\
&= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.82

$$
\frac{\sqrt{a+bx}\,(-2a+3bx)}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x]),x]
[Out] (Sqrt[a + b*x]*(-2*a + 3*b*x))/(4*a^2*x^2) - (3*b^2*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(4*a^(5/2))
```

Maple [A]
time = 0.11, size = 66, normalized size = 0.97

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3bx+2a)}{4a^2x^2}-\frac{3b^2\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	45
derivativedivides	$2b^2\left(-\frac{\sqrt{bx+a}}{4a\,b^2x^2}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66

default	$2b^{2}\left(-\frac{\sqrt{bx+a}}{4a\,b^{2}x^{2}}-\frac{3\left(-\frac{\sqrt{bx+a}}{2abx}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/4*(b*x+a)^(1/2)/a/b^2/x^2-3/4/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)))
```

Maxima [A]
time = 0.48, size = 92, normalized size = 1.35

$$\frac{3\,b^{2}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{3}{2}}b^{2}-5\,\sqrt{bx+a}\,ab^{2}}{4\left((bx+a)^{2}a^{2}-2\,(bx+a)a^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
1/4*(3*(b*x + a)^(3/2)*b^2 - 5*sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a^2 - 2*(b
*x + a)*a^3 + a^4)
```

Fricas [A]
time = 0.45, size = 123, normalized size = 1.81

$$\left[\frac{3\,\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx-2\,a^{2})\sqrt{bx+a}}{8\,a^{3}x^{2}},\ \frac{3\,\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx-2\,a^{2})\sqrt{bx+a}}{4\,a^{3}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

Sympy [A]
time = 2.47, size = 102, normalized size = 1.50

$$-\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(b
*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

Giac [A]
time = 1.01, size = 69, normalized size = 1.01

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx+a}\,ab^3}{a^2b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.06, size = 51, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{3/2}}{4\,a^2\,x^2}-\frac{5\,\sqrt{a+b\,x}}{4\,a\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/2)),x)
```

```
[Out] (3*(a + b*x)^(3/2))/(4*a^2*x^2) - (5*(a + b*x)^(1/2))/(4*a*x^2) - (3*b^2*at
anh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

3.342 $\int \frac{1}{x^4\sqrt{a+bx}}\,dx$

Optimal. Leaf size=90

$$-\frac{\sqrt{a+bx}}{3ax^3}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

```
[Out] 5/8*b^3*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)-1/3*(b*x+a)^(1/2)/x^3/a+5/12
*b*(b*x+a)^(1/2)/a^2/x^2-5/8*b^2*(b*x+a)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 214}

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}-\frac{5b^2\sqrt{a+bx}}{8a^3x}+\frac{5b\sqrt{a+bx}}{12a^2x^2}-\frac{\sqrt{a+bx}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -1/3*Sqrt[a + b*x]/(a*x^3) + (5*b*Sqrt[a + b*x])/(12*a^2*x^2) - (5*b^2*Sqrt
[a + b*x])/(8*a^3*x) + (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{3ax^3} - \frac{(5b)\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{6a} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} + \frac{(5b^2)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a^2} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{(5b^3)\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^3} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{(5b^2)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{8a^3} \\
&= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.74

$$-\frac{\sqrt{a+bx}\,(8a^2-10abx+15b^2x^2)}{24a^3x^3} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(8*a^2 - 10*a*b*x + 15*b^2*x^2))/(a^3*x^3) + (5*b^3*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Maple [A]
time = 0.10, size = 90, normalized size = 1.00

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(15x^2b^2-10abx+8a^2)}{24a^3x^3}+\frac{5b^3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{7}{2}}}$	56

derivativedivides	$2b^{3}\left(-\frac{\sqrt{bx+a}}{6a\,b^{3}x^{3}}+\frac{\frac{5\sqrt{bx+a}}{24a\,b^{2}x^{2}}+\frac{5\left(-\frac{3\sqrt{bx+a}}{8abx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}}{a}\right)$	90
default	$2b^{3}\left(-\frac{\sqrt{bx+a}}{6a\,b^{3}x^{3}}+\frac{\frac{5\sqrt{bx+a}}{24a\,b^{2}x^{2}}+\frac{5\left(-\frac{3\sqrt{bx+a}}{8abx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{6a}}{a}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3*(-1/6*(b*x+a)^(1/2)/a/b^3/x^3+5/6/a*(1/4*(b*x+a)^(1/2)/a/b^2/x^2+3/4/
a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))))
```

Maxima [A]
time = 0.48, size = 121, normalized size = 1.34

$$-\frac{5\,b^{3}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{5}{2}}b^{3}-40\,(bx+a)^{\frac{3}{2}}ab^{3}+33\,\sqrt{bx+a}\,a^{2}b^{3}}{24\left((bx+a)^{3}a^{3}-3\,(bx+a)^{2}a^{4}+3\,(bx+a)a^{5}-a^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
- 1/24*(15*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 33*sqrt(b*x + a
)*a^2*b^3)/((b*x + a)^3*a^3 - 3*(b*x + a)^2*a^4 + 3*(b*x + a)*a^5 - a^6)
```

Fricas [A]
time = 0.42, size = 145, normalized size = 1.61

$$\left[\frac{15\,\sqrt{a}\,b^{3}x^{3}\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3})\sqrt{bx+a}}{48\,a^{4}x^{3}},-\frac{15\,\sqrt{-a}\,b^{3}x^{3}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3})\sqrt{bx+a}}{24\,a^{4}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

Sympy [A]
time = 7.66, size = 129, normalized size = 1.43

$$-\frac{1}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{3}{2}}}{24a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{5}{2}}}{8a^{3}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**(1/2),x)
```

```
[Out] -1/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(12*a*x**(5/2)*sqrt(a/(
b*x) + 1)) - 5*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(5/2)/(
8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/(8*a**(7/2))
```

Giac [A]
time = 1.36, size = 84, normalized size = 0.93

$$-\frac{\frac{15\,b^{4}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}}+\frac{15\,(bx+a)^{\frac{5}{2}}b^{4}-40\,(bx+a)^{\frac{3}{2}}ab^{4}+33\,\sqrt{bx+a}\,a^{2}b^{4}}{a^{3}b^{3}x^{3}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

Mupad [B]
time = 0.05, size = 69, normalized size = 0.77

$$\frac{5\,{\left(a+b\,x\right)}^{3/2}}{3\,a^2\,x^3}-\frac{11\,\sqrt{a+b\,x}}{8\,a\,x^3}-\frac{5\,{\left(a+b\,x\right)}^{5/2}}{8\,a^3\,x^3}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^(1/2)),x)
```

```
[Out] (5*(a + b*x)^(3/2))/(3*a^2*x^3) - (11*(a + b*x)^(1/2))/(8*a*x^3) - (5*(a +
b*x)^(5/2))/(8*a^3*x^3) - (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^
(7/2))
```

3.343 $\int \frac{x^4}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

```
[Out] 4*a^2*(b*x+a)^(3/2)/b^5-8/5*a*(b*x+a)^(5/2)/b^5+2/7*(b*x+a)^(7/2)/b^5-2*a^4
/b^5/(b*x+a)^(1/2)-8*a^3*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x)
^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{3/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.67

$$\frac{2(-128a^4-64a^3bx+16a^2b^2x^2-8ab^3x^3+5b^4x^4)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-128*a^4 - 64*a^3*b*x + 16*a^2*b^2*x^2 - 8*a*b^3*x^3 + 5*b^4*x^4))/(35*
b^5*Sqrt[a + b*x])
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.73

method	result	size
gosper	$-\frac{2\left(-5 b^{4} x^{4}+8 a \,b^{3} x^{3}-16 a^{2} b^{2} x^{2}+64 a^{3} b x +128 a^{4}\right)}{35 \sqrt{b x +a}\, b^{5}}$	54
trager	$-\frac{2\left(-5 b^{4} x^{4}+8 a \,b^{3} x^{3}-16 a^{2} b^{2} x^{2}+64 a^{3} b x +128 a^{4}\right)}{35 \sqrt{b x +a}\, b^{5}}$	54
risch	$-\frac{2\left(-5 b^{3} x^{3}+13 a \,b^{2} x^{2}-29 a^{2} b x +93 a^{3}\right) \sqrt{b x +a}}{35 b^{5}}-\frac{2 a^{4}}{b^{5} \sqrt{b x +a}}$	59
derivativedivides	$\frac{\frac{2 \left(b x +a \right)^{\frac{7}{2}}}{7}-\frac{8 a \left(b x +a \right)^{\frac{5}{2}}}{5}+4 a^{2} \left(b x +a \right)^{\frac{3}{2}}-8 a^{3} \sqrt{b x +a}-\frac{2 a^{4}}{\sqrt{b x +a}}}{b^{5}}$	62
default	$\frac{\frac{2 \left(b x +a \right)^{\frac{7}{2}}}{7}-\frac{8 a \left(b x +a \right)^{\frac{5}{2}}}{5}+4 a^{2} \left(b x +a \right)^{\frac{3}{2}}-8 a^{3} \sqrt{b x +a}-\frac{2 a^{4}}{\sqrt{b x +a}}}{b^{5}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^5*(1/7*(b*x+a)^(7/2)-4/5*a*(b*x+a)^(5/2)+2*a^2*(b*x+a)^(3/2)-4*a^3*(b*x
+a)^(1/2)-a^4/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^5}-\frac{8\,(bx+a)^{\frac{5}{2}}a}{5\,b^5}+\frac{4\,(bx+a)^{\frac{3}{2}}a^2}{b^5}-\frac{8\,\sqrt{bx+a}\,a^3}{b^5}-\frac{2\,a^4}{\sqrt{bx+a}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^5 - 8/5*(b*x + a)^(5/2)*a/b^5 + 4*(b*x + a)^(3/2)*a^2
/b^5 - 8*sqrt(b*x + a)*a^3/b^5 - 2*a^4/(sqrt(b*x + a)*b^5)
```

Fricas [A]
time = 0.42, size = 63, normalized size = 0.74

$$\frac{2\,(5\,b^4x^4-8\,ab^3x^3+16\,a^2b^2x^2-64\,a^3bx-128\,a^4)\sqrt{bx+a}}{35\,(b^6x+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3606 vs. $2(82) = 164$.
time = 2.01, size = 3606, normalized size = 42.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(3/2),x)
```

```
[Out] -256*a**(87/2)*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**
38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**
10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*
x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(87/2)/(35*a**
40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 +
7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200
*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30
*b**15*x**10) - 2432*a**(85/2)*b*x*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**
39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x
**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 2
560*a**(85/2)*b*x/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2
+ 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 735
0*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a*
*31*b**14*x**9 + 35*a**30*b**15*x**10) - 10336*a**(83/2)*b**2*x**2*sqrt(1 +
 b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**
37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b*
*11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*
x**9 + 35*a**30*b**15*x**10) + 11520*a**(83/2)*b**2*x**2/(35*a**40*b**5 + 3
50*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*
b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**1
2*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**1
0) - 25840*a**(81/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b
**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4
+ 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1
575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 30720
*a**(81/2)*b**3*x**3/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x*
*2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 +
7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350
*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 41990*a**(79/2)*b**4*x**4*sqrt(
```

```
1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*
a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34
*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**
14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(79/2)*b**4*x**4/(35*a**40*b**5
+ 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**
36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b
**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x
**10) - 46182*a**(77/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**3
9*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x*
*4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
+ 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 64
512*a**(77/2)*b**5*x**5/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7
*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5
 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 +
350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 34584*a**(75/2)*b**6*x**6*sq
rt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 42
00*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a*
*34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*
b**14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(75/2)*b**6*x**6/(35*a**40*b*
*5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*
a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**3
3*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**1
5*x**10) - 17112*a**(73/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 30720*a**(73/2)*b**7*x**7/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b
**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x
**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 4980*a**(71/2)*b**8*x**8*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 11520*a**(71/2)*b**8*x**8/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a*...
```

Giac [A]
time = 1.37, size = 77, normalized size = 0.91

$$-\frac{2\,a^4}{\sqrt{bx+a}\,b^5}+\frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30}-28\,(bx+a)^{\frac{5}{2}}ab^{30}+70\,(bx+a)^{\frac{3}{2}}a^2b^{30}-140\,\sqrt{bx+a}\,a^3b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.84

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b^5}-\frac{8\,a^3\,\sqrt{a+b\,x}}{b^5}+\frac{4\,a^2\,{\left(a+b\,x\right)}^{3/2}}{b^5}-\frac{2\,a^4}{b^5\,\sqrt{a+b\,x}}-\frac{8\,a\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^5) - (8*a^3*(a + b*x)^(1/2))/b^5 + (4*a^2*(a + b*x
)^(3/2))/b^5 - (2*a^4)/(b^5*(a + b*x)^(1/2)) - (8*a*(a + b*x)^(5/2))/(5*b^5
)
```

3.344 $\int \frac{x^3}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

```
[Out] -2*a*(b*x+a)^(3/2)/b^4+2/5*(b*x+a)^(5/2)/b^4+2*a^3/b^4/(b*x+a)^(1/2)+6*a^2*
(b*x+a)^(1/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(3
/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{3/2}}\,dx&=\int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx\\&=\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.68

$$\frac{2(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(3/2),x]
[Out] (2*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[a + b*x])
```

Maple [A]
time = 0.11, size = 49, normalized size = 0.74

method	result	size
gosper	$\frac{\frac{2}{5}b^{3}x^{3}-\frac{4}{5}a\,b^{2}x^{2}+\frac{16}{5}a^{2}bx+\frac{32}{5}a^{3}}{b^{4}\sqrt{bx+a}}$	42
trager	$\frac{\frac{2}{5}b^{3}x^{3}-\frac{4}{5}a\,b^{2}x^{2}+\frac{16}{5}a^{2}bx+\frac{32}{5}a^{3}}{b^{4}\sqrt{bx+a}}$	42
risch	$\frac{2\left(x^{2}b^{2}-3abx+11a^{2}\right)\sqrt{bx+a}}{5b^{4}}+\frac{2a^{3}}{b^{4}\sqrt{bx+a}}$	47
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a(bx+a)^{\frac{3}{2}}+6a^{2}\sqrt{bx+a}+\frac{2a^{3}}{\sqrt{bx+a}}}{b^{4}}$	49
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-2a(bx+a)^{\frac{3}{2}}+6a^{2}\sqrt{bx+a}+\frac{2a^{3}}{\sqrt{bx+a}}}{b^{4}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b^4*(1/5*(b*x+a)^(5/2)-a*(b*x+a)^(3/2)+3*a^2*(b*x+a)^(1/2)+a^3/(b*x+a)^(1
/2))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.85

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^{4}}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{b^{4}}+\frac{6\,\sqrt{bx+a}\,a^{2}}{b^{4}}+\frac{2\,a^{3}}{\sqrt{bx+a}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] 2/5*(b*x + a)^(5/2)/b^4 - 2*(b*x + a)^(3/2)*a/b^4 + 6*sqrt(b*x + a)*a^2/b^4
 + 2*a^3/(sqrt(b*x + a)*b^4)
```

Fricas [A]
time = 0.44, size = 51, normalized size = 0.77

$$\frac{2\,(b^{3}x^{3}-2\,ab^{2}x^{2}+8\,a^{2}bx+16\,a^{3})\sqrt{bx+a}}{5\,(b^{5}x+ab^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(63) = 126$.
time = 1.25, size = 1538, normalized size = 23.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(3/2),x)
```

```
[Out] 32*a**(45/2)*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**
6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*
a**14*b**10*x**6) - 32*a**(45/2)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18
*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5
+ 5*a**14*b**10*x**6) + 176*a**(43/2)*b*x*sqrt(1 + b*x/a)/(5*a**20*b**4 + 3
0*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x
**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 192*a**(43/2)*b*x/(5*a**20
*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**
16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 396*a**(41/2)*b**
2*x**2*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) - 480*a**(41/2)*b**2*x**2/(5*a**20*b**4 + 30*a**19*b**5*x + 75*
a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*
x**5 + 5*a**14*b**10*x**6) + 462*a**(39/2)*b**3*x**3*sqrt(1 + b*x/a)/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 640*a**(39/2)*b
**3*x**3/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b
**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) +
290*a**(37/2)*b**4*x**4*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 7
5*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**
9*x**5 + 5*a**14*b**10*x**6) - 480*a**(37/2)*b**4*x**4/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 92*a**(35/2)*b**5*x**5*sqrt(1
 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*
b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) -
 192*a**(35/2)*b**5*x**5/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x*
*2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**1
4*b**10*x**6) + 16*a**(33/2)*b**6*x**6*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 32*a**(33/2)*b**6*x**6/(5*a**
```

```
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 6*a**(31/2)*b**
7*x**7*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) + 2*a**(29/2)*b**8*x**8*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**1
9*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 +
30*a**15*b**9*x**5 + 5*a**14*b**10*x**6)
```

Giac [A]
time = 1.20, size = 61, normalized size = 0.92

$$\frac{2\,a^3}{\sqrt{bx+a}\,b^4}+\frac{2\left((bx+a)^{\frac{5}{2}}b^{16}-5\,(bx+a)^{\frac{3}{2}}ab^{16}+15\,\sqrt{bx+a}\,a^2b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.85

$$\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^4}+\frac{6\,a^2\,\sqrt{a+b\,x}}{b^4}+\frac{2\,a^3}{b^4\,\sqrt{a+b\,x}}-\frac{2\,a\,{\left(a+b\,x\right)}^{3/2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(1/2))/b^4 + (2*a^3)/(b^4*(a
 + b*x)^(1/2)) - (2*a*(a + b*x)^(3/2))/b^4
```

3.345 $\int \frac{x^2}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b^3-2*a^2/b^3/(b*x+a)^(1/2)-4*a*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2)
)/(3*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{3/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx\\ &= -\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.69

$$\frac{2(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(3/2),x]
[Out] (2*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[a + b*x])
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.78

method	result	size
gosper	$-\frac{2\left(-x^{2} b^{2}+4 a b x+8 a^{2}\right)}{3 \sqrt{b x+a}\, b^{3}}$	32
trager	$-\frac{2\left(-x^{2} b^{2}+4 a b x+8 a^{2}\right)}{3 \sqrt{b x+a}\, b^{3}}$	32
risch	$-\frac{2(-b x+5 a) \sqrt{b x+a}}{3 b^{3}}-\frac{2 a^{2}}{b^{3} \sqrt{b x+a}}$	37
derivativedivides	$\frac{\frac{2(b x+a)^{\frac{3}{2}}}{3}-4 a \sqrt{b x+a}-\frac{2 a^{2}}{\sqrt{b x+a}}}{b^{3}}$	38
default	$\frac{\frac{2(b x+a)^{\frac{3}{2}}}{3}-4 a \sqrt{b x+a}-\frac{2 a^{2}}{\sqrt{b x+a}}}{b^{3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b^3*(1/3*(b*x+a)^(3/2)-2*a*(b*x+a)^(1/2)-a^2/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^{3}}-\frac{4\,\sqrt{bx+a}\,a}{b^{3}}-\frac{2\,a^{2}}{\sqrt{bx+a}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b^3 - 4*sqrt(b*x + a)*a/b^3 - 2*a^2/(sqrt(b*x + a)*b^3)
```

Fricas [A]
time = 0.42, size = 40, normalized size = 0.82

$$\frac{2\,(b^{2}x^{2}-4\,abx-8\,a^{2})\sqrt{bx+a}}{3\,(b^{4}x+ab^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(46) = 92$.
time = 0.87, size = 534, normalized size = 10.90

$$-\frac{16a^{\frac{19}{2}}\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{16a^{\frac{19}{2}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{40a^{\frac{17}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{48a^{\frac{17}{2}}bx}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{30a^{\frac{15}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{48a^{\frac{15}{2}}b^2x^2}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}-\frac{4a^{\frac{13}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{16a^{\frac{13}{2}}b^3x^3}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}+\frac{2a^{\frac{11}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(3/2),x)
```

```
[Out] -16*a**(19/2)*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x*
*2 + 3*a**5*b**6*x**3) + 16*a**(19/2)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6
*b**5*x**2 + 3*a**5*b**6*x**3) - 40*a**(17/2)*b*x*sqrt(1 + b*x/a)/(3*a**8*b
**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(17/2)*b
*x/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 30
*a**(15/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*
b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(15/2)*b**2*x**2/(3*a**8*b**3 + 9*a**
7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 4*a**(13/2)*b**3*x**3*sqr
t(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*
x**3) + 16*a**(13/2)*b**3*x**3/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x
**2 + 3*a**5*b**6*x**3) + 2*a**(11/2)*b**4*x**4*sqrt(1 + b*x/a)/(3*a**8*b**
3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
```

Giac [A]
time = 1.33, size = 46, normalized size = 0.94

$$-\frac{2\,a^2}{\sqrt{bx+a}\,b^3}+\frac{2\left((bx+a)^{\frac{3}{2}}b^6-6\,\sqrt{bx+a}\,ab^6\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$-\frac{12\,a\,(a+b\,x)-2\,(a+b\,x)^2+6\,a^2}{3\,b^3\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(3/2),x)
```

```
[Out] -(12*a*(a + b*x) - 2*(a + b*x)^2 + 6*a^2)/(3*b^3*(a + b*x)^(1/2))
```

3.346 $\int \frac{x}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

[Out] 2*a/b^2/(b*x+a)^(1/2)+2*(b*x+a)^(1/2)/b^2

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x)^(3/2),x]

[Out] (2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx\\ &= \frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 21, normalized size = 0.70

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(3/2),x]
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.77

method	result	size
gosper	$\frac{2bx+4a}{b^2\sqrt{bx+a}}$	20
trager	$\frac{2bx+4a}{b^2\sqrt{bx+a}}$	20
derivativedivides	$\frac{2\sqrt{bx+a}+\frac{2a}{\sqrt{bx+a}}}{b^2}$	23
default	$\frac{2\sqrt{bx+a}+\frac{2a}{\sqrt{bx+a}}}{b^2}$	23
risch	$\frac{2a}{b^2\sqrt{bx+a}}+\frac{2\sqrt{bx+a}}{b^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*((b*x+a)^(1/2)+a/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.87

$$\frac{2\sqrt{bx+a}}{b^2}+\frac{2a}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] 2*sqrt(b*x + a)/b^2 + 2*a/(sqrt(b*x + a)*b^2)
```

Fricas [A]
time = 0.50, size = 29, normalized size = 0.97

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.26, size = 37, normalized size = 1.23

$$\begin{cases} \frac{4a}{b^2\sqrt{a+bx}} + \frac{2x}{b\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((4*a/(b**2*sqrt(a + b*x)) + 2*x/(b*sqrt(a + b*x)), Ne(b, 0)), (x*
*2/(2*a**(3/2)), True))
```

Giac [A]
time = 0.81, size = 29, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{bx+a}}{b} + \frac{a}{\sqrt{bx+a}\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

Mupad [B]
time = 0.09, size = 19, normalized size = 0.63

$$\frac{4\,a+2\,b\,x}{b^2\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(3/2),x)
```

```
[Out] (4*a + 2*b*x)/(b^2*(a + b*x)^(1/2))
```

3.347 $\int \frac{1}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{b\sqrt{a+bx}}$$

```
[Out] -2/b/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}}\,dx = -\frac{2}{b\sqrt{a+bx}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3/2),x]
```

```
[Out] -2/(b*Sqrt[a + b*x])
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{2}{b\sqrt{bx+a}}$	13
derivativedivides	$-\frac{2}{b\sqrt{bx+a}}$	13
default	$-\frac{2}{b\sqrt{bx+a}}$	13
trager	$-\frac{2}{b\sqrt{bx+a}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

Fricas [A]
time = 0.48, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.86

$$-\frac{2}{b\sqrt{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2),x)
```

```
[Out] -2/(b*sqrt(a + b*x))
```

Giac [**A**]
time = 0.68, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(b*x + a)*b)
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{b\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(3/2),x)
[Out] -2/(b*(a + b*x)^(1/2))
```

3.348 $\int \frac{1}{x(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=38

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 214}

$$\frac{2}{a\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{a+bx}}+\frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{ab}\\ &= \frac{2}{a\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{2}{a\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(3/2)),x]
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.13, size = 31, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{bx+a}}$	31
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{bx+a}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.48, size = 45, normalized size = 1.18

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) + 2/(sqrt(
b*x + a)*a)
```

Fricas [A]
time = 0.46, size = 110, normalized size = 2.89

$$\left[\frac{(bx+a)\sqrt{a}\,\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a}{a^2bx+a^3}, \frac{2\left((bx+a)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.
time = 0.78, size = 146, normalized size = 3.84

$$\frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/2
) + a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*
x) + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt(1
+ b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
```

Giac [A]
time = 0.72, size = 37, normalized size = 0.97

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="giac")
```

[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)

Mupad [B]
time = 0.04, size = 30, normalized size = 0.79

$$\frac{2}{a\,\sqrt{a+b\,x}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*x)^(3/2)),x)

[Out] 2/(a*(a + b*x)^(1/2)) - (2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2)

3.349 $\int \frac{1}{x^2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 3*b*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)-3*b/a^2/(b*x+a)^(1/2)-1/a/x/(b*x
+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 214}

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (-3*b)/(a^2*Sqrt[a + b*x]) - 1/(a*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(a+bx)^{3/2}}\,dx &= \frac{2}{ax\sqrt{a+bx}}+\frac{3\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a}\\
&= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}-\frac{(3b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^2}\\
&= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}-\frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^2}\\
&= \frac{2}{ax\sqrt{a+bx}}-\frac{3\sqrt{a+bx}}{a^2x}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.86

$$
\frac{-a-3bx}{a^2x\sqrt{a+bx}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (-a - 3*b*x)/(a^2*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a
^(5/2)
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.95

method	result	size

risch	$-\frac{\sqrt{bx+a}}{a^{2}x}-\frac{b\left(\frac{4}{\sqrt{bx+a}}-\frac{6\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{2a^{2}}$	50
derivativedivides	$2b\left(-\frac{1}{a^{2}\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}\right)$	54
default	$2b\left(-\frac{1}{a^{2}\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/a^2/(b*x+a)^(1/2)+1/a^2*(-1/2*(b*x+a)^(1/2)/b/x+3/2*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.47, size = 76, normalized size = 1.33

$$-\frac{3\,(bx+a)b-2\,ab}{(bx+a)^{\frac{3}{2}}a^{2}-\sqrt{bx+a}\,a^{3}}-\frac{3\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x + a)*b - 2*a*b)/((b*x + a)^(3/2)*a^2 - sqrt(b*x + a)*a^3) - 3/2*b*
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
```

Fricas [A]
time = 0.46, size = 151, normalized size = 2.65

$$\left[\frac{3\,(b^{2}x^{2}+abx)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(3\,abx+a^{2})\sqrt{bx+a}}{2\,(a^{3}bx^{2}+a^{4}x)},\,-\frac{3\,(b^{2}x^{2}+abx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx+a^{2})\sqrt{bx+a}}{a^{3}bx^{2}+a^{4}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

Sympy [A]
time = 1.77, size = 73, normalized size = 1.28

$$-\frac{1}{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^{2}\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a/
(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)
```

Giac [A]
time = 0.74, size = 64, normalized size = 1.12

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}-\frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

Mupad [B]
time = 0.12, size = 60, normalized size = 1.05

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,b}{a}-\frac{3\,b\,(a+b\,x)}{a^2}}{a\,\sqrt{a+b\,x}-(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*b)/a - (3*b*(a + b*x))/a
^2)/(a*(a + b*x)^(1/2) - (a + b*x)^(3/2))
```

3.350 $\int \frac{1}{x^3(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{15b^2}{4a^3\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(7/2)+15/4*b^2/a^3/(b*x+a)^(1/2)
-1/2/a/x^2/(b*x+a)^(1/2)+5/4*b/a^2/x/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {44, 53, 65, 214}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15b^2}{4a^3\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (15*b^2)/(4*a^3*Sqrt[a + b*x]) - 1/(2*a*x^2*Sqrt[a + b*x]) + (5*b)/(4*a^2*x
*Sqrt[a + b*x]) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx)^{3/2}}\,dx &= \frac{2}{ax^2\sqrt{a+bx}} + \frac{5\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{a} \\
&= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} - \frac{(15b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^2} \\
&= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} + \frac{(15b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^3} \\
&= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} + \frac{(15b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a^3} \\
&= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 0.77

$$\frac{-2a^2+5abx+15b^2x^2}{4a^3x^2\sqrt{a+bx}} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*a^2 + 5*a*b*x + 15*b^2*x^2)/(4*a^3*x^2*Sqrt[a + b*x]) - (15*b^2*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Maple [A]
time = 0.12, size = 68, normalized size = 0.78

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-7bx+2a)}{4a^3x^2}+\frac{b^2\left(\frac{16}{\sqrt{bx+a}}-\frac{30\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{8a^3}$	60
derivativedivides	$2b^2\left(\frac{1}{a^3\sqrt{bx+a}}-\frac{\frac{-\frac{7(bx+a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}\right)$	68
default	$2b^2\left(\frac{1}{a^3\sqrt{bx+a}}-\frac{\frac{-\frac{7(bx+a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^3}\right)$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/a^3/(b*x+a)^(1/2)-1/a^3*((-7/8*(b*x+a)^(3/2)+9/8*a*(b*x+a)^(1/2))/
b^2/x^2+15/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.51, size = 108, normalized size = 1.24

$$\frac{15\,(bx+a)^2b^2-25\,(bx+a)ab^2+8\,a^2b^2}{4\left((bx+a)^{\frac{5}{2}}a^3-2\,(bx+a)^{\frac{3}{2}}a^4+\sqrt{bx+a}\,a^5\right)}+\frac{15\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(b*x + a)^2*b^2 - 25*(b*x + a)*a*b^2 + 8*a^2*b^2)/((b*x + a)^(5/2)*
a^3 - 2*(b*x + a)^(3/2)*a^4 + sqrt(b*x + a)*a^5) + 15/8*b^2*log((sqrt(b*x +
 a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
```

Fricas [A]
time = 0.43, size = 189, normalized size = 2.17

$$\left[\frac{15\,(b^3x^3+ab^2x^2)\sqrt{a}\,\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\,(15\,ab^2x^2+5\,a^2bx-2\,a^3)\sqrt{bx+a}}{8\,(a^4bx^3+a^5x^2)},\ \frac{15\,(b^3x^3+ab^2x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^2x^2+5\,a^2bx-2\,a^3)\sqrt{bx+a}}{4\,(a^4bx^3+a^5x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

Sympy [A]
time = 3.99, size = 107, normalized size = 1.23

$$-\frac{1}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{15b^{\frac{3}{2}}}{4a^{3}\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{15b^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*sq
rt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))
```

Giac [A]
time = 1.36, size = 80, normalized size = 0.92

$$\frac{15\,b^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}\,a^{3}}+\frac{2\,b^{2}}{\sqrt{bx+a}\,a^{3}}+\frac{7\,(bx+a)^{\frac{3}{2}}b^{2}-9\sqrt{bx+a}\,ab^{2}}{4\,a^{3}b^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B]
time = 0.06, size = 90, normalized size = 1.03

$$\frac{\frac{2\,b^{2}}{a}+\frac{15\,b^{2}\,(a+b\,x)^{2}}{4\,a^{3}}-\frac{25\,b^{2}\,(a+b\,x)}{4\,a^{2}}}{(a+b\,x)^{5/2}-2\,a\,(a+b\,x)^{3/2}+a^{2}\,\sqrt{a+b\,x}}-\frac{15\,b^{2}\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(3/2)),x)
```

```
[Out] ((2*b^2)/a + (15*b^2*(a + b*x)^2)/(4*a^3) - (25*b^2*(a + b*x))/(4*a^2))/((a
 + b*x)^(5/2) - 2*a*(a + b*x)^(3/2) + a^2*(a + b*x)^(1/2)) - (15*b^2*atanh(
(a + b*x)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

3.351 $\int \frac{x^4}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

```
[Out] -2/3*a^4/b^5/(b*x+a)^(3/2)-8/3*a*(b*x+a)^(3/2)/b^5+2/5*(b*x+a)^(5/2)/b^5+8*
a^3/b^5/(b*x+a)^(1/2)+12*a^2*(b*x+a)^(1/2)/b^5
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*Sq
rt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5*b
^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.66

$$\frac{2(128a^4+192a^3bx+48a^2b^2x^2-8ab^3x^3+3b^4x^4)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a + b*x)^(5/2),x]

[Out] (2*(128*a^4 + 192*a^3*b*x + 48*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4))/(15*b^5*(a + b*x)^(3/2))

Maple [A]
time = 0.12, size = 62, normalized size = 0.71

method	result	size
gosper	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,b^3x^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}b^5}$	54
trager	$\frac{\frac{2}{5}b^4x^4-\frac{16}{15}a\,b^3x^3+\frac{32}{5}a^2b^2x^2+\frac{128}{5}a^3bx+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}b^5}$	54
risch	$\frac{2(3x^2b^2-14abx+73a^2)\sqrt{bx+a}}{15b^5}+\frac{2a^3(12bx+11a)}{3b^5(bx+a)^{\frac{3}{2}}}$	56
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}+12a^2\sqrt{bx+a}-\frac{2a^4}{3(bx+a)^{\frac{3}{2}}}+\frac{8a^3}{\sqrt{bx+a}}}{b^5}$	62
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{8a(bx+a)^{\frac{3}{2}}}{3}+12a^2\sqrt{bx+a}-\frac{2a^4}{3(bx+a)^{\frac{3}{2}}}+\frac{8a^3}{\sqrt{bx+a}}}{b^5}$	62

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/b^5*(1/5*(b*x+a)^(5/2)-4/3*a*(b*x+a)^(3/2)+6*a^2*(b*x+a)^(1/2)-1/3*a^4/(b*x+a)^(3/2)+4*a^3/(b*x+a)^(1/2))

Maxima [A]
time = 0.28, size = 71, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a}{3\,b^5}+\frac{12\,\sqrt{bx+a}\,a^2}{b^5}+\frac{8\,a^3}{\sqrt{bx+a}\,b^5}-\frac{2\,a^4}{3\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="maxima")

[Out] 2/5*(b*x + a)^(5/2)/b^5 - 8/3*(b*x + a)^(3/2)*a/b^5 + 12*sqrt(b*x + a)*a^2/b^5 + 8*a^3/(sqrt(b*x + a)*b^5) - 2/3*a^4/((b*x + a)^(3/2)*b^5)

Fricas [A]
time = 0.42, size = 74, normalized size = 0.85

$$\frac{2\,(3\,b^4x^4-8\,ab^3x^3+48\,a^2b^2x^2+192\,a^3bx+128\,a^4)\sqrt{bx+a}}{15\,(b^7x^2+2\,ab^6x+a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3456 vs. $2(83) = 166$.
time = 2.02, size = 3456, normalized size = 39.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(5/2),x)
```

```
[Out] 256*a**(85/2)*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(85/2)/(15*a**40*
b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150
*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**
33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**1
5*x**10) + 2432*a**(83/2)*b*x*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**
(83/2)*b*x/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a
**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*
b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14
*x**9 + 15*a**30*b**15*x**10) + 10336*a**(81/2)*b**2*x**2*sqrt(1 + b*x/a)/(
15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x*
*3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 +
 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a
**30*b**15*x**10) - 11520*a**(81/2)*b**2*x**2/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 25840*a*
*(79/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a
**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b
**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13
*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(79/2)*b**3
*x**3/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*
b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11
*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9
 + 15*a**30*b**15*x**10) + 41990*a**(77/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a*
```

```
*40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 +
3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800
*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*
b**15*x**10) - 53760*a**(77/2)*b**4*x**4/(15*a**40*b**5 + 150*a**39*b**6*x
+ 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*
a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**3
2*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 46192*a**(75/
2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*
b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*
x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8
 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 64512*a**(75/2)*b**5*x**5
/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*
x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6
 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15
*a**30*b**15*x**10) + 34664*a**(73/2)*b**6*x**6*sqrt(1 + b*x/a)/(15*a**40*b
**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*
a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**3
3*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15
*x**10) - 53760*a**(73/2)*b**6*x**6/(15*a**40*b**5 + 150*a**39*b**6*x + 675
*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35
*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**
13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 17392*a**(71/2)*b*
*7*x**7*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*
x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5
+ 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 15
0*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(71/2)*b**7*x**7/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10) + 5540*a**(69/2)*b**8*x**8*sqrt(1 + b*x/a)/(15*a**40*b**5 +
150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*
b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**1
2*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10
) - 11520*a**(69/2)*b**8*x**8/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10...
```

Giac [A]
time = 1.88, size = 75, normalized size = 0.86

$$\frac{2\,(12\,(bx+a)a^3-a^4)}{3\,(bx+a)^{\frac{3}{2}}b^5}+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20}-20\,(bx+a)^{\frac{3}{2}}ab^{20}+90\,\sqrt{bx+a}\,a^2b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

Mupad [B]
time = 0.05, size = 68, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{5/2}}{5\,b^5}+\frac{8\,a^3\,\left(a+b\,x\right)-\frac{2\,a^4}{3}}{b^5\,{\left(a+b\,x\right)}^{3/2}}+\frac{12\,a^2\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{3/2}}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^5) + (8*a^3*(a + b*x) - (2*a^4)/3)/(b^5*(a + b*x)^
(3/2)) + (12*a^2*(a + b*x)^(1/2))/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5)
```

3.352 $\int \frac{x^3}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

```
[Out] 2/3*a^3/b^4/(b*x+a)^(3/2)+2/3*(b*x+a)^(3/2)/b^4-6*a^2/b^4/(b*x+a)^(1/2)-6*a
*(b*x+a)^(1/2)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[a
 + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{5/2}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{5/2}}+\frac{3a^2}{b^3(a+bx)^{3/2}}-\frac{3a}{b^3\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^3}\right)dx\\ &= \frac{2a^3}{3b^4(a+bx)^{3/2}}-\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{6a\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{3/2}}{3b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.69

$$\frac{2(a^3-9a^2(a+bx)-9a(a+bx)^2+(a+bx)^3)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(a^3 - 9*a^2*(a + b*x) - 9*a*(a + b*x)^2 + (a + b*x)^3))/(3*b^4*(a + b*x
)^(3/2))
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.74

method	result	size
trager	$-\frac{2(bx+2a)\left(-x^2b^2+8abx+8a^2\right)}{3b^4(bx+a)^{\frac{3}{2}}}$	39
gosper	$-\frac{2\left(-b^3x^3+6a\,b^2x^2+24a^2bx+16a^3\right)}{3(bx+a)^{\frac{3}{2}}b^4}$	43
risch	$-\frac{2(-bx+8a)\sqrt{bx+a}}{3b^4}-\frac{2a^2(9bx+8a)}{3b^4(bx+a)^{\frac{3}{2}}}$	45
derivativedivides	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-6a\sqrt{bx+a}+\frac{2a^3}{3(bx+a)^{\frac{3}{2}}}-\frac{6a^2}{\sqrt{bx+a}}}{b^4}$	50
default	$\frac{\frac{2(bx+a)^{\frac{3}{2}}}{3}-6a\sqrt{bx+a}+\frac{2a^3}{3(bx+a)^{\frac{3}{2}}}-\frac{6a^2}{\sqrt{bx+a}}}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/3*(b*x+a)^(3/2)-3*a*(b*x+a)^(1/2)+1/3*a^3/(b*x+a)^(3/2)-3*a^2/(b*x
+a)^(1/2))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^4}-\frac{6\,\sqrt{bx+a}\,a}{b^4}-\frac{6\,a^2}{\sqrt{bx+a}\,b^4}+\frac{2\,a^3}{3\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^4 - 6*sqrt(b*x + a)*a/b^4 - 6*a^2/(sqrt(b*x + a)*b^4)
 + 2/3*a^3/((b*x + a)^(3/2)*b^4)
```

Fricas [A]
time = 0.43, size = 62, normalized size = 0.91

$$\frac{2\,(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3)\sqrt{bx+a}}{3\,(b^6x^2+2\,ab^5x+a^2b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(65) = 130$.

time = 0.38, size = 163, normalized size = 2.40

$$\begin{cases} -\frac{32a^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{48a^2bx}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{12ab^2x^2}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} + \frac{2b^3x^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**3/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48*
a**2*b*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*a*b**2*x**2
/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*b**3*x**3/(3*a*b**4*
sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), (x**4/(4*a**(5/2)), Tru
e))
```

Giac [A]

time = 1.12, size = 59, normalized size = 0.87

$$-\frac{2\left(9\left(bx+a\right)a^2-a^3\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^4}+\frac{2\left(\left(bx+a\right)^{\frac{3}{2}}b^8-9\sqrt{bx+a}\,ab^8\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

Mupad [B]

time = 0.04, size = 47, normalized size = 0.69

$$-\frac{18\,a\,{\left(a+b\,x\right)}^2+18\,a^2\,\left(a+b\,x\right)-2\,{\left(a+b\,x\right)}^3-2\,a^3}{3\,b^4\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^2 + 18*a^2*(a + b*x) - 2*(a + b*x)^3 - 2*a^3)/(3*b^4*(a +
b*x)^(3/2))
```

3.353 $\int \frac{x^2}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

```
[Out] -2/3*a^2/b^3/(b*x+a)^(3/2)+4*a/b^3/(b*x+a)^(1/2)+2*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a +
b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx\\ &= -\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.71

$$\frac{2(8a^2+12abx+3b^2x^2)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2))
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.73

method	result	size
gosper	$\frac{2x^2b^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
trager	$\frac{2x^2b^2+8abx+\frac{16}{3}a^2}{b^3(bx+a)^{\frac{3}{2}}}$	32
risch	$\frac{2\sqrt{bx+a}}{b^3}+\frac{2a(6bx+5a)}{3b^3(bx+a)^{\frac{3}{2}}}$	35
derivativedivides	$\frac{2\sqrt{bx+a}+\frac{4a}{\sqrt{bx+a}}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}}}{b^3}$	36
default	$\frac{2\sqrt{bx+a}+\frac{4a}{\sqrt{bx+a}}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}}}{b^3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*((b*x+a)^(1/2)+2*a/(b*x+a)^(1/2)-1/3*a^2/(b*x+a)^(3/2))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.84

$$\frac{2\sqrt{bx+a}}{b^3}+\frac{4a}{\sqrt{bx+a}\,b^3}-\frac{2a^2}{3(bx+a)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 4*a/(sqrt(b*x + a)*b^3) - 2/3*a^2/((b*x + a)^(3/2)*b^
3)
```

Fricas [A]
time = 0.42, size = 52, normalized size = 1.06

$$\frac{2\left(3b^2x^2+12abx+8a^2\right)\sqrt{bx+a}}{3\left(b^5x^2+2ab^4x+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(46) = 92$.
time = 0.40, size = 121, normalized size = 2.47

$$\begin{cases} \frac{16a^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{24abx}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} + \frac{6b^2x^2}{3ab^3\sqrt{a+bx}+3b^4x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**2/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*a
*b*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*b**2*x**2/(3*a*b
**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), (x**3/(3*a**(5/2)),
 True))
```

Giac [A]
time = 0.63, size = 39, normalized size = 0.80

$$\frac{2\sqrt{bx+a}}{b^3} + \frac{2\left(6\left(bx+a\right)a - a^2\right)}{3\left(bx+a\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

Mupad [B]
time = 0.08, size = 35, normalized size = 0.71

$$\frac{6\left(a+b\,x\right)^2+12\,a\left(a+b\,x\right)-2\,a^2}{3\,b^3\left(a+b\,x\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(5/2),x)
```

```
[Out] (6*(a + b*x)^2 + 12*a*(a + b*x) - 2*a^2)/(3*b^3*(a + b*x)^(3/2))
```

3.354 $\int \frac{x}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=32

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

[Out] 2/3*a/b^2/(b*x+a)^(3/2)-2/b^2/(b*x+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x)^(5/2),x]

[Out] (2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{5/2}} \, dx &= \int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right) \, dx\\ &= \frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x)^(5/2),x]

```
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

Maple [A]
time = 0.09, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}}b^2}$	21
trager	$-\frac{2(3bx+2a)}{3(bx+a)^{\frac{3}{2}}b^2}$	21
derivativedivides	$\frac{-\frac{2}{\sqrt{bx+a}}+\frac{2a}{3(bx+a)^{\frac{3}{2}}}}{b^2}$	26
default	$\frac{-\frac{2}{\sqrt{bx+a}}+\frac{2a}{3(bx+a)^{\frac{3}{2}}}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/b^2*(-1/(b*x+a)^(1/2)+1/3*a/(b*x+a)^(3/2))
```

Maxima [A]
time = 0.26, size = 26, normalized size = 0.81

$$-\frac{2}{\sqrt{bx+a}\,b^2}+\frac{2\,a}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b^2) + 2/3*a/((b*x + a)^(3/2)*b^2)
```

Fricas [A]
time = 0.40, size = 41, normalized size = 1.28

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(29) = 58.
time = 0.35, size = 80, normalized size = 2.50

$$\begin{cases} -\frac{4a}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}}-\frac{6bx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*b*x/(
3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), (x**2/(2*a**(5
/2)), True))
```

Giac [A]
time = 0.66, size = 20, normalized size = 0.62

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.62

$$-\frac{4\,a+6\,b\,x}{3\,b^2\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(5/2),x)
```

```
[Out] -(4*a + 6*b*x)/(3*b^2*(a + b*x)^(3/2))
```

3.355 $\int \frac{1}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=16

$$-\frac{2}{3b(a+bx)^{3/2}}$$

```
[Out] -2/3/b/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5/2),x]
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}} dx = -\frac{2}{3b(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5/2),x]
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Maple [A]
time = 0.09, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
default	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13
trager	$-\frac{2}{3b(bx+a)^{\frac{3}{2}}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b/(b*x+a)^(3/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.48, size = 31, normalized size = 1.94

$$-\frac{2\sqrt{bx+a}}{3\,(b^3x^2+2\,ab^2x+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.88

$$-\frac{2}{3b\,(a+bx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2),x)
[Out] -2/(3*b*(a + b*x)**(3/2))
```

Giac [A]
time = 1.26, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="giac")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$-\frac{2}{3\,b\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(5/2),x)
[Out] -2/(3*b*(a + b*x)^(3/2))
```

3.356 $\int \frac{1}{x(a+bx)^{5/2}} dx$

Optimal. Leaf size=54

$$\frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x+a)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {53, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*(a + b*x)^(3/2)) + 2/(a^2*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{5/2}}\,dx &= \frac{2}{3a(a+bx)^{3/2}}+\frac{\int \frac{1}{x(a+bx)^{3/2}}\,dx}{a}\\ &= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a^2}\\ &= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^2b}\\ &= \frac{2}{3a(a+bx)^{3/2}}+\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.91

$$\frac{2(a+3(a+bx))}{3a^2(a+bx)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(5/2)),x]
[Out] (2*(a + 3*(a + b*x)))/(3*a^2*(a + b*x)^(3/2)) - (2*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/a^(5/2)
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2}{3a(bx+a)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^2\sqrt{bx+a}}$	43
default	$\frac{2}{3a(bx+a)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^2\sqrt{bx+a}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x+a)^
(1/2)
```

Maxima [A]
time = 0.49, size = 53, normalized size = 0.98

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) + 2/3*(3*b
*x + 4*a)/((b*x + a)^(3/2)*a^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(42) = 84.
time = 0.51, size = 177, normalized size = 3.28

$$\left[\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(3\,abx+4\,a^2)\sqrt{bx+a}}{3\,(a^3b^2x^2+2\,a^4bx+a^5)},\ \frac{2\left(3\,(b^2x^2+2\,abx+a^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(3\,abx+4\,a^2)\sqrt{bx+a}\right)}{3\,(a^3b^2x^2+2\,a^4bx+a^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(48) = 96.
time = 1.47, size = 697, normalized size = 12.91

$$\frac{8a^7\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{3a^7\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{6a^7\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{14a^6bx\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{9a^6bx\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{18a^6bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{6a^5b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{9a^5b^2x^2\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{18a^5b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}+\frac{3a^4b^3x^3\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}-\frac{6a^4b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^2x^2+3a^{\frac{13}{2}}b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x*
*2 + 3*a**(13/2)*b**3*x**3) + 3*a**7*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*
b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**7*log(sqrt(1 +
b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(
13/2)*b**3*x**3) + 14*a**6*b*x*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b
*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*a**6*b*x*log(b*x/a)
/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) - 18*a**6*b*x*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(1
 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 9*a**5*b**2*x**2*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**5*b**2*x**2*log(s
qrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 3*a**4*b**3*x**3*log(b*x/a)/(3*a**(19/2) + 9*a**
(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x
**3*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3)
```

Giac [A]
time = 0.84, size = 45, normalized size = 0.83

$$\frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a^2} + \frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

Mupad [B]
time = 0.05, size = 42, normalized size = 0.78

$$\frac{\frac{2\,(a+b\,x)}{a^2}+\frac{2}{3\,a}}{(a+b\,x)^{3/2}} - \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*(a + b*x))/a^2 + 2/(3*a))/(a + b*x)^(3/2) - (2*atanh((a + b*x)^(1/2)/a^
(1/2)))/a^(5/2)
```

3.357 $\int \frac{1}{x^2(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=74

$$-\frac{5b}{3a^2(a+bx)^{3/2}}-\frac{1}{ax(a+bx)^{3/2}}-\frac{5b}{a^3\sqrt{a+bx}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3*b/a^2/(b*x+a)^(3/2)-1/a/x/(b*x+a)^(3/2)+5*b*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(7/2)-5*b/a^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 214}

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{a+bx}}-\frac{5b}{3a^2(a+bx)^{3/2}}-\frac{1}{ax(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-5*b)/(3*a^2*(a + b*x)^(3/2)) - 1/(a*x*(a + b*x)^(3/2)) - (5*b)/(a^3*Sqrt[
a + b*x]) + (5*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{5/2}}\,dx &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{5\int \frac{1}{x^2(a+bx)^{3/2}}\,dx}{3a}\\ &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a^2}\\ &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{(5b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^3}\\ &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}-\frac{5\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a^3}\\ &= \frac{2}{3ax(a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{a+bx}}-\frac{5\sqrt{a+bx}}{a^3x}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 63, normalized size = 0.85

$$\frac{-3a^2-20abx-15b^2x^2}{3a^3x(a+bx)^{3/2}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-3*a^2 - 20*a*b*x - 15*b^2*x^2)/(3*a^3*x*(a + b*x)^(3/2)) + (5*b*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.10, size = 66, normalized size = 0.89

method	result	size
risch	$-\frac{\sqrt{bx+a}}{a^3x}-\frac{b\left(-\frac{10\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{8}{\sqrt{bx+a}}+\frac{4a}{3(bx+a)^{\frac{3}{2}}}\right)}{2a^3}$	60
derivativedivides	$2b\left(-\frac{1}{3a^2(bx+a)^{\frac{3}{2}}}-\frac{2}{a^3\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^3}\right)$	66
default	$2b\left(-\frac{1}{3a^2(bx+a)^{\frac{3}{2}}}-\frac{2}{a^3\sqrt{bx+a}}+\frac{-\frac{\sqrt{bx+a}}{2bx}+\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^3}\right)$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/3/a^2/(b*x+a)^(3/2)-2/a^3/(b*x+a)^(1/2)+1/a^3*(-1/2*(b*x+a)^(1/2)/b
/x+5/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

Maxima [A]
time = 0.50, size = 89, normalized size = 1.20

$$-\frac{15\,(bx+a)^2b-10\,(bx+a)ab-2\,a^2b}{3\left((bx+a)^{\frac{5}{2}}a^3-(bx+a)^{\frac{3}{2}}a^4\right)}-\frac{5\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(15*(b*x + a)^2*b - 10*(b*x + a)*a*b - 2*a^2*b)/((b*x + a)^(5/2)*a^3 -
 (b*x + a)^(3/2)*a^4) - 5/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a)
+ sqrt(a)))/a^(7/2)
```

Fricas [A]
time = 0.50, size = 221, normalized size = 2.99

$$\left[\frac{15\,(b^3x^3+2\,ab^2x^2+a^2bx)\sqrt{a}\,\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(15\,ab^2x^2+20\,a^2bx+3\,a^3)\sqrt{bx+a}}{6\,(a^4b^2x^3+2\,a^5bx^2+a^6x)},-\frac{15\,(b^3x^3+2\,ab^2x^2+a^2bx)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(15\,ab^2x^2+20\,a^2bx+3\,a^3)\sqrt{bx+a}}{3\,(a^4b^2x^3+2\,a^5bx^2+a^6x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(68) = 136$.
time = 2.88, size = 818, normalized size = 11.05

$$-\frac{6a^{17}\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{46a^{16}bx\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{15a^{16}bx\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{30a^{16}bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{70a^{15}b^2x^2\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{45a^{15}b^2x^2\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{90a^{15}b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{30a^{14}b^3x^3\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{45a^{14}b^3x^3\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{90a^{14}b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{15a^{13}b^4x^4\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{30a^{13}b^4x^4\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a**(
39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x
**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 + b*x/
a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a
**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*a
**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log(sqrt(1 + b*x
/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*
a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6*a**(39/2)*x +
18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*
a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35
/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*log(sqrt(1 + b*
x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6
*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(6*a**(39/2)*x + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**1
3*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
```

Giac [A]
time = 0.97, size = 65, normalized size = 0.88

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx+a}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

Mupad [B]
time = 0.11, size = 73, normalized size = 0.99

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2\,b}{3\,a}+\frac{10\,b\,(a+b\,x)}{3\,a^2}-\frac{5\,b\,(a+b\,x)^2}{a^3}}{a\,(a+b\,x)^{3/2}-(a+b\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(7/2) - ((2*b)/(3*a) + (10*b*(a + b*
x))/(3*a^2) - (5*b*(a + b*x)^2)/a^3)/(a*(a + b*x)^(3/2) - (a + b*x)^(5/2))
```

3.358 $\int \frac{1}{x^3(a+bx)^{5/2}}\, dx$

Optimal. Leaf size=106

$$\frac{35b^2}{12a^3(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}+\frac{35b^2}{4a^4\sqrt{a+bx}}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] 35/12*b^2/a^3/(b*x+a)^(3/2)-1/2/a/x^2/(b*x+a)^(3/2)+7/4*b/a^2/x/(b*x+a)^(3/
2)-35/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(9/2)+35/4*b^2/a^4/(b*x+a)^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 214}

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{a+bx}}+\frac{35b^2}{12a^3(a+bx)^{3/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (35*b^2)/(12*a^3*(a + b*x)^(3/2)) - 1/(2*a*x^2*(a + b*x)^(3/2)) + (7*b)/(4*
a^2*x*(a + b*x)^(3/2)) + (35*b^2)/(4*a^4*Sqrt[a + b*x]) - (35*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{5/2}}\,dx &= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{7\int \frac{1}{x^3(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}+\frac{35\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{3a^2}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}-\frac{(35b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^3}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{(35b^2)\int \frac{1}{x\sqrt{a+b}}}{8a^4}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{(35b)\text{Subst}\left(\int \frac{}{-}\right.}{}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}-\frac{35b^2\tanh^{-1}\left(\sqrt{\right.}{4a^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 78, normalized size = 0.74

$$\frac{-6a^3+21a^2bx+140ab^2x^2+105b^3x^3}{12a^4x^2(a+bx)^{3/2}}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (-6*a^3 + 21*a^2*b*x + 140*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^2*(a + b*x)^(
3/2)) - (35*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Maple [A]
time = 0.11, size = 81, normalized size = 0.76

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-11bx+2a)}{4a^4x^2}+\frac{b^2\left(-\frac{70\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{48}{\sqrt{bx+a}}+\frac{16a}{3(bx+a)^{\frac{3}{2}}}\right)}{8a^4}$	70
derivativedivides	$2b^2\left(-\frac{\frac{-\frac{11(bx+a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{35\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}+\frac{3}{a^4\sqrt{bx+a}}+\frac{1}{3a^3(bx+a)^{\frac{3}{2}}}\right)$	81
default	$2b^2\left(-\frac{\frac{-\frac{11(bx+a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx+a}}{8}}{b^2x^2}+\frac{35\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^4}+\frac{3}{a^4\sqrt{bx+a}}+\frac{1}{3a^3(bx+a)^{\frac{3}{2}}}\right)$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/a^4*((-11/8*(b*x+a)^(3/2)+13/8*a*(b*x+a)^(1/2))/b^2/x^2+35/8*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))+3/a^4/(b*x+a)^(1/2)+1/3/a^3/(b*x+a)^(3/
2))
```

Maxima [A]
time = 0.48, size = 123, normalized size = 1.16

$$\frac{105\,(bx+a)^3b^2-175\,(bx+a)^2ab^2+56\,(bx+a)a^2b^2+8\,a^3b^2}{12\left((bx+a)^{\frac{7}{2}}a^4-2\,(bx+a)^{\frac{5}{2}}a^5+(bx+a)^{\frac{3}{2}}a^6\right)}+\frac{35\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x + a)^3*b^2 - 175*(b*x + a)^2*a*b^2 + 56*(b*x + a)*a^2*b^2 +
8*a^3*b^2)/((b*x + a)^(7/2)*a^4 - 2*(b*x + a)^(5/2)*a^5 + (b*x + a)^(3/2)*a
^6) + 35/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(
9/2)
```

Fricas [A]
time = 0.49, size = 255, normalized size = 2.41

$$\left[\frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4)\sqrt{bx+a}}{24\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)},\ \frac{105\,(b^4x^4+2\,ab^3x^3+a^2b^2x^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4)\sqrt{bx+a}}{12\,(a^5b^2x^4+2\,a^6bx^3+a^7x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(99) = 198$.
time = 10.56, size = 464, normalized size = 4.38

$$-\frac{6a^{\frac{89}{2}}b^{75}x^{75}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{21a^{\frac{87}{2}}b^{76}x^{76}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{140a^{\frac{85}{2}}b^{77}x^{77}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{105a^{\frac{83}{2}}b^{78}x^{78}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{105a^{42}b^{\frac{155}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{105a^{41}b^{\frac{157}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) +
 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(87/2)*
b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(
91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/
(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(15
3/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12*a**(93/2
)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) + 1)*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a
/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a
**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**
(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))
```

Giac [A]
time = 0.91, size = 93, normalized size = 0.88

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^4}+\frac{2\,(9\,(bx+a)b^2+ab^2)}{3\,(bx+a)^{\frac{3}{2}}a^4}+\frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

Mupad [B]
time = 0.12, size = 105, normalized size = 0.99

$$\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,{\left(a+b\,x\right)}^2}{12\,a^3}+\frac{35\,b^2\,{\left(a+b\,x\right)}^3}{4\,a^4}+\frac{14\,b^2\,\left(a+b\,x\right)}{3\,a^2}}{{\left(a+b\,x\right)}^{7/2}-2\,a\,{\left(a+b\,x\right)}^{5/2}+a^2\,{\left(a+b\,x\right)}^{3/2}}-\frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*b^2)/(3*a) - (175*b^2*(a + b*x)^2)/(12*a^3) + (35*b^2*(a + b*x)^3)/(4*a
^4) + (14*b^2*(a + b*x))/(3*a^2))/((a + b*x)^(7/2) - 2*a*(a + b*x)^(5/2) +
a^2*(a + b*x)^(3/2)) - (35*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(9/2))
```

3.359 $\int \frac{1}{x\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {65, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.10, size = 20, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
default	$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.49, size = 19, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

Fricas [A]
time = 0.48, size = 58, normalized size = 2.32

$$\left[-\frac{\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)}{a}, \frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x)/a, 2*arctan(sqrt(b
*x - a)/sqrt(a))/sqrt(a)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 54, normalized size = 2.16

$$\begin{cases} \frac{2i \operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2 \operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1),
 (-2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), True))
```

Giac [A]
time = 0.73, size = 19, normalized size = 0.76

$$\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.76

$$\frac{2 \operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/2)),x)
```

```
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2)
```

3.360 $\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{-a+bx}}{ax}+\frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)+(b*x-a)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {44, 65, 211}

$$\frac{b\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-a+bx}}\,dx = \frac{\sqrt{-a+bx}}{ax} + \frac{b\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a}$$
$$= \frac{\sqrt{-a+bx}}{ax} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 1.00

$$\frac{\sqrt{-a+bx}}{ax} + \frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-a + b*x]),x]
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.13, size = 44, normalized size = 1.00

method	result	size
derivativedivides	$2b\left(\frac{\sqrt{bx-a}}{2abx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	44
default	$2b\left(\frac{\sqrt{bx-a}}{2abx}+\frac{\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}\right)$	44
risch	$-\frac{-bx+a}{ax\sqrt{bx-a}}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*b*(1/2*(b*x-a)^(1/2)/a/b/x+1/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2))
```

Maxima [A]
time = 0.49, size = 46, normalized size = 1.05

$$\frac{\sqrt{bx-a}\,b}{(bx-a)a+a^2}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x - a)*b/((b*x - a)*a + a^2) + b*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/
2)
```

Fricas [A]
time = 0.50, size = 97, normalized size = 2.20

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\sqrt{bx-a}\,a}{2a^2x},\ \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*sqrt(
b*x - a)*a)/(a^2*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) + sqrt(b*x
- a)*a)/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.13, size = 121, normalized size = 2.75

$$\begin{cases} \frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a\sqrt{x}}+\frac{ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{1}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{\sqrt{b}}{a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(b)*sqrt(a/(b*x) - 1)/(a*sqrt(x)) + I*b*acosh(sqrt(a)/(sqr
t(b)*sqrt(x)))/a**(3/2), Abs(a/(b*x)) > 1), (-1/(sqrt(b)*x**(3/2)*sqrt(-a/(
b*x) + 1)) + sqrt(b)/(a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(
b)*sqrt(x)))/a**(3/2), True))
```

Giac [A]
time = 0.84, size = 43, normalized size = 0.98

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{bx-a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + sqrt(b*x - a)*b/(a*x))/b
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.82

$$\frac{\sqrt{b\,x-a}}{a\,x} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/2)),x)
```

```
[Out] (b*x - a)^(1/2)/(a*x) + (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2)
```

3.361 $\int \frac{1}{x^3\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

```
[Out] 3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(b*x-a)^(1/2)/a/x^2+3/4*b
*(b*x-a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {44, 65, 211}

$$\frac{3b^2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{bx-a}}{4a^2x}+\frac{\sqrt{bx-a}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(2*a*x^2) + (3*b*Sqrt[-a + b*x])/(4*a^2*x) + (3*b^2*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{-a+bx}}\,dx &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{(3b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{(3b^2)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^2}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{(3b)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a^2}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 0.81

$$\frac{\sqrt{-a+bx}\,(2a+3bx)}{4a^2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (Sqrt[-a + b*x]*(2*a + 3*b*x))/(4*a^2*x^2) + (3*b^2*ArcTan[Sqrt[-a + b*x]/S
qrt[a]])/(4*a^(5/2))
```

Maple [A]
time = 0.09, size = 72, normalized size = 0.97

method	result	size
risch	$-\frac{(-bx+a)(3bx+2a)}{4a^2x^2\sqrt{bx-a}}+\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	55
derivativedivides	$2b^2\left(\frac{\sqrt{bx-a}}{4a\,b^2x^2}+\frac{\frac{3\sqrt{bx-a}}{8abx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}}{a}\right)$	72

default	$2b^2\left(\frac{\sqrt{bx-a}}{4a\,b^2x^2}+\frac{\frac{3\sqrt{bx-a}}{8abx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}}{a}\right)$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(1/4*(b*x-a)^(1/2)/a/b^2/x^2+3/4/a*(1/2*(b*x-a)^(1/2)/a/b/x+1/2*arcta
n((b*x-a)^(1/2)/a^(1/2))/a^(3/2)))
```

Maxima [A]
time = 0.49, size = 86, normalized size = 1.16

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}+\frac{3\,(bx-a)^{\frac{3}{2}}b^2+5\,\sqrt{bx-a}\,ab^2}{4\,\left((bx-a)^2a^2+2\,(bx-a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 1/4*(3*(b*x - a)^(3/2)*b^2
+ 5*sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a^4)
```

Fricas [A]
time = 0.65, size = 128, normalized size = 1.73

$$\left[-\frac{3\,\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(3\,abx+2\,a^2)\sqrt{bx-a}}{8\,a^3x^2},\frac{3\,\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(3\,abx+2\,a^2)\sqrt{bx-a}}{4\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2
*(3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(
sqrt(b*x - a)/sqrt(a)) + (3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.47, size = 216, normalized size = 2.92

$$\begin{cases} \frac{i}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{i\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{3ib^{\frac{3}{2}}}{4a^2\sqrt{x}\,\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\,\sqrt{-\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(4*a*x**(3/
2)*sqrt(a/(b*x) - 1)) - 3*I*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) - 1)) + 3
*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), Abs(a/(b*x)) > 1), (
-1/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - sqrt(b)/(4*a*x**(3/2)*sqrt(-a/
(b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) - 3*b**2*asin(
sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), True))
```

Giac [**A**]
time = 0.62, size = 68, normalized size = 0.92

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{3\,(bx-a)^{\frac{3}{2}}b^3+5\,\sqrt{bx-a}\,ab^3}{a^2b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + (3*(b*x - a)^(3/2)*b^3 +
 5*sqrt(b*x - a)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [**B**]
time = 0.05, size = 57, normalized size = 0.77

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{5/2}}+\frac{5\,\sqrt{b\,x-a}}{4\,a\,x^2}+\frac{3\,{\left(b\,x-a\right)}^{3/2}}{4\,a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/2)),x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*(b*x - a)^(1/2))/(4*
a*x^2) + (3*(b*x - a)^(3/2))/(4*a^2*x^2)
```

3.362 $\int \frac{1}{x(-a+bx)^{3/2}} \, dx$

Optimal. Leaf size=42

$$-\frac{2}{a\sqrt{-a+bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 65, 211}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(-a+bx)^{3/2}}\,dx = -\frac{2}{a\sqrt{-a+bx}} - \frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a}$$
$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{ab}$$
$$= -\frac{2}{a\sqrt{-a+bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2}{a\sqrt{-a+bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(3/2)),x]
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.09, size = 35, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{a\sqrt{bx-a}}$	35
default	$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{a\sqrt{bx-a}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

Maxima [A]
time = 0.48, size = 34, normalized size = 0.81

$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} - \frac{2}{\sqrt{bx-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

Fricas [A]
time = 0.72, size = 124, normalized size = 2.95

$$\left[-\frac{(bx-a)\sqrt{-a}\,\log\left(\frac{bx+2\sqrt{bx-a}\,\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}\,a}{a^2bx-a^3},\,-\frac{2\left((bx-a)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a\right)}{a^2bx-a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqr
t(b*x - a)*a)/(a^2*b*x - a^3), -2*((b*x - a)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + sqrt(b*x - a)*a)/(a^2*b*x - a^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 437, normalized size = 10.40

$$\begin{cases} -\frac{2a^3\sqrt{-1+\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{2ia^3\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2ia^2bx\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{2a^2bx\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2ia^3\sqrt{1-\frac{bx}{a}}}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{ia^3\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{2ia^3\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{\pi a^3}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{ia^2bx\log\left(\frac{bx}{a}\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2ia^2bx\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{\pi a^2bx}{-a^{\frac{9}{2}}+a^{\frac{7}{2}}bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*sqrt(-1 + b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*log
(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(b)*sqrt(x)/sqrt(a))/
(-a**(9/2) + a**(7/2)*b*x) - 2*a**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9
/2) + a**(7/2)*b*x) + I*a**2*b*x*log(b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*
I*a**2*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-a**(9/2) + a**(7/2)*b*x) + 2*a**2
*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-a**(9/2) + a**(7/2)*b*x), Abs(b*x/a)
 > 1), (-2*I*a**3*sqrt(1 - b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - I*a**3*log(b
*x/a)/(-a**(9/2) + a**(7/2)*b*x) + 2*I*a**3*log(sqrt(1 - b*x/a) + 1)/(-a**(
9/2) + a**(7/2)*b*x) - pi*a**3/(-a**(9/2) + a**(7/2)*b*x) + I*a**2*b*x*log(
b*x/a)/(-a**(9/2) + a**(7/2)*b*x) - 2*I*a**2*b*x*log(sqrt(1 - b*x/a) + 1)/(
-a**(9/2) + a**(7/2)*b*x) + pi*a**2*b*x/(-a**(9/2) + a**(7/2)*b*x), True))
```

Giac [A]
time = 0.86, size = 34, normalized size = 0.81

$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{\sqrt{bx-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

Mupad [B]
time = 0.10, size = 34, normalized size = 0.81

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\,\sqrt{b\,x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(3/2)),x)
```

```
[Out] - (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(b*x - a)^(1/2))
```

3.363 $\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{3b}{a^2\sqrt{-a+bx}}+\frac{1}{ax\sqrt{-a+bx}}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)-3*b/a^2/(b*x-a)^(1/2)+1/a/x/(b*x
-a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 211}

$$-\frac{3b\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{bx-a}}+\frac{1}{ax\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] (-3*b)/(a^2*Sqrt[-a + b*x]) + 1/(a*x*Sqrt[-a + b*x]) - (3*b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a}\\
&= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{(3b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^2}\\
&= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{3\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^2}\\
&= -\frac{2}{ax\sqrt{-a+bx}}-\frac{3\sqrt{-a+bx}}{a^2x}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.82

$$\frac{a-3bx}{a^2x\sqrt{-a+bx}}-\frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] (a - 3*b*x)/(a^2*x*Sqrt[-a + b*x]) - (3*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a
^(5/2)
```

Maple [A]
time = 0.10, size = 61, normalized size = 0.98

method	result	size

risch	$\frac{-bx+a}{a^{2}x\sqrt{bx-a}}-\frac{2b}{a^{2}\sqrt{bx-a}}-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	59
derivativedivides	$2b\left(-\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}-\frac{1}{a^{2}\sqrt{bx-a}}\right)$	61
default	$2b\left(-\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{2}}-\frac{1}{a^{2}\sqrt{bx-a}}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(-1/a^2*(1/2*(b*x-a)^(1/2)/b/x+3/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2
))-1/a^2/(b*x-a)^(1/2))
```

Maxima [A]
time = 0.50, size = 67, normalized size = 1.08

$$-\frac{3\,(bx-a)b+2\,ab}{(bx-a)^{\frac{3}{2}}a^{2}+\sqrt{bx-a}\,a^{3}}-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x - a)*b + 2*a*b)/((b*x - a)^(3/2)*a^2 + sqrt(b*x - a)*a^3) - 3*b*ar
ctan(sqrt(b*x - a)/sqrt(a))/a^(5/2)
```

Fricas [A]
time = 1.54, size = 164, normalized size = 2.65

$$\left[-\frac{3\,(b^{2}x^{2}-abx)\sqrt{-a}\,\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,(3\,abx-a^{2})\sqrt{bx-a}}{2\,(a^{3}bx^{2}-a^{4}x)},\,-\frac{3\,(b^{2}x^{2}-abx)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(3\,abx-a^{2})\sqrt{bx-a}}{a^{3}bx^{2}-a^{4}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^2*x^2 - a*b*x)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2
*a)/x) + 2*(3*a*b*x - a^2)*sqrt(b*x - a))/(a^3*b*x^2 - a^4*x), -(3*(b^2*x^2
 - a*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (3*a*b*x - a^2)*sqrt(b*x
- a))/(a^3*b*x^2 - a^4*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.72, size = 156, normalized size = 2.52

$$\begin{cases} -\frac{i}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{3i\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{3ib\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{1}{a\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{3b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(a**2*sq
rt(x)*sqrt(a/(b*x) - 1)) - 3*I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2),
 Abs(a/(b*x)) > 1), (1/(a*sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/
(a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) + 3*b*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/a**
(5/2), True))
```

Giac [A]
time = 1.56, size = 64, normalized size = 1.03

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{3\,(bx-a)b + 2\,ab}{\left((bx-a)^{\frac{3}{2}} + \sqrt{bx-a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) - (3*(b*x - a)*b + 2*a*b)/(((b*x
 - a)^(3/2) + sqrt(b*x - a)*a)*a^2)
```

Mupad [B]
time = 0.06, size = 52, normalized size = 0.84

$$\frac{1}{a\,x\,\sqrt{b\,x-a}} - \frac{3\,b}{a^2\,\sqrt{b\,x-a}} - \frac{3\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(3/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(1/2)) - (3*b)/(a^2*(b*x - a)^(1/2)) - (3*b*atan((b*x - a)
^(1/2)/a^(1/2)))/a^(5/2)
```

3.364 $\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{15b^2}{4a^3\sqrt{-a+bx}}+\frac{1}{2ax^2\sqrt{-a+bx}}+\frac{5b}{4a^2x\sqrt{-a+bx}}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)-15/4*b^2/a^3/(b*x-a)^(1/2)+
1/2/a/x^2/(b*x-a)^(1/2)+5/4*b/a^2/x/(b*x-a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 211}

$$-\frac{15b^2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15b^2}{4a^3\sqrt{bx-a}}+\frac{5b}{4a^2x\sqrt{bx-a}}+\frac{1}{2ax^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] (-15*b^2)/(4*a^3*Sqrt[-a + b*x]) + 1/(2*a*x^2*Sqrt[-a + b*x]) + (5*b)/(4*a^
2*x*Sqrt[-a + b*x]) - (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{a}\\ &= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{(15b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^2}\\ &= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{15b\sqrt{-a+bx}}{4a^3x}-\frac{(15b^2)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^3}\\ &= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{15b\sqrt{-a+bx}}{4a^3x}-\frac{(15b)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{}\right.}{4a^3}\\ &= -\frac{2}{ax^2\sqrt{-a+bx}}-\frac{5\sqrt{-a+bx}}{2a^2x^2}-\frac{15b\sqrt{-a+bx}}{4a^3x}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.75

$$\frac{2a^2+5abx-15b^2x^2}{4a^3x^2\sqrt{-a+bx}}-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] (2*a^2 + 5*a*b*x - 15*b^2*x^2)/(4*a^3*x^2*Sqrt[-a + b*x]) - (15*b^2*ArcTan[
Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Maple [A]
time = 0.10, size = 77, normalized size = 0.81

method	result	size
risch	$\frac{(-bx+a)(7bx+2a)}{4a^{3}x^{2}\sqrt{bx-a}}-\frac{2b^{2}}{a^{3}\sqrt{bx-a}}-\frac{15b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$	72
derivativedivides	$2b^{2}\left(-\frac{\frac{\frac{7(bx-a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{3}}-\frac{1}{a^{3}\sqrt{bx-a}}\right)$	77
default	$2b^{2}\left(-\frac{\frac{\frac{7(bx-a)^{\frac{3}{2}}}{8}+\frac{9a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{15\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{3}}-\frac{1}{a^{3}\sqrt{bx-a}}\right)$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/a^3*((7/8*(b*x-a)^(3/2)+9/8*a*(b*x-a)^(1/2))/b^2/x^2+15/8*arctan(
(b*x-a)^(1/2)/a^(1/2))/a^(1/2))-1/a^3/(b*x-a)^(1/2))
```

Maxima [A]
time = 0.48, size = 104, normalized size = 1.09

$$-\frac{15\,(bx-a)^{2}b^{2}+25\,(bx-a)ab^{2}+8\,a^{2}b^{2}}{4\left((bx-a)^{\frac{5}{2}}a^{3}+2\,(bx-a)^{\frac{3}{2}}a^{4}+\sqrt{bx-a}\,a^{5}\right)}-\frac{15\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(15*(b*x - a)^2*b^2 + 25*(b*x - a)*a*b^2 + 8*a^2*b^2)/((b*x - a)^(5/2)
*a^3 + 2*(b*x - a)^(3/2)*a^4 + sqrt(b*x - a)*a^5) - 15/4*b^2*arctan(sqrt(b*
x - a)/sqrt(a))/a^(7/2)
```

Fricas [A]
time = 1.03, size = 198, normalized size = 2.08

$$\left[-\frac{15\,(b^{3}x^{3}-ab^{2}x^{2})\sqrt{-a}\,\log\left(\frac{bx+2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)+2\,(15\,ab^{2}x^{2}-5\,a^{2}bx-2\,a^{3})\sqrt{bx-a}}{8\,(a^{4}bx^{3}-a^{5}x^{2})},-\frac{15\,(b^{3}x^{3}-ab^{2}x^{2})\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(15\,ab^{2}x^{2}-5\,a^{2}bx-2\,a^{3})\sqrt{bx-a}}{4\,(a^{4}bx^{3}-a^{5}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a
) - 2*a)/x) + 2*(15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^
3 - a^5*x^2), -1/4*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + (15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^3 - a^
5*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.98, size = 226, normalized size = 2.38

$$\begin{cases} -\frac{i}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{5i\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{15ib^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}-1}} - \frac{15ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ \frac{1}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{-\frac{a}{bx}+1}} + \frac{15b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 5*I*sqrt(b)/(4*a**
2*x**(3/2)*sqrt(a/(b*x) - 1)) + 15*I*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x)
- 1)) - 15*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), Abs(a/(b*x
)) > 1), (1/(2*a*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x
**(3/2)*sqrt(-a/(b*x) + 1)) - 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(-a/(b*x) + 1
)) + 15*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), True))
```

Giac [A]
time = 1.51, size = 81, normalized size = 0.85

$$-\frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}} - \frac{2\,b^2}{\sqrt{bx-a}\,a^3} - \frac{7\,(bx-a)^{\frac{3}{2}}b^2 + 9\,\sqrt{bx-a}\,ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) - 2*b^2/(sqrt(b*x - a)*a^3)
 - 1/4*(7*(b*x - a)^(3/2)*b^2 + 9*sqrt(b*x - a)*a*b^2)/(a^3*b^2*x^2)
```

Mupad [B]
time = 0.13, size = 101, normalized size = 1.06

$$-\frac{\frac{2\,b^2}{a} + \frac{15\,b^2\,(a-b\,x)^2}{4\,a^3} - \frac{25\,b^2\,(a-b\,x)}{4\,a^2}}{2\,a\,(b\,x-a)^{3/2} + (b\,x-a)^{5/2} + a^2\,\sqrt{b\,x-a}} - \frac{15\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(3/2)),x)
```

```
[Out] - ((2*b^2)/a + (15*b^2*(a - b*x)^2)/(4*a^3) - (25*b^2*(a - b*x))/(4*a^2))/(
2*a*(b*x - a)^(3/2) + (b*x - a)^(5/2) + a^2*(b*x - a)^(1/2)) - (15*b^2*atan
((b*x - a)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

3.365 $\int \frac{1}{x(-a+bx)^{5/2}} dx$

Optimal. Leaf size=60

$$-\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 65, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{bx-a}}-\frac{2}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*(-a + b*x)^(3/2)) + 2/(a^2*Sqrt[-a + b*x]) + (2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(-a+bx)^{5/2}}\,dx &= -\frac{2}{3a(-a+bx)^{3/2}}-\frac{\int \frac{1}{x(-a+bx)^{3/2}}\,dx}{a}\\
&= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a^2}\\
&= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^2b}\\
&= -\frac{2}{3a(-a+bx)^{3/2}}+\frac{2}{a^2\sqrt{-a+bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.87

$$-\frac{8a-6bx}{3a^2(-a+bx)^{3/2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(5/2)),x]
[Out] -1/3*(8*a - 6*b*x)/(a^2*(-a + b*x)^(3/2)) + (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a
]])/a^(5/2)
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2}{3a(bx-a)^{\frac{3}{2}}}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^2\sqrt{bx-a}}$	49
default	$-\frac{2}{3a(bx-a)^{\frac{3}{2}}}+\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^2\sqrt{bx-a}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

Maxima [A]
time = 0.49, size = 42, normalized size = 0.70

$$\frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

Fricas [A]
time = 0.52, size = 182, normalized size = 3.03

$$\left[-\frac{3\,(b^{2}x^{2}-2\,abx+a^{2})\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(3\,abx-4\,a^{2})\sqrt{bx-a}}{3\,(a^{3}b^{2}x^{2}-2\,a^{4}bx+a^{5})},\ \frac{2\left(3\,(b^{2}x^{2}-2\,abx+a^{2})\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(3\,abx-4\,a^{2})\sqrt{bx-a}\right)}{3\,(a^{3}b^{2}x^{2}-2\,a^{4}bx+a^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt
(-a) - 2*a)/x) - 2*(3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*
x + a^5), 2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqr
t(a)) + (3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*x + a^5)]
```

Sympy [C] Result contains complex when optimal does not.
time = 104.59, size = 1950, normalized size = 32.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((8*a**7*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(
15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(b*x/a)/(-3*a**(19/2
```

```
) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*I*
a**7*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(1
5/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**7*asin(sqrt(a)/(sqrt(b)*sqrt
(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)
*b**3*x**3) - 14*a**6*b*x*sqrt(-1 + b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x
 - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log(b*x/a)/
(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) + 18*I*a**6*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17
/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**6*b*x*asin
(sqrt(a)/(sqrt(b)*sqrt(x)))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(-1 + b*x/a)/(-3*a
**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3)
 + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15
/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*a**5*b**2*x**2*log(sqrt(b)*sq
rt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*
a**(13/2)*b**3*x**3) + 18*a**5*b**2*x**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(-
3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x*
*3) - 3*I*a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**
(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**4*b**3*x**3*log(sqrt(b)*
sqrt(x)/sqrt(a))/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 +
3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(
-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x
**3), Abs(b*x/a) > 1), (8*I*a**7*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2
)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*I*a**7*log(b*x/a
)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**
3*x**3) - 6*I*a**7*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x
 - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 3*pi*a**7/(-3*a**(19/2)
 + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 14*I*
a**6*b*x*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2
*x**2 + 3*a**(13/2)*b**3*x**3) - 9*I*a**6*b*x*log(b*x/a)/(-3*a**(19/2) + 9*
a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 18*I*a**6*
b*x*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*
b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 9*pi*a**6*b*x/(-3*a**(19/2) + 9*a**(17
/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**5*b**2*x*
*2*sqrt(1 - b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 9*I*a**5*b**2*x**2*log(b*x/a)/(-3*a**(19/2) + 9*
a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*I*a**5*
b**2*x**2*log(sqrt(1 - b*x/a) + 1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(
15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*pi*a**5*b**2*x**2/(-3*a**(19/2
) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 3*I*
a**4*b**3*x**3*log(b*x/a)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**
2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*I*a**4*b**3*x**3*log(sqrt(1 - b*x/a) +
1)/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b*
*3*x**3) - 3*pi*a**4*b**3*x**3/(-3*a**(19/2) + 9*a**(17/2)*b*x - 9*a**(15/2
)*b**2*x**2 + 3*a**(13/2)*b**3*x**3), True))
```

Giac [A]
time = 1.02, size = 42, normalized size = 0.70

$$\frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

Mupad [B]
time = 0.09, size = 48, normalized size = 0.80

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,(a-b\,x)}{a^2}+\frac{2}{3\,a}}{(b\,x-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(5/2)),x)
```

```
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a - b*x))/a^2 + 2/(3*a))/(
b*x - a)^(3/2)
```

3.366 $\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{5b}{3a^2(-a+bx)^{3/2}}+\frac{1}{ax(-a+bx)^{3/2}}+\frac{5b}{a^3\sqrt{-a+bx}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3*b/a^2/(b*x-a)^(3/2)+1/a/x/(b*x-a)^(3/2)+5*b*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(7/2)+5*b/a^3/(b*x-a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {44, 53, 65, 211}

$$\frac{5b\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{bx-a}}-\frac{5b}{3a^2(bx-a)^{3/2}}+\frac{1}{ax(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (-5*b)/(3*a^2*(-a + b*x)^(3/2)) + 1/(a*x*(-a + b*x)^(3/2)) + (5*b)/(a^3*Sqr
t[-a + b*x]) + (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx &= -\frac{2}{3ax(-a+bx)^{3/2}}-\frac{5\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx}{3a}\\ &= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a^2}\\ &= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\sqrt{-a+bx}}{a^3x}+\frac{(5b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^3}\\ &= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\sqrt{-a+bx}}{a^3x}+\frac{5\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-}\right.}{a^3}\\ &= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\sqrt{-a+bx}}{a^3x}+\frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.83

$$\frac{3a^2-20abx+15b^2x^2}{3a^3x(-a+bx)^{3/2}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (3*a^2 - 20*a*b*x + 15*b^2*x^2)/(3*a^3*x*(-a + b*x)^(3/2)) + (5*b*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.91

method	result	size
derivativedivides	$2b\left(\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}-\frac{1}{3a^{2}(bx-a)^{\frac{3}{2}}}+\frac{2}{a^{3}\sqrt{bx-a}}\right)$	74
default	$2b\left(\frac{\frac{\sqrt{bx-a}}{2bx}+\frac{5\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{2\sqrt{a}}}{a^{3}}-\frac{1}{3a^{2}(bx-a)^{\frac{3}{2}}}+\frac{2}{a^{3}\sqrt{bx-a}}\right)$	74
risch	$-\frac{-bx+a}{a^{3}x\sqrt{bx-a}}+\frac{5b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{4b}{a^{3}\sqrt{bx-a}}-\frac{2b}{3a^{2}(bx-a)^{\frac{3}{2}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b*(1/a^3*(1/2*(b*x-a)^(1/2)/b/x+5/2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
)-1/3/a^2/(b*x-a)^(3/2)+2/a^3/(b*x-a)^(1/2))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 1.01

$$\frac{15\,(bx-a)^{2}b+10\,(bx-a)ab-2\,a^{2}b}{3\left((bx-a)^{\frac{5}{2}}a^{3}+(bx-a)^{\frac{3}{2}}a^{4}\right)}+\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(b*x - a)^2*b + 10*(b*x - a)*a*b - 2*a^2*b)/((b*x - a)^(5/2)*a^3 +
(b*x - a)^(3/2)*a^4) + 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2)
```

Fricas [A]
time = 0.50, size = 226, normalized size = 2.79

$$\left[-\frac{15\,(b^{3}x^{3}-2\,ab^{2}x^{2}+a^{2}bx)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(15\,ab^{2}x^{2}-20\,a^{2}bx+3\,a^{3})\sqrt{bx-a}}{6\,(a^{4}b^{2}x^{3}-2\,a^{5}bx^{2}+a^{6}x)},\ \frac{15\,(b^{3}x^{3}-2\,ab^{2}x^{2}+a^{2}bx)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(15\,ab^{2}x^{2}-20\,a^{2}bx+3\,a^{3})\sqrt{bx-a}}{3\,(a^{4}b^{2}x^{3}-2\,a^{5}bx^{2}+a^{6}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^3*x^3 - 2*a*b^2*x^2 + a^2*b*x)*sqrt(-a)*log((b*x - 2*sqrt(b*x
- a)*sqrt(-a) - 2*a)/x) - 2*(15*a*b^2*x^2 - 20*a^2*b*x + 3*a^3)*sqrt(b*x -
a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x), 1/3*(15*(b^3*x^3 - 2*a*b^2*x^2 + a
^2*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (15*a*b^2*x^2 - 20*a^2*b*x
+ 3*a^3)*sqrt(b*x - a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.14, size = 66, normalized size = 0.81

$$\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\,(6\,(bx-a)b-ab)}{3\,(bx-a)^{\frac{3}{2}}a^{3}}+\frac{\sqrt{bx-a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) + 2/3*(6*(b*x - a)*b - a*b)/((b*x
 - a)^(3/2)*a^3) + sqrt(b*x - a)/(a^3*x)
```

Mupad [**B**]
time = 0.12, size = 70, normalized size = 0.86

$$\frac{1}{a\,x\,{\left(b\,x-a\right)}^{3/2}}-\frac{20\,b}{3\,a^2\,{\left(b\,x-a\right)}^{3/2}}+\frac{5\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5\,b^2\,x}{a^3\,{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(5/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(3/2)) - (20*b)/(3*a^2*(b*x - a)^(3/2)) + (5*b*atan((b*x -
 a)^(1/2)/a^(1/2)))/a^(7/2) + (5*b^2*x)/(a^3*(b*x - a)^(3/2))
```

3.367 $\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=116

$$-\frac{35b^2}{12a^3(-a+bx)^{3/2}}+\frac{1}{2ax^2(-a+bx)^{3/2}}+\frac{7b}{4a^2x(-a+bx)^{3/2}}+\frac{35b^2}{4a^4\sqrt{-a+bx}}+\frac{35b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] -35/12*b^2/a^3/(b*x-a)^(3/2)+1/2/a/x^2/(b*x-a)^(3/2)+7/4*b/a^2/x/(b*x-a)^(3
/2)+35/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(9/2)+35/4*b^2/a^4/(b*x-a)^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 211}

$$\frac{35b^2\text{ArcTan}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{bx-a}}-\frac{35b^2}{12a^3(bx-a)^{3/2}}+\frac{7b}{4a^2x(bx-a)^{3/2}}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (-35*b^2)/(12*a^3*(-a + b*x)^(3/2)) + 1/(2*a*x^2*(-a + b*x)^(3/2)) + (7*b)/
(4*a^2*x*(-a + b*x)^(3/2)) + (35*b^2)/(4*a^4*Sqrt[-a + b*x]) + (35*b^2*ArcT
an[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx = -\frac{2}{3ax^2(-a+bx)^{3/2}} - \frac{7\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx}{3a}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{3a^2}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{(35b)\int \frac{1}{x^2\sqrt{-a+bx}}\,d}{4a^3}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{(35b}{}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{(35b}{}$$

$$= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{35b^2}{}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 0.71

$$\frac{6a^3+21a^2bx-140ab^2x^2+105b^3x^3}{12a^4x^2(-a+bx)^{3/2}} + \frac{35b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (6*a^3 + 21*a^2*b*x - 140*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^2*(-a + b*x)^(
3/2)) + (35*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Maple [A]
time = 0.12, size = 90, normalized size = 0.78

method	result	size
risch	$-\frac{(-bx+a)(11bx+2a)}{4a^{4}x^{2}\sqrt{bx-a}}+\frac{35b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{9}{2}}}+\frac{6b^{2}}{a^{4}\sqrt{bx-a}}-\frac{2b^{2}}{3a^{3}(bx-a)^{\frac{3}{2}}}$	89
derivativedivides	$2b^{2}\left(-\frac{1}{3a^{3}(bx-a)^{\frac{3}{2}}}+\frac{3}{a^{4}\sqrt{bx-a}}+\frac{\frac{\frac{11(bx-a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{35\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{4}}\right)$	90
default	$2b^{2}\left(-\frac{1}{3a^{3}(bx-a)^{\frac{3}{2}}}+\frac{3}{a^{4}\sqrt{bx-a}}+\frac{\frac{\frac{11(bx-a)^{\frac{3}{2}}}{8}+\frac{13a\sqrt{bx-a}}{8}}{b^{2}x^{2}}+\frac{35\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{8\sqrt{a}}}{a^{4}}\right)$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2*(-1/3/a^3/(b*x-a)^(3/2)+3/a^4/(b*x-a)^(1/2)+1/a^4*((11/8*(b*x-a)^(3/2
)+13/8*a*(b*x-a)^(1/2))/b^2/x^2+35/8*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2))
)
```

Maxima [A]
time = 0.48, size = 121, normalized size = 1.04

$$\frac{105\,(bx-a)^{3}b^{2}+175\,(bx-a)^{2}ab^{2}+56\,(bx-a)a^{2}b^{2}-8\,a^{3}b^{2}}{12\left((bx-a)^{\frac{7}{2}}a^{4}+2\,(bx-a)^{\frac{5}{2}}a^{5}+(bx-a)^{\frac{3}{2}}a^{6}\right)}+\frac{35\,b^{2}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x - a)^3*b^2 + 175*(b*x - a)^2*a*b^2 + 56*(b*x - a)*a^2*b^2 -
8*a^3*b^2)/((b*x - a)^(7/2)*a^4 + 2*(b*x - a)^(5/2)*a^5 + (b*x - a)^(3/2)*a
^6) + 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2)
```

Fricas [A]
time = 0.71, size = 260, normalized size = 2.24

$$\left[-\frac{105\,(b^{4}x^{4}-2\,ab^{3}x^{3}+a^{2}b^{2}x^{2})\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\,(105\,ab^{3}x^{3}-140\,a^{2}b^{2}x^{2}+21\,a^{3}bx+6\,a^{4})\sqrt{bx-a}}{24\,(a^{5}b^{2}x^{4}-2\,a^{6}bx^{3}+a^{7}x^{2})},\ \frac{105\,(b^{4}x^{4}-2\,ab^{3}x^{3}+a^{2}b^{2}x^{2})\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+(105\,ab^{3}x^{3}-140\,a^{2}b^{2}x^{2}+21\,a^{3}bx+6\,a^{4})\sqrt{bx-a}}{12\,(a^{5}b^{2}x^{4}-2\,a^{6}bx^{3}+a^{7}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*log((b*x - 2*sqr
t(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3
*b*x + 6*a^4)*sqrt(b*x - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
05*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(
a)) + (105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3*b*x + 6*a^4)*sqrt(b*x - a))
/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2)]
```

<u>Sympy</u> [**C**] Result contains complex when optimal does not.
time = 12.86, size = 1108, normalized size = 9.55

$$\begin{cases} \frac{12 i a^{\frac{89}{2}} b^{75} x^{75}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} + \frac{42 i a^{\frac{87}{2}} b^{76} x^{76}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} - \frac{280 i a^{\frac{85}{2}} b^{77} x^{77}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} + \frac{210 i a^{\frac{83}{2}} b^{78} x^{78}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} + \frac{210 i a^{42} b^{\frac{155}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} \operatorname{acosh}{\left(\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right)}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} - \frac{105 \pi a^{42} b^{\frac{155}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} - \frac{210 i a^{41} b^{\frac{157}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1} \operatorname{acosh}{\left(\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right)}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} + \frac{105 \pi a^{41} b^{\frac{157}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}}{24 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{\frac{a}{bx} - 1} - 24 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{\frac{a}{bx} - 1}} & \text{for}\: \left|{\frac{a}{bx}}\right| > 1 \\- \frac{6 a^{\frac{89}{2}} b^{75} x^{75}}{12 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} - 12 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1}} - \frac{21 a^{\frac{87}{2}} b^{76} x^{76}}{12 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} - 12 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1}} + \frac{140 a^{\frac{85}{2}} b^{77} x^{77}}{12 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} - 12 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1}} - \frac{105 a^{\frac{83}{2}} b^{78} x^{78}}{12 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} - 12 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1}} - \frac{105 a^{42} b^{\frac{155}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} \operatorname{asin}{\left(\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right)}}{12 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} - 12 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1}} + \frac{105 a^{41} b^{\frac{157}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1} \operatorname{asin}{\left(\frac{\sqrt{a}}{\sqrt{b} \sqrt{x}} \right)}}{12 a^{\frac{93}{2}} b^{\frac{151}{2}} x^{\frac{155}{2}} \sqrt{- \frac{a}{bx} + 1} - 12 a^{\frac{91}{2}} b^{\frac{153}{2}} x^{\frac{157}{2}} \sqrt{- \frac{a}{bx} + 1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((12*I*a**(89/2)*b**75*x**75/(24*a**(93/2)*b**(151/2)*x**(155/2)*s
qrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) +
42*I*a**(87/2)*b**76*x**76/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x)
 - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 280*I*a**(8
5/2)*b**77*x**77/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24
*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**(83/2)*b**78
*x**78/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)
*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**42*b**(155/2)*x**(155/
2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151
/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(
a/(b*x) - 1)) - 105*pi*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) - 1)/(24*a*
*(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x
**(157/2)*sqrt(a/(b*x) - 1)) - 210*I*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*
x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1))
 + 105*pi*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) - 1)/(24*a**(93/2)*b**(1
51/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqr
t(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-6*a**(89/2)*b**75*x**75/(12*a**(93/2)
*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(-a/(b*x) + 1)) - 21*a**(87/2)*b**76*x**76/(12*a**(93/2)*b**(151/2)
*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a
/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/(12*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1
)) - 105*a**(83/2)*b**78*x**78/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/
(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) - 105*a
**42*b**(155/2)*x**(155/2)*sqrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)
```

```
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b*
*(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) + 105*a**41*b**(157/2)*x**(157/2)*s
qrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*
x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/
(b*x) + 1)), True))
```

Giac [A]
time = 1.04, size = 97, normalized size = 0.84

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}+\frac{2\,(9\,(bx-a)b^2-ab^2)}{3\,(bx-a)^{\frac{3}{2}}a^4}+\frac{11\,(bx-a)^{\frac{3}{2}}b^2+13\,\sqrt{bx-a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2) + 2/3*(9*(b*x - a)*b^2 - a*b
^2)/((b*x - a)^(3/2)*a^4) + 1/4*(11*(b*x - a)^(3/2)*b^2 + 13*sqrt(b*x - a)*
a*b^2)/(a^4*b^2*x^2)
```

Mupad [B]
time = 0.07, size = 117, normalized size = 1.01

$$\frac{35\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{9/2}}-\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,(a-b\,x)^2}{12\,a^3}+\frac{35\,b^2\,(a-b\,x)^3}{4\,a^4}+\frac{14\,b^2\,(a-b\,x)}{3\,a^2}}{2\,a\,(b\,x-a)^{5/2}+(b\,x-a)^{7/2}+a^2\,(b\,x-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(5/2)),x)
```

```
[Out] (35*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(9/2)) - ((2*b^2)/(3*a) - (175*
b^2*(a - b*x)^2)/(12*a^3) + (35*b^2*(a - b*x)^3)/(4*a^4) + (14*b^2*(a - b*x
))/(3*a^2))/(2*a*(b*x - a)^(5/2) + (b*x - a)^(7/2) + a^2*(b*x - a)^(3/2))
```

3.368 $\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

```
[Out] x^m/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {12, 75}

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}} \, dx &= \frac{1}{2} \int \frac{x^{-1+m}(2am+b(-1+2m)x)}{(a+bx)^{3/2}} \, dx \\ &= \frac{x^m}{\sqrt{a+bx}} \end{aligned}$$

Mathematica [A]

time = 0.18, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Maple [A]
time = 0.09, size = 12, normalized size = 0.92

method	result	size
gosper	$\frac{x^{m}}{\sqrt{b x +a}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x^m/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.33, size = 11, normalized size = 0.85

$$\frac{x^{m}}{\sqrt{b x + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="max
ima")
```

```
[Out] x^m/sqrt(b*x + a)
```

Fricas [A]
time = 0.79, size = 14, normalized size = 1.08

$$\frac{x x^{m - 1}}{\sqrt{b x + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="fri
cas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x + a)
```

Sympy [C] Result contains complex when optimal does not.
time = 35.50, size = 78, normalized size = 6.00

$$\frac{m x^{m} \Gamma\left(m\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, m \\ m + 1 \end{matrix}\middle| {\frac{b x e^{i \pi}}{a}} \right)}}{\sqrt{a} \Gamma\left(m + 1\right)} + \frac{b x x^{m} \left(2 m - 1\right) \Gamma\left(m + 1\right) {{}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, m + 1 \\ m + 2 \end{matrix}\middle| {\frac{b x e^{i \pi}}{a}} \right)}}{2 a^{\frac{3}{2}} \Gamma\left(m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((3/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) + b*x*x**m*(2*m - 1)*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,)
, b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/2*(b*(2*m - 1)*x + 2*a*m)*x^(m - 1)/(b*x + a)^(3/2), x)
```

Mupad [B]
time = 0.41, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x*(2*m - 1)))/(2*(a + b*x)^(3/2)),x)
```

```
[Out] x^m/(a + b*x)^(1/2)
```

3.369 $\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right)\,dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

```
[Out] x^m/(b*x+a)^(1/2)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 7.08, number of steps used = 5, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {69, 67}

$$\frac{x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{\sqrt{a+bx}}-\frac{2mx^m\sqrt{a+bx}\,\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] (x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/((-((b*x)/a))^m*Sqrt[a
+ b*x]) - (2*m*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*
x)/a])/(a*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx = -\left(\frac{1}{2}b\int \frac{x^m}{(a+bx)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx$$

$$= -\left(\frac{1}{2}\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}\,dx\right)-\frac{\left(bmx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int}{a}$$

$$= \frac{x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{\sqrt{a+bx}}-\frac{2mx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2}{a}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int -\frac{b\,x^m}{2\,(bx+a)^{\frac{3}{2}}}+\frac{m\,x^{-1+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

Maxima [A]
time = 0.35, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] x^m/sqrt(b*x + a)
```

Fricas [A]
time = 0.99, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] x^m/sqrt(b*x + a)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.40, size = 73, normalized size = 5.62

$$\frac{mx^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m\\ m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}-\frac{bxx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x**m/(b*x+a)**(3/2)+m*x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) - b*x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_
polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x + a) - 1/2*b*x^m/(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x}}-\frac{b\,x^m}{2\,{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)),x)
```

```
[Out] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)), x)
```

3.370 $\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {7, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx = \int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.11, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.48, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.60, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A]
time = 0.44, size = 24, normalized size = 1.04

$$-\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A]
time = 0.98, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 0.00, size = 17, normalized size = 0.74

$$-\frac{2\,\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.371 $\int x^3 \sqrt[3]{a+bx}\, dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}$$

```
[Out] -3/4*a^3*(b*x+a)^(4/3)/b^4+9/7*a^2*(b*x+a)^(7/3)/b^4-9/10*a*(b*x+a)^(10/3)/
b^4+3/13*(b*x+a)^(13/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*(
a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx\\&=-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{4/3}\left(-81a^3+108a^2bx-126ab^2x^2+140b^3x^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-81*a^3 + 108*a^2*b*x - 126*a*b^2*x^2 + 140*b^3*x^3))/(
1820*b^4)
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{4}{3}}\left(-140b^3x^3+126a\,b^2x^2-108a^2bx+81a^3\right)}{1820b^4}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{9a(bx+a)^{\frac{10}{3}}}{10}+\frac{9a^2(bx+a)^{\frac{7}{3}}}{7}-\frac{3a^3(bx+a)^{\frac{4}{3}}}{4}}{b^4}$	50
default	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{9a(bx+a)^{\frac{10}{3}}}{10}+\frac{9a^2(bx+a)^{\frac{7}{3}}}{7}-\frac{3a^3(bx+a)^{\frac{4}{3}}}{4}}{b^4}$	50
trager	$-\frac{3\left(-140b^4x^4-14a\,b^3x^3+18a^2b^2x^2-27a^3bx+81a^4\right)(bx+a)^{\frac{1}{3}}}{1820b^4}$	54
risch	$-\frac{3\left(-140b^4x^4-14a\,b^3x^3+18a^2b^2x^2-27a^3bx+81a^4\right)(bx+a)^{\frac{1}{3}}}{1820b^4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/13*(b*x+a)^(13/3)-3/10*a*(b*x+a)^(10/3)+3/7*a^2*(b*x+a)^(7/3)-1/4*
a^3*(b*x+a)^(4/3))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^4}-\frac{9\,(bx+a)^{\frac{10}{3}}a}{10\,b^4}+\frac{9\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^4}-\frac{3\,(bx+a)^{\frac{4}{3}}a^3}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^4 - 9/10*(b*x + a)^(10/3)*a/b^4 + 9/7*(b*x + a)^(7/
3)*a^2/b^4 - 3/4*(b*x + a)^(4/3)*a^3/b^4
```

Fricas [A]
time = 0.52, size = 53, normalized size = 0.74

$$\frac{3\,(140\,b^4x^4+14\,ab^3x^3-18\,a^2b^2x^2+27\,a^3bx-81\,a^4)(bx+a)^{\frac{1}{3}}}{1820\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^4*x^4 + 14*a*b^3*x^3 - 18*a^2*b^2*x^2 + 27*a^3*b*x - 81*a^4)*
(b*x + a)^(1/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.
time = 1.24, size = 1742, normalized size = 24.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 2
7300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 1092
0*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(73/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 1377*
a**(70/3)*b*x*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27
300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920
*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1458*a**(70/3)*b*x/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 3
213*a**(67/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b
**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x*
*4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 3645*a**(67/3)*b**2*x
**2/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a
**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14
*b**10*x**6) - 3927*a**(64/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 4860*
a**(64/3)*b**3*x**3/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**
6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x
**5 + 1820*a**14*b**10*x**6) - 2163*a**(61/3)*b**4*x**4*(1 + b*x/a)**(1/3)/
(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17
*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**
10*x**6) + 3645*a**(61/3)*b**4*x**4/(1820*a**20*b**4 + 10920*a**19*b**5*x +
 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10
920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1827*a**(58/3)*b**5*x**5*(1
+ b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x*
*2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5
+ 1820*a**14*b**10*x**6) + 1458*a**(58/3)*b**5*x**5/(1820*a**20*b**4 + 1092
0*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**1
6*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6573*a**(55/
3)*b**6*x**6*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 273
```

```
00*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*
a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(55/3)*b**6*x**6/(1820*a*
*20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x*
*3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
 + 8787*a**(52/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**
19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**
8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6498*a**(49/3)*b*
*8*x**8*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a*
*18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15
*b**9*x**5 + 1820*a**14*b**10*x**6) + 2562*a**(46/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 3640
0*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a*
*14*b**10*x**6) + 420*a**(43/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.
time = 2.10, size = 117, normalized size = 1.62

$$\frac{3\left(\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a}{b^3}+\frac{4\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3+455\,(bx+a)^{\frac{1}{3}}a^4\right)}{b^3}\right)}{1820\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/1820*(13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^3 + 4*(35*(b*x + a)^(13/3) - 182*(b*x
+ a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*(b*
x + a)^(1/3)*a^4)/b^3)/b
```

Mupad [**B**]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{13/3}}{13\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{4/3}}{4\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(13/3))/(13*b^4) - (3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a
 + b*x)^(7/3))/(7*b^4) - (9*a*(a + b*x)^(10/3))/(10*b^4)
```

3.372 $\int x^2 \sqrt[3]{a+bx}\, dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}$$

```
[Out] 3/4*a^2*(b*x+a)^(4/3)/b^3-6/7*a*(b*x+a)^(7/3)/b^3+3/10*(b*x+a)^(10/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a + b
*x)^(10/3))/(10*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt[3]{a+bx}\,dx &= \int \left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{4/3}\left(9a^2-12abx+14b^2x^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3)*(9*a^2 - 12*a*b*x + 14*b^2*x^2))/(140*b^3)
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{4}{3}}\left(14x^2b^2-12abx+9a^2\right)}{140b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{6a(bx+a)^{\frac{7}{3}}}{7}+\frac{3a^2(bx+a)^{\frac{4}{3}}}{4}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{6a(bx+a)^{\frac{7}{3}}}{7}+\frac{3a^2(bx+a)^{\frac{4}{3}}}{4}}{b^3}$	38
trager	$\frac{3\left(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3\right)(bx+a)^{\frac{1}{3}}}{140b^3}$	43
risch	$\frac{3\left(14b^3x^3+2a\,b^2x^2-3a^2bx+9a^3\right)(bx+a)^{\frac{1}{3}}}{140b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/b^3*(1/10*(b*x+a)^(10/3)-2/7*a*(b*x+a)^(7/3)+1/4*a^2*(b*x+a)^(4/3))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^3}-\frac{6\,(bx+a)^{\frac{7}{3}}a}{7\,b^3}+\frac{3\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/10*(b*x + a)^(10/3)/b^3 - 6/7*(b*x + a)^(7/3)*a/b^3 + 3/4*(b*x + a)^(4/3)
*a^2/b^3
```

Fricas [A]
time = 0.45, size = 42, normalized size = 0.79

$$\frac{3\,(14\,b^3x^3+2\,ab^2x^2-3\,a^2bx+9\,a^3)(bx+a)^{\frac{1}{3}}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/140*(14*b^3*x^3 + 2*a*b^2*x^2 - 3*a^2*b*x + 9*a^3)*(b*x + a)^(1/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.
time = 0.81, size = 666, normalized size = 12.57

$$\frac{27 a^{\frac{34}{3}} \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} - \frac{27 a^{\frac{34}{3}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} + \frac{72 a^{\frac{31}{3}} bx \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} - \frac{81 a^{\frac{31}{3}} bx}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} + \frac{60 a^{\frac{28}{3}} b^{2}x^{2} \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} - \frac{81 a^{\frac{28}{3}} b^{2}x^{2}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} + \frac{60 a^{\frac{25}{3}} b^{3}x^{3} \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} - \frac{27 a^{\frac{25}{3}} b^{3}x^{3}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} + \frac{135 a^{\frac{22}{3}} b^{4}x^{4} \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} + \frac{132 a^{\frac{19}{3}} b^{5}x^{5} \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}} + \frac{42 a^{\frac{16}{3}} b^{6}x^{6} \sqrt[3]{1+\frac{bx}{a}}}{140a^{8}b^{3}+420a^{7}b^{4}x+420a^{6}b^{5}x^{2}+140a^{5}b^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) - 27*a**(34/3)/(140*a**8*b**3 + 420*a**7*b
**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 72*a**(31/3)*b*x*(1 + b*
x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**
5*b**6*x**3) - 81*a**(31/3)*b*x/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(28/3)*b**2*x**2*(1 + b*x/a)**(1/3
)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**
3) - 81*a**(28/3)*b**2*x**2/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**
5*x**2 + 140*a**5*b**6*x**3) + 60*a**(25/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1
40*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) -
 27*a**(25/3)*b**3*x**3/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x*
*2 + 140*a**5*b**6*x**3) + 135*a**(22/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(140*
a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 13
2*a**(19/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x +
 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 42*a**(16/3)*b**6*x**6*(1 + b*x
/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5
*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 1.24, size = 92, normalized size = 1.74

$$\frac{3 \left(\frac{10 \left(2 \left(bx+a\right)^{\frac{7}{3}}-7 \left(bx+a\right)^{\frac{4}{3}}a+14 \left(bx+a\right)^{\frac{1}{3}}a^{2}\right)a}{b^{2}}+\frac{14 \left(bx+a\right)^{\frac{10}{3}}-60 \left(bx+a\right)^{\frac{7}{3}}a+105 \left(bx+a\right)^{\frac{4}{3}}a^{2}-140 \left(bx+a\right)^{\frac{1}{3}}a^{3}}{b^{2}}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/140*(10*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a/b^2 + (14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)
*a^2 - 140*(b*x + a)^(1/3)*a^3)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{42\,{\left(a+b\,x\right)}^{10/3}-120\,a\,{\left(a+b\,x\right)}^{7/3}+105\,a^2\,{\left(a+b\,x\right)}^{4/3}}{140\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/3),x)
```

```
[Out] (42*(a + b*x)^(10/3) - 120*a*(a + b*x)^(7/3) + 105*a^2*(a + b*x)^(4/3))/(14
0*b^3)
```

3.373 $\int x\sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}$$

```
[Out] -3/4*a*(b*x+a)^(4/3)/b^2+3/7*(b*x+a)^(7/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}\,(-3a^2+abx+4b^2x^2)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-3*a^2 + a*b*x + 4*b^2*x^2))/(28*b^2)
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{4}{3}}(-4bx+3a)}{28b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{4}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{4}}{b^2}$	26
trager	$-\frac{3(-4x^2b^2-abx+3a^2)(bx+a)^{\frac{1}{3}}}{28b^2}$	32
risch	$-\frac{3(-4x^2b^2-abx+3a^2)(bx+a)^{\frac{1}{3}}}{28b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/7*(b*x+a)^(7/3)-1/4*a*(b*x+a)^(4/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^2}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/7*(b*x + a)^(7/3)/b^2 - 3/4*(b*x + a)^(4/3)*a/b^2
```

Fricas [A]
time = 0.47, size = 30, normalized size = 0.88

$$\frac{3\,(4\,b^2x^2+abx-3\,a^2)(bx+a)^{\frac{1}{3}}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/28*(4*b^2*x^2 + a*b*x - 3*a^2)*(b*x + a)^(1/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.
time = 0.54, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{13}{3}}}{28a^2b^2+28ab^3x}-\frac{6a^{\frac{10}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{10}{3}}bx}{28a^2b^2+28ab^3x}+\frac{15a^{\frac{7}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{12a^{\frac{4}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 9*a**(13/3)/
(28*a**2*b**2 + 28*a*b**3*x) - 6*a**(10/3)*b*x*(1 + b*x/a)**(1/3)/(28*a**2*
b**2 + 28*a*b**3*x) + 9*a**(10/3)*b*x/(28*a**2*b**2 + 28*a*b**3*x) + 15*a**
(7/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 12*a**(4/
3)*b**3*x**3*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(26) = 52$.
time = 1.08, size = 67, normalized size = 1.97

$$\frac{3\left(\frac{7\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}\,a\right)a}{b}+\frac{2\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}\,a+14\,(bx+a)^{\frac{1}{3}}\,a^2\right)}{b}\right)}{28\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/28*(7*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a/b + 2*(2*(b*x + a)^(7/3)
- 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b)/b
```

Mupad [**B**]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{21\,a\,(a+b\,x)^{4/3}-12\,(a+b\,x)^{7/3}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/3),x)
```

```
[Out] -(21*a*(a + b*x)^(4/3) - 12*(a + b*x)^(7/3))/(28*b^2)
```

3.374 $\int \sqrt[3]{a+bx}\, dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{4/3}}{4b}$$

```
[Out] 3/4*(b*x+a)^(4/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\, dx = \frac{3(a+bx)^{4/3}}{4b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
default	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
trager	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13
risch	$\frac{3(bx+a)^{\frac{4}{3}}}{4b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(b*x+a)^(4/3)/b
```

Maxima [A]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

Fricas [A]
time = 0.44, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{3(a+bx)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3),x)
```

```
[Out] 3*(a + b*x)**(4/3)/(4*b)
```

Giac [A]
time = 0.76, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{4/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

3.375 $\int \frac{\sqrt[3]{a+bx}}{x}\,dx$

Optimal. Leaf size=91

$$3\sqrt[3]{a+bx}-\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)+\frac{3}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 3*(b*x+a)^(1/3)-1/2*a^(1/3)*ln(x)+3/2*a^(1/3)*ln(a^(1/3)-(b*x+a)^(1/3))-a^(
1/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 59, 631, 210, 31}

$$-\sqrt{3}\,\sqrt[3]{a}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)+3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}\sqrt[3]{a}\,\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx}}{x}\,dx &= 3\sqrt[3]{a+bx}+a\int \frac{1}{x(a+bx)^{2/3}}\,dx\\ &= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{1}{2}\left(3a^{2/3}\right)\text{Su}\\ &= 3\sqrt[3]{a+bx}-\frac{1}{2}\sqrt[3]{a}\log(x)+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.\\ &= 3\sqrt[3]{a+bx}-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.\end{aligned}$$

Mathematica [A]
time = 0.08, size = 113, normalized size = 1.24

$$3\sqrt[3]{a+bx}-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3)
)/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Maple [A]
time = 0.10, size = 90, normalized size = 0.99

method	result
derivativedivides	$3(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$
default	$3(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+\right.}{3}\right.}{3a^{\frac{2}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(b*x+a)^(1/3)+3*(1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(2/3)*ln((b*
x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))*a
```

Maxima [A]
time = 0.51, size = 86, normalized size = 0.95

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Fricas [A]
time = 0.47, size = 91, normalized size = 1.00

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.00, size = 180, normalized size = 1.98

$$\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma(
7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pol
ar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)
/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Giac [A]
time = 1.13, size = 87, normalized size = 0.96

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

Mupad [B]
time = 0.12, size = 107, normalized size = 1.18

$$a^{1/3}\ln\left(9\,a\,(a+b\,x)^{1/3}-9\,a^{4/3}\right)+3\,(a+b\,x)^{1/3}+\frac{a^{1/3}\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{1/3}\ln\left(9\,a\,(a+b\,x)^{1/3}+\frac{9\,a^{4/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x,x)
```

```
[Out] a^(1/3)*log(9*a*(a + b*x)^(1/3) - 9*a^(4/3)) + 3*(a + b*x)^(1/3) + (a^(1/3)
*log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/2 - (a^(1/3)*log(9*a*(a + b*x)^(1/3) + (9*a^(4/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/2
```

3.376 $\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt[3]{a+bx}}{x}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

```
[Out] -(b*x+a)^(1/3)/x-1/6*b*ln(x)/a^(2/3)+1/2*b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3
)-1/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 59, 631, 210, 31}

$$-\frac{b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx &= -\frac{\sqrt[3]{a+bx}}{x}+\frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\log(x)}{6a^{2/3}}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,d\right.}{2\sqrt[3]{a}}\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}+\frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right.}{a^{2/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{x}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 119, normalized size = 1.23

$$-\frac{6a^{2/3}\sqrt[3]{a+bx}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^2,x]
```

[Out] -1/6*(6*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(2/3)*x)

Maple [A]
time = 0.15, size = 95, normalized size = 0.98

method	result
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}\right.$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3bx}+\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/3)/x^2,x,method=_RETURNVERBOSE)

[Out] 3*b*(-1/3*(b*x+a)^(1/3)/b/x+1/9/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/18/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/9/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))

Maxima [A]
time = 0.50, size = 93, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}}+\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")

[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3) - 1/6*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + 1/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - (b*x + a)^(1/3)/x

Fricas [A]
time = 0.50, size = 139, normalized size = 1.43

$$-\frac{2\sqrt{3}\,(a^2)^{\frac{1}{6}}abx\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+(a^2)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\,(a^2)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+6\,(bx+a)^{\frac{1}{3}}a^2}{6\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*b*x*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/
3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b
*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2
/3)*b*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*
x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 643, normalized size = 6.63

$$\frac{4a^{\frac{7}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{12a^2b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**2,x)
```

```
[Out] 4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*
gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*
pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2
*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(
4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(
7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b
+ x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a
**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*e
xp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*
gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi
/3)*gamma(7/3))
```

Giac [A]
time = 1.05, size = 105, normalized size = 1.08

$$-\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6\,(bx+a)^{\frac{1}{3}}b}{x}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(2/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1
/3)*b/x)/b
```

Mupad [B]
time = 0.07, size = 117, normalized size = 1.21

$$\frac{b\ln\left(3\,b\,{\left(a+b\,x\right)}^{1/3}-3\,a^{1/3}\,b\right)}{3\,a^{2/3}}-\frac{{\left(a+b\,x\right)}^{1/3}}{x}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{2}+3\,b\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{2/3}}-\frac{\ln\left(\frac{3\,a^{1/3}\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{2}+3\,b\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{6\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^2,x)
```

```
[Out] (b*log(3*b*(a + b*x)^(1/3) - 3*a^(1/3)*b))/(3*a^(2/3)) - (a + b*x)^(1/3)/x
 - (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)^(1/3))*(b - 3^(1/2)
*b*1i))/(6*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)
^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(2/3))
```

3.377 $\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}$$

```
[Out] -1/2*(b*x+a)^(1/3)/x^2-1/6*b*(b*x+a)^(1/3)/a/x+1/18*b^2*ln(x)/a^(5/3)-1/6*b
^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/3)+1/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^
(1/3))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$,
Rules used = {43, 44, 59, 631, 210, 31}

$$\frac{b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x)^(1/3)/x^2 - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx &= -\frac{\sqrt[3]{a+bx}}{2x^2}+\frac{1}{6}b\int \frac{1}{x^2(a+bx)^{2/3}}\,dx\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}-\frac{b^2\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a}\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}+\frac{b^2\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\mathrm{Subst}(
\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{b^2\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3
\\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a\right.}{6a^{5/3}}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 129, normalized size = 1.02

$$\frac{-\frac{3a^{2/3}\sqrt[3]{a+bx}\,(3a+bx)}{x^2}+2\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(1/3)/x^3,x]

[Out] ((-3*a^(2/3)*(a + b*x)^(1/3)*(3*a + b*x))/x^2 + 2*Sqrt[3]*b^2*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b^2*Log[a^(1/3) - (a + b*x)^(1/3)] + b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(5/3))

Maple [A]
time = 0.14, size = 118, normalized size = 0.93

method	result
derivativedivides	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{3a^{\frac{2}{3}}}}{9a}\right.$
default	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{4}{3}}}{18a}+\frac{(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right.}{3}\right.}{3a^{\frac{2}{3}}}}{9a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/3)/x^3,x,method=_RETURNVERBOSE)

[Out] 3*b^2*(-(1/18/a*(b*x+a)^(4/3)+1/9*(b*x+a)^(1/3))/b^2/x^2-1/9/a*(1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))

Maxima [A]

time = 0.52, size = 139, normalized size = 1.09

$$\frac{\sqrt{3}\, b^2 \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{4}{3}}b^2+2\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(5/3) - 1/6*((b*x + a)^(
4/3)*b^2 + 2*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3)
```

Fricas [A]
time = 0.47, size = 187, normalized size = 1.47

$$\frac{2\sqrt{3}\,ab^2x^2\sqrt{-(-a^2)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\,(-a^2)^{\frac{1}{3}}a-2\sqrt{3}\,(-a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-(-a^2)^{\frac{1}{3}}}}{3\,a^2}\right)+(-a^2)^{\frac{2}{3}}\,b^2x^2\log\left((bx+a)^{\frac{2}{3}}a-(-a^2)^{\frac{1}{3}}\,a+(-a^2)^{\frac{2}{3}}\,(bx+a)^{\frac{1}{3}}\right)-2\,(-a^2)^{\frac{2}{3}}\,b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-(-a^2)^{\frac{2}{3}}\right)-3\,(a^2bx+3\,a^3)(bx+a)^{\frac{1}{3}}}{18\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) +
 (-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)
*(b*x + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(
2/3)) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.66, size = 2266, normalized size = 17.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**3,x)
```

```
[Out] -4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a*
*7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)
+ 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*
```

```
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)
*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(
a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi
/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(
7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x
)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3
)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*
(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*p
i/3)*gamma(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/
3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)*
*2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma
(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*e
xp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*e
xp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3
)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*g
amma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/
b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
) + 4*a**(7/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**
6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*
pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 4*
a**(7/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3)
 - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2
*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7
/3)) - 12*a**5*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**4*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(
2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**3*b**(13/3)*(a/
```

```
b + x)**(7/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
)
```

Giac [A]
time = 1.73, size = 128, normalized size = 1.01

$$\frac{\frac{2\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left((bx+a)^{\frac{4}{3}}b^3+2\,(bx+a)^{\frac{1}{3}}ab^3\right)}{ab^2x^2}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a)^(
4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.23, size = 196, normalized size = 1.54

$$\frac{b^2\ln\left(\frac{b^2}{(-a)^{2/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a}\right)}{9\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{2\,(-a)^{2/3}}+\frac{b^2\,(a+b\,x)^{1/3}}{a}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18\,(-a)^{5/3}}-\frac{\frac{b^2\,(a+b\,x)^{1/3}}{3}+\frac{b^2\,(a+b\,x)^{4/3}}{6\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2}+\frac{b^2\ln\left(\frac{b^2\,(a+b\,x)^{1/3}}{a}-\frac{b^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{2/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^3,x)
```

```
[Out] (b^2*log(b^2/(-a)^(2/3) - (b^2*(a + b*x)^(1/3))/a))/(9*(-a)^(5/3)) - (log((
3^(1/2)*b^2*1i + b^2)/(2*(-a)^(2/3)) + (b^2*(a + b*x)^(1/3))/a)*(3^(1/2)*b^
2*1i + b^2))/(18*(-a)^(5/3)) - ((b^2*(a + b*x)^(1/3))/3 + (b^2*(a + b*x)^(4
/3))/(6*a))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + (b^2*log((b^2*(a + b*x)^(
1/3))/a - (b^2*((3^(1/2)*1i)/2 - 1/2))/(-a)^(2/3))*((3^(1/2)*1i)/2 - 1/2))/
(9*(-a)^(5/3))
```

3.378 $\int x^3(a+bx)^{2/3}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}$$

```
[Out] -3/5*a^3*(b*x+a)^(5/3)/b^4+9/8*a^2*(b*x+a)^(8/3)/b^4-9/11*a*(b*x+a)^(11/3)/
b^4+3/14*(b*x+a)^(14/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*(
a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{2/3}\,dx &= \int\left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{5/3}\left(-81a^3+135a^2bx-180ab^2x^2+220b^3x^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-81*a^3 + 135*a^2*b*x - 180*a*b^2*x^2 + 220*b^3*x^3))/(
3080*b^4)
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{5}{3}}\left(-220b^{3}x^{3}+180a\,b^{2}x^{2}-135a^{2}bx+81a^{3}\right)}{3080b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{14}{3}}}{14}-\frac{9a(bx+a)^{\frac{11}{3}}}{11}+\frac{9a^{2}(bx+a)^{\frac{8}{3}}}{8}-\frac{3a^{3}(bx+a)^{\frac{5}{3}}}{5}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{14}{3}}}{14}-\frac{9a(bx+a)^{\frac{11}{3}}}{11}+\frac{9a^{2}(bx+a)^{\frac{8}{3}}}{8}-\frac{3a^{3}(bx+a)^{\frac{5}{3}}}{5}}{b^{4}}$	50
trager	$-\frac{3\left(-220b^{4}x^{4}-40a\,b^{3}x^{3}+45a^{2}b^{2}x^{2}-54a^{3}bx+81a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080b^{4}}$	54
risch	$-\frac{3\left(-220b^{4}x^{4}-40a\,b^{3}x^{3}+45a^{2}b^{2}x^{2}-54a^{3}bx+81a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080b^{4}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/14*(b*x+a)^(14/3)-3/11*a*(b*x+a)^(11/3)+3/8*a^2*(b*x+a)^(8/3)-1/5*
a^3*(b*x+a)^(5/3))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{14}{3}}}{14\,b^{4}}-\frac{9\,(bx+a)^{\frac{11}{3}}a}{11\,b^{4}}+\frac{9\,(bx+a)^{\frac{8}{3}}a^{2}}{8\,b^{4}}-\frac{3\,(bx+a)^{\frac{5}{3}}a^{3}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x + a)^(14/3)/b^4 - 9/11*(b*x + a)^(11/3)*a/b^4 + 9/8*(b*x + a)^(8/
3)*a^2/b^4 - 3/5*(b*x + a)^(5/3)*a^3/b^4
```

Fricas [A]
time = 0.43, size = 53, normalized size = 0.74

$$\frac{3\,(220\,b^{4}x^{4}+40\,ab^{3}x^{3}-45\,a^{2}b^{2}x^{2}+54\,a^{3}bx-81\,a^{4})(bx+a)^{\frac{2}{3}}}{3080\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^4*x^4 + 40*a*b^3*x^3 - 45*a^2*b^2*x^2 + 54*a^3*b*x - 81*a^4)*
(b*x + a)^(2/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(68) = 136$.
time = 1.27, size = 1742, normalized size = 24.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 4
6200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 1848
0*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(74/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 1296*
a**(71/3)*b*x*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 1458*a**(71/3)*b*x/(3080*a**20*
b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 +
 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 2
808*a**(68/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b
**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x*
*4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 3645*a**(68/3)*b**2*x
**2/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a
**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14
*b**10*x**6) - 3120*a**(65/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4860*
a**(65/3)*b**3*x**3/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**
6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x
**5 + 3080*a**14*b**10*x**6) - 1050*a**(62/3)*b**4*x**4*(1 + b*x/a)**(2/3)/
(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17
*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**
10*x**6) + 3645*a**(62/3)*b**4*x**4/(3080*a**20*b**4 + 18480*a**19*b**5*x +
 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18
480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4032*a**(59/3)*b**5*x**5*(1
+ b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x*
*2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5
+ 3080*a**14*b**10*x**6) + 1458*a**(59/3)*b**5*x**5/(3080*a**20*b**4 + 1848
0*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**1
6*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 11004*a**(56
/3)*b**6*x**6*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
```

```
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(56/3)*b**6*x**6/(3080*a
**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x
**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6
) + 14352*a**(53/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a
**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b
**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 10485*a**(50/3)
*b**8*x**8*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200
*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a*
*15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4080*a**(47/3)*b**9*x**9*(1 + b*x/
a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 6
1600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080
*a**14*b**10*x**6) + 660*a**(44/3)*b**10*x**10*(1 + b*x/a)**(2/3)/(3080*a**
20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**
3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(56) = 112.
time = 1.12, size = 117, normalized size = 1.62

$$\frac{3\left(\frac{7\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^{2}-220\,(bx+a)^{\frac{2}{3}}a^{3}\right)a}{b^{3}}+\frac{2\left(110\,(bx+a)^{\frac{14}{3}}-560\,(bx+a)^{\frac{11}{3}}a+1155\,(bx+a)^{\frac{8}{3}}a^{2}-1232\,(bx+a)^{\frac{5}{3}}a^{3}+770\,(bx+a)^{\frac{2}{3}}a^{4}\right)}{b^{3}}\right)}{3080\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/3080*(7*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3
)*a^2 - 220*(b*x + a)^(2/3)*a^3)*a/b^3 + 2*(110*(b*x + a)^(14/3) - 560*(b*x
 + a)^(11/3)*a + 1155*(b*x + a)^(8/3)*a^2 - 1232*(b*x + a)^(5/3)*a^3 + 770*
(b*x + a)^(2/3)*a^4)/b^3)/b
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{14/3}}{14\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(14/3))/(14*b^4) - (3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a
 + b*x)^(8/3))/(8*b^4) - (9*a*(a + b*x)^(11/3))/(11*b^4)
```

3.379 $\int x^2(a+bx)^{2/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}$$

```
[Out] 3/5*a^2*(b*x+a)^(5/3)/b^3-3/4*a*(b*x+a)^(8/3)/b^3+3/11*(b*x+a)^(11/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a + b
*x)^(11/3))/(11*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{2/3}\,dx&=\int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{5/3}\left(9a^2-15abx+20b^2x^2\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(2/3),x]
[Out] (3*(a + b*x)^(5/3)*(9*a^2 - 15*a*b*x + 20*b^2*x^2))/(220*b^3)
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{5}{3}}\left(20x^2b^2-15abx+9a^2\right)}{220b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{3a(bx+a)^{\frac{8}{3}}}{4}+\frac{3a^2(bx+a)^{\frac{5}{3}}}{5}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{3a(bx+a)^{\frac{8}{3}}}{4}+\frac{3a^2(bx+a)^{\frac{5}{3}}}{5}}{b^3}$	38
trager	$\frac{3\left(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3\right)(bx+a)^{\frac{2}{3}}}{220b^3}$	43
risch	$\frac{3\left(20b^3x^3+5a\,b^2x^2-6a^2bx+9a^3\right)(bx+a)^{\frac{2}{3}}}{220b^3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/b^3*(1/11*(b*x+a)^(11/3)-1/4*a*(b*x+a)^(8/3)+1/5*a^2*(b*x+a)^(5/3))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^3}-\frac{3\,(bx+a)^{\frac{8}{3}}a}{4\,b^3}+\frac{3\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="maxima")
[Out] 3/11*(b*x + a)^(11/3)/b^3 - 3/4*(b*x + a)^(8/3)*a/b^3 + 3/5*(b*x + a)^(5/3)
*a^2/b^3
```

Fricas [A]
time = 0.49, size = 42, normalized size = 0.79

$$\frac{3\,(20\,b^3x^3+5\,ab^2x^2-6\,a^2bx+9\,a^3)(bx+a)^{\frac{2}{3}}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="fricas")
[Out] 3/220*(20*b^3*x^3 + 5*a*b^2*x^2 - 6*a^2*b*x + 9*a^3)*(b*x + a)^(2/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(49) = 98$.
time = 0.85, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{35}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{63a^{\frac{32}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{32}{3}}bx}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{42a^{\frac{29}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{29}{3}}b^2x^2}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{78a^{\frac{26}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{26}{3}}b^3x^3}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{207a^{\frac{23}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{195a^{\frac{20}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{60a^{\frac{17}{3}}b^6x^6\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) - 27*a**(35/3)/(220*a**8*b**3 + 660*a**7*b
**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 63*a**(32/3)*b*x*(1 + b*
x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**
5*b**6*x**3) - 81*a**(32/3)*b*x/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) + 42*a**(29/3)*b**2*x**2*(1 + b*x/a)**(2/3
)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**
3) - 81*a**(29/3)*b**2*x**2/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**
5*x**2 + 220*a**5*b**6*x**3) + 78*a**(26/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(2
20*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) -
 27*a**(26/3)*b**3*x**3/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x*
*2 + 220*a**5*b**6*x**3) + 207*a**(23/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(220*
a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 19
5*a**(20/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x +
 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 60*a**(17/3)*b**6*x**6*(1 + b*x
/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5
*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 1.13, size = 92, normalized size = 1.74

$$\frac{3\left(\frac{11\left(5(bx+a)^{\frac{8}{3}}-16(bx+a)^{\frac{5}{3}}a+20(bx+a)^{\frac{2}{3}}a^2\right)a}{b^2}+\frac{40(bx+a)^{\frac{11}{3}}-165(bx+a)^{\frac{8}{3}}a+264(bx+a)^{\frac{5}{3}}a^2-220(bx+a)^{\frac{2}{3}}a^3}{b^2}\right)}{440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/440*(11*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^
2)*a/b^2 + (40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/
3)*a^2 - 220*(b*x + a)^(2/3)*a^3)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{60\,(a+b\,x)^{11/3}-165\,a\,(a+b\,x)^{8/3}+132\,a^2\,(a+b\,x)^{5/3}}{220\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(2/3),x)
```

```
[Out] (60*(a + b*x)^(11/3) - 165*a*(a + b*x)^(8/3) + 132*a^2*(a + b*x)^(5/3))/(22
0*b^3)
```

3.380 $\int x(a+bx)^{2/3}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}$$

```
[Out] -3/5*a*(b*x+a)^(5/3)/b^2+3/8*(b*x+a)^(8/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{2/3}\,dx&=\int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.03

$$\frac{3(a+bx)^{2/3}\left(-3a^2+2abx+5b^2x^2\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-3*a^2 + 2*a*b*x + 5*b^2*x^2))/(40*b^2)
```

Maple [A]
time = 0.09, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{5}{3}}(-5bx+3a)}{40b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{3a(bx+a)^{\frac{5}{3}}}{5}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{3a(bx+a)^{\frac{5}{3}}}{5}}{b^2}$	26
trager	$-\frac{3(-5x^2b^2-2abx+3a^2)(bx+a)^{\frac{2}{3}}}{40b^2}$	32
risch	$-\frac{3(-5x^2b^2-2abx+3a^2)(bx+a)^{\frac{2}{3}}}{40b^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/8*(b*x+a)^(8/3)-1/5*a*(b*x+a)^(5/3))
```

Maxima [A]
time = 0.30, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^2}-\frac{3\,(bx+a)^{\frac{5}{3}}a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="maxima")
[Out] 3/8*(b*x + a)^(8/3)/b^2 - 3/5*(b*x + a)^(5/3)*a/b^2
```

Fricas [A]
time = 0.49, size = 31, normalized size = 0.91

$$\frac{3\,(5\,b^2x^2+2\,abx-3\,a^2)(bx+a)^{\frac{2}{3}}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="fricas")
[Out] 3/40*(5*b^2*x^2 + 2*a*b*x - 3*a^2)*(b*x + a)^(2/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(31) = 62$.
time = 0.56, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{14}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{14}{3}}}{40a^2b^2+40ab^3x}-\frac{3a^{\frac{11}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{11}{3}}bx}{40a^2b^2+40ab^3x}+\frac{21a^{\frac{8}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{15a^{\frac{5}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(14/3)*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 9*a**(14/3)/
(40*a**2*b**2 + 40*a*b**3*x) - 3*a**(11/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**2*
b**2 + 40*a*b**3*x) + 9*a**(11/3)*b*x/(40*a**2*b**2 + 40*a*b**3*x) + 21*a**
(8/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 15*a**(5/
3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.
time = 0.75, size = 68, normalized size = 2.00

$$\frac{3\left(\frac{4\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}\,a\right)a}{b}+\frac{5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}\,a+20\,(bx+a)^{\frac{2}{3}}\,a^2}{b}\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/40*(4*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)*a/b + (5*(b*x + a)^(8/3)
- 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{24\,a\,(a+b\,x)^{5/3}-15\,(a+b\,x)^{8/3}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(2/3),x)
```

```
[Out] -(24*a*(a + b*x)^(5/3) - 15*(a + b*x)^(8/3))/(40*b^2)
```

3.381 $\int (a+bx)^{2/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{5/3}}{5b}$$

[Out] 3/5*(b*x+a)^(5/3)/b

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(2/3),x]

[Out] (3*(a + b*x)^(5/3))/(5*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(2/3),x]

[Out] (3*(a + b*x)^(5/3))/(5*b)

Maple [A]
time = 0.09, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
default	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
trager	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13
risch	$\frac{3(bx+a)^{\frac{5}{3}}}{5b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*(b*x+a)^(5/3)/b
```

Maxima [A]
time = 0.26, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

Fricas [A]
time = 0.50, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{3(a+bx)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3),x)
[Out] 3*(a + b*x)**(5/3)/(5*b)
```

Giac [A]
time = 0.78, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="giac")
[Out] 3/5*(b*x + a)^(5/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{5/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3),x)
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

3.382 $\int \frac{(a+bx)^{2/3}}{x}\,dx$

Optimal. Leaf size=92

$$\frac{3}{2}(a+bx)^{2/3}+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 3/2*(b*x+a)^(2/3)-1/2*a^(2/3)*ln(x)+3/2*a^(2/3)*ln(a^(1/3)-(b*x+a)^(1/3))+a
^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 57, 631, 210, 31}

$$\sqrt{3}\,a^{2/3}\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)+\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3)
)/(Sqrt[3]*a^(1/3))] - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3) - (a + b
*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{2/3}}{x}\,dx &= \frac{3}{2}(a+bx)^{2/3} + a\int \frac{1}{x\sqrt[3]{a+bx}}\,dx\\ &= \frac{3}{2}(a+bx)^{2/3} - \frac{1}{2}a^{2/3}\log(x) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) + \frac{1}{2}(3a)\text{S}\\ &= \frac{3}{2}(a+bx)^{2/3} - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-\right.\\ &= \frac{3}{2}(a+bx)^{2/3} + \sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{\right.\end{aligned}$$

Mathematica [A]
time = 0.06, size = 114, normalized size = 1.24

$$\frac{3}{2}(a+bx)^{2/3} + \sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \frac{1}{2}a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(
1/3))/Sqrt[3]] + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(2/3)*Log[a^(2
/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Maple [A]
time = 0.09, size = 90, normalized size = 0.98

method	result
derivativedivides	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}\right.$
default	$\frac{3(bx+a)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(b*x+a)^(2/3)+3*(1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((
b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*
3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))*a
```

Maxima [A]
time = 0.48, size = 85, normalized size = 0.92

$$\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/2*(b*x + a)^(2/3)
```

Fricas [A]
time = 0.48, size = 110, normalized size = 1.20

$$\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{2}\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)+\left(a^{2}\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(a^2)^(1/3)*(b*x + a)
^(1/3))/a) - 1/2*(a^2)^(1/3)*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^
(2/3)*(b*x + a)^(1/3)) + (a^2)^(1/3)*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) +
 3/2*(b*x + a)^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.07, size = 182, normalized size = 1.98

$$\frac{5a^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x,x)
```

```
[Out] 5*a**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(3*gamma(
8/3)) + 5*a**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(5/3)/(3*gamma(8/3)) + 5*a**(2/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)
/(3*gamma(8/3)) + 5*b**(2/3)*(a/b + x)**(2/3)*gamma(5/3)/(2*gamma(8/3))
```

Giac [A]
time = 1.29, size = 86, normalized size = 0.93

$$\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/2*(b*x + a)^(2/3)
```

Mupad [B]
time = 0.11, size = 117, normalized size = 1.27

$$\frac{3\,{\left(a+b\,x\right)}^{2/3}}{2}+a^{2/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{7/3}\right)+\frac{a^{2/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{7/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{2/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{7/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x)^(2/3))/2 + a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - 9*a^(7/3)) + (a
^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/
2)*1i - 1))/2 - (a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i
 + 1)^2)/4)*(3^(1/2)*1i + 1))/2
```

3.383 $\int \frac{(a+bx)^{2/3}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{(a+bx)^{2/3}}{x}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

```
[Out] -(b*x+a)^(2/3)/x-1/3*b*ln(x)/a^(1/3)+b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(1/3)+2/
3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 57, 631, 210, 31}

$$\frac{2b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3) - (a
+ b*x)^(1/3)])/a^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = -\frac{(a+bx)^{2/3}}{x} + \frac{1}{3}(2b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$

$$= -\frac{(a+bx)^{2/3}}{x} - \frac{b\log(x)}{3\sqrt[3]{a}} + b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{b\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{(a+bx)^{2/3}}{x} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}} - \frac{(2b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cdots}{\cdots}\right.}{\sqrt[3]{a}}$$

$$= -\frac{(a+bx)^{2/3}}{x} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

Mathematica [**A**]
time = 0.14, size = 120, normalized size = 1.28

$$\frac{-3\sqrt[3]{a}\,(a+bx)^{2/3}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-bx\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3\sqrt[3]{a}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^2,x]
```

[Out] (-3*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(1/3)*x)

Maple [A]
time = 0.10, size = 95, normalized size = 1.01

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{x}+\frac{2b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{2b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3bx}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{1}{3}}}\right.$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3bx}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{1}{3}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(2/3)/x^2,x,method=_RETURNVERBOSE)

[Out] 3*b*(-1/3*(b*x+a)^(2/3)/b/x+2/9/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/9/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+2/9*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))

Maxima [A]
time = 0.48, size = 93, normalized size = 0.99

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{2b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="maxima")

[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 1/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + 2/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(1/3) - (b*x + a)^(2/3)/x

Fricas [A]
time = 0.75, size = 252, normalized size = 2.68

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3(bx+a)^{\frac{2}{3}}a}{3ax},\ \frac{6\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}bx\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3(bx+a)^{\frac{2}{3}}a}{3ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^
(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3
*(b*x + a)^(2/3)*a)/(a*x), 1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/3)*(2
*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b
*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1
/3)) - 3*(b*x + a)^(2/3)*a)/(a*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.15, size = 643, normalized size = 6.84

$$\frac{10a^{\frac{8}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{15a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**2,x)
```

```
[Out] 10*a**(8/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gam
ma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)
*gamma(8/3)) + 10*a**(8/3)*b*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3
) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*ex
p(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 10*
a**(5/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**
(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*
exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 1
0*a**(5/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(8/3)) + 15*a**2*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*
pi/3)*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(
2*I*pi/3)*gamma(8/3))
```

Giac [A]
time = 2.08, size = 106, normalized size = 1.13

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3\,(bx+a)^{\frac{2}{3}}b}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(1/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x + a)^(2/
3)*b/x)/b
```

Mupad [B]
time = 0.11, size = 127, normalized size = 1.35

$$\frac{2\,b\,\ln\left(4\,a^{1/3}\,b^2-4\,b^2\,{(a+b\,x)}^{1/3}\right)}{3\,a^{1/3}}-\frac{{(a+b\,x)}^{2/3}}{x}-\frac{\ln\left(a^{1/3}\,{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,b^2\,{(a+b\,x)}^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{1/3}}-\frac{\ln\left(a^{1/3}\,{\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,b^2\,{(a+b\,x)}^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^2,x)
```

```
[Out] (2*b*log(4*a^(1/3)*b^2 - 4*b^2*(a + b*x)^(1/3)))/(3*a^(1/3)) - (a + b*x)^(2
/3)/x - (log(a^(1/3)*(b - 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*x)^(1/3))*(b - 3^(
1/2)*b*1i))/(3*a^(1/3)) - (log(a^(1/3)*(b + 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*
x)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(1/3))
```

3.384 $\int \frac{(a+bx)^{2/3}}{x^3} dx$

Optimal. Leaf size=127

$$-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}$$

```
[Out] -1/2*(b*x+a)^(2/3)/x^2-1/3*b*(b*x+a)^(2/3)/a/x+1/18*b^2*ln(x)/a^(4/3)-1/6*b
^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)-1/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^
(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 57, 631, 210, 31}

$$-\frac{b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -1/2*(a + b*x)^(2/3)/x^2 - (b*(a + b*x)^(2/3))/(3*a*x) - (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b^2*Log[
x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{2/3}}{x^3}\,dx &= -\frac{(a+bx)^{2/3}}{2x^2}+\frac{1}{3}b\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx \\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a} \\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}+\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\text{Sub}}{} \\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}+\frac{b^2\text{Subst}\left(\int \frac{1}{-3-x}\right.}{} \\
&= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\right.}{6a^{4/3}}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 147, normalized size = 1.16

$$-\frac{(a+bx)^{2/3}(a+2(a+bx))}{6ax^2}-\frac{b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{4/3}}+\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(a + 2*(a + b*x)))/(a*x^2) - (b^2*ArcTan[1/Sqrt[3] +
(2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) - (b^2*Log[a^(1
/3) - (a + b*x)^(1/3)])/(9*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^
(1/3) + (a + b*x)^(2/3)])/(18*a^(4/3))
```

Maple [A]
time = 0.12, size = 118, normalized size = 0.93

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(2bx+3a)}{6x^2a}-\frac{b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{4}{3}}}+\frac{b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{4}{3}}}-\frac{b^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2(\ldots\right.}{\ldots}\right)}{9a^{\frac{4}{3}}}$
derivativedivides	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{5}{3}}}{9a}+\frac{(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\ldots\right.}{3}\right)}{3a^{\frac{1}{3}}}}{9a}\right.$
default	$3b^2\left(-\frac{\frac{(bx+a)^{\frac{5}{3}}}{9a}+\frac{(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}\ldots\right.}{3}\right)}{3a^{\frac{1}{3}}}}{9a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-(1/9/a*(b*x+a)^(5/3)+1/18*(b*x+a)^(2/3))/b^2/x^2-1/9/a*(1/3/a^(1/3)
*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/
3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)
+1))))
```

Maxima [A]
time = 0.51, size = 139, normalized size = 1.09

$$-\frac{\sqrt{3}\, b^{2} \arctan\left(\frac{\sqrt{3}\, \left(2\, {\left(b x + a\right)}^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)}{9\, a^{\frac{4}{3}}} + \frac{b^{2} \log\left({\left(b x + a\right)}^{\frac{2}{3}} + {\left(b x + a\right)}^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18\, a^{\frac{4}{3}}} - \frac{b^{2} \log\left({\left(b x + a\right)}^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{9\, a^{\frac{4}{3}}} - \frac{2\, {\left(b x + a\right)}^{\frac{5}{3}} b^{2} + {\left(b x + a\right)}^{\frac{2}{3}} a b^{2}}{6\, \left({\left(b x + a\right)}^{2} a - 2\, {\left(b x + a\right)} a^{2} + a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(4/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(4/3) - 1/6*(2*(b*x + a
)^(5/3)*b^2 + (b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3
)
```

Fricas [A]
time = 0.85, size = 350, normalized size = 2.76

$$\left[\frac{3\, \sqrt{\frac{1}{3}}\, a b^{2} x^{2} \sqrt{\frac{\left(-a\right)^{\frac{1}{3}}}{a}} \log\left(\frac{2\, b x - 3\, \sqrt{\frac{1}{3}}\, \left(2\, {\left(b x + a\right)}^{\frac{2}{3}} \left(-a\right)^{\frac{2}{3}} - {\left(b x + a\right)}^{\frac{1}{3}} a + \left(-a\right)^{\frac{1}{3}} a\right) \sqrt{\frac{\left(-a\right)^{\frac{1}{3}}}{a}} - 3\, {\left(b x + a\right)}^{\frac{1}{3}} \left(-a\right)^{\frac{2}{3}} + 3\, a}{x}\right) + \left(-a\right)^{\frac{2}{3}} b^{2} x^{2} \log\left({\left(b x + a\right)}^{\frac{2}{3}} - {\left(b x + a\right)}^{\frac{1}{3}} \left(-a\right)^{\frac{1}{3}} + \left(-a\right)^{\frac{2}{3}}\right) - 2\, \left(-a\right)^{\frac{2}{3}} b^{2} x^{2} \log\left({\left(b x + a\right)}^{\frac{1}{3}} + \left(-a\right)^{\frac{1}{3}}\right) - 3\, {\left(2\, a b x + 3\, a^{2}\right)} {\left(b x + a\right)}^{\frac{2}{3}}}{18\, a^{2} x^{2}}, -\frac{6\, \sqrt{\frac{1}{3}}\, a b^{2} x^{2} \sqrt{-\frac{\left(-a\right)^{\frac{1}{3}}}{a}} \arctan\left(\sqrt{\frac{1}{3}}\, \left(2\, {\left(b x + a\right)}^{\frac{1}{3}} - \left(-a\right)^{\frac{1}{3}}\right) \sqrt{-\frac{\left(-a\right)^{\frac{1}{3}}}{a}}\right) - \left(-a\right)^{\frac{2}{3}} b^{2} x^{2} \log\left({\left(b x + a\right)}^{\frac{2}{3}} - {\left(b x + a\right)}^{\frac{1}{3}} \left(-a\right)^{\frac{1}{3}} + \left(-a\right)^{\frac{2}{3}}\right) + 2\, \left(-a\right)^{\frac{2}{3}} b^{2} x^{2} \log\left({\left(b x + a\right)}^{\frac{1}{3}} + \left(-a\right)^{\frac{1}{3}}\right) + 3\, {\left(2\, a b x + 3\, a^{2}\right)} {\left(b x + a\right)}^{\frac{2}{3}}}{18\, a^{2} x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2
*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b^2*x^2*log((
b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b^
2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x + 3*a^2)*(b*x + a)^(2/
3))/(a^2*x^2), -1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-
a)^(2/3)*b^2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(2*a*b*x + 3*a^2)*(b
*x + a)^(2/3))/(a^2*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.72, size = 2266, normalized size = 17.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**3,x)
```

```
[Out] -10*a**(17/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
```

```
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(2*I
*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(
2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3
*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a**7
*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) +
 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2*I
*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**
5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*ex
p(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(2*I*
pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2
*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a
**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*
exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
```

```
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**5*b**(8/3)*(a/b + x)**(2/3)*exp(
2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b +
x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamm
a(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**4*b**(
11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamm
a(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)
*gamma(8/3)) + 30*a**3*b**(14/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(5/3)/
(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamm
a(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3))
```

Giac [A]
time = 1.51, size = 129, normalized size = 1.02

$$-\frac{\frac{2\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\,(bx+a)^{\frac{5}{3}}b^3+(bx+a)^{\frac{2}{3}}ab^3\right)}{ab^2x^2}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) - b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x + a
)^(5/3)*b^3 + (b*x + a)^(2/3)*a*b^3)/(a*b^2*x^2))/b
```

Mupad [B]
time = 0.33, size = 194, normalized size = 1.53

$$\frac{(-1)^{1/3}\,b^2\,\ln\left((a+b\,x)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{9\,a^{4/3}}-\frac{\frac{b^2\,(a+b\,x)^{2/3}}{6}+\frac{b^2\,(a+b\,x)^{5/3}}{3\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2}+\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{4/3}}-\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^3,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x)^(1/3) - (-1)^(2/3)*a^(1/3)))/(9*a^(4/3)) - ((
b^2*(a + b*x)^(2/3))/6 + (b^2*(a + b*x)^(5/3))/(3*a))/((a + b*x)^2 - 2*a*(a
 + b*x) + a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^
(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(9
*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^(2/3)
*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(4
/3))
```

3.385 $\int x^3(a+bx)^{4/3}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}$$

```
[Out] -3/7*a^3*(b*x+a)^(7/3)/b^4+9/10*a^2*(b*x+a)^(10/3)/b^4-9/13*a*(b*x+a)^(13/3
)/b^4+3/16*(b*x+a)^(16/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*a
*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{4/3}\,dx &= \int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{7/3}\left(-81a^3+189a^2bx-315ab^2x^2+455b^3x^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-81*a^3 + 189*a^2*b*x - 315*a*b^2*x^2 + 455*b^3*x^3))/(
7280*b^4)
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{7}{3}}\left(-455b^3x^3+315a\,b^2x^2-189a^2bx+81a^3\right)}{7280b^4}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{16}{3}}}{16}-\frac{9a(bx+a)^{\frac{13}{3}}}{13}+\frac{9a^2(bx+a)^{\frac{10}{3}}}{10}-\frac{3a^3(bx+a)^{\frac{7}{3}}}{7}}{b^4}$	50
default	$\frac{\frac{3(bx+a)^{\frac{16}{3}}}{16}-\frac{9a(bx+a)^{\frac{13}{3}}}{13}+\frac{9a^2(bx+a)^{\frac{10}{3}}}{10}-\frac{3a^3(bx+a)^{\frac{7}{3}}}{7}}{b^4}$	50
trager	$-\frac{3\left(-455b^5x^5-595a\,b^4x^4-14a^2b^3x^3+18a^3b^2x^2-27a^4bx+81a^5\right)(bx+a)^{\frac{1}{3}}}{7280b^4}$	65
risch	$-\frac{3\left(-455b^5x^5-595a\,b^4x^4-14a^2b^3x^3+18a^3b^2x^2-27a^4bx+81a^5\right)(bx+a)^{\frac{1}{3}}}{7280b^4}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/16*(b*x+a)^(16/3)-3/13*a*(b*x+a)^(13/3)+3/10*a^2*(b*x+a)^(10/3)-1/
7*a^3*(b*x+a)^(7/3))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{16}{3}}}{16\,b^4}-\frac{9\,(bx+a)^{\frac{13}{3}}a}{13\,b^4}+\frac{9\,(bx+a)^{\frac{10}{3}}a^2}{10\,b^4}-\frac{3\,(bx+a)^{\frac{7}{3}}a^3}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x + a)^(16/3)/b^4 - 9/13*(b*x + a)^(13/3)*a/b^4 + 9/10*(b*x + a)^(1
0/3)*a^2/b^4 - 3/7*(b*x + a)^(7/3)*a^3/b^4
```

Fricas [A]
time = 0.68, size = 64, normalized size = 0.89

$$\frac{3\,(455\,b^5x^5+595\,ab^4x^4+14\,a^2b^3x^3-18\,a^3b^2x^2+27\,a^4bx-81\,a^5)(bx+a)^{\frac{1}{3}}}{7280\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^5*x^5 + 595*a*b^4*x^4 + 14*a^2*b^3*x^3 - 18*a^3*b^2*x^2 + 27*
a^4*b*x - 81*a^5)*(b*x + a)^(1/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1844 vs. $2(68) = 136$.
time = 1.40, size = 1844, normalized size = 25.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(4/3),x)
```

```
[Out] -243*a**(76/3)*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 1
09200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 4
3680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 243*a**(76/3)/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) -
 1377*a**(73/3)*b*x*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*
x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**
4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(73/3)*b*x/(72
80*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*
b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**
10*x**6) - 3213*a**(70/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 4
3680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 10920
0*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a
**(70/3)*b**2*x**2/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**
6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9
*x**5 + 7280*a**14*b**10*x**6) - 3927*a**(67/3)*b**3*x**3*(1 + b*x/a)**(1/3
)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a
**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**1
4*b**10*x**6) + 4860*a**(67/3)*b**3*x**3/(7280*a**20*b**4 + 43680*a**19*b**
5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x
**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) - 798*a**(64/3)*b**4*x
**4*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18
*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*
b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a**(64/3)*b**4*x**4/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) +
 11382*a**(61/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**1
9*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b
**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(61/3)*
b**5*x**5/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 +
145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7
280*a**14*b**10*x**6) + 35238*a**(58/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(7280*
```

```
a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**
7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*
x**6) + 243*a**(58/3)*b**6*x**6/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109
200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 436
80*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 56562*a**(55/3)*b**7*x**7*(1
+ b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x
**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x*
*5 + 7280*a**14*b**10*x**6) + 54273*a**(52/3)*b**8*x**8*(1 + b*x/a)**(1/3)/
(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**
17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*
b**10*x**6) + 31227*a**(49/3)*b**9*x**9*(1 + b*x/a)**(1/3)/(7280*a**20*b**4
 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 1
09200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 99
75*a**(46/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*
b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**
8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1365*a**(43/3)*b*
*11*x**11*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200
*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*
a**15*b**9*x**5 + 7280*a**14*b**10*x**6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(56) = 112.

time = 0.85, size = 193, normalized size = 2.68

$$\frac{3\left(\frac{52\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a^2}{b^3}+\frac{32\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3+455\,(bx+a)^{\frac{1}{3}}a^4\right)a}{b^3}+\frac{5\left(91\,(bx+a)^{\frac{16}{3}}-560\,(bx+a)^{\frac{13}{3}}a+1456\,(bx+a)^{\frac{10}{3}}a^2-2080\,(bx+a)^{\frac{7}{3}}a^3+1820\,(bx+a)^{\frac{4}{3}}a^4-1456\,(bx+a)^{\frac{1}{3}}a^5\right)}{b^3}\right)}{7280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/7280*(52*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a^2/b^3 + 32*(35*(b*x + a)^(13/3) - 182*(b
*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*
(b*x + a)^(1/3)*a^4)*a/b^3 + 5*(91*(b*x + a)^(16/3) - 560*(b*x + a)^(13/3)*
a + 1456*(b*x + a)^(10/3)*a^2 - 2080*(b*x + a)^(7/3)*a^3 + 1820*(b*x + a)^(
4/3)*a^4 - 1456*(b*x + a)^(1/3)*a^5)/b^3)/b
```

Mupad [B]

time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{16/3}}{16\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{7/3}}{7\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{10/3}}{10\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{13/3}}{13\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(16/3))/(16*b^4) - (3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a
 + b*x)^(10/3))/(10*b^4) - (9*a*(a + b*x)^(13/3))/(13*b^4)
```

3.386 $\int x^2(a+bx)^{4/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}$$

```
[Out] 3/7*a^2*(b*x+a)^(7/3)/b^3-3/5*a*(b*x+a)^(10/3)/b^3+3/13*(b*x+a)^(13/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a + b*x)^(13/3))/(13*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{4/3}\,dx &= \int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{7/3}\left(9a^2-21abx+35b^2x^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(9*a^2 - 21*a*b*x + 35*b^2*x^2))/(455*b^3)
```

Maple [A]
time = 0.10, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{7}{3}}(35x^2b^2-21abx+9a^2)}{455b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{3a(bx+a)^{\frac{10}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{7}{3}}}{7}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{13}{3}}}{13}-\frac{3a(bx+a)^{\frac{10}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{7}{3}}}{7}}{b^3}$	38
trager	$\frac{3(35b^4x^4+49a\,b^3x^3+2a^2b^2x^2-3a^3bx+9a^4)(bx+a)^{\frac{1}{3}}}{455b^3}$	54
risch	$\frac{3(35b^4x^4+49a\,b^3x^3+2a^2b^2x^2-3a^3bx+9a^4)(bx+a)^{\frac{1}{3}}}{455b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/13*(b*x+a)^(13/3)-1/5*a*(b*x+a)^(10/3)+1/7*a^2*(b*x+a)^(7/3))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^3}-\frac{3\,(bx+a)^{\frac{10}{3}}a}{5\,b^3}+\frac{3\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^3 - 3/5*(b*x + a)^(10/3)*a/b^3 + 3/7*(b*x + a)^(7/3
)*a^2/b^3
```

Fricas [A]
time = 0.74, size = 53, normalized size = 1.00

$$\frac{3\,(35\,b^4x^4+49\,ab^3x^3+2\,a^2b^2x^2-3\,a^3bx+9\,a^4)(bx+a)^{\frac{1}{3}}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^4*x^4 + 49*a*b^3*x^3 + 2*a^2*b^2*x^2 - 3*a^3*b*x + 9*a^4)*(b*x
+ a)^(1/3)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(49) = 98$.

time = 0.94, size = 733, normalized size = 13.83

$$\frac{27a^{\frac{37}{3}}\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{37}{3}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{72a^{\frac{34}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{81a^{\frac{34}{3}}bx}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{60a^{\frac{31}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{81a^{\frac{31}{3}}b^2x^2}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{165a^{\frac{28}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{28}{3}}b^3x^3}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{555a^{\frac{25}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{762a^{\frac{22}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{462a^{\frac{19}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{105a^{\frac{16}{3}}b^7x^7\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(4/3),x)
```

```
[Out] 27*a**(37/3)*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a*
*6*b**5*x**2 + 455*a**5*b**6*x**3) - 27*a**(37/3)/(455*a**8*b**3 + 1365*a**
7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 72*a**(34/3)*b*x*(1
+ b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 4
55*a**5*b**6*x**3) - 81*a**(34/3)*b*x/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 60*a**(31/3)*b**2*x**2*(1 + b*x/
a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**
5*b**6*x**3) - 81*a**(31/3)*b**2*x**2/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 165*a**(28/3)*b**3*x**3*(1 + b*x
/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a*
*5*b**6*x**3) - 27*a**(28/3)*b**3*x**3/(455*a**8*b**3 + 1365*a**7*b**4*x +
1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 555*a**(25/3)*b**4*x**4*(1 + b*
x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a
**5*b**6*x**3) + 762*a**(22/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(455*a**8*b**3
+ 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 462*a**(19
/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a
**6*b**5*x**2 + 455*a**5*b**6*x**3) + 105*a**(16/3)*b**7*x**7*(1 + b*x/a)**
(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b*
*6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(41) = 82$.

time = 0.84, size = 157, normalized size = 2.96

$$\frac{3\left(\frac{65\left(2(bx+a)^{\frac{7}{3}}-7(bx+a)^{\frac{4}{3}}a+14(bx+a)^{\frac{1}{3}}a^2\right)a^2}{b^2}+\frac{13\left(14(bx+a)^{\frac{10}{3}}-60(bx+a)^{\frac{7}{3}}a+105(bx+a)^{\frac{4}{3}}a^2-140(bx+a)^{\frac{1}{3}}a^3\right)a}{b^2}+\frac{2\left(35(bx+a)^{\frac{13}{3}}-182(bx+a)^{\frac{10}{3}}a+390(bx+a)^{\frac{7}{3}}a^2-455(bx+a)^{\frac{4}{3}}a^3+455(bx+a)^{\frac{1}{3}}a^4\right)}{b^2}\right)}{910b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/910*(65*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a^2/b^2 + 13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^
(4/3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^2 + 2*(35*(b*x + a)^(13/3) - 182*(
b*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455
*(b*x + a)^(1/3)*a^4)/b^2)/b
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{105\,{\left(a+b\,x\right)}^{13/3}-273\,a\,{\left(a+b\,x\right)}^{10/3}+195\,a^2\,{\left(a+b\,x\right)}^{7/3}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(4/3),x)
[Out] (105*(a + b*x)^(13/3) - 273*a*(a + b*x)^(10/3) + 195*a^2*(a + b*x)^(7/3))/(
455*b^3)
```

3.387 $\int x(a+bx)^{4/3}\,dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}$$

```
[Out] -3/7*a*(b*x+a)^(7/3)/b^2+3/10*(b*x+a)^(10/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{4/3}\,dx &= \int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}(-3a+7bx)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-3*a + 7*b*x))/(70*b^2)
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{7}{3}}(-7bx+3a)}{70b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{3a(bx+a)^{\frac{7}{3}}}{7}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{3a(bx+a)^{\frac{7}{3}}}{7}}{b^2}$	26
trager	$-\frac{3(-7b^3x^3-11a\,b^2x^2-a^2bx+3a^3)(bx+a)^{\frac{1}{3}}}{70b^2}$	43
risch	$-\frac{3(-7b^3x^3-11a\,b^2x^2-a^2bx+3a^3)(bx+a)^{\frac{1}{3}}}{70b^2}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/10*(b*x+a)^(10/3)-1/7*a*(b*x+a)^(7/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^2}-\frac{3\,(bx+a)^{\frac{7}{3}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="maxima")
[Out] 3/10*(b*x + a)^(10/3)/b^2 - 3/7*(b*x + a)^(7/3)*a/b^2
```

Fricas [A]
time = 1.10, size = 41, normalized size = 1.21

$$\frac{3\,(7\,b^3x^3+11\,ab^2x^2+a^2bx-3\,a^3)(bx+a)^{\frac{1}{3}}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="fricas")
[Out] 3/70*(7*b^3*x^3 + 11*a*b^2*x^2 + a^2*b*x - 3*a^3)*(b*x + a)^(1/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

time = 0.19, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{9a^3\sqrt[3]{a+bx}}{70b^2} + \frac{3a^2x\sqrt[3]{a+bx}}{70b} + \frac{33ax^2\sqrt[3]{a+bx}}{70} + \frac{3bx^3\sqrt[3]{a+bx}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x)**(1/3)/(70*b**2) + 3*a**2*x*(a + b*x)**(1/3)/(
70*b) + 33*a*x**2*(a + b*x)**(1/3)/70 + 3*b*x**3*(a + b*x)**(1/3)/10, Ne(b,
 0)), (a**(4/3)*x**2/2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(26) = 52.
time = 0.67, size = 118, normalized size = 3.47

$$\frac{3\left(\frac{35\left((bx+a)^{\frac{4}{3}}-4(bx+a)^{\frac{1}{3}}a\right)a^2}{b}+\frac{20\left(2(bx+a)^{\frac{7}{3}}-7(bx+a)^{\frac{4}{3}}a+14(bx+a)^{\frac{1}{3}}a^2\right)a}{b}+\frac{14(bx+a)^{\frac{10}{3}}-60(bx+a)^{\frac{7}{3}}a+105(bx+a)^{\frac{4}{3}}a^2-140(bx+a)^{\frac{1}{3}}a^3}{b}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/140*(35*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a^2/b + 20*(2*(b*x + a)^(
7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)*a/b + (14*(b*x + a)^(1
0/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2 - 140*(b*x + a)^(1/3)
*a^3)/b)/b
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{30\,a\,(a+b\,x)^{7/3}-21\,(a+b\,x)^{10/3}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(4/3),x)
```

```
[Out] -(30*a*(a + b*x)^(7/3) - 21*(a + b*x)^(10/3))/(70*b^2)
```

3.388 $\int (a+bx)^{4/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{7/3}}{7b}$$

```
[Out] 3/7*(b*x+a)^(7/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3),x]
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{4/3}\,dx = \frac{3(a+bx)^{7/3}}{7b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3),x]
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
default	$\frac{3(bx+a)^{\frac{7}{3}}}{7b}$	13
trager	$\frac{3(x^2b^2+2abx+a^2)(bx+a)^{\frac{1}{3}}}{7b}$	29
risch	$\frac{3(x^2b^2+2abx+a^2)(bx+a)^{\frac{1}{3}}}{7b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*(b*x+a)^(7/3)/b
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.77, size = 28, normalized size = 1.75

$$\frac{3\,(b^2x^2+2\,abx+a^2)(bx+a)^{\frac{1}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{3(a+bx)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3),x)
```

```
[Out] 3*(a + b*x)**(7/3)/(7*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.
time = 1.42, size = 58, normalized size = 3.62

$$\frac{3\left(2\,(bx+a)^{\frac{7}{3}} - 7\,(bx+a)^{\frac{4}{3}}a + 28\,(bx+a)^{\frac{1}{3}}a^2 + 7\left((bx+a)^{\frac{4}{3}} - 4\,(bx+a)^{\frac{1}{3}}a\right)a\right)}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 28*(b*x + a)^(1/3)*a^2 + 7*
((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{7/3}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

3.389 $\int \frac{(a+bx)^{4/3}}{x} \, dx$

Optimal. Leaf size=105

$$3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 3*a*(b*x+a)^(1/3)+3/4*(b*x+a)^(4/3)-1/2*a^(4/3)*ln(x)+3/2*a^(4/3)*ln(a^(1/3
)-(b*x+a)^(1/3))-a^(4/3)*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/
2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 59, 631, 210, 31}

$$-\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x,x]
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 - Sqrt[3]*a^(4/3)*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] - (a^(4/3)*Log[x])/2 + (3*a^(4/3
)*Log[a^(1/3) - (a + b*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{x}\,dx &= \frac{3}{4}(a+bx)^{4/3}+a\int \frac{\sqrt[3]{a+bx}}{x}\,dx\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}+a^2\int \frac{1}{x(a+bx)^{2/3}}\,dx\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\frac{1}{2}a^{4/3}\log(x)-\frac{1}{2}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right.\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(3a^{4/3}\right)\text{Sub}\\ &= 3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{2}a^{4/3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 1.16

$$\frac{3}{4}\sqrt[3]{a+bx}\,(5a+bx)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x,x]
```

```
[Out] (3*(a + b*x)^(1/3)*(5*a + b*x))/4 - Sqrt[3]*a^(4/3)*ArcTan[(1 + (2*(a + b*x
)^(1/3))/a^(1/3))/Sqrt[3]] + a^(4/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(4
/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

Maple [A]
time = 0.09, size = 102, normalized size = 0.97

method	result
derivativedivides	$\frac{3(bx+a)^{\frac{4}{3}}}{4}+3a(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\sqrt{\right.}}{3}\right.$
default	$\frac{3(bx+a)^{\frac{4}{3}}}{4}+3a(bx+a)^{\frac{1}{3}}+3\left(\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\sqrt{\right.}}{3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(b*x+a)^(4/3)+3*a*(b*x+a)^(1/3)+3*(1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3)
)-1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3/a^(2/3)*3
^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1)))*a^2
```

Maxima [A]
time = 0.50, size = 96, normalized size = 0.91

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+a)^{\frac{4}{3}}+3\,(bx+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + a)^(4/3) + 3*(b*x + a)^(1
/3)*a
```

Fricas [A]
time = 0.84, size = 98, normalized size = 0.93

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+5\,a)(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + 5*a)*(b*x + a)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.30, size = 209, normalized size = 1.99

$$\frac{7a^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{7}{3}\right)}{\Gamma\left(\frac{10}{3}\right)}+\frac{7b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}{4\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x,x)
```

```
[Out] 7*a**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(3*gamma(
10/3)) + 7*a**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(3*gamma(10/3)) + 7*a**(4/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/
3)/(3*gamma(10/3)) + 7*a*b**(1/3)*(a/b + x)**(1/3)*gamma(7/3)/gamma(10/3) +
 7*b**(4/3)*(a/b + x)**(4/3)*gamma(7/3)/(4*gamma(10/3))
```

Giac [A]
time = 1.83, size = 97, normalized size = 0.92

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{4}(bx+a)^{\frac{4}{3}}+3\,(bx+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/4*(b*x + a)^(4/3) + 3*(b*x +
a)^(1/3)*a
```

Mupad [B]
time = 0.06, size = 123, normalized size = 1.17

$$3\,a\,{\left(a+b\,x\right)}^{1/3}+\frac{3\,{\left(a+b\,x\right)}^{4/3}}{4}+a^{4/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{7/3}\right)+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-9\,a^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+9\,a^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x,x)
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 + a^(4/3)*log(9*a^2*(a + b*x)^(
1/3) - 9*a^(7/3)) + (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i - 1))/2 - 9*a^2*(a
+ b*x)^(1/3))*(3^(1/2)*1i - 1))/2 - (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i + 1
))/2 + 9*a^2*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/2
```

3.390 $\int \frac{(a+bx)^{4/3}}{x^2} \, dx$

Optimal. Leaf size=107

$$4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

```
[Out] 4*b*(b*x+a)^(1/3)-(b*x+a)^(4/3)/x-2/3*a^(1/3)*b*ln(x)+2*a^(1/3)*b*ln(a^(1/3
)-(b*x+a)^(1/3))-4/3*a^(1/3)*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)
*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 59, 631, 210, 31}

$$-\frac{4\sqrt[3]{a}\,b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}}-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] 4*b*(a + b*x)^(1/3) - (a + b*x)^(4/3)/x - (4*a^(1/3)*b*ArcTan[(a^(1/3) + 2*
(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log[x])/3 + 2*a
^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{x^2}\,dx &= -\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4b)\int \frac{\sqrt[3]{a+bx}}{x}\,dx\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4ab)\int \frac{1}{x(a+bx)^{2/3}}\,dx\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)-\left(2\sqrt[3]{a}\,b\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right.\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(4\sqrt[3]{a}\,b\right)\mathrm{S}\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\end{aligned}$$

Mathematica [A]

time = 0.17, size = 128, normalized size = 1.20

$$\frac{1}{3}\left(-\frac{3(a-3bx)\sqrt[3]{a+bx}}{x}-4\sqrt{3}\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-2\sqrt[3]{a}\,b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^2,x]
[Out] ((-3*(a - 3*b*x)*(a + b*x)^(1/3))/x - 4*Sqrt[3]*a^(1/3)*b*ArcTan[(1 + (2*(a
 + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3
)] - 2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/
3
```

Maple [A]
time = 0.15, size = 106, normalized size = 0.99

method	result
derivativedivides	$3b\left((bx+a)^{\frac{1}{3}}-a\left(\frac{(bx+a)^{\frac{1}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{4\sqrt{3}\ \mathrm{arctan}}{}\right.\right.$
default	$3b\left((bx+a)^{\frac{1}{3}}-a\left(\frac{(bx+a)^{\frac{1}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{4\sqrt{3}\ \mathrm{arctan}}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^2,x,method=_RETURNVERBOSE)
[Out] 3*b*((b*x+a)^(1/3)-a*(1/3*(b*x+a)^(1/3)/b/x-4/9/a^(2/3)*ln((b*x+a)^(1/3)-a^
(1/3))+2/9/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+4/9/a^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.49, size = 104, normalized size = 0.97

$$-\frac{4}{3}\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{2}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{4}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}b-\frac{(bx+a)^{\frac{1}{3}}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(
1/3)) - 2/3*a^(1/3)*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/
3)) + 4/3*a^(1/3)*b*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)*b -
(b*x + a)^(1/3)*a/x
```

Fricas [A]
time = 0.82, size = 111, normalized size = 1.04

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\,a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(3\,bx-a)(bx+a)^{\frac{1}{3}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + 2*a^(1/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3
) + a^(2/3)) - 4*a^(1/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3*(3*b*x - a)
*(b*x + a)^(1/3))/x
```

Sympy [C] Result contains complex when optimal does not.
time = 1.40, size = 719, normalized size = 6.72

$$\frac{28a^{\frac{10}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{84a^3b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{63a^2b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**2,x)
```

```
[Out] 28*a**(10/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*ga
mma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/
3)*gamma(10/3)) + 28*a**(10/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2
*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(10/3)*b*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*
a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/
3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**
(1/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*log(1 - b**(
1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(
2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(7/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) -
9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 84*a**3*b**(4/3)*(a/b + x)*
*(1/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*
b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 63*a**2*b**(7/3)*(a/b + x)**(4/3)*
```

```
exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3))
```

Giac [A]
time = 1.26, size = 119, normalized size = 1.11

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)-9\,(bx+a)^{\frac{1}{3}}b^2+\frac{3\,(bx+a)^{\frac{1}{3}}ab}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3)
)/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x + a)^(1/3) - a^(1/3))) - 9*(b*x + a)^
(1/3)*b^2 + 3*(b*x + a)^(1/3)*a*b/x)/b
```

Mupad [B]
time = 0.07, size = 131, normalized size = 1.22

$$3\,b\,(a+b\,x)^{1/3}+\frac{4\,a^{1/3}\,b\,\ln\left(12\,a^{4/3}\,b-12\,a\,b\,(a+b\,x)^{1/3}\right)}{3}-\frac{a\,(a+b\,x)^{1/3}}{x}+\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}-6\,a^{4/3}\,b\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}+6\,a^{4/3}\,b\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^2,x)
```

```
[Out] 3*b*(a + b*x)^(1/3) + (4*a^(1/3)*b*log(12*a^(4/3)*b - 12*a*b*(a + b*x)^(1/3
)))/3 - (a*(a + b*x)^(1/3))/x + (2*a^(1/3)*b*log(12*a*b*(a + b*x)^(1/3) - 6
*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (2*a^(1/3)*b*log(12*a*b*
(a + b*x)^(1/3) + 6*a^(4/3)*b*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/3
```

3.391 $\int \frac{(a+bx)^{4/3}}{x^3} \, dx$

Optimal. Leaf size=124

$$-\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}$$

```
[Out] -2/3*b*(b*x+a)^(1/3)/x-1/2*(b*x+a)^(4/3)/x^2-1/9*b^2*ln(x)/a^(2/3)+1/3*b^2*
ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3)-2/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/
3))/a^(1/3)*3^(1/2))/a^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 59, 631, 210, 31}

$$-\frac{2b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-2*b*(a + b*x)^(1/3))/(3*x) - (a + b*x)^(4/3)/(2*x^2) - (2*b^2*ArcTan[(a^(
1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^2*Lo
g[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{4/3}}{x^3}\,dx &= -\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{3}(2b)\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx \\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}+\frac{1}{9}\left(2b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx \\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{b^2\text{Su}}{} \\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}+\frac{\left(2b^2\right)\text{Subst}\left(\int\right.}{} \\
&= -\frac{2b\sqrt[3]{a+bx}}{3x}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\right.}{3a^{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 136, normalized size = 1.10

$$
\frac{1}{18}\left(-\frac{3\sqrt[3]{a+bx}\,(3a+7bx)}{x^2}-\frac{4\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{2/3}}-\frac{2b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{a^{2/3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] ((-3*(a + b*x)^(1/3)*(3*a + 7*b*x))/x^2 - (4*Sqrt[3]*b^2*ArcTan[(1 + (2*(a
+ b*x)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*b^2*Log[a^(1/3) - (a + b*x)^(
1/3)])/a^(2/3) - (2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(
2/3)])/a^(2/3))/18
```

Maple [A]
time = 0.14, size = 110, normalized size = 0.89

method	result
derivativedivides	$3b^2\left(-\frac{\frac{7(bx+a)^{\frac{4}{3}}}{18}-\frac{2a(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{\ldots}\right.}{27a\ldots}\right.$
default	$3b^2\left(-\frac{\frac{7(bx+a)^{\frac{4}{3}}}{18}-\frac{2a(bx+a)^{\frac{1}{3}}}{9}}{b^2x^2}+\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{2}{3}}}-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{\ldots}\right.}{27a\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-(7/18*(b*x+a)^(4/3)-2/9*a*(b*x+a)^(1/3))/b^2/x^2+2/27/a^(2/3)*ln((b
*x+a)^(1/3)-a^(1/3))-1/27/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^
(2/3))-2/27/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))
)
```

Maxima [A]
time = 0.51, size = 136, normalized size = 1.10

$$-\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{2}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{7(bx+a)^{\frac{4}{3}}b^2-4(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2-2(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
```

```
a^(2/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - 1/6*(7*(b*x + a)
^(4/3)*b^2 - 4*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

Fricas [A]
time = 0.61, size = 162, normalized size = 1.31

$$-\frac{4\sqrt{3}\left(a^{2}\right)^{\frac{1}{6}}ab^{2}x^{2}\arctan\left(\frac{\left(a^{2}\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^{2}\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^{2}}\right)+2\left(a^{2}\right)^{\frac{2}{3}}b^{2}x^{2}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^{2}\right)^{\frac{1}{3}}a+\left(a^{2}\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-4\left(a^{2}\right)^{\frac{2}{3}}b^{2}x^{2}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^{2}\right)^{\frac{2}{3}}\right)+3\left(7a^{2}bx+3a^{3}\right)(bx+a)^{\frac{1}{3}}}{18a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 2*(a^2)^(2/3)*b^2
*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) -
 4*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x
+ 3*a^3)*(b*x + a)^(1/3))/(a^2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.52, size = 2266, normalized size = 18.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**3,x)
```

```
[Out] 28*a**(19/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)
/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*ga
mma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*
b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3
)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a
**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b +
 x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma
(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**
3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(54
*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*exp(
```

```
-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*
gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*
I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
- 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*
(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a/b
 + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*ga
mma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*
pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) -
54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a
/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(
a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/
3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(10/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*g
amma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(
10/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)
*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a*
*(10/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3)
 - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)
**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(10/3)) + 84*a**6*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(7/3)/(54*
a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(1
0/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*
(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 231*a**5*b**(10/3)*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b
*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*p
i/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 1
47*a**4*b**(13/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*
I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**
3*exp(2*I*pi/3)*gamma(10/3))
```

Giac [**A**]

time = 1.25, size = 127, normalized size = 1.02

$$-\frac{\frac{4\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2\,b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7\,(bx+a)^{\frac{4}{3}}b^3-4\,(bx+a)^{\frac{1}{3}}ab^3\right)}{b^2x^2}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*x +
 a)^(4/3)*b^3 - 4*(b*x + a)^(1/3)*a*b^3)/(b^2*x^2))/b
```

Mupad [B]
time = 0.12, size = 174, normalized size = 1.40

$$\frac{2\,b^2\,\ln\left(2\,b^2\,{\left(a+b\,x\right)}^{1/3}-2\,a^{1/3}\,b^2\right)}{9\,a^{2/3}}-\frac{\frac{7\,b^2\,{\left(a+b\,x\right)}^{4/3}}{6}-\frac{2\,a\,b^2\,{\left(a+b\,x\right)}^{1/3}}{3}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}-\frac{\ln\left(2\,b^2\,{\left(a+b\,x\right)}^{1/3}+a^{1/3}\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,a^{2/3}}+\frac{b^2\,\ln\left(2\,b^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{1/3}\,b^2\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^3,x)
```

```
[Out] (2*b^2*log(2*b^2*(a + b*x)^(1/3) - 2*a^(1/3)*b^2))/(9*a^(2/3)) - ((7*b^2*(a
 + b*x)^(4/3))/6 - (2*a*b^2*(a + b*x)^(1/3))/3)/((a + b*x)^2 - 2*a*(a + b*x
) + a^2) - (log(2*b^2*(a + b*x)^(1/3) + a^(1/3)*(3^(1/2)*b^2*1i + b^2))*(3^
(1/2)*b^2*1i + b^2))/(9*a^(2/3)) + (b^2*log(2*b^2*(a + b*x)^(1/3) - 9*a^(1/
3)*b^2*((3^(1/2)*1i)/9 - 1/9))*((3^(1/2)*1i)/9 - 1/9))/a^(2/3)
```

3.392 $\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}$$

```
[Out] -3/2*a^3*(b*x+a)^(2/3)/b^4+9/5*a^2*(b*x+a)^(5/3)/b^4-9/8*a*(b*x+a)^(8/3)/b^
4+3/11*(b*x+a)^(11/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*(
a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{2/3}\left(-81a^3+54a^2bx-45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-81*a^3 + 54*a^2*b*x - 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.69

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$	43
trager	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$	43
risch	$-\frac{3(bx+a)^{\frac{2}{3}}\left(-40b^3x^3+45a\,b^2x^2-54a^2bx+81a^3\right)}{440b^4}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{9a(bx+a)^{\frac{8}{3}}}{8}+\frac{9a^2(bx+a)^{\frac{5}{3}}}{5}-\frac{3a^3(bx+a)^{\frac{2}{3}}}{2}}{b^4}$	50
default	$\frac{\frac{3(bx+a)^{\frac{11}{3}}}{11}-\frac{9a(bx+a)^{\frac{8}{3}}}{8}+\frac{9a^2(bx+a)^{\frac{5}{3}}}{5}-\frac{3a^3(bx+a)^{\frac{2}{3}}}{2}}{b^4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/11*(b*x+a)^(11/3)-3/8*a*(b*x+a)^(8/3)+3/5*a^2*(b*x+a)^(5/3)-1/2*a^
3*(b*x+a)^(2/3))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^4}-\frac{9\,(bx+a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx+a)^{\frac{5}{3}}a^2}{5\,b^4}-\frac{3\,(bx+a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^4 - 9/8*(b*x + a)^(8/3)*a/b^4 + 9/5*(b*x + a)^(5/3)
*a^2/b^4 - 3/2*(b*x + a)^(2/3)*a^3/b^4
```

Fricas [A]
time = 0.66, size = 42, normalized size = 0.58

$$\frac{3\,(40\,b^3x^3-45\,ab^2x^2+54\,a^2bx-81\,a^3)(bx+a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 - 45*a*b^2*x^2 + 54*a^2*b*x - 81*a^3)*(b*x + a)^(2/3)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(68) = 136$.

time = 1.20, size = 1640, normalized size = 22.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 660
0*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**1
5*b**9*x**5 + 440*a**14*b**10*x**6) + 243*a**(71/3)/(440*a**20*b**4 + 2640*
a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**
8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1296*a**(68/3)*b*x*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(68/3)*b*x/(440*a**20*b**4 + 2640*a**19*b**5
*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2
640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 2808*a**(65/3)*b**2*x**2*(1 +
 b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 +
 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a
**14*b**10*x**6) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4 + 2640*a**19*b*
*5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 +
 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 3120*a**(62/3)*b**3*x**3*(1
 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2
 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440
*a**14*b**10*x**6) + 4860*a**(62/3)*b**3*x**3/(440*a**20*b**4 + 2640*a**19*
b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4
 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1710*a**(59/3)*b**4*x**4*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 72*a**(56/3)*b**5*x**5*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1104*a**(53/3)*b**6*x**
6*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*
x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 +
 440*a**14*b**10*x**6) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4 + 2640*a**
```

```
19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x
**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1152*a**(50/3)*b**7*x*
*7*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6
*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5
+ 440*a**14*b**10*x**6) + 585*a**(47/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(440*a
**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3
 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 12
0*a**(44/3)*b**9*x**9*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*
x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 26
40*a**15*b**9*x**5 + 440*a**14*b**10*x**6)
```

Giac [A]
time = 1.01, size = 49, normalized size = 0.68

$$\frac{3\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^{2}-220\,(bx+a)^{\frac{2}{3}}a^{3}\right)}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a^
2 - 220*(b*x + a)^(2/3)*a^3)/b^4
```

Mupad [B]
time = 0.04, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{2/3}}{2\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(11/3))/(11*b^4) - (3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a
 + b*x)^(5/3))/(5*b^4) - (9*a*(a + b*x)^(8/3))/(8*b^4)
```

3.393 $\int \frac{x^2}{\sqrt[3]{a+bx}} \, dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{2/3}}{2b^3} - \frac{6a(a+bx)^{5/3}}{5b^3} + \frac{3(a+bx)^{8/3}}{8b^3}$$

```
[Out] 3/2*a^2*(b*x+a)^(2/3)/b^3-6/5*a*(b*x+a)^(5/3)/b^3+3/8*(b*x+a)^(8/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3a^2(a+bx)^{2/3}}{2b^3} + \frac{3(a+bx)^{8/3}}{8b^3} - \frac{6a(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a + b
*x)^(8/3))/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt[3]{a+bx}} \, dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}} - \frac{2a(a+bx)^{2/3}}{b^2} + \frac{(a+bx)^{5/3}}{b^2}\right) dx \\ &= \frac{3a^2(a+bx)^{2/3}}{2b^3} - \frac{6a(a+bx)^{5/3}}{5b^3} + \frac{3(a+bx)^{8/3}}{8b^3} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{2/3}\left(9a^2-6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.72

method	result	size
gosper	$\frac{3(bx+a)^{\frac{2}{3}}(5x^2b^2-6abx+9a^2)}{40b^3}$	32
trager	$\frac{3(bx+a)^{\frac{2}{3}}(5x^2b^2-6abx+9a^2)}{40b^3}$	32
risch	$\frac{3(bx+a)^{\frac{2}{3}}(5x^2b^2-6abx+9a^2)}{40b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{6a(bx+a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{2}{3}}}{2}}{b^3}$	38
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{6a(bx+a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx+a)^{\frac{2}{3}}}{2}}{b^3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/8*(b*x+a)^(8/3)-2/5*a*(b*x+a)^(5/3)+1/2*a^2*(b*x+a)^(2/3))
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^3}-\frac{6\,(bx+a)^{\frac{5}{3}}a}{5\,b^3}+\frac{3\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^3 - 6/5*(b*x + a)^(5/3)*a/b^3 + 3/2*(b*x + a)^(2/3)*a
^2/b^3
```

Fricas [A]
time = 0.54, size = 31, normalized size = 0.58

$$\frac{3\,(5\,b^2x^2-6\,abx+9\,a^2)(bx+a)^{\frac{2}{3}}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

[Out] 3/40*(5*b^2*x^2 - 6*a*b*x + 9*a^2)*(b*x + a)^(2/3)/b^3

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(49) = 98$.
time = 0.82, size = 600, normalized size = 11.32

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{32}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{63a^{\frac{29}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{29}{3}}bx}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{42a^{\frac{26}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{26}{3}}b^2x^2}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{18a^{\frac{23}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{23}{3}}b^3x^3}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{27a^{\frac{20}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}+\frac{15a^{\frac{17}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^8b^3+120a^7b^4x+120a^6b^5x^2+40a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x+a)**(1/3),x)

[Out] 27*a**(32/3)*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(32/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 42*a**(26/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(26/3)*b**2*x**2/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*b**3*x**3/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(20/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3)

Giac [A]
time = 1.72, size = 37, normalized size = 0.70

$$\frac{3\left(5\left(bx+a\right)^{\frac{8}{3}}-16\left(bx+a\right)^{\frac{5}{3}}a+20\left(bx+a\right)^{\frac{2}{3}}a^2\right)}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="giac")

[Out] 3/40*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b^3

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{15\,{\left(a+b\,x\right)}^{8/3}-48\,a\,{\left(a+b\,x\right)}^{5/3}+60\,a^2\,{\left(a+b\,x\right)}^{2/3}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*x)^(1/3),x)

[Out] (15*(a + b*x)^(8/3) - 48*a*(a + b*x)^(5/3) + 60*a^2*(a + b*x)^(2/3))/(40*b^3)

3.394 $\int \frac{x}{\sqrt[3]{a+bx}} \, dx$

Optimal. Leaf size=34

$$-\frac{3a(a+bx)^{2/3}}{2b^2}+\frac{3(a+bx)^{5/3}}{5b^2}$$

```
[Out] -3/2*a*(b*x+a)^(2/3)/b^2+3/5*(b*x+a)^(5/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3(a+bx)^{5/3}}{5b^2}-\frac{3a(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{a+bx}} \, dx &= \int \left(-\frac{a}{b\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b}\right) dx\\ &= -\frac{3a(a+bx)^{2/3}}{2b^2}+\frac{3(a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}(-3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-3*a + 2*b*x))/(10*b^2)
```

Maple [A]
time = 0.12, size = 26, normalized size = 0.76

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
trager	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
risch	$-\frac{3(bx+a)^{\frac{2}{3}}(-2bx+3a)}{10b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-\frac{3a(bx+a)^{\frac{2}{3}}}{2}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{5}{3}}}{5}-\frac{3a(bx+a)^{\frac{2}{3}}}{2}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/5*(b*x+a)^(5/3)-1/2*a*(b*x+a)^(2/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^2}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/5*(b*x + a)^(5/3)/b^2 - 3/2*(b*x + a)^(2/3)*a/b^2
```

Fricas [A]
time = 0.75, size = 20, normalized size = 0.59

$$\frac{3\,(2\,bx-3\,a)(bx+a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/10*(2*b*x - 3*a)*(b*x + a)^(2/3)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(31) = 62$.
time = 0.57, size = 162, normalized size = 4.76

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{11}{3}}}{10a^2b^2+10ab^3x}-\frac{3a^{\frac{8}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{8}{3}}bx}{10a^2b^2+10ab^3x}+\frac{6a^{\frac{5}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x) + 9*a**(11/3)/
(10*a**2*b**2 + 10*a*b**3*x) - 3*a**(8/3)*b*x*(1 + b*x/a)**(2/3)/(10*a**2*b
**2 + 10*a*b**3*x) + 9*a**(8/3)*b*x/(10*a**2*b**2 + 10*a*b**3*x) + 6*a**(5/
3)*b**2*x**2*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x)
```

Giac [A]
time = 1.12, size = 25, normalized size = 0.74

$$\frac{3\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)/b^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.74

$$-\frac{15\,a\,(a+b\,x)^{2/3}-6\,(a+b\,x)^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/3),x)
```

```
[Out] -(15*a*(a + b*x)^(2/3) - 6*(a + b*x)^(5/3))/(10*b^2)
```

3.395 $\int \frac{1}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{2/3}}{2b}$$

```
[Out] 3/2*(b*x+a)^(2/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1/3),x]
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1/3),x]
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
default	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
trager	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13
risch	$\frac{3(bx+a)^{\frac{2}{3}}}{2b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(b*x+a)^(2/3)/b
```

Maxima [A]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Fricas [A]
time = 0.79, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{3(a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3),x)
```

```
[Out] 3*(a + b*x)**(2/3)/(2*b)
```

Giac [**A**]
time = 0.96, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

3.396 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

```
[Out] -1/2*ln(x)/a^(1/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(1/3)+arctan(1/3*(a^(1/3
)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt[3]{a+bx}}\,dx &= -\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{\cdots}\right)}{2\sqrt[3]{a}} \\
&= -\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}} \\
&= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 95, normalized size = 1.20

$$
\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2\sqrt[3]{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3
) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2
/3)])/(2*a^(1/3))
```

Maple [A]
time = 0.11, size = 75, normalized size = 0.95

method	result	size

derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75
default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^
(1/3)+1))
```

Maxima [A]
time = 0.48, size = 76, normalized size = 0.96

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(1/3)
```

Fricas [A]
time = 0.90, size = 213, normalized size = 2.70

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx+\sqrt{3}\,\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\,\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a^
(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a
^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*a^(2
```

```
/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*log
((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x
+ a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.88, size = 155, normalized size = 1.96

$$\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/3),x)
```

```
[Out] 2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(
5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a**(1
/3)*gamma(5/3))
```

Giac [A]
time = 1.15, size = 77, normalized size = 0.97

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left(\left(bx+a\right)^{\frac{2}{3}}+\left(bx+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\log\left(\left|\left(bx+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B]
time = 0.09, size = 99, normalized size = 1.25

$$\frac{\ln\left(9\left(a+b\,x\right)^{1/3}-9\,a^{1/3}\right)}{a^{1/3}}+\frac{\ln\left(9\left(a+b\,x\right)^{1/3}-\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}-\frac{\ln\left(9\left(a+b\,x\right)^{1/3}-\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(1/3) + (log(9*(a + b*x)^(1/3) - (9*a^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a^(1/3)) - (log(9*(a + b*
x)^(1/3) - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a^(1/3))
```

3.397 $\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=100

$$-\frac{(a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

```
[Out] -(b*x+a)^(2/3)/a/x+1/6*b*ln(x)/a^(4/3)-1/2*b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4
/3)-1/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 57, 631, 210, 31}

$$-\frac{b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx &= -\frac{(a+bx)^{2/3}}{ax}-\frac{b\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a}\\
&= -\frac{(a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}+\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+\cdots}\right.}{2a\cdots}\\
&= -\frac{(a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a\cdots}}{\sqrt[3]{\cdots}}\right.}{a^{4/3}}\\
&= -\frac{(a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 120, normalized size = 1.20

$$-\frac{6\sqrt[3]{a}\,(a+bx)^{2/3}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-bx\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*(6*a^(1/3)*(a + b*x)^(2/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1
/3))/a^(1/3))/Sqrt[3]] + 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] - b*x*Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(a^(4/3)*x)
```

Maple [A]
time = 0.12, size = 104, normalized size = 1.04

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{ax}-\frac{b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{4}{3}}}$
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/3*(b*x+a)^(2/3)/a/b/x+1/3/a*(-1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))
+1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-1/3*3^(1/2)/a^
(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.50, size = 106, normalized size = 1.06

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}b}{(bx+a)a-a^{2}}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}}-\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - (b*x + a)^(2/3)*b/((b*x + a)*a - a^2) + 1/6*b*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x + a)^(1/3) - a
^(1/3))/a^(4/3)
```

Fricas [A]
time = 0.72, size = 306, normalized size = 3.06

$$\left[\frac{3\,\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx-3\,\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x}\right)+(-a)^{\frac{2}{3}}\,bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\,bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6\,(bx+a)^{\frac{2}{3}}a}{6\,a^{2}x},\ -\frac{6\,\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}\,bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2\,(-a)^{\frac{2}{3}}\,bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+6\,(bx+a)^{\frac{2}{3}}a}{6\,a^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*x
 + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x + a)^
(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqrt(1/3
)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3
))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x + a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3) + (-a)^
(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.14, size = 831, normalized size = 8.31

$$-\frac{2a^{\frac{5}{3}}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{5}{3}}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{5}{3}}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{2a^{\frac{2}{3}}b^{\frac{10}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{2a^{\frac{2}{3}}b^{\frac{10}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{2a^{\frac{2}{3}}b^{\frac{10}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{6ab^{3}\left(\frac{a}{b}+x\right)^{2}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}{9a^{3}b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)-9a^{2}b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*p
i/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a
/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x
)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**
3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7
```

```
/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a*
*(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/
3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3
)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*
I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +
 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3))
```

Giac [A]
time = 0.99, size = 109, normalized size = 1.09

$$-\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6\,(bx+a)^{\frac{2}{3}}b}{ax}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2
/3)*b/(a*x))/b
```

Mupad [B]
time = 0.14, size = 130, normalized size = 1.30

$$-\frac{(a+b\,x)^{2/3}}{a\,x}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}-\frac{b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b - 3^(
1/2)*b*1i))/(6*a^(4/3)) - (a + b*x)^(2/3)/(a*x) + (log((b + 3^(1/2)*b*1i)^2
/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3)) -
 (b*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(4/3))
```

3.398 $\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=130

$$-\frac{(a+bx)^{2/3}}{2ax^2}+\frac{2b(a+bx)^{2/3}}{3a^2x}+\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}$$

```
[Out] -1/2*(b*x+a)^(2/3)/a/x^2+2/3*b*(b*x+a)^(2/3)/a^2/x-1/9*b^2*ln(x)/a^(7/3)+1/
3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(7/3)+2/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x+
a)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 57, 631, 210, 31}

$$\frac{2b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{(a+bx)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/2*(a + b*x)^(2/3)/(a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) -
 (b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx = -\frac{(a+bx)^{2/3}}{2ax^2} - \frac{(2b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{(2b^2)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^2}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{b^2}{}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}} - \frac{(2b^2)\text{Subst}\left(\right.}{}$$

$$= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}\right.}{3}$$

Mathematica [A]
time = 0.14, size = 149, normalized size = 1.15

$$-\frac{(a+bx)^{2/3}(7a-4(a+bx))}{6a^2x^2} + \frac{2b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{7/3}} - \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(7*a - 4*(a + b*x)))/(a^2*x^2) + (2*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(7/3))
```

Maple [A]
time = 0.11, size = 130, normalized size = 1.00

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}(-4bx+3a)}{6a^2x^2}+\frac{2b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\right.}{}\right.}{9a^{\frac{7}{3}}}$
derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+}{a^{\frac{1}{3}}}\right.}{3}\right.}{3a^{\frac{1}{3}}}}{3a}\right.}{3a}\right.$

default	$3b^2\left(-\frac{(bx+a)^{\frac{2}{3}}}{6a\,b^2x^2}-\frac{2\left(-\frac{(bx+a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(ba}{3}\right.}{3a^{\frac{1}{3}}}\right.}{3a}\right.}{3a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-1/6/a*(b*x+a)^(2/3)/b^2/x^2-2/3/a*(-1/3*(b*x+a)^(2/3)/a/b/x+1/3/a*(
-1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)
*(b*x+a)^(1/3)+a^(2/3))-1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(
b*x+a)^(1/3)+1)))))
```

Maxima [A]
time = 0.48, size = 142, normalized size = 1.09

$$\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{2\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx+a)^{\frac{5}{3}}b^2-7\,(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3) + 1/6*(4*(b*x + a)^
(5/3)*b^2 - 7*(b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 + a
^4)
```

Fricas [A]
time = 0.92, size = 296, normalized size = 2.28

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-2\,a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4\,a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(4\,abx-3\,a^2)(bx+a)^{\frac{2}{3}}}{18\,a^3x^2},\ \frac{12\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}b^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-2\,a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4\,a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(4\,abx-3\,a^2)(bx+a)^{\frac{2}{3}}}{18\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(
b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*
(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3)
 - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt
(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^
2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

<u>**Sympy**</u> **[C]** Result contains complex when optimal does not.
time = 1.98, size = 2730, normalized size = 21.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/3),x)
```

```
[Out] 4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**
4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b**(1
0/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b
**(10/3)*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 8
1*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(1
3/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(
a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*
a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)
*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**
(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(7/3)*
exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/
3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3)
- 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(
10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b +
```

```
 x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(
7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*
b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3
)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(2
*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*ex
p(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a*
*7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*p
i/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3
)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(
7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I
*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(5/3)*b**(19/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3
)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3)
 - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/
3)*b**(19/3)*(a/b + x)**(13/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a*
*(5/3)*b**(19/3)*(a/b + x)**(13/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a
**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 21*a**4*b**4*(a
/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*
a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(
a/b + x)**3*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)...
```

Giac [A]

time = 0.86, size = 130, normalized size = 1.00

$$\frac{\frac{4\sqrt{3}\,b^{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2\,b^{3}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4\,b^{3}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4\,(bx+a)^{\frac{5}{3}}b^{3}-7\,(bx+a)^{\frac{2}{3}}ab^{3}\right)}{a^{2}b^{2}x^{2}}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x +
a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.23, size = 182, normalized size = 1.40

$$\frac{2\,b^2\,\ln\left({\left(a+b\,x\right)}^{1/3}-a^{1/3}\right)}{9\,a^{7/3}}-\frac{\frac{7\,b^2\,{\left(a+b\,x\right)}^{2/3}}{6\,a}-\frac{2\,b^2\,{\left(a+b\,x\right)}^{5/3}}{3\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}-\frac{\ln\left(\frac{4\,b^4\,{\left(a+b\,x\right)}^{1/3}}{9\,a^4}-\frac{{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}^2}{9\,a^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,a^{7/3}}+\frac{b^2\,\ln\left(\frac{4\,b^4\,{\left(a+b\,x\right)}^{1/3}}{9\,a^4}-\frac{9\,b^4\,{\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}^2}{a^{11/3}}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/3)),x)
```

```
[Out] (2*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(7/3)) - ((7*b^2*(a + b*x)^(2/3
))/(6*a) - (2*b^2*(a + b*x)^(5/3))/(3*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) - (log((4*b^4*(a + b*x)^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(9*a
^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*a^(7/3)) + (b^2*log((4*b^4*(a + b*x)^(
1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1i)/9 - 1/9)^2)/a^(11/3))*((3^(1/2)*1i)/9
- 1/9))/a^(7/3)
```

3.399 $\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}$$

```
[Out] 3/2*a^3*(b*x-a)^(2/3)/b^4+9/5*a^2*(b*x-a)^(5/3)/b^4+9/8*a*(b*x-a)^(8/3)/b^4
+3/11*(b*x-a)^(11/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a*
(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^3}{b^3\sqrt[3]{-a+bx}}+\frac{3a^2(-a+bx)^{2/3}}{b^3}+\frac{3a(-a+bx)^{5/3}}{b^3}+\frac{(-a+bx)^{8/3}}{b^3}\right)dx\\ &= \frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.60

$$\frac{3(-a+bx)^{2/3}\left(81a^3+54a^2bx+45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(81*a^3 + 54*a^2*b*x + 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

Maple [A]
time = 0.11, size = 58, normalized size = 0.72

method	result	size
gosper	$\frac{3\left(40 b^{3} x^{3}+45 a\,b^{2} x^{2}+54 a^{2} b x +81 a^{3}\right) \left(b x -a \right)^{\frac{2}{3}}}{440 b^{4}}$	45
trager	$\frac{3\left(40 b^{3} x^{3}+45 a\,b^{2} x^{2}+54 a^{2} b x +81 a^{3}\right) \left(b x -a \right)^{\frac{2}{3}}}{440 b^{4}}$	45
risch	$-\frac{3\left(-b x +a \right) \left(40 b^{3} x^{3}+45 a\,b^{2} x^{2}+54 a^{2} b x +81 a^{3}\right)}{440 b^{4} \left(b x -a \right)^{\frac{1}{3}}}$	51
derivativedivides	$\frac{\frac{3 \left(b x -a \right)^{\frac{11}{3}}}{11}+\frac{9 a \left(b x -a \right)^{\frac{8}{3}}}{8}+\frac{9 a^{2} \left(b x -a \right)^{\frac{5}{3}}}{5}+\frac{3 a^{3} \left(b x -a \right)^{\frac{2}{3}}}{2}}{b^{4}}$	58
default	$\frac{\frac{3 \left(b x -a \right)^{\frac{11}{3}}}{11}+\frac{9 a \left(b x -a \right)^{\frac{8}{3}}}{8}+\frac{9 a^{2} \left(b x -a \right)^{\frac{5}{3}}}{5}+\frac{3 a^{3} \left(b x -a \right)^{\frac{2}{3}}}{2}}{b^{4}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/11*(b*x-a)^(11/3)+3/8*a*(b*x-a)^(8/3)+3/5*a^2*(b*x-a)^(5/3)+1/2*a^
3*(b*x-a)^(2/3))
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{11}{3}}}{11\,b^4}+\frac{9\,(bx-a)^{\frac{8}{3}}a}{8\,b^4}+\frac{9\,(bx-a)^{\frac{5}{3}}a^2}{5\,b^4}+\frac{3\,(bx-a)^{\frac{2}{3}}a^3}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x - a)^(11/3)/b^4 + 9/8*(b*x - a)^(8/3)*a/b^4 + 9/5*(b*x - a)^(5/3)
*a^2/b^4 + 3/2*(b*x - a)^(2/3)*a^3/b^4
```

Fricas [A]
time = 0.70, size = 44, normalized size = 0.55

$$\frac{3\,(40\,b^3x^3+45\,ab^2x^2+54\,a^2bx+81\,a^3)(bx-a)^{\frac{2}{3}}}{440\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 45*a*b^2*x^2 + 54*a^2*b*x + 81*a^3)*(b*x - a)^(2/3)/b^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.32, size = 4974, normalized size = 62.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*ex
p(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3
) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2
640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a
**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600
*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**1
6*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**1
0*x**6*exp(I*pi/3)) - 1296*a**(68/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(4
40*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6
*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4
*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(
I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5
*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*ex
p(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*p
i/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2808*a**(65/3)*b**2*x**2*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b
**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(
I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/
3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) -
 3120*a**(62/3)*b**3*x**3*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/
3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) -
2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860
*a**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*
pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3)
 + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 44
0*a**14*b**10*x**6*exp(I*pi/3)) + 1710*a**(59/3)*b**4*x**4*(-1 + b*x/a)**(2
/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
 + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 66
00*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**
14*b**10*x**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I
```

```
*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640
*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 72*a**(5
6/3)*b**5*x**5*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3)
- 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a
**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*
b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*
b**5*x**5/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 660
0*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**
16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**
10*x**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*x**6*(-1 + b*x/a)**(2/3)*exp(I*p
i/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**
18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b*
*8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x*
*6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4*exp(I*pi/3) - 264
0*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*
b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*
x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*
x**7*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**
19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*
x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*
exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*x**8*(
-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 120*a**(44/3)*b**9*x**9*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)), Abs(b*x/a) > 1), (-243*a**(71/3)*(1 -
b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*...
```

Giac [**A**]
time = 1.11, size = 57, normalized size = 0.71

$$\frac{3\left(40\,(bx-a)^{\frac{11}{3}}+165\,(bx-a)^{\frac{8}{3}}a+264\,(bx-a)^{\frac{5}{3}}a^{2}+220\,(bx-a)^{\frac{2}{3}}a^{3}\right)}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x - a)^(11/3) + 165*(b*x - a)^(8/3)*a + 264*(b*x - a)^(5/3)*a^
2 + 220*(b*x - a)^(2/3)*a^3)/b^4
```

Mupad [B]
time = 0.05, size = 64, normalized size = 0.80

$$\frac{3\,{\left(b\,x-a\right)}^{11/3}}{11\,b^4}+\frac{9\,a\,{\left(b\,x-a\right)}^{8/3}}{8\,b^4}+\frac{3\,a^3\,{\left(b\,x-a\right)}^{2/3}}{2\,b^4}+\frac{9\,a^2\,{\left(b\,x-a\right)}^{5/3}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x - a)^(1/3),x)
```

```
[Out] (3*(b*x - a)^(11/3))/(11*b^4) + (9*a*(b*x - a)^(8/3))/(8*b^4) + (3*a^3*(b*x
 - a)^(2/3))/(2*b^4) + (9*a^2*(b*x - a)^(5/3))/(5*b^4)
```

3.400 $\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}$$

```
[Out] 3/2*a^2*(b*x-a)^(2/3)/b^3+6/5*a*(b*x-a)^(5/3)/b^3+3/8*(b*x-a)^(8/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
+ b*x)^(8/3))/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{-a+bx}}+\frac{2a(-a+bx)^{2/3}}{b^2}+\frac{(-a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.63

$$\frac{3(-a+bx)^{2/3}\left(9a^2+6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-a + b*x)^(1/3),x]
[Out] (3*(-a + b*x)^(2/3)*(9*a^2 + 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

Maple [A]
time = 0.11, size = 44, normalized size = 0.75

method	result	size
gosper	$\frac{3(5x^2b^2+6abx+9a^2)(bx-a)^{\frac{2}{3}}}{40b^3}$	34
trager	$\frac{3(5x^2b^2+6abx+9a^2)(bx-a)^{\frac{2}{3}}}{40b^3}$	34
risch	$-\frac{3(-bx+a)(5x^2b^2+6abx+9a^2)}{40b^3(bx-a)^{\frac{1}{3}}}$	40
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{8}{3}}}{8}+\frac{6a(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx-a)^{\frac{2}{3}}}{2}}{b^3}$	44
default	$\frac{\frac{3(bx-a)^{\frac{8}{3}}}{8}+\frac{6a(bx-a)^{\frac{5}{3}}}{5}+\frac{3a^2(bx-a)^{\frac{2}{3}}}{2}}{b^3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/b^3*(1/8*(b*x-a)^(8/3)+2/5*a*(b*x-a)^(5/3)+1/2*a^2*(b*x-a)^(2/3))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{8}{3}}}{8\,b^3}+\frac{6\,(bx-a)^{\frac{5}{3}}a}{5\,b^3}+\frac{3\,(bx-a)^{\frac{2}{3}}a^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="maxima")
[Out] 3/8*(b*x - a)^(8/3)/b^3 + 6/5*(b*x - a)^(5/3)*a/b^3 + 3/2*(b*x - a)^(2/3)*a
^2/b^3
```

Fricas [A]
time = 0.69, size = 33, normalized size = 0.56

$$\frac{3\,(5\,b^2x^2+6\,abx+9\,a^2)(bx-a)^{\frac{2}{3}}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 6*a*b*x + 9*a^2)*(b*x - a)^(2/3)/b^3
```

Sympy [C] Result contains complex when optimal does not.
time = 1.10, size = 1326, normalized size = 22.47

$$\begin{cases} -\frac{27a^{\frac{32}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{27a^{\frac{32}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{63a^{\frac{29}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{29}{3}}bxe^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{42a^{\frac{26}{3}}b^2x^2\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{81a^{\frac{26}{3}}b^2x^2e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{18a^{\frac{23}{3}}b^3x^3\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{23}{3}}b^3x^3e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{20}{3}}b^4x^4\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{15a^{\frac{17}{3}}b^5x^5\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3} & \text{for}\: \left|\frac{bx}{a}\right| > 1 \\ -\frac{27a^{\frac{32}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{27a^{\frac{32}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{63a^{\frac{29}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{81a^{\frac{29}{3}}bxe^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{42a^{\frac{26}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{81a^{\frac{26}{3}}b^2x^2e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{18a^{\frac{23}{3}}b^3x^3\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{23}{3}}b^3x^3e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}-\frac{27a^{\frac{20}{3}}b^4x^4\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3}+\frac{15a^{\frac{17}{3}}b^5x^5\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}}{-40a^8b^3+120a^7b^4x-120a^6b^5x^2+40a^5b^6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4
*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(
-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) +
 63*a**(29/3)*b*x*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 12
0*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40
*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42
*a**(26/3)*b**2*x**2*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x -
 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*p
i/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x
**3) + 18*a**(23/3)*b**3*x**3*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7
*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*b**3*x**3*
exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a*
*5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b
**5*x**5*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b*
*5*x**2 + 40*a**5*b**6*x**3), Abs(b*x/a) > 1), (-27*a**(32/3)*(1 - b*x/a)**
(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*
b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 - b*
x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*
x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42*a**(26/3)*b
**2*x**2*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x
- 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*
pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*
x**3) + 18*a**(23/3)*b**3*x**3*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b
**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23
/3)*b**3*x**3*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**
5*x**2 + 40*a**5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(1 - b*x/a)**(2/3)*exp
(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*
b**6*x**3) + 15*a**(17/3)*b**5*x**5*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a
**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3), True)
)
```

Giac [A]

time = 1.45, size = 43, normalized size = 0.73

$$\frac{3 \left(5 \left(bx-a\right)^{\frac{8}{3}}+16 \left(bx-a\right)^{\frac{5}{3}} a+20 \left(bx-a\right)^{\frac{2}{3}} a^{2}\right)}{40 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/40*(5*(b*x - a)^(8/3) + 16*(b*x - a)^(5/3)*a + 20*(b*x - a)^(2/3)*a^2)/b^
3
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.73

$$\frac{48\,a\,{\left(b\,x-a\right)}^{5/3}+15\,{\left(b\,x-a\right)}^{8/3}+60\,a^2\,{\left(b\,x-a\right)}^{2/3}}{40\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x - a)^(1/3),x)
```

```
[Out] (48*a*(b*x - a)^(5/3) + 15*(b*x - a)^(8/3) + 60*a^2*(b*x - a)^(2/3))/(40*b^
3)
```

3.401 $\int \frac{x}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=38

$$\frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}$$

```
[Out] 3/2*a*(b*x-a)^(2/3)/b^2+3/5*(b*x-a)^(5/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a}{b\sqrt[3]{-a+bx}}+\frac{(-a+bx)^{2/3}}{b}\right)dx\\ &= \frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.68

$$\frac{3(-a+bx)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.79

method	result	size
gosper	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^2}$	23
trager	$\frac{3(2bx+3a)(bx-a)^{\frac{2}{3}}}{10b^2}$	23
risch	$-\frac{3(-bx+a)(2bx+3a)}{10b^2(bx-a)^{\frac{1}{3}}}$	29
derivativedivides	$\frac{\frac{3(bx-a)^{\frac{5}{3}}}{5}+\frac{3a(bx-a)^{\frac{2}{3}}}{2}}{b^2}$	30
default	$\frac{\frac{3(bx-a)^{\frac{5}{3}}}{5}+\frac{3a(bx-a)^{\frac{2}{3}}}{2}}{b^2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/5*(b*x-a)^(5/3)+1/2*a*(b*x-a)^(2/3))
```

Maxima [A]
time = 0.26, size = 30, normalized size = 0.79

$$\frac{3\,(bx-a)^{\frac{5}{3}}}{5\,b^2}+\frac{3\,(bx-a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="maxima")
[Out] 3/5*(b*x - a)^(5/3)/b^2 + 3/2*(b*x - a)^(2/3)*a/b^2
```

Fricas [A]
time = 0.72, size = 22, normalized size = 0.58

$$\frac{3\,(2\,bx+3\,a)(bx-a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="fricas")
[Out] 3/10*(2*b*x + 3*a)*(b*x - a)^(2/3)/b^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 486, normalized size = 12.79

$$\begin{cases} -\frac{9a^{\frac{11}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{3a^{\frac{8}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{6a^{\frac{5}{3}}b^2x^2\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-9*a**(11/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(
I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*pi/3)
 + 10*a*b**3*x*exp(I*pi/3)) + 3*a**(8/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3
)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-
10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 6*a**(5/3)*b**2*x**2*
(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*ex
p(I*pi/3)), Abs(b*x/a) > 1), (9*a**(11/3)*(1 - b*x/a)**(2/3)/(-10*a**2*b**2
*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*
pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 3*a**(8/3)*b*x*(1 - b*x/a)**(2/3)/(-10*a
**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-10*a**2*
b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 6*a**(5/3)*b**2*x**2*(1 - b*x
/a)**(2/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)), True))
```

Giac [A]
time = 0.94, size = 29, normalized size = 0.76

$$\frac{3\left(2\,(bx-a)^{\frac{5}{3}}+5\,(bx-a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(2*(b*x - a)^(5/3) + 5*(b*x - a)^(2/3)*a)/b^2
```

Mupad [B]
time = 0.03, size = 29, normalized size = 0.76

$$\frac{15\,a\,(b\,x-a)^{2/3}+6\,(b\,x-a)^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x - a)^(1/3),x)
```

```
[Out] (15*a*(b*x - a)^(2/3) + 6*(b*x - a)^(5/3))/(10*b^2)
```

3.402 $\int \frac{1}{\sqrt[3]{-a+bx}} \, dx$

Optimal. Leaf size=18

$$\frac{3(-a+bx)^{2/3}}{2b}$$

```
[Out] 3/2*(b*x-a)^(2/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{32\}$

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx}} \, dx = \frac{3(-a+bx)^{2/3}}{2b}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$\frac{3(-a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Maple [A]
time = 0.11, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
derivativedivides	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
default	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
trager	$\frac{3(bx-a)^{\frac{2}{3}}}{2b}$	15
risch	$-\frac{3(-bx+a)}{2b(bx-a)^{\frac{1}{3}}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(b*x-a)^(2/3)/b
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="maxima")
[Out] 3/2*(b*x - a)^(2/3)/b
```

Fricas [A]
time = 0.86, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="fricas")
[Out] 3/2*(b*x - a)^(2/3)/b
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.67

$$\frac{3(-a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**(1/3),x)
```

```
[Out] 3*(-a + b*x)**(2/3)/(2*b)
```

Giac [A]
time = 0.90, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

Mupad [B]
time = 0.02, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x-a\right)}^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x - a)^(1/3),x)
```

```
[Out] (3*(b*x - a)^(2/3))/(2*b)
```

3.403 $\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=82

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

```
[Out] 1/2*ln(x)/a^(1/3)-3/2*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)-arctan(1/3*(a^(1/3)
-2*(b*x-a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {58, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3
)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{-a+bx}\right) - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x\right.}{2\sqrt[3]{a}}$$

$$= \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [A]
time = 0.06, size = 100, normalized size = 1.22

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*(-a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) + (-a + b*x)^(1/3)] + Log[a^(2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*
x)^(2/3)])/(2*a^(1/3))
```

Maple [A]
time = 0.11, size = 83, normalized size = 1.01

method	result	size
derivativedivides	$-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}$	83

default	$-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/2/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*
x-a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(
1/3)-1))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 1.05

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}+\frac{\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(1/3) +
 1/2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - log
((b*x - a)^(1/3) + a^(1/3))/a^(1/3)
```

Fricas [A]
time = 0.72, size = 285, normalized size = 3.48

$$\left[\frac{\sqrt{3}\,a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+\sqrt{3}\,\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\,\sqrt{3}\,a\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt((-a)^(1/3)/a)*log((2*b*x + sqrt(3)*(2*(b*x - a)^(2/3)*
(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x
- a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a
)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)^(1/3) - (-a)^
(1/3)))/a, 1/2*(2*sqrt(3)*a*sqrt(-(-a)^(1/3)/a)*arctan(1/3*sqrt(3)*(2*(b*x
- a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*log((b*x - a)^(2
/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)
^(1/3) - (-a)^(1/3)))/a]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.88, size = 160, normalized size = 1.95

$$-\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/3),x)
```

```
[Out] -2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(
1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*log(1 - b**(1/3)*(-a/b + x)**(
1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*exp(2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(3*a**(1/3)*gamma(5/3))
```

Giac [A]
time = 1.24, size = 112, normalized size = 1.37

$$-\frac{\sqrt{3}\,(-a)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{a}+\frac{(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{2\,a}-\frac{(-a)^{\frac{2}{3}}\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/a + 1/2*(-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3
) + (-a)^(2/3))/a - (-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)^(1/3)))/a
```

Mupad [B]
time = 0.09, size = 117, normalized size = 1.43

$$\frac{\ln\left(9\,(b\,x-a)^{1/3}-9\,(-a)^{1/3}\right)}{(-a)^{1/3}}+\frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}-\frac{\ln\left(9\,(b\,x-a)^{1/3}-\frac{9\,(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/3)),x)
```

```
[Out] log(9*(b*x - a)^(1/3) - 9*(-a)^(1/3))/(-a)^(1/3) + (log(9*(b*x - a)^(1/3) -
 (9*(-a)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-a)^(1/3)) - (l
og(9*(b*x - a)^(1/3) - (9*(-a)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1
))/(2*(-a)^(1/3))
```

3.404 $\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=103

$$\frac{(-a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}$$

```
[Out] (b*x-a)^(2/3)/a/x+1/6*b*ln(x)/a^(4/3)-1/2*b*ln(a^(1/3)+(b*x-a)^(1/3))/a^(4/
3)-1/3*b*arctan(1/3*(a^(1/3)-2*(b*x-a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)*3^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {44, 58, 631, 210, 31}

$$-\frac{b\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2a^{4/3}}+\frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) + (-
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx = \frac{(-a+bx)^{2/3}}{ax} + \frac{b\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{3a}$$

$$= \frac{(-a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{2a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{}}\right.}{}$$

$$= \frac{(-a+bx)^{2/3}}{ax} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}} + \frac{b\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{a^{4/3}}$$

$$= \frac{(-a+bx)^{2/3}}{ax} - \frac{b\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}$$

Mathematica [A]
time = 0.12, size = 128, normalized size = 1.24

$$\frac{6\sqrt[3]{a}\,(-a+bx)^{2/3} - 2\sqrt{3}\,bx\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2bx\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right) + bx\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{6a^{4/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(1/3)),x]
```

[Out] (6*a^(1/3)*(-a + b*x)^(2/3) - 2*Sqrt[3]*b*x*ArcTan[(1 - (2*(-a + b*x)^(1/3)
)/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) + (-a + b*x)^(1/3)] + b*x*Log[a^(2/
3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(6*a^(4/3)*x)

Maple [A]
time = 0.12, size = 113, normalized size = 1.10

method	result
risch	$-\frac{-bx+a}{ax(bx-a)^{\frac{1}{3}}}-\frac{b\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{4}{3}}}$
derivativedivides	$3b\left(\frac{(bx-a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$
default	$3b\left(\frac{(bx-a)^{\frac{2}{3}}}{3abx}+\frac{-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{1}{3}}}}{3a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)

[Out] 3*b*(1/3*(b*x-a)^(2/3)/a/b/x+1/3/a*(-1/3*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+
1/6/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*x-a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(
1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(1/3)-1))))

Maxima [A]
time = 0.51, size = 116, normalized size = 1.13

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{4}{3}}}+\frac{(bx-a)^{\frac{2}{3}}b}{(bx-a)a+a^2}+\frac{b\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{4}{3}}}-\frac{b\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="maxima")

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(
4/3) + (b*x - a)^(2/3)*b/((b*x - a)*a + a^2) + 1/6*b*log((b*x - a)^(2/3) -
(b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x - a)^(1/3) + a^
(1/3))/a^(4/3)
```

Fricas [A]
time = 0.66, size = 328, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+(-a)^{\frac{2}{3}}\,bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\,bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6\,(bx-a)^{\frac{2}{3}}a}{6\,a^2x},\ \frac{6\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}\,bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2\,(-a)^{\frac{2}{3}}\,bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6\,(bx-a)^{\frac{2}{3}}a}{6\,a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2*(b*x
 - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*b*x*log((b*x - a)^
(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x - a)^(1/3) - (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x), 1/6*(6*sqrt(1/3)
*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3)
)*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*b*x*log((b*x - a)^(2/3) + (b*x - a)^(1/
3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*x - a)^(1/3) - (-a)^(
1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.24, size = 838, normalized size = 8.14

$$-\frac{2a^{\frac{5}{3}}b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{5}{3}}b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{5}{3}}b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{2}{3}}b^{\frac{10}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{2}{3}}b^{\frac{10}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}-\frac{2a^{\frac{2}{3}}b^{\frac{10}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{6ab^3\left(-\frac{a}{b}+x\right)^2e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}{9a^3b^{\frac{4}{3}}\left(-\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)+9a^2b^{\frac{7}{3}}\left(-\frac{a}{b}+x\right)^{\frac{7}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*e
xp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**
(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)*
*(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3
))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) +
 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b
**(10/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*p
i/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) -
```

```
2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b
 + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(
2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2
/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 6*a*b**3*(-a/b + x)**2
*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3))
```

Giac [A]
time = 2.45, size = 144, normalized size = 1.40

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a}-\frac{b^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a}-\frac{2\,(-a)^{\frac{2}{3}}b^2\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{6\,(bx-a)^{\frac{2}{3}}b}{ax}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a)
^(1/3))/((-a)^(1/3)*a) - b^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/
3) + (-a)^(2/3))/((-a)^(1/3)*a) - 2*(-a)^(2/3)*b^2*log(abs((b*x - a)^(1/3)
- (-a)^(1/3)))/a^2 + 6*(b*x - a)^(2/3)*b/(a*x))/b
```

Mupad [B]
time = 0.18, size = 133, normalized size = 1.29

$$\frac{(b\,x-a)^{2/3}}{a\,x}-\frac{b\,\ln\left((b\,x-a)^{1/3}+a^{1/3}\right)}{3\,a^{4/3}}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/3)),x)
```

```
[Out] (b*x - a)^(2/3)/(a*x) - (b*log((b*x - a)^(1/3) + a^(1/3)))/(3*a^(4/3)) + (l
og((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x - a)^(1/3))/a^2)*(b - 3^(1/
2)*b*1i))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x -
 a)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3))
```

3.405 $\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=136

$$\frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}-\frac{2b^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}}$$

```
[Out] 1/2*(b*x-a)^(2/3)/a/x^2+2/3*b*(b*x-a)^(2/3)/a^2/x+1/9*b^2*ln(x)/a^(7/3)-1/3
*b^2*ln(a^(1/3)+(b*x-a)^(1/3))/a^(7/3)-2/9*b^2*arctan(1/3*(a^(1/3)-2*(b*x-a
)^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {44, 58, 631, 210, 31}

$$-\frac{2b^2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{3a^{7/3}}+\frac{2b(bx-a)^{2/3}}{3a^2x}+\frac{(bx-a)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(2*a*x^2) + (2*b*(-a + b*x)^(2/3))/(3*a^2*x) - (2*b^2*ArcT
an[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) +
 (b^2*Log[x])/(9*a^(7/3)) - (b^2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(7/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx &= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{(2b)\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx}{3a}\\ &= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}+\frac{(2b^2)\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{9a^2}\\ &= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx,x,\sqrt[3]{-a+bx}\right)}{3a^{7/3}}\\ &= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}}+\frac{(2b^2)\,\text{Sul}}{}\\ &= \frac{(-a+bx)^{2/3}}{2ax^2}+\frac{2b(-a+bx)^{2/3}}{3a^2x}-\frac{2b^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{b^2\log(x)}{9a^{7/3}}-\frac{b^2\log\left(\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 160, normalized size = 1.18

$$\frac{(-a+bx)^{2/3}(7a+4(-a+bx))}{6a^2x^2}-\frac{2b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}-\frac{2b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{9a^{7/3}}+\frac{b^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{-a+bx}+(-a+bx)^{2/3}\right)}{9a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] ((-a + b*x)^(2/3)*(7*a + 4*(-a + b*x)))/(6*a^2*x^2) - (2*b^2*ArcTan[1/Sqrt[
3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (2*b^2*
Log[a^(1/3) + (-a + b*x)^(1/3)])/(9*a^(7/3)) + (b^2*Log[a^(2/3) - a^(1/3)*(
-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(9*a^(7/3))
```

Maple [A]
time = 0.10, size = 141, normalized size = 1.04

method	result
risch	$-\frac{(-bx+a)(4bx+3a)}{6a^2x^2(bx-a)^{\frac{1}{3}}}-\frac{2b^2\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{b^2\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(}{}\right)}{9a^{\frac{7}{3}}}$
derivativedivides	$3b^2\left(\frac{(bx-a)^{\frac{2}{3}}}{6a\,b^2x^2}+\frac{\frac{2(bx-a)^{\frac{2}{3}}}{9abx}+\frac{2\left(-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)}{a^{\frac{1}{3}}}\right.}{3}\right)}{3a^{\frac{1}{3}}}\right.}{9a}}{a}\right.$
default	$3b^2\left(\frac{(bx-a)^{\frac{2}{3}}}{6a\,b^2x^2}+\frac{\frac{2(bx-a)^{\frac{2}{3}}}{9abx}+\frac{2\left(-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{\ln\left((bx-a)^{\frac{2}{3}}-a^{\frac{1}{3}}(bx-a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)}{a^{\frac{1}{3}}}\right.}{3}\right)}{3a^{\frac{1}{3}}}\right.}{9a}}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(1/6/a*(b*x-a)^(2/3)/b^2/x^2+2/3/a*(1/3*(b*x-a)^(2/3)/a/b/x+1/3/a*(-1
/3*ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/6/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(
b*x-a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*
x-a)^(1/3)-1)))))
```

Maxima [A]
time = 0.51, size = 159, normalized size = 1.17

$$\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}+\frac{b^2\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}}-\frac{2\,b^2\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx-a)^{\frac{5}{3}}b^2+7\,(bx-a)^{\frac{2}{3}}ab^2}{6\left((bx-a)^2a^2+2\,(bx-a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a
^(7/3) + 1/9*b^2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) - 2/9*b^2*log((b*x - a)^(1/3) + a^(1/3))/a^(7/3) + 1/6*(4*(b*x - a)^
(5/3)*b^2 + 7*(b*x - a)^(2/3)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a
^4)
```

Fricas [A]
time = 0.72, size = 374, normalized size = 2.75

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx+3\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3\,a}{x}\right)+2\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\,(4\,abx+3\,a^2)(bx-a)^{\frac{2}{3}}}{18\,a^3x^2},\ \frac{12\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+2\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\,(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\,(4\,abx+3\,a^2)(bx-a)^{\frac{2}{3}}}{18\,a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2
*(b*x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + 2*(-a)^(2/3)*b^2*x^2*log
((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(-a)^(2/3)*
b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2)*(b*x - a)^(
2/3))/(a^3*x^2), 1/18*(12*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sq
rt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 2*(-a)^(2/3
)*b^2*x^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) -
4*(-a)^(2/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2
)*(b*x - a)^(2/3))/(a^3*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.01, size = 2744, normalized size = 20.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/3),x)

[Out] -4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)
*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)
*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3
) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(
14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)
**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I
*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamm
a(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4
*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/
b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b +
x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp
(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
 - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*log(1 - b**(1/
3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)
**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp
(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*log(1
- b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3
)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b*
```

```
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/
3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*log(
1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a
**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*
(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**
(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(
2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/
(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(
7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b +
 x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)
*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(-2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81
*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/
3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b +
x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 21*a**4*b**4*(-a/b + x)**2*exp(2*I*p
i/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3
) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b
**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-
a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(-a/b + x)**3*exp
(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*ex...
```

Giac [A]
time = 2.88, size = 167, normalized size = 1.23

$$\frac{\frac{4\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a^2}-\frac{2\,b^3\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a^2}-\frac{4\,(-a)^{\frac{2}{3}}b^3\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^3}+\frac{3\left(4\,(bx-a)^{\frac{5}{3}}b^3+7\,(bx-a)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^2}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/((-a)^(1/3)*a^2) - 2*b^3*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3))/((-a)^(1/3)*a^2) - 4*(-a)^(2/3)*b^3*log(abs((b*x - a)
^(1/3) - (-a)^(1/3)))/a^3 + 3*(4*(b*x - a)^(5/3)*b^3 + 7*(b*x - a)^(2/3)*a*
b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.22, size = 216, normalized size = 1.59

$$\frac{\frac{7\,b^2\,(b\,x-a)^{2/3}}{6\,a}+\frac{2\,b^2\,(b\,x-a)^{5/3}}{3\,a^2}}{(a-b\,x)^2-2\,a\,(a-b\,x)+a^2}-\frac{\ln\left(\frac{4\,b^4\,(b\,x-a)^{1/3}}{9\,a^4}-\frac{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)^2}{9\,(-a)^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{9\,(-a)^{7/3}}+\frac{2\,b^2\,\ln\left(\frac{4\,b^4\,(b\,x-a)^{1/3}}{9\,a^4}-\frac{4\,b^4}{9\,(-a)^{11/3}}\right)}{9\,(-a)^{7/3}}+\frac{b^2\,\ln\left(\frac{4\,b^4\,(b\,x-a)^{1/3}}{9\,a^4}-\frac{9\,b^4\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)^2}{(-a)^{11/3}}\right)\,\left(-\frac{1}{9}+\frac{\sqrt{3}\,1{}\mathrm{i}}{9}\right)}{(-a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/3)),x)
```

```
[Out] ((7*b^2*(b*x - a)^(2/3))/(6*a) + (2*b^2*(b*x - a)^(5/3))/(3*a^2))/((a - b*x
)^2 - 2*a*(a - b*x) + a^2) - (log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (3^(1/2
)*b^2*1i + b^2)^2/(9*(-a)^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*(-a)^(7/3)) +
 (2*b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (4*b^4)/(9*(-a)^(11/3))))/(9*
(-a)^(7/3)) + (b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1
i)/9 - 1/9)^2)/(-a)^(11/3))*((3^(1/2)*1i)/9 - 1/9))/(-a)^(7/3)
```

3.406 $\int \frac{x^3}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=70

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}$$

```
[Out] -3*a^3*(b*x+a)^(1/3)/b^4+9/4*a^2*(b*x+a)^(4/3)/b^4-9/7*a*(b*x+a)^(7/3)/b^4+
3/10*(b*x+a)^(10/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a +
b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx\\ &= -\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx}\,(-81a^3+27a^2bx-18ab^2x^2+14b^3x^3)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-81*a^3 + 27*a^2*b*x - 18*a*b^2*x^2 + 14*b^3*x^3))/(140
*b^4)
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.71

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
trager	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
risch	$-\frac{3(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$	43
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{9a(bx+a)^{\frac{7}{3}}}{7}+\frac{9a^{2}(bx+a)^{\frac{4}{3}}}{4}-3a^{3}(bx+a)^{\frac{1}{3}}}{b^{4}}$	50
default	$\frac{\frac{3(bx+a)^{\frac{10}{3}}}{10}-\frac{9a(bx+a)^{\frac{7}{3}}}{7}+\frac{9a^{2}(bx+a)^{\frac{4}{3}}}{4}-3a^{3}(bx+a)^{\frac{1}{3}}}{b^{4}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/10*(b*x+a)^(10/3)-3/7*a*(b*x+a)^(7/3)+3/4*a^2*(b*x+a)^(4/3)-a^3*(b
*x+a)^(1/3))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^4}-\frac{9\,(bx+a)^{\frac{7}{3}}a}{7\,b^4}+\frac{9\,(bx+a)^{\frac{4}{3}}a^2}{4\,b^4}-\frac{3\,(bx+a)^{\frac{1}{3}}a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^4 - 9/7*(b*x + a)^(7/3)*a/b^4 + 9/4*(b*x + a)^(4/3)
*a^2/b^4 - 3*(b*x + a)^(1/3)*a^3/b^4
```

Fricas [A]
time = 0.68, size = 42, normalized size = 0.60

$$\frac{3\,(14\,b^3x^3-18\,ab^2x^2+27\,a^2bx-81\,a^3)(bx+a)^{\frac{1}{3}}}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 - 18*a*b^2*x^2 + 27*a^2*b*x - 81*a^3)*(b*x + a)^(1/3)/b^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1640 vs. $2(66) = 132$.
time = 1.20, size = 1640, normalized size = 23.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) + 243*a**(70/3)/(140*a**20*b**4 + 840*a**
19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x
**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 1377*a**(67/3)*b*x*(1 +
 b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 +
2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**
14*b**10*x**6) + 1458*a**(67/3)*b*x/(140*a**20*b**4 + 840*a**19*b**5*x + 21
00*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**1
5*b**9*x**5 + 140*a**14*b**10*x**6) - 3213*a**(64/3)*b**2*x**2*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6) + 3645*a**(64/3)*b**2*x**2/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) - 3927*a**(61/3)*b**3*x**3*(1 + b*x/a)**(
1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17
*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x
**6) + 4860*a**(61/3)*b**3*x**3/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a
**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b*
*9*x**5 + 140*a**14*b**10*x**6) - 2583*a**(58/3)*b**4*x**4*(1 + b*x/a)**(1/
3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b
**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**
6) + 3645*a**(58/3)*b**4*x**4/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**
18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9
*x**5 + 140*a**14*b**10*x**6) - 693*a**(55/3)*b**5*x**5*(1 + b*x/a)**(1/3)/
(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7
*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6)
+ 1458*a**(55/3)*b**5*x**5/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*
b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x*
*5 + 140*a**14*b**10*x**6) + 273*a**(52/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(14
0*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x*
*3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 2
43*a**(52/3)*b**6*x**6/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6
```

```
*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
 140*a**14*b**10*x**6) + 387*a**(49/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(140*a*
*20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 +
 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 198*a
**(46/3)*b**8*x**8*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x +
2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a*
*15*b**9*x**5 + 140*a**14*b**10*x**6) + 42*a**(43/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6)
```

Giac [A]
time = 1.34, size = 49, normalized size = 0.70

$$\frac{3\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)}{140\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/140*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2
 - 140*(b*x + a)^(1/3)*a^3)/b^4
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,(a+b\,x)^{10/3}}{10\,b^4}-\frac{3\,a^3\,(a+b\,x)^{1/3}}{b^4}+\frac{9\,a^2\,(a+b\,x)^{4/3}}{4\,b^4}-\frac{9\,a\,(a+b\,x)^{7/3}}{7\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(10/3))/(10*b^4) - (3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b
*x)^(4/3))/(4*b^4) - (9*a*(a + b*x)^(7/3))/(7*b^4)
```

3.407 $\int \frac{x^2}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=51

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}$$

```
[Out] 3*a^2*(b*x+a)^(1/3)/b^3-3/2*a*(b*x+a)^(4/3)/b^3+3/7*(b*x+a)^(7/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)^
(7/3))/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{2/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx\\ &= \frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 0.69

$$\frac{3\sqrt[3]{a+bx}\,(9a^2-3abx+2b^2x^2)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3)
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.73

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}(2x^2b^2-3abx+9a^2)}{14b^3}$	32
trager	$\frac{3(bx+a)^{\frac{1}{3}}(2x^2b^2-3abx+9a^2)}{14b^3}$	32
risch	$\frac{3(bx+a)^{\frac{1}{3}}(2x^2b^2-3abx+9a^2)}{14b^3}$	32
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{2}+3a^2(bx+a)^{\frac{1}{3}}}{b^3}$	37
default	$\frac{\frac{3(bx+a)^{\frac{7}{3}}}{7}-\frac{3a(bx+a)^{\frac{4}{3}}}{2}+3a^2(bx+a)^{\frac{1}{3}}}{b^3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/7*(b*x+a)^(7/3)-1/2*a*(b*x+a)^(4/3)+a^2*(b*x+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^3}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{2\,b^3}+\frac{3\,(bx+a)^{\frac{1}{3}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^3 - 3/2*(b*x + a)^(4/3)*a/b^3 + 3*(b*x + a)^(1/3)*a^2
/b^3
```

Fricas [A]
time = 0.95, size = 31, normalized size = 0.61

$$\frac{3\,(2\,b^2x^2-3\,abx+9\,a^2)(bx+a)^{\frac{1}{3}}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 - 3*a*b*x + 9*a^2)*(b*x + a)^(1/3)/b^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(48) = 96$.
time = 0.79, size = 600, normalized size = 11.76

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{31}{3}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{72a^{\frac{28}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{28}{3}}bx}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{60a^{\frac{25}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{25}{3}}b^2x^2}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{18a^{\frac{22}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{22}{3}}b^3x^3}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{9a^{\frac{19}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{6a^{\frac{16}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b*
*5*x**2 + 14*a**5*b**6*x**3) - 27*a**(31/3)/(14*a**8*b**3 + 42*a**7*b**4*x
+ 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 72*a**(28/3)*b*x*(1 + b*x/a)**(1
/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
 - 81*a**(28/3)*b*x/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14
*a**5*b**6*x**3) + 60*a**(25/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(14*a**8*b**3
+ 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 81*a**(25/3)*b*
*2*x**2/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x
**3) + 18*a**(22/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b*
*4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(22/3)*b**3*x**3/(14*
a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 9*a**
(19/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**
6*b**5*x**2 + 14*a**5*b**6*x**3) + 6*a**(16/3)*b**5*x**5*(1 + b*x/a)**(1/3)
/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
```

Giac [**A**]
time = 1.02, size = 37, normalized size = 0.73

$$\frac{3\left(2\left(bx+a\right)^{\frac{7}{3}}-7\left(bx+a\right)^{\frac{4}{3}}a+14\left(bx+a\right)^{\frac{1}{3}}a^2\right)}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b^3
```

Mupad [**B**]
time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,{\left(a+b\,x\right)}^{7/3}-21\,a\,{\left(a+b\,x\right)}^{4/3}+42\,a^2\,{\left(a+b\,x\right)}^{1/3}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(2/3),x)
```

```
[Out] (6*(a + b*x)^(7/3) - 21*a*(a + b*x)^(4/3) + 42*a^2*(a + b*x)^(1/3))/(14*b^3
)
```

3.408 $\int \frac{x}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=32

$$-\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}$$

```
[Out] -3*a*(b*x+a)^(1/3)/b^2+3/4*(b*x+a)^(4/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx\\ &= -\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(-3a+bx)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(2/3),x]
```

```
[Out] (3*(-3*a + b*x)*(a + b*x)^(1/3))/(4*b^2)
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.81

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^2}$	21
trager	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^2}$	21
risch	$-\frac{3(bx+a)^{\frac{1}{3}}(-bx+3a)}{4b^2}$	21
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{4}{3}}}{4}-3a(bx+a)^{\frac{1}{3}}}{b^2}$	26
default	$\frac{\frac{3(bx+a)^{\frac{4}{3}}}{4}-3a(bx+a)^{\frac{1}{3}}}{b^2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^2*(1/4*(b*x+a)^(4/3)-a*(b*x+a)^(1/3))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b^2} - \frac{3\,(bx+a)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b^2 - 3*(b*x + a)^(1/3)*a/b^2
```

Fricas [A]
time = 1.03, size = 19, normalized size = 0.59

$$\frac{3\,(bx+a)^{\frac{1}{3}}(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(1/3)*(b*x - 3*a)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.
time = 0.52, size = 162, normalized size = 5.06

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{10}{3}}}{4a^2b^2+4ab^3x}-\frac{6a^{\frac{7}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{7}{3}}bx}{4a^2b^2+4ab^3x}+\frac{3a^{\frac{4}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x) + 9*a**(10/3)/(4
*a**2*b**2 + 4*a*b**3*x) - 6*a**(7/3)*b*x*(1 + b*x/a)**(1/3)/(4*a**2*b**2 +
 4*a*b**3*x) + 9*a**(7/3)*b*x/(4*a**2*b**2 + 4*a*b**3*x) + 3*a**(4/3)*b**2*
x**2*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x)
```

Giac [A]
time = 0.85, size = 23, normalized size = 0.72

$$\frac{3\left((bx+a)^{\frac{4}{3}}-4(bx+a)^{\frac{1}{3}}a\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)/b^2
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.78

$$-\frac{12\,a\,(a+b\,x)^{1/3}-3\,(a+b\,x)^{4/3}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(2/3),x)
```

```
[Out] -(12*a*(a + b*x)^(1/3) - 3*(a + b*x)^(4/3))/(4*b^2)
```

3.409 $\int \frac{1}{(a+bx)^{2/3}} \, dx$

Optimal. Leaf size=14

$$\frac{3\sqrt[3]{a+bx}}{b}$$

```
[Out] 3*(b*x+a)^(1/3)/b
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}} \, dx = \frac{3\sqrt[3]{a+bx}}{b}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
derivativedivides	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
default	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
trager	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13
risch	$\frac{3(bx+a)^{\frac{1}{3}}}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*(b*x+a)^(1/3)/b
```

Maxima [**A**]
time = 0.28, size = 12, normalized size = 0.86

$$\frac{3\left(bx+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Fricas [**A**]
time = 0.76, size = 12, normalized size = 0.86

$$\frac{3\left(bx+a\right)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Sympy [**A**]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3),x)
```

```
[Out] 3*(a + b*x)**(1/3)/b
```

Giac [A]
time = 0.78, size = 12, normalized size = 0.86

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3*(b*x + a)^(1/3)/b
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{3\,(a+b\,x)^{1/3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

3.410 $\int \frac{1}{x(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

```
[Out] -1/2*ln(x)/a^(2/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(2/3)-arctan(1/3*(a^(1/3
)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.308$, Rules used = {59, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/3)
) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{\log(x)}{2a^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+\right.}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 1.16

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*
x)^(2/3)])/a^(2/3)
```

Maple [A]
time = 0.11, size = 76, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}} - \frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}$	76

default	$\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b
*x+a)^(1/3)+a^(2/3))-1/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a
)^(1/3)+1))
```

Maxima [A]
time = 0.52, size = 77, normalized size = 0.96

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g((b*x + a)^(1/3) - a^(1/3))/a^(2/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.
time = 0.60, size = 115, normalized size = 1.44

$$-\frac{2\,\sqrt{3}\,(a^2)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\,(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^2}\right)+(a^2)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\,(a^2)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*log((b*x + a)^(2/3)*a
+ (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*log((b*x + a
)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.94, size = 150, normalized size = 1.88

$$\frac{\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(2/3),x)
```

```
[Out] log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/
3)) + exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/
a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b**(1/
3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*ga
mma(4/3))
```

Giac [A]
time = 0.90, size = 78, normalized size = 0.98

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3)
```

Mupad [B]
time = 0.17, size = 95, normalized size = 1.19

$$\frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,\,1\mathrm{i}\right)}{2}-9\,(a+b\,x)^{1/3}\right)\,\left(-1+\sqrt{3}\,\,1\mathrm{i}\right)}{2\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,\,1\mathrm{i}\right)}{2}+9\,(a+b\,x)^{1/3}\right)\,\left(1+\sqrt{3}\,\,1\mathrm{i}\right)}{2\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(2/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(2/3) + (log((9*a^(1/3)*(3^(1/2)*1i -
1))/2 - 9*(a + b*x)^(1/3))*(3^(1/2)*1i - 1))/(2*a^(2/3)) - (log((9*a^(1/3)*
(3^(1/2)*1i + 1))/2 + 9*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/(2*a^(2/3))
```

3.411 $\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=98

$$-\frac{\sqrt[3]{a+bx}}{ax}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}$$

```
[Out] -(b*x+a)^(1/3)/a/x+1/3*b*ln(x)/a^(5/3)-b*ln(a^(1/3)-(b*x+a)^(1/3))/a^(5/3)+
2/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 59, 631, 210, 31}

$$\frac{2b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/3) -
 (a + b*x)^(1/3)])/a^(5/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{ax}-\frac{(2b)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{3a}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{b\log(x)}{3a^{5/3}}+\frac{b\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{b\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+}\right.}{a^{4/}}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}-\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{}}{}\right.}{a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 119, normalized size = 1.21

$$
\frac{-3a^{2/3}\sqrt[3]{a+bx}+2\sqrt{3}\,bx\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+bx\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{5/3}x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] (-3*a^(2/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*b*x*ArcTan[(1 + (2*(a + b*x)^(1/3))
/a^(1/3))/Sqrt[3]] - 2*b*x*Log[a^(1/3) - (a + b*x)^(1/3)] + b*x*Log[a^(2/3)
 + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(5/3)*x)
```

Maple [A]
time = 0.13, size = 104, normalized size = 1.06

method	result
derivativedivides	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}}{a}\right)$
default	$3b\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}}{a}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/3*(b*x+a)^(1/3)/a/b/x+2/3/a*(-1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))
+1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.48, size = 106, normalized size = 1.08

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}b}{(bx+a)a-a^2}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{5}{3}}}-\frac{2\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
5/3) - (b*x + a)^(1/3)*b/((b*x + a)*a - a^2) + 1/3*b*log((b*x + a)^(2/3) +
(b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/3*b*log((b*x + a)^(1/3) - a^
(1/3))/a^(5/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(75) = 150$.
time = 0.50, size = 166, normalized size = 1.69

$$\frac{2\sqrt{3}\,abx\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3(bx+a)^{\frac{1}{3}}a^2}{3a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*a*b*x*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*
a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^
2)^(2/3)*b*x*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x + a
)^(1/3)) - 2*(-a^2)^(2/3)*b*x*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)) - 3*(b*
x + a)^(1/3)*a^2)/(a^3*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.21, size = 830, normalized size = 8.47

$$-\frac{2a^{\frac{4}{3}}b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{4}{3}}b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{4}{3}}b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}\,b^{\frac{8}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}\,b^{\frac{8}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}\,b^{\frac{8}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{3ab^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{1}{3}\right)}{9a^3b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)-9a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{5}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(2/3),x)
```

```
[Out] -2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*p
i/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)
) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*g
amma(4/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b*
*(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x
)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3
*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b
+ x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3
)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5
/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b +
 x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*g
amma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 3*
a*b**2*(a/b + x)*exp(2*I*pi/3)*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)
```

```
*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*
gamma(4/3))
```

Giac [A]
time = 0.82, size = 108, normalized size = 1.10

$$\frac{\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\,(bx+a)^{\frac{1}{3}}b}{ax}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(5/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x + a)^(1/
3)*b/(a*x))/b
```

Mupad [B]
time = 0.13, size = 122, normalized size = 1.24

$$-\frac{(a+b\,x)^{1/3}}{a\,x}+\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3\,a^{5/3}}-\frac{2\,b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/a^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b - 3^(1/2)
*b*1i))/(3*a^(5/3)) - (a + b*x)^(1/3)/(a*x) + (log((3*(b + 3^(1/2)*b*1i))/a
^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(3*a^(5/3)) - (2*b*lo
g((a + b*x)^(1/3) - a^(1/3)))/(3*a^(5/3))
```

3.412 $\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}$$

```
[Out] -1/2*(b*x+a)^(1/3)/a/x^2+5/6*b*(b*x+a)^(1/3)/a^2/x-5/18*b^2*ln(x)/a^(8/3)+5
/6*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(8/3)-5/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x
+a)^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 59, 631, 210, 31}

$$-\frac{5b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -1/2*(a + b*x)^(1/3)/(a*x^2) + (5*b*(a + b*x)^(1/3))/(6*a^2*x) - (5*b^2*Arc
Tan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) -
 (5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^
(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{2ax^2}-\frac{(5b)\int \frac{1}{x^2(a+bx)^{2/3}}\,dx}{6a}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}+\frac{(5b^2)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a^2}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}+\frac{(5b^2)\,\text{Subst}\,(}{}\\ &= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}\right.}{6}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 149, normalized size = 1.15

$$-\frac{\sqrt[3]{a+bx}\,(8a-5(a+bx))}{6a^2x^2}-\frac{5b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{8/3}}-\frac{5b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(8*a - 5*(a + b*x)))/(a^2*x^2) - (5*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (5*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(8/3)) - (5*b^2*Log[a^(2/3) + a^(1/3)*
(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(8/3))
```

Maple [A]
time = 0.15, size = 130, normalized size = 1.00

method	result
derivativedivides	$3b^2\left(-\frac{(bx+a)^{\frac{1}{3}}}{6a\,b^2x^2}-\frac{5\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(b}{3}\right.}{}\right.}{9a^{\frac{2}{3}}}}{a}\right.}{6a}\right.$
default	$3b^2\left(-\frac{(bx+a)^{\frac{1}{3}}}{6a\,b^2x^2}-\frac{5\left(-\frac{(bx+a)^{\frac{1}{3}}}{3abx}+\frac{-\frac{2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(b}{3}\right.}{}\right.}{9a^{\frac{2}{3}}}}{a}\right.}{6a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(-1/6/a*(b*x+a)^(1/3)/b^2/x^2-5/6/a*(-1/3*(b*x+a)^(1/3)/a/b/x+2/3/a*(
-1/3/a^(2/3)*ln((b*x+a)^(1/3)-a^(1/3))+1/6/a^(2/3)*ln((b*x+a)^(2/3)+a^(1/3)
*(b*x+a)^(1/3)+a^(2/3))+1/3/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(
b*x+a)^(1/3)+1)))))
```

Maxima [A]
time = 0.48, size = 142, normalized size = 1.09

$$-\frac{5\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{8}{3}}}-\frac{5\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{8}{3}}}+\frac{5\,b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{8}{3}}}+\frac{5\,(bx+a)^{\frac{4}{3}}b^2-8\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -5/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(8/3) - 5/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(8/3) + 5/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(8/3) + 1/6*(5*(b*x + a
)^(4/3)*b^2 - 8*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 +
 a^4)
```

Fricas [A]
time = 0.50, size = 162, normalized size = 1.25

$$-\frac{10\sqrt{3}\,(a^2)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{(a^2)^{\frac{1}{6}}\left(\sqrt{3}\,(a^2)^{\frac{1}{3}}a+2\sqrt{3}\,(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3\,a^2}\right)+5\,(a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(a^2)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-10\,(a^2)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)-3\,(5\,a^2bx-3\,a^3)(bx+a)^{\frac{1}{3}}}{18\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/18*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 5*(a^2)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3))
- 10*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^2*b*
x - 3*a^3)*(b*x + a)^(1/3))/(a^4*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.03, size = 2728, normalized size = 20.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(2/3),x)
```

```
[Out] 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
```

4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(
8/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)
/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamm
a(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*
a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)
*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8/3)*(a/b +
x)**(2/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*
gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) +
162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(1
1/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(
a/b + x)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162
*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3
)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**
(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*
log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/
(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(
5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x
)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp
(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6
*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/
b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3
)*exp(2*I*pi/3)*gamma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(2*
I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**
(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b +
x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp
(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*g
amma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b
+ x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*
gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
+ 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(
a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/
3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi
/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/
3)) - 10*a**(4/3)*b**(17/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)
*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) -

```
 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3
)*b**(17/3)*(a/b + x)**(11/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi
/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3
) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b
**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3)*b**(17/3
)*(a/b + x)**(11/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 24*a**4*b**3*(a
/b + x)*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 39*a**3*b**4*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*
I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma
(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a*
*4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*ga...
```

Giac [A]
time = 0.99, size = 130, normalized size = 1.00

$$-\frac{\frac{10\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5\,b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5\,(bx+a)^{\frac{4}{3}}b^3-8\,(bx+a)^{\frac{1}{3}}ab^3\right)}{a^2b^2x^2}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/18*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(8/3) + 5*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(8/3) - 10*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(b*x
 + a)^(4/3)*b^3 - 8*(b*x + a)^(1/3)*a*b^3)/(a^2*b^2*x^2))/b
```

Mupad [B]
time = 0.13, size = 175, normalized size = 1.35

$$\frac{5\,b^2\,\ln\left({\left(a+b\,x\right)}^{1/3}-a^{1/3}\right)}{9\,a^{8/3}}-\frac{\frac{4\,b^2\,{\left(a+b\,x\right)}^{1/3}}{3\,a}-\frac{5\,b^2\,{\left(a+b\,x\right)}^{4/3}}{6\,a^2}}{{\left(a+b\,x\right)}^2-2\,a\,\left(a+b\,x\right)+a^2}+\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,{\left(a+b\,x\right)}^{1/3}}{a^2}-\frac{5\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}}-\frac{5\,b^2\,\ln\left(\frac{5\,b^2\,{\left(a+b\,x\right)}^{1/3}}{a^2}+\frac{5\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(2/3)),x)
```

```
[Out] (5*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(8/3)) - ((4*b^2*(a + b*x)^(1/3
))/(3*a) - (5*b^2*(a + b*x)^(4/3))/(6*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
```

```
a^2) + (5*b^2*log((5*b^2*(a + b*x)^(1/3))/a^2 - (5*b^2*((3^(1/2)*1i)/2 - 1/
2))/a^(5/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) - (5*b^2*log((5*b^2*(a + b
*x)^(1/3))/a^2 + (5*b^2*((3^(1/2)*1i)/2 + 1/2))/a^(5/3))*((3^(1/2)*1i)/2 +
1/2))/(9*a^(8/3))
```

3.413 $\int \frac{x^3}{(a+bx)^{4/3}} \, dx$

Optimal. Leaf size=70

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

```
[Out] 3*a^3/b^4/(b*x+a)^(1/3)+9/2*a^2*(b*x+a)^(2/3)/b^4-9/5*a*(b*x+a)^(5/3)/b^4+3
/8*(b*x+a)^(8/3)/b^4
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a +
 b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{(a+bx)^{5/3}}{b^3}\right)dx\\ &= \frac{3a^3}{b^4\sqrt[3]{a+bx}}+\frac{9a^2(a+bx)^{2/3}}{2b^4}-\frac{9a(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{8/3}}{8b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.66

$$\frac{3(81a^3+27a^2bx-9ab^2x^2+5b^3x^3)}{40b^4\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x - 9*a*b^2*x^2 + 5*b^3*x^3))/(40*b^4*(a + b*x)^(1/3)
)
```

Maple [A]
time = 0.12, size = 49, normalized size = 0.70

method	result	size
gosper	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
trager	$\frac{\frac{3}{8}b^3x^3-\frac{27}{40}a\,b^2x^2+\frac{81}{40}a^2bx+\frac{243}{40}a^3}{(bx+a)^{\frac{1}{3}}b^4}$	43
risch	$\frac{3(5x^2b^2-14abx+41a^2)(bx+a)^{\frac{2}{3}}}{40b^4}+\frac{3a^3}{b^4(bx+a)^{\frac{1}{3}}}$	48
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{9a(bx+a)^{\frac{5}{3}}}{5}+\frac{9a^2(bx+a)^{\frac{2}{3}}}{2}+\frac{3a^3}{(bx+a)^{\frac{1}{3}}}}{b^4}$	49
default	$\frac{\frac{3(bx+a)^{\frac{8}{3}}}{8}-\frac{9a(bx+a)^{\frac{5}{3}}}{5}+\frac{9a^2(bx+a)^{\frac{2}{3}}}{2}+\frac{3a^3}{(bx+a)^{\frac{1}{3}}}}{b^4}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*(1/8*(b*x+a)^(8/3)-3/5*a*(b*x+a)^(5/3)+3/2*a^2*(b*x+a)^(2/3)+a^3/(b*x
+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^4}-\frac{9\,(bx+a)^{\frac{5}{3}}a}{5\,b^4}+\frac{9\,(bx+a)^{\frac{2}{3}}a^2}{2\,b^4}+\frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^4 - 9/5*(b*x + a)^(5/3)*a/b^4 + 9/2*(b*x + a)^(2/3)*a
^2/b^4 + 3*a^3/((b*x + a)^(1/3)*b^4)
```

Fricas [A]
time = 0.43, size = 52, normalized size = 0.74

$$\frac{3\,(5\,b^3x^3-9\,ab^2x^2+27\,a^2bx+81\,a^3)(bx+a)^{\frac{2}{3}}}{40\,(b^5x+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 - 9*a*b^2*x^2 + 27*a^2*b*x + 81*a^3)*(b*x + a)^(2/3)/(b^5*x
 + a*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1538 vs. $2(66) = 132$.
time = 1.27, size = 1538, normalized size = 21.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a*
*18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*
x**5 + 40*a**14*b**10*x**6) - 243*a**(68/3)/(40*a**20*b**4 + 240*a**19*b**5
*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*
a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1296*a**(65/3)*b*x*(1 + b*x/a)**(2
/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**
7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) -
 1458*a**(65/3)*b*x/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2
 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**
14*b**10*x**6) + 2808*a**(62/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**20*b**4
 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16
*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(62/3)*b*
*2*x**2/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17
*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**
6) + 3120*a**(59/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19
*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 +
 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 4860*a**(59/3)*b**3*x**3/(40*
a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1830*a*
*(56/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 60
0*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b
**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(56/3)*b**4*x**4/(40*a**20*b**4 +
 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b
**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 528*a**(53/3)*b**5*
x**5*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*
x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40
*a**14*b**10*x**6) - 1458*a**(53/3)*b**5*x**5/(40*a**20*b**4 + 240*a**19*b*
*5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 24
0*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 96*a**(50/3)*b**6*x**6*(1 + b*x/
a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**
```

```
17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x
**6) - 243*a**(50/3)*b**6*x**6/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**1
8*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x*
*5 + 40*a**14*b**10*x**6) + 48*a**(47/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(40*a
**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 15*a**(4
4/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a
**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9
*x**5 + 40*a**14*b**10*x**6)
```

Giac [A]
time = 1.56, size = 62, normalized size = 0.89

$$\frac{3\,a^3}{(bx+a)^{\frac{1}{3}}b^4}+\frac{3\left(5\,(bx+a)^{\frac{8}{3}}b^{28}-24\,(bx+a)^{\frac{5}{3}}ab^{28}+60\,(bx+a)^{\frac{2}{3}}a^2b^{28}\right)}{40\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3*a^3/((b*x + a)^(1/3)*b^4) + 3/40*(5*(b*x + a)^(8/3)*b^28 - 24*(b*x + a)^(
5/3)*a*b^28 + 60*(b*x + a)^(2/3)*a^2*b^28)/b^32
```

Mupad [B]
time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{2/3}}{2\,b^4}+\frac{3\,a^3}{b^4\,{\left(a+b\,x\right)}^{1/3}}-\frac{9\,a\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(8/3))/(8*b^4) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) + (3*a^3)/(b^
4*(a + b*x)^(1/3)) - (9*a*(a + b*x)^(5/3))/(5*b^4)
```

3.414 $\int \frac{x^2}{(a+bx)^{4/3}} dx$

Optimal. Leaf size=49

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

```
[Out] -3*a^2/b^3/(b*x+a)^(1/3)-3*a*(b*x+a)^(2/3)/b^3+3/5*(b*x+a)^(5/3)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^(
5/3))/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{4/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx\\ &= -\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 0.69

$$\frac{3(-9a^2-3abx+b^2x^2)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (3*(-9*a^2 - 3*a*b*x + b^2*x^2))/(5*b^3*(a + b*x)^(1/3))
```

Maple [A]
time = 0.13, size = 38, normalized size = 0.78

method	result	size
gosper	$-\frac{3\left(-x^{2} b^{2}+3 a b x+9 a^{2}\right)}{5\left(b x+a\right)^{\frac{1}{3}} b^{3}}$	32
trager	$-\frac{3\left(-x^{2} b^{2}+3 a b x+9 a^{2}\right)}{5\left(b x+a\right)^{\frac{1}{3}} b^{3}}$	32
risch	$-\frac{3\left(-b x+4 a\right)\left(b x+a\right)^{\frac{2}{3}}}{5 b^{3}}-\frac{3 a^{2}}{b^{3}\left(b x+a\right)^{\frac{1}{3}}}$	37
derivativedivides	$\frac{\frac{3\left(b x+a\right)^{\frac{5}{3}}}{5}-3 a\left(b x+a\right)^{\frac{2}{3}}-\frac{3 a^{2}}{\left(b x+a\right)^{\frac{1}{3}}}}{b^{3}}$	38
default	$\frac{\frac{3\left(b x+a\right)^{\frac{5}{3}}}{5}-3 a\left(b x+a\right)^{\frac{2}{3}}-\frac{3 a^{2}}{\left(b x+a\right)^{\frac{1}{3}}}}{b^{3}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^3*(1/5*(b*x+a)^(5/3)-a*(b*x+a)^(2/3)-a^2/(b*x+a)^(1/3))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.84

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^3}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{b^3}-\frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^3 - 3*(b*x + a)^(2/3)*a/b^3 - 3*a^2/((b*x + a)^(1/3)*
b^3)
```

Fricas [A]
time = 0.46, size = 40, normalized size = 0.82

$$\frac{3\,(b^2x^2-3\,abx-9\,a^2)(bx+a)^{\frac{2}{3}}}{5\,(b^4x+ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b^2*x^2 - 3*a*b*x - 9*a^2)*(b*x + a)^(2/3)/(b^4*x + a*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(46) = 92$.
time = 0.83, size = 534, normalized size = 10.90

$$-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{27a^{\frac{29}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{63a^{\frac{26}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{81a^{\frac{26}{3}}bx}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{42a^{\frac{23}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{81a^{\frac{23}{3}}b^2x^2}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}-\frac{3a^{\frac{20}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{27a^{\frac{20}{3}}b^3x^3}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}+\frac{3a^{\frac{17}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{5a^8b^3+15a^7b^4x+15a^6b^5x^2+5a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(4/3),x)
```

```
[Out] -27*a**(29/3)*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b*
*5*x**2 + 5*a**5*b**6*x**3) + 27*a**(29/3)/(5*a**8*b**3 + 15*a**7*b**4*x +
15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 63*a**(26/3)*b*x*(1 + b*x/a)**(2/3)
/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81
*a**(26/3)*b*x/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b
**6*x**3) - 42*a**(23/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**
7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81*a**(23/3)*b**2*x**2/(
5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 3*a*
*(20/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**
6*b**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(20/3)*b**3*x**3/(5*a**8*b**3 + 15*
a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 3*a**(17/3)*b**4*x**4
*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a
**5*b**6*x**3)
```

Giac [A]
time = 1.41, size = 46, normalized size = 0.94

$$-\frac{3\,a^2}{(bx+a)^{\frac{1}{3}}b^3}+\frac{3\left((bx+a)^{\frac{5}{3}}b^{12}-5\,(bx+a)^{\frac{2}{3}}ab^{12}\right)}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3*a^2/((b*x + a)^(1/3)*b^3) + 3/5*((b*x + a)^(5/3)*b^12 - 5*(b*x + a)^(2/3
)*a*b^12)/b^15
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$-\frac{15\,a\,(a+b\,x)-3\,(a+b\,x)^2+15\,a^2}{5\,b^3\,(a+b\,x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x) - 3*(a + b*x)^2 + 15*a^2)/(5*b^3*(a + b*x)^(1/3))
```

3.415 $\int \frac{x}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=32

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

```
[Out] 3*a/b^2/(b*x+a)^(1/3)+3/2*(b*x+a)^(2/3)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx\\ &= \frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(4/3),x]
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

Maple [A]
time = 0.12, size = 25, normalized size = 0.78

method	result	size
gosper	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}} b^2}$	20
trager	$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}} b^2}$	20
derivativedivides	$\frac{\frac{3(bx+a)^{\frac{2}{3}}}{2}+\frac{3a}{(bx+a)^{\frac{1}{3}}}}{b^2}$	25
default	$\frac{\frac{3(bx+a)^{\frac{2}{3}}}{2}+\frac{3a}{(bx+a)^{\frac{1}{3}}}}{b^2}$	25
risch	$\frac{3a}{b^2(bx+a)^{\frac{1}{3}}}+\frac{3(bx+a)^{\frac{2}{3}}}{2b^2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
[Out] 3/b^2*(1/2*(b*x+a)^(2/3)+a/(b*x+a)^(1/3))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b^2}+\frac{3\,a}{(bx+a)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="maxima")
[Out] 3/2*(b*x + a)^(2/3)/b^2 + 3*a/((b*x + a)^(1/3)*b^2)
```

Fricas [A]
time = 0.51, size = 29, normalized size = 0.91

$$\frac{3\,(bx+3\,a)(bx+a)^{\frac{2}{3}}}{2\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] 3/2*(b*x + 3*a)*(b*x + a)^(2/3)/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.27, size = 41, normalized size = 1.28

$$\begin{cases} \frac{9a}{2b^2\sqrt[3]{a+bx}} + \frac{3x}{2b\sqrt[3]{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(2*b**2*(a + b*x)**(1/3)) + 3*x/(2*b*(a + b*x)**(1/3)), Ne(b
, 0)), (x**2/(2*a**(4/3)), True))
```

Giac [A]
time = 1.03, size = 30, normalized size = 0.94

$$\frac{3\left(\frac{(bx+a)^{\frac{2}{3}}}{b} + \frac{2a}{(bx+a)^{\frac{1}{3}}b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*((b*x + a)^(2/3)/b + 2*a/((b*x + a)^(1/3)*b))/b
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.62

$$\frac{9a + 3bx}{2b^2(a + bx)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(4/3),x)
```

```
[Out] (9*a + 3*b*x)/(2*b^2*(a + b*x)^(1/3))
```

3.416 $\int \frac{1}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=14

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

[Out] -3/b/(b*x+a)^(1/3)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-4/3),x]

[Out] -3/(b*(a + b*x)^(1/3))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}}\,dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-4/3),x]

[Out] -3/(b*(a + b*x)^(1/3))

Maple [A]
time = 0.11, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
derivativedivides	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
default	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13
trager	$-\frac{3}{b(bx+a)^{\frac{1}{3}}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b/(b*x+a)^(1/3)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -3/((b*x + a)^(1/3)*b)
```

Fricas [A]
time = 0.96, size = 20, normalized size = 1.43

$$-\frac{3\,(bx+a)^{\frac{2}{3}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -3*(b*x + a)^(2/3)/(b^2*x + a*b)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.86

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3),x)
```

```
[Out] -3/(b*(a + b*x)**(1/3))
```

Giac [A]
time = 0.70, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3/((b*x + a)^(1/3)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{3}{b\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(4/3),x)
```

```
[Out] -3/(b*(a + b*x)^(1/3))
```

3.417 $\int \frac{1}{x(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=93

$$\frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}$$

```
[Out] 3/a/(b*x+a)^(1/3)-1/2*ln(x)/a^(4/3)+3/2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(4/3)+a
rctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(4/3)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {53, 57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt
[3]*a^(1/3))])/a^(4/3) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a + b*x)^(1
/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{4/3}}\,dx &= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{a}\\ &= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{3\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\right.}{2a}\\ &= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+}}{\sqrt[3]{a}}\right.}{a^{4/3}}\\ &= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 111, normalized size = 1.19

$$\frac{\frac{6\sqrt[3]{a}}{\sqrt[3]{a+bx}}+2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] ((6*a^(1/3))/(a + b*x)^(1/3) + 2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^
(1/3))/Sqrt[3]] + 2*Log[a^(1/3) - (a + b*x)^(1/3)] - Log[a^(2/3) + a^(1/3)*
(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(2*a^(4/3))
```

Maple [A]
time = 0.11, size = 95, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}}{a}+\frac{3}{a(bx+a)^{\frac{1}{3}}}$	95
default	$\frac{\frac{\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}}{a}+\frac{3}{a(bx+a)^{\frac{1}{3}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/a*(1/3/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/6/a^(1/3)*ln((b*x+a)^(2/3)+a^(
1/3)*(b*x+a)^(1/3)+a^(2/3))+1/3*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/
3)*(b*x+a)^(1/3)+1)))+3/a/(b*x+a)^(1/3)
```

Maxima [A]
time = 0.49, size = 88, normalized size = 0.95

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

Fricas [A]
time = 1.89, size = 285, normalized size = 3.06

$$\left[\frac{\sqrt{3}\,(abx+a^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6\,(bx+a)^{\frac{2}{3}}a}{2\,(a^2bx+a^3)},\ -\frac{(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}\,(abx+a^2)\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-6\,(bx+a)^{\frac{2}{3}}a}{2\,(a^2bx+a^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(a*b*x + a^2)*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - (b*x + a)*a^(2/3)*log((b*x + a)^(2/3) + (b*x
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x + a)*a^(2/3)*log((b*x + a)^(1/3) -
a^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*b*x + a^3), -1/2*((b*x + a)*a^(2/3)*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x + a)*a^(2/3
)*log((b*x + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x + a^2)*arctan(1/3*sqrt(
3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x + a)^(2/3)*a)/(a
^2*b*x + a^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.00, size = 184, normalized size = 1.98

$$-\frac{\Gamma\left(-\frac{1}{3}\right)}{a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(4/3),x)
```

```
[Out] -gamma(-1/3)/(a*b**(1/3)*(a/b + x)**(1/3)*gamma(2/3)) - log(1 - b**(1/3)*(a
/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(2*I*pi/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/
3)/(3*a**(4/3)*gamma(2/3)) - exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3))
```

Giac [A]
time = 1.05, size = 89, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

Mupad [B]
time = 0.06, size = 114, normalized size = 1.23

$$\frac{\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-9\,a^{4/3}\right)}{a^{4/3}}+\frac{3}{a\,{\left(a+b\,x\right)}^{1/3}}+\frac{\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{4/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{4/3}}-\frac{\ln\left(9\,a\,{\left(a+b\,x\right)}^{1/3}-\frac{9\,a^{4/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(4/3)),x)
```

```
[Out] log(9*a*(a + b*x)^(1/3) - 9*a^(4/3))/a^(4/3) + 3/(a*(a + b*x)^(1/3)) + (log
(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/
(2*a^(4/3)) - (log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i + 1)^2)/4)*
(3^(1/2)*1i + 1))/(2*a^(4/3))
```

3.418 $\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=113

$$-\frac{4b}{a^2\sqrt[3]{a+bx}}-\frac{1}{ax\sqrt[3]{a+bx}}-\frac{4b\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}$$

```
[Out] -4*b/a^2/(b*x+a)^(1/3)-1/a/x/(b*x+a)^(1/3)+2/3*b*ln(x)/a^(7/3)-2*b*ln(a^(1/
3)-(b*x+a)^(1/3))/a^(7/3)-4/3*b*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3
)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {44, 53, 57, 631, 210, 31}

$$-\frac{4b\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b}{a^2\sqrt[3]{a+bx}}-\frac{1}{ax\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] (-4*b)/(a^2*(a + b*x)^(1/3)) - 1/(a*x*(a + b*x)^(1/3)) - (4*b*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b*Log[x]
)/(3*a^(7/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \frac{3}{ax\sqrt[3]{a+bx}}+\frac{4\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}}-\frac{4(a+bx)^{2/3}}{a^2x}-\frac{(4b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a^2}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}}-\frac{4(a+bx)^{2/3}}{a^2x}+\frac{2b\log(x)}{3a^{7/3}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{a^{7/3}}-$$

$$= \frac{3}{ax\sqrt[3]{a+bx}}-\frac{4(a+bx)^{2/3}}{a^2x}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}+\frac{(4b)\text{Subst}\left(\int\right.}{}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}}-\frac{4(a+bx)^{2/3}}{a^2x}-\frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}\right.}{a^{7}}$$

Mathematica [A]
time = 0.21, size = 123, normalized size = 1.09

$$\frac{-\frac{3\sqrt[3]{a}\,(a+4bx)}{x\sqrt[3]{a+bx}}-4\sqrt{3}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2b\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] ((-3*a^(1/3)*(a + 4*b*x))/(x*(a + b*x)^(1/3)) - 4*Sqrt[3]*b*ArcTan[(1 + (2*
(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 4*b*Log[a^(1/3) - (a + b*x)^(1/3)] + 2
*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(7/3))
```

Maple [A]
time = 0.11, size = 112, normalized size = 0.99

method	result
risch	$-\frac{(bx+a)^{\frac{2}{3}}}{a^{2}x}-\frac{3b}{a^{2}(bx+a)^{\frac{1}{3}}}-\frac{4b\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{4b\sqrt{3}\,\arctan\left(\ldots\right)}{3a\ldots}$
derivativedivides	$3b\left(-\frac{1}{a^{2}(bx+a)^{\frac{1}{3}}}+\frac{-\frac{(bx+a)^{\frac{2}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx\ldots}{\ldots}\right)}{3}\right)}{9a^{\frac{1}{3}}}}{a^{2}}\ldots\right.$
default	$3b\left(-\frac{1}{a^{2}(bx+a)^{\frac{1}{3}}}+\frac{-\frac{(bx+a)^{\frac{2}{3}}}{3bx}-\frac{4\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{1}{3}}}+\frac{2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{1}{3}}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx\ldots}{\ldots}\right)}{3}\right)}{9a^{\frac{1}{3}}}}{a^{2}}\ldots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b*(-1/a^2/(b*x+a)^(1/3)+1/a^2*(-1/3*(b*x+a)^(2/3)/b/x-4/9/a^(1/3)*ln((b*x
+a)^(1/3)-a^(1/3))+2/9/a^(1/3)*ln((b*x+a)^(2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/
3))-4/9*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]
time = 0.52, size = 122, normalized size = 1.08

$$-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx+a)b-3\,ab}{(bx+a)^{\frac{4}{3}}a^{2}-(bx+a)^{\frac{1}{3}}a^{3}}+\frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) - (4*(b*x + a)*b - 3*a*b)/((b*x + a)^(4/3)*a^2 - (b*x + a)^(1/3)*a^3)
 + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7/3) -
 4/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(88) = 176.
time = 1.26, size = 407, normalized size = 3.60

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^{2}x^{2}+a^{2}bx\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+2\left(b^{2}x^{2}+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\left(b^{2}x^{2}+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(4abx+a^{2}\right)(bx+a)^{\frac{2}{3}}}{3\left(a^{3}bx^{2}+a^{4}x\right)},\ -\frac{12\sqrt{\frac{1}{3}}\left(ab^{2}x^{2}+a^{2}bx\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\left(b^{2}x^{2}+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+4\left(b^{2}x^{2}+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(4abx+a^{2}\right)(bx+a)^{\frac{2}{3}}}{3\left(a^{3}bx^{2}+a^{4}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*s
qrt(1/3)*(2*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*
sqrt((-a)^(1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + 2*(b^2*x^2 +
a*b*x)*(-a)^(2/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(
2/3)) - 4*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) -
3*(4*a*b*x + a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x), -1/3*(12*sqrt(1/3)*
(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log(
(b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^2 + a
*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x + a^2)*(b*x
 + a)^(2/3))/(a^3*b*x^2 + a^4*x)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.38, size = 857, normalized size = 7.58

$$-\frac{9a^{\frac{4}{3}}b^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}+\frac{12\sqrt[3]{a}\,b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{4ab\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{4ab\sqrt[3]{\frac{a}{b}+x}\,e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{4ab\sqrt[3]{\frac{a}{b}+x}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}+\frac{4b^{2}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}+\frac{4b^{2}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}+\frac{4b^{2}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{-9a^{\frac{10}{3}}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)+9a^{\frac{7}{3}}b\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(4/3),x)
```

```
[Out] -9*a**(4/3)*b**(2/3)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*g
```

```
amma(2/3)) + 12*a**(1/3)*b**(5/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a
**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)
**(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*
pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a
/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*log(1 - b**(1/3)*(a/b + x)*
*(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi
/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3)) +
4*b**2*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/
3)) + 4*b**2*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)
*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3))
```

Giac [A]
time = 0.92, size = 120, normalized size = 1.06

$$-\frac{4\,\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\,\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}+\frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,b\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx+a)b-3\,ab}{\left((bx+a)^{\frac{4}{3}}-(bx+a)^{\frac{1}{3}}a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7
/3) - 4/3*b*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) - (4*(b*x + a)*b -
3*a*b)/(((b*x + a)^(4/3) - (b*x + a)^(1/3)*a)*a^2)
```

Mupad [B]
time = 0.07, size = 173, normalized size = 1.53

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\,(a+b\,x)}{a^2}}{a\,(a+b\,x)^{1/3}-(a+b\,x)^{4/3}}+\frac{\ln\left(a^{7/3}\,\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\,\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\,\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\,\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}-\frac{4\,b\,\ln\left(16\,a^{7/3}\,b^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(2*b - 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a + b*x)^(1/3))*(2*b - 3^
(1/2)*b*2i))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x))/a^2)/(a*(a + b*x)^(1/
```

```
3) - (a + b*x)^(4/3)) + (log(a^(7/3)*(2*b + 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a
 + b*x)^(1/3))*(2*b + 3^(1/2)*b*2i))/(3*a^(7/3)) - (4*b*log(16*a^(7/3)*b^2
- 16*a^2*b^2*(a + b*x)^(1/3)))/(3*a^(7/3))
```

3.419 $\int \frac{1}{x^3(a+bx)^{4/3}} dx$

Optimal. Leaf size=149

$$\frac{14b^2}{3a^3\sqrt[3]{a+bx}}-\frac{1}{2ax^2\sqrt[3]{a+bx}}+\frac{7b}{6a^2x\sqrt[3]{a+bx}}+\frac{14b^2\tan^{-1}\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\ldots\right)}{3a^{10}\ldots}$$

```
[Out] 14/3*b^2/a^3/(b*x+a)^(1/3)-1/2/a/x^2/(b*x+a)^(1/3)+7/6*b/a^2/x/(b*x+a)^(1/3
)-7/9*b^2*ln(x)/a^(10/3)+7/3*b^2*ln(a^(1/3)-(b*x+a)^(1/3))/a^(10/3)+14/9*b^
2*arctan(1/3*(a^(1/3)+2*(b*x+a)^(1/3))/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 57, 631, 210, 31}

$$\frac{14b^2\text{ArcTan}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\frac{14b^2}{3a^3\sqrt[3]{a+bx}}+\frac{7b}{6a^2x\sqrt[3]{a+bx}}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] (14*b^2)/(3*a^3*(a + b*x)^(1/3)) - 1/(2*a*x^2*(a + b*x)^(1/3)) + (7*b)/(6*a
^2*x*(a + b*x)^(1/3)) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) + (7*b^2*L
og[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{4/3}}\,dx = \frac{3}{ax^2\sqrt[3]{a+bx}} + \frac{7\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} - \frac{(14b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a^2}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{(14b^2)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^3}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{(7b^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\right.}{3a^{10/}}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+\right.}{3a^{10/3}}$$

$$= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{7b^2}{9}$$

Mathematica [A]
time = 0.28, size = 142, normalized size = 0.95

$$\frac{\frac{3\sqrt[3]{a}\,(-3a^2+7abx+28b^2x^2)}{x^2\sqrt[3]{a+bx}} + 28\sqrt{3}\,b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - 14b^2\log\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] ((3*a^(1/3)*(-3*a^2 + 7*a*b*x + 28*b^2*x^2))/(x^2*(a + b*x)^(1/3)) + 28*Sqr
t[3]*b^2*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^2*Log[a^(
1/3) - (a + b*x)^(1/3)] - 14*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a
 + b*x)^(2/3)])/(18*a^(10/3))
```

Maple [A]
time = 0.13, size = 126, normalized size = 0.85

method	result

risch	$-\frac{(bx+a)^{\frac{2}{3}}(-10bx+3a)}{6a^3x^2}+\frac{3b^2}{a^3(bx+a)^{\frac{1}{3}}}+\frac{14b^2\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^2\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2\sqrt{\ }}{}$
derivativedivides	$3b^2\left(\frac{1}{a^3(bx+a)^{\frac{1}{3}}}-\frac{\frac{-\frac{5(bx+a)^{\frac{5}{3}}}{9}+\frac{13a(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{1}{3}}}+\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{1}{3}}}-\frac{14\sqrt{3}\ \arctan}{}}{a^3}\right.$
default	$3b^2\left(\frac{1}{a^3(bx+a)^{\frac{1}{3}}}-\frac{\frac{-\frac{5(bx+a)^{\frac{5}{3}}}{9}+\frac{13a(bx+a)^{\frac{2}{3}}}{18}}{b^2x^2}-\frac{14\ln\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{27a^{\frac{1}{3}}}+\frac{7\ln\left((bx+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{27a^{\frac{1}{3}}}-\frac{14\sqrt{3}\ \arctan}{}}{a^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3*b^2*(1/a^3/(b*x+a)^(1/3)-1/a^3*((-5/9*(b*x+a)^(5/3)+13/18*a*(b*x+a)^(2/3)
)/b^2/x^2-14/27/a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))+7/27/a^(1/3)*ln((b*x+a)^(
2/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-14/27*3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2
)*(2/a^(1/3)*(b*x+a)^(1/3)+1))))
```

Maxima [A]

time = 0.49, size = 158, normalized size = 1.06

$$\frac{14\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}}+\frac{28(bx+a)^2b^2-49(bx+a)ab^2+18a^2b^2}{6\left((bx+a)^{\frac{7}{3}}a^3-2(bx+a)^{\frac{4}{3}}a^4+(bx+a)^{\frac{1}{3}}a^5\right)}-\frac{7b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) + 1/6*(28*(b*x + a)^2*b^2 - 49*(b*x + a)*a*b^2 + 18*a^2*b^2)/((b*x
 + a)^(7/3)*a^3 - 2*(b*x + a)^(4/3)*a^4 + (b*x + a)^(1/3)*a^5) - 7/9*b^2*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(10/3) + 14/9*b^2*
log((b*x + a)^(1/3) - a^(1/3))/a^(10/3)
```

Fricas [A]
time = 1.25, size = 407, normalized size = 2.73

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-14\left(b^{3}x^{3}+ab^{2}x^{2}\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+28\left(b^{3}x^{3}+ab^{2}x^{2}\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(28ab^{2}x^{2}+7a^{2}bx-3a^{3}\right)(bx+a)^{\frac{2}{3}}}{18\left(a^{4}bx^{3}+a^{5}x^{2}\right)},-\frac{14\left(b^{3}x^{3}+ab^{2}x^{2}\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-28\left(b^{3}x^{3}+ab^{2}x^{2}\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{84\sqrt{\frac{1}{3}}\left(ab^{3}x^{3}+a^{2}b^{2}x^{2}\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-3\left(28ab^{2}x^{2}+7a^{2}bx-3a^{3}\right)(bx+a)^{\frac{2}{3}}}{18\left(a^{4}bx^{3}+a^{5}x^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/18*(42*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-1/a^(2/3))*log((2*b*x +
 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt
(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 14*(b^3*x^3 + a*b^2*x^
2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 28*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(28*a*b^2*x^
2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2), -1/18*(14*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
 a^(2/3)) - 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3))
 - 84*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x
+ a)^(2/3))/(a^4*b*x^3 + a^5*x^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.99, size = 2793, normalized size = 18.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(4/3),x)
```

```
[Out] 54*a**(13/3)*b**(5/3)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/
3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 201*a*
*(10/3)*b**(8/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 2
31*a**(7/3)*b**(11/3)*(a/b + x)**2*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) - 84*a**(4/3)*b**(14/3)*(a/b + x)**3*exp(2*I*pi/3)*gamma(-1/3)/(-54*a*
*(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*ex
p(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*
```

```
gamma(2/3)) + 28*a**4*b**2*(a/b + x)**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma
(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a
**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a
/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3
)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) -
162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)
*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a/b + x)*
*(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(
19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)
*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*b**3*(a/b + x)**(4/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b +
 x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) -
84*a**3*b**3*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1
/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3)
 + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*
b**3*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)
/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(
2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(
16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a
/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a
**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
 x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)
*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2
*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(
2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a*
*(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b +
 x)**(10/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma
(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(1
```

```
9/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
+ x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*
exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(
1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-5
4*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/
b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a*...
```

Giac [A]

time = 0.81, size = 140, normalized size = 0.94

$$\frac{14\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}}-\frac{7\,b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{10}{3}}}+\frac{14\,b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{10}{3}}}+\frac{3\,b^2}{(bx+a)^{\frac{1}{3}}a^3}+\frac{10\,(bx+a)^{\frac{5}{3}}b^2-13\,(bx+a)^{\frac{2}{3}}ab^2}{6\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) - 7/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(10/3) + 14/9*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(10/3) + 3*b^2/(
(b*x + a)^(1/3)*a^3) + 1/6*(10*(b*x + a)^(5/3)*b^2 - 13*(b*x + a)^(2/3)*a*b
^2)/(a^3*b^2*x^2)
```

Mupad [B]

time = 0.13, size = 221, normalized size = 1.48

$$\frac{\frac{3\,b^2}{a}+\frac{14\,b^2\,(a+b\,x)^2}{3\,a^3}-\frac{49\,b^2\,(a+b\,x)}{6\,a^2}}{(a+b\,x)^{7/3}-2\,a\,(a+b\,x)^{4/3}+a^2\,(a+b\,x)^{1/3}}+\frac{\ln\left(588\,a^3\,b^4\,(a+b\,x)^{1/3}-3\,a^{10/3}\left(-7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)^2\right)\left(-7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)}{9\,a^{10/3}}-\frac{\ln\left(588\,a^3\,b^4\,(a+b\,x)^{1/3}-3\,a^{10/3}\left(7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)^2\right)\left(7\,b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)}{9\,a^{10/3}}+\frac{14\,b^2\,\ln\left(588\,a^3\,b^4\,(a+b\,x)^{1/3}-588\,a^{10/3}\,b^4\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(4/3)),x)
```

```
[Out] ((3*b^2)/a + (14*b^2*(a + b*x)^2)/(3*a^3) - (49*b^2*(a + b*x))/(6*a^2))/((a
 + b*x)^(7/3) - 2*a*(a + b*x)^(4/3) + a^2*(a + b*x)^(1/3)) + (log(588*a^3*b
^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^(1/2)*b^2*7i - 7*b^2)^2)*(3^(1/2)*b^2*7i
 - 7*b^2))/(9*a^(10/3)) - (log(588*a^3*b^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^
(1/2)*b^2*7i + 7*b^2)^2)*(3^(1/2)*b^2*7i + 7*b^2))/(9*a^(10/3)) + (14*b^2*l
og(588*a^3*b^4*(a + b*x)^(1/3) - 588*a^(10/3)*b^4))/(9*a^(10/3))
```

3.420 $\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

```
[Out] -1/2*ln(x)/a+3/2*ln(a-(b^3*x+a^3)^(1/3))/a+arctan(1/3*(a+2*(b^3*x+a^3)^(1/3
))/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = -\frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3+b^3x}\right)-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3+\ldots}\right.}{2a}$$

$$=-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a}$$

$$=\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 1.37

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)+2\log\left(a-\sqrt[3]{a^3+b^3x}\right)-\log\left(a^2+a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] + 2*Log[a - (a^3
 + b^3*x)^(1/3)] - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/3)])/
(2*a)
```

Maple [A]
time = 0.12, size = 86, normalized size = 1.21

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^2+a\left(b^3x+a^3\right)^{\frac{1}{3}}+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	86

default	$\frac{\ln\left(a-(b^3x+a^3)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^2+a(b^3x+a^3)^{\frac{1}{3}}+(b^3x+a^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2(b^3x+a^3)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(b^3*x+a^3)^(1/3))/a+1/a*(-1/2*ln(a^2+a*(b^3*x+a^3)^(1/3)+(b^3*x+a^3)^
(2/3))+3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.48, size = 86, normalized size = 1.21

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(a+2\,(b^3x+a^3)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}-\frac{\log\left(a^2+(b^3x+a^3)^{\frac{1}{3}}a+(b^3x+a^3)^{\frac{2}{3}}\right)}{2\,a}+\frac{\log\left(-a+(b^3x+a^3)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(-a + (b^3*x + a^3)^(1
/3))/a
```

Fricas [A]
time = 1.02, size = 88, normalized size = 1.24

$$\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,(b^3x+a^3)^{\frac{1}{3}}}{3\,a}\right)-\log\left(a^2+(b^3x+a^3)^{\frac{1}{3}}a+(b^3x+a^3)^{\frac{2}{3}}\right)+2\,\log\left(-a+(b^3x+a^3)^{\frac{1}{3}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) -
log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) + 2*log(-a + (b^3*x
+ a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.88, size = 138, normalized size = 1.94

$$\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(1/3),x)
```

```
[Out] exp(I*pi/3)*log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamm
a(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(2*I
*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.56, size = 87, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(abs(-a + (b^3*x + a^3
)^(1/3)))/a
```

Mupad [B]
time = 0.10, size = 105, normalized size = 1.48

$$\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-9\,a\right)}{a}+\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}-\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(1/3)),x)
```

```
[Out] log(9*(b^3*x + a^3)^(1/3) - 9*a)/a + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(b^3*x + a^3)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

3.421 $\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

```
[Out] -1/2*ln(x)/a+3/2*ln(a-(-b^3*x+a^3)^(1/3))/a+arctan(1/3*(a+2*(-b^3*x+a^3)^(1
/3))/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = -\frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-\right.}{2a}$$

$$=-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a}$$

$$=\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 1.38

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)+2\log\left(a-\sqrt[3]{a^3-b^3x}\right)-\log\left(a^2+a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] + 2*Log[a - (a^3
 - b^3*x)^(1/3)] - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/3)])/
(2*a)
```

Maple [A]
time = 0.12, size = 90, normalized size = 1.23

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^2+a\left(-b^3x+a^3\right)^{\frac{1}{3}}+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	90

default	$\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(-b^3*x+a^3)^(1/3))/a+1/a*(-1/2*ln(a^2+a*(-b^3*x+a^3)^(1/3)+(-b^3*x+a^
3)^(2/3))+3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.48, size = 90, normalized size = 1.23

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(a+2\,\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a}+\frac{\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(-a + (-b^3*x + a^3
)^(1/3))/a
```

Fricas [A]
time = 1.10, size = 92, normalized size = 1.26

$$\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)-\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)+2\,\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a) -
 log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) + 2*log(-a + (-b^
3*x + a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 136, normalized size = 1.86

$$-\frac{e^{-\frac{2i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(1/3),x)
```

```
[Out] -exp(-2*I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*g
amma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(I*pi)/(b*(-a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(5*I
*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.63, size = 91, normalized size = 1.25

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(abs(-a + (-b^3*x +
 a^3)^(1/3)))/a
```

Mupad [B]
time = 0.13, size = 108, normalized size = 1.48

$$\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-9a\right)}{a}+\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a}-\frac{\ln\left(9\left(a^3-b^3x\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(1/3)),x)
```

```
[Out] log(9*(a^3 - b^3*x)^(1/3) - 9*a)/a + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(a^3 - b^3*x)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

3.422 $\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

```
[Out] 1/2*ln(x)/a-3/2*ln(a+(b^3*x-a^3)^(1/3))/a-arctan(1/3*(a-2*(b^3*x-a^3)^(1/3)
)/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {58, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)-\frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-\right.}{2a}$$

$$=\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a}$$

$$=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.06, size = 102, normalized size = 1.38

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)-2\log\left(a+\sqrt[3]{-a^3+b^3x}\right)+\log\left(a^2-a\sqrt[3]{-a^3+b^3x}+(-a^3+b^3x)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]
[Out] (-2*Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a + (-
a^3 + b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x)^(2/
3)])/(2*a)
```

Maple [A]
time = 0.10, size = 96, normalized size = 1.30

method	result	size
derivativedivides	$-\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}+\frac{\frac{\ln\left(a^2-a\left(b^3x-a^3\right)^{\frac{1}{3}}+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	96
default	$-\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}+\frac{\frac{\ln\left(a^2-a\left(b^3x-a^3\right)^{\frac{1}{3}}+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(a+(b^3*x-a^3)^(1/3))/a+1/a*(1/2*ln(a^2-a*(b^3*x-a^3)^(1/3)+(b^3*x-a^3)^
(2/3))+3^(1/2)*arctan(1/3*(-a+2*(b^3*x-a^3)^(1/3))*3^(1/2)/a))
```

Maxima [A]
time = 0.49, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\left(a-2\,(b^3x-a^3)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}+\frac{\log\left(a^2-(b^3x-a^3)^{\frac{1}{3}}a+(b^3x-a^3)^{\frac{2}{3}}\right)}{2\,a}-\frac{\log\left(a+(b^3x-a^3)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(a + (b^3*x - a^3)^(1
/3))/a
```

Fricas [A]
time = 0.93, size = 93, normalized size = 1.26

$$\frac{2\,\sqrt{3}\,\arctan\left(-\frac{\sqrt{3}\,a-2\,\sqrt{3}\,(b^3x-a^3)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^2-(b^3x-a^3)^{\frac{1}{3}}a+(b^3x-a^3)^{\frac{2}{3}}\right)-2\,\log\left(a+(b^3x-a^3)^{\frac{1}{3}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) +
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) - 2*log(a + (b^3*x
- a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.92, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gam
ma(-1/3)/(3*a*gamma(2/3)) + log(-a*exp_polar(I*pi)/(b*(-a**3/b**3 + x)**(1/
3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(5*I*pi
/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.60, size = 95, normalized size = 1.28

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(abs(a + (b^3*x - a^3
)^(1/3)))/a
```

Mupad [B]
time = 0.11, size = 112, normalized size = 1.51

$$-\frac{\ln\left(9a+9\left(b^3x-a^3\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\left(b^3x-a^3\right)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a}+\frac{\ln\left(\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\left(b^3x-a^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1))/
(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1
i - 1))/(2*a) - log(9*a + 9*(b^3*x - a^3)^(1/3))/a
```

3.423 $\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

```
[Out] 1/2*ln(x)/a-3/2*ln(a+(-b^3*x-a^3)^(1/3))/a-arctan(1/3*(a-2*(-b^3*x-a^3)^(1/
3))/a*3^(1/2))*3^(1/2)/a
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {58, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\log(x)}{2a} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3-b^3x}\right) - \frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-\right.}}{2a}$$

$$= \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a} + \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

Mathematica [A]
time = 0.07, size = 106, normalized size = 1.39

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right) - 2\log\left(a+\sqrt[3]{-a^3-b^3x}\right) + \log\left(a^2-a\sqrt[3]{-a^3-b^3x}+(-a^3-b^3x)^{2/3}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a + (-
a^3 - b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x)^(2/
3)])/(2*a)
```

Maple [A]
time = 0.12, size = 100, normalized size = 1.32

method	result	size
derivativedivides	$\frac{\frac{\ln\left(a^2-a\left(-b^3x-a^3\right)^{\frac{1}{3}}+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(-a+2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a} - \frac{\ln\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$	100
default	$\frac{\frac{\ln\left(a^2-a\left(-b^3x-a^3\right)^{\frac{1}{3}}+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\left(-a+2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a} - \frac{\ln\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*ln(a^2-a*(-b^3*x-a^3)^(1/3)+(-b^3*x-a^3)^(2/3))+3^(1/2)*arctan(1/3
*(-a+2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a))-ln(a+(-b^3*x-a^3)^(1/3))/a
```

Maxima [A]
time = 0.50, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(a + (-b^3*x - a^3
)^(1/3))/a
```

Fricas [A]
time = 0.87, size = 97, normalized size = 1.28

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
+ log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) - 2*log(a + (-b^
3*x - a^3)^(1/3)))/a
```

Sympy [C] Result contains complex when optimal does not.
time = 0.94, size = 139, normalized size = 1.83

$$\frac{\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^3}{b^3}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(1/3),x)
```

```
[Out] log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a
*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3/b**3 + x)**(
1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) + exp(2*I*pi/3)*log(-a*exp_polar(2*
I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

Giac [A]
time = 0.80, size = 99, normalized size = 1.30

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(abs(a + (-b^3*x -
 a^3)^(1/3)))/a
```

Mupad [B]
time = 0.07, size = 115, normalized size = 1.51

$$-\frac{\ln\left(9a+9\left(-a^3-x\,b^3\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\left(-a^3-x\,b^3\right)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a}+\frac{\ln\left(\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}+9\left(-a^3-x\,b^3\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1)
)/(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/
2)*1i - 1))/(2*a) - log(9*a + 9*(- b^3*x - a^3)^(1/3))/a
```

3.424 $\int \frac{1}{x(a^3+b^3x)^{2/3}}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a-(b^3*x+a^3)^(1/3))/a^2-arctan(1/3*(a+2*(b^3*x+a^3)^
(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {59, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx &= -\frac{\log(x)}{2a^2}-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{3\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3+b^3x}\right)}{2a}\\ &= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a^2}\\ &= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 95, normalized size = 1.32

$$-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3+b^3x}}{\sqrt{3}\,a}\right)-2\log\left(a-\sqrt[3]{a^3+b^3x}\right)+\log\left(a^2+a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 + b^3*x)^(1/3)] + Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/
3)])/a^2
```

Maple [A]
time = 0.12, size = 87, normalized size = 1.21

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}+\frac{-\frac{\ln\left(a^2+a\left(b^3x+a^3\right)^{\frac{1}{3}}+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}$	87

default	$\frac{\ln\left(a-(b^3x+a^3)^{\frac{1}{3}}\right)}{a^2}+\frac{-\frac{\ln\left(a^2+a\left(b^3x+a^3\right)^{\frac{1}{3}}+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(b^3*x+a^3)^(1/3))/a^2+1/a^2*(-1/2*ln(a^2+a*(b^3*x+a^3)^(1/3)+(b^3*x+a
^3)^(2/3))-3^(1/2)*arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.48, size = 87, normalized size = 1.21

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(a+2\,(b^3x+a^3)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2+(b^3x+a^3)^{\frac{1}{3}}a+(b^3x+a^3)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(-a+(b^3x+a^3)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(-a + (b^3*x + a^
3)^(1/3))/a^2
```

Fricas [A]
time = 0.70, size = 86, normalized size = 1.19

$$-\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,(b^3x+a^3)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^2+(b^3x+a^3)^{\frac{1}{3}}a+(b^3x+a^3)^{\frac{2}{3}}\right)-2\,\log\left(-a+(b^3x+a^3)^{\frac{1}{3}}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) +
 log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) - 2*log(-a + (b^3*x
 + a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 134, normalized size = 1.86

$$\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(-2
*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/a)*gamma(1/3)
/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_p
olar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 1.12, size = 88, normalized size = 1.22

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (b^3*x
+ a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.14, size = 101, normalized size = 1.40

$$\frac{\ln\left(9a-9\left(a^3+x\,b^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}-\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}-\frac{\ln\left(9\left(a^3+x\,b^3\right)^{1/3}+\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(2/3)),x)
```

```
[Out] log(9*a - 9*(b^3*x + a^3)^(1/3))/a^2 + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x + a^3)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.425 $\int \frac{1}{x(a^3-b^3x)^{2/3}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a-(-b^3*x+a^3)^(1/3))/a^2-arctan(1/3*(a+2*(-b^3*x+a^3
)^(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {59, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}-\frac{3\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{3\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a}$$
$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a^2}$$
$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

Mathematica [A]
time = 0.05, size = 99, normalized size = 1.34

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{a+2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}\right)-2\log\left(a-\sqrt[3]{a^3-b^3x}\right)+\log\left(a^2+a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 - b^3*x)^(1/3)] + Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/
3)])/a^2
```

Maple [A]
time = 0.12, size = 91, normalized size = 1.23

method	result	size
derivativedivides	$\frac{\ln\left(a-\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{a^2}+\frac{-\frac{\ln\left(a^2+a\left(-b^3x+a^3\right)^{\frac{1}{3}}+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}$	91

default	$\frac{\ln\left(a-\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}+\frac{-\frac{\ln\left(a^{2}+a\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(a-(-b^3*x+a^3)^(1/3))/a^2+1/a^2*(-1/2*ln(a^2+a*(-b^3*x+a^3)^(1/3)+(-b^3*
x+a^3)^(2/3))-3^(1/2)*arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2)))
```

Maxima [A]
time = 0.48, size = 91, normalized size = 1.23

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(a+2\,\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(-a + (-b^3*x
+ a^3)^(1/3))/a^2
```

Fricas [A]
time = 0.50, size = 90, normalized size = 1.22

$$-\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,a+2\,\sqrt{3}\,\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}}{3\,a}\right)+\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)-2\,\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a)
+ log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) - 2*log(-a + (-b
^3*x + a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.90, size = 136, normalized size = 1.84

$$\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(1/3)/(3*a**2*g
amma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 1.38, size = 92, normalized size = 1.24

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^2}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2\,a^2}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (-b^
3*x + a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.11, size = 104, normalized size = 1.41

$$\frac{\ln\left(9\,a-9\,{\left(a^3-b^3\,x\right)}^{1/3}\right)}{a^2}+\frac{\ln\left(9\,{\left(a^3-b^3\,x\right)}^{1/3}-\frac{9\,a\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\,{\left(a^3-b^3\,x\right)}^{1/3}+\frac{9\,a\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(2/3)),x)
```

```
[Out] log(9*a - 9*(a^3 - b^3*x)^(1/3))/a^2 + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(a^3 - b^3*x)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.426 $\int \frac{1}{x(-a^3+b^3x)^{2/3}}\,dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}\,a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a+(b^3*x-a^3)^(1/3))/a^2-arctan(1/3*(a-2*(b^3*x-a^3)^
(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {60, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}\,a}\right)}{a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3\right.}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right.}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

Mathematica [A]
time = 0.04, size = 102, normalized size = 1.38

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3+b^3x}}{\sqrt{3}a}\right)-2\log\left(a+\sqrt[3]{-a^3+b^3x}\right)+\log\left(a^2-a\sqrt[3]{-a^3+b^3x}+\left(-a^3+b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
+ (-a^3 + b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x)
^(2/3)])/a^2
```

Maple [A]
time = 0.12, size = 95, normalized size = 1.28

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(a^2-a\left(b^3x-a^3\right)^{\frac{1}{3}}+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$	95
default	$\frac{-\frac{\ln\left(a^2-a\left(b^3x-a^3\right)^{\frac{1}{3}}+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*ln(a^2-a*(b^3*x-a^3)^(1/3)+(b^3*x-a^3)^(2/3))+3^(1/2)*arctan(1/
3*(-a+2*(b^3*x-a^3)^(1/3))*3^(1/2)/a))+ln(a+(b^3*x-a^3)^(1/3))/a^2
```

Maxima [A]
time = 0.48, size = 93, normalized size = 1.26

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(a + (b^3*x - a^3
)^(1/3))/a^2
```

Fricas [A]
time = 0.44, size = 95, normalized size = 1.28

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)+2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) -
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) + 2*log(a + (b^3*x
- a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.91, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(2/3),x)
```

```
[Out] -exp(-I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(
1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 0.88, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (b^3*x -
 a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.16, size = 107, normalized size = 1.45

$$\frac{\ln\left(9a+9\left(b^3x-a^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(b^3x-a^3\right)^{1/3}+\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}-\frac{\ln\left(9\left(b^3x-a^3\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(b^3*x - a^3)^(1/3))/a^2 + (log(9*(b^3*x - a^3)^(1/3) + (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x - a^3)^(1/3) -
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.427 $\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

```
[Out] -1/2*ln(x)/a^2+3/2*ln(a+(-b^3*x-a^3)^(1/3))/a^2-arctan(1/3*(a-2*(-b^3*x-a^3
)^(1/3))/a*3^(1/2))*3^(1/2)/a^2
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {60, 631, 210, 31}

$$-\frac{\log(x)}{2a^2}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3\right.}}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}+\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right.}{a^2}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

Mathematica [A]
time = 0.05, size = 106, normalized size = 1.39

$$-\frac{2\sqrt{3}\,\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}\,a}\right)-2\log\left(a+\sqrt[3]{-a^3-b^3x}\right)+\log\left(a^2-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a
+ (-a^3 - b^3*x)^(1/3)] + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x)
^(2/3)])/a^2
```

Maple [A]
time = 0.12, size = 99, normalized size = 1.30

method	result	siz
derivativedivides	$\frac{-\frac{\ln\left(a^2-a\left(-b^3x-a^3\right)^{\frac{1}{3}}+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$	99
default	$\frac{-\frac{\ln\left(a^2-a\left(-b^3x-a^3\right)^{\frac{1}{3}}+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/2*ln(a^2-a*(-b^3*x-a^3)^(1/3)+(-b^3*x-a^3)^(2/3))+3^(1/2)*arctan(
1/3*(-a+2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a))+ln(a+(-b^3*x-a^3)^(1/3))/a^2
```

Maxima [A]
time = 0.48, size = 97, normalized size = 1.28

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(a + (-b^3*x -
 a^3)^(1/3))/a^2
```

Fricas [A]
time = 0.49, size = 99, normalized size = 1.30

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)+2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
- log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) + 2*log(a + (-b^
3*x - a^3)^(1/3)))/a^2
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 133, normalized size = 1.75

$$\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^3}{b^3}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(2/3),x)
```

```
[Out] exp(-2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma
(4/3)) - exp(-I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(a**3/b**3 + x)**(1/3)*exp_po
lar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

Giac [A]
time = 1.26, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (-b^3
*x - a^3)^(1/3)))/a^2
```

Mupad [B]
time = 0.16, size = 110, normalized size = 1.45

$$\frac{\ln\left(9a+9\left(-a^3-x\,b^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(-a^3-x\,b^3\right)^{1/3}+\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}-\frac{\ln\left(9\left(-a^3-x\,b^3\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(- b^3*x - a^3)^(1/3))/a^2 + (log(9*(- b^3*x - a^3)^(1/3) + (9*
a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(- b^3*x - a^3)^(
1/3) - (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

3.428 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}$$

[Out] a*x^(1+m)/(1+m)+b*x^(2+m)/(2+m)

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a + b*x),x]

[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx&=\int\left(ax^m+bx^{1+m}\right)\,dx\\&=\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.88

$$x^{1+m}\left(\frac{a}{1+m}+\frac{bx}{2+m}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a + b*x),x]

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

Maple [A]
time = 0.01, size = 30, normalized size = 1.20

method	result	size
norman	$\frac{ax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b\,x^2\mathrm{e}^{m\ln(x)}}{2+m}$	30
risch	$\frac{x(bmx+am+bx+2a)x^m}{(2+m)(1+m)}$	30
gosper	$\frac{x^{1+m}(bmx+am+bx+2a)}{(2+m)(1+m)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] a/(1+m)*x*exp(m*ln(x))+b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.27, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.46, size = 33, normalized size = 1.32

$$\frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(19) = 38$.
time = 0.10, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x}+b\log(x) & \text{for } m=-2 \\ a\log(x)+bx & \text{for } m=-1 \\ \frac{amxx^m}{m^2+3m+2}+\frac{2axx^m}{m^2+3m+2}+\frac{bmx^2x^m}{m^2+3m+2}+\frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

Giac [A]
time = 1.66, size = 43, normalized size = 1.72

$$\frac{bmx^2x^m + amxx^m + bx^2x^m + 2\,axx^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [B]
time = 0.31, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,(2\,a + a\,m + b\,x + b\,m\,x)}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
```

```
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

3.429 $\int x^{5/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

```
[Out] 2/7*a*x^(7/2)+2/9*b*x^(9/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x),x]
[Out] (2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)\,dx &= \int \left(ax^{5/2}+bx^{7/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{63}x^{7/2}(9a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x),x]
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
trager	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14
risch	$\frac{2x^{\frac{7}{2}}(7bx+9a)}{63}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2/7*a*x^(7/2)+2/9*b*x^(9/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Fricas [A]
time = 0.46, size = 18, normalized size = 0.86

$$\frac{2}{63}\,\left(7\,bx^4+9\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

Sympy [A]
time = 0.20, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(7/2)/7 + 2*b*x**(9/2)/9
```

Giac [A]
time = 1.12, size = 13, normalized size = 0.62

$$\frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

Mupad [B]
time = 0.09, size = 13, normalized size = 0.62

$$\frac{2\,x^{7/2}\,\left(9\,a+7\,b\,x\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x),x)
```

```
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

3.430 $\int x^{3/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

```
[Out] 2/5*a*x^(5/2)+2/7*b*x^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)\,dx &= \int \left(ax^{3/2}+bx^{5/2}\right)\,dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{35}x^{5/2}(7a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{7}{2}}}{7}$	14
trager	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14
risch	$\frac{2x^{\frac{5}{2}}(5bx+7a)}{35}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2/5*a*x^(5/2)+2/7*b*x^(7/2)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.62

$$\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Fricas [A]
time = 0.39, size = 18, normalized size = 0.86

$$\frac{2}{35}\left(5\,bx^{3}+7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

Sympy [A]
time = 0.11, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(7/2)/7
```

Giac [A]
time = 1.20, size = 13, normalized size = 0.62

$$\frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{5}ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.62

$$\frac{2\,x^{5/2}\,(7\,a+5\,b\,x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x),x)
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

3.431 $\int \sqrt{x}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

```
[Out] 2/3*a*x^(3/2)+2/5*b*x^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)\,dx &= \int \left(a\sqrt{x}+bx^{3/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{2}{15}x^{3/2}(5a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
trager	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14
risch	$\frac{2x^{\frac{3}{2}}(3bx+5a)}{15}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*x^(3/2)+2/5*b*x^(5/2)
```

Maxima [A]
time = 0.31, size = 13, normalized size = 0.62

$$\frac{2}{5}\,bx^{\frac{5}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Fricas [A]
time = 0.39, size = 16, normalized size = 0.76

$$\frac{2}{15}\left(3\,bx^2+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

Sympy [A]
time = 0.66, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(5/2)/5
```

Giac [A]
time = 1.39, size = 13, normalized size = 0.62

$$\frac{2}{5}\,bx^{\frac{5}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{2\,x^{3/2}\,(5\,a+3\,b\,x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x),x)
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

3.432 $\int \frac{a+bx}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

```
[Out] 2/3*b*x^(3/2)+2*a*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{\sqrt{x}} \, dx &= \int \left(\frac{a}{\sqrt{x}}+b\sqrt{x}\right) \, dx \\ &= 2a\sqrt{x}+\frac{2}{3}bx^{3/2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{x}\,(3a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/3
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
gosper	$\frac{2\sqrt{x}\,(bx+3a)}{3}$	13
trager	$\left(\frac{2bx}{3}+2a\right)\sqrt{x}$	13
risch	$\frac{2\sqrt{x}\,(bx+3a)}{3}$	13
derivativedivides	$\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}$	14
default	$\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*b*x^(3/2)+2*a*x^(1/2)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}}+2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="maxima")
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Fricas [A]
time = 0.46, size = 12, normalized size = 0.63

$$\frac{2}{3}\,(bx+3\,a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.89

$$2a\sqrt{x}+\frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(3/2)/3
```

Giac [A]
time = 0.92, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.63

$$\frac{2\,\sqrt{x}\,\left(3\,a+b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(3*a + b*x))/3
```

3.433 $\int \frac{a+bx}{x^{3/2}} dx$

Optimal. Leaf size=17

$$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

```
[Out] -2*a/x^(1/2)+2*b*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 0.76

$$-\frac{2(a-bx)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*(a - b*x))/Sqrt[x]
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
trager	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
risch	$-\frac{2(-bx+a)}{\sqrt{x}}$	12
derivativedivides	$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$	14
default	$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*a/x^(1/2)+2*b*x^(1/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$2\,b\sqrt{x}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="maxima")
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

Fricas [A]
time = 0.41, size = 12, normalized size = 0.71

$$\frac{2\,(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="fricas")
[Out] 2*(b*x - a)/sqrt(x)
```

Sympy [A]
time = 0.13, size = 15, normalized size = 0.88

$$-\frac{2a}{\sqrt{x}}+2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*sqrt(x)
```

Giac [A]
time = 0.55, size = 13, normalized size = 0.76

$$2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.65

$$-\frac{2\,(a - b\,x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(3/2),x)
```

```
[Out] -(2*(a - b*x))/x^(1/2)
```

3.434 $\int \frac{a+bx}{x^{5/2}} dx$

Optimal. Leaf size=19

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

```
[Out] -2/3*a/x^(3/2)-2*b/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{3/2}}\right) dx\\ &= -\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/2),x]
```

```
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
trager	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
risch	$-\frac{2(3bx+a)}{3x^{\frac{3}{2}}}$	12
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$	14
default	$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a/x^(3/2)-2*b/x^(1/2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="maxima")
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Fricas [A]
time = 0.43, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Sympy [A]
time = 0.18, size = 19, normalized size = 1.00

$$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) - 2*b/sqrt(x)
```

Giac [A]
time = 0.64, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$-\frac{2\,a+6\,b\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/2),x)
```

```
[Out] -(2*a + 6*b*x)/(3*x^(3/2))
```

3.435 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43
$$\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(2+m)/(2+m)+b^2*x^(3+m)/(3+m)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = $\{45\}$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx&=\int\left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)\,dx\\&=\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.88
$$x^{1+m}\left(\frac{a^2}{1+m}+\frac{2abx}{2+m}+\frac{b^2x^2}{3+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.19

method	result	size
norman	$\frac{a^{2} x\,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{2} x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}+\frac{2 a b\,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}$	51
risch	$\frac{x \left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x+3 m\,x^{2} b^{2}+a^{2} m^{2}+8 m x a b+2 x^{2} b^{2}+5 m\,a^{2}+6 a b x+6 a^{2}\right) x^{m}}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$	86
gosper	$\frac{x^{1+m} \left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x+3 m\,x^{2} b^{2}+a^{2} m^{2}+8 m x a b+2 x^{2} b^{2}+5 m\,a^{2}+6 a b x+6 a^{2}\right)}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(1+m)*x*exp(m*ln(x))+b^2/(3+m)*x^3*exp(m*ln(x))+2*a*b/(2+m)*x^2*exp(m*l
n(x))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 1.00

$$\frac{b^{2} x^{m+3}}{m+3}+\frac{2\,abx^{m+2}}{m+2}+\frac{a^{2} x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.51, size = 85, normalized size = 1.98

$$\frac{((b^{2} m^{2}+3\,b^{2} m+2\,b^{2}) x^{3}+2\,(abm^{2}+4\,abm+3\,ab) x^{2}+(a^{2} m^{2}+5\,a^{2} m+6\,a^{2}) x) x^{m}}{m^{3}+6\,m^{2}+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(36) = 72.
time = 0.17, size = 299, normalized size = 6.95

$$\begin{cases} - \frac{a^{2}}{2 x^{2}} - \frac{2 a b}{x} + b^{2} \log{\left(x \right)} & \text{for}\: m = -3 \\- \frac{a^{2}}{x} + 2 a b \log{\left(x \right)} + b^{2} x & \text{for}\: m = -2 \\a^{2} \log{\left(x \right)} + 2 a b x + \frac{b^{2} x^{2}}{2} & \text{for}\: m = -1 \\\frac{a^{2} m^{2} x x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{5 a^{2} m x x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{6 a^{2} x x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{2 a b m^{2} x^{2} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{8 a b m x^{2} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{6 a b x^{2} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{b^{2} m^{2} x^{3} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{3 b^{2} m x^{3} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{2 b^{2} x^{3} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.
time = 0.61, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^3x^m + 2\,abm^2x^2x^m + 3\,b^2mx^3x^m + a^2m^2xx^m + 8\,abmx^2x^m + 2\,b^2x^3x^m + 5\,a^2mxx^m + 6\,abx^2x^m + 6\,a^2xx^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

Mupad [B]
time = 0.42, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

3.436 $\int x^{5/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

```
[Out] 2/7*a^2*x^(7/2)+4/9*a*b*x^(9/2)+2/11*b^2*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{7/2}+b^2x^{9/2}\right)\,dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.78

$$\frac{2}{693}x^{7/2}\left(99a^2+154abx+63b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*(99*a^2 + 154*a*b*x + 63*b^2*x^2))/693
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}(63x^2b^2+154abx+99a^2)}{693}$	25
derivativedivides	$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{11}{2}}}{11}$	25
default	$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{11}{2}}}{11}$	25
trager	$\frac{2x^{\frac{7}{2}}(63x^2b^2+154abx+99a^2)}{693}$	25
risch	$\frac{2x^{\frac{7}{2}}(63x^2b^2+154abx+99a^2)}{693}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 2/7*a^2*x^(7/2)+4/9*a*b*x^(9/2)+2/11*b^2*x^(11/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Fricas [A]
time = 0.55, size = 29, normalized size = 0.81

$$\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

Sympy [A]
time = 0.28, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(11/2)/11
```

Giac [A]
time = 0.59, size = 24, normalized size = 0.67

$$\frac{2}{11}\,b^2x^{\frac{11}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

Mupad [B]
time = 0.10, size = 24, normalized size = 0.67

$$\frac{2\,x^{7/2}\,(99\,a^2 + 154\,a\,b\,x + 63\,b^2\,x^2)}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(7/2)*(99*a^2 + 63*b^2*x^2 + 154*a*b*x))/693
```

3.437 $\int x^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

```
[Out] 2/5*a^2*x^(5/2)+4/7*a*b*x^(7/2)+2/9*b^2*x^(9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^2\,dx &= \int \left(a^2x^{3/2}+2abx^{5/2}+b^2x^{7/2}\right)\,dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{315}x^{5/2}\left(63a^2+90abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*(63*a^2 + 90*a*b*x + 35*b^2*x^2))/315
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}(35x^2b^2+90abx+63a^2)}{315}$	25
derivativedivides	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
default	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
trager	$\frac{2x^{\frac{5}{2}}(35x^2b^2+90abx+63a^2)}{315}$	25
risch	$\frac{2x^{\frac{5}{2}}(35x^2b^2+90abx+63a^2)}{315}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 2/5*a^2*x^(5/2)+4/7*a*b*x^(7/2)+2/9*b^2*x^(9/2)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Fricas [A]
time = 0.46, size = 29, normalized size = 0.81

$$\frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

Sympy [A]
time = 0.16, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(9/2)/9
```

Giac [A]
time = 0.57, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{5/2}\,\left(63\,a^2+90\,a\,b\,x+35\,b^2\,x^2\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(5/2)*(63*a^2 + 35*b^2*x^2 + 90*a*b*x))/315
```

3.438 $\int \sqrt{x} (a+bx)^2 dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

```
[Out] 2/3*a^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*b^2*x^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{3/2}+b^2x^{5/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{105}x^{3/2}\left(35a^2+42abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*x^(3/2)*(35*a^2 + 42*a*b*x + 15*b^2*x^2))/105
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}(15x^2b^2+42abx+35a^2)}{105}$	25
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{7}{2}}}{7}$	25
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{7}{2}}}{7}$	25
trager	$\frac{2x^{\frac{3}{2}}(15x^2b^2+42abx+35a^2)}{105}$	25
risch	$\frac{2x^{\frac{3}{2}}(15x^2b^2+42abx+35a^2)}{105}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a^2*x^(3/2)+4/5*a*b*x^(5/2)+2/7*b^2*x^(7/2)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Fricas [A]
time = 0.42, size = 27, normalized size = 0.75

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

Sympy [C] Result contains complex when optimal does not.
time = 63.51, size = 1851, normalized size = 51.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*x**(1/2),x)

[Out] Piecewise((16*a**(23/2)*sqrt(-1 + b*(a/b + x)/a)/(-105*a**8*b**(3/2) + 315*
a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2
)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a
/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3)
- 40*a**(21/2)*b*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)/(-105*a**8*b**(3/2) + 3
15*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(
9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**8*b**(3/2) + 315*a
**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)
*(a/b + x)**3) + 30*a**(19/2)*b**2*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**2/(-
105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b +
x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)*b**2*(a/b + x)**2/
(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b
+ x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*a**(17/2)*b**3*sqrt(-1 + b*(
a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x)
- 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 16*I*a
**(17/2)*b**3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x
) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 100*
a**(15/2)*b**4*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)**4/(-105*a**8*b**(3/2) +
315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**
(9/2)*(a/b + x)**3) - 96*a**(13/2)*b**5*sqrt(-1 + b*(a/b + x)/a)*(a/b + x)*
*5/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a
/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 30*a**(11/2)*b**6*sqrt(-1 +
b*(a/b + x)/a)*(a/b + x)**6/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b +
x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3), Abs(
b*(a/b + x)/a) > 1), (16*I*a**(23/2)*sqrt(1 - b*(a/b + x)/a)/(-105*a**8*b**
(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*
a**5*b**(9/2)*(a/b + x)**3) - 16*I*a**(23/2)/(-105*a**8*b**(3/2) + 315*a**7
*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a
/b + x)**3) - 40*I*a**(21/2)*b*sqrt(1 - b*(a/b + x)/a)*(a/b + x)/(-105*a**8
*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 +
105*a**5*b**(9/2)*(a/b + x)**3) + 48*I*a**(21/2)*b*(a/b + x)/(-105*a**8*b**
(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*
a**5*b**(9/2)*(a/b + x)**3) + 30*I*a**(19/2)*b**2*sqrt(1 - b*(a/b + x)/a)*(
a/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b*
*(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 48*I*a**(19/2)*b**2
*(a/b + x)**2/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*
b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 40*I*a**(17/2)*b*
*3*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b**(
5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b +
x)**3) + 16*I*a**(17/2)*b**3*(a/b + x)**3/(-105*a**8*b**(3/2) + 315*a**7*b*
*(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b
+ x)**3) + 100*I*a**(15/2)*b**4*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**4/(-105*
a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**
```

```
2 + 105*a**5*b**(9/2)*(a/b + x)**3) - 96*I*a**(13/2)*b**5*sqrt(1 - b*(a/b +
 x)/a)*(a/b + x)**5/(-105*a**8*b**(3/2) + 315*a**7*b**(5/2)*(a/b + x) - 315
*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(a/b + x)**3) + 30*I*a**(11
/2)*b**6*sqrt(1 - b*(a/b + x)/a)*(a/b + x)**6/(-105*a**8*b**(3/2) + 315*a**
7*b**(5/2)*(a/b + x) - 315*a**6*b**(7/2)*(a/b + x)**2 + 105*a**5*b**(9/2)*(
a/b + x)**3), True))
```

Giac [A]
time = 0.52, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^{2}x^{\frac{7}{2}}+\frac{4}{5}\,abx^{\frac{5}{2}}+\frac{2}{3}\,a^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="giac")
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{3/2}\,\left(35\,a^2+42\,a\,b\,x+15\,b^2\,x^2\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^2,x)
[Out] (2*x^(3/2)*(35*a^2 + 15*b^2*x^2 + 42*a*b*x))/105
```

3.439 $\int \frac{(a+bx)^2}{\sqrt{x}} dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}$$

```
[Out] 4/3*a*b*x^(3/2)+2/5*b^2*x^(5/2)+2*a^2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{x}} dx &= \int \left(\frac{a^2}{\sqrt{x}} + 2ab\sqrt{x} + b^2x^{3/2}\right) dx \\ &= 2a^2\sqrt{x} + \frac{4}{3}abx^{3/2} + \frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$\frac{2}{15}\sqrt{x}\left(15a^2 + 10abx + 3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 10*a*b*x + 3*b^2*x^2))/15
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.74

method	result	size
trager	$\left(\frac{2}{5}x^2b^2+\frac{4}{3}abx+2a^2\right)\sqrt{x}$	24
gosper	$\frac{2\sqrt{x}\,(3x^2b^2+10abx+15a^2)}{15}$	25
derivativedivides	$\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}+2a^2\sqrt{x}$	25
default	$\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}+2a^2\sqrt{x}$	25
risch	$\frac{2\sqrt{x}\,(3x^2b^2+10abx+15a^2)}{15}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/3*a*b*x^(3/2)+2/5*b^2*x^(5/2)+2*a^2*x^(1/2)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Fricas [A]
time = 0.46, size = 24, normalized size = 0.71

$$\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

Sympy [A]
time = 0.07, size = 32, normalized size = 0.94

$$2a^2\sqrt{x}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(5/2)/5
```

Giac [A]
time = 0.62, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}+2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.71

$$\frac{2\,\sqrt{x}\,\left(15\,a^2+10\,a\,b\,x+3\,b^2\,x^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/2),x)
```

```
[Out] (2*x^(1/2)*(15*a^2 + 3*b^2*x^2 + 10*a*b*x))/15
```

3.440 $\int \frac{(a+bx)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

```
[Out] 2/3*b^2*x^(3/2)-2*a^2/x^(1/2)+4*a*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{3/2}}\,dx&=\int \left(\frac{a^2}{x^{3/2}}+\frac{2ab}{\sqrt{x}}+b^2\sqrt{x}\right)dx\\&=-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.88

$$-\frac{2(3a^2-6abx-b^2x^2)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(3/2),x]
[Out] (-2*(3*a^2 - 6*a*b*x - b^2*x^2))/(3*Sqrt[x])
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.78

method	result	size
gosper	$-\frac{2(-x^2b^2-6abx+3a^2)}{3\sqrt{x}}$	25
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3}-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}$	25
default	$\frac{2b^2x^{\frac{3}{2}}}{3}-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}$	25
trager	$-\frac{2(-x^2b^2-6abx+3a^2)}{3\sqrt{x}}$	25
risch	$-\frac{2(-x^2b^2-6abx+3a^2)}{3\sqrt{x}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*b^2*x^(3/2)-2*a^2/x^(1/2)+4*a*b*x^(1/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^2x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="maxima")
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Fricas [A]
time = 0.47, size = 23, normalized size = 0.72

$$\frac{2\left(b^2x^2+6\,abx-3\,a^2\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

Sympy [A]
time = 0.15, size = 31, normalized size = 0.97

$$-\frac{2a^2}{\sqrt{x}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(3/2),x)
[Out] -2*a**2/sqrt(x) + 4*a*b*sqrt(x) + 2*b**2*x**(3/2)/3
```

Giac [A]
time = 0.54, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^2x^{\frac{3}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="giac")
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.75

$$\frac{-6\,a^2 + 12\,a\,b\,x + 2\,b^2\,x^2}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(3/2),x)
[Out] (2*b^2*x^2 - 6*a^2 + 12*a*b*x)/(3*x^(1/2))
```

3.441 $\int \frac{(a+bx)^2}{x^{5/2}} dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

```
[Out] -2/3*a^2/x^(3/2)-4*a*b/x^(1/2)+2*b^2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/2}}\,dx &= \int \left(\frac{a^2}{x^{5/2}} + \frac{2ab}{x^{3/2}} + \frac{b^2}{\sqrt{x}}\right) dx \\ &= -\frac{2a^2}{3x^{3/2}} - \frac{4ab}{\sqrt{x}} + 2b^2\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 0.81

$$-\frac{2(a^2 + 6abx - 3b^2x^2)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/2),x]
```

```
[Out] (-2*(a^2 + 6*a*b*x - 3*b^2*x^2))/(3*x^(3/2))
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.78

method	result	size
gosper	$-\frac{2(-3x^2b^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
trager	$-\frac{2(-3x^2b^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
risch	$-\frac{2(-3x^2b^2+6abx+a^2)}{3x^{\frac{3}{2}}}$	23
derivativedivides	$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$	25
default	$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a^2/x^(3/2)-4*a*b/x^(1/2)+2*b^2*x^(1/2)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.72

$$2\,b^2\sqrt{x}-\frac{2\,(6\,abx+a^2)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Fricas [A]
time = 0.44, size = 24, normalized size = 0.75

$$\frac{2\,(3\,b^2x^2-6\,abx-a^2)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

Sympy [A]
time = 0.22, size = 31, normalized size = 0.97

$$-\frac{2a^2}{3x^{\frac{3}{2}}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) - 4*a*b/sqrt(x) + 2*b**2*sqrt(x)
```

Giac [A]
time = 0.55, size = 23, normalized size = 0.72

$$2\,b^2\sqrt{x} - \frac{2\,(6\,abx + a^2)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.75

$$-\frac{2\,a^2 + 12\,a\,b\,x - 6\,b^2\,x^2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/2),x)
```

```
[Out] -(2*a^2 - 6*b^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

3.442 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(2+m)/(2+m)+3*a*b^2*x^(3+m)/(3+m)+b^3*x^(4+m)/(
4+m)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx&=\int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)\,dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 0.89

$$x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx}{2+m}+\frac{3ab^2x^2}{3+m}+\frac{b^3x^3}{4+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

Maple [A]
time = 0.09, size = 72, normalized size = 1.18

method	result
norman	$\frac{a^3 x\, e^{m \ln(x)}}{1+m}+\frac{b^3 x^4 e^{m \ln(x)}}{4+m}+\frac{3a\, b^2 x^3 e^{m \ln(x)}}{3+m}+\frac{3a^2 b\, x^2 e^{m \ln(x)}}{2+m}$
risch	$\frac{x\left(b^3 m^3 x^3+3a\, b^2 m^3 x^2+6b^3 m^2 x^3+3a^2 b\, m^3 x+21a\, b^2 m^2 x^2+11m\, x^3 b^3+a^3 m^3+24a^2 b\, m^2 x+42m\, x^2 a\, b^2+6b^3 x^3+9a^3 m^2+57mx\, a^2 b+2\right.}{(4+m)(3+m)(2+m)(1+m)}$
gosper	$\frac{x^{1+m}\left(b^3 m^3 x^3+3a\, b^2 m^3 x^2+6b^3 m^2 x^3+3a^2 b\, m^3 x+21a\, b^2 m^2 x^2+11m\, x^3 b^3+a^3 m^3+24a^2 b\, m^2 x+42m\, x^2 a\, b^2+6b^3 x^3+9a^3 m^2+57mx\, a^2\right.}{(4+m)(3+m)(2+m)(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/(1+m)*x*exp(m*ln(x))+b^3/(4+m)*x^4*exp(m*ln(x))+3*a*b^2/(3+m)*x^3*exp(m
*ln(x))+3*a^2*b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.31, size = 61, normalized size = 1.00

$$\frac{b^3 x^{m+4}}{m+4}+\frac{3\, ab^2 x^{m+3}}{m+3}+\frac{3\, a^2 b x^{m+2}}{m+2}+\frac{a^3 x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.
time = 0.51, size = 157, normalized size = 2.57

$$\frac{((b^3 m^3+6\, b^3 m^2+11\, b^3 m+6\, b^3)x^4+3\,(ab^2 m^3+7\, ab^2 m^2+14\, ab^2 m+8\, ab^2)x^3+3\,(a^2 bm^3+8\, a^2 bm^2+19\, a^2 bm+12\, a^2 b)x^2+(a^3 m^3+9\, a^3 m^2+26\, a^3 m+24\, a^3)x)x^m}{m^4+10\, m^3+35\, m^2+50\, m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
```

```
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(53) = 106$.
time = 0.24, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x) & \text{for } m = -4 \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x & \text{for } m = -3 \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} & \text{for } m = -2 \\ a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3} & \text{for } m = -1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(61) = 122$.
time = 0.58, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^4x^m + 3\,ab^2m^3x^3x^m + 6\,b^3m^2x^4x^m + 3\,a^2bm^3x^2x^m + 21\,ab^2m^2x^3x^m + 11\,b^3mx^4x^m + a^3m^3xx^m + 24\,a^2bm^2x^2x^m + 42\,ab^2mx^3x^m + 6\,b^3x^4x^m + 9\,a^3m^2xx^m + 57\,a^2bmx^2x^m + 24\,ab^2x^3x^m + 26\,a^3mxx^m + 36\,a^2bx^2x^m + 24\,a^3xx^m}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B]
time = 0.39, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

3.443 $\int x^{5/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

```
[Out] 2/7*a^3*x^(7/2)+2/3*a^2*b*x^(9/2)+6/11*a*b^2*x^(11/2)+2/13*b^3*x^(13/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(13/2))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^3\,dx&=\int\left(a^3x^{5/2}+3a^2bx^{7/2}+3ab^2x^{9/2}+b^3x^{11/2}\right)\,dx\\&=\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{2x^{7/2}\left(429a^3+1001a^2bx+819ab^2x^2+231b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^3,x]
[Out] (2*x^(7/2)*(429*a^3 + 1001*a^2*b*x + 819*a*b^2*x^2 + 231*b^3*x^3))/3003
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36
derivativedivides	$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{2a^{2}b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{13}{2}}}{13}$	36
default	$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{2a^{2}b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{13}{2}}}{13}$	36
trager	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36
risch	$\frac{2x^{\frac{7}{2}}\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)}{3003}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 2/7*a^3*x^(7/2)+2/3*a^2*b*x^(9/2)+6/11*a*b^2*x^(11/2)+2/13*b^3*x^(13/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^{3}x^{\frac{13}{2}}+\frac{6}{11}\,ab^{2}x^{\frac{11}{2}}+\frac{2}{3}\,a^{2}bx^{\frac{9}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="maxima")
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

Fricas [A]
time = 0.48, size = 40, normalized size = 0.78

$$\frac{2}{3003}\left(231\,b^{3}x^{6}+819\,ab^{2}x^{5}+1001\,a^{2}bx^{4}+429\,a^{3}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="fricas")
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

Sympy [A]
time = 0.37, size = 49, normalized size = 0.96

$$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{2a^{2}bx^{\frac{9}{2}}}{3}+\frac{6ab^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(13/2)/13
```

Giac [A]
time = 0.56, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^3x^{\frac{13}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,b^3\,x^{13/2}}{13}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,a\,b^2\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(13/2))/13 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(11/2))/11
```

3.444 $\int x^{3/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

```
[Out] 2/5*a^3*x^(5/2)+6/7*a^2*b*x^(7/2)+2/3*a*b^2*x^(9/2)+2/11*b^3*x^(11/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(1
1/2))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^3\,dx &= \int \left(a^3x^{3/2}+3a^2bx^{5/2}+3ab^2x^{7/2}+b^3x^{9/2}\right)\,dx\\ &= \frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{2x^{5/2}(231a^3+495a^2bx+385ab^2x^2+105b^3x^3)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^3,x]
[Out] (2*x^(5/2)*(231*a^3 + 495*a^2*b*x + 385*a*b^2*x^2 + 105*b^3*x^3))/1155
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)}{1155}$	36
derivativedivides	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2b\,x^{\frac{7}{2}}}{7}+\frac{2a\,b^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
default	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2b\,x^{\frac{7}{2}}}{7}+\frac{2a\,b^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
trager	$\frac{2x^{\frac{5}{2}}\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)}{1155}$	36
risch	$\frac{2x^{\frac{5}{2}}\left(105b^3x^3+385a\,b^2x^2+495a^2bx+231a^3\right)}{1155}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 2/5*a^3*x^(5/2)+6/7*a^2*b*x^(7/2)+2/3*a*b^2*x^(9/2)+2/11*b^3*x^(11/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="maxima")
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

Fricas [A]
time = 0.55, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(105\,b^3x^5+385\,ab^2x^4+495\,a^2bx^3+231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="fricas")
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

Sympy [A]
time = 0.23, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{2ab^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 6*a**2*b*x**(7/2)/7 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(
11/2)/11
```

Giac [A]
time = 0.56, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^3x^{\frac{11}{2}}+\frac{2}{3}\,ab^2x^{\frac{9}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{11/2}}{11}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{2\,a\,b^2\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(11/2))/11 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^
(9/2))/3
```

3.445 $\int \sqrt{x}\,(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

```
[Out] 2/3*a^3*x^(3/2)+6/5*a^2*b*x^(5/2)+6/7*a*b^2*x^(7/2)+2/9*b^3*x^(9/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(9
/2))/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^3\,dx &= \int \left(a^3\sqrt{x}+3a^2bx^{3/2}+3ab^2x^{5/2}+b^3x^{7/2}\right)\,dx\\ &= \frac{2}{3}a^3x^{3/2}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{9}b^3x^{9/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{2}{315}x^{3/2}\left(105a^3+189a^2bx+135ab^2x^2+35b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^3,x]
[Out] (2*x^(3/2)*(105*a^3 + 189*a^2*b*x + 135*a*b^2*x^2 + 35*b^3*x^3))/315
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
derivativedivides	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
default	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{5}{2}}}{5}+\frac{6a\,b^{2}x^{\frac{7}{2}}}{7}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
trager	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36
risch	$\frac{2x^{\frac{3}{2}}\left(35b^{3}x^{3}+135a\,b^{2}x^{2}+189a^{2}bx+105a^{3}\right)}{315}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*a^3*x^(3/2)+6/5*a^2*b*x^(5/2)+6/7*a*b^2*x^(7/2)+2/9*b^3*x^(9/2)
```

Maxima [A]
time = 0.38, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^{3}x^{\frac{9}{2}}+\frac{6}{7}\,ab^{2}x^{\frac{7}{2}}+\frac{6}{5}\,a^{2}bx^{\frac{5}{2}}+\frac{2}{3}\,a^{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="maxima")
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Fricas [A]
time = 0.59, size = 38, normalized size = 0.75

$$\frac{2}{315}\left(35\,b^{3}x^{4}+135\,ab^{2}x^{3}+189\,a^{2}bx^{2}+105\,a^{3}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="fricas")
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*x**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{9/2}}{9}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{6\,a\,b^2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(9/2))/9 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7
/2))/7
```

3.446 $\int \frac{(a+bx)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=47

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

```
[Out] 2*a^2*b*x^(3/2)+6/5*a*b^2*x^(5/2)+2/7*b^3*x^(7/2)+2*a^3*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2b\sqrt{x}+3ab^2x^{3/2}+b^3x^{5/2}\right)dx\\ &= 2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.83

$$\frac{2}{35}\sqrt{x}\left(35a^3+35a^2bx+21ab^2x^2+5b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[x],x]
[Out] (2*Sqrt[x]*(35*a^3 + 35*a^2*b*x + 21*a*b^2*x^2 + 5*b^3*x^3))/35
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.77

method	result	size
trager	$\left(\frac{2}{7}b^{3}x^{3}+\frac{6}{5}a\,b^{2}x^{2}+2a^{2}bx+2a^{3}\right)\sqrt{x}$	35
gosper	$\frac{2\sqrt{x}\,\left(5b^{3}x^{3}+21a\,b^{2}x^{2}+35a^{2}bx+35a^{3}\right)}{35}$	36
derivativedivides	$2a^{2}b\,x^{\frac{3}{2}}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{7}{2}}}{7}+2a^{3}\sqrt{x}$	36
default	$2a^{2}b\,x^{\frac{3}{2}}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{7}{2}}}{7}+2a^{3}\sqrt{x}$	36
risch	$\frac{2\sqrt{x}\,\left(5b^{3}x^{3}+21a\,b^{2}x^{2}+35a^{2}bx+35a^{3}\right)}{35}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a^2*b*x^(3/2)+6/5*a*b^2*x^(5/2)+2/7*b^3*x^(7/2)+2*a^3*x^(1/2)
```

Maxima [A]
time = 0.57, size = 35, normalized size = 0.74

$$\frac{2}{7}\,b^{3}x^{\frac{7}{2}}+\frac{6}{5}\,ab^{2}x^{\frac{5}{2}}+2\,a^{2}bx^{\frac{3}{2}}+2\,a^{3}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="maxima")
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Fricas [A]
time = 1.16, size = 35, normalized size = 0.74

$$\frac{2}{35}\left(5\,b^{3}x^{3}+21\,ab^{2}x^{2}+35\,a^{2}bx+35\,a^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

Sympy [A]
time = 0.10, size = 46, normalized size = 0.98

$$2a^{3}\sqrt{x}+2a^{2}bx^{\frac{3}{2}}+\frac{6ab^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(7/2)/
7
```

Giac [A]
time = 0.51, size = 35, normalized size = 0.74

$$\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+2\,a^2bx^{\frac{3}{2}}+2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.74

$$2\,a^3\,\sqrt{x}+\frac{2\,b^3\,x^{7/2}}{7}+2\,a^2\,b\,x^{3/2}+\frac{6\,a\,b^2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/2),x)
```

```
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(7/2))/7 + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5
```

3.447 $\int \frac{(a+bx)^3}{x^{3/2}} \, dx$

Optimal. Leaf size=45

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

```
[Out] 2*a*b^2*x^(3/2)+2/5*b^3*x^(5/2)-2*a^3/x^(1/2)+6*a^2*b*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{3/2}} \, dx &= \int \left(\frac{a^3}{x^{3/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2\sqrt{x}+b^3x^{3/2}\right) dx\\ &= -\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.87

$$-\frac{2(5a^3-15a^2bx-5ab^2x^2-b^3x^3)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(3/2),x]
[Out] (-2*(5*a^3 - 15*a^2*b*x - 5*a*b^2*x^2 - b^3*x^3))/(5*Sqrt[x])
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.80

method	result	size
gosper	$-\frac{2\left(-b^{3} x^{3}-5 a\,b^{2} x^{2}-15 a^{2} b x+5 a^{3}\right)}{5 \sqrt{x}}$	36
derivativedivides	$2 a\,b^{2} x^{\frac{3}{2}}+\frac{2 b^{3} x^{\frac{5}{2}}}{5}-\frac{2 a^{3}}{\sqrt{x}}+6 a^{2} b \sqrt{x}$	36
default	$2 a\,b^{2} x^{\frac{3}{2}}+\frac{2 b^{3} x^{\frac{5}{2}}}{5}-\frac{2 a^{3}}{\sqrt{x}}+6 a^{2} b \sqrt{x}$	36
trager	$-\frac{2\left(-b^{3} x^{3}-5 a\,b^{2} x^{2}-15 a^{2} b x+5 a^{3}\right)}{5 \sqrt{x}}$	36
risch	$-\frac{2\left(-b^{3} x^{3}-5 a\,b^{2} x^{2}-15 a^{2} b x+5 a^{3}\right)}{5 \sqrt{x}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*a*b^2*x^(3/2)+2/5*b^3*x^(5/2)-2*a^3/x^(1/2)+6*a^2*b*x^(1/2)
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^{3}x^{\frac{5}{2}}+2\,ab^{2}x^{\frac{3}{2}}+6\,a^{2}b\sqrt{x}-\frac{2\,a^{3}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="maxima")
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Fricas [A]
time = 0.69, size = 34, normalized size = 0.76

$$\frac{2\left(b^{3}x^{3}+5\,ab^{2}x^{2}+15\,a^{2}bx-5\,a^{3}\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

Sympy [A]
time = 0.19, size = 44, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + 6a^2b\sqrt{x} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(3/2),x)
[Out] -2*a**3/sqrt(x) + 6*a**2*b*sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(5/2)/5
```

Giac [A]
time = 0.56, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="giac")
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.78

$$\frac{2\,b^3\,x^{5/2}}{5} - \frac{2\,a^3}{\sqrt{x}} + 6\,a^2\,b\,\sqrt{x} + 2\,a\,b^2\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(3/2),x)
[Out] (2*b^3*x^(5/2))/5 - (2*a^3)/x^(1/2) + 6*a^2*b*x^(1/2) + 2*a*b^2*x^(3/2)
```

3.448 $\int \frac{(a+bx)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

```
[Out] -2/3*a^3/x^(3/2)+2/3*b^3*x^(3/2)-6*a^2*b/x^(1/2)+6*a*b^2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2)
)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/2}}\,dx &= \int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+b^3\sqrt{x}\right)dx\\ &= -\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.81

$$\frac{2(-a^3-9a^2bx+9ab^2x^2+b^3x^3)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/2),x]
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.77

method	result	size
gosper	$-\frac{2(-b^3x^3-9a\,b^2x^2+9a^2bx+a^3)}{3x^{\frac{3}{2}}}$	34
trager	$-\frac{2(-b^3x^3-9a\,b^2x^2+9a^2bx+a^3)}{3x^{\frac{3}{2}}}$	34
risch	$-\frac{2(-b^3x^3-9a\,b^2x^2+9a^2bx+a^3)}{3x^{\frac{3}{2}}}$	34
derivativedivides	$-\frac{2a^3}{3x^{\frac{3}{2}}}+\frac{2b^3x^{\frac{3}{2}}}{3}-\frac{6a^2b}{\sqrt{x}}+6a\,b^2\sqrt{x}$	36
default	$-\frac{2a^3}{3x^{\frac{3}{2}}}+\frac{2b^3x^{\frac{3}{2}}}{3}-\frac{6a^2b}{\sqrt{x}}+6a\,b^2\sqrt{x}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a^3/x^(3/2)+2/3*b^3*x^(3/2)-6*a^2*b/x^(1/2)+6*a*b^2*x^(1/2)
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}}+6\,ab^2\sqrt{x}-\frac{2\,(9\,a^2bx+a^3)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="maxima")
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Fricas [A]
time = 0.61, size = 34, normalized size = 0.72

$$\frac{2\,(b^3x^3+9\,ab^2x^2-9\,a^2bx-a^3)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

Sympy [A]
time = 0.22, size = 46, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} - \frac{6a^2b}{\sqrt{x}} + 6ab^2\sqrt{x} + \frac{2b^3x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) - 6*a**2*b/sqrt(x) + 6*a*b**2*sqrt(x) + 2*b**3*x**(3/2
)/3
```

Giac [A]
time = 0.53, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\,(9\,a^2bx + a^3)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.74

$$-\frac{2\,a^3 + 18\,a^2\,b\,x - 18\,a\,b^2\,x^2 - 2\,b^3\,x^3}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/2),x)
```

```
[Out] -(2*a^3 - 2*b^3*x^3 - 18*a*b^2*x^2 + 18*a^2*b*x)/(3*x^(3/2))
```

3.449 $\int \frac{x^{5/2}}{a+bx} \, dx$

Optimal. Leaf size=68

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

[Out] -2/3*a*x^(3/2)/b^2+2/5*x^(5/2)/b-2*a^(5/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(7/2)+2*a^2*x^(1/2)/b^3

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 211}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[x^(5/2)/(a + b*x),x]

[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{a+bx}\,dx &= \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx}\,dx}{b}\\
&= -\frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{a+bx}\,dx}{b^2}\\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{a^3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.90

$$
\frac{2\sqrt{x}\,(15a^2-5abx+3b^2x^2)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x),x]
[Out] (2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.79

method	result	size
risch	$\frac{2(3x^2b^2-5abx+15a^2)\sqrt{x}}{15b^3} - \frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5} - \frac{2abx^{\frac{3}{2}}}{3} + 2a^2\sqrt{x}}{b^3} - \frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5} - \frac{2abx^{\frac{3}{2}}}{3} + 2a^2\sqrt{x}}{b^3} - \frac{2a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)-1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 54, normalized size = 0.79

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}-5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Fricas [A]
time = 0.65, size = 132, normalized size = 1.94

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}}{15\,b^3},\,-\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(3\,b^2x^2-5\,abx+15\,a^2)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [A]
time = 2.84, size = 122, normalized size = 1.79

$$\begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for}\: a=0\wedge b=0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for}\: a=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for}\: b=0 \\ -\frac{a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}}+\frac{a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^4\sqrt{-\frac{a}{b}}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*b), Eq(a, 0))
, (2*x**(7/2)/(7*a), Eq(b, 0)), (-a**3*log(sqrt(x) - sqrt(-a/b))/(b**4*sqrt
(-a/b)) + a**3*log(sqrt(x) + sqrt(-a/b))/(b**4*sqrt(-a/b)) + 2*a**2*sqrt(x)
/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Giac [A]
time = 0.60, size = 59, normalized size = 0.87

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^4x^{\frac{5}{2}}-5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B]
time = 0.06, size = 48, normalized size = 0.71

$$\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,x^{3/2}}{3\,b^2}+\frac{2\,a^2\,\sqrt{x}}{b^3}-\frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 - (2*a^(5/2
)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

3.450 $\int \frac{x^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=53

$$-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b+2*a^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)-2*a*x^(1/2)
/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 211}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{a+bx}\,dx &= \frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{a+bx}\,dx}{b}\\
&= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{a^2\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{3/2}}{3b} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.92

$$
\frac{2\sqrt{x}\,(-3a+bx)}{3b^2} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x),x]
[Out] (2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Maple [A]
time = 0.10, size = 43, normalized size = 0.81

method	result	size
risch	$-\frac{2(-bx+3a)\sqrt{x}}{3b^2} + \frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	42
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2} + \frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+a\sqrt{x}\right)}{b^2} + \frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2*(-1/3*b*x^(3/2)+a*x^(1/2))+2*a^2/b^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a
*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 42, normalized size = 0.79

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

Fricas [A]
time = 0.72, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(49) = 98$.
time = 0.69, size = 107, normalized size = 2.02

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\ a=0\wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for}\ a=0 \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for}\ b=0 \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}}-\frac{a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^3\sqrt{-\frac{a}{b}}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0))
, (2*x**(5/2)/(5*a), Eq(b, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(b**3*sqrt(
-a/b)) - a**2*log(sqrt(x) + sqrt(-a/b))/(b**3*sqrt(-a/b)) - 2*a*sqrt(x)/b**
2 + 2*x**(3/2)/(3*b), True))
```

Giac [A]
time = 0.60, size = 45, normalized size = 0.85

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(b^2x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b}-\frac{2\,a\,\sqrt{x}}{b^2}+\frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) - (2*a*x^(1/2))/b^2 + (2*a^(3/2)*atan((b^(1/2)*x^(1/2))/a
^(1/2)))/b^(5/2)
```

3.451 $\int \frac{\sqrt{x}}{a+bx}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)+2*x^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 65, 211}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{a+bx}\,dx = \frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b}-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Fricas [A]
time = 0.51, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,\sqrt{x}}{b},\,-\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(36) = 72.
time = 0.32, size = 88, normalized size = 2.20

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}}+\frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{b^2\sqrt{-\frac{a}{b}}}+\frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)),
 (2*sqrt(x)/b, Eq(a, 0)), (-a*log(sqrt(x) - sqrt(-a/b))/(b**2*sqrt(-a/b)) +
 a*log(sqrt(x) + sqrt(-a/b))/(b**2*sqrt(-a/b)) + 2*sqrt(x)/b, True))
```

Giac [**A**]
time = 0.52, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="giac")
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

Mupad [**B**]
time = 0.04, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{x}}{b}-\frac{2\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x),x)
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

3.452 $\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] 2*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx &= 2\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.10, size = 19, normalized size = 0.66

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 18, normalized size = 0.62

$$\frac{2\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="maxima")
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Fricas [A]
time = 0.70, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.
time = 0.41, size = 73, normalized size = 2.52

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a, Eq(b, 0)), (-2/
(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - sqrt(-a/b))/(b*sqrt(-a/b)) - log(sqr
t(x) + sqrt(-a/b))/(b*sqrt(-a/b)), True))
```

Giac [A]
time = 0.57, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

Mupad [B]
time = 0.04, size = 19, normalized size = 0.66

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)),x)
```

```
[Out] (2*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.453 $\int \frac{1}{x^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=40

$$-\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)-2/a/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 211}

$$-\frac{2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)}\,dx &= -\frac{2}{a\sqrt{x}}-\frac{b\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a}\\ &= -\frac{2}{a\sqrt{x}}-\frac{(2b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a}\\ &= -\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$-\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)),x]
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{2}{a\sqrt{x}}$	32
default	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{2}{a\sqrt{x}}$	32
risch	$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{2}{a\sqrt{x}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -2*b/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))-2/a/x^(1/2)
```

Maxima [A]
time = 0.49, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Fricas [A]
time = 0.48, size = 93, normalized size = 2.32

$$\left[\frac{x\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,\sqrt{x}}{ax},\ \frac{2\left(x\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(37) = 74$.
time = 0.97, size = 85, normalized size = 2.12

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a=0\wedge b=0\\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\ a=0\\ -\frac{2}{a\sqrt{x}} & \text{for}\ b=0\\ -\frac{\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{a\sqrt{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0)
), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - sqrt(-a/b))/(a*sqrt(-a/b)) +
 log(sqrt(x) + sqrt(-a/b))/(a*sqrt(-a/b)) - 2/(a*sqrt(x)), True))
```

Giac [A]
time = 0.57, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="giac")
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

Mupad [B]
time = 0.04, size = 28, normalized size = 0.70

$$-\frac{2}{a\,\sqrt{x}}-\frac{2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)),x)
[Out] - 2/(a*x^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

3.454 $\int \frac{1}{x^{5/2}(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -2/3/a/x^(3/2)+2*b^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)+2*b/a^2/x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {53, 65, 211}

$$\frac{2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)}\,dx &= -\frac{2}{3ax^{3/2}} - \frac{b\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a} \\
&= -\frac{2}{3ax^{3/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{b^2\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^2} \\
&= -\frac{2}{3ax^{3/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^2} \\
&= -\frac{2}{3ax^{3/2}} + \frac{2b}{a^2\sqrt{x}} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.91

$$-\frac{2(a-3bx)}{3a^2x^{3/2}} + \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] (-2*(a - 3*b*x))/(3*a^2*x^(3/2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

Maple [A]
time = 0.11, size = 43, normalized size = 0.81

method	result	size
risch	$-\frac{2(-3bx+a)}{3a^2x^{\frac{3}{2}}} + \frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	40
derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}} + \frac{2b}{a^2\sqrt{x}} + \frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	43

default	$-\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^{2}\sqrt{x}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)+2*b/a^2/x^(1/2)+2*b^2/a^2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)
^(1/2))
```

Maxima [A]
time = 0.49, size = 41, normalized size = 0.77

$$\frac{2\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}+\frac{2\,(3\,bx-a)}{3\,a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

Fricas [A]
time = 0.51, size = 118, normalized size = 2.23

$$\left[\frac{3\,bx^{2}\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^{2}x^{2}},\,-\frac{2\left(3\,bx^{2}\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^{2}x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(49) = 98$.

time = 3.52, size = 107, normalized size = 2.02

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{2}{3ax^{\frac{3}{2}}} + \frac{b\log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}} - \frac{b\log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{a^2\sqrt{-\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)
), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + b*log(sqrt(x) - sqrt
(-a/b))/(a**2*sqrt(-a/b)) - b*log(sqrt(x) + sqrt(-a/b))/(a**2*sqrt(-a/b)) +
 2*b/(a**2*sqrt(x)), True))
```

Giac [A]
time = 0.74, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} + \frac{2\,(3\,bx - a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

Mupad [B]
time = 0.10, size = 38, normalized size = 0.72

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2}{3\,a} - \frac{2\,b\,x}{a^2}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/(3*a) - (2*b*x)/a^
2)/x^(3/2)
```

3.455 $\int \frac{1}{x^{7/2}(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)-2*b^(5/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/
a^(7/2)-2*b^2/a^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {53, 65, 211}

$$-\frac{2b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) - (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)}\,dx &= -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{5/2}(a+bx)}\,dx}{a} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} + \frac{b^2\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a^2} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{b^3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{(2b^3)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.90

$$-\frac{2(3a^2-5abx+15b^2x^2)}{15a^3x^{5/2}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] (-2*(3*a^2 - 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.11, size = 54, normalized size = 0.79

method	result	size
risch	$-\frac{2(15x^2b^2-5abx+3a^2)}{15a^3x^{\frac{5}{2}}} - \frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	53
derivativedivides	$-\frac{2b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}} - \frac{2}{5a\,x^{\frac{5}{2}}} - \frac{2b^2}{a^3\sqrt{x}} + \frac{2b}{3a^2x^{\frac{3}{2}}}$	54

default	$-\frac{2b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}-\frac{2}{5a\,x^{\frac{5}{2}}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^3/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))-2/5/a/x^(5/2)-2*b^2/a^
3/x^(1/2)+2/3*b/a^2/x^(3/2)
```

Maxima [A]
time = 0.50, size = 52, normalized size = 0.76

$$-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^3}-\frac{2\,(15\,b^2x^2-5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

Fricas [A]
time = 0.61, size = 144, normalized size = 2.12

$$\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\;\frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^2x^2-5\,abx+3\,a^2)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [A]

time = 14.78, size = 126, normalized size = 1.85

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} - \frac{b^2 \log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}} + \frac{b^2 \log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{a^3\sqrt{-\frac{a}{b}}} - \frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2)
) - b**2*log(sqrt(x) - sqrt(-a/b))/(a**3*sqrt(-a/b)) + b**2*log(sqrt(x) + s
qrt(-a/b))/(a**3*sqrt(-a/b)) - 2*b**2/(a**3*sqrt(x)), True))
```

Giac [A]
time = 0.66, size = 52, normalized size = 0.76

$$-\frac{2\,b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} - \frac{2\,(15\,b^2x^2 - 5\,abx + 3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B]
time = 0.11, size = 49, normalized size = 0.72

$$-\frac{\frac{2}{5\,a} + \frac{2\,b^2\,x^2}{a^3} - \frac{2\,b\,x}{3\,a^2}}{x^{5/2}} - \frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)),x)
```

```
[Out] - (2/(5*a) + (2*b^2*x^2)/a^3 - (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atan((
b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

3.456 $\int \frac{x^{5/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] 5/3*x^(3/2)/b^2-x^(5/2)/b/(b*x+a)+5*a^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))
/b^(7/2)-5*a*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$\frac{5a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^2}\,dx &= -\frac{x^{5/2}}{b(a+bx)}+\frac{5\int \frac{x^{3/2}}{a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}-\frac{(5a)\int \frac{\sqrt{x}}{a+bx}\,dx}{2b^2}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{(5a^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.97

$$\frac{\sqrt{x}\,(-15a^2-10abx+2b^2x^2)}{3b^3(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(-15*a^2 - 10*a*b*x + 2*b^2*x^2))/(3*b^3*(a + b*x)) + (5*a^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.12, size = 59, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	59
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}\right)}{b^{3}}+\frac{2a^{2}\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$	59
risch	$-\frac{2(-bx+6a)\sqrt{x}}{3b^{3}}-\frac{a^{2}\sqrt{x}}{b^{3}(bx+a)}+\frac{5a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^3*(-1/3*b*x^(3/2)+2*a*x^(1/2))+2/b^3*a^2*(-1/2*x^(1/2)/(b*x+a)+5/2/(a*
b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 63, normalized size = 0.90

$$-\frac{a^{2}\sqrt{x}}{b^{4}x+ab^{3}}+\frac{5\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{2\left(bx^{\frac{3}{2}}-6\,a\sqrt{x}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x + a*b^3) + 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^3) + 2/3*(b*x^(3/2) - 6*a*sqrt(x))/b^3
```

Fricas [A]
time = 0.57, size = 161, normalized size = 2.30

$$\left[\frac{15\,(abx+a^{2})\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\,\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,b^{2}x^{2}-10\,abx-15\,a^{2})\sqrt{x}}{6\,(b^{4}x+ab^{3})},\ \frac{15\,(abx+a^{2})\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{\frac{a}{b}}}{a}\right)+(2\,b^{2}x^{2}-10\,abx-15\,a^{2})\sqrt{x}}{3\,(b^{4}x+ab^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(63) = 126$.
time = 14.30, size = 389, normalized size = 5.56

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^2} & \text{for } a = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^2} & \text{for } b = 0 \\ \frac{15a^3 \log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} - \frac{15a^3 \log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} - \frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} + \frac{15a^2bx \log\left(\sqrt{x} - \sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} - \frac{15a^2bx \log\left(\sqrt{x} + \sqrt{-\frac{a}{b}}\right)}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} - \frac{20ab^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} + \frac{4b^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{6ab^4\sqrt{-\frac{a}{b}} + 6b^5x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**2), Eq(a,
0)), (2*x**(7/2)/(7*a**2), Eq(b, 0)), (15*a**3*log(sqrt(x) - sqrt(-a/b))/(6
*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**3*log(sqrt(x) + sqrt(-a/b
))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 30*a**2*b*sqrt(x)*sqrt(-a/
b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 15*a**2*b*x*log(sqrt(x) -
sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 15*a**2*b*x*log(s
qrt(x) + sqrt(-a/b))/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) - 20*a*b**
2*x**(3/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)) + 4*b**3*
x**(5/2)*sqrt(-a/b)/(6*a*b**4*sqrt(-a/b) + 6*b**5*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.61, size = 65, normalized size = 0.93

$$\frac{5\,a^2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} - \frac{a^2\sqrt{x}}{(bx+a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} - 6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

Mupad [B]
time = 0.11, size = 58, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3\,b^2} - \frac{4\,a\,\sqrt{x}}{b^3} - \frac{a^2\,\sqrt{x}}{x\,b^4 + a\,b^3} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*a*x^(1/2))/b^3 - (a^2*x^(1/2))/(a*b^3 + b^4*x) + (
5*a^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

3.457 $\int \frac{x^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x}}{b^2} - \frac{x^{3/2}}{b(a+bx)} - \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] -x^(3/2)/b/(b*x+a)-3*arctan(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)+3*x^(1
/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {43, 52, 65, 211}

$$-\frac{3\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}} - \frac{x^{3/2}}{b(a+bx)} + \frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx)^2}\,dx &= -\frac{x^{3/2}}{b(a+bx)}+\frac{3\int \frac{\sqrt{x}}{a+bx}\,dx}{2b}\\ &= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b^2}\\ &= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.95

$$\frac{\sqrt{x}\,(3a+2bx)}{b^2(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(3*a + 2*b*x))/(b^2*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Maple [A]
time = 0.13, size = 47, normalized size = 0.82

method	result	size

derivativedivides	$\frac{2\sqrt{x}}{b^{2}}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	47
default	$\frac{2\sqrt{x}}{b^{2}}-\frac{2a\left(-\frac{\sqrt{x}}{2(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{2}}$	47
risch	$\frac{2\sqrt{x}}{b^{2}}+\frac{a\sqrt{x}}{b^{2}(bx+a)}-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^2-2*a/b^2*(-1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/2
)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 49, normalized size = 0.86

$$\frac{a\sqrt{x}}{b^{3}x+ab^{2}}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\,\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*sqrt(x)/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + 2*sqrt(x)/b^2
```

Fricas [A]
time = 0.56, size = 134, normalized size = 2.35

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\,(b^{3}x+ab^{2})},\,-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^{3}x+ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(49) = 98$.
time = 4.03, size = 332, normalized size = 5.82

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b^2} & \text{for } a = 0 \\ -\frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} - \frac{3abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{3abx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} + \frac{4b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{2ab^3\sqrt{-\frac{a}{b}}+2b^4x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (-3*a**2*log(sqrt(x) - sqrt(-a/b))/(2*a*b**
3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a**2*log(sqrt(x) + sqrt(-a/b))/(2*a
*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 6*a*b*sqrt(x)*sqrt(-a/b)/(2*a*b**
3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) - 3*a*b*x*log(sqrt(x) - sqrt(-a/b))/(2*
a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 3*a*b*x*log(sqrt(x) + sqrt(-a/b)
)/(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)) + 4*b**2*x**(3/2)*sqrt(-a/b)/
(2*a*b**3*sqrt(-a/b) + 2*b**4*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.53, size = 46, normalized size = 0.81

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{a\sqrt{x}}{(bx+a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

Mupad [B]
time = 0.12, size = 46, normalized size = 0.81

$$\frac{2\,\sqrt{x}}{b^2} + \frac{a\,\sqrt{x}}{x\,b^3 + a\,b^2} - \frac{3\,\sqrt{a}\,\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 + b^3*x) - (3*a^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/b^(5/2)
```

3.458 $\int \frac{\sqrt{x}}{(a+bx)^2} dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{x}}{b(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)-x^(1/2)/b/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^2}\,dx = -\frac{\sqrt{x}}{b(a+bx)} + \frac{\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}}{b(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.00

$$-\frac{\sqrt{x}}{b(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^2,x]
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Maple [A]
time = 0.10, size = 37, normalized size = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{b(bx+a)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37
default	$-\frac{\sqrt{x}}{b(bx+a)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -x^(1/2)/b/(b*x+a)+1/b/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 37, normalized size = 0.80

$$-\frac{\sqrt{x}}{b^2x+ab} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.50, size = 115, normalized size = 2.50

$$\left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\,(ab^3x+a^2b^2)},-\frac{ab\sqrt{x}+\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(37) = 74$.
time = 1.59, size = 269, normalized size = 5.85

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for } b=0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for } a=0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}-\frac{2b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}+\frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}}-\frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2ab^2\sqrt{-\frac{a}{b}}+2b^3x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*s
qrt(-a/b) + 2*b**3*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*sq
rt(-a/b) + 2*b**3*x*sqrt(-a/b)) - 2*b*sqrt(x)*sqrt(-a/b)/(2*a*b**2*sqrt(-a/
b) + 2*b**3*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a*b**2*sqrt(-a
/b) + 2*b**3*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a*b**2*sqrt(-
a/b) + 2*b**3*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.61, size = 36, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

Mupad [B]
time = 0.04, size = 34, normalized size = 0.74

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^2,x)
```

```
[Out] atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2)) - x^(1/2)/(b*(a + b*x))
```

3.459 $\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\sqrt{x}}{a(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[Out] arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)+x^(1/2)/a/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {44, 65, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.00

$$\frac{\sqrt{x}}{a(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(bx+a)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36
default	$\frac{\sqrt{x}}{a(bx+a)} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.57, size = 35, normalized size = 0.78

$$\frac{\sqrt{x}}{abx+a^2} + \frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.47, size = 116, normalized size = 2.58

$$\left[\frac{2\,ab\sqrt{x} - \sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\,(a^2b^2x+a^3b)}, \frac{ab\sqrt{x} - \sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(37) = 74.
time = 2.65, size = 277, normalized size = 6.16

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for}\: b = 0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{2b\sqrt{x}\sqrt{-\frac{a}{b}}}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^2b\sqrt{-\frac{a}{b}}+2ab^2x\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*sq
rt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - a*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b*s
qrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) + 2*b*sqrt(x)*sqrt(-a/b)/(2*a**2*b*sqrt(
-a/b) + 2*a*b**2*x*sqrt(-a/b)) + b*x*log(sqrt(x) - sqrt(-a/b))/(2*a**2*b*sq
rt(-a/b) + 2*a*b**2*x*sqrt(-a/b)) - b*x*log(sqrt(x) + sqrt(-a/b))/(2*a**2*b
*sqrt(-a/b) + 2*a*b**2*x*sqrt(-a/b)), True))
```

Giac [A]
time = 0.58, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} + \frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="giac")
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

Mupad [B]
time = 0.09, size = 33, normalized size = 0.73

$$\frac{\sqrt{x}}{a\,\left(a+b\,x\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^2),x)
[Out] x^(1/2)/(a*(a + b*x)) + atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

3.460 $\int \frac{1}{x^{3/2}(a+bx)^2} \, dx$

Optimal. Leaf size=56

$$-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -3*arctan(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)-3/a^2/x^(1/2)+1/a/(b*x+a
)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$-\frac{3\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^2}\,dx &= \frac{1}{a\sqrt{x}\,(a+bx)}+\frac{3\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^2}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 54, normalized size = 0.96

$$\frac{-2a-3bx}{a^2\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a - 3*b*x)/(a^2*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Maple [**A**]
time = 0.11, size = 47, normalized size = 0.84

method	result	size

derivativedivides	$-\frac{2b\left(\frac{\sqrt{x}}{2bx+2a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	47
default	$-\frac{2b\left(\frac{\sqrt{x}}{2bx+2a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	47
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{b\sqrt{x}}{a^{2}(bx+a)}-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a^2*(1/2*x^(1/2)/(b*x+a)+3/2/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2))
)-2/a^2/x^(1/2)
```

Maxima [A]
time = 0.55, size = 51, normalized size = 0.91

$$-\frac{3\,bx+2\,a}{a^{2}bx^{\frac{3}{2}}+a^{3}\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.49, size = 147, normalized size = 2.62

$$\left[\frac{3\,(bx^{2}+ax)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\,(a^{2}bx^{2}+a^{3}x)},\ \frac{3\,(bx^{2}+ax)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx+2\,a)\sqrt{x}}{a^{2}bx^{2}+a^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(51) = 102$.
time = 7.54, size = 384, normalized size = 6.86

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5b^2x^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a^2\sqrt{x}} & \text{for } b = 0 \\ -\frac{3a\sqrt{x}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} + \frac{3a\sqrt{x}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} - \frac{4a\sqrt{-\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} - \frac{3bx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} + \frac{3bx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} - \frac{6bx\sqrt{-\frac{a}{b}}}{2a^3\sqrt{x}\sqrt{-\frac{a}{b}}+2a^2bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-3*a*sqrt(x)*log(sqrt(x) - sqrt(-a/b)
)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) + 3*a*sqrt(x)*
log(sqrt(x) + sqrt(-a/b))/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sq
rt(-a/b)) - 4*a*sqrt(-a/b)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*s
qrt(-a/b)) - 3*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(2*a**3*sqrt(x)*sqrt(-a
/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) + 3*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b)
)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)) - 6*b*x*sqrt(-
a/b)/(2*a**3*sqrt(x)*sqrt(-a/b) + 2*a**2*b*x**(3/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 0.51, size = 49, normalized size = 0.88

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2} - \frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

Mupad [B]
time = 0.12, size = 48, normalized size = 0.86

$$-\frac{\frac{2}{a}+\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}+b\,x^{3/2}} - \frac{3\,\sqrt{b}\,\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^2),x)
```

```
[Out] - (2/a + (3*b*x)/a^2)/(a*x^(1/2) + b*x^(3/2)) - (3*b^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/a^(5/2)
```

3.461 $\int \frac{1}{x^{5/2}(a+bx)^2} dx$

Optimal. Leaf size=69

$$-\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3/a^2/x^(3/2)+1/a/x^(3/2)/(b*x+a)+5*b^(3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/
2))/a^(7/2)+5*b/a^3/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$\frac{5b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) + (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a + b*x)) + (5*b^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a+bx)} + \frac{5\int \frac{1}{x^{5/2}(a+bx)}\,dx}{2a} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a+bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a^2} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{(5b^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^3} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{5}{3a^2x^{3/2}} + \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a+bx)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.99

$$\frac{-2a^2+10abx+15b^2x^2}{3a^3x^{3/2}(a+bx)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a^2 + 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a + b*x)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.11, size = 58, normalized size = 0.84

method	result	size

risch	$-\frac{2(-6bx+a)}{3a^{3}x^{\frac{3}{2}}}+\frac{b^{2}\sqrt{x}}{a^{3}(bx+a)}+\frac{5b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^{3}\sqrt{ab}}$	57
derivativedivides	$-\frac{2}{3a^{2}x^{\frac{3}{2}}}+\frac{4b}{a^{3}\sqrt{x}}+\frac{2b^{2}\left(\frac{\sqrt{x}}{2bx+2a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}$	58
default	$-\frac{2}{3a^{2}x^{\frac{3}{2}}}+\frac{4b}{a^{3}\sqrt{x}}+\frac{2b^{2}\left(\frac{\sqrt{x}}{2bx+2a}+\frac{5\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^2/x^(3/2)+4*b/a^3/x^(1/2)+2*b^2/a^3*(1/2*x^(1/2)/(b*x+a)+5/2/(a*b)^(
1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 64, normalized size = 0.93

$$\frac{15\,b^{2}x^{2}+10\,abx-2\,a^{2}}{3\left(a^{3}bx^{\frac{5}{2}}+a^{4}x^{\frac{3}{2}}\right)}+\frac{5\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.50, size = 184, normalized size = 2.67

$$\left[\frac{15\,(b^{2}x^{3}+abx^{2})\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(15\,b^{2}x^{2}+10\,abx-2\,a^{2})\sqrt{x}}{6\,(a^{3}bx^{3}+a^{4}x^{2})},-\frac{15\,(b^{2}x^{3}+abx^{2})\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(15\,b^{2}x^{2}+10\,abx-2\,a^{2})\sqrt{x}}{3\,(a^{3}bx^{3}+a^{4}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(65) = 130$.
time = 29.41, size = 452, normalized size = 6.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{3a^2x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{7b^2x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ -\frac{4a^2\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{20abx\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} - \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} + \frac{30b^2x^2\sqrt{-\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}+6a^3bx^{\frac{5}{2}}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**2*x**(3/2)), Eq(b,
 0)), (-2/(7*b**2*x**(7/2)), Eq(a, 0)), (-4*a**2*sqrt(-a/b)/(6*a**4*x**(3/2
)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*a*b*x**(3/2)*log(sqrt(x)
- sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) -
 15*a*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*
a**3*b*x**(5/2)*sqrt(-a/b)) + 20*a*b*x*sqrt(-a/b)/(6*a**4*x**(3/2)*sqrt(-a/
b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) + 15*b**2*x**(5/2)*log(sqrt(x) - sqrt(-a
/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x**(5/2)*sqrt(-a/b)) - 15*b**2*
x**(5/2)*log(sqrt(x) + sqrt(-a/b))/(6*a**4*x**(3/2)*sqrt(-a/b) + 6*a**3*b*x
**(5/2)*sqrt(-a/b)) + 30*b**2*x**2*sqrt(-a/b)/(6*a**4*x**(3/2)*sqrt(-a/b) +
 6*a**3*b*x**(5/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 0.96, size = 58, normalized size = 0.84

$$\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} + \frac{b^2\sqrt{x}}{(bx+a)a^3} + \frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

Mupad [B]
time = 0.15, size = 58, normalized size = 0.84

$$\frac{\frac{5\,b^2\,x^2}{a^3}-\frac{2}{3\,a}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}+b\,x^{5/2}}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^2),x)
```

```
[Out] ((5*b^2*x^2)/a^3 - 2/(3*a) + (10*b*x)/(3*a^2))/(a*x^(3/2) + b*x^(5/2)) + (5
*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

3.462 $\int \frac{x^{7/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=95

$$-\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

```
[Out] 35/12*x^(3/2)/b^3-1/2*x^(7/2)/b/(b*x+a)^2-7/4*x^(5/2)/b^2/(b*x+a)+35/4*a^(3
/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)-35/4*a*x^(1/2)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$\frac{35a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{35a\sqrt{x}}{4b^4}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a + b*x)^2)
 - (7*x^(5/2))/(4*b^2*(a + b*x)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{(a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a+bx)^2}+\frac{7\int \frac{x^{5/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35\int \frac{x^{3/2}}{a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{(35a)\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^3}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{(35a^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{(35a^2)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 81, normalized size = 0.85

$$
\frac{\sqrt{x}\,\left(-105a^3-175a^2bx-56ab^2x^2+8b^3x^3\right)}{12b^4(a+bx)^2}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x)^3,x]
[Out] (Sqrt[x]*(-105*a^3 - 175*a^2*b*x - 56*a*b^2*x^2 + 8*b^3*x^3))/(12*b^4*(a +
b*x)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Maple [A]
time = 0.13, size = 68, normalized size = 0.72

method	result	size
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}\right)}{b^{4}}+\frac{2a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$	68
default	$-\frac{2\left(-\frac{b\,x^{\frac{3}{2}}}{3}+3a\sqrt{x}\right)}{b^{4}}+\frac{2a^{2}\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{4}}$	68
risch	$-\frac{2(-bx+9a)\sqrt{x}}{3b^{4}}-\frac{13a^{2}x^{\frac{3}{2}}}{4b^{3}(bx+a)^{2}}-\frac{11a^{3}\sqrt{x}}{4b^{4}(bx+a)^{2}}+\frac{35a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^{4}\sqrt{ab}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^4*(-1/3*b*x^(3/2)+3*a*x^(1/2))+2/b^4*a^2*((-13/8*b*x^(3/2)-11/8*a*x^(1
/2))/(b*x+a)^2+35/8/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 86, normalized size = 0.91

$$-\frac{13\,a^{2}bx^{\frac{3}{2}}+11\,a^{3}\sqrt{x}}{4\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})}+\frac{35\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^{4}}+\frac{2\,\left(bx^{\frac{3}{2}}-9\,a\sqrt{x}\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) + 11*a^3*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(b*x^(3/2) - 9*a
*sqrt(x))/b^4
```

Fricas [A]
time = 0.56, size = 227, normalized size = 2.39

$$\left[\frac{105\,(ab^{2}x^{2}+2\,a^{2}bx+a^{3})\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^{3}x^{3}-56\,ab^{2}x^{2}-175\,a^{2}bx-105\,a^{3})\sqrt{x}}{24\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})},\ \frac{105\,(ab^{2}x^{2}+2\,a^{2}bx+a^{3})\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(8\,b^{3}x^{3}-56\,ab^{2}x^{2}-175\,a^{2}bx-105\,a^{3})\sqrt{x}}{12\,(b^{6}x^{2}+2\,ab^{5}x+a^{2}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(87) = 174$.
time = 96.96, size = 762, normalized size = 8.02

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{9}{2}}}{9a^3} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^3} & \text{for}\: a = 0 \\ \frac{105a^4\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} - \frac{105a^4\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} - \frac{210a^3b\sqrt{x}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} + \frac{210a^3bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} - \frac{210a^3bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} - \frac{350a^2b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} + \frac{105a^2b^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} - \frac{105a^2b^2x^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} - \frac{112ab^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} + \frac{16b^4x^{\frac{7}{2}}\sqrt{-\frac{a}{b}}}{24a^2b^5\sqrt{-\frac{a}{b}}+48ab^6x\sqrt{-\frac{a}{b}}+24b^7x^2\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a**3), Eq(b,
0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (105*a**4*log(sqrt(x) - sqrt(-a/b))/(
24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b))
 - 105*a**4*log(sqrt(x) + sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*
x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*a**3*b*sqrt(x)*sqrt(-a/b)/(24
*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) +
 210*a**3*b*x*log(sqrt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**
6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 210*a**3*b*x*log(sqrt(x) + sqrt
(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sq
rt(-a/b)) - 350*a**2*b**2*x**(3/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(-a/b) + 48
*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) + 105*a**2*b**2*x**2*log(sq
rt(x) - sqrt(-a/b))/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*
b**7*x**2*sqrt(-a/b)) - 105*a**2*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(24*a*
*2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)) - 11
2*a*b**3*x**(5/2)*sqrt(-a/b)/(24*a**2*b**5*sqrt(-a/b) + 48*a*b**6*x*sqrt(-a
/b) + 24*b**7*x**2*sqrt(-a/b)) + 16*b**4*x**(7/2)*sqrt(-a/b)/(24*a**2*b**5*
sqrt(-a/b) + 48*a*b**6*x*sqrt(-a/b) + 24*b**7*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 1.38, size = 77, normalized size = 0.81

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^4} - \frac{13\,a^2bx^{\frac{3}{2}} + 11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4} + \frac{2\left(b^6x^{\frac{3}{2}} - 9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

Mupad [B]
time = 0.12, size = 81, normalized size = 0.85

$$\frac{2\,x^{3/2}}{3\,b^3}-\frac{\frac{11\,a^3\,\sqrt{x}}{4}+\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4+2\,a\,b^5\,x+b^6\,x^2}-\frac{6\,a\,\sqrt{x}}{b^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x)^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) - ((11*a^3*x^(1/2))/4 + (13*a^2*b*x^(3/2))/4)/(a^2*b^4
+ b^6*x^2 + 2*a*b^5*x) - (6*a*x^(1/2))/b^4 + (35*a^(3/2)*atan((b^(1/2)*x^(1
/2))/a^(1/2)))/(4*b^(9/2))
```

3.463 $\int \frac{x^{5/2}}{(a+bx)^3}\,dx$

Optimal. Leaf size=82

$$\frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

```
[Out] -1/2*x^(5/2)/b/(b*x+a)^2-5/4*x^(3/2)/b^2/(b*x+a)-15/4*arctan(b^(1/2)*x^(1/2
)/a^(1/2))*a^(1/2)/b^(7/2)+15/4*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 52, 65, 211}

$$-\frac{15\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a+bx)^2}+\frac{5\int \frac{x^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}+\frac{15\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 70, normalized size = 0.85

$$\frac{\sqrt{x}\,(15a^2+25abx+8b^2x^2)}{4b^3(a+bx)^2}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(15*a^2 + 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a + b*x)^2) - (15*Sqrt[a]
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Maple [**A**]
time = 0.12, size = 56, normalized size = 0.68

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^3} - \frac{2a\left(\frac{-\frac{9bx^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	56
default	$\frac{2\sqrt{x}}{b^3} - \frac{2a\left(\frac{-\frac{9bx^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(bx+a)^2}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	56
risch	$\frac{2\sqrt{x}}{b^3}+\frac{9a\,x^{\frac{3}{2}}}{4b^2(bx+a)^2}+\frac{7a^2\sqrt{x}}{4b^3(bx+a)^2}-\frac{15a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^3\sqrt{ab}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^3-2/b^3*a*((-9/8*b*x^(3/2)-7/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)^
(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 73, normalized size = 0.89

$$\frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3}+\frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 15/4*
a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3
```

Fricas [A]
time = 0.73, size = 200, normalized size = 2.44

$$\left[\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)},\ -\frac{15\,(b^2x^2+2\,abx+a^2)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(8\,b^2x^2+25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(73) = 146$.
time = 38.62, size = 683, normalized size = 8.33

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^3} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b^3} & \text{for } a = 0 \\ -\frac{15a^3\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{15a^3\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{30a^2b\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} - \frac{30a^2bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{30a^2bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{50ab^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} - \frac{15ab^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{15ab^2x^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} + \frac{16b^3x^{\frac{5}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^4\sqrt{-\frac{a}{b}}+16ab^5x\sqrt{-\frac{a}{b}}+8b^6x^2\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**3), Eq(b, 0
)), (2*sqrt(x)/b**3, Eq(a, 0)), (-15*a**3*log(sqrt(x) - sqrt(-a/b))/(8*a**2
*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15*a*
*3*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/
b) + 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**4*sq
rt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) - 30*a**2*b*x*l
og(sqrt(x) - sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) +
 8*b**6*x**2*sqrt(-a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 50*a*b**
2*x**(3/2)*sqrt(-a/b)/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*
b**6*x**2*sqrt(-a/b)) - 15*a*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b*
*4*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 15*a*b**
2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**4*sqrt(-a/b) + 16*a*b**5*x*sqrt
(-a/b) + 8*b**6*x**2*sqrt(-a/b)) + 16*b**3*x**(5/2)*sqrt(-a/b)/(8*a**2*b**4
*sqrt(-a/b) + 16*a*b**5*x*sqrt(-a/b) + 8*b**6*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 1.92, size = 59, normalized size = 0.72

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3} + \frac{2\,\sqrt{x}}{b^3} + \frac{9\,abx^{\frac{3}{2}} + 7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

Mupad [B]
time = 0.14, size = 69, normalized size = 0.84

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

3.464 $\int \frac{x^{3/2}}{(a+bx)^3} dx$

Optimal. Leaf size=70

$$-\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

```
[Out] -1/2*x^(3/2)/b/(b*x+a)^2+3/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)/a^(1/2
)-3/4*x^(1/2)/b^2/(b*x+a)
```

Rubi [A]

time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{3\sqrt{x}}{4b^2(a+bx)}-\frac{x^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a+bx)^2}+\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^2}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.84

$$
-\frac{\sqrt{x}\,(3a+5bx)}{4b^2(a+bx)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(3*a + 5*b*x))/(b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.10, size = 50, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	50
default	$\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(b
*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 61, normalized size = 0.87

$$-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.72, size = 185, normalized size = 2.64

$$\left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)+2\,(5\,ab^2x+3\,a^2b)\sqrt{x}}{8\,(ab^5x^2+2\,a^2b^4x+a^3b^3)},-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+(5\,ab^2x+3\,a^2b)\sqrt{x}}{4\,(ab^5x^2+2\,a^2b^4x+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(61) = 122$.
time = 15.88, size = 605, normalized size = 8.64

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{b^3\sqrt{x}} & \text{for}\: a = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^3} & \text{for}\: b = 0 \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{6ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} + \frac{6abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{6abx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{10b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} + \frac{3b^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} - \frac{3b^2x^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^2b^3\sqrt{-\frac{a}{b}}+16ab^4x\sqrt{-\frac{a}{b}}+8b^5x^2\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**3*sqrt(x)), Eq(a, 0))
, (2*x**(5/2)/(5*a**3), Eq(b, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a**
2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*a*
*2*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/
b) + 8*b**5*x**2*sqrt(-a/b)) - 6*a*b*sqrt(x)*sqrt(-a/b)/(8*a**2*b**3*sqrt(-
a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) + 6*a*b*x*log(sqrt(
x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*
```

```
x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b
) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 10*b**2*x**(3/2)*sqr
t(-a/b)/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt
(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 1
6*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2*sqrt(-a/b)) - 3*b**2*x**2*log(sqrt(x) +
 sqrt(-a/b))/(8*a**2*b**3*sqrt(-a/b) + 16*a*b**4*x*sqrt(-a/b) + 8*b**5*x**2
*sqrt(-a/b)), True))
```

Giac [A]
time = 1.37, size = 47, normalized size = 0.67

$$\frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^2} - \frac{5\,bx^{\frac{3}{2}} + 3\,a\sqrt{x}}{4\,(bx+a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.13, size = 58, normalized size = 0.83

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}} - \frac{\frac{5\,x^{3/2}}{4\,b} + \frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2)) - ((5*x^(3/2))/(4*b
) + (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.465 $\int \frac{\sqrt{x}}{(a+bx)^3}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[Out] 1/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/2*x^(1/2)/b/(b*x+a)^2
+1/4*x^(1/2)/a/b/(b*x+a)
```

Rubi [A]

time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*
Sqrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx}{4b}\\ &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8ab}\\ &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\ &= -\frac{\sqrt{x}}{2b(a+bx)^2}+\frac{\sqrt{x}}{4ab(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.82

$$-\frac{\sqrt{x}\,(a-bx)}{4ab(a+bx)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^3,x]
[Out] -1/4*(Sqrt[x]*(a - b*x))/(a*b*(a + b*x)^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]]/(4*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.71

method	result	size

derivativedivides	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	52
default	$\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/8/a*x^(3/2)-1/8*x^(1/2)/b)/(b*x+a)^2+1/4/a/b/(a*b)^(1/2)*arctan(b*x^(1
/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 64, normalized size = 0.88

$$\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\left(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b\right)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.93, size = 186, normalized size = 2.55

$$\left[-\frac{\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\,\left(ab^{2}x-a^{2}b\right)\sqrt{x}}{8\,\left(a^{2}b^{4}x^{2}+2\,a^{3}b^{3}x+a^{4}b^{2}\right)},-\frac{\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(ab^{2}x-a^{2}b\right)\sqrt{x}}{4\,\left(a^{2}b^{4}x^{2}+2\,a^{3}b^{3}x+a^{4}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(58) = 116.
time = 7.04, size = 627, normalized size = 8.59

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^{3}} & \text{for}\: b = 0 \\ -\frac{2}{3b^{3}x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{a^{2}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}-\frac{a^{2}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}-\frac{2ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}+\frac{2abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}-\frac{2abx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}+\frac{2b^{2}x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}+\frac{b^{2}x^{2}\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}}-\frac{b^{2}x^{2}\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{-\frac{a}{b}}+16a^{2}b^{3}x\sqrt{-\frac{a}{b}}+8ab^{4}x^{2}\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(-a/b))/(8*a
**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b))
 - a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*
sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*sqrt(x)*sqrt(-a/b)/(8*a**3*b
**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + 2*
a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sq
rt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b
)) + 2*b**2*x**(3/2)*sqrt(-a/b)/(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sq
rt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)) + b**2*x**2*log(sqrt(x) - sqrt(-a/b))/
(8*a**3*b**2*sqrt(-a/b) + 16*a**2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a
/b)) - b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**3*b**2*sqrt(-a/b) + 16*a**
2*b**3*x*sqrt(-a/b) + 8*a*b**4*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 1.21, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab}+\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\,(bx+a)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

Mupad [B]
time = 0.13, size = 56, normalized size = 0.77

$$\frac{\frac{x^{3/2}}{4\,a}-\frac{\sqrt{x}}{4\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^3,x)
```

```
[Out] (x^(3/2)/(4*a) - x^(1/2)/(4*b))/(a^2 + b^2*x^2 + 2*a*b*x) + atan((b^(1/2)*x
^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2))
```

3.466 $\int \frac{1}{\sqrt{x}\,(a+bx)^3}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

```
[Out] 3/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)+1/2*x^(1/2)/a/(b*x+a)^2
+3/4*x^(1/2)/a^2/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {44, 65, 211}

$$\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\,(a+bx)^3}\,dx &= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)^2}\,dx}{4a}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 59, normalized size = 0.84

$$
\frac{\sqrt{x}\,(5a+3bx)}{4a^2(a+bx)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (Sqrt[x]*(5*a + 3*b*x))/(4*a^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Maple [**A**]
time = 0.11, size = 59, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2a(bx+a)^2}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59
default	$\frac{\sqrt{x}}{2a(bx+a)^2}+\frac{\frac{3\sqrt{x}}{4a(bx+a)}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a\sqrt{ab}}}{a}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/2*x^(1/2)/a/(b*x+a)^2+3/2/a*(1/2*x^(1/2)/a/(b*x+a)+1/2/a/(a*b)^(1/2)*arct
an(b*x^(1/2)/(a*b)^(1/2)))

Maxima [A]
time = 0.52, size = 60, normalized size = 0.86

$$\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="maxima")

[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)

Fricas [A]
time = 0.73, size = 186, normalized size = 2.66

$$\left[-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\,(3\,ab^2x+5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2+2\,a^4b^2x+a^5b)},-\frac{3\,(b^2x^2+2\,abx+a^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-(3\,ab^2x+5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="fricas")

[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(61) = 122.
time = 11.41, size = 632, normalized size = 9.03

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a=0\wedge b=0\\ \frac{2\sqrt{x}}{a^3} & \text{for}\: b=0\\ -\frac{2}{5b^3x^{\frac{5}{2}}} & \text{for}\: a=0\\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}-\frac{3a^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{10ab\sqrt{x}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{6abx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}-\frac{6abx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{6b^2x^{\frac{3}{2}}\sqrt{-\frac{a}{b}}}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}+\frac{3b^2x^2\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}}-\frac{3b^2x^2\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{8a^4b\sqrt{-\frac{a}{b}}+16a^3b^2x\sqrt{-\frac{a}{b}}+8a^2b^3x^2\sqrt{-\frac{a}{b}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**3/x**(1/2),x)

[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0)),
(-2/(5*b**3*x**(5/2)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(-a/b))/(8*a**4
*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) -
3*a**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt
(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 10*a*b*sqrt(x)*sqrt(-a/b)/(8*a**4*b

```
*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 6*
a*b*x*log(sqrt(x) - sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(
-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) - 6*a*b*x*log(sqrt(x) + sqrt(-a/b))/(8
*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b
)) + 6*b**2*x**(3/2)*sqrt(-a/b)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(
-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(-a/b)
)/(8*a**4*b*sqrt(-a/b) + 16*a**3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(
-a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(-a/b))/(8*a**4*b*sqrt(-a/b) + 16*a*
*3*b**2*x*sqrt(-a/b) + 8*a**2*b**3*x**2*sqrt(-a/b)), True))
```

Giac [A]
time = 1.87, size = 47, normalized size = 0.67

$$\frac{3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^2} + \frac{3\,bx^{\frac{3}{2}} + 5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

Mupad [B]
time = 0.13, size = 57, normalized size = 0.81

$$\frac{\frac{5\sqrt{x}}{4\,a} + \frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^3),x)
```

```
[Out] ((5*x^(1/2))/(4*a) + (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (3*
atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

3.467 $\int \frac{1}{x^{3/2}(a+bx)^3}\,dx$

Optimal. Leaf size=82

$$-\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*arctan(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)-15/4/a^3/x^(1/2)+1/2/
a/(b*x+a)^2/x^(1/2)+5/4/a^2/(b*x+a)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$-\frac{15\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15}{4a^3\sqrt{x}}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] -15/(4*a^3*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x)^2) + 5/(4*a^2*Sqrt[x]*(a + b
*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^3}\,dx &= \frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5\int \frac{1}{x^{3/2}(a+bx)^2}\,dx}{4a}\\ &= \frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{15\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^2}\\ &= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{(15b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^3}\\ &= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{(15b)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^3}\\ &= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.85

$$\frac{-8a^2-25abx-15b^2x^2}{4a^3\sqrt{x}\,(a+bx)^2}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^3),x]
[Out] (-8*a^2 - 25*a*b*x - 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a + b*x)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Maple [A]
time = 0.12, size = 56, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{2}{a^{3}\sqrt{x}}-\frac{2b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	56
default	$-\frac{2}{a^{3}\sqrt{x}}-\frac{2b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{15\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}$	56
risch	$-\frac{2}{a^{3}\sqrt{x}}-\frac{7b^{2}x^{\frac{3}{2}}}{4a^{3}(bx+a)^{2}}-\frac{9b\sqrt{x}}{4a^{2}(bx+a)^{2}}-\frac{15b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^{3}\sqrt{ab}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3/x^(1/2)-2/a^3*b*((7/8*b*x^(3/2)+9/8*a*x^(1/2))/(b*x+a)^2+15/8/(a*b)^
(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 73, normalized size = 0.89

$$-\frac{15\,b^{2}x^{2}+25\,abx+8\,a^{2}}{4\left(a^{3}b^{2}x^{\frac{5}{2}}+2\,a^{4}bx^{\frac{3}{2}}+a^{5}\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.58, size = 214, normalized size = 2.61

$$\left[\frac{15\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^{2}x^{2}+25\,abx+8\,a^{2}\right)\sqrt{x}}{8\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)},\ \frac{15\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^{2}x^{2}+25\,abx+8\,a^{2}\right)\sqrt{x}}{4\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(75) = 150$.
time = 30.95, size = 779, normalized size = 9.50

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{a^{3} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{7 b^{3} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ - \frac{15 a^{2} \sqrt{x} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} + \frac{15 a^{2} \sqrt{x} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} - \frac{16 a^{2} \sqrt{- \frac{a}{b}}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} - \frac{30 a b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} + \frac{30 a b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} - \frac{50 a b x \sqrt{- \frac{a}{b}}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} - \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} + \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} - \frac{30 b^{2} x^{2} \sqrt{- \frac{a}{b}}}{8 a^{5} \sqrt{x} \sqrt{- \frac{a}{b}} + 16 a^{4} b x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 8 a^{3} b^{2} x^{\frac{5}{2}} \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**3*sqrt(x)), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-15*a**2*sqrt(x)*log(sqrt(x) - sqrt(-
a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b
**2*x**(5/2)*sqrt(-a/b)) + 15*a**2*sqrt(x)*log(sqrt(x) + sqrt(-a/b))/(8*a**
5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)
*sqrt(-a/b)) - 16*a**2*sqrt(-a/b)/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x*
*(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 30*a*b*x**(3/2)*log(
sqrt(x) - sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(
-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) + 30*a*b*x**(3/2)*log(sqrt(x) + sq
rt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a*
*3*b**2*x**(5/2)*sqrt(-a/b)) - 50*a*b*x*sqrt(-a/b)/(8*a**5*sqrt(x)*sqrt(-a/
b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 15*
b**2*x**(5/2)*log(sqrt(x) - sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**
4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) + 15*b**2*x**(5/
2)*log(sqrt(x) + sqrt(-a/b))/(8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2
)*sqrt(-a/b) + 8*a**3*b**2*x**(5/2)*sqrt(-a/b)) - 30*b**2*x**2*sqrt(-a/b)/(
8*a**5*sqrt(x)*sqrt(-a/b) + 16*a**4*b*x**(3/2)*sqrt(-a/b) + 8*a**3*b**2*x**
(5/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 1.01, size = 59, normalized size = 0.72

$$-\frac{15 \, b \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{4 \, \sqrt{a b} a^{3}} - \frac{2}{a^{3} \sqrt{x}} - \frac{7 \, b^{2} x^{\frac{3}{2}} + 9 \, a b \sqrt{x}}{4 \, {\left(b x + a\right)}^{2} a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

Mupad [B]
time = 0.15, size = 70, normalized size = 0.85

$$-\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}+\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}+2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^3),x)
```

```
[Out] - (2/a + (15*b^2*x^2)/(4*a^3) + (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2
) + 2*a*b*x^(3/2)) - (15*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2
))
```

3.468 $\int \frac{1}{x^{5/2}(a+bx)^3} dx$

Optimal. Leaf size=95

$$-\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] -35/12/a^3/x^(3/2)+1/2/a/x^(3/2)/(b*x+a)^2+7/4/a^2/x^(3/2)/(b*x+a)+35/4*b^(
3/2)*arctan(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)+35/4*b/a^4/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {44, 53, 65, 211}

$$\frac{35b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] -35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) + 1/(2*a*x^(3/2)*(a + b*x)^2)
 + 7/(4*a^2*x^(3/2)*(a + b*x)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^3}\,dx &= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7\int \frac{1}{x^{5/2}(a+bx)^2}\,dx}{4a}\\
&= \frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35\int \frac{1}{x^{5/2}(a+bx)}\,dx}{8a^2}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}-\frac{(35b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^3}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{(35b^2)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^4}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{(35b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\right.}{4a^4}\\
&= -\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{1}{2ax^{3/2}(a+bx)^2}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.85

$$\frac{-8a^3+56a^2bx+175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a+bx)^2}+\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] (-8*a^3 + 56*a^2*b*x + 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a + b*
x)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Maple [A]
time = 0.13, size = 67, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2}{3a^{3}x^{\frac{3}{2}}}+\frac{6b}{a^{4}\sqrt{x}}+\frac{2b^{2}\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	67
default	$-\frac{2}{3a^{3}x^{\frac{3}{2}}}+\frac{6b}{a^{4}\sqrt{x}}+\frac{2b^{2}\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(bx+a)^{2}}+\frac{35\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}$	67
risch	$-\frac{2(-9bx+a)}{3a^{4}x^{\frac{3}{2}}}+\frac{11b^{3}x^{\frac{3}{2}}}{4a^{4}(bx+a)^{2}}+\frac{13b^{2}\sqrt{x}}{4a^{3}(bx+a)^{2}}+\frac{35b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4a^{4}\sqrt{ab}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)+2/a^4*b^2*((11/8*b*x^(3/2)+13/8*a*x^(1/2))
/(b*x+a)^2+35/8/(a*b)^(1/2)*arctan(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 86, normalized size = 0.91

$$\frac{105\,b^{3}x^{3}+175\,ab^{2}x^{2}+56\,a^{2}bx-8\,a^{3}}{12\left(a^{4}b^{2}x^{\frac{7}{2}}+2\,a^{5}bx^{\frac{5}{2}}+a^{6}x^{\frac{3}{2}}\right)}+\frac{35\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
[Out] 1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

Fricas [A]
time = 0.64, size = 250, normalized size = 2.63

$$\left[\frac{105\,(b^{3}x^{4}+2\,ab^{2}x^{3}+a^{2}bx^{2})\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(105\,b^{3}x^{3}+175\,ab^{2}x^{2}+56\,a^{2}bx-8\,a^{3})\sqrt{x}}{24\,(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2})},-\frac{105\,(b^{3}x^{4}+2\,ab^{2}x^{3}+a^{2}bx^{2})\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(105\,b^{3}x^{3}+175\,ab^{2}x^{2}+56\,a^{2}bx-8\,a^{3})\sqrt{x}}{12\,(a^{4}b^{2}x^{4}+2\,a^{5}bx^{3}+a^{6}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sq
rt(x)*sqrt(-b/a) - a)/(b*x + a)) + 2*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*
b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2), -1/12*(105*(b^
3*x^4 + 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x)))
- (105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)*sqrt(x))/(a^4*b^2*x^4
+ 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 869 vs. $2(88) = 176$.
time = 83.77, size = 869, normalized size = 9.15

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{3 a^{3} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ - \frac{16 a^{3} \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} - \frac{105 a^{2} b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{112 a^{2} b x \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} - \frac{210 a b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{350 a b^{2} x^{2} \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{x} - \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} - \frac{105 b^{3} x^{\frac{7}{2}} \log{\left(\sqrt{x} + \sqrt{- \frac{a}{b}} \right)}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} + \frac{210 b^{3} x^{3} \sqrt{- \frac{a}{b}}}{24 a^{6} x^{\frac{3}{2}} \sqrt{- \frac{a}{b}} + 48 a^{5} b x^{\frac{5}{2}} \sqrt{- \frac{a}{b}} + 24 a^{4} b^{2} x^{\frac{7}{2}} \sqrt{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**3*x**(3/2)), Eq(b,
 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-16*a**3*sqrt(-a/b)/(24*a**6*x**(3
/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt
(-a/b)) + 105*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/2)*s
qrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b
)) - 105*a**2*b*x**(3/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-
a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) +
112*a**2*b*x*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*s
qrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 210*a*b**2*x**(5/2)*log(sqr
t(x) - sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-
a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) - 210*a*b**2*x**(5/2)*log(sqrt(x)
+ sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b)
+ 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) + 350*a*b**2*x**2*sqrt(-a/b)/(24*a**6*x
**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*
sqrt(-a/b)) + 105*b**3*x**(7/2)*log(sqrt(x) - sqrt(-a/b))/(24*a**6*x**(3/2)
*sqrt(-a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a
/b)) - 105*b**3*x**(7/2)*log(sqrt(x) + sqrt(-a/b))/(24*a**6*x**(3/2)*sqrt(-
a/b) + 48*a**5*b*x**(5/2)*sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)) +
210*b**3*x**3*sqrt(-a/b)/(24*a**6*x**(3/2)*sqrt(-a/b) + 48*a**5*b*x**(5/2)*
sqrt(-a/b) + 24*a**4*b**2*x**(7/2)*sqrt(-a/b)), True))
```

Giac [A]
time = 0.73, size = 71, normalized size = 0.75

$$\frac{35\, b^{2} \arctan\left(\frac{b \sqrt{x}}{\sqrt{a b}}\right)}{4\, \sqrt{a b}\, a^{4}} + \frac{2\, {\left(9\, b x - a\right)}}{3\, a^{4} x^{\frac{3}{2}}} + \frac{11\, b^{3} x^{\frac{3}{2}} + 13\, a b^{2} \sqrt{x}}{4\, {\left(b x + a\right)}^{2} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

Mupad [B]
time = 0.16, size = 80, normalized size = 0.84

$$\frac{\frac{175\,b^2\,x^2}{12\,a^3}-\frac{2}{3\,a}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}+2\,a\,b\,x^{5/2}}+\frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^3),x)
```

```
[Out] ((175*b^2*x^2)/(12*a^3) - 2/(3*a) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) + 2*a*b*x^(5/2)) + (35*b^(3/2)*atan((b^(1/2)*x
^(1/2))/a^(1/2)))/(4*a^(9/2))
```

3.469 $\int \frac{x^{5/2}}{-a+bx} \, dx$

Optimal. Leaf size=68

$$\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] 2/3*a*x^(3/2)/b^2+2/5*x^(5/2)/b-2*a^(5/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/
b^(7/2)+2*a^2*x^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 65, 214}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 + (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{-a+bx}\,dx &= \frac{2x^{5/2}}{5b}+\frac{a\int \frac{x^{3/2}}{-a+bx}\,dx}{b}\\
&= \frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{a^2\int \frac{\sqrt{x}}{-a+bx}\,dx}{b^2}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{a^3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}+\frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.90

$$\frac{2\sqrt{x}\,(15a^2+5abx+3b^2x^2)}{15b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.79

method	result	size
risch	$\frac{2(3x^2b^2+5abx+15a^2)\sqrt{x}}{15b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	53
derivativedivides	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54
default	$\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)+1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 70, normalized size = 1.03

$$\frac{a^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}+5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) +
2/15*(3*b^2*x^(5/2) + 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

Fricas [A]
time = 0.51, size = 131, normalized size = 1.93

$$\left[\frac{15\,a^2\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3},\ \frac{2\left(15\,a^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) +
2*(3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3, 2/15*(15*a^2*sqrt(-a/b)*arct
an(b*sqrt(x)*sqrt(-a/b)/a) + (3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

Sympy [A]
time = 2.92, size = 117, normalized size = 1.72

$$\begin{cases} \tilde{\infty}x^{\frac{5}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{7}{2}}}{7a} & \text{for}\: b = 0 \\ \frac{2x^{\frac{5}{2}}}{5b} & \text{for}\: a = 0 \\ \frac{a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^4\sqrt{\frac{a}{b}}}-\frac{a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^4\sqrt{\frac{a}{b}}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(7/2)/(7*a), Eq(b, 0)
), (2*x**(5/2)/(5*b), Eq(a, 0)), (a**3*log(sqrt(x) - sqrt(a/b))/(b**4*sqrt(
a/b)) - a**3*log(sqrt(x) + sqrt(a/b))/(b**4*sqrt(a/b)) + 2*a**2*sqrt(x)/b**
3 + 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), True))
```

Giac [A]
time = 0.58, size = 61, normalized size = 0.90

$$\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^3}+\frac{2\left(3\,b^4x^{\frac{5}{2}}+5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2/15*(3*b^4*x^(5/2) +
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

Mupad [B]
time = 0.15, size = 51, normalized size = 0.75

$$\frac{2\,x^{5/2}}{5\,b}+\frac{2\,a\,x^{3/2}}{3\,b^2}+\frac{2\,a^2\,\sqrt{x}}{b^3}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) + (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 + (a^(5/2)*
atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*2i)/b^(7/2)
```

3.470 $\int \frac{x^{3/2}}{-a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b-2*a^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)+2*a*x^(1/2
)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 65, 214}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{-a+bx}\,dx &= \frac{2x^{3/2}}{3b}+\frac{a\int \frac{\sqrt{x}}{-a+bx}\,dx}{b}\\ &= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{a^2\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b^2}\\ &= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.92

$$\frac{2\sqrt{x}\,(3a+bx)}{3b^2}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x),x]
[Out] (2*Sqrt[x]*(3*a + b*x))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

Maple [A]
time = 0.12, size = 43, normalized size = 0.81

method	result	size
risch	$\frac{2(bx+3a)\sqrt{x}}{3b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	41
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}}{b^2}-\frac{2a^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(1/3*b*x^(3/2)+a*x^(1/2))-2*a^2/b^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*
b)^(1/2))
```

Maxima [A]
time = 0.49, size = 58, normalized size = 1.09

$$\frac{a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}+3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2) +
2/3*(b*x^(3/2) + 3*a*sqrt(x))/b^2
```

Fricas [A]
time = 0.55, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(bx+3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(bx+3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(b
*x + 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a)
+ (b*x + 3*a)*sqrt(x))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.
time = 0.70, size = 102, normalized size = 1.92

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0 \wedge b=0 \\ -\frac{2x^{\frac{5}{2}}}{5a} & \text{for } b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ \frac{a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^3\sqrt{\frac{a}{b}}}-\frac{a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^3\sqrt{\frac{a}{b}}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a), Eq(b, 0)
), (2*x**(3/2)/(3*b), Eq(a, 0)), (a**2*log(sqrt(x) - sqrt(a/b))/(b**3*sqrt(
a/b)) - a**2*log(sqrt(x) + sqrt(a/b))/(b**3*sqrt(a/b)) + 2*a*sqrt(x)/b**2 +
 2*x**(3/2)/(3*b), True))
```

Giac [A]
time = 0.55, size = 47, normalized size = 0.89

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^2}+\frac{2\left(b^2x^{\frac{3}{2}}+3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) + 2/3*(b^2*x^(3/2) + 3*
a*b*sqrt(x))/b^3
```

Mupad [B]
time = 0.11, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b}+\frac{2\,a\,\sqrt{x}}{b^2}-\frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) + (2*a*x^(1/2))/b^2 - (2*a^(3/2)*atanh((b^(1/2)*x^(1/2))/
a^(1/2)))/b^(5/2)
```

3.471 $\int \frac{\sqrt{x}}{-a+bx}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)+2*x^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 65, 214}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-a+bx}\,dx = \frac{2\sqrt{x}}{b} + \frac{a\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
default	$\frac{2\sqrt{x}}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32
risch	$\frac{2\sqrt{x}}{b}-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.50, size = 47, normalized size = 1.18

$$\frac{a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b) + 2*sq
rt(x)/b
```

Fricas [A]
time = 0.53, size = 83, normalized size = 2.08

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\sqrt{x}}{b}, \frac{2\left(\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*sqrt(x))/b
, 2*(sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + sqrt(x))/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.
time = 0.32, size = 83, normalized size = 2.08

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a=0\wedge b=0 \\ -\frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{b^2\sqrt{\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{b^2\sqrt{\frac{a}{b}}}+\frac{2\sqrt{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(3/2)/(3*a), Eq(b, 0))
, (2*sqrt(x)/b, Eq(a, 0)), (a*log(sqrt(x) - sqrt(a/b))/(b**2*sqrt(a/b)) - a
*log(sqrt(x) + sqrt(a/b))/(b**2*sqrt(a/b)) + 2*sqrt(x)/b, True))
```

Giac [**A**]
time = 0.57, size = 33, normalized size = 0.82

$$\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) + 2*sqrt(x)/b
```

Mupad [**B**]
time = 0.11, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{x}}{b}-\frac{2\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

3.472 $\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx &= 2\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)),x]
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Maple [A]
time = 0.10, size = 19, normalized size = 0.66

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.48, size = 34, normalized size = 1.17

$$\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="maxima")
[Out] log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/sqrt(a*b)
```

Fricas [A]
time = 0.47, size = 67, normalized size = 2.31

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{ab}, \frac{2\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a))/(a*b), 2*sqrt(-a*
b)*arctan(sqrt(-a*b)/(b*sqrt(x)))/(a*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(29) = 58$.
time = 0.42, size = 68, normalized size = 2.34

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ \frac{\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{b\sqrt{\frac{a}{b}}} - \frac{\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{b\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*sqrt(x)/a, Eq(b, 0)), (-2
/(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - sqrt(a/b))/(b*sqrt(a/b)) - log(sqrt
(x) + sqrt(a/b))/(b*sqrt(a/b)), True))
```

Giac [A]
time = 0.61, size = 20, normalized size = 0.69

$$\frac{2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B]
time = 0.13, size = 19, normalized size = 0.66

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)),x)
```

```
[Out] -(2*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.473 $\int \frac{1}{x^{3/2}(-a+bx)}\,dx$

Optimal. Leaf size=40

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)+2/a/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 65, 214}

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)),x]
```

```
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{3/2}(-a+bx)}\,dx = \frac{2}{a\sqrt{x}} + \frac{b\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a}$$
$$= \frac{2}{a\sqrt{x}} + \frac{(2b)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 1.00

$$\frac{2}{a\sqrt{x}} - \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)),x]
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2}{a\sqrt{x}}-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
default	$\frac{2}{a\sqrt{x}}-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32
risch	$\frac{2}{a\sqrt{x}}-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a),x,method=_RETURNVERBOSE)
[Out] 2/a/x^(1/2)-2*b/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 47, normalized size = 1.18

$$\frac{b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a) + 2/(a
*sqrt(x))
```

Fricas [A]
time = 0.45, size = 91, normalized size = 2.28

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\sqrt{x}}{ax},\frac{2\left(x\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [(x*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*sqrt(x))
/(a*x), 2*(x*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + sqrt(x))/(a*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.
time = 1.02, size = 76, normalized size = 1.90

$$\begin{cases}\frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0\wedge b=0\\ \frac{2}{a\sqrt{x}} & \text{for } b=0\\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0\\ \frac{\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{a\sqrt{\frac{a}{b}}}-\frac{\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{a\sqrt{\frac{a}{b}}}+\frac{2}{a\sqrt{x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2/(a*sqrt(x)), Eq(b, 0)), (
-2/(3*b*x**(3/2)), Eq(a, 0)), (log(sqrt(x) - sqrt(a/b))/(a*sqrt(a/b)) - log
(sqrt(x) + sqrt(a/b))/(a*sqrt(a/b)) + 2/(a*sqrt(x)), True))
```

Giac [A]
time = 0.59, size = 33, normalized size = 0.82

$$\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) + 2/(a*sqrt(x))
```

Mupad [B]
time = 0.06, size = 28, normalized size = 0.70

$$\frac{2}{a\,\sqrt{x}}-\frac{2\,\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)),x)
```

```
[Out] 2/(a*x^(1/2)) - (2*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

3.474 $\int \frac{1}{x^{5/2}(-a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 2/3/a/x^(3/2)-2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)+2*b/a^2/x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 65, 214}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] 2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(-a+bx)}\,dx &= \frac{2}{3ax^{3/2}}+\frac{b\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a^2}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.91

$$\frac{2(a+3bx)}{3a^2x^{3/2}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] (2*(a + 3*b*x))/(3*a^2*x^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

Maple [A]
time = 0.10, size = 43, normalized size = 0.81

method	result	size
risch	$\frac{2bx+\frac{2a}{3}}{a^2x^{\frac{3}{2}}}-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	40
derivativedivides	$-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}+\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}$	43

default	$-\frac{2b^2 \operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}+\frac{2}{3a\,x^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2/a^2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))+2/3/a/x^(3/2)+2*b/a^2
/x^(1/2)
```

Maxima [A]
time = 0.50, size = 55, normalized size = 1.04

$$\frac{b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2) +
2/3*(3*b*x + a)/(a^2*x^(3/2))
```

Fricas [A]
time = 0.38, size = 113, normalized size = 2.13

$$\left[\frac{3\,bx^2\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(3\,bx+a)\sqrt{x}}{3\,a^2x^2},\ \frac{2\left(3\,bx^2\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx+a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) +
2*(3*b*x + a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(-b/a)*arctan(a*sqrt(-b/
a)/(b*sqrt(x))) + (3*b*x + a)*sqrt(x))/(a^2*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

time = 3.43, size = 99, normalized size = 1.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0 \wedge b=0 \\ \frac{2}{3ax^{\frac{3}{2}}} & \text{for } b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0 \\ \frac{2}{3ax^{\frac{3}{2}}} + \frac{b\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{a^2\sqrt{\frac{a}{b}}} - \frac{b\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{a^2\sqrt{\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a*x**(3/2)), Eq(b, 0))
, (-2/(5*b*x**(5/2)), Eq(a, 0)), (2/(3*a*x**(3/2)) + b*log(sqrt(x) - sqrt(a
/b))/(a**2*sqrt(a/b)) - b*log(sqrt(x) + sqrt(a/b))/(a**2*sqrt(a/b)) + 2*b/(
a**2*sqrt(x)), True))
```

Giac [A]
time = 0.51, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^2} + \frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 2/3*(3*b*x + a)/(a^2*
x^(3/2))
```

Mupad [B]
time = 0.12, size = 37, normalized size = 0.70

$$\frac{\frac{2}{3\,a}+\frac{2\,b\,x}{a^2}}{x^{3/2}} - \frac{2\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)),x)
```

```
[Out] (2/(3*a) + (2*b*x)/a^2)/x^(3/2) - (2*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2
)))/a^(5/2)
```

3.475 $\int \frac{1}{x^{7/2}(-a+bx)}\,dx$

Optimal. Leaf size=68

$$\frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] 2/5/a/x^(5/2)+2/3*b/a^2/x^(3/2)-2*b^(5/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/
a^(7/2)+2*b^2/a^3/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 65, 214}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] 2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) + (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}(-a+bx)}\,dx &= \frac{2}{5ax^{5/2}}+\frac{b\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{a}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{b^2\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a^2}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{b^3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a^3}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{(2b^3)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\ &= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.90

$$\frac{2(3a^2+5abx+15b^2x^2)}{15a^3x^{5/2}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(-a + b*x)),x]
[Out] (2*(3*a^2 + 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.79

method	result	size
risch	$\frac{2x^2b^2+\frac{2}{3}abx+\frac{2}{5}a^2}{a^3x^{\frac{5}{2}}}-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}$	53
derivativedivides	$-\frac{2b^3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}+\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}$	54

default	$-\frac{2b^3 \operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}+\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x-a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3/a^3/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))+2/5/a/x^(5/2)+2/3*b/a
^2/x^(3/2)+2*b^2/a^3/x^(1/2)
```

Maxima [A]
time = 0.50, size = 68, normalized size = 1.00

$$\frac{b^3 \log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{2\,(15\,b^2x^2+5\,abx+3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3) +
2/15*(15*b^2*x^2 + 5*a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Fricas [A]
time = 0.41, size = 143, normalized size = 2.10

$$\left[\frac{15\,b^2x^3\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(15\,b^2x^2+5\,abx+3\,a^2)\sqrt{x}}{15\,a^3x^3},\ \frac{2\left(15\,b^2x^3\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(15\,b^2x^2+5\,abx+3\,a^2)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*b^2*x^3*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)
) + 2*(15*b^2*x^2 + 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3*s
qrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 + 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

Sympy [A]

time = 14.83, size = 117, normalized size = 1.72

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for } a = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} + \frac{2b}{3a^2x^{\frac{3}{2}}} + \frac{b^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{a^3\sqrt{\frac{a}{b}}} - \frac{b^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{a^3\sqrt{\frac{a}{b}}} + \frac{2b^2}{a^3\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2/(5*a*x**(5/2)), Eq(b, 0))
, (-2/(7*b*x**(7/2)), Eq(a, 0)), (2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2))
+ b**2*log(sqrt(x) - sqrt(a/b))/(a**3*sqrt(a/b)) - b**2*log(sqrt(x) + sqrt(
a/b))/(a**3*sqrt(a/b)) + 2*b**2/(a**3*sqrt(x)), True))
```

Giac [A]
time = 0.50, size = 54, normalized size = 0.79

$$\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^3} + \frac{2\,(15\,b^2x^2 + 5\,abx + 3\,a^2)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/15*(15*b^2*x^2 + 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

Mupad [B]
time = 0.13, size = 48, normalized size = 0.71

$$\frac{\frac{2}{5\,a} + \frac{2\,b^2\,x^2}{a^3} + \frac{2\,b\,x}{3\,a^2}}{x^{5/2}} - \frac{2\,b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(7/2)*(a - b*x)),x)
```

```
[Out] (2/(5*a) + (2*b^2*x^2)/a^3 + (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atanh((b
^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

3.476 $\int \frac{x^{5/2}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

```
[Out] 5/3*x^(3/2)/b^2+x^(5/2)/b/(-b*x+a)-5*a^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2
))/b^(7/2)+5*a*x^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) + x^(5/2)/(b*(a - b*x)) - (5*a^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^2}\,dx &= \frac{x^{5/2}}{b(a-bx)}+\frac{5\int \frac{x^{3/2}}{-a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b^2}\\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b^3}\\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 70, normalized size = 1.00

$$\frac{\sqrt{x}\,(-15a^2+10abx+2b^2x^2)}{3b^3(-a+bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(-15*a^2 + 10*a*b*x + 2*b^2*x^2))/(3*b^3*(-a + b*x)) - (5*a^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Maple [A]
time = 0.12, size = 60, normalized size = 0.86

method	result	size
risch	$\frac{2(bx+6a)\sqrt{x}}{3b^3}+\frac{a^2\left(-\frac{\sqrt{x}}{bx-a}-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^3}$	57
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^3}-\frac{2a^2\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	60
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^3}-\frac{2a^2\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/3*b*x^(3/2)+2*a*x^(1/2))-2/b^3*a^2*(-1/2*x^(1/2)/(-b*x+a)+5/2/(a*b
)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.49, size = 81, normalized size = 1.16

$$-\frac{a^2\sqrt{x}}{b^4x-ab^3}+\frac{5\,a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{2\left(bx^{\frac{3}{2}}+6\,a\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x - a*b^3) + 5/2*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(
x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2/3*(b*x^(3/2) + 6*a*sqrt(x))/b^3
```

Fricas [A]
time = 0.44, size = 167, normalized size = 2.39

$$\left[\frac{15\,(abx-a^2)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,b^2x^2+10\,abx-15\,a^2)\sqrt{x}}{6\,(b^4x-ab^3)},\ \frac{15\,(abx-a^2)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(2\,b^2x^2+10\,abx-15\,a^2)\sqrt{x}}{3\,(b^4x-ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x - a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x
 - a)) + 2*(2*b^2*x^2 + 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3), 1/3*(1
5*(a*b*x - a^2)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b^2*x^2 + 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(63) = 126$.
time = 15.26, size = 354, normalized size = 5.06

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^2} & \text{for } a = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^2} & \text{for } b = 0 \\ -\frac{15a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} + \frac{15a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} - \frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} + \frac{15a^2bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} - \frac{15a^2bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} + \frac{20ab^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} + \frac{4b^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{-6ab^4\sqrt{\frac{a}{b}}+6b^5x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**2), Eq(a,
0)), (2*x**(7/2)/(7*a**2), Eq(b, 0)), (-15*a**3*log(sqrt(x) - sqrt(a/b))/(-
6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 15*a**3*log(sqrt(x) + sqrt(a/b))
/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) - 30*a**2*b*sqrt(x)*sqrt(a/b)/(
-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 15*a**2*b*x*log(sqrt(x) - sqrt(
a/b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) - 15*a**2*b*x*log(sqrt(x)
+ sqrt(a/b))/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 20*a*b**2*x**(3/2
)*sqrt(a/b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)) + 4*b**3*x**(5/2)*sq
rt(a/b)/(-6*a*b**4*sqrt(a/b) + 6*b**5*x*sqrt(a/b)), True))
```

Giac [A]
time = 0.92, size = 69, normalized size = 0.99

$$\frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^3} - \frac{a^2\sqrt{x}}{(bx-a)b^3} + \frac{2\left(b^4x^{\frac{3}{2}} + 6\,ab^3\sqrt{x}\right)}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) - a^2*sqrt(x)/((b*x - a
)*b^3) + 2/3*(b^4*x^(3/2) + 6*a*b^3*sqrt(x))/b^6
```

Mupad [B]
time = 0.07, size = 61, normalized size = 0.87

$$\frac{2\,x^{3/2}}{3\,b^2} + \frac{4\,a\,\sqrt{x}}{b^3} + \frac{a^2\,\sqrt{x}}{a\,b^3 - b^4\,x} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) + (4*a*x^(1/2))/b^3 + (a^2*x^(1/2))/(a*b^3 - b^4*x) + (
a^(3/2)*atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*5i)/b^(7/2)
```

3.477 $\int \frac{x^{3/2}}{(-a+bx)^2} \, dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} - \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

```
[Out] x^(3/2)/b/(-b*x+a)-3*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)+3*x^(
1/2)/b^2
```

Rubi [A]

time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}} + \frac{x^{3/2}}{b(a-bx)} + \frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 + x^(3/2)/(b*(a - b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(-a+bx)^2}\,dx &= \frac{x^{3/2}}{b(a-bx)}+\frac{3\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b}\\ &= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}+\frac{(3a)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b^2}\\ &= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}+\frac{(3a)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{3\sqrt{x}}{b^2}+\frac{x^{3/2}}{b(a-bx)}-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 56, normalized size = 0.98

$$\frac{\sqrt{x}\,(-3a+2bx)}{b^2(-a+bx)}-\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(-3*a + 2*b*x))/(b^2*(-a + b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/b^(5/2)
```

Maple [**A**]
time = 0.13, size = 48, normalized size = 0.84

method	result	size

derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	48
default	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{2(-bx+a)}+\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^2}$	48
risch	$\frac{2\sqrt{x}}{b^2}+\frac{a\left(-\frac{\sqrt{x}}{bx-a}-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^2-2*a/b^2*(-1/2*x^(1/2)/(-b*x+a)+3/2/(a*b)^(1/2)*arctanh(b*x^(1
/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 68, normalized size = 1.19

$$-\frac{a\sqrt{x}}{b^3x-ab^2}+\frac{3\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a*sqrt(x)/(b^3*x - a*b^2) + 3/2*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) +
 sqrt(a*b)))/(sqrt(a*b)*b^2) + 2*sqrt(x)/b^2
```

Fricas [A]
time = 0.44, size = 138, normalized size = 2.42

$$\left[\frac{3\,(bx-a)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,bx-3\,a)\sqrt{x}}{2\,(b^3x-ab^2)},\frac{3\,(bx-a)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(2\,bx-3\,a)\sqrt{x}}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x - a)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)
) + 2*(2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2), (3*(b*x - a)*sqrt(-a/b)*arcta
n(b*sqrt(x)*sqrt(-a/b)/a) + (2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(49) = 98$.
time = 3.76, size = 301, normalized size = 5.28

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b^2} & \text{for } a = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \text{for } b = 0 \\ -\frac{3a^2 \log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}} + 2b^4x\sqrt{\frac{a}{b}}} + \frac{3a^2 \log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}} + 2b^4x\sqrt{\frac{a}{b}}} - \frac{6ab\sqrt{x}\sqrt{\frac{a}{b}}}{-2ab^3\sqrt{\frac{a}{b}} + 2b^4x\sqrt{\frac{a}{b}}} + \frac{3abx \log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}} + 2b^4x\sqrt{\frac{a}{b}}} - \frac{3abx \log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{-2ab^3\sqrt{\frac{a}{b}} + 2b^4x\sqrt{\frac{a}{b}}} + \frac{4b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{-2ab^3\sqrt{\frac{a}{b}} + 2b^4x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**2, Eq(a, 0)), (
2*x**(5/2)/(5*a**2), Eq(b, 0)), (-3*a**2*log(sqrt(x) - sqrt(a/b))/(-2*a*b**
3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 3*a**2*log(sqrt(x) + sqrt(a/b))/(-2*a*b
**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) - 6*a*b*sqrt(x)*sqrt(a/b)/(-2*a*b**3*sq
rt(a/b) + 2*b**4*x*sqrt(a/b)) + 3*a*b*x*log(sqrt(x) - sqrt(a/b))/(-2*a*b**3
*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) - 3*a*b*x*log(sqrt(x) + sqrt(a/b))/(-2*a*b
**3*sqrt(a/b) + 2*b**4*x*sqrt(a/b)) + 4*b**2*x**(3/2)*sqrt(a/b)/(-2*a*b**3*
sqrt(a/b) + 2*b**4*x*sqrt(a/b)), True))
```

Giac [A]
time = 0.59, size = 51, normalized size = 0.89

$$\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^2} - \frac{a\sqrt{x}}{(bx-a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 3*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - a*sqrt(x)/((b*x - a)*b^
2) + 2*sqrt(x)/b^2
```

Mupad [B]
time = 0.11, size = 47, normalized size = 0.82

$$\frac{2\,\sqrt{x}}{b^2} + \frac{a\,\sqrt{x}}{a\,b^2 - b^3\,x} - \frac{3\,\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 - b^3*x) - (3*a^(1/2)*atanh((b^(1/2)*x
^(1/2))/a^(1/2)))/b^(5/2)
```

3.478 $\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

```
[Out] -arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)+x^(1/2)/b/(-b*x+a)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {43, 65, 214}

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2)
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx = \frac{\sqrt{x}}{b(a-bx)} + \frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b}$$
$$= \frac{\sqrt{x}}{b(a-bx)} + \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}}{b(a-bx)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 1.04

$$-\frac{\sqrt{x}}{b(-a+bx)} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^2,x]
[Out] -(Sqrt[x]/(b*(-a + b*x))) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(
3/2))
```

Maple [A]
time = 0.12, size = 38, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\sqrt{x}}{b(-bx+a)} - \frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	38
default	$\frac{\sqrt{x}}{b(-bx+a)} - \frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
[Out] x^(1/2)/b/(-b*x+a)-1/b/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 1.19

$$-\frac{\sqrt{x}}{b^2x-ab} + \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
[Out] -sqrt(x)/(b^2*x - a*b) + 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.42, size = 123, normalized size = 2.62

$$\left[-\frac{2\,ab\sqrt{x} - \sqrt{ab}\,(bx-a)\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{2\,(ab^3x-a^2b^2)}, -\frac{ab\sqrt{x} - \sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab^3x-a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a - 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a*b^3*x - a^2*b^2), -(a*b*sqrt(x) - sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a*b^3*x - a^2*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(37) = 74$.
time = 1.63, size = 243, normalized size = 5.17

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for}\: a = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for}\: b = 0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2ab^2\sqrt{\frac{a}{b}}+2b^3x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**2,x)
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0))
, (2*x**(3/2)/(3*a**2), Eq(b, 0)), (-a*log(sqrt(x) - sqrt(a/b))/(-2*a*b**2*
sqrt(a/b) + 2*b**3*x*sqrt(a/b)) + a*log(sqrt(x) + sqrt(a/b))/(-2*a*b**2*sqr
t(a/b) + 2*b**3*x*sqrt(a/b)) - 2*b*sqrt(x)*sqrt(a/b)/(-2*a*b**2*sqrt(a/b) +
 2*b**3*x*sqrt(a/b)) + b*x*log(sqrt(x) - sqrt(a/b))/(-2*a*b**2*sqrt(a/b) +
2*b**3*x*sqrt(a/b)) - b*x*log(sqrt(x) + sqrt(a/b))/(-2*a*b**2*sqrt(a/b) + 2
*b**3*x*sqrt(a/b)), True))
```

Giac [A]
time = 0.94, size = 40, normalized size = 0.85

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b} - \frac{\sqrt{x}}{(bx-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) - sqrt(x)/((b*x - a)*b)
```

Mupad [B]
time = 0.11, size = 35, normalized size = 0.74

$$\frac{\sqrt{x}}{b\,\left(a-b\,x\right)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^2,x)
```

```
[Out] x^(1/2)/(b*(a - b*x)) - atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2))
```

3.479 $\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{x}}{a(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

```
[Out] arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)+x^(1/2)/a/(-b*x+a)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {44, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx = \frac{\sqrt{x}}{a(a-bx)} - \frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} - \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{\sqrt{x}}{a(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.00

$$\frac{\sqrt{x}}{a(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{x}}{a(-bx+a)} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37
default	$\frac{\sqrt{x}}{a(-bx+a)} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/a/(-b*x+a)+1/a/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 56, normalized size = 1.22

$$-\frac{\sqrt{x}}{abx-a^2} - \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(a*b*x - a^2) - 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*a)
```

Fricas [A]
time = 0.42, size = 122, normalized size = 2.65

$$\left[-\frac{2\,ab\sqrt{x} - \sqrt{ab}\,(bx-a)\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)}{2\,(a^2b^2x-a^3b)}, -\frac{ab\sqrt{x} + \sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{a^2b^2x-a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a + 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a^2*b^2*x - a^3*b), -(a*b*sqrt(x) + sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a^2*b^2*x - a^3*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(37) = 74.
time = 2.64, size = 252, normalized size = 5.48

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for}\ b = 0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for}\ a = 0 \\ \frac{a\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{2b\sqrt{x}\sqrt{\frac{a}{b}}}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} - \frac{bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} + \frac{bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{-2a^2b\sqrt{\frac{a}{b}}+2ab^2x\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (a*log(sqrt(x) - sqrt(a/b))/(-2*a**2*b*sq
rt(a/b) + 2*a*b**2*x*sqrt(a/b)) - a*log(sqrt(x) + sqrt(a/b))/(-2*a**2*b*sqr
t(a/b) + 2*a*b**2*x*sqrt(a/b)) - 2*b*sqrt(x)*sqrt(a/b)/(-2*a**2*b*sqrt(a/b)
 + 2*a*b**2*x*sqrt(a/b)) - b*x*log(sqrt(x) - sqrt(a/b))/(-2*a**2*b*sqrt(a/b
) + 2*a*b**2*x*sqrt(a/b)) + b*x*log(sqrt(x) + sqrt(a/b))/(-2*a**2*b*sqrt(a/
b) + 2*a*b**2*x*sqrt(a/b)), True))
```

Giac [A]
time = 1.34, size = 41, normalized size = 0.89

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a} - \frac{\sqrt{x}}{(bx-a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="giac")
[Out] -arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) - sqrt(x)/((b*x - a)*a)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}}{a\,\left(a-b\,x\right)}+\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^2),x)
[Out] x^(1/2)/(a*(a - b*x)) + atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

3.480 $\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 3*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)-3/a^2/x^(1/2)+1/a/(-b*x+
a)/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {44, 53, 65, 214}

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx &= \frac{1}{a\sqrt{x}\,(a-bx)}-\frac{3\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^2}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}-\frac{(3b)\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.96

$$\frac{-2a+3bx}{a^2\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a + 3*b*x)/(a^2*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.84

method	result	size

derivativedivides	$-\frac{2}{a^{2}\sqrt{x}}+\frac{2b\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}$	48
default	$-\frac{2}{a^{2}\sqrt{x}}+\frac{2b\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}$	48
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{b\left(\frac{\sqrt{x}}{bx-a}-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^{2}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2/x^(1/2)+2*b/a^2*(1/2*x^(1/2)/(-b*x+a)+3/2/(a*b)^(1/2)*arctanh(b*x^(1
/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 69, normalized size = 1.21

$$-\frac{3\,bx-2\,a}{a^{2}bx^{\frac{3}{2}}-a^{3}\sqrt{x}}-\frac{3\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x - 2*a)/(a^2*b*x^(3/2) - a^3*sqrt(x)) - 3/2*b*log((b*sqrt(x) - sqrt(
a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.39, size = 151, normalized size = 2.65

$$\left[\frac{3\,(bx^{2}-ax)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(3\,bx-2\,a)\sqrt{x}}{2\,(a^{2}bx^{2}-a^{3}x)},\,-\frac{3\,(bx^{2}-ax)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx-2\,a)\sqrt{x}}{a^{2}bx^{2}-a^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 - a*x)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x
- a)) - 2*(3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 - a^3*x), -(3*(b*x^2 - a*x)*sqr
t(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (3*b*x - 2*a)*sqrt(x))/(a^2*b*x^
2 - a^3*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(51) = 102$.
time = 7.12, size = 354, normalized size = 6.21

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{2}{5 b^{2} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{3 a \sqrt{x} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} + \frac{3 a \sqrt{x} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} - \frac{4 a \sqrt{\frac{a}{b}}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} + \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{x} - \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} - \frac{3 b x^{\frac{3}{2}} \log{\left(\sqrt{x} + \sqrt{\frac{a}{b}} \right)}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} + \frac{6 b x \sqrt{\frac{a}{b}}}{2 a^{3} \sqrt{x} \sqrt{\frac{a}{b}} - 2 a^{2} b x^{\frac{3}{2}} \sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)
), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-3*a*sqrt(x)*log(sqrt(x) - sqrt(a/b))
/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)) + 3*a*sqrt(x)*log
(sqrt(x) + sqrt(a/b))/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/
b)) - 4*a*sqrt(a/b)/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)
) + 3*b*x**(3/2)*log(sqrt(x) - sqrt(a/b))/(2*a**3*sqrt(x)*sqrt(a/b) - 2*a**
2*b*x**(3/2)*sqrt(a/b)) - 3*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(2*a**3*sqr
t(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)) + 6*b*x*sqrt(a/b)/(2*a**3*sqr
t(x)*sqrt(a/b) - 2*a**2*b*x**(3/2)*sqrt(a/b)), True))
```

Giac [A]
time = 1.03, size = 52, normalized size = 0.91

$$-\frac{3 \, b \arctan\left(\frac{b \sqrt{x}}{\sqrt{-a b}}\right)}{\sqrt{-a b} a^{2}} - \frac{3 \, b x - 2 \, a}{{\left(b x^{\frac{3}{2}} - a \sqrt{x}\right)} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) - (3*b*x - 2*a)/((b*x^(3
/2) - a*sqrt(x))*a^2)
```

Mupad [B]
time = 0.07, size = 49, normalized size = 0.86

$$\frac{3\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2}{a}-\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}-b\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^2),x)
```

```
[Out] (3*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/a - (3*b*x)/a^2)/
(a*x^(1/2) - b*x^(3/2))
```

3.481 $\int \frac{1}{x^{5/2}(-a+bx)^2} \, dx$

Optimal. Leaf size=70

$$-\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[Out] -5/3/a^2/x^(3/2)+1/a/x^(3/2)/(-b*x+a)+5*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(
1/2))/a^(7/2)-5*b/a^3/x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 214}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{5b}{a^3\sqrt{x}} - \frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) - (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a - b*x)) + (5*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a-bx)} - \frac{5\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{2a}\\
&= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a^2}\\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^3}\\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3}\\
&= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 0.99

$$\frac{-2a^2-10abx+15b^2x^2}{3a^3x^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a^2 - 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a - b*x)) + (5*b^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.11, size = 59, normalized size = 0.84

method	result	size

risch	$-\frac{2(6bx+a)}{3a^{3}x^{\frac{3}{2}}}-\frac{b^{2}\left(\frac{\sqrt{x}}{bx-a}-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{a^{3}}$	56
derivativedivides	$\frac{2b^{2}\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}-\frac{2}{3a^{2}x^{\frac{3}{2}}}-\frac{4b}{a^{3}\sqrt{x}}$	59
default	$\frac{2b^{2}\left(\frac{\sqrt{x}}{-2bx+2a}+\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}}-\frac{2}{3a^{2}x^{\frac{3}{2}}}-\frac{4b}{a^{3}\sqrt{x}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/a^3*(1/2*x^(1/2)/(-b*x+a)+5/2/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/
2)))-2/3/a^2/x^(3/2)-4*b/a^3/x^(1/2)
```

Maxima [A]
time = 0.56, size = 82, normalized size = 1.17

$$-\frac{15\,b^{2}x^{2}-10\,abx-2\,a^{2}}{3\left(a^{3}bx^{\frac{5}{2}}-a^{4}x^{\frac{3}{2}}\right)}-\frac{5\,b^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*b^2*x^2 - 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) - a^4*x^(3/2)) - 5/2*b^
2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

Fricas [A]
time = 0.40, size = 187, normalized size = 2.67

$$\left[\frac{15\,(b^{2}x^{3}-abx^{2})\sqrt{\frac{b}{a}}\,\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(15\,b^{2}x^{2}-10\,abx-2\,a^{2})\sqrt{x}}{6\,(a^{3}bx^{3}-a^{4}x^{2})},\ -\frac{15\,(b^{2}x^{3}-abx^{2})\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(15\,b^{2}x^{2}-10\,abx-2\,a^{2})\sqrt{x}}{3\,(a^{3}bx^{3}-a^{4}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 - a*b*x^2)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a
)/(b*x - a)) - 2*(15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*
x^2), -1/3*(15*(b^2*x^3 - a*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x
))) + (15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(65) = 130$.

time = 30.59, size = 416, normalized size = 5.94

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{7b^2x^{\frac{7}{2}}} & \text{for}\ a = 0 \\ -\frac{2}{3a^2x^{\frac{3}{2}}} & \text{for}\ b = 0 \\ -\frac{4a^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15abx^{\frac{3}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{20abx\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30b^2x^2\sqrt{\frac{a}{b}}}{6a^4x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 6a^3bx^{\frac{5}{2}}\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*a**2*sqrt(a/b)/(6*a**4*x**(3/2)
*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) - 15*a*b*x**(3/2)*log(sqrt(x) - s
qrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) + 15*a*
b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x
**(5/2)*sqrt(a/b)) - 20*a*b*x*sqrt(a/b)/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3
*b*x**(5/2)*sqrt(a/b)) + 15*b**2*x**(5/2)*log(sqrt(x) - sqrt(a/b))/(6*a**4*
x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)) - 15*b**2*x**(5/2)*log(sq
rt(x) + sqrt(a/b))/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*sqrt(a/b)
) + 30*b**2*x**2*sqrt(a/b)/(6*a**4*x**(3/2)*sqrt(a/b) - 6*a**3*b*x**(5/2)*s
qrt(a/b)), True))
```

Giac [A]

time = 1.37, size = 61, normalized size = 0.87

$$-\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^3} - \frac{b^2\sqrt{x}}{(bx-a)a^3} - \frac{2\,(6\,bx+a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -5*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) - b^2*sqrt(x)/((b*x -
a)*a^3) - 2/3*(6*b*x + a)/(a^3*x^(3/2))
```

Mupad [B]

time = 0.14, size = 60, normalized size = 0.86

$$\frac{5\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{\frac{2}{3\,a} - \frac{5\,b^2\,x^2}{a^3} + \frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2} - b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^2),x)
```

```
[Out] (5*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2) - (2/(3*a) - (5*b^2*x^
2)/a^3 + (10*b*x)/(3*a^2))/(a*x^(3/2) - b*x^(5/2))
```

3.482 $\int \frac{x^{7/2}}{(-a+bx)^3} \, dx$

Optimal. Leaf size=97

$$\frac{35a\sqrt{x}}{4b^4} + \frac{35x^{3/2}}{12b^3} - \frac{x^{7/2}}{2b(a-bx)^2} + \frac{7x^{5/2}}{4b^2(a-bx)} - \frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

```
[Out] 35/12*x^(3/2)/b^3-1/2*x^(7/2)/b/(-b*x+a)^2+7/4*x^(5/2)/b^2/(-b*x+a)-35/4*a^
(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(9/2)+35/4*a*x^(1/2)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}} + \frac{35a\sqrt{x}}{4b^4} + \frac{7x^{5/2}}{4b^2(a-bx)} - \frac{x^{7/2}}{2b(a-bx)^2} + \frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a - b*x)^2)
+ (7*x^(5/2))/(4*b^2*(a - b*x)) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{(-a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7\int \frac{x^{5/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35\int \frac{x^{3/2}}{-a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^3}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a^2)\operatorname{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.85

$$\frac{\sqrt{x}\,(105a^3-175a^2bx+56ab^2x^2+8b^3x^3)}{12b^4(a-bx)^2}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(-a + b*x)^3,x]
[Out] (Sqrt[x]*(105*a^3 - 175*a^2*b*x + 56*a*b^2*x^2 + 8*b^3*x^3))/(12*b^4*(a - b
*x)^2) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(9/2))
```

Maple [A]
time = 0.11, size = 69, normalized size = 0.71

method	result	size
risch	$\frac{2(bx+9a)\sqrt{x}}{3b^4}+\frac{a^2\left(\frac{-\frac{13b\,x^{\frac{3}{2}}}{4}+\frac{11a\sqrt{x}}{4}}{(bx-a)^2}-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^4}$	67
derivativedivides	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}-\frac{2a^2\left(\frac{\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	69
default	$\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^4}-\frac{2a^2\left(\frac{\frac{13b\,x^{\frac{3}{2}}}{8}-\frac{11a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^4*(1/3*b*x^(3/2)+3*a*x^(1/2))-2/b^4*a^2*((13/8*b*x^(3/2)-11/8*a*x^(1/2)
)/(-b*x+a)^2+35/8/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.61, size = 103, normalized size = 1.06

$$-\frac{13\,a^2bx^{\frac{3}{2}}-11\,a^3\sqrt{x}}{4\,(b^6x^2-2\,ab^5x+a^2b^4)}+\frac{35\,a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^4}+\frac{2\left(bx^{\frac{3}{2}}+9\,a\sqrt{x}\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) - 11*a^3*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4) +
35/8*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^
4) + 2/3*(b*x^(3/2) + 9*a*sqrt(x))/b^4
```

Fricas [A]
time = 0.39, size = 227, normalized size = 2.34

$$\left[\frac{105\,(ab^2x^2-2\,a^2bx+a^3)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(8\,b^3x^3+56\,ab^2x^2-175\,a^2bx+105\,a^3)\sqrt{x}}{24\,(b^6x^2-2\,ab^5x+a^2b^4)},\ \frac{105\,(ab^2x^2-2\,a^2bx+a^3)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(8\,b^3x^3+56\,ab^2x^2-175\,a^2bx+105\,a^3)\sqrt{x}}{12\,(b^6x^2-2\,ab^5x+a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*s
qrt(a/b) + a)/(b*x - a)) + 2*(8*b^3*x^3 + 56*a*b^2*x^2 - 175*a^2*b*x + 105*
a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^3*x^3 + 56*a*b
^2*x^2 - 175*a^2*b*x + 105*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(87) = 174$.

time = 105.64, size = 695, normalized size = 7.16

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^3} & \text{for}\: a = 0 \\ -\frac{2x^{\frac{9}{2}}}{9a^3} & \text{for}\: b = 0 \\ \frac{105a^4\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{105a^4\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3b\sqrt{x}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{210a^3bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{210a^3bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{350a^2b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{105a^2b^2x^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} - \frac{105a^2b^2x^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{112ab^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} + \frac{16b^4x^{\frac{7}{2}}\sqrt{\frac{a}{b}}}{24a^2b^5\sqrt{\frac{a}{b}}-48ab^6x\sqrt{\frac{a}{b}}+24b^7x^2\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*b**3), Eq(a,
0)), (-2*x**(9/2)/(9*a**3), Eq(b, 0)), (105*a**4*log(sqrt(x) - sqrt(a/b))/(
24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) -
105*a**4*log(sqrt(x) + sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqr
t(a/b) + 24*b**7*x**2*sqrt(a/b)) + 210*a**3*b*sqrt(x)*sqrt(a/b)/(24*a**2*b*
*5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 210*a**3*b
*x*log(sqrt(x) - sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b)
 + 24*b**7*x**2*sqrt(a/b)) + 210*a**3*b*x*log(sqrt(x) + sqrt(a/b))/(24*a**2
*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 350*a**
2*b**2*x**(3/2)*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) +
 24*b**7*x**2*sqrt(a/b)) + 105*a**2*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(24*
a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) - 105
*a**2*b**2*x**2*log(sqrt(x) + sqrt(a/b))/(24*a**2*b**5*sqrt(a/b) - 48*a*b**
6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 112*a*b**3*x**(5/2)*sqrt(a/b)/(24
*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) + 24*b**7*x**2*sqrt(a/b)) + 16
*b**4*x**(7/2)*sqrt(a/b)/(24*a**2*b**5*sqrt(a/b) - 48*a*b**6*x*sqrt(a/b) +
24*b**7*x**2*sqrt(a/b)), True))
```

Giac [**A**]
time = 0.72, size = 81, normalized size = 0.84

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,b^4} - \frac{13\,a^2bx^{\frac{3}{2}} - 11\,a^3\sqrt{x}}{4\,(bx-a)^2b^4} + \frac{2\left(b^6x^{\frac{3}{2}} + 9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^4) - 1/4*(13*a^2*b*x^(3
/2) - 11*a^3*sqrt(x))/((b*x - a)^2*b^4) + 2/3*(b^6*x^(3/2) + 9*a*b^5*sqrt(x
))/b^9
```

Mupad [B]
time = 0.14, size = 83, normalized size = 0.86

$$\frac{\frac{11\,a^3\,\sqrt{x}}{4}-\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4-2\,a\,b^5\,x+b^6\,x^2}+\frac{2\,x^{3/2}}{3\,b^3}+\frac{6\,a\,\sqrt{x}}{b^4}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,35{}\mathrm{i}}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(7/2)/(a - b*x)^3,x)
```

```
[Out] ((11*a^3*x^(1/2))/4 - (13*a^2*b*x^(3/2))/4)/(a^2*b^4 + b^6*x^2 - 2*a*b^5*x)
 + (2*x^(3/2))/(3*b^3) + (6*a*x^(1/2))/b^4 + (a^(3/2)*atan((b^(1/2)*x^(1/2)
*1i)/a^(1/2))*35i)/(4*b^(9/2))
```

3.483 $\int \frac{x^{5/2}}{(-a+bx)^3} \, dx$

Optimal. Leaf size=84

$$\frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

```
[Out] -1/2*x^(5/2)/b/(-b*x+a)^2+5/4*x^(3/2)/b^2/(-b*x+a)-15/4*arctanh(b^(1/2)*x^(
1/2)/a^(1/2))*a^(1/2)/b^(7/2)+15/4*x^(1/2)/b^3
```

Rubi [A]

time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 52, 65, 214}

$$-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a - b*x)^2) + (5*x^(3/2))/(4*b^2*(a -
b*x)) - (15*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5\int \frac{x^{3/2}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{15\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^2}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}+\frac{(15a)\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 71, normalized size = 0.85

$$\frac{\sqrt{x}\,(15a^2-25abx+8b^2x^2)}{4b^3(a-bx)^2}-\frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(15*a^2 - 25*a*b*x + 8*b^2*x^2))/(4*b^3*(a - b*x)^2) - (15*Sqrt[a]
*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Maple [A]
time = 0.14, size = 57, normalized size = 0.68

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	57
default	$\frac{2\sqrt{x}}{b^3}-\frac{2a\left(\frac{\frac{9b\,x^{\frac{3}{2}}}{8}-\frac{7a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	57
risch	$\frac{2\sqrt{x}}{b^3}+\frac{a\left(\frac{-\frac{9b\,x^{\frac{3}{2}}}{4}+\frac{7a\sqrt{x}}{4}}{(bx-a)^2}-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{b^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)/b^3-2/b^3*a*((9/8*b*x^(3/2)-7/8*a*x^(1/2))/(-b*x+a)^2+15/8/(a*b)^
(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 90, normalized size = 1.07

$$-\frac{9\,abx^{\frac{3}{2}}-7\,a^2\sqrt{x}}{4\,(b^5x^2-2\,ab^4x+a^2b^3)}+\frac{15\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/(b^5*x^2 - 2*a*b^4*x + a^2*b^3) + 15/8
*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2
*sqrt(x)/b^3
```

Fricas [A]
time = 0.41, size = 199, normalized size = 2.37

$$\left[\frac{15\,(b^2x^2-2\,abx+a^2)\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(8\,b^2x^2-25\,abx+15\,a^2)\sqrt{x}}{8\,(b^5x^2-2\,ab^4x+a^2b^3)},\ \frac{15\,(b^2x^2-2\,abx+a^2)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+(8\,b^2x^2-25\,abx+15\,a^2)\sqrt{x}}{4\,(b^5x^2-2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/
b) + a)/(b*x - a)) + 2*(8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2 -
2*a*b^4*x + a^2*b^3), 1/4*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a/b)*arctan(b
*sqrt(x)*sqrt(-a/b)/a) + (8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
- 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(73) = 146$.
time = 40.13, size = 624, normalized size = 7.43

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^3} & \text{for}\: a = 0 \\ \frac{15a^3\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{15a^3\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{30a^2b\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{30a^2bx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{30a^2bx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{50ab^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{15ab^2x^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} - \frac{15ab^2x^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} + \frac{16b^3x^{\frac{5}{2}}\sqrt{\frac{a}{b}}}{8a^2b^4\sqrt{\frac{a}{b}}-16ab^5x\sqrt{\frac{a}{b}}+8b^6x^2\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(7/2)/(7*a**3), Eq(b,
0)), (2*sqrt(x)/b**3, Eq(a, 0)), (15*a**3*log(sqrt(x) - sqrt(a/b))/(8*a**2*
b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 15*a**3*l
og(sqrt(x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*
b**6*x**2*sqrt(a/b)) + 30*a**2*b*sqrt(x)*sqrt(a/b)/(8*a**2*b**4*sqrt(a/b) -
 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 30*a**2*b*x*log(sqrt(x) -
 sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sq
rt(a/b)) + 30*a**2*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16
*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 50*a*b**2*x**(3/2)*sqrt(a/b)
/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) +
15*a*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*
x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b)) - 15*a*b**2*x**2*log(sqrt(x) + sqrt(a/
b))/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b) + 8*b**6*x**2*sqrt(a/b))
 + 16*b**3*x**(5/2)*sqrt(a/b)/(8*a**2*b**4*sqrt(a/b) - 16*a*b**5*x*sqrt(a/b
) + 8*b**6*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.75, size = 63, normalized size = 0.75

$$\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,b^3} + \frac{2\,\sqrt{x}}{b^3} - \frac{9\,abx^{\frac{3}{2}} - 7\,a^2\sqrt{x}}{4\,(bx-a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2*sqrt(x)/b^3 - 1/4*
(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/((b*x - a)^2*b^3)
```

Mupad [B]
time = 0.06, size = 69, normalized size = 0.82

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4}-\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3-2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 - (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 - 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

3.484 $\int \frac{x^{3/2}}{(-a+bx)^3} \, dx$

Optimal. Leaf size=72

$$-\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

```
[Out] -1/2*x^(3/2)/b/(-b*x+a)^2-3/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/b^(5/2)/a^(1
/2)+3/4*x^(1/2)/b^2/(-b*x+a)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {43, 65, 214}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{x^{3/2}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] -1/2*x^(3/2)/(b*(a - b*x)^2) + (3*Sqrt[x])/(4*b^2*(a - b*x)) - (3*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(-a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx}{4b}\\ &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8b^2}\\ &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\ &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 0.83

$$\frac{\sqrt{x}\,(3a-5bx)}{4b^2(a-bx)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(3*a - 5*b*x))/(4*b^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Maple [A]
time = 0.10, size = 51, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2\left(\frac{5x^{\frac{3}{2}}}{8b}-\frac{3a\sqrt{x}}{8b^2}\right)}{(-bx+a)^2}-\frac{3\,\text{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	51
default	$-\frac{2\left(\frac{5x^{\frac{3}{2}}}{8b}-\frac{3a\sqrt{x}}{8b^2}\right)}{(-bx+a)^2}-\frac{3\,\text{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4b^2\sqrt{ab}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(5/8*x^(3/2)/b-3/8*a*x^(1/2)/b^2)/(-b*x+a)^2-3/4/b^2/(a*b)^(1/2)*arctanh
(b*x^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.56, size = 78, normalized size = 1.08

$$-\frac{5\,bx^{\frac{3}{2}} - 3\,a\sqrt{x}}{4\,(b^4x^2 - 2\,ab^3x + a^2b^2)} + \frac{3\,\log\left(\frac{b\sqrt{x} - \sqrt{ab}}{b\sqrt{x} + \sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) - 3*a*sqrt(x))/(b^4*x^2 - 2*a*b^3*x + a^2*b^2) + 3/8*log(
(b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 0.47, size = 186, normalized size = 2.58

$$\left[\frac{3\,(b^2x^2 - 2\,abx + a^2)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right) - 2\,(5\,ab^2x - 3\,a^2b)\sqrt{x}}{8\,(ab^5x^2 - 2\,a^2b^4x + a^3b^3)}, \frac{3\,(b^2x^2 - 2\,abx + a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right) - (5\,ab^2x - 3\,a^2b)\sqrt{x}}{4\,(ab^5x^2 - 2\,a^2b^4x + a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) - 2*(5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x
 + a^3*b^3), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) - (5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x + a^
3*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(61) = 122.
time = 17.91, size = 552, normalized size = 7.67

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{5}{2}}}{5a^3} & \text{for } b = 0 \\ -\frac{2}{b^3\sqrt{x}} & \text{for } a = 0 \\ \frac{3a^2\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} + \frac{6ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} - \frac{6abx\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} + \frac{6abx\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} - \frac{10b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} + \frac{3b^2x^2\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} - \frac{3b^2x^2\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{8a^2b^3\sqrt{\frac{a}{b}} - 16ab^4x\sqrt{\frac{a}{b}} + 8b^5x^2\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a**3), Eq(b,
0)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (3*a**2*log(sqrt(x) - sqrt(a/b))/(8*a**
2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 3*a**2*
log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8
*b**5*x**2*sqrt(a/b)) + 6*a*b*sqrt(x)*sqrt(a/b)/(8*a**2*b**3*sqrt(a/b) - 16
*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 6*a*b*x*log(sqrt(x) - sqrt(a
```

```
/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)
) + 6*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*s
qrt(a/b) + 8*b**5*x**2*sqrt(a/b)) - 10*b**2*x**(3/2)*sqrt(a/b)/(8*a**2*b**3
*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)) + 3*b**2*x**2*l
og(sqrt(x) - sqrt(a/b))/(8*a**2*b**3*sqrt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*
b**5*x**2*sqrt(a/b)) - 3*b**2*x**2*log(sqrt(x) + sqrt(a/b))/(8*a**2*b**3*sq
rt(a/b) - 16*a*b**4*x*sqrt(a/b) + 8*b**5*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.68, size = 51, normalized size = 0.71

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,b^2}-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\,(bx-a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - 1/4*(5*b*x^(3/2) - 3*a*
sqrt(x))/((b*x - a)^2*b^2)
```

Mupad [B]
time = 0.14, size = 58, normalized size = 0.81

$$-\frac{\frac{5\,x^{3/2}}{4\,b}-\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x)^3,x)
```

```
[Out] - ((5*x^(3/2))/(4*b) - (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2))
```

3.485 $\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

```
[Out] 1/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)-1/2*x^(1/2)/b/(-b*x+a)
^2+1/4*x^(1/2)/a/b/(-b*x+a)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {43, 44, 65, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[x]/(b*(a - b*x)^2) + Sqrt[x]/(4*a*b*(a - b*x)) + ArcTanh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx}{4b}\\ &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8ab}\\ &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\operatorname{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4ab}\\ &= -\frac{\sqrt{x}}{2b(a-bx)^2}+\frac{\sqrt{x}}{4ab(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 0.80

$$-\frac{\sqrt{x}\,(a+bx)}{4ab(a-bx)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^3,x]
[Out] -1/4*(Sqrt[x]*(a + b*x))/(a*b*(a - b*x)^2) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]]/(4*a^(3/2)*b^(3/2))
```

Maple [A]
time = 0.11, size = 53, normalized size = 0.71

method	result	size

derivativedivides	$-\frac{2\left(\frac{x^{\frac{3}{2}}}{8a}+\frac{\sqrt{x}}{8b}\right)}{(-bx+a)^{2}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	53
default	$-\frac{2\left(\frac{x^{\frac{3}{2}}}{8a}+\frac{\sqrt{x}}{8b}\right)}{(-bx+a)^{2}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4ab\sqrt{ab}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/8/a*x^(3/2)+1/8*x^(1/2)/b)/(-b*x+a)^2+1/4/a/b/(a*b)^(1/2)*arctanh(b*x
^(1/2)/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 80, normalized size = 1.07

$$-\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\left(ab^{3}x^{2}-2\,a^{2}b^{2}x+a^{3}b\right)}-\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^(3/2) + a*sqrt(x))/(a*b^3*x^2 - 2*a^2*b^2*x + a^3*b) - 1/8*log((b
*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 0.42, size = 183, normalized size = 2.44

$$\left[\frac{(b^{2}x^{2}-2\,abx+a^{2})\sqrt{ab}\,\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)-2\,(ab^{2}x+a^{2}b)\sqrt{x}}{8\,(a^{2}b^{4}x^{2}-2\,a^{3}b^{3}x+a^{4}b^{2})},\ -\frac{(b^{2}x^{2}-2\,abx+a^{2})\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+(ab^{2}x+a^{2}b)\sqrt{x}}{4\,(a^{2}b^{4}x^{2}-2\,a^{3}b^{3}x+a^{4}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a + 2*sqrt(a*b)*sqrt(x
))/(b*x - a)) - 2*(a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a
^4*b^2), -1/4*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b*sq
rt(x))) + (a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a^4*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(58) = 116.
time = 8.19, size = 575, normalized size = 7.67

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{3b^{3}x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ -\frac{2x^{\frac{3}{2}}}{3a^{3}} & \text{for}\: b = 0 \\ -\frac{a^{2}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}+\frac{a^{2}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}-\frac{2ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}+\frac{2abx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}-\frac{2abx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}-\frac{2b^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}-\frac{b^{2}x^{2}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}}+\frac{b^{2}x^{2}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^{3}b^{2}\sqrt{\frac{a}{b}}-16a^{2}b^{3}x\sqrt{\frac{a}{b}}+8ab^{4}x^{2}\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b**3*x**(3/2)), Eq(a,
 0)), (-2*x**(3/2)/(3*a**3), Eq(b, 0)), (-a**2*log(sqrt(x) - sqrt(a/b))/(8*
a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) +
 a**2*log(sqrt(x) + sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt
(a/b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*a*b*sqrt(x)*sqrt(a/b)/(8*a**3*b**2*sqr
t(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) + 2*a*b*x*log(
sqrt(x) - sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*
a*b**4*x**2*sqrt(a/b)) - 2*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**3*b**2*sqrt
(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) - 2*b**2*x**(3/
2)*sqrt(a/b)/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x
**2*sqrt(a/b)) - b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**3*b**2*sqrt(a/b)
- 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**4*x**2*sqrt(a/b)) + b**2*x**2*log(sqrt(
x) + sqrt(a/b))/(8*a**3*b**2*sqrt(a/b) - 16*a**2*b**3*x*sqrt(a/b) + 8*a*b**
4*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.61, size = 55, normalized size = 0.73

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,ab}-\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\,(bx-a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] -1/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a*b) - 1/4*(b*x^(3/2) + a*sqr
t(x))/((b*x - a)^2*a*b)
```

Mupad [B]
time = 0.14, size = 57, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}-\frac{\frac{x^{3/2}}{4\,a}+\frac{\sqrt{x}}{4\,b}}{a^2-2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x)^3,x)
```

```
[Out] atanh((b^(1/2)*x^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(3/2)/(4*a) + x^(
1/2)/(4*b))/(a^2 + b^2*x^2 - 2*a*b*x)
```

3.486 $\int \frac{1}{\sqrt{x}(-a+bx)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{x}}{2a(a-bx)^2}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

```
[Out] -3/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)-1/2*x^(1/2)/a/(-b*x+a
)^2-3/4*x^(1/2)/a^2/(-b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {44, 65, 214}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] -1/2*Sqrt[x]/(a*(a - b*x)^2) - (3*Sqrt[x])/(4*a^2*(a - b*x)) - (3*ArcTanh[(
Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(-a+bx)^3}\,dx = -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)^2}\,dx}{4a}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^2}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} + \frac{3\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^2}$$
$$= -\frac{\sqrt{x}}{2a(a-bx)^2} - \frac{3\sqrt{x}}{4a^2(a-bx)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.83

$$\frac{\sqrt{x}\,(-5a+3bx)}{4a^2(a-bx)^2} - \frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] (Sqrt[x]*(-5*a + 3*b*x))/(4*a^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2a(-bx+a)^2} - \frac{3\left(\frac{\sqrt{x}}{2a(-bx+a)} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{2a}$	61
default	$-\frac{\sqrt{x}}{2a(-bx+a)^2} - \frac{3\left(\frac{\sqrt{x}}{2a(-bx+a)} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{2a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/a/(-b*x+a)^2-3/2/a*(1/2*x^(1/2)/a/(-b*x+a)+1/2/a/(a*b)^(1/2)*a
rctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 77, normalized size = 1.07

$$\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(a^2b^2x^2-2\,a^3bx+a^4)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) - 5*a*sqrt(x))/(a^2*b^2*x^2 - 2*a^3*b*x + a^4) + 3/8*log((
b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 0.44, size = 185, normalized size = 2.57

$$\left[\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)+2\,(3\,ab^2x-5\,a^2b)\sqrt{x}}{8\,(a^3b^3x^2-2\,a^4b^2x+a^5b)},\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+(3\,ab^2x-5\,a^2b)\sqrt{x}}{4\,(a^3b^3x^2-2\,a^4b^2x+a^5b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) + 2*(3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2
*x + a^5*b), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) + (3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*x +
a^5*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(63) = 126.
time = 12.33, size = 580, normalized size = 8.06

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{5b^3x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{2\sqrt{x}}{a^3} & \text{for}\: b = 0 \\ \frac{3a^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{3a^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{10ab\sqrt{x}\sqrt{\frac{a}{b}}}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{6abx\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{6abx\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{6b^2x^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} + \frac{3b^2x^2\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} - \frac{3b^2x^2\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^4b\sqrt{\frac{a}{b}}-16a^3b^2x\sqrt{\frac{a}{b}}+8a^2b^3x^2\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**3*x**(5/2)), Eq(a,
 0)), (-2*sqrt(x)/a**3, Eq(b, 0)), (3*a**2*log(sqrt(x) - sqrt(a/b))/(8*a**4
*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 3*a
```

```
**2*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b)
 + 8*a**2*b**3*x**2*sqrt(a/b)) - 10*a*b*sqrt(x)*sqrt(a/b)/(8*a**4*b*sqrt(a/
b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 6*a*b*x*log(s
qrt(x) - sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2
*b**3*x**2*sqrt(a/b)) + 6*a*b*x*log(sqrt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b
) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) + 6*b**2*x**(3/2
)*sqrt(a/b)/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x*
*2*sqrt(a/b)) + 3*b**2*x**2*log(sqrt(x) - sqrt(a/b))/(8*a**4*b*sqrt(a/b) -
16*a**3*b**2*x*sqrt(a/b) + 8*a**2*b**3*x**2*sqrt(a/b)) - 3*b**2*x**2*log(sq
rt(x) + sqrt(a/b))/(8*a**4*b*sqrt(a/b) - 16*a**3*b**2*x*sqrt(a/b) + 8*a**2*
b**3*x**2*sqrt(a/b)), True))
```

Giac [A]
time = 0.96, size = 51, normalized size = 0.71

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^2}+\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(bx-a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 1/4*(3*b*x^(3/2) - 5*a*
sqrt(x))/((b*x - a)^2*a^2)
```

Mupad [B]
time = 0.13, size = 58, normalized size = 0.81

$$-\frac{\frac{5\,\sqrt{x}}{4\,a}-\frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)^3),x)
```

```
[Out] - ((5*x^(1/2))/(4*a) - (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

3.487 $\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx$

Optimal. Leaf size=84

$$\frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*arctanh(b^(1/2)*x^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)+15/4/a^3/x^(1/2)-1/2
/a/(-b*x+a)^2/x^(1/2)-5/4/a^2/(-b*x+a)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 214}

$$-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] 15/(4*a^3*Sqrt[x]) - 1/(2*a*Sqrt[x]*(a - b*x)^2) - 5/(4*a^2*Sqrt[x]*(a - b*
x)) - (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx &= -\frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx}{4a} \\
&= -\frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{15\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^2} \\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{(15b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^3} \\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} + \frac{(15b)\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4a^3} \\
&= \frac{15}{4a^3\sqrt{x}} - \frac{1}{2a\sqrt{x}\,(a-bx)^2} - \frac{5}{4a^2\sqrt{x}\,(a-bx)} - \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.85

$$
\frac{8a^2-25abx+15b^2x^2}{4a^3\sqrt{x}\,(a-bx)^2} - \frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] (8*a^2 - 25*a*b*x + 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a - b*x)^2) - (15*Sqrt[b]*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Maple [A]
time = 0.12, size = 57, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{2b\left(\frac{-\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(-bx+a)^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}+\frac{2}{a^{3}\sqrt{x}}$	57
default	$-\frac{2b\left(\frac{-\frac{7b\,x^{\frac{3}{2}}}{8}+\frac{9a\sqrt{x}}{8}}{(-bx+a)^{2}}+\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}+\frac{2}{a^{3}\sqrt{x}}$	57
risch	$\frac{2}{a^{3}\sqrt{x}}+\frac{b\left(\frac{\frac{7b\,x^{\frac{3}{2}}}{4}-\frac{9a\sqrt{x}}{4}}{(bx-a)^{2}}-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^3*b*((-7/8*b*x^(3/2)+9/8*a*x^(1/2))/(-b*x+a)^2+15/8/(a*b)^(1/2)*arctan
h(b*x^(1/2)/(a*b)^(1/2)))+2/a^3/x^(1/2)
```

Maxima [A]
time = 0.52, size = 90, normalized size = 1.07

$$\frac{15\,b^{2}x^{2}-25\,abx+8\,a^{2}}{4\left(a^{3}b^{2}x^{\frac{5}{2}}-2\,a^{4}bx^{\frac{3}{2}}+a^{5}\sqrt{x}\right)}+\frac{15\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(15*b^2*x^2 - 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) - 2*a^4*b*x^(3/2) + a^
5*sqrt(x)) + 15/8*b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(s
qrt(a*b)*a^3)
```

Fricas [A]
time = 0.44, size = 213, normalized size = 2.54

$$\left[\frac{15\,(b^{2}x^{3}-2\,abx^{2}+a^{2}x)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(15\,b^{2}x^{2}-25\,abx+8\,a^{2})\sqrt{x}}{8\,(a^{3}b^{2}x^{3}-2\,a^{4}bx^{2}+a^{5}x)},\ \frac{15\,(b^{2}x^{3}-2\,abx^{2}+a^{2}x)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(15\,b^{2}x^{2}-25\,abx+8\,a^{2})\sqrt{x}}{4\,(a^{3}b^{2}x^{3}-2\,a^{4}bx^{2}+a^{5}x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqr
t(b/a) + a)/(b*x - a)) + 2*(15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*b^
2*x^3 - 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(-b
/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 - 2*a^4*b*x^2 + a^5*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(75) = 150$.
time = 35.10, size = 716, normalized size = 8.52

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{7b^3x^{\frac{7}{2}}} & \text{for } a = 0 \\ \frac{2}{a^3\sqrt{x}} & \text{for } b = 0 \\ \frac{15a^2\sqrt{x}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15a^2\sqrt{x}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{16a^2\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{50abx\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x}-\sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} - \frac{15b^2x^{\frac{5}{2}}\log\left(\sqrt{x}+\sqrt{\frac{a}{b}}\right)}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} + \frac{30b^2x^2\sqrt{\frac{a}{b}}}{8a^5\sqrt{x}\sqrt{\frac{a}{b}}-16a^4bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}+8a^3b^2x^{\frac{5}{2}}\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**3*x**(7/2)), Eq(a,
 0)), (2/(a**3*sqrt(x)), Eq(b, 0)), (15*a**2*sqrt(x)*log(sqrt(x) - sqrt(a/b
))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x
**(5/2)*sqrt(a/b)) - 15*a**2*sqrt(x)*log(sqrt(x) + sqrt(a/b))/(8*a**5*sqrt(
x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b
)) + 16*a**2*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(
a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) - 30*a*b*x**(3/2)*log(sqrt(x) - sqrt
(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b*
*2*x**(5/2)*sqrt(a/b)) + 30*a*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(8*a**5*s
qrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt
(a/b)) - 50*a*b*x*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*
sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)) + 15*b**2*x**(5/2)*log(sqrt(x)
- sqrt(a/b))/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a
**3*b**2*x**(5/2)*sqrt(a/b)) - 15*b**2*x**(5/2)*log(sqrt(x) + sqrt(a/b))/(8
*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/
2)*sqrt(a/b)) + 30*b**2*x**2*sqrt(a/b)/(8*a**5*sqrt(x)*sqrt(a/b) - 16*a**4*
b*x**(3/2)*sqrt(a/b) + 8*a**3*b**2*x**(5/2)*sqrt(a/b)), True))
```

Giac [A]
time = 1.19, size = 63, normalized size = 0.75

$$\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^3} + \frac{2}{a^3\sqrt{x}} + \frac{7\,b^2x^{\frac{3}{2}} - 9\,ab\sqrt{x}}{4\,(bx-a)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/(a^3*sqrt(x)) + 1/
4*(7*b^2*x^(3/2) - 9*a*b*sqrt(x))/((b*x - a)^2*a^3)
```

Mupad [B]
time = 0.15, size = 69, normalized size = 0.82

$$\frac{\frac{2}{a}+\frac{15\,b^2\,x^2}{4\,a^3}-\frac{25\,b\,x}{4\,a^2}}{a^2\,\sqrt{x}+b^2\,x^{5/2}-2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)^3),x)
```

```
[Out] (2/a + (15*b^2*x^2)/(4*a^3) - (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2)
- 2*a*b*x^(3/2)) - (15*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2)
)
```

3.488 $\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx$

Optimal. Leaf size=97

$$\frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

```
[Out] 35/12/a^3/x^(3/2)-1/2/a/x^(3/2)/(-b*x+a)^2-7/4/a^2/x^(3/2)/(-b*x+a)-35/4*b^
(3/2)*arctanh(b^(1/2)*x^(1/2)/a^(1/2))/a^(9/2)+35/4*b/a^4/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {44, 53, 65, 214}

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] 35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) - 1/(2*a*x^(3/2)*(a - b*x)^2)
- 7/(4*a^2*x^(3/2)*(a - b*x)) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx &= -\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx}{4a}\\ &= -\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{35\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{8a^2}\\ &= \frac{35}{12a^3x^{3/2}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{(35b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^3}\\ &= \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{(35b^2)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^4}\\ &= \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{(35b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\right.}{4a^4}\\ &= \frac{35}{12a^3x^{3/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{1}{2ax^{3/2}(a-bx)^2}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.85

$$\frac{8a^3+56a^2bx-175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a-bx)^2}-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] (8*a^3 + 56*a^2*b*x - 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a - b*x
)^2) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

Maple [A]
time = 0.13, size = 68, normalized size = 0.70

method	result	size
risch	$\frac{6bx+\frac{2a}{3}}{a^4x^{\frac{3}{2}}}+\frac{b^2\left(\frac{\frac{11b\,x^{\frac{3}{2}}}{4}-\frac{13a\sqrt{x}}{4}}{(bx-a)^2}-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}}\right)}{a^4}$	66
derivativedivides	$\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}-\frac{2b^2\left(\frac{-\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	68
default	$\frac{2}{3a^3x^{\frac{3}{2}}}+\frac{6b}{a^4\sqrt{x}}-\frac{2b^2\left(\frac{-\frac{11b\,x^{\frac{3}{2}}}{8}+\frac{13a\sqrt{x}}{8}}{(-bx+a)^2}+\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)-2/a^4*b^2*((-11/8*b*x^(3/2)+13/8*a*x^(1/2))
/(-b*x+a)^2+35/8/(a*b)^(1/2)*arctanh(b*x^(1/2)/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 103, normalized size = 1.06

$$\frac{105\,b^3x^3-175\,ab^2x^2+56\,a^2bx+8\,a^3}{12\left(a^4b^2x^{\frac{7}{2}}-2\,a^5bx^{\frac{5}{2}}+a^6x^{\frac{3}{2}}\right)}+\frac{35\,b^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)/(a^4*b^2*x^(7/2) -
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/8*b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sq
rt(x) + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 0.43, size = 249, normalized size = 2.57

$$\left[\frac{105\,(b^3x^4-2\,ab^2x^3+a^2bx^2)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(105\,b^3x^3-175\,ab^2x^2+56\,a^2bx+8\,a^3)\sqrt{x}}{24\,(a^4b^2x^4-2\,a^5bx^3+a^6x^2)},\ \frac{105\,(b^3x^4-2\,ab^2x^3+a^2bx^2)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(105\,b^3x^3-175\,ab^2x^2+56\,a^2bx+8\,a^3)\sqrt{x}}{12\,(a^4b^2x^4-2\,a^5bx^3+a^6x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*log((b*x - 2*a*sqr
t(x)*sqrt(b/a) + a)/(b*x - a)) + 2*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*
x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(b^3*x
^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) +
 (105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 -
 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(88) = 176$.
time = 94.62, size = 799, normalized size = 8.24

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{3a^3x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{9b^3x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ \frac{16a^3\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} + \frac{105a^2bx^{\frac{3}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} - \frac{105a^2bx^{\frac{3}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} + \frac{112a^2bx\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} - \frac{210ab^2x^{\frac{5}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} + \frac{210ab^2x^{\frac{5}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} - \frac{350ab^2x^2\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} + \frac{105b^3x^{\frac{7}{2}}\log\left(\sqrt{x} - \sqrt{\frac{a}{b}}\right)}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} - \frac{105b^3x^{\frac{7}{2}}\log\left(\sqrt{x} + \sqrt{\frac{a}{b}}\right)}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} + \frac{210b^3x^3\sqrt{\frac{a}{b}}}{24a^6x^{\frac{3}{2}}\sqrt{\frac{a}{b}} - 48a^5bx^{\frac{5}{2}}\sqrt{\frac{a}{b}} + 24a^4b^2x^{\frac{7}{2}}\sqrt{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a**3*x**(3/2)), Eq(b,
0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (16*a**3*sqrt(a/b)/(24*a**6*x**(3/2)
*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)
) + 105*a**2*b*x**(3/2)*log(sqrt(x) - sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b
) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) - 105*a
**2*b*x**(3/2)*log(sqrt(x) + sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a*
*5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 112*a**2*b*x*s
qrt(a/b)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a*
*4*b**2*x**(7/2)*sqrt(a/b)) - 210*a*b**2*x**(5/2)*log(sqrt(x) - sqrt(a/b))/
(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x
**(7/2)*sqrt(a/b)) + 210*a*b**2*x**(5/2)*log(sqrt(x) + sqrt(a/b))/(24*a**6*
x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*s
qrt(a/b)) - 350*a*b**2*x**2*sqrt(a/b)/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5
*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 105*b**3*x**(7/2
)*log(sqrt(x) - sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)
*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a/b)) - 105*b**3*x**(7/2)*log(sqrt(
x) + sqrt(a/b))/(24*a**6*x**(3/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b)
+ 24*a**4*b**2*x**(7/2)*sqrt(a/b)) + 210*b**3*x**3*sqrt(a/b)/(24*a**6*x**(3
/2)*sqrt(a/b) - 48*a**5*b*x**(5/2)*sqrt(a/b) + 24*a**4*b**2*x**(7/2)*sqrt(a
/b)), True))
```

Giac [A]
time = 1.46, size = 73, normalized size = 0.75

$$\frac{35\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^4} + \frac{2\,(9\,bx + a)}{3\,a^4x^{\frac{3}{2}}} + \frac{11\,b^3x^{\frac{3}{2}} - 13\,ab^2\sqrt{x}}{4\,(bx - a)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^4) + 2/3*(9*b*x + a)/(a
^4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) - 13*a*b^2*sqrt(x))/((b*x - a)^2*a^4)
```

Mupad [B]
time = 0.17, size = 80, normalized size = 0.82

$$\frac{\frac{2}{3\,a}-\frac{175\,b^2\,x^2}{12\,a^3}+\frac{35\,b^3\,x^3}{4\,a^4}+\frac{14\,b\,x}{3\,a^2}}{a^2\,x^{3/2}+b^2\,x^{7/2}-2\,a\,b\,x^{5/2}}-\frac{35\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)^3),x)
```

```
[Out] (2/(3*a) - (175*b^2*x^2)/(12*a^3) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) - 2*a*b*x^(5/2)) - (35*b^(3/2)*atanh((b^(1/2)*
x^(1/2))/a^(1/2)))/(4*a^(9/2))
```

3.489 $\int x^{5/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=122

$$\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

```
[Out] -5/64*a^4*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-5/96*a^2*x^(3/2)*(
b*x+a)^(1/2)/b^2+1/24*a*x^(5/2)*(b*x+a)^(1/2)/b+1/4*x^(7/2)*(b*x+a)^(1/2)+5
/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a + b*x])/(96*
b^2) + (a*x^(5/2)*Sqrt[a + b*x])/(24*b) + (x^(7/2)*Sqrt[a + b*x])/4 - (5*a^
4*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^2)\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{48b}$$

$$= -\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}+\frac{(5a^3)\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b^2}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\int}{}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\,\mathrm{Su}}{}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{(5a^4)\,\mathrm{Su}}{}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}-\frac{5a^4\tanh}{}$$

Mathematica [A]
time = 0.10, size = 88, normalized size = 0.72

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(15a^3-10a^2bx+8ab^2x^2+48b^3x^3)+15a^4\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3*
x^3) + 15*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(192*b^(7/2))
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.05

method	result
risch	$\frac{(48b^3x^3+8ab^2x^2-10a^2bx+15a^3)\sqrt{x}\,\sqrt{bx+a}}{192b^3}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{128b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}}{4b}-\frac{5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+a)^(3/2)-5/8*a/b*(1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/2*a/b*(1
/2/b*x^(1/2)*(b*x+a)^(3/2)-1/4*a/b*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))
^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1
/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(88) = 176.
time = 0.50, size = 178, normalized size = 1.46

$$\frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}+\frac{73\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{55\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^7-\frac{4\,(bx+a)b^6}{x}+\frac{6\,(bx+a)^2b^5}{x^2}-\frac{4\,(bx+a)^3b^4}{x^3}+\frac{(bx+a)^4b^3}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) + 73*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) - 55*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^7 - 4*(b*x + a)*b^6/x + 6*(b*x + a)^2*b^5/x^2 - 4*(b*x +
a)^3*b^4/x^3 + (b*x + a)^4*b^3/x^4)
```

Fricas [A]
time = 0.41, size = 162, normalized size = 1.33

$$\left[\frac{15\,a^{4}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(48\,b^{4}x^{3}+8\,ab^{3}x^{2}-10\,a^{2}b^{2}x+15\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{384\,b^{4}}, \frac{15\,a^{4}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(48\,b^{4}x^{3}+8\,ab^{3}x^{2}-10\,a^{2}b^{2}x+15\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{192\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^4, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^4]
```

Sympy [A]
time = 24.06, size = 153, normalized size = 1.25

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*a**(5/2)*x**(3/2)/(192*b**
2*sqrt(1 + b*x/a)) - a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*sqrt(a)*x
**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b*
*(7/2)) + b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
```

,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(1/2), x)
```

3.490 $\int x^{3/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)+1/12*a*x^(3/2)*(b*x+
a)^(1/2)/b+1/3*x^(5/2)*(b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] -1/8*(a^2*Sqrt[x]*Sqrt[a + b*x])/b^2 + (a*x^(3/2)*Sqrt[a + b*x])/(12*b) + (
x^(5/2)*Sqrt[a + b*x])/3 + (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
8*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\sqrt{a+bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\
&= \frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}-\frac{a^2\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\int\frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^2}}\,dx,x,\right.}{8b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{}}{\sqrt{a+}}\right.}{8b^2}\\
&= -\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.79

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(-3a^2+2abx+8b^2x^2)-3a^3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) - 3*a^3*Log[-
(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(24*b^(5/2))
```

Maple [A]
time = 0.12, size = 106, normalized size = 1.08

method	result	size
risch	$-\frac{(-8x^2b^2-2abx+3a^2)\sqrt{x}\,\sqrt{bx+a}}{24b^2}+\frac{a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}{3b}-\frac{a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/2*a/b*(1/2/b*x^(1/2)*(b*x+a)^(3/2)-1/4*a/b*(x
^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*
a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(70) = 140$.
time = 0.50, size = 146, normalized size = 1.49

$$-\frac{a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{5}{2}}}-\frac{\frac{3\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}+\frac{8\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^5-\frac{3\,(bx+a)b^4}{x}+\frac{3\,(bx+a)^2b^3}{x^2}-\frac{(bx+a)^3b^2}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(5/2) - 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) + 8*(b*x + a)^(3/2
)*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x + a)*b^4/x +
 3*(b*x + a)^2*b^3/x^2 - (b*x + a)^3*b^2/x^3)
```

Fricas [A]
time = 0.47, size = 141, normalized size = 1.44

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2+2\,ab^2x-3\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^3},\,-\frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^3x^2+2\,ab^2x-3\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [**A**]
time = 5.99, size = 122, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}+\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(1/2),x)
```

```
[Out] -a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - a**(3/2)*x**(3/2)/(24*b*sqrt(1
 + b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(5/2)) + b*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(1/2), x)
```

3.491 $\int \sqrt{x} \sqrt{a+bx} \, dx$

Optimal. Leaf size=74

$$\frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}$$

```
[Out] -1/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)+1/2*x^(3/2)*(b*x+a)
^(1/2)+1/4*a*x^(1/2)*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (a*Sqrt[x]*Sqrt[a + b*x])/(4*b) + (x^(3/2)*Sqrt[a + b*x])/2 - (a^2*ArcTanh[
(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\sqrt{a+bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b}\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{4b}\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b}\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.85

$$\frac{\sqrt{x}\sqrt{a+bx}\,(a+2bx)}{4b}+\frac{a^2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(a + 2*b*x))/(4*b) + (a^2*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/(4*b^(3/2))
```

Maple [A]
time = 0.14, size = 84, normalized size = 1.14

method	result	size

risch	$\frac{(2bx+a)\sqrt{x}\,\sqrt{bx+a}}{4b}-\frac{a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	74
default	$\frac{\sqrt{x}\,(bx+a)^{\frac{3}{2}}}{2b}-\frac{a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*x^(1/2)*(b*x+a)^(3/2)-1/4*a/b*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a)
)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(
1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(52) = 104$.
time = 0.49, size = 108, normalized size = 1.46

$$\frac{a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{bx+a}\,a^2b}{\sqrt{x}}+\frac{(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^3-\frac{2\,(bx+a)b^2}{x}+\frac{(bx+a)^2b}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqr
t(x)))/b^(3/2) + 1/4*(sqrt(b*x + a)*a^2*b/sqrt(x) + (b*x + a)^(3/2)*a^2/x^(
3/2))/(b^3 - 2*(b*x + a)*b^2/x + (b*x + a)^2*b/x^2)
```

Fricas [A]
time = 0.46, size = 114, normalized size = 1.54

$$\left[\frac{a^2\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^2x+ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b^2},\frac{a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(2\,b^2x+ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [A]
time = 1.96, size = 97, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)/(4*b*sqrt(1 + b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 + b*x
/a)) - a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) + b*x**(5/2)/(2*sqr
t(a)*sqrt(1 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.15, size = 52, normalized size = 0.70

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)\,\sqrt{a+b\,x}-\frac{a^2\,\ln\left(a+2\,b\,x+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{a+b\,x}\right)}{8\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + a/(4*b))*(a + b*x)^(1/2) - (a^2*log(a + 2*b*x + 2*b^(1/2)*x^
(1/2)*(a + b*x)^(1/2)))/(8*b^(3/2))
```

3.492 $\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=44

$$\sqrt{x}\,\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

```
[Out] a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)+x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\sqrt{x}\,\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b
]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx &= \sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}a\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx \\
&= \sqrt{x}\,\sqrt{a+bx}+a\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right) \\
&= \sqrt{x}\,\sqrt{a+bx}+a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right) \\
&= \sqrt{x}\,\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 1.07

$$\sqrt{x}\,\sqrt{a+bx}-\frac{a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[a + b*x] - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

Maple [A]
time = 0.12, size = 62, normalized size = 1.41

method	result	size
default	$\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}$	62
risch	$\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2
*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.50, size = 70, normalized size = 1.59

$$-\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx+a}\,a}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt
(x)))/sqrt(b) - sqrt(b*x + a)*a/((b - (b*x + a)/x)*sqrt(x))
```

Fricas [A]
time = 0.46, size = 93, normalized size = 2.11

$$\left[\frac{a\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b},\,-\frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

Sympy [A]
time = 1.07, size = 42, normalized size = 0.95

$$\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.68, size = 41, normalized size = 0.93

$$\sqrt{x}\,\sqrt{a+b\,x}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a + b*x)^(1/2) + (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2) - a
^(1/2))))/b^(1/2)
```

3.493 $\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2\sqrt{a+bx}}{\sqrt{x}}+2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))*b^(1/2)-2*(b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b
*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\
&= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+(2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+(2b)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= -\frac{2\sqrt{a+bx}}{\sqrt{x}}+2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 47, normalized size = 1.04

$$-\frac{2\sqrt{a+bx}}{\sqrt{x}}-2\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(3/2),x]
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*
x]]
```

Maple [**A**]
time = 0.11, size = 61, normalized size = 1.36

method	result	size
risch	$-\frac{2\sqrt{bx+a}}{\sqrt{x}}+\frac{\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x+a)^(1/2)/x^(1/2)+b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*
(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.51, size = 54, normalized size = 1.20

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)/sqrt(x)
```

Fricas [A]
time = 0.49, size = 89, normalized size = 1.98

$$\left[\frac{\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,\sqrt{x}}{x},\,-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 0.82, size = 68, normalized size = 1.51

$$-\frac{2\sqrt{a}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}+2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{2b\sqrt{x}}{\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt
(a)) - 2*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/x^(3/2), x)
```

3.494 $\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

```
[Out] -2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 1.00

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.12, size = 49, normalized size = 2.33

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
risch	$-\frac{2(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	16
default	$-\frac{\sqrt{bx+a}}{x^{\frac{3}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^{2}\sqrt{x}}\right)}{2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1/2)/x^(3/2)-1/2*a*(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1
/2)/a^2/x^(1/2))
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A]
time = 0.43, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 0.79, size = 41, normalized size = 1.95

$$-\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.93, size = 33, normalized size = 1.57

$$-\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{3\,((bx+a)b-ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(b*x + a)^(3/2)*b^4/(((b*x + a)*b - a*b)^(3/2)*a*abs(b))
```

Mupad [B]
time = 0.24, size = 21, normalized size = 1.00

$$-\frac{\left(\frac{2\,b\,x}{3\,a}+\frac{2}{3}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(5/2),x)
```

```
[Out] -(((2*b*x)/(3*a) + 2/3)*(a + b*x)^(1/2))/x^(3/2)
```

3.495 $\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}$$

```
[Out] -2/5*(b*x+a)^(3/2)/a/x^(5/2)+4/15*b*(b*x+a)^(3/2)/a^2/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}-\frac{(2b)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}}+\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 39, normalized size = 0.89

$$-\frac{2\sqrt{a+bx}\,(3a^2+abx-2b^2x^2)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*a^2 + a*b*x - 2*b^2*x^2))/(15*a^2*x^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.
time = 0.10, size = 71, normalized size = 1.61

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(-2bx+3a)}{15x^{\frac{5}{2}}a^2}$	24
risch	$-\frac{2\sqrt{bx+a}\,(-2x^2b^2+abx+3a^2)}{15x^{\frac{5}{2}}a^2}$	34
default	$-\frac{\sqrt{bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{4}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x+a)^(1/2)/x^(5/2)-1/4*a*(-2/5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*(-2/
3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)))
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b*x + a)^(3/2)*b/x^(3/2) - 3*(b*x + a)^(5/2)/x^(5/2))/a^2
```

Fricas [A]
time = 0.58, size = 34, normalized size = 0.77

$$\frac{2\,(2\,b^2x^2-abx-3\,a^2)\sqrt{bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

Sympy [A]
time = 2.89, size = 65, normalized size = 1.48

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a*
x) + 4*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2)
```

Giac [A]
time = 0.75, size = 50, normalized size = 1.14

$$\frac{2\left(\frac{2(bx+a)b^5}{a^2}-\frac{5b^5}{a}\right)(bx+a)^{\frac{3}{2}}b}{15\left((bx+a)b-ab\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*(b*x + a)*b^5/a^2 - 5*b^5/a)*(b*x + a)^(3/2)*b/(((b*x + a)*b - a*b)
^(5/2)*abs(b))
```

Mupad [B]
time = 0.25, size = 32, normalized size = 0.73

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,b\,x}{15\,a}-\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(7/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((2*b*x)/(15*a) - (4*b^2*x^2)/(15*a^2) + 2/5))/x^(5/2)
```

3.496 $\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}$$

```
[Out] -2/7*(b*x+a)^(3/2)/a/x^(7/2)+8/35*b*(b*x+a)^(3/2)/a^2/x^(5/2)-16/105*b^2*(b
*x+a)^(3/2)/a^3/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) + (8*b*(a + b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a + b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{9/2}}\,dx = -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} - \frac{(4b)\int \frac{\sqrt{a+bx}}{x^{7/2}}\,dx}{7a}$$
$$= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} + \frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}} + \frac{(8b^2)\int \frac{\sqrt{a+bx}}{x^{5/2}}\,dx}{35a^2}$$
$$= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}} + \frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.75

$$-\frac{2\sqrt{a+bx}\,(15a^3+3a^2bx-4ab^2x^2+8b^3x^3)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(9/2),x]
[Out] (-2*Sqrt[a + b*x]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

Maple [A]
time = 0.11, size = 93, normalized size = 1.37

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}(8x^2b^2-12abx+15a^2)}{105x^{\frac{7}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}\,(8b^3x^3-4a\,b^2x^2+3a^2bx+15a^3)}{105x^{\frac{7}{2}}a^3}$	46
default	$-\frac{\sqrt{bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(b*x+a)^(1/2)/x^(7/2)-1/6*a*(-2/7*(b*x+a)^(1/2)/a/x^(7/2)-6/7*b/a*(-2/
5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+
a)^(1/2)/a^2/x^(1/2))))
```

Maxima [A]
time = 0.30, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{42\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(b*x + a)^(3/2)*b^2/x^(3/2) - 42*(b*x + a)^(5/2)*b/x^(5/2) + 15*
(b*x + a)^(7/2)/x^(7/2))/a^3
```

Fricas [A]
time = 0.53, size = 45, normalized size = 0.66

$$-\frac{2\,(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(63) = 126$.
time = 8.65, size = 347, normalized size = 5.10

$$-\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{24ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{16b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] -30*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*x**
2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b*x) +
1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 16*b**(
19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105
*a**3*b**6*x**5)
```

Giac [A]
time = 0.93, size = 66, normalized size = 0.97

$$-\frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx+a)b^7}{a^3}-\frac{7\,b^7}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{3}{2}}b}{105\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -2/105*(35*b^7/a + 4*(2*(b*x + a)*b^7/a^3 - 7*b^7/a^2)*(b*x + a))*(b*x + a)
^(3/2)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.26, size = 43, normalized size = 0.63

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{16\,b^3\,x^3}{105\,a^3}-\frac{8\,b^2\,x^2}{105\,a^2}+\frac{2\,b\,x}{35\,a}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(9/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((16*b^3*x^3)/(105*a^3) - (8*b^2*x^2)/(105*a^2) + (2*b*x)
/(35*a) + 2/7))/x^(7/2)
```

3.497 $\int x^{5/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=127

$$-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}$$

```
[Out] 5/64*a^4*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)-5/96*a^2*x^(3/2)*(-
b*x+a)^(1/2)/b^2-1/24*a*x^(5/2)*(-b*x+a)^(1/2)/b+1/4*x^(7/2)*(-b*x+a)^(1/2)
-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^4\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a - b*x])/(96
*b^2) - (a*x^(5/2)*Sqrt[a - b*x])/(24*b) + (x^(7/2)*Sqrt[a - b*x])/4 + (5*a
^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a-bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{1}{8}a\int\frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^2)\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{48b}$$
$$= -\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^3)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b^2}$$
$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^4)\int}{}$$
$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^4)\,\mathrm{S}}{}$$
$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{(5a^4)\,\mathrm{S}}{}$$
$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}+\frac{5a^4\tan}{}$$

Mathematica [A]
time = 0.14, size = 92, normalized size = 0.72

$$\frac{1}{192}\left(\frac{\sqrt{x}\sqrt{a-bx}\,(-15a^3-10a^2bx-8ab^2x^2+48b^3x^3)}{b^3}+\frac{15a^4\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-15*a^3 - 10*a^2*b*x - 8*a*b^2*x^2 + 48*b^3*x^3))/
b^3 + (15*a^4*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2))/192
```

Maple [A]
time = 0.13, size = 135, normalized size = 1.06

method	result
risch	$-\frac{\left(-48 b^{3} x^{3}+8 a\,b^{2} x^{2}+10 a^{2} b x +15 a^{3}\right) \sqrt{x}\,\sqrt{-b x +a}}{192 b^{3}}+\frac{5 a^{4} \arctan\left(\frac{\sqrt{b}\,\left(x -\frac{a}{2 b}\right)}{\sqrt{-x^{2} b +a x}}\right) \sqrt{x \left(-b x +a \right)}}{128 b^{\frac{7}{2}} \sqrt{x}\,\sqrt{-b x +a}}$
default	$-\frac{x^{\frac{5}{2}} \left(-b x +a \right)^{\frac{3}{2}}}{4 b}+\frac{5 a \left(-\frac{x^{\frac{3}{2}} \left(-b x +a \right)^{\frac{3}{2}}}{3 b}+\frac{a \left(-\frac{\sqrt{x}\,\left(-b x +a \right)^{\frac{3}{2}}}{2 b}+\frac{a \left(\sqrt{x}\,\sqrt{-b x +a}+\frac{a \sqrt{x \left(-b x +a \right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x -\frac{a}{2 b}\right)}{\sqrt{-x^{2} b +a x}}\right)}{2 \sqrt{-b x +a}\,\sqrt{x}\,\sqrt{b}}\right)}{4 b}\right)}{2 b}\right)}{8 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+a)^(3/2)+5/8*a/b*(-1/3/b*x^(3/2)*(-b*x+a)^(3/2)+1/2*a/
b*(-1/2/b*x^(1/2)*(-b*x+a)^(3/2)+1/4*a/b*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(
-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-
b*x^2+a*x)^(1/2)))))
```

Maxima [A]
time = 0.50, size = 170, normalized size = 1.34

$$-\frac{5\,a^{4} \arctan\left(\frac{\sqrt{-b x+a}}{\sqrt{b} \sqrt{x}}\right)}{64\,b^{\frac{7}{2}}} + \frac{\frac{15\,\sqrt{-b x+a}\,a^{4} b^{3}}{\sqrt{x}} - \frac{73\,(-b x+a)^{\frac{3}{2}} a^{4} b^{2}}{x^{\frac{3}{2}}} - \frac{55\,(-b x+a)^{\frac{5}{2}} a^{4} b}{x^{\frac{5}{2}}} - \frac{15\,(-b x+a)^{\frac{7}{2}} a^{4}}{x^{\frac{7}{2}}}}{192\left(b^{7} - \frac{4\,(b x-a) b^{6}}{x} + \frac{6\,(b x-a)^{2} b^{5}}{x^{2}} - \frac{4\,(b x-a)^{3} b^{4}}{x^{3}} + \frac{(b x-a)^{4} b^{3}}{x^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) - 73*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 55*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^7 - 4*(b*x
- a)*b^6/x + 6*(b*x - a)^2*b^5/x^2 - 4*(b*x - a)^3*b^4/x^3 + (b*x - a)^4*b^
3/x^4)
```

Fricas [A]
time = 0.54, size = 164, normalized size = 1.29

$$\left[-\frac{15\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^4},-\frac{15\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(48\,b^4x^3-8\,ab^3x^2-10\,a^2b^2x-15\,a^3b)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*s
qrt(x))/b^4, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))
) - (48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 22.56, size = 323, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{-1+\frac{bx}{a}}}-\frac{7i\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{ibx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}+\frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}-\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(5/2)*x
**(3/2)/(192*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(5/2)/(96*b*sqrt(-1 + b
*x/a)) - 7*I*sqrt(a)*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqrt(b
)*sqrt(x)/sqrt(a))/(64*b**(7/2)) + I*b*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a)
), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 - b*x/a)) + 5*a**(
5/2)*x**(3/2)/(192*b**2*sqrt(1 - b*x/a)) + a**(3/2)*x**(5/2)/(96*b*sqrt(1 -
 b*x/a)) + 7*sqrt(a)*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(7/2)) - b*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True
))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(1/2), x)
```

3.498 $\int x^{3/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=102

$$-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)-1/12*a*x^(3/2)*(-b*x
+a)^(1/2)/b+1/3*x^(5/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] -1/8*(a^2*Sqrt[x]*Sqrt[a - b*x])/b^2 - (a*x^(3/2)*Sqrt[a - b*x])/(12*b) + (
x^(5/2)*Sqrt[a - b*x])/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8
*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}\sqrt{a-bx}\,dx = \frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{1}{6}a\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^2\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b}$$

$$= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\int\frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{16b^2}$$

$$= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\text{Subst}\left(\int\frac{1}{\sqrt{a-bx^2}}\,dx,\ldots\right.}{8b^2}$$

$$= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\text{Subst}\left(\int\frac{1}{1+bx^2}\,dx,x,\ldots\right.}{8b^2}$$

$$= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.79

$$\frac{1}{24}\left(\frac{\sqrt{x}\,\sqrt{a-bx}\,(-3a^2-2abx+8b^2x^2)}{b^2}-\frac{3a^3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-3*a^2 - 2*a*b*x + 8*b^2*x^2))/b^2 - (3*a^3*Log[-(
Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(5/2))/24
```

Maple [A]
time = 0.13, size = 112, normalized size = 1.10

method	result	siz
risch	$-\frac{(-8x^2b^2+2abx+3a^2)\sqrt{x}\,\sqrt{-bx+a}}{24b^2}+\frac{a^3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{16b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	91
default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3b}+\frac{a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}\right)}{2b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+a)^(3/2)+1/2*a/b*(-1/2/b*x^(1/2)*(-b*x+a)^(3/2)+1/4*a/
b*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b
^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.50, size = 135, normalized size = 1.32

$$-\frac{a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{3\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}-\frac{8\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^5-\frac{3\,(bx-a)b^4}{x}+\frac{3\,(bx-a)^2b^3}{x^2}-\frac{(bx-a)^3b^2}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) - 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x - a)*b^4/x + 3*(b*x - a)^2*b^3/x^2 - (b*x -
 a)^3*b^2/x^3)
```

Fricas [A]
time = 0.47, size = 142, normalized size = 1.39

$$\left[-\frac{3\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(8\,b^3x^2-2\,ab^2x-3\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^3},-\frac{3\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(8\,b^3x^2-2\,ab^2x-3\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
- 2*(8*b^3*x^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/24*(3
*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 2*a*b^
2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 5.56, size = 260, normalized size = 2.55

$$\begin{cases} \frac{i a^{\frac{5}{2}} \sqrt{x}}{8 b^{2} \sqrt{-1 + \frac{b x}{a}}} - \frac{i a^{\frac{3}{2}} x^{\frac{3}{2}}}{24 b \sqrt{-1 + \frac{b x}{a}}} - \frac{5 i \sqrt{a} x^{\frac{5}{2}}}{12 \sqrt{-1 + \frac{b x}{a}}} - \frac{i a^{3} \operatorname{acosh}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{8 b^{\frac{5}{2}}} + \frac{i b x^{\frac{7}{2}}}{3 \sqrt{a} \sqrt{-1 + \frac{b x}{a}}} & \text{for}\: \left|{\frac{b x}{a}}\right| > 1 \\- \frac{a^{\frac{5}{2}} \sqrt{x}}{8 b^{2} \sqrt{1 - \frac{b x}{a}}} + \frac{a^{\frac{3}{2}} x^{\frac{3}{2}}}{24 b \sqrt{1 - \frac{b x}{a}}} + \frac{5 \sqrt{a} x^{\frac{5}{2}}}{12 \sqrt{1 - \frac{b x}{a}}} + \frac{a^{3} \operatorname{asin}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{8 b^{\frac{5}{2}}} - \frac{b x^{\frac{7}{2}}}{3 \sqrt{a} \sqrt{1 - \frac{b x}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(3/
2)/(24*b*sqrt(-1 + b*x/a)) - 5*I*sqrt(a)*x**(5/2)/(12*sqrt(-1 + b*x/a)) - I
*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + I*b*x**(7/2)/(3*sqrt(a)
*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 - b*
x/a)) + a**(3/2)*x**(3/2)/(24*b*sqrt(1 - b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*s
qrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) - b*x**(7
/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(1/2), x)
```

3.499 $\int \sqrt{x}\,\sqrt{a-bx}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{x}\,\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}$$

```
[Out] 1/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)+1/2*x^(3/2)*(-b*x+a)
^(1/2)-1/4*a*x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\,\sqrt{a-bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] -1/4*(a*Sqrt[x]*Sqrt[a - b*x])/b + (x^(3/2)*Sqrt[a - b*x])/2 + (a^2*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{a-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b}\\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a-bx}+\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.92

$$\frac{\sqrt{x}\sqrt{a-bx}\,(-a+2bx)}{4b}+\frac{a^2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{4(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a - b*x]*(-a + 2*b*x))/(4*b) + (a^2*Log[-(Sqrt[-b]*Sqrt[x]) +
 Sqrt[a - b*x]])/(4*(-b)^(3/2))
```

Maple [A]
time = 0.11, size = 89, normalized size = 1.16

method	result	size

risch	$-\frac{(-2bx+a)\sqrt{x}\,\sqrt{-bx+a}}{4b}+\frac{a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\,(-bx+a)}}{8b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	78
default	$-\frac{\sqrt{x}\,(-bx+a)^{\frac{3}{2}}}{2b}+\frac{a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*x^(1/2)*(-b*x+a)^(3/2)+1/4*a/b*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*
x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x
^2+a*x)^(1/2)))
```

Maxima [A]
time = 0.53, size = 95, normalized size = 1.23

$$-\frac{a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}-\frac{(-bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx-a)b^{2}}{x}+\frac{(bx-a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/4*(sqrt(-b*x
 + a)*a^2*b/sqrt(x) - (-b*x + a)^(3/2)*a^2/x^(3/2))/(b^3 - 2*(b*x - a)*b^2/x
 + (b*x - a)^2*b/x^2)
```

Fricas [A]
time = 0.45, size = 118, normalized size = 1.53

$$\left[-\frac{a^{2}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(2\,b^{2}x-ab)\sqrt{-bx+a}\,\sqrt{x}}{8\,b^{2}},\ -\frac{a^{2}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(2\,b^{2}x-ab)\sqrt{-bx+a}\,\sqrt{x}}{4\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2
*(2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/4*(a^2*sqrt(b)*arctan(sqrt
(-b*x + a)/(sqrt(b)*sqrt(x))) - (2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2
]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.30, size = 207, normalized size = 2.69

$$\begin{cases} \frac{ia^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{-1+\frac{bx}{a}}} - \frac{3i\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}} - \frac{ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + \frac{ibx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1-\frac{bx}{a}}} + \frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}} + \frac{a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} - \frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(3/2)*sqrt(x)/(4*b*sqrt(-1 + b*x/a)) - 3*I*sqrt(a)*x**(3/2)
/(4*sqrt(-1 + b*x/a)) - I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2))
+ I*b*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(3/2)*sq
rt(x)/(4*b*sqrt(1 - b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 - b*x/a)) + a**2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) - b*x**(5/2)/(2*sqrt(a)*sqrt(1
- b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.08, size = 58, normalized size = 0.75

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{a}{4\,b}\right)\,\sqrt{a-b\,x}-\frac{a^2\,\ln\left(a-2\,b\,x+2\,\sqrt{-b}\,\sqrt{x}\,\sqrt{a-b\,x}\right)}{8\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - a/(4*b))*(a - b*x)^(1/2) - (a^2*log(a - 2*b*x + 2*(-b)^(1/2)
*x^(1/2)*(a - b*x)^(1/2)))/(8*(-b)^(3/2))
```

3.500 $\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=46

$$\sqrt{x}\,\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

[Out] a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)+x^(1/2)*(-b*x+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}+\sqrt{x}\,\sqrt{a-bx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b*x]/Sqrt[x],x]

[Out] Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}a\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \sqrt{x}\,\sqrt{a-bx}+a\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}\,\sqrt{a-bx}+a\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \sqrt{x}\,\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.15

$$\sqrt{x}\,\sqrt{a-bx}+\frac{ab\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*b*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)
^(3/2)
```

Maple [A]
time = 0.12, size = 66, normalized size = 1.43

method	result	size
default	$\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}$	66

risch	$\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1
/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))
```

Maxima [A]
time = 0.53, size = 52, normalized size = 1.13

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\sqrt{-bx+a}\,a}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + sqrt(-b*x + a)*a/((b
- (b*x - a)/x)*sqrt(x))
```

Fricas [A]
time = 0.49, size = 94, normalized size = 2.04

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*s
qrt(-b*x + a)*b*sqrt(x))/b, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt
(x))) - sqrt(-b*x + a)*b*sqrt(x))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.98, size = 119, normalized size = 2.59

$$\begin{cases} -\frac{i\sqrt{a}\,\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}-\frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{ibx^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \sqrt{a}\,\sqrt{x}\,\sqrt{1-\frac{bx}{a}}+\frac{a\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)/sqrt(-1 + b*x/a) - I*a*acosh(sqrt(b)*sqrt(x)/
sqrt(a))/sqrt(b) + I*b*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (sqrt(a)*sqrt(x)*sqrt(1 - b*x/a) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b
), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.59, size = 43, normalized size = 0.93

$$\sqrt{x}\,\sqrt{a-b\,x}+\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a - b*x)^(1/2) + (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^
(1/2))))/b^(1/2)
```

3.501 $\int \frac{\sqrt{a-bx}}{x^{3/2}} \, dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}}-2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))*b^(1/2)-2*(-b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 65, 223, 209}

$$-2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2\sqrt{a-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\ &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 1.13

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(3/2),x]
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a -
b*x]]
```

Maple [A]
time = 0.13, size = 66, normalized size = 1.40

method	result	size
risch	$-\frac{2\sqrt{-bx+a}}{\sqrt{x}} - \frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-b*x+a)^(1/2)/x^(1/2)-b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(
1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.50, size = 35, normalized size = 0.74

$$2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\frac{2\sqrt{-bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)/sqrt(
x)
```

Fricas [A]
time = 0.55, size = 91, normalized size = 1.94

$$\left[\frac{\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,\sqrt{x}}{x},\frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*sqrt(-
b*x + a)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))
- sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.91, size = 148, normalized size = 3.15

$$\begin{cases}\frac{2i\sqrt{a}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}+2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{2ib\sqrt{x}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}-2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(a)/(sqrt(x)*sqrt(-1 + b*x/a)) + 2*I*sqrt(b)*acosh(sqrt(
b)*sqrt(x)/sqrt(a)) - 2*I*b*sqrt(x)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a)
> 1), (-2*sqrt(a)/(sqrt(x)*sqrt(1 - b*x/a)) - 2*sqrt(b)*asin(sqrt(b)*sqrt(x
)/sqrt(a)) + 2*b*sqrt(x)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(1/2)/x^(3/2), x)
```

3.502 $\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

```
[Out] -2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = -\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A]
time = 0.07, size = 22, normalized size = 1.00

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.
time = 0.11, size = 52, normalized size = 2.36

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
risch	$-\frac{2(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$	17
default	$-\frac{\sqrt{-bx+a}}{x^{\frac{3}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^{2}\sqrt{x}}\right)}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+a)^(1/2)/x^(3/2)-1/2*a*(-2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)
^(1/2)/a^2/x^(1/2))
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.73

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-b*x + a)^(3/2)/(a*x^(3/2))
```

Fricas [A]
time = 0.46, size = 23, normalized size = 1.05

$$\frac{2\,(bx-a)\sqrt{-bx+a}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)/(a*x^(3/2))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.86, size = 88, normalized size = 4.00

$$\begin{cases} -\frac{2\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{3x}+\frac{2b^{\frac{3}{2}}\,\sqrt{\frac{a}{bx}-1}}{3a} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{2i\sqrt{b}\,\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{2ib^{\frac{3}{2}}\,\sqrt{-\frac{a}{bx}+1}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 2*b**(3/2)*sqrt(a/(b*x) - 1
)/(3*a), Abs(a/(b*x)) > 1), (-2*I*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 2*I*b*
*(3/2)*sqrt(-a/(b*x) + 1)/(3*a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.
time = 1.24, size = 42, normalized size = 1.91

$$\frac{2\,(bx-a)\sqrt{-bx+a}\,b^4}{3\,((bx-a)b+ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)*b^4/(((b*x - a)*b + a*b)^(3/2)*a*abs(b))
```

Mupad [B]
time = 0.24, size = 21, normalized size = 0.95

$$\frac{\left(\frac{2\,b\,x}{3\,a}-\frac{2}{3}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(5/2),x)
```

```
[Out] (((2*b*x)/(3*a) - 2/3)*(a - b*x)^(1/2))/x^(3/2)
```

3.503 $\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}$$

```
[Out] -2/5*(-b*x+a)^(3/2)/a/x^(5/2)-4/15*b*(-b*x+a)^(3/2)/a^2/x^(3/2)
```

Rubi [A]

time = 0.00, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {47, 37}

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}+\frac{(2b)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.89

$$-\frac{2\sqrt{a-bx}\,(3a^2-abx-2b^2x^2)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(3*a^2 - a*b*x - 2*b^2*x^2))/(15*a^2*x^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.12, size = 75, normalized size = 1.63

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}(2bx+3a)}{15x^{\frac{5}{2}}a^2}$	25
risch	$-\frac{2\sqrt{-bx+a}\,(-2x^2b^2-abx+3a^2)}{15x^{\frac{5}{2}}a^2}$	36
default	$-\frac{\sqrt{-bx+a}}{2x^{\frac{5}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{4}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-b*x+a)^(1/2)/x^(5/2)-1/4*a*(-2/5/a/x^(5/2)*(-b*x+a)^(1/2)+4/5*b/a*(-
2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)))
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.72

$$-\frac{2\left(\frac{5\,(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(-b*x + a)^(5/2)/x^(5/2))/a^2
```

Fricas [A]
time = 0.46, size = 34, normalized size = 0.74

$$\frac{2\,(2\,b^2x^2+abx-3\,a^2)\sqrt{-bx+a}}{15\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 + a*b*x - 3*a^2)*sqrt(-b*x + a)/(a^2*x^(5/2))
```

Sympy [C] Result contains complex when optimal does not.
time = 3.15, size = 241, normalized size = 5.24

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{5x^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}{15a^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{6ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{x(-15a^3bx+15a^2b^2x^2)}-\frac{8ia^2b^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}-\frac{2iab^{\frac{7}{2}}x\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2}+\frac{4ib^{\frac{9}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(5*x**2) + 2*b**(3/2)*sqrt(a/(b*x)
- 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) - 1)/(15*a**2), Abs(a/(b*x)) > 1),
(6*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1)/(x*(-15*a**3*b*x + 15*a**2*b**2*x**2)
) - 8*I*a**2*b**(5/2)*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2)
 - 2*I*a*b**(7/2)*x*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2) +
 4*I*b**(9/2)*x**2*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2), T
rue))
```

Giac [A]
time = 0.79, size = 61, normalized size = 1.33

$$\frac{2\left(\frac{2(bx-a)b^5}{a^2}+\frac{5b^5}{a}\right)(bx-a)\sqrt{-bx+a}\,b}{15\left((bx-a)b+ab\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*(b*x - a)*b^5/a^2 + 5*b^5/a)*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a)
*b + a*b)^(5/2)*abs(b))
```

Mupad [B]
time = 0.25, size = 32, normalized size = 0.70

$$\frac{\sqrt{a-b\,x}\,\left(\frac{4\,b^2\,x^2}{15\,a^2}+\frac{2\,b\,x}{15\,a}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(7/2),x)
```

```
[Out] ((a - b*x)^(1/2)*((4*b^2*x^2)/(15*a^2) + (2*b*x)/(15*a) - 2/5))/x^(5/2)
```

3.504 $\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}$$

```
[Out] -2/7*(-b*x+a)^(3/2)/a/x^(7/2)-8/35*b*(-b*x+a)^(3/2)/a^2/x^(5/2)-16/105*b^2*
(-b*x+a)^(3/2)/a^3/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) - (8*b*(a - b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a - b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx = -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} + \frac{(4b)\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx}{7a}$$
$$= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} + \frac{(8b^2)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{35a^2}$$
$$= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}} - \frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}} - \frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.73

$$-\frac{2\sqrt{a-bx}\,(15a^3-3a^2bx-4ab^2x^2-8b^3x^3)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(15*a^3 - 3*a^2*b*x - 4*a*b^2*x^2 - 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

Maple [A]
time = 0.10, size = 98, normalized size = 1.38

method	result	size
gosper	$-\frac{2(-bx+a)^{\frac{3}{2}}(8x^2b^2+12abx+15a^2)}{105x^{\frac{7}{2}}a^3}$	36
risch	$-\frac{2\sqrt{-bx+a}\,(-8b^3x^3-4a\,b^2x^2-3a^2bx+15a^3)}{105x^{\frac{7}{2}}a^3}$	47
default	$-\frac{\sqrt{-bx+a}}{3x^{\frac{7}{2}}}-\frac{a\left(-\frac{2\sqrt{-bx+a}}{7a\,x^{\frac{7}{2}}}+\frac{6b\left(-\frac{2\sqrt{-bx+a}}{5a\,x^{\frac{5}{2}}}+\frac{4b\left(-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}\right)}{6}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b*x+a)^(1/2)/x^(7/2)-1/6*a*(-2/7/a/x^(7/2)*(-b*x+a)^(1/2)+6/7*b/a*(-
2/5/a/x^(5/2)*(-b*x+a)^(1/2)+4/5*b/a*(-2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(
-b*x+a)^(1/2)/a^2/x^(1/2))))
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.69

$$-\frac{2\left(\frac{35\,(-bx+a)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}+\frac{42\,(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 42*(-b*x + a)^(5/2)*b/x^(5/2) + 1
5*(-b*x + a)^(7/2)/x^(7/2))/a^3
```

Fricas [A]
time = 0.50, size = 46, normalized size = 0.65

$$\frac{2\,(8\,b^{3}x^{3}+4\,ab^{2}x^{2}+3\,a^{2}bx-15\,a^{3})\sqrt{-bx+a}}{105\,a^{3}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*b^3*x^3 + 4*a*b^2*x^2 + 3*a^2*b*x - 15*a^3)*sqrt(-b*x + a)/(a^3*x^
(7/2))
```

Sympy [C] Result contains complex when optimal does not.
time = 11.47, size = 707, normalized size = 9.96

$$\begin{cases} -\frac{30a^{5}b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}+\frac{66a^{4}b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{34a^{3}b^{\frac{13}{2}}x^{2}\sqrt{\frac{a}{bx}-1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}+\frac{6a^{2}b^{\frac{15}{2}}x^{3}\sqrt{\frac{a}{bx}-1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{24ab^{\frac{17}{2}}x^{4}\sqrt{\frac{a}{bx}-1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}+\frac{16b^{\frac{19}{2}}x^{5}\sqrt{\frac{a}{bx}-1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{30ia^{5}b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}+\frac{66ia^{4}b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{34ia^{3}b^{\frac{13}{2}}x^{2}\sqrt{-\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}+\frac{6ia^{2}b^{\frac{15}{2}}x^{3}\sqrt{-\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}-\frac{24iab^{\frac{17}{2}}x^{4}\sqrt{-\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}}+\frac{16ib^{\frac{19}{2}}x^{5}\sqrt{-\frac{a}{bx}+1}}{105a^{5}b^{4}x^{3}-210a^{4}b^{5}x^{4}+105a^{3}b^{6}x^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((-30*a**5*b**(9/2)*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a*
*4*b**5*x**4 + 105*a**3*b**6*x**5) + 66*a**4*b**(11/2)*x*sqrt(a/(b*x) - 1)/
(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b*
*(13/2)*x**2*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 1
05*a**3*b**6*x**5) + 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(105*a**5*b**4
*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt
(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5
) + 16*b**(19/2)*x**5*sqrt(a/(b*x) - 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5
*x**4 + 105*a**3*b**6*x**5), Abs(a/(b*x)) > 1), (-30*I*a**5*b**(9/2)*sqrt(-
a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5)
 + 66*I*a**4*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*
b**5*x**4 + 105*a**3*b**6*x**5) - 34*I*a**3*b**(13/2)*x**2*sqrt(-a/(b*x) +
```

```
1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 6*I*a**
2*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 24*I*a*b**(17/2)*x**4*sqrt(-a/(b*x) + 1)/(105*a**
5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) + 16*I*b**(19/2)*x**
5*sqrt(-a/(b*x) + 1)/(105*a**5*b**4*x**3 - 210*a**4*b**5*x**4 + 105*a**3*b*
*6*x**5), True))
```

Giac [A]
time = 0.79, size = 79, normalized size = 1.11

$$\frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx-a)b^7}{a^3}+\frac{7\,b^7}{a^2}\right)(bx-a)\right)(bx-a)\sqrt{-bx+a}\,b}{105\left((bx-a)b+ab\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*b^7/a + 4*(2*(b*x - a)*b^7/a^3 + 7*b^7/a^2)*(b*x - a))*(b*x - a)*
sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.27, size = 43, normalized size = 0.61

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,b^2\,x^2}{105\,a^2}+\frac{16\,b^3\,x^3}{105\,a^3}+\frac{2\,b\,x}{35\,a}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((a - b*x)^(1/2)*((8*b^2*x^2)/(105*a^2) + (16*b^3*x^3)/(105*a^3) + (2*b*x)/
(35*a) - 2/7))/x^(7/2)
```

3.505 $\int x^{5/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=108

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] -5/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/24*x^(3/2)*(b*x+2)^(1/2
)/b^2+1/12*x^(5/2)*(b*x+2)^(1/2)/b+1/4*x^(7/2)*(b*x+2)^(1/2)+5/8*x^(1/2)*(b
*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{8b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(24*b^2) + (x
^(5/2)*Sqrt[2 + b*x])/(12*b) + (x^(7/2)*Sqrt[2 + b*x])/4 - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}\sqrt{2+bx}\,dx &= \frac{1}{4}x^{7/2}\sqrt{2+bx}+\frac{1}{4}\int\frac{x^{5/2}}{\sqrt{2+bx}}\,dx\\
&= \frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\int\frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{12b}\\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}+\frac{5\int\frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b^2}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\int\frac{1}{\sqrt{x}\sqrt{2+b}}}{8b^3}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\text{Subst}\left(\int\frac{}{\sqrt{2}}\right.}{}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{24b^2}+\frac{x^{5/2}\sqrt{2+bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2+bx}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}}{\sqrt{2}}\right.}{4b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.70

$$
\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2+6b^3x^3)}{24b^3}+\frac{5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5*
Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(7/2))
```

Maple [A]
time = 0.14, size = 121, normalized size = 1.12

method	result	size
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}\left(42b^3x^3+14x^2b^2-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{1344}+\frac{5\sqrt{\pi}\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{32}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	71

risch	$\frac{(6b^3x^3+2x^2b^2-5bx+15)\sqrt{x}\,\sqrt{bx+2}}{24b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	85
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{3}{2}}}{4b}-\frac{5\left(\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}\right)}{4b}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+2)^(3/2)-5/4/b*(1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/b*(1/2/b*x
^(1/2)*(b*x+2)^(3/2)-1/2/b*(x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)
^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(75) = 150$.
time = 0.54, size = 163, normalized size = 1.51

$$\frac{\frac{15\sqrt{bx+2}\,b^3}{\sqrt{x}}+\frac{73(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4(bx+2)b^6}{x}+\frac{6(bx+2)^2b^5}{x^2}-\frac{4(bx+2)^3b^4}{x^3}+\frac{(bx+2)^4b^3}{x^4}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) + 73*(b*x + 2)^(3/2)*b^2/x^(3/2) - 55*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x + 2)*b^
6/x + 6*(b*x + 2)^2*b^5/x^2 - 4*(b*x + 2)^3*b^4/x^3 + (b*x + 2)^4*b^3/x^4)
+ 5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(
x)))/b^(7/2)
```

Fricas [A]
time = 0.48, size = 140, normalized size = 1.30

$$\left[\frac{(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^4},\frac{(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*
sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/24*((6*b^4*x^3
 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

Sympy [A]
time = 22.82, size = 117, normalized size = 1.08

$$\frac{bx^{\frac{9}{2}}}{4\sqrt{bx+2}} + \frac{7x^{\frac{7}{2}}}{12\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{24b\sqrt{bx+2}} + \frac{5x^{\frac{3}{2}}}{24b^2\sqrt{bx+2}} + \frac{5\sqrt{x}}{4b^3\sqrt{bx+2}} - \frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(9/2)/(4*sqrt(b*x + 2)) + 7*x**(7/2)/(12*sqrt(b*x + 2)) - x**(5/2)/(24
*b*sqrt(b*x + 2)) + 5*x**(3/2)/(24*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(4*b**3*
sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{b\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(1/2), x)
```

3.506 $\int x^{3/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+1/6*x^(3/2)*(b*x+2)^(1/2)/b+1/
3*x^(5/2)*(b*x+2)^(1/2)-1/2*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\,\sqrt{bx+2}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 + b*x])/b^2 + (x^(3/2)*Sqrt[2 + b*x])/(6*b) + (x^(5/2)
*Sqrt[2 + b*x])/3 + ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2+bx}\,dx=\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{1}{3}\int\frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$=\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}-\frac{\int\frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}$$
$$=-\frac{\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\int\frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^2}$$
$$=-\frac{\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}$$
$$=-\frac{\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 0.77

$$\frac{\sqrt{x}\,\sqrt{2+bx}\,(-3+bx+2b^2x^2)}{6b^2}-\frac{\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 + b*x],x]
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 2*b^2*x^2))/(6*b^2) - Log[-(Sqrt[b]*Sqrt
[x]) + Sqrt[2 + b*x]]/b^(5/2)
```

Maple [A]
time = 0.10, size = 100, normalized size = 1.19

method	result	size
meijerg	$-\frac{4\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(-10x^2b^2-5bx+15)\sqrt{\frac{bx}{2}+1}}{120}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2+bx-3)\sqrt{x}\,\sqrt{bx+2}}{6b^2}+\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	76

default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3b}-\frac{\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}$	100

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/b*(1/2/b*x^(1/2)*(b*x+2)^(3/2)-1/2/b*(x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)))

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(59) = 118$.
time = 0.52, size = 134, normalized size = 1.60

$$-\frac{\frac{3\sqrt{bx+2}\,b^2}{\sqrt{x}}+\frac{8(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5-\frac{3(bx+2)b^4}{x}+\frac{3(bx+2)^2b^3}{x^2}-\frac{(bx+2)^3b^2}{x^3}\right)}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="maxima")

[Out] -1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) + 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x + 2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x + 2)*b^4/x + 3*(b*x + 2)^2*b^3/x^2 - (b*x + 2)^3*b^2/x^3) - 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)

Fricas [**A**]
time = 0.53, size = 121, normalized size = 1.44

$$\left[\frac{(2\,b^3x^2+b^2x-3\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^3},\ \frac{(2\,b^3x^2+b^2x-3\,b)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="fricas")

[Out] [1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]

Sympy [A]
time = 5.14, size = 90, normalized size = 1.07

$$\frac{bx^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{5x^{\frac{5}{2}}}{6\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{6b\sqrt{bx+2}}-\frac{\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(7/2)/(3*sqrt(b*x + 2)) + 5*x**(5/2)/(6*sqrt(b*x + 2)) - x**(3/2)/(6*b
*sqrt(b*x + 2)) - sqrt(x)/(b**2*sqrt(b*x + 2)) + asinh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/b**(5/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{b\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(1/2), x)
```

3.507 $\int \sqrt{x}\,\sqrt{2+bx}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] -arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*x^(3/2)*(b*x+2)^(1/2)+1/2
*x^(1/2)*(b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 - ArcSinh[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,\sqrt{2+bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{2+bx} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx \\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b} \\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b} \\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2+bx} - \frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]

time = 0.06, size = 56, normalized size = 0.88

$$\frac{\sqrt{x}\,(1+bx)\sqrt{2+bx}}{2b} + \frac{\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(1 + b*x)*Sqrt[2 + b*x])/(2*b) + Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 +
 b*x]]/b^(3/2)
```

Maple [A]

time = 0.11, size = 79, normalized size = 1.23

method	result	size
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(3bx+3)\sqrt{\frac{bx}{2}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	55
risch	$\frac{(bx+1)\sqrt{x}\,\sqrt{bx+2}}{2b}-\frac{\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+2}}$	68
default	$\frac{\sqrt{x}\,(bx+2)^{\frac{3}{2}}}{2b}-\frac{\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*x^(1/2)*(b*x+2)^(3/2)-1/2/b*(x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/
(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.
time = 0.53, size = 98, normalized size = 1.53

$$\frac{\frac{\sqrt{bx+2}\,b}{\sqrt{x}} + \frac{(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3 - \frac{2\,(bx+2)b^2}{x} + \frac{(bx+2)^2 b}{x^2}} + \frac{\log\left(-\frac{\sqrt{b} - \frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b} + \frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b*x + 2)*b/sqrt(x) + (b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x + 2)*b^2/
x + (b*x + 2)^2*b/x^2) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b
) + sqrt(b*x + 2)/sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.49, size = 101, normalized size = 1.58

$$\left[\frac{(b^2x+b)\sqrt{bx+2}\,\sqrt{x} + \sqrt{b}\,\log\left(bx - \sqrt{bx+2}\,\sqrt{b}\,\sqrt{x} + 1\right)}{2\,b^2}, \frac{(b^2x+b)\sqrt{bx+2}\,\sqrt{x} + 2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*s
qrt(b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + 2*sqrt(-
b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A]
time = 2.09, size = 71, normalized size = 1.11

$$\frac{bx^{\frac{5}{2}}}{2\sqrt{bx+2}} + \frac{3x^{\frac{3}{2}}}{2\sqrt{bx+2}} + \frac{\sqrt{x}}{b\sqrt{bx+2}} - \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(5/2)/(2*sqrt(b*x + 2)) + 3*x**(3/2)/(2*sqrt(b*x + 2)) + sqrt(x)/(b*sq
rt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [B]
time = 0.10, size = 46, normalized size = 0.72

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{1}{2\,b}\right)\,\sqrt{b\,x+2}-\frac{\ln\left(b\,x+\sqrt{b}\,\sqrt{x}\,\sqrt{b\,x+2}+1\right)}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + 1/(2*b))*(b*x + 2)^(1/2) - log(b*x + b^(1/2)*x^(1/2)*(b*x +
2)^(1/2) + 1)/(2*b^(3/2))
```

3.508 $\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=40

$$\sqrt{x}\,\sqrt{2+bx}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+x^(1/2)*(b*x+2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + b*x]/Sqrt[x],x]

[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{2+bx} + \int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= \sqrt{x}\,\sqrt{2+bx} + 2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\,\sqrt{2+bx} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 1.15

$$\sqrt{x}\,\sqrt{2+bx} - \frac{2\log\left(-\sqrt{b}\,\sqrt{x} + \sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 + b*x] - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Maple [A]
time = 0.13, size = 58, normalized size = 1.45

method	result	size
meijerg	$-\frac{-\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\sqrt{\frac{bx}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{\pi}}$	49
default	$\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	58
risch	$\sqrt{x}\,\sqrt{bx+2}+\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] x^(1/2)*(b*x+2)^(1/2)+(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^
(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(29) = 58$.
time = 0.53, size = 68, normalized size = 1.70

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}-\frac{2\sqrt{bx+2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b) - 2*sqrt(b*x + 2)/((b - (b*x + 2)/x)*sqrt(x))
```

Fricas [A]
time = 0.55, size = 86, normalized size = 2.15

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x}+\sqrt{b}\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{b},\ \frac{\sqrt{bx+2}\,b\sqrt{x}-2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b, (sqrt(b*x + 2)*b*sqrt(x) - 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-
b)/(b*sqrt(x))))/b]
```

Sympy [A]
time = 0.88, size = 37, normalized size = 0.92

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(b*x + 2) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [B]
time = 0.62, size = 40, normalized size = 1.00

$$\sqrt{x}\,\sqrt{b\,x+2}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x + 2)^(1/2) - (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(
1/2))))/b^(1/2)
```

3.509 $\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2\sqrt{2+bx}}{\sqrt{x}}+2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{49, 56, 221\}$

$$2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+(2b)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+2\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 1.15

$$-\frac{2\sqrt{2+bx}}{\sqrt{x}}-2\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*
x]]
```

Maple [A]
time = 0.12, size = 49, normalized size = 1.20

method	result	size
meijerg	$-\frac{\sqrt{b}\left(\frac{4\sqrt{\pi}\,\sqrt{2}\,\sqrt{\frac{bx}{2}+1}}{\sqrt{x}\,\sqrt{b}}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	49
risch	$-\frac{2\sqrt{bx+2}}{\sqrt{x}}+\frac{\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b^(1/2)/Pi^(1/2)*(4*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(1/2*b*x+1)^(1/2)
-4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 54, normalized size = 1.32

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.49, size = 87, normalized size = 2.12

$$\left[\frac{\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-2\,\sqrt{bx+2}\,\sqrt{x}}{x},\,-\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b*x + 2)*
sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + sqr
t(b*x + 2)*sqrt(x))/x]
```

Sympy [A]
time = 0.82, size = 48, normalized size = 1.17

$$-2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}-\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2\sqrt{b}\,\log\left(\sqrt{1+\frac{2}{bx}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(1 + 2/(b*x)) - sqrt(b)*log(1/(b*x)) + 2*sqrt(b)*log(sqrt(1
+ 2/(b*x)) + 1)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x+2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(1/2)/x^(3/2), x)
```

3.510 $\int \frac{\sqrt{2+bx}}{x^{5/2}} \, dx$

Optimal. Leaf size=18

$$-\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

```
[Out] -1/3*(b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{5/2}} \, dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$-\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.12, size = 27, normalized size = 1.50

method	result	size
gosper	$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	13
meijerg	$-\frac{2\sqrt{2}\,\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	17
risch	$-\frac{x^{2}b^{2}+4bx+4}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	26
default	$-\frac{2\sqrt{bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{bx+2}}{3\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+2)^(1/2)/x^(3/2)-1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.51, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.72, size = 37, normalized size = 2.06

$$-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(5/2),x)
```

```
[Out] -b**(3/2)*sqrt(1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 1.07, size = 29, normalized size = 1.61

$$-\frac{(bx+2)^{\frac{3}{2}}b^4}{3\left((bx+2)b-2\,b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(b*x + 2)^(3/2)*b^4/(((b*x + 2)*b - 2*b)^(3/2)*abs(b))
```

Mupad [B]
time = 0.21, size = 18, normalized size = 1.00

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}+\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(5/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/3 + 2/3))/x^(3/2)
```

3.511 $\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}$$

```
[Out] -1/5*(b*x+2)^(3/2)/x^(5/2)+1/15*b*(b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(7/2),x]
```

```
[Out] -1/5*(2 + b*x)^(3/2)/x^(5/2) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}-\frac{1}{5}b\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.82

$$\frac{\sqrt{2+bx}\,(-6-bx+b^2x^2)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(7/2),x]
[Out] (Sqrt[2 + b*x]*(-6 - b*x + b^2*x^2))/(15*x^(5/2))
```

Maple [A]
time = 0.12, size = 43, normalized size = 1.13

method	result	size
gosper	$\frac{(bx+2)^{\frac{3}{2}}(bx-3)}{15x^{\frac{5}{2}}}$	18
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{6}x^2b^2+\frac{1}{6}bx+1\right)\sqrt{\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
risch	$\frac{b^3x^3+x^2b^2-8bx-12}{15x^{\frac{5}{2}}\sqrt{bx+2}}$	33
default	$-\frac{2\sqrt{bx+2}}{5x^{\frac{5}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(b*x+2)^(1/2)/x^(5/2)+1/5*b*(-1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^
(1/2)/x^(1/2))
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.68

$$\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.54, size = 25, normalized size = 0.66

$$\frac{(b^2x^2-bx-6)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 - b*x - 6)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [A]
time = 3.22, size = 56, normalized size = 1.47

$$\frac{b^{\frac{5}{2}}\sqrt{1+\frac{2}{bx}}}{15}-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{15x}-\frac{2\sqrt{b}\sqrt{1+\frac{2}{bx}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] b**(5/2)*sqrt(1 + 2/(b*x))/15 - b**(3/2)*sqrt(1 + 2/(b*x))/(15*x) - 2*sqrt(
b)*sqrt(1 + 2/(b*x))/(5*x**2)
```

Giac [A]
time = 0.75, size = 42, normalized size = 1.11

$$\frac{((bx+2)b^5-5\,b^5)(bx+2)^{\frac{3}{2}}b}{15\,((bx+2)b-2\,b)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(5/2)*a
bs(b))
```

Mupad [B]
time = 0.22, size = 26, normalized size = 0.68

$$-\frac{\sqrt{b\,x+2}\,\left(-\frac{b^2\,x^2}{15}+\frac{b\,x}{15}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(7/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/15 - (b^2*x^2)/15 + 2/5))/x^(5/2)
```

3.512 $\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=59

$$-\frac{(2+bx)^{3/2}}{7x^{7/2}}+\frac{2b(2+bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}$$

```
[Out] -1/7*(b*x+2)^(3/2)/x^(7/2)+2/35*b*(b*x+2)^(3/2)/x^(5/2)-2/105*b^2*(b*x+2)^(
3/2)/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -1/7*(2 + b*x)^(3/2)/x^(7/2) + (2*b*(2 + b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 + b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{9/2}}\,dx &= -\frac{(2+bx)^{3/2}}{7x^{7/2}} - \frac{1}{7}(2b)\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx \\ &= -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} + \frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx \\ &= -\frac{(2+bx)^{3/2}}{7x^{7/2}} + \frac{2b(2+bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.68

$$\frac{\sqrt{2+bx}\,(-30-3bx+2b^2x^2-2b^3x^3)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-30 - 3*b*x + 2*b^2*x^2 - 2*b^3*x^3))/(105*x^(7/2))
```

Maple [A]
time = 0.13, size = 59, normalized size = 1.00

method	result	size
gosper	$-\frac{(bx+2)^{\frac{3}{2}}(2x^2b^2-6bx+15)}{105x^{\frac{7}{2}}}$	27
meijerg	$-\frac{2\sqrt{2}\,\left(\frac{1}{15}b^3x^3-\frac{1}{15}x^2b^2+\frac{1}{10}bx+1\right)\sqrt{\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
risch	$-\frac{2b^4x^4+2b^3x^3-x^2b^2+36bx+60}{105x^{\frac{7}{2}}\sqrt{bx+2}}$	43
default	$-\frac{2\sqrt{bx+2}}{7x^{\frac{7}{2}}}+\frac{b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x+2)^(1/2)/x^(7/2)+1/7*b*(-1/5*(b*x+2)^(1/2)/x^(5/2)-2/5*b*(-1/3*(b
*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)))
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.69

$$-\frac{(bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b*x + 2)^(3/2)*b^2/x^(3/2) + 1/10*(b*x + 2)^(5/2)*b/x^(5/2) - 1/28*(
b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A]
time = 0.48, size = 34, normalized size = 0.58

$$-\frac{(2\,b^3x^3-2\,b^2x^2+3\,bx+30)\sqrt{bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x + 30)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(53) = 106$.
time = 8.05, size = 270, normalized size = 4.58

$$-\frac{2b^{\frac{19}{2}}x^5\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{3b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{132b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{120b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] -2*b**(19/2)*x**5*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b*
*4*x**3) - 6*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**
4 + 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 42
0*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(105*b**
6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 132*b**(11/2)*x*sqrt(1 + 2/(b*x))
/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(1 + 2/
(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3)
```

Giac [A]
time = 1.14, size = 55, normalized size = 0.93

$$-\frac{(35\,b^7+2\,((bx+2)b^7-7\,b^7)(bx+2))(bx+2)^{\frac{3}{2}}b}{105\,((bx+2)b-2\,b)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2)^(3/2)*b/(((
b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 34, normalized size = 0.58

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{105}-\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(9/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/35 - (2*b^2*x^2)/105 + (2*b^3*x^3)/105 + 2/7))/x^(
7/2)
```

3.513 $\int x^{5/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=112

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] 5/4*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/24*x^(3/2)*(-b*x+2)^(1/2)
/b^2-1/12*x^(5/2)*(-b*x+2)^(1/2)/b+1/4*x^(7/2)*(-b*x+2)^(1/2)-5/8*x^(1/2)*(
-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {52, 56, 222}

$$\frac{5\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(24*b^2) - (
x^(5/2)*Sqrt[2 - b*x])/(12*b) + (x^(7/2)*Sqrt[2 - b*x])/4 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{5/2}\sqrt{2-bx}\,dx=\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{1}{4}\int\frac{x^{5/2}}{\sqrt{2-bx}}\,dx$$
$$=-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int\frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{12b}$$
$$=-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int\frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2}$$
$$=-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\int\frac{1}{\sqrt{x}\sqrt{2-}}}{8b^3}$$
$$=-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\text{Subst}\left(\int\frac{}{\sqrt{}}\right.}{}$$
$$=-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{x^{5/2}\sqrt{2-bx}}{12b}+\frac{1}{4}x^{7/2}\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{4b^{7/2}}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{2-bx}\left(-15-5bx-2b^2x^2+6b^3x^3\right)}{24b^3}+\frac{5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(7/2))
```

Maple [A]
time = 0.12, size = 128, normalized size = 1.14

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\,(-b)^{\frac{7}{2}}\left(-42b^3x^3+14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{168b^3}-\frac{5\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{4b^{\frac{7}{2}}}}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$

risch	$-\frac{(6b^3x^3-2x^2b^2-5bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{24b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+2)^{\frac{3}{2}}}{4b}+\frac{-\frac{5x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{12b}+\frac{5\left(-\frac{\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\,\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}\right.}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+2)^(3/2)+5/4/b*(-1/3/b*x^(3/2)*(-b*x+2)^(3/2)+1/b*(-1/
2/b*x^(1/2)*(-b*x+2)^(3/2)+1/2/b*(x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)
/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))
))
```

Maxima [A]

time = 0.52, size = 147, normalized size = 1.31

$$\frac{\frac{15\sqrt{-bx+2}\,b^3}{\sqrt{x}}-\frac{73\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4\,(bx-2)b^6}{x}+\frac{6\,(bx-2)^2b^5}{x^2}-\frac{4\,(bx-2)^3b^4}{x^3}+\frac{(bx-2)^4b^3}{x^4}\right)}-\frac{5\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) - 73*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 55*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x - 2
)*b^6/x + 6*(b*x - 2)^2*b^5/x^2 - 4*(b*x - 2)^3*b^4/x^3 + (b*x - 2)^4*b^3/x
^4) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]

time = 0.50, size = 141, normalized size = 1.26

$$\left[\frac{(6\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-15\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{24\,b^4},\frac{(6\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-30\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 15
*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, 1/24*((6*b^
4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arc
tan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 22.59, size = 250, normalized size = 2.23

$$\begin{cases} \frac{ibx^{\frac{9}{2}}}{4\sqrt{bx-2}} - \frac{7ix^{\frac{7}{2}}}{12\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{24b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{24b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{4b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for}\ |bx| > 2 \\ -\frac{bx^{\frac{9}{2}}}{4\sqrt{-bx+2}} + \frac{7x^{\frac{7}{2}}}{12\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{24b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{24b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{4b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(9/2)/(4*sqrt(b*x - 2)) - 7*I*x**(7/2)/(12*sqrt(b*x - 2))
 - I*x**(5/2)/(24*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(24*b**2*sqrt(b*x - 2)) +
 5*I*sqrt(x)/(4*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(4*b**(7/2)), Abs(b*x) > 2), (-b*x**(9/2)/(4*sqrt(-b*x + 2)) + 7*x**(7/2)/(
12*sqrt(-b*x + 2)) + x**(5/2)/(24*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(24*b**2*s
qrt(-b*x + 2)) - 5*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)
*sqrt(x)/2)/(4*b**(7/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{2-b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(1/2), x)
```

3.514 $\int x^{3/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

[Out] arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-1/6*x^(3/2)*(-b*x+2)^(1/2)/b+1/3*x^(5/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b^2

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*Sqrt[2 - b*x],x]

[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x])/b^2 - (x^(3/2)*Sqrt[2 - b*x])/(6*b) + (x^(5/2)*Sqrt[2 - b*x])/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}\sqrt{2-bx}\,dx=\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{1}{3}\int\frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$=-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\int\frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}$$
$$=-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\int\frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}$$
$$=-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}$$
$$=-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.83

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-3-bx+2b^2x^2)}{6b^2}+\frac{b\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-3 - b*x + 2*b^2*x^2))/(6*b^2) + (b*Log[-(Sqrt[-b]*
Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Maple [A]
time = 0.12, size = 106, normalized size = 1.22

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(-10x^2b^2+5bx+15\right)\sqrt{-\frac{bx}{2}+1}}{30b^2}-\frac{\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}}}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$	81
default	$-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{3b}+\frac{-\frac{\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\,\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{2b}}{b}$	106

risch	$-\frac{(2x^2b^2-bx-3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+2)^(3/2)+1/b*(-1/2/b*x^(1/2)*(-b*x+2)^(3/2)+1/2/b*(x^(
1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arcta
n(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))))
```

Maxima [A]
time = 0.54, size = 117, normalized size = 1.34

$$\frac{\frac{3\sqrt{-bx+2}\,b^2}{\sqrt{x}}-\frac{8\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5-\frac{3\,(bx-2)b^4}{x}+\frac{3\,(bx-2)^2b^3}{x^2}-\frac{(bx-2)^3b^2}{x^3}\right)}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) - 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x - 2)*b^4/x + 3*(b*x - 2)^2*b^3/x^2 - (b*x
 - 2)^3*b^2/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.45, size = 125, normalized size = 1.44

$$\left[\frac{(2\,b^3x^2-b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}\,-3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}\,+1\right)}{6\,b^3},\frac{(2\,b^3x^2-b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}\,-6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x) - 3*sqrt(-b)*log(-b*
x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 - b^2*x - 3*
b)*sqrt(-b*x + 2)*sqrt(x) - 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x
))))/b^3]
```

Sympy [C] Result contains complex when optimal does not.

time = 5.37, size = 194, normalized size = 2.23

$$\begin{cases} \frac{ibx^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{5ix^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{6b\sqrt{bx-2}} + \frac{i\sqrt{x}}{b^2\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ -\frac{bx^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{5x^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{6b\sqrt{-bx+2}} - \frac{\sqrt{x}}{b^2\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(7/2)/(3*sqrt(b*x - 2)) - 5*I*x**(5/2)/(6*sqrt(b*x - 2))
- I*x**(3/2)/(6*b*sqrt(b*x - 2)) + I*sqrt(x)/(b**2*sqrt(b*x - 2)) - I*acosh
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x) > 2), (-b*x**(7/2)/(3*sqrt(-
b*x + 2)) + 5*x**(5/2)/(6*sqrt(-b*x + 2)) + x**(3/2)/(6*b*sqrt(-b*x + 2)) -
 sqrt(x)/(b**2*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2),
True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{2-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(1/2), x)
```

3.515 $\int \sqrt{x}\sqrt{2-bx}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*
x^(1/2)*(-b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x])/b + (x^(3/2)*Sqrt[2 - b*x])/2 + ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\sqrt{2-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}\\ &= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.95

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-1+bx)}{2b}+\frac{\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 - b*x],x]
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-1 + b*x))/(2*b) + Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2
 - b*x]]/(-b)^(3/2)
```

Maple [A]
time = 0.12, size = 85, normalized size = 1.31

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\,(-b)^{\frac{3}{2}}(-3bx+3)\sqrt{-\frac{bx}{2}+1}}{6b}-\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}}}{\sqrt{-b}\sqrt{\pi}\,b}$	73
default	$-\frac{\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}{2b}+\frac{\sqrt{x}\sqrt{-bx+2}+\frac{\sqrt{(-bx+2)x}\arctan\left(\frac{\sqrt{b}\,(x-\frac{1}{b})}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\sqrt{x}\sqrt{b}}}{2b}$	85
risch	$-\frac{(bx-1)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)x}}{2b\sqrt{-x(bx-2)}\sqrt{-bx+2}}+\frac{\arctan\left(\frac{\sqrt{b}\,(x-\frac{1}{b})}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)x}}{2b^{\frac{3}{2}}\sqrt{x}\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*x^(1/2)*(-b*x+2)^(3/2)+1/2/b*(x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1
/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2
)))
```

Maxima [A]
time = 0.50, size = 81, normalized size = 1.25

$$\frac{\frac{\sqrt{-bx+2}\,b}{\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3-\frac{2\,(bx-2)b^2}{x}+\frac{(bx-2)^2b}{x^2}}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(-b*x + 2)*b/sqrt(x) - (-b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x - 2)*b^
2/x + (b*x - 2)^2*b/x^2) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.48, size = 107, normalized size = 1.65

$$\left[\frac{(b^2x-b)\sqrt{-bx+2}\,\sqrt{x}-\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b^2},\frac{(b^2x-b)\sqrt{-bx+2}\,\sqrt{x}-2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x +
2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - 2*
sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.82, size = 155, normalized size = 2.38

$$\begin{cases}\frac{ibx^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{3ix^{\frac{3}{2}}}{2\sqrt{bx-2}}+\frac{i\sqrt{x}}{b\sqrt{bx-2}}-\frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx|>2\\ -\frac{bx^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{3x^{\frac{3}{2}}}{2\sqrt{-bx+2}}-\frac{\sqrt{x}}{b\sqrt{-bx+2}}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(5/2)/(2*sqrt(b*x - 2)) - 3*I*x**(3/2)/(2*sqrt(b*x - 2))
+ I*sqrt(x)/(b*sqrt(b*x - 2)) - I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
, Abs(b*x) > 2), (-b*x**(5/2)/(2*sqrt(-b*x + 2)) + 3*x**(3/2)/(2*sqrt(-b*x
+ 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/
2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [B]
time = 0.10, size = 53, normalized size = 0.82

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{1}{2\,b}\right)\,\sqrt{2-b\,x}-\frac{\ln\left(\sqrt{-b}\,\sqrt{x}\,\sqrt{2-b\,x}-b\,x+1\right)}{2\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(2*b))*(2 - b*x)^(1/2) - log((-b)^(1/2)*x^(1/2)*(2 - b*x)^
(1/2) - b*x + 1)/(2*(-b)^(3/2))
```

3.516 $\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{x}\,\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

[Out] 2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+x^(1/2)*(-b*x+2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{2\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\sqrt{x}\,\sqrt{2-bx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 - b*x]/Sqrt[x],x]

[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\,\sqrt{2-bx} + \int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= \sqrt{x}\,\sqrt{2-bx} + 2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x}\,\sqrt{2-bx} + \frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 1.27

$$\sqrt{x}\,\sqrt{2-bx} - \frac{2\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{2-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 - b*x] - (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/Sqrt[-
b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(30) = 60$.
time = 0.11, size = 63, normalized size = 1.54

method	result	size
default	$\sqrt{x}\,\sqrt{-bx+2} + \frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	63
meijerg	$\frac{\sqrt{-b}\left(-\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{-b}\,\sqrt{-\frac{bx}{2}+1} - \frac{2\sqrt{\pi}\,\sqrt{-b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}\right)}{\sqrt{\pi}\,b}$	63
risch	$-\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}} + \frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*ar
ctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.50, size = 49, normalized size = 1.20

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}+\frac{2\sqrt{-bx+2}}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 2*sqrt(-b*x + 2)/((b
- (b*x - 2)/x)*sqrt(x))
```

Fricas [A]
time = 0.45, size = 89, normalized size = 2.17

$$\left[\frac{\sqrt{-bx+2}\,b\sqrt{x}-\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b},\ \frac{\sqrt{-bx+2}\,b\sqrt{x}-2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b*x + 2)*b*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sq
rt(x) + 1))/b, (sqrt(-b*x + 2)*b*sqrt(x) - 2*sqrt(b)*arctan(sqrt(-b*x + 2)/
(sqrt(b)*sqrt(x))))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.90, size = 119, normalized size = 2.90

$$\begin{cases} \frac{ibx^{\frac{3}{2}}}{\sqrt{bx-2}}-\frac{2i\sqrt{x}}{\sqrt{bx-2}}-\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx|>2 \\ -\frac{bx^{\frac{3}{2}}}{\sqrt{-bx+2}}+\frac{2\sqrt{x}}{\sqrt{-bx+2}}+\frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(3/2)/sqrt(b*x - 2) - 2*I*sqrt(x)/sqrt(b*x - 2) - 2*I*aco
sh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x) > 2), (-b*x**(3/2)/sqrt(-b*
```

```
x + 2) + 2*sqrt(x)/sqrt(-b*x + 2) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(
b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [B]
time = 0.56, size = 42, normalized size = 1.02

$$\sqrt{x}\,\sqrt{2-b\,x}-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(2 - b*x)^(1/2) - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1
/2))))/b^(1/2)
```

3.517 $\int \frac{\sqrt{2-bx}}{x^{3/2}} \, dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$-2\sqrt{b}\,\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - (2b)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.26

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.
time = 0.11, size = 64, normalized size = 1.52

method	result	size
meijerg	$\frac{(-b)^{\frac{3}{2}}\left(\frac{4\sqrt{\pi}\,\sqrt{2}\,\sqrt{-\frac{bx}{2}+1}}{\sqrt{x}\,\sqrt{-b}}+\frac{4\sqrt{\pi}\,\sqrt{b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{-b}}\right)}{2\sqrt{\pi}\,b}$	64
risch	$\frac{2(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b)^(3/2)/Pi^(1/2)/b*(4*Pi^(1/2)/x^(1/2)*2^(1/2)/(-b)^(1/2)*(-1/2*b*x+1)^(1/2)+4*Pi^(1/2)/(-b)^(1/2)*b^(1/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.52, size = 35, normalized size = 0.83

$$2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\frac{2\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + 2)/sqrt(
x)
```

Fricas [A]
time = 0.45, size = 90, normalized size = 2.14

$$\left[\frac{\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-2\sqrt{-bx+2}\,\sqrt{x}}{x},\frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - 2*sqrt(-b*x
+ 2)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - sq
rt(-b*x + 2)*sqrt(x))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.85, size = 122, normalized size = 2.90

$$\begin{cases} 2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{2ib\sqrt{x}}{\sqrt{bx-2}}+\frac{4i}{\sqrt{x}\sqrt{bx-2}} & \text{for } |bx|>2 \\ -2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{2b\sqrt{x}}{\sqrt{-bx+2}}-\frac{4}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - 2*I*b*sqrt(x)/sqr
t(b*x - 2) + 4*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x) > 2), (-2*sqrt(b)*asin(s
qrt(2)*sqrt(b)*sqrt(x)/2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 4/(sqrt(x)*sqrt(-b
*x + 2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(1/2)/x^(3/2), x)
```

3.518 $\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

```
[Out] -1/3*(-b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(5/2),x]
[Out] -1/3*(2 - b*x)^(3/2)/x^(3/2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx = -\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A]
time = 0.06, size = 19, normalized size = 1.00

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(5/2),x]
[Out] -1/3*(2 - b*x)^(3/2)/x^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.14, size = 29, normalized size = 1.53

method	result	size
gosper	$-\frac{(-bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	14
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$	17
default	$-\frac{2\sqrt{-bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{-bx+2}}{3\sqrt{x}}$	29
risch	$-\frac{\sqrt{(-bx+2)\,x}\,\left(x^{2} b^{2}-4bx+4\right)}{3x^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{-x\,(bx-2)}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(-b*x+2)^(1/2)/x^(3/2)+1/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.68

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -1/3*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.48, size = 18, normalized size = 0.95

$$\frac{(bx-2)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)/x^(3/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 83, normalized size = 4.37

$$\begin{cases} \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} - \frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} - \frac{2i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(3/2)*sqrt(-1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 1/Abs(b*x) > 1/2), (I*b**(3/2)*sqrt(1 - 2/(b*x))/3 - 2*I*sqrt(b)*sqrt(1
 - 2/(b*x))/(3*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.
time = 1.42, size = 35, normalized size = 1.84

$$\frac{(bx-2)\sqrt{-bx+2}\,b^4}{3\left((bx-2)b+2\,b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)*b^4/(((b*x - 2)*b + 2*b)^(3/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 18, normalized size = 0.95

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}-\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(5/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/3 - 2/3))/x^(3/2)
```

3.519 $\int \frac{\sqrt{2-bx}}{x^{7/2}} \, dx$

Optimal. Leaf size=40

$$-\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}$$

```
[Out] -1/5*(-b*x+2)^(3/2)/x^(5/2)-1/15*b*(-b*x+2)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -1/5*(2 - b*x)^(3/2)/x^(5/2) - (b*(2 - b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{7/2}} \, dx &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}+\frac{1}{5}b\int \frac{\sqrt{2-bx}}{x^{5/2}} \, dx\\ &= -\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 31, normalized size = 0.78

$$\frac{\sqrt{2-bx}\,(-6+bx+b^2x^2)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(7/2),x]
[Out] (Sqrt[2 - b*x]*(-6 + b*x + b^2*x^2))/(15*x^(5/2))
```

Maple [A]
time = 0.11, size = 46, normalized size = 1.15

method	result	size
gosper	$-\frac{(bx+3)(-bx+2)^{\frac{3}{2}}}{15x^{\frac{5}{2}}}$	19
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{6}x^2b^2-\frac{1}{6}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
default	$-\frac{2\sqrt{-bx+2}}{5x^{\frac{5}{2}}}-\frac{b\left(-\frac{\sqrt{-bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{-bx+2}}{3\sqrt{x}}\right)}{5}$	46
risch	$-\frac{\sqrt{(-bx+2)\,x}\,(b^3x^3-x^2b^2-8bx+12)}{15x^{\frac{5}{2}}\sqrt{-bx+2}\,\sqrt{-x\,(bx-2)}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(-b*x+2)^(1/2)/x^(5/2)-1/5*b*(-1/3*(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+
2)^(1/2)/x^(1/2))
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.70

$$-\frac{(-bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}}{10\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] -1/6*(-b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(-b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.50, size = 25, normalized size = 0.62

$$\frac{(b^2x^2+bx-6)\sqrt{-bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 + b*x - 6)*sqrt(-b*x + 2)/x^(5/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.86, size = 194, normalized size = 4.85

$$\begin{cases} \frac{b^{\frac{9}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{15b^2x^2-30bx} - \frac{b^{\frac{7}{2}}x\sqrt{-1+\frac{2}{bx}}}{15b^2x^2-30bx} - \frac{8b^{\frac{5}{2}}\sqrt{-1+\frac{2}{bx}}}{15b^2x^2-30bx} + \frac{12b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{x(15b^2x^2-30bx)} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ \frac{ib^{\frac{5}{2}}\sqrt{1-\frac{2}{bx}}}{15} + \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{15x} - \frac{2i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{5x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((b**(9/2)*x**2*sqrt(-1 + 2/(b*x))/(15*b**2*x**2 - 30*b*x) - b**(7
/2)*x*sqrt(-1 + 2/(b*x))/(15*b**2*x**2 - 30*b*x) - 8*b**(5/2)*sqrt(-1 + 2/(
b*x))/(15*b**2*x**2 - 30*b*x) + 12*b**(3/2)*sqrt(-1 + 2/(b*x))/(x*(15*b**2*
x**2 - 30*b*x)), 1/Abs(b*x) > 1/2), (I*b**(5/2)*sqrt(1 - 2/(b*x))/15 + I*b*
*(3/2)*sqrt(1 - 2/(b*x))/(15*x) - 2*I*sqrt(b)*sqrt(1 - 2/(b*x))/(5*x**2), T
rue))
```

Giac [A]
time = 1.38, size = 48, normalized size = 1.20

$$\frac{((bx-2)b^5+5\,b^5)(bx-2)\sqrt{-bx+2}\,b}{15\,((bx-2)b+2\,b)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x - 2)*b^5 + 5*b^5)*(b*x - 2)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b
)^(5/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 26, normalized size = 0.65

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b^2\,x^2}{15}+\frac{b\,x}{15}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(7/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/15 + (b^2*x^2)/15 - 2/5))/x^(5/2)
```

3.520 $\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{(2-bx)^{3/2}}{7x^{7/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}$$

```
[Out] -1/7*(-b*x+2)^(3/2)/x^(7/2)-2/35*b*(-b*x+2)^(3/2)/x^(5/2)-2/105*b^2*(-b*x+2
)^(3/2)/x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -1/7*(2 - b*x)^(3/2)/x^(7/2) - (2*b*(2 - b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 - b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx = -\frac{(2-bx)^{3/2}}{7x^{7/2}} + \frac{1}{7}(2b)\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx$$
$$= -\frac{(2-bx)^{3/2}}{7x^{7/2}} - \frac{2b(2-bx)^{3/2}}{35x^{5/2}} + \frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx$$
$$= -\frac{(2-bx)^{3/2}}{7x^{7/2}} - \frac{2b(2-bx)^{3/2}}{35x^{5/2}} - \frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.66

$$\frac{\sqrt{2-bx}\,\left(-30+3bx+2b^2x^2+2b^3x^3\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-30 + 3*b*x + 2*b^2*x^2 + 2*b^3*x^3))/(105*x^(7/2))
```

Maple [A]
time = 0.13, size = 63, normalized size = 1.02

method	result	size
gosper	$-\frac{\left(2x^2b^2+6bx+15\right)\left(-bx+2\right)^{\frac{3}{2}}}{105x^{\frac{7}{2}}}$	28
meijerg	$-\frac{2\sqrt{2}\,\left(-\frac{1}{15}b^3x^3-\frac{1}{15}x^2b^2-\frac{1}{10}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
default	$-\frac{2\sqrt{-bx+2}}{7x^{\frac{7}{2}}}-\frac{b\left(-\frac{\sqrt{-bx+2}}{5x^{\frac{5}{2}}}+\frac{2b\left(-\frac{\sqrt{-bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{-bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	63
risch	$-\frac{\sqrt{\left(-bx+2\right)x}\,\left(2b^4x^4-2b^3x^3-x^2b^2-36bx+60\right)}{105x^{\frac{7}{2}}\sqrt{-bx+2}\,\sqrt{-x\left(bx-2\right)}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(-b*x+2)^(1/2)/x^(7/2)-1/7*b*(-1/5*(-b*x+2)^(1/2)/x^(5/2)+2/5*b*(-1/3*
(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+2)^(1/2)/x^(1/2)))
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.71

$$-\frac{(-bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(-bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -1/12*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 1/10*(-b*x + 2)^(5/2)*b/x^(5/2) - 1/28
*(-b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A]
time = 0.49, size = 35, normalized size = 0.56

$$\frac{(2\,b^3x^3+2\,b^2x^2+3\,bx-30)\sqrt{-bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*b^3*x^3 + 2*b^2*x^2 + 3*b*x - 30)*sqrt(-b*x + 2)/x^(7/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.83, size = 556, normalized size = 8.97

$$\begin{cases} \frac{2b^{\frac{19}{2}}x^5\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{3b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{132b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{120b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2} \\ \frac{2ib^{\frac{19}{2}}x^5\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{6ib^{\frac{17}{2}}x^4\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{3ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{34ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}+\frac{132ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3}-\frac{120ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{105b^6x^5-420b^5x^4+420b^4x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((2*b**(19/2)*x**5*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x*
*4 + 420*b**4*x**3) - 6*b**(17/2)*x**4*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 -
420*b**5*x**4 + 420*b**4*x**3) + 3*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(105*b
**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(-1 + 2/(
b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 132*b**(11/2)*x*sqr
t(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9
/2)*sqrt(-1 + 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3), 1/A
bs(b*x) > 1/2), (2*I*b**(19/2)*x**5*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*
b**5*x**4 + 420*b**4*x**3) - 6*I*b**(17/2)*x**4*sqrt(1 - 2/(b*x))/(105*b**6
*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 3*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x
))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) - 34*I*b**(13/2)*x**2*sq
rt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3) + 132*I*b**
```

```
(11/2)*x*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**4*x**3)
- 120*I*b**(9/2)*sqrt(1 - 2/(b*x))/(105*b**6*x**5 - 420*b**5*x**4 + 420*b**
4*x**3), True))
```

Giac [A]
time = 1.29, size = 61, normalized size = 0.98

$$\frac{(35\,b^7+2\,((bx-2)b^7+7\,b^7)(bx-2))(bx-2)\sqrt{-bx+2}\,b}{105\,((bx-2)b+2\,b)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(35*b^7 + 2*((b*x - 2)*b^7 + 7*b^7)*(b*x - 2))*(b*x - 2)*sqrt(-b*x +
2)*b/(((b*x - 2)*b + 2*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.22, size = 34, normalized size = 0.55

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,b^3\,x^3}{105}+\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/35 + (2*b^2*x^2)/105 + (2*b^3*x^3)/105 - 2/7))/x^(7
/2)
```

3.521 $\int x^{5/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=143

$$\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}$$

```
[Out] 1/5*x^(7/2)*(b*x+a)^(3/2)-3/128*a^5*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/
b^(7/2)-1/64*a^3*x^(3/2)*(b*x+a)^(1/2)/b^2+1/80*a^2*x^(5/2)*(b*x+a)^(1/2)/b
+3/40*a*x^(7/2)*(b*x+a)^(1/2)+3/128*a^4*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a + b*x])/(64*b
^2) + (a^2*x^(5/2)*Sqrt[a + b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a + b*x])/40 +
 (x^(7/2)*(a + b*x)^(3/2))/5 - (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(128*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a+bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a+bx)^{3/2} + \frac{1}{10}(3a)\int x^{5/2}\sqrt{a+bx}\,dx$$

$$= \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}(a+bx)^{3/2} + \frac{1}{80}\left(3a^2\right)\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{a^2x^{5/2}\sqrt{a+bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}(a+bx)^{3/2} - \frac{a^3\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{32b}$$

$$= -\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{5/2}\sqrt{a+bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}(a+bx)^{3/2} + \frac{(3a^4)}{}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{5/2}\sqrt{a+bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{5/2}\sqrt{a+bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{5/2}\sqrt{a+bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}$$

$$= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{5/2}\sqrt{a+bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a+bx} + \frac{1}{5}x^{7/2}$$

Mathematica [A]
time = 0.13, size = 99, normalized size = 0.69

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\,(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4)+15a^5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{640b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(3/2),x]
```

[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*a
*b^3*x^3 + 128*b^4*x^4) + 15*a^5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(
640*b^(7/2))

Maple [A]
time = 0.12, size = 144, normalized size = 1.01

method	result
risch	$\frac{(128b^4x^4+176a\,b^3x^3+8a^2b^2x^2-10a^3bx+15a^4)\sqrt{x}\,\sqrt{bx+a}}{640b^3}-\frac{3a^5\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{256b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{5}{2}}(bx+a)^{\frac{5}{2}}}{5b}-\frac{a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b}-\frac{a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{\cdots}\right)}{2\sqrt{bx+a}\,\sqrt{x}\cdots}\right)}{4}\right)}{6b}\right)}{8b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/5/b*x^(5/2)*(b*x+a)^(5/2)-1/2*a/b*(1/4/b*x^(3/2)*(b*x+a)^(5/2)-3/8*a/b*(1
/3/b*x^(1/2)*(b*x+a)^(5/2)-1/6*a/b*(1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2

```
)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x
)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(103) = 206$.
time = 0.50, size = 212, normalized size = 1.48

$$\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^5b^4}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}-\frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8-\frac{5\,(bx+a)b^7}{x}+\frac{10\,(bx+a)^2b^6}{x^2}-\frac{10\,(bx+a)^3b^5}{x^3}+\frac{5\,(bx+a)^4b^4}{x^4}-\frac{(bx+a)^5b^3}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^(
3/2)*a^5*b^3/x^(3/2) - 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^(
7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x + a)*b^7
/x + 10*(b*x + a)^2*b^6/x^2 - 10*(b*x + a)^3*b^5/x^3 + 5*(b*x + a)^4*b^4/x^
4 - (b*x + a)^5*b^3/x^5)
```

Fricas [A]
time = 0.51, size = 184, normalized size = 1.29

$$\left[\frac{15\,a^5\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b)\sqrt{bx+a}\,\sqrt{x}}{1280\,b^4},\ \frac{15\,a^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b)\sqrt{bx+a}\,\sqrt{x}}{640\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)*s
qrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt
(-b)/(b*sqrt(x))) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b
^2*x + 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [A]
time = 64.17, size = 178, normalized size = 1.24

$$\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}-\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(3/2),x)
```

```
[Out] 3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + a**(7/2)*x**(3/2)/(128*b**2
*sqrt(1 + b*x/a)) - a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*a**(3/2)
*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*x/a))
 - 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(5
*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(3/2), x)
```

3.522 $\int x^{3/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=119

$$-\frac{3a^3\sqrt{x}\,\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(b*x+a)^(3/2)+3/64*a^4*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b
^(5/2)+1/32*a^2*x^(3/2)*(b*x+a)^(1/2)/b+1/8*a*x^(5/2)*(b*x+a)^(1/2)-3/64*a^
3*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\,\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^2) + (a^2*x^(3/2)*Sqrt[a + b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a + b*x])/8 + (x^(5/2)*(a + b*x)^(3/2))/4 + (3*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{1}{8}(3a)\int x^{3/2}\sqrt{a+bx}\,dx\\ &= \frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{1}{16}a^2\int\frac{x^{3/2}}{\sqrt{a+bx}}\,dx\\ &= \frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}-\frac{(3a^3)\int\frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{(3a}{}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{(3a}{}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{(3a}{}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}+\frac{3a^4}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 0.74

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\left(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3\right)-3a^4\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3*
x^3) - 3*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64*b^(5/2))
```

Maple [A]
time = 0.10, size = 122, normalized size = 1.03

method	result
risch	$-\frac{(-16b^3x^3-24a\,b^2x^2-2a^2bx+3a^3)\sqrt{x}\,\sqrt{bx+a}}{64b^2}+\frac{3a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{128b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{5}{2}}}{3b}-\frac{a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+a)^(5/2)-3/8*a/b*(1/3/b*x^(1/2)*(b*x+a)^(5/2)-1/6*a/b*(1
/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/
2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2))
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(85) = 170.
time = 0.52, size = 178, normalized size = 1.50

$$-\frac{3\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{5}{2}}}-\frac{\frac{3\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{11\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx+a)b^5}{x}+\frac{6\,(bx+a)^2b^4}{x^2}-\frac{4\,(bx+a)^3b^3}{x^3}+\frac{(bx+a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/64*(3*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 11*(b*x + a)^(3
/2)*a^4*b^2/x^(3/2) - 11*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 3*(b*x + a)^(7/2)*
a^4/x^(7/2))/(b^6 - 4*(b*x + a)*b^5/x + 6*(b*x + a)^2*b^4/x^2 - 4*(b*x + a)
^3*b^3/x^3 + (b*x + a)^4*b^2/x^4)
```

Fricas [A]
time = 0.46, size = 163, normalized size = 1.37

$$\left[\frac{3\,a^{4}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(16\,b^{4}x^{3}+24\,ab^{3}x^{2}+2\,a^{2}b^{2}x-3\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{128\,b^{3}},-\frac{3\,a^{4}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(16\,b^{4}x^{3}+24\,ab^{3}x^{2}+2\,a^{2}b^{2}x-3\,a^{3}b)\sqrt{bx+a}\,\sqrt{x}}{64\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

Sympy [A]
time = 13.32, size = 153, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}}+\frac{3a^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^{2}x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - a**(5/2)*x**(3/2)/(64*b*sqr
t(1 + b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*sqrt(a)*b*x**
(7/2)/(8*sqrt(1 + b*x/a)) + 3*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(5
/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} (a + b x)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(3/2),x)
[Out] int(x^(3/2)*(a + b*x)^(3/2), x)
```

3.523 $\int \sqrt{x}\,(a+bx)^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)-1/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^
(3/2)+1/4*a*x^(3/2)*(b*x+a)^(1/2)+1/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b) + (a*x^(3/2)*Sqrt[a + b*x])/4 + (x^(3/2)*
(a + b*x)^(3/2))/3 - (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8*b^(3
/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(a+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\,\sqrt{a+bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b}\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,\right.}{8b}\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\right.}{8b}\\
&= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 0.80

$$\frac{\sqrt{x}\,\sqrt{a+bx}\,(3a^2+14abx+8b^2x^2)}{24b}+\frac{a^3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(3*a^2 + 14*a*b*x + 8*b^2*x^2))/(24*b) + (a^3*Log[-(
Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(3/2))
```

Maple [A]
time = 0.11, size = 97, normalized size = 1.02

method	result	size
risch	$\frac{(8x^2b^2+14abx+3a^2)\sqrt{x}\,\sqrt{bx+a}}{24b}-\frac{a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2}+\frac{a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4}\right)}{2}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)+1/2*a*(1/2*x^(3/2)*(b*x+a)^(1/2)+1/4*a*(x^(1/2)*(
b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/
2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(67) = 134$.
time = 0.50, size = 144, normalized size = 1.52

$$\frac{a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\frac{3\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{8\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^4-\frac{3\,(bx+a)b^3}{x}+\frac{3\,(bx+a)^2b^2}{x^2}-\frac{(bx+a)^3b}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(3/2) + 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 8*(b*x + a)^(3/2)
*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x + a)*b^3/x +
3*(b*x + a)^2*b^2/x^2 - (b*x + a)^3*b/x^3)
```

Fricas [A]
time = 0.46, size = 140, normalized size = 1.47

$$\left[\frac{3\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2+14\,ab^2x+3\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^2},\ \frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^3x^2+14\,ab^2x+3\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

Sympy [**A**]
time = 4.11, size = 124, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^{2}x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 +
b*x/a)) + 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 + b*x/a)) - a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(3/2), x)
```

3.524 $\int \frac{(a+bx)^{3/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=71

$$\frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}$$

```
[Out] 3/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)+1/2*(b*x+a)^(3/2)*x^
(1/2)+3/4*a*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a + b*x])/4 + (Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a^2*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx}+\frac{1}{2}\sqrt{x}\,(a+bx)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.87

$$\frac{1}{4}\sqrt{x}\,\sqrt{a+bx}\,(5a+2bx)-\frac{3a^2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(5*a + 2*b*x))/4 - (3*a^2*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/(4*Sqrt[b])
```

Maple [A]
time = 0.12, size = 78, normalized size = 1.10

method	result	size

risch	$\frac{(2bx+5a)\sqrt{x}\,\sqrt{bx+a}}{4}+\frac{3a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	73
default	$\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1
/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(49) = 98$.
time = 0.49, size = 107, normalized size = 1.51

$$-\frac{3\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,\sqrt{b}}-\frac{\frac{3\,\sqrt{bx+a}\,a^2b}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/sqrt(b) - 1/4*(3*sqrt(b*x + a)*a^2*b/sqrt(x) - 5*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x + a)^2/x^2)
```

Fricas [A]
time = 0.44, size = 119, normalized size = 1.68

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^2x+5\,ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b},-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^2x+5\,ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
```

```
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

Sympy [A]
time = 1.87, size = 75, normalized size = 1.06

$$\frac{5a^{\frac{3}{2}}\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{4}+\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(1/2),x)
[Out] 5*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a)/4 + sqrt(a)*b*x**(3/2)*sqrt(1 + b*x/a)/2
 + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(1/2),x)
[Out] int((a + b*x)^(3/2)/x^(1/2), x)
```

3.525 $\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=63

$$3b\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 3*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))*b^(1/2)-2*(b*x+a)^(3/2)/x^(1/2)+
3*b*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {49, 52, 65, 223, 212}

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[a + b*x] - (2*(a + b*x)^(3/2))/Sqrt[x] + 3*a*Sqrt[b]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx \\
&= 3b\sqrt{x}\,\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + \frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx \\
&= 3b\sqrt{x}\,\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + (3ab)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right) \\
&= 3b\sqrt{x}\,\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + (3ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right) \\
&= 3b\sqrt{x}\,\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + 3a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 54, normalized size = 0.86

$$\frac{(-2a+bx)\sqrt{a+bx}}{\sqrt{x}} - 3a\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(3/2),x]
[Out] ((-2*a + b*x)*Sqrt[a + b*x])/Sqrt[x] - 3*a*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]]
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.13

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-bx+2a)}{\sqrt{x}}+\frac{3a\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\,(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1/2)*(-b*x+2*a)/x^(1/2)+3/2*a*b^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x
^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.51, size = 84, normalized size = 1.33

$$-\frac{3}{2}\,a\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*a*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 2*sqrt(b*x + a)*a/sqrt(x) - sqrt(b*x + a)*a*b/((b - (b*x +
a)/x)*sqrt(x))
```

Fricas [A]
time = 0.47, size = 109, normalized size = 1.73

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{2\,x},\,-\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

Sympy [A]
time = 1.49, size = 92, normalized size = 1.46

$$-\frac{2a^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}}+3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^{2}x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] -2*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) - sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a) +
 3*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1
 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{x^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(3/2), x)
```

3.526 $\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] -2/3*(b*x+a)^(3/2)/x^(3/2)+2*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))
-2*b*(b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[a + b*x])/Sqrt[x] - (2*(a + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx\\
&= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\
&= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= -\frac{2b\sqrt{a+bx}}{\sqrt{x}}-\frac{2(a+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 55, normalized size = 0.86

$$-\frac{2\sqrt{a+bx}\,(a+4bx)}{3x^{3/2}}-2b^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(a + 4*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]]
```

Maple [**A**]
time = 0.11, size = 67, normalized size = 1.05

method	result	size
risch	$-\frac{2\sqrt{bx+a}\,(4bx+a)}{3x^{\frac{3}{2}}}+\frac{b^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{\sqrt{x}\,\sqrt{bx+a}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(4*b*x+a)/x^(3/2)+b^(3/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+
a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.53, size = 67, normalized size = 1.05

$$-b^{\frac{3}{2}} \log \left(-\frac{\sqrt{b} - \frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b} + \frac{\sqrt{bx+a}}{\sqrt{x}}}\right) - \frac{2\sqrt{bx+a}\, b}{\sqrt{x}} - \frac{2\,(bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)*b/sqrt(x) - 2/3*(b*x + a)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.43, size = 109, normalized size = 1.70

$$\left[\frac{3\,b^{\frac{3}{2}}x^{2}\log\left(2\,bx + 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) - 2\,(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{3\,x^{2}}, -\frac{2\left(3\,\sqrt{-b}\,bx^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right) + (4\,bx+a)\sqrt{bx+a}\,\sqrt{x}\right)}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A]
time = 1.77, size = 71, normalized size = 1.11

$$-\frac{2a\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{3x} - \frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3} - b^{\frac{3}{2}}\log\left(\frac{a}{bx}\right) + 2b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(5/2),x)
```

[Out] -2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 8*b**(3/2)*sqrt(a/(b*x) + 1)/3 - b**(3/2)*log(a/(b*x)) + 2*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,1,0]%%%}+%%%{4,[

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(3/2)/x^(5/2),x)

[Out] int((a + b*x)^(3/2)/x^(5/2), x)

3.527 $\int x^{5/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=149

$$-\frac{3a^4\sqrt{x}\,\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}+\frac{3a^5\tan^{-1}\left(\cdots\right)}{128b\cdots}$$

```
[Out] 1/5*x^(7/2)*(-b*x+a)^(3/2)+3/128*a^5*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))
/b^(7/2)-1/64*a^3*x^(3/2)*(-b*x+a)^(1/2)/b^2-1/80*a^2*x^(5/2)*(-b*x+a)^(1/2
)/b+3/40*a*x^(7/2)*(-b*x+a)^(1/2)-3/128*a^4*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{3a^5\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{3a^4\sqrt{x}\,\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b^2) - (a^2*x^(5/2)*Sqrt[a - b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a - b*x])/40
+ (x^(7/2)*(a - b*x)^(3/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/(128*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{1}{10}(3a)\int x^{5/2}\sqrt{a-bx}\,dx$$
$$= \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{1}{80}\left(3a^2\right)\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{a^3\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{32b}$$
$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{(3a^4}{}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7}$$

Mathematica [A]
time = 0.18, size = 104, normalized size = 0.70

$$\frac{1}{640}\left(-\frac{\sqrt{x}\sqrt{a-bx}\left(15a^4+10a^3bx+8a^2b^2x^2-176ab^3x^3+128b^4x^4\right)}{b^3}+\frac{15a^5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-((Sqrt[x]*Sqrt[a - b*x]*(15*a^4 + 10*a^3*b*x + 8*a^2*b^2*x^2 - 176*a*b^3*
x^3 + 128*b^4*x^4))/b^3) + (15*a^5*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
)/(-b)^(7/2))/640
```

Maple [A]

time = 0.11, size = 152, normalized size = 1.02

method	result
risch	$-\frac{\left(128 b^{4} x^{4}-176 a\,b^{3} x^{3}+8 a^{2} b^{2} x^{2}+10 a^{3} b x+15 a^{4}\right) \sqrt{x}\,\sqrt{-b x+a}}{640 b^{3}}+\frac{3 a^{5} \arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right) \sqrt{x\left(-b x+a\right)}}{256 b^{\frac{7}{2}} \sqrt{x}\,\sqrt{-b x+a}}$
default	$-\frac{x^{\frac{5}{2}} \left(-b x+a\right)^{\frac{5}{2}}}{5 b}+\frac{a\left(-\frac{x^{\frac{3}{2}} \left(-b x+a\right)^{\frac{5}{2}}}{4 b}+\frac{3 a\left(-\frac{\sqrt{x}\,\left(-b x+a\right)^{\frac{5}{2}}}{3 b}+\frac{a\left(\frac{\left(-b x+a\right)^{\frac{3}{2}} \sqrt{x}}{2}+\frac{3 a\left(\sqrt{x}\,\sqrt{-b x+a}+\frac{a\sqrt{x\left(-b x+\right.}}{2\sqrt{-}}\right.}{4}\right.}{6 b}\right.}{8 b}\right.}{2 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+a)^(5/2)+1/2*a/b*(-1/4/b*x^(3/2)*(-b*x+a)^(5/2)+3/8*a/
b*(-1/3/b*x^(1/2)*(-b*x+a)^(5/2)+1/6*a/b*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*
(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(
1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))))
```

Maxima [A]
time = 0.52, size = 207, normalized size = 1.39

$$-\frac{3\,a^5\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^5b^4}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}-\frac{128\,(-bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8-\frac{5\,(bx-a)b^7}{x}+\frac{10\,(bx-a)^2b^6}{x^2}-\frac{10\,(bx-a)^3b^5}{x^3}+\frac{5\,(bx-a)^4b^4}{x^4}-\frac{(bx-a)^5b^3}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) - 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x - a)*b^7/x + 10*(b*x - a)^2*b^6/x^2
 - 10*(b*x - a)^3*b^5/x^3 + 5*(b*x - a)^4*b^4/x^4 - (b*x - a)^5*b^3/x^5)
```

Fricas [A]
time = 0.46, size = 185, normalized size = 1.24

$$\left[-\frac{15\,a^5\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+10\,a^3b^2x+15\,a^4b)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^4},\ -\frac{15\,a^5\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(128\,b^5x^4-176\,ab^4x^3+8\,a^2b^3x^2+10\,a^3b^2x+15\,a^4b)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a^3*b^2*x + 15*a^4
*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) + (128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a
^3*b^2*x + 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 66.96, size = 376, normalized size = 2.52

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{-1+\frac{bx}{a}}}-\frac{23ia^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}+\frac{19i\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}-\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}-\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**3*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**(5/2)/(320*b*sqrt(-1 + b
*x/a)) - 23*I*a**(3/2)*x**(7/2)/(80*sqrt(-1 + b*x/a)) + 19*I*sqrt(a)*b*x**(
9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b
**(7/2)) - I*b**2*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1),
(-3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(128*b*
*2*sqrt(1 - b*x/a)) + a**(5/2)*x**(5/2)/(320*b*sqrt(1 - b*x/a)) + 23*a**(3/
2)*x**(7/2)/(80*sqrt(1 - b*x/a)) - 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 - b*x/a
)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(
5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a-b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(3/2), x)
```

3.528 $\int x^{3/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{3a^3\sqrt{x}\,\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(-b*x+a)^(3/2)+3/64*a^4*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/
b^(5/2)-1/32*a^2*x^(3/2)*(-b*x+a)^(1/2)/b+1/8*a*x^(5/2)*(-b*x+a)^(1/2)-3/64
*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{3a^4\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\,\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^2) - (a^2*x^(3/2)*Sqrt[a - b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a - b*x])/8 + (x^(5/2)*(a - b*x)^(3/2))/4 + (3*a^4*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a-bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{8}(3a)\int x^{3/2}\sqrt{a-bx}\,dx\\ &= \frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{16}a^2\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx\\ &= -\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a^3)\int\frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a}{}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a}{}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{(3a}{}\\ &= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{3a}{}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 0.75

$$\frac{1}{64}\left(-\frac{\sqrt{x}\sqrt{a-bx}\,(3a^3+2a^2bx-24ab^2x^2+16b^3x^3)}{b^2}-\frac{3a^4\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-((Sqrt[x]*Sqrt[a - b*x]*(3*a^3 + 2*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/
b^2) - (3*a^4*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(5/2))/64
```

Maple [A]
time = 0.11, size = 129, normalized size = 1.04

method	result
risch	$-\frac{(16b^3x^3-24a\,b^2x^2+2a^2bx+3a^3)\sqrt{x}\,\sqrt{-bx+a}}{64b^2}+\frac{3a^4\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{128b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4b}+\frac{3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{5}{2}}}{3b}+\frac{a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\arctan\left(\frac{\sqrt{b}\,(}{\sqrt{-x^2}}\right.}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right.}{4}\right.}{6b}\right.}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+a)^(5/2)+3/8*a/b*(-1/3/b*x^(1/2)*(-b*x+a)^(5/2)+1/6*a/
b*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+
a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2
+a*x)^(1/2)))))
```

Maxima [A]
time = 0.52, size = 170, normalized size = 1.37

$$-\frac{3\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{5}{2}}}+\frac{\frac{3\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{11\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx-a)b^5}{x}+\frac{6\,(bx-a)^2b^4}{x^2}-\frac{4\,(bx-a)^3b^3}{x^3}+\frac{(bx-a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/64*(3*sqrt(-
b*x + a)*a^4*b^3/sqrt(x) + 11*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 11*(-b*x +
 a)^(5/2)*a^4*b/x^(5/2) - 3*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^6 - 4*(b*x - a
)*b^5/x + 6*(b*x - a)^2*b^4/x^2 - 4*(b*x - a)^3*b^3/x^3 + (b*x - a)^4*b^2/x
^4)
```

Fricas [A]
time = 0.48, size = 163, normalized size = 1.31

$$\left[-\frac{3\,a^{4}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(16\,b^{4}x^{3}-24\,ab^{3}x^{2}+2\,a^{2}b^{2}x+3\,a^{3}b)\sqrt{-bx+a}\,\sqrt{x}}{128\,b^{3}},\ -\frac{3\,a^{4}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(16\,b^{4}x^{3}-24\,ab^{3}x^{2}+2\,a^{2}b^{2}x+3\,a^{3}b)\sqrt{-bx+a}\,\sqrt{x}}{64\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(3*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^3, -1/64*(3*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) +
(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqrt(x))
/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 12.59, size = 323, normalized size = 2.60

$$\begin{cases} \frac{3ia^{\frac{7}{2}}\sqrt{x}}{64b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{13ia^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{-1+\frac{bx}{a}}}+\frac{5i\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{4}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}-\frac{ib^{2}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}}+\frac{3a^{4}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^{2}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(7/2)*sqrt(x)/(64*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**
(3/2)/(64*b*sqrt(-1 + b*x/a)) - 13*I*a**(3/2)*x**(5/2)/(32*sqrt(-1 + b*x/a)
) + 5*I*sqrt(a)*b*x**(7/2)/(8*sqrt(-1 + b*x/a)) - 3*I*a**4*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(5/2)) - I*b**2*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a))
, Abs(b*x/a) > 1), (-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 - b*x/a)) + a**(5/2
)*x**(3/2)/(64*b*sqrt(1 - b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 - b*x/a
)) - 5*sqrt(a)*b*x**(7/2)/(8*sqrt(1 - b*x/a)) + 3*a**4*asin(sqrt(b)*sqrt(x)
/sqrt(a))/(64*b**(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} (a - b x)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(3/2), x)
```

3.529 $\int \sqrt{x} (a-bx)^{3/2} dx$

Optimal. Leaf size=99

$$-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b
^(3/2)+1/4*a*x^(3/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] -1/8*(a^2*Sqrt[x]*Sqrt[a - b*x])/b + (a*x^(3/2)*Sqrt[a - b*x])/4 + (x^(3/2)
*(a - b*x)^(3/2))/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*b^(3
/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(a-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{2}a\int \sqrt{x}\,\sqrt{a-bx}\,dx\\
&= \frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{16b}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,d\right.}{8b}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\right.}{8b}\\
&= -\frac{a^2\sqrt{x}\,\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.83

$$-\frac{\sqrt{x}\,\sqrt{a-bx}\,(3a^2-14abx+8b^2x^2)}{24b}+\frac{a^3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{8(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] -1/24*(Sqrt[x]*Sqrt[a - b*x]*(3*a^2 - 14*a*b*x + 8*b^2*x^2))/b + (a^3*Log[-
(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*(-b)^(3/2))
```

Maple [A]
time = 0.11, size = 104, normalized size = 1.05

method	result	s
risch	$-\frac{(8x^2b^2-14abx+3a^2)\sqrt{x}\,\sqrt{-bx+a}}{24b}+\frac{a^3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{16b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	9
default	$\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2}+\frac{a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4}\right)}{2}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/2*a*(1/2*x^(3/2)*(-b*x+a)^(1/2)+1/4*a*(-x^(1/2
)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*
arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.52, size = 133, normalized size = 1.34

$$-\frac{a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{3\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}+\frac{8\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^4-\frac{3\,(bx-a)b^3}{x}+\frac{3\,(bx-a)^2b^2}{x^2}-\frac{(bx-a)^3b}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) + 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x - a)*b^3/x + 3*(b*x - a)^2*b^2/x^2 - (b*x -
 a)^3*b/x^3)
```

Fricas [A]
time = 0.47, size = 141, normalized size = 1.42

$$\left[-\frac{3\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2-14\,ab^2x+3\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^2},-\frac{3\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(8\,b^3x^2-14\,ab^2x+3\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/24*(
3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 - 14*a*
b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 4.02, size = 264, normalized size = 2.67

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{-1+\frac{bx}{a}}} - \frac{17ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}} + \frac{11i\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} - \frac{ib^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}} + \frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}} - \frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{b^2x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b*sqrt(-1 + b*x/a)) - 17*I*a**(3/2)*x**(3/
2)/(24*sqrt(-1 + b*x/a)) + 11*I*sqrt(a)*b*x**(5/2)/(12*sqrt(-1 + b*x/a)) -
I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) - I*b**2*x**(7/2)/(3*sqr
t(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b*sqrt(1 - b
*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 - b*x/a)) - 11*sqrt(a)*b*x**(5/2)/
(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) + b*
*2*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a-b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a - b*x)^(3/2), x)
```

3.530 $\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=74

$$\frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}$$

```
[Out] 3/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)+1/2*(-b*x+a)^(3/2)*x
^(1/2)+3/4*a*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{3a^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a - b*x])/4 + (Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a^2*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.92

$$
-\frac{1}{4}\sqrt{x}\,\sqrt{a-bx}\,(-5a+2bx)-\frac{3a^2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{4\sqrt{-b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] -1/4*(Sqrt[x]*Sqrt[a - b*x]*(-5*a + 2*b*x)) - (3*a^2*Log[-(Sqrt[-b]*Sqrt[x]
) + Sqrt[a - b*x]])/(4*Sqrt[-b])
```

Maple [A]
time = 0.10, size = 83, normalized size = 1.12

method	result	size

risch	$\frac{(-2bx+5a)\sqrt{x}\,\sqrt{-bx+a}}{4}+\frac{3a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\,(-bx+a)}}{8\sqrt{b}\,\sqrt{x}\,\sqrt{-bx+a}}$	77
default	$\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))
^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*
x)^(1/2)))
```

Maxima [A]
time = 0.52, size = 93, normalized size = 1.26

$$-\frac{3\,a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,\sqrt{b}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{2}-\frac{2\,(bx-a)b}{x}+\frac{(bx-a)^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/4*(3*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 5*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^2 - 2*(b*x - a)*b
/x + (b*x - a)^2/x^2)
```

Fricas [A]
time = 0.56, size = 119, normalized size = 1.61

$$\left[-\frac{3\,a^{2}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x-5\,ab)\sqrt{-bx+a}\,\sqrt{x}}{8\,b},-\frac{3\,a^{2}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^{2}x-5\,ab)\sqrt{-bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(b)*arctan(
sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x
))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.78, size = 190, normalized size = 2.57

$$\begin{cases} -\frac{5ia^{\frac{3}{2}}\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}} + \frac{7i\sqrt{a}\,bx^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} - \frac{ib^2x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{4} - \frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2} + \frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-5*I*a**(3/2)*sqrt(x)/(4*sqrt(-1 + b*x/a)) + 7*I*sqrt(a)*b*x**(3
/2)/(4*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(
b)) - I*b**2*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (5*a**
(3/2)*sqrt(x)*sqrt(1 - b*x/a)/4 - sqrt(a)*b*x**(3/2)*sqrt(1 - b*x/a)/2 + 3*
a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(1/2), x)
```

3.531 $\int \frac{(a-bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=66

$$-3b\sqrt{x}\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - 3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -3*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))*b^(1/2)-2*(-b*x+a)^(3/2)/x^(1/2
)-3*b*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-3a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right) - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - 3b\sqrt{x}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[a - b*x] - (2*(a - b*x)^(3/2))/Sqrt[x] - 3*a*Sqrt[b]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{\sqrt{x}} - (3b)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx \\
&= -3b\sqrt{x}\,\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - \frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= -3b\sqrt{x}\,\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - (3ab)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= -3b\sqrt{x}\,\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - (3ab)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= -3b\sqrt{x}\,\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - 3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.92

$$\frac{(-2a-bx)\sqrt{a-bx}}{\sqrt{x}} - 3a\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(3/2),x]
[Out] ((-2*a - b*x)*Sqrt[a - b*x])/Sqrt[x] - 3*a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x])
 + Sqrt[a - b*x]]
```

Maple [A]
time = 0.10, size = 74, normalized size = 1.12

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(bx+2a)}{\sqrt{x}}-\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{2\sqrt{x}\,\sqrt{-bx+a}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+a)^(1/2)*(b*x+2*a)/x^(1/2)-3/2*a*b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/
(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.51, size = 68, normalized size = 1.03

$$3\,a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{-bx+a}\,ab}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 3*a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)*a/s
qrt(x) - sqrt(-b*x + a)*a*b/((b - (b*x - a)/x)*sqrt(x))
```

Fricas [A]
time = 0.63, size = 109, normalized size = 1.65

$$\left[\frac{3\,a\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)-2\,(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{2\,x},\frac{3\,a\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
2*(b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x, (3*a*sqrt(b)*x*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) - (b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.67, size = 197, normalized size = 2.98

$$\begin{cases}\frac{2ia^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,b\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}+3ia\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{ib^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}}-3a\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(3/2)/(sqrt(x)*sqrt(-1 + b*x/a)) - I*sqrt(a)*b*sqrt(x)/sq
rt(-1 + b*x/a) + 3*I*a*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a)) - I*b**2*x**(
3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(3/2)/(sqrt(x)*sqr
t(1 - b*x/a)) + sqrt(a)*b*sqrt(x)/sqrt(1 - b*x/a) - 3*a*sqrt(b)*asin(sqrt(b
)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(3/2), x)
```

3.532 $\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2b\sqrt{a-bx}}{\sqrt{x}}-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -2/3*(-b*x+a)^(3/2)/x^(3/2)+2*b^(3/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2)
)+2*b*(-b*x+a)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 65, 223, 209}

$$2b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[a - b*x])/Sqrt[x] - (2*(a - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx\\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 0.96

$$\frac{2\sqrt{a-bx}\,(-a+4bx)}{3x^{3/2}} + 2\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x} + \sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(5/2),x]
[Out] (2*Sqrt[a - b*x]*(-a + 4*b*x))/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[a - b*x]]
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.06

method	result	size
risch	$-\frac{2\sqrt{-bx+a}\,(-4bx+a)}{3x^{\frac{3}{2}}} + \frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{\sqrt{x}\,\sqrt{-bx+a}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-b*x+a)^(1/2)*(-4*b*x+a)/x^(3/2)+b^(3/2)*arctan(b^(1/2)*(x-1/2*a/b)/(
-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.53, size = 49, normalized size = 0.73

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+a}\,b}{\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + a)*b/sq
rt(x) - 2/3*(-b*x + a)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.59, size = 115, normalized size = 1.72

$$\left[\frac{3\,\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2},-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqr
t(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.29, size = 187, normalized size = 2.79

$$\begin{cases}-\frac{2a\sqrt{b}\,\sqrt{\frac{a}{bx}-1}}{3x}+\frac{8b^{\frac{3}{2}}\,\sqrt{\frac{a}{bx}-1}}{3}-2ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right) & \text{for}\ \left|\frac{a}{bx}\right|>1\\ -\frac{2ia\sqrt{b}\,\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{8ib^{\frac{3}{2}}\,\sqrt{-\frac{a}{bx}+1}}{3}+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 8*b**(3/2)*sqrt(a/(b*x) -
 1)/3 - 2*I*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + I*b**(3/2)*log(a/(b*x
```

```
)) + 2*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)), Abs(a/(b*x)) > 1), (-2*I*a*s
qrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 8*I*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + I*b**
(3/2)*log(a/(b*x)) - 2*I*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(5/2), x)
```

3.533 $\int x^{5/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{3\sqrt{x}\,\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] 1/5*x^(7/2)*(b*x+2)^(3/2)-3/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-
1/8*x^(3/2)*(b*x+2)^(1/2)/b^2+1/20*x^(5/2)*(b*x+2)^(1/2)/b+3/20*x^(7/2)*(b*
x+2)^(1/2)+3/8*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{8b^3}-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 + b*x])/(8*b^2) + (x^(5
/2)*Sqrt[2 + b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 + b*x])/20 + (x^(7/2)*(2 + b*
x)^(3/2))/5 - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(2+bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3}{5}\int x^{5/2}\sqrt{2+bx}\,dx\\
&= \frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx\\
&= \frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{4b}\\
&= -\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2+b}}}{8b^2}\\
&= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)\\
&= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)\\
&= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.67

$$
\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2+22b^3x^3+8b^4x^4)}{40b^3}+\frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 22*b^3*x^3 + 8*b^4*x^4))/(
40*b^3) + (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(7/2))
```

Maple [A]
time = 0.12, size = 135, normalized size = 1.07

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(56b^4x^4+154b^3x^3+14x^2b^2-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{280}-\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4}}{b^{\frac{7}{2}}\sqrt{\pi}}$

risch	$\frac{(8b^4x^4+22b^3x^3+2x^2b^2-5bx+15)\sqrt{x}\,\sqrt{bx+2}}{40b^3}-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{5}{2}}}{5b}-\frac{\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3b}-\frac{\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}\right)}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+2)^(5/2)-1/b*(1/4/b*x^(3/2)*(b*x+2)^(5/2)-3/4/b*(1/3/b*x
^(1/2)*(b*x+2)^(5/2)-1/3/b*(1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(
1/2)+3/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+
2*x)^(1/2))/b^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(87) = 174$.
time = 0.52, size = 194, normalized size = 1.54

$$\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70\,(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128\,(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5\,(bx+2)b^7}{x}+\frac{10\,(bx+2)^2b^6}{x^2}-\frac{10\,(bx+2)^3b^5}{x^3}+\frac{5\,(bx+2)^4b^4}{x^4}-\frac{(bx+2)^5b^3}{x^5}\right)}+\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) - 128*(
b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(9
/2)/x^(9/2))/(b^8 - 5*(b*x + 2)*b^7/x + 10*(b*x + 2)^2*b^6/x^2 - 10*(b*x +
2)^3*b^5/x^3 + 5*(b*x + 2)^4*b^4/x^4 - (b*x + 2)^5*b^3/x^5) + 3/8*log(-(sqr
t(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.56, size = 156, normalized size = 1.24

$$\left[\frac{(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{40\,b^4},\ \frac{(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*
sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/4
0*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt
(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

Sympy [A]
time = 65.07, size = 136, normalized size = 1.08

$$\frac{b^2x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{19bx^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{40b\sqrt{bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{bx+2}}+\frac{3\sqrt{x}}{4b^3\sqrt{bx+2}}-\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(11/2)/(5*sqrt(b*x + 2)) + 19*b*x**(9/2)/(20*sqrt(b*x + 2)) + 23*x*
*(7/2)/(20*sqrt(b*x + 2)) - x**(5/2)/(40*b*sqrt(b*x + 2)) + x**(3/2)/(8*b**
2*sqrt(b*x + 2)) + 3*sqrt(x)/(4*b**3*sqrt(b*x + 2)) - 3*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(b\,x+2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(3/2), x)
```

3.534 $\int x^{3/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=105

$$-\frac{3\sqrt{x}\,\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(b*x+2)^(3/2)+3/4*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+
1/8*x^(3/2)*(b*x+2)^(1/2)/b+1/4*x^(5/2)*(b*x+2)^(1/2)-3/8*x^(1/2)*(b*x+2)^(
1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{bx+2}}{8b^2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (3*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}(2+bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\int \frac{}{\sqrt{x}\sqrt{}}}{8}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\text{Subst}\left(\int\right.}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\sinh^{-1}\left(\right.}{4b}$$

Mathematica [A]
time = 0.09, size = 75, normalized size = 0.71

$$\frac{\sqrt{x}\sqrt{2+bx}\,(-3+bx+6b^2x^2+2b^3x^3)}{8b^2} - \frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(3/2),x]
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 6*b^2*x^2 + 2*b^3*x^3))/(8*b^2) - (3*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(5/2))
```

Maple [A]
time = 0.11, size = 114, normalized size = 1.09

method	result	siz
meijerg	$\frac{-\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}\,(-10b^3x^3-30x^2b^2-5bx+15)\sqrt{\frac{bx}{2}+1}}{40}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{4}}{b^{\frac{5}{2}}\sqrt{\pi}}$	71
risch	$\frac{(2b^3x^3+6x^2b^2+bx-3)\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{5}{2}}\sqrt{x}\sqrt{bx+2}}$	84

default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{5}{2}}}{3b}-\frac{\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{3b}\right)}{4b}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+2)^(5/2)-3/4/b*(1/3/b*x^(1/2)*(b*x+2)^(5/2)-1/3/b*(1/2*(
b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(1/2)+3/2*(x*(b*x+2))^(1/2)/(b*x+2
)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(72) = 144$.
time = 0.53, size = 163, normalized size = 1.55

$$-\frac{\frac{3\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{11\,(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}-\frac{11\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{3\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^{6}-\frac{4\,(bx+2)b^{5}}{x}+\frac{6\,(bx+2)^{2}b^{4}}{x^{2}}-\frac{4\,(bx+2)^{3}b^{3}}{x^{3}}+\frac{(bx+2)^{4}b^{2}}{x^{4}}\right)}-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*sqrt(b*x + 2)*b^3/sqrt(x) - 11*(b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(b*
x + 2)^(5/2)*b/x^(5/2) + 3*(b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x + 2)*b^5/
x + 6*(b*x + 2)^2*b^4/x^2 - 4*(b*x + 2)^3*b^3/x^3 + (b*x + 2)^4*b^2/x^4) -
3/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(5/2)
```

Fricas [A]
time = 0.47, size = 137, normalized size = 1.30

$$\left[\frac{(2\,b^{4}x^{3}+6\,b^{3}x^{2}+b^{2}x-3\,b)\sqrt{bx+2}\,\sqrt{x}+3\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{8\,b^{3}},\ \frac{(2\,b^{4}x^{3}+6\,b^{3}x^{2}+b^{2}x-3\,b)\sqrt{bx+2}\,\sqrt{x}-6\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{8\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*b^4*x^3 + 6*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(
b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/8*((2*b^4*x^3 + 6*b
```

```
^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A]
time = 12.62, size = 117, normalized size = 1.11

$$\frac{b^2x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{5bx^{\frac{7}{2}}}{4\sqrt{bx+2}}+\frac{13x^{\frac{5}{2}}}{8\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^2\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(9/2)/(4*sqrt(b*x + 2)) + 5*b*x**(7/2)/(4*sqrt(b*x + 2)) + 13*x**(5
/2)/(8*sqrt(b*x + 2)) - x**(3/2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sq
rt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(b\,x+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(3/2), x)
```

3.535 $\int \sqrt{x}\,(2+bx)^{3/2}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)-arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*
x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b
*x)^(3/2))/3 - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(2+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2+bx)^{3/2}+\int \sqrt{x}\,\sqrt{2+bx}\,dx\\ &= \frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx\\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b}\\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 0.79

$$\frac{\sqrt{x}\,\sqrt{2+bx}\,(3+7bx+2b^2x^2)}{6b}+\frac{\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(3 + 7*b*x + 2*b^2*x^2))/(6*b) + Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[2 + b*x]]/b^(3/2)
```

Maple [A]
time = 0.12, size = 87, normalized size = 1.06

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(2x^2b^2+7bx+3)\sqrt{\frac{bx}{2}+1}}{6}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2+7bx+3)\sqrt{x}\,\sqrt{bx+2}}{6b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	77
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{3}+\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)+1/2*x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/
2)/b-1/2/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)
+(b*x^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.
time = 0.52, size = 132, normalized size = 1.61

$$\frac{\frac{3\sqrt{bx+2}\,b^2}{\sqrt{x}} - \frac{8\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} - \frac{3\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^4 - \frac{3\,(bx+2)b^3}{x} + \frac{3\,(bx+2)^2b^2}{x^2} - \frac{(bx+2)^3b}{x^3}\right)} + \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) - 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x + 2
)^(5/2)/x^(5/2))/(b^4 - 3*(b*x + 2)*b^3/x + 3*(b*x + 2)^2*b^2/x^2 - (b*x +
2)^3*b/x^3) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*
x + 2)/sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.60, size = 124, normalized size = 1.51

$$\left[\frac{(2\,b^3x^2+7\,b^2x+3\,b)\sqrt{bx+2}\,\sqrt{x}\,+3\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)}{6\,b^2}, \frac{(2\,b^3x^2+7\,b^2x+3\,b)\sqrt{bx+2}\,\sqrt{x}\,+6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 7*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x
 - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/6*((2*b^3*x^2 + 7*b^2*x + 3*b
)*sqrt(b*x + 2)*sqrt(x) + 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))))/b^2]
```

Sympy [A]
time = 3.74, size = 92, normalized size = 1.12

$$\frac{b^2x^{\frac{7}{2}}}{3\sqrt{bx+2}} + \frac{11bx^{\frac{5}{2}}}{6\sqrt{bx+2}} + \frac{17x^{\frac{3}{2}}}{6\sqrt{bx+2}} + \frac{\sqrt{x}}{b\sqrt{bx+2}} - \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] b**2*x**(7/2)/(3*sqrt(b*x + 2)) + 11*b*x**(5/2)/(6*sqrt(b*x + 2)) + 17*x**(
3/2)/(6*sqrt(b*x + 2)) + sqrt(x)/(b*sqrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*
sqrt(x)/2)/b**(3/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(b\,x+2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(3/2), x)
```

3.536 $\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$\frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 3*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+1/2*(b*x+2)^(3/2)*x^(1/2)+3/
2*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{1}{2}\sqrt{x}\,(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/2 + (Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*ArcSinh[(Sqr
t[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx \\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx \\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+3\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.89

$$\frac{1}{2}\sqrt{x}\,\sqrt{2+bx}\,(5+bx)-\frac{3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(5 + b*x))/2 - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Maple [A]
time = 0.12, size = 72, normalized size = 1.18

method	result	size
meijerg	$\frac{4\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\left(\frac{bx}{8}+\frac{5}{8}\right)\sqrt{\frac{bx}{2}+1}+3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{\pi}}$	54
risch	$\frac{(bx+5)\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	65
default	$\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{3\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(b*x+2)^(1/2)+3/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(42) = 84$.
time = 0.52, size = 98, normalized size = 1.61

$$-\frac{3 \log \left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2 \sqrt{b}}-\frac{\frac{3 \sqrt{bx+2}\, b}{\sqrt{x}}-\frac{5 (bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^2-\frac{2 (bx+2) b}{x}+\frac{(bx+2)^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - (3*sqrt(b*x + 2)*b/sqrt(x) - 5*(b*x + 2)^(3/2)/x^(3/2))/(b^2
- 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2)
```

Fricas [A]
time = 0.49, size = 105, normalized size = 1.72

$$\left[\frac{(b^2x+5\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b}, \frac{(b^2x+5\,b)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b, 1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) - 6*sq
rt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b]
```

Sympy [A]
time = 1.78, size = 76, normalized size = 1.25

$$\frac{b^2x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{7bx^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{5\sqrt{x}}{\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] b**2*x**(5/2)/(2*sqrt(b*x + 2)) + 7*b*x**(3/2)/(2*sqrt(b*x + 2)) + 5*sqrt(x
)/sqrt(b*x + 2) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(1/2), x)
```

3.537 $\int \frac{(2+bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=58

$$3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + 6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 6*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(b*x+2)^(3/2)/x^(1/2)+3*b*
x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 52, 56, 221}

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}} + 3b\sqrt{x}\sqrt{bx+2} + 6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[2 + b*x] - (2*(2 + b*x)^(3/2))/Sqrt[x] + 6*Sqrt[b]*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= 3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+(3b)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= 3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+(6b)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= 3b\sqrt{x}\,\sqrt{2+bx}-\frac{2(2+bx)^{3/2}}{\sqrt{x}}+6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.88

$$\frac{(-4+bx)\sqrt{2+bx}}{\sqrt{x}}-6\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-4 + b*x)*Sqrt[2 + b*x])/Sqrt[x] - 6*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[2 + b*x]]
```

Maple [A]
time = 0.12, size = 55, normalized size = 0.95

method	result	size
meijerg	$\frac{3\sqrt{b}\left(-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{bx}{4}+1\right)\sqrt{\frac{bx}{2}+1}}{3\sqrt{x}\,\sqrt{b}}+4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	55

risch	$\frac{x^2b^2-2bx-8}{\sqrt{x}\,\sqrt{bx+2}}+\frac{3\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*b^(1/2)/Pi^(1/2)*(-8/3*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(-1/4*b*x+1)*(1
/2*b*x+1)^(1/2)+4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.53, size = 81, normalized size = 1.40

$$-3\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+2}}{\sqrt{x}}-\frac{2\sqrt{bx+2}\,b}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 4*sqrt(b*x + 2)/sqrt(x) - 2*sqrt(b*x + 2)*b/((b - (b*x + 2)/x)*
sqrt(x))
```

Fricas [A]
time = 0.47, size = 99, normalized size = 1.71

$$\left[\frac{3\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x},\,-\frac{6\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + sqrt(b*x + 2)*
(b*x - 4)*sqrt(x))/x, -(6*sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))) - sqrt(b*x + 2)*(b*x - 4)*sqrt(x))/x]
```

Sympy [A]
time = 1.51, size = 73, normalized size = 1.26

$$6\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{bx+2}}-\frac{2b\sqrt{x}}{\sqrt{bx+2}}-\frac{8}{\sqrt{x}\,\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] 6*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/sqrt(b*x + 2) -
2*b*sqrt(x)/sqrt(b*x + 2) - 8/(sqrt(x)*sqrt(b*x + 2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(3/2), x)
```

3.538 $\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{2b\sqrt{2+bx}}{\sqrt{x}}-\frac{2(2+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(b*x+2)^(3/2)/x^(3/2)+2*b^(3/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))-2
*b*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 56, 221}

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[2 + b*x])/Sqrt[x] - (2*(2 + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2(2+bx)^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + (2b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2b\sqrt{2+bx}}{\sqrt{x}} - \frac{2(2+bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 0.92

$$-\frac{4\sqrt{2+bx}\,(1+2bx)}{3x^{3/2}} - 2b^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-4*Sqrt[2 + b*x]*(1 + 2*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[2 + b*x]]
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.92

method	result	size
meijerg	$\frac{3b^{\frac{3}{2}}\left(-\frac{16\sqrt{\pi}\,\sqrt{2}\,(2bx+1)\sqrt{\frac{bx}{2}+1}}{9x^{\frac{3}{2}}b^{\frac{3}{2}}}+\frac{8\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3}\right)}{4\sqrt{\pi}}$	55
risch	$-\frac{4(2x^2b^2+5bx+2)}{3x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^{\frac{3}{2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*b^(3/2)/Pi^(1/2)*(-16/9*Pi^(1/2)/x^(3/2)*2^(1/2)/b^(3/2)*(2*b*x+1)*(1/2
*b*x+1)^(1/2)+8/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 67, normalized size = 1.12

$$-b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)*b/sqrt(x) - 2/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.49, size = 108, normalized size = 1.80

$$\left[\frac{3\,b^{\frac{3}{2}}x^{2}\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-4\,(2\,bx+1)\sqrt{bx+2}\,\sqrt{x}}{3\,x^{2}},-\frac{2\left(3\,\sqrt{-b}\,bx^{2}\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,bx+1)\sqrt{bx+2}\,\sqrt{x}\right)}{3\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 4*(2*b*x
 + 1)*sqrt(b*x + 2)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + 2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2]
```

Sympy [A]
time = 1.68, size = 70, normalized size = 1.17

$$-\frac{8b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{4\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] -8*b**(3/2)*sqrt(1 + 2/(b*x))/3 - b**(3/2)*log(1/(b*x)) + 2*b**(3/2)*log(sq
rt(1 + 2/(b*x)) + 1) - 4*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(5/2), x)
```

3.539 $\int x^{5/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{x^{5/2}\sqrt{2-bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2-bx}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

```
[Out] 1/5*x^(7/2)*(-b*x+2)^(3/2)+3/4*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-
1/8*x^(3/2)*(-b*x+2)^(1/2)/b^2-1/20*x^(5/2)*(-b*x+2)^(1/2)/b+3/20*x^(7/2)*(
-b*x+2)^(1/2)-3/8*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{3\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 - b*x])/(8*b^2) - (x^(
5/2)*Sqrt[2 - b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 - b*x])/20 + (x^(7/2)*(2 - b
*x)^(3/2))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{5/2}(2-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{5}\int x^{5/2}\sqrt{2-bx}\,dx$$
$$= \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{4b}$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-}}}{8b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-b$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-b$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-b$$

Mathematica [A]
time = 0.15, size = 90, normalized size = 0.69

$$-\frac{\sqrt{x}\sqrt{2-bx}\,(15+5bx+2b^2x^2-22b^3x^3+8b^4x^4)}{40b^3} + \frac{3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/40*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2 - 22*b^3*x^3 + 8*b^4*x
^4))/b^3 + (3*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(7/2))
```

Maple [A]
time = 0.11, size = 143, normalized size = 1.09

method	result

meijerg	$-\frac{24\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(56b^4x^4-154b^3x^3+14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{6720b^3}+\frac{\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{32b^{\frac{7}{2}}}\right)}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$\frac{\left(8b^4x^4-22b^3x^3+2x^2b^2+5bx+15\right)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{40b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+2)^{\frac{5}{2}}}{5b}+\frac{-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}}{4b}+\frac{-\frac{\sqrt{x}\,(-bx+2)^{\frac{5}{2}}}{4b}+\frac{\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\cdots\right)}{2\sqrt{-bx+2}\,\sqrt{\cdots}}}{b}}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+2)^(5/2)+1/b*(-1/4/b*x^(3/2)*(-b*x+2)^(5/2)+3/4/b*(-1/
3/b*x^(1/2)*(-b*x+2)^(5/2)+1/3/b*(1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-
b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b
^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))))
```

Maxima [A]
time = 0.53, size = 179, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^4}{\sqrt{x}}+\frac{70\,(-bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128\,(-bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}-\frac{70\,(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5\,(bx-2)b^7}{x}+\frac{10\,(bx-2)^2b^6}{x^2}-\frac{10\,(bx-2)^3b^5}{x^3}+\frac{5\,(bx-2)^4b^4}{x^4}-\frac{(bx-2)^5b^3}{x^5}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) - 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^8 - 5*(b*x - 2)*b^7/x + 10*(b*x - 2)^2*b^6/x^2 - 10*(b
*x - 2)^3*b^5/x^3 + 5*(b*x - 2)^4*b^4/x^4 - (b*x - 2)^5*b^3/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.45, size = 157, normalized size = 1.20

$$\left[-\frac{\left(8\,b^5x^4-22\,b^4x^3+2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{40\,b^4},\ -\frac{\left(8\,b^5x^4-22\,b^4x^3+2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{40\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2
)*sqrt(x) + 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
4, -1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x +
 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 62.73, size = 289, normalized size = 2.21

$$\begin{cases} -\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{bx-2}} + \frac{19ibx^{\frac{9}{2}}}{20\sqrt{bx-2}} - \frac{23ix^{\frac{7}{2}}}{20\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{40b\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b^2\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^3\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } |bx| > 2 \\ \frac{b^2x^{\frac{11}{2}}}{5\sqrt{-bx+2}} - \frac{19bx^{\frac{9}{2}}}{20\sqrt{-bx+2}} + \frac{23x^{\frac{7}{2}}}{20\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{40b\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b^2\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^3\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(11/2)/(5*sqrt(b*x - 2)) + 19*I*b*x**(9/2)/(20*sqrt(b
*x - 2)) - 23*I*x**(7/2)/(20*sqrt(b*x - 2)) - I*x**(5/2)/(40*b*sqrt(b*x - 2
)) - I*x**(3/2)/(8*b**2*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**3*sqrt(b*x - 2))
 - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2)), Abs(b*x) > 2), (b**2*
x**(11/2)/(5*sqrt(-b*x + 2)) - 19*b*x**(9/2)/(20*sqrt(-b*x + 2)) + 23*x**(7
/2)/(20*sqrt(-b*x + 2)) + x**(5/2)/(40*b*sqrt(-b*x + 2)) + x**(3/2)/(8*b**2
*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(2-b\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(3/2),x)
[Out] int(x^(5/2)*(2 - b*x)^(3/2), x)
```

3.540 $\int x^{3/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(-b*x+2)^(3/2)+3/4*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-
1/8*x^(3/2)*(-b*x+2)^(1/2)/b+1/4*x^(5/2)*(-b*x+2)^(1/2)-3/8*x^(1/2)*(-b*x+2
)^(1/2)/b^2
```

Rubi [A]

time = 0.02, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{3\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (3*ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}(2-bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\int \frac{}{\sqrt{x}}}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\text{Subst}\Big(}{}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\sin^{-1}\Big(}{4}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.74

$$-\frac{\sqrt{x}\sqrt{2-bx}\,(3+bx-6b^2x^2+2b^3x^3)}{8b^2} - \frac{3\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/8*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x - 6*b^2*x^2 + 2*b^3*x^3))/b^2 - (3*Log
[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(5/2))
```

Maple [A]
time = 0.11, size = 122, normalized size = 1.12

method	result
meijerg	$-\frac{12\left(-\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\,(-b)^{\frac{5}{2}}\left(10b^3x^3-30x^2b^2+5bx+15\right)\sqrt{-\frac{bx}{2}+1}}{480b^2}+\frac{\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{16b^{\frac{5}{2}}}\right)}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$

risch	$\frac{(2b^3x^3-6x^2b^2+bx+3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{8b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}}{4b}+\frac{-\frac{\sqrt{x}\,(-bx+2)^{\frac{5}{2}}}{4b}+\frac{\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+2)^(5/2)+3/4/b*(-1/3/b*x^(1/2)*(-b*x+2)^(5/2)+1/3/b*(1
/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2)
/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))
)
```

Maxima [A]
time = 0.53, size = 147, normalized size = 1.35

$$\frac{\frac{3\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{11\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^6-\frac{4\,(bx-2)b^5}{x}+\frac{6\,(bx-2)^2b^4}{x^2}-\frac{4\,(bx-2)^3b^3}{x^3}+\frac{(bx-2)^4b^2}{x^4}\right)}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*sqrt(-b*x + 2)*b^3/sqrt(x) + 11*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(-
b*x + 2)^(5/2)*b/x^(5/2) - 3*(-b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x - 2)*b
^5/x + 6*(b*x - 2)^2*b^4/x^2 - 4*(b*x - 2)^3*b^3/x^3 + (b*x - 2)^4*b^2/x^4)
 - 3/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.44, size = 139, normalized size = 1.28

$$\left[-\frac{(2\,b^4x^3-6\,b^3x^2+b^2x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{8\,b^3},-\frac{(2\,b^4x^3-6\,b^3x^2+b^2x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*b^4*x^3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqr
t(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/8*((2*b^4*x^
3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt
(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 12.59, size = 250, normalized size = 2.29

$$\begin{cases} -\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{bx-2}} + \frac{5ibx^{\frac{7}{2}}}{4\sqrt{bx-2}} - \frac{13ix^{\frac{5}{2}}}{8\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ \frac{b^2x^{\frac{9}{2}}}{4\sqrt{-bx+2}} - \frac{5bx^{\frac{7}{2}}}{4\sqrt{-bx+2}} + \frac{13x^{\frac{5}{2}}}{8\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(9/2)/(4*sqrt(b*x - 2)) + 5*I*b*x**(7/2)/(4*sqrt(b*x
- 2)) - 13*I*x**(5/2)/(8*sqrt(b*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) +
3*I*sqrt(x)/(4*b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(
4*b**(5/2)), Abs(b*x) > 2), (b**2*x**(9/2)/(4*sqrt(-b*x + 2)) - 5*b*x**(7/2
)/(4*sqrt(-b*x + 2)) + 13*x**(5/2)/(8*sqrt(-b*x + 2)) + x**(3/2)/(8*b*sqrt(
-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqr
t(x)/2)/(4*b**(5/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(2-b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(3/2), x)
```

3.541 $\int \sqrt{x}\,(2-bx)^{3/2}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+1/2*
x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\,\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x])/b + (x^(3/2)*Sqrt[2 - b*x])/2 + (x^(3/2)*(2 -
b*x)^(3/2))/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}\,(2-bx)^{3/2}\,dx = \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \int \sqrt{x}\,\sqrt{2-bx}\,dx$$
$$= \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{\sqrt{x}\,\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\,\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x,\right.}{b}$$
$$= -\frac{\sqrt{x}\,\sqrt{2-bx}}{2b} + \frac{1}{2}x^{3/2}\sqrt{2-bx} + \frac{1}{3}x^{3/2}(2-bx)^{3/2} + \frac{\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.85

$$-\frac{\sqrt{x}\,\sqrt{2-bx}\,(3-7bx+2b^2x^2)}{6b} + \frac{\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(3 - 7*b*x + 2*b^2*x^2))/b + Log[-(Sqrt[-b]*Sqr
t[x]) + Sqrt[2 - b*x]]/(-b)^(3/2)
```

Maple [A]
time = 0.11, size = 94, normalized size = 1.12

method	result	size
meijerg	$-\frac{6\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}\left(2x^2b^2-7bx+3\right)\sqrt{-\frac{bx}{2}+1}}{36b}+\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{6b^{\frac{3}{2}}}\right)}{\sqrt{-b}\,\sqrt{\pi}\,b}$	81
default	$\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{3}+\frac{x^{\frac{3}{2}}\sqrt{-bx+2}}{2}-\frac{\sqrt{x}\,\sqrt{-bx+2}}{2b}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	94

risch	$\frac{(2x^2b^2-7bx+3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^
(1/2)/b+1/2/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2
)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.51, size = 115, normalized size = 1.37

$$\frac{\frac{3\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{8(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^4-\frac{3(bx-2)b^3}{x}+\frac{3(bx-2)^2b^2}{x^2}-\frac{(bx-2)^3b}{x^3}\right)}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) + 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^4 - 3*(b*x - 2)*b^3/x + 3*(b*x - 2)^2*b^2/x^2 - (b*x
 - 2)^3*b/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.53, size = 125, normalized size = 1.49

$$\left[-\frac{(2b^3x^2-7b^2x+3b)\sqrt{-bx+2}\,\sqrt{x}+3\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6b^2},-\frac{(2b^3x^2-7b^2x+3b)\sqrt{-bx+2}\,\sqrt{x}+6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 - 7*b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(
-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, -1/6*((2*b^3*x^2 - 7*b^2*
x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x))))/b^2]
```

Sympy [C] Result contains complex when optimal does not.

time = 3.60, size = 197, normalized size = 2.35

$$\begin{cases} -\frac{ib^2x^{\frac{7}{2}}}{3\sqrt{bx-2}} + \frac{11ibx^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{17ix^{\frac{3}{2}}}{6\sqrt{bx-2}} + \frac{i\sqrt{x}}{b\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx| > 2 \\ \frac{b^2x^{\frac{7}{2}}}{3\sqrt{-bx+2}} - \frac{11bx^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{17x^{\frac{3}{2}}}{6\sqrt{-bx+2}} - \frac{\sqrt{x}}{b\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(7/2)/(3*sqrt(b*x - 2)) + 11*I*b*x**(5/2)/(6*sqrt(b*x
 - 2)) - 17*I*x**(3/2)/(6*sqrt(b*x - 2)) + I*sqrt(x)/(b*sqrt(b*x - 2)) - I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x) > 2), (b**2*x**(7/2)/(3
*sqrt(-b*x + 2)) - 11*b*x**(5/2)/(6*sqrt(-b*x + 2)) + 17*x**(3/2)/(6*sqrt(-
b*x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(3/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(2-b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(3/2), x)
```

3.542 $\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=63

$$\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 3*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+1/2*(-b*x+2)^(3/2)*x^(1/2)+3/
2*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{3\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 - b*x])/2 + (Sqrt[x]*(2 - b*x)^(3/2))/2 + (3*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+3\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]

time = 0.08, size = 60, normalized size = 0.95

$$-\frac{1}{2}\sqrt{x}\,\sqrt{2-bx}\,(-5+bx)-\frac{3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(-5 + b*x)) - (3*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt
[2 - b*x]])/Sqrt[-b]
```

Maple [A]

time = 0.11, size = 78, normalized size = 1.24

method	result	size
meijerg	$-\frac{3\sqrt{-b}\left(\frac{4\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{-b}\,\left(-\frac{bx}{8}+\frac{5}{8}\right)\sqrt{-\frac{bx}{2}+1}}{3}+\frac{\sqrt{\pi}\,\sqrt{-b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}\right)}{\sqrt{\pi}\,b}$	69
default	$\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	78
risch	$\frac{(bx-5)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	95

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*x^(1/2)*(-b*x+2)^(1/2)+3/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))

Maxima [A]
time = 0.50, size = 79, normalized size = 1.25

$$-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{3\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{5(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^2-\frac{2(bx-2)b}{x}+\frac{(bx-2)^2}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")

[Out] -3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + (3*sqrt(-b*x + 2)*b/sqrt(x) + 5*(-b*x + 2)^(3/2)/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)^2/x^2)

Fricas [A]
time = 0.52, size = 107, normalized size = 1.70

$$\left[-\frac{(b^2x-5b)\sqrt{-bx+2}\sqrt{x}+3\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{2b},-\frac{(b^2x-5b)\sqrt{-bx+2}\sqrt{x}+6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")

[Out] [-1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, -1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b]

Sympy [C] Result contains complex when optimal does not.
time = 1.68, size = 165, normalized size = 2.62

$$\begin{cases} -\frac{ib^2x^{\frac{5}{2}}}{2\sqrt{bx-2}}+\frac{7ibx^{\frac{3}{2}}}{2\sqrt{bx-2}}-\frac{5i\sqrt{x}}{\sqrt{bx-2}}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx|>2 \\ \frac{b^2x^{\frac{5}{2}}}{2\sqrt{-bx+2}}-\frac{7bx^{\frac{3}{2}}}{2\sqrt{-bx+2}}+\frac{5\sqrt{x}}{\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(5/2)/(2*sqrt(b*x - 2)) + 7*I*b*x**(3/2)/(2*sqrt(b*x
- 2)) - 5*I*sqrt(x)/sqrt(b*x - 2) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sq
rt(b), Abs(b*x) > 2), (b**2*x**(5/2)/(2*sqrt(-b*x + 2)) - 7*b*x**(3/2)/(2*s
qrt(-b*x + 2)) + 5*sqrt(x)/sqrt(-b*x + 2) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x)/
2)/sqrt(b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(1/2), x)
```

3.543 $\int \frac{(2-bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=60

$$-3b\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-6\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -6*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(-b*x+2)^(3/2)/x^(1/2)-3*b
*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$-6\sqrt{b}\,\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\sqrt{2-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[2 - b*x] - (2*(2 - b*x)^(3/2))/Sqrt[x] - 6*Sqrt[b]*ArcSin
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(6b)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -3b\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 0.97

$$\frac{(-4-bx)\sqrt{2-bx}}{\sqrt{x}}-6\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(3/2),x]
[Out] ((-4 - b*x)*Sqrt[2 - b*x])/Sqrt[x] - 6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + S
qrt[2 - b*x]]
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.17

method	result	size
meijerg	$-\frac{3(-b)^{\frac{3}{2}}\left(-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(\frac{bx}{4}+1\right)\sqrt{-\frac{bx}{2}+1}}{3\sqrt{x}\,\sqrt{-b}}-\frac{4\sqrt{\pi}\,\sqrt{b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{-b}}\right)}{2\sqrt{\pi}\,b}$	70

risch	$\frac{(x^2b^2+2bx-8)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}-\frac{3\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*(-b)^(3/2)/Pi^(1/2)/b*(-8/3*Pi^(1/2)/x^(1/2)*2^(1/2)/(-b)^(1/2)*(1/4*b
*x+1)*(-1/2*b*x+1)^(1/2)-4*Pi^(1/2)/(-b)^(1/2)*b^(1/2)*arcsin(1/2*b^(1/2)*x
^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.52, size = 63, normalized size = 1.05

$$6\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{4\sqrt{-bx+2}}{\sqrt{x}}-\frac{2\sqrt{-bx+2}\,b}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 4*sqrt(-b*x + 2)/sqrt(
x) - 2*sqrt(-b*x + 2)*b/((b - (b*x - 2)/x)*sqrt(x))
```

Fricas [A]
time = 0.48, size = 101, normalized size = 1.68

$$\left[\frac{3\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(bx+4)\sqrt{-bx+2}\,\sqrt{x}}{x},\frac{6\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+4)\sqrt{-bx+2}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (b*x + 4)*
sqrt(-b*x + 2)*sqrt(x))/x, (6*sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt
(x))) - (b*x + 4)*sqrt(-b*x + 2)*sqrt(x))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.46, size = 158, normalized size = 2.63

$$\begin{cases} 6i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)-\frac{ib^2x^{\frac{3}{2}}}{\sqrt{bx-2}}-\frac{2ib\sqrt{x}}{\sqrt{bx-2}}+\frac{8i}{\sqrt{x}\,\sqrt{bx-2}} & \text{for}\ |bx|>2 \\ -6\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{-bx+2}}+\frac{2b\sqrt{x}}{\sqrt{-bx+2}}-\frac{8}{\sqrt{x}\,\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((6*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - I*b**2*x**(3/2)/s
qrt(b*x - 2) - 2*I*b*sqrt(x)/sqrt(b*x - 2) + 8*I/(sqrt(x)*sqrt(b*x - 2)), A
bs(b*x) > 2), (-6*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/s
qrt(-b*x + 2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 8/(sqrt(x)*sqrt(-b*x + 2)), Tr
ue))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(3/2), x)
```

3.544 $\int \frac{(2-bx)^{3/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2b\sqrt{2-bx}}{\sqrt{x}}-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(-b*x+2)^(3/2)/x^(3/2)+2*b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))+2
*b*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$2b^{3/2}\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[2 - b*x])/Sqrt[x] - (2*(2 - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{2-bx}}{x^{3/2}}\,dx \\
&= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\
&= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right) \\
&= \frac{2b\sqrt{2-bx}}{\sqrt{x}} - \frac{2(2-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 62, normalized size = 1.00

$$\frac{4\sqrt{2-bx}\,(-1+2bx)}{3x^{3/2}} + 2\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (4*Sqrt[2 - b*x]*(-1 + 2*b*x))/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[2 - b*x]]
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.13

method	result	size
meijerg	$-\frac{3(-b)^{\frac{5}{2}}\left(-\frac{16\sqrt{\pi}\,\sqrt{2}\,(-2bx+1)\sqrt{-\frac{bx}{2}+1}}{9x^{\frac{3}{2}}(-b)^{\frac{3}{2}}}+\frac{8\sqrt{\pi}\,b^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3(-b)^{\frac{3}{2}}}\right)}{4\sqrt{\pi}\,b}$	70
risch	$-\frac{4(2x^2b^2-5bx+2)\sqrt{(-bx+2)\,x}}{3x^{\frac{3}{2}}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*(-b)^(5/2)/Pi^(1/2)/b*(-16/9*Pi^(1/2)/x^(3/2)*2^(1/2)/(-b)^(3/2)*(-2*b
*x+1)*(-1/2*b*x+1)^(1/2)+8/3*Pi^(1/2)/(-b)^(3/2)*b^(3/2)*arcsin(1/2*b^(1/2)
*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.51, size = 49, normalized size = 0.79

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+2}\,b}{\sqrt{x}}-\frac{2\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + 2)*b/sq
rt(x) - 2/3*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.46, size = 111, normalized size = 1.79

$$\left[\frac{3\,\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+4\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2},-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 4*
(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqrt(-b
*x + 2)/(sqrt(b)*sqrt(x))) - 2*(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.08, size = 184, normalized size = 2.97

$$\begin{cases}\frac{8b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}+ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\,\sqrt{x}}\right)+2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)-\frac{4\sqrt{b}\,\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2}\\ \frac{8ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right)-\frac{4i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((8*b**(3/2)*sqrt(-1 + 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*
b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 2*b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2) - 4*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x), 1/Abs(b*x) > 1/2), (8*I*b**(3/2)*
sqrt(1 - 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*b**(3/2)*log(sqrt(1 - 2
/(b*x)) + 1) - 4*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(5/2), x)
```

3.545 $\int x^{5/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=164

$$\frac{5a^5\sqrt{x}\,\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx$$

```
[Out] 1/12*a*x^(7/2)*(b*x+a)^(3/2)+1/6*x^(7/2)*(b*x+a)^(5/2)-5/512*a^6*arctanh(b^
(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-5/768*a^4*x^(3/2)*(b*x+a)^(1/2)/b^2+1/
192*a^3*x^(5/2)*(b*x+a)^(1/2)/b+1/32*a^2*x^(7/2)*(b*x+a)^(1/2)+5/512*a^5*x^
(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{5a^5\sqrt{x}\,\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^5*Sqrt[x]*Sqrt[a + b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a + b*x])/(76
8*b^2) + (a^3*x^(5/2)*Sqrt[a + b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a + b*x])/
32 + (a*x^(7/2)*(a + b*x)^(3/2))/12 + (x^(7/2)*(a + b*x)^(5/2))/6 - (5*a^6*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(512*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a+bx)^{5/2}+\frac{1}{12}(5a)\int x^{5/2}(a+bx)^{3/2}\,dx$$

$$= \frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}+\frac{1}{8}a^2\int x^{5/2}\sqrt{a+bx}\,dx$$

$$= \frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}+\frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}- -$$

$$= -\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}$$

$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}$$

Mathematica [A]
time = 0.12, size = 110, normalized size = 0.67

$$\frac{\sqrt{b}\sqrt{x}\sqrt{a+bx}\left(15a^5-10a^4bx+8a^3b^2x^2+432a^2b^3x^3+640ab^4x^4+256b^5x^5\right)+15a^6\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*a
^2*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) + 15*a^6*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]])/(1536*b^(7/2))
```

Maple [A]
time = 0.10, size = 160, normalized size = 0.98

method	result
risch	$\frac{\left(256b^5x^5+640a\,b^4x^4+432a^2b^3x^3+8a^3b^2x^2-10a^4bx+15a^5\right)\sqrt{x}\,\sqrt{bx+a}}{1536b^3}-\frac{5a^6\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\left(bx+a\right)}}{1024b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$

$$5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{7}{2}}}{5b}-\frac{3a\left(\frac{\sqrt{x}\,(bx+a)^{\frac{7}{2}}}{4b}-\frac{a\left(\frac{(bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx}{}\right.}{}\right.}{6}\right.}{8b}\right.}{10b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+a)^(7/2)-5/12*a/b*(1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/10*a/b*
(1/4/b*x^(1/2)*(b*x+a)^(7/2)-1/8*a/b*(1/3*(b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*
(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/
(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))))
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(118) = 236$.
time = 0.51, size = 244, normalized size = 1.49

$$\frac{5\,a^{6}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^{6}b^{5}}{\sqrt{x}}-\frac{85\,(bx+a)^{\frac{3}{2}}a^{6}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(bx+a)^{\frac{5}{2}}a^{6}b^{3}}{x^{\frac{5}{2}}}+\frac{198\,(bx+a)^{\frac{7}{2}}a^{6}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(bx+a)^{\frac{9}{2}}a^{6}b}{x^{\frac{9}{2}}}+\frac{15\,(bx+a)^{\frac{11}{2}}a^{6}}{x^{\frac{11}{2}}}}{1536\left(b^{9}-\frac{6\,(bx+a)b^{8}}{x}+\frac{15\,(bx+a)^{2}b^{7}}{x^{2}}-\frac{20\,(bx+a)^{3}b^{6}}{x^{3}}+\frac{15\,(bx+a)^{4}b^{5}}{x^{4}}-\frac{6\,(bx+a)^{5}b^{4}}{x^{5}}+\frac{(bx+a)^{6}b^{3}}{x^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/1024*a^6*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(7/2) + 1/1536*(15*sqrt(b*x + a)*a^6*b^5/sqrt(x) - 85*(b*x + a)
^(3/2)*a^6*b^4/x^(3/2) + 198*(b*x + a)^(5/2)*a^6*b^3/x^(5/2) + 198*(b*x + a
)^(7/2)*a^6*b^2/x^(7/2) - 85*(b*x + a)^(9/2)*a^6*b/x^(9/2) + 15*(b*x + a)^(
11/2)*a^6/x^(11/2))/(b^9 - 6*(b*x + a)*b^8/x + 15*(b*x + a)^2*b^7/x^2 - 20*
(b*x + a)^3*b^6/x^3 + 15*(b*x + a)^4*b^5/x^4 - 6*(b*x + a)^5*b^4/x^5 + (b*x
 + a)^6*b^3/x^6)
```

Fricas [A]
time = 0.39, size = 206, normalized size = 1.26

$$\left[\frac{15\,a^{6}\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(256\,b^{6}x^{5}+640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}+8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x+15\,a^{5}b)\sqrt{bx+a}\,\sqrt{x}}{3072\,b^{4}},\ \frac{15\,a^{6}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(256\,b^{6}x^{5}+640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}+8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x+15\,a^{5}b)\sqrt{bx+a}\,\sqrt{x}}{1536\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*a^6*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b
^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan
(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a
^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x)
)/b^4]
```

Sympy [A]
time = 191.95, size = 207, normalized size = 1.26

$$\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1+\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{5a^6\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}+\frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(5/2),x)
```

```
[Out] 5*a**(11/2)*sqrt(x)/(512*b**3*sqrt(1 + b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*
b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(5/2)/(768*b*sqrt(1 + b*x/a)) + 55*a**(
5/2)*x**(7/2)/(192*sqrt(1 + b*x/a)) + 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 + b
*x/a)) + 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 + b*x/a)) - 5*a**6*asinh(sqrt(
b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 + b*x
/a))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(5/2), x)
```

3.546 $\int x^{3/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{3a^5\tanh^{-1}\Big(}{128b}$$

```
[Out] 1/8*a*x^(5/2)*(b*x+a)^(3/2)+1/5*x^(5/2)*(b*x+a)^(5/2)+3/128*a^5*arctanh(b^(
1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)+1/64*a^3*x^(3/2)*(b*x+a)^(1/2)/b+1/16*a
^2*x^(5/2)*(b*x+a)^(1/2)-3/128*a^4*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]

time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^2) + (a^3*x^(3/2)*Sqrt[a + b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a + b*x])/16 + (a*x^(5/2)*(a + b*x)^(3/2))/8 + (x^(5
/2)*(a + b*x)^(5/2))/5 + (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
128*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(a+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}\,dx \\
&= \frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a+bx}\,dx \\
&= \frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{32}a^3\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx \\
&= \frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}-\frac{(3
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+ \\
&= -\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+ \\
&= -\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+ \\
&= -\frac{3a^4\sqrt{x}\,\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 99, normalized size = 0.71

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{a+bx}\,\left(-15a^4+10a^3bx+248a^2b^2x^2+336ab^3x^3+128b^4x^4\right)-15a^5\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(5/2),x]
```

[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 336*a*b^3*x^3 + 128*b^4*x^4) - 15*a^5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(640*b^(5/2))

Maple [A]
time = 0.12, size = 138, normalized size = 0.99

method	result
risch	$-\frac{\left(-128 b^{4} x^{4}-336 a\,b^{3} x^{3}-248 a^{2} b^{2} x^{2}-10 a^{3} b x +15 a^{4}\right) \sqrt{x}\,\sqrt{b x +a}}{640 b^{2}}+\frac{3 a^{5} \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{x^{2} b +a x}\right) \sqrt{x \left(b x +a \right)}}{256 b^{\frac{5}{2}} \sqrt{x}\,\sqrt{b x +a}}$
default	$\frac{x^{\frac{3}{2}} \left(b x +a \right)^{\frac{7}{2}}}{5 b}-\frac{3 a \left(\frac{\sqrt{x}\,\left(b x +a \right)^{\frac{7}{2}}}{4 b}-\frac{a \left(\frac{\left(b x +a \right)^{\frac{5}{2}} \sqrt{x}}{3}+\frac{5 a \left(\frac{\left(b x +a \right)^{\frac{3}{2}} \sqrt{x}}{2}+\frac{3 a \left(\sqrt{x}\,\sqrt{b x +a}+\frac{a \sqrt{x \left(b x +a \right)}\, \ln \left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+ \cdots\right.}{2 \sqrt{b x +a}\,\sqrt{x} \cdots}\right)}{4}\right)}{6}\right)}{8 b}\right)}{10 b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(b*x+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/10*a/b*(1/4/b*x^(1/2)*(b*x+a)^(7/2)-1/8*a/b*(1/3*(b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(

```
b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b
^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(100) = 200$.
time = 0.52, size = 212, normalized size = 1.51

$$-\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{5}{2}}}-\frac{\frac{15\sqrt{bx+a}\,a^5b^4}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}+\frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^7-\frac{5\,(bx+a)b^6}{x}+\frac{10\,(bx+a)^2b^5}{x^2}-\frac{10\,(bx+a)^3b^4}{x^3}+\frac{5\,(bx+a)^4b^3}{x^4}-\frac{(bx+a)^5b^2}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="maxima")
[Out] -3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^
(3/2)*a^5*b^3/x^(3/2) + 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^
(7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x + a)*b^
6/x + 10*(b*x + a)^2*b^5/x^2 - 10*(b*x + a)^3*b^4/x^3 + 5*(b*x + a)^4*b^3/x
^4 - (b*x + a)^5*b^2/x^5)
```

Fricas [A]
time = 0.42, size = 185, normalized size = 1.32

$$\left[\frac{15\,a^5\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b)\sqrt{bx+a}\,\sqrt{x}}{1280\,b^3},-\frac{15\,a^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b)\sqrt{bx+a}\,\sqrt{x}}{640\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="fricas")
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)
*sqrt(b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*
a^3*b^2*x - 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

Sympy [A]
time = 33.31, size = 180, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1+\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1+\frac{bx}{a}}}+\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^2x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}+\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(3/2)/(128*b*s
qrt(1 + b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 + b*x/a)) + 73*a**(3/2)
*b*x**(7/2)/(80*sqrt(1 + b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1 + b*
x/a)) + 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) + b**3*x**(11/
2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(5/2), x)
```

3.547 $\int \sqrt{x}\,(a+bx)^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}$$

```
[Out] 5/24*a*x^(3/2)*(b*x+a)^(3/2)+1/4*x^(3/2)*(b*x+a)^(5/2)-5/64*a^4*arctanh(b^(
1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)+5/32*a^2*x^(3/2)*(b*x+a)^(1/2)+5/64*a^3
*x^(1/2)*(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {52, 65, 223, 212}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a + b*x])/32 + (
5*a*x^(3/2)*(a + b*x)^(3/2))/24 + (x^(3/2)*(a + b*x)^(5/2))/4 - (5*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}\,(a+bx)^{3/2}\,dx\\
&= \frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{16}\left(5a^2\right)\int \sqrt{x}\,\sqrt{a+bx}\,dx\\
&= \frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.75

$$\frac{\sqrt{x}\,\sqrt{a+bx}\,\left(15a^3+118a^2bx+136ab^2x^2+48b^3x^3\right)}{192b}+\frac{5a^4\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*b^3*x^3))
/(192*b) + (5*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64*b^(3/2))
```

Maple [A]
time = 0.12, size = 113, normalized size = 0.97

method	result
risch	$\frac{(48b^3x^3+136a\,b^2x^2+118a^2bx+15a^3)\sqrt{x}\,\sqrt{bx+a}}{192b}-\frac{5a^4\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{128b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$
default	$\frac{x^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}}{4}+\frac{5a\left(\frac{x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2}+\frac{a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4}\right)}{2}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(b*x+a)^(5/2)+5/8*a*(1/3*x^(3/2)*(b*x+a)^(3/2)+1/2*a*(1/2*x^(3/
2)*(b*x+a)^(1/2)+1/4*a*(x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(
1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.
time = 0.52, size = 176, normalized size = 1.52

$$\frac{5\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{3}{2}}}+\frac{\frac{15\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{55\,(bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx+a)b^4}{x}+\frac{6\,(bx+a)^2b^3}{x^2}-\frac{4\,(bx+a)^3b^2}{x^3}+\frac{(bx+a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(3/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 55*(b*x + a)^(
```

```
3/2)*a^4*b^2/x^(3/2) + 73*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^5 - 4*(b*x + a)*b^4/x + 6*(b*x + a)^2*b^3/x^2 - 4*(b*x +
a)^3*b^2/x^3 + (b*x + a)^4*b/x^4)
```

Fricas [A]
time = 0.43, size = 162, normalized size = 1.40

$$\left[\frac{15\,a^4\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{384\,b^2},\ \frac{15\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b)\sqrt{bx+a}\,\sqrt{x}}{192\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

Sympy [A]
time = 8.52, size = 155, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqrt
(1 + b*x/a)) + 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*sqrt(a)*b*
*2*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(6
4*b**(3/2)) + b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(5/2), x)
```

3.548 $\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=92

$$\frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

```
[Out] 5/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)+5/12*a*(b*x+a)^(3/2)
*x^(1/2)+1/3*(b*x+a)^(5/2)*x^(1/2)+5/8*a^2*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/8 + (5*a*Sqrt[x]*(a + b*x)^(3/2))/12 + (Sqrt[
x]*(a + b*x)^(5/2))/3 + (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8
*Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{6}(5a)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}(5a^2)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{16}(5a^3)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}(5a^3)\,\mathrm{Subst}\Bigg(\int \frac{1}{\sqrt{a+}}$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{1}{8}(5a^3)\,\mathrm{Subst}\Bigg(\int \frac{1}{1-bx}$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx} + \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2} + \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.79

$$\frac{1}{24}\sqrt{x}\,\sqrt{a+bx}\,\left(33a^2+26abx+8b^2x^2\right) - \frac{5a^3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(33*a^2 + 26*a*b*x + 8*b^2*x^2))/24 - (5*a^3*Log[-(S
qrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*Sqrt[b])
```

Maple [A]
time = 0.10, size = 94, normalized size = 1.02

method	result	size
risch	$\frac{(8x^2b^2+26abx+33a^2)\sqrt{x}\,\sqrt{bx+a}}{24}+\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x\,(bx+a)}}{16\sqrt{b}\,\sqrt{x}\,\sqrt{bx+a}}$	84
default	$\frac{(bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{bx+a}+\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2\sqrt{bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(
b*x+a)^(1/2)+1/2*a*(x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b
^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.
time = 0.51, size = 141, normalized size = 1.53

$$-\frac{5\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,\sqrt{b}}-\frac{\frac{15\,\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{33\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3-\frac{3\,(bx+a)b^2}{x}+\frac{3\,(bx+a)^2b}{x^2}-\frac{(bx+a)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/sqrt(b) - 1/24*(15*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3
/2)*a^3*b/x^(3/2) + 33*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x + a)*b^2/
x + 3*(b*x + a)^2*b/x^2 - (b*x + a)^3/x^3)
```

Fricas [A]
time = 0.41, size = 141, normalized size = 1.53

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2+26\,ab^2x+33\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{48\,b},-\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(8\,b^3x^2+26\,ab^2x+33\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

Sympy [**A**]
time = 3.75, size = 102, normalized size = 1.11

$$\frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{8}+\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^2x^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 11*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a)/8 + 13*a**(3/2)*b*x**(3/2)*sqrt(1 + b*x
/a)/12 + sqrt(a)*b**2*x**(5/2)*sqrt(1 + b*x/a)/3 + 5*a**3*asinh(sqrt(b)*sqr
t(x)/sqrt(a))/(8*sqrt(b))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(1/2), x)
```

3.549 $\int \frac{(a+bx)^{5/2}}{x^{3/2}} dx$

Optimal. Leaf size=89

$$\frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] 15/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))*b^(1/2)-2*(b*x+a)^(5/2)/x^(
1/2)+5/2*b*(b*x+a)^(3/2)*x^(1/2)+15/4*a*b*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*a*b*Sqrt[x]*Sqrt[a + b*x])/4 + (5*b*Sqrt[x]*(a + b*x)^(3/2))/2 - (2*(a
+ b*x)^(5/2))/Sqrt[x] + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]])/4
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{\sqrt{x}}+(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}(15ab)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,d\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+ba}}\right.\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\right.\\
&= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx}+\frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2}-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right.\end{aligned}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.82

$$\frac{\sqrt{a+bx}\,\left(-8a^2+9abx+2b^2x^2\right)}{4\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-8*a^2 + 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[b
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/4
```

Maple [A]
time = 0.10, size = 84, normalized size = 0.94

method	result	size
risch	$-\frac{\sqrt{bx+a}\,\left(-2x^{2}b^{2}-9abx+8a^{2}\right)}{4\sqrt{x}}+\frac{15a^{2}\sqrt{b}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\left(bx+a\right)}}{8\sqrt{x}\,\sqrt{bx+a}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(-2*b^2*x^2-9*a*b*x+8*a^2)/x^(1/2)+15/8*a^2*b^(1/2)*ln((
1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/
2)
```

Maxima [A]
time = 0.50, size = 125, normalized size = 1.40

$$-\frac{15}{8}\,a^{2}\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,a^{2}}{\sqrt{x}}-\frac{\frac{7\,\sqrt{bx+a}\,a^{2}b^{2}}{\sqrt{x}}-\frac{9\,(bx+a)^{\frac{3}{2}}\,a^{2}b}{x^{\frac{3}{2}}}}{4\left(b^{2}-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/8*a^2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*
x + a)/sqrt(x))) - 2*sqrt(b*x + a)*a^2/sqrt(x) - 1/4*(7*sqrt(b*x + a)*a^2*b
^2/sqrt(x) - 9*(b*x + a)^(3/2)*a^2*b/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x
 + a)^2/x^2)
```

Fricas [A]
time = 0.55, size = 137, normalized size = 1.54

$$\left[\frac{15\,a^{2}\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x^{2}+9\,abx-8\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{8\,x},-\frac{15\,a^{2}\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{2}x^{2}+9\,abx-8\,a^{2})\sqrt{bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
```

```
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

Sympy [A]
time = 3.72, size = 126, normalized size = 1.42

$$-\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] -2*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + a**(3/2)*b*sqrt(x)/(4*sqrt(1 + b*x/
a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sqrt(1 + b*x/a)) + 15*a**2*sqrt(b)*asinh(
sqrt(b)*sqrt(x)/sqrt(a))/4 + b**3*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{x^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(3/2), x)
```

3.550 $\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=86

$$5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

```
[Out] -2/3*(b*x+a)^(5/2)/x^(3/2)+5*a*b^(3/2)*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2
))-10/3*b*(b*x+a)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)+5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a + b*x] - (10*b*(a + b*x)^(3/2))/(3*Sqrt[x]) - (2*(a +
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx\\
&= -\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x\right.\\
&= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\right.\\
&= 5b^2\sqrt{x}\,\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 69, normalized size = 0.80

$$\frac{\sqrt{a+bx}\left(-2a^2-14abx+3b^2x^2\right)}{3x^{3/2}}-5ab^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-2*a^2 - 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 5*a*b^(3/2)*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

Maple [A]
time = 0.11, size = 82, normalized size = 0.95

method	result	size
risch	$-\frac{\sqrt{bx+a}\,(-3x^2b^2+14abx+2a^2)}{3x^{\frac{3}{2}}}+\frac{5ab^{\frac{3}{2}}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{2\sqrt{x}\,\sqrt{bx+a}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x+a)^(1/2)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*ln((1/
2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.51, size = 100, normalized size = 1.16

$$-\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+a}\,ab}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab^2}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5/2*a*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 4*sqrt(b*x + a)*a*b/sqrt(x) - sqrt(b*x + a)*a*b^2/((b - (b*
x + a)/x)*sqrt(x)) - 2/3*(b*x + a)^(3/2)*a/x^(3/2)
```

Fricas [A]
time = 0.48, size = 138, normalized size = 1.60

$$\left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(3\,b^2x^2-14\,abx-2\,a^2)\sqrt{bx+a}\,\sqrt{x}}{6\,x^2},\,-\frac{15\,a\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(3\,b^2x^2-14\,abx-2\,a^2)\sqrt{bx+a}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

Sympy [A]
time = 3.40, size = 99, normalized size = 1.15

$$-\frac{2a^{2}\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] -2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 14*a*b**(3/2)*sqrt(a/(b*x) + 1)/3
 - 5*a*b**(3/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1) +
b**(5/2)*x*sqrt(a/(b*x) + 1)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(5/2), x)
```

3.551 $\int x^{5/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=171

$$-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-$$

```
[Out] 1/12*a*x^(7/2)*(-b*x+a)^(3/2)+1/6*x^(7/2)*(-b*x+a)^(5/2)+5/512*a^6*arctan(b
^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)-5/768*a^4*x^(3/2)*(-b*x+a)^(1/2)/b^2
-1/192*a^3*x^(5/2)*(-b*x+a)^(1/2)/b+1/32*a^2*x^(7/2)*(-b*x+a)^(1/2)-5/512*a
^5*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^6\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^5*Sqrt[x]*Sqrt[a - b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a - b*x])/(7
68*b^2) - (a^3*x^(5/2)*Sqrt[a - b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a - b*x])
/32 + (a*x^(7/2)*(a - b*x)^(3/2))/12 + (x^(7/2)*(a - b*x)^(5/2))/6 + (5*a^6
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(512*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a-bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{1}{12}(5a)\int x^{5/2}(a-bx)^{3/2}\,dx$$

$$= \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{1}{8}a^2\int x^{5/2}\sqrt{a-bx}\,dx$$

$$= \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-bx)^{5/2} +$$

$$= -\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-bx)^{3/2} +$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} +$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} +$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} +$$

$$= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} +$$

Mathematica [A]
time = 0.19, size = 114, normalized size = 0.67

$$\frac{\frac{\sqrt{x}\sqrt{a-bx}\left(-15a^5-10a^4bx-8a^3b^2x^2+432a^2b^3x^3-640ab^4x^4+256b^5x^5\right)}{b^3}+\frac{15a^6\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-15*a^5 - 10*a^4*b*x - 8*a^3*b^2*x^2 + 432*a^2*b^3
*x^3 - 640*a*b^4*x^4 + 256*b^5*x^5))/b^3 + (15*a^6*Log[-(Sqrt[-b]*Sqrt[x])
+ Sqrt[a - b*x]])/(-b)^(7/2))/1536
```

Maple [A]
time = 0.11, size = 169, normalized size = 0.99

method	result
risch	$-\frac{\left(-256 b^{5} x^{5}+640 a\,b^{4} x^{4}-432 a^{2} b^{3} x^{3}+8 a^{3} b^{2} x^{2}+10 a^{4} b x+15 a^{5}\right) \sqrt{x}\,\sqrt{-b x+a}}{1536 b^{3}}+\frac{5 a^{6} \arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right) \sqrt{x\,(-}{1024 b^{\frac{7}{2}} \sqrt{x}\,\sqrt{-b x+a}}$

$$3a\left(\sqrt{x}\,\sqrt{-bx+\right.$$

$$5a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{\quad}{\quad}\right.$$

$$a\left(\frac{(-bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{\quad}{\quad}\right.$$

$$3a\left(-\frac{\sqrt{x}\,(-bx+a)^{\frac{7}{2}}}{4b}+\frac{\quad}{8b}\right.$$

$$5a\left(-\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{7}{2}}}{5b}+\frac{\quad}{10b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+a)^(7/2)+5/12*a/b*(-1/5/b*x^(3/2)*(-b*x+a)^(7/2)+3/10*
a/b*(-1/4/b*x^(1/2)*(-b*x+a)^(7/2)+1/8*a/b*(1/3*(-b*x+a)^(5/2)*x^(1/2)+5/6*
a*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+
a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2
+a*x)^(1/2)))))))
```

Maxima [A]
time = 0.49, size = 242, normalized size = 1.42

$$-\frac{5\,a^{6}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{512\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{6}b^{5}}{\sqrt{x}}+\frac{85\,(-bx+a)^{\frac{3}{2}}a^{6}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(-bx+a)^{\frac{5}{2}}a^{6}b^{3}}{x^{\frac{5}{2}}}-\frac{198\,(-bx+a)^{\frac{7}{2}}a^{6}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(-bx+a)^{\frac{9}{2}}a^{6}b}{x^{\frac{9}{2}}}-\frac{15\,(-bx+a)^{\frac{11}{2}}a^{6}}{x^{\frac{11}{2}}}}{1536\left(b^{9}-\frac{6\,(bx-a)b^{8}}{x}+\frac{15\,(bx-a)^{2}b^{7}}{x^{2}}-\frac{20\,(bx-a)^{3}b^{6}}{x^{3}}+\frac{15\,(bx-a)^{4}b^{5}}{x^{4}}-\frac{6\,(bx-a)^{5}b^{4}}{x^{5}}+\frac{(bx-a)^{6}b^{3}}{x^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -5/512*a^6*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/1536*(15*sq
rt(-b*x + a)*a^6*b^5/sqrt(x) + 85*(-b*x + a)^(3/2)*a^6*b^4/x^(3/2) + 198*(-
b*x + a)^(5/2)*a^6*b^3/x^(5/2) - 198*(-b*x + a)^(7/2)*a^6*b^2/x^(7/2) - 85*
(-b*x + a)^(9/2)*a^6*b/x^(9/2) - 15*(-b*x + a)^(11/2)*a^6/x^(11/2))/(b^9 -
6*(b*x - a)*b^8/x + 15*(b*x - a)^2*b^7/x^2 - 20*(b*x - a)^3*b^6/x^3 + 15*(b
*x - a)^4*b^5/x^4 - 6*(b*x - a)^5*b^4/x^5 + (b*x - a)^6*b^3/x^6)
```

Fricas [A]
time = 0.50, size = 208, normalized size = 1.22

$$\left[-\frac{15\,a^{6}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(256\,b^{6}x^{5}-640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}-8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x-15\,a^{5}b)\sqrt{-bx+a}\,\sqrt{x}}{3072\,b^{4}},-\frac{15\,a^{6}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(256\,b^{6}x^{5}-640\,ab^{5}x^{4}+432\,a^{2}b^{4}x^{3}-8\,a^{3}b^{3}x^{2}-10\,a^{4}b^{2}x-15\,a^{5}b)\sqrt{-bx+a}\,\sqrt{x}}{1536\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(15*a^6*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(256*b^6*x^5 - 640*a*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*
a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/1536*(15*a^6*sqrt(b)*
arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (256*b^6*x^5 - 640*a*b^5*x^4 + 4
32*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sq
rt(x))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 184.16, size = 435, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{-1+\frac{bx}{a}}} - \frac{55ia^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{67ia^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^6\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} + \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}} + \frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{5a^6\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} - \frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((5*I*a**(11/2)*sqrt(x)/(512*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(9/2)
*x**(3/2)/(1536*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x**(5/2)/(768*b*sqrt(-1
 + b*x/a)) - 55*I*a**(5/2)*x**(7/2)/(192*sqrt(-1 + b*x/a)) + 67*I*a**(3/2)*
b*x**(9/2)/(96*sqrt(-1 + b*x/a)) - 7*I*sqrt(a)*b**2*x**(11/2)/(12*sqrt(-1 +
 b*x/a)) - 5*I*a**6*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + I*b**3*
x**(13/2)/(6*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(11/2)*sqrt
(x)/(512*b**3*sqrt(1 - b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*b**2*sqrt(1 - b*
x/a)) + a**(7/2)*x**(5/2)/(768*b*sqrt(1 - b*x/a)) + 55*a**(5/2)*x**(7/2)/(1
92*sqrt(1 - b*x/a)) - 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 - b*x/a)) + 7*sqrt(
a)*b**2*x**(11/2)/(12*sqrt(1 - b*x/a)) + 5*a**6*asin(sqrt(b)*sqrt(x)/sqrt(a
))/(512*b**(7/2)) - b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a-b\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(5/2), x)
```

3.552 $\int x^{3/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{3a^5\tan^{-1}\left(\cdots\right)}{128b\cdots}$$

```
[Out] 1/8*a*x^(5/2)*(-b*x+a)^(3/2)+1/5*x^(5/2)*(-b*x+a)^(5/2)+3/128*a^5*arctan(b^
(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)-1/64*a^3*x^(3/2)*(-b*x+a)^(1/2)/b+1/1
6*a^2*x^(5/2)*(-b*x+a)^(1/2)-3/128*a^4*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{3a^5\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^2) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a - b*x])/16 + (a*x^(5/2)*(a - b*x)^(3/2))/8 + (x^(5
/2)*(a - b*x)^(5/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(1
28*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}(a-bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a-bx)^{3/2}\,dx$$

$$= \frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a-bx}\,dx$$

$$= \frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+\frac{1}{32}a^3\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}+$$

$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+$$

$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+$$

$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+$$

$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+$$

Mathematica [A]
time = 0.18, size = 103, normalized size = 0.71

$$\frac{1}{640}\left(\frac{\sqrt{x}\sqrt{a-bx}\left(-15a^4-10a^3bx+248a^2b^2x^2-336ab^3x^3+128b^4x^4\right)}{b^2}-\frac{15a^5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] ((Sqrt[x]*Sqrt[a - b*x]*(-15*a^4 - 10*a^3*b*x + 248*a^2*b^2*x^2 - 336*a*b^3
*x^3 + 128*b^4*x^4))/b^2 - (15*a^5*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
)/(-b)^(5/2))/640
```

Maple [A]

time = 0.14, size = 146, normalized size = 1.00

method	result
risch	$-\frac{\left(-128 b^{4} x^{4}+336 a\,b^{3} x^{3}-248 a^{2} b^{2} x^{2}+10 a^{3} b x+15 a^{4}\right) \sqrt{x}\,\sqrt{-b x+a}}{640 b^{2}}+\frac{3 a^{5} \arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2 b}\right)}{\sqrt{-x^{2} b+a x}}\right) \sqrt{x\left(-b x+a\right)}}{256 b^{\frac{5}{2}} \sqrt{x}\,\sqrt{-b x+a}}$
default	$-\frac{x^{\frac{3}{2}} \left(-b x+a\right)^{\frac{7}{2}}}{5 b}+\frac{3 a \left(-\frac{\sqrt{x}\,\left(-b x+a\right)^{\frac{7}{2}}}{4 b}+\frac{a \left(\frac{\left(-b x+a\right)^{\frac{5}{2}} \sqrt{x}}{3}+\frac{5 a \left(\frac{\left(-b x+a\right)^{\frac{3}{2}} \sqrt{x}}{2}+\frac{3 a \left(\sqrt{x}\,\sqrt{-b x+a}+\frac{a \sqrt{x\left(-b x+\right.}}{2 \sqrt{-}}\right.}{4}\right.}{6}\right.}{8 b}\right.}{10 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+a)^(7/2)+3/10*a/b*(-1/4/b*x^(1/2)*(-b*x+a)^(7/2)+1/8*a
/b*(1/3*(-b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(
1/2)*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)
*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))))
```

Maxima [A]
time = 0.53, size = 207, normalized size = 1.42

$$-\frac{3\,a^{5}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{5}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{5}b^{4}}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}\,a^{5}b^{3}}{x^{\frac{3}{2}}}+\frac{128\,(-bx+a)^{\frac{5}{2}}\,a^{5}b^{2}}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}\,a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}\,a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{7}-\frac{5\,(bx-a)b^{6}}{x}+\frac{10\,(bx-a)^{2}b^{5}}{x^{2}}-\frac{10\,(bx-a)^{3}b^{4}}{x^{3}}+\frac{5\,(bx-a)^{4}b^{3}}{x^{4}}-\frac{(bx-a)^{5}b^{2}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) + 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x - a)*b^6/x + 10*(b*x - a)^2*b^5/x^2
 - 10*(b*x - a)^3*b^4/x^3 + 5*(b*x - a)^4*b^3/x^4 - (b*x - a)^5*b^2/x^5)
```

Fricas [A]
time = 0.51, size = 186, normalized size = 1.27

$$\left[-\frac{15\,a^{5}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(128\,b^{5}x^{4}-336\,ab^{4}x^{3}+248\,a^{2}b^{3}x^{2}-10\,a^{3}b^{2}x-15\,a^{4}b)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^{3}},\ -\frac{15\,a^{5}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(128\,b^{5}x^{4}-336\,ab^{4}x^{3}+248\,a^{2}b^{3}x^{2}-10\,a^{3}b^{2}x-15\,a^{4}b)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 - 10*a^3*b^2*x - 15*a
^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x
+ a)/(sqrt(b)*sqrt(x))) - (128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 -
10*a^3*b^2*x - 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 33.93, size = 379, normalized size = 2.60

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{-1+\frac{bx}{a}}}-\frac{129ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{-1+\frac{bx}{a}}}+\frac{73ia^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}-\frac{29i\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{5}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{ib^{3}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^{2}x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^{5}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b*sqrt(-1 + b*x/a)) - 129*I*a**(5/2)*x**(5/2)/(320*sqrt(-1 + b*
x/a)) + 73*I*a**(3/2)*b*x**(7/2)/(80*sqrt(-1 + b*x/a)) - 29*I*sqrt(a)*b**2*
x**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(1
28*b**(5/2)) + I*b**3*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(12
8*b*sqrt(1 - b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 - b*x/a)) - 73*a**
(3/2)*b*x**(7/2)/(80*sqrt(1 - b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1
 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) - b**3*x**
(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(5/2), x)
```

3.553 $\int \sqrt{x}\,(a-bx)^{5/2}\,dx$

Optimal. Leaf size=121

$$-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}$$

```
[Out] 5/24*a*x^(3/2)*(-b*x+a)^(3/2)+1/4*x^(3/2)*(-b*x+a)^(5/2)+5/64*a^4*arctan(b^
(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)+5/32*a^2*x^(3/2)*(-b*x+a)^(1/2)-5/64*
a^3*x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{5a^4\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a - b*x])/32 +
(5*a*x^(3/2)*(a - b*x)^(3/2))/24 + (x^(3/2)*(a - b*x)^(5/2))/4 + (5*a^4*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}\,(a-bx)^{3/2}\,dx\\ &= \frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{16}(5a^2)\int \sqrt{x}\,\sqrt{a-bx}\,dx\\ &= \frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\frac{1}{64}(5a^3)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\\ &= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\\ &= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\\ &= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\\ &= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}+\end{aligned}$$

Mathematica [A]
time = 0.15, size = 93, normalized size = 0.77

$$\frac{\sqrt{x}\,\sqrt{a-bx}\,(-15a^3+118a^2bx-136ab^2x^2+48b^3x^3)}{192b}+\frac{5a^4\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{64(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a - b*x]*(-15*a^3 + 118*a^2*b*x - 136*a*b^2*x^2 + 48*b^3*x^3)
)/(192*b) + (5*a^4*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(64*(-b)^(3/2)
)
```

Maple [A]

time = 0.11, size = 121, normalized size = 1.00

method	result
risch	$-\frac{(-48b^3x^3+136a\,b^2x^2-118a^2bx+15a^3)\sqrt{x}\,\sqrt{-bx+a}}{192b}+\frac{5a^4\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{128b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{5}{2}}}{4}+\frac{5a\left(\frac{x^{\frac{3}{2}}(-bx+a)^{\frac{3}{2}}}{3}+\frac{a\left(\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2}+\frac{a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,(x}{\sqrt{-x^2b}}\right.}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right.}{4}}{2}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(-b*x+a)^(5/2)+5/8*a*(1/3*x^(3/2)*(-b*x+a)^(3/2)+1/2*a*(1/2*x^(
3/2)*(-b*x+a)^(1/2)+1/4*a*(-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x
+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(
1/2)))))
```

Maxima [A]

time = 0.51, size = 168, normalized size = 1.39

$$-\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{3}{2}}}+\frac{\frac{15\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{55\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx-a)b^4}{x}+\frac{6\,(bx-a)^2b^3}{x^2}-\frac{4\,(bx-a)^3b^2}{x^3}+\frac{(bx-a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) + 55*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) + 73*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^5 - 4*(b*x
- a)*b^4/x + 6*(b*x - a)^2*b^3/x^2 - 4*(b*x - a)^3*b^2/x^3 + (b*x - a)^4*b/
x^4)
```

Fricas [A]
time = 0.48, size = 164, normalized size = 1.36

$$\left[-\frac{15\,a^{4}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(48\,b^{4}x^{3}-136\,ab^{3}x^{2}+118\,a^{2}b^{2}x-15\,a^{3}b)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^{2}},-\frac{15\,a^{4}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(48\,b^{4}x^{3}-136\,ab^{3}x^{2}+118\,a^{2}b^{2}x-15\,a^{3}b)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a
)*sqrt(x))/b^2, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(
x))) - (48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x +
a)*sqrt(x))/b^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 8.79, size = 326, normalized size = 2.69

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{-1+\frac{bx}{a}}} - \frac{133ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{127ia^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{23i\sqrt{a}\,b^{2}x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{4}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} + \frac{ib^{3}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\: \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}} + \frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{23\sqrt{a}\,b^{2}x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}} + \frac{5a^{4}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}} - \frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b*sqrt(-1 + b*x/a)) - 133*I*a**(5/2)*x*
*(3/2)/(192*sqrt(-1 + b*x/a)) + 127*I*a**(3/2)*b*x**(5/2)/(96*sqrt(-1 + b*x
/a)) - 23*I*sqrt(a)*b**2*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sq
rt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) + I*b**3*x**(9/2)/(4*sqrt(a)*sqrt(-1 +
 b*x/a)), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 - b*x/a)) + 13
3*a**(5/2)*x**(3/2)/(192*sqrt(1 - b*x/a)) - 127*a**(3/2)*b*x**(5/2)/(96*sqr
t(1 - b*x/a)) + 23*sqrt(a)*b**2*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin
(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) - b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 -
 b*x/a)), True))
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a-b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a - b*x)^(5/2), x)
```

3.554 $\int \frac{(a-bx)^{5/2}}{\sqrt{x}} dx$

Optimal. Leaf size=96

$$\frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}$$

```
[Out] 5/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)+5/12*a*(-b*x+a)^(3/2
)*x^(1/2)+1/3*(-b*x+a)^(5/2)*x^(1/2)+5/8*a^2*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {52, 65, 223, 209}

$$\frac{5a^3\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a - b*x])/8 + (5*a*Sqrt[x]*(a - b*x)^(3/2))/12 + (Sqrt[
x]*(a - b*x)^(5/2))/3 + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*
Sqrt[b])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{6}(5a)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-}}\right.\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\mathrm{Subst}\left(\int \frac{1}{1+b}\right.\\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a-bx)^{5/2}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 79, normalized size = 0.82

$$\frac{1}{24}\sqrt{x}\,\sqrt{a-bx}\,\left(33a^2-26abx+8b^2x^2\right)-\frac{5a^3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{8\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a - b*x]*(33*a^2 - 26*a*b*x + 8*b^2*x^2))/24 - (5*a^3*Log[-(S
qrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*Sqrt[-b])
```

Maple [**A**]
time = 0.10, size = 100, normalized size = 1.04

method	result	siz
risch	$\frac{(8x^2b^2-26abx+33a^2)\sqrt{x}\,\sqrt{-bx+a}}{24}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{16\sqrt{b}\,\sqrt{x}\,\sqrt{-bx+a}}$	88
default	$\frac{(-bx+a)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5a\left(\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3a\left(\sqrt{x}\,\sqrt{-bx+a}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{x}\,\sqrt{b}}\right)}{4}\right)}{6}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-b*x+a)^(5/2)*x^(1/2)+5/6*a*(1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*(x^(1/2)
*(-b*x+a)^(1/2)+1/2*a*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arc
tan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.51, size = 130, normalized size = 1.35

$$-\frac{5\,a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,\sqrt{b}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{33\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^3-\frac{3\,(bx-a)b^2}{x}+\frac{3\,(bx-a)^2b}{x^2}-\frac{(bx-a)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/24*(15*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 33*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x - a)*b^2/x + 3*(b*x - a)^2*b/x^2 - (b*x
- a)^3/x^3)
```

Fricas [A]
time = 0.57, size = 142, normalized size = 1.48

$$\left[-\frac{15\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(8\,b^3x^2-26\,ab^2x+33\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b},-\frac{15\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(8\,b^3x^2-26\,ab^2x+33\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 - 2*(8*b^3*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/24*(
15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 26*a
*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.91, size = 246, normalized size = 2.56

$$\begin{cases} - \frac{11 i a^{\frac{5}{2}} \sqrt{x}}{8 \sqrt{-1 + \frac{b x}{a}}} + \frac{59 i a^{\frac{3}{2}} b x^{\frac{3}{2}}}{24 \sqrt{-1 + \frac{b x}{a}}} - \frac{17 i \sqrt{a} b^{2} x^{\frac{5}{2}}}{12 \sqrt{-1 + \frac{b x}{a}}} - \frac{5 i a^{3} \operatorname{acosh}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{8 \sqrt{b}} + \frac{i b^{3} x^{\frac{7}{2}}}{3 \sqrt{a} \sqrt{-1 + \frac{b x}{a}}} & \text{for}\: \left|{\frac{b x}{a}}\right| > 1 \\\frac{11 a^{\frac{5}{2}} \sqrt{x} \sqrt{1 - \frac{b x}{a}}}{8} - \frac{13 a^{\frac{3}{2}} b x^{\frac{3}{2}} \sqrt{1 - \frac{b x}{a}}}{12} + \frac{\sqrt{a} b^{2} x^{\frac{5}{2}} \sqrt{1 - \frac{b x}{a}}}{3} + \frac{5 a^{3} \operatorname{asin}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{8 \sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-11*I*a**(5/2)*sqrt(x)/(8*sqrt(-1 + b*x/a)) + 59*I*a**(3/2)*b*x*
*(3/2)/(24*sqrt(-1 + b*x/a)) - 17*I*sqrt(a)*b**2*x**(5/2)/(12*sqrt(-1 + b*x
/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + I*b**3*x**(7/2
)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (11*a**(5/2)*sqrt(x)*sqrt(
1 - b*x/a)/8 - 13*a**(3/2)*b*x**(3/2)*sqrt(1 - b*x/a)/12 + sqrt(a)*b**2*x**
(5/2)*sqrt(1 - b*x/a)/3 + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)),
 True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x\right)}^{5/2}}{\sqrt{x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(1/2),x)
[Out] int((a - b*x)^(5/2)/x^(1/2), x)
```

3.555 $\int \frac{(a-bx)^{5/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=93

$$-\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -15/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))*b^(1/2)-2*(-b*x+a)^(5/2)/x
^(1/2)-5/2*b*(-b*x+a)^(3/2)*x^(1/2)-15/4*a*b*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-\frac{15}{4}a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*a*b*Sqrt[x]*Sqrt[a - b*x])/4 - (5*b*Sqrt[x]*(a - b*x)^(3/2))/2 - (2*(a
 - b*x)^(5/2))/Sqrt[x] - (15*a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a -
b*x]])/4
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx \\
&= -\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}(15ab)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx \\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}} \\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-b}}\right. \\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\right. \\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right.
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 79, normalized size = 0.85

$$\frac{\sqrt{a-bx}\,\left(-8a^2-9abx+2b^2x^2\right)}{4\sqrt{x}}-\frac{15}{4}a^2\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(3/2),x]
```

[Out] (Sqrt[a - b*x]*(-8*a^2 - 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/4

Maple [A]
time = 0.12, size = 88, normalized size = 0.95

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(-2x^2b^2+9abx+8a^2)}{4\sqrt{x}}-\frac{15a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{8\sqrt{x}\,\sqrt{-bx+a}}$	88

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x+a)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4*(-b*x+a)^(1/2)*(-2*b^2*x^2+9*a*b*x+8*a^2)/x^(1/2)-15/8*a^2*b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)

Maxima [A]
time = 0.50, size = 112, normalized size = 1.20

$$\frac{15}{4}\,a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\sqrt{-bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\sqrt{-bx+a}\,a^2b^2}{\sqrt{x}}+\frac{9\,(-bx+a)^{\frac{3}{2}}a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx-a)b}{x}+\frac{(bx-a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")

[Out] 15/4*a^2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)*a^2/sqrt(x) - 1/4*(7*sqrt(-b*x + a)*a^2*b^2/sqrt(x) + 9*(-b*x + a)^(3/2)*a^2*b/x^(3/2))/(b^2 - 2*(b*x - a)*b/x + (b*x - a)^2/x^2)

Fricas [A]
time = 0.52, size = 137, normalized size = 1.47

$$\left[\frac{15\,a^2\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)+2\,(2\,b^2x^2-9\,abx-8\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{8\,x},\frac{15\,a^2\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^2x^2-9\,abx-8\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")

[Out] [1/8*(15*a^2*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*(2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x, 1/4*(15*a^2*sqrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x]

Sympy [C] Result contains complex when optimal does not.
time = 3.81, size = 267, normalized size = 2.87

$$\begin{cases} \frac{2ia^{\frac{5}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} + \frac{ia^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}} - \frac{11i\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}} + \frac{15ia^2\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4} + \frac{ib^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} - \frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}} + \frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}} - \frac{15a^2\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4} - \frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(5/2)/(sqrt(x)*sqrt(-1 + b*x/a)) + I*a**(3/2)*b*sqrt(x)/(
4*sqrt(-1 + b*x/a)) - 11*I*sqrt(a)*b**2*x**(3/2)/(4*sqrt(-1 + b*x/a)) + 15*
I*a**2*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/4 + I*b**3*x**(5/2)/(2*sqrt(a
)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(5/2)/(sqrt(x)*sqrt(1 - b*x/a)
) - a**(3/2)*b*sqrt(x)/(4*sqrt(1 - b*x/a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sq
rt(1 - b*x/a)) - 15*a**2*sqrt(b)*asin(sqrt(b)*sqrt(x)/sqrt(a))/4 - b**3*x**
(5/2)/(2*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(3/2), x)
```

3.556 $\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=90

$$5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

```
[Out] -2/3*(-b*x+a)^(5/2)/x^(3/2)+5*a*b^(3/2)*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/
2))+10/3*b*(-b*x+a)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$5ab^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)+5b^2\sqrt{x}\,\sqrt{a-bx}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a - b*x] + (10*b*(a - b*x)^(3/2))/(3*Sqrt[x]) - (2*(a -
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x
]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx \\
&= \frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x, \right. \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{}{\sqrt{a}} \right. \\
&= 5b^2\sqrt{x}\,\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.15, size = 76, normalized size = 0.84

$$\frac{\sqrt{a-bx}\,\left(-2a^2+14abx+3b^2x^2\right)}{3x^{3/2}}+5a\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-2*a^2 + 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 5*a*Sqrt[-b]*
b*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.96

method	result	size
risch	$-\frac{\sqrt{-bx+a}\,(-3x^2b^2-14abx+2a^2)}{3x^{\frac{3}{2}}}+\frac{5ab^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{2\sqrt{x}\,\sqrt{-bx+a}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-b*x+a)^(1/2)*(-3*b^2*x^2-14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*arcta
n(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+
a)^(1/2)
```

Maxima [A]
time = 0.50, size = 84, normalized size = 0.93

$$-5\,ab^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{4\,\sqrt{-bx+a}\,ab}{\sqrt{x}}+\frac{\sqrt{-bx+a}\,ab^2}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*a*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 4*sqrt(-b*x + a)*a*
b/sqrt(x) + sqrt(-b*x + a)*a*b^2/((b - (b*x - a)/x)*sqrt(x)) - 2/3*(-b*x +
a)^(3/2)*a/x^(3/2)
```

Fricas [A]
time = 0.60, size = 139, normalized size = 1.54

$$\left[\frac{15\,a\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(3\,b^2x^2+14\,abx-2\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{6\,x^2},\,-\frac{15\,ab^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(3\,b^2x^2+14\,abx-2\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(3*b^2*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2, -1/3*(15
*a*b^(3/2)*x^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 14*a
*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.58, size = 245, normalized size = 2.72

$$\begin{cases} -\frac{2a^2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-5iab^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}-1} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia^2\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{14iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}-5iab^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right)+ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 14*a*b**(3/2)*sqrt(a/(
b*x) - 1)/3 - 5*I*a*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + 5*I*a*b**(3/2
)*log(a/(b*x))/2 + 5*a*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)) + b**(5/2)*x*
sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (-2*I*a**2*sqrt(b)*sqrt(-a/(b*x) + 1)
/(3*x) + 14*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + 5*I*a*b**(3/2)*log(a/(b*x))
/2 - 5*I*a*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1) + I*b**(5/2)*x*sqrt(-a/(b*x
) + 1), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(5/2), x)
```

3.557 $\int x^{5/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=144

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{48b^2}+\frac{x^{5/2}\sqrt{2+bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2+bx}+\frac{1}{6}x^{7/2}(2+bx)^{3/2}+\frac{1}{6}x^{7/2}(2+bx)^{5/2}-\frac{5\sinh}{}$$

```
[Out] 1/6*x^(7/2)*(b*x+2)^(3/2)+1/6*x^(7/2)*(b*x+2)^(5/2)-5/8*arcsinh(1/2*b^(1/2)
*x^(1/2)*2^(1/2))/b^(7/2)-5/48*x^(3/2)*(b*x+2)^(1/2)/b^2+1/24*x^(5/2)*(b*x+
2)^(1/2)/b+1/8*x^(7/2)*(b*x+2)^(1/2)+5/16*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]

time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{16b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(48*b^2) + (
x^(5/2)*Sqrt[2 + b*x])/(24*b) + (x^(7/2)*Sqrt[2 + b*x])/8 + (x^(7/2)*(2 + b
*x)^(3/2))/6 + (x^(7/2)*(2 + b*x)^(5/2))/6 - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}(2+bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{5}{6}\int x^{5/2}(2+bx)^{3/2}\,dx$$
$$= \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{1}{2}\int x^{5/2}\sqrt{2+bx}\,dx$$
$$= \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+b}}}{24b}$$
$$= -\frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)$$

Mathematica [A]
time = 0.11, size = 92, normalized size = 0.64

$$\frac{\sqrt{x}\sqrt{2+bx}\,(15-5bx+2b^2x^2+54b^3x^3+40b^4x^4+8b^5x^5)}{48b^3} + \frac{5\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 54*b^3*x^3 + 40*b^4*x^4 +
8*b^5*x^5))/(48*b^3) + (5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(8*b^(7/
2))
```

Maple [A]
time = 0.11, size = 147, normalized size = 1.02

method	result

meijerg	$-\frac{120\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(56b^5x^5+280b^4x^4+378b^3x^3+14x^2b^2-35bx+105\right)\sqrt{\frac{bx}{2}+1}}{40320}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{192}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$
risch	$\frac{\left(8b^5x^5+40b^4x^4+54b^3x^3+2x^2b^2-5bx+15\right)\sqrt{x}\,\sqrt{bx+2}}{48b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{5}{2}}(bx+2)^{\frac{7}{2}}}{6b}-\frac{5\left(\frac{x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}}{5b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x\,(bx+2)}\,\ln(\cdots}{2\sqrt{bx+\cdots}}}{4b}\cdots\right.}{5b}\cdots\right.}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+2)^(7/2)-5/6/b*(1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/5/b*(1/4/b
*x^(1/2)*(b*x+2)^(7/2)-1/4/b*(1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x
^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/2)+5/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2
)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(99) = 198$.
time = 0.50, size = 223, normalized size = 1.55

$$\frac{\frac{15\sqrt{bx+2}\,b^5}{\sqrt{x}}-\frac{85\,(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{198\,(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}+\frac{198\,(bx+2)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}}-\frac{85\,(bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}}+\frac{15\,(bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^9-\frac{6\,(bx+2)b^8}{x}+\frac{15\,(bx+2)^2b^7}{x^2}-\frac{20\,(bx+2)^3b^6}{x^3}+\frac{15\,(bx+2)^4b^5}{x^4}-\frac{6\,(bx+2)^5b^4}{x^5}+\frac{(bx+2)^6b^3}{x^6}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(b*x + 2)*b^5/sqrt(x) - 85*(b*x + 2)^(3/2)*b^4/x^(3/2) + 198*(
b*x + 2)^(5/2)*b^3/x^(5/2) + 198*(b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(b*x + 2)
```

```
^(9/2)*b/x^(9/2) + 15*(b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x + 2)*b^8/x +
 15*(b*x + 2)^2*b^7/x^2 - 20*(b*x + 2)^3*b^6/x^3 + 15*(b*x + 2)^4*b^5/x^4 -
 6*(b*x + 2)^5*b^4/x^5 + (b*x + 2)^6*b^3/x^6) + 5/16*log(-(sqrt(b) - sqrt(b
*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.61, size = 172, normalized size = 1.19

$$\left[\frac{(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{48\,b^4},\ \frac{(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{48\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*s
qrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) +
 1))/b^4, 1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x
+ 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))))/b^4]
```

Sympy [A]
time = 184.70, size = 158, normalized size = 1.10

$$\frac{b^3x^{\frac{13}{2}}}{6\sqrt{bx+2}}+\frac{7b^2x^{\frac{11}{2}}}{6\sqrt{bx+2}}+\frac{67bx^{\frac{9}{2}}}{24\sqrt{bx+2}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{48b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{48b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{8b^3\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(13/2)/(6*sqrt(b*x + 2)) + 7*b**2*x**(11/2)/(6*sqrt(b*x + 2)) + 67*
b*x**(9/2)/(24*sqrt(b*x + 2)) + 55*x**(7/2)/(24*sqrt(b*x + 2)) - x**(5/2)/(
48*b*sqrt(b*x + 2)) + 5*x**(3/2)/(48*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(8*b**
3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2} (b x + 2)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(5/2), x)
```

3.558 $\int x^{3/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{3\sqrt{x}\,\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2+bx)^{5/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(b*x+2)^(3/2)+1/5*x^(5/2)*(b*x+2)^(5/2)+3/4*arcsinh(1/2*b^(1/2)
*x^(1/2)*2^(1/2))/b^(5/2)+1/8*x^(3/2)*(b*x+2)^(1/2)/b+1/4*x^(5/2)*(b*x+2)^(
1/2)-3/8*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2
))/5 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{3/2}(2+bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(2+bx)^{5/2}+\int x^{3/2}(2+bx)^{3/2}\,dx$$
$$= \frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2+bx)^{5/2}+\frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2+bx)^{5/2}+\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2+bx)^{5/2}-\frac{3\int\frac{\sqrt{x}}{\sqrt{2+}}}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2-$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2-$$
$$= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2}+\frac{x^{3/2}\sqrt{2+bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2+bx}+\frac{1}{4}x^{5/2}(2+bx)^{3/2}+\frac{1}{5}x^{5/2}(2-$$

Mathematica [A]
time = 0.12, size = 84, normalized size = 0.68

$$\frac{\sqrt{x}\sqrt{2+bx}\,(-15+5bx+62b^2x^2+42b^3x^3+8b^4x^4)}{40b^2}-\frac{3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(5/2),x]
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-15 + 5*b*x + 62*b^2*x^2 + 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(5/2))
```

Maple [A]
time = 0.11, size = 126, normalized size = 1.02

method	result
meijerg	$-\frac{60\left(\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}\left(-8b^4x^4-42b^3x^3-62x^2b^2-5bx+15\right)\sqrt{\frac{bx}{2}+1}}{2400}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{80}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$

risch	$\frac{(8b^4x^4+42b^3x^3+62x^2b^2+5bx-15)\sqrt{x}\,\sqrt{bx+2}}{40b^2}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x(bx+2)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{7}{2}}}{5b}-\frac{3\left(\frac{\sqrt{x}\,(bx+2)^{\frac{7}{2}}}{4b}-\frac{\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x(bx+2)}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+\right.}}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}}{4b}\right.}{5b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/5/b*(1/4/b*x^(1/2)*(b*x+2)^(7/2)-1/4/b*(1/3*(
b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/2)+5/
2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1
/2))/b^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(84) = 168$.
time = 0.51, size = 194, normalized size = 1.58

$$-\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{128(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7-\frac{5(bx+2)b^6}{x}+\frac{10(bx+2)^2b^5}{x^2}-\frac{10(bx+2)^3b^4}{x^3}+\frac{5(bx+2)^4b^3}{x^4}-\frac{(bx+2)^5b^2}{x^5}\right)}-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) + 128*
(b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(
9/2)/x^(9/2))/(b^7 - 5*(b*x + 2)*b^6/x + 10*(b*x + 2)^2*b^5/x^2 - 10*(b*x +
 2)^3*b^4/x^3 + 5*(b*x + 2)^4*b^3/x^4 - (b*x + 2)^5*b^2/x^5) - 3/8*log(-(sq
rt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.57, size = 155, normalized size = 1.26

$$\left[\frac{(8b^5x^4+42b^4x^3+62b^3x^2+5b^2x-15b)\sqrt{bx+2}\,\sqrt{x}+15\sqrt{b}\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{40b^3},\frac{(8b^5x^4+42b^4x^3+62b^3x^2+5b^2x-15b)\sqrt{bx+2}\,\sqrt{x}-30\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)
*sqrt(x) + 15*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/
40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)*sq
rt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A]
time = 32.70, size = 138, normalized size = 1.12

$$\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{29b^{2}x^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{73bx^{\frac{7}{2}}}{20\sqrt{bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(11/2)/(5*sqrt(b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(b*x + 2)) + 73
*b*x**(7/2)/(20*sqrt(b*x + 2)) + 129*x**(5/2)/(40*sqrt(b*x + 2)) - x**(3/2)
/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*s
qrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(b\,x+2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(5/2), x)
```

3.559 $\int \sqrt{x}\,(2+bx)^{5/2}\,dx$

Optimal. Leaf size=102

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2+bx}+\frac{5}{12}x^{3/2}(2+bx)^{3/2}+\frac{1}{4}x^{3/2}(2+bx)^{5/2}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

```
[Out] 5/12*x^(3/2)*(b*x+2)^(3/2)+1/4*x^(3/2)*(b*x+2)^(5/2)-5/4*arcsinh(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(3/2)+5/8*x^(3/2)*(b*x+2)^(1/2)+5/8*x^(1/2)*(b*x+2)^(1
/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 + b*x])/8 + (5*x^(3/2)*
(2 + b*x)^(3/2))/12 + (x^(3/2)*(2 + b*x)^(5/2))/4 - (5*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{x}\,(2+bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(2+bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,(2+bx)^{3/2}\,dx$$
$$= \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,\sqrt{2+bx}\,dx$$
$$= \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$$
$$= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} - \frac{5\int \ldots}{\ldots}$$
$$= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} - \frac{5\mathrm{Subst}\ldots}{\ldots}$$
$$= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} - \frac{5\sinh\ldots}{\ldots}$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 0.75

$$\frac{\sqrt{x}\,\sqrt{2+bx}\,(15+59bx+34b^2x^2+6b^3x^3)}{24b} + \frac{5\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 + 59*b*x + 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5*
Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(3/2))
```

Maple [A]
time = 0.11, size = 99, normalized size = 0.97

method	result
meijerg	$-\frac{30\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(6b^3x^3+34x^2b^2+59bx+15)\sqrt{\frac{bx}{2}+1}}{720}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{24}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$

risch	$\frac{(6b^3x^3+34x^2b^2+59bx+15)\sqrt{x}\,\sqrt{bx+2}}{24b}-\frac{5\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{8b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$
default	$\frac{x^{\frac{3}{2}}(bx+2)^{\frac{5}{2}}}{4}+\frac{5x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}{12}+\frac{5x^{\frac{3}{2}}\sqrt{bx+2}}{8}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{8b}-\frac{5\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{8b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(b*x+2)^(5/2)+5/12*x^(3/2)*(b*x+2)^(3/2)+5/8*x^(3/2)*(b*x+2)^(1
/2)+5/8*x^(1/2)*(b*x+2)^(1/2)/b-5/8/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(69) = 138$.
time = 0.60, size = 161, normalized size = 1.58

$$\frac{\frac{15\sqrt{bx+2}\,b^3}{\sqrt{x}}-\frac{55\,(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{73\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^5-\frac{4\,(bx+2)b^4}{x}+\frac{6\,(bx+2)^2b^3}{x^2}-\frac{4\,(bx+2)^3b^2}{x^3}+\frac{(bx+2)^4b}{x^4}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) - 55*(b*x + 2)^(3/2)*b^2/x^(3/2) + 73*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x + 2)*b^
4/x + 6*(b*x + 2)^2*b^3/x^2 - 4*(b*x + 2)^3*b^2/x^3 + (b*x + 2)^4*b/x^4) +
5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(3/2)
```

Fricas [A]
time = 0.93, size = 140, normalized size = 1.37

$$\left[\frac{(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^2},\ \frac{(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 1
5*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/24*((6*b^4*x
```

```
^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arct
an(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A]
time = 8.33, size = 119, normalized size = 1.17

$$\frac{b^3x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{23b^2x^{\frac{7}{2}}}{12\sqrt{bx+2}}+\frac{127bx^{\frac{5}{2}}}{24\sqrt{bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b\sqrt{bx+2}}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] b**3*x**(9/2)/(4*sqrt(b*x + 2)) + 23*b**2*x**(7/2)/(12*sqrt(b*x + 2)) + 127
*b*x**(5/2)/(24*sqrt(b*x + 2)) + 133*x**(3/2)/(24*sqrt(b*x + 2)) + 5*sqrt(x
)/(4*b*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(b\,x+2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(5/2), x)
```

3.560 $\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=79

$$\frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 5*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+1/
3*(b*x+2)^(5/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{1}{3}\sqrt{x}\,(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}\,(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*Sqrt[x]*(2 + b*x)^(3/2))/6 + (Sqrt[x]*(2 +
 b*x)^(5/2))/3 + (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{3}\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx\\ &= \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+5\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\right.\\ &= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.80

$$\frac{1}{6}\sqrt{x}\,\sqrt{2+bx}\,\left(33+13bx+2b^2x^2\right)-\frac{5\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(33 + 13*b*x + 2*b^2*x^2))/6 - (5*Log[-(Sqrt[b]*Sqrt
[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Maple [A]
time = 0.11, size = 84, normalized size = 1.06

method	result	size
meijerg	$-\frac{15\left(-\frac{8\sqrt{\pi}\,\sqrt{b}\,\sqrt{x}\,\sqrt{2}\,\left(\frac{1}{24}x^2b^2+\frac{13}{48}bx+\frac{11}{16}\right)\sqrt{\frac{bx}{2}+1}}{15}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3}\right)}{\sqrt{b}\,\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2+13bx+33)\sqrt{x}\,\sqrt{bx+2}}{6}+\frac{5\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	74

default	$\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2}+\frac{5\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(b*x+2)^(1/
2)+5/2*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*
x)^(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(54) = 108$.
time = 0.51, size = 129, normalized size = 1.63

$$-\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\sqrt{b}}-\frac{\frac{15\sqrt{bx+2}\,b^{2}}{\sqrt{x}}-\frac{40\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{3}-\frac{3\,(bx+2)b^{2}}{x}+\frac{3\,(bx+2)^{2}b}{x^{2}}-\frac{(bx+2)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - 1/3*(15*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3
/2) + 33*(b*x + 2)^(5/2)/x^(5/2))/(b^3 - 3*(b*x + 2)*b^2/x + 3*(b*x + 2)^2*
b/x^2 - (b*x + 2)^3/x^3)
```

Fricas [A]
time = 0.93, size = 123, normalized size = 1.56

$$\left[\frac{(2\,b^{3}x^{2}+13\,b^{2}x+33\,b)\sqrt{bx+2}\,\sqrt{x}\,+15\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)}{6\,b},\frac{(2\,b^{3}x^{2}+13\,b^{2}x+33\,b)\sqrt{bx+2}\,\sqrt{x}\,-30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 13*b^2*x + 33*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(
b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 + 13*b^2*x + 3
3*b)*sqrt(b*x + 2)*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b]
```

Sympy [A]
time = 3.71, size = 97, normalized size = 1.23

$$\frac{b^3x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{17b^2x^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{59bx^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{11\sqrt{x}}{\sqrt{bx+2}}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b**3*x**(7/2)/(3*sqrt(b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(b*x + 2)) + 59*b
*x**(3/2)/(6*sqrt(b*x + 2)) + 11*sqrt(x)/sqrt(b*x + 2) + 5*asinh(sqrt(2)*sq
rt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(1/2), x)
```

3.561 $\int \frac{(2+bx)^{5/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=79

$$\frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] 15*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(b*x+2)^(5/2)/x^(1/2)+5/2
*b*(b*x+2)^(3/2)*x^(1/2)+15/2*b*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\,\sqrt{bx+2}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*b*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*b*Sqrt[x]*(2 + b*x)^(3/2))/2 - (2*(2 +
b*x)^(5/2))/Sqrt[x] + 15*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{\sqrt{x}}+(5b)\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx\\ &= \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+\frac{1}{2}(15b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+\frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+(15b)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,\right.\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 62, normalized size = 0.78

$$\frac{\sqrt{2+bx}\,(-16+9bx+b^2x^2)}{2\sqrt{x}}-15\sqrt{b}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(3/2),x]
[Out] (Sqrt[2 + b*x]*(-16 + 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[b]*Log[-(Sqrt
[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

Maple [A]
time = 0.11, size = 63, normalized size = 0.80

method	result	size
meijerg	$-\frac{15\sqrt{b}\left(\frac{16\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{1}{16}x^2b^2-\frac{9}{16}bx+1\right)\sqrt{\frac{bx}{2}+1}}{15\sqrt{x}\,\sqrt{b}}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)\right)}{2\sqrt{\pi}}$	63

risch	$\frac{b^3x^3+11x^2b^2+2bx-32}{2\sqrt{x}\,\sqrt{bx+2}}+\frac{15\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2\sqrt{x}\,\sqrt{bx+2}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/2*b^(1/2)/Pi^(1/2)*(16/15*Pi^(1/2)/x^(1/2)*2^(1/2)/b^(1/2)*(-1/16*x^2*b
^2-9/16*b*x+1)*(1/2*b*x+1)^(1/2)-2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(
1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(56) = 112$.
time = 0.52, size = 113, normalized size = 1.43

$$-\frac{15}{2}\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{\frac{7\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{9\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2-\frac{2\,(bx+2)b}{x}+\frac{(bx+2)^2}{x^2}}-\frac{8\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x +
2)/sqrt(x))) - (7*sqrt(b*x + 2)*b^2/sqrt(x) - 9*(b*x + 2)^(3/2)*b/x^(3/2))/
(b^2 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2) - 8*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 1.13, size = 116, normalized size = 1.47

$$\left[\frac{15\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(b^2x^2+9\,bx-16)\sqrt{bx+2}\,\sqrt{x}}{2\,x},-\frac{30\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-(b^2x^2+9\,bx-16)\sqrt{bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x^2
+ 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/x, -1/2*(30*sqrt(-b)*x*arctan(sqrt(b*x
 + 2)*sqrt(-b)/(b*sqrt(x))) - (b^2*x^2 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))
/x]
```

Sympy [A]
time = 3.59, size = 94, normalized size = 1.19

$$15\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{b\sqrt{x}}{\sqrt{bx+2}}-\frac{16}{\sqrt{x}\,\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] 15*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**3*x**(5/2)/(2*sqrt(b*x + 2
)) + 11*b**2*x**(3/2)/(2*sqrt(b*x + 2)) + b*sqrt(x)/sqrt(b*x + 2) - 16/(sqr
t(x)*sqrt(b*x + 2))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(3/2), x)
```

3.562 $\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=81

$$5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(b*x+2)^(5/2)/x^(3/2)+10*b^(3/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))-
10/3*b*(b*x+2)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(b*x+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 52, 56, 221}

$$10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\,\sqrt{bx+2}-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 + b*x] - (10*b*(2 + b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 +
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx\\ &= -\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\right.\\ &= 5b^2\sqrt{x}\,\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.11, size = 63, normalized size = 0.78

$$\frac{\sqrt{2+bx}\,\left(-8-28bx+3b^2x^2\right)}{3x^{3/2}}-10b^{3/2}\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(5/2),x]
[Out] (Sqrt[2 + b*x]*(-8 - 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 10*b^(3/2)*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

Maple [A]
time = 0.14, size = 63, normalized size = 0.78

method	result	size

meijerg	$-\frac{15b^{\frac{3}{2}}\left(\frac{32\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{3}{8}x^2b^2+\frac{7}{2}bx+1\right)\sqrt{\frac{bx}{2}+1}}{45x^{\frac{3}{2}}b^{\frac{3}{2}}}-\frac{8\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3}\right)}{4\sqrt{\pi}}$	63
risch	$\frac{3b^3x^3-22x^2b^2-64bx-16}{3x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{5b^{\frac{3}{2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -15/4*b^(3/2)/Pi^(1/2)*(32/45*Pi^(1/2)/x^(3/2)*2^(1/2)/b^(3/2)*(-3/8*x^2*b^
2+7/2*b*x+1)*(1/2*b*x+1)^(1/2)-8/3*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(
1/2)))
```

Maxima [A]
time = 0.50, size = 96, normalized size = 1.19

$$-5\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{8\,\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\,\sqrt{bx+2}\,b^2}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}-\frac{4\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 8*sqrt(b*x + 2)*b/sqrt(x) - 2*sqrt(b*x + 2)*b^2/((b - (b*x + 2)
/x)*sqrt(x)) - 4/3*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.79, size = 123, normalized size = 1.52

$$\left[\frac{15\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\left(3\,b^2x^2-28\,bx-8\right)\sqrt{bx+2}\,\sqrt{x}}{3\,x^2},-\frac{30\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(3\,b^2x^2-28\,bx-8\right)\sqrt{bx+2}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (3*b^2*
x^2 - 28*b*x - 8)*sqrt(b*x + 2)*sqrt(x))/x^2, -1/3*(30*sqrt(-b)*b*x^2*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 28*b*x - 8)*sqrt(b*x +
 2)*sqrt(x))/x^2]
```

Sympy [A]
time = 3.42, size = 88, normalized size = 1.09

$$b^{\frac{5}{2}}x\sqrt{1+\frac{2}{bx}}-\frac{28b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-5b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+10b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{8\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] b**(5/2)*x*sqrt(1 + 2/(b*x)) - 28*b**(3/2)*sqrt(1 + 2/(b*x))/3 - 5*b**(3/2)
*log(1/(b*x)) + 10*b**(3/2)*log(sqrt(1 + 2/(b*x)) + 1) - 8*sqrt(b)*sqrt(1 +
 2/(b*x))/(3*x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(5/2), x)
```

3.563 $\int x^{5/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5\sin}{}$$

```
[Out] 1/6*x^(7/2)*(-b*x+2)^(3/2)+1/6*x^(7/2)*(-b*x+2)^(5/2)+5/8*arcsin(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(7/2)-5/48*x^(3/2)*(-b*x+2)^(1/2)/b^2-1/24*x^(5/2)*(-b
*x+2)^(1/2)/b+1/8*x^(7/2)*(-b*x+2)^(1/2)-5/16*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{5\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(48*b^2) -
(x^(5/2)*Sqrt[2 - b*x])/(24*b) + (x^(7/2)*Sqrt[2 - b*x])/8 + (x^(7/2)*(2 -
b*x)^(3/2))/6 + (x^(7/2)*(2 - b*x)^(5/2))/6 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(2-bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5}{6}\int x^{5/2}(2-bx)^{3/2}\,dx\\ &= \frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{2}\int x^{5/2}\sqrt{2-bx}\,dx\\ &= \frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{8}\int\frac{x^{5/2}}{\sqrt{2-bx}}\,dx\\ &= -\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{5\int\frac{x}{\sqrt{2}}}{2}\\ &= -\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{6}x^{7/2}(2-\\ &= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-\\ &= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-\\ &= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{x^{5/2}\sqrt{2-bx}}{24b}+\frac{1}{8}x^{7/2}\sqrt{2-bx}+\frac{1}{6}x^{7/2}(2-\end{aligned}$$

Mathematica [A]
time = 0.16, size = 98, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{2-bx}\,(-15-5bx-2b^2x^2+54b^3x^3-40b^4x^4+8b^5x^5)}{48b^3}+\frac{5\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{8(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(5/2),x]
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 54*b^3*x^3 - 40*b^4*x^4 +
 8*b^5*x^5))/(48*b^3) + (5*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(8*(-b
)^(7/2))
```

Maple [A]
time = 0.13, size = 157, normalized size = 1.05

method	result

meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(-56b^5x^5+280b^4x^4-378b^3x^3+14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{336b^3}-\frac{5\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{8b^{\frac{7}{2}}}}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{(8b^5x^5-40b^4x^4+54b^3x^3-2x^2b^2-5bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{48b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}(-bx+2)^{\frac{7}{2}}}{6b}+\frac{-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{7}{2}}}{6b}+\frac{5\left(-\frac{3\sqrt{x}\,(-bx+2)^{\frac{7}{2}}}{20b}+\frac{3\left(\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{5\sqrt{(-}}{}\right.}{20b}\right.}{6b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+2)^(7/2)+5/6/b*(-1/5/b*x^(3/2)*(-b*x+2)^(7/2)+3/5/b*(-
1/4/b*x^(1/2)*(-b*x+2)^(7/2)+1/4/b*(1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)
^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(
1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2)))))
```

Maxima [A]

time = 0.54, size = 209, normalized size = 1.39

$$\frac{\frac{15\sqrt{-bx+2}\,b^5}{\sqrt{x}}+\frac{85(-bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{198(-bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}-\frac{198(-bx+2)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}}-\frac{85(-bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}}-\frac{15(-bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^9-\frac{6(bx-2)b^8}{x}+\frac{15(bx-2)^2b^7}{x^2}-\frac{20(bx-2)^3b^6}{x^3}+\frac{15(bx-2)^4b^5}{x^4}-\frac{6(bx-2)^5b^4}{x^5}+\frac{(bx-2)^6b^3}{x^6}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(-b*x + 2)*b^5/sqrt(x) + 85*(-b*x + 2)^(3/2)*b^4/x^(3/2) + 198
*(-b*x + 2)^(5/2)*b^3/x^(5/2) - 198*(-b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(-b*x
 + 2)^(9/2)*b/x^(9/2) - 15*(-b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x - 2)*b
^8/x + 15*(b*x - 2)^2*b^7/x^2 - 20*(b*x - 2)^3*b^6/x^3 + 15*(b*x - 2)^4*b^5
/x^4 - 6*(b*x - 2)^5*b^4/x^5 + (b*x - 2)^6*b^3/x^6) - 5/8*arctan(sqrt(-b*x
+ 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 1.01, size = 173, normalized size = 1.15

$$\left[\frac{\left(8\,b^{6}x^{5}-40\,b^{5}x^{4}+54\,b^{4}x^{3}-2\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{48\,b^{4}},\frac{\left(8\,b^{6}x^{5}-40\,b^{5}x^{4}+54\,b^{4}x^{3}-2\,b^{3}x^{2}-5\,b^{2}x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{48\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*s
qrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt
(x) + 1))/b^4, 1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b
^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 183.34, size = 335, normalized size = 2.23

$$\begin{cases} \frac{ib^{3}x^{\frac{13}{2}}}{6\sqrt{bx-2}}-\frac{7ib^{2}x^{\frac{11}{2}}}{6\sqrt{bx-2}}+\frac{67ibx^{\frac{9}{2}}}{24\sqrt{bx-2}}-\frac{55ix^{\frac{7}{2}}}{24\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{48b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{48b^{2}\sqrt{bx-2}}+\frac{5i\sqrt{x}}{8b^{3}\sqrt{bx-2}}-\frac{5i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{for}\ |bx|>2 \\ -\frac{b^{3}x^{\frac{13}{2}}}{6\sqrt{-bx+2}}+\frac{7b^{2}x^{\frac{11}{2}}}{6\sqrt{-bx+2}}-\frac{67bx^{\frac{9}{2}}}{24\sqrt{-bx+2}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{48b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{48b^{2}\sqrt{-bx+2}}-\frac{5\sqrt{x}}{8b^{3}\sqrt{-bx+2}}+\frac{5\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(13/2)/(6*sqrt(b*x - 2)) - 7*I*b**2*x**(11/2)/(6*sqrt(
b*x - 2)) + 67*I*b*x**(9/2)/(24*sqrt(b*x - 2)) - 55*I*x**(7/2)/(24*sqrt(b*x
 - 2)) - I*x**(5/2)/(48*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(48*b**2*sqrt(b*x -
 2)) + 5*I*sqrt(x)/(8*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/(8*b**(7/2)), Abs(b*x) > 2), (-b**3*x**(13/2)/(6*sqrt(-b*x + 2)) + 7*
b**2*x**(11/2)/(6*sqrt(-b*x + 2)) - 67*b*x**(9/2)/(24*sqrt(-b*x + 2)) + 55*
x**(7/2)/(24*sqrt(-b*x + 2)) + x**(5/2)/(48*b*sqrt(-b*x + 2)) + 5*x**(3/2)/
(48*b**2*sqrt(-b*x + 2)) - 5*sqrt(x)/(8*b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
```

%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(5/2), x)
```

3.564 $\int x^{3/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

```
[Out] 1/4*x^(5/2)*(-b*x+2)^(3/2)+1/5*x^(5/2)*(-b*x+2)^(5/2)+3/4*arcsin(1/2*b^(1/2
)*x^(1/2)*2^(1/2))/b^(5/2)-1/8*x^(3/2)*(-b*x+2)^(1/2)/b+1/4*x^(5/2)*(-b*x+2
)^(1/2)-3/8*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{3\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (x^(5/2)*(2 - b*x)^(5/2
))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2-bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2-bx)^{5/2}+\int x^{3/2}(2-bx)^{3/2}\,dx\\
&= \frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx\\
&= \frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{3\int \frac{\sqrt{}}{\sqrt{2-}}}{8b}\\
&= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-\\
&= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-\\
&= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{8b^2}-\frac{x^{3/2}\sqrt{2-bx}}{8b}+\frac{1}{4}x^{5/2}\sqrt{2-bx}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{5}x^{5/2}(2-
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 90, normalized size = 0.70

$$
\frac{\sqrt{x}\,\sqrt{2-bx}\,(-15-5bx+62b^2x^2-42b^3x^3+8b^4x^4)}{40b^2}-\frac{3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x + 62*b^2*x^2 - 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) - (3*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(5/2))
```

Maple [A]
time = 0.12, size = 135, normalized size = 1.05

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(-8b^4x^4+42b^3x^3-62x^2b^2+5bx+15\right)\sqrt{-\frac{bx}{2}+1}}{40b^2}-\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{5}{2}}}}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$

risch	$-\frac{(8b^4x^4-42b^3x^3+62x^2b^2-5bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{40b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{7}{2}}}{5b}+\frac{-\frac{3\sqrt{x}\,(-bx+2)^{\frac{7}{2}}}{20b}+\frac{3\left(\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{5\sqrt{(-bx+2)\,x}\;\mathrm{arc}}{2\sqrt{-bx+}}\right.}{20b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+2)^(7/2)+3/5/b*(-1/4/b*x^(1/2)*(-b*x+2)^(7/2)+1/4/b*(1
/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^(
1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(
x-1/b)/(-b*x^2+2*x)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(89) = 178$.
time = 0.54, size = 179, normalized size = 1.40

$$\frac{\frac{15\sqrt{-bx+2}\,b^4}{\sqrt{x}}+\frac{70(-bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{128(-bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}-\frac{70(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7-\frac{5(bx-2)b^6}{x}+\frac{10(bx-2)^2b^5}{x^2}-\frac{10(bx-2)^3b^4}{x^3}+\frac{5(bx-2)^4b^3}{x^4}-\frac{(bx-2)^5b^2}{x^5}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) + 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^7 - 5*(b*x - 2)*b^6/x + 10*(b*x - 2)^2*b^5/x^2 - 10*(b
*x - 2)^3*b^4/x^3 + 5*(b*x - 2)^4*b^3/x^4 - (b*x - 2)^5*b^2/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.71, size = 157, normalized size = 1.23

$$\left[\frac{(8b^5x^4-42b^4x^3+62b^3x^2-5b^2x-15b)\sqrt{-bx+2}\,\sqrt{x}-15\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{40b^3},\ \frac{(8b^5x^4-42b^4x^3+62b^3x^2-5b^2x-15b)\sqrt{-bx+2}\,\sqrt{x}-30\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{40b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2
)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
```

```
3, 1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x +
 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 33.12, size = 292, normalized size = 2.28

$$\begin{cases} \frac{ib^3x^{\frac{11}{2}}}{5\sqrt{bx-2}} - \frac{29ib^2x^{\frac{9}{2}}}{20\sqrt{bx-2}} + \frac{73ibx^{\frac{7}{2}}}{20\sqrt{bx-2}} - \frac{129ix^{\frac{5}{2}}}{40\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ -\frac{b^3x^{\frac{11}{2}}}{5\sqrt{-bx+2}} + \frac{29b^2x^{\frac{9}{2}}}{20\sqrt{-bx+2}} - \frac{73bx^{\frac{7}{2}}}{20\sqrt{-bx+2}} + \frac{129x^{\frac{5}{2}}}{40\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(11/2)/(5*sqrt(b*x - 2)) - 29*I*b**2*x**(9/2)/(20*sqrt
(b*x - 2)) + 73*I*b*x**(7/2)/(20*sqrt(b*x - 2)) - 129*I*x**(5/2)/(40*sqrt(b
*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**2*sqrt(b*x -
2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), Abs(b*x) > 2), (-b
**3*x**(11/2)/(5*sqrt(-b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(-b*x + 2)) - 7
3*b*x**(7/2)/(20*sqrt(-b*x + 2)) + 129*x**(5/2)/(40*sqrt(-b*x + 2)) + x**(3
/2)/(8*b*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(2-b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(5/2), x)
```

3.565 $\int \sqrt{x}\,(2-bx)^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}+\frac{5}{8}x^{3/2}\sqrt{2-bx}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

```
[Out] 5/12*x^(3/2)*(-b*x+2)^(3/2)+1/4*x^(3/2)*(-b*x+2)^(5/2)+5/4*arcsin(1/2*b^(1/
2)*x^(1/2)*2^(1/2))/b^(3/2)+5/8*x^(3/2)*(-b*x+2)^(1/2)-5/8*x^(1/2)*(-b*x+2)
^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 - b*x])/8 + (5*x^(3/2)
*(2 - b*x)^(3/2))/12 + (x^(3/2)*(2 - b*x)^(5/2))/4 + (5*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{x}\,(2-bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,(2-bx)^{3/2}\,dx$$
$$= \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,\sqrt{2-bx}\,dx$$
$$= \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\int \frac{}{\sqrt{}}}{}$$
$$= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\text{Subs}}{}$$
$$= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\sin^{-}}{}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.77

$$\frac{\sqrt{x}\,\sqrt{2-bx}\,(-15+59bx-34b^2x^2+6b^3x^3)}{24b} + \frac{5\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{4(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 + 59*b*x - 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*(-b)^(3/2))
```

Maple [A]
time = 0.13, size = 107, normalized size = 1.01

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}\left(-6b^3x^3+34x^2b^2-59bx+15\right)\sqrt{-\frac{bx}{2}+1}}{24b}-\frac{5\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{3}{2}}}}{\sqrt{-b}\,\sqrt{\pi}\,b}$
default	$\frac{x^{\frac{3}{2}}(-bx+2)^{\frac{5}{2}}}{4}+\frac{5x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}{12}+\frac{5x^{\frac{3}{2}}\sqrt{-bx+2}}{8}-\frac{5\sqrt{x}\,\sqrt{-bx+2}}{8b}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,(x-}{\sqrt{-x^2b-}}\right)}{8b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$

risch	$-\frac{(6b^3x^3-34x^2b^2+59bx-15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{24b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{8b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^(3/2)*(-b*x+2)^(5/2)+5/12*x^(3/2)*(-b*x+2)^(3/2)+5/8*x^(3/2)*(-b*x+2)
^(1/2)-5/8*x^(1/2)*(-b*x+2)^(1/2)/b+5/8/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)
^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.50, size = 145, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{55\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{73\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^5-\frac{4\,(bx-2)b^4}{x}+\frac{6\,(bx-2)^2b^3}{x^2}-\frac{4\,(bx-2)^3b^2}{x^3}+\frac{(bx-2)^4b}{x^4}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) + 55*(-b*x + 2)^(3/2)*b^2/x^(3/2) + 73*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x - 2
)*b^4/x + 6*(b*x - 2)^2*b^3/x^2 - 4*(b*x - 2)^3*b^2/x^3 + (b*x - 2)^4*b/x^4
) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

Fricas [A]
time = 0.73, size = 141, normalized size = 1.33

$$\left[\frac{(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{24\,b^2},\ \frac{(6\,b^4x^3-34\,b^3x^2+59\,b^2x-15\,b)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) -
15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/24*((6*
b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)
*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 8.50, size = 253, normalized size = 2.39

$$\begin{cases} \frac{ib^3x^{\frac{9}{2}}}{4\sqrt{bx-2}}-\frac{23ib^2x^{\frac{7}{2}}}{12\sqrt{bx-2}}+\frac{127ibx^{\frac{5}{2}}}{24\sqrt{bx-2}}-\frac{133ix^{\frac{3}{2}}}{24\sqrt{bx-2}}+\frac{5i\sqrt{x}}{4b\sqrt{bx-2}}-\frac{5i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{for}\ |bx|>2 \\ -\frac{b^3x^{\frac{9}{2}}}{4\sqrt{-bx+2}}+\frac{23b^2x^{\frac{7}{2}}}{12\sqrt{-bx+2}}-\frac{127bx^{\frac{5}{2}}}{24\sqrt{-bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{-bx+2}}-\frac{5\sqrt{x}}{4b\sqrt{-bx+2}}+\frac{5\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(9/2)/(4*sqrt(b*x - 2)) - 23*I*b**2*x**(7/2)/(12*sqrt(
b*x - 2)) + 127*I*b*x**(5/2)/(24*sqrt(b*x - 2)) - 133*I*x**(3/2)/(24*sqrt(b
*x - 2)) + 5*I*sqrt(x)/(4*b*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/(4*b**(3/2)), Abs(b*x) > 2), (-b**3*x**(9/2)/(4*sqrt(-b*x + 2)) + 23
*b**2*x**(7/2)/(12*sqrt(-b*x + 2)) - 127*b*x**(5/2)/(24*sqrt(-b*x + 2)) + 1
33*x**(3/2)/(24*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b*sqrt(-b*x + 2)) + 5*asin(s
qrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(2-b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(5/2), x)
```

3.566 $\int \frac{(2-bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=82

$$\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 5*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+1/
3*(-b*x+2)^(5/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 - b*x])/2 + (5*Sqrt[x]*(2 - b*x)^(3/2))/6 + (Sqrt[x]*(2 -
 b*x)^(5/2))/3 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{3}\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx\\ &= \frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+5\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{\right.\\ &= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 0.84

$$\frac{1}{6}\sqrt{x}\,\sqrt{2-bx}\,\left(33-13bx+2b^2x^2\right)-\frac{5\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(33 - 13*b*x + 2*b^2*x^2))/6 - (5*Log[-(Sqrt[-b]*Sqr
t[x]) + Sqrt[2 - b*x]])/Sqrt[-b]
```

Maple [A]
time = 0.14, size = 91, normalized size = 1.11

method	result	size
meijerg	$\frac{15\sqrt{-b}\left(-\frac{8\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{-b}\,\left(\frac{1}{24}x^2b^2-\frac{13}{48}bx+\frac{11}{16}\right)\sqrt{-\frac{bx}{2}+1}}{15}-\frac{\sqrt{\pi}\,\sqrt{-b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3\sqrt{b}}\right)}{\sqrt{\pi}\,b}$	78
default	$\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{x}\,\sqrt{-bx+2}}{2}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	91

risch	$-\frac{(2x^2b^2-13bx+33)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*x^(1/2)*(-b*x+2)^
(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*
(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.51, size = 112, normalized size = 1.37

$$-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{15\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{40\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^3-\frac{3\,(bx-2)b^2}{x}+\frac{3\,(bx-2)^2b}{x^2}-\frac{(bx-2)^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/3*(15*sqrt(-b*x + 2
)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 33*(-b*x + 2)^(5/2)/x^(5/2)
)/(b^3 - 3*(b*x - 2)*b^2/x + 3*(b*x - 2)^2*b/x^2 - (b*x - 2)^3/x^3)
```

Fricas [A]
time = 0.65, size = 125, normalized size = 1.52

$$\left[\frac{(2\,b^3x^2-13\,b^2x+33\,b)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b},\ \frac{(2\,b^3x^2-13\,b^2x+33\,b)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 13*b^2*x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 - 13*b^2*
x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b
)*sqrt(x))))/b]
```

Sympy [C] Result contains complex when optimal does not.

time = 3.70, size = 207, normalized size = 2.52

$$\begin{cases} \frac{ib^3x^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{17ib^2x^{\frac{5}{2}}}{6\sqrt{bx-2}} + \frac{59ibx^{\frac{3}{2}}}{6\sqrt{bx-2}} - \frac{11i\sqrt{x}}{\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } |bx| > 2 \\ -\frac{b^3x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{17b^2x^{\frac{5}{2}}}{6\sqrt{-bx+2}} - \frac{59bx^{\frac{3}{2}}}{6\sqrt{-bx+2}} + \frac{11\sqrt{x}}{\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(7/2)/(3*sqrt(b*x - 2)) - 17*I*b**2*x**(5/2)/(6*sqrt(b
*x - 2)) + 59*I*b*x**(3/2)/(6*sqrt(b*x - 2)) - 11*I*sqrt(x)/sqrt(b*x - 2) -
 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x) > 2), (-b**3*x**(7/
2)/(3*sqrt(-b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(-b*x + 2)) - 59*b*x**(3/2)
/(6*sqrt(-b*x + 2)) + 11*sqrt(x)/sqrt(-b*x + 2) + 5*asin(sqrt(2)*sqrt(b)*sq
rt(x)/2)/sqrt(b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(1/2), x)
```

3.567 $\int \frac{(2-bx)^{5/2}}{x^{3/2}} dx$

Optimal. Leaf size=82

$$-\frac{15}{2}b\sqrt{x}\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-15\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -15*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))*b^(1/2)-2*(-b*x+2)^(5/2)/x^(1/2)-5/
2*b*(-b*x+2)^(3/2)*x^(1/2)-15/2*b*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 52, 56, 222}

$$-15\sqrt{b}\,\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\sqrt{2-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*b*Sqrt[x]*Sqrt[2 - b*x])/2 - (5*b*Sqrt[x]*(2 - b*x)^(3/2))/2 - (2*(2 -
 b*x)^(5/2))/Sqrt[x] - 15*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx \\
&= -\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{1}{2}(15b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx \\
&= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx \\
&= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-(15b)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx \right. \\
&= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 68, normalized size = 0.83

$$\frac{\sqrt{2-bx}\,(-16-9bx+b^2x^2)}{2\sqrt{x}}-15\sqrt{-b}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-16 - 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[-b]*Log[-(Sqr
t[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

Maple [A]
time = 0.12, size = 78, normalized size = 0.95

method	result	size

meijerg	$\frac{15(-b)^{\frac{3}{2}}\left(\frac{16\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{1}{16}x^2b^2+\frac{9}{16}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{15\sqrt{x}\,\sqrt{-b}}+\frac{2\sqrt{\pi}\,\sqrt{b}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{-b}}\right)}{2\sqrt{\pi}\,b}$	78
risch	$-\frac{(b^3x^3-11x^2b^2+2bx+32)\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}-\frac{15\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{2\sqrt{x}\,\sqrt{-bx+2}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15/2*(-b)^(3/2)/Pi^(1/2)/b*(16/15*Pi^(1/2)/x^(1/2)*2^(1/2)/(-b)^(1/2)*(-1/1
6*x^2*b^2+9/16*b*x+1)*(-1/2*b*x+1)^(1/2)+2*Pi^(1/2)/(-b)^(1/2)*b^(1/2)*arcs
in(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 96, normalized size = 1.17

$$15\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{\frac{7\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{9\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2-\frac{2\,(bx-2)b}{x}+\frac{(bx-2)^2}{x^2}}-\frac{8\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 15*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (7*sqrt(-b*x + 2)*b^2
/sqrt(x) + 9*(-b*x + 2)^(3/2)*b/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)
^2/x^2) - 8*sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.62, size = 117, normalized size = 1.43

$$\left[\frac{15\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+(b^2x^2-9\,bx-16)\sqrt{-bx+2}\,\sqrt{x}}{2\,x},\frac{30\,\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(b^2x^2-9\,bx-16)\sqrt{-bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (b^2*
x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x))/x, 1/2*(30*sqrt(b)*x*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x
))/x]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.58, size = 201, normalized size = 2.45

$$\begin{cases} 15i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)+\frac{ib^3x^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{11ib^2x^{\frac{3}{2}}}{2\sqrt{bx-2}}+\frac{ib\sqrt{x}}{\sqrt{bx-2}}+\frac{16i}{\sqrt{x}\,\sqrt{bx-2}} & \text{for } |bx|>2 \\ -15\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{-bx+2}}-\frac{b\sqrt{x}}{\sqrt{-bx+2}}-\frac{16}{\sqrt{x}\,\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((15*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) + I*b**3*x**(5/2)/
(2*sqrt(b*x - 2)) - 11*I*b**2*x**(3/2)/(2*sqrt(b*x - 2)) + I*b*sqrt(x)/sqrt
(b*x - 2) + 16*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x) > 2), (-15*sqrt(b)*asin(
sqrt(2)*sqrt(b)*sqrt(x)/2) - b**3*x**(5/2)/(2*sqrt(-b*x + 2)) + 11*b**2*x**
(3/2)/(2*sqrt(-b*x + 2)) - b*sqrt(x)/sqrt(-b*x + 2) - 16/(sqrt(x)*sqrt(-b*x
 + 2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(3/2), x)
```

3.568 $\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=84

$$5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

```
[Out] -2/3*(-b*x+2)^(5/2)/x^(3/2)+10*b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))+
10/3*b*(-b*x+2)^(3/2)/x^(1/2)+5*b^2*x^(1/2)*(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 52, 56, 222}

$$10b^{3/2}\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\,\sqrt{2-bx}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 - b*x] + (10*b*(2 - b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 -
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx\\
&= \frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\
&= 5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\right.\\
&= 5b^2\sqrt{x}\,\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 0.83

$$\frac{\sqrt{2-bx}\,\left(-8+28bx+3b^2x^2\right)}{3x^{3/2}}+10\sqrt{-b}\,b\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-8 + 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 10*Sqrt[-b]*b*Log[-
(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

Maple [A]
time = 0.15, size = 78, normalized size = 0.93

method	result	size

meijerg	$\frac{15(-b)^{\frac{5}{2}}\left(\frac{32\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{3}{8}x^2b^2-\frac{7}{2}bx+1\right)\sqrt{-\frac{bx}{2}+1}}{45x^{\frac{3}{2}}(-b)^{\frac{3}{2}}}-\frac{8\sqrt{\pi}\,b^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{3(-b)^{\frac{3}{2}}}\right)}{4\sqrt{\pi}\,b}$	78
risch	$-\frac{(3b^3x^3+22x^2b^2-64bx+16)\sqrt{(-bx+2)\,x}}{3x^{\frac{3}{2}}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{x}\,\sqrt{-bx+2}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15/4*(-b)^(5/2)/Pi^(1/2)/b*(32/45*Pi^(1/2)/x^(3/2)*2^(1/2)/(-b)^(3/2)*(-3/8
*x^2*b^2-7/2*b*x+1)*(-1/2*b*x+1)^(1/2)-8/3*Pi^(1/2)/(-b)^(3/2)*b^(3/2)*arcs
in(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.51, size = 79, normalized size = 0.94

$$-10\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{8\,\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{2\,\sqrt{-bx+2}\,b^2}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}-\frac{4\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -10*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 8*sqrt(-b*x + 2)*b/s
qrt(x) + 2*sqrt(-b*x + 2)*b^2/((b - (b*x - 2)/x)*sqrt(x)) - 4/3*(-b*x + 2)^
(3/2)/x^(3/2)
```

Fricas [A]
time = 0.67, size = 126, normalized size = 1.50

$$\left[\frac{15\,\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+(3\,b^2x^2+28\,bx-8)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2},-\frac{30\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-(3\,b^2x^2+28\,bx-8)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (
3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*x + 2)*sqrt(x))/x^2, -1/3*(30*b^(3/2)*x^2*a
rctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*
x + 2)*sqrt(x))/x^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 3.53, size = 223, normalized size = 2.65

$$\begin{cases} b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}+\frac{28b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right)+10b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{8\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{1}{|bx|}>\frac{1}{2} \\ ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}+\frac{28ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right)-\frac{8i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(5/2)*x*sqrt(-1 + 2/(b*x)) + 28*b**(3/2)*sqrt(-1 + 2/(b*x))/3
 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 10*
b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) - 8*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 1/Abs(b*x) > 1/2), (I*b**(5/2)*x*sqrt(1 - 2/(b*x)) + 28*I*b**(3/2)*sqrt
(1 - 2/(b*x))/3 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(sqrt(1 - 2/
(b*x)) + 1) - 8*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(5/2), x)
```

3.569 $\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=101

$$\frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

```
[Out] -5/8*a^3*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-5/12*a*x^(3/2)*(b*x
+a)^(1/2)/b^2+1/3*x^(5/2)*(b*x+a)^(1/2)/b+5/8*a^2*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a + b*x])/(12*b^2
) + (x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= -\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} + \frac{(5a^2)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a^3)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{16b^3}$$

$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \ldots\right)}{8b^3}$$

$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+\ldots}}\right)}{8b^3}$$

$$= \frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.75

$$\frac{\sqrt{x}\sqrt{a+bx}\,(15a^2-10abx+8b^2x^2)}{24b^3} + \frac{5a^3\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2))/(24*b^3) + (5*a^3*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(7/2))
```

Maple [A]
time = 0.12, size = 109, normalized size = 1.08

method	result	size
risch	$\frac{(8x^2b^2-10abx+15a^2)\sqrt{x}\,\sqrt{bx+a}}{24b^3}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+a}}$	87
default	$\frac{x^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5a\left(\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4b}\right)}{6b}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(5/2)*(b*x+a)^(1/2)/b-5/6*a/b*(1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a/b*(x
^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2
)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.51, size = 146, normalized size = 1.45

$$\frac{5\,a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{15\,(bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^6-\frac{3\,(bx+a)b^5}{x}+\frac{3\,(bx+a)^2b^4}{x^2}-\frac{(bx+a)^3b^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(7/2) - 1/24*(33*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3/
2)*a^3*b/x^(3/2) + 15*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x + a)*b^5/x
 + 3*(b*x + a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3)
```

Fricas [A]
time = 0.47, size = 140, normalized size = 1.39

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{48\,b^4},\ \frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

Sympy [**A**]
time = 9.21, size = 128, normalized size = 1.27

$$\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^2\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}}-\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*
sqrt(1 + b*x/a)) - sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*a**3*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(1/2), x)
```

3.570 $\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=77

$$-\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)+1/2*x^(3/2)*(b*x+a)^
(1/2)/b-3/4*a*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 65, 223, 212}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^2) + (x^(3/2)*Sqrt[a + b*x])/(2*b) + (3*a
^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx &= \frac{x^{3/2}\sqrt{a+bx}}{2b} - \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b^2} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^2} \\
&= -\frac{3a\sqrt{x}\,\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.86

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{a+bx}\,(-3a+2bx) - 3a^2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-3*a + 2*b*x) - 3*a^2*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[a + b*x]])/(4*b^(5/2))
```

Maple [A]
time = 0.10, size = 87, normalized size = 1.13

method	result	size

risch	$-\frac{(-2bx+3a)\sqrt{x}\,\sqrt{bx+a}}{4b^{2}}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)\sqrt{x\,(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+a}}$	76
default	$\frac{x^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3a\left(\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}\right)}{4b}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a/b*(x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*
(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^
(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(55) = 110$.
time = 0.50, size = 112, normalized size = 1.45

$$-\frac{3\,a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{5\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}-\frac{3\,(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{4}-\frac{2\,(bx+a)b^{3}}{x}+\frac{(bx+a)^{2}b^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(5/2) + 1/4*(5*sqrt(b*x + a)*a^2*b/sqrt(x) - 3*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2)
```

Fricas [A]
time = 0.51, size = 119, normalized size = 1.55

$$\left[\frac{3\,a^{2}\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x-3\,ab)\sqrt{bx+a}\,\sqrt{x}}{8\,b^{3}},-\frac{3\,a^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^{2}x-3\,ab)\sqrt{bx+a}\,\sqrt{x}}{4\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
```

```
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

Sympy [A]
time = 2.82, size = 100, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - sqrt(a)*x**(3/2)/(4*b*sqrt(1
 + b*x/a)) + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/
(2*sqrt(a)*sqrt(1 + b*x/a))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(1/2), x)
```

3.571 $\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

[Out] -a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)+x^(1/2)*(b*x+a)^(1/2)/b

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {52, 65, 223, 212}

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[a + b*x],x]

[Out] (Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(3/2)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx &= \frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{2b}\\ &= \frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= \frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}\\ &= \frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.02

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}+\frac{a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a + b*x],x]
[Out] (Sqrt[x]*Sqrt[a + b*x])/b + (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(
3/2)
```

Maple [A]
time = 0.11, size = 65, normalized size = 1.35

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	65

risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b}-\frac{a\sqrt{x\,(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{bx+a}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/
2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

time = 0.50, size = 73, normalized size = 1.52

$$\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}-\frac{\sqrt{bx+a}\,a}{\left(b^2-\frac{(bx+a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(
x)))/b^(3/2) - sqrt(b*x + a)*a/((b^2 - (b*x + a)*b/x)*sqrt(x))
```

Fricas [A]

time = 0.61, size = 91, normalized size = 1.90

$$\left[\frac{a\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)+2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b^2},\ \frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

Sympy [A]

time = 1.13, size = 44, normalized size = 0.92

$$\frac{\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a)/b - a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/
2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.55, size = 44, normalized size = 0.92

$$\frac{\sqrt{x}\,\sqrt{a+b\,x}}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (x^(1/2)*(a + b*x)^(1/2))/b - (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2)
 - a^(1/2))))/b^(3/2)
```

3.572 $\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.07

$$-\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.
time = 0.12, size = 48, normalized size = 1.71

method	result	size
default	$\frac{\sqrt{x(bx+a)}\,\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{\sqrt{x}\sqrt{bx+a}\sqrt{b}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^
(1/2))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.
time = 0.49, size = 41, normalized size = 1.46

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
sqrt(b)
```

Fricas [A]
time = 0.54, size = 57, normalized size = 2.04

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}\,+a\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [A]
time = 0.49, size = 22, normalized size = 0.79

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-4,[0,1,1]%%%}+%%%{-4,[0,1
,0]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{12,[1,1,1]%%%}+%%%{4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.03, size = 30, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

3.573 $\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=19

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

```
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Maple [A]
time = 0.12, size = 16, normalized size = 0.84

method	result	size
gosper	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
default	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16
risch	$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Fricas [A]
time = 0.69, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

Sympy [A]
time = 0.44, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 1.61, size = 33, normalized size = 1.74

$$-\frac{2\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a*abs(b))
```

Mupad [B]
time = 0.35, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{a+b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*(a + b*x)^(1/2))/(a*x^(1/2))
```

3.574 $\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\sqrt{a+bx}}{3ax^{3/2}}+\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}$$

```
[Out] -2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {47, 37}

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}+\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.61

$$-\frac{2(a-2bx)\sqrt{a+bx}}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]
[Out] (-2*(a - 2*b*x)*Sqrt[a + b*x])/(3*a^2*x^(3/2))
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.75

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^2}$	22
risch	$-\frac{2\sqrt{bx+a}\,(-2bx+a)}{3x^{\frac{3}{2}}a^2}$	22
default	$-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{3\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^2
```

Fricas [A]
time = 0.75, size = 23, normalized size = 0.52

$$\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

Sympy [A]
time = 0.96, size = 42, normalized size = 0.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax}+\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**2
)
```

Giac [A]
time = 1.26, size = 50, normalized size = 1.14

$$\frac{2\left(\frac{2(bx+a)b^3}{a^2}-\frac{3b^3}{a}\right)\sqrt{bx+a}\,b}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(b*x + a)*b^3/a^2 - 3*b^3/a)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(3
/2)*abs(b))
```

Mupad [B]
time = 0.34, size = 25, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}-\frac{4\,b\,x}{3\,a^2}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) - (4*b*x)/(3*a^2))*(a + b*x)^(1/2))/x^(3/2)
```

3.575 $\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}$$

```
[Out] -2/5*(b*x+a)^(1/2)/a/x^(5/2)+8/15*b*(b*x+a)^(1/2)/a^2/x^(3/2)-16/15*b^2*(b*
x+a)^(1/2)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(5*a*x^(5/2)) + (8*b*Sqrt[a + b*x])/(15*a^2*x^(3/2)) - (
16*b^2*Sqrt[a + b*x])/(15*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{5ax^{5/2}} - \frac{(4b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a}$$
$$= -\frac{2\sqrt{a+bx}}{5ax^{5/2}} + \frac{8b\sqrt{a+bx}}{15a^2x^{3/2}} + \frac{(8b^2)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{15a^2}$$
$$= -\frac{2\sqrt{a+bx}}{5ax^{5/2}} + \frac{8b\sqrt{a+bx}}{15a^2x^{3/2}} - \frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\,(3a^2-4abx+8b^2x^2)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

Maple [A]
time = 0.12, size = 55, normalized size = 0.81

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(8x^2b^2-4abx+3a^2)}{15x^{\frac{5}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}\,(8x^2b^2-4abx+3a^2)}{15x^{\frac{5}{2}}a^3}$	35
default	$-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}} - \frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b
*x+a)^(1/2)/a^2/x^(1/2))
```

Maxima [A]
time = 0.27, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{15\sqrt{bx+a}\,b^2}{\sqrt{x}} - \frac{10\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} + \frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*sqrt(b*x + a)*b^2/sqrt(x) - 10*(b*x + a)^(3/2)*b/x^(3/2) + 3*(b*x
 + a)^(5/2)/x^(5/2))/a^3
```

Fricas [A]
time = 0.69, size = 34, normalized size = 0.50

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(63) = 126.
time = 3.50, size = 287, normalized size = 4.22

$$-\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{4a^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{24ab^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{16b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 +
 15*a**3*b**6*x**4) - 4*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2)*x**2*sqrt(a/
(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 24
*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3
+ 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

Giac [A]
time = 1.20, size = 66, normalized size = 0.97

$$-\frac{2\left(\frac{15\,b^5}{a}+4\left(\frac{2\,(bx+a)b^5}{a^3}-\frac{5\,b^5}{a^2}\right)(bx+a)\right)\sqrt{bx+a}\,b}{15\left((bx+a)b-ab\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*b^5/a + 4*(2*(b*x + a)*b^5/a^3 - 5*b^5/a^2)*(b*x + a))*sqrt(b*x +
 a)*b/(((b*x + a)*b - a*b)^(5/2)*abs(b))
```

Mupad [B]
time = 0.35, size = 36, normalized size = 0.53

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a}+\frac{16\,b^2\,x^2}{15\,a^3}-\frac{8\,b\,x}{15\,a^2}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(1/2)),x)
[Out] -((a + b*x)^(1/2)*(2/(5*a) + (16*b^2*x^2)/(15*a^3) - (8*b*x)/(15*a^2)))/x^(
5/2)
```

3.576 $\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=92

$$-\frac{2\sqrt{a+bx}}{7ax^{7/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}$$

```
[Out] -2/7*(b*x+a)^(1/2)/a/x^(7/2)+12/35*b*(b*x+a)^(1/2)/a^2/x^(5/2)-16/35*b^2*(b
*x+a)^(1/2)/a^3/x^(3/2)+32/35*b^3*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(7*a*x^(7/2)) + (12*b*Sqrt[a + b*x])/(35*a^2*x^(5/2)) -
(16*b^2*Sqrt[a + b*x])/(35*a^3*x^(3/2)) + (32*b^3*Sqrt[a + b*x])/(35*a^4*Sq
rt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} - \frac{(6b)\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{7a} \\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} + \frac{(24b^2)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{35a^2} \\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} - \frac{(16b^3)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{35a^3} \\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}} + \frac{12b\sqrt{a+bx}}{35a^2x^{5/2}} - \frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}} + \frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.55

$$-\frac{2\sqrt{a+bx}\,(5a^3-6a^2bx+8ab^2x^2-16b^3x^3)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

Maple [A]
time = 0.12, size = 77, normalized size = 0.84

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
risch	$-\frac{2\sqrt{bx+a}\,(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3)}{35x^{\frac{7}{2}}a^4}$	46
default	$-\frac{2\sqrt{bx+a}}{7a\,x^{\frac{7}{2}}}-\frac{6b\left(-\frac{2\sqrt{bx+a}}{5a\,x^{\frac{5}{2}}}-\frac{4b\left(-\frac{2\sqrt{bx+a}}{3a\,x^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3a^2\sqrt{x}}\right)}{5a}\right)}{7a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x+a)^(1/2)/a/x^(7/2)-6/7*b/a*(-2/5*(b*x+a)^(1/2)/a/x^(5/2)-4/5*b/a*
(-2/3*(b*x+a)^(1/2)/a/x^(3/2)+4/3*b*(b*x+a)^(1/2)/a^2/x^(1/2)))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.66

$$\frac{2\left(\frac{35\sqrt{bx+a}\,b^3}{\sqrt{x}}-\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{21\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(b*x + a)*b^3/sqrt(x) - 35*(b*x + a)^(3/2)*b^2/x^(3/2) + 21*(b
*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^4
```

Fricas [A]
time = 1.49, size = 45, normalized size = 0.49

$$\frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(87) = 174.
time = 9.87, size = 488, normalized size = 5.30

$$-\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{18a^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{10a^4b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{10a^3b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{60a^2b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{80ab^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{32b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x*
*4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(21/2)*x*sqrt(a
/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5
+ 35*a**4*b**12*x**6) - 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(35*a**7*b
**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
+ 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b*
*10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*a**2*b**(27/2)*x*
*4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b*
*11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(35*
a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*
x**6) + 32*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b
**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
```

Giac [A]
time = 0.99, size = 82, normalized size = 0.89

$$-\frac{2\left(\frac{35\,b^7}{a}-2\left(\frac{35\,b^7}{a^2}+4\left(\frac{2\,(bx+a)b^7}{a^4}-\frac{7\,b^7}{a^3}\right)(bx+a)\right)(bx+a)\right)\sqrt{bx+a}\,b}{35\left((bx+a)b-ab\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(35*b^7/a - 2*(35*b^7/a^2 + 4*(2*(b*x + a)*b^7/a^4 - 7*b^7/a^3)*(b*x
+ a))*(b*x + a))*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.38, size = 47, normalized size = 0.51

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{7\,a}+\frac{16\,b^2\,x^2}{35\,a^3}-\frac{32\,b^3\,x^3}{35\,a^4}-\frac{12\,b\,x}{35\,a^2}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(7*a) + (16*b^2*x^2)/(35*a^3) - (32*b^3*x^3)/(35*a^4)
- (12*b*x)/(35*a^2)))/x^(7/2)
```

3.577 $\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}$$

```
[Out] 15/4*a^2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(7/2)-2*x^(5/2)/b/(b*x+a)
^(1/2)+5/2*x^(3/2)*(b*x+a)^(1/2)/b^2-15/4*a*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]

time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[a + b*x]) - (15*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^3) + (5*
x^(3/2)*Sqrt[a + b*x])/(2*b^2) + (15*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/(4*b^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{(15a^2)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\right.}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+}}\right.}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.13, size = 76, normalized size = 0.79

$$\frac{\sqrt{x}\,(-15a^2-5abx+2b^2x^2)}{4b^3\sqrt{a+bx}}-\frac{15a^2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(-15*a^2 - 5*a*b*x + 2*b^2*x^2))/(4*b^3*Sqrt[a + b*x]) - (15*a^2*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(7/2))
```

Maple [A]
time = 0.13, size = 119, normalized size = 1.24

method	result
risch	$-\frac{(-2bx+7a)\sqrt{x}\,\sqrt{bx+a}}{4b^3}+\frac{\left(\frac{15a^2\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{8b^{\frac{7}{2}}}-\frac{2a^2\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{b^4\left(x+\frac{a}{b}\right)}\right)\sqrt{x\,(bx+}}{\sqrt{x}\,\sqrt{bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*b*x+7*a)*x^(1/2)*(b*x+a)^(1/2)/b^3+(15/8/b^(7/2)*a^2*ln((1/2*a+b*x
)/b^(1/2)+(b*x^2+a*x)^(1/2))-2/b^4*a^2/(x+a/b)*((x+a/b)^2*b-a*(x+a/b))^(1/2
))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.49, size = 131, normalized size = 1.36

$$-\frac{8\,a^2b^2-\frac{25\,(bx+a)a^2b}{x}+\frac{15\,(bx+a)^2a^2}{x^2}}{4\left(\frac{\sqrt{bx+a}\,b^5}{\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{15\,a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*b^2 - 25*(b*x + a)*a^2*b/x + 15*(b*x + a)^2*a^2/x^2)/(sqrt(b*x
+ a)*b^5/sqrt(x) - 2*(b*x + a)^(3/2)*b^4/x^(3/2) + (b*x + a)^(5/2)*b^3/x^(5
/2)) - 15/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
+ a)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.93, size = 175, normalized size = 1.82

$$\left[\frac{15\,(a^2bx+a^3)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(2\,b^3x^2-5\,ab^2x-15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{8\,(b^5x+ab^4)},-\frac{15\,(a^2bx+a^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-(2\,b^3x^2-5\,ab^2x-15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{4\,(b^5x+ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

Sympy [A]
time = 5.46, size = 105, normalized size = 1.09

$$-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1+\frac{bx}{a}}}-\frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2*
sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) + x*
*(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))
```

Giac [A]
time = 19.08, size = 131, normalized size = 1.36

$$\frac{\left(2\,\sqrt{(bx+a)b-ab}\,\sqrt{bx+a}\,\left(\frac{2\,(bx+a)}{b^3}-\frac{9\,a}{b^3}\right)-\frac{32\,a^3}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{3}{2}}}-\frac{15\,a^2\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{b^{\frac{5}{2}}}\right)|b|}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b^3 - 9*a/b^3) -
32*a^3/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(3/2)
) - 15*a^2*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
)*abs(b)/b^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(3/2),x)
[Out] int(x^(5/2)/(a + b*x)^(3/2), x)
```

3.578 $\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

```
[Out] -3*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/2)-2*x^(3/2)/b/(b*x+a)^(1/
2)+3*x^(1/2)*(b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {49, 52, 65, 223, 212}

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*Sqrt[x]*Sqrt[a + b*x])/b^2 - (3*a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{(3a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\,\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 57, normalized size = 0.84

$$
\frac{\sqrt{x}\,(3a+bx)}{b^2\sqrt{a+bx}}+\frac{3a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(3*a + b*x))/(b^2*Sqrt[a + b*x]) + (3*a*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[a + b*x]])/b^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(52) = 104$.
time = 0.13, size = 106, normalized size = 1.56

method	result	size
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b^{2}}+\frac{\left(-\frac{3a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^{2}b+ax}\right)}{2b^{\frac{5}{2}}}+\frac{2a\sqrt{\left(x+\frac{a}{b}\right)^{2}b-a\left(x+\frac{a}{b}\right)}}{b^{3}\left(x+\frac{a}{b}\right)}\right)\sqrt{x\left(bx+a\right)}}{\sqrt{x}\,\sqrt{bx+a}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))+2*a/b^3/(x+a/b)*((x+a/b)^2*b-a*(x+a/b))^(1/2))*(x*(b*x+a))^(1/2)/x
^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.51, size = 92, normalized size = 1.35

$$\frac{2\,ab-\frac{3\,(bx+a)a}{x}}{\frac{\sqrt{bx+a}\,b^{3}}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x + a)*a/x)/(sqrt(b*x + a)*b^3/sqrt(x) - (b*x + a)^(3/2)*b^2/
x^(3/2)) + 3/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
 + a)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.88, size = 145, normalized size = 2.13

$$\left[\frac{3\,(abx+a^{2})\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(b^{2}x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{2\,(b^{4}x+ab^{3})},\ \frac{3\,(abx+a^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(b^{2}x+3\,ab)\sqrt{bx+a}\,\sqrt{x}}{b^{4}x+ab^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

Sympy [A]
time = 1.91, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sqrt(a
))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(52) = 104.
time = 18.42, size = 115, normalized size = 1.69

$$\frac{\left(\frac{8\,a^2\sqrt{b}}{\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{3\,a\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}+\frac{2\,\sqrt{(bx+a)b-ab}\,\sqrt{bx+a}}{b}\right)|b|}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(8*a^2*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a
*b) + 3*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b)
+ 2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*abs(b)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(3/2), x)
```

3.579 $\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

```
[Out] 2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(3/2)-2*x^(1/2)/b/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}\\
&= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}\\
&= -\frac{2\sqrt{x}}{b\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 1.04

$$
-\frac{2\sqrt{x}}{b\sqrt{a+bx}}-\frac{2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
)/b^(3/2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x+a)^(3/2),x)
```

Maxima [A]
time = 0.50, size = 57, normalized size = 1.19

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + a)*b)
```

Fricas [A]
time = 0.76, size = 119, normalized size = 2.48

$$\left[\frac{(bx+a)\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,b\sqrt{x}}{b^3x+ab^2},-\frac{2\left((bx+a)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

Sympy [A]
time = 0.81, size = 46, normalized size = 0.96

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 + b
*x/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(36) = 72.
time = 18.57, size = 85, normalized size = 1.77

$$-\frac{\left(\frac{4\,a\sqrt{b}}{\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b) +
 log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b))*abs(b)/b
^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a + b*x)^(3/2), x)
```

3.580 $\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(a + b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a+bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])

Maple [A]
time = 0.12, size = 16, normalized size = 0.84

method	result	size
gosper	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16
default	$\frac{2\sqrt{x}}{a\sqrt{bx+a}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.79

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

Fricas [A]
time = 0.81, size = 22, normalized size = 1.16

$$\frac{2\sqrt{bx+a}\,\sqrt{x}}{abx+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

Sympy [A]
time = 0.44, size = 17, normalized size = 0.89

$$\frac{2}{a\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 2/(a*sqrt(b)*sqrt(a/(b*x) + 1))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(15) = 30.
time = 0.97, size = 45, normalized size = 2.37

$$\frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*abs(
b))
```

Mupad [B]
time = 0.33, size = 22, normalized size = 1.16

$$\frac{2\,\sqrt{x}\,\sqrt{a+b\,x}}{a^2+b\,x\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a + b*x)^(1/2))/(a^2 + a*b*x)
```

3.581 $\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{a\sqrt{x}\,\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

```
[Out] 2/a/x^(1/2)/(b*x+a)^(1/2)-4*(b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {47, 37}

$$\frac{2}{a\sqrt{x}\,\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{x}\,\sqrt{a+bx}}+\frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{x}\,\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

Maple [A]
time = 0.14, size = 33, normalized size = 0.85

method	result	size
gosper	$-\frac{2(2bx+a)}{\sqrt{x}\,\sqrt{bx+a}\,a^2}$	22
default	$-\frac{2}{a\sqrt{x}\,\sqrt{bx+a}}-\frac{4b\sqrt{x}}{a^2\sqrt{bx+a}}$	33
risch	$-\frac{2\sqrt{bx+a}}{a^2\sqrt{x}}-\frac{2b\sqrt{x}}{a^2\sqrt{bx+a}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/a/x^(1/2)/(b*x+a)^(1/2)-4*b/a^2*x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.82

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}\,a^2}-\frac{2\,\sqrt{bx+a}}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) - 2*sqrt(b*x + a)/(a^2*sqrt(x))
```

Fricas [A]
time = 0.86, size = 34, normalized size = 0.87

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

Sympy [A]
time = 0.79, size = 41, normalized size = 1.05

$$-\frac{2}{a\sqrt{b}\,x\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(31) = 62.
time = 1.63, size = 82, normalized size = 2.10

$$-\frac{4\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a|b|}-\frac{2\,\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(5/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a*a
bs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^2*abs(b))
```

Mupad [B]
time = 0.39, size = 39, normalized size = 1.00

$$-\frac{2\,a\,\sqrt{a+b\,x}+4\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(a^3+b\,x\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a + b*x)^(1/2) + 4*b*x*(a + b*x)^(1/2))/(x^(1/2)*(a^3 + a^2*b*x))
```

3.582 $\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2}{ax^{3/2}\sqrt{a+bx}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/a/x^(3/2)/(b*x+a)^(1/2)-8/3*(b*x+a)^(1/2)/a^2/x^(3/2)+16/3*b*(b*x+a)^(1/2
)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a + b*x]) - (8*Sqrt[a + b*x])/(3*a^2*x^(3/2)) + (16*b*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a+bx}} + \frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} - \frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{ax^{3/2}\sqrt{a+bx}} - \frac{8\sqrt{a+bx}}{3a^2x^{3/2}} + \frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}$$

Mathematica [A]
time = 0.08, size = 38, normalized size = 0.60

$$-\frac{2(a^2-4abx-8b^2x^2)}{3a^3x^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.87

method	result	size
gosper	$-\frac{2(-8x^2b^2-4abx+a^2)}{3x^{\frac{3}{2}}\sqrt{bx+a}\,a^3}$	33
risch	$-\frac{2\sqrt{bx+a}\,(-5bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{2b^2\sqrt{x}}{a^3\sqrt{bx+a}}$	41
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4b\left(-\frac{2}{a\sqrt{x}\,\sqrt{bx+a}}-\frac{4b\sqrt{x}}{a^2\sqrt{bx+a}}\right)}{3a}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(b*x+a)^(1/2)-4/3*b/a*(-2/a/x^(1/2)/(b*x+a)^(1/2)-4*b/a^2*x^
(1/2)/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 50, normalized size = 0.79

$$\frac{2\,b^2\sqrt{x}}{\sqrt{bx+a}\,a^3}+\frac{2\left(\frac{6\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) + 2/3*(6*sqrt(b*x + a)*b/sqrt(x) - (b*x +
 a)^(3/2)/x^(3/2))/a^3
```

Fricas [A]
time = 0.73, size = 49, normalized size = 0.78

$$\frac{2\,(8\,b^2x^2+4\,abx-a^2)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^3bx^3+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(58) = 116$.
time = 2.18, size = 219, normalized size = 3.48

$$-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a*
*3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**
4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*
a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x**3*sqrt
(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.
time = 1.35, size = 98, normalized size = 1.56

$$\frac{2\,\sqrt{bx+a}\,\left(\frac{5\,(bx+a)b^2|b|}{a^3}-\frac{6\,b^2|b|}{a^2}\right)}{3\,((bx+a)b-ab)^{\frac{3}{2}}}+\frac{4\,b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(5*(b*x + a)*b^2*abs(b)/a^3 - 6*b^2*abs(b)/a^2)/((b*x + a
)*b - a*b)^(3/2) + 4*b^(7/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b -
a*b))^2 + a*b)*a^2*abs(b))
```

Mupad [B]
time = 0.41, size = 46, normalized size = 0.73

$$\frac{\sqrt{a+b\,x}\,\left(\frac{8\,x}{3\,a^2}-\frac{2}{3\,a\,b}+\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(3/2)),x)
```

```
[Out] ((a + b*x)^(1/2)*((8*x)/(3*a^2) - 2/(3*a*b) + (16*b*x^2)/(3*a^3)))/(x^(5/2)
 + (a*x^(3/2))/b)
```

3.583 $\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}$$

```
[Out] 2/a/x^(5/2)/(b*x+a)^(1/2)-12/5*(b*x+a)^(1/2)/a^2/x^(5/2)+16/5*b*(b*x+a)^(1/
2)/a^3/x^(3/2)-32/5*b^2*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(5/2)*Sqrt[a + b*x]) - (12*Sqrt[a + b*x])/(5*a^2*x^(5/2)) + (16*b*Sq
rt[a + b*x])/(5*a^3*x^(3/2)) - (32*b^2*Sqrt[a + b*x])/(5*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx = \frac{2}{ax^{5/2}\sqrt{a+bx}} + \frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} - \frac{(24b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a^2}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} + \frac{(16b^2)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{5a^3}$$
$$= \frac{2}{ax^{5/2}\sqrt{a+bx}} - \frac{12\sqrt{a+bx}}{5a^2x^{5/2}} + \frac{16b\sqrt{a+bx}}{5a^3x^{3/2}} - \frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.56

$$-\frac{2(a^3-2a^2bx+8ab^2x^2+16b^3x^3)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

Maple [A]
time = 0.13, size = 77, normalized size = 0.89

method	result	size
gosper	$-\frac{2(16b^3x^3+8a\,b^2x^2-2a^2bx+a^3)}{5x^{\frac{5}{2}}\sqrt{bx+a}\;a^4}$	44
risch	$-\frac{2\sqrt{bx+a}\;(11x^2b^2-3abx+a^2)}{5a^4x^{\frac{5}{2}}}-\frac{2b^3\sqrt{x}}{a^4\sqrt{bx+a}}$	52
default	$-\frac{2}{5a\,x^{\frac{5}{2}}\sqrt{bx+a}}-\frac{6b\left(-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4b\left(-\frac{2}{a\sqrt{x}\;\sqrt{bx+a}}-\frac{4b\sqrt{x}}{a^2\sqrt{bx+a}}\right)}{3a}\right)}{5a}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/5/a/x^(5/2)/(b*x+a)^(1/2)-6/5*b/a*(-2/3/a/x^(3/2)/(b*x+a)^(1/2)-4/3*b/a*
(-2/a/x^(1/2)/(b*x+a)^(1/2)-4*b/a^2*x^(1/2)/(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.74

$$-\frac{2\,b^3\sqrt{x}}{\sqrt{bx+a}\,a^4}-\frac{2\left(\frac{15\,\sqrt{bx+a}\,b^2}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) - 2/5*(15*sqrt(b*x + a)*b^2/sqrt(x) - 5*
(b*x + a)^(3/2)*b/x^(3/2) + (b*x + a)^(5/2)/x^(5/2))/a^4
```

Fricas [A]
time = 0.61, size = 58, normalized size = 0.67

$$-\frac{2\,(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3)\sqrt{bx+a}\,\sqrt{x}}{5\,(a^4bx^4+a^5x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(82) = 164$.
time = 6.76, size = 348, normalized size = 4.00

$$-\frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{60a^2b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{80ab^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{32b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3
+ 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqrt(a/(
b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a
**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**
2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 80*a*b**
(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a
**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(
5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x
**5)
```

Giac [A]
time = 1.48, size = 121, normalized size = 1.39

$$-\frac{4\,b^{\frac{9}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)a^{3}|b|}-\frac{2\left(\frac{15\,b^{6}}{a^{2}|b|}+(bx+a)\left(\frac{11\,(bx+a)b^{6}}{a^{4}|b|}-\frac{25\,b^{6}}{a^{3}|b|}\right)\right)\sqrt{bx+a}}{5\,((bx+a)b-ab)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(9/2)/((((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a^3
*abs(b)) - 2/5*(15*b^6/(a^2*abs(b)) + (b*x + a)*(11*(b*x + a)*b^6/(a^4*abs(
b)) - 25*b^6/(a^3*abs(b))))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(5/2)
```

Mupad [B]
time = 0.43, size = 58, normalized size = 0.67

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a\,b}-\frac{4\,x}{5\,a^{2}}+\frac{16\,b\,x^{2}}{5\,a^{3}}+\frac{32\,b^{2}\,x^{3}}{5\,a^{4}}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(3/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(5*a*b) - (4*x)/(5*a^2) + (16*b*x^2)/(5*a^3) + (32*b^2
*x^3)/(5*a^4)))/(x^(7/2) + (a*x^(5/2))/b)
```

3.584 $\int \frac{x^{5/2}}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=91

$$-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}$$

```
[Out] -2/3*x^(5/2)/b/(b*x+a)^(3/2)-5*a*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(
7/2)-10/3*x^(3/2)/b^2/(b*x+a)^(1/2)+5*x^(1/2)*(b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{a+bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a + b*x]) + (
5*Sqrt[x]*Sqrt[a + b*x])/b^3 - (5*a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b^2}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.14, size = 72, normalized size = 0.79

$$\frac{\sqrt{x}\,(15a^2+20abx+3b^2x^2)}{3b^3(a+bx)^{3/2}}+\frac{5a\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*(15*a^2 + 20*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2)) + (5*a*Lo
g[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(67) = 134.
time = 0.14, size = 147, normalized size = 1.62

method	result
risch	$\frac{\sqrt{x}\,\sqrt{bx+a}}{b^3}+\frac{\left(-\frac{5a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{x^2b+ax}\right)}{2b^{\frac{7}{2}}}-\frac{2a^2\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x+\frac{a}{b}\right)}}{3b^5\left(x+\frac{a}{b}\right)^2}+\frac{14a\sqrt{\left(x+\frac{a}{b}\right)^2b-a\left(x\right.}}{3b^4\left(x+\frac{a}{b}\right)}\right.}{\sqrt{x}\,\sqrt{bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^3+(-5/2/b^(7/2)*a*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x
)^(1/2))-2/3/b^5*a^2/(x+a/b)^2*((x+a/b)^2*b-a*(x+a/b))^(1/2)+14/3/b^4*a/(x+
a/b)*((x+a/b)^2*b-a*(x+a/b))^(1/2))*(x*(b*x+a))^(1/2)/x^(1/2)/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.50, size = 109, normalized size = 1.20

$$\frac{2\,ab^2+\frac{10\,(bx+a)ab}{x}-\frac{15\,(bx+a)^2a}{x^2}}{3\left(\frac{(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{5\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x + a)*a*b/x - 15*(b*x + a)^2*a/x^2)/((b*x + a)^(3/2)*
b^4/x^(3/2) - (b*x + a)^(5/2)*b^3/x^(5/2)) + 5/2*a*log(-(sqrt(b) - sqrt(b*x
 + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.64, size = 214, normalized size = 2.35

$$\left[\frac{15\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,(3\,b^3x^2+20\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{6\,(b^6x^2+2\,ab^5x+a^2b^4)},\ \frac{15\,(ab^2x^2+2\,a^2bx+a^3)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(3\,b^3x^2+20\,ab^2x+15\,a^2b)\sqrt{bx+a}\,\sqrt{x}}{3\,(b^6x^2+2\,ab^5x+a^2b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(85) = 170$.
time = 4.79, size = 396, normalized size = 4.35

$$-\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a)
)/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*
x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 + b*x/a)*a
sinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x
/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 15*a**40*b**(45/2)
*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(5
3/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**
(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 +
 b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1
+ b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(67) = 134$.
time = 21.51, size = 197, normalized size = 2.16

$$\frac{\left(\frac{15\,a\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{b^{\frac{5}{2}}}+\frac{6\,\sqrt{(bx+a)b-ab}\,\sqrt{bx+a}}{b^3}+\frac{8\left(9\,a^2\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4\sqrt{b}+12\,a^3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{3}{2}}+7\,a^4b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3b^2}\right)|b|}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
+ 6*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b^3 + 8*(9*a^2*(sqrt(b*x + a)*sqr
t(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 12*a^3*(sqrt(b*x + a)*sqrt(b) -
 sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 7*a^4*b^(5/2))/(((sqrt(b*x + a)*sqrt(
b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*b^2))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(5/2), x)
```

3.585 $\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

```
[Out] -2/3*x^(3/2)/b/(b*x+a)^(3/2)+2*arctanh(b^(1/2)*x^(1/2)/(b*x+a)^(1/2))/b^(5/
2)-2*x^(1/2)/b^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a + b*x]) + (2*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}+\frac{\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{b^2}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 60, normalized size = 0.87

$$-\frac{2\sqrt{x}\,(3a+4bx)}{3b^2(a+bx)^{3/2}}-\frac{2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[x]*(3*a + 4*b*x))/(3*b^2*(a + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]])/b^(5/2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x+a)^(5/2),x)
```

Maxima [A]
time = 0.49, size = 69, normalized size = 1.00

$$-\frac{2\left(b+\frac{3(bx+a)}{x}\right)x^{\frac{3}{2}}}{3(bx+a)^{\frac{3}{2}}b^2}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.61, size = 186, normalized size = 2.70

$$\left[\frac{3\left(b^2x^2+2abx+a^2\right)\sqrt{b}\log\left(2bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\left(4b^2x+3ab\right)\sqrt{bx+a}\sqrt{x}}{3\left(b^5x^2+2ab^4x+a^2b^3\right)},-\frac{2\left(3\left(b^2x^2+2abx+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(4b^2x+3ab\right)\sqrt{bx+a}\sqrt{x}\right)}{3\left(b^5x^2+2ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(63) = 126.
time = 2.04, size = 328, normalized size = 4.75

$$\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] 6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/
(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x*
```

```
*(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a)*asin
h(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a)
 + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(23/2)*x**
14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)
*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2
)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/
a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(51) = 102$.

time = 21.86, size = 165, normalized size = 2.39

$$-\frac{\left(\frac{3 \log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}+\frac{8\left(3a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4\sqrt{b}+3a^2\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{3}{2}}+2a^3b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3}\right)|b|}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b) +
8*(3*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 3*a^2*
(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 2*a^3*b^(5/2)
)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3)*abs(b)/b^3
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(5/2), x)
```

3.586 $\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

```
[Out] 2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.12, size = 54, normalized size = 2.57

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3a(bx+a)^{\frac{3}{2}}}$	16
default	$-\frac{\sqrt{x}}{b(bx+a)^{\frac{3}{2}}}+\frac{a\left(\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{bx+a}}\right)}{2b}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*x^(1/2)/(b*x+a)^(3/2)+1/2*a/b*(2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)
/a^2/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.71

$$\frac{2\,x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.81, size = 33, normalized size = 1.57

$$\frac{2\,\sqrt{bx+a}\,x^{\frac{3}{2}}}{3\,(ab^2x^2+2\,a^2bx+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.
time = 0.70, size = 42, normalized size = 2.00

$$\frac{2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] 2*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(15) = 30.
time = 1.75, size = 86, normalized size = 4.10

$$\frac{4\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+a^{2}b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + a^2*b^
(5/2))*abs(b)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^
3*b^2)
```

Mupad [B]
time = 0.24, size = 36, normalized size = 1.71

$$\frac{2\,x^{3/2}\,\sqrt{a+b\,x}}{3\,\left(a^3+2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*x^(3/2)*(a + b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 + 2*a^2*b*x))
```

3.587 $\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}$$

```
[Out] 2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.74

method	result	size
gosper	$\frac{2\sqrt{x}\,(2bx+3a)}{3(bx+a)^{\frac{3}{2}}a^2}$	24
default	$\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{bx+a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.63

$$-\frac{2\left(b-\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
[Out] -2/3*(b - 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*a^2)
```

Fricas [A]
time = 0.77, size = 43, normalized size = 1.00

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^2b^2x^2+2\,a^3bx+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(37) = 74$.
time = 0.97, size = 92, normalized size = 2.14

$$\frac{6a}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}+\frac{4bx}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)
) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x
) + 1))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(31) = 62$.
time = 1.67, size = 81, normalized size = 1.88

$$\frac{8\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(5/2)/(
((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*abs(b))
```

Mupad [B]
time = 0.40, size = 54, normalized size = 1.26

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a+b\,x}+4\,b\,x^{3/2}\,\sqrt{a+b\,x}}{3\,a^4+6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a + b*x)^(1/2) + 4*b*x^(3/2)*(a + b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 + 6*a^3*b*x)
```

3.588 $\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=64

$$\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/3/a/(b*x+a)^(3/2)/x^(1/2)+8/3/a^2/x^(1/2)/(b*x+a)^(1/2)-16/3*(b*x+a)^(1/2
)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a + b*x]) - (16*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx = \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}} + \frac{8\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}$$
$$= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 40, normalized size = 0.62

$$-\frac{2(3a^2+12abx+8b^2x^2)}{3a^3\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

Maple [A]
time = 0.14, size = 54, normalized size = 0.84

method	result	size
gosper	$-\frac{2(8x^2b^2+12abx+3a^2)}{3\sqrt{x}\,(bx+a)^{\frac{3}{2}}a^3}$	35
risch	$-\frac{2\sqrt{bx+a}}{a^3\sqrt{x}} - \frac{2b(5bx+6a)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}a^3}$	41
default	$-\frac{2}{a(bx+a)^{\frac{3}{2}}\sqrt{x}} - \frac{4b\left(\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{bx+a}}\right)}{a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/(b*x+a)^(3/2)/x^(1/2)-4*b/a*(2/3*x^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a
^2/(b*x+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.72

$$\frac{2\left(b^2-\frac{6\,(bx+a)b}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^3} - \frac{2\,\sqrt{bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x + a)*b/x)*x^(3/2)/((b*x + a)^(3/2)*a^3) - 2*sqrt(b*x + a)
/(a^3*sqrt(x))
```

Fricas [A]
time = 0.91, size = 58, normalized size = 0.91

$$-\frac{2\,(8\,b^2x^2+12\,abx+3\,a^2)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^3b^2x^3+2\,a^4bx^2+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(58) = 116.
time = 2.12, size = 153, normalized size = 2.39

$$-\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b*
*6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x + 3*a**3*b**6*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(46) = 92.
time = 1.81, size = 159, normalized size = 2.48

$$-\frac{2\,\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a^3|b|}-\frac{4\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{5}{2}}+12\,a\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{7}{2}}+5\,a^2b^{\frac{9}{2}}\right)}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(b*
x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(5/2) + 12*a*(sqrt(b*x + a)*s
```

```
qrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) + 5*a^2*b^(9/2))/(((sqrt(b*x +
a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*a^2*abs(b))
```

Mupad [B]
time = 0.42, size = 71, normalized size = 1.11

$$-\frac{6\,a^2\,\sqrt{a+b\,x}+16\,b^2\,x^2\,\sqrt{a+b\,x}+24\,a\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b+3\,x\,a^3\,b^2\right)+3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(5/2)),x)
[Out] -(6*a^2*(a + b*x)^(1/2) + 16*b^2*x^2*(a + b*x)^(1/2) + 24*a*b*x*(a + b*x)^(
1/2))/(x^(1/2)*(x*(6*a^4*b + 3*a^3*b^2*x) + 3*a^5))
```

3.589 $\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}$$

```
[Out] 2/3/a/x^(3/2)/(b*x+a)^(3/2)+4/a^2/x^(3/2)/(b*x+a)^(1/2)-16/3*(b*x+a)^(1/2)/
a^3/x^(3/2)+32/3*b*(b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a + b*x]) - (16*Sqrt[
a + b*x])/(3*a^3*x^(3/2)) + (32*b*Sqrt[a + b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx = \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3x^{3/2}} - \frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^3}$$
$$= \frac{2}{3ax^{3/2}(a+bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a+bx}} - \frac{16\sqrt{a+bx}}{3a^3x^{3/2}} + \frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.58

$$-\frac{2(a^3-6a^2bx-24ab^2x^2-16b^3x^3)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 - 6*a^2*b*x - 24*a*b^2*x^2 - 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

Maple [A]
time = 0.13, size = 76, normalized size = 0.90

method	result	size
gosper	$-\frac{2(-16b^3x^3-24a\,b^2x^2-6a^2bx+a^3)}{3x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}a^4}$	44
risch	$-\frac{2\sqrt{bx+a}\,(-8bx+a)}{3a^4x^{\frac{3}{2}}}+\frac{2b^2(8bx+9a)\sqrt{x}}{3(bx+a)^{\frac{3}{2}}a^4}$	49
default	$-\frac{2}{3a\,x^{\frac{3}{2}}(bx+a)^{\frac{3}{2}}}-\frac{2b\left(-\frac{2}{a(bx+a)^{\frac{3}{2}}\sqrt{x}}-\frac{4b\left(\frac{2\sqrt{x}}{3a(bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{bx+a}}\right)}{a}\right)}{a}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(b*x+a)^(3/2)-2*b/a*(-2/a/(b*x+a)^(3/2)/x^(1/2)-4*b/a*(2/3*x
^(1/2)/a/(b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.76

$$\frac{2\left(\frac{9\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}-\frac{2\left(b^3-\frac{9\,(bx+a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(9*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^4 - 2/3*(b^3 -
9*(b*x + a)*b^2/x)*x^(3/2)/((b*x + a)^(3/2)*a^4)
```

Fricas [A]
time = 0.86, size = 71, normalized size = 0.85

$$\frac{2\,(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3)\sqrt{bx+a}\,\sqrt{x}}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(78) = 156.
time = 4.03, size = 337, normalized size = 4.01

$$-\frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*
a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x) + 1
)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**
4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10
*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**3*sqrt(a
/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4
*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*
b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(62) = 124.

time = 1.26, size = 175, normalized size = 2.08

$$\frac{2\sqrt{bx+a}\left(\frac{8(bx+a)b^2|b|}{a^4}-\frac{9b^2|b|}{a^3}\right)}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}}+\frac{8\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4 b^{\frac{7}{2}}+9a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2 b^{\frac{9}{2}}+4a^2b^{\frac{11}{2}}\right)}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3 a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b*x + a)*b^2*abs(b)/a^4 - 9*b^2*abs(b)/a^3)/((b*x + a
)*b - a*b)^(3/2) + 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))
^4*b^(7/2) + 9*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(9/2
) + 4*a^2*b^(11/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 +
 a*b)^3*a^3*abs(b))
```

Mupad [B]
time = 0.47, size = 88, normalized size = 1.05

$$\frac{32\,b^3\,x^3\,\sqrt{a+b\,x}-2\,a^3\,\sqrt{a+b\,x}+12\,a^2\,b\,x\,\sqrt{a+b\,x}+48\,a\,b^2\,x^2\,\sqrt{a+b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b+3\,x\,a^4\,b^2\right)+3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (32*b^3*x^3*(a + b*x)^(1/2) - 2*a^3*(a + b*x)^(1/2) + 12*a^2*b*x*(a + b*x)^
(1/2) + 48*a*b^2*x^2*(a + b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b + 3*a^4*b^2*x) +
 3*a^6))
```

3.590 $\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=105

$$-\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}$$

```
[Out] 5/8*a^3*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)-5/12*a*x^(3/2)*(-b*x
+a)^(1/2)/b^2-1/3*x^(5/2)*(-b*x+a)^(1/2)/b-5/8*a^2*x^(1/2)*(-b*x+a)^(1/2)/b
^3
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{5a^3\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}-\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (-5*a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a - b*x])/(12*b^
2) - (x^(5/2)*Sqrt[a - b*x])/(3*b) + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
 - b*x]])/(8*b^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx = -\frac{x^{5/2}\sqrt{a-bx}}{3b} + \frac{(5a)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{6b}$$

$$= -\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b} + \frac{(5a^2)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b^2}$$

$$= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b} + \frac{(5a^3)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{16b^3}$$

$$= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b} + \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x\right.}{8b^3}$$

$$= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b} + \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{}}{\sqrt{a}}\right.}{8b^3}$$

$$= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b} + \frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.78

$$-\frac{\sqrt{x}\,\sqrt{a-bx}\,(15a^2+10abx+8b^2x^2)}{24b^3} + \frac{5a^3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{8(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] -1/24*(Sqrt[x]*Sqrt[a - b*x]*(15*a^2 + 10*a*b*x + 8*b^2*x^2))/b^3 + (5*a^3*
Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*(-b)^(7/2))
```

Maple [A]
time = 0.12, size = 116, normalized size = 1.10

method	result
risch	$-\frac{(8x^2b^2+10abx+15a^2)\sqrt{x}\,\sqrt{-bx+a}}{24b^3}+\frac{5a^3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)\sqrt{x\,(-bx+a)}}{16b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+a}}$
default	$-\frac{x^{\frac{5}{2}}\sqrt{-bx+a}}{3b}+\frac{5a\left(-\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2b}+\frac{3a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right.}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right.}{4b}\right.}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^(5/2)*(-b*x+a)^(1/2)/b+5/6*a/b*(-1/2*x^(3/2)*(-b*x+a)^(1/2)/b+3/4*a/
b*(-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x
+a)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))))
```

Maxima [A]
time = 0.48, size = 135, normalized size = 1.29

$$-\frac{5\,a^3\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{-bx+a}\,a^3b^2}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}a^3b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+a)^{\frac{5}{2}}a^3}{x^{\frac{5}{2}}}}{24\left(b^6-\frac{3\,(bx-a)b^5}{x}+\frac{3\,(bx-a)^2b^4}{x^2}-\frac{(bx-a)^3b^3}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) - 1/24*(33*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 15*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x - a)*b^5/x + 3*(b*x - a)^2*b^4/x^2 - (b*
x - a)^3*b^3/x^3)
```

Fricas [A]
time = 0.90, size = 141, normalized size = 1.34

$$\left[-\frac{15\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(8\,b^3x^2+10\,ab^2x+15\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^4},\,-\frac{15\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(8\,b^3x^2+10\,ab^2x+15\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(8*b^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/24
*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 + 10
*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 9.08, size = 270, normalized size = 2.57

$$\begin{cases} \frac{5ia^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}} + \frac{5a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(5/2)*sqrt(x)/(8*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(3/2)*x**
*(3/2)/(24*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(5/2)/(12*b*sqrt(-1 + b*x/
a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) - I*x**(7/2)/(3*
sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(5/2)*sqrt(x)/(8*b**3*sq
rt(1 - b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**
*(5/2)/(12*b*sqrt(1 - b*x/a)) + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**
(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(1/2),x)
[Out] int(x^(5/2)/(a - b*x)^(1/2), x)
```

3.591 $\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=80

$$-\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)-1/2*x^(3/2)*(-b*x+a)
^(1/2)/b-3/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{3a^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^2) - (x^(3/2)*Sqrt[a - b*x])/(2*b) + (3*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b}\\ &= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a^2)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{8b^2}\\ &= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^2}\\ &= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^2}\\ &= -\frac{3a\sqrt{x}\,\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.89

$$-\frac{\sqrt{x}\,\sqrt{a-bx}\,(3a+2bx)}{4b^2}-\frac{3a^2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{4(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] -1/4*(Sqrt[x]*Sqrt[a - b*x]*(3*a + 2*b*x))/b^2 - (3*a^2*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[a - b*x]])/(4*(-b)^(5/2))
```

Maple [A]
time = 0.13, size = 93, normalized size = 1.16

method	result	size

risch	$-\frac{(2bx+3a)\sqrt{x}\,\sqrt{-bx+a}}{4b^{2}}+\frac{3a^{2}\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)\sqrt{x\,(-bx+a)}}{8b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	80
default	$-\frac{x^{\frac{3}{2}}\sqrt{-bx+a}}{2b}+\frac{3a\left(-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}\right)}{4b}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)*(-b*x+a)^(1/2)/b+3/4*a/b*(-x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3
/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-
b*x^2+a*x)^(1/2)))
```

Maxima [A]
time = 0.49, size = 98, normalized size = 1.22

$$-\frac{3\,a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}-\frac{\frac{5\,\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}+\frac{3\,(-bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{4}-\frac{2\,(bx-a)b^{3}}{x}+\frac{(bx-a)^{2}b^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) - 1/4*(5*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 3*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^4 - 2*(b*x - a)*b
^3/x + (b*x - a)^2*b^2/x^2)
```

Fricas [A]
time = 0.72, size = 119, normalized size = 1.49

$$\left[-\frac{3\,a^{2}\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(2\,b^{2}x+3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{8\,b^{3}},-\frac{3\,a^{2}\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^{2}x+3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{4\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(b)*arcta
n(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt
(x))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.81, size = 214, normalized size = 2.68

$$\begin{cases} \frac{3ia^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{-1+\frac{bx}{a}}} - \frac{i\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{-1+\frac{bx}{a}}} - \frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^2\sqrt{1-\frac{bx}{a}}} + \frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1-\frac{bx}{a}}} + \frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((3*I*a**(3/2)*sqrt(x)/(4*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(3
/2)/(4*b*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**
(5/2)) - I*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-3*a**(
3/2)*sqrt(x)/(4*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**(3/2)/(4*b*sqrt(1 - b*x/
a)) + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/(2*sqrt(
a)*sqrt(1 - b*x/a)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(1/2), x)
```

3.592 $\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=50

$$-\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

```
[Out] a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)-x^(1/2)*(-b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {52, 65, 223, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/
b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b}\\ &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 1.12

$$-\frac{\sqrt{x}\,\sqrt{a-bx}}{b}+\frac{a\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a - b*x],x]
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])
/(-b)^(3/2)
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.40

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\,(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	70

risch	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b}+\frac{a\sqrt{x\left(-bx+a\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2b^{\frac{3}{2}}\sqrt{x}\,\sqrt{-bx+a}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)
^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))
```

Maxima [A]
time = 0.49, size = 56, normalized size = 1.12

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{\sqrt{-bx+a}\,a}{\left(b^{2}-\frac{(bx-a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) - sqrt(-b*x + a)*a/((b^
2 - (b*x - a)*b/x)*sqrt(x))
```

Fricas [A]
time = 0.69, size = 93, normalized size = 1.86

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b^{2}},\,-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\sqrt{-bx+a}\,b\sqrt{x}}{b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*s
qrt(-b*x + a)*b*sqrt(x))/b^2, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sq
rt(x))) + sqrt(-b*x + a)*b*sqrt(x))/b^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.23, size = 121, normalized size = 2.42

$$\begin{cases} \frac{i\sqrt{a}\,\sqrt{x}}{b\sqrt{-1+\frac{bx}{a}}}-\frac{ia\,\mathrm{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{ix^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{\sqrt{a}\,\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{b}+\frac{a\,\mathrm{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(a)*sqrt(x)/(b*sqrt(-1 + b*x/a)) - I*a*acosh(sqrt(b)*sqrt(
x)/sqrt(a))/b**(3/2) - I*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (-sqrt(a)*sqrt(x)*sqrt(1 - b*x/a)/b + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/b
**(3/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[
```

Mupad [B]
time = 0.52, size = 47, normalized size = 0.94

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{a-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(1/2),x)
```

```
[Out] (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^(1/2))))/b^(3/2) - (x^(1/2
)*(a - b*x)^(1/2))/b
```

3.593 $\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {65, 223, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 1.21

$$-\frac{2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{a-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]
[Out] (-2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/Sqrt[-b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(21) = 42$.
time = 0.11, size = 51, normalized size = 1.76

method	result	size
default	$\frac{\sqrt{x(-bx+a)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{\sqrt{x}\,\sqrt{-bx+a}\,\sqrt{b}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2*a/b
)/(-b*x^2+a*x)^(1/2))
```

Maxima [A]
time = 0.50, size = 21, normalized size = 0.72

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Fricas [A]
time = 1.18, size = 57, normalized size = 1.97

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)}{b}, -\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)/b, -2*arctan
(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 54, normalized size = 1.86

$$\begin{cases} -\frac{2i\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{2\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), Abs(b*x/a) > 1), (2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{4,[0,1,1]%%%}+%%%{4,[0,1,0
```

]%%%}+%%%{-4,[0,0,1]%%%},0,%%%{6,[2,0,0]%%%}+%%%{-12,[1,1,1]%%%}+%%%{-4,[1,
1,0]%%%}+%%%{4,[

Mupad [B]
time = 0.03, size = 27, normalized size = 0.93

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a-b\,x}-\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a - b*x)^(1/2) - a^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

3.594 $\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

```
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Maple [A]
time = 0.13, size = 17, normalized size = 0.85

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
default	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17
risch	$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

Fricas [A]
time = 1.07, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 46, normalized size = 2.30

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/a, Abs(a/(b*x)) > 1), (-2*I*sqrt(b)
*sqrt(-a/(b*x) + 1)/a, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.
time = 1.34, size = 35, normalized size = 1.75

$$-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a*abs(b))
```

Mupad [B]
time = 0.40, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{a-b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -(2*(a - b*x)^(1/2))/(a*x^(1/2))
```

3.595 $\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{a-bx}}{3ax^{3/2}}-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}$$

```
[Out] -2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}+\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}\,(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.76

method	result	size
gosper	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^2}$	23
risch	$-\frac{2\sqrt{-bx+a}\,(2bx+a)}{3x^{\frac{3}{2}}a^2}$	23
default	$-\frac{2\sqrt{-bx+a}}{3a\,x^{\frac{3}{2}}}-\frac{4b\sqrt{-bx+a}}{3a^2\sqrt{x}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(-b*x+a)^(1/2)/a/x^(3/2)-4/3*b*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Maxima [A]
time = 0.29, size = 32, normalized size = 0.70

$$-\frac{2\left(\frac{3\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2/3*(3*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^2
```

Fricas [A]
time = 0.84, size = 22, normalized size = 0.48

$$-\frac{2\,(2\,bx+a)\sqrt{-bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x + a)*sqrt(-b*x + a)/(a^2*x^(3/2))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.05, size = 177, normalized size = 3.85

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3ax} - \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a^2} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} + \frac{2iab^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} - \frac{4ib^{\frac{7}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*a*x) - 4*b**(3/2)*sqrt(a/(b*x) -
 1)/(3*a**2), Abs(a/(b*x)) > 1), (2*I*a**2*b**(3/2)*sqrt(-a/(b*x) + 1)/(-3*
a**3*b*x + 3*a**2*b**2*x**2) + 2*I*a*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-3*a**3
*b*x + 3*a**2*b**2*x**2) - 4*I*b**(7/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**3*b*
x + 3*a**2*b**2*x**2), True))
```

Giac [A]
time = 1.19, size = 54, normalized size = 1.17

$$-\frac{2\left(\frac{2(bx-a)b^3}{a^2}+\frac{3b^3}{a}\right)\sqrt{-bx+a}\,b}{3\left((bx-a)b+ab\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*(b*x - a)*b^3/a^2 + 3*b^3/a)*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^
(3/2)*abs(b))
```

Mupad [B]
time = 0.35, size = 26, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}+\frac{4\,b\,x}{3\,a^2}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) + (4*b*x)/(3*a^2))*(a - b*x)^(1/2))/x^(3/2)
```

3.596 $\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}$$

```
[Out] -15/4*a^2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(7/2)+2*x^(5/2)/b/(-b*x+
a)^(1/2)+5/2*x^(3/2)*(-b*x+a)^(1/2)/b^2+15/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {49, 52, 65, 223, 209}

$$-\frac{15a^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[a - b*x]) + (15*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^3) + (5*x
^(3/2)*Sqrt[a - b*x])/(2*b^2) - (15*a^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]])/(4*b^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx = \frac{2x^{5/2}}{b\sqrt{a-bx}} - \frac{5\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{b}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b^2}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a^2)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{8b^3}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \right.}{4b^3}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{(15a^2)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-}}\right.}{4b^3}$$

$$= \frac{2x^{5/2}}{b\sqrt{a-bx}} + \frac{15a\sqrt{x}\,\sqrt{a-bx}}{4b^3} + \frac{5x^{3/2}\sqrt{a-bx}}{2b^2} - \frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}$$

Mathematica [A]

time = 0.16, size = 81, normalized size = 0.81

$$\frac{1}{4}\left(\frac{\sqrt{x}\,(15a^2-5abx-2b^2x^2)}{b^3\sqrt{a-bx}}-\frac{15a^2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] ((Sqrt[x]*(15*a^2 - 5*a*b*x - 2*b^2*x^2))/(b^3*Sqrt[a - b*x]) - (15*a^2*Log
[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2))/4
```

Maple [A]
time = 0.13, size = 127, normalized size = 1.27

method	result
risch	$\frac{(2bx+7a)\sqrt{x}\,\sqrt{-bx+a}}{4b^3}+\frac{\left(-\frac{15a^2\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{8b^{\frac{7}{2}}}-\frac{2a^2\sqrt{-\left(-\frac{a}{b}+x\right)^2b-a\left(-\frac{a}{b}+x\right)}}{b^4\left(-\frac{a}{b}+x\right)}\right)\sqrt{x\,(-}}{\sqrt{x}\,\sqrt{-bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*b*x+7*a)/b^3*x^(1/2)*(-b*x+a)^(1/2)+(-15/8/b^(7/2)*a^2*arctan(b^(1/2
)*(x-1/2*a/b)/(-b*x^2+a*x)^(1/2))-2/b^4*a^2/(-a/b+x)*(-(-a/b+x)^2*b-a*(-a/b
+x))^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.52, size = 118, normalized size = 1.18

$$\frac{8\,a^2b^2-\frac{25\,(bx-a)a^2b}{x}+\frac{15\,(bx-a)^2a^2}{x^2}}{4\left(\frac{\sqrt{-bx+a}\,b^5}{\sqrt{x}}+\frac{2\,(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{15\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*b^2 - 25*(b*x - a)*a^2*b/x + 15*(b*x - a)^2*a^2/x^2)/(sqrt(-b*x
+ a)*b^5/sqrt(x) + 2*(-b*x + a)^(3/2)*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^
(5/2)) + 15/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 1.29, size = 181, normalized size = 1.81

$$\left[-\frac{15\,(a^2bx-a^3)\sqrt{-b}\,\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(2\,b^3x^2+5\,ab^2x-15\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{8\,(b^5x-ab^4)},\ \frac{15\,(a^2bx-a^3)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(2\,b^3x^2+5\,ab^2x-15\,a^2b)\sqrt{-bx+a}\,\sqrt{x}}{4\,(b^5x-ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(a^2*b*x - a^3)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*s
qrt(x) + a) - 2*(2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/
(b^5*x - a*b^4), 1/4*(15*(a^2*b*x - a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqr
t(b)*sqrt(x))) + (2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))
/(b^5*x - a*b^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 5.68, size = 224, normalized size = 2.24

$$\begin{cases} -\frac{15ia^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{-1+\frac{bx}{a}}}+\frac{5i\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{-1+\frac{bx}{a}}}+\frac{15ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1-\frac{bx}{a}}}-\frac{15a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}-\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-15*I*a**(3/2)*sqrt(x)/(4*b**3*sqrt(-1 + b*x/a)) + 5*I*sqrt(a)*x
**(3/2)/(4*b**2*sqrt(-1 + b*x/a)) + 15*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a)
)/(4*b**(7/2)) + I*x**(5/2)/(2*sqrt(a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 - b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**
2*sqrt(1 - b*x/a)) - 15*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) - x
**(5/2)/(2*sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(74) = 148.
time = 20.88, size = 154, normalized size = 1.54

$$\frac{\left(2\,\sqrt{(bx-a)b+ab}\,\sqrt{-bx+a}\,\left(\frac{2\,(bx-a)}{b^3}+\frac{9\,a}{b^3}\right)+\frac{32\,a^3}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}\,b}-\frac{15\,a^2\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}\,b^2}\right)|b|}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b^3 + 9*a/b^3) +
 32*a^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt
(-b)*b) - 15*a^2*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)
/(sqrt(-b)*b^2))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(3/2), x)
```

3.597 $\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

```
[Out] -3*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/2)+2*x^(3/2)/b/(-b*x+a)^(1
/2)+3*x^(1/2)*(-b*x+a)^(1/2)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$-\frac{3a\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[a - b*x]) + (3*Sqrt[x]*Sqrt[a - b*x])/b^2 - (3*a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{a-bx}}-\frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b^2}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}-\frac{(3a)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}-\frac{(3a)\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\
&= \frac{2x^{3/2}}{b\sqrt{a-bx}}+\frac{3\sqrt{x}\,\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 64, normalized size = 0.90

$$
\frac{\sqrt{x}\,(3a-bx)}{b^2\sqrt{a-bx}}+\frac{3a\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(3*a - b*x))/(b^2*Sqrt[a - b*x]) + (3*a*Log[-(Sqrt[-b]*Sqrt[x]) +
Sqrt[a - b*x]])/(-b)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.
time = 0.13, size = 114, normalized size = 1.61

method	result
risch	$\frac{\sqrt{x}\,\sqrt{-bx+a}}{b^{2}}+\frac{\left(-\frac{3a\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^{2}b+ax}}\right)}{2b^{\frac{5}{2}}}-\frac{2a\sqrt{-\left(-\frac{a}{b}+x\right)^{2}b-a\left(-\frac{a}{b}+x\right)}}{b^{3}\left(-\frac{a}{b}+x\right)}\right)\sqrt{x\left(-bx+a\right)}}{\sqrt{x}\,\sqrt{-bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x
^2+a*x)^(1/2))-2*a/b^3/(-a/b+x)*(-(-a/b+x)^2*b-a*(-a/b+x))^(1/2))*(x*(-b*x+
a))^(1/2)/x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.50, size = 75, normalized size = 1.06

$$\frac{2\,ab-\frac{3\,(bx-a)a}{x}}{\frac{\sqrt{-bx+a}\,b^{3}}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{3\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x - a)*a/x)/(sqrt(-b*x + a)*b^3/sqrt(x) + (-b*x + a)^(3/2)*b^
2/x^(3/2)) + 3*a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 1.00, size = 152, normalized size = 2.14

$$\left[-\frac{3\,(abx-a^{2})\sqrt{-b}\,\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(b^{2}x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{2\,(b^{4}x-ab^{3})},\ \frac{3\,(abx-a^{2})\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+(b^{2}x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{b^{4}x-ab^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(a*b*x - a^2)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt
(x) + a) - 2*(b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3), (3*(a
*b*x - a^2)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (b^2*x - 3*a
*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.93, size = 155, normalized size = 2.18

$$\begin{cases} -\frac{3i\sqrt{a}\sqrt{x}}{b^2\sqrt{-1+\frac{bx}{a}}} + \frac{3ia\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} + \frac{ix^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{3\sqrt{a}\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}} - \frac{3a\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}} - \frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*sqrt(x)/(b**2*sqrt(-1 + b*x/a)) + 3*I*a*acosh(sqrt(
b)*sqrt(x)/sqrt(a))/b**(5/2) + I*x**(3/2)/(sqrt(a)*b*sqrt(-1 + b*x/a)), Abs
(b*x/a) > 1), (3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 - b*x/a)) - 3*a*asin(sqrt(b)*
sqrt(x)/sqrt(a))/b**(5/2) - x**(3/2)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(55) = 110.
time = 22.16, size = 130, normalized size = 1.83

$$-\frac{\left(\frac{8a^2\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab} + \frac{3a\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}} - \frac{2\sqrt{(bx-a)b+ab}\sqrt{-bx+a}}{b}\right)|b|}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(8*a^2*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2
 - a*b) + 3*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sq
rt(-b) - 2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*abs(b)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(3/2), x)
```

3.598 $\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

```
[Out] -2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(3/2)+2*x^(1/2)/b/(-b*x+a)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 65, 223, 209}

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{b}\\ &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}\\ &= \frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.10, size = 56, normalized size = 1.12

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
)/(-b)^(3/2)
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(-bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(3/2),x)
[Out] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

Maxima [A]
time = 0.49, size = 38, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{-bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 a)*b)
```

Fricas [A]
time = 0.90, size = 128, normalized size = 2.56

$$\left[-\frac{(bx-a)\sqrt{-b}\log\left(-2\,bx-2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\sqrt{-bx+a}\,b\sqrt{x}}{b^3x-ab^2},\frac{2\left((bx-a)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}\right)}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
[Out] [-((b*x - a)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2), 2*((b*x - a)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2
)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.93, size = 102, normalized size = 2.04

$$\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2i\sqrt{x}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{2\,\mathrm{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*I*sqrt(x)/(sqrt(
a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*asin(sqrt(b)*sqrt(x)/sqrt(a))/
b**(3/2) + 2*sqrt(x)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(38) = 76$.
time = 20.15, size = 98, normalized size = 1.96

$$-\frac{\left(\frac{4\,a\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
) + log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b))*ab
s(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{{(a-b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a - b*x)^(3/2), x)
```

3.599 $\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(a - b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]

[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])

Maple [A]
time = 0.12, size = 17, normalized size = 0.85

method	result	size
gosper	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17
default	$\frac{2\sqrt{x}}{a\sqrt{-bx+a}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.80

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)/(sqrt(-b*x + a)*a)
```

Fricas [A]
time = 0.95, size = 25, normalized size = 1.25

$$-\frac{2\sqrt{-bx+a}\,\sqrt{x}}{abx-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(-b*x + a)*sqrt(x)/(a*b*x - a^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 44, normalized size = 2.20

$$\begin{cases} \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i}{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((2/(a*sqrt(b)*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-2*I/(a*sqr
t(b)*sqrt(-a/(b*x) + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(16) = 32$.
time = 0.89, size = 53, normalized size = 2.65

$$-\frac{4\sqrt{-b}\,b}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
)*abs(b))
```

Mupad [B]
time = 0.34, size = 24, normalized size = 1.20

$$\frac{2\sqrt{x}\,\sqrt{a-b\,x}}{a^2-a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a - b*x)^(1/2))/(a^2 - a*b*x)
```

3.600 $\int \frac{1}{x^{3/2}(a-bx)^{3/2}} \, dx$

Optimal. Leaf size=41

$$\frac{2}{a\sqrt{x}\,\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

```
[Out] 2/a/x^(1/2)/(-b*x+a)^(1/2)-4*(-b*x+a)^(1/2)/a^2/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$\frac{2}{a\sqrt{x}\,\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{x}\,\sqrt{a-bx}}+\frac{2\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{x}\,\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 26, normalized size = 0.63

$$-\frac{2(a-2bx)}{a^2\sqrt{x}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a - 2*b*x))/(a^2*Sqrt[x]*Sqrt[a - b*x])
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.85

method	result	size
gosper	$-\frac{2(-2bx+a)}{\sqrt{x}\,\sqrt{-bx+a}\,a^{2}}$	23
default	$-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^{2}\sqrt{-bx+a}}$	35
risch	$-\frac{2\sqrt{-bx+a}}{a^{2}\sqrt{x}}+\frac{2b\sqrt{x}}{a^{2}\sqrt{-bx+a}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/x^(1/2)/(-b*x+a)^(1/2)+4*b/a^2*x^(1/2)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.83

$$\frac{2\,b\sqrt{x}}{\sqrt{-bx+a}\,a^{2}}-\frac{2\,\sqrt{-bx+a}}{a^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x)/(sqrt(-b*x + a)*a^2) - 2*sqrt(-b*x + a)/(a^2*sqrt(x))
```

Fricas [A]
time = 1.02, size = 38, normalized size = 0.93

$$-\frac{2\,(2\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{a^{2}bx^{2}-a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x - a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b*x^2 - a^3*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.83, size = 112, normalized size = 2.73

$$\begin{cases} -\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}-1}} + \frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{2iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-a^3b+a^2b^2x} - \frac{4ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-a^3b+a^2b^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2/(a*sqrt(b)*x*sqrt(a/(b*x) - 1)) + 4*sqrt(b)/(a**2*sqrt(a/(b*x
) - 1)), Abs(a/(b*x)) > 1), (2*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/(-a**3*b + a
**2*b**2*x) - 4*I*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-a**3*b + a**2*b**2*x), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(33) = 66.
time = 1.17, size = 94, normalized size = 2.29

$$-\frac{4\sqrt{-b}\,b^2}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b} - \sqrt{(bx-a)b+ab}\right)^2 - ab\right)a|b|} - \frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)*a*abs(b)) - 2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^2*abs(b))
```

Mupad [B]
time = 0.40, size = 42, normalized size = 1.02

$$-\frac{2\,a\,\sqrt{a-b\,x} - 4\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(a^3-a^2\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a - b*x)^(1/2) - 4*b*x*(a - b*x)^(1/2))/(x^(1/2)*(a^3 - a^2*b*x))
```

3.601 $\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/a/x^(3/2)/(-b*x+a)^(1/2)-8/3*(-b*x+a)^(1/2)/a^2/x^(3/2)-16/3*b*(-b*x+a)^(
1/2)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a - b*x]) - (8*Sqrt[a - b*x])/(3*a^2*x^(3/2)) - (16*b*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx = \frac{2}{ax^{3/2}\sqrt{a-bx}} + \frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a}$$
$$= \frac{2}{ax^{3/2}\sqrt{a-bx}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} + \frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}$$
$$= \frac{2}{ax^{3/2}\sqrt{a-bx}} - \frac{8\sqrt{a-bx}}{3a^2x^{3/2}} - \frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}$$

Mathematica [A]
time = 0.10, size = 39, normalized size = 0.59

$$-\frac{2(a^2+4abx-8b^2x^2)}{3a^3x^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a^2 + 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a - b*x])
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.88

method	result	size
gosper	$-\frac{2(-8x^2b^2+4abx+a^2)}{3x^{\frac{3}{2}}\sqrt{-bx+a}\,a^3}$	34
risch	$-\frac{2\sqrt{-bx+a}\,(5bx+a)}{3a^3x^{\frac{3}{2}}}+\frac{2b^2\sqrt{x}}{a^3\sqrt{-bx+a}}$	43
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\sqrt{-bx+a}}+\frac{4b\left(-\frac{2}{a\sqrt{x}\,\sqrt{-bx+a}}+\frac{4b\sqrt{x}}{a^2\sqrt{-bx+a}}\right)}{3a}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(-b*x+a)^(1/2)+4/3*b/a*(-2/a/x^(1/2)/(-b*x+a)^(1/2)+4*b/a^2*
x^(1/2)/(-b*x+a)^(1/2))
```

Maxima [A]
time = 0.30, size = 52, normalized size = 0.79

$$\frac{2\,b^2\sqrt{x}}{\sqrt{-bx+a}\,a^3} - \frac{2\left(\frac{6\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(-b*x + a)*a^3) - 2/3*(6*sqrt(-b*x + a)*b/sqrt(x) + (-b*
x + a)^(3/2)/x^(3/2))/a^3
```

Fricas [A]
time = 1.07, size = 51, normalized size = 0.77

$$-\frac{2\,(8\,b^2x^2-4\,abx-a^2)\sqrt{-bx+a}\,\sqrt{x}}{3\,(a^3bx^3-a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 4*a*b*x - a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b*x^3 - a^4*x^
2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.43, size = 452, normalized size = 6.85

$$\begin{cases} -\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{6ia^2b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}+\frac{24iab^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3}-\frac{16ib^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{3a^5b^4x-6a^4b^5x^2+3a^3b^6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2*a**3*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*
x**2 + 3*a**3*b**6*x**3) - 6*a**2*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**
4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*
x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 16*b**(15/2
)*x**3*sqrt(a/(b*x) - 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x*
*3), Abs(a/(b*x)) > 1), (-2*I*a**3*b**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4
*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) - 6*I*a**2*b**(11/2)*x*sqrt(-a/(b
*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*I*a*b**
(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*b**5*x**2 + 3*a**3*b
**6*x**3) - 16*I*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(3*a**5*b**4*x - 6*a**4*
b**5*x**2 + 3*a**3*b**6*x**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(50) = 100.
time = 0.90, size = 112, normalized size = 1.70

$$-\frac{2\sqrt{-bx+a}\left(\frac{5\,(bx-a)b^2|b|}{a^3}+\frac{6\,b^2|b|}{a^2}\right)}{3\,((bx-a)b+ab)^{\frac{3}{2}}}-\frac{4\sqrt{-b}\,b^3}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")

[Out] -2/3*sqrt(-b*x + a)*(5*(b*x - a)*b^2*abs(b)/a^3 + 6*b^2*abs(b)/a^2)/((b*x - a)*b + a*b)^(3/2) - 4*sqrt(-b)*b^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*a^2*abs(b))

Mupad [B]
time = 0.43, size = 48, normalized size = 0.73

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,x}{3\,a^2}+\frac{2}{3\,a\,b}-\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}-\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(5/2)*(a - b*x)^(3/2)),x)

[Out] ((a - b*x)^(1/2)*((8*x)/(3*a^2) + 2/(3*a*b) - (16*b*x^2)/(3*a^3)))/(x^(5/2) - (a*x^(3/2))/b)

3.602 $\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}$$

```
[Out] 2/3*x^(5/2)/b/(-b*x+a)^(3/2)+5*a*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(
7/2)-10/3*x^(3/2)/b^2/(-b*x+a)^(1/2)-5*x^(1/2)*(-b*x+a)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {49, 52, 65, 223, 209}

$$\frac{5a\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a - b*x]) - (5
*Sqrt[x]*Sqrt[a - b*x])/b^3 + (5*a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx}{3b}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{(5a)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{(5a)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{(5a)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\,\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 78, normalized size = 0.82

$$-\frac{\sqrt{x}\,(15a^2-20abx+3b^2x^2)}{3b^3(a-bx)^{3/2}}+\frac{5a\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[x]*(15*a^2 - 20*a*b*x + 3*b^2*x^2))/(b^3*(a - b*x)^(3/2)) + (5*a
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(-b)^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(71) = 142.
time = 0.12, size = 160, normalized size = 1.68

method	result
risch	$-\frac{\sqrt{x}\,\sqrt{-bx+a}}{b^3}+\frac{\left(\frac{5a\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{a}{2b}\right)}{\sqrt{-x^2b+ax}}\right)}{2b^{\frac{7}{2}}}+\frac{2a^2\sqrt{-\left(-\frac{a}{b}+x\right)^2b-a\left(-\frac{a}{b}+x\right)}}{3b^5\left(-\frac{a}{b}+x\right)^2}+\frac{14a\sqrt{-\left(-\frac{a}{b}\right.}}{}\right.}{\sqrt{x}\,\sqrt{-bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b^3+(5/2/b^(7/2)*a*arctan(b^(1/2)*(x-1/2*a/b)/(-b*x
^2+a*x)^(1/2))+2/3/b^5*a^2/(-a/b+x)^2*(-(-a/b+x)^2*b-a*(-a/b+x))^(1/2)+14/3
/b^4*a/(-a/b+x)*(-(-a/b+x)^2*b-a*(-a/b+x))^(1/2))*(x*(-b*x+a))^(1/2)/x^(1/2
)/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.49, size = 94, normalized size = 0.99

$$\frac{2\,ab^2+\frac{10\,(bx-a)ab}{x}-\frac{15\,(bx-a)^2a}{x^2}}{3\left(\frac{(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{5\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x - a)*a*b/x - 15*(b*x - a)^2*a/x^2)/((-b*x + a)^(3/2)
*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^(5/2)) - 5*a*arctan(sqrt(-b*x + a)/(s
qrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.70, size = 215, normalized size = 2.26

$$\left[-\frac{15\,(ab^{2}x^{2} - 2\,a^{2}bx + a^{3})\sqrt{-b}\,\log\left(-2\,bx + 2\,\sqrt{-bx + a}\,\sqrt{-b}\,\sqrt{x} + a\right) + 2\,(3\,b^{3}x^{2} - 20\,ab^{2}x + 15\,a^{2}b)\sqrt{-bx + a}\,\sqrt{x}}{6\,(b^{6}x^{2} - 2\,ab^{5}x + a^{2}b^{4})}, -\frac{15\,(ab^{2}x^{2} - 2\,a^{2}bx + a^{3})\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx + a}}{\sqrt{b}\,\sqrt{x}}\right) + (3\,b^{3}x^{2} - 20\,ab^{2}x + 15\,a^{2}b)\sqrt{-bx + a}\,\sqrt{x}}{3\,(b^{6}x^{2} - 2\,ab^{5}x + a^{2}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x +
 a)*sqrt(-b)*sqrt(x) + a) + 2*(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x
 + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), -1/3*(15*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 -
 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*
b^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 5.33, size = 971, normalized size = 10.22

$$\begin{cases} -\frac{30ia^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} + \frac{15\pi a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} + \frac{30ia^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} - \frac{15\pi a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} + \frac{30ia^{40}b^{\frac{45}{2}}x^{26}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} - \frac{40ia^{39}b^{\frac{47}{2}}x^{27}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} + \frac{6ia^{38}b^{\frac{49}{2}}x^{28}}{6a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{-1+\frac{bx}{a}}-6a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{-1+\frac{bx}{a}}} & \text{for}\: \left|\frac{bx}{a}\right| > 1 \\ \frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}} - \frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}} - \frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}} + \frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}} - \frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-30*I*a**(81/2)*b**22*x**(51/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*s
qrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(7
7/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 15*pi*a**(81/2)*b**22*x**(51/2
)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a*
*(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 30*I*a**(79/2)*b**23*x**(53
/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*
x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/
a)) - 15*pi*a**(79/2)*b**23*x**(53/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/
2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b
*x/a)) + 30*I*a**40*b**(45/2)*x**26/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-
1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) - 40*I*a**39
*b**(47/2)*x**27/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(
77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 6*I*a**38*b**(49/2)*x**28/(6*
a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(
53/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (15*a**(81/2)*b**22*x**(51/2)*sqr
t(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)
*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 15*a*
*(79/2)*b**23*x**(53/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a*
*(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/
2)*sqrt(1 - b*x/a)) - 15*a**40*b**(45/2)*x**26/(3*a**(79/2)*b**(51/2)*x**(5
```

```
1/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) + 2
0*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) -
3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 3*a**38*b**(49/2)*x**28/
(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x*
*(53/2)*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(71) = 142$.
time = 20.98, size = 221, normalized size = 2.33

$$\frac{\left(\frac{15\,a\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\right)}{\sqrt{-b}\,b^{2}}-\frac{6\,\sqrt{(bx-a)b+ab}\,\sqrt{-bx+a}}{b^{3}}-\frac{8\left(9\,a^{2}\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{4}-12\,a^{3}\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}b+7\,a^{4}b^{2}\right)}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}\sqrt{-b}\,b}\right)|b|}{6\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/(sqrt(
-b)*b^2) - 6*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b^3 - 8*(9*a^2*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4 - 12*a^3*(sqrt(-b*x + a)*sqrt
(-b) - sqrt((b*x - a)*b + a*b))^2*b + 7*a^4*b^2)/(((sqrt(-b*x + a)*sqrt(-b)
 - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(5/2), x)
```

3.603 $\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b/(-b*x+a)^(3/2)+2*arctan(b^(1/2)*x^(1/2)/(-b*x+a)^(1/2))/b^(5/
2)-2*x^(1/2)/b^2/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 65, 223, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a - b*x]) + (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx}{b}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(a-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 66, normalized size = 0.92

$$\frac{2\sqrt{x}\,(-3a+4bx)}{3b^2(a-bx)^{3/2}}-\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{a-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(5/2),x]
[Out] (2*Sqrt[x]*(-3*a + 4*b*x))/(3*b^2*(a - b*x)^(3/2)) - (2*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[a - b*x]])/(-b)^(5/2)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(-bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

Maxima [A]
time = 0.49, size = 52, normalized size = 0.72

$$\frac{2\left(b+\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}b^2}-\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.77, size = 188, normalized size = 2.61

$$\left[-\frac{3\,(b^2x^2-2\,abx+a^2)\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,(4\,b^2x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}}{3\,(b^5x^2-2\,ab^4x+a^2b^3)},\,-\frac{2\left(3\,(b^2x^2-2\,abx+a^2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(4\,b^2x-3\,ab)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\,(b^5x^2-2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*s
qrt(-b)*sqrt(x) + a) - 2*(4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^5*x^2
 - 2*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x)
)/(b^5*x^2 - 2*a*b^4*x + a^2*b^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.28, size = 833, normalized size = 11.57

$$\begin{cases} -\frac{6ia^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{3\pi a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{3\pi a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{8ia^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}} & \text{for}\: \left|\frac{bx}{a}\right|>1 \\ \frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}+\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*I*a**(39/2)*b**11*x**(27/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37
/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 3*pi*a**(39/2)*b**11*x**(27/2)*
```

```
sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(
37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 6*I*a**(37/2)*b**12*x**(29/2)
*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**
(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a))
 - 3*pi*a**(37/2)*b**12*x**(29/2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x
**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a
)) + 6*I*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b
*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) - 8*I*a**18*b**(2
5/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (6*a**(39/2)*b**11*
x**(27/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27
/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*
x/a)) - 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sq
rt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(2
9/2)*x**(29/2)*sqrt(1 - b*x/a)) - 6*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(
27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 -
b*x/a)) + 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 -
 b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(54) = 108$.

time = 21.70, size = 194, normalized size = 2.69

$$\frac{\left(\frac{3 \log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\right)}{\sqrt{-b}}+\frac{8\left(3a\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{4}\sqrt{-b}-3a^{2}\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\sqrt{-b}\,b+2a^{3}\sqrt{-b}\,b^{2}\right)}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}}\right)|b|}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b)
+ 8*(3*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) - 3
*a^2*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b + 2*a
^3*sqrt(-b)*b^2)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)^3)*abs(b)/b^3
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(5/2), x)
```

3.604 $\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

```
[Out] 2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(16) = 32$.
time = 0.11, size = 56, normalized size = 2.55

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}}{3a(-bx+a)^{\frac{3}{2}}}$	17
default	$\frac{\sqrt{x}}{b(-bx+a)^{\frac{3}{2}}}-\frac{a\left(\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^{2}\sqrt{-bx+a}}\right)}{2b}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*x^(1/2)/(-b*x+a)^(3/2)-1/2*a/b*(2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2
)/a^2/(-b*x+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.73

$$\frac{2\,x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((-b*x + a)^(3/2)*a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 0.55, size = 34, normalized size = 1.55

$$\frac{2\,\sqrt{-bx+a}\,x^{\frac{3}{2}}}{3\,(ab^2x^2-2\,a^2bx+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*x^(3/2)/(a*b^2*x^2 - 2*a^2*b*x + a^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 95, normalized size = 4.32

$$\begin{cases} \frac{2ix^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*I*x**(3/2)/(-3*a**(5/2)*sqrt(-1 + b*x/a) + 3*a**(3/2)*b*x*sqrt
(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*x**(3/2)/(-3*a**(5/2)*sqrt(1 - b*x/a) +
 3*a**(3/2)*b*x*sqrt(1 - b*x/a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(16) = 32.
time = 1.81, size = 102, normalized size = 4.64

$$\frac{4\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{4}\sqrt{-b}+a^{2}\sqrt{-b}\,b^{2}\right)|b|}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) + a^2
*sqrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))
^2 - a*b)^3*b^2)
```

Mupad [B]
time = 0.25, size = 37, normalized size = 1.68

$$\frac{2\,x^{3/2}\,\sqrt{a-b\,x}}{3\,\left(a^3-2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(5/2),x)
```

```
[Out] (2*x^(3/2)*(a - b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 - 2*a^2*b*x))
```

3.605 $\int \frac{1}{\sqrt{x}\,(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}$$

[Out] 2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(a - b*x)^(5/2)),x]

[Out] (2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a-bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}\,(a-bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 30, normalized size = 0.67

$$\frac{2\sqrt{x}\,(3a-2bx)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x]*(3*a - 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.76

method	result	size
gosper	$\frac{2\sqrt{x}\,(-2bx+3a)}{3(-bx+a)^{\frac{3}{2}}a^2}$	25
default	$\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{-bx+a}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.67

$$\frac{2\left(b-\frac{3(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b - 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*a^2)
```

Fricas [A]
time = 0.54, size = 44, normalized size = 0.98

$$-\frac{2\left(2\,bx-3\,a\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^2b^2x^2-2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x - 3*a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b^2*x^2 - 2*a^3*b*x + a^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 197, normalized size = 4.38

$$\begin{cases} -\frac{6a}{-3a^3\sqrt{b}\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}}+\frac{4bx}{-3a^3\sqrt{b}\sqrt{\frac{a}{bx}-1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{6iab}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}-\frac{4ib^2x}{-3a^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}+3a^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-6*a/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)*x*sqrt
(a/(b*x) - 1)) + 4*b*x/(-3*a**3*sqrt(b)*sqrt(a/(b*x) - 1) + 3*a**2*b**(3/2)
*x*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (6*I*a*b/(-3*a**3*b**(3/2)*sqrt(-
a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)) - 4*I*b**2*x/(-3*a**3*
b**(3/2)*sqrt(-a/(b*x) + 1) + 3*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.
time = 2.38, size = 96, normalized size = 2.13

$$\frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}\,b^2}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*abs(
b))
```

Mupad [B]
time = 0.41, size = 56, normalized size = 1.24

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a-b\,x}-4\,b\,x^{3/2}\,\sqrt{a-b\,x}}{3\,a^4-6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a - b*x)^(1/2) - 4*b*x^(3/2)*(a - b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 - 6*a^3*b*x)
```

3.606 $\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}$$

```
[Out] 2/3/a/(-b*x+a)^(3/2)/x^(1/2)+8/3/a^2/x^(1/2)/(-b*x+a)^(1/2)-16/3*(-b*x+a)^(
1/2)/a^3/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a - b*x]) - (16*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx = \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{4\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}} + \frac{8\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}$$
$$= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}} + \frac{8}{3a^2\sqrt{x}\,\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}$$

Mathematica [**A**]
time = 0.10, size = 41, normalized size = 0.61

$$-\frac{2(3a^2-12abx+8b^2x^2)}{3a^3\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

Maple [**A**]
time = 0.16, size = 57, normalized size = 0.85

method	result	size
gosper	$-\frac{2(8x^2b^2-12abx+3a^2)}{3\sqrt{x}\,(-bx+a)^{\frac{3}{2}}a^3}$	36
risch	$-\frac{2\sqrt{-bx+a}}{a^3\sqrt{x}}+\frac{2b(-5bx+6a)\sqrt{x}}{3(-bx+a)^{\frac{3}{2}}a^3}$	43
default	$-\frac{2}{a(-bx+a)^{\frac{3}{2}}\sqrt{x}}+\frac{4b\left(\frac{2\sqrt{x}}{3a(-bx+a)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{-bx+a}}\right)}{a}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/a/(-b*x+a)^(3/2)/x^(1/2)+4*b/a*(2/3*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)
/a^2/(-b*x+a)^(1/2))
```

Maxima [**A**]
time = 0.31, size = 50, normalized size = 0.75

$$\frac{2\left(b^2-\frac{6\,(bx-a)b}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^3}-\frac{2\,\sqrt{-bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x - a)*b/x)*x^(3/2)/((-b*x + a)^(3/2)*a^3) - 2*sqrt(-b*x +
a)/(a^3*sqrt(x))
```

Fricas [A]
time = 0.55, size = 59, normalized size = 0.88

$$-\frac{2\,(8\,b^2x^2-12\,abx+3\,a^2)\sqrt{-bx+a}\,\sqrt{x}}{3\,(a^3b^2x^3-2\,a^4bx^2+a^5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 12*a*b*x + 3*a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b^2*x^3 - 2
*a^4*b*x^2 + a^5*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.24, size = 314, normalized size = 4.69

$$\begin{cases} -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{6ia^2b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{24iab^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16ib^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*a**2*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) + 24*a*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a
**4*b**5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(3*a**
5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), Abs(a/(b*x)) > 1), (-6*I*a**2*b
**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2)
 + 24*I*a*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a
**3*b**6*x**2) - 16*I*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a*
*4*b**5*x + 3*a**3*b**6*x**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(49) = 98.
time = 1.48, size = 189, normalized size = 2.82

$$-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a^3|b|}-\frac{4\left(3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}\,b^2-12\,a\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}\,b^3+5\,a^2\sqrt{-b}\,b^4\right)}{3\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(-
b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b)*b^2 - 12*a*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b^3 + 5*a^2*sqrt(-b)
*b^4)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*a^2*
abs(b))
```

Mupad [B]
time = 0.44, size = 73, normalized size = 1.09

$$\frac{6\,a^2\,\sqrt{a-b\,x}+16\,b^2\,x^2\,\sqrt{a-b\,x}-24\,a\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b-3\,a^3\,b^2\,x\right)-3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a^2*(a - b*x)^(1/2) + 16*b^2*x^2*(a - b*x)^(1/2) - 24*a*b*x*(a - b*x)^(1
/2))/(x^(1/2)*(x*(6*a^4*b - 3*a^3*b^2*x) - 3*a^5))
```

3.607 $\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}$$

```
[Out] 2/3/a/x^(3/2)/(-b*x+a)^(3/2)+4/a^2/x^(3/2)/(-b*x+a)^(1/2)-16/3*(-b*x+a)^(1/
2)/a^3/x^(3/2)-32/3*b*(-b*x+a)^(1/2)/a^4/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a - b*x]) - (16*Sqrt[
a - b*x])/(3*a^3*x^(3/2)) - (32*b*Sqrt[a - b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx = \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{2\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3x^{3/2}} + \frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^3}$$
$$= \frac{2}{3ax^{3/2}(a-bx)^{3/2}} + \frac{4}{a^2x^{3/2}\sqrt{a-bx}} - \frac{16\sqrt{a-bx}}{3a^3x^{3/2}} - \frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}$$

Mathematica [A]
time = 0.11, size = 50, normalized size = 0.57

$$-\frac{2(a^3+6a^2bx-24ab^2x^2+16b^3x^3)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

Maple [A]
time = 0.14, size = 80, normalized size = 0.91

method	result	size
gosper	$-\frac{2\left(16b^3x^3-24a\,b^2x^2+6a^2bx+a^3\right)}{3x^{\frac{3}{2}}\left(-bx+a\right)^{\frac{3}{2}}a^4}$	45
risch	$-\frac{2\sqrt{-bx+a}\,(8bx+a)}{3a^4x^{\frac{3}{2}}}+\frac{2b^2(-8bx+9a)\sqrt{x}}{3(-bx+a)^{\frac{3}{2}}a^4}$	51
default	$-\frac{2}{3a\,x^{\frac{3}{2}}\left(-bx+a\right)^{\frac{3}{2}}}+\frac{2b\left(-\frac{2}{a\left(-bx+a\right)^{\frac{3}{2}}\sqrt{x}}+\frac{4b\left(\frac{2\sqrt{x}}{3a\left(-bx+a\right)^{\frac{3}{2}}}+\frac{4\sqrt{x}}{3a^2\sqrt{-bx+a}}\right)}{a}\right)}{a}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a/x^(3/2)/(-b*x+a)^(3/2)+2*b/a*(-2/a/(-b*x+a)^(3/2)/x^(1/2)+4*b/a*(2/3
*x^(1/2)/a/(-b*x+a)^(3/2)+4/3*x^(1/2)/a^2/(-b*x+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 68, normalized size = 0.77

$$-\frac{2\left(\frac{9\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}+\frac{2\left(b^3-\frac{9\,(bx-a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(9*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^4 + 2/3*(b^3
 - 9*(b*x - a)*b^2/x)*x^(3/2)/((-b*x + a)^(3/2)*a^4)
```

Fricas [A]
time = 0.64, size = 70, normalized size = 0.80

$$-\frac{2\,(16\,b^3x^3-24\,ab^2x^2+6\,a^2bx+a^3)\sqrt{-bx+a}\,\sqrt{x}}{3\,(a^4b^2x^4-2\,a^5bx^3+a^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*b^3*x^3 - 24*a*b^2*x^2 + 6*a^2*b*x + a^3)*sqrt(-b*x + a)*sqrt(x)/(
a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.35, size = 688, normalized size = 7.82

$$\begin{cases} \frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}-\frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}-\frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{2ia^4b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10ia^3b^{\frac{21}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}-\frac{60ia^2b^{\frac{23}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{80iab^{\frac{25}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4}-\frac{32ib^{\frac{27}{2}}x^4\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*a**4*b**(19/2)*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**1
0*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(
a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a*
*4*b**12*x**4) - 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x +
 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2
)*x**3*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11
*x**3 + 3*a**4*b**12*x**4) - 32*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-3*a**7*b
**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4), Abs(a/(
b*x)) > 1), (2*I*a**4*b**(19/2)*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6
*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*I*a**3*b**(21/2)*
x*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**
```

```
3 + 3*a**4*b**12*x**4) - 60*I*a**2*b**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a*
*7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80
*I*a*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2
- 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) - 32*I*b**(27/2)*x**4*sqrt(-a/(b*x
) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**
12*x**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(66) = 132.
time = 1.72, size = 207, normalized size = 2.35

$$-\frac{2\sqrt{-bx+a}\left(\frac{8(bx-a)b^2|b|}{a^4}+\frac{9b^2|b|}{a^3}\right)}{3((bx-a)b+ab)^{\frac{3}{2}}}-\frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}\,b^3-9a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}\,b^4+4a^2\sqrt{-b}\,b^5\right)}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(-b*x + a)*(8*(b*x - a)*b^2*abs(b)/a^4 + 9*b^2*abs(b)/a^3)/((b*x -
 a)*b + a*b)^(3/2) - 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a
*b))^4*sqrt(-b)*b^3 - 9*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b
))^2*sqrt(-b)*b^4 + 4*a^2*sqrt(-b)*b^5)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((
b*x - a)*b + a*b))^2 - a*b)^3*a^3*abs(b))
```

Mupad [B]
time = 0.47, size = 92, normalized size = 1.05

$$\frac{2\,a^3\,\sqrt{a-b\,x}+32\,b^3\,x^3\,\sqrt{a-b\,x}+12\,a^2\,b\,x\,\sqrt{a-b\,x}-48\,a\,b^2\,x^2\,\sqrt{a-b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b-3\,a^4\,b^2\,x\right)-3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (2*a^3*(a - b*x)^(1/2) + 32*b^3*x^3*(a - b*x)^(1/2) + 12*a^2*b*x*(a - b*x)^
(1/2) - 48*a*b^2*x^2*(a - b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b - 3*a^4*b^2*x) -
 3*a^6))
```

3.608 $\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=88

$$\frac{5\sqrt{x}\,\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] -5*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/6*x^(3/2)*(b*x+2)^(1/2)/b
^2+1/3*x^(5/2)*(b*x+2)^(1/2)/b+5/2*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{2b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(6*b^2) + (x^
(5/2)*Sqrt[2 + b*x])/(3*b) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx &= \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{3b}\\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\
&= \frac{5\sqrt{x}\,\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^3}\\
&= \frac{5\sqrt{x}\,\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}\\
&= \frac{5\sqrt{x}\,\sqrt{2+bx}}{2b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{6b^2} + \frac{x^{5/2}\sqrt{2+bx}}{3b} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.75

$$\frac{\sqrt{x}\,\sqrt{2+bx}\,(15-5bx+2b^2x^2)}{6b^3} + \frac{5\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 + b*x],x]
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2))/(6*b^3) + (5*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

Maple [A]
time = 0.13, size = 104, normalized size = 1.18

method	result	size
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(14x^2b^2-35bx+105)\sqrt{\frac{bx}{2}+1}}{42}-5\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{(2x^2b^2-5bx+15)\sqrt{x}\,\sqrt{bx+2}}{6b^3}-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{7}{2}}\sqrt{x}\,\sqrt{bx+2}}$	77

default	$\frac{x^{\frac{5}{2}}\sqrt{bx+2}}{3b}-\frac{5\left(\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}\right)}{2b}\right)}{3b}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(5/2)*(b*x+2)^(1/2)/b-5/3/b*(1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2/b*(x^(1/
2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*
x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(63) = 126$.
time = 0.50, size = 134, normalized size = 1.52

$$-\frac{\frac{33\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{40(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{15(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^6-\frac{3(bx+2)b^5}{x}+\frac{3(bx+2)^2b^4}{x^2}-\frac{(bx+2)^3b^3}{x^3}\right)}+\frac{5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3/2) + 15*(b*x
 + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x + 2)*b^5/x + 3*(b*x + 2)^2*b^4/x^2 - (b*
x + 2)^3*b^3/x^3) + 5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + s
qrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.55, size = 124, normalized size = 1.41

$$\left[\frac{(2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^4},\ \frac{(2\,b^3x^2-5\,b^2x+15\,b)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b
*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/6*((2*b^3*x^2 - 5*b^2*x + 1
```

```
5*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b^4]
```

Sympy [**A**]
time = 8.56, size = 95, normalized size = 1.08

$$\frac{x^{\frac{7}{2}}}{3\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{6b\sqrt{bx+2}} + \frac{5x^{\frac{3}{2}}}{6b^2\sqrt{bx+2}} + \frac{5\sqrt{x}}{b^3\sqrt{bx+2}} - \frac{5\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(7/2)/(3*sqrt(b*x + 2)) - x**(5/2)/(6*b*sqrt(b*x + 2)) + 5*x**(3/2)/(6*b
**2*sqrt(b*x + 2)) + 5*sqrt(x)/(b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(7/2)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(1/2), x)
```

3.609 $\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=67

$$-\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{2b}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] 3*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+1/2*x^(3/2)*(b*x+2)^(1/2)/b-
3/2*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\,\sqrt{bx+2}}{2b^2}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(2*b) + (3*Arc
Sinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx &= \frac{x^{3/2}\sqrt{2+bx}}{2b} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}\\ &= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^2}\\ &= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}\\ &= -\frac{3\sqrt{x}\,\sqrt{2+bx}}{2b^2} + \frac{x^{3/2}\sqrt{2+bx}}{2b} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.85

$$\frac{\sqrt{x}\,(-3+bx)\sqrt{2+bx}}{2b^2} - \frac{3\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 + b*x],x]
[Out] (Sqrt[x]*(-3 + b*x)*Sqrt[2 + b*x])/(2*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sq
rt[2 + b*x]])/b^(5/2)
```

Maple [A]
time = 0.11, size = 83, normalized size = 1.24

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,(-5bx+15)\sqrt{\frac{bx}{2}+1}}{10}+3\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55
risch	$\frac{(bx-3)\sqrt{x}\,\sqrt{bx+2}}{2b^2}+\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)\sqrt{x\,(bx+2)}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{bx+2}}$	68

default	$\frac{x^{\frac{3}{2}}\sqrt{bx+2}}{2b}-\frac{3\left(\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}\right)}{2b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2/b*(x^(1/2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*
x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(48) = 96$.
time = 0.51, size = 102, normalized size = 1.52

$$\frac{\frac{5\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{3(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{4}-\frac{2(bx+2)b^{3}}{x}+\frac{(bx+2)^{2}b^{2}}{x^{2}}}-\frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] (5*sqrt(b*x + 2)*b/sqrt(x) - 3*(b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x + 2)*
b^3/x + (b*x + 2)^2*b^2/x^2) - 3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(
sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.48, size = 105, normalized size = 1.57

$$\left[\frac{(b^{2}x-3\,b)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b^{3}},\frac{(b^{2}x-3\,b)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] [1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*
sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

Sympy [A]
time = 2.49, size = 75, normalized size = 1.12

$$\frac{x^{\frac{5}{2}}}{2\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{2b\sqrt{bx+2}}-\frac{3\sqrt{x}}{b^{2}\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(5/2)/(2*sqrt(b*x + 2)) - x**(3/2)/(2*b*sqrt(b*x + 2)) - 3*sqrt(x)/(b**2
*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(1/2), x)
```

3.610 $\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] -2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+x^(1/2)*(b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 56, 221}

$$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx = \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b}$$
$$= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\,\sqrt{2+bx}}{b} - \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.14

$$\frac{\sqrt{x}\,\sqrt{2+bx}}{b} + \frac{2\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b + (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(
3/2)
```

Maple [A]
time = 0.12, size = 62, normalized size = 1.44

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\sqrt{\frac{bx}{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	49
default	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	62
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b}-\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{3}{2}}\sqrt{bx+2}\,\sqrt{x}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)/b-1/b^(3/2)*(x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*l
n((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.51, size = 70, normalized size = 1.63

$$\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{bx+2}}{\left(b^2-\frac{(bx+2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b
^(3/2) - 2*sqrt(b*x + 2)/((b^2 - (b*x + 2)*b/x)*sqrt(x))
```

Fricas [A]
time = 0.54, size = 87, normalized size = 2.02

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x}+\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{b^2},\frac{\sqrt{bx+2}\,b\sqrt{x}+2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b^2, (sqrt(b*x + 2)*b*sqrt(x) + 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt
(-b)/(b*sqrt(x))))/b^2]
```

Sympy [A]
time = 1.11, size = 54, normalized size = 1.26

$$\frac{x^{\frac{3}{2}}}{\sqrt{bx+2}}+\frac{2\sqrt{x}}{b\sqrt{bx+2}}-\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(3/2)/sqrt(b*x + 2) + 2*sqrt(x)/(b*sqrt(b*x + 2)) - 2*asinh(sqrt(2)*sqrt
(b)*sqrt(x)/2)/b**(3/2)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [B]
time = 0.59, size = 43, normalized size = 1.00

$$\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{b^{3/2}}+\frac{\sqrt{x}\,\sqrt{b\,x+2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(1/2),x)
```

```
[Out] (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(1/2))))/b^(3/2) + (x^(1/2)
*(b*x + 2)^(1/2))/b
```

3.611 $\int \frac{1}{\sqrt{x}\sqrt{2+bx}} dx$

Optimal. Leaf size=24

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {56, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.25

$$-\frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(17) = 34$.
time = 0.12, size = 46, normalized size = 1.92

method	result	size
meijerg	$\frac{2\,\mathrm{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}$	18
default	$\frac{\sqrt{x\,(bx+2)}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}\,\sqrt{b}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(b*x+2))^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2
))/b^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.50, size = 41, normalized size = 1.71

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b)
```

Fricas [A]
time = 0.49, size = 55, normalized size = 2.29

$$\left[\frac{\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1)/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))/b]
```

Sympy [A]
time = 0.44, size = 24, normalized size = 1.00

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28
```

Mupad [B]
time = 0.04, size = 30, normalized size = 1.25

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{b\,x+2}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (b*x + 2)^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

3.612 $\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{2+bx}}{\sqrt{x}}$$

```
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{\sqrt{2+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Maple [A]
time = 0.10, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
default	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
risch	$-\frac{\sqrt{bx+2}}{\sqrt{x}}$	13
meijerg	$-\frac{\sqrt{2}\,\sqrt{\frac{bx}{2}+1}}{\sqrt{x}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.44, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(b*x + 2)/sqrt(x)
```

Sympy [A]
time = 0.45, size = 15, normalized size = 0.94

$$-\sqrt{b}\,\sqrt{1+\frac{2}{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(1 + 2/(b*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.
time = 0.97, size = 29, normalized size = 1.81

$$-\frac{\sqrt{bx+2}\,b^2}{\sqrt{(bx+2)b-2\,b}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b))
```

Mupad [B]
time = 0.33, size = 12, normalized size = 0.75

$$-\frac{\sqrt{b\,x+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(b*x + 2)^(1/2)/x^(1/2)
```

3.613 $\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=38

$$-\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] -1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/3*Sqrt[2 + b*x]/x^(3/2) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.61

$$\frac{(-1+bx)\sqrt{2+bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

Maple [A]
time = 0.14, size = 27, normalized size = 0.71

method	result	size
gosper	$\frac{\sqrt{bx+2}\,(bx-1)}{3x^{\frac{3}{2}}}$	18
meijerg	$-\frac{\sqrt{2}\,(-bx+1)\sqrt{\frac{bx}{2}+1}}{3x^{\frac{3}{2}}}$	23
risch	$\frac{x^2b^2+bx-2}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	25
default	$-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.68

$$\frac{\sqrt{bx+2}\,b}{2\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/6*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.40, size = 17, normalized size = 0.45

$$\frac{\sqrt{bx+2}\,(bx-1)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x + 2)*(b*x - 1)/x^(3/2)
```

Sympy [A]
time = 1.01, size = 34, normalized size = 0.89

$$\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] b**(3/2)*sqrt(1 + 2/(b*x))/3 - sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

Giac [A]
time = 1.29, size = 42, normalized size = 1.11

$$\frac{((bx+2)b^3-3b^3)\sqrt{bx+2}\,b}{3\left((bx+2)b-2b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + 2)*b^3 - 3*b^3)*sqrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(3/2)*abs(
b))
```

Mupad [B]
time = 0.32, size = 17, normalized size = 0.45

$$\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}-\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((b*x)/3 - 1/3))/x^(3/2)
```

3.614 $\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}$$

```
[Out] -1/5*(b*x+2)^(1/2)/x^(5/2)+2/15*b*(b*x+2)^(1/2)/x^(3/2)-2/15*b^2*(b*x+2)^(1
/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/5*Sqrt[2 + b*x]/x^(5/2) + (2*b*Sqrt[2 + b*x])/(15*x^(3/2)) - (2*b^2*Sqrt
[2 + b*x])/(15*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{5x^{5/2}}-\frac{1}{5}(2b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}+\frac{1}{15}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 32, normalized size = 0.54

$$\frac{\sqrt{2+bx}\left(-3+2bx-2b^2x^2\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*(-3 + 2*b*x - 2*b^2*x^2))/(15*x^(5/2))
```

Maple [**A**]
time = 0.12, size = 43, normalized size = 0.73

method	result	size
gosper	$-\frac{\sqrt{bx+2}\,(2x^2b^2-2bx+3)}{15x^{\frac{5}{2}}}$	27
meijerg	$-\frac{\sqrt{2}\,\left(\frac{2}{3}x^2b^2-\frac{2}{3}bx+1\right)\sqrt{\frac{bx}{2}+1}}{5x^{\frac{5}{2}}}$	31
risch	$-\frac{2b^3x^3+2x^2b^2-bx+6}{15x^{\frac{5}{2}}\sqrt{bx+2}}$	35
default	$-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(b*x+2)^(1/2)/x^(5/2)-2/5*b*(-1/3*(b*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^
(1/2)/x^(1/2))
```

Maxima [**A**]
time = 0.27, size = 41, normalized size = 0.69

$$-\frac{\sqrt{bx+2}\,b^2}{4\,\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{20\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/sqrt(x) + 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/20*(b*x
+ 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 0.44, size = 26, normalized size = 0.44

$$-\frac{(2\,b^2x^2-2\,bx+3)\,\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*b^2*x^2 - 2*b*x + 3)*sqrt(b*x + 2)/x^(5/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(53) = 106.
time = 3.48, size = 224, normalized size = 3.80

$$-\frac{2b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{6b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{3b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{4b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{12b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 60*b**4*
x**2) - 6*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 6
0*b**4*x**2) - 3*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x
**3 + 60*b**4*x**2) - 4*b**(11/2)*x*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b*
*5*x**3 + 60*b**4*x**2) - 12*b**(9/2)*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*
b**5*x**3 + 60*b**4*x**2)
```

Giac [A]
time = 0.96, size = 55, normalized size = 0.93

$$-\frac{(15\,b^5+2\,((bx+2)b^5-5\,b^5)(bx+2))\,\sqrt{bx+2}\,b}{15\,((bx+2)b-2\,b)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^5 + 2*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2))*sqrt(b*x + 2)*b/(((b*x
 + 2)*b - 2*b)^(5/2)*abs(b))
```

Mupad [B]
time = 0.32, size = 26, normalized size = 0.44

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^2\,x^2}{15}-\frac{2\,b\,x}{15}+\frac{1}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(1/2)),x)
[Out] -((b*x + 2)^(1/2)*((2*b^2*x^2)/15 - (2*b*x)/15 + 1/5))/x^(5/2)
```

3.615 $\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}+\frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}$$

```
[Out] -1/7*(b*x+2)^(1/2)/x^(7/2)+3/35*b*(b*x+2)^(1/2)/x^(5/2)-2/35*b^2*(b*x+2)^(1
/2)/x^(3/2)+2/35*b^3*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/7*Sqrt[2 + b*x]/x^(7/2) + (3*b*Sqrt[2 + b*x])/(35*x^(5/2)) - (2*b^2*Sqrt
[2 + b*x])/(35*x^(3/2)) + (2*b^3*Sqrt[2 + b*x])/(35*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{7x^{7/2}} - \frac{1}{7}(3b)\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx \\ &= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} + \frac{1}{35}\left(6b^2\right)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx \\ &= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} - \frac{2b^2\sqrt{2+bx}}{35x^{3/2}} - \frac{1}{35}\left(2b^3\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\ &= -\frac{\sqrt{2+bx}}{7x^{7/2}} + \frac{3b\sqrt{2+bx}}{35x^{5/2}} - \frac{2b^2\sqrt{2+bx}}{35x^{3/2}} + \frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.50

$$\frac{\sqrt{2+bx}\,\left(-5+3bx-2b^2x^2+2b^3x^3\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

Maple [A]
time = 0.13, size = 59, normalized size = 0.74

method	result	size
gosper	$\frac{\sqrt{bx+2}\,\left(2b^3x^3-2x^2b^2+3bx-5\right)}{35x^{\frac{7}{2}}}$	35
meijerg	$-\frac{\sqrt{2}\,\left(-\frac{2}{5}b^3x^3+\frac{2}{5}x^2b^2-\frac{3}{5}bx+1\right)\sqrt{\frac{bx}{2}+1}}{7x^{\frac{7}{2}}}$	39
risch	$\frac{2b^4x^4+2b^3x^3-x^2b^2+bx-10}{35x^{\frac{7}{2}}\sqrt{bx+2}}$	42
default	$-\frac{\sqrt{bx+2}}{7x^{\frac{7}{2}}}-\frac{3b\left(-\frac{\sqrt{bx+2}}{5x^{\frac{5}{2}}}-\frac{2b\left(-\frac{\sqrt{bx+2}}{3x^{\frac{3}{2}}}+\frac{b\sqrt{bx+2}}{3\sqrt{x}}\right)}{5}\right)}{7}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/7*(b*x+2)^(1/2)/x^(7/2)-3/7*b*(-1/5*(b*x+2)^(1/2)/x^(5/2)-2/5*b*(-1/3*(b
*x+2)^(1/2)/x^(3/2)+1/3*b*(b*x+2)^(1/2)/x^(1/2)))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.70

$$\frac{\sqrt{bx+2}\,b^3}{8\,\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^2}{8\,x^{\frac{3}{2}}}+\frac{3\,(bx+2)^{\frac{5}{2}}b}{40\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{56\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x + 2)*b^3/sqrt(x) - 1/8*(b*x + 2)^(3/2)*b^2/x^(3/2) + 3/40*(b*x
 + 2)^(5/2)*b/x^(5/2) - 1/56*(b*x + 2)^(7/2)/x^(7/2)
```

Fricas [A]
time = 0.41, size = 34, normalized size = 0.42

$$\frac{(2\,b^3x^3-2\,b^2x^2+3\,bx-5)\sqrt{bx+2}}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x - 5)*sqrt(b*x + 2)/x^(7/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(73) = 146.
time = 9.81, size = 374, normalized size = 4.68

$$\frac{2b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{10b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{15b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{5b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{10b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{36b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{40b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b*
*10*x**4 + 280*b**9*x**3) + 10*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(35*b**12*x
**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) + 15*b**(27/2)*x**4*
sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b*
*9*x**3) + 5*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x*
*5 + 420*b**10*x**4 + 280*b**9*x**3) - 10*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/
(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) - 36*b**(
21/2)*x*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4
+ 280*b**9*x**3) - 40*b**(19/2)*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**1
1*x**5 + 420*b**10*x**4 + 280*b**9*x**3)
```

Giac [A]
time = 0.85, size = 68, normalized size = 0.85

$$-\frac{(35\,b^7-(35\,b^7+2\,((bx+2)b^7-7\,b^7)(bx+2))(bx+2))\sqrt{bx+2}\,b}{35\,((bx+2)b-2\,b)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*b^7 - (35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2))*s
qrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

Mupad [B]
time = 0.33, size = 33, normalized size = 0.41

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{35}-\frac{2\,b^2\,x^2}{35}+\frac{3\,b\,x}{35}-\frac{1}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((3*b*x)/35 - (2*b^2*x^2)/35 + (2*b^3*x^3)/35 - 1/7))/x^(7
/2)
```

3.616 $\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] 15*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-2*x^(5/2)/b/(b*x+2)^(1/2)+5
/2*x^(3/2)*(b*x+2)^(1/2)/b^2-15/2*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]

time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 52, 56, 221}

$$\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{15\sqrt{x}\,\sqrt{bx+2}}{2b^3}+\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[2 + b*x]) - (15*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) + (5*x^
(3/2)*Sqrt[2 + b*x])/(2*b^2) + (15*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{b}\\ &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}-\frac{15\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\ &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^3}\\ &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\ &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.76

$$\frac{\sqrt{x}\,(-30-5bx+b^2x^2)}{2b^3\sqrt{2+bx}}-\frac{15\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(3/2),x]
[Out] (Sqrt[x]*(-30 - 5*b*x + b^2*x^2))/(2*b^3*Sqrt[2 + b*x]) - (15*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

Maple [A]
time = 0.13, size = 63, normalized size = 0.73

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(-\frac{7}{2}x^2b^2+\frac{35}{2}bx+105\right)}{14\sqrt{\frac{bx}{2}+1}}+15\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$	63
risch	$\frac{(bx-7)\sqrt{x}\,\sqrt{bx+2}}{2b^3}+\frac{\left(\frac{15\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{2b^{\frac{7}{2}}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{b^4\left(x+\frac{2}{b}\right)}\right)\sqrt{x\,(bx+2)}}{\sqrt{x}\,\sqrt{bx+2}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/b^(7/2)/Pi^(1/2)*(-1/112*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(-7/2*x^2*b^2+3
5/2*b*x+105)/(1/2*b*x+1)^(1/2)+15/8*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^
(1/2)))
```

Maxima [A]
time = 0.49, size = 119, normalized size = 1.38

$$-\frac{8\,b^2-\frac{25\,(bx+2)b}{x}+\frac{15\,(bx+2)^2}{x^2}}{\frac{\sqrt{bx+2}\,b^5}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}}-\frac{15\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(8*b^2 - 25*(b*x + 2)*b/x + 15*(b*x + 2)^2/x^2)/(sqrt(b*x + 2)*b^5/sqrt(x)
 - 2*(b*x + 2)^(3/2)*b^4/x^(3/2) + (b*x + 2)^(5/2)*b^3/x^(5/2)) - 15/2*log(
-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/
2)
```

Fricas [A]
time = 0.43, size = 152, normalized size = 1.77

$$\left[\frac{15\,(bx+2)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(b^3x^2-5\,b^2x-30\,b)\sqrt{bx+2}\,\sqrt{x}}{2\,(b^5x+2\,b^4)},\,-\frac{30\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-(b^3x^2-5\,b^2x-30\,b)\sqrt{bx+2}\,\sqrt{x}}{2\,(b^5x+2\,b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*(b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (
b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4), -1/2*(30*
(b*x + 2)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (b^3*x^2 -
5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4)]
```

Sympy [A]
time = 5.02, size = 80, normalized size = 0.93

$$\frac{x^{\frac{5}{2}}}{2b\sqrt{bx+2}} - \frac{5x^{\frac{3}{2}}}{2b^2\sqrt{bx+2}} - \frac{15\sqrt{x}}{b^3\sqrt{bx+2}} + \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(5/2)/(2*b*sqrt(b*x + 2)) - 5*x**(3/2)/(2*b**2*sqrt(b*x + 2)) - 15*sqrt(
x)/(b**3*sqrt(b*x + 2)) + 15*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2)
```

Giac [A]
time = 2.61, size = 119, normalized size = 1.38

$$\frac{\left(\sqrt{(bx+2)b-2\,b}\,\sqrt{bx+2}\,\left(\frac{bx+2}{b^3}-\frac{9}{b^3}\right)-\frac{15\,\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{b^{\frac{5}{2}}}-\frac{64}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)b^{\frac{3}{2}}}\right)|b|}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)/b^3 - 9/b^3) - 15*log
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/b^(5/2) - 64/(((sqrt(
b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(3/2)))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{{\left(b\,x+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(3/2), x)
```

3.617 $\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] -6*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-2*x^(3/2)/b/(b*x+2)^(1/2)+3
*x^(1/2)*(b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {49, 52, 56, 221}

$$-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\,\sqrt{bx+2}}{b^2}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*Sqrt[x]*Sqrt[2 + b*x])/b^2 - (6*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b}\\ &= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b^2}\\ &= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= -\frac{2x^{3/2}}{b\sqrt{2+bx}}+\frac{3\sqrt{x}\,\sqrt{2+bx}}{b^2}-\frac{6\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 0.86

$$\frac{\sqrt{x}\,(6+bx)}{b^2\sqrt{2+bx}}+\frac{6\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(6 + b*x))/(b^2*Sqrt[2 + b*x]) + (6*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[
2 + b*x]])/b^(5/2)
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.87

method	result	size

meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(\frac{5bx}{2}+15\right)}{5\sqrt{\frac{bx}{2}+1}}-6\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b^{2}}+\frac{\left(-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^{2}b+2x}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{\left(x+\frac{2}{b}\right)^{2}b-2x-\frac{4}{b}}}{b^{3}\left(x+\frac{2}{b}\right)}\right)\sqrt{x\left(bx+2\right)}}{\sqrt{x}\,\sqrt{bx+2}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b^(5/2)/Pi^(1/2)*(1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(5/2*b*x+15)/(1/2
*b*x+1)^(1/2)-3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 90, normalized size = 1.43

$$\frac{2\left(2\,b-\frac{3\,(bx+2)}{x}\right)}{\frac{\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x + 2)/x)/(sqrt(b*x + 2)*b^3/sqrt(x) - (b*x + 2)^(3/2)*b^2/x^
(3/2)) + 3*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 0.49, size = 134, normalized size = 2.13

$$\left[\frac{3\,(bx+2)\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(b^{2}x+6\,b)\sqrt{bx+2}\,\sqrt{x}}{b^{4}x+2\,b^{3}},\ \frac{6\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+(b^{2}x+6\,b)\sqrt{bx+2}\,\sqrt{x}}{b^{4}x+2\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*(b*x + 2)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x
 + 6*b)*sqrt(b*x + 2)*sqrt(x))/(b^4*x + 2*b^3), (6*(b*x + 2)*sqrt(-b)*arcta
```

```
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 6*b)*sqrt(b*x + 2)*sqrt(x)
)/(b^4*x + 2*b^3)]
```

Sympy [A]
time = 1.73, size = 58, normalized size = 0.92

$$\frac{x^{\frac{3}{2}}}{b\sqrt{bx+2}} + \frac{6\sqrt{x}}{b^2\sqrt{bx+2}} - \frac{6\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(3/2)/(b*sqrt(b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(b*x + 2)) - 6*asinh(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(48) = 96.
time = 2.81, size = 106, normalized size = 1.68

$$\frac{\left(\frac{3\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{\sqrt{b}} + \frac{\sqrt{(bx+2)b-2b}\,\sqrt{bx+2}}{b} + \frac{16\sqrt{b}}{\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] (3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + sqrt(
(b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b + 16*sqrt(b)/((sqrt(b*x + 2)*sqrt(b) - s
qrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(3/2), x)
```

3.618 $\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\sqrt{x}}{b\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)-2*x^(1/2)/b/(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 56, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 1.14

$$-\frac{2\sqrt{x}}{b\sqrt{2+bx}} - \frac{2\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
)/b^(3/2)
```

Maple [A]
time = 0.12, size = 48, normalized size = 1.09

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}}{\sqrt{\frac{bx}{2}+1}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}\sqrt{\pi}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^(3/2)/Pi^(1/2)*(-1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)/(1/2*b*x+1)^(1/2)
+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 57, normalized size = 1.30

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + 2)*b)
```

Fricas [A]
time = 0.58, size = 117, normalized size = 2.66

$$\left[\frac{(bx+2)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-2\sqrt{bx+2}\,b\sqrt{x}}{b^3x+2\,b^2},-\frac{2\left((bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\,b\sqrt{x}\right)}{b^3x+2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b
*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2), -2*((b*x + 2)*sqrt(-b)*arctan(sqrt(b*x
+ 2)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2)]
```

Sympy [A]
time = 0.77, size = 41, normalized size = 0.93

$$-\frac{2\sqrt{x}}{b\sqrt{bx+2}}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*sqrt(x)/(b*sqrt(b*x + 2)) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(33) = 66.
time = 2.75, size = 82, normalized size = 1.86

$$-\frac{\left(\frac{\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{\sqrt{b}}+\frac{8\sqrt{b}}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + 8*sqrt
(b)/((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x + 2)^(3/2), x)
```

3.619 $\int \frac{1}{\sqrt{x}\,(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{x}}{\sqrt{2+bx}}$$

[Out] x^(1/2)/(b*x+2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]

[Out] Sqrt[x]/Sqrt[2 + b*x]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2+bx}}$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]

[Out] Sqrt[x]/Sqrt[2 + b*x]

Maple [A]
time = 0.13, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
default	$\frac{\sqrt{x}}{\sqrt{bx+2}}$	12
meijerg	$\frac{\sqrt{x}\,\sqrt{2}}{2\sqrt{\frac{bx}{2}+1}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A]
time = 0.30, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

Fricas [A]
time = 0.55, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

Sympy [A]
time = 0.48, size = 15, normalized size = 1.00

$$\frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] 1/(sqrt(b)*sqrt(1 + 2/(b*x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(11) = 22.
time = 1.73, size = 44, normalized size = 2.93

$$\frac{4\,b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}\,-\,\sqrt{(bx+2)b-2\,b}\,\right)^{2}+2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(
b))
```

Mupad [B]
time = 0.31, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] x^(1/2)/(b*x + 2)^(1/2)
```

3.620 $\int \frac{1}{x^{3/2}(2+bx)^{3/2}} dx$

Optimal. Leaf size=32

$$\frac{1}{\sqrt{x}\sqrt{2+bx}} - \frac{\sqrt{2+bx}}{\sqrt{x}}$$

```
[Out] 1/x^(1/2)/(b*x+2)^(1/2)-(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{1}{\sqrt{x}\sqrt{bx+2}} - \frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(2+bx)^{3/2}} dx &= \frac{1}{\sqrt{x}\sqrt{2+bx}} + \int \frac{1}{x^{3/2}\sqrt{2+bx}} dx \\
&= \frac{1}{\sqrt{x}\sqrt{2+bx}} - \frac{\sqrt{2+bx}}{\sqrt{x}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 0.66

$$\frac{-1-bx}{\sqrt{x}\,\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

Maple [A]
time = 0.14, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{bx+1}{\sqrt{x}\,\sqrt{bx+2}}$	18
meijerg	$-\frac{\sqrt{2}\,(bx+1)}{2\sqrt{x}\,\sqrt{\frac{bx}{2}+1}}$	22
default	$-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}$	27
risch	$-\frac{\sqrt{bx+2}}{2\sqrt{x}}-\frac{b\sqrt{x}}{2\sqrt{bx+2}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^(1/2)/(b*x+2)^(1/2)-b*x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.81

$$-\frac{b\sqrt{x}}{2\,\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{2\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt(x)/sqrt(b*x + 2) - 1/2*sqrt(b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.54, size = 28, normalized size = 0.88

$$-\frac{\sqrt{bx+2}\,(bx+1)\sqrt{x}}{bx^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x + 2)*(b*x + 1)*sqrt(x)/(b*x^2 + 2*x)
```

Sympy [A]
time = 0.76, size = 34, normalized size = 1.06

$$-\frac{\sqrt{b}}{\sqrt{1+\frac{2}{bx}}}-\frac{1}{\sqrt{b}\,x\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] -sqrt(b)/sqrt(1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(1 + 2/(b*x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(24) = 48.
time = 1.16, size = 74, normalized size = 2.31

$$-\frac{\sqrt{bx+2}\,b^2}{2\,\sqrt{(bx+2)b-2\,b}\,|b|}-\frac{2\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}\;-\;\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 2*b^(5/2)/(((sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b))
```

Mupad [B]
time = 0.35, size = 17, normalized size = 0.53

$$-\frac{b\,x+1}{\sqrt{x}\,\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] -(b*x + 1)/(x^(1/2)*(b*x + 2)^(1/2))
```

3.621 $\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] 1/x^(3/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(3/2)+2/3*b*(b*x+2)^(1/2)/x^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x])/(3*x^(3/2)) + (2*b*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx = \frac{1}{x^{3/2}\sqrt{2+bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} - \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx$$
$$= \frac{1}{x^{3/2}\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3x^{3/2}} + \frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 0.60

$$\frac{-1+2bx+2b^2x^2}{3x^{3/2}\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

Maple [A]
time = 0.13, size = 43, normalized size = 0.81

method	result	size
gosper	$\frac{2x^2b^2+2bx-1}{3x^{\frac{3}{2}}\sqrt{bx+2}}$	27
meijerg	$-\frac{\sqrt{2}\,(-2x^2b^2-2bx+1)}{6x^{\frac{3}{2}}\sqrt{\frac{bx}{2}+1}}$	31
default	$-\frac{1}{3x^{\frac{3}{2}}\sqrt{bx+2}}-\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}\right)}{3}$	43
risch	$\frac{5x^2b^2+8bx-4}{12x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^2\sqrt{x}}{4\sqrt{bx+2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3/x^(3/2)/(b*x+2)^(1/2)-2/3*b*(-1/x^(1/2)/(b*x+2)^(1/2)-b*x^(1/2)/(b*x+2
)^(1/2))
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.77

$$\frac{b^2\sqrt{x}}{4\sqrt{bx+2}} + \frac{\sqrt{bx+2}\,b}{2\sqrt{x}} - \frac{(bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(b*x + 2) + 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/12*(b*x + 2
)^(3/2)/x^(3/2)
```

Fricas [A]
time = 0.73, size = 39, normalized size = 0.74

$$\frac{(2\,b^2x^2+2\,bx-1)\,\sqrt{bx+2}\,\sqrt{x}}{3\,(bx^3+2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*x^2 + 2*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^3 + 2*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(49) = 98.
time = 2.20, size = 170, normalized size = 3.21

$$\frac{2b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{3b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] 2*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
 + 6*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) + 3*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) - 2*b**(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(37) = 74.
time = 1.29, size = 86, normalized size = 1.62

$$\frac{b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}+\frac{(5\,(bx+2)b^2|b|-12\,b^2|b|)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] b^(7/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b)
) + 1/12*(5*(b*x + 2)*b^2*abs(b) - 12*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*
b - 2*b)^(3/2)
```

Mupad [B]
time = 0.38, size = 37, normalized size = 0.70

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,x}{3}+\frac{2\,b\,x^2}{3}-\frac{1}{3\,b}\right)}{x^{5/2}+\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(3/2)),x)
[Out] ((b*x + 2)^(1/2)*((2*x)/3 + (2*b*x^2)/3 - 1/(3*b)))/(x^(5/2) + (2*x^(3/2))/
b)
```

3.622 $\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}$$

```
[Out] 1/x^(5/2)/(b*x+2)^(1/2)-3/5*(b*x+2)^(1/2)/x^(5/2)+2/5*b*(b*x+2)^(1/2)/x^(3/
2)-2/5*b^2*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(5/2)*Sqrt[2 + b*x]) - (3*Sqrt[2 + b*x])/(5*x^(5/2)) + (2*b*Sqrt[2 + b
*x])/(5*x^(3/2)) - (2*b^2*Sqrt[2 + b*x])/(5*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{5/2}\sqrt{2+bx}}+3\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}-\frac{1}{5}(6b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}+\frac{1}{5}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 39, normalized size = 0.53

$$\frac{-1+bx-2b^2x^2-2b^3x^3}{5x^{5/2}\sqrt{2+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

Maple [A]
time = 0.14, size = 59, normalized size = 0.80

method	result	size
gosper	$-\frac{2b^3x^3+2x^2b^2-bx+1}{5x^{\frac{5}{2}}\sqrt{bx+2}}$	35
meijerg	$-\frac{\sqrt{2}\,(2b^3x^3+2x^2b^2-bx+1)}{10x^{\frac{5}{2}}\sqrt{\frac{bx}{2}+1}}$	39
risch	$-\frac{11b^3x^3+16x^2b^2-8bx+8}{40x^{\frac{5}{2}}\sqrt{bx+2}}-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}$	51
default	$-\frac{1}{5x^{\frac{5}{2}}\sqrt{bx+2}}-\frac{3b\left(-\frac{1}{3x^{\frac{3}{2}}\sqrt{bx+2}}-\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{b\sqrt{x}}{\sqrt{bx+2}}\right)}{3}\right)}{5}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^(5/2)/(b*x+2)^(1/2)-3/5*b*(-1/3/x^(3/2)/(b*x+2)^(1/2)-2/3*b*(-1/x^(1
/2)/(b*x+2)^(1/2)-b*x^(1/2)/(b*x+2)^(1/2)))
```

Maxima [A]
time = 0.30, size = 56, normalized size = 0.76

$$-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}-\frac{3\sqrt{bx+2}\,b^2}{8\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{8\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{40\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*sqrt(x)/sqrt(b*x + 2) - 3/8*sqrt(b*x + 2)*b^2/sqrt(x) + 1/8*(b*x +
 2)^(3/2)*b/x^(3/2) - 1/40*(b*x + 2)^(5/2)/x^(5/2)
```

Fricas [A]
time = 1.25, size = 47, normalized size = 0.64

$$-\frac{(2\,b^3x^3+2\,b^2x^2-bx+1)\sqrt{bx+2}\,\sqrt{x}}{5\,(bx^4+2\,x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*b^3*x^3 + 2*b^2*x^2 - b*x + 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^4 + 2*x^3
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(70) = 140$.
time = 6.66, size = 269, normalized size = 3.64

$$-\frac{2b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{10b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{15b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{5b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{4b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**1
0*x**3 + 40*b**9*x**2) - 10*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(5*b**12*x**5
+ 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 15*b**(25/2)*x**3*sqrt(1
+ 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) -
5*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10
*x**3 + 40*b**9*x**2) - 4*b**(19/2)*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b*
*11*x**4 + 60*b**10*x**3 + 40*b**9*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(52) = 104.
time = 1.35, size = 107, normalized size = 1.45

$$-\frac{b^{\frac{9}{2}}}{2\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)|b|}-\frac{\left(\frac{60\,b^{6}}{|b|}+\left(\frac{11\,(bx+2)b^{6}}{|b|}-\frac{50\,b^{6}}{|b|}\right)(bx+2)\right)\sqrt{bx+2}}{40\,((bx+2)b-2\,b)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*a
bs(b)) - 1/40*(60*b^6/abs(b) + (11*(b*x + 2)*b^6/abs(b) - 50*b^6/abs(b))*(b
*x + 2))*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(5/2)
```

Mupad [B]
time = 0.43, size = 46, normalized size = 0.62

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b\,x^2}{5}-\frac{x}{5}+\frac{1}{5\,b}+\frac{2\,b^2\,x^3}{5}\right)}{x^{7/2}+\frac{2\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] -((b*x + 2)^(1/2)*((2*b*x^2)/5 - x/5 + 1/(5*b) + (2*b^2*x^3)/5))/(x^(7/2) +
 (2*x^(5/2))/b)
```

3.623 $\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] -2/3*x^(5/2)/b/(b*x+2)^(3/2)-10*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2
)-10/3*x^(3/2)/b^2/(b*x+2)^(1/2)+5*x^(1/2)*(b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {49, 52, 56, 221}

$$-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\,\sqrt{bx+2}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 + b*x]) + (
5*Sqrt[x]*Sqrt[2 + b*x])/b^3 - (10*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx}{3b}\\ &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b^2}\\ &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{b^3}\\ &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{10\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\ &= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2+bx}}+\frac{5\sqrt{x}\,\sqrt{2+bx}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 0.77

$$\frac{\sqrt{x}\,(60+40bx+3b^2x^2)}{3b^3(2+bx)^{3/2}}+\frac{10\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{2+bx}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*(60 + 40*b*x + 3*b^2*x^2))/(3*b^3*(2 + b*x)^(3/2)) + (10*Log[-(Sqr
t[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.73

method	result
meijerg	$\frac{\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{b}\,\left(\frac{21}{4}x^2b^2+70bx+105\right)}{21\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}-10\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{7}{2}}\sqrt{\pi}}$
risch	$\frac{\sqrt{x}\,\sqrt{bx+2}}{b^3}+\frac{\left(-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{x^2b+2x}\right)}{b^{\frac{7}{2}}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{3b^5\left(x+\frac{2}{b}\right)^2}+\frac{28\sqrt{\left(x+\frac{2}{b}\right)^2b-2x-\frac{4}{b}}}{3b^4\left(x+\frac{2}{b}\right)}\right)}{\sqrt{x}\,\sqrt{bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/3/b^(7/2)/Pi^(1/2)*(1/56*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(21/4*x^2*b^2+7
0*b*x+105)/(1/2*b*x+1)^(3/2)-15/4*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1
/2)))
```

Maxima [A]
time = 0.49, size = 105, normalized size = 1.22

$$\frac{2\left(2\,b^2+\frac{10\,(bx+2)b}{x}-\frac{15\,(bx+2)^2}{x^2}\right)}{3\left(\frac{(bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x + 2)*b/x - 15*(b*x + 2)^2/x^2)/((b*x + 2)^(3/2)*b^4/x^
(3/2) - (b*x + 2)^(5/2)*b^3/x^(5/2)) + 5*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt
(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 1.34, size = 186, normalized size = 2.16

$$\left[\frac{15\,(b^2x^2+4\,bx+4)\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+(3\,b^3x^2+40\,b^2x+60\,b)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^6x^2+4\,b^5x+4\,b^4)},\ \frac{30\,(b^2x^2+4\,bx+4)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+(3\,b^3x^2+40\,b^2x+60\,b)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^6x^2+4\,b^5x+4\,b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt
(x) + 1) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt(x))/(b^6*x^2 +
```

```
4*b^5*x + 4*b^4), 1/3*(30*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt
(x))/(b^6*x^2 + 4*b^5*x + 4*b^4)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.
time = 3.82, size = 308, normalized size = 3.58

$$\frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{30b^{10}x^{\frac{27}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25
/2)*sqrt(b*x + 2)) + 40*b**(21/2)*x**14/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2
) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) + 60*b**(19/2)*x**13/(3*b**(27/2)*
x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 30*b**10*x
**(27/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27
/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 60*b**9*x**(25/2
)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqr
t(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(63) = 126$.
time = 2.88, size = 182, normalized size = 2.12

$$\frac{\left(\frac{15\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}\right)}{b^{\frac{5}{2}}}+\frac{3\sqrt{(bx+2)b-2b}\sqrt{bx+2}}{b^{3}}+\frac{16\left(9\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{4}\sqrt{b}+24\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}b^{\frac{3}{2}}+28b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}+2b\right)^{3}b^{2}}\right)|b|}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/b^(5/2) +
3*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b^3 + 16*(9*(sqrt(b*x + 2)*sqrt(b)
- sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 24*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*
x + 2)*b - 2*b))^2*b^(3/2) + 28*b^(5/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b
*x + 2)*b - 2*b))^2 + 2*b)^3*b^2))*abs(b)/b^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(5/2), x)
```

3.624 $\int \frac{x^{3/2}}{(2+bx)^{5/2}} dx$

Optimal. Leaf size=65

$$-\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] -2/3*x^(3/2)/b/(b*x+2)^(3/2)+2*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)
-2*x^(1/2)/b^2/(b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 56, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 + b*x]) + (2*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx &= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx}{b} \\
&= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2} \\
&= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{2+bx}} + \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 58, normalized size = 0.89

$$-\frac{4\sqrt{x}\,(3+2bx)}{3b^2(2+bx)^{3/2}} - \frac{2\log\left(-\sqrt{b}\sqrt{x} + \sqrt{2+bx}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*Sqrt[x]*(3 + 2*b*x))/(3*b^2*(2 + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[2 + b*x]])/b^(5/2)
```

Maple [A]
time = 0.13, size = 55, normalized size = 0.85

method	result	size
meijerg	$\frac{-\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\sqrt{b}\,(10bx+15)}{15\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}+2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{b^{\frac{5}{2}}\sqrt{\pi}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3/b^(5/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(10*b*x+15)/(1
/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 69, normalized size = 1.06

$$-\frac{2\left(b+\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}b^{2}}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + 2)/x)*x^(3/2)/((b*x + 2)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 1.22, size = 171, normalized size = 2.63

$$\left[\frac{3\,(b^{2}x^{2}+4\,bx+4)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-4\,(2\,b^{2}x+3\,b)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^{5}x^{2}+4\,b^{4}x+4\,b^{3})},-\frac{2\left(3\,(b^{2}x^{2}+4\,bx+4)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,b^{2}x+3\,b)\sqrt{bx+2}\,\sqrt{x}\right)}{3\,(b^{5}x^{2}+4\,b^{4}x+4\,b^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(
x) + 1) - 4*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x + 4*b
^3), -2/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))) + 2*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x +
 4*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(61) = 122.
time = 1.89, size = 257, normalized size = 3.95

$$-\frac{8b^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}-\frac{12b^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{6b^{5}x^{\frac{15}{2}}\sqrt{bx+2}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{12b^{4}x^{\frac{13}{2}}\sqrt{bx+2}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -8*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x**(13
/2)*sqrt(b*x + 2)) - 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2)
+ 6*b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 6*b**5*x**(15/2)*sqrt(b*x + 2)*asi
nh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(
13/2)*x**(13/2)*sqrt(b*x + 2)) + 12*b**4*x**(13/2)*sqrt(b*x + 2)*asinh(sqrt
```

```
(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x
**(13/2)*sqrt(b*x + 2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(48) = 96$.

time = 3.36, size = 154, normalized size = 2.37

$$-\frac{\left(\frac{3\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{\sqrt{b}}+\frac{16\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^4\sqrt{b}+6\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2b^{\frac{3}{2}}+8b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)^3}\right)|b|}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) +
16*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 6*(sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(3/2) + 8*b^(5/2))/((sqrt(
b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3)*abs(b)/b^3
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(5/2), x)
```

3.625 $\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=18

$$\frac{x^{3/2}}{3(2+bx)^{3/2}}$$

```
[Out] 1/3*x^(3/2)/(b*x+2)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {37}

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}}\,dx = \frac{x^{3/2}}{3(2+bx)^{3/2}}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{x^{3/2}}{3(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]
```

[Out] x^(3/2)/(3*(2 + b*x)^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(12) = 24$.
time = 0.10, size = 46, normalized size = 2.56

method	result	size
gosper	$\frac{x^{\frac{3}{2}}}{3(bx+2)^{\frac{3}{2}}}$	13
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}}{12\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	17
default	$-\frac{\sqrt{x}}{b(bx+2)^{\frac{3}{2}}}+\frac{\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}}{b}$	46

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/b*x^(1/2)/(b*x+2)^(3/2)+1/b*(1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2))

Maxima [A]
time = 0.28, size = 12, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")

[Out] 1/3*x^(3/2)/(b*x + 2)^(3/2)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.
time = 0.68, size = 27, normalized size = 1.50

$$\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{3\,(b^2x^2+4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")

[Out] 1/3*sqrt(b*x + 2)*x^(3/2)/(b^2*x^2 + 4*b*x + 4)

Sympy [A]
time = 0.74, size = 27, normalized size = 1.50

$$\frac{x^{\frac{3}{2}}}{3bx\sqrt{bx+2}+6\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(5/2),x)
[Out] x**(3/2)/(3*b*x*sqrt(b*x + 2) + 6*sqrt(b*x + 2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(12) = 24$.
time = 2.05, size = 82, normalized size = 4.56

$$\frac{4\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{4}\sqrt{b}+4\,b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^{2}+2\,b\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 4*b^(5
/2))*abs(b)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*
b^2)
```

Mupad [B]
time = 0.25, size = 12, normalized size = 0.67

$$\frac{x^{3/2}}{3\,{\left(b\,x+2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(5/2),x)
[Out] x^(3/2)/(3*(b*x + 2)^(3/2))
```

3.626 $\int \frac{1}{\sqrt{x}\,(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}$$

```
[Out] 1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(2+bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}\,(2+bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}\,(3+bx)}{3(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

Maple [A]
time = 0.12, size = 26, normalized size = 0.70

method	result	size
gosper	$\frac{\sqrt{x}\,(bx+3)}{3(bx+2)^{\frac{3}{2}}}$	18
meijerg	$\frac{\sqrt{x}\,\sqrt{2}\,(bx+3)}{12\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	22
default	$\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.65

$$-\frac{\left(b-\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{6\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(b - 3*(b*x + 2)/x)*x^(3/2)/(b*x + 2)^(3/2)
```

Fricas [A]
time = 0.96, size = 32, normalized size = 0.86

$$\frac{(bx+3)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^2x^2+4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^2 + 4*b*x + 4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(29) = 58$.
time = 1.03, size = 75, normalized size = 2.03

$$\frac{bx}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}+\frac{3}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b*x/(3*b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x))) + 3/(3*b
**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.
time = 2.13, size = 79, normalized size = 2.14

$$\frac{8\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(5/2)/(
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

Mupad [B]
time = 0.36, size = 42, normalized size = 1.14

$$\frac{3\sqrt{x}\sqrt{b\,x+2}+b\,x^{3/2}\sqrt{b\,x+2}}{3\,b^2\,x^2+12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(b*x + 2)^(1/2) + b*x^(3/2)*(b*x + 2)^(1/2))/(12*b*x + 3*b^2*x^2
 + 12)
```

3.627 $\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{3\sqrt{x}\,(2+bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] 1/3/(b*x+2)^(3/2)/x^(1/2)+2/3/x^(1/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(1/
2)
```

Rubi [A]
time = 0.00, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{bx+2}}+\frac{1}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 + b*x]) - (2*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx \\ &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2+bx}} + \frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx \\ &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}} + \frac{2}{3\sqrt{x}\,\sqrt{2+bx}} - \frac{2\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 32, normalized size = 0.58

$$\frac{-3-6bx-2b^2x^2}{3\sqrt{x}\,(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.76

method	result	size
gosper	$-\frac{2x^2b^2+6bx+3}{3\sqrt{x}\,(bx+2)^{\frac{3}{2}}}$	27
meijerg	$-\frac{\sqrt{2}\,(2x^2b^2+6bx+3)}{12\sqrt{x}\,\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	31
risch	$-\frac{\sqrt{bx+2}}{4\sqrt{x}}-\frac{b(5bx+12)\sqrt{x}}{12(bx+2)^{\frac{3}{2}}}$	33
default	$-\frac{1}{(bx+2)^{\frac{3}{2}}\sqrt{x}}-2b\left(\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}\right)$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/(b*x+2)^(3/2)/x^(1/2)-2*b*(1/3*x^(1/2)/(b*x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)
^(1/2))
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.73

$$\frac{\left(b^2-\frac{6\,(bx+2)b}{x}\right)x^{\frac{3}{2}}}{12\,(bx+2)^{\frac{3}{2}}}-\frac{\sqrt{bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x + 2)*b/x)*x^(3/2)/(b*x + 2)^(3/2) - 1/4*sqrt(b*x + 2)/sq
rt(x)
```

Fricas [A]
time = 0.89, size = 45, normalized size = 0.82

$$-\frac{(2\,b^2x^2+6\,bx+3)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^2x^3+4\,bx^2+4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 + 6*b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^3 + 4*b*x^2 + 4*x
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.
time = 2.08, size = 117, normalized size = 2.13

$$-\frac{2b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{6b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{3b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -2*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 6
*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 3*b**(
9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(37) = 74$.
time = 1.64, size = 145, normalized size = 2.64

$$-\frac{\sqrt{bx+2}\,b^2}{4\sqrt{(bx+2)b-2\,b}\,|b|}-\frac{3\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^4b^{\frac{5}{2}}+24\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2b^{\frac{7}{2}}+20\,b^{\frac{9}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 1/3*(3*(sqrt(b*x
+ 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(5/2) + 24*(sqrt(b*x + 2)*sqrt(
```

```
b) - sqrt((b*x + 2)*b - 2*b))^2*b^(7/2) + 20*b^(9/2))/(((sqrt(b*x + 2)*sqrt
(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

Mupad [B]
time = 0.38, size = 57, normalized size = 1.04

$$-\frac{3\sqrt{b\,x+2}\,+6\,b\,x\,\sqrt{b\,x+2}\,+2\,b^2\,x^2\,\sqrt{b\,x+2}}{\sqrt{x}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] -(3*(b*x + 2)^(1/2) + 6*b*x*(b*x + 2)^(1/2) + 2*b^2*x^2*(b*x + 2)^(1/2))/(x
^(1/2)*(x*(12*b + 3*b^2*x) + 12))
```

3.628 $\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}$$

```
[Out] 1/3/x^(3/2)/(b*x+2)^(3/2)+1/x^(3/2)/(b*x+2)^(1/2)-2/3*(b*x+2)^(1/2)/x^(3/2)
+2/3*b*(b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x
])/(3*x^(3/2)) + (2*b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx\\ &= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}+2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.56

$$\frac{-1+3bx+6b^2x^2+2b^3x^3}{3x^{3/2}(2+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

Maple [A]
time = 0.13, size = 58, normalized size = 0.82

method	result	size
gosper	$\frac{2b^3x^3+6x^2b^2+3bx-1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}$	35
meijerg	$-\frac{\sqrt{2}\,(-2b^3x^3-6x^2b^2-3bx+1)}{12x^{\frac{3}{2}}\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	39
risch	$\frac{4x^2b^2+7bx-2}{12x^{\frac{3}{2}}\sqrt{bx+2}}+\frac{b^2(4bx+9)\sqrt{x}}{12(bx+2)^{\frac{3}{2}}}$	49
default	$-\frac{1}{3x^{\frac{3}{2}}(bx+2)^{\frac{3}{2}}}-b\left(-\frac{1}{(bx+2)^{\frac{3}{2}}\sqrt{x}}-2b\left(\frac{\sqrt{x}}{3(bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{bx+2}}\right)\right)$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)/(b*x+2)^(3/2)-b*(-1/(b*x+2)^(3/2)/x^(1/2)-2*b*(1/3*x^(1/2)/(b*
x+2)^(3/2)+1/3*x^(1/2)/(b*x+2)^(1/2)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.77

$$\frac{3\sqrt{bx+2}\,b}{8\sqrt{x}} - \frac{\left(b^3 - \frac{9\,(bx+2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(bx+2)^{\frac{3}{2}}} - \frac{(bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(b*x + 2)*b/sqrt(x) - 1/24*(b^3 - 9*(b*x + 2)*b^2/x)*x^(3/2)/(b*x +
 2)^(3/2) - 1/24*(b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 1.08, size = 55, normalized size = 0.77

$$\frac{(2\,b^3x^3 + 6\,b^2x^2 + 3\,bx - 1)\sqrt{bx+2}\,\sqrt{x}}{3\,(b^2x^4 + 4\,bx^3 + 4\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^3*x^3 + 6*b^2*x^2 + 3*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^4 + 4*
b*x^3 + 4*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(66) = 132.
time = 3.98, size = 257, normalized size = 3.62

$$\frac{2b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} + \frac{10b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} + \frac{15b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} + \frac{5b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x} - \frac{2b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 2*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10
*x**2 + 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18
*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2*sqrt(1 + 2/(b*
x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 5*b**(21/2
)*x*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b*
*9*x) - 2*b**(19/2)*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b*
*10*x**2 + 24*b**9*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(49) = 98.
time = 1.88, size = 158, normalized size = 2.23

$$\frac{(4\,(bx+2)b^2|b| - 9\,b^2|b|)\sqrt{bx+2}}{12\,((bx+2)b - 2\,b)^{\frac{3}{2}}} + \frac{3\left(\sqrt{bx+2}\,\sqrt{b} - \sqrt{(bx+2)b - 2\,b}\right)^4 b^{\frac{7}{2}} + 18\left(\sqrt{bx+2}\,\sqrt{b} - \sqrt{(bx+2)b - 2\,b}\right)^2 b^{\frac{9}{2}} + 16\,b^{\frac{11}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b} - \sqrt{(bx+2)b - 2\,b}\right)^2 + 2\,b\right)^3 |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*(b*x + 2)*b^2*abs(b) - 9*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*b - 2
*b)^(3/2) + 1/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(7
/2) + 18*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(9/2) + 16*b
^(11/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs
(b))
```

Mupad [B]
time = 0.42, size = 71, normalized size = 1.00

$$\frac{3\,b\,x\,\sqrt{b\,x+2}-\sqrt{b\,x+2}+6\,b^2\,x^2\,\sqrt{b\,x+2}+2\,b^3\,x^3\,\sqrt{b\,x+2}}{x^{3/2}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*b*x*(b*x + 2)^(1/2) - (b*x + 2)^(1/2) + 6*b^2*x^2*(b*x + 2)^(1/2) + 2*b^
3*x^3*(b*x + 2)^(1/2))/(x^(3/2)*(x*(12*b + 3*b^2*x) + 12))
```

3.629 $\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=91

$$-\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] 5*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)-5/6*x^(3/2)*(-b*x+2)^(1/2)/b^
2-1/3*x^(5/2)*(-b*x+2)^(1/2)/b-5/2*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{5\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(6*b^2) - (x
^(5/2)*Sqrt[2 - b*x])/(3*b) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx &= -\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{3b}\\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}\\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^3}\\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.79

$$
-\frac{\sqrt{x}\,\sqrt{2-bx}\,(15+5bx+2b^2x^2)}{6b^3}+\frac{5\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2))/b^3 + (5*Log[-(Sqrt[-
b]*Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Maple [A]
time = 0.13, size = 111, normalized size = 1.22

method	result
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(14x^2b^2+35bx+105\right)\sqrt{-\frac{bx}{2}+1}}{336b^3}+\frac{5\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{8b^{\frac{7}{2}}}\right)}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$

risch	$\frac{(2x^2b^2+5bx+15)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{6b^3\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{7}{2}}\sqrt{x}\,\sqrt{-bx+2}}$
default	$-\frac{x^{\frac{5}{2}}\sqrt{-bx+2}}{3b}+\frac{-\frac{5x^{\frac{3}{2}}\sqrt{-bx+2}}{6b}+\frac{5\left(-\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2b}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}\right)}{3b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^(5/2)*(-b*x+2)^(1/2)/b+5/3/b*(-1/2*x^(3/2)*(-b*x+2)^(1/2)/b+3/2/b*(-
x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2
)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))))
```

Maxima [A]
time = 0.50, size = 117, normalized size = 1.29

$$-\frac{\frac{33\sqrt{-bx+2}\,b^2}{\sqrt{x}}+\frac{40\,(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^6-\frac{3\,(bx-2)b^5}{x}+\frac{3\,(bx-2)^2b^4}{x^2}-\frac{(bx-2)^3b^3}{x^3}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(-b*x + 2)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 15*(-
b*x + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x - 2)*b^5/x + 3*(b*x - 2)^2*b^4/x^2 -
(b*x - 2)^3*b^3/x^3) - 5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 0.86, size = 125, normalized size = 1.37

$$\left[-\frac{(2\,b^3x^2+5\,b^2x+15\,b)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^4},-\frac{(2\,b^3x^2+5\,b^2x+15\,b)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, -1/6*((2*b^3*x^2 + 5*b^
2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt
(b)*sqrt(x))))/b^4]
```

Sympy [C] Result contains complex when optimal does not.
time = 8.60, size = 204, normalized size = 2.24

$$\begin{cases} -\frac{ix^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{6b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{6b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for } |bx| > 2 \\ \frac{x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{6b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{6b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(7/2)/(3*sqrt(b*x - 2)) - I*x**(5/2)/(6*b*sqrt(b*x - 2)) -
 5*I*x**(3/2)/(6*b**2*sqrt(b*x - 2)) + 5*I*sqrt(x)/(b**3*sqrt(b*x - 2)) - 5
*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), Abs(b*x) > 2), (x**(7/2)/(3*s
qrt(-b*x + 2)) + x**(5/2)/(6*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(6*b**2*sqrt(-b
*x + 2)) - 5*sqrt(x)/(b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2)/b**(7/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(1/2), x)
```

3.630 $\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=69

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] 3*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-1/2*x^(3/2)*(-b*x+2)^(1/2)/b-
3/2*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {52, 56, 222}

$$\frac{3\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(2*b) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx = -\frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^2}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{3\sqrt{x}\,\sqrt{2-bx}}{2b^2} - \frac{x^{3/2}\sqrt{2-bx}}{2b} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [A]
time = 0.08, size = 63, normalized size = 0.91

$$-\frac{\sqrt{x}\,\sqrt{2-bx}\,(3+bx)}{2b^2} - \frac{3\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x))/b^2 - (3*Log[-(Sqrt[-b]*Sqrt[x]) + S
qrt[2 - b*x]])/(-b)^(5/2)
```

Maple [A]
time = 0.11, size = 89, normalized size = 1.29

method	result	size
meijerg	$-\frac{4\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}(5bx+15)\sqrt{-\frac{bx}{2}+1}}{40b^2}+\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{5}{2}}}\right)}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$	73
default	$-\frac{x^{\frac{3}{2}}\sqrt{-bx+2}}{2b}+\frac{-\frac{3\sqrt{x}\,\sqrt{-bx+2}}{2b}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{2b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}}{b}$	89

risch	$\frac{(bx+3)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)\sqrt{(-bx+2)\,x}}{2b^{\frac{5}{2}}\sqrt{x}\,\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(3/2)*(-b*x+2)^(1/2)/b+3/2/b*(-x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((
-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)
^(1/2)))
```

Maxima [A]
time = 0.50, size = 85, normalized size = 1.23

$$-\frac{\frac{5\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{3\,(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^4-\frac{2\,(bx-2)b^3}{x}+\frac{(bx-2)^2b^2}{x^2}}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(5*sqrt(-b*x + 2)*b/sqrt(x) + 3*(-b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x -
2)*b^3/x + (b*x - 2)^2*b^2/x^2) - 3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))
)/b^(5/2)
```

Fricas [A]
time = 1.49, size = 107, normalized size = 1.55

$$\left[-\frac{(b^2x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b^3},-\frac{(b^2x+3\,b)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt
(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

Sympy [C] Result contains complex when optimal does not.
time = 2.48, size = 162, normalized size = 2.35

$$\begin{cases} -\frac{ix^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{2b\sqrt{bx-2}}+\frac{3i\sqrt{x}}{b^2\sqrt{bx-2}}-\frac{3i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } |bx|>2 \\ \frac{x^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{2b\sqrt{-bx+2}}-\frac{3\sqrt{x}}{b^2\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(5/2)/(2*sqrt(b*x - 2)) - I*x**(3/2)/(2*b*sqrt(b*x - 2)) +
 3*I*sqrt(x)/(b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(5/2), Abs(b*x) > 2), (x**(5/2)/(2*sqrt(-b*x + 2)) + x**(3/2)/(2*b*sqrt(-b
*x + 2)) - 3*sqrt(x)/(b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2)/b**(5/2), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(1/2), x)
```

3.631 $\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=45

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{b}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] 2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)-x^(1/2)*(-b*x+2)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {52, 56, 222}

$$\frac{2\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx &= -\frac{\sqrt{x}\,\sqrt{2-bx}}{b}+\frac{\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{2-bx}}{b}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{\sqrt{x}\,\sqrt{2-bx}}{b}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.24

$$-\frac{\sqrt{x}\,\sqrt{2-bx}}{b}+\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 - b*x],x]
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])
/(-b)^(3/2)
```

Maple [A]
time = 0.13, size = 67, normalized size = 1.49

method	result	size
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{3}{2}}\sqrt{-\frac{bx}{2}+1}}{2b}+\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}}\right)}{\sqrt{-b}\,\sqrt{\pi}\,b}$	66
default	$-\frac{\sqrt{x}\,\sqrt{-bx+2}}{b}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	67
risch	$\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{b\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{b^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{x}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)*(-b*x+2)^(1/2)/b+1/b^(3/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/
2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.49, size = 52, normalized size = 1.16

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{-bx+2}}{\left(b^2-\frac{(bx-2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) - 2*sqrt(-b*x + 2)/((b^
2 - (b*x - 2)*b/x)*sqrt(x))
```

Fricas [A]
time = 1.54, size = 90, normalized size = 2.00

$$\left[-\frac{\sqrt{-bx+2}\,b\sqrt{x}+\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b^2},-\frac{\sqrt{-bx+2}\,b\sqrt{x}+2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-b*x + 2)*b*sqrt(x) + sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*s
qrt(x) + 1))/b^2, -(sqrt(-b*x + 2)*b*sqrt(x) + 2*sqrt(b)*arctan(sqrt(-b*x +
 2)/(sqrt(b)*sqrt(x))))/b^2]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 119, normalized size = 2.64

$$\begin{cases} -\frac{ix^{\frac{3}{2}}}{\sqrt{bx-2}}+\frac{2i\sqrt{x}}{b\sqrt{bx-2}}-\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx|>2 \\ \frac{x^{\frac{3}{2}}}{\sqrt{-bx+2}}-\frac{2\sqrt{x}}{b\sqrt{-bx+2}}+\frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(3/2)/sqrt(b*x - 2) + 2*I*sqrt(x)/(b*sqrt(b*x - 2)) - 2*I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x) > 2), (x**(3/2)/sqrt(-b
*x + 2) - 2*sqrt(x)/(b*sqrt(-b*x + 2)) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
b**(3/2), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [**B**]
time = 0.52, size = 46, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{2-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(1/2),x)
```

```
[Out] - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1/2))))/b^(3/2) - (x^(1/2
)*(2 - b*x)^(1/2))/b
```

3.632 $\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {56, 222}

$$\frac{2\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.46

$$-\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
[Out] (-2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/Sqrt[-b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.
time = 0.12, size = 50, normalized size = 2.08

method	result	size
meijerg	$\frac{2\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}}$	18
default	$\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}\,\sqrt{b}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/b)/(-
b*x^2+2*x)^(1/2))
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.88

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Fricas [A]
time = 1.05, size = 56, normalized size = 2.33

$$\left[-\frac{\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b},\,-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan(sqr
t(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 56, normalized size = 2.33

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ |bx| > 2 \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x) > 2), (2
*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28
```

Mupad [B]
time = 0.03, size = 27, normalized size = 1.12

$$\frac{4\operatorname{atan}\left(\frac{\sqrt{2}-\sqrt{2-b\,x}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (2 - b*x)^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

3.633 $\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

```
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx = -\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Maple [A]
time = 0.11, size = 14, normalized size = 0.82

method	result	size
gosper	$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$	14
default	$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$	14
meijerg	$-\frac{\sqrt{2}\,\sqrt{-\frac{bx}{2}+1}}{\sqrt{x}}$	17
risch	$\frac{(bx-2)\sqrt{(-bx+2)\,x}}{\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A]
time = 0.99, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 41, normalized size = 2.41

$$\begin{cases} -\sqrt{b}\sqrt{-1+\frac{2}{bx}} & \text{for } \frac{1}{|bx|}>\frac{1}{2} \\ -i\sqrt{b}\sqrt{1-\frac{2}{bx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(-1 + 2/(b*x)), 1/Abs(b*x) > 1/2), (-I*sqrt(b)*sqrt
(1 - 2/(b*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.
time = 2.33, size = 30, normalized size = 1.76

$$-\frac{\sqrt{-bx+2}\,b^2}{\sqrt{(bx-2)b+2\,b}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b))
```

Mupad [B]
time = 0.31, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2-b\,x}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(2 - b*x)^(1/2)/x^(1/2)
```

3.634 $\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] -1/3*(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {47, 37}

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -1/3*Sqrt[2 - b*x]/x^(3/2) - (b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx&=-\frac{\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\&=-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 25, normalized size = 0.62

$$\frac{(-1-bx)\sqrt{2-bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 - b*x]),x]
[Out] ((-1 - b*x)*Sqrt[2 - b*x])/(3*x^(3/2))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.72

method	result	size
gosper	$-\frac{(bx+1)\sqrt{-bx+2}}{3x^{\frac{3}{2}}}$	19
meijerg	$-\frac{\sqrt{2}\,(bx+1)\sqrt{-\frac{bx}{2}+1}}{3x^{\frac{3}{2}}}$	22
default	$-\frac{\sqrt{-bx+2}}{3x^{\frac{3}{2}}}-\frac{b\sqrt{-bx+2}}{3\sqrt{x}}$	29
risch	$\frac{\sqrt{(-bx+2)\,x}\,(x^{2}b^{2}-bx-2)}{3x^{\frac{3}{2}}\sqrt{-bx+2}\,\sqrt{-x\,(bx-2)}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-b*x+2)^(1/2)/x^(3/2)-1/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.70

$$-\frac{\sqrt{-bx+2}\,b}{2\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/6*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 1.25, size = 18, normalized size = 0.45

$$-\frac{(bx+1)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x + 1)*sqrt(-b*x + 2)/x^(3/2)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.03, size = 139, normalized size = 3.48

$$\begin{cases} -\frac{b^{\frac{7}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^2x^2-6bx} + \frac{b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^2x^2-6bx} + \frac{2b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^2x^2-6bx} & \text{for } \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} - \frac{i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-b**(7/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**2*x**2 - 6*b*x) + b**(5/
2)*x*sqrt(-1 + 2/(b*x))/(3*b**2*x**2 - 6*b*x) + 2*b**(3/2)*sqrt(-1 + 2/(b*x
))/(3*b**2*x**2 - 6*b*x), 1/Abs(b*x) > 1/2), (-I*b**(3/2)*sqrt(1 - 2/(b*x))
/3 - I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

Giac [A]
time = 1.72, size = 43, normalized size = 1.08

$$-\frac{((bx-2)b^3+3b^3)\sqrt{-bx+2}\,b}{3\left((bx-2)b+2b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((b*x - 2)*b^3 + 3*b^3)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b)^(3/2)*ab
s(b))
```

Mupad [B]
time = 0.29, size = 19, normalized size = 0.48

$$-\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}+\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -((2 - b*x)^(1/2)*((b*x)/3 + 1/3))/x^(3/2)
```

3.635 $\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2x^{5/2}}{b\sqrt{2-bx}}+\frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] -15*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)+2*x^(5/2)/b/(-b*x+2)^(1/2)+
5/2*x^(3/2)*(-b*x+2)^(1/2)/b^2+15/2*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 52, 56, 222}

$$-\frac{15\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[2 - b*x]) + (15*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) + (5*x^(
3/2)*Sqrt[2 - b*x])/(2*b^2) - (15*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{2-bx}} - \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{b} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{2b^3} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\,\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 71, normalized size = 0.80

$$-\frac{\sqrt{x}\,(-30+5bx+b^2x^2)}{2b^3\sqrt{2-bx}} - \frac{15\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] -1/2*(Sqrt[x]*(-30 + 5*b*x + b^2*x^2))/(b^3*Sqrt[2 - b*x]) - (15*Log[-(Sqrt
[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Maple [A]
time = 0.14, size = 81, normalized size = 0.91

method	result
meijerg	$-\frac{8\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(-\frac{7}{2}x^{2}b^{2}-\frac{35}{2}bx+105\right)}{112b^{3}\sqrt{-\frac{bx}{2}+1}}-\frac{15\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{8b^{\frac{7}{2}}}\right)}{(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{(bx+7)\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{2b^{3}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}-\frac{\left(\frac{15\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{2b^{\frac{7}{2}}}+\frac{8\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{b^{4}\left(x-\frac{2}{b}\right)}\right)\sqrt{(-bx}}{\sqrt{x}\,\sqrt{-bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/(-b)^(5/2)/Pi^(1/2)/b*(1/112*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(7/2)*(-7/2*x
^2*b^2-35/2*b*x+105)/b^3/(-1/2*b*x+1)^(1/2)-15/8*Pi^(1/2)*(-b)^(7/2)/b^(7/2
)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 101, normalized size = 1.13

$$\frac{8\,b^{2}-\frac{25\,(bx-2)b}{x}+\frac{15\,(bx-2)^{2}}{x^{2}}}{\frac{\sqrt{-bx+2}\,b^{5}}{\sqrt{x}}+\frac{2\,(-bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}}+\frac{15\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] (8*b^2 - 25*(b*x - 2)*b/x + 15*(b*x - 2)^2/x^2)/(sqrt(-b*x + 2)*b^5/sqrt(x)
 + 2*(-b*x + 2)^(3/2)*b^4/x^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) + 15*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 1.38, size = 155, normalized size = 1.74

$$\left[-\frac{15\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(b^{3}x^{2}+5\,b^{2}x-30\,b)\sqrt{-bx+2}\,\sqrt{x}}{2\,(b^{5}x-2\,b^{4})},\ \frac{30\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(b^{3}x^{2}+5\,b^{2}x-30\,b)\sqrt{-bx+2}\,\sqrt{x}}{2\,(b^{5}x-2\,b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(15*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1
) - (b^3*x^2 + 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4), 1/2
*(30*(b*x - 2)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^3*x^2
+ 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 5.08, size = 172, normalized size = 1.93

$$\begin{cases} \frac{ix^{\frac{5}{2}}}{2b\sqrt{bx-2}} + \frac{5ix^{\frac{3}{2}}}{2b^2\sqrt{bx-2}} - \frac{15i\sqrt{x}}{b^3\sqrt{bx-2}} + \frac{15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for}\ |bx| > 2 \\ -\frac{x^{\frac{5}{2}}}{2b\sqrt{-bx+2}} - \frac{5x^{\frac{3}{2}}}{2b^2\sqrt{-bx+2}} + \frac{15\sqrt{x}}{b^3\sqrt{-bx+2}} - \frac{15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(5/2)/(2*b*sqrt(b*x - 2)) + 5*I*x**(3/2)/(2*b**2*sqrt(b*x -
 2)) - 15*I*sqrt(x)/(b**3*sqrt(b*x - 2)) + 15*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), Abs(b*x) > 2), (-x**(5/2)/(2*b*sqrt(-b*x + 2)) - 5*x**(3/2)
/(2*b**2*sqrt(-b*x + 2)) + 15*sqrt(x)/(b**3*sqrt(-b*x + 2)) - 15*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(7/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(66) = 132.
time = 4.56, size = 136, normalized size = 1.53

$$\frac{\left(\sqrt{(bx-2)b+2b}\sqrt{-bx+2}\left(\frac{bx-2}{b^3}+\frac{9}{b^3}\right)-\frac{15\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}\,b^2}+\frac{64}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b\right)\sqrt{-b}\,b}\right)|b|}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)/b^3 + 9/b^3) - 15*lo
g((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b)*b^2) + 6
4/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b)*b
))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(3/2),x)
[Out] int(x^(5/2)/(2 - b*x)^(3/2), x)
```

3.636 $\int \frac{x^{3/2}}{(2-bx)^{3/2}} dx$

Optimal. Leaf size=65

$$\frac{2x^{3/2}}{b\sqrt{2-bx}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] -6*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)+2*x^(3/2)/b/(-b*x+2)^(1/2)+3
*x^(1/2)*(-b*x+2)^(1/2)/b^2
```

Rubi [A]

time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {49, 52, 56, 222}

$$-\frac{6\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[2 - b*x]) + (3*Sqrt[x]*Sqrt[2 - b*x])/b^2 - (6*ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{2-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b} \\
&= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2} - \frac{3\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2} - \frac{6\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2} \\
&= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\,\sqrt{2-bx}}{b^2} - \frac{6\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 61, normalized size = 0.94

$$\frac{\sqrt{x}\,(6-bx)}{b^2\sqrt{2-bx}} + \frac{6\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*(6 - b*x))/(b^2*Sqrt[2 - b*x]) + (6*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt
[2 - b*x]])/(-b)^(5/2)
```

Maple [A]
time = 0.14, size = 73, normalized size = 1.12

method	result

meijerg	$-\frac{4\left(\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}\left(-\frac{5bx}{2}+15\right)}{20b^2\sqrt{-\frac{bx}{2}+1}}-\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2b^{\frac{5}{2}}}\right)}{(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$
risch	$-\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{b^2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}-\frac{\left(\frac{3\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^2b+2x}}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{-\left(x-\frac{2}{b}\right)^2b-2x+\frac{4}{b}}}{b^3\left(x-\frac{2}{b}\right)}\right)\sqrt{(-bx+}}{\sqrt{x}\,\sqrt{-bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(-b)^(3/2)/Pi^(1/2)/b*(1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-5/2*b*
x+15)/b^2/(-1/2*b*x+1)^(1/2)-3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*b^(
1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 71, normalized size = 1.09

$$\frac{2\left(2\,b-\frac{3\,(bx-2)}{x}\right)}{\frac{\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{6\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x - 2)/x)/(sqrt(-b*x + 2)*b^3/sqrt(x) + (-b*x + 2)^(3/2)*b^2/
x^(3/2)) + 6*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 1.00, size = 138, normalized size = 2.12

$$\left[-\frac{3\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-(b^2x-6\,b)\sqrt{-bx+2}\,\sqrt{x}}{b^4x-2\,b^3},\ \frac{6\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(b^2x-6\,b)\sqrt{-bx+2}\,\sqrt{x}}{b^4x-2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(3*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (
b^2*x - 6*b)*sqrt(-b*x + 2)*sqrt(x))/(b^4*x - 2*b^3), (6*(b*x - 2)*sqrt(b)*
```

```
arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x - 6*b)*sqrt(-b*x + 2)*sqr
t(x))/(b^4*x - 2*b^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.72, size = 126, normalized size = 1.94

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{b\sqrt{bx-2}} - \frac{6i\sqrt{x}}{b^2\sqrt{bx-2}} + \frac{6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } |bx| > 2 \\ -\frac{x^{\frac{3}{2}}}{b\sqrt{-bx+2}} + \frac{6\sqrt{x}}{b^2\sqrt{-bx+2}} - \frac{6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(b*sqrt(b*x - 2)) - 6*I*sqrt(x)/(b**2*sqrt(b*x - 2))
+ 6*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x) > 2), (-x**(3/2)/
(b*sqrt(-b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(-b*x + 2)) - 6*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(5/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(50) = 100.
time = 4.50, size = 119, normalized size = 1.83

$$-\frac{\left(\frac{3\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}} - \frac{\sqrt{(bx-2)b+2b}\sqrt{-bx+2}}{b} + \frac{16\sqrt{-b}}{\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) - s
qrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b + 16*sqrt(-b)/((sqrt(-b*x + 2)*sqrt
(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(b)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(3/2), x)
```

3.637 $\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

```
[Out] -2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(3/2)+2*x^(1/2)/b/(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}}-\frac{2\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Mathematica [A]
time = 0.10, size = 56, normalized size = 1.24

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
)/(-b)^(3/2)
```

Maple [A]
time = 0.10, size = 67, normalized size = 1.49

method	result	size
meijerg	$-\frac{2\left(\frac{\sqrt{\pi}\sqrt{x}\sqrt{2}\,(-b)^{\frac{3}{2}}}{2b\sqrt{-\frac{bx}{2}+1}}-\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{b^{\frac{3}{2}}}\right)}{\sqrt{-b}\sqrt{\pi}\,b}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-b)^(1/2)/Pi^(1/2)/b*(1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(3/2)/b/(-1/2*b
*x+1)^(1/2)-Pi^(1/2)*(-b)^(3/2)/b^(3/2)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))
)
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.84

$$\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{-bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 2)*b)
```

Fricas [A]
time = 1.09, size = 122, normalized size = 2.71

$$\left[-\frac{(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+2\sqrt{-bx+2}\,b\sqrt{x}}{b^3x-2b^2},\frac{2\left((bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+2}\,b\sqrt{x}\right)}{b^3x-2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 2*s
qrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2), 2*((b*x - 2)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.85, size = 90, normalized size = 2.00

$$\begin{cases} -\frac{2i\sqrt{x}}{b\sqrt{bx-2}}+\frac{2i\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for } |bx|>2 \\ \frac{2\sqrt{x}}{b\sqrt{-bx+2}}-\frac{2\,\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*I*sqrt(x)/(b*sqrt(b*x - 2)) + 2*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(3/2), Abs(b*x) > 2), (2*sqrt(x)/(b*sqrt(-b*x + 2)) - 2*asin(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(3/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(34) = 68$.
time = 5.23, size = 92, normalized size = 2.04

$$-\frac{\left(\frac{\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}}+\frac{8\sqrt{-b}}{\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) + 8*s
qrt(-b)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(
b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(2 - b*x)^(3/2), x)
```

3.638 $\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

```
[Out] x^(1/2)/(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2-bx}}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{\sqrt{x}}{\sqrt{-bx+2}}$	13
default	$\frac{\sqrt{x}}{\sqrt{-bx+2}}$	13
meijerg	$\frac{\sqrt{x}\,\sqrt{2}}{2\sqrt{-\frac{bx}{2}+1}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] sqrt(x)/sqrt(-b*x + 2)
```

Fricas [A]
time = 1.07, size = 20, normalized size = 1.25

$$-\frac{\sqrt{-bx+2}\,\sqrt{x}}{bx-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 41, normalized size = 2.56

$$\begin{cases} \frac{1}{\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for}\ \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{i}{\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((1/(sqrt(b)*sqrt(-1 + 2/(b*x))), 1/Abs(b*x) > 1/2), (-I/(sqrt(b)*
sqrt(1 - 2/(b*x))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(12) = 24$.
time = 2.26, size = 50, normalized size = 3.12

$$-\frac{4\sqrt{-b}\,b}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b))
```

Mupad [B]
time = 0.30, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] x^(1/2)/(2 - b*x)^(1/2)
```

3.639 $\int \frac{1}{x^{3/2}(2-bx)^{3/2}} \, dx$

Optimal. Leaf size=34

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

[Out] 1/x^(1/2)/(-b*x+2)^(1/2)-(-b*x+2)^(1/2)/x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(2 - b*x)^(3/2)),x]

[Out] 1/(Sqrt[x]*Sqrt[2 - b*x]) - Sqrt[2 - b*x]/Sqrt[x]

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{3/2}} \, dx &= \frac{1}{\sqrt{x}\sqrt{2-bx}}+\int \frac{1}{x^{3/2}\sqrt{2-bx}} \, dx\\ &= \frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 0.62

$$\frac{-1+bx}{\sqrt{x}\,\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x)/(Sqrt[x]*Sqrt[2 - b*x])
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.82

method	result	size
gosper	$\frac{bx-1}{\sqrt{x}\,\sqrt{-bx+2}}$	18
meijerg	$-\frac{\sqrt{2}\,(-bx+1)}{2\sqrt{x}\,\sqrt{-\frac{bx}{2}+1}}$	23
default	$-\frac{1}{\sqrt{x}\,\sqrt{-bx+2}}+\frac{b\sqrt{x}}{\sqrt{-bx+2}}$	28
risch	$\frac{(bx-2)\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}+\frac{b\sqrt{x}\,\sqrt{(-bx+2)\,x}}{2\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^(1/2)/(-b*x+2)^(1/2)+b*x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.82

$$\frac{b\sqrt{x}}{2\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)/sqrt(x)
```

Fricas [A]
time = 1.17, size = 29, normalized size = 0.85

$$-\frac{(bx-1)\sqrt{-bx+2}\,\sqrt{x}}{bx^2-2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^2 - 2*x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.82, size = 90, normalized size = 2.65

$$\begin{cases} -\frac{b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}}{b^2x-2b}+\frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{b^2x-2b} & \text{for } \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{i\sqrt{b}}{\sqrt{1-\frac{2}{bx}}}+\frac{i}{\sqrt{b}\,x\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-b**(5/2)*x*sqrt(-1 + 2/(b*x))/(b**2*x - 2*b) + b**(3/2)*sqrt(-1
 + 2/(b*x))/(b**2*x - 2*b), 1/Abs(b*x) > 1/2), (-I*sqrt(b)/sqrt(1 - 2/(b*x)
) + I/(sqrt(b)*x*sqrt(1 - 2/(b*x))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(26) = 52.
time = 2.20, size = 83, normalized size = 2.44

$$-\frac{\sqrt{-bx+2}\,b^2}{2\sqrt{(bx-2)b+2\,b}\,|b|}-\frac{2\sqrt{-b}\,b^2}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 2*sqrt(-b)*b^2/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*abs(b))
```

Mupad [B]
time = 0.32, size = 27, normalized size = 0.79

$$\frac{b\sqrt{x}}{\sqrt{2-b\,x}}-\frac{1}{\sqrt{x}\,\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] (b*x^(1/2))/(2 - b*x)^(1/2) - 1/(x^(1/2)*(2 - b*x)^(1/2))
```

3.640 $\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/x^(3/2)/(-b*x+2)^(1/2)-2/3*(-b*x+2)^(1/2)/x^(3/2)-2/3*b*(-b*x+2)^(1/2)/x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {47, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x])/(3*x^(3/2)) - (2*b*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2-bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} - \frac{2b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 33, normalized size = 0.59

$$\frac{-1-2bx+2b^2x^2}{3x^{3/2}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
[Out] (-1 - 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 - b*x])
```

Maple [A]
time = 0.14, size = 45, normalized size = 0.80

method	result	size
gosper	$\frac{2x^2b^2-2bx-1}{3x^{\frac{3}{2}}\sqrt{-bx+2}}$	28
meijerg	$-\frac{\sqrt{2}\,(-2x^2b^2+2bx+1)}{6x^{\frac{3}{2}}\sqrt{-\frac{bx}{2}+1}}$	31
default	$-\frac{1}{3x^{\frac{3}{2}}\sqrt{-bx+2}}+\frac{2b\left(-\frac{1}{\sqrt{x}\,\sqrt{-bx+2}}+\frac{b\sqrt{x}}{\sqrt{-bx+2}}\right)}{3}$	45
risch	$\frac{(5x^2b^2-8bx-4)\sqrt{(-bx+2)\,x}}{12x^{\frac{3}{2}}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{b^2\sqrt{x}\,\sqrt{(-bx+2)\,x}}{4\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/3/x^(3/2)/(-b*x+2)^(1/2)+2/3*b*(-1/x^(1/2)/(-b*x+2)^(1/2)+b*x^(1/2)/(-b*
x+2)^(1/2))
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.79

$$\frac{b^2\sqrt{x}}{4\sqrt{-bx+2}} - \frac{\sqrt{-bx+2}\,b}{2\sqrt{x}} - \frac{(-bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/12*(-b*x
+ 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 1.01, size = 40, normalized size = 0.71

$$-\frac{(2\,b^2x^2-2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}}{3\,(bx^3-2\,x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^3 - 2*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.33, size = 355, normalized size = 6.34

$$\begin{cases} -\frac{2b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{3b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{2ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}+\frac{6ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{3ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x}-\frac{2ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^3-12b^5x^2+12b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*x**2
 + 12*b**4*x) + 6*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*
x**2 + 12*b**4*x) - 3*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5
*x**2 + 12*b**4*x) - 2*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**3 - 12*b**5*x
**2 + 12*b**4*x), 1/Abs(b*x) > 1/2), (-2*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))
/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) + 6*I*b**(13/2)*x**2*sqrt(1 - 2/(
b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) - 3*I*b**(11/2)*x*sqrt(1 - 2
/(b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x) - 2*I*b**(9/2)*sqrt(1 - 2/
(b*x))/(3*b**6*x**3 - 12*b**5*x**2 + 12*b**4*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(40) = 80.
time = 1.79, size = 96, normalized size = 1.71

$$-\frac{\sqrt{-b}\,b^3}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}-\frac{(5\,(bx-2)b^2|b|+12\,b^2|b|)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b)*b^3/((((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b)) - 1/12*(5*(b*x - 2)*b^2*abs(b) + 12*b^2*abs(b))*sqrt(-b*x + 2)/((
b*x - 2)*b + 2*b)^(3/2)
```

Mupad [B]
time = 0.36, size = 38, normalized size = 0.68

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,x}{3}-\frac{2\,b\,x^2}{3}+\frac{1}{3\,b}\right)}{x^{5/2}-\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] ((2 - b*x)^(1/2)*((2*x)/3 - (2*b*x^2)/3 + 1/(3*b)))/(x^(5/2) - (2*x^(3/2))/
b)
```

3.641 $\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

```
[Out] 2/3*x^(5/2)/b/(-b*x+2)^(3/2)+10*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(7/2)
-10/3*x^(3/2)/b^2/(-b*x+2)^(1/2)-5*x^(1/2)*(-b*x+2)^(1/2)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {49, 52, 56, 222}

$$\frac{10\text{ArcSin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(2 - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 - b*x]) - (5
*Sqrt[x]*Sqrt[2 - b*x])/b^3 + (10*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{5\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx}{3b}\\ &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b^2}\\ &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}+\frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b^3}\\ &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}+\frac{10\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\ &= \frac{2x^{5/2}}{3b(2-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 72, normalized size = 0.81

$$-\frac{\sqrt{x}\,(60-40bx+3b^2x^2)}{3b^3(2-bx)^{3/2}}+\frac{10\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] -1/3*(Sqrt[x]*(60 - 40*b*x + 3*b^2*x^2))/(b^3*(2 - b*x)^(3/2)) + (10*Log[-(
Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(-b)^(7/2)
```

Maple [A]
time = 0.12, size = 81, normalized size = 0.91

method	result
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{7}{2}}\left(\frac{21}{4}x^{2}b^{2}-70bx+105\right)}{56b^{3}\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}+\frac{15\sqrt{\pi}\,(-b)^{\frac{7}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{4b^{\frac{7}{2}}}\right)}{3(-b)^{\frac{5}{2}}\sqrt{\pi}\,b}$
risch	$\frac{\sqrt{x}\,(bx-2)\sqrt{(-bx+2)\,x}}{b^{3}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{\left(\frac{5\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{b}\right)}{\sqrt{-x^{2}b+2x}}\right)}{b^{\frac{7}{2}}}+\frac{28\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{3b^{4}\left(x-\frac{2}{b}\right)}+\frac{8\sqrt{-(x-}}{3b}\right.}{\sqrt{x}\,\sqrt{-bx+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3/(-b)^(5/2)/Pi^(1/2)/b*(-1/56*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(7/2)*(21/4
*x^2*b^2-70*b*x+105)/b^3/(-1/2*b*x+1)^(3/2)+15/4*Pi^(1/2)*(-b)^(7/2)/b^(7/2
)*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.48, size = 86, normalized size = 0.97

$$\frac{2\left(2\,b^{2}+\frac{10\,(bx-2)b}{x}-\frac{15\,(bx-2)^{2}}{x^{2}}\right)}{3\left(\frac{(-bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}\right)}-\frac{10\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x - 2)*b/x - 15*(b*x - 2)^2/x^2)/((-b*x + 2)^(3/2)*b^4/x
^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) - 10*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x)))/b^(7/2)
```

Fricas [A]
time = 1.31, size = 187, normalized size = 2.10

$$\left[-\frac{15\,(b^{2}x^{2}-4\,bx+4)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+(3\,b^{3}x^{2}-40\,b^{2}x+60\,b)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^{6}x^{2}-4\,b^{5}x+4\,b^{4})},\,-\frac{30\,(b^{2}x^{2}-4\,bx+4)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+(3\,b^{3}x^{2}-40\,b^{2}x+60\,b)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^{6}x^{2}-4\,b^{5}x+4\,b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(15*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)
*sqrt(x) + 1) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)*sqrt(x))/(b^6*
x^2 - 4*b^5*x + 4*b^4), -1/3*(30*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)
*sqrt(x))/(b^6*x^2 - 4*b^5*x + 4*b^4)]
```

Sympy [C] Result contains complex when optimal does not.
time = 4.08, size = 751, normalized size = 8.44

$$\begin{cases} -\frac{3ib^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{40ib^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{60ib^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{30ib^{10}x^{\frac{27}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{15\pi b^{10}x^{\frac{27}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{60ib^{9}x^{\frac{25}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{30\pi b^{9}x^{\frac{25}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} & \text{for}\: |bx|>2 \\ \frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}-\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}+\frac{30b^{10}x^{\frac{27}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-3*I*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b*
*(25/2)*x**(25/2)*sqrt(b*x - 2)) + 40*I*b**(21/2)*x**14/(3*b**(27/2)*x**(27
/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 60*I*b**(19/2)*x
**13/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x
- 2)) - 30*I*b**10*x**(27/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)
/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)
) + 15*pi*b**10*x**(27/2)*sqrt(b*x - 2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2
) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) + 60*I*b**9*x**(25/2)*sqrt(b*x - 2
)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6
*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 30*pi*b**9*x**(25/2)*sqrt(b*x - 2)/(3
*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)),
Abs(b*x) > 2), (3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6
*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) - 40*b**(21/2)*x**14/(3*b**(27/2)*x**(
27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) + 60*b**(19/2)
*x**13/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-
b*x + 2)) + 30*b**10*x**(27/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x
+ 2)) - 60*b**9*x**(25/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3
*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2))
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(66) = 132.
time = 5.75, size = 200, normalized size = 2.25

$$\frac{\left(\frac{15\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}\right)}{\sqrt{-b}\,b^{2}}-\frac{3\sqrt{(bx-2)b+2b}\sqrt{-bx+2}}{b^{3}}-\frac{16\left(9\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{4}-24\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}b+28b^{2}\right)}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}-2b\right)^{3}\sqrt{-b}\,b}\right)|b|}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b
)*b^2) - 3*sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b^3 - 16*(9*(sqrt(-b*x +
2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4 - 24*(sqrt(-b*x + 2)*sqrt(-b) - sq
rt((b*x - 2)*b + 2*b))^2*b + 28*b^2)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x
 - 2)*b + 2*b))^2 - 2*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(5/2), x)
```

3.642 $\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

```
[Out] 2/3*x^(3/2)/b/(-b*x+2)^(3/2)+2*arcsin(1/2*b^(1/2)*x^(1/2)*2^(1/2))/b^(5/2)-
2*x^(1/2)/b^2/(-b*x+2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {49, 56, 222}

$$\frac{2\text{ArcSin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(2 - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 - b*x]) + (2*Ar
cSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx}{b}\\ &= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2}\\ &= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 64, normalized size = 0.96

$$\frac{4\sqrt{x}\,(-3+2bx)}{3b^2(2-bx)^{3/2}}-\frac{2\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{(-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[x]*(-3 + 2*b*x))/(3*b^2*(2 - b*x)^(3/2)) - (2*Log[-(Sqrt[-b]*Sqrt[x
]) + Sqrt[2 - b*x]])/(-b)^(5/2)
```

Maple [A]
time = 0.13, size = 73, normalized size = 1.09

method	result	size
meijerg	$-\frac{4\left(-\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,(-b)^{\frac{5}{2}}(-10bx+15)}{20b^2\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}+\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{2b^{\frac{5}{2}}}\right)}{3(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/(-b)^(3/2)/Pi^(1/2)/b*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-10*
b*x+15)/b^2/(-1/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*b
^(1/2)*x^(1/2)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 50, normalized size = 0.75

$$\frac{2\left(b+\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+2)^{\frac{3}{2}}b^{2}}-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - 2)/x)*x^(3/2)/((-b*x + 2)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

Fricas [A]
time = 1.47, size = 173, normalized size = 2.58

$$\left[-\frac{3\,(b^2x^2-4\,bx+4)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-4\,(2\,b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^5x^2-4\,b^4x+4\,b^3)},-\frac{2\left(3\,(b^2x^2-4\,bx+4)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\,(2\,b^2x-3\,b)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\,(b^5x^2-4\,b^4x+4\,b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*
sqrt(x) + 1) - 4*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4*x
 + 4*b^3), -2/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))) - 2*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4
*x + 4*b^3)]
```

Sympy [C] Result contains complex when optimal does not.
time = 1.89, size = 648, normalized size = 9.67

$$\begin{cases} \frac{8ib^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{12ib^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6ib^{5}x^{\frac{15}{2}}\sqrt{bx-2}\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{3\pi b^{5}x^{\frac{15}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{12ib^{4}x^{\frac{13}{2}}\sqrt{bx-2}\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6\pi b^{4}x^{\frac{13}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} & \text{for}\: |bx| > 2 \\ -\frac{8b^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{12b^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{6b^{5}x^{\frac{15}{2}}\sqrt{-bx+2}\,\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}-\frac{12b^{4}x^{\frac{13}{2}}\sqrt{-bx+2}\,\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((8*I*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(
13/2)*x**(13/2)*sqrt(b*x - 2)) - 12*I*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*
sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) - 6*I*b**5*x**(15/2)*s
qrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b
*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) + 3*pi*b**5*x**(15/2)*sqrt(b
*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b
*x - 2)) + 12*I*b**4*x**(13/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x -
```

```
2)) - 6*pi*b**4*x**(13/2)*sqrt(b*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2
) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)), Abs(b*x) > 2), (-8*b**(11/2)*x**8
/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*sqrt(-b*x +
2)) + 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*
x**(13/2)*sqrt(-b*x + 2)) + 6*b**5*x**(15/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sq
rt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13
/2)*sqrt(-b*x + 2)) - 12*b**4*x**(13/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)
*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*s
qrt(-b*x + 2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(50) = 100$.
time = 4.90, size = 178, normalized size = 2.66

$$\frac{\left(\frac{3\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}\right)}{\sqrt{-b}}+\frac{16\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{4}\sqrt{-b}-6\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}\sqrt{-b}\,b+8\sqrt{-b}\,b^{2}\right)}{\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}-2b\right)^{3}}\right)|b|}{3b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b)
+ 16*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) - 6*
(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b + 8*sqrt(-
b)*b^2)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3)*ab
s(b)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(5/2), x)
```

3.643 $\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

```
[Out] 1/3*x^(3/2)/(-b*x+2)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{37\}$

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx = \frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Mathematica [A]
time = 0.07, size = 19, normalized size = 1.00

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(5/2),x]
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(13) = 26$.
time = 0.13, size = 49, normalized size = 2.58

method	result	size
gosper	$\frac{x^{\frac{3}{2}}}{3(-bx+2)^{\frac{3}{2}}}$	14
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}}{12\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	17
default	$\frac{\sqrt{x}}{b(-bx+2)^{\frac{3}{2}}}-\frac{\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}}{b}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*x^(1/2)/(-b*x+2)^(3/2)-1/b*(1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*
x+2)^(1/2))
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.68

$$\frac{x^{\frac{3}{2}}}{3\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)/(-b*x + 2)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.88, size = 28, normalized size = 1.47

$$\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{3\,(b^2x^2-4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-b*x + 2)*x^(3/2)/(b^2*x^2 - 4*b*x + 4)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.78, size = 63, normalized size = 3.32

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{3bx\sqrt{bx-2}-6\sqrt{bx-2}} & \text{for } |bx| > 2 \\ -\frac{x^{\frac{3}{2}}}{3bx\sqrt{-bx+2}-6\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(3*b*x*sqrt(b*x - 2) - 6*sqrt(b*x - 2)), Abs(b*x) > 2
), (-x**(3/2)/(3*b*x*sqrt(-b*x + 2) - 6*sqrt(-b*x + 2)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(13) = 26.
time = 3.73, size = 95, normalized size = 5.00

$$\frac{4\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{4}\sqrt{-b}+4\sqrt{-b}\,b^{2}\right)|b|}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}-2b\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) + 4*s
qrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2
 - 2*b)^3*b^2)
```

Mupad [B]
time = 0.23, size = 13, normalized size = 0.68

$$\frac{x^{3/2}}{3\,(2-b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(5/2),x)
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

3.644 $\int \frac{1}{\sqrt{x}\,(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}$$

[Out] 1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x+2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {47, 37}

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]

[Out] Sqrt[x]/(3*(2 - b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 - b*x])

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(2-bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}\,(2-bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 24, normalized size = 0.62

$$-\frac{\sqrt{x}\,(-3+bx)}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
[Out] -1/3*(Sqrt[x]*(-3 + b*x))/(2 - b*x)^(3/2)
```

Maple [A]
time = 0.11, size = 28, normalized size = 0.72

method	result	size
gosper	$-\frac{\sqrt{x}\,(bx-3)}{3(-bx+2)^{\frac{3}{2}}}$	19
meijerg	$\frac{\sqrt{x}\,\sqrt{2}\,(-bx+3)}{12\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	23
default	$\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x+2)^(1/2)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.64

$$\frac{\left(b-\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{6\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
[Out] 1/6*(b - 3*(b*x - 2)/x)*x^(3/2)/(-b*x + 2)^(3/2)
```

Fricas [A]
time = 0.73, size = 33, normalized size = 0.85

$$-\frac{(bx-3)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^2x^2-4\,bx+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x - 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^2 - 4*b*x + 4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.04, size = 165, normalized size = 4.23

$$\begin{cases} \frac{b^2x}{3b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}-6b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}-\frac{3b}{3b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}-6b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}} & \text{for } \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{ibx}{3b^{\frac{3}{2}}x\sqrt{1-\frac{2}{bx}}-6\sqrt{b}\sqrt{1-\frac{2}{bx}}}+\frac{3i}{3b^{\frac{3}{2}}x\sqrt{1-\frac{2}{bx}}-6\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((b**2*x/(3*b**(5/2)*x*sqrt(-1 + 2/(b*x)) - 6*b**(3/2)*sqrt(-1 + 2
/(b*x))) - 3*b/(3*b**(5/2)*x*sqrt(-1 + 2/(b*x)) - 6*b**(3/2)*sqrt(-1 + 2/(b
*x))), 1/Abs(b*x) > 1/2), (-I*b*x/(3*b**(3/2)*x*sqrt(1 - 2/(b*x)) - 6*sqrt(
b)*sqrt(1 - 2/(b*x))) + 3*I/(3*b**(3/2)*x*sqrt(1 - 2/(b*x)) - 6*sqrt(b)*sqr
t(1 - 2/(b*x))), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(27) = 54$.
time = 1.72, size = 90, normalized size = 2.31

$$\frac{8\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b\right)\sqrt{-b}\,b^2}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(
b))
```

Mupad [B]
time = 0.36, size = 45, normalized size = 1.15

$$\frac{3\sqrt{x}\,\sqrt{2-b\,x}-b\,x^{3/2}\,\sqrt{2-b\,x}}{3\,b^2\,x^2-12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(2 - b*x)^(1/2) - b*x^(3/2)*(2 - b*x)^(1/2))/(3*b^2*x^2 - 12*b*x
 + 12)
```

3.645 $\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/3/(-b*x+2)^(3/2)/x^(1/2)+2/3/x^(1/2)/(-b*x+2)^(1/2)-2/3*(-b*x+2)^(1/2)/x^
(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {47, 37}

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 - b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 - b*x]) - (2*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx \\ &= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 33, normalized size = 0.57

$$-\frac{3-6bx+2b^2x^2}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
[Out] -1/3*(3 - 6*b*x + 2*b^2*x^2)/(Sqrt[x]*(2 - b*x)^(3/2))
```

Maple [A]
time = 0.13, size = 45, normalized size = 0.78

method	result	size
gosper	$-\frac{2x^2b^2-6bx+3}{3\sqrt{x}\,(-bx+2)^{\frac{3}{2}}}$	28
meijerg	$-\frac{\sqrt{2}\,(2x^2b^2-6bx+3)}{12\sqrt{x}\,\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	31
default	$-\frac{1}{(-bx+2)^{\frac{3}{2}}\sqrt{x}}+2b\left(\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}\right)$	45
risch	$\frac{(bx-2)\sqrt{(-bx+2)\,x}}{4\sqrt{-x\,(bx-2)}\,\sqrt{x}\,\sqrt{-bx+2}}+\frac{b(5bx-12)\sqrt{x}\,\sqrt{(-bx+2)\,x}}{12\sqrt{-x\,(bx-2)}\,(bx-2)\sqrt{-bx+2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/(-b*x+2)^(3/2)/x^(1/2)+2*b*(1/3*x^(1/2)/(-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x
+2)^(1/2))
```

Maxima [A]
time = 0.27, size = 42, normalized size = 0.72

$$\frac{\left(b^2-\frac{6\,(bx-2)b}{x}\right)x^{\frac{3}{2}}}{12\,(-bx+2)^{\frac{3}{2}}}-\frac{\sqrt{-bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x - 2)*b/x)*x^(3/2)/(-b*x + 2)^(3/2) - 1/4*sqrt(-b*x + 2)/
sqrt(x)
```

Fricas [A]
time = 1.29, size = 46, normalized size = 0.79

$$-\frac{(2 b^2x^2 - 6 bx + 3)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^2x^3 - 4 bx^2 + 4 x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 6*b*x + 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^3 - 4*b*x^2 + 4*
x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.20, size = 245, normalized size = 4.22

$$\begin{cases} -\frac{2b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{for}\ \frac{1}{|bx|} > \frac{1}{2} \\ -\frac{2ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} + \frac{6ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} - \frac{3ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4) + 6*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b
**4) - 3*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b**4), 1
/Abs(b*x) > 1/2), (-2*I*b**(13/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*
b**5*x + 12*b**4) + 6*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**
5*x + 12*b**4) - 3*I*b**(9/2)*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(40) = 80.
time = 2.46, size = 170, normalized size = 2.93

$$-\frac{\sqrt{-bx+2}\,b^2}{4\sqrt{(bx-2)b+2b}\,|b|} - \frac{3\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^4\sqrt{-b}\,b^2 - 24\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^2\sqrt{-b}\,b^3 + 20\sqrt{-b}\,b^4}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^2 - 2b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 1/3*(3*(sqrt(-b*
x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b)*b^2 - 24*(sqrt(-b*x +
 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b^3 + 20*sqrt(-b)*b^4)/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(b))
```

Mupad [B]
time = 0.37, size = 59, normalized size = 1.02

$$\frac{3\,\sqrt{2-b\,x}\,-6\,b\,x\,\sqrt{2-b\,x}\,+2\,b^2\,x^2\,\sqrt{2-b\,x}}{\sqrt{x}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*(2 - b*x)^(1/2) - 6*b*x*(2 - b*x)^(1/2) + 2*b^2*x^2*(2 - b*x)^(1/2))/(x^
(1/2)*(x*(12*b - 3*b^2*x) - 12))
```

3.646 $\int \frac{1}{x^{5/2}(2-bx)^{5/2}} dx$

Optimal. Leaf size=75

$$\frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

```
[Out] 1/3/x^(3/2)/(-b*x+2)^(3/2)+1/x^(3/2)/(-b*x+2)^(1/2)-2/3*(-b*x+2)^(1/2)/x^(3
/2)-2/3*b*(-b*x+2)^(1/2)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {47, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 - b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x
])/(3*x^(3/2)) - (2*b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx \\ &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} + 2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} + \frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx \\ &= \frac{1}{3x^{3/2}(2-bx)^{3/2}} + \frac{1}{x^{3/2}\sqrt{2-bx}} - \frac{2\sqrt{2-bx}}{3x^{3/2}} - \frac{2b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 41, normalized size = 0.55

$$-\frac{1+3bx-6b^2x^2+2b^3x^3}{3x^{3/2}(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(1 + 3*b*x - 6*b^2*x^2 + 2*b^3*x^3)/(x^(3/2)*(2 - b*x)^(3/2))
```

Maple [A]
time = 0.12, size = 61, normalized size = 0.81

method	result	size
gosper	$-\frac{2b^3x^3-6x^2b^2+3bx+1}{3x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}$	36
meijerg	$-\frac{\sqrt{2}\,(2b^3x^3-6x^2b^2+3bx+1)}{12x^{\frac{3}{2}}\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}}$	39
default	$-\frac{1}{3x^{\frac{3}{2}}(-bx+2)^{\frac{3}{2}}}+b\left(-\frac{1}{(-bx+2)^{\frac{3}{2}}\sqrt{x}}+2b\left(\frac{\sqrt{x}}{3(-bx+2)^{\frac{3}{2}}}+\frac{\sqrt{x}}{3\sqrt{-bx+2}}\right)\right)$	61
risch	$\frac{(4x^2b^2-7bx-2)\sqrt{(-bx+2)\,x}}{12x^{\frac{3}{2}}\sqrt{-x\,(bx-2)}\,\sqrt{-bx+2}}+\frac{b^2(4bx-9)\sqrt{x}\,\sqrt{(-bx+2)\,x}}{12\sqrt{-x\,(bx-2)}\,(bx-2)\sqrt{-bx+2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^(3/2)/(-b*x+2)^(3/2)+b*(-1/(-b*x+2)^(3/2)/x^(1/2)+2*b*(1/3*x^(1/2)/(
-b*x+2)^(3/2)+1/3*x^(1/2)/(-b*x+2)^(1/2)))
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.77

$$-\frac{3\sqrt{-bx+2}\,b}{8\sqrt{x}}+\frac{\left(b^{3}-\frac{9\,(bx-2)b^{2}}{x}\right)x^{\frac{3}{2}}}{24\,(-bx+2)^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*sqrt(-b*x + 2)*b/sqrt(x) + 1/24*(b^3 - 9*(b*x - 2)*b^2/x)*x^(3/2)/(-b*
x + 2)^(3/2) - 1/24*(-b*x + 2)^(3/2)/x^(3/2)
```

Fricas [A]
time = 1.48, size = 56, normalized size = 0.75

$$-\frac{(2\,b^{3}x^{3}-6\,b^{2}x^{2}+3\,bx+1)\sqrt{-bx+2}\,\sqrt{x}}{3\,(b^{2}x^{4}-4\,bx^{3}+4\,x^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^3*x^3 - 6*b^2*x^2 + 3*b*x + 1)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^4 -
4*b*x^3 + 4*x^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.98, size = 530, normalized size = 7.07

$$\begin{cases} -\frac{2b^{\frac{27}{2}}x^{4}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{10b^{\frac{25}{2}}x^{3}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}-\frac{15b^{\frac{23}{2}}x^{2}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{5b^{\frac{21}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{2b^{\frac{19}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x} & \text{for}\ \frac{1}{|bx|}>\frac{1}{2} \\ -\frac{2ib^{\frac{27}{2}}x^{4}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{10ib^{\frac{25}{2}}x^{3}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}-\frac{15ib^{\frac{23}{2}}x^{2}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{5ib^{\frac{21}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x}+\frac{2ib^{\frac{19}{2}}\sqrt{1-\frac{2}{bx}}}{3b^{12}x^{4}-18b^{11}x^{3}+36b^{10}x^{2}-24b^{9}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(27/2)*x**4*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**
*3 + 36*b**10*x**2 - 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(-1 + 2/(b*x))/(3*b
**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) - 15*b**(23/2)*x**2*
sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*
x) + 5*b**(21/2)*x*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b*
*10*x**2 - 24*b**9*x) + 2*b**(19/2)*sqrt(-1 + 2/(b*x))/(3*b**12*x**4 - 18*b
**11*x**3 + 36*b**10*x**2 - 24*b**9*x), 1/Abs(b*x) > 1/2), (-2*I*b**(27/2)*
x**4*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b
**9*x) + 10*I*b**(25/2)*x**3*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**
3 + 36*b**10*x**2 - 24*b**9*x) - 15*I*b**(23/2)*x**2*sqrt(1 - 2/(b*x))/(3*b
**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x) + 5*I*b**(21/2)*x*sq
```

```
rt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*x**2 - 24*b**9*x)
+ 2*I*b**(19/2)*sqrt(1 - 2/(b*x))/(3*b**12*x**4 - 18*b**11*x**3 + 36*b**10*
x**2 - 24*b**9*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(53) = 106.
time = 2.78, size = 183, normalized size = 2.44

$$-\frac{\left(4\,(bx-2)b^2|b|+9\,b^2|b|\right)\sqrt{-bx+2}}{12\,\left((bx-2)b+2\,b\right)^{\frac{3}{2}}}-\frac{3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}\,b^3-18\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b^4+16\sqrt{-b}\,b^5}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(4*(b*x - 2)*b^2*abs(b) + 9*b^2*abs(b))*sqrt(-b*x + 2)/((b*x - 2)*b +
 2*b)^(3/2) - 1/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*
sqrt(-b)*b^3 - 18*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqr
t(-b)*b^4 + 16*sqrt(-b)*b^5)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b
+ 2*b))^2 - 2*b)^3*abs(b))
```

Mupad [B]
time = 0.44, size = 73, normalized size = 0.97

$$\frac{\sqrt{2-b\,x}+3\,b\,x\,\sqrt{2-b\,x}-6\,b^2\,x^2\,\sqrt{2-b\,x}+2\,b^3\,x^3\,\sqrt{2-b\,x}}{x^{3/2}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] ((2 - b*x)^(1/2) + 3*b*x*(2 - b*x)^(1/2) - 6*b^2*x^2*(2 - b*x)^(1/2) + 2*b^
3*x^3*(2 - b*x)^(1/2))/(x^(3/2)*(x*(12*b - 3*b^2*x) - 12))
```

3.647 $\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=27

$$-\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

```
[Out] 1/2*arcsin(-1+2*x)-(1-x)^(1/2)*x^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {52, 55, 633, 222}

$$-\frac{1}{2}\text{ArcSin}(1-2x)-\sqrt{1-x}\,\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 - x],x]
[Out] -(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx &= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx\\
&= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\
&= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 1.30

$$-\sqrt{-((-1+x)x)}+2\tan^{-1}\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 - x],x]
[Out] -Sqrt[-((-1 + x)*x)] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

Maple [A]
time = 0.13, size = 41, normalized size = 1.52

method	result	size
meijerg	$\frac{i\left(i\sqrt{\pi}\,\sqrt{x}\,\sqrt{1-x}-i\sqrt{\pi}\,\arcsin\left(\sqrt{x}\right)\right)}{\sqrt{\pi}}$	34
default	$-\sqrt{1-x}\,\sqrt{x}+\frac{\sqrt{x\,(1-x)}\,\arcsin(2x-1)}{2\sqrt{x}\,\sqrt{1-x}}$	41
risch	$\frac{\sqrt{x}\,(-1+x)\sqrt{x\,(1-x)}}{\sqrt{-x\,(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{x\,(1-x)}\,\arcsin(2x-1)}{2\sqrt{x}\,\sqrt{1-x}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(1-x)^(1/2)*x^(1/2)+1/2*(x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(2*x-1)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.37

$$\frac{\sqrt{-x+1}}{\sqrt{x}\left(\frac{x-1}{x}-1\right)}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] sqrt(-x + 1)/(sqrt(x)*((x - 1)/x - 1)) - arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [A]
time = 1.01, size = 27, normalized size = 1.00

$$-\sqrt{x}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.85, size = 54, normalized size = 2.00

$$\begin{cases}-i\sqrt{x}\,\sqrt{x-1}-i\operatorname{acosh}\left(\sqrt{x}\right) & \text{for } |x|>1\\ \frac{x^{\frac{3}{2}}}{\sqrt{1-x}}-\frac{\sqrt{x}}{\sqrt{1-x}}+\operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1-x)**(1/2),x)
[Out] Piecewise((-I*sqrt(x)*sqrt(x - 1) - I*acosh(sqrt(x)), Abs(x) > 1), (x**(3/2
)/sqrt(1 - x) - sqrt(x)/sqrt(1 - x) + asin(sqrt(x)), True))
```

Giac [A]
time = 2.27, size = 17, normalized size = 0.63

$$-\sqrt{x}\,\sqrt{-x+1}+\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] -sqrt(x)*sqrt(-x + 1) + arcsin(sqrt(x))
```

Mupad [B]
time = 0.57, size = 31, normalized size = 1.15

$$2\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-\sqrt{x}\,\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1 - x)^(1/2),x)
[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - x^(1/2)*(1 - x)^(1/2)
```

3.648 $\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] arcsin(-1+2*x)
```

Rubi [A]

time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 633, 222}

$$-\text{ArcSin}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right)\\
&= -\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(8) = 16$.
time = 0.01, size = 38, normalized size = 4.75

$$\frac{2\sqrt{-1+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-1+x}}\right)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[-1 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-1 + x]])/Sqrt[-((-1 + x)*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.
time = 0.12, size = 27, normalized size = 3.38

method	result	size
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{x\left(1-x\right)}\,\arcsin(2x-1)}{\sqrt{x}\,\sqrt{1-x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(2*x-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.49, size = 14, normalized size = 1.75

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 1.15, size = 14, normalized size = 1.75

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.41, size = 20, normalized size = 2.50

$$\begin{cases} -2i \operatorname{acosh}\left(\sqrt{x}\right) & \text{for}\ |x| > 1 \\ 2 \operatorname{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x)), Abs(x) > 1), (2*asin(sqrt(x)), True))
```

Giac [A]
time = 1.91, size = 6, normalized size = 0.75

$$2 \arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(x))
```

Mupad [B]
time = 0.05, size = 16, normalized size = 2.00

$$-4 \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - x)^(1/2)),x)
```

```
[Out] -4*atan(((1 - x)^(1/2) - 1)/x^(1/2))
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

```
[Out] 2*arcsin(b^(1/2)*x^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {56, 222}

$$\frac{2\text{ArcSin}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.84

$$-\frac{2\log\left(-\sqrt{-b}\sqrt{x}+\sqrt{1-bx}\right)}{\sqrt{-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[1 - b*x]])/Sqrt[-b]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(13) = 26$.
time = 0.13, size = 48, normalized size = 2.53

method	result	size
meijerg	$\frac{2 \arcsin\left(\sqrt{b}\,\sqrt{x}\right)}{\sqrt{b}}$	14
default	$\frac{\sqrt{x\left(-bx+1\right)}\,\arctan\left(\frac{\sqrt{b}\,\left(x-\frac{1}{2b}\right)}{\sqrt{-x^{2}b+x}}\right)}{\sqrt{x}\,\sqrt{-bx+1}\,\sqrt{b}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(-b*x+1))^(1/2)/x^(1/2)/(-b*x+1)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b)/
(-b*x^2+x)^(1/2))
```

Maxima [A]
time = 0.49, size = 21, normalized size = 1.11

$$-\frac{2 \arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

Fricas [A]
time = 0.94, size = 57, normalized size = 3.00

$$\left[-\frac{\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+1}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b}, -\frac{2 \arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + 1)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan
(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 42, normalized size = 2.21

$$\begin{cases} -\frac{2i \operatorname{acosh}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}} & \text{for } |bx| > 1 \\ \frac{2 \operatorname{asin}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+1)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x))/sqrt(b), Abs(b*x) > 1), (2*asin(sqrt
(b)*sqrt(x))/sqrt(b), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16
```

Mupad [B]
time = 0.13, size = 23, normalized size = 1.21

$$-\frac{4 \operatorname{atan}\left(\frac{\sqrt{1-b\,x}-1}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((1 - b*x)^(1/2) - 1)/(b^(1/2)*x^(1/2))))/b^(1/2)
```

3.650 $\int x^{5/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

```
[Out] 3/8*a*x^(8/3)+3/11*b*x^(11/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(8/3))/8 + (3*b*x^(11/3))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)\,dx&=\int\left(ax^{5/3}+bx^{8/3}\right)\,dx\\&=\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{88}x^{8/3}(11a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x),x]
```

```
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14
derivativedivides	$\frac{3a\,x^{\frac{8}{3}}}{8}+\frac{3b\,x^{\frac{11}{3}}}{11}$	14
default	$\frac{3a\,x^{\frac{8}{3}}}{8}+\frac{3b\,x^{\frac{11}{3}}}{11}$	14
trager	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14
risch	$\frac{3x^{\frac{8}{3}}(8bx+11a)}{88}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 3/8*a*x^(8/3)+3/11*b*x^(11/3)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Fricas [A]
time = 0.75, size = 18, normalized size = 0.86

$$\frac{3}{88}\left(8\,bx^{3}+11\,ax^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/88*(8*b*x^3 + 11*a*x^2)*x^(2/3)
```

Sympy [A]
time = 0.28, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{8}{3}}}{8}+\frac{3bx^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a),x)
[Out] 3*a*x**(8/3)/8 + 3*b*x**(11/3)/11
```

Giac [A]
time = 2.02, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}} + \frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="giac")
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.62

$$\frac{3\,x^{8/3}\,\left(11\,a+8\,b\,x\right)}{88}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x),x)
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

3.651 $\int x^{4/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

```
[Out] 3/7*a*x^(7/3)+3/10*b*x^(10/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x),x]
[Out] (3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)\,dx&=\int\left(ax^{4/3}+bx^{7/3}\right)\,dx\\&=\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{70}x^{7/3}(10a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x),x]
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
derivativedivides	$\frac{3a\,x^{\frac{7}{3}}}{7}+\frac{3b\,x^{\frac{10}{3}}}{10}$	14
default	$\frac{3a\,x^{\frac{7}{3}}}{7}+\frac{3b\,x^{\frac{10}{3}}}{10}$	14
trager	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14
risch	$\frac{3x^{\frac{7}{3}}(7bx+10a)}{70}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 3/7*a*x^(7/3)+3/10*b*x^(10/3)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{3}{10}\,bx^{\frac{10}{3}}+\frac{3}{7}\,ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Fricas [A]
time = 0.80, size = 18, normalized size = 0.86

$$\frac{3}{70}\left(7\,bx^{3}+10\,ax^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/70*(7*b*x^3 + 10*a*x^2)*x^(1/3)
```

Sympy [A]
time = 0.20, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{7}{3}}}{7}+\frac{3bx^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(7/3)/7 + 3*b*x**(10/3)/10
```

Giac [A]
time = 2.04, size = 13, normalized size = 0.62

$$\frac{3}{10}bx^{\frac{10}{3}}+\frac{3}{7}ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{7/3}\,\left(10\,a+7\,b\,x\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x),x)
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

3.652 $\int x^{2/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

```
[Out] 3/5*a*x^(5/3)+3/8*b*x^(8/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x),x]
[Out] (3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)\,dx &= \int\left(ax^{2/3}+bx^{5/3}\right)\,dx\\ &= \frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{40}x^{5/3}(8a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x),x]
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
derivativedivides	$\frac{3a\,x^{\frac{5}{3}}}{5}+\frac{3b\,x^{\frac{8}{3}}}{8}$	14
default	$\frac{3a\,x^{\frac{5}{3}}}{5}+\frac{3b\,x^{\frac{8}{3}}}{8}$	14
trager	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14
risch	$\frac{3x^{\frac{5}{3}}(5bx+8a)}{40}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 3/5*a*x^(5/3)+3/8*b*x^(8/3)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.62

$$\frac{3}{8}\,bx^{\frac{8}{3}}+\frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Fricas [A]
time = 0.91, size = 16, normalized size = 0.76

$$\frac{3}{40}\left(5\,bx^2+8\,ax\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/40*(5*b*x^2 + 8*a*x)*x^(2/3)
```

Sympy [A]
time = 0.11, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{5}{3}}}{5}+\frac{3bx^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a),x)
[Out] 3*a*x**(5/3)/5 + 3*b*x**(8/3)/8
```

Giac [A]
time = 1.44, size = 13, normalized size = 0.62

$$\frac{3}{8}\,bx^{\frac{8}{3}} + \frac{3}{5}\,ax^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="giac")
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{5/3}\,\left(8\,a+5\,b\,x\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x),x)
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

3.653 $\int \sqrt[3]{x}\,(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

```
[Out] 3/4*a*x^(4/3)+3/7*b*x^(7/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = $\{45\}$

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}\,(a+bx)\,dx &= \int \left(a\sqrt[3]{x}+bx^{4/3}\right)\,dx\\ &= \frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{28}x^{4/3}(7a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

Maple [A]
time = 0.07, size = 14, normalized size = 0.67

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
derivativedivides	$\frac{3a\,x^{\frac{4}{3}}}{4}+\frac{3b\,x^{\frac{7}{3}}}{7}$	14
default	$\frac{3a\,x^{\frac{4}{3}}}{4}+\frac{3b\,x^{\frac{7}{3}}}{7}$	14
trager	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14
risch	$\frac{3x^{\frac{4}{3}}(4bx+7a)}{28}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*a*x^(4/3)+3/7*b*x^(7/3)
```

Maxima [A]
time = 0.26, size = 13, normalized size = 0.62

$$\frac{3}{7}\,bx^{\frac{7}{3}}+\frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Fricas [A]
time = 0.85, size = 16, normalized size = 0.76

$$\frac{3}{28}\,(4\,bx^2+7\,ax)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b*x^2 + 7*a*x)*x^(1/3)
```

Sympy [A]
time = 0.64, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{4}{3}}}{4}+\frac{3bx^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a),x)
```

```
[Out] 3*a*x**(4/3)/4 + 3*b*x**(7/3)/7
```

Giac [A]
time = 1.62, size = 13, normalized size = 0.62

$$\frac{3}{7}\,bx^{\frac{7}{3}} + \frac{3}{4}\,ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="giac")
```

```
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{4/3}\,(7\,a+4\,b\,x)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x),x)
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

3.654 $\int \frac{a+bx}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=21

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

```
[Out] 3/2*a*x^(2/3)+3/5*b*x^(5/3)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a}{\sqrt[3]{x}}+bx^{2/3}\right)\,dx\\ &= \frac{3}{2}ax^{2/3}+\frac{3}{5}bx^{5/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{10}x^{2/3}(5a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
trager	$\left(\frac{3bx}{5}+\frac{3a}{2}\right)x^{\frac{2}{3}}$	13
gosper	$\frac{3x^{\frac{2}{3}}(2bx+5a)}{10}$	14
derivativedivides	$\frac{3a\,x^{\frac{2}{3}}}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	14
default	$\frac{3a\,x^{\frac{2}{3}}}{2}+\frac{3b\,x^{\frac{5}{3}}}{5}$	14
risch	$\frac{3x^{\frac{2}{3}}(2bx+5a)}{10}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*a*x^(2/3)+3/5*b*x^(5/3)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="maxima")
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Fricas [A]
time = 0.68, size = 13, normalized size = 0.62

$$\frac{3}{10}\,(2\,bx+5\,a)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="fricas")
[Out] 3/10*(2*b*x + 5*a)*x^(2/3)
```

Sympy [A]
time = 0.72, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{2}{3}}}{2}+\frac{3bx^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/3),x)
```

```
[Out] 3*a*x**(2/3)/2 + 3*b*x**(5/3)/5
```

Giac [A]
time = 1.75, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}}+\frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{2/3}\,\left(5\,a+2\,b\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

3.655 $\int \frac{a+bx}{x^{2/3}}\,dx$

Optimal. Leaf size=19

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

```
[Out] 3*a*x^(1/3)+3/4*b*x^(4/3)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(2/3),x]
```

```
[Out] 3*a*x^(1/3) + (3*b*x^(4/3))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{2/3}}\,dx &= \int \left(\frac{a}{x^{2/3}}+b\sqrt[3]{x}\right)\,dx\\ &= 3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.84

$$\frac{3}{4}\sqrt[3]{x}\,(4a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
trager	$\left(\frac{3bx}{4}+3a\right)x^{\frac{1}{3}}$	13
risch	$\frac{3x^{\frac{1}{3}}(bx+4a)}{4}$	13
derivativedivides	$3a\,x^{\frac{1}{3}}+\frac{3b\,x^{\frac{4}{3}}}{4}$	14
default	$3a\,x^{\frac{1}{3}}+\frac{3b\,x^{\frac{4}{3}}}{4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(2/3),x,method=_RETURNVERBOSE)
[Out] 3*a*x^(1/3)+3/4*b*x^(4/3)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$\frac{3}{4}bx^{\frac{4}{3}}+3\,ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="maxima")
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Fricas [A]
time = 0.52, size = 12, normalized size = 0.63

$$\frac{3}{4}\,(bx+4\,a)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="fricas")
[Out] 3/4*(b*x + 4*a)*x^(1/3)
```

Sympy [A]
time = 0.46, size = 17, normalized size = 0.89

$$3a\sqrt[3]{x}+\frac{3bx^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(2/3),x)
```

```
[Out] 3*a*x**(1/3) + 3*b*x**(4/3)/4
```

Giac [**A**]
time = 1.32, size = 13, normalized size = 0.68

$$\frac{3}{4}\,bx^{\frac{4}{3}} + 3\,ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.63

$$\frac{3\,x^{1/3}\,(4\,a+b\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(2/3),x)
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

3.656 $\int \frac{a+bx}{x^{4/3}}\,dx$

Optimal. Leaf size=19

$$-\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}$$

```
[Out] -3*a/x^(1/3)+3/2*b*x^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(4/3),x]
```

```
[Out] (-3*a)/x^(1/3) + (3*b*x^(2/3))/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{4/3}}\,dx &= \int \left(\frac{a}{x^{4/3}}+\frac{b}{\sqrt[3]{x}}\right)dx\\ &= -\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.89

$$-\frac{3(2a-bx)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(4/3),x]
```

```
[Out] (-3*(2*a - b*x))/(2*x^(1/3))
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
derivativedivides	$-\frac{3a}{x^{\frac{1}{3}}}+\frac{3b\,x^{\frac{2}{3}}}{2}$	14
default	$-\frac{3a}{x^{\frac{1}{3}}}+\frac{3b\,x^{\frac{2}{3}}}{2}$	14
trager	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14
risch	$-\frac{3(-bx+2a)}{2x^{\frac{1}{3}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(4/3),x,method=_RETURNVERBOSE)
[Out] -3*a/x^(1/3)+3/2*b*x^(2/3)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.68

$$\frac{3}{2}\,bx^{\frac{2}{3}}-\frac{3\,a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="maxima")
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Fricas [A]
time = 0.83, size = 12, normalized size = 0.63

$$\frac{3\,(bx-2\,a)}{2\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="fricas")
[Out] 3/2*(b*x - 2*a)/x^(1/3)
```

Sympy [A]
time = 0.15, size = 17, normalized size = 0.89

$$-\frac{3a}{\sqrt[3]{x}}+\frac{3bx^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(4/3),x)
```

```
[Out] -3*a/x**(1/3) + 3*b*x**(2/3)/2
```

Giac [A]
time = 1.89, size = 13, normalized size = 0.68

$$\frac{3}{2}\,bx^{\frac{2}{3}} - \frac{3\,a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$-\frac{6\,a-3\,b\,x}{2\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(4/3),x)
```

```
[Out] -(6*a - 3*b*x)/(2*x^(1/3))
```

3.657 $\int \frac{a+bx}{x^{5/3}} \, dx$

Optimal. Leaf size=19

$$-\frac{3a}{2x^{2/3}}+3b\sqrt[3]{x}$$

```
[Out] -3/2*a/x^(2/3)+3*b*x^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$3b\sqrt[3]{x}-\frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/3),x]
```

```
[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/3}} \, dx &= \int \left(\frac{a}{x^{5/3}}+\frac{b}{x^{2/3}}\right) \, dx\\ &= -\frac{3a}{2x^{2/3}}+3b\sqrt[3]{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.79

$$-\frac{3(a-2bx)}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/3),x]
```

```
[Out] (-3*(a - 2*b*x))/(2*x^(2/3))
```

Maple [A]
time = 0.03, size = 14, normalized size = 0.74

method	result	size
gosper	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
trager	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
risch	$-\frac{3(-2bx+a)}{2x^{\frac{2}{3}}}$	12
derivativedivides	$-\frac{3a}{2x^{\frac{2}{3}}}+3b\,x^{\frac{1}{3}}$	14
default	$-\frac{3a}{2x^{\frac{2}{3}}}+3b\,x^{\frac{1}{3}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/3),x,method=_RETURNVERBOSE)
[Out] -3/2*a/x^(2/3)+3*b*x^(1/3)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}}-\frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="maxima")
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Fricas [A]
time = 0.79, size = 13, normalized size = 0.68

$$\frac{3\,(2\,bx-a)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="fricas")
[Out] 3/2*(2*b*x - a)/x^(2/3)
```

Sympy [A]
time = 0.17, size = 17, normalized size = 0.89

$$-\frac{3a}{2x^{\frac{2}{3}}}+3b\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/3),x)
[Out] -3*a/(2*x**(2/3)) + 3*b*x**(1/3)
```

Giac [A]
time = 2.17, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="giac")
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$-\frac{3\,a - 6\,b\,x}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/3),x)
[Out] -(3*a - 6*b*x)/(2*x^(2/3))
```

3.658 $\int x^{5/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

```
[Out] 3/8*a^2*x^(8/3)+6/11*a*b*x^(11/3)+3/14*b^2*x^(14/3)
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^2\,dx&=\int\left(a^2x^{5/3}+2abx^{8/3}+b^2x^{11/3}\right)\,dx\\&=\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}\left(44x^{2}b^{2}+112abx+77a^{2}\right)}{616}$	25
derivativedivides	$\frac{3a^{2}x^{\frac{8}{3}}}{8}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{3b^{2}x^{\frac{14}{3}}}{14}$	25
default	$\frac{3a^{2}x^{\frac{8}{3}}}{8}+\frac{6ab\,x^{\frac{11}{3}}}{11}+\frac{3b^{2}x^{\frac{14}{3}}}{14}$	25
trager	$\frac{3x^{\frac{8}{3}}\left(44x^{2}b^{2}+112abx+77a^{2}\right)}{616}$	25
risch	$\frac{3x^{\frac{8}{3}}\left(44x^{2}b^{2}+112abx+77a^{2}\right)}{616}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/8*a^2*x^(8/3)+6/11*a*b*x^(11/3)+3/14*b^2*x^(14/3)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^{2}x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^{2}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Fricas [A]
time = 0.65, size = 29, normalized size = 0.81

$$\frac{3}{616}\left(44\,b^{2}x^{4}+112\,abx^{3}+77\,a^{2}x^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/616*(44*b^2*x^4 + 112*a*b*x^3 + 77*a^2*x^2)*x^(2/3)
```

Sympy [A]
time = 0.42, size = 34, normalized size = 0.94

$$\frac{3a^{2}x^{\frac{8}{3}}}{8}+\frac{6abx^{\frac{11}{3}}}{11}+\frac{3b^{2}x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(8/3)/8 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(14/3)/14
```

Giac [A]
time = 1.63, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^2x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^2x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{8/3}\,\left(77\,a^2+112\,a\,b\,x+44\,b^2\,x^2\right)}{616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(8/3)*(77*a^2 + 44*b^2*x^2 + 112*a*b*x))/616
```

3.659 $\int x^{4/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

```
[Out] 3/7*a^2*x^(7/3)+3/5*a*b*x^(10/3)+3/13*b^2*x^(13/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^2\,dx &= \int \left(a^2x^{4/3}+2abx^{7/3}+b^2x^{10/3}\right)\,dx\\ &= \frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}\left(35x^{2}b^{2}+91abx+65a^{2}\right)}{455}$	25
derivativedivides	$\frac{3a^{2}x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{10}{3}}}{5}+\frac{3b^{2}x^{\frac{13}{3}}}{13}$	25
default	$\frac{3a^{2}x^{\frac{7}{3}}}{7}+\frac{3ab\,x^{\frac{10}{3}}}{5}+\frac{3b^{2}x^{\frac{13}{3}}}{13}$	25
trager	$\frac{3x^{\frac{7}{3}}\left(35x^{2}b^{2}+91abx+65a^{2}\right)}{455}$	25
risch	$\frac{3x^{\frac{7}{3}}\left(35x^{2}b^{2}+91abx+65a^{2}\right)}{455}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/7*a^2*x^(7/3)+3/5*a*b*x^(10/3)+3/13*b^2*x^(13/3)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^{2}x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^{2}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Fricas [A]
time = 0.52, size = 29, normalized size = 0.81

$$\frac{3}{455}\left(35\,b^{2}x^{4}+91\,abx^{3}+65\,a^{2}x^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/455*(35*b^2*x^4 + 91*a*b*x^3 + 65*a^2*x^2)*x^(1/3)
```

Sympy [A]
time = 0.34, size = 34, normalized size = 0.94

$$\frac{3a^{2}x^{\frac{7}{3}}}{7}+\frac{3abx^{\frac{10}{3}}}{5}+\frac{3b^{2}x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(7/3)/7 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(13/3)/13
```

Giac [A]
time = 2.00, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^2x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^2x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{7/3}\,\left(65\,a^2+91\,a\,b\,x+35\,b^2\,x^2\right)}{455}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(7/3)*(65*a^2 + 35*b^2*x^2 + 91*a*b*x))/455
```

3.660 $\int x^{2/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

```
[Out] 3/5*a^2*x^(5/3)+3/4*a*b*x^(8/3)+3/11*b^2*x^(11/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^2\,dx &= \int \left(a^2x^{2/3}+2abx^{5/3}+b^2x^{8/3}\right)\,dx\\ &= \frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{220}x^{5/3}\left(44a^2+55abx+20b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*(44*a^2 + 55*a*b*x + 20*b^2*x^2))/220
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}\left(20x^{2}b^{2}+55abx+44a^{2}\right)}{220}$	25
derivativedivides	$\frac{3a^{2}x^{\frac{5}{3}}}{5}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{3b^{2}x^{\frac{11}{3}}}{11}$	25
default	$\frac{3a^{2}x^{\frac{5}{3}}}{5}+\frac{3ab\,x^{\frac{8}{3}}}{4}+\frac{3b^{2}x^{\frac{11}{3}}}{11}$	25
trager	$\frac{3x^{\frac{5}{3}}\left(20x^{2}b^{2}+55abx+44a^{2}\right)}{220}$	25
risch	$\frac{3x^{\frac{5}{3}}\left(20x^{2}b^{2}+55abx+44a^{2}\right)}{220}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 3/5*a^2*x^(5/3)+3/4*a*b*x^(8/3)+3/11*b^2*x^(11/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^{2}x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^{2}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Fricas [A]
time = 0.86, size = 27, normalized size = 0.75

$$\frac{3}{220}\left(20\,b^{2}x^{3}+55\,abx^{2}+44\,a^{2}x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/220*(20*b^2*x^3 + 55*a*b*x^2 + 44*a^2*x)*x^(2/3)
```

Sympy [A]
time = 0.18, size = 34, normalized size = 0.94

$$\frac{3a^{2}x^{\frac{5}{3}}}{5}+\frac{3abx^{\frac{8}{3}}}{4}+\frac{3b^{2}x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(5/3)/5 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(11/3)/11
```

Giac [A]
time = 1.58, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^2x^{\frac{11}{3}}+\frac{3}{4}\,abx^{\frac{8}{3}}+\frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{5/3}\,\left(44\,a^2+55\,a\,b\,x+20\,b^2\,x^2\right)}{220}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(5/3)*(44*a^2 + 20*b^2*x^2 + 55*a*b*x))/220
```

3.661 $\int \sqrt[3]{x} (a+bx)^2 dx$

Optimal. Leaf size=36

$$\frac{3}{4}a^2x^{4/3} + \frac{6}{7}abx^{7/3} + \frac{3}{10}b^2x^{10/3}$$

```
[Out] 3/4*a^2*x^(4/3)+6/7*a*b*x^(7/3)+3/10*b^2*x^(10/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{4}a^2x^{4/3} + \frac{6}{7}abx^{7/3} + \frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)^2\,dx &= \int \left(a^2\sqrt[3]{x} + 2abx^{4/3} + b^2x^{7/3}\right) dx\\ &= \frac{3}{4}a^2x^{4/3} + \frac{6}{7}abx^{7/3} + \frac{3}{10}b^2x^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{140}x^{4/3}\left(35a^2 + 40abx + 14b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*(35*a^2 + 40*a*b*x + 14*b^2*x^2))/140
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.69

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}\left(14x^{2}b^{2}+40abx+35a^{2}\right)}{140}$	25
derivativedivides	$\frac{3a^{2}x^{\frac{4}{3}}}{4}+\frac{6abx^{\frac{7}{3}}}{7}+\frac{3b^{2}x^{\frac{10}{3}}}{10}$	25
default	$\frac{3a^{2}x^{\frac{4}{3}}}{4}+\frac{6abx^{\frac{7}{3}}}{7}+\frac{3b^{2}x^{\frac{10}{3}}}{10}$	25
trager	$\frac{3x^{\frac{4}{3}}\left(14x^{2}b^{2}+40abx+35a^{2}\right)}{140}$	25
risch	$\frac{3x^{\frac{4}{3}}\left(14x^{2}b^{2}+40abx+35a^{2}\right)}{140}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*a^2*x^(4/3)+6/7*a*b*x^(7/3)+3/10*b^2*x^(10/3)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^{2}x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^{2}x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Fricas [A]
time = 1.32, size = 27, normalized size = 0.75

$$\frac{3}{140}\left(14\,b^{2}x^{3}+40\,abx^{2}+35\,a^{2}x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^2*x^3 + 40*a*b*x^2 + 35*a^2*x)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.01, size = 2633, normalized size = 73.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**2,x)
```

```
[Out] Piecewise((27*a**(34/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(-140*a**
8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*
a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3
*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7
*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I
*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 72*a**(31/3)*b*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2
*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(
a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3))
- 81*a**(31/3)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(28/3)*b**2*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp
(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)
*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)
) + 81*a**(28/3)*b**2*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*
a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp
(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(25/3)*
b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-140*a**8*b**(
4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b
**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2
*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3
) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x
)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 135*a
**(22/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(2*I*pi/3)/(-140*
a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 4
20*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)
**3*exp(2*I*pi/3)) - 132*a**(19/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b +
x)**5*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(
a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 14
0*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 42*a**(16/3)*b**6*(-1 + b*(a
/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi
/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b +
 x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), Abs(
b*(a/b + x)/a) > 1), (-27*a**(34/3)*(1 - b*(a/b + x)/a)**(1/3)/(-140*a**8*b
**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**
6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*ex
p(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b*
*(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi
/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 72*a**(31/3)*b*(1 -
b*(a/b + x)/a)**(1/3)*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**
7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*
I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(31/3)*b*(
```

```
a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*ex
p(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(
13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(28/3)*b**2*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a
/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140
*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 81*a**(28/3)*b**2*(a/b + x)**
2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*p
i/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(25/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(
a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)
*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b
**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140
*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) -
420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x
)**3*exp(2*I*pi/3)) - 135*a**(22/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b +
x)**4/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2
*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/
3)*(a/b + x)**3*exp(2*I*pi/3)) + 132*a**(19/3)*b**5*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)**5/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b
 + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a
**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 42*a**(16/3)*b**6*(1 - b*(a/b +
 x)/a)**(1/3)*(a/b + x)**6/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi...
```

Giac [A]
time = 1.98, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^2x^{\frac{10}{3}}+\frac{6}{7}\,abx^{\frac{7}{3}}+\frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="giac")
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{4/3}\,\left(35\,a^2+40\,a\,b\,x+14\,b^2\,x^2\right)}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^2,x)
[Out] (3*x^(4/3)*(35*a^2 + 14*b^2*x^2 + 40*a*b*x))/140
```

3.662 $\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=36

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

```
[Out] 3/2*a^2*x^(2/3)+6/5*a*b*x^(5/3)+3/8*b^2*x^(8/3)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt[3]{x}}\,dx&=\int \left(\frac{a^2}{\sqrt[3]{x}}+2abx^{2/3}+b^2x^{5/3}\right)dx\\&=\frac{3}{2}a^2x^{2/3}+\frac{6}{5}abx^{5/3}+\frac{3}{8}b^2x^{8/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{40}x^{2/3}\left(20a^2+16abx+5b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(20*a^2 + 16*a*b*x + 5*b^2*x^2))/40
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.69

method	result	size
trager	$\left(\frac{3}{8}x^2b^2+\frac{6}{5}abx+\frac{3}{2}a^2\right)x^{\frac{2}{3}}$	24
gosper	$\frac{3x^{\frac{2}{3}}\left(5x^2b^2+16abx+20a^2\right)}{40}$	25
derivativedivides	$\frac{3a^2x^{\frac{2}{3}}}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+\frac{3b^2x^{\frac{8}{3}}}{8}$	25
default	$\frac{3a^2x^{\frac{2}{3}}}{2}+\frac{6ab\,x^{\frac{5}{3}}}{5}+\frac{3b^2x^{\frac{8}{3}}}{8}$	25
risch	$\frac{3x^{\frac{2}{3}}\left(5x^2b^2+16abx+20a^2\right)}{40}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*a^2*x^(2/3)+6/5*a*b*x^(5/3)+3/8*b^2*x^(8/3)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^2x^{\frac{8}{3}}+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{2}\,a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="maxima")
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Fricas [A]
time = 1.07, size = 24, normalized size = 0.67

$$\frac{3}{40}\left(5\,b^2x^2+16\,abx+20\,a^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="fricas")
[Out] 3/40*(5*b^2*x^2 + 16*a*b*x + 20*a^2)*x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 1765, normalized size = 49.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2/x**(1/3),x)

[Out] Piecewise((-27*a**(32/3)*(-1 + b*(a/b + x)/a)**(2/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 63*a**(29/3)*b*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 18*a**(23/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(20/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 15*a**(17/3)*b**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3), Abs(b*(a/b + x)/a) > 1), (-27*a**(32/3)*(1 - b*(a/b + x)/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 63*a**(29/3)*b*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 18*a**(23/3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(

```
20/3)*b**4*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(-40*a**8*
b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 4
0*a**5*b**(11/3)*(a/b + x)**3) + 15*a**(17/3)*b**5*(1 - b*(a/b + x)/a)**(2/
3)*(a/b + x)**5*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b +
 x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3), Tru
e))
```

Giac [A]
time = 1.65, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^2x^{\frac{8}{3}}+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{3}{2}\,a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="giac")
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{2/3}\,\left(20\,a^2+16\,a\,b\,x+5\,b^2\,x^2\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/3),x)
[Out] (3*x^(2/3)*(20*a^2 + 5*b^2*x^2 + 16*a*b*x))/40
```

3.663 $\int \frac{(a+bx)^2}{x^{2/3}}\, dx$

Optimal. Leaf size=34

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

[Out] 3*a^2*x^(1/3)+3/2*a*b*x^(4/3)+3/7*b^2*x^(7/3)

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/x^(2/3),x]

[Out] 3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{2/3}}\, dx &= \int \left(\frac{a^2}{x^{2/3}}+2ab\sqrt[3]{x}+b^2x^{4/3}\right) dx\\ &= 3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.82

$$\frac{3}{14}\sqrt[3]{x}\left(14a^2+7abx+2b^2x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2/x^(2/3),x]

```
[Out] (3*x^(1/3)*(14*a^2 + 7*a*b*x + 2*b^2*x^2))/14
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.74

method	result	size
trager	$\left(\frac{3}{7}x^{2}b^{2}+\frac{3}{2}abx+3a^{2}\right)x^{\frac{1}{3}}$	24
gosper	$\frac{3x^{\frac{1}{3}}\left(2x^{2}b^{2}+7abx+14a^{2}\right)}{14}$	25
derivativedivides	$3a^{2}x^{\frac{1}{3}}+\frac{3abx^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{7}{3}}}{7}$	25
default	$3a^{2}x^{\frac{1}{3}}+\frac{3abx^{\frac{4}{3}}}{2}+\frac{3b^{2}x^{\frac{7}{3}}}{7}$	25
risch	$\frac{3x^{\frac{1}{3}}\left(2x^{2}b^{2}+7abx+14a^{2}\right)}{14}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(2/3),x,method=_RETURNVERBOSE)
[Out] 3*a^2*x^(1/3)+3/2*a*b*x^(4/3)+3/7*b^2*x^(7/3)
```

Maxima [A]
time = 0.26, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^{2}x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^{2}x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="maxima")
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Fricas [A]
time = 0.96, size = 24, normalized size = 0.71

$$\frac{3}{14}\left(2\,b^{2}x^{2}+7\,abx+14\,a^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="fricas")
[Out] 3/14*(2*b^2*x^2 + 7*a*b*x + 14*a^2)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.98, size = 1741, normalized size = 51.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2/x**(2/3),x)

[Out] Piecewise((-27*a**(31/3)*(-1 + b*(a/b + x)/a)**(1/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 72*a**(28/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 9*a**(19/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3), Abs(b*(a/b + x)/a) > 1), (-27*a**(31/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 72*a**(28/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 9*a**(19/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*pi/3)/

```
(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x
)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(1 - b*(a/b + x)/
a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a
/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3),
True))
```

Giac [A]
time = 1.72, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="giac")
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.71

$$\frac{3\,x^{1/3}\,\left(14\,a^2+7\,a\,b\,x+2\,b^2\,x^2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(2/3),x)
[Out] (3*x^(1/3)*(14*a^2 + 2*b^2*x^2 + 7*a*b*x))/14
```

3.664 $\int \frac{(a+bx)^2}{x^{4/3}} \, dx$

Optimal. Leaf size=32

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

```
[Out] -3*a^2/x^(1/3)+3*a*b*x^(2/3)+3/5*b^2*x^(5/3)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(4/3),x]
```

```
[Out] (-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{4/3}} \, dx &= \int \left(\frac{a^2}{x^{4/3}}+\frac{2ab}{\sqrt[3]{x}}+b^2x^{2/3}\right) \, dx\\ &= -\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.88

$$-\frac{3(5a^2-5abx-b^2x^2)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(4/3),x]
[Out] (-3*(5*a^2 - 5*a*b*x - b^2*x^2))/(5*x^(1/3))
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.78

method	result	size
gosper	$-\frac{3(-x^2b^2-5abx+5a^2)}{5x^{\frac{1}{3}}}$	25
derivativedivides	$-\frac{3a^2}{x^{\frac{1}{3}}}+3ab\,x^{\frac{2}{3}}+\frac{3b^2x^{\frac{5}{3}}}{5}$	25
default	$-\frac{3a^2}{x^{\frac{1}{3}}}+3ab\,x^{\frac{2}{3}}+\frac{3b^2x^{\frac{5}{3}}}{5}$	25
trager	$-\frac{3(-x^2b^2-5abx+5a^2)}{5x^{\frac{1}{3}}}$	25
risch	$-\frac{3(-x^2b^2-5abx+5a^2)}{5x^{\frac{1}{3}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(4/3),x,method=_RETURNVERBOSE)
[Out] -3*a^2/x^(1/3)+3*a*b*x^(2/3)+3/5*b^2*x^(5/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="maxima")
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Fricas [A]
time = 0.71, size = 23, normalized size = 0.72

$$\frac{3\,(b^2x^2+5\,abx-5\,a^2)}{5\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="fricas")
[Out] 3/5*(b^2*x^2 + 5*a*b*x - 5*a^2)/x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.06, size = 1826, normalized size = 57.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(4/3),x)
```

```
[Out] Piecewise((-27*a**(29/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(
-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
 x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/3)*b
**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b*
*2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 63*a*
*(26/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(-5*a**8
*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*
exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)
*(a/b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6
*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 42
*a**(23/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(I*pi/3)/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b*
*(7/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3)
- 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi
/3)) + 3*a**(20/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I
*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**
2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**
(20/3)*b**(10/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*ex
p(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**
3*exp(I*pi/3)) + 3*a**(17/3)*b**(13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x
)**4*exp(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 1
5*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)
), Abs(b*(a/b + x)/a) > 1), (27*a**(29/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/
3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a
/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/
3)*b**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**
6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 6
3*a**(26/3)*b**(4/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)/(-5*a**8*exp(I*pi
/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/
3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)*(a/b + x
)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/
b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 42*a**(23/3
)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 1
5*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*
a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b**(7/3)*(a/b + x)**2/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(20/3)*b**
(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a*
*7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5
*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**(20/3)*b**(10/3)*(a/b + x)**3/(-5*a
```

```
**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)*
*2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(17/3)*b**(13
/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-5*a**8*exp(I*pi/3) + 15*a**7*
b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b*
*3*(a/b + x)**3*exp(I*pi/3)), True))
```

Giac [A]
time = 2.45, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}} + 3\,abx^{\frac{2}{3}} - \frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="giac")
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.75

$$\frac{-15\,a^2+15\,a\,b\,x+3\,b^2\,x^2}{5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(4/3),x)
[Out] (3*b^2*x^2 - 15*a^2 + 15*a*b*x)/(5*x^(1/3))
```

3.665 $\int \frac{(a+bx)^2}{x^{5/3}} dx$

Optimal. Leaf size=34

$$-\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3}$$

```
[Out] -3/2*a^2/x^(2/3)+6*a*b*x^(1/3)+3/4*b^2*x^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/3),x]
```

```
[Out] (-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{x^{5/3}} dx &= \int \left(\frac{a^2}{x^{5/3}} + \frac{2ab}{x^{2/3}} + b^2\sqrt[3]{x}\right) dx \\ &= -\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3} \end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.82

$$-\frac{3(2a^2 - 8abx - b^2x^2)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/3),x]
```

```
[Out] (-3*(2*a^2 - 8*a*b*x - b^2*x^2))/(4*x^(2/3))
```

Maple [A]
time = 0.10, size = 25, normalized size = 0.74

method	result	size
gosper	$-\frac{3(-x^2b^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
derivativedivides	$-\frac{3a^2}{2x^{\frac{2}{3}}}+6ab\,x^{\frac{1}{3}}+\frac{3b^2x^{\frac{4}{3}}}{4}$	25
default	$-\frac{3a^2}{2x^{\frac{2}{3}}}+6ab\,x^{\frac{1}{3}}+\frac{3b^2x^{\frac{4}{3}}}{4}$	25
trager	$-\frac{3(-x^2b^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25
risch	$-\frac{3(-x^2b^2-8abx+2a^2)}{4x^{\frac{2}{3}}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/3),x,method=_RETURNVERBOSE)
[Out] -3/2*a^2/x^(2/3)+6*a*b*x^(1/3)+3/4*b^2*x^(4/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}}+6\,abx^{\frac{1}{3}}-\frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="maxima")
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Fricas [A]
time = 0.59, size = 23, normalized size = 0.68

$$\frac{3\,(b^2x^2+8\,abx-2\,a^2)}{4\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="fricas")
[Out] 3/4*(b^2*x^2 + 8*a*b*x - 2*a^2)/x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.97, size = 1957, normalized size = 57.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/3),x)
```

```
[Out] Piecewise((-27*a**(28/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)
/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*
(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 27*a
**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3
) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(
2*I*pi/3)) + 72*a**(25/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*ex
p(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12
*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi
/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(
a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b
**3*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(22/3)*b**(8/3)*(-1 + b*(a/b + x)/a
)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b
 + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3
*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*
exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)*
*2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 12*a**(19/3)*b
**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-4*a**8*ex
p(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2
*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**
(11/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi
/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*ex
p(2*I*pi/3)) + 3*a**(16/3)*b**(14/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*
*4*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
 - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2
*I*pi/3)), Abs(b*(a/b + x)/a) > 1), (27*a**(28/3)*b**(2/3)*(1 - b*(a/b + x)
/a)**(1/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*
a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/
3)) - 27*a**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*ex
p(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
x)**3*exp(2*I*pi/3)) - 72*a**(25/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/
b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6
*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3))
+ 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b +
 x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(22/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
- 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*
I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a
**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4
*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 12*a**(19/3)*b**(11/3)*(1 - b*(a/b
 + x)/a)**(1/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*e
```

```
xp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
 x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**(11/3)*(a/b + x)**3/(-4*a**8*exp(2*
I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp
(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 3*a**(16/3)*b**(14/3
)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(-4*a**8*exp(2*I*pi/3) + 12*a**7*
b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**
5*b**3*(a/b + x)**3*exp(2*I*pi/3)), True))
```

Giac [A]
time = 2.62, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}} + 6\,abx^{\frac{1}{3}} - \frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="giac")
```

```
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.71

$$\frac{-6\,a^2+24\,a\,b\,x+3\,b^2\,x^2}{4\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/3),x)
```

```
[Out] (3*b^2*x^2 - 6*a^2 + 24*a*b*x)/(4*x^(2/3))
```

3.666 $\int x^{5/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

```
[Out] 3/8*a^3*x^(8/3)+9/11*a^2*b*x^(11/3)+9/14*a*b^2*x^(14/3)+3/17*b^3*x^(17/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3*
x^(17/3))/17
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^3\,dx&=\int\left(a^3x^{5/3}+3a^2bx^{8/3}+3ab^2x^{11/3}+b^3x^{14/3}\right)dx\\&=\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{3x^{8/3}\left(1309a^3+2856a^2bx+2244ab^2x^2+616b^3x^3\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^3,x]
[Out] (3*x^(8/3)*(1309*a^3 + 2856*a^2*b*x + 2244*a*b^2*x^2 + 616*b^3*x^3))/10472
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{8}{3}}}{8}+\frac{9a^{2}b\,x^{\frac{11}{3}}}{11}+\frac{9a\,b^{2}x^{\frac{14}{3}}}{14}+\frac{3b^{3}x^{\frac{17}{3}}}{17}$	36
default	$\frac{3a^{3}x^{\frac{8}{3}}}{8}+\frac{9a^{2}b\,x^{\frac{11}{3}}}{11}+\frac{9a\,b^{2}x^{\frac{14}{3}}}{14}+\frac{3b^{3}x^{\frac{17}{3}}}{17}$	36
trager	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36
risch	$\frac{3x^{\frac{8}{3}}\left(616b^{3}x^{3}+2244a\,b^{2}x^{2}+2856a^{2}bx+1309a^{3}\right)}{10472}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 3/8*a^3*x^(8/3)+9/11*a^2*b*x^(11/3)+9/14*a*b^2*x^(14/3)+3/17*b^3*x^(17/3)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^{3}x^{\frac{17}{3}}+\frac{9}{14}\,ab^{2}x^{\frac{14}{3}}+\frac{9}{11}\,a^{2}bx^{\frac{11}{3}}+\frac{3}{8}\,a^{3}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="maxima")
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

Fricas [A]
time = 0.61, size = 40, normalized size = 0.78

$$\frac{3}{10472}\left(616\,b^{3}x^{5}+2244\,ab^{2}x^{4}+2856\,a^{2}bx^{3}+1309\,a^{3}x^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="fricas")
[Out] 3/10472*(616*b^3*x^5 + 2244*a*b^2*x^4 + 2856*a^2*b*x^3 + 1309*a^3*x^2)*x^(2
/3)
```

Sympy [A]
time = 0.59, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(8/3)/8 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(14/3)/14 + 3*b**3*
x**(17/3)/17
```

Giac [A]
time = 1.87, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{8/3}}{8}+\frac{3\,b^3\,x^{17/3}}{17}+\frac{9\,a^2\,b\,x^{11/3}}{11}+\frac{9\,a\,b^2\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(8/3))/8 + (3*b^3*x^(17/3))/17 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*
x^(14/3))/14
```

3.667 $\int x^{4/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

```
[Out] 3/7*a^3*x^(7/3)+9/10*a^2*b*x^(10/3)+9/13*a*b^2*x^(13/3)+3/16*b^3*x^(16/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3*
x^(16/3))/16
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^3\,dx &= \int \left(a^3x^{4/3}+3a^2bx^{7/3}+3ab^2x^{10/3}+b^3x^{13/3}\right)\,dx\\ &= \frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{3x^{7/3}\left(1040a^3+2184a^2bx+1680ab^2x^2+455b^3x^3\right)}{7280}$$

Antiderivative was successfully verified.

[In] Integrate[x^(4/3)*(a + b*x)^3,x]

[Out] (3*x^(7/3)*(1040*a^3 + 2184*a^2*b*x + 1680*a*b^2*x^2 + 455*b^3*x^3))/7280

Maple [A]
time = 0.12, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{7}{3}}}{7}+\frac{9a^{2}b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^{2}x^{\frac{13}{3}}}{13}+\frac{3b^{3}x^{\frac{16}{3}}}{16}$	36
default	$\frac{3a^{3}x^{\frac{7}{3}}}{7}+\frac{9a^{2}b\,x^{\frac{10}{3}}}{10}+\frac{9a\,b^{2}x^{\frac{13}{3}}}{13}+\frac{3b^{3}x^{\frac{16}{3}}}{16}$	36
trager	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36
risch	$\frac{3x^{\frac{7}{3}}\left(455b^{3}x^{3}+1680a\,b^{2}x^{2}+2184a^{2}bx+1040a^{3}\right)}{7280}$	36

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(4/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)

[Out] 3/7*a^3*x^(7/3)+9/10*a^2*b*x^(10/3)+9/13*a*b^2*x^(13/3)+3/16*b^3*x^(16/3)

Maxima [A]
time = 0.26, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^{3}x^{\frac{16}{3}}+\frac{9}{13}\,ab^{2}x^{\frac{13}{3}}+\frac{9}{10}\,a^{2}bx^{\frac{10}{3}}+\frac{3}{7}\,a^{3}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="maxima")

[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)

Fricas [A]
time = 0.76, size = 40, normalized size = 0.78

$$\frac{3}{7280}\,\left(455\,b^{3}x^{5}+1680\,ab^{2}x^{4}+2184\,a^{2}bx^{3}+1040\,a^{3}x^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="fricas")

[Out] 3/7280*(455*b^3*x^5 + 1680*a*b^2*x^4 + 2184*a^2*b*x^3 + 1040*a^3*x^2)*x^(1/
3)

Sympy [A]
time = 0.47, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{7}{3}}}{7}+\frac{9a^2bx^{\frac{10}{3}}}{10}+\frac{9ab^2x^{\frac{13}{3}}}{13}+\frac{3b^3x^{\frac{16}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(7/3)/7 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(13/3)/13 + 3*b**3*
x**(16/3)/16
```

Giac [A]
time = 1.75, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^3x^{\frac{16}{3}}+\frac{9}{13}\,ab^2x^{\frac{13}{3}}+\frac{9}{10}\,a^2bx^{\frac{10}{3}}+\frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{7/3}}{7}+\frac{3\,b^3\,x^{16/3}}{16}+\frac{9\,a^2\,b\,x^{10/3}}{10}+\frac{9\,a\,b^2\,x^{13/3}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(7/3))/7 + (3*b^3*x^(16/3))/16 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*
x^(13/3))/13
```

3.668 $\int x^{2/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

```
[Out] 3/5*a^3*x^(5/3)+9/8*a^2*b*x^(8/3)+9/11*a*b^2*x^(11/3)+3/14*b^3*x^(14/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {45}

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x^
(14/3))/14
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^3\,dx&=\int\left(a^3x^{2/3}+3a^2bx^{5/3}+3ab^2x^{8/3}+b^3x^{11/3}\right)\,dx\\&=\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{3x^{5/3}(616a^3+1155a^2bx+840ab^2x^2+220b^3x^3)}{3080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^3,x]
[Out] (3*x^(5/3)*(616*a^3 + 1155*a^2*b*x + 840*a*b^2*x^2 + 220*b^3*x^3))/3080
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36
derivativedivides	$\frac{3a^{3}x^{\frac{5}{3}}}{5}+\frac{9a^{2}b\,x^{\frac{8}{3}}}{8}+\frac{9a\,b^{2}x^{\frac{11}{3}}}{11}+\frac{3b^{3}x^{\frac{14}{3}}}{14}$	36
default	$\frac{3a^{3}x^{\frac{5}{3}}}{5}+\frac{9a^{2}b\,x^{\frac{8}{3}}}{8}+\frac{9a\,b^{2}x^{\frac{11}{3}}}{11}+\frac{3b^{3}x^{\frac{14}{3}}}{14}$	36
trager	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36
risch	$\frac{3x^{\frac{5}{3}}\left(220b^{3}x^{3}+840a\,b^{2}x^{2}+1155a^{2}bx+616a^{3}\right)}{3080}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 3/5*a^3*x^(5/3)+9/8*a^2*b*x^(8/3)+9/11*a*b^2*x^(11/3)+3/14*b^3*x^(14/3)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^{3}x^{\frac{14}{3}}+\frac{9}{11}\,ab^{2}x^{\frac{11}{3}}+\frac{9}{8}\,a^{2}bx^{\frac{8}{3}}+\frac{3}{5}\,a^{3}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="maxima")
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

Fricas [A]
time = 0.94, size = 38, normalized size = 0.75

$$\frac{3}{3080}\left(220\,b^{3}x^{4}+840\,ab^{2}x^{3}+1155\,a^{2}bx^{2}+616\,a^{3}x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="fricas")
[Out] 3/3080*(220*b^3*x^4 + 840*a*b^2*x^3 + 1155*a^2*b*x^2 + 616*a^3*x)*x^(2/3)
```

Sympy [A]
time = 0.29, size = 49, normalized size = 0.96

$$\frac{3a^{3}x^{\frac{5}{3}}}{5}+\frac{9a^{2}bx^{\frac{8}{3}}}{8}+\frac{9ab^{2}x^{\frac{11}{3}}}{11}+\frac{3b^{3}x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(5/3)/5 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(11/3)/11 + 3*b**3*x*
*(14/3)/14
```

Giac [A]
time = 1.50, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^{3}x^{\frac{14}{3}}+\frac{9}{11}\,ab^{2}x^{\frac{11}{3}}+\frac{9}{8}\,a^{2}bx^{\frac{8}{3}}+\frac{3}{5}\,a^{3}x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{5/3}}{5}+\frac{3\,b^3\,x^{14/3}}{14}+\frac{9\,a^2\,b\,x^{8/3}}{8}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(5/3))/5 + (3*b^3*x^(14/3))/14 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^
(11/3))/11
```

3.669 $\int \sqrt[3]{x} (a+bx)^3 \, dx$

Optimal. Leaf size=51

$$\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

```
[Out] 3/4*a^3*x^(4/3)+9/7*a^2*b*x^(7/3)+9/10*a*b^2*x^(10/3)+3/13*b^3*x^(13/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x^
(13/3))/13
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)^3\,dx &= \int \left(a^3\sqrt[3]{x}+3a^2bx^{4/3}+3ab^2x^{7/3}+b^3x^{10/3}\right)dx\\ &= \frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{3x^{4/3}(455a^3+780a^2bx+546ab^2x^2+140b^3x^3)}{1820}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*(455*a^3 + 780*a^2*b*x + 546*a*b^2*x^2 + 140*b^3*x^3))/1820
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.71

method	result	size
gosper	$\frac{3x^{\frac{4}{3}}\left(140b^3x^3+546a\,b^2x^2+780a^2bx+455a^3\right)}{1820}$	36
derivativedivides	$\frac{3a^3x^{\frac{4}{3}}}{4}+\frac{9a^2b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^2x^{\frac{10}{3}}}{10}+\frac{3b^3x^{\frac{13}{3}}}{13}$	36
default	$\frac{3a^3x^{\frac{4}{3}}}{4}+\frac{9a^2b\,x^{\frac{7}{3}}}{7}+\frac{9a\,b^2x^{\frac{10}{3}}}{10}+\frac{3b^3x^{\frac{13}{3}}}{13}$	36
trager	$\frac{3x^{\frac{4}{3}}\left(140b^3x^3+546a\,b^2x^2+780a^2bx+455a^3\right)}{1820}$	36
risch	$\frac{3x^{\frac{4}{3}}\left(140b^3x^3+546a\,b^2x^2+780a^2bx+455a^3\right)}{1820}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*a^3*x^(4/3)+9/7*a^2*b*x^(7/3)+9/10*a*b^2*x^(10/3)+3/13*b^3*x^(13/3)
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^3x^{\frac{13}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

Fricas [A]
time = 1.38, size = 38, normalized size = 0.75

$$\frac{3}{1820}\left(140\,b^3x^4+546\,ab^2x^3+780\,a^2bx^2+455\,a^3x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^3*x^4 + 546*a*b^2*x^3 + 780*a^2*b*x^2 + 455*a^3*x)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.50, size = 5012, normalized size = 98.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/3)*(b*x+a)**3,x)

[Out] Piecewise((-243*a**(73/3)*(-1 + b*(a/b + x)/a)**(1/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(73/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 3213*a**(67/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3927*a**(64/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 2163*a**(61/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1827*a**(58/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/

```
3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3
)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(
a/b + x)**5*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b +
x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**
3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5
 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 6573*a**(55/3)*b**6*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**6/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)
**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)*
*5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*e
xp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a*
*18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14
*b**(22/3)*(a/b + x)**6) - 8787*a**(52/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*
(a/b + x)**7/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*
a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a
**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**
14*b**(22/3)*(a/b + x)**6) + 6498*a**(49/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3
)*(a/b + x)**8/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 2730
0*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300
*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a
**14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(-1 + b*(a/b + x)/a)**(1
/3)*(a/b + x)**9/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*...
```

Giac [A]
time = 2.35, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^3x^{\frac{13}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="giac")
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{4/3}}{4}+\frac{3\,b^3\,x^{13/3}}{13}+\frac{9\,a^2\,b\,x^{7/3}}{7}+\frac{9\,a\,b^2\,x^{10/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(4/3))/4 + (3*b^3*x^(13/3))/13 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^
(10/3))/10
```

3.670 $\int \frac{(a+bx)^3}{\sqrt[3]{x}} dx$

Optimal. Leaf size=51

$$\frac{3}{2}a^3x^{2/3} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}$$

```
[Out] 3/2*a^3*x^(2/3)+9/5*a^2*b*x^(5/3)+9/8*a*b^2*x^(8/3)+3/11*b^3*x^(11/3)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{2}a^3x^{2/3} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(1
1/3))/11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^3}{\sqrt[3]{x}} + 3a^2bx^{2/3} + 3ab^2x^{5/3} + b^3x^{8/3}\right) dx\\ &= \frac{3}{2}a^3x^{2/3} + \frac{9}{5}a^2bx^{5/3} + \frac{9}{8}ab^2x^{8/3} + \frac{3}{11}b^3x^{11/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.76

$$\frac{3}{440}x^{2/3}\left(220a^3 + 264a^2bx + 165ab^2x^2 + 40b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(1/3),x]
[Out] (3*x^(2/3)*(220*a^3 + 264*a^2*b*x + 165*a*b^2*x^2 + 40*b^3*x^3))/440
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.71

method	result	size
trager	$\left(\frac{3}{11}b^3x^3+\frac{9}{8}a\,b^2x^2+\frac{9}{5}a^2bx+\frac{3}{2}a^3\right)x^{\frac{2}{3}}$	35
gosper	$\frac{3x^{\frac{2}{3}}\left(40b^3x^3+165a\,b^2x^2+264a^2bx+220a^3\right)}{440}$	36
derivativedivides	$\frac{3a^3x^{\frac{2}{3}}}{2}+\frac{9a^2b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^2x^{\frac{8}{3}}}{8}+\frac{3b^3x^{\frac{11}{3}}}{11}$	36
default	$\frac{3a^3x^{\frac{2}{3}}}{2}+\frac{9a^2b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^2x^{\frac{8}{3}}}{8}+\frac{3b^3x^{\frac{11}{3}}}{11}$	36
risch	$\frac{3x^{\frac{2}{3}}\left(40b^3x^3+165a\,b^2x^2+264a^2bx+220a^3\right)}{440}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*a^3*x^(2/3)+9/5*a^2*b*x^(5/3)+9/8*a*b^2*x^(8/3)+3/11*b^3*x^(11/3)
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^3x^{\frac{11}{3}}+\frac{9}{8}\,ab^2x^{\frac{8}{3}}+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+\frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="maxima")
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

Fricas [A]
time = 0.63, size = 35, normalized size = 0.69

$$\frac{3}{440}\left(40\,b^3x^3+165\,ab^2x^2+264\,a^2bx+220\,a^3\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="fricas")
[Out] 3/440*(40*b^3*x^3 + 165*a*b^2*x^2 + 264*a^2*b*x + 220*a^3)*x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.46, size = 6246, normalized size = 122.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(440*a**20
*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a*
*18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*e
xp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**
(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi
/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1296*a**(68/3)*b*(-1 + b*(a/b + x)/a)
**(2/3)*(a/b + x)*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(
440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*
a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6
*exp(I*pi/3)) + 2808*a**(65/3)*b**2*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**
2*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b +
x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*
b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(
I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/
3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)) - 3120*a**(62/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I*pi
/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*p
i/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*
(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
x)**6*exp(I*pi/3)) - 4860*a**(62/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*e
xp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3
)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3)
+ 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b
 + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1710
*a**(59/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(I*pi/3)/(440*a
**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
```

```
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3)
 - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x
)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**
16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*e
xp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 72*a**(56/3)*b
**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5*exp(I*pi/3)/(440*a**20*b**(2/3
)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(
8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/
3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(
a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1
458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**1
9*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*p
i/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)
*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) +
 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*(-1 +
b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi
/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*
a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**
5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a**(53/
3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 660...
```

Giac [A]

time = 1.62, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^{3}x^{\frac{11}{3}}+\frac{9}{8}\,ab^{2}x^{\frac{8}{3}}+\frac{9}{5}\,a^{2}bx^{\frac{5}{3}}+\frac{3}{2}\,a^{3}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="giac")
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

Mupad [B]

time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{2/3}}{2}+\frac{3\,b^3\,x^{11/3}}{11}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/3),x)
[Out] (3*a^3*x^(2/3))/2 + (3*b^3*x^(11/3))/11 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(8/3))/8
```

3.671 $\int \frac{(a+bx)^3}{x^{2/3}}\,dx$

Optimal. Leaf size=49

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

```
[Out] 3*a^3*x^(1/3)+9/4*a^2*b*x^(4/3)+9/7*a*b^2*x^(7/3)+3/10*b^3*x^(10/3)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(2/3),x]
```

```
[Out] 3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3)
)/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{2/3}}\,dx &= \int \left(\frac{a^3}{x^{2/3}}+3a^2b\sqrt[3]{x}+3ab^2x^{4/3}+b^3x^{7/3}\right)dx\\ &= 3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$\frac{3}{140}\sqrt[3]{x}\left(140a^3+105a^2bx+60ab^2x^2+14b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(2/3),x]
[Out] (3*x^(1/3)*(140*a^3 + 105*a^2*b*x + 60*a*b^2*x^2 + 14*b^3*x^3))/140
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.73

method	result	size
trager	$\left(\frac{3}{10}b^{3}x^{3}+\frac{9}{7}a\,b^{2}x^{2}+\frac{9}{4}a^{2}bx+3a^{3}\right)x^{\frac{1}{3}}$	35
gosper	$\frac{3x^{\frac{1}{3}}\left(14b^{3}x^{3}+60a\,b^{2}x^{2}+105a^{2}bx+140a^{3}\right)}{140}$	36
derivativedivides	$3a^{3}x^{\frac{1}{3}}+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{7}{3}}}{7}+\frac{3b^{3}x^{\frac{10}{3}}}{10}$	36
default	$3a^{3}x^{\frac{1}{3}}+\frac{9a^{2}b\,x^{\frac{4}{3}}}{4}+\frac{9a\,b^{2}x^{\frac{7}{3}}}{7}+\frac{3b^{3}x^{\frac{10}{3}}}{10}$	36
risch	$\frac{3x^{\frac{1}{3}}\left(14b^{3}x^{3}+60a\,b^{2}x^{2}+105a^{2}bx+140a^{3}\right)}{140}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(2/3),x,method=_RETURNVERBOSE)
[Out] 3*a^3*x^(1/3)+9/4*a^2*b*x^(4/3)+9/7*a*b^2*x^(7/3)+3/10*b^3*x^(10/3)
```

Maxima [A]
time = 0.37, size = 35, normalized size = 0.71

$$\frac{3}{10}\,b^{3}x^{\frac{10}{3}}+\frac{9}{7}\,ab^{2}x^{\frac{7}{3}}+\frac{9}{4}\,a^{2}bx^{\frac{4}{3}}+3\,a^{3}x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="maxima")
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Fricas [A]
time = 0.57, size = 35, normalized size = 0.71

$$\frac{3}{140}\left(14\,b^{3}x^{3}+60\,ab^{2}x^{2}+105\,a^{2}bx+140\,a^{3}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="fricas")
[Out] 3/140*(14*b^3*x^3 + 60*a*b^2*x^2 + 105*a^2*b*x + 140*a^3)*x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.53, size = 6667, normalized size = 136.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(2/3),x)
```

```
[Out] Piecewise((243*a**(70/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*
a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*e
xp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16
*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*ex
p(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1377*a**(67
/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(140*a**20*b**(1/
3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*
b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp
(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b*
*(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2
*I*pi/3)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)*
*2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a
**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**
5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3213*a*
*(64/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(140*a*
*20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2
100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
 x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 84
0*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x
)**6*exp(2*I*pi/3)) + 3645*a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 3927*a**(61/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2
*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*e
xp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*
b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*ex
p(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b*
*(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860*a**(61/3)*b**3*(a/b + x)**3/(140
*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3)
+ 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/
b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b
+ x)**6*exp(2*I*pi/3)) + 2583*a**(58/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a
```

```
/b + x)**4*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(
4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/
3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)
*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3
) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*a**(58/3)*b**4*(
a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x
)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**
17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4
*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14
*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 693*a**(55/3)*b**5*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**5*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) -
 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)
**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*
a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)*
*5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1458*a
**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
*(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 273*a**(52/3)*b**6*
(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) + 243*a**(52/3)*b**6*(a/b + x)**6/(140*a...
```

Giac [A]
time = 1.57, size = 35, normalized size = 0.71

$$\frac{3}{10}\,b^3x^{\frac{10}{3}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+3\,a^3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="giac")
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$3\,a^3\,x^{1/3}+\frac{3\,b^3\,x^{10/3}}{10}+\frac{9\,a^2\,b\,x^{4/3}}{4}+\frac{9\,a\,b^2\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(2/3),x)
[Out] 3*a^3*x^(1/3) + (3*b^3*x^(10/3))/10 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3
))/7
```

3.672 $\int \frac{(a+bx)^3}{x^{4/3}} \, dx$

Optimal. Leaf size=49

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

```
[Out] -3*a^3/x^(1/3)+9/2*a^2*b*x^(2/3)+9/5*a*b^2*x^(5/3)+3/8*b^3*x^(8/3)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8/3))/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{4/3}}\,dx &= \int \left(\frac{a^3}{x^{4/3}}+\frac{3a^2b}{\sqrt[3]{x}}+3ab^2x^{2/3}+b^3x^{5/3}\right)dx\\ &= -\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$-\frac{3(40a^3-60a^2bx-24ab^2x^2-5b^3x^3)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*(40*a^3 - 60*a^2*b*x - 24*a*b^2*x^2 - 5*b^3*x^3))/(40*x^(1/3))
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.73

method	result	size
gosper	$-\frac{3\left(-5 b^{3} x^{3}-24 a\,b^{2} x^{2}-60 a^{2} b x+40 a^{3}\right)}{40 x^{\frac{1}{3}}}$	36
derivativedivides	$-\frac{3 a^{3}}{x^{\frac{1}{3}}}+\frac{9 a^{2} b\,x^{\frac{2}{3}}}{2}+\frac{9 a\,b^{2} x^{\frac{5}{3}}}{5}+\frac{3 b^{3} x^{\frac{8}{3}}}{8}$	36
default	$-\frac{3 a^{3}}{x^{\frac{1}{3}}}+\frac{9 a^{2} b\,x^{\frac{2}{3}}}{2}+\frac{9 a\,b^{2} x^{\frac{5}{3}}}{5}+\frac{3 b^{3} x^{\frac{8}{3}}}{8}$	36
trager	$-\frac{3\left(-5 b^{3} x^{3}-24 a\,b^{2} x^{2}-60 a^{2} b x+40 a^{3}\right)}{40 x^{\frac{1}{3}}}$	36
risch	$-\frac{3\left(-5 b^{3} x^{3}-24 a\,b^{2} x^{2}-60 a^{2} b x+40 a^{3}\right)}{40 x^{\frac{1}{3}}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*a^3/x^(1/3)+9/2*a^2*b*x^(2/3)+9/5*a*b^2*x^(5/3)+3/8*b^3*x^(8/3)
```

Maxima [A]
time = 0.33, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Fricas [A]
time = 0.47, size = 35, normalized size = 0.71

$$\frac{3\,(5\,b^3x^3+24\,ab^2x^2+60\,a^2bx-40\,a^3)}{40\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 + 24*a*b^2*x^2 + 60*a^2*b*x - 40*a^3)/x^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.61, size = 4004, normalized size = 81.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(4/3),x)
```

```
[Out] Piecewise((243*a**(68/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)/(40*a**20 - 2
40*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b +
x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**1
4*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b +
x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**
15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(62/3)*b**(7/3
)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(40*a**20 - 240*a**19*b*(a/b + x
) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b
**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6
) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**1
9*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*
(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b +
 x)**3/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 80
0*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a
/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b
 + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*
a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(
13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(40*a**20 - 240*a**19*b*(a/b
 + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**
16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x
)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**5/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*
```

```
(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 96*a**(50/3)*b*
*(19/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(40*a**20 - 240*a**19*b*(a
/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a
**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
 x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**7/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(-1 +
 b*(a/b + x)/a)**(2/3)*(a/b + x)**8/(40*a**20 - 240*a**19*b*(a/b + x) + 600
*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/
b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), Abs(
b*(a/b + x)/a) > 1), (243*a**(68/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/3)*exp
(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2
*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 -
800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*
(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(1 - b
*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b +
 x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16
*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)*
*6) + 1458*a**(65/3)*b**(4/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19
*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(
a/b + x)**6) + 2808*a**(62/3)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)
**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b ...
```

Giac [A]
time = 1.00, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="giac")
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{8/3}}{8}-\frac{3\,a^3}{x^{1/3}}+\frac{9\,a^2\,b\,x^{2/3}}{2}+\frac{9\,a\,b^2\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(4/3),x)
```

```
[Out] (3*b^3*x^(8/3))/8 - (3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3
))/5
```

3.673 $\int \frac{(a+bx)^3}{x^{5/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

```
[Out] -3/2*a^3/x^(2/3)+9*a^2*b*x^(1/3)+9/4*a*b^2*x^(4/3)+3/7*b^3*x^(7/3)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7/
3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/3}}\,dx &= \int \left(\frac{a^3}{x^{5/3}}+\frac{3a^2b}{x^{2/3}}+3ab^2\sqrt[3]{x}+b^3x^{4/3}\right)dx\\ &= -\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 0.80

$$-\frac{3(14a^3-84a^2bx-21ab^2x^2-4b^3x^3)}{28x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/3),x]
[Out] (-3*(14*a^3 - 84*a^2*b*x - 21*a*b^2*x^2 - 4*b^3*x^3))/(28*x^(2/3))
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.73

method	result	size
gosper	$-\frac{3\left(-4b^3x^3-21a\,b^2x^2-84a^2bx+14a^3\right)}{28x^{\frac{2}{3}}}$	36
derivativedivides	$-\frac{3a^3}{2x^{\frac{2}{3}}}+9a^2b\,x^{\frac{1}{3}}+\frac{9a\,b^2x^{\frac{4}{3}}}{4}+\frac{3b^3x^{\frac{7}{3}}}{7}$	36
default	$-\frac{3a^3}{2x^{\frac{2}{3}}}+9a^2b\,x^{\frac{1}{3}}+\frac{9a\,b^2x^{\frac{4}{3}}}{4}+\frac{3b^3x^{\frac{7}{3}}}{7}$	36
trager	$-\frac{3\left(-4b^3x^3-21a\,b^2x^2-84a^2bx+14a^3\right)}{28x^{\frac{2}{3}}}$	36
risch	$-\frac{3\left(-4b^3x^3-21a\,b^2x^2-84a^2bx+14a^3\right)}{28x^{\frac{2}{3}}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/3),x,method=_RETURNVERBOSE)
[Out] -3/2*a^3/x^(2/3)+9*a^2*b*x^(1/3)+9/4*a*b^2*x^(4/3)+3/7*b^3*x^(7/3)
```

Maxima [A]
time = 0.35, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="maxima")
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Fricas [A]
time = 0.39, size = 35, normalized size = 0.71

$$\frac{3\left(4\,b^3x^3+21\,ab^2x^2+84\,a^2bx-14\,a^3\right)}{28\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="fricas")
[Out] 3/28*(4*b^3*x^3 + 21*a*b^2*x^2 + 84*a^2*b*x - 14*a^3)/x^(2/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.55, size = 3964, normalized size = 80.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**3/x**(5/3),x)

[Out] Piecewise((243*a**(67/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)/(28*a**20 - 1
68*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b +
x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**1
4*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 168*a
19*b*(a/b + x) + 420*a18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**
3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b*
6(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
 + x)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560
*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/
b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)*(a/b + x)
*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**
2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b
5*(a/b + x)5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3)*b**(8/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a
/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a
16*b4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
 x)**6) - 3927*a**(58/3)*b**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
5(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(55/3)*b**(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(
a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*
a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b
+ x)**6) - 903*a**(52/3)*b**(17/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
5(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(

```
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(2
8*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b
**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**
5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(-1 + b*(a/b + x)/
a)**(1/3)*(a/b + x)**8/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(
a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 1
68*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6), Abs(b*(a/b + x)/a
) > 1), (243*a**(67/3)*b**(2/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(28*
a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**
3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5
+ 28*a**14*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**2
0 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a
/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28
*a**14*b**6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**
2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4
- 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3
)*b**(5/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a*
*18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b +
 x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a
**(61/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(28*a
**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(...
```

Giac [A]
time = 0.98, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="giac")
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{7/3}}{7}-\frac{3\,a^3}{2\,x^{2/3}}+9\,a^2\,b\,x^{1/3}+\frac{9\,a\,b^2\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/3),x)
```

```
[Out] (3*b^3*x^(7/3))/7 - (3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3
))/4
```

3.674 $\int \frac{x^{5/3}}{a+bx} \, dx$

Optimal. Leaf size=125

$$-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}-\frac{\sqrt{3}\, a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}$$

```
[Out] -3/2*a*x^(2/3)/b^2+3/5*x^(5/3)/b-3/2*a^(5/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^
(8/3)+1/2*a^(5/3)*ln(b*x+a)/b^(8/3)-a^(5/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x
^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/b^(8/3)
```

Rubi [A]
time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 58, 631, 210, 31}

$$-\frac{\sqrt{3}\, a^{5/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(2/3))/(2*b^2) + (3*x^(5/3))/(5*b) - (Sqrt[3]*a^(5/3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(8/3) - (3*a^(5/3)*Log[a^(1/3
) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) + (a^(5/3)*Log[a + b*x])/(2*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/3}}{a+bx}\,dx &= \frac{3x^{5/3}}{5b}-\frac{a\int \frac{x^{2/3}}{a+bx}\,dx}{b}\\
&=-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}+\frac{a^2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b^2}\\
&=-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{2b^3}-\frac{(3a^{5/3}\end{aligned}$$

$$=-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}+\frac{(3a^{5/3})\,\text{Subst}\left(\int \frac{1}{-3-}\right.}{b}$$

$$=-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}}-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log}{2}$$

Mathematica [A]

time = 0.09, size = 140, normalized size = 1.12

$$\frac{-15ab^{2/3}x^{2/3}+6b^{5/3}x^{5/3}-10\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)+5a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{10b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x),x]
```

```
[Out] (-15*a*b^(2/3)*x^(2/3) + 6*b^(5/3)*x^(5/3) - 10*Sqrt[3]*a^(5/3)*ArcTan[(1 -
 (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 10*a^(5/3)*Log[a^(1/3) + b^(1/3)*x
^(1/3)] + 5*a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)
])/(10*b^(8/3))
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.99

method	result
risch	$-\frac{3(-2bx+5a)x^{\frac{2}{3}}}{10b^{2}}-\frac{a^{2}\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^{2}\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$-\frac{3\left(-\frac{b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{\frac{2}{3}}}{2}\right)}{b^{2}}+\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^{2}}{b^{2}}$
default	$-\frac{3\left(-\frac{b\,x^{\frac{5}{3}}}{5}+\frac{a\,x^{\frac{2}{3}}}{2}\right)}{b^{2}}+\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a^{2}}{b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b^2*(-1/5*b*x^(5/3)+1/2*a*x^(2/3))+3*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b
)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*
3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a^2/b^
2
```

Maxima [A]
time = 0.57, size = 130, normalized size = 1.04

$$\frac{\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^2\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a^2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\left(2\,bx^{\frac{5}{3}}-5\,ax^{\frac{2}{3}}\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(1/3)) + 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(1/3)) - a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/10*(2
*b*x^(5/3) - 5*a*x^(2/3))/b^2
```

Fricas [A]
time = 0.52, size = 147, normalized size = 1.18

$$\frac{10\,\sqrt{3}\,a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3\,a}\right)-5\,a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+10\,a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)+3\,(2\,bx-5\,a)x^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/10*(10*sqrt(3)*a*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a^2/b
^2)^(1/3) + sqrt(3)*a)/a) - 5*a*(-a^2/b^2)^(1/3)*log(-b*x^(1/3)*(-a^2/b^2)^
(2/3) + a*x^(2/3) - a*(-a^2/b^2)^(1/3)) + 10*a*(-a^2/b^2)^(1/3)*log(b*(-a^2
/b^2)^(2/3) + a*x^(1/3)) + 3*(2*b*x - 5*a)*x^(2/3))/b^2
```

Sympy [A]
time = 17.69, size = 180, normalized size = 1.44

$$\begin{cases} \tilde{\infty}x^{\frac{5}{3}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{8}{3}}}{8a} & \text{for}\: b = 0 \\ \frac{3x^{\frac{5}{3}}}{5b} & \text{for}\: a = 0 \\ -\frac{a^3\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^4\left(-\frac{a}{b}\right)^{\frac{4}{3}}}+\frac{a^3\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\,\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^4\left(-\frac{a}{b}\right)^{\frac{4}{3}}}-\frac{\sqrt{3}\,a^3\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b^4\left(-\frac{a}{b}\right)^{\frac{4}{3}}}-\frac{3ax^{\frac{2}{3}}}{2b^2}+\frac{3x^{\frac{5}{3}}}{5b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a), Eq(b, 0))
, (3*x**(5/3)/(5*b), Eq(a, 0)), (-a**3*log(x**(1/3) - (-a/b)**(1/3))/(b**4*
(-a/b)**(4/3)) + a**3*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)*
```

```
*(2/3))/(2*b**4*(-a/b)**(4/3)) - sqrt(3)*a**3*atan(2*sqrt(3)*x**(1/3)/(3*(-
a/b)**(1/3)) + sqrt(3)/3)/(b**4*(-a/b)**(4/3)) - 3*a*x**(2/3)/(2*b**2) + 3*
x**(5/3)/(5*b), True))
```

Giac [A]
time = 1.61, size = 138, normalized size = 1.10

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^{2}}-\frac{\sqrt{3}\,\left(-ab^{2}\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{4}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{4}}+\frac{3\left(2\,b^{4}x^{\frac{5}{3}}-5\,ab^{3}x^{\frac{2}{3}}\right)}{10\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - sqrt(3)*(-a*b^2)^(2/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/2*
(-a*b^2)^(2/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 3
/10*(2*b^4*x^(5/3) - 5*a*b^3*x^(2/3))/b^5
```

Mupad [B]
time = 0.24, size = 151, normalized size = 1.21

$$\frac{3\,x^{5/3}}{5\,b}+\frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3}-\frac{9\,{\left(-a\right)}^{13/3}}{b^{10/3}}\right)}{b^{8/3}}-\frac{3\,a\,x^{2/3}}{2\,b^2}+\frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3}-\frac{9\,{\left(-a\right)}^{13/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{8/3}}-\frac{{\left(-a\right)}^{5/3}\,\ln\left(\frac{9\,a^4\,x^{1/3}}{b^3}-\frac{9\,{\left(-a\right)}^{13/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x),x)
```

```
[Out] (3*x^(5/3))/(5*b) + ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3))/b
^(10/3)))/b^(8/3) - (3*a*x^(2/3))/(2*b^2) + ((-a)^(5/3)*log((9*a^4*x^(1/3))
/b^3 - (9*(-a)^(13/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 -
 1/2))/b^(8/3) - ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3)*((3^(
1/2)*1i)/2 + 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 + 1/2))/b^(8/3)
```

3.675 $\int \frac{x^{4/3}}{a+bx}\,dx$

Optimal. Leaf size=123

$$-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{7/3}}+\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}$$

```
[Out] -3*a*x^(1/3)/b^2+3/4*x^(4/3)/b+3/2*a^(4/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(7
/3)-1/2*a^(4/3)*ln(b*x+a)/b^(7/3)-a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(
1/3))/a^(1/3)*3^(1/2))*3^(1/2)/b^(7/3)
```

Rubi [A]
time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 60, 631, 210, 31}

$$-\frac{\sqrt{3}\,a^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{7/3}}+\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(4/3))/(4*b) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (3*a^(4/3)*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*b^(7/3)) - (a^(4/3)*Log[a + b*x])/(2*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{3x^{4/3}}{4b} - \frac{a\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{b}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{a^2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b^2}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{8/3}} + \frac{\left(3a^{4/}\right.}{}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.}{b^7}$$

$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log}{2b}$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 1.14

$$\frac{-12a\sqrt[3]{b}\,\sqrt[3]{x} + 3b^{4/3}x^{4/3} - 4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - 2a^{4/3}\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{4b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x),x]
```

```
[Out] (-12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x^(
1/3)] - 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])
/(4*b^(7/3))
```

Maple [A]

time = 0.17, size = 123, normalized size = 1.00

method	result	s
derivativedivides	$-\frac{3\left(-\frac{b x^{\frac{4}{3}}}{4}+a x^{\frac{1}{3}}\right)}{b^{2}}+\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) a^{2}}{b^{2}}$	1
default	$-\frac{3\left(-\frac{b x^{\frac{4}{3}}}{4}+a x^{\frac{1}{3}}\right)}{b^{2}}+\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right) a^{2}}{b^{2}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b^2*(-1/4*b*x^(4/3)+a*x^(1/3))+3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/
3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a^2/b^2
```

Maxima [A]

time = 0.58, size = 128, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\left(bx^{\frac{4}{3}}-4\,ax^{\frac{1}{3}}\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(4/3)/(b*x+a),x, algorithm="maxima")

[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3/4*(b*x^(4/3) - 4*a*x^(1/3))/b^2

Fricas [A]
time = 0.50, size = 116, normalized size = 0.94

$$\frac{4\sqrt{3}\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3\,(bx-4\,a)x^{\frac{1}{3}}}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(4/3)/(b*x+a),x, algorithm="fricas")

[Out] 1/4*(4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a/b)^(2/3) - sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3)) + 4*a*(a/b)^(1/3)*log(x^(1/3) + (a/b)^(1/3)) + 3*(b*x - 4*a)*x^(1/3))/b^2

Sympy [A]
time = 9.27, size = 173, normalized size = 1.41

$$\begin{cases} \tilde{\infty}x^{\frac{4}{3}} & \text{for}\: a=0 \wedge b=0 \\ \frac{3x^{\frac{7}{3}}}{7a} & \text{for}\: b=0 \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{for}\: a=0 \\ -\frac{3a\sqrt[3]{x}}{b^{2}}-\frac{a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^{2}}+\frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{2}}+\frac{\sqrt{3}\,a\sqrt[3]{-\frac{a}{b}}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b^{2}}+\frac{3x^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(4/3)/(b*x+a),x)

[Out] Piecewise((zoo*x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a), Eq(b, 0)), (3*x**(4/3)/(4*b), Eq(a, 0)), (-3*a*x**(1/3)/b**2 - a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/b**2 + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b**2) + sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/b**2 + 3*x**(4/3)/(4*b), True))

Giac [A]
time = 1.54, size = 136, normalized size = 1.11

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^{2}}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}}+\frac{3\left(b^{3}x^{\frac{4}{3}}-4\,ab^{2}x^{\frac{1}{3}}\right)}{4\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + sqrt(3)*(-a*b^2)^(1/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*
(-a*b^2)^(1/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 3
/4*(b^3*x^(4/3) - 4*a*b^2*x^(1/3))/b^4
```

Mupad [B]
time = 0.07, size = 126, normalized size = 1.02

$$\frac{3\,x^{4/3}}{4\,b}-\frac{3\,a\,x^{1/3}}{b^2}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{1/3}}+9\,a^2\,x^{1/3}\right)}{b^{7/3}}+\frac{a^{4/3}\,\ln\left(9\,a^2\,x^{1/3}+\frac{9\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{7/3}}-\frac{a^{4/3}\,\ln\left(9\,a^2\,x^{1/3}-\frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x),x)
```

```
[Out] (3*x^(4/3))/(4*b) - (3*a*x^(1/3))/b^2 + (a^(4/3)*log((9*a^(7/3))/b^(1/3) +
9*a^2*x^(1/3)))/b^(7/3) + (a^(4/3)*log(9*a^2*x^(1/3) + (9*a^(7/3)*((3^(1/2)
*1i)/2 - 1/2))/b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(7/3) - (a^(4/3)*log(9*a^
2*x^(1/3) - (9*a^(7/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1
/2))/b^(7/3)
```

3.676 $\int \frac{x^{2/3}}{a+bx}\,dx$

Optimal. Leaf size=111

$$\frac{3x^{2/3}}{2b}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

```
[Out] 3/2*x^(2/3)/b+3/2*a^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(5/3)-1/2*a^(2/3)*l
n(b*x+a)/b^(5/3)+a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(
1/2))*3^(1/2)/b^(5/3)
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {52, 58, 631, 210, 31}

$$\frac{\sqrt{3}\,a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(
Sqrt[3]*a^(1/3))])/b^(5/3) + (3*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*
b^(5/3)) - (a^(2/3)*Log[a + b*x])/(2*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{3x^{2/3}}{2b} - \frac{a\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b}$$

$$= \frac{3x^{2/3}}{2b} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{(3a)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^2} + \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \cdots\right)}{2b^{5\ldots}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{3a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \cdots\right)}{b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + \frac{3a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

Mathematica [A]
time = 0.07, size = 127, normalized size = 1.14

$$\frac{3b^{2/3}x^{2/3} + 2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) - a^{2/3}\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{2b^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2/3)/(a + b*x),x]

[Out] (3*b^(2/3)*x^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(5/3))

Maple [A]
time = 0.11, size = 112, normalized size = 1.01

method	result	size
risch	$\frac{3x^{\frac{2}{3}}}{2b}+\frac{a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	107
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2b}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	112
default	$\frac{3x^{\frac{2}{3}}}{2b}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	112

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2/3)/(b*x+a),x,method=_RETURNVERBOSE)

[Out] 3/2*x^(2/3)/b-3*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a/b

Maxima [A]

time = 0.48, size = 114, normalized size = 1.03

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(1/3)) + 3/2*x^(2/3)/b - 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/
b)^(2/3))/(b^2*(a/b)^(1/3)) + a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(1/3)
)
```

Fricas [A]
time = 0.45, size = 128, normalized size = 1.15

$$-\frac{2\sqrt{3}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)-2\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3\,x^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^(
1/3) - sqrt(3)*a)/a) + (a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a*x
^(2/3) + a*(a^2/b^2)^(1/3)) - 2*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x
^(1/3)) - 3*x^(2/3))/b
```

Sympy [A]
time = 3.15, size = 162, normalized size = 1.46

$$\begin{cases} \tilde{\infty}x^{\frac{2}{3}} & \text{for}\: a=0 \wedge b=0 \\ \frac{3x^{\frac{5}{3}}}{5a} & \text{for}\: b=0 \\ \frac{3x^{\frac{2}{3}}}{2b} & \text{for}\: a=0 \\ -\frac{a\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b^{2}\sqrt[3]{-\frac{a}{b}}}+\frac{a\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\,\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{2}\sqrt[3]{-\frac{a}{b}}}-\frac{\sqrt{3}\,a\,\mathrm{atan}\left(\frac{2\sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b^{2}\sqrt[3]{-\frac{a}{b}}}+\frac{3x^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a), Eq(b, 0))
, (3*x**(2/3)/(2*b), Eq(a, 0)), (-a*log(x**(1/3) - (-a/b)**(1/3))/(b**2*(-a
```

```
/b)**(1/3)) + a*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3)
)/(2*b**2*(-a/b)**(1/3)) - sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/
3)) + sqrt(3)/3)/(b**2*(-a/b)**(1/3)) + 3*x**(2/3)/(2*b), True))
```

Giac [A]
time = 1.18, size = 118, normalized size = 1.06

$$\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}+\frac{3\,x^{\frac{2}{3}}}{2\,b}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b + 3/2*x^(2/3)/b + sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^3 - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
b^3
```

Mupad [B]
time = 0.15, size = 130, normalized size = 1.17

$$\frac{3\,x^{2/3}}{2\,b}+\frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{4/3}}+\frac{9\,a^2\,x^{1/3}}{b}\right)}{b^{5/3}}+\frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{4/3}}+\frac{9\,a^2\,x^{1/3}}{b}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{5/3}}-\frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{4/3}}+\frac{9\,a^2\,x^{1/3}}{b}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x),x)
```

```
[Out] (3*x^(2/3))/(2*b) + (a^(2/3)*log((9*a^(7/3))/b^(4/3) + (9*a^2*x^(1/3))/b))/
b^(5/3) + (a^(2/3)*log((9*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(4/3) + (9*a^
2*x^(1/3))/b)*((3^(1/2)*1i)/2 - 1/2))/b^(5/3) - (a^(2/3)*log((9*a^(7/3)*((3
^(1/2)*1i)/2 + 1/2)^2)/b^(4/3) + (9*a^2*x^(1/3))/b)*((3^(1/2)*1i)/2 + 1/2))
/b^(5/3)
```

3.677 $\int \frac{\sqrt[3]{x}}{a+bx}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt[3]{x}}{b}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}$$

```
[Out] 3*x^(1/3)/b-3/2*a^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(4/3)+1/2*a^(1/3)*ln(
b*x+a)/b^(4/3)+a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/
2))*3^(1/2)/b^(4/3)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {52, 60, 631, 210, 31}

$$\frac{\sqrt{3}\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt
[3]*a^(1/3))])/b^(4/3) - (3*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(4
/3)) + (a^(1/3)*Log[a + b*x])/(2*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{3\sqrt[3]{x}}{b} - \frac{a\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{a}\,\log(a+bx)}{2b^{4/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\sqrt[3]{x}}{b} - \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\,\log(a+bx)}{2b^{4/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt{3}\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\,\log(a+bx)}{2b^{4/3}}$$

Mathematica [A]
time = 0.06, size = 126, normalized size = 1.16

$$\frac{6\sqrt[3]{b}\sqrt[3]{x}+2\sqrt{3}\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x),x]
```

```
[Out] (6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1
/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + a^(1/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(4/3))
```

Maple [A]
time = 0.15, size = 112, normalized size = 1.03

method	result	size
derivativedivides	$\frac{3x^{\frac{1}{3}}}{b}-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	112
default	$\frac{3x^{\frac{1}{3}}}{b}-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)/b-3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*
ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*a/b
```

Maxima [A]
time = 0.49, size = 115, normalized size = 1.06

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\,x^{\frac{1}{3}}}{b}+\frac{a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(b*x+a),x, algorithm="maxima")

[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) + 3*x^(1/3)/b + 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(2/3))

Fricas [A]
time = 0.52, size = 114, normalized size = 1.05

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6\,x^{\frac{1}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(b*x+a),x, algorithm="fricas")

[Out] 1/2*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b)^(2/3) - sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^(1/3)) + 6*x^(1/3))/b

Sympy [A]
time = 2.09, size = 148, normalized size = 1.36

$$\begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a} & \text{for}\: b = 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{for}\: a = 0 \\ \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{b} - \frac{\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b} - \frac{\sqrt{3}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/3)/(b*x+a),x)

[Out] Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a), Eq(b, 0)), (3*x**(1/3)/b, Eq(a, 0)), (3*x**(1/3)/b + (-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/b - (-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b) - sqrt(3)*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/b, True))

Giac [A]
time = 1.47, size = 119, normalized size = 1.09

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}}+\frac{3\,x^{\frac{1}{3}}}{b}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(b*x+a),x, algorithm="giac")

```
[Out] (-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b - sqrt(3)*(-a*b^2)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 3*x^(1/3)/b
 - 1/2*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
2
```

Mupad [B]
time = 0.07, size = 126, normalized size = 1.16

$$\frac{3\,x^{1/3}}{b}+\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}+9\,a\,b\,x^{1/3}\right)}{b^{4/3}}+\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+9\,a\,b\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}}-\frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-9\,a\,b\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x),x)
```

```
[Out] (3*x^(1/3))/b + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3) + 9*a*b*x^(1/3)))/b^(4
/3) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 - 1/2) + 9*a*b*x
^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(4/3) - ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2
/3)*((3^(1/2)*1i)/2 + 1/2) - 9*a*b*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/b^(4/3)
```

3.678 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}$$

```
[Out] -3/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(2/3)+1/2*ln(b*x+a)/a^(1/3)/b^(2
/3)-arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(1/3)
/b^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {58, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3
)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[
a + b*x]/(2*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx = \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}\,b^{2/3}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}$$

Mathematica [A]
time = 0.06, size = 103, normalized size = 1.03

$$\frac{-2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) + \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/
3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(
2/3)])/(2*a^(1/3)*b^(2/3))
```

Maple [A]
time = 0.12, size = 96, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	96
default	$-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)
^(1/3)*x^(1/3)+(a/b)^(2/3))+3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x^(1/3)-1))
```

Maxima [A]
time = 0.49, size = 103, normalized size = 1.03

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(1/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3
)) - log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Fricas [A]
time = 0.51, size = 313, normalized size = 3.13

$$\left[\frac{\sqrt{3}\,ab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+\sqrt{3}\,\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-2\,(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{2\,ab^2},\ \frac{2\,\sqrt{3}\,ab\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{3}\,\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{3\,b}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-2\,(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{2\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x - a*b + sqrt(3)*(a*b*
x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x + a)) + (-a*b^2)^(2/3)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/
3) - (-a*b^2)^(1/3)))/(a*b^2), 1/2*(2*sqrt(3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*a
rctan(1/3*sqrt(3)*(2*b*x^(1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 + (-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/
3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A]

time = 2.37, size = 141, normalized size = 1.41

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a} & \text{for}\: b = 0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for}\: a = 0 \\ \frac{\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{b\sqrt[3]{-\frac{a}{b}}} - \frac{\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\,\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{b\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a), Eq(b, 0))
, (-3/(b*x**(1/3)), Eq(a, 0)), (log(x**(1/3) - (-a/b)**(1/3))/(b*(-a/b)**(1
/3)) - log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b*(-
a/b)**(1/3)) + sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/
3)/(b*(-a/b)**(1/3)), True))
```

Giac [A]

time = 1.62, size = 118, normalized size = 1.18

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} - \frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2} + \frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/2*(-
a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B]

time = 0.11, size = 120, normalized size = 1.20

$$\frac{\ln\left(9\,b\,x^{1/3} - 9\,{\left(-a\right)}^{1/3}\,b^{2/3}\right)}{{\left(-a\right)}^{1/3}\,b^{2/3}} + \frac{\ln\left(9\,b\,x^{1/3} - \frac{9\,{\left(-a\right)}^{1/3}\,b^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{\left(-a\right)}^{1/3}\,b^{2/3}} - \frac{\ln\left(9\,b\,x^{1/3} - \frac{9\,{\left(-a\right)}^{1/3}\,b^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{\left(-a\right)}^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)),x)
```

```
[Out] log(9*b*x^(1/3) - 9*(-a)^(1/3)*b^(2/3))/((-a)^(1/3)*b^(2/3)) + (log(9*b*x^(
1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-
a)^(1/3)*b^(2/3)) - (log(9*b*x^(1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i +
1)^2)/4)*(3^(1/2)*1i + 1))/(2*(-a)^(1/3)*b^(2/3))
```

3.679 $\int \frac{1}{x^{2/3}(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

```
[Out] 3/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(1/3)-1/2*ln(b*x+a)/a^(2/3)/b^(1/
3)-arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))*3^(1/2)/a^(2/3)/
b^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {60, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}+\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2/3
)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[
a + b*x]/(2*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = -\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}$$

$$= \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}} + \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

Mathematica [**A**]
time = 0.06, size = 103, normalized size = 1.03

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)
*x^(2/3)])/(a^(2/3)*b^(1/3))
```

Maple [**A**]
time = 0.10, size = 95, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	95
default	$\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^
(1/3)*x^(1/3)+(a/b)^(2/3))+1/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x^(1/3)-1))
```

Maxima [A]
time = 0.48, size = 102, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(2/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3
)) + log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.52, size = 307, normalized size = 3.07

$$\left[\frac{\sqrt{3}\,ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+\sqrt{3}\left(2\,abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{2\,a^2b},\ \frac{2\sqrt{3}\,ab\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{3}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{3\,a^2}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + sqrt(3)*(2*a*
b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b)
 - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) - (a^2*b)^(2/3)*log(a*b*x^(2/3) +
(a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3)
+ (a^2*b)^(2/3)))/(a^2*b), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(
-1/3*sqrt(3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)
/b)/a^2) - (a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*
x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A]
time = 3.98, size = 141, normalized size = 1.41

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a} & \text{for}\ b = 0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for}\ a = 0 \\ \frac{\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a, Eq(b, 0)), (-
3/(2*b*x**(2/3)), Eq(a, 0)), (log(x**(1/3) - (-a/b)**(1/3))/(b*(-a/b)**(2/3
)) - log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*b*(-a/
b)**(2/3)) - sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)
/(b*(-a/b)**(2/3)), True))
```

Giac [A]
time = 1.66, size = 117, normalized size = 1.17

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a} + \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab} + \frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a + sqrt(3)*(-a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/2*(-a*
b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B]
time = 0.21, size = 110, normalized size = 1.10

$$\frac{\ln\left(9\,a^{1/3}\,b^{5/3} + 9\,b^2\,x^{1/3}\right)}{a^{2/3}\,b^{1/3}} + \frac{\ln\left(9\,b^2\,x^{1/3} + \frac{9\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(9\,b^2\,x^{1/3} - \frac{9\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)),x)
```

```
[Out] log(9*a^(1/3)*b^(5/3) + 9*b^2*x^(1/3))/(a^(2/3)*b^(1/3)) + (log(9*b^2*x^(1/
3) + (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^(2/3)*b
^(1/3)) - (log(9*b^2*x^(1/3) - (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(
1/2)*1i + 1))/(2*a^(2/3)*b^(1/3))
```

3.680 $\int \frac{1}{x^{4/3}(a+bx)}\,dx$

Optimal. Leaf size=109

$$-\frac{3}{a\sqrt[3]{x}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}+\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}$$

```
[Out] -3/a/x^(1/3)+3/2*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(4/3)-1/2*b^(1/3)*ln
(b*x+a)/a^(4/3)+b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1
/2))*3^(1/2)/a^(4/3)
```

Rubi [A]
time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {53, 58, 631, 210, 31}

$$\frac{\sqrt{3}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}+\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqr
t[3]*a^(1/3))])/a^(4/3) + (3*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
4/3)) - (b^(1/3)*Log[a + b*x])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{3}{a\sqrt[3]{x}} - \frac{b\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{a}$$

$$= -\frac{3}{a\sqrt[3]{x}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a} + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}}{}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\right.}{a^{4/3}}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a}{2a^{4/3}}$$

Mathematica [A]
time = 0.09, size = 127, normalized size = 1.17

$$\frac{-\frac{6\sqrt[3]{a}}{\sqrt[3]{x}} + 2\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - \sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(4/3)*(a + b*x)),x]

[Out] ((-6*a^(1/3))/x^(1/3) + 2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a^(4/3))

Maple [A]
time = 0.12, size = 112, normalized size = 1.03

method	result	size
risch	$-\frac{3}{a\,x^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	104
derivativedivides	$-\frac{3}{a\,x^{\frac{1}{3}}}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	112
default	$-\frac{3}{a\,x^{\frac{1}{3}}}-\frac{3\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	112

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(4/3)/(b*x+a),x,method=_RETURNVERBOSE)

[Out] -3/a/x^(1/3)-3*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*b/a

Maxima [A]

time = 0.48, size = 111, normalized size = 1.02

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3}{ax^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(1/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/
3)) + log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 3/(a*x^(1/3))
```

Fricas [A]
time = 0.63, size = 113, normalized size = 1.04

$$-\frac{2\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+6\,x^{\frac{2}{3}}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^(1/3) - 1/3*
sqrt(3)) + x*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(
1/3)) - 2*x*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 6*x^(2/3))/(a*x)
```

Sympy [A]
time = 8.90, size = 151, normalized size = 1.39

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{3}{a\sqrt[3]{x}} & \text{for}\: b = 0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for}\: a = 0 \\ -\frac{\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{a\sqrt[3]{-\frac{a}{b}}}+\frac{\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\,\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\sqrt[3]{-\frac{a}{b}}}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a\sqrt[3]{-\frac{a}{b}}}-\frac{3}{a\sqrt[3]{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(a*x**(1/3)), Eq(b, 0)),
 (-3/(4*b*x**(4/3)), Eq(a, 0)), (-log(x**(1/3) - (-a/b)**(1/3))/(a*(-a/b)**
(1/3)) + log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*a*
(-a/b)**(1/3)) - sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3
)/3)/(a*(-a/b)**(1/3)) - 3/(a*x**(1/3)), True))
```

Giac [A]
time = 1.39, size = 125, normalized size = 1.15

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b}-\frac{3}{ax^{\frac{1}{3}}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + sqrt(3)*(-a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 3/(
a*x^(1/3)) - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a^2*b)
```

Mupad [B]
time = 0.15, size = 124, normalized size = 1.14

$$\frac{b^{1/3}\,\ln\left(9\,a^{4/3}\,b^{8/3}+9\,a\,b^3\,x^{1/3}\right)}{a^{4/3}}-\frac{3}{a\,x^{1/3}}+\frac{b^{1/3}\,\ln\left(9\,a\,b^3\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{4/3}}-\frac{b^{1/3}\,\ln\left(9\,a\,b^3\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)),x)
```

```
[Out] (b^(1/3)*log(9*a^(4/3)*b^(8/3) + 9*a*b^3*x^(1/3)))/a^(4/3) - 3/(a*x^(1/3))
+ (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2)^2
)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3) - (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3
)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 + 1/2))/a^(4/3)
```

3.681 $\int \frac{1}{x^{5/3}(a+bx)}\,dx$

Optimal. Leaf size=111

$$-\frac{3}{2ax^{2/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}$$

```
[Out] -3/2/a/x^(2/3)-3/2*b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(5/3)+1/2*b^(2/3)*
ln(b*x+a)/a^(5/3)+b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^
(1/2))*3^(1/2)/a^(5/3)
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {53, 60, 631, 210, 31}

$$\frac{\sqrt{3}\,b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(5/3) - (3*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a
^(5/3)) + (b^(2/3)*Log[a + b*x])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = -\frac{3}{2ax^{2/3}} - \frac{b\int \frac{1}{x^{2/3}(a+bx)}\,dx}{a}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\left(3b^{2/3}\right)\text{S}}{}$$

$$= -\frac{3}{2ax^{2/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{a^{5/3}}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+}{2a^{5/3}}$$

Mathematica [A]
time = 0.09, size = 126, normalized size = 1.14

$$\frac{-\frac{3a^{2/3}}{x^{2/3}} + 2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] ((-3*a^(2/3))/x^(2/3) + 2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a
^(1/3))/Sqrt[3]] - 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + b^(2/3)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a^(5/3))
```

Maple [A]
time = 0.17, size = 112, normalized size = 1.01

method	result	size
derivativedivides	$-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}-\frac{3}{2a\,x^{\frac{2}{3}}}$	112
default	$-\frac{3\left(\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}-\frac{3}{2a\,x^{\frac{2}{3}}}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -3*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-
(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x^(1/3)-1)))*b/a-3/2/a/x^(2/3)
```

Maxima [A]
time = 0.49, size = 112, normalized size = 1.01

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3}{2\,ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(2/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/
3)) - log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 3/2/(a*x^(2/3))
```

Fricas [A]
time = 0.56, size = 147, normalized size = 1.32

$$\frac{2\sqrt{3}\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+2\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,x^{\frac{1}{3}}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2
)^(2/3) - sqrt(3)*b)/b) - x*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*
(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x*(-b^2/a^2)^(1/3)*log(b*x^(1/
3) - a*(-b^2/a^2)^(1/3)) - 3*x^(1/3))/(a*x)
```

Sympy [A]
time = 11.68, size = 155, normalized size = 1.40

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}} & \text{for}\: b = 0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for}\: a = 0 \\ -\frac{\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\,\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(-\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3}{2ax^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*a*x**(2/3)), Eq(b, 0)
), (-3/(5*b*x**(5/3)), Eq(a, 0)), (-log(x**(1/3) - (-a/b)**(1/3))/(a*(-a/b)
**(2/3)) + log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(2*
a*(-a/b)**(2/3)) + sqrt(3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt
(3)/3)/(a*(-a/b)**(2/3)) - 3/(2*a*x**(2/3)), True))
```

Giac [A]
time = 2.09, size = 120, normalized size = 1.08

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a^2}-\frac{3}{2\,ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 - sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/2*(-a
*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/2/(a
*x^(2/3))
```

Mupad [B]
time = 0.07, size = 138, normalized size = 1.24

$$\frac{b^{2/3}\,\ln\left(9\,{\left(-a\right)}^{7/3}\,b^{8/3}-9\,a^2\,b^3\,x^{1/3}\right)}{{\left(-a\right)}^{5/3}}-\frac{3}{2\,a\,x^{2/3}}+\frac{b^{2/3}\,\ln\left(9\,{\left(-a\right)}^{7/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-9\,a^2\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{{\left(-a\right)}^{5/3}}-\frac{b^{2/3}\,\ln\left(9\,{\left(-a\right)}^{7/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+9\,a^2\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{{\left(-a\right)}^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)),x)
```

```
[Out] (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3) - 9*a^2*b^3*x^(1/3)))/(-a)^(5/3) - 3/(2*a
*x^(2/3)) + (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2) - 9*a^
2*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(-a)^(5/3) - (b^(2/3)*log(9*(-a)^(7/
3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 9*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/
2))/(-a)^(5/3)
```

3.682 $\int \frac{x^{5/3}}{(a+bx)^2}\,dx$

Optimal. Leaf size=129

$$\frac{5x^{2/3}}{2b^2}-\frac{x^{5/3}}{b(a+bx)}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}$$

```
[Out] 5/2*x^(2/3)/b^2-x^(5/3)/b/(b*x+a)+5/2*a^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b
^(8/3)-5/6*a^(2/3)*ln(b*x+a)/b^(8/3)+5/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x^(1/3))/a^(1/3)*3^(1/2))/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 58, 631, 210, 31}

$$\frac{5a^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}+\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (5*x^(2/3))/(2*b^2) - x^(5/3)/(b*(a + b*x)) + (5*a^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) - (5*a^(2/3)*Log[a + b*x])/(6*b^(8/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = -\frac{x^{5/3}}{b(a+bx)} + \frac{5\int \frac{x^{2/3}}{a+bx}\,dx}{3b}$$

$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} - \frac{(5a)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b^2}$$

$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}} - \frac{(5a)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^3} + \frac{(5a^2}{}$$

$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} + \frac{5a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}} - \frac{(5a^{2/3})\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} + \frac{5a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{5a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3}\text{lo}}{6}$$

Mathematica [A]
time = 0.20, size = 142, normalized size = 1.10

$$\frac{\frac{3b^{2/3}x^{2/3}(5a+3bx)}{a+bx} + 10\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) - 5a^{2/3}\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{6b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] ((3*b^(2/3)*x^(2/3)*(5*a + 3*b*x))/(a + b*x) + 10*Sqrt[3]*a^(2/3)*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 10*a^(2/3)*Log[a^(1/3) + b^(1/3)
*x^(1/3)] - 5*a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/
3)])/(6*b^(8/3))
```

Maple [A]
time = 0.12, size = 124, normalized size = 0.96

method	result

risch	$\frac{3x^{\frac{2}{3}}}{2b^2}+\frac{a\,x^{\frac{2}{3}}}{b^2(bx+a)}+\frac{5a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5a\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2b^2}-\frac{3a\left(-\frac{x^{\frac{2}{3}}}{3(bx+a)}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2}$
default	$\frac{3x^{\frac{2}{3}}}{2b^2}-\frac{3a\left(-\frac{x^{\frac{2}{3}}}{3(bx+a)}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*x^(2/3)/b^2-3*a/b^2*(-1/3*x^(2/3)/(b*x+a)-5/9/b/(a/b)^(1/3)*ln(x^(1/3)+
(a/b)^(1/3))+5/18/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))
+5/9*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]

time = 0.49, size = 133, normalized size = 1.03

$$\frac{ax^{\frac{2}{3}}}{b^3x+ab^2}-\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\,x^{\frac{2}{3}}}{2\,b^2}-\frac{5\,a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(2/3)/(b^3*x + a*b^2) - 5/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/2*x^(2/3)/b^2 - 5/6*a*log(x^
```

```
(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) + 5/3*a*log(x^
(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Fricas [A]
time = 0.60, size = 162, normalized size = 1.26

$$-\frac{10\,\sqrt{3}\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+5\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10\,(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3\,(3\,bx+5\,a)x^{\frac{2}{3}}}{6\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(10*sqrt(3)*(b*x + a)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*
(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(b*x + a)*(a^2/b^2)^(1/3)*log(-b*x^(1/3
)*(a^2/b^2)^(2/3) + a*x^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(b*x + a)*(a^2/b^2)
^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x^(1/3)) - 3*(3*b*x + 5*a)*x^(2/3))/(b^3*x
 + a*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(124) = 248$.
time = 89.40, size = 595, normalized size = 4.61

$$\begin{cases} \tilde{\infty} x^{\frac{2}{3}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{8}{3}}}{8a^2} & \text{for}\: b = 0 \\ \frac{3x^{\frac{2}{3}}}{2b^2} & \text{for}\: a = 0 \\ -\frac{10a^2\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}+\frac{5a^2\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}-\frac{10\sqrt{3}a^2\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}-\frac{10a^2\log(2)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}+\frac{15abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}-\frac{10abx\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}+\frac{5abx\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}-\frac{10\sqrt{3}abx\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}-\frac{10abx\log(2)}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}}+\frac{9b^2x^{\frac{5}{3}}\sqrt[3]{-\frac{a}{b}}}{6ab^3\sqrt[3]{-\frac{a}{b}}+6b^4x\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a**2), Eq(b,
0)), (3*x**(2/3)/(2*b**2), Eq(a, 0)), (-10*a**2*log(x**(1/3) - (-a/b)**(1/3
))/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) + 5*a**2*log(4*x**(2/3
) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a*b**3*(-a/b)**(1/3) + 6
*b**4*x*(-a/b)**(1/3)) - 10*sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)*
*(1/3)) + sqrt(3)/3)/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) - 10
*a**2*log(2)/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) + 15*a*b*x**
(2/3)*(-a/b)**(1/3)/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) - 10*
a*b*x*log(x**(1/3) - (-a/b)**(1/3))/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/
b)**(1/3)) + 5*a*b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**
(2/3))/(6*a*b**3*(-a/b)**(1/3) + 6*b**4*x*(-a/b)**(1/3)) - 10*sqrt(3)*a*b*x
*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a*b**3*(-a/b)**(
1/3) + 6*b**4*x*(-a/b)**(1/3)) - 10*a*b*x*log(2)/(6*a*b**3*(-a/b)**(1/3) +
6*b**4*x*(-a/b)**(1/3)) + 9*b**2*x**(5/3)*(-a/b)**(1/3)/(6*a*b**3*(-a/b)**(
1/3) + 6*b**4*x*(-a/b)**(1/3)), True))
```

Giac [A]
time = 1.08, size = 135, normalized size = 1.05

$$\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b^{2}}+\frac{ax^{\frac{2}{3}}}{(bx+a)b^{2}}+\frac{3\,x^{\frac{2}{3}}}{2\,b^{2}}+\frac{5\,\sqrt{3}\,(-ab^{2})^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4}}-\frac{5\,(-ab^{2})^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + a*x^(2/3)/((b*x + a
)*b^2) + 3/2*x^(2/3)/b^2 + 5/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2
*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 5/6*(-a*b^2)^(2/3)*log(x^(2/3)
 + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
```

Mupad [B]
time = 0.26, size = 150, normalized size = 1.16

$$\frac{3\,x^{2/3}}{2\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)}{3\,b^{8/3}}+\frac{a\,x^{2/3}}{x\,b^3+a\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}-\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(2/3))/(2*b^2) + (5*a^(2/3)*log((25*a^(7/3))/b^(10/3) + (25*a^2*x^(1/3
))/b^3))/(3*b^(8/3)) + (a*x^(2/3))/(a*b^2 + b^3*x) + (5*a^(2/3)*log((25*a^(
7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1
i)/2 - 1/2))/(3*b^(8/3)) - (5*a^(2/3)*log((25*a^(7/3)*((3^(1/2)*1i)/2 + 1/2
)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(8/3))
```

3.683 $\int \frac{x^{4/3}}{(a+bx)^2} \, dx$

Optimal. Leaf size=125

$$\frac{4\sqrt[3]{x}}{b^2}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}$$

```
[Out] 4*x^(1/3)/b^2-x^(4/3)/b/(b*x+a)-2*a^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/b^(7/
3)+2/3*a^(1/3)*ln(b*x+a)/b^(7/3)+4/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x^(1/3))/a^(1/3)*3^(1/2))/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 52, 60, 631, 210, 31}

$$\frac{4\sqrt[3]{a}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (4*x^(1/3))/b^2 - x^(4/3)/(b*(a + b*x)) + (4*a^(1/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (2*a^(1/3)*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/b^(7/3) + (2*a^(1/3)*Log[a + b*x])/(3*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = -\frac{x^{4/3}}{b(a+bx)} + \frac{4\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{3b}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{(4a)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b^2}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(2a^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{b^{8/3}} -$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(4\sqrt[3]{a}\right)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{4\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\text{l}}{}$$

Mathematica [A]
time = 0.20, size = 142, normalized size = 1.14

$$\frac{\frac{3\sqrt[3]{b}\,\sqrt[3]{x}\,(4a+3bx)}{a+bx} + 4\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4\sqrt[3]{a}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right) + 2\sqrt[3]{a}\,\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{3b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] ((3*b^(1/3)*x^(1/3)*(4*a + 3*b*x))/(a + b*x) + 4*Sqrt[3]*a^(1/3)*ArcTan[(1
- (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 4*a^(1/3)*Log[a^(1/3) + b^(1/3)*x
^(1/3)] + 2*a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)
])/(3*b^(7/3))
```

Maple [A]
time = 0.26, size = 124, normalized size = 0.99

method	result	size

derivativedivides	$\frac{3x^{\frac{1}{3}}}{b^{2}}-\frac{3a\left(-\frac{x^{\frac{1}{3}}}{3(bx+a)}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$	12
default	$\frac{3x^{\frac{1}{3}}}{b^{2}}-\frac{3a\left(-\frac{x^{\frac{1}{3}}}{3(bx+a)}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)/b^2-3*a/b^2*(-1/3*x^(1/3)/(b*x+a)+4/9/b/(a/b)^(2/3)*ln(x^(1/3)+(a
/b)^(1/3))-2/9/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+4/
9/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]

time = 0.48, size = 133, normalized size = 1.06

$$\frac{ax^{\frac{1}{3}}}{b^{3}x+ab^{2}}-\frac{4\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\,x^{\frac{1}{3}}}{b^{2}}+\frac{2\,a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4\,a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(1/3)/(b^3*x + a*b^2) - 4/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3*x^(1/3)/b^2 + 2/3*a*log(x^(2
/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4/3*a*log(x^(1
/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [A]

time = 0.81, size = 147, normalized size = 1.18

$$\frac{4\sqrt{3}\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-2\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+4\,(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+3\,(3\,bx+4\,a)x^{\frac{1}{3}}}{3\,(b^{3}x+ab^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*x + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b
)^(2/3) - sqrt(3)*a)/a) - 2*(b*x + a)*(-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-
a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x + a)*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^
(1/3)) + 3*(3*b*x + 4*a)*x^(1/3))/(b^3*x + a*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(121) = 242$.

time = 68.81, size = 457, normalized size = 3.66

$$\begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{7}{3}}}{7a^2} & \text{for } b = 0 \\ \frac{3\sqrt[3]{x}}{b^2} & \text{for } a = 0 \\ \frac{12a\sqrt[3]{x}}{3ab^2+3b^3x} + \frac{4a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x} - \frac{4\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2+3b^3x} + \frac{4a\sqrt[3]{-\frac{a}{b}}\log(2)}{3ab^2+3b^3x} + \frac{9bx^{\frac{4}{3}}}{3ab^2+3b^3x} + \frac{4bx\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x} - \frac{2bx\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x} - \frac{4\sqrt{3}bx\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2+3b^3x} + \frac{4bx\sqrt[3]{-\frac{a}{b}}\log(2)}{3ab^2+3b^3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a**2), Eq(b,
0)), (3*x**(1/3)/b**2, Eq(a, 0)), (12*a*x**(1/3)/(3*a*b**2 + 3*b**3*x) + 4*
a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x) - 2*a*(
-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(
3*a*b**2 + 3*b**3*x) - 4*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3
*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a*b**2 + 3*b**3*x) + 4*a*(-a/b)**(1/3)*log(
2)/(3*a*b**2 + 3*b**3*x) + 9*b*x**(4/3)/(3*a*b**2 + 3*b**3*x) + 4*b*x*(-a/b
)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x) - 2*b*x*(-a/b)
**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b
**2 + 3*b**3*x) - 4*sqrt(3)*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-
a/b)**(1/3)) + sqrt(3)/3)/(3*a*b**2 + 3*b**3*x) + 4*b*x*(-a/b)**(1/3)*log(2
)/(3*a*b**2 + 3*b**3*x), True))
```

Giac [A]
time = 0.86, size = 135, normalized size = 1.08

$$\frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b^2} - \frac{4\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{ax^{\frac{1}{3}}}{(bx+a)b^2} + \frac{3\,x^{\frac{1}{3}}}{b^2} - \frac{2\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - 4/3*sqrt(3)*(-a*b^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + a
*x^(1/3)/((b*x + a)*b^2) + 3*x^(1/3)/b^2 - 2/3*(-a*b^2)^(1/3)*log(x^(2/3) +
 x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

Mupad [B]
time = 0.15, size = 142, normalized size = 1.14

$$\frac{3\,x^{1/3}}{b^2}+\frac{a\,x^{1/3}}{x\,b^3+a\,b^2}+\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{12\,{\left(-a\right)}^{4/3}}{b^{1/3}}+12\,a\,x^{1/3}\right)}{3\,b^{7/3}}-\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}-\frac{12\,{\left(-a\right)}^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{7/3}}+\frac{{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}+\frac{9\,{\left(-a\right)}^{4/3}\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{1/3}}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(1/3))/b^2 + (a*x^(1/3))/(a*b^2 + b^3*x) + (4*(-a)^(1/3)*log((12*(-a)^
(4/3))/b^(1/3) + 12*a*x^(1/3)))/(3*b^(7/3)) - (4*(-a)^(1/3)*log(12*a*x^(1/3
) - (12*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1/2))
/(3*b^(7/3)) + ((-a)^(1/3)*log(12*a*x^(1/3) + (9*(-a)^(4/3)*((3^(1/2)*2i)/3
 - 2/3))/b^(1/3))*((3^(1/2)*2i)/3 - 2/3))/b^(7/3)
```

3.684 $\int \frac{x^{2/3}}{(a+bx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{x^{2/3}}{b(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}$$

```
[Out] -x^(2/3)/b/(b*x+a)-ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(1/3)/b^(5/3)+1/3*ln(b*x+a
)/a^(1/3)/b^(5/3)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2
))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 58, 631, 210, 31}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(
1/3)*b^(5/3)) + Log[a + b*x]/(3*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
```

```
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{(a+bx)^2}\,dx = -\frac{x^{2/3}}{b(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b}$$

$$= -\frac{x^{2/3}}{b(a+bx)} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{b^2} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,d\right.}{\sqrt[3]{a}\,b^{5/3}}$$

$$= -\frac{x^{2/3}}{b(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{}}{\sqrt[3]{a}}\right.}{\sqrt[3]{a}\,b^{5/3}}$$

$$= -\frac{x^{2/3}}{b(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}} + \frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}$$

Mathematica [A]
time = 0.17, size = 133, normalized size = 1.16

$$\frac{-\frac{3b^{2/3}x^{2/3}}{a+bx} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{a}}}{3b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] ((-3*b^(2/3)*x^(2/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3)
)/a^(1/3))/Sqrt[3]])/a^(1/3) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(1/3) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/a^(1/3))/(3*b^(5/
3))
```

Maple [A]
time = 0.11, size = 118, normalized size = 1.03

method	result	size
derivativedivides	$-\frac{x^{\frac{2}{3}}}{b(bx+a)}+\frac{-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	118
default	$-\frac{x^{\frac{2}{3}}}{b(bx+a)}+\frac{-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(2/3)/b/(b*x+a)+2/b*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a
/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.48, size = 120, normalized size = 1.04

$$-\frac{x^{\frac{2}{3}}}{b^2x+ab}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(2/3)/(b^2*x + a*b) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/
```

3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) - 2/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(1/3))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(84) = 168$.

time = 0.98, size = 394, normalized size = 3.43

$$\left[-\frac{3ab^2x^{\frac{2}{3}} - 3\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right) - (-ab^2)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right) + 2(-ab^2)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{3(ab^4x+a^2b^3)}, -\frac{3ab^2x^{\frac{2}{3}} - 6\sqrt{\frac{1}{3}}(ab^2x+a^2b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right) - (-ab^2)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right) + 2(-ab^2)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{3(ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a*b^2*x^(2/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*
x + a)) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/
3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^
(1/3)))/(a*b^4*x + a^2*b^3), -1/3*(3*a*b^2*x^(2/3) - 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^4*x + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(107) = 214$.

time = 39.06, size = 527, normalized size = 4.58

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{3}{b^2\sqrt[3]{x}} & \text{for}\ a = 0 \\ \frac{3x^{\frac{5}{3}}}{5a^2} & \text{for}\ b = 0 \\ \frac{2a\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{a\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}\,a\,\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2a\log(2)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{3bx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} - \frac{bx\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}\,bx\,\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log(2)}{3ab^2\sqrt[3]{-\frac{a}{b}}+3b^3x\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (-3/(b**2*x**(1/3)), Eq(a, 0
)), (3*x**(5/3)/(5*a**2), Eq(b, 0)), (2*a*log(x**(1/3) - (-a/b)**(1/3))/(3*
a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - a*log(4*x**(2/3) + 4*x**(1
/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a
/b)**(1/3)) + 2*sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(
3)/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*a*log(2)/(3*a*b
**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - 3*b*x**(2/3)*(-a/b)**(1/3)/(3
*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) + 2*b*x*log(x**(1/3) - (-a/
b)**(1/3))/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) - b*x*log(4*x*
*(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2*(-a/b)**(1/3
```

```
) + 3*b**3*x*(-a/b)**(1/3)) + 2*sqrt(3)*b*x*atan(2*sqrt(3)*x**(1/3)/(3*(-a/
b)**(1/3)) + sqrt(3)/3)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)) +
 2*b*x*log(2)/(3*a*b**2*(-a/b)**(1/3) + 3*b**3*x*(-a/b)**(1/3)), True))
```

Giac [A]
time = 1.00, size = 136, normalized size = 1.18

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}-\frac{x^{\frac{2}{3}}}{(bx+a)b}-\frac{2\sqrt{3}\,(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^3}+\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) - x^(2/3)/((b*x +
a)*b) - 2/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

Mupad [B]
time = 0.24, size = 142, normalized size = 1.23

$$\frac{2\ln\left(\frac{4\,x^{1/3}}{b}-\frac{4\,(-a)^{1/3}}{b^{4/3}}\right)}{3\,(-a)^{1/3}\,b^{5/3}}-\frac{x^{2/3}}{b\,(a+b\,x)}+\frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{b^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3\,(-a)^{1/3}\,b^{5/3}}-\frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{b^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3\,(-a)^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^2,x)
```

```
[Out] (2*log((4*x^(1/3))/b - (4*(-a)^(1/3))/b^(4/3)))/(3*(-a)^(1/3)*b^(5/3)) - x^
(2/3)/(b*(a + b*x)) + (log((4*x^(1/3))/b - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2)/
b^(4/3))*(3^(1/2)*1i - 1))/(3*(-a)^(1/3)*b^(5/3)) - (log((4*x^(1/3))/b - ((
-a)^(1/3)*(3^(1/2)*1i + 1)^2)/b^(4/3))*(3^(1/2)*1i + 1))/(3*(-a)^(1/3)*b^(5
/3))
```

3.685 $\int \frac{\sqrt[3]{x}}{(a+bx)^2} dx$

Optimal. Leaf size=117

$$-\frac{\sqrt[3]{x}}{b(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

```
[Out] -x^(1/3)/b/(b*x+a)+1/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(2/3)/b^(4/3)-1/6*ln(b
*x+a)/a^(2/3)/b^(4/3)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^
(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 60, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(2/
3)*b^(4/3)) - Log[a + b*x]/(6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{\sqrt[3]{x}}{b(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, \ldots\right)}{2a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}} \ldots\right)}{a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

Mathematica [A]
time = 0.17, size = 134, normalized size = 1.15

$$\frac{-\frac{6\sqrt[3]{b}\,\sqrt[3]{x}}{a+bx} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{2/3}} - \frac{\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{a^{2/3}}}{6b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] ((-6*b^(1/3)*x^(1/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3)
)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(2/3) -
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/a^(2/3))/(6*b^(4/
3))
```

Maple [A]
time = 0.10, size = 117, normalized size = 1.00

method	result	size
derivativedivides	$-\frac{x^{\frac{1}{3}}}{b(bx+a)}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b}$	117
default	$-\frac{x^{\frac{1}{3}}}{b(bx+a)}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/3)/b/(b*x+a)+1/b*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/
b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.48, size = 120, normalized size = 1.03

$$-\frac{x^{\frac{1}{3}}}{b^2x+ab}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(1/3)/(b^2*x + a*b) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/
```

```
3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(84) = 168$.

time = 1.02, size = 389, normalized size = 3.32

$$\left[-\frac{6\,a^{2}bx^{\frac{1}{3}}-3\,\sqrt{\frac{1}{3}}\,(ab^{2}x+a^{2}b)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-(a^{2}b)^{\frac{1}{3}}a+(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}-3\,(a^{2}b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)+(a^{2}b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+(a^{2}b)^{\frac{1}{3}}a-(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)-2\,(a^{2}b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+(a^{2}b)^{\frac{2}{3}}\right)}{6\,(a^{2}b^{3}x+a^{3}b^{2})},\ -\frac{6\,a^{2}bx^{\frac{1}{3}}-6\,\sqrt{\frac{1}{3}}\,(ab^{2}x+a^{2}b)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^{2}b)^{\frac{1}{3}}a-2\,(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)+(a^{2}b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+(a^{2}b)^{\frac{1}{3}}a-(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)-2\,(a^{2}b)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+(a^{2}b)^{\frac{2}{3}}\right)}{6\,(a^{2}b^{3}x+a^{3}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a^2*b*x^(1/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/
b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2
*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
 + a)) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b
)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3
)))/(a^2*b^3*x + a^3*b^2), -1/6*(6*a^2*b*x^(1/3) - 6*sqrt(1/3)*(a*b^2*x + a
^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^
(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(b*x + a)*log(a*b
*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x
+ a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(107) = 214$.

time = 25.54, size = 450, normalized size = 3.85

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a^{2}} & \text{for}\: b = 0 \\ -\frac{3}{2b^{2}x^{\frac{2}{3}}} & \text{for}\: a = 0 \\ -\frac{6a\sqrt[3]{x}}{6a^{2}b+6ab^{2}x}-\frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^{2}b+6ab^{2}x}+\frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{2}b+6ab^{2}x}+\frac{2\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^{2}b+6ab^{2}x}-\frac{2a\sqrt[3]{-\frac{a}{b}}\log(2)}{6a^{2}b+6ab^{2}x}-\frac{2bx\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^{2}b+6ab^{2}x}+\frac{bx\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{2}b+6ab^{2}x}+\frac{2\sqrt{3}bx\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^{2}b+6ab^{2}x}-\frac{2bx\sqrt[3]{-\frac{a}{b}}\log(2)}{6a^{2}b+6ab^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**2), Eq(b,
0)), (-3/(2*b**2*x**(2/3)), Eq(a, 0)), (-6*a*x**(1/3)/(6*a**2*b + 6*a*b**2*
x) - 2*a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(6*a**2*b + 6*a*b**2*x
) + a*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(
2/3))/(6*a**2*b + 6*a*b**2*x) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x*
*(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**2*b + 6*a*b**2*x) - 2*a*(-a/b)*
*(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x) - 2*b*x*(-a/b)**(1/3)*log(x**(1/3) -
(-a/b)**(1/3))/(6*a**2*b + 6*a*b**2*x) + b*x*(-a/b)**(1/3)*log(4*x**(2/3) +
 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b + 6*a*b**2*x) + 2*sq
rt(3)*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)
```

```
/3)/(6*a**2*b + 6*a*b**2*x) - 2*b*x*(-a/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b*
*2*x), True))
```

Giac [A]
time = 0.97, size = 136, normalized size = 1.16

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}+\frac{\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2}-\frac{x^{\frac{1}{3}}}{(bx+a)b}+\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^2) - x^(1/3)/((b*x + a)*b) + 1/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B]
time = 0.06, size = 120, normalized size = 1.03

$$\frac{\ln\left(3\,b\,x^{1/3}+3\,a^{1/3}\,b^{2/3}\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{x^{1/3}}{b\,\left(a+b\,x\right)}+\frac{\ln\left(3\,b\,x^{1/3}+\frac{3\,a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(3\,b\,x^{1/3}-\frac{3\,a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^2,x)
```

```
[Out] log(3*b*x^(1/3) + 3*a^(1/3)*b^(2/3))/(3*a^(2/3)*b^(4/3)) - x^(1/3)/(b*(a +
b*x)) + (log(3*b*x^(1/3) + (3*a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)
*1i - 1))/(6*a^(2/3)*b^(4/3)) - (log(3*b*x^(1/3) - (3*a^(1/3)*b^(2/3)*(3^(1
/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(4/3))
```

3.686 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx$

Optimal. Leaf size=116

$$\frac{x^{2/3}}{a(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}$$

```
[Out] x^(2/3)/a/(b*x+a)-1/2*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(4/3)/b^(2/3)+1/6*ln(b*
x+a)/a^(4/3)/b^(2/3)-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(
1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 58, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/
3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(4/3)*
b^(2/3)) + Log[a + b*x]/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx = \frac{x^{2/3}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a}$$

$$= \frac{x^{2/3}}{a(a+bx)} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2ab} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+\cdots}\right.}{2a^{4/3}b\cdots}$$

$$= \frac{x^{2/3}}{a(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\cdots}{\sqrt[3]{a}\cdots}\right.}{a^{4/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{a(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}}$$

Mathematica [A]
time = 0.16, size = 133, normalized size = 1.15

$$\frac{\frac{6\sqrt[3]{a}\,x^{2/3}}{a+bx} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{b^{2/3}}}{6a^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] ((6*a^(1/3)*x^(2/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))
/a^(1/3))/Sqrt[3]])/b^(2/3) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(2/3) +
Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/b^(2/3))/(6*a^(4/3
))
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.00

method	result	size
derivativedivides	$\frac{x^{\frac{2}{3}}}{a(bx+a)}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{a}$	116
default	$\frac{x^{\frac{2}{3}}}{a(bx+a)}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(2/3)/a/(b*x+a)+1/a*(-1/3/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/
b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.50, size = 127, normalized size = 1.09

$$\frac{x^{\frac{2}{3}}}{abx+a^2}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(2/3)/(a*b*x + a^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
```

```
) + (a/b)^(2/3))/(a*b*(a/b)^(1/3)) - 1/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(83) = 166$.

time = 0.71, size = 396, normalized size = 3.41

$$\left[\frac{6ab^2x^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x - ab + 3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}}a + 2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx + a}\right) + \left(-ab^2\right)^{\frac{2}{3}}(bx + a)\log\left(b^2x^{\frac{2}{3}} + \left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{2}{3}}\right) - 2\left(-ab^2\right)^{\frac{2}{3}}(bx + a)\log\left(bx^{\frac{1}{3}} - \left(-ab^2\right)^{\frac{1}{3}}\right)}{6\left(a^2b^3x + a^3b^2\right)}, \frac{6ab^2x^{\frac{2}{3}} + 6\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right) + \left(-ab^2\right)^{\frac{2}{3}}(bx + a)\log\left(b^2x^{\frac{2}{3}} + \left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{2}{3}}\right) - 2\left(-ab^2\right)^{\frac{2}{3}}(bx + a)\log\left(bx^{\frac{1}{3}} - \left(-ab^2\right)^{\frac{1}{3}}\right)}{6\left(a^2b^3x + a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*b^2*x^(2/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3
) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^(
1/3)))/(a^2*b^3*x + a^3*b^2), 1/6*(6*a*b^2*x^(2/3) + 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a^2*b^3*x + a^3*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(107) = 214$.

time = 27.07, size = 544, normalized size = 4.69

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{3}{4b^2x^{\frac{4}{3}}} & \text{for}\: a = 0 \\ \frac{3x^{\frac{2}{3}}}{2a^2} & \text{for}\: b = 0 \\ \frac{2a\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} - \frac{a\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}a\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2a\log(2)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{6bx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} - \frac{bx\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2\sqrt{3}bx\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} + \frac{2bx\log(2)}{6a^2b\sqrt[3]{-\frac{a}{b}} + 6ab^2x\sqrt[3]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b**2*x**(4/3)), Eq(a,
 0)), (3*x**(2/3)/(2*a**2), Eq(b, 0)), (2*a*log(x**(1/3) - (-a/b)**(1/3))/(
6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) - a*log(4*x**(2/3) + 4*x
**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2
*x*(-a/b)**(1/3)) + 2*sqrt(3)*a*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) +
 sqrt(3)/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*a*log(2
)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 6*b*x**(2/3)*(-a/b)
**(1/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*b*x*log(x**
(1/3) - (-a/b)**(1/3))/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3))
- b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*
b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)**(1/3)) + 2*sqrt(3)*b*x*atan(2*sqrt(3)*
```

```
x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*
x*(-a/b)**(1/3)) + 2*b*x*log(2)/(6*a**2*b*(-a/b)**(1/3) + 6*a*b**2*x*(-a/b)
**(1/3)), True))
```

Giac [A]
time = 1.26, size = 132, normalized size = 1.14

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{x^{\frac{2}{3}}}{(bx+a)a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + x^(2/3)/((b*x + a)
*a) - 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

Mupad [B]
time = 0.36, size = 144, normalized size = 1.24

$$\frac{x^{2/3}}{a\,\left(a+b\,x\right)}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{{\left(-1\right)}^{2/3}\,b^{2/3}}{a^{5/3}}+\frac{b\,x^{1/3}}{a^2}\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{4/3}\,b^{2/3}}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{9\,{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{a^{5/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{4/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^2),x)
```

```
[Out] x^(2/3)/(a*(a + b*x)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/a^(5/3) + (b*x
^(1/3))/a^2))/(3*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((b*x^(1/3))/a^2 + ((-1)
^(2/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(5/3))*((3^(1/2)*1i)/2 + 1/2))/(
3*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*log((b*x^(1/3))/a^2 + (9*(-1)^(2/3)*b^(2/3
)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(5/3))*((3^(1/2)*1i)/6 - 1/6))/(a^(4/3)*b^(2/
3))
```

3.687 $\int \frac{1}{x^{2/3}(a+bx)^2}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt[3]{x}}{a(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}$$

```
[Out] x^(1/3)/a/(b*x+a)+ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(5/3)/b^(1/3)-1/3*ln(b*x+a)
/a^(5/3)/b^(1/3)-2/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2)
)/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 60, 631, 210, 31}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(5/3
)*b^(1/3)) - Log[a + b*x]/(3*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{2/3}(a+bx)^2}\,dx &= \frac{\sqrt[3]{x}}{a(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a}\\ &= \frac{\sqrt[3]{x}}{a(a+bx)}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{a^{4/3}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,d\right.}{a^{5/3}\sqrt[3]{b}}\\ &= \frac{\sqrt[3]{x}}{a(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\,\sqrt[3]{}}{\sqrt[3]{a}}\right.}{a^{5/3}\sqrt[3]{b}}\\ &= \frac{\sqrt[3]{x}}{a(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 134, normalized size = 1.19

$$\frac{\frac{3a^{2/3}\sqrt[3]{x}}{a+bx}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{b}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{b}}}{3a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] ((3*a^(2/3)*x^(1/3))/(a + b*x) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))
/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(1/3) -
Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/b^(1/3))/(3*a^(5/3
))
```

Maple [A]
time = 0.11, size = 117, normalized size = 1.04

method	result	size
derivativedivides	$\frac{x^{\frac{1}{3}}}{a(bx+a)}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	117
default	$\frac{x^{\frac{1}{3}}}{a(bx+a)}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/3)/a/(b*x+a)+2/a*(1/3/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b
)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.49, size = 127, normalized size = 1.12

$$\frac{x^{\frac{1}{3}}}{abx+a^2}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(1/3)/(a*b*x + a^2) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
```

```
) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(82) = 164$.
time = 0.87, size = 387, normalized size = 3.42

$$\left[\frac{3 a^2 b x^{\frac{1}{3}} + 3 \sqrt{\frac{1}{3}} (a b^2 x + a^2 b) \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}} \log\left(\frac{2 a b x - a^2 + 3 \sqrt{\frac{1}{3}} \left(2 a b x^{\frac{2}{3}} - (a^2 b)^{\frac{1}{3}} a + (a^2 b)^{\frac{2}{3}} x^{\frac{1}{3}}\right) \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}} - 3 (a^2 b)^{\frac{1}{3}} a x^{\frac{1}{3}}}{b x + a}\right) - (a^2 b)^{\frac{2}{3}} (b x + a) \log\left(a b x^{\frac{2}{3}} + (a^2 b)^{\frac{1}{3}} a - (a^2 b)^{\frac{2}{3}} x^{\frac{1}{3}}\right) + 2 (a^2 b)^{\frac{2}{3}} (b x + a) \log\left(a b x^{\frac{1}{3}} + (a^2 b)^{\frac{2}{3}}\right)}{3 (a^3 b^2 x + a^4 b)}, \frac{3 a^2 b x^{\frac{1}{3}} + 6 \sqrt{\frac{1}{3}} (a b^2 x + a^2 b) \sqrt{\frac{(a^2 b)^{\frac{1}{3}}}{b}} \arctan\left(-\frac{\sqrt{\frac{1}{3}} \left((a^2 b)^{\frac{1}{3}} a - 2 (a^2 b)^{\frac{2}{3}} x^{\frac{1}{3}}\right) \sqrt{\frac{(a^2 b)^{\frac{1}{3}}}{b}}}{a^2}\right) - (a^2 b)^{\frac{2}{3}} (b x + a) \log\left(a b x^{\frac{2}{3}} + (a^2 b)^{\frac{1}{3}} a - (a^2 b)^{\frac{2}{3}} x^{\frac{1}{3}}\right) + 2 (a^2 b)^{\frac{2}{3}} (b x + a) \log\left(a b x^{\frac{1}{3}} + (a^2 b)^{\frac{2}{3}}\right)}{3 (a^3 b^2 x + a^4 b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^2*b*x^(1/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)
^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)
))/(a^3*b^2*x + a^4*b), 1/3*(3*a^2*b*x^(1/3) + 6*sqrt(1/3)*(a*b^2*x + a^2*b
)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3
)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(
2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)
*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^3*b^2*x + a^4*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(107) = 214$.
time = 39.31, size = 434, normalized size = 3.84

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a^2} & \text{for } b = 0 \\ -\frac{3}{5b^2x^{\frac{5}{3}}} & \text{for } a = 0 \\ \frac{3a\sqrt[3]{x}}{3a^3+3a^2bx} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{3a^3+3a^2bx} + \frac{a\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3a^3+3a^2bx} + \frac{2\sqrt{3}a\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3a^3+3a^2bx} - \frac{2a\sqrt[3]{-\frac{a}{b}}\log(2)}{3a^3+3a^2bx} - \frac{2bx\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x} - \sqrt[3]{-\frac{a}{b}}\right)}{3a^3+3a^2bx} + \frac{bx\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}} + 4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}} + 4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3a^3+3a^2bx} + \frac{2\sqrt{3}bx\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}} + \frac{\sqrt{3}}{3}\right)}{3a^3+3a^2bx} - \frac{2bx\sqrt[3]{-\frac{a}{b}}\log(2)}{3a^3+3a^2bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a**2, Eq(b, 0)),
 (-3/(5*b**2*x**(5/3)), Eq(a, 0)), (3*a*x**(1/3)/(3*a**3 + 3*a**2*b*x) - 2*
a*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a**3 + 3*a**2*b*x) + a*(-a
/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*
a**3 + 3*a**2*b*x) + 2*sqrt(3)*a*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(
-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3 + 3*a**2*b*x) - 2*a*(-a/b)**(1/3)*log(2)
/(3*a**3 + 3*a**2*b*x) - 2*b*x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/
(3*a**3 + 3*a**2*b*x) + b*x*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b
)**(1/3) + 4*(-a/b)**(2/3))/(3*a**3 + 3*a**2*b*x) + 2*sqrt(3)*b*x*(-a/b)**(
1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3 + 3*a**
2*b*x) - 2*b*x*(-a/b)**(1/3)*log(2)/(3*a**3 + 3*a**2*b*x), True))
```

Giac [A]
time = 1.76, size = 132, normalized size = 1.17

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{2}}+\frac{2\,\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}b}+\frac{x^{\frac{1}{3}}}{(bx+a)a}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + 2/3*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b
) + x^(1/3)/((b*x + a)*a) + 1/3*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^2*b)
```

Mupad [B]
time = 0.22, size = 134, normalized size = 1.19

$$\frac{2\,\ln\left(\frac{6\,b^{5/3}}{a^{2/3}}+\frac{6\,b^2\,x^{1/3}}{a}\right)}{3\,a^{5/3}\,b^{1/3}}+\frac{x^{1/3}}{a\,\left(a+b\,x\right)}+\frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}+\frac{3\,b^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{3\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}-\frac{3\,b^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{3\,a^{5/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^2),x)
```

```
[Out] (2*log((6*b^(5/3))/a^(2/3) + (6*b^2*x^(1/3))/a))/(3*a^(5/3)*b^(1/3)) + x^(1
/3)/(a*(a + b*x)) + (log((6*b^2*x^(1/3))/a + (3*b^(5/3)*(3^(1/2)*1i - 1))/a
^(2/3))*(3^(1/2)*1i - 1))/(3*a^(5/3)*b^(1/3)) - (log((6*b^2*x^(1/3))/a - (3
*b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(3*a^(5/3)*b^(1/3))
```

3.688 $\int \frac{1}{x^{4/3}(a+bx)^2} \, dx$

Optimal. Leaf size=124

$$-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}\,(a+bx)}+\frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}}$$

```
[Out] -4/a^2/x^(1/3)+1/a/x^(1/3)/(b*x+a)+2*b^(1/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^
(7/3)-2/3*b^(1/3)*ln(b*x+a)/a^(7/3)+4/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 58, 631, 210, 31}

$$\frac{4\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] -4/(a^2*x^(1/3)) + 1/(a*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/a^(7/3) - (2*b^(1/3)*Log[a + b*x])/(3*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\int \frac{1}{x^{4/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{(4b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right.}{a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{\left(4\sqrt[3]{b}\right.}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right.}{a^{7/3}}$$

Mathematica [A]
time = 0.21, size = 142, normalized size = 1.15

$$\frac{-\frac{3\sqrt[3]{a}\,(3a+4bx)}{\sqrt[3]{x}\,(a+bx)} + 4\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - 2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] ((-3*a^(1/3)*(3*a + 4*b*x))/(x^(1/3)*(a + b*x)) + 4*Sqrt[3]*b^(1/3)*ArcTan[
(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3
)*x^(1/3)] - 2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2
/3)])/(3*a^(7/3))
```

Maple [A]
time = 0.14, size = 124, normalized size = 1.00

method	result

risch	$-\frac{3}{a^{2} x^{\frac{1}{3}}}-\frac{b\, x^{\frac{2}{3}}}{a^{2}(b x+a)}+\frac{4 \ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2 \ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3 a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$-\frac{3}{a^{2} x^{\frac{1}{3}}}-\frac{3 b\left(\frac{x^{\frac{2}{3}}}{3 b x+3 a}-\frac{4 \ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2 \ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$
default	$-\frac{3}{a^{2} x^{\frac{1}{3}}}-\frac{3 b\left(\frac{x^{\frac{2}{3}}}{3 b x+3 a}-\frac{4 \ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2 \ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(\frac{2 x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/a^2/x^(1/3)-3*b/a^2*(1/3*x^(2/3)/(b*x+a)-4/9/b/(a/b)^(1/3)*ln(x^(1/3)+(a
/b)^(1/3))+2/9/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+4/
9*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]

time = 0.50, size = 132, normalized size = 1.06

$$-\frac{4\, b x+3\, a}{a^{2} b x^{\frac{4}{3}}+a^{3} x^{\frac{1}{3}}}-\frac{4\, \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(2\, x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\, a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\, \log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\, a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\, \log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x + 3*a)/(a^2*b*x^(4/3) + a^3*x^(1/3)) - 4/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/3*log(x^(2/
```

```
3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/3*log(x^(1/3)
 + (a/b)^(1/3))/(a^2*(a/b)^(1/3))
```

Fricas [A]
time = 1.15, size = 156, normalized size = 1.26

$$-\frac{4\sqrt{3}\left(bx^{2}+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\left(bx^{2}+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-4\left(bx^{2}+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\left(4bx+3a\right)x^{\frac{2}{3}}}{3\left(a^{2}bx^{2}+a^{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*(b*x^2 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^
(1/3) - 1/3*sqrt(3)) + 2*(b*x^2 + a*x)*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/
3) + b*x^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^2 + a*x)*(b/a)^(1/3)*log(a*(b/a)^(
2/3) + b*x^(1/3)) + 3*(4*b*x + 3*a)*x^(2/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(122) = 244$.
time = 88.31, size = 690, normalized size = 5.56

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{3}{a^{2} \sqrt[3]{x}} & \text{for}\: b = 0 \\ - \frac{3}{7 b^{2} x^{\frac{7}{3}}} & \text{for}\: a = 0 \\ - \frac{4 a \sqrt[3]{x} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} + \frac{2 a \sqrt[3]{x} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{4 \sqrt{3} a \sqrt[3]{x} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{4 a \sqrt[3]{x} \log{\left(2 \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{9 a \sqrt[3]{- \frac{a}{b}}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{4 b x^{\frac{4}{3}} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} + \frac{2 b x^{\frac{4}{3}} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{4 \sqrt{3} b x^{\frac{4}{3}} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{4 b x^{\frac{4}{3}} \log{\left(2 \right)}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} - \frac{12 b x \sqrt[3]{- \frac{a}{b}}}{3 a^{3} \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 3 a^{2} b x^{\frac{4}{3}} \sqrt[3]{- \frac{a}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (-3/(a**2*x**(1/3)), Eq(b, 0
)), (-3/(7*b**2*x**(7/3)), Eq(a, 0)), (-4*a*x**(1/3)*log(x**(1/3) - (-a/b)*
*(1/3))/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) +
 2*a*x**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/
(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*sqrt(
3)*a*x**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**
3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*a*x**(1/3)*
log(2)/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) -
9*a*(-a/b)**(1/3)/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)
**(1/3)) - 4*b*x**(4/3)*log(x**(1/3) - (-a/b)**(1/3))/(3*a**3*x**(1/3)*(-a/
b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) + 2*b*x**(4/3)*log(4*x**(2/3)
+ 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a**3*x**(1/3)*(-a/b)**(1/3
) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*sqrt(3)*b*x**(4/3)*atan(2*sqrt(3)*
x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(3*a**3*x**(1/3)*(-a/b)**(1/3) + 3*
a**2*b*x**(4/3)*(-a/b)**(1/3)) - 4*b*x**(4/3)*log(2)/(3*a**3*x**(1/3)*(-a/b
)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)) - 12*b*x*(-a/b)**(1/3)/(3*a**3*
x**(1/3)*(-a/b)**(1/3) + 3*a**2*b*x**(4/3)*(-a/b)**(1/3)), True))
```

Giac [A]
time = 1.52, size = 145, normalized size = 1.17

$$\frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}+\frac{4\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b}-\frac{4\,bx+3\,a}{\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)a^2}-\frac{2\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 4/3*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*
b) - (4*b*x + 3*a)/((b*x^(4/3) + a*x^(1/3))*a^2) - 2/3*(-a*b^2)^(2/3)*log(x
^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

Mupad [B]
time = 0.15, size = 151, normalized size = 1.22

$$\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}+16\,a^2\,b^3\,x^{1/3}\right)}{3\,a^{7/3}}-\frac{\frac{3}{a}+\frac{4\,b\,x}{a^2}}{a\,x^{1/3}+b\,x^{4/3}}-\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+16\,a^2\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{7/3}}+\frac{b^{1/3}\,\ln\left(9\,a^{7/3}\,b^{8/3}\,{\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}^2+16\,a^2\,b^3\,x^{1/3}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^2),x)
```

```
[Out] (4*b^(1/3)*log(16*a^(7/3)*b^(8/3) + 16*a^2*b^3*x^(1/3)))/(3*a^(7/3)) - (3/a
 + (4*b*x)/a^2)/(a*x^(1/3) + b*x^(4/3)) - (4*b^(1/3)*log(16*a^(7/3)*b^(8/3)
*((3^(1/2)*1i)/2 + 1/2)^2 + 16*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*
a^(7/3)) + (b^(1/3)*log(9*a^(7/3)*b^(8/3)*((3^(1/2)*2i)/3 - 2/3)^2 + 16*a^2
*b^3*x^(1/3))*((3^(1/2)*2i)/3 - 2/3))/a^(7/3)
```

3.689 $\int \frac{1}{x^{5/3}(a+bx)^2}\,dx$

Optimal. Leaf size=128

$$-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}$$

```
[Out] -5/2/a^2/x^(2/3)+1/a/x^(2/3)/(b*x+a)-5/2*b^(2/3)*ln(a^(1/3)+b^(1/3)*x^(1/3)
)/a^(8/3)+5/6*b^(2/3)*ln(b*x+a)/a^(8/3)+5/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 60, 631, 210, 31}

$$\frac{5b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] -5/(2*a^2*x^(2/3)) + 1/(a*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(8/3)) + (5*b^(2/3)*Log[a + b*x])/(6*a^(8/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)^2}\,dx = \frac{1}{ax^{2/3}(a+bx)} + \frac{5\int \frac{1}{x^{5/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{(5b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a^2}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx,\right.}{2a^{7/3}}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{(5b^{2/3}}{}$$

$$= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right.}{2a^{8/3}}$$

Mathematica [A]
time = 0.22, size = 142, normalized size = 1.11

$$\frac{-\frac{3a^{2/3}(3a+5bx)}{x^{2/3}(a+bx)} + 10\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + 5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{6a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] ((-3*a^(2/3)*(3*a + 5*b*x))/(x^(2/3)*(a + b*x)) + 10*Sqrt[3]*b^(2/3)*ArcTan
[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 10*b^(2/3)*Log[a^(1/3) + b^(1
/3)*x^(1/3)] + 5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^
(2/3)])/(6*a^(8/3))
```

Maple [A]
time = 0.22, size = 124, normalized size = 0.97

method	result	siz

derivativedivides	$-\frac{3}{2a^{2}x^{\frac{2}{3}}}-\frac{3b\left(\frac{x^{\frac{1}{3}}}{3bx+3a}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{2}}$	1
default	$-\frac{3}{2a^{2}x^{\frac{2}{3}}}-\frac{3b\left(\frac{x^{\frac{1}{3}}}{3bx+3a}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{2}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/a^2/x^(2/3)-3*b/a^2*(1/3*x^(1/3)/(b*x+a)+5/9/b/(a/b)^(2/3)*ln(x^(1/3)+
(a/b)^(1/3))-5/18/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))
+5/9/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1)))
```

Maxima [A]
time = 0.50, size = 132, normalized size = 1.03

$$-\frac{5\,bx+3\,a}{2\,\left(a^{2}bx^{\frac{5}{3}}+a^{3}x^{\frac{2}{3}}\right)}-\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*b*x + 3*a)/(a^2*b*x^(5/3) + a^3*x^(2/3)) - 5/3*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/6*log(x
^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/3*log(x^(
1/3) + (a/b)^(1/3))/(a^2*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(91) = 182$.

time = 1.64, size = 189, normalized size = 1.48

$$\frac{10\,\sqrt{3}\,(bx^2+ax)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-5\,(bx^2+ax)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\,(bx^2+ax)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\,(5\,bx+3\,a)x^{\frac{1}{3}}}{6\,(a^2bx^2+a^3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*sqrt(3)*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1
/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*log
(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 10*(b
*x^2 + a*x)*(-b^2/a^2)^(1/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(5*b*x
 + 3*a)*x^(1/3))/(a^2*b*x^2 + a^3*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(126) = 252$.
time = 130.18, size = 590, normalized size = 4.61

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{8}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{3}{2a^2x^{\frac{2}{3}}} & \text{for}\: b = 0 \\ - \frac{3}{8b^2x^{\frac{8}{3}}} & \text{for}\: a = 0 \\ - \frac{9a^2}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} + \frac{10abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} - \frac{5abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} - \frac{10\sqrt{3}abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} + \frac{10abx^{\frac{2}{3}}\sqrt[3]{-\frac{a}{b}}\log(2)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} - \frac{15abx}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} + \frac{10b^2x^{\frac{5}{3}}\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} - \frac{5b^2x^{\frac{5}{3}}\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} - \frac{10\sqrt{3}b^2x^{\frac{5}{3}}\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} + \frac{10b^2x^{\frac{5}{3}}\sqrt[3]{-\frac{a}{b}}\log(2)}{6a^4x^{\frac{2}{3}}+6a^3bx^{\frac{5}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*a**2*x**(2/3)), Eq(b,
 0)), (-3/(8*b**2*x**(8/3)), Eq(a, 0)), (-9*a**2/(6*a**4*x**(2/3) + 6*a**3*
b*x**(5/3)) + 10*a*b*x**(2/3)*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(
6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) - 5*a*b*x**(2/3)*(-a/b)**(1/3)*log(4*x
**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**4*x**(2/3) + 6*
a**3*b*x**(5/3)) - 10*sqrt(3)*a*b*x**(2/3)*(-a/b)**(1/3)*atan(2*sqrt(3)*x**
(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3))
 + 10*a*b*x**(2/3)*(-a/b)**(1/3)*log(2)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)
) - 15*a*b*x/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) + 10*b**2*x**(5/3)*(-a/b
)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)
) - 5*b**2*x**(5/3)*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3)
 + 4*(-a/b)**(2/3))/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) - 10*sqrt(3)*b**2
*x**(5/3)*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)
/3)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)) + 10*b**2*x**(5/3)*(-a/b)**(1/3)*
log(2)/(6*a**4*x**(2/3) + 6*a**3*b*x**(5/3)), True))
```

Giac [A]
time = 1.32, size = 137, normalized size = 1.07

$$\frac{5\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}-\frac{5\,\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3}-\frac{bx^{\frac{1}{3}}}{(bx+a)a^2}-\frac{5\,(-ab^2)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^3}-\frac{3}{2\,a^2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 5/3*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 -
 b*x^(1/3)/((b*x + a)*a^2) - 5/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^3 - 3/2/(a^2*x^(2/3))
```

Mupad [B]
time = 0.17, size = 166, normalized size = 1.30

$$\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}-15\,a^4\,b^3\,x^{1/3}\right)}{3\,a^{8/3}}-\frac{\frac{3}{2\,a}+\frac{5\,b\,x}{2\,a^2}}{a\,x^{2/3}+b\,x^{5/3}}+\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}-15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}-\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}+15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^2),x)
```

```
[Out] (5*(-1)^(1/3)*b^(2/3)*log(15*(-1)^(1/3)*a^(13/3)*b^(8/3) - 15*a^4*b^3*x^(1/
3)))/(3*a^(8/3)) - (3/(2*a) + (5*b*x)/(2*a^2))/(a*x^(2/3) + b*x^(5/3)) + (5
*(-1)^(1/3)*b^(2/3)*log(15*a^4*b^3*x^(1/3) - 15*(-1)^(1/3)*a^(13/3)*b^(8/3)
*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(8/3)) - (5*(-1)^(1/3
)*b^(2/3)*log(15*a^4*b^3*x^(1/3) + 15*(-1)^(1/3)*a^(13/3)*b^(8/3)*((3^(1/2)
*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3))
```

3.690 $\int \frac{x^{5/3}}{(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}$$

```
[Out] -1/2*x^(5/3)/b/(b*x+a)^2-5/6*x^(2/3)/b^2/(b*x+a)-5/6*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(1/3)/b^(8/3)+5/18*ln(b*x+a)/a^(1/3)/b^(8/3)-5/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {43, 58, 631, 210, 31}

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{x^{5/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(5/3)/(b*(a + b*x)^2) - (5*x^(2/3))/(6*b^2*(a + b*x)) - (5*ArcTan[(a
^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3))
 - (5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(1/3)*b^(8/3)) + (5*Log[a + b*x]
)/(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/3}}{(a+bx)^3}\,dx &= -\frac{x^{5/3}}{2b(a+bx)^2}+\frac{5\int \frac{x^{2/3}}{(a+bx)^2}\,dx}{6b}\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}+\frac{5\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9b^2}\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}+\frac{5\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{6b^3}-\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}+\frac{5\text{Subst}\left(\int \frac{1}{-3-}\right.}{}\\
&= -\frac{x^{5/3}}{2b(a+bx)^2}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}}+\frac{5\log}{18}
\end{aligned}$$

Mathematica [A]

time = 0.23, size = 142, normalized size = 1.01

$$\frac{-\frac{3b^{2/3}x^{2/3}(5a+8bx)}{(a+bx)^2}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}}+\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{a}}}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] ((-3*b^(2/3)*x^(2/3)*(5*a + 8*b*x))/(a + b*x)^2 - (10*Sqrt[3]*ArcTan[(1 - (
2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - (10*Log[a^(1/3) + b^(1/3)*x
^(1/3)])/a^(1/3) + (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/
3)])/a^(1/3))/(18*b^(8/3))
```

Maple [A]
time = 0.12, size = 130, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^2}}{(bx+a)^2}+\frac{-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^2}$	130
default	$\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^2}}{(bx+a)^2}+\frac{-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^2}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(-4/9*x^(5/3)/b-5/18*a*x^(2/3)/b^2)/(b*x+a)^2+5/3/b^2*(-1/3/b/(a/b)^(1/3)
*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(
a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(
1/3)-1)))
```

Maxima [A]
time = 0.49, size = 143, normalized size = 1.02

$$-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 5/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1
/3)) + 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/
3)) - 5/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(99) = 198$.
time = 0.71, size = 506, normalized size = 3.61

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+5\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-10\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)-3\left(8ab^3x+5a^2b^2\right)x^{\frac{2}{3}}}{18\left(ab^6x^2+2a^2b^5x+a^3b^4\right)}, \frac{30\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+5\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-10\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)-3\left(8ab^3x+5a^2b^2\right)x^{\frac{2}{3}}}{18\left(ab^6x^2+2a^2b^5x+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b
^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^
2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3
))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
 - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.16, size = 146, normalized size = 1.04

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^2}-\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4}-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\left(bx+a\right)^2b^2}+\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) - 5/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^4) - 1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/((b*x + a)^2*b^2) + 5/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

Mupad [B]
time = 0.17, size = 165, normalized size = 1.18

$$\frac{5\,\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{25\,{\left(-a\right)}^{1/3}}{9\,b^{10/3}}\right)}{9\,{\left(-a\right)}^{1/3}\,b^{8/3}}-\frac{\frac{4\,x^{5/3}}{3\,b}+\frac{5\,a\,x^{2/3}}{6\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{{\left(-a\right)}^{1/3}\,{\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}^2}{36\,b^{10/3}}\right)\,\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,{\left(-a\right)}^{1/3}\,b^{8/3}}-\frac{\ln\left(\frac{25\,x^{1/3}}{9\,b^3}-\frac{{\left(-a\right)}^{1/3}\,{\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}^2}{36\,b^{10/3}}\right)\,\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,{\left(-a\right)}^{1/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^3,x)
```

```
[Out] (5*log((25*x^(1/3))/(9*b^3) - (25*(-a)^(1/3))/(9*b^(10/3))))/(9*(-a)^(1/3)*
b^(8/3)) - ((4*x^(5/3))/(3*b) + (5*a*x^(2/3))/(6*b^2))/(a^2 + b^2*x^2 + 2*a
*b*x) + (log((25*x^(1/3))/(9*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(36*b^(
10/3)))*(3^(1/2)*5i - 5))/(18*(-a)^(1/3)*b^(8/3)) - (log((25*x^(1/3))/(9*b^
3) - ((-a)^(1/3)*(3^(1/2)*5i + 5)^2)/(36*b^(10/3)))*(3^(1/2)*5i + 5))/(18*(
-a)^(1/3)*b^(8/3))
```

3.691 $\int \frac{x^{4/3}}{(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}$$

```
[Out] -1/2*x^(4/3)/b/(b*x+a)^2-2/3*x^(1/3)/b^2/(b*x+a)+1/3*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(2/3)/b^(7/3)-1/9*ln(b*x+a)/a^(2/3)/b^(7/3)-2/9*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {43, 60, 631, 210, 31}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(4/3)/(b*(a + b*x)^2) - (2*x^(1/3))/(3*b^2*(a + b*x)) - (2*ArcTan[(a
^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3))
 + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(7/3)) - Log[a + b*x]/(9*a^(
2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{4/3}}{(a+bx)^3}\,dx &= -\frac{x^{4/3}}{2b(a+bx)^2}+\frac{2\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx}{3b} \\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9b^2} \\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{3\sqrt[3]{a}\,b^{8/3}}+ \\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}+\frac{2\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{3a^{2/}} \\
&= -\frac{x^{4/3}}{2b(a+bx)^2}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a}{9a^{2/3}}
\end{aligned}$$

Mathematica [A]

time = 0.23, size = 142, normalized size = 1.01

$$\frac{-\frac{3\sqrt[3]{b}\sqrt[3]{x}(4a+7bx)}{(a+bx)^2}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{2/3}}-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{a^{2/3}}}{18b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] ((-3*b^(1/3)*x^(1/3)*(4*a + 7*b*x))/(a + b*x)^2 - (4*Sqrt[3]*ArcTan[(1 - (2
*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*Log[a^(1/3) + b^(1/3)*x^(
1/3)])/a^(2/3) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)
])/a^(2/3))/(18*b^(7/3))
```

Maple [A]
time = 0.12, size = 130, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^{2}}}{(bx+a)^{2}}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^{2}}$	130
default	$\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^{2}}}{(bx+a)^{2}}+\frac{\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^{2}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(-7/18*x^(4/3)/b-2/9*a*x^(1/3)/b^2)/(b*x+a)^2+2/3/b^2*(1/3/b/(a/b)^(2/3)*
ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a
/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1
/3)-1)))
```

Maxima [A]
time = 0.51, size = 143, normalized size = 1.02

$$-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\,(b^4x^2+2\,ab^3x+a^2b^2)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 2/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2
/3)) - 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3
)) + 2/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(99) = 198$.
time = 1.05, size = 503, normalized size = 3.59

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^{2}+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-(a^{2}b)^{\frac{1}{3}}a+(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}-3(a^{2}b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-2\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^{2}b)^{\frac{1}{3}}a-(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+4\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^{2}b)^{\frac{2}{3}}\right)-3\left(7a^{2}b^{2}x+4a^{3}b\right)x^{\frac{1}{3}}}{18\left(a^{2}b^{5}x^{2}+2a^{3}b^{4}x+a^{4}b^{3}\right)},\ \frac{12\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^{2}b)^{\frac{1}{3}}a-2(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-2\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^{2}b)^{\frac{1}{3}}a-(a^{2}b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+4\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^{2}b)^{\frac{2}{3}}\right)-3\left(7a^{2}b^{2}x+4a^{3}b\right)x^{\frac{1}{3}}}{18\left(a^{2}b^{5}x^{2}+2a^{3}b^{4}x+a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^
(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3
)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2
/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*
b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.79, size = 146, normalized size = 1.04

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^{3}}-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\left(bx+a\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) + 2/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^3) + 1/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a*b^3) - 1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.07, size = 139, normalized size = 0.99

$$\frac{2\,\ln\left(2\,x^{1/3}+\frac{2\,a^{1/3}}{b^{1/3}}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\frac{7\,x^{4/3}}{6\,b}+\frac{2\,a\,x^{1/3}}{3\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(2\,x^{1/3}+\frac{a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\ln\left(2\,x^{1/3}-\frac{a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^3,x)
```

```
[Out] (2*log(2*x^(1/3) + (2*a^(1/3))/b^(1/3)))/(9*a^(2/3)*b^(7/3)) - ((7*x^(4/3))
/(6*b) + (2*a*x^(1/3))/(3*b^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (log(2*x^(1/3)
+ (a^(1/3)*(3^(1/2)*1i - 1))/b^(1/3))*(3^(1/2)*1i - 1))/(9*a^(2/3)*b^(7/3))
 - (log(2*x^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1))/b^(1/3))*(3^(1/2)*1i + 1))/(
9*a^(2/3)*b^(7/3))
```

3.692 $\int \frac{x^{2/3}}{(a+bx)^3}\, dx$

Optimal. Leaf size=143

$$-\frac{x^{2/3}}{2b(a+bx)^2}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}$$

```
[Out] -1/2*x^(2/3)/b/(b*x+a)^2+1/3*x^(2/3)/a/b/(b*x+a)-1/6*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(4/3)/b^(5/3)+1/18*ln(b*x+a)/a^(4/3)/b^(5/3)-1/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 58, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(2/3)/(b*(a + b*x)^2) + x^(2/3)/(3*a*b*(a + b*x)) - ArcTan[(a^(1/3)
- 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a
^(1/3) + b^(1/3)*x^(1/3)]/(6*a^(4/3)*b^(5/3)) + Log[a + b*x]/(18*a^(4/3)*b^
(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx}{3b}$$
$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9ab}$$
$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6ab^2} - \text{—} \quad \text{Sub}$$
$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{3a^{4/}}$$
$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+}{18a^{4/3}}$$

Mathematica [A]
time = 0.22, size = 133, normalized size = 0.93

$$\frac{-\frac{3\sqrt[3]{a}\,b^{2/3}x^{2/3}(a-2bx)}{(a+bx)^2} - 2\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) + \log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] ((-3*a^(1/3)*b^(2/3)*x^(2/3)*(a - 2*b*x))/(a + b*x)^2 - 2*Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) + b^(1/3)*x^(1/3)]
 + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(4/3)*b^
(5/3))
```

Maple [A]
time = 0.10, size = 132, normalized size = 0.92

method	result	size

derivativedivides	$\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^2}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3ab}$	132
default	$\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^2}+\frac{-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3ab}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(1/9/a*x^(5/3)-1/18*x^(2/3)/b)/(b*x+a)^2+1/3/a/b*(-1/3/b/(a/b)^(1/3)*ln(x
^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^
(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-
1)))
```

Maxima [A]
time = 0.49, size = 153, normalized size = 1.07

$$\frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\,(ab^3x^2+2\,a^2b^2x+a^3b)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x^(5/3) - a*x^(2/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
1/3)) + 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(1/3)) - 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(102) = 204.
time = 1.20, size = 508, normalized size = 3.55

$$\left[\frac{3\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-2\,(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)+3\,(2\,ab^3x-a^2b^2)x^{\frac{2}{3}}}{18\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)},\ \frac{6\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-2\,(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)+3\,(2\,ab^3x-a^2b^2)x^{\frac{2}{3}}}{18\,(a^2b^5x^2+2\,a^3b^4x+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)
^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^
2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^
2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*
x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*
a*b^3*x - a^2*b^2)*x^(2/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(126) = 252$.
time = 159.62, size = 1171, normalized size = 8.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a**3), Eq(b,
0)), (-3/(4*b**3*x**(4/3)), Eq(a, 0)), (2*a**2*log(x**(1/3) - (-a/b)**(1/3)
)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x*
*2*(-a/b)**(1/3)) - a**2*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/
b)**(2/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*
a*b**4*x**2*(-a/b)**(1/3)) + 2*sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)/(3*(-a/
b)**(1/3)) + sqrt(3)/3)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)
**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 2*a**2*log(2)/(18*a**3*b**2*(-a/b
)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) - 3
*a*b*x**(2/3)*(-a/b)**(1/3)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-
a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 4*a*b*x*log(x**(1/3) - (-a/b)
**(1/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*
b**4*x**2*(-a/b)**(1/3)) - 2*a*b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3
) + 4*(-a/b)**(2/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(
1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 4*sqrt(3)*a*b*x*atan(2*sqrt(3)*x**(1
/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b*
*3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 4*a*b*x*log(2)/(18*a**
3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)
**(1/3)) + 6*b**2*x**(5/3)*(-a/b)**(1/3)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a
**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 2*b**2*x**2*log(
x**(1/3) - (-a/b)**(1/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/
b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) - b**2*x**2*log(4*x**(2/3) + 4*x*
```

```
*(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**2*(-a/b)**(1/3) + 36*a*
*2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) + 2*sqrt(3)*b**2*x*
*2*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**3*b**2*(-a
/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1/3) + 18*a*b**4*x**2*(-a/b)**(1/3)) +
 2*b**2*x**2*log(2)/(18*a**3*b**2*(-a/b)**(1/3) + 36*a**2*b**3*x*(-a/b)**(1
/3) + 18*a*b**4*x**2*(-a/b)**(1/3)), True))
```

Giac [A]
time = 1.28, size = 149, normalized size = 1.04

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^3}+\frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\,(bx+a)^2ab}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) - 1/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^3) + 1/6*(2*b*x^(5/3) - a*x^(2/3))/((b*x + a)^2*a*b) + 1/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

Mupad [B]
time = 0.26, size = 172, normalized size = 1.20

$$\frac{\frac{x^{5/3}}{3\,a}-\frac{x^{2/3}}{6\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{1}{9\,a^{5/3}\,{(-b)}^{4/3}}+\frac{x^{1/3}}{9\,a^2\,b}\right)}{9\,a^{4/3}\,{(-b)}^{5/3}}+\frac{\ln\left(\frac{x^{1/3}}{9\,a^2\,b}+\frac{{\left(-1+\sqrt{3}\,1\mathrm{i}\right)}^2}{36\,a^{5/3}\,{(-b)}^{4/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,{(-b)}^{5/3}}-\frac{\ln\left(\frac{x^{1/3}}{9\,a^2\,b}+\frac{{\left(1+\sqrt{3}\,1\mathrm{i}\right)}^2}{36\,a^{5/3}\,{(-b)}^{4/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,{(-b)}^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^3,x)
```

```
[Out] (x^(5/3)/(3*a) - x^(2/3)/(6*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(1/(9*a^(5/3
)*(-b)^(4/3)) + x^(1/3)/(9*a^2*b))/(9*a^(4/3)*(-b)^(5/3)) + (log(x^(1/3)/(9
*a^2*b) + (3^(1/2)*1i - 1)^2/(36*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/(18
*a^(4/3)*(-b)^(5/3)) - (log(x^(1/3)/(9*a^2*b) + (3^(1/2)*1i + 1)^2/(36*a^(5
/3)*(-b)^(4/3)))*(3^(1/2)*1i + 1))/(18*a^(4/3)*(-b)^(5/3))
```

3.693 $\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt[3]{x}}{2b(a+bx)^2}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}$$

```
[Out] -1/2*x^(1/3)/b/(b*x+a)^2+1/6*x^(1/3)/a/b/(b*x+a)+1/6*ln(a^(1/3)+b^(1/3)*x^(
1/3))/a^(5/3)/b^(4/3)-1/18*ln(b*x+a)/a^(5/3)/b^(4/3)-1/9*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {43, 44, 60, 631, 210, 31}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] -1/2*x^(1/3)/(b*(a + b*x)^2) + x^(1/3)/(6*a*b*(a + b*x)) - ArcTan[(a^(1/3)
- 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a
^(1/3) + b^(1/3)*x^(1/3)]/(6*a^(5/3)*b^(4/3)) - Log[a + b*x]/(18*a^(5/3)*b^
(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
]))] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6b}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9ab}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \text{Sub}\ldots$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx \ldots\right.}{3a^{5/}\ldots}$$

$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+\ldots}{18a^{5/3}\ldots}$$

Mathematica [A]
time = 0.22, size = 136, normalized size = 0.95

$$\frac{\frac{3a^{2/3}\sqrt[3]{b}\sqrt[3]{x}(-2a+bx)}{(a+bx)^2} - 2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right) - \log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] ((3*a^(2/3)*b^(1/3)*x^(1/3)*(-2*a + b*x))/(a + b*x)^2 - 2*Sqrt[3]*ArcTan[(1
 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + b^(1/3)*x^(1/3)]
 - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(5/3)*b^
(4/3))
```

Maple [A]
time = 0.11, size = 132, normalized size = 0.92

method	result	size

derivativedivides	$\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^2}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3ab}$	132
default	$\frac{\frac{x^{\frac{4}{3}}}{6a}-\frac{x^{\frac{1}{3}}}{3b}}{(bx+a)^2}+\frac{\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3ab}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*(1/18/a*x^(4/3)-1/9*x^(1/3)/b)/(b*x+a)^2+1/3/a/b*(1/3/b/(a/b)^(2/3)*ln(x^
(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1
)))
```

Maxima [A]
time = 0.49, size = 152, normalized size = 1.06

$$\frac{bx^{\frac{4}{3}}-2\,ax^{\frac{1}{3}}}{6\,(ab^3x^2+2\,a^2b^2x+a^3b)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^(4/3) - 2*a*x^(1/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
2/3)) - 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(2/3)) + 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(102) = 204$.
time = 1.05, size = 501, normalized size = 3.50

$$\left[\frac{3\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3\,(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\,(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\,(a^2b^2x-2\,a^3b)x^{\frac{1}{3}}}{18\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)},\ \frac{6\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a-2\,(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\,(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\,(a^2b^2x-2\,a^3b)x^{\frac{1}{3}}}{18\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - (b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1
/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*
log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^
4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x
+ a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*
b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^2 + 2*a*b*x + a^2)*(a
^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*
(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) +
3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(126) = 252$.

time = 129.92, size = 899, normalized size = 6.29

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3 x^{\frac{4}{3}}}{4 a^{3}} & \text{for}\: b = 0 \\ - \frac{3}{5 b^{3} x^{\frac{5}{3}}} & \text{for}\: a = 0 \\ - \frac{6 a^{2} \sqrt[3]{x}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} - \frac{2 a^{2} \sqrt[3]{- \frac{a}{b}} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{a^{2} \sqrt[3]{- \frac{a}{b}} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{2 \sqrt{3} a^{2} \sqrt[3]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} - \frac{2 a^{2} \sqrt[3]{- \frac{a}{b}} \log{\left(2 \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{3 a b x^{\frac{4}{3}}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} - \frac{4 a b x \sqrt[3]{- \frac{a}{b}} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{2 a b x \sqrt[3]{- \frac{a}{b}} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{4 \sqrt{3} a b x \sqrt[3]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} - \frac{4 a b x \sqrt[3]{- \frac{a}{b}} \log{\left(2 \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} - \frac{2 b^{2} x^{2} \sqrt[3]{- \frac{a}{b}} \log{\left(\sqrt[3]{x} - \sqrt[3]{- \frac{a}{b}} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{b^{2} x^{2} \sqrt[3]{- \frac{a}{b}} \log{\left(4 x^{\frac{2}{3}} + 4 \sqrt[3]{x} \sqrt[3]{- \frac{a}{b}} + 4 \left(- \frac{a}{b}\right)^{\frac{2}{3}} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} + \frac{2 \sqrt{3} b^{2} x^{2} \sqrt[3]{- \frac{a}{b}} \operatorname{atan}{\left(\frac{2 \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{- \frac{a}{b}}} + \frac{\sqrt{3}}{3} \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} - \frac{2 b^{2} x^{2} \sqrt[3]{- \frac{a}{b}} \log{\left(2 \right)}}{18 a^{4} b + 36 a^{3} b^{2} x + 18 a^{2} b^{3} x^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**3), Eq(b,
0)), (-3/(5*b**3*x**(5/3)), Eq(a, 0)), (-6*a**2*x**(1/3)/(18*a**4*b + 36*a*
*3*b**2*x + 18*a**2*b**3*x**2) - 2*a**2*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)
**(1/3))/(18*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + a**2*(-a/b)**(1
/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**4*b
 + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 2*sqrt(3)*a**2*(-a/b)**(1/3)*atan(
2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b + 36*a**3*b**2
*x + 18*a**2*b**3*x**2) - 2*a**2*(-a/b)**(1/3)*log(2)/(18*a**4*b + 36*a**3*
b**2*x + 18*a**2*b**3*x**2) + 3*a*b*x**(4/3)/(18*a**4*b + 36*a**3*b**2*x +
18*a**2*b**3*x**2) - 4*a*b*x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(1
8*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 2*a*b*x*(-a/b)**(1/3)*log(
4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**4*b + 36*a*
*3*b**2*x + 18*a**2*b**3*x**2) + 4*sqrt(3)*a*b*x*(-a/b)**(1/3)*atan(2*sqrt(
3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b + 36*a**3*b**2*x + 18
*a**2*b**3*x**2) - 4*a*b*x*(-a/b)**(1/3)*log(2)/(18*a**4*b + 36*a**3*b**2*x
 + 18*a**2*b**3*x**2) - 2*b**2*x**2*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1
/3))/(18*a**4*b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + b**2*x**2*(-a/b)**(
1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**4*
b + 36*a**3*b**2*x + 18*a**2*b**3*x**2) + 2*sqrt(3)*b**2*x**2*(-a/b)**(1/3)
*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b + 36*a**
3*b**2*x + 18*a**2*b**3*x**2) - 2*b**2*x**2*(-a/b)**(1/3)*log(2)/(18*a**4*b
 + 36*a**3*b**2*x + 18*a**2*b**3*x**2), True))
```

Giac [A]
time = 1.16, size = 148, normalized size = 1.03

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{2}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{2}}+\frac{bx^{\frac{4}{3}}-2\,ax^{\frac{1}{3}}}{6\left(bx+a\right)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^2*b^2) + 1/6*(b*x^(4/3) - 2*a*x^(1/3))/((b*x + a)^2*a*b)
```

Mupad [B]
time = 0.24, size = 146, normalized size = 1.02

$$\frac{\frac{x^{4/3}}{6\,a}-\frac{x^{1/3}}{3\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(\frac{b^{2/3}}{a^{2/3}}+\frac{b\,x^{1/3}}{a}\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(\frac{b\,x^{1/3}}{a}+\frac{b^{2/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(\frac{b\,x^{1/3}}{a}-\frac{b^{2/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^3,x)
```

```
[Out] (x^(4/3)/(6*a) - x^(1/3)/(3*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(b^(2/3)/a^(
2/3) + (b*x^(1/3))/a)/(9*a^(5/3)*b^(4/3)) + (log((b*x^(1/3))/a + (b^(2/3)*(
3^(1/2)*1i - 1))/(2*a^(2/3)))*(3^(1/2)*1i - 1))/(18*a^(5/3)*b^(4/3)) - (log
((b*x^(1/3))/a - (b^(2/3)*(3^(1/2)*1i + 1))/(2*a^(2/3)))*(3^(1/2)*1i + 1))/
(18*a^(5/3)*b^(4/3))
```

3.694 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{x^{2/3}}{2a(a+bx)^2}+\frac{2x^{2/3}}{3a^2(a+bx)}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}$$

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/3*x^(2/3)/a^2/(b*x+a)-1/3*ln(a^(1/3)+b^(1/3)*x^(1
/3))/a^(7/3)/b^(2/3)+1/9*ln(b*x+a)/a^(7/3)/b^(2/3)-2/9*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {44, 58, 631, 210, 31}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] x^(2/3)/(2*a*(a + b*x)^2) + (2*x^(2/3))/(3*a^2*(a + b*x)) - (2*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) -
Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(7/3)*b^(2/3)) + Log[a + b*x]/(9*a^(7/3
)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}\,(a+bx)^3}\,dx = \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9a^2}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{3a^2b} -$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{2\text{Subst}\left(\int \frac{1}{-3-}\right.}{}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log}{9}$$

Mathematica [A]

time = 0.16, size = 142, normalized size = 1.01

$$\frac{\frac{3\sqrt[3]{a}\,x^{2/3}(7a+4bx)}{(a+bx)^2}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{2/3}}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{b^{2/3}}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] ((3*a^(1/3)*x^(2/3)*(7*a + 4*b*x))/(a + b*x)^2 - (4*Sqrt[3]*ArcTan[(1 - (2*
b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/b^(2/3) - (4*Log[a^(1/3) + b^(1/3)*x^(1
/3)])/b^(2/3) + (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]
)/b^(2/3))/(18*a^(7/3))
```

Maple [A]
time = 0.11, size = 139, normalized size = 0.99

method	result	siz
derivativedivides	$\frac{x^{\frac{2}{3}}}{2a(bx+a)^2}+\frac{\frac{2x^{\frac{2}{3}}}{3a(bx+a)}+\frac{2\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a}}{a}$	139
default	$\frac{x^{\frac{2}{3}}}{2a(bx+a)^2}+\frac{\frac{2x^{\frac{2}{3}}}{3a(bx+a)}+\frac{2\left(-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a}}{a}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/a*(1/3*x^(2/3)/a/(b*x+a)+1/3/a*(-1/3/b/(a/b)^(1/3
)*ln(x^(1/3)+(a/b)^(1/3))+1/6/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+
```

(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^
(1/3)-1))))

Maxima [A]
time = 0.49, size = 151, normalized size = 1.08

$$\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 2/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
1/3)) + 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(
1/3)) - 2/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(99) = 198.
time = 0.88, size = 510, normalized size = 3.64

$$\left[\frac{6\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2\,(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+2\,(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-4\,(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)+3\,(4\,ab^3x+7\,a^2b^2)x^{\frac{2}{3}}}{18\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)},\ \frac{12\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\right)+2\,(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-4\,(b^2x^2+2\,abx+a^2)(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)+3\,(4\,ab^3x+7\,a^2b^2)x^{\frac{2}{3}}}{18\,(a^3b^4x^2+2\,a^4b^3x+a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^
2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)
^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))
/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
+ 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(129) = 258.
time = 140.71, size = 1175, normalized size = 8.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**3), Eq(b,
0)), (-3/(7*b**3*x**(7/3)), Eq(a, 0)), (4*a**2*log(x**(1/3) - (-a/b)**(1/3)
)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x*
*2*(-a/b)**(1/3)) - 2*a**2*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-
a/b)**(2/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a
**2*b**3*x**2*(-a/b)**(1/3)) + 4*sqrt(3)*a**2*atan(2*sqrt(3)*x**(1/3)/(3*(-
a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)*
*(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 4*a**2*log(2)/(18*a**4*b*(-a/b)
**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) +
 21*a*b*x**(2/3)*(-a/b)**(1/3)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-
a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 8*a*b*x*log(x**(1/3) - (-a
/b)**(1/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a*
*2*b**3*x**2*(-a/b)**(1/3)) - 4*a*b*x*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(
1/3) + 4*(-a/b)**(2/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(
1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 8*sqrt(3)*a*b*x*atan(2*sqrt(3)*x*
*(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b*
*2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 8*a*b*x*log(2)/(18*
a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a
/b)**(1/3)) + 12*b**2*x**(5/3)*(-a/b)**(1/3)/(18*a**4*b*(-a/b)**(1/3) + 36*
a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 4*b**2*x**2*
log(x**(1/3) - (-a/b)**(1/3))/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a
/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) - 2*b**2*x**2*log(4*x**(2/3)
+ 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**4*b*(-a/b)**(1/3) + 36
*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)) + 4*sqrt(3)*b
**2*x**2*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**4*b*
(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)**(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1
/3)) + 4*b**2*x**2*log(2)/(18*a**4*b*(-a/b)**(1/3) + 36*a**3*b**2*x*(-a/b)*
*(1/3) + 18*a**2*b**3*x**2*(-a/b)**(1/3)), True))
```

Giac [A]
time = 2.09, size = 143, normalized size = 1.02

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}-\frac{2\,\sqrt{3}\,\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2}+\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\,(bx+a)^2a^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 2/9*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^2) + 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/((b*x + a)^2*a^2) + 1/9*(-a*b^2)^(2/3
)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

Mupad [B]
time = 0.19, size = 167, normalized size = 1.19

$$\frac{\frac{7x^{2/3}}{6a}+\frac{2bx^{5/3}}{3a^2}}{a^2+2abx+b^2x^2}+\frac{2\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{4b^{2/3}}{9(-a)^{11/3}}\right)}{9(-a)^{7/3}b^{2/3}}+\frac{\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{11/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{7/3}b^{2/3}}-\frac{\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{11/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{7/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^3),x)
```

```
[Out] ((7*x^(2/3))/(6*a) + (2*b*x^(5/3))/(3*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (2*
log((4*b*x^(1/3))/(9*a^4) - (4*b^(2/3))/(9*(-a)^(11/3))))/(9*(-a)^(7/3)*b^(
2/3)) + (log((4*b*x^(1/3))/(9*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(9*(-a)^(
11/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(7/3)*b^(2/3)) - (log((4*b*x^(1/3))/(9*a^
4) - (b^(2/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(9*(-a
)^(7/3)*b^(2/3))
```

3.695 $\int \frac{1}{x^{2/3}(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}$$

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/6*x^(1/3)/a^2/(b*x+a)+5/6*ln(a^(1/3)+b^(1/3)*x^(1
/3))/a^(8/3)/b^(1/3)-5/18*ln(b*x+a)/a^(8/3)/b^(1/3)-5/9*arctan(1/3*(a^(1/3)
-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {44, 60, 631, 210, 31}

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] x^(1/3)/(2*a*(a + b*x)^2) + (5*x^(1/3))/(6*a^2*(a + b*x)) - (5*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) +
(5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(8/3)*b^(1/3)) - (5*Log[a + b*x])/(
18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)^3}\,dx = \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6a}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} + \frac{5\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^2}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} - \frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6a^{7/3}b^{2/3}}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} + \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}} - \frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}} + \frac{5\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt[3]{x}}{2a(a+bx)^2} + \frac{5\sqrt[3]{x}}{6a^2(a+bx)} - \frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}} + \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}} -$$

Mathematica [A]

time = 0.15, size = 142, normalized size = 1.01

$$\frac{\frac{3a^{2/3}\sqrt[3]{x}\,(8a+5bx)}{(a+bx)^2}-\frac{10\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{b}}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{\sqrt[3]{b}}}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] ((3*a^(2/3)*x^(1/3)*(8*a + 5*b*x))/(a + b*x)^2 - (10*Sqrt[3]*ArcTan[(1 - (2
*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*Log[a^(1/3) + b^(1/3)*x^
(1/3)])/b^(1/3) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3
)])/b^(1/3))/(18*a^(8/3))
```

Maple [A]

time = 0.13, size = 139, normalized size = 0.99

method	result	siz
derivativedivides	$\frac{x^{\frac{1}{3}}}{2a(bx+a)^2}+\frac{\frac{5x^{\frac{1}{3}}}{6a(bx+a)}+\frac{5\left(\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2a}}{a}$	13
default	$\frac{x^{\frac{1}{3}}}{2a(bx+a)^2}+\frac{\frac{5x^{\frac{1}{3}}}{6a(bx+a)}+\frac{5\left(\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{2a}}{a}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/2/a*(1/3*x^(1/3)/a/(b*x+a)+2/3/a*(1/3/b/(a/b)^(2/
3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)
```

```
+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
^(1/3)-1))))
```

Maxima [A]
time = 0.48, size = 151, normalized size = 1.08

$$\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\,(a^2b^2x^2+2\,a^3bx+a^4)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 5/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
2/3)) - 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(2/3)) + 5/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(99) = 198.
time = 1.06, size = 499, normalized size = 3.56

$$\left[\frac{15\,\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3\,(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-5\,(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\,(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\,(5\,a^2b^2x+8\,a^3b)x^{\frac{1}{3}}}{18\,(a^4b^3x^2+2\,a^5b^2x+a^6b)},\ \frac{30\,\sqrt{\frac{1}{3}}\,(ab^3x^2+2\,a^2b^2x+a^3b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a-2\,(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-5\,(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\,(b^2x^2+2\,abx+a^2)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\,(5\,a^2b^2x+8\,a^3b)x^{\frac{1}{3}}}{18\,(a^4b^3x^2+2\,a^5b^2x+a^6b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - 5*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)
^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2
/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(
a^4*b^3*x^2 + 2*a^5*b^2*x + a^6*b), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(
2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(a^4*b^3*x^2 + 2*a^5*b^2*x + a^6
*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(133) = 266.
time = 199.84, size = 853, normalized size = 6.09

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{3}{8b^3x^{\frac{8}{3}}} & \text{for}\: a = 0 \\ \frac{3\sqrt[3]{x}}{a^3} & \text{for}\: b = 0 \\ \frac{24a^2\sqrt[3]{x}}{18a^5+36a^4bx+18a^3b^2x^2} - \frac{10a^2\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{5a^2\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{10\sqrt{3}a^2\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{18a^5+36a^4bx+18a^3b^2x^2} - \frac{10a^2\sqrt[3]{-\frac{a}{b}}\log(2)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{15abx^{\frac{4}{3}}}{18a^5+36a^4bx+18a^3b^2x^2} - \frac{20abx\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{10abx\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{20\sqrt{3}abx\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{18a^5+36a^4bx+18a^3b^2x^2} - \frac{20abx\sqrt[3]{-\frac{a}{b}}\log(2)}{18a^5+36a^4bx+18a^3b^2x^2} - \frac{10b^2x^2\sqrt[3]{-\frac{a}{b}}\log\left(\sqrt[3]{x}-\sqrt[3]{-\frac{a}{b}}\right)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{5b^2x^2\sqrt[3]{-\frac{a}{b}}\log\left(4x^{\frac{2}{3}}+4\sqrt[3]{x}\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18a^5+36a^4bx+18a^3b^2x^2} + \frac{10\sqrt{3}b^2x^2\sqrt[3]{-\frac{a}{b}}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{-\frac{a}{b}}}+\frac{\sqrt{3}}{3}\right)}{18a^5+36a^4bx+18a^3b^2x^2} - \frac{10b^2x^2\sqrt[3]{-\frac{a}{b}}\log(2)}{18a^5+36a^4bx+18a^3b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(8/3), Eq(a, 0) & Eq(b, 0)), (-3/(8*b**3*x**(8/3)), Eq(a,
 0)), (3*x**(1/3)/a**3, Eq(b, 0)), (24*a**2*x**(1/3)/(18*a**5 + 36*a**4*b*x
 + 18*a**3*b**2*x**2) - 10*a**2*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))
/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) + 5*a**2*(-a/b)**(1/3)*log(4*x
**(2/3) + 4*x**(1/3)*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**5 + 36*a**4*b*
x + 18*a**3*b**2*x**2) + 10*sqrt(3)*a**2*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1
/3)/(3*(-a/b)**(1/3)) + sqrt(3)/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x*
*2) - 10*a**2*(-a/b)**(1/3)*log(2)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x*
*2) + 15*a*b*x**(4/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - 20*a*b*
x*(-a/b)**(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(18*a**5 + 36*a**4*b*x + 18*a
**3*b**2*x**2) + 10*a*b*x*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)*
*(1/3) + 4*(-a/b)**(2/3))/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) + 20*
sqrt(3)*a*b*x*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqr
t(3)/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - 20*a*b*x*(-a/b)**(1/3
)*log(2)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - 10*b**2*x**2*(-a/b)*
*(1/3)*log(x**(1/3) - (-a/b)**(1/3))/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*
x**2) + 5*b**2*x**2*(-a/b)**(1/3)*log(4*x**(2/3) + 4*x**(1/3)*(-a/b)**(1/3)
 + 4*(-a/b)**(2/3))/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) + 10*sqrt(3
)*b**2*x**2*(-a/b)**(1/3)*atan(2*sqrt(3)*x**(1/3)/(3*(-a/b)**(1/3)) + sqrt(
3)/3)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2) - 10*b**2*x**2*(-a/b)**(1
/3)*log(2)/(18*a**5 + 36*a**4*b*x + 18*a**3*b**2*x**2), True))
```

Giac [A]
time = 1.54, size = 143, normalized size = 1.02

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{3}}+\frac{5\,\sqrt{3}\,\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b}+\frac{5\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b}+\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\left(bx+a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 5/9*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
) + 5/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a^3*b) + 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/((b*x + a)^2*a^2)
```

Mupad [B]
time = 0.24, size = 157, normalized size = 1.12

$$\frac{\frac{4\,x^{1/3}}{3\,a}+\frac{5\,b\,x^{4/3}}{6\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{5\,\ln\left(\frac{5\,b^{5/3}}{a^{5/3}}+\frac{5\,b^2\,x^{1/3}}{a^2}\right)}{9\,a^{8/3}\,b^{1/3}}+\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}+\frac{b^{5/3}\,\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(-5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}-\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}-\frac{b^{5/3}\,\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(5+\sqrt{3}\,5{}\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^3),x)
```

```
[Out] ((4*x^(1/3))/(3*a) + (5*b*x^(4/3))/(6*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (5*
log((5*b^(5/3))/a^(5/3) + (5*b^2*x^(1/3))/a^2))/(9*a^(8/3)*b^(1/3)) + (log(
(5*b^2*x^(1/3))/a^2 + (b^(5/3)*(3^(1/2)*5i - 5))/(2*a^(5/3)))*(3^(1/2)*5i -
 5))/(18*a^(8/3)*b^(1/3)) - (log((5*b^2*x^(1/3))/a^2 - (b^(5/3)*(3^(1/2)*5i
 + 5))/(2*a^(5/3)))*(3^(1/2)*5i + 5))/(18*a^(8/3)*b^(1/3))
```

3.696 $\int \frac{1}{x^{4/3}(a+bx)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}\,(a+bx)^2}+\frac{7}{6a^2\sqrt[3]{x}\,(a+bx)}+\frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}}$$

```
[Out] -14/3/a^3/x^(1/3)+1/2/a/x^(1/3)/(b*x+a)^2+7/6/a^2/x^(1/3)/(b*x+a)+7/3*b^(1/
3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(10/3)-7/9*b^(1/3)*ln(b*x+a)/a^(10/3)+14/9
*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(10/3)*3
^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 58, 631, 210, 31}

$$\frac{14\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\,\log(a+bx)}{9a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{7}{6a^2\sqrt[3]{x}\,(a+bx)}+\frac{1}{2a\sqrt[3]{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] -14/(3*a^3*x^(1/3)) + 1/(2*a*x^(1/3)*(a + b*x)^2) + 7/(6*a^2*x^(1/3)*(a + b
*x)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(10/3)) + (7*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(10
/3)) - (7*b^(1/3)*Log[a + b*x])/(9*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^3}\,dx = \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7\int \frac{1}{x^{4/3}(a+bx)^2}\,dx}{6a}$$

$$= \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\int \frac{1}{x^{4/3}(a+bx)}\,dx}{9a^2}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{(14b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9a^3}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{7\sqrt[3]{b}\,\log(a+bx)}{9a^{10/3}} - \frac{7\text{Subst}\left(\int \frac{\quad}{\frac{a^{2/}}{b^{2/}}}\right.}{}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}} - \frac{7\sqrt[3]{}}{}$$

$$= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{7}{}$$

Mathematica [A]
time = 0.27, size = 153, normalized size = 1.01

$$\frac{-\frac{3\sqrt[3]{a}\,(18a^2+49abx+28b^2x^2)}{\sqrt[3]{x}\,(a+bx)^2} + 28\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 28\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right) - 14\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] ((-3*a^(1/3)*(18*a^2 + 49*a*b*x + 28*b^2*x^2))/(x^(1/3)*(a + b*x)^2) + 28*S
qrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^(1/
3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(10/3))
```

Maple [A]
time = 0.14, size = 133, normalized size = 0.88

method	result

derivativedivides	$-\frac{3b\left(\frac{\frac{5b\,x^{\frac{5}{3}}}{9}+\frac{13a\,x^{\frac{2}{3}}}{18}}{(bx+a)^2}-\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3}-$
default	$-\frac{3b\left(\frac{\frac{5b\,x^{\frac{5}{3}}}{9}+\frac{13a\,x^{\frac{2}{3}}}{18}}{(bx+a)^2}-\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3}-$
risch	$-\frac{3}{a^3x^{\frac{1}{3}}}-\frac{5b^2x^{\frac{5}{3}}}{3a^3(bx+a)^2}-\frac{13b\,x^{\frac{2}{3}}}{6a^2(bx+a)^2}+\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{14\sqrt{3}\,\arctan}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/a^3*b*((5/9*b*x^(5/3)+13/18*a*x^(2/3))/(b*x+a)^2-14/27/b/(a/b)^(1/3)*ln(
x^(1/3)+(a/b)^(1/3))+7/27/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b
)^(2/3))+14/27*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1
/3)-1)))-3/a^3/x^(1/3)
```

Maxima [A]

time = 0.50, size = 154, normalized size = 1.01

$$-\frac{28\,b^2x^2+49\,abx+18\,a^2}{6\left(a^3b^2x^{\frac{7}{3}}+2\,a^4bx^{\frac{4}{3}}+a^5x^{\frac{1}{3}}\right)}-\frac{14\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(28*b^2*x^2 + 49*a*b*x + 18*a^2)/(a^3*b^2*x^(7/3) + 2*a^4*b*x^(4/3) +
a^5*x^(1/3)) - 14/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a
```

```
/b)^(1/3))/(a^3*(a/b)^(1/3)) - 7/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) + 14/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(1/
3))
```

Fricas [A]
time = 0.79, size = 211, normalized size = 1.39

$$-\frac{28\sqrt{3}\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\left(b^{2}x^{3}+2\,abx^{2}+a^{2}x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\left(28\,b^{2}x^{2}+49\,abx+18\,a^{2}\right)x^{\frac{2}{3}}}{18\left(a^{3}b^{2}x^{3}+2\,a^{4}bx^{2}+a^{5}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*arctan(2/3*sqrt
(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(
b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 28*(b^
2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 3*(
28*b^2*x^2 + 49*a*b*x + 18*a^2)*x^(2/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.71, size = 155, normalized size = 1.02

$$\frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{4}}+\frac{14\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}b}-\frac{3}{a^{3}x^{\frac{1}{3}}}-\frac{7\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^{4}b}-\frac{10\,b^{2}x^{\frac{5}{3}}+13\,abx^{\frac{2}{3}}}{6\left(bx+a\right)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 14/9*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 + 14/9*sqrt(3)*(-a
*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
4*b) - 3/(a^3*x^(1/3)) - 7/9*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^4*b) - 1/6*(10*b^2*x^(5/3) + 13*a*b*x^(2/3))/((b*x +
 a)^2*a^3)
```

Mupad [B]
time = 0.09, size = 174, normalized size = 1.14

$$\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}+588\,a^3\,b^3\,x^{1/3}\right)}{9\,a^{10/3}}-\frac{\frac{3}{a}+\frac{14\,b^2\,x^2}{3\,a^3}+\frac{49\,b\,x}{6\,a^2}}{a^2\,x^{1/3}+b^2\,x^{7/3}+2\,a\,b\,x^{4/3}}+\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+588\,a^3\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2+588\,a^3\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^3),x)
```

```
[Out] (14*b^(1/3)*log(588*a^(10/3)*b^(8/3) + 588*a^3*b^3*x^(1/3)))/(9*a^(10/3)) -
 (3/a + (14*b^2*x^2)/(3*a^3) + (49*b*x)/(6*a^2))/(a^2*x^(1/3) + b^2*x^(7/3)
 + 2*a*b*x^(4/3)) + (14*b^(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 -
1/2)^2 + 588*a^3*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (14*b^
(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2 + 588*a^3*b^3*x^(1/
3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

3.697 $\int \frac{1}{x^{5/3}(a+bx)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}$$

```
[Out] -10/3/a^3/x^(2/3)+1/2/a/x^(2/3)/(b*x+a)^2+4/3/a^2/x^(2/3)/(b*x+a)-10/3*b^(2
/3)*ln(a^(1/3)+b^(1/3)*x^(1/3))/a^(11/3)+10/9*b^(2/3)*ln(b*x+a)/a^(11/3)+20
/9*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x^(1/3))/a^(1/3)*3^(1/2))/a^(11/3)
*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {44, 53, 60, 631, 210, 31}

$$\frac{20b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}-\frac{10}{3a^3x^{2/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] -10/(3*a^3*x^(2/3)) + 1/(2*a*x^(2/3)*(a + b*x)^2) + 4/(3*a^2*x^(2/3)*(a + b
*x)) + (20*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(11/3)) - (10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(1
1/3)) + (10*b^(2/3)*Log[a + b*x])/(9*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)^3}\,dx = \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4\int \frac{1}{x^{5/3}(a+bx)^2}\,dx}{3a}$$

$$= \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20\int \frac{1}{x^{5/3}(a+bx)}\,dx}{9a^2}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} - \frac{(20b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^3}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{10b^{2/3}\log(a+bx)}{9a^{11/3}} - \frac{\left(10\sqrt[3]{b}\right)\text{Subs}}{}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} - \frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}} + \frac{10b}{}$$

$$= -\frac{10}{3a^3x^{2/3}} + \frac{1}{2ax^{2/3}(a+bx)^2} + \frac{4}{3a^2x^{2/3}(a+bx)} + \frac{20b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{11/3}} - \frac{10b}{}$$

Mathematica [A]
time = 0.28, size = 153, normalized size = 1.01

$$\frac{-\frac{3a^{2/3}(9a^2+32abx+20b^2x^2)}{x^{2/3}(a+bx)^2} + 40\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 40b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right) + 20b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] ((-3*a^(2/3)*(9*a^2 + 32*a*b*x + 20*b^2*x^2))/(x^(2/3)*(a + b*x)^2) + 40*Sq
rt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 40*b^(2/3
)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + 20*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(11/3))
```

Maple [A]
time = 0.16, size = 133, normalized size = 0.88

method	result

derivativedivides	$-\frac{3b\left(\frac{\frac{11b\,x^{\frac{4}{3}}}{18}+\frac{7a\,x^{\frac{1}{3}}}{9}}{(bx+a)^2}+\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{20\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^3}-$
default	$-\frac{3b\left(\frac{\frac{11b\,x^{\frac{4}{3}}}{18}+\frac{7a\,x^{\frac{1}{3}}}{9}}{(bx+a)^2}+\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{20\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/a^3*b*((11/18*b*x^(4/3)+7/9*a*x^(1/3))/(b*x+a)^2+20/27/b/(a/b)^(2/3)*ln(
x^(1/3)+(a/b)^(1/3))-10/27/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/
b)^(2/3))+20/27/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(
1/3)-1)))-3/2/a^3/x^(2/3)
```

Maxima [A]
time = 0.49, size = 154, normalized size = 1.01

$$-\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(a^3b^2x^{\frac{8}{3}}+2\,a^4bx^{\frac{5}{3}}+a^5x^{\frac{2}{3}}\right)}-\frac{20\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/(a^3*b^2*x^(8/3) + 2*a^4*b*x^(5/3) + a
^5*x^(2/3)) - 20/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*(a/b)^(2/3)) + 10/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(2/3)) - 20/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(2/
3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(107) = 214$.
time = 0.73, size = 244, normalized size = 1.61

$$\frac{40\sqrt{3}\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-20\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+40\left(b^2x^3+2\,abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\left(20\,b^2x^2+32\,abx+9\,a^2\right)x^{\frac{1}{3}}}{18\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(40*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(b^2*x^3 + 2*a*b
*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/
3) + a^2*(-b^2/a^2)^(2/3)) + 40*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1
/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(20*b^2*x^2 + 32*a*b*x + 9*a^2)
*x^(1/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.72, size = 150, normalized size = 0.99

$$\frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}-\frac{20\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}-\frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4}-\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 20/9*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 - 20/9*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4
 - 10/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a
^4 - 1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/((b*x^(4/3) + a*x^(1/3))^2*a^3)
```

Mupad [B]
time = 0.17, size = 182, normalized size = 1.20

$$\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}-540\,a^6\,b^3\,x^{1/3}\right)}{9\,{\left(-a\right)}^{11/3}}-\frac{\frac{3}{2\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{16\,b\,x}{3\,a^2}}{a^2\,x^{2/3}+b^2\,x^{8/3}+2\,a\,b\,x^{5/3}}+\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-540\,a^6\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{\left(-a\right)}^{11/3}}-\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+540\,a^6\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{\left(-a\right)}^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^3),x)
```

```
[Out] (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3) - 540*a^6*b^3*x^(1/3)))/(9*(-a)^(11
/3)) - (3/(2*a) + (10*b^2*x^2)/(3*a^3) + (16*b*x)/(3*a^2))/(a^2*x^(2/3) + b
^2*x^(8/3) + 2*a*b*x^(5/3)) + (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(
1/2)*1i)/2 - 1/2) - 540*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*(-a)^(1
1/3)) - (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 54
0*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*(-a)^(11/3))
```

3.698 $\int \frac{\sqrt[4]{1-x}}{1+x}\,dx$

Optimal. Leaf size=58

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

```
[Out] 4*(1-x)^(1/4)-2*2^(1/4)*arctan(1/2*(1-x)^(1/4)*2^(3/4))-2*2^(1/4)*arctanh(1
/2*(1-x)^(1/4)*2^(3/4))
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {52, 65, 218, 212, 209}

$$-2\sqrt[4]{2}\,\text{ArcTan}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)+4\sqrt[4]{1-x}-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{1-x}}{1+x}\,dx &= 4\sqrt[4]{1-x}+2\int \frac{1}{(1-x)^{3/4}(1+x)}\,dx \\
&= 4\sqrt[4]{1-x}-8\text{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \sqrt[4]{1-x}\right) \\
&= 4\sqrt[4]{1-x}-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \sqrt[4]{1-x}\right)-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\right. \\
&= 4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 1.00

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Maple [A]
time = 1.21, size = 60, normalized size = 1.03

method	result
derivativedivides	$4(1-x)^{\frac{1}{4}}-2^{\frac{1}{4}}\left(\ln\left(\frac{(1-x)^{\frac{1}{4}}+2^{\frac{1}{4}}}{(1-x)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right)+2\arctan\left(\frac{(1-x)^{\frac{1}{4}}2^{\frac{3}{4}}}{2}\right)\right)$
default	$4(1-x)^{\frac{1}{4}}-2^{\frac{1}{4}}\left(\ln\left(\frac{(1-x)^{\frac{1}{4}}+2^{\frac{1}{4}}}{(1-x)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right)+2\arctan\left(\frac{(1-x)^{\frac{1}{4}}2^{\frac{3}{4}}}{2}\right)\right)$
trager	$4(1-x)^{\frac{1}{4}}+\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(\frac{-x\,\mathrm{RootOf}\left(\_Z^4-2\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{Roo}\right.}{}\right.$
risch	$-\frac{4(-1+x)}{(1-x)^{\frac{3}{4}}}+\frac{\left(\mathrm{RootOf}\left(\_Z^4-2\right)\ln\left(\frac{2\sqrt{-x^3+3x^2-3x+1}\,\mathrm{RootOf}\left(\_Z^4-2\right)^3x-2\sqrt{-x^3+3x^2-3}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/4)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(1-x)^(1/4)-2^(1/4)*(ln(((1-x)^(1/4)+2^(1/4))/((1-x)^(1/4)-2^(1/4)))+2*ar
ctan(1/2*(1-x)^(1/4)*2^(3/4)))
```

Maxima [A]
time = 0.51, size = 61, normalized size = 1.05

$$-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}-(-x+1)^{\frac{1}{4}}}{2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="maxima")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) + 2^(1/4)*log(-(2^(1/4) - (-x
 + 1)^(1/4))/(2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

Fricas [A]
time = 0.58, size = 82, normalized size = 1.41

$$4\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-x+1}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 4*2^(1/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-x + 1)) - 1/2*2^(3/4)*(-x
 + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x + 1)^(1/4)) + 2^(1/4)*log(-2^(1/4)
 + (-x + 1)^(1/4)) + 4*(-x + 1)^(1/4)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.20, size = 243, normalized size = 4.19

$$\frac{5\sqrt[4]{-1}\,\sqrt[4]{x-1}\,\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5(-1)^{\frac{3}{4}}\cdot\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{3i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{5i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5(-1)^{\frac{3}{4}}\cdot\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{7i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/4)/(1+x),x)
```

```
[Out] 5*(-1)**(1/4)*(x - 1)**(1/4)*gamma(5/4)/gamma(9/4) + 5*(-2)**(1/4)*exp(-I*p
i/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(I*pi/4)/2 + 1)*gamma(5/4)/(4*ga
mma(9/4)) - 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4
)*exp_polar(3*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4)) - 5*(-2)**(1/4)*exp(
-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(5*I*pi/4)/2 + 1)*gamma(5/4)
/(4*gamma(9/4)) + 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)
**(1/4)*exp_polar(7*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4))
```

Giac [A]
time = 1.27, size = 64, normalized size = 1.10

$$-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right|\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="giac")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x +
 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

Mupad [B]
time = 0.07, size = 46, normalized size = 0.79

$$4\,{\left(1-x\right)}^{1/4}-2\,2^{1/4}\,\mathrm{atanh}\left(\frac{2^{3/4}\,{\left(1-x\right)}^{1/4}}{2}\right)-2\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(1-x\right)}^{1/4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/4)/(x + 1),x)
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*atanh((2^(3/4)*(1 - x)^(1/4))/2) - 2*2^(1/4)*at
an((2^(3/4)*(1 - x)^(1/4))/2)
```

3.699 $\int x^m(a+bx)^{10}\,dx$

Optimal. Leaf size=187

$$\frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}+\frac{210a^4b^6x^{7+m}}{7+m}+\frac{120a^3b^7x^{8+m}}{8+m}$$

```
[Out] a^10*x^(1+m)/(1+m)+10*a^9*b*x^(2+m)/(2+m)+45*a^8*b^2*x^(3+m)/(3+m)+120*a^7*
b^3*x^(4+m)/(4+m)+210*a^6*b^4*x^(5+m)/(5+m)+252*a^5*b^5*x^(6+m)/(6+m)+210*a
^4*b^6*x^(7+m)/(7+m)+120*a^3*b^7*x^(8+m)/(8+m)+45*a^2*b^8*x^(9+m)/(9+m)+10*
a*b^9*x^(10+m)/(10+m)+b^10*x^(11+m)/(11+m)
```

Rubi [A]
time = 0.06, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^{10}x^{m+1}}{m+1}+\frac{10a^9bx^{m+2}}{m+2}+\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\frac{45a^2b^8x^{m+9}}{m+9}+\frac{10ab^9x^{m+10}}{m+10}+\frac{b^{10}x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^10,x]
```

```
[Out] (a^10*x^(1 + m))/(1 + m) + (10*a^9*b*x^(2 + m))/(2 + m) + (45*a^8*b^2*x^(3
+ m))/(3 + m) + (120*a^7*b^3*x^(4 + m))/(4 + m) + (210*a^6*b^4*x^(5 + m))/(
5 + m) + (252*a^5*b^5*x^(6 + m))/(6 + m) + (210*a^4*b^6*x^(7 + m))/(7 + m)
+ (120*a^3*b^7*x^(8 + m))/(8 + m) + (45*a^2*b^8*x^(9 + m))/(9 + m) + (10*a*
b^9*x^(10 + m))/(10 + m) + (b^10*x^(11 + m))/(11 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{10}\,dx &= \int \left(a^{10}x^m+10a^9bx^{1+m}+45a^8b^2x^{2+m}+120a^7b^3x^{3+m}+210a^6b^4x^{4+m}+252a^5b^5x^{5+m}+\right.\\ &= \frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 166, normalized size = 0.89

$$x^{1+m}\left(\frac{a^{10}}{1+m}+\frac{10a^9bx}{2+m}+\frac{45a^8b^2x^2}{3+m}+\frac{120a^7b^3x^3}{4+m}+\frac{210a^6b^4x^4}{5+m}+\frac{252a^5b^5x^5}{6+m}+\frac{210a^4b^6x^6}{7+m}+\frac{120a^3b^7x^7}{8+m}+\frac{45a^2b^8x^8}{9+m}+\frac{10ab^9x^9}{10+m}+\frac{b^{10}x^{10}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^10,x]
```

```
[Out] x^(1 + m)*(a^10/(1 + m) + (10*a^9*b*x)/(2 + m) + (45*a^8*b^2*x^2)/(3 + m) +
 (120*a^7*b^3*x^3)/(4 + m) + (210*a^6*b^4*x^4)/(5 + m) + (252*a^5*b^5*x^5)/
(6 + m) + (210*a^4*b^6*x^6)/(7 + m) + (120*a^3*b^7*x^7)/(8 + m) + (45*a^2*b
^8*x^8)/(9 + m) + (10*a*b^9*x^9)/(10 + m) + (b^10*x^10)/(11 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1533 vs. $2(187) = 374$.
time = 0.11, size = 1534, normalized size = 8.20

method	result	size
risch	Expression too large to display	1534
gosper	Expression too large to display	1535

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^10*m^10*x^10+10*a*b^9*m^10*x^9+55*b^10*m^9*x^10+45*a^2*b^8*m^10*x^8+56
0*a*b^9*m^9*x^9+1320*b^10*m^8*x^10+120*a^3*b^7*m^10*x^7+2565*a^2*b^8*m^9*x^
8+13650*a*b^9*m^8*x^9+18150*b^10*m^7*x^10+210*a^4*b^6*m^10*x^6+6960*a^3*b^7
*m^9*x^7+63540*a^2*b^8*m^8*x^8+190200*a*b^9*m^7*x^9+157773*b^10*m^6*x^10+25
2*a^5*b^5*m^10*x^5+12390*a^4*b^6*m^9*x^6+175320*a^3*b^7*m^8*x^7+898290*a^2*
b^8*m^7*x^8+1672230*a*b^9*m^6*x^9+902055*b^10*m^5*x^10+210*a^6*b^4*m^10*x^4
+15120*a^5*b^5*m^9*x^5+317520*a^4*b^6*m^8*x^6+2517840*a^3*b^7*m^7*x^7+79994
25*a^2*b^8*m^6*x^8+9653280*a*b^9*m^5*x^9+3416930*b^10*m^4*x^10+120*a^7*b^3*
m^10*x^3+12810*a^6*b^4*m^9*x^4+394380*a^5*b^5*m^8*x^5+4638060*a^4*b^6*m^7*x
^6+22748040*a^3*b^7*m^6*x^7+46695285*a^2*b^8*m^5*x^8+36862550*a*b^9*m^4*x^9
+8409500*b^10*m^3*x^10+45*a^8*b^2*m^10*x^2+7440*a^7*b^3*m^9*x^3+340200*a^6*
b^4*m^8*x^4+5866560*a^5*b^5*m^7*x^5+42592410*a^4*b^6*m^6*x^6+134522640*a^3*
b^7*m^5*x^7+180021510*a^2*b^8*m^4*x^8+91331800*a*b^9*m^3*x^9+12753576*b^10*
m^2*x^10+10*a^9*b*m^10*x+2835*a^8*b^2*m^9*x^2+201240*a^7*b^3*m^8*x^3+515970
0*a^6*b^4*m^7*x^4+54871236*a^5*b^5*m^6*x^5+255740310*a^4*b^6*m^5*x^6+524563
080*a^3*b^7*m^4*x^7+449614260*a^2*b^8*m^3*x^8+139262760*a*b^9*m^2*x^9+10628
640*b^10*m*x^10+a^10*m^10+640*a^9*b*m^9*x+78120*a^8*b^2*m^8*x^2+3115440*a^7
*b^3*m^7*x^3+49260330*a^6*b^4*m^6*x^4+335437200*a^5*b^5*m^5*x^5+1011120180*
a^4*b^6*m^4*x^6+1322982960*a^3*b^7*m^3*x^7+690085080*a^2*b^8*m^2*x^8+116552
160*a*b^9*m*x^9+3628800*b^10*x^10+65*a^10*m^9+17970*a^9*b*m^8*x+1235790*a^8
*b^2*m^7*x^2+30429000*a^7*b^3*m^6*x^3+307585530*a^6*b^4*m^5*x^4+1348939620*
a^5*b^5*m^4*x^5+2581262040*a^4*b^6*m^3*x^6+2047105440*a^3*b^7*m^2*x^7+58054
3200*a^2*b^8*m*x^8+39916800*a*b^9*x^9+1860*a^10*m^8+290760*a^9*b*m^7*x+1237
6665*a^8*b^2*m^6*x^2+194790960*a^7*b^3*m^5*x^3+1263374700*a^6*b^4*m^4*x^4+3
497286240*a^5*b^5*m^3*x^5+4035361680*a^4*b^6*m^2*x^6+1733313600*a^3*b^7*m*x
```

```
^7+199584000*a^2*b^8*x^8+30810*a^10*m^7+2992710*a^9*b*m^6*x+81560115*a^8*b^
2*m^5*x^2+821580360*a^7*b^3*m^4*x^3+3342229800*a^6*b^4*m^3*x^4+5541317712*a
^5*b^5*m^2*x^5+3445243200*a^4*b^6*m*x^6+598752000*a^3*b^7*x^7+326613*a^10*m
^6+20390160*a^9*b*m^5*x+355598730*a^8*b^2*m^4*x^2+2233166160*a^7*b^3*m^3*x^
3+5393046960*a^6*b^4*m^2*x^4+4783423680*a^5*b^5*m*x^5+1197504000*a^4*b^6*x^
6+2310945*a^10*m^5+92615030*a^9*b*m^4*x+1003011660*a^8*b^2*m^3*x^2+36983044
80*a^7*b^3*m^2*x^3+4727540160*a^6*b^4*m*x^4+1676505600*a^5*b^5*x^5+11028590
*a^10*m^4+274727240*a^9*b*m^3*x+1727578440*a^8*b^2*m^2*x^2+3316939200*a^7*b
^3*m*x^3+1676505600*a^6*b^4*x^4+34967140*a^10*m^3+503126280*a^9*b*m^2*x+160
8573600*a^8*b^2*m*x^2+1197504000*a^7*b^3*x^3+70290936*a^10*m^2+502927200*a^
9*b*m*x+598752000*a^8*b^2*x^2+80627040*a^10*m+199584000*a^9*b*x+39916800*a^
10)*x^m/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(4+m)/(3+m)/(2+m)/(1+m)
```

Maxima [A]
time = 0.26, size = 187, normalized size = 1.00

$$\frac{b^{10}x^{m+11}}{m+11}+\frac{10\,ab^9x^{m+10}}{m+10}+\frac{45\,a^2b^8x^{m+9}}{m+9}+\frac{120\,a^3b^7x^{m+8}}{m+8}+\frac{210\,a^4b^6x^{m+7}}{m+7}+\frac{252\,a^5b^5x^{m+6}}{m+6}+\frac{210\,a^6b^4x^{m+5}}{m+5}+\frac{120\,a^7b^3x^{m+4}}{m+4}+\frac{45\,a^8b^2x^{m+3}}{m+3}+\frac{10\,a^9bx^{m+2}}{m+2}+\frac{a^{10}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] b^10*x^(m + 11)/(m + 11) + 10*a*b^9*x^(m + 10)/(m + 10) + 45*a^2*b^8*x^(m +
 9)/(m + 9) + 120*a^3*b^7*x^(m + 8)/(m + 8) + 210*a^4*b^6*x^(m + 7)/(m + 7)
 + 252*a^5*b^5*x^(m + 6)/(m + 6) + 210*a^6*b^4*x^(m + 5)/(m + 5) + 120*a^7*
b^3*x^(m + 4)/(m + 4) + 45*a^8*b^2*x^(m + 3)/(m + 3) + 10*a^9*b*x^(m + 2)/(
m + 2) + a^10*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(187) = 374$.
time = 0.62, size = 1277, normalized size = 6.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] ((b^10*m^10 + 55*b^10*m^9 + 1320*b^10*m^8 + 18150*b^10*m^7 + 157773*b^10*m^
6 + 902055*b^10*m^5 + 3416930*b^10*m^4 + 8409500*b^10*m^3 + 12753576*b^10*m
^2 + 10628640*b^10*m + 3628800*b^10)*x^11 + 10*(a*b^9*m^10 + 56*a*b^9*m^9 +
 1365*a*b^9*m^8 + 19020*a*b^9*m^7 + 167223*a*b^9*m^6 + 965328*a*b^9*m^5 + 3
686255*a*b^9*m^4 + 9133180*a*b^9*m^3 + 13926276*a*b^9*m^2 + 11655216*a*b^9*
m + 3991680*a*b^9)*x^10 + 45*(a^2*b^8*m^10 + 57*a^2*b^8*m^9 + 1412*a^2*b^8*
m^8 + 19962*a^2*b^8*m^7 + 177765*a^2*b^8*m^6 + 1037673*a^2*b^8*m^5 + 400047
8*a^2*b^8*m^4 + 9991428*a^2*b^8*m^3 + 15335224*a^2*b^8*m^2 + 12900960*a^2*b
^8*m + 4435200*a^2*b^8)*x^9 + 120*(a^3*b^7*m^10 + 58*a^3*b^7*m^9 + 1461*a^3
*b^7*m^8 + 20982*a^3*b^7*m^7 + 189567*a^3*b^7*m^6 + 1121022*a^3*b^7*m^5 + 4
```

```
371359*a^3*b^7*m^4 + 11024858*a^3*b^7*m^3 + 17059212*a^3*b^7*m^2 + 14444280
*a^3*b^7*m + 4989600*a^3*b^7)*x^8 + 210*(a^4*b^6*m^10 + 59*a^4*b^6*m^9 + 15
12*a^4*b^6*m^8 + 22086*a^4*b^6*m^7 + 202821*a^4*b^6*m^6 + 1217811*a^4*b^6*m
^5 + 4814858*a^4*b^6*m^4 + 12291724*a^4*b^6*m^3 + 19216008*a^4*b^6*m^2 + 16
405920*a^4*b^6*m + 5702400*a^4*b^6)*x^7 + 252*(a^5*b^5*m^10 + 60*a^5*b^5*m^
9 + 1565*a^5*b^5*m^8 + 23280*a^5*b^5*m^7 + 217743*a^5*b^5*m^6 + 1331100*a^5
*b^5*m^5 + 5352935*a^5*b^5*m^4 + 13878120*a^5*b^5*m^3 + 21989356*a^5*b^5*m^
2 + 18981840*a^5*b^5*m + 6652800*a^5*b^5)*x^6 + 210*(a^6*b^4*m^10 + 61*a^6*
b^4*m^9 + 1620*a^6*b^4*m^8 + 24570*a^6*b^4*m^7 + 234573*a^6*b^4*m^6 + 14646
93*a^6*b^4*m^5 + 6016070*a^6*b^4*m^4 + 15915380*a^6*b^4*m^3 + 25681176*a^6*
b^4*m^2 + 22512096*a^6*b^4*m + 7983360*a^6*b^4)*x^5 + 120*(a^7*b^3*m^10 + 6
2*a^7*b^3*m^9 + 1677*a^7*b^3*m^8 + 25962*a^7*b^3*m^7 + 253575*a^7*b^3*m^6 +
 1623258*a^7*b^3*m^5 + 6846503*a^7*b^3*m^4 + 18609718*a^7*b^3*m^3 + 3081920
4*a^7*b^3*m^2 + 27641160*a^7*b^3*m + 9979200*a^7*b^3)*x^4 + 45*(a^8*b^2*m^1
0 + 63*a^8*b^2*m^9 + 1736*a^8*b^2*m^8 + 27462*a^8*b^2*m^7 + 275037*a^8*b^2*
m^6 + 1812447*a^8*b^2*m^5 + 7902194*a^8*b^2*m^4 + 22289148*a^8*b^2*m^3 + 38
390632*a^8*b^2*m^2 + 35746080*a^8*b^2*m + 13305600*a^8*b^2)*x^3 + 10*(a^9*b
*m^10 + 64*a^9*b*m^9 + 1797*a^9*b*m^8 + 29076*a^9*b*m^7 + 299271*a^9*b*m^6
+ 2039016*a^9*b*m^5 + 9261503*a^9*b*m^4 + 27472724*a^9*b*m^3 + 50312628*a^9
*b*m^2 + 50292720*a^9*b*m + 19958400*a^9*b)*x^2 + (a^10*m^10 + 65*a^10*m^9
+ 1860*a^10*m^8 + 30810*a^10*m^7 + 326613*a^10*m^6 + 2310945*a^10*m^5 + 110
28590*a^10*m^4 + 34967140*a^10*m^3 + 70290936*a^10*m^2 + 80627040*a^10*m +
39916800*a^10)*x)*x^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 +
 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2
+ 120543840*m + 39916800)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 9996 vs. $2(172) = 344$.
time = 1.13, size = 9996, normalized size = 53.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**10,x)
```

```
[Out] Piecewise((-a**10/(10*x**10) - 10*a**9*b/(9*x**9) - 45*a**8*b**2/(8*x**8) -
 120*a**7*b**3/(7*x**7) - 35*a**6*b**4/x**6 - 252*a**5*b**5/(5*x**5) - 105*
a**4*b**6/(2*x**4) - 40*a**3*b**7/x**3 - 45*a**2*b**8/(2*x**2) - 10*a*b**9/
x + b**10*log(x), Eq(m, -11)), (-a**10/(9*x**9) - 5*a**9*b/(4*x**8) - 45*a*
*8*b**2/(7*x**7) - 20*a**7*b**3/x**6 - 42*a**6*b**4/x**5 - 63*a**5*b**5/x**
4 - 70*a**4*b**6/x**3 - 60*a**3*b**7/x**2 - 45*a**2*b**8/x + 10*a*b**9*log(
x) + b**10*x, Eq(m, -10)), (-a**10/(8*x**8) - 10*a**9*b/(7*x**7) - 15*a**8*
b**2/(2*x**6) - 24*a**7*b**3/x**5 - 105*a**6*b**4/(2*x**4) - 84*a**5*b**5/x
**3 - 105*a**4*b**6/x**2 - 120*a**3*b**7/x + 45*a**2*b**8*log(x) + 10*a*b**
9*x + b**10*x**2/2, Eq(m, -9)), (-a**10/(7*x**7) - 5*a**9*b/(3*x**6) - 9*a*
```

```
*8*b**2/x**5 - 30*a**7*b**3/x**4 - 70*a**6*b**4/x**3 - 126*a**5*b**5/x**2 -
 210*a**4*b**6/x + 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 +
b**10*x**3/3, Eq(m, -8)), (-a**10/(6*x**6) - 2*a**9*b/x**5 - 45*a**8*b**2/(
4*x**4) - 40*a**7*b**3/x**3 - 105*a**6*b**4/x**2 - 252*a**5*b**5/x + 210*a*
*4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x**3/3 +
 b**10*x**4/4, Eq(m, -7)), (-a**10/(5*x**5) - 5*a**9*b/(2*x**4) - 15*a**8*b
**2/x**3 - 60*a**7*b**3/x**2 - 210*a**6*b**4/x + 252*a**5*b**5*log(x) + 210
*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x**3 + 5*a*b**9*x**4/2 + b*
*10*x**5/5, Eq(m, -6)), (-a**10/(4*x**4) - 10*a**9*b/(3*x**3) - 45*a**8*b**
2/(2*x**2) - 120*a**7*b**3/x + 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105
*a**4*b**6*x**2 + 40*a**3*b**7*x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 +
 b**10*x**6/6, Eq(m, -5)), (-a**10/(3*x**3) - 5*a**9*b/x**2 - 45*a**8*b**2/
x + 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b
**6*x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x
**7/7, Eq(m, -4)), (-a**10/(2*x**2) - 10*a**9*b/x + 45*a**8*b**2*log(x) + 1
20*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x**3 + 105*a**4*b**6*x**
4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a*b**9*x**7/7 + b**10*x*
*8/8, Eq(m, -3)), (-a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b
**3*x**2 + 70*a**6*b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a
**3*b**7*x**6 + 45*a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9, Eq(m,
 -2)), (a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**
3 + 105*a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a
**3*b**7*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10,
Eq(m, -1)), (a**10*m**10*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 65*a**10*m**9*x*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1860*a**10*m**8*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m
**3 + 150917976*m**2 + 120543840*m + 39916800) + 30810*a**10*m**7*x*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 326613*a**10*m**6*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2310945*a**10*m**5*x
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 11028590*a**10*m**4*x*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*a
**10*m**3*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 70290936*a**10*m**2*x*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
```

```
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 80627040*a**10*m*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35742
3*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15
0917976*m**2 + 120543840*m + 39916800) + 39916800*a**10*x*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 10*a**9*b*m**10*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 640*a**9*b*m**9*x**2*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 17970*a**9*b*m**8*x**2*x**m/(m**11...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(187) = 374$.

time = 1.27, size = 1925, normalized size = 10.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] (b^10*m^10*x^11*x^m + 10*a*b^9*m^10*x^10*x^m + 55*b^10*m^9*x^11*x^m + 45*a^
2*b^8*m^10*x^9*x^m + 560*a*b^9*m^9*x^10*x^m + 1320*b^10*m^8*x^11*x^m + 120*
a^3*b^7*m^10*x^8*x^m + 2565*a^2*b^8*m^9*x^9*x^m + 13650*a*b^9*m^8*x^10*x^m
+ 18150*b^10*m^7*x^11*x^m + 210*a^4*b^6*m^10*x^7*x^m + 6960*a^3*b^7*m^9*x^8
*x^m + 63540*a^2*b^8*m^8*x^9*x^m + 190200*a*b^9*m^7*x^10*x^m + 157773*b^10*
m^6*x^11*x^m + 252*a^5*b^5*m^10*x^6*x^m + 12390*a^4*b^6*m^9*x^7*x^m + 17532
0*a^3*b^7*m^8*x^8*x^m + 898290*a^2*b^8*m^7*x^9*x^m + 1672230*a*b^9*m^6*x^10
*x^m + 902055*b^10*m^5*x^11*x^m + 210*a^6*b^4*m^10*x^5*x^m + 15120*a^5*b^5*
m^9*x^6*x^m + 317520*a^4*b^6*m^8*x^7*x^m + 2517840*a^3*b^7*m^7*x^8*x^m + 79
99425*a^2*b^8*m^6*x^9*x^m + 9653280*a*b^9*m^5*x^10*x^m + 3416930*b^10*m^4*x
^11*x^m + 120*a^7*b^3*m^10*x^4*x^m + 12810*a^6*b^4*m^9*x^5*x^m + 394380*a^5
*b^5*m^8*x^6*x^m + 4638060*a^4*b^6*m^7*x^7*x^m + 22748040*a^3*b^7*m^6*x^8*x
^m + 46695285*a^2*b^8*m^5*x^9*x^m + 36862550*a*b^9*m^4*x^10*x^m + 8409500*b
^10*m^3*x^11*x^m + 45*a^8*b^2*m^10*x^3*x^m + 7440*a^7*b^3*m^9*x^4*x^m + 340
200*a^6*b^4*m^8*x^5*x^m + 5866560*a^5*b^5*m^7*x^6*x^m + 42592410*a^4*b^6*m^
6*x^7*x^m + 134522640*a^3*b^7*m^5*x^8*x^m + 180021510*a^2*b^8*m^4*x^9*x^m +
 91331800*a*b^9*m^3*x^10*x^m + 12753576*b^10*m^2*x^11*x^m + 10*a^9*b*m^10*x
^2*x^m + 2835*a^8*b^2*m^9*x^3*x^m + 201240*a^7*b^3*m^8*x^4*x^m + 5159700*a^
6*b^4*m^7*x^5*x^m + 54871236*a^5*b^5*m^6*x^6*x^m + 255740310*a^4*b^6*m^5*x^
7*x^m + 524563080*a^3*b^7*m^4*x^8*x^m + 449614260*a^2*b^8*m^3*x^9*x^m + 139
262760*a*b^9*m^2*x^10*x^m + 10628640*b^10*m*x^11*x^m + a^10*m^10*x*x^m + 64
0*a^9*b*m^9*x^2*x^m + 78120*a^8*b^2*m^8*x^3*x^m + 3115440*a^7*b^3*m^7*x^4*x
^m + 49260330*a^6*b^4*m^6*x^5*x^m + 335437200*a^5*b^5*m^5*x^6*x^m + 1011120
```

```
180*a^4*b^6*m^4*x^7*x^m + 1322982960*a^3*b^7*m^3*x^8*x^m + 690085080*a^2*b^
8*m^2*x^9*x^m + 116552160*a*b^9*m*x^10*x^m + 3628800*b^10*x^11*x^m + 65*a^1
0*m^9*x*x^m + 17970*a^9*b*m^8*x^2*x^m + 1235790*a^8*b^2*m^7*x^3*x^m + 30429
000*a^7*b^3*m^6*x^4*x^m + 307585530*a^6*b^4*m^5*x^5*x^m + 1348939620*a^5*b^
5*m^4*x^6*x^m + 2581262040*a^4*b^6*m^3*x^7*x^m + 2047105440*a^3*b^7*m^2*x^8
*x^m + 580543200*a^2*b^8*m*x^9*x^m + 39916800*a*b^9*x^10*x^m + 1860*a^10*m^
8*x*x^m + 290760*a^9*b*m^7*x^2*x^m + 12376665*a^8*b^2*m^6*x^3*x^m + 1947909
60*a^7*b^3*m^5*x^4*x^m + 1263374700*a^6*b^4*m^4*x^5*x^m + 3497286240*a^5*b^
5*m^3*x^6*x^m + 4035361680*a^4*b^6*m^2*x^7*x^m + 1733313600*a^3*b^7*m*x^8*x
^m + 199584000*a^2*b^8*x^9*x^m + 30810*a^10*m^7*x*x^m + 2992710*a^9*b*m^6*x
^2*x^m + 81560115*a^8*b^2*m^5*x^3*x^m + 821580360*a^7*b^3*m^4*x^4*x^m + 334
2229800*a^6*b^4*m^3*x^5*x^m + 5541317712*a^5*b^5*m^2*x^6*x^m + 3445243200*a
^4*b^6*m*x^7*x^m + 598752000*a^3*b^7*x^8*x^m + 326613*a^10*m^6*x*x^m + 2039
0160*a^9*b*m^5*x^2*x^m + 355598730*a^8*b^2*m^4*x^3*x^m + 2233166160*a^7*b^3
*m^3*x^4*x^m + 5393046960*a^6*b^4*m^2*x^5*x^m + 4783423680*a^5*b^5*m*x^6*x^
m + 1197504000*a^4*b^6*x^7*x^m + 2310945*a^10*m^5*x*x^m + 92615030*a^9*b*m^
4*x^2*x^m + 1003011660*a^8*b^2*m^3*x^3*x^m + 3698304480*a^7*b^3*m^2*x^4*x^m
 + 4727540160*a^6*b^4*m*x^5*x^m + 1676505600*a^5*b^5*x^6*x^m + 11028590*a^1
0*m^4*x*x^m + 274727240*a^9*b*m^3*x^2*x^m + 1727578440*a^8*b^2*m^2*x^3*x^m
+ 3316939200*a^7*b^3*m*x^4*x^m + 1676505600*a^6*b^4*x^5*x^m + 34967140*a^10
*m^3*x*x^m + 503126280*a^9*b*m^2*x^2*x^m + 1608573600*a^8*b^2*m*x^3*x^m + 1
197504000*a^7*b^3*x^4*x^m + 70290936*a^10*m^2*x*x^m + 502927200*a^9*b*m*x^2
*x^m + 598752000*a^8*b^2*x^3*x^m + 80627040*a^10*m*x*x^m + 199584000*a^9*b*
x^2*x^m + 39916800*a^10*x*x^m)/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357
423*m^7 + 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 15091
7976*m^2 + 120543840*m + 39916800)
```

Mupad [B]
time = 1.37, size = 1274, normalized size = 6.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^10,x)
```

```
[Out] (a^10*x*x^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590*m^4 + 2310
945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39916800))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (b^10*x^m*x^11*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 +
 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (45*a^2*b^8*x^m*x^9*(12900960*m + 15335224*m^2 + 9991428*m^3 + 40004
78*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 + 57*m^9 + m^10 +
```

4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133 39535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^1 1 + 39916800) + (120*a^3*b^7*x^m*x^8*(14444280*m + 17059212*m^2 + 11024858* m^3 + 4371359*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^ 9 + m^10 + 4989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599573 0*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66 *m^10 + m^11 + 39916800) + (210*a^4*b^6*x^m*x^7*(16405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821*m^6 + 22086*m^7 + 1512* m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 150917976*m^2 + 105258076*m^ 3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 19 25*m^9 + 66*m^10 + m^11 + 39916800) + (252*a^5*b^5*x^m*x^6*(18981840*m + 21 989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1331100*m^5 + 217743*m^6 + 23280* m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/(120543840*m + 150917976*m^2 + 1 05258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 326 70*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (210*a^6*b^4*x^m*x^5*(2251 2096*m + 25681176*m^2 + 15915380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m ^6 + 24570*m^7 + 1620*m^8 + 61*m^9 + m^10 + 7983360))/(120543840*m + 150917 976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742 3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (120*a^7*b^3*x^ m*x^4*(27641160*m + 30819204*m^2 + 18609718*m^3 + 6846503*m^4 + 1623258*m^5 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9 + m^10 + 9979200))/(120543840 *m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558* m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (45* a^8*b^2*x^m*x^3*(35746080*m + 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1 812447*m^5 + 275037*m^6 + 27462*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600)) /(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916 800) + (10*a*b^9*x^m*x^10*(11655216*m + 13926276*m^2 + 9133180*m^3 + 368625 5*m^4 + 965328*m^5 + 167223*m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 39 91680))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339 535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (10*a^9*b*x^m*x^2*(50292720*m + 50312628*m^2 + 27472724*m^3 + 9261503*m^4 + 2039016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m ^10 + 19958400))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^ 4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^1 0 + m^11 + 39916800)

3.700 $\int x^m(a+bx)^7\,dx$

Optimal. Leaf size=133

$$\frac{a^7x^{1+m}}{1+m}+\frac{7a^6bx^{2+m}}{2+m}+\frac{21a^5b^2x^{3+m}}{3+m}+\frac{35a^4b^3x^{4+m}}{4+m}+\frac{35a^3b^4x^{5+m}}{5+m}+\frac{21a^2b^5x^{6+m}}{6+m}+\frac{7ab^6x^{7+m}}{7+m}+\frac{b^7x^{8+m}}{8+m}$$

```
[Out] a^7*x^(1+m)/(1+m)+7*a^6*b*x^(2+m)/(2+m)+21*a^5*b^2*x^(3+m)/(3+m)+35*a^4*b^3
*x^(4+m)/(4+m)+35*a^3*b^4*x^(5+m)/(5+m)+21*a^2*b^5*x^(6+m)/(6+m)+7*a*b^6*x^
(7+m)/(7+m)+b^7*x^(8+m)/(8+m)
```

Rubi [A]
time = 0.04, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$\frac{a^7x^{m+1}}{m+1}+\frac{7a^6bx^{m+2}}{m+2}+\frac{21a^5b^2x^{m+3}}{m+3}+\frac{35a^4b^3x^{m+4}}{m+4}+\frac{35a^3b^4x^{m+5}}{m+5}+\frac{21a^2b^5x^{m+6}}{m+6}+\frac{7ab^6x^{m+7}}{m+7}+\frac{b^7x^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^7,x]
```

```
[Out] (a^7*x^(1 + m))/(1 + m) + (7*a^6*b*x^(2 + m))/(2 + m) + (21*a^5*b^2*x^(3 +
m))/(3 + m) + (35*a^4*b^3*x^(4 + m))/(4 + m) + (35*a^3*b^4*x^(5 + m))/(5 +
m) + (21*a^2*b^5*x^(6 + m))/(6 + m) + (7*a*b^6*x^(7 + m))/(7 + m) + (b^7*x^
(8 + m))/(8 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^7\,dx &= \int \left(a^7x^m+7a^6bx^{1+m}+21a^5b^2x^{2+m}+35a^4b^3x^{3+m}+35a^3b^4x^{4+m}+21a^2b^5x^{5+m}+7ab^6x\right.\\ &= \frac{a^7x^{1+m}}{1+m}+\frac{7a^6bx^{2+m}}{2+m}+\frac{21a^5b^2x^{3+m}}{3+m}+\frac{35a^4b^3x^{4+m}}{4+m}+\frac{35a^3b^4x^{5+m}}{5+m}+\frac{21a^2b^5x^{6+m}}{6+m}+\frac{7ab^6}{7}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 118, normalized size = 0.89

$$x^{1+m}\left(\frac{a^7}{1+m}+\frac{7a^6bx}{2+m}+\frac{21a^5b^2x^2}{3+m}+\frac{35a^4b^3x^3}{4+m}+\frac{35a^3b^4x^4}{5+m}+\frac{21a^2b^5x^5}{6+m}+\frac{7ab^6x^6}{7+m}+\frac{b^7x^7}{8+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^7,x]
```

```
[Out] x^(1 + m)*(a^7/(1 + m) + (7*a^6*b*x)/(2 + m) + (21*a^5*b^2*x^2)/(3 + m) + (
35*a^4*b^3*x^3)/(4 + m) + (35*a^3*b^4*x^4)/(5 + m) + (21*a^2*b^5*x^5)/(6 +
m) + (7*a*b^6*x^6)/(7 + m) + (b^7*x^7)/(8 + m))
```

Maple [A]
time = 0.10, size = 156, normalized size = 1.17

method	result
norman	$\frac{a^{7} x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{7} x^{8} {\mathrm e}^{m \ln \left(x \right)}}{8+m}+\frac{7 a \,b^{6} x^{7} {\mathrm e}^{m \ln \left(x \right)}}{7+m}+\frac{21 a^{2} b^{5} x^{6} {\mathrm e}^{m \ln \left(x \right)}}{6+m}+\frac{35 a^{3} b^{4} x^{5} {\mathrm e}^{m \ln \left(x \right)}}{5+m}+\frac{35 a^{4} b^{3} x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}+\frac{21 a^{5} b^{2} x^{3} {\mathrm e}^{m}}{3+m}$
risch	$x \left(b^{7} m^{7} x^{7}+7 a \,b^{6} m^{7} x^{6}+28 b^{7} m^{6} x^{7}+21 a^{2} b^{5} m^{7} x^{5}+203 a \,b^{6} m^{6} x^{6}+322 b^{7} m^{5} x^{7}+35 a^{3} b^{4} m^{7} x^{4}+630 a^{2} b^{5} m^{6} x^{5}+2401 a \,b^{6} m^{5} x^{6}+1960 b^{7}\right.$
gosper	$x^{1+m} \left(b^{7} m^{7} x^{7}+7 a \,b^{6} m^{7} x^{6}+28 b^{7} m^{6} x^{7}+21 a^{2} b^{5} m^{7} x^{5}+203 a \,b^{6} m^{6} x^{6}+322 b^{7} m^{5} x^{7}+35 a^{3} b^{4} m^{7} x^{4}+630 a^{2} b^{5} m^{6} x^{5}+2401 a \,b^{6} m^{5} x^{6}+19\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] a^7/(1+m)*x*exp(m*ln(x))+b^7/(8+m)*x^8*exp(m*ln(x))+7*a*b^6/(7+m)*x^7*exp(m
*ln(x))+21*a^2*b^5/(6+m)*x^6*exp(m*ln(x))+35*a^3*b^4/(5+m)*x^5*exp(m*ln(x))
+35*a^4*b^3/(4+m)*x^4*exp(m*ln(x))+21*a^5*b^2/(3+m)*x^3*exp(m*ln(x))+7*a^6*
b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.27, size = 133, normalized size = 1.00

$$\frac{b^{7} x^{m+8}}{m+8}+\frac{7 \, a b^{6} x^{m+7}}{m+7}+\frac{21 \, a^{2} b^{5} x^{m+6}}{m+6}+\frac{35 \, a^{3} b^{4} x^{m+5}}{m+5}+\frac{35 \, a^{4} b^{3} x^{m+4}}{m+4}+\frac{21 \, a^{5} b^{2} x^{m+3}}{m+3}+\frac{7 \, a^{6} b x^{m+2}}{m+2}+\frac{a^{7} x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] b^7*x^(m + 8)/(m + 8) + 7*a*b^6*x^(m + 7)/(m + 7) + 21*a^2*b^5*x^(m + 6)/(m
 + 6) + 35*a^3*b^4*x^(m + 5)/(m + 5) + 35*a^4*b^3*x^(m + 4)/(m + 4) + 21*a^
5*b^2*x^(m + 3)/(m + 3) + 7*a^6*b*x^(m + 2)/(m + 2) + a^7*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(133) = 266.
time = 0.84, size = 665, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] ((b^7*m^7 + 28*b^7*m^6 + 322*b^7*m^5 + 1960*b^7*m^4 + 6769*b^7*m^3 + 13132*
b^7*m^2 + 13068*b^7*m + 5040*b^7)*x^8 + 7*(a*b^6*m^7 + 29*a*b^6*m^6 + 343*a
*b^6*m^5 + 2135*a*b^6*m^4 + 7504*a*b^6*m^3 + 14756*a*b^6*m^2 + 14832*a*b^6*
m + 5760*a*b^6)*x^7 + 21*(a^2*b^5*m^7 + 30*a^2*b^5*m^6 + 366*a^2*b^5*m^5 +
2340*a^2*b^5*m^4 + 8409*a^2*b^5*m^3 + 16830*a^2*b^5*m^2 + 17144*a^2*b^5*m +
 6720*a^2*b^5)*x^6 + 35*(a^3*b^4*m^7 + 31*a^3*b^4*m^6 + 391*a^3*b^4*m^5 + 2
581*a^3*b^4*m^4 + 9544*a^3*b^4*m^3 + 19564*a^3*b^4*m^2 + 20304*a^3*b^4*m +
8064*a^3*b^4)*x^5 + 35*(a^4*b^3*m^7 + 32*a^4*b^3*m^6 + 418*a^4*b^3*m^5 + 28
64*a^4*b^3*m^4 + 10993*a^4*b^3*m^3 + 23312*a^4*b^3*m^2 + 24876*a^4*b^3*m +
10080*a^4*b^3)*x^4 + 21*(a^5*b^2*m^7 + 33*a^5*b^2*m^6 + 447*a^5*b^2*m^5 + 3
195*a^5*b^2*m^4 + 12864*a^5*b^2*m^3 + 28692*a^5*b^2*m^2 + 32048*a^5*b^2*m +
 13440*a^5*b^2)*x^3 + 7*(a^6*b*m^7 + 34*a^6*b*m^6 + 478*a^6*b*m^5 + 3580*a^
6*b*m^4 + 15289*a^6*b*m^3 + 36706*a^6*b*m^2 + 44712*a^6*b*m + 20160*a^6*b)*
x^2 + (a^7*m^7 + 35*a^7*m^6 + 511*a^7*m^5 + 4025*a^7*m^4 + 18424*a^7*m^3 +
48860*a^7*m^2 + 69264*a^7*m + 40320*a^7)*x)*x^m/(m^8 + 36*m^7 + 546*m^6 + 4
536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4257 vs. $2(121) = 242$.
time = 0.64, size = 4257, normalized size = 32.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**7,x)
```

```
[Out] Piecewise((-a**7/(7*x**7) - 7*a**6*b/(6*x**6) - 21*a**5*b**2/(5*x**5) - 35*
a**4*b**3/(4*x**4) - 35*a**3*b**4/(3*x**3) - 21*a**2*b**5/(2*x**2) - 7*a*b*
*6/x + b**7*log(x), Eq(m, -8)), (-a**7/(6*x**6) - 7*a**6*b/(5*x**5) - 21*a*
*5*b**2/(4*x**4) - 35*a**4*b**3/(3*x**3) - 35*a**3*b**4/(2*x**2) - 21*a**2*
b**5/x + 7*a*b**6*log(x) + b**7*x, Eq(m, -7)), (-a**7/(5*x**5) - 7*a**6*b/(
4*x**4) - 7*a**5*b**2/x**3 - 35*a**4*b**3/(2*x**2) - 35*a**3*b**4/x + 21*a*
*2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2, Eq(m, -6)), (-a**7/(4*x**4) - 7*
a**6*b/(3*x**3) - 21*a**5*b**2/(2*x**2) - 35*a**4*b**3/x + 35*a**3*b**4*log
(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3, Eq(m, -5)), (-a**7/(3
*x**3) - 7*a**6*b/(2*x**2) - 21*a**5*b**2/x + 35*a**4*b**3*log(x) + 35*a**3
*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/3 + b**7*x**4/4, Eq(m, -4)),
(-a**7/(2*x**2) - 7*a**6*b/x + 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a*
*3*b**4*x**2/2 + 7*a**2*b**5*x**3 + 7*a*b**6*x**4/4 + b**7*x**5/5, Eq(m, -3
)), (-a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*
a**3*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6, Eq(
m, -2)), (a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**
3/3 + 35*a**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x*
*7/7, Eq(m, -1)), (a**7*m**7*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**7*m**6*
```

x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 511*a**7*m**5*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 4025*a**7*m**4*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 18424*a**7*m**3*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 48860*a**7*m**2*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 69264*a**7*m*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a**7*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 7*a**6*b*m**7*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 238*a**6*b*m**6*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 3346*a**6*b*m**5*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 25060*a**6*b*m**4*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 107023*a**6*b*m**3*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 256942*a**6*b*m**2*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 312984*a**6*b*m*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 141120*a**6*b*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**5*b**2*m**7*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 693*a**5*b**2*m**6*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 9387*a**5*b**2*m**5*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 67095*a**5*b**2*m**4*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 270144*a**5*b**2*m**3*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 602532*a**5*b**2*m**2*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 673008*a**5*b**2*m*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 282240*a**5*b**2*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**4*b**3*m**7*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 1120*a**4*b**3*m**6*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14630*a**4*b**3*m**5*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 100240*a**4*b**3*m**4*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22

```
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 384755*a**4*b**3*
m**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 815920...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(133) = 266.
time = 1.47, size = 992, normalized size = 7.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] (b^7*m^7*x^8*x^m + 7*a*b^6*m^7*x^7*x^m + 28*b^7*m^6*x^8*x^m + 21*a^2*b^5*m^
7*x^6*x^m + 203*a*b^6*m^6*x^7*x^m + 322*b^7*m^5*x^8*x^m + 35*a^3*b^4*m^7*x^
5*x^m + 630*a^2*b^5*m^6*x^6*x^m + 2401*a*b^6*m^5*x^7*x^m + 1960*b^7*m^4*x^8
*x^m + 35*a^4*b^3*m^7*x^4*x^m + 1085*a^3*b^4*m^6*x^5*x^m + 7686*a^2*b^5*m^5
*x^6*x^m + 14945*a*b^6*m^4*x^7*x^m + 6769*b^7*m^3*x^8*x^m + 21*a^5*b^2*m^7*
x^3*x^m + 1120*a^4*b^3*m^6*x^4*x^m + 13685*a^3*b^4*m^5*x^5*x^m + 49140*a^2*
b^5*m^4*x^6*x^m + 52528*a*b^6*m^3*x^7*x^m + 13132*b^7*m^2*x^8*x^m + 7*a^6*b
*m^7*x^2*x^m + 693*a^5*b^2*m^6*x^3*x^m + 14630*a^4*b^3*m^5*x^4*x^m + 90335*
a^3*b^4*m^4*x^5*x^m + 176589*a^2*b^5*m^3*x^6*x^m + 103292*a*b^6*m^2*x^7*x^m
 + 13068*b^7*m*x^8*x^m + a^7*m^7*x*x^m + 238*a^6*b*m^6*x^2*x^m + 9387*a^5*b
^2*m^5*x^3*x^m + 100240*a^4*b^3*m^4*x^4*x^m + 334040*a^3*b^4*m^3*x^5*x^m +
353430*a^2*b^5*m^2*x^6*x^m + 103824*a*b^6*m*x^7*x^m + 5040*b^7*x^8*x^m + 35
*a^7*m^6*x*x^m + 3346*a^6*b*m^5*x^2*x^m + 67095*a^5*b^2*m^4*x^3*x^m + 38475
5*a^4*b^3*m^3*x^4*x^m + 684740*a^3*b^4*m^2*x^5*x^m + 360024*a^2*b^5*m*x^6*x
^m + 40320*a*b^6*x^7*x^m + 511*a^7*m^5*x*x^m + 25060*a^6*b*m^4*x^2*x^m + 27
0144*a^5*b^2*m^3*x^3*x^m + 815920*a^4*b^3*m^2*x^4*x^m + 710640*a^3*b^4*m*x^
5*x^m + 141120*a^2*b^5*x^6*x^m + 4025*a^7*m^4*x*x^m + 107023*a^6*b*m^3*x^2*
x^m + 602532*a^5*b^2*m^2*x^3*x^m + 870660*a^4*b^3*m*x^4*x^m + 282240*a^3*b^
4*x^5*x^m + 18424*a^7*m^3*x*x^m + 256942*a^6*b*m^2*x^2*x^m + 673008*a^5*b^2
*m*x^3*x^m + 352800*a^4*b^3*x^4*x^m + 48860*a^7*m^2*x*x^m + 312984*a^6*b*m*
x^2*x^m + 282240*a^5*b^2*x^3*x^m + 69264*a^7*m*x*x^m + 141120*a^6*b*x^2*x^m
 + 40320*a^7*x*x^m)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*
m^3 + 118124*m^2 + 109584*m + 40320)
```

Mupad [B]
time = 0.78, size = 683, normalized size = 5.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^7,x)
```

```
[Out] (a^7*x*x^m*(69264*m + 48860*m^2 + 18424*m^3 + 4025*m^4 + 511*m^5 + 35*m^6 +
 m^7 + 40320))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (b^7*x^m*x^8*(13068*m + 13132*m^2 + 6769*
m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(109584*m + 118124*m^2 + 6
7284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^2
*b^5*x^m*x^6*(17144*m + 16830*m^2 + 8409*m^3 + 2340*m^4 + 366*m^5 + 30*m^6
+ m^7 + 6720))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (35*a^3*b^4*x^m*x^5*(20304*m + 19564*m^2
+ 9544*m^3 + 2581*m^4 + 391*m^5 + 31*m^6 + m^7 + 8064))/(109584*m + 118124*
m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) +
(35*a^4*b^3*x^m*x^4*(24876*m + 23312*m^2 + 10993*m^3 + 2864*m^4 + 418*m^5 +
 32*m^6 + m^7 + 10080))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 45
36*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^5*b^2*x^m*x^3*(32048*m + 2
8692*m^2 + 12864*m^3 + 3195*m^4 + 447*m^5 + 33*m^6 + m^7 + 13440))/(109584*
m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8
+ 40320) + (7*a*b^6*x^m*x^7*(14832*m + 14756*m^2 + 7504*m^3 + 2135*m^4 + 34
3*m^5 + 29*m^6 + m^7 + 5760))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^
4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (7*a^6*b*x^m*x^2*(44712*m
+ 36706*m^2 + 15289*m^3 + 3580*m^4 + 478*m^5 + 34*m^6 + m^7 + 20160))/(1095
84*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m
^8 + 40320)
```

3.701 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}$$

```
[Out] a^3*x^(1+m)/(1+m)+3*a^2*b*x^(2+m)/(2+m)+3*a*b^2*x^(3+m)/(3+m)+b^3*x^(4+m)/(
4+m)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {45}

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx&=\int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)\,dx\\&=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.89

$$x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx}{2+m}+\frac{3ab^2x^2}{3+m}+\frac{b^3x^3}{4+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

Maple [A]
time = 0.10, size = 72, normalized size = 1.18

method	result
norman	$\frac{a^{3}x\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b^{3}x^{4}\mathrm{e}^{m\ln(x)}}{4+m}+\frac{3a\,b^{2}x^{3}\mathrm{e}^{m\ln(x)}}{3+m}+\frac{3a^{2}b\,x^{2}\mathrm{e}^{m\ln(x)}}{2+m}$
risch	$\frac{x\left(b^{3}m^{3}x^{3}+3a\,b^{2}m^{3}x^{2}+6b^{3}m^{2}x^{3}+3a^{2}b\,m^{3}x+21a\,b^{2}m^{2}x^{2}+11b^{3}m\,x^{3}+a^{3}m^{3}+24a^{2}b\,m^{2}x+42a\,b^{2}m\,x^{2}+6b^{3}x^{3}+9a^{3}m^{2}+57a^{2}bmx+\right.}{(4+m)(3+m)(2+m)(1+m)}$
gosper	$\frac{x^{1+m}\left(b^{3}m^{3}x^{3}+3a\,b^{2}m^{3}x^{2}+6b^{3}m^{2}x^{3}+3a^{2}b\,m^{3}x+21a\,b^{2}m^{2}x^{2}+11b^{3}m\,x^{3}+a^{3}m^{3}+24a^{2}b\,m^{2}x+42a\,b^{2}m\,x^{2}+6b^{3}x^{3}+9a^{3}m^{2}+57a^{2}b\right.}{(4+m)(3+m)(2+m)(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/(1+m)*x*exp(m*ln(x))+b^3/(4+m)*x^4*exp(m*ln(x))+3*a*b^2/(3+m)*x^3*exp(m
*ln(x))+3*a^2*b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+4}}{m+4}+\frac{3\,ab^2x^{m+3}}{m+3}+\frac{3\,a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(61) = 122$.
time = 1.04, size = 157, normalized size = 2.57

$$\frac{\left((b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3)x^4+3\,(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2)x^3+3\,(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b)x^2+(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3)x\right)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
```

```
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(53) = 106$.
time = 0.24, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x) & \text{for } m = -4 \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x & \text{for } m = -3 \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} & \text{for } m = -2 \\ a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3} & \text{for } m = -1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(61) = 122$.
time = 1.09, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^4x^m + 3\,ab^2m^3x^3x^m + 6\,b^3m^2x^4x^m + 3\,a^2bm^3x^2x^m + 21\,ab^2m^2x^3x^m + 11\,b^3mx^4x^m + a^3m^3xx^m + 24\,a^2bm^2x^2x^m + 42\,ab^2mx^3x^m + 6\,b^3x^4x^m + 9\,a^3m^2xx^m + 57\,a^2bmx^2x^m + 24\,ab^2x^3x^m + 26\,a^3mxx^m + 36\,a^2bx^2x^m + 24\,a^3xx^m}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

Mupad [B]
time = 0.44, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

3.702 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}$$

```
[Out] a^2*x^(1+m)/(1+m)+2*a*b*x^(2+m)/(2+m)+b^2*x^(3+m)/(3+m)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx&=\int\left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)\,dx\\&=\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.88

$$x^{1+m}\left(\frac{a^2}{1+m}+\frac{2abx}{2+m}+\frac{b^2x^2}{3+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

Maple [A]
time = 0.12, size = 51, normalized size = 1.19

method	result	size
norman	$\frac{a^{2} x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{2} x^{3} {\mathrm e}^{m \ln \left(x \right)}}{3+m}+\frac{2 a b \,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}$	51
risch	$\frac{x \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 a b m x +2 x^{2} b^{2}+5 a^{2} m +6 a b x +6 a^{2}\right) x^{m}}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$	86
gosper	$\frac{x^{1+m} \left(b^{2} m^{2} x^{2}+2 a b \,m^{2} x +3 b^{2} m \,x^{2}+a^{2} m^{2}+8 a b m x +2 x^{2} b^{2}+5 a^{2} m +6 a b x +6 a^{2}\right)}{\left(3+m \right) \left(2+m \right) \left(1+m \right)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(1+m)*x*exp(m*ln(x))+b^2/(3+m)*x^3*exp(m*ln(x))+2*a*b/(2+m)*x^2*exp(m*l
n(x))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 1.00

$$\frac{b^{2} x^{m+3}}{m+3}+\frac{2 \, a b x^{m+2}}{m+2}+\frac{a^{2} x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 1.16, size = 85, normalized size = 1.98

$$\frac{{\left({\left(b^{2} m^{2} + 3 \, b^{2} m + 2 \, b^{2}\right)} x^{3} + 2 \, {\left(a b m^{2} + 4 \, a b m + 3 \, a b\right)} x^{2} + {\left(a^{2} m^{2} + 5 \, a^{2} m + 6 \, a^{2}\right)} x\right)} x^{m}}{m^{3} + 6 \, m^{2} + 11 \, m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(36) = 72.
time = 0.17, size = 299, normalized size = 6.95

$$\begin{cases} - \frac{a^{2}}{2 x^{2}} - \frac{2 a b}{x} + b^{2} \log{\left(x \right)} & \text{for}\: m = -3 \\- \frac{a^{2}}{x} + 2 a b \log{\left(x \right)} + b^{2} x & \text{for}\: m = -2 \\a^{2} \log{\left(x \right)} + 2 a b x + \frac{b^{2} x^{2}}{2} & \text{for}\: m = -1 \\\frac{a^{2} m^{2} x x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{5 a^{2} m x x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{6 a^{2} x x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{2 a b m^{2} x^{2} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{8 a b m x^{2} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{6 a b x^{2} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{b^{2} m^{2} x^{3} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{3 b^{2} m x^{3} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} + \frac{2 b^{2} x^{3} x^{m}}{m^{3} + 6 m^{2} + 11 m + 6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.
time = 1.92, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^3x^m + 2\,abm^2x^2x^m + 3\,b^2mx^3x^m + a^2m^2xx^m + 8\,abmx^2x^m + 2\,b^2x^3x^m + 5\,a^2mxx^m + 6\,abx^2x^m + 6\,a^2xx^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

Mupad [B]
time = 0.37, size = 93, normalized size = 2.16

$$x^m\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

3.703 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}$$

```
[Out] a*x^(1+m)/(1+m)+b*x^(2+m)/(2+m)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx &= \int \left(ax^m+bx^{1+m}\right)\,dx\\ &= \frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.88

$$x^{1+m}\left(\frac{a}{1+m}+\frac{bx}{2+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
```

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

Maple [A]
time = 0.01, size = 30, normalized size = 1.20

method	result	size
norman	$\frac{ax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b\,x^{2}\mathrm{e}^{m\ln(x)}}{2+m}$	30
risch	$\frac{x(bmx+am+bx+2a)x^{m}}{(2+m)(1+m)}$	30
gosper	$\frac{x^{1+m}(bmx+am+bx+2a)}{(2+m)(1+m)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x,method=_RETURNVERBOSE)
[Out] a/(1+m)*x*exp(m*ln(x))+b/(2+m)*x^2*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2}+\frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.92, size = 33, normalized size = 1.32

$$\frac{((bm+b)x^{2}+(am+2\,a)x)x^{m}}{m^{2}+3\,m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(19) = 38$.
time = 0.10, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x}+b\log{(x)} & \text{for}\: m=-2 \\ a\log{(x)}+bx & \text{for}\: m=-1 \\ \frac{amxx^{m}}{m^{2}+3m+2}+\frac{2axx^{m}}{m^{2}+3m+2}+\frac{bmx^{2}x^{m}}{m^{2}+3m+2}+\frac{bx^{2}x^{m}}{m^{2}+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
```

```
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

Giac [A]
time = 2.10, size = 43, normalized size = 1.72

$$\frac{bmx^2x^m + amxx^m + bx^2x^m + 2\,axx^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
```

```
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

Mupad [B]
time = 0.30, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,(2\,a + a\,m + b\,x + b\,m\,x)}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
```

```
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

3.704 $\int \frac{x^m}{a+bx} \, dx$

Optimal. Leaf size=29

$$\frac{x^{1+m} {}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

```
[Out] x^(1+m)*hypergeom([1, 1+m],[2+m],-b*x/a)/a/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {66}

$$\frac{x^{m+1} {}_2F_1\left(1,m+1;m+2;-\frac{bx}{a}\right)}{a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{a+bx} \, dx = \frac{x^{1+m} {}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.00

$$\frac{x^{1+m} {}_2F_1\left(1,1+m;2+m;-\frac{bx}{a}\right)}{a(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((b*x)/a)])/(a*(1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a),x)
```

```
[Out] int(x^m/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m/(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.34, size = 61, normalized size = 2.10

$$\frac{mxx^m\Phi\left(\frac{bxe^{i\pi}}{a}, 1, m + 1\right)\Gamma(m + 1)}{a\Gamma\left(m + 2\right)} + \frac{xx^m\Phi\left(\frac{bxe^{i\pi}}{a}, 1, m + 1\right)\Gamma(m + 1)}{a\Gamma\left(m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a),x)
```

```
[Out] m*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*gamma(m
+ 2)) + x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a*ga
mma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x),x)
```

```
[Out] int(x^m/(a + b*x), x)
```

3.705 $\int \frac{x^m}{(a+bx)^2}\,dx$

Optimal. Leaf size=29

$$\frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{a}\right)}{a^2(1+m)}$$

```
[Out] x^(1+m)*hypergeom([2, 1+m],[2+m],-b*x/a)/a^2/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{66\}$

$$\frac{x^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{bx}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^2,x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{(a+bx)^2}\,dx = \frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{a}\right)}{a^2(1+m)}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(2,1+m;2+m;-\frac{bx}{a}\right)}{a^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^2,x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((b*x)/a)])/(a^2*(1 + m))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(bx+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^2,x)
```

```
[Out] int(x^m/(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 262, normalized size = 9.03

$$-\frac{am^2xx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma\left(m+2\right)+a^2bx\Gamma\left(m+2\right)}-\frac{amxx^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma\left(m+2\right)+a^2bx\Gamma\left(m+2\right)}+\frac{amxx^m\Gamma(m+1)}{a^3\Gamma\left(m+2\right)+a^2bx\Gamma\left(m+2\right)}+\frac{axx^m\Gamma(m+1)}{a^3\Gamma\left(m+2\right)+a^2bx\Gamma\left(m+2\right)}-\frac{bm^2x^2x^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma\left(m+2\right)+a^2bx\Gamma\left(m+2\right)}-\frac{bmx^2x^m\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{a^3\Gamma\left(m+2\right)+a^2bx\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**2,x)
```

```
[Out] -a*m**2*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3
*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - a*m*x*x**m*lerchphi(b*x*exp_polar(
I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
 + a*m*x*x**m*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) + a*
x*x**m*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2)) - b*m**2*x*
*2*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(
```

```
m + 2) + a**2*b*x*gamma(m + 2)) - b*m*x**2*x**m*lerchphi(b*x*exp_polar(I*pi
)/a, 1, m + 1)*gamma(m + 1)/(a**3*gamma(m + 2) + a**2*b*x*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m/(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^2,x)
[Out] int(x^m/(a + b*x)^2, x)
```

3.706 $\int \frac{x^m}{(a+bx)^3} dx$

Optimal. Leaf size=29

$$\frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

```
[Out] x^(1+m)*hypergeom([3, 1+m],[2+m],-b*x/a)/a^3/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {66}

$$\frac{x^{m+1} {}_2F_1\left(3,m+1;m+2;-\frac{bx}{a}\right)}{a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^3,x]
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{(a+bx)^3} dx = \frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.00

$$\frac{x^{1+m} {}_2F_1\left(3,1+m;2+m;-\frac{bx}{a}\right)}{a^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^3,x]
[Out] (x^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((b*x)/a)])/(a^3*(1 + m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^3,x)
```

```
[Out] int(x^m/(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 717, normalized size = 24.72

$$\frac{a^{2}m^{3}xx^{m}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}-\frac{a^{2}m^{2}xx^{m}\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}-\frac{a^{2}mxx^{m}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}+\frac{a^{2}mxx^{m}\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}+\frac{2a^{2}xx^{m}\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}+\frac{2abm^{3}x^{2}x^{m}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}-\frac{abm^{2}x^{2}x^{m}\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}-\frac{2abmx^{2}x^{m}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}+\frac{abx^{2}x^{m}\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}+\frac{b^{2}m^{3}x^{3}x^{m}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}-\frac{b^{2}mx^{3}x^{m}\Phi\left(\frac{bxe^{i\pi}}{a},1,m+1\right)\Gamma(m+1)}{2a^{5}\Gamma(m+2)+4a^{4}bx\Gamma(m+2)+2a^{3}b^{2}x^{2}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**3,x)
```

```
[Out] a**2*m**3*x*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*
a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)
) - a**2*m**2*x*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m
 + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a**2*m*x*x**m*lerchphi(b*x*exp_pol
ar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(
m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + a**2*m*x*x**m*gamma(m + 1)/(2*a**
5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) +
```

```
2*a**2*x*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2)
+ 2*a**3*b**2*x**2*gamma(m + 2)) + 2*a*b*m**3*x**2*x**m*lerchphi(b*x*exp_po
lar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma
(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - a*b*m**2*x**2*x**m*gamma(m + 1)/
(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m +
 2)) - 2*a*b*m*x**2*x**m*lerchphi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m
+ 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamm
a(m + 2)) + a*b*x**2*x**m*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4*b*x*ga
mma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) + b**2*m**3*x**3*x**m*lerchphi(
b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*a**4
*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2)) - b**2*m*x**3*x**m*lerch
phi(b*x*exp_polar(I*pi)/a, 1, m + 1)*gamma(m + 1)/(2*a**5*gamma(m + 2) + 4*
a**4*b*x*gamma(m + 2) + 2*a**3*b**2*x**2*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{{\left(a+b\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^3,x)
```

```
[Out] int(x^m/(a + b*x)^3, x)
```

3.707 $\int x^m(a+bx)^{5/2}\,dx$

Optimal. Leaf size=48

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}$$

```
[Out] 2/7*x^m*(b*x+a)^(7/2)*hypergeom([7/2, -m],[9/2],1+b*x/a)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m(a+bx)^{7/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};\frac{bx}{a}+1\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^(5/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{5/2}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m(a+bx)^{5/2}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 48, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},-m;\frac{9}{2};1+\frac{bx}{a}\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^(5/2),x]
[Out] (2*x^m*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, -m, 9/2, 1 + (b*x)/a])/(7*b*(
-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(5/2),x)
[Out] int(x^m*(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.22, size = 37, normalized size = 0.77

$$\frac{a^{\frac{5}{2}}xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(5/2),x)
```

```
[Out] a**(5/2)*x*x**m*gamma(m + 1)*hyper((-5/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^(5/2),x)
```

```
[Out] int(x^m*(a + b*x)^(5/2), x)
```

3.708 $\int x^m(a+bx)^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}$$

```
[Out] 2/5*x^m*(b*x+a)^(5/2)*hypergeom([5/2, -m],[7/2],1+b*x/a)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m(a+bx)^{5/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};\frac{bx}{a}+1\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^(3/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{3/2}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m(a+bx)^{3/2}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},-m;\frac{7}{2};1+\frac{bx}{a}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^(3/2),x]
```

```
[Out] (2*x^m*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, -m, 7/2, 1 + (b*x)/a])/(5*b*(
-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^m(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(3/2),x)
```

```
[Out] int(x^m*(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.67, size = 37, normalized size = 0.77

$$\frac{a^{\frac{3}{2}}xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(3/2),x)
```

```
[Out] a**(3/2)*x*x**m*gamma(m + 1)*hyper((-3/2, m + 1), (m + 2,), b*x*exp_polar(I
*pi)/a)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^(3/2),x)
```

```
[Out] int(x^m*(a + b*x)^(3/2), x)
```

3.709 $\int x^m \sqrt{a+bx}\, dx$

Optimal. Leaf size=48

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}$$

[Out] 2/3*x^m*(b*x+a)^(3/2)*hypergeom([3/2, -m],[5/2],1+b*x/a)/b/((-b*x/a)^m)

Rubi [A]

time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m(a+bx)^{3/2}\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};\frac{bx}{a}+1\right)}{3b}$$

Antiderivative was successfully verified.

[In] Int[x^m*Sqrt[a + b*x],x]

[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m)

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int x^m\sqrt{a+bx}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int\left(-\frac{bx}{a}\right)^m\sqrt{a+bx}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},-m;\frac{5}{2};1+\frac{bx}{a}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Sqrt[a + b*x],x]
[Out] (2*x^m*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, -m, 5/2, 1 + (b*x)/a])/(3*b*(
-((b*x)/a))^m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^m\sqrt{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^(1/2),x)
[Out] int(x^m*(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.75, size = 37, normalized size = 0.77

$$\frac{\sqrt{a}\,xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*x**m*gamma(m + 1)*hyper((-1/2, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*x^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^(1/2),x)
```

```
[Out] int(x^m*(a + b*x)^(1/2), x)
```

3.710 $\int \frac{x^m}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=46

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

```
[Out] 2*x^m*hypergeom([1/2, -m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx}}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x],x]
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(1/2),x)
[Out] int(x^m/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 36, normalized size = 0.78

$$\frac{xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(s
qrt(a)*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x)^(1/2), x)
```

3.711 $\int \frac{x^m}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}$$

```
[Out] -2*x^m*hypergeom([-1/2, -m],[1/2],1+b*x/a)/b/((-b*x/a)^m)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{(a+bx)^{3/2}}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}\,dx\\ &= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 1.00

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^(3/2),x]
[Out] (-2*x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m*Sq
rt[a + b*x])
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(3/2),x)
[Out] int(x^m/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*x^m/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 36, normalized size = 0.78

$$\frac{xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(3/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(a
**(3/2)*gamma(m + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(3/2),x)
```

```
[Out] int(x^m/(a + b*x)^(3/2), x)
```

3.712 $\int \frac{x^m}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=48

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}$$

```
[Out] -2/3*x^m*hypergeom([-3/2, -m],[-1/2],1+b*x/a)/b/((-b*x/a)^m)/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};\frac{bx}{a}+1\right)}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{(a+bx)^{5/2}} \, dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{5/2}} \, dx \\
&= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m} {}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.00

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\,{}_2F_1\left(-\frac{3}{2},-m;-\frac{1}{2};1+\frac{bx}{a}\right)}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a + b*x)^(5/2),x]
[Out] (-2*x^m*Hypergeometric2F1[-3/2, -m, -1/2, 1 + (b*x)/a])/(3*b*(-((b*x)/a))^m
*(a + b*x)^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(5/2),x)
[Out] int(x^m/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x^m/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*x^m/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.68, size = 36, normalized size = 0.75

$$\frac{xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{a^{\frac{5}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(5/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((5/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(a
**(5/2)*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(5/2),x)
```

```
[Out] int(x^m/(a + b*x)^(5/2), x)
```

3.713 $\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}$$

```
[Out] 2*a^2*x^m*hypergeom([1/2, -2-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b^3/((-b*x/a)^
m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$\frac{2a^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\ {}_2F_1\left(\frac{1}{2},-m-2;\frac{3}{2};\frac{bx}{a}+1\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{2+m}}{\sqrt{a+bx}}\,dx &= \frac{\left(a^2x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{2+m}}{\sqrt{a+bx}}\,dx}{b^2}\\ &= \frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 1.00

$$\frac{2a^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)/Sqrt[a + b*x],x]
[Out] (2*a^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -2 - m, 3/2, 1 + (b*x)/a])/
(b^3*(-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{2+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)/(b*x+a)^(1/2),x)
[Out] int(x^(2+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m + 2)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m + 2)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.61, size = 37, normalized size = 0.73

$$\frac{x^3x^m\Gamma(m+3)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+3\\m+4\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**3*x**m*gamma(m + 3)*hyper((1/2, m + 3), (m + 4,), b*x*exp_polar(I*pi)/a)
/(sqrt(a)*gamma(m + 4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m + 2)/(a + b*x)^(1/2), x)
```

3.714 $\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=49

$$-\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}$$

```
[Out] -2*a*x^m*hypergeom([1/2, -1-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b^2/((-b*x/a)^m
)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$-\frac{2ax^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m-1;\frac{3}{2};\frac{bx}{a}+1\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{1+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(ax^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{1+m}}{\sqrt{a+bx}}\,dx}{b}\\ &= -\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.00

$$-\frac{2ax^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)/Sqrt[a + b*x],x]
[Out] (-2*a*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -1 - m, 3/2, 1 + (b*x)/a])/(
b^2*(-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{1+m}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)/(b*x+a)^(1/2),x)
[Out] int(x^(1+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m + 1)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m + 1)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.21, size = 37, normalized size = 0.76

$$\frac{x^2x^m\Gamma(m+2)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+2\\ m+3\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**2*x**m*gamma(m + 2)*hyper((1/2, m + 2), (m + 3,), b*x*exp_polar(I*pi)/a)
/(sqrt(a)*gamma(m + 3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m+1}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m + 1)/(a + b*x)^(1/2), x)
```

3.715 $\int \frac{x^m}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=46

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

```
[Out] 2*x^m*hypergeom([1/2, -m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {69, 67}

$$\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{\sqrt{a+bx}}\,dx &= \left(x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}\,dx\\ &= \frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[a + b*x],x]
```

```
[Out] (2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b
*x)/a))^m)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(b*x+a)^(1/2),x)
```

```
[Out] int(x^m/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.74, size = 36, normalized size = 0.78

$$\frac{xx^m\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a)/(s
qrt(a)*gamma(m + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x)^(1/2),x)
```

```
[Out] int(x^m/(a + b*x)^(1/2), x)
```

3.716 $\int \frac{x^{-1+m}}{\sqrt{a+bx}} dx$

Optimal. Leaf size=48

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}$$

```
[Out] -2*x^m*hypergeom([1/2, 1-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/a/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$-\frac{2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\ {}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-1+m}}{\sqrt{a+bx}}\,dx}{a}\\ &= -\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 48, normalized size = 1.00

$$-\frac{2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)/Sqrt[a + b*x],x]
[Out] (-2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*x)/a])/(a*(
-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)/(b*x+a)^(1/2),x)
[Out] int(x^(-1+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m - 1)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m - 1)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.92, size = 31, normalized size = 0.65

$$\frac{x^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*gam
ma(m + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-1}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m - 1)/(a + b*x)^(1/2), x)
```

3.717 $\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=49

$$\frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}$$

```
[Out] 2*b*x^m*hypergeom([1/2, 2-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/a^2/((-b*x/a)^m)
```

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$\frac{2bx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)/Sqrt[a + b*x],x]
```

```
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-2+m}}{\sqrt{a+bx}}\,dx &= \frac{\left(b^2x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-2+m}}{\sqrt{a+bx}}\,dx}{a^2}\\ &= \frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 1.00

$$\frac{2bx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},2-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)/Sqrt[a + b*x],x]
[Out] (2*b*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 2 - m, 3/2, 1 + (b*x)/a])/(a^
2*(-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-2+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)/(b*x+a)^(1/2),x)
[Out] int(x^(-2+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m - 2)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m - 2)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.02, size = 32, normalized size = 0.65

$$\frac{x^m\Gamma(m-1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m-1\\ m\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,x\Gamma(m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m - 1)*hyper((1/2, m - 1), (m,), b*x*exp_polar(I*pi)/a)/(sqrt(a)
*x*gamma(m))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m - 2)/(a + b*x)^(1/2), x)
```

3.718 $\int \frac{x^{-3+m}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$-\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}$$

```
[Out] -2*b^2*x^m*hypergeom([1/2, 3-m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/a^3/((-b*x/a)^
m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$-\frac{2b^2x^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\ {}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)/Sqrt[a + b*x],x]
```

```
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-3+m}}{\sqrt{a+bx}}\,dx &= -\frac{\left(b^3x^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-3+m}}{\sqrt{a+bx}}\,dx}{a^3}\\ &= -\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 1.00

$$-\frac{2b^2x^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},3-m;\frac{3}{2};1+\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)/Sqrt[a + b*x],x]
[Out] (-2*b^2*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3 - m, 3/2, 1 + (b*x)/a])/
(a^3*(-((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-3+m}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)/(b*x+a)^(1/2),x)
[Out] int(x^(-3+m)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(m - 3)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(x^(m - 3)/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 13.75, size = 37, normalized size = 0.73

$$\frac{x^m\Gamma(m-2)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},m-2\\m-1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,x^2\Gamma\left(m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)/(b*x+a)**(1/2),x)
```

```
[Out] x**m*gamma(m - 2)*hyper((1/2, m - 2), (m - 1,), b*x*exp_polar(I*pi)/a)/(sqr
t(a)*x**2*gamma(m - 1))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)/sqrt(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{m-3}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(m - 3)/(a + b*x)^(1/2), x)
```

3.719 $\int \frac{x^m}{\sqrt{2+3x}}\,dx$

Optimal. Leaf size=31

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] 1/2*x^(1+m)*hypergeom([1/2, 1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{66\}$

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[2 + 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{\sqrt{2+3x}}\,dx=\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[2 + 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m)
)
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.94

method	result	size
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)\sqrt{2}}{2+2m}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(3*x + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.54, size = 37, normalized size = 1.19

$$\frac{\sqrt{2}\, x x^{m} \Gamma(m+1)\, {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, m+1 \\ m+2 \end{matrix}\middle| \frac{3xe^{i\pi}}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(2+3*x)**(1/2),x)
```

```
[Out] sqrt(2)*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x*exp_polar(I*p
i)/2)/(2*gamma(m + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(3*x + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(3*x + 2)^(1/2),x)
```

```
[Out] int(x^m/(3*x + 2)^(1/2), x)
```

3.720 $\int \frac{x^m}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=31

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] 1/2*x^(1+m)*hypergeom([1/2, 1+m],[2+m],3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {66}

$$\frac{x^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[2 - 3*x],x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{x^m}{\sqrt{2-3x}}\,dx = \frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{x^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[2 - 3*x],x]
```

[Out] (x^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))

Maple [A]
time = 0.12, size = 29, normalized size = 0.94

method	result	size
meijerg	$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)\sqrt{2}}{2+2m}$	29

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*x^(1+m)*hypergeom([1/2,1+m],[2+m],3/2*x)/(1+m)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^m/sqrt(-3*x + 2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="fricas")

[Out] integral(-x^m*sqrt(-3*x + 2)/(3*x - 2), x)

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 46, normalized size = 1.48

$$-\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}i\sqrt{x-\frac{2}{3}}\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\right|\frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m/(2-3*x)**(1/2),x)

[Out] -2*2**m*sqrt(3)*I*sqrt(x - 2/3)*hyper((1/2, -m), (3/2,), 3*(x - 2/3)*exp_po
lar(I*pi)/2)/(3*3**m)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(2-3*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt(-3*x + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{2-3\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2 - 3*x)^(1/2),x)
[Out] int(x^m/(2 - 3*x)^(1/2), x)
```

3.721 $\int \frac{x^m}{\sqrt{-2+3x}}\,dx$

Optimal. Leaf size=36

$$\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

```
[Out] (3/2)^(-1-m)*hypergeom([1/2, -m],[3/2],1-3/2*x)*(-2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{67\}$

$$\left(\frac{3}{2}\right)^{-m-1}\sqrt{3x-2}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[-2 + 3*x],x]
[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{x^m}{\sqrt{-2+3x}}\,dx = \left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 1.00

$$\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2+3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (3/2)^(-1 - m)*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 - (3*x)/2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.13, size = 43, normalized size = 1.19

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\mathrm{signum}\left(x-\frac{2}{3}\right)}\,x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2\sqrt{\mathrm{signum}\left(x-\frac{2}{3}\right)}\,(1+m)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)/signum(x-2/3)^(1/2)*(-signum(x-2/3))^(1/2)/(1+m)*x^(1+m)*hyperg
eom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(3*x - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/sqrt(3*x - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.63, size = 36, normalized size = 1.00

$$-\frac{\sqrt{2}\,ixx^{m}\Gamma(m+1)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},m+1\\m+2\end{matrix}\middle|\frac{3x}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-2+3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x/2)/(2*gamm
a(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2+3*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt(3*x - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\sqrt{3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(3*x - 2)^(1/2),x)
[Out] int(x^m/(3*x - 2)^(1/2), x)
```

3.722 $\int \frac{x^m}{\sqrt{-2-3x}}\,dx$

Optimal. Leaf size=50

$$-2^{1+m}3^{-1-m}\sqrt{-2-3x}\,(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

```
[Out] -2^(1+m)*3^(-1-m)*x^m*hypergeom([1/2, -m],[3/2],1+3/2*x)*(-2-3*x)^(1/2)/((-
x)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {69, 12, 67}

$$-2^{m+1}3^{-m-1}\sqrt{-3x-2}\,(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^m/Sqrt[-2 - 3*x],x]
```

```
[Out] -((2^(1 + m)*3^(-1 - m)*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2,
1 + (3*x)/2])/(-x)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{-2-3x}}\,dx = \left(\left(\frac{2}{3}\right)^m (-x)^{-m}x^m\right)\int \frac{\left(\frac{3}{2}\right)^m (-x)^m}{\sqrt{-2-3x}}\,dx$$
$$= \left((-x)^{-m}x^m\right)\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$$
$$= -2^{1+m}3^{-1-m}\sqrt{-2-3x}\,(-x)^{-m}x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.96

$$-\frac{2}{3}\left(1+\frac{1}{2}(-2-3x)\right)^{-m}\sqrt{-2-3x}\,x^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/Sqrt[-2 - 3*x],x]
```

```
[Out] (-2*Sqrt[-2 - 3*x]*x^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(3*(1
+ (-2 - 3*x)/2)^m)
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 30, normalized size = 0.60

method	result	size
meijerg	$-\frac{i x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)\sqrt{2}}{2(1+m)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x^(1+m)*hypergeom([1/2,1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m/sqrt(-3*x - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-x^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.61, size = 41, normalized size = 0.82

$$-\frac{\sqrt{2}\,ixx^{m}\Gamma(m+1)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},m+1\\ m+2\end{matrix}\middle|\frac{3xe^{i\pi}}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-2-3*x)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*x**m*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*x*exp_polar(
I*pi)/2)/(2*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^m/sqrt(-3*x - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m}{\sqrt{-3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(- 3*x - 2)^(1/2),x)
```

```
[Out] int(x^m/(- 3*x - 2)^(1/2), x)
```

3.723 $\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$\frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

```
[Out] 2*(-x)^m*hypergeom([1/2, -m],[3/2],1+b*x/a)*(b*x+a)^(1/2)/b/((-b*x/a)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {69, 67}

$$\frac{2(-x)^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[a + b*x],x]
```

```
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-((b*x)/a))^m)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(-x)^m}{\sqrt{a+bx}}\,dx &= \left((-x)^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{\sqrt{a+bx}}\,dx\\ &= \frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$\frac{2(-x)^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\ {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[a + b*x],x]
[Out] (2*(-x)^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (b*x)/a])/(b*(-
((b*x)/a))^m)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-x)^m}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(b*x+a)^(1/2),x)
[Out] int((-x)^m/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((-x)^m/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((-x)^m/sqrt(b*x + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.70, size = 42, normalized size = 0.88

$$\frac{xx^m e^{i\pi m}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(b*x+a)**(1/2),x)
```

```
[Out] x*x**m*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), b*x*exp_polar
(I*pi)/a)/(sqrt(a)*gamma(m + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(-x)^m}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(a + b*x)^(1/2),x)
```

```
[Out] int((-x)^m/(a + b*x)^(1/2), x)
```

3.724 $\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx$

Optimal. Leaf size=34

$$-\frac{(-x)^{1+m}\,_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] -1/2*(-x)^(1+m)*hypergeom([1/2, 1+m],[2+m],-3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {66}

$$-\frac{(-x)^{m+1}\,_2F_1\left(\frac{1}{2},m+1;m+2;-\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[2 + 3*x],x]
```

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1
 + m)))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{2+3x}}\,dx = -\frac{(-x)^{1+m}\,_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.94

$$\frac{(-x)^m x\,_2F_1\left(\frac{1}{2},1+m;2+m;-\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[2 + 3*x],x]
```

```
[Out] ((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (-3*x)/2])/(Sqrt[2]*(1 + m))
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.88

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^{m} x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)}{2+2m}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(3*x + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-x)^m/sqrt(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.55, size = 44, normalized size = 1.29

$$\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}\sqrt{x+\frac{2}{3}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(2+3*x)**(1/2),x)
```

[Out] 2*2**m*sqrt(3)*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*exp_polar(2*I*pi)/2)/(3*3**m)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)^m/(2+3*x)^(1/2),x, algorithm="giac")

[Out] integrate((-x)^m/sqrt(3*x + 2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(-x)^m}{\sqrt{3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x)^m/(3*x + 2)^(1/2),x)

[Out] int((-x)^m/(3*x + 2)^(1/2), x)

3.725 $\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=34

$$-\frac{(-x)^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

```
[Out] -1/2*(-x)^(1+m)*hypergeom([1/2, 1+m],[2+m],3/2*x)/(1+m)*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{66\}$

$$-\frac{(-x)^{m+1}\,{}_2F_1\left(\frac{1}{2},m+1;m+2;\frac{3x}{2}\right)}{\sqrt{2}\,(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[2 - 3*x],x]
```

```
[Out] -(((-x)^(1 + m)*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1
+ m)))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{2-3x}}\,dx=-\frac{(-x)^{1+m}\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.94

$$\frac{(-x)^m x\,{}_2F_1\left(\frac{1}{2},1+m;2+m;\frac{3x}{2}\right)}{\sqrt{2}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[2 - 3*x],x]
[Out] ((-x)^m*x*Hypergeometric2F1[1/2, 1 + m, 2 + m, (3*x)/2])/(Sqrt[2]*(1 + m))
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.88

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^{m} x \,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2+2m}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2-3*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="maxima")
[Out] integrate((-x)^m/sqrt(-3*x + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="fricas")
[Out] integral(-(-x)^m*sqrt(-3*x + 2)/(3*x - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 53, normalized size = 1.56

$$-\frac{2 \cdot 2^{m} \sqrt{3} \cdot 3^{-m} i \sqrt{x - \frac{2}{3}}\, e^{i\pi m} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, -m \\ \frac{3}{2} \end{matrix}\middle| \frac{3\left(x-\frac{2}{3}\right)e^{i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*I*sqrt(x - 2/3)*exp(I*pi*m)*hyper((1/2, -m), (3/2,), 3*(x -
 2/3)*exp_polar(I*pi)/2)/(3*3**m)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(2-3*x)^(1/2),x, algorithm="giac")
[Out] integrate((-x)^m/sqrt(-3*x + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(-x)^m}{\sqrt{2-3\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(2 - 3*x)^(1/2),x)
[Out] int((-x)^m/(2 - 3*x)^(1/2), x)
```

3.726 $\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx$

Optimal. Leaf size=49

$$2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

```
[Out] 2^(1+m)*3^(-1-m)*(-x)^m*hypergeom([1/2, -m],[3/2],1-3/2*x)*(-2+3*x)^(1/2)/(
x^m)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {69, 12, 67}

$$2^{m+1}3^{-m-1}\sqrt{3x-2}\,(-x)^m x^{-m}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2,
 1 - (3*x)/2])/x^m
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 69

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m]), Int[((-d)*(x/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{-2+3x}}\,dx = \left(\left(\frac{2}{3}\right)^m (-x)^m x^{-m}\right)\int \frac{\left(\frac{3}{2}\right)^m x^m}{\sqrt{-2+3x}}\,dx$$
$$= \left((-x)^m x^{-m}\right)\int \frac{x^m}{\sqrt{-2+3x}}\,dx$$
$$= 2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.00

$$2^{1+m}3^{-1-m}(-x)^m x^{-m}\sqrt{-2+3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1-\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[-2 + 3*x],x]
```

```
[Out] (2^(1 + m)*3^(-1 - m)*(-x)^m*Sqrt[-2 + 3*x]*Hypergeometric2F1[1/2, -m, 3/2,
 1 - (3*x)/2])/x^m
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 5.
time = 0.13, size = 44, normalized size = 0.90

method	result	size
meijerg	$\frac{\sqrt{2}\,(-x)^m\sqrt{-\mathrm{signum}\left(x-\frac{2}{3}\right)}\;x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],\frac{3x}{2}\right)}{2\sqrt{\mathrm{signum}\left(x-\frac{2}{3}\right)}\,(1+m)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(-2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-x)^m/signum(x-2/3)^(1/2)*(-signum(x-2/3))^(1/2)/(1+m)*x*hyper
geom([1/2,1+m],[2+m],3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(3*x - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="fricas")
[Out] integral((-x)^m/sqrt(3*x - 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 42, normalized size = 0.86

$$-\frac{\sqrt{2}\,ixx^m e^{i\pi m}\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, m+1\\ m+2\end{matrix}\middle|\frac{3x}{2}\right)}{2\Gamma\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(-2+3*x)**(1/2),x)
[Out] -sqrt(2)*I*x*x**m*exp(I*pi*m)*gamma(m + 1)*hyper((1/2, m + 1), (m + 2,), 3*
x/2)/(2*gamma(m + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2+3*x)^(1/2),x, algorithm="giac")
[Out] integrate((-x)^m/sqrt(3*x - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(-x)^m}{\sqrt{3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(3*x - 2)^(1/2),x)
[Out] int((-x)^m/(3*x - 2)^(1/2), x)
```

3.727 $\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx$

Optimal. Leaf size=37

$$-\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2-3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

```
[Out] -(3/2)^(-1-m)*hypergeom([1/2, -m],[3/2],1+3/2*x)*(-2-3*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{67\}$

$$-\left(\frac{3}{2}\right)^{-m-1}\sqrt{-3x-2}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{3x}{2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x)^m/Sqrt[-2 - 3*x],x]
```

```
[Out] -((3/2)^(-1 - m)*Sqrt[-2 - 3*x]*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2
])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(-x)^m}{\sqrt{-2-3x}}\,dx=-\left(\frac{3}{2}\right)^{-1-m}\sqrt{-2-3x}\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.54

$$-\frac{2}{3}\left(1+\frac{1}{2}(-2-3x)\right)^{-m}\sqrt{-2-3x}\;x^{-m}\left(-x^2\right)^m\;{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};1+\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x)^m/Sqrt[-2 - 3*x],x]
```

```
[Out] (-2*Sqrt[-2 - 3*x]*(-x^2)^m*Hypergeometric2F1[1/2, -m, 3/2, 1 + (3*x)/2])/(
3*(1 + (-2 - 3*x)/2)^m*x^m)
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 31, normalized size = 0.84

method	result	size
meijerg	$-\frac{i\sqrt{2}\,(-x)^{m}x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+m\right],[2+m],-\frac{3x}{2}\right)}{2(1+m)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(-2-3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*2^(1/2)*(-x)^m/(1+m)*x*hypergeom([1/2,1+m],[2+m],-3/2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-x)^m/sqrt(-3*x - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(-x)^m*sqrt(-3*x - 2)/(3*x + 2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.54, size = 48, normalized size = 1.30

$$-\frac{2\cdot 2^{m}\sqrt{3}\cdot 3^{-m}i\sqrt{x+\frac{2}{3}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},-m\\ \frac{3}{2}\end{matrix}\middle|\frac{3\left(x+\frac{2}{3}\right)e^{2i\pi}}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**m/(-2-3*x)**(1/2),x)
```

```
[Out] -2*2**m*sqrt(3)*I*sqrt(x + 2/3)*hyper((1/2, -m), (3/2,), 3*(x + 2/3)*exp_po
lar(2*I*pi)/2)/(3*3**m)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^m/(-2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-x)^m/sqrt(-3*x - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(-x)^m}{\sqrt{-3\,x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^m/(- 3*x - 2)^(1/2),x)
```

```
[Out] int((-x)^m/(- 3*x - 2)^(1/2), x)
```

3.728 $\int \frac{x^n}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=26

$$-2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

```
[Out] -2*hypergeom([1/2, -n],[3/2],1-x)*(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{67\}$

$$-2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^n/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{x^n}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$-2\sqrt{1-x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/Sqrt[1 - x],x]
```

```
[Out] -2*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x]
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.88

method	result	size
meijerg	$\frac{x^{1+n}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+n\right],[2+n],x\right)}{1+n}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(1+n)*x^(1+n)*hypergeom([1/2,1+n],[2+n],x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^n/sqrt(-x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="fricas")
[Out] integral(-x^n*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.48, size = 26, normalized size = 1.00

$$-2i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/(1-x)**(1/2),x)
[Out] -2*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^n/sqrt(-x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^n}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(1 - x)^(1/2),x)
```

```
[Out] int(x^n/(1 - x)^(1/2), x)
```

3.729 $\int \frac{x^n}{\sqrt{a-ax}} dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{a-ax}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

```
[Out] -2*hypergeom([1/2, -n],[3/2],1-x)*(-a*x+a)^(1/2)/a
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {67}

$$-\frac{2\sqrt{a-ax}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^n/Sqrt[a - a*x],x]
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{x^n}{\sqrt{a-ax}} dx = -\frac{2\sqrt{a-ax}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Mathematica [A]
time = 0.11, size = 30, normalized size = 1.00

$$-\frac{2\sqrt{a-ax}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-x\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/Sqrt[a - a*x],x]
[Out] (-2*Sqrt[a - a*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - x])/a
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^n}{\sqrt{-ax+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(-a*x+a)^(1/2),x)
[Out] int(x^n/(-a*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x^n/sqrt(-a*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a*x + a)*x^n/(a*x - a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.56, size = 31, normalized size = 1.03

$$-\frac{2i\sqrt{x-1}\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2}, -n\\ \frac{3}{2}\end{matrix}\middle|(x-1)\,e^{i\pi}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/(-a*x+a)**(1/2),x)
[Out] -2*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi))/sqrt(a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/(-a*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^n/sqrt(-a*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^n}{\sqrt{a-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(a - a*x)^(1/2),x)
```

```
[Out] int(x^n/(a - a*x)^(1/2), x)
```

3.730 $\int x^m(a+bx)^n\,dx$

Optimal. Leaf size=47

$$\frac{x^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}$$

```
[Out] x^(1+m)*(b*x+a)^n*hypergeom([-n, 1+m],[2+m],-b*x/a)/(1+m)/((1+b*x/a)^n)
```

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {68, 66}

$$\frac{x^{m+1}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^m\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{x^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$\frac{x^{1+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^m(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^n,x)
```

```
[Out] int(x^m*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*x^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.77, size = 34, normalized size = 0.72

$$\frac{a^n x x^m \Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-n,m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**n,x)
```

```
[Out] a**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(I*pi)/a
)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(a+b\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^n,x)
```

```
[Out] int(x^m*(a + b*x)^n, x)
```

3.731 $\int (cx)^m (a+bx)^n \, dx$

Optimal. Leaf size=52

$$\frac{(cx)^{1+m}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{c(1+m)}$$

```
[Out] (c*x)^(1+m)*(b*x+a)^n*hypergeom([-n, 1+m],[2+m],-b*x/a)/c/(1+m)/((1+b*x/a)^
n)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{(cx)^{m+1}(a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{bx}{a}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a + b*x)^n,x]
```

```
[Out] ((c*x)^(1 + m)*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])
/(c*(1 + m)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (cx)^m (a+bx)^n \, dx &= \left((a+bx)^n \left(1+\frac{bx}{a}\right)^{-n}\right) \int (cx)^m \left(1+\frac{bx}{a}\right)^n \, dx\\ &= \frac{(cx)^{1+m}(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{c(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.92

$$\frac{x(cx)^m(a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{bx}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a + b*x)^n,x]
[Out] (x*(c*x)^m*(a + b*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((b*x)/a)])/((1
 + m)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (cx)^m (bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(b*x+a)^n,x)
[Out] int((c*x)^m*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*(c*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="fricas")
[Out] integral((b*x + a)^n*(c*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.43, size = 37, normalized size = 0.71

$$\frac{a^n c^m x x^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(b*x+a)**n,x)
```

```
[Out] a**n*c**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(I*
pi)/a)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(c*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,x)^m\,(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a + b*x)^n,x)
```

```
[Out] int((c*x)^m*(a + b*x)^n, x)
```

3.732 $\int x^3(a+bx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}$$

```
[Out] -a^3*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*(b*x+a)^(2+n)/b^4/(2+n)-3*a*(b*x+a)^(3+n
)/b^4/(3+n)+(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {45}

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^n,x]
```

```
[Out] -((a^3*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (3*a^2*(a + b*x)^(2 + n))/(b^4*(
2 + n)) - (3*a*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (a + b*x)^(4 + n)/(b^4*(4
 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^n\,dx &= \int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 67, normalized size = 0.81

$$\frac{(a+bx)^{1+n}\left(-\frac{a^3}{1+n}+\frac{3a^2(a+bx)}{2+n}-\frac{3a(a+bx)^2}{3+n}+\frac{(a+bx)^3}{4+n}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-(a^3/(1 + n)) + (3*a^2*(a + b*x))/(2 + n) - (3*a*(a +
b*x)^2)/(3 + n) + (a + b*x)^3/(4 + n)))/b^4
```

Maple [A]
time = 0.11, size = 126, normalized size = 1.52

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3)}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4)(bx+a)^n}{(3+n)(4+n)(2+n)(1+n)b^4}$
norman	$\frac{x^4e^{n\ln(bx+a)}}{4+n}+\frac{an\,x^3e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{6a^4e^{n\ln(bx+a)}}{b^4(n^4+10n^3+35n^2+50n+24)}-\frac{3a^2n\,x^2e^{n\ln(bx+a)}}{b^2(n^3+9n^2+26n+24)}+\frac{6n\,a^3x\,e^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9*a
*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/b^4/(n^4+10*n
^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.28, size = 101, normalized size = 1.22

$$\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.90, size = 143, normalized size = 1.72

$$\frac{(6\,a^3bnx+(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4)x^4-6\,a^4+(ab^3n^3+3\,ab^3n^2+2\,ab^3n)x^3-3\,(a^2b^2n^2+a^2b^2n)x^2)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*(b*
x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(71) = 142$.
time = 0.58, size = 1318, normalized size = 15.88

$$\begin{cases} \frac{a^{n}x^{4}}{4} & \text{for}\: b = 0 \\ \frac{6a^{3}\log\left(\frac{a}{b}+x\right)}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} + \frac{11a^{3}}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} + \frac{18a^{2}bx\log\left(\frac{a}{b}+x\right)}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} + \frac{27a^{2}bx}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} + \frac{18ab^{2}x^{2}\log\left(\frac{a}{b}+x\right)}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} + \frac{18ab^{2}x^{2}}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} + \frac{6b^{3}x^{3}\log\left(\frac{a}{b}+x\right)}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}} & \text{for}\: n = -4 \\ -\frac{6a^{3}\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}} - \frac{9a^{3}}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}} - \frac{12a^{2}bx\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}} - \frac{12a^{2}bx}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}} - \frac{6ab^{2}x^{2}\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}} + \frac{2b^{3}x^{3}}{2a^{2}b^{4}+4ab^{5}x+2b^{6}x^{2}} & \text{for}\: n = -3 \\ \frac{6a^{3}\log\left(\frac{a}{b}+x\right)}{2ab^{4}+2b^{5}x} + \frac{6a^{3}}{2ab^{4}+2b^{5}x} + \frac{6a^{2}bx\log\left(\frac{a}{b}+x\right)}{2ab^{4}+2b^{5}x} - \frac{3ab^{2}x^{2}}{2ab^{4}+2b^{5}x} + \frac{b^{3}x^{3}}{2ab^{4}+2b^{5}x} & \text{for}\: n = -2 \\ -\frac{a^{3}\log\left(\frac{a}{b}+x\right)}{b^{4}} + \frac{a^{2}x}{b^{3}} - \frac{ax^{2}}{2b^{2}} + \frac{x^{3}}{3b} & \text{for}\: n = -1 \\ -\frac{6a^{4}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{6a^{3}bnx(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} - \frac{3a^{2}b^{2}n^{2}x^{2}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} - \frac{3a^{2}b^{2}nx^{2}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{ab^{3}n^{3}x^{3}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{3ab^{3}n^{2}x^{3}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{2ab^{3}nx^{3}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{b^{4}n^{3}x^{4}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{6b^{4}n^{2}x^{4}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{11b^{4}nx^{4}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} + \frac{6b^{4}x^{4}(a+bx)^{n}}{b^{4}n^{4}+10b^{4}n^{3}+35b^{4}n^{2}+50b^{4}n+24b^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a
**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2
*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log
(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 1
8*a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2
+ 6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2
*b*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a
**3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) -
 3*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(
n, -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3
*b), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*
a**2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(
b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n
*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)*
*n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4
```

```
*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(83) = 166$.
time = 1.73, size = 226, normalized size = 2.72

$$\frac{(bx+a)^n b^4 n^3 x^4 + (bx+a)^n a b^3 n^3 x^3 + 6\,(bx+a)^n b^4 n^2 x^4 + 3\,(bx+a)^n a b^3 n^2 x^3 + 11\,(bx+a)^n b^4 n x^4 - 3\,(bx+a)^n a^2 b^2 n^2 x^2 + 2\,(bx+a)^n a b^3 n x^3 + 6\,(bx+a)^n b^4 x^4 - 3\,(bx+a)^n a^2 b^2 n x^2 + 6\,(bx+a)^n a^3 b n x - 6\,(bx+a)^n a^4}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50\,b^4 n + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [B]
time = 0.53, size = 176, normalized size = 2.12

$$(a+b\,x)^n\,\left(\frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^2\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 +
 24) - (6*a^4)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x)/(b^3
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^3*(3*n + n^2 + 2))/(b*(50*n
+ 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2
+ 10*n^3 + n^4 + 24)))
```

3.733 $\int x^2(a+bx)^n\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}$$

```
[Out] a^2*(b*x+a)^(1+n)/b^3/(1+n)-2*a*(b*x+a)^(2+n)/b^3/(2+n)+(b*x+a)^(3+n)/b^3/(
3+n)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {45}

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n,x]
```

```
[Out] (a^2*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*x)^(2 + n))/(b^3*(2 + n
)) + (a + b*x)^(3 + n)/(b^3*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\,dx &= \int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx\\ &= \frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2\right)}{b^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

Maple [A]
time = 0.11, size = 73, normalized size = 1.22

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^{2} n^{2} x^{2}+3 b^{2} n\, x^{2}-2 a b n x+2 x^{2} b^{2}-2 a b x+2 a^{2}\right)}{b^{3} \left(n^{3}+6 n^{2}+11 n+6\right)}$	73
risch	$\frac{\left(b^{3} n^{2} x^{3}+a\, b^{2} n^{2} x^{2}+3 b^{3} n\, x^{3}+a\, b^{2} n\, x^{2}+2 b^{3} x^{3}-2 a^{2} b n x+2 a^{3}\right) (bx+a)^{n}}{(2+n) (3+n) (1+n) b^{3}}$	88
norman	$\frac{x^{3} \mathrm{e}^{n \ln (bx+a)}}{3+n}+\frac{a n\, x^{2} \mathrm{e}^{n \ln (bx+a)}}{b \left(n^{2}+5 n+6\right)}+\frac{2 a^{3} \mathrm{e}^{n \ln (bx+a)}}{b^{3} \left(n^{3}+6 n^{2}+11 n+6\right)}-\frac{2 n\, a^{2} x\, \mathrm{e}^{n \ln (bx+a)}}{b^{2} \left(n^{3}+6 n^{2}+11 n+6\right)}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 68, normalized size = 1.13

$$\frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 1.24, size = 96, normalized size = 1.60

$$-\frac{(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2)(bx+a)^n}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(51) = 102$.

time = 0.37, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^n x^3}{3} & \text{for } b = 0 \\ \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for } n = -3 \\ -\frac{2a^2 \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} & \text{for } n = -2 \\ \frac{a^2 \log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} & \text{for } n = -1 \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3n^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{3b^3nx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{2b^3x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b**3 +
b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x)
 + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x/b
**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*
n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3
*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b
**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(60) = 120$.

time = 2.25, size = 140, normalized size = 2.33

$$\frac{(bx+a)^n b^3 n^2 x^3 + (bx+a)^n a b^2 n^2 x^2 + 3\,(bx+a)^n b^3 n x^3 + (bx+a)^n a b^2 n x^2 + 2\,(bx+a)^n b^3 x^3 - 2\,(bx+a)^n a^2 b n x + 2\,(bx+a)^n a^3}{b^3 n^3 + 6\,b^3 n^2 + 11\,b^3 n + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

Mupad [B]

time = 0.56, size = 192, normalized size = 3.20

$$\begin{cases} \frac{2a^2 \ln(a+bx)+b^2x^2-2abx}{2b^3} & \text{if } n=-1 \\ \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\ln(a+bx)}{b^3} & \text{if } n=-2 \\ \frac{\ln(a+bx)+\frac{2a}{a+bx}-\frac{a^2}{2(a+bx)^2}}{b^3} & \text{if } n=-3 \\ \frac{2(a+bx)^{n+1}(8a^2-8abnx-8abx+4b^2n^2x^2+12b^2nx^2+8b^2x^2)}{b^3(8n^3+48n^2+88n+48)} & \text{if } n\neq -1 \wedge n\neq -2 \wedge n\neq -3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), n == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, n == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, n ~= -1 & n ~= -2 & n ~=
 -3, (2*(a + b*x)^(n + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*n*x^2 - 8*a*b*x + 4*b
^2*n^2*x^2 - 8*a*b*n*x))/(b^3*(88*n + 48*n^2 + 8*n^3 + 48)))
```

3.734 $\int x(a+bx)^n\,dx$

Optimal. Leaf size=39

$$-\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}$$

```
[Out] -a*(b*x+a)^(1+n)/b^2/(1+n)+(b*x+a)^(2+n)/b^2/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {45}

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n,x]
[Out] -((a*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (a + b*x)^(2 + n)/(b^2*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\,dx &= \int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx\\ &= -\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n,x]
```

[Out] ((a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n))

Maple [A]
time = 0.12, size = 36, normalized size = 0.92

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(-xnb-bx+a)}{b^2(n^2+3n+2)}$	36
risch	$-\frac{(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{b^2(2+n)(1+n)}$	50
norman	$\frac{x^2 \mathrm{e}^{n\ln(bx+a)}}{2+n}+\frac{nax\,\mathrm{e}^{n\ln(bx+a)}}{b(n^2+3n+2)}-\frac{a^2\mathrm{e}^{n\ln(bx+a)}}{b^2(n^2+3n+2)}$	73

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x+a)^n,x,method=_RETURNVERBOSE)

[Out] -(b*x+a)^(1+n)*(-b*n*x-b*x+a)/b^2/(n^2+3*n+2)

Maxima [A]
time = 0.27, size = 42, normalized size = 1.08

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n,x, algorithm="maxima")

[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)

Fricas [A]
time = 1.21, size = 53, normalized size = 1.36

$$\frac{(abnx+(b^2n+b^2)x^2-a^2)(bx+a)^n}{b^2n^2+3\,b^2n+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n,x, algorithm="fricas")

[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*(b*x + a)^n/(b^2*n^2 + 3*b^2*n + 2*b^2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(31) = 62.
time = 0.22, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^n x^2}{2} & \text{for}\ b=0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{a}{ab^2+b^3x}+\frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ n=-2 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{x}{b} & \text{for}\ n=-1 \\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2}+\frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n +
 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*n*x*
*2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2), True))
```

Giac [A]
time = 1.03, size = 76, normalized size = 1.95

$$\frac{(bx+a)^n b^2 n x^2 + (bx+a)^n a b n x + (bx+a)^n b^2 x^2 - (bx+a)^n a^2}{b^2 n^2 + 3\,b^2 n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x +
 a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

Mupad [B]
time = 0.38, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if } n=-1 \\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } n=-2 \\ \frac{2\left(\frac{(a+b\,x)^{n+2}}{2\,n+4}-\frac{a\,(a+b\,x)^{n+1}}{2\,n+2}\right)}{b^2} & \text{if } n\neq -1 \wedge n\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, -(a*log(a + b*x) - b*x)/b^2, n == -2, (log(a + b*x) + a/
(a + b*x))/b^2, n ~= -1 & n ~= -2, (2*((a + b*x)^(n + 2)/(2*n + 4) - (a*(a
+ b*x)^(n + 1))/(2*n + 2)))/b^2)
```

3.735 $\int (a+bx)^n \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{1+n}}{b(1+n)}$$

```
[Out] (b*x+a)^(1+n)/b/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^n \, dx = \frac{(a+bx)^{1+n}}{b(1+n)}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{1+n}}{b+bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b + b*n)
```

Maple [A]
time = 0.11, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{(bx+a)^{1+n}}{b(1+n)}$	19
default	$\frac{(bx+a)^{1+n}}{b(1+n)}$	19
risch	$\frac{(bx+a)(bx+a)^{n}}{b(1+n)}$	22
norman	$\frac{x\,e^{n\ln(bx+a)}}{1+n}+\frac{a\,e^{n\ln(bx+a)}}{b(1+n)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

Fricas [A]
time = 1.25, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^{n}}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^n/(b*n + b)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.11

$$\frac{\begin{cases}\frac{(a+bx)^{n+1}}{n+1} & \text{for } n\neq -1\\ \log(a+bx) & \text{otherwise}\end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n,x)
[Out] Piecewise(((a + b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*x), True))/b
```

Giac [A]
time = 1.69, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="giac")
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

Mupad [B]
time = 0.20, size = 18, normalized size = 1.00

$$\frac{(a+b\,x)^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n,x)
[Out] (a + b*x)^(n + 1)/(b*(n + 1))
```

3.736 $\int \frac{(a+bx)^n}{x} dx$

Optimal. Leaf size=35

$$-\frac{(a+bx)^{1+n}\,_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] -(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {67}

$$-\frac{(a+bx)^{n+1}\,_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x}\,dx = -\frac{(a+bx)^{1+n}\,_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$-\frac{(a+bx)^{1+n}\,_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x,x]
```

```
[Out] -(((a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1
 + n)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x,x)
```

```
[Out] int((b*x+a)^n/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.
time = 0.61, size = 83, normalized size = 2.37

$$-\frac{bb^n n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n \Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma\left(n+2\right)}-\frac{bb^n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n \Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma\left(n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x,x)
```

```
[Out] -b*b**n*n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n
+ 1)/(a*gamma(n + 2)) - b*b**n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/
a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x,x, algorithm="giac")
[Out] integrate((b*x + a)^n/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,x)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x,x)
[Out] int((a + b*x)^n/x, x)
```

3.737 $\int \frac{(a+bx)^n}{x^2} \, dx$

Optimal. Leaf size=35

$$\frac{b(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

[Out] b*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/(1+n)

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{67\}$

$$\frac{b(a+bx)^{n+1} {}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^n/x^2,x]

[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*(1 + n))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x^2} \, dx = \frac{b(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

Mathematica [A]
time = 0.02, size = 35, normalized size = 1.00

$$\frac{b(a+bx)^{1+n} {}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^n/x^2,x]

```
[Out] (b*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^2*
(1 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^2,x)
```

```
[Out] int((b*x+a)^n/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/x^2, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(27) = 54$.
time = 0.90, size = 354, normalized size = 10.11

$$\frac{ab^2b^nn^2\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma\left(n+2\right)+a^2b\left(\frac{a}{b}+x\right)\Gamma\left(n+2\right)}+\frac{ab^2b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma\left(n+2\right)+a^2b\left(\frac{a}{b}+x\right)\Gamma\left(n+2\right)}-\frac{ab^2b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{-a^3\Gamma\left(n+2\right)+a^2b\left(\frac{a}{b}+x\right)\Gamma\left(n+2\right)}-\frac{ab^2b^n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{-a^3\Gamma\left(n+2\right)+a^2b\left(\frac{a}{b}+x\right)\Gamma\left(n+2\right)}-\frac{b^3b^nn^2\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma\left(n+2\right)+a^2b\left(\frac{a}{b}+x\right)\Gamma\left(n+2\right)}-\frac{b^3b^nn\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{-a^3\Gamma\left(n+2\right)+a^2b\left(\frac{a}{b}+x\right)\Gamma\left(n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**2,x)
```

```
[Out] a*b**2*b**n*n**2*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*g
amma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma(n + 2)) + a*b**2*b
**n*n*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)
/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma(n + 2)) - a*b**2*b**n*n*(a/b
```

```
+ x)*(a/b + x)**n*gamma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b + x)*gamma
(n + 2)) - a*b**2*b**n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(-a**3*gamma(n +
 2) + a**2*b*(a/b + x)*gamma(n + 2)) - b**3*b**n*n**2*(a/b + x)**2*(a/b + x
)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(-a**3*gamma(n + 2) + a
**2*b*(a/b + x)*gamma(n + 2)) - b**3*b**n*n*(a/b + x)**2*(a/b + x)**n*lerch
phi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(-a**3*gamma(n + 2) + a**2*b*(a/b
 + x)*gamma(n + 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,x)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^2,x)
```

```
[Out] int((a + b*x)^n/x^2, x)
```

3.738 $\int \frac{(a+bx)^n}{x^3} dx$

Optimal. Leaf size=38

$$-\frac{b^2(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

```
[Out] -b^2*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {67}

$$-\frac{b^2(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^3,x]
```

```
[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(a+bx)^n}{x^3}\,dx = -\frac{b^2(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$-\frac{b^2(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^3,x]
```

[Out] -((b^2*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^3*(1 + n)))

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^n/x^3,x)

[Out] int((b*x+a)^n/x^3,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n/x^3,x, algorithm="maxima")

[Out] integrate((b*x + a)^n/x^3, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n/x^3,x, algorithm="fricas")

[Out] integral((b*x + a)^n/x^3, x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(31) = 62.
time = 2.02, size = 918, normalized size = 24.16

$$-\frac{a^2b^3b^nn^3\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}+\frac{a^2b^3b^nn^2\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}+\frac{a^2b^3b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}-\frac{a^2b^3b^nn\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}-\frac{2a^2b^3b^n\left(\frac{a}{b}+x\right)\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}+\frac{2ab^4b^nn^3\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}-\frac{ab^4b^nn^2\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}-\frac{2ab^4b^nn\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}+\frac{ab^4b^n\left(\frac{a}{b}+x\right)^2\left(\frac{a}{b}+x\right)^n\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}-\frac{b^5b^nn^3\left(\frac{a}{b}+x\right)^3\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}+\frac{b^5b^nn\left(\frac{a}{b}+x\right)^3\left(\frac{a}{b}+x\right)^n\Phi\left(\frac{b\left(\frac{a}{b}+x\right)}{a},1,n+1\right)\Gamma(n+1)}{2a^5\Gamma(n+2)-4a^4b\left(\frac{a}{b}+x\right)\Gamma(n+2)+2a^3b^2\left(\frac{a}{b}+x\right)^2\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**n/x**3,x)

[Out] -a**2*b**3*b**n*n**3*(a/b + x)*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n +
1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*
a**3*b**2*(a/b + x)**2*gamma(n + 2)) + a**2*b**3*b**n*n**2*(a/b + x)*(a/b +
x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2)

```
+ 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) + a**2*b**3*b**n*n*(a/b + x)*(a/b
+ x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2)
 - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2))
 - a**2*b**3*b**n*n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2
) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)
) - 2*a**2*b**3*b**n*(a/b + x)*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n +
2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2
)) + 2*a*b**4*b**n*n**3*(a/b + x)**2*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1
, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2
) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) - a*b**4*b**n*n**2*(a/b + x)**2*
(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n
 + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) - 2*a*b**4*b**n*n*(a/b + x)*
*2*(a/b + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamm
a(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma
(n + 2)) + a*b**4*b**n*(a/b + x)**2*(a/b + x)**n*gamma(n + 1)/(2*a**5*gamma
(n + 2) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(
n + 2)) - b**5*b**n*n**3*(a/b + x)**3*(a/b + x)**n*lerchphi(b*(a/b + x)/a,
1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2) - 4*a**4*b*(a/b + x)*gamma(n +
2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)) + b**5*b**n*n*(a/b + x)**3*(a/b
 + x)**n*lerchphi(b*(a/b + x)/a, 1, n + 1)*gamma(n + 1)/(2*a**5*gamma(n + 2
) - 4*a**4*b*(a/b + x)*gamma(n + 2) + 2*a**3*b**2*(a/b + x)**2*gamma(n + 2)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^3,x, algorithm="giac")
[Out] integrate((b*x + a)^n/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,x\right)}^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^3,x)
[Out] int((a + b*x)^n/x^3, x)
```

3.739 $\int x^{-4+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=110

$$-\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}+\frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}$$

```
[Out] -x^(-3+n)*(b*x+a)^(1-n)/a/(3-n)+2*b*x^(-2+n)*(b*x+a)^(1-n)/a^2/(2-n)/(3-n)-
2*b^2*x^(-1+n)*(b*x+a)^(1-n)/a^3/(3-n)/(n^2-3*n+2)
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-3 + n)*(a + b*x)^(1 - n))/(a*(3 - n))) + (2*b*x^(-2 + n)*(a + b*x)^(
1 - n))/(a^2*(2 - n)*(3 - n)) - (2*b^2*x^(-1 + n)*(a + b*x)^(1 - n))/(a^3*(
1 - n)*(2 - n)*(3 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-4+n}(a+bx)^{-n}\,dx &= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} - \frac{(2b)\int x^{-3+n}(a+bx)^{-n}\,dx}{a(3-n)} \\ &= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} + \frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)} + \frac{(2b^2)\int x^{-2+n}(a+bx)^{-n}\,dx}{a^2(2-n)(3-n)} \\ &= -\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)} + \frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)} - \frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.58

$$\frac{x^{-3+n}(a+bx)^{1-n}\left(a^2(2-3n+n^2)+2ab(-1+n)x+2b^2x^2\right)}{a^3(-3+n)(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-3 + n)*(a + b*x)^(1 - n)*(a^2*(2 - 3*n + n^2) + 2*a*b*(-1 + n)*x + 2*b
^2*x^2))/(a^3*(-3 + n)*(-2 + n)*(-1 + n))
```

Maple [A]
time = 0.13, size = 77, normalized size = 0.70

method	result	size
gosper	$\frac{(bx+a)x^{-3+n}\left(a^2n^2+2abnx+2x^2b^2-3a^2n-2abx+2a^2\right)(bx+a)^{-n}}{(-3+n)(-2+n)(-1+n)a^3}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4+n)/((b*x+a)^n),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)*x^(-3+n)*(a^2*n^2+2*a*b*n*x+2*b^2*x^2-3*a^2*n-2*a*b*x+2*a^2)/((b*x+
a)^n)/(-3+n)/(-2+n)/(-1+n)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

Fricas [A]
time = 1.55, size = 104, normalized size = 0.95

$$\frac{(2\,ab^2nx^3 + 2\,b^3x^4 + (a^2bn^2 - a^2bn)x^2 + (a^3n^2 - 3\,a^3n + 2\,a^3)x)x^{n-4}}{(a^3n^3 - 6\,a^3n^2 + 11\,a^3n - 6\,a^3)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] (2*a*b^2*n*x^3 + 2*b^3*x^4 + (a^2*b*n^2 - a^2*b*n)*x^2 + (a^3*n^2 - 3*a^3*n
 + 2*a^3)*x)*x^(n - 4)/((a^3*n^3 - 6*a^3*n^2 + 11*a^3*n - 6*a^3)*(b*x + a)^
n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4+n)/((b*x+a)**n),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="giac")
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

Mupad [B]
time = 0.52, size = 136, normalized size = 1.24

$$\frac{\frac{x\,x^{n-4}\,\left(n^2-3\,n+2\right)}{n^3-6\,n^2+11\,n-6}+\frac{2\,b^3\,x^{n-4}\,x^4}{a^3\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{2\,b^2\,n\,x^{n-4}\,x^3}{a^2\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{b\,n\,x^{n-4}\,x^2\,\left(n-1\right)}{a\,\left(n^3-6\,n^2+11\,n-6\right)}}{{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 4)/(a + b*x)^n,x)
[Out] ((x*x^(n - 4)*(n^2 - 3*n + 2))/(11*n - 6*n^2 + n^3 - 6) + (2*b^3*x^(n - 4)*
x^4)/(a^3*(11*n - 6*n^2 + n^3 - 6)) + (2*b^2*n*x^(n - 4)*x^3)/(a^2*(11*n -
6*n^2 + n^3 - 6)) + (b*n*x^(n - 4)*x^2*(n - 1))/(a*(11*n - 6*n^2 + n^3 - 6)
))/(a + b*x)^n
```

3.740 $\int x^{-3+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=64

$$-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}$$

```
[Out] -x^(-2+n)*(b*x+a)^(1-n)/a/(2-n)+b*x^(-1+n)*(b*x+a)^(1-n)/a^2/(1-n)/(2-n)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-2 + n)*(a + b*x)^(1 - n))/(a*(2 - n))) + (b*x^(-1 + n)*(a + b*x)^(1
- n))/(a^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3+n}(a+bx)^{-n}\,dx &= -\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}-\frac{b\int x^{-2+n}(a+bx)^{-n}\,dx}{a(2-n)}\\ &= -\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.61

$$\frac{x^{-2+n}(a+bx)^{1-n}(a(-1+n)+bx)}{a^2(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + n)/(a + b*x)^n,x]
[Out] (x^(-2 + n)*(a + b*x)^(1 - n)*(a*(-1 + n) + b*x))/(a^2*(-2 + n)*(-1 + n))
```

Maple [A]
time = 0.12, size = 44, normalized size = 0.69

method	result	size
gosper	$\frac{x^{-2+n}(an+bx-a)(bx+a)(bx+a)^{-n}}{(-2+n)(-1+n)a^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+n)/((b*x+a)^n),x,method=_RETURNVERBOSE)
[Out] x^(-2+n)*(a*n+b*x-a)*(b*x+a)/((b*x+a)^n)/(-2+n)/(-1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="maxima")
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

Fricas [A]
time = 1.45, size = 64, normalized size = 1.00

$$\frac{(abnx^2+b^2x^3+(a^2n-a^2)x)x^{n-3}}{(a^2n^2-3\,a^2n+2\,a^2)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] (a*b*n*x^2 + b^2*x^3 + (a^2*n - a^2)*x)*x^(n - 3)/((a^2*n^2 - 3*a^2*n + 2*a
^2)*(b*x + a)^n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+n)/((b*x+a)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

Mupad [B]
time = 0.45, size = 80, normalized size = 1.25

$$\frac{\frac{x\,x^{n-3}\,\left(n-1\right)}{n^2-3\,n+2}+\frac{b^2\,x^{n-3}\,x^3}{a^2\,\left(n^2-3\,n+2\right)}+\frac{b\,n\,x^{n-3}\,x^2}{a\,\left(n^2-3\,n+2\right)}}{{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 3)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 3)*(n - 1))/(n^2 - 3*n + 2) + (b^2*x^(n - 3)*x^3)/(a^2*(n^2 - 3*
n + 2)) + (b*n*x^(n - 3)*x^2)/(a*(n^2 - 3*n + 2)))/(a + b*x)^n
```

3.741 $\int x^{-2+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=28

$$-\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

```
[Out] -x^(-1+n)*(b*x+a)^(1-n)/a/(1-n)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-1 + n)*(a + b*x)^(1 - n))/(a*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-2+n}(a+bx)^{-n}\,dx = -\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.89

$$\frac{x^{-1+n}(a+bx)^{1-n}}{a(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-1 + n)*(a + b*x)^(1 - n))/(a*(-1 + n))
```

Maple [A]
time = 0.12, size = 29, normalized size = 1.04

method	result	size
gosper	$\frac{x^{-1+n}(bx+a)(bx+a)^{-n}}{a(-1+n)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+n)/((b*x+a)^n),x,method=_RETURNVERBOSE)
[Out] x^(-1+n)*(b*x+a)/a/(-1+n)/((b*x+a)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="maxima")
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

Fricas [A]
time = 1.12, size = 33, normalized size = 1.18

$$\frac{(bx^2 + ax)x^{n-2}}{(an - a)(bx + a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*x^(n - 2)/((a*n - a)*(b*x + a)^n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(19) = 38.
time = 201.00, size = 85, normalized size = 3.04

$$\begin{cases} -\frac{1}{bx} & \text{for}\ a = 0 \wedge n = 1 \\ -\frac{x^n (bx)^{-n}}{x} & \text{for}\ a = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x\right)}{a} & \text{for}\ n = 1 \\ \frac{ax^n}{anx(a+bx)^n - ax(a+bx)^n} + \frac{bxx^n}{anx(a+bx)^n - ax(a+bx)^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+n)/((b*x+a)**n),x)
```

```
[Out] Piecewise((-1/(b*x), Eq(a, 0) & Eq(n, 1)), (-x**n/(x*(b*x)**n), Eq(a, 0)),
(log(x)/a - log(a/b + x)/a, Eq(n, 1)), (a*x**n/(a*n*x*(a + b*x)**n - a*x*(a
 + b*x)**n) + b*x*x**n/(a*n*x*(a + b*x)**n - a*x*(a + b*x)**n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

Mupad [B]
time = 0.35, size = 29, normalized size = 1.04

$$\frac{x^n\,\left(a+b\,x\right)}{a\,x\,\left(n-1\right)\,{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 2)/(a + b*x)^n,x)
```

```
[Out] (x^n*(a + b*x))/(a*x*(n - 1)*(a + b*x)^n)
```

3.742 $\int x^{-1+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=39

$$\frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}$$

```
[Out] x^n*(1+b*x/a)^n*hypergeom([n, n],[1+n],-b*x/a)/n/((b*x+a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {68, 66}

$$\frac{x^n(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n;n+1;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x)^n,x]
```

```
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*
x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^{-1+n}\left(1+\frac{bx}{a}\right)^{-n}dx\\ &= \frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{x^n(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,n;1+n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x)^n,x]
[Out] (x^n*(1 + (b*x)/a)^n*Hypergeometric2F1[n, n, 1 + n, -((b*x)/a)])/(n*(a + b*x)^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1+n}(bx+a)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/((b*x+a)^n),x)
[Out] int(x^(-1+n)/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="maxima")
[Out] integrate(x^(n - 1)/(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] integral(x^(n - 1)/(b*x + a)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 185.55, size = 27, normalized size = 0.69

$$\frac{a^{-n}x^n\Gamma(n)\,{}_2F_1\left(\begin{matrix} n,n \\ n+1 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/((b*x+a)**n),x)
```

```
[Out] x**n*gamma(n)*hyper((n, n), (n + 1,), b*x*exp_polar(I*pi)/a)/(a**n*gamma(n
+ 1))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 1)/(b*x + a)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{n-1}}{{\left(a+b\,x\right)}^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x)^n,x)
```

```
[Out] int(x^(n - 1)/(a + b*x)^n, x)
```

3.743 $\int x^n(a+bx)^{-n}\,dx$

Optimal. Leaf size=45

$$\frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}$$

```
[Out] x^(1+n)*(1+b*x/a)^n*hypergeom([n, 1+n],[2+n],-b*x/a)/(1+n)/((b*x+a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{x^{n+1}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{bx}{a}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n/(a + b*x)^n,x]
```

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^n(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n\right)\int x^n\left(1+\frac{bx}{a}\right)^{-n}dx\\ &= \frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x^{1+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{bx}{a}\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n/(a + b*x)^n,x]
```

```
[Out] (x^(1 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 1 + n, 2 + n, -((b*x)/a)])/
((1 + n)*(a + b*x)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int x^{n}(bx+a)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/((b*x+a)^n),x)
```

```
[Out] int(x^n/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^n/(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral(x^n/(b*x + a)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.33, size = 32, normalized size = 0.71

$$\frac{a^{-n}xx^{n}\Gamma(n+1)\,{}_2F_1\left(\begin{matrix} n,n+1 \\ n+2 \end{matrix}\middle| \frac{bxe^{i\pi}}{a}\right)}{\Gamma(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n/((b*x+a)**n),x)
```

```
[Out] x*x**n*gamma(n + 1)*hyper((n, n + 1), (n + 2,), b*x*exp_polar(I*pi)/a)/(a**
n*gamma(n + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^n/(b*x + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^n}{{\left(a+b\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n/(a + b*x)^n,x)
```

```
[Out] int(x^n/(a + b*x)^n, x)
```

3.744 $\int x^{1+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=45

$$\frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\,{}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}$$

[Out] x^(2+n)*(1+b*x/a)^n*hypergeom([n, 2+n],[3+n],-b*x/a)/(2+n)/((b*x+a)^n)

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {68, 66}

$$\frac{x^{n+2}(a+bx)^{-n}\left(\frac{bx}{a}+1\right)^{n}\,{}_2F_1\left(n,n+2;n+3;-\frac{bx}{a}\right)}{n+2}$$

Antiderivative was successfully verified.

[In] Int[x^(1 + n)/(a + b*x)^n,x]

[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/((2 + n)*(a + b*x)^n)

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^{1+n}(a+bx)^{-n}\,dx &= \left((a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\right)\int x^{1+n}\left(1+\frac{bx}{a}\right)^{-n}dx\\ &= \frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}\,{}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{x^{2+n}(a+bx)^{-n}\left(1+\frac{bx}{a}\right)^{n}{}_2F_1\left(n,2+n;3+n;-\frac{bx}{a}\right)}{2+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + n)/(a + b*x)^n,x]
[Out] (x^(2 + n)*(1 + (b*x)/a)^n*Hypergeometric2F1[n, 2 + n, 3 + n, -((b*x)/a)])/
((2 + n)*(a + b*x)^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{1+n}(bx+a)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+n)/((b*x+a)^n),x)
[Out] int(x^(1+n)/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="maxima")
[Out] integrate(x^(n + 1)/(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] integral(x^(n + 1)/(b*x + a)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 108.86, size = 34, normalized size = 0.76

$$\frac{a^{-n}x^2x^n\Gamma(n+2)\,{}_2F_1\left(\begin{matrix}n,n+2\\n+3\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\Gamma(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+n)/((b*x+a)**n),x)
```

```
[Out] x**2*x**n*gamma(n + 2)*hyper((n, n + 2), (n + 3,), b*x*exp_polar(I*pi)/a)/(
a**n*gamma(n + 3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n + 1)/(b*x + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{n+1}}{(a+b\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n + 1)/(a + b*x)^n,x)
```

```
[Out] int(x^(n + 1)/(a + b*x)^n, x)
```

3.745 $\int x^{3/2}(a+bx)^n\,dx$

Optimal. Leaf size=45

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

```
[Out] 2/5*x^(5/2)*(b*x+a)^n*hypergeom([5/2, -n],[7/2],-b*x/a)/((1+b*x/a)^n)
```

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^n,x]
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^n\,dx&=\left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{3/2}\left(1+\frac{bx}{a}\right)^n\,dx\\&=\frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 1.00

$$\frac{2}{5}x^{5/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{5}{2},-n;\frac{7}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^n,x]
[Out] (2*x^(5/2)*(a + b*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((b*x)/a)])/(5*(1 +
 (b*x)/a)^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^n,x)
[Out] int(x^(3/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="fricas")
[Out] integral((b*x + a)^n*x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 78.93, size = 27, normalized size = 0.60

$$\frac{2a^nx^{\frac{5}{2}}{}_2F_1\left(\begin{matrix}\frac{5}{2},-n\\ \frac{7}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**n,x)
```

```
[Out] 2*a**n*x**(5/2)*hyper((5/2, -n), (7/2,), b*x*exp_polar(I*pi)/a)/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{3/2}\,{\left(a+b\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^n,x)
```

```
[Out] int(x^(3/2)*(a + b*x)^n, x)
```

3.746 $\int \sqrt{x}\,(a+bx)^n\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

```
[Out] 2/3*x^(3/2)*(b*x+a)^n*hypergeom([3/2, -n],[5/2],-b*x/a)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^n,x]
```

```
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^n\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \sqrt{x}\left(1+\frac{bx}{a}\right)^n dx\\ &= \frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.00

$$\frac{2}{3}x^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(\frac{3}{2},-n;\frac{5}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^n,x]
[Out] (2*x^(3/2)*(a + b*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((b*x)/a)])/(3*(1 +
 (b*x)/a)^n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^n,x)
[Out] int(x^(1/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="fricas")
[Out] integral((b*x + a)^n*sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.58, size = 27, normalized size = 0.60

$$\frac{2a^n x^{\frac{3}{2}}{}_2F_1\left(\begin{matrix}\frac{3}{2},-n\\ \frac{5}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**n,x)
```

```
[Out] 2*a**n*x**(3/2)*hyper((3/2, -n), (5/2,), b*x*exp_polar(I*pi)/a)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x}\,\left(a+b\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^n,x)
```

```
[Out] int(x^(1/2)*(a + b*x)^n, x)
```

3.747 $\int \frac{(a+bx)^n}{\sqrt{x}}\,dx$

Optimal. Leaf size=43

$$2\sqrt{x}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

```
[Out] 2*(b*x+a)^n*hypergeom([1/2, -n],[3/2],-b*x/a)*x^(1/2)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$2\sqrt{x}\,(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{\sqrt{x}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{\sqrt{x}}\,dx\\ &= 2\sqrt{x}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.00

$$2\sqrt{x}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{bx}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/Sqrt[x],x]
[Out] (2*Sqrt[x]*(a + b*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((b*x)/a)])/(1 + (b
*x)/a)^n
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^n}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(1/2),x)
[Out] int((b*x+a)^n/x^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/sqrt(x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="fricas")
[Out] integral((b*x + a)^n/sqrt(x), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.36, size = 26, normalized size = 0.60

$$2a^n\sqrt{x}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-n\\ \frac{3}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(1/2),x)
[Out] 2*a**n*sqrt(x)*hyper((1/2, -n), (3/2,), b*x*exp_polar(I*pi)/a)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(1/2),x)
[Out] int((a + b*x)^n/x^(1/2), x)
```

3.748 $\int \frac{(a+bx)^n}{x^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

```
[Out] -2*(b*x+a)^n*hypergeom([-1/2, -n],[1/2],-b*x/a)/((1+b*x/a)^n)/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^(3/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^{3/2}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{x^{3/2}}\,dx\\ &= -\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 43, normalized size = 1.00

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{1}{2},-n;\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^(3/2),x]
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((b*x)/a)])/(Sqrt[x]*(1 +
 (b*x)/a)^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(3/2),x)
[Out] int((b*x+a)^n/x^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/x^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="fricas")
[Out] integral((b*x + a)^n/x^(3/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.23, size = 29, normalized size = 0.67

$$-\frac{2a^n{}_2F_1\left(\begin{matrix}-\frac{1}{2},-n\\ \frac{1}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(3/2),x)
```

```
[Out] -2*a**n*hyper((-1/2, -n), (1/2,), b*x*exp_polar(I*pi)/a)/sqrt(x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(3/2),x)
```

```
[Out] int((a + b*x)^n/x^(3/2), x)
```

3.749 $\int \frac{(a+bx)^n}{x^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

```
[Out] -2/3*(b*x+a)^n*hypergeom([-3/2, -n],[-1/2],-b*x/a)/x^(3/2)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$-\frac{2(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^{5/2}}\,dx &= \left((a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int \frac{\left(1+\frac{bx}{a}\right)^n}{x^{5/2}}\,dx\\ &= -\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 1.00

$$-\frac{2(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-\frac{3}{2},-n;-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/x^(5/2),x]
[Out] (-2*(a + b*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((b*x)/a)])/(3*x^(3/2)*(
1 + (b*x)/a)^n)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^(5/2),x)
[Out] int((b*x+a)^n/x^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/x^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="fricas")
[Out] integral((b*x + a)^n/x^(5/2), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 186.75, size = 32, normalized size = 0.71

$$-\frac{2a^n{}_2F_1\left(\begin{matrix}-\frac{3}{2},-n\\-\frac{1}{2}\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**(5/2),x)
```

```
[Out] -2*a**n*hyper((-3/2, -n), (-1/2,), b*x*exp_polar(I*pi)/a)/(3*x**(3/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/x^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(5/2),x)
```

```
[Out] int((a + b*x)^n/x^(5/2), x)
```

3.750 $\int (bx)^m(2+dx)^n\,dx$

Optimal. Leaf size=35

$$\frac{2^n(bx)^{1+m}\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{b(1+m)}$$

```
[Out] 2^n*(b*x)^(1+m)*hypergeom([-n, 1+m],[2+m],-1/2*d*x)/b/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {66}

$$\frac{2^n(bx)^{m+1}\,{}_2F_1\left(m+1,-n;m+2;-\frac{dx}{2}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(2 + d*x)^n,x]
```

```
[Out] (2^n*(b*x)^(1 + m)*Hypergeometric2F1[1 + m, -n, 2 + m, -1/2*(d*x)])/(b*(1 +
 m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rubi steps

$$\int (bx)^m(2+dx)^n\,dx=\frac{2^n(bx)^{1+m}\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{b(1+m)}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 0.89

$$\frac{2^n x(bx)^m\,{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{2}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(2 + d*x)^n,x]
```

[Out] (2^n*x*(b*x)^m*Hypergeometric2F1[1 + m, -n, 2 + m, -1/2*(d*x)])/(1 + m)

Maple [A]
time = 0.13, size = 32, normalized size = 0.91

method	result	size
meijerg	$\frac{2^{n} (b x)^{m} x \,\mathrm{hypergeom}\left(\left[-n , 1+m \right], \left[2+m \right], -\frac{d x}{2}\right)}{1+m}$	32

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x)^m*(d*x+2)^n,x,method=_RETURNVERBOSE)

[Out] 2^n*(b*x)^m/(1+m)*x*hypergeom([-n,1+m],[2+m],-1/2*d*x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="maxima")

[Out] integrate((b*x)^m*(d*x + 2)^n, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="fricas")

[Out] integral((b*x)^m*(d*x + 2)^n, x)

Sympy [C] Result contains complex when optimal does not.
time = 1.14, size = 37, normalized size = 1.06

$$\frac{2^{n} b^{m} x x^{m} \Gamma\left(m + 1\right) {{}_{2}F_{1}\left(\begin{matrix} - n, m + 1 \\ m + 2 \end{matrix}\middle| {\frac{d x e^{i \pi}}{2}} \right)}}{\Gamma\left(m + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)**m*(d*x+2)**n,x)

[Out] 2**n*b**m*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), d*x*exp_polar(I*pi)/2)/gamma(m + 2)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+2)^n,x, algorithm="giac")
[Out] integrate((b*x)^m*(d*x + 2)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int (b\,x)^m\,(d\,x+2)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(d*x + 2)^n,x)
[Out] int((b*x)^m*(d*x + 2)^n, x)
```

3.751 $\int (bx)^m(c-bcx)^n\,dx$

Optimal. Leaf size=40

$$-\frac{(c-bcx)^{1+n}\,{}_2F_1(-m,1+n;2+n;1-bx)}{bc(1+n)}$$

```
[Out] -(-b*c*x+c)^(1+n)*hypergeom([-m, 1+n],[2+n],-b*x+1)/b/c/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {67}

$$-\frac{(c-bcx)^{n+1}\,{}_2F_1(-m,n+1;n+2;1-bx)}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(c - b*c*x)^n,x]
```

```
[Out] -(((c - b*c*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, 1 - b*x])/(b*c*(
1 + n)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int (bx)^m(c-bcx)^n\,dx = -\frac{(c-bcx)^{1+n}\,{}_2F_1(-m,1+n;2+n;1-bx)}{bc(1+n)}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 1.10

$$\frac{x(bx)^m(1-bx)^{-n}(c-bcx)^n\,{}_2F_1(1+m,-n;2+m;bx)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(c - b*c*x)^n,x]
```

```
[Out] (x*(b*x)^m*(c - b*c*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, b*x])/((1 + m)
*(1 - b*x)^n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx)^m (-bcx + c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(-b*c*x+c)^n,x)
[Out] int((b*x)^m*(-b*c*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="maxima")
[Out] integrate((-b*c*x + c)^n*(b*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="fricas")
[Out] integral((-b*c*x + c)^n*(b*x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.14, size = 37, normalized size = 0.92

$$\frac{b^m c^n x x^m \Gamma(m+1) \, {}_2F_1\left(\begin{matrix} -n, m+1 \\ m+2 \end{matrix} \middle| bxe^{2i\pi}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(-b*c*x+c)**n,x)
[Out] b**m*c**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), b*x*exp_polar(2*
I*pi))/gamma(m + 2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(-b*c*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((-b*c*x + c)^n*(b*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,x)^m\,(c-b\,c\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(c - b*c*x)^n,x)
```

```
[Out] int((b*x)^m*(c - b*c*x)^n, x)
```

3.752 $\int (bx)^m (c+dx)^n \, dx$

Optimal. Leaf size=52

$$\frac{(bx)^{1+m}(c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{b(1+m)}$$

```
[Out] (b*x)^(1+m)*(d*x+c)^n*hypergeom([-n, 1+m],[2+m],-d*x/c)/b/(1+m)/((1+d*x/c)^
n)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {68, 66}

$$\frac{(bx)^{m+1}(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} {}_2F_1\left(m+1,-n;m+2;-\frac{dx}{c}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])
/(b*(1 + m)*(1 + (d*x)/c)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (bx)^m (c+dx)^n \, dx &= \left((c+dx)^n \left(1+\frac{dx}{c}\right)^{-n}\right) \int (bx)^m \left(1+\frac{dx}{c}\right)^n \, dx\\ &= \frac{(bx)^{1+m}(c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{b(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.92

$$\frac{x(bx)^m(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}{}_2F_1\left(1+m,-n;2+m;-\frac{dx}{c}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^m*(c + d*x)^n,x]
```

```
[Out] (x*(b*x)^m*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*x)/c)])/((1
 + m)*(1 + (d*x)/c)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (bx)^m (dx+c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(d*x+c)^n,x)
```

```
[Out] int((b*x)^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x)^m*(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*(d*x + c)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.43, size = 37, normalized size = 0.71

$$\frac{b^m c^n x x^m \Gamma(m+1)\,{}_2F_1\left(\begin{matrix}-n, m+1\\ m+2\end{matrix}\middle|\frac{dxe^{i\pi}}{c}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)**m*(d*x+c)**n,x)
```

```
[Out] b**m*c**n*x*x**m*gamma(m + 1)*hyper((-n, m + 1), (m + 2,), d*x*exp_polar(I*
pi)/c)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*(d*x + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,x)^m\,{(c+d\,x)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((b*x)^m*(c + d*x)^n, x)
```

3.753 $\int x^{-1+n}(a+bx)^{-1-n}\,dx$

Optimal. Leaf size=19

$$\frac{x^n(a+bx)^{-n}}{an}$$

```
[Out] x^n/a/n/((b*x+a)^n)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx = \frac{x^n(a+bx)^{-n}}{an}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.00

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Maple [A]
time = 0.11, size = 20, normalized size = 1.05

method	result	size
gosper	$\frac{x^n(bx+a)^{-n}}{an}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b*x+a)^(-1-n),x,method=_RETURNVERBOSE)
```

```
[Out] x^n*(b*x+a)^(-n)/a/n
```

Maxima [A]
time = 0.27, size = 22, normalized size = 1.16

$$\frac{e^{(-n\log(bx+a)+n\log(x))}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="maxima")
```

```
[Out] e^(-n*log(b*x + a) + n*log(x))/(a*n)
```

Fricas [A]
time = 0.98, size = 32, normalized size = 1.68

$$\frac{(bx^2+ax)(bx+a)^{-n-1}x^{n-1}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-n - 1)*x^(n - 1)/(a*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(12) = 24$.
time = 208.88, size = 197, normalized size = 10.37

$$\begin{cases} -\frac{x^n(bx)^{-n}}{bx} & \text{for } a=0 \\ \frac{0^{-n-1}x^n}{n} & \text{for } a=-bx \\ \frac{x^n\left(0^{\frac{1}{n}}\right)^{-n-1}}{n} & \text{for } a=0^{\frac{1}{n}}-bx \\ \frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for } n=0 \\ \frac{a^2x^n}{a^3n(a+bx)^n+2a^2bnx(a+bx)^n+ab^2nx^2(a+bx)^n}+\frac{abxx^n}{a^3n(a+bx)^n+2a^2bnx(a+bx)^n+ab^2nx^2(a+bx)^n}+\frac{bxx^n}{a^2n(a+bx)^n+abnx(a+bx)^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b*x+a)**(-1-n),x)
```

```
[Out] Piecewise((-x**n/(b*x*(b*x)**n), Eq(a, 0)), (0**(-n - 1)*x**n/n, Eq(a, -b*x
)), (x**n*(0**(1/n))**(-n - 1)/n, Eq(a, 0**(1/n) - b*x)), (log(x)/a - log(a
/b + x)/a, Eq(n, 0)), (a**2*x**n/(a**3*n*(a + b*x)**n + 2*a**2*b*n*x*(a + b
*x)**n + a*b**2*n*x**2*(a + b*x)**n) + a*b*x*x**n/(a**3*n*(a + b*x)**n + 2*
a**2*b*n*x*(a + b*x)**n + a*b**2*n*x**2*(a + b*x)**n) + b*x*x**n/(a**2*n*(a
 + b*x)**n + a*b*n*x*(a + b*x)**n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 1)*x^(n - 1), x)
```

Mupad [B]
time = 0.50, size = 19, normalized size = 1.00

$$\frac{x^n}{a\,n\,{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x)^(n + 1),x)
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

3.754 $\int x^{-3-n}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

```
[Out] -x^(-2-n)*(b*x+a)^(1+n)/a/(2+n)+b*x^(-1-n)*(b*x+a)^(1+n)/a^2/(1+n)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3-n}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.69

$$-\frac{x^{-2-n}(a+an-bx)(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - n)*(a + b*x)^n,x]
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^{-2-n}(an-bx+a)}{(2+n)(1+n)a^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x^(-2-n)*(a*n-b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Fricas [A]
time = 0.91, size = 64, normalized size = 1.10

$$-\frac{(abnx^2 - b^2x^3 + (a^2n + a^2)x)(bx+a)^n x^{-n-3}}{a^2n^2 + 3\,a^2n + 2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(46) = 92$.

time = 16.28, size = 328, normalized size = 5.66

$$\begin{cases} -\frac{x^{-n}(bx)^n}{2x^2} & \text{for } a = 0 \\ \frac{a\log(x)}{a^3+a^2bx} - \frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} + \frac{a}{a^3+a^2bx} + \frac{bx\log(x)}{a^3+a^2bx} - \frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for } n = -2 \\ -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for } n = -1 \\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} + \frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-(b*x)**n/(2*x**2*x**n), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x)
- a*log(a/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3
 + a**2*b*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) -
b*log(x)/a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**
2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)*
*n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*
(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n)
+ b**2*x**2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2
*x**2*x**n), True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Mupad [B]

time = 0.50, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\left(\frac{x\,(n+1)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

3.755 $\int x^{2n-3(1+n)}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

```
[Out] -x^(-2-n)*(b*x+a)^(1+n)/a/(2+n)+b*x^(-1-n)*(b*x+a)^(1+n)/a^2/(n^2+3*n+2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{2n-3(1+n)}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 0.69

$$-\frac{x^{-2-n}(a+an-bx)(a+bx)^{1+n}}{a^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

Maple [A]
time = 0.12, size = 41, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^{-2-n}(an-bx+a)}{(2+n)(1+n)a^2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x^(-2-n)*(a*n-b*x+a)/(2+n)/(1+n)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Fricas [A]
time = 1.02, size = 64, normalized size = 1.10

$$-\frac{(abnx^2-b^2x^3+(a^2n+a^2)x)(bx+a)^n x^{-n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(48) = 96$.
time = 16.75, size = 328, normalized size = 5.66

$$\begin{cases} -\frac{x^{-n}(bx)^n}{2x^2} & \text{for } a = 0 \\ \frac{a\log(x)}{a^3+a^2bx} - \frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} + \frac{a}{a^3+a^2bx} + \frac{bx\log(x)}{a^3+a^2bx} - \frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for } n = -2 \\ -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for } n = -1 \\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} + \frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-(b*x)**n/(2*x**2*x**n), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x)
- a*log(a/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3
 + a**2*b*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) -
b*log(x)/a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**
2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)*
*n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*
(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n)
+ b**2*x**2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2
*x**2*x**n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

Mupad [B]
time = 0.00, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\left(\frac{x\,(n+1)}{x^{n+3}\,(n^2+3\,n+2)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,(n^2+3\,n+2)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,(n^2+3\,n+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

3.756 $\int x^3 \sqrt{cx^2} (a + bx) \, dx$

Optimal. Leaf size=35

$$\frac{1}{5} a x^4 \sqrt{cx^2} + \frac{1}{6} b x^5 \sqrt{cx^2}$$

```
[Out] 1/5*a*x^4*(c*x^2)^(1/2)+1/6*b*x^5*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{5} a x^4 \sqrt{cx^2} + \frac{1}{6} b x^5 \sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^4*Sqrt[c*x^2])/5 + (b*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{cx^2} (a + bx) \, dx &= \frac{\sqrt{cx^2} \int x^4 (a + bx) \, dx}{x} \\
&= \frac{\sqrt{cx^2} \int \left(a x^4 + b x^5\right) \, dx}{x} \\
&= \frac{1}{5} a x^4 \sqrt{cx^2} + \frac{1}{6} b x^5 \sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}\,(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^4(5bx+6a)\sqrt{c\,x^2}}{30}$	21
default	$\frac{x^4(5bx+6a)\sqrt{c\,x^2}}{30}$	21
risch	$\frac{a\,x^4\sqrt{c\,x^2}}{5}+\frac{b\,x^5\sqrt{c\,x^2}}{6}$	28
trager	$\frac{(5b\,x^5+6a\,x^4+5b\,x^4+6a\,x^3+5b\,x^3+6a\,x^2+5x^2b+6ax+5bx+6a+5b)(-1+x)\sqrt{c\,x^2}}{30x}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*x^4*(5*b*x+6*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.94

$$\frac{(cx^2)^{\frac{3}{2}}\,bx^3}{6\,c}+\frac{(cx^2)^{\frac{3}{2}}\,ax^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(3/2)*b*x^3/c + 1/5*(c*x^2)^(3/2)*a*x^2/c
```

Fricas [A]
time = 0.94, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^5+6\,ax^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*x^5 + 6*a*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.11, size = 29, normalized size = 0.83

$$\frac{ax^4\sqrt{cx^2}}{5}+\frac{bx^5\sqrt{cx^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*x**4*sqrt(c*x**2)/5 + b*x**5*sqrt(c*x**2)/6
```

Giac [A]
time = 2.26, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x)+6\,ax^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x), x)
```

3.757 $\int x^2 \sqrt{cx^2} (a+bx) \, dx$

Optimal. Leaf size=35

$$\frac{1}{4}ax^3\sqrt{cx^2} + \frac{1}{5}bx^4\sqrt{cx^2}$$

```
[Out] 1/4*a*x^3*(c*x^2)^(1/2)+1/5*b*x^4*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{4}ax^3\sqrt{cx^2} + \frac{1}{5}bx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^3*Sqrt[c*x^2])/4 + (b*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int x^2\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\,\int x^3(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\int\left(ax^3+bx^4\right)\,dx}{x}\\ &= \frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}\,(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^3(4bx+5a)\sqrt{c\,x^2}}{20}$	21
default	$\frac{x^3(4bx+5a)\sqrt{c\,x^2}}{20}$	21
risch	$\frac{a\,x^3\sqrt{c\,x^2}}{4}+\frac{b\,x^4\sqrt{c\,x^2}}{5}$	28
trager	$\frac{(4b\,x^4+5a\,x^3+4b\,x^3+5a\,x^2+4x^2b+5ax+4bx+5a+4b)(-1+x)\sqrt{c\,x^2}}{20x}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^3*(4*b*x+5*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.33, size = 31, normalized size = 0.89

$$\frac{(cx^2)^{\frac{3}{2}}\,bx^2}{5\,c}+\frac{(cx^2)^{\frac{3}{2}}\,ax}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(3/2)*b*x^2/c + 1/4*(c*x^2)^(3/2)*a*x/c
```

Fricas [A]
time = 0.83, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^4+5\,ax^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*x^4 + 5*a*x^3)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.10, size = 29, normalized size = 0.83

$$\frac{ax^3\sqrt{cx^2}}{4}+\frac{bx^4\sqrt{cx^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*x**3*sqrt(c*x**2)/4 + b*x**4*sqrt(c*x**2)/5
```

Giac [**A**]
time = 1.09, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x)+5\,ax^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x), x)
```

3.758 $\int x\sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

```
[Out] 1/3*a*x^2*(c*x^2)^(1/2)+1/4*b*x^3*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 45}

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^2*Sqrt[c*x^2])/3 + (b*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x^2(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(ax^2+bx^3\right)\,dx}{x}\\ &= \frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^2(3bx+4a)\sqrt{c\,x^2}}{12}$	21
default	$\frac{x^2(3bx+4a)\sqrt{c\,x^2}}{12}$	21
risch	$\frac{a\,x^2\sqrt{c\,x^2}}{3}+\frac{b\,x^3\sqrt{c\,x^2}}{4}$	28
trager	$\frac{(3b\,x^3+4a\,x^2+3x^2b+4ax+3bx+4a+3b)(-1+x)\sqrt{c\,x^2}}{12x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x^2*(3*b*x+4*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.80

$$\frac{(cx^2)^{\frac{3}{2}}\,bx}{4\,c}+\frac{(cx^2)^{\frac{3}{2}}\,a}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*b*x/c + 1/3*(c*x^2)^(3/2)*a/c
```

Fricas [A]
time = 0.84, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^3+4\,ax^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.08, size = 29, normalized size = 0.83

$$\frac{ax^2\sqrt{cx^2}}{3}+\frac{bx^3\sqrt{cx^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x**2)**(1/2),x)
[Out] a*x**2*sqrt(c*x**2)/3 + b*x**3*sqrt(c*x**2)/4
```

Giac [A]
time = 3.13, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x)+4\,ax^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x),x)
[Out] int(x*(c*x^2)^(1/2)*(a + b*x), x)
```

3.759 $\int \sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

```
[Out] 1/2*a*x*(c*x^2)^(1/2)+1/3*b*x^2*(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x*Sqrt[c*x^2])/2 + (b*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.58

method	result	size
gosper	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
default	$\frac{x(2bx+3a)\sqrt{c\,x^2}}{6}$	19
risch	$\frac{ax\sqrt{c\,x^2}}{2}+\frac{b\,x^2\sqrt{c\,x^2}}{3}$	26
trager	$\frac{(2x^2b+3ax+2bx+3a+2b)(-1+x)\sqrt{c\,x^2}}{6x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(2*b*x+3*a)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.32, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{cx^2}\,ax+\frac{(cx^2)^{\frac{3}{2}}b}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2)*a*x + 1/3*(c*x^2)^(3/2)*b/c
```

Fricas [A]
time = 1.63, size = 20, normalized size = 0.61

$$\frac{1}{6}\left(2\,bx^2+3\,ax\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.07, size = 27, normalized size = 0.82

$$\frac{ax\sqrt{cx^2}}{2} + \frac{bx^2\sqrt{cx^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2),x)
```

```
[Out] a*x*sqrt(c*x**2)/2 + b*x**2*sqrt(c*x**2)/3
```

Giac [A]
time = 1.27, size = 22, normalized size = 0.67

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x) + 3\,ax^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.54, size = 20, normalized size = 0.61

$$\frac{\sqrt{c}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] (c^(1/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

3.760 $\int \frac{\sqrt{cx^2}(a+bx)}{x} dx$

Optimal. Leaf size=27

$$a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}$$

```
[Out] a*(c*x^2)^(1/2)+1/2*b*x*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {15}

$$a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] a*Sqrt[c*x^2] + (b*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x} dx &= \frac{\sqrt{cx^2}\int (a+bx)\,dx}{x}\\ &= a\sqrt{cx^2}+\frac{1}{2}bx\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.89

$$\frac{cx^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

[Out] (c*x^2*(2*a + b*x))/(2*Sqrt[c*x^2])

Maple [A]
time = 0.02, size = 17, normalized size = 0.63

method	result	size
gosper	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
default	$\frac{(bx+2a)\sqrt{c\,x^2}}{2}$	17
risch	$a\sqrt{c\,x^2}+\frac{bx\sqrt{c\,x^2}}{2}$	22
trager	$\frac{(bx+2a+b)(-1+x)\sqrt{c\,x^2}}{2x}$	24

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(c*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*(b*x+2*a)*(c*x^2)^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 0.89, size = 16, normalized size = 0.59

$$\frac{1}{2}\sqrt{cx^2}\,(bx+2\,a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="fricas")

[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)

Sympy [A]
time = 0.07, size = 22, normalized size = 0.81

$$a\sqrt{cx^2}+\frac{bx\sqrt{cx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x,x)
```

```
[Out] a*sqrt(c*x**2) + b*x*sqrt(c*x**2)/2
```

Giac [A]
time = 1.90, size = 17, normalized size = 0.63

$$\frac{1}{2}\left(bx^2+2\,ax\right)\sqrt{c}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sqrt(c)*sgn(x)
```

Mupad [B]
time = 0.19, size = 14, normalized size = 0.52

$$\frac{\sqrt{c}\,\left|x\right|\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x,x)
```

```
[Out] (c^(1/2)*abs(x)*(2*a + b*x))/2
```

3.761 $\int \frac{\sqrt{cx^2}(a+bx)}{x^2} dx$

Optimal. Leaf size=28

$$b\sqrt{cx^2} + \frac{a\sqrt{cx^2}\log(x)}{x}$$

```
[Out] b*(c*x^2)^(1/2)+a*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{a\sqrt{cx^2}\log(x)}{x} + b\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] b*Sqrt[c*x^2] + (a*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)}{x^2} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(b+\frac{a}{x}\right) dx}{x} \\
&= b\sqrt{cx^2} + \frac{a\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.71

$$\frac{cx(bx + a\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.71

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(bx+a\ln(x))}{x}$	20
risch	$b\sqrt{c\,x^2}+\frac{a\ln(x)\sqrt{c\,x^2}}{x}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)/x*(b*x+a*ln(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.09, size = 19, normalized size = 0.68

$$\frac{\sqrt{cx^2}\,(bx + a\log(x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**2, x)
```

Giac [**A**]
time = 1.87, size = 17, normalized size = 0.61

$$(bx\mathrm{sgn}(x) + a\log(|x|)\,\mathrm{sgn}(x))\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^2, x)
```

3.762 $\int \frac{\sqrt{cx^2}\,(a+bx)}{x^3}\,dx$

Optimal. Leaf size=32

$$-\frac{a\sqrt{cx^2}}{x^2}+\frac{b\sqrt{cx^2}\,\log(x)}{x}$$

```
[Out] -a*(c*x^2)^(1/2)/x^2+b*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{b\sqrt{cx^2}\,\log(x)}{x}-\frac{a\sqrt{cx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

```
[Out] -((a*Sqrt[c*x^2])/x^2) + (b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)}{x^3}\,dx &= \frac{\sqrt{cx^2}\,\int \frac{a+bx}{x^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{x}\\ &= -\frac{a\sqrt{cx^2}}{x^2}+\frac{b\sqrt{cx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.62

$$\frac{c(-a+bx\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
[Out] (c*(-a + b*x*Log[x]))/Sqrt[c*x^2]
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.66

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(bx\ln(x)-a)}{x^2}$	21
risch	$-\frac{a\sqrt{c\,x^2}}{x^2}+\frac{b\ln(x)\sqrt{c\,x^2}}{x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(1/2)*(b*x*ln(x)-a)/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.47, size = 20, normalized size = 0.62

$$\frac{\sqrt{cx^2}\,(bx\log(x)-a)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**3, x)
```

Giac [A]
time = 2.70, size = 20, normalized size = 0.62

$$\left(b\log\left(|x|\right)\operatorname{sgn}(x) - \frac{a\operatorname{sgn}(x)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b*log(abs(x))*sgn(x) - a*sgn(x)/x)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^3,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^3, x)
```

3.763 $\int \frac{\sqrt{cx^2}\,(a+bx)}{x^4}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{cx^2}\,(a+bx)^2}{2ax^3}$$

```
[Out] -1/2*(b*x+a)^2*(c*x^2)^(1/2)/x^3/a
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{\sqrt{cx^2}\,(a+bx)^2}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -1/2*(Sqrt[c*x^2]*(a + b*x)^2)/(a*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)}{x^4}\,dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^3}\,dx}{x}\\ &= -\frac{\sqrt{cx^2}\,(a+bx)^2}{2ax^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.85

$$-\frac{\sqrt{cx^2}\,(a+2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -1/2*(Sqrt[c*x^2]*(a + 2*b*x))/x^3
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.73

method	result	size
gosper	$-\frac{(2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	19
default	$-\frac{(2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	19
risch	$\frac{\left(-bx-\frac{a}{2}\right)\sqrt{c\,x^{2}}}{x^{3}}$	20
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^{2}}}{2x^{3}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b*x+a)*(c*x^2)^(1/2)/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.98, size = 18, normalized size = 0.69

$$-\frac{\sqrt{cx^{2}}\,(2\,bx+a)}{2\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/x^3
```

Sympy [A]
time = 0.17, size = 29, normalized size = 1.12

$$-\frac{a\sqrt{cx^2}}{2x^3}-\frac{b\sqrt{cx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**4,x)
[Out] -a*sqrt(c*x**2)/(2*x**3) - b*sqrt(c*x**2)/x**2
```

Giac [A]
time = 2.23, size = 19, normalized size = 0.73

$$-\frac{(2\,bx\mathrm{sgn}(x)+a\mathrm{sgn}(x))\sqrt{c}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/2*(2*b*x*sgn(x) + a*sgn(x))*sqrt(c)/x^2
```

Mupad [B]
time = 0.14, size = 28, normalized size = 1.08

$$-\frac{a\,\sqrt{c}\,x^2+2\,b\,\sqrt{c}\,x^3}{2\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^4,x)
[Out] -(a*c^(1/2)*x^2 + 2*b*c^(1/2)*x^3)/(2*x*(x^2)^(3/2))
```

3.764 $\int x^3(cx^2)^{3/2}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

```
[Out] 1/7*a*c*x^6*(c*x^2)^(1/2)+1/8*b*c*x^7*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^6*Sqrt[c*x^2])/7 + (b*c*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{3/2}(a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)\,dx}{x}\\ &= \frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.57

method	result	size
gosper	$\frac{x^{4}(7bx+8a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{56}$	21
default	$\frac{x^{4}(7bx+8a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{56}$	21
risch	$\frac{a c\,x^{6}\sqrt{c\,x^{2}}}{7}+\frac{b c\,x^{7}\sqrt{c\,x^{2}}}{8}$	30
trager	$\frac{c\left(7b\,x^{7}+8a\,x^{6}+7b\,x^{6}+8a\,x^{5}+7b\,x^{5}+8a\,x^{4}+7b\,x^{4}+8a\,x^{3}+7b\,x^{3}+8a\,x^{2}+7x^{2}b+8ax+7bx+8a+7b\right)(-1+x)\sqrt{c\,x^{2}}}{56x}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*x^4*(7*b*x+8*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.89

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^3}{8\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b*x^3/c + 1/7*(c*x^2)^(5/2)*a*x^2/c
```

Fricas [A]
time = 0.63, size = 24, normalized size = 0.65

$$\frac{1}{56}\left(7\,bcx^7+8\,acx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/56*(7*b*c*x^7 + 8*a*c*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.23, size = 29, normalized size = 0.78

$$\frac{ax^4(cx^2)^{\frac{3}{2}}}{7} + \frac{bx^5(cx^2)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*x**4*(c*x**2)**(3/2)/7 + b*x**5*(c*x**2)**(3/2)/8
```

Giac [A]
time = 1.60, size = 22, normalized size = 0.59

$$\frac{1}{56}\left(7\,bx^8\mathrm{sgn}(x) + 8\,ax^7\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\left(c\,x^2\right)^{3/2}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x), x)
```

3.765 $\int x^2(cx^2)^{3/2}\,(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

```
[Out] 1/6*a*c*x^5*(c*x^2)^(1/2)+1/7*b*c*x^6*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^5*Sqrt[c*x^2])/6 + (b*c*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^5+bx^6\right)\,dx}{x}\\&=\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.57

method	result	size
gosper	$\frac{x^{3}(6bx+7a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
default	$\frac{x^{3}(6bx+7a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{42}$	21
risch	$\frac{ac\,x^{5}\sqrt{c\,x^{2}}}{6}+\frac{bc\,x^{6}\sqrt{c\,x^{2}}}{7}$	30
trager	$\frac{c\left(6b\,x^{6}+7a\,x^{5}+6b\,x^{5}+7a\,x^{4}+6b\,x^{4}+7a\,x^{3}+6b\,x^{3}+7a\,x^{2}+6x^{2}b+7ax+6bx+7a+6b\right)(-1+x)\sqrt{c\,x^{2}}}{42x}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/42*x^3*(6*b*x+7*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.84

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/7*(c*x^2)^(5/2)*b*x^2/c + 1/6*(c*x^2)^(5/2)*a*x/c
```

Fricas [A]
time = 1.17, size = 24, normalized size = 0.65

$$\frac{1}{42}\left(6\,bcx^6+7\,acx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/42*(6*b*c*x^6 + 7*a*c*x^5)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.19, size = 29, normalized size = 0.78

$$\frac{ax^3(cx^2)^{\frac{3}{2}}}{6}+\frac{bx^4(cx^2)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*x**3*(c*x**2)**(3/2)/6 + b*x**4*(c*x**2)**(3/2)/7
```

Giac [**A**]
time = 1.96, size = 22, normalized size = 0.59

$$\frac{1}{42}\left(6\,bx^7\mathrm{sgn}(x)+7\,ax^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x), x)
```

3.766 $\int x(cx^2)^{3/2}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

```
[Out] 1/5*a*c*x^4*(c*x^2)^(1/2)+1/6*b*c*x^5*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {15, 45}

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^4*Sqrt[c*x^2])/5 + (b*c*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^4+bx^5\right)\,dx}{x}\\&=\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.57

method	result	size
gosper	$\frac{x^{2}(5bx+6a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{30}$	21
default	$\frac{x^{2}(5bx+6a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{30}$	21
risch	$\frac{ac\,x^{4}\sqrt{c\,x^{2}}}{5}+\frac{bc\,x^{5}\sqrt{c\,x^{2}}}{6}$	30
trager	$\frac{c\left(5b\,x^{5}+6a\,x^{4}+5b\,x^{4}+6a\,x^{3}+5b\,x^{3}+6a\,x^{2}+5x^{2}b+6ax+5bx+6a+5b\right)(-1+x)\sqrt{c\,x^{2}}}{30x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/30*x^2*(5*b*x+6*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.31, size = 28, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx}{6\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b*x/c + 1/5*(c*x^2)^(5/2)*a/c
```

Fricas [A]
time = 0.83, size = 24, normalized size = 0.65

$$\frac{1}{30}\left(5\,bcx^5+6\,acx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(5*b*c*x^5 + 6*a*c*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.16, size = 29, normalized size = 0.78

$$\frac{ax^2(cx^2)^{\frac{3}{2}}}{5}+\frac{bx^3(cx^2)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*x**2*(c*x**2)**(3/2)/5 + b*x**3*(c*x**2)**(3/2)/6
```

Giac [A]
time = 1.32, size = 22, normalized size = 0.59

$$\frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x)+6\,ax^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x\left(c\,x^2\right)^{3/2}(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(3/2)*(a + b*x), x)
```

3.767 $\int (cx^2)^{3/2} (a+bx) \, dx$

Optimal. Leaf size=37

$$\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}$$

```
[Out] 1/4*a*c*x^3*(c*x^2)^(1/2)+1/5*b*c*x^4*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^3*Sqrt[c*x^2])/4 + (b*c*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (cx^2)^{3/2} (a+bx) \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx) \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right) \, dx}{x} \\
&= \frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.59

$$\frac{1}{20}x\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.51

method	result	size
gosper	$\frac{x(4bx+5a)\left(c\,x^2\right)^{\frac{3}{2}}}{20}$	19
default	$\frac{x(4bx+5a)\left(c\,x^2\right)^{\frac{3}{2}}}{20}$	19
risch	$\frac{ac\,x^3\sqrt{c\,x^2}}{4}+\frac{bc\,x^4\sqrt{c\,x^2}}{5}$	30
trager	$\frac{c\left(4b\,x^4+5a\,x^3+4b\,x^3+5a\,x^2+4x^2b+5ax+4bx+5a+4b\right)(-1+x)\sqrt{c\,x^2}}{20x}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/20*x*(4*b*x+5*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.68

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}ax+\frac{\left(cx^2\right)^{\frac{5}{2}}b}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*a*x + 1/5*(c*x^2)^(5/2)*b/c
```

Fricas [A]
time = 0.73, size = 24, normalized size = 0.65

$$\frac{1}{20}\left(4\,bcx^4+5\,acx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/20*(4*b*c*x^4 + 5*a*c*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.14, size = 27, normalized size = 0.73

$$\frac{ax(cx^2)^{\frac{3}{2}}}{4} + \frac{bx^2(cx^2)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a),x)
[Out] a*x*(c*x**2)**(3/2)/4 + b*x**2*(c*x**2)**(3/2)/5
```

Giac [A]
time = 1.09, size = 22, normalized size = 0.59

$$\frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x) + 5\,ax^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x),x)
[Out] int((c*x^2)^(3/2)*(a + b*x), x)
```

3.768 $\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

[Out] 1/3*a*c*x^2*(c*x^2)^(1/2)+1/4*b*c*x^3*(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

[In] Int[((c*x^2)^(3/2)*(a + b*x))/x,x]

[Out] (a*c*x^2*Sqrt[c*x^2])/3 + (b*c*x^3*Sqrt[c*x^2])/4

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)\,dx}{x}\\ &= \frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.68

$$\frac{1}{12}cx^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]
[Out] (c*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.49

method	result	size
gosper	$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{3}{2}}}{12}$	18
default	$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{3}{2}}}{12}$	18
risch	$\frac{ac\,x^2\sqrt{c\,x^2}}{3}+\frac{bc\,x^3\sqrt{c\,x^2}}{4}$	30
trager	$\frac{c\left(3b\,x^3+4a\,x^2+3x^2b+4ax+3bx+4a+3b\right)(-1+x)\sqrt{c\,x^2}}{12x}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/12*(3*b*x+4*a)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.59

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}bx+\frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*b*x + 1/3*(c*x^2)^(3/2)*a
```

Fricas [A]
time = 1.24, size = 24, normalized size = 0.65

$$\frac{1}{12}\left(3\,bcx^3+4\,acx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="fricas")
[Out] 1/12*(3*b*c*x^3 + 4*a*c*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.14, size = 24, normalized size = 0.65

$$\frac{a(cx^2)^{\frac{3}{2}}}{3}+\frac{bx(cx^2)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x,x)
[Out] a*(c*x**2)**(3/2)/3 + b*x*(c*x**2)**(3/2)/4
```

Giac [A]
time = 0.96, size = 22, normalized size = 0.59

$$\frac{1}{12}\left(3\,bx^4\mathrm{sgn}(x)+4\,ax^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="giac")
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x, x)
```

3.769 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx$

Optimal. Leaf size=35

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

```
[Out] 1/2*a*c*x*(c*x^2)^(1/2)+1/3*b*c*x^2*(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c*x*Sqrt[c*x^2])/2 + (b*c*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int (ax+bx^2)\,dx}{x} \\
&= \frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.66

$$\frac{1}{6}cx\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{(2bx+3a)(c\,x^2)^{\frac{3}{2}}}{6x}$	21
default	$\frac{(2bx+3a)(c\,x^2)^{\frac{3}{2}}}{6x}$	21
risch	$\frac{acx\sqrt{c\,x^2}}{2}+\frac{bc\,x^2\sqrt{c\,x^2}}{3}$	28
trager	$\frac{c(2x^2b+3ax+2bx+3a+2b)(-1+x)\sqrt{c\,x^2}}{6x}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/x*(2*b*x+3*a)*(c*x^2)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.93, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bcx^2+3\,acx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="fricas")
[Out] 1/6*(2*b*c*x^2 + 3*a*c*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.14, size = 24, normalized size = 0.69

$$\frac{a(cx^2)^{\frac{3}{2}}}{2x} + \frac{b(cx^2)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**2,x)
[Out] a*(c*x**2)**(3/2)/(2*x) + b*(c*x**2)**(3/2)/3
```

Giac [A]
time = 1.03, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x) + 3\,ax^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="giac")
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(3/2)
```

Mupad [B]
time = 0.27, size = 20, normalized size = 0.57

$$\frac{c^{3/2}\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^2,x)
[Out] (c^(3/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

3.770 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx$

Optimal. Leaf size=29

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

```
[Out] a*c*(c*x^2)^(1/2)+1/2*b*c*x*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{15\}$

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] a*c*Sqrt[c*x^2] + (b*c*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)\, dx}{x}\\ &= ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.72

$$\frac{1}{2}c\sqrt{cx^2}\,(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

[Out] (c*Sqrt[c*x^2]*(2*a + b*x))/2

Maple [A]
time = 0.02, size = 20, normalized size = 0.69

method	result	size
gosper	$\frac{(bx+2a)(cx^2)^{\frac{3}{2}}}{2x^2}$	20
default	$\frac{(bx+2a)(cx^2)^{\frac{3}{2}}}{2x^2}$	20
risch	$ac\sqrt{cx^2} + \frac{bcx\sqrt{cx^2}}{2}$	24
trager	$\frac{c(bx+2a+b)(-1+x)\sqrt{cx^2}}{2x}$	25

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(3/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)

[Out] 1/2/x^2*(b*x+2*a)*(c*x^2)^(3/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 1.29, size = 18, normalized size = 0.62

$$\frac{1}{2}\left(bcx + 2\,ac\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="fricas")

[Out] 1/2*(b*c*x + 2*a*c)*sqrt(c*x^2)

Sympy [A]
time = 0.21, size = 26, normalized size = 0.90

$$\frac{a(cx^2)^{\frac{3}{2}}}{x^2} + \frac{b(cx^2)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**3,x)
[Out] a*(c*x**2)**(3/2)/x**2 + b*(c*x**2)**(3/2)/(2*x)
```

Giac [A]
time = 2.65, size = 17, normalized size = 0.59

$$\frac{1}{2}\left(bx^2+2\,ax\right)c^{\frac{3}{2}}\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)*sgn(x)
```

Mupad [B]
time = 0.22, size = 14, normalized size = 0.48

$$\frac{c^{3/2}\,|x|\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^3,x)
[Out] (c^(3/2)*abs(x)*(2*a + b*x))/2
```

3.771 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^4} dx$

Optimal. Leaf size=30

$$bc\sqrt{cx^2} + \frac{ac\sqrt{cx^2}\log(x)}{x}$$

```
[Out] b*c*(c*x^2)^(1/2)+a*c*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{ac\sqrt{cx^2}\log(x)}{x} + bc\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] b*c*Sqrt[c*x^2] + (a*c*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{a+bx}{x}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(b+\frac{a}{x}\right)\,dx}{x}\\
&= bc\sqrt{cx^2} + \frac{ac\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.70

$$\frac{(cx^2)^{3/2}(bx+a\log(x))}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x + a*Log[x]))/x^3
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.67

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(bx+a\ln(x))}{x^3}$	20
risch	$bc\sqrt{c\,x^2}+\frac{ac\ln(x)\sqrt{c\,x^2}}{x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)/x^3*(b*x+a*ln(x))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.05, size = 21, normalized size = 0.70

$$\frac{(bcx+ac\log(x))\sqrt{cx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c*log(x))*sqrt(c*x^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}(a+bx)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)/x**4, x)
```

Giac [A]
time = 1.91, size = 17, normalized size = 0.57

$$(bx\text{sgn}(x) + a\log(|x|)\,\text{sgn}(x))c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,x^2)^{3/2}\,(a+b\,x)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^4,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x^4, x)
```

3.772 $\int x^3(cx^2)^{5/2}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

```
[Out] 1/9*a*c^2*x^8*(c*x^2)^(1/2)+1/10*b*c^2*x^9*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^8*Sqrt[c*x^2])/9 + (b*c^2*x^9*Sqrt[c*x^2])/10
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^8(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^8+bx^9\right)\,dx}{x}\\&=\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result
gosper	$\frac{x^4(9bx+10a)(c\,x^2)^{\frac{5}{2}}}{90}$
default	$\frac{x^4(9bx+10a)(c\,x^2)^{\frac{5}{2}}}{90}$
risch	$\frac{a\,c^2x^8\sqrt{c\,x^2}}{9}+\frac{b\,c^2x^9\sqrt{c\,x^2}}{10}$
trager	$\frac{c^2\left(9b\,x^9+10a\,x^8+9b\,x^8+10a\,x^7+9b\,x^7+10a\,x^6+9b\,x^6+10a\,x^5+9b\,x^5+10a\,x^4+9b\,x^4+10a\,x^3+9b\,x^3+10a\,x^2+9x^2b+10ax+9bx+10a\right.}{90x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/90*x^4*(9*b*x+10*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.80

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^3}{10\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}ax^2}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/10*(c*x^2)^(7/2)*b*x^3/c + 1/9*(c*x^2)^(7/2)*a*x^2/c
```

Fricas [A]
time = 1.39, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^9+10\,ac^2x^8\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/90*(9*b*c^2*x^9 + 10*a*c^2*x^8)*sqrt(c*x^2)
```

Sympy [A]
time = 0.42, size = 29, normalized size = 0.71

$$\frac{ax^4(cx^2)^{\frac{5}{2}}}{9} + \frac{bx^5(cx^2)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*x**4*(c*x**2)**(5/2)/9 + b*x**5*(c*x**2)**(5/2)/10
```

Giac [A]
time = 1.72, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^{10}\mathrm{sgn}(x) + 10\,ac^2x^9\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/90*(9*b*c^2*x^10*sgn(x) + 10*a*c^2*x^9*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int(x^3*(c*x^2)^(5/2)*(a + b*x), x)
```

3.773 $\int x^2(cx^2)^{5/2}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

```
[Out] 1/8*a*c^2*x^7*(c*x^2)^(1/2)+1/9*b*c^2*x^8*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^7*Sqrt[c*x^2])/8 + (b*c^2*x^8*Sqrt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^7(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^7+bx^8\right)\,dx}{x}\\&=\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	s
gosper	$\frac{x^{3}(8bx+9a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{72}$	2
default	$\frac{x^{3}(8bx+9a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{72}$	2
risch	$\frac{a\,c^{2}x^{7}\sqrt{c\,x^{2}}}{8}+\frac{b\,c^{2}x^{8}\sqrt{c\,x^{2}}}{9}$	3
trager	$\frac{c^{2}\left(8b\,x^{8}+9a\,x^{7}+8b\,x^{7}+9a\,x^{6}+8b\,x^{6}+9a\,x^{5}+8b\,x^{5}+9a\,x^{4}+8b\,x^{4}+9a\,x^{3}+8b\,x^{3}+9a\,x^{2}+8x^{2}b+9ax+8bx+9a+8b\right)(-1+x)\sqrt{c\,x^{2}}}{72x}$	[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72*x^3*(8*b*x+9*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^2}{9\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}ax}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(7/2)*b*x^2/c + 1/8*(c*x^2)^(7/2)*a*x/c
```

Fricas [A]
time = 0.98, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^8+9\,ac^2x^7\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/72*(8*b*c^2*x^8 + 9*a*c^2*x^7)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.36, size = 29, normalized size = 0.71

$$\frac{ax^3(cx^2)^{\frac{5}{2}}}{8} + \frac{bx^4(cx^2)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(5/2)*(b*x+a),x)
[Out] a*x**3*(c*x**2)**(5/2)/8 + b*x**4*(c*x**2)**(5/2)/9
```

Giac [**A**]
time = 1.35, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^9\mathrm{sgn}(x) + 9\,ac^2x^8\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/72*(8*b*c^2*x^9*sgn(x) + 9*a*c^2*x^8*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(a + b*x),x)
[Out] int(x^2*(c*x^2)^(5/2)*(a + b*x), x)
```

3.774 $\int x(cx^2)^{5/2}(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

```
[Out] 1/7*a*c^2*x^6*(c*x^2)^(1/2)+1/8*b*c^2*x^7*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {15, 45}

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^6*Sqrt[c*x^2])/7 + (b*c^2*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(cx^2)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)\,dx}{x}\\&=\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$\frac{x^{2}(7bx+8a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{56}$	21
default	$\frac{x^{2}(7bx+8a)\left(c\,x^{2}\right)^{\frac{5}{2}}}{56}$	21
risch	$\frac{a\,c^{2}x^{6}\sqrt{c\,x^{2}}}{7}+\frac{b\,c^{2}x^{7}\sqrt{c\,x^{2}}}{8}$	34
trager	$\frac{c^{2}\left(7b\,x^{7}+8a\,x^{6}+7b\,x^{6}+8a\,x^{5}+7b\,x^{5}+8a\,x^{4}+7b\,x^{4}+8a\,x^{3}+7b\,x^{3}+8a\,x^{2}+7x^{2}b+8ax+7bx+8a+7b\right)(-1+x)\sqrt{c\,x^{2}}}{56x}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*x^2*(7*b*x+8*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.68

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx}{8\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}a}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(7/2)*b*x/c + 1/7*(c*x^2)^(7/2)*a/c
```

Fricas [A]
time = 1.33, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^2x^7+8\,ac^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/56*(7*b*c^2*x^7 + 8*a*c^2*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.30, size = 29, normalized size = 0.71

$$\frac{ax^2(cx^2)^{\frac{5}{2}}}{7} + \frac{bx^3(cx^2)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*x**2*(c*x**2)**(5/2)/7 + b*x**3*(c*x**2)**(5/2)/8
```

Giac [A]
time = 2.10, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^2x^8\mathrm{sgn}(x) + 8\,ac^2x^7\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/56*(7*b*c^2*x^8*sgn(x) + 8*a*c^2*x^7*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{5/2}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x), x)
```

3.775 $\int (cx^2)^{5/2} (a + bx) \, dx$

Optimal. Leaf size=41

$$\frac{1}{6}ac^2x^5\sqrt{cx^2} + \frac{1}{7}bc^2x^6\sqrt{cx^2}$$

```
[Out] 1/6*a*c^2*x^5*(c*x^2)^(1/2)+1/7*b*c^2*x^6*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{1}{6}ac^2x^5\sqrt{cx^2} + \frac{1}{7}bc^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^5*Sqrt[c*x^2])/6 + (b*c^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (cx^2)^{5/2} (a + bx) \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x} \\
&= \frac{1}{6}ac^2x^5\sqrt{cx^2} + \frac{1}{7}bc^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

Maple [A]
time = 0.03, size = 19, normalized size = 0.46

method	result	size
gosper	$\frac{x(6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}}{42}$	19
default	$\frac{x(6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}}{42}$	19
risch	$\frac{a\,c^2x^5\sqrt{c\,x^2}}{6}+\frac{b\,c^2x^6\sqrt{c\,x^2}}{7}$	34
trager	$\frac{c^2\left(6b\,x^6+7a\,x^5+6b\,x^5+7a\,x^4+6b\,x^4+7a\,x^3+6b\,x^3+7a\,x^2+6x^2b+7ax+6bx+7a+6b\right)(-1+x)\sqrt{c\,x^2}}{42x}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/42*x*(6*b*x+7*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.61

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}ax+\frac{\left(cx^2\right)^{\frac{7}{2}}b}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*a*x + 1/7*(c*x^2)^(7/2)*b/c
```

Fricas [A]
time = 1.17, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^2x^6+7\,ac^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/42*(6*b*c^2*x^6 + 7*a*c^2*x^5)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.26, size = 27, normalized size = 0.66

$$\frac{ax(cx^2)^{\frac{5}{2}}}{6} + \frac{bx^2(cx^2)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a),x)
[Out] a*x*(c*x**2)**(5/2)/6 + b*x**2*(c*x**2)**(5/2)/7
```

Giac [**A**]
time = 2.44, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^2x^7\mathrm{sgn}(x) + 7\,ac^2x^6\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/42*(6*b*c^2*x^7*sgn(x) + 7*a*c^2*x^6*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x),x)
[Out] int((c*x^2)^(5/2)*(a + b*x), x)
```

3.776 $\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx$

Optimal. Leaf size=41

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

```
[Out] 1/5*a*c^2*x^4*(c*x^2)^(1/2)+1/6*b*c^2*x^5*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (a*c^2*x^4*Sqrt[c*x^2])/5 + (b*c^2*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^4+bx^5\right)\,dx}{x}\\ &= \frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.61

$$\frac{1}{30}cx^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]
[Out] (c*x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.44

method	result	size
gosper	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
default	$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$	18
risch	$\frac{a\,c^2x^4\sqrt{c\,x^2}}{5}+\frac{b\,c^2x^5\sqrt{c\,x^2}}{6}$	34
trager	$\frac{c^2\left(5b\,x^5+6a\,x^4+5b\,x^4+6a\,x^3+5b\,x^3+6a\,x^2+5x^2b+6ax+5bx+6a+5b\right)(-1+x)\sqrt{c\,x^2}}{30x}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x,x,method=_RETURNVERBOSE)
[Out] 1/30*(5*b*x+6*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.54

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}bx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b*x + 1/5*(c*x^2)^(5/2)*a
```

Fricas [A]
time = 1.27, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^5+6\,ac^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="fricas")
[Out] 1/30*(5*b*c^2*x^5 + 6*a*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.33, size = 24, normalized size = 0.59

$$\frac{a(cx^2)^{\frac{5}{2}}}{5} + \frac{bx(cx^2)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x,x)
[Out] a*(c*x**2)**(5/2)/5 + b*x*(c*x**2)**(5/2)/6
```

Giac [A]
time = 1.80, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^6\mathrm{sgn}(x) + 6\,ac^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="giac")
[Out] 1/30*(5*b*c^2*x^6*sgn(x) + 6*a*c^2*x^5*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x))/x, x)
```

3.777 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=41

$$\frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

```
[Out] 1/4*a*c^2*x^3*(c*x^2)^(1/2)+1/5*b*c^2*x^4*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c^2*x^3*Sqrt[c*x^2])/4 + (b*c^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right)\,dx}{x}\\ &= \frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.56

$$\frac{1}{20}cx\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
[Out] (c*x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.51

method	result	size
gosper	$\frac{(4bx+5a)\left(c\,x^2\right)^{\frac{5}{2}}}{20x}$	21
default	$\frac{(4bx+5a)\left(c\,x^2\right)^{\frac{5}{2}}}{20x}$	21
risch	$\frac{a\,c^2x^3\sqrt{c\,x^2}}{4}+\frac{b\,c^2x^4\sqrt{c\,x^2}}{5}$	34
trager	$\frac{c^2\left(4b\,x^4+5a\,x^3+4b\,x^3+5a\,x^2+4x^2b+5ax+4bx+5a+4b\right)(-1+x)\sqrt{c\,x^2}}{20x}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/20/x*(4*b*x+5*a)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.59

$$\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}b+\frac{\left(cx^2\right)^{\frac{5}{2}}a}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(5/2)*b + 1/4*(c*x^2)^(5/2)*a/x
```

Fricas [A]
time = 1.10, size = 28, normalized size = 0.68

$$\frac{1}{20}\left(4\,bc^2x^4+5\,ac^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="fricas")
[Out] 1/20*(4*b*c^2*x^4 + 5*a*c^2*x^3)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.27, size = 24, normalized size = 0.59

$$\frac{a(cx^2)^{\frac{5}{2}}}{4x} + \frac{b(cx^2)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**2,x)
[Out] a*(c*x**2)**(5/2)/(4*x) + b*(c*x**2)**(5/2)/5
```

Giac [**A**]
time = 1.02, size = 28, normalized size = 0.68

$$\frac{1}{20}\left(4\,bc^2x^5\mathrm{sgn}(x) + 5\,ac^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="giac")
[Out] 1/20*(4*b*c^2*x^5*sgn(x) + 5*a*c^2*x^4*sgn(x))*sqrt(c)
```

Mupad [**B**]
time = 0.28, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\left(4\,b\,\sqrt{x^{10}} + 5\,a\,x^3\,\sqrt{x^2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^2,x)
[Out] (c^(5/2)*(4*b*(x^10)^(1/2) + 5*a*x^3*(x^2)^(1/2)))/20
```

3.778 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

[Out] 1/3*a*c^2*x^2*(c*x^2)^(1/2)+1/4*b*c^2*x^3*(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^3,x]

[Out] (a*c^2*x^2*Sqrt[c*x^2])/3 + (b*c^2*x^3*Sqrt[c*x^2])/4

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^2+bx^3\right)dx}{x}\\&=\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.66

$$\frac{1}{12}c^2x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.51

method	result	size
gosper	$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{5}{2}}}{12x^2}$	21
default	$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{5}{2}}}{12x^2}$	21
risch	$\frac{a\,c^2x^2\sqrt{c\,x^2}}{3}+\frac{b\,c^2x^3\sqrt{c\,x^2}}{4}$	34
trager	$\frac{c^2\left(3b\,x^3+4a\,x^2+3x^2b+4ax+3bx+4a+3b\right)(-1+x)\sqrt{c\,x^2}}{12x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/x^2*(3*b*x+4*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.87, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^3+4\,ac^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="fricas")
[Out] 1/12*(3*b*c^2*x^3 + 4*a*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.33, size = 27, normalized size = 0.66

$$\frac{a(cx^2)^{\frac{5}{2}}}{3x^2} + \frac{b(cx^2)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**3,x)
[Out] a*(c*x**2)**(5/2)/(3*x**2) + b*(c*x**2)**(5/2)/(4*x)
```

Giac [A]
time = 1.76, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^4\mathrm{sgn}(x) + 4\,ac^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="giac")
[Out] 1/12*(3*b*c^2*x^4*sgn(x) + 4*a*c^2*x^3*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.27, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\left(4\,a\,\sqrt{x^6} + 3\,b\,x^3\,\sqrt{x^2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^3,x)
[Out] (c^(5/2)*(4*a*(x^6)^(1/2) + 3*b*x^3*(x^2)^(1/2)))/12
```

3.779 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx$

Optimal. Leaf size=39

$$\frac{1}{2}ac^2x\sqrt{cx^2} + \frac{1}{3}bc^2x^2\sqrt{cx^2}$$

```
[Out] 1/2*a*c^2*x*(c*x^2)^(1/2)+1/3*b*c^2*x^2*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{2}ac^2x\sqrt{cx^2} + \frac{1}{3}bc^2x^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (a*c^2*x*Sqrt[c*x^2])/2 + (b*c^2*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)}{x^4} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x} \\
&= \frac{1}{2}ac^2x\sqrt{cx^2} + \frac{1}{3}bc^2x^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.64

$$\frac{1}{6}c^2x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.54

method	result	size
gosper	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
default	$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$	21
risch	$\frac{a\,c^2x\sqrt{c\,x^2}}{2}+\frac{b\,c^2x^2\sqrt{c\,x^2}}{3}$	32
trager	$\frac{c^2\left(2x^2b+3ax+2bx+3a+2b\right)(-1+x)\sqrt{c\,x^2}}{6x}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/x^3*(2*b*x+3*a)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.02, size = 26, normalized size = 0.67

$$\frac{1}{6}\left(2\,bc^2x^2+3\,ac^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^2*x^2 + 3*a*c^2*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.34, size = 29, normalized size = 0.74

$$\frac{a(cx^2)^{\frac{5}{2}}}{2x^3} + \frac{b(cx^2)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**4,x)
```

```
[Out] a*(c*x**2)**(5/2)/(2*x**3) + b*(c*x**2)**(5/2)/(3*x**2)
```

Giac [A]
time = 1.92, size = 28, normalized size = 0.72

$$\frac{1}{6}\left(2\,bc^2x^3\mathrm{sgn}(x) + 3\,ac^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^2*x^3*sgn(x) + 3*a*c^2*x^2*sgn(x))*sqrt(c)
```

Mupad [B]
time = 0.26, size = 20, normalized size = 0.51

$$\frac{c^{5/2}\left(2\,b\,\sqrt{x^6} + 3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^4,x)
```

```
[Out] (c^(5/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

3.780 $\int \frac{x^3(a+bx)}{\sqrt{cx^2}} dx$

Optimal. Leaf size=35

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

```
[Out] 1/3*a*x^4/(c*x^2)^(1/2)+1/4*b*x^5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^4)/(3*Sqrt[c*x^2]) + (b*x^5)/(4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax^2+bx^3\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^4(4a+3bx)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(4*a + 3*b*x))/(12*Sqrt[c*x^2])
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^{4}\left(3bx+4a\right)}{12\sqrt{c\,x^{2}}}$	21
default	$\frac{x^{4}\left(3bx+4a\right)}{12\sqrt{c\,x^{2}}}$	21
risch	$\frac{a\,x^{4}}{3\sqrt{c\,x^{2}}}+\frac{b\,x^{5}}{4\sqrt{c\,x^{2}}}$	28
trager	$\frac{\left(3b\,x^{3}+4a\,x^{2}+3x^{2}b+4ax+3bx+4a+3b\right)\left(-1+x\right)\sqrt{c\,x^{2}}}{12cx}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^4*(3*b*x+4*a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.94

$$\frac{\sqrt{cx^2}\,bx^3}{4\,c}+\frac{\sqrt{cx^2}\,ax^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2)*b*x^3/c + 1/3*sqrt(c*x^2)*a*x^2/c
```

Fricas [A]
time = 0.99, size = 25, normalized size = 0.71

$$\frac{\left(3\,bx^3+4\,ax^2\right)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.21, size = 29, normalized size = 0.83

$$\frac{ax^4}{3\sqrt{cx^2}} + \frac{bx^5}{4\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**4/(3*sqrt(c*x**2)) + b*x**5/(4*sqrt(c*x**2))
```

Giac [A]
time = 2.67, size = 22, normalized size = 0.63

$$\frac{3\,bx^4 + 4\,ax^3}{12\,\sqrt{c}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*x^4 + 4*a*x^3)/(sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(1/2), x)
```

3.781 $\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

```
[Out] 1/2*a*x^3/(c*x^2)^(1/2)+1/3*b*x^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^3)/(2*Sqrt[c*x^2]) + (b*x^4)/(3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax+bx^2\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^3}{2\sqrt{cx^2}}+\frac{bx^4}{3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^3(3a+2bx)}{6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x^3*(3*a + 2*b*x))/(6*Sqrt[c*x^2])
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
gosper	$\frac{x^3(2bx+3a)}{6\sqrt{c\,x^2}}$	21
default	$\frac{x^3(2bx+3a)}{6\sqrt{c\,x^2}}$	21
risch	$\frac{a\,x^3}{2\sqrt{c\,x^2}}+\frac{b\,x^4}{3\sqrt{c\,x^2}}$	28
trager	$\frac{(2x^2b+3ax+2bx+3a+2b)(-1+x)\sqrt{c\,x^2}}{6cx}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*x^3*(2*b*x+3*a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.74

$$\frac{\sqrt{cx^2}\,bx^2}{3\,c}+\frac{ax^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(c*x^2)*b*x^2/c + 1/2*a*x^2/sqrt(c)
```

Fricas [A]
time = 0.86, size = 23, normalized size = 0.66

$$\frac{(2\,bx^2+3\,ax)\sqrt{cx^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.20, size = 29, normalized size = 0.83

$$\frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(1/2),x)
[Out] a*x**3/(2*sqrt(c*x**2)) + b*x**4/(3*sqrt(c*x**2))
```

Giac [A]
time = 2.24, size = 22, normalized size = 0.63

$$\frac{2\,bx^3 + 3\,ax^2}{6\,\sqrt{c}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/6*(2*b*x^3 + 3*a*x^2)/(sqrt(c)*sgn(x))
```

Mupad [B]
time = 0.25, size = 23, normalized size = 0.66

$$\frac{2\,b\,\sqrt{x^6} + 3\,a\,x\,\sqrt{x^2}}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(1/2),x)
[Out] (2*b*(x^6)^(1/2) + 3*a*x*(x^2)^(1/2))/(6*c^(1/2))
```

3.782 $\int \frac{x(a+bx)}{\sqrt{cx^2}} dx$

Optimal. Leaf size=32

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

[Out] a*x^2/(c*x^2)^(1/2)+1/2*b*x^3/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {15}

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*x))/Sqrt[c*x^2],x]

[Out] (a*x^2)/Sqrt[c*x^2] + (b*x^3)/(2*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{\sqrt{cx^2}} dx &= \frac{x\int (a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.72

$$\frac{x^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*x))/Sqrt[c*x^2],x]

```
[Out] (x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.62

method	result	size
gosper	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
default	$\frac{x^{2}(bx+2a)}{2\sqrt{c\,x^{2}}}$	20
trager	$\frac{(bx+2a+b)(-1+x)\sqrt{c\,x^{2}}}{2cx}$	27
risch	$\frac{a\,x^{2}}{\sqrt{c\,x^{2}}}+\frac{b\,x^{3}}{2\sqrt{c\,x^{2}}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*(b*x+2*a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.69

$$\frac{bx^2}{2\sqrt{c}} + \frac{\sqrt{cx^2}\,a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2/sqrt(c) + sqrt(c*x^2)*a/c
```

Fricas [A]
time = 1.26, size = 19, normalized size = 0.59

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c
```

Sympy [A]
time = 0.18, size = 27, normalized size = 0.84

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**2/sqrt(c*x**2) + b*x**3/(2*sqrt(c*x**2))
```

Giac [A]
time = 2.48, size = 19, normalized size = 0.59

$$\frac{bx^2 + 2\,ax}{2\,\sqrt{c}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/(sqrt(c)*sgn(x))
```

Mupad [B]
time = 0.22, size = 19, normalized size = 0.59

$$\frac{2\,a\,|x| + b\,x\,\sqrt{x^2}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (2*a*abs(x) + b*x*(x^2)^(1/2))/(2*c^(1/2))
```

3.783 $\int \frac{a+bx}{\sqrt{cx^2}} dx$

Optimal. Leaf size=29

$$\frac{bx^2}{\sqrt{cx^2}} + \frac{ax\log(x)}{\sqrt{cx^2}}$$

```
[Out] b*x^2/(c*x^2)^(1/2)+a*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$\frac{ax\log(x)}{\sqrt{cx^2}} + \frac{bx^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (b*x^2)/Sqrt[c*x^2] + (a*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{bx^2}{\sqrt{cx^2}} + \frac{ax\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.66

$$\frac{x(bx + a\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c*x^2],x]
[Out] (x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

Maple [A]
time = 0.03, size = 18, normalized size = 0.62

method	result	size
default	$\frac{x(bx+a\ln(x))}{\sqrt{c\,x^2}}$	18
risch	$\frac{b\,x^2}{\sqrt{c\,x^2}}+\frac{ax\ln(x)}{\sqrt{c\,x^2}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(c*x^2)^(1/2)*x*(b*x+a*ln(x))
```

Maxima [A]
time = 0.28, size = 20, normalized size = 0.69

$$\frac{a\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] a*log(x)/sqrt(c) + sqrt(c*x^2)*b/c
```

Fricas [A]
time = 1.08, size = 22, normalized size = 0.76

$$\frac{\sqrt{cx^2}\,(bx + a\log(x))}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)/sqrt(c*x**2), x)
```

Giac [A]
time = 2.02, size = 23, normalized size = 0.79

$$\frac{bx}{\sqrt{c}\,\mathrm{sgn}\,(x)}+\frac{a\log\left(|x|\right)}{\sqrt{c}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] b*x/(sqrt(c)*sgn(x)) + a*log(abs(x))/(sqrt(c)*sgn(x))
```

Mupad [B]
time = 0.51, size = 17, normalized size = 0.59

$$\frac{b\,|x|+a\,\ln\left(c\,x\right)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(1/2),x)
[Out] (b*abs(x) + a*log(c*x)*sign(x))/c^(1/2)
```

3.784 $\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}$$

[Out] -a/(c*x^2)^(1/2)+b*x*ln(x)/(c*x^2)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(x*Sqrt[c*x^2]),x]

[Out] -(a/Sqrt[c*x^2]) + (b*x*Log[x])/Sqrt[c*x^2]

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{\sqrt{cx^2}}\\ &= -\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.85

$$\frac{cx^2(-a+bx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*Sqrt[c*x^2]),x]
[Out] (c*x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.67

method	result	size
default	$\frac{bx\ln(x)-a}{\sqrt{c\,x^2}}$	18
risch	$-\frac{a}{\sqrt{c\,x^2}}+\frac{bx\ln(x)}{\sqrt{c\,x^2}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (b*x*ln(x)-a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.63

$$\frac{b\log(x)}{\sqrt{c}}-\frac{a}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b*log(x)/sqrt(c) - a/(sqrt(c)*x)
```

Fricas [A]
time = 0.86, size = 23, normalized size = 0.85

$$\frac{\sqrt{cx^2}\,(bx\log(x)-a)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)/(x*sqrt(c*x**2)), x)
```

Giac [A]
time = 2.69, size = 26, normalized size = 0.96

$$\frac{b\log\left(|x|\right)}{\sqrt{c}\,\mathrm{sgn}\left(x\right)}-\frac{a}{\sqrt{c}\,x\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(x))/(sqrt(c)*sgn(x)) - a/(sqrt(c)*x*sgn(x))
```

Mupad [B]
time = 1.22, size = 22, normalized size = 0.81

$$-\frac{\frac{a}{\sqrt{x^2}}-b\,\ln\left(c\,x\right)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(1/2)),x)
```

```
[Out] -(a/(x^2)^(1/2) - b*log(c*x)*sign(x))/c^(1/2)
```

3.785 $\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

```
[Out] -1/2*(b*x+a)^2/a/x/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -1/2*(a + b*x)^2/(a*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ax\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 23, normalized size = 0.88

$$\frac{cx(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
[Out] (c*x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.73

method	result	size
gosper	$-\frac{2bx+a}{2x\sqrt{c\,x^2}}$	19
default	$-\frac{2bx+a}{2x\sqrt{c\,x^2}}$	19
risch	$\frac{-bx-\frac{a}{2}}{x\sqrt{c\,x^2}}$	20
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^2}}{2c\,x^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(2*b*x+a)/x/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.73

$$-\frac{b}{\sqrt{c}\,x}-\frac{a}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -b/(sqrt(c)*x) - 1/2*a/(sqrt(c)*x^2)
```

Fricas [A]
time = 1.38, size = 21, normalized size = 0.81

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c*x^3)
```

Sympy [A]
time = 0.20, size = 24, normalized size = 0.92

$$-\frac{a}{2x\sqrt{cx^2}}-\frac{b}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(1/2),x)
```

```
[Out] -a/(2*x*sqrt(c*x**2)) - b/sqrt(c*x**2)
```

Giac [A]
time = 2.02, size = 18, normalized size = 0.69

$$-\frac{2\,bx+a}{2\,\sqrt{c}\,x^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/(sqrt(c)*x^2*sgn(x))
```

Mupad [B]
time = 0.16, size = 25, normalized size = 0.96

$$-\frac{2\,b\,x^3+a\,x^2}{2\,\sqrt{c}\,\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(1/2)*x*(x^2)^(3/2))
```

3.786 $\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

```
[Out] -1/3*a/x^2/(c*x^2)^(1/2)-1/2*b/x/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -1/3*a/(x^2*Sqrt[c*x^2]) - b/(2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx&=\frac{x\int \frac{a+bx}{x^4}\,dx}{\sqrt{cx^2}}\\&=\frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)\,dx}{\sqrt{cx^2}}\\&=-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.63

$$\frac{c(-2a-3bx)}{6\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] (c*(-2*a - 3*b*x))/(6*(c*x^2)^(3/2))
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{x^2\sqrt{c\,x^2}}$	20
gosper	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
default	$-\frac{3bx+2a}{6x^2\sqrt{c\,x^2}}$	21
trager	$\frac{(-1+x)\left(2a\,x^2+3x^2b+2ax+3bx+2a\right)\sqrt{c\,x^2}}{6c\,x^4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(3*b*x+2*a)/x^2/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.54

$$-\frac{b}{2\sqrt{c}\,x^2}-\frac{a}{3\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(sqrt(c)*x^2) - 1/3*a/(sqrt(c)*x^3)
```

Fricas [A]
time = 1.14, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c*x^4)
```

Sympy [A]
time = 0.22, size = 29, normalized size = 0.83

$$-\frac{a}{3x^2\sqrt{cx^2}} - \frac{b}{2x\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(1/2),x)
[Out] -a/(3*x**2*sqrt(c*x**2)) - b/(2*x*sqrt(c*x**2))
```

Giac [A]
time = 2.69, size = 20, normalized size = 0.57

$$-\frac{3\,bx + 2\,a}{6\,\sqrt{c}\,x^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -1/6*(3*b*x + 2*a)/(sqrt(c)*x^3*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.74

$$-\frac{2\,a\,\sqrt{x^2}\,+3\,b\,x\,\sqrt{x^2}}{6\,\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(1/2)),x)
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(1/2)*x^4)
```

3.787 $\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

```
[Out] -1/4*a/x^3/(c*x^2)^(1/2)-1/3*b/x^2/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -1/4*a/(x^3*Sqrt[c*x^2]) - b/(3*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)\,dx}{\sqrt{cx^2}}\\ &= -\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.69

$$\frac{-3a-4bx}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a - 4*b*x)/(12*x^3*Sqrt[c*x^2])
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.60

method	result	size
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{x^3\sqrt{c\,x^2}}$	20
gosper	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
default	$-\frac{4bx+3a}{12x^3\sqrt{c\,x^2}}$	21
trager	$\frac{(-1+x)(3a\,x^3+4b\,x^3+3a\,x^2+4x^2b+3ax+4bx+3a)\sqrt{c\,x^2}}{12c\,x^5}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(4*b*x+3*a)/x^3/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.54

$$-\frac{b}{3\sqrt{c}\,x^3}-\frac{a}{4\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(sqrt(c)*x^3) - 1/4*a/(sqrt(c)*x^4)
```

Fricas [A]
time = 0.84, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c*x^5)
```

Sympy [A]
time = 0.25, size = 31, normalized size = 0.89

$$-\frac{a}{4x^3\sqrt{cx^2}} - \frac{b}{3x^2\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a/(4*x**3*sqrt(c*x**2)) - b/(3*x**2*sqrt(c*x**2))
```

Giac [A]
time = 2.33, size = 20, normalized size = 0.57

$$-\frac{4\,bx + 3\,a}{12\,\sqrt{c}\,x^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(4*b*x + 3*a)/(sqrt(c)*x^4*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.74

$$-\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(1/2)),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(1/2)*x^5)
```

3.788 $\int \frac{x^3(a+bx)}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{ax^2}{c\sqrt{cx^2}} + \frac{bx^3}{2c\sqrt{cx^2}}$$

```
[Out] a*x^2/c/(c*x^2)^(1/2)+1/2*b*x^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{15\}$

$$\frac{ax^2}{c\sqrt{cx^2}} + \frac{bx^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (a*x^2)/(c*Sqrt[c*x^2]) + (b*x^3)/(2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)\,dx}{c\sqrt{cx^2}}\\ &= \frac{ax^2}{c\sqrt{cx^2}} + \frac{bx^3}{2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))

Maple [A]
time = 0.02, size = 20, normalized size = 0.53

method	result	size
gosper	$\frac{x^{4}(bx+2a)}{2(c\,x^{2})^{\frac{3}{2}}}$	20
default	$\frac{x^{4}(bx+2a)}{2(c\,x^{2})^{\frac{3}{2}}}$	20
trager	$\frac{(bx+2a+b)(-1+x)\sqrt{c\,x^{2}}}{2c^{2}x}$	27
risch	$\frac{a\,x^{2}}{c\sqrt{c\,x^{2}}}+\frac{b\,x^{3}}{2c\sqrt{c\,x^{2}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*x^4*(b*x+2*a)/(c*x^2)^(3/2)

Maxima [A]
time = 0.29, size = 32, normalized size = 0.84

$$\frac{bx^3}{2\sqrt{cx^2}\,c}+\frac{ax^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")

[Out] 1/2*b*x^3/(sqrt(c*x^2)*c) + a*x^2/(sqrt(c*x^2)*c)

Fricas [A]
time = 1.19, size = 19, normalized size = 0.50

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c^2

Sympy [A]
time = 0.23, size = 27, normalized size = 0.71

$$\frac{ax^4}{(cx^2)^{\frac{3}{2}}}+\frac{bx^5}{2\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] a*x**4/(c*x**2)**(3/2) + b*x**5/(2*(c*x**2)**(3/2))
```

Giac [A]
time = 2.42, size = 19, normalized size = 0.50

$$\frac{bx^2 + 2\,ax}{2\,c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/(c^(3/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{{\left(c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(3/2), x)
```

3.789 $\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{bx^2}{c\sqrt{cx^2}}+\frac{ax\log(x)}{c\sqrt{cx^2}}$$

```
[Out] b*x^2/c/(c*x^2)^(1/2)+a*x*ln(x)/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^2)/(c*Sqrt[c*x^2]) + (a*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)\,dx}{c\sqrt{cx^2}}\\ &= \frac{bx^2}{c\sqrt{cx^2}}+\frac{ax\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.60

$$\frac{x^3(bx + a\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x + a*Log[x]))/(c*x^2)^(3/2)
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.57

method	result	size
default	$\frac{x^{3}\left(bx+a\ln\left(x\right)\right)}{\left(c\,x^{2}\right)^{\frac{3}{2}}}$	20
risch	$\frac{b\,x^{2}}{c\sqrt{c\,x^{2}}}+\frac{ax\ln\left(x\right)}{c\sqrt{c\,x^{2}}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*(b*x+a*ln(x))
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.66

$$\frac{bx^2}{\sqrt{cx^2}\,c} + \frac{a\log\left(x\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(c*x^2)*c) + a*log(x)/c^(3/2)
```

Fricas [A]
time = 1.10, size = 22, normalized size = 0.63

$$\frac{\sqrt{cx^2}\,(bx + a\log\left(x\right))}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx)}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)/(c*x**2)**(3/2), x)
```

Giac [**A**]
time = 1.68, size = 27, normalized size = 0.77

$$\frac{\frac{bx}{\sqrt{c}\,\mathrm{sgn}(x)} + \frac{a\log(|x|)}{\sqrt{c}\,\mathrm{sgn}(x)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b*x/(sqrt(c)*sgn(x)) + a*log(abs(x))/(sqrt(c)*sgn(x)))/c
```

Mupad [**B**]
time = 0.32, size = 30, normalized size = 0.86

$$\frac{b\,|x|}{c^{3/2}}+\frac{a\,\ln\left(x+|x|\right)}{c^{3/2}}-\frac{a\,x}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] (b*abs(x))/c^(3/2) + (a*log(x + abs(x)))/c^(3/2) - (a*x)/(c^(3/2)*(x^2)^(1/
2))
```

3.790 $\int \frac{x(a+bx)}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=33

$$-\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}$$

```
[Out] -a/c/(c*x^2)^(1/2)+b*x*ln(x)/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 45}

$$\frac{bx\log(x)}{c\sqrt{cx^2}}-\frac{a}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] -(a/(c*Sqrt[c*x^2])) + (b*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{c\sqrt{cx^2}}+\frac{bx\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.67

$$\frac{x^2(-a+bx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.64

method	result	size
default	$\frac{x^2(bx\ln(x)-a)}{(cx^2)^{\frac{3}{2}}}$	21
risch	$-\frac{a}{c\sqrt{cx^2}}+\frac{bx\ln(x)}{c\sqrt{cx^2}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] x^2*(b*x*ln(x)-a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.64

$$\frac{b\log(x)}{c^{\frac{3}{2}}}-\frac{a}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b*log(x)/c^(3/2) - a/(sqrt(c*x^2)*c)
```

Fricas [A]
time = 0.91, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\,(bx\log(x)-a)}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx)}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)/(c*x**2)**(3/2), x)
```

Giac [A]
time = 2.36, size = 30, normalized size = 0.91

$$\frac{\frac{b\log(|x|)}{\sqrt{c}\,\mathrm{sgn}(x)} - \frac{a}{\sqrt{c}\,x\mathrm{sgn}(x)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b*log(abs(x))/(sqrt(c)*sgn(x)) - a/(sqrt(c)*x*sgn(x)))/c
```

Mupad [B]
time = 0.25, size = 28, normalized size = 0.85

$$-\frac{a+b\,x-b\,\ln\left(x+|x|\right)\,\sqrt{x^2}}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] -(a + b*x - b*log(x + abs(x))*(x^2)^(1/2))/(c^(3/2)*(x^2)^(1/2))
```

3.791 $\int \frac{a+bx}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

[Out] -1/2*(b*x+a)^2/a/c/x/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c*x^2)^(3/2),x]

[Out] -1/2*(a + b*x)^2/(a*c*x*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2acx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.76

$$\frac{x(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.59

method	result	size
gosper	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
default	$-\frac{x(2bx+a)}{2(c\,x^2)^{\frac{3}{2}}}$	17
risch	$\frac{-bx-\frac{a}{2}}{cx\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^2}}{2c^2x^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(2*b*x+a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.79

$$-\frac{b}{\sqrt{cx^2}\,c}-\frac{a}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*c) - 1/2*a/(c^(3/2)*x^2)
```

Fricas [A]
time = 1.16, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^2*x^3)
```

Sympy [A]
time = 0.20, size = 27, normalized size = 0.93

$$-\frac{ax}{2\left(cx^{2}\right)^{\frac{3}{2}}}-\frac{bx^{2}}{\left(cx^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] -a*x/(2*(c*x**2)**(3/2)) - b*x**2/(c*x**2)**(3/2)
```

Giac [A]
time = 3.15, size = 18, normalized size = 0.62

$$-\frac{2\,bx+a}{2\,c^{\frac{3}{2}}x^{2}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/(c^(3/2)*x^2*sgn(x))
```

Mupad [B]
time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^{3}+a\,x^{2}}{2\,c^{3/2}\,x\,\left(x^{2}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(3/2),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(3/2)*x*(x^2)^(3/2))
```

3.792 $\int \frac{a+bx}{x(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

[Out] -1/3*a/c/x^2/(c*x^2)^(1/2)-1/2*b/c/x/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(x*(c*x^2)^(3/2)),x]

[Out] -1/3*a/(c*x^2*Sqrt[c*x^2]) - b/(2*c*x*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)\,dx}{c\sqrt{cx^2}}\\
&= -\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.61

$$\frac{cx^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.44

method	result	size
gosper	$-\frac{3bx+2a}{6\left(c\,x^2\right)^{\frac{3}{2}}}$	18
default	$-\frac{3bx+2a}{6\left(c\,x^2\right)^{\frac{3}{2}}}$	18
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{c\,x^2\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)\left(2a\,x^2+3x^2b+2ax+3bx+2a\right)\sqrt{c\,x^2}}{6c^2x^4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(3*b*x+2*a)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.46

$$-\frac{b}{2\,c^{\frac{3}{2}}x^2}-\frac{a}{3\,c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b/(c^(3/2)*x^2) - 1/3*a/(c^(3/2)*x^3)
```

Fricas [A]
time = 1.00, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^2*x^4)
```

Sympy [A]
time = 0.24, size = 26, normalized size = 0.63

$$-\frac{a}{3\left(cx^2\right)^{\frac{3}{2}}}-\frac{bx}{2\left(cx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(3/2),x)
[Out] -a/(3*(c*x**2)**(3/2)) - b*x/(2*(c*x**2)**(3/2))
```

Giac [A]
time = 3.53, size = 20, normalized size = 0.49

$$-\frac{3\,bx+2\,a}{6\,c^{\frac{3}{2}}x^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/6*(3*b*x + 2*a)/(c^(3/2)*x^3*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{3/2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(3/2)),x)
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(3/2)*x^4)
```

3.793 $\int \frac{a+bx}{x^2(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

```
[Out] -1/4*a/c/x^3/(c*x^2)^(1/2)-1/3*b/c/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/4*a/(c*x^3*Sqrt[c*x^2]) - b/(3*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^2*x^5)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{4bx+3a}{12x(c\,x^2)^{\frac{3}{2}}}$	21
default	$-\frac{4bx+3a}{12x(c\,x^2)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{c\,x^3\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(3a\,x^3+4b\,x^3+3a\,x^2+4x^2b+3ax+4bx+3a)\sqrt{c\,x^2}}{12c^2x^5}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(4*b*x+3*a)/x/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.46

$$-\frac{b}{3\,c^{\frac{3}{2}}x^3}-\frac{a}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(c^(3/2)*x^3) - 1/4*a/(c^(3/2)*x^4)
```

Fricas [A]
time = 0.82, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^2*x^5)
```

Sympy [**A**]
time = 0.24, size = 26, normalized size = 0.63

$$-\frac{a}{4x\left(cx^{2}\right)^{\frac{3}{2}}}-\frac{b}{3\left(cx^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(3/2),x)
[Out] -a/(4*x*(c*x**2)**(3/2)) - b/(3*(c*x**2)**(3/2))
```

Giac [**A**]
time = 2.63, size = 20, normalized size = 0.49

$$-\frac{4\,bx+3\,a}{12\,c^{\frac{3}{2}}x^{4}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/12*(4*b*x + 3*a)/(c^(3/2)*x^4*sgn(x))
```

Mupad [**B**]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^{2}}+4\,b\,x\,\sqrt{x^{2}}}{12\,c^{3/2}\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(3/2)),x)
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(3/2)*x^5)
```

3.794 $\int \frac{a+bx}{x^3(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

[Out] -1/5*a/c/x^4/(c*x^2)^(1/2)-1/4*b/c/x^3/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]

[Out] -1/5*a/(c*x^4*Sqrt[c*x^2]) - b/(4*c*x^3*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-4a-5bx)}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] (c*(-4*a - 5*b*x))/(20*(c*x^2)^(5/2))
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{5bx+4a}{20x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
default	$-\frac{5bx+4a}{20x^{2}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{4}-\frac{a}{5}}{c\,x^{4}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)\left(4a\,x^{4}+5b\,x^{4}+4a\,x^{3}+5b\,x^{3}+4a\,x^{2}+5x^{2}b+4ax+5bx+4a\right)\sqrt{c\,x^{2}}}{20c^{2}x^{6}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(5*b*x+4*a)/x^2/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{3}{2}}x^{4}}-\frac{a}{5\,c^{\frac{3}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(3/2)*x^4) - 1/5*a/(c^(3/2)*x^5)
```

Fricas [A]
time = 1.20, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^2*x^6)
```

Sympy [A]
time = 0.28, size = 29, normalized size = 0.71

$$-\frac{a}{5x^2\left(cx^2\right)^{\frac{3}{2}}}-\frac{b}{4x\left(cx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(3/2),x)
[Out] -a/(5*x**2*(c*x**2)**(3/2)) - b/(4*x*(c*x**2)**(3/2))
```

Giac [A]
time = 2.49, size = 20, normalized size = 0.49

$$-\frac{5\,bx+4\,a}{20\,c^{\frac{3}{2}}x^5\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/20*(5*b*x + 4*a)/(c^(3/2)*x^5*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2}+5\,b\,x\,\sqrt{x^2}}{20\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(3/2)),x)
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(3/2)*x^6)
```

3.795 $\int \frac{a+bx}{x^4(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

```
[Out] -1/6*a/c/x^5/(c*x^2)^(1/2)-1/5*b/c/x^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -1/6*a/(c*x^5*Sqrt[c*x^2]) - b/(5*c*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{6bx+5a}{30x^{3}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
default	$-\frac{6bx+5a}{30x^{3}\left(c\,x^{2}\right)^{\frac{3}{2}}}$	21
risch	$\frac{-\frac{bx}{5}-\frac{a}{6}}{c\,x^{5}\sqrt{c\,x^{2}}}$	23
trager	$\frac{\left(-1+x\right)\left(5a\,x^{5}+6b\,x^{5}+5a\,x^{4}+6b\,x^{4}+5a\,x^{3}+6b\,x^{3}+5a\,x^{2}+6x^{2}b+5ax+6bx+5a\right)\sqrt{c\,x^{2}}}{30c^{2}x^{7}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(6*b*x+5*a)/x^3/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{3}{2}}x^{5}}-\frac{a}{6\,c^{\frac{3}{2}}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(3/2)*x^5) - 1/6*a/(c^(3/2)*x^6)
```

Fricas [A]
time = 1.13, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^2*x^7)
```

Sympy [A]
time = 0.33, size = 31, normalized size = 0.76

$$-\frac{a}{6x^3\left(cx^2\right)^{\frac{3}{2}}}-\frac{b}{5x^2\left(cx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a/(6*x**3*(c*x**2)**(3/2)) - b/(5*x**2*(c*x**2)**(3/2))
```

Giac [A]
time = 2.91, size = 20, normalized size = 0.49

$$-\frac{6\,bx+5\,a}{30\,c^{\frac{3}{2}}x^6\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/30*(6*b*x + 5*a)/(c^(3/2)*x^6*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(3/2)),x)
```

```
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(3/2)*x^7)
```

3.796 $\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] -a/c^2/(c*x^2)^(1/2)+b*x*ln(x)/c^2/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a/(c^2*Sqrt[c*x^2])) + (b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.67

$$\frac{-a+bx\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (-a + b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.64

method	result	size
default	$\frac{x^{4}\left(b x \ln \left(x \right)-a \right)}{\left(c \,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$-\frac{a}{c^{2} \sqrt{c \,x^{2}}}+\frac{b x \ln \left(x \right)}{c^{2} \sqrt{c \,x^{2}}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] x^4*(b*x*ln(x)-a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.73

$$-\frac{ax^2}{\left(cx^2\right)^{\frac{3}{2}}c}+\frac{b\log\left(x\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -a*x^2/((c*x^2)^(3/2)*c) + b*log(x)/c^(5/2)
```

Fricas [A]
time = 1.24, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\left(bx\log\left(x\right)-a\right)}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)/(c*x**2)**(5/2), x)
```

Giac [A]
time = 2.41, size = 26, normalized size = 0.79

$$\frac{b\log(|x|)}{c^{\frac{5}{2}}\operatorname{sgn}(x)} - \frac{a}{c^{\frac{5}{2}}x\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] b*log(abs(x))/(c^(5/2)*sgn(x)) - a/(c^(5/2)*x*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,(a+b\,x)}{{(c\,x^2)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(5/2), x)
```

3.797 $\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

[Out] -1/2*(b*x+a)^2/a/c^2/x/(c*x^2)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*x))/(c*x^2)^(5/2),x]

[Out] -1/2*(a + b*x)^2/(a*c^2*x*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.83

$$\frac{x^3(-a-2bx)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-a - 2*b*x))/(2*(c*x^2)^(5/2))
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.66

method	result	size
gosper	$-\frac{x^3(2bx+a)}{2(c\,x^2)^{\frac{5}{2}}}$	19
default	$-\frac{x^3(2bx+a)}{2(c\,x^2)^{\frac{5}{2}}}$	19
risch	$\frac{-bx-\frac{a}{2}}{c^2x\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(ax+2bx+a)\sqrt{c\,x^2}}{2c^3x^3}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^3*(2*b*x+a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.90

$$-\frac{bx^2}{\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b*x^2/((c*x^2)^(3/2)*c) - 1/2*a/(c^(5/2)*x^2)
```

Fricas [A]
time = 1.21, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^3*x^3)
```

Sympy [**A**]
time = 0.28, size = 29, normalized size = 1.00

$$-\frac{ax^3}{2\left(cx^2\right)^{\frac{5}{2}}} - \frac{bx^4}{\left(cx^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**3/(2*(c*x**2)**(5/2)) - b*x**4/(c*x**2)**(5/2)
```

Giac [**A**]
time = 1.52, size = 18, normalized size = 0.62

$$-\frac{2\,bx + a}{2\,c^{\frac{5}{2}}x^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/(c^(5/2)*x^2*sgn(x))
```

Mupad [**B**]
time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3 + a\,x^2}{2\,c^{5/2}\,x\,\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(5/2)*x*(x^2)^(3/2))
```

3.798 $\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

```
[Out] -1/3*a/c^2/x^2/(c*x^2)^(1/2)-1/2*b/c^2/x/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {15, 45}

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -1/3*a/(c^2*x^2*Sqrt[c*x^2]) - b/(2*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.59

$$\frac{x^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{x^2(3bx+2a)}{6(c\,x^2)^{\frac{5}{2}}}$	21
default	$-\frac{x^2(3bx+2a)}{6(c\,x^2)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{2}-\frac{a}{3}}{c^2x^2\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(2a\,x^2+3x^2b+2ax+3bx+2a)\sqrt{c\,x^2}}{6c^3x^4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x^2*(3*b*x+2*a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.30, size = 23, normalized size = 0.56

$$-\frac{a}{3\,\left(cx^2\right)^{\frac{3}{2}}c}-\frac{b}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a/((c*x^2)^(3/2)*c) - 1/2*b/(c^(5/2)*x^2)
```

Fricas [A]
time = 1.14, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^3*x^4)
```

Sympy [A]
time = 0.28, size = 31, normalized size = 0.76

$$-\frac{ax^2}{3\,(cx^2)^{\frac{5}{2}}}-\frac{bx^3}{2\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(5/2),x)
[Out] -a*x**2/(3*(c*x**2)**(5/2)) - b*x**3/(2*(c*x**2)**(5/2))
```

Giac [A]
time = 1.60, size = 20, normalized size = 0.49

$$-\frac{3\,bx+2\,a}{6\,c^{\frac{5}{2}}x^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/6*(3*b*x + 2*a)/(c^(5/2)*x^3*sgn(x))
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{5/2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(5/2),x)
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(5/2)*x^4)
```

3.799 $\int \frac{a+bx}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

[Out] -1/4*a/c^2/x^3/(c*x^2)^(1/2)-1/3*b/c^2/x^2/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 45}

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c*x^2)^(5/2),x]

[Out] -1/4*a/(c^2*x^3*Sqrt[c*x^2]) - b/(3*c^2*x^2*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(5/2),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^3*x^5)
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.46

method	result	size
gosper	$-\frac{x(4bx+3a)}{12(c\,x^2)^{\frac{5}{2}}}$	19
default	$-\frac{x(4bx+3a)}{12(c\,x^2)^{\frac{5}{2}}}$	19
risch	$\frac{-\frac{bx}{3}-\frac{a}{4}}{c^2x^3\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)(3a\,x^3+4b\,x^3+3a\,x^2+4x^2b+3ax+4bx+3a)\sqrt{c\,x^2}}{12c^3x^5}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12*x*(4*b*x+3*a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.56

$$-\frac{b}{3\,(cx^2)^{\frac{3}{2}}\,c}-\frac{a}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*b/((c*x^2)^(3/2)*c) - 1/4*a/(c^(5/2)*x^4)
```

Fricas [A]
time = 0.94, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^3*x^5)
```

Sympy [A]
time = 0.28, size = 29, normalized size = 0.71

$$-\frac{ax}{4\left(cx^{2}\right)^{\frac{5}{2}}}-\frac{bx^{2}}{3\left(cx^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(5/2),x)
[Out] -a*x/(4*(c*x**2)**(5/2)) - b*x**2/(3*(c*x**2)**(5/2))
```

Giac [A]
time = 1.85, size = 20, normalized size = 0.49

$$-\frac{4\,bx+3\,a}{12\,c^{\frac{5}{2}}x^{4}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/12*(4*b*x + 3*a)/(c^(5/2)*x^4*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}+4\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(5/2),x)
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(5/2)*x^5)
```

3.800 $\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

```
[Out] -1/5*a/c^2/x^4/(c*x^2)^(1/2)-1/4*b/c^2/x^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/5*a/(c^2*x^4*Sqrt[c*x^2]) - b/(4*c^2*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4a+5bx)}{20c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/20*(Sqrt[c*x^2]*(4*a + 5*b*x))/(c^3*x^6)
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.44

method	result	size
gosper	$-\frac{5bx+4a}{20\left(c\,x^{2}\right)^{\frac{5}{2}}}$	18
default	$-\frac{5bx+4a}{20\left(c\,x^{2}\right)^{\frac{5}{2}}}$	18
risch	$\frac{-\frac{bx}{4}-\frac{a}{5}}{c^{2}x^{4}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)\left(4a\,x^{4}+5b\,x^{4}+4a\,x^{3}+5b\,x^{3}+4a\,x^{2}+5x^{2}b+4ax+5bx+4a\right)\sqrt{c\,x^{2}}}{20c^{3}x^{6}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(5*b*x+4*a)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{5}{2}}x^4}-\frac{a}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*b/(c^(5/2)*x^4) - 1/5*a/(c^(5/2)*x^5)
```

Fricas [A]
time = 1.11, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^3*x^6)
```

Sympy [A]
time = 0.31, size = 26, normalized size = 0.63

$$-\frac{a}{5\left(cx^2\right)^{\frac{5}{2}}} - \frac{bx}{4\left(cx^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(5/2),x)
[Out] -a/(5*(c*x**2)**(5/2)) - b*x/(4*(c*x**2)**(5/2))
```

Giac [A]
time = 1.83, size = 20, normalized size = 0.49

$$-\frac{5\,bx+4\,a}{20\,c^{\frac{5}{2}}x^5\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/20*(5*b*x + 4*a)/(c^(5/2)*x^5*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2}+5\,b\,x\,\sqrt{x^2}}{20\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(5/2)),x)
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(5/2)*x^6)
```

3.801 $\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

```
[Out] -1/6*a/c^2/x^5/(c*x^2)^(1/2)-1/5*b/c^2/x^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/6*a/(c^2*x^5*Sqrt[c*x^2]) - b/(5*c^2*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(5a+6bx)}{30c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/30*(Sqrt[c*x^2]*(5*a + 6*b*x))/(c^3*x^7)
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{6bx+5a}{30x\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
default	$-\frac{6bx+5a}{30x\left(c\,x^{2}\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{5}-\frac{a}{6}}{c^{2}x^{5}\sqrt{c\,x^{2}}}$	23
trager	$\frac{(-1+x)\left(5a\,x^{5}+6b\,x^{5}+5a\,x^{4}+6b\,x^{4}+5a\,x^{3}+6b\,x^{3}+5a\,x^{2}+6x^{2}b+5ax+6bx+5a\right)\sqrt{c\,x^{2}}}{30c^{3}x^{7}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(6*b*x+5*a)/x/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{5}{2}}x^5}-\frac{a}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*b/(c^(5/2)*x^5) - 1/6*a/(c^(5/2)*x^6)
```

Fricas [A]
time = 1.16, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^3*x^7)
```

Sympy [A]
time = 0.39, size = 26, normalized size = 0.63

$$-\frac{a}{6x\left(cx^2\right)^{\frac{5}{2}}}-\frac{b}{5\left(cx^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(5/2),x)
[Out] -a/(6*x*(c*x**2)**(5/2)) - b/(5*(c*x**2)**(5/2))
```

Giac [A]
time = 2.67, size = 20, normalized size = 0.49

$$-\frac{6\,bx+5\,a}{30\,c^{\frac{5}{2}}x^6\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/30*(6*b*x + 5*a)/(c^(5/2)*x^6*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}+6\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(5/2)),x)
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(5/2)*x^7)
```

3.802 $\int \frac{a+bx}{x^3(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

```
[Out] -1/7*a/c^2/x^6/(c*x^2)^(1/2)-1/6*b/c^2/x^5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -1/7*a/(c^2*x^6*Sqrt[c*x^2]) - b/(6*c^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^8}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^8}+\frac{b}{x^7}\right)\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-6a-7bx)}{42\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
[Out] (c*(-6*a - 7*b*x))/(42*(c*x^2)^(7/2))
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{7bx+6a}{42x^2(c\,x^2)^{\frac{5}{2}}}$	21
default	$-\frac{7bx+6a}{42x^2(c\,x^2)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{6}-\frac{a}{7}}{c^2x^6\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)\left(6a\,x^6+7b\,x^6+6a\,x^5+7b\,x^5+6a\,x^4+7b\,x^4+6a\,x^3+7b\,x^3+6a\,x^2+7x^2b+6ax+7bx+6a\right)\sqrt{c\,x^2}}{42c^3x^8}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/42*(7*b*x+6*a)/x^2/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{6\,c^{\frac{5}{2}}x^6}-\frac{a}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/6*b/(c^(5/2)*x^6) - 1/7*a/(c^(5/2)*x^7)
```

Fricas [A]
time = 0.86, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(7\,bx+6\,a)}{42\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/42*sqrt(c*x^2)*(7*b*x + 6*a)/(c^3*x^8)
```

Sympy [A]
time = 0.40, size = 29, normalized size = 0.71

$$-\frac{a}{7x^2\left(cx^2\right)^{\frac{5}{2}}}-\frac{b}{6x\left(cx^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(5/2),x)
[Out] -a/(7*x**2*(c*x**2)**(5/2)) - b/(6*x*(c*x**2)**(5/2))
```

Giac [A]
time = 2.07, size = 20, normalized size = 0.49

$$-\frac{7\,bx+6\,a}{42\,c^{\frac{5}{2}}x^7\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/42*(7*b*x + 6*a)/(c^(5/2)*x^7*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{6\,a\,\sqrt{x^2}+7\,b\,x\,\sqrt{x^2}}{42\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(5/2)),x)
[Out] -(6*a*(x^2)^(1/2) + 7*b*x*(x^2)^(1/2))/(42*c^(5/2)*x^8)
```

3.803 $\int \frac{a+bx}{x^4(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

```
[Out] -1/8*a/c^2/x^7/(c*x^2)^(1/2)-1/7*b/c^2/x^6/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -1/8*a/(c^2*x^7*Sqrt[c*x^2]) - b/(7*c^2*x^6*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^9}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^9}+\frac{b}{x^8}\right)\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

Maple [A]
time = 0.03, size = 21, normalized size = 0.51

method	result	size
gosper	$-\frac{8bx+7a}{56x^3\left(c\,x^2\right)^{\frac{5}{2}}}$	21
default	$-\frac{8bx+7a}{56x^3\left(c\,x^2\right)^{\frac{5}{2}}}$	21
risch	$\frac{-\frac{bx}{7}-\frac{a}{8}}{c^2x^7\sqrt{c\,x^2}}$	23
trager	$\frac{(-1+x)\left(7a\,x^7+8b\,x^7+7a\,x^6+8b\,x^6+7a\,x^5+8b\,x^5+7a\,x^4+8b\,x^4+7a\,x^3+8b\,x^3+7a\,x^2+8x^2b+7ax+8bx+7a\right)\sqrt{c\,x^2}}{56c^3x^9}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/56*(8*b*x+7*a)/x^3/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.46

$$-\frac{b}{7\,c^{\frac{5}{2}}x^7}-\frac{a}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/7*b/(c^(5/2)*x^7) - 1/8*a/(c^(5/2)*x^8)
```

Fricas [A]
time = 0.96, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(8\,bx+7\,a)}{56\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/56*sqrt(c*x^2)*(8*b*x + 7*a)/(c^3*x^9)
```

Sympy [A]
time = 0.45, size = 31, normalized size = 0.76

$$-\frac{a}{8x^3\left(cx^2\right)^{\frac{5}{2}}}-\frac{b}{7x^2\left(cx^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(5/2),x)
[Out] -a/(8*x**3*(c*x**2)**(5/2)) - b/(7*x**2*(c*x**2)**(5/2))
```

Giac [A]
time = 1.39, size = 20, normalized size = 0.49

$$-\frac{8\,bx+7\,a}{56\,c^{\frac{5}{2}}x^8\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/56*(8*b*x + 7*a)/(c^(5/2)*x^8*sgn(x))
```

Mupad [B]
time = 0.16, size = 26, normalized size = 0.63

$$-\frac{7\,a\,\sqrt{x^2}+8\,b\,x\,\sqrt{x^2}}{56\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(5/2)),x)
[Out] -(7*a*(x^2)^(1/2) + 8*b*x*(x^2)^(1/2))/(56*c^(5/2)*x^9)
```

3.804 $\int x^3 \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=57

$$\frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}$$

```
[Out] 1/5*a^2*x^4*(c*x^2)^(1/2)+1/3*a*b*x^5*(c*x^2)^(1/2)+1/7*b^2*x^6*(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4*Sqrt[c*x^2])/5 + (a*b*x^5*Sqrt[c*x^2])/3 + (b^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^4(a+bx)^2\,dx}{x} \\
&= \frac{\sqrt{cx^2}\int\left(a^2x^4+2abx^5+b^2x^6\right)dx}{x} \\
&= \frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.56

method	result
gosper	$\frac{x^4(15x^2b^2+35abx+21a^2)\sqrt{c\,x^2}}{105}$
default	$\frac{x^4(15x^2b^2+35abx+21a^2)\sqrt{c\,x^2}}{105}$
risch	$\frac{a^2x^4\sqrt{c\,x^2}}{5}+\frac{ab\,x^5\sqrt{c\,x^2}}{3}+\frac{b^2x^6\sqrt{c\,x^2}}{7}$
trager	$\frac{(15b^2x^6+35ab\,x^5+15b^2x^5+21a^2x^4+35ab\,x^4+15b^2x^4+21a^2x^3+35ab\,x^3+15b^2x^3+21a^2x^2+35ab\,x^2+15x^2b^2+21a^2x+35abx+15b^2x-}{105x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*x^4*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.95

$$\frac{(cx^2)^{\frac{3}{2}}\,b^2x^4}{7\,c}+\frac{(cx^2)^{\frac{3}{2}}\,abx^3}{3\,c}+\frac{(cx^2)^{\frac{3}{2}}\,a^2x^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(3/2)*b^2*x^4/c + 1/3*(c*x^2)^(3/2)*a*b*x^3/c + 1/5*(c*x^2)^(3/
2)*a^2*x^2/c
```

Fricas [A]
time = 0.70, size = 33, normalized size = 0.58

$$\frac{1}{105}\left(15\,b^2x^6+35\,abx^5+21\,a^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*x^6 + 35*a*b*x^5 + 21*a^2*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.14, size = 49, normalized size = 0.86

$$\frac{a^2x^4\sqrt{cx^2}}{5}+\frac{abx^5\sqrt{cx^2}}{3}+\frac{b^2x^6\sqrt{cx^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4*sqrt(c*x**2)/5 + a*b*x**5*sqrt(c*x**2)/3 + b**2*x**6*sqrt(c*x**2)
/7
```

Giac [A]
time = 1.20, size = 35, normalized size = 0.61

$$\frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x)+35\,abx^6\mathrm{sgn}(x)+21\,a^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.805 $\int x^2 \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=57

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

```
[Out] 1/4*a^2*x^3*(c*x^2)^(1/2)+2/5*a*b*x^4*(c*x^2)^(1/2)+1/6*b^2*x^5*(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*x^4*Sqrt[c*x^2])/5 + (b^2*x^5*Sqrt[c*x^2])
/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^3(a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\ &= \frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.56

method	result
gosper	$\frac{x^3\left(10x^2b^2+24abx+15a^2\right)\sqrt{c\,x^2}}{60}$
default	$\frac{x^3\left(10x^2b^2+24abx+15a^2\right)\sqrt{c\,x^2}}{60}$
risch	$\frac{a^2x^3\sqrt{c\,x^2}}{4}+\frac{2ab\,x^4\sqrt{c\,x^2}}{5}+\frac{b^2x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{\left(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24ab\,x^3+10b^2x^3+15a^2x^2+24ab\,x^2+10x^2b^2+15a^2x+24abx+10b^2x+15a^2+24ab+10b^2\right)(-1+x)\sqrt{}}{60x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*x^3*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.91

$$\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x^3}{6\,c}+\frac{2\left(cx^2\right)^{\frac{3}{2}}abx^2}{5\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}a^2x}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(3/2)*b^2*x^3/c + 2/5*(c*x^2)^(3/2)*a*b*x^2/c + 1/4*(c*x^2)^(3/
2)*a^2*x/c
```

Fricas [A]
time = 0.63, size = 33, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^5+24\,abx^4+15\,a^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*x^5 + 24*a*b*x^4 + 15*a^2*x^3)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.12, size = 51, normalized size = 0.89

$$\frac{a^2x^3\sqrt{cx^2}}{4}+\frac{2abx^4\sqrt{cx^2}}{5}+\frac{b^2x^5\sqrt{cx^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3*sqrt(c*x**2)/4 + 2*a*b*x**4*sqrt(c*x**2)/5 + b**2*x**5*sqrt(c*x**
2)/6
```

Giac [**A**]
time = 1.61, size = 35, normalized size = 0.61

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x)+24\,abx^5\mathrm{sgn}(x)+15\,a^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.806 $\int x\sqrt{cx^2}\,(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

```
[Out] 1/3*a^2*x^2*(c*x^2)^(1/2)+1/2*a*b*x^3*(c*x^2)^(1/2)+1/5*b^2*x^4*(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2*Sqrt[c*x^2])/3 + (a*b*x^3*Sqrt[c*x^2])/2 + (b^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}\,(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\&=\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.56

method	result	size
gosper	$\frac{x^2(6x^2b^2+15abx+10a^2)\sqrt{c\,x^2}}{30}$	32
default	$\frac{x^2(6x^2b^2+15abx+10a^2)\sqrt{c\,x^2}}{30}$	32
risch	$\frac{a^2x^2\sqrt{c\,x^2}}{3}+\frac{ab\,x^3\sqrt{c\,x^2}}{2}+\frac{b^2x^4\sqrt{c\,x^2}}{5}$	46
trager	$\frac{(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2)(-1+x)\sqrt{c\,x^2}}{30x}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/30*x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.86

$$\frac{(cx^2)^{\frac{3}{2}}\,b^2x^2}{5\,c}+\frac{(cx^2)^{\frac{3}{2}}\,abx}{2\,c}+\frac{(cx^2)^{\frac{3}{2}}\,a^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(3/2)*b^2*x^2/c + 1/2*(c*x^2)^(3/2)*a*b*x/c + 1/3*(c*x^2)^(3/2)
*a^2/c
```

Fricas [A]
time = 0.71, size = 33, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^4+15\,abx^3+10\,a^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.11, size = 49, normalized size = 0.86

$$\frac{a^2x^2\sqrt{cx^2}}{3}+\frac{abx^3\sqrt{cx^2}}{2}+\frac{b^2x^4\sqrt{cx^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**2*sqrt(c*x**2)/3 + a*b*x**3*sqrt(c*x**2)/2 + b**2*x**4*sqrt(c*x**2)
/5
```

Giac [A]
time = 1.40, size = 35, normalized size = 0.61

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x)+15\,abx^4\mathrm{sgn}(x)+10\,a^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.807 $\int \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2x\sqrt{cx^2} + \frac{2}{3}abx^2\sqrt{cx^2} + \frac{1}{4}b^2x^3\sqrt{cx^2}$$

```
[Out] 1/2*a^2*x*(c*x^2)^(1/2)+2/3*a*b*x^2*(c*x^2)^(1/2)+1/4*b^2*x^3*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{1}{2}a^2x\sqrt{cx^2} + \frac{2}{3}abx^2\sqrt{cx^2} + \frac{1}{4}b^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x*Sqrt[c*x^2])/2 + (2*a*b*x^2*Sqrt[c*x^2])/3 + (b^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx^2} (a+bx)^2 \, dx &= \frac{\sqrt{cx^2} \int x(a+bx)^2 \, dx}{x} \\
&= \frac{\sqrt{cx^2} \int (a^2x + 2abx^2 + b^2x^3) \, dx}{x} \\
&= \frac{1}{2}a^2x\sqrt{cx^2} + \frac{2}{3}abx^2\sqrt{cx^2} + \frac{1}{4}b^2x^3\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.60

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Maple [A]
time = 0.11, size = 30, normalized size = 0.55

method	result	size
gosper	$\frac{x\left(3x^2b^2+8abx+6a^2\right)\sqrt{c\,x^2}}{12}$	30
default	$\frac{x\left(3x^2b^2+8abx+6a^2\right)\sqrt{c\,x^2}}{12}$	30
risch	$\frac{a^2x\sqrt{c\,x^2}}{2}+\frac{2ab\,x^2\sqrt{c\,x^2}}{3}+\frac{b^2x^3\sqrt{c\,x^2}}{4}$	44
trager	$\frac{\left(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2\right)(-1+x)\sqrt{c\,x^2}}{12x}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(1/2)
```

Maxima [A]
time = 0.31, size = 44, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2}\,a^2x+\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x}{4\,c}+\frac{2\left(cx^2\right)^{\frac{3}{2}}ab}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(c*x^2)*a^2*x + 1/4*(c*x^2)^(3/2)*b^2*x/c + 2/3*(c*x^2)^(3/2)*a*b/c
```

Fricas [A]
time = 0.51, size = 31, normalized size = 0.56

$$\frac{1}{12}\left(3\,b^2x^3+8\,abx^2+6\,a^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.11, size = 49, normalized size = 0.89

$$\frac{a^2x\sqrt{cx^2}}{2}+\frac{2abx^2\sqrt{cx^2}}{3}+\frac{b^2x^3\sqrt{cx^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*x*sqrt(c*x**2)/2 + 2*a*b*x**2*sqrt(c*x**2)/3 + b**2*x**3*sqrt(c*x**2)/
4
```

Giac [**A**]
time = 1.05, size = 35, normalized size = 0.64

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x)+8\,abx^3\mathrm{sgn}(x)+6\,a^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(1/2)*(a + b*x)^2, x)
```

3.808 $\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x}\,dx$

Optimal. Leaf size=26

$$\frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

[Out] 1/3*(b*x+a)^3*(c*x^2)^(1/2)/b/x

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]

[Out] (Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x}\,dx &= \frac{\sqrt{cx^2}\,\int (a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.96

$$\frac{cx(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (c*x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Maple [A]
time = 0.10, size = 23, normalized size = 0.88

method	result	size
default	$\frac{(bx+a)^3\sqrt{c\,x^2}}{3bx}$	23
risch	$\frac{(bx+a)^3\sqrt{c\,x^2}}{3bx}$	23
gosper	$\frac{(x^2b^2+3abx+3a^2)\sqrt{c\,x^2}}{3}$	28
trager	$\frac{(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c\,x^2}}{3x}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.51, size = 27, normalized size = 1.04

$$\frac{1}{3}\left(b^2x^2 + 3\,abx + 3\,a^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 0.08, size = 41, normalized size = 1.58

$$a^2\sqrt{cx^2} + abx\sqrt{cx^2} + \frac{b^2x^2\sqrt{cx^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x,x)
[Out] a**2*sqrt(c*x**2) + a*b*x*sqrt(c*x**2) + b**2*x**2*sqrt(c*x**2)/3
```

Giac [A]
time = 1.16, size = 29, normalized size = 1.12

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}(x)}{b} - \frac{a^3\mathrm{sgn}(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="giac")
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x, x)
```

3.809 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}+\frac{a^2\sqrt{cx^2}\log(x)}{x}$$

```
[Out] 2*a*b*(c*x^2)^(1/2)+1/2*b^2*x*(c*x^2)^(1/2)+a^2*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c*x^2] + (b^2*x*Sqrt[c*x^2])/2 + (a^2*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx}{x}\\ &= 2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}+\frac{a^2\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.67

$$\frac{cx(bx(4a+bx)+2a^2\log(x))}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
[Out] (c*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.67

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(x^2b^2+2a^2\ln(x)+4abx)}{2x}$	33
risch	$\frac{\sqrt{c\,x^2}\,b(\frac{1}{2}x^2b+2ax)}{x}+\frac{a^2\ln(x)\sqrt{c\,x^2}}{x}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(1/2)*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.71, size = 32, normalized size = 0.65

$$\frac{(b^2x^2+4\,abx+2\,a^2\log{(x)})\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**2, x)
```

Giac [A]
time = 1.18, size = 32, normalized size = 0.65

$$\frac{1}{2}\left(b^2x^2\text{sgn}(x)+4\,abx\text{sgn}(x)+2\,a^2\log\left(|x|\right)\text{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,(a+b\,x)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2, x)
```

3.810 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=49

$$b^2\sqrt{cx^2} - \frac{a^2\sqrt{cx^2}}{x^2} + \frac{2ab\sqrt{cx^2}\log(x)}{x}$$

```
[Out] b^2*(c*x^2)^(1/2)-a^2*(c*x^2)^(1/2)/x^2+2*a*b*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2\sqrt{cx^2}}{x^2} + \frac{2ab\sqrt{cx^2}\log(x)}{x} + b^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
```

```
[Out] b^2*Sqrt[c*x^2] - (a^2*Sqrt[c*x^2])/x^2 + (2*a*b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3} dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^2} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(b^2 + \frac{a^2}{x^2} + \frac{2ab}{x}\right) dx}{x} \\
&= b^2\sqrt{cx^2} - \frac{a^2\sqrt{cx^2}}{x^2} + \frac{2ab\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.63

$$\frac{c(-a^2+b^2x^2+2abx\log(x))}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
[Out] (c*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/Sqrt[c*x^2]
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.65

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(2ab\ln(x)x+x^2b^2-a^2)}{x^2}$	32
risch	$b^2\sqrt{c\,x^2}-\frac{a^2\sqrt{c\,x^2}}{x^2}+\frac{2ab\ln(x)\sqrt{c\,x^2}}{x}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(1/2)*(2*a*b*ln(x)*x+x^2*b^2-a^2)/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.63, size = 31, normalized size = 0.63

$$\frac{(b^2x^2+2\,abx\log{(x)}-a^2)\sqrt{cx^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**3, x)
```

Giac [A]
time = 1.21, size = 31, normalized size = 0.63

$$\left(b^2 x\mathrm{sgn}(x) + 2\,ab\log\left(|x|\right)\mathrm{sgn}(x) - \frac{a^2\mathrm{sgn}(x)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*x*sgn(x) + 2*a*b*log(abs(x))*sgn(x) - a^2*sgn(x)/x)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3, x)
```

3.811 $\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\,\log(x)}{x}$$

```
[Out] -1/2*a^2*(c*x^2)^(1/2)/x^3-2*a*b*(c*x^2)^(1/2)/x^2+b^2*ln(x)*(c*x^2)^(1/2)/
x
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\,\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] -1/2*(a^2*Sqrt[c*x^2])/x^3 - (2*a*b*Sqrt[c*x^2])/x^2 + (b^2*Sqrt[c*x^2]*Log
[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^4}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^3}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{x}\\
&= -\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\,\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.67

$$\frac{\sqrt{cx^2}\,(-a(a+4bx)+2b^2x^2\log(x))}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*x^3)
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.63

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(2b^2\ln(x)x^2-4abx-a^2)}{2x^3}$	34
risch	$\frac{\sqrt{c\,x^2}\,(-\frac{1}{2}a^2-2abx)}{x^3}+\frac{b^2\ln(x)\sqrt{c\,x^2}}{x}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.68, size = 33, normalized size = 0.61

$$\frac{(2\,b^2x^2\log{(x)}-4\,abx-a^2)\sqrt{cx^2}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**4, x)
```

Giac [A]
time = 1.58, size = 35, normalized size = 0.65

$$\frac{1}{2}\left(2\,b^2\log\left(|x|\right)\mathrm{sgn}(x) - \frac{4\,abx\mathrm{sgn}(x) + a^2\mathrm{sgn}(x)}{x^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(x))*sgn(x) - (4*a*b*x*sgn(x) + a^2*sgn(x))/x^2)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,(a+b\,x)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4, x)
```

3.812 $\int x^3(cx^2)^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

```
[Out] 1/7*a^2*c*x^6*(c*x^2)^(1/2)+1/4*a*b*c*x^7*(c*x^2)^(1/2)+1/9*b^2*c*x^8*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^6*Sqrt[c*x^2])/7 + (a*b*c*x^7*Sqrt[c*x^2])/4 + (b^2*c*x^8*Sqrt[c*x
^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^6+2abx^7+b^2x^8\right)dx}{x}\\&=\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.53

method	result
gosper	$\frac{x^4(28x^2b^2+63abx+36a^2)(cx^2)^{\frac{3}{2}}}{252}$
default	$\frac{x^4(28x^2b^2+63abx+36a^2)(cx^2)^{\frac{3}{2}}}{252}$
risch	$\frac{a^2cx^6\sqrt{cx^2}}{7}+\frac{abcx^7\sqrt{cx^2}}{4}+\frac{b^2cx^8\sqrt{cx^2}}{9}$
trager	$\frac{c(28b^2x^8+63abx^7+28b^2x^7+36a^2x^6+63abx^6+28b^2x^6+36a^2x^5+63abx^5+28b^2x^5+36a^2x^4+63abx^4+28b^2x^4+36a^2x^3+63abx^3+28b}{252x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/252*x^4*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.90

$$\frac{(cx^2)^{\frac{5}{2}}b^2x^4}{9c}+\frac{(cx^2)^{\frac{5}{2}}abx^3}{4c}+\frac{(cx^2)^{\frac{5}{2}}a^2x^2}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/9*(c*x^2)^(5/2)*b^2*x^4/c + 1/4*(c*x^2)^(5/2)*a*b*x^3/c + 1/7*(c*x^2)^(5/
2)*a^2*x^2/c
```

Fricas [A]
time = 0.65, size = 36, normalized size = 0.60

$$\frac{1}{252}\left(28\,b^2cx^8+63\,abcx^7+36\,a^2cx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c*x^8 + 63*a*b*c*x^7 + 36*a^2*c*x^6)*sqrt(c*x^2)
```

Sympy [A]
time = 0.26, size = 49, normalized size = 0.82

$$\frac{a^2x^4(cx^2)^{\frac{3}{2}}}{7}+\frac{abx^5(cx^2)^{\frac{3}{2}}}{4}+\frac{b^2x^6(cx^2)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**4*(c*x**2)**(3/2)/7 + a*b*x**5*(c*x**2)**(3/2)/4 + b**2*x**6*(c*x**
2)**(3/2)/9
```

Giac [A]
time = 1.54, size = 35, normalized size = 0.58

$$\frac{1}{252}\left(28\,b^2x^9\mathrm{sgn}(x)+63\,abx^8\mathrm{sgn}(x)+36\,a^2x^7\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.813 $\int x^2(cx^2)^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

```
[Out] 1/6*a^2*c*x^5*(c*x^2)^(1/2)+2/7*a*b*c*x^6*(c*x^2)^(1/2)+1/8*b^2*c*x^7*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^5*Sqrt[c*x^2])/6 + (2*a*b*c*x^6*Sqrt[c*x^2])/7 + (b^2*c*x^7*Sqrt[c
*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^5+2abx^6+b^2x^7\right)\,dx}{x}\\&=\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.53

method	result
gosper	$\frac{x^{3}\left(21x^{2}b^{2}+48abx+28a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{168}$
default	$\frac{x^{3}\left(21x^{2}b^{2}+48abx+28a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{168}$
risch	$\frac{a^{2}c\,x^{5}\sqrt{c\,x^{2}}}{6}+\frac{2abc\,x^{6}\sqrt{c\,x^{2}}}{7}+\frac{b^{2}c\,x^{7}\sqrt{c\,x^{2}}}{8}$
trager	$\frac{c\left(21b^{2}x^{7}+48ab\,x^{6}+21b^{2}x^{6}+28a^{2}x^{5}+48ab\,x^{5}+21b^{2}x^{5}+28a^{2}x^{4}+48ab\,x^{4}+21b^{2}x^{4}+28a^{2}x^{3}+48ab\,x^{3}+21b^{2}x^{3}+28a^{2}x^{2}+48ab\,x^{2}+21x^{2}\right.}{168x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/168*x^3*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.87

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^3}{8\,c}+\frac{2\left(cx^2\right)^{\frac{5}{2}}abx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2x}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b^2*x^3/c + 2/7*(c*x^2)^(5/2)*a*b*x^2/c + 1/6*(c*x^2)^(5/
2)*a^2*x/c
```

Fricas [A]
time = 0.61, size = 36, normalized size = 0.60

$$\frac{1}{168}\left(21\,b^2cx^7+48\,abcx^6+28\,a^2cx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c*x^7 + 48*a*b*c*x^6 + 28*a^2*c*x^5)*sqrt(c*x^2)
```

Sympy [A]
time = 0.23, size = 51, normalized size = 0.85

$$\frac{a^2x^3(cx^2)^{\frac{3}{2}}}{6}+\frac{2abx^4(cx^2)^{\frac{3}{2}}}{7}+\frac{b^2x^5(cx^2)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**3*(c*x**2)**(3/2)/6 + 2*a*b*x**4*(c*x**2)**(3/2)/7 + b**2*x**5*(c*x
**2)**(3/2)/8
```

Giac [A]
time = 0.85, size = 35, normalized size = 0.58

$$\frac{1}{168}\left(21\,b^2x^8\mathrm{sgn}(x)+48\,abx^7\mathrm{sgn}(x)+28\,a^2x^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.814 $\int x(cx^2)^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

```
[Out] 1/5*a^2*c*x^4*(c*x^2)^(1/2)+1/3*a*b*c*x^5*(c*x^2)^(1/2)+1/7*b^2*c*x^6*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^4*Sqrt[c*x^2])/5 + (a*b*c*x^5*Sqrt[c*x^2])/3 + (b^2*c*x^6*Sqrt[c*x
^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^4+2abx^5+b^2x^6\right)\,dx}{x}\\&=\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.53

method	result
gosper	$\frac{x^2(15x^2b^2+35abx+21a^2)(cx^2)^{\frac{3}{2}}}{105}$
default	$\frac{x^2(15x^2b^2+35abx+21a^2)(cx^2)^{\frac{3}{2}}}{105}$
risch	$\frac{a^2cx^4\sqrt{cx^2}}{5}+\frac{abcx^5\sqrt{cx^2}}{3}+\frac{b^2cx^6\sqrt{cx^2}}{7}$
trager	$\frac{c(15b^2x^6+35abx^5+15b^2x^5+21a^2x^4+35abx^4+15b^2x^4+21a^2x^3+35abx^3+15b^2x^3+21a^2x^2+35abx^2+15x^2b^2+21a^2x+35abx+15b^2x}{105x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/105*x^2*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.82

$$\frac{(cx^2)^{\frac{5}{2}}b^2x^2}{7c}+\frac{(cx^2)^{\frac{5}{2}}abx}{3c}+\frac{(cx^2)^{\frac{5}{2}}a^2}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/7*(c*x^2)^(5/2)*b^2*x^2/c + 1/3*(c*x^2)^(5/2)*a*b*x/c + 1/5*(c*x^2)^(5/2)
*a^2/c
```

Fricas [A]
time = 0.47, size = 36, normalized size = 0.60

$$\frac{1}{105}\left(15\,b^2cx^6+35\,abcx^5+21\,a^2cx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*c*x^6 + 35*a*b*c*x^5 + 21*a^2*c*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.19, size = 49, normalized size = 0.82

$$\frac{a^2x^2(cx^2)^{\frac{3}{2}}}{5}+\frac{abx^3(cx^2)^{\frac{3}{2}}}{3}+\frac{b^2x^4(cx^2)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**2*(c*x**2)**(3/2)/5 + a*b*x**3*(c*x**2)**(3/2)/3 + b**2*x**4*(c*x**
2)**(3/2)/7
```

Giac [A]
time = 1.48, size = 35, normalized size = 0.58

$$\frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x)+35\,abx^6\mathrm{sgn}(x)+21\,a^2x^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.815 $\int (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}$$

```
[Out] 1/4*a^2*c*x^3*(c*x^2)^(1/2)+2/5*a*b*c*x^4*(c*x^2)^(1/2)+1/6*b^2*c*x^5*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^3*Sqrt[c*x^2])/4 + (2*a*b*c*x^4*Sqrt[c*x^2])/5 + (b^2*c*x^5*Sqrt[c
*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^2 \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right) \, dx}{x} \\
&= \frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.50

method	result
gosper	$\frac{x\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{60}$
default	$\frac{x\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{60}$
risch	$\frac{a^2c\,x^3\sqrt{c\,x^2}}{4}+\frac{2abc\,x^4\sqrt{c\,x^2}}{5}+\frac{b^2c\,x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{c\left(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24ab\,x^3+10b^2x^3+15a^2x^2+24ab\,x^2+10x^2b^2+15a^2x+24abx+10b^2x+15a^2+24ab+10b^2\right)(-1+x)}{60x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/60*x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.73

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}}a^2x+\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x}{6\,c}+\frac{2\left(cx^2\right)^{\frac{5}{2}}ab}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*a^2*x + 1/6*(c*x^2)^(5/2)*b^2*x/c + 2/5*(c*x^2)^(5/2)*a*b
/c
```

Fricas [A]
time = 0.46, size = 36, normalized size = 0.60

$$\frac{1}{60}\left(10\,b^2cx^5+24\,abcx^4+15\,a^2cx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c*x^5 + 24*a*b*c*x^4 + 15*a^2*c*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.17, size = 49, normalized size = 0.82

$$\frac{a^2x(cx^2)^{\frac{3}{2}}}{4}+\frac{2abx^2(cx^2)^{\frac{3}{2}}}{5}+\frac{b^2x^3(cx^2)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x*(c*x**2)**(3/2)/4 + 2*a*b*x**2*(c*x**2)**(3/2)/5 + b**2*x**3*(c*x**2
)**(3/2)/6
```

Giac [A]
time = 1.30, size = 35, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x)+24\,abx^5\mathrm{sgn}(x)+15\,a^2x^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)*(a + b*x)^2, x)
```

3.816 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=60

$$\frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}$$

```
[Out] 1/3*a^2*c*x^2*(c*x^2)^(1/2)+1/2*a*b*c*x^3*(c*x^2)^(1/2)+1/5*b^2*c*x^4*(c*x^
2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c*x^2*Sqrt[c*x^2])/3 + (a*b*c*x^3*Sqrt[c*x^2])/2 + (b^2*c*x^4*Sqrt[c*x
^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\ &= \frac{1}{3}a^2cx^2\sqrt{cx^2}+\frac{1}{2}abcx^3\sqrt{cx^2}+\frac{1}{5}b^2cx^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.60

$$\frac{1}{30}cx^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
[Out] (c*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.48

method	result	size
gosper	$\frac{(6x^2b^2+15abx+10a^2)(cx^2)^{\frac{3}{2}}}{30}$	29
default	$\frac{(6x^2b^2+15abx+10a^2)(cx^2)^{\frac{3}{2}}}{30}$	29
risch	$\frac{a^2cx^2\sqrt{cx^2}}{3}+\frac{abcx^3\sqrt{cx^2}}{2}+\frac{b^2cx^4\sqrt{cx^2}}{5}$	49
trager	$\frac{c(6b^2x^4+15abx^3+6b^2x^3+10a^2x^2+15abx^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2)(-1+x)\sqrt{cx^2}}{30x}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/30*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.67

$$\frac{1}{2}\left(cx^2\right)^{\frac{3}{2}}abx+\frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a^2+\frac{\left(cx^2\right)^{\frac{5}{2}}b^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*(c*x^2)^(3/2)*a*b*x + 1/3*(c*x^2)^(3/2)*a^2 + 1/5*(c*x^2)^(5/2)*b^2/c
```

Fricas [A]
time = 0.46, size = 36, normalized size = 0.60

$$\frac{1}{30}\left(6\,b^2cx^4+15\,abcx^3+10\,a^2cx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*c*x^4 + 15*a*b*c*x^3 + 10*a^2*c*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.17, size = 44, normalized size = 0.73

$$\frac{a^2(cx^2)^{\frac{3}{2}}}{3}+\frac{abx(cx^2)^{\frac{3}{2}}}{2}+\frac{b^2x^2(cx^2)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*(c*x**2)**(3/2)/3 + a*b*x*(c*x**2)**(3/2)/2 + b**2*x**2*(c*x**2)**(3/2
)/5
```

Giac [A]
time = 1.24, size = 35, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x)+15\,abx^4\mathrm{sgn}(x)+10\,a^2x^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x, x)
```

3.817 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

```
[Out] 1/2*a^2*c*x*(c*x^2)^(1/2)+2/3*a*b*c*x^2*(c*x^2)^(1/2)+1/4*b^2*c*x^3*(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c*x*Sqrt[c*x^2])/2 + (2*a*b*c*x^2*Sqrt[c*x^2])/3 + (b^2*c*x^3*Sqrt[c*x
^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\ &= \frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.59

$$\frac{1}{12}cx\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.55

method	result	size
gosper	$\frac{(3x^2b^2+8abx+6a^2)(c\,x^2)^{\frac{3}{2}}}{12x}$	32
default	$\frac{(3x^2b^2+8abx+6a^2)(c\,x^2)^{\frac{3}{2}}}{12x}$	32
risch	$\frac{a^2cx\sqrt{c\,x^2}}{2}+\frac{2abc\,x^2\sqrt{c\,x^2}}{3}+\frac{b^2c\,x^3\sqrt{c\,x^2}}{4}$	47
trager	$\frac{c(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2)(-1+x)\sqrt{c\,x^2}}{12x}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(3/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.43, size = 34, normalized size = 0.59

$$\frac{1}{12}\left(3\,b^2cx^3+8\,abcx^2+6\,a^2cx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c*x^3 + 8*a*b*c*x^2 + 6*a^2*c*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.17, size = 44, normalized size = 0.76

$$\frac{a^2(cx^2)^{\frac{3}{2}}}{2x} + \frac{2ab(cx^2)^{\frac{3}{2}}}{3} + \frac{b^2x(cx^2)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*(c*x**2)**(3/2)/(2*x) + 2*a*b*(c*x**2)**(3/2)/3 + b**2*x*(c*x**2)**(3/
2)/4
```

Giac [A]
time = 1.07, size = 35, normalized size = 0.60

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x) + 8\,abx^3\mathrm{sgn}(x) + 6\,a^2x^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2, x)
```

3.818 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx$

Optimal. Leaf size=27

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

```
[Out] 1/3*c*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}\,(a+bx)^2}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\,(a+bx)^3}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^3)/(3*b*x^3)
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.85

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(bx+a\right)^{3}}{3x^{3}b}$	23
risch	$\frac{c\left(bx+a\right)^{3}\sqrt{c\,x^{2}}}{3bx}$	24
gosper	$\frac{\left(x^{2}b^{2}+3abx+3a^{2}\right)\left(c\,x^{2}\right)^{\frac{3}{2}}}{3x^{2}}$	31
trager	$\frac{c\left(x^{2}b^{2}+3abx+b^{2}x+3a^{2}+3ab+b^{2}\right)\left(-1+x\right)\sqrt{c\,x^{2}}}{3x}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2)^(3/2)/x^3*(b*x+a)^3/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.46, size = 30, normalized size = 1.11

$$\frac{1}{3}\left(b^{2}cx^{2}+3\,abcx+3\,a^{2}c\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c*x^2 + 3*a*b*c*x + 3*a^2*c)*sqrt(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.
time = 0.25, size = 41, normalized size = 1.52

$$\frac{a^2(cx^2)^{\frac{3}{2}}}{x^2}+\frac{ab(cx^2)^{\frac{3}{2}}}{x}+\frac{b^2(cx^2)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**3,x)
[Out] a**2*(c*x**2)**(3/2)/x**2 + a*b*(c*x**2)**(3/2)/x + b**2*(c*x**2)**(3/2)/3
```

Giac [A]
time = 1.47, size = 29, normalized size = 1.07

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}(x)}{b}-\frac{a^3\mathrm{sgn}(x)}{b}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="giac")
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c\,x^2)^{3/2}\,(a+b\,x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3, x)
```

3.819 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=52

$$2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}+\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}$$

```
[Out] 2*a*b*c*(c*x^2)^(1/2)+1/2*b^2*c*x*(c*x^2)^(1/2)+a^2*c*ln(x)*(c*x^2)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}+2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] 2*a*b*c*Sqrt[c*x^2] + (b^2*c*x*Sqrt[c*x^2])/2 + (a^2*c*Sqrt[c*x^2]*Log[x])/
x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}\\
&= 2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}+\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.65

$$\frac{(cx^2)^{3/2}\left(bx(4a+bx)+2a^2\log(x)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*x^3)
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.63

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(x^2b^2+2a^2\ln(x)+4abx\right)}{2x^3}$	33
risch	$\frac{c\sqrt{c\,x^2}\;b\left(\frac{1}{2}x^2b+2ax\right)}{x}+\frac{a^2c\ln(x)\sqrt{c\,x^2}}{x}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/x^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.49, size = 35, normalized size = 0.67

$$\frac{\left(b^2cx^2+4\,abcx+2\,a^2c\log\left(x\right)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*x^2 + 4*a*b*c*x + 2*a^2*c*log(x))*sqrt(c*x^2)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}} (a+bx)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**2/x**4, x)
```

Giac [**A**]
time = 1.66, size = 32, normalized size = 0.62

$$\frac{1}{2} \left(b^2x^2\text{sgn}(x) + 4\,abx\text{sgn}(x) + 2\,a^2 \log\left(|x|\right)\text{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4, x)
```

3.820 $\int x(cx^2)^{5/2}(a+bx)^2\,dx$

Optimal. Leaf size=66

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

```
[Out] 1/7*a^2*c^2*x^6*(c*x^2)^(1/2)+1/4*a*b*c^2*x^7*(c*x^2)^(1/2)+1/9*b^2*c^2*x^8
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^6*Sqrt[c*x^2])/7 + (a*b*c^2*x^7*Sqrt[c*x^2])/4 + (b^2*c^2*x^8*Sq
rt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{5/2}(a+bx)^2\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int\left(a^2x^6+2abx^7+b^2x^8\right)dx}{x}\\ &= \frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result
gosper	$\frac{x^2\left(28x^2b^2+63abx+36a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{252}$
default	$\frac{x^2\left(28x^2b^2+63abx+36a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{252}$
risch	$\frac{a^2c^2x^6\sqrt{c\,x^2}}{7}+\frac{ab\,c^2x^7\sqrt{c\,x^2}}{4}+\frac{b^2c^2x^8\sqrt{c\,x^2}}{9}$
trager	$\frac{c^2\left(28b^2x^8+63ab\,x^7+28b^2x^7+36a^2x^6+63ab\,x^6+28b^2x^6+36a^2x^5+63ab\,x^5+28b^2x^5+36a^2x^4+63ab\,x^4+28b^2x^4+36a^2x^3+63ab\,x^3+28\right.}{252x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/252*x^2*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.74

$$\frac{\left(cx^2\right)^{\frac{7}{2}}b^2x^2}{9\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}abx}{4\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}a^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(7/2)*b^2*x^2/c + 1/4*(c*x^2)^(7/2)*a*b*x/c + 1/7*(c*x^2)^(7/2)
*a^2/c
```

Fricas [A]
time = 0.42, size = 42, normalized size = 0.64

$$\frac{1}{252}\left(28\,b^2c^2x^8+63\,abc^2x^7+36\,a^2c^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c^2*x^8 + 63*a*b*c^2*x^7 + 36*a^2*c^2*x^6)*sqrt(c*x^2)
```

Sympy [**A**]
time = 0.36, size = 49, normalized size = 0.74

$$\frac{a^2x^2(cx^2)^{\frac{5}{2}}}{7}+\frac{abx^3(cx^2)^{\frac{5}{2}}}{4}+\frac{b^2x^4(cx^2)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x**2*(c*x**2)**(5/2)/7 + a*b*x**3*(c*x**2)**(5/2)/4 + b**2*x**4*(c*x**
2)**(5/2)/9
```

Giac [**A**]
time = 1.47, size = 44, normalized size = 0.67

$$\frac{1}{252}\left(28\,b^2c^2x^9\mathrm{sgn}(x)+63\,abc^2x^8\mathrm{sgn}(x)+36\,a^2c^2x^7\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*b^2*c^2*x^9*sgn(x) + 63*a*b*c^2*x^8*sgn(x) + 36*a^2*c^2*x^7*sgn(x
))*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{5/2}\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x)^2, x)
```

3.821 $\int (cx^2)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=66

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

```
[Out] 1/6*a^2*c^2*x^5*(c*x^2)^(1/2)+2/7*a*b*c^2*x^6*(c*x^2)^(1/2)+1/8*b^2*c^2*x^7
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^5*Sqrt[c*x^2])/6 + (2*a*b*c^2*x^6*Sqrt[c*x^2])/7 + (b^2*c^2*x^7*
Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(cx^2\right)^{5/2}(a+bx)^2\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^2\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^5+2abx^6+b^2x^7\right)dx}{x}\\
&= \frac{1}{6}a^2c^2x^5\sqrt{cx^2}+\frac{2}{7}abc^2x^6\sqrt{cx^2}+\frac{1}{8}b^2c^2x^7\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.50

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

Maple [A]
time = 0.10, size = 30, normalized size = 0.45

method	result
gosper	$\frac{x\left(21x^2b^2+48abx+28a^2\right)\left(cx^2\right)^{\frac{5}{2}}}{168}$
default	$\frac{x\left(21x^2b^2+48abx+28a^2\right)\left(cx^2\right)^{\frac{5}{2}}}{168}$
risch	$\frac{a^2c^2x^5\sqrt{cx^2}}{6}+\frac{2ab\,c^2x^6\sqrt{cx^2}}{7}+\frac{b^2c^2x^7\sqrt{cx^2}}{8}$
trager	$\frac{c^2\left(21b^2x^7+48ab\,x^6+21b^2x^6+28a^2x^5+48ab\,x^5+21b^2x^5+28a^2x^4+48ab\,x^4+21b^2x^4+28a^2x^3+48ab\,x^3+21b^2x^3+28a^2x^2+48ab\,x^2+21x\right.}{168x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/168*x*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.67

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}a^2x+\frac{\left(cx^2\right)^{\frac{7}{2}}b^2x}{8\,c}+\frac{2\left(cx^2\right)^{\frac{7}{2}}ab}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*a^2*x + 1/8*(c*x^2)^(7/2)*b^2*x/c + 2/7*(c*x^2)^(7/2)*a*b
/c
```

Fricas [A]
time = 0.42, size = 42, normalized size = 0.64

$$\frac{1}{168}\left(21\,b^2c^2x^7+48\,abc^2x^6+28\,a^2c^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c^2*x^7 + 48*a*b*c^2*x^6 + 28*a^2*c^2*x^5)*sqrt(c*x^2)
```

Sympy [A]
time = 0.30, size = 49, normalized size = 0.74

$$\frac{a^2x(cx^2)^{\frac{5}{2}}}{6}+\frac{2abx^2(cx^2)^{\frac{5}{2}}}{7}+\frac{b^2x^3(cx^2)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*x*(c*x**2)**(5/2)/6 + 2*a*b*x**2*(c*x**2)**(5/2)/7 + b**2*x**3*(c*x**2
)**(5/2)/8
```

Giac [A]
time = 1.81, size = 44, normalized size = 0.67

$$\frac{1}{168}\left(21\,b^2c^2x^8\mathrm{sgn}(x)+48\,abc^2x^7\mathrm{sgn}(x)+28\,a^2c^2x^6\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*c^2*x^8*sgn(x) + 48*a*b*c^2*x^7*sgn(x) + 28*a^2*c^2*x^6*sgn(x
))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(5/2)*(a + b*x)^2, x)
```

3.822 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx$

Optimal. Leaf size=66

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

```
[Out] 1/5*a^2*c^2*x^4*(c*x^2)^(1/2)+1/3*a*b*c^2*x^5*(c*x^2)^(1/2)+1/7*b^2*c^2*x^6
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c^2*x^4*Sqrt[c*x^2])/5 + (a*b*c^2*x^5*Sqrt[c*x^2])/3 + (b^2*c^2*x^6*Sq
rt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^4+2abx^5+b^2x^6\right)\,dx}{x}\\
&= \frac{1}{5}a^2c^2x^4\sqrt{cx^2}+\frac{1}{3}abc^2x^5\sqrt{cx^2}+\frac{1}{7}b^2c^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.55

$$\frac{1}{105}cx^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (c*x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

Maple [A]
time = 0.11, size = 29, normalized size = 0.44

method	result
gosper	$\frac{\left(15x^2b^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
default	$\frac{\left(15x^2b^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{105}$
risch	$\frac{a^2c^2x^4\sqrt{c\,x^2}}{5}+\frac{ab\,c^2x^5\sqrt{c\,x^2}}{3}+\frac{b^2c^2x^6\sqrt{c\,x^2}}{7}$
trager	$\frac{c^2\left(15b^2x^6+35ab\,x^5+15b^2x^5+21a^2x^4+35ab\,x^4+15b^2x^4+21a^2x^3+35ab\,x^3+15b^2x^3+21a^2x^2+35ab\,x^2+15x^2b^2+21a^2x+35abx+15b^2\right.}{105x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.61

$$\frac{1}{3}\left(cx^2\right)^{\frac{5}{2}}abx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a^2+\frac{\left(cx^2\right)^{\frac{7}{2}}b^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^2)^(5/2)*a*b*x + 1/5*(c*x^2)^(5/2)*a^2 + 1/7*(c*x^2)^(7/2)*b^2/c
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.64

$$\frac{1}{105}\left(15\,b^2c^2x^6+35\,abc^2x^5+21\,a^2c^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^2*c^2*x^6 + 35*a*b*c^2*x^5 + 21*a^2*c^2*x^4)*sqrt(c*x^2)
```

Sympy [A]
time = 0.31, size = 44, normalized size = 0.67

$$\frac{a^2(cx^2)^{\frac{5}{2}}}{5}+\frac{abx(cx^2)^{\frac{5}{2}}}{3}+\frac{b^2x^2(cx^2)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/5 + a*b*x*(c*x**2)**(5/2)/3 + b**2*x**2*(c*x**2)**(5/2
)/7
```

Giac [A]
time = 0.97, size = 44, normalized size = 0.67

$$\frac{1}{105}\left(15\,b^2c^2x^7\mathrm{sgn}(x)+35\,abc^2x^6\mathrm{sgn}(x)+21\,a^2c^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*c^2*x^7*sgn(x) + 35*a*b*c^2*x^6*sgn(x) + 21*a^2*c^2*x^5*sgn(x
))*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x, x)
```

3.823 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

```
[Out] 1/4*a^2*c^2*x^3*(c*x^2)^(1/2)+2/5*a*b*c^2*x^4*(c*x^2)^(1/2)+1/6*b^2*c^2*x^5
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*c^2*x^4*Sqrt[c*x^2])/5 + (b^2*c^2*x^5*
Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^2\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\
&= \frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.52

$$\frac{1}{60}cx\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
[Out] (c*x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.48

method	result
gosper	$\frac{\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{60x}$
default	$\frac{\left(10x^2b^2+24abx+15a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{60x}$
risch	$\frac{a^2c^2x^3\sqrt{c\,x^2}}{4}+\frac{2ab\,c^2x^4\sqrt{c\,x^2}}{5}+\frac{b^2c^2x^5\sqrt{c\,x^2}}{6}$
trager	$\frac{c^2\left(10b^2x^5+24ab\,x^4+10b^2x^4+15a^2x^3+24ab\,x^3+10b^2x^3+15a^2x^2+24ab\,x^2+10x^2b^2+15a^2x+24abx+10b^2x+15a^2+24ab+10b^2\right)(-1+x}{60x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/60/x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.61

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}b^2x+\frac{2}{5}\left(cx^2\right)^{\frac{5}{2}}ab+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b^2*x + 2/5*(c*x^2)^(5/2)*a*b + 1/4*(c*x^2)^(5/2)*a^2/x
```

Fricas [A]
time = 0.39, size = 42, normalized size = 0.64

$$\frac{1}{60}\left(10\,b^2c^2x^5+24\,abc^2x^4+15\,a^2c^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c^2*x^5 + 24*a*b*c^2*x^4 + 15*a^2*c^2*x^3)*sqrt(c*x^2)
```

Sympy [A]
time = 0.33, size = 44, normalized size = 0.67

$$\frac{a^2(cx^2)^{\frac{5}{2}}}{4x} + \frac{2ab(cx^2)^{\frac{5}{2}}}{5} + \frac{b^2x(cx^2)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/(4*x) + 2*a*b*(c*x**2)**(5/2)/5 + b**2*x*(c*x**2)**(5/
2)/6
```

Giac [A]
time = 1.25, size = 44, normalized size = 0.67

$$\frac{1}{60}\left(10\,b^2c^2x^6\mathrm{sgn}(x) + 24\,abc^2x^5\mathrm{sgn}(x) + 15\,a^2c^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*c^2*x^6*sgn(x) + 24*a*b*c^2*x^5*sgn(x) + 15*a^2*c^2*x^4*sgn(x)
)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2, x)
```

3.824 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=66

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

```
[Out] 1/3*a^2*c^2*x^2*(c*x^2)^(1/2)+1/2*a*b*c^2*x^3*(c*x^2)^(1/2)+1/5*b^2*c^2*x^4
*(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (a^2*c^2*x^2*Sqrt[c*x^2])/3 + (a*b*c^2*x^3*Sqrt[c*x^2])/2 + (b^2*c^2*x^4*Sq
rt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^2 dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right) dx}{x} \\
&= \frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 0.58

$$\frac{1}{30}c^2x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result	size
gosper	$\frac{\left(6x^2b^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{30x^2}$	32
default	$\frac{\left(6x^2b^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{30x^2}$	32
risch	$\frac{a^2c^2x^2\sqrt{c\,x^2}}{3}+\frac{ab\,c^2x^3\sqrt{c\,x^2}}{2}+\frac{b^2c^2x^4\sqrt{c\,x^2}}{5}$	55
trager	$\frac{c^2\left(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2\right)(-1+x)\sqrt{c\,x^2}}{30x}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.42, size = 42, normalized size = 0.64

$$\frac{1}{30}\left(6\,b^2c^2x^4+15\,abc^2x^3+10\,a^2c^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
[Out] 1/30*(6*b^2*c^2*x^4 + 15*a*b*c^2*x^3 + 10*a^2*c^2*x^2)*sqrt(c*x^2)
```

Sympy [A]
time = 0.38, size = 44, normalized size = 0.67

$$\frac{a^2(cx^2)^{\frac{5}{2}}}{3x^2} + \frac{ab(cx^2)^{\frac{5}{2}}}{2x} + \frac{b^2(cx^2)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**3,x)
[Out] a**2*(c*x**2)**(5/2)/(3*x**2) + a*b*(c*x**2)**(5/2)/(2*x) + b**2*(c*x**2)**
(5/2)/5
```

Giac [A]
time = 1.18, size = 44, normalized size = 0.67

$$\frac{1}{30}\left(6\,b^2c^2x^5\mathrm{sgn}(x) + 15\,abc^2x^4\mathrm{sgn}(x) + 10\,a^2c^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="giac")
[Out] 1/30*(6*b^2*c^2*x^5*sgn(x) + 15*a*b*c^2*x^4*sgn(x) + 10*a^2*c^2*x^3*sgn(x))
*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3, x)
```

3.825 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4} dx$

Optimal. Leaf size=64

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

```
[Out] 1/2*a^2*c^2*x*(c*x^2)^(1/2)+2/3*a*b*c^2*x^2*(c*x^2)^(1/2)+1/4*b^2*c^2*x^3*(
c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c*x^2])/2 + (2*a*b*c^2*x^2*Sqrt[c*x^2])/3 + (b^2*c^2*x^3*Sq
rt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)\,dx}{x}\\ &= \frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.56

$$\frac{1}{12}c^2x\sqrt{cx^2}\,(6a^2+8abx+3b^2x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
[Out] (c^2*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.50

method	result	size
gosper	$\frac{(3x^2b^2+8abx+6a^2)(c\,x^2)^{\frac{5}{2}}}{12x^3}$	32
default	$\frac{(3x^2b^2+8abx+6a^2)(c\,x^2)^{\frac{5}{2}}}{12x^3}$	32
risch	$\frac{a^2c^2x\sqrt{c\,x^2}}{2}+\frac{2ab\,c^2x^2\sqrt{c\,x^2}}{3}+\frac{b^2c^2x^3\sqrt{c\,x^2}}{4}$	53
trager	$\frac{c^2(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2)(-1+x)\sqrt{c\,x^2}}{12x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x,method=_RETURNVERBOSE)
[Out] 1/12/x^3*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(5/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.44, size = 40, normalized size = 0.62

$$\frac{1}{12}\left(3\,b^2c^2x^3+8\,abc^2x^2+6\,a^2c^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^2*x^3 + 8*a*b*c^2*x^2 + 6*a^2*c^2*x)*sqrt(c*x^2)
```

Sympy [A]
time = 0.38, size = 49, normalized size = 0.77

$$\frac{a^2(cx^2)^{\frac{5}{2}}}{2x^3} + \frac{2ab(cx^2)^{\frac{5}{2}}}{3x^2} + \frac{b^2(cx^2)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**4,x)
```

```
[Out] a**2*(c*x**2)**(5/2)/(2*x**3) + 2*a*b*(c*x**2)**(5/2)/(3*x**2) + b**2*(c*x*
*2)**(5/2)/(4*x)
```

Giac [A]
time = 1.94, size = 44, normalized size = 0.69

$$\frac{1}{12} \left(3 \, b^2 c^2 x^4 \mathrm{sgn}(x) + 8 \, abc^2 x^3 \mathrm{sgn}(x) + 6 \, a^2 c^2 x^2 \mathrm{sgn}(x)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*c^2*x^4*sgn(x) + 8*a*b*c^2*x^3*sgn(x) + 6*a^2*c^2*x^2*sgn(x))*s
qrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4, x)
```

3.826 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5} dx$

Optimal. Leaf size=29

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

```
[Out] 1/3*c^2*(b*x+a)^3*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}\,(a+bx)^2}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.90

$$\frac{(cx^2)^{5/2}\,(a+bx)^3}{3bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(a + b*x)^3)/(3*b*x^5)
```

Maple [A]
time = 0.12, size = 23, normalized size = 0.79

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(bx+a\right)^{3}}{3x^{5}b}$	23
risch	$\frac{c^{2}\left(bx+a\right)^{3}\sqrt{c\,x^{2}}}{3bx}$	26
gosper	$\frac{\left(x^{2}b^{2}+3abx+3a^{2}\right)\left(c\,x^{2}\right)^{\frac{5}{2}}}{3x^{4}}$	31
trager	$\frac{c^{2}\left(x^{2}b^{2}+3abx+b^{2}x+3a^{2}+3ab+b^{2}\right)\left(-1+x\right)\sqrt{c\,x^{2}}}{3x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^2)^(5/2)/x^5*(b*x+a)^3/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.42, size = 36, normalized size = 1.24

$$\frac{1}{3}\left(b^{2}c^{2}x^{2}+3\,abc^{2}x+3\,a^{2}c^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^2 + 3*a*b*c^2*x + 3*a^2*c^2)*sqrt(c*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.39, size = 46, normalized size = 1.59

$$\frac{a^2(cx^2)^{\frac{5}{2}}}{x^4} + \frac{ab(cx^2)^{\frac{5}{2}}}{x^3} + \frac{b^2(cx^2)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**5,x)
[Out] a**2*(c*x**2)**(5/2)/x**4 + a*b*(c*x**2)**(5/2)/x**3 + b**2*(c*x**2)**(5/2)
/(3*x**2)
```

Giac [A]
time = 1.34, size = 41, normalized size = 1.41

$$\frac{1}{3}\left(b^2c^2x^3\mathrm{sgn}(x) + 3\,abc^2x^2\mathrm{sgn}(x) + 3\,a^2c^2x\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="giac")
[Out] 1/3*(b^2*c^2*x^3*sgn(x) + 3*a*b*c^2*x^2*sgn(x) + 3*a^2*c^2*x*sgn(x))*sqrt(c
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(c\,x^2)^{5/2}\,(a+b\,x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5, x)
```

3.827 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6} dx$

Optimal. Leaf size=58

$$2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2} + \frac{a^2c^2\sqrt{cx^2}\log(x)}{x}$$

```
[Out] 2*a*b*c^2*(c*x^2)^(1/2)+1/2*b^2*c^2*x*(c*x^2)^(1/2)+a^2*c^2*ln(x)*(c*x^2)^(
1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c^2\sqrt{cx^2}\log(x)}{x} + 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] 2*a*b*c^2*Sqrt[c*x^2] + (b^2*c^2*x*Sqrt[c*x^2])/2 + (a^2*c^2*Sqrt[c*x^2]*Lo
g[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x} dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(2ab + \frac{a^2}{x} + b^2x\right) dx}{x} \\
&= 2abc^2\sqrt{cx^2} + \frac{1}{2}b^2c^2x\sqrt{cx^2} + \frac{a^2c^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.60

$$\frac{c^3x(bx(4a+bx)+2a^2\log(x))}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] (c^3*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.57

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(x^{2}b^{2}+2a^{2}\ln\left(x\right)+4abx\right)}{2x^{5}}$	33
risch	$\frac{c^{2}\sqrt{c\,x^{2}}\;b\left(\frac{1}{2}x^{2}b+2ax\right)}{x}+\frac{a^{2}c^{2}\ln\left(x\right)\sqrt{c\,x^{2}}}{x}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(5/2)*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/x^5
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.44, size = 41, normalized size = 0.71

$$\frac{\left(b^2c^2x^2+4\,abc^2x+2\,a^2c^2\log\left(x\right)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 + 4*a*b*c^2*x + 2*a^2*c^2*log(x))*sqrt(c*x^2)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}} (a+bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**2/x**6, x)
```

Giac [**A**]
time = 1.15, size = 41, normalized size = 0.71

$$\frac{1}{2}\left(b^2c^2x^2\mathrm{sgn}(x) + 4\,abc^2x\mathrm{sgn}(x) + 2\,a^2c^2\log\left(|x|\right)\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2*x^2*sgn(x) + 4*a*b*c^2*x*sgn(x) + 2*a^2*c^2*log(abs(x))*sgn(x)
)*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^6, x)
```

3.828 $\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

```
[Out] 1/3*a^2*x^4/(c*x^2)^(1/2)+1/2*a*b*x^5/(c*x^2)^(1/2)+1/5*b^2*x^6/(c*x^2)^(1/
2)
```

Rubi [A]

time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^4)/(3*Sqrt[c*x^2]) + (a*b*x^5)/(2*Sqrt[c*x^2]) + (b^2*x^6)/(5*Sqrt[c
*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}}\,dx&=\frac{x\int x^2(a+bx)^2\,dx}{\sqrt{cx^2}}\\&=\frac{x\int\left(a^2x^2+2abx^3+b^2x^4\right)\,dx}{\sqrt{cx^2}}\\&=\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{x^4(10a^2+15abx+6b^2x^2)}{30\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*Sqrt[c*x^2])
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.56

method	result	size
gosper	$\frac{x^4(6x^2b^2+15abx+10a^2)}{30\sqrt{c\,x^2}}$	32
default	$\frac{x^4(6x^2b^2+15abx+10a^2)}{30\sqrt{c\,x^2}}$	32
risch	$\frac{a^2x^4}{3\sqrt{c\,x^2}}+\frac{ab\,x^5}{2\sqrt{c\,x^2}}+\frac{b^2x^6}{5\sqrt{c\,x^2}}$	46
trager	$\frac{(6b^2x^4+15ab\,x^3+6b^2x^3+10a^2x^2+15ab\,x^2+6x^2b^2+10a^2x+15abx+6b^2x+10a^2+15ab+6b^2)(-1+x)\sqrt{c\,x^2}}{30cx}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*x^4*(6*b^2*x^2+15*a*b*x+10*a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.95

$$\frac{\sqrt{cx^2}\,b^2x^4}{5\,c}+\frac{\sqrt{cx^2}\,abx^3}{2\,c}+\frac{\sqrt{cx^2}\,a^2x^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2)*b^2*x^4/c + 1/2*sqrt(c*x^2)*a*b*x^3/c + 1/3*sqrt(c*x^2)*a^2
*x^2/c
```

Fricas [A]
time = 0.44, size = 36, normalized size = 0.63

$$\frac{(6\,b^2x^4+15\,abx^3+10\,a^2x^2)\sqrt{cx^2}}{30\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.24, size = 49, normalized size = 0.86

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4/(3*sqrt(c*x**2)) + a*b*x**5/(2*sqrt(c*x**2)) + b**2*x**6/(5*sqrt(
c*x**2))
```

Giac [A]
time = 1.02, size = 42, normalized size = 0.74

$$\frac{6\,b^2\sqrt{c}\,x^5+15\,ab\sqrt{c}\,x^4+10\,a^2\sqrt{c}\,x^3}{30\,c\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*sqrt(c)*x^5 + 15*a*b*sqrt(c)*x^4 + 10*a^2*sqrt(c)*x^3)/(c*sgn(x
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

3.829 $\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}} dx$

Optimal. Leaf size=57

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

```
[Out] 1/2*a^2*x^3/(c*x^2)^(1/2)+2/3*a*b*x^4/(c*x^2)^(1/2)+1/4*b^2*x^5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^3)/(2*Sqrt[c*x^2]) + (2*a*b*x^4)/(3*Sqrt[c*x^2]) + (b^2*x^5)/(4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}} dx &= \frac{x \int x(a+bx)^2 dx}{\sqrt{cx^2}} \\
&= \frac{x \int (a^2x + 2abx^2 + b^2x^3) dx}{\sqrt{cx^2}} \\
&= \frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.61

$$\frac{x^3(6a^2 + 8abx + 3b^2x^2)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^3*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*Sqrt[c*x^2])
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.56

method	result	size
gosper	$\frac{x^3(3x^2b^2+8abx+6a^2)}{12\sqrt{c\,x^2}}$	32
default	$\frac{x^3(3x^2b^2+8abx+6a^2)}{12\sqrt{c\,x^2}}$	32
risch	$\frac{a^2x^3}{2\sqrt{c\,x^2}}+\frac{2ab\,x^4}{3\sqrt{c\,x^2}}+\frac{b^2x^5}{4\sqrt{c\,x^2}}$	46
trager	$\frac{(3b^2x^3+8ab\,x^2+3x^2b^2+6a^2x+8abx+3b^2x+6a^2+8ab+3b^2)(-1+x)\sqrt{c\,x^2}}{12cx}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*x^3*(3*b^2*x^2+8*a*b*x+6*a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,b^2x^3}{4\,c} + \frac{2\,\sqrt{cx^2}\,abx^2}{3\,c} + \frac{a^2x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(c*x^2)*b^2*x^3/c + 2/3*sqrt(c*x^2)*a*b*x^2/c + 1/2*a^2*x^2/sqrt(c)
```

Fricas [A]
time = 0.44, size = 34, normalized size = 0.60

$$\frac{(3\,b^2x^3 + 8\,abx^2 + 6\,a^2x)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)/c
```

Sympy [A]
time = 0.22, size = 51, normalized size = 0.89

$$\frac{a^2x^3}{2\sqrt{cx^2}}+\frac{2abx^4}{3\sqrt{cx^2}}+\frac{b^2x^5}{4\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3/(2*sqrt(c*x**2)) + 2*a*b*x**4/(3*sqrt(c*x**2)) + b**2*x**5/(4*sqr
t(c*x**2))
```

Giac [A]
time = 0.87, size = 42, normalized size = 0.74

$$\frac{3\,b^2\sqrt{c}\,x^4+8\,ab\sqrt{c}\,x^3+6\,a^2\sqrt{c}\,x^2}{12\,c\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*sqrt(c)*x^4 + 8*a*b*sqrt(c)*x^3 + 6*a^2*sqrt(c)*x^2)/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

3.830 $\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

```
[Out] 1/3*x*(b*x+a)^3/b/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.88

method	result	size
default	$\frac{x(bx+a)^3}{3b\sqrt{c\,x^2}}$	21
risch	$\frac{x(bx+a)^3}{3b\sqrt{c\,x^2}}$	21
gosper	$\frac{x^2(x^2b^2+3abx+3a^2)}{3\sqrt{c\,x^2}}$	31
trager	$\frac{(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c\,x^2}}{3cx}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(b*x+a)^3/b/(c*x^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 0.26, size = 42, normalized size = 1.75

$$\frac{\sqrt{cx^2}\,b^2x^2}{3\,c}+\frac{abx^2}{\sqrt{c}}+\frac{\sqrt{cx^2}\,a^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(c*x^2)*b^2*x^2/c + a*b*x^2/sqrt(c) + sqrt(c*x^2)*a^2/c
```

Fricas [A]
time = 0.41, size = 30, normalized size = 1.25

$$\frac{(b^2x^2+3\,abx+3\,a^2)\sqrt{cx^2}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.20, size = 46, normalized size = 1.92

$$\frac{a^2x^2}{\sqrt{cx^2}} + \frac{abx^3}{\sqrt{cx^2}} + \frac{b^2x^4}{3\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] a**2*x**2/sqrt(c*x**2) + a*b*x**3/sqrt(c*x**2) + b**2*x**4/(3*sqrt(c*x**2))
```

Giac [A]
time = 0.93, size = 39, normalized size = 1.62

$$\frac{b^2\sqrt{c}\,x^3 + 3\,ab\sqrt{c}\,x^2 + 3\,a^2\sqrt{c}\,x}{3\,c\text{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(b^2*sqrt(c)*x^3 + 3*a*b*sqrt(c)*x^2 + 3*a^2*sqrt(c)*x)/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(1/2),x)
[Out] int((x*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

3.831 $\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=52

$$\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}$$

```
[Out] 2*a*b*x^2/(c*x^2)^(1/2)+1/2*b^2*x^3/(c*x^2)^(1/2)+a^2*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{a^2x\log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (2*a*b*x^2)/Sqrt[c*x^2] + (b^2*x^3)/(2*Sqrt[c*x^2]) + (a^2*x*Log[x])/Sqrt[c
*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 0.62

$$\frac{x(bx(4a+bx)+2a^2\log(x))}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c*x^2],x]
[Out] (x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 31, normalized size = 0.60

method	result	size
default	$\frac{x(x^2b^2+2a^2\ln(x)+4abx)}{2\sqrt{c\,x^2}}$	31
risch	$\frac{xb(\frac{1}{2}x^2b+2ax)}{\sqrt{c\,x^2}}+\frac{a^2x\ln(x)}{\sqrt{c\,x^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*x*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.67

$$\frac{b^2x^2}{2\sqrt{c}}+\frac{a^2\log(x)}{\sqrt{c}}+\frac{2\sqrt{cx^2}\,ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*b^2*x^2/sqrt(c) + a^2*log(x)/sqrt(c) + 2*sqrt(c*x^2)*a*b/c
```

Fricas [A]
time = 0.40, size = 35, normalized size = 0.67

$$\frac{(b^2x^2+4\,abx+2\,a^2\log(x))\sqrt{cx^2}}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/sqrt(c*x**2), x)
```

Giac [A]
time = 0.99, size = 43, normalized size = 0.83

$$\frac{a^2\log(|x|)}{\sqrt{c}\,\mathrm{sgn}(x)} + \frac{b^2c^{\frac{3}{2}}x^2\mathrm{sgn}(x) + 4\,abc^{\frac{3}{2}}x\mathrm{sgn}(x)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^2*log(abs(x))/(sqrt(c)*sgn(x)) + 1/2*(b^2*c^(3/2)*x^2*sgn(x) + 4*a*b*c^(3
/2)*x*sgn(x))/c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(1/2),x)
```

```
[Out] int((a + b*x)^2/(c*x^2)^(1/2), x)
```

3.832 $\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}$$

```
[Out] -a^2/(c*x^2)^(1/2)+b^2*x^2/(c*x^2)^(1/2)+2*a*b*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a^2/Sqrt[c*x^2]) + (b^2*x^2)/Sqrt[c*x^2] + (2*a*b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.72

$$\frac{cx^2(-a^2 + b^2x^2 + 2abx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
[Out] (c*x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.62

method	result	size
default	$\frac{2ab\ln(x)x+x^2b^2-a^2}{\sqrt{c\,x^2}}$	29
risch	$-\frac{a^2}{\sqrt{c\,x^2}}+\frac{b^2x^2}{\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{\sqrt{c\,x^2}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] (2*a*b*ln(x)*x+x^2*b^2-a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.74

$$\frac{2\,ab\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}\,b^2}{c} - \frac{a^2}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 2*a*b*log(x)/sqrt(c) + sqrt(c*x^2)*b^2/c - a^2/(sqrt(c)*x)
```

Fricas [A]
time = 0.44, size = 34, normalized size = 0.72

$$\frac{(b^2x^2 + 2\,abx\log(x) - a^2)\sqrt{cx^2}}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x*sqrt(c*x**2)), x)
```

Giac [A]
time = 1.08, size = 42, normalized size = 0.89

$$\frac{b^2x}{\sqrt{c}\,\mathrm{sgn}\,(x)}+\frac{2\,ab\log\left(|x|\right)}{\sqrt{c}\,\mathrm{sgn}\,(x)}-\frac{a^2}{\sqrt{c}\,x\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b^2*x/(sqrt(c)*sgn(x)) + 2*a*b*log(abs(x))/(sqrt(c)*sgn(x)) - a^2/(sqrt(c)*
x*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^2}{x\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^2/(x*(c*x^2)^(1/2)), x)
```

3.833 $\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

```
[Out] -2*a*b/(c*x^2)^(1/2)-1/2*a^2/x/(c*x^2)^(1/2)+b^2*x*ln(x)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (-2*a*b)/Sqrt[c*x^2] - a^2/(2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.71

$$\frac{cx(-a(a+4bx)+2b^2x^2\log(x))}{2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
[Out] (c*x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 34, normalized size = 0.69

method	result	size
default	$\frac{2b^2\ln(x)x^2-4abx-a^2}{2x\sqrt{c\,x^2}}$	34
risch	$\frac{-\frac{1}{2}a^2-2abx}{\sqrt{c\,x^2}\,x}+\frac{b^2x\ln(x)}{\sqrt{c\,x^2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 0.63

$$\frac{b^2\log(x)}{\sqrt{c}}-\frac{2\,ab}{\sqrt{c}\,x}-\frac{a^2}{2\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b^2*log(x)/sqrt(c) - 2*a*b/(sqrt(c)*x) - 1/2*a^2/(sqrt(c)*x^2)
```

Fricas [A]
time = 0.39, size = 36, normalized size = 0.73

$$\frac{(2\,b^2x^2\log(x)-4\,abx-a^2)\sqrt{cx^2}}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x**2*sqrt(c*x**2)), x)
```

Giac [A]
time = 1.75, size = 43, normalized size = 0.88

$$\frac{b^2\log\left(|x|\right)}{\sqrt{c}\,\mathrm{sgn}\left(x\right)}-\frac{4\,ab\sqrt{c}\,x+a^2\sqrt{c}}{2\,cx^2\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b^2*log(abs(x))/(sqrt(c)*sgn(x)) - 1/2*(4*a*b*sqrt(c)*x + a^2*sqrt(c))/(c*x
^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^2}{x^2\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)), x)
```

3.834 $\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

[Out] -1/3*(b*x+a)^3/a/x^2/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]

[Out] -1/3*(a + b*x)^3/(a*x^2*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.27

$$\frac{c(-a^2-3abx-3b^2x^2)}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
[Out] (c*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(3/2))
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.15

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
default	$-\frac{3x^2b^2+3abx+a^2}{3x^2\sqrt{c\,x^2}}$	30
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{x^2\sqrt{c\,x^2}}$	31
trager	$\frac{(-1+x)\left(a^2x^2+3ab\,x^2+3x^2b^2+a^2x+3abx+a^2\right)\sqrt{c\,x^2}}{3c\,x^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^2/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 1.27

$$-\frac{b^2}{\sqrt{c}\,x}-\frac{ab}{\sqrt{c}\,x^2}-\frac{a^2}{3\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -b^2/(sqrt(c)*x) - a*b/(sqrt(c)*x^2) - 1/3*a^2/(sqrt(c)*x^3)
```

Fricas [A]
time = 0.41, size = 32, normalized size = 1.23

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c*x^4)
```

Sympy [A]
time = 0.23, size = 42, normalized size = 1.62

$$-\frac{a^2}{3x^2\sqrt{cx^2}}-\frac{ab}{x\sqrt{cx^2}}-\frac{b^2}{\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(3*x**2*sqrt(c*x**2)) - a*b/(x*sqrt(c*x**2)) - b**2/sqrt(c*x**2)
```

Giac [A]
time = 1.36, size = 39, normalized size = 1.50

$$-\frac{3\,b^2\sqrt{c}\,x^2+3\,ab\sqrt{c}\,x+a^2\sqrt{c}}{3\,cx^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x + a^2*sqrt(c))/(c*x^3*sgn(x))
```

Mupad [B]
time = 0.18, size = 33, normalized size = 1.27

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,\sqrt{c}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(1/2)),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(1/2)*(x^2)^(5/2))
```

3.835 $\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

```
[Out] -1/4*a^2/x^3/(c*x^2)^(1/2)-2/3*a*b/x^2/(c*x^2)^(1/2)-1/2*b^2/x/(c*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -1/4*a^2/(x^3*Sqrt[c*x^2]) - (2*a*b)/(3*x^2*Sqrt[c*x^2]) - b^2/(2*x*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{a^2}{4x^3\sqrt{cx^2}} - \frac{2ab}{3x^2\sqrt{cx^2}} - \frac{b^2}{2x\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.61

$$\frac{-3a^2-8abx-6b^2x^2}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x^3*Sqrt[c*x^2])
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.56

method	result	size
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{x^3\sqrt{c\,x^2}}$	31
gosper	$-\frac{6x^2b^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32
default	$-\frac{6x^2b^2+8abx+3a^2}{12x^3\sqrt{c\,x^2}}$	32
trager	$\frac{(-1+x)\left(3a^2x^3+8ab\,x^3+6b^2x^3+3a^2x^2+8ab\,x^2+6x^2b^2+3a^2x+8abx+3a^2\right)\sqrt{c\,x^2}}{12c\,x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x^3/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.58

$$-\frac{b^2}{2\sqrt{c}\,x^2} - \frac{2\,ab}{3\sqrt{c}\,x^3} - \frac{a^2}{4\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*b^2/(sqrt(c)*x^2) - 2/3*a*b/(sqrt(c)*x^3) - 1/4*a^2/(sqrt(c)*x^4)
```

Fricas [A]
time = 0.47, size = 34, normalized size = 0.60

$$-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c*x^5)
```

Sympy [A]
time = 0.26, size = 51, normalized size = 0.89

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(1/2),x)
[Out] -a**2/(4*x**3*sqrt(c*x**2)) - 2*a*b/(3*x**2*sqrt(c*x**2)) - b**2/(2*x*sqrt(
c*x**2))
```

Giac [A]
time = 1.39, size = 40, normalized size = 0.70

$$-\frac{6\,b^2\sqrt{c}\,x^2+8\,ab\sqrt{c}\,x+3\,a^2\sqrt{c}}{12\,cx^4\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -1/12*(6*b^2*sqrt(c)*x^2 + 8*a*b*sqrt(c)*x + 3*a^2*sqrt(c))/(c*x^4*sgn(x))
```

Mupad [B]
time = 0.19, size = 42, normalized size = 0.74

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(1/2)),x)
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(1
/2)*x^5)
```

3.836 $\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=27

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

```
[Out] 1/3*x*(b*x+a)^3/b/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)^2\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 0.96

$$\frac{x^3(a+bx)^3}{3b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^3)/(3*b*(c*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 23, normalized size = 0.85

method	result	size
default	$\frac{(bx+a)^3 x^3}{3(c x^2)^{\frac{3}{2}} b}$	23
risch	$\frac{x(bx+a)^3}{3bc\sqrt{c x^2}}$	24
gosper	$\frac{x^4(x^2b^2+3abx+3a^2)}{3(c x^2)^{\frac{3}{2}}}$	31
trager	$\frac{(x^2b^2+3abx+b^2x+3a^2+3ab+b^2)(-1+x)\sqrt{c x^2}}{3c^2x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/(c*x^2)^(3/2)*x^3/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.
time = 0.27, size = 52, normalized size = 1.93

$$\frac{b^2x^4}{3\sqrt{cx^2}\,c}+\frac{abx^3}{\sqrt{cx^2}\,c}+\frac{a^2x^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^4/(sqrt(c*x^2)*c) + a*b*x^3/(sqrt(c*x^2)*c) + a^2*x^2/(sqrt(c*x^2
)*c)
```

Fricas [A]
time = 0.43, size = 30, normalized size = 1.11

$$\frac{(b^2x^2+3\,abx+3\,a^2)\sqrt{cx^2}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.
time = 0.25, size = 46, normalized size = 1.70

$$\frac{a^2x^4}{(cx^2)^{\frac{3}{2}}} + \frac{abx^5}{(cx^2)^{\frac{3}{2}}} + \frac{b^2x^6}{3\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] a**2*x**4/(c*x**2)**(3/2) + a*b*x**5/(c*x**2)**(3/2) + b**2*x**6/(3*(c*x**2
)**(3/2))
```

Giac [**A**]
time = 1.47, size = 39, normalized size = 1.44

$$\frac{b^2\sqrt{c}\,x^3 + 3\,ab\sqrt{c}\,x^2 + 3\,a^2\sqrt{c}\,x}{3\,c^2\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(b^2*sqrt(c)*x^3 + 3*a*b*sqrt(c)*x^2 + 3*a^2*sqrt(c)*x)/(c^2*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3\,(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

3.837 $\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}$$

```
[Out] 2*a*b*x^2/c/(c*x^2)^(1/2)+1/2*b^2*x^3/c/(c*x^2)^(1/2)+a^2*x*ln(x)/c/(c*x^2)
^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (2*a*b*x^2)/(c*Sqrt[c*x^2]) + (b^2*x^3)/(2*c*Sqrt[c*x^2]) + (a^2*x*Log[x])/
(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{c\sqrt{cx^2}}\\
&= \frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.56

$$\frac{x^3(bx(4a+bx)+2a^2\log(x))}{2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.54

method	result	size
default	$\frac{x^3(x^2b^2+2a^2\ln(x)+4abx)}{2(cx^2)^{\frac{3}{2}}}$	33
risch	$\frac{xb(\frac{1}{2}x^2b+2ax)}{c\sqrt{cx^2}}+\frac{a^2x\ln(x)}{c\sqrt{cx^2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(x^2*b^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.74

$$\frac{b^2x^3}{2\sqrt{cx^2}\,c}+\frac{2\,abx^2}{\sqrt{cx^2}\,c}+\frac{a^2\log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^3/(sqrt(c*x^2)*c) + 2*a*b*x^2/(sqrt(c*x^2)*c) + a^2*log(x)/c^(3/2
)
```

Fricas [A]
time = 0.47, size = 35, normalized size = 0.57

$$\frac{(b^2x^2+4\,abx+2\,a^2\log(x))\sqrt{cx^2}}{2\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c^2*x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(3/2),x)

[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(3/2), x)

Giac [**A**]
time = 0.84, size = 48, normalized size = 0.79

$$\frac{\frac{2\,a^2\log(|x|)}{\sqrt{c}\,\mathrm{sgn}(x)}+\frac{b^2c^{\frac{3}{2}}x^2\mathrm{sgn}(x)+4\,abc^{\frac{3}{2}}x\mathrm{sgn}(x)}{c^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")

[Out] 1/2*(2*a^2*log(abs(x))/(sqrt(c)*sgn(x)) + (b^2*c^(3/2)*x^2*sgn(x) + 4*a*b*c^(3/2)*x*sgn(x))/c^2)/c

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^2}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2),x)

[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2), x)

3.838 $\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}$$

```
[Out] -a^2/c/(c*x^2)^(1/2)+b^2*x^2/c/(c*x^2)^(1/2)+2*a*b*x*ln(x)/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -(a^2/(c*Sqrt[c*x^2])) + (b^2*x^2)/(c*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(c*Sq
rt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx}{c\sqrt{cx^2}}\\ &= -\frac{a^2}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 0.59

$$\frac{x^2(-a^2+b^2x^2+2abx\log(x))}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.57

method	result	size
default	$\frac{x^2(2ab\ln(x)x+x^2b^2-a^2)}{(c\,x^2)^{\frac{3}{2}}}$	32
risch	$-\frac{a^2}{c\sqrt{c\,x^2}}+\frac{b^2x^2}{c\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{c\sqrt{c\,x^2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(2*a*b*ln(x)*x+x^2*b^2-a^2)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.75

$$\frac{b^2x^2}{\sqrt{cx^2}\,c}+\frac{2\,ab\log(x)}{c^{\frac{3}{2}}}-\frac{a^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*x^2/(sqrt(c*x^2)*c) + 2*a*b*log(x)/c^(3/2) - a^2/(sqrt(c*x^2)*c)
```

Fricas [A]
time = 0.47, size = 34, normalized size = 0.61

$$\frac{(b^2x^2+2\,abx\log(x)-a^2)\sqrt{cx^2}}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**2/(c*x**2)**(3/2), x)
```

Giac [A]
time = 0.84, size = 46, normalized size = 0.82

$$\frac{\frac{b^2x}{\sqrt{c}\,\mathrm{sgn}(x)} + \frac{2\,ab\log(|x|)}{\sqrt{c}\,\mathrm{sgn}(x)} - \frac{a^2}{\sqrt{c}\,x\mathrm{sgn}(x)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*x/(sqrt(c)*sgn(x)) + 2*a*b*log(abs(x))/(sqrt(c)*sgn(x)) - a^2/(sqrt(c)
*x*sgn(x)))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\left(a+b\,x\right)}^2}{{\left(c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

3.839 $\int \frac{(a+bx)^2}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=58

$$-\frac{2ab}{c\sqrt{cx^2}}-\frac{a^2}{2cx\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

```
[Out] -2*a*b/c/(c*x^2)^(1/2)-1/2*a^2/c/x/(c*x^2)^(1/2)+b^2*x*ln(x)/c/(c*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (-2*a*b)/(c*Sqrt[c*x^2]) - a^2/(2*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{2ab}{c\sqrt{cx^2}} - \frac{a^2}{2cx\sqrt{cx^2}} + \frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 0.59

$$\frac{x(-a(a+4bx)+2b^2x^2\log(x))}{2\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.55

method	result	size
default	$\frac{x(2b^2\ln(x)x^2-4abx-a^2)}{2(c\,x^2)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{2}a^2-2abx}{cx\sqrt{c\,x^2}}+\frac{b^2x\ln(x)}{c\sqrt{c\,x^2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.60

$$\frac{b^2\log(x)}{c^{\frac{3}{2}}} - \frac{2\,ab}{\sqrt{cx^2}\,c} - \frac{a^2}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

[Out] b^2*log(x)/c^(3/2) - 2*a*b/(sqrt(c*x^2)*c) - 1/2*a^2/(c^(3/2)*x^2)

Fricas [A]
time = 0.45, size = 36, normalized size = 0.62

$$\frac{\left(2\, b^2 x^2 \log\left(x\right) - 4\, abx - a^2\right)\sqrt{cx^2}}{2\, c^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")

[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^2*x^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx\right)^2}{\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**2/(c*x**2)**(3/2),x)

[Out] Integral((a + b*x)**2/(c*x**2)**(3/2), x)

Giac [A]
time = 1.43, size = 49, normalized size = 0.84

$$\frac{\frac{2\, b^2 \log\left(|x|\right)}{\sqrt{c}\ \mathrm{sgn}\left(x\right)} - \frac{4\, ab\sqrt{c}\ x + a^2\sqrt{c}}{cx^2 \mathrm{sgn}\left(x\right)}}{2\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")

[Out] 1/2*(2*b^2*log(abs(x))/(sqrt(c)*sgn(x)) - (4*a*b*sqrt(c)*x + a^2*sqrt(c))/(c*x^2*sgn(x)))/c

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a + b\, x\right)}^2}{{\left(c\, x^2\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^2/(c*x^2)^(3/2),x)

[Out] int((a + b*x)^2/(c*x^2)^(3/2), x)

3.840 $\int \frac{(a+bx)^2}{x(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

```
[Out] -1/3*(b*x+a)^3/a/c/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 37\}$

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -1/3*(a + b*x)^3/(a*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.24

$$\frac{cx^2(-a^2 - 3abx - 3b^2x^2)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 27, normalized size = 0.93

method	result	size
gosper	$-\frac{3x^2b^2+3abx+a^2}{3(cx^2)^{\frac{3}{2}}}$	27
default	$-\frac{3x^2b^2+3abx+a^2}{3(cx^2)^{\frac{3}{2}}}$	27
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{cx^2\sqrt{cx^2}}$	34
trager	$\frac{(-1+x)(a^2x^2+3ab\,x^2+3x^2b^2+a^2x+3abx+a^2)\sqrt{cx^2}}{3c^2x^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 1.28

$$-\frac{b^2}{\sqrt{cx^2}\,c} - \frac{ab}{c^{\frac{3}{2}}x^2} - \frac{a^2}{3\,c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -b^2/(sqrt(c*x^2)*c) - a*b/(c^(3/2)*x^2) - 1/3*a^2/(c^(3/2)*x^3)
```

Fricas [A]
time = 0.44, size = 32, normalized size = 1.10

$$-\frac{(3\,b^2x^2 + 3\,abx + a^2)\sqrt{cx^2}}{3\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^2*x^4)
```

Sympy [A]
time = 0.22, size = 42, normalized size = 1.45

$$-\frac{a^2}{3\left(cx^2\right)^{\frac{3}{2}}}-\frac{abx}{\left(cx^2\right)^{\frac{3}{2}}}-\frac{b^2x^2}{\left(cx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(3*(c*x**2)**(3/2)) - a*b*x/(c*x**2)**(3/2) - b**2*x**2/(c*x**2)**(3/
2)
```

Giac [A]
time = 1.37, size = 39, normalized size = 1.34

$$-\frac{3\,b^2\sqrt{c}\,x^2+3\,ab\sqrt{c}\,x+a^2\sqrt{c}}{3\,c^2x^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x + a^2*sqrt(c))/(c^2*x^3*sgn(x))
```

Mupad [B]
time = 0.19, size = 33, normalized size = 1.14

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{3/2}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(3/2)),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(3/2)*(x^2)^(5/2))
```

3.841 $\int \frac{(a+bx)^2}{x^2(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

```
[Out] -1/4*a^2/c/x^3/(c*x^2)^(1/2)-2/3*a*b/c/x^2/(c*x^2)^(1/2)-1/2*b^2/c/x/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/4*a^2/(c*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c*x^2*Sqrt[c*x^2]) - b^2/(2*c*x*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^2 \left(cx^2\right)^{3/2}}\, dx = \frac{x \int \frac{(a+bx)^2}{x^5}\, dx}{c\sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^5} + \frac{2ab}{x^4} + \frac{b^2}{x^3}\right) dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{4cx^3\sqrt{cx^2}} - \frac{2ab}{3cx^2\sqrt{cx^2}} - \frac{b^2}{2cx\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(3a^2 + 8abx + 6b^2x^2)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^2*x^5)
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.48

method	result	size
gosper	$-\frac{6x^2b^2+8abx+3a^2}{12x(c\,x^2)^{\frac{3}{2}}}$	32
default	$-\frac{6x^2b^2+8abx+3a^2}{12x(c\,x^2)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{c\,x^3\sqrt{c\,x^2}}$	34
trager	$\frac{(-1+x)(3a^2x^3+8ab\,x^3+6b^2x^3+3a^2x^2+8ab\,x^2+6x^2b^2+3a^2x+8abx+3a^2)\sqrt{c\,x^2}}{12c^2x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.26, size = 33, normalized size = 0.50

$$-\frac{b^2}{2\,c^{\frac{3}{2}}x^2} - \frac{2\,ab}{3\,c^{\frac{3}{2}}x^3} - \frac{a^2}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b^2/(c^(3/2)*x^2) - 2/3*a*b/(c^(3/2)*x^3) - 1/4*a^2/(c^(3/2)*x^4)
```

Fricas [A]
time = 0.43, size = 34, normalized size = 0.52

$$-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^2*x^5)
```

Sympy [A]
time = 0.25, size = 46, normalized size = 0.70

$$-\frac{a^2}{4x\,(cx^2)^{\frac{3}{2}}}-\frac{2ab}{3\,(cx^2)^{\frac{3}{2}}}-\frac{b^2x}{2\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(3/2),x)
[Out] -a**2/(4*x*(c*x**2)**(3/2)) - 2*a*b/(3*(c*x**2)**(3/2)) - b**2*x/(2*(c*x**2
)**(3/2))
```

Giac [A]
time = 1.56, size = 40, normalized size = 0.61

$$-\frac{6\,b^2\sqrt{c}\,x^2+8\,ab\sqrt{c}\,x+3\,a^2\sqrt{c}}{12\,c^2x^4\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/12*(6*b^2*sqrt(c)*x^2 + 8*a*b*sqrt(c)*x + 3*a^2*sqrt(c))/(c^2*x^4*sgn(x)
)
```

Mupad [B]
time = 0.19, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(3/2)),x)
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(3
/2)*x^5)
```

3.842 $\int \frac{(a+bx)^2}{x^3 (cx^2)^{3/2}} dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

```
[Out] -1/5*a^2/c/x^4/(c*x^2)^(1/2)-1/2*a*b/c/x^3/(c*x^2)^(1/2)-1/3*b^2/c/x^2/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -1/5*a^2/(c*x^4*Sqrt[c*x^2]) - (a*b)/(2*c*x^3*Sqrt[c*x^2]) - b^2/(3*c*x^2*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{5cx^4\sqrt{cx^2}} - \frac{ab}{2cx^3\sqrt{cx^2}} - \frac{b^2}{3cx^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.50

$$\frac{c(-6a^2-15abx-10b^2x^2)}{30\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] (c*(-6*a^2 - 15*a*b*x - 10*b^2*x^2))/(30*(c*x^2)^(5/2))
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result	size
gosper	$-\frac{10x^2b^2+15abx+6a^2}{30x^2\left(c\,x^2\right)^{\frac{3}{2}}}$	32
default	$-\frac{10x^2b^2+15abx+6a^2}{30x^2\left(c\,x^2\right)^{\frac{3}{2}}}$	32
risch	$\frac{-\frac{1}{3}x^2b^2-\frac{1}{2}abx-\frac{1}{5}a^2}{c\,x^4\sqrt{c\,x^2}}$	34
trager	$\frac{(-1+x)\left(6a^2x^4+15ab\,x^4+10b^2x^4+6a^2x^3+15ab\,x^3+10b^2x^3+6a^2x^2+15ab\,x^2+10x^2b^2+6a^2x+15abx+6a^2\right)\sqrt{c\,x^2}}{30c^2x^6}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/x^2/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.50

$$-\frac{b^2}{3\,c^{\frac{3}{2}}x^3} - \frac{ab}{2\,c^{\frac{3}{2}}x^4} - \frac{a^2}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/3*b^2/(c^(3/2)*x^3) - 1/2*a*b/(c^(3/2)*x^4) - 1/5*a^2/(c^(3/2)*x^5)
```

Fricas [A]
time = 0.44, size = 34, normalized size = 0.52

$$-\frac{(10\,b^2x^2+15\,abx+6\,a^2)\sqrt{cx^2}}{30\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^2*x^6)
```

Sympy [A]
time = 0.29, size = 46, normalized size = 0.70

$$-\frac{a^2}{5x^2\,(cx^2)^{\frac{3}{2}}}-\frac{ab}{2x\,(cx^2)^{\frac{3}{2}}}-\frac{b^2}{3\,(cx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(3/2),x)
[Out] -a**2/(5*x**2*(c*x**2)**(3/2)) - a*b/(2*x*(c*x**2)**(3/2)) - b**2/(3*(c*x**
2)**(3/2))
```

Giac [A]
time = 1.42, size = 40, normalized size = 0.61

$$-\frac{10\,b^2\sqrt{c}\,x^2+15\,ab\sqrt{c}\,x+6\,a^2\sqrt{c}}{30\,c^2x^5\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/30*(10*b^2*sqrt(c)*x^2 + 15*a*b*sqrt(c)*x + 6*a^2*sqrt(c))/(c^2*x^5*sgn(
x))
```

Mupad [B]
time = 0.20, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(3/2)),x)
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(3/2)*x^6)
```

3.843 $\int \frac{(a+bx)^2}{x^4(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

```
[Out] -1/6*a^2/c/x^5/(c*x^2)^(1/2)-2/5*a*b/c/x^4/(c*x^2)^(1/2)-1/4*b^2/c/x^3/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -1/6*a^2/(c*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c*x^4*Sqrt[c*x^2]) - b^2/(4*c*x^3
*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2}{6cx^5\sqrt{cx^2}} - \frac{2ab}{5cx^4\sqrt{cx^2}} - \frac{b^2}{4cx^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.53

$$\frac{-10a^2-24abx-15b^2x^2}{60x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{15x^2b^2+24abx+10a^2}{60x^3(c\,x^2)^{\frac{3}{2}}}$
default	$-\frac{15x^2b^2+24abx+10a^2}{60x^3(c\,x^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{1}{4}x^2b^2-\frac{2}{5}abx-\frac{1}{6}a^2}{c\,x^5\sqrt{c\,x^2}}$
trager	$\frac{(-1+x)(10a^2x^5+24ab\,x^5+15b^2x^5+10a^2x^4+24ab\,x^4+15b^2x^4+10a^2x^3+24ab\,x^3+15b^2x^3+10a^2x^2+24ab\,x^2+15x^2b^2+10a^2x+24abx+}{60c^2x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x^3/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{3}{2}}x^4}-\frac{2\,ab}{5\,c^{\frac{3}{2}}x^5}-\frac{a^2}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(3/2)*x^4) - 2/5*a*b/(c^(3/2)*x^5) - 1/6*a^2/(c^(3/2)*x^6)
```

Fricas [A]
time = 0.49, size = 34, normalized size = 0.52

$$-\frac{(15\,b^2x^2+24\,abx+10\,a^2)\sqrt{cx^2}}{60\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^2*x^7)
```

Sympy [A]
time = 0.32, size = 51, normalized size = 0.77

$$-\frac{a^2}{6x^3\left(cx^2\right)^{\frac{3}{2}}}-\frac{2ab}{5x^2\left(cx^2\right)^{\frac{3}{2}}}-\frac{b^2}{4x\left(cx^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(3/2),x)
[Out] -a**2/(6*x**3*(c*x**2)**(3/2)) - 2*a*b/(5*x**2*(c*x**2)**(3/2)) - b**2/(4*x
*(c*x**2)**(3/2))
```

Giac [A]
time = 0.88, size = 40, normalized size = 0.61

$$-\frac{15\,b^2\sqrt{c}\,x^2+24\,ab\sqrt{c}\,x+10\,a^2\sqrt{c}}{60\,c^2x^6\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/60*(15*b^2*sqrt(c)*x^2 + 24*a*b*sqrt(c)*x + 10*a^2*sqrt(c))/(c^2*x^6*sgn
(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}+15\,b^2\,x^2\,\sqrt{x^2}+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(3/2)),x)
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(3/2)*x^7)
```

3.844 $\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] -a^2/c^2/(c*x^2)^(1/2)+b^2*x^2/c^2/(c*x^2)^(1/2)+2*a*b*x*ln(x)/c^2/(c*x^2)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a^2/(c^2*Sqrt[c*x^2])) + (b^2*x^2)/(c^2*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(
c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)\,dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.59

$$\frac{-a^2+b^2x^2+2abx\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-a^2 + b^2*x^2 + 2*a*b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.57

method	result	size
default	$\frac{x^4(2ab\ln(x)x+x^2b^2-a^2)}{(c\,x^2)^{\frac{5}{2}}}$	32
risch	$-\frac{a^2}{c^2\sqrt{c\,x^2}}+\frac{b^2x^2}{c^2\sqrt{c\,x^2}}+\frac{2abx\ln(x)}{c^2\sqrt{c\,x^2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^4*(2*a*b*ln(x)*x+x^2*b^2-a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.80

$$\frac{b^2x^4}{(cx^2)^{\frac{3}{2}}\,c}-\frac{a^2x^2}{(cx^2)^{\frac{3}{2}}\,c}+\frac{2\,ab\log{(x)}}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*x^4/((c*x^2)^(3/2)*c) - a^2*x^2/((c*x^2)^(3/2)*c) + 2*a*b*log(x)/c^(5/2
)
```

Fricas [A]
time = 0.59, size = 34, normalized size = 0.61

$$\frac{(b^2x^2 + 2\,abx\log{(x)} - a^2)\sqrt{cx^2}}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^3*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)^2}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**2/(c*x**2)**(5/2), x)
```

Giac [A]
time = 1.03, size = 42, normalized size = 0.75

$$\frac{b^2x}{c^{\frac{5}{2}}\operatorname{sgn}{(x)}} + \frac{2\,ab\log{(|x|)}}{c^{\frac{5}{2}}\operatorname{sgn}{(x)}} - \frac{a^2}{c^{\frac{5}{2}}x\operatorname{sgn}{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] b^2*x/(c^(5/2)*sgn(x)) + 2*a*b*log(abs(x))/(c^(5/2)*sgn(x)) - a^2/(c^(5/2)*
x*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(a+b\,x\right)}^2}{{\left(c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

3.845 $\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2ab}{c^2\sqrt{cx^2}}-\frac{a^2}{2c^2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

```
[Out] -2*a*b/c^2/(c*x^2)^(1/2)-1/2*a^2/c^2/x/(c*x^2)^(1/2)+b^2*x*ln(x)/c^2/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-2*a*b)/(c^2*Sqrt[c*x^2]) - a^2/(2*c^2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c^
2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{2ab}{c^2\sqrt{cx^2}} - \frac{a^2}{2c^2x\sqrt{cx^2}} + \frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.62

$$\frac{x^3(-a(a+4bx)+2b^2x^2\log(x))}{2\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(5/2))
```

Maple [A]
time = 0.11, size = 34, normalized size = 0.59

method	result	size
default	$\frac{x^3(2b^2\ln(x)x^2-4abx-a^2)}{2(c\,x^2)^{\frac{5}{2}}}$	34
risch	$\frac{-\frac{1}{2}a^2-2abx}{c^2x\sqrt{c\,x^2}}+\frac{b^2x\ln(x)}{c^2\sqrt{c\,x^2}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.66

$$-\frac{2\,abx^2}{(cx^2)^{\frac{3}{2}}\,c}+\frac{b^2\log(x)}{c^{\frac{5}{2}}}-\frac{a^2}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

[Out] -2*a*b*x^2/((c*x^2)^(3/2)*c) + b^2*log(x)/c^(5/2) - 1/2*a^2/(c^(5/2)*x^2)

Fricas [A]
time = 1.03, size = 36, normalized size = 0.62

$$\frac{\left(2 b^2 x^2 \log\left(x\right) - 4 abx - a^2\right) \sqrt{cx^2}}{2 c^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx)^2}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(5/2), x)
```

Giac [A]
time = 0.82, size = 43, normalized size = 0.74

$$\frac{b^2 \log\left(|x|\right)}{c^{\frac{5}{2}} \operatorname{sgn}\left(x\right)} - \frac{4 ab\sqrt{c}\, x + a^2 \sqrt{c}}{2 c^3 x^2 \operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] b^2*log(abs(x))/(c^(5/2)*sgn(x)) - 1/2*(4*a*b*sqrt(c)*x + a^2*sqrt(c))/(c^3
*x^2*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2 \left(a + b\, x\right)^2}{{\left(c\, x^2\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

3.846 $\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

```
[Out] -1/3*(b*x+a)^3/a/c^2/x^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -1/3*(a + b*x)^3/(a*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{(cx^2)^{5/2}} dx &= \frac{x \int \frac{(a+bx)^2}{x^4} dx}{c^2\sqrt{cx^2}} \\ &= -\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.21

$$\frac{x^2(-a^2 - 3abx - 3b^2x^2)}{3\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 30, normalized size = 1.03

method	result	size
gosper	$-\frac{x^2(3x^2b^2+3abx+a^2)}{3(c\,x^2)^{\frac{5}{2}}}$	30
default	$-\frac{x^2(3x^2b^2+3abx+a^2)}{3(c\,x^2)^{\frac{5}{2}}}$	30
risch	$\frac{-x^2b^2-abx-\frac{1}{3}a^2}{c^2x^2\sqrt{c\,x^2}}$	34
trager	$\frac{(-1+x)(a^2x^2+3ab\,x^2+3x^2b^2+a^2x+3abx+a^2)\sqrt{c\,x^2}}{3c^3x^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3*x^2*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 1.52

$$-\frac{b^2x^2}{(cx^2)^{\frac{3}{2}}\,c} - \frac{a^2}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{ab}{c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -b^2*x^2/((c*x^2)^(3/2)*c) - 1/3*a^2/((c*x^2)^(3/2)*c) - a*b/(c^(5/2)*x^2)
```

Fricas [A]
time = 1.16, size = 32, normalized size = 1.10

$$-\frac{(3\,b^2x^2 + 3\,abx + a^2)\sqrt{cx^2}}{3\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^3*x^4)
```

Sympy [A]
time = 0.28, size = 48, normalized size = 1.66

$$-\frac{a^2x^2}{3\left(cx^2\right)^{\frac{5}{2}}}-\frac{abx^3}{\left(cx^2\right)^{\frac{5}{2}}}-\frac{b^2x^4}{\left(cx^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x**2/(3*(c*x**2)**(5/2)) - a*b*x**3/(c*x**2)**(5/2) - b**2*x**4/(c*x*
*2)**(5/2)
```

Giac [A]
time = 0.97, size = 39, normalized size = 1.34

$$-\frac{3\,b^2\sqrt{c}\,x^2+3\,ab\sqrt{c}\,x+a^2\sqrt{c}}{3\,c^3x^3\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x + a^2*sqrt(c))/(c^3*x^3*sgn(x))
```

Mupad [B]
time = 0.18, size = 33, normalized size = 1.14

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{5/2}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(5/2)*(x^2)^(5/2))
```

3.847 $\int \frac{(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

```
[Out] -1/4*a^2/c^2/x^3/(c*x^2)^(1/2)-2/3*a*b/c^2/x^2/(c*x^2)^(1/2)-1/2*b^2/c^2/x/
(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 45}

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(5/2),x]
```

```
[Out] -1/4*a^2/(c^2*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c^2*x^2*Sqrt[c*x^2]) - b^2/(2*c
^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(3a^2+8abx+6b^2x^2)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^3*x^5)
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.45

method	result	size
gosper	$-\frac{x(6x^2b^2+8abx+3a^2)}{12(c\,x^2)^{\frac{5}{2}}}$	30
default	$-\frac{x(6x^2b^2+8abx+3a^2)}{12(c\,x^2)^{\frac{5}{2}}}$	30
risch	$\frac{-\frac{1}{2}x^2b^2-\frac{2}{3}abx-\frac{1}{4}a^2}{c^2x^3\sqrt{c\,x^2}}$	34
trager	$\frac{(-1+x)(3a^2x^3+8ab\,x^3+6b^2x^3+3a^2x^2+8ab\,x^2+6x^2b^2+3a^2x+8abx+3a^2)\sqrt{c\,x^2}}{12c^3x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12*x*(6*b^2*x^2+8*a*b*x+3*a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.56

$$-\frac{2\,ab}{3\,(cx^2)^{\frac{3}{2}}\,c}-\frac{b^2}{2\,c^{\frac{5}{2}}x^2}-\frac{a^2}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -2/3*a*b/((c*x^2)^(3/2)*c) - 1/2*b^2/(c^(5/2)*x^2) - 1/4*a^2/(c^(5/2)*x^4)
```

Fricas [A]
time = 0.90, size = 34, normalized size = 0.52

$$-\frac{(6\,b^2x^2+8\,abx+3\,a^2)\sqrt{cx^2}}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^3*x^5)
```

Sympy [A]
time = 0.28, size = 51, normalized size = 0.77

$$-\frac{a^2x}{4\,(cx^2)^{\frac{5}{2}}}-\frac{2abx^2}{3\,(cx^2)^{\frac{5}{2}}}-\frac{b^2x^3}{2\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(5/2),x)
[Out] -a**2*x/(4*(c*x**2)**(5/2)) - 2*a*b*x**2/(3*(c*x**2)**(5/2)) - b**2*x**3/(2
*(c*x**2)**(5/2))
```

Giac [A]
time = 0.81, size = 40, normalized size = 0.61

$$-\frac{6\,b^2\sqrt{c}\,x^2+8\,ab\sqrt{c}\,x+3\,a^2\sqrt{c}}{12\,c^3x^4\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/12*(6*b^2*sqrt(c)*x^2 + 8*a*b*sqrt(c)*x + 3*a^2*sqrt(c))/(c^3*x^4*sgn(x)
)
```

Mupad [B]
time = 0.17, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}+6\,b^2\,x^2\,\sqrt{x^2}+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(5/2),x)
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(5
/2)*x^5)
```

3.848 $\int \frac{(a+bx)^2}{x(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

```
[Out] -1/5*a^2/c^2/x^4/(c*x^2)^(1/2)-1/2*a*b/c^2/x^3/(c*x^2)^(1/2)-1/3*b^2/c^2/x^
2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/5*a^2/(c^2*x^4*Sqrt[c*x^2]) - (a*b)/(2*c^2*x^3*Sqrt[c*x^2]) - b^2/(3*c^2
*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{5c^2x^4\sqrt{cx^2}} - \frac{ab}{2c^2x^3\sqrt{cx^2}} - \frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(6a^2+15abx+10b^2x^2)}{30c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/30*(Sqrt[c*x^2]*(6*a^2 + 15*a*b*x + 10*b^2*x^2))/(c^3*x^6)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.44

method	result	size
gosper	$-\frac{10x^2b^2+15abx+6a^2}{30\left(c\,x^2\right)^{\frac{5}{2}}}$	29
default	$-\frac{10x^2b^2+15abx+6a^2}{30\left(c\,x^2\right)^{\frac{5}{2}}}$	29
risch	$\frac{-\frac{1}{3}x^2b^2-\frac{1}{2}abx-\frac{1}{5}a^2}{c^2x^4\sqrt{c\,x^2}}$	34
trager	$\frac{(-1+x)\left(6a^2x^4+15ab\,x^4+10b^2x^4+6a^2x^3+15ab\,x^3+10b^2x^3+6a^2x^2+15ab\,x^2+10x^2b^2+6a^2x+15abx+6a^2\right)\sqrt{c\,x^2}}{30c^3x^6}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.56

$$-\frac{b^2}{3\,(cx^2)^{\frac{3}{2}}\,c} - \frac{ab}{2\,c^{\frac{5}{2}}x^4} - \frac{a^2}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*b^2/((c*x^2)^(3/2)*c) - 1/2*a*b/(c^(5/2)*x^4) - 1/5*a^2/(c^(5/2)*x^5)
```

Fricas [A]
time = 0.61, size = 34, normalized size = 0.52

$$-\frac{(10\,b^2x^2+15\,abx+6\,a^2)\sqrt{cx^2}}{30\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^3*x^6)
```

Sympy [A]
time = 0.31, size = 46, normalized size = 0.70

$$-\frac{a^2}{5\,(cx^2)^{\frac{5}{2}}}-\frac{abx}{2\,(cx^2)^{\frac{5}{2}}}-\frac{b^2x^2}{3\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(5/2),x)
[Out] -a**2/(5*(c*x**2)**(5/2)) - a*b*x/(2*(c*x**2)**(5/2)) - b**2*x**2/(3*(c*x**
2)**(5/2))
```

Giac [A]
time = 1.02, size = 40, normalized size = 0.61

$$-\frac{10\,b^2\sqrt{c}\,x^2+15\,ab\sqrt{c}\,x+6\,a^2\sqrt{c}}{30\,c^3x^5\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/30*(10*b^2*sqrt(c)*x^2 + 15*a*b*sqrt(c)*x + 6*a^2*sqrt(c))/(c^3*x^5*sgn(
x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(5/2)),x)
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(5/2)*x^6)
```

3.849 $\int \frac{(a+bx)^2}{x^2 (cx^2)^{5/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

```
[Out] -1/6*a^2/c^2/x^5/(c*x^2)^(1/2)-2/5*a*b/c^2/x^4/(c*x^2)^(1/2)-1/4*b^2/c^2/x^
3/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/6*a^2/(c^2*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c^2*x^4*Sqrt[c*x^2]) - b^2/(4*c
^2*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^2(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2}{6c^2x^5\sqrt{cx^2}} - \frac{2ab}{5c^2x^4\sqrt{cx^2}} - \frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\,(10a^2+24abx+15b^2x^2)}{60c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/60*(Sqrt[c*x^2]*(10*a^2 + 24*a*b*x + 15*b^2*x^2))/(c^3*x^7)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{15x^2b^2+24abx+10a^2}{60x(cx^2)^{\frac{5}{2}}}$
default	$-\frac{15x^2b^2+24abx+10a^2}{60x(cx^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{4}x^2b^2-\frac{2}{5}abx-\frac{1}{6}a^2}{c^2x^5\sqrt{cx^2}}$
trager	$\frac{(-1+x)(10a^2x^5+24ab\,x^5+15b^2x^5+10a^2x^4+24ab\,x^4+15b^2x^4+10a^2x^3+24ab\,x^3+15b^2x^3+10a^2x^2+24ab\,x^2+15x^2b^2+10a^2x+24abx+}{60c^3x^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{5}{2}}x^4} - \frac{2\,ab}{5\,c^{\frac{5}{2}}x^5} - \frac{a^2}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(5/2)*x^4) - 2/5*a*b/(c^(5/2)*x^5) - 1/6*a^2/(c^(5/2)*x^6)
```

Fricas [A]
time = 0.47, size = 34, normalized size = 0.52

$$-\frac{(15\,b^2x^2+24\,abx+10\,a^2)\sqrt{cx^2}}{60\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^3*x^7)
```

Sympy [A]
time = 0.35, size = 46, normalized size = 0.70

$$-\frac{a^2}{6x\,(cx^2)^{\frac{5}{2}}}-\frac{2ab}{5\,(cx^2)^{\frac{5}{2}}}-\frac{b^2x}{4\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(5/2),x)
[Out] -a**2/(6*x*(c*x**2)**(5/2)) - 2*a*b/(5*(c*x**2)**(5/2)) - b**2*x/(4*(c*x**2
)**(5/2))
```

Giac [A]
time = 0.99, size = 40, normalized size = 0.61

$$-\frac{15\,b^2\sqrt{c}\,x^2+24\,ab\sqrt{c}\,x+10\,a^2\sqrt{c}}{60\,c^3x^6\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/60*(15*b^2*sqrt(c)*x^2 + 24*a*b*sqrt(c)*x + 10*a^2*sqrt(c))/(c^3*x^6*sgn
(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}+15\,b^2\,x^2\,\sqrt{x^2}+24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(5/2)),x)
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(5/2)*x^7)
```

3.850 $\int \frac{(a+bx)^2}{x^3(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

```
[Out] -1/7*a^2/c^2/x^6/(c*x^2)^(1/2)-1/3*a*b/c^2/x^5/(c*x^2)^(1/2)-1/5*b^2/c^2/x^
4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -1/7*a^2/(c^2*x^6*Sqrt[c*x^2]) - (a*b)/(3*c^2*x^5*Sqrt[c*x^2]) - b^2/(5*c^2
*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^8}\,dx}{c^2\sqrt{cx^2}} \\
&= \frac{x\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx}{c^2\sqrt{cx^2}} \\
&= -\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.50

$$\frac{c(-15a^2-35abx-21b^2x^2)}{105\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] (c*(-15*a^2 - 35*a*b*x - 21*b^2*x^2))/(105*(c*x^2)^(7/2))
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{21x^2b^2+35abx+15a^2}{105x^2\left(c\,x^2\right)^{\frac{5}{2}}}$
default	$-\frac{21x^2b^2+35abx+15a^2}{105x^2\left(c\,x^2\right)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{5}x^2b^2-\frac{1}{3}abx-\frac{1}{7}a^2}{c^2x^6\sqrt{c\,x^2}}$
trager	$\frac{(-1+x)\left(15a^2x^6+35ab\,x^6+21b^2x^6+15a^2x^5+35ab\,x^5+21b^2x^5+15a^2x^4+35ab\,x^4+21b^2x^4+15a^2x^3+35ab\,x^3+21b^2x^3+15a^2x^2+35ab\right.}{105c^3x^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(21*b^2*x^2+35*a*b*x+15*a^2)/x^2/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.50

$$-\frac{b^2}{5\,c^{\frac{5}{2}}x^5}-\frac{ab}{3\,c^{\frac{5}{2}}x^6}-\frac{a^2}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/5*b^2/(c^(5/2)*x^5) - 1/3*a*b/(c^(5/2)*x^6) - 1/7*a^2/(c^(5/2)*x^7)
```

Fricas [A]
time = 0.40, size = 34, normalized size = 0.52

$$-\frac{(21\,b^2x^2+35\,abx+15\,a^2)\sqrt{cx^2}}{105\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)*sqrt(c*x^2)/(c^3*x^8)
```

Sympy [A]
time = 0.38, size = 46, normalized size = 0.70

$$-\frac{a^2}{7x^2\,(cx^2)^{\frac{5}{2}}}-\frac{ab}{3x\,(cx^2)^{\frac{5}{2}}}-\frac{b^2}{5\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(5/2),x)
[Out] -a**2/(7*x**2*(c*x**2)**(5/2)) - a*b/(3*x*(c*x**2)**(5/2)) - b**2/(5*(c*x**
2)**(5/2))
```

Giac [A]
time = 0.87, size = 40, normalized size = 0.61

$$-\frac{21\,b^2\sqrt{c}\,x^2+35\,ab\sqrt{c}\,x+15\,a^2\sqrt{c}}{105\,c^3x^7\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -1/105*(21*b^2*sqrt(c)*x^2 + 35*a*b*sqrt(c)*x + 15*a^2*sqrt(c))/(c^3*x^7*sg
n(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{15\,a^2\,\sqrt{x^2}+21\,b^2\,x^2\,\sqrt{x^2}+35\,a\,b\,x\,\sqrt{x^2}}{105\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(5/2)),x)
[Out] -(15*a^2*(x^2)^(1/2) + 21*b^2*x^2*(x^2)^(1/2) + 35*a*b*x*(x^2)^(1/2))/(105*
c^(5/2)*x^8)
```

3.851 $\int \frac{(a+bx)^2}{x^4 (cx^2)^{5/2}} \, dx$

Optimal. Leaf size=66

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

```
[Out] -1/8*a^2/c^2/x^7/(c*x^2)^(1/2)-2/7*a*b/c^2/x^6/(c*x^2)^(1/2)-1/6*b^2/c^2/x^
5/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -1/8*a^2/(c^2*x^7*Sqrt[c*x^2]) - (2*a*b)/(7*c^2*x^6*Sqrt[c*x^2]) - b^2/(6*c
^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4 \left(cx^2\right)^{5/2}} \, dx = \frac{x \int \frac{(a+bx)^2}{x^9} \, dx}{c^2 \sqrt{cx^2}}$$
$$= \frac{x \int \left(\frac{a^2}{x^9} + \frac{2ab}{x^8} + \frac{b^2}{x^7}\right) dx}{c^2 \sqrt{cx^2}}$$
$$= -\frac{a^2}{8c^2 x^7 \sqrt{cx^2}} - \frac{2ab}{7c^2 x^6 \sqrt{cx^2}} - \frac{b^2}{6c^2 x^5 \sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.53

$$\frac{-21a^2 - 48abx - 28b^2x^2}{168x^3 \left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 32, normalized size = 0.48

method	result
gosper	$-\frac{28x^2b^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
default	$-\frac{28x^2b^2+48abx+21a^2}{168x^3(c\,x^2)^{\frac{5}{2}}}$
risch	$\frac{-\frac{1}{6}x^2b^2-\frac{2}{7}abx-\frac{1}{8}a^2}{c^2x^7\sqrt{c\,x^2}}$
trager	$\frac{(-1+x)\left(21a^2x^7+48ab\,x^7+28b^2x^7+21a^2x^6+48ab\,x^6+28b^2x^6+21a^2x^5+48ab\,x^5+28b^2x^5+21a^2x^4+48ab\,x^4+28b^2x^4+21a^2x^3+48ab\,x^3}{168c^3x^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/168*(28*b^2*x^2+48*a*b*x+21*a^2)/x^3/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.50

$$-\frac{b^2}{6\,c^{\frac{5}{2}}x^6} - \frac{2\,ab}{7\,c^{\frac{5}{2}}x^7} - \frac{a^2}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b^2/(c^(5/2)*x^6) - 2/7*a*b/(c^(5/2)*x^7) - 1/8*a^2/(c^(5/2)*x^8)
```

Fricas [A]
time = 0.42, size = 34, normalized size = 0.52

$$-\frac{(28\,b^2x^2+48\,abx+21\,a^2)\sqrt{cx^2}}{168\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)*sqrt(c*x^2)/(c^3*x^9)
```

Sympy [A]
time = 0.42, size = 51, normalized size = 0.77

$$-\frac{a^2}{8x^3\,(cx^2)^{\frac{5}{2}}}-\frac{2ab}{7x^2\,(cx^2)^{\frac{5}{2}}}-\frac{b^2}{6x\,(cx^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(8*x**3*(c*x**2)**(5/2)) - 2*a*b/(7*x**2*(c*x**2)**(5/2)) - b**2/(6*x
*(c*x**2)**(5/2))
```

Giac [A]
time = 0.82, size = 40, normalized size = 0.61

$$-\frac{28\,b^2\sqrt{c}\,x^2+48\,ab\sqrt{c}\,x+21\,a^2\sqrt{c}}{168\,c^3x^8\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/168*(28*b^2*sqrt(c)*x^2 + 48*a*b*sqrt(c)*x + 21*a^2*sqrt(c))/(c^3*x^8*sg
n(x))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.64

$$-\frac{21\,a^2\,\sqrt{x^2}+28\,b^2\,x^2\,\sqrt{x^2}+48\,a\,b\,x\,\sqrt{x^2}}{168\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(21*a^2*(x^2)^(1/2) + 28*b^2*x^2*(x^2)^(1/2) + 48*a*b*x*(x^2)^(1/2))/(168*
c^(5/2)*x^9)
```

3.852 $\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=102

$$-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] -a^3*(c*x^2)^(1/2)/b^4+1/2*a^2*x*(c*x^2)^(1/2)/b^3-1/3*a*x^2*(c*x^2)^(1/2)/
b^2+1/4*x^3*(c*x^2)^(1/2)/b+a^4*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a^3*Sqrt[c*x^2])/b^4) + (a^2*x*Sqrt[c*x^2])/(2*b^3) - (a*x^2*Sqrt[c*x^2]
)/(3*b^2) + (x^3*Sqrt[c*x^2])/(4*b) + (a^4*Sqrt[c*x^2]*Log[a + b*x])/(b^5*x
)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx = \frac{\sqrt{cx^2}\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.62

$$\frac{cx(bx(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3)+12a^4\log(a+bx))}{12b^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^4*Log[a +
b*x]))/(12*b^5*Sqrt[c*x^2])
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.62

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx)}{12b^5x}$	63
risch	$\frac{\sqrt{c\,x^2}\,(\frac{1}{4}b^3x^4-\frac{1}{3}a\,b^2x^3+\frac{1}{2}a^2b\,x^2-a^3x)}{x\,b^4}+\frac{a^4\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*x^2)^(1/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x
```

Maxima [A]
time = 0.29, size = 128, normalized size = 1.25

$$\frac{(-1)^{\frac{2cx}{b}}a^4\sqrt{c}\,\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}a^4\sqrt{c}\,\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{\sqrt{cx^2}\,a^2x}{2b^3}+\frac{(cx^2)^{\frac{3}{2}}x}{4bc}-\frac{\sqrt{cx^2}\,a^3}{b^4}-\frac{(cx^2)^{\frac{3}{2}}a}{3b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*sqrt(c)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/2*sqrt(c*x^2)*a^2*x/b^3 + 1/4*(c*x^2
)^(3/2)*x/(b*c) - sqrt(c*x^2)*a^3/b^4 - 1/3*(c*x^2)^(3/2)*a/(b^2*c)
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.61

$$\frac{\left(3\,b^4x^4 - 4\,ab^3x^3 + 6\,a^2b^2x^2 - 12\,a^3bx + 12\,a^4\log\left(bx + a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{cx^2}}{a + bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 0.61, size = 81, normalized size = 0.79

$$\frac{1}{12}\,\sqrt{c}\left(\frac{12\,a^4\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^5} - \frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}(x)}{b^5} + \frac{3\,b^3x^4\mathrm{sgn}(x) - 4\,ab^2x^3\mathrm{sgn}(x) + 6\,a^2bx^2\mathrm{sgn}(x) - 12\,a^3x\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c\,x^2}}{a + b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x), x)
```

3.853 $\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=80

$$\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] a^2*(c*x^2)^(1/2)/b^3-1/2*a*x*(c*x^2)^(1/2)/b^2+1/3*x^2*(c*x^2)^(1/2)/b-a^3
*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (a^2*Sqrt[c*x^2])/b^3 - (a*x*Sqrt[c*x^2])/(2*b^2) + (x^2*Sqrt[c*x^2])/(3*b)
 - (a^3*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\,\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.65

$$\frac{cx(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx))}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[
c*x^2])
```

Maple [A]
time = 0.14, size = 52, normalized size = 0.65

method	result	size
default	$-\frac{\sqrt{c\,x^2}\,\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6x\,b^4}$	52
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{x\,b^3}-\frac{a^3\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^2)^(1/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x/b^4
```

Maxima [A]
time = 0.29, size = 110, normalized size = 1.38

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ax}{2\,b^2}+\frac{\sqrt{cx^2}\,a^2}{b^3}+\frac{(cx^2)^{\frac{3}{2}}}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*sqrt(c)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*x/b^2 + sqrt(c*x^2)
*a^2/b^3 + 1/3*(c*x^2)^(3/2)/(b*c)
```

Fricas [A]
time = 0.36, size = 51, normalized size = 0.64

$$\frac{(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log(bx+a))\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 0.93, size = 69, normalized size = 0.86

$$-\frac{1}{6}\sqrt{c}\left(\frac{6\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2x^3\mathrm{sgn}(x)-3\,abx^2\mathrm{sgn}(x)+6\,a^2x\mathrm{sgn}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(c)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x), x)
```

3.854 $\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=58

$$-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] -a*(c*x^2)^(1/2)/b^2+1/2*x*(c*x^2)^(1/2)/b+a^2*ln(b*x+a)*(c*x^2)^(1/2)/b^3/
x
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 45}

$$\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a*Sqrt[c*x^2])/b^2) + (x*Sqrt[c*x^2])/(2*b) + (a^2*Sqrt[c*x^2]*Log[a + b
*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^2}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\ &= -\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.69

$$\frac{cx(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]
[Out] (c*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.69

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(x^2b^2+2a^2\ln(bx+a)-2abx)}{2b^3x}$	40
risch	$\frac{\sqrt{c\,x^2}\,(\frac{1}{2}x^2b-ax)}{x\,b^2}+\frac{a^2\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(1/2)*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x
```

Maxima [A]
time = 0.29, size = 91, normalized size = 1.57

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,x}{2\,b}-\frac{\sqrt{cx^2}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] (-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*x/b - sqrt(c*x^2)*a/b^
2
```

Fricas [A]
time = 0.39, size = 39, normalized size = 0.67

$$\frac{(b^2x^2-2\,abx+2\,a^2\log(bx+a))\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 1.04, size = 54, normalized size = 0.93

$$\frac{1}{2}\sqrt{c}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bx^2\mathrm{sgn}(x)-2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x), x)
```

3.855 $\int \frac{\sqrt{cx^2}}{a+bx} dx$

Optimal. Leaf size=38

$$\frac{\sqrt{cx^2}}{b} - \frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] (c*x^2)^(1/2)/b-a*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$\frac{\sqrt{cx^2}}{b} - \frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]/b - (a*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\ &= \frac{\sqrt{cx^2}}{b} - \frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.74

$$\frac{cx(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] (c*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.76

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,(a\ln(bx+a)-bx)}{b^{2}x}$	29
risch	$\frac{\sqrt{c\,x^{2}}}{b}-\frac{a\ln(bx+a)\sqrt{c\,x^{2}}}{b^{2}x}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(a*ln(b*x+a)-b*x)/b^2/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.
time = 0.29, size = 74, normalized size = 1.95

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*sqrt(c)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)/b
```

Fricas [A]
time = 0.38, size = 27, normalized size = 0.71

$$\frac{\sqrt{cx^2}\,(bx - a\log(bx + a))}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a),x)
[Out] Integral(sqrt(c*x**2)/(a + b*x), x)
```

Giac [A]
time = 2.06, size = 37, normalized size = 0.97

$$\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b}-\frac{a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^2}+\frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
[Out] sqrt(c)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x),x)
[Out] int((c*x^2)^(1/2)/(a + b*x), x)
```

3.856 $\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c*x^2]/(x*(a + b*x)),x]

[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.95

$$\frac{cx\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

```
[Out] (c*x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.95

method	result	size
default	$\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{bx}$	21
risch	$\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{bx}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 0.45, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^2}\,\log\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x*(a + b*x)), x)
```

Giac [A]
time = 1.68, size = 28, normalized size = 1.27

$$\sqrt{c}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b}-\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] sqrt(c)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{\sqrt{c\,x^2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x*(a + b*x)), x)
```

3.857 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{cx^2}\,\log(x)}{ax}-\frac{\sqrt{cx^2}\,\log(a+bx)}{ax}$$

```
[Out] ln(x)*(c*x^2)^(1/2)/a/x-ln(b*x+a)*(c*x^2)^(1/2)/a/x
```

Rubi [A]
time = 0.00, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 36, 29, 31}

$$\frac{\sqrt{cx^2}\,\log(x)}{ax}-\frac{\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[x])/(a*x) - (Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \frac{1}{x}\,dx}{ax} - \frac{\left(b\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{\sqrt{cx^2}\log(x)}{ax} - \frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.62

$$\frac{cx(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
[Out] (c*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 26, normalized size = 0.62

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(\ln(x)-\ln(bx+a))}{ax}$	26
risch	$\frac{\sqrt{c\,x^{2}}\,\ln(-x)}{xa}-\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{ax}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(1/2)*(ln(x)-ln(b*x+a))/a/x
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.57

$$-\frac{\sqrt{c}\,\log\left(bx+a\right)}{a}+\frac{\sqrt{c}\,\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="maxima")
[Out] -sqrt(c)*log(b*x + a)/a + sqrt(c)*log(x)/a
```

Fricas [A]
time = 0.42, size = 64, normalized size = 1.52

$$\left[\frac{\sqrt{cx^2}\,\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x +
 a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)), x)
```

3.858 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=61

$$-\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}$$

```
[Out] -(c*x^2)^(1/2)/a/x^2-b*ln(x)*(c*x^2)^(1/2)/a^2/x+b*ln(b*x+a)*(c*x^2)^(1/2)/
a^2/x
```

Rubi [A]

time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -(Sqrt[c*x^2]/(a*x^2)) - (b*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*Sqrt[c*x^2]*Lo
g[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^3(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\ &= -\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.52

$$-\frac{c(a + bx\log(x) - bx\log(a + bx))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
[Out] -((c*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.54

method	result	size
default	$-\frac{\sqrt{c\,x^{2}}\,(bx\ln(x)-b\ln(bx+a)x+a)}{a^{2}x^{2}}$	33
risch	$-\frac{\sqrt{c\,x^{2}}}{a\,x^{2}}+\frac{\sqrt{c\,x^{2}}\,b\ln(-bx-a)}{x\,a^{2}}-\frac{b\ln(x)\sqrt{c\,x^{2}}}{a^{2}x}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(1/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x^2
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.61

$$\frac{b\sqrt{c}\,\log\left(bx+a\right)}{a^2}-\frac{b\sqrt{c}\,\log\left(x\right)}{a^2}-\frac{\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="maxima")
[Out] b*sqrt(c)*log(b*x + a)/a^2 - b*sqrt(c)*log(x)/a^2 - sqrt(c)/(a*x)
```

Fricas [A]
time = 0.41, size = 31, normalized size = 0.51

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)), x)
```

3.859 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}$$

```
[Out] -1/2*(c*x^2)^(1/2)/x^3/a+b*(c*x^2)^(1/2)/a^2/x^2+b^2*ln(x)*(c*x^2)^(1/2)/a^
3/x-b^2*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 46}

$$\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] -1/2*Sqrt[c*x^2]/(a*x^3) + (b*Sqrt[c*x^2])/(a^2*x^2) + (b^2*Sqrt[c*x^2]*Log
[x])/(a^3*x) - (b^2*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx&=\frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}\,dx}{x}\\&=\frac{\sqrt{cx^2}\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\&=-\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.63

$$\frac{\sqrt{cx^2}\,(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx))}{2a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x])
)/(2*a^3*x^3)
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.61

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2)}{2a^3x^3}$	51
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{bx}{a^2}-\frac{1}{2a}\right)}{x^3}+\frac{\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^3}-\frac{b^2\ln(bx+a)\sqrt{c\,x^2}}{a^3x}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^3
```

Maxima [A]
time = 0.26, size = 52, normalized size = 0.62

$$-\frac{b^2\sqrt{c}\,\log(bx+a)}{a^3}+\frac{b^2\sqrt{c}\,\log(x)}{a^3}+\frac{2\,b\sqrt{c}\,x-a\sqrt{c}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*sqrt(c)*log(b*x + a)/a^3 + b^2*sqrt(c)*log(x)/a^3 + 1/2*(2*b*sqrt(c)*x
 - a*sqrt(c))/(a^2*x^2)
```

Fricas [A]
time = 0.40, size = 44, normalized size = 0.52

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^4 (a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^4\,\left(a+b\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)), x)
```

3.860 $\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=107

$$-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] -a^3*c*(c*x^2)^(1/2)/b^4+1/2*a^2*c*x*(c*x^2)^(1/2)/b^3-1/3*a*c*x^2*(c*x^2)^
(1/2)/b^2+1/4*c*x^3*(c*x^2)^(1/2)/b+a^4*c*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{a^4c\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2])/b^4) + (a^2*c*x*Sqrt[c*x^2])/(2*b^3) - (a*c*x^2*Sqrt[
c*x^2])/(3*b^2) + (c*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c*Sqrt[c*x^2]*Log[a + b*
x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.60

$$\frac{(cx^2)^{3/2}\left(bx(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3)+12a^4\log(a+bx)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^
4*Log[a + b*x]))/(12*b^5*x^3)
```

Maple [A]
time = 0.13, size = 63, normalized size = 0.59

method	result	size
default	$\frac{(cx^2)^{\frac{3}{2}}(3b^4x^4-4ab^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx)}{12b^5x^3}$	63
risch	$\frac{c\sqrt{cx^2}\left(\frac{1}{4}b^3x^4-\frac{1}{3}ab^2x^3+\frac{1}{2}a^2bx^2-a^3x\right)}{xb^4}+\frac{a^4c\ln(bx+a)\sqrt{cx^2}}{b^5x}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*x^2)^(3/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x^3
```

Maxima [A]
time = 0.29, size = 124, normalized size = 1.16

$$\frac{(-1)^{\frac{2cx}{b}}a^4c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}a^4c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{(cx^2)^{\frac{3}{2}}x}{4b}+\frac{\sqrt{cx^2}a^2cx}{2b^3}-\frac{(cx^2)^{\frac{3}{2}}a}{3b^2}-\frac{\sqrt{cx^2}a^3c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(3/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*x/b + 1/2*sqrt(c*x^2
)*a^2*c*x/b^3 - 1/3*(c*x^2)^(3/2)*a/b^2 - sqrt(c*x^2)*a^3*c/b^4
```

Fricas [A]
time = 0.45, size = 67, normalized size = 0.63

$$\frac{\left(3\, b^{4} c x^{4} - 4\, a b^{3} c x^{3} + 6\, a^{2} b^{2} c x^{2} - 12\, a^{3} b c x + 12\, a^{4} c \log\left(b x + a\right)\right) \sqrt{c x^{2}}}{12\, b^{5} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c*x^4 - 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 - 12*a^3*b*c*x + 12*a^4
*c*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c x^{2}\right)^{\frac{3}{2}}}{a + b x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x), x)
```

Giac [A]
time = 2.30, size = 81, normalized size = 0.76

$$\frac{1}{12}\, c^{\frac{3}{2}} \left(\frac{12\, a^{4} \log\left(|bx + a|\right) \mathrm{sgn}\left(x\right)}{b^{5}} - \frac{12\, a^{4} \log\left(|a|\right) \mathrm{sgn}\left(x\right)}{b^{5}} + \frac{3\, b^{3} x^{4} \mathrm{sgn}\left(x\right) - 4\, a b^{2} x^{3} \mathrm{sgn}\left(x\right) + 6\, a^{2} b x^{2} \mathrm{sgn}\left(x\right) - 12\, a^{3} x \mathrm{sgn}\left(x\right)}{b^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*c^(3/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(c\,x^2\right)}^{3/2}}{a+b\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x), x)
```

3.861 $\int \frac{(cx^2)^{3/2}}{a+bx} dx$

Optimal. Leaf size=84

$$\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] a^2*c*(c*x^2)^(1/2)/b^3-1/2*a*c*x*(c*x^2)^(1/2)/b^2+1/3*c*x^2*(c*x^2)^(1/2)
/b-a^3*c*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]

time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}+\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (a^2*c*Sqrt[c*x^2])/b^3 - (a*c*x*Sqrt[c*x^2])/(2*b^2) + (c*x^2*Sqrt[c*x^2])
/(3*b) - (a^3*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{a+bx}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\
&= \frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\,\log(a+bx)}{b^4x}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.63

$$\frac{(cx^2)^{3/2}\left(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6
*b^4*x^3)
```

Maple [A]
time = 0.11, size = 52, normalized size = 0.62

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6x^3b^4}$	52
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{x\,b^3}-\frac{a^3c\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^3/b
^4
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.30

$$-\frac{(-1)^{\frac{2cx}{b}}a^3c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2acx}{b}}a^3c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,acx}{2\,b^2}+\frac{(cx^2)^{\frac{3}{2}}}{3\,b}+\frac{\sqrt{cx^2}\,a^2c}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(3/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c*x/b^2 + 1/3*(c*x^
2)^(3/2)/b + sqrt(c*x^2)*a^2*c/b^3
```

Fricas [A]
time = 0.46, size = 55, normalized size = 0.65

$$\frac{\left(2\,b^3cx^3-3\,ab^2cx^2+6\,a^2bcx-6\,a^3c\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")

[Out] 1/6*(2*b^3*c*x^3 - 3*a*b^2*c*x^2 + 6*a^2*b*c*x - 6*a^3*c*log(b*x + a))*sqrt(c*x^2)/(b^4*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(3/2)/(b*x+a),x)

[Out] Integral((c*x**2)**(3/2)/(a + b*x), x)

Giac [A]
time = 2.02, size = 69, normalized size = 0.82

$$-\frac{1}{6}\,c^{\frac{3}{2}}\left(\frac{6\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2x^3\mathrm{sgn}(x)-3\,abx^2\mathrm{sgn}(x)+6\,a^2x\mathrm{sgn}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")

[Out] -1/6*c^(3/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(3/2)/(a + b*x),x)

[Out] int((c*x^2)^(3/2)/(a + b*x), x)

3.862 $\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx$

Optimal. Leaf size=61

$$-\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}+\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] -a*c*(c*x^2)^(1/2)/b^2+1/2*c*x*(c*x^2)^(1/2)/b+a^2*c*ln(b*x+a)*(c*x^2)^(1/2
)/b^3/x
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] -((a*c*Sqrt[c*x^2])/b^2) + (c*x*Sqrt[c*x^2])/(2*b) + (a^2*c*Sqrt[c*x^2]*Log
[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}$$
$$= -\frac{ac\sqrt{cx^2}}{b^2}+\frac{cx\sqrt{cx^2}}{2b}+\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}$$

Mathematica [**A**]
time = 0.00, size = 42, normalized size = 0.69

$$\frac{c^2x(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]
[Out] (c^2*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [**A**]
time = 0.11, size = 40, normalized size = 0.66

method	result	size
default	$\frac{(cx^2)^{\frac{3}{2}}(x^2b^2+2a^2\ln(bx+a)-2abx)}{2b^3x^3}$	40
risch	$\frac{c\sqrt{cx^2}\,(\frac{1}{2}x^2b-ax)}{x\,b^2}+\frac{a^2c\ln(bx+a)\sqrt{cx^2}}{b^3x}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(3/2)*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^3
```

Maxima [**A**]
time = 0.31, size = 93, normalized size = 1.52

$$\frac{(-1)^{\frac{2cx}{b}}a^2c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2acx}{b}}a^2c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,cx}{2b}-\frac{\sqrt{cx^2}\,ac}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c*x/b - sqrt(c*x^2)*a*
c/b^2
```

Fricas [A]
time = 0.43, size = 42, normalized size = 0.69

$$\frac{\left(b^2cx^2 - 2\,abcx + 2\,a^2c\log\left(bx + a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c*x^2 - 2*a*b*c*x + 2*a^2*c*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x\left(a + bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)), x)
```

Giac [A]
time = 1.59, size = 54, normalized size = 0.89

$$\frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{2\,a^2\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^3} - \frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3} + \frac{bx^2\mathrm{sgn}(x) - 2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x\,\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)), x)
```

3.863 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=40

$$\frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] c*(c*x^2)^(1/2)/b-a*c*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2])/b - (a*c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx}{x}\\ &= \frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 0.75

$$\frac{c^2x(bx - a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
[Out] (c^2*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.72

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}(a\ln(bx+a)-bx)}{b^2x^3}$	29
risch	$\frac{c\sqrt{c\,x^2}}{b} - \frac{ac\ln(bx+a)\sqrt{c\,x^2}}{b^2x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(3/2)*(a*ln(b*x+a)-b*x)/b^2/x^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(36) = 72.
time = 0.29, size = 75, normalized size = 1.88

$$-\frac{(-1)^{\frac{2cx}{b}}ac^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}ac^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}\,c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="maxima")
[Out] -(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(3/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c/b
```

Fricas [A]
time = 0.40, size = 29, normalized size = 0.72

$$\frac{(bcx - ac\log(bx+a))\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x - a*c*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.44, size = 37, normalized size = 0.92

$$c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b}-\frac{a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^2}+\frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)), x)
```

3.864 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)} dx$

Optimal. Leaf size=23

$$\frac{c\sqrt{cx^2} \log(a+bx)}{bx}$$

```
[Out] c*ln(b*x+a)*(c*x^2)^(1/2)/b/x
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{c\sqrt{cx^2} \log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)} dx &= \frac{\left(c\sqrt{cx^2}\right) \int \frac{1}{a+bx} dx}{x}\\ &= \frac{c\sqrt{cx^2} \log(a+bx)}{bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2} \log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

Maple [A]
time = 0.13, size = 21, normalized size = 0.91

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\ln\left(bx+a\right)}{x^{3}b}$	21
risch	$\frac{c\ln\left(bx+a\right)\sqrt{c\,x^{2}}}{bx}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)/x^3*ln(b*x+a)/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.57

$$\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] c^(3/2)*log(b*x + a)/b
```

Fricas [A]
time = 0.42, size = 21, normalized size = 0.91

$$\frac{\sqrt{cx^{2}}\,c\log\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c*log(b*x + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{3}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x)
```

Giac [A]
time = 1.15, size = 28, normalized size = 1.22

$$c^{\frac{3}{2}}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b}-\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] c^(3/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^3\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^3*(a + b*x)), x)
```

3.865 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{c\sqrt{cx^2}\,\log(x)}{ax}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

```
[Out] c*ln(x)*(c*x^2)^(1/2)/a/x-c*ln(b*x+a)*(c*x^2)^(1/2)/a/x
```

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c\sqrt{cx^2}\,\log(x)}{ax}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[x])/(a*x) - (c*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax} - \frac{\left(bc\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{c\sqrt{cx^2}\log(x)}{ax} - \frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.61

$$\frac{(cx^2)^{3/2}(\log(x)-\log(a+bx))}{ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*(Log[x] - Log[a + b*x]))/(a*x^3)
```

Maple [A]
time = 0.11, size = 26, normalized size = 0.59

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(\ln(x)-\ln(bx+a))}{a\,x^3}$	26
risch	$\frac{c\sqrt{c\,x^2}\;\ln(-x)}{xa} - \frac{c\ln(bx+a)\sqrt{c\,x^2}}{ax}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(3/2)*(ln(x)-ln(b*x+a))/a/x^3
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.55

$$-\frac{c^{\frac{3}{2}}\log(bx+a)}{a} + \frac{c^{\frac{3}{2}}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(3/2)*log(b*x + a)/a + c^(3/2)*log(x)/a
```

Fricas [A]
time = 0.45, size = 66, normalized size = 1.50

$$\left[\frac{\sqrt{cx^2}\,c\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^4\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)), x)
```

3.866 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx$

Optimal. Leaf size=64

$$-\frac{c\sqrt{cx^2}}{ax^2}-\frac{bc\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] -c*(c*x^2)^(1/2)/a/x^2-b*c*ln(x)*(c*x^2)^(1/2)/a^2/x+b*c*ln(b*x+a)*(c*x^2)^
(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{bc\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a*x^2)) - (b*c*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c*Sqrt[c
*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{ax^2}-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.53

$$-\frac{c^2(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
[Out] -((c^2*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Maple [A]
time = 0.11, size = 33, normalized size = 0.52

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}(bx\ln(x)-b\ln(bx+a)x+a)}{a^2x^4}$	33
risch	$-\frac{c\sqrt{c\,x^2}}{a\,x^2}+\frac{c\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^2}-\frac{bc\ln(x)\sqrt{c\,x^2}}{a^2x}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(3/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x^4
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.58

$$\frac{bc^{\frac{3}{2}}\log(bx+a)}{a^2}-\frac{bc^{\frac{3}{2}}\log(x)}{a^2}-\frac{c^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*log(b*x + a)/a^2 - b*c^(3/2)*log(x)/a^2 - c^(3/2)/(a*x)
```

Fricas [A]
time = 0.48, size = 33, normalized size = 0.52

$$\frac{\left(bcx\log\left(\frac{bx+a}{x}\right)-ac\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x*log((b*x + a)/x) - a*c)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{3}{2}}}{x^5\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(c\,x^2\right)}^{3/2}}{x^5\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)), x)
```

3.867 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)} dx$

Optimal. Leaf size=88

$$-\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}$$

```
[Out] -1/2*c*(c*x^2)^(1/2)/x^3/a+b*c*(c*x^2)^(1/2)/a^2/x^2+b^2*c*ln(x)*(c*x^2)^(1
/2)/a^3/x-b^2*c*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] -1/2*(c*Sqrt[c*x^2])/(a*x^3) + (b*c*Sqrt[c*x^2])/(a^2*x^2) + (b^2*c*Sqrt[c*
x^2]*Log[x])/(a^3*x) - (b^2*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.60

$$\frac{(cx^2)^{3/2}\left(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx)\right)}{2a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
[Out] ((c*x^2)^(3/2)*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x
]))/(2*a^3*x^5)
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.58

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2\right)}{2a^3x^5}$	51
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{bx}{a^2}-\frac{1}{2a}\right)}{x^3}+\frac{c\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^3}-\frac{b^2c\ln(bx+a)\sqrt{c\,x^2}}{a^3x}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(3/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^5
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.59

$$-\frac{b^2c^{\frac{3}{2}}\log(bx+a)}{a^3}+\frac{b^2c^{\frac{3}{2}}\log(x)}{a^3}+\frac{2\,bc^{\frac{3}{2}}x-ac^{\frac{3}{2}}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*c^(3/2)*log(b*x + a)/a^3 + b^2*c^(3/2)*log(x)/a^3 + 1/2*(2*b*c^(3/2)*x
 - a*c^(3/2))/(a^2*x^2)
```

Fricas [A]
time = 0.46, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2cx^2\log\left(\frac{x}{bx+a}\right)+2\,abcx-a^2c\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*c*x^2*log(x/(b*x + a)) + 2*a*b*c*x - a^2*c)*sqrt(c*x^2)/(a^3*x^3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^2\right)}^{3/2}}{x^6\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)), x)
```

3.868 $\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=112

$$-\frac{c\sqrt{cx^2}}{3ax^4}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

```
[Out] -1/3*c*(c*x^2)^(1/2)/a/x^4+1/2*b*c*(c*x^2)^(1/2)/a^2/x^3-b^2*c*(c*x^2)^(1/2
)/a^3/x^2-b^3*c*ln(x)*(c*x^2)^(1/2)/a^4/x+b^3*c*ln(b*x+a)*(c*x^2)^(1/2)/a^4
/x
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{b^3c\sqrt{cx^2}\,\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\,\log(a+bx)}{a^4x}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -1/3*(c*Sqrt[c*x^2])/(a*x^4) + (b*c*Sqrt[c*x^2])/(2*a^2*x^3) - (b^2*c*Sqrt[
c*x^2])/(a^3*x^2) - (b^3*c*Sqrt[c*x^2]*Log[x])/(a^4*x) + (b^3*c*Sqrt[c*x^2]
*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^4(a+bx)}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{3ax^4} + \frac{bc\sqrt{cx^2}}{2a^2x^3} - \frac{b^2c\sqrt{cx^2}}{a^3x^2} - \frac{b^3c\sqrt{cx^2}\log(x)}{a^4x} + \frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.58

$$-\frac{(cx^2)^{3/2}\left(a(2a^2-3abx+6b^2x^2)+6b^3x^3\log(x)-6b^3x^3\log(a+bx)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -1/6*((c*x^2)^(3/2)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] - 6
*b^3*x^3*Log[a + b*x]))/(a^4*x^6)
```

Maple [A]
time = 0.13, size = 62, normalized size = 0.55

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3)}{6a^4x^6}$	62
risch	$\frac{c\sqrt{c\,x^2}\left(-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}\right)}{x^4} - \frac{b^3c\ln(x)\sqrt{c\,x^2}}{a^4x} + \frac{c\sqrt{c\,x^2}\,b^3\ln(-bx-a)}{x\,a^4}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^7/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b
*x+2*a^3)/a^4/x^6
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.59

$$\frac{b^3c^{\frac{3}{2}}\log(bx+a)}{a^4} - \frac{b^3c^{\frac{3}{2}}\log(x)}{a^4} - \frac{6\,b^2c^{\frac{3}{2}}x^2 - 3\,abc^{\frac{3}{2}}x + 2\,a^2c^{\frac{3}{2}}}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*c^(3/2)*log(b*x + a)/a^4 - b^3*c^(3/2)*log(x)/a^4 - 1/6*(6*b^2*c^(3/2)*
x^2 - 3*a*b*c^(3/2)*x + 2*a^2*c^(3/2))/(a^3*x^3)
```

Fricas [A]
time = 0.44, size = 59, normalized size = 0.53

$$\frac{\left(6\,b^3cx^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2cx^2+3\,a^2bcx-2\,a^3c\right)\sqrt{cx^2}}{6\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*c*x^3*log((b*x + a)/x) - 6*a*b^2*c*x^2 + 3*a^2*b*c*x - 2*a^3*c)*
sqrt(c*x^2)/(a^4*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^7\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**7*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^2\right)}^{3/2}}{x^7\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^7*(a + b*x)), x)
```

3.869 $\int \frac{(cx^2)^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=142

$$\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}-\frac{a^5c^2\sqrt{cx^2}\,\log(a+bx)}{b^6x}$$

```
[Out] a^4*c^2*(c*x^2)^(1/2)/b^5-1/2*a^3*c^2*x*(c*x^2)^(1/2)/b^4+1/3*a^2*c^2*x^2*(
c*x^2)^(1/2)/b^3-1/4*a*c^2*x^3*(c*x^2)^(1/2)/b^2+1/5*c^2*x^4*(c*x^2)^(1/2)/
b-a^5*c^2*ln(b*x+a)*(c*x^2)^(1/2)/b^6/x
```

Rubi [A]
time = 0.03, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{15, 45\}$

$$-\frac{a^5c^2\sqrt{cx^2}\,\log(a+bx)}{b^6x}+\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2])/b^5 - (a^3*c^2*x*Sqrt[c*x^2])/(2*b^4) + (a^2*c^2*x^2*
Sqrt[c*x^2])/(3*b^3) - (a*c^2*x^3*Sqrt[c*x^2])/(4*b^2) + (c^2*x^4*Sqrt[c*x^
2])/(5*b) - (a^5*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^6*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{a+bx}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^5}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a^5}{b^5(a+bx)}\right)dx}{x}$$
$$= \frac{a^4c^2\sqrt{cx^2}}{b^5} - \frac{a^3c^2x\sqrt{cx^2}}{2b^4} + \frac{a^2c^2x^2\sqrt{cx^2}}{3b^3} - \frac{ac^2x^3\sqrt{cx^2}}{4b^2} + \frac{c^2x^4\sqrt{cx^2}}{5b} - \frac{a^5c^2\sqrt{cx^2}\,\log(a}{b^6x}$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 0.54

$$\frac{c^3x(bx(60a^4 - 30a^3bx + 20a^2b^2x^2 - 15ab^3x^3 + 12b^4x^4) - 60a^5\log(a+bx))}{60b^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(a + b*x),x]
[Out] (c^3*x*(b*x*(60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^4*x
^4) - 60*a^5*Log[a + b*x]))/(60*b^6*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.52

method	result	size
default	$-\frac{(cx^2)^{\frac{5}{2}}(-12b^5x^5+15a\,b^4x^4-20a^2b^3x^3+30a^3b^2x^2+60a^5\ln(bx+a)-60a^4bx)}{60x^5b^6}$	74
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{5}b^4x^5-\frac{1}{4}a\,b^3x^4+\frac{1}{3}a^2b^2x^3-\frac{1}{2}a^3b\,x^2+a^4x\right)}{x\,b^5}-\frac{a^5c^2\ln(bx+a)\sqrt{c\,x^2}}{b^6x}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/60*(c*x^2)^(5/2)*(-12*b^5*x^5+15*a*b^4*x^4-20*a^2*b^3*x^3+30*a^3*b^2*x^2
+60*a^5*ln(b*x+a)-60*a^4*b*x)/x^5/b^6
```

Maxima [A]
time = 0.29, size = 146, normalized size = 1.03

$$-\frac{(-1)^{\frac{2cx}{b}}a^5c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^6} - \frac{(-1)^{\frac{2acx}{b}}a^5c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^6} - \frac{(cx^2)^{\frac{3}{2}}acx}{4b^2} - \frac{\sqrt{cx^2}\,a^3c^2x}{2b^4} + \frac{(cx^2)^{\frac{5}{2}}}{5b} + \frac{(cx^2)^{\frac{3}{2}}a^2c}{3b^3} + \frac{\sqrt{cx^2}\,a^4c^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^5*c^(5/2)*log(2*c*x/b)/b^6 - (-1)^(2*a*c*x/b)*a^5*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^6 - 1/4*(c*x^2)^(3/2)*a*c*x/b^2 - 1/2*sqr
t(c*x^2)*a^3*c^2*x/b^4 + 1/5*(c*x^2)^(5/2)/b + 1/3*(c*x^2)^(3/2)*a^2*c/b^3
+ sqrt(c*x^2)*a^4*c^2/b^5
```

Fricas [**A**]
time = 0.92, size = 91, normalized size = 0.64

$$\frac{\left(12\,b^5c^2x^5-15\,ab^4c^2x^4+20\,a^2b^3c^2x^3-30\,a^3b^2c^2x^2+60\,a^4bc^2x-60\,a^5c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{60\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*c^2*x^5 - 15*a*b^4*c^2*x^4 + 20*a^2*b^3*c^2*x^3 - 30*a^3*b^2*c
^2*x^2 + 60*a^4*b*c^2*x - 60*a^5*c^2*log(b*x + a))*sqrt(c*x^2)/(b^6*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(a + b*x), x)
```

Giac [**A**]
time = 1.40, size = 116, normalized size = 0.82

$$-\frac{1}{60}\left(\frac{60\,a^5c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^6}-\frac{60\,a^5c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^6}-\frac{12\,b^4c^2x^5\mathrm{sgn}(x)-15\,ab^3c^2x^4\mathrm{sgn}(x)+20\,a^2b^2c^2x^3\mathrm{sgn}(x)-30\,a^3bc^2x^2\mathrm{sgn}(x)+60\,a^4c^2x\mathrm{sgn}(x)}{b^5}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/60*(60*a^5*c^2*log(abs(b*x + a))*sgn(x)/b^6 - 60*a^5*c^2*log(abs(a))*sgn
(x)/b^6 - (12*b^4*c^2*x^5*sgn(x) - 15*a*b^3*c^2*x^4*sgn(x) + 20*a^2*b^2*c^2
*x^3*sgn(x) - 30*a^3*b*c^2*x^2*sgn(x) + 60*a^4*c^2*x*sgn(x))/b^5)*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^2\right)}^{5/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(5/2)/(a + b*x), x)
```

3.870 $\int \frac{(cx^2)^{5/2}}{x(a+bx)} dx$

Optimal. Leaf size=117

$$-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}$$

```
[Out] -a^3*c^2*(c*x^2)^(1/2)/b^4+1/2*a^2*c^2*x*(c*x^2)^(1/2)/b^3-1/3*a*c^2*x^2*(c
*x^2)^(1/2)/b^2+1/4*c^2*x^3*(c*x^2)^(1/2)/b+a^4*c^2*ln(b*x+a)*(c*x^2)^(1/2)
/b^5/x
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{a^4c^2\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2])/b^4) + (a^2*c^2*x*Sqrt[c*x^2])/(2*b^3) - (a*c^2*x^2
*Sqrt[c*x^2])/(3*b^2) + (c^2*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c^2*Sqrt[c*x^2]*
Log[a + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}$$
$$= -\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.00, size = 65, normalized size = 0.56

$$\frac{c(cx^2)^{3/2}\left(bx(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3)+12a^4\log(a+bx)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]
[Out] (c*(c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*
a^4*Log[a + b*x]))/(12*b^5*x^3)
```

Maple [A]
time = 0.15, size = 63, normalized size = 0.54

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx)}{12b^5x^5}$	63
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{4}b^3x^4-\frac{1}{3}a\,b^2x^3+\frac{1}{2}a^2b\,x^2-a^3x\right)}{x\,b^4}+\frac{a^4c^2\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/12*(c*x^2)^(5/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x^5
```

Maxima [A]
time = 0.29, size = 130, normalized size = 1.11

$$\frac{(-1)^{\frac{2cx}{b}}\,a^4c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}\,a^4c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{(cx^2)^{\frac{3}{2}}\,cx}{4\,b}+\frac{\sqrt{cx^2}\,a^2c^2x}{2\,b^3}-\frac{(cx^2)^{\frac{3}{2}}\,ac}{3\,b^2}-\frac{\sqrt{cx^2}\,a^3c^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(5/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*c*x/b + 1/2*sqrt(c*x
^2)*a^2*c^2*x/b^3 - 1/3*(c*x^2)^(3/2)*a*c/b^2 - sqrt(c*x^2)*a^3*c^2/b^4
```

Fricas [A]
time = 0.74, size = 77, normalized size = 0.66

$$\frac{\left(3\,b^{4}c^{2}x^{4}-4\,ab^{3}c^{2}x^{3}+6\,a^{2}b^{2}c^{2}x^{2}-12\,a^{3}bc^{2}x+12\,a^{4}c^{2}\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{12\,b^{5}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c^2*x^4 - 4*a*b^3*c^2*x^3 + 6*a^2*b^2*c^2*x^2 - 12*a^3*b*c^2*x
+ 12*a^4*c^2*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x*(a + b*x)), x)
```

Giac [A]
time = 1.64, size = 99, normalized size = 0.85

$$\frac{1}{12}\left(\frac{12\,a^{4}c^{2}\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b^{5}}-\frac{12\,a^{4}c^{2}\log\left(|a|\right)\operatorname{sgn}(x)}{b^{5}}+\frac{3\,b^{3}c^{2}x^{4}\operatorname{sgn}(x)-4\,ab^{2}c^{2}x^{3}\operatorname{sgn}(x)+6\,a^{2}bc^{2}x^{2}\operatorname{sgn}(x)-12\,a^{3}c^{2}x\operatorname{sgn}(x)}{b^{4}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(12*a^4*c^2*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*c^2*log(abs(a))*sgn(
x)/b^5 + (3*b^3*c^2*x^4*sgn(x) - 4*a*b^2*c^2*x^3*sgn(x) + 6*a^2*b*c^2*x^2*s
gn(x) - 12*a^3*c^2*x*sgn(x))/b^4)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^{2}\right)}^{5/2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x*(a + b*x)), x)
```

3.871 $\int \frac{(cx^2)^{5/2}}{x^2(a+bx)} dx$

Optimal. Leaf size=92

$$\frac{a^2c^2\sqrt{cx^2}}{b^3} - \frac{ac^2x\sqrt{cx^2}}{2b^2} + \frac{c^2x^2\sqrt{cx^2}}{3b} - \frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

```
[Out] a^2*c^2*(c*x^2)^(1/2)/b^3-1/2*a*c^2*x*(c*x^2)^(1/2)/b^2+1/3*c^2*x^2*(c*x^2)
^(1/2)/b-a^3*c^2*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x} + \frac{a^2c^2\sqrt{cx^2}}{b^3} - \frac{ac^2x\sqrt{cx^2}}{2b^2} + \frac{c^2x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2])/b^3 - (a*c^2*x*Sqrt[c*x^2])/(2*b^2) + (c^2*x^2*Sqrt[c
*x^2])/(3*b) - (a^3*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}$$
$$= \frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 0.59

$$\frac{c(cx^2)^{3/2}\left(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/
(6*b^4*x^3)
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.57

method	result	size
default	$-\frac{(c\,x^2)^{\frac{5}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6b^4x^5}$	52
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{x\,b^3}-\frac{a^3c^2\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(c*x^2)^(5/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/b^4/x
^5
```

Maxima [A]
time = 0.28, size = 114, normalized size = 1.24

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a^3c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^3c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ac^2x}{2\,b^2}+\frac{(cx^2)^{\frac{3}{2}}\,c}{3\,b}+\frac{\sqrt{cx^2}\,a^2c^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(5/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c^2*x/b^2 + 1/3*(c*
x^2)^(3/2)*c/b + sqrt(c*x^2)*a^2*c^2/b^3
```

Fricas [A]
time = 0.78, size = 63, normalized size = 0.68

$$\frac{\left(2\,b^{3}c^{2}x^{3}-3\,ab^{2}c^{2}x^{2}+6\,a^{2}bc^{2}x-6\,a^{3}c^{2}\log\left(bx+a\right)\right)\sqrt{cx^{2}}}{6\,b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c^2*x^3 - 3*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - 6*a^3*c^2*log(b*x +
a))*sqrt(c*x^2)/(b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**2*(a + b*x)), x)
```

Giac [A]
time = 1.09, size = 84, normalized size = 0.91

$$-\frac{1}{6}\left(\frac{6\,a^{3}c^{2}\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^{4}}-\frac{6\,a^{3}c^{2}\log\left(|a|\right)\mathrm{sgn}(x)}{b^{4}}-\frac{2\,b^{2}c^{2}x^{3}\mathrm{sgn}(x)-3\,abc^{2}x^{2}\mathrm{sgn}(x)+6\,a^{2}c^{2}x\mathrm{sgn}(x)}{b^{3}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*c^2*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*c^2*log(abs(a))*sgn(x)
/b^4 - (2*b^2*c^2*x^3*sgn(x) - 3*a*b*c^2*x^2*sgn(x) + 6*a^2*c^2*x*sgn(x))/b
^3)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^{2}\right)}^{5/2}}{x^{2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^2*(a + b*x)), x)
```

3.872 $\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=67

$$-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] -a*c^2*(c*x^2)^(1/2)/b^2+1/2*c^2*x*(c*x^2)^(1/2)/b+a^2*c^2*ln(b*x+a)*(c*x^2
)^(1/2)/b^3/x
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2])/b^2) + (c^2*x*Sqrt[c*x^2])/(2*b) + (a^2*c^2*Sqrt[c*x^
2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)\,dx}{x}$$
$$= -\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.63

$$\frac{c^3x(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
[Out] (c^3*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A]
time = 0.17, size = 40, normalized size = 0.60

method	result	size
default	$\frac{(c\,x^2)^{\frac{5}{2}}\,(x^2b^2+2a^2\ln(bx+a)-2abx)}{2b^3x^5}$	40
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(\frac{1}{2}x^2b-ax\right)}{x\,b^2}+\frac{a^2c^2\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^2)^(5/2)*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^5
```

Maxima [A]
time = 0.28, size = 97, normalized size = 1.45

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,c^2x}{2\,b}-\frac{\sqrt{cx^2}\,ac^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="maxima")
```

[Out] (-1)^(2*c*x/b)*a^2*c^(5/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(5/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c^2*x/b - sqrt(c*x^2)*a*c^2/b^2

Fricas [A]
time = 0.58, size = 48, normalized size = 0.72

$$\frac{(b^2c^2x^2 - 2\,abc^2x + 2\,a^2c^2\log{(bx + a)})\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="fricas")

[Out] 1/2*(b^2*c^2*x^2 - 2*a*b*c^2*x + 2*a^2*c^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}}{x^3\,(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(5/2)/x**3/(b*x+a),x)

[Out] Integral((c*x**2)**(5/2)/(x**3*(a + b*x)), x)

Giac [A]
time = 1.83, size = 66, normalized size = 0.99

$$\frac{1}{2}\left(\frac{2\,a^2c^2\log{(|bx + a|)}\,\mathrm{sgn}(x)}{b^3} - \frac{2\,a^2c^2\log{(|a|)}\,\mathrm{sgn}(x)}{b^3} + \frac{bc^2x^2\mathrm{sgn}(x) - 2\,ac^2x\mathrm{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="giac")

[Out] 1/2*(2*a^2*c^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*c^2*log(abs(a))*sgn(x)/b^3 + (b*c^2*x^2*sgn(x) - 2*a*c^2*x*sgn(x))/b^2)*sqrt(c)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{5/2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(5/2)/(x^3*(a + b*x)),x)

[Out] int((c*x^2)^(5/2)/(x^3*(a + b*x)), x)

3.873 $\int \frac{(cx^2)^{5/2}}{x^4(a+bx)} \, dx$

Optimal. Leaf size=44

$$\frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

```
[Out] c^2*(c*x^2)^(1/2)/b-a*c^2*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2])/b - (a*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^4(a+bx)} \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x}{a+bx} \, dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx}{x} \\
&= \frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 0.68

$$\frac{c^3x(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
[Out] (c^3*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.66

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(a\ln\left(bx+a\right)-bx\right)}{b^{2}x^{5}}$	29
risch	$\frac{c^{2}\sqrt{c\,x^{2}}}{b}-\frac{a\,c^{2}\ln\left(bx+a\right)\sqrt{c\,x^{2}}}{b^{2}x}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^4/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(5/2)*(a*ln(b*x+a)-b*x)/b^2/x^5
```

Maxima [A]
time = 0.29, size = 77, normalized size = 1.75

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^2}-\frac{(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}+\frac{\sqrt{cx^2}\,c^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="maxima")
[Out] -(-1)^(2*c*x/b)*a*c^(5/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(5/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c^2/b
```

Fricas [A]
time = 0.73, size = 33, normalized size = 0.75

$$\frac{(bc^2x - ac^2\log(bx + a))\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x - a*c^2*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}}{x^4\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**4*(a + b*x)), x)
```

Giac [**A**]
time = 1.46, size = 46, normalized size = 1.05

$$\left(\frac{c^2x\mathrm{sgn}(x)}{b} - \frac{ac^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^2} + \frac{ac^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] (c^2*x*sgn(x)/b - a*c^2*log(abs(b*x + a))*sgn(x)/b^2 + a*c^2*log(abs(a))*sg
n(x)/b^2)*sqrt(c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^4*(a + b*x)), x)
```

3.874 $\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx$

Optimal. Leaf size=25

$$\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}$$

[Out] c^2*ln(b*x+a)*(c*x^2)^(1/2)/b/x

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

[In] Int[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]

[Out] (c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^5(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

Maple [A]
time = 0.13, size = 21, normalized size = 0.84

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}} \ln \left(b x +a \right)}{x^{5} b}$	21
risch	$\frac{c^{2} \ln \left(b x +a \right) \sqrt{c\,x^{2}}}{b x}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^5/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(5/2)/x^5*ln(b*x+a)/b
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.52

$$\frac{c^{\frac{5}{2}} \log \left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] c^(5/2)*log(b*x + a)/b
```

Fricas [A]
time = 0.57, size = 23, normalized size = 0.92

$$\frac{\sqrt{cx^2}\, c^2 \log \left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*c^2*log(b*x + a)/(b*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^5 \left(a+bx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**5/(b*x+a),x)
[Out] Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x)
```

Giac [A]
time = 1.16, size = 34, normalized size = 1.36

$$\left(\frac{c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b}-\frac{c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="giac")
[Out] (c^2*log(abs(b*x + a))*sgn(x)/b - c^2*log(abs(a))*sgn(x)/b)*sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^5\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^5*(a + b*x)),x)
[Out] int((c*x^2)^(5/2)/(x^5*(a + b*x)), x)
```

3.875 $\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx$

Optimal. Leaf size=48

$$\frac{c^2\sqrt{cx^2}\,\log(x)}{ax}-\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

```
[Out] c^2*ln(x)*(c*x^2)^(1/2)/a/x-c^2*ln(b*x+a)*(c*x^2)^(1/2)/a/x
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c^2\sqrt{cx^2}\,\log(x)}{ax}-\frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[x])/(a*x) - (c^2*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^6(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax} - \frac{\left(bc^2\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}$$
$$= \frac{c^2\sqrt{cx^2}\,\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\,\log(a+bx)}{ax}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.58

$$\frac{c^3x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^3*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 26, normalized size = 0.54

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(\ln(x)-\ln(bx+a)\right)}{a\,x^{5}}$	26
risch	$\frac{c^{2}\sqrt{c\,x^{2}}\,\ln(-x)}{xa}-\frac{c^{2}\ln(bx+a)\sqrt{c\,x^{2}}}{ax}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^6/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(5/2)*(ln(x)-ln(b*x+a))/a/x^5
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.50

$$-\frac{c^{\frac{5}{2}}\log\left(bx+a\right)}{a}+\frac{c^{\frac{5}{2}}\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="maxima")
```

[Out] -c^(5/2)*log(b*x + a)/a + c^(5/2)*log(x)/a

Fricas [A]
time = 0.81, size = 70, normalized size = 1.46

$$\left[\frac{\sqrt{cx^2}\,c^2\log\left(\frac{x}{bx+a}\right)}{ax},\ \frac{2\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="fricas")

[Out] [sqrt(c*x^2)*c^2*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c^2*arctan(sqrt(c*x^2)*(2*b*x + a)*sqrt(-c)/(a*c*x))/a]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}}{x^6\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(5/2)/x**6/(b*x+a),x)

[Out] Integral((c*x**2)**(5/2)/(x**6*(a + b*x)), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sa

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(c\,x^2)^{5/2}}{x^6\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(5/2)/(x^6*(a + b*x)),x)

[Out] int((c*x^2)^(5/2)/(x^6*(a + b*x)), x)

3.876 $\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{c^2\sqrt{cx^2}}{ax^2}-\frac{bc^2\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] -c^2*(c*x^2)^(1/2)/a/x^2-b*c^2*ln(x)*(c*x^2)^(1/2)/a^2/x+b*c^2*ln(b*x+a)*(c
*x^2)^(1/2)/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{bc^2\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\,\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^2*Sqrt[c*x^2])/(a*x^2)) - (b*c^2*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c^2*
Sqrt[c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}$$
$$= -\frac{c^2\sqrt{cx^2}}{ax^2}-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [**A**]
time = 0.01, size = 34, normalized size = 0.49

$$-\frac{c^3(a+bx\log(x)-bx\log(a+bx))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
[Out] -((c^3*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

Maple [**A**]
time = 0.13, size = 33, normalized size = 0.47

method	result	size
default	$-\frac{\left(c\,x^{2}\right)^{\frac{5}{2}}\left(bx\ln\left(x\right)-b\ln\left(bx+a\right)x+a\right)}{a^{2}x^{6}}$	33
risch	$-\frac{c^{2}\sqrt{c\,x^{2}}}{a\,x^{2}}+\frac{c^{2}\sqrt{c\,x^{2}}\,b\ln\left(-bx-a\right)}{x\,a^{2}}-\frac{b\,c^{2}\ln\left(x\right)\sqrt{c\,x^{2}}}{a^{2}x}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^7/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(5/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/a^2/x^6
```

Maxima [**A**]
time = 0.30, size = 37, normalized size = 0.53

$$\frac{bc^{\frac{5}{2}}\log\left(bx+a\right)}{a^2}-\frac{bc^{\frac{5}{2}}\log\left(x\right)}{a^2}-\frac{c^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*log(b*x + a)/a^2 - b*c^(5/2)*log(x)/a^2 - c^(5/2)/(a*x)
```

Fricas [A]
time = 0.90, size = 37, normalized size = 0.53

$$\frac{\left(bc^2x\log\left(\frac{bx+a}{x}\right)-ac^2\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x*log((b*x + a)/x) - a*c^2)*sqrt(c*x^2)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}}{x^7\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**7*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{(c\,x^2)}^{5/2}}{x^7\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^7*(a + b*x)), x)
```

3.877 $\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=83

$$\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}$$

```
[Out] a^2*x^2/b^3/(c*x^2)^(1/2)-1/2*a*x^3/b^2/(c*x^2)^(1/2)+1/3*x^4/b/(c*x^2)^(1/
2)-a^3*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}+\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*Sqrt[c*x^2]) + x^4/(3*b*Sqrt[c
*x^2]) - (a^3*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{x^3}{a+bx}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} - \frac{a^3}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x^2}{b^3\sqrt{cx^2}} - \frac{ax^3}{2b^2\sqrt{cx^2}} + \frac{x^4}{3b\sqrt{cx^2}} - \frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.61

$$\frac{x(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx))}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[c*
x^2])
```

Maple [A]
time = 0.12, size = 50, normalized size = 0.60

method	result	size
default	$-\frac{x\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx\right)}{6\sqrt{c\,x^2}\,b^4}$	50
risch	$\frac{x\left(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x\right)}{\sqrt{c\,x^2}\,b^3}-\frac{a^3x\ln(bx+a)}{b^4\sqrt{c\,x^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(1/2)/b^4
```

Maxima [A]
time = 0.29, size = 142, normalized size = 1.71

$$\frac{\sqrt{cx^2}\,x^2}{3\,bc} - \frac{7\,ax^2}{6\,b^2\sqrt{c}} - \frac{(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}} + \frac{2\,\sqrt{cx^2}\,ax}{3\,b^2c} - \frac{14\,a^2x}{3\,b^3\sqrt{c}} - \frac{a^3\log(bx)}{b^4\sqrt{c}} + \frac{17\,\sqrt{cx^2}\,a^2}{3\,b^3c} - \frac{7\,a^3}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*x^2/(b*c) - 7/6*a*x^2/(b^2*sqrt(c)) - (-1)^(2*a*c*x/b)*a^3*
log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^2*c)
- 14/3*a^2*x/(b^3*sqrt(c)) - a^3*log(b*x)/(b^4*sqrt(c)) + 17/3*sqrt(c*x^2)*
a^2/(b^3*c) - 7/2*a^3/(b^4*sqrt(c))
```

Fricas [**A**]
time = 0.94, size = 54, normalized size = 0.65

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{cx^2}\,\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [**A**]
time = 0.74, size = 75, normalized size = 0.90

$$\frac{a^3\log\left(|a|\right)\operatorname{sgn}(x)}{b^4\sqrt{c}}-\frac{a^3\log\left(|bx+a|\right)}{b^4\sqrt{c}\,\operatorname{sgn}\left(x\right)}+\frac{2\,b^2cx^3-3\,abcx^2+6\,a^2cx}{6\,b^3c^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a^3*log(abs(a))*sgn(x)/(b^4*sqrt(c)) - a^3*log(abs(b*x + a))/(b^4*sqrt(c)*s
gn(x)) + 1/6*(2*b^2*c*x^3 - 3*a*b*c*x^2 + 6*a^2*c*x)/(b^3*c^(3/2)*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.878 $\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=61

$$-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}$$

```
[Out] -a*x^2/b^2/(c*x^2)^(1/2)+1/2*x^3/b/(c*x^2)^(1/2)+a^2*x*ln(b*x+a)/b^3/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[c*x^2])) + x^3/(2*b*Sqrt[c*x^2]) + (a^2*x*Log[a + b*x])
/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{x^2}{a+bx}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.64

$$\frac{x(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

Maple [A]
time = 0.11, size = 38, normalized size = 0.62

method	result	size
default	$\frac{x\left(x^{2} b^{2}+2 a^{2} \ln \left(b x+a\right)-2 a b x\right)}{2 \sqrt{c\,x^{2}}\, b^{3}}$	38
risch	$\frac{x\left(\frac{1}{2} x^{2} b-a x\right)}{\sqrt{c\,x^{2}}\, b^{2}}+\frac{a^{2} x \ln \left(b x+a\right)}{b^{3} \sqrt{c\,x^{2}}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(1/2)/b^3
```

Maxima [A]
time = 0.29, size = 100, normalized size = 1.64

$$\frac{x^2}{2\,b\sqrt{c}}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}}+\frac{2\,ax}{b^2\sqrt{c}}+\frac{a^2\log\left(bx\right)}{b^3\sqrt{c}}-\frac{3\,\sqrt{cx^2}\,a}{b^2c}+\frac{3\,a^2}{2\,b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/(b*sqrt(c)) + (-1)^(2*a*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^3*sqrt(c)) + 2*a*x/(b^2*sqrt(c)) + a^2*log(b*x)/(b^3*sqrt(c)) - 3*sqrt(c*
x^2)*a/(b^2*c) + 3/2*a^2/(b^3*sqrt(c))
```

Fricas [A]
time = 0.65, size = 42, normalized size = 0.69

$$\frac{\left(b^2 x^2 - 2\,abx + 2\,a^2 \log\left(bx + a\right)\right)\sqrt{cx^2}}{2\,b^3 cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^2}\,(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.63, size = 66, normalized size = 1.08

$$-\frac{a^2 \log\left(|a|\right)\mathrm{sgn}(x)}{b^3\sqrt{c}} + \frac{a^2 \log\left(|bx + a|\right)}{b^3\sqrt{c}\,\mathrm{sgn}\left(x\right)} + \frac{b\sqrt{c}\,x^2\mathrm{sgn}(x) - 2\,a\sqrt{c}\,x\mathrm{sgn}(x)}{2\,b^2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a))*sgn(x)/(b^3*sqrt(c)) + a^2*log(abs(b*x + a))/(b^3*sqrt(c)*
sgn(x)) + 1/2*(b*sqrt(c)*x^2*sgn(x) - 2*a*sqrt(c)*x*sgn(x))/(b^2*c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{c\,x^2}\,\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.879 $\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

```
[Out] x^2/b/(c*x^2)^(1/2)-a*x*ln(b*x+a)/b^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] x^2/(b*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.69

$$\frac{x(bx - a\log(a + bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 27, normalized size = 0.69

method	result	size
default	$-\frac{x(a\ln(bx+a)-bx)}{\sqrt{c\,x^2}\,b^2}$	27
risch	$\frac{x^2}{b\sqrt{c\,x^2}} - \frac{ax\ln(bx+a)}{b^2\sqrt{c\,x^2}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(a*ln(b*x+a)-b*x)/(c*x^2)^(1/2)/b^2
```

Maxima [A]
time = 0.29, size = 64, normalized size = 1.64

$$-\frac{(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}} - \frac{a\log(bx)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^2*sqrt(c)) - a*log(b*
x)/(b^2*sqrt(c)) + sqrt(c*x^2)/(b*c)
```

Fricas [A]
time = 0.49, size = 30, normalized size = 0.77

$$\frac{\sqrt{cx^2}\,(bx - a\log(bx + a))}{b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.58, size = 46, normalized size = 1.18

$$\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{b^2\sqrt{c}}+\frac{x}{b\sqrt{c}\operatorname{sgn}(x)}-\frac{a\log\left(|bx+a|\right)}{b^2\sqrt{c}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] a*log(abs(a))*sgn(x)/(b^2*sqrt(c)) + x/(b*sqrt(c)*sgn(x)) - a*log(abs(b*x +
 a))/(b^2*sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{c\,x^2}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.880 $\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=20

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[c*x^2]*(a + b*x)),x]

[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 19, normalized size = 0.95

method	result	size
default	$\frac{x\ln(bx+a)}{b\sqrt{c\,x^2}}$	19
risch	$\frac{x\ln(bx+a)}{b\sqrt{c\,x^2}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 0.28, size = 46, normalized size = 2.30

$$\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b\sqrt{c}}+\frac{\log(bx)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*sqrt(c)) + log(b*x)/(b*s
qrt(c))
```

Fricas [A]
time = 0.43, size = 23, normalized size = 1.15

$$\frac{\sqrt{cx^2}\,\log(bx+a)}{bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [A]
time = 0.50, size = 32, normalized size = 1.60

$$-\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{b\sqrt{c}}+\frac{\log\left(|bx+a|\right)}{b\sqrt{c}\operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(a))*sgn(x)/(b*sqrt(c)) + log(abs(b*x + a))/(b*sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.881 $\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

```
[Out] x*ln(x)/a/(c*x^2)^(1/2)-x*ln(b*x+a)/a/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \frac{1}{x}\,dx}{a\sqrt{cx^2}} - \frac{(bx)\int \frac{1}{a+bx}\,dx}{a\sqrt{cx^2}}$$
$$= \frac{x\log(x)}{a\sqrt{cx^2}} - \frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 0.66

$$\frac{x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

Maple [A]
time = 0.11, size = 24, normalized size = 0.63

method	result	size
default	$\frac{x(\ln(x)-\ln(bx+a))}{\sqrt{c\,x^2}\,a}$	24
risch	$\frac{x\ln(-x)}{\sqrt{c\,x^2}\,a}-\frac{x\ln(bx+a)}{a\sqrt{c\,x^2}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(ln(x)-ln(b*x+a))/(c*x^2)^(1/2)/a
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.92

$$-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*sqrt(c))
```

Fricas [A]
time = 0.43, size = 70, normalized size = 1.84

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{acx}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x
 + a)*sqrt(-c)/(a*c*x))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^2}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{c\,x^2}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)), x)
```

3.882 $\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}$$

```
[Out] -1/a/(c*x^2)^(1/2)-b*x*ln(x)/a^2/(c*x^2)^(1/2)+b*x*ln(b*x+a)/a^2/(c*x^2)^(1
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 46}

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(1/(a*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*Sqrt[c*x^2]) + (b*x*Log[a + b*x])/
(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.67

$$\frac{cx^2(-a-bx\log(x)+bx\log(a+bx))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
[Out] (c*x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.56

method	result	size
default	$-\frac{bx\ln(x)-b\ln(bx+a)x+a}{\sqrt{c\,x^2}\;a^2}$	30
risch	$-\frac{1}{a\sqrt{c\,x^2}}+\frac{xb\ln(-bx-a)}{\sqrt{c\,x^2}\;a^2}-\frac{bx\ln(x)}{a^2\sqrt{c\,x^2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x*ln(x)-b*ln(b*x+a)*x+a)/(c*x^2)^(1/2)/a^2
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.69

$$\frac{b\log(bx+a)}{a^2\sqrt{c}}-\frac{b\log(x)}{a^2\sqrt{c}}-\frac{1}{a\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b*log(b*x + a)/(a^2*sqrt(c)) - b*log(x)/(a^2*sqrt(c)) - 1/(a*sqrt(c)*x)
```

Fricas [A]
time = 0.41, size = 34, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)), x)
```

3.883 $\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=77

$$\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}$$

```
[Out] b/a^2/(c*x^2)^(1/2)-1/2/a/x/(c*x^2)^(1/2)+b^2*x*ln(x)/a^3/(c*x^2)^(1/2)-b^2
*x*ln(b*x+a)/a^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}+\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] b/(a^2*Sqrt[c*x^2]) - 1/(2*a*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*Sqrt[c*x^
2]) - (b^2*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{b}{a^2\sqrt{cx^2}} - \frac{1}{2ax\sqrt{cx^2}} + \frac{b^2x\log(x)}{a^3\sqrt{cx^2}} - \frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.68

$$\frac{cx(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx))}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3
*(c*x^2)^(3/2))
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.66

method	result	size
default	$\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2}{2x\sqrt{c\,x^2}\,a^3}$	51
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{\sqrt{c\,x^2}\,x}+\frac{x\,b^2\ln(-x)}{\sqrt{c\,x^2}\,a^3}-\frac{b^2x\ln(bx+a)}{a^3\sqrt{c\,x^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/(c*x^2)^(1/2)/a^3
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.71

$$-\frac{b^2\log\left(bx+a\right)}{a^3\sqrt{c}} + \frac{b^2\log\left(x\right)}{a^3\sqrt{c}} + \frac{2\,b\sqrt{c}\,x - a\sqrt{c}}{2\,a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/(a^3*sqrt(c)) + b^2*log(x)/(a^3*sqrt(c)) + 1/2*(2*b*sqrt(
c)*x - a*sqrt(c))/(a^2*c*x^2)
```

Fricas [A]
time = 0.50, size = 47, normalized size = 0.61

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)), x)
```

3.884 $\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{b^2}{a^3\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}$$

```
[Out] -b^2/a^3/(c*x^2)^(1/2)-1/3/a/x^2/(c*x^2)^(1/2)+1/2*b/a^2/x/(c*x^2)^(1/2)-b^
3*x*ln(x)/a^4/(c*x^2)^(1/2)+b^3*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}-\frac{b^2}{a^3\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*Sqrt[c*x^2])) - 1/(3*a*x^2*Sqrt[c*x^2]) + b/(2*a^2*x*Sqrt[c*x^2]
) - (b^3*x*Log[x])/(a^4*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*Sqrt[c*x^2
])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{b^2}{a^3\sqrt{cx^2}} - \frac{1}{3ax^2\sqrt{cx^2}} + \frac{b}{2a^2x\sqrt{cx^2}} - \frac{b^3x\log(x)}{a^4\sqrt{cx^2}} + \frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.63

$$\frac{c(a(-2a^2+3abx-6b^2x^2)-6b^3x^3\log(x)+6b^3x^3\log(a+bx))}{6a^4\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
[Out] (c*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log[a +
 b*x]))/(6*a^4*(c*x^2)^(3/2))
```

Maple [A]
time = 0.14, size = 62, normalized size = 0.62

method	result	size
default	$-\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3}{6x^2\sqrt{c\,x^2}\,a^4}$	62
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{\sqrt{c\,x^2}\,x^2} - \frac{b^3x\ln(x)}{a^4\sqrt{c\,x^2}} + \frac{x\,b^3\ln(-bx-a)}{\sqrt{c\,x^2}\,a^4}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6/x^2*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/
(c*x^2)^(1/2)/a^4
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.69

$$\frac{b^3\log(bx+a)}{a^4\sqrt{c}} - \frac{b^3\log(x)}{a^4\sqrt{c}} - \frac{6\,b^2\sqrt{c}\,x^2 - 3\,ab\sqrt{c}\,x + 2\,a^2\sqrt{c}}{6\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*sqrt(c)) - b^3*log(x)/(a^4*sqrt(c)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c*x^3)
```

Fricas [A]
time = 0.43, size = 58, normalized size = 0.58

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^3\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c*x**2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^3\,\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)), x)
```

3.885 $\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=95

$$\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}$$

```
[Out] a^2*x^2/b^3/c/(c*x^2)^(1/2)-1/2*a*x^3/b^2/c/(c*x^2)^(1/2)+1/3*x^4/b/c/(c*x^
2)^(1/2)-a^3*x*ln(b*x+a)/b^4/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}+\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*c*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*c*Sqrt[c*x^2]) + x^4/(3*b*c*
Sqrt[c*x^2]) - (a^3*x*Log[a + b*x])/(b^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{x^3}{a+bx}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{a^2x^2}{b^3c\sqrt{cx^2}} - \frac{ax^3}{2b^2c\sqrt{cx^2}} + \frac{x^4}{3bc\sqrt{cx^2}} - \frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.56

$$\frac{x^3(bx(6a^2-3abx+2b^2x^2)-6a^3\log(a+bx))}{6b^4(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*(c*x^
2)^(3/2))
```

Maple [A]
time = 0.13, size = 52, normalized size = 0.55

method	result	size
default	$-\frac{x^3(-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx)}{6(c\,x^2)^{\frac{3}{2}}b^4}$	52
risch	$\frac{x(\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x)}{c\sqrt{c\,x^2}\,b^3}-\frac{a^3x\ln(bx+a)}{b^4c\sqrt{c\,x^2}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/6*x^3*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(3/2)/b
^4
```

Maxima [A]
time = 0.30, size = 162, normalized size = 1.71

$$\frac{x^4}{3\sqrt{cx^2}\,bc}-\frac{ax^3}{2\sqrt{cx^2}\,b^2c}+\frac{a^2x^2}{\sqrt{cx^2}\,b^3c}-\frac{(-1)^{\frac{2acx}{b}}a^3\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4c^{\frac{3}{2}}}+\frac{29\,a^3x}{6\sqrt{cx^2}\,b^4c}-\frac{a^3\log(bx)}{b^4c^{\frac{3}{2}}}-\frac{2\,a^4}{\sqrt{cx^2}\,b^5c}+\frac{2\,a^4}{b^5c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/(sqrt(c*x^2)*b*c) - 1/2*a*x^3/(sqrt(c*x^2)*b^2*c) + a^2*x^2/(sqrt(c
*x^2)*b^3*c) - (-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*c^(
3/2)) + 29/6*a^3*x/(sqrt(c*x^2)*b^4*c) - a^3*log(b*x)/(b^4*c^(3/2)) - 2*a^4
/(sqrt(c*x^2)*b^5*c) + 2*a^4/(b^5*c^(3/2)*x)
```

Fricas [A]
time = 0.40, size = 54, normalized size = 0.57

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^6}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 1.30, size = 80, normalized size = 0.84

$$\frac{\frac{6\,a^3\log\left(|a|\right)\mathrm{sgn}\left(x\right)}{b^4\sqrt{c}}-\frac{6\,a^3\log\left(|bx+a|\right)}{b^4\sqrt{c}\,\mathrm{sgn}\left(x\right)}+\frac{2\,b^2cx^3-3\,abcx^2+6\,a^2cx}{b^3c^{\frac{3}{2}}\,\mathrm{sgn}\left(x\right)}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(a))*sgn(x)/(b^4*sqrt(c)) - 6*a^3*log(abs(b*x + a))/(b^4*
sqrt(c)*sgn(x)) + (2*b^2*c*x^3 - 3*a*b*c*x^2 + 6*a^2*c*x)/(b^3*c^(3/2)*sgn(
x)))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^6}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c*x^2)^(3/2)*(a + b*x)),x)
[Out] int(x^6/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.886 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}$$

```
[Out] -a*x^2/b^2/c/(c*x^2)^(1/2)+1/2*x^3/b/c/(c*x^2)^(1/2)+a^2*x*ln(b*x+a)/b^3/c/
(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*c*Sqrt[c*x^2])) + x^3/(2*b*c*Sqrt[c*x^2]) + (a^2*x*Log[a + b
*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{x^2}{a+bx}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 0.59

$$\frac{x^3(bx(-2a+bx)+2a^2\log(a+bx))}{2b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*(c*x^2)^(3/2))
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.57

method	result	size
default	$\frac{x^3(x^2b^2+2a^2\ln(bx+a)-2abx)}{2(cx^2)^{\frac{3}{2}}b^3}$	40
risch	$\frac{x\left(\frac{1}{2}x^2b-ax\right)}{c\sqrt{cx^2}\,b^2}+\frac{a^2x\ln(bx+a)}{b^3c\sqrt{cx^2}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*x^3*(x^2*b^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(3/2)/b^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(62) = 124.
time = 0.31, size = 140, normalized size = 2.00

$$\frac{x^3}{2\sqrt{cx^2}\,bc}-\frac{ax^2}{\sqrt{cx^2}\,b^2c}+\frac{(-1)^{\frac{2acx}{b}}a^2\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}-\frac{7a^2x}{2\sqrt{cx^2}\,b^3c}+\frac{a^2\log(bx)}{b^3c^{\frac{3}{2}}}+\frac{2a^3}{\sqrt{cx^2}\,b^4c}-\frac{2a^3}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(c*x^2)*b*c) - a*x^2/(sqrt(c*x^2)*b^2*c) + (-1)^(2*a*c*x/b)*a^
2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) - 7/2*a^2*x/(sqrt(c*x^2)*b^3
*c) + a^2*log(b*x)/(b^3*c^(3/2)) + 2*a^3/(sqrt(c*x^2)*b^4*c) - 2*a^3/(b^4*c
^(3/2)*x)
```

Fricas [A]
time = 0.37, size = 42, normalized size = 0.60

$$\frac{\left(b^{2} x^{2}-2 \, a b x+2 \, a^{2} \log \left(b x+a\right)\right) \sqrt{c x^{2}}}{2 \, b^{3} c^{2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(c x^{2}\right)^{\frac{3}{2}} \left(a+b x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 1.48, size = 72, normalized size = 1.03

$$-\frac{\frac{2 \, a^{2} \log \left(|a|\right) \mathrm{sgn}\left(x\right)}{b^{3} \sqrt{c}}-\frac{2 \, a^{2} \log \left(|b x+a|\right)}{b^{3} \sqrt{c} \mathrm{sgn}\left(x\right)}-\frac{b \sqrt{c} x^{2} \mathrm{sgn}\left(x\right)-2 \, a \sqrt{c} x \mathrm{sgn}\left(x\right)}{b^{2} c}}{2 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(a))*sgn(x)/(b^3*sqrt(c)) - 2*a^2*log(abs(b*x + a))/(b^3
*sqrt(c)*sgn(x)) - (b*sqrt(c)*x^2*sgn(x) - 2*a*sqrt(c)*x*sgn(x))/(b^2*c))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.887 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=45

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

```
[Out] x^2/b/c/(c*x^2)^(1/2)-a*x*ln(b*x+a)/b^2/c/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] x^2/(b*c*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.64

$$\frac{x^3(bx - a\log(a + bx))}{b^2 (cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x - a*Log[a + b*x]))/(b^2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.64

method	result	size
default	$-\frac{x^{3} \left(a \ln \left(b x +a \right)-b x \right)}{\left(c \,x^{2}\right)^{\frac{3}{2}} b^{2}}$	29
risch	$\frac{x^{2}}{b c \sqrt{c \,x^{2}}}-\frac{a x \ln \left(b x +a \right)}{b^{2} c \sqrt{c \,x^{2}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -x^3*(a*ln(b*x+a)-b*x)/(c*x^2)^(3/2)/b^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(41) = 82.
time = 0.29, size = 116, normalized size = 2.58

$$\frac{x^2}{\sqrt{cx^2}\,bc} - \frac{(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}} + \frac{2\,ax}{\sqrt{cx^2}\,b^2c} - \frac{a\log(bx)}{b^2c^{\frac{3}{2}}} - \frac{2\,a^2}{\sqrt{cx^2}\,b^3c} + \frac{2\,a^2}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] x^2/(sqrt(c*x^2)*b*c) - (-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^2*c^(3/2)) + 2*a*x/(sqrt(c*x^2)*b^2*c) - a*log(b*x)/(b^2*c^(3/2)) - 2*a^2
/(sqrt(c*x^2)*b^3*c) + 2*a^2/(b^3*c^(3/2)*x)
```

Fricas [A]
time = 0.37, size = 30, normalized size = 0.67

$$\frac{\sqrt{cx^2}\,(bx - a\log(bx + a))}{b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 1.21, size = 50, normalized size = 1.11

$$\frac{\frac{a\log(|a|)\operatorname{sgn}(x)}{b^2\sqrt{c}}+\frac{x}{b\sqrt{c}\,\operatorname{sgn}(x)}-\frac{a\log(|bx+a|)}{b^2\sqrt{c}\,\operatorname{sgn}(x)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] (a*log(abs(a))*sgn(x)/(b^2*sqrt(c)) + x/(b*sqrt(c)*sgn(x)) - a*log(abs(b*x
+ a))/(b^2*sqrt(c)*sgn(x)))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.888 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)} \, dx$

Optimal. Leaf size=23

$$\frac{x \log(a+bx)}{bc\sqrt{cx^2}}$$

[Out] x*ln(b*x+a)/b/c/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 31}

$$\frac{x \log(a+bx)}{bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)),x]

[Out] (x*Log[a + b*x])/(b*c*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(cx^2)^{3/2}(a+bx)} \, dx &= \frac{x \int \frac{1}{a+bx} \, dx}{c\sqrt{cx^2}} \\ &= \frac{x \log(a+bx)}{bc\sqrt{cx^2}} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 0.96

$$\frac{x^3 \log(a+bx)}{b\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

Maple [A]
time = 0.13, size = 21, normalized size = 0.91

method	result	size
default	$\frac{x^{3} \ln \left(b x +a \right)}{\left(c \,x^{2}\right)^{\frac{3}{2}} b}$	21
risch	$\frac{x \ln \left(b x +a \right)}{b c \sqrt{c \,x^{2}}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*ln(b*x+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(21) = 42.
time = 0.30, size = 74, normalized size = 3.22

$$\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{bc^{\frac{3}{2}}}+\frac{\log\left(bx\right)}{bc^{\frac{3}{2}}}+\frac{2\,a}{\sqrt{cx^2}\,b^2c}-\frac{2\,a}{b^2c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*c^(3/2)) + log(b*x)/(b*c
^(3/2)) + 2*a/(sqrt(c*x^2)*b^2*c) - 2*a/(b^2*c^(3/2)*x)
```

Fricas [A]
time = 0.52, size = 23, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,\log\left(bx+a\right)}{bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [A]
time = 0.97, size = 37, normalized size = 1.61

$$-\frac{\frac{\log\left(|a|\right)\mathrm{sgn}\left(x\right)}{b\sqrt{c}}-\frac{\log\left(|bx+a|\right)}{b\sqrt{c}\,\mathrm{sgn}\left(x\right)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(log(abs(a))*sgn(x)/(b*sqrt(c)) - log(abs(b*x + a))/(b*sqrt(c)*sgn(x)))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.889 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

```
[Out] x*ln(x)/a/c/(c*x^2)^(1/2)-x*ln(b*x+a)/a/c/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*c*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx = \frac{x\int \frac{1}{x(a+bx)}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \frac{1}{x}\,dx}{ac\sqrt{cx^2}} - \frac{(bx)\int \frac{1}{a+bx}\,dx}{ac\sqrt{cx^2}}$$
$$= \frac{x\log(x)}{ac\sqrt{cx^2}} - \frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.61

$$\frac{x^3(\log(x)-\log(a+bx))}{a\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(Log[x] - Log[a + b*x]))/(a*(c*x^2)^(3/2))
```

Maple [A]
time = 0.13, size = 26, normalized size = 0.59

method	result	size
default	$\frac{x^3(\ln(x)-\ln(bx+a))}{(c\,x^2)^{\frac{3}{2}}\,a}$	26
risch	$\frac{x\ln(-x)}{c\sqrt{c\,x^2}\,a} - \frac{x\ln(bx+a)}{ac\sqrt{c\,x^2}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(ln(x)-ln(b*x+a))/(c*x^2)^(3/2)/a
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.80

$$-\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*c^(3/2))
```

Fricas [A]
time = 0.51, size = 70, normalized size = 1.59

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ac^2x}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c^2*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/(a*c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{{(c\,x^2)}^{3/2}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.890 $\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx$

Optimal. Leaf size=63

$$-\frac{1}{ac\sqrt{cx^2}}-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}$$

```
[Out] -1/a/c/(c*x^2)^(1/2)-b*x*ln(x)/a^2/c/(c*x^2)^(1/2)+b*x*ln(b*x+a)/a^2/c/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 46}

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(1/(a*c*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*c*Sqrt[c*x^2]) + (b*x*Log[a + b*
x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{1}{ac\sqrt{cx^2}}-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 0.56

$$\frac{x^2(-a-bx\log(x)+bx\log(a+bx))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 33, normalized size = 0.52

method	result	size
default	$-\frac{x^2(bx\ln(x)-b\ln(bx+a)x+a)}{(c\,x^2)^{\frac{3}{2}}a^2}$	33
risch	$-\frac{1}{ac\sqrt{c\,x^2}}+\frac{xb\ln(-bx-a)}{c\sqrt{c\,x^2}\,a^2}-\frac{bx\ln(x)}{a^2c\sqrt{c\,x^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] -x^2*(b*x*ln(x)-b*ln(b*x+a)*x+a)/(c*x^2)^(3/2)/a^2
```

Maxima [A]
time = 0.30, size = 51, normalized size = 0.81

$$\frac{(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}}-\frac{1}{\sqrt{cx^2}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] (-1)^(2*a*c*x/b)*b*log(-2*a*c*x/(b*abs(b*x + a)))/(a^2*c^(3/2)) - 1/(sqrt(c
*x^2)*a*c)
```

Fricas [A]
time = 0.43, size = 34, normalized size = 0.54

$$\frac{\sqrt{cx^2}\,\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c^2*x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a),x)
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)),x)
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.891 $\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=89

$$\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}$$

```
[Out] b/a^2/c/(c*x^2)^(1/2)-1/2/a/c/x/(c*x^2)^(1/2)+b^2*x*ln(x)/a^3/c/(c*x^2)^(1/
2)-b^2*x*ln(b*x+a)/a^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 46}

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] b/(a^2*c*Sqrt[c*x^2]) - 1/(2*a*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*c*Sqr
t[c*x^2]) - (b^2*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.57

$$\frac{x(-a(a-2bx)+2b^2x^2\log(x)-2b^2x^2\log(a+bx))}{2a^3(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3*(
c*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 49, normalized size = 0.55

method	result	size
default	$\frac{x\left(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2\right)}{2\left(cx^2\right)^{\frac{3}{2}}a^3}$	49
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{cx\sqrt{cx^2}}+\frac{x\,b^2\ln(-x)}{c\sqrt{cx^2}\,a^3}-\frac{b^2x\ln(bx+a)}{a^3c\sqrt{cx^2}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/(c*x^2)^(3/2)/a^3
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.73

$$-\frac{(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}+\frac{b}{\sqrt{cx^2}\,a^2c}-\frac{1}{2\,ac^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*b^2*log(-2*a*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) + b/(sqr
t(c*x^2)*a^2*c) - 1/2/(a*c^(3/2)*x^2)
```

Fricas [A]
time = 0.45, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)), x)
```

3.892 $\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=115

$$-\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

```
[Out] -b^2/a^3/c/(c*x^2)^(1/2)-1/3/a/c/x^2/(c*x^2)^(1/2)+1/2*b/a^2/c/x/(c*x^2)^(1
/2)-b^3*x*ln(x)/a^4/c/(c*x^2)^(1/2)+b^3*x*ln(b*x+a)/a^4/c/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}-\frac{b^2}{a^3c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*c*Sqrt[c*x^2])) - 1/(3*a*c*x^2*Sqrt[c*x^2]) + b/(2*a^2*c*x*Sqrt[
c*x^2]) - (b^3*x*Log[x])/(a^4*c*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*c*
Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\,(cx^2)^{3/2}\,(a+bx)}\,dx = \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{c\sqrt{cx^2}}$$

$$= \frac{x\int \left(\frac{1}{ax^4} - \frac{b}{a^2x^3} + \frac{b^2}{a^3x^2} - \frac{b^3}{a^4x} + \frac{b^4}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}$$

$$= -\frac{b^2}{a^3c\sqrt{cx^2}} - \frac{1}{3acx^2\sqrt{cx^2}} + \frac{b}{2a^2cx\sqrt{cx^2}} - \frac{b^3x\log(x)}{a^4c\sqrt{cx^2}} + \frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 0.57

$$\frac{cx^2(a(-2a^2+3abx-6b^2x^2)-6b^3x^3\log(x)+6b^3x^3\log(a+bx))}{6a^4\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (c*x^2*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log
[a + b*x]))/(6*a^4*(c*x^2)^(5/2))
```

Maple [A]
time = 0.14, size = 59, normalized size = 0.51

method	result	size
default	$-\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3}{6(c\,x^2)^{\frac{3}{2}}a^4}$	59
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{c\,x^2\sqrt{c\,x^2}} - \frac{b^3x\ln(x)}{a^4c\sqrt{c\,x^2}} + \frac{x\,b^3\ln(-bx-a)}{c\sqrt{c\,x^2}\;a^4}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/(c*x
^2)^(3/2)/a^4
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.60

$$\frac{b^3\log(bx+a)}{a^4c^{\frac{3}{2}}} - \frac{b^3\log(x)}{a^4c^{\frac{3}{2}}} - \frac{6\,b^2\sqrt{c}\,x^2 - 3\,ab\sqrt{c}\,x + 2\,a^2\sqrt{c}}{6\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*c^(3/2)) - b^3*log(x)/(a^4*c^(3/2)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c^2*x^3)
```

Fricas [A]
time = 0.45, size = 58, normalized size = 0.50

$$\frac{\left(6\, b^3 x^3 \log\left(\frac{bx+a}{x}\right) - 6\, ab^2 x^2 + 3\, a^2 bx - 2\, a^3\right)\sqrt{cx^2}}{6\, a^4 c^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(cx^2)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/(x*(c*x**2)**(3/2)*(a + b*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c\,x^2\right)}^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(3/2)*(a + b*x)), x)
```

3.893 $\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=106

$$\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}$$

```
[Out] 3*a^2*(c*x^2)^(1/2)/b^4-a*x*(c*x^2)^(1/2)/b^3+1/3*x^2*(c*x^2)^(1/2)/b^2-a^4
*(c*x^2)^(1/2)/b^5/x/(b*x+a)-4*a^3*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\,\log(a+bx)}{b^5x}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (3*a^2*Sqrt[c*x^2])/b^4 - (a*x*Sqrt[c*x^2])/b^3 + (x^2*Sqrt[c*x^2])/(3*b^2)
 - (a^4*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*Sqrt[c*x^2]*Log[a + b*x])/(
b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{x}$$
$$= \frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.02, size = 81, normalized size = 0.76

$$\frac{cx(-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4-12a^3(a+bx)\log(a+bx))}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(
a + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.14, size = 88, normalized size = 0.83

method	result	size
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-ab\,x^2+3a^2x\right)}{x\,b^4}-\frac{a^4\sqrt{c\,x^2}}{b^5x(bx+a)}-\frac{4a^3\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	87
default	$-\frac{\sqrt{c\,x^2}\,\left(-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)-9a^3bx+3a^4\right)}{3x\,b^5(bx+a)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^2)^(1/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x/b^5/(b*x+a)
```

Maxima [A]
time = 0.29, size = 135, normalized size = 1.27

$$\frac{\sqrt{cx^2}\,a^3}{b^5x+ab^4}-\frac{4\,(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^5}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}-\frac{\sqrt{cx^2}\,ax}{b^3}+\frac{3\,\sqrt{cx^2}\,a^2}{b^4}+\frac{(cx^2)^{\frac{3}{2}}}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*x + a*b^4) - 4*(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
sqrt(c*x^2)*a*x/b^3 + 3*sqrt(c*x^2)*a^2/b^4 + 1/3*(c*x^2)^(3/2)/(b^2*c)
```

Fricas [A]
time = 0.46, size = 83, normalized size = 0.78

$$\frac{\left(b^{4} x^{4} - 2\, a b^{3} x^{3} + 6\, a^{2} b^{2} x^{2} + 9\, a^{3} b x - 3\, a^{4} - 12\, \left(a^{3} b x + a^{4}\right) \log\left(b x + a\right)\right) \sqrt{c x^{2}}}{3\, \left(b^{6} x^{2} + a b^{5} x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \sqrt{c x^{2}}}{\left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 3.01, size = 96, normalized size = 0.91

$$-\frac{1}{3}\, \sqrt{c} \left(\frac{12\, a^{3} \log\left(|bx + a|\right) \mathrm{sgn}\left(x\right)}{b^{5}} + \frac{3\, a^{4} \mathrm{sgn}\left(x\right)}{(bx + a) b^{5}} - \frac{3\, \left(4\, a^{3} \log\left(|a|\right) + a^{3}\right) \mathrm{sgn}\left(x\right)}{b^{5}} - \frac{b^{4} x^{3} \mathrm{sgn}\left(x\right) - 3\, a b^{3} x^{2} \mathrm{sgn}\left(x\right) + 9\, a^{2} b^{2} x \mathrm{sgn}\left(x\right)}{b^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(c)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\, \sqrt{c\, x^2}}{{\left(a + b\, x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

3.894 $\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] -2*a*(c*x^2)^(1/2)/b^3+1/2*x*(c*x^2)^(1/2)/b^2+a^3*(c*x^2)^(1/2)/b^4/x/(b*x
+a)+3*a^2*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (-2*a*Sqrt[c*x^2])/b^3 + (x*Sqrt[c*x^2])/(2*b^2) + (a^3*Sqrt[c*x^2])/(b^4*x
*(a + b*x)) + (3*a^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}$$
$$= -\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.82

$$\frac{cx(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx))}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b
*x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.12, size = 76, normalized size = 0.89

method	result	size
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{1}{2}x^2b-2ax\right)}{x\,b^3}+\frac{a^3\sqrt{c\,x^2}}{b^4x(bx+a)}+\frac{3a^2\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	75
default	$\frac{\sqrt{c\,x^2}\,\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2x\,b^4(bx+a)}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x/b^4/(b*x+a)
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.39

$$-\frac{\sqrt{cx^2}\,a^2}{b^4x+ab^3}+\frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^4}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}+\frac{\sqrt{cx^2}\,x}{2\,b^2}-\frac{2\,\sqrt{cx^2}\,a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*x + a*b^3) + 3*(-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b
)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 +
 1/2*sqrt(c*x^2)*x/b^2 - 2*sqrt(c*x^2)*a/b^3
```

Fricas [A]
time = 0.48, size = 72, normalized size = 0.85

$$\frac{\left(b^{3} x^{3} - 3\,a b^{2} x^{2} - 4\,a^{2} b x + 2\,a^{3} + 6\,\left(a^{2} b x + a^{3}\right) \log\left(b x + a\right)\right) \sqrt{c x^{2}}}{2\,\left(b^{5} x^{2} + a b^{4} x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{c x^{2}}}{\left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 1.29, size = 80, normalized size = 0.94

$$\frac{1}{2}\,\sqrt{c}\left(\frac{6\,a^{2} \log\left(|bx + a|\right) \mathrm{sgn}\left(x\right)}{b^{4}} + \frac{2\,a^{3} \mathrm{sgn}\left(x\right)}{(bx + a) b^{4}} - \frac{2\,\left(3\,a^{2} \log\left(|a|\right) + a^{2}\right) \mathrm{sgn}\left(x\right)}{b^{4}} + \frac{b^{2} x^{2} \mathrm{sgn}\left(x\right) - 4\,a b x \mathrm{sgn}\left(x\right)}{b^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

3.895 $\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx^2}}{b^2}-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] (c*x^2)^(1/2)/b^2-a^2*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*ln(b*x+a)*(c*x^2)^(1/
2)/b^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]/b^2 - (a^2*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*Sqrt[c*x^2]*Lo
g[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}$$
$$= \frac{\sqrt{cx^2}}{b^2}-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.82

$$\frac{cx(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2
]*(a + b*x))
```

Maple [A]
time = 0.12, size = 62, normalized size = 0.95

method	result	size
risch	$\frac{\sqrt{c\,x^2}}{b^2}-\frac{a^2\sqrt{c\,x^2}}{b^3x(bx+a)}-\frac{2a\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	60
default	$-\frac{\sqrt{c\,x^2}\,(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2)}{x\,b^3(bx+a)}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x/b^3/
(b*x+a)
```

Maxima [A]
time = 0.29, size = 96, normalized size = 1.48

$$\frac{\sqrt{cx^2}\,a}{b^3x+ab^2}-\frac{2\,(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^3
 - 2*(-1)^(2*a*c*x/b)*a*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt(c
*x^2)/b^2
```

Fricas [A]
time = 0.43, size = 57, normalized size = 0.88

$$\frac{\left(b^{2} x^{2}+a b x-a^{2}-2 \left(a b x+a^{2}\right) \log \left(b x+a\right)\right) \sqrt{c x^{2}}}{b^{4} x^{2}+a b^{3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*x^2
 + a*b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{c x^{2}}}{\left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [A]
time = 1.00, size = 58, normalized size = 0.89

$$\sqrt{c} \left(\frac{x \mathrm{sgn}(x)}{b^{2}}-\frac{2 \, a \log \left(|bx+a|\right) \mathrm{sgn}(x)}{b^{3}}+\frac{(2 \, a \log \left(|a|\right)+a) \mathrm{sgn}(x)}{b^{3}}-\frac{a^{2} \mathrm{sgn}(x)}{(bx+a) b^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(c)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

3.896 $\int \frac{\sqrt{cx^2}}{(a+bx)^2} dx$

Optimal. Leaf size=47

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

[Out] a*(c*x^2)^(1/2)/b^2/x/(b*x+a)+ln(b*x+a)*(c*x^2)^(1/2)/b^2/x

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 45}

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c*x^2]/(a + b*x)^2,x]

[Out] (a*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{x}\\ &= \frac{a\sqrt{cx^2}}{b^2x(a+bx)}+\frac{\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.77

$$\frac{cx(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.87

method	result	size
default	$\frac{\sqrt{c\,x^{2}}\,(b\ln(bx+a)x+a\ln(bx+a)+a)}{x\,b^{2}(bx+a)}$	41
risch	$\frac{a\sqrt{c\,x^{2}}}{b^{2}x(bx+a)}+\frac{\ln(bx+a)\sqrt{c\,x^{2}}}{b^{2}x}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x/b^2/(b*x+a)
```

Maxima [A]
time = 0.28, size = 79, normalized size = 1.68

$$\frac{(-1)^{\frac{2\,cx}{b}}\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^2}+\frac{(-1)^{\frac{2\,acx}{b}}\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}-\frac{\sqrt{cx^2}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*sqrt(c)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*sqrt(c)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)/(b^2*x + a*b)
```

Fricas [A]
time = 0.49, size = 38, normalized size = 0.81

$$\frac{\sqrt{cx^2}\,((bx+a)\log(bx+a)+a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*x^2 + a*b^2*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(a + b*x)**2, x)
```

Giac [**A**]
time = 0.79, size = 46, normalized size = 0.98

$$-\sqrt{c}\left(\frac{(\log(|a|)+1)\mathrm{sgn}(x)}{b^2}-\frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2}-\frac{a\mathrm{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(c)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{{(a+b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(1/2)/(a + b*x)^2, x)
```

3.897 $\int \frac{\sqrt{cx^2}}{x(a+bx)^2} dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

```
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{(a+bx)^2}\,dx}{x}\\ &= -\frac{\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.96

$$-\frac{cx}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -((c*x)/(b*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A]
time = 0.12, size = 23, normalized size = 0.96

method	result	size
gosper	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
default	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
risch	$-\frac{\sqrt{c\,x^{2}}}{bx(bx+a)}$	23
trager	$\frac{(-1+x)\sqrt{c\,x^{2}}}{(bx+a)(a+b)x}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.67

$$-\frac{\sqrt{c}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c)/(b^2*x + a*b)
```

Fricas [A]
time = 0.52, size = 23, normalized size = 0.96

$$-\frac{\sqrt{cx^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [A]
time = 0.28, size = 32, normalized size = 1.33

$$\begin{cases} -\frac{\sqrt{cx^2}}{abx+b^2x^2} & \text{for } b \neq 0 \\ \frac{\sqrt{cx^2}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] Piecewise((-sqrt(c*x**2)/(a*b*x + b**2*x**2), Ne(b, 0)), (sqrt(c*x**2)/a**2
, True))
```

Giac [A]
time = 0.86, size = 29, normalized size = 1.21

$$-\sqrt{c}\left(\frac{\text{sgn}(x)}{(bx+a)b}-\frac{\text{sgn}(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(c)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

Mupad [B]
time = 0.16, size = 22, normalized size = 0.92

$$-\frac{\sqrt{c\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*x*(a + b*x))
```

3.898 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx^2}}{ax(a+bx)}+\frac{\sqrt{cx^2}\,\log(x)}{a^2x}-\frac{\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] (c*x^2)^(1/2)/a/x/(b*x+a)+ln(x)*(c*x^2)^(1/2)/a^2/x-ln(b*x+a)*(c*x^2)^(1/2)
/a^2/x
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{\sqrt{cx^2}\,\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]/(a*x*(a + b*x)) + (Sqrt[c*x^2]*Log[x])/(a^2*x) - (Sqrt[c*x^2]*L
og[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{x}$$
$$= \frac{\sqrt{cx^2}}{ax(a+bx)} + \frac{\sqrt{cx^2}\log(x)}{a^2x} - \frac{\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 0.69

$$\frac{cx(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
[Out] (c*x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a +
 b*x))
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.80

method	result	size
default	$\frac{\sqrt{c\,x^2}\,(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{x\,a^2(bx+a)}$	52
risch	$\frac{\sqrt{c\,x^2}}{ax(bx+a)}+\frac{\sqrt{c\,x^2}\,\ln(-x)}{x\,a^2}-\frac{\ln(bx+a)\sqrt{c\,x^2}}{a^2x}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(1/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/x/a^2/(b*x+a)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.58

$$\frac{\sqrt{c}}{abx+a^2} - \frac{\sqrt{c}\,\log(bx+a)}{a^2} + \frac{\sqrt{c}\,\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

[Out] sqrt(c)/(a*b*x + a^2) - sqrt(c)*log(b*x + a)/a^2 + sqrt(c)*log(x)/a^2

Fricas [A]
time = 0.45, size = 42, normalized size = 0.65

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="fricas")

[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*x^2 + a^3*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^2\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a)**2,x)

[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)**2), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sa

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{c\,x^2}}{x^2\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2),x)

[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2), x)

3.899 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{cx^2}}{a^2x^2}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

```
[Out] -(c*x^2)^(1/2)/a^2/x^2-b*(c*x^2)^(1/2)/a^2/x/(b*x+a)-2*b*ln(x)*(c*x^2)^(1/2
)/a^3/x+2*b*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 46\}$

$$-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(a^2*x^2)) - (b*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*Sqrt[c*
x^2]*Log[x])/(a^3*x) + (2*b*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{\sqrt{cx^2}}{a^2x^2} - \frac{b\sqrt{cx^2}}{a^2x(a+bx)} - \frac{2b\sqrt{cx^2}\log(x)}{a^3x} + \frac{2b\sqrt{cx^2}\log(a+bx)}{a^3x}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.66

$$-\frac{c(a(a+2bx)+2bx(a+bx)\log(x)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
[Out] -((c*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*x]
))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A]
time = 0.11, size = 74, normalized size = 0.85

method	result	size
default	$-\frac{\sqrt{c\,x^2}\,\left(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2\right)}{x^2a^3(bx+a)}$	74
risch	$\frac{\sqrt{c\,x^2}\,\left(-\frac{2bx}{a^2}-\frac{1}{a}\right)}{x^2(bx+a)} - \frac{2b\ln(x)\sqrt{c\,x^2}}{a^3x} + \frac{2\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+
a)*a*b*x+2*a*b*x+a^2)/x^2/a^3/(b*x+a)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.67

$$-\frac{2\,b\sqrt{c}\,x+a\sqrt{c}}{a^2bx^2+a^3x} + \frac{2\,b\sqrt{c}\,\log\left(bx+a\right)}{a^3} - \frac{2\,b\sqrt{c}\,\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="maxima")

[Out] -(2*b*sqrt(c)*x + a*sqrt(c))/(a^2*b*x^2 + a^3*x) + 2*b*sqrt(c)*log(b*x + a)/a^3 - 2*b*sqrt(c)*log(x)/a^3

Fricas [A]
time = 0.45, size = 60, normalized size = 0.69

$$-\frac{\left(2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3+a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="fricas")

[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*x^3 + a^4*x^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{cx^2}}{x^3\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a)**2,x)

[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)**2), x)

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sa

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{c\,x^2}}{x^3\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2),x)

[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2), x)

3.900 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2} dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{cx^2}}{2a^2x^3}+\frac{2b\sqrt{cx^2}}{a^3x^2}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

```
[Out] -1/2*(c*x^2)^(1/2)/a^2/x^3+2*b*(c*x^2)^(1/2)/a^3/x^2+b^2*(c*x^2)^(1/2)/a^3/
x/(b*x+a)+3*b^2*ln(x)*(c*x^2)^(1/2)/a^4/x-3*b^2*ln(b*x+a)*(c*x^2)^(1/2)/a^4
/x
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/2*Sqrt[c*x^2]/(a^2*x^3) + (2*b*Sqrt[c*x^2])/(a^3*x^2) + (b^2*Sqrt[c*x^2]
)/(a^3*x*(a + b*x)) + (3*b^2*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*Sqrt[c*x^
2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx = \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)^2}\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{\sqrt{cx^2}}{2a^2x^3} + \frac{2b\sqrt{cx^2}}{a^3x^2} + \frac{b^2\sqrt{cx^2}}{a^3x(a+bx)} + \frac{3b^2\sqrt{cx^2}\log(x)}{a^4x} - \frac{3b^2\sqrt{cx^2}\log(a+bx)}{a^4x}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 0.73

$$\frac{\sqrt{cx^2}\left(a(-a^2+3abx+6b^2x^2)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] (Sqrt[c*x^2]*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] -
 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^3*(a + b*x))
```

Maple [A]
time = 0.13, size = 95, normalized size = 0.85

method	result	size
risch	$\frac{\sqrt{c\,x^2}\,\left(\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}\right)}{x^3(bx+a)} - \frac{3b^2\ln(bx+a)\sqrt{c\,x^2}}{a^4x} + \frac{3\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^4}$	90
default	$\frac{\sqrt{c\,x^2}\,\left(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3\right)}{2x^3a^4(bx+a)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*
ln(b*x+a)*a*b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^3/a^4/(b*x+a)
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.71

$$\frac{6\,b^2\sqrt{c}\,x^2+3\,ab\sqrt{c}\,x-a^2\sqrt{c}}{2\,(a^3bx^3+a^4x^2)} - \frac{3\,b^2\sqrt{c}\,\log(bx+a)}{a^4} + \frac{3\,b^2\sqrt{c}\,\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x - a^2*sqrt(c))/(a^3*b*x^3 + a^4*x^
2) - 3*b^2*sqrt(c)*log(b*x + a)/a^4 + 3*b^2*sqrt(c)*log(x)/a^4
```

Fricas [A]
time = 0.48, size = 77, normalized size = 0.69

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\,(b^3x^3+ab^2x^2)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\,(a^4bx^4+a^5x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^4\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^4\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2), x)
```

3.901 $\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=111

$$\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

```
[Out] 3*a^2*c*(c*x^2)^(1/2)/b^4-a*c*x*(c*x^2)^(1/2)/b^3+1/3*c*x^2*(c*x^2)^(1/2)/b
^2-a^4*c*(c*x^2)^(1/2)/b^5/x/(b*x+a)-4*a^3*c*ln(b*x+a)*(c*x^2)^(1/2)/b^5/x
```

Rubi [A]
time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (3*a^2*c*Sqrt[c*x^2])/b^4 - (a*c*x*Sqrt[c*x^2])/b^3 + (c*x^2*Sqrt[c*x^2])/(
3*b^2) - (a^4*c*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*c*Sqrt[c*x^2]*Log[a
 + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right)dx}{x}$$
$$= \frac{3a^2c\sqrt{cx^2}}{b^4} - \frac{acx\sqrt{cx^2}}{b^3} + \frac{cx^2\sqrt{cx^2}}{3b^2} - \frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)} - \frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 0.74

$$\frac{(cx^2)^{3/2}\left(-3a^4 + 9a^3bx + 6a^2b^2x^2 - 2ab^3x^3 + b^4x^4 - 12a^3(a+bx)\log(a+bx)\right)}{3b^5x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4
- 12*a^3*(a + b*x)*Log[a + b*x]))/(3*b^5*x^3*(a + b*x))
```

Maple [A]
time = 0.16, size = 88, normalized size = 0.79

method	result	size
default	$-\frac{(cx^2)^{\frac{3}{2}}\left(-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)-9a^3bx+3a^4\right)}{3x^3b^5(bx+a)}$	88
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{3}b^2x^3-ab\,x^2+3a^2x\right)}{x\,b^4} - \frac{a^4c\sqrt{c\,x^2}}{b^5x(bx+a)} - \frac{4a^3c\ln(bx+a)\sqrt{c\,x^2}}{b^5x}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c*x^2)^(3/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x^3/b^5/(b*x+a)
```

Maxima [A]
time = 0.29, size = 132, normalized size = 1.19

$$\frac{(cx^2)^{\frac{3}{2}}a}{b^3x+ab^2} - \frac{4\,(-1)^{\frac{2cx}{b}}\,a^3c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^5} - \frac{4\,(-1)^{\frac{2acx}{b}}\,a^3c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5} - \frac{2\sqrt{cx^2}\,acx}{b^3} + \frac{(cx^2)^{\frac{3}{2}}}{3\,b^2} + \frac{4\sqrt{cx^2}\,a^2c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (c*x^2)^(3/2)*a/(b^3*x + a*b^2) - 4*(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
2*sqrt(c*x^2)*a*c*x/b^3 + 1/3*(c*x^2)^(3/2)/b^2 + 4*sqrt(c*x^2)*a^2*c/b^4
```

Fricas [A]
time = 0.44, size = 91, normalized size = 0.82

$$\frac{\left(b^{4} c x^{4} - 2 \, a b^{3} c x^{3} + 6 \, a^{2} b^{2} c x^{2} + 9 \, a^{3} b c x - 3 \, a^{4} c - 12 \, {\left(a^{3} b c x + a^{4} c\right)} \log\left(b x + a\right)\right)} \sqrt{c x^{2}}}{3 \, {\left(b^{6} x^{2} + a b^{5} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*c*x^4 - 2*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 9*a^3*b*c*x - 3*a^4*c -
12*(a^3*b*c*x + a^4*c)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(c x^{2}\right)^{\frac{3}{2}}}{\left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.83, size = 96, normalized size = 0.86

$$-\frac{1}{3} \, c^{\frac{3}{2}} {\left(\frac{12 \, a^{3} \log\left({\left| b x + a \right|}\right) \mathrm{sgn}\left(x\right)}{b^{5}} + \frac{3 \, a^{4} \mathrm{sgn}\left(x\right)}{{\left(b x + a\right)} b^{5}} - \frac{3 \, {\left(4 \, a^{3} \log\left({\left| a \right|}\right) + a^{3}\right)} \mathrm{sgn}\left(x\right)}{b^{5}} - \frac{b^{4} x^{3} \mathrm{sgn}\left(x\right) - 3 \, a b^{3} x^{2} \mathrm{sgn}\left(x\right) + 9 \, a^{2} b^{2} x \mathrm{sgn}\left(x\right)}{b^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*c^(3/2)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(c\,x^2\right)}^{3/2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x)^2, x)
```

3.902 $\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

```
[Out] -2*a*c*(c*x^2)^(1/2)/b^3+1/2*c*x*(c*x^2)^(1/2)/b^2+a^3*c*(c*x^2)^(1/2)/b^4/
x/(b*x+a)+3*a^2*c*ln(b*x+a)*(c*x^2)^(1/2)/b^4/x
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {15, 45}

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\,\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c*Sqrt[c*x^2])/b^3 + (c*x*Sqrt[c*x^2])/(2*b^2) + (a^3*c*Sqrt[c*x^2])/
(b^4*x*(a + b*x)) + (3*a^2*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}$$
$$= -\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\,\log(a+bx)}{b^4x}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 0.80

$$\frac{(cx^2)^{3/2}\left(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx)\right)}{2b^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)
*Log[a + b*x]))/(2*b^4*x^3*(a + b*x))
```

Maple [A]
time = 0.14, size = 76, normalized size = 0.85

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2x^3b^4(bx+a)}$	76
risch	$\frac{c\sqrt{c\,x^2}\,\left(\frac{1}{2}x^2b-2ax\right)}{x\,b^3}+\frac{a^3c\sqrt{c\,x^2}}{b^4x(bx+a)}+\frac{3a^2c\ln(bx+a)\sqrt{c\,x^2}}{b^4x}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x^3/b^4/(b*x+a)
```

Maxima [A]
time = 0.31, size = 115, normalized size = 1.29

$$\frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{(cx^2)^{\frac{3}{2}}}{b^2x+ab}+\frac{3\,\sqrt{cx^2}\,cx}{2\,b^2}-\frac{3\,\sqrt{cx^2}\,ac}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3*(-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*c^(3
/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - (c*x^2)^(3/2)/(b^2*x + a*b) + 3/2*
sqrt(c*x^2)*c*x/b^2 - 3*sqrt(c*x^2)*a*c/b^3
```

Fricas [A]
time = 0.47, size = 79, normalized size = 0.89

$$\frac{\left(b^{3} c x^{3} - 3 \, a b^{2} c x^{2} - 4 \, a^{2} b c x + 2 \, a^{3} c + 6 \, \left(a^{2} b c x + a^{3} c\right) \log \left(b x + a\right)\right) \sqrt{c x^{2}}}{2 \, \left(b^{5} x^{2} + a b^{4} x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 2*a^3*c + 6*(a^2*b*c*x + a^3
*c)*log(b*x + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c x^{2}\right)^{\frac{3}{2}}}{\left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x)**2, x)
```

Giac [A]
time = 0.56, size = 80, normalized size = 0.90

$$\frac{1}{2} \, c^{\frac{3}{2}} {\left(\frac{6 \, a^{2} \log \left({\left| b x + a \right|}\right) \mathrm{sgn}\left(x\right)}{b^{4}} + \frac{2 \, a^{3} \mathrm{sgn}\left(x\right)}{{\left(b x + a\right)} b^{4}} - \frac{2 \, {\left(3 \, a^{2} \log \left({\left| a \right|}\right) + a^{2}\right)} \mathrm{sgn}\left(x\right)}{b^{4}} + \frac{b^{2} x^{2} \mathrm{sgn}\left(x\right) - 4 \, a b x \mathrm{sgn}\left(x\right)}{b^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)/(a + b*x)^2, x)
```

3.903 $\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx$

Optimal. Leaf size=68

$$\frac{c\sqrt{cx^2}}{b^2}-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

```
[Out] c*(c*x^2)^(1/2)/b^2-a^2*c*(c*x^2)^(1/2)/b^3/x/(b*x+a)-2*a*c*ln(b*x+a)*(c*x^
2)^(1/2)/b^3/x
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/b^2 - (a^2*c*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*c*Sqrt[c
*x^2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}$$
$$= \frac{c\sqrt{cx^2}}{b^2} - \frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)} - \frac{2ac\sqrt{cx^2}\,\log(a+bx)}{b^3x}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 0.81

$$\frac{c^2x(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
[Out] (c^2*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x
^2]*(a + b*x))
```

Maple [A]
time = 0.12, size = 62, normalized size = 0.91

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2)}{x^3b^3(bx+a)}$	62
risch	$\frac{c\sqrt{c\,x^2}}{b^2}-\frac{a^2c\sqrt{c\,x^2}}{b^3x(bx+a)}-\frac{2ac\ln(bx+a)\sqrt{c\,x^2}}{b^3x}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -(c*x^2)^(3/2)*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x^3/b^
3/(b*x+a)
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.44

$$\frac{\sqrt{cx^2}\,ac}{b^3x+ab^2} - \frac{2\,(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3} - \frac{2\,(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3} + \frac{\sqrt{cx^2}\,c}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a*c/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b
^3 - 2*(-1)^(2*a*c*x/b)*a*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt
(c*x^2)*c/b^2
```

Fricas [A]
time = 0.46, size = 63, normalized size = 0.93

$$\frac{\left(b^2cx^2 + abcx - a^2c - 2\left(abcx + a^2c\right)\log\left(bx + a\right)\right)\sqrt{cx^2}}{b^4x^2 + ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x^2 + a*b*c*x - a^2*c - 2*(a*b*c*x + a^2*c)*log(b*x + a))*sqrt(c*x^2
)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x\left(a + bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)**2), x)
```

Giac [A]
time = 0.54, size = 58, normalized size = 0.85

$$c^{\frac{3}{2}}\left(\frac{x\operatorname{sgn}(x)}{b^2} - \frac{2\,a\log\left(|bx + a|\right)\operatorname{sgn}(x)}{b^3} + \frac{(2\,a\log\left(|a|\right) + a)\operatorname{sgn}(x)}{b^3} - \frac{a^2\operatorname{sgn}(x)}{(bx + a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^2\right)}^{3/2}}{x\,{\left(a + b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)^2), x)
```

3.904 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=49

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

```
[Out] a*c*(c*x^2)^(1/2)/b^2/x/(b*x+a)+c*ln(b*x+a)*(c*x^2)^(1/2)/b^2/x
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\,\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (a*c*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{(a+bx)^2}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{x}\\ &= \frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\,\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.78

$$\frac{c^2x(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
[Out] (c^2*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.13, size = 41, normalized size = 0.84

method	result	size
default	$\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}\left(b\ln\left(bx+a\right)x+a\ln\left(bx+a\right)+a\right)}{x^{3}b^{2}\left(bx+a\right)}$	41
risch	$\frac{ac\sqrt{c\,x^{2}}}{b^{2}x\left(bx+a\right)}+\frac{c\ln\left(bx+a\right)\sqrt{c\,x^{2}}}{b^{2}x}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(3/2)*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/x^3/b^2/(b*x+a)
```

Maxima [A]
time = 0.30, size = 80, normalized size = 1.63

$$\frac{(-1)^{\frac{2\,cx}{b}}\,c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^2}+\frac{(-1)^{\frac{2\,acx}{b}}\,c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2}-\frac{\sqrt{cx^2}\,c}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
[Out] (-1)^(2*c*x/b)*c^(3/2)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*c^(3/2)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)*c/(b^2*x + a*b)
```

Fricas [A]
time = 0.42, size = 43, normalized size = 0.88

$$\frac{\sqrt{cx^2}\,(ac+(bcx+ac)\log(bx+a))}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(b*x + a))/(b^3*x^2 + a*b^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^2 (a+bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)**2), x)
```

Giac [A]
time = 0.85, size = 46, normalized size = 0.94

$$-c^{\frac{3}{2}}\left(\frac{(\log(|a|)+1)\mathrm{sgn}(x)}{b^2} - \frac{\log(|bx+a|)\,\mathrm{sgn}(x)}{b^2} - \frac{a\mathrm{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="giac")
[Out] -c^(3/2)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x^2)^{3/2}}{x^2\,(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2),x)
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2), x)
```

3.905 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

```
[Out] -c*(c*x^2)^(1/2)/b/x/(b*x+a)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{(a+bx)^2}\,dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

Maple [A]
time = 0.12, size = 23, normalized size = 0.92

method	result	size
gosper	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}}{(bx+a)b\,x^{3}}$	23
default	$-\frac{\left(c\,x^{2}\right)^{\frac{3}{2}}}{(bx+a)b\,x^{3}}$	23
risch	$-\frac{c\sqrt{c\,x^{2}}}{bx(bx+a)}$	24
trager	$\frac{c(-1+x)\sqrt{c\,x^{2}}}{(bx+a)(a+b)x}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/(b*x+a)/b*(c*x^2)^(3/2)/x^3
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.64

$$-\frac{c^{\frac{3}{2}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
[Out] -c^(3/2)/(b^2*x + a*b)
```

Fricas [A]
time = 0.44, size = 24, normalized size = 0.96

$$-\frac{\sqrt{cx^2}\,c}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
[Out] -sqrt(c*x^2)*c/(b^2*x^2 + a*b*x)
```

Sympy [A]
time = 0.58, size = 37, normalized size = 1.48

$$\begin{cases} -\frac{(cx^2)^{\frac{3}{2}}}{abx^3+b^2x^4} & \text{for } b \neq 0 \\ \frac{(cx^2)^{\frac{3}{2}}}{a^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Piecewise((-(c*x**2)**(3/2)/(a*b*x**3 + b**2*x**4), Ne(b, 0)), ((c*x**2)**(
3/2)/(a**2*x**2), True))
```

Giac [A]
time = 0.81, size = 29, normalized size = 1.16

$$-c^{\frac{3}{2}}\left(\frac{\operatorname{sgn}(x)}{(bx+a)b}-\frac{\operatorname{sgn}(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -c^(3/2)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

Mupad [B]
time = 0.15, size = 24, normalized size = 0.96

$$-\frac{c^{3/2}\,\sqrt{x^2}}{b^2\,x^2+a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)^2),x)
```

```
[Out] -(c^(3/2)*(x^2)^(1/2))/(b^2*x^2 + a*b*x)
```

3.906 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx$

Optimal. Leaf size=68

$$\frac{c\sqrt{cx^2}}{ax(a+bx)}+\frac{c\sqrt{cx^2}\,\log(x)}{a^2x}-\frac{c\sqrt{cx^2}\,\log(a+bx)}{a^2x}$$

```
[Out] c*(c*x^2)^(1/2)/a/x/(b*x+a)+c*ln(x)*(c*x^2)^(1/2)/a^2/x-c*ln(b*x+a)*(c*x^2)
^(1/2)/a^2/x
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{c\sqrt{cx^2}\,\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\,\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/(a*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[x])/(a^2*x) - (c*Sqrt[
c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{x}$$
$$= \frac{c\sqrt{cx^2}}{ax(a+bx)} + \frac{c\sqrt{cx^2}\log(x)}{a^2x} - \frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.68

$$\frac{(cx^2)^{3/2}(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
[Out] ((c*x^2)^(3/2)*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*x^3*(a
 + b*x))
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.76

method	result	size
default	$\frac{(c\,x^2)^{\frac{3}{2}}(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{x^3a^2(bx+a)}$	52
risch	$\frac{c\sqrt{c\,x^2}}{ax(bx+a)}+\frac{c\sqrt{c\,x^2}\,\ln(-x)}{x\,a^2}-\frac{c\ln(bx+a)\sqrt{c\,x^2}}{a^2x}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] (c*x^2)^(3/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/x^3/a^2/(b*x+
a)
```

Maxima [A]
time = 0.38, size = 38, normalized size = 0.56

$$\frac{c^{\frac{3}{2}}}{abx+a^2} - \frac{c^{\frac{3}{2}}\log(bx+a)}{a^2} + \frac{c^{\frac{3}{2}}\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] c^(3/2)/(a*b*x + a^2) - c^(3/2)*log(b*x + a)/a^2 + c^(3/2)*log(x)/a^2
```

Fricas [**A**]
time = 0.48, size = 47, normalized size = 0.69

$$\frac{\sqrt{cx^2}\,\left(ac+(bcx+ac)\log\left(\frac{x}{bx+a}\right)\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(x/(b*x + a)))/(a^2*b*x^2 + a^3*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{3}{2}}}{x^4\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(c\,x^2)^{3/2}}{x^4\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2), x)
```

3.907 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{c\sqrt{cx^2}}{a^2x^2}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

```
[Out] -c*(c*x^2)^(1/2)/a^2/x^2-b*c*(c*x^2)^(1/2)/a^2/x/(b*x+a)-2*b*c*ln(x)*(c*x^2
)^(1/2)/a^3/x+2*b*c*ln(b*x+a)*(c*x^2)^(1/2)/a^3/x
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a^2*x^2)) - (b*c*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*c
*Sqrt[c*x^2]*Log[x])/(a^3*x) + (2*b*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{a^2x^2}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\,\log(a+bx)}{a^3x}$$

Mathematica [A]
time = 0.02, size = 59, normalized size = 0.65

$$-\frac{c^2(a(a+2bx)+2bx(a+bx)\log(x)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c^2*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*
x]))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.81

method	result	size
default	$-\frac{(c\,x^2)^{\frac{3}{2}}\left(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2\right)}{x^4a^3(bx+a)}$	74
risch	$\frac{c\sqrt{c\,x^2}\,\left(-\frac{2bx}{a^2}-\frac{1}{a}\right)}{x^2(bx+a)}-\frac{2bc\ln(x)\sqrt{c\,x^2}}{a^3x}+\frac{2c\sqrt{c\,x^2}\,b\ln(-bx-a)}{x\,a^3}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(3/2)*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+
a)*a*b*x+2*a*b*x+a^2)/x^4/a^3/(b*x+a)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.64

$$\frac{2\,bc^{\frac{3}{2}}\log(bx+a)}{a^3}-\frac{2\,bc^{\frac{3}{2}}\log(x)}{a^3}-\frac{2\,bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*b*c^(3/2)*log(b*x + a)/a^3 - 2*b*c^(3/2)*log(x)/a^3 - (2*b*c^(3/2)*x + a*
c^(3/2))/(a^2*b*x^2 + a^3*x)
```

Fricas [A]
time = 0.43, size = 65, normalized size = 0.71

$$-\frac{\left(2\,abcx+a^{2}c-2\left(b^{2}cx^{2}+abcx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^{2}}}{a^{3}bx^{3}+a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*c*x + a^2*c - 2*(b^2*c*x^2 + a*b*c*x)*log((b*x + a)/x))*sqrt(c*x^2)
/(a^3*b*x^3 + a^4*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{5}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^{2}\right)}^{3/2}}{x^{5}\,{\left(a+b\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2), x)
```

3.908 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2} dx$

Optimal. Leaf size=117

$$-\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}$$

```
[Out] -1/2*c*(c*x^2)^(1/2)/a^2/x^3+2*b*c*(c*x^2)^(1/2)/a^3/x^2+b^2*c*(c*x^2)^(1/2
)/a^3/x/(b*x+a)+3*b^2*c*ln(x)*(c*x^2)^(1/2)/a^4/x-3*b^2*c*ln(b*x+a)*(c*x^2)
^(1/2)/a^4/x
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$\frac{3b^2c\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] -1/2*(c*Sqrt[c*x^2])/(a^2*x^3) + (2*b*c*Sqrt[c*x^2])/(a^3*x^2) + (b^2*c*Sqr
t[c*x^2])/(a^3*x*(a + b*x)) + (3*b^2*c*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2
*c*Sqrt[c*x^2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)^2}\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{x}$$
$$= -\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 0.70

$$\frac{(cx^2)^{3/2}\left(a(-a^2+3abx+6b^2x^2)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x]
 - 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^5*(a + b*x))
```

Maple [A]
time = 0.14, size = 95, normalized size = 0.81

method	result	size
risch	$\frac{c\sqrt{c\,x^2}\left(\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}\right)}{x^3(bx+a)}-\frac{3b^2c\ln(bx+a)\sqrt{c\,x^2}}{a^4x}+\frac{3c\sqrt{c\,x^2}\,b^2\ln(-x)}{x\,a^4}$	93
default	$\frac{(c\,x^2)^{\frac{3}{2}}(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3)}{2x^5a^4(bx+a)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*
ln(b*x+a)*a*b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^5/a^4/(b*x+a)
```

Maxima [A]
time = 0.29, size = 79, normalized size = 0.68

$$-\frac{3\,b^2c^{\frac{3}{2}}\log(bx+a)}{a^4}+\frac{3\,b^2c^{\frac{3}{2}}\log(x)}{a^4}+\frac{6\,b^2c^{\frac{3}{2}}x^2+3\,abc^{\frac{3}{2}}x-a^2c^{\frac{3}{2}}}{2\,(a^3bx^3+a^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*b^2*c^(3/2)*log(b*x + a)/a^4 + 3*b^2*c^(3/2)*log(x)/a^4 + 1/2*(6*b^2*c^(
3/2)*x^2 + 3*a*b*c^(3/2)*x - a^2*c^(3/2))/(a^3*b*x^3 + a^4*x^2)
```

Fricas [A]
time = 0.44, size = 82, normalized size = 0.70

$$\frac{\left(6\,ab^2cx^2+3\,a^2bcx-a^3c+6\left(b^3cx^3+ab^2cx^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*c*x^2 + 3*a^2*b*c*x - a^3*c + 6*(b^3*c*x^3 + a*b^2*c*x^2)*log(
x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(c\,x^2\right)}^{3/2}}{x^6\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2), x)
```

3.909 $\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=107

$$\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}$$

```
[Out] 3*a^2*x^2/b^4/(c*x^2)^(1/2)-a*x^3/b^3/(c*x^2)^(1/2)+1/3*x^4/b^2/(c*x^2)^(1/
2)-a^4*x/b^5/(b*x+a)/(c*x^2)^(1/2)-4*a^3*x*ln(b*x+a)/b^5/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}+\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (3*a^2*x^2)/(b^4*Sqrt[c*x^2]) - (a*x^3)/(b^3*Sqrt[c*x^2]) + x^4/(3*b^2*Sqrt
[c*x^2]) - (a^4*x)/(b^5*Sqrt[c*x^2]*(a + b*x)) - (4*a^3*x*Log[a + b*x])/(b^
5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^4}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{3a^2}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{b^2} + \frac{a^4}{b^4(a+bx)^2} - \frac{4a^3}{b^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{3a^2x^2}{b^4\sqrt{cx^2}} - \frac{ax^3}{b^3\sqrt{cx^2}} + \frac{x^4}{3b^2\sqrt{cx^2}} - \frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)} - \frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 80, normalized size = 0.75

$$\frac{x(-3a^4 + 9a^3bx + 6a^2b^2x^2 - 2ab^3x^3 + b^4x^4 - 12a^3(a+bx)\log(a+bx))}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(a
+ b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.12, size = 86, normalized size = 0.80

method	result	size
risch	$\frac{x\left(\frac{1}{3}b^2x^3 - ab\,x^2 + 3a^2x\right)}{\sqrt{c\,x^2}\,b^4} - \frac{a^4x}{b^5(bx+a)\sqrt{c\,x^2}} - \frac{4a^3x\ln(bx+a)}{b^5\sqrt{c\,x^2}}$	81
default	$-\frac{x\left(-b^4x^4 + 2a\,b^3x^3 + 12\ln(bx+a)a^3bx - 6a^2b^2x^2 + 12a^4\ln(bx+a) - 9a^3bx + 3a^4\right)}{3\sqrt{c\,x^2}\,b^5(bx+a)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*a^3*b*x-6*a^2*b^2*x^2+12*a^4*ln(b
*x+a)-9*a^3*b*x+3*a^4)/(c*x^2)^(1/2)/b^5/(b*x+a)
```

Maxima [A]
time = 0.31, size = 168, normalized size = 1.57

$$\frac{\sqrt{cx^2}\,a^3}{b^5cx + ab^4c} + \frac{\sqrt{cx^2}\,x^2}{3\,b^2c} - \frac{5\,ax^2}{3\,b^3\sqrt{c}} - \frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5\sqrt{c}} + \frac{2\,\sqrt{cx^2}\,ax}{3\,b^3c} - \frac{20\,a^2x}{3\,b^4\sqrt{c}} - \frac{4\,a^3\log(bx)}{b^5\sqrt{c}} + \frac{29\,\sqrt{cx^2}\,a^2}{3\,b^4c} - \frac{5\,a^3}{b^5\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*c*x + a*b^4*c) + 1/3*sqrt(c*x^2)*x^2/(b^2*c) - 5/3*a*x
^2/(b^3*sqrt(c)) - 4*(-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b
^5*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^3*c) - 20/3*a^2*x/(b^4*sqrt(c)) - 4*a^
3*log(b*x)/(b^5*sqrt(c)) + 29/3*sqrt(c*x^2)*a^2/(b^4*c) - 5*a^3/(b^5*sqrt(c
))
```

Fricas [A]
time = 0.49, size = 85, normalized size = 0.79

$$\frac{\left(b^{4} x^{4}-2 a b^{3} x^{3}+6 a^{2} b^{2} x^{2}+9 a^{3} b x-3 a^{4}-12\left(a^{3} b x+a^{4}\right) \log \left(b x+a\right)\right) \sqrt{c x^{2}}}{3\left(b^{6} c x^{2}+a b^{5} c x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*c*x^2 + a*b^5*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{c x^{2}}\left(a+b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.63, size = 107, normalized size = 1.00

$$-\frac{4 a^{3} \log \left(\left|bx+a\right|\right)}{b^{5} \sqrt{c} \operatorname{sgn}\left(x\right)}-\frac{a^{4}}{(bx+a) b^{5} \sqrt{c} \operatorname{sgn}\left(x\right)}+\frac{\left(4 a^{3} \log \left(\left|a\right|\right)+a^{3}\right) \operatorname{sgn}(x)}{b^{5} \sqrt{c}}+\frac{b^{4} c x^{3}-3 a b^{3} c x^{2}+9 a^{2} b^{2} c x}{3 b^{6} c^{\frac{3}{2}} \operatorname{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/(b^5*sqrt(c)*sgn(x)) - a^4/((b*x + a)*b^5*sqrt(c)*
sgn(x)) + (4*a^3*log(abs(a)) + a^3)*sgn(x)/(b^5*sqrt(c)) + 1/3*(b^4*c*x^3 -
 3*a*b^3*c*x^2 + 9*a^2*b^2*c*x)/(b^6*c^(3/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}}{\sqrt{c\,x^{2}}\,{\left(a+b\,x\right)}^{2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.910 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=86

$$-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}$$

```
[Out] -2*a*x^2/b^3/(c*x^2)^(1/2)+1/2*x^3/b^2/(c*x^2)^(1/2)+a^3*x/b^4/(b*x+a)/(c*x
^2)^(1/2)+3*a^2*x*ln(b*x+a)/b^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (-2*a*x^2)/(b^3*Sqrt[c*x^2]) + x^3/(2*b^2*Sqrt[c*x^2]) + (a^3*x)/(b^4*Sqrt[
c*x^2]*(a + b*x)) + (3*a^2*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^3}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 0.80

$$\frac{x(2a^3-4a^2bx-3ab^2x^2+b^3x^3+6a^2(a+bx)\log(a+bx))}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b*x
]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.12, size = 74, normalized size = 0.86

method	result	size
risch	$\frac{x\left(\frac{1}{2}x^2b-2ax\right)}{\sqrt{c\,x^2}\;b^3}+\frac{a^3x}{b^4(bx+a)\sqrt{c\,x^2}}+\frac{3a^2x\ln(bx+a)}{b^4\sqrt{c\,x^2}}$	69
default	$\frac{x\left(b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)-4a^2bx+2a^3\right)}{2\sqrt{c\,x^2}\;b^4(bx+a)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b^3*x^3+6*ln(b*x+a)*a^2*b*x-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*
a^3)/(c*x^2)^(1/2)/b^4/(b*x+a)
```

Maxima [A]
time = 0.30, size = 129, normalized size = 1.50

$$-\frac{\sqrt{cx^2}\,a^2}{b^4cx+ab^3c}+\frac{x^2}{2\,b^2\sqrt{c}}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}}+\frac{2\,ax}{b^3\sqrt{c}}+\frac{3\,a^2\log\left(bx\right)}{b^4\sqrt{c}}-\frac{4\,\sqrt{cx^2}\,a}{b^3c}+\frac{3\,a^2}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] -sqrt(c*x^2)*a^2/(b^4*c*x + a*b^3*c) + 1/2*x^2/(b^2*sqrt(c)) + 3*(-1)^(2*a*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2*a*x/(b^3*sqrt(c)) + 3*a^2*log(b*x)/(b^4*sqrt(c)) - 4*sqrt(c*x^2)*a/(b^3*c) + 3/2*a^2/(b^4*sqrt(c))

Fricas [A]
time = 0.44, size = 74, normalized size = 0.86

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,(b^5cx^2+ab^4cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x + a))*sqrt(c*x^2)/(b^5*c*x^2 + a*b^4*c*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(b*x+a)**2/(c*x**2)**(1/2),x)

[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x)

Giac [A]
time = 0.66, size = 97, normalized size = 1.13

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4\sqrt{c}\,\mathrm{sgn}\left(x\right)}-\frac{(3\,a^2\log\left(|a|\right)+a^2)\mathrm{sgn}(x)}{b^4\sqrt{c}}+\frac{a^3}{(bx+a)b^4\sqrt{c}\,\mathrm{sgn}\left(x\right)}+\frac{b^2\sqrt{c}\,x^2\mathrm{sgn}(x)-4\,ab\sqrt{c}\,x\mathrm{sgn}(x)}{2\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")

[Out] 3*a^2*log(abs(b*x + a))/(b^4*sqrt(c)*sgn(x)) - (3*a^2*log(abs(a)) + a^2)*sgn(x)/(b^4*sqrt(c)) + a^3/((b*x + a)*b^4*sqrt(c)*sgn(x)) + 1/2*(b^2*sqrt(c)*x^2*sgn(x) - 4*a*b*sqrt(c)*x*sgn(x))/(b^4*c)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.911 $\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^2}{b^2\sqrt{cx^2}}-\frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}$$

```
[Out] x^2/b^2/(c*x^2)^(1/2)-a^2*x/b^3/(b*x+a)/(c*x^2)^(1/2)-2*a*x*ln(b*x+a)/b^3/(
c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*Sqrt[c*x^2]) - (a^2*x)/(b^3*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Log[a
+ b*x])/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{x^2}{b^2\sqrt{cx^2}} - \frac{a^2x}{b^3\sqrt{cx^2}\,(a+bx)} - \frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 0.81

$$\frac{x(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2]*
(a + b*x))
```

Maple [A]
time = 0.14, size = 60, normalized size = 0.94

method	result	size
risch	$\frac{x^2}{b^2\sqrt{c\,x^2}} - \frac{a^2x}{b^3(bx+a)\sqrt{c\,x^2}} - \frac{2ax\ln(bx+a)}{b^3\sqrt{c\,x^2}}$	59
default	$-\frac{x\left(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2\right)}{\sqrt{c\,x^2}\;b^3(bx+a)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(1/2)/b^3/
(b*x+a)
```

Maxima [A]
time = 0.31, size = 88, normalized size = 1.38

$$\frac{\sqrt{cx^2}\,a}{b^3cx+ab^2c} - \frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}} - \frac{2\,a\log(bx)}{b^3\sqrt{c}} + \frac{\sqrt{cx^2}}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*c*x + a*b^2*c) - 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*ab
s(b*x + a)))/(b^3*sqrt(c)) - 2*a*log(b*x)/(b^3*sqrt(c)) + sqrt(c*x^2)/(b^2*
c)
```

Fricas [**A**]

time = 0.42, size = 59, normalized size = 0.92

$$\frac{(b^2x^2 + abx - a^2 - 2\,(abx + a^2)\log{(bx + a)})\sqrt{cx^2}}{b^4cx^2 + ab^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c*x
^2 + a*b^3*c*x)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^2}\,(a + bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [**A**]

time = 0.58, size = 72, normalized size = 1.12

$$\frac{(2\,a\log{(|a|)} + a)\mathrm{sgn}(x)}{b^3\sqrt{c}} + \frac{x}{b^2\sqrt{c}\,\mathrm{sgn}\,(x)} - \frac{2\,a\log{(|bx + a|)}}{b^3\sqrt{c}\,\mathrm{sgn}\,(x)} - \frac{a^2}{(bx + a)b^3\sqrt{c}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*log(abs(a)) + a)*sgn(x)/(b^3*sqrt(c)) + x/(b^2*sqrt(c)*sgn(x)) - 2*a*l
og(abs(b*x + a))/(b^3*sqrt(c)*sgn(x)) - a^2/((b*x + a)*b^3*sqrt(c)*sgn(x))
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{c\,x^2}\,{(a + b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.912 $\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=43

$$\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

```
[Out] a*x/b^2/(b*x+a)/(c*x^2)^(1/2)+x*ln(b*x+a)/b^2/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx&=\frac{x\int \frac{x}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\&=\frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx}{\sqrt{cx^2}}\\&=\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.81

$$\frac{x(a+(a+bx)\log(a+bx))}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

Maple [A]
time = 0.13, size = 39, normalized size = 0.91

method	result	size
default	$\frac{x(b\ln(bx+a)x+a\ln(bx+a)+a)}{\sqrt{c\,x^2}\,b^2(bx+a)}$	39
risch	$\frac{ax}{b^2(bx+a)\sqrt{c\,x^2}}+\frac{x\ln(bx+a)}{b^2\sqrt{c\,x^2}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(1/2)/b^2/(b*x+a)
```

Maxima [A]
time = 0.30, size = 68, normalized size = 1.58

$$-\frac{\sqrt{cx^2}}{b^2cx+abc}+\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}}+\frac{\log(bx)}{b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)/(b^2*c*x + a*b*c) + (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x +
 a)))/(b^2*sqrt(c)) + log(b*x)/(b^2*sqrt(c))
```

Fricas [A]
time = 0.46, size = 40, normalized size = 0.93

$$\frac{\sqrt{cx^2}\,((bx+a)\log(bx+a)+a)}{b^3cx^2+ab^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c*x^2 + a*b^2*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.58, size = 53, normalized size = 1.23

$$-\frac{(\log(|a|)+1)\mathrm{sgn}(x)}{b^2\sqrt{c}}+\frac{\log(|bx+a|)}{b^2\sqrt{c}\,\mathrm{sgn}(x)}+\frac{a}{(bx+a)b^2\sqrt{c}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -(log(abs(a)) + 1)*sgn(x)/(b^2*sqrt(c)) + log(abs(b*x + a))/(b^2*sqrt(c)*sg
n(x)) + a/((b*x + a)*b^2*sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{c\,x^2}\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2),x)
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.913 $\int \frac{x}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=22

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 32}

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= -\frac{x}{b\sqrt{cx^2}\,(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.95

method	result	size
gosper	$-\frac{x}{b(bx+a)\sqrt{c\,x^2}}$	21
default	$-\frac{x}{b(bx+a)\sqrt{c\,x^2}}$	21
risch	$-\frac{x}{b(bx+a)\sqrt{c\,x^2}}$	21
trager	$\frac{(-1+x)\sqrt{c\,x^2}}{c(bx+a)(a+b)x}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.95

$$\frac{\sqrt{cx^2}}{abcx + a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)/(a*b*c*x + a^2*c)
```

Fricas [A]
time = 0.40, size = 25, normalized size = 1.14

$$-\frac{\sqrt{cx^2}}{b^2cx^2 + abcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c*x^2 + a*b*c*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.
time = 0.40, size = 68, normalized size = 3.09

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{cx^2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^2}{\sqrt{cx^2}} & \text{for } a = -bx \\ \frac{x^2}{a^2\sqrt{cx^2}} & \text{for } b = 0 \\ -\frac{x}{ab\sqrt{cx^2}+b^2x\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(c*x**2), Eq(a, 0) & Eq(b, 0)), (zoo*x**2/sqrt(c*x**2),
Eq(a, -b*x)), (x**2/(a**2*sqrt(c*x**2)), Eq(b, 0)), (-x/(a*b*sqrt(c*x**2) +
 b**2*x*sqrt(c*x**2)), True))
```

Giac [A]
time = 0.66, size = 32, normalized size = 1.45

$$\frac{\mathrm{sgn}(x)}{ab\sqrt{c}} - \frac{1}{(bx+a)b\sqrt{c}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sgn(x)/(a*b*sqrt(c)) - 1/((b*x + a)*b*sqrt(c)*sgn(x))
```

Mupad [B]
time = 0.16, size = 25, normalized size = 1.14

$$-\frac{\sqrt{c\,x^2}}{b\,c\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*c*x*(a + b*x))
```

3.914 $\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=59

$$\frac{x}{a\sqrt{cx^2}(a+bx)} + \frac{x\log(x)}{a^2\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2\sqrt{cx^2}}$$

```
[Out] x/a/(b*x+a)/(c*x^2)^(1/2)+x*ln(x)/a^2/(c*x^2)^(1/2)-x*ln(b*x+a)/a^2/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 46}

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}} + \frac{x\log(x)}{a^2\sqrt{cx^2}} + \frac{x}{a\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x/(a*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*Sqrt[c*x^2]) - (x*Log[a + b*x
])/(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{x}{a\sqrt{cx^2}(a+bx)}+\frac{x\log(x)}{a^2\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}$$

Mathematica [A]
time = 0.00, size = 44, normalized size = 0.75

$$\frac{x(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a + b
*x))
```

Maple [A]
time = 0.13, size = 50, normalized size = 0.85

method	result	size
default	$\frac{x(bx\ln(x)-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)+a)}{\sqrt{c\,x^2}\,a^2(bx+a)}$	50
risch	$\frac{x}{a(bx+a)\sqrt{c\,x^2}}+\frac{x\ln(-x)}{\sqrt{c\,x^2}\,a^2}-\frac{x\ln(bx+a)}{a^2\sqrt{c\,x^2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2/(b*x+a)
```

Maxima [A]
time = 0.29, size = 61, normalized size = 1.03

$$-\frac{\sqrt{cx^2}\,b}{a^2bcx+a^3c}-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*b/(a^2*b*c*x + a^3*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b
*x + a)))/(a^2*sqrt(c))
```

Fricas [A]
time = 0.46, size = 44, normalized size = 0.75

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bcx^2+a^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c*x^2 + a^3*c*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.915 $\int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{1}{a^2\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}$$

```
[Out] -1/a^2/(c*x^2)^(1/2)-b*x/a^2/(b*x+a)/(c*x^2)^(1/2)-2*b*x*ln(x)/a^3/(c*x^2)^
(1/2)+2*b*x*ln(b*x+a)/a^3/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 46}

$$-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*Sqrt[c*x^2])) - (b*x)/(a^2*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log[x])
/(a^3*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{1}{a^2\sqrt{cx^2}} - \frac{bx}{a^2\sqrt{cx^2}\,(a+bx)} - \frac{2bx\log(x)}{a^3\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}$$

Mathematica [**A**]
time = 0.02, size = 60, normalized size = 0.77

$$\frac{cx^2(-a(a+2bx) - 2bx(a+bx)\log(x) + 2bx(a+bx)\log(a+bx))}{a^3\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a +
 b*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

Maple [**A**]
time = 0.11, size = 71, normalized size = 0.91

method	result	size
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{\sqrt{c\,x^2}\,(bx+a)} - \frac{2bx\ln(x)}{a^3\sqrt{c\,x^2}} + \frac{2xb\ln(-bx-a)}{\sqrt{c\,x^2}\,a^3}$	69
default	$-\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2}{\sqrt{c\,x^2}\,a^3(bx+a)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+a)*a*b*x+2*a*b
*x+a^2)/(c*x^2)^(1/2)/a^3/(b*x+a)
```

Maxima [**A**]
time = 0.29, size = 57, normalized size = 0.73

$$-\frac{2\,bx+a}{a^2b\sqrt{c}\,x^2+a^3\sqrt{c}\,x} + \frac{2\,b\log\left(bx+a\right)}{a^3\sqrt{c}} - \frac{2\,b\log\left(x\right)}{a^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*sqrt(c)*x^2 + a^3*sqrt(c)*x) + 2*b*log(b*x + a)/(a^3*sq
rt(c)) - 2*b*log(x)/(a^3*sqrt(c))
```

Fricas [A]
time = 0.48, size = 62, normalized size = 0.79

$$-\frac{\left(2\,abx+a^2-2\,(b^2x^2+abx)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bcx^3+a^4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c*x^3 + a^4*c*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.916 $\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=103

$$\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}$$

```
[Out] 2*b/a^3/(c*x^2)^(1/2)-1/2/a^2/x/(c*x^2)^(1/2)+b^2*x/a^3/(b*x+a)/(c*x^2)^(1/
2)+3*b^2*x*ln(x)/a^4/(c*x^2)^(1/2)-3*b^2*x*ln(b*x+a)/a^4/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*Sqrt[c*x^2]) - 1/(2*a^2*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*Sqrt[c*x^2
]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*Sqrt[c*x^2]) - (3*b^2*x*Log[a + b*x])/
(a^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{cx^2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{2b}{a^3\sqrt{cx^2}} - \frac{1}{2a^2x\sqrt{cx^2}} + \frac{b^2x}{a^3\sqrt{cx^2}(a+bx)} + \frac{3b^2x\log(x)}{a^4\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 0.79

$$\frac{cx(a(-a^2+3abx+6b^2x^2)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx))}{2a^4(cx^2)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x
^2*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A]
time = 0.14, size = 95, normalized size = 0.92

method	result	size
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{\sqrt{c\,x^2}\,x(bx+a)} - \frac{3b^2x\ln(bx+a)}{a^4\sqrt{c\,x^2}} + \frac{3x\,b^2\ln(-x)}{\sqrt{c\,x^2}\,a^4}$	86
default	$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3}{2x\sqrt{c\,x^2}\,a^4(bx+a)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*ln(b*x+a)*a*
b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(1/2)/a^4/(b*x+a)
```

Maxima [A]
time = 0.29, size = 76, normalized size = 0.74

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3b\sqrt{c}\,x^3+a^4\sqrt{c}\,x^2\right)} - \frac{3\,b^2\log(bx+a)}{a^4\sqrt{c}} + \frac{3\,b^2\log(x)}{a^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*sqrt(c)*x^3 + a^4*sqrt(c)*x^2) - 3*b
^2*log(b*x + a)/(a^4*sqrt(c)) + 3*b^2*log(x)/(a^4*sqrt(c))
```

Fricas [**A**]
time = 0.40, size = 79, normalized size = 0.77

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bcx^4+a^5cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c*x^4 + a^5*c*x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\sqrt{cx^2}\,\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^2\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

3.917 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=73

$$\frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}$$

```
[Out] x^2/b^2/c/(c*x^2)^(1/2)-a^2*x/b^3/c/(b*x+a)/(c*x^2)^(1/2)-2*a*x*ln(b*x+a)/b
^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {15, 45}

$$-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*c*Sqrt[c*x^2]) - (a^2*x)/(b^3*c*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Lo
g[a + b*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{x^2}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{x^2}{b^2c\sqrt{cx^2}} - \frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)} - \frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.74

$$\frac{x^3(-a^2+abx+b^2x^2-2a(a+bx)\log(a+bx))}{b^3\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*(c*x^2)^(3
/2)*(a + b*x))
```

Maple [A]
time = 0.13, size = 62, normalized size = 0.85

method	result	size
default	$-\frac{x^3\left(2\ln(bx+a)abx-x^2b^2+2a^2\ln(bx+a)-abx+a^2\right)}{(c\,x^2)^{\frac{3}{2}}b^3(bx+a)}$	62
risch	$\frac{x^2}{b^2c\sqrt{c\,x^2}}-\frac{a^2x}{b^3c(bx+a)\sqrt{c\,x^2}}-\frac{2ax\ln(bx+a)}{b^3c\sqrt{c\,x^2}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^3*(2*ln(b*x+a)*a*b*x-x^2*b^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(3/2)/b^
3/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(67) = 134.
time = 0.32, size = 149, normalized size = 2.04

$$\frac{a^3}{\sqrt{cx^2}\,b^5cx+\sqrt{cx^2}\,ab^4c}+\frac{x^2}{\sqrt{cx^2}\,b^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{cx^2}\,b^3c}-\frac{2\,a\log\left(bx\right)}{b^3c^{\frac{3}{2}}}-\frac{5\,a^2}{\sqrt{cx^2}\,b^4c}+\frac{4\,a^2}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(sqrt(c*x^2)*b^5*c*x + sqrt(c*x^2)*a*b^4*c) + x^2/(sqrt(c*x^2)*b^2*c) -
 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) + 2*a*x/
(sqrt(c*x^2)*b^3*c) - 2*a*log(b*x)/(b^3*c^(3/2)) - 5*a^2/(sqrt(c*x^2)*b^4*c
) + 4*a^2/(b^4*c^(3/2)*x)
```

Fricas [A]
time = 0.41, size = 63, normalized size = 0.86

$$\frac{\left(b^{2} x^{2}+a b x-a^{2}-2\left(a b x+a^{2}\right) \log \left(b x+a\right)\right) \sqrt{c x^{2}}}{b^{4} c^{2} x^{2}+a b^{3} c^{2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c^2
*x^2 + a*b^3*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(c x^{2}\right)^{\frac{3}{2}} \left(a+b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.62, size = 76, normalized size = 1.04

$$\frac{\frac{\left(2 a \log\left(|a|\right)+a\right) \mathrm{sgn}\left(x\right)}{b^{3} \sqrt{c}}+\frac{x}{b^{2} \sqrt{c}\, \mathrm{sgn}\left(x\right)}-\frac{2 a \log\left(|bx+a|\right)}{b^{3} \sqrt{c}\, \mathrm{sgn}\left(x\right)}-\frac{a^{2}}{\left(bx+a\right) b^{3} \sqrt{c}\, \mathrm{sgn}\left(x\right)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] ((2*a*log(abs(a)) + a)*sgn(x)/(b^3*sqrt(c)) + x/(b^2*sqrt(c)*sgn(x)) - 2*a*
log(abs(b*x + a))/(b^3*sqrt(c)*sgn(x)) - a^2/((b*x + a)*b^3*sqrt(c)*sgn(x))
)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.918 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=49

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

```
[Out] a*x/b^2/c/(b*x+a)/(c*x^2)^(1/2)+x*ln(b*x+a)/b^2/c/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(cx^2)^{3/2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{x}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.76

$$\frac{x^3(a+(a+bx)\log(a+bx))}{b^2\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x^3*(a + (a + b*x)*Log[a + b*x]))/(b^2*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A]
time = 0.14, size = 41, normalized size = 0.84

method	result	size
default	$\frac{x^3(b\ln(bx+a)x+a\ln(bx+a)+a)}{(c\,x^2)^{\frac{3}{2}}\,b^2(bx+a)}$	41
risch	$\frac{ax}{b^2c(bx+a)\sqrt{c\,x^2}}+\frac{x\ln(bx+a)}{b^2c\sqrt{c\,x^2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] x^3*(b*ln(b*x+a)*x+a*ln(b*x+a)+a)/(c*x^2)^(3/2)/b^2/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(45) = 90.
time = 0.29, size = 108, normalized size = 2.20

$$-\frac{a^2}{\sqrt{cx^2}\,b^4cx+\sqrt{cx^2}\,ab^3c}+\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}}+\frac{\log(bx)}{b^2c^{\frac{3}{2}}}+\frac{3\,a}{\sqrt{cx^2}\,b^3c}-\frac{2\,a}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] -a^2/(sqrt(c*x^2)*b^4*c*x + sqrt(c*x^2)*a*b^3*c) + (-1)^(2*a*c*x/b)*log(-2*
a*c*x/(b*abs(b*x + a)))/(b^2*c^(3/2)) + log(b*x)/(b^2*c^(3/2)) + 3*a/(sqrt(
c*x^2)*b^3*c) - 2*a/(b^3*c^(3/2)*x)
```

Fricas [A]
time = 0.45, size = 44, normalized size = 0.90

$$\frac{\sqrt{cx^2}\,((bx+a)\log(bx+a)+a)}{b^3c^2x^2+ab^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c^2*x^2 + a*b^2*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [A]
time = 0.69, size = 59, normalized size = 1.20

$$-\frac{\frac{(\log(|a|)+1)\operatorname{sgn}(x)}{b^2\sqrt{c}} - \frac{\log(|bx+a|)}{b^2\sqrt{c}\,\operatorname{sgn}(x)} - \frac{a}{(bx+a)b^2\sqrt{c}\,\operatorname{sgn}(x)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -((log(abs(a)) + 1)*sgn(x)/(b^2*sqrt(c)) - log(abs(b*x + a))/(b^2*sqrt(c)*s
gn(x)) - a/((b*x + a)*b^2*sqrt(c)*sgn(x)))/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.919 $\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=25

$$-\frac{x}{bc\sqrt{cx^2}(a+bx)}$$

[Out] -x/b/c/(b*x+a)/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$-\frac{x}{bc\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]

[Out] -(x/(b*c*Sqrt[c*x^2]*(a + b*x)))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(cx^2)^{3/2}(a+bx)^2}dx &= \frac{x\int \frac{1}{(a+bx)^2}dx}{c\sqrt{cx^2}}\\ &= -\frac{x}{bc\sqrt{cx^2}(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.96

$$-\frac{x^3}{b(cx^2)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

Maple [A]
time = 0.14, size = 23, normalized size = 0.92

method	result	size
gosper	$-\frac{x^{3}}{\left(b x +a \right) b \left(c \,x^{2}\right)^{\frac{3}{2}}}$	23
default	$-\frac{x^{3}}{\left(b x +a \right) b \left(c \,x^{2}\right)^{\frac{3}{2}}}$	23
risch	$-\frac{x}{b c \left(b x +a \right) \sqrt{c \,x^{2}}}$	24
trager	$\frac{\left(-1+x \right) \sqrt{c \,x^{2}}}{c^{2} \left(b x +a \right) \left(a +b \right) x}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*x+a)/b*x^3/(c*x^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.
time = 0.29, size = 47, normalized size = 1.88

$$\frac{a}{\sqrt{cx^2}\,b^3cx + \sqrt{cx^2}\,ab^2c} - \frac{1}{\sqrt{cx^2}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a/(sqrt(c*x^2)*b^3*c*x + sqrt(c*x^2)*a*b^2*c) - 1/(sqrt(c*x^2)*b^2*c)
```

Fricas [A]
time = 0.46, size = 29, normalized size = 1.16

$$-\frac{\sqrt{cx^2}}{b^2c^2x^2 + abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c^2*x^2 + a*b*c^2*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(19) = 38$.

time = 0.55, size = 73, normalized size = 2.92

$$\begin{cases} \frac{\tilde{\infty}x^2}{(cx^2)^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^4}{(cx^2)^{\frac{3}{2}}} & \text{for } a = -bx \\ \frac{x^4}{a^2(cx^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{x^3}{ab(cx^2)^{\frac{3}{2}} + b^2x(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2/(c*x**2)**(3/2), Eq(a, 0) & Eq(b, 0)), (zoo*x**4/(c*x**
2)**(3/2), Eq(a, -b*x)), (x**4/(a**2*(c*x**2)**(3/2)), Eq(b, 0)), (-x**3/(a
*b*(c*x**2)**(3/2) + b**2*x*(c*x**2)**(3/2)), True))
```

Giac [A]

time = 1.16, size = 36, normalized size = 1.44

$$\frac{\frac{\text{sgn}(x)}{ab\sqrt{c}} - \frac{1}{(bx+a)b\sqrt{c}\,\text{sgn}(x)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (sgn(x)/(a*b*sqrt(c)) - 1/((b*x + a)*b*sqrt(c)*sgn(x)))/c
```

Mupad [B]

time = 0.17, size = 25, normalized size = 1.00

$$-\frac{\sqrt{c\,x^2}}{b\,c^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*c^2*x*(a + b*x))
```

3.920 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=68

$$\frac{x}{ac\sqrt{cx^2}\,(a+bx)}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}$$

```
[Out] x/a/c/(b*x+a)/(c*x^2)^(1/2)+x*ln(x)/a^2/c/(c*x^2)^(1/2)-x*ln(b*x+a)/a^2/c/(
c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 46}

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x/(a*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*c*Sqrt[c*x^2]) - (x*Log[a +
 b*x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{x^2}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x} - \frac{b}{a(a+bx)^2} - \frac{b}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{x}{ac\sqrt{cx^2}\,(a+bx)} + \frac{x\log(x)}{a^2c\sqrt{cx^2}} - \frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.68

$$\frac{x^3(a+(a+bx)\log(x)-(a+bx)\log(a+bx))}{a^2\,(cx^2)^{3/2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] (x^3*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*(c*x^2)^(3/2)*(a
 + b*x))
```

Maple [A]
time = 0.12, size = 52, normalized size = 0.76

method	result	size
default	$\frac{x^{3}\left(bx\ln\left(x\right)-b\ln\left(bx+a\right)x+a\ln\left(x\right)-a\ln\left(bx+a\right)+a\right)}{\left(c\,x^{2}\right)^{\frac{3}{2}}a^{2}\left(bx+a\right)}$	52
risch	$\frac{x}{ac\left(bx+a\right)\sqrt{c\,x^{2}}}+\frac{x\ln\left(-x\right)}{c\sqrt{c\,x^{2}}\,a^{2}}-\frac{x\ln\left(bx+a\right)}{a^{2}c\sqrt{c\,x^{2}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] x^3*(b*x*ln(x)-b*ln(b*x+a)*x+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2/(b*x+
a)
```

Maxima [A]
time = 0.29, size = 82, normalized size = 1.21

$$-\frac{1}{\sqrt{cx^2}\,b^2cx+\sqrt{cx^2}\,abc} - \frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}} + \frac{1}{\sqrt{cx^2}\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2)*b^2*c*x + sqrt(c*x^2)*a*b*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*
x/(b*abs(b*x + a)))/(a^2*c^(3/2)) + 1/(sqrt(c*x^2)*a*b*c)
```

Fricas [A]
time = 0.44, size = 48, normalized size = 0.71

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bc^2x^2+a^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c^2*x^2 + a^3*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.921 $\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{1}{a^2c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}$$

```
[Out] -1/a^2/c/(c*x^2)^(1/2)-b*x/a^2/c/(b*x+a)/(c*x^2)^(1/2)-2*b*x*ln(x)/a^3/c/(c
*x^2)^(1/2)+2*b*x*ln(b*x+a)/a^3/c/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 46}

$$-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*c*Sqrt[c*x^2])) - (b*x)/(a^2*c*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log
[x])/(a^3*c*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{1}{a^2c\sqrt{cx^2}} - \frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)} - \frac{2bx\log(x)}{a^3c\sqrt{cx^2}} + \frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 0.66

$$\frac{x^2(-a(a+2bx) - 2bx(a+bx)\log(x) + 2bx(a+bx)\log(a+bx))}{a^3(cx^2)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a + b
*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.82

method	result	size
default	$-\frac{x^2(2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)abx+2abx+a^2)}{(cx^2)^{\frac{3}{2}}a^3(bx+a)}$	74
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{c\sqrt{cx^2}\,(bx+a)} - \frac{2bx\ln(x)}{a^3c\sqrt{cx^2}} + \frac{2xb\ln(-bx-a)}{c\sqrt{cx^2}\,a^3}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x^2*(2*b^2*ln(x)*x^2-2*b^2*ln(b*x+a)*x^2+2*a*b*ln(x)*x-2*ln(b*x+a)*a*b*x+2
*a*b*x+a^2)/(c*x^2)^(3/2)/a^3/(b*x+a)
```

Maxima [A]
time = 0.31, size = 79, normalized size = 0.88

$$\frac{1}{\sqrt{cx^2}\,abcx + \sqrt{cx^2}\,a^2c} + \frac{2(-1)^{\frac{2acx}{b}}\,b\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}} - \frac{2}{\sqrt{cx^2}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] 1/(sqrt(c*x^2)*a*b*c*x + sqrt(c*x^2)*a^2*c) + 2*(-1)^(2*a*c*x/b)*b*log(-2*a
*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) - 2/(sqrt(c*x^2)*a^2*c)
```

Fricas [A]
time = 0.51, size = 66, normalized size = 0.73

$$-\frac{\left(2\,abx+a^2-2\,(b^2x^2+abx)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bc^2x^3+a^4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c^2*x^3 + a^4*c^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{(cx^2)^{\frac{3}{2}}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a)**2,x)
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x}{{(c\,x^2)}^{3/2}\,{(a+b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.922 $\int \frac{1}{(cx^2)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=118

$$\frac{2b}{a^3c\sqrt{cx^2}} - \frac{1}{2a^2cx\sqrt{cx^2}} + \frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)} + \frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

```
[Out] 2*b/a^3/c/(c*x^2)^(1/2)-1/2/a^2/c/x/(c*x^2)^(1/2)+b^2*x/a^3/c/(b*x+a)/(c*x^
2)^(1/2)+3*b^2*x*ln(x)/a^4/c/(c*x^2)^(1/2)-3*b^2*x*ln(b*x+a)/a^4/c/(c*x^2)^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 46}

$$\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}} + \frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)} + \frac{2b}{a^3c\sqrt{cx^2}} - \frac{1}{2a^2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*c*Sqrt[c*x^2]) - 1/(2*a^2*c*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*c*Sqrt
[c*x^2]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*c*Sqrt[c*x^2]) - (3*b^2*x*Log[a
+ b*x])/(a^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(cx^2)^{3/2}(a+bx)^2}\,dx = \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{2b}{a^3c\sqrt{cx^2}} - \frac{1}{2a^2cx\sqrt{cx^2}} + \frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)} + \frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}} - \frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}$$

Mathematica [A]
time = 0.01, size = 80, normalized size = 0.68

$$\frac{x(a(-a^2+3abx+6b^2x^2)+6b^2x^2(a+bx)\log(x)-6b^2x^2(a+bx)\log(a+bx))}{2a^4(cx^2)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x^2
*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

Maple [A]
time = 0.14, size = 93, normalized size = 0.79

method	result	size
default	$\frac{x\left(6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6\ln(x)a\,b^2x^2-6\ln(bx+a)a\,b^2x^2+6a\,b^2x^2+3a^2bx-a^3\right)}{2(c\,x^2)^{\frac{3}{2}}a^4(bx+a)}$	93
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{cx\sqrt{c\,x^2}\,(bx+a)} - \frac{3b^2x\ln(bx+a)}{a^4c\sqrt{c\,x^2}} + \frac{3x\,b^2\ln(-x)}{c\sqrt{c\,x^2}\,a^4}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*ln(x)*a*b^2*x^2-6*ln(b*x+a)*a*
b^2*x^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(3/2)/a^4/(b*x+a)
```

Maxima [A]
time = 0.29, size = 98, normalized size = 0.83

$$-\frac{b}{\sqrt{cx^2}\,a^2bcx+\sqrt{cx^2}\,a^3c} - \frac{3\,(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^4c^{\frac{3}{2}}} + \frac{3\,b}{\sqrt{cx^2}\,a^3c} - \frac{1}{2\,a^2c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*a^2*b*c*x + sqrt(c*x^2)*a^3*c) - 3*(-1)^(2*a*c*x/b)*b^2*log
(-2*a*c*x/(b*abs(b*x + a)))/(a^4*c^(3/2)) + 3*b/(sqrt(c*x^2)*a^3*c) - 1/2/(
a^2*c^(3/2)*x^2)
```

Fricas [A]
time = 0.59, size = 83, normalized size = 0.70

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bc^2x^4+a^5c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c^2*x^4 + a^5*c^2*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

3.923 $\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}$$

```
[Out] -a^3*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^4/(1+n)/x+3*a^2*(b*x+a)^(2+n)*(c*x^2)^(1
/2)/b^4/(2+n)/x-3*a*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^4/(3+n)/x+(b*x+a)^(4+n)*(
c*x^2)^(1/2)/b^4/(4+n)/x
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a^3*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(
b^4*(3 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\int x^3(a+bx)^n\,dx}{x} \\
&= \frac{\sqrt{cx^2}\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x} \\
&= -\frac{a^3\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}}{b^4(}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.74

$$
\frac{cx(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)\sqrt{cx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 136, normalized size = 1.04

method	result
gosper	$-\frac{\sqrt{c\,x^2}\,(bx+a)^{1+n}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{x\,b^4\left(n^4+10n^3+35n^2+50n+24\right)}$
risch	$-\frac{\sqrt{c\,x^2}\,\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)}{x(3+n)(4+n)(2+n)(1+n)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(1+n)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.29, size = 116, normalized size = 0.89

$$
\frac{\left((n^3+6\,n^2+11\,n+6)b^4\sqrt{c}\,x^4+(n^3+3\,n^2+2\,n)ab^3\sqrt{c}\,x^3-3\,(n^2+n)a^2b^2\sqrt{c}\,x^2+6\,a^3b\sqrt{c}\,nx-6\,a^4\sqrt{c}\,\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*sqrt(c)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*sqrt(
c)*x^3 - 3*(n^2 + n)*a^2*b^2*sqrt(c)*x^2 + 6*a^3*b*sqrt(c)*n*x - 6*a^4*sqrt
(c))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.52, size = 153, normalized size = 1.17

$$\frac{\left(6\,a^{3}bnx + (b^{4}n^{3} + 6\,b^{4}n^{2} + 11\,b^{4}n + 6\,b^{4})x^{4} - 6\,a^{4} + (ab^{3}n^{3} + 3\,ab^{3}n^{2} + 2\,ab^{3}n)x^{3} - 3\,(a^{2}b^{2}n^{2} + a^{2}b^{2}n)x^{2}\right)\sqrt{cx^{2}}\,(bx + a)^{n}}{(b^{4}n^{4} + 10\,b^{4}n^{3} + 35\,b^{4}n^{2} + 50\,b^{4}n + 24\,b^{4})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{n} x^{3} \sqrt{c x^{2}}}{4} & \text{for}\: b = 0 \\ \int \frac{x^{2} \sqrt{c x^{2}}}{\left(a + b x\right)^{4}}\, dx & \text{for}\: n = -4 \\ \int \frac{x^{2} \sqrt{c x^{2}}}{\left(a + b x\right)^{3}}\, dx & \text{for}\: n = -3 \\ \int \frac{x^{2} \sqrt{c x^{2}}}{\left(a + b x\right)^{2}}\, dx & \text{for}\: n = -2 \\ \int \frac{x^{2} \sqrt{c x^{2}}}{a + b x}\, dx & \text{for}\: n = -1 \\ - \frac{6 a^{4} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{6 a^{3} b n x \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} - \frac{3 a^{2} b^{2} n^{2} x^{2} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} - \frac{3 a^{2} b^{2} n x^{2} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{a b^{3} n^{3} x^{3} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{3 a b^{3} n^{2} x^{3} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{2 a b^{3} n x^{3} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{b^{4} n^{3} x^{4} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{6 b^{4} n^{2} x^{4} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{11 b^{4} n x^{4} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} + \frac{6 b^{4} x^{4} \sqrt{c x^{2}} \left(a + b x\right)^{n}}{b^{4} n^{4} x + 10 b^{4} n^{3} x + 35 b^{4} n^{2} x + 50 b^{4} n x + 24 b^{4} x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**3*sqrt(c*x**2)/4, Eq(b, 0)), (Integral(x**2*sqrt(c*x**2)
/(a + b*x)**4, x), Eq(n, -4)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**3, x)
, Eq(n, -3)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x), Eq(n, -2)), (In
tegral(x**2*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (-6*a**4*sqrt(c*x**2)*(
a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x +
24*b**4*x) + 6*a**3*b*n*x*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*
n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) - 3*a**2*b**2*n**2*x**2*
sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x +
50*b**4*n*x + 24*b**4*x) - 3*a**2*b**2*n*x**2*sqrt(c*x**2)*(a + b*x)**n/(b*
*4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + a*
b**3*n**3*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35
*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 3*a*b**3*n**2*x**3*sqrt(c*x**2)*(
a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x +
24*b**4*x) + 2*a*b**3*n*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b*
*4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + b**4*n**3*x**4*sqrt
```

```
(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b
**4*n*x + 24*b**4*x) + 6*b**4*n**2*x**4*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**
4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 11*b**4*
n*x**4*sqrt(c*x**2)*(a + b*x)**n/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n*
*2*x + 50*b**4*n*x + 24*b**4*x) + 6*b**4*x**4*sqrt(c*x**2)*(a + b*x)**n/(b*
*4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(123) = 246.
time = 0.67, size = 300, normalized size = 2.29

$$\left(\frac{6\,a^4a^n\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}+\frac{(bx+a)^n b^4n^3x^4\mathrm{sgn}(x)+(bx+a)^n ab^3n^3x^3\mathrm{sgn}(x)+6\,(bx+a)^n b^4n^2x^4\mathrm{sgn}(x)+3\,(bx+a)^n ab^3n^2x^3\mathrm{sgn}(x)+11\,(bx+a)^n b^4nx^4\mathrm{sgn}(x)-3\,(bx+a)^n a^2b^2n^2x^2\mathrm{sgn}(x)+2\,(bx+a)^n ab^3nx^3\mathrm{sgn}(x)+6\,(bx+a)^n b^4x^4\mathrm{sgn}(x)-3\,(bx+a)^n a^2b^2nx^2\mathrm{sgn}(x)+6\,(bx+a)^n a^3bnx\mathrm{sgn}(x)-6\,(bx+a)^n a^4\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*sqrt(c)
```

Mupad [B]
time = 0.35, size = 214, normalized size = 1.63

$$\frac{(a+b\,x)^n\left(\frac{x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) - (6*a^4*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^3 +
n^4 + 24)) + (6*a^3*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (a*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3
+ n^4 + 24)) - (3*a^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24))))/x
```

3.924 $\int x\sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=96

$$\frac{a^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2a\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

```
[Out] a^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^3/(1+n)/x-2*a*(b*x+a)^(2+n)*(c*x^2)^(1/2)
/b^3/(2+n)/x+(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^3/(3+n)/x
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {15, 45}

$$\frac{a^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (a^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*Sqrt[c*x^2]*(a +
 b*x)^(2 + n))/(b^3*(2 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3*(3 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\sqrt{cx^2}(a+bx)^n\,dx = \frac{\sqrt{cx^2}\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\int\left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2a\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x} + \frac{\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.71

$$\frac{cx(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.11, size = 83, normalized size = 0.86

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)\sqrt{c\,x^2}}{x\,b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{\sqrt{c\,x^2}\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{x(2+n)(3+n)(1+n)b^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(1/2)/x/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.29, size = 80, normalized size = 0.83

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.50, size = 106, normalized size = 1.10

$$-\frac{(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^2 \sqrt{cx^2}}{3} & \text{for}\: b = 0 \\ \int \frac{x\sqrt{cx^2}}{(a+bx)^3}\, dx & \text{for}\: n = -3 \\ \int \frac{x\sqrt{cx^2}}{(a+bx)^2}\, dx & \text{for}\: n = -2 \\ \int \frac{x\sqrt{cx^2}}{a+bx}\, dx & \text{for}\: n = -1 \\ \frac{2a^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} - \frac{2a^2bnx\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{ab^2n^2x^2\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{ab^2nx^2\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{b^3n^2x^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{3b^3nx^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{2b^3x^3\sqrt{cx^2}\,(a+bx)^n}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**2*sqrt(c*x**2)/3, Eq(b, 0)), (Integral(x*sqrt(c*x**2)/(a
 + b*x)**3, x), Eq(n, -3)), (Integral(x*sqrt(c*x**2)/(a + b*x)**2, x), Eq(n
, -2)), (Integral(x*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (2*a**3*sqrt(c*
x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) -
 2*a**2*b*n*x*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b
**3*n*x + 6*b**3*x) + a*b**2*n**2*x**2*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3
*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + a*b**2*n*x**2*sqrt(c*x**2)*(
a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + b**3*n
**2*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n
*x + 6*b**3*x) + 3*b**3*n*x**3*sqrt(c*x**2)*(a + b*x)**n/(b**3*n**3*x + 6*b
**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + 2*b**3*x**3*sqrt(c*x**2)*(a + b*x)**
n/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(90) = 180.
time = 0.71, size = 200, normalized size = 2.08

$$-\left(\frac{2\,a^3a^n\mathrm{sgn}(x)}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}-\frac{(bx+a)^nb^3n^2x^3\mathrm{sgn}(x)+(bx+a)^nab^2n^2x^2\mathrm{sgn}(x)+3\,(bx+a)^nb^3nx^3\mathrm{sgn}(x)+(bx+a)^nab^2nx^2\mathrm{sgn}(x)+2\,(bx+a)^nb^3x^3\mathrm{sgn}(x)-2\,(bx+a)^na^2bnx\mathrm{sgn}(x)+2\,(bx+a)^na^3\mathrm{sgn}(x)}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^3*a^n*sgn(x)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3) - ((b*x + a)^n*
b^3*n^2*x^3*sgn(x) + (b*x + a)^n*a*b^2*n^2*x^2*sgn(x) + 3*(b*x + a)^n*b^3*n
*x^3*sgn(x) + (b*x + a)^n*a*b^2*n*x^2*sgn(x) + 2*(b*x + a)^n*b^3*x^3*sgn(x)
 - 2*(b*x + a)^n*a^2*b*n*x*sgn(x) + 2*(b*x + a)^n*a^3*sgn(x))/(b^3*n^3 + 6*
b^3*n^2 + 11*b^3*n + 6*b^3))*sqrt(c)
```

Mupad [B]
time = 0.25, size = 142, normalized size = 1.48

$$\frac{(a+b\,x)^n\,\left(\frac{2\,a^3\,\sqrt{c\,x^2}}{b^3\,(n^3+6\,n^2+11\,n+6)}+\frac{x^3\,\sqrt{c\,x^2}\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,(n^3+6\,n^2+11\,n+6)}+\frac{a\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,(n^3+6\,n^2+11\,n+6)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (x^3*(
c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*n*x*(c*x^2)
^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b
*(11*n + 6*n^2 + n^3 + 6))))/x
```

3.925 $\int \sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=63

$$-\frac{a\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

```
[Out] -a*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^2/(1+n)/x+(b*x+a)^(2+n)*(c*x^2)^(1/2)/b^2/
(2+n)/x
```

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 45}

$$\frac{\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (Sqrt[c*x^2]*(a + b*
x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\,\int x(a+bx)^n\,dx}{x}\\
&= \frac{\sqrt{cx^2}\,\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\
&= -\frac{a\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.70

$$\frac{cx(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2]
)
```

Maple [A]
time = 0.12, size = 46, normalized size = 0.73

method	result	size
gosper	$-\frac{\sqrt{c\,x^2}\,(bx+a)^{1+n}(-bnx-bx+a)}{x\,b^2(n^2+3n+2)}$	46
risch	$-\frac{\sqrt{c\,x^2}\,(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{x\,b^2(2+n)(1+n)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(1+n)*(-b*n*x-b*x+a)/x/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.33, size = 51, normalized size = 0.81

$$\frac{\left(b^2\sqrt{c}\,(n+1)x^2+ab\sqrt{c}\,nx-a^2\sqrt{c}\,\right)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*sqrt(c)*(n + 1)*x^2 + a*b*sqrt(c)*n*x - a^2*sqrt(c))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.53, size = 63, normalized size = 1.00

$$\frac{(abnx+(b^2n+b^2)x^2-a^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2n^2+3\,b^2n+2\,b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n^2 + 3*b
^2*n + 2*b^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{n}x\sqrt{cx^{2}}}{2} & \text{for}\: b = 0 \\ \int \frac{\sqrt{cx^{2}}}{(a+bx)^{2}}\, dx & \text{for}\: n = -2 \\ \int \frac{\sqrt{cx^{2}}}{a+bx}\, dx & \text{for}\: n = -1 \\ -\frac{a^{2}\sqrt{cx^{2}}\,(a+bx)^{n}}{b^{2}n^{2}x+3b^{2}nx+2b^{2}x} + \frac{abnx\sqrt{cx^{2}}\,(a+bx)^{n}}{b^{2}n^{2}x+3b^{2}nx+2b^{2}x} + \frac{b^{2}nx^{2}\sqrt{cx^{2}}\,(a+bx)^{n}}{b^{2}n^{2}x+3b^{2}nx+2b^{2}x} + \frac{b^{2}x^{2}\sqrt{cx^{2}}\,(a+bx)^{n}}{b^{2}n^{2}x+3b^{2}nx+2b^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x*sqrt(c*x**2)/2, Eq(b, 0)), (Integral(sqrt(c*x**2)/(a + b*
x)**2, x), Eq(n, -2)), (Integral(sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (-
a**2*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x) + a*b*
n*x*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x) + b**2*
n*x**2*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x) + b*
*2*x**2*sqrt(c*x**2)*(a + b*x)**n/(b**2*n**2*x + 3*b**2*n*x + 2*b**2*x), Tr
ue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(59) = 118.
time = 1.05, size = 119, normalized size = 1.89

$$\left(\frac{a^{2}a^{n}\mathrm{sgn}(x)}{b^{2}n^{2}+3\,b^{2}n+2\,b^{2}} + \frac{(bx+a)^{n}b^{2}nx^{2}\mathrm{sgn}(x) + (bx+a)^{n}abnx\mathrm{sgn}(x) + (bx+a)^{n}b^{2}x^{2}\mathrm{sgn}(x) - (bx+a)^{n}a^{2}\mathrm{sgn}(x)}{b^{2}n^{2}+3\,b^{2}n+2\,b^{2}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*sqrt(c)
```

Mupad [B]
time = 0.22, size = 85, normalized size = 1.35

$$\frac{(a+b\,x)^{n}\left(\frac{x^{2}\,\sqrt{c\,x^{2}}\,(n+1)}{n^{2}+3\,n+2} - \frac{a^{2}\,\sqrt{c\,x^{2}}}{b^{2}\,(n^{2}+3\,n+2)} + \frac{a\,n\,x\,\sqrt{c\,x^{2}}}{b\,(n^{2}+3\,n+2)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*(c*x^2)^(1
/2))/(b^2*(3*n + n^2 + 2)) + (a*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2))))/x
```

3.926 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}$$

[Out] (b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]

[Out] (Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x}\,dx &= \frac{\sqrt{cx^2}\,\int (a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.97

$$\frac{cx(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.97

method	result	size
gosper	$\frac{(bx+a)^{1+n}\sqrt{c\,x^{2}}}{b(1+n)x}$	29
risch	$\frac{(bx+a)(bx+a)^{n}\sqrt{c\,x^{2}}}{b(1+n)x}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x
```

Maxima [A]
time = 0.29, size = 28, normalized size = 0.93

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\,\right)(bx+a)^{n}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A]
time = 0.49, size = 30, normalized size = 1.00

$$\frac{\sqrt{cx^{2}}\,(bx+a)(bx+a)^{n}}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\sqrt{cx^2}}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n\sqrt{cx^2} & \text{for } b = 0 \\ \int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx & \text{for } n = -1 \\ \frac{a\sqrt{cx^2}\,(a+bx)^n}{bnx+bx} + \frac{bx\sqrt{cx^2}\,(a+bx)^n}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(c*x**2)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sqrt(c*x**2), Eq(b,
 0)), (Integral(sqrt(c*x**2)/(x*(a + b*x)), x), Eq(n, -1)), (a*sqrt(c*x**2)
*(a + b*x)**n/(b*n*x + b*x) + b*x*sqrt(c*x**2)*(a + b*x)**n/(b*n*x + b*x),
True))
```

Giac [A]
time = 1.68, size = 42, normalized size = 1.40

$$-\sqrt{c}\left(\frac{a^{n+1}\mathrm{sgn}(x)}{bn+b} - \frac{(bx+a)^{n+1}\mathrm{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -sqrt(c)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

Mupad [B]
time = 0.23, size = 31, normalized size = 1.03

$$\frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^n\,\left(a+b\,x\right)}{b\,x\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((c*x^2)^(1/2)*(a + b*x)^n*(a + b*x))/(b*x*(n + 1))
```

3.927 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}$$

```
[Out] -(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a/(1+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]
```

```
[Out] -((Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*
x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^2}\,dx &= \frac{\sqrt{cx^2}\,\int \frac{(a+bx)^n}{x}\,dx}{x}\\ &= -\frac{\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.98

$$-\frac{cx(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^2,x]
[Out] -((c*x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*Sqrt[c*x^2]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\sqrt{c\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^2, x)
```

3.928 $\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=47

$$\frac{b\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

```
[Out] b*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^2/(1+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{b\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]
```

```
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^3}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{b\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.00

$$\frac{b\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^3,x]
```

```
[Out] (b*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
x)/a])/(a^2*(1 + n)*x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n}\sqrt{c\,x^{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^{n}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^3,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^3, x)
```

3.929 $\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{b^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}$$

```
[Out] -b^2*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^3/(1+n
)/x
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]
```

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx &= \frac{\sqrt{cx^2}\,\int \frac{(a+bx)^n}{x^3}\,dx}{x}\\ &= -\frac{b^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.00

$$-\frac{b^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x^4,x]
```

```
[Out] -((b^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^3*(1 + n)*x))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\,\sqrt{c\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] int((b*x+a)^n*(c*x^2)^(1/2)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**n/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\,{\left(a+b\,x\right)}^n}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^4,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^n)/x^4, x)
```

3.930 $\int x(cx^2)^{3/2}(a+bx)^n\,dx$

Optimal. Leaf size=169

$$\frac{a^4c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac\sqrt{cx^2}\,(a+bx)^{4+n}}{b^5(4+n)x}+\frac{c\sqrt{cx^2}\,(a+}{b^5(5+n}$$

```
[Out] a^4*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^5/(1+n)/x-4*a^3*c*(b*x+a)^(2+n)*(c*x^2)
^(1/2)/b^5/(2+n)/x+6*a^2*c*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^5/(3+n)/x-4*a*c*(b
*x+a)^(4+n)*(c*x^2)^(1/2)/b^5/(4+n)/x+c*(b*x+a)^(5+n)*(c*x^2)^(1/2)/b^5/(5+
n)/x
```

Rubi [A]
time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 45}

$$\frac{a^4c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (a^4*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c*Sqrt[c*x^2
]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c*Sqrt[c*x^2]*(a + b*x)^(3 +
n))/(b^5*(3 + n)*x) - (4*a*c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4 + n)*x)
 + (c*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}$$
$$=\frac{\left(c\sqrt{cx^2}\right)\int\left(\frac{a^4(a+bx)^n}{b^4}-\frac{4a^3(a+bx)^{1+n}}{b^4}+\frac{6a^2(a+bx)^{2+n}}{b^4}-\frac{4a(a+bx)^{3+n}}{b^4}+\frac{(a+bx)^{4+n}}{b^4}\right)d}{x}$$
$$=\frac{a^4c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac}{}$$

Mathematica [A]
time = 0.07, size = 132, normalized size = 0.78

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{1+n}\left(24a^4-24a^3b(1+n)x+12a^2b^2\left(2+3n+n^2\right)x^2-4ab^3\left(6+11n+6n^2+n^3\right)x^3+b^4\left(24+50n+35n^2+10n^3+n^4\right)x^4\right)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^2*
(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50*n
 + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n
)*x^3)
```

Maple [A]
time = 0.13, size = 199, normalized size = 1.18

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44x^3an\,b^3+24b^4x^4+36a^2n\,x^2b^2-24a\,b\right.}{x^3b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}$
risch	$\frac{c\sqrt{c\,x^2}\,\left(b^5n^4x^5+a\,b^4n^4x^4+10b^5n^3x^5+6a\,b^4n^3x^4+35b^5n^2x^5-4a^2b^3n^3x^3+11a\,b^4n^2x^4+50b^5n\,x^5-12a^2b^3n^2x^3+6x^4an\,b^4+24b^5\right.}{x(4+n)(5+n)(3+n)(2+n)(1+n)b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(3/2)/x^3/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A]
time = 0.34, size = 157, normalized size = 0.93

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5c^{\frac{3}{2}}x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4c^{\frac{3}{2}}x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3c^{\frac{3}{2}}x^3+12\left(n^2+n\right)a^3b^2c^{\frac{3}{2}}x^2-24\,a^4bc^{\frac{3}{2}}nx+24\,a^5c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(3/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(3/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(3/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(3/2)*x^2 - 24*a^4*b*c^(3/2)*n*x + 24*a^5*c^(3/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [**A**]
time = 0.81, size = 233, normalized size = 1.38

$$-\frac{\left(24\,a^{4}bcnx-24\,a^{5}c-\left(b^{5}cn^{4}+10\,b^{5}cn^{3}+35\,b^{5}cn^{2}+50\,b^{5}cn+24\,b^{5}c\right)x^{5}-\left(ab^{4}cn^{4}+6\,ab^{4}cn^{3}+11\,ab^{4}cn^{2}+6\,ab^{4}cn\right)x^{4}+4\,\left(a^{2}b^{3}cn^{3}+3\,a^{2}b^{3}cn^{2}+2\,a^{2}b^{3}cn\right)x^{3}-12\,\left(a^{3}b^{2}cn^{2}+a^{3}b^{2}cn\right)x^{2}\right)\sqrt{cx^{2}}\,\left(bx+a\right)^{n}}{\left(b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c*n*x - 24*a^5*c - (b^5*c*n^4 + 10*b^5*c*n^3 + 35*b^5*c*n^2 + 50
*b^5*c*n + 24*b^5*c)*x^5 - (a*b^4*c*n^4 + 6*a*b^4*c*n^3 + 11*a*b^4*c*n^2 +
6*a*b^4*c*n)*x^4 + 4*(a^2*b^3*c*n^3 + 3*a^2*b^3*c*n^2 + 2*a^2*b^3*c*n)*x^3
- 12*(a^3*b^2*c*n^2 + a^3*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^5*n^5 +
 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)*(a + b*x)**n, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(159) = 318.
time = 1.00, size = 426, normalized size = 2.52

$$-\left(\frac{24\,a^{5}a^{n}\mathrm{sgn}\left(x\right)}{b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}}-\frac{\left(bx+a\right)^{n}b^{5}n^{4}x^{5}\mathrm{sgn}\left(x\right)+\left(bx+a\right)^{n}ab^{4}n^{4}x^{4}\mathrm{sgn}\left(x\right)+10\,\left(bx+a\right)^{n}b^{5}n^{3}x^{5}\mathrm{sgn}\left(x\right)+6\,\left(bx+a\right)^{n}ab^{4}n^{3}x^{4}\mathrm{sgn}\left(x\right)+35\,\left(bx+a\right)^{n}b^{5}n^{2}x^{5}\mathrm{sgn}\left(x\right)-4\,\left(bx+a\right)^{n}a^{2}b^{3}n^{3}x^{3}\mathrm{sgn}\left(x\right)+11\,\left(bx+a\right)^{n}ab^{4}n^{2}x^{4}\mathrm{sgn}\left(x\right)+50\,\left(bx+a\right)^{n}b^{5}nx^{5}\mathrm{sgn}\left(x\right)-12\,\left(bx+a\right)^{n}a^{2}b^{3}n^{2}x^{3}\mathrm{sgn}\left(x\right)+6\,\left(bx+a\right)^{n}ab^{4}nx^{4}\mathrm{sgn}\left(x\right)+24\,\left(bx+a\right)^{n}b^{5}x^{5}\mathrm{sgn}\left(x\right)+12\,\left(bx+a\right)^{n}a^{3}b^{2}n^{2}x^{2}\mathrm{sgn}\left(x\right)-8\,\left(bx+a\right)^{n}a^{2}b^{3}nx^{3}\mathrm{sgn}\left(x\right)+12\,\left(bx+a\right)^{n}a^{3}b^{2}nx^{2}\mathrm{sgn}\left(x\right)-24\,\left(bx+a\right)^{n}a^{4}bnx\mathrm{sgn}\left(x\right)+24\,\left(bx+a\right)^{n}a^{5}\mathrm{sgn}\left(x\right)}{b^{5}n^{5}+15\,b^{5}n^{4}+85\,b^{5}n^{3}+225\,b^{5}n^{2}+274\,b^{5}n+120\,b^{5}}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] -(24*a^5*a^n*sgn(x)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*
b^5*n + 120*b^5) - ((b*x + a)^n*b^5*n^4*x^5*sgn(x) + (b*x + a)^n*a*b^4*n^4*
x^4*sgn(x) + 10*(b*x + a)^n*b^5*n^3*x^5*sgn(x) + 6*(b*x + a)^n*a*b^4*n^3*x^
4*sgn(x) + 35*(b*x + a)^n*b^5*n^2*x^5*sgn(x) - 4*(b*x + a)^n*a^2*b^3*n^3*x^
```

```
3*sgn(x) + 11*(b*x + a)^n*a*b^4*n^2*x^4*sgn(x) + 50*(b*x + a)^n*b^5*n*x^5*s
gn(x) - 12*(b*x + a)^n*a^2*b^3*n^2*x^3*sgn(x) + 6*(b*x + a)^n*a*b^4*n*x^4*s
gn(x) + 24*(b*x + a)^n*b^5*x^5*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n^2*x^2*sgn(
x) - 8*(b*x + a)^n*a^2*b^3*n*x^3*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n*x^2*sgn(
x) - 24*(b*x + a)^n*a^4*b*n*x*sgn(x) + 24*(b*x + a)^n*a^5*sgn(x))/(b^5*n^5
+ 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5))*c^(3/2)
```

Mupad [B]
time = 0.41, size = 307, normalized size = 1.82

$$\frac{(a+b\,x)^n\,\left(\frac{24\,a^5\,c\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{c\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}-\frac{24\,a^4\,c\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((24*a^5*c*(c*x^2)^(1/2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120)) + (c*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24
))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) - (24*a^4*c*n*x*(c*x^2)^
(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c*n*x^4*(
c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^
4 + n^5 + 120)) + (12*a^3*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c*n*x^3*(c*x^2)^(1/2)*(3*n + n
^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))))/x
```

3.931 $\int (cx^2)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=135

$$-\frac{a^3c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}$$

```
[Out] -a^3*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^4/(1+n)/x+3*a^2*c*(b*x+a)^(2+n)*(c*x^2
)^(1/2)/b^4/(2+n)/x-3*a*c*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^4/(3+n)/x+c*(b*x+a)
^(4+n)*(c*x^2)^(1/2)/b^4/(4+n)/x
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^3c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c*Sqrt[c*
x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c*Sqrt[c*x^2]*(a + b*x)^(3 +
 n))/(b^4*(3 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(cx^2\right)^{3/2}(a+bx)^n\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c\sqrt{}}{}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 0.73

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 +
 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(
3 + n)*(4 + n)*x^3)
```

Maple [A]
time = 0.14, size = 136, normalized size = 1.01

method	result
gosper	$-\frac{(bx+a)^{1+n}\left(c\,x^2\right)^{\frac{3}{2}}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{x^3b^4\left(n^4+10n^3+35n^2+50n+24\right)}$
risch	$-\frac{c\sqrt{c\,x^2}\,\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)}{x(3+n)(4+n)(2+n)(1+n)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(3/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^3/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.86

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4c^{\frac{3}{2}}x^4+(n^3+3\,n^2+2\,n)ab^3c^{\frac{3}{2}}x^3-3\,(n^2+n)a^2b^2c^{\frac{3}{2}}x^2+6\,a^3bc^{\frac{3}{2}}nx-6\,a^4c^{\frac{3}{2}}\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(3/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(3/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(3/2)*x^2 + 6*a^3*b*c^(3/2)*n*x - 6*a^4*c^(3
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.46, size = 164, normalized size = 1.21

$$\frac{\left(6\,a^3bcnx - 6\,a^4c + (b^4cn^3 + 6\,b^4cn^2 + 11\,b^4cn + 6\,b^4c)x^4 + (ab^3cn^3 + 3\,ab^3cn^2 + 2\,ab^3cn)x^3 - 3\,(a^2b^2cn^2 + a^2b^2cn)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^4n^4 + 10\,b^4n^3 + 35\,b^4n^2 + 50\,b^4n + 24\,b^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c*n*x - 6*a^4*c + (b^4*c*n^3 + 6*b^4*c*n^2 + 11*b^4*c*n + 6*b^4*c)
*x^4 + (a*b^3*c*n^3 + 3*a*b^3*c*n^2 + 2*a*b^3*c*n)*x^3 - 3*(a^2*b^2*c*n^2 +
 a^2*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*
n^2 + 50*b^4*n + 24*b^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{3}{2}} (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(127) = 254.
time = 0.73, size = 300, normalized size = 2.22

$$\left(\frac{6\,a^4a^n\mathrm{sgn}(x)}{b^4n^4 + 10\,b^4n^3 + 35\,b^4n^2 + 50\,b^4n + 24\,b^4} + \frac{(bx+a)^nb^4n^3x^4\mathrm{sgn}(x) + (bx+a)^nab^3n^3x^3\mathrm{sgn}(x) + 6\,(bx+a)^nb^4n^2x^4\mathrm{sgn}(x) + 3\,(bx+a)^nab^3n^2x^3\mathrm{sgn}(x) + 11\,(bx+a)^nb^4nx^4\mathrm{sgn}(x) - 3\,(bx+a)^na^2b^2n^2x^2\mathrm{sgn}(x) + 2\,(bx+a)^nab^3nx^3\mathrm{sgn}(x) + 6\,(bx+a)^nb^4x^4\mathrm{sgn}(x) - 3\,(bx+a)^na^2b^2nx^2\mathrm{sgn}(x) + 6\,(bx+a)^na^3bnx\mathrm{sgn}(x) - 6\,(bx+a)^na^4\mathrm{sgn}(x)}{b^4n^4 + 10\,b^4n^3 + 35\,b^4n^2 + 50\,b^4n + 24\,b^4}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
```

```
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*c^(3/2)
```

Mupad [B]
time = 0.32, size = 219, normalized size = 1.62

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2
 + 10*n^3 + n^4 + 24) - (6*a^4*c*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^
3 + n^4 + 24)) + (6*a^3*c*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (3*a^2*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (a*c*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24))))/x
```

3.932 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x} dx$

Optimal. Leaf size=99

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

```
[Out] a^2*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^3/(1+n)/x-2*a*c*(b*x+a)^(2+n)*(c*x^2)^(
1/2)/b^3/(2+n)/x+c*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^3/(3+n)/x
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c*Sqrt[c*x^2]*
(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3
*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2ac\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x} + \frac{c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.71

$$\frac{c^2x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 83, normalized size = 0.84

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{x^3b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{c\sqrt{c\,x^2}\,\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{x(2+n)(3+n)(1+n)b^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(3/2)/x^3/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.29, size = 80, normalized size = 0.81

$$\frac{\left((n^2+3\,n+2)b^3c^{\frac{3}{2}}x^3+(n^2+n)ab^2c^{\frac{3}{2}}x^2-2\,a^2bc^{\frac{3}{2}}nx+2\,a^3c^{\frac{3}{2}}\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(3/2)*x^3 + (n^2 + n)*a*b^2*c^(3/2)*x^2 - 2*a^2*b*c^
(3/2)*n*x + 2*a^3*c^(3/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.52, size = 113, normalized size = 1.14

$$-\frac{\left(2\,a^2bcnx-2\,a^3c-\left(b^3cn^2+3\,b^3cn+2\,b^3c\right)x^3-\left(ab^2cn^2+ab^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c*n*x - 2*a^3*c - (b^3*c*n^2 + 3*b^3*c*n + 2*b^3*c)*x^3 - (a*b^2*
c*n^2 + a*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*
b^3*n + 6*b^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}\,(a+bx)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x, x)
```

Mupad [B]
time = 0.26, size = 146, normalized size = 1.47

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,c\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,c\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6
) + (2*a^3*c*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*c*n*x*(
c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c*n*x^2*(c*x^2)^(1/2)*(n
+ 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

3.933 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx$

Optimal. Leaf size=65

$$-\frac{ac\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

```
[Out] -a*c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^2/(1+n)/x+c*(b*x+a)^(2+n)*(c*x^2)^(1/2)/
b^2/(2+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c*Sqrt[c*x^2]*(a
+ b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}$$
$$= -\frac{ac\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.71

$$\frac{c^2x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
[Out] (c^2*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Maple [A]
time = 0.11, size = 46, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(cx^2)^{\frac{3}{2}}(-bnx-bx+a)}{x^3b^2(n^2+3n+2)}$	46
risch	$-\frac{c\sqrt{cx^2}\,(-b^2nx^2-abnx-x^2b^2+a^2)(bx+a)^n}{x\,b^2(2+n)(1+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^2,x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*(c*x^2)^(3/2)*(-b*n*x-b*x+a)/x^3/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.32, size = 51, normalized size = 0.78

$$\frac{\left(b^2c^{\frac{3}{2}}(n+1)x^2+abc^{\frac{3}{2}}nx-a^2c^{\frac{3}{2}}\right)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^(3/2)*(n + 1)*x^2 + a*b*c^(3/2)*n*x - a^2*c^(3/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.61, size = 68, normalized size = 1.05

$$\frac{(abcnx - a^2c + (b^2cn + b^2c)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2n^2 + 3\,b^2n + 2\,b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (a*b*c*n*x - a^2*c + (b^2*c*n + b^2*c)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n
^2 + 3*b^2*n + 2*b^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n (cx^2)^{\frac{3}{2}}}{2x} & \text{for}\: b = 0 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a+bx)^2}\,dx & \text{for}\: n = -2 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^2(a+bx)}\,dx & \text{for}\: n = -1 \\ -\frac{a^2 (cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3} + \frac{abnx (cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3} + \frac{b^2nx^2 (cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3} + \frac{b^2x^2 (cx^2)^{\frac{3}{2}}(a+bx)^n}{b^2n^2x^3+3b^2nx^3+2b^2x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Piecewise((a**n*(c*x**2)**(3/2)/(2*x), Eq(b, 0)), (Integral((c*x**2)**(3/2)
/(x**2*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(3/2)/(x**2*(a +
b*x)), x), Eq(n, -1)), (-a**2*(c*x**2)**(3/2)*(a + b*x)**n/(b**2*n**2*x**3
+ 3*b**2*n*x**3 + 2*b**2*x**3) + a*b*n*x*(c*x**2)**(3/2)*(a + b*x)**n/(b**2
*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + b**2*n*x**2*(c*x**2)**(3/2)*(a
+ b*x)**n/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + b**2*x**2*(c*x**
2)**(3/2)*(a + b*x)**n/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3), True
))
```

Giac [A]
time = 0.53, size = 119, normalized size = 1.83

$$\left(\frac{a^2a^n\mathrm{sgn}(x)}{b^2n^2 + 3\,b^2n + 2\,b^2} + \frac{(bx+a)^n b^2nx^2\mathrm{sgn}(x) + (bx+a)^n abnx\mathrm{sgn}(x) + (bx+a)^n b^2x^2\mathrm{sgn}(x) - (bx+a)^n a^2\mathrm{sgn}(x)}{b^2n^2 + 3\,b^2n + 2\,b^2}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*c^(3/2)
```

Mupad [B]
time = 0.23, size = 88, normalized size = 1.35

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c\,\sqrt{c\,x^2}}{b^2\,(n^2+3\,n+2)}+\frac{a\,c\,n\,x\,\sqrt{c\,x^2}}{b\,(n^2+3\,n+2)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c*(c*x^2
)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2)
)))/x
```

3.934 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3} dx$

Optimal. Leaf size=31

$$\frac{c\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}$$

[Out] c*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]

[Out] (c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.97

$$\frac{(cx^2)^{3/2}(a+bx)^{1+n}}{b(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n))/(b*(1 + n)*x^3)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.94

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(c\,x^{2}\right)^{\frac{3}{2}}}{b(1+n)x^{3}}$	29
risch	$\frac{c\sqrt{c\,x^{2}}\,(bx+a)(bx+a)^{n}}{xb(1+n)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*(c*x^2)^(3/2)/x^3
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.90

$$\frac{\left(bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}\right)(bx+a)^{n}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] (b*c^(3/2)*x + a*c^(3/2))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A]
time = 0.51, size = 33, normalized size = 1.06

$$\frac{(bcx+ac)\sqrt{cx^{2}}\,(bx+a)^{n}}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{(cx^2)^{\frac{3}{2}}}{ax^2} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n (cx^2)^{\frac{3}{2}}}{x^2} & \text{for } b = 0 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^3(a+bx)}\, dx & \text{for } n = -1 \\ \frac{a(cx^2)^{\frac{3}{2}}(a+bx)^n}{bnx^3+bx^3} + \frac{bx(cx^2)^{\frac{3}{2}}(a+bx)^n}{bnx^3+bx^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Piecewise((((c*x**2)**(3/2)/(a*x**2), Eq(b, 0) & Eq(n, -1)), (a**n*(c*x**2)*
*(3/2)/x**2, Eq(b, 0)), (Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x), Eq(
n, -1)), (a*(c*x**2)**(3/2)*(a + b*x)**n/(b*n*x**3 + b*x**3) + b*x*(c*x**2)
**(3/2)*(a + b*x)**n/(b*n*x**3 + b*x**3), True))
```

Giac [A]
time = 0.82, size = 42, normalized size = 1.35

$$-c^{\frac{3}{2}}\left(\frac{a^{n+1}\mathrm{sgn}(x)}{bn+b} - \frac{(bx+a)^{n+1}\mathrm{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] -c^(3/2)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

Mupad [B]
time = 0.23, size = 45, normalized size = 1.45

$$\frac{\left(\frac{c\,x\,\sqrt{c\,x^2}}{n+1}+\frac{a\,c\,\sqrt{c\,x^2}}{b\,(n+1)}\right)\,{\left(a+b\,x\right)}^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] (((c*x*(c*x^2)^(1/2))/(n + 1) + (a*c*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b*x)^
n)/x
```

3.935 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^4} dx$

Optimal. Leaf size=48

$$-\frac{c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}$$

```
[Out] -c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a/(1+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x}\,dx}{x}\\ &= -\frac{c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$-\frac{(cx^2)^{3/2}(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x]
[Out] -(((c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x^3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2)^{\frac{3}{2}}\,(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}\,(a+bx)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^4,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^4, x)
```

3.936 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^5} dx$

Optimal. Leaf size=48

$$\frac{bc\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

```
[Out] b*c*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^2/(1+n)
/x
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bc\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (b*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^5}\,dx = \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^2}\,dx}{x}$$
$$= \frac{bc\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$\frac{b(cx^2)^{3/2}(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x]
[Out] (b*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x^3)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2)^{\frac{3}{2}}\,(bx+a)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}\,(a+bx)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^5,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^5, x)
```

3.937 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^6} dx$

Optimal. Leaf size=51

$$-\frac{b^2 c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}$$

```
[Out] -b^2*c*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^3/(1
+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 67}

$$-\frac{b^2 c\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((b^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}\,(a+bx)^n}{x^6}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^3}\,dx}{x}\\ &= -\frac{b^2 c\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.98

$$-\frac{b^2(cx^2)^{3/2}(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((b^2*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1
 + (b*x)/a])/(a^3*(1 + n)*x^3))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}(bx+a)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

```
[Out] int((c*x^2)^(3/2)*(b*x+a)^n/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}(a+bx)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**6,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^n}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^6,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^6, x)
```

3.938 $\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=217

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^6(1+n)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^6(2+n)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^6(3+n)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{4+n}}{b^6(4+n)x}-5ac$$

```
[Out] -a^5*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^6/(1+n)/x+5*a^4*c^2*(b*x+a)^(2+n)*(c
*x^2)^(1/2)/b^6/(2+n)/x-10*a^3*c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^6/(3+n)/x+
10*a^2*c^2*(b*x+a)^(4+n)*(c*x^2)^(1/2)/b^6/(4+n)/x-5*a*c^2*(b*x+a)^(5+n)*(c
*x^2)^(1/2)/b^6/(5+n)/x+c^2*(b*x+a)^(6+n)*(c*x^2)^(1/2)/b^6/(6+n)/x
```

Rubi [A]
time = 0.06, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 45}

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^6(n+5)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+6}}{b^6(n+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] -((a^5*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^6*(1 + n)*x)) + (5*a^4*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^6*(2 + n)*x) - (10*a^3*c^2*Sqrt[c*x^2]*(a +
b*x)^(3 + n))/(b^6*(3 + n)*x) + (10*a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/
(b^6*(4 + n)*x) - (5*a*c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^6*(5 + n)*x) +
 (c^2*Sqrt[c*x^2]*(a + b*x)^(6 + n))/(b^6*(6 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx = \frac{\left(c^2\sqrt{cx^2}\right) \int x^5 (a+bx)^n \, dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int \left(-\frac{a^5(a+bx)^n}{b^5} + \frac{5a^4(a+bx)^{1+n}}{b^5} - \frac{10a^3(a+bx)^{2+n}}{b^5} + \frac{10a^2(a+bx)^{3+n}}{b^5} - \frac{5a(a+bx)^4}{b^5}\right.}{x}$$
$$= -\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^6(1+n)x} + \frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^6(2+n)x} - \frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^6(3+n)x}$$

Mathematica [A]
time = 0.09, size = 172, normalized size = 0.79

$$\frac{c^3x(a+bx)^{1+n}\left(-120a^5+120a^4b(1+n)x-60a^3b^2(2+3n+n^2)x^2+20a^2b^3(6+11n+6n^2+n^3)x^3-5ab^4(24+50n+35n^2+10n^3+n^4)x^4+b^5(120+274n+225n^2+85n^3+15n^4+n^5)x^5\right)}{b^6(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-120*a^5 + 120*a^4*b*(1 + n)*x - 60*a^3*b^2*(2 +
3*n + n^2)*x^2 + 20*a^2*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 - 5*a*b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4 + b^5*(120 + 274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5)*x^5))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*Sqrt[
c*x^2])
```

Maple [A]
time = 0.14, size = 280, normalized size = 1.29

method	result
gosper	$-\frac{(bx+a)^{1+n}\,(c\,x^2)^{\frac{5}{2}}\,(-b^5n^5x^5-15b^5n^4x^5+5a\,b^4n^4x^4-85b^5n^3x^5+50a\,b^4n^3x^4-225b^5n^2x^5-20a^2b^3n^3x^3+175a\,b^4n^2x^4-274b^5n\,x^5}{x^5b^6(n^6+21n^5+1}$
risch	$-\frac{c^2\sqrt{c\,x^2}\,(-b^6n^5x^6-a\,b^5n^5x^5-15b^6n^4x^6-10a\,b^5n^4x^5-85b^6n^3x^6+5a^2b^4n^4x^4-35a\,b^5n^3x^5-225b^6n^2x^6+30a^2b^4n^3x^4-50a\,b^5}{x(5+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b^5*n^5*x^5-15*b^5*n^4*x^5+5*a*b^4*n^4*x^4-8
5*b^5*n^3*x^5+50*a*b^4*n^3*x^4-225*b^5*n^2*x^5-20*a^2*b^3*n^3*x^3+175*a*b^4
*n^2*x^4-274*b^5*n*x^5-120*a^2*b^3*n^2*x^3+250*a*b^4*n*x^4-120*b^5*x^5+60*a
^3*b^2*n^2*x^2-220*a^2*b^3*n*x^3+120*a*b^4*x^4+180*a^3*b^2*n*x^2-120*a^2*b^
3*x^3-120*a^4*b*n*x+120*a^3*b^2*x^2-120*a^4*b*x+120*a^5)/x^5/b^6/(n^6+21*n^
5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

Maxima [A]
time = 0.30, size = 203, normalized size = 0.94

$$\frac{\left((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6c^{\frac{5}{2}}x^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5c^{\frac{5}{2}}x^5-5\,(n^4+6\,n^3+11\,n^2+6\,n)a^2b^4c^{\frac{5}{2}}x^4+20\,(n^3+3\,n^2+2\,n)a^3b^3c^{\frac{5}{2}}x^3-60\,(n^2+n)a^4b^2c^{\frac{5}{2}}x^2+120\,a^5bc^{\frac{5}{2}}nx-120\,a^6c^{\frac{5}{2}}\right)(bx+a)^n}{(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*c^(5/2)*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*c^(5/2)*x^5 - 5*(n^4 + 6*n^3 + 11*n^2
 + 6*n)*a^2*b^4*c^(5/2)*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*c^(5/2)*x^3 -
60*(n^2 + n)*a^4*b^2*c^(5/2)*x^2 + 120*a^5*b*c^(5/2)*n*x - 120*a^6*c^(5/2))
*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6)
```

Fricas [A]
time = 0.73, size = 352, normalized size = 1.62

$$\frac{\left(120\,a^5bc^2nx-120\,a^6c^2+(b^6c^2n^5+15\,b^6c^2n^4+85\,b^6c^2n^3+225\,b^6c^2n^2+274\,b^6c^2n+120\,b^6c^2)x^6+(ab^5c^2n^5+10\,ab^5c^2n^4+35\,ab^5c^2n^3+50\,ab^5c^2n^2+24\,ab^5c^2n)x^5-5\,(a^2b^4c^2n^4+6\,a^2b^4c^2n^3+11\,a^2b^4c^2n^2+6\,a^2b^4c^2n)x^4+20\,(a^3b^3c^2n^3+3\,a^3b^3c^2n^2+2\,a^3b^3c^2n)x^3-60\,(a^4b^2c^2n^2+a^4b^2c^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^6n^6+21\,b^6n^5+175\,b^6n^4+735\,b^6n^3+1624\,b^6n^2+1764\,b^6n+720\,b^6)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (120*a^5*b*c^2*n*x - 120*a^6*c^2 + (b^6*c^2*n^5 + 15*b^6*c^2*n^4 + 85*b^6*c
^2*n^3 + 225*b^6*c^2*n^2 + 274*b^6*c^2*n + 120*b^6*c^2)*x^6 + (a*b^5*c^2*n^
5 + 10*a*b^5*c^2*n^4 + 35*a*b^5*c^2*n^3 + 50*a*b^5*c^2*n^2 + 24*a*b^5*c^2*n
)*x^5 - 5*(a^2*b^4*c^2*n^4 + 6*a^2*b^4*c^2*n^3 + 11*a^2*b^4*c^2*n^2 + 6*a^2
*b^4*c^2*n)*x^4 + 20*(a^3*b^3*c^2*n^3 + 3*a^3*b^3*c^2*n^2 + 2*a^3*b^3*c^2*n
)*x^3 - 60*(a^4*b^2*c^2*n^2 + a^4*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/(
(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6
*n + 720*b^6)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{5}{2}}(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(205) = 410.
time = 0.92, size = 640, normalized size = 2.95

$$\left(\frac{120\,a^6a^nc^2\mathrm{sgn}(x)}{b^6n^6+21\,b^6n^5+175\,b^6n^4+735\,b^6n^3+1624\,b^6n^2+1764\,b^6n+720\,b^6}+\frac{(bx+a)^nb^6c^2n^5x^6\mathrm{sgn}(x)+(bx+a)^nab^5c^2n^5x^5\mathrm{sgn}(x)+15\,(bx+a)^nb^6c^2n^4x^6\mathrm{sgn}(x)+10\,(bx+a)^nab^5c^2n^4x^5\mathrm{sgn}(x)+85\,(bx+a)^nb^6c^2n^3x^6\mathrm{sgn}(x)-5\,(bx+a)^na^2b^4c^2n^4x^4\mathrm{sgn}(x)+35\,(bx+a)^nab^5c^2n^3x^5\mathrm{sgn}(x)+225\,(bx+a)^nb^6c^2n^2x^6\mathrm{sgn}(x)-30\,(bx+a)^na^2b^4c^2n^3x^4\mathrm{sgn}(x)+50\,(bx+a)^nab^5c^2n^2x^5\mathrm{sgn}(x)+274\,(bx+a)^nb^6c^2nx^6\mathrm{sgn}(x)+20\,(bx+a)^na^3b^3c^2n^3x^3\mathrm{sgn}(x)-55\,(bx+a)^na^2b^4c^2n^2x^4\mathrm{sgn}(x)+24\,(bx+a)^nab^5c^2nx^5\mathrm{sgn}(x)+120\,(bx+a)^nb^6c^2x^6\mathrm{sgn}(x)+60\,(bx+a)^na^3b^3c^2n^2x^3\mathrm{sgn}(x)-30\,(bx+a)^na^2b^4c^2nx^4\mathrm{sgn}(x)-60\,(bx+a)^na^4b^2c^2n^2x^2\mathrm{sgn}(x)+40\,(bx+a)^na^3b^3c^2nx^3\mathrm{sgn}(x)-60\,(bx+a)^na^4b^2c^2nx^2\mathrm{sgn}(x)+120\,(bx+a)^na^5bc^2nx\mathrm{sgn}(x)-120\,(bx+a)^na^6c^2\mathrm{sgn}(x)}{b^6n^6+21\,b^6n^5+175\,b^6n^4+735\,b^6n^3+1624\,b^6n^2+1764\,b^6n+720\,b^6}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (120*a^6*a^n*c^2*sgn(x)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 +
 1624*b^6*n^2 + 1764*b^6*n + 720*b^6) + ((b*x + a)^n*b^6*c^2*n^5*x^6*sgn(x)
 + (b*x + a)^n*a*b^5*c^2*n^5*x^5*sgn(x) + 15*(b*x + a)^n*b^6*c^2*n^4*x^6*sg
n(x) + 10*(b*x + a)^n*a*b^5*c^2*n^4*x^5*sgn(x) + 85*(b*x + a)^n*b^6*c^2*n^3
*x^6*sgn(x) - 5*(b*x + a)^n*a^2*b^4*c^2*n^4*x^4*sgn(x) + 35*(b*x + a)^n*a*b
^5*c^2*n^3*x^5*sgn(x) + 225*(b*x + a)^n*b^6*c^2*n^2*x^6*sgn(x) - 30*(b*x +
a)^n*a^2*b^4*c^2*n^3*x^4*sgn(x) + 50*(b*x + a)^n*a*b^5*c^2*n^2*x^5*sgn(x) +
 274*(b*x + a)^n*b^6*c^2*n*x^6*sgn(x) + 20*(b*x + a)^n*a^3*b^3*c^2*n^3*x^3*
sgn(x) - 55*(b*x + a)^n*a^2*b^4*c^2*n^2*x^4*sgn(x) + 24*(b*x + a)^n*a*b^5*c
^2*n*x^5*sgn(x) + 120*(b*x + a)^n*b^6*c^2*x^6*sgn(x) + 60*(b*x + a)^n*a^3*b
^3*c^2*n^2*x^3*sgn(x) - 30*(b*x + a)^n*a^2*b^4*c^2*n*x^4*sgn(x) - 60*(b*x +
 a)^n*a^4*b^2*c^2*n^2*x^2*sgn(x) + 40*(b*x + a)^n*a^3*b^3*c^2*n*x^3*sgn(x)
- 60*(b*x + a)^n*a^4*b^2*c^2*n*x^2*sgn(x) + 120*(b*x + a)^n*a^5*b*c^2*n*x*s
gn(x) - 120*(b*x + a)^n*a^6*c^2*sgn(x))/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6))*sqrt(c)
```

Mupad [B]
time = 0.50, size = 424, normalized size = 1.95

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^6\,\sqrt{c\,x^2}\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}-\frac{120\,a^6\,c^2\,\sqrt{c\,x^2}}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{120\,a^5\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{5\,a^2\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{60\,a^4\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,c^2\,n\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{20\,a^3\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^6*(c*x^2)^(1/2)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n
^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (
120*a^6*c^2*(c*x^2)^(1/2))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21
*n^5 + n^6 + 720)) + (120*a^5*c^2*n*x*(c*x^2)^(1/2))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (5*a^2*c^2*n*x^4*(c*x^2)^(1/
2)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
21*n^5 + n^6 + 720)) - (60*a^4*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^4*(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*c^2*n*x^5*(c*x
^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (20*a^3*c^2*n*x^3*(c*x^2)^(1/2)*(3*n
 + n^2 + 2))/(b^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 7
20))))/x
```

3.939 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=179

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{4+n}}{b^5(4+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{5+n}}{b^5(5+n)x}$$

```
[Out] a^4*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^5/(1+n)/x-4*a^3*c^2*(b*x+a)^(2+n)*(c*
x^2)^(1/2)/b^5/(2+n)/x+6*a^2*c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^5/(3+n)/x-4*
a*c^2*(b*x+a)^(4+n)*(c*x^2)^(1/2)/b^5/(4+n)/x+c^2*(b*x+a)^(5+n)*(c*x^2)^(1/
2)/b^5/(5+n)/x
```

Rubi [A]
time = 0.04, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c^2*Sqrt[c
*x^2]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c^2*Sqrt[c*x^2]*(a + b*x)
^(3 + n))/(b^5*(3 + n)*x) - (4*a*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4
 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int\left(\frac{a^4(a+bx)^n}{b^4} - \frac{4a^3(a+bx)^{1+n}}{b^4} + \frac{6a^2(a+bx)^{2+n}}{b^4} - \frac{4a(a+bx)^{3+n}}{b^4} + \frac{(a+bx)^{4+n}}{b^4}\right) d}{x}$$
$$= \frac{a^4c^2\sqrt{cx^2}(a+bx)^{1+n}}{b^5(1+n)x} - \frac{4a^3c^2\sqrt{cx^2}(a+bx)^{2+n}}{b^5(2+n)x} + \frac{6a^2c^2\sqrt{cx^2}(a+bx)^{3+n}}{b^5(3+n)x} - 4$$

Mathematica [A]
time = 0.02, size = 133, normalized size = 0.74

$$\frac{c(cx^2)^{3/2}(a+bx)^{1+n}\left(24a^4-24a^3b(1+n)x+12a^2b^2(2+3n+n^2)x^2-4ab^3(6+11n+6n^2+n^3)x^3+b^4(24+50n+35n^2+10n^3+n^4)x^4\right)}{b^5(1+n)(2+n)(3+n)(4+n)(5+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^
2*(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50
*n + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*x^3)
```

Maple [A]
time = 0.14, size = 199, normalized size = 1.11

method	result
gosper	$\frac{(bx+a)^{1+n}\left(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44x^3an\,b^3+24b^4x^4+36a^2n\,x^2b^2-24a\,b\right.}{x^5b^5(n^5+15n^4+85n^3+225n^2+274n+120)}$
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(b^5n^4x^5+a\,b^4n^4x^4+10b^5n^3x^5+6a\,b^4n^3x^4+35b^5n^2x^5-4a^2b^3n^3x^3+11a\,b^4n^2x^4+50b^5n\,x^5-12a^2b^3n^2x^3+6x^4an\,b^4+24b\right.}{x(4+n)(5+n)(3+n)(2+n)(1+n)b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x,x,method=_RETURNVERBOSE)
[Out] (b*x+a)^(1+n)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(5/2)/x^5/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A]
time = 0.30, size = 157, normalized size = 0.88

$$\frac{\left((n^4+10n^3+35n^2+50n+24)b^5c^{\frac{5}{2}}x^5+(n^4+6n^3+11n^2+6n)ab^4c^{\frac{5}{2}}x^4-4(n^3+3n^2+2n)a^2b^3c^{\frac{5}{2}}x^3+12(n^2+n)a^3b^2c^{\frac{5}{2}}x^2-24a^4bc^{\frac{5}{2}}nx+24a^5c^{\frac{5}{2}}\right)(bx+a)^n}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(5/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(5/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(5/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(5/2)*x^2 - 24*a^4*b*c^(5/2)*n*x + 24*a^5*c^(5/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

Fricas [A]
time = 0.57, size = 265, normalized size = 1.48

$$-\frac{\left(24\,a^4bc^2nx-24\,a^5c^2-\left(b^5c^2n^4+10\,b^5c^2n^3+35\,b^5c^2n^2+50\,b^5c^2n+24\,b^5c^2\right)x^5-\left(ab^4c^2n^4+6\,ab^4c^2n^3+11\,ab^4c^2n^2+6\,ab^4c^2n\right)x^4+4\,\left(a^2b^3c^2n^3+3\,a^2b^3c^2n^2+2\,a^2b^3c^2n\right)x^3-12\,\left(a^3b^2c^2n^2+a^3b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c^2*n*x - 24*a^5*c^2 - (b^5*c^2*n^4 + 10*b^5*c^2*n^3 + 35*b^5*c^
2*n^2 + 50*b^5*c^2*n + 24*b^5*c^2)*x^5 - (a*b^4*c^2*n^4 + 6*a*b^4*c^2*n^3 +
 11*a*b^4*c^2*n^2 + 6*a*b^4*c^2*n)*x^4 + 4*(a^2*b^3*c^2*n^3 + 3*a^2*b^3*c^2
*n^2 + 2*a^2*b^3*c^2*n)*x^3 - 12*(a^3*b^2*c^2*n^2 + a^3*b^2*c^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 27
4*b^5*n + 120*b^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}\left(a+bx\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x, x)
```

Mupad [B]
time = 0.38, size = 319, normalized size = 1.78

$$\frac{{\left(a+b\,x\right)}^n\,\left(\frac{c^2\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{24\,a^5\,c^2\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{24\,a^4\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(
274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (24*a^5*c^2*(c*x^2)^(1/2))
/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (24*a^4*c^2*n*x*(c
*x^2)^(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c^2
*n*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3
 + 15*n^4 + n^5 + 120)) + (12*a^3*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c^2*n*x^3*(c*x^2)^(1
/2)*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
)/x
```

3.940 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2} dx$

Optimal. Leaf size=143

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}$$

```
[Out] -a^3*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^4/(1+n)/x+3*a^2*c^2*(b*x+a)^(2+n)*(c
*x^2)^(1/2)/b^4/(2+n)/x-3*a*c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^4/(3+n)/x+c^2
*(b*x+a)^(4+n)*(c*x^2)^(1/2)/b^4/(4+n)/x
```

Rubi [A]
time = 0.03, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c^2*Sqrt[c*x^2]*(a + b*x
)^(3 + n))/(b^4*(3 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}$$
$$= -\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+$$

Mathematica [A]
time = 0.01, size = 99, normalized size = 0.69

$$\frac{c(cx^2)^{3/2}(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2(2+3n+n^2)x^2+b^3(6+11n+6n^2+n^3)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2
 + 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)
*(3 + n)*(4 + n)*x^3)
```

Maple [A]
time = 0.14, size = 136, normalized size = 0.95

method	result
gosper	$-\frac{(bx+a)^{1+n}(cx^2)^{\frac{5}{2}}(-b^3n^3x^3-6b^3n^2x^3+3ab^2n^2x^2-11b^3nx^3+9ab^2nx^2-6b^3x^3-6a^2bnx+6ab^2x^2-6a^2bx+6a^3)}{x^5b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{c^2\sqrt{cx^2}\,(-b^4n^3x^4-ab^3n^3x^3-6b^4n^2x^4-3ab^3n^2x^3-11b^4nx^4+3a^2b^2n^2x^2-2x^3anb^3-6b^4x^4+3a^2nx^2b^2-6a^3bnx+6a^4)(bx+}{x(3+n)(4+n)(2+n)(1+n)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^5/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.81

$$\frac{\left((n^3+6n^2+11n+6)b^4c^{\frac{5}{2}}x^4+(n^3+3n^2+2n)ab^3c^{\frac{5}{2}}x^3-3(n^2+n)a^2b^2c^{\frac{5}{2}}x^2+6a^3bc^{\frac{5}{2}}nx-6a^4c^{\frac{5}{2}}\right)(bx+a)^n}{(n^4+10n^3+35n^2+50n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(5/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(5/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(5/2)*x^2 + 6*a^3*b*c^(5/2)*n*x - 6*a^4*c^(5
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [A]
time = 0.74, size = 186, normalized size = 1.30

$$\frac{\left(6\,a^3bc^2nx-6\,a^4c^2+\left(b^4c^2n^3+6\,b^4c^2n^2+11\,b^4c^2n+6\,b^4c^2\right)x^4+\left(ab^3c^2n^3+3\,ab^3c^2n^2+2\,ab^3c^2n\right)x^3-3\left(a^2b^2c^2n^2+a^2b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c^2*n*x - 6*a^4*c^2 + (b^4*c^2*n^3 + 6*b^4*c^2*n^2 + 11*b^4*c^2*n
+ 6*b^4*c^2)*x^4 + (a*b^3*c^2*n^3 + 3*a*b^3*c^2*n^2 + 2*a*b^3*c^2*n)*x^3 -
3*(a^2*b^2*c^2*n^2 + a^2*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4
+ 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^2, x)
```

Mupad [B]
time = 0.32, size = 229, normalized size = 1.60

$$\frac{(a+b\,x)^n\left(\frac{c^2\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c^2\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n
^2 + 10*n^3 + n^4 + 24) - (6*a^4*c^2*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (6*a^3*c^2*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24)) + (a*c^2*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n +
35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/x
```

3.941 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=105

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

```
[Out] a^2*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^3/(1+n)/x-2*a*c^2*(b*x+a)^(2+n)*(c*x^
2)^(1/2)/b^3/(2+n)/x+c^2*(b*x+a)^(3+n)*(c*x^2)^(1/2)/b^3/(3+n)/x
```

Rubi [A]
time = 0.02, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c^2*Sqrt[c*x
^2]*(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(3 + n)
)/(b^3*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}$$
$$= \frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x} - \frac{2ac^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x} + \frac{c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.67

$$\frac{c^3x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 83, normalized size = 0.79

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{x^5b^3\left(n^3+6n^2+11n+6\right)}$	83
risch	$\frac{c^2\sqrt{c\,x^2}\,\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{x(2+n)(3+n)(1+n)b^3}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(5/2)/x^5/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 80, normalized size = 0.76

$$\frac{\left((n^2+3\,n+2)b^3c^{\frac{5}{2}}x^3+(n^2+n)ab^2c^{\frac{5}{2}}x^2-2\,a^2bc^{\frac{5}{2}}nx+2\,a^3c^{\frac{5}{2}}\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(5/2)*x^3 + (n^2 + n)*a*b^2*c^(5/2)*x^2 - 2*a^2*b*c^
(5/2)*n*x + 2*a^3*c^(5/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

Fricas [A]
time = 0.66, size = 127, normalized size = 1.21

$$-\frac{(2\,a^2bc^2nx-2\,a^3c^2-(b^3c^2n^2+3\,b^3c^2n+2\,b^3c^2)x^3-(ab^2c^2n^2+ab^2c^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c^2*n*x - 2*a^3*c^2 - (b^3*c^2*n^2 + 3*b^3*c^2*n + 2*b^3*c^2)*x^3
 - (a*b^2*c^2*n^2 + a*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6
*b^3*n^2 + 11*b^3*n + 6*b^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx^2)^{\frac{5}{2}}\,(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^3, x)
```

Mupad [B]
time = 0.27, size = 154, normalized size = 1.47

$$\frac{(a+b\,x)^n\left(\frac{2\,a^3\,c^2\,\sqrt{c\,x^2}}{b^3\,(n^3+6\,n^2+11\,n+6)}+\frac{c^2\,x^3\,\sqrt{c\,x^2}\,\,(n^2+3\,n+2)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,(n^3+6\,n^2+11\,n+6)}+\frac{a\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\,(n+1)}{b\,(n^3+6\,n^2+11\,n+6)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*c^2*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (c
^2*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*c^2
*n*x*(c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c^2*n*x^2*(c*x^2)^(
1/2)*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

3.942 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^4} dx$

Optimal. Leaf size=69

$$-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

```
[Out] -a*c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b^2/(1+n)/x+c^2*(b*x+a)^(2+n)*(c*x^2)^(1
/2)/b^2/(2+n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(cx^2)^{5/2} (a+bx)^n}{x^4} dx = \frac{\left(c^2\sqrt{cx^2}\right) \int x(a+bx)^n \, dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int \left(-\frac{a(a+bx)^n}{b} + \frac{(a+bx)^{1+n}}{b}\right) dx}{x}$$
$$= -\frac{ac^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x} + \frac{c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.67

$$\frac{c^3x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Maple [A]
time = 0.16, size = 46, normalized size = 0.67

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(c\,x^2)^{\frac{5}{2}}(-bnx-bx+a)}{x^5b^2(n^2+3n+2)}$	46
risch	$-\frac{c^2\sqrt{c\,x^2}\,(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{x\,b^2(2+n)(1+n)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(c*x^2)^(5/2)*(-b*n*x-b*x+a)/x^5/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.74

$$\frac{\left(b^2c^{\frac{5}{2}}(n+1)x^2 + abc^{\frac{5}{2}}nx - a^2c^{\frac{5}{2}}\right)(bx+a)^n}{(n^2+3\,n+2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*c^(5/2)*(n + 1)*x^2 + a*b*c^(5/2)*n*x - a^2*c^(5/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

Fricas [A]
time = 0.72, size = 76, normalized size = 1.10

$$\frac{(abc^2nx - a^2c^2 + (b^2c^2n + b^2c^2)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^2n^2 + 3\,b^2n + 2\,b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
```

```
[Out] (a*b*c^2*n*x - a^2*c^2 + (b^2*c^2*n + b^2*c^2)*x^2)*sqrt(c*x^2)*(b*x + a)^n
/((b^2*n^2 + 3*b^2*n + 2*b^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n (cx^2)^{\frac{5}{2}}}{2x^3} & \text{for}\ b = 0 \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^4(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^4(a+bx)}\,dx & \text{for}\ n = -1 \\ -\frac{a^2(cx^2)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5} + \frac{abnx(cx^2)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5} + \frac{b^2nx^2(cx^2)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5} + \frac{b^2x^2(cx^2)^{\frac{5}{2}}(a+bx)^n}{b^2n^2x^5+3b^2nx^5+2b^2x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Piecewise((a**n*(c*x**2)**(5/2)/(2*x**3), Eq(b, 0)), (Integral((c*x**2)**(5
/2)/(x**4*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(5/2)/(x**4*(a
 + b*x)), x), Eq(n, -1)), (-a**2*(c*x**2)**(5/2)*(a + b*x)**n/(b**2*n**2*x*
*5 + 3*b**2*n*x**5 + 2*b**2*x**5) + a*b*n*x*(c*x**2)**(5/2)*(a + b*x)**n/(b
**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + b**2*n*x**2*(c*x**2)**(5/2)*
(a + b*x)**n/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + b**2*x**2*(c*
x**2)**(5/2)*(a + b*x)**n/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5), T
rue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^4, x)
```

Mupad [B]
time = 0.24, size = 94, normalized size = 1.36

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c^2\,\sqrt{c\,x^2}}{b^2\,(n^2+3\,n+2)}+\frac{a\,c^2\,n\,x\,\sqrt{c\,x^2}}{b\,(n^2+3\,n+2)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c^2*(c
*x^2)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c^2*n*x*(c*x^2)^(1/2))/(b*(3*n + n^
2 + 2))))/x
```

3.943 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5} dx$

Optimal. Leaf size=33

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}$$

```
[Out] c^2*(b*x+a)^(1+n)*(c*x^2)^(1/2)/b/(1+n)/x
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.94

$$\frac{c^3x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.88

method	result	size
gosper	$\frac{(bx+a)^{1+n}(c\,x^{2})^{\frac{5}{2}}}{b(1+n)x^{5}}$	29
risch	$\frac{c^{2}\sqrt{c\,x^{2}}\,(bx+a)(bx+a)^{n}}{xb(1+n)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*(c*x^2)^(5/2)/x^5
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.85

$$\frac{\left(bc^{\frac{5}{2}}x+ac^{\frac{5}{2}}\right)(bx+a)^{n}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
```

```
[Out] (b*c^(5/2)*x + a*c^(5/2))*(b*x + a)^n/(b*(n + 1))
```

Fricas [A]
time = 0.76, size = 37, normalized size = 1.12

$$\frac{(bc^{2}x+ac^{2})\sqrt{cx^{2}}\,(bx+a)^{n}}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
```

```
[Out] (b*c^2*x + a*c^2)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{(cx^2)^{\frac{5}{2}}}{ax^4} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n (cx^2)^{\frac{5}{2}}}{x^4} & \text{for } b = 0 \\ \int \frac{(cx^2)^{\frac{5}{2}}}{x^5(a+bx)}\, dx & \text{for } n = -1 \\ \frac{a(cx^2)^{\frac{5}{2}}(a+bx)^n}{bnx^5+bx^5} + \frac{bx(cx^2)^{\frac{5}{2}}(a+bx)^n}{bnx^5+bx^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Piecewise((((c*x**2)**(5/2)/(a*x**4), Eq(b, 0) & Eq(n, -1)), (a**n*(c*x**2)*
*(5/2)/x**4, Eq(b, 0)), (Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x), Eq(
n, -1)), (a*(c*x**2)**(5/2)*(a + b*x)**n/(b*n*x**5 + b*x**5) + b*x*(c*x**2)
**(5/2)*(a + b*x)**n/(b*n*x**5 + b*x**5), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^5, x)
```

Mupad [B]
time = 0.23, size = 49, normalized size = 1.48

$$\frac{\left(\frac{c^2\,x\,\sqrt{c\,x^2}}{n+1}+\frac{a\,c^2\,\sqrt{c\,x^2}}{b\,(n+1)}\right)\,{(a+b\,x)}^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x)
```

```
[Out] (((c^2*x*(c*x^2)^(1/2))/(n + 1) + (a*c^2*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b
*x)^n)/x
```

3.944 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^6}\,dx$

Optimal. Leaf size=50

$$-\frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}$$

```
[Out] -c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a/(1+n)/
x
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]
```

```
[Out] -((c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a*(1 + n)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^6}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x}\,dx}{x}\\ &= -\frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.94

$$-\frac{(cx^2)^{5/2}(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x]
[Out] -(((c*x^2)^(5/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (
b*x)/a])/(a*(1 + n)*x^5))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^2)^{\frac{5}{2}}(bx+a)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
[Out] int((c*x^2)^(5/2)*(b*x+a)^n/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="maxima")
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c^2/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}(a+bx)^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^n}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^6,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^6, x)
```

3.945 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^7} dx$

Optimal. Leaf size=50

$$\frac{bc^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}$$

```
[Out] b*c^2*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)*(c*x^2)^(1/2)/a^2/(1+
n)/x
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bc^2\sqrt{cx^2}\,(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]
```

```
[Out] (b*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a^2*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}\,(a+bx)^n}{x^7}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^n}{x^2}\,dx}{x}\\ &= \frac{bc^2\sqrt{cx^2}\,(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.94

$$\frac{b(cx^2)^{5/2}(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x]
[Out] (b*(c*x^2)^(5/2)*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (
b*x)/a])/(a^2*(1 + n)*x^5)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(bx+a\right)^n}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
[Out] int((c*x^2)^(5/2)*(b*x+a)^n/x^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="maxima")
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^7, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*c^2/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}\left(a+bx\right)^n}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**7,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^n}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^7,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^7, x)
```

3.946 $\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}$$

```
[Out] -a^3*x*(b*x+a)^(1+n)/b^4/(1+n)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+n)/b^4/(2+n
)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+n)/b^4/(3+n)/(c*x^2)^(1/2)+x*(b*x+a)^(4+n)
/b^4/(4+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*x)
^(2 + n))/(b^4*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*(3 + n
)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 0.78

$$\frac{x(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2(2+3n+n^2)\,x^2+b^3(6+11n+6n^2+n^3)\,x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 134, normalized size = 1.09

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{\sqrt{c\,x^2}\;b^4(n^4+10n^3+35n^2+50n+24)}$	134
risch	$-\frac{x\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{\sqrt{c\,x^2}\;(3+n)(4+n)(2+n)(1+n)b^4}$	154

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9
*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(1/
2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.85

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*sqrt(c))
```

Fricas [A]
time = 1.29, size = 158, normalized size = 1.28

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\,\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4cn^4+10\,b^4cn^3+35\,b^4cn^2+50\,b^4cn+24\,b^4c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c*n^4 + 10*b^4*c*n^3 + 35*b^4*c*n^2 + 50*b^4*c*n
 + 24*b^4*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^5}{4\sqrt{cx^2}} & \text{for}\: b = 0 \\ \int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^4}\,dx & \text{for}\: n = -4 \\ \int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^3}\,dx & \text{for}\: n = -3 \\ \int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^2}\,dx & \text{for}\: n = -2 \\ \int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for}\: n = -1 \\ -\frac{6a^4x(a+bx)^n}{b^4n^4\sqrt{cx^2}+10b^4n^3\sqrt{cx^2}+35b^4n^2\sqrt{cx^2}+50b^4n\sqrt{cx^2}+24b^4\sqrt{cx^2}} + \frac{6a^3bnx^2(a+bx)^n}{b^4n^4\sqrt{cx^2}+10b^4n^3\sqrt{cx^2}+35b^4n^2\sqrt{cx^2}+50b^4n\sqrt{cx^2}+24b^4\sqrt{cx^2}} - \frac{3a^2b^2n^2x^3(a+bx)^n}{b^4n^4\sqrt{cx^2}+10b^4n^3\sqrt{cx^2}+35b^4n^2\sqrt{cx^2}+50b^4n\sqrt{cx^2}+24b^4\sqrt{cx^2}} - \frac{3a^2b^2nx^3(a+bx)^n}{b^4n^4\sqrt{cx^2}+10b^4n^3\sqrt{cx^2}+35b^4n^2\sqrt{cx^2}+50b^4n\sqrt{cx^2}+24b^4\sqrt{cx^2}} + \frac{ab^3n^3x^4(a+bx)^n}{b^4n^4\sqrt{cx^2}+10b^4n^3\sqrt{cx^2}+35b^4n^2\sqrt{cx^2}+50b^4n\sqrt{cx^2}+24b^4\sqrt{cx^2}} + \frac{3ab^3n^2x^4(a+bx)^n}{b^4n^4\sqrt{cx^2}+10b^4n^3\sqrt{cx^2}+35b^4n^2\sqrt{cx^2}+50b^4n\sqrt{cx^2}+24b^4\sqrt{cx^2}} + \cdots & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**5/(4*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**4/(sqrt(c*x*
*2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**
3), x), Eq(n, -3)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x), Eq(n, -
2)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1)), (-6*a**4*x*(a
 + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n*
*2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 6*a**3*b
*n*x**2*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) +
35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2))
- 3*a**2*b**2*n**2*x**3*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3
*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**
4*sqrt(c*x**2)) - 3*a**2*b**2*n*x**3*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) +
 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x
**2) + 24*b**4*sqrt(c*x**2)) + a*b**3*n**3*x**4*(a + b*x)**n/(b**4*n**4*sqr
t(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4
*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 3*a*b**3*n**2*x**4*(a + b*x)**n/(
b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x*
```

```
*2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 2*a*b**3*n*x**4*(a +
 b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*b**4*n**2
*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + b**4*n**3*
x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*x**2) + 35*
b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c*x**2)) + 6
*b**4*n**2*x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**3*sqrt(c*
x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b**4*sqrt(c
*x**2)) + 11*b**4*n*x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b**4*n**
3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) + 24*b*
*4*sqrt(c*x**2)) + 6*b**4*x**5*(a + b*x)**n/(b**4*n**4*sqrt(c*x**2) + 10*b*
*4*n**3*sqrt(c*x**2) + 35*b**4*n**2*sqrt(c*x**2) + 50*b**4*n*sqrt(c*x**2) +
 24*b**4*sqrt(c*x**2)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.37, size = 186, normalized size = 1.51

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,x}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
+ 24) - (6*a^4*x)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x^2)
/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 + 2))/(b*(
50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

3.947 $\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}} dx$

Optimal. Leaf size=90

$$\frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}$$

```
[Out] a^2*x*(b*x+a)^(1+n)/b^3/(1+n)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+n)/b^3/(2+n)/(
c*x^2)^(1/2)+x*(b*x+a)^(3+n)/b^3/(3+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}} - \frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}} + \frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2 +
 n))/(b^3*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*(3 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 0.74

$$\frac{x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2\right)}{b^3(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 81, normalized size = 0.90

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)x}{\sqrt{c\,x^2}\;b^3\left(n^3+6n^2+11n+6\right)}$	81
risch	$\frac{x\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{\sqrt{c\,x^2}\;(2+n)(3+n)(1+n)b^3}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
/(c*x^2)^(1/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.29, size = 83, normalized size = 0.92

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c)
```

Fricas [A]
time = 0.65, size = 110, normalized size = 1.22

$$-\frac{\left(2\,a^{2}bnx-\left(b^{3}n^{2}+3\,b^{3}n+2\,b^{3}\right)x^{3}-2\,a^{3}-\left(ab^{2}n^{2}+ab^{2}n\right)x^{2}\right)\sqrt{cx^{2}}\,(bx+a)^{n}}{\left(b^{3}cn^{3}+6\,b^{3}cn^{2}+11\,b^{3}cn+6\,b^{3}c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c*n^3 + 6*b^3*c*n^2 + 11*b^3*c*n +
6*b^3*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{n} x^{4}}{3 \sqrt{c x^{2}}} & \text{for}\: b = 0 \\ \int \frac{x^{3}}{\sqrt{c x^{2}} \left(a + b x\right)^{3}}\, dx & \text{for}\: n = -3 \\ \int \frac{x^{3}}{\sqrt{c x^{2}} \left(a + b x\right)^{2}}\, dx & \text{for}\: n = -2 \\ \int \frac{x^{3}}{\sqrt{c x^{2}} \left(a + b x\right)}\, dx & \text{for}\: n = -1 \\ \frac{2 a^{3} x \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} - \frac{2 a^{2} b n x^{2} \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} + \frac{a b^{2} n^{2} x^{3} \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} + \frac{a b^{2} n x^{3} \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} + \frac{b^{3} n^{2} x^{4} \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} + \frac{3 b^{3} n x^{4} \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} + \frac{2 b^{3} x^{4} \left(a + b x\right)^{n}}{b^{3} n^{3} \sqrt{c x^{2}} + 6 b^{3} n^{2} \sqrt{c x^{2}} + 11 b^{3} n \sqrt{c x^{2}} + 6 b^{3} \sqrt{c x^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**4/(3*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**3/(sqrt(c*x*
*2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)**
2), x), Eq(n, -2)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1))
, (2*a**3*x*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2)
 + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) - 2*a**2*b*n*x**2*(a + b*x
)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*
x**2) + 6*b**3*sqrt(c*x**2)) + a*b**2*n**2*x**3*(a + b*x)**n/(b**3*n**3*sqr
t(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt
(c*x**2)) + a*b**2*n*x**3*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**
2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) + b**3*n**2*
x**4*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b
**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)) + 3*b**3*n*x**4*(a + b*x)**n/(b**
3*n**3*sqrt(c*x**2) + 6*b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6
*b**3*sqrt(c*x**2)) + 2*b**3*x**4*(a + b*x)**n/(b**3*n**3*sqrt(c*x**2) + 6*
b**3*n**2*sqrt(c*x**2) + 11*b**3*n*sqrt(c*x**2) + 6*b**3*sqrt(c*x**2)), Tru
e))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.29, size = 121, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,x}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,\left(n+1\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (2*a^3*x)/(b
^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*(11*n + 6*n^2 + n^3 + 6))
 + (a*n*x^3*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

3.948 $\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}} \, dx$

Optimal. Leaf size=59

$$-\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}$$

```
[Out] -a*x*(b*x+a)^(1+n)/b^2/(1+n)/(c*x^2)^(1/2)+x*(b*x+a)^(2+n)/b^2/(2+n)/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n)
)/(b^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.73

$$\frac{x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.75

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x(-bnx-bx+a)}{\sqrt{c\,x^2}\,b^2(n^2+3n+2)}$	44
risch	$-\frac{x(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{\sqrt{c\,x^2}\,b^2(2+n)(1+n)}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x*(-b*n*x-b*x+a)/(c*x^2)^(1/2)/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.76

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n}{(n^2+3\,n+2)b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*sqrt(c))
```

Fricas [A]
time = 0.92, size = 66, normalized size = 1.12

$$\frac{(abnx + (b^2n + b^2)x^2 - a^2)\sqrt{cx^2}\,(bx + a)^n}{(b^2cn^2 + 3\,b^2cn + 2\,b^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c*n^2 + 3
*b^2*c*n + 2*b^2*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^3}{2\sqrt{cx^2}} & \text{for}\: b = 0 \\ \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\, dx & \text{for}\: n = -2 \\ \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\, dx & \text{for}\: n = -1 \\ -\frac{a^2x(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{abnx^2(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{b^2nx^3(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} + \frac{b^2x^3(a+bx)^n}{b^2n^2\sqrt{cx^2}+3b^2n\sqrt{cx^2}+2b^2\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**3/(2*sqrt(c*x**2)), Eq(b, 0)), (Integral(x**2/(sqrt(c*x*
*2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**2/(sqrt(c*x**2)*(a + b*x)),
 x), Eq(n, -1)), (-a**2*x*(a + b*x)**n/(b**2*n**2*sqrt(c*x**2) + 3*b**2*n*s
qrt(c*x**2) + 2*b**2*sqrt(c*x**2)) + a*b*n*x**2*(a + b*x)**n/(b**2*n**2*sqr
t(c*x**2) + 3*b**2*n*sqrt(c*x**2) + 2*b**2*sqrt(c*x**2)) + b**2*n*x**3*(a +
 b*x)**n/(b**2*n**2*sqrt(c*x**2) + 3*b**2*n*sqrt(c*x**2) + 2*b**2*sqrt(c*x*
*2)) + b**2*x**3*(a + b*x)**n/(b**2*n**2*sqrt(c*x**2) + 3*b**2*n*sqrt(c*x**
2) + 2*b**2*sqrt(c*x**2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.28, size = 71, normalized size = 1.20

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,x}{b^2\,(n^2+3\,n+2)}+\frac{a\,n\,x^2}{b\,(n^2+3\,n+2)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(1/2),x)
[Out] ((a + b*x)^n*((x^3*(n + 1))/(3*n + n^2 + 2) - (a^2*x)/(b^2*(3*n + n^2 + 2))
 + (a*n*x^2)/(b*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

3.949 $\int \frac{x(a+bx)^n}{\sqrt{cx^2}} dx$

Optimal. Leaf size=28

$$\frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

```
[Out] x*(b*x+a)^(1+n)/b/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 1.00

$$\frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 27, normalized size = 0.96

method	result	size
gosper	$\frac{x(bx+a)^{1+n}}{b(1+n)\sqrt{c\,x^2}}$	27
risch	$\frac{(bx+a)x(bx+a)^n}{b(1+n)\sqrt{c\,x^2}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*x+a)^(1+n)/b/(1+n)/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 1.11

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\,\right)(bx+a)^n}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c*(n + 1))
```

Fricas [A]
time = 1.11, size = 33, normalized size = 1.18

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bcn+bc)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c*n + b*c)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{a\sqrt{cx^2}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^2}{\sqrt{cx^2}} & \text{for } b = 0 \\ \int \frac{x}{\sqrt{cx^2}(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ax(a+bx)^n}{bn\sqrt{cx^2}+b\sqrt{cx^2}} + \frac{bx^2(a+bx)^n}{bn\sqrt{cx^2}+b\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((x**2/(a*sqrt(c*x**2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**2/sqrt(c*
x**2), Eq(b, 0)), (Integral(x/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1)), (a*
x*(a + b*x)**n/(b*n*sqrt(c*x**2) + b*sqrt(c*x**2)) + b*x**2*(a + b*x)**n/(b
*n*sqrt(c*x**2) + b*sqrt(c*x**2)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.22, size = 36, normalized size = 1.29

$$\frac{\left(\frac{x^2}{n+1}+\frac{a\,x}{b\,(n+1)}\right)\,{(a+b\,x)}^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((x^2/(n + 1) + (a*x)/(b*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

3.950 $\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}$$

```
[Out] -x*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 67}

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/Sqrt[c*x^2],x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x}\,dx}{\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 45, normalized size = 1.00

$$-\frac{x(a+bx)^{1+n}{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/Sqrt[c*x^2],x]
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
(1 + n)*Sqrt[c*x^2]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/(c*x^2)^(1/2),x)
[Out] int((b*x+a)^n/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/sqrt(c*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/sqrt(c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/sqrt(c*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c*x^2)^(1/2),x)
```

```
[Out] int((a + b*x)^n/(c*x^2)^(1/2), x)
```

3.951 $\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=45

$$\frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\sqrt{cx^2}}$$

```
[Out] b*x*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x*Sqrt[c*x^2]),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.07

$$\frac{bcx^3(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x*Sqrt[c*x^2]),x]
```

```
[Out] (b*c*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^2*(1 + n)*(c*x^2)^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

```
[Out] int((b*x+a)^n/x/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)**n/(x*sqrt(c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(x*(c*x^2)^(1/2)),x)
[Out] int((a + b*x)^n/(x*(c*x^2)^(1/2)), x)
```

3.952 $\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\sqrt{cx^2}}$$

```
[Out] -b^2*x*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/(1+n)/(c*x^2)^(1
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.06

$$-\frac{b^2cx^3(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x^2*Sqrt[c*x^2]),x]
[Out] -((b^2*c*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)
/a])/(a^3*(1 + n)*(c*x^2)^(3/2)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x^2\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
[Out] int((b*x+a)^n/x^2/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**n/(x**2*sqrt(c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(sqrt(c*x^2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x^2\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^n/(x^2*(c*x^2)^(1/2)), x)
```

3.953 $\int \frac{x^6(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}$$

```
[Out] -a^3*x*(b*x+a)^(1+n)/b^4/c/(1+n)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+n)/b^4/c/
(2+n)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+n)/b^4/c/(3+n)/(c*x^2)^(1/2)+x*(b*x+a)
^(4+n)/b^4/c/(4+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*
x)^(2 + n))/(b^4*c*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*c*
(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int x^3(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.73

$$\frac{x^3(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2(2+3n+n^2)x^2+b^3(6+11n+6n^2+n^3)x^3\right)}{b^4(1+n)(2+n)(3+n)(4+n)(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(c*x^2)^(3/2))
```

Maple [A]
time = 0.14, size = 136, normalized size = 1.01

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^3\left(-b^3n^3x^3-6b^3n^2x^3+3ab^2n^2x^2-11b^3nx^3+9ab^2nx^2-6b^3x^3-6a^2bnx+6ab^2x^2-6a^2bx+6a^3\right)}{(cx^2)^{\frac{3}{2}}b^4(n^4+10n^3+35n^2+50n+24)}$	136
risch	$-\frac{x\left(-b^4n^3x^4-ab^3n^3x^3-6b^4n^2x^4-3ab^3n^2x^3-11b^4nx^4+3a^2b^2n^2x^2-2x^3anb^3-6b^4x^4+3a^2nx^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{c\sqrt{cx^2}\,(3+n)(4+n)(2+n)(1+n)b^4}$	157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x^3*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
3/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.28, size = 104, normalized size = 0.77

$$\frac{((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4)(bx+a)^n}{(n^4+10n^3+35n^2+50n+24)b^4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(3/2))
```

Fricas [A]
time = 0.73, size = 168, normalized size = 1.24

$$\frac{(6\,a^3bnx + (b^4n^3 + 6\,b^4n^2 + 11\,b^4n + 6\,b^4)x^4 - 6\,a^4 + (ab^3n^3 + 3\,ab^3n^2 + 2\,ab^3n)x^3 - 3\,(a^2b^2n^2 + a^2b^2n)x^2)\sqrt{cx^2}\,(bx + a)^n}{(b^4c^2n^4 + 10\,b^4c^2n^3 + 35\,b^4c^2n^2 + 50\,b^4c^2n + 24\,b^4c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^2*n^4 + 10*b^4*c^2*n^3 + 35*b^4*c^2*n^2 + 50*b
^4*c^2*n + 24*b^4*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^7}{4(cx^2)^{\frac{3}{2}}} & \text{for}\: b = 0 \\ \int \frac{x^6}{(cx^2)^{\frac{3}{2}}(a+bx)^4}\, dx & \text{for}\: n = -4 \\ \int \frac{x^6}{(cx^2)^{\frac{3}{2}}(a+bx)^3}\, dx & \text{for}\: n = -3 \\ \int \frac{x^6}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\, dx & \text{for}\: n = -2 \\ \int \frac{x^6}{(cx^2)^{\frac{3}{2}}(a+bx)}\, dx & \text{for}\: n = -1 \\ - \frac{6a^4x^3(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{6a^3bnx^4(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} - \frac{3a^2b^2n^2x^5(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} - \frac{3a^2b^2nx^5(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{ab^3n^3x^6(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{3ab^3n^2x^6(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{2ab^3nx^6(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{b^4n^3x^7(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{6b^4n^2x^7(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{11b^4nx^7(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} + \frac{6b^4x^7(a+bx)^n}{b^4n^4(cx^2)^{\frac{3}{2}} + 10b^4n^3(cx^2)^{\frac{3}{2}} + 35b^4n^2(cx^2)^{\frac{3}{2}} + 50b^4n(cx^2)^{\frac{3}{2}} + 24b^4(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**7/(4*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**6/((c*x**
2)**(3/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**6/((c*x**2)**(3/2)*(a
 + b*x)**3), x), Eq(n, -3)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)**2),
 x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x), Eq(n, -1))
, (-6*a**4*x**3*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x
**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24
*b**4*(c*x**2)**(3/2)) + 6*a**3*b*n*x**4*(a + b*x)**n/(b**4*n**4*(c*x**2)**
(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b*
*4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) - 3*a**2*b**2*n**2*x**5*(a
+ b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b*
*4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/
2)) - 3*a**2*b**2*n*x**5*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*
n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(
3/2) + 24*b**4*(c*x**2)**(3/2)) + a*b**3*n**3*x**6*(a + b*x)**n/(b**4*n**4*
(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/
2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + 3*a*b**3*n**2*x
```

```
**6*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2)
+ 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**
2)**(3/2)) + 2*a*b**3*n*x**6*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b
**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2
)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + b**4*n**3*x**7*(a + b*x)**n/(b**4*n**
4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(
3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)) + 6*b**4*n**2*x
**7*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2)
+ 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**
2)**(3/2)) + 11*b**4*n*x**7*(a + b*x)**n/(b**4*n**4*(c*x**2)**(3/2) + 10*b*
*4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2) + 50*b**4*n*(c*x**2)
**(3/2) + 24*b**4*(c*x**2)**(3/2)) + 6*b**4*x**7*(a + b*x)**n/(b**4*n**4*(c
*x**2)**(3/2) + 10*b**4*n**3*(c*x**2)**(3/2) + 35*b**4*n**2*(c*x**2)**(3/2)
 + 50*b**4*n*(c*x**2)**(3/2) + 24*b**4*(c*x**2)**(3/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.40, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,\left(n+1\right)}{b^2\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24)) - (6*a^4*x)/(b^4*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*
n*x^2)/(b^3*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 +
2))/(b*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*
c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

3.954 $\int \frac{x^5(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}$$

```
[Out] a^2*x*(b*x+a)^(1+n)/b^3/c/(1+n)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+n)/b^3/c/(2+
n)/(c*x^2)^(1/2)+x*(b*x+a)^(3+n)/b^3/c/(3+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2
 + n))/(b^3*c*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c*(3 + n)*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{c\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 0.70

$$\frac{x^3(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2\right)}{b^3(1+n)(2+n)(3+n)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*(c*x^2)^(3/2))
```

Maple [A]
time = 0.13, size = 83, normalized size = 0.84

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)x^3}{(c\,x^2)^{\frac{3}{2}}b^3(n^3+6n^2+11n+6)}$	83
risch	$\frac{x\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{c\sqrt{c\,x^2}\,(2+n)(3+n)(1+n)b^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^3/(c*x^2)^(3/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.84

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^2)
```

Fricas [A]
time = 0.66, size = 118, normalized size = 1.19

$$-\frac{(2\,a^2bnx - (b^3n^2 + 3\,b^3n + 2\,b^3)x^3 - 2\,a^3 - (ab^2n^2 + ab^2n)x^2)\sqrt{cx^2}\,(bx+a)^n}{(b^3c^2n^3 + 6\,b^3c^2n^2 + 11\,b^3c^2n + 6\,b^3c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^2*n^3 + 6*b^3*c^2*n^2 + 11*b^3*c^
2*n + 6*b^3*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^6}{3(cx^2)^{\frac{3}{2}}} & \text{for}\ b = 0 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^3}\, dx & \text{for}\ n = -3 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\, dx & \text{for}\ n = -2 \\ \int \frac{x^5}{(cx^2)^{\frac{3}{2}}(a+bx)}\, dx & \text{for}\ n = -1 \\ \frac{2a^3x^3(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} - \frac{2a^2bnx^4(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{ab^2n^2x^5(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{ab^2nx^5(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{b^3n^2x^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{3b^3nx^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} + \frac{2b^3x^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{3}{2}}+6b^3n^2(cx^2)^{\frac{3}{2}}+11b^3n(cx^2)^{\frac{3}{2}}+6b^3(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**6/(3*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**5/((c*x**
2)**(3/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**5/((c*x**2)**(3/2)*(a
 + b*x)**2), x), Eq(n, -2)), (Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
, Eq(n, -1)), (2*a**3*x**3*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3
*n**2*(c*x**2)**(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2))
 - 2*a**2*b*n*x**4*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c
*x**2)**(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + a*b**
2*n**2*x**5*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)*
*(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + a*b**2*n*x**
5*(a + b*x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 1
1*b**3*n*(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + b**3*n**2*x**6*(a + b*
x)**n/(b**3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 11*b**3*n*
(c*x**2)**(3/2) + 6*b**3*(c*x**2)**(3/2)) + 3*b**3*n*x**6*(a + b*x)**n/(b**
3*n**3*(c*x**2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 11*b**3*n*(c*x**2)**
(3/2) + 6*b**3*(c*x**2)**(3/2)) + 2*b**3*x**6*(a + b*x)**n/(b**3*n**3*(c*x*
*2)**(3/2) + 6*b**3*n**2*(c*x**2)**(3/2) + 11*b**3*n*(c*x**2)**(3/2) + 6*b*
*3*(c*x**2)**(3/2)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.31, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{c\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,\left(n+1\right)}{b\,c\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3*x
)/(b^3*c*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c*(11*n + 6*n^2 + n
^3 + 6)) + (a*n*x^3*(n + 1))/(b*c*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

3.955 $\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=65

$$-\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}$$

```
[Out] -a*x*(b*x+a)^(1+n)/b^2/c/(1+n)/(c*x^2)^(1/2)+x*(b*x+a)^(2+n)/b^2/c/(2+n)/(c
*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 45}

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 +
n))/(b^2*c*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{c\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}} + \frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.69

$$\frac{x^3(a+bx)^{1+n}(-a+b(1+n)x)}{b^2(1+n)(2+n)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] (x^3*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*(c*x^2)^(3/
2))
```

Maple [A]
time = 0.13, size = 46, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^3(-bnx-bx+a)}{(cx^2)^{\frac{3}{2}}b^2(n^2+3n+2)}$	46
risch	$-\frac{x(-b^2n\,x^2-abnx-x^2b^2+a^2)(bx+a)^n}{c\sqrt{c\,x^2}\,b^2(2+n)(1+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x^3*(-b*n*x-b*x+a)/(c*x^2)^(3/2)/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.69

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n}{(n^2+3\,n+2)b^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(3/2))
```

Fricas [A]
time = 0.79, size = 72, normalized size = 1.11

$$\frac{(abnx + (b^2n + b^2)x^2 - a^2)\sqrt{cx^2}\,(bx + a)^n}{(b^2c^2n^2 + 3\,b^2c^2n + 2\,b^2c^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^2*n^2 +
 3*b^2*c^2*n + 2*b^2*c^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^5}{2(cx^2)^{\frac{3}{2}}} & \text{for}\ b = 0 \\ \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{x^4}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for}\ n = -1 \\ -\frac{a^2x^3(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{abnx^4(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{b^2nx^5(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} + \frac{b^2x^5(a+bx)^n}{b^2n^2(cx^2)^{\frac{3}{2}}+3b^2n(cx^2)^{\frac{3}{2}}+2b^2(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**5/(2*(c*x**2)**(3/2)), Eq(b, 0)), (Integral(x**4/((c*x**
2)**(3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**4/((c*x**2)**(3/2)*(a
 + b*x)), x), Eq(n, -1)), (-a**2*x**3*(a + b*x)**n/(b**2*n**2*(c*x**2)**(3/
2) + 3*b**2*n*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)) + a*b*n*x**4*(a + b
*x)**n/(b**2*n**2*(c*x**2)**(3/2) + 3*b**2*n*(c*x**2)**(3/2) + 2*b**2*(c*x*
*2)**(3/2)) + b**2*n*x**5*(a + b*x)**n/(b**2*n**2*(c*x**2)**(3/2) + 3*b**2*
n*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)) + b**2*x**5*(a + b*x)**n/(b**2*
n**2*(c*x**2)**(3/2) + 3*b**2*n*(c*x**2)**(3/2) + 2*b**2*(c*x**2)**(3/2)),
True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{c\,(n^2+3\,n+2)}-\frac{a^2\,x}{b^2\,c\,(n^2+3\,n+2)}+\frac{a\,n\,x^2}{b\,c\,(n^2+3\,n+2)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c*(3*n + n^2
 + 2)) + (a*n*x^2)/(b*c*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

3.956 $\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}$$

[Out] x*(b*x+a)^(1+n)/b/c/(1+n)/(c*x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]

[Out] (x*(a + b*x)^(1 + n))/(b*c*(1 + n)*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.97

$$\frac{x^3(a+bx)^{1+n}}{b(1+n)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n))/(b*(1 + n)*(c*x^2)^(3/2))
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.94

method	result	size
gosper	$\frac{(bx+a)^{1+n}x^3}{b(1+n)(cx^2)^{\frac{3}{2}}}$	29
risch	$\frac{x(bx+a)(bx+a)^n}{c\sqrt{cx^2}\,b(1+n)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*x^3/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x + a\sqrt{c}\,\right)(bx+a)^n}{bc^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^2*(n + 1))
```

Fricas [A]
time = 0.64, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bc^2n+bc^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^2*n + b*c^2)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4}{a(cx^2)^{\frac{3}{2}}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^4}{(cx^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ \int \frac{x^3}{(cx^2)^{\frac{3}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ \frac{ax^3(a+bx)^n}{bn(cx^2)^{\frac{3}{2}}+b(cx^2)^{\frac{3}{2}}} + \frac{bx^4(a+bx)^n}{bn(cx^2)^{\frac{3}{2}}+b(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((x**4/(a*(c*x**2)**(3/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**4/(c*x
**2)**(3/2), Eq(b, 0)), (Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x), Eq(
n, -1)), (a*x**3*(a + b*x)**n/(b*n*(c*x**2)**(3/2) + b*(c*x**2)**(3/2)) + b
*x**4*(a + b*x)**n/(b*n*(c*x**2)**(3/2) + b*(c*x**2)**(3/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c\,(n+1)}+\frac{a\,x}{b\,c\,(n+1)}\right)\,{\left(a+b\,x\right)}^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((x^2/(c*(n + 1)) + (a*x)/(b*c*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

3.957 $\int \frac{x^2(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac(1+n)\sqrt{cx^2}}$$

```
[Out] -x*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/c/(1+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$-\frac{x^3 (a+bx)^{1+n} {}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] -((x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*(c*x^2)^(3/2)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2 (bx+a)^n}{\left(c\,x^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
[Out] int(x^2*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a+bx)^n}{\left(cx^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*x)^n)/(c*x^2)^(3/2), x)
```

3.958 $\int \frac{x(a+bx)^n}{(cx^2)^{3/2}} dx$

Optimal. Leaf size=48

$$\frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c(1+n)\sqrt{cx^2}}$$

```
[Out] b*x*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/c/(1+n)/(c*x^2)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 67}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 47, normalized size = 0.98

$$\frac{bx^3(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^2*(1 + n)*(c*x^2)^(3/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x(bx+a)^n}{(c\,x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int(x*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(3/2),x)
[Out] Integral(x*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*x/(c*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(3/2),x)
[Out] int((x*(a + b*x)^n)/(c*x^2)^(3/2), x)
```

3.959 $\int \frac{(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c(1+n)\sqrt{cx^2}}$$

```
[Out] -b^2*x*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/c/(1+n)/(c*x^2)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 67}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c*x^2)^(3/2),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 0.98

$$-\frac{b^2x^3(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c*x^2)^(3/2),x]
```

```
[Out] -((b^2*x^3*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^3*(1 + n)*(c*x^2)^(3/2)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{(c\,x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(c*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{{(c\,x^2)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((a + b*x)^n/(c*x^2)^(3/2), x)
```

3.960 $\int \frac{(a+bx)^n}{x\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c(1+n)\sqrt{cx^2}}$$

```
[Out] b^3*x*(b*x+a)^(1+n)*hypergeom([4, 1+n],[2+n],1+b*x/a)/a^4/c/(1+n)/(c*x^2)^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= \frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 1.00

$$\frac{b^3cx^5(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (b^3*c*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^4*(1 + n)*(c*x^2)^(5/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n}{x\left(c\,x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

```
[Out] int((b*x+a)^n/x/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n/((c*x^2)^(3/2)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^2*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^n}{x\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/x/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**n/(x*(c*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/x/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/((c*x^2)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^n}{x\,{\left(c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(x*(c*x^2)^(3/2)),x)
```

```
[Out] int((a + b*x)^n/(x*(c*x^2)^(3/2)), x)
```

3.961 $\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}$$

```
[Out] -a^3*x*(b*x+a)^(1+n)/b^4/c^2/(1+n)/(c*x^2)^(1/2)+3*a^2*x*(b*x+a)^(2+n)/b^4/
c^2/(2+n)/(c*x^2)^(1/2)-3*a*x*(b*x+a)^(3+n)/b^4/c^2/(3+n)/(c*x^2)^(1/2)+x*(
b*x+a)^(4+n)/b^4/c^2/(4+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c^2*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a +
b*x)^(2 + n))/(b^4*c^2*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^
4*c^2*(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c^2*(4 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int x^3(a+bx)^n\,dx}{c^2\sqrt{cx^2}} \\
&= \frac{x\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c^2\sqrt{cx^2}} \\
&= -\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 99, normalized size = 0.73

$$\frac{x(a+bx)^{1+n}\left(-6a^3+6a^2b(1+n)x-3ab^2\left(2+3n+n^2\right)x^2+b^3\left(6+11n+6n^2+n^3\right)x^3\right)}{b^4c^2(1+n)(2+n)(3+n)(4+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*c^2*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.12, size = 136, normalized size = 1.01

method	result	siz
gosper	$-\frac{(bx+a)^{1+n}x^5\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)}{(c\,x^2)^{\frac{5}{2}}b^4(n^4+10n^3+35n^2+50n+24)}$	13
risch	$-\frac{x\left(-b^4n^3x^4-a\,b^3n^3x^3-6b^4n^2x^4-3a\,b^3n^2x^3-11b^4n\,x^4+3a^2b^2n^2x^2-2x^3an\,b^3-6b^4x^4+3a^2n\,x^2b^2-6a^3bnx+6a^4\right)(bx+a)^n}{c^2\sqrt{c\,x^2}\,(3+n)(4+n)(2+n)(1+n)b^4}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*x^5*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
5/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.77

$$\frac{\left((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(5/2))
```

Fricas [A]
time = 0.84, size = 168, normalized size = 1.24

$$\frac{\left(6\,a^{3}bnx+\left(b^{4}n^{3}+6\,b^{4}n^{2}+11\,b^{4}n+6\,b^{4}\right)x^{4}-6\,a^{4}+\left(ab^{3}n^{3}+3\,ab^{3}n^{2}+2\,ab^{3}n\right)x^{3}-3\,\left(a^{2}b^{2}n^{2}+a^{2}b^{2}n\right)x^{2}\right)\sqrt{cx^{2}}\,\left(bx+a\right)^{n}}{\left(b^{4}c^{3}n^{4}+10\,b^{4}c^{3}n^{3}+35\,b^{4}c^{3}n^{2}+50\,b^{4}c^{3}n+24\,b^{4}c^{3}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^3*n^4 + 10*b^4*c^3*n^3 + 35*b^4*c^3*n^2 + 50*b
^4*c^3*n + 24*b^4*c^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^{n}x^{9}}{4\left(cx^{2}\right)^{\frac{5}{2}}} & \text{for}\: b = 0 \\ \int \frac{x^{8}}{\left(cx^{2}\right)^{\frac{5}{2}}\left(a+bx\right)^{4}}\, dx & \text{for}\: n = -4 \\ \int \frac{x^{8}}{\left(cx^{2}\right)^{\frac{5}{2}}\left(a+bx\right)^{3}}\, dx & \text{for}\: n = -3 \\ \int \frac{x^{8}}{\left(cx^{2}\right)^{\frac{5}{2}}\left(a+bx\right)^{2}}\, dx & \text{for}\: n = -2 \\ \int \frac{x^{8}}{\left(cx^{2}\right)^{\frac{5}{2}}\left(a+bx\right)}\, dx & \text{for}\: n = -1 \\ -\frac{6a^{4}x^{5}(a+bx)^{n}}{b^{4}n^{4}(cx^{2})^{\frac{5}{2}}+10b^{4}n^{3}(cx^{2})^{\frac{5}{2}}+35b^{4}n^{2}(cx^{2})^{\frac{5}{2}}+50b^{4}n(cx^{2})^{\frac{5}{2}}+24b^{4}(cx^{2})^{\frac{5}{2}}} + \frac{6a^{3}bnx^{6}(a+bx)^{n}}{b^{4}n^{4}(cx^{2})^{\frac{5}{2}}+10b^{4}n^{3}(cx^{2})^{\frac{5}{2}}+35b^{4}n^{2}(cx^{2})^{\frac{5}{2}}+50b^{4}n(cx^{2})^{\frac{5}{2}}+24b^{4}(cx^{2})^{\frac{5}{2}}} - \frac{3a^{2}b^{2}n^{2}x^{7}(a+bx)^{n}}{b^{4}n^{4}(cx^{2})^{\frac{5}{2}}+10b^{4}n^{3}(cx^{2})^{\frac{5}{2}}+35b^{4}n^{2}(cx^{2})^{\frac{5}{2}}+50b^{4}n(cx^{2})^{\frac{5}{2}}+24b^{4}(cx^{2})^{\frac{5}{2}}} - \frac{3a^{2}b^{2}nx^{7}(a+bx)^{n}}{b^{4}n^{4}(cx^{2})^{\frac{5}{2}}+10b^{4}n^{3}(cx^{2})^{\frac{5}{2}}+35b^{4}n^{2}(cx^{2})^{\frac{5}{2}}+50b^{4}n(cx^{2})^{\frac{5}{2}}+24b^{4}(cx^{2})^{\frac{5}{2}}} + \frac{ab^{3}n^{3}x^{8}(a+bx)^{n}}{b^{4}n^{4}(cx^{2})^{\frac{5}{2}}+10b^{4}n^{3}(cx^{2})^{\frac{5}{2}}+35b^{4}n^{2}(cx^{2})^{\frac{5}{2}}+50b^{4}n(cx^{2})^{\frac{5}{2}}+24b^{4}(cx^{2})^{\frac{5}{2}}} + \text{[illegible]} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**9/(4*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**8/((c*x**
2)**(5/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**8/((c*x**2)**(5/2)*(a
 + b*x)**3), x), Eq(n, -3)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)**2),
 x), Eq(n, -2)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)), x), Eq(n, -1))
, (-6*a**4*x**5*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x
**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24
*b**4*(c*x**2)**(5/2)) + 6*a**3*b*n*x**6*(a + b*x)**n/(b**4*n**4*(c*x**2)**
(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b*
*4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) - 3*a**2*b**2*n**2*x**7*(a
+ b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b*
*4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/
2)) - 3*a**2*b**2*n*x**7*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*
n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(
5/2) + 24*b**4*(c*x**2)**(5/2)) + a*b**3*n**3*x**8*(a + b*x)**n/(b**4*n**4*
(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/
2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + 3*a*b**3*n**2*x
```

```
**8*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2)
+ 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**
2)**(5/2)) + 2*a*b**3*n*x**8*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b
**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2
)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + b**4*n**3*x**9*(a + b*x)**n/(b**4*n**
4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(
5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)) + 6*b**4*n**2*x
**9*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2)
+ 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**
2)**(5/2)) + 11*b**4*n*x**9*(a + b*x)**n/(b**4*n**4*(c*x**2)**(5/2) + 10*b*
*4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2) + 50*b**4*n*(c*x**2)
**(5/2) + 24*b**4*(c*x**2)**(5/2)) + 6*b**4*x**9*(a + b*x)**n/(b**4*n**4*(c
*x**2)**(5/2) + 10*b**4*n**3*(c*x**2)**(5/2) + 35*b**4*n**2*(c*x**2)**(5/2)
 + 50*b**4*n*(c*x**2)**(5/2) + 24*b**4*(c*x**2)**(5/2)), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.41, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c^2*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (6*a^4*x)/(b^4*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*
a^3*n*x^2)/(b^3*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n +
n^2 + 2))/(b*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1
))/(b^2*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

3.962 $\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}$$

```
[Out] a^2*x*(b*x+a)^(1+n)/b^3/c^2/(1+n)/(c*x^2)^(1/2)-2*a*x*(b*x+a)^(2+n)/b^3/c^2
/(2+n)/(c*x^2)^(1/2)+x*(b*x+a)^(3+n)/b^3/c^2/(3+n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{15, 45\}$

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c^2*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^
(2 + n))/(b^3*c^2*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c^2*(3
+ n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int x^2(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int\left(\frac{a^2(a+bx)^n}{b^2} - \frac{2a(a+bx)^{1+n}}{b^2} + \frac{(a+bx)^{2+n}}{b^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= \frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}} - \frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}} + \frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.71

$$\frac{x(a+bx)^{1+n}\left(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2\right)}{b^3c^2(1+n)(2+n)(3+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*c^2*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.14, size = 83, normalized size = 0.84

method	result	size
gosper	$\frac{(bx+a)^{1+n}\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2x^2b^2-2abx+2a^2\right)x^5}{(c\,x^2)^{\frac{5}{2}}b^3(n^3+6n^2+11n+6)}$	83
risch	$\frac{x\left(b^3n^2x^3+a\,b^2n^2x^2+3b^3n\,x^3+a\,b^2n\,x^2+2b^3x^3-2a^2bnx+2a^3\right)(bx+a)^n}{c^2\sqrt{c\,x^2}\,(2+n)(3+n)(1+n)b^3}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] (b*x+a)^(1+n)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^5/(c*x^2)^(5/2)/b^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.84

$$\frac{\left((n^2+3\,n+2)b^3\sqrt{c}\,x^3+(n^2+n)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\,\right)(bx+a)^n}{(n^3+6\,n^2+11\,n+6)b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^3)
```

Fricas [A]
time = 0.71, size = 118, normalized size = 1.19

$$-\frac{\left(2\,a^2bnx-(b^3n^2+3\,b^3n+2\,b^3)x^3-2\,a^3-(ab^2n^2+ab^2n)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{(b^3c^3n^3+6\,b^3c^3n^2+11\,b^3c^3n+6\,b^3c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^3*n^3 + 6*b^3*c^3*n^2 + 11*b^3*c^
3*n + 6*b^3*c^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^8}{3(cx^2)^{\frac{5}{2}}} & \text{for}\ b = 0 \\ \int \frac{x^7}{(cx^2)^{\frac{5}{2}}(a+bx)^3}\,dx & \text{for}\ n = -3 \\ \int \frac{x^7}{(cx^2)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{x^7}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx & \text{for}\ n = -1 \\ \frac{2a^3x^5(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} - \frac{2a^2bnx^6(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{ab^2n^2x^7(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{ab^2nx^7(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{b^3n^2x^8(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{3b^3nx^8(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} + \frac{2b^3x^8(a+bx)^n}{b^3n^3(cx^2)^{\frac{5}{2}}+6b^3n^2(cx^2)^{\frac{5}{2}}+11b^3n(cx^2)^{\frac{5}{2}}+6b^3(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**8/(3*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**7/((c*x**
2)**(5/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**7/((c*x**2)**(5/2)*(a
 + b*x)**2), x), Eq(n, -2)), (Integral(x**7/((c*x**2)**(5/2)*(a + b*x)), x)
, Eq(n, -1)), (2*a**3*x**5*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3
*n**2*(c*x**2)**(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2))
 - 2*a**2*b*n*x**6*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c
*x**2)**(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + a*b**
2*n**2*x**7*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)*
*(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + a*b**2*n*x**
7*(a + b*x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 1
1*b**3*n*(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + b**3*n**2*x**8*(a + b*
x)**n/(b**3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 11*b**3*n*
(c*x**2)**(5/2) + 6*b**3*(c*x**2)**(5/2)) + 3*b**3*n*x**8*(a + b*x)**n/(b**
3*n**3*(c*x**2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 11*b**3*n*(c*x**2)**
(5/2) + 6*b**3*(c*x**2)**(5/2)) + 2*b**3*x**8*(a + b*x)**n/(b**3*n**3*(c*x*
*2)**(5/2) + 6*b**3*n**2*(c*x**2)**(5/2) + 11*b**3*n*(c*x**2)**(5/2) + 6*b*
*3*(c*x**2)**(5/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.35, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\left(\frac{x^4\,(n^2+3\,n+2)}{c^2\,(n^3+6\,n^2+11\,n+6)}+\frac{2\,a^3\,x}{b^3\,c^2\,(n^3+6\,n^2+11\,n+6)}-\frac{2\,a^2\,n\,x^2}{b^2\,c^2\,(n^3+6\,n^2+11\,n+6)}+\frac{a\,n\,x^3\,(n+1)}{b\,c^2\,(n^3+6\,n^2+11\,n+6)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c^2*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3
*x)/(b^3*c^2*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c^2*(11*n + 6*n
^2 + n^3 + 6)) + (a*n*x^3*(n + 1))/(b*c^2*(11*n + 6*n^2 + n^3 + 6))))/(c*x^
2)^(1/2)
```

3.963 $\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=65

$$-\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}$$

```
[Out] -a*x*(b*x+a)^(1+n)/b^2/c^2/(1+n)/(c*x^2)^(1/2)+x*(b*x+a)^(2+n)/b^2/c^2/(2+n
)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 45}

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2
+ n))/(b^2*c^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6(a+bx)^n}{(cx^2)^{5/2}}\,dx = \frac{x\int x(a+bx)^n\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{x\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.71

$$\frac{x(a+bx)^{1+n}(-a+b(1+n)x)}{b^2c^2(1+n)(2+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*c^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

Maple [A]
time = 0.14, size = 46, normalized size = 0.71

method	result	size
gosper	$-\frac{(bx+a)^{1+n}x^5(-bnx-bx+a)}{(cx^2)^{\frac{5}{2}}b^2(n^2+3n+2)}$	46
risch	$-\frac{x(-b^2nx^2-abnx-x^2b^2+a^2)(bx+a)^n}{c^2\sqrt{cx^2}\,b^2(2+n)(1+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*x^5*(-b*n*x-b*x+a)/(c*x^2)^(5/2)/b^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.69

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n}{(n^2+3\,n+2)b^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(5/2))
```

Fricas [A]
time = 0.83, size = 72, normalized size = 1.11

$$\frac{(abnx + (b^2n + b^2)x^2 - a^2)\sqrt{cx^2}\,(bx + a)^n}{(b^2c^3n^2 + 3\,b^2c^3n + 2\,b^2c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^3*n^2 +
 3*b^2*c^3*n + 2*b^2*c^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^7}{2(cx^2)^{\frac{5}{2}}} & \text{for}\ b = 0 \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx & \text{for}\ n = -1 \\ -\frac{a^2x^5(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} + \frac{abnx^6(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} + \frac{b^2nx^7(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} + \frac{b^2x^7(a+bx)^n}{b^2n^2(cx^2)^{\frac{5}{2}}+3b^2n(cx^2)^{\frac{5}{2}}+2b^2(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**7/(2*(c*x**2)**(5/2)), Eq(b, 0)), (Integral(x**6/((c*x**
2)**(5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(5/2)*(a
 + b*x)), x), Eq(n, -1)), (-a**2*x**5*(a + b*x)**n/(b**2*n**2*(c*x**2)**(5/
2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + a*b*n*x**6*(a + b
*x)**n/(b**2*n**2*(c*x**2)**(5/2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x*
*2)**(5/2)) + b**2*n*x**7*(a + b*x)**n/(b**2*n**2*(c*x**2)**(5/2) + 3*b**2*
n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)) + b**2*x**7*(a + b*x)**n/(b**2*
n**2*(c*x**2)**(5/2) + 3*b**2*n*(c*x**2)**(5/2) + 2*b**2*(c*x**2)**(5/2)),
True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{c^2\,(n^2+3\,n+2)}-\frac{a^2\,x}{b^2\,c^2\,(n^2+3\,n+2)}+\frac{a\,n\,x^2}{b\,c^2\,(n^2+3\,n+2)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c^2*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c^2*(3*n +
 n^2 + 2)) + (a*n*x^2)/(b*c^2*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

3.964 $\int \frac{x^5(a+bx)^n}{(cx^2)^{5/2}} dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}$$

[Out] x*(b*x+a)^(1+n)/b/c^2/(1+n)/(c*x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]

[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(a+bx)^n}{(cx^2)^{5/2}} dx &= \frac{x\int (a+bx)^n dx}{c^2\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.94

method	result	size
gosper	$\frac{(bx+a)^{1+n} x^{5}}{b(1+n)(c\,x^{2})^{\frac{5}{2}}}$	29
risch	$\frac{x(bx+a)(bx+a)^{n}}{c^{2}\sqrt{c\,x^{2}}\;b(1+n)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)/b/(1+n)*x^5/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 1.00

$$\frac{(b\sqrt{c}\,x + a\sqrt{c}\,)(bx + a)^{n}}{bc^{3}(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^3*(n + 1))
```

Fricas [A]
time = 1.82, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^{2}}\,(bx + a)(bx + a)^{n}}{(bc^{3}n + bc^{3})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^3*n + b*c^3)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6}{a(cx^2)^{\frac{5}{2}}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^6}{(cx^2)^{\frac{5}{2}}} & \text{for } b = 0 \\ \int \frac{x^5}{(cx^2)^{\frac{5}{2}}(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ax^5(a+bx)^n}{bn(cx^2)^{\frac{5}{2}}+b(cx^2)^{\frac{5}{2}}} + \frac{bx^6(a+bx)^n}{bn(cx^2)^{\frac{5}{2}}+b(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((x**6/(a*(c*x**2)**(5/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**6/(c*x
**2)**(5/2), Eq(b, 0)), (Integral(x**5/((c*x**2)**(5/2)*(a + b*x)), x), Eq(
n, -1)), (a*x**5*(a + b*x)**n/(b*n*(c*x**2)**(5/2) + b*(c*x**2)**(5/2)) + b
*x**6*(a + b*x)**n/(b*n*(c*x**2)**(5/2) + b*(c*x**2)**(5/2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c^2\,(n+1)}+\frac{a\,x}{b\,c^2\,(n+1)}\right)\,{(a+b\,x)}^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((x^2/(c^2*(n + 1)) + (a*x)/(b*c^2*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

3.965 $\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac^2(1+n)\sqrt{cx^2}}$$

```
[Out] -x*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/c^2/(1+n)/(c*x^2)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 67}

$$-\frac{x(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{ac^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((x*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*
c^2*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{x(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{ac^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$-\frac{x^5(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] -((x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(
a*(1 + n)*(c*x^2)^(5/2)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^4(bx+a)^n}{\left(c\,x^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
[Out] int(x^4*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx)^n}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.966 $\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c^2(1+n)\sqrt{cx^2}}$$

```
[Out] b*x*(b*x+a)^(1+n)*hypergeom([2, 1+n],[2+n],1+b*x/a)/a^2/c^2/(1+n)/(c*x^2)^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$\frac{bx(a+bx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{bx}{a}+1\right)}{a^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b*x*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(a^
2*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^2}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{bx(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 0.98

$$\frac{bx^5(a+bx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{bx}{a}\right)}{a^2(1+n)\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (b*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^2*(1 + n)*(c*x^2)^(5/2))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^3(bx+a)^n}{(c\,x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
[Out] int(x^3*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)^n}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.967 $\int \frac{x^2(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c^2(1+n)\sqrt{cx^2}}$$

```
[Out] -b^2*x*(b*x+a)^(1+n)*hypergeom([3, 1+n],[2+n],1+b*x/a)/a^3/c^2/(1+n)/(c*x^2
)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 67}

$$-\frac{b^2x(a+bx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{bx}{a}+1\right)}{a^3c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((b^2*x*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^3*c^2*(1 + n)*Sqrt[c*x^2]))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^3}\,dx}{c^2\sqrt{cx^2}}\\
&= -\frac{b^2x(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3c^2(1+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 0.98

$$-\frac{b^2x^5(a+bx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;1+\frac{bx}{a}\right)}{a^3(1+n)\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((b^2*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, 1 + (b*x)/a
])/(a^3*(1 + n)*(c*x^2)^(5/2)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2(bx+a)^n}{(c\,x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int(x^2*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx)^n}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^2/(c*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.968 $\int \frac{x(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c^2(1+n)\sqrt{cx^2}}$$

```
[Out] b^3*x*(b*x+a)^(1+n)*hypergeom([4, 1+n],[2+n],1+b*x/a)/a^4/c^2/(1+n)/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 67}

$$\frac{b^3x(a+bx)^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{bx}{a}+1\right)}{a^4c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (b^3*x*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])/(
a^4*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^n}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{b^3x(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4c^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 0.98

$$\frac{b^3x^5(a+bx)^{1+n}\,{}_2F_1\left(4,1+n;2+n;1+\frac{bx}{a}\right)}{a^4(1+n)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] (b^3*x^5*(a + b*x)^(1 + n)*Hypergeometric2F1[4, 1 + n, 2 + n, 1 + (b*x)/a])
/(a^4*(1 + n)*(c*x^2)^(5/2))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x(bx+a)^n}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(5/2),x)
[Out] int(x*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n/(c^3*x^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx)^n}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x/(c*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,{\left(a+b\,x\right)}^n}{{\left(c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.969 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}$$

```
[Out] a*c^2*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x+b*c^2*(d*x)^(7+m)*(c*x^2)^(1/2)
/d^7/(7+m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{ac^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (b*c^2*(d*x)^(7 + m)*Sq
rt[c*x^2])/(d^7*(7 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx = \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)\,dx}{d^5x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a(dx)^{5+m}+\frac{b(dx)^{6+m}}{d}\right)dx}{d^5x}$$
$$= \frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x} + \frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.58

$$\frac{x(dx)^m \left(cx^2\right)^{5/2} (a(7+m)+b(6+m)x)}{(6+m)(7+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a*(7 + m) + b*(6 + m)*x))/((6 + m)*(7 + m))
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.62

method	result	size
gosper	$\frac{x(bmx+am+6bx+7a)(dx)^m\left(cx^2\right)^{\frac{5}{2}}}{(7+m)(6+m)}$	40
risch	$\frac{c^2x^5\sqrt{cx^2}\,(bmx+am+6bx+7a)(dx)^m}{(7+m)(6+m)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x+a*m+6*b*x+7*a)*(d*x)^m*(c*x^2)^(5/2)/(7+m)/(6+m)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.60

$$\frac{bc^{\frac{5}{2}}d^m x^7 x^m}{m+7} + \frac{ac^{\frac{5}{2}}d^m x^6 x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a*c^(5/2)*d^m*x^6*x^m/(m + 6)
```

Fricas [A]
time = 0.81, size = 58, normalized size = 0.89

$$\frac{\left(\left(bc^{2}m+6\,bc^{2}\right)x^{6}+\left(ac^{2}m+7\,ac^{2}\right)x^{5}\right)\sqrt{cx^{2}}\,(dx)^{m}}{m^{2}+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c^2*m + 6*b*c^2)*x^6 + (a*c^2*m + 7*a*c^2)*x^5)*sqrt(c*x^2)*(d*x)^m/(m^
2 + 13*m + 42)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\left(cx^{2}\right)^{\frac{5}{2}}}{x^{7}}\,dx+\int \frac{b\left(cx^{2}\right)^{\frac{5}{2}}}{x^{6}}\,dx}{d^{7}} & \text{for}\: m = -7 \\ \frac{\int \frac{a\left(cx^{2}\right)^{\frac{5}{2}}}{x^{6}}\,dx+\int \frac{b\left(cx^{2}\right)^{\frac{5}{2}}}{x^{5}}\,dx}{d^{6}} & \text{for}\: m = -6 \\ \frac{amx\left(cx^{2}\right)^{\frac{5}{2}}(dx)^{m}}{m^{2}+13m+42} + \frac{7ax\left(cx^{2}\right)^{\frac{5}{2}}(dx)^{m}}{m^{2}+13m+42} + \frac{bmx^{2}\left(cx^{2}\right)^{\frac{5}{2}}(dx)^{m}}{m^{2}+13m+42} + \frac{6bx^{2}\left(cx^{2}\right)^{\frac{5}{2}}(dx)^{m}}{m^{2}+13m+42} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(5/2)/x**7, x) + Integral(b*(c*x**2)**(5/2
)/x**6, x))/d**7, Eq(m, -7)), ((Integral(a*(c*x**2)**(5/2)/x**6, x) + Integ
ral(b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a*m*x*(c*x**2)**(5/2)*(d
*x)**m/(m**2 + 13*m + 42) + 7*a*x*(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m + 4
2) + b*m*x**2*(c*x**2)**(5/2)*(d*x)**m/(m**2 + 13*m + 42) + 6*b*x**2*(c*x**
2)**(5/2)*(d*x)**m/(m**2 + 13*m + 42), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.27, size = 44, normalized size = 0.68

$$\frac{c^2\,x^5\,{\left(d\,x\right)}^m\,\sqrt{c\,x^2}\,\left(7\,a+a\,m+6\,b\,x+b\,m\,x\right)}{m^2+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] (c^2*x^5*(d*x)^m*(c*x^2)^(1/2)*(7*a + a*m + 6*b*x + b*m*x))/(13*m + m^2 + 4
2)
```

3.970 $\int (dx)^m (cx^2)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}$$

```
[Out] a*c*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x+b*c*(d*x)^(5+m)*(c*x^2)^(1/2)/d^5
/(5+m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{ac\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (b*c*(d*x)^(5 + m)*Sqrt[c
*x^2])/(d^5*(5 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m (cx^2)^{3/2} (a+bx) \, dx = \frac{\left(c\sqrt{cx^2}\right) \int x^3(dx)^m(a+bx) \, dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right) \int (dx)^{3+m}(a+bx) \, dx}{d^3x}$$
$$= \frac{\left(c\sqrt{cx^2}\right) \int \left(a(dx)^{3+m} + \frac{b(dx)^{4+m}}{d}\right) \, dx}{d^3x}$$
$$= \frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x} + \frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.62

$$\frac{x(dx)^m (cx^2)^{3/2} (a(5+m)+b(4+m)x)}{(4+m)(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a*(5 + m) + b*(4 + m)*x))/((4 + m)*(5 + m))
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.66

method	result	size
gosper	$\frac{x(bmx+am+4bx+5a)(dx)^m (c x^2)^{\frac{3}{2}}}{(5+m)(4+m)}$	40
risch	$\frac{c x^3 \sqrt{c x^2}\, (bmx+am+4bx+5a)(dx)^m}{(5+m)(4+m)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x+a*m+4*b*x+5*a)*(d*x)^m*(c*x^2)^(3/2)/(5+m)/(4+m)
```

Maxima [A]
time = 0.29, size = 39, normalized size = 0.64

$$\frac{bc^{\frac{3}{2}}d^m x^5 x^m}{m+5} + \frac{ac^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a*c^(3/2)*d^m*x^4*x^m/(m + 4)
```

Fricas [A]
time = 0.63, size = 50, normalized size = 0.82

$$\frac{((bcm + 4\,bc)x^4 + (acm + 5\,ac)x^3)\sqrt{cx^2}\,(dx)^m}{m^2 + 9\,m + 20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c*m + 4*b*c)*x^4 + (a*c*m + 5*a*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^2 + 9*m
+ 20)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^5}\,dx + \int \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx}{d^5} & \text{for}\: m = -5 \\ \frac{\int \frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx + \int \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^3}\,dx}{d^4} & \text{for}\: m = -4 \\ \frac{amx\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} + \frac{5ax\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} + \frac{bmx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} + \frac{4bx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^2+9m+20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(3/2)/x**5, x) + Integral(b*(c*x**2)**(3/2
)/x**4, x))/d**5, Eq(m, -5)), ((Integral(a*(c*x**2)**(3/2)/x**4, x) + Integ
ral(b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a*m*x*(c*x**2)**(3/2)*(d
*x)**m/(m**2 + 9*m + 20) + 5*a*x*(c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20)
 + b*m*x**2*(c*x**2)**(3/2)*(d*x)**m/(m**2 + 9*m + 20) + 4*b*x**2*(c*x**2)*
*(3/2)*(d*x)**m/(m**2 + 9*m + 20), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa

Mupad [B]
time = 0.24, size = 42, normalized size = 0.69

$$\frac{c\,x^3\,{\left(d\,x\right)}^m\,\sqrt{c\,x^2}\,\left(5\,a+a\,m+4\,b\,x+b\,m\,x\right)}{m^2+9\,m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x),x)
[Out] (c*x^3*(d*x)^m*(c*x^2)^(1/2)*(5*a + a*m + 4*b*x + b*m*x))/(9*m + m^2 + 20)
```

3.971 $\int (dx)^m \sqrt{cx^2} (a+bx) \, dx$

Optimal. Leaf size=59

$$\frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}$$

```
[Out] a*(d*x)^(2+m)*(c*x^2)^(1/2)/d^2/(2+m)/x+b*(d*x)^(3+m)*(c*x^2)^(1/2)/d^3/(3+
m)/x
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{a\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x}+\frac{b\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (b*(d*x)^(3 + m)*Sqrt[c*x^2
])/(d^3*(3 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{cx^2}\,(a+bx)\,dx = \frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)\,dx}{x}$$
$$= \frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)\,dx}{dx}$$
$$= \frac{\sqrt{cx^2}\,\int \left(a(dx)^{1+m}+\frac{b(dx)^{2+m}}{d}\right)dx}{dx}$$
$$= \frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.64

$$\frac{x(dx)^m\sqrt{cx^2}\,(a(3+m)+b(2+m)x)}{(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a*(3 + m) + b*(2 + m)*x))/((2 + m)*(3 + m))
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.68

method	result	size
gosper	$\frac{x(bmx+am+2bx+3a)(dx)^m\sqrt{c\,x^2}}{(3+m)(2+m)}$	40
risch	$\frac{x(bmx+am+2bx+3a)(dx)^m\sqrt{c\,x^2}}{(3+m)(2+m)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x+a*m+2*b*x+3*a)*(d*x)^m*(c*x^2)^(1/2)/(3+m)/(2+m)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.66

$$\frac{b\sqrt{c}\,d^m x^3 x^m}{m+3}+\frac{a\sqrt{c}\,d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a*sqrt(c)*d^m*x^2*x^m/(m + 2)
```

Fricas [A]
time = 0.64, size = 44, normalized size = 0.75

$$\frac{((bm+2\,b)x^2+(am+3\,a)x)\sqrt{cx^2}\,(dx)^m}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*m + 2*b)*x^2 + (a*m + 3*a)*x)*sqrt(c*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\sqrt{cx^2}}{x^3}\,dx+\int \frac{b\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for } m=-3 \\ \frac{\int \frac{a\sqrt{cx^2}}{x^2}\,dx+\int \frac{b\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for } m=-2 \\ \frac{amx\sqrt{cx^2}\,(dx)^m}{m^2+5m+6}+\frac{3ax\sqrt{cx^2}\,(dx)^m}{m^2+5m+6}+\frac{bmx^2\sqrt{cx^2}\,(dx)^m}{m^2+5m+6}+\frac{2bx^2\sqrt{cx^2}\,(dx)^m}{m^2+5m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*sqrt(c*x**2)/x**3, x) + Integral(b*sqrt(c*x**2)/x**2
, x))/d**3, Eq(m, -3)), ((Integral(a*sqrt(c*x**2)/x**2, x) + Integral(b*sqr
t(c*x**2)/x, x))/d**2, Eq(m, -2)), (a*m*x*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m
 + 6) + 3*a*x*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m + 6) + b*m*x**2*sqrt(c*x**2
)*(d*x)**m/(m**2 + 5*m + 6) + 2*b*x**2*sqrt(c*x**2)*(d*x)**m/(m**2 + 5*m +
6), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.21, size = 39, normalized size = 0.66

$$\frac{x\,(d\,x)^m\,\sqrt{c\,x^2}\,\left(3\,a+a\,m+2\,b\,x+b\,m\,x\right)}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x),x)
[Out] (x*(d*x)^m*(c*x^2)^(1/2)*(3*a + a*m + 2*b*x + b*m*x))/(5*m + m^2 + 6)
```

3.972 $\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}} dx$

Optimal. Leaf size=48

$$\frac{ax(dx)^m}{m\sqrt{cx^2}} + \frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}$$

```
[Out] a*x*(d*x)^m/m/(c*x^2)^(1/2)+b*x*(d*x)^(1+m)/d/(1+m)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$\frac{ax(dx)^m}{m\sqrt{cx^2}} + \frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int (dx)^{-1+m}(a+bx)\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int \left(a(dx)^{-1+m}+\frac{b(dx)^m}{d}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{ax(dx)^m}{m\sqrt{cx^2}} + \frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.69

$$\frac{x(dx)^m(a+am+bmx)}{m(1+m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x*(d*x)^m*(a + a*m + b*m*x))/(m*(1 + m)*Sqrt[c*x^2])
```

Maple [A]
time = 0.01, size = 32, normalized size = 0.67

method	result	size
gosper	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32
risch	$\frac{x(bmx+am+a)(dx)^m}{(1+m)m\sqrt{c\,x^2}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x+a*m+a)*(d*x)^m/(1+m)/m/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.67

$$\frac{bd^m x x^m}{\sqrt{c}\,(m+1)} + \frac{ad^m x^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a*d^m*x^m/(sqrt(c)*m)
```

Fricas [A]
time = 2.03, size = 36, normalized size = 0.75

$$\frac{(bmx + am + a)\sqrt{cx^2}\,(dx)^m}{(cm^2 + cm)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b*m*x + a*m + a)*sqrt(c*x^2)*(d*x)^m/((c*m^2 + c*m)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b}{\sqrt{cx^2}}\, dx + \int \frac{a}{x\sqrt{cx^2}}\, dx}{d} & \text{for}\ m = -1 \\ \int \frac{a+bx}{\sqrt{cx^2}}\, dx & \text{for}\ m = 0 \\ \frac{amx(dx)^m}{m^2\sqrt{cx^2} + m\sqrt{cx^2}} + \frac{ax(dx)^m}{m^2\sqrt{cx^2} + m\sqrt{cx^2}} + \frac{bmx^2(dx)^m}{m^2\sqrt{cx^2} + m\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b/sqrt(c*x**2), x) + Integral(a/(x*sqrt(c*x**2)), x))/
d, Eq(m, -1)), (Integral((a + b*x)/sqrt(c*x**2), x), Eq(m, 0)), (a*m*x*(d*x
)**m/(m**2*sqrt(c*x**2) + m*sqrt(c*x**2)) + a*x*(d*x)**m/(m**2*sqrt(c*x**2)
 + m*sqrt(c*x**2)) + b*m*x**2*(d*x)**m/(m**2*sqrt(c*x**2) + m*sqrt(c*x**2))
, True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.21, size = 30, normalized size = 0.62

$$\frac{\left(\frac{a\,x}{m}+\frac{b\,x^2}{m+1}\right)\,{\left(d\,x\right)}^m}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (((a*x)/m + (b*x^2)/(m + 1))*(d*x)^m)/(c*x^2)^(1/2)
```

3.973 $\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

```
[Out] -a*d^2*x*(d*x)^(-2+m)/c/(2-m)/(c*x^2)^(1/2)-b*d*x*(d*x)^(-1+m)/c/(1-m)/(c*x
^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -((a*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (b*d*x*(d*x)^(-1 + m)
)/(c*(1 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int \left(a(dx)^{-3+m} + \frac{b(dx)^{-2+m}}{d}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}} - \frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.58

$$\frac{x(dx)^m(a(-1+m)+b(-2+m)x)}{(-2+m)(-1+m)\,(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x*(d*x)^m*(a*(-1 + m) + b*(-2 + m)*x))/((-2 + m)*(-1 + m)*(c*x^2)^(3/2))
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.62

method	result	size
gosper	$\frac{x(bmx+am-2bx-a)(dx)^m}{(-1+m)(-2+m)(cx^2)^{\frac{3}{2}}}$	40
risch	$\frac{(bmx+am-2bx-a)(dx)^m}{cx\sqrt{cx^2}\,(-1+m)(-2+m)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x+a*m-2*b*x-a)*(d*x)^m/(-1+m)/(-2+m)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.30, size = 39, normalized size = 0.60

$$\frac{bd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{ad^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a*d^m*x^m/(c^(3/2)*(m - 2)*x^2)
```

Fricas [A]
time = 1.49, size = 53, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,(am+(bm-2\,b)x-a)(dx)^m}{(c^2m^2-3\,c^2m+2\,c^2)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 2*b)*x - a)*(d*x)^m/((c^2*m^2 - 3*c^2*m + 2*c^2)*
x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d\left(\int \frac{ax}{(cx^2)^{\frac{3}{2}}}\,dx+\int \frac{bx^2}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for}\ m=1\\ d^2\left(\int \frac{ax^2}{(cx^2)^{\frac{3}{2}}}\,dx+\int \frac{bx^3}{(cx^2)^{\frac{3}{2}}}\,dx\right) & \text{for}\ m=2\\ \frac{amx(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}}-\frac{ax(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}}+\frac{bmx^2(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}}-\frac{2bx^2(dx)^m}{m^2(cx^2)^{\frac{3}{2}}-3m(cx^2)^{\frac{3}{2}}+2(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((d*(Integral(a*x/(c*x**2)**(3/2), x) + Integral(b*x**2/(c*x**2)**
(3/2), x)), Eq(m, 1)), (d**2*(Integral(a*x**2/(c*x**2)**(3/2), x) + Integra
l(b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a*m*x*(d*x)**m/(m**2*(c*x**2)**(
3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) - a*x*(d*x)**m/(m**2*(c*x**
2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) + b*m*x**2*(d*x)**m/(m
**2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)) - 2*b*x**2*(
d*x)**m/(m**2*(c*x**2)**(3/2) - 3*m*(c*x**2)**(3/2) + 2*(c*x**2)**(3/2)), T
rue))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(3/2), x)
```

Mupad [B]
time = 0.26, size = 48, normalized size = 0.74

$$\frac{b\,{\left(d\,x\right)}^m}{c\,\sqrt{c\,x^2}\,\left(m-1\right)}+\frac{a\,{\left(d\,x\right)}^m}{c\,x\,\sqrt{c\,x^2}\,\left(m-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x)
[Out] (b*(d*x)^m)/(c*(c*x^2)^(1/2)*(m - 1)) + (a*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m -
 2))
```

3.974 $\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

```
[Out] -a*d^4*x*(d*x)^(-4+m)/c^2/(4-m)/(c*x^2)^(1/2)-b*d^3*x*(d*x)^(-3+m)/c^2/(3-m
)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {15, 16, 45}

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -((a*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (b*d^3*x*(d*x)^(-3
+ m))/(c^2*(3 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int \left(a(dx)^{-5+m}+\frac{b(dx)^{-4+m}}{d}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}} - \frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.57

$$\frac{x(dx)^m(a(-3+m)+b(-4+m)x)}{(-4+m)(-3+m)\,(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (x*(d*x)^m*(a*(-3 + m) + b*(-4 + m)*x))/((-4 + m)*(-3 + m)*(c*x^2)^(5/2))
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.60

method	result	size
gosper	$\frac{x(bmx+am-4bx-3a)(dx)^m}{(-3+m)(-4+m)(c\,x^2)^{\frac{5}{2}}}$	40
risch	$\frac{(bmx+am-4bx-3a)(dx)^m}{c^2x^3\sqrt{c\,x^2}\,(-3+m)(-4+m)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] x*(b*m*x+a*m-4*b*x-3*a)*(d*x)^m/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.58

$$\frac{bd^mx^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{ad^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Fricas [A]
time = 0.95, size = 53, normalized size = 0.79

$$\frac{\sqrt{cx^2}\,(am + (bm - 4\,b)x - 3\,a)(dx)^m}{(c^3m^2 - 7\,c^3m + 12\,c^3)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 4*b)*x - 3*a)*(d*x)^m/((c^3*m^2 - 7*c^3*m + 12*c^
3)*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d^3\left(\int \frac{ax^3}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{bx^4}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for}\: m = 3 \\ d^4\left(\int \frac{ax^4}{(cx^2)^{\frac{5}{2}}}\,dx + \int \frac{bx^5}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for}\: m = 4 \\ \frac{amx(dx)^m}{m^2(cx^2)^{\frac{5}{2}} - 7m(cx^2)^{\frac{5}{2}} + 12(cx^2)^{\frac{5}{2}}} - \frac{3ax(dx)^m}{m^2(cx^2)^{\frac{5}{2}} - 7m(cx^2)^{\frac{5}{2}} + 12(cx^2)^{\frac{5}{2}}} + \frac{bmx^2(dx)^m}{m^2(cx^2)^{\frac{5}{2}} - 7m(cx^2)^{\frac{5}{2}} + 12(cx^2)^{\frac{5}{2}}} - \frac{4bx^2(dx)^m}{m^2(cx^2)^{\frac{5}{2}} - 7m(cx^2)^{\frac{5}{2}} + 12(cx^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**3*(Integral(a*x**3/(c*x**2)**(5/2), x) + Integral(b*x**4/(c*x
**2)**(5/2), x)), Eq(m, 3)), (d**4*(Integral(a*x**4/(c*x**2)**(5/2), x) + I
ntegral(b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a*m*x*(d*x)**m/(m**2*(c*x*
*2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) - 3*a*x*(d*x)**m/(m*
*2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) + b*m*x**2*(
d*x)**m/(m**2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2)**(5/2)) -
 4*b*x**2*(d*x)**m/(m**2*(c*x**2)**(5/2) - 7*m*(c*x**2)**(5/2) + 12*(c*x**2
)**(5/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(5/2), x)
```

Mupad [B]
time = 0.28, size = 47, normalized size = 0.70

$$-\frac{(d\,x)^m\,(3\,a-a\,m+4\,b\,x-b\,m\,x)}{c^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2-7\,m+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x)
[Out] -((d*x)^m*(3*a - a*m + 4*b*x - b*m*x))/(c^2*x^3*(c*x^2)^(1/2)*(m^2 - 7*m +
12))
```

3.975 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=103

$$\frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}+\frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

```
[Out] a^2*c^2*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x+2*a*b*c^2*(d*x)^(7+m)*(c*x^2)
^(1/2)/d^7/(7+m)/x+b^2*c^2*(d*x)^(8+m)*(c*x^2)^(1/2)/d^8/(8+m)/x
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2c^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}\,(dx)^{m+8}}{d^8(m+8)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (2*a*b*c^2*(d*x)^(7 +
 m)*Sqrt[c*x^2])/(d^7*(7 + m)*x) + (b^2*c^2*(d*x)^(8 + m)*Sqrt[c*x^2])/(d^8
*(8 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx = \frac{\left(c^2\sqrt{cx^2}\right) \int x^5 (dx)^m (a+bx)^2 \, dx}{x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int (dx)^{5+m} (a+bx)^2 \, dx}{d^5 x}$$
$$= \frac{\left(c^2\sqrt{cx^2}\right) \int \left(a^2 (dx)^{5+m} + \frac{2ab(dx)^{6+m}}{d} + \frac{b^2(dx)^{7+m}}{d^2}\right) dx}{d^5 x}$$
$$= \frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x} + \frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x} + \frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 0.47

$$x(dx)^m \left(cx^2\right)^{5/2} \left(\frac{a^2}{6+m} + \frac{2abx}{7+m} + \frac{b^2x^2}{8+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(5/2)*(a^2/(6 + m) + (2*a*b*x)/(7 + m) + (b^2*x^2)/(8 + m
))
```

Maple [A]
time = 0.10, size = 95, normalized size = 0.92

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+13m\,x^2b^2+a^2m^2+28abmx+42x^2b^2+15a^2m+96abx+56a^2\right)(dx)^m\left(c\,x^2\right)^{\frac{5}{2}}}{(8+m)(7+m)(6+m)}$	95
risch	$\frac{c^2x^5\sqrt{c\,x^2}\,\left(b^2m^2x^2+2ab\,m^2x+13m\,x^2b^2+a^2m^2+28abmx+42x^2b^2+15a^2m+96abx+56a^2\right)(dx)^m}{(8+m)(7+m)(6+m)}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+13*b^2*m*x^2+a^2*m^2+28*a*b*m*x+42*b^2*x^2+15*a^
2*m+96*a*b*x+56*a^2)*(d*x)^m*(c*x^2)^(5/2)/(8+m)/(7+m)/(6+m)
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.62

$$\frac{b^2c^{\frac{5}{2}}d^m x^8 x^m}{m+8} + \frac{2\,abc^{\frac{5}{2}}d^m x^7 x^m}{m+7} + \frac{a^2c^{\frac{5}{2}}d^m x^6 x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(5/2)*d^m*x^8*x^m/(m + 8) + 2*a*b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a^2*c
^(5/2)*d^m*x^6*x^m/(m + 6)
```

Fricas [A]
time = 0.78, size = 123, normalized size = 1.19

$$\frac{\left(\left(b^2c^2m^2+13\,b^2c^2m+42\,b^2c^2\right)x^7+2\,\left(abc^2m^2+14\,abc^2m+48\,abc^2\right)x^6+\left(a^2c^2m^2+15\,a^2c^2m+56\,a^2c^2\right)x^5\right)\sqrt{cx^2}\,(dx)^m}{m^3+21\,m^2+146\,m+336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*m^2 + 13*b^2*c^2*m + 42*b^2*c^2)*x^7 + 2*(a*b*c^2*m^2 + 14*a*b*c^
2*m + 48*a*b*c^2)*x^6 + (a^2*c^2*m^2 + 15*a^2*c^2*m + 56*a^2*c^2)*x^5)*sqrt
(c*x^2)*(d*x)^m/(m^3 + 21*m^2 + 146*m + 336)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^8}\,dx+\int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^6}\,dx+\int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^7}\,dx}{d^8} & \text{for}\: m=-8 \\ \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^7}\,dx+\int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^5}\,dx+\int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^6}\,dx}{d^7} & \text{for}\: m=-7 \\ \frac{\int \frac{a^2(cx^2)^{\frac{5}{2}}}{x^6}\,dx+\int \frac{b^2(cx^2)^{\frac{5}{2}}}{x^4}\,dx+\int \frac{2ab(cx^2)^{\frac{5}{2}}}{x^5}\,dx}{d^6} & \text{for}\: m=-6 \\ \frac{a^2m^2x(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{15a^2mx(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{56a^2x(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{2abm^2x^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{28abmx^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{96abx^2(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{b^2m^2x^3(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{13b^2mx^3(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336}+\frac{42b^2x^3(cx^2)^{\frac{5}{2}}(dx)^m}{m^3+21m^2+146m+336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise((((Integral(a**2*(c*x**2)**(5/2)/x**8, x) + Integral(b**2*(c*x**2)
**(5/2)/x**6, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**7, x))/d**8, Eq(m, -8)
), ((Integral(a**2*(c*x**2)**(5/2)/x**7, x) + Integral(b**2*(c*x**2)**(5/2)
/x**5, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**6, x))/d**7, Eq(m, -7)), ((In
tegral(a**2*(c*x**2)**(5/2)/x**6, x) + Integral(b**2*(c*x**2)**(5/2)/x**4,
x) + Integral(2*a*b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a**2*m**2*
x*(c*x**2)**(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 15*a**2*m*x*(c*
x**2)**(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 56*a**2*x*(c*x**2)**
(5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 2*a*b*m**2*x**2*(c*x**2)**(
5/2)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 28*a*b*m*x**2*(c*x**2)**(5/2
)*(d*x)**m/(m**3 + 21*m**2 + 146*m + 336) + 96*a*b*x**2*(c*x**2)**(5/2)*(d*
x)**m/(m**3 + 21*m**2 + 146*m + 336) + b**2*m**2*x**3*(c*x**2)**(5/2)*(d*x)
**m/(m**3 + 21*m**2 + 146*m + 336) + 13*b**2*m*x**3*(c*x**2)**(5/2)*(d*x)**
m/(m**3 + 21*m**2 + 146*m + 336) + 42*b**2*x**3*(c*x**2)**(5/2)*(d*x)**m/(m
**3 + 21*m**2 + 146*m + 336), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.31, size = 127, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,c^2\,x^5\,\sqrt{c\,x^2}\,\left(m^2+15\,m+56\right)}{m^3+21\,m^2+146\,m+336}+\frac{b^2\,c^2\,x^7\,\sqrt{c\,x^2}\,\left(m^2+13\,m+42\right)}{m^3+21\,m^2+146\,m+336}+\frac{2\,a\,b\,c^2\,x^6\,\sqrt{c\,x^2}\,\left(m^2+14\,m+48\right)}{m^3+21\,m^2+146\,m+336}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c^2*x^5*(c*x^2)^(1/2)*(15*m + m^2 + 56))/(146*m + 21*m^2 + m^
3 + 336) + (b^2*c^2*x^7*(c*x^2)^(1/2)*(13*m + m^2 + 42))/(146*m + 21*m^2 +
m^3 + 336) + (2*a*b*c^2*x^6*(c*x^2)^(1/2)*(14*m + m^2 + 48))/(146*m + 21*m^
2 + m^3 + 336))
```

3.976 $\int (dx)^m (cx^2)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=97

$$\frac{a^2c(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}+\frac{b^2c(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}$$

```
[Out] a^2*c*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x+2*a*b*c*(d*x)^(5+m)*(c*x^2)^(1/
2)/d^5/(5+m)/x+b^2*c*(d*x)^(6+m)*(c*x^2)^(1/2)/d^6/(6+m)/x
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2c\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (2*a*b*c*(d*x)^(5 + m)*
Sqrt[c*x^2])/(d^5*(5 + m)*x) + (b^2*c*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 +
m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2\,dx = \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)^2\,dx}{x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)^2\,dx}{d^3x}$$
$$= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2(dx)^{3+m} + \frac{2ab(dx)^{4+m}}{d} + \frac{b^2(dx)^{5+m}}{d^2}\right)dx}{d^3x}$$
$$= \frac{a^2c(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x} + \frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x} + \frac{b^2c(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 0.49

$$x(dx)^m \left(cx^2\right)^{3/2}\left(\frac{a^2}{4+m} + \frac{2abx}{5+m} + \frac{b^2x^2}{6+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(3/2)*(a^2/(4 + m) + (2*a*b*x)/(5 + m) + (b^2*x^2)/(6 + m
))
```

Maple [A]
time = 0.11, size = 95, normalized size = 0.98

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+9 m\, x^{2} b^{2}+a^{2} m^{2}+20 a b m x+20 x^{2} b^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(d x\right)^{m} \left(c\,x^{2}\right)^{\frac{3}{2}}}{\left(6+m\right) \left(5+m\right) \left(4+m\right)}$	95
risch	$\frac{c\,x^{3} \sqrt{c\,x^{2}}\, \left(b^{2} m^{2} x^{2}+2 a b\, m^{2} x+9 m\, x^{2} b^{2}+a^{2} m^{2}+20 a b m x+20 x^{2} b^{2}+11 a^{2} m+48 a b x+30 a^{2}\right) \left(d x\right)^{m}}{\left(6+m\right) \left(5+m\right) \left(4+m\right)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+9*b^2*m*x^2+a^2*m^2+20*a*b*m*x+20*b^2*x^2+11*a^2
*m+48*a*b*x+30*a^2)*(d*x)^m*(c*x^2)^(3/2)/(6+m)/(5+m)/(4+m)
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.66

$$\frac{b^2c^{\frac{3}{2}}d^m x^6 x^m}{m+6} + \frac{2\,abc^{\frac{3}{2}}d^m x^5 x^m}{m+5} + \frac{a^2c^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(3/2)*d^m*x^6*x^m/(m + 6) + 2*a*b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a^2*c
^(3/2)*d^m*x^4*x^m/(m + 4)
```

Fricas [A]
time = 0.65, size = 105, normalized size = 1.08

$$\frac{\left((b^2cm^2 + 9\,b^2cm + 20\,b^2c)x^5 + 2\,(abcm^2 + 10\,abcm + 24\,abc)x^4 + (a^2cm^2 + 11\,a^2cm + 30\,a^2c)x^3\right)\sqrt{cx^2}\,(dx)^m}{m^3 + 15\,m^2 + 74\,m + 120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c*m^2 + 9*b^2*c*m + 20*b^2*c)*x^5 + 2*(a*b*c*m^2 + 10*a*b*c*m + 24*a*
b*c)*x^4 + (a^2*c*m^2 + 11*a^2*c*m + 30*a^2*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^
3 + 15*m^2 + 74*m + 120)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2\left(cx^2\right)^{\frac{3}{2}}}{x^6}\,dx + \int \frac{b^2\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx + \int \frac{2ab\left(cx^2\right)^{\frac{3}{2}}}{x^5}\,dx}{d^6} & \text{for}\: m = -6 \\ \frac{\int \frac{a^2\left(cx^2\right)^{\frac{3}{2}}}{x^5}\,dx + \int \frac{b^2\left(cx^2\right)^{\frac{3}{2}}}{x^3}\,dx + \int \frac{2ab\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx}{d^5} & \text{for}\: m = -5 \\ \frac{\int \frac{a^2\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx + \int \frac{b^2\left(cx^2\right)^{\frac{3}{2}}}{x^2}\,dx + \int \frac{2ab\left(cx^2\right)^{\frac{3}{2}}}{x^3}\,dx}{d^4} & \text{for}\: m = -4 \\ \frac{a^2m^2x\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{11a^2mx\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{30a^2x\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{2abm^2x^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{20abmx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{48abx^2\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{b^2m^2x^3\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{9b^2mx^3\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} + \frac{20b^2x^3\left(cx^2\right)^{\frac{3}{2}}(dx)^m}{m^3+15m^2+74m+120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(3/2)/x**6, x) + Integral(b**2*(c*x**2)
**(3/2)/x**4, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**5, x))/d**6, Eq(m, -6)
), ((Integral(a**2*(c*x**2)**(3/2)/x**5, x) + Integral(b**2*(c*x**2)**(3/2)
/x**3, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**4, x))/d**5, Eq(m, -5)), ((In
tegral(a**2*(c*x**2)**(3/2)/x**4, x) + Integral(b**2*(c*x**2)**(3/2)/x**2,
x) + Integral(2*a*b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a**2*m**2*
x*(c*x**2)**(3/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 11*a**2*m*x*(c*x
**2)**(3/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 30*a**2*x*(c*x**2)**(3
/2)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 2*a*b*m**2*x**2*(c*x**2)**(3/2
)*(d*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 20*a*b*m*x**2*(c*x**2)**(3/2)*(d
*x)**m/(m**3 + 15*m**2 + 74*m + 120) + 48*a*b*x**2*(c*x**2)**(3/2)*(d*x)**m
/(m**3 + 15*m**2 + 74*m + 120) + b**2*m**2*x**3*(c*x**2)**(3/2)*(d*x)**m/(m
**3 + 15*m**2 + 74*m + 120) + 9*b**2*m*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3
+ 15*m**2 + 74*m + 120) + 20*b**2*x**3*(c*x**2)**(3/2)*(d*x)**m/(m**3 + 15*
m**2 + 74*m + 120), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.28, size = 121, normalized size = 1.25

$$(d\,x)^m\,\left(\frac{a^2\,c\,x^3\,\sqrt{c\,x^2}\,\left(m^2+11\,m+30\right)}{m^3+15\,m^2+74\,m+120}+\frac{b^2\,c\,x^5\,\sqrt{c\,x^2}\,\left(m^2+9\,m+20\right)}{m^3+15\,m^2+74\,m+120}+\frac{2\,a\,b\,c\,x^4\,\sqrt{c\,x^2}\,\left(m^2+10\,m+24\right)}{m^3+15\,m^2+74\,m+120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c*x^3*(c*x^2)^(1/2)*(11*m + m^2 + 30))/(74*m + 15*m^2 + m^3 +
 120) + (b^2*c*x^5*(c*x^2)^(1/2)*(9*m + m^2 + 20))/(74*m + 15*m^2 + m^3 + 1
20) + (2*a*b*c*x^4*(c*x^2)^(1/2)*(10*m + m^2 + 24))/(74*m + 15*m^2 + m^3 +
120))
```

3.977 $\int (dx)^m \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=94

$$\frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}+\frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}$$

```
[Out] a^2*(d*x)^(2+m)*(c*x^2)^(1/2)/d^2/(2+m)/x+2*a*b*(d*x)^(3+m)*(c*x^2)^(1/2)/d
^3/(3+m)/x+b^2*(d*x)^(4+m)*(c*x^2)^(1/2)/d^4/(4+m)/x
```

Rubi [A]

time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2\sqrt{cx^2}(dx)^{m+2}}{d^2(m+2)x}+\frac{2ab\sqrt{cx^2}(dx)^{m+3}}{d^3(m+3)x}+\frac{b^2\sqrt{cx^2}(dx)^{m+4}}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (2*a*b*(d*x)^(3 + m)*Sqrt
[c*x^2])/(d^3*(3 + m)*x) + (b^2*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (dx)^m\sqrt{cx^2}\,(a+bx)^2\,dx=\frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)^2\,dx}{x}$$
$$=\frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)^2\,dx}{dx}$$
$$=\frac{\sqrt{cx^2}\,\int \left(a^2(dx)^{1+m}+\frac{2ab(dx)^{2+m}}{d}+\frac{b^2(dx)^{3+m}}{d^2}\right)dx}{dx}$$
$$=\frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}+\frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.77

$$\frac{x(dx)^m\sqrt{cx^2}\,(a^2(12+7m+m^2)+2ab(8+6m+m^2)\,x+b^2(6+5m+m^2)\,x^2)}{(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a^2*(12 + 7*m + m^2) + 2*a*b*(8 + 6*m + m^2)*x + b^
2*(6 + 5*m + m^2)*x^2))/((2 + m)*(3 + m)*(4 + m))
```

Maple [A]
time = 0.12, size = 95, normalized size = 1.01

method	result	size
gosper	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x+5 m\,x^{2} b^{2}+a^{2} m^{2}+12 a b m x+6 x^{2} b^{2}+7 a^{2} m+16 a b x+12 a^{2}\right) \left(d x\right)^{m} \sqrt{c\,x^{2}}}{\left(4+m\right) \left(3+m\right) \left(2+m\right)}$	95
risch	$\frac{x\left(b^{2} m^{2} x^{2}+2 a b\,m^{2} x+5 m\,x^{2} b^{2}+a^{2} m^{2}+12 a b m x+6 x^{2} b^{2}+7 a^{2} m+16 a b x+12 a^{2}\right) \left(d x\right)^{m} \sqrt{c\,x^{2}}}{\left(4+m\right) \left(3+m\right) \left(2+m\right)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+5*b^2*m*x^2+a^2*m^2+12*a*b*m*x+6*b^2*x^2+7*a^2*m
+16*a*b*x+12*a^2)*(d*x)^m*(c*x^2)^(1/2)/(4+m)/(3+m)/(2+m)
```

Maxima [A]
time = 0.29, size = 64, normalized size = 0.68

$$\frac{b^2\sqrt{c}\,d^m x^4 x^m}{m+4}+\frac{2\,ab\sqrt{c}\,d^m x^3 x^m}{m+3}+\frac{a^2\sqrt{c}\,d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(c)*d^m*x^4*x^m/(m + 4) + 2*a*b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a^2*s
qrt(c)*d^m*x^2*x^m/(m + 2)
```

Fricas [A]
time = 0.70, size = 94, normalized size = 1.00

$$\frac{((b^2m^2 + 5\,b^2m + 6\,b^2)x^3 + 2\,(abm^2 + 6\,abm + 8\,ab)x^2 + (a^2m^2 + 7\,a^2m + 12\,a^2)x)\sqrt{cx^2}\,(dx)^m}{m^3 + 9\,m^2 + 26\,m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 6*b^2)*x^3 + 2*(a*b*m^2 + 6*a*b*m + 8*a*b)*x^2 + (a^2
*m^2 + 7*a^2*m + 12*a^2)*x)*sqrt(c*x^2)*(d*x)^m/(m^3 + 9*m^2 + 26*m + 24)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2\sqrt{cx^2}}{x^4}\,dx + \int \frac{b^2\sqrt{cx^2}}{x^2}\,dx + \int \frac{2ab\sqrt{cx^2}}{x^3}\,dx}{d^4} & \text{for}\: m = -4 \\ \frac{\int \frac{a^2\sqrt{cx^2}}{x^3}\,dx + \int \frac{b^2\sqrt{cx^2}}{x}\,dx + \int \frac{2ab\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for}\: m = -3 \\ \frac{\int b^2\sqrt{cx^2}\,dx + \int \frac{a^2\sqrt{cx^2}}{x^2}\,dx + \int \frac{2ab\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for}\: m = -2 \\ \frac{a^2m^2x\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{7a^2mx\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{12a^2x\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{2abm^2x^2\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{12abmx^2\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{16abx^2\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{b^2m^2x^3\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{5b^2mx^3\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} + \frac{6b^2x^3\sqrt{cx^2}\,(dx)^m}{m^3+9m^2+26m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*sqrt(c*x**2)/x**4, x) + Integral(b**2*sqrt(c*x**2
)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)/x**3, x))/d**4, Eq(m, -4)), ((Inte
gral(a**2*sqrt(c*x**2)/x**3, x) + Integral(b**2*sqrt(c*x**2)/x, x) + Integr
al(2*a*b*sqrt(c*x**2)/x**2, x))/d**3, Eq(m, -3)), ((Integral(b**2*sqrt(c*x*
*2), x) + Integral(a**2*sqrt(c*x**2)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)
/x, x))/d**2, Eq(m, -2)), (a**2*m**2*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2
 + 26*m + 24) + 7*a**2*m*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24
) + 12*a**2*x*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 2*a*b*m**
2*x**2*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 12*a*b*m*x**2*sq
rt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 16*a*b*x**2*sqrt(c*x**2)*
(d*x)**m/(m**3 + 9*m**2 + 26*m + 24) + b**2*m**2*x**3*sqrt(c*x**2)*(d*x)**m
/(m**3 + 9*m**2 + 26*m + 24) + 5*b**2*m*x**3*sqrt(c*x**2)*(d*x)**m/(m**3 +
9*m**2 + 26*m + 24) + 6*b**2*x**3*sqrt(c*x**2)*(d*x)**m/(m**3 + 9*m**2 + 26
*m + 24), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.26, size = 116, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,x\,\sqrt{c\,x^2}\,\left(m^2+7\,m+12\right)}{m^3+9\,m^2+26\,m+24}+\frac{b^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2+5\,m+6\right)}{m^3+9\,m^2+26\,m+24}+\frac{2\,a\,b\,x^2\,\sqrt{c\,x^2}\,\left(m^2+6\,m+8\right)}{m^3+9\,m^2+26\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*x*(c*x^2)^(1/2)*(7*m + m^2 + 12))/(26*m + 9*m^2 + m^3 + 24) +
 (b^2*x^3*(c*x^2)^(1/2)*(5*m + m^2 + 6))/(26*m + 9*m^2 + m^3 + 24) + (2*a*b
*x^2*(c*x^2)^(1/2)*(6*m + m^2 + 8))/(26*m + 9*m^2 + m^3 + 24))
```

3.978 $\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}+\frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}$$

```
[Out] a^2*x*(d*x)^m/m/(c*x^2)^(1/2)+2*a*b*x*(d*x)^(1+m)/d/(1+m)/(c*x^2)^(1/2)+b^2
*x*(d*x)^(2+m)/d^2/(2+m)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (2*a*b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c
*x^2]) + (b^2*x*(d*x)^(2 + m))/(d^2*(2 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int (dx)^{-1+m}(a+bx)^2\,dx}{\sqrt{cx^2}}$$
$$= \frac{(dx)\int \left(a^2(dx)^{-1+m} + \frac{2ab(dx)^m}{d} + \frac{b^2(dx)^{1+m}}{d^2}\right)dx}{\sqrt{cx^2}}$$
$$= \frac{a^2x(dx)^m}{m\sqrt{cx^2}} + \frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}} + \frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.77

$$\frac{x(dx)^m\left(a^2(2+3m+m^2)+2abm(2+m)x+b^2m(1+m)x^2\right)}{m(1+m)(2+m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a^2*(2 + 3*m + m^2) + 2*a*b*m*(2 + m)*x + b^2*m*(1 + m)*x^2))/(
m*(1 + m)*(2 + m)*Sqrt[c*x^2])
```

Maple [A]
time = 0.13, size = 79, normalized size = 0.98

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+m\,x^2b^2+a^2m^2+4abmx+3a^2m+2a^2\right)(dx)^m}{(2+m)(1+m)m\sqrt{c\,x^2}}$	79
risch	$\frac{x\left(b^2m^2x^2+2ab\,m^2x+m\,x^2b^2+a^2m^2+4abmx+3a^2m+2a^2\right)(dx)^m}{(2+m)(1+m)m\sqrt{c\,x^2}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+b^2*m*x^2+a^2*m^2+4*a*b*m*x+3*a^2*m+2*a^2)*(d*x)
^m/(2+m)/(1+m)/m/(c*x^2)^(1/2)
```

Maxima [A]
time = 0.31, size = 57, normalized size = 0.70

$$\frac{b^2d^mx^2x^m}{\sqrt{c}\,(m+2)} + \frac{2\,abd^mxx^m}{\sqrt{c}\,(m+1)} + \frac{a^2d^mx^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^2*x^m/(sqrt(c)*(m + 2)) + 2*a*b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a^2
*d^m*x^m/(sqrt(c)*m)
```

Fricas [A]
time = 0.72, size = 85, normalized size = 1.05

$$\frac{(a^2m^2 + 3\,a^2m + (b^2m^2 + b^2m)x^2 + 2\,a^2 + 2\,(abm^2 + 2\,abm)x)\sqrt{cx^2}\,(dx)^m}{(cm^3 + 3\,cm^2 + 2\,cm)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 + 3*a^2*m + (b^2*m^2 + b^2*m)*x^2 + 2*a^2 + 2*(a*b*m^2 + 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c*m^3 + 3*c*m^2 + 2*c*m)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b^2}{\sqrt{cx^2}}\, dx + \int \frac{a^2}{x^2\sqrt{cx^2}}\, dx + \int \frac{2ab}{x\sqrt{cx^2}}\, dx}{d^2} & \text{for}\: m = -2 \\ \frac{\int \frac{2ab}{\sqrt{cx^2}}\, dx + \int \frac{a^2}{x\sqrt{cx^2}}\, dx + \int \frac{b^2x}{\sqrt{cx^2}}\, dx}{d} & \text{for}\: m = -1 \\ \int \frac{(a+bx)^2}{\sqrt{cx^2}}\, dx & \text{for}\: m = 0 \\ \frac{a^2m^2x(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{3a^2mx(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{2a^2x(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{2abm^2x^2(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{4abmx^2(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{b^2m^2x^3(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} + \frac{b^2mx^3(dx)^m}{m^3\sqrt{cx^2}+3m^2\sqrt{cx^2}+2m\sqrt{cx^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b**2/sqrt(c*x**2), x) + Integral(a**2/(x**2*sqrt(c*x**
2)), x) + Integral(2*a*b/(x*sqrt(c*x**2)), x))/d**2, Eq(m, -2)), ((Integral
(2*a*b/sqrt(c*x**2), x) + Integral(a**2/(x*sqrt(c*x**2)), x) + Integral(b**
2*x/sqrt(c*x**2), x))/d, Eq(m, -1)), (Integral((a + b*x)**2/sqrt(c*x**2), x
), Eq(m, 0)), (a**2*m**2*x*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2
) + 2*m*sqrt(c*x**2)) + 3*a**2*m*x*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqr
t(c*x**2) + 2*m*sqrt(c*x**2)) + 2*a**2*x*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m*
*2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 2*a*b*m**2*x**2*(d*x)**m/(m**3*sqrt(c
*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + 4*a*b*m*x**2*(d*x)**m/(m
**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + b**2*m**2*x**3
*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x**2)) + b*
*2*m*x**3*(d*x)**m/(m**3*sqrt(c*x**2) + 3*m**2*sqrt(c*x**2) + 2*m*sqrt(c*x*
*2)), True))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 0.26, size = 62, normalized size = 0.77

$$\frac{(d\,x)^m\,\left(\frac{a^2\,x}{m}+\frac{b^2\,x^3\,(m+1)}{m^2+3\,m+2}+\frac{2\,a\,b\,x^2\,(m+2)}{m^2+3\,m+2}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] ((d*x)^m*((a^2*x)/m + (b^2*x^3*(m + 1))/(3*m + m^2 + 2) + (2*a*b*x^2*(m + 2
))/(3*m + m^2 + 2)))/(c*x^2)^(1/2)
```

3.979 $\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

```
[Out] -a^2*d^2*x*(d*x)^(-2+m)/c/(2-m)/(c*x^2)^(1/2)-2*a*b*d*x*(d*x)^(-1+m)/c/(1-m
)/(c*x^2)^(1/2)+b^2*x*(d*x)^m/c/m/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^2*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (2*a*b*d*x*(d*x)^(-
1 + m))/(c*(1 - m)*Sqrt[c*x^2]) + (b^2*x*(d*x)^m)/(c*m*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{3/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)^2\,dx}{c\sqrt{cx^2}}$$
$$= \frac{(d^3x)\int \left(a^2(dx)^{-3+m} + \frac{2ab(dx)^{-2+m}}{d} + \frac{b^2(dx)^{-1+m}}{d^2}\right)dx}{c\sqrt{cx^2}}$$
$$= -\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}} - \frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}} + \frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.67

$$\frac{x(dx)^m\left(a^2(-1+m)m+2ab(-2+m)mx+b^2\left(2-3m+m^2\right)x^2\right)}{(-2+m)(-1+m)m\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(-1 + m)*m + 2*a*b*(-2 + m)*m*x + b^2*(2 - 3*m + m^2)*x^2))
/((-2 + m)*(-1 + m)*m*(c*x^2)^(3/2))
```

Maple [A]
time = 0.12, size = 83, normalized size = 0.89

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x-3m\,x^2b^2+a^2m^2-4abmx+2x^2b^2-a^2m\right)(dx)^m}{m(-1+m)(-2+m)(c\,x^2)^{\frac{3}{2}}}$	83
risch	$\frac{\left(b^2m^2x^2+2ab\,m^2x-3m\,x^2b^2+a^2m^2-4abmx+2x^2b^2-a^2m\right)(dx)^m}{cx\sqrt{c\,x^2}\;m(-1+m)(-2+m)}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-3*b^2*m*x^2+a^2*m^2-4*a*b*m*x+2*b^2*x^2-a^2*m)*(
d*x)^m/m/(-1+m)/(-2+m)/(c*x^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.63

$$\frac{b^2d^mx^m}{c^{\frac{3}{2}}m} + \frac{2\,abd^mx^m}{c^{\frac{3}{2}}(m-1)x} + \frac{a^2d^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(3/2)*m) + 2*a*b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a^2*d^m*x^m/(
c^(3/2)*(m - 2)*x^2)
```

Fricas [A]
time = 1.66, size = 92, normalized size = 0.99

$$\frac{\left(a^{2} m^{2}-a^{2} m+\left(b^{2} m^{2}-3 b^{2} m+2 b^{2}\right) x^{2}+2\left(a b m^{2}-2 a b m\right) x\right) \sqrt{c x^{2}}\,(d x)^{m}}{\left(c^{2} m^{3}-3 c^{2} m^{2}+2 c^{2} m\right) x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - a^2*m + (b^2*m^2 - 3*b^2*m + 2*b^2)*x^2 + 2*(a*b*m^2 - 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c^2*m^3 - 3*c^2*m^2 + 2*c^2*m)*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \frac{(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\, dx & \text{for}\ m=0 \\ d\left(\int \frac{a^2x}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{b^2x^3}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{2abx^2}{(cx^2)^{\frac{3}{2}}}\, dx\right) & \text{for}\ m=1 \\ d^2\left(\int \frac{a^2x^2}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{b^2x^4}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{2abx^3}{(cx^2)^{\frac{3}{2}}}\, dx\right) & \text{for}\ m=2 \\ \frac{a^2m^2x(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} - \frac{a^2mx(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} + \frac{2abm^2x^2(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} - \frac{4abmx^2(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} + \frac{b^2m^2x^3(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} - \frac{3b^2mx^3(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} + \frac{2b^2x^3(dx)^m}{m^3(cx^2)^{\frac{3}{2}}-3m^2(cx^2)^{\frac{3}{2}}+2m(cx^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((Integral((a + b*x)**2/(c*x**2)**(3/2), x), Eq(m, 0)), (d*(Integr
al(a**2*x/(c*x**2)**(3/2), x) + Integral(b**2*x**3/(c*x**2)**(3/2), x) + In
tegral(2*a*b*x**2/(c*x**2)**(3/2), x)), Eq(m, 1)), (d**2*(Integral(a**2*x**
2/(c*x**2)**(3/2), x) + Integral(b**2*x**4/(c*x**2)**(3/2), x) + Integral(2
*a*b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a**2*m**2*x*(d*x)**m/(m**3*(c*x
**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) - a**2*m*x*(d*x
)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2))
+ 2*a*b*m**2*x**2*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) +
 2*m*(c*x**2)**(3/2)) - 4*a*b*m*x**2*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**
2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) + b**2*m**2*x**3*(d*x)**m/(m**3*(c
*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**(3/2)) - 3*b**2*m*x*
*3*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2) + 2*m*(c*x**2)**
(3/2)) + 2*b**2*x**3*(d*x)**m/(m**3*(c*x**2)**(3/2) - 3*m**2*(c*x**2)**(3/2
) + 2*m*(c*x**2)**(3/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(3/2), x)
```

Mupad [B]
time = 0.32, size = 66, normalized size = 0.71

$$\frac{a^2\,{\left(d\,x\right)}^m}{c\,x\,\sqrt{c\,x^2}\,\left(m-2\right)}+\frac{b\,{\left(d\,x\right)}^m\,\left(2\,a\,m-b\,x+b\,m\,x\right)}{c\,m\,\sqrt{c\,x^2}\,\left(m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m - 2)) + (b*(d*x)^m*(2*a*m - b*x + b*m*x
))/(c*m*(c*x^2)^(1/2)*(m - 1))
```

3.980 $\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

```
[Out] -a^2*d^4*x*(d*x)^(-4+m)/c^2/(4-m)/(c*x^2)^(1/2)-2*a*b*d^3*x*(d*x)^(-3+m)/c^
2/(3-m)/(c*x^2)^(1/2)-b^2*d^2*x*(d*x)^(-2+m)/c^2/(2-m)/(c*x^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {15, 16, 45}

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^2*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (2*a*b*d^3*x*(d*x
)^(-3 + m))/(c^2*(3 - m)*Sqrt[c*x^2]) - (b^2*d^2*x*(d*x)^(-2 + m))/(c^2*(2
- m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(dx)^m(a+bx)^2}{(cx^2)^{5/2}}\,dx = \frac{x\int \frac{(dx)^m(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)^2\,dx}{c^2\sqrt{cx^2}}$$
$$= \frac{(d^5x)\int \left(a^2(dx)^{-5+m}+\frac{2ab(dx)^{-4+m}}{d}+\frac{b^2(dx)^{-3+m}}{d^2}\right)dx}{c^2\sqrt{cx^2}}$$
$$= -\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.69

$$\frac{x(dx)^m\left(a^2(6-5m+m^2)+2ab(8-6m+m^2)\,x+b^2(12-7m+m^2)\,x^2\right)}{(-4+m)(-3+m)(-2+m)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(6 - 5*m + m^2) + 2*a*b*(8 - 6*m + m^2)*x + b^2*(12 - 7*m +
 m^2)*x^2))/((-4 + m)*(-3 + m)*(-2 + m)*(c*x^2)^(5/2))
```

Maple [A]
time = 0.11, size = 95, normalized size = 0.90

method	result	size
gosper	$\frac{x\left(b^2m^2x^2+2ab\,m^2x-7m\,x^2b^2+a^2m^2-12abmx+12x^2b^2-5a^2m+16abx+6a^2\right)(dx)^m}{(-2+m)(-3+m)(-4+m)\left(c\,x^2\right)^{\frac{5}{2}}}$	95
risch	$\frac{\left(b^2m^2x^2+2ab\,m^2x-7m\,x^2b^2+a^2m^2-12abmx+12x^2b^2-5a^2m+16abx+6a^2\right)(dx)^m}{c^2x^3\sqrt{c\,x^2}\,(-2+m)(-3+m)(-4+m)}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-7*b^2*m*x^2+a^2*m^2-12*a*b*m*x+12*b^2*x^2-5*a^2*
m+16*a*b*x+6*a^2)*(d*x)^m/(-2+m)/(-3+m)/(-4+m)/(c*x^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.61

$$\frac{b^2d^mx^m}{c^{\frac{5}{2}}(m-2)x^2}+\frac{2\,abd^mx^m}{c^{\frac{5}{2}}(m-3)x^3}+\frac{a^2d^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(5/2)*(m - 2)*x^2) + 2*a*b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a
^2*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

Fricas [A]
time = 1.19, size = 106, normalized size = 1.01

$$\frac{(a^2m^2 - 5\,a^2m + (b^2m^2 - 7\,b^2m + 12\,b^2)x^2 + 6\,a^2 + 2\,(abm^2 - 6\,abm + 8\,ab)x)\sqrt{cx^2}\,(dx)^m}{(c^3m^3 - 9\,c^3m^2 + 26\,c^3m - 24\,c^3)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - 5*a^2*m + (b^2*m^2 - 7*b^2*m + 12*b^2)*x^2 + 6*a^2 + 2*(a*b*m^2
- 6*a*b*m + 8*a*b)*x)*sqrt(c*x^2)*(d*x)^m/((c^3*m^3 - 9*c^3*m^2 + 26*c^3*m
- 24*c^3)*x^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d^{2} \left(\int \frac{a^{2} x^{2}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx + \int \frac{b^{2} x^{4}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx + \int \frac{2 a b x^{3}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx\right) & \text{for}\: m = 2 \\ d^{3} \left(\int \frac{a^{2} x^{3}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx + \int \frac{b^{2} x^{5}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx + \int \frac{2 a b x^{4}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx\right) & \text{for}\: m = 3 \\ d^{4} \left(\int \frac{a^{2} x^{4}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx + \int \frac{b^{2} x^{6}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx + \int \frac{2 a b x^{5}}{\left(c x^{2}\right)^{\frac{5}{2}}}\, dx\right) & \text{for}\: m = 4 \\ \frac{a^{2} m^{2} x \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} - \frac{5 a^{2} m x \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} + \frac{6 a^{2} x \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} + \frac{2 a b m^{2} x^{2} \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} - \frac{12 a b m x^{2} \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} + \frac{16 a b x^{2} \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} + \frac{b^{2} m^{2} x^{3} \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} - \frac{7 b^{2} m x^{3} \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} + \frac{12 b^{2} x^{3} \left(d x\right)^{m}}{m^{3} \left(c x^{2}\right)^{\frac{5}{2}} - 9 m^{2} \left(c x^{2}\right)^{\frac{5}{2}} + 26 m \left(c x^{2}\right)^{\frac{5}{2}} - 24 \left(c x^{2}\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**2*(Integral(a**2*x**2/(c*x**2)**(5/2), x) + Integral(b**2*x**
4/(c*x**2)**(5/2), x) + Integral(2*a*b*x**3/(c*x**2)**(5/2), x)), Eq(m, 2))
, (d**3*(Integral(a**2*x**3/(c*x**2)**(5/2), x) + Integral(b**2*x**5/(c*x**
2)**(5/2), x) + Integral(2*a*b*x**4/(c*x**2)**(5/2), x)), Eq(m, 3)), (d**4*
(Integral(a**2*x**4/(c*x**2)**(5/2), x) + Integral(b**2*x**6/(c*x**2)**(5/2
), x) + Integral(2*a*b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a**2*m**2*x*(
d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/
2) - 24*(c*x**2)**(5/2)) - 5*a**2*m*x*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m*
*2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) + 6*a**2*x*
(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5
/2) - 24*(c*x**2)**(5/2)) + 2*a*b*m**2*x**2*(d*x)**m/(m**3*(c*x**2)**(5/2)
- 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) - 12*
a*b*m*x**2*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(
c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) + 16*a*b*x**2*(d*x)**m/(m**3*(c*x**2)*
*(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)
) + b**2*m**2*x**3*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2)
+ 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)) - 7*b**2*m*x**3*(d*x)**m/(m**3
```

```
*(c*x**2)**(5/2) - 9*m**2*(c*x**2)**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x*
*2)**(5/2)) + 12*b**2*x**3*(d*x)**m/(m**3*(c*x**2)**(5/2) - 9*m**2*(c*x**2)
**(5/2) + 26*m*(c*x**2)**(5/2) - 24*(c*x**2)**(5/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(5/2), x)
```

Mupad [B]
time = 0.34, size = 82, normalized size = 0.78

$$\frac{a^2\,{\left(d\,x\right)}^m}{c^2\,x^3\,\sqrt{c\,x^2}\,\left(m-4\right)}+\frac{b^2\,{\left(d\,x\right)}^m}{c^2\,x\,\sqrt{c\,x^2}\,\left(m-2\right)}+\frac{2\,a\,b\,{\left(d\,x\right)}^m}{c^2\,x^2\,\sqrt{c\,x^2}\,\left(m-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x)
[Out] (a^2*(d*x)^m)/(c^2*x^3*(c*x^2)^(1/2)*(m - 4)) + (b^2*(d*x)^m)/(c^2*x*(c*x^2
)^(1/2)*(m - 2)) + (2*a*b*(d*x)^m)/(c^2*x^2*(c*x^2)^(1/2)*(m - 3))
```

3.981 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=67

$$\frac{c^2 (dx)^{6+m} \sqrt{cx^2} (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{d^6 (6+m) x}$$

```
[Out] c^2*(d*x)^(6+m)*(b*x+a)^n*hypergeom([-n, 6+m],[7+m],-b*x/a)*(c*x^2)^(1/2)/d
^6/(6+m)/x/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{c^2 \sqrt{cx^2} (dx)^{m+6} (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(m+6,-n;m+7;-\frac{bx}{a}\right)}{d^6 (m+6) x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^2*(d*x)^(6 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 +
 m, -((b*x)/a)])/(d^6*(6 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^n \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)^n \, dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)^n \, dx}{d^5x}\\ &= \frac{\left(c^2\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{5+m}\left(1+\frac{bx}{a}\right)^n \, dx}{d^5x}\\ &= \frac{c^2(dx)^{6+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{d^6(6+m)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.85

$$\frac{x(dx)^m \left(cx^2\right)^{5/2} (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(6+m,-n;7+m;-\frac{bx}{a}\right)}{6+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n*Hypergeometric2F1[6 + m, -n, 7 + m, -(
(b*x)/a)])/((6 + m)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^2\right)^{\frac{5}{2}} (bx+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m*c^2*x^4, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^n, x)
```

3.982 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=65

$$\frac{c(dx)^{4+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{d^4(4+m)x}$$

```
[Out] c*(d*x)^(4+m)*(b*x+a)^n*hypergeom([-n, 4+m],[5+m],-b*x/a)*(c*x^2)^(1/2)/d^4
/(4+m)/x/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{c\sqrt{cx^2}\,(dx)^{m+4}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m+4,-n;m+5;-\frac{bx}{a}\right)}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (c*(d*x)^(4 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m
, -((b*x)/a)])/(d^4*(4 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^n\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)^n\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)^n\,dx}{d^3x}\\ &= \frac{\left(c\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{3+m}\left(1+\frac{bx}{a}\right)^n\,dx}{d^3x}\\ &= \frac{c(dx)^{4+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{d^4(4+m)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.88

$$\frac{x(dx)^m\left(cx^2\right)^{3/2}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(4+m,-n;5+m;-\frac{bx}{a}\right)}{4+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n*Hypergeometric2F1[4 + m, -n, 5 + m, -(
(b*x)/a)])/((4 + m)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^2\right)^{\frac{3}{2}} (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m*c*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{3}{2}} (dx)^m (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**n,x)
[Out] Integral((c*x**2)**(3/2)*(d*x)**m*(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\,x)^m\,\left(c\,x^2\right)^{3/2}\,(a+b\,x)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n,x)
[Out] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^n, x)
```

3.983 $\int (dx)^m \sqrt{cx^2} (a+bx)^n \, dx$

Optimal. Leaf size=64

$$\frac{(dx)^{2+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{d^2(2+m)x}$$

```
[Out] (d*x)^(2+m)*(b*x+a)^n*hypergeom([-n, 2+m],[3+m],-b*x/a)*(c*x^2)^(1/2)/d^2/(
2+m)/x/((1+b*x/a)^n)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{\sqrt{cx^2}\,(dx)^{m+2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m+2,-n;m+3;-\frac{bx}{a}\right)}{d^2(m+2)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] ((d*x)^(2 + m)*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m,
-((b*x)/a)])/(d^2*(2 + m)*x*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int (dx)^m\sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\,\int x(dx)^m(a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,\int (dx)^{1+m}(a+bx)^n\,dx}{dx}\\ &= \frac{\left(\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{1+m}\left(1+\frac{bx}{a}\right)^n\,dx}{dx}\\ &= \frac{(dx)^{2+m}\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{d^2(2+m)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.89

$$\frac{x(dx)^m\sqrt{cx^2}\,(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(2+m,-n;3+m;-\frac{bx}{a}\right)}{2+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a + b*x)^n*Hypergeometric2F1[2 + m, -n, 3 + m, -((b
*x)/a)])/((2 + m)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{c\,x^2}\,(bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2}\,(dx)^m\,(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**n,x)
[Out] Integral(sqrt(c*x**2)*(d*x)**m*(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^n,x, algorithm="giac")
[Out] integrate(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\,x)^m\,\sqrt{c\,x^2}\,(a+b\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^n,x)
[Out] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^n, x)
```

3.984 $\int \frac{(dx)^m(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

```
[Out] x*(d*x)^m*(b*x+a)^n*hypergeom([m, -n],[1+m],-b*x/a)/m/((1+b*x/a)^n)/(c*x^2)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$\frac{x(dx)^m(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(m,-n;m+1;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int (dx)^{-1+m}(a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{\left(dx(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-1+m}\left(1+\frac{bx}{a}\right)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.00

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(m,-n;1+m;-\frac{bx}{a}\right)}{m\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[m, -n, 1 + m, -((b*x)/a)])/(m*Sqrt
[c*x^2]*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\,(bx+a)^n}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*(d*x)^m/sqrt(c*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (a+bx)^n}{\sqrt{cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(1/2),x)
[Out] Integral((d*x)**m*(a + b*x)**n/sqrt(c*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*(d*x)^m/sqrt(c*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x)^m\,(a+b\,x)^n}{\sqrt{c\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(1/2),x)
[Out] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(1/2), x)
```

3.985 $\int \frac{(dx)^m(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{d^2x(dx)^{-2+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

```
[Out] -d^2*x*(d*x)^(-2+m)*(b*x+a)^n*hypergeom([-n, -2+m],[-1+m],-b*x/a)/c/(2-m)/(
(1+b*x/a)^n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$-\frac{d^2x(dx)^{m-2}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-2,-n;m-1;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((d^2*x*(d*x)^(-2 + m)*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -
((b*x)/a)])/(c*(2 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x^3}\,dx}{c\sqrt{cx^2}}\\
&= \frac{(d^3x)\int (dx)^{-3+m}(a+bx)^n\,dx}{c\sqrt{cx^2}}\\
&= \frac{\left(d^3x(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-3+m}\left(1+\frac{bx}{a}\right)^n\,dx}{c\sqrt{cx^2}}\\
&= -\frac{d^2x(dx)^{-2+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{c(2-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.84

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-2+m,-n;-1+m;-\frac{bx}{a}\right)}{(-2+m)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[-2 + m, -n, -1 + m, -((b*x)/a)])/(
(-2 + m)*(c*x^2)^(3/2)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\,(bx+a)^n}{(c\,x^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c^2*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (a+bx)^n}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/(c*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m\,(a+b\,x)^n}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(3/2), x)
```

3.986 $\int \frac{(dx)^m(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{d^4x(dx)^{-4+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

```
[Out] -d^4*x*(d*x)^(-4+m)*(b*x+a)^n*hypergeom([-n, -4+m],[-3+m],-b*x/a)/c^2/(4-m)
/((1+b*x/a)^n)/(c*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 16, 68, 66}

$$-\frac{d^4x(dx)^{m-4}(a+bx)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(m-4,-n;m-3;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((d^4*x*(d*x)^(-4 + m)*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -
((b*x)/a)])/(c^2*(4 - m)*Sqrt[c*x^2]*(1 + (b*x)/a)^n))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
```

```
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^n}{x^5}\,dx}{c^2\sqrt{cx^2}} \\
&= \frac{(d^5x)\int (dx)^{-5+m}(a+bx)^n\,dx}{c^2\sqrt{cx^2}} \\
&= \frac{\left(d^5x(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int (dx)^{-5+m}\left(1+\frac{bx}{a}\right)^n\,dx}{c^2\sqrt{cx^2}} \\
&= -\frac{d^4x(dx)^{-4+m}(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{c^2(4-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.84

$$\frac{x(dx)^m(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\,{}_2F_1\left(-4+m,-n;-3+m;-\frac{bx}{a}\right)}{(-4+m)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x)^n*Hypergeometric2F1[-4 + m, -n, -3 + m, -((b*x)/a)])/(
(-4 + m)*(c*x^2)^(5/2)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\,(bx+a)^n}{(c\,x^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

```
[Out] int((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2)*(b*x + a)^n*(d*x)^m/(c^3*x^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m (a + bx)^n}{(cx^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x)**n/(c*x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x)^m/(c*x^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m\,(a + b\,x)^n}{(c\,x^2)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] int(((d*x)^m*(a + b*x)^n)/(c*x^2)^(5/2), x)
```

3.987 $\int x^3 (cx^2)^p (a+bx)^{-5-2p} \, dx$

Optimal. Leaf size=33

$$\frac{x^4 (cx^2)^p (a+bx)^{-2(2+p)}}{2a(2+p)}$$

```
[Out] 1/2*x^4*(c*x^2)^p/a/(2+p)/((b*x+a)^(4+2*p))
```

Rubi [A]

time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$\frac{x^4 (cx^2)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p)/(2*a*(2 + p)*(a + b*x)^(2*(2 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int x^3 (cx^2)^p (a+bx)^{-5-2p} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{3+2p} (a+bx)^{-5-2p} \, dx \\ &= \frac{x^4 (cx^2)^p (a+bx)^{-2(2+p)}}{2a(2+p)} \end{aligned}$$

Mathematica [A]

time = 0.04, size = 32, normalized size = 0.97

$$\frac{x^4 (cx^2)^p (a+bx)^{-4-2p}}{a(4+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p*(a + b*x)^(-4 - 2*p))/(a*(4 + 2*p))
```

Maple [A]
time = 0.14, size = 32, normalized size = 0.97

method	result	size
gosper	$\frac{x^{4}\left(b x +a \right)^{-4-2 p} \left(c \,x^{2}\right)^{p}}{2 a \left(2+p \right)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^4*(b*x+a)^(-4-2*p)/a/(2+p)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 5)*x^3, x)
```

Fricas [A]
time = 0.95, size = 40, normalized size = 1.21

$$\frac{(bx^5 + ax^4)(cx^2)^p\,(bx + a)^{-2\,p-5}}{2\,(ap + 2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^5 + a*x^4)*(c*x^2)^p*(b*x + a)^(-2*p - 5)/(a*p + 2*a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**p*(b*x+a)**(-5-2*p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.
time = 1.60, size = 74, normalized size = 2.24

$$\frac{\left(cx^2\right)^p bx^5 e^{\left(-2\,p\log(bx+a)-5\,\log(bx+a)\right)} + \left(cx^2\right)^p ax^4 e^{\left(-2\,p\log(bx+a)-5\,\log(bx+a)\right)}}{2\,(ap+2\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2)^p*b*x^5*e^(-2*p*log(b*x + a) - 5*log(b*x + a)) + (c*x^2)^p*a*x
^4*e^(-2*p*log(b*x + a) - 5*log(b*x + a)))/(a*p + 2*a)
```

Mupad [B]
time = 0.27, size = 33, normalized size = 1.00

$$\frac{x^4\,{\left(c\,x^2\right)}^p}{2\,a\,\left(p+2\right)\,{\left(a+b\,x\right)}^{2\,p+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^p)/(a + b*x)^(2*p + 5),x)
```

```
[Out] (x^4*(c*x^2)^p)/(2*a*(p + 2)*(a + b*x)^(2*p + 4))
```

3.988 $\int x^2(cx^2)^p (a+bx)^{-4-2p} dx$

Optimal. Leaf size=32

$$\frac{x^3(cx^2)^p (a+bx)^{-3-2p}}{a(3+2p)}$$

```
[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$\frac{x^3(cx^2)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(-3 - 2*p))/(a*(3 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^2(cx^2)^p (a+bx)^{-4-2p} dx &= \left(x^{-2p}(cx^2)^p\right)\int x^{2+2p}(a+bx)^{-4-2p} dx\\ &= \frac{x^3(cx^2)^p (a+bx)^{-3-2p}}{a(3+2p)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 1.06

$$\frac{x^3(cx^2)^p (a+bx)^{1-2(2+p)}}{a(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
[Out] (x^3*(c*x^2)^p*(a + b*x)^(1 - 2*(2 + p)))/(a*(3 + 2*p))
```

Maple [A]
time = 0.14, size = 33, normalized size = 1.03

method	result	size
gosper	$\frac{x^{3} \left(c\,x^{2}\right)^{p} \left(b x +a \right)^{-3-2 p}}{a \left(3+2 p \right)}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x,method=_RETURNVERBOSE)
[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

Fricas [A]
time = 1.04, size = 40, normalized size = 1.25

$$\frac{(bx^4 + ax^3)(cx^2)^p\,(bx+a)^{-2\,p-4}}{2\,ap + 3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="fricas")
[Out] (b*x^4 + a*x^3)*(c*x^2)^p*(b*x + a)^(-2*p - 4)/(2*a*p + 3*a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**p*(b*x+a)**(-4-2*p),x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

Mupad [B]
time = 0.24, size = 34, normalized size = 1.06

$$\frac{x^3\,{\left(c\,x^2\right)}^p}{a\,\left(2\,p+3\right)\,{\left(a+b\,x\right)}^{2\,p+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^p)/(a + b*x)^(2*p + 4),x)
```

```
[Out] (x^3*(c*x^2)^p)/(a*(2*p + 3)*(a + b*x)^(2*p + 3))
```

3.989 $\int x(cx^2)^p (a+bx)^{-3-2p} dx$

Optimal. Leaf size=33

$$\frac{x^2(cx^2)^p (a+bx)^{-2(1+p)}}{2a(1+p)}$$

```
[Out] 1/2*x^2*(c*x^2)^p/a/(1+p)/((b*x+a)^(2+2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$\frac{x^2(cx^2)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p)/(2*a*(1 + p)*(a + b*x)^(2*(1 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x(cx^2)^p (a+bx)^{-3-2p} dx &= \left(x^{-2p}(cx^2)^p\right)\int x^{1+2p}(a+bx)^{-3-2p} dx\\ &= \frac{x^2(cx^2)^p (a+bx)^{-2(1+p)}}{2a(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.97

$$\frac{x^2(cx^2)^p (a+bx)^{-2-2p}}{a(2+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p*(a + b*x)^(-2 - 2*p))/(a*(2 + 2*p))
```

Maple [A]
time = 0.14, size = 32, normalized size = 0.97

method	result	size
gosper	$\frac{x^{2}\left(b x+a\right)^{-2-2 p} \left(c \,x^{2}\right)^{p}}{2 a \left(1+p \right)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*(b*x+a)^(-2-2*p)/a/(1+p)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 3)*x, x)
```

Fricas [A]
time = 0.87, size = 38, normalized size = 1.15

$$\frac{(bx^3 + ax^2)(cx^2)^p\,(bx + a)^{-2\,p-3}}{2\,(ap + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 + a*x^2)*(c*x^2)^p*(b*x + a)^(-2*p - 3)/(a*p + a)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(27) = 54.
time = 145.45, size = 1409, normalized size = 42.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**p*(b*x+a)**(-3-2*p),x)
```

```
[Out] Piecewise((-(c*x**2)**p/(b**3*x*(b*x)**(2*p)), Eq(a, 0)), (0**(-2*p - 3)*x*
*2*(c*x**2)**p/(2*p + 2), Eq(a, -b*x)), (x**2*(c*x**2)**p*(0**(1/p))**(-2*p
 - 3)/(2*p + 2), Eq(a, 0**(1/p) - b*x)), ((log(x)/a - log(a/b + x)/a)/c, Eq
(p, -1)), (a**4*x**2*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b
*x)**(2*p) + 12*a**6*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p)
+ 30*a**5*b**2*p*x**2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p)
 + 40*a**4*b**3*p*x**3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p
) + 30*a**3*b**4*p*x**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*
p) + 12*a**2*b**5*p*x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2
*p) + 2*a*b**6*p*x**6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) +
3*a**3*b*x**3*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b*x)**(2
*p) + 12*a**6*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p) + 30*a*
*5*b**2*p*x**2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p) + 40*a
**4*b**3*p*x**3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p) + 30*
a**3*b**4*p*x**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*p) + 12
*a**2*b**5*p*x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2*p) + 2
*a*b**6*p*x**6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) + 3*a**2*
b**2*x**4*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b*x)**(2*p)
+ 12*a**6*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p) + 30*a**5*b
**2*p*x**2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p) + 40*a**4*
b**3*p*x**3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p) + 30*a**3
*b**4*p*x**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*p) + 12*a**
2*b**5*p*x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2*p) + 2*a*b
**6*p*x**6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) + a*b**3*x**5
*(c*x**2)**p/(2*a**7*p*(a + b*x)**(2*p) + 2*a**7*(a + b*x)**(2*p) + 12*a**6
*b*p*x*(a + b*x)**(2*p) + 12*a**6*b*x*(a + b*x)**(2*p) + 30*a**5*b**2*p*x**
2*(a + b*x)**(2*p) + 30*a**5*b**2*x**2*(a + b*x)**(2*p) + 40*a**4*b**3*p*x*
*3*(a + b*x)**(2*p) + 40*a**4*b**3*x**3*(a + b*x)**(2*p) + 30*a**3*b**4*p*x
**4*(a + b*x)**(2*p) + 30*a**3*b**4*x**4*(a + b*x)**(2*p) + 12*a**2*b**5*p*
x**5*(a + b*x)**(2*p) + 12*a**2*b**5*x**5*(a + b*x)**(2*p) + 2*a*b**6*p*x**
6*(a + b*x)**(2*p) + 2*a*b**6*x**6*(a + b*x)**(2*p)) + b*x**3*(c*x**2)**p/(
2*a**4*p*(a + b*x)**(2*p) + 2*a**4*(a + b*x)**(2*p) + 6*a**3*b*p*x*(a + b*x
)**(2*p) + 6*a**3*b*x*(a + b*x)**(2*p) + 6*a**2*b**2*p*x**2*(a + b*x)**(2*p
) + 6*a**2*b**2*x**2*(a + b*x)**(2*p) + 2*a*b**3*p*x**3*(a + b*x)**(2*p) +
2*a*b**3*x**3*(a + b*x)**(2*p)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(33) = 66$. time = 0.79, size = 72, normalized size = 2.18

$$\frac{(cx^2)^p\,bx^3e^{(-2\,p\log(bx+a)-3\,\log(bx+a))}+(cx^2)^p\,ax^2e^{(-2\,p\log(bx+a)-3\,\log(bx+a))}}{2\,(ap+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="giac")
```

```
[Out] 1/2*((c*x^2)^p*b*x^3*e^(-2*p*log(b*x + a) - 3*log(b*x + a)) + (c*x^2)^p*a*x
^2*e^(-2*p*log(b*x + a) - 3*log(b*x + a)))/(a*p + a)
```

Mupad [B]
time = 0.22, size = 33, normalized size = 1.00

$$\frac{x^2\,{\left(c\,x^2\right)}^p}{2\,a\,\left(p+1\right)\,{\left(a+b\,x\right)}^{2\,p+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^p)/(a + b*x)^(2*p + 3),x)
[Out] (x^2*(c*x^2)^p)/(2*a*(p + 1)*(a + b*x)^(2*p + 2))
```

3.990 $\int (cx^2)^p (a+bx)^{-2-2p} dx$

Optimal. Leaf size=30

$$\frac{x(cx^2)^p (a+bx)^{-1-2p}}{a(1+2p)}$$

```
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 37}

$$\frac{x(cx^2)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int (cx^2)^p (a+bx)^{-2-2p} dx &= \left(x^{-2p}(cx^2)^p\right) \int x^{2p}(a+bx)^{-2-2p} dx\\ &= \frac{x(cx^2)^p (a+bx)^{-1-2p}}{a(1+2p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.93

$$\frac{x(cx^2)^p (a+bx)^{-1-2p}}{a+2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a + 2*a*p)
```

Maple [A]
time = 0.13, size = 31, normalized size = 1.03

method	result	size
gosper	$\frac{x\left(c\,x^{2}\right)^{p}\left(bx+a\right)^{-1-2p}}{a\left(1+2p\right)}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

Fricas [A]
time = 1.17, size = 36, normalized size = 1.20

$$\frac{(bx^2+ax)(cx^2)^p\,(bx+a)^{-2\,p-2}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(c*x^2)^p*(b*x + a)^(-2*p - 2)/(2*a*p + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{(bx)^{-2p}(cx^2)^p}{b^2x} & \text{for}\ a=0 \\ \frac{0^{-2p-2}x(cx^2)^p}{2p+1} & \text{for}\ a=-bx \\ \frac{x(cx^2)^p\left(0^{\frac{1}{p}}\right)^{-2p-2}}{2p+1} & \text{for}\ a=0^{\frac{1}{p}}-bx \\ \int\frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for}\ p=-\frac{1}{2} \\ \frac{a^3x(cx^2)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}}+\frac{2a^2bx^2(cx^2)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}}+\frac{ab^2x^3(cx^2)^p}{2a^5p(a+bx)^{2p}+a^5(a+bx)^{2p}+8a^4bpx(a+bx)^{2p}+4a^4bx(a+bx)^{2p}+12a^3b^2px^2(a+bx)^{2p}+6a^3b^2x^2(a+bx)^{2p}+8a^2b^3px^3(a+bx)^{2p}+4a^2b^3x^3(a+bx)^{2p}+2ab^4px^4(a+bx)^{2p}+ab^4x^4(a+bx)^{2p}}+\frac{bx^2(cx^2)^p}{2a^3p(a+bx)^{2p}+a^3(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2a^2bx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}+ab^2x^2(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-2-2*p),x)
```

```
[Out] Piecewise((-(c*x**2)**p/(b**2*x*(b*x)**(2*p)), Eq(a, 0)), (0**(-2*p - 2)*x*
(c*x**2)**p/(2*p + 1), Eq(a, -b*x)), (x*(c*x**2)**p*(0**(1/p))**(-2*p - 2)/
(2*p + 1), Eq(a, 0**(1/p) - b*x)), (Integral(1/(sqrt(c*x**2)*(a + b*x)), x)
, Eq(p, -1/2)), (a**3*x*(c*x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a +
b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) +
 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) +
 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) +
2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + 2*a**2*b
*x**2*(c*x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) + 8*a*
*4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2*p*x*
*2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**
3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a
 + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + a*b**2*x**3*(c*x**2)**p/(2
*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**
(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p)
+ 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) +
 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b
**4*x**4*(a + b*x)**(2*p)) + b*x**2*(c*x**2)**p/(2*a**3*p*(a + b*x)**(2*p)
+ a**3*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a**2*b*x*(a + b
*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p) + a*b**2*x**2*(a + b*x)**(2*p
)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

Mupad [B]
time = 0.20, size = 32, normalized size = 1.07

$$\frac{x\,{\left(c\,x^2\right)}^p}{a\,\left(2\,p+1\right)\,{\left(a+b\,x\right)}^{2\,p+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(a + b*x)^(2*p + 2),x)
[Out] (x*(c*x^2)^p)/(a*(2*p + 1)*(a + b*x)^(2*p + 1))
```

3.991 $\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} \, dx$

Optimal. Leaf size=26

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

```
[Out] 1/2*(c*x^2)^p/a/p/((b*x+a)^(2*p))
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-1+2p} (a+bx)^{-1-2p} \, dx \\ &= \frac{(cx^2)^p (a+bx)^{-2p}}{2ap} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Maple [A]
time = 0.12, size = 25, normalized size = 0.96

method	result	size
gosper	$\frac{(bx+a)^{-2p}(c\,x^2)^p}{2ap}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(-2*p)/a/p*(c*x^2)^p
```

Maxima [A]
time = 0.29, size = 27, normalized size = 1.04

$$\frac{c^p e^{(-2\,p\log(bx+a)+2\,p\log(x))}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c^p*e^(-2*p*log(b*x + a) + 2*p*log(x))/(a*p)
```

Fricas [A]
time = 1.06, size = 31, normalized size = 1.19

$$\frac{(bx+a)(cx^2)^p\,(bx+a)^{-2\,p-1}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p - 1)/(a*p)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(19) = 38$.
time = 17.62, size = 250, normalized size = 9.62

$$\begin{cases} -\frac{(bx)^{-2p}(cx^2)^p}{bx} & \text{for}\ a=0 \\ \frac{0^{-2p-1}(cx^2)^p}{2p} & \text{for}\ a=-bx \\ \frac{(cx^2)^p\left(0^{\frac{1}{p}}\right)^{-2p-1}}{2p} & \text{for}\ a=0^{\frac{1}{p}}-bx \\ \frac{\log(x)}{a}-\frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for}\ p=0 \\ \frac{a^2(cx^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}}+\frac{abx(cx^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}}+\frac{bx(cx^2)^p}{2a^2p(a+bx)^{2p}+2abpx(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**p*(b*x+a)**(-1-2*p)/x,x)

[Out] Piecewise((-(c*x**2)**p/(b*x*(b*x)**(2*p)), Eq(a, 0)), (0**(-2*p - 1)*(c*x*
*2)**p/(2*p), Eq(a, -b*x)), ((c*x**2)**p*(0**(1/p))**(-2*p - 1)/(2*p), Eq(a
, 0**(1/p) - b*x)), (log(x)/a - log(a/b + x)/a, Eq(p, 0)), (a**2*(c*x**2)**
p/(2*a**3*p*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x
2*(a + b*x)(2*p)) + a*b*x*(c*x**2)**p/(2*a**3*p*(a + b*x)**(2*p) + 4*a*
*2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p)) + b*x*(c*x**2
)**p/(2*a**2*p*(a + b*x)**(2*p) + 2*a*b*p*x*(a + b*x)**(2*p)), True))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="giac")

[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 1)/x, x)

Mupad [B]
time = 0.26, size = 26, normalized size = 1.00

$$\frac{{\left(c\,x^2\right)}^p}{2\,a\,p\,{\left(a+b\,x\right)}^{2\,p}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^p/(x*(a + b*x)^(2*p + 1)),x)

[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))

3.992 $\int \frac{(cx^2)^p(a+bx)^{-2p}}{x^2} dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

```
[Out] -(c*x^2)^p*(b*x+a)^(1-2*p)/a/(1-2*p)/x
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(1 - 2*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-2+2p}(a+bx)^{-2p} dx \\ &= -\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{1-2p}}{a(-1+2p)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(-1 + 2*p)*x)
```

Maple [A]
time = 0.15, size = 38, normalized size = 1.15

method	result
gosper	$\frac{(bx+a)\left(c\,x^{2}\right)^{p}(bx+a)^{-2p}}{xa(2p-1)}$
risch	$\frac{(bx+a)(bx+a)^{-2p}\mathrm{e}^{\frac{p\left(-i\pi \mathrm{csgn}\left(ix^{2}\right)^{3}+2i\pi \mathrm{csgn}\left(ix^{2}\right)^{2}\mathrm{csgn}(ix)-i\pi\ \mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}(ix)^{2}+i\pi\ \mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}\left(ic\,x^{2}\right)^{2}-i\pi\ \mathrm{csgn}\left(ix^{2}\right)\mathrm{csgn}\left(ic\,x^{2}\right)\mathrm{csgn}(i}{2}}}{(2p-1)ax}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/x^2/((b*x+a)^(2*p)),x,method=_RETURNVERBOSE)
[Out] 1/x*(b*x+a)/a/(2*p-1)*(c*x^2)^p/((b*x+a)^(2*p))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="maxima")
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

Fricas [A]
time = 0.57, size = 37, normalized size = 1.12

$$\frac{(bx+a)(cx^2)^p}{(2\,ap-a)(bx+a)^{2\,p}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="fricas")
[Out] (b*x + a)*(c*x^2)^p/((2*a*p - a)*(b*x + a)^(2*p)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{\sqrt{cx^2}}{bx^2} & \text{for}\: a = 0 \wedge p = \frac{1}{2} \\ -\frac{(bx)^{-2p}\left(cx^2\right)^p}{x} & \text{for}\: a = 0 \\ \int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx & \text{for}\: p = \frac{1}{2} \\ \frac{a\left(cx^2\right)^p}{2apx(a+bx)^{2p}-ax(a+bx)^{2p}} + \frac{bx\left(cx^2\right)^p}{2apx(a+bx)^{2p}-ax(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p/x**2/((b*x+a)**(2*p)),x)
```

```
[Out] Piecewise((-sqrt(c*x**2)/(b*x**2), Eq(a, 0) & Eq(p, 1/2)), (-(c*x**2)**p/(x
*(b*x)**(2*p)), Eq(a, 0)), (Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x), Eq(
p, 1/2)), (a*(c*x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(a + b*x)**(2*p))
+ b*x*(c*x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(a + b*x)**(2*p)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

Mupad [B]
time = 0.24, size = 32, normalized size = 0.97

$$\frac{(c\,x^2)^p\,(a+b\,x)^{1-2\,p}}{a\,x\,\left(2\,p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x^2*(a + b*x)^(2*p)),x)
```

```
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*x*(2*p - 1))
```

3.993 $\int \frac{(cx^2)^p(a+bx)^{1-2p}}{x^3}\,dx$

Optimal. Leaf size=35

$$-\frac{(cx^2)^p\,(a+bx)^{2-2p}}{2a(1-p)x^2}$$

```
[Out] -1/2*(c*x^2)^p*(b*x+a)^(2-2*p)/a/(1-p)/x^2
```

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$-\frac{(cx^2)^p\,(a+bx)^{2-2p}}{2a(1-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] -1/2*((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(1 - p)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p\,(a+bx)^{1-2p}}{x^3}\,dx &= \left(x^{-2p}(cx^2)^p\right)\int x^{-3+2p}(a+bx)^{1-2p}\,dx\\ &= -\frac{(cx^2)^p\,(a+bx)^{2-2p}}{2a(1-p)x^2}\end{aligned}$$

Mathematica [A]

time = 0.04, size = 32, normalized size = 0.91

$$\frac{(cx^2)^p\,(a+bx)^{2-2p}}{a(-2+2p)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
[Out] ((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(-2 + 2*p)*x^2)
```

Maple [A]
time = 0.15, size = 32, normalized size = 0.91

method	result
gosper	$\frac{(bx+a)^{2-2p}(c\,x^2)^p}{2x^2a(p-1)}$
risch	$\frac{(bx+a)^{1-2p}(bx+a)\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix^2\right)^3+2i\pi\mathrm{csgn}\left(ix^2\right)^2\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}(ix)^2+i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}\left(ic\,x^2\right)^2-i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}\left(ic\,x^2\right)\mathrm{csg}}{2}}}{2x^2a(p-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x^2*(b*x+a)^(2-2*p)/a/(p-1)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

Fricas [A]
time = 0.53, size = 37, normalized size = 1.06

$$\frac{(bx+a)(cx^2)^p\,(bx+a)^{-2\,p+1}}{2\,(ap-a)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="fricas")
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 1)/((a*p - a)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p\,(a+bx)^{1-2p}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(1-2*p)/x**3,x)
```

```
[Out] Integral((c*x**2)**p*(a + b*x)**(1 - 2*p)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

Mupad [B]
time = 0.25, size = 50, normalized size = 1.43

$$\frac{\left(\frac{(c\,x^2)^p}{2\,(p-1)}+\frac{b\,x\,(c\,x^2)^p}{2\,a\,(p-1)}\right)\,(a+b\,x)^{1-2\,p}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x)
```

```
[Out] (((c*x^2)^p/(2*(p - 1)) + (b*x*(c*x^2)^p)/(2*a*(p - 1)))*(a + b*x)^(1 - 2*p
))/x^2
```

3.994 $\int \frac{(cx^2)^p(a+bx)^{2-2p}}{x^4}\,dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(3-2p)x^3}$$

```
[Out] -(c*x^2)^p*(b*x+a)^(3-2*p)/a/(3-2*p)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 37}

$$-\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(3-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(3 - 2*p)*x^3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p\,(a+bx)^{2-2p}}{x^4}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{-4+2p}(a+bx)^{2-2p}\,dx\\ &= -\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(3-2p)x^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p\,(a+bx)^{3-2p}}{a(-3+2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(-3 + 2*p)*x^3)
```

Maple [A]
time = 0.19, size = 33, normalized size = 1.00

method	result
gosper	$\frac{(bx+a)^{3-2p}(c\,x^2)^p}{x^3 a(2p-3)}$
risch	$\frac{(bx+a)^{2-2p}(bx+a)\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}\left(ix^2\right)^3+2i\pi\mathrm{csgn}\left(ix^2\right)^2\mathrm{csgn}(ix)-i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}(ix)^2+i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}\left(ic\,x^2\right)^2-i\pi\,\mathrm{csgn}\left(ix^2\right)\mathrm{csgn}\left(ic\,x^2\right)\mathrm{csgn}(}{2}}}{x^3 a(2p-3)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/x^3*(b*x+a)^(3-2*p)/a/(2*p-3)*(c*x^2)^p
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

Fricas [A]
time = 0.90, size = 37, normalized size = 1.12

$$\frac{(bx+a)(cx^2)^p\,(bx+a)^{-2\,p+2}}{(2\,ap-3\,a)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 2)/((2*a*p - 3*a)*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p\,(a+bx)^{2-2p}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(2-2*p)/x**4,x)
```

```
[Out] Integral((c*x**2)**p*(a + b*x)**(2 - 2*p)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

Mupad [B]
time = 0.25, size = 51, normalized size = 1.55

$$\frac{\left(\frac{(c\,x^2)^p}{2\,p-3}+\frac{b\,x\,(c\,x^2)^p}{a\,(2\,p-3)}\right)\,(a+b\,x)^{2-2\,p}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x)
```

```
[Out] (((c*x^2)^p/(2*p - 3) + (b*x*(c*x^2)^p)/(a*(2*p - 3)))*(a + b*x)^(2 - 2*p))
/x^3
```

3.995 $\int x^m(cx^2)^p (a+bx)^{-2-m-2p} dx$

Optimal. Leaf size=38

$$\frac{x^{1+m}(cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {15, 37}

$$\frac{x^{m+1}(cx^2)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]

[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^m\left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx\\ &= \frac{x^{1+m}(cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 1.00

$$\frac{x^{1+m}(cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Maple [A]
time = 0.16, size = 39, normalized size = 1.03

method	result	size
gosper	$\frac{x^{1+m} \left(c\,x^{2}\right)^{p} \left(b x +a \right)^{-1-m-2 p}}{a \left(1+m +2 p \right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

Fricas [A]
time = 0.65, size = 49, normalized size = 1.29

$$\frac{\left(b x^{2} + a x\right) \left(b x + a\right)^{-m - 2 \, p - 2} x^{m} e^{\left(p \log\left(c\right) + 2 \, p \log\left(x\right)\right)}}{a m + 2 \, a p + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*x^m*e^(p*log(c) + 2*p*log(x))/(a*m +
 2*a*p + a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8012 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

Mupad [**B**]
time = 0.34, size = 50, normalized size = 1.32

$$\frac{x\,x^m\,{\left(c\,x^2\right)}^p}{a\,{\left(a+b\,x\right)}^m\,{\left(a+b\,x\right)}^{2\,p}\,\left(a+b\,x\right)\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
```

```
[Out] (x*x^m*(c*x^2)^p)/(a*(a + b*x)^m*(a + b*x)^(2*p)*(a + b*x)*(m + 2*p + 1))
```

3.996 $\int (dx)^m (cx^2)^p (a+bx)^{-2-m-2p} \, dx$

Optimal. Leaf size=39

$$\frac{x(dx)^m (cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

```
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Rubi [A]

time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 20, 37}

$$\frac{x(cx^2)^p (dx)^m (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx = \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^{-2-m-2p}\,dx$$
$$= \left(x^{-m-2p}(dx)^m \left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx$$
$$= \frac{x(dx)^m \left(cx^2\right)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{x(dx)^m \left(cx^2\right)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Maple [A]
time = 0.21, size = 40, normalized size = 1.03

method	result	size
gosper	$\frac{x(dx)^m\left(c\,x^2\right)^p(bx+a)^{-1-m-2p}}{a(1+m+2p)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x,method=_RETURNVERBOSE)
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

Fricas [A]
time = 0.62, size = 57, normalized size = 1.46

$$\frac{(bx^2+ax)(bx+a)^{-m-2\,p-2}(dx)^m\,e^{\left(2\,p\log(dx)+p\log\left(\frac{c}{d^2}\right)\right)}}{am+2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*(d*x)^m*e^(2*p*log(d*x) + p*log(c/d^
2))/(a*m + 2*a*p + a)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8012 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

Mupad [B]
time = 0.26, size = 39, normalized size = 1.00

$$\frac{x\,{\left(d\,x\right)}^m\,{\left(c\,x^2\right)}^p}{a\,{\left(a+b\,x\right)}^{m+2\,p+1}\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
```

```
[Out] (x*(d*x)^m*(c*x^2)^p)/(a*(a + b*x)^(m + 2*p + 1)*(m + 2*p + 1))
```

3.997 $\int x^m(cx^2)^p (a+bx)^n \, dx$

Optimal. Leaf size=63

$$\frac{x^{1+m}(cx^2)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

```
[Out] x^(1+m)*(c*x^2)^p*(b*x+a)^n*hypergeom([-n, 1+m+2*p],[2+m+2*p],-b*x/a)/(1+m+
2*p)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {15, 68, 66}

$$\frac{x^{m+1}(cx^2)^p (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}\int x^m (cx^2)^p (a+bx)^n\,dx &= \left(x^{-2p}(cx^2)^p\right)\int x^{m+2p}(a+bx)^n\,dx\\ &= \left(x^{-2p}(cx^2)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{m+2p}\left(1+\frac{bx}{a}\right)^n dx\\ &= \frac{x^{1+m}(cx^2)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$\frac{x^{1+m}(cx^2)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^n,x]
[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int x^m \left(c\,x^2\right)^p (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^n,x)
[Out] int(x^m*(c*x^2)^p*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\left(cx^2\right)^p\left(a+bx\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**n,x)
```

```
[Out] Integral(x**m*(c*x**2)**p*(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^p*(b*x + a)^n*x^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\,{\left(c\,x^2\right)}^p\,{\left(a+b\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(a + b*x)^n,x)
```

```
[Out] int(x^m*(c*x^2)^p*(a + b*x)^n, x)
```

3.998 $\int (dx)^m (cx^2)^p (a+bx)^n \, dx$

Optimal. Leaf size=68

$$\frac{(dx)^{1+m} (cx^2)^p (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} {}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{d(1+m+2p)}$$

```
[Out] (d*x)^(1+m)*(c*x^2)^p*(b*x+a)^n*hypergeom([-n, 1+m+2*p],[2+m+2*p],-b*x/a)/d
/(1+m+2*p)/((1+b*x/a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 20, 68, 66}

$$\frac{x(cx^2)^p (dx)^m (a+bx)^n \left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,m+2p+1;m+2p+2;-\frac{bx}{a}\right)}{m+2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^p (a+bx)^n\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^n\,dx \\
&= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^n\,dx \\
&= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}\right)\int x^{m+2p}\left(1+\frac{bx}{a}\right)^n\,dx \\
&= \frac{x(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 64, normalized size = 0.94

$$\frac{x(dx)^m\left(cx^2\right)^p(a+bx)^n\left(1+\frac{bx}{a}\right)^{-n}{}_2F_1\left(-n,1+m+2p;2+m+2p;-\frac{bx}{a}\right)}{1+m+2p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^n,x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^n*Hypergeometric2F1[-n, 1 + m + 2*p, 2 + m +
 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*x)/a)^n)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^2\right)^p (bx+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)
```

```
[Out] int((d*x)^m*(c*x^2)^p*(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="fricas")
[Out] integral((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^2)^p (dx)^m (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**n,x)
[Out] Integral((c*x**2)**p*(d*x)**m*(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^n,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^n*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,(c\,x^2)^p\,(a+b\,x)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(a + b*x)^n,x)
[Out] int((d*x)^m*(c*x^2)^p*(a + b*x)^n, x)
```

3.999 $\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$\frac{b^2(a+bx)^3}{3d^3}$$

[Out] 1/3*b^2*(b*x+a)^3/d^3

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/((a*d)/b + d*x)^3,x]

[Out] (b^2*(a + b*x)^3)/(3*d^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int (a+bx)^2\,dx}{d^3}\\ &= \frac{b^2(a+bx)^3}{3d^3}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Maple [A]
time = 0.13, size = 16, normalized size = 0.94

method	result	size
default	$\frac{b^2(bx+a)^3}{3d^3}$	16
gosper	$\frac{b^3x(x^2b^2+3abx+3a^2)}{3d^3}$	28
risch	$\frac{b^5x^3}{3d^3}+\frac{b^4a\,x^2}{d^3}+\frac{b^3a^2x}{d^3}+\frac{b^2a^3}{3d^3}$	46
norman	$\frac{\frac{b^7x^5}{3d}+\frac{5a\,b^6x^4}{3d}+\frac{10a^2b^5x^3}{3d}-\frac{3a^5b^2}{d}-\frac{5a^4b^3x}{d}}{d^2(bx+a)^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^2*(b*x+a)^3/d^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.29, size = 31, normalized size = 1.82

$$\frac{b^5x^3+3\,ab^4x^2+3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.48, size = 31, normalized size = 1.82

$$\frac{b^5x^3+3\,ab^4x^2+3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.03, size = 34, normalized size = 2.00

$$\frac{a^2b^3x}{d^3} + \frac{ab^4x^2}{d^3} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*d/b+d*x)**3,x)
```

```
[Out] a**2*b**3*x/d**3 + a*b**4*x**2/d**3 + b**5*x**3/(3*d**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 2.02, size = 31, normalized size = 1.82

$$\frac{b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

Mupad [B]
time = 0.05, size = 27, normalized size = 1.59

$$\frac{b^3\,x\,\left(3\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*d^3)
```

3.1000 $\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=23

$$\frac{ab^3x}{d^3}+\frac{b^4x^2}{2d^3}$$

[Out] a*b^3*x/d^3+1/2*b^4*x^2/d^3

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{ab^3x}{d^3}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^4/((a*d)/b + d*x)^3,x]

[Out] (a*b^3*x)/d^3 + (b^4*x^2)/(2*d^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx)\, dx}{d^3}\\ &= \frac{ab^3x}{d^3}+\frac{b^4x^2}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^3\left(ax+\frac{bx^2}{2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] (b^3*(a*x + (b*x^2)/2))/d^3
```

Maple [A]
time = 0.14, size = 18, normalized size = 0.78

method	result	size
gosper	$\frac{b^{3} x \left(b x +2 a \right)}{2 d^{3}}$	17
default	$\frac{b^{3} \left(\frac{1}{2} x^{2} b +a x \right)}{d^{3}}$	18
risch	$\frac{a\,b^{3} x}{d^{3}}+\frac{b^{4} x^{2}}{2 d^{3}}$	22
norman	$\frac{\frac{b^{6} x^{4}}{2 d}+\frac{2 a\,b^{5} x^{3}}{d}-\frac{5 a^{4} b^{2}}{2 d}-\frac{4 a^{3} b^{3} x}{d}}{d^{2} \left(b x +a \right)^{2}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
[Out] b^3/d^3*(1/2*x^2*b+a*x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Fricas [A]
time = 0.45, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.87

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*d/b+d*x)**3,x)
```

```
[Out] a*b**3*x/d**3 + b**4*x**2/(2*d**3)
```

Giac [A]
time = 1.31, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^3\,x\,(2\,a+b\,x)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x*(2*a + b*x))/(2*d^3)
```

3.1001 $\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

[Out] b^3*x/d^3

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^3/((a*d)/b + d*x)^3,x]

[Out] (b^3*x)/d^3

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int 1\, dx}{d^3} \\ &= \frac{b^3x}{d^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Maple [A]
time = 0.15, size = 9, normalized size = 1.12

method	result	size
default	$\frac{b^{3} x}{d^{3}}$	9
risch	$\frac{b^{3} x}{d^{3}}$	9
norman	$\frac{\frac{b^{5} x^{3}}{d}-\frac{2 a^{3} b^{2}}{d}-\frac{3 a^{2} b^{3} x}{d}}{d^{2} \left(b x +a \right)^{2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3
```

Maxima [A]
time = 0.28, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

Fricas [A]
time = 0.39, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] b**3*x/d**3
```

Giac [A]
time = 0.77, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3
```

Mupad [B]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^3*x)/d^3
```

3.1002 $\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3} \, dx$

Optimal. Leaf size=13

$$\frac{b^2 \log(a+bx)}{d^3}$$

[Out] b^2*ln(b*x+a)/d^3

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 31}

$$\frac{b^2 \log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2/((a*d)/b + d*x)^3,x]

[Out] (b^2*Log[a + b*x])/d^3

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3} \, dx &= \frac{b^3 \int \frac{1}{a+bx} \, dx}{d^3} \\
&= \frac{b^2 \log(a+bx)}{d^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2 \log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Maple [A]
time = 0.15, size = 14, normalized size = 1.08

method	result	size
default	$\frac{b^2 \ln(bx+a)}{d^3}$	14
norman	$\frac{b^2 \ln(bx+a)}{d^3}$	14
risch	$\frac{b^2 \ln(bx+a)}{d^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(b*x+a)/d^3
```

Maxima [A]
time = 0.29, size = 13, normalized size = 1.00

$$\frac{b^2 \log(bx+a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/d^3
```

Fricas [A]
time = 0.46, size = 13, normalized size = 1.00

$$\frac{b^2 \log(bx+a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(b*x + a)/d^3
```

Sympy [A]
time = 0.03, size = 19, normalized size = 1.46

$$\frac{b^2 \log(ad^3+bd^3x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] b**2*log(a*d**3 + b*d**3*x)/d**3
```

Giac [A]
time = 1.07, size = 14, normalized size = 1.08

$$\frac{b^2 \log(|bx + a|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*x + a))/d^3
```

Mupad [B]
time = 0.05, size = 13, normalized size = 1.00

$$\frac{b^2\,\ln(a + b\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^2*log(a + b*x))/d^3
```

3.1003 $\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{b^2}{d^3(a+bx)}$$

```
[Out] -b^2/d^3/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((a*d)/b + d*x)^3,x]
```

```
[Out] -(b^2/(d^3*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^2}\,dx}{d^3}\\ &= -\frac{b^2}{d^3(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((a*d)/b + d*x)^3,x]
[Out] -(b^2/(d^3*(a + b*x)))
```

Maple [A]
time = 0.13, size = 16, normalized size = 1.07

method	result	size
gosper	$-\frac{b^{2}}{d^{3}\left(b x+a\right)}$	16
default	$-\frac{b^{2}}{d^{3}\left(b x+a\right)}$	16
risch	$-\frac{b^{2}}{d^{3}\left(b x+a\right)}$	16
norman	$\frac{-\frac{a\,b^{2}}{d}-\frac{b^{3} x}{d}}{d^{2} \left(b x+a\right)^{2}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
[Out] -b^2/d^3/(b*x+a)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] -b^2/(b*d^3*x + a*d^3)
```

Fricas [A]
time = 0.41, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -b^2/(b*d^3*x + a*d^3)
```

Sympy [A]
time = 0.06, size = 19, normalized size = 1.27

$$-\frac{b^3}{abd^3 + b^2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(a*b*d**3 + b**2*d**3*x)
```

Giac [A]
time = 1.08, size = 15, normalized size = 1.00

$$-\frac{b^2}{(bx+a)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -b^2/((b*x + a)*d^3)
```

Mupad [B]
time = 0.04, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d*x + (a*d)/b)^3,x)
```

```
[Out] -b^2/(d^3*(a + b*x))
```

3.1004 $\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{3d^3(a+bx)^3}$$

```
[Out] -1/3*b^2/d^3/(b*x+a)^3
```

Rubi [A]

time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/3*b^2/(d^3*(a + b*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^4}\,dx}{d^3}\\ &= -\frac{b^2}{3d^3(a+bx)^3}\end{aligned}$$

Mathematica [A]

time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*((a*d)/b + d*x)^3),x]
[Out] -1/3*b^2/(d^3*(a + b*x)^3)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{b^2}{3d^3(bx+a)^3}$	16
default	$-\frac{b^2}{3d^3(bx+a)^3}$	16
norman	$-\frac{b^2}{3d^3(bx+a)^3}$	16
risch	$-\frac{b^2}{3d^3(bx+a)^3}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/3*b^2/d^3/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.31, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.44, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.
time = 0.12, size = 53, normalized size = 3.12

$$-\frac{b^3}{3a^3bd^3 + 9a^2b^2d^3x + 9ab^3d^3x^2 + 3b^4d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(3*a**3*b*d**3 + 9*a**2*b**2*d**3*x + 9*a*b**3*d**3*x**2 + 3*b**4*d**
3*x**3)
```

Giac [A]
time = 1.32, size = 15, normalized size = 0.88

$$-\frac{b^2}{3(bx+a)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/3*b^2/((b*x + a)^3*d^3)
```

Mupad [B]
time = 0.15, size = 49, normalized size = 2.88

$$-\frac{b^2}{3\left(a^3\,d^3+3\,a^2\,b\,d^3\,x+3\,a\,b^2\,d^3\,x^2+b^3\,d^3\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)),x)
```

```
[Out] -b^2/(3*(a^3*d^3 + b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x))
```

3.1005 $\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{4d^3(a+bx)^4}$$

```
[Out] -1/4*b^2/d^3/(b*x+a)^4
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/4*b^2/(d^3*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^5}\,dx}{d^3}\\ &= -\frac{b^2}{4d^3(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/4*b^2/(d^3*(a + b*x)^4)
```

Maple [A]
time = 0.14, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{b^{2}}{4 d^{3} \left(b x +a \right)^{4}}$	16
default	$-\frac{b^{2}}{4 d^{3} \left(b x +a \right)^{4}}$	16
norman	$-\frac{b^{2}}{4 d^{3} \left(b x +a \right)^{4}}$	16
risch	$-\frac{b^{2}}{4 d^{3} \left(b x +a \right)^{4}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*b^2/d^3/(b*x+a)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.
time = 0.27, size = 61, normalized size = 3.59

$$-\frac{b^{2}}{4\left(b^{4} d^{3} x^{4}+4\, a b^{3} d^{3} x^{3}+6\, a^{2} b^{2} d^{3} x^{2}+4\, a^{3} b d^{3} x+a^{4} d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.
time = 0.49, size = 61, normalized size = 3.59

$$-\frac{b^{2}}{4\left(b^{4} d^{3} x^{4}+4\, a b^{3} d^{3} x^{3}+6\, a^{2} b^{2} d^{3} x^{2}+4\, a^{3} b d^{3} x+a^{4} d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(15) = 30$.
time = 0.14, size = 68, normalized size = 4.00

$$-\frac{b^3}{4a^4bd^3 + 16a^3b^2d^3x + 24a^2b^3d^3x^2 + 16ab^4d^3x^3 + 4b^5d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(4*a**4*b*d**3 + 16*a**3*b**2*d**3*x + 24*a**2*b**3*d**3*x**2 + 16*a*
b**4*d**3*x**3 + 4*b**5*d**3*x**4)
```

Giac [A]
time = 1.05, size = 15, normalized size = 0.88

$$-\frac{b^2}{4\,(bx+a)^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + a)^4*d^3)
```

Mupad [B]
time = 0.06, size = 63, normalized size = 3.71

$$-\frac{b^2}{4\,\left(a^4\,d^3+4\,a^3\,b\,d^3\,x+6\,a^2\,b^2\,d^3\,x^2+4\,a\,b^3\,d^3\,x^3+b^4\,d^3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^2),x)
```

```
[Out] -b^2/(4*(a^4*d^3 + b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^
3*b*d^3*x))
```

3.1006 $\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{5d^3(a+bx)^5}$$

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/5*b^2/(d^3*(a + b*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^6}\,dx}{d^3}\\ &= -\frac{b^2}{5d^3(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/5*b^2/(d^3*(a + b*x)^5)
```

Maple [A]
time = 0.14, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{b^{2}}{5 d^{3} \left(b x +a \right)^{5}}$	16
default	$-\frac{b^{2}}{5 d^{3} \left(b x +a \right)^{5}}$	16
norman	$-\frac{b^{2}}{5 d^{3} \left(b x +a \right)^{5}}$	16
risch	$-\frac{b^{2}}{5 d^{3} \left(b x +a \right)^{5}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(a*d/b+d*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.30, size = 75, normalized size = 4.41

$$-\frac{b^{2}}{5 \left(b^{5} d^{3} x^{5}+5 a b^{4} d^{3} x^{4}+10 a^{2} b^{3} d^{3} x^{3}+10 a^{3} b^{2} d^{3} x^{2}+5 a^{4} b d^{3} x+a^{5} d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.44, size = 75, normalized size = 4.41

$$-\frac{b^{2}}{5 \left(b^{5} d^{3} x^{5}+5 a b^{4} d^{3} x^{4}+10 a^{2} b^{3} d^{3} x^{3}+10 a^{3} b^{2} d^{3} x^{2}+5 a^{4} b d^{3} x+a^{5} d^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(15) = 30$.
time = 0.18, size = 83, normalized size = 4.88

$$-\frac{b^3}{5a^5bd^3 + 25a^4b^2d^3x + 50a^3b^3d^3x^2 + 50a^2b^4d^3x^3 + 25ab^5d^3x^4 + 5b^6d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(5*a**5*b*d**3 + 25*a**4*b**2*d**3*x + 50*a**3*b**3*d**3*x**2 + 50*a*
*2*b**4*d**3*x**3 + 25*a*b**5*d**3*x**4 + 5*b**6*d**3*x**5)
```

Giac [A]
time = 2.50, size = 15, normalized size = 0.88

$$-\frac{b^2}{5\left(bx+a\right)^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/5*b^2/((b*x + a)^5*d^3)
```

Mupad [B]
time = 0.05, size = 77, normalized size = 4.53

$$-\frac{b^2}{5\,\left(a^5\,d^3+5\,a^4\,b\,d^3\,x+10\,a^3\,b^2\,d^3\,x^2+10\,a^2\,b^3\,d^3\,x^3+5\,a\,b^4\,d^3\,x^4+b^5\,d^3\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^3),x)
```

```
[Out] -b^2/(5*(a^5*d^3 + b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^3*b^2*d^3*x^2 + 10*
a^2*b^3*d^3*x^3 + 5*a^4*b*d^3*x))
```

3.1007 $\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{b^5(c+dx)^3}{3d^6}$$

[Out] 1/3*b^5*(d*x+c)^3/d^6

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

[In] Int[((b*c)/d + b*x)^5/(c + d*x)^3,x]

[Out] (b^5*(c + d*x)^3)/(3*d^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3}\,dx &= \frac{b^5\int (c+dx)^2\,dx}{d^5}\\ &= \frac{b^5(c+dx)^3}{3d^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{b^{5}(dx+c)^{3}}{3d^{6}}$	16
gosper	$\frac{b^{5}x\left(d^{2}x^{2}+3cdx+3c^{2}\right)}{3d^{5}}$	28
risch	$\frac{b^{5}x^{3}}{3d^{3}}+\frac{b^{5}c\,x^{2}}{d^{4}}+\frac{b^{5}c^{2}x}{d^{5}}+\frac{b^{5}c^{3}}{3d^{6}}$	46
norman	$\frac{\frac{b^{5}d^{3}x^{5}}{3}+\frac{5c^{3}b^{5}x^{2}}{2}-\frac{c^{5}b^{5}}{2d^{2}}+\frac{5b^{5}c\,d^{2}x^{4}}{3}+\frac{10b^{5}c^{2}d\,x^{3}}{3}}{d^{4}(dx+c)^{2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^5/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b^5*(d*x+c)^3/d^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.
time = 0.29, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3+3\,b^5cdx^2+3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.
time = 0.58, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3+3\,b^5cdx^2+3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.04, size = 34, normalized size = 2.00

$$\frac{b^5c^2x}{d^5} + \frac{b^5cx^2}{d^4} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**5/(d*x+c)**3,x)
```

```
[Out] b**5*c**2*x/d**5 + b**5*c*x**2/d**4 + b**5*x**3/(3*d**3)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 1.87, size = 35, normalized size = 2.06

$$\frac{b^5d^2x^3 + 3\,b^5cdx^2 + 3\,b^5c^2x}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

Mupad [**B**]
time = 0.16, size = 27, normalized size = 1.59

$$\frac{b^5\,x\,\left(3\,c^2 + 3\,c\,d\,x + d^2\,x^2\right)}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^5/(c + d*x)^3,x)
```

```
[Out] (b^5*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/(3*d^5)
```

3.1008 $\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3} \, dx$

Optimal. Leaf size=23

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

```
[Out] b^4*c*x/d^4+1/2*b^4*x^2/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3} \, dx &= \frac{b^4 \int (c+dx) \, dx}{d^4}\\ &= \frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^4\left(cx+\frac{dx^2}{2}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*(c*x + (d*x^2)/2))/d^4
```

Maple [A]
time = 0.13, size = 18, normalized size = 0.78

method	result	size
gosper	$\frac{b^{4} x \left(d x+2 c\right)}{2 d^{4}}$	17
default	$\frac{b^{4} \left(c x+\frac{1}{2} d \,x^{2}\right)}{d^{4}}$	18
risch	$\frac{b^{4} c x}{d^{4}}+\frac{b^{4} x^{2}}{2 d^{3}}$	22
norman	$\frac{\frac{b^{4} d^{2} x^{4}}{2}-\frac{5 c^{4} b^{4}}{2 d^{2}}+2 b^{4} c d \,x^{3}-\frac{4 c^{3} b^{4} x}{d}}{d^{3} \left(d x+c\right)^{2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^4/d^4*(c*x+1/2*d*x^2)
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Fricas [A]
time = 0.46, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.87

$$\frac{b^4cx}{d^4} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**4/(d*x+c)**3,x)
```

```
[Out] b**4*c*x/d**4 + b**4*x**2/(2*d**3)
```

Giac [A]
time = 1.89, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^4\,x\,(2\,c+d\,x)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^4/(c + d*x)^3,x)
```

```
[Out] (b^4*x*(2*c + d*x))/(2*d^4)
```

3.1009 $\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

[Out] b^3*x/d^3

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((b*c)/d + b*x)^3/(c + d*x)^3,x]

[Out] (b^3*x)/d^3

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx &= \frac{b^3\int 1\,dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Maple [A]
time = 0.14, size = 9, normalized size = 1.12

method	result	size
default	$\frac{b^3 x}{d^3}$	9
risch	$\frac{b^3 x}{d^3}$	9
norman	$\frac{b^3 d\,x^3-\frac{2c^3 b^3}{d^2}-\frac{3c^2 b^3 x}{d}}{d^2 (dx+c)^2}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3
```

Maxima [A]
time = 0.29, size = 8, normalized size = 1.00

$$\frac{b^3 x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3
```

Fricas [A]
time = 0.52, size = 8, normalized size = 1.00

$$\frac{b^3 x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^3*x/d^3
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.88

$$\frac{b^3 x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3
```

Giac [A]
time = 1.53, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3
```

Mupad [B]
time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^3/(c + d*x)^3,x)
```

```
[Out] (b^3*x)/d^3
```

3.1010 $\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=13

$$\frac{b^2\log(c+dx)}{d^3}$$

```
[Out] b^2*ln(d*x+c)/d^3
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 31}

$$\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3}\,dx &= \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2}\\ &= \frac{b^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Maple [A]
time = 0.16, size = 14, normalized size = 1.08

method	result	size
default	$\frac{b^2 \ln(dx+c)}{d^3}$	14
norman	$\frac{b^2 \ln(dx+c)}{d^3}$	14
risch	$\frac{b^2 \ln(dx+c)}{d^3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*ln(d*x+c)/d^3
```

Maxima [A]
time = 0.28, size = 13, normalized size = 1.00

$$\frac{b^2 \log(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(d*x + c)/d^3
```

Fricas [A]
time = 0.55, size = 13, normalized size = 1.00

$$\frac{b^2 \log(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(d*x + c)/d^3
```

Sympy [A]
time = 0.02, size = 17, normalized size = 1.31

$$\frac{b^2 \log(cd^2 + d^3x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c*d**2 + d**3*x)/d**3
```

Giac [A]
time = 2.14, size = 14, normalized size = 1.08

$$\frac{b^2 \log(|dx + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3
```

Mupad [B]
time = 0.14, size = 13, normalized size = 1.00

$$\frac{b^2 \ln(c + d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3
```

3.1011 $\int \frac{\frac{bc}{d}+bx}{(c+dx)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{b}{d^2(c+dx)}$$

[Out] -b/d^2/(d*x+c)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*c)/d + b*x)/(c + d*x)^3,x]

[Out] -(b/(d^2*(c + d*x)))

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\frac{bc}{d}+bx}{(c+dx)^3}\,dx &= \frac{b\int \frac{1}{(c+dx)^2}\,dx}{d}\\ &= -\frac{b}{d^2(c+dx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

Maple [A]
time = 0.13, size = 14, normalized size = 1.08

method	result	size
gosper	$-\frac{b}{d^2(dx+c)}$	14
default	$-\frac{b}{d^2(dx+c)}$	14
risch	$-\frac{b}{d^2(dx+c)}$	14
norman	$\frac{-\frac{cb}{d^2}-\frac{bx}{d}}{(dx+c)^2}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/d^2/(d*x+c)
```

Maxima [A]
time = 0.33, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -b/(d^3*x + c*d^2)
```

Fricas [A]
time = 0.55, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -b/(d^3*x + c*d^2)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.92

$$-\frac{b}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)**3,x)
```

```
[Out] -b/(c*d**2 + d**3*x)
```

Giac [A]
time = 1.60, size = 13, normalized size = 1.00

$$-\frac{b}{(dx+c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -b/((d*x + c)*d^2)
```

Mupad [B]
time = 0.04, size = 13, normalized size = 1.00

$$-\frac{b}{d^2\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)/(c + d*x)^3,x)
```

```
[Out] -b/(d^2*(c + d*x))
```

3.1012 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(c+dx)^3}$$

```
[Out] -1/3/b/(d*x+c)^3
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/3*1/(b*(c + d*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx &= \frac{d\int \frac{1}{(c+dx)^4}\,dx}{b}\\ &= -\frac{1}{3b(c+dx)^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/3*1/(b*(c + d*x)^3)
```

Maple [A]
time = 0.15, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{3b(dx+c)^3}$	13
default	$-\frac{1}{3b(dx+c)^3}$	13
norman	$-\frac{1}{3b(dx+c)^3}$	13
risch	$-\frac{1}{3b(dx+c)^3}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b/(d*x+c)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.
time = 0.29, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.
time = 0.67, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.
time = 0.11, size = 44, normalized size = 3.14

$$-\frac{d}{3bc^3d + 9bc^2d^2x + 9bcd^3x^2 + 3bd^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)**3,x)
[Out] -d/(3*b*c**3*d + 9*b*c**2*d**2*x + 9*b*c*d**3*x**2 + 3*b*d**4*x**3)
```

Giac [A]
time = 2.13, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(dx + c)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
[Out] -1/3/((d*x + c)^3*b)
```

Mupad [B]
time = 0.05, size = 38, normalized size = 2.71

$$-\frac{1}{3\,b\,c^3 + 9\,b\,c^2\,d\,x + 9\,b\,c\,d^2\,x^2 + 3\,b\,d^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)*(c + d*x)^3),x)
[Out] -1/(3*b*c^3 + 3*b*d^3*x^3 + 9*b*c^2*d*x + 9*b*c*d^2*x^2)
```

3.1013 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2(c+dx)^3}\,dx$

Optimal. Leaf size=15

$$-\frac{d}{4b^2(c+dx)^4}$$

[Out] -1/4*d/b^2/(d*x+c)^4

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

[In] Int[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]

[Out] -1/4*d/(b^2*(c + d*x)^4)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2(c+dx)^3}\,dx &= \frac{d^2\int \frac{1}{(c+dx)^5}\,dx}{b^2}\\ &= -\frac{d}{4b^2(c+dx)^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/4*d/(b^2*(c + d*x)^4)
```

Maple [A]
time = 0.13, size = 14, normalized size = 0.93

method	result	size
gosper	$-\frac{d}{4b^2(dx+c)^4}$	14
default	$-\frac{d}{4b^2(dx+c)^4}$	14
norman	$-\frac{d}{4b^2(dx+c)^4}$	14
risch	$-\frac{d}{4b^2(dx+c)^4}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d/b^2/(d*x+c)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.
time = 0.29, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.
time = 0.80, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(14) = 28$.
time = 0.19, size = 68, normalized size = 4.53

$$-\frac{d^2}{4b^2c^4d + 16b^2c^3d^2x + 24b^2c^2d^3x^2 + 16b^2cd^4x^3 + 4b^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] -d**2/(4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2 + 16*b*
*2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

Giac [A]
time = 1.21, size = 20, normalized size = 1.33

$$-\frac{b^2}{4\left(bx + \frac{bc}{d}\right)^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + b*c/d)^4*d^3)
```

Mupad [B]
time = 0.05, size = 61, normalized size = 4.07

$$-\frac{d}{4\left(b^2\,c^4 + 4\,b^2\,c^3\,d\,x + 6\,b^2\,c^2\,d^2\,x^2 + 4\,b^2\,c\,d^3\,x^3 + b^2\,d^4\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^2*(c + d*x)^3),x)
```

```
[Out] -d/(4*(b^2*c^4 + b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*
c^3*d*x))
```

3.1014 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3} dx$

Optimal. Leaf size=17

$$-\frac{d^2}{5b^3(c+dx)^5}$$

```
[Out] -1/5*d^2/b^3/(d*x+c)^5
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/5*d^2/(b^3*(c + d*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3} dx &= \frac{d^3 \int \frac{1}{(c+dx)^6} dx}{b^3}\\ &= -\frac{d^2}{5b^3(c+dx)^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/5*d^2/(b^3*(c + d*x)^5)
```

Maple [A]
time = 0.16, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{d^{2}}{5b^{3}(dx+c)^{5}}$	16
default	$-\frac{d^{2}}{5b^{3}(dx+c)^{5}}$	16
norman	$-\frac{d^{2}}{5b^{3}(dx+c)^{5}}$	16
risch	$-\frac{d^{2}}{5b^{3}(dx+c)^{5}}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*d^2/b^3/(d*x+c)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(15) = 30.
time = 0.28, size = 75, normalized size = 4.41

$$-\frac{d^{2}}{5\left(b^{3}d^{5}x^{5}+5\,b^{3}cd^{4}x^{4}+10\,b^{3}c^{2}d^{3}x^{3}+10\,b^{3}c^{3}d^{2}x^{2}+5\,b^{3}c^{4}dx+b^{3}c^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(15) = 30.
time = 0.71, size = 75, normalized size = 4.41

$$-\frac{d^{2}}{5\left(b^{3}d^{5}x^{5}+5\,b^{3}cd^{4}x^{4}+10\,b^{3}c^{2}d^{3}x^{3}+10\,b^{3}c^{3}d^{2}x^{2}+5\,b^{3}c^{4}dx+b^{3}c^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(15) = 30$.
time = 0.18, size = 83, normalized size = 4.88

$$-\frac{d^3}{5b^3c^5d + 25b^3c^4d^2x + 50b^3c^3d^3x^2 + 50b^3c^2d^4x^3 + 25b^3cd^5x^4 + 5b^3d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] -d**3/(5*b**3*c**5*d + 25*b**3*c**4*d**2*x + 50*b**3*c**3*d**3*x**2 + 50*b*
*3*c**2*d**4*x**3 + 25*b**3*c*d**5*x**4 + 5*b**3*d**6*x**5)
```

Giac [A]
time = 1.96, size = 15, normalized size = 0.88

$$-\frac{d^2}{5\,(dx+c)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/5*d^2/((d*x + c)^5*b^3)
```

Mupad [B]
time = 0.17, size = 77, normalized size = 4.53

$$-\frac{d^2}{5\,\left(b^3\,c^5+5\,b^3\,c^4\,d\,x+10\,b^3\,c^3\,d^2\,x^2+10\,b^3\,c^2\,d^3\,x^3+5\,b^3\,c\,d^4\,x^4+b^3\,d^5\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^3*(c + d*x)^3),x)
```

```
[Out] -d^2/(5*(b^3*c^5 + b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^3*d^2*x^2 + 10*
b^3*c^2*d^3*x^3 + 5*b^3*c^4*d*x))
```

3.1015 $\int (a+bx)^5(ac+bcx)^n\,dx$

Optimal. Leaf size=24

$$\frac{(ac+bcx)^{6+n}}{bc^6(6+n)}$$

```
[Out] (b*c*x+a*c)^(6+n)/b/c^6/(6+n)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] (a*c + b*c*x)^(6 + n)/(b*c^6*(6 + n))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^n\,dx &= \frac{\int (ac+bcx)^{5+n}\,dx}{c^5}\\ &= \frac{(ac+bcx)^{6+n}}{bc^6(6+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.04

$$\frac{(a+bx)^6(c(a+bx))^n}{b(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] ((a + b*x)^6*(c*(a + b*x))^n)/(b*(6 + n))
```

Maple [A]
time = 0.17, size = 27, normalized size = 1.12

method	result
gosper	$\frac{(bx+a)^6(bcx+ac)^n}{b(6+n)}$
risch	$\frac{(x^6b^6+6a\,x^5b^5+15a^2x^4b^4+20a^3b^3x^3+15a^4x^2b^2+6a^5xb+a^6)(c(bx+a))^n}{b(6+n)}$
norman	$\frac{a^6 e^{n\ln(bcx+ac)}}{b(6+n)}+\frac{b^5x^6e^{n\ln(bcx+ac)}}{6+n}+\frac{6a^5x\,e^{n\ln(bcx+ac)}}{6+n}+\frac{6a\,b^4x^5e^{n\ln(bcx+ac)}}{6+n}+\frac{15a^2b^3x^4e^{n\ln(bcx+ac)}}{6+n}+\frac{20a^3b^2x^3e^{n\ln(b}}{6+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^6/b/(6+n)*(b*c*x+a*c)^n
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(24) = 48$.
time = 0.33, size = 649, normalized size = 27.04

$$\frac{5\,(b^2c^n(n+1)x^2+abc^nnx-a^2c^n)(bx+a)^na^4}{(n^2+3\,n+2)b}+\frac{10\,((n^2+3\,n+2)b^3c^nx^3+(n^2+n)ab^2c^nx^2-2\,a^2bc^nnx+2\,a^3c^n)(bx+a)^na^3}{(n^3+6\,n^2+11\,n+6)b}+\frac{(bcx+ac)^{n+1}a^5}{bc(n+1)}+\frac{10\,((n^3+6\,n^2+11\,n+6)b^4c^nx^4+(n^3+3\,n^2+2\,n)ab^3c^nx^3-3\,(n^2+n)a^2b^2c^nx^2+6\,a^3bc^nnx-6\,a^4c^n)(bx+a)^na^2}{(n^4+10\,n^3+35\,n^2+50\,n+24)b}+\frac{5\,((n^4+10\,n^3+35\,n^2+50\,n+24)b^5c^nx^5+(n^4+6\,n^3+11\,n^2+6\,n)ab^4c^nx^4-4\,(n^3+3\,n^2+2\,n)a^2b^3c^nx^3+12\,(n^2+n)a^3b^2c^nx^2-24\,a^4bc^nnx+24\,a^5c^n)(bx+a)^na}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b}+\frac{((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6c^nx^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5c^nx^5-5\,(n^4+6\,n^3+11\,n^2+6\,n)a^2b^4c^nx^4+20\,(n^3+3\,n^2+2\,n)a^3b^3c^nx^3-60\,(n^2+n)a^4b^2c^nx^2+120\,a^5bc^nnx-120\,a^6c^n)(bx+a)^n}{(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 5*(b^2*c^n*(n + 1)*x^2 + a*b*c^n*n*x - a^2*c^n)*(b*x + a)^n*a^4/((n^2 + 3*n
 + 2)*b) + 10*((n^2 + 3*n + 2)*b^3*c^n*x^3 + (n^2 + n)*a*b^2*c^n*x^2 - 2*a^
2*b*c^n*n*x + 2*a^3*c^n)*(b*x + a)^n*a^3/((n^3 + 6*n^2 + 11*n + 6)*b) + (b*
c*x + a*c)^(n + 1)*a^5/(b*c*(n + 1)) + 10*((n^3 + 6*n^2 + 11*n + 6)*b^4*c^n
*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^n*x^3 - 3*(n^2 + n)*a^2*b^2*c^n*x^2 + 6*
a^3*b*c^n*n*x - 6*a^4*c^n)*(b*x + a)^n*a^2/((n^4 + 10*n^3 + 35*n^2 + 50*n +
 24)*b) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^n*x^5 + (n^4 + 6*n^3
 + 11*n^2 + 6*n)*a*b^4*c^n*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^n*x^3 + 12
*(n^2 + n)*a^3*b^2*c^n*x^2 - 24*a^4*b*c^n*n*x + 24*a^5*c^n)*(b*x + a)^n*a/(
(n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b) + ((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^6*c^n*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 +
24*n)*a*b^5*c^n*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*c^n*x^4 + 20*(
n^3 + 3*n^2 + 2*n)*a^3*b^3*c^n*x^3 - 60*(n^2 + n)*a^4*b^2*c^n*x^2 + 120*a^5
*b*c^n*n*x - 120*a^6*c^n)*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.
time = 0.81, size = 80, normalized size = 3.33

$$\frac{\left(b^{6} x^{6} + 6 \, a b^{5} x^{5} + 15 \, a^{2} b^{4} x^{4} + 20 \, a^{3} b^{3} x^{3} + 15 \, a^{4} b^{2} x^{2} + 6 \, a^{5} b x + a^{6}\right)\left(b c x + a c\right)^{n}}{b n + 6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] (b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 +
 6*a^5*b*x + a^6)*(b*c*x + a*c)^n/(b*n + 6*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(19) = 38$.
time = 0.67, size = 212, normalized size = 8.83

$$\begin{cases} \frac{x}{a c^{6}} & \text{for}\: b = 0 \wedge n = -6 \\ a^{5} x \left(a c\right)^{n} & \text{for}\: b = 0 \\ \frac{\log{\left(\frac{a}{b} + x \right)}}{b c^{6}} & \text{for}\: n = -6 \\ \frac{a^{6} \left(a c + b c x\right)^{n}}{b n + 6 b} + \frac{6 a^{5} b x \left(a c + b c x\right)^{n}}{b n + 6 b} + \frac{15 a^{4} b^{2} x^{2} \left(a c + b c x\right)^{n}}{b n + 6 b} + \frac{20 a^{3} b^{3} x^{3} \left(a c + b c x\right)^{n}}{b n + 6 b} + \frac{15 a^{2} b^{4} x^{4} \left(a c + b c x\right)^{n}}{b n + 6 b} + \frac{6 a b^{5} x^{5} \left(a c + b c x\right)^{n}}{b n + 6 b} + \frac{b^{6} x^{6} \left(a c + b c x\right)^{n}}{b n + 6 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((x/(a*c**6), Eq(b, 0) & Eq(n, -6)), (a**5*x*(a*c)**n, Eq(b, 0)),
(log(a/b + x)/(b*c**6), Eq(n, -6)), (a**6*(a*c + b*c*x)**n/(b*n + 6*b) + 6*
a**5*b*x*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**4*b**2*x**2*(a*c + b*c*x)**n/
(b*n + 6*b) + 20*a**3*b**3*x**3*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**2*b**4
*x**4*(a*c + b*c*x)**n/(b*n + 6*b) + 6*a*b**5*x**5*(a*c + b*c*x)**n/(b*n +
6*b) + b**6*x**6*(a*c + b*c*x)**n/(b*n + 6*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(24) = 48$.
time = 2.17, size = 141, normalized size = 5.88

$$\frac{\left(b c x + a c\right)^{n} b^{6} x^{6} + 6 \, \left(b c x + a c\right)^{n} a b^{5} x^{5} + 15 \, \left(b c x + a c\right)^{n} a^{2} b^{4} x^{4} + 20 \, \left(b c x + a c\right)^{n} a^{3} b^{3} x^{3} + 15 \, \left(b c x + a c\right)^{n} a^{4} b^{2} x^{2} + 6 \, \left(b c x + a c\right)^{n} a^{5} b x + \left(b c x + a c\right)^{n} a^{6}}{b n + 6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((b*c*x + a*c)^n*b^6*x^6 + 6*(b*c*x + a*c)^n*a*b^5*x^5 + 15*(b*c*x + a*c)^n
*a^2*b^4*x^4 + 20*(b*c*x + a*c)^n*a^3*b^3*x^3 + 15*(b*c*x + a*c)^n*a^4*b^2*
x^2 + 6*(b*c*x + a*c)^n*a^5*b*x + (b*c*x + a*c)^n*a^6)/(b*n + 6*b)
```

Mupad [B]
time = 0.33, size = 107, normalized size = 4.46

$$(a\,c+b\,c\,x)^n\,\left(\frac{a^6}{b\,\left(n+6\right)}+\frac{b^5\,x^6}{n+6}+\frac{6\,a^5\,x}{n+6}+\frac{15\,a^4\,b\,x^2}{n+6}+\frac{6\,a\,b^4\,x^5}{n+6}+\frac{20\,a^3\,b^2\,x^3}{n+6}+\frac{15\,a^2\,b^3\,x^4}{n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^n*(a + b*x)^5,x)
```

```
[Out] (a*c + b*c*x)^n*(a^6/(b*(n + 6)) + (b^5*x^6)/(n + 6) + (6*a^5*x)/(n + 6) +
(15*a^4*b*x^2)/(n + 6) + (6*a*b^4*x^5)/(n + 6) + (20*a^3*b^2*x^3)/(n + 6) +
 (15*a^2*b^3*x^4)/(n + 6))
```

3.1016 $\int (a+bx)^5(ac+bcx)^3\,dx$

Optimal. Leaf size=17

$$\frac{c^3(a+bx)^9}{9b}$$

```
[Out] 1/9*c^3*(b*x+a)^9/b
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^3\,dx &= c^3\int (a+bx)^8\,dx\\ &= \frac{c^3(a+bx)^9}{9b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^3,x]
```

```
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.12, size = 114, normalized size = 6.71

method	result
gosper	$\frac{x\left(b^{8} x^{8}+9 a\, b^{7} x^{7}+36 a^{2} x^{6} b^{6}+84 a^{3} x^{5} b^{5}+126 a^{4} x^{4} b^{4}+126 a^{5} x^{3} b^{3}+84 a^{6} x^{2} b^{2}+36 a^{7} x b+9 a^{8}\right) c^{3}}{9}$
default	$\frac{1}{9} b^{8} c^{3} x^{9}+a\, b^{7} c^{3} x^{8}+4 a^{2} b^{6} c^{3} x^{7}+\frac{28}{3} a^{3} b^{5} c^{3} x^{6}+14 a^{4} b^{4} c^{3} x^{5}+14 a^{5} b^{3} c^{3} x^{4}+\frac{28}{3} a^{6} c^{3} b^{2} x^{3}+4 a^{7} c^{3} b\, x$
norman	$\frac{1}{9} b^{8} c^{3} x^{9}+a\, b^{7} c^{3} x^{8}+4 a^{2} b^{6} c^{3} x^{7}+\frac{28}{3} a^{3} b^{5} c^{3} x^{6}+14 a^{4} b^{4} c^{3} x^{5}+14 a^{5} b^{3} c^{3} x^{4}+\frac{28}{3} a^{6} c^{3} b^{2} x^{3}+4 a^{7} c^{3} b\, x$
risch	$\frac{b^{8} c^{3} x^{9}}{9}+a\, b^{7} c^{3} x^{8}+4 a^{2} b^{6} c^{3} x^{7}+\frac{28 a^{3} b^{5} c^{3} x^{6}}{3}+14 a^{4} b^{4} c^{3} x^{5}+14 a^{5} b^{3} c^{3} x^{4}+\frac{28 a^{6} c^{3} b^{2} x^{3}}{3}+4 a^{7} c^{3} b\, x^{2}+a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^8*c^3*x^9+a*b^7*c^3*x^8+4*a^2*b^6*c^3*x^7+28/3*a^3*b^5*c^3*x^6+14*a^4
*b^4*c^3*x^5+14*a^5*b^3*c^3*x^4+28/3*a^6*c^3*b^2*x^3+4*a^7*c^3*b*x^2+a^8*c^
3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.29, size = 113, normalized size = 6.65

$$\frac{1}{9}\, b^{8} c^{3} x^{9}+a b^{7} c^{3} x^{8}+4\, a^{2} b^{6} c^{3} x^{7}+\frac{28}{3}\, a^{3} b^{5} c^{3} x^{6}+14\, a^{4} b^{4} c^{3} x^{5}+14\, a^{5} b^{3} c^{3} x^{4}+\frac{28}{3}\, a^{6} b^{2} c^{3} x^{3}+4\, a^{7} b c^{3} x^{2}+a^{8} c^{3} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 0.92, size = 113, normalized size = 6.65

$$\frac{1}{9}\, b^{8} c^{3} x^{9}+a b^{7} c^{3} x^{8}+4\, a^{2} b^{6} c^{3} x^{7}+\frac{28}{3}\, a^{3} b^{5} c^{3} x^{6}+14\, a^{4} b^{4} c^{3} x^{5}+14\, a^{5} b^{3} c^{3} x^{4}+\frac{28}{3}\, a^{6} b^{2} c^{3} x^{3}+4\, a^{7} b c^{3} x^{2}+a^{8} c^{3} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(12) = 24$.
time = 0.02, size = 124, normalized size = 7.29

$$a^8c^3x + 4a^7bc^3x^2 + \frac{28a^6b^2c^3x^3}{3} + 14a^5b^3c^3x^4 + 14a^4b^4c^3x^5 + \frac{28a^3b^5c^3x^6}{3} + 4a^2b^6c^3x^7 + ab^7c^3x^8 + \frac{b^8c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**3,x)
```

```
[Out] a**8*c**3*x + 4*a**7*b*c**3*x**2 + 28*a**6*b**2*c**3*x**3/3 + 14*a**5*b**3*
c**3*x**4 + 14*a**4*b**4*c**3*x**5 + 28*a**3*b**5*c**3*x**6/3 + 4*a**2*b**6
*c**3*x**7 + a*b**7*c**3*x**8 + b**8*c**3*x**9/9
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(15) = 30$.
time = 1.54, size = 113, normalized size = 6.65

$$\frac{1}{9}b^8c^3x^9 + ab^7c^3x^8 + 4a^2b^6c^3x^7 + \frac{28}{3}a^3b^5c^3x^6 + 14a^4b^4c^3x^5 + 14a^5b^3c^3x^4 + \frac{28}{3}a^6b^2c^3x^3 + 4a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

Mupad [B]
time = 0.05, size = 113, normalized size = 6.65

$$a^8c^3x + 4a^7bc^3x^2 + \frac{28a^6b^2c^3x^3}{3} + 14a^5b^3c^3x^4 + 14a^4b^4c^3x^5 + \frac{28a^3b^5c^3x^6}{3} + 4a^2b^6c^3x^7 + ab^7c^3x^8 + \frac{b^8c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^3*(a + b*x)^5,x)
```

```
[Out] a^8*c^3*x + (b^8*c^3*x^9)/9 + 4*a^7*b*c^3*x^2 + a*b^7*c^3*x^8 + (28*a^6*b^2
*c^3*x^3)/3 + 14*a^5*b^3*c^3*x^4 + 14*a^4*b^4*c^3*x^5 + (28*a^3*b^5*c^3*x^6
)/3 + 4*a^2*b^6*c^3*x^7
```

3.1017 $\int (a+bx)^5(ac+bcx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(a+bx)^8}{8b}$$

```
[Out] 1/8*c^2*(b*x+a)^8/b
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^2\,dx &= c^2\int (a+bx)^7\,dx\\ &= \frac{c^2(a+bx)^8}{8b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.14, size = 100, normalized size = 5.88

method	result	size
gosper	$\frac{x\left(b^{7} x^{7}+8 a\,b^{6} x^{6}+28 a^{2} b^{5} x^{5}+56 a^{3} b^{4} x^{4}+70 a^{4} b^{3} x^{3}+56 a^{5} b^{2} x^{2}+28 a^{6} b x +8 a^{7}\right) c^{2}}{8}$	80
default	$\frac{1}{8} b^{7} c^{2} x^{8}+a\,b^{6} c^{2} x^{7}+\frac{7}{2} a^{2} b^{5} c^{2} x^{6}+7 a^{3} b^{4} c^{2} x^{5}+\frac{35}{4} a^{4} b^{3} c^{2} x^{4}+7 a^{5} b^{2} c^{2} x^{3}+\frac{7}{2} a^{6} b\,c^{2} x^{2}+a^{7} c^{2} x$	100
norman	$\frac{1}{8} b^{7} c^{2} x^{8}+a\,b^{6} c^{2} x^{7}+\frac{7}{2} a^{2} b^{5} c^{2} x^{6}+7 a^{3} b^{4} c^{2} x^{5}+\frac{35}{4} a^{4} b^{3} c^{2} x^{4}+7 a^{5} b^{2} c^{2} x^{3}+\frac{7}{2} a^{6} b\,c^{2} x^{2}+a^{7} c^{2} x$	100
risch	$\frac{b^{7} c^{2} x^{8}}{8}+a\,b^{6} c^{2} x^{7}+\frac{7 a^{2} b^{5} c^{2} x^{6}}{2}+7 a^{3} b^{4} c^{2} x^{5}+\frac{35 a^{4} b^{3} c^{2} x^{4}}{4}+7 a^{5} b^{2} c^{2} x^{3}+\frac{7 a^{6} b\,c^{2} x^{2}}{2}+a^{7} c^{2} x +\frac{c^{2} a^{8}}{8 b}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^7*c^2*x^8+a*b^6*c^2*x^7+7/2*a^2*b^5*c^2*x^6+7*a^3*b^4*c^2*x^5+35/4*a^
4*b^3*c^2*x^4+7*a^5*b^2*c^2*x^3+7/2*a^6*b*c^2*x^2+a^7*c^2*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.29, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^{7}c^{2}x^{8}+ab^{6}c^{2}x^{7}+\frac{7}{2}\,a^{2}b^{5}c^{2}x^{6}+7\,a^{3}b^{4}c^{2}x^{5}+\frac{35}{4}\,a^{4}b^{3}c^{2}x^{4}+7\,a^{5}b^{2}c^{2}x^{3}+\frac{7}{2}\,a^{6}bc^{2}x^{2}+a^{7}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.61, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^{7}c^{2}x^{8}+ab^{6}c^{2}x^{7}+\frac{7}{2}\,a^{2}b^{5}c^{2}x^{6}+7\,a^{3}b^{4}c^{2}x^{5}+\frac{35}{4}\,a^{4}b^{3}c^{2}x^{4}+7\,a^{5}b^{2}c^{2}x^{3}+\frac{7}{2}\,a^{6}bc^{2}x^{2}+a^{7}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(12) = 24$.
time = 0.02, size = 110, normalized size = 6.47

$$a^7c^2x + \frac{7a^6bc^2x^2}{2} + 7a^5b^2c^2x^3 + \frac{35a^4b^3c^2x^4}{4} + 7a^3b^4c^2x^5 + \frac{7a^2b^5c^2x^6}{2} + ab^6c^2x^7 + \frac{b^7c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**2,x)
```

```
[Out] a**7*c**2*x + 7*a**6*b*c**2*x**2/2 + 7*a**5*b**2*c**2*x**3 + 35*a**4*b**3*c
**2*x**4/4 + 7*a**3*b**4*c**2*x**5 + 7*a**2*b**5*c**2*x**6/2 + a*b**6*c**2*
x**7 + b**7*c**2*x**8/8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(15) = 30$.
time = 0.89, size = 99, normalized size = 5.82

$$\frac{1}{8}\,b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}\,a^2b^5c^2x^6 + 7\,a^3b^4c^2x^5 + \frac{35}{4}\,a^4b^3c^2x^4 + 7\,a^5b^2c^2x^3 + \frac{7}{2}\,a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

Mupad [B]
time = 0.04, size = 99, normalized size = 5.82

$$a^7\,c^2\,x + \frac{7\,a^6\,b\,c^2\,x^2}{2} + 7\,a^5\,b^2\,c^2\,x^3 + \frac{35\,a^4\,b^3\,c^2\,x^4}{4} + 7\,a^3\,b^4\,c^2\,x^5 + \frac{7\,a^2\,b^5\,c^2\,x^6}{2} + a\,b^6\,c^2\,x^7 + \frac{b^7\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^2*(a + b*x)^5,x)
```

```
[Out] a^7*c^2*x + (b^7*c^2*x^8)/8 + (7*a^6*b*c^2*x^2)/2 + a*b^6*c^2*x^7 + 7*a^5*b
^2*c^2*x^3 + (35*a^4*b^3*c^2*x^4)/4 + 7*a^3*b^4*c^2*x^5 + (7*a^2*b^5*c^2*x^
6)/2
```

3.1018 $\int (a+bx)^5(ac+bcx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^7}{7b}$$

```
[Out] 1/7*c*(b*x+a)^7/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{21, 32\}$

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)\,dx &= c\int (a+bx)^6\,dx\\ &= \frac{c(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.13, size = 72, normalized size = 4.80

method	result	size
gosper	$\frac{cx\left(x^{6} b^{6}+7 a\,x^{5} b^{5}+21 a^{2} x^{4} b^{4}+35 a^{3} b^{3} x^{3}+35 a^{4} x^{2} b^{2}+21 a^{5} x b+7 a^{6}\right)}{7}$	67
default	$\frac{1}{7} b^{6} c\,x^{7}+a\,b^{5} c\,x^{6}+3 a^{2} b^{4} c\,x^{5}+5 a^{3} b^{3} c\,x^{4}+5 a^{4} b^{2} c\,x^{3}+3 a^{5} b c\,x^{2}+a^{6} c x$	72
norman	$\frac{1}{7} b^{6} c\,x^{7}+a\,b^{5} c\,x^{6}+3 a^{2} b^{4} c\,x^{5}+5 a^{3} b^{3} c\,x^{4}+5 a^{4} b^{2} c\,x^{3}+3 a^{5} b c\,x^{2}+a^{6} c x$	72
risch	$\frac{b^{6} c\,x^{7}}{7}+a\,b^{5} c\,x^{6}+3 a^{2} b^{4} c\,x^{5}+5 a^{3} b^{3} c\,x^{4}+5 a^{4} b^{2} c\,x^{3}+3 a^{5} b c\,x^{2}+a^{6} c x+\frac{c\,a^{7}}{7 b}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^6*c*x^7+a*b^5*c*x^6+3*a^2*b^4*c*x^5+5*a^3*b^3*c*x^4+5*a^4*b^2*c*x^3+3
*a^5*b*c*x^2+a^6*c*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.29, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^{6}cx^{7}+ab^{5}cx^{6}+3\,a^{2}b^{4}cx^{5}+5\,a^{3}b^{3}cx^{4}+5\,a^{4}b^{2}cx^{3}+3\,a^{5}bcx^{2}+a^{6}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 0.67, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^{6}cx^{7}+ab^{5}cx^{6}+3\,a^{2}b^{4}cx^{5}+5\,a^{3}b^{3}cx^{4}+5\,a^{4}b^{2}cx^{3}+3\,a^{5}bcx^{2}+a^{6}cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(10) = 20$.
time = 0.02, size = 78, normalized size = 5.20

$$a^6cx + 3a^5bcx^2 + 5a^4b^2cx^3 + 5a^3b^3cx^4 + 3a^2b^4cx^5 + ab^5cx^6 + \frac{b^6cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c),x)
```

```
[Out] a**6*c*x + 3*a**5*b*c*x**2 + 5*a**4*b**2*c*x**3 + 5*a**3*b**3*c*x**4 + 3*a*
*2*b**4*c*x**5 + a*b**5*c*x**6 + b**6*c*x**7/7
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(13) = 26$.
time = 1.58, size = 71, normalized size = 4.73

$$\frac{1}{7}b^6cx^7 + ab^5cx^6 + 3a^2b^4cx^5 + 5a^3b^3cx^4 + 5a^4b^2cx^3 + 3a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

Mupad [B]
time = 0.03, size = 71, normalized size = 4.73

$$c\,a^6\,x + 3\,c\,a^5\,b\,x^2 + 5\,c\,a^4\,b^2\,x^3 + 5\,c\,a^3\,b^3\,x^4 + 3\,c\,a^2\,b^4\,x^5 + c\,a\,b^5\,x^6 + \frac{c\,b^6\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)*(a + b*x)^5,x)
```

```
[Out] (b^6*c*x^7)/7 + a^6*c*x + 5*a^4*b^2*c*x^3 + 5*a^3*b^3*c*x^4 + 3*a^2*b^4*c*x
^5 + 3*a^5*b*c*x^2 + a*b^5*c*x^6
```

3.1019 $\int \frac{(a+bx)^5}{ac+bcx}\,dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^5}{5bc}$$

[Out] 1/5*(b*x+a)^5/b/c

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x),x]

[Out] (a + b*x)^5/(5*b*c)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{ac+bcx}\,dx &= \frac{\int (a+bx)^4\,dx}{c}\\ &= \frac{(a+bx)^5}{5bc}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{(bx+a)^5}{5bc}$	16
gosper	$\frac{x(b^4x^4+5a\,b^3x^3+10a^2b^2x^2+10a^3bx+5a^4)}{5c}$	47
norman	$\frac{a^4x}{c}+\frac{a\,b^3x^4}{c}+\frac{b^4x^5}{5c}+\frac{2a^3b\,x^2}{c}+\frac{2b^2a^2x^3}{c}$	58
risch	$\frac{b^4x^5}{5c}+\frac{a\,b^3x^4}{c}+\frac{2b^2a^2x^3}{c}+\frac{2a^3b\,x^2}{c}+\frac{a^4x}{c}+\frac{a^5}{5bc}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(b*x+a)^5/b/c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.
time = 0.28, size = 48, normalized size = 2.82

$$\frac{b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.
time = 0.55, size = 48, normalized size = 2.82

$$\frac{b^4x^5+5\,ab^3x^4+10\,a^2b^2x^3+10\,a^3bx^2+5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(10) = 20$.
time = 0.02, size = 51, normalized size = 3.00

$$\frac{a^4x}{c} + \frac{2a^3bx^2}{c} + \frac{2a^2b^2x^3}{c} + \frac{ab^3x^4}{c} + \frac{b^4x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c),x)
```

```
[Out] a**4*x/c + 2*a**3*b*x**2/c + 2*a**2*b**2*x**3/c + a*b**3*x**4/c + b**4*x**5
/(5*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 2.23, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

Mupad [B]
time = 0.03, size = 57, normalized size = 3.35

$$\frac{a^4\,x}{c} + \frac{b^4\,x^5}{5\,c} + \frac{2\,a^3\,b\,x^2}{c} + \frac{a\,b^3\,x^4}{c} + \frac{2\,a^2\,b^2\,x^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x),x)
```

```
[Out] (a^4*x)/c + (b^4*x^5)/(5*c) + (2*a^3*b*x^2)/c + (a*b^3*x^4)/c + (2*a^2*b^2*
x^3)/c
```

3.1020 $\int \frac{(a+bx)^5}{(ac+bcx)^2} dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^4}{4bc^2}$$

```
[Out] 1/4*(b*x+a)^4/b/c^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^2} dx &= \frac{\int (a+bx)^3 dx}{c^2} \\ &= \frac{(a+bx)^4}{4bc^2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{(bx+a)^4}{4b\,c^2}$	16
gosper	$\frac{x(b^3x^3+4a\,b^2x^2+6a^2bx+4a^3)}{4c^2}$	36
risch	$\frac{b^3x^4}{4c^2}+\frac{b^2a\,x^3}{c^2}+\frac{3b\,a^2x^2}{2c^2}+\frac{a^3x}{c^2}+\frac{a^4}{4b\,c^2}$	55
norman	$\frac{\frac{a^4x}{c}+\frac{b^4x^5}{4c}+\frac{5a\,b^3x^4}{4c}+\frac{5a^3b\,x^2}{2c}+\frac{5b^2a^2x^3}{2c}}{c(bx+a)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^4/b/c^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.29, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.
time = 0.45, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.
time = 0.03, size = 46, normalized size = 2.71

$$\frac{a^3x}{c^2} + \frac{3a^2bx^2}{2c^2} + \frac{ab^2x^3}{c^2} + \frac{b^3x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**2,x)
```

```
[Out] a**3*x/c**2 + 3*a**2*b*x**2/(2*c**2) + a*b**2*x**3/c**2 + b**3*x**4/(4*c**2
)
```

Giac [A]
time = 1.89, size = 18, normalized size = 1.06

$$\frac{(bcx + ac)^4}{4\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*c*x + a*c)^4/(b*c^6)
```

Mupad [B]
time = 0.05, size = 43, normalized size = 2.53

$$\frac{a^3\,x}{c^2} + \frac{b^3\,x^4}{4\,c^2} + \frac{3\,a^2\,b\,x^2}{2\,c^2} + \frac{a\,b^2\,x^3}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^2,x)
```

```
[Out] (a^3*x)/c^2 + (b^3*x^4)/(4*c^2) + (3*a^2*b*x^2)/(2*c^2) + (a*b^2*x^3)/c^2
```

3.1021 $\int \frac{(a+bx)^5}{(ac+bcx)^3} \, dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^3}{3bc^3}$$

[Out] 1/3*(b*x+a)^3/b/c^3

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^3,x]

[Out] (a + b*x)^3/(3*b*c^3)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^3} \, dx &= \frac{\int (a+bx)^2 \, dx}{c^3} \\ &= \frac{(a+bx)^3}{3bc^3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] (a + b*x)^3/(3*b*c^3)
```

Maple [A]
time = 0.15, size = 16, normalized size = 0.94

method	result	size
default	$\frac{(bx+a)^3}{3b\,c^3}$	16
gosper	$\frac{x(x^2b^2+3abx+3a^2)}{3c^3}$	25
risch	$\frac{b^2x^3}{3c^3}+\frac{ba\,x^2}{c^3}+\frac{a^2x}{c^3}+\frac{a^3}{3b\,c^3}$	41
norman	$\frac{\frac{b^4x^5}{3c}+\frac{5a\,b^3x^4}{3c}-\frac{3a^5}{bc}+\frac{10b^2a^2x^3}{3c}-\frac{5a^4x}{c}}{c^2(bx+a)^2}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/b/c^3
```

Maxima [A]
time = 0.31, size = 26, normalized size = 1.53

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Fricas [A]
time = 0.41, size = 26, normalized size = 1.53

$$\frac{b^2x^3+3\,abx^2+3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.03, size = 29, normalized size = 1.71

$$\frac{a^2x}{c^3}+\frac{abx^2}{c^3}+\frac{b^2x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**3,x)
```

```
[Out] a**2*x/c**3 + a*b*x**2/c**3 + b**2*x**3/(3*c**3)
```

Giac [A]
time = 1.92, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

Mupad [B]
time = 0.04, size = 24, normalized size = 1.41

$$\frac{x\,\left(3\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^3,x)
```

```
[Out] (x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*c^3)
```

3.1022 $\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx$

Optimal. Leaf size=18

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

[Out] a*x/c^4+1/2*b*x^2/c^4

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{21\}$

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] (a*x)/c^4 + (b*x^2)/(2*c^4)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^4}\,dx &= \frac{\int (a+bx)\,dx}{c^4}\\ &= \frac{ax}{c^4}+\frac{bx^2}{2c^4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 0.89

$$\frac{ax+\frac{bx^2}{2}}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^4,x]

```
[Out] (a*x + (b*x^2)/2)/c^4
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{x(bx+2a)}{2c^4}$	14
default	$\frac{\frac{1}{2}x^2b+ax}{c^4}$	15
risch	$\frac{ax}{c^4}+\frac{b\,x^2}{2c^4}$	17
norman	$\frac{\frac{b^4x^5}{2c}+\frac{5a\,b^3x^4}{2c}-\frac{9a^5}{2bc}-\frac{10a^3b\,x^2}{c}-\frac{25a^4x}{2c}}{c^3(bx+a)^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*x^2*b+a*x)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Fricas [A]
time = 0.46, size = 15, normalized size = 0.83

$$\frac{bx^2+2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.83

$$\frac{ax}{c^4}+\frac{bx^2}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**4,x)
```

```
[Out] a*x/c**4 + b*x**2/(2*c**4)
```

Giac [A]
time = 2.56, size = 15, normalized size = 0.83

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,(2\,a + b\,x)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^4,x)
```

```
[Out] (x*(2*a + b*x))/(2*c^4)
```

3.1023 $\int \frac{(a+bx)^5}{(ac+bcx)^5} dx$

Optimal. Leaf size=5

$$\frac{x}{c^5}$$

[Out] x/c^5

Rubi [A]
time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 8}

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^5,x]

[Out] x/c^5

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^5} dx &= \frac{\int 1 \, dx}{c^5} \\ &= \frac{x}{c^5}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^5,x]
```

```
[Out] x/c^5
```

Maple [A]
time = 0.15, size = 6, normalized size = 1.20

method	result	size
default	$\frac{x}{c^5}$	6
risch	$\frac{x}{c^5}$	6
norman	$\frac{\frac{b^4x^5}{c}+\frac{a^4x}{c}+\frac{4a\,b^3x^4}{c}+\frac{4a^3b\,x^2}{c}+\frac{6b^2a^2x^3}{c}}{c^4(bx+a)^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^5
```

Maxima [A]
time = 0.30, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] x/c^5
```

Fricas [A]
time = 0.45, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] x/c^5
```

Sympy [A]
time = 0.03, size = 3, normalized size = 0.60

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**5,x)
```

```
[Out] x/c**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.
time = 2.82, size = 15, normalized size = 3.00

$$\frac{bcx + ac}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] (b*c*x + a*c)/(b*c^6)
```

Mupad [B]
time = 0.01, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^5,x)
```

```
[Out] x/c^5
```

3.1024 $\int \frac{(a+bx)^5}{(ac+bcx)^6} dx$

Optimal. Leaf size=13

$$\frac{\log(a+bx)}{bc^6}$$

```
[Out] ln(b*x+a)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 31}

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^6} dx &= \frac{\int \frac{1}{a+bx} dx}{c^6}\\ &= \frac{\log(a+bx)}{bc^6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Maple [A]
time = 0.14, size = 14, normalized size = 1.08

method	result	size
default	$\frac{\ln(bx+a)}{b\,c^{6}}$	14
norman	$\frac{\ln(bx+a)}{b\,c^{6}}$	14
risch	$\frac{\ln(bx+a)}{b\,c^{6}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+a)/b/c^6
```

Maxima [A]
time = 0.29, size = 13, normalized size = 1.00

$$\frac{\log(bx+a)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c^6)
```

Fricas [A]
time = 0.50, size = 13, normalized size = 1.00

$$\frac{\log(bx+a)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] log(b*x + a)/(b*c^6)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 1.31

$$\frac{\log(ac^6+bc^6x)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**6,x)
```

```
[Out] log(a*c**6 + b*c**6*x)/(b*c**6)
```

Giac [A]
time = 1.33, size = 14, normalized size = 1.08

$$\frac{\log(|bx + a|)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/(b*c^6)
```

Mupad [B]
time = 0.04, size = 13, normalized size = 1.00

$$\frac{\ln(a + b\,x)}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^6,x)
```

```
[Out] log(a + b*x)/(b*c^6)
```

3.1025 $\int \frac{(a+bx)^5}{(ac+bcx)^7}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{bc^7(a+bx)}$$

```
[Out] -1/b/c^7/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^7}\,dx &= \frac{\int \frac{1}{(a+bx)^2}\,dx}{c^7}\\ &= -\frac{1}{bc^7(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

Maple [A]
time = 0.15, size = 16, normalized size = 1.07

method	result	size
gosper	$-\frac{1}{b\,c^{7}\left(bx+a\right)}$	16
default	$-\frac{1}{b\,c^{7}\left(bx+a\right)}$	16
risch	$-\frac{1}{b\,c^{7}\left(bx+a\right)}$	16
norman	$\frac{-\frac{a^{5}}{bc}-\frac{b^{4}x^{5}}{c}-\frac{5a^{4}x}{c}-\frac{5a\,b^{3}x^{4}}{c}-\frac{10b^{2}a^{2}x^{3}}{c}-\frac{10a^{3}b\,x^{2}}{c}}{c^{6}\left(bx+a\right)^{6}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/c^7/(b*x+a)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 1.27

$$-\frac{1}{b^{2}c^{7}x+abc^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

Fricas [A]
time = 0.48, size = 19, normalized size = 1.27

$$-\frac{1}{b^{2}c^{7}x+abc^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

Sympy [A]
time = 0.07, size = 17, normalized size = 1.13

$$-\frac{1}{abc^{7}+b^{2}c^{7}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**7,x)
```

```
[Out] -1/(a*b*c**7 + b**2*c**7*x)
```

Giac [A]
time = 2.79, size = 15, normalized size = 1.00

$$-\frac{1}{(bx+a)bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b*c^7)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 1.27

$$-\frac{1}{x\,b^2\,c^7+a\,b\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^7,x)
```

```
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

3.1026 $\int \frac{(a+bx)^5}{(ac+bcx)^8} dx$

Optimal. Leaf size=17

$$-\frac{1}{2bc^8(a+bx)^2}$$

[Out] -1/2/b/c^8/(b*x+a)^2

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^8,x]

[Out] -1/2*1/(b*c^8*(a + b*x)^2)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^8} dx &= \frac{\int \frac{1}{(a+bx)^3} dx}{c^8}\\ &= -\frac{1}{2bc^8(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/2*1/(b*c^8*(a + b*x)^2)
```

Maple [A]
time = 0.12, size = 16, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{2b\,c^{8}\left(bx+a\right)^{2}}$	16
default	$-\frac{1}{2b\,c^{8}\left(bx+a\right)^{2}}$	16
risch	$-\frac{1}{2b\,c^{8}\left(bx+a\right)^{2}}$	16
norman	$\frac{-\frac{5a^{3}b\,x^{2}}{c}-\frac{a^{5}}{2bc}-\frac{b^{4}x^{5}}{2c}-\frac{5a\,b^{3}x^{4}}{2c}-\frac{5b^{2}a^{2}x^{3}}{c}-\frac{5a^{4}x}{2c}}{c^{7}\left(bx+a\right)^{7}}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/c^8/(b*x+a)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.29, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^{3}c^{8}x^{2}+2\,ab^{2}c^{8}x+a^{2}bc^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="maxima")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.43, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^{3}c^{8}x^{2}+2\,ab^{2}c^{8}x+a^{2}bc^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.
time = 0.10, size = 36, normalized size = 2.12

$$-\frac{1}{2a^2bc^8 + 4ab^2c^8x + 2b^3c^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**8,x)
```

```
[Out] -1/(2*a**2*b*c**8 + 4*a*b**2*c**8*x + 2*b**3*c**8*x**2)
```

Giac [A]
time = 2.14, size = 15, normalized size = 0.88

$$-\frac{1}{2\,(bx+a)^2bc^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="giac")
```

```
[Out] -1/2/((b*x + a)^2*b*c^8)
```

Mupad [B]
time = 0.15, size = 35, normalized size = 2.06

$$-\frac{1}{2\,a^2\,b\,c^8+4\,a\,b^2\,c^8\,x+2\,b^3\,c^8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^8,x)
```

```
[Out] -1/(2*a^2*b*c^8 + 2*b^3*c^8*x^2 + 4*a*b^2*c^8*x)
```

3.1027 $\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{2+3x}\,\log(2+3x)}{3\sqrt{-2-3x}}$$

```
[Out] 1/3*ln(2+3*x)*(2+3*x)^(1/2)/(-2-3*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {23, 31}

$$\frac{\sqrt{3x+2}\,\log(3x+2)}{3\sqrt{-3x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
[Out] (Sqrt[2 + 3*x]*Log[2 + 3*x])/(3*Sqrt[-2 - 3*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx &= \frac{\sqrt{2+3x}\,\int \frac{1}{2+3x}\,dx}{\sqrt{-2-3x}}\\ &= \frac{\sqrt{2+3x}\,\log(2+3x)}{3\sqrt{-2-3x}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{(2+3x)\log(2+3x)}{3\sqrt{-(2+3x)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.82

method	result	size
meijerg	$-\frac{i \ln\left(1+\frac{3x}{2}\right)}{3}$	10
default	$\frac{\ln(2+3x)\sqrt{2+3x}}{3\sqrt{-2-3x}}$	23
risch	$-\frac{i\sqrt{\frac{-2-3x}{2+3x}}\,\sqrt{2+3x}\,\ln(2+3x)}{3\sqrt{-2-3x}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(2+3*x)*(2+3*x)^(1/2)/(-2-3*x)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.51, size = 6, normalized size = 0.21

$$\frac{1}{3}i\,\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x + 2/3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 0.54, size = 66, normalized size = 2.36

$$\begin{cases} 0 & \text{for } \frac{1}{\left|x+\frac{2}{3}\right|} < 1 \wedge \left|x+\frac{2}{3}\right| < 1 \\ -\frac{i\log\left(x+\frac{2}{3}\right)}{3} & \text{for } \left|x+\frac{2}{3}\right| < 1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{3}}\right)}{3} & \text{for } \frac{1}{\left|x+\frac{2}{3}\right|} < 1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x+\frac{2}{3}\right)}{3} - \frac{iG_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x+\frac{2}{3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((0, (Abs(x + 2/3) < 1) & (1/Abs(x + 2/3) < 1)), (-I*log(x + 2/3)/
3, Abs(x + 2/3) < 1), (I*log(1/(x + 2/3))/3, 1/Abs(x + 2/3) < 1), (I*meijer
g(((), (1, 1)), ((0, 0), ()), x + 2/3)/3 - I*meijerg(((1, 1), ()), ((), (0,
 0)), x + 2/3)/3, True))
```

Giac [C] Result contains complex when optimal does not.
time = 2.01, size = 11, normalized size = 0.39

$$-\frac{1}{3}i\,\log\left(|3\,x+2|\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*I*log(abs(3*x + 2))*sgn(x)
```

Mupad [B]
time = 0.22, size = 35, normalized size = 1.25

$$-\frac{4\,\mathrm{atan}\left(\frac{-\sqrt{-3\,x-2}+\sqrt{2}\,1\mathrm{i}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3*x - 2)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((2^(1/2)*1i - (- 3*x - 2)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

3.1028 $\int (a+bx)(ac-bcx)^3\,dx$

Optimal. Leaf size=38

$$-\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}$$

```
[Out] -1/2*a*c^3*(-b*x+a)^4/b+1/5*c^3*(-b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {45}

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] -1/2*(a*c^3*(a - b*x)^4)/b + (c^3*(a - b*x)^5)/(5*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^3\,dx &= \int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right)dx\\ &= -\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.05

$$c^3\left(a^4x-a^3bx^2+\frac{1}{2}ab^3x^4-\frac{b^4x^5}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

Maple [A]
time = 0.14, size = 45, normalized size = 1.18

method	result	size
gosper	$\frac{x\left(-2b^{4}x^{4}+5a\,b^{3}x^{3}-10a^{3}bx+10a^{4}\right)c^{3}}{10}$	37
default	$-\frac{1}{5}b^{4}c^{3}x^{5}+\frac{1}{2}a\,b^{3}c^{3}x^{4}-a^{3}b\,c^{3}x^{2}+a^{4}c^{3}x$	45
norman	$-\frac{1}{5}b^{4}c^{3}x^{5}+\frac{1}{2}a\,b^{3}c^{3}x^{4}-a^{3}b\,c^{3}x^{2}+a^{4}c^{3}x$	45
risch	$-\frac{1}{5}b^{4}c^{3}x^{5}+\frac{1}{2}a\,b^{3}c^{3}x^{4}-a^{3}b\,c^{3}x^{2}+a^{4}c^{3}x$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/5*b^4*c^3*x^5+1/2*a*b^3*c^3*x^4-a^3*b*c^3*x^2+a^4*c^3*x
```

Maxima [A]
time = 0.28, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^{4}c^{3}x^{5}+\frac{1}{2}\,ab^{3}c^{3}x^{4}-a^{3}bc^{3}x^{2}+a^{4}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Fricas [A]
time = 0.90, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^{4}c^{3}x^{5}+\frac{1}{2}\,ab^{3}c^{3}x^{4}-a^{3}bc^{3}x^{2}+a^{4}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Sympy [A]
time = 0.01, size = 44, normalized size = 1.16

$$a^{4}c^{3}x-a^{3}bc^{3}x^{2}+\frac{ab^{3}c^{3}x^{4}}{2}-\frac{b^{4}c^{3}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**3,x)
[Out] a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

Giac [A]
time = 2.59, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

Mupad [B]
time = 0.16, size = 44, normalized size = 1.16

$$a^4\,c^3\,x-a^3\,b\,c^3\,x^2+\frac{a\,b^3\,c^3\,x^4}{2}-\frac{b^4\,c^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x),x)
[Out] a^4*c^3*x - (b^4*c^3*x^5)/5 - a^3*b*c^3*x^2 + (a*b^3*c^3*x^4)/2
```

3.1029 $\int (a+bx)(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$-\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}$$

```
[Out] -2/3*a*c^2*(-b*x+a)^3/b+1/4*c^2*(-b*x+a)^4/b
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^2,x]
```

```
[Out] (-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^2\,dx &= \int \left(2a(ac-bcx)^2-\frac{(ac-bcx)^3}{c}\right)dx\\ &= -\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.11

$$c^2\left(a^3x-\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^2,x]
```

```
[Out] c^2*(a^3*x - (a^2*b*x^2)/2 - (a*b^2*x^3)/3 + (b^3*x^4)/4)
```

Maple [A]
time = 0.11, size = 45, normalized size = 1.18

method	result	size
gosper	$\frac{x\left(3b^{3}x^{3}-4a\,b^{2}x^{2}-6a^{2}bx+12a^{3}\right)c^{2}}{12}$	37
default	$\frac{1}{4}b^{3}c^{2}x^{4}-\frac{1}{3}a\,b^{2}c^{2}x^{3}-\frac{1}{2}a^{2}b\,c^{2}x^{2}+a^{3}c^{2}x$	45
norman	$\frac{1}{4}b^{3}c^{2}x^{4}-\frac{1}{3}a\,b^{2}c^{2}x^{3}-\frac{1}{2}a^{2}b\,c^{2}x^{2}+a^{3}c^{2}x$	45
risch	$\frac{1}{4}b^{3}c^{2}x^{4}-\frac{1}{3}a\,b^{2}c^{2}x^{3}-\frac{1}{2}a^{2}b\,c^{2}x^{2}+a^{3}c^{2}x$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*b^3*c^2*x^4-1/3*a*b^2*c^2*x^3-1/2*a^2*b*c^2*x^2+a^3*c^2*x
```

Maxima [A]
time = 0.29, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^{3}c^{2}x^{4}-\frac{1}{3}\,ab^{2}c^{2}x^{3}-\frac{1}{2}\,a^{2}bc^{2}x^{2}+a^{3}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="maxima")
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Fricas [A]
time = 0.66, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^{3}c^{2}x^{4}-\frac{1}{3}\,ab^{2}c^{2}x^{3}-\frac{1}{2}\,a^{2}bc^{2}x^{2}+a^{3}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="fricas")
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Sympy [A]
time = 0.01, size = 46, normalized size = 1.21

$$a^{3}c^{2}x-\frac{a^{2}bc^{2}x^{2}}{2}-\frac{ab^{2}c^{2}x^{3}}{3}+\frac{b^{3}c^{2}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**2,x)
```

```
[Out] a**3*c**2*x - a**2*b*c**2*x**2/2 - a*b**2*c**2*x**3/3 + b**3*c**2*x**4/4
```

Giac [A]
time = 2.53, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^3c^2x^4 - \frac{1}{3}\,ab^2c^2x^3 - \frac{1}{2}\,a^2bc^2x^2 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

Mupad [B]
time = 0.05, size = 44, normalized size = 1.16

$$a^3\,c^2\,x - \frac{a^2\,b\,c^2\,x^2}{2} - \frac{a\,b^2\,c^2\,x^3}{3} + \frac{b^3\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x),x)
```

```
[Out] a^3*c^2*x + (b^3*c^2*x^4)/4 - (a^2*b*c^2*x^2)/2 - (a*b^2*c^2*x^3)/3
```

3.1030 $\int (a+bx)(ac-bcx)\,dx$

Optimal. Leaf size=18

$$a^2cx-\frac{1}{3}b^2cx^3$$

[Out] a^2*c*x-1/3*b^2*c*x^3

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {41}

$$a^2cx-\frac{1}{3}b^2cx^3$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)*(a*c - b*c*x),x]

[Out] a^2*c*x - (b^2*c*x^3)/3

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)\,dx &= \int \left(a^2c-b^2cx^2\right)\,dx\\ &= a^2cx-\frac{1}{3}b^2cx^3\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$c\left(a^2x-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)*(a*c - b*c*x),x]

[Out] c*(a^2*x - (b^2*x^3)/3)

Maple [A]
time = 0.04, size = 17, normalized size = 0.94

method	result	size
default	$a^2cx - \frac{1}{3}b^2c\,x^3$	17
norman	$a^2cx - \frac{1}{3}b^2c\,x^3$	17
risch	$a^2cx - \frac{1}{3}b^2c\,x^3$	17
gosper	$\frac{cx(-x^2b^2+3a^2)}{3}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] a^2*c*x-1/3*b^2*c*x^3
```

Maxima [A]
time = 0.29, size = 16, normalized size = 0.89

$$-\frac{1}{3}\,b^2cx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="maxima")
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Fricas [A]
time = 0.66, size = 16, normalized size = 0.89

$$-\frac{1}{3}\,b^2cx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.83

$$a^2cx - \frac{b^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x)
```

```
[Out] a**2*c*x - b**2*c*x**3/3
```

Giac [A]
time = 1.76, size = 16, normalized size = 0.89

$$-\frac{1}{3}b^2cx^3 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

Mupad [B]
time = 0.02, size = 18, normalized size = 1.00

$$\frac{c\,x\,\left(3\,a^2-b^2\,x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x),x)
```

```
[Out] (c*x*(3*a^2 - b^2*x^2))/3
```

3.1031 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax + \frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx = ax + \frac{bx^2}{2}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
gosper	$\frac{1}{2}x^2b+ax$	11
default	$\frac{1}{2}x^2b+ax$	11

norman	$\frac{1}{2}x^2b + ax$	11
risch	$\frac{1}{2}x^2b + ax$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*b+a*x
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.70, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

Giac [A]
time = 2.02, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
[Out] 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
[Out] a*x + (b*x^2)/2
```

3.1032 $\int \frac{a+bx}{ac-bcx}\,dx$

Optimal. Leaf size=23

$$-\frac{x}{c}-\frac{2a\log(a-bx)}{bc}$$

```
[Out] -x/c-2*a*ln(-b*x+a)/b/c
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = $\{45\}$

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{ac-bcx}\,dx &= \int \left(-\frac{1}{c}+\frac{2a}{c(a-bx)}\right)dx\\ &= -\frac{x}{c}-\frac{2a\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$-\frac{x}{c}-\frac{2a\log(a-bx)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Maple [A]
time = 0.11, size = 22, normalized size = 0.96

method	result	size
default	$\frac{-x-\frac{2a\ln(-bx+a)}{b}}{c}$	22
norman	$-\frac{x}{c}-\frac{2a\ln(-bx+a)}{bc}$	24
risch	$-\frac{x}{c}-\frac{2a\ln(-bx+a)}{bc}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] 1/c*(-x-2*a/b*ln(-b*x+a))
```

Maxima [A]
time = 0.28, size = 24, normalized size = 1.04

$$-\frac{x}{c}-\frac{2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
[Out] -x/c - 2*a*log(b*x - a)/(b*c)
```

Fricas [A]
time = 1.06, size = 23, normalized size = 1.00

$$-\frac{bx+2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.74

$$-\frac{2a\log\left(-a+bx\right)}{bc}-\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -2*a*log(-a + b*x)/(b*c) - x/c
```

Giac [A]
time = 2.19, size = 25, normalized size = 1.09

$$-\frac{x}{c} - \frac{2\,a\log\left(|bx - a|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -x/c - 2*a*log(abs(b*x - a))/(b*c)
```

Mupad [B]
time = 0.05, size = 23, normalized size = 1.00

$$-\frac{b\,x+2\,a\,\ln\left(b\,x-a\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x),x)
```

```
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

3.1033 $\int \frac{a+bx}{(ac-bcx)^2} \, dx$

Optimal. Leaf size=32

$$\frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2}$$

```
[Out] 2*a/b/c^2/(-b*x+a)+ln(-b*x+a)/b/c^2
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] (2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{(ac-bcx)^2} \, dx &= \int \left(\frac{2a}{c^2(a-bx)^2} - \frac{1}{c^2(a-bx)}\right) dx \\
&= \frac{2a}{bc^2(a-bx)} + \frac{\log(a-bx)}{bc^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.88

$$\frac{\frac{2a}{a-bx} + \log(c(a-bx))}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^2,x]
```

[Out] ((2*a)/(a - b*x) + Log[c*(a - b*x)])/(b*c^2)

Maple [A]
time = 0.12, size = 31, normalized size = 0.97

method	result	size
default	$\frac{\frac{2a}{b(-bx+a)}+\frac{\ln(-bx+a)}{b}}{c^2}$	31
norman	$\frac{2a}{b\,c^2(-bx+a)}+\frac{\ln(-bx+a)}{b\,c^2}$	33
risch	$\frac{2a}{b\,c^2(-bx+a)}+\frac{\ln(-bx+a)}{b\,c^2}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)

[Out] 1/c^2*(2*a/b/(-b*x+a)+1/b*ln(-b*x+a))

Maxima [A]
time = 0.29, size = 37, normalized size = 1.16

$$-\frac{2\,a}{b^2c^2x-abc^2}+\frac{\log\left(bx-a\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")

[Out] -2*a/(b^2*c^2*x - a*b*c^2) + log(b*x - a)/(b*c^2)

Fricas [A]
time = 0.63, size = 39, normalized size = 1.22

$$\frac{(bx-a)\log\left(bx-a\right)-2\,a}{b^2c^2x-abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")

[Out] ((b*x - a)*log(b*x - a) - 2*a)/(b^2*c^2*x - a*b*c^2)

Sympy [A]
time = 0.07, size = 29, normalized size = 0.91

$$-\frac{2a}{-abc^2+b^2c^2x}+\frac{\log\left(-a+bx\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)**2,x)

```
[Out] -2*a/(-a*b*c**2 + b**2*c**2*x) + log(-a + b*x)/(b*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(33) = 66$.
time = 2.07, size = 81, normalized size = 2.53

$$-\frac{\frac{a}{(bcx-ac)b}+\frac{\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc}}{c}-\frac{a}{(bcx-ac)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -(a/((b*c*x - a*c)*b) + log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(b)*abs(c)
))/(b*c))/c - a/((b*c*x - a*c)*b*c)
```

Mupad [B]
time = 0.05, size = 37, normalized size = 1.16

$$\frac{\ln\left(b\,x-a\right)}{b\,c^2}+\frac{2\,a}{b\,\left(a\,c^2-b\,c^2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^2,x)
```

```
[Out] log(b*x - a)/(b*c^2) + (2*a)/(b*(a*c^2 - b*c^2*x))
```

3.1034 $\int \frac{a+bx}{(ac-bcx)^3} dx$

Optimal. Leaf size=13

$$\frac{x}{c^3(a-bx)^2}$$

```
[Out] x/c^3/(-b*x+a)^2
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {34}

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^3,x]
[Out] x/(c^3*(a - b*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{a+bx}{(ac-bcx)^3} dx = \frac{x}{c^3(a-bx)^2}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^3,x]
[Out] x/(c^3*(a - b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.
time = 0.14, size = 32, normalized size = 2.46

method	result	size
gosper	$\frac{x}{c^3(-bx+a)^2}$	14
norman	$\frac{x}{c^3(-bx+a)^2}$	14
risch	$\frac{x}{c^3(-bx+a)^2}$	14
default	$\frac{\frac{a}{b(-bx+a)^2}-\frac{1}{b(-bx+a)}}{c^3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/b/(-b*x+a)^2-1/b/(-b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.29, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.60, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.
time = 0.09, size = 27, normalized size = 2.08

$$\frac{x}{a^2c^3 - 2abc^3x + b^2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] x/(a**2*c**3 - 2*a*b*c**3*x + b**2*c**3*x**2)
```

Giac [A]
time = 1.62, size = 14, normalized size = 1.08

$$\frac{x}{(bx-a)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] x/((b*x - a)^2*c^3)
```

Mupad [B]
time = 0.15, size = 13, normalized size = 1.00

$$\frac{x}{c^3\,{\left(a-b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^3,x)
```

```
[Out] x/(c^3*(a - b*x)^2)
```

3.1035 $\int \frac{a+bx}{(ac-bcx)^4} dx$

Optimal. Leaf size=38

$$\frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

```
[Out] 2/3*a/b/c^4/(-b*x+a)^3-1/2/b/c^4/(-b*x+a)^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] (2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{(ac-bcx)^4} dx &= \int \left(\frac{2a}{c^4(a-bx)^4} - \frac{1}{c^4(a-bx)^3}\right) dx \\ &= \frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.66

$$-\frac{a+3bx}{6bc^4(-a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^4,x]
```

[Out] -1/6*(a + 3*b*x)/(b*c^4*(-a + b*x)^3)

Maple [A]
time = 0.11, size = 33, normalized size = 0.87

method	result	size
gosper	$\frac{3bx+a}{6(-bx+a)^{3} c^{4} b}$	23
risch	$\frac{\frac{x}{2}+\frac{a}{6b}}{c^{4} (-bx+a)^{3}}$	23
norman	$\frac{\frac{a}{6bc}+\frac{x}{2c}}{c^{3} (-bx+a)^{3}}$	29
default	$\frac{-\frac{1}{2b(-bx+a)^{2}}+\frac{2a}{3b(-bx+a)^{3}}}{c^{4}}$	33

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)/(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)

[Out] 1/c^4*(-1/2/b/(-b*x+a)^2+2/3*a/b/(-b*x+a)^3)

Maxima [A]
time = 0.30, size = 54, normalized size = 1.42

$$-\frac{3\,bx+a}{6\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="maxima")

[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c^4)

Fricas [A]
time = 0.46, size = 54, normalized size = 1.42

$$-\frac{3\,bx+a}{6\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="fricas")

[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c^4)

Sympy [A]
time = 0.13, size = 56, normalized size = 1.47

$$\frac{-a-3bx}{-6a^3bc^4+18a^2b^2c^4x-18ab^3c^4x^2+6b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a - 3*b*x)/(-6*a**3*b*c**4 + 18*a**2*b**2*c**4*x - 18*a*b**3*c**4*x**2 +
6*b**4*c**4*x**3)
```

Giac [A]
time = 1.76, size = 23, normalized size = 0.61

$$-\frac{3\,bx+a}{6\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + a)/((b*x - a)^3*b*c^4)
```

Mupad [B]
time = 0.05, size = 54, normalized size = 1.42

$$\frac{\frac{x}{2}+\frac{a}{6\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^4,x)
```

```
[Out] (x/2 + a/(6*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*x)
```

3.1036 $\int \frac{a+bx}{(ac-bcx)^5} \, dx$

Optimal. Leaf size=38

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

[Out] 1/2*a/b/c^5/(-b*x+a)^4-1/3/b/c^5/(-b*x+a)^3

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(a*c - b*c*x)^5,x]

[Out] a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{(ac-bcx)^5} \, dx &= \int \left(\frac{2a}{c^5(a-bx)^5}-\frac{1}{c^5(a-bx)^4}\right) \, dx\\
&= \frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.63

$$\frac{a+2bx}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(a*c - b*c*x)^5,x]

```
[Out] (a + 2*b*x)/(6*b*c^5*(a - b*x)^4)
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.87

method	result	size
gosper	$\frac{2bx+a}{6(-bx+a)^{4}c^{5}b}$	23
risch	$\frac{\frac{x}{3}+\frac{a}{6b}}{c^{5}(-bx+a)^{4}}$	23
norman	$\frac{\frac{a}{6bc}+\frac{x}{3c}}{c^{4}(-bx+a)^{4}}$	29
default	$\frac{-\frac{1}{3b(-bx+a)^{3}}+\frac{a}{2b(-bx+a)^{4}}}{c^{5}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-1/3/b/(-b*x+a)^3+1/2*a/b/(-b*x+a)^4)
```

Maxima [A]
time = 0.29, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\,(b^{5}c^{5}x^{4}-4\,ab^{4}c^{5}x^{3}+6\,a^{2}b^{3}c^{5}x^{2}-4\,a^{3}b^{2}c^{5}x+a^{4}bc^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

Fricas [A]
time = 0.42, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\,(b^{5}c^{5}x^{4}-4\,ab^{4}c^{5}x^{3}+6\,a^{2}b^{3}c^{5}x^{2}-4\,a^{3}b^{2}c^{5}x+a^{4}bc^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.
time = 0.17, size = 73, normalized size = 1.92

$$-\frac{-a-2bx}{6a^{4}bc^{5}-24a^{3}b^{2}c^{5}x+36a^{2}b^{3}c^{5}x^{2}-24ab^{4}c^{5}x^{3}+6b^{5}c^{5}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a - 2*b*x)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a**2*b**3*c**5*x**2
 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Giac [A]
time = 1.53, size = 40, normalized size = 1.05

$$\frac{a}{2\,(bcx-ac)^4bc}+\frac{1}{3\,(bcx-ac)^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/2*a/((b*c*x - a*c)^4*b*c) + 1/3/((b*c*x - a*c)^3*b*c^2)
```

Mupad [B]
time = 0.17, size = 67, normalized size = 1.76

$$\frac{\frac{x}{3}+\frac{a}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^5,x)
```

```
[Out] (x/3 + a/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3 + 6*a^2*b^2*c^5*x^
2 - 4*a^3*b*c^5*x)
```

3.1037 $\int \frac{a+bx}{(ac-bcx)^6}\,dx$

Optimal. Leaf size=38

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

```
[Out] 2/5*a/b/c^6/(-b*x+a)^5-1/4/b/c^6/(-b*x+a)^4
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] (2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^6}\,dx &= \int \left(\frac{2a}{c^6(a-bx)^6}-\frac{1}{c^6(a-bx)^5}\right)dx\\ &= \frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{3a+5bx}{20bc^6(-a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^6,x]
```

```
[Out] -1/20*(3*a + 5*b*x)/(b*c^6*(-a + b*x)^5)
```

Maple [A]
time = 0.14, size = 33, normalized size = 0.87

method	result	size
risch	$\frac{\frac{x}{4}+\frac{3a}{20b}}{c^6(-bx+a)^5}$	23
gosper	$\frac{5bx+3a}{20(-bx+a)^5 c^6 b}$	25
norman	$\frac{\frac{3a}{20bc}+\frac{x}{4c}}{c^5(-bx+a)^5}$	29
default	$\frac{-\frac{1}{4b(-bx+a)^4}+\frac{2a}{5b(-bx+a)^5}}{c^6}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(-1/4/b/(-b*x+a)^4+2/5*a/b/(-b*x+a)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.
time = 0.28, size = 84, normalized size = 2.21

$$-\frac{5\,bx+3\,a}{20\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.
time = 0.44, size = 84, normalized size = 2.21

$$-\frac{5\,bx+3\,a}{20\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(31) = 62$.
time = 0.24, size = 88, normalized size = 2.32

$$\frac{-3a - 5bx}{-20a^5bc^6 + 100a^4b^2c^6x - 200a^3b^3c^6x^2 + 200a^2b^4c^6x^3 - 100ab^5c^6x^4 + 20b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**6,x)
```

```
[Out] (-3*a - 5*b*x)/(-20*a**5*b*c**6 + 100*a**4*b**2*c**6*x - 200*a**3*b**3*c**6
*x**2 + 200*a**2*b**4*c**6*x**3 - 100*a*b**5*c**6*x**4 + 20*b**6*c**6*x**5)
```

Giac [A]
time = 1.46, size = 25, normalized size = 0.66

$$-\frac{5\,bx + 3\,a}{20\,(bx - a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/20*(5*b*x + 3*a)/((b*x - a)^5*b*c^6)
```

Mupad [B]
time = 0.08, size = 82, normalized size = 2.16

$$\frac{\frac{x}{4} + \frac{3\,a}{20\,b}}{a^5\,c^6 - 5\,a^4\,b\,c^6\,x + 10\,a^3\,b^2\,c^6\,x^2 - 10\,a^2\,b^3\,c^6\,x^3 + 5\,a\,b^4\,c^6\,x^4 - b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^6,x)
```

```
[Out] (x/4 + (3*a)/(20*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6*x^4 + 10*a^3*b^2*
c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

3.1038 $\int (a+bx)^2(ac-bcx)^3\,dx$

Optimal. Leaf size=57

$$-\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}$$

```
[Out] -a^2*c^3*(-b*x+a)^4/b+4/5*a*c^3*(-b*x+a)^5/b-1/6*c^3*(-b*x+a)^6/b
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {45}

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] -((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^6
)/(6*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^3\,dx &= \int \left(4a^2(ac-bcx)^3-\frac{4a(ac-bcx)^4}{c}+\frac{(ac-bcx)^5}{c^2}\right)dx\\ &= -\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 68, normalized size = 1.19

$$c^3\left(a^5x-\frac{1}{2}a^4bx^2-\frac{2}{3}a^3b^2x^3+\frac{1}{2}a^2b^3x^4+\frac{1}{5}ab^4x^5-\frac{b^5x^6}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^5*x - (a^4*b*x^2)/2 - (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 + (a*b^4*x
^5)/5 - (b^5*x^6)/6)
```

Maple [A]
time = 0.17, size = 73, normalized size = 1.28

method	result	size
gosper	$\frac{x\left(-5b^{5}x^{5}+6a\,b^{4}x^{4}+15a^{2}b^{3}x^{3}-20a^{3}b^{2}x^{2}-15a^{4}bx+30a^{5}\right)c^{3}}{30}$	59
default	$-\frac{1}{6}b^{5}c^{3}x^{6}+\frac{1}{5}a\,b^{4}c^{3}x^{5}+\frac{1}{2}a^{2}b^{3}c^{3}x^{4}-\frac{2}{3}a^{3}c^{3}b^{2}x^{3}-\frac{1}{2}a^{4}c^{3}b\,x^{2}+a^{5}c^{3}x$	73
norman	$-\frac{1}{6}b^{5}c^{3}x^{6}+\frac{1}{5}a\,b^{4}c^{3}x^{5}+\frac{1}{2}a^{2}b^{3}c^{3}x^{4}-\frac{2}{3}a^{3}c^{3}b^{2}x^{3}-\frac{1}{2}a^{4}c^{3}b\,x^{2}+a^{5}c^{3}x$	73
risch	$-\frac{1}{6}b^{5}c^{3}x^{6}+\frac{1}{5}a\,b^{4}c^{3}x^{5}+\frac{1}{2}a^{2}b^{3}c^{3}x^{4}-\frac{2}{3}a^{3}c^{3}b^{2}x^{3}-\frac{1}{2}a^{4}c^{3}b\,x^{2}+a^{5}c^{3}x$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b^5*c^3*x^6+1/5*a*b^4*c^3*x^5+1/2*a^2*b^3*c^3*x^4-2/3*a^3*c^3*b^2*x^3-
1/2*a^4*c^3*b*x^2+a^5*c^3*x
```

Maxima [A]
time = 0.28, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^{5}c^{3}x^{6}+\frac{1}{5}\,ab^{4}c^{3}x^{5}+\frac{1}{2}\,a^{2}b^{3}c^{3}x^{4}-\frac{2}{3}\,a^{3}b^{2}c^{3}x^{3}-\frac{1}{2}\,a^{4}bc^{3}x^{2}+a^{5}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Fricas [A]
time = 0.45, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^{5}c^{3}x^{6}+\frac{1}{5}\,ab^{4}c^{3}x^{5}+\frac{1}{2}\,a^{2}b^{3}c^{3}x^{4}-\frac{2}{3}\,a^{3}b^{2}c^{3}x^{3}-\frac{1}{2}\,a^{4}bc^{3}x^{2}+a^{5}c^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Sympy [A]
time = 0.02, size = 78, normalized size = 1.37

$$a^{5}c^{3}x-\frac{a^{4}bc^{3}x^{2}}{2}-\frac{2a^{3}b^{2}c^{3}x^{3}}{3}+\frac{a^{2}b^{3}c^{3}x^{4}}{2}+\frac{ab^{4}c^{3}x^{5}}{5}-\frac{b^{5}c^{3}x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**3,x)
```

```
[Out] a**5*c**3*x - a**4*b*c**3*x**2/2 - 2*a**3*b**2*c**3*x**3/3 + a**2*b**3*c**3
*x**4/2 + a*b**4*c**3*x**5/5 - b**5*c**3*x**6/6
```

Giac [A]
time = 2.09, size = 72, normalized size = 1.26

$$-\frac{1}{6}\,b^5c^3x^6+\frac{1}{5}\,ab^4c^3x^5+\frac{1}{2}\,a^2b^3c^3x^4-\frac{2}{3}\,a^3b^2c^3x^3-\frac{1}{2}\,a^4bc^3x^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

Mupad [B]
time = 0.03, size = 72, normalized size = 1.26

$$a^5\,c^3\,x-\frac{a^4\,b\,c^3\,x^2}{2}-\frac{2\,a^3\,b^2\,c^3\,x^3}{3}+\frac{a^2\,b^3\,c^3\,x^4}{2}+\frac{a\,b^4\,c^3\,x^5}{5}-\frac{b^5\,c^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x)^2,x)
```

```
[Out] a^5*c^3*x - (b^5*c^3*x^6)/6 - (a^4*b*c^3*x^2)/2 + (a*b^4*c^3*x^5)/5 - (2*a^
3*b^2*c^3*x^3)/3 + (a^2*b^3*c^3*x^4)/2
```

3.1039 $\int (a+bx)^2(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

```
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {41, 200}

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^2\,dx &= \int \left(a^2c-b^2cx^2\right)^2\,dx\\ &= \int \left(a^4c^2-2a^2b^2c^2x^2+b^4c^2x^4\right)\,dx\\ &= a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.00

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Maple [A]
time = 0.13, size = 35, normalized size = 0.92

method	result	size
gosper	$\frac{x\left(3b^4x^4-10a^2b^2x^2+15a^4\right)c^2}{15}$	32
default	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
norman	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35
risch	$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5-\frac{2}{3}\,a^2b^2c^2x^3+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Fricas [A]
time = 0.53, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5-\frac{2}{3}\,a^2b^2c^2x^3+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Sympy [A]
time = 0.01, size = 36, normalized size = 0.95

$$a^4c^2x-\frac{2a^2b^2c^2x^3}{3}+\frac{b^4c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**2,x)
```

```
[Out] a**4*c**2*x - 2*a**2*b**2*c**2*x**3/3 + b**4*c**2*x**5/5
```

Giac [A]
time = 3.11, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5-\frac{2}{3}\,a^2b^2c^2x^3+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

Mupad [B]
time = 0.04, size = 31, normalized size = 0.82

$$\frac{c^2\,x\,\left(15\,a^4-10\,a^2\,b^2\,x^2+3\,b^4\,x^4\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x)^2,x)
```

```
[Out] (c^2*x*(15*a^4 + 3*b^4*x^4 - 10*a^2*b^2*x^2))/15
```

3.1040 $\int (a+bx)^2(ac-bcx)\,dx$

Optimal. Leaf size=32

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

```
[Out] 2/3*a*c*(b*x+a)^3/b-1/4*c*(b*x+a)^4/b
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {45}

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] (2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)\,dx &= \int \left(2ac(a+bx)^2-c(a+bx)^3\right)\,dx\\ &= \frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.25

$$c\left(a^3x+\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x),x]
```

[Out] c*(a^3*x + (a^2*b*x^2)/2 - (a*b^2*x^3)/3 - (b^3*x^4)/4)

Maple [A]
time = 0.12, size = 37, normalized size = 1.16

method	result	size
gosper	$\frac{cx\left(-3b^{3}x^{3}-4a\,b^{2}x^{2}+6a^{2}bx+12a^{3}\right)}{12}$	35
default	$-\frac{1}{4}b^{3}c\,x^{4}-\frac{1}{3}a\,b^{2}c\,x^{3}+\frac{1}{2}a^{2}bc\,x^{2}+a^{3}cx$	37
norman	$-\frac{1}{4}b^{3}c\,x^{4}-\frac{1}{3}a\,b^{2}c\,x^{3}+\frac{1}{2}a^{2}bc\,x^{2}+a^{3}cx$	37
risch	$-\frac{1}{4}b^{3}c\,x^{4}-\frac{1}{3}a\,b^{2}c\,x^{3}+\frac{1}{2}a^{2}bc\,x^{2}+a^{3}cx$	37

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^2*(-b*c*x+a*c),x,method=_RETURNVERBOSE)

[Out] -1/4*b^3*c*x^4-1/3*a*b^2*c*x^3+1/2*a^2*b*c*x^2+a^3*c*x

Maxima [A]
time = 0.28, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^{3}cx^{4}-\frac{1}{3}\,ab^{2}cx^{3}+\frac{1}{2}\,a^{2}bcx^{2}+a^{3}cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="maxima")

[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x

Fricas [A]
time = 0.47, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^{3}cx^{4}-\frac{1}{3}\,ab^{2}cx^{3}+\frac{1}{2}\,a^{2}bcx^{2}+a^{3}cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="fricas")

[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x

Sympy [A]
time = 0.01, size = 39, normalized size = 1.22

$$a^{3}cx+\frac{a^{2}bcx^{2}}{2}-\frac{ab^{2}cx^{3}}{3}-\frac{b^{3}cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c),x)
```

```
[Out] a**3*c*x + a**2*b*c*x**2/2 - a*b**2*c*x**3/3 - b**3*c*x**4/4
```

Giac [**A**]
time = 2.69, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

Mupad [**B**]
time = 0.05, size = 36, normalized size = 1.12

$$c\,a^3\,x+\frac{c\,a^2\,b\,x^2}{2}-\frac{c\,a\,b^2\,x^3}{3}-\frac{c\,b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x)^2,x)
```

```
[Out] a^3*c*x - (b^3*c*x^4)/4 + (a^2*b*c*x^2)/2 - (a*b^2*c*x^3)/3
```

3.1041 $\int (a+bx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

[Out] 1/3*(b*x+a)^3/b

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2 \, dx = \frac{(a+bx)^3}{3b}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^2,x]

[Out] (a + b*x)^3/(3*b)

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(bx+a)^3}{3b}$	13
gosper	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
norman	$\frac{1}{3}b^2x^3 + ab\,x^2 + a^2x$	21
risch	$\frac{b^2x^3}{3} + ab\,x^2 + a^2x + \frac{a^3}{3b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^3/b
```

Maxima [A]
time = 0.29, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Fricas [A]
time = 0.39, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3 + abx^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.01, size = 19, normalized size = 1.36

$$a^2x + abx^2 + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
```

```
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

Giac [A]
time = 1.73, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b*x + a)^3/b
```

Mupad [B]
time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
```

```
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

3.1042 $\int \frac{(a+bx)^2}{ac-bcx}\,dx$

Optimal. Leaf size=43

$$-\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}$$

```
[Out] -2*a*x/c-1/2*(b*x+a)^2/b/c-4*a^2*ln(-b*x+a)/b/c
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.053$, Rules used = $\{45\}$

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x),x]
[Out] (-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{ac-bcx}\,dx &= \int \left(-\frac{2a}{c}-\frac{a+bx}{c}+\frac{4a^2}{ac-bcx}\right)dx\\ &= -\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.86

$$-\frac{3ax}{c}-\frac{bx^2}{2c}-\frac{4a^2\log(a-bx)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x),x]
[Out] (-3*a*x)/c - (b*x^2)/(2*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Maple [A]
time = 0.18, size = 31, normalized size = 0.72

method	result	size
default	$\frac{-\frac{x^2 b}{2}-3ax-\frac{4a^2 \ln(-bx+a)}{b}}{c}$	31
norman	$-\frac{3ax}{c}-\frac{b\,x^2}{2c}-\frac{4a^2 \ln(-bx+a)}{bc}$	36
risch	$-\frac{3ax}{c}-\frac{b\,x^2}{2c}-\frac{4a^2 \ln(-bx+a)}{bc}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
[Out] 1/c*(-1/2*x^2*b-3*a*x-4*a^2/b*ln(-b*x+a))
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.81

$$-\frac{4\,a^2 \log\left(bx-a\right)}{bc}-\frac{bx^2+6\,ax}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
[Out] -4*a^2*log(b*x - a)/(b*c) - 1/2*(b*x^2 + 6*a*x)/c
```

Fricas [A]
time = 0.51, size = 34, normalized size = 0.79

$$-\frac{b^2x^2+6\,abx+8\,a^2 \log\left(bx-a\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/(b*c)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 0.72

$$-\frac{4a^2 \log\left(-a+bx\right)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c),x)
[Out] -4*a**2*log(-a + b*x)/(b*c) - 3*a*x/c - b*x**2/(2*c)
```

Giac [A]
time = 2.07, size = 46, normalized size = 1.07

$$-\frac{4\,a^2\log\left(|bx-a|\right)}{bc}-\frac{b^3cx^2+6\,ab^2cx}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
[Out] -4*a^2*log(abs(b*x - a))/(b*c) - 1/2*(b^3*c*x^2 + 6*a*b^2*c*x)/(b^2*c^2)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.79

$$-\frac{8\,a^2\,\ln\left(b\,x-a\right)+b^2\,x^2+6\,a\,b\,x}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x),x)
[Out] -(8*a^2*log(b*x - a) + b^2*x^2 + 6*a*b*x)/(2*b*c)
```

3.1043 $\int \frac{(a+bx)^2}{(ac-bcx)^2} \, dx$

Optimal. Leaf size=41

$$\frac{x}{c^2}+\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}$$

```
[Out] x/c^2+4*a^2/b/c^2/(-b*x+a)+4*a*ln(-b*x+a)/b/c^2
```

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^2}\,dx &= \int \left(\frac{1}{c^2}+\frac{4a^2}{c^2(a-bx)^2}-\frac{4a}{c^2(a-bx)}\right)dx\\ &= \frac{x}{c^2}+\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}\end{aligned}$$

Mathematica [A]

time = 0.02, size = 35, normalized size = 0.85

$$\frac{x+\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]
[Out] (x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b)/c^2
```

Maple [A]
time = 0.16, size = 36, normalized size = 0.88

method	result	size
default	$\frac{x+\frac{4a^{2}}{b(-bx+a)}+\frac{4a\ln(-bx+a)}{b}}{c^{2}}$	36
risch	$\frac{x}{c^{2}}+\frac{4a^{2}}{b\,c^{2}(-bx+a)}+\frac{4a\ln(-bx+a)}{b\,c^{2}}$	42
norman	$\frac{\frac{5a^{2}}{bc}-\frac{b\,x^{2}}{c}}{c(-bx+a)}+\frac{4a\ln(-bx+a)}{b\,c^{2}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(x+4*a^2/b/(-b*x+a)+4*a/b*ln(-b*x+a))
```

Maxima [A]
time = 0.29, size = 46, normalized size = 1.12

$$-\frac{4\,a^{2}}{b^{2}c^{2}x-abc^{2}}+\frac{x}{c^{2}}+\frac{4\,a\log\left(bx-a\right)}{bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
[Out] -4*a^2/(b^2*c^2*x - a*b*c^2) + x/c^2 + 4*a*log(b*x - a)/(b*c^2)
```

Fricas [A]
time = 0.55, size = 57, normalized size = 1.39

$$\frac{b^{2}x^{2}-abx-4\,a^{2}+4\left(abx-a^{2}\right)\log\left(bx-a\right)}{b^{2}c^{2}x-abc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
[Out] (b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*c^2*x - a*b*c
^2)
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.95

$$-\frac{4a^{2}}{-abc^{2}+b^{2}c^{2}x}+\frac{4a\log\left(-a+bx\right)}{bc^{2}}+\frac{x}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -4*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*log(-a + b*x)/(b*c**2) + x/c**2
```

Giac [A]
time = 1.92, size = 79, normalized size = 1.93

$$-\frac{4a^2}{(bcx-ac)bc} - \frac{4a\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc^2} + \frac{bcx-ac}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -4*a^2/((b*c*x - a*c)*b*c) - 4*a*log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(
b)*abs(c)))/(b*c^2) + (b*c*x - a*c)/(b*c^3)
```

Mupad [B]
time = 0.15, size = 46, normalized size = 1.12

$$\frac{x}{c^2} + \frac{4a^2}{b\,(a\,c^2 - b\,c^2\,x)} + \frac{4\,a\,\ln(b\,x-a)}{b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^2,x)
```

```
[Out] x/c^2 + (4*a^2)/(b*(a*c^2 - b*c^2*x)) + (4*a*log(b*x - a))/(b*c^2)
```

3.1044 $\int \frac{(a+bx)^2}{(ac-bcx)^3} \, dx$

Optimal. Leaf size=52

$$\frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}$$

```
[Out] 2*a^2/b/c^3/(-b*x+a)^2-4*a/b/c^3/(-b*x+a)-ln(-b*x+a)/b/c^3
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] (2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^3
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^3} \, dx &= \int \left(\frac{4a^2}{c^3(a-bx)^3} - \frac{4a}{c^3(a-bx)^2} + \frac{1}{c^3(a-bx)}\right) dx \\ &= \frac{2a^2}{bc^3(a-bx)^2} - \frac{4a}{bc^3(a-bx)} - \frac{\log(a-bx)}{bc^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.63

$$-\frac{\frac{2a(a-2bx)}{(a-bx)^2} + \log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] -(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/(b*c^3))
```

Maple [A]
time = 0.13, size = 48, normalized size = 0.92

method	result	size
risch	$\frac{4ax-\frac{2a^2}{b}}{c^3(-bx+a)^2}-\frac{\ln(-bx+a)}{b\,c^3}$	42
default	$\frac{-\frac{4a}{b(-bx+a)}-\frac{\ln(-bx+a)}{b}+\frac{2a^2}{b(-bx+a)^2}}{c^3}$	48
norman	$\frac{-\frac{2a^2}{bc}+\frac{4ax}{c}}{c^2(-bx+a)^2}-\frac{\ln(-bx+a)}{b\,c^3}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-4*a/b/(-b*x+a)-1/b*ln(-b*x+a)+2*a^2/b/(-b*x+a)^2)
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.17

$$\frac{2\,(2\,abx-a^2)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}-\frac{\log(bx-a)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*x - a^2)/(b^3*c^3*x^2 - 2*a*b^2*c^3*x + a^2*b*c^3) - log(b*x - a)/
(b*c^3)
```

Fricas [A]
time = 0.46, size = 69, normalized size = 1.33

$$\frac{4\,abx-2\,a^2-(b^2x^2-2\,abx+a^2)\log(bx-a)}{b^3c^3x^2-2\,ab^2c^3x+a^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] (4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*c^3*x^2 - 2
*a*b^2*c^3*x + a^2*b*c^3)
```

Sympy [A]
time = 0.13, size = 54, normalized size = 1.04

$$-\frac{2a^2-4abx}{a^2bc^3-2ab^2c^3x+b^3c^3x^2}-\frac{\log\left(-a+bx\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(2*a**2 - 4*a*b*x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) - log(
-a + b*x)/(b*c**3)
```

Giac [A]
time = 1.93, size = 46, normalized size = 0.88

$$-\frac{\log\left(|bx-a|\right)}{bc^3}+\frac{2\left(2\,abx-a^2\right)}{\left(bx-a\right)^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x - a))/(b*c^3) + 2*(2*a*b*x - a^2)/((b*x - a)^2*b*c^3)
```

Mupad [B]
time = 0.17, size = 59, normalized size = 1.13

$$\frac{4\,a\,x-\frac{2\,a^2}{b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}-\frac{\ln\left(b\,x-a\right)}{b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^3,x)
```

```
[Out] (4*a*x - (2*a^2)/b)/(a^2*c^3 + b^2*c^3*x^2 - 2*a*b*c^3*x) - log(b*x - a)/(b
*c^3)
```

3.1045 $\int \frac{(a+bx)^2}{(ac-bcx)^4} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

```
[Out] 1/6*(b*x+a)^3/a/b/c^4/(-b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] (a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} dx = \frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.11

$$-\frac{a^2+3b^2x^2}{3bc^4(-a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*b^2*x^2)/(b*c^4*(-a + b*x)^3)
```

Maple [A]
time = 0.14, size = 48, normalized size = 1.71

method	result	size
risch	$\frac{x^2b+\frac{a^2}{3b}}{c^4(-bx+a)^3}$	27
gosper	$\frac{3x^2b^2+a^2}{3(-bx+a)^3c^4b}$	29
norman	$\frac{\frac{a^2}{3bc}+\frac{b\,x^2}{c}}{c^3(-bx+a)^3}$	33
default	$\frac{-\frac{2a}{b(-bx+a)^2}+\frac{4a^2}{3b(-bx+a)^3}+\frac{1}{b(-bx+a)}}{c^4}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-2*a/b/(-b*x+a)^2+4/3*a^2/b/(-b*x+a)^3+1/b/(-b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.30, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.41, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\,(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

time = 0.15, size = 61, normalized size = 2.18

$$\frac{-a^2 - 3b^2x^2}{-3a^3bc^4 + 9a^2b^2c^4x - 9ab^3c^4x^2 + 3b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a**2 - 3*b**2*x**2)/(-3*a**3*b*c**4 + 9*a**2*b**2*c**4*x - 9*a*b**3*c**4*
x**2 + 3*b**4*c**4*x**3)
```

Giac [A]
time = 2.08, size = 29, normalized size = 1.04

$$-\frac{3\,b^2x^2 + a^2}{3\,(bx - a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/((b*x - a)^3*b*c^4)
```

Mupad [B]
time = 0.05, size = 58, normalized size = 2.07

$$\frac{b\,x^2 + \frac{a^2}{3\,b}}{a^3\,c^4 - 3\,a^2\,b\,c^4\,x + 3\,a\,b^2\,c^4\,x^2 - b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^4,x)
```

```
[Out] (b*x^2 + a^2/(3*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*
x)
```

3.1046 $\int \frac{(a+bx)^2}{(ac-bcx)^5} dx$

Optimal. Leaf size=56

$$\frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2}$$

```
[Out] a^2/b/c^5/(-b*x+a)^4-4/3*a/b/c^5/(-b*x+a)^3+1/2/b/c^5/(-b*x+a)^2
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$\frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*x
)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^2}{(ac-bcx)^5} dx &= \int \left(\frac{4a^2}{c^5(a-bx)^5} - \frac{4a}{c^5(a-bx)^4} + \frac{1}{c^5(a-bx)^3} \right) dx \\ &= \frac{a^2}{bc^5(a-bx)^4} - \frac{4a}{3bc^5(a-bx)^3} + \frac{1}{2bc^5(a-bx)^2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.62

$$\frac{a^2+2abx+3b^2x^2}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]
```

```
[Out] (a^2 + 2*a*b*x + 3*b^2*x^2)/(6*b*c^5*(a - b*x)^4)
```

Maple [A]
time = 0.15, size = 48, normalized size = 0.86

method	result	size
risch	$\frac{\frac{x^2 b}{2}+\frac{a x}{3}+\frac{a^2}{6 b}}{c^5 (-b x+a)^4}$	32
gosper	$\frac{3 x^2 b^2+2 a b x+a^2}{6 (-b x+a)^4 c^5 b}$	34
norman	$\frac{\frac{a^2}{6 b c}+\frac{b\, x^2}{2 c}+\frac{a x}{3 c}}{c^4 (-b x+a)^4}$	41
default	$\frac{\frac{1}{2 b (-b x+a)^2}-\frac{4 a}{3 b (-b x+a)^3}+\frac{a^2}{b (-b x+a)^4}}{c^5}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/2/b/(-b*x+a)^2-4/3*a/b/(-b*x+a)^3+a^2/b/(-b*x+a)^4)
```

Maxima [A]
time = 0.28, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Fricas [A]
time = 0.46, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\,(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

Sympy [A]
time = 0.19, size = 85, normalized size = 1.52

$$-\frac{-a^2 - 2abx - 3b^2x^2}{6a^4bc^5 - 24a^3b^2c^5x + 36a^2b^3c^5x^2 - 24ab^4c^5x^3 + 6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a**2 - 2*a*b*x - 3*b**2*x**2)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*
a**2*b**3*c**5*x**2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

Giac [A]
time = 1.41, size = 64, normalized size = 1.14

$$\frac{\frac{6\,a^2}{(bcx-ac)^4b} + \frac{8\,a}{(bcx-ac)^3bc} + \frac{3}{(bcx-ac)^2bc^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^2/((b*c*x - a*c)^4*b) + 8*a/((b*c*x - a*c)^3*b*c) + 3/((b*c*x - a*
c)^2*b*c^2))/c
```

Mupad [B]
time = 0.05, size = 76, normalized size = 1.36

$$\frac{\frac{a\,x}{3} + \frac{b\,x^2}{2} + \frac{a^2}{6\,b}}{a^4\,c^5 - 4\,a^3\,b\,c^5\,x + 6\,a^2\,b^2\,c^5\,x^2 - 4\,a\,b^3\,c^5\,x^3 + b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^5,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/2 + a^2/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3
+ 6*a^2*b^2*c^5*x^2 - 4*a^3*b*c^5*x)
```

3.1047 $\int \frac{(a+bx)^2}{(ac-bcx)^6} dx$

Optimal. Leaf size=57

$$\frac{4a^2}{5bc^6(a-bx)^5} - \frac{a}{bc^6(a-bx)^4} + \frac{1}{3bc^6(a-bx)^3}$$

```
[Out] 4/5*a^2/b/c^6/(-b*x+a)^5-a/b/c^6/(-b*x+a)^4+1/3/b/c^6/(-b*x+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{4a^2}{5bc^6(a-bx)^5} - \frac{a}{bc^6(a-bx)^4} + \frac{1}{3bc^6(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] (4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*x
)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^6} dx &= \int \left(\frac{4a^2}{c^6(a-bx)^6} - \frac{4a}{c^6(a-bx)^5} + \frac{1}{c^6(a-bx)^4}\right) dx\\ &= \frac{4a^2}{5bc^6(a-bx)^5} - \frac{a}{bc^6(a-bx)^4} + \frac{1}{3bc^6(a-bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.67

$$-\frac{2a^2+5abx+5b^2x^2}{15bc^6(-a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] -1/15*(2*a^2 + 5*a*b*x + 5*b^2*x^2)/(b*c^6*(-a + b*x)^5)
```

Maple [A]
time = 0.15, size = 49, normalized size = 0.86

method	result	size
risch	$\frac{\frac{x^2 b}{3}+\frac{a x}{3}+\frac{2 a^2}{15 b}}{c^6 (-b x+a)^5}$	32
gosper	$\frac{5 x^2 b^2+5 a b x+2 a^2}{15 (-b x+a)^5 c^6 b}$	36
norman	$\frac{\frac{2 a^2}{15 b c}+\frac{b\,x^2}{3 c}+\frac{a x}{3 c}}{c^5 (-b x+a)^5}$	41
default	$\frac{\frac{1}{3 b (-b x+a)^3}+\frac{4 a^2}{5 b (-b x+a)^5}-\frac{a}{b (-b x+a)^4}}{c^6}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/3/b/(-b*x+a)^3+4/5*a^2/b/(-b*x+a)^5-a/b/(-b*x+a)^4)
```

Maxima [A]
time = 0.29, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Fricas [A]
time = 0.42, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.
time = 0.26, size = 100, normalized size = 1.75

$$\frac{-2a^2 - 5abx - 5b^2x^2}{-15a^5bc^6 + 75a^4b^2c^6x - 150a^3b^3c^6x^2 + 150a^2b^4c^6x^3 - 75ab^5c^6x^4 + 15b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**6,x)
```

```
[Out] (-2*a**2 - 5*a*b*x - 5*b**2*x**2)/(-15*a**5*b*c**6 + 75*a**4*b**2*c**6*x -
150*a**3*b**3*c**6*x**2 + 150*a**2*b**4*c**6*x**3 - 75*a*b**5*c**6*x**4 + 1
5*b**6*c**6*x**5)
```

Giac [A]
time = 1.33, size = 36, normalized size = 0.63

$$-\frac{5\,b^2x^2 + 5\,abx + 2\,a^2}{15\,(bx-a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x - a)^5*b*c^6)
```

Mupad [B]
time = 0.19, size = 91, normalized size = 1.60

$$\frac{\frac{a\,x}{3} + \frac{b\,x^2}{3} + \frac{2\,a^2}{15\,b}}{a^5\,c^6 - 5\,a^4\,b\,c^6\,x + 10\,a^3\,b^2\,c^6\,x^2 - 10\,a^2\,b^3\,c^6\,x^3 + 5\,a\,b^4\,c^6\,x^4 - b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^6,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/3 + (2*a^2)/(15*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6
*x^4 + 10*a^3*b^2*c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

3.1048 $\int \frac{(a+bx)^2}{(ac-bcx)^7} dx$

Optimal. Leaf size=59

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

```
[Out] 2/3*a^2/b/c^7/(-b*x+a)^6-4/5*a/b/c^7/(-b*x+a)^5+1/4/b/c^7/(-b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(a
 - b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^7}\,dx &= \int \left(\frac{4a^2}{c^7(a-bx)^7}-\frac{4a}{c^7(a-bx)^6}+\frac{1}{c^7(a-bx)^5}\right)dx\\ &= \frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.63

$$\frac{7a^2+18abx+15b^2x^2}{60bc^7(a-bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (7*a^2 + 18*a*b*x + 15*b^2*x^2)/(60*b*c^7*(a - b*x)^6)
```

Maple [A]
time = 0.16, size = 49, normalized size = 0.83

method	result	size
risch	$\frac{\frac{x^2 b}{4}+\frac{3ax}{10}+\frac{7a^2}{60b}}{c^7(-bx+a)^6}$	32
gosper	$\frac{15x^2b^2+18abx+7a^2}{60(-bx+a)^6 c^7 b}$	36
norman	$\frac{\frac{7a^2}{60bc}+\frac{b\,x^2}{4c}+\frac{3ax}{10c}}{c^6(-bx+a)^6}$	41
default	$\frac{\frac{1}{4b(-bx+a)^4}+\frac{2a^2}{3b(-bx+a)^6}-\frac{4a}{5b(-bx+a)^5}}{c^7}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^7*(1/4/b/(-b*x+a)^4+2/3*a^2/b/(-b*x+a)^6-4/5*a/b/(-b*x+a)^5)
```

Maxima [A]
time = 0.29, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

Fricas [A]
time = 0.46, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.
time = 0.28, size = 117, normalized size = 1.98

$$-\frac{-7a^2 - 18abx - 15b^2x^2}{60a^6bc^7 - 360a^5b^2c^7x + 900a^4b^3c^7x^2 - 1200a^3b^4c^7x^3 + 900a^2b^5c^7x^4 - 360ab^6c^7x^5 + 60b^7c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**7,x)
```

```
[Out] -(-7*a**2 - 18*a*b*x - 15*b**2*x**2)/(60*a**6*b*c**7 - 360*a**5*b**2*c**7*x
 + 900*a**4*b**3*c**7*x**2 - 1200*a**3*b**4*c**7*x**3 + 900*a**2*b**5*c**7*
x**4 - 360*a*b**6*c**7*x**5 + 60*b**7*c**7*x**6)
```

Giac [A]
time = 1.14, size = 36, normalized size = 0.61

$$\frac{15\,b^2x^2 + 18\,abx + 7\,a^2}{60\,(bx - a)^6bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="giac")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/((b*x - a)^6*b*c^7)
```

Mupad [B]
time = 0.11, size = 104, normalized size = 1.76

$$\frac{\frac{3\,a\,x}{10} + \frac{b\,x^2}{4} + \frac{7\,a^2}{60\,b}}{a^6\,c^7 - 6\,a^5\,b\,c^7\,x + 15\,a^4\,b^2\,c^7\,x^2 - 20\,a^3\,b^3\,c^7\,x^3 + 15\,a^2\,b^4\,c^7\,x^4 - 6\,a\,b^5\,c^7\,x^5 + b^6\,c^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^7,x)
```

```
[Out] ((3*a*x)/10 + (b*x^2)/4 + (7*a^2)/(60*b))/(a^6*c^7 + b^6*c^7*x^6 - 6*a*b^5*
c^7*x^5 + 15*a^4*b^2*c^7*x^2 - 20*a^3*b^3*c^7*x^3 + 15*a^2*b^4*c^7*x^4 - 6*
a^5*b*c^7*x)
```

3.1049 $\int \frac{(ac-bcx)^3}{a+bx}\,dx$

Optimal. Leaf size=61

$$-4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}$$

```
[Out] -4*a^2*c^3*x+a*c^3*(-b*x+a)^2/b+1/3*c^3*(-b*x+a)^3/b+8*a^3*c^3*ln(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] -4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^3
*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{a+bx}\,dx &= \int \left(-4a^2c^3+\frac{8a^3c^3}{a+bx}-2ac^2(ac-bcx)-c(ac-bcx)^2\right)dx\\ &= -4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.69

$$c^3\left(-7a^2x+2abx^2-\frac{b^2x^3}{3}+\frac{8a^3\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] c^3*(-7*a^2*x + 2*a*b*x^2 - (b^2*x^3)/3 + (8*a^3*Log[a + b*x])/b)
```

Maple [A]
time = 0.15, size = 41, normalized size = 0.67

method	result	size
default	$c^3\left(-\frac{b^2x^3}{3}+2ab\,x^2-7a^2x+\frac{8a^3\ln(bx+a)}{b}\right)$	41
norman	$-7a^2c^3x-\frac{b^2c^3x^3}{3}+2a\,c^3b\,x^2+\frac{8a^3c^3\ln(bx+a)}{b}$	49
risch	$-7a^2c^3x-\frac{b^2c^3x^3}{3}+2a\,c^3b\,x^2+\frac{8a^3c^3\ln(bx+a)}{b}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*b^2*x^3+2*a*b*x^2-7*a^2*x+8/b*a^3*ln(b*x+a))
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.79

$$-\frac{1}{3}\,b^2c^3x^3+2\,abc^3x^2-7\,a^2c^3x+\frac{8\,a^3c^3\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*c^3*x^3 + 2*a*b*c^3*x^2 - 7*a^2*c^3*x + 8*a^3*c^3*log(b*x + a)/b
```

Fricas [A]
time = 0.48, size = 52, normalized size = 0.85

$$-\frac{b^3c^3x^3-6\,ab^2c^3x^2+21\,a^2bc^3x-24\,a^3c^3\log\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 21*a^2*b*c^3*x - 24*a^3*c^3*log(b*x +
 a))/b
```

Sympy [A]
time = 0.06, size = 49, normalized size = 0.80

$$\frac{8a^3c^3\log\left(a+bx\right)}{b}-7a^2c^3x+2abc^3x^2-\frac{b^2c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a),x)
```

```
[Out] 8*a**3*c**3*log(a + b*x)/b - 7*a**2*c**3*x + 2*a*b*c**3*x**2 - b**2*c**3*x*
*3/3
```

Giac [A]
time = 1.42, size = 59, normalized size = 0.97

$$\frac{8\,a^3c^3\log\left(|bx+a|\right)}{b}-\frac{b^5c^3x^3-6\,ab^4c^3x^2+21\,a^2b^3c^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 8*a^3*c^3*log(abs(b*x + a))/b - 1/3*(b^5*c^3*x^3 - 6*a*b^4*c^3*x^2 + 21*a^2
*b^3*c^3*x)/b^3
```

Mupad [B]
time = 0.05, size = 48, normalized size = 0.79

$$\frac{8\,a^3\,c^3\,\ln\left(a+b\,x\right)}{b}-\frac{b^2\,c^3\,x^3}{3}-7\,a^2\,c^3\,x+2\,a\,b\,c^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x),x)
```

```
[Out] (8*a^3*c^3*log(a + b*x))/b - (b^2*c^3*x^3)/3 - 7*a^2*c^3*x + 2*a*b*c^3*x^2
```

3.1050 $\int \frac{(ac-bcx)^2}{a+bx} \, dx$

Optimal. Leaf size=43

$$-2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b}$$

```
[Out] -2*a*c^2*x+1/2*c^2*(-b*x+a)^2/b+4*a^2*c^2*ln(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {45}

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] -2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{a+bx}\,dx &= \int \left(-2ac^2+\frac{4a^2c^2}{a+bx}-c(ac-bcx)\right)dx\\ &= -2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 0.72

$$c^2\left(-3ax+\frac{bx^2}{2}+\frac{4a^2\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x),x]
[Out] c^2*(-3*a*x + (b*x^2)/2 + (4*a^2*Log[a + b*x])/b)
```

Maple [A]
time = 0.15, size = 30, normalized size = 0.70

method	result	size
default	$c^{2} \left(\frac{x^{2} b}{2}-3 a x +\frac{4 a^{2} \ln \left(b x +a \right)}{b}\right)$	30
norman	$-3 a \,c^{2} x +\frac{b \,c^{2} x^{2}}{2}+\frac{4 a^{2} c^{2} \ln \left(b x +a \right)}{b}$	35
risch	$-3 a \,c^{2} x +\frac{b \,c^{2} x^{2}}{2}+\frac{4 a^{2} c^{2} \ln \left(b x +a \right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a),x,method=_RETURNVERBOSE)
[Out] c^2*(1/2*x^2*b-3*a*x+4*a^2/b*ln(b*x+a))
```

Maxima [A]
time = 0.29, size = 34, normalized size = 0.79

$$\frac{1}{2} \, b c^{2} x^{2} - 3 \, a c^{2} x + \frac{4 \, a^{2} c^{2} \log \left(b x + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="maxima")
[Out] 1/2*b*c^2*x^2 - 3*a*c^2*x + 4*a^2*c^2*log(b*x + a)/b
```

Fricas [A]
time = 0.54, size = 38, normalized size = 0.88

$$\frac{b^{2} c^{2} x^{2} - 6 \, a b c^{2} x + 8 \, a^{2} c^{2} \log \left(b x + a\right)}{2 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*c^2*x^2 - 6*a*b*c^2*x + 8*a^2*c^2*log(b*x + a))/b
```

Sympy [A]
time = 0.05, size = 34, normalized size = 0.79

$$\frac{4 a^{2} c^{2} \log{\left(a + b x \right)}}{b} - 3 a c^{2} x + \frac{b c^{2} x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a),x)
[Out] 4*a**2*c**2*log(a + b*x)/b - 3*a*c**2*x + b*c**2*x**2/2
```

Giac [A]
time = 1.42, size = 45, normalized size = 1.05

$$\frac{4\,a^2c^2\log\left(|bx+a|\right)}{b}+\frac{b^3c^2x^2-6\,ab^2c^2x}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="giac")
[Out] 4*a^2*c^2*log(abs(b*x + a))/b + 1/2*(b^3*c^2*x^2 - 6*a*b^2*c^2*x)/b^2
```

Mupad [B]
time = 0.15, size = 32, normalized size = 0.74

$$\frac{c^2\,\left(8\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-6\,a\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x),x)
[Out] (c^2*(8*a^2*log(a + b*x) + b^2*x^2 - 6*a*b*x))/(2*b)
```

3.1051 $\int \frac{ac-bcx}{a+bx}\,dx$

Optimal. Leaf size=18

$$-cx+\frac{2ac\log(a+bx)}{b}$$

[Out] -c*x+2*a*c*ln(b*x+a)/b

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$\frac{2ac\log(a+bx)}{b}-cx$$

Antiderivative was successfully verified.

[In] Int[(a*c - b*c*x)/(a + b*x),x]

[Out] -(c*x) + (2*a*c*Log[a + b*x])/b

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{a+bx}\,dx &= \int \left(-c+\frac{2ac}{a+bx}\right)dx\\ &= -cx+\frac{2ac\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$c\left(-x+\frac{2a\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a*c - b*c*x)/(a + b*x),x]

```
[Out] c*(-x + (2*a*Log[a + b*x])/b)
```

Maple [A]
time = 0.14, size = 19, normalized size = 1.06

method	result	size
default	$c\left(-x+\frac{2a\ln(bx+a)}{b}\right)$	19
norman	$-cx+\frac{2ac\ln(bx+a)}{b}$	19
risch	$-cx+\frac{2ac\ln(bx+a)}{b}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] c*(-x+2*a/b*ln(b*x+a))
```

Maxima [A]
time = 0.28, size = 18, normalized size = 1.00

$$-cx+\frac{2\,ac\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="maxima")
[Out] -c*x + 2*a*c*log(b*x + a)/b
```

Fricas [A]
time = 0.49, size = 20, normalized size = 1.11

$$-\frac{bcx-2\,ac\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="fricas")
[Out] -(b*c*x - 2*a*c*log(b*x + a))/b
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.83

$$\frac{2ac\log\left(a+bx\right)}{b}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x)
```

```
[Out] 2*a*c*log(a + b*x)/b - c*x
```

Giac [A]
time = 1.01, size = 19, normalized size = 1.06

$$-cx + \frac{2\,ac\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -c*x + 2*a*c*log(abs(b*x + a))/b
```

Mupad [B]
time = 0.04, size = 18, normalized size = 1.00

$$\frac{2\,a\,c\,\ln\left(a+b\,x\right)}{b}-c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x),x)
```

```
[Out] (2*a*c*log(a + b*x))/b - c*x
```

3.1052 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Maple [A]
time = 0.13, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(b*x+a)/b
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
[Out] log(b*x + a)/b
```

Fricas [A]
time = 0.51, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
[Out] log(b*x + a)/b
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
[Out] log(a + b*x)/b
```

Giac [A]
time = 1.49, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

3.1053 $\int \frac{1}{(a+bx)(ac-bcx)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

```
[Out] arctanh(b*x/a)/a/b/c
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {35, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)}\,dx &= \int \frac{1}{a^2c-b^2cx^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Maple [A]
time = 0.16, size = 35, normalized size = 2.06

method	result	size
default	$\frac{-\frac{\ln(-bx+a)}{2ab}+\frac{\ln(bx+a)}{2ab}}{c}$	35
norman	$-\frac{\ln(-bx+a)}{2abc}+\frac{\ln(bx+a)}{2abc}$	37
risch	$-\frac{\ln(-bx+a)}{2abc}+\frac{\ln(bx+a)}{2abc}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2/a/b*ln(-b*x+a)+1/2*ln(b*x+a)/a/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.28, size = 37, normalized size = 2.18

$$\frac{\log(bx+a)}{2\,abc} - \frac{\log(bx-a)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)/(a*b*c) - 1/2*log(b*x - a)/(a*b*c)
```

Fricas [A]
time = 0.53, size = 28, normalized size = 1.65

$$\frac{\log(bx+a) - \log(bx-a)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.08, size = 22, normalized size = 1.29

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2} - \frac{\log\left(\frac{a}{b}+x\right)}{2}}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x)
```

```
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(17) = 34$.
time = 1.68, size = 39, normalized size = 2.29

$$\frac{\log(|bx + a|)}{2\,abc} - \frac{\log(|bx - a|)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x + a))/(a*b*c) - 1/2*log(abs(b*x - a))/(a*b*c)
```

Mupad [B]
time = 0.17, size = 17, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)),x)
```

```
[Out] atanh((b*x)/a)/(a*b*c)
```

3.1054 $\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx$

Optimal. Leaf size=42

$$\frac{1}{2abc^2(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}$$

```
[Out] 1/2/a/b/c^2/(-b*x+a)+1/2*arctanh(b*x/a)/a^2/b/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {46, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] 1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx &= \int \left(\frac{1}{2ac^2(a-bx)^2}+\frac{1}{2ac^2\left(a^2-b^2x^2\right)}\right)\,dx\\
&= \frac{1}{2abc^2(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac^2}\\
&= \frac{1}{2abc^2(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.26

$$\frac{2a + (-a + bx)\log(a - bx) + (a - bx)\log(a + bx)}{4a^2bc^2(a - bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*c^2*(a - b*x))
```

Maple [A]
time = 0.15, size = 51, normalized size = 1.21

method	result	size
default	$\frac{-\frac{\ln(-bx+a)}{4a^2b}+\frac{1}{2ab(-bx+a)}+\frac{\ln(bx+a)}{4a^2b}}{c^2}$	51
norman	$\frac{1}{2ab\,c^2(-bx+a)} - \frac{\ln(-bx+a)}{4a^2b\,c^2} + \frac{\ln(bx+a)}{4a^2b\,c^2}$	56
risch	$\frac{1}{2ab\,c^2(-bx+a)} - \frac{\ln(-bx+a)}{4a^2b\,c^2} + \frac{\ln(bx+a)}{4a^2b\,c^2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4/a^2/b*ln(-b*x+a)+1/2/a/b/(-b*x+a)+1/4/a^2/b*ln(b*x+a))
```

Maxima [A]
time = 0.28, size = 60, normalized size = 1.43

$$-\frac{1}{2\,(ab^2c^2x - a^2bc^2)} + \frac{\log\left(bx + a\right)}{4\,a^2bc^2} - \frac{\log\left(bx - a\right)}{4\,a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c^2*x - a^2*b*c^2) + 1/4*log(b*x + a)/(a^2*b*c^2) - 1/4*log(b*x - a)/(a^2*b*c^2)
```

Fricas [A]
time = 0.63, size = 60, normalized size = 1.43

$$\frac{(bx - a)\log\left(bx + a\right) - (bx - a)\log\left(bx - a\right) - 2\,a}{4\,(a^2b^2c^2x - a^3bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*c^2*x
 - a^3*b*c^2)
```

Sympy [A]
time = 0.12, size = 48, normalized size = 1.14

$$-\frac{1}{-2a^2bc^2+2ab^2c^2x}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -1/(-2*a**2*b*c**2 + 2*a*b**2*c**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)
/(a**2*b*c**2)
```

Giac [A]
time = 2.39, size = 53, normalized size = 1.26

$$-\frac{1}{2\,(bcx-ac)abc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/2/((b*c*x - a*c)*a*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^2*b*
c^2)
```

Mupad [B]
time = 0.07, size = 42, normalized size = 1.00

$$\frac{1}{2\,a\,b\,\left(a\,c^2-b\,c^2\,x\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)),x)
```

```
[Out] 1/(2*a*b*(a*c^2 - b*c^2*x)) + atanh((b*x)/a)/(2*a^2*b*c^2)
```

3.1055 $\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}$$

```
[Out] 1/4/a/b/c^3/(-b*x+a)^2+1/4/a^2/b/c^3/(-b*x+a)+1/4*arctanh(b*x/a)/a^3/b/c^3
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {46, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4a^2bc^3(a-bx)}+\frac{1}{4abc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(4
*a^3*b*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx &= \int \left(\frac{1}{2ac^3(a-bx)^3}+\frac{1}{4a^2c^3(a-bx)^2}+\frac{1}{4a^2c^3\left(a^2-b^2x^2\right)}\right)dx\\
&= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2c^3}\\
&= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.03

$$\frac{2a(2a-bx)-(a-bx)^2\log(a-bx)+(a-bx)^2\log(a+bx)}{8a^3bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] (2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8*
a^3*b*c^3*(a - b*x)^2)
```

Maple [A]
time = 0.18, size = 67, normalized size = 1.06

method	result	size
risch	$\frac{-\frac{x}{4a^2}+\frac{1}{2ab}}{c^3(-bx+a)^2}-\frac{\ln(-bx+a)}{8a^3b\,c^3}+\frac{\ln(bx+a)}{8a^3b\,c^3}$	64
default	$\frac{-\frac{\ln(-bx+a)}{8a^3b}+\frac{1}{4a^2b(-bx+a)}+\frac{1}{4ab(-bx+a)^2}+\frac{\ln(bx+a)}{8a^3b}}{c^3}$	67
norman	$\frac{\frac{3x}{4a^2c}-\frac{b\,x^2}{2a^3c}}{c^2(-bx+a)^2}-\frac{\ln(-bx+a)}{8a^3b\,c^3}+\frac{\ln(bx+a)}{8a^3b\,c^3}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/8/a^3/b*ln(-b*x+a)+1/4/a^2/b/(-b*x+a)+1/4/a/b/(-b*x+a)^2+1/8/a^3/
b*ln(b*x+a))
```

Maxima [A]
time = 0.30, size = 82, normalized size = 1.30

$$-\frac{bx-2\,a}{4\,(a^2b^3c^3x^2-2\,a^3b^2c^3x+a^4bc^3)}+\frac{\log\left(bx+a\right)}{8\,a^3bc^3}-\frac{\log\left(bx-a\right)}{8\,a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x - 2*a)/(a^2*b^3*c^3*x^2 - 2*a^3*b^2*c^3*x + a^4*b*c^3) + 1/8*log(
b*x + a)/(a^3*b*c^3) - 1/8*log(b*x - a)/(a^3*b*c^3)
```

Fricas [A]
time = 0.48, size = 98, normalized size = 1.56

$$-\frac{2\,abx-4\,a^2-(b^2x^2-2\,abx+a^2)\log\left(bx+a\right)+(b^2x^2-2\,abx+a^2)\log\left(bx-a\right)}{8\,(a^3b^3c^3x^2-2\,a^4b^2c^3x+a^5bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 -
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*c^3*x^2 - 2*a^4*b^2*c^3*x + a^5*b*c^
3)
```

Sympy [A]
time = 0.16, size = 71, normalized size = 1.13

$$-\frac{-2a+bx}{4a^4bc^3-8a^3b^2c^3x+4a^2b^3c^3x^2}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8}-\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a + b*x)/(4*a**4*b*c**3 - 8*a**3*b**2*c**3*x + 4*a**2*b**3*c**3*x**2)
- (log(-a/b + x)/8 - log(a/b + x)/8)/(a**3*b*c**3)
```

Giac [A]
time = 2.56, size = 69, normalized size = 1.10

$$\frac{\log\left(|bx+a|\right)}{8\,a^3bc^3}-\frac{\log\left(|bx-a|\right)}{8\,a^3bc^3}-\frac{abx-2\,a^2}{4\,(bx-a)^2a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(b*x + a))/(a^3*b*c^3) - 1/8*log(abs(b*x - a))/(a^3*b*c^3) - 1/4
*(a*b*x - 2*a^2)/((b*x - a)^2*a^3*b*c^3)
```

Mupad [B]
time = 0.08, size = 64, normalized size = 1.02

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b\,c^3}-\frac{\frac{x}{4\,a^2}-\frac{1}{2\,a\,b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)),x)
```

```
[Out] atanh((b*x)/a)/(4*a^3*b*c^3) - (x/(4*a^2) - 1/(2*a*b))/(a^2*c^3 + b^2*c^3*x
^2 - 2*a*b*c^3*x)
```

3.1056 $\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx$

Optimal. Leaf size=54

$$5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}$$

```
[Out] 5*a*c^3*x-1/2*b*c^3*x^2-8*a^3*c^3/b/(b*x+a)-12*a^2*c^3*ln(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {45}

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a +
 b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx &= \int \left(5ac^3-bc^3x+\frac{8a^3c^3}{(a+bx)^2}-\frac{12a^2c^3}{a+bx}\right)dx\\ &= 5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 0.85

$$c^3\left(5ax-\frac{bx^2}{2}-\frac{8a^3}{b(a+bx)}-\frac{12a^2\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x)^2,x]
[Out] c^3*(5*a*x - (b*x^2)/2 - (8*a^3)/(b*(a + b*x)) - (12*a^2*Log[a + b*x])/b)
```

Maple [A]
time = 0.15, size = 45, normalized size = 0.83

method	result	size
default	$c^{3} \left(-\frac{x^{2} b}{2}+5 a x -\frac{8 a^{3}}{b \left(b x +a \right)}-\frac{12 a^{2} \ln \left(b x +a \right)}{b}\right)$	45
risch	$5 a \,c^{3} x -\frac{b \,c^{3} x^{2}}{2}-\frac{8 a^{3} c^{3}}{b \left(b x +a \right)}-\frac{12 a^{2} c^{3} \ln \left(b x +a \right)}{b}$	53
norman	$\frac{13 a^{2} c^{3} x -\frac{1}{2} b^{2} c^{3} x^{3}+\frac{9}{2} a \,c^{3} b \,x^{2}}{b x +a}-\frac{12 a^{2} c^{3} \ln \left(b x +a \right)}{b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] c^3*(-1/2*x^2*b+5*a*x-8/b*a^3/(b*x+a)-12*a^2/b*ln(b*x+a))
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.98

$$-\frac{1}{2} \, b c^{3} x^{2} - \frac{8 \, a^{3} c^{3}}{b^{2} x + a b} + 5 \, a c^{3} x - \frac{12 \, a^{2} c^{3} \log \left(b x + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="maxima")
[Out] -1/2*b*c^3*x^2 - 8*a^3*c^3/(b^2*x + a*b) + 5*a*c^3*x - 12*a^2*c^3*log(b*x +
 a)/b
```

Fricas [A]
time = 0.78, size = 79, normalized size = 1.46

$$-\frac{b^{3} c^{3} x^{3} - 9 \, a b^{2} c^{3} x^{2} - 10 \, a^{2} b c^{3} x + 16 \, a^{3} c^{3} + 24 \, {\left(a^{2} b c^{3} x + a^{3} c^{3}\right)} \log \left(b x + a\right)}{2 \, {\left(b^{2} x + a b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="fricas")
[Out] -1/2*(b^3*c^3*x^3 - 9*a*b^2*c^3*x^2 - 10*a^2*b*c^3*x + 16*a^3*c^3 + 24*(a^2
*b*c^3*x + a^3*c^3)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A]
time = 0.10, size = 51, normalized size = 0.94

$$- \frac{8 a^{3} c^{3}}{a b + b^{2} x} - \frac{12 a^{2} c^{3} \log{\left(a + b x \right)}}{b} + 5 a c^{3} x - \frac{b c^{3} x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a)**2,x)
```

```
[Out] -8*a**3*c**3/(a*b + b**2*x) - 12*a**2*c**3*log(a + b*x)/b + 5*a*c**3*x - b*
c**3*x**2/2
```

Giac [A]
time = 1.51, size = 80, normalized size = 1.48

$$\frac{12\,a^2c^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{8\,a^3c^3}{(bx+a)b}+\frac{\left(\frac{12\,ac^3}{bx+a}-c^3\right)(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 12*a^2*c^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - 8*a^3*c^3/((b*x + a)*
b) + 1/2*(12*a*c^3/(b*x + a) - c^3)*(b*x + a)^2/b
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.96

$$5\,a\,c^3\,x-\frac{b\,c^3\,x^2}{2}-\frac{12\,a^2\,c^3\,\ln\left(a+b\,x\right)}{b}-\frac{8\,a^3\,c^3}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x)^2,x)
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (12*a^2*c^3*log(a + b*x))/b - (8*a^3*c^3)/(b*(a
 + b*x))
```

3.1057 $\int \frac{(ac-bcx)^2}{(a+bx)^2} \, dx$

Optimal. Leaf size=39

$$c^2x-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}$$

```
[Out] c^2*x-4*a^2*c^2/b/(b*x+a)-4*a*c^2*ln(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx &= \int \left(c^2+\frac{4a^2c^2}{(a+bx)^2}-\frac{4ac^2}{a+bx}\right)dx\\ &= c^2x-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.85

$$c^2\left(x-\frac{4a^2}{b(a+bx)}-\frac{4a\log(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x)^2,x]
```

```
[Out] c^2*(x - (4*a^2)/(b*(a + b*x)) - (4*a*Log[a + b*x])/b)
```

Maple [A]
time = 0.15, size = 34, normalized size = 0.87

method	result	size
default	$c^{2} \left(x -\frac{4 a^{2}}{b \left(b x +a \right)}-\frac{4 a \ln \left(b x +a \right)}{b}\right)$	34
risch	$c^{2} x -\frac{4 a^{2} c^{2}}{b \left(b x +a \right)}-\frac{4 a \,c^{2} \ln \left(b x +a \right)}{b}$	40
norman	$\frac{b \,c^{2} x^{2}+5 a \,c^{2} x}{b x +a}-\frac{4 a \,c^{2} \ln \left(b x +a \right)}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(x-4*a^2/b/(b*x+a)-4*a/b*ln(b*x+a))
```

Maxima [A]
time = 0.28, size = 40, normalized size = 1.03

$$-\frac{4 \, a^{2} c^{2}}{b^{2} x + a b} + c^{2} x - \frac{4 \, a c^{2} \log \left(b x + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2*c^2/(b^2*x + a*b) + c^2*x - 4*a*c^2*log(b*x + a)/b
```

Fricas [A]
time = 0.51, size = 61, normalized size = 1.56

$$\frac{b^{2} c^{2} x^{2} + a b c^{2} x - 4 \, a^{2} c^{2} - 4 \, {\left(a b c^{2} x + a^{2} c^{2}\right)} \log \left(b x + a\right)}{b^{2} x + a b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*x^2 + a*b*c^2*x - 4*a^2*c^2 - 4*(a*b*c^2*x + a^2*c^2)*log(b*x + a)
)/(b^2*x + a*b)
```

Sympy [A]
time = 0.07, size = 36, normalized size = 0.92

$$-\frac{4a^{2}c^{2}}{ab + b^{2}x} - \frac{4ac^{2}\log{\left(a + bx \right)}}{b} + c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a)**2,x)
```

```
[Out] -4*a**2*c**2/(a*b + b**2*x) - 4*a*c**2*log(a + b*x)/b + c**2*x
```

Giac [A]
time = 1.30, size = 59, normalized size = 1.51

$$\frac{4\,ac^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}+\frac{(bx+a)c^2}{b}-\frac{4\,a^2c^2}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*a*c^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b + (b*x + a)*c^2/b - 4*a^2*
c^2/((b*x + a)*b)
```

Mupad [B]
time = 0.17, size = 39, normalized size = 1.00

$$c^2\,x-\frac{4\,a\,c^2\,\ln\left(a+b\,x\right)}{b}-\frac{4\,a^2\,c^2}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x)^2,x)
```

```
[Out] c^2*x - (4*a*c^2*log(a + b*x))/b - (4*a^2*c^2)/(b*(a + b*x))
```

3.1058 $\int \frac{ac-bcx}{(a+bx)^2} \, dx$

Optimal. Leaf size=27

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

[Out] -2*a*c/b/(b*x+a)-c*ln(b*x+a)/b

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a*c - b*c*x)/(a + b*x)^2,x]

[Out] (-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{ac-bcx}{(a+bx)^2} \, dx &= \int \left(\frac{2ac}{(a+bx)^2}-\frac{c}{a+bx}\right) dx \\
&= -\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.85

$$-\frac{c\left(\frac{2a}{a+bx}+\log(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*c - b*c*x)/(a + b*x)^2,x]

```
[Out] -((c*((2*a)/(a + b*x) + Log[a + b*x]))/b)
```

Maple [A]
time = 0.14, size = 28, normalized size = 1.04

method	result	size
norman	$\frac{2cx}{bx+a} - \frac{c\ln(bx+a)}{b}$	25
default	$c\left(-\frac{2a}{b(bx+a)} - \frac{\ln(bx+a)}{b}\right)$	28
risch	$-\frac{2ac}{b(bx+a)} - \frac{c\ln(bx+a)}{b}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] c*(-2*a/b/(b*x+a)-ln(b*x+a)/b)
```

Maxima [A]
time = 0.30, size = 28, normalized size = 1.04

$$-\frac{2\,ac}{b^2x+ab} - \frac{c\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="maxima")
[Out] -2*a*c/(b^2*x + a*b) - c*log(b*x + a)/b
```

Fricas [A]
time = 0.65, size = 33, normalized size = 1.22

$$-\frac{2\,ac + (bcx + ac)\log\left(bx+a\right)}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="fricas")
[Out] -(2*a*c + (b*c*x + a*c)*log(b*x + a))/(b^2*x + a*b)
```

Sympy [A]
time = 0.06, size = 24, normalized size = 0.89

$$-\frac{2ac}{ab+b^2x} - \frac{c\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)**2,x)
```

```
[Out] -2*a*c/(a*b + b**2*x) - c*log(a + b*x)/b
```

Giac [A]
time = 1.81, size = 54, normalized size = 2.00

$$c\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx+a)b}\right) - \frac{ac}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b)) - a*c/((b*x
+ a)*b)
```

Mupad [B]
time = 0.04, size = 27, normalized size = 1.00

$$-\frac{c\ln\left(a+b\,x\right)}{b}-\frac{2\,a\,c}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x)^2,x)
```

```
[Out] - (c*log(a + b*x))/b - (2*a*c)/(b*(a + b*x))
```

3.1059 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A]
time = 0.13, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A]
time = 0.59, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A]
time = 2.52, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 1.00

$$-\frac{1}{b\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
[Out] -1/(b*(a + b*x))
```

3.1060 $\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}$$

```
[Out] -1/2/a/b/c/(b*x+a)+1/2*arctanh(b*x/a)/a^2/b/c
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {46, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] -1/2*1/(a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(ac-bcx)}\,dx &= \int \left(\frac{1}{2ac(a+bx)^2}+\frac{1}{2ac\left(a^2-b^2x^2\right)}\right)dx\\ &= -\frac{1}{2abc(a+bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac}\\ &= -\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.22

$$\frac{-2a-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)}{4a^2bc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] (-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*c*(a + b*
x))
```

Maple [A]
time = 0.15, size = 50, normalized size = 1.22

method	result	size
default	$\frac{-\frac{\ln(-bx+a)}{4a^2b}+\frac{\ln(bx+a)}{4a^2b}-\frac{1}{2ab(bx+a)}}{c}$	50
norman	$-\frac{1}{2abc(bx+a)}-\frac{\ln(-bx+a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$	55
risch	$-\frac{1}{2abc(bx+a)}-\frac{\ln(-bx+a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/4/a^2/b*ln(-b*x+a)+1/4/a^2/b*ln(b*x+a)-1/2/a/b/(b*x+a))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.34

$$-\frac{1}{2\,(ab^2cx+a^2bc)}+\frac{\log{(bx+a)}}{4\,a^2bc}-\frac{\log{(bx-a)}}{4\,a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c*x + a^2*b*c) + 1/4*log(b*x + a)/(a^2*b*c) - 1/4*log(b*x - a)/
(a^2*b*c)
```

Fricas [A]
time = 0.58, size = 51, normalized size = 1.24

$$\frac{(bx+a)\log{(bx+a)}-(bx+a)\log{(bx-a)}-2\,a}{4\,(a^2b^2cx+a^3bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*c*x +
a^3*b*c)
```

Sympy [A]
time = 0.11, size = 44, normalized size = 1.07

$$-\frac{1}{2a^2bc+2ab^2cx}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4}-\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -1/(2*a**2*b*c + 2*a*b**2*c*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b
*c)
```

Giac [A]
time = 1.69, size = 44, normalized size = 1.07

$$-\frac{\log\left(\left|-\frac{2a}{bx+a}+1\right|\right)}{4a^2bc}-\frac{1}{2(bx+a)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(-2*a/(b*x + a) + 1))/(a^2*b*c) - 1/2/((b*x + a)*a*b*c)
```

Mupad [B]
time = 0.18, size = 37, normalized size = 0.90

$$\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c}-\frac{1}{2\,a\,b\,\left(a\,c+b\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)^2),x)
```

```
[Out] atanh((b*x)/a)/(2*a^2*b*c) - 1/(2*a*b*(a*c + b*c*x))
```

3.1061 $\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

```
[Out] 1/2*x/a^2/c^2/(-b^2*x^2+a^2)+1/2*arctanh(b*x/a)/a^3/b/c^2
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {41, 205, 214}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}+\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx = \int \frac{1}{\left(a^2c-b^2cx^2\right)^2}\,dx$$
$$= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\int \frac{1}{a^2c-b^2cx^2}\,dx}{2a^2c}$$
$$= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)} + \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.61

$$\frac{2abx+\left(-a^2+b^2x^2\right)\log(a-bx)+\left(a^2-b^2x^2\right)\log(a+bx)}{4a^3bc^2(a-bx)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a*b*x + (-a^2 + b^2*x^2)*Log[a - b*x] + (a^2 - b^2*x^2)*Log[a + b*x])/(4
*a^3*b*c^2*(a - b*x)*(a + b*x))
```

Maple [A]
time = 0.16, size = 66, normalized size = 1.43

method	result	size
norman	$\frac{x}{2a^2c^2(bx+a)(-bx+a)}-\frac{\ln(-bx+a)}{4a^3b\,c^2}+\frac{\ln(bx+a)}{4a^3b\,c^2}$	61
risch	$\frac{x}{2a^2c^2(bx+a)(-bx+a)}-\frac{\ln(-bx+a)}{4a^3b\,c^2}+\frac{\ln(bx+a)}{4a^3b\,c^2}$	61
default	$\frac{-\frac{\ln(-bx+a)}{4a^3b}+\frac{1}{4a^2b(-bx+a)}+\frac{\ln(bx+a)}{4a^3b}-\frac{1}{4a^2b(bx+a)}}{c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4/a^3/b*ln(-b*x+a)+1/4/a^2/b/(-b*x+a)+1/4/a^3/b*ln(b*x+a)-1/4/a^2
/b/(b*x+a))
```

Maxima [A]
time = 0.34, size = 64, normalized size = 1.39

$$-\frac{x}{2\left(a^2b^2c^2x^2-a^4c^2\right)}+\frac{\log\left(bx+a\right)}{4\,a^3bc^2}-\frac{\log\left(bx-a\right)}{4\,a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(a^2*b^2*c^2*x^2 - a^4*c^2) + 1/4*log(b*x + a)/(a^3*b*c^2) - 1/4*log
(b*x - a)/(a^3*b*c^2)
```

Fricas [A]
time = 0.78, size = 76, normalized size = 1.65

$$-\frac{2\,abx-(b^2x^2-a^2)\log{(bx+a)}+(b^2x^2-a^2)\log{(bx-a)}}{4\,(a^3b^3c^2x^2-a^5bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a)
)/(a^3*b^3*c^2*x^2 - a^5*b*c^2)
```

Sympy [A]
time = 0.11, size = 49, normalized size = 1.07

$$-\frac{x}{-2a^4c^2+2a^2b^2c^2x^2}+\frac{-\frac{\log{\left(-\frac{a}{b}+x\right)}}{4}+\frac{\log{\left(\frac{a}{b}+x\right)}}{4}}{a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -x/(-2*a**4*c**2 + 2*a**2*b**2*c**2*x**2) + (-log(-a/b + x)/4 + log(a/b + x
)/4)/(a**3*b*c**2)
```

Giac [A]
time = 1.18, size = 83, normalized size = 1.80

$$-\frac{1}{4\,(bcx-ac)a^2bc}+\frac{\log{\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}}{4\,a^3bc^2}+\frac{1}{8\,a^3b\left(\frac{2\,ac}{bcx-ac}+1\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/4/((b*c*x - a*c)*a^2*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^3*
b*c^2) + 1/8/(a^3*b*(2*a*c/(b*c*x - a*c) + 1)*c^2)
```

Mupad [B]
time = 0.18, size = 46, normalized size = 1.00

$$\frac{x}{2\,a^2\,\left(a^2\,c^2-b^2\,c^2\,x^2\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)^2),x)
```

```
[Out] x/(2*a^2*(a^2*c^2 - b^2*c^2*x^2)) + atanh((b*x)/a)/(2*a^3*b*c^2)
```

3.1062 $\int \frac{1}{(a+bx)^2(ac-bcx)^3} dx$

Optimal. Leaf size=83

$$\frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

```
[Out] 1/8/a^2/b/c^3/(-b*x+a)^2+1/4/a^3/b/c^3/(-b*x+a)-1/8/a^3/b/c^3/(b*x+a)+3/8*a
rctanh(b*x/a)/a^4/b/c^3
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {46, 214}

$$\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(a
 + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx &= \int \left(\frac{1}{4a^2c^3(a-bx)^3}+\frac{1}{4a^3c^3(a-bx)^2}+\frac{1}{8a^3c^3(a+bx)^2}+\frac{3}{8a^3c^3\left(a^2-b^2x^2\right)}\right)dx\\ &= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3c^3}\\ &= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.82

$$\frac{\frac{2a(2a^2+3abx-3b^2x^2)}{(a-bx)^2(a+bx)}-3\log(a-bx)+3\log(a+bx)}{16a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] ((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b*
x] + 3*Log[a + b*x])/(16*a^4*b*c^3)
```

Maple [A]
time = 0.18, size = 82, normalized size = 0.99

method	result	size
risch	$\frac{-\frac{3b\,x^{2}}{8a^{3}}+\frac{3x}{8a^{2}}+\frac{1}{4ab}}{(bx+a)c^{3}(-bx+a)^{2}}-\frac{3\ln(-bx+a)}{16a^{4}c^{3}b}+\frac{3\ln(bx+a)}{16a^{4}c^{3}b}$	80
default	$\frac{-\frac{3\ln(-bx+a)}{16a^{4}b}+\frac{1}{4a^{3}b(-bx+a)}+\frac{1}{8a^{2}b(-bx+a)^{2}}+\frac{3\ln(bx+a)}{16a^{4}b}-\frac{1}{8a^{3}b(bx+a)}}{c^{3}}$	82
norman	$\frac{\frac{1}{4abc}+\frac{3x}{8a^{2}c}-\frac{3b\,x^{2}}{8a^{3}c}}{(bx+a)c^{2}(-bx+a)^{2}}-\frac{3\ln(-bx+a)}{16a^{4}c^{3}b}+\frac{3\ln(bx+a)}{16a^{4}c^{3}b}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-3/16/a^4/b*ln(-b*x+a)+1/4/a^3/b/(-b*x+a)+1/8/a^2/b/(-b*x+a)^2+3/16/
a^4/b*ln(b*x+a)-1/8/a^3/b/(b*x+a))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 1.30

$$-\frac{3\,b^2x^2-3\,abx-2\,a^2}{8\,(a^3b^4c^3x^3-a^4b^3c^3x^2-a^5b^2c^3x+a^6bc^3)}+\frac{3\,\log(bx+a)}{16\,a^4bc^3}-\frac{3\,\log(bx-a)}{16\,a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*c^3*x^3 - a^4*b^3*c^3*x^2 - a^5
*b^2*c^3*x + a^6*b*c^3) + 3/16*log(b*x + a)/(a^4*b*c^3) - 3/16*log(b*x - a)
/(a^4*b*c^3)
```

Fricas [A]
time = 0.95, size = 146, normalized size = 1.76

$$-\frac{6\,ab^2x^2-6\,a^2bx-4\,a^3-3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log(bx+a)+3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log(bx-a)}{16\,(a^4b^4c^3x^3-a^5b^3c^3x^2-a^6b^2c^3x+a^7bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a))/
(a^4*b^4*c^3*x^3 - a^5*b^3*c^3*x^2 - a^6*b^2*c^3*x + a^7*b*c^3)
```

Sympy [A]
time = 0.22, size = 104, normalized size = 1.25

$$-\frac{-2a^2-3abx+3b^2x^2}{8a^6bc^3-8a^5b^2c^3x-8a^4b^3c^3x^2+8a^3b^4c^3x^3}-\frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b*c**3 - 8*a**5*b**2*c**3*x - 8*
a**4*b**3*c**3*x**2 + 8*a**3*b**4*c**3*x**3) - (3*log(-a/b + x)/16 - 3*log(
a/b + x)/16)/(a**4*b*c**3)
```

Giac [A]
time = 1.70, size = 81, normalized size = 0.98

$$-\frac{3\log\left(\left|-\frac{2a}{bx+a}+1\right|\right)}{16a^4bc^3}-\frac{1}{8(bx+a)a^3bc^3}+\frac{\frac{12a}{bx+a}-5}{32a^4bc^3\left(\frac{2a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -3/16*log(abs(-2*a/(b*x + a) + 1))/(a^4*b*c^3) - 1/8/((b*x + a)*a^3*b*c^3)
+ 1/32*(12*a/(b*x + a) - 5)/(a^4*b*c^3*(2*a/(b*x + a) - 1)^2)
```

Mupad [B]
time = 0.10, size = 86, normalized size = 1.04

$$\frac{\frac{3x}{8a^2}+\frac{1}{4ab}-\frac{3bx^2}{8a^3}}{a^3c^3-a^2bc^3x-ab^2c^3x^2+b^3c^3x^3}+\frac{3\,\mathrm{atanh}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)^2),x)
```

```
[Out] ((3*x)/(8*a^2) + 1/(4*a*b) - (3*b*x^2)/(8*a^3))/(a^3*c^3 + b^3*c^3*x^3 - a*
b^2*c^3*x^2 - a^2*b*c^3*x) + (3*atanh((b*x)/a))/(8*a^4*b*c^3)
```

3.1063 $\int (1-x)^{9/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=108

$$\frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1-x)^{9/2}(1+x)^{3/}$$

```
[Out] 7/8*(1-x)^(3/2)*(1+x)^(3/2)+21/40*(1-x)^(5/2)*(1+x)^(3/2)+3/10*(1-x)^(7/2)*
(1+x)^(3/2)+1/6*(1-x)^(9/2)*(1+x)^(3/2)+21/16*arcsin(x)+21/16*x*(1-x)^(1/2)
*(1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{21\text{ArcSin}(x)}{16}+\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (21*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/8 + (21
*(1 - x)^(5/2)*(1 + x)^(3/2))/40 + (3*(1 - x)^(7/2)*(1 + x)^(3/2))/10 + ((1
 - x)^(9/2)*(1 + x)^(3/2))/6 + (21*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{9/2}\sqrt{1+x}\,dx &= \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}+\frac{3}{2}\int (1-x)^{7/2}\sqrt{1+x}\,dx \\
&= \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}+\frac{21}{10}\int (1-x)^{5/2}\sqrt{1+x}\,dx \\
&= \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1-x)^{9/2}(1+x)^{3/2}+\frac{21}{8}\int (\\
&= \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-x)^{7/2}(1+x)^{3/2}+\frac{1}{6}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{8}(1-x)^{3/2}(1+x)^{3/2}+\frac{21}{40}(1-x)^{5/2}(1+x)^{3/2}+\frac{3}{10}(1-
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.68

$$-\frac{\sqrt{1+x}\,(-448+523x+181x^2-606x^3+542x^4-232x^5+40x^6)}{240\sqrt{1-x}}+\frac{21}{8}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*Sqrt[1 + x],x]
[Out] -1/240*(Sqrt[1 + x]*(-448 + 523*x + 181*x^2 - 606*x^3 + 542*x^4 - 232*x^5 +
 40*x^6))/Sqrt[1 - x] + (21*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/8
```

Maple [A]
time = 0.16, size = 113, normalized size = 1.05

method	result
risch	$-\frac{(40x^5-192x^4+350x^3-256x^2-75x+448)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{240\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{21\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}}}{6}+\frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}{10}+\frac{21(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{40}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{8}+\frac{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{16}-\frac{21\sqrt{1-x}\,\sqrt{1}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)^(9/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*(1-x)^(9/2)*(1+x)^(3/2)+3/10*(1-x)^(7/2)*(1+x)^(3/2)+21/40*(1-x)^(5/2)*(1+x)^(3/2)+7/8*(1-x)^(3/2)*(1+x)^(3/2)+21/16*(1-x)^(1/2)*(1+x)^(3/2)-21/16*(1-x)^(1/2)*(1+x)^(1/2)+21/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)

Maxima [A]
time = 0.50, size = 68, normalized size = 0.63

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{3}{2}}x^3+\frac{4}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{13}{8}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{28}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{21}{16}\sqrt{-x^2+1}\,x+\frac{21}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="maxima")

[Out] -1/6*(-x^2 + 1)^(3/2)*x^3 + 4/5*(-x^2 + 1)^(3/2)*x^2 - 13/8*(-x^2 + 1)^(3/2)*x + 28/15*(-x^2 + 1)^(3/2) + 21/16*sqrt(-x^2 + 1)*x + 21/16*arcsin(x)

Fricas [A]
time = 1.07, size = 62, normalized size = 0.57

$$\frac{1}{240}\left(40\,x^5-192\,x^4+350\,x^3-256\,x^2-75\,x+448\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{21}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="fricas")

[Out] 1/240*(40*x^5 - 192*x^4 + 350*x^3 - 256*x^2 - 75*x + 448)*sqrt(x + 1)*sqrt(-x + 1) - 21/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

Sympy [C] Result contains complex when optimal does not.
time = 89.42, size = 287, normalized size = 2.66

$$\begin{cases} -\frac{21i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}-\frac{59i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}}+\frac{1151i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}}-\frac{2947i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}}+\frac{8171i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}}-\frac{1045i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{21i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{21\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{59(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}}-\frac{1151(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}}+\frac{2947(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}}-\frac{8171(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}}+\frac{1045(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{21\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(9/2)*(1+x)**(1/2),x)

[Out] Piecewise((-21*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt(x - 1)) - 59*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) + 1151*I*(x + 1)**(9/2)/(1

```
20*sqrt(x - 1)) - 2947*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) + 8171*I*(x + 1)*
*(5/2)/(240*sqrt(x - 1)) - 1045*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 21*I*sq
rt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1) > 2), (21*asin(sqrt(2)*sqrt(x + 1)/2)
/8 - (x + 1)**(13/2)/(6*sqrt(1 - x)) + 59*(x + 1)**(11/2)/(30*sqrt(1 - x))
- 1151*(x + 1)**(9/2)/(120*sqrt(1 - x)) + 2947*(x + 1)**(7/2)/(120*sqrt(1 -
 x)) - 8171*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 1045*(x + 1)**(3/2)/(48*sqrt
(1 - x)) - 21*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(76) = 152.
time = 1.87, size = 185, normalized size = 1.71

$$\frac{1}{240}\left(\left(2\left(\left(4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{40}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{3}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{21}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)*
(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10
)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/3*((2*x - 5)*(x
+ 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) +
 sqrt(x + 1)*sqrt(-x + 1) + 21/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,\sqrt{x+1}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(1/2), x)
```

3.1064 $\int (1-x)^{7/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=88

$$\frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{8}\sin^{-1}(x)$$

```
[Out] 7/12*(1-x)^(3/2)*(1+x)^(3/2)+7/20*(1-x)^(5/2)*(1+x)^(3/2)+1/5*(1-x)^(7/2)*(
1+x)^(3/2)+7/8*arcsin(x)+7/8*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{7\text{ArcSin}(x)}{8}+\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + (7*(
1 - x)^(5/2)*(1 + x)^(3/2))/20 + ((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*ArcSi
n[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{7/2}\sqrt{1+x}\,dx &= \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{5}\int (1-x)^{5/2}\sqrt{1+x}\,dx\\
&= \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\
&= \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x)^{7/2}(1+x)^{3/2}+\frac{7}{4}\int \surd\\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x\\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x\\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{7}{20}(1-x)^{5/2}(1+x)^{3/2}+\frac{1}{5}(1-x
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.77

$$\frac{\sqrt{1+x}\,(136-121x-127x^2+202x^3-114x^4+24x^5)}{120\sqrt{1-x}}+\frac{7}{4}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(136 - 121*x - 127*x^2 + 202*x^3 - 114*x^4 + 24*x^5))/(120*Sqr
t[1 - x]) + (7*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

Maple [A]
time = 0.15, size = 99, normalized size = 1.12

method	result
risch	$\frac{(24x^4-90x^3+112x^2-15x-136)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{120\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{7\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}{5}+\frac{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{20}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{12}+\frac{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{7\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{7\sqrt{(1+x)}}{8\sqrt{1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1-x)^(7/2)*(1+x)^(3/2)+7/20*(1-x)^(5/2)*(1+x)^(3/2)+7/12*(1-x)^(3/2)*(
1+x)^(3/2)+7/8*(1-x)^(1/2)*(1+x)^(3/2)-7/8*(1-x)^(1/2)*(1+x)^(1/2)+7/8*((1+
x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 54, normalized size = 0.61

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{-x^2+1}\,x+\frac{7}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2)
+ 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

Fricas [A]
time = 0.63, size = 57, normalized size = 0.65

$$-\frac{1}{120}\left(24\,x^4-90\,x^3+112\,x^2-15\,x-136\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{7}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x + 1)*sqrt(-x + 1) -
7/4*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 29.45, size = 252, normalized size = 2.86

$$\begin{cases} -\frac{7i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{39i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{449i(x+1)^{\frac{7}{2}}}{60\sqrt{x-1}}+\frac{1657i(x+1)^{\frac{5}{2}}}{120\sqrt{x-1}}-\frac{263i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{7i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{7\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{39(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{449(x+1)^{\frac{7}{2}}}{60\sqrt{1-x}}-\frac{1657(x+1)^{\frac{5}{2}}}{120\sqrt{1-x}}+\frac{263(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{7\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 39*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 449*I*(x + 1)**(7/2)/(60*s
qrt(x - 1)) + 1657*I*(x + 1)**(5/2)/(120*sqrt(x - 1)) - 263*I*(x + 1)**(3/2
)/(24*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (7*a
sin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 39*(x + 1)
```

```
**(9/2)/(20*sqrt(1 - x)) + 449*(x + 1)**(7/2)/(60*sqrt(1 - x)) - 1657*(x +
1)**(5/2)/(120*sqrt(1 - x)) + 263*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 7*sqrt(
x + 1)/(4*sqrt(1 - x)), True))
```

Giac [A]
time = 2.66, size = 115, normalized size = 1.31

$$-\frac{1}{120}\,\left(\left(2\,\left(3\,\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{12}\,\left(\left(2\,\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\,\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{7}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(
x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 7/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(1/2), x)
```

3.1065 $\int (1-x)^{5/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=68

$$\frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x)$$

```
[Out] 5/12*(1-x)^(3/2)*(1+x)^(3/2)+1/4*(1-x)^(5/2)*(1+x)^(3/2)+5/8*arcsin(x)+5/8*
x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{5\text{ArcSin}(x)}{8}+\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + ((1
- x)^(5/2)*(1 + x)^(3/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{5/2}\sqrt{1+x}\,dx &= \frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\ &= \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{}{\sqrt{1}}\\ &= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{}{\sqrt{1}}\\ &= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x\end{aligned}$$

Mathematica [A]
time = 0.08, size = 63, normalized size = 0.93

$$-\frac{\sqrt{1+x}\,(-16+7x+25x^2-22x^3+6x^4)}{24\sqrt{1-x}}+\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] -1/24*(Sqrt[1 + x]*(-16 + 7*x + 25*x^2 - 22*x^3 + 6*x^4))/Sqrt[1 - x] + (5*
ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

Maple [A]
time = 0.16, size = 85, normalized size = 1.25

method	result
risch	$-\frac{(6x^3-16x^2+9x+16)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}{4}+\frac{5(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{12}+\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1-x)^(5/2)*(1+x)^(3/2)+5/12*(1-x)^(3/2)*(1+x)^(3/2)+5/8*(1-x)^(1/2)*(1
+x)^(3/2)-5/8*(1-x)^(1/2)*(1+x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(
1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^{2}+1}\,x+\frac{5}{8}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

Fricas [A]
time = 0.57, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6\,x^{3}-16\,x^{2}+9\,x+16\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.19, size = 216, normalized size = 3.18

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}+\frac{127i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{133i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}-\frac{127(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{133(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 23*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) + 127*I*(x + 1)**(5/2)/(24*sq
rt(x - 1)) - 133*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqr
t(x - 1)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/
2)/(4*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(12*sqrt(1 - x)) - 127*(x + 1)**(5/2
)/(24*sqrt(1 - x)) + 133*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4
*sqrt(1 - x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(48) = 96.
time = 1.21, size = 101, normalized size = 1.49

$$\frac{1}{24}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{6}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x -
 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqrt(x
 + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(1/2), x)
```

3.1066 $\int (1-x)^{3/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*arcsin(x)+1/2*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {51, 38, 41, 222}

$$\frac{\text{ArcSin}(x)}{2}+\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{3/2}\sqrt{1+x}\,dx &= \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 52, normalized size = 1.08

$$\frac{\sqrt{1+x}\,(2+x-5x^2+2x^3)}{6\sqrt{1-x}}+\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*Sqrt[1 + x],x]
[Out] (Sqrt[1 + x]*(2 + x - 5*x^2 + 2*x^3))/(6*Sqrt[1 - x]) + ArcTan[Sqrt[1 + x]/
Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.
time = 0.16, size = 71, normalized size = 1.48

method	result	size
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}{3}+\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$\frac{(2x^2-3x-2)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+
x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 28, normalized size = 0.58

$$\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.46, size = 47, normalized size = 0.98

$$-\frac{1}{6}\left(2\,x^{2}-3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 - 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.38, size = 167, normalized size = 3.48

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}+\frac{11i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{17i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}-\frac{11(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{17(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) + 11*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 17*I*(x + 1)**(3/2)/(6*sqrt(x -
1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (asin(sqrt(2)*sqrt(x + 1)
/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) - 11*(x + 1)**(5/2)/(6*sqrt(1 - x)) +
17*(x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.41, size = 50, normalized size = 1.04

$$-\frac{1}{6}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x
 + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(1/2),x)
[Out] int((1 - x)^(3/2)*(x + 1)^(1/2), x)
```

3.1067 $\int \sqrt{1-x}\,\sqrt{1+x}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {38, 41, 222}

$$\frac{\text{ArcSin}(x)}{2}+\frac{1}{2}\sqrt{1-x}\,\sqrt{x+1}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*Sqrt[1 + x],x]
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ArcSin[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\,\sqrt{1+x}\,dx &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.32

$$\frac{1}{2}x\sqrt{1-x^2}-\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[1 + x],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20) = 40$.
time = 0.16, size = 57, normalized size = 2.04

method	result	size
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$-\frac{x\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1+x)*(1-x))^(
1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 17, normalized size = 0.61

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.50, size = 38, normalized size = 1.36

$$\frac{1}{2}\sqrt{x+1}\,x\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.60, size = 131, normalized size = 4.68

$$\begin{cases} -i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{3i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{3(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x - 1
)) - 3*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x
+ 1) > 2), (asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) +
3*(x + 1)**(3/2)/(2*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 1.33, size = 42, normalized size = 1.50

$$\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1} + \sqrt{x+1}\,\sqrt{-x+1} + \arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/
2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 0.20, size = 37, normalized size = 1.32

$$\frac{x\sqrt{1-x}\,\sqrt{x+1}}{2} - \frac{\ln\left(x - \sqrt{1-x}\,\sqrt{x+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] (x*(1 - x)^(1/2)*(x + 1)^(1/2))/2 - (log(x - (1 - x)^(1/2)*(x + 1)^(1/2)*1i
)*1i)/2
```

3.1068 $\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=21

$$-\sqrt{1-x}\,\sqrt{1+x}\,+\sin^{-1}(x)$$

```
[Out] arcsin(x)-(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\text{ArcSin}(x)-\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x],x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = -\sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\sqrt{1-x}\,\sqrt{1+x} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\sqrt{1-x}\,\sqrt{1+x} + \sin^{-1}(x)$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.62

$$-\sqrt{1-x^2} + 2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x],x]
[Out] -Sqrt[1 - x^2] + 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.17, size = 42, normalized size = 2.00

method	result	size
default	$-\sqrt{1-x}\,\sqrt{1+x} + \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	42
risch	$\frac{\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin
(x)
```

Maxima [A]
time = 0.51, size = 14, normalized size = 0.67

$$-\sqrt{-x^2+1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.56, size = 37, normalized size = 1.76

$$-\sqrt{x+1}\,\sqrt{-x+1} - 2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.93, size = 99, normalized size = 4.71

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} + \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
 + 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (2*asin(sqrt(2)*sqrt(x + 1
)/2) + (x + 1)**(3/2)/sqrt(1 - x) - 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.29, size = 28, normalized size = 1.33

$$-\sqrt{x+1}\,\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 0.14, size = 14, normalized size = 0.67

$$\operatorname{asin}(x) - \sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(1/2),x)
[Out] asin(x) - (1 - x^2)^(1/2)
```

3.1069 $\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x)$$

[Out] -arcsin(x)+2*(1+x)^(1/2)/(1-x)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}} - \text{ArcSin}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x]/(1 - x)^(3/2),x]

[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx = \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x)$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 1.70

$$\frac{2\sqrt{1+x}}{\sqrt{1-x}} - 2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(3/2),x]
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.
time = 0.19, size = 64, normalized size = 2.78

method	result	size
risch	$\frac{2\sqrt{1+x}\,\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} - \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*(1+x)^(1/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)-((1+x)*
(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x-1} - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(19) = 38$.
time = 1.29, size = 48, normalized size = 2.09

$$\frac{2\left((x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.79, size = 70, normalized size = 3.04

$$\begin{cases} 2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ -2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1),
Abs(x + 1) > 2), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x
), True))
```

Giac [A]
time = 0.91, size = 33, normalized size = 1.43

$$-\frac{2\sqrt{x+1}\sqrt{-x+1}}{x-1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(3/2),x)
[Out] int((x + 1)^(1/2)/(1 - x)^(3/2), x)
```

3.1070 $\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

```
[Out] 1/3*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

```
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$\frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(5/2),x]
```

[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 0.16, size = 30, normalized size = 1.50

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}}{3(1-x)^{\frac{3}{2}}}$	15
default	$\frac{2\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{3\sqrt{1-x}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^2+2x+1)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	49

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(1/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/3*(1+x)^(1/2)/(1-x)^(3/2)-1/3*(1+x)^(1/2)/(1-x)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.
time = 0.27, size = 38, normalized size = 1.90

$$\frac{2\sqrt{-x^2+1}}{3(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="maxima")

[Out] 2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 1.08, size = 33, normalized size = 1.65

$$\frac{x^2+(x+1)^{\frac{3}{2}}\sqrt{-x+1}-2x+1}{3(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="fricas")

[Out] 1/3*(x^2 + (x + 1)^(3/2)*sqrt(-x + 1) - 2*x + 1)/(x^2 - 2*x + 1)

Sympy [C] Result contains complex when optimal does not.
time = 0.84, size = 60, normalized size = 3.00

$$\begin{cases} \frac{i(x+1)^{\frac{3}{2}}}{3\sqrt{x-1}\,(x+1)-6\sqrt{x-1}} & \text{for } |x+1|>2 \\ -\frac{(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}\,(x+1)-6\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(3/2)/(3*sqrt(x - 1)*(x + 1) - 6*sqrt(x - 1)), Abs(x
+ 1) > 2), (-(x + 1)**(3/2)/(3*sqrt(1 - x)*(x + 1) - 6*sqrt(1 - x)), True))
```

Giac [A]
time = 0.99, size = 19, normalized size = 0.95

$$\frac{(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^2
```

Mupad [B]
time = 0.27, size = 34, normalized size = 1.70

$$\frac{\left(\frac{x\sqrt{x+1}}{3}+\frac{\sqrt{x+1}}{3}\right)\sqrt{1-x}}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(5/2),x)
```

```
[Out] (((x*(x + 1)^(1/2))/3 + (x + 1)^(1/2)/3)*(1 - x)^(1/2))/(x^2 - 2*x + 1)
```

3.1071 $\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx$

Optimal. Leaf size=41

$$\frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}$$

```
[Out] 1/5*(1+x)^(3/2)/(1-x)^(5/2)+1/15*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]

time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(7/2),x]
[Out] (1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.56

$$-\frac{(-4+x)(1+x)^{3/2}}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

```
[Out] -1/15*((-4 + x)*(1 + x)^(3/2))/(1 - x)^(5/2)
```

Maple [A]
time = 0.15, size = 44, normalized size = 1.07

method	result	size
gosper	$-\frac{(1+x)^{\frac{3}{2}}(x-4)}{15(1-x)^{\frac{5}{2}}}$	18
default	$\frac{2\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{15(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{15\sqrt{1-x}}$	44
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^3-2x^2-7x-4)}{15\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(1+x)^(1/2)/(1-x)^(5/2)-1/15*(1+x)^(1/2)/(1-x)^(3/2)-1/15*(1+x)^(1/2)/(
1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.
time = 0.29, size = 64, normalized size = 1.56

$$-\frac{2\sqrt{-x^2+1}}{5(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{15(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{15(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.54, size = 53, normalized size = 1.29

$$\frac{4x^3-12x^2+(x^2-3x-4)\sqrt{x+1}\sqrt{-x+1}+12x-4}{15(x^3-3x^2+3x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x - 4)
/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.01, size = 172, normalized size = 4.20

$$\begin{cases} \frac{i(x+1)^{\frac{5}{2}}}{15\sqrt{x-1}\,(x+1)^2-60\sqrt{x-1}\,(x+1)+60\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{15\sqrt{x-1}\,(x+1)^2-60\sqrt{x-1}\,(x+1)+60\sqrt{x-1}} & \text{for } |x+1|>2 \\ -\frac{(x+1)^{\frac{5}{2}}}{15\sqrt{1-x}\,(x+1)^2-60\sqrt{1-x}\,(x+1)+60\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{15\sqrt{1-x}\,(x+1)^2-60\sqrt{1-x}\,(x+1)+60\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(5/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60*sqrt(x - 1)*(x
+ 1) + 60*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60
*sqrt(x - 1)*(x + 1) + 60*sqrt(x - 1)), Abs(x + 1) > 2), (-(x + 1)**(5/2)/(
15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(1 - x)) + 5*(x
 + 1)**(3/2)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(
1 - x)), True))
```

Giac [A]
time = 1.20, size = 22, normalized size = 0.54

$$\frac{(x+1)^{\frac{3}{2}}(x-4)\sqrt{-x+1}}{15\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x + 1)^(3/2)*(x - 4)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B]
time = 0.24, size = 50, normalized size = 1.22

$$-\frac{\sqrt{1-x}\,\left(\frac{x\sqrt{x+1}}{5}+\frac{4\sqrt{x+1}}{15}-\frac{x^2\sqrt{x+1}}{15}\right)}{x^3-3\,x^2+3\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((x*(x + 1)^(1/2))/5 + (4*(x + 1)^(1/2))/15 - (x^2*(x + 1)^
(1/2))/15))/(3*x - 3*x^2 + x^3 - 1)
```

3.1072 $\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=61

$$\frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{35(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}$$

```
[Out] 1/7*(1+x)^(3/2)/(1-x)^(7/2)+2/35*(1+x)^(3/2)/(1-x)^(5/2)+2/105*(1+x)^(3/2)/
(1-x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*(1 + x)^(3/2))/(35*(1 - x)^(5/2)) + (2
*(1 + x)^(3/2))/(105*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2}{7}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2}{35}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{35(1-x)^{5/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 0.49

$$\frac{(1+x)^{3/2}\left(23-10x+2x^2\right)}{105(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(9/2),x]
[Out] ((1 + x)^(3/2)*(23 - 10*x + 2*x^2))/(105*(1 - x)^(7/2))
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.95

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}\left(2x^2-10x+23\right)}{105(1-x)^{\frac{7}{2}}}$	25
default	$\frac{2\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{35(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{105(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{105\sqrt{1-x}}$	58
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(2x^4-6x^3+5x^2+36x+23\right)}{105\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(1+x)^(1/2)/(1-x)^(7/2)-1/35*(1+x)^(1/2)/(1-x)^(5/2)-2/105*(1+x)^(1/2)/
(1-x)^(3/2)-2/105*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(43) = 86.
time = 0.28, size = 95, normalized size = 1.56

$$\frac{2\sqrt{-x^2+1}}{7\left(x^4-4x^3+6x^2-4x+1\right)}+\frac{\sqrt{-x^2+1}}{35\left(x^3-3x^2+3x-1\right)}-\frac{2\sqrt{-x^2+1}}{105\left(x^2-2x+1\right)}+\frac{2\sqrt{-x^2+1}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt(-
x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.61, size = 70, normalized size = 1.15

$$\frac{23\,x^4-92\,x^3+138\,x^2+(2\,x^3-8\,x^2+13\,x+23)\sqrt{x+1}\,\sqrt{-x+1}-92\,x+23}{105\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x + 1)*
sqrt(-x + 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.86, size = 566, normalized size = 9.28

$$\begin{cases} \frac{2i(x+1)^{\frac{9}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} - \frac{18i(x+1)^{\frac{7}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} + \frac{63i(x+1)^{\frac{5}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} - \frac{70i(x+1)^{\frac{3}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ -\frac{2(x+1)^{\frac{9}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} + \frac{18(x+1)^{\frac{7}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} - \frac{63(x+1)^{\frac{5}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} + \frac{70(x+1)^{\frac{3}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(9/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(x - 1)
*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1) + 1680
*sqrt(x - 1)) - 18*I*(x + 1)**(7/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(
x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1)
+ 1680*sqrt(x - 1)) + 63*I*(x + 1)**(5/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840
*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x
 + 1) + 1680*sqrt(x - 1)) - 70*I*(x + 1)**(3/2)/(105*sqrt(x - 1)*(x + 1)**4
 - 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x -
 1)*(x + 1) + 1680*sqrt(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(9/2)/(105*s
qrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(x +
1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 18*(x + 1)**(7/2)/(1
05*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(
x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) - 63*(x + 1)**(5/2
)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 -
x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 70*(x + 1)**
(3/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(
1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)), True))
```

Giac [A]
time = 1.08, size = 29, normalized size = 0.48

$$\frac{(2(x+1)(x-6)+35)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{105(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(2*(x + 1)*(x - 6) + 35)*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.27, size = 64, normalized size = 1.05

$$\frac{\sqrt{1-x}\left(\frac{13x\sqrt{x+1}}{105}+\frac{23\sqrt{x+1}}{105}-\frac{8x^2\sqrt{x+1}}{105}+\frac{2x^3\sqrt{x+1}}{105}\right)}{x^4-4x^3+6x^2-4x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((13*x*(x + 1)^(1/2))/105 + (23*(x + 1)^(1/2))/105 - (8*x^2*
(x + 1)^(1/2))/105 + (2*x^3*(x + 1)^(1/2))/105))/(6*x^2 - 4*x - 4*x^3 + x^4
 + 1)
```

3.1073 $\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=81

$$\frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}$$

```
[Out] 1/9*(1+x)^(3/2)/(1-x)^(9/2)+1/21*(1+x)^(3/2)/(1-x)^(7/2)+2/105*(1+x)^(3/2)/
(1-x)^(5/2)+2/315*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(3/2)/(9*(1 - x)^(9/2)) + (1 + x)^(3/2)/(21*(1 - x)^(7/2)) + (2*(1
+ x)^(3/2))/(105*(1 - x)^(5/2)) + (2*(1 + x)^(3/2))/(315*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}} + \frac{1}{3}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx \\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}} + \frac{(1+x)^{3/2}}{21(1-x)^{7/2}} + \frac{2}{21}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx \\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}} + \frac{(1+x)^{3/2}}{21(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{5/2}} + \frac{2}{105}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx \\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}} + \frac{(1+x)^{3/2}}{21(1-x)^{7/2}} + \frac{2(1+x)^{3/2}}{105(1-x)^{5/2}} + \frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 0.43

$$\frac{(1+x)^{3/2}\left(58-33x+12x^2-2x^3\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(11/2),x]
[Out] ((1 + x)^(3/2)*(58 - 33*x + 12*x^2 - 2*x^3))/(315*(1 - x)^(9/2))
```

Maple [A]
time = 0.17, size = 72, normalized size = 0.89

method	result	size
gosper	$-\frac{(1+x)^{\frac{3}{2}}\left(2x^3-12x^2+33x-58\right)}{315(1-x)^{\frac{9}{2}}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(2x^5-8x^4+11x^3-4x^2-83x-58\right)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{2\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}-\frac{\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{315\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(1+x)^(1/2)/(1-x)^(9/2)-1/63*(1+x)^(1/2)/(1-x)^(7/2)-1/105*(1+x)^(1/2)/
(1-x)^(5/2)-2/315*(1+x)^(1/2)/(1-x)^(3/2)-2/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(57) = 114.
time = 0.28, size = 131, normalized size = 1.62

$$-\frac{2\sqrt{-x^2+1}}{9\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}-\frac{\sqrt{-x^2+1}}{63\left(x^4-4x^3+6x^2-4x+1\right)}+\frac{\sqrt{-x^2+1}}{105\left(x^3-3x^2+3x-1\right)}-\frac{2\sqrt{-x^2+1}}{315\left(x^2-2x+1\right)}+\frac{2\sqrt{-x^2+1}}{315\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 1/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/315*sqrt(-x^2 + 1)/
(x - 1)
```

Fricas [A]
time = 0.54, size = 85, normalized size = 1.05

$$\frac{58\,x^5-290\,x^4+580\,x^3-580\,x^2+(2\,x^4-10\,x^3+21\,x^2-25\,x-58)\sqrt{x+1}\,\sqrt{-x+1}\,+290\,x-58}{315\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(58*x^5 - 290*x^4 + 580*x^3 - 580*x^2 + (2*x^4 - 10*x^3 + 21*x^2 - 25
*x - 58)*sqrt(x + 1)*sqrt(-x + 1) + 290*x - 58)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 47.66, size = 1561, normalized size = 19.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(15/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x -
1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4
 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*s
qrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 30*I*(x + 1)**(13/2)/(315*sqrt(x
- 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**
5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*s
qrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1
95*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)
**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*
sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)
*(x + 1) - 40320*sqrt(x - 1)) - 715*I*(x + 1)**(9/2)/(315*sqrt(x - 1)*(x +
1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*
sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)
*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1530*I*(x +
 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 2646
0*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1
```

```
)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) -
 40320*sqrt(x - 1)) - 1764*I*(x + 1)**(5/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4
410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x -
1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)*
*2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 840*I*(x + 1)**(3/2)
/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x -
 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)*
*3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqr
t(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(15/2)/(315*sqrt(1 - x)*(x + 1)**7
 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(
1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x +
 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 30*(x + 1)**(13/
2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1
 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1
)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*s
qrt(1 - x)) - 195*(x + 1)**(11/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1
 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)
**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 14112
0*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 715*(x + 1)**(9/2)/(315*sqrt(1
 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)*
*5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*
sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) -
1530*(x + 1)**(7/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)*
*6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*s
qrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*
(x + 1) - 40320*sqrt(1 - x)) + 1764*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)
**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sq
rt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(
x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) - 840*(x + 1)**
(3/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqr
t(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x
+ 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 4032
0*sqrt(1 - x)), True))
```

Giac [A]
time = 1.03, size = 35, normalized size = 0.43

$$\frac{((2(x+1)(x-8)+63)(x+1)-105)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{315(x-1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1)^(3/2)*sqrt(-x + 1)/(
x - 1)^5
```

Mupad [B]
time = 0.28, size = 80, normalized size = 0.99

$$-\frac{\sqrt{1-x}\,\left(\frac{5\,x\,\sqrt{x+1}}{63}+\frac{58\,\sqrt{x+1}}{315}-\frac{x^2\,\sqrt{x+1}}{15}+\frac{2\,x^3\,\sqrt{x+1}}{63}-\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((5*x*(x + 1)^(1/2))/63 + (58*(x + 1)^(1/2))/315 - (x^2*(x
+ 1)^(1/2))/15 + (2*x^3*(x + 1)^(1/2))/63 - (2*x^4*(x + 1)^(1/2))/315))/(5*
x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

3.1074 $\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}}+\frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}$$

```
[Out] 1/11*(1+x)^(3/2)/(1-x)^(11/2)+4/99*(1+x)^(3/2)/(1-x)^(9/2)+4/231*(1+x)^(3/2
)/(1-x)^(7/2)+8/1155*(1+x)^(3/2)/(1-x)^(5/2)+8/3465*(1+x)^(3/2)/(1-x)^(3/2)
```

Rubi [A]

time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*(1 + x)^(3/2))/(99*(1 - x)^(9/2)) +
(4*(1 + x)^(3/2))/(231*(1 - x)^(7/2)) + (8*(1 + x)^(3/2))/(1155*(1 - x)^(5/
2)) + (8*(1 + x)^(3/2))/(3465*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4}{11}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4}{33}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8}{231}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx}{1155} \\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}} + \frac{4(1+x)^{3/2}}{99(1-x)^{9/2}} + \frac{4(1+x)^{3/2}}{231(1-x)^{7/2}} + \frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}} + \frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.40

$$
\frac{(1+x)^{3/2}\left(547-364x+180x^2-56x^3+8x^4\right)}{3465(1-x)^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(3/2)*(547 - 364*x + 180*x^2 - 56*x^3 + 8*x^4))/(3465*(1 - x)^(11/
2))
```

Maple [A]
time = 0.16, size = 86, normalized size = 0.85

method	result	size
gosper	$\frac{(1+x)^{\frac{3}{2}}\left(8x^4-56x^3+180x^2-364x+547\right)}{3465(1-x)^{\frac{11}{2}}}$	35
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(8x^6-40x^5+76x^4-60x^3-x^2+730x+547\right)}{3465\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^5\sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{2\sqrt{1+x}}{11(1-x)^{\frac{11}{2}}}-\frac{\sqrt{1+x}}{99(1-x)^{\frac{9}{2}}}-\frac{4\sqrt{1+x}}{693(1-x)^{\frac{7}{2}}}-\frac{4\sqrt{1+x}}{1155(1-x)^{\frac{5}{2}}}-\frac{8\sqrt{1+x}}{3465(1-x)^{\frac{3}{2}}}-\frac{8\sqrt{1+x}}{3465\sqrt{1-x}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*(1+x)^(1/2)/(1-x)^(11/2)-1/99*(1+x)^(1/2)/(1-x)^(9/2)-4/693*(1+x)^(1/2
)/(1-x)^(7/2)-4/1155*(1+x)^(1/2)/(1-x)^(5/2)-8/3465*(1+x)^(1/2)/(1-x)^(3/2)
-8/3465*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(71) = 142$.
time = 0.33, size = 172, normalized size = 1.70

$$\frac{2\sqrt{-x^2+1}}{11\left(x^6-6x^5+15x^4-20x^3+15x^2-6x+1\right)}+\frac{\sqrt{-x^2+1}}{99\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}-\frac{4\sqrt{-x^2+1}}{693\left(x^4-4x^3+6x^2-4x+1\right)}+\frac{4\sqrt{-x^2+1}}{1155\left(x^3-3x^2+3x-1\right)}-\frac{8\sqrt{-x^2+1}}{3465\left(x^2-2x+1\right)}+\frac{8\sqrt{-x^2+1}}{3465\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 2/11*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 1/
99*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 4/693*sqrt(-x
^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/1155*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 8/3465*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/3465*sqrt(-x^2 + 1
)/(x - 1)
```

Fricas [A]
time = 0.57, size = 100, normalized size = 0.99

$$\frac{547x^6-3282x^5+8205x^4-10940x^3+8205x^2+\left(8x^5-48x^4+124x^3-184x^2+183x+547\right)\sqrt{x+1}\sqrt{-x+1}-3282x+547}{3465\left(x^6-6x^5+15x^4-20x^3+15x^2-6x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(547*x^6 - 3282*x^5 + 8205*x^4 - 10940*x^3 + 8205*x^2 + (8*x^5 - 48*
x^4 + 124*x^3 - 184*x^2 + 183*x + 547)*sqrt(x + 1)*sqrt(-x + 1) - 3282*x +
547)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 134.79, size = 3648, normalized size = 36.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((8*I*(x + 1)**(23/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 184*I*(x + 1)**(21/2)/(3465*s
qrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)
*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)
**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 -
146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574
```

400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x
- 1)) + 1932*I*(x + 1)**(19/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
sqrt(x - 1)(x + 1) - 7096320*sqrt(x - 1)) - 12236*I*(x + 1)**(17/2)/(3465
sqrt(x - 1)(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x -
1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x +
1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5
- 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 975
74400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(
x - 1)) + 52003*I*(x + 1)**(15/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqr
t(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*
(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)
6 + 102453120*sqrt(x - 1)*(x + 1)5 - 146361600*sqrt(x - 1)*(x + 1)**4 +
 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029
760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 155316*I*(x + 1)**(13/2)/(
3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(
x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(
x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)
5 - 146361600*sqrt(x - 1)*(x + 1)4 + 146361600*sqrt(x - 1)*(x + 1)**3 -
 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*s
qrt(x - 1)) + 329588*I*(x + 1)**(11/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7623
0*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x
- 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x
 + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)
4 + 146361600*sqrt(x - 1)*(x + 1)3 - 97574400*sqrt(x - 1)*(x + 1)**2 +
39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 488224*I*(x + 1)**(9/
2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*s
qrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x -
1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x
+ 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)*
*3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70963
20*sqrt(x - 1)) + 479952*I*(x + 1)**(7/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7
6230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt
(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)
*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x +
 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2
 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 280896*I*(x + 1)**
(5/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 76230
0*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x
 - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*
(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x +
1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70

```
96320*sqrt(x - 1)) + 73920*I*(x + 1)**(3/2)/(3465*sqrt(x - 1)*(x + 1)**11 -
 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sq
rt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x -
1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x
 + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)*
*2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)), Abs(x + 1) > 2),
(-8*(x + 1)**(23/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x +
1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 1
8295200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 10245312
0*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqr
t(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x
)*(x + 1) - 7096320*sqrt(1 - x)) + 184*(x + 1)*...
```

Giac [A]
time = 0.92, size = 42, normalized size = 0.42

$$\frac{(4\,((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)+1155)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3465\,(x-1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(4*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1) + 1155)*(x + 1)
^(3/2)*sqrt(-x + 1)/(x - 1)^6
```

Mupad [B]
time = 0.29, size = 94, normalized size = 0.93

$$\frac{\sqrt{1-x}\,\left(\frac{61\,x\,\sqrt{x+1}}{1155}+\frac{547\,\sqrt{x+1}}{3465}-\frac{184\,x^2\,\sqrt{x+1}}{3465}+\frac{124\,x^3\,\sqrt{x+1}}{3465}-\frac{16\,x^4\,\sqrt{x+1}}{1155}+\frac{8\,x^5\,\sqrt{x+1}}{3465}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((61*x*(x + 1)^(1/2))/1155 + (547*(x + 1)^(1/2))/3465 - (184
*x^2*(x + 1)^(1/2))/3465 + (124*x^3*(x + 1)^(1/2))/3465 - (16*x^4*(x + 1)^(
1/2))/1155 + (8*x^5*(x + 1)^(1/2))/3465))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 -
 6*x^5 + x^6 + 1)
```

3.1075 $\int (1-x)^{9/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=109

$$\frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{10}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{14}(1-x)^{7/2}(1+x)^{5/2}+\frac{1}{7}(1-x)^{9/2}(1+x)^{5}$$

```
[Out] 3/8*(1-x)^(3/2)*x*(1+x)^(3/2)+3/10*(1-x)^(5/2)*(1+x)^(5/2)+3/14*(1-x)^(7/2)
*(1+x)^(5/2)+1/7*(1-x)^(9/2)*(1+x)^(5/2)+9/16*arcsin(x)+9/16*x*(1-x)^(1/2)*
(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{9\text{ArcSin}(x)}{16}+\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (9*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (3*(1 - x)^(3/2)*x*(1 + x)^(3/2))/8 + (3
*(1 - x)^(5/2)*(1 + x)^(5/2))/10 + (3*(1 - x)^(7/2)*(1 + x)^(5/2))/14 + ((1
 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{9/2}(1+x)^{3/2}\,dx &= \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{9}{7}\int (1-x)^{7/2}(1+x)^{3/2}\,dx \\
&= \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{5/2}(1+x)^{3/2}\,dx \\
&= \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int \\
&= \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1 \\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1 \\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1 \\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 78, normalized size = 0.72

$$\frac{\sqrt{1-x}\,(368+613x-411x^2-306x^3+558x^4-72x^5-200x^6+80x^7)}{560\sqrt{1+x}} - \frac{9}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x]*(368 + 613*x - 411*x^2 - 306*x^3 + 558*x^4 - 72*x^5 - 200*x^6
+ 80*x^7))/(560*Sqrt[1 + x]) - (9*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

Maple [A]
time = 0.14, size = 127, normalized size = 1.17

method	result
risch	$-\frac{(80x^6-280x^5+208x^4+350x^3-656x^2+245x+368)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{560\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{9\sqrt{(1+x)(1-x)}\ \text{arcs}}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{5}{2}}}{7} + \frac{3(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{14} + \frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{10} + \frac{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{8} + \frac{3\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{8} - \frac{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1-x)^(9/2)*(1+x)^(5/2)+3/14*(1-x)^(7/2)*(1+x)^(5/2)+3/10*(1-x)^(5/2)*(
1+x)^(5/2)+3/8*(1-x)^(3/2)*(1+x)^(5/2)+3/8*(1-x)^(1/2)*(1+x)^(5/2)-3/16*(1-
x)^(1/2)*(1+x)^(3/2)-9/16*(1-x)^(1/2)*(1+x)^(1/2)+9/16*((1+x)*(1-x))^(1/2)/
(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [**A**]
time = 0.57, size = 66, normalized size = 0.61

$$\frac{1}{7}\left(-x^{2}+1\right)^{\frac{5}{2}}x^{2}-\frac{1}{2}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{23}{35}\left(-x^{2}+1\right)^{\frac{5}{2}}+\frac{3}{8}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{9}{16}\sqrt{-x^{2}+1}\,x+\frac{9}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(5/2)*x^2 - 1/2*(-x^2 + 1)^(5/2)*x + 23/35*(-x^2 + 1)^(5/2)
+ 3/8*(-x^2 + 1)^(3/2)*x + 9/16*sqrt(-x^2 + 1)*x + 9/16*arcsin(x)
```

Fricas [**A**]
time = 0.53, size = 67, normalized size = 0.61

$$\frac{1}{560}\left(80\,x^{6}-280\,x^{5}+208\,x^{4}+350\,x^{3}-656\,x^{2}+245\,x+368\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{9}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(80*x^6 - 280*x^5 + 208*x^4 + 350*x^3 - 656*x^2 + 245*x + 368)*sqrt(x
 + 1)*sqrt(-x + 1) - 9/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(77) = 154.
time = 1.00, size = 237, normalized size = 2.17

$$\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6\,x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{120}\left(\left(2\left(\left(4\left(5\,x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{120}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{6}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{6}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{9}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)
*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/120*((2*((4*(
5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(
x + 1)*sqrt(-x + 1) - 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)
*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/6*((2*(3*x - 10)*(x + 1) + 43)
*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(
x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 9/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2} (x+1)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(3/2), x)
```

3.1076 $\int (1-x)^{7/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{16}\sin^{-1}(x)$$

```
[Out] 7/24*(1-x)^(3/2)*x*(1+x)^(3/2)+7/30*(1-x)^(5/2)*(1+x)^(5/2)+1/6*(1-x)^(7/2)
*(1+x)^(5/2)+7/16*arcsin(x)+7/16*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{7\text{ArcSin}(x)}{16}+\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
7*(1 - x)^(5/2)*(1 + x)^(5/2))/30 + ((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*Ar
cSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{7/2}(1+x)^{3/2}\,dx &= \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{6}\int (1-x)^{5/2}(1+x)^{3/2}\,dx \\
&= \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx \\
&= \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1-x)^{7/2}(1+x)^{5/2}+\frac{7}{8}\int \\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1 \\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1 \\
&= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{7}{30}(1-x)^{5/2}(1+x)^{5/2}+\frac{1}{6}(1
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.82

$$\frac{\sqrt{1-x}\,(96+231x-57x^2-182x^3+106x^4+56x^5-40x^6)}{240\sqrt{1+x}}-\frac{7}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x]*(96 + 231*x - 57*x^2 - 182*x^3 + 106*x^4 + 56*x^5 - 40*x^6))/(
240*Sqrt[1 + x]) - (7*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

Maple [A]
time = 0.16, size = 113, normalized size = 1.27

method	result
risch	$\frac{(40x^5-96x^4-10x^3+192x^2-135x-96)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{240\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{7\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{5}{2}}}{6}+\frac{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{30}+\frac{7(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{24}+\frac{7\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{7\sqrt{1-x}\,\sqrt{1}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(7/2)*(1+x)^(5/2)+7/30*(1-x)^(5/2)*(1+x)^(5/2)+7/24*(1-x)^(3/2)*(
1+x)^(5/2)+7/24*(1-x)^(1/2)*(1+x)^(5/2)-7/48*(1-x)^(1/2)*(1+x)^(3/2)-7/16*(
1-x)^(1/2)*(1+x)^(1/2)+7/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arc
sin(x)
```

Maxima [A]
time = 0.50, size = 52, normalized size = 0.58

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{2}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{7}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{7}{16}\sqrt{-x^2+1}\,x+\frac{7}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)*x + 2/5*(-x^2 + 1)^(5/2) + 7/24*(-x^2 + 1)^(3/2)*x +
7/16*sqrt(-x^2 + 1)*x + 7/16*arcsin(x)
```

Fricas [A]
time = 0.43, size = 62, normalized size = 0.70

$$-\frac{1}{240}\left(40\,x^5-96\,x^4-10\,x^3+192\,x^2-135\,x-96\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{7}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(40*x^5 - 96*x^4 - 10*x^3 + 192*x^2 - 135*x - 96)*sqrt(x + 1)*sqrt(-
x + 1) - 7/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 109.66, size = 287, normalized size = 3.22

$$\begin{cases} -\frac{7i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}+\frac{47i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}}-\frac{683i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}}+\frac{1151i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}}-\frac{1543i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}}-\frac{7i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{7i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}-\frac{47(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}}+\frac{683(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}}-\frac{1151(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}}+\frac{1543(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}}+\frac{7(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{7\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) + 47*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) - 683*I*(x + 1)**(9/2)/(120
*sqrt(x - 1)) + 1151*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) - 1543*I*(x + 1)**(
5/2)/(240*sqrt(x - 1)) - 7*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 7*I*sqrt(x +
 1)/(8*sqrt(x - 1)), Abs(x + 1) > 2), (7*asin(sqrt(2)*sqrt(x + 1)/2)/8 + (x
```

```
 + 1)**(13/2)/(6*sqrt(1 - x)) - 47*(x + 1)**(11/2)/(30*sqrt(1 - x)) + 683*(
x + 1)**(9/2)/(120*sqrt(1 - x)) - 1151*(x + 1)**(7/2)/(120*sqrt(1 - x)) + 1
543*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 7*(x + 1)**(3/2)/(48*sqrt(1 - x)) -
7*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(63) = 126.
time = 2.04, size = 185, normalized size = 2.08

$$-\frac{1}{240}\left(\left(2\left(\left(4\left(5x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{120}\left(\left(2\left(3\left(4x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{12}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{7}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 133
)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x -
10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(
x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 7/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(3/2), x)
```

3.1077 $\int (1-x)^{5/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] 1/4*(1-x)^(3/2)*x*(1+x)^(3/2)+1/5*(1-x)^(5/2)*(1+x)^(5/2)+3/8*arcsin(x)+3/8
*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{3\text{ArcSin}(x)}{8}+\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{5/2}(1+x)^{3/2}\,dx &= \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\ &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{}{\sqrt{}}\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{}{\sqrt{}}\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 68, normalized size = 0.99

$$\frac{\sqrt{1-x}\,(8+33x+9x^2-26x^3-2x^4+8x^5)}{40\sqrt{1+x}}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(8 + 33*x + 9*x^2 - 26*x^3 - 2*x^4 + 8*x^5))/(40*Sqrt[1 + x])
- (3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

Maple [A]
time = 0.14, size = 99, normalized size = 1.43

method	result
risch	$-\frac{(8x^4-10x^3-16x^2+25x+8)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{40\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{5}{2}}}{5}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)}}{8\sqrt{1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1-x)^(5/2)*(1+x)^(5/2)+1/4*(1-x)^(3/2)*(1+x)^(5/2)+1/4*(1-x)^(1/2)*(1+
x)^(5/2)-1/8*(1-x)^(1/2)*(1+x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+x)
*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.63, size = 40, normalized size = 0.58

$$\frac{1}{5}\left(-x^{2}+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^{2}+1}\,x+\frac{3}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*
arcsin(x)
```

Fricas [A]
time = 0.42, size = 57, normalized size = 0.83

$$\frac{1}{40}\left(8\,x^{4}-10\,x^{3}-16\,x^{2}+25\,x+8\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 10*x^3 - 16*x^2 + 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*ar
ctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 34.68, size = 248, normalized size = 3.59

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}-\frac{29i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}+\frac{73i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{129i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{29(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}-\frac{73(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{129(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) - 29*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) + 73*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - 129*I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sq
rt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (3*asin(sqrt
(2)*sqrt(x + 1)/2)/4 - (x + 1)**(11/2)/(5*sqrt(1 - x)) + 29*(x + 1)**(9/2)/
(20*sqrt(1 - x)) - 73*(x + 1)**(7/2)/(20*sqrt(1 - x)) + 129*(x + 1)**(5/2)/
(40*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1
 - x)), True))
```

Giac [A]
time = 2.30, size = 91, normalized size = 1.32

$$\frac{1}{120}\left(\left(2\left(3\left(4x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x
 + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) +
 sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(3/2), x)
```

3.1078 $\int (1-x)^{3/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] 1/4*(1-x)^(3/2)*x*(1+x)^(3/2)+3/8*arcsin(x)+3/8*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {38, 41, 222}

$$\frac{3\text{ArcSin}(x)}{8}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + (3*Ar
cSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{3/2}(1+x)^{3/2}\,dx &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx \\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{8}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 46, normalized size = 0.94

$$
-\frac{1}{8}x\sqrt{1-x^2}\left(-5+2x^2\right)-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] -1/8*(x*Sqrt[1 - x^2]*(-5 + 2*x^2)) - (3*ArcTan[Sqrt[1 - x^2]/(1 + x)])/4
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.
time = 0.14, size = 85, normalized size = 1.73

method	result
risch	$\frac{x(2x^2-5)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{8\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1-x)^(3/2)*(1+x)^(5/2)+1/4*(1-x)^(1/2)*(1+x)^(5/2)-1/8*(1-x)^(1/2)*(1+
x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1
-x)^(1/2)*arcsin(x)
```

Maxima [**A**]
time = 0.55, size = 29, normalized size = 0.59

$$
\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin\left(x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*arcsin(x)
```

Fricas [A]
time = 0.44, size = 46, normalized size = 0.94

$$-\frac{1}{8}\left(2\,x^{3}-5\,x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 - 5*x)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 11.28, size = 212, normalized size = 4.33

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}+\frac{5i(x+1)^{\frac{7}{2}}}{4\sqrt{x-1}}-\frac{13i(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}-\frac{5(x+1)^{\frac{7}{2}}}{4\sqrt{1-x}}+\frac{13(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) + 5*I*(x + 1)**(7/2)/(4*sqrt(x - 1)) - 13*I*(x + 1)**(5/2)/(8*sqrt(x
 - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1))
, Abs(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*sqr
t(1 - x)) - 5*(x + 1)**(7/2)/(4*sqrt(1 - x)) + 13*(x + 1)**(5/2)/(8*sqrt(1
- x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)), Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(35) = 70.
time = 1.06, size = 101, normalized size = 2.06

$$-\frac{1}{24}\left(\left(2\left(3\,x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{6}\left(\left(2\,x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\,\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{3}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(
x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(3/2), x)
```

3.1079 $\int \sqrt{1-x}\,(1+x)^{3/2}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x} - \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2} + \frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/3*(1-x)^(3/2)*(1+x)^(3/2)+1/2*arcsin(x)+1/2*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{\text{ArcSin}(x)}{2} - \frac{1}{3}(1-x)^{3/2}(x+1)^{3/2} + \frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 - ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\,(1+x)^{3/2}\,dx &= -\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 1.12

$$\frac{\sqrt{1-x}\,(-2+x+5x^2+2x^3)}{6\sqrt{1+x}}-\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x]*(-2 + x + 5*x^2 + 2*x^3))/(6*Sqrt[1 + x]) - ArcTan[Sqrt[1 - x]
/Sqrt[1 + x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.
time = 0.14, size = 71, normalized size = 1.48

method	result	size
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{3}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{6}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$-\frac{(2x^2+3x-2)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(1-x)^(1/2)*(1+x)^(5/2)-1/6*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+
x)^(1/2)+1/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 28, normalized size = 0.58

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.47, size = 47, normalized size = 0.98

$$\frac{1}{6}\left(2\,x^{2}+3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.55, size = 163, normalized size = 3.40

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{5i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ \operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{5(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) - 5*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - I*(x + 1)**(3/2)/(6*sqrt(x - 1))
+ I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (asin(sqrt(2)*sqrt(x + 1)/2)
- (x + 1)**(7/2)/(3*sqrt(1 - x)) + 5*(x + 1)**(5/2)/(6*sqrt(1 - x)) + (x +
1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 0.98, size = 66, normalized size = 1.38

$$\frac{1}{6}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{1-x}\,\left(x+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(3/2),x)
[Out] int((1 - x)^(1/2)*(x + 1)^(3/2), x)
```

3.1080 $\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=47

$$-\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}\,-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)-1/2*(1-x)^(1/2)*(1+x)^(3/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\frac{3\text{ArcSin}(x)}{2}-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx = -\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{3}{2}\sin^{-1}(x)$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 1.04

$$-\frac{\sqrt{1-x}\,(4+5x+x^2)}{2\sqrt{1+x}} - 3\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/Sqrt[1 - x],x]
[Out] -1/2*(Sqrt[1 - x]*(4 + 5*x + x^2))/Sqrt[1 + x] - 3*ArcTan[Sqrt[1 - x]/Sqrt[
1 + x]]
```

Maple [A]
time = 0.16, size = 57, normalized size = 1.21

method	result	size
default	$-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2} - \frac{3\sqrt{1-x}\,\sqrt{1+x}}{2} + \frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$\frac{(4+x)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(1-x)^(1/2)*(1+x)^(3/2)-3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^
(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.51, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A]
time = 0.48, size = 40, normalized size = 0.85

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.23, size = 134, normalized size = 2.85

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{3\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 3*I*sqrt(x + 1)/sqrt(x - 1), Abs(
x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 - x)
) + (x + 1)**(3/2)/(2*sqrt(1 - x)) - 3*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.15, size = 31, normalized size = 0.66

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1}+3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(1/2),x)
[Out] int((x + 1)^(3/2)/(1 - x)^(1/2), x)
```

3.1081 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=41

$$3\sqrt{1-x}\,\sqrt{1+x}+\frac{2(1+x)^{3/2}}{\sqrt{1-x}}-3\sin^{-1}(x)$$

```
[Out] -3*arcsin(x)+2*(1+x)^(3/2)/(1-x)^(1/2)+3*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-3\text{ArcSin}(x)+\frac{2(x+1)^{3/2}}{\sqrt{1-x}}+3\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] 3*Sqrt[1 - x]*Sqrt[1 + x] + (2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\ &= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.11, size = 41, normalized size = 1.00

$$\frac{(-5+x)\sqrt{1-x^2}}{-1+x} - 6\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] ((-5 + x)*Sqrt[1 - x^2])/(-1 + x) - 6*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.
time = 0.16, size = 72, normalized size = 1.76

method	result	size
risch	$-\frac{(x^2-4x-5)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} - \frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(x^2-4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1
/2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.49, size = 42, normalized size = 1.02

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{x^2-2x+1}-\frac{6\sqrt{-x^2+1}}{x-1}-3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="maxima")
[Out] -(-x^2 + 1)^(3/2)/(x^2 - 2*x + 1) - 6*sqrt(-x^2 + 1)/(x - 1) - 3*arcsin(x)
```

Fricas [A]
time = 0.53, size = 52, normalized size = 1.27

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1}+6\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+5\,x-5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="fricas")
[Out] (sqrt(x + 1)*(x - 5)*sqrt(-x + 1) + 6*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x +
 1) - 1)/x) + 5*x - 5)/(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.59, size = 99, normalized size = 2.41

$$\begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{6i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{6\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2),x)
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
- 6*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (-6*asin(sqrt(2)*sqrt(x + 1
)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 6*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.04, size = 35, normalized size = 0.85

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1}}{x-1}-6\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 5)*sqrt(-x + 1)/(x - 1) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1)
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(3/2), x)
```

3.1082 $\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2\sqrt{1+x}}{\sqrt{1-x}}+\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\sin^{-1}(x)$$

```
[Out] 2/3*(1+x)^(3/2)/(1-x)^(3/2)+arcsin(x)-2*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {49, 41, 222}

$$\text{ArcSin}(x)+\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} - \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx \\
&= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2\sqrt{1+x}}{\sqrt{1-x}} + \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.12

$$
\frac{4\sqrt{1+x}\,(-1+2x)}{3(1-x)^{3/2}} - 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (4*Sqrt[1 + x]*(-1 + 2*x))/(3*(1 - x)^(3/2)) - 2*ArcTan[Sqrt[1 - x]/Sqrt[1
+ x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.
time = 0.16, size = 76, normalized size = 1.85

method	result	size
risch	$-\frac{4(2x^2+x-1)\sqrt{(1+x)(1-x)}}{3(-1+x)\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} + \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(2*x^2+x-1)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/
2)/(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 0.51, size = 66, normalized size = 1.61

$$
-\frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)} + \frac{7\,\sqrt{-x^2+1}}{3\,(x-1)} + \arcsin\left(x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2 + 3*x - 1) + 2/3*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x - 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 0.54, size = 71, normalized size = 1.73

$$-\frac{2\left(2x^{2}-2(2x-1)\sqrt{x+1}\sqrt{-x+1}+3\left(x^{2}-2x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-4x+2\right)}{3\left(x^{2}-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*x^2 - 2*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*(x^2 - 2*x + 1)*arct
an((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.01, size = 498, normalized size = 12.15

$$\begin{cases} -\frac{6i\sqrt{x-1}(x+1)^{\frac{15}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{3\pi\sqrt{x-1}(x+1)^{\frac{15}{2}}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{12i\sqrt{x-1}(x+1)^{\frac{13}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}-\frac{6\pi\sqrt{x-1}(x+1)^{\frac{13}{2}}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}+\frac{8i(x+1)^{8}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}}-\frac{12i(x+1)^{7}}{3\sqrt{x-1}(x+1)^{\frac{15}{2}}-6\sqrt{x-1}(x+1)^{\frac{13}{2}}} & \text{for}\ |x+1|>2 \\ \frac{6\sqrt{1-x}(x+1)^{\frac{15}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}-\frac{12\sqrt{1-x}(x+1)^{\frac{13}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}-\frac{8(x+1)^{8}}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}}+\frac{12(x+1)^{7}}{3\sqrt{1-x}(x+1)^{\frac{15}{2}}-6\sqrt{1-x}(x+1)^{\frac{13}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((-6*I*sqrt(x - 1)*(x + 1)**(15/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(3
*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) + 3*pi*sqrt(x
 - 1)*(x + 1)**(15/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1
)**(13/2)) + 12*I*sqrt(x - 1)*(x + 1)**(13/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/
(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x + 1)**(13/2)) - 6*pi*sqrt
(x - 1)*(x + 1)**(13/2)/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1)*(x +
 1)**(13/2)) + 8*I*(x + 1)**8/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqrt(x - 1
)*(x + 1)**(13/2)) - 12*I*(x + 1)**7/(3*sqrt(x - 1)*(x + 1)**(15/2) - 6*sqr
t(x - 1)*(x + 1)**(13/2)), Abs(x + 1) > 2), (6*sqrt(1 - x)*(x + 1)**(15/2)*
asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*
(x + 1)**(13/2)) - 12*sqrt(1 - x)*(x + 1)**(13/2)*asin(sqrt(2)*sqrt(x + 1)/
2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) - 8*(x +
 1)**8/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) + 12
*(x + 1)**7/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2))
, True))
```

Giac [**A**]
time = 1.12, size = 38, normalized size = 0.93

$$\frac{4\,(2\,x-1)\sqrt{x+1}\,\sqrt{-x+1}}{3\,(x-1)^2}+2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 2*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(5/2), x)
```

3.1083 $\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx$

Optimal. Leaf size=20

$$\frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

```
[Out] 1/5*(1+x)^(5/2)/(1-x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$\frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(14) = 28$.
time = 0.13, size = 57, normalized size = 2.85

method	result	size
gosper	$\frac{(1+x)^{\frac{5}{2}}}{5(1-x)^{\frac{5}{2}}}$	15
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^3+3x^2+3x+1)}{5\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54
default	$\frac{(1+x)^{\frac{3}{2}}}{(1-x)^{\frac{5}{2}}}-\frac{6\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}+\frac{\sqrt{1+x}}{5(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{5\sqrt{1-x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)^(3/2)/(1-x)^(5/2)-6/5*(1+x)^(1/2)/(1-x)^(5/2)+1/5*(1+x)^(1/2)/(1-x)^(
3/2)+1/5*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(14) = 28$.
time = 0.31, size = 94, normalized size = 4.70

$$\frac{(-x^2+1)^{\frac{3}{2}}}{x^4-4\,x^3+6\,x^2-4\,x+1}+\frac{6\,\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)}+\frac{\sqrt{-x^2+1}}{5\,(x^2-2\,x+1)}-\frac{\sqrt{-x^2+1}}{5\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 6/5*sqrt(-x^2 + 1)/(x^3
- 3*x^2 + 3*x - 1) + 1/5*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/5*sqrt(-x^2 + 1
)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(14) = 28$.
time = 0.48, size = 52, normalized size = 2.60

$$\frac{x^3-3\,x^2-(x^2+2\,x+1)\sqrt{x+1}\,\sqrt{-x+1}+3\,x-1}{5\,(x^3-3\,x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(x^3 - 3*x^2 - (x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*x - 1)/(x^3
 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.01, size = 87, normalized size = 4.35

$$\begin{cases} -\frac{i(x+1)^{\frac{5}{2}}}{5\sqrt{x-1}\,(x+1)^2-20\sqrt{x-1}\,(x+1)+20\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{(x+1)^{\frac{5}{2}}}{5\sqrt{1-x}\,(x+1)^2-20\sqrt{1-x}\,(x+1)+20\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(5/2)/(5*sqrt(x - 1)*(x + 1)**2 - 20*sqrt(x - 1)*(x
+ 1) + 20*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**(5/2)/(5*sqrt(1 - x)*(x
+ 1)**2 - 20*sqrt(1 - x)*(x + 1) + 20*sqrt(1 - x)), True))
```

Giac [A]
time = 1.05, size = 19, normalized size = 0.95

$$-\frac{(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{5\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B]
time = 0.25, size = 50, normalized size = 2.50

$$-\frac{\sqrt{1-x}\,\left(\frac{2\,x\,\sqrt{x+1}}{5}+\frac{\sqrt{x+1}}{5}+\frac{x^2\,\sqrt{x+1}}{5}\right)}{x^3-3\,x^2+3\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((2*x*(x + 1)^(1/2))/5 + (x + 1)^(1/2)/5 + (x^2*(x + 1)^(1/
2))/5))/(3*x - 3*x^2 + x^3 - 1)
```

3.1084 $\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=41

$$\frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}}$$

```
[Out] 1/7*(1+x)^(5/2)/(1-x)^(7/2)+1/35*(1+x)^(5/2)/(1-x)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(9/2),x]
[Out] (1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{1}{7}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.56

$$-\frac{(-6+x)(1+x)^{5/2}}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] -1/35*((-6 + x)*(1 + x)^(5/2))/(1 - x)^(7/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(29) = 58.
time = 0.14, size = 72, normalized size = 1.76

method	result	size
gosper	$-\frac{(1+x)^{\frac{5}{2}}(-6+x)}{35(1-x)^{\frac{7}{2}}}$	18
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^4-3x^3-15x^2-17x-6)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59
default	$\frac{(1+x)^{\frac{3}{2}}}{2(1-x)^{\frac{7}{2}}}-\frac{3\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{70(1-x)^{\frac{5}{2}}}+\frac{\sqrt{1+x}}{35(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{35\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(3/2)/(1-x)^(7/2)-3/7*(1+x)^(1/2)/(1-x)^(7/2)+3/70*(1+x)^(1/2)/(1
-x)^(5/2)+1/35*(1+x)^(1/2)/(1-x)^(3/2)+1/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(29) = 58.
time = 0.28, size = 131, normalized size = 3.20

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{2(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{3\sqrt{-x^2+1}}{7(x^4-4x^3+6x^2-4x+1)}-\frac{3\sqrt{-x^2+1}}{70(x^3-3x^2+3x-1)}+\frac{\sqrt{-x^2+1}}{35(x^2-2x+1)}-\frac{\sqrt{-x^2+1}}{35(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 3/7*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/70*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 1/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/35*sqrt(-x^2 + 1)/(x
 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 0.58, size = 69, normalized size = 1.68

$$\frac{6\,x^4 - 24\,x^3 + 36\,x^2 - (x^3 - 4\,x^2 - 11\,x - 6)\sqrt{x+1}\,\sqrt{-x+1} - 24\,x + 6}{35\,(x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(6*x^4 - 24*x^3 + 36*x^2 - (x^3 - 4*x^2 - 11*x - 6)*sqrt(x + 1)*sqrt(-
x + 1) - 24*x + 6)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.46, size = 226, normalized size = 5.51

$$\begin{cases} -\frac{i(x+1)^{\frac{7}{2}}}{35\sqrt{x-1}\,(x+1)^3-210\sqrt{x-1}\,(x+1)^2+420\sqrt{x-1}\,(x+1)-280\sqrt{x-1}} + \frac{7i(x+1)^{\frac{5}{2}}}{35\sqrt{x-1}\,(x+1)^3-210\sqrt{x-1}\,(x+1)^2+420\sqrt{x-1}\,(x+1)-280\sqrt{x-1}} & \text{for } |x+1| > 2 \\ \frac{(x+1)^{\frac{7}{2}}}{35\sqrt{1-x}\,(x+1)^3-210\sqrt{1-x}\,(x+1)^2+420\sqrt{1-x}\,(x+1)-280\sqrt{1-x}} - \frac{7(x+1)^{\frac{5}{2}}}{35\sqrt{1-x}\,(x+1)^3-210\sqrt{1-x}\,(x+1)^2+420\sqrt{1-x}\,(x+1)-280\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(7/2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(
x + 1)**2 + 420*sqrt(x - 1)*(x + 1) - 280*sqrt(x - 1)) + 7*I*(x + 1)**(5/2)
/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(x + 1)**2 + 420*sqrt(x - 1)*
(x + 1) - 280*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**(7/2)/(35*sqrt(1 - x
)*(x + 1)**3 - 210*sqrt(1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*s
qrt(1 - x)) - 7*(x + 1)**(5/2)/(35*sqrt(1 - x)*(x + 1)**3 - 210*sqrt(1 - x)
*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*sqrt(1 - x)), True))
```

Giac [A]
time = 1.49, size = 22, normalized size = 0.54

$$-\frac{(x+1)^{\frac{5}{2}}(x-6)\sqrt{-x+1}}{35\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(x + 1)^(5/2)*(x - 6)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.27, size = 64, normalized size = 1.56

$$\frac{\sqrt{1-x}\,\left(\frac{11\,x\,\sqrt{x+1}}{35}+\frac{6\,\sqrt{x+1}}{35}+\frac{4\,x^2\,\sqrt{x+1}}{35}-\frac{x^3\,\sqrt{x+1}}{35}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11*x*(x + 1)^(1/2))/35 + (6*(x + 1)^(1/2))/35 + (4*x^2*(x
+ 1)^(1/2))/35 - (x^3*(x + 1)^(1/2))/35))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

3.1085 $\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}} \, dx$

Optimal. Leaf size=61

$$\frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}$$

```
[Out] 1/9*(1+x)^(5/2)/(1-x)^(9/2)+2/63*(1+x)^(5/2)/(1-x)^(7/2)+2/315*(1+x)^(5/2)/
(1-x)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*(1 + x)^(5/2))/(63*(1 - x)^(7/2)) + (2
*(1 + x)^(5/2))/(315*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2}{63}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 0.49

$$\frac{(1+x)^{5/2}\left(47-14x+2x^2\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(5/2)*(47 - 14*x + 2*x^2))/(315*(1 - x)^(9/2))
```

Maple [A]
time = 0.18, size = 86, normalized size = 1.41

method	result	size
gosper	$\frac{(1+x)^{\frac{5}{2}}\left(2x^{2}-14x+47\right)}{315(1-x)^{\frac{9}{2}}}$	25
risch	$\frac{\sqrt{(1+x)\,(1-x)}\,\left(2x^{5}-8x^{4}+11x^{3}+101x^{2}+127x+47\right)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^{4}\sqrt{-(1+x)\,(-1+x)}}$	66
default	$\frac{(1+x)^{\frac{3}{2}}}{3(1-x)^{\frac{9}{2}}}-\frac{2\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}+\frac{\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}+\frac{\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{315\sqrt{1-x}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+x)^(3/2)/(1-x)^(9/2)-2/9*(1+x)^(1/2)/(1-x)^(9/2)+1/63*(1+x)^(1/2)/(1
-x)^(7/2)+1/105*(1+x)^(1/2)/(1-x)^(5/2)+2/315*(1+x)^(1/2)/(1-x)^(3/2)+2/315
*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(43) = 86$.
time = 0.28, size = 172, normalized size = 2.82

$$\frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}+\frac{2\,\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{\sqrt{-x^2+1}}{63\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{\sqrt{-x^2+1}}{105\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{315\,(x^2-2\,x+1)}-\frac{2\,\sqrt{-x^2+1}}{315\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 2
/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/63*sqrt(-x^
2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^2
+ 3*x - 1) + 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/315*sqrt(-x^2 + 1)/(x
 - 1)
```

Fricas [A]
time = 0.80, size = 86, normalized size = 1.41

$$\frac{47\,x^5-235\,x^4+470\,x^3-470\,x^2-\left(2\,x^4-10\,x^3+21\,x^2+80\,x+47\right)\sqrt{x+1}\,\sqrt{-x+1}+235\,x-47}{315\,\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(47*x^5 - 235*x^4 + 470*x^3 - 470*x^2 - (2*x^4 - 10*x^3 + 21*x^2 + 80
*x + 47)*sqrt(x + 1)*sqrt(-x + 1) + 235*x - 47)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 46.30, size = 675, normalized size = 11.07

$$\begin{cases} - \frac{2 i \left(x + 1\right)^{\frac{11}{2}}}{315 \sqrt{x - 1} \left(x + 1\right)^{5} - 3150 \sqrt{x - 1} \left(x + 1\right)^{4} + 12600 \sqrt{x - 1} \left(x + 1\right)^{3} - 25200 \sqrt{x - 1} \left(x + 1\right)^{2} + 25200 \sqrt{x - 1} \left(x + 1\right) - 10080 \sqrt{x - 1}} + \frac{22 i \left(x + 1\right)^{\frac{9}{2}}}{315 \sqrt{x - 1} \left(x + 1\right)^{5} - 3150 \sqrt{x - 1} \left(x + 1\right)^{4} + 12600 \sqrt{x - 1} \left(x + 1\right)^{3} - 25200 \sqrt{x - 1} \left(x + 1\right)^{2} + 25200 \sqrt{x - 1} \left(x + 1\right) - 10080 \sqrt{x - 1}} - \frac{99 i \left(x + 1\right)^{\frac{7}{2}}}{315 \sqrt{x - 1} \left(x + 1\right)^{5} - 3150 \sqrt{x - 1} \left(x + 1\right)^{4} + 12600 \sqrt{x - 1} \left(x + 1\right)^{3} - 25200 \sqrt{x - 1} \left(x + 1\right)^{2} + 25200 \sqrt{x - 1} \left(x + 1\right) - 10080 \sqrt{x - 1}} + \frac{126 i \left(x + 1\right)^{\frac{5}{2}}}{315 \sqrt{x - 1} \left(x + 1\right)^{5} - 3150 \sqrt{x - 1} \left(x + 1\right)^{4} + 12600 \sqrt{x - 1} \left(x + 1\right)^{3} - 25200 \sqrt{x - 1} \left(x + 1\right)^{2} + 25200 \sqrt{x - 1} \left(x + 1\right) - 10080 \sqrt{x - 1}} & \text{for}\: \left|{x + 1}\right| > 2 \\\frac{2 \left(x + 1\right)^{\frac{11}{2}}}{315 \sqrt{1 - x} \left(x + 1\right)^{5} - 3150 \sqrt{1 - x} \left(x + 1\right)^{4} + 12600 \sqrt{1 - x} \left(x + 1\right)^{3} - 25200 \sqrt{1 - x} \left(x + 1\right)^{2} + 25200 \sqrt{1 - x} \left(x + 1\right) - 10080 \sqrt{1 - x}} - \frac{22 \left(x + 1\right)^{\frac{9}{2}}}{315 \sqrt{1 - x} \left(x + 1\right)^{5} - 3150 \sqrt{1 - x} \left(x + 1\right)^{4} + 12600 \sqrt{1 - x} \left(x + 1\right)^{3} - 25200 \sqrt{1 - x} \left(x + 1\right)^{2} + 25200 \sqrt{1 - x} \left(x + 1\right) - 10080 \sqrt{1 - x}} + \frac{99 \left(x + 1\right)^{\frac{7}{2}}}{315 \sqrt{1 - x} \left(x + 1\right)^{5} - 3150 \sqrt{1 - x} \left(x + 1\right)^{4} + 12600 \sqrt{1 - x} \left(x + 1\right)^{3} - 25200 \sqrt{1 - x} \left(x + 1\right)^{2} + 25200 \sqrt{1 - x} \left(x + 1\right) - 10080 \sqrt{1 - x}} - \frac{126 \left(x + 1\right)^{\frac{5}{2}}}{315 \sqrt{1 - x} \left(x + 1\right)^{5} - 3150 \sqrt{1 - x} \left(x + 1\right)^{4} + 12600 \sqrt{1 - x} \left(x + 1\right)^{3} - 25200 \sqrt{1 - x} \left(x + 1\right)^{2} + 25200 \sqrt{1 - x} \left(x + 1\right) - 10080 \sqrt{1 - x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x -
 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**
2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 22*I*(x + 1)**(9/2)/(3
15*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)
*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10
080*sqrt(x - 1)) - 99*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*s
qrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x
 + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 126*I*(x + 1)**
(5/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqr
t(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x +
 1) - 10080*sqrt(x - 1)), Abs(x + 1) > 2), (2*(x + 1)**(11/2)/(315*sqrt(1 -
 x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3
 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1
- x)) - 22*(x + 1)**(9/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x
 + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25
```

```
200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)) + 99*(x + 1)**(7/2)/(315*sqrt(
1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)
**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt
(1 - x)) - 126*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x
)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2
+ 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)), True))
```

Giac [A]
time = 1.37, size = 29, normalized size = 0.48

$$-\frac{(2(x+1)(x-8)+63)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{315(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] -1/315*(2*(x + 1)*(x - 8) + 63)*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.32, size = 80, normalized size = 1.31

$$-\frac{\sqrt{1-x}\left(\frac{16x\sqrt{x+1}}{63}+\frac{47\sqrt{x+1}}{315}+\frac{x^2\sqrt{x+1}}{15}-\frac{2x^3\sqrt{x+1}}{63}+\frac{2x^4\sqrt{x+1}}{315}\right)}{x^5-5x^4+10x^3-10x^2+5x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((16*x*(x + 1)^(1/2))/63 + (47*(x + 1)^(1/2))/315 + (x^2*(x
 + 1)^(1/2))/15 - (2*x^3*(x + 1)^(1/2))/63 + (2*x^4*(x + 1)^(1/2))/315))/(5
*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

3.1086 $\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx$

Optimal. Leaf size=81

$$\frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}$$

```
[Out] 1/11*(1+x)^(5/2)/(1-x)^(11/2)+1/33*(1+x)^(5/2)/(1-x)^(9/2)+2/231*(1+x)^(5/2
)/(1-x)^(7/2)+2/1155*(1+x)^(5/2)/(1-x)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (1 + x)^(5/2)/(33*(1 - x)^(9/2)) + (2*(
1 + x)^(5/2))/(231*(1 - x)^(7/2)) + (2*(1 + x)^(5/2))/(1155*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2}{33}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2}{231}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 35, normalized size = 0.43

$$\frac{(1+x)^{5/2}\left(152-61x+16x^2-2x^3\right)}{1155(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(5/2)*(152 - 61*x + 16*x^2 - 2*x^3))/(1155*(1 - x)^(11/2))
```

Maple [A]
time = 0.16, size = 100, normalized size = 1.23

method	result	size
gosper	$-\frac{(1+x)^{\frac{5}{2}}\left(2x^3-16x^2+61x-152\right)}{1155(1-x)^{\frac{11}{2}}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,\left(2x^6-10x^5+19x^4-15x^3-289x^2-395x-152\right)}{1155\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^5\sqrt{-(1+x)(-1+x)}}$	71
default	$\frac{(1+x)^{\frac{3}{2}}}{4(1-x)^{\frac{11}{2}}}-\frac{3\sqrt{1+x}}{22(1-x)^{\frac{11}{2}}}+\frac{\sqrt{1+x}}{132(1-x)^{\frac{9}{2}}}+\frac{\sqrt{1+x}}{231(1-x)^{\frac{7}{2}}}+\frac{\sqrt{1+x}}{385(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{1155(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{1155\sqrt{1-x}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1+x)^(3/2)/(1-x)^(11/2)-3/22*(1+x)^(1/2)/(1-x)^(11/2)+1/132*(1+x)^(1/2
)/(1-x)^(9/2)+1/231*(1+x)^(1/2)/(1-x)^(7/2)+1/385*(1+x)^(1/2)/(1-x)^(5/2)+2
/1155*(1+x)^(1/2)/(1-x)^(3/2)+2/1155*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(57) = 114$.
time = 0.27, size = 218, normalized size = 2.69

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{4\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}-\frac{3\,\sqrt{-x^2+1}}{22\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}-\frac{\sqrt{-x^2+1}}{132\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{\sqrt{-x^2+1}}{231\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{\sqrt{-x^2+1}}{385\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{1155\,(x^2-2\,x+1)}-\frac{2\,\sqrt{-x^2+1}}{1155\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 3/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
+ 1) - 1/132*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/2
31*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/385*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/1155*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/1155*sqrt
(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.86, size = 101, normalized size = 1.25

$$\frac{152\,x^{6}-912\,x^{5}+2280\,x^{4}-3040\,x^{3}+2280\,x^{2}-\left(2\,x^{5}-12\,x^{4}+31\,x^{3}-46\,x^{2}-243\,x-152\right)\sqrt{x+1}\,\sqrt{-x+1}\,-912\,x+152}{1155\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(152*x^6 - 912*x^5 + 2280*x^4 - 3040*x^3 + 2280*x^2 - (2*x^5 - 12*x^
4 + 31*x^3 - 46*x^2 - 243*x - 152)*sqrt(x + 1)*sqrt(-x + 1) - 912*x + 152)/
(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 133.33, size = 1751, normalized size = 21.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(17/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x
 - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x +
1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2
069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x
 - 1)) + 34*I*(x + 1)**(15/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)
**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206
9760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
 1)) - 255*I*(x + 1)**(13/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) + 1105*I*(x + 1)**(11/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
```

```
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) - 2750*I*(x + 1)**(9/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1
)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**
5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 20697
60*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1
)) + 3564*I*(x + 1)**(7/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)
*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206976
0*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1)
) - 1848*I*(x + 1)**(5/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)*
(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
+ 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069760
*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1))
, Abs(x + 1) > 2), (2*(x + 1)**(17/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*
sqrt(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)
*(x + 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)*
*3 + 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*
sqrt(1 - x)) - 34*(x + 1)**(15/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt
(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x
+ 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 +
 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt
(1 - x)) + 255*(x + 1)**(13/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1
- x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1
)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 20
69760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1
- x)) - 1105*(x + 1)**(11/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 -
x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)*
*5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069
760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 -
x)) + 2750*(x + 1)**(9/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*
(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5
+ 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760
*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x))
 - 3564*(x + 1)**(7/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(x
+ 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 + 1
293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*sq
rt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x)) +
1848*(x + 1)**(5/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(x + 1
)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 + 1293
600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*sqrt(
1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x)), True
))
```

Giac [A]

time = 1.07, size = 35, normalized size = 0.43

$$-\frac{\left(\left(2\left(x+1\right)\left(x-10\right)+99\right)\left(x+1\right)-231\right)\left(x+1\right)^{\frac{5}{2}}\sqrt{-x+1}}{1155\left(x-1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1)^(5/2)*sqrt(-x + 1
)/(x - 1)^6
```

Mupad [B]
time = 0.31, size = 94, normalized size = 1.16

$$\frac{\sqrt{1-x}\,\left(\frac{81\,x\,\sqrt{x+1}}{385}+\frac{152\,\sqrt{x+1}}{1155}+\frac{46\,x^2\,\sqrt{x+1}}{1155}-\frac{31\,x^3\,\sqrt{x+1}}{1155}+\frac{4\,x^4\,\sqrt{x+1}}{385}-\frac{2\,x^5\,\sqrt{x+1}}{1155}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((81*x*(x + 1)^(1/2))/385 + (152*(x + 1)^(1/2))/1155 + (46*x
^2*(x + 1)^(1/2))/1155 - (31*x^3*(x + 1)^(1/2))/1155 + (4*x^4*(x + 1)^(1/2)
)/385 - (2*x^5*(x + 1)^(1/2))/1155))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^
5 + x^6 + 1)
```

3.1087 $\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4(1+x)^{5/2}}{143(1-x)^{11/2}}+\frac{4(1+x)^{5/2}}{429(1-x)^{9/2}}+\frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}}+\frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}$$

```
[Out] 1/13*(1+x)^(5/2)/(1-x)^(13/2)+4/143*(1+x)^(5/2)/(1-x)^(11/2)+4/429*(1+x)^(5
/2)/(1-x)^(9/2)+8/3003*(1+x)^(5/2)/(1-x)^(7/2)+8/15015*(1+x)^(5/2)/(1-x)^(5
/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*(1 + x)^(5/2))/(143*(1 - x)^(11/2))
+ (4*(1 + x)^(5/2))/(429*(1 - x)^(9/2)) + (8*(1 + x)^(5/2))/(3003*(1 - x)^(
7/2)) + (8*(1 + x)^(5/2))/(15015*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}}\,dx = \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4}{13}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{12}{143}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8}{429}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx}{3003}$$
$$= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}} + \frac{4(1+x)^{5/2}}{143(1-x)^{11/2}} + \frac{4(1+x)^{5/2}}{429(1-x)^{9/2}} + \frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}} + \frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}$$

Mathematica [A]
time = 0.07, size = 40, normalized size = 0.40

$$\frac{(1+x)^{5/2}\left(1763-852x+308x^2-72x^3+8x^4\right)}{15015(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(5/2)*(1763 - 852*x + 308*x^2 - 72*x^3 + 8*x^4))/(15015*(1 - x)^(1
3/2))
```

Maple [A]
time = 0.14, size = 114, normalized size = 1.13

method	result
gosper	$\frac{(1+x)^{\frac{5}{2}}\left(8x^4-72x^3+308x^2-852x+1763\right)}{15015(1-x)^{\frac{13}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,\left(8x^7-48x^6+116x^5-136x^4+59x^3+3041x^2+4437x+1763\right)}{15015\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^6\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{3}{2}}}{5(1-x)^{\frac{13}{2}}}-\frac{6\sqrt{1+x}}{65(1-x)^{\frac{13}{2}}}+\frac{3\sqrt{1+x}}{715(1-x)^{\frac{11}{2}}}+\frac{\sqrt{1+x}}{429(1-x)^{\frac{9}{2}}}+\frac{4\sqrt{1+x}}{3003(1-x)^{\frac{7}{2}}}+\frac{4\sqrt{1+x}}{5005(1-x)^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{15015(1-x)^{\frac{3}{2}}}+\frac{8\sqrt{1+x}}{15015\sqrt{1-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1+x)^(3/2)/(1-x)^(13/2)-6/65*(1+x)^(1/2)/(1-x)^(13/2)+3/715*(1+x)^(1/2
)/(1-x)^(11/2)+1/429*(1+x)^(1/2)/(1-x)^(9/2)+4/3003*(1+x)^(1/2)/(1-x)^(7/2)
```

```
+4/5005*(1+x)^(1/2)/(1-x)^(5/2)+8/15015*(1+x)^(1/2)/(1-x)^(3/2)+8/15015*(1+
x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(71) = 142$.
time = 0.33, size = 269, normalized size = 2.66

$$\frac{(-x^2+1)^{\frac{3}{2}}}{5\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}+\frac{6\,\sqrt{-x^2+1}}{65\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}+\frac{3\,\sqrt{-x^2+1}}{715\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}-\frac{\sqrt{-x^2+1}}{429\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{4\,\sqrt{-x^2+1}}{3003\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{4\,\sqrt{-x^2+1}}{5005\,(x^3-3\,x^2+3\,x-1)}+\frac{8\,\sqrt{-x^2+1}}{15015\,(x^2-2\,x+1)}-\frac{8\,\sqrt{-x^2+1}}{15015\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 6/65*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^
3 - 21*x^2 + 7*x - 1) + 3/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3
 + 15*x^2 - 6*x + 1) - 1/429*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
+ 5*x - 1) + 4/3003*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/5005
*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 8/15015*sqrt(-x^2 + 1)/(x^2 - 2*x
 + 1) - 8/15015*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.79, size = 116, normalized size = 1.15

$$\frac{1763\,x^7-12341\,x^6+37023\,x^5-61705\,x^4+61705\,x^3-37023\,x^2-(8\,x^6-56\,x^5+172\,x^4-308\,x^3+367\,x^2+2674\,x+1763)\sqrt{x+1}\,\sqrt{-x+1}+12341\,x-1763}{15015\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1763*x^7 - 12341*x^6 + 37023*x^5 - 61705*x^4 + 61705*x^3 - 37023*x
^2 - (8*x^6 - 56*x^5 + 172*x^4 - 308*x^3 + 367*x^2 + 2674*x + 1763)*sqrt(x
+ 1)*sqrt(-x + 1) + 12341*x - 1763)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3
 - 21*x^2 + 7*x - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [A]
time = 1.41, size = 42, normalized size = 0.42

$$-\frac{(4\,((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)+3003)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{15015\,(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] -1/15015*(4*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1) + 3003)*(x +
 1)^(5/2)*sqrt(-x + 1)/(x - 1)^7
```

Mupad [B]
time = 0.33, size = 110, normalized size = 1.09

$$-\frac{\sqrt{1-x}\,\left(\frac{382\,x\,\sqrt{x+1}}{2145}+\frac{1763\,\sqrt{x+1}}{15015}+\frac{367\,x^2\,\sqrt{x+1}}{15015}-\frac{4\,x^3\,\sqrt{x+1}}{195}+\frac{172\,x^4\,\sqrt{x+1}}{15015}-\frac{8\,x^5\,\sqrt{x+1}}{2145}+\frac{8\,x^6\,\sqrt{x+1}}{15015}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((382*x*(x + 1)^(1/2))/2145 + (1763*(x + 1)^(1/2))/15015 +
(367*x^2*(x + 1)^(1/2))/15015 - (4*x^3*(x + 1)^(1/2))/195 + (172*x^4*(x + 1
)^(1/2))/15015 - (8*x^5*(x + 1)^(1/2))/2145 + (8*x^6*(x + 1)^(1/2))/15015))
/(7*x - 21*x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

3.1088 $\int (1-x)^{11/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2}+\frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+\frac{11}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{11}{72}(1-x)^{9/2}($$

```
[Out] 55/192*(1-x)^(3/2)*x*(1+x)^(3/2)+11/48*(1-x)^(5/2)*x*(1+x)^(5/2)+11/56*(1-x
)^(7/2)*(1+x)^(7/2)+11/72*(1-x)^(9/2)*(1+x)^(7/2)+1/9*(1-x)^(11/2)*(1+x)^(7
/2)+55/128*arcsin(x)+55/128*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {51, 38, 41, 222}

$$\frac{55\text{ArcSin}(x)}{128}+\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}+\frac{55}{128}x\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (55*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (55*(1 - x)^(3/2)*x*(1 + x)^(3/2))/192
 + (11*(1 - x)^(5/2)*x*(1 + x)^(5/2))/48 + (11*(1 - x)^(7/2)*(1 + x)^(7/2))
/56 + (11*(1 - x)^(9/2)*(1 + x)^(7/2))/72 + ((1 - x)^(11/2)*(1 + x)^(7/2))/
9 + (55*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{11/2}(1+x)^{5/2}\,dx &= \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2}+\frac{11}{9}\int (1-x)^{9/2}(1+x)^{5/2}\,dx \\
&= \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2}+\frac{1}{9}(1-x)^{11/2}(1+x)^{7/2}+\frac{11}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx \\
&= \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{11}{72}(1-x)^{9/2}(1+x)^{7/2}+\frac{1}{9}(1-x)^{11/2}(1+x)^{7/2}+\frac{11}{8} \\
&= \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+\frac{11}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{11}{72}(1-x)^{9/2}(1+x)^{7/2}+\frac{1}{9} \\
&= \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2}+\frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+\frac{11}{56}(1-x)^{7/2}(1+x)^{7/2}+ \\
&= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2}+\frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+ \\
&= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2}+\frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+ \\
&= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2}+\frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2}+
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 88, normalized size = 0.68

$$\frac{\sqrt{1-x}\,(3712+8311x-5641x^2-7174x^3+11514x^4+1224x^5-8248x^6+2000x^7+2128x^8-896x^9)}{8064\sqrt{1+x}}-\frac{55}{64}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]
[Out] (Sqrt[1 - x]*(3712 + 8311*x - 5641*x^2 - 7174*x^3 + 11514*x^4 + 1224*x^5 -
8248*x^6 + 2000*x^7 + 2128*x^8 - 896*x^9))/(8064*Sqrt[1 + x]) - (55*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]])/64
```

Maple [A]
time = 0.14, size = 155, normalized size = 1.19

method	result
risch	$\frac{(896x^8-3024x^7+1024x^6+7224x^5-8448x^4-3066x^3+10240x^2-4599x-3712)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{8064\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{55\sqrt{(}}{1}$

default	$\frac{(1-x)^{\frac{11}{2}}(1+x)^{\frac{7}{2}}}{9}+\frac{11(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{72}+\frac{11(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56}+\frac{11(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{48}+\frac{11(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{48}+\frac{11\sqrt{1-x}\,(1+}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(11/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*(1-x)^(11/2)*(1+x)^(7/2)+11/72*(1-x)^(9/2)*(1+x)^(7/2)+11/56*(1-x)^(7/2
)*(1+x)^(7/2)+11/48*(1-x)^(5/2)*(1+x)^(7/2)+11/48*(1-x)^(3/2)*(1+x)^(7/2)+1
1/64*(1-x)^(1/2)*(1+x)^(7/2)-11/192*(1-x)^(1/2)*(1+x)^(5/2)-55/384*(1-x)^(1
/2)*(1+x)^(3/2)-55/128*(1-x)^(1/2)*(1+x)^(1/2)+55/128*((1+x)*(1-x))^(1/2)/(
1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.52, size = 78, normalized size = 0.60

$$\frac{1}{9}\left(-x^2+1\right)^{\frac{7}{2}}x^2-\frac{3}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{29}{63}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{11}{48}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{55}{192}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{55}{128}\sqrt{-x^2+1}\,x+\frac{55}{128}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(-x^2 + 1)^(7/2)*x^2 - 3/8*(-x^2 + 1)^(7/2)*x + 29/63*(-x^2 + 1)^(7/2)
+ 11/48*(-x^2 + 1)^(5/2)*x + 55/192*(-x^2 + 1)^(3/2)*x + 55/128*sqrt(-x^2 +
 1)*x + 55/128*arcsin(x)
```

Fricas [A]
time = 0.91, size = 77, normalized size = 0.59

$$-\frac{1}{8064}\left(896\,x^8-3024\,x^7+1024\,x^6+7224\,x^5-8448\,x^4-3066\,x^3+10240\,x^2-4599\,x-3712\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{55}{64}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*x^8 - 3024*x^7 + 1024*x^6 + 7224*x^5 - 8448*x^4 - 3066*x^3 + 1
0240*x^2 - 4599*x - 3712)*sqrt(x + 1)*sqrt(-x + 1) - 55/64*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(11/2)*(1+x)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4060 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(92) = 184.
time = 1.59, size = 323, normalized size = 2.48

$$-\frac{1}{40320}\left(\left(2\left(\left(4\left(5\left(2\left(7\left(8x-65\right)\left(x+1\right)+2073\right)\left(x+1\right)-9833\right)\left(x+1\right)+75293\right)\left(x+1\right)-310203\right)\left(x+1\right)+216993\right)\left(x+1\right)-205275\right)\left(x+1\right)+69615\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7x-50\right)\left(x+1\right)+1219\right)\left(x+1\right)-12463\right)\left(x+1\right)+64233\right)\left(x+1\right)-53963\right)\left(x+1\right)+59465\right)\left(x+1\right)-23205\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{840}\left(\left(2\left(\left(4\left(5\left(6x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{40}\left(\left(2\left(\left(4\left(5x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{4}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{55}{64}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/40320*((2*((4*(5*(2*(7*(8*x - 65)*(x + 1) + 2073)*(x + 1) - 9833)*(x + 1
) + 75293)*(x + 1) - 310203)*(x + 1) + 216993)*(x + 1) - 205275)*(x + 1) +
69615)*sqrt(x + 1)*sqrt(-x + 1) + 1/6720*((2*((4*(5*(6*(7*x - 50)*(x + 1) +
 1219)*(x + 1) - 12463)*(x + 1) + 64233)*(x + 1) - 53963)*(x + 1) + 59465)*
(x + 1) - 23205)*sqrt(x + 1)*sqrt(-x + 1) + 1/840*((2*((4*(5*(6*x - 37)*(x
+ 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)
*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1)
- 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/4*((2*(3*
x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x -
5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 55/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{11/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(11/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(11/2)*(x + 1)^(5/2), x)
```

3.1089 $\int (1-x)^{9/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=110

$$\frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1-$$

```
[Out] 15/64*(1-x)^(3/2)*x*(1+x)^(3/2)+3/16*(1-x)^(5/2)*x*(1+x)^(5/2)+9/56*(1-x)^(
7/2)*(1+x)^(7/2)+1/8*(1-x)^(9/2)*(1+x)^(7/2)+45/128*arcsin(x)+45/128*x*(1-x
)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{45\text{ArcSin}(x)}{128}+\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (45*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (15*(1 - x)^(3/2)*x*(1 + x)^(3/2))/64
+ (3*(1 - x)^(5/2)*x*(1 + x)^(5/2))/16 + (9*(1 - x)^(7/2)*(1 + x)^(7/2))/56
 + ((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (45*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{9/2}(1+x)^{5/2}\,dx &= \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{9}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx\\ &= \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{9}{8}\int (1-x)^{5/2}(1+x)^{5/2}\,dx\\ &= \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{15}{16}\\ &= \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\\ &= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\\ &= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\\ &= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\end{aligned}$$

Mathematica [A]
time = 0.16, size = 82, normalized size = 0.75

$$\frac{1}{896}\left(\frac{\sqrt{1-x}\,(256+837x-187x^2-978x^3+558x^4+600x^5-424x^6-144x^7+112x^8)}{\sqrt{1+x}}-630\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] ((Sqrt[1 - x]*(256 + 837*x - 187*x^2 - 978*x^3 + 558*x^4 + 600*x^5 - 424*x^
6 - 144*x^7 + 112*x^8))/Sqrt[1 + x] - 630*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/
896
```

Maple [A]
time = 0.14, size = 141, normalized size = 1.28

method	result
risch	$-\frac{(112x^7-256x^6-168x^5+768x^4-210x^3-768x^2+581x+256)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{896\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{45\sqrt{(1+x)(1-}}{128\sqrt{1+x}}$

default	$\frac{(1-x)^{\frac{9}{2}}(1+x)^{\frac{7}{2}}}{8}+\frac{9(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{56}+\frac{3(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{16}+\frac{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{16}+\frac{9\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{64}-\frac{3\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(1-x)^(9/2)*(1+x)^(7/2)+9/56*(1-x)^(7/2)*(1+x)^(7/2)+3/16*(1-x)^(5/2)*(
1+x)^(7/2)+3/16*(1-x)^(3/2)*(1+x)^(7/2)+9/64*(1-x)^(1/2)*(1+x)^(7/2)-3/64*(
1-x)^(1/2)*(1+x)^(5/2)-15/128*(1-x)^(1/2)*(1+x)^(3/2)-45/128*(1-x)^(1/2)*(1
+x)^(1/2)+45/128*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.49, size = 64, normalized size = 0.58

$$-\frac{1}{8}\left(-x^{2}+1\right)^{\frac{7}{2}}x+\frac{2}{7}\left(-x^{2}+1\right)^{\frac{7}{2}}+\frac{3}{16}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{15}{64}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{45}{128}\sqrt{-x^{2}+1}\,x+\frac{45}{128}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^2 + 1)^(7/2)*x + 2/7*(-x^2 + 1)^(7/2) + 3/16*(-x^2 + 1)^(5/2)*x +
15/64*(-x^2 + 1)^(3/2)*x + 45/128*sqrt(-x^2 + 1)*x + 45/128*arcsin(x)
```

Fricas [A]
time = 0.77, size = 72, normalized size = 0.65

$$\frac{1}{896}\left(112\,x^{7}-256\,x^{6}-168\,x^{5}+768\,x^{4}-210\,x^{3}-768\,x^{2}+581\,x+256\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{45}{64}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/896*(112*x^7 - 256*x^6 - 168*x^5 + 768*x^4 - 210*x^3 - 768*x^2 + 581*x +
256)*sqrt(x + 1)*sqrt(-x + 1) - 45/64*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(78) = 156$.
time = 2.01, size = 296, normalized size = 2.69

$$\frac{1}{13440}\left(\left(2\left(\left(4\left(5\left(6\left(7x-50\right)\left(x+1\right)+1219\right)\left(x+1\right)-12463\right)\left(x+1\right)+64233\right)\left(x+1\right)-53963\right)\left(x+1\right)+59465\right)\left(x+1\right)-23205\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{80}\left(\left(2\left(\left(4\left(5x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{40}\left(\left(2\left(3\left(4x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{8}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{45}{64}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/13440*((2*((4*(5*(6*(7*x - 50)*(x + 1) + 1219)*(x + 1) - 12463)*(x + 1) +
 64233)*(x + 1) - 53963)*(x + 1) + 59465)*(x + 1) - 23205)*sqrt(x + 1)*sqrt
(-x + 1) - 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x +
 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/80
*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) -
405)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)*(x +
1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/8*((2*(3*x - 10)*(x +
 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*((2*x - 5)*(x + 1) +
 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(
x + 1)*sqrt(-x + 1) + 45/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(5/2), x)
```

3.1090 $\int (1-x)^{7/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=90

$$\frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\frac{5}{16}\sin^{-1}(x)$$

```
[Out] 5/24*(1-x)^(3/2)*x*(1+x)^(3/2)+1/6*(1-x)^(5/2)*x*(1+x)^(5/2)+1/7*(1-x)^(7/2
)*(1+x)^(7/2)+5/16*arcsin(x)+5/16*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{5\text{ArcSin}(x)}{16}+\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + ((1 - x)^(7/2)*(1 + x)^(7/2))/7 + (5*Arc
Sin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{7/2}(1+x)^{5/2}\,dx &= \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\int (1-x)^{5/2}(1+x)^{5/2}\,dx\\
&= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1-x)^{7/2}(1+x)^{7/2}+\frac{5}{8}\int\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{1}{7}(1
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 78, normalized size = 0.87

$$\frac{\sqrt{1-x}\,(48+279x+87x^2-326x^3-38x^4+200x^5+8x^6-48x^7)}{336\sqrt{1+x}}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]
[Out] (Sqrt[1 - x]*(48 + 279*x + 87*x^2 - 326*x^3 - 38*x^4 + 200*x^5 + 8*x^6 - 48
*x^7))/(336*Sqrt[1 + x]) - (5*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

Maple [A]
time = 0.16, size = 127, normalized size = 1.41

method	result
risch	$\frac{(48x^6-56x^5-144x^4+182x^3+144x^2-231x-48)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{336\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}(1+x)^{\frac{7}{2}}}{7}+\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{8}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(1-x)^(7/2)*(1+x)^(7/2)+1/6*(1-x)^(5/2)*(1+x)^(7/2)+1/6*(1-x)^(3/2)*(1+
x)^(7/2)+1/8*(1-x)^(1/2)*(1+x)^(7/2)-1/24*(1-x)^(1/2)*(1+x)^(5/2)-5/48*(1-x
)^(1/2)*(1+x)^(3/2)-5/16*(1-x)^(1/2)*(1+x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(
1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [**A**]
time = 0.51, size = 52, normalized size = 0.58

$$\frac{1}{7}\left(-x^{2}+1\right)^{\frac{7}{2}}+\frac{1}{6}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^{2}+1}\,x+\frac{5}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(7/2) + 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5
/16*sqrt(-x^2 + 1)*x + 5/16*arcsin(x)
```

Fricas [**A**]
time = 0.84, size = 67, normalized size = 0.74

$$-\frac{1}{336}\left(48\,x^{6}-56\,x^{5}-144\,x^{4}+182\,x^{3}+144\,x^{2}-231\,x-48\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/336*(48*x^6 - 56*x^5 - 144*x^4 + 182*x^3 + 144*x^2 - 231*x - 48)*sqrt(x
+ 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(64) = 128.
time = 2.17, size = 143, normalized size = 1.59

$$-\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6\,x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{40}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781
)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4
*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x +
 1) - 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sq
rt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2} (x+1)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(5/2), x)
```

3.1091 $\int (1-x)^{5/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\sin^{-1}(x)$$

```
[Out] 5/24*(1-x)^(3/2)*x*(1+x)^(3/2)+1/6*(1-x)^(5/2)*x*(1+x)^(5/2)+5/16*arcsin(x)
+5/16*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {38, 41, 222}

$$\frac{5\text{ArcSin}(x)}{16}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + (5*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int(1-x)^{5/2}(1+x)^{5/2}\,dx &= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{6}\int(1-x)^{3/2}(1+x)^{3/2}\,dx \\ &= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{8}\int\sqrt{1-x}\,\sqrt{1+x}\,dx \\ &= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16} \\ &= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16} \\ &= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 51, normalized size = 0.73

$$\frac{1}{48}x\sqrt{1-x^2}\,(33-26x^2+8x^4)-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (x*Sqrt[1 - x^2]*(33 - 26*x^2 + 8*x^4))/48 - (5*ArcTan[Sqrt[1 - x^2]/(1 + x
)])/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(50) = 100.
time = 0.15, size = 113, normalized size = 1.61

method	result
risch	$-\frac{x(8x^4-26x^2+33)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{48\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{16\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{6}+\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{8}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{24}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{48}-\frac{5\sqrt{1-x}\,\sqrt{1}}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(5/2)*(1+x)^(7/2)+1/6*(1-x)^(3/2)*(1+x)^(7/2)+1/8*(1-x)^(1/2)*(1+
x)^(7/2)-1/24*(1-x)^(1/2)*(1+x)^(5/2)-5/48*(1-x)^(1/2)*(1+x)^(3/2)-5/16*(1-
x)^(1/2)*(1+x)^(1/2)+5/16*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Maxima [A]
time = 0.50, size = 41, normalized size = 0.59

$$\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}\,x+\frac{5}{16}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5/16*sqrt(-x^2 + 1)*x +
5/16*arcsin(x)
```

Fricas [A]
time = 0.72, size = 51, normalized size = 0.73

$$\frac{1}{48}\left(8\,x^5-26\,x^3+33\,x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 26*x^3 + 33*x)*sqrt(x + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 186.20, size = 284, normalized size = 4.06

$$\begin{cases} -\frac{5i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{8}+\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}-\frac{7i(x+1)^{\frac{11}{2}}}{6\sqrt{x-1}}+\frac{67i(x+1)^{\frac{9}{2}}}{24\sqrt{x-1}}-\frac{55i(x+1)^{\frac{7}{2}}}{24\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{5\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{8}-\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{7(x+1)^{\frac{11}{2}}}{6\sqrt{1-x}}-\frac{67(x+1)^{\frac{9}{2}}}{24\sqrt{1-x}}+\frac{55(x+1)^{\frac{7}{2}}}{24\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{5\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) - 7*I*(x + 1)**(11/2)/(6*sqrt(x - 1)) + 67*I*(x + 1)**(9/2)/(24*sqr
t(x - 1)) - 55*I*(x + 1)**(7/2)/(24*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*sqr
t(x - 1)) - 5*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sqrt(x
 - 1)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(13/2)
/(6*sqrt(1 - x)) + 7*(x + 1)**(11/2)/(6*sqrt(1 - x)) - 67*(x + 1)**(9/2)/(2
4*sqrt(1 - x)) + 55*(x + 1)**(7/2)/(24*sqrt(1 - x)) + (x + 1)**(5/2)/(48*sq
rt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqrt(x + 1)/(8*sqrt(1 -
x)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(50) = 100$.
time = 1.69, size = 185, normalized size = 2.64

$$\frac{1}{240}\left(\left(2\left(\left(4\left(5x-26\right)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{120}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{3}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{5}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)
*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 1
0)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x
 + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
+ sqrt(x + 1)*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(5/2), x)
```

3.1092 $\int (1-x)^{3/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)$$

```
[Out] 1/4*(1-x)^(3/2)*x*(1+x)^(3/2)-1/5*(1-x)^(5/2)*(1+x)^(5/2)+3/8*arcsin(x)+3/8
*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{3\text{ArcSin}(x)}{8}-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 - ((1 -
 x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{3/2}(1+x)^{5/2}\,dx &= -\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\ &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{}{\sqrt{}}\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{}{\sqrt{}}\\ &= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.99

$$\frac{\sqrt{1-x}\,(-8+17x+41x^2+6x^3-18x^4-8x^5)}{40\sqrt{1+x}}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-8 + 17*x + 41*x^2 + 6*x^3 - 18*x^4 - 8*x^5))/(40*Sqrt[1 + x]
) - (3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

Maple [A]
time = 0.16, size = 99, normalized size = 1.43

method	result
risch	$\frac{(8x^4+10x^3-16x^2-25x+8)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{40\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{3}{2}}(1+x)^{\frac{7}{2}}}{5}+\frac{3\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{20}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{20}-\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{8}-\frac{3\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{3\sqrt{(1-}}{8\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1-x)^(3/2)*(1+x)^(7/2)+3/20*(1-x)^(1/2)*(1+x)^(7/2)-1/20*(1-x)^(1/2)*(
1+x)^(5/2)-1/8*(1-x)^(1/2)*(1+x)^(3/2)-3/8*(1-x)^(1/2)*(1+x)^(1/2)+3/8*((1+
x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.53, size = 40, normalized size = 0.58

$$-\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8
*arcsin(x)
```

Fricas [A]
time = 0.75, size = 57, normalized size = 0.83

$$-\frac{1}{40}\left(8\,x^4+10\,x^3-16\,x^2-25\,x+8\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/40*(8*x^4 + 10*x^3 - 16*x^2 - 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*a
rctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 60.64, size = 245, normalized size = 3.55

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{19i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{19(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 19*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 23*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x
 - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), (3*asin(sqrt(2)*
sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 19*(x + 1)**(9/2)/(20*
sqrt(1 - x)) + 23*(x + 1)**(7/2)/(20*sqrt(1 - x)) + (x + 1)**(5/2)/(40*sqrt
(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)),
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(49) = 98.
time = 1.24, size = 114, normalized size = 1.65

$$-\frac{1}{120}\left(\left(2\left(3\left(4x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(
x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{3/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(5/2), x)
```

3.1093 $\int \sqrt{1-x}\,(1+x)^{5/2}\,dx$

Optimal. Leaf size=68

$$\frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\sin^{-1}(x)$$

```
[Out] -5/12*(1-x)^(3/2)*(1+x)^(3/2)-1/4*(1-x)^(3/2)*(1+x)^(5/2)+5/8*arcsin(x)+5/8
*x*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {51, 38, 41, 222}

$$\frac{5\text{ArcSin}(x)}{8}-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}\,x\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 - (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 - ((1
- x)^(3/2)*(1 + x)^(5/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 51

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a
 + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[2*c*(n/(m + n + 1)
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x}\,(1+x)^{5/2}\,dx &= -\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\,(1+x)^{3/2}\,dx \\
&= -\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx \\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{}{\sqrt{1}} \\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{}{\sqrt{1}} \\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\sin^{-1}(
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.93

$$\frac{\sqrt{1-x}\,(-16-7x+25x^2+22x^3+6x^4)}{24\sqrt{1+x}}-\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-16 - 7*x + 25*x^2 + 22*x^3 + 6*x^4))/(24*Sqrt[1 + x]) - (5*A
rcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

Maple [A]
time = 0.14, size = 85, normalized size = 1.25

method	result
risch	$-\frac{(6x^3+16x^2+9x-16)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{\sqrt{1-x}\,(1+x)^{\frac{7}{2}}}{4}-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{12}-\frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{24}-\frac{5\sqrt{1-x}\,\sqrt{1+x}}{8}+\frac{5\sqrt{(1+x)(1-x)}\,\mathrm{arc}}{8\sqrt{1+x}\,\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1-x)^(1/2)*(1+x)^(7/2)-1/12*(1-x)^(1/2)*(1+x)^(5/2)-5/24*(1-x)^(1/2)*(
1+x)^(3/2)-5/8*(1-x)^(1/2)*(1+x)^(1/2)+5/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^{2}+1\right)^{\frac{3}{2}}x-\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^{2}+1}\,x+\frac{5}{8}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

Fricas [A]
time = 0.82, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6\,x^{3}+16\,x^{2}+9\,x-16\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 20.76, size = 212, normalized size = 3.12

$$\begin{cases} -\frac{5i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{7i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ \frac{5\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{7(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 7*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - I*(x + 1)**(5/2)/(24*sqrt(x
- 1)) - 5*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqrt(x - 1
)), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/2)/(4*s
qrt(1 - x)) + 7*(x + 1)**(7/2)/(12*sqrt(1 - x)) + (x + 1)**(5/2)/(24*sqrt(1
 - x)) + 5*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4*sqrt(1 - x)),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(48) = 96.

time = 1.18, size = 101, normalized size = 1.49

$$\frac{1}{24}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{3}{2}\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*sqrt(x + 1)*(x -
 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqrt(x
 + 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-x}\,\left(x+1\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(5/2), x)
```

3.1094 $\int \frac{(1+x)^{5/2}}{\sqrt{1-x}} \, dx$

Optimal. Leaf size=67

$$-\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}\,-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] 5/2*arcsin(x)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-1/3*(1-x)^(1/2)*(1+x)^(5/2)-5/2*(
1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\frac{5\text{ArcSin}(x)}{2}-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
- x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx = -\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2} + \frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$$
$$= -\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2} + \frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}$$
$$= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2} + \frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2} - \frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2} + \frac{5}{2}\sin^{-1}(x)$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.84

$$-\frac{\sqrt{1-x}\,(22+31x+11x^2+2x^3)}{6\sqrt{1+x}} - 5\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/Sqrt[1 - x],x]
[Out] -1/6*(Sqrt[1 - x]*(22 + 31*x + 11*x^2 + 2*x^3))/Sqrt[1 + x] - 5*ArcTan[Sqrt
[1 - x]/Sqrt[1 + x]]
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.06

method	result	size
default	$-\frac{\sqrt{1-x}\,(1+x)^{\frac{5}{2}}}{3} - \frac{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{6} - \frac{5\sqrt{1-x}\,\sqrt{1+x}}{2} + \frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$\frac{(2x^2+9x+22)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(1-x)^(1/2)*(1+x)^(5/2)-5/6*(1-x)^(1/2)*(1+x)^(3/2)-5/2*(1-x)^(1/2)*(1
+x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.51, size = 42, normalized size = 0.63

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2 - \frac{3}{2}\sqrt{-x^2+1}\,x - \frac{11}{3}\sqrt{-x^2+1} + \frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

Fricas [A]
time = 0.95, size = 47, normalized size = 0.70

$$-\frac{1}{6}\left(2\,x^2+9\,x+22\right)\sqrt{x+1}\,\sqrt{-x+1}-5\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sq
rt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 9.02, size = 170, normalized size = 2.54

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(6*sqrt(x - 1)
) + 5*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt(x +
1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) + (x + 1)**(5/2)/(6*sqrt(1 - x)) + 5
*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 5*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.44, size = 39, normalized size = 0.58

$$-\frac{1}{6}\left((2\,x+7)(x+1)+15\right)\sqrt{x+1}\,\sqrt{-x+1}+5\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x + 7)*(x + 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(
2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{5/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(1/2),x)
[Out] int((x + 1)^(5/2)/(1 - x)^(1/2), x)
```

3.1095 $\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{2(1+x)^{5/2}}{\sqrt{1-x}}-\frac{15}{2}\sin^{-1}(x)$$

```
[Out] -15/2*arcsin(x)+2*(1+x)^(5/2)/(1-x)^(1/2)+5/2*(1-x)^(1/2)*(1+x)^(3/2)+15/2*
(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{15\text{ArcSin}(x)}{2}+\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}\,(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(3/2),x]
```

```
[Out] (15*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*Sqrt[1 - x]*(1 + x)^(3/2))/2 + (2*(1 +
x)^(5/2))/Sqrt[1 - x] - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - 5\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\
&= \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.75

$$-\frac{\sqrt{1+x}\,(-24+7x+x^2)}{2\sqrt{1-x}} + 15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]
[Out] -1/2*(Sqrt[1 + x]*(-24 + 7*x + x^2))/Sqrt[1 - x] + 15*ArcTan[Sqrt[1 - x]/Sq
rt[1 + x]]
```

Maple [A]
time = 0.18, size = 77, normalized size = 1.18

method	result	size
risch	$-\frac{(x^3+8x^2-17x-24)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} - \frac{15\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^3+8*x^2-17*x-24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.54, size = 56, normalized size = 0.86

$$-\frac{x^3}{2\sqrt{-x^2+1}} - \frac{4x^2}{\sqrt{-x^2+1}} + \frac{17x}{2\sqrt{-x^2+1}} + \frac{12}{\sqrt{-x^2+1}} - \frac{15}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) - 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) + 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Fricas [A]
time = 0.62, size = 58, normalized size = 0.89

$$\frac{(x^2+7x-24)\sqrt{x+1}\sqrt{-x+1} + 30(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 24x - 24}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x - 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) + 24*x - 24)/(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.57, size = 138, normalized size = 2.12

$$\begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} + \frac{5i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} - \frac{15i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ |x+1| > 2 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} - \frac{5(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} + \frac{15\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 5*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 15*I*sqrt(x + 1)/sqrt(x - 1), A
bs(x + 1) > 2), (-15*asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1
 - x)) - 5*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 15*sqrt(x + 1)/sqrt(1 - x), Tru
e))
```

Giac [A]
time = 1.22, size = 42, normalized size = 0.65

$$\frac{((x+6)(x+1)-30)\sqrt{x+1}\,\sqrt{-x+1}}{2\,(x-1)} - 15\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((x + 6)*(x + 1) - 30)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 15*arcsin(1/2
*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(3/2), x)
```

3.1096 $\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-5\sqrt{1-x}\,\sqrt{1+x}-\frac{10(1+x)^{3/2}}{3\sqrt{1-x}}+\frac{2(1+x)^{5/2}}{3(1-x)^{3/2}}+5\sin^{-1}(x)$$

```
[Out] 2/3*(1+x)^(5/2)/(1-x)^(3/2)+5*arcsin(x)-10/3*(1+x)^(3/2)/(1-x)^(1/2)-5*(1-x
)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$5\text{ArcSin}(x)+\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}}-\frac{10(x+1)^{3/2}}{3\sqrt{1-x}}-5\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] -5*Sqrt[1 - x]*Sqrt[1 + x] - (10*(1 + x)^(3/2))/(3*Sqrt[1 - x]) + (2*(1 + x
)^(5/2))/(3*(1 - x)^(3/2)) + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} - \frac{5}{3}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx \\
&= -\frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\
&= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 51, normalized size = 0.81

$$-\frac{\sqrt{1-x^2}\,(23-34x+3x^2)}{3(-1+x)^2} + 10\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]
[Out] -1/3*(Sqrt[1 - x^2]*(23 - 34*x + 3*x^2))/(-1 + x)^2 + 10*ArcTan[Sqrt[1 + x]
/Sqrt[1 - x]]
```

Maple [A]
time = 0.16, size = 84, normalized size = 1.33

method	result	size
risch	$\frac{(3x^3-31x^2-11x+23)\sqrt{(1+x)(1-x)}}{3(-1+x)\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} + \frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*x^3-31*x^2-11*x+23)/(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)
/(1-x)^(1/2)/(1+x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcs
in(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(47) = 94$.
time = 0.50, size = 99, normalized size = 1.57

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{x^4-4x^3+6x^2-4x+1}-\frac{5(-x^2+1)^{\frac{3}{2}}}{3(x^3-3x^2+3x-1)}+\frac{10\sqrt{-x^2+1}}{3(x^2-2x+1)}+\frac{35\sqrt{-x^2+1}}{3(x-1)}+5\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(5/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x
^3 - 3*x^2 + 3*x - 1) + 10/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 35/3*sqrt(-x^
2 + 1)/(x - 1) + 5*arcsin(x)
```

Fricas [A]
time = 0.59, size = 75, normalized size = 1.19

$$-\frac{23x^2+(3x^2-34x+23)\sqrt{x+1}\sqrt{-x+1}+30(x^2-2x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-46x+23}{3(x^2-2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(23*x^2 + (3*x^2 - 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x^2 - 2*x
 + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 46*x + 23)/(x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.53, size = 575, normalized size = 9.13

$$\begin{cases} -\frac{30i\sqrt{x-1}(x+1)^{\frac{27}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{15\pi\sqrt{x-1}(x+1)^{\frac{27}{2}}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{60i\sqrt{x-1}(x+1)^{\frac{25}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{30\pi\sqrt{x-1}(x+1)^{\frac{25}{2}}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{3i(x+1)^{15}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{40i(x+1)^{14}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{60i(x+1)^{13}}{3\sqrt{x-1}(x+1)^{\frac{27}{2}}-6\sqrt{x-1}(x+1)^{\frac{25}{2}}} & \text{for}\ |x+1|>2 \\ \frac{30\sqrt{1-x}(x+1)^{\frac{27}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}-\frac{60\sqrt{1-x}(x+1)^{\frac{25}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}+\frac{3(x+1)^{15}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}-\frac{40(x+1)^{14}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}+\frac{60(x+1)^{13}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((-30*I*sqrt(x - 1)*(x + 1)**(27/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(
3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) + 15*pi*sqrt
(x - 1)*(x + 1)**(27/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x +
 1)**(25/2)) + 60*I*sqrt(x - 1)*(x + 1)**(25/2)*acosh(sqrt(2)*sqrt(x + 1)/2
)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)) - 30*pi*s
```

```
qrt(x - 1)*(x + 1)**(25/2)/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x - 1)*(
x + 1)**(25/2)) - 3*I*(x + 1)**15/(3*sqrt(x - 1)*(x + 1)**(27/2) - 6*sqrt(x
 - 1)*(x + 1)**(25/2)) + 40*I*(x + 1)**14/(3*sqrt(x - 1)*(x + 1)**(27/2) -
6*sqrt(x - 1)*(x + 1)**(25/2)) - 60*I*(x + 1)**13/(3*sqrt(x - 1)*(x + 1)**(
27/2) - 6*sqrt(x - 1)*(x + 1)**(25/2)), Abs(x + 1) > 2), (30*sqrt(1 - x)*(x
 + 1)**(27/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) -
6*sqrt(1 - x)*(x + 1)**(25/2)) - 60*sqrt(1 - x)*(x + 1)**(25/2)*asin(sqrt(2
)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x + 1)**(2
5/2)) + 3*(x + 1)**15/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x + 1
)**(25/2)) - 40*(x + 1)**14/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*
(x + 1)**(25/2)) + 60*(x + 1)**13/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1
 - x)*(x + 1)**(25/2)), True))
```

Giac [A]
time = 1.32, size = 44, normalized size = 0.70

$$-\frac{((3x-37)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{3\,(x-1)^2}+10\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((3*x - 37)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 10*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(5/2), x)
```

3.1097 $\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{1+x}}{\sqrt{1-x}}-\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}-\sin^{-1}(x)$$

```
[Out] -2/3*(1+x)^(3/2)/(1-x)^(3/2)+2/5*(1+x)^(5/2)/(1-x)^(5/2)-arcsin(x)+2*(1+x)^
(1/2)/(1-x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$-\text{ArcSin}(x)+\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1 +
 x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx &= \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx \\
&= -\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} + \int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}} - \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}} + \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}} - \sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.81

$$\frac{2\sqrt{1+x}\,(13-24x+23x^2)}{15(1-x)^{5/2}} + 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x]*(13 - 24*x + 23*x^2))/(15*(1 - x)^(5/2)) + 2*ArcTan[Sqrt[1 -
 x]/Sqrt[1 + x]]
```

Maple [A]
time = 0.16, size = 84, normalized size = 1.33

method	result	size
risch	$\frac{2(23x^3-x^2-11x+13)\sqrt{(1+x)(1-x)}}{15(-1+x)^2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} - \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(23*x^3-x^2-11*x+13)/(-1+x)^2/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2
)/(1-x)^(1/2)/(1+x)^(1/2)-((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsi
n(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(47) = 94.
time = 0.54, size = 160, normalized size = 2.54

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{5(x^5-5x^4+10x^3-10x^2+5x-1)} + \frac{(-x^2+1)^{\frac{3}{2}}}{x^4-4x^3+6x^2-4x+1} + \frac{(-x^2+1)^{\frac{3}{2}}}{3(x^3-3x^2+3x-1)} + \frac{6\sqrt{-x^2+1}}{5(x^3-3x^2+3x-1)} - \frac{7\sqrt{-x^2+1}}{15(x^2-2x+1)} - \frac{38\sqrt{-x^2+1}}{15(x-1)} - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + (-x^2 + 1
)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2
 + 3*x - 1) + 6/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 7/15*sqrt(-x^2 +
 1)/(x^2 - 2*x + 1) - 38/15*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Fricas [A]
time = 0.94, size = 91, normalized size = 1.44

$$\frac{2\left(13\,x^{3}-39\,x^{2}-\left(23\,x^{2}-24\,x+13\right)\sqrt{x+1}\,\sqrt{-x+1}+15\left(x^{3}-3\,x^{2}+3\,x-1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+39\,x-13\right)}{15\left(x^{3}-3\,x^{2}+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(13*x^3 - 39*x^2 - (23*x^2 - 24*x + 13)*sqrt(x + 1)*sqrt(-x + 1) + 15*
(x^3 - 3*x^2 + 3*x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 39*x - 1
3)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.28, size = 1606, normalized size = 25.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(35/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(1
5*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 15*pi*sqrt(x - 1
)*(x + 1)**(35/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)*
*(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)
) - 180*I*sqrt(x - 1)*(x + 1)**(33/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt
(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*
(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 90*pi*sqrt(x - 1)*(x +
 1)**(33/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2
) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 36
0*I*sqrt(x - 1)*(x + 1)**(31/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1
)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1
)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 180*pi*sqrt(x - 1)*(x + 1)**
(31/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 1
80*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 240*I*s
qrt(x - 1)*(x + 1)**(29/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1)*(x
```

```
+ 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(3
1/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 120*pi*sqrt(x - 1)*(x + 1)**(29/2
)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sq
rt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 46*I*(x + 1)
**18/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180
*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 232*I*(x
+ 1)**17/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) +
 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 400*I
*(x + 1)**16/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/
2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 2
40*I*(x + 1)**15/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**
(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2))
, Abs(x + 1) > 2), (-30*sqrt(1 - x)*(x + 1)**(35/2)*asin(sqrt(2)*sqrt(x + 1
)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180
*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 180*sqrt(
1 - x)*(x + 1)**(33/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)*
*(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2)
- 120*sqrt(1 - x)*(x + 1)**(29/2)) - 360*sqrt(1 - x)*(x + 1)**(31/2)*asin(s
qrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x +
 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(2
9/2)) + 240*sqrt(1 - x)*(x + 1)**(29/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqr
t(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)
*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 46*(x + 1)**18/(15*sq
rt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x
)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 232*(x + 1)**17/(15*
sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
 x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 400*(x + 1)**16/(1
5*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1
 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 240*(x + 1)**15/
(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt
(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)), True))
```

Giac [A]

time = 1.24, size = 44, normalized size = 0.70

$$-\frac{2\,((23\,x-47)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{15\,(x-1)^3}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*((23*x - 47)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3 - 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(7/2),x)
[Out] int((x + 1)^(5/2)/(1 - x)^(7/2), x)
```

3.1098 $\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

```
[Out] 1/7*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$\frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(14) = 28$.
time = 0.14, size = 85, normalized size = 4.25

method	result	size
gosper	$\frac{(1+x)^{\frac{7}{2}}}{7(1-x)^{\frac{7}{2}}}$	15
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^4+4x^3+6x^2+4x+1)}{7\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	59
default	$\frac{(1+x)^{\frac{5}{2}}}{(1-x)^{\frac{7}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{2(1-x)^{\frac{7}{2}}}+\frac{15\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}-\frac{3\sqrt{1+x}}{14(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{7(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{7\sqrt{1-x}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)^(5/2)/(1-x)^(7/2)-5/2*(1+x)^(3/2)/(1-x)^(7/2)+15/7*(1+x)^(1/2)/(1-x)^
(7/2)-3/14*(1+x)^(1/2)/(1-x)^(5/2)-1/7*(1+x)^(1/2)/(1-x)^(3/2)-1/7*(1+x)^(1
/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(14) = 28$.
time = 0.28, size = 171, normalized size = 8.55

$$\frac{(-x^2+1)^{\frac{5}{2}}}{x^6-6x^5+15x^4-20x^3+15x^2-6x+1}+\frac{5(-x^2+1)^{\frac{3}{2}}}{2(x^5-5x^4+10x^3-10x^2+5x-1)}+\frac{15\sqrt{-x^2+1}}{7(x^4-4x^3+6x^2-4x+1)}+\frac{3\sqrt{-x^2+1}}{14(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{7(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{7(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/2*(
-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 15/7*sqrt(-x^2
+ 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 3/14*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3
*x - 1) - 1/7*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/7*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(14) = 28$.
time = 1.12, size = 66, normalized size = 3.30

$$\frac{x^4-4x^3+6x^2+(x^3+3x^2+3x+1)\sqrt{x+1}\,\sqrt{-x+1}-4x+1}{7(x^4-4x^3+6x^2-4x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*(x^4 - 4*x^3 + 6*x^2 + (x^3 + 3*x^2 + 3*x + 1)*sqrt(x + 1)*sqrt(-x + 1)
 - 4*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.69, size = 114, normalized size = 5.70

$$\begin{cases} \frac{i(x+1)^{\frac{7}{2}}}{7\sqrt{x-1}\,(x+1)^3-42\sqrt{x-1}\,(x+1)^2+84\sqrt{x-1}\,(x+1)-56\sqrt{x-1}} & \text{for } |x+1|>2 \\ -\frac{(x+1)^{\frac{7}{2}}}{7\sqrt{1-x}\,(x+1)^3-42\sqrt{1-x}\,(x+1)^2+84\sqrt{1-x}\,(x+1)-56\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(7/2)/(7*sqrt(x - 1)*(x + 1)**3 - 42*sqrt(x - 1)*(x +
 1)**2 + 84*sqrt(x - 1)*(x + 1) - 56*sqrt(x - 1)), Abs(x + 1) > 2), (-(x +
1)**(7/2)/(7*sqrt(1 - x)*(x + 1)**3 - 42*sqrt(1 - x)*(x + 1)**2 + 84*sqrt(1
 - x)*(x + 1) - 56*sqrt(1 - x)), True))
```

Giac [A]
time = 1.23, size = 19, normalized size = 0.95

$$\frac{(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{7\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.28, size = 64, normalized size = 3.20

$$\frac{\sqrt{1-x}\,\left(\frac{3\,x\,\sqrt{x+1}}{7}+\frac{\sqrt{x+1}}{7}+\frac{3\,x^2\,\sqrt{x+1}}{7}+\frac{x^3\,\sqrt{x+1}}{7}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((3*x*(x + 1)^(1/2))/7 + (x + 1)^(1/2)/7 + (3*x^2*(x + 1)^(1
/2))/7 + (x^3*(x + 1)^(1/2))/7))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

3.1099 $\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx$

Optimal. Leaf size=41

$$\frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}$$

```
[Out] 1/9*(1+x)^(7/2)/(1-x)^(9/2)+1/63*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(11/2),x]
[Out] (1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{1}{9}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 23, normalized size = 0.56

$$-\frac{(-8+x)(1+x)^{7/2}}{63(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]
```

```
[Out] -1/63*((-8 + x)*(1 + x)^(7/2))/(1 - x)^(9/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(29) = 58.
time = 0.14, size = 100, normalized size = 2.44

method	result	size
gosper	$-\frac{(1+x)^{\frac{7}{2}}(x-8)}{63(1-x)^{\frac{9}{2}}}$	18
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^5-4x^4-26x^3-44x^2-31x-8)}{63\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	64
default	$\frac{(1+x)^{\frac{5}{2}}}{2(1-x)^{\frac{9}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{6(1-x)^{\frac{9}{2}}}+\frac{5\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{126(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{42(1-x)^{\frac{5}{2}}}-\frac{\sqrt{1+x}}{63(1-x)^{\frac{3}{2}}}-\frac{\sqrt{1+x}}{63\sqrt{1-x}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1+x)^(5/2)/(1-x)^(9/2)-5/6*(1+x)^(3/2)/(1-x)^(9/2)+5/9*(1+x)^(1/2)/(1-
x)^(9/2)-5/126*(1+x)^(1/2)/(1-x)^(7/2)-1/42*(1+x)^(1/2)/(1-x)^(5/2)-1/63*(1
+x)^(1/2)/(1-x)^(3/2)-1/63*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(29) = 58.
time = 0.30, size = 218, normalized size = 5.32

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{2(x^7-7x^6+21x^5-35x^4+35x^3-21x^2+7x-1)}-\frac{5(-x^2+1)^{\frac{3}{2}}}{6(x^6-6x^5+15x^4-20x^3+15x^2-6x+1)}-\frac{5\sqrt{-x^2+1}}{9(x^5-5x^4+10x^3-10x^2+5x-1)}-\frac{5\sqrt{-x^2+1}}{126(x^4-4x^3+6x^2-4x+1)}+\frac{\sqrt{-x^2+1}}{42(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{63(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{63(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(5/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 5/6*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
 + 1) - 5/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/12
6*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/42*sqrt(-x^2 + 1)/(x^3
 - 3*x^2 + 3*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/63*sqrt(-x^2
+ 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.
time = 1.61, size = 83, normalized size = 2.02

$$\frac{8\,x^{5}-40\,x^{4}+80\,x^{3}-80\,x^{2}+\left(x^{4}-5\,x^{3}-21\,x^{2}-23\,x-8\right)\sqrt{x+1}\,\sqrt{-x+1}+40\,x-8}{63\,\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/63*(8*x^5 - 40*x^4 + 80*x^3 - 80*x^2 + (x^4 - 5*x^3 - 21*x^2 - 23*x - 8)*
sqrt(x + 1)*sqrt(-x + 1) + 40*x - 8)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x -
 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 50.92, size = 280, normalized size = 6.83

$$\begin{cases} \frac{i\left(x+1\right)^{\frac{9}{2}}}{63\sqrt{x-1}\left(x+1\right)^{4}-504\sqrt{x-1}\left(x+1\right)^{3}+1512\sqrt{x-1}\left(x+1\right)^{2}-2016\sqrt{x-1}\left(x+1\right)+1008\sqrt{x-1}} - \frac{9i\left(x+1\right)^{\frac{7}{2}}}{63\sqrt{x-1}\left(x+1\right)^{4}-504\sqrt{x-1}\left(x+1\right)^{3}+1512\sqrt{x-1}\left(x+1\right)^{2}-2016\sqrt{x-1}\left(x+1\right)+1008\sqrt{x-1}} & \text{for}\: \left|{x+1}\right| > 2 \\ -\frac{\left(x+1\right)^{\frac{9}{2}}}{63\sqrt{1-x}\left(x+1\right)^{4}-504\sqrt{1-x}\left(x+1\right)^{3}+1512\sqrt{1-x}\left(x+1\right)^{2}-2016\sqrt{1-x}\left(x+1\right)+1008\sqrt{1-x}} + \frac{9\left(x+1\right)^{\frac{7}{2}}}{63\sqrt{1-x}\left(x+1\right)^{4}-504\sqrt{1-x}\left(x+1\right)^{3}+1512\sqrt{1-x}\left(x+1\right)^{2}-2016\sqrt{1-x}\left(x+1\right)+1008\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(9/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1)*(x
 + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 1008*sq
rt(x - 1)) - 9*I*(x + 1)**(7/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1
)*(x + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 100
8*sqrt(x - 1)), Abs(x + 1) > 2), (-(x + 1)**(9/2)/(63*sqrt(1 - x)*(x + 1)**
4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqrt(1
- x)*(x + 1) + 1008*sqrt(1 - x)) + 9*(x + 1)**(7/2)/(63*sqrt(1 - x)*(x + 1)
**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqrt(
1 - x)*(x + 1) + 1008*sqrt(1 - x)), True))
```

Giac [A]
time = 1.12, size = 22, normalized size = 0.54

$$\frac{\left(x+1\right)^{\frac{7}{2}}\left(x-8\right)\sqrt{-x+1}}{63\left(x-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] 1/63*(x + 1)^(7/2)*(x - 8)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.30, size = 80, normalized size = 1.95

$$-\frac{\sqrt{1-x}\,\left(\frac{23\,x\,\sqrt{x+1}}{63}+\frac{8\,\sqrt{x+1}}{63}+\frac{x^2\,\sqrt{x+1}}{3}+\frac{5\,x^3\,\sqrt{x+1}}{63}-\frac{x^4\,\sqrt{x+1}}{63}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((23*x*(x + 1)^(1/2))/63 + (8*(x + 1)^(1/2))/63 + (x^2*(x +
 1)^(1/2))/3 + (5*x^3*(x + 1)^(1/2))/63 - (x^4*(x + 1)^(1/2))/63))/(5*x - 1
0*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

3.1100 $\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} dx$

Optimal. Leaf size=61

$$\frac{(1+x)^{7/2}}{11(1-x)^{11/2}} + \frac{2(1+x)^{7/2}}{99(1-x)^{9/2}} + \frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}$$

```
[Out] 1/11*(1+x)^(7/2)/(1-x)^(11/2)+2/99*(1+x)^(7/2)/(1-x)^(9/2)+2/693*(1+x)^(7/2
)/(1-x)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}} + \frac{2(x+1)^{7/2}}{99(1-x)^{9/2}} + \frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*(1 + x)^(7/2))/(99*(1 - x)^(9/2)) +
(2*(1 + x)^(7/2))/(693*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2}{11}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{99(1-x)^{9/2}}+\frac{2}{99}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx \\
&= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{99(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 30, normalized size = 0.49

$$\frac{(1+x)^{7/2}\left(79-18x+2x^2\right)}{693(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]
[Out] ((1 + x)^(7/2)*(79 - 18*x + 2*x^2))/(693*(1 - x)^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(43) = 86$.
time = 0.14, size = 114, normalized size = 1.87

method	result
gosper	$\frac{(1+x)^{\frac{7}{2}}\left(2x^2-18x+79\right)}{693(1-x)^{\frac{11}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\left(2x^6-10x^5+19x^4+216x^3+404x^2+298x+79\right)}{693\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^5\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{3(1-x)^{\frac{11}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{12(1-x)^{\frac{11}{2}}}+\frac{5\sqrt{1+x}}{22(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{396(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{693(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{231(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{693(1-x)^{\frac{3}{2}}}-\frac{2\sqrt{1+x}}{693\sqrt{1-x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(13/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(1+x)^(5/2)/(1-x)^(11/2)-5/12*(1+x)^(3/2)/(1-x)^(11/2)+5/22*(1+x)^(1/2)
/(1-x)^(11/2)-5/396*(1+x)^(1/2)/(1-x)^(9/2)-5/693*(1+x)^(1/2)/(1-x)^(7/2)-1
/231*(1+x)^(1/2)/(1-x)^(5/2)-2/693*(1+x)^(1/2)/(1-x)^(3/2)-2/693*(1+x)^(1/2
)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(43) = 86$.
time = 0.28, size = 269, normalized size = 4.41

$$\frac{(-x^2+1)^{\frac{5}{2}}}{3\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}+\frac{5\,(-x^2+1)^{\frac{3}{2}}}{12\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}+\frac{5\,\sqrt{-x^2+1}}{22\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}+\frac{5\,\sqrt{-x^2+1}}{396\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}-\frac{5\,\sqrt{-x^2+1}}{693\,(x^4-4\,x^3+6\,x^2-4\,x+1)}+\frac{\sqrt{-x^2+1}}{231\,(x^3-3\,x^2+3\,x-1)}-\frac{2\,\sqrt{-x^2+1}}{693\,(x^2-2\,x+1)}+\frac{2\,\sqrt{-x^2+1}}{693\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(5/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 5/12*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1) + 5/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^
3 + 15*x^2 - 6*x + 1) + 5/396*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
 + 5*x - 1) - 5/693*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/231*
sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/693*sqrt(-x^2 + 1)/(x^2 - 2*x +
1) + 2/693*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(43) = 86$.
time = 0.78, size = 100, normalized size = 1.64

$$\frac{79\,x^{6}-474\,x^{5}+1185\,x^{4}-1580\,x^{3}+1185\,x^{2}+(2\,x^{5}-12\,x^{4}+31\,x^{3}+185\,x^{2}+219\,x+79)\sqrt{x+1}\,\sqrt{-x+1}\,-474\,x+79}{693\,(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/693*(79*x^6 - 474*x^5 + 1185*x^4 - 1580*x^3 + 1185*x^2 + (2*x^5 - 12*x^4
+ 31*x^3 + 185*x^2 + 219*x + 79)*sqrt(x + 1)*sqrt(-x + 1) - 474*x + 79)/(x^
6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 148.97, size = 784, normalized size = 12.85

$$\begin{cases} \frac{2i(x+1)^{\frac{13}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} - \frac{26i(x+1)^{\frac{11}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} + \frac{143i(x+1)^{\frac{9}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} - \frac{198i(x+1)^{\frac{7}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} & \text{for}\ |x+1|>2 \\ -\frac{2(x+1)^{\frac{13}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} + \frac{26(x+1)^{\frac{11}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} - \frac{143(x+1)^{\frac{9}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} + \frac{198(x+1)^{\frac{7}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(13/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)**
3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sqrt
(x - 1)) - 26*I*(x + 1)**(11/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)*
*3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sqr
t(x - 1)) + 143*I*(x + 1)**(9/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x
- 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)
**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sq
rt(x - 1)) - 198*I*(x + 1)**(7/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x
 - 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1
)**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*s
qrt(x - 1)), Abs(x + 1) > 2), (-2*(x + 1)**(13/2)/(693*sqrt(1 - x)*(x + 1)*
```

```
*6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sq
rt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(
x + 1) + 44352*sqrt(1 - x)) + 26*(x + 1)**(11/2)/(693*sqrt(1 - x)*(x + 1)**
6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sqr
t(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(x
 + 1) + 44352*sqrt(1 - x)) - 143*(x + 1)**(9/2)/(693*sqrt(1 - x)*(x + 1)**6
 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sqrt
(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(x
+ 1) + 44352*sqrt(1 - x)) + 198*(x + 1)**(7/2)/(693*sqrt(1 - x)*(x + 1)**6
- 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sqrt(
1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(x +
 1) + 44352*sqrt(1 - x)), True))
```

Giac [A]
time = 1.32, size = 29, normalized size = 0.48

$$\frac{(2\,(x+1)(x-10)+99)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{693\,(x-1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/693*(2*(x + 1)*(x - 10) + 99)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^6
```

Mupad [B]
time = 0.31, size = 94, normalized size = 1.54

$$\frac{\sqrt{1-x}\,\left(\frac{73\,x\,\sqrt{x+1}}{231}+\frac{79\,\sqrt{x+1}}{693}+\frac{185\,x^2\,\sqrt{x+1}}{693}+\frac{31\,x^3\,\sqrt{x+1}}{693}-\frac{4\,x^4\,\sqrt{x+1}}{231}+\frac{2\,x^5\,\sqrt{x+1}}{693}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((73*x*(x + 1)^(1/2))/231 + (79*(x + 1)^(1/2))/693 + (185*x^
2*(x + 1)^(1/2))/693 + (31*x^3*(x + 1)^(1/2))/693 - (4*x^4*(x + 1)^(1/2))/2
31 + (2*x^5*(x + 1)^(1/2))/693))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^5 +
x^6 + 1)
```

3.1101 $\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx$

Optimal. Leaf size=81

$$\frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}$$

```
[Out] 1/13*(1+x)^(7/2)/(1-x)^(13/2)+3/143*(1+x)^(7/2)/(1-x)^(11/2)+2/429*(1+x)^(7
/2)/(1-x)^(9/2)+2/3003*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*(1 + x)^(7/2))/(143*(1 - x)^(11/2))
 + (2*(1 + x)^(7/2))/(429*(1 - x)^(9/2)) + (2*(1 + x)^(7/2))/(3003*(1 - x)^(
7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx &= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3}{13}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{6}{143}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2}{429}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 35, normalized size = 0.43

$$\frac{(1+x)^{7/2}\left(310-97x+20x^2-2x^3\right)}{3003(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]
[Out] ((1 + x)^(7/2)*(310 - 97*x + 20*x^2 - 2*x^3))/(3003*(1 - x)^(13/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.
time = 0.16, size = 128, normalized size = 1.58

method	result
gosper	$-\frac{(1+x)^{\frac{7}{2}}\left(2x^3-20x^2+97x-310\right)}{3003(1-x)^{\frac{13}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(2x^7-12x^6+29x^5-34x^4-736x^3-1492x^2-1143x-310\right)}{3003\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^6\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{4(1-x)^{\frac{13}{2}}}-\frac{(1+x)^{\frac{3}{2}}}{4(1-x)^{\frac{13}{2}}}+\frac{3\sqrt{1+x}}{26(1-x)^{\frac{13}{2}}}-\frac{3\sqrt{1+x}}{572(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{1716(1-x)^{\frac{9}{2}}}-\frac{5\sqrt{1+x}}{3003(1-x)^{\frac{7}{2}}}-\frac{\sqrt{1+x}}{1001(1-x)^{\frac{5}{2}}}-\frac{2\sqrt{1+x}}{3003(1-x)^{\frac{3}{2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(15/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(1+x)^(5/2)/(1-x)^(13/2)-1/4*(1+x)^(3/2)/(1-x)^(13/2)+3/26*(1+x)^(1/2)/
(1-x)^(13/2)-3/572*(1+x)^(1/2)/(1-x)^(11/2)-5/1716*(1+x)^(1/2)/(1-x)^(9/2)-
5/3003*(1+x)^(1/2)/(1-x)^(7/2)-1/1001*(1+x)^(1/2)/(1-x)^(5/2)-2/3003*(1+x)^
(1/2)/(1-x)^(3/2)-2/3003*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(57) = 114$.
time = 0.27, size = 325, normalized size = 4.01

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{4\,(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1)}-\frac{(-x^2+1)^{\frac{3}{2}}}{4\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}-\frac{3\,\sqrt{-x^2+1}}{26\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}-\frac{3\,\sqrt{-x^2+1}}{572\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}+\frac{5\,\sqrt{-x^2+1}}{1716\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}-\frac{5\,\sqrt{-x^2+1}}{3003\,(x^4-4\,x^3+6\,x^2-4\,x+1)}+\frac{\sqrt{-x^2+1}}{1001\,(x^3-3\,x^2+3\,x-1)}-\frac{2\,\sqrt{-x^2+1}}{3003\,(x^2-2\,x+1)}+\frac{2\,\sqrt{-x^2+1}}{3003\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(5/2)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 +
84*x^3 - 36*x^2 + 9*x - 1) - 1/4*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 5
6*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) - 3/26*sqrt(-x^2 + 1)/(x^7 - 7*
x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1) - 3/572*sqrt(-x^2 + 1)/(
x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/1716*sqrt(-x^2 + 1)/(
x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/3003*sqrt(-x^2 + 1)/(x^4 - 4*x
^3 + 6*x^2 - 4*x + 1) + 1/1001*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/3
003*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/3003*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.
time = 1.35, size = 115, normalized size = 1.42

$$\frac{310\,x^7-2170\,x^6+6510\,x^5-10850\,x^4+10850\,x^3-6510\,x^2+(2\,x^6-14\,x^5+43\,x^4-77\,x^3-659\,x^2-833\,x-310)\sqrt{x+1}\,\sqrt{-x+1}\,+2170\,x-310}{3003\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(310*x^7 - 2170*x^6 + 6510*x^5 - 10850*x^4 + 10850*x^3 - 6510*x^2 +
(2*x^6 - 14*x^5 + 43*x^4 - 77*x^3 - 659*x^2 - 833*x - 310)*sqrt(x + 1)*sqrt
(-x + 1) + 2170*x - 310)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 +
 7*x - 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [A]
time = 1.96, size = 35, normalized size = 0.43

$$\frac{((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{3003\,(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] 1/3003*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1)^(7/2)*sqrt(-x + 1
)/(x - 1)^7
```

Mupad [B]
time = 0.31, size = 110, normalized size = 1.36

$$-\frac{\sqrt{1-x}\,\left(\frac{119\,x\,\sqrt{x+1}}{429}+\frac{310\,\sqrt{x+1}}{3003}+\frac{659\,x^2\,\sqrt{x+1}}{3003}+\frac{x^3\,\sqrt{x+1}}{39}-\frac{43\,x^4\,\sqrt{x+1}}{3003}+\frac{2\,x^5\,\sqrt{x+1}}{429}-\frac{2\,x^6\,\sqrt{x+1}}{3003}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((119*x*(x + 1)^(1/2))/429 + (310*(x + 1)^(1/2))/3003 + (65
9*x^2*(x + 1)^(1/2))/3003 + (x^3*(x + 1)^(1/2))/39 - (43*x^4*(x + 1)^(1/2))
/3003 + (2*x^5*(x + 1)^(1/2))/429 - (2*x^6*(x + 1)^(1/2))/3003))/(7*x - 21*
x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

3.1102 $\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx$

Optimal. Leaf size=101

$$\frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}$$

```
[Out] 1/15*(1+x)^(7/2)/(1-x)^(15/2)+4/195*(1+x)^(7/2)/(1-x)^(13/2)+4/715*(1+x)^(7
/2)/(1-x)^(11/2)+8/6435*(1+x)^(7/2)/(1-x)^(9/2)+8/45045*(1+x)^(7/2)/(1-x)^(
7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] (1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*(1 + x)^(7/2))/(195*(1 - x)^(13/2))
+ (4*(1 + x)^(7/2))/(715*(1 - x)^(11/2)) + (8*(1 + x)^(7/2))/(6435*(1 - x)^
(9/2)) + (8*(1 + x)^(7/2))/(45045*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4}{15}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx \\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4}{65}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx \\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8}{715}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx \\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}} + \frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx}{6435} \\ &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{195(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{715(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}} + \frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.40

$$\frac{(1+x)^{7/2}\left(4243-1628x+468x^2-88x^3+8x^4\right)}{45045(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]
[Out] ((1 + x)^(7/2)*(4243 - 1628*x + 468*x^2 - 88*x^3 + 8*x^4))/(45045*(1 - x)^(
15/2))
```

Maple [A]
time = 0.16, size = 142, normalized size = 1.41

method	result
gosper	$\frac{(1+x)^{\frac{7}{2}}\left(8x^4-88x^3+468x^2-1628x+4243\right)}{45045(1-x)^{\frac{15}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(8x^8-56x^7+164x^6-252x^5+195x^4+8988x^3+19414x^2+15344x+4243\right)}{45045\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^7\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{5(1-x)^{\frac{15}{2}}}-\frac{(1+x)^{\frac{3}{2}}}{6(1-x)^{\frac{15}{2}}}+\frac{\sqrt{1+x}}{15(1-x)^{\frac{15}{2}}}-\frac{\sqrt{1+x}}{390(1-x)^{\frac{13}{2}}}-\frac{\sqrt{1+x}}{715(1-x)^{\frac{11}{2}}}-\frac{\sqrt{1+x}}{1287(1-x)^{\frac{9}{2}}}-\frac{4\sqrt{1+x}}{9009(1-x)^{\frac{7}{2}}}-\frac{4\sqrt{1+x}}{15015(1-x)^{\frac{5}{2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(17/2),x,method=_RETURNVERBOSE)
[Out] 1/5*(1+x)^(5/2)/(1-x)^(15/2)-1/6*(1+x)^(3/2)/(1-x)^(15/2)+1/15/(1-x)^(15/2)
*(1+x)^(1/2)-1/390*(1+x)^(1/2)/(1-x)^(13/2)-1/715*(1+x)^(1/2)/(1-x)^(11/2)-
```

```
1/1287*(1+x)^(1/2)/(1-x)^(9/2)-4/9009*(1+x)^(1/2)/(1-x)^(7/2)-4/15015*(1+x)
^(1/2)/(1-x)^(5/2)-8/45045*(1+x)^(1/2)/(1-x)^(3/2)-8/45045*(1+x)^(1/2)/(1-x
)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(71) = 142$.
time = 0.28, size = 386, normalized size = 3.82

$$\frac{\left(-x^{2}+1\right)^{\frac{5}{2}}}{5\left(x^{10}-10x^{9}+45x^{8}-120x^{7}+210x^{6}-252x^{5}+210x^{4}-120x^{3}+45x^{2}-10x+1\right)}+\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{6\left(x^{9}-9x^{8}+36x^{7}-84x^{6}+126x^{5}-126x^{4}+84x^{3}-36x^{2}+9x-1\right)}+\frac{\sqrt{-x^{2}+1}}{15\left(x^{8}-8x^{7}+28x^{6}-56x^{5}+70x^{4}-56x^{3}+28x^{2}-8x+1\right)}+\frac{\sqrt{-x^{2}+1}}{390\left(x^{7}-7x^{6}+21x^{5}-35x^{4}+35x^{3}-21x^{2}+7x-1\right)}-\frac{\sqrt{-x^{2}+1}}{715\left(x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1\right)}+\frac{\sqrt{-x^{2}+1}}{1287\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}-\frac{4\sqrt{-x^{2}+1}}{9009\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}+\frac{4\sqrt{-x^{2}+1}}{15015\left(x^{3}-3x^{2}+3x-1\right)}-\frac{8\sqrt{-x^{2}+1}}{45045\left(x^{2}-2x+1\right)}+\frac{8\sqrt{-x^{2}+1}}{45045\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5
+ 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1) + 1/6*(-x^2 + 1)^(3/2)/(x^9 - 9*x^
8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) + 1/15
*sqrt(-x^2 + 1)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 -
 8*x + 1) + 1/390*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 -
21*x^2 + 7*x - 1) - 1/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 1
5*x^2 - 6*x + 1) + 1/1287*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5
*x - 1) - 4/9009*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/15015*s
qrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 8/45045*sqrt(-x^2 + 1)/(x^2 - 2*x +
 1) + 8/45045*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.94, size = 130, normalized size = 1.29

$$\frac{4243x^{8}-33944x^{7}+118804x^{6}-237608x^{5}+297010x^{4}-237608x^{3}+118804x^{2}+\left(8x^{7}-64x^{6}+228x^{5}-480x^{4}+675x^{3}+8313x^{2}+11101x+4243\right)\sqrt{x+1}\,\sqrt{-x+1}-33944x+4243}{45045\left(x^{8}-8x^{7}+28x^{6}-56x^{5}+70x^{4}-56x^{3}+28x^{2}-8x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="fricas")
```

```
[Out] 1/45045*(4243*x^8 - 33944*x^7 + 118804*x^6 - 237608*x^5 + 297010*x^4 - 2376
08*x^3 + 118804*x^2 + (8*x^7 - 64*x^6 + 228*x^5 - 480*x^4 + 675*x^3 + 8313*
x^2 + 11101*x + 4243)*sqrt(x + 1)*sqrt(-x + 1) - 33944*x + 4243)/(x^8 - 8*x
^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(17/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [A]
time = 1.84, size = 42, normalized size = 0.42

$$\frac{(4\,((2\,(x+1)(x-14)+195)(x+1)-715)(x+1)+6435)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{45045\,(x-1)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="giac")
```

```
[Out] 1/45045*(4*((2*(x + 1)*(x - 14) + 195)*(x + 1) - 715)*(x + 1) + 6435)*(x +
1)^(7/2)*sqrt(-x + 1)/(x - 1)^8
```

Mupad [B]
time = 0.35, size = 124, normalized size = 1.23

$$\frac{\sqrt{1-x}\,\left(\frac{11101\,x\,\sqrt{x+1}}{45045}+\frac{4243\,\sqrt{x+1}}{45045}+\frac{2771\,x^2\,\sqrt{x+1}}{15015}+\frac{15\,x^3\,\sqrt{x+1}}{1001}-\frac{32\,x^4\,\sqrt{x+1}}{3003}+\frac{76\,x^5\,\sqrt{x+1}}{15015}-\frac{64\,x^6\,\sqrt{x+1}}{45045}+\frac{8\,x^7\,\sqrt{x+1}}{45045}\right)}{x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(17/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11101*x*(x + 1)^(1/2))/45045 + (4243*(x + 1)^(1/2))/45045
+ (2771*x^2*(x + 1)^(1/2))/15015 + (15*x^3*(x + 1)^(1/2))/1001 - (32*x^4*(x
 + 1)^(1/2))/3003 + (76*x^5*(x + 1)^(1/2))/15015 - (64*x^6*(x + 1)^(1/2))/4
5045 + (8*x^7*(x + 1)^(1/2))/45045))/(28*x^2 - 8*x - 56*x^3 + 70*x^4 - 56*x
^5 + 28*x^6 - 8*x^7 + x^8 + 1)
```

3.1103 $\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}} \, dx$

Optimal. Leaf size=121

$$\frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{153153(1-x)^{7/2}}$$

```
[Out] 1/17*(1+x)^(7/2)/(1-x)^(17/2)+1/51*(1+x)^(7/2)/(1-x)^(15/2)+4/663*(1+x)^(7/
2)/(1-x)^(13/2)+4/2431*(1+x)^(7/2)/(1-x)^(11/2)+8/21879*(1+x)^(7/2)/(1-x)^(
9/2)+8/153153*(1+x)^(7/2)/(1-x)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] (1 + x)^(7/2)/(17*(1 - x)^(17/2)) + (1 + x)^(7/2)/(51*(1 - x)^(15/2)) + (4*
(1 + x)^(7/2))/(663*(1 - x)^(13/2)) + (4*(1 + x)^(7/2))/(2431*(1 - x)^(11/2
)) + (8*(1 + x)^(7/2))/(21879*(1 - x)^(9/2)) + (8*(1 + x)^(7/2))/(153153*(1
 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}}\,dx = \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{5}{17}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4}{51}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4}{221}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx}{2431}$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}} +$$
$$= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}} + \frac{(1+x)^{7/2}}{51(1-x)^{15/2}} + \frac{4(1+x)^{7/2}}{663(1-x)^{13/2}} + \frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}} + \frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}} +$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.37

$$\frac{(1+x)^{7/2}\left(13252-5871x+2096x^2-556x^3+96x^4-8x^5\right)}{153153(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(7/2)*(13252 - 5871*x + 2096*x^2 - 556*x^3 + 96*x^4 - 8*x^5))/(153
153*(1 - x)^(17/2))
```

Maple [A]
time = 0.18, size = 156, normalized size = 1.29

method	result
gosper	$-\frac{(1+x)^{\frac{7}{2}}\left(8x^5-96x^4+556x^3-2096x^2+5871x-13252\right)}{153153(1-x)^{\frac{17}{2}}}$
risch	$-\frac{\sqrt{(1+x)(1-x)}\,\left(8x^9-64x^8+220x^7-416x^6+447x^5-216x^4-25610x^3-58124x^2-47137x-13252\right)}{153153\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^8\sqrt{-(1+x)(-1+x)}}$
default	$\frac{(1+x)^{\frac{5}{2}}}{6(1-x)^{\frac{17}{2}}}-\frac{5(1+x)^{\frac{3}{2}}}{42(1-x)^{\frac{17}{2}}}+\frac{5\sqrt{1+x}}{119(1-x)^{\frac{17}{2}}}-\frac{\sqrt{1+x}}{714(1-x)^{\frac{15}{2}}}-\frac{\sqrt{1+x}}{1326(1-x)^{\frac{13}{2}}}-\frac{\sqrt{1+x}}{2431(1-x)^{\frac{11}{2}}}-\frac{5\sqrt{1+x}}{21879(1-x)^{\frac{9}{2}}}-\frac{20\sqrt{1+x}}{153153(1-x)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(5/2)/(1-x)^(19/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1+x)^(5/2)/(1-x)^(17/2)-5/42*(1+x)^(3/2)/(1-x)^(17/2)+5/119/(1-x)^(17/
2)*(1+x)^(1/2)-1/714/(1-x)^(15/2)*(1+x)^(1/2)-1/1326*(1+x)^(1/2)/(1-x)^(13/
2)-1/2431*(1+x)^(1/2)/(1-x)^(11/2)-5/21879*(1+x)^(1/2)/(1-x)^(9/2)-20/15315
3*(1+x)^(1/2)/(1-x)^(7/2)-4/51051*(1+x)^(1/2)/(1-x)^(5/2)-8/153153*(1+x)^(1
/2)/(1-x)^(3/2)-8/153153*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(85) = 170$.
time = 0.28, size = 452, normalized size = 3.74

$$-\frac{(-x^2+1)^{\frac{5}{2}}}{6\,(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1)}-\frac{5\,(-x^2+1)^{\frac{3}{2}}}{42\,(x^{10}-10\,x^9+45\,x^8-120\,x^7+210\,x^6-252\,x^5+210\,x^4-120\,x^3+45\,x^2-10\,x+1)}-\frac{5\,\sqrt{-x^2+1}}{119\,(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1)}-\frac{\sqrt{-x^2+1}}{714\,(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1)}+\frac{\sqrt{-x^2+1}}{1326\,(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1)}-\frac{\sqrt{-x^2+1}}{2431\,(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1)}+\frac{5\,\sqrt{-x^2+1}}{21879\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}-\frac{20\,\sqrt{-x^2+1}}{153153\,(x^4-4\,x^3+6\,x^2-4\,x+1)}+\frac{4\,\sqrt{-x^2+1}}{51051\,(x^3-3\,x^2+3\,x-1)}-\frac{8\,\sqrt{-x^2+1}}{153153\,(x^2-2\,x+1)}+\frac{8\,\sqrt{-x^2+1}}{153153\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^
6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1) - 5/42*(-x^2 + 1)^(3/2
)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5 + 210*x^4 - 120*x^3
 + 45*x^2 - 10*x + 1) - 5/119*sqrt(-x^2 + 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6
 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) - 1/714*sqrt(-x^2 + 1)/(x
^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) + 1/1326
*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1)
 - 1/2431*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
 + 5/21879*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 20/15
3153*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/51051*sqrt(-x^2 + 1
)/(x^3 - 3*x^2 + 3*x - 1) - 8/153153*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/153
153*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [**A**]
time = 0.88, size = 145, normalized size = 1.20

$$\frac{13252\,x^9-119268\,x^8+477072\,x^7-1113168\,x^6+1669752\,x^5-1669752\,x^4+1113168\,x^3-477072\,x^2+(8\,x^8-72\,x^7+292\,x^6-708\,x^5+1155\,x^4-1371\,x^3-24239\,x^2-33885\,x-13252)\sqrt{x+1}\,\sqrt{-x+1}+119268\,x-13252}{153153\,(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="fricas")
```

```
[Out] 1/153153*(13252*x^9 - 119268*x^8 + 477072*x^7 - 1113168*x^6 + 1669752*x^5 -
 1669752*x^4 + 1113168*x^3 - 477072*x^2 + (8*x^8 - 72*x^7 + 292*x^6 - 708*x
^5 + 1155*x^4 - 1371*x^3 - 24239*x^2 - 33885*x - 13252)*sqrt(x + 1)*sqrt(-x
 + 1) + 119268*x - 13252)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^
4 + 84*x^3 - 36*x^2 + 9*x - 1)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(19/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8437 deep
```

Giac [A]
time = 2.51, size = 48, normalized size = 0.40

$$\frac{((4((2(x+1)(x-16)+255)(x+1)-1105)(x+1)+12155)(x+1)-21879)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{153153(x-1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="giac")
```

```
[Out] 1/153153*((4*((2*(x + 1)*(x - 16) + 255)*(x + 1) - 1105)*(x + 1) + 12155)*(
x + 1) - 21879)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^9
```

Mupad [B]
time = 0.37, size = 140, normalized size = 1.16

$$-\frac{\sqrt{1-x}\left(\frac{3765x\sqrt{x+1}}{17017}+\frac{13252\sqrt{x+1}}{153153}+\frac{24239x^2\sqrt{x+1}}{153153}+\frac{457x^3\sqrt{x+1}}{51051}-\frac{5x^4\sqrt{x+1}}{663}+\frac{236x^5\sqrt{x+1}}{51051}-\frac{292x^6\sqrt{x+1}}{153153}+\frac{8x^7\sqrt{x+1}}{17017}-\frac{8x^8\sqrt{x+1}}{153153}\right)}{x^9-9x^8+36x^7-84x^6+126x^5-126x^4+84x^3-36x^2+9x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(19/2),x)
```

```
[Out] -((1 - x)^(1/2)*((3765*x*(x + 1)^(1/2))/17017 + (13252*(x + 1)^(1/2))/15315
3 + (24239*x^2*(x + 1)^(1/2))/153153 + (457*x^3*(x + 1)^(1/2))/51051 - (5*x
^4*(x + 1)^(1/2))/663 + (236*x^5*(x + 1)^(1/2))/51051 - (292*x^6*(x + 1)^(1
/2))/153153 + (8*x^7*(x + 1)^(1/2))/17017 - (8*x^8*(x + 1)^(1/2))/153153))/
(9*x - 36*x^2 + 84*x^3 - 126*x^4 + 126*x^5 - 84*x^6 + 36*x^7 - 9*x^8 + x^9
- 1)
```

3.1104 $\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=64

$$-\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

```
[Out] 3/2*arcsin(a*x)/a-1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*
x+1)^(1/2)/a
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {52, 41, 222}

$$\frac{3\text{ArcSin}(ax)}{2a}-\frac{\sqrt{1-ax}\,(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\,\sqrt{ax+1}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] (-3*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a) - (Sqrt[1 - a*x]*(1 + a*x)^(3/2))/(2
*a) + (3*ArcSin[a*x])/(2*a)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx = -\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a} - \frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a} + \frac{3\sin^{-1}(ax)}{2a}$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 1.00

$$-\frac{\frac{\sqrt{1-ax}\,(4+5ax+a^2x^2)}{\sqrt{1+ax}} + 6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] -1/2*((Sqrt[1 - a*x]*(4 + 5*a*x + a^2*x^2))/Sqrt[1 + a*x] + 6*ArcTan[Sqrt[1
 - a*x]/Sqrt[1 + a*x]])/a
```

Maple [A]
time = 0.15, size = 98, normalized size = 1.53

method	result
default	$-\frac{(ax+1)^{\frac{3}{2}}\sqrt{-ax+1}}{2a} - \frac{3\sqrt{-ax+1}\,\sqrt{ax+1}}{2a} + \frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}$
risch	$\frac{(ax+4)\sqrt{ax+1}\,(ax-1)\sqrt{(ax+1)(-ax+1)}}{2a\sqrt{-(ax+1)(ax-1)}\,\sqrt{-ax+1}} + \frac{3\sqrt{(ax+1)(-ax+1)}\,\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\,\sqrt{-ax+1}\,\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a+3/2*
((a*x+1)*(-a*x+1))^(1/2)/(a*x+1)^(1/2)/(-a*x+1)^(1/2)/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

Maxima [A]
time = 0.49, size = 42, normalized size = 0.66

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Fricas [A]
time = 1.04, size = 55, normalized size = 0.86

$$-\frac{(ax+4)\sqrt{ax+1}\,\sqrt{-ax+1}+6\arctan\left(\frac{\sqrt{ax+1}\,\sqrt{-ax+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) + 6*arctan((sqrt(a*x + 1)*sqrt
(-a*x + 1) - 1)/(a*x)))/a
```

Sympy [A]
time = 16.60, size = 88, normalized size = 1.38

$$\begin{cases} \frac{2\left(\begin{cases} -\frac{ax\sqrt{-ax+1}\,\sqrt{ax+1}}{4}-\sqrt{-ax+1}\,\sqrt{ax+1}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{ax+1}}{2}\right)}{2} & \text{for}\ \sqrt{ax+1}>-\sqrt{2}\wedge\sqrt{ax+1}<\sqrt{2} \end{cases}\right)}{a} & \text{for}\ a\neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/(-a*x+1)**(1/2),x)
[Out] Piecewise((2*Piecewise((-a*x*sqrt(-a*x + 1)*sqrt(a*x + 1)/4 - sqrt(-a*x + 1
)*sqrt(a*x + 1) + 3*asin(sqrt(2)*sqrt(a*x + 1)/2)/2, (sqrt(a*x + 1) < sqrt(
2)) & (sqrt(a*x + 1) > -sqrt(2))))/a, Ne(a, 0)), (x, True))
```

Giac [A]
time = 1.64, size = 42, normalized size = 0.66

$$-\frac{(ax+4)\sqrt{ax+1}\,\sqrt{-ax+1}-6\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{ax+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 6*arcsin(1/2*sqrt(2)*sqrt(a*
x + 1)))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,x+1)^{3/2}}{\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2),x)
```

```
[Out] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2), x)
```

3.1105 $\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx$

Optimal. Leaf size=62

$$-\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}+\frac{3\sin^{-1}(ax)}{2a}$$

```
[Out] -1/2*(-a^2*x^2+1)^(3/2)/a/(-a*x+1)+3/2*arcsin(a*x)/a-3/2*(-a^2*x^2+1)^(1/2)
/a
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {809, 679, 222}

$$-\frac{(1-a^2x^2)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\text{ArcSin}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*a) - (1 - a^2*x^2)^(3/2)/(2*a*(1 - a*x)) + (3*Arc
Sin[a*x])/(2*a)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 809

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx \\
&= -\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3\sin^{-1}(ax)}{2a}
\end{aligned}
$$

Mathematica [**A**]
time = 0.17, size = 69, normalized size = 1.11

$$
\frac{(-4-ax)\sqrt{1-a^2x^2}}{2a}-\frac{3\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{2\sqrt{-a^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] ((-4 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) - (3*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^
2*x^2]])/(2*Sqrt[-a^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(52) = 104.
time = 0.16, size = 120, normalized size = 1.94

method	result
risch	$\frac{(ax+4)(a^2x^2-1)}{2a\sqrt{-a^2x^2+1}}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}$
default	$-\frac{x\sqrt{-a^2x^2+1}}{2}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-\frac{2\left(\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}-\frac{a\arctan\left(\frac{}{\sqrt{-a^2\left(x-\right.}}\right.}{\sqrt{}}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x*(-a^2*x^2+1)^(1/2)-1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)
^(1/2))-2/a*((-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^
(1/2)*x/(-a^2*(x-1/a)^2-2*a*(x-1/a))^(1/2)))
```

Maxima [A]
time = 0.49, size = 42, normalized size = 0.68

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

Fricas [A]
time = 1.10, size = 48, normalized size = 0.77

$$-\frac{\sqrt{-a^2x^2+1}\,(ax+4)+6\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a
```

Sympy [A]
time = 2.99, size = 76, normalized size = 1.23

$$-\begin{cases}-\frac{-\sqrt{-a^2x^2+1}+\operatorname{asin}(ax)}{a} & \text{for}\ ax>-1\wedge ax<1\end{cases}-\begin{cases}-\frac{-\frac{ax\sqrt{-a^2x^2+1}}{2}-\sqrt{-a^2x^2+1}+\frac{\operatorname{asin}(ax)}{2}}{a} & \text{for}\ ax>-1\wedge ax<1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

```
[Out] -Piecewise((-(-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, (a*x > -1) & (a*x < 1))
) - Piecewise((-(-a*x*sqrt(-a**2*x**2 + 1)/2 - sqrt(-a**2*x**2 + 1) + asin(
a*x)/2)/a, (a*x > -1) & (a*x < 1)))
```

Giac [A]
time = 1.16, size = 34, normalized size = 0.55

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\left(x+\frac{4}{a}\right)+\frac{3\arcsin(ax)\operatorname{sgn}(a)}{2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

Mupad [B]
time = 0.15, size = 55, normalized size = 0.89

$$\frac{\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2}+\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a}{\sqrt{-a^2}}-\frac{x\,\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - a^2*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] ((3*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a)/(-a^2)^(1/2) - (x
*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

3.1106 $\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=87

$$\frac{35}{8}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{24}(1-x)^{3/2}\sqrt{1+x}+\frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{8}\sin^{-1}(x)$$

```
[Out] 35/8*arcsin(x)+35/24*(1-x)^(3/2)*(1+x)^(1/2)+7/12*(1-x)^(5/2)*(1+x)^(1/2)+1
/4*(1-x)^(7/2)*(1+x)^(1/2)+35/8*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 41, 222}

$$\frac{35\text{ArcSin}(x)}{8}+\frac{1}{4}\sqrt{x+1}\,(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (35*Sqrt[1 - x]*Sqrt[1 + x])/8 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/24 + (7*(1
- x)^(5/2)*Sqrt[1 + x])/12 + ((1 - x)^(7/2)*Sqrt[1 + x])/4 + (35*ArcSin[x])
/8
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx &= \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{7}{4}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{12}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} + \frac{35}{8}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} \\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x} \\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{24}(1-x)^{3/2}\sqrt{1+x} + \frac{7}{12}(1-x)^{5/2}\sqrt{1+x} + \frac{1}{4}(1-x)^{7/2}\sqrt{1+x}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 63, normalized size = 0.72

$$
\frac{\sqrt{1+x}\,(160-241x+113x^2-38x^3+6x^4)}{24\sqrt{1-x}} + \frac{35}{4}\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/Sqrt[1 + x],x]
[Out] (Sqrt[1 + x]*(160 - 241*x + 113*x^2 - 38*x^3 + 6*x^4))/(24*Sqrt[1 - x]) + (
35*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

Maple [**A**]
time = 0.14, size = 85, normalized size = 0.98

method	result
risch	$\frac{(6x^3-32x^2+81x-160)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{24\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{8\sqrt{1+x}\,\sqrt{1-x}}$
default	$\frac{(1-x)^{\frac{7}{2}}\sqrt{1+x}}{4} + \frac{7(1-x)^{\frac{5}{2}}\sqrt{1+x}}{12} + \frac{35(1-x)^{\frac{3}{2}}\sqrt{1+x}}{24} + \frac{35\sqrt{1-x}\,\sqrt{1+x}}{8} + \frac{35\sqrt{(1+x)(1-x)}}{8\sqrt{1+x}\,\sqrt{1-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(1-x)^(7/2)*(1+x)^(1/2)+7/12*(1-x)^(5/2)*(1+x)^(1/2)+35/24*(1-x)^(3/2)*
(1+x)^(1/2)+35/8*(1-x)^(1/2)*(1+x)^(1/2)+35/8*((1+x)*(1-x))^(1/2)/(1+x)^(1/
2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.51, size = 56, normalized size = 0.64

$$-\frac{1}{4}\sqrt{-x^2+1}\,x^3+\frac{4}{3}\sqrt{-x^2+1}\,x^2-\frac{27}{8}\sqrt{-x^2+1}\,x+\frac{20}{3}\sqrt{-x^2+1}+\frac{35}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 + 4/3*sqrt(-x^2 + 1)*x^2 - 27/8*sqrt(-x^2 + 1)*x +
20/3*sqrt(-x^2 + 1) + 35/8*arcsin(x)
```

Fricas [A]
time = 1.04, size = 52, normalized size = 0.60

$$-\frac{1}{24}\left(6x^3-32x^2+81x-160\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{35}{4}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 - 32*x^2 + 81*x - 160)*sqrt(x + 1)*sqrt(-x + 1) - 35/4*arctan(
(sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.70, size = 197, normalized size = 2.26

$$\begin{cases} -\frac{i\sqrt{x-1}\,(x+1)^{\frac{7}{2}}}{4}+\frac{25i\sqrt{x-1}\,(x+1)^{\frac{5}{2}}}{12}-\frac{163i\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{24}+\frac{93i\sqrt{x-1}\,\sqrt{x+1}}{8}-\frac{35i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4} & \text{for}\ |x+1|>2 \\ \frac{35\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}-\frac{31(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{263(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}-\frac{605(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}+\frac{93\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x - 1)*(x + 1)**(7/2)/4 + 25*I*sqrt(x - 1)*(x + 1)**(5/2
)/12 - 163*I*sqrt(x - 1)*(x + 1)**(3/2)/24 + 93*I*sqrt(x - 1)*sqrt(x + 1)/8
 - 35*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4, Abs(x + 1) > 2), (35*asin(sqrt(2)*s
qrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*sqrt(1 - x)) - 31*(x + 1)**(7/2)/(12*sq
rt(1 - x)) + 263*(x + 1)**(5/2)/(24*sqrt(1 - x)) - 605*(x + 1)**(3/2)/(24*s
qrt(1 - x)) + 93*sqrt(x + 1)/(4*sqrt(1 - x)), True))
```

Giac [A]
time = 1.27, size = 101, normalized size = 1.16

$$-\frac{1}{24}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{35}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 35/4*arcsin(1/2*sqrt(2)*sqrt
(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(1/2), x)
```

3.1107 $\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=67

$$\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}\;+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}\;+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}\;+\frac{5}{2}\sin^{-1}(x)$$

```
[Out] 5/2*arcsin(x)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+1/3*(1-x)^(5/2)*(1+x)^(1/2)+5/2*(
1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 41, 222}

$$\frac{5\text{ArcSin}(x)}{2}+\frac{1}{3}\sqrt{x+1}\,(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*(1 - x)^(3/2)*Sqrt[1 + x])/6 + ((1 - x)^
(5/2)*Sqrt[1 + x])/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx &= \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}} \\
&= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{1}{3}(1-x)^{5/2}\sqrt{1+x} + \frac{5}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.84

$$\frac{\sqrt{1+x}\,(22-31x+11x^2-2x^3)}{6\sqrt{1-x}} + 5\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(22 - 31*x + 11*x^2 - 2*x^3))/(6*Sqrt[1 - x]) + 5*ArcTan[Sqrt[
1 + x]/Sqrt[1 - x]]
```

Maple [A]
time = 0.16, size = 71, normalized size = 1.06

method	result	size
default	$\frac{(1-x)^{\frac{5}{2}}\sqrt{1+x}}{3}+\frac{5(1-x)^{\frac{3}{2}}\sqrt{1+x}}{6}+\frac{5\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	71
risch	$-\frac{(2x^2-9x+22)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1-x)^(5/2)*(1+x)^(1/2)+5/6*(1-x)^(3/2)*(1+x)^(1/2)+5/2*(1-x)^(1/2)*(1+
x)^(1/2)+5/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.63

$$\frac{1}{3}\sqrt{-x^2+1}\,x^2 - \frac{3}{2}\sqrt{-x^2+1}\,x + \frac{11}{3}\sqrt{-x^2+1} + \frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x + 11/3*sqrt(-x^2 + 1) + 5/2*a
rcsin(x)
```

Fricas [A]
time = 0.92, size = 47, normalized size = 0.70

$$\frac{1}{6}\left(2\,x^{2}-9\,x+22\right)\sqrt{x+1}\,\sqrt{-x+1}-5\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sqr
t(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.11, size = 173, normalized size = 2.58

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{17i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{59i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}-\frac{11i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{17(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{59(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}+\frac{11\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - 17*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 59*I*(x + 1)**(3/2)/(6*sqrt(x
- 1)) - 11*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (5*asin(sqrt(2)*sqrt
(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 17*(x + 1)**(5/2)/(6*sqrt(1 -
 x)) - 59*(x + 1)**(3/2)/(6*sqrt(1 - x)) + 11*sqrt(x + 1)/sqrt(1 - x), True
))
```

Giac [A]
time = 1.50, size = 69, normalized size = 1.03

$$\frac{1}{6}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+5\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(2)*sqrt(x + 1))

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(5/2)/(x + 1)^(1/2),x)

[Out] int((1 - x)^(5/2)/(x + 1)^(1/2), x)

3.1108 $\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=47

$$\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\sin^{-1}(x)$$

```
[Out] 3/2*arcsin(x)+1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\frac{3\text{ArcSin}(x)}{2}+\frac{1}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 - x]*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*ArcSin[x
])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx &= \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{1}{2}(1-x)^{3/2}\sqrt{1+x} + \frac{3}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 1.04

$$\frac{\sqrt{1+x}\,(4-5x+x^2)}{2\sqrt{1-x}} + 3\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(4 - 5*x + x^2))/(2*Sqrt[1 - x]) + 3*ArcTan[Sqrt[1 + x]/Sqrt[1
 - x]]
```

Maple [A]
time = 0.16, size = 57, normalized size = 1.21

method	result	size
default	$\frac{(1-x)^{\frac{3}{2}}\sqrt{1+x}}{2} + \frac{3\sqrt{1-x}\,\sqrt{1+x}}{2} + \frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	57
risch	$\frac{(x-4)\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1-x)^(3/2)*(1+x)^(1/2)+3/2*(1-x)^(1/2)*(1+x)^(1/2)+3/2*((1+x)*(1-x))^(
1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.49, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x + 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x + 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

Fricas [A]
time = 1.26, size = 40, normalized size = 0.85

$$-\frac{1}{2}\sqrt{x+1}\,(x-4)\sqrt{-x+1}-3\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.41, size = 138, normalized size = 2.94

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{7i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}-\frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1|>2 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{7(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 7*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 5*I*sqrt(x + 1)/sqrt(x - 1), Ab
s(x + 1) > 2), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 -
x)) - 7*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 5*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.70, size = 44, normalized size = 0.94

$$-\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+3\,\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(1/2),x)
[Out] int((1 - x)^(3/2)/(x + 1)^(1/2), x)
```

3.1109 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{1-x}\,\sqrt{1+x}\,+\sin^{-1}(x)$$

```
[Out] arcsin(x)+(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 41, 222}

$$\text{ArcSin}(x)+\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x]*Sqrt[1 + x] + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx &= \sqrt{1-x}\,\sqrt{1+x}+\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= \sqrt{1-x}\,\sqrt{1+x}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x}\,\sqrt{1+x}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.60

$$\sqrt{1-x^2}+2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 - x^2] + 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(16) = 32$.
time = 0.16, size = 41, normalized size = 2.05

method	result	size
default	$\sqrt{1-x}\,\sqrt{1+x}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	41
risch	$-\frac{\sqrt{1+x}\,(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1-x)^(1/2)*(1+x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(
x)
```

Maxima [A]
time = 0.50, size = 12, normalized size = 0.60

$$\sqrt{-x^2+1}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 1.04, size = 36, normalized size = 1.80

$$\sqrt{x+1}\,\sqrt{-x+1} - 2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.75, size = 99, normalized size = 4.95

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } |x+1| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
 - 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1) > 2), (2*asin(sqrt(2)*sqrt(x + 1
)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(x + 1)/sqrt(1 - x), True))
```

Giac [A]
time = 1.66, size = 27, normalized size = 1.35

$$\sqrt{x+1}\,\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 0.12, size = 12, normalized size = 0.60

$$\operatorname{asin}(x) + \sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(1/2),x)
[Out] asin(x) + (1 - x^2)^(1/2)
```

3.1110 $\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

```
[Out] arcsin(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {41, 222}

$$\text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14 vs. 2(2) = 4.
time = 0.00, size = 14, normalized size = 7.00

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
[Out] ArcTan[x/Sqrt[1 - x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(2) = 4$.
time = 0.14, size = 27, normalized size = 13.50

method	result	size
default	$\frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.51, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(2) = 4$.
time = 0.81, size = 22, normalized size = 11.00

$$-2 \arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 39, normalized size = 19.50

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for } |x+1| > 2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1) > 2), (2*asin(sqrt
(2)*sqrt(x + 1)/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 1.65, size = 13, normalized size = 6.50

$$2 \arcsin\left(\frac{1}{2} \sqrt{2} \sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 0.08, size = 22, normalized size = 11.00

$$-4 \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] -4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1))
```

3.1111 $\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{1+x}}{\sqrt{1-x}}$$

```
[Out] (1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Maple [A]
time = 0.13, size = 14, normalized size = 0.82

method	result	size
gosper	$\frac{\sqrt{1+x}}{\sqrt{1-x}}$	14
default	$\frac{\sqrt{1+x}}{\sqrt{1-x}}$	14
risch	$\frac{\sqrt{(1+x)(1-x)}\,\sqrt{1+x}}{\sqrt{1-x}\,\sqrt{-(1+x)(-1+x)}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] (1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.53, size = 16, normalized size = 0.94

$$-\frac{\sqrt{-x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.88, size = 23, normalized size = 1.35

$$\frac{x-\sqrt{x+1}\,\sqrt{-x+1}-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] (x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.44, size = 31, normalized size = 1.82

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)), 1/Abs(x + 1) > 1/2), (-I/sqrt(1 - 2/(x +
 1)), True))
```

Giac [A]
time = 1.00, size = 19, normalized size = 1.12

$$-\frac{\sqrt{x+1}\,\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1)/(x - 1)
```

Mupad [B]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x + 1)^(1/2)/(1 - x)^(1/2)
```

3.1112 $\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}$$

```
[Out] 1/3*(1+x)^(1/2)/(1-x)^(3/2)+1/3*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
[Out] Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.56

$$-\frac{(-2+x)\sqrt{1+x}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
[Out] -1/3*((-2 + x)*Sqrt[1 + x])/(1 - x)^(3/2)
```

Maple [A]
time = 0.13, size = 30, normalized size = 0.73

method	result	size
gosper	$-\frac{\sqrt{1+x}\,(-2+x)}{3(1-x)^{\frac{3}{2}}}$	18
default	$\frac{\sqrt{1+x}}{3(1-x)^{\frac{3}{2}}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}$	30
risch	$\frac{\sqrt{(1+x)(1-x)}\,(x^{2}-x-2)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(1+x)^(1/2)/(1-x)^(3/2)+1/3*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.49, size = 38, normalized size = 0.93

$$\frac{\sqrt{-x^2+1}}{3\,(x^2-2\,x+1)}-\frac{\sqrt{-x^2+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 0.91, size = 39, normalized size = 0.95

$$\frac{2\,x^2-\sqrt{x+1}\,(x-2)\sqrt{-x+1}-4\,x+2}{3\,(x^2-2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x^2 - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 4*x + 2)/(x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.16, size = 128, normalized size = 3.12

$$\begin{cases} \frac{x+1}{3\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6\sqrt{-1+\frac{2}{x+1}}} - \frac{3}{3\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6\sqrt{-1+\frac{2}{x+1}}} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i(x+1)}{3\sqrt{1-\frac{2}{x+1}}\,(x+1)-6\sqrt{1-\frac{2}{x+1}}} + \frac{3i}{3\sqrt{1-\frac{2}{x+1}}\,(x+1)-6\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise(((x + 1)/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))
) - 3/(3*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*sqrt(-1 + 2/(x + 1))), 1/Abs(x +
1) > 1/2), (-I*(x + 1)/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1
))) + 3*I/(3*sqrt(1 - 2/(x + 1))*(x + 1) - 6*sqrt(1 - 2/(x + 1))), True))
```

Giac [A]
time = 1.13, size = 22, normalized size = 0.54

$$-\frac{\sqrt{x+1}\,(x-2)\sqrt{-x+1}}{3\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)/(x - 1)^2
```

Mupad [B]
time = 0.31, size = 43, normalized size = 1.05

$$\frac{x\sqrt{1-x}+2\sqrt{1-x}-x^2\sqrt{1-x}}{3\,(x-1)^2\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x + 1
)^(1/2))
```

3.1113 $\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=61

$$\frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}$$

```
[Out] 1/5*(1+x)^(1/2)/(1-x)^(5/2)+2/15*(1+x)^(1/2)/(1-x)^(3/2)+2/15*(1+x)^(1/2)/(
1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*Sqrt[1 + x])/(15*(1 - x)^(3/2)) + (2*Sqr
t[1 + x])/(15*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{15(1-x)^{3/2}} + \frac{2}{15}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx \\
&= \frac{\sqrt{1+x}}{5(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{15(1-x)^{3/2}} + \frac{2\sqrt{1+x}}{15\sqrt{1-x}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.49

$$\frac{\sqrt{1+x}\,(7-6x+2x^2)}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(7 - 6*x + 2*x^2))/(15*(1 - x)^(5/2))
```

Maple [A]
time = 0.14, size = 44, normalized size = 0.72

method	result	size
gosper	$\frac{\sqrt{1+x}\,(2x^2-6x+7)}{15(1-x)^{\frac{5}{2}}}$	25
default	$\frac{\sqrt{1+x}}{5(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{15(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}$	44
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^3-4x^2+x+7)}{15\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(1+x)^(1/2)/(1-x)^(5/2)+2/15*(1+x)^(1/2)/(1-x)^(3/2)+2/15*(1+x)^(1/2)/(
1-x)^(1/2)
```

Maxima [A]
time = 0.49, size = 64, normalized size = 1.05

$$-\frac{\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)}+\frac{2\,\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)}-\frac{2\,\sqrt{-x^2+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 2/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) - 2/15*sqrt(-x^2 + 1)/(x - 1)
```

Fricas [A]
time = 1.74, size = 56, normalized size = 0.92

$$\frac{7\,x^3-21\,x^2-\left(2\,x^2-6\,x+7\right)\sqrt{x+1}\,\sqrt{-x+1}\,+21\,x-7}{15\,\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3 - 21*x^2 - (2*x^2 - 6*x + 7)*sqrt(x + 1)*sqrt(-x + 1) + 21*x -
7)/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.44, size = 303, normalized size = 4.97

$$\begin{cases} \frac{2(x+1)^2}{15\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}\,(x+1)+60\sqrt{-1+\frac{2}{x+1}}} - \frac{10(x+1)}{15\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}\,(x+1)+60\sqrt{-1+\frac{2}{x+1}}} + \frac{15}{15\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2-60\sqrt{-1+\frac{2}{x+1}}\,(x+1)+60\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i(x+1)^2}{15\sqrt{1-\frac{2}{x+1}}\,(x+1)^2-60\sqrt{1-\frac{2}{x+1}}\,(x+1)+60\sqrt{1-\frac{2}{x+1}}} + \frac{10i(x+1)}{15\sqrt{1-\frac{2}{x+1}}\,(x+1)^2-60\sqrt{1-\frac{2}{x+1}}\,(x+1)+60\sqrt{1-\frac{2}{x+1}}} - \frac{15i}{15\sqrt{1-\frac{2}{x+1}}\,(x+1)^2-60\sqrt{1-\frac{2}{x+1}}\,(x+1)+60\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*(x + 1)**2/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 +
2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x + 1))) - 10*(x + 1)/(15*sqrt(-1 + 2/
(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x + 1) + 60*sqrt(-1 + 2/(x +
 1))) + 15/(15*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 60*sqrt(-1 + 2/(x + 1))*(x
 + 1) + 60*sqrt(-1 + 2/(x + 1))), 1/Abs(x + 1) > 1/2), (-2*I*(x + 1)**2/(15
*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(
1 - 2/(x + 1))) + 10*I*(x + 1)/(15*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*sqrt
(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1))) - 15*I/(15*sqrt(1 - 2/(x
+ 1))*(x + 1)**2 - 60*sqrt(1 - 2/(x + 1))*(x + 1) + 60*sqrt(1 - 2/(x + 1)))
, True))
```

Giac [A]
time = 0.92, size = 29, normalized size = 0.48

$$-\frac{\left(2\,\left(x+1\right)\left(x-4\right)+15\right)\sqrt{x+1}\,\sqrt{-x+1}}{15\,\left(x-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -1/15*(2*(x + 1)*(x - 4) + 15)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B]
time = 0.32, size = 55, normalized size = 0.90

$$-\frac{x\sqrt{1-x}+7\sqrt{1-x}-4x^2\sqrt{1-x}+2x^3\sqrt{1-x}}{15(x-1)^3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(1/2)),x)
[Out] -(x*(1 - x)^(1/2) + 7*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(15*(x - 1)^3*(x + 1)^(1/2))
```

3.1114 $\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}$$

```
[Out] 1/7*(1+x)^(1/2)/(1-x)^(7/2)+3/35*(1+x)^(1/2)/(1-x)^(5/2)+2/35*(1+x)^(1/2)/(
1-x)^(3/2)+2/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*Sqrt[1 + x])/(35*(1 - x)^(5/2)) + (2*Sqr
t[1 + x])/(35*(1 - x)^(3/2)) + (2*Sqrt[1 + x])/(35*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{6}{35}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{35(1-x)^{3/2}} + \frac{2}{35}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{7(1-x)^{7/2}} + \frac{3\sqrt{1+x}}{35(1-x)^{5/2}} + \frac{2\sqrt{1+x}}{35(1-x)^{3/2}} + \frac{2\sqrt{1+x}}{35\sqrt{1-x}}$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 0.43

$$\frac{\sqrt{1+x}\,(12-13x+8x^2-2x^3)}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(12 - 13*x + 8*x^2 - 2*x^3))/(35*(1 - x)^(7/2))
```

Maple [A]
time = 0.14, size = 58, normalized size = 0.72

method	result	size
gosper	$-\frac{\sqrt{1+x}\,(2x^3-8x^2+13x-12)}{35(1-x)^{\frac{7}{2}}}$	30
default	$\frac{\sqrt{1+x}}{7(1-x)^{\frac{7}{2}}}+\frac{3\sqrt{1+x}}{35(1-x)^{\frac{5}{2}}}+\frac{2\sqrt{1+x}}{35(1-x)^{\frac{3}{2}}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}$	58
risch	$\frac{\sqrt{(1+x)\,(1-x)}\,(2x^4-6x^3+5x^2+x-12)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)\,(-1+x)}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/7*(1+x)^(1/2)/(1-x)^(7/2)+3/35*(1+x)^(1/2)/(1-x)^(5/2)+2/35*(1+x)^(1/2)/(
1-x)^(3/2)+2/35*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.49, size = 95, normalized size = 1.17

$$\frac{\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)} - \frac{3\,\sqrt{-x^2+1}}{35\,(x^3-3\,x^2+3\,x-1)} + \frac{2\,\sqrt{-x^2+1}}{35\,(x^2-2\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{35\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/35*sqrt(-x^
2 + 1)/(x - 1)
```

Fricas [A]
time = 1.06, size = 71, normalized size = 0.88

$$\frac{12\,x^4-48\,x^3+72\,x^2-(2\,x^3-8\,x^2+13\,x-12)\sqrt{x+1}\,\sqrt{-x+1}\,-48\,x+12}{35\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(12*x^4 - 48*x^3 + 72*x^2 - (2*x^3 - 8*x^2 + 13*x - 12)*sqrt(x + 1)*sq
rt(-x + 1) - 48*x + 12)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.42, size = 542, normalized size = 6.69

$$\begin{cases} \frac{2(x+1)^3}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} - \frac{14(x+1)^2}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} + \frac{35(x+1)}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} - \frac{35}{35\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420\sqrt{-1+\frac{2}{x+1}}(x+1)-280\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{2i(x+1)^3}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} + \frac{14i(x+1)^2}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} - \frac{35i(x+1)}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} + \frac{35i}{35\sqrt{1-\frac{2}{x+1}}(x+1)^3-210\sqrt{1-\frac{2}{x+1}}(x+1)^2+420\sqrt{1-\frac{2}{x+1}}(x+1)-280\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*(x + 1)**3/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1 +
 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 + 2
/(x + 1))) - 14*(x + 1)**2/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-
1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))) + 35*(x + 1)/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-
1 + 2/(x + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1
+ 2/(x + 1))) - 35/(35*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*sqrt(-1 + 2/(x
 + 1))*(x + 1)**2 + 420*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*sqrt(-1 + 2/(x +
 1))), 1/Abs(x + 1) > 1/2), (-2*I*(x + 1)**3/(35*sqrt(1 - 2/(x + 1))*(x + 1
)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1)
 - 280*sqrt(1 - 2/(x + 1))) + 14*I*(x + 1)**2/(35*sqrt(1 - 2/(x + 1))*(x +
1)**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1
) - 280*sqrt(1 - 2/(x + 1))) - 35*I*(x + 1)/(35*sqrt(1 - 2/(x + 1))*(x + 1)
**3 - 210*sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1)
- 280*sqrt(1 - 2/(x + 1))) + 35*I/(35*sqrt(1 - 2/(x + 1))*(x + 1)**3 - 210*
sqrt(1 - 2/(x + 1))*(x + 1)**2 + 420*sqrt(1 - 2/(x + 1))*(x + 1) - 280*sqrt
(1 - 2/(x + 1))), True))
```

Giac [A]
time = 0.91, size = 35, normalized size = 0.43

$$-\frac{((2(x+1)(x-6)+35)(x+1)-35)\sqrt{x+1}\sqrt{-x+1}}{35(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*(x + 1)*(x - 6) + 35)*(x + 1) - 35)*sqrt(x + 1)*sqrt(-x + 1)/(x -
 1)^4
```

Mupad [B]
time = 0.34, size = 67, normalized size = 0.83

$$-\frac{x\sqrt{1-x}-12\sqrt{1-x}+5x^2\sqrt{1-x}-6x^3\sqrt{1-x}+2x^4\sqrt{1-x}}{35(x-1)^4\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(1/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) - 12*(1 - x)^(1/2) + 5*x^2*(1 - x)^(1/2) - 6*x^3*(1 - x)^
(1/2) + 2*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

3.1115 $\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=101

$$\frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}$$

```
[Out] 1/9*(1+x)^(1/2)/(1-x)^(9/2)+4/63*(1+x)^(1/2)/(1-x)^(7/2)+4/105*(1+x)^(1/2)/
(1-x)^(5/2)+8/315*(1+x)^(1/2)/(1-x)^(3/2)+8/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*Sqrt[1 + x])/(63*(1 - x)^(7/2)) + (4*Sqr
t[1 + x])/(105*(1 - x)^(5/2)) + (8*Sqrt[1 + x])/(315*(1 - x)^(3/2)) + (8*Sq
rt[1 + x])/(315*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4}{9}\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4}{21}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8}{105}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$$
$$= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8\sqrt{1+x}}{315(1-x)^{3/2}} + \frac{8}{315}\int \frac{}{(1-x)^{3/}}$$
$$= \frac{\sqrt{1+x}}{9(1-x)^{9/2}} + \frac{4\sqrt{1+x}}{63(1-x)^{7/2}} + \frac{4\sqrt{1+x}}{105(1-x)^{5/2}} + \frac{8\sqrt{1+x}}{315(1-x)^{3/2}} + \frac{8\sqrt{1+x}}{315\sqrt{1-x}}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.40

$$\frac{\sqrt{1+x}\,(83-100x+84x^2-40x^3+8x^4)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(83 - 100*x + 84*x^2 - 40*x^3 + 8*x^4))/(315*(1 - x)^(9/2))
```

Maple [A]
time = 0.16, size = 72, normalized size = 0.71

method	result	size
gosper	$\frac{\sqrt{1+x}\,(8x^4-40x^3+84x^2-100x+83)}{315(1-x)^{\frac{9}{2}}}$	35
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-32x^4+44x^3-16x^2-17x+83)}{315\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{\sqrt{1+x}}{9(1-x)^{\frac{9}{2}}}+\frac{4\sqrt{1+x}}{63(1-x)^{\frac{7}{2}}}+\frac{4\sqrt{1+x}}{105(1-x)^{\frac{5}{2}}}+\frac{8\sqrt{1+x}}{315(1-x)^{\frac{3}{2}}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/9*(1+x)^(1/2)/(1-x)^(9/2)+4/63*(1+x)^(1/2)/(1-x)^(7/2)+4/105*(1+x)^(1/2)/
(1-x)^(5/2)+8/315*(1+x)^(1/2)/(1-x)^(3/2)+8/315*(1+x)^(1/2)/(1-x)^(1/2)
```

Maxima [A]
time = 0.53, size = 131, normalized size = 1.30

$$-\frac{\sqrt{-x^2+1}}{9\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}+\frac{4\,\sqrt{-x^2+1}}{63\,(x^4-4\,x^3+6\,x^2-4\,x+1)}-\frac{4\,\sqrt{-x^2+1}}{105\,(x^3-3\,x^2+3\,x-1)}+\frac{8\,\sqrt{-x^2+1}}{315\,(x^2-2\,x+1)}-\frac{8\,\sqrt{-x^2+1}}{315\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 4/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 8/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 8/315*sqrt(-x^2 + 1)/
(x - 1)
```

Fricas [A]
time = 1.37, size = 86, normalized size = 0.85

$$\frac{83\,x^5-415\,x^4+830\,x^3-830\,x^2-(8\,x^4-40\,x^3+84\,x^2-100\,x+83)\sqrt{x+1}\,\sqrt{-x+1}+415\,x-83}{315\,(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(83*x^5 - 415*x^4 + 830*x^3 - 830*x^2 - (8*x^4 - 40*x^3 + 84*x^2 - 10
0*x + 83)*sqrt(x + 1)*sqrt(-x + 1) + 415*x - 83)/(x^5 - 5*x^4 + 10*x^3 - 10
*x^2 + 5*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 41.47, size = 850, normalized size = 8.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((8*(x + 1)**4/(315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1
 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqr
t(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1))) - 72*(x + 1)**3/(315
*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7
560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) +
5040*sqrt(-1 + 2/(x + 1))) + 252*(x + 1)**2/(315*sqrt(-1 + 2/(x + 1))*(x +
1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x
 + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1)))
- 420*(x + 1)/(315*sqrt(-1 + 2/(x + 1))*(x + 1)**4 - 2520*sqrt(-1 + 2/(x +
1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x + 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(
x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x + 1))) + 315/(315*sqrt(-1 + 2/(x + 1)
```

```
)*(x + 1)**4 - 2520*sqrt(-1 + 2/(x + 1))*(x + 1)**3 + 7560*sqrt(-1 + 2/(x +
 1))*(x + 1)**2 - 10080*sqrt(-1 + 2/(x + 1))*(x + 1) + 5040*sqrt(-1 + 2/(x
+ 1))), 1/Abs(x + 1) > 1/2), (-8*I*(x + 1)**4/(315*sqrt(1 - 2/(x + 1))*(x +
 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x
+ 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) + 7
2*I*(x + 1)**3/(315*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1
))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x +
 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) - 252*I*(x + 1)**2/(315*sqrt(1 - 2
/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 -
2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2
/(x + 1))) + 420*I*(x + 1)/(315*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*sqrt(
1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*sqr
t(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 - 2/(x + 1))) - 315*I/(315*sqrt(1 -
2/(x + 1))*(x + 1)**4 - 2520*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*sqrt(1 -
 2/(x + 1))*(x + 1)**2 - 10080*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*sqrt(1 -
2/(x + 1))), True))
```

Giac [A]
time = 0.81, size = 42, normalized size = 0.42

$$-\frac{(4\,((2\,(x+1)(x-8)+63)(x+1)-105)(x+1)+315)\sqrt{x+1}\,\sqrt{-x+1}}{315\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/315*(4*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1) + 315)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.36, size = 80, normalized size = 0.79

$$\frac{17\,x\,\sqrt{1-x}\,-83\,\sqrt{1-x}\,+16\,x^2\,\sqrt{1-x}\,-44\,x^3\,\sqrt{1-x}\,+32\,x^4\,\sqrt{1-x}\,-8\,x^5\,\sqrt{1-x}}{315\,(x-1)^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(1/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 83*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(315*(x - 1)^5*(x +
 1)^(1/2))
```

3.1116 $\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=85

$$-\frac{2(1-x)^{7/2}}{\sqrt{1+x}}-\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}-\frac{7}{3}(1-x)^{5/2}\sqrt{1+x}-\frac{35}{2}\sin^{-1}(x)$$

```
[Out] -35/2*arcsin(x)-2*(1-x)^(7/2)/(1+x)^(1/2)-35/6*(1-x)^(3/2)*(1+x)^(1/2)-7/3*
(1-x)^(5/2)*(1+x)^(1/2)-35/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{35\text{ArcSin}(x)}{2}-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}\,(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/Sqrt[1 + x] - (35*Sqrt[1 - x]*Sqrt[1 + x])/2 - (35*(1 -
x)^(3/2)*Sqrt[1 + x])/6 - (7*(1 - x)^(5/2)*Sqrt[1 + x])/3 - (35*ArcSin[x])/
2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - 7\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\\ &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\\ &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 56, normalized size = 0.66

$$-\frac{\sqrt{1-x}\,(166+55x-13x^2+2x^3)}{6\sqrt{1+x}} + 35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]
[Out] -1/6*(Sqrt[1 - x]*(166 + 55*x - 13*x^2 + 2*x^3))/Sqrt[1 + x] + 35*ArcTan[Sq
rt[1 - x]/Sqrt[1 + x]]
```

Maple [A]
time = 0.14, size = 84, normalized size = 0.99

method	result	size

risch	$\frac{(2x^4-15x^3+68x^2+111x-166)\sqrt{(1+x)(1-x)}}{6\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*x^4-15*x^3+68*x^2+111*x-166)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/
2)/(1-x)^(1/2)/(1+x)^(1/2)-35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Maxima [A]
time = 0.51, size = 70, normalized size = 0.82

$$\frac{x^4}{3\sqrt{-x^2+1}}-\frac{5\,x^3}{2\sqrt{-x^2+1}}+\frac{34\,x^2}{3\sqrt{-x^2+1}}+\frac{37\,x}{2\sqrt{-x^2+1}}-\frac{83}{3\sqrt{-x^2+1}}-\frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/sqrt(-x^2 + 1) - 5/2*x^3/sqrt(-x^2 + 1) + 34/3*x^2/sqrt(-x^2 + 1) +
 37/2*x/sqrt(-x^2 + 1) - 83/3/sqrt(-x^2 + 1) - 35/2*arcsin(x)
```

Fricas [A]
time = 0.98, size = 65, normalized size = 0.76

$$-\frac{(2\,x^3-13\,x^2+55\,x+166)\sqrt{x+1}\,\sqrt{-x+1}-210\,(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+166\,x+166}{6\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*((2*x^3 - 13*x^2 + 55*x + 166)*sqrt(x + 1)*sqrt(-x + 1) - 210*(x + 1)*
arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 166*x + 166)/(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.48, size = 206, normalized size = 2.42

$$\begin{cases} 35i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}+\frac{23i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{125i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{13i\sqrt{x+1}}{\sqrt{x-1}}+\frac{32i}{\sqrt{x-1}\,\sqrt{x+1}} & \text{for } |x+1|>2 \\ -35\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}-\frac{23(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{125(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{13\sqrt{x+1}}{\sqrt{1-x}}-\frac{32}{\sqrt{1-x}\,\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) + 23*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 125*I*(x + 1)**(3/2)/(6*sqrt(x
 - 1)) + 13*I*sqrt(x + 1)/sqrt(x - 1) + 32*I/(sqrt(x - 1)*sqrt(x + 1)), Abs
(x + 1) > 2), (-35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 -
 x)) - 23*(x + 1)**(5/2)/(6*sqrt(1 - x)) + 125*(x + 1)**(3/2)/(6*sqrt(1 - x
)) - 13*sqrt(x + 1)/sqrt(1 - x) - 32/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [A]
time = 1.31, size = 81, normalized size = 0.95

$$-\frac{1}{6}\left((2x-17)(x+1)+87\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{8\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{8\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-35\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 17)*(x + 1) + 87)*sqrt(x + 1)*sqrt(-x + 1) + 8*(sqrt(2) - sqrt
(-x + 1))/sqrt(x + 1) - 8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 35*arcsin(
1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(3/2), x)
```

3.1117 $\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2(1-x)^{5/2}}{\sqrt{1+x}}-\frac{15}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{2}(1-x)^{3/2}\sqrt{1+x}-\frac{15}{2}\sin^{-1}(x)$$

```
[Out] -15/2*arcsin(x)-2*(1-x)^(5/2)/(1+x)^(1/2)-5/2*(1-x)^(3/2)*(1+x)^(1/2)-15/2*
(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-\frac{15\text{ArcSin}(x)}{2}-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}\,(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/Sqrt[1 + x] - (15*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*(1 - x
)^(3/2)*Sqrt[1 + x])/2 - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - 5\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.75

$$\frac{\sqrt{1-x}\,(-24-7x+x^2)}{2\sqrt{1+x}} + 15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x]*(-24 - 7*x + x^2))/(2*Sqrt[1 + x]) + 15*ArcTan[Sqrt[1 - x]/Sqr
t[1 + x]]
```

Maple [A]
time = 0.16, size = 77, normalized size = 1.18

method	result	size
risch	$-\frac{(x^3-8x^2-17x+24)\sqrt{(1+x)(1-x)}}{2\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} - \frac{15\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^3-8*x^2-17*x+24)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1
/2)/(1+x)^(1/2)-15/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 56, normalized size = 0.86

$$-\frac{x^3}{2\sqrt{-x^2+1}}+\frac{4\,x^2}{\sqrt{-x^2+1}}+\frac{17\,x}{2\sqrt{-x^2+1}}-\frac{12}{\sqrt{-x^2+1}}-\frac{15}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) + 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) - 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

Fricas [A]
time = 0.91, size = 58, normalized size = 0.89

$$\frac{(x^2-7\,x-24)\sqrt{x+1}\,\sqrt{-x+1}+30\,(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-24\,x-24}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x + 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) - 24*x - 24)/(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.41, size = 167, normalized size = 2.57

$$\begin{cases} 15i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{11i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}}+\frac{16i}{\sqrt{x-1}\,\sqrt{x+1}} & \text{for}\ |x+1|>2 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{11(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}}-\frac{16}{\sqrt{1-x}\,\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - 11*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1) + 16
*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) > 2), (-15*asin(sqrt(2)*sqrt(x + 1
)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) + 11*(x + 1)**(3/2)/(2*sqrt(1 - x)) -
 sqrt(x + 1)/sqrt(1 - x) - 16/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [A]
time = 1.30, size = 73, normalized size = 1.12

$$\frac{1}{2}\sqrt{x+1}\,(x-8)\sqrt{-x+1}+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-15\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 8)*sqrt(-x + 1) + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x +
1) - 4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 15*arcsin(1/2*sqrt(2)*sqrt(x
+ 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(3/2), x)
```

3.1118 $\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\sin^{-1}(x)$$

```
[Out] -3*arcsin(x)-2*(1-x)^(3/2)/(1+x)^(1/2)-3*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$-3\text{ArcSin}(x)-\frac{2(1-x)^{3/2}}{\sqrt{x+1}}-3\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*Sqrt[1 - x]*Sqrt[1 + x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}}-3\sqrt{1-x}\,\sqrt{1+x}-3\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 43, normalized size = 1.05

$$\frac{(-5-x)\sqrt{1-x}}{\sqrt{1+x}}-6\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] ((-5 - x)*Sqrt[1 - x])/Sqrt[1 + x] - 6*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(33) = 66$.
time = 0.16, size = 71, normalized size = 1.73

method	result	size
risch	$\frac{(x^2+4x-5)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}-\frac{3\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x^2+4*x-5)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/
2)-3*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 41, normalized size = 1.00

$$\frac{(-x^2+1)^{\frac{3}{2}}}{x^2+2x+1}-\frac{6\sqrt{-x^2+1}}{x+1}-3\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^2 + 2*x + 1) - 6*sqrt(-x^2 + 1)/(x + 1) - 3*arcsin(x)
```

Fricas [A]
time = 1.41, size = 53, normalized size = 1.29

$$-\frac{(x+5)\sqrt{x+1}\sqrt{-x+1}-6(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+5x+5}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x + 5)*sqrt(x + 1)*sqrt(-x + 1) - 6*(x + 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + 5*x + 5)/(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.31, size = 131, normalized size = 3.20

$$\begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{2i\sqrt{x+1}}{\sqrt{x-1}}+\frac{8i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1|>2 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\frac{8}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
- 2*I*sqrt(x + 1)/sqrt(x - 1) + 8*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) >
 2), (-6*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(
x + 1)/sqrt(1 - x) - 8/(sqrt(1 - x)*sqrt(x + 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(33) = 66.
time = 1.60, size = 70, normalized size = 1.71

$$-\sqrt{x+1}\sqrt{-x+1}+\frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{2\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 2*sqrt
(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(3/2), x)
```

3.1119 $\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \sin^{-1}(x)$$

```
[Out] -arcsin(x)-2*(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {49, 41, 222}

$$-\text{ArcSin}(x) - \frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx = -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2\sqrt{1-x}}{\sqrt{1+x}} - \sin^{-1}(x)$$

<u>**Mathematica**</u> [**A**]
time = 0.04, size = 39, normalized size = 1.70

$$-\frac{2\sqrt{1-x}}{\sqrt{1+x}} + 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] + 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

<u>**Maple**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.
time = 0.14, size = 67, normalized size = 2.91

method	result	size
risch	$\frac{2(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}} - \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1+x)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)/(1+x)^(1/2)-
((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

<u>**Maxima**</u> [**A**]
time = 0.48, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x+1} - \arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

[Out] -2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.
time = 1.34, size = 50, normalized size = 2.17

$$\frac{2\left((x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")

[Out] 2*((x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x + 1)

Sympy [C] Result contains complex when optimal does not.
time = 0.73, size = 102, normalized size = 4.43

$$\begin{cases} 2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{2i\sqrt{x+1}}{\sqrt{x-1}}+\frac{4i}{\sqrt{x-1}\sqrt{x+1}} & \text{for } |x+1|>2 \\ -2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\frac{4}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(1/2)/(1+x)**(3/2),x)

[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1) + 4*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1) > 2), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x) - 4/(sqrt(1 - x)*sqrt(x + 1)), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.
time = 1.26, size = 55, normalized size = 2.39

$$\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")

[Out] (sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-x}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(3/2),x)
[Out] int((1 - x)^(1/2)/(x + 1)^(3/2), x)
```

3.1120 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

```
[Out] -(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.83

method	result	size
gosper	$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	15
default	$-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	15
risch	$\frac{(-1+x)\sqrt{(1+x)(1-x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}\,\sqrt{1+x}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.50, size = 16, normalized size = 0.89

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A]
time = 1.29, size = 23, normalized size = 1.28

$$-\frac{x+\sqrt{x+1}\,\sqrt{-x+1}+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
[Out] -(x + sqrt(x + 1)*sqrt(-x + 1) + 1)/(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 31, normalized size = 1.72

$$\begin{cases} -\sqrt{-1+\frac{2}{x+1}} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -i\sqrt{1-\frac{2}{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1)), 1/Abs(x + 1) > 1/2), (-I*sqrt(1 - 2/(x +
1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.
time = 1.63, size = 43, normalized size = 2.39

$$\frac{\sqrt{2} - \sqrt{-x+1}}{2\sqrt{x+1}} - \frac{\sqrt{x+1}}{2\left(\sqrt{2} - \sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)/(sqrt(2) - sqrt(
-x + 1))
```

Mupad [B]
time = 0.36, size = 14, normalized size = 0.78

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(1 - x)^(1/2)/(x + 1)^(1/2)
```

3.1121 $\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=18

$$\frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {39}

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$$

Mathematica [A]
time = 0.03, size = 13, normalized size = 0.72

$$\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/Sqrt[1 - x^2]
```

Maple [A]
time = 0.15, size = 29, normalized size = 1.61

method	result	size
gosper	$\frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$	15
default	$\frac{1}{\sqrt{1-x}\,\sqrt{1+x}}-\frac{\sqrt{1-x}}{\sqrt{1+x}}$	29
risch	$\frac{\sqrt{(1+x)\,(1-x)}\,x}{\sqrt{1-x}\,\sqrt{1+x}\,\sqrt{-(1+x)\,(-1+x)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1-x)^(1/2)/(1+x)^(1/2)-(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.61

$$\frac{x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 1)
```

Fricas [A]
time = 1.07, size = 22, normalized size = 1.22

$$-\frac{\sqrt{x+1}\,x\sqrt{-x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1)/(x^2 - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.86, size = 63, normalized size = 3.50

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{x-1}+\frac{\sqrt{-1+\frac{2}{x+1}}}{x-1} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}}+\frac{i}{\sqrt{1-\frac{2}{x+1}}\,(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))*(x + 1)/(x - 1) + sqrt(-1 + 2/(x + 1))/(x
- 1), 1/Abs(x + 1) > 1/2), (-I/sqrt(1 - 2/(x + 1)) + I/(sqrt(1 - 2/(x + 1))
*(x + 1)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(14) = 28$.
time = 1.80, size = 62, normalized size = 3.44

$$\frac{\sqrt{2} - \sqrt{-x+1}}{4\sqrt{x+1}} - \frac{\sqrt{x+1}\sqrt{-x+1}}{2(x-1)} - \frac{\sqrt{x+1}}{4\left(\sqrt{2} - \sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)*sqrt(-x + 1)/(x
- 1) - 1/4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

Mupad [**B**]
time = 0.31, size = 14, normalized size = 0.78

$$\frac{x}{\sqrt{1-x}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] x/((1 - x)^(1/2)*(x + 1)^(1/2))
```

3.1122 $\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/3/(1-x)^(3/2)/(1+x)^(1/2)+2/3*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 39}

$$\frac{2x}{3\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.71

$$\frac{-1-2x+2x^2}{3(-1+x)\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-1 - 2*x + 2*x^2)/(3*(-1 + x)*Sqrt[1 - x^2])
```

Maple [A]
time = 0.14, size = 44, normalized size = 1.05

method	result	size
gosper	$-\frac{2x^2-2x-1}{3\sqrt{1+x}\,(1-x)^{\frac{3}{2}}}$	25
default	$\frac{1}{3(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{2}{3\sqrt{1-x}\,\sqrt{1+x}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	44
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^2-2x-1)}{3\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)\sqrt{-(1+x)(-1+x)}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(1-x)^(3/2)/(1+x)^(1/2)+2/3/(1-x)^(1/2)/(1+x)^(1/2)-2/3*(1-x)^(1/2)/(1+
x)^(1/2)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.95

$$\frac{2\,x}{3\,\sqrt{-x^2+1}}-\frac{1}{3\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.82, size = 54, normalized size = 1.29

$$\frac{x^3-x^2-(2\,x^2-2\,x-1)\sqrt{x+1}\,\sqrt{-x+1}-x+1}{3\,(x^3-x^2-x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - x^2 - (2*x^2 - 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x + 1)/(x^3 -
 x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.02, size = 160, normalized size = 3.81

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-12x+3(x+1)^2}+\frac{6\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^2}-\frac{3\sqrt{-1+\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-12x+3(x+1)^2}+\frac{6i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^2}-\frac{3i\sqrt{1-\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*sq
rt(-1 + 2/(x + 1))*(x + 1)/(-12*x + 3*(x + 1)**2) - 3*sqrt(-1 + 2/(x + 1))/
(-12*x + 3*(x + 1)**2), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x +
 1)**2/(-12*x + 3*(x + 1)**2) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-12*x + 3*
(x + 1)**2) - 3*I*sqrt(1 - 2/(x + 1))/(-12*x + 3*(x + 1)**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.
time = 1.27, size = 67, normalized size = 1.60

$$\frac{\sqrt{2}-\sqrt{-x+1}}{8\sqrt{x+1}}-\frac{(5x-7)\sqrt{x+1}\,\sqrt{-x+1}}{12\,(x-1)^2}-\frac{\sqrt{x+1}}{8\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/12*(5*x - 7)*sqrt(x + 1)*sqrt(
-x + 1)/(x - 1)^2 - 1/8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

Mupad [B]
time = 0.32, size = 42, normalized size = 1.00

$$\frac{2\,x\,\sqrt{1-x}+\sqrt{1-x}-2\,x^2\,\sqrt{1-x}}{3\,(x-1)^2\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(3/2)),x)
```

```
[Out] (2*x*(1 - x)^(1/2) + (1 - x)^(1/2) - 2*x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x +
 1)^(1/2))
```

3.1123 $\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}} dx$

Optimal. Leaf size=62

$$\frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{5\sqrt{1-x}\sqrt{1+x}}$$

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(1/2)+1/5/(1-x)^(3/2)/(1+x)^(1/2)+2/5*x/(1-x)^(1/2)/(
1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 39}

$$\frac{2x}{5\sqrt{1-x}\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + 1/(5*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(
5*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx &= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{3}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx \\
&= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{1}{5(1-x)^{3/2}\sqrt{1+x}} + \frac{2}{5}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx \\
&= \frac{1}{5(1-x)^{5/2}\sqrt{1+x}} + \frac{1}{5(1-x)^{3/2}\sqrt{1+x}} + \frac{2x}{5\sqrt{1-x}\,\sqrt{1+x}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 33, normalized size = 0.53

$$\frac{2+x-4x^2+2x^3}{5(-1+x)^2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] (2 + x - 4*x^2 + 2*x^3)/(5*(-1 + x)^2*Sqrt[1 - x^2])
```

Maple [A]
time = 0.16, size = 58, normalized size = 0.94

method	result	size
gosper	$\frac{2x^3-4x^2+x+2}{5\sqrt{1+x}\,(1-x)^{\frac{5}{2}}}$	28
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^3-4x^2+x+2)}{5\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	54
default	$\frac{1}{5(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{1}{5(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{2}{5\sqrt{1-x}\,\sqrt{1+x}}-\frac{2\sqrt{1-x}}{5\sqrt{1+x}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(1/2)+1/5/(1-x)^(3/2)/(1+x)^(1/2)+2/5/(1-x)^(1/2)/(1+
x)^(1/2)-2/5*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 1.27

$$\frac{2\,x}{5\,\sqrt{-x^2+1}}+\frac{1}{5\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{1}{5\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/sqrt(-x^2 + 1) + 1/5/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(
-x^2 + 1)) - 1/5/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 0.94, size = 59, normalized size = 0.95

$$\frac{2\,x^4-4\,x^3-(2\,x^3-4\,x^2+x+2)\sqrt{x+1}\,\sqrt{-x+1}\,+4\,x-2}{5\,(x^4-2\,x^3+2\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4 - 4*x^3 - (2*x^3 - 4*x^2 + x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x
 - 2)/(x^4 - 2*x^3 + 2*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.20, size = 284, normalized size = 4.58

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{10\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20}-\frac{15\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{5\sqrt{-1+\frac{2}{x+1}}}{60x+5(x+1)^3-30(x+1)^2+20} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{10i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{60x+5(x+1)^3-30(x+1)^2+20}-\frac{15i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{60x+5(x+1)^3-30(x+1)^2+20}+\frac{5i\sqrt{1-\frac{2}{x+1}}}{60x+5(x+1)^3-30(x+1)^2+20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(60*x + 5*(x + 1)**3 - 30*(x
+ 1)**2 + 20) + 10*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(60*x + 5*(x + 1)**3 - 3
0*(x + 1)**2 + 20) - 15*sqrt(-1 + 2/(x + 1))*(x + 1)/(60*x + 5*(x + 1)**3 -
 30*(x + 1)**2 + 20) + 5*sqrt(-1 + 2/(x + 1))/(60*x + 5*(x + 1)**3 - 30*(x
+ 1)**2 + 20), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(6
0*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 10*I*sqrt(1 - 2/(x + 1))*(x + 1)
**2/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) - 15*I*sqrt(1 - 2/(x + 1))*(
x + 1)/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20) + 5*I*sqrt(1 - 2/(x + 1))
/(60*x + 5*(x + 1)**3 - 30*(x + 1)**2 + 20), True))
```

Giac [A]
time = 1.05, size = 73, normalized size = 1.18

$$\frac{\sqrt{2}-\sqrt{-x+1}}{16\sqrt{x+1}}-\frac{\sqrt{x+1}}{16\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((11\,x-39)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{40\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/16*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/40*((11*x - 39)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x -
1)^3
```

Mupad [B]
time = 0.34, size = 55, normalized size = 0.89

$$-\frac{x\sqrt{1-x}+2\sqrt{1-x}-4x^2\sqrt{1-x}+2x^3\sqrt{1-x}}{5(x-1)^3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(5*(x - 1)^3*(x + 1)^(1/2))
```

3.1124 $\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}} \, dx$

Optimal. Leaf size=82

$$\frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8x}{35\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(1/2)+4/35/(1-x)^(5/2)/(1+x)^(1/2)+4/35/(1-x)^(3/2)/(
1+x)^(1/2)+8/35*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{47, 39\}$

$$\frac{8x}{35\sqrt{1-x}\,\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(35*(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(35*
(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(35*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx = \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{12}{35}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{3/2}\sqrt{1+x}} + \frac{8}{35}\int$$
$$= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{35(1-x)^{3/2}\sqrt{1+x}} + \frac{}{35\sqrt{1}}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.49

$$\frac{-13+4x+20x^2-24x^3+8x^4}{35(-1+x)^3\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
[Out] (-13 + 4*x + 20*x^2 - 24*x^3 + 8*x^4)/(35*(-1 + x)^3*Sqrt[1 - x^2])
```

Maple [A]
time = 0.14, size = 72, normalized size = 0.88

method	result	size
gosper	$-\frac{8x^4-24x^3+20x^2+4x-13}{35\sqrt{1+x}\,(1-x)^{\frac{7}{2}}}$	35
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^4-24x^3+20x^2+4x-13)}{35\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	61
default	$\frac{1}{7(1-x)^{\frac{7}{2}}\sqrt{1+x}}+\frac{4}{35(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{4}{35(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{8}{35\sqrt{1-x}\,\sqrt{1+x}}-\frac{8\sqrt{1-x}}{35\sqrt{1+x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/7/(1-x)^(7/2)/(1+x)^(1/2)+4/35/(1-x)^(5/2)/(1+x)^(1/2)+4/35/(1-x)^(3/2)/(
1+x)^(1/2)+8/35/(1-x)^(1/2)/(1+x)^(1/2)-8/35*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(58) = 116$.
time = 0.28, size = 134, normalized size = 1.63

$$\frac{8x}{35\sqrt{-x^2+1}} - \frac{1}{7\left(\sqrt{-x^2+1}\,x^3 - 3\sqrt{-x^2+1}\,x^2 + 3\sqrt{-x^2+1}\,x - \sqrt{-x^2+1}\right)} + \frac{4}{35\left(\sqrt{-x^2+1}\,x^2 - 2\sqrt{-x^2+1}\,x + \sqrt{-x^2+1}\right)} - \frac{4}{35\left(\sqrt{-x^2+1}\,x - \sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/35*x/sqrt(-x^2 + 1) - 1/7/(sqrt(-x^2 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*
sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4/35/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2
 + 1)*x + sqrt(-x^2 + 1)) - 4/35/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 1.45, size = 86, normalized size = 1.05

$$\frac{13\,x^{5}-39\,x^{4}+26\,x^{3}+26\,x^{2}-\left(8\,x^{4}-24\,x^{3}+20\,x^{2}+4\,x-13\right)\sqrt{x+1}\,\sqrt{-x+1}-39\,x+13}{35\,\left(x^{5}-3\,x^{4}+2\,x^{3}+2\,x^{2}-3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(13*x^5 - 39*x^4 + 26*x^3 + 26*x^2 - (8*x^4 - 24*x^3 + 20*x^2 + 4*x -
13)*sqrt(x + 1)*sqrt(-x + 1) - 39*x + 13)/(x^5 - 3*x^4 + 2*x^3 + 2*x^2 - 3*
x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 30.97, size = 425, normalized size = 5.18

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{56\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{35\sqrt{-1+\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{56i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{35i\sqrt{1-\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 28
0*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(
-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*sqrt
(-1 + 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840
*(x + 1)**2 - 560) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)/(-1120*x + 35*(x + 1)
**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*sqrt(-1 + 2/(x + 1))/(-11
20*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), 1/Abs(x + 1)
 > 1/2), (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 28
0*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/
(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*I*s
qrt(1 - 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 8
40*(x + 1)**2 - 560) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-1120*x + 35*(x +
 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*I*sqrt(1 - 2/(x + 1))/
(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), True))
```

Giac [A]
time = 1.19, size = 79, normalized size = 0.96

$$\frac{\sqrt{2}-\sqrt{-x+1}}{32\sqrt{x+1}}-\frac{\sqrt{x+1}}{32\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{(((93\,x-523)(x+1)+1400)(x+1)-1120)\sqrt{x+1}\,\sqrt{-x+1}}{560\,(x-1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/32*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/560*(((93*x - 523)*(x + 1) + 1400)*(x + 1) - 1120)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.36, size = 68, normalized size = 0.83

$$-\frac{4\,x\,\sqrt{1-x}-13\,\sqrt{1-x}+20\,x^{2}\,\sqrt{1-x}-24\,x^{3}\,\sqrt{1-x}+8\,x^{4}\,\sqrt{1-x}}{35\,(x-1)^{4}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(4*x*(1 - x)^(1/2) - 13*(1 - x)^(1/2) + 20*x^2*(1 - x)^(1/2) - 24*x^3*(1 -
 x)^(1/2) + 8*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

3.1125 $\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{3/2}\sqrt{1+x}}+\frac{8x}{63\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(1/2)+5/63/(1-x)^(7/2)/(1+x)^(1/2)+4/63/(1-x)^(5/2)/(
1+x)^(1/2)+4/63/(1-x)^(3/2)/(1+x)^(1/2)+8/63*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 39}

$$\frac{8x}{63\sqrt{1-x}\,\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*Sqrt[1 + x]) + 5/(63*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(63*
(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(63*(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(63*S
qrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx = \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{9}\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{20}{63}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{4}{21}\int$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{}{63(1-}$$
$$= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}} + \frac{5}{63(1-x)^{7/2}\sqrt{1+x}} + \frac{4}{63(1-x)^{5/2}\sqrt{1+x}} + \frac{}{63(1-}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.44

$$\frac{20-17x-16x^2+44x^3-32x^4+8x^5}{63(-1+x)^4\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] (20 - 17*x - 16*x^2 + 44*x^3 - 32*x^4 + 8*x^5)/(63*(-1 + x)^4*Sqrt[1 - x^2]
)
```

Maple [A]
time = 0.14, size = 86, normalized size = 0.84

method	result
gosper	$\frac{8x^5-32x^4+44x^3-16x^2-17x+20}{63\sqrt{1+x}\,(1-x)^{\frac{9}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-32x^4+44x^3-16x^2-17x+20)}{63\sqrt{1-x}\,\sqrt{1+x}\,(-1+x)^4\sqrt{-(1+x)(-1+x)}}$
default	$\frac{1}{9(1-x)^{\frac{9}{2}}\sqrt{1+x}}+\frac{5}{63(1-x)^{\frac{7}{2}}\sqrt{1+x}}+\frac{4}{63(1-x)^{\frac{5}{2}}\sqrt{1+x}}+\frac{4}{63(1-x)^{\frac{3}{2}}\sqrt{1+x}}+\frac{8}{63\sqrt{1-x}\,\sqrt{1+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(1/2)+5/63/(1-x)^(7/2)/(1+x)^(1/2)+4/63/(1-x)^(5/2)/(
1+x)^(1/2)+4/63/(1-x)^(3/2)/(1+x)^(1/2)+8/63/(1-x)^(1/2)/(1+x)^(1/2)-8/63*(
1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(72) = 144$.
time = 0.28, size = 201, normalized size = 1.97

$$\frac{8\,x}{63\,\sqrt{-x^2+1}}+\frac{1}{9\,\left(\sqrt{-x^2+1}\,x^4-4\,\sqrt{-x^2+1}\,x^3+6\,\sqrt{-x^2+1}\,x^2-4\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{5}{63\,\left(\sqrt{-x^2+1}\,x^3-3\,\sqrt{-x^2+1}\,x^2+3\,\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}+\frac{4}{63\,\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{4}{63\,\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/63*x/sqrt(-x^2 + 1) + 1/9/(sqrt(-x^2 + 1)*x^4 - 4*sqrt(-x^2 + 1)*x^3 + 6*
sqrt(-x^2 + 1)*x^2 - 4*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 5/63/(sqrt(-x^2
 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4
/63/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 4/63/(sqrt
(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

Fricas [A]
time = 1.02, size = 91, normalized size = 0.89

$$\frac{20\,x^6-80\,x^5+100\,x^4-100\,x^2-\left(8\,x^5-32\,x^4+44\,x^3-16\,x^2-17\,x+20\right)\sqrt{x+1}\,\sqrt{-x+1}+80\,x-20}{63\,\left(x^6-4\,x^5+5\,x^4-5\,x^2+4\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*(20*x^6 - 80*x^5 + 100*x^4 - 100*x^2 - (8*x^5 - 32*x^4 + 44*x^3 - 16*x
^2 - 17*x + 20)*sqrt(x + 1)*sqrt(-x + 1) + 80*x - 20)/(x^6 - 4*x^5 + 5*x^4
- 5*x^2 + 4*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 82.95, size = 593, normalized size = 5.81

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^5}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}+\frac{72\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}-\frac{252\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}+\frac{420\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}-\frac{315\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}+\frac{63\sqrt{-1+\frac{2}{x+1}}}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^5}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}+\frac{72i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}-\frac{252i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}+\frac{420i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}-\frac{315i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024}+\frac{63i\sqrt{1-\frac{2}{x+1}}}{5040x+63(x+1)^5-630(x+1)^4+2520(x+1)^3-5040(x+1)^2+3024} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(5040*x + 63*(x + 1)**5 - 630
*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) + 72*sqrt(-1 + 2/(x
 + 1))*(x + 1)**4/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**
3 - 5040*(x + 1)**2 + 3024) - 252*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(5040*x +
 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024)
 + 420*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(5040*x + 63*(x + 1)**5 - 630*(x + 1
)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) - 315*sqrt(-1 + 2/(x + 1))
*(x + 1)/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*
(x + 1)**2 + 3024) + 63*sqrt(-1 + 2/(x + 1))/(5040*x + 63*(x + 1)**5 - 630*
```

```
(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024), 1/Abs(x + 1) > 1/2)
, (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**5/(5040*x + 63*(x + 1)**5 - 630*(x + 1
)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) + 72*I*sqrt(1 - 2/(x + 1))
*(x + 1)**4/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 50
40*(x + 1)**2 + 3024) - 252*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(5040*x + 63*(
x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) + 42
0*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4
 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024) - 315*I*sqrt(1 - 2/(x + 1))*(x
 + 1)/(5040*x + 63*(x + 1)**5 - 630*(x + 1)**4 + 2520*(x + 1)**3 - 5040*(x
+ 1)**2 + 3024) + 63*I*sqrt(1 - 2/(x + 1))/(5040*x + 63*(x + 1)**5 - 630*(x
 + 1)**4 + 2520*(x + 1)**3 - 5040*(x + 1)**2 + 3024), True))
```

Giac [A]
time = 1.24, size = 85, normalized size = 0.83

$$\frac{\sqrt{2}-\sqrt{-x+1}}{64\sqrt{x+1}}-\frac{\sqrt{x+1}}{64\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((((193\,x-1481)(x+1)+5544)(x+1)-8400)(x+1)+5040)\sqrt{x+1}\,\sqrt{-x+1}}{2016\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/64*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/2016*((((193*x - 1481)*(x + 1) + 5544)*(x + 1) - 8400)*(x +
1) + 5040)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.36, size = 80, normalized size = 0.78

$$\frac{17\,x\,\sqrt{1-x}-20\,\sqrt{1-x}+16\,x^2\,\sqrt{1-x}-44\,x^3\,\sqrt{1-x}+32\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{63\,(x-1)^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(3/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 20*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(63*(x - 1)^5*(x +
1)^(1/2))
```

3.1126 $\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{1+x}}+\frac{105}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{2}(1-x)^{3/2}\sqrt{1+x}+7(1-x)^{5/2}\sqrt{1+x}+\frac{105}{2}\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(9/2)/(1+x)^(3/2)+105/2*arcsin(x)+6*(1-x)^(7/2)/(1+x)^(1/2)+35/2
*(1-x)^(3/2)*(1+x)^(1/2)+7*(1-x)^(5/2)*(1+x)^(1/2)+105/2*(1-x)^(1/2)*(1+x)^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$\frac{105\text{ArcSin}(x)}{2}-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) + (6*(1 - x)^(7/2))/Sqrt[1 + x] + (105
*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/2 + 7*(1 - x)^
(5/2)*Sqrt[1 + x] + (105*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} - 3\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 21\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 7(1-x)^{5/2}\sqrt{1+x} + 35\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\int \frac{\sqrt{1}}{\sqrt{1}} \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5} \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5} \\
&= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 61, normalized size = 0.59

$$\frac{\sqrt{1-x}\,(494+679x+102x^2-17x^3+2x^4)}{6(1+x)^{3/2}} - 105\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(494 + 679*x + 102*x^2 - 17*x^3 + 2*x^4))/(6*(1 + x)^(3/2)) -
105*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

Maple [A]
time = 0.17, size = 89, normalized size = 0.86

method	result	size
risch	$-\frac{(2x^5-19x^4+119x^3+577x^2-185x-494)\sqrt{(1+x)(1-x)}}{6(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{105\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(9/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*x^5-19*x^4+119*x^3+577*x^2-185*x-494)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(
1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+105/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/
(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.51, size = 125, normalized size = 1.21

$$\frac{x^6}{3(-x^2+1)^{\frac{3}{2}}}-\frac{7x^5}{2(-x^2+1)^{\frac{3}{2}}}+\frac{23x^4}{(-x^2+1)^{\frac{3}{2}}}+\frac{35}{2}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{143x}{6\sqrt{-x^2+1}}-\frac{127x^2}{(-x^2+1)^{\frac{3}{2}}}+\frac{22x}{3(-x^2+1)^{\frac{3}{2}}}+\frac{247}{3(-x^2+1)^{\frac{3}{2}}}+\frac{105}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^6/(-x^2 + 1)^(3/2) - 7/2*x^5/(-x^2 + 1)^(3/2) + 23*x^4/(-x^2 + 1)^(3/
2) + 35/2*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 143/6*x/sqrt(-x
^2 + 1) - 127*x^2/(-x^2 + 1)^(3/2) + 22/3*x/(-x^2 + 1)^(3/2) + 247/3/(-x^2
+ 1)^(3/2) + 105/2*arcsin(x)
```

Fricas [A]
time = 0.96, size = 85, normalized size = 0.83

$$\frac{494x^2+(2x^4-17x^3+102x^2+679x+494)\sqrt{x+1}\,\sqrt{-x+1}-630(x^2+2x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+988x+494}{6(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(494*x^2 + (2*x^4 - 17*x^3 + 102*x^2 + 679*x + 494)*sqrt(x + 1)*sqrt(-x
 + 1) - 630*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 988*
x + 494)/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 47.55, size = 248, normalized size = 2.41

$$\begin{cases} -105i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{29i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{215i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{43i\sqrt{x+1}}{3\sqrt{x-1}}-\frac{448i}{3\sqrt{x-1}\,\sqrt{x+1}}+\frac{64i}{3\sqrt{x-1}\,(x+1)^{\frac{3}{2}}} & \text{for } |x+1|>2 \\ 105\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{29(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{215(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{43\sqrt{x+1}}{3\sqrt{1-x}}+\frac{448}{3\sqrt{1-x}\,\sqrt{x+1}}-\frac{64}{3\sqrt{1-x}\,(x+1)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-105*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x
 - 1)) - 29*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 215*I*(x + 1)**(3/2)/(6*sqrt
(x - 1)) + 43*I*sqrt(x + 1)/(3*sqrt(x - 1)) - 448*I/(3*sqrt(x - 1)*sqrt(x +
 1)) + 64*I/(3*sqrt(x - 1)*(x + 1)**(3/2)), Abs(x + 1) > 2), (105*asin(sqrt
(2)*sqrt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 29*(x + 1)**(5/2)/(6*
sqrt(1 - x)) - 215*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 43*sqrt(x + 1)/(3*sqrt(
1 - x)) + 448/(3*sqrt(1 - x)*sqrt(x + 1)) - 64/(3*sqrt(1 - x)*(x + 1)**(3/2
)), True))
```

Giac [A]
time = 2.13, size = 127, normalized size = 1.23

$$\frac{1}{6}\left((2\,x-23)(x+1)+165\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{3\,(x+1)^{\frac{3}{2}}}-\frac{34\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{2\,(x+1)^{\frac{3}{2}}\left(\frac{51\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}+105\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 23)*(x + 1) + 165)*sqrt(x + 1)*sqrt(-x + 1) + 2/3*(sqrt(2) - sq
rt(-x + 1))^3/(x + 1)^(3/2) - 34*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 2/3
*(x + 1)^(3/2)*(51*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(
-x + 1))^3 + 105*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{9/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)/(x + 1)^(5/2), x)
```

3.1127 $\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{1+x}}+\frac{35}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{35}{2}\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(7/2)/(1+x)^(3/2)+35/2*arcsin(x)+14/3*(1-x)^(5/2)/(1+x)^(1/2)+35
/6*(1-x)^(3/2)*(1+x)^(1/2)+35/2*(1-x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {49, 52, 41, 222}

$$\frac{35\text{ArcSin}(x)}{2}-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) + (14*(1 - x)^(5/2))/(3*Sqrt[1 + x]) +
 (35*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/6 + (35*Ar
cSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} - \frac{7}{3}\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{}{\sqrt{1}} \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{}{\sqrt{1}} \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\sin^{-1}(
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.64

$$\frac{\sqrt{1-x}\,(164+229x+30x^2-3x^3)}{6(1+x)^{3/2}} - 35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(164 + 229*x + 30*x^2 - 3*x^3))/(6*(1 + x)^(3/2)) - 35*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]]
```

Maple [A]
time = 0.16, size = 84, normalized size = 0.97

method	result	size

risch	$\frac{(3x^4-33x^3-199x^2+65x+164)\sqrt{(1+x)(1-x)}}{6(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{35\sqrt{(1+x)(1-x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(3*x^4-33*x^3-199*x^2+65*x+164)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x
)*(1-x))^(1/2)/(1-x)^(1/2)+35/2*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)
*arcsin(x)
```

Maxima [A]
time = 0.49, size = 111, normalized size = 1.28

$$-\frac{x^5}{2(-x^2+1)^{\frac{3}{2}}}+\frac{6x^4}{(-x^2+1)^{\frac{3}{2}}}+\frac{35}{6}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}}-\frac{2}{(-x^2+1)^{\frac{3}{2}}}\right)-\frac{61x}{6\sqrt{-x^2+1}}-\frac{44x^2}{(-x^2+1)^{\frac{3}{2}}}+\frac{16x}{3(-x^2+1)^{\frac{3}{2}}}+\frac{82}{3(-x^2+1)^{\frac{3}{2}}}+\frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/(-x^2 + 1)^(3/2) + 6*x^4/(-x^2 + 1)^(3/2) + 35/6*x*(3*x^2/(-x^2 +
1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 61/6*x/sqrt(-x^2 + 1) - 44*x^2/(-x^2 + 1)^
(3/2) + 16/3*x/(-x^2 + 1)^(3/2) + 82/3/(-x^2 + 1)^(3/2) + 35/2*arcsin(x)
```

Fricas [A]
time = 1.11, size = 81, normalized size = 0.93

$$\frac{164x^2-(3x^3-30x^2-229x-164)\sqrt{x+1}\,\sqrt{-x+1}-210(x^2+2x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+328x+164}{6(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(164*x^2 - (3*x^3 - 30*x^2 - 229*x - 164)*sqrt(x + 1)*sqrt(-x + 1) - 21
0*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 328*x + 164)/(
x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 14.61, size = 212, normalized size = 2.44

$$\begin{cases}-35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{15i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{41i\sqrt{x+1}}{3\sqrt{x-1}}-\frac{176i}{3\sqrt{x-1}\,\sqrt{x+1}}+\frac{32i}{3\sqrt{x-1}\,(x+1)^{\frac{3}{2}}} & \text{for } |x+1|>2\\ 35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{15(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{41\sqrt{x+1}}{3\sqrt{1-x}}+\frac{176}{3\sqrt{1-x}\,\sqrt{x+1}}-\frac{32}{3\sqrt{1-x}\,(x+1)^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x
- 1)) + 15*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 41*I*sqrt(x + 1)/(3*sqrt(x -
1)) - 176*I/(3*sqrt(x - 1)*sqrt(x + 1)) + 32*I/(3*sqrt(x - 1)*(x + 1)**(3/2
)), Abs(x + 1) > 2), (35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sq
rt(1 - x)) - 15*(x + 1)**(3/2)/(2*sqrt(1 - x)) - 41*sqrt(x + 1)/(3*sqrt(1 -
 x)) + 176/(3*sqrt(1 - x)*sqrt(x + 1)) - 32/(3*sqrt(1 - x)*(x + 1)**(3/2)),
 True))
```

Giac [A]
time = 1.37, size = 119, normalized size = 1.37

$$-\frac{1}{2}\,\sqrt{x+1}\,(x-12)\sqrt{-x+1}+\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{3\,(x+1)^{\frac{3}{2}}}-\frac{13\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{39\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}+35\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 12)*sqrt(-x + 1) + 1/3*(sqrt(2) - sqrt(-x + 1))^3/(x
+ 1)^(3/2) - 13*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/3*(x + 1)^(3/2)*(3
9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 35*a
rcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(5/2), x)
```

3.1128 $\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(5/2)/(1+x)^(3/2)+5*arcsin(x)+10/3*(1-x)^(3/2)/(1+x)^(1/2)+5*(1-
x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 52, 41, 222}

$$5\text{ArcSin}(x)-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\,\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) + (10*(1 - x)^(3/2))/(3*Sqrt[1 + x]) +
 5*Sqrt[1 - x]*Sqrt[1 + x] + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}-\frac{5}{3}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{1+x}}+5\sqrt{1-x}\,\sqrt{1+x}+5\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 51, normalized size = 0.81

$$\frac{\sqrt{1-x}\,(23+34x+3x^2)}{3(1+x)^{3/2}}+10\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]
[Out] (Sqrt[1 - x]*(23 + 34*x + 3*x^2))/(3*(1 + x)^(3/2)) + 10*ArcTan[Sqrt[1 + x]
/Sqrt[1 - x]]
```

Maple [A]
time = 0.17, size = 79, normalized size = 1.25

method	result	size
risch	$-\frac{(3x^3+31x^2-11x-23)\sqrt{(1+x)(1-x)}}{3(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}+\frac{5\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(5/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(3*x^3+31*x^2-11*x-23)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))
^(1/2)/(1-x)^(1/2)+5*((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(47) = 94$.
time = 0.51, size = 98, normalized size = 1.56

$$\frac{(-x^2+1)^{\frac{5}{2}}}{x^4+4\,x^3+6\,x^2+4\,x+1} - \frac{5\,(-x^2+1)^{\frac{3}{2}}}{3\,(x^3+3\,x^2+3\,x+1)} - \frac{10\,\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)} + \frac{35\,\sqrt{-x^2+1}}{3\,(x+1)} + 5\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x^
3 + 3*x^2 + 3*x + 1) - 10/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 35/3*sqrt(-x^2
 + 1)/(x + 1) + 5*arcsin(x)
```

Fricas [A]
time = 1.02, size = 75, normalized size = 1.19

$$\frac{23\,x^2+(3\,x^2+34\,x+23)\sqrt{x+1}\,\sqrt{-x+1}-30\,(x^2+2\,x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+46\,x+23}{3\,(x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(23*x^2 + (3*x^2 + 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) - 30*(x^2 + 2*x
+ 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 46*x + 23)/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 4.73, size = 162, normalized size = 2.57

$$\begin{cases} \sqrt{-1+\frac{2}{x+1}}\,(x+1)+\frac{28\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{8\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)+5i\log\left(x+1\right)+10\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ i\sqrt{1-\frac{2}{x+1}}\,(x+1)+\frac{28i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{8i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)-10i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))*(x + 1) + 28*sqrt(-1 + 2/(x + 1))/3 - 8*sqr
t(-1 + 2/(x + 1))/(3*(x + 1)) + 5*I*log(1/(x + 1)) + 5*I*log(x + 1) + 10*as
in(sqrt(2)*sqrt(x + 1)/2), 1/Abs(x + 1) > 1/2), (I*sqrt(1 - 2/(x + 1))*(x +
 1) + 28*I*sqrt(1 - 2/(x + 1))/3 - 8*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) + 5*
I*log(1/(x + 1)) - 10*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(47) = 94.

time = 2.20, size = 115, normalized size = 1.83

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{6\,(x+1)^{\frac{3}{2}}}+\sqrt{x+1}\,\sqrt{-x+1}-\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)}{2\,\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{27\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{6\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}+10\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + sqrt(x + 1)*sqrt(-x + 1) - 9
/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/6*(x + 1)^(3/2)*(27*(sqrt(2) -
sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 10*arcsin(1/2*sqr
t(2)*sqrt(x + 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(5/2), x)
```

3.1129 $\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{1+x}}+\sin^{-1}(x)$$

```
[Out] -2/3*(1-x)^(3/2)/(1+x)^(3/2)+arcsin(x)+2*(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {49, 41, 222}

$$\text{ArcSin}(x)-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} - \int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx \\ &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}} + \frac{2\sqrt{1-x}}{\sqrt{1+x}} + \sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.12

$$\frac{4\sqrt{1-x}\,(1+2x)}{3(1+x)^{3/2}} - 2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]
[Out] (4*Sqrt[1 - x]*(1 + 2*x))/(3*(1 + x)^(3/2)) - 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.
time = 0.16, size = 73, normalized size = 1.78

method	result	size
risch	$-\frac{4(2x^2-x-1)\sqrt{(1+x)(1-x)}}{3(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}} + \frac{\sqrt{(1+x)(1-x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(3/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -4/3*(2*x^2-x-1)/(1+x)^(3/2)/(-(1+x)*(-1+x))^(1/2)*((1+x)*(1-x))^(1/2)/(1-x)^(1/2)+((1+x)*(1-x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 0.51, size = 66, normalized size = 1.61

$$-\frac{(-x^2+1)^{\frac{3}{2}}}{3\,(x^3+3\,x^2+3\,x+1)} - \frac{2\,\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)} + \frac{7\,\sqrt{-x^2+1}}{3\,(x+1)} + \arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 + 3*x^2 + 3*x + 1) - 2/3*sqrt(-x^2 + 1)/(x^2 + 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 1.54, size = 71, normalized size = 1.73

$$\frac{2\left(2x^2+2(2x+1)\sqrt{x+1}\sqrt{-x+1}-3(x^2+2x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+4x+2\right)}{3(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + 2*(2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) - 3*(x^2 + 2*x + 1)*arcta
n((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 1.92, size = 128, normalized size = 3.12

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{4\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+i\log\left(\frac{1}{x+1}\right)+i\log(x+1)+2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ \frac{8i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{4i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+i\log\left(\frac{1}{x+1}\right)-2i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))/3 - 4*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
I*log(1/(x + 1)) + I*log(x + 1) + 2*asin(sqrt(2)*sqrt(x + 1)/2), 1/Abs(x +
1) > 1/2), (8*I*sqrt(1 - 2/(x + 1))/3 - 4*I*sqrt(1 - 2/(x + 1))/(3*(x + 1))
 + I*log(1/(x + 1)) - 2*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.
time = 2.00, size = 102, normalized size = 2.49

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{12(x+1)^{\frac{3}{2}}}-\frac{5\left(\sqrt{2}-\sqrt{-x+1}\right)}{4\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{12\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 5/4*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/12*(x + 1)^(3/2)*(15*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
 1)/(sqrt(2) - sqrt(-x + 1))^3 + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(5/2), x)
```

3.1130 $\int \frac{\sqrt{1-x}}{(1+x)^{5/2}} \, dx$

Optimal. Leaf size=20

$$-\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

```
[Out] -1/3*(1-x)^(3/2)/(1+x)^(3/2)
```

Rubi [A]

time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{37\}$

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

```
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}} \, dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

Mathematica [A]

time = 0.05, size = 20, normalized size = 1.00

$$-\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(5/2),x]
```

```
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.
time = 0.16, size = 30, normalized size = 1.50

method	result	size
gosper	$-\frac{(1-x)^{\frac{3}{2}}}{3(1+x)^{\frac{3}{2}}}$	15
default	$-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{1+x}}$	30
risch	$-\frac{\sqrt{(1+x)(1-x)}\,(x^{2}-2x+1)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1-x)^(1/2)/(1+x)^(3/2)+1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.
time = 0.30, size = 38, normalized size = 1.90

$$-\frac{2\sqrt{-x^{2}+1}}{3(x^{2}+2x+1)}+\frac{\sqrt{-x^{2}+1}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 1.09, size = 37, normalized size = 1.85

$$-\frac{x^{2}-\sqrt{x+1}\,(x-1)\sqrt{-x+1}+2x+1}{3(x^{2}+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^2 - sqrt(x + 1)*(x - 1)*sqrt(-x + 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.86, size = 66, normalized size = 3.30

$$\begin{cases} \frac{\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{2\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ \frac{i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{2i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))/3 - 2*sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 1/A
bs(x + 1) > 1/2), (I*sqrt(1 - 2/(x + 1))/3 - 2*I*sqrt(1 - 2/(x + 1))/(3*(x
+ 1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(14) = 28$.
time = 1.09, size = 89, normalized size = 4.45

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{24\,(x+1)^{\frac{3}{2}}} - \frac{\sqrt{2}-\sqrt{-x+1}}{8\,\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{24\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 1/8*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/24*(x + 1)^(3/2)*(3*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
1)/(sqrt(2) - sqrt(-x + 1))^3
```

Mupad [B]
time = 0.26, size = 32, normalized size = 1.60

$$\frac{x\,\sqrt{1-x}-\sqrt{1-x}}{(3\,x+3)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(5/2),x)
```

```
[Out] (x*(1 - x)^(1/2) - (1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

3.1131 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}$$

```
[Out] -1/3*(1-x)^(1/2)/(1+x)^(3/2)-1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {47, 37}

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -1/3*Sqrt[1 - x]/(1 + x)^(3/2) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx\\ &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.56

$$-\frac{\sqrt{1-x}\,(2+x)}{3(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
[Out] -1/3*(Sqrt[1 - x]*(2 + x))/(1 + x)^(3/2)
```

Maple [A]
time = 0.16, size = 30, normalized size = 0.73

method	result	size
gosper	$-\frac{(2+x)\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}$	18
default	$-\frac{\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}$	30
risch	$\frac{\sqrt{(1+x)\,(1-x)}\,(x^{2}+x-2)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)\,(-1+x)}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(1-x)^(1/2)/(1+x)^(3/2)-1/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.53, size = 38, normalized size = 0.93

$$-\frac{\sqrt{-x^2+1}}{3\,(x^2+2\,x+1)}-\frac{\sqrt{-x^2+1}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Fricas [A]
time = 1.59, size = 38, normalized size = 0.93

$$-\frac{2\,x^2+(x+2)\sqrt{x+1}\,\sqrt{-x+1}\,+4\,x+2}{3\,(x^2+2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^2 + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x + 2)/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.33, size = 66, normalized size = 1.61

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))/3 - sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 1/Ab
s(x + 1) > 1/2), (-I*sqrt(1 - 2/(x + 1))/3 - I*sqrt(1 - 2/(x + 1))/(3*(x +
1)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(29) = 58.
time = 1.19, size = 89, normalized size = 2.17

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{48\,(x+1)^{\frac{3}{2}}}+\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)}{16\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{48\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 3/16*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/48*(x + 1)^(3/2)*(9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) +
 1)/(sqrt(2) - sqrt(-x + 1))^3
```

Mupad [B]
time = 0.31, size = 33, normalized size = 0.80

$$-\frac{x\sqrt{1-x}+2\sqrt{1-x}}{(3\,x+3)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

3.1132 $\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$$

```
[Out] 1/(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(Sqrt[1 - x]*(1 + x)^(3/2)) - (2*Sqrt[1 - x])/(3*(1 + x)^(3/2)) - (2*Sqrt
[1 - x])/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}}\,dx = \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} + 2\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx$$
$$= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3(1+x)^{3/2}} + \frac{2}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx$$
$$= \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3(1+x)^{3/2}} - \frac{2\sqrt{1-x}}{3\sqrt{1+x}}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 0.52

$$\frac{-1+2x+2x^2}{3\sqrt{1-x}\,(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] (-1 + 2*x + 2*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2))
```

Maple [A]
time = 0.16, size = 43, normalized size = 0.74

method	result	size
gosper	$\frac{2x^2+2x-1}{3(1+x)^{\frac{3}{2}}\sqrt{1-x}}$	25
default	$\frac{1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}} - \frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}} - \frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	43
risch	$\frac{\sqrt{(1+x)(1-x)}\,(2x^2+2x-1)}{3\sqrt{1-x}\,(1+x)^{\frac{3}{2}}\sqrt{-(1+x)(-1+x)}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(3/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)
^(1/2)
```

Maxima [A]
time = 0.29, size = 38, normalized size = 0.66

$$\frac{2\,x}{3\,\sqrt{-x^2+1}} - \frac{1}{3\left(\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1))
```

Fricas [A]
time = 1.06, size = 49, normalized size = 0.84

$$-\frac{x^3 + x^2 + (2\,x^2 + 2\,x - 1)\sqrt{x+1}\,\sqrt{-x+1} - x - 1}{3\,(x^3 + x^2 - x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + x^2 + (2*x^2 + 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x - 1)/(x^3
+ x^2 - x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.86, size = 167, normalized size = 2.88

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-6x+3(x+1)^2-6} + \frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-6x+3(x+1)^2-6} + \frac{\sqrt{-1+\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-6x+3(x+1)^2-6} + \frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-6x+3(x+1)^2-6} + \frac{i\sqrt{1-\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2
*sqrt(-1 + 2/(x + 1))*(x + 1)/(-6*x + 3*(x + 1)**2 - 6) + sqrt(-1 + 2/(x +
1))/(-6*x + 3*(x + 1)**2 - 6), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1
))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2*I*sqrt(1 - 2/(x + 1))*(x + 1)/(
-6*x + 3*(x + 1)**2 - 6) + I*sqrt(1 - 2/(x + 1))/(-6*x + 3*(x + 1)**2 - 6),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(42) = 84.
time = 1.57, size = 108, normalized size = 1.86

$$\frac{\left(\sqrt{2} - \sqrt{-x+1}\right)^3}{96\,(x+1)^{\frac{3}{2}}} + \frac{7\left(\sqrt{2} - \sqrt{-x+1}\right)}{32\,\sqrt{x+1}} - \frac{\sqrt{x+1}\,\sqrt{-x+1}}{4\,(x-1)} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{21\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} + 1\right)}{96\left(\sqrt{2} - \sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 7/32*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 1/96*(x + 1)^(3/2)*
(21*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3
```

Mupad [B]
time = 0.34, size = 48, normalized size = 0.83

$$-\frac{2\,x\,\sqrt{1-x}-\sqrt{1-x}+2\,x^2\,\sqrt{1-x}}{\left(3\,x^2-3\right)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(2*x*(1 - x)^(1/2) - (1 - x)^(1/2) + 2*x^2*(1 - x)^(1/2))/((3*x^2 - 3)*(x
+ 1)^(1/2))
```

3.1133 $\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/3*x/(1-x)^(3/2)/(1+x)^(3/2)+2/3*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {40, 39}

$$\frac{2x}{3\sqrt{1-x}\,\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] x/(3*(1 - x)^(3/2)*(1 + x)^(3/2)) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 0.56

$$\frac{3x-2x^3}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
[Out] (3*x - 2*x^3)/(3*(1 - x^2)^(3/2))
```

Maple [A]
time = 0.15, size = 57, normalized size = 1.33

method	result	size
gosper	$-\frac{x(2x^2-3)}{3(1+x)^{\frac{3}{2}}(1-x)^{\frac{3}{2}}}$	23
default	$\frac{1}{3(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{1}{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3(1+x)^{\frac{3}{2}}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(5/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/(1-x)^(3/2)/(1+x)^(3/2)+1/(1-x)^(1/2)/(1+x)^(3/2)-2/3*(1-x)^(1/2)/(1+x)
^(3/2)-2/3*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.31, size = 25, normalized size = 0.58

$$\frac{2\,x}{3\,\sqrt{-x^2+1}}+\frac{x}{3\,(-x^2+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
[Out] 2/3*x/sqrt(-x^2 + 1) + 1/3*x/(-x^2 + 1)^(3/2)
```

Fricas [A]
time = 1.18, size = 35, normalized size = 0.81

$$-\frac{(2\,x^3-3\,x)\sqrt{x+1}\,\sqrt{-x+1}}{3\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(2*x^3 - 3*x)*sqrt(x + 1)*sqrt(-x + 1)/(x^4 - 2*x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.75, size = 280, normalized size = 6.51

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12}+\frac{6\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{3\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{\sqrt{-1+\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12}+\frac{6i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{3i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{12x+3(x+1)^3-12(x+1)^2+12}-\frac{i\sqrt{1-\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x
+ 1)**2 + 12) + 6*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 - 12
*(x + 1)**2 + 12) - 3*sqrt(-1 + 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3 - 1
2*(x + 1)**2 + 12) - sqrt(-1 + 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x + 1)
**2 + 12), 1/Abs(x + 1) > 1/2), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(12*x
+ 3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(
12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*I*sqrt(1 - 2/(x + 1))*(x + 1)
/(12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - I*sqrt(1 - 2/(x + 1))/(12*x +
 3*(x + 1)**3 - 12*(x + 1)**2 + 12), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(31) = 62$.
time = 1.69, size = 113, normalized size = 2.63

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{192\,(x+1)^{\frac{3}{2}}}+\frac{11\left(\sqrt{2}-\sqrt{-x+1}\right)}{64\,\sqrt{x+1}}-\frac{(4\,x-5)\sqrt{x+1}\,\sqrt{-x+1}}{12\,(x-1)^{2}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{192\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 11/64*(sqrt(2) - sqrt(-x +
 1))/sqrt(x + 1) - 1/12*(4*x - 5)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 - 1/19
2*(x + 1)^(3/2)*(33*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt
(-x + 1))^3
```

Mupad [B]
time = 0.37, size = 41, normalized size = 0.95

$$\frac{3\,x\,\sqrt{1-x}\,-2\,x^3\,\sqrt{1-x}}{(3\,x+3)\,\left(x-1\right)^2\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(5/2)),x)
```

```
[Out] (3*x*(1 - x)^(1/2) - 2*x^3*(1 - x)^(1/2))/((3*x + 3)*(x - 1)^2*(x + 1)^(1/2
))
```

3.1134 $\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{15\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(3/2)+4/15*x/(1-x)^(3/2)/(1+x)^(3/2)+8/15*x/(1-x)^(1/
2)/(1+x)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {47, 40, 39}

$$\frac{8x}{15\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*x)/(15*(1 - x)^(3/2)*(1 + x)^(3/2))
+ (8*x)/(15*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{15}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{15\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.63

$$\frac{3+12x-12x^2-8x^3+8x^4}{15(1-x)^{5/2}(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] (3 + 12*x - 12*x^2 - 8*x^3 + 8*x^4)/(15*(1 - x)^(5/2)*(1 + x)^(3/2))
```

Maple [A]
time = 0.16, size = 72, normalized size = 1.14

method	result	size
gosper	$\frac{8x^4-8x^3-12x^2+12x+3}{15(1+x)^{\frac{3}{2}}(1-x)^{\frac{5}{2}}}$	35
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^4-8x^3-12x^2+12x+3)}{15\sqrt{1-x}\,(1+x)^{\frac{3}{2}}(-1+x)^2\sqrt{-(1+x)(-1+x)}}$	61
default	$\frac{1}{5(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{15(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{5\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{15(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{15\sqrt{1+x}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(1-x)^(5/2)/(1+x)^(3/2)+4/15/(1-x)^(3/2)/(1+x)^(3/2)+4/5/(1-x)^(1/2)/(1
+x)^(3/2)-8/15*(1-x)^(1/2)/(1+x)^(3/2)-8/15*(1-x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.35, size = 52, normalized size = 0.83

$$\frac{8\,x}{15\,\sqrt{-x^2+1}}+\frac{4\,x}{15\,(-x^2+1)^{\frac{3}{2}}}-\frac{1}{5\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-x^2 + 1) + 4/15*x/(-x^2 + 1)^(3/2) - 1/5/((-x^2 + 1)^(3/2)*x -
 (-x^2 + 1)^(3/2))
```

Fricas [A]
time = 0.99, size = 84, normalized size = 1.33

$$\frac{3\,x^{5}-3\,x^{4}-6\,x^{3}+6\,x^{2}-\left(8\,x^{4}-8\,x^{3}-12\,x^{2}+12\,x+3\right)\sqrt{x+1}\,\sqrt{-x+1}\,+3\,x-3}{15\,\left(x^{5}-x^{4}-2\,x^{3}+2\,x^{2}+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5 - 3*x^4 - 6*x^3 + 6*x^2 - (8*x^4 - 8*x^3 - 12*x^2 + 12*x + 3)*s
qrt(x + 1)*sqrt(-x + 1) + 3*x - 3)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.17, size = 425, normalized size = 6.75

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{40\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}-\frac{60\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{20\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{5\sqrt{-1+\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{40i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}-\frac{60i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{20i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120}+\frac{5i\sqrt{1-\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 40*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-1
20*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*sqrt(-1 +
 2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1
)**2 - 120) + 20*sqrt(-1 + 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*sqrt(-1 + 2/(x + 1))/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), 1/Abs(x + 1) > 1/2), (-8*
I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 +
180*(x + 1)**2 - 120) + 40*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*I*sqrt(1 - 2/(x + 1))
*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120)
 + 20*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3
 + 180*(x + 1)**2 - 120) + 5*I*sqrt(1 - 2/(x + 1))/(-120*x + 15*(x + 1)**4
- 90*(x + 1)**3 + 180*(x + 1)**2 - 120), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.

time = 1.32, size = 119, normalized size = 1.89

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{384\,(x+1)^{\frac{3}{2}}}+\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)}{128\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{45\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{384\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}-\frac{((73\,x-247)(x+1)+360)\sqrt{x+1}\,\sqrt{-x+1}}{240\,(x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 15/128*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/384*(x + 1)^(3/2)*(45*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/240*((73*x - 247)*(x + 1) + 360)*sq
rt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

Mupad [B]
time = 0.38, size = 75, normalized size = 1.19

$$-\frac{12\,x\,\sqrt{1-x}+3\,\sqrt{1-x}-12\,x^2\,\sqrt{1-x}-8\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{\left(15\,x+15\right)\,{\left(x-1\right)}^3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(12*x*(1 - x)^(1/2) + 3*(1 - x)^(1/2) - 12*x^2*(1 - x)^(1/2) - 8*x^3*(1 -
x)^(1/2) + 8*x^4*(1 - x)^(1/2))/((15*x + 15)*(x - 1)^3*(x + 1)^(1/2))
```

3.1135 $\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{21\sqrt{1-x}\,\sqrt{1+x}}$$

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(3/2)+1/7/(1-x)^(5/2)/(1+x)^(3/2)+4/21*x/(1-x)^(3/2)/
(1+x)^(3/2)+8/21*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {47, 40, 39}

$$\frac{8x}{21\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + 1/(7*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*
x)/(21*(1 - x)^(3/2)*(1 + x)^(3/2)) + (8*x)/(21*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{5}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{7}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{21}\int\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{}{21\sqrt{}}\end{aligned}$$

Mathematica [**A**]
time = 0.07, size = 45, normalized size = 0.54

$$\frac{6+9x-24x^2+4x^3+16x^4-8x^5}{21(1-x)^{7/2}(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] (6 + 9*x - 24*x^2 + 4*x^3 + 16*x^4 - 8*x^5)/(21*(1 - x)^(7/2)*(1 + x)^(3/2)
)
```

Maple [**A**]
time = 0.16, size = 86, normalized size = 1.04

method	result	size
gosper	$-\frac{8x^5-16x^4-4x^3+24x^2-9x-6}{21(1+x)^{\frac{3}{2}}(1-x)^{\frac{7}{2}}}$	40
risch	$\frac{\sqrt{(1+x)(1-x)}\,(8x^5-16x^4-4x^3+24x^2-9x-6)}{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}(-1+x)^3\sqrt{-(1+x)(-1+x)}}$	66
default	$\frac{1}{7(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}+\frac{1}{7(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{21(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{4}{7\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{21(1+x)^{\frac{3}{2}}}-\frac{8\sqrt{1-x}}{21\sqrt{1+x}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(9/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(1-x)^(7/2)/(1+x)^(3/2)+1/7/(1-x)^(5/2)/(1+x)^(3/2)+4/21/(1-x)^(3/2)/(1
+x)^(3/2)+4/7/(1-x)^(1/2)/(1+x)^(3/2)-8/21*(1-x)^(1/2)/(1+x)^(3/2)-8/21*(1-
x)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.55, size = 91, normalized size = 1.10

$$\frac{8\,x}{21\,\sqrt{-x^2+1}}+\frac{4\,x}{21\,\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{1}{7\,\left(\left(-x^2+1\right)^{\frac{3}{2}}x^2-2\,\left(-x^2+1\right)^{\frac{3}{2}}x+\left(-x^2+1\right)^{\frac{3}{2}}\right)}-\frac{1}{7\,\left(\left(-x^2+1\right)^{\frac{3}{2}}x-\left(-x^2+1\right)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/21*x/sqrt(-x^2 + 1) + 4/21*x/(-x^2 + 1)^(3/2) + 1/7/((-x^2 + 1)^(3/2)*x^2
 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 1/7/((-x^2 + 1)^(3/2)*x - (-x
^2 + 1)^(3/2))
```

Fricas [A]
time = 1.45, size = 101, normalized size = 1.22

$$\frac{6\,x^6-12\,x^5-6\,x^4+24\,x^3-6\,x^2-\left(8\,x^5-16\,x^4-4\,x^3+24\,x^2-9\,x-6\right)\sqrt{x+1}\,\sqrt{-x+1}-12\,x+6}{21\,\left(x^6-2\,x^5-x^4+4\,x^3-x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(6*x^6 - 12*x^5 - 6*x^4 + 24*x^3 - 6*x^2 - (8*x^5 - 16*x^4 - 4*x^3 + 2
4*x^2 - 9*x - 6)*sqrt(x + 1)*sqrt(-x + 1) - 12*x + 6)/(x^6 - 2*x^5 - x^4 +
4*x^3 - x^2 - 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 51.38, size = 593, normalized size = 7.14

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^5}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}+\frac{56\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}-\frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}+\frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}-\frac{35\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}-\frac{7\sqrt{-1+\frac{2}{x+1}}}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^5}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}+\frac{56i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}-\frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}+\frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}-\frac{35i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336}-\frac{7i\sqrt{1-\frac{2}{x+1}}}{336x+21(x+1)^5-168(x+1)^4+504(x+1)^3-672(x+1)^2+336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(336*x + 21*(x + 1)**5 - 168*
(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 56*sqrt(-1 + 2/(x + 1
))*(x + 1)**4/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 67
2*(x + 1)**2 + 336) - 140*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(336*x + 21*(x +
1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) + 140*sqrt(
-1 + 2/(x + 1))*(x + 1)**2/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x
 + 1)**3 - 672*(x + 1)**2 + 336) - 35*sqrt(-1 + 2/(x + 1))*(x + 1)/(336*x +
 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) -
7*sqrt(-1 + 2/(x + 1))/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1
)**3 - 672*(x + 1)**2 + 336), 1/Abs(x + 1) > 1/2), (-8*I*sqrt(1 - 2/(x + 1)
)*(x + 1)**5/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672
```

```
*(x + 1)**2 + 336) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(336*x + 21*(x + 1
)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) - 140*I*sqrt
(1 - 2/(x + 1))*(x + 1)**3/(336*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x
 + 1)**3 - 672*(x + 1)**2 + 336) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(33
6*x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 33
6) - 35*I*sqrt(1 - 2/(x + 1))*(x + 1)/(336*x + 21*(x + 1)**5 - 168*(x + 1)*
*4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336) - 7*I*sqrt(1 - 2/(x + 1))/(336*
x + 21*(x + 1)**5 - 168*(x + 1)**4 + 504*(x + 1)**3 - 672*(x + 1)**2 + 336)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(59) = 118$.
time = 1.12, size = 125, normalized size = 1.51

$$\frac{\left(\sqrt{2} - \sqrt{-x+1}\right)^{3}}{768\,(x+1)^{\frac{3}{2}}} + \frac{19\left(\sqrt{2} - \sqrt{-x+1}\right)}{256\,\sqrt{x+1}} - \frac{(x+1)^{\frac{3}{2}}\left(\frac{57\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1} + 1\right)}{768\left(\sqrt{2} - \sqrt{-x+1}\right)^{3}} - \frac{(((79\,x-432)(x+1)+1120)(x+1)-840)\sqrt{x+1}\,\sqrt{-x+1}}{336\,(x-1)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/768*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 19/256*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/768*(x + 1)^(3/2)*(57*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/336*(((79*x - 432)*(x + 1) + 1120)*
(x + 1) - 840)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^4
```

Mupad [B]
time = 0.41, size = 86, normalized size = 1.04

$$\frac{9\,x\,\sqrt{1-x} + 6\,\sqrt{1-x} - 24\,x^2\,\sqrt{1-x} + 4\,x^3\,\sqrt{1-x} + 16\,x^4\,\sqrt{1-x} - 8\,x^5\,\sqrt{1-x}}{(21\,x+21)\,(x-1)^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(5/2)),x)
```

```
[Out] (9*x*(1 - x)^(1/2) + 6*(1 - x)^(1/2) - 24*x^2*(1 - x)^(1/2) + 4*x^3*(1 - x)
^(1/2) + 16*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/((21*x + 21)*(x - 1)^4
*(x + 1)^(1/2))
```

3.1136 $\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}} dx$

Optimal. Leaf size=103

$$\frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63(1-x)^{3/2}(1+x)^{3/2}}+\frac{16}{63\sqrt{1-x}}$$

```
[Out] 1/9/(1-x)^(9/2)/(1+x)^(3/2)+2/21/(1-x)^(7/2)/(1+x)^(3/2)+2/21/(1-x)^(5/2)/(
1+x)^(3/2)+8/63*x/(1-x)^(3/2)/(1+x)^(3/2)+16/63*x/(1-x)^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {47, 40, 39}

$$\frac{16x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + 2/(21*(1 - x)^(7/2)*(1 + x)^(3/2)) + 2/
(21*(1 - x)^(5/2)*(1 + x)^(3/2)) + (8*x)/(63*(1 - x)^(3/2)*(1 + x)^(3/2)) +
 (16*x)/(63*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{10}{21}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8}{21}\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{}{63}\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{}{63}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.49

$$\frac{19+6x-66x^2+56x^3+24x^4-48x^5+16x^6}{63(1-x)^{9/2}(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] (19 + 6*x - 66*x^2 + 56*x^3 + 24*x^4 - 48*x^5 + 16*x^6)/(63*(1 - x)^(9/2)*(
1 + x)^(3/2))
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.97

method	result
gosper	$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63(1+x)^{\frac{3}{2}}(1-x)^{\frac{9}{2}}}$
risch	$\frac{\sqrt{(1+x)(1-x)}\,(16x^6-48x^5+24x^4+56x^3-66x^2+6x+19)}{63\sqrt{1-x}\,(1+x)^{\frac{3}{2}}(-1+x)^4\sqrt{-(1+x)(-1+x)}}$
default	$\frac{1}{9(1-x)^{\frac{9}{2}}(1+x)^{\frac{3}{2}}}+\frac{2}{21(1-x)^{\frac{7}{2}}(1+x)^{\frac{3}{2}}}+\frac{2}{21(1-x)^{\frac{5}{2}}(1+x)^{\frac{3}{2}}}+\frac{8}{63(1-x)^{\frac{3}{2}}(1+x)^{\frac{3}{2}}}+\frac{8}{21\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}-\frac{16\sqrt{1-x}}{63(1+x)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(11/2)/(1+x)^(5/2),x,method=_RETURNVERBOSE)
```

[Out] 1/9/(1-x)^(9/2)/(1+x)^(3/2)+2/21/(1-x)^(7/2)/(1+x)^(3/2)+2/21/(1-x)^(5/2)/(1+x)^(3/2)+8/63/(1-x)^(3/2)/(1+x)^(3/2)+8/21/(1-x)^(1/2)/(1+x)^(3/2)-16/63*(1-x)^(1/2)/(1+x)^(3/2)-16/63*(1-x)^(1/2)/(1+x)^(1/2)

Maxima [A]
time = 0.28, size = 146, normalized size = 1.42

$$\frac{16\,x}{63\,\sqrt{-x^2+1}}+\frac{8\,x}{63\,(-x^2+1)^{\frac{3}{2}}}-\frac{1}{9\left((-x^2+1)^{\frac{3}{2}}x^3-3\,(-x^2+1)^{\frac{3}{2}}x^2+3\,(-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}+\frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x^2-2\,(-x^2+1)^{\frac{3}{2}}x+(-x^2+1)^{\frac{3}{2}}\right)}-\frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="maxima")

[Out] 16/63*x/sqrt(-x^2 + 1) + 8/63*x/(-x^2 + 1)^(3/2) - 1/9/((-x^2 + 1)^(3/2)*x^3 - 3*(-x^2 + 1)^(3/2)*x^2 + 3*(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2)) + 2/21/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 2/21/((-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))

Fricas [A]
time = 1.83, size = 114, normalized size = 1.11

$$\frac{19\,x^7-57\,x^6+19\,x^5+95\,x^4-95\,x^3-19\,x^2-(16\,x^6-48\,x^5+24\,x^4+56\,x^3-66\,x^2+6\,x+19)\sqrt{x+1}\,\sqrt{-x+1}+57\,x-19}{63\,(x^7-3\,x^6+x^5+5\,x^4-5\,x^3-x^2+3\,x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="fricas")

[Out] 1/63*(19*x^7 - 57*x^6 + 19*x^5 + 95*x^4 - 95*x^3 - 19*x^2 - (16*x^6 - 48*x^5 + 24*x^4 + 56*x^3 - 66*x^2 + 6*x + 19)*sqrt(x + 1)*sqrt(-x + 1) + 57*x - 19)/(x^7 - 3*x^6 + x^5 + 5*x^4 - 5*x^3 - x^2 + 3*x - 1)

Sympy [C] Result contains complex when optimal does not.
time = 162.01, size = 789, normalized size = 7.66

$$\begin{cases} -\frac{16\sqrt{-1+\frac{2}{x+1}}(x+1)^6}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{144\sqrt{-1+\frac{2}{x+1}}(x+1)^5}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}-\frac{504\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{840\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}-\frac{630\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{126\sqrt{-1+\frac{2}{x+1}}(x+1)}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{21\sqrt{-1+\frac{2}{x+1}}}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016} & \text{for}\ \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{16i\sqrt{1-\frac{2}{x+1}}(x+1)^6}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{144i\sqrt{1-\frac{2}{x+1}}(x+1)^5}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}-\frac{504i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{840i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}-\frac{630i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{126i\sqrt{1-\frac{2}{x+1}}(x+1)}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016}+\frac{21i\sqrt{1-\frac{2}{x+1}}}{-2016x+63(x+1)^6-630(x+1)^5+2520(x+1)^4-5040(x+1)^3+5040(x+1)^2-2016} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)**(11/2)/(1+x)**(5/2),x)

[Out] Piecewise((-16*sqrt(-1 + 2/(x + 1))*(x + 1)**6/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 144*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) - 504*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 840*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) - 630*sqrt(-1 + 2/(x +

```
1))*(x + 1)**2/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4
- 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 126*sqrt(-1 + 2/(x + 1))*(x +
 1)/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x +
 1)**3 + 5040*(x + 1)**2 - 2016) + 21*sqrt(-1 + 2/(x + 1))/(-2016*x + 63*(x
 + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1
)**2 - 2016), 1/Abs(x + 1) > 1/2), (-16*I*sqrt(1 - 2/(x + 1))*(x + 1)**6/(-
2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3
 + 5040*(x + 1)**2 - 2016) + 144*I*sqrt(1 - 2/(x + 1))*(x + 1)**5/(-2016*x
+ 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040
*(x + 1)**2 - 2016) - 504*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-2016*x + 63*(x
 + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1
)**2 - 2016) + 840*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-2016*x + 63*(x + 1)**
6 - 630*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 -
2016) - 630*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(-2016*x + 63*(x + 1)**6 - 630
*(x + 1)**5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) +
 126*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**
5 + 2520*(x + 1)**4 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016) + 21*I*sqrt
(1 - 2/(x + 1))/(-2016*x + 63*(x + 1)**6 - 630*(x + 1)**5 + 2520*(x + 1)**4
 - 5040*(x + 1)**3 + 5040*(x + 1)**2 - 2016), True))
```

Giac [A]
time = 0.99, size = 131, normalized size = 1.27

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{1536\,(x+1)^{\frac{3}{2}}}+\frac{23\left(\sqrt{2}-\sqrt{-x+1}\right)}{512\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{69\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{1536\left(\sqrt{2}-\sqrt{-x+1}\right)^3}-\frac{((((667\,x-5021)(x+1)+18396)(x+1)-26880)(x+1)+15120)\sqrt{x+1}\,\sqrt{-x+1}}{4032\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 23/512*(sqrt(2) - sqrt(-x
 + 1))/sqrt(x + 1) - 1/1536*(x + 1)^(3/2)*(69*(sqrt(2) - sqrt(-x + 1))^2/(x
 + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/4032*((((667*x - 5021)*(x + 1) +
18396)*(x + 1) - 26880)*(x + 1) + 15120)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

Mupad [B]
time = 0.42, size = 99, normalized size = 0.96

$$-\frac{6\,x\,\sqrt{1-x}+19\,\sqrt{1-x}-66\,x^2\,\sqrt{1-x}+56\,x^3\,\sqrt{1-x}+24\,x^4\,\sqrt{1-x}-48\,x^5\,\sqrt{1-x}+16\,x^6\,\sqrt{1-x}}{(63\,x+63)\,(x-1)^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(6*x*(1 - x)^(1/2) + 19*(1 - x)^(1/2) - 66*x^2*(1 - x)^(1/2) + 56*x^3*(1 -
 x)^(1/2) + 24*x^4*(1 - x)^(1/2) - 48*x^5*(1 - x)^(1/2) + 16*x^6*(1 - x)^(1
/2))/((63*x + 63)*(x - 1)^5*(x + 1)^(1/2))
```

3.1137 $\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx$

Optimal. Leaf size=126

$$\frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2}+\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a}}{\sqrt{a}\,\sqrt{c}}\right.$$

```
[Out] 5/24*a*c*x*(a*x+a)^(3/2)*(-c*x+c)^(3/2)+1/6*x*(a*x+a)^(5/2)*(-c*x+c)^(5/2)+
5/8*a^(5/2)*c^(5/2)*arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))+5/
16*a^2*c^2*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {38, 65, 223, 209}

$$\frac{5}{8}a^{5/2}c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{5}{16}a^2c^2x\sqrt{ax+a}\,\sqrt{c-cx}+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (5*a^2*c^2*x*Sqrt[a + a*x]*Sqrt[c - c*x])/16 + (5*a*c*x*(a + a*x)^(3/2)*(c
- c*x)^(3/2))/24 + (x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a^(5/2)*c^(5/
2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx &= \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2}+\frac{1}{6}(5ac)\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx\\ &= \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2}+\frac{1}{8}\left(5a^2c^2\right)\int \sqrt{a+\\ &= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c\\ &= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c\\ &= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c\\ &= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(a+ax)^{5/2}(c\end{aligned}$$

Mathematica [A]
time = 0.25, size = 86, normalized size = 0.68

$$\frac{c^2(a(1+x))^{5/2}\left(x\sqrt{1+x}\,\sqrt{c-cx}\,(33-26x^2+8x^4)-30\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{c}\,\sqrt{1+x}}\right)\right)}{48(1+x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
[Out] (c^2*(a*(1 + x))^(5/2)*(x*Sqrt[1 + x]*Sqrt[c - c*x]*(33 - 26*x^2 + 8*x^4) -
 30*Sqrt[c]*ArcTan[Sqrt[c - c*x]/(Sqrt[c]*Sqrt[1 + x])]))/(48*(1 + x)^(5/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(94) = 188.
time = 0.19, size = 198, normalized size = 1.57

method	result

risch	$-\frac{x(8x^4-26x^2+33)(1+x)(-1+x)a^3c^3}{48\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}+\frac{5\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)a^3c^3\sqrt{-a(1+x)\,c(-1+x)}}{16\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$

$$5a\left(-\frac{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}}}{5c}+\frac{3a\left(-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}}}{4c}+\frac{a\left(\frac{(-cx+c)^{\frac{5}{2}}\sqrt{ax+a}}{3a}+\frac{5c\left(\frac{(-cx+c)^{\frac{3}{2}}\sqrt{ax}}{2a}\right.}{}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/c*(a*x+a)^(5/2)*(-c*x+c)^(7/2)+5/6*a*(-1/5/c*(a*x+a)^(3/2)*(-c*x+c)^(7
/2)+3/5*a*(-1/4/c*(a*x+a)^(1/2)*(-c*x+c)^(7/2)+1/4*a*(1/3/a*(-c*x+c)^(5/2)*
(a*x+a)^(1/2)+5/3*c*(1/2/a*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+3/2*c*(1/a*(-c*x+c)
^(1/2)*(a*x+a)^(1/2)+c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2
)/(a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))))))
```

Maxima [A]
time = 0.50, size = 72, normalized size = 0.57

$$\frac{5\,a^3c^3\arcsin\left(x\right)}{16\,\sqrt{ac}}+\frac{5}{16}\,\sqrt{-acx^2+ac}\,a^2c^2x+\frac{5}{24}\left(-acx^2+ac\right)^{\frac{3}{2}}acx+\frac{1}{6}\left(-acx^2+ac\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*c^3*arcsin(x)/sqrt(a*c) + 5/16*sqrt(-a*c*x^2 + a*c)*a^2*c^2*x + 5/
24*(-a*c*x^2 + a*c)^(3/2)*a*c*x + 1/6*(-a*c*x^2 + a*c)^(5/2)*x
```

Fricas [A]
time = 0.72, size = 201, normalized size = 1.60

$$\left[\frac{5}{32}\,\sqrt{-ac}\,a^2c^2\log\left(2\,acx^2+2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)+\frac{1}{48}\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c}\,,-\frac{5}{16}\,\sqrt{ac}\,a^2c^2\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)+\frac{1}{48}\left(8\,a^2c^2x^5-26\,a^2c^2x^3+33\,a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [5/32*sqrt(-a*c)*a^2*c^2*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c
*x + c)*x - a*c) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2*x^3 + 33*a^2*c^2*x)*sqr
t(a*x + a)*sqrt(-c*x + c), -5/16*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt(a*
x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a*c)) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2
*x^3 + 33*a^2*c^2*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(x+1)\right)^{\frac{5}{2}}\left(-c(x-1)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(5/2)*(-c*x+c)**(5/2),x)
```

```
[Out] Integral((a*(x + 1))**(5/2)*(-c*(x - 1))**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(94) = 188.
time = 1.48, size = 679, normalized size = 5.39

$$\frac{1}{240}\left(\frac{150\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^{2}c}\left(\left(2\left((ax+a)\left(4\,(ax+a)\left(\frac{5\,(ax+a)}{a^{5}}-\frac{31}{a^{4}}\right)+\frac{321}{a^{3}}\right)-\frac{451}{a^{2}}\right)(ax+a)+\frac{745}{a}\right)(ax+a)-405\right)\sqrt{ax+a}\right)c^{2}|a|-\frac{1}{120}\left(\frac{90\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^{2}c}\left(\left(2\,(ax+a)\left(3\,(ax+a)\left(\frac{4\,(ax+a)}{a^{4}}-\frac{21}{a^{3}}\right)+\frac{133}{a^{2}}\right)-\frac{295}{a}\right)(ax+a)+195\right)\sqrt{ax+a}\right)c^{2}|a|-\frac{1}{12}\left(\frac{18\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^{2}c}\left((ax+a)\left(2\,(ax+a)\left(\frac{3\,(ax+a)}{a^{3}}-\frac{13}{a^{2}}\right)+\frac{43}{a}\right)-39\right)\sqrt{ax+a}\right)c^{2}|a|+\frac{1}{3}\left(\frac{6\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\left((ax+a)\left(\frac{2\,(ax+a)}{a^{2}}-\frac{7}{a}\right)+9\right)\right)c^{2}|a|-\left(\frac{2\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\right)c^{2}|a|+\frac{\left(\frac{2\,a^{3}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\,(ax-2\,a)\right)c^{2}|a|}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*(150*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*((a*x + a)*(4*(a
*x + a)*(5*(a*x + a)/a^5 - 31/a^4) + 321/a^3) - 451/a^2)*(a*x + a) + 745/a)
*(a*x + a) - 405)*sqrt(a*x + a))*c^2*abs(a) - 1/120*(90*a^2*c*log(abs(-sqrt
(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-
(a*x + a)*a*c + 2*a^2*c)*((2*(a*x + a)*(3*(a*x + a)*(4*(a*x + a)/a^4 - 21/a
^3) + 133/a^2) - 295/a)*(a*x + a) + 195)*sqrt(a*x + a))*c^2*abs(a) - 1/12*(
18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)
))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x + a)*(3*(
a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c^2*abs(a) + 1/3*(6*a^2
*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqr
t(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2*(a*x +
 a)/a^2 - 7/a) + 9))*c^2*abs(a) - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a
) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a
^2*c)*sqrt(a*x + a))*c^2*abs(a) + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x
 + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c +
 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c^2*abs(a)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,x)^{5/2}\,(c-c\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(5/2)*(c - c*x)^(5/2),x)
```

```
[Out] int((a + a*x)^(5/2)*(c - c*x)^(5/2), x)
```

3.1138 $\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)$$

```
[Out] 1/4*x*(a*x+a)^(3/2)*(-c*x+c)^(3/2)+3/4*a^(3/2)*c^(3/2)*arctan(c^(1/2)*(a*x+
a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))+3/8*a*c*x*(a*x+a)^(1/2)*(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {38, 65, 223, 209}

$$\frac{3}{4}a^{3/2}c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\,\sqrt{c-cx}+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (3*a*c*x*Sqrt[a + a*x]*Sqrt[c - c*x])/8 + (x*(a + a*x)^(3/2)*(c - c*x)^(3/2
))/4 + (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx = \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}(3ac)\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx$$
$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{8}(3a^2c^2)\int \frac{1}{\sqrt{a+ax}\ \cdots}$$
$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}(3ac^2)\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{2\cdots}}\right.$$
$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{4}(3ac^2)\,\text{Subst}\left(\int \frac{1}{1+\cdots}\right.$$
$$= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx} + \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2} + \frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\cdots}{\sqrt{a}\,\cdots}\right.$$

Mathematica [A]
time = 0.17, size = 79, normalized size = 0.82

$$-\frac{c(a(1+x))^{3/2}\left(x\sqrt{1+x}\,\sqrt{c-cx}\,(-5+2x^2)+6\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{c}\,\sqrt{1+x}}\right)\right)}{8(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] -1/8*(c*(a*(1 + x))^(3/2)*(x*Sqrt[1 + x]*Sqrt[c - c*x]*(-5 + 2*x^2) + 6*Sqr
t[c]*ArcTan[Sqrt[c - c*x]/(Sqrt[c]*Sqrt[1 + x])]))/(1 + x)^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.
time = 0.18, size = 150, normalized size = 1.56

method	result
risch	$\frac{x(2x^2-5)(1+x)(-1+x)a^2c^2}{8\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}} + \frac{3\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-acx^2+ac}}\right)a^2c^2\sqrt{-a(1+x)c(-1+x)}}{8\sqrt{ac}\,\sqrt{a(1+x)}\,\sqrt{-c(-1+x)}}$

default	$-\frac{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}}}{4c}+\frac{3a\left(-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{5}{2}}}{3c}+\frac{a\left(\frac{(-cx+c)^{\frac{3}{2}}\sqrt{ax+a}}{2a}+\frac{3c\left(\frac{\sqrt{-cx+c}\,\sqrt{ax+a}}{a}+\frac{c\sqrt{(-c}}{}\right.}{}\right.}{3}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/c*(a*x+a)^(3/2)*(-c*x+c)^(5/2)+3/4*a*(-1/3/c*(a*x+a)^(1/2)*(-c*x+c)^(5
/2)+1/3*a*(1/2/a*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+3/2*c*(1/a*(-c*x+c)^(1/2)*(a*
x+a)^(1/2)+c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1
/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2)))))
```

Maxima [A]
time = 0.52, size = 50, normalized size = 0.52

$$\frac{3\,a^2c^2\arcsin\left(x\right)}{8\,\sqrt{ac}}+\frac{3}{8}\,\sqrt{-acx^2+ac}\,acx+\frac{1}{4}\left(-acx^2+ac\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^2*c^2*arcsin(x)/sqrt(a*c) + 3/8*sqrt(-a*c*x^2 + a*c)*a*c*x + 1/4*(-a*
c*x^2 + a*c)^(3/2)*x
```

Fricas [A]
time = 0.70, size = 155, normalized size = 1.61

$$\left[\frac{3}{16}\sqrt{-ac}\,ac\log\left(2\,acx^2+2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\,\sqrt{-cx+c}\,,-\frac{3}{8}\,\sqrt{ac}\,ac\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)-\frac{1}{8}\left(2\,acx^3-5\,acx\right)\sqrt{ax+a}\,\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/16*sqrt(-a*c)*a*c*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - a*c) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c), -3/8
```

```
*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a
*c)) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(x+1))^{\frac{3}{2}} (-c(x-1))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(3/2)*(-c*x+c)**(3/2),x)
```

```
[Out] Integral((a*(x + 1))**(3/2)*(-c*(x - 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(70) = 140.
time = 1.72, size = 403, normalized size = 4.20

$$-\frac{\left(\frac{18\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^{2}c}\,\left((ax+a)\left(2\,(ax+a)\left(\frac{3\,(ax+a)}{a^{3}}-\frac{13}{a^{2}}\right)+\frac{43}{a}\right)-39\right)\sqrt{ax+a}\right)c|a|}{24\,a}+\frac{\left(\frac{6\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\,\left((ax+a)\left(\frac{2\,(ax+a)}{a^{2}}-\frac{7}{a}\right)+9\right)\right)c|a|}{6\,a}-\frac{\left(\frac{2\,a^{2}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\right)c|a|}{a}+\frac{\left(\frac{2\,a^{3}c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^{2}c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\,(ax-2\,a)\right)c|a|}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2
*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x +
a)*(3*(a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c*abs(a)/a + 1/6
*(6*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2
*(a*x + a)/a^2 - 7/a) + 9))*c*abs(a)/a - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(
a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*
c + 2*a^2*c)*sqrt(a*x + a))*c*abs(a)/a + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*s
qrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a
)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c*abs(a)/a^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+a\,x\right)}^{3/2}\,{\left(c-c\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(3/2)*(c - c*x)^(3/2),x)
```

```
[Out] int((a + a*x)^(3/2)*(c - c*x)^(3/2), x)
```

3.1139 $\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)$$

```
[Out] arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))*a^(1/2)*c^(1/2)+1/2*x*
(a*x+a)^(1/2)*(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 65, 223, 209}

$$\sqrt{a}\,\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{ax+a}}{\sqrt{a}\,\sqrt{c-cx}}\right)+\frac{1}{2}x\sqrt{ax+a}\,\sqrt{c-cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{2}(ac)\int \frac{1}{\sqrt{a+ax}\,\sqrt{c-cx}}\,dx\\ &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+c\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx,x,\sqrt{a+ax}\right)\\ &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+c\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx,x,\frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)\\ &= \frac{1}{2}x\sqrt{a+ax}\,\sqrt{c-cx}+\sqrt{a}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)\end{aligned}$$

Mathematica [A]
time = 0.18, size = 69, normalized size = 1.03

$$\frac{1}{2}\sqrt{c}\,\sqrt{a(1+x)}\left(\frac{x\sqrt{c-cx}}{\sqrt{c}}+\frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{1+x}}{\sqrt{c-cx}}\right)}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (Sqrt[c]*Sqrt[a*(1 + x)]*((x*Sqrt[c - c*x])/Sqrt[c] + (2*ArcTan[(Sqrt[c]*Sq
rt[1 + x])/Sqrt[c - c*x]])/Sqrt[1 + x]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(49) = 98$.
time = 0.14, size = 102, normalized size = 1.52

method	result
risch	$-\frac{x(1+x)(-1+x)ac}{2\sqrt{a\,(1+x)}\,\sqrt{-c\,(-1+x)}}+\frac{\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)ac\sqrt{-a\,(1+x)\,c\,(-1+x)}}{2\sqrt{ac}\,\sqrt{a\,(1+x)}\,\sqrt{-c\,(-1+x)}}$

default	$-\frac{\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}}{2c}+\frac{a\left(\frac{\sqrt{-cx+c}\,\sqrt{ax+a}}{a}+\frac{c\sqrt{(-cx+c)(ax+a)}\,\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{-cx+c}\,\sqrt{ax+a}\,\sqrt{ac}}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c*(a*x+a)^(1/2)*(-c*x+c)^(3/2)+1/2*a*(1/a*(-c*x+c)^(1/2)*(a*x+a)^(1/2)
+c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*arctan
((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2)))
```

Maxima [A]
time = 0.53, size = 28, normalized size = 0.42

$$\frac{ac\arcsin(x)}{2\sqrt{ac}}+\frac{1}{2}\sqrt{-acx^2+ac}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*arcsin(x)/sqrt(a*c) + 1/2*sqrt(-a*c*x^2 + a*c)*x
```

Fricas [A]
time = 0.68, size = 127, normalized size = 1.90

$$\left[\frac{1}{2}\sqrt{ax+a}\,\sqrt{-cx+c}\,x+\frac{1}{4}\sqrt{-ac}\,\log\left(2\,acx^2+2\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right),\frac{1}{2}\sqrt{ax+a}\,\sqrt{-cx+c}\,x-\frac{1}{2}\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*x + a)*sqrt(-c*x + c)*x + 1/4*sqrt(-a*c)*log(2*a*c*x^2 + 2*sqrt
(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x - a*c), 1/2*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - 1/2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*
x^2 - a*c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(x+1)}\,\sqrt{-c(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(1/2)*(-c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(x + 1))*sqrt(-c*(x - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(49) = 98$.

time = 1.29, size = 173, normalized size = 2.58

$$-\frac{\left(\frac{2\,a^{2} c \log\left(\left|-\sqrt{-a c} \sqrt{a x+a}+\sqrt{-(a x+a) a c+2\,a^{2} c}\right|\right)}{\sqrt{-a c}}-\sqrt{-(a x+a) a c+2\,a^{2} c} \sqrt{a x+a}\right)|a|}{a^{2}}+\frac{\left(\frac{2\,a^{3} c \log\left(\left|-\sqrt{-a c} \sqrt{a x+a}+\sqrt{-(a x+a) a c+2\,a^{2} c}\right|\right)}{\sqrt{-a c}}+\sqrt{-(a x+a) a c+2\,a^{2} c} \sqrt{a x+a}\,(a x-2\,a)\right)|a|}{2\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*abs(a)/a^2 +
 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a
^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*
a))*abs(a)/a^3
```

Mupad [B]

time = 0.30, size = 59, normalized size = 0.88

$$\frac{x\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}{2}-\frac{\sqrt{a}\,\sqrt{-c}\,\ln\left(\sqrt{-c}\,\sqrt{a\,\left(x+1\right)}\,\sqrt{-c\,\left(x-1\right)}-\sqrt{a}\,c\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(1/2)*(c - c*x)^(1/2),x)
```

```
[Out] (x*(a + a*x)^(1/2)*(c - c*x)^(1/2))/2 - (a^(1/2)*(-c)^(1/2)*log((-c)^(1/2)*
(a*x + 1))^(1/2)*(-c*(x - 1))^(1/2) - a^(1/2)*c*x))/2
```

3.1140 $\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] 2*arctan(c^(1/2)*(a*x+a)^(1/2)/a^(1/2)/(-c*x+c)^(1/2))/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {65, 223, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[c
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+ax}\,\sqrt{c-cx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right)}{a}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)}{a}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 1.09

$$\frac{2\sqrt{1+x}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{1+x}}{\sqrt{c-cx}}\right)}{\sqrt{c}\,\sqrt{a(1+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
[Out] (2*Sqrt[1 + x]*ArcTan[(Sqrt[c]*Sqrt[1 + x])/Sqrt[c - c*x]])/(Sqrt[c]*Sqrt[a
*(1 + x)])
```

Maple [A]
time = 0.14, size = 57, normalized size = 1.33

method	result	size
default	$\frac{\sqrt{(-cx+c)\,(ax+a)}\;\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{ax+a}\,\sqrt{-cx+c}\,\sqrt{ac}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-c*x+c)*(a*x+a))^(1/2)/(a*x+a)^(1/2)/(-c*x+c)^(1/2)/(a*c)^(1/2)*arctan((a
*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 8, normalized size = 0.19

$$\frac{\arcsin(x)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(a*c)
```

Fricas [A]
time = 1.20, size = 101, normalized size = 2.35

$$\left[-\frac{\sqrt{-ac}\,\log\left(2\,acx^2-2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)}{2\,ac},-\frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(2*a*c*x^2 - 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*
x - a*c)/(a*c), -sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/
(a*c*x^2 - a*c))/(a*c)]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.23, size = 85, normalized size = 1.98

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\,\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(1/2)/(-c*x+c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
x**(-2))/(4*pi**(3/2)*sqrt(a)*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**2)/(4*pi**(
3/2)*sqrt(a)*sqrt(c))
```

Giac [A]
time = 1.18, size = 49, normalized size = 1.14

$$-\frac{2\,a\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/(
sqrt(-a*c)*abs(a))
```

Mupad [B]
time = 0.18, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{c-c\,x}-\sqrt{c}\right)}{\sqrt{a\,c}\,\left(\sqrt{a+a\,x}-\sqrt{a}\right)}\right)}{\sqrt{a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(1/2)*(c - c*x)^(1/2)),x)
```

```
[Out] -(4*atan((a*((c - c*x)^(1/2) - c^(1/2)))/((a*c)^(1/2)*((a + a*x)^(1/2) - a^
(1/2)))))/(a*c)^(1/2)
```

3.1141 $\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{ac\sqrt{a+ax}\,\sqrt{c-cx}}$$

```
[Out] x/a/c/(a*x+a)^(1/2)/(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{39\}$

$$\frac{x}{ac\sqrt{ax+a}\,\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{ac\sqrt{a+ax}\,\sqrt{c-cx}}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 1.00

$$\frac{x(1+x)}{c(a(1+x))^{3/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] (x*(1 + x))/(c*(a*(1 + x))^(3/2)*Sqrt[c - c*x])
```

Maple [A]
time = 0.14, size = 47, normalized size = 1.74

method	result	size
risch	$\frac{x}{ac\sqrt{a\left(1+x\right)}\,\sqrt{-c\left(-1+x\right)}}$	24
gosper	$-\frac{(1+x)(-1+x)x}{(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}$	25
default	$-\frac{1}{ac\sqrt{ax+a}\,\sqrt{-cx+c}}+\frac{\sqrt{ax+a}}{c\,a^{2}\sqrt{-cx+c}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/c/(a*x+a)^(1/2)/(-c*x+c)^(1/2)+1/c/a^2/(-c*x+c)^(1/2)*(a*x+a)^(1/2)
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.78

$$\frac{x}{\sqrt{-acx^2+ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-a*c*x^2 + a*c)*a*c)
```

Fricas [A]
time = 1.44, size = 39, normalized size = 1.44

$$-\frac{\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{a^2c^2x^2-a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*x + a)*sqrt(-c*x + c)*x/(a^2*c^2*x^2 - a^2*c^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.19, size = 82, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(3/2)/(-c*x+c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), x
**(-2))/(2*pi**(3/2)*a**(3/2)*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4,
 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/x**2)/(2*pi**(3
/2)*a**(3/2)*c**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(23) = 46$.
time = 1.01, size = 116, normalized size = 4.30

$$-\frac{2\sqrt{-ac}\,a}{\left(2a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2\right)c|a|}-\frac{\sqrt{-(ax+a)ac+2a^2c}\,\sqrt{ax+a}}{2\left((ax+a)ac-2a^2c\right)c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-a*c)*a/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)*c*abs(a)) - 1/2*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a
)/(((a*x + a)*a*c - 2*a^2*c)*c*abs(a))
```

Mupad [B]
time = 0.39, size = 23, normalized size = 0.85

$$\frac{x}{a\,c\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x)
```

```
[Out] x/(a*c*(a + a*x)^(1/2)*(c - c*x)^(1/2))
```

3.1142 $\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\sqrt{c-cx}}$$

```
[Out] 1/3*x/a/c/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+2/3*x/a^2/c^2/(a*x+a)^(1/2)/(-c*x+c)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*
Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{3ac}\\ &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\sqrt{c-cx}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.69

$$\frac{x(1+x)\left(-3+2x^2\right)}{3c^2(-1+x)(a(1+x))^{5/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
[Out] (x*(1 + x)*(-3 + 2*x^2))/(3*c^2*(-1 + x)*(a*(1 + x))^(5/2)*Sqrt[c - c*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.
time = 0.14, size = 105, normalized size = 1.72

method	result	size
gosper	$\frac{(1+x)(-1+x)x(2x^2-3)}{3(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}$	32
default	$-\frac{1}{3ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{3}{2}}}+\frac{-\frac{1}{ac\sqrt{ax+a}\,(-cx+c)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{ax+a}}{3ac(-cx+c)^{\frac{3}{2}}}+\frac{2\sqrt{ax+a}}{3a\,c^2\sqrt{-cx+c}}}{a}}{a}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/a/c/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+1/a*(-1/a/c/(a*x+a)^(1/2)/(-c*x+c)^(3
/2)+2/a*(1/3/a/c/(-c*x+c)^(3/2)*(a*x+a)^(1/2)+1/3/a/c^2/(-c*x+c)^(1/2)*(a*x
+a)^(1/2)))
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.74

$$\frac{x}{3\left(-acx^2+ac\right)^{\frac{3}{2}}ac}+\frac{2\,x}{3\,\sqrt{-acx^2+ac}\,a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="maxima")
[Out] 1/3*x/((-a*c*x^2 + a*c)^(3/2)*a*c) + 2/3*x/(sqrt(-a*c*x^2 + a*c)*a^2*c^2)
```

Fricas [A]
time = 0.97, size = 57, normalized size = 0.93

$$-\frac{\left(2\,x^3-3\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{3\left(a^3c^3x^4-2\,a^3c^3x^2+a^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^3*c^3*x^4 - 2*a^3*c^3*x^
2 + a^3*c^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 7.26, size = 82, normalized size = 1.34

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{1}{2}, \frac{5}{2}, 3 \\ \frac{5}{4}, \frac{7}{4}, 2, \frac{5}{2}, 3 & 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, 1 & \\ \frac{3}{4}, \frac{5}{4} & -\frac{1}{2}, 0, 2, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(5/2)/(-c*x+c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), x*
*(-2))/(3*pi**(3/2)*a**(5/2)*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4, 5/4,
1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), exp_polar(-2*I*pi)/x**2)/(3*pi**(3/
2)*a**(5/2)*c**(5/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(49) = 98.
time = 1.12, size = 237, normalized size = 3.89

$$-\frac{\sqrt{-(ax+a)ac+2a^2c}\,\sqrt{ax+a}\left(\frac{4(ax+a)|a|}{a^2c}-\frac{9|a|}{ac}\right)}{12\left((ax+a)ac-2a^2c\right)^2}-\frac{16\sqrt{-ac}\,a^4c^2-18\sqrt{-ac}\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2a^2c+3\sqrt{-ac}\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^4}{3\left(2a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2\right)^3c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(4*(a*x + a)*abs(a)/(a^2
*c) - 9*abs(a)/(a*c))/((a*x + a)*a*c - 2*a^2*c)^2 - 1/3*(16*sqrt(-a*c)*a^4*
c^2 - 18*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2
*c))^2*a^2*c + 3*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^4)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)^3*c^2*abs(a))
```

Mupad [B]
time = 0.41, size = 62, normalized size = 1.02

$$-\frac{3\,x\,\sqrt{c-c\,x}-2\,x^3\,\sqrt{c-c\,x}}{\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^2\,\left(3\,a^2\,\left(c-c\,x\right)-6\,a^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x)
```

```
[Out] -(3*x*(c - c*x)^(1/2) - 2*x^3*(c - c*x)^(1/2))/((a + a*x)^(1/2)*(c - c*x)^2
*(3*a^2*(c - c*x) - 6*a^2*c))
```

3.1143 $\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}\,\sqrt{c-cx}}$$

```
[Out] 1/5*x/a/c/(a*x+a)^(5/2)/(-c*x+c)^(5/2)+4/15*x/a^2/c^2/(a*x+a)^(3/2)/(-c*x+c
)^(3/2)+8/15*x/a^3/c^3/(a*x+a)^(1/2)/(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {40, 39}

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*x)/(15*a^2*c^2*(a + a*x)^(3/
2)*(c - c*x)^(3/2)) + (8*x)/(15*a^3*c^3*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{5ac}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8\int \frac{1}{(a+ax)^{3/2}(c-c}}{15a^2c^2}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.54

$$\frac{x(15-20x^2+8x^4)}{15a^3c^3\sqrt{a(1+x)}\,\sqrt{c-cx}\,(-1+x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] (x*(15 - 20*x^2 + 8*x^4))/(15*a^3*c^3*Sqrt[a*(1 + x)]*Sqrt[c - c*x]*(-1 + x
^2)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(73) = 146.
time = 0.16, size = 163, normalized size = 1.79

method	result
gosper	$-\frac{(1+x)(-1+x)x(8x^4-20x^2+15)}{15(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}$
default	$-\frac{1}{5ac(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{5}{2}}}+\frac{-\frac{1}{3ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{5}{2}}}+\frac{-\frac{4}{3ac\sqrt{ax+a}\,(-cx+c)^{\frac{5}{2}}}+\frac{4\left(\frac{3\sqrt{ax+a}}{5ac(-cx+c)^{\frac{5}{2}}}+\frac{3\left(\frac{2\sqrt{ax+a}}{15ac(-cx+c)^{\frac{3}{2}}}+\frac{2\sqrt{a}}{15a\,c^2\sqrt{}}\right)}{c}\right)}{3a}}{a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/c/(a*x+a)^(5/2)/(-c*x+c)^(5/2)+1/a*(-1/3/a/c/(a*x+a)^(3/2)/(-c*x+c)^
(5/2)+4/3/a*(-1/a/c/(a*x+a)^(1/2)/(-c*x+c)^(5/2)+3/a*(1/5/a/c/(-c*x+c)^(5/2
)*(a*x+a)^(1/2)+2/5/c*(1/3/a/c/(-c*x+c)^(3/2)*(a*x+a)^(1/2)+1/3/a/c^2/(-c*x
+c)^(1/2)*(a*x+a)^(1/2)))))
```

Maxima [A]
time = 0.31, size = 67, normalized size = 0.74

$$\frac{x}{5\,(-acx^2+ac)^{\frac{5}{2}}ac}+\frac{4\,x}{15\,(-acx^2+ac)^{\frac{3}{2}}a^2c^2}+\frac{8\,x}{15\,\sqrt{-acx^2+ac}\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-a*c*x^2 + a*c)^(5/2)*a*c) + 4/15*x/((-a*c*x^2 + a*c)^(3/2)*a^2*c^2
) + 8/15*x/(sqrt(-a*c*x^2 + a*c)*a^3*c^3)
```

Fricas [A]
time = 1.21, size = 74, normalized size = 0.81

$$-\frac{(8x^5-20x^3+15x)\sqrt{ax+a}\,\sqrt{-cx+c}}{15\,(a^4c^4x^6-3\,a^4c^4x^4+3\,a^4c^4x^2-a^4c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*x^5 - 20*x^3 + 15*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^4*c^4*x^6 - 3
*a^4*c^4*x^4 + 3*a^4*c^4*x^2 - a^4*c^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 35.64, size = 85, normalized size = 0.93

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\,\frac{1}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(7/2)/(-c*x+c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 x**(-2))/(15*pi**(3/2)*a**(7/2)*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/2, 5/4,
 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), exp_polar(-2*I*pi)/x**2)/(15*
pi**(3/2)*a**(7/2)*c**(7/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(73) = 146.
time = 1.54, size = 333, normalized size = 3.66

$$-\frac{\sqrt{-(ax+a)ac+2a^2c}\,\sqrt{ax+a}\left((ax+a)\left(\frac{64(ax+a)}{c|a|}-\frac{275a}{c|a|}\right)+\frac{300a^2}{c|a|}\right)}{240((ax+a)ac-2a^2c)^3}+\frac{1024a^8c^4-2200\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2a^6c^3+1660\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^4a^4c^2-450\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^6a^2c+45\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^8}{60\left(2a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2\right)^5\sqrt{-ac}\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(64*(a*x + a
)/(c*abs(a)) - 275*a/(c*abs(a))) + 300*a^2/(c*abs(a)))/((a*x + a)*a*c - 2*a
^2*c)^3 + 1/60*(1024*a^8*c^4 - 2200*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
+ a)*a*c + 2*a^2*c))^2*a^6*c^3 + 1660*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*
x + a)*a*c + 2*a^2*c))^4*a^4*c^2 - 450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a
*x + a)*a*c + 2*a^2*c))^6*a^2*c + 45*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^8)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^2)^5*sqrt(-a*c)*c^2*abs(a))
```

Mupad [B]
time = 0.44, size = 50, normalized size = 0.55

$$\frac{x\,\left(8\,x^4-20\,x^2+15\right)}{15\,a^3\,\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^{5/2}\,\left(c+3\,c\,x-x\,\left(c-c\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x)
```

```
[Out] (x*(8*x^4 - 20*x^2 + 15))/(15*a^3*(a + a*x)^(1/2)*(c - c*x)^(5/2)*(c + 3*c*
x - x*(c - c*x)))
```

3.1144 $\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{16x}{35a^4c^4\sqrt{a+ax}\,\sqrt{c-cx}}$$

```
[Out] 1/7*x/a/c/(a*x+a)^(7/2)/(-c*x+c)^(7/2)+6/35*x/a^2/c^2/(a*x+a)^(5/2)/(-c*x+c
)^(5/2)+8/35*x/a^3/c^3/(a*x+a)^(3/2)/(-c*x+c)^(3/2)+16/35*x/a^4/c^4/(a*x+a)
^(1/2)/(-c*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {40, 39}

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*x)/(35*a^2*c^2*(a + a*x)^(5/
2)*(c - c*x)^(5/2)) + (8*x)/(35*a^3*c^3*(a + a*x)^(3/2)*(c - c*x)^(3/2)) +
(16*x)/(35*a^4*c^4*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx = \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx}{7ac}$$
$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{24\int \frac{1}{(a+ax)^{5/2}(c-c}{35a^2c^2}$$
$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)^{3/}}$$
$$= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}} + \frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}} + \frac{8x}{35a^3c^3(a+ax)^{3/}}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.45

$$\frac{x(-35+70x^2-56x^4+16x^6)}{35a^4c^4\sqrt{a(1+x)}\,\sqrt{c-cx}\,(-1+x^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] (x*(-35 + 70*x^2 - 56*x^4 + 16*x^6))/(35*a^4*c^4*Sqrt[a*(1 + x)]*Sqrt[c - c
*x]*(-1 + x^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(97) = 194.
time = 0.14, size = 221, normalized size = 1.83

method	result
gosper	$\frac{(1+x)(-1+x)x(16x^6-56x^4+70x^2-35)}{35(ax+a)^{\frac{9}{2}}(-cx+c)^{\frac{9}{2}}}$

default	$-\frac{1}{7ac(ax+a)^{\frac{7}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{-\frac{1}{5ac(ax+a)^{\frac{5}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{-\frac{2}{5ac(ax+a)^{\frac{3}{2}}(-cx+c)^{\frac{7}{2}}}+\frac{6\left(-\frac{5}{3ac\sqrt{ax+a}\,(-cx+c)^{\frac{7}{2}}}+\frac{5\left(\frac{4\sqrt{ax+a}}{7ac(-cx+c)^{\frac{7}{2}}}\cdots\right.}{\cdots}\right.}{\cdots}}{a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/a/c/(a*x+a)^(7/2)/(-c*x+c)^(7/2)+1/a*(-1/5/a/c/(a*x+a)^(5/2)/(-c*x+c)^
(7/2)+6/5/a*(-1/3/a/c/(a*x+a)^(3/2)/(-c*x+c)^(7/2)+5/3/a*(-1/a/c/(a*x+a)^(1
/2)/(-c*x+c)^(7/2)+4/a*(1/7/a/c/(-c*x+c)^(7/2)*(a*x+a)^(1/2)+3/7/c*(1/5/a/c
/(-c*x+c)^(5/2)*(a*x+a)^(1/2)+2/5/c*(1/3/a/c/(-c*x+c)^(3/2)*(a*x+a)^(1/2)+1
/3/a/c^2/(-c*x+c)^(1/2)*(a*x+a)^(1/2))))))
```

Maxima [A]
time = 0.28, size = 89, normalized size = 0.74

$$\frac{x}{7\left(-acx^2+ac\right)^{\frac{7}{2}}ac}+\frac{6\,x}{35\left(-acx^2+ac\right)^{\frac{5}{2}}a^2c^2}+\frac{8\,x}{35\left(-acx^2+ac\right)^{\frac{3}{2}}a^3c^3}+\frac{16\,x}{35\,\sqrt{-acx^2+ac}\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-a*c*x^2 + a*c)^(7/2)*a*c) + 6/35*x/((-a*c*x^2 + a*c)^(5/2)*a^2*c^2
) + 8/35*x/((-a*c*x^2 + a*c)^(3/2)*a^3*c^3) + 16/35*x/(sqrt(-a*c*x^2 + a*c)
*a^4*c^4)
```

Fricas [A]
time = 1.01, size = 89, normalized size = 0.74

$$-\frac{\left(16\,x^7-56\,x^5+70\,x^3-35\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{35\left(a^5c^5x^8-4\,a^5c^5x^6+6\,a^5c^5x^4-4\,a^5c^5x^2+a^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*x^7 - 56*x^5 + 70*x^3 - 35*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^5*c
^5*x^8 - 4*a^5*c^5*x^6 + 6*a^5*c^5*x^4 - 4*a^5*c^5*x^2 + a^5*c^5)
```

Sympy [C] Result contains complex when optimal does not.
time = 180.07, size = 85, normalized size = 0.70

$$\frac{4iG_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 & \frac{1}{2}, \frac{9}{2}, 5 \\ \frac{9}{4}, \frac{11}{4}, 4, \frac{9}{2}, 5 & 0 \end{matrix}\middle| {\frac{1}{x^{2}}} \right)}{105\pi^{\frac{3}{2}}a^{\frac{9}{2}}c^{\frac{9}{2}}} + \frac{4G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{7}{4}, \frac{9}{4}, 1 & \\ \frac{7}{4}, \frac{9}{4} & -\frac{1}{2}, 0, 4, 0 \end{matrix}\middle| {\frac{e^{-2i\pi}}{x^{2}}} \right)}{105\pi^{\frac{3}{2}}a^{\frac{9}{2}}c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(9/2)/(-c*x+c)**(9/2),x)
```

```
[Out] 4*I*meijerg(((9/4, 11/4, 1), (1/2, 9/2, 5)), ((9/4, 11/4, 4, 9/2, 5), (0,))
, x**(-2))/(105*pi**(3/2)*a**(9/2)*c**(9/2)) + 4*meijerg(((-1/2, 0, 1/2, 7/
4, 9/4, 1), ()), ((7/4, 9/4), (-1/2, 0, 4, 0)), exp_polar(-2*I*pi)/x**2)/(1
05*pi**(3/2)*a**(9/2)*c**(9/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(97) = 194.
time = 1.40, size = 437, normalized size = 3.61

$$-\frac{\sqrt{-(ax+a)ac+2a^2c}\left((ax+a)\left((ax+a)\left(\frac{256(ax+a)|a|}{a^2c}-\frac{1617|a|}{ac}\right)+\frac{3430|a|}{c}\right)-\frac{2450a|a|}{c}\right)\sqrt{ax+a}}{1120\left((ax+a)ac-2a^2c\right)^4}+\frac{16384a^{12}c^6-51744\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2a^{10}c^5+66416\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^4a^8c^4-43120\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^6a^6c^3+14280\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^8a^4c^2-2450\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^{10}a^2c+175\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^{12}}{280\left(2a^2c-\left(\sqrt{-ac}\sqrt{ax+a}-\sqrt{-(ax+a)ac+2a^2c}\right)^2\right)^7\sqrt{-ac}\,ac^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/1120*sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*((a*x + a)*(256*(a*x + a)
*abs(a)/(a^2*c) - 1617*abs(a)/(a*c)) + 3430*abs(a)/c) - 2450*a*abs(a)/c)*sq
rt(a*x + a)/((a*x + a)*a*c - 2*a^2*c)^4 + 1/280*(16384*a^12*c^6 - 51744*(sq
rt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2*a^10*c^5 + 66416
*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^4*a^8*c^4 - 43
120*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^6*a^6*c^3 +
 14280*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^8*a^4*c^
2 - 2450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^10*a^2
*c + 175*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^12)/((
2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2)^7*
sqrt(-a*c)*a*c^3*abs(a))
```

Mupad [B]
time = 0.48, size = 66, normalized size = 0.55

$$-\frac{x\,\left(16\,x^6-56\,x^4+70\,x^2-35\right)}{35\,a^4\,\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^{7/2}\,\left(c-x^2\,\left(c-c\,x\right)+7\,c\,x-4\,x\,\left(c-c\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x)
```

```
[Out] -(x*(70*x^2 - 56*x^4 + 16*x^6 - 35))/(35*a^4*(a + a*x)^(1/2)*(c - c*x)^(7/2
)*(c - x^2*(c - c*x) + 7*c*x - 4*x*(c - c*x)))
```

3.1145 $\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}+\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{c}}\right.}{8b}$$

```
[Out] 5/24*a^2*c*x*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)+1/6*x*(b*x+a)^(5/2)*(-b*c*x+a
*c)^(5/2)+5/8*a^6*c^(5/2)*arctan(c^(1/2)*(b*x+a)^(1/2)/(c*(-b*x+a))^(1/2))/
b+5/16*a^4*c^2*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {38, 65, 223, 209}

$$\frac{5a^6c^{5/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (5*a^4*c^2*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/16 + (5*a^2*c*x*(a + b*x)^(3/
2)*(a*c - b*c*x)^(3/2))/24 + (x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5
*a^6*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(8*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx = \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{1}{6}(5a^2c)\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$$
$$= \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2} + \frac{1}{8}(5a^4c^2)$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+$$
$$= \frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{6}x(a+$$

Mathematica [A]
time = 0.18, size = 103, normalized size = 0.76

$$\frac{(c(a-bx))^{5/2}\left(bx\sqrt{a-bx}\sqrt{a+bx}\,(33a^4-26a^2b^2x^2+8b^4x^4)+30a^6\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{48b(a-bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] ((c*(a - b*x))^(5/2)*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(33*a^4 - 26*a^2*b^2*
x^2 + 8*b^4*x^4) + 30*a^6*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(48*b*(a -
b*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(107) = 214$.
time = 0.16, size = 242, normalized size = 1.79

method	result
risch	$\frac{x\left(8b^{4}x^{4}-26a^{2}b^{2}x^{2}+33a^{4}\right)\sqrt{bx+a}\,\left(-bx+a\right)c^{3}}{48\sqrt{-c\left(bx-a\right)}}+\frac{5a^{6}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)\sqrt{-\left(bx+a\right)c\left(bx-a\right)}\,c^{3}}{16\sqrt{b^{2}c}\,\sqrt{bx+a}\,\sqrt{-c\left(bx-a\right)}}$

$$5ac\left(\frac{(-bcx\cdots}{}\right.$$

$$a\left(\frac{(-bcx+ac)^{\frac{5}{2}}\sqrt{bx+a}}{3b}+\frac{\cdots}{}\right.$$

$$3a\left(-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{7}{2}}}{4bc}+\frac{\cdots}{}\right.$$

$$5a\left(-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}}{5bc}+\frac{\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b/c*(b*x+a)^(5/2)*(-b*c*x+a*c)^(7/2)+5/6*a*(-1/5/b/c*(b*x+a)^(3/2)*(-b
*c*x+a*c)^(7/2)+3/5*a*(-1/4/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(7/2)+1/4*a*(1/3
/b*(-b*c*x+a*c)^(5/2)*(b*x+a)^(1/2)+5/3*a*c*(1/2/b*(-b*c*x+a*c)^(3/2)*(b*x+
a)^(1/2)+3/2*a*c*(1/b*(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+a*c*((b*x+a)*(-b*c*x
+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^
(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2)))))))
```

Maxima [A]
time = 0.50, size = 89, normalized size = 0.66

$$\frac{5\,a^6c^{\frac{5}{2}}\arcsin\left(\frac{bx}{a}\right)}{16\,b}+\frac{5}{16}\,\sqrt{-b^2cx^2+a^2c}\,a^4c^2x+\frac{5}{24}\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^2cx+\frac{1}{6}\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*c^(5/2)*arcsin(b*x/a)/b + 5/16*sqrt(-b^2*c*x^2 + a^2*c)*a^4*c^2*x
+ 5/24*(-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c*x + 1/6*(-b^2*c*x^2 + a^2*c)^(5/2)*
x
```

Fricas [A]
time = 1.59, size = 232, normalized size = 1.72

$$\left[\frac{15\,a^6\sqrt{-c}\,c^2\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{96\,b},-\frac{15\,a^6c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^6*sqrt(-c)*c^2*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(8*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4
*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b, -1/48*(15*a^6*c^(5/2)*arctan
(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (8*b^5
*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c(-a+bx)\right)^{\frac{5}{2}}\left(a+bx\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(-b*c*x+a*c)**(5/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(5/2)*(a + b*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(109) = 218$.
time = 1.28, size = 622, normalized size = 4.61

$$-\frac{240\left(\frac{2ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}\right)a^5c^2-120\left(\frac{2a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}+\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}(bx-2a)\right)a^4c^2-80\left(\frac{6a^3c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-((2bx-5a)(bx+a)+9a^2)\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}\right)a^3c^2+20\left(\frac{18a^4c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-(39a^3-(2(3bx-10a)(bx+a)+43a^2)(bx+a))\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}\right)a^2c^2+2\left(\frac{90a^5c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-(195a^4-(295a^3-2(3(4bx-17a)(bx+a)+133a^2)(bx+a))(bx+a))\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}\right)ac^2-\left(\frac{150a^6c\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2ac}\right|\right)}{\sqrt{-c}}-(405a^5-(745a^4-2(451a^3-(4(5bx-26a)(bx+a)+321a^2)(bx+a))(bx+a))(bx+a))\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}\right)c^2}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*
a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^5*c^2 - 120*(
2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*a^4*c^2 - 80*(
6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*
x + a))*a^3*c^2 + 20*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
 + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^
2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^2*c^2 + 2*(90*a^5
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
- (195*a^4 - (295*a^3 - 2*(3*(4*b*x - 17*a)*(b*x + a) + 133*a^2)*(b*x + a))
*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a*c^2 - (150*a^6*c*lo
g(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (40
5*a^5 - (745*a^4 - 2*(451*a^3 - (4*(5*b*x - 26*a)*(b*x + a) + 321*a^2)*(b*x
 + a))*(b*x + a))*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*c^2)
/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{5/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2), x)
```

3.1146 $\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}$$

```
[Out] 1/4*x*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)+3/4*a^4*c^(3/2)*arctan(c^(1/2)*(b*x+
a)^(1/2)/(c*(-b*x+a))^(1/2))/b+3/8*a^2*c*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {38, 65, 223, 209}

$$\frac{3a^4c^{3/2}\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (3*a^2*c*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 + (x*(a + b*x)^(3/2)*(a*c - b
*c*x)^(3/2))/4 + (3*a^4*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a -
b*x)]])/(4*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx &= \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{4}(3a^2c)\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx\\ &= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{8}(3a^4c^2)\int \frac{}{\sqrt{}}\\ &= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{(3a^4c^2)\,\mathrm{Subst}\big(}{}\\ &= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{(3a^4c^2)\,\mathrm{Subst}\big(}{}\\ &= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{3a^4c^{3/2}\tan^{-1}\big(}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 0.90

$$\frac{(c(a-bx))^{3/2}\left(bx\sqrt{a-bx}\,\sqrt{a+bx}\,(5a^2-2b^2x^2)+6a^4\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{8b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] ((c*(a - b*x))^(3/2)*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(5*a^2 - 2*b^2*x^2) +
 6*a^4*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(8*b*(a - b*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(80) = 160.
time = 0.17, size = 184, normalized size = 1.80

method	result

risch	$\frac{x(-2x^2b^2+5a^2)\sqrt{bx+a}\,(-bx+a)c^2}{8\sqrt{-c(bx-a)}} + \frac{3a^4\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)\,c\,(bx-a)}\,c^2}{8\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$-\frac{(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}}{4bc} + \frac{3a\left(-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}}{3bc} + \frac{a\left(\frac{(-bcx+ac)^{\frac{3}{2}}\sqrt{bx+a}}{2b} + \frac{3ac\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}}{b} + \frac{ac}{\cdots}\right)}{\cdots}\right)}{3\cdots}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b/c*(b*x+a)^(3/2)*(-b*c*x+a*c)^(5/2)+3/4*a*(-1/3/b/c*(b*x+a)^(1/2)*(-b
*c*x+a*c)^(5/2)+1/3*a*(1/2/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+3/2*a*c*(1/b*
(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+a*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a
*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^
2*c)^(1/2)))))
```

Maxima [A]

time = 0.50, size = 63, normalized size = 0.62

$$\frac{3\,a^4c^{\frac{3}{2}}\arcsin\left(\frac{bx}{a}\right)}{8\,b} + \frac{3}{8}\sqrt{-b^2cx^2+a^2c}\,a^2cx + \frac{1}{4}\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^4*c^(3/2)*arcsin(b*x/a)/b + 3/8*sqrt(-b^2*c*x^2 + a^2*c)*a^2*c*x + 1/
4*(-b^2*c*x^2 + a^2*c)^(3/2)*x
```

Fricas [A]

time = 1.28, size = 193, normalized size = 1.89

$$\left[\frac{3\,a^4\sqrt{-c}\,c\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)-2\,(2\,b^3cx^3-5\,a^2bcx)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{16\,b}, -\frac{3\,a^4c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+(2\,b^3cx^3-5\,a^2bcx)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^4*sqrt(-c)*c*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) - 2*(2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/b, -1/8*(3*a^4*c^(3/2)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c
*x + a*c)*sqrt(b*x + a))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (-c(-a+bx))^{\frac{3}{2}} (a+bx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**(3/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(3/2)*(a + b*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(82) = 164.
time = 1.49, size = 354, normalized size = 3.47

$$-\frac{24\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\right)a^3c-12\left(\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}+\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\,(bx-2\,a)\right)a^2c-4\left(\frac{6\,a^3c\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-((2\,bx-5\,a)(bx+a)+9\,a^2)\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\right)ac+\left(\frac{18\,a^4c\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-(39\,a^3-(2\,(3\,bx-10\,a)(bx+a)+43\,a^2)(bx+a))\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\right)c}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a^3*c - 12*(2*a^2
*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c)
+ sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*a^2*c - 4*(6*a^3*c*
log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (
(2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*
a*c + (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c
)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*x + a) + 43*a^2)*(b*x + a))*s
qrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*c)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{3/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2),x)
```

```
[Out] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2), x)
```

3.1147 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx$

Optimal. Leaf size=68

$$\frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$

```
[Out] a^2*arctan(c^(1/2)*(b*x+a)^(1/2)/(c*(-b*x+a))^(1/2))*c^(1/2)/b+1/2*x*(b*x+a
)^(1/2)*(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 65, 223, 209}

$$\frac{a^2\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\,\sqrt{ac-bcx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + b*x])/Sqrt[c*(a - b*x)]])/b
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{1}{2}\left(a^2c\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx\\ &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{\left(a^2c\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{\left(a^2c\right)\mathrm{Subst}\left(\int \frac{1}{1+cx^2}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\\ &= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 1.15

$$\frac{\sqrt{c(a-bx)}\left(bx\sqrt{a-bx}\sqrt{a+bx}+2a^2\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)\right)}{2b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*x*Sqrt[a - b*x]*Sqrt[a + b*x] + 2*a^2*ArcTan[Sqrt[a +
 b*x]/Sqrt[a - b*x]]))/(2*b*Sqrt[a - b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.
time = 0.18, size = 126, normalized size = 1.85

method	result
risch	$\frac{x(-bx+a)\sqrt{bx+a}\,c}{2\sqrt{-c\,(bx-a)}}+\frac{a^2\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-\left(bx+a\right)c\left(bx-a\right)}\,c}{2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c\,(bx-a)}}$

default	$-\frac{\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}{2bc}+\frac{a\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}}{b}+\frac{ac\sqrt{(bx+a)(-bcx+ac)}\,\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a}}\right)}{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,\sqrt{b^2c}}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(3/2)+1/2*a*(1/b*(-b*c*x+a*c)^(1/2)*(b*
x+a)^(1/2)+a*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2
)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2)))
```

Maxima [A]
time = 0.52, size = 39, normalized size = 0.57

$$\frac{a^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^2cx^2+a^2c}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*sqrt(c)*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*x
```

Fricas [A]
time = 1.32, size = 159, normalized size = 2.34

$$\left[\frac{a^2\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,bx}{4\,b},-\frac{a^2\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\sqrt{-bcx+ac}\,\sqrt{bx+a}\,bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a^2*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(-c)*x - a^2*c) + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b, -1/2*(a^2*s
qrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2
*c)) - sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-c\,(-a+bx)}\,\sqrt{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(56) = 112$.
time = 1.06, size = 148, normalized size = 2.18

$$\frac{\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}+\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\,(bx-2\,a)-2\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\right)a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a) - 2*(2*a*c
*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*a)/b
```

Mupad [B]
time = 0.20, size = 72, normalized size = 1.06

$$\frac{x\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2}-\frac{a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,\left(a-b\,x\right)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{\left(-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] (x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (a^2*b^(1/2)*c^2*log((-b*c)^(1/
2)*(c*(a - b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

3.1148 $\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

[Out] 2*arctan(c^(1/2)*(b*x+a)^(1/2)/(c*(-b*x+a))^(1/2))/b/c^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {65, 223, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]

[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(b*Sqrt[c])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Mathematica [A]
time = 0.05, size = 48, normalized size = 1.26

$$\frac{2\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (2*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(b*Sqrt[c*(a - b*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.
time = 0.19, size = 71, normalized size = 1.87

method	result	size
default	$\frac{\sqrt{(bx+a)(-bcx+ac)}\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2cx^2+a^2c}}\right)}{\sqrt{bx+a}\sqrt{-bcx+ac}\sqrt{b^2c}}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(-b*c*x+a*c))^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(b^2*c)^(1/2)
*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

Maxima [A]
time = 0.50, size = 14, normalized size = 0.37

$$\frac{\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/a)/(b*sqrt(c))
```

Fricas [A]
time = 0.96, size = 108, normalized size = 2.84

$$\left[-\frac{\sqrt{-c}\log\left(2\,b^{2}cx^{2}-2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^{2}c\right)}{2\,bc},-\frac{\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^{2}cx^{2}-a^{2}c}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
-c)*x - a^2*c)/(b*c), -arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/
(b^2*c*x^2 - a^2*c))/(b*sqrt(c))]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.23, size = 90, normalized size = 2.37

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{a^{2}}{b^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{a^{2}e^{-2i\pi}}{b^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b**2*
x**2))/(4*pi**(3/2)*b*sqrt(c))
```

Giac [A]
time = 1.88, size = 42, normalized size = 1.11

$$-\frac{2\,\log\left(\left|-\sqrt{bx+a}\,\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{b\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/(b*sqrt(-
c))
```

Mupad [B]
time = 0.18, size = 53, normalized size = 1.39

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
[Out] -(4*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(b^2*c)^(1/2)
```

3.1149 $\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

```
[Out] x/a^2/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{39\}$

$$\frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
[Out] x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx = \frac{x}{a^2c\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Mathematica [A]
time = 0.06, size = 29, normalized size = 0.97

$$\frac{x}{a^2c\sqrt{c(a-bx)}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
[Out] x/(a^2*c*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.
time = 0.15, size = 59, normalized size = 1.97

method	result	size
gosper	$\frac{(-bx+a)x}{\sqrt{bx+a}\,a^{2}(-bcx+ac)^{\frac{3}{2}}}$	30
default	$-\frac{1}{abc\sqrt{bx+a}\,\sqrt{-bcx+ac}}+\frac{\sqrt{bx+a}}{bc\,a^{2}\sqrt{-bcx+ac}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/b/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)+1/b/c/a^2/(-b*c*x+a*c)^(1/2)*(b*x
+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.83

$$\frac{x}{\sqrt{-b^{2}cx^{2}+a^{2}c}\,a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-b^2*c*x^2 + a^2*c)*a^2*c)
```

Fricas [A]
time = 0.89, size = 45, normalized size = 1.50

$$-\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,x}{a^{2}b^{2}c^{2}x^{2}-a^{4}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*c*x + a*c)*sqrt(b*x + a)*x/(a^2*b^2*c^2*x^2 - a^4*c^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.35, size = 94, normalized size = 3.13

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{a^{2}}{b^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}a^{2}bc^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{a^{2}e^{-2i\pi}}{b^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}a^{2}bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(-b*c*x+a*c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), a
**2/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), a**2*exp_polar(-2*I*pi)/(b*
*2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(26) = 52.
time = 1.75, size = 103, normalized size = 3.43

$$\frac{\frac{4\sqrt{-c}}{\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2-2\,ac\right)ac}-\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}}{((bx+a)c-2\,ac)a^2c}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(-c)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2 -
 2*a*c)*a*c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)/(((b*x + a)*c - 2*a
*c)*a^2*c))/b
```

Mupad [B]
time = 0.50, size = 26, normalized size = 0.87

$$\frac{x}{a^2\,c\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2)),x)
```

```
[Out] x/(a^2*c*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))
```

3.1150 $\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

```
[Out] 1/3*x/a^2/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+2/3*x/a^4/c^2/(b*x+a)^(1/2)/(-
b*c*x+a*c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {40, 39}

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{3a^2c}\\ &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.69

$$\frac{3a^2x - 2b^2x^3}{3a^4c(c(a-bx))^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] (3*a^2*x - 2*b^2*x^3)/(3*a^4*c*(c*(a - b*x))^(3/2)*(a + b*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(55) = 110$.
time = 0.14, size = 129, normalized size = 1.93

method	result	size
gosper	$\frac{(-bx+a)x(-2x^2b^2+3a^2)}{3(bx+a)^{\frac{3}{2}}a^4(-bcx+ac)^{\frac{5}{2}}}$	45
default	$-\frac{1}{3abc(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{3}{2}}}+\frac{-\frac{1}{abc\sqrt{bx+a}\,(-bcx+ac)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{bx+a}}{3abc(-bcx+ac)^{\frac{3}{2}}}+\frac{2\sqrt{bx+a}}{3b\,a^2c^2\sqrt{-bcx+ac}}}{a}}{a}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/b/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+1/a*(-1/a/b/c/(b*x+a)^(1/2)/(-b
*c*x+a*c)^(3/2)+2/a*(1/3/a/b/c/(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+1/3/b/a^2/c
^2/(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.79

$$\frac{x}{3\,(-b^2cx^2+a^2c)^{\frac{3}{2}}a^2c}+\frac{2\,x}{3\,\sqrt{-b^2cx^2+a^2c}\,a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c) + 2/3*x/(sqrt(-b^2*c*x^2 + a^2*c)*
a^4*c^2)
```

Fricas [A]
time = 1.74, size = 72, normalized size = 1.07

$$-\frac{(2\,b^2x^3-3\,a^2x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{3\,(a^4b^4c^3x^4-2\,a^6b^2c^3x^2+a^8c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^3 - 3*a^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^4*b^4*c^3*x^
4 - 2*a^6*b^2*c^3*x^2 + a^8*c^3)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.06, size = 94, normalized size = 1.40

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3 \\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0 \end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0 \end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(-b*c*x+a*c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), a*
*2/(b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4
, 5/4, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), a**2*exp_polar(-2*I*pi)/(b**
2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(55) = 110.
time = 1.46, size = 199, normalized size = 2.97

$$-\frac{\frac{\sqrt{-(bx+a)c+2ac}\sqrt{bx+a}\left(\frac{4(bx+a)}{a^4c}-\frac{9}{a^3c}\right)}{((bx+a)c-2ac)^2}+\frac{4\left(3\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2ac}\right)^4-18a\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2ac}\right)^2c+16a^2c^2\right)}{\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2ac}\right)^2-2ac\right)^3a^3\sqrt{-c}\,c}}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(4*(b*x + a)/(a^4*c) - 9/(a
^3*c))/((b*x + a)*c - 2*a*c)^2 + 4*(3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x
+ a)*c + 2*a*c))^4 - 18*a*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^2*c + 16*a^2*c^2)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*
c))^2 - 2*a*c)^3*a^3*sqrt(-c)*c))/b
```

Mupad [B]
time = 0.58, size = 80, normalized size = 1.19

$$-\frac{3\,a^2\,x\,\sqrt{a\,c-b\,c\,x}-2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^2\,\left(3\,a^4\,\left(a\,c-b\,c\,x\right)-6\,a^5\,c\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(3*a^2*x*(a*c - b*c*x)^(1/2) - 2*b^2*x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*
x)^2*(3*a^4*(a*c - b*c*x) - 6*a^5*c)*(a + b*x)^(1/2))
```

3.1151 $\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}} dx$

Optimal. Leaf size=100

$$\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}$$

```
[Out] 1/5*x/a^2/c/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2)+4/15*x/a^4/c^2/(b*x+a)^(3/2)/(
-b*c*x+a*c)^(3/2)+8/15*x/a^6/c^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*x)/(15*a^4*c^2*(a + b*
x)^(3/2)*(a*c - b*c*x)^(3/2)) + (8*x)/(15*a^6*c^3*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx}{5a^2c}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8\int \frac{1}{(a+bx)^{3}}}{1}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{}{15a^6c^3\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.57

$$\frac{15a^4x - 20a^2b^2x^3 + 8b^4x^5}{15a^6c(c(a-bx))^{5/2}(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] (15*a^4*x - 20*a^2*b^2*x^3 + 8*b^4*x^5)/(15*a^6*c*(c*(a - b*x))^(5/2)*(a +
b*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(82) = 164.
time = 0.17, size = 202, normalized size = 2.02

method	result
gosper	$\frac{(-bx+a)x(8b^4x^4-20a^2b^2x^2+15a^4)}{15(bx+a)^{\frac{5}{2}}a^6(-bcx+ac)^{\frac{7}{2}}}$
default	$-\frac{1}{5abc(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{5}{2}}}+\frac{-\frac{1}{3abc(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{5}{2}}}+\frac{-\frac{4}{3abc\sqrt{bx+a}\,(-bcx+ac)^{\frac{5}{2}}}+\frac{4\left(\frac{3\sqrt{bx+a}}{5abc(-bcx+ac)^{\frac{5}{2}}}+\frac{3\left(\frac{2\sqrt{bx+\cdots}}{15abc(-bcx+a\cdots}\cdots\right)}{\cdots}\right)}{3\cdots}}{a}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/b/c/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2)+1/a*(-1/3/a/b/c/(b*x+a)^(3/2)/(
-b*c*x+a*c)^(5/2)+4/3/a*(-1/a/b/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(5/2)+3/a*(1/5
/a/b/c/(-b*c*x+a*c)^(5/2)*(b*x+a)^(1/2)+2/5/a/c*(1/3/a/b/c/(-b*c*x+a*c)^(3/
2)*(b*x+a)^(1/2)+1/3/b/a^2/c^2/(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)))))
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.79

$$\frac{x}{5\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^2c}+\frac{4\,x}{15\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^4c^2}+\frac{8\,x}{15\,\sqrt{-b^2cx^2+a^2c}\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-b^2*c*x^2 + a^2*c)^(5/2)*a^2*c) + 4/15*x/((-b^2*c*x^2 + a^2*c)^(3/
2)*a^4*c^2) + 8/15*x/(sqrt(-b^2*c*x^2 + a^2*c)*a^6*c^3)
```

Fricas [A]
time = 1.07, size = 98, normalized size = 0.98

$$-\frac{\left(8\,b^{4}x^{5}-20\,a^{2}b^{2}x^{3}+15\,a^{4}x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{15\,\left(a^{6}b^{6}c^{4}x^{6}-3\,a^{8}b^{4}c^{4}x^{4}+3\,a^{10}b^{2}c^{4}x^{2}-a^{12}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^4*x^5 - 20*a^2*b^2*x^3 + 15*a^4*x)*sqrt(-b*c*x + a*c)*sqrt(b*x +
 a)/(a^6*b^6*c^4*x^6 - 3*a^8*b^4*c^4*x^4 + 3*a^10*b^2*c^4*x^2 - a^12*c^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 39.18, size = 97, normalized size = 0.97

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\frac{a^{2}}{b^{2}x^{2}}\right)}{15\pi^{\frac{3}{2}}a^{6}bc^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\frac{a^{2}e^{-2i\pi}}{b^{2}x^{2}}\right)}{15\pi^{\frac{3}{2}}a^{6}bc^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(-b*c*x+a*c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 a**2/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/
2, 5/4, 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), a**2*exp_polar(-2*I*pi
)/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(82) = 164.
time = 1.79, size = 296, normalized size = 2.96

$$-\frac{\frac{\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}\,\left((bx+a)\left(\frac{64\,(bx+a)}{a^{6}c}-\frac{275}{a^{5}c}\right)+\frac{300}{a^{4}c}\right)}{\left((bx+a)c-2\,ac\right)^{3}}+\frac{4\left(45\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{8}-450\,a\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{6}c+1660\,a^{2}\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{4}c^{2}-2200\,a^{3}\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{2}c^{3}+1024\,a^{4}c^{4}\right)}{\left(\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{2}-2\,ac\right)^{5}a^{5}\sqrt{-c}\,c^{2}}}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*(64*(b*x + a)/(
a^6*c) - 275/(a^5*c)) + 300/(a^4*c))/((b*x + a)*c - 2*a*c)^3 + 4*(45*(sqrt(
b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^8 - 450*a*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*c + 1660*a^2*(sqrt(b*x + a)*sqrt(-c)
 - sqrt(-(b*x + a)*c + 2*a*c))^4*c^2 - 2200*a^3*(sqrt(b*x + a)*sqrt(-c) - s
qrt(-(b*x + a)*c + 2*a*c))^2*c^3 + 1024*a^4*c^4)/(((sqrt(b*x + a)*sqrt(-c)
- sqrt(-(b*x + a)*c + 2*a*c))^2 - 2*a*c)^5*a^5*sqrt(-c)*c^2))/b
```

Mupad [B]
time = 0.65, size = 111, normalized size = 1.11

$$\frac{15\,a^4\,x\,\sqrt{a\,c-b\,c\,x}+8\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}-20\,a^2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^3\,\left(60\,a^8\,c-\left(a\,c-b\,c\,x\right)\,\left(45\,a^7+15\,b\,x\,a^6\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(7/2)*(a + b*x)^(7/2)),x)
```

```
[Out] (15*a^4*x*(a*c - b*c*x)^(1/2) + 8*b^4*x^5*(a*c - b*c*x)^(1/2) - 20*a^2*b^2*
x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*x)^3*(60*a^8*c - (a*c - b*c*x)*(45*a^7
 + 15*a^6*b*x))*(a + b*x)^(1/2))
```

3.1152 $\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx$

Optimal. Leaf size=133

$$\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{}{35a^8c^4\sqrt{a+}}$$

```
[Out] 1/7*x/a^2/c/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2)+6/35*x/a^4/c^2/(b*x+a)^(5/2)/(
-b*c*x+a*c)^(5/2)+8/35*x/a^6/c^3/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2)+16/35*x/a
^8/c^4/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {40, 39}

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*x)/(35*a^4*c^2*(a + b*
x)^(5/2)*(a*c - b*c*x)^(5/2)) + (8*x)/(35*a^6*c^3*(a + b*x)^(3/2)*(a*c - b*
c*x)^(3/2)) + (16*x)/(35*a^8*c^4*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx = \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx}{7a^2c}$$

$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{24\int \frac{}{(a+}}{}$$

$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{}{35a^6c^3(}$$

$$= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}} + \frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}} + \frac{}{35a^6c^3(}$$

Mathematica [A]
time = 0.10, size = 76, normalized size = 0.57

$$\frac{\sqrt{c(a-bx)}\,(35a^6x - 70a^4b^2x^3 + 56a^2b^4x^5 - 16b^6x^7)}{35a^8c^5(a-bx)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(35*a^6*x - 70*a^4*b^2*x^3 + 56*a^2*b^4*x^5 - 16*b^6*x^7
))/(35*a^8*c^5*(a - b*x)^4*(a + b*x)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(109) = 218$.
time = 0.15, size = 275, normalized size = 2.07

method	result
gosper	$\frac{(-bx+a)x(-16x^6b^6+56a^2x^4b^4-70a^4x^2b^2+35a^6)}{35(bx+a)^{\frac{7}{2}}a^8(-bcx+ac)^{\frac{9}{2}}}$

default	$-\frac{1}{7abc(bx+a)^{\frac{7}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{-\frac{1}{5abc(bx+a)^{\frac{5}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{-\frac{2}{5abc(bx+a)^{\frac{3}{2}}(-bcx+ac)^{\frac{7}{2}}}+\frac{6\left(-\frac{5}{3abc\sqrt{bx+a}\,(-bcx+ac)^{\frac{7}{2}}}+\frac{5\left(\frac{4\sqrt{(}}{7abc(}\right.}{}\right.}{}}{}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/a/b/c/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2)+1/a*(-1/5/a/b/c/(b*x+a)^(5/2)/(
-b*c*x+a*c)^(7/2)+6/5/a*(-1/3/a/b/c/(b*x+a)^(3/2)/(-b*c*x+a*c)^(7/2)+5/3/a*
(-1/a/b/c/(b*x+a)^(1/2)/(-b*c*x+a*c)^(7/2)+4/a*(1/7/a/b/c/(-b*c*x+a*c)^(7/2
)*(b*x+a)^(1/2)+3/7/a/c*(1/5/a/b/c/(-b*c*x+a*c)^(5/2)*(b*x+a)^(1/2)+2/5/a/c
*(1/3/a/b/c/(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+1/3/b/a^2/c^2/(-b*c*x+a*c)^(1/
2)*(b*x+a)^(1/2)))))))
```

Maxima [A]
time = 0.27, size = 105, normalized size = 0.79

$$\frac{x}{7\,(-b^2cx^2+a^2c)^{\frac{7}{2}}a^2c}+\frac{6\,x}{35\,(-b^2cx^2+a^2c)^{\frac{5}{2}}a^4c^2}+\frac{8\,x}{35\,(-b^2cx^2+a^2c)^{\frac{3}{2}}a^6c^3}+\frac{16\,x}{35\,\sqrt{-b^2cx^2+a^2c}\,a^8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-b^2*c*x^2 + a^2*c)^(7/2)*a^2*c) + 6/35*x/((-b^2*c*x^2 + a^2*c)^(5/
2)*a^4*c^2) + 8/35*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^6*c^3) + 16/35*x/(sqrt(-
b^2*c*x^2 + a^2*c)*a^8*c^4)
```

Fricas [A]
time = 1.28, size = 122, normalized size = 0.92

$$-\frac{(16\,b^6x^7-56\,a^2b^4x^5+70\,a^4b^2x^3-35\,a^6x)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{35\,(a^8b^8c^5x^8-4\,a^{10}b^6c^5x^6+6\,a^{12}b^4c^5x^4-4\,a^{14}b^2c^5x^2+a^{16}c^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="fricas")

[Out] -1/35*(16*b^6*x^7 - 56*a^2*b^4*x^5 + 70*a^4*b^2*x^3 - 35*a^6*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^8*b^8*c^5*x^8 - 4*a^10*b^6*c^5*x^6 + 6*a^12*b^4*c^5*x^4 - 4*a^14*b^2*c^5*x^2 + a^16*c^5)

Sympy [C] Result contains complex when optimal does not.
time = 185.11, size = 97, normalized size = 0.73

$$\frac{4iG_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{1}{2},\frac{9}{2},5 \\ \frac{9}{4},\frac{11}{4},4,\frac{9}{2},5 & 0 \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{105\pi^{\frac{3}{2}}a^8bc^{\frac{9}{2}}}+\frac{4G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{7}{4},\frac{9}{4},1 & \\ \frac{7}{4},\frac{9}{4} & -\frac{1}{2},0,4,0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{105\pi^{\frac{3}{2}}a^8bc^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(9/2)/(-b*c*x+a*c)**(9/2),x)

[Out] 4*I*meijerg(((9/4, 11/4, 1), (1/2, 9/2, 5)), ((9/4, 11/4, 4, 9/2, 5), (0,)), a**2/(b**2*x**2))/(105*pi**(3/2)*a**8*b*c**(9/2)) + 4*meijerg(((-1/2, 0, 1/2, 7/4, 9/4, 1), ()), ((7/4, 9/4), (-1/2, 0, 4, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(105*pi**(3/2)*a**8*b*c**(9/2))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(109) = 218.
time = 2.52, size = 393, normalized size = 2.95

$$-\frac{\frac{\left((bx+a)\left((bx+a)\left(\frac{256\,(bx+a)}{a^8c}-\frac{1617}{a^7c}\right)+\frac{3430}{a^6c}\right)-\frac{2450}{a^5c}\right)\sqrt{-(bx+a)c+2\,ac}\,\sqrt{bx+a}}{((bx+a)c-2\,ac)^4}+\frac{4\left(175\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{12}-2450\,a\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{10}c+14280\,a^2\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{8}c^2-43120\,a^3\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{6}c^3+66416\,a^4\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{4}c^4-51744\,a^5\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^{2}c^5+16384\,a^6c^6\right)}{\left(\left(\sqrt{bx+a}\,\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2-2\,ac\right)^7a^7\sqrt{-c}\,c^3}}{1120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="giac")

[Out] -1/1120*(((b*x + a)*((b*x + a)*(256*(b*x + a)/(a^8*c) - 1617/(a^7*c)) + 3430/(a^6*c)) - 2450/(a^5*c))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)/((b*x + a)*c - 2*a*c)^4 + 4*(175*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^12 - 2450*a*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^10*c + 14280*a^2*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^8*c^2 - 43120*a^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*c^3 + 66416*a^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*c^4 - 51744*a^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c^5 + 16384*a^6*c^6)/(((sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2 - 2*a*c)^7*a^7*sqrt(-c)*c^3))/b

Mupad [B]
time = 0.71, size = 170, normalized size = 1.28

$$-\frac{35\,a^6\,x\,\sqrt{a\,c-b\,c\,x}-16\,b^6\,x^7\,\sqrt{a\,c-b\,c\,x}-70\,a^4\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}+56\,a^2\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}}{\left(\left(70\,a^9\,{\left(a\,c-b\,c\,x\right)}^5+35\,a^8\,{\left(a\,c-b\,c\,x\right)}^5\,\left(a+b\,x\right)\right)\,\left(a+b\,x\right)+{\left(a\,c-b\,c\,x\right)}^4\,\left(140\,a^{10}\,\left(a\,c-b\,c\,x\right)-280\,a^{11}\,c\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(9/2)*(a + b*x)^(9/2)),x)
```

```
[Out] -(35*a^6*x*(a*c - b*c*x)^(1/2) - 16*b^6*x^7*(a*c - b*c*x)^(1/2) - 70*a^4*b^
2*x^3*(a*c - b*c*x)^(1/2) + 56*a^2*b^4*x^5*(a*c - b*c*x)^(1/2))/(((70*a^9*(
a*c - b*c*x)^5 + 35*a^8*(a*c - b*c*x)^5*(a + b*x))*(a + b*x) + (a*c - b*c*x
)^4*(140*a^10*(a*c - b*c*x) - 280*a^11*c))*(a + b*x)^(1/2))
```

3.1153 $\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+6\sqrt{6}(1-2x)^{5/2}x(1+2x)^{5/2}+\frac{45}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] 15/2*(1-2*x)^(3/2)*x*(1+2*x)^(3/2)*6^(1/2)+45/8*arcsin(2*x)*6^(1/2)+6*(1-2*
x)^(5/2)*x*(1+2*x)^(5/2)*6^(1/2)+45/4*x*6^(1/2)*(1-2*x)^(1/2)*(1+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 222}

$$\frac{45}{4}\sqrt{\frac{3}{2}}\text{ArcSin}(2x)+6\sqrt{6}(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{2x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (45*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 15*Sqrt[3/2]*(1 - 2*x)^(3/
2)*x*(1 + 2*x)^(3/2) + 6*Sqrt[6]*(1 - 2*x)^(5/2)*x*(1 + 2*x)^(5/2) + (45*Sq
rt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx = 6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2} + 5\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$$

$$= 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2} + \frac{45}{2}\int \sqrt{3-$$

$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x$$

$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x$$

$$= \frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + 6\sqrt{6}\,(1-2x$$

Mathematica [A]
time = 1.28, size = 193, normalized size = 1.93

$$-\frac{3\sqrt{3-6x}\,x(33-104x^2+128x^4)\left(8119+45112x+91052x^2+80768x^3+30160x^4+3712x^5+64x^6-\sqrt{2+4x}\,(5741+26158x+41096x^2+26224x^3+6160x^4+352x^5)\right)}{2\left(11482+52316x+82192x^2+52448x^3+12320x^4+704x^5-\sqrt{2+4x}\,(8119+28874x+33304x^2+14160x^3+1840x^4+32x^5)\right)}+45\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{-\sqrt{2}+\sqrt{1+2x}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[3 - 6*x]*x*(33 - 104*x^2 + 128*x^4)*(8119 + 45112*x + 91052*x^2 +
80768*x^3 + 30160*x^4 + 3712*x^5 + 64*x^6 - Sqrt[2 + 4*x]*(5741 + 26158*x +
 41096*x^2 + 26224*x^3 + 6160*x^4 + 352*x^5)))/(2*(11482 + 52316*x + 82192*
x^2 + 52448*x^3 + 12320*x^4 + 704*x^5 - Sqrt[2 + 4*x]*(8119 + 28874*x + 333
04*x^2 + 14160*x^3 + 1840*x^4 + 32*x^5))) + 45*Sqrt[3/2]*ArcTan[(-Sqrt[2] +
 Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

Maple [A]
time = 0.18, size = 134, normalized size = 1.34

method	result
risch	$-\frac{3x(128x^4-104x^2+33)(2x-1)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{4\sqrt{-(2x-1)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{45\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{8\sqrt{2+4x}\,\sqrt{3-6x}}$
default	$\frac{(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{7}{2}}}{24}+\frac{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{7}{2}}}{8}+\frac{9\sqrt{3-6x}\,(2+4x)^{\frac{7}{2}}}{32}-\frac{3(2+4x)^{\frac{5}{2}}\sqrt{3-6x}}{16}-\frac{15(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{16}-\frac{45}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(5/2)*(2+4*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(3-6*x)^(5/2)*(2+4*x)^(7/2)+1/8*(3-6*x)^(3/2)*(2+4*x)^(7/2)+9/32*(3-6*
x)^(1/2)*(2+4*x)^(7/2)-3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-15/16*(2+4*x)^(3/2)
```

```
*(3-6*x)^(1/2)-45/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+45/8*((2+4*x)*(3-6*x))^(1/2
)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2)
```

Maxima [A]
time = 0.51, size = 46, normalized size = 0.46

$$\frac{1}{6}\left(-24\,x^2+6\right)^{\frac{5}{2}}x+\frac{5}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x+\frac{45}{4}\,\sqrt{-24\,x^2+6}\,x+\frac{45}{8}\,\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-24*x^2 + 6)^(5/2)*x + 5/4*(-24*x^2 + 6)^(3/2)*x + 45/4*sqrt(-24*x^2 +
 6)*x + 45/8*sqrt(6)*arcsin(2*x)
```

Fricas [A]
time = 1.00, size = 65, normalized size = 0.65

$$\frac{3}{4}\left(128\,x^5-104\,x^3+33\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{45}{8}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 3/4*(128*x^5 - 104*x^3 + 33*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 45/8*sqrt(3)*
sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(5/2)*(4*x+2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(70) = 140.
time = 1.52, size = 227, normalized size = 2.27

$$\frac{3}{40}\,\sqrt{3}\,\sqrt{2}\left(\left(\left(2\left(\left(8\left(5\,x-13\right)\left(2\,x+1\right)+321\right)\left(2\,x+1\right)-451\right)\left(2\,x+1\right)+745\right)\left(2\,x+1\right)-405\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+2\left(\left(2\left(3\left(8\,x-17\right)\left(2\,x+1\right)+133\right)\left(2\,x+1\right)-295\right)\left(2\,x+1\right)+195\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-20\left(\left(4\left(3\,x-5\right)\left(2\,x+1\right)+43\right)\left(2\,x+1\right)-39\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-80\left(\left(4\,x-5\right)\left(2\,x+1\right)+9\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+240\,\sqrt{2\,x+1}\left(x-1\right)\sqrt{-2\,x+1}+240\,\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+150\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{2\,x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(3)*sqrt(2)*(((2*((8*(5*x - 13)*(2*x + 1) + 321)*(2*x + 1) - 451)*
(2*x + 1) + 745)*(2*x + 1) - 405)*sqrt(2*x + 1)*sqrt(-2*x + 1) + 2*((2*(3*(
8*x - 17)*(2*x + 1) + 133)*(2*x + 1) - 295)*(2*x + 1) + 195)*sqrt(2*x + 1)*
sqrt(-2*x + 1) - 20*((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2*x
+ 1)*sqrt(-2*x + 1) - 80*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
+ 1) + 240*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + 240*sqrt(2*x + 1)*sqrt(-2
*x + 1) + 150*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (4\,x+2)^{5/2}\,(3-6\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2),x)
```

```
[Out] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2), x)
```

3.1154 $\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$

Optimal. Leaf size=74

$$\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

```
[Out] 3/2*(1-2*x)^(3/2)*x*(1+2*x)^(3/2)*6^(1/2)+9/8*arcsin(2*x)*6^(1/2)+9/4*x*6^(
1/2)*(1-2*x)^(1/2)*(1+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {38, 41, 222}

$$\frac{9}{4}\sqrt{\frac{3}{2}}\,\text{ArcSin}(2x)+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{2x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (9*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 3*Sqrt[3/2]*(1 - 2*x)^(3/2)
*x*(1 + 2*x)^(3/2) + (9*Sqrt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx = 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{9}{2}\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{27}{2}\int \frac{}{\sqrt{3-6x}$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{27}{2}\int \frac{1}{\sqrt{6-24}$$
$$= \frac{9}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2} + \frac{9}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(74) = 148.
time = 0.82, size = 179, normalized size = 2.42

$$\frac{-12\sqrt{6}\,\sqrt{1-2x}\,x\sqrt{1+2x}\,(-5+8x^2)\,(-169-490x-364x^2-56x^3)-12\sqrt{3}\,\sqrt{1-2x}\,x(-5+8x^2)\,(239+932x+1088x^2+368x^3+16x^4)}{-2704-7840x-5824x^2-896x^3+\sqrt{2}\,\sqrt{1+2x}\,(1912+3632x+1440x^2+64x^3)}+9\sqrt{\frac{3}{2}}\,\tan^{-1}\left(\frac{-\sqrt{2}+\sqrt{1+2x}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (-12*Sqrt[6]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x]*(-5 + 8*x^2)*(-169 - 490*x - 364
*x^2 - 56*x^3) - 12*Sqrt[3]*Sqrt[1 - 2*x]*x*(-5 + 8*x^2)*(239 + 932*x + 108
8*x^2 + 368*x^3 + 16*x^4))/(-2704 - 7840*x - 5824*x^2 - 896*x^3 + Sqrt[2]*S
qrt[1 + 2*x]*(1912 + 3632*x + 1440*x^2 + 64*x^3)) + 9*Sqrt[3/2]*ArcTan[(-Sq
rt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.
time = 0.14, size = 102, normalized size = 1.38

method	result
default	$\frac{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}}{16}+\frac{3(2+4x)^{\frac{5}{2}}\sqrt{3-6x}}{16}-\frac{3(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{16}-\frac{9\sqrt{3-6x}\,\sqrt{2+4x}}{8}+\frac{9\sqrt{(2+4x)(3-}}{8\sqrt{2+4x}}$
risch	$\frac{3x(8x^2-5)(2x-1)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{4\sqrt{-(2x-1)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}}+\frac{9\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{8\sqrt{2+4x}\,\sqrt{3-6x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(3/2)*(2+4*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(3-6*x)^(3/2)*(2+4*x)^(5/2)+3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-3/16*(2+4
*x)^(3/2)*(3-6*x)^(1/2)-9/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+9/8*((2+4*x)*(3-6*x
))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2)
```

Maxima [**A**]
time = 0.49, size = 34, normalized size = 0.46

$$\frac{1}{4}\left(-24\,x^{2}+6\right)^{\frac{3}{2}}x+\frac{9}{4}\,\sqrt{-24\,x^{2}+6}\,x+\frac{9}{8}\,\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-24*x^2 + 6)^(3/2)*x + 9/4*sqrt(-24*x^2 + 6)*x + 9/8*sqrt(6)*arcsin(2*
x)
```

Fricas [**A**]
time = 1.26, size = 60, normalized size = 0.81

$$-\frac{3}{4}\left(8\,x^{3}-5\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{9}{8}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -3/4*(8*x^3 - 5*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 9/8*sqrt(3)*sqrt(2)*arcta
n(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(3/2)*(4*x+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(50) = 100.
time = 1.66, size = 125, normalized size = 1.69

$$-\frac{1}{8}\,\sqrt{3}\,\sqrt{2}\,\left(\left(\left(4\left(3\,x-5\right)\left(2\,x+1\right)+43\right)\left(2\,x+1\right)-39\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+4\left(\left(4\,x-5\right)\left(2\,x+1\right)+9\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-24\,\sqrt{2\,x+1}\left(x-1\right)\sqrt{-2\,x+1}-24\,\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-18\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{2\,x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*sqrt(2)*(((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2*
x + 1)*sqrt(-2*x + 1) + 4*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
```

```
 + 1) - 24*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) - 24*sqrt(2*x + 1)*sqrt(-2*
x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (4x+2)^{3/2}(3-6x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2),x)
[Out] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2), x)
```

3.1155 $\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}\,+\frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

```
[Out] 1/4*arcsin(2*x)*6^(1/2)+1/2*x*6^(1/2)*(1-2*x)^(1/2)*(1+2*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {38, 41, 222}

$$\frac{1}{2}\sqrt{\frac{3}{2}}\,\text{ArcSin}(2x)+\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,\sqrt{2x+1}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + (Sqrt[3/2]*ArcSin[2*x])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx = \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx$$
$$= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + 3\int \frac{1}{\sqrt{6-24x^2}}\,dx$$
$$= \sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x} + \frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.
time = 0.66, size = 108, normalized size = 2.51

$$\frac{\sqrt{3-6x}\,x\left(7+4x^2-5\sqrt{2+4x}+x\left(16-6\sqrt{2+4x}\right)\right)}{-10+7\sqrt{2+4x}+2x\left(-6+\sqrt{2+4x}\right)} + \sqrt{6}\,\tan^{-1}\left(\frac{-\sqrt{2}+\sqrt{1+2x}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (Sqrt[3 - 6*x]*x*(7 + 4*x^2 - 5*Sqrt[2 + 4*x] + x*(16 - 6*Sqrt[2 + 4*x])))/
(-10 + 7*Sqrt[2 + 4*x] + 2*x*(-6 + Sqrt[2 + 4*x])) + Sqrt[6]*ArcTan[(-Sqrt[
2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 0.18, size = 70, normalized size = 1.63

method	result	size
default	$\frac{(2+4x)^{\frac{3}{2}}\sqrt{3-6x}}{8} - \frac{\sqrt{3-6x}\,\sqrt{2+4x}}{4} + \frac{\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{4\sqrt{2+4x}\,\sqrt{3-6x}}$	70
risch	$-\frac{x(2x-1)(1+2x)\sqrt{(2+4x)(3-6x)}\,\sqrt{6}}{2\sqrt{-(2x-1)(1+2x)}\,\sqrt{3-6x}\,\sqrt{2+4x}} + \frac{\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{4\sqrt{2+4x}\,\sqrt{3-6x}}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(1/2)*(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2+4*x)^(3/2)*(3-6*x)^(1/2)-1/4*(3-6*x)^(1/2)*(2+4*x)^(1/2)+1/4*((2+4*x
)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2)
```

Maxima [A]
time = 0.48, size = 22, normalized size = 0.51

$$\frac{1}{2}\sqrt{-24\,x^2+6}\,x + \frac{1}{4}\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-24*x^2 + 6)*x + 1/4*sqrt(6)*arcsin(2*x)
```

Fricas [A]
time = 1.17, size = 52, normalized size = 1.21

$$\frac{1}{2}\sqrt{4x+2}\,x\sqrt{-6x+3} - \frac{1}{4}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x + 2)*x*sqrt(-6*x + 3) - 1/4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3
)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.10, size = 187, normalized size = 4.35

$$\begin{cases} -\frac{\sqrt{6}\,i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{2} + \frac{\sqrt{6}\,i\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{x-\frac{1}{2}}} - \frac{3\sqrt{6}\,i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{x-\frac{1}{2}}} + \frac{\sqrt{6}\,i\sqrt{x+\frac{1}{2}}}{2\sqrt{x-\frac{1}{2}}} & \text{for}\ \left|x+\frac{1}{2}\right| > 1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{2} - \frac{\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}} + \frac{3\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}} - \frac{\sqrt{6}\sqrt{x+\frac{1}{2}}}{2\sqrt{\frac{1}{2}-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(1/2)*(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/2 + sqrt(6)*I*(x + 1/2)**(5/2)/s
qrt(x - 1/2) - 3*sqrt(6)*I*(x + 1/2)**(3/2)/(2*sqrt(x - 1/2)) + sqrt(6)*I*s
qrt(x + 1/2)/(2*sqrt(x - 1/2)), Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x + 1
/2))/2 - sqrt(6)*(x + 1/2)**(5/2)/sqrt(1/2 - x) + 3*sqrt(6)*(x + 1/2)**(3/2
)/(2*sqrt(1/2 - x)) - sqrt(6)*sqrt(x + 1/2)/(2*sqrt(1/2 - x)), True))
```

Giac [A]
time = 1.30, size = 55, normalized size = 1.28

$$\frac{1}{2}\sqrt{3}\sqrt{2}\left(\sqrt{2x+1}\,(x-1)\sqrt{-2x+1} + \sqrt{2x+1}\sqrt{-2x+1} + \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*sqrt(2)*(sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + sqrt(2*x + 1)*s
qrt(-2*x + 1) + arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

Mupad [B]
time = 0.26, size = 44, normalized size = 1.02

$$\frac{x\,\sqrt{4\,x+2}\,\sqrt{3-6\,x}}{2}-\frac{\sqrt{6}\,\ln\left(x-\frac{\sqrt{1-2\,x}\,\sqrt{2\,x+1}\,1{}\mathrm{i}}{2}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(1/2)*(3 - 6*x)^(1/2),x)
```

```
[Out] (x*(4*x + 2)^(1/2)*(3 - 6*x)^(1/2))/2 - (6^(1/2)*log(x - ((1 - 2*x)^(1/2)*(
2*x + 1)^(1/2)*1i)/2)*1i)/4
```

3.1156 $\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx$

Optimal. Leaf size=13

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

```
[Out] 1/12*arcsin(2*x)*6^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {41, 222}

$$\frac{\text{ArcSin}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] ArcSin[2*x]/(2*Sqrt[6])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx &= \int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \frac{\sin^{-1}(2x)}{2\sqrt{6}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.
time = 0.04, size = 27, normalized size = 2.08

$$-\frac{\tan^{-1}\left(\frac{\sqrt{1-4x^2}}{1+2x}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] -(ArcTan[Sqrt[1 - 4*x^2]/(1 + 2*x)]/Sqrt[6])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(9) = 18$.
time = 0.16, size = 37, normalized size = 2.85

method	result	size
default	$\frac{\sqrt{(2+4x)(3-6x)}\,\arcsin(2x)\sqrt{6}}{12\sqrt{2+4x}\,\sqrt{3-6x}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(1/2)/(2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((2+4*x)*(3-6*x))^(1/2)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*arcsin(2*x)*6^(1/2
)
```

Maxima [A]
time = 0.48, size = 9, normalized size = 0.69

$$\frac{1}{12}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*arcsin(2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(9) = 18$.
time = 1.20, size = 28, normalized size = 2.15

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.00, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{\sqrt{6}\, i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{for}\ \left|x+\frac{1}{2}\right| > 1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(1/2)/(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/6, Abs(x + 1/2) > 1), (sqrt(6)*a
sin(sqrt(x + 1/2))/6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 1.05, size = 21, normalized size = 1.62

$$\frac{1}{6}\sqrt{3}\sqrt{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*sqrt(2)*arcsin(1/2*sqrt(2)*sqrt(2*x + 1))
```

Mupad [B]
time = 0.05, size = 40, normalized size = 3.08

$$-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{24}\left(\sqrt{3}-\sqrt{3-6x}\right)}{6\left(\sqrt{2}-\sqrt{4x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(1/2)*(3 - 6*x)^(1/2)),x)
```

```
[Out] -(6^(1/2)*atan((24^(1/2)*(3^(1/2) - (3 - 6*x)^(1/2)))/(6*(2^(1/2) - (4*x +
2)^(1/2)))))/3
```

3.1157 $\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{x}{6\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}$$

[Out] 1/36*x*6^(1/2)/(1-2*x)^(1/2)/(1+2*x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {39}

$$\frac{x}{6\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]

[Out] x/(6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.
time = 0.69, size = 59, normalized size = 2.11

$$\frac{x\left(3+2x-2\sqrt{2+4x}\right)}{6\sqrt{3-6x}\,\left(-4+3\sqrt{2+4x}+2x\left(-4+\sqrt{2+4x}\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]

```
[Out] (x*(3 + 2*x - 2*Sqrt[2 + 4*x]))/(6*Sqrt[3 - 6*x]*(-4 + 3*Sqrt[2 + 4*x] + 2*
x*(-4 + Sqrt[2 + 4*x])))
```

Maple [A]
time = 0.15, size = 34, normalized size = 1.21

method	result	size
gosper	$-\frac{(2x-1)(1+2x)x}{(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}$	28
default	$\frac{1}{12\sqrt{3-6x}\,\sqrt{2+4x}}-\frac{\sqrt{3-6x}}{36\sqrt{2+4x}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(3/2)/(2+4*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/12/(3-6*x)^(1/2)/(2+4*x)^(1/2)-1/36/(2+4*x)^(1/2)*(3-6*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.43

$$\frac{x}{6\sqrt{-24\,x^2+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="maxima")
[Out] 1/6*x/sqrt(-24*x^2 + 6)
```

Fricas [A]
time = 1.27, size = 26, normalized size = 0.93

$$-\frac{\sqrt{4\,x+2}\,x\sqrt{-6\,x+3}}{36\,(4\,x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="fricas")
[Out] -1/36*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 37.34, size = 156, normalized size = 5.57

$$\begin{cases} -\frac{2\sqrt{6}\,i\sqrt{x-\frac{1}{2}}\,(x+\frac{1}{2})}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\,i\sqrt{x-\frac{1}{2}}}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{for } \left|x+\frac{1}{2}\right|>1 \\ -\frac{2\sqrt{6}\,\sqrt{\frac{1}{2}-x}\,(x+\frac{1}{2})}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\,\sqrt{\frac{1}{2}-x}}{144(x+\frac{1}{2})^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(3/2)/(4*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(6)*I*sqrt(x - 1/2)*(x + 1/2)/(144*(x + 1/2)**(3/2) - 144
*sqrt(x + 1/2)) + sqrt(6)*I*sqrt(x - 1/2)/(144*(x + 1/2)**(3/2) - 144*sqrt(
x + 1/2)), Abs(x + 1/2) > 1), (-2*sqrt(6)*sqrt(1/2 - x)*(x + 1/2)/(144*(x +
 1/2)**(3/2) - 144*sqrt(x + 1/2)) + sqrt(6)*sqrt(1/2 - x)/(144*(x + 1/2)**(
3/2) - 144*sqrt(x + 1/2)), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(20) = 40$.
time = 2.38, size = 79, normalized size = 2.82

$$\frac{\sqrt{6}\left(\sqrt{2}-\sqrt{-2\,x+1}\right)}{288\,\sqrt{2\,x+1}}-\frac{\sqrt{6}\,\sqrt{2\,x+1}\,\sqrt{-2\,x+1}}{144\,(2\,x-1)}-\frac{\sqrt{6}\,\sqrt{2\,x+1}}{288\left(\sqrt{2}-\sqrt{-2\,x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/288*sqrt(6)*(sqrt(2) - sqrt(-2*x + 1))/sqrt(2*x + 1) - 1/144*sqrt(6)*sqrt
(2*x + 1)*sqrt(-2*x + 1)/(2*x - 1) - 1/288*sqrt(6)*sqrt(2*x + 1)/(sqrt(2) -
 sqrt(-2*x + 1))
```

Mupad [**B**]
time = 0.46, size = 24, normalized size = 0.86

$$-\frac{x\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(36\,x-18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(3/2)*(3 - 6*x)^(3/2)),x)
```

```
[Out] -(x*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(36*x - 18))
```

3.1158 $\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}$$

```
[Out] 1/648*x/(1-2*x)^(3/2)/(1+2*x)^(3/2)*6^(1/2)+1/324*x*6^(1/2)/(1-2*x)^(1/2)/(
1+2*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {40, 39}

$$\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] x/(108*Sqrt[6]*(1 - 2*x)^(3/2)*(1 + 2*x)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 2*
x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx &= \frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{1}{9}\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx\\ &= \frac{x}{108\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}\end{aligned}$$

Mathematica [A]
time = 0.79, size = 107, normalized size = 1.88

$$\frac{x(-3+8x^2)\left(99+8x^3-70\sqrt{2+4x}+x\left(246-104\sqrt{2+4x}\right)+x^2\left(132-24\sqrt{2+4x}\right)\right)}{54\sqrt{3-6x}\,(-1+2x)\left(-4+3\sqrt{2+4x}+2x\left(-4+\sqrt{2+4x}\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] (x*(-3 + 8*x^2)*(99 + 8*x^3 - 70*Sqrt[2 + 4*x] + x*(246 - 104*Sqrt[2 + 4*x]
) + x^2*(132 - 24*Sqrt[2 + 4*x])))/(54*Sqrt[3 - 6*x]*(-1 + 2*x)*(-4 + 3*Sqr
t[2 + 4*x] + 2*x*(-4 + Sqrt[2 + 4*x]))^3)
```

Maple [A]
time = 0.15, size = 66, normalized size = 1.16

method	result	size
gosper	$\frac{(2x-1)(1+2x)x(8x^2-3)}{3(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}$	35
default	$\frac{1}{36(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{3}{2}}}+\frac{1}{36\sqrt{3-6x}\,(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{162(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{324\sqrt{2+4x}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(5/2)/(2+4*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36/(3-6*x)^(3/2)/(2+4*x)^(3/2)+1/36/(3-6*x)^(1/2)/(2+4*x)^(3/2)-1/162/(2+
4*x)^(3/2)*(3-6*x)^(1/2)-1/324/(2+4*x)^(1/2)*(3-6*x)^(1/2)
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.44

$$\frac{x}{54\sqrt{-24x^2+6}}+\frac{x}{18\left(-24x^2+6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/54*x/sqrt(-24*x^2 + 6) + 1/18*x/(-24*x^2 + 6)^(3/2)
```

Fricas [A]
time = 1.05, size = 39, normalized size = 0.68

$$-\frac{(8x^3-3x)\sqrt{4x+2}\,\sqrt{-6x+3}}{648\left(16x^4-8x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/648*(8*x^3 - 3*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(16*x^4 - 8*x^2 + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(5/2)/(4*x+2)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(41) = 82.
time = 1.57, size = 144, normalized size = 2.53

$$\frac{1}{82944}\sqrt{6}\left(\frac{\left(\sqrt{2}-\sqrt{-2x+1}\right)^{3}}{(2x+1)^{\frac{3}{2}}}+\frac{33\left(\sqrt{2}-\sqrt{-2x+1}\right)}{\sqrt{2x+1}}\right)-\frac{\left(4\sqrt{6}(2x+1)-9\sqrt{6}\right)\sqrt{2x+1}\sqrt{-2x+1}}{5184(2x-1)^{2}}-\frac{\sqrt{6}(2x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-2x+1}\right)^{2}}{2x+1}+1\right)}{82944\left(\sqrt{2}-\sqrt{-2x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/82944*sqrt(6)*((sqrt(2) - sqrt(-2*x + 1))^3/(2*x + 1)^(3/2) + 33*(sqrt(2)
 - sqrt(-2*x + 1))/sqrt(2*x + 1)) - 1/5184*(4*sqrt(6)*(2*x + 1) - 9*sqrt(6)
)*sqrt(2*x + 1)*sqrt(-2*x + 1)/(2*x - 1)^2 - 1/82944*sqrt(6)*(2*x + 1)^(3/2
)*(33*(sqrt(2) - sqrt(-2*x + 1))^2/(2*x + 1) + 1)/(sqrt(2) - sqrt(-2*x + 1)
)^3
```

Mupad [B]
time = 0.31, size = 49, normalized size = 0.86

$$-\frac{3x\sqrt{3-6x}-8x^{3}\sqrt{3-6x}}{\sqrt{4x+2}\left(-2592x^{3}+1296x^{2}+648x-324\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(5/2)*(3 - 6*x)^(5/2)),x)
```

```
[Out] -(3*x*(3 - 6*x)^(1/2) - 8*x^3*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(648*x + 12
96*x^2 - 2592*x^3 - 324))
```

3.1159 $\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx$

Optimal. Leaf size=85

$$\frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,\sqrt{1+2x}}$$

```
[Out] 1/6480*x/(1-2*x)^(5/2)/(1+2*x)^(5/2)*6^(1/2)+1/4860*x/(1-2*x)^(3/2)/(1+2*x)
^(3/2)*6^(1/2)+1/2430*x*6^(1/2)/(1-2*x)^(1/2)/(1+2*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {40, 39}

$$\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] x/(1080*Sqrt[6]*(1 - 2*x)^(5/2)*(1 + 2*x)^(5/2)) + x/(810*Sqrt[6]*(1 - 2*x)
^(3/2)*(1 + 2*x)^(3/2)) + x/(405*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(-
x)*(a + b*x)^(m + 1)*((c + d*x)^(m + 1)/(2*a*c*(m + 1))), x] + Dist[(2*m +
3)/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{2}{15}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx\\ &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{2}{135}\int \frac{}{(3}\\ &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{}{405\sqrt{6}\,\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 1.09, size = 149, normalized size = 1.75

$$\frac{x(15-80x^2+128x^4)\left(3363+32x^5-2378\sqrt{2+4x}+x\left(13930-7472\sqrt{2+4x}\right)-80x^4\left(-19+2\sqrt{2+4x}\right)-80x^3\left(-121+28\sqrt{2+4x}\right)-8x^2\left(-2375+894\sqrt{2+4x}\right)\right)}{810\sqrt{3-6x}\,(1-2x)^2\left(-4+3\sqrt{2+4x}+2x\left(-4+\sqrt{2+4x}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] (x*(15 - 80*x^2 + 128*x^4)*(3363 + 32*x^5 - 2378*Sqrt[2 + 4*x] + x*(13930 -
 7472*Sqrt[2 + 4*x]) - 80*x^4*(-19 + 2*Sqrt[2 + 4*x]) - 80*x^3*(-121 + 28*S
qrt[2 + 4*x]) - 8*x^2*(-2375 + 894*Sqrt[2 + 4*x])))/(810*Sqrt[3 - 6*x]*(1 -
 2*x)^2*(-4 + 3*Sqrt[2 + 4*x] + 2*x*(-4 + Sqrt[2 + 4*x]))^5)
```

Maple [A]
time = 0.14, size = 98, normalized size = 1.15

method	result
gosper	$-\frac{(2x-1)(1+2x)x(128x^4-80x^2+15)}{15(3-6x)^{\frac{7}{2}}(2+4x)^{\frac{7}{2}}}$
default	$\frac{1}{60(3-6x)^{\frac{5}{2}}(2+4x)^{\frac{5}{2}}}+\frac{1}{108(3-6x)^{\frac{3}{2}}(2+4x)^{\frac{5}{2}}}+\frac{1}{81\sqrt{3-6x}\,(2+4x)^{\frac{5}{2}}}-\frac{\sqrt{3-6x}}{405(2+4x)^{\frac{5}{2}}}-\frac{\sqrt{3-6x}}{1215(2+4x)^{\frac{3}{2}}}-\frac{\sqrt{3-6x}}{2430\sqrt{2+4x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-6*x)^(7/2)/(2+4*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60/(3-6*x)^(5/2)/(2+4*x)^(5/2)+1/108/(3-6*x)^(3/2)/(2+4*x)^(5/2)+1/81/(3-
6*x)^(1/2)/(2+4*x)^(5/2)-1/405/(2+4*x)^(5/2)*(3-6*x)^(1/2)-1/1215/(2+4*x)^(
3/2)*(3-6*x)^(1/2)-1/2430/(2+4*x)^(1/2)*(3-6*x)^(1/2)
```

Maxima [A]
time = 0.26, size = 37, normalized size = 0.44

$$\frac{x}{405\sqrt{-24x^2+6}}+\frac{x}{135\left(-24x^2+6\right)^{\frac{3}{2}}}+\frac{x}{30\left(-24x^2+6\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/405*x/sqrt(-24*x^2 + 6) + 1/135*x/(-24*x^2 + 6)^(3/2) + 1/30*x/(-24*x^2 +
 6)^(5/2)
```

Fricas [A]
time = 1.24, size = 49, normalized size = 0.58

$$-\frac{(128x^5-80x^3+15x)\sqrt{4x+2}\,\sqrt{-6x+3}}{19440\left(64x^6-48x^4+12x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/19440*(128*x^5 - 80*x^3 + 15*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(64*x^6 - 4
8*x^4 + 12*x^2 - 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(7/2)/(4*x+2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(61) = 122.
time = 1.76, size = 205, normalized size = 2.41

$$\frac{1}{39813120}\sqrt{6}\left(\frac{3\left(\sqrt{2}-\sqrt{-2x+1}\right)^5}{(2x+1)^{\frac{5}{2}}}+\frac{85\left(\sqrt{2}-\sqrt{-2x+1}\right)^3}{(2x+1)^{\frac{3}{2}}}+\frac{2130\left(\sqrt{2}-\sqrt{-2x+1}\right)}{\sqrt{2x+1}}\right)-\frac{\left(\left(64\sqrt{6}(2x+1)-275\sqrt{6}\right)(2x+1)+300\sqrt{6}\right)\sqrt{2x+1}\sqrt{-2x+1}}{622080(2x-1)^3}-\frac{\sqrt{6}\left(\frac{2130\left(\sqrt{2}-\sqrt{-2x+1}\right)^4}{(2x+1)^2}+\frac{85\left(\sqrt{2}-\sqrt{-2x+1}\right)^2}{2x+1}+3\right)(2x+1)^{\frac{5}{2}}}{39813120\left(\sqrt{2}-\sqrt{-2x+1}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/39813120*sqrt(6)*(3*(sqrt(2) - sqrt(-2*x + 1))^5/(2*x + 1)^(5/2) + 85*(sq
rt(2) - sqrt(-2*x + 1))^3/(2*x + 1)^(3/2) + 2130*(sqrt(2) - sqrt(-2*x + 1))
/sqrt(2*x + 1)) - 1/622080*((64*sqrt(6)*(2*x + 1) - 275*sqrt(6))*(2*x + 1)
+ 300*sqrt(6))*sqrt(2*x + 1)*sqrt(-2*x + 1)/(2*x - 1)^3 - 1/39813120*sqrt(6
)*(2130*(sqrt(2) - sqrt(-2*x + 1))^4/(2*x + 1)^2 + 85*(sqrt(2) - sqrt(-2*x
+ 1))^2/(2*x + 1) + 3)*(2*x + 1)^(5/2)/(sqrt(2) - sqrt(-2*x + 1))^5
```

Mupad [**B**]
time = 0.45, size = 66, normalized size = 0.78

$$-\frac{15\,x\,\sqrt{3-6\,x}-80\,x^3\,\sqrt{3-6\,x}+128\,x^5\,\sqrt{3-6\,x}}{\left(\left(6\,x-3\right)\,\left(240\,x+360\right)+1440\right)\,\sqrt{4\,x+2}\,{\left(6\,x-3\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(7/2)*(3 - 6*x)^(7/2)),x)
```

```
[Out] -(15*x*(3 - 6*x)^(1/2) - 80*x^3*(3 - 6*x)^(1/2) + 128*x^5*(3 - 6*x)^(1/2))/
(((6*x - 3)*(240*x + 360) + 1440)*(4*x + 2)^(1/2)*(6*x - 3)^3)
```

3.1160 $\int (3-x)^{3/2}(-2+x)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{3}{64}\sqrt{3-x}\,\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}-\frac{3}{128}\sin^{-1}(5-2$$

[Out] -1/4*(3-x)^(5/2)*(-2+x)^(3/2)+3/128*arcsin(-5+2*x)+1/32*(3-x)^(3/2)*(-2+x)^(1/2)-1/8*(3-x)^(5/2)*(-2+x)^(1/2)+3/64*(3-x)^(1/2)*(-2+x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {52, 55, 633, 222}

$$-\frac{3}{128}\text{ArcSin}(5-2x)-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}\,(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}\,(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\,\sqrt{3-x}$$

Antiderivative was successfully verified.

[In] Int[(3 - x)^(3/2)*(-2 + x)^(3/2),x]

[Out] (3*Sqrt[3 - x]*Sqrt[-2 + x])/64 + ((3 - x)^(3/2)*Sqrt[-2 + x])/32 - ((3 - x)^(5/2)*Sqrt[-2 + x])/8 - ((3 - x)^(5/2)*(-2 + x)^(3/2))/4 - (3*ArcSin[5 - 2*x])/128

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int (3-x)^{3/2}(-2+x)^{3/2}\,dx = -\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{8}\int (3-x)^{3/2}\sqrt{-2+x}\,dx$$
$$= -\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{1}{16}\int \frac{(3-x)^{3/2}}{\sqrt{-2+x}}\,dx$$
$$= \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{6}$$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}($$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}($$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}($$
$$= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}($$

Mathematica [A]
time = 0.09, size = 82, normalized size = 0.90

$$-\frac{\sqrt{-6+5x-x^2}\left(\sqrt{-2+x}\,(675-1095x+650x^2-168x^3+16x^4)+3\sqrt{-3+x}\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-3+x}{-2+x}}}\right)\right)}{64(-3+x)\sqrt{-2+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
[Out] -1/64*(Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(675 - 1095*x + 650*x^2 - 168*x^3
 + 16*x^4) + 3*Sqrt[-3 + x]*ArcTanh[1/Sqrt[(-3 + x)/(-2 + x)]]))/((-3 + x)*
Sqrt[-2 + x])
```

Maple [A]
time = 0.16, size = 89, normalized size = 0.98

method	result
risch	$\frac{(16x^3-120x^2+290x-225)(-3+x)\sqrt{-2+x}\,\sqrt{(-2+x)(3-x)}}{64\sqrt{-(-3+x)(-2+x)}\,\sqrt{3-x}}+\frac{3\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{128\sqrt{-2+x}\,\sqrt{3-x}}$

default	$\frac{(3-x)^{\frac{3}{2}}(-2+x)^{\frac{5}{2}}}{4}+\frac{\sqrt{3-x}\,(-2+x)^{\frac{5}{2}}}{8}-\frac{\sqrt{3-x}\,(-2+x)^{\frac{3}{2}}}{32}-\frac{3\sqrt{3-x}\,\sqrt{-2+x}}{64}+\frac{3\sqrt{(-2+x)\,(3-}}{128\sqrt{-2+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(3/2)*(-2+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(3-x)^(3/2)*(-2+x)^(5/2)+1/8*(3-x)^(1/2)*(-2+x)^(5/2)-1/32*(3-x)^(1/2)*
(-2+x)^(3/2)-3/64*(3-x)^(1/2)*(-2+x)^(1/2)+3/128*((-2+x)*(3-x))^(1/2)/(-2+x
)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A]
time = 0.50, size = 67, normalized size = 0.74

$$\frac{1}{4}\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}x-\frac{5}{8}\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^{2}+5\,x-6}\,x-\frac{15}{64}\sqrt{-x^{2}+5\,x-6}+\frac{3}{128}\arcsin\left(2\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 5*x - 6)^(3/2)*x - 5/8*(-x^2 + 5*x - 6)^(3/2) + 3/32*sqrt(-x^2
+ 5*x - 6)*x - 15/64*sqrt(-x^2 + 5*x - 6) + 3/128*arcsin(2*x - 5)
```

Fricas [A]
time = 1.46, size = 62, normalized size = 0.68

$$-\frac{1}{64}\left(16\,x^{3}-120\,x^{2}+290\,x-225\right)\sqrt{x-2}\,\sqrt{-x+3}-\frac{3}{128}\arctan\left(\frac{\left(2\,x-5\right)\sqrt{x-2}\,\sqrt{-x+3}}{2\left(x^{2}-5\,x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 120*x^2 + 290*x - 225)*sqrt(x - 2)*sqrt(-x + 3) - 3/128*arc
tan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [C] Result contains complex when optimal does not.
time = 11.55, size = 199, normalized size = 2.19

$$\begin{cases}-\frac{3i\operatorname{acosh}\left(\sqrt{x-2}\right)}{64}-\frac{i(x-2)^{\frac{9}{2}}}{4\sqrt{x-3}}+\frac{5i(x-2)^{\frac{7}{2}}}{8\sqrt{x-3}}-\frac{13i(x-2)^{\frac{5}{2}}}{32\sqrt{x-3}}-\frac{i(x-2)^{\frac{3}{2}}}{64\sqrt{x-3}}+\frac{3i\sqrt{x-2}}{64\sqrt{x-3}} & \text{for}\ |x-2|>1\\ \frac{3\operatorname{asin}\left(\sqrt{x-2}\right)}{64}+\frac{(x-2)^{\frac{9}{2}}}{4\sqrt{3-x}}-\frac{5(x-2)^{\frac{7}{2}}}{8\sqrt{3-x}}+\frac{13(x-2)^{\frac{5}{2}}}{32\sqrt{3-x}}+\frac{(x-2)^{\frac{3}{2}}}{64\sqrt{3-x}}-\frac{3\sqrt{x-2}}{64\sqrt{3-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(3/2)*(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(x - 2))/64 - I*(x - 2)**(9/2)/(4*sqrt(x - 3)) +
5*I*(x - 2)**(7/2)/(8*sqrt(x - 3)) - 13*I*(x - 2)**(5/2)/(32*sqrt(x - 3)) -
 I*(x - 2)**(3/2)/(64*sqrt(x - 3)) + 3*I*sqrt(x - 2)/(64*sqrt(x - 3)), Abs(
x - 2) > 1), (3*asin(sqrt(x - 2))/64 + (x - 2)**(9/2)/(4*sqrt(3 - x)) - 5*(
x - 2)**(7/2)/(8*sqrt(3 - x)) + 13*(x - 2)**(5/2)/(32*sqrt(3 - x)) + (x - 2
)**(3/2)/(64*sqrt(3 - x)) - 3*sqrt(x - 2)/(64*sqrt(3 - x)), True))
```

Giac [A]
time = 1.25, size = 101, normalized size = 1.11

$$-\frac{1}{192}\left(2\left(4\left(6x+35\right)(x-2)+523\right)(x-2)+801\right)\sqrt{x-2}\,\sqrt{-x+3}+\frac{7}{24}\left(2\left(4x+15\right)(x-2)+69\right)\sqrt{x-2}\,\sqrt{-x+3}-4\left(2x+3\right)\sqrt{x-2}\,\sqrt{-x+3}+12\sqrt{x-2}\,\sqrt{-x+3}+\frac{3}{64}\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/192*(2*(4*(6*x + 35)*(x - 2) + 523)*(x - 2) + 801)*sqrt(x - 2)*sqrt(-x +
 3) + 7/24*(2*(4*x + 15)*(x - 2) + 69)*sqrt(x - 2)*sqrt(-x + 3) - 4*(2*x +
3)*sqrt(x - 2)*sqrt(-x + 3) + 12*sqrt(x - 2)*sqrt(-x + 3) + 3/64*arcsin(sqr
t(x - 2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (x-2)^{3/2}\,(3-x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(3/2)*(3 - x)^(3/2),x)
```

```
[Out] int((x - 2)^(3/2)*(3 - x)^(3/2), x)
```

3.1161 $\int \sqrt{3-x}\,\sqrt{-2+x}\,dx$

Optimal. Leaf size=51

$$\frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x} - \frac{1}{2}(3-x)^{3/2}\sqrt{-2+x} - \frac{1}{8}\sin^{-1}(5-2x)$$

```
[Out] 1/8*arcsin(-5+2*x)-1/2*(3-x)^(3/2)*(-2+x)^(1/2)+1/4*(3-x)^(1/2)*(-2+x)^(1/2
)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {52, 55, 633, 222}

$$-\frac{1}{8}\text{ArcSin}(5-2x) - \frac{1}{2}\sqrt{x-2}\,(3-x)^{3/2} + \frac{1}{4}\sqrt{x-2}\,\sqrt{3-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[3 - x]*Sqrt[-2 + x])/4 - ((3 - x)^(3/2)*Sqrt[-2 + x])/2 - ArcSin[5 -
2*x]/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x}\,\sqrt{-2+x}\,dx &= -\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{4}\int \frac{\sqrt{3-x}}{\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-2\right.\\ &= \frac{1}{4}\sqrt{3-x}\,\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\sin^{-1}(5-2x)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 1.41

$$\frac{\sqrt{-6+5x-x^2}\left(\sqrt{-2+x}\,(15-11x+2x^2)-\sqrt{-3+x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-3+x}{-2+x}}}\right)\right)}{4(-3+x)\sqrt{-2+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x]*Sqrt[-2 + x],x]
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(15 - 11*x + 2*x^2) - Sqrt[-3 + x]*ArcT
anh[1/Sqrt[(-3 + x)/(-2 + x)]]))/(4*(-3 + x)*Sqrt[-2 + x])
```

Maple [A]
time = 0.17, size = 61, normalized size = 1.20

method	result	size
default	$\frac{\sqrt{3-x}\,(-2+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{3-x}\,\sqrt{-2+x}}{4}+\frac{\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{8\sqrt{-2+x}\,\sqrt{3-x}}$	61
risch	$-\frac{(2x-5)(-3+x)\sqrt{-2+x}\,\sqrt{(-2+x)(3-x)}}{4\sqrt{-(-3+x)(-2+x)}\,\sqrt{3-x}}+\frac{\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{8\sqrt{-2+x}\,\sqrt{3-x}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(1/2)*(-2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(3-x)^(1/2)*(-2+x)^(3/2)-1/4*(3-x)^(1/2)*(-2+x)^(1/2)+1/8*((-2+x)*(3-x)
)^(1/2)/(-2+x)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A]
time = 0.48, size = 38, normalized size = 0.75

$$\frac{1}{2}\sqrt{-x^2+5x-6}\,x-\frac{5}{4}\sqrt{-x^2+5x-6}+\frac{1}{8}\arcsin\left(2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 5*x - 6)*x - 5/4*sqrt(-x^2 + 5*x - 6) + 1/8*arcsin(2*x - 5)
```

Fricas [A]
time = 1.71, size = 52, normalized size = 1.02

$$\frac{1}{4}\left(2x-5\right)\sqrt{x-2}\,\sqrt{-x+3}-\frac{1}{8}\arctan\left(\frac{(2x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2\left(x^2-5x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3) - 1/8*arctan(1/2*(2*x - 5)*sqrt(x -
2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.83, size = 124, normalized size = 2.43

$$\begin{cases}-\frac{i\operatorname{acosh}\left(\sqrt{x-2}\right)}{4}+\frac{i(x-2)^{\frac{5}{2}}}{2\sqrt{x-3}}-\frac{3i(x-2)^{\frac{3}{2}}}{4\sqrt{x-3}}+\frac{i\sqrt{x-2}}{4\sqrt{x-3}} & \text{for } |x-2|>1\\ \frac{\operatorname{asin}\left(\sqrt{x-2}\right)}{4}-\frac{(x-2)^{\frac{5}{2}}}{2\sqrt{3-x}}+\frac{3(x-2)^{\frac{3}{2}}}{4\sqrt{3-x}}-\frac{\sqrt{x-2}}{4\sqrt{3-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(1/2)*(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(x - 2))/4 + I*(x - 2)**(5/2)/(2*sqrt(x - 3)) - 3*I
*(x - 2)**(3/2)/(4*sqrt(x - 3)) + I*sqrt(x - 2)/(4*sqrt(x - 3)), Abs(x - 2)
 > 1), (asin(sqrt(x - 2))/4 - (x - 2)**(5/2)/(2*sqrt(3 - x)) + 3*(x - 2)**(
3/2)/(4*sqrt(3 - x)) - sqrt(x - 2)/(4*sqrt(3 - x)), True))
```

Giac [A]
time = 2.26, size = 42, normalized size = 0.82

$$\frac{1}{4}\left(2x+3\right)\sqrt{x-2}\,\sqrt{-x+3}-2\sqrt{x-2}\,\sqrt{-x+3}+\frac{1}{4}\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x - 2)*sqrt(-x + 3) - 2*sqrt(x - 2)*sqrt(-x + 3) + 1/4*a
rcsin(sqrt(x - 2))
```

Mupad [B]
time = 0.21, size = 41, normalized size = 0.80

$$\left(\frac{x}{2}-\frac{5}{4}\right)\,\sqrt{x-2}\,\sqrt{3-x}-\frac{\ln\left(x-\frac{5}{2}-\sqrt{x-2}\,\sqrt{3-x}\,1\mathrm{i}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(1/2)*(3 - x)^(1/2),x)
```

```
[Out] (x/2 - 5/4)*(x - 2)^(1/2)*(3 - x)^(1/2) - (log(x - (x - 2)^(1/2)*(3 - x)^(1
/2)*1i - 5/2)*1i)/8
```

3.1162 $\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(5-2x)$$

```
[Out] arcsin(-5+2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 633, 222}

$$-\text{ArcSin}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
```

```
[Out] -ArcSin[5 - 2*x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x}\,\sqrt{-2+x}}\,dx &= \int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-2x\right)\\
&= -\sin^{-1}(5-2x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(8) = 16$.
time = 0.03, size = 44, normalized size = 5.50

$$\frac{2\sqrt{-3+x}\,\sqrt{-2+x}\,\tanh^{-1}\left(\frac{\sqrt{-2+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(-2+x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
```

```
[Out] (2*Sqrt[-3 + x]*Sqrt[-2 + x]*ArcTanh[Sqrt[-2 + x]/Sqrt[-3 + x]])/Sqrt[-((-3
 + x)*(-2 + x))]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.
time = 0.17, size = 31, normalized size = 3.88

method	result	size
default	$\frac{\sqrt{(-2+x)(3-x)}\,\arcsin(2x-5)}{\sqrt{-2+x}\,\sqrt{3-x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(1/2)/(-2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-2+x)*(3-x))^(1/2)/(-2+x)^(1/2)/(3-x)^(1/2)*arcsin(2*x-5)
```

Maxima [A]
time = 0.47, size = 6, normalized size = 0.75

$$\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(6) = 12$.
time = 1.16, size = 32, normalized size = 4.00

$$-\arctan\left(\frac{(2x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2(x^2-5x+6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.75, size = 26, normalized size = 3.25

$$\begin{cases} -2i \operatorname{acosh}\left(\sqrt{x-2}\right) & \text{for } |x-2| > 1 \\ 2 \operatorname{asin}\left(\sqrt{x-2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(-2+x)**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(x - 2)), Abs(x - 2) > 1), (2*asin(sqrt(x - 2)),
True))
```

Giac [A]
time = 1.48, size = 8, normalized size = 1.00

$$2 \arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(sqrt(x - 2))
```

Mupad [B]
time = 0.18, size = 31, normalized size = 3.88

$$-4 \operatorname{atan}\left(\frac{\sqrt{x-2} - \sqrt{2}\,1\mathrm{i}}{\sqrt{3} - \sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(1/2)*(3 - x)^(1/2)),x)
[Out] -4*atan(((x - 2)^(1/2) - 2^(1/2)*1i)/(3^(1/2) - (3 - x)^(1/2)))
```

3.1163 $\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}$$

```
[Out] 2/(3-x)^(1/2)/(-2+x)^(1/2)-4*(3-x)^(1/2)/(-2+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$\frac{2}{\sqrt{3-x}\,\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] 2/(Sqrt[3 - x]*Sqrt[-2 + x]) - (4*Sqrt[3 - x])/Sqrt[-2 + x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx &= \frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}+2\int \frac{1}{\sqrt{3-x}\,(-2+x)^{3/2}}\,dx\\ &= \frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 0.57

$$\frac{2(-5+2x)}{\sqrt{-6+5x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] (2*(-5 + 2*x))/Sqrt[-6 + 5*x - x^2]
```

Maple [A]
time = 0.16, size = 30, normalized size = 0.81

method	result	size
gosper	$\frac{4x-10}{\sqrt{3-x}\,\sqrt{-2+x}}$	20
default	$\frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}$	30
risch	$\frac{2\sqrt{(-2+x)\,(3-x)}\,(2x-5)}{\sqrt{3-x}\,\sqrt{-2+x}\,\sqrt{-(-3+x)\,(-2+x)}}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(3/2)/(-2+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(3-x)^(1/2)/(-2+x)^(1/2)-4*(3-x)^(1/2)/(-2+x)^(1/2)
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.81

$$\frac{4\,x}{\sqrt{-x^2+5\,x-6}}-\frac{10}{\sqrt{-x^2+5\,x-6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 5*x - 6) - 10/sqrt(-x^2 + 5*x - 6)
```

Fricas [A]
time = 1.28, size = 29, normalized size = 0.78

$$-\frac{2\,(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}}{x^2-5\,x+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.22, size = 100, normalized size = 2.70

$$\begin{cases} -\frac{4i\sqrt{x-3}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} + \frac{2i\sqrt{x-3}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{for } |x-2|>1 \\ -\frac{4\sqrt{3-x}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} + \frac{2\sqrt{3-x}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-4*I*sqrt(x - 3)*(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*I*sq
rt(x - 3)/((x - 2)**(3/2) - sqrt(x - 2)), Abs(x - 2) > 1), (-4*sqrt(3 - x)*
(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*sqrt(3 - x)/((x - 2)**(3/2) - sq
rt(x - 2)), True))
```

Giac [A]
time = 1.78, size = 53, normalized size = 1.43

$$-\frac{\sqrt{-x+3}-1}{\sqrt{x-2}} - \frac{2\sqrt{x-2}\sqrt{-x+3}}{x-3} + \frac{\sqrt{x-2}}{\sqrt{-x+3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-x + 3) - 1)/sqrt(x - 2) - 2*sqrt(x - 2)*sqrt(-x + 3)/(x - 3) + sqrt
(x - 2)/(sqrt(-x + 3) - 1)
```

Mupad [B]
time = 0.25, size = 32, normalized size = 0.86

$$-\frac{4\,x\,\sqrt{3-x}-10\,\sqrt{3-x}}{\sqrt{x-2}\,\left(x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(3/2)*(3 - x)^(3/2)),x)
```

```
[Out] -(4*x*(3 - x)^(1/2) - 10*(3 - x)^(1/2))/((x - 2)^(1/2)*(x - 3))
```

3.1164 $\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}} \, dx$

Optimal. Leaf size=79

$$\frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}}-\frac{16\sqrt{3-x}}{3(-2+x)^{3/2}}-\frac{32\sqrt{3-x}}{3\sqrt{-2+x}}$$

```
[Out] 2/3/(3-x)^(3/2)/(-2+x)^(3/2)+4/(-2+x)^(3/2)/(3-x)^(1/2)-16/3*(3-x)^(1/2)/(-
2+x)^(3/2)-32/3*(3-x)^(1/2)/(-2+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {47, 37}

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] 2/(3*(3 - x)^(3/2)*(-2 + x)^(3/2)) + 4/(Sqrt[3 - x]*(-2 + x)^(3/2)) - (16*S
qrt[3 - x])/(3*(-2 + x)^(3/2)) - (32*Sqrt[3 - x])/(3*Sqrt[-2 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx = \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + 2\int \frac{1}{(3-x)^{3/2}(-2+x)^{5/2}}\,dx$$
$$= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} + 8\int \frac{1}{\sqrt{3-x}\,(-2+x)^{5/2}}\,dx$$
$$= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} - \frac{16\sqrt{3-x}}{3(-2+x)^{3/2}} + \frac{16}{3}\int \frac{1}{\sqrt{3-}}$$
$$= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}} + \frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}} - \frac{16\sqrt{3-x}}{3(-2+x)^{3/2}} - \frac{32\sqrt{3-x}}{3\sqrt{-2+x}}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.54

$$\frac{2(-235+294x-120x^2+16x^3)}{3(-3+x)(-2+x)\sqrt{-6+5x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (2*(-235 + 294*x - 120*x^2 + 16*x^3))/(3*(-3 + x)*(-2 + x)*Sqrt[-6 + 5*x -
x^2])
```

Maple [A]
time = 0.17, size = 58, normalized size = 0.73

method	result	size
gosper	$-\frac{2\left(16x^{3}-120x^{2}+294x-235\right)}{3\left(-2+x\right)^{\frac{3}{2}}\left(3-x\right)^{\frac{3}{2}}}$	30
default	$\frac{2}{3\left(3-x\right)^{\frac{3}{2}}\left(-2+x\right)^{\frac{3}{2}}}+\frac{4}{\left(-2+x\right)^{\frac{3}{2}}\sqrt{3-x}}-\frac{16\sqrt{3-x}}{3\left(-2+x\right)^{\frac{3}{2}}}-\frac{32\sqrt{3-x}}{3\sqrt{-2+x}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(5/2)/(-2+x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(3-x)^(3/2)/(-2+x)^(3/2)+4/(-2+x)^(3/2)/(3-x)^(1/2)-16/3*(3-x)^(1/2)/(-
2+x)^(3/2)-32/3*(3-x)^(1/2)/(-2+x)^(1/2)
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.75

$$\frac{32\,x}{3\,\sqrt{-x^2+5\,x-6}} - \frac{80}{3\,\sqrt{-x^2+5\,x-6}} + \frac{4\,x}{3\,(-x^2+5\,x-6)^{\frac{3}{2}}} - \frac{10}{3\,(-x^2+5\,x-6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x/sqrt(-x^2 + 5*x - 6) - 80/3/sqrt(-x^2 + 5*x - 6) + 4/3*x/(-x^2 + 5*x
 - 6)^(3/2) - 10/3/(-x^2 + 5*x - 6)^(3/2)
```

Fricas [A]
time = 1.10, size = 49, normalized size = 0.62

$$-\frac{2\,(16\,x^3-120\,x^2+294\,x-235)\sqrt{x-2}\,\sqrt{-x+3}}{3\,(x^4-10\,x^3+37\,x^2-60\,x+36)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*x^3 - 120*x^2 + 294*x - 235)*sqrt(x - 2)*sqrt(-x + 3)/(x^4 - 10*x^
3 + 37*x^2 - 60*x + 36)
```

Sympy [C] Result contains complex when optimal does not.
time = 5.93, size = 282, normalized size = 3.57

$$\begin{cases} -\frac{32\sqrt{-1+\frac{1}{x-2}}\,(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6}+\frac{48\sqrt{-1+\frac{1}{x-2}}\,(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{12\sqrt{-1+\frac{1}{x-2}}\,(x-2)}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{2\sqrt{-1+\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{for } \frac{1}{|x-2|}>1 \\ -\frac{32i\sqrt{1-\frac{1}{x-2}}\,(x-2)^3}{3x+3(x-2)^3-6(x-2)^2-6}+\frac{48i\sqrt{1-\frac{1}{x-2}}\,(x-2)^2}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{12i\sqrt{1-\frac{1}{x-2}}\,(x-2)}{3x+3(x-2)^3-6(x-2)^2-6}-\frac{2i\sqrt{1-\frac{1}{x-2}}}{3x+3(x-2)^3-6(x-2)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(5/2)/(-2+x)**(5/2),x)
```

```
[Out] Piecewise((-32*sqrt(-1 + 1/(x - 2))*(x - 2)**3/(3*x + 3*(x - 2)**3 - 6*(x -
 2)**2 - 6) + 48*sqrt(-1 + 1/(x - 2))*(x - 2)**2/(3*x + 3*(x - 2)**3 - 6*(x
 - 2)**2 - 6) - 12*sqrt(-1 + 1/(x - 2))*(x - 2)/(3*x + 3*(x - 2)**3 - 6*(x
- 2)**2 - 6) - 2*sqrt(-1 + 1/(x - 2))/(3*x + 3*(x - 2)**3 - 6*(x - 2)**2 -
6), 1/Abs(x - 2) > 1), (-32*I*sqrt(1 - 1/(x - 2))*(x - 2)**3/(3*x + 3*(x -
2)**3 - 6*(x - 2)**2 - 6) + 48*I*sqrt(1 - 1/(x - 2))*(x - 2)**2/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 12*I*sqrt(1 - 1/(x - 2))*(x - 2)/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 2*I*sqrt(1 - 1/(x - 2))/(3*x + 3*(x - 2)**3
- 6*(x - 2)**2 - 6), True))
```

Giac [A]
time = 1.10, size = 97, normalized size = 1.23

$$-\frac{\left(\sqrt{-x+3}-1\right)^3}{12\,(x-2)^{\frac{3}{2}}}-\frac{11\left(\sqrt{-x+3}-1\right)}{4\sqrt{x-2}}-\frac{2\,(8\,x-25)\sqrt{x-2}\,\sqrt{-x+3}}{3\,(x-3)^2}+\frac{(x-2)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{-x+3}-1\right)^2}{x-2}+1\right)}{12\left(\sqrt{-x+3}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(-x + 3) - 1)^3/(x - 2)^(3/2) - 11/4*(sqrt(-x + 3) - 1)/sqrt(x -
 2) - 2/3*(8*x - 25)*sqrt(x - 2)*sqrt(-x + 3)/(x - 3)^2 + 1/12*(x - 2)^(3/2
)*(33*(sqrt(-x + 3) - 1)^2/(x - 2) + 1)/(sqrt(-x + 3) - 1)^3
```

Mupad [B]
time = 0.37, size = 69, normalized size = 0.87

$$-\frac{32\,{\left(x-2\right)}^3\,\sqrt{3-x}-48\,{\left(x-2\right)}^2\,\sqrt{3-x}+2\,\sqrt{3-x}+12\,\left(x-2\right)\,\sqrt{3-x}}{\left(3\,x-6\right)\,\sqrt{x-2}\,{\left(x-3\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(5/2)*(3 - x)^(5/2)),x)
```

```
[Out] -(32*(x - 2)^3*(3 - x)^(1/2) - 48*(x - 2)^2*(3 - x)^(1/2) + 2*(3 - x)^(1/2)
 + 12*(x - 2)*(3 - x)^(1/2))/((3*x - 6)*(x - 2)^(1/2)*(x - 3)^2)
```

3.1165 $\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}} dx$

Optimal. Leaf size=21

$$\frac{x}{9\sqrt{3-x}\sqrt{3+x}}$$

```
[Out] 1/9*x/(3-x)^(1/2)/(3+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{39\}$

$$\frac{x}{9\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
[Out] x/(9*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}} dx = \frac{x}{9\sqrt{3-x}\sqrt{3+x}}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{9-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
[Out] x/(9*Sqrt[9 - x^2])
```

Maple [A]
time = 0.16, size = 30, normalized size = 1.43

method	result	size
gosper	$\frac{x}{9\sqrt{3-x}\,\sqrt{3+x}}$	16
default	$\frac{1}{3\sqrt{3-x}\,\sqrt{3+x}}-\frac{\sqrt{3-x}}{9\sqrt{3+x}}$	30
risch	$\frac{\sqrt{(3+x)(3-x)}\,x}{9\sqrt{3-x}\,\sqrt{3+x}\,\sqrt{-(-3+x)(3+x)}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-x)^(3/2)/(3+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(3-x)^(1/2)/(3+x)^(1/2)-1/9/(3+x)^(1/2)*(3-x)^(1/2)
```

Maxima [A]
time = 0.34, size = 12, normalized size = 0.57

$$\frac{x}{9\sqrt{-x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-x^2 + 9)
```

Fricas [A]
time = 1.09, size = 22, normalized size = 1.05

$$-\frac{\sqrt{x+3}\,x\sqrt{-x+3}}{9\,(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(x + 3)*x*sqrt(-x + 3)/(x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 71, normalized size = 3.38

$$\begin{cases} -\frac{\sqrt{-1+\frac{6}{x+3}}\,(x+3)}{9x-27}+\frac{3\sqrt{-1+\frac{6}{x+3}}}{9x-27} & \text{for}\ \frac{1}{|x+3|}>\frac{1}{6} \\ -\frac{i}{9\sqrt{1-\frac{6}{x+3}}}+\frac{i}{3\sqrt{1-\frac{6}{x+3}}\,(x+3)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 6/(x + 3))*(x + 3)/(9*x - 27) + 3*sqrt(-1 + 6/(x + 3)
)/(9*x - 27), 1/Abs(x + 3) > 1/6), (-I/(9*sqrt(1 - 6/(x + 3))) + I/(3*sqrt(
1 - 6/(x + 3))*(x + 3)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(15) = 30$.
time = 1.35, size = 62, normalized size = 2.95

$$\frac{\sqrt{6} - \sqrt{-x+3}}{36\sqrt{x+3}} - \frac{\sqrt{x+3}\sqrt{-x+3}}{18(x-3)} - \frac{\sqrt{x+3}}{36\left(\sqrt{6} - \sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/18*sqrt(x + 3)*sqrt(-x + 3)/(
x - 3) - 1/36*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3))
```

Mupad [B]
time = 0.36, size = 22, normalized size = 1.05

$$-\frac{x\sqrt{3-x}}{(9x-27)\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - x)^(1/2))/((9*x - 27)*(x + 3)^(1/2))
```

3.1166 $\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}} dx$

Optimal. Leaf size=24

$$\frac{x}{9\sqrt{3-bx}\sqrt{3+bx}}$$

[Out] 1/9*x/(-b*x+3)^(1/2)/(b*x+3)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {39}

$$\frac{x}{9\sqrt{3-bx}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

[In] Int[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]

[Out] x/(9*Sqrt[3 - b*x]*Sqrt[3 + b*x])

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}} dx = \frac{x}{9\sqrt{3-bx}\sqrt{3+bx}}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{9-b^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]

[Out] x/(9*Sqrt[9 - b^2*x^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(18) = 36.
time = 0.15, size = 42, normalized size = 1.75

method	result	size
gosper	$\frac{x}{9\sqrt{-bx+3}\,\sqrt{bx+3}}$	19
default	$\frac{1}{3b\sqrt{-bx+3}\,\sqrt{bx+3}}-\frac{\sqrt{-bx+3}}{9b\sqrt{bx+3}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3/b/(-b*x+3)^(1/2)/(b*x+3)^(1/2)-1/9/b/(b*x+3)^(1/2)*(-b*x+3)^(1/2)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.62

$$\frac{x}{9\sqrt{-b^2x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
[Out] 1/9*x/sqrt(-b^2*x^2 + 9)
```

Fricas [A]
time = 1.64, size = 29, normalized size = 1.21

$$-\frac{\sqrt{bx+3}\,\sqrt{-bx+3}\,x}{9\,(b^2x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
[Out] -1/9*sqrt(b*x + 3)*sqrt(-b*x + 3)*x/(b^2*x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.40, size = 73, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(3/2)/(b*x+3)**(3/2),x)
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), 9
/(b**2*x**2))/(18*pi**(3/2)*b) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()),
```

```
((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*x**2))/(18*pi**(
3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(18) = 36$.
time = 1.40, size = 77, normalized size = 3.21

$$\frac{\frac{\sqrt{6}-\sqrt{-bx+3}}{\sqrt{bx+3}} - \frac{2\sqrt{bx+3}\sqrt{-bx+3}}{bx-3} - \frac{\sqrt{bx+3}}{\sqrt{6}-\sqrt{-bx+3}}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*((sqrt(6) - sqrt(-b*x + 3))/sqrt(b*x + 3) - 2*sqrt(b*x + 3)*sqrt(-b*x
+ 3)/(b*x - 3) - sqrt(b*x + 3)/(sqrt(6) - sqrt(-b*x + 3)))/b
```

Mupad [B]
time = 0.46, size = 26, normalized size = 1.08

$$-\frac{x\sqrt{3-b\,x}}{\sqrt{b\,x+3}\,\left(9\,b\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - b*x)^(3/2)*(b*x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - b*x)^(1/2))/((b*x + 3)^(1/2)*(9*b*x - 27))
```

3.1167 $\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{18\sqrt{2}\,\sqrt{3-x}\,\sqrt{3+x}}$$

```
[Out] 1/36*x*2^(1/2)/(3-x)^(1/2)/(3+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{39\}$

$$\frac{x}{18\sqrt{2}\,\sqrt{3-x}\,\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
[Out] x/(18*Sqrt[2]*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{18\sqrt{2}\,\sqrt{3-x}\,\sqrt{3+x}}$$

Mathematica [A]
time = 0.10, size = 21, normalized size = 0.81

$$\frac{x}{18\sqrt{6-2x}\,\sqrt{3+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
[Out] x/(18*Sqrt[6 - 2*x]*Sqrt[3 + x])
```

Maple [A]
time = 0.14, size = 30, normalized size = 1.15

method	result	size
gosper	$-\frac{(-3+x)x}{9\sqrt{3+x}\,(6-2x)^{\frac{3}{2}}}$	19
default	$\frac{1}{6\sqrt{6-2x}\,\sqrt{3+x}}-\frac{\sqrt{6-2x}}{36\sqrt{3+x}}$	30
risch	$\frac{\sqrt{(3+x)\,(6-2x)}\,\sqrt{2}\,x}{36\sqrt{3+x}\,\sqrt{6-2x}\,\sqrt{-(-3+x)\,(3+x)}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6-2*x)^(3/2)/(3+x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(6-2*x)^(1/2)/(3+x)^(1/2)-1/36/(3+x)^(1/2)*(6-2*x)^(1/2)
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.46

$$\frac{x}{18\sqrt{-2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*x^2 + 18)
```

Fricas [A]
time = 1.53, size = 22, normalized size = 0.85

$$-\frac{\sqrt{x+3}\,x\sqrt{-2x+6}}{36\,(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(x + 3)*x*sqrt(-2*x + 6)/(x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 8.37, size = 92, normalized size = 3.54

$$\begin{cases} -\frac{\sqrt{2}\sqrt{-1+\frac{6}{x+3}}\,(x+3)}{36x-108}+\frac{3\sqrt{2}\sqrt{-1+\frac{6}{x+3}}}{36x-108} & \text{for}\ \frac{1}{|x+3|}>\frac{1}{6} \\ -\frac{\sqrt{2}\,i}{36\sqrt{1-\frac{6}{x+3}}}+\frac{\sqrt{2}\,i}{12\sqrt{1-\frac{6}{x+3}}\,(x+3)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(2)*sqrt(-1 + 6/(x + 3))*(x + 3)/(36*x - 108) + 3*sqrt(2)*s
qrt(-1 + 6/(x + 3))/(36*x - 108), 1/Abs(x + 3) > 1/6), (-sqrt(2)*I/(36*sqrt
(1 - 6/(x + 3))) + sqrt(2)*I/(12*sqrt(1 - 6/(x + 3))*(x + 3)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(18) = 36$.
time = 1.25, size = 71, normalized size = 2.73

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-x+3}\right)}{144\sqrt{x+3}}-\frac{\sqrt{2}\sqrt{x+3}\sqrt{-x+3}}{72\,(x-3)}-\frac{\sqrt{2}\sqrt{x+3}}{144\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/72*sqrt(2)*sqrt(x +
3)*sqrt(-x + 3)/(x - 3) - 1/144*sqrt(2)*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3)
)
```

Mupad [B]
time = 0.37, size = 22, normalized size = 0.85

$$-\frac{x\sqrt{6-2\,x}}{(36\,x-108)\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((6 - 2*x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*x)^(1/2))/((36*x - 108)*(x + 3)^(1/2))
```

3.1168 $\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx$

Optimal. Leaf size=29

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{3+bx}}$$

```
[Out] 1/36*x*2^(1/2)/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {39}

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx = \frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{3+bx}}$$

Mathematica [A]
time = 0.17, size = 19, normalized size = 0.66

$$\frac{x}{18\sqrt{18-2b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[18 - 2*b^2*x^2])
```

Maple [A]
time = 0.18, size = 42, normalized size = 1.45

method	result	size
gosper	$-\frac{(bx-3)x}{9\sqrt{bx+3}\,(-2bx+6)^{\frac{3}{2}}}$	24
default	$\frac{1}{6b\sqrt{-2bx+6}\,\sqrt{bx+3}}-\frac{\sqrt{-2bx+6}}{36b\sqrt{bx+3}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b/(-2*b*x+6)^(1/2)/(b*x+3)^(1/2)-1/36/b/(b*x+3)^(1/2)*(-2*b*x+6)^(1/2)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.52

$$\frac{x}{18\sqrt{-2\,b^2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*b^2*x^2 + 18)
```

Fricas [A]
time = 0.80, size = 29, normalized size = 1.00

$$-\frac{\sqrt{bx+3}\,\sqrt{-2\,bx+6}\,x}{36\,(b^2x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(b*x + 3)*sqrt(-2*b*x + 6)*x/(b^2*x^2 - 9)
```

Sympy [C] Result contains complex when optimal does not.
time = 10.81, size = 83, normalized size = 2.86

$$-\frac{\sqrt{2}\,iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b}+\frac{\sqrt{2}\,G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -sqrt(2)*I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2),
(0,)), 9/(b**2*x**2))/(72*pi**(3/2)*b) + sqrt(2)*meijerg(((-1/2, 0, 1/4, 1/
```

```
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*
x**2))/(72*pi**(3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(21) = 42$.
time = 0.96, size = 79, normalized size = 2.72

$$-\frac{\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{bx+3}\right)}{\sqrt{-bx+3}}+\frac{2\sqrt{2}\sqrt{-bx+3}}{\sqrt{bx+3}}-\frac{\sqrt{2}\sqrt{-bx+3}}{\sqrt{6}-\sqrt{bx+3}}}{144\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(2)*(sqrt(6) - sqrt(b*x + 3))/sqrt(-b*x + 3) + 2*sqrt(2)*sqrt(-
b*x + 3)/sqrt(b*x + 3) - sqrt(2)*sqrt(-b*x + 3)/(sqrt(6) - sqrt(b*x + 3)))/
b
```

Mupad [B]
time = 0.32, size = 26, normalized size = 0.90

$$-\frac{x\,\sqrt{6-2\,b\,x}}{\sqrt{b\,x+3}\,\left(36\,b\,x-108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 3)^(3/2)*(6 - 2*b*x)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*b*x)^(1/2))/((b*x + 3)^(1/2)*(36*b*x - 108))
```

3.1169 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{-ad+bdx}}\,dx$

Optimal. Leaf size=39

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2))/b/d^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]

[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{-ad+bdx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{-2ad+dx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}$$

Mathematica [A]
time = 0.07, size = 39, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-ad+bdx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-(a*d) + b*d*x]/(Sqrt[d]*Sqrt[a + b*x])])/(b*Sqrt[d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.
time = 0.17, size = 76, normalized size = 1.95

method	result	size
default	$\frac{\sqrt{(bx+a)(bdx-ad)}\,\ln\left(\frac{b^2 dx}{\sqrt{b^2 d}}+\sqrt{b^2 d\,x^2-a^2 d}\right)}{\sqrt{bx+a}\,\sqrt{bdx-ad}\,\sqrt{b^2 d}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(b*d*x-a*d))^(1/2)/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2)*ln(b^2*d*x/(b^2
*d)^(1/2)+(b^2*d*x^2-a^2*d)^(1/2))/(b^2*d)^(1/2)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 1.00

$$\frac{\log\left(2\,b^2dx + 2\,\sqrt{b^2dx^2-a^2d}\,b\sqrt{d}\right)}{b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*d*x + 2*sqrt(b^2*d*x^2 - a^2*d)*b*sqrt(d))/(b*sqrt(d))
```

Fricas [A]
time = 1.11, size = 108, normalized size = 2.77

$$\left[\frac{\log\left(2\,b^{2}dx^{2}+2\,\sqrt{bdx-ad}\,\sqrt{bx+a}\,b\sqrt{d}\,x-a^{2}d\right)}{2\,b\sqrt{d}},-\frac{\sqrt{-d}\,\arctan\left(\frac{\sqrt{bdx-ad}\,\sqrt{bx+a}\,b\sqrt{-d}\,x}{b^{2}dx^{2}-a^{2}d}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b^2*d*x^2 + 2*sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(d)*x - a^2*
d)/(b*sqrt(d)), -sqrt(-d)*arctan(sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(-d)
*x/(b^2*d*x^2 - a^2*d))/(b*d)]
```

Sympy [C] Result contains complex when optimal does not.
time = 13.31, size = 88, normalized size = 2.26

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\,\frac{a^{2}}{b^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\,\frac{a^{2}e^{2i\pi}}{b^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*d*x-a*d)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a**
2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(d)) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(2*I*pi)/(b**2*x*
*2))/(4*pi**(3/2)*b*sqrt(d))
```

Giac [A]
time = 1.71, size = 37, normalized size = 0.95

$$-\frac{2\,\log\left(\left|-\sqrt{bx+a}\,\sqrt{d}+\sqrt{(bx+a)d-2\,ad}\right|\right)}{b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b*x + a)*sqrt(d) + sqrt((b*x + a)*d - 2*a*d)))/(b*sqrt(d))
```

Mupad [B]
time = 0.22, size = 56, normalized size = 1.44

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,d\,x-a\,d}-\sqrt{-a\,d}\right)}{\sqrt{-b^2\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*d*x - a*d)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*d*x - a*d)^(1/2) - (-a*d)^(1/2)))/((-b^2*d)^(1/2)*((a + b*x
)^(1/2) - a^(1/2)))))/(-b^2*d)^(1/2)
```

3.1170 $\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}\,e}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}\,e}-\frac{\log\left(\frac{\sqrt{6-3ex}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{2+e}}{\sqrt{2+ex}}\right.}{\sqrt{2}\sqrt[4]{3}\,e}$$

```
[Out] -1/6*ln(3^(1/2)-(-e*x+2)^(1/4)*6^(1/2)/(e*x+2)^(1/4)+3^(1/2)*(-e*x+2)^(1/2)
/(e*x+2)^(1/2))*3^(3/4)/e*2^(1/2)+1/6*ln(3^(1/2)+(-e*x+2)^(1/4)*6^(1/2)/(e*
x+2)^(1/4)+3^(1/2)*(-e*x+2)^(1/2)/(e*x+2)^(1/2))*3^(3/4)/e*2^(1/2)-1/3*arct
an(-1+(-e*x+2)^(1/4)*2^(1/2)/(e*x+2)^(1/4))*2^(1/2)*3^(3/4)/e-1/3*arctan(1+
(-e*x+2)^(1/4)*2^(1/2)/(e*x+2)^(1/4))*2^(1/2)*3^(3/4)/e
```

Rubi [A]
time = 0.18, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}\,e}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt[4]{3}\,e}-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}\,e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx = -\frac{4\text{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}$$
$$= -\frac{4\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= \frac{2\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}\,x}\right.}{e}$$
$$= -\frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\log\left(\sqrt{2-ex}+\sqrt{2}\right.}{}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\log\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, size = 42, normalized size = 0.17

$$-\frac{\sqrt{2}(6-3ex)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{12}(6-3ex)\right)}{9e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
[Out] -1/9*(Sqrt[2]*(6 - 3*e*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (6 - 3*e*x
)/12])/e
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3ex+6)^{\frac{1}{4}}\,(ex+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

```
[Out] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x*e + 2)^(3/4)*(-3*x*e + 6)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(184) = 368$.
time = 1.06, size = 452, normalized size = 1.88

$$2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe^{4}-2e^{3})\sqrt{\frac{3\sqrt{\frac{1}{3}}(xe^{3}-2e^{2})e^{(-2)}+\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}}{xe-2}}e^{(-3)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-xe+2}{xe-2}\right)e^{(-1)}+2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe^{4}-2e^{3})\sqrt{\frac{3\sqrt{\frac{1}{3}}(xe^{3}-2e^{2})e^{(-2)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}}{xe-2}}e^{(-3)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}+xe-2}{xe-2}\right)e^{(-1)}-\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}e^{(-1)}\log\left(\frac{3\left(3\sqrt{\frac{1}{3}}(xe^{3}-2e^{2})e^{(-2)}+\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}\right)}{xe-2}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}e^{(-1)}\log\left(\frac{3\left(3\sqrt{\frac{1}{3}}(xe^{3}-2e^{2})e^{(-2)}-\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(xe+2)^{\frac{1}{4}}(-3xe+6)^{\frac{3}{4}}-\sqrt{xe+2}\sqrt{-3xe+6}\right)}{xe-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(1/3)^(1/4)*arctan((sqrt(3)*sqrt(2)*(1/3)^(3/4)*(x*e^4 - 2*e^3)*s
qrt((3*sqrt(1/3)*(x*e^3 - 2*e^2)*e^(-2) + sqrt(2)*(1/3)^(1/4)*(x*e + 2)^(1/
4)*(-3*x*e + 6)^(3/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e - 2))*e^(-3) -
 sqrt(2)*(1/3)^(3/4)*(x*e + 2)^(1/4)*(-3*x*e + 6)^(3/4) - x*e + 2)/(x*e - 2
))*e^(-1) + 2*sqrt(2)*(1/3)^(1/4)*arctan((sqrt(3)*sqrt(2)*(1/3)^(3/4)*(x*e^
4 - 2*e^3)*sqrt((3*sqrt(1/3)*(x*e^3 - 2*e^2)*e^(-2) - sqrt(2)*(1/3)^(1/4)*(
x*e + 2)^(1/4)*(-3*x*e + 6)^(3/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e -
2))*e^(-3) - sqrt(2)*(1/3)^(3/4)*(x*e + 2)^(1/4)*(-3*x*e + 6)^(3/4) + x*e -
 2)/(x*e - 2))*e^(-1) - 1/2*sqrt(2)*(1/3)^(1/4)*e^(-1)*log(3*(3*sqrt(1/3)*(
x*e^3 - 2*e^2)*e^(-2) + sqrt(2)*(1/3)^(1/4)*(x*e + 2)^(1/4)*(-3*x*e + 6)^(3
/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e - 2)) + 1/2*sqrt(2)*(1/3)^(1/4)*
e^(-1)*log(3*(3*sqrt(1/3)*(x*e^3 - 2*e^2)*e^(-2) - sqrt(2)*(1/3)^(1/4)*(x*e
 + 2)^(1/4)*(-3*x*e + 6)^(3/4) - sqrt(x*e + 2)*sqrt(-3*x*e + 6))/(x*e - 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{3^{\frac{3}{4}}\int\frac{1}{\sqrt[4]{-ex+2}\,(ex+2)^{\frac{3}{4}}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)**(1/4)/(e*x+2)**(3/4),x)
```

```
[Out] 3**(3/4)*Integral(1/((-e*x + 2)**(1/4)*(e*x + 2)**(3/4)), x)/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="giac")
[Out] integrate(1/((x*e + 2)^(3/4)*(-3*x*e + 6)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x+2)^{3/4}\,(6-3\,e\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)),x)
[Out] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)), x)
```

3.1171 $\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=144

$$\frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-\frac{14a^2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\right|}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] 14/5*a^2*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-14/15*I*(a-I*a*x)^(3/4)*(a+I*a*x
)^(3/4)-2/5*I*(a-I*a*x)^(7/4)*(a+I*a*x)^(3/4)/a-14/5*a^2*(x^2+1)^(1/4)*(cos
(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^
(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 42, 235, 233, 202}

$$-\frac{14a^2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{14a^2x}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (14*a^2*x)/(5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - ((14*I)/15)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(3/4) - (((2*I)/5)*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
/a - (14*a^2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4
)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{7/4}}{\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{1}{5}(7a)\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx\\
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{1}{5}\left(7a^2\right)\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a
\end{aligned}$$

$$\begin{aligned}
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{\left(7a^2\sqrt[4]{a^2+a^2x^2}\right)\int \frac{}{\sqrt[4]{}}}{5\sqrt[4]{a-iax}\,\sqrt[4]{a-}}\\
&= -\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}+\frac{\left(7a^2\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+}}}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+ia}}\\
&= \frac{14a^2x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-\\
&= \frac{14a^2x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{14}{15}i(a-iax)^{3/4}(a+iax)^{3/4}-\frac{2i(a-iax)^{7/4}(a+iax)^{3/4}}{5a}-
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.49

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{11/4}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(1/4),x]

[Out] (((2*I)/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[1/4, 11/4, 15/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.22, size = 104, normalized size = 0.72

method	result	size
risch	$-\frac{2(10i+3x)(x-i)(x+i)a^{2}}{15(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)a^{2}\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)

[Out] -2/15*(10*I+3*x)*(x-I)*(x+I)*a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+7/5/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(1/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")

[Out] -1/15*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 10*I*x - 21) - 15*x*integral(14/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(x^4 + x^2), x))/x

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia(x+i)\right)^{\frac{7}{4}}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(1/4), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%{i
,[0,1]%%%}] at parameters values [99,84]ext_reduce Error: Bad Argument Type
integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{7/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(1/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(1/4), x)
```

3.1172 $\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=106

$$\frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{2a\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2*a*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(3/4)
/a-2*a*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*Ellipt
icE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {52, 42, 235, 233, 202}

$$-\frac{2a\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (2*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (((2*I)/3)*(a - I*a*x)^(3/4
)*(a + I*a*x)^(3/4))/a - (2*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((
a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+a\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx \\
&= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= -\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{\left(a\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{\left(a\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{2ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}-\frac{2a\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.66

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{7/4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[1/4,
 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 94, normalized size = 0.89

method	result	size
risch	$-\frac{2i(x-i)(x+i)a}{3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)a\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*(x-I)*(x+I)*a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+1/(a^2)^(1/4)*x*
hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x
))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^2),
 x) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(I*x - 3))/(a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{3}{4}}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(1/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(4*i*((sageVA
Ra+(-i)*sageVARa*sageVARx)^(1/4))^6/(-((sageVARa+(-i)*sageVARa*sageVARx)^(1
/4))^4+2*
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(1/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(1/4), x)
```

3.1173 $\int \frac{1}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} dx$

Optimal. Leaf size=71

$$\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] 2*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)-2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^
(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1
/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {42, 235, 233, 202}

$$\frac{2x}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}} - \frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) - (2*(1 + x^2)^(1/4)*EllipticE[
ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx &= \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{\sqrt[4]{1+x^2}\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.99

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)),x]
[Out] (((2*I)/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{1}{4}}\,(iax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
[Out] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (a^2*x*integral(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2),
 x) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^2*x)
```

Sympy [A]
time = 1.71, size = 102, normalized size = 1.44

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{8},\frac{5}{8},1 & \frac{1}{4},\frac{1}{2},\frac{3}{4}\\ -\frac{1}{4},\frac{1}{8},\frac{1}{4},\frac{5}{8},\frac{3}{4} & 0\end{matrix}\middle|\frac{e^{-3i\pi}}{x^2}\right)e^{\frac{i\pi}{4}}}{4\pi\sqrt{a}\,\Gamma\left(\frac{1}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{3}{8},0,\frac{1}{8},\frac{1}{2},1 & \\ -\frac{3}{8},\frac{1}{8} & -\frac{1}{2},-\frac{1}{4},0,0\end{matrix}\middle|\frac{e^{-i\pi}}{x^2}\right)}{4\pi\sqrt{a}\,\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] -I*meijerg(((1/8, 5/8, 1), (1/4, 1/2, 3/4)), ((-1/4, 1/8, 1/4, 5/8, 3/4), (
0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*sqrt(a)*gamma(1/4)) + I*me
ijerg(((-1/2, -3/8, 0, 1/8, 1/2, 1), ()), ((-3/8, 1/8), (-1/2, -1/4, 0, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*sqrt(a)*gamma(1/4))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a-a\,x\,1\mathrm{i}\right)^{1/4}\,\left(a+a\,x\,1\mathrm{i}\right)^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(1/4)), x)
```

3.1174 $\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=78

$$-\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^
2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*
x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {50, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (-2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ellipti
cE[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 50

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{5/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+a\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 68, normalized size = 0.87

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)),x]
[Out] ((-2*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, 1/2 - (I/
2)*x])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 94, normalized size = 1.21

method	result	size
risch	$\frac{2x-2i}{a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x-I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] ((a^3*x^2 + I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x) - 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 + I
*a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,\left(-ia\,(x+i)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{5/4}\,(a + a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(1/4)), x)
```

3.1175 $\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=82

$$-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -4/5*I/a/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+2/5*(x^2+1)^(1/4)*(cos(1/2*arctan(
x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^2/(
a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/5)/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Ell
ipticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx &= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx \\
&= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}} \\
&= -\frac{4i}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\sqrt[4]{a+iax}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.85

$$
-\frac{2i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a(a-iax)^{5/4}\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 105, normalized size = 1.28

method	result	size
risch	$\frac{\frac{2}{5}x^2+\frac{2}{5}ix+\frac{4}{5}}{(x+i)a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(x^2+2+I*x)/(x+I)/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-
a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x + 2*I) + 5*(a^4*x^2 + 2*I*a^
4*x - a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^
4), x))/(a^4*x^2 + 2*I*a^4*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\,\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{9/4}\,(a + a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(1/4)), x)
```

3.1176 $\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=115

$$-\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -4/15*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)-2/9*I*(a+I*a*x)^(3/4)/a^2/(a-I*
a*x)^(9/4)+2/15*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x
))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4
)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{15a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}}-\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/15)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/9)*(a + I*a
*x)^(3/4))/(a^2*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]
/2, 2])/(15*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}} + \frac{\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx}{3a}$$

$$= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}} + \frac{\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{15a}$$

$$= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}} + \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)}}{15a\sqrt[4]{a-iax}\sqrt[4]{a+i}}$$

$$= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}} + \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,d}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+ia}}$$

$$= -\frac{4i}{15a^2(a-iax)^{5/4}\sqrt[4]{a+iax}} - \frac{2i(a+iax)^{3/4}}{9a^2(a-iax)^{9/4}} + \frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x\right.}{15a^3\sqrt[4]{a-iax}\sqrt[4]{a+i}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.61

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{9}{4},\frac{1}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/9)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, 1/2
- (I/2)*x])/(a*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 113, normalized size = 0.98

method	result	size
risch	$\frac{\frac{2}{15}x^3+\frac{4}{15}ix^2-\frac{4}{45}x+\frac{22}{45}i}{(x+i)^2 a^3 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}} - \frac{x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{15(a^2)^{\frac{1}{4}} a^3 (-a(ix-1))^{\frac{1}{4}} (a(ix+1))^{\frac{1}{4}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(6*I*x^2+3*x^3-2*x+11*I)/(x+I)^2/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(
1/4)-1/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*
(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(13/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 9*I*x - 11) + 45*(a^5
*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I*a^5)*integral(-1/15*(I*a*x + a)^(3/4)*(-I*
a*x + a)^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I*a^
5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,\left(-ia\,(x+i)\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(13/4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(1/4)), x)
```

3.1177 $\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=148

$$-\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{39a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -4/39*I/a^3/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)-2/13*I*(a+I*a*x)^(3/4)/a^2/(a-I
*a*x)^(13/4)-10/117*I*(a+I*a*x)^(3/4)/a^3/(a-I*a*x)^(9/4)+2/39*(x^2+1)^(1/4
)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(
x)),2^(1/2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{39a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}-\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-4*I)/39)/(a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) - (((2*I)/13)*(a + I*
a*x)^(3/4))/(a^2*(a - I*a*x)^(13/4)) - (((10*I)/117)*(a + I*a*x)^(3/4))/(a^
3*(a - I*a*x)^(9/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(39*a^
4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{17/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}+\frac{5\int \frac{1}{(a-iax)^{13/4}\sqrt[4]{a+iax}}\,dx}{13a}\\ &= -\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\frac{5\int \frac{1}{(a-iax)^{9/4}\sqrt[4]{a+iax}}\,dx}{39a^2}\\ &= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\\ &= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\\ &= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\\ &= -\frac{4i}{39a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i(a+iax)^{3/4}}{13a^2(a-iax)^{13/4}}-\frac{10i(a+iax)^{3/4}}{117a^3(a-iax)^{9/4}}+\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.47

$$-\frac{2i2^{3/4}\sqrt[4]{1+ix}\ {}_2F_1\left(-\frac{13}{4},\frac{1}{4};-\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13a(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/13)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, 1/
2 - (I/2)*x])/(a*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 114, normalized size = 0.77

method	result	size
risch	$\frac{\frac{2}{39}x^4+\frac{2}{13}ix^3-\frac{16}{117}x^2-\frac{40}{117}}{(x+i)^3a^4(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{39(a^2)^{\frac{1}{4}}a^4(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/117*(9*I*x^3+3*x^4-20-8*x^2)/(x+I)^3/a^4/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^
(1/4)-1/39/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(-1+I*x)
*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(17/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/117*(2*(3*x^3 + 12*I*x^2 - 20*x - 20*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3
/4) + 117*(a^6*x^4 + 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)*integral(-1
/39*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(a^6*x^4 + 4*
I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5458 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{17/4}\,(a + a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(1/4)), x)
```

3.1178 $\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=256

$$-\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{i\log\left(1+\frac{\sqrt{a-}}{\sqrt{a+}}\right.}{}$$

[Out] -I*(a-I*a*x)^(1/4)*(a+I*a*x)^(3/4)/a-1/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)+1/2*I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-1/4*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)+1/4*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)

Rubi [A]

time = 0.12, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{i\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]

[Out] ((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (I*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$
$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+2i\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx,x,\sqrt[4]{a-iax}\right)$$
$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+2i\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$
$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+i\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d$$
$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{1}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{1}{2}i\text{Subst}\Big($$
$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}+\frac{i\log\left(1+\frac{\sqrt{a-}}{\sqrt{a-}}\right.}{}$$
$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{\sqrt{2}}$$

Mathematica [A]
time = 0.34, size = 111, normalized size = 0.43

$$\frac{\sqrt[4]{-i+x}\,\sqrt[4]{a-iax}\left((-i+x)^{3/4}\sqrt[4]{i+x}+i\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+i\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{\sqrt[4]{i+x}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
[Out] ((-I + x)^(1/4)*(a - I*a*x)^(1/4)*((-I + x)^(3/4)*(I + x)^(1/4) + I*ArcTan[
(I + x)^(1/4)/(-I + x)^(1/4)] + I*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)]))/(
(I + x)^(1/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.43, size = 479, normalized size = 1.87

method	result
risch	$\frac{i(x-i)(x+i)(-a(ix-1))^{\frac{1}{4}}}{(ix-1)(a(ix+1))^{\frac{1}{4}}}-\left(\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(-\frac{\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)x^2+i\,\mathrm{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2i\right.}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] I*(x-I)*(x+I)*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)-(1/2*RootOf(_Z
^2-I)*ln(-((1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2+I*RootOf(_Z^2-I)*
(1-x^4-2*I*x^3-2*I*x)^(3/4)+x^3+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+2*I*RootOf(
_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+2*I*x^2-(1-x^4-2*I*x^3-2*I*x)^(1/2)-R
ootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-x)/(-1+I*x)^2)+1/2*I*RootOf(_Z^2-
I)*ln(-(I*(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(
1-x^4-2*I*x^3-2*I*x)^(1/4)*x+x^3+RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)
-I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x-I*RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/
4)+2*I*x^2+(1-x^4-2*I*x^3-2*I*x)^(1/2)-x)/(-1+I*x)^2))*(-a*(-1+I*x))^(1/4)/
(-1+I*x)*(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

Fricas [A]
time = 0.74, size = 194, normalized size = 0.76

$$\frac{\sqrt{i}\,a\log\left(\frac{\sqrt{i}\,(ax-ia)+(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{i}\,a\log\left(-\frac{\sqrt{i}\,(ax-ia)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)+\sqrt{-i}\,a\log\left(\frac{\sqrt{-i}\,(ax-ia)+(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{-i}\,a\log\left(-\frac{\sqrt{-i}\,(ax-ia)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-2i\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(I)*a*log((sqrt(I)*(a*x - I*a) + (I*a*x + a)^(3/4)*(-I*a*x + a)^(1
/4))/(x - I)) - sqrt(I)*a*log(-(sqrt(I)*(a*x - I*a) - (I*a*x + a)^(3/4)*(-I
```

```
*a*x + a)^(1/4))/(x - I)) + sqrt(-I)*a*log((sqrt(-I)*(a*x - I*a) + (I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-I)*a*log(-(sqrt(-I)*(a*x - I
*a) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 2*I*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(1/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(4*i*((sageVA
Ra+(-i)*sageVARa*sageVARx)^(1/4))^4/(-((sageVARa+(-i)*sageVARa*sageVARx)^(1
/4))^4+2*
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4),x)
```

```
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4), x)
```

3.1179 $\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=233

$$-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{a+}}\right.}{\sqrt{2}\,a}$$

```
[Out] -1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*
x)^(1/2))/a*2^(1/2)+1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I
*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(
a+I*a*x)^(1/4))*2^(1/2)/a+I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4
))*2^(1/2)/a
```

Rubi [A]

time = 0.09, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{i\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a -
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$
$$= \frac{(4i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{a}$$
$$= \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a}}{\sqrt[4]{a}}\right.}{a}$$
$$= -\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{\sqrt{2}\,a}$$
$$= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} -$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.33

$$\frac{2\sqrt[4]{-1}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)-i\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
[Out] (2*(-1)^(1/4)*(ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)] -
I*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]))/a
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{3}{4}}\,(iax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

```
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A]
time = 0.80, size = 221, normalized size = 0.95

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x-i\,a^2)\sqrt{\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x-i\,a^2)\sqrt{-\frac{4i}{a^2}}+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x-i\,a^2)\sqrt{-\frac{4i}{a^2}}-2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{2\,(x-i)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3
/4)*(-I*a*x + a)^(1/4))/(x - I)) - 1/2*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I*a
^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 1/2*
sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/(x - I)) - 1/2*sqrt(-4*I/a^2)*log(-1/2*((a^2*x - I*a^
2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,\left(-ia\,(x+i)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)), x)
```

3.1180 $\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

```
[Out] -2/3*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Mathematica [A]
time = 0.09, size = 33, normalized size = 1.00

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Maple [A]
time = 0.16, size = 31, normalized size = 0.94

method	result	size
risch	$\frac{\frac{2x}{3}-\frac{2i}{3}}{a(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
[Out] 2/3/a/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(x-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [A]
time = 0.86, size = 31, normalized size = 0.94

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,(a^3x+i\,a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x + I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[4]{ia\,(x-i)}\,\left(-ia\,(x+i)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(x-\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{3\,a^2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -(2*(x - 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^2*(x*1i - 1)*(a*(x*1i + 1))^(1/4))
```

3.1181 $\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=67

$$-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}$$

```
[Out] -2/7*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(7/4)-4/21*I*(a+I*a*x)^(3/4)/a^3/(a-I*
a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {47, 37}

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I
*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}} + \frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{7a}$$
$$= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}} - \frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.67

$$\frac{2(5-2ix)(a+iax)^{3/4}}{21a^3(i+x)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
[Out] (2*(5 - (2*I)*x)*(a + I*a*x)^(3/4))/(21*a^3*(I + x)*(a - I*a*x)^(3/4))
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.66

method	result	size
risch	$\frac{\frac{4}{21}x^2+\frac{2}{7}ix+\frac{10}{21}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
[Out] 2/21/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(2*x^2+5+3*I*x)/(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [A]
time = 2.39, size = 44, normalized size = 0.66

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+5i)}{21\,(a^4x^2+2i\,a^4x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + 5*I)/(a^4*x^2 + 2*I*a^4*x
- a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\left(-ia\left(x+i\right)\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.67, size = 46, normalized size = 0.69

$$-\frac{\left(-a\left(-1+x\,1{}\mathrm{i}\right)\right)^{1/4}\left(2\,x^2+x\,3{}\mathrm{i}+5\right)\,2{}\mathrm{i}}{21\,a^3\,{\left(-1+x\,1{}\mathrm{i}\right)}^2\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -((-a*(x*1i - 1))^(1/4)*(x*3i + 2*x^2 + 5)*2i)/(21*a^3*(x*1i - 1)^2*(a*(x*1
i + 1))^(1/4))
```

3.1182 $\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}$$

```
[Out] -2/11*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(11/4)-8/77*I*(a+I*a*x)^(3/4)/a^3/(a-
I*a*x)^(7/4)-16/231*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) - (((8*I)/77)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(7/4)) - (((16*I)/231)*(a + I*a*x)^(3/4))/(
a^4*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} + \frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{11a}$$
$$= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} - \frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}} + \frac{8\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{77a^2}$$
$$= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}} - \frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}} - \frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.52

$$\frac{2(a+iax)^{3/4}\,(41i+28x-8ix^2)}{231a^4(i+x)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(41*I + 28*x - (8*I)*x^2))/(231*a^4*(I + x)^2*(a - I*a
*x)^(3/4))
```

Maple [A]
time = 0.16, size = 50, normalized size = 0.50

method	result	size
risch	$\frac{\frac{16}{231}x^3+\frac{40}{231}ix^2-\frac{26}{231}x+\frac{82}{231}i}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(20*I*x^2+8*x^3-13*x+41*I)/
(x+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)

Fricas [A]
time = 1.14, size = 57, normalized size = 0.57

$$\frac{2 \left(i\, a x+a\right)^{\frac{3}{4}} \left(-i\, a x+a\right)^{\frac{1}{4}} \left(8\, x^{2}+28 i\, x-41\right)}{231 \left(a^{5} x^{3}+3 i\, a^{5} x^{2}-3\, a^{5} x-i\, a^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")

[Out] 2/231*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 28*I*x - 41)/(a^5*x^3 + 3*I*a^5*x^2 - 3*a^5*x - I*a^5)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(1/4),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDone

Mupad [B]
time = 0.75, size = 51, normalized size = 0.51

$$\frac{{\left(x-\mathrm{i}\right)}^4\,{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)}^{1/4}\,\left(8\,x^2+x\,28{}\mathrm{i}-41\right)\,2{}\mathrm{i}}{231\,a^4\,{\left(x^2+1\right)}^3\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(1/4)),x)

[Out] ((x - 1i)^4*(-a*(x*1i - 1))^(1/4)*(x*28i + 8*x^2 - 41)*2i)/(231*a^4*(x^2 + 1)^3*(a*(x*1i + 1))^(1/4))

3.1183 $\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=133

$$-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}$$

```
[Out] -2/15*I*(a+I*a*x)^(3/4)/a^2/(a-I*a*x)^(15/4)-4/55*I*(a+I*a*x)^(3/4)/a^3/(a-
I*a*x)^(11/4)-16/385*I*(a+I*a*x)^(3/4)/a^4/(a-I*a*x)^(7/4)-32/1155*I*(a+I*a
*x)^(3/4)/a^5/(a-I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) - (((4*I)/55)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(11/4)) - (((16*I)/385)*(a + I*a*x)^(3/4))/
(a^4*(a - I*a*x)^(7/4)) - (((32*I)/1155)*(a + I*a*x)^(3/4))/(a^5*(a - I*a*x
)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} + \frac{2\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx}{5a}$$
$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} + \frac{8\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{55a^2}$$
$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} + \frac{16\int \frac{1}{(a-iax)}}{3}$$
$$= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}} - \frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}} - \frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}} - \frac{32i(a+i}{1155a^5(a-}$$

Mathematica [A]
time = 0.10, size = 57, normalized size = 0.43

$$\frac{2(a+iax)^{3/4}(-159+138ix+72x^2-16ix^3)}{1155a^5(i+x)^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(-159 + (138*I)*x + 72*x^2 - (16*I)*x^3))/(1155*a^5*(I
 + x)^3*(a - I*a*x)^(3/4))
```

Maple [A]
time = 0.17, size = 55, normalized size = 0.41

method	result	size
risch	$\frac{\frac{32}{1155}x^4+\frac{16}{165}ix^3-\frac{4}{35}x^2-\frac{2}{55}ix-\frac{106}{385}}{a^4(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/a^4/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(56*I*x^3+16*x^4-21*I*x-15
9-66*x^2)/(x+I)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

Fricas [A]
time = 1.03, size = 68, normalized size = 0.51

$$\frac{2\,(16\,x^3+72i\,x^2-138\,x-159i)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{1155\,(a^6x^4+4i\,a^6x^3-6\,a^6x^2-4i\,a^6x+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 2/1155*(16*x^3 + 72*I*x^2 - 138*x - 159*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(
1/4)/(a^6*x^4 + 4*I*a^6*x^3 - 6*a^6*x^2 - 4*I*a^6*x + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(19/4)/(a+I*a*x)**(1/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7772 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.79, size = 57, normalized size = 0.43

$$-\frac{{(x-\mathrm{i})}^5\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}\,\left(-16\,x^3-x^2\,72\mathrm{i}+138\,x+159\mathrm{i}\right)\,2\mathrm{i}}{1155\,a^5\,{\left(x^2+1\right)}^4\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(19/4)*(a + a*x*1i)^(1/4)),x)
[Out] -((x - 1i)^5*(-a*(x*1i - 1))^(1/4)*(138*x - x^2*72i - 16*x^3 + 159i)*2i)/(1
155*a^5*(x^2 + 1)^4*(a*(x*1i + 1))^(1/4))
```

3.1184 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=256

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\log\left(1+\frac{\sqrt{\ }}{\sqrt{\ }}\right.}{}$$

```
[Out] -I*(a-I*a*x)^(3/4)*(a+I*a*x)^(1/4)/a-3/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)
/(a+I*a*x)^(1/4))*2^(1/2)+3/2*I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^
(1/4))*2^(1/2)+3/4*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)
^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)-3/4*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x
)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$,
Rules used = {52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3i\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((3*I)*ArcTan[1 + (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((3*I)/2)*Log[1 + Sqrt[a
- I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
Sqrt[2] - (((3*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a
- I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{1}{2}(3a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx\\ &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx,x,\sqrt[4]{a-iax}\right)\\ &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\\ &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-3i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+3i\text{Subst}\left(\int \frac{1+}{1+}\right.\\ &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{3}{2}i\text{Subst}\\ &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}-\frac{3i\log\left(1+\right.}{}\\ &= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 111, normalized size = 0.43

$$\frac{(-i+x)^{3/4}(a-iax)^{3/4}\left(\sqrt[4]{-i+x}\,(i+x)^{3/4}-3i\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+3i\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{(i+x)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I + x)^(3/4)*(a - I*a*x)^(3/4)*((-I + x)^(1/4)*(I + x)^(3/4) - (3*I)*Arc
Tan[(I + x)^(1/4)/(-I + x)^(1/4)] + (3*I)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1
/4)]))/((I + x)^(3/4)*(a + I*a*x)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.60, size = 465, normalized size = 1.82

method	result
risch	$-\frac{i(x-i)(x+i)a}{(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}+\frac{\left(-\frac{3\,\mathrm{RootOf}\left(\_\_Z^2-i\right)\ln\left(-\frac{-\mathrm{RootOf}\left(\_\_Z^2-i\right)\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2-i\,\mathrm{RootOf}\left(\_\_Z^2-i\right)\left(-x^4+2\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(x-I)*(x+I)/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*a+(-3/2*RootOf(_Z^2-I)
*ln(-(-RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x^2-I*RootOf(_Z^2-I)*(1-x
^4+2*I*x^3+2*I*x)^(3/4)+x^3+2*I*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*
x+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x-2*I*x^2+RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I
*x)^(1/4)+(1-x^4+2*I*x^3+2*I*x)^(1/2)-x)/(1+I*x)^2)-3/2*I*RootOf(_Z^2-I)*ln
(-(-I*(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(1-x^
4+2*I*x^3+2*I*x)^(1/4)*x+x^3-RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)-I*(
1-x^4+2*I*x^3+2*I*x)^(1/2)*x+I*RootOf(_Z^2-I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)-2
*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)-x)/(1+I*x)^2))/(a*(1+I*x))^(3/4)*(-(-1+I
*x)*(1+I*x)^3)^(1/4)/(-a*(-1+I*x))^(1/4)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

Fricas [A]
time = 0.84, size = 198, normalized size = 0.77

$$\frac{\sqrt{9i}\,a\log\left(\frac{\sqrt{9i}\,(ax+ia)+3\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-\sqrt{9i}\,a\log\left(-\frac{\sqrt{9i}\,(ax+ia)-3\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)+\sqrt{-9i}\,a\log\left(\frac{\sqrt{-9i}\,(ax+ia)+3\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-\sqrt{-9i}\,a\log\left(-\frac{\sqrt{-9i}\,(ax+ia)-3\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(x+i)}\right)-2i\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(9*I)*a*log(1/3*(sqrt(9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a
*x + a)^(3/4))/(x + I)) - sqrt(9*I)*a*log(-1/3*(sqrt(9*I)*(a*x + I*a) - 3*(
```

```
I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) + sqrt(-9*I)*a*log(1/3*(sqrt(
-9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - sqrt
(-9*I)*a*log(-1/3*(sqrt(-9*I)*(a*x + I*a) - 3*(I*a*x + a)^(1/4)*(-I*a*x + a
)^(3/4))/(x + I)) - 2*I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia(x-i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(3/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(4*i*((sageVA
Ra+(-i)*sageVARa*sageVARx)^(1/4))^6/((-((sageVARa+(-i)*sageVARa*sageVARx)^(
1/4))^4+2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4), x)
```

3.1185 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=233

$$-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{a+}}\right.}{\sqrt{2}\,a}$$

```
[Out] 1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x
)^(1/2))/a*2^(1/2)-1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*
a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a
+I*a*x)^(1/4))*2^(1/2)/a+I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)
)*2^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$,
Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$-\frac{i\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a +
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx = \frac{(4i)\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$
$$= \frac{(4i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$
$$= -\frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-ia}}{\sqrt[4]{a+ia}}\right.}{a}$$
$$= \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a}}{\sqrt[4]{a}}\right.}{a}$$
$$= \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{\sqrt{2}\,a}$$
$$= -\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} +$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.33

$$-\frac{2\sqrt[4]{-1}\left(\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)-i\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (-2*(-1)^(1/4)*(ArcTanh[((-1)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)] -
 I*ArcTanh[((-1)^(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)]))/a
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{1}{4}}\,(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [A]
time = 0.64, size = 221, normalized size = 0.95

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x+ia^2)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2(x+i)}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x+ia^2)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2(x+i)}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x+ia^2)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2(x+i)}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x+ia^2)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2(x+i)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(1
/4)*(-I*a*x + a)^(3/4))/(x + I)) - 1/2*sqrt(4*I/a^2)*log(-1/2*((a^2*x + I*a
^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) + 1/2*
sqrt(-4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4
)*(-I*a*x + a)^(3/4))/(x + I)) - 1/2*sqrt(-4*I/a^2)*log(-1/2*((a^2*x + I*a^
2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{3}{4}}\sqrt[4]{-ia(x+i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)), x)
```

3.1186 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

```
[Out] -2*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Maple [A]
time = 0.16, size = 31, normalized size = 1.00

method	result	size
risch	$\frac{2x-2i}{a(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
[Out] 2/a/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(x-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [A]
time = 0.88, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x+i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x + I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)), x)
```

3.1187 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}$$

```
[Out] -2/5*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(5/4)-4/5*I*(a+I*a*x)^(1/4)/a^3/(a-I*a
*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}} + \frac{2\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{5a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}} - \frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.67

$$\frac{2(3-2ix)\sqrt[4]{a+iax}}{5a^3(i+x)\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
[Out] (2*(3 - (2*I)*x)*(a + I*a*x)^(1/4))/(5*a^3*(I + x)*(a - I*a*x)^(1/4))
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.66

method	result	size
risch	$\frac{\frac{4}{5}x^2+\frac{2}{5}ix+\frac{6}{5}}{a^2(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
[Out] 2/5/a^2/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(2*x^2+3+I*x)/(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [A]
time = 0.96, size = 44, normalized size = 0.66

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x+3i)}{5\,(a^4x^2+2i\,a^4x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(2*x + 3*I)/(a^4*x^2 + 2*I*a^4*x -
 a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}}\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{9/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)), x)
```

3.1188 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}$$

```
[Out] -2/9*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(9/4)-8/45*I*(a+I*a*x)^(1/4)/a^3/(a-I*
a*x)^(5/4)-16/45*I*(a+I*a*x)^(1/4)/a^4/(a-I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) - (((8*I)/45)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(5/4)) - (((16*I)/45)*(a + I*a*x)^(1/4))/(a^4
*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx = -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} + \frac{4\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx}{9a}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} - \frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{45a^2}$$
$$= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}} - \frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}} - \frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.52

$$\frac{2\sqrt[4]{a+iax}\,(17i+20x-8ix^2)}{45a^4(i+x)^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(a + I*a*x)^(1/4)*(17*I + 20*x - (8*I)*x^2))/(45*a^4*(I + x)^2*(a - I*a*
x)^(1/4))
```

Maple [A]
time = 0.16, size = 50, normalized size = 0.50

method	result	size
risch	$\frac{\frac{16}{45}x^3+\frac{8}{15}ix^2+\frac{2}{15}x+\frac{34}{45}i}{a^3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/a^3/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(12*I*x^2+8*x^3+3*x+17*I)/(x
+I)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

Fricas [A]
time = 0.75, size = 57, normalized size = 0.57

$$\frac{2\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}\left(8\,x^{2}+20i\,x-17\right)}{45\left(a^{5}x^{3}+3i\,a^{5}x^{2}-3\,a^{5}x-i\,a^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2/45*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 20*I*x - 17)/(a^5*x^3 +
3*I*a^5*x^2 - 3*a^5*x - I*a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}}\left(-ia\left(x+i\right)\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(13/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{13/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)), x)
```

3.1189 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}} dx$

Optimal. Leaf size=112

$$-\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{10a^2(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -10/3*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4)-2/3*I*(a-I*a*x)^(5/4)*(a+I*a*x)^(1/
4)/a+10/3*a^2*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))
*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {52, 42, 239, 237}

$$\frac{10a^2(x^2+1)^{3/4}\,F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-10*I)/3)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4) - (((2*I)/3)*(a - I*a*x)^(5
/4)*(a + I*a*x)^(1/4))/a + (10*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2
])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx &= -\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{1}{3}(5a)\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx\\
&= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{1}{3}\left(5a^2\right)\int \frac{1}{(a-iax)^{3/4}(a+ia\ldots}\\
&= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{\left(5a^2\left(a^2+a^2x^2\right)^{3/4}\right)\int \frac{1}{(a^2+a^2x^{\ldots}}}{3(a-iax)^{3/4}(a+iax)^{3/\ldots}}\\
&= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{\left(5a^2\left(1+x^2\right)^{3/4}\right)\int \frac{1}{(1+x^2)^{3/4}}\,d\ldots}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= -\frac{10}{3}i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}-\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{3a}+\frac{10a^2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\ldots\right.}{3(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.62

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{9/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[3/4,
 9/4, 13/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{5}{4}}}{(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] -2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x + 6*I) + integral(5/3*(I*a*x +
 a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia(x+i)\right)^{\frac{5}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(3/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(1/4*16*i*((s
ageVARa+(-i)*sageVARa*sageVARx)^(1/4))^8/((-((sageVARa+(-i)*sageVARa*sageVA
Rx)^(1/4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{5/4}}{(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(3/4), x)
```

3.1190 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=76

$$-\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{2a(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4)/a+2*a*(x^2+1)^(3/4)*(cos(1/2*arctan(x)
)^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*
x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {52, 42, 239, 237}

$$\frac{2a(x^2+1)^{3/4}\,F\left(\frac{\text{ArcTan}(x)}{2}\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-2*I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))/a + (2*a*(1 + x^2)^(3/4)*Ellip
ticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+a\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx\\ &= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{\left(a(a^2+a^2x^2)^{3/4}\right)\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{\left(a(1+x^2)^{3/4}\right)\int \frac{1}{(1+x^2)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{a}+\frac{2a(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.92

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(3/4),x]
[Out] (((2*I)/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[3/4,
 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] (a*integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^2 + a), x) - 2*I*(I*a
*x + a)^(1/4)*(-I*a*x + a)^(1/4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(3/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(4*i*((sageVA
Ra+(-i)*sageVARa*sageVARx)^(1/4))^4/((-((sageVARa+(-i)*sageVARa*sageVARx)^(
1/4))^4+2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{1/4}}{(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(3/4),x)
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(3/4), x)
```

3.1191 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=43

$$\frac{2(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 2*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(s
in(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {42, 239, 237}

$$\frac{2(x^2+1)^{3/4}\,F\left(\frac{\text{ArcTan}(x)}{2}\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a*x
)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx = \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{2\left(1+x^2\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 68, normalized size = 1.58

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}\sqrt[4]{a-iax}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((2*I)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 3/4
, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{3}{4}}\,(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] integral((I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a^2*x^2 + a^2), x)
```

Sympy [A]
time = 2.49, size = 100, normalized size = 2.33

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{8}, \frac{7}{8}, 1 & \frac{1}{2}, \frac{3}{4}, \frac{5}{4} \\ \frac{1}{4}, \frac{3}{8}, \frac{3}{4}, \frac{7}{8}, \frac{5}{4} & 0 \end{matrix} \middle| {\frac{e^{-3i\pi}}{x^{2}}} \right)e^{\frac{3i\pi}{4}}}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{8}, 0, \frac{3}{8}, \frac{1}{2}, 1 & \\ -\frac{1}{8}, \frac{3}{8} & -\frac{1}{2}, 0, \frac{1}{4}, 0 \end{matrix} \middle| {\frac{e^{-i\pi}}{x^{2}}} \right)}{4\pi a^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
[Out] -I*meijerg(((3/8, 7/8, 1), (1/2, 3/4, 5/4)), ((1/4, 3/8, 3/4, 7/8, 5/4), (0
,)), exp_polar(-3*I*pi)/x**2)*exp(3*I*pi/4)/(4*pi*a**(3/2)*gamma(3/4)) + I*
meijerg(((-1/2, -1/8, 0, 3/8, 1/2, 1), ()), ((-1/8, 3/8), (-1/2, 0, 1/4, 0)
), exp_polar(-I*pi)/x**2)/(4*pi*a**(3/2)*gamma(3/4))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{3/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(3/4)), x)
```

3.1192 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2/3*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*(cos(1/2*arcta
n(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a/(
a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 239, 237}

$$\frac{2(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)
*EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{3a}\\ &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a+iax}}{3a^2(a-iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.85

$$-\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)),x]
[Out] (((-2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, 1/2 -
 (I/2)*x])/(a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{7}{4}}\,(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

```
[Out] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 1/3*(3*(a^3*x + I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x + I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(3/4),x)
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{7/4}\,(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(3/4)), x)
```

3.1193 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=115

$$-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2/7*I*(a+I*a*x)^(1/4)/a^2/(a-I*a*x)^(7/4)-2/7*I*(a+I*a*x)^(1/4)/a^3/(a-I*a
*x)^(3/4)+2/7*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))
*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]

time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 239, 237}

$$-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{2(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(7/4)) - (((2*I)/7)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x
]/2, 2])/(7*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{11/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}+\frac{3\int \frac{1}{(a-iax)^{7/4}(a+iax)^{3/4}}\,dx}{7a}\\ &= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{7a^2}\\ &= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{(1+x^2)^{3/4}}\,dx}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= -\frac{2i\sqrt[4]{a+iax}}{7a^2(a-iax)^{7/4}}-\frac{2i\sqrt[4]{a+iax}}{7a^3(a-iax)^{3/4}}+\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{7a^2(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.61

$$-\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, 1/2
 - (I/2)*x])/(a*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{11}{4}}\left(iax+a\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
[Out] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 1/7*(7*(a^4*x^2 + 2*I*a^4*x - a^4)*integral(1/7*(I*a*x + a)^(1/4)*(-I*a*x +
 a)^(1/4)/(a^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x +
 2*I))/(a^4*x^2 + 2*I*a^4*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{3}{4}}(-ia(x+i))^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(3/4),x)
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{11/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(3/4)), x)
```

3.1194 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=291

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}+\frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

```
[Out] 4/3*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(3/4)+7/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(1/
4)/a+7/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-7/2*I*
arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-7/4*I*ln(1-(a-I*a
*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)+
7/4*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x
)^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {49, 52, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{7i\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}\,(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) + (((7*I)/3)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + ((7*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4)
)/(a + I*a*x)^(1/4)])/Sqrt[2] - ((7*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((7*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a
+ I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((7*I
)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} - \frac{7}{3}\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{1}{2}(7a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right.$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - 14i\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + 7i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) -$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-}}{\sqrt[4]{a+}}\right.$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} - \frac{7i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}$$

$$= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}} + \frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a} + \frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{7i\tan^{-1}}{}$$

Mathematica [A]

time = 0.70, size = 121, normalized size = 0.42

$$-\frac{(a-iax)^{3/4}\left((i+x)^{3/4}(-11i+3x)-21i(-i+x)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+21i(-i+x)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{3(i+x)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] -1/3*((a - I*a*x)^(3/4)*((I + x)^(3/4)*(-11*I + 3*x) - (21*I)*(-I + x)^(3/4
)*ArcTan[(I + x)^(1/4)/(-I + x)^(1/4)] + (21*I)*(-I + x)^(3/4)*ArcTanh[(I +
 x)^(1/4)/(-I + x)^(1/4)]))/((I + x)^(3/4)*(a + I*a*x)^(3/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.61, size = 465, normalized size = 1.60

method	result
risch	$\frac{i(3x^2-8ix+11)a}{3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}+\frac{\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2-x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4+\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*(-8*I*x+3*x^2+11)/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*a+(-7/2*RootO
f(_Z^2+I)*ln((-(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-x^3+I*RootOf(
_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+2*I*Roo
tOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x+2*I*x^2+(1-x^4+2*I*x^3+2*I*x)^(1/
2)+RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)+x)/(1+I*x)^2)-7/2*I*RootOf(_Z
^2+I)*ln((-I*(1-x^4+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I
)*(1-x^4+2*I*x^3+2*I*x)^(1/4)*x-x^3-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+RootOf(
_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(3/4)+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^
(1/4)+2*I*x^2-(1-x^4+2*I*x^3+2*I*x)^(1/2)+x)/(1+I*x)^2))/(a*(1+I*x))^(3/4)*
(-(-1+I*x)*(1+I*x)^3)^(1/4)/(-a*(-1+I*x))^(1/4)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(7/4), x)
```

Fricas [A]
time = 0.72, size = 235, normalized size = 0.81

$$-\frac{3\sqrt{49i}\,(ax-ia)\log\left(\frac{\sqrt{49i}\,(ax+ia)+7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)-3\sqrt{49i}\,(ax-ia)\log\left(-\frac{\sqrt{49i}\,(ax+ia)-7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)+3\sqrt{-49i}\,(ax-ia)\log\left(\frac{\sqrt{-49i}\,(ax+ia)+7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)-3\sqrt{-49i}\,(ax-ia)\log\left(-\frac{\sqrt{-49i}\,(ax+ia)-7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7\,(x+i)}\right)+2\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}(-3ix-11)}{6\,(ax-ia)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(49*I)*(a*x - I*a)*log(1/7*(sqrt(49*I)*(a*x + I*a) + 7*(I*a*x +
 a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*sqrt(49*I)*(a*x - I*a)*log(-1/7*
(sqrt(49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
+ 3*sqrt(-49*I)*(a*x - I*a)*log(1/7*(sqrt(-49*I)*(a*x + I*a) + 7*(I*a*x + a
)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*sqrt(-49*I)*(a*x - I*a)*log(-1/7*(
sqrt(-49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I))
+ 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(-3*I*x - 11))/(a*x - I*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(7/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%{i
,[0,1]%%%}] at parameters values [44,93]ext_reduce Error: Bad Argument Type
integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(a-a\,x\,1{}\mathrm{i})}^{7/4}}{{(a+a\,x\,1{}\mathrm{i})}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4),x)
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4), x)
```

3.1195 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=266

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log\left(1+\frac{\sqrt{a-ia}}{\sqrt{a+ia}}\right.}{}$$

[Out] 4/3*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(3/4)-1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)+1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)+I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a-I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a

Rubi [A]
time = 0.10, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {49, 65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{i\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx \\ &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\text{Subst}\left(\int \frac{x^2}{(2a-x^4)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a} \\ &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\ &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-}}{\sqrt[4]{a-}}\right.}{a} \\ &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx,\right.}{a} \\ &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\right.}{\sqrt{2}\,a} \\ &= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right.}{a}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 112, normalized size = 0.42

$$\frac{2\left(\frac{2i(a-iax)^{3/4}}{(a+iax)^{3/4}} + 3\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right) - 3(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (2*(((2*I)*(a - I*a*x)^(3/4))/(a + I*a*x)^(3/4) + 3*(-1)^(1/4)*ArcTanh[((-1
)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)] - 3*(-1)^(3/4)*ArcTanh[((-1)^
(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)]))/(3*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.30, size = 455, normalized size = 1.71

method	result
risch	$\frac{\frac{4x}{3}+\frac{4i}{3}}{(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}-\frac{\left(\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2-x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4+2ix\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*(x+I)/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)-(RootOf(_Z^2+I)*ln((-(1-x^4
+2*I*x^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-x^3+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^
3+2*I*x)^(3/4)+I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+2*I*RootOf(_Z^2+I)*(1-x^4+2*
I*x^3+2*I*x)^(1/4)*x+2*I*x^2+(1-x^4+2*I*x^3+2*I*x)^(1/2)+RootOf(_Z^2+I)*(1-
x^4+2*I*x^3+2*I*x)^(1/4)+x)/(1+I*x)^2)+I*RootOf(_Z^2+I)*ln((-I*(1-x^4+2*I*x
^3+2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(
1/4)*x-x^3-I*(1-x^4+2*I*x^3+2*I*x)^(1/2)*x+RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*
I*x)^(3/4)+I*RootOf(_Z^2+I)*(1-x^4+2*I*x^3+2*I*x)^(1/4)+2*I*x^2-(1-x^4+2*I*
x^3+2*I*x)^(1/2)+x)/(1+I*x)^2))/(a*(1+I*x))^(3/4)*(-(-1+I*x)*(1+I*x)^3)^(1/
4)/(-a*(-1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

Fricas [A]
time = 0.70, size = 298, normalized size = 1.12

$$-\frac{3\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-3\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)+3\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-3\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{2\left(x+i\right)}\right)-8\left(iax+a\right)^{\frac{1}{4}}\left(-iax+a\right)^{\frac{3}{4}}}{6\left(a^2x-ia^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(a^2*x - I*a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2
) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 3*(a^2*x - I*a^2)*sq
rt(4*I/a^2)*log(-1/2*((a^2*x + I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(
-I*a*x + a)^(3/4))/(x + I)) + 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(1/2*((a^
```

```
2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x +
I)) - 3*(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x + I*a^2)*sqrt(-4*I/
a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(x + I)) - 8*(I*a*x + a)^(1/
4)*(-I*a*x + a)^(3/4))/(a^2*x - I*a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(7/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(a-a\,x\,1{}\mathrm{i})}^{3/4}}{{(a+a\,x\,1{}\mathrm{i})}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4), x)
```

3.1196 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

```
[Out] 2/3*I*(a-I*a*x)^(3/4)/a^2/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx = \frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Mathematica [A]
time = 0.06, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Maple [A]
time = 0.15, size = 31, normalized size = 0.94

method	result	size
risch	$\frac{\frac{2x}{3}+\frac{2i}{3}}{a\left(a(ix+1)\right)^{\frac{3}{4}}\left(-a(ix-1)\right)^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [A]
time = 0.74, size = 31, normalized size = 0.94

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,(a^3x-i\,a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x - I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)), x)
```

3.1197 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=65

$$-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}$$

```
[Out] -2*I/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4)+4/3*I*(a-I*a*x)^(3/4)/a^3/(a+I*a*x
)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(
3/4))/(a^3*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{2\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx}{a}$$
$$= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.69

$$\frac{2(1+2ix)(a-iax)^{3/4}}{3a^3(i+x)(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
[Out] (2*(1 + (2*I)*x)*(a - I*a*x)^(3/4))/(3*a^3*(I + x)*(a + I*a*x)^(3/4))
```

Maple [A]
time = 0.15, size = 33, normalized size = 0.51

method	result	size
risch	$\frac{-\frac{2i}{3}+\frac{4x}{3}}{a^2(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
[Out] 2/3/a^2/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(-I+2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

Fricas [A]
time = 0.63, size = 36, normalized size = 0.55

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(2\,x-i)}{3\,(a^4x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(2*x - I)/(a^4*x^2 + a^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)), x)
```

3.1198 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}-\frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}$$

```
[Out] -2/5*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4)-8/5*I/a^3/(a-I*a*x)^(1/4)/(a+I*a
*x)^(3/4)+16/15*I*(a-I*a*x)^(3/4)/a^4/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) - ((8*I)/5)/(a^3*(a -
I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((16*I)/15)*(a - I*a*x)^(3/4))/(a^4*(a +
 I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} + \frac{4\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx}{5a}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{8\int \frac{}{\sqrt[4]{a-iax}}}{5a}$$
$$= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}} - \frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}} + \frac{16i(a-iax)}{15a^4(a+iax}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 0.52

$$\frac{2i(a-iax)^{3/4}\left(7+4ix+8x^2\right)}{15a^4(i+x)^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
[Out] (((2*I)/15)*(a - I*a*x)^(3/4)*(7 + (4*I)*x + 8*x^2))/(a^4*(I + x)^2*(a + I*
a*x)^(3/4))
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.44

method	result	size
risch	$\frac{\frac{16}{15}x^2+\frac{8}{15}ix+\frac{14}{15}}{a^3(a(ix+1))^{\frac{3}{4}}(-a(ix-1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x,method=_RETURNVERBOSE)
[Out] 2/15/a^3/(a*(1+I*x))^(3/4)/(-a*(-1+I*x))^(1/4)*(8*x^2+4*I*x+7)/(x+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.61, size = 56, normalized size = 0.56

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(8\,x^2+4i\,x+7)}{15\,(a^5x^3+i\,a^5x^2+a^5x+i\,a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2/15*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 4*I*x + 7)/(a^5*x^3 + I*
a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{(a-a\,x\,1\mathrm{i})}^{9/4}\,{(a+a\,x\,1\mathrm{i})}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)), x)
```

3.1199 $\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=139

$$\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}+\frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a}-\frac{10a^2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 4/3*I*(a-I*a*x)^(9/4)/a/(a+I*a*x)^(3/4)+10*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1/4
)+2*I*(a-I*a*x)^(5/4)*(a+I*a*x)^(1/4)/a-10*a^2*(x^2+1)^(3/4)*(cos(1/2*arcta
n(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-
I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 52, 42, 239, 237}

$$-\frac{10a^2(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}}+\frac{2i\sqrt[4]{a+iax}\,(a-iax)^{5/4}}{a}+10i\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(9/4))/(a*(a + I*a*x)^(3/4)) + (10*I)*(a - I*a*x)^(1
/4)*(a + I*a*x)^(1/4) + ((2*I)*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))/a - (10
*a^2*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/((a - I*a*x)^(3/4)*(a + I*a
*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-iax)^{9/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} - 3\int \frac{(a-iax)^{5/4}}{(a+iax)^{3/4}}\,dx \\ &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - (5a)\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx \\ &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - (5a^2)\int \frac{}{(a-} \\ &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{\left(5a^2(a^2+a^2\right.}{(a-ia} \\ &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{\left(5a^2(1+x^2)\right.}{(a-iax)} \\ &= \frac{4i(a-iax)^{9/4}}{3a(a+iax)^{3/4}} + 10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax} + \frac{2i(a-iax)^{5/4}\sqrt[4]{a+iax}}{a} - \frac{10a^2(1+x^2)}{(a-iax}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.50

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{13/4}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{17}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(9/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/13)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(13/4)*Hypergeometric2F1[7/4, 1
3/4, 17/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{9}{4}}}{(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(9/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(x - I)*integral(-5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(x^2 + 1),
x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(x^2 + 11*I*x + 20))/(x - I)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{9}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(9/4)/(I*a*(x - I))**(7/4), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(-(-i)/4*16*(
(sageVARa+(-i)*sageVARa*sageVARx)^(1/4))^12/(-((sageVARa+(-i)*sageVARa*sage
VARx)^(1/
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{9/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(9/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(9/4)/(a + a*x*1i)^(7/4), x)
```

3.1200 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}} dx$

Optimal. Leaf size=113

$$\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\sqrt[4]{a+iax}}{3a}-\frac{10a(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 4/3*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(3/4)+10/3*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(1
/4)/a-10/3*a*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*
EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 52, 42, 239, 237}

$$-\frac{10a(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{3(a+iax)^{3/4}(a-iax)^{3/4}}+\frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a+iax}\sqrt[4]{a-iax}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(3/4)) + (((10*I)/3)*(a - I*a*
x)^(1/4)*(a + I*a*x)^(1/4))/a - (10*a*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}-\frac{5}{3}\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{1}{3}(5a)\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{\left(5a(a^2+a^2x^2)^{3/4}\right)\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{\left(5a(1+x^2)^{3/4}\right)\int \frac{1}{(1+x^2)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}}\\
&= \frac{4i(a-iax)^{5/4}}{3a(a+iax)^{3/4}}+\frac{10i\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}{3a}-\frac{10a(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 70, normalized size = 0.62

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{9/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/9)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[7/4, 9/4
, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{5}{4}}}{(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
[Out] int((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 1/3*(3*(a*x - I*a)*integral(-5/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a*x^
2 + a), x) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*(-3*I*x - 7))/(a*x - I*
a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(7/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(-(-i)/4*16*(
(sageVARa+(-i)*sageVARa*sageVARx)^(1/4))^8/(-((sageVARa+(-i)*sageVARa*sageV
ARx)^(1/4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(7/4), x)
```

3.1201 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=79

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 4/3*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(3/4)-2/3*(x^2+1)^(3/4)*(cos(1/2*arctan(x
))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/(a-I*a
*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {49, 42, 239, 237}

$$\frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2(x^2+1)^{3/4}\,F\left(\frac{\text{ArcTan}(x)}{2}\middle|\,2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(3/4)) - (2*(1 + x^2)^(3/4)*El
lipticF[ArcTan[x]/2, 2])/(3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{7/4}}\,dx &= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{1}{3}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx \\
&= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= \frac{4i\sqrt[4]{a-iax}}{3a(a+iax)^{3/4}}-\frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3(a-iax)^{3/4}(a+iax)^{3/4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.89

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/5)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 7/4
, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-iax+a)^{\frac{1}{4}}}{(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)
```

[Out] int((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(7/4), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")

[Out] 1/3*(3*(a^2*x - I*a^2)*integral(-1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a^2*x^2 + a^2), x) + 4*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^2*x - I*a^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\left(x+i\right)}}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)

[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(7/4), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")

[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(7/4), x)

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{1/4}}{(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(7/4),x)
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(7/4), x)
```

3.1202 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2(1+x^2)^{3/4}\,F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 2/3*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*(cos(1/2*arctan
(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a/(a
-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 239, 237}

$$\frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2(x^2+1)^{3/4}\,F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (((2*I)/3)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*
EllipticF[ArcTan[x]/2, 2])/(3*a*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{7/4}}\,dx &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{\int \frac{1}{(a-iax)^{3/4}(a+iax)^{3/4}}\,dx}{3a}\\ &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{(a^2+a^2x^2)^{3/4}\int \frac{1}{(a^2+a^2x^2)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{(1+x^2)^{3/4}\int \frac{1}{(1+x^2)^{3/4}}\,dx}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\\ &= \frac{2i\sqrt[4]{a-iax}}{3a^2(a+iax)^{3/4}}+\frac{2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a(a-iax)^{3/4}(a+iax)^{3/4}}\end{aligned}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 68, normalized size = 0.83

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\sqrt[4]{a-iax}\;{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(7/4)),x]
[Out] (I*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 7/4, 5/
4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{3}{4}}\,(iax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^3*x - I*a^3)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^3*x^2 + a^3), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^3*x - I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}\left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{3/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(7/4)),x)
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(7/4)), x)
```

3.1203 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=81

$$\frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] 2/3*x/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)+2/3*(x^2+1)^(3/4)*(cos(1/2*arctan
(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a^2/
(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {42, 205, 239, 237}

$$\frac{2(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (2*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (2*(1 + x^2)^(3/4)*Elli
pticF[ArcTan[x]/2, 2])/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx = \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{7/4}}\,dx}{(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(a^2+a^2x^2\right)^{3/4}\int \frac{1}{\left(a^2+a^2x^2\right)^{3/4}}\,dx}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(1+x^2\right)^{3/4}\int \frac{1}{\left(1+x^2\right)^{3/4}}\,dx}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$
$$= \frac{2x}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{2\left(1+x^2\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.86

$$-\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a^2(a-iax)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(7/4)),x]
[Out] ((-1/3*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-3/4, 7/4, 1/4, 1/2 - (
I/2)*x])/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{7}{4}}\left(iax+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
[Out] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 1/3*(3*(a^4*x^2 + a^4)*integral(1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)/(a
^4*x^2 + a^4), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4)*x)/(a^4*x^2 + a^
4)
```

Sympy [A]
time = 21.95, size = 95, normalized size = 1.17

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{8},\frac{11}{8},1 & \frac{1}{2},\frac{7}{4},\frac{9}{4} \\ \frac{7}{8},\frac{5}{4},\frac{11}{8},\frac{7}{4},\frac{9}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{-\frac{i\pi}{4}}}{4\pi a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{3}{8},\frac{1}{2},\frac{7}{8},1 & \\ \frac{3}{8},\frac{7}{8} & -\frac{1}{2},0,\frac{5}{4},0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
[Out] -I*meijerg(((7/8, 11/8, 1), (1/2, 7/4, 9/4)), ((7/8, 5/4, 11/8, 7/4, 9/4),
(0,)), exp_polar(-3*I*pi)/x**2)*exp(-I*pi/4)/(4*pi*a**(7/2)*gamma(7/4)) + I
*meijerg(((-1/2, 0, 3/8, 1/2, 7/8, 1), ()), ((3/8, 7/8), (-1/2, 0, 5/4, 0))
, exp_polar(-I*pi)/x**2)/(4*pi*a**(7/2)*gamma(7/4))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{7/4}\,(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(7/4)),x)
[Out] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(7/4)), x)
```

3.1204 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=114

$$-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10(1+x^2)^{3/4}F\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}$$

```
[Out] -2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(3/4)+10/21*x/a^3/(a-I*a*x)^(3/4)/(a+I
*a*x)^(3/4)+10/21*(x^2+1)^(3/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan
(x))*EllipticF(sin(1/2*arctan(x)),2^(1/2))/a^3/(a-I*a*x)^(3/4)/(a+I*a*x)^(3
/4)
```

Rubi [A]
time = 0.02, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 239, 237}

$$\frac{10(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(21*a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2
, 2])/(21*a^3*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{5\int \frac{1}{(a-iax)^{7/4}(a+iax)^{7/4}}\,dx}{7a} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{\left(5(a^2+a^2x^2)^{3/4}\right)\int \frac{1}{(a^2+a^2x^2)^{7/4}}\,dx}{7a(a-iax)^{3/4}(a+iax)^{3/4}} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(5(a^2+\cdots\right.}{21a^3(\cdots} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{\left(5(1+\cdots\right.}{21a^3(a-\cdots} \\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{10x}{21a^3(a-iax)^{3/4}(a+iax)^{3/4}} + \frac{10(1+\cdots}{21a^3(a\cdots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.61

$$
-\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{7}{4};-\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a-iax)^{7/4}(a+iax)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-1/7*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-7/4, 7/4, -3/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{11}{4}} \left(iax+a\right)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/21*(21*(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)*integral(5/21*(I*a*x + a)^(1
/4)*(-I*a*x + a)^(1/4)/(a^5*x^2 + a^5), x) + 2*(I*a*x + a)^(1/4)*(-I*a*x +
a)^(1/4)*(5*x^2 + 5*I*x + 3))/(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}} \left(-ia\left(x+i\right)\right)^{\frac{11}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a-a\,x\,1\mathrm{i}\right)^{11/4}\,\left(a+a\,x\,1\mathrm{i}\right)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(7/4)), x)
```

3.1205 $\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}} \, dx$

Optimal. Leaf size=147

$$-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}-\frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}}+\frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10(1+x^2)^{3/4}}{33a^4(a-ia}$$

```
[Out] -2/11*I/a^2/(a-I*a*x)^(11/4)/(a+I*a*x)^(3/4)-2/11*I/a^3/(a-I*a*x)^(7/4)/(a+
I*a*x)^(3/4)+10/33*x/a^4/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)+10/33*(x^2+1)^(3/4
)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticF(sin(1/2*arctan(
x)),2^(1/2))/a^4/(a-I*a*x)^(3/4)/(a+I*a*x)^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 239, 237}

$$\frac{10(x^2+1)^{3/4}F\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}+\frac{10x}{33a^4(a-iax)^{3/4}(a+iax)^{3/4}}-\frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}}-\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/11)/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4)) - ((2*I)/11)/(a^3*(a
 - I*a*x)^(7/4)*(a + I*a*x)^(3/4)) + (10*x)/(33*a^4*(a - I*a*x)^(3/4)*(a +
I*a*x)^(3/4)) + (10*(1 + x^2)^(3/4)*EllipticF[ArcTan[x]/2, 2])/(33*a^4*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(3/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{15/4}(a+iax)^{7/4}}\,dx = -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} + \frac{7\int \frac{1}{(a-iax)^{11/4}(a+iax)^{7/4}}\,dx}{11a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{5\int \frac{}{(a-}}{}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{\left(5(a^2\right.}{11a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{}{33a^4(a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{}{33a^4(a}$$

$$= -\frac{2i}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}} - \frac{2i}{11a^3(a-iax)^{7/4}(a+iax)^{3/4}} + \frac{}{33a^4(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.48

$$-\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}\,{}_2F_1\left(-\frac{11}{4},\frac{7}{4};-\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a-iax)^{11/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-1/11*I)*2^(1/4)*(1 + I*x)^(3/4)*Hypergeometric2F1[-11/4, 7/4, -7/4, 1/2
- (I/2)*x])/(a^2*(a - I*a*x)^(11/4)*(a + I*a*x)^(3/4))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{15}{4}} (iax+a)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

```
[Out] int(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/33*(33*(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*x - a^6)*integral(5/33*(I*a*x + a
)^(1/4)*(-I*a*x + a)^(1/4)/(a^6*x^2 + a^6), x) + 2*(5*x^3 + 10*I*x^2 - 2*x
+ 6*I)*(I*a*x + a)^(1/4)*(-I*a*x + a)^(1/4))/(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a
^6*x - a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{15/4}\,(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(7/4)), x)
```

3.1206 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=137

$$-\frac{14ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a}+\frac{14a\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -14*a*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+4*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(1/
4)+14/3*I*(a-I*a*x)^(3/4)*(a+I*a*x)^(3/4)/a+14*a*(x^2+1)^(1/4)*(cos(1/2*arc
tan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/(
a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]

time = 0.02, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {49, 52, 42, 235, 233, 202}

$$\frac{14a\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}}+\frac{14i(a+iax)^{3/4}(a-iax)^{3/4}}{3a}-\frac{14ax}{\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (-14*a*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(7/4))
/(a*(a + I*a*x)^(1/4)) + (((14*I)/3)*(a - I*a*x)^(3/4)*(a + I*a*x)^(3/4))/a
 + (14*a*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I*a*x)^(1/4)*(a +
 I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{5/4}}\,dx = \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} - 7\int \frac{(a-iax)^{3/4}}{\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - (7a)\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - \frac{\left(7a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

$$= \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} - \frac{\left(7a\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

$$= -\frac{14ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} + \frac{\left(7a\sqrt[4]{1+x\right.}{\sqrt[4]{a-i\ldots}}$$

$$= -\frac{14ax}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{7/4}}{a\sqrt[4]{a+iax}} + \frac{14i(a-iax)^{3/4}(a+iax)^{3/4}}{3a} + \frac{14a\sqrt[4]{1+x\ldots}}{\sqrt[4]{a-\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.51

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{11/4}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/11)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[5/4, 1
1/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 96, normalized size = 0.70

method	result	size
risch	$\frac{2i(x^2-12ix+13)a}{3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)a\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(x^2+13-12*I*x)*a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-7/(a^2)^(1/4)
*x*hypergeom([1/4,1/2],[3/2],-x^2)*a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+
I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(-I*x^2 + 8*x - 21*I) - 3*(a*x
^2 - I*a*x)*integral(-14*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a*x^4 + a*x^
2), x))/(a*x^2 - I*a*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(5/4), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%{i
,[0,1]%%%}] at parameters values [44,93]ext_reduce Error: Bad Argument Type
integrate
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{7/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(5/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(5/4), x)
```

3.1207 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=102

$$-\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -6*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+4*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(1/4)+
6*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(s
in(1/2*arctan(x)),2^(1/2))/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 42, 235, 233, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (-6*x)/((a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + ((4*I)*(a - I*a*x)^(3/4))/(a
*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/((a - I
*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} - 3\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} - \frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} - \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= -\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} + \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= -\frac{6x}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} + \frac{4i(a-iax)^{3/4}}{a\sqrt[4]{a+iax}} + \frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.69

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{7/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(5/4),x]
[Out] ((I/7)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[5/4, 7/4
, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 88, normalized size = 0.86

method	result	size
risch	$\frac{4x+4i}{(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^{2}\right)\left(-a^{2}(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^{2})^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(x+I)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+
I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x - 3*I) - (a^2*x^2 - I*a^2*x)*in
tegral(-6*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^2*x^4 + a^2*x^2), x))/(a^
2*x^2 - I*a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia(x+i)\right)^{\frac{3}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{3/4}}{(a + a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(5/4),x)
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(5/4), x)
```

3.1208 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=78

$$\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)+2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2
)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*x
)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {50, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|\,2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*I)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*Elliptic
E[ArcTan[x]/2, 2])/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 50

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{5/4}}\,dx &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+a\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx\\ &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\left(a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2i}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.90

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((I/3)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 5/4
, 7/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.17, size = 94, normalized size = 1.21

method	result	size
risch	$\frac{2x+2i}{a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x+I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([
1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] ((a^3*x^2 - I*a^3*x)*integral(-2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*
x^4 + a^3*x^2), x) + 2*I*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4))/(a^3*x^2 - I
*a^3*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\sqrt[4]{-ia\left(x+i\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(5/4)),x)
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(5/4)), x)
```

3.1209 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(s
in(1/2*arctan(x)),2^(1/2))/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|\,2\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(a^2*(a - I*a*x)^(1/4)*(a + I
*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx = \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$
$$= \frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$
$$= \frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|\,2\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 68, normalized size = 1.48

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, 1/2 - (I/2)
*x])/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 91, normalized size = 1.98

method	result	size
risch	$\frac{2x}{a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/(a^2)^(1/4)*x*hypergeom([1/
4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a
*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")

[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(5/4)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")

[Out] (2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*x + (a^4*x^2 + a^4)*integral(-(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^4), x))/(a^4*x^2 + a^4)

Sympy [A]
time = 5.65, size = 97, normalized size = 2.11

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{8},\frac{9}{8},1 & \frac{1}{2},\frac{5}{4},\frac{7}{4} \\ \frac{5}{8},\frac{3}{4},\frac{9}{8},\frac{5}{4},\frac{7}{4} & 0 \end{matrix}\middle| \frac{e^{-3i\pi}}{x^2}\right)e^{-\frac{3i\pi}{4}}}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{8},\frac{1}{2},\frac{5}{8},1 & \\ \frac{1}{8},\frac{5}{8} & -\frac{1}{2},0,\frac{3}{4},0 \end{matrix}\middle| \frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)

[Out] -I*meijerg(((5/8, 9/8, 1), (1/2, 5/4, 7/4)), ((5/8, 3/4, 9/8, 5/4, 7/4), (0,)), exp_polar(-3*I*pi)/x**2)*exp(-3*I*pi/4)/(4*pi*a**(5/2)*gamma(5/4)) + I*meijerg(((-1/2, 0, 1/8, 1/2, 5/8, 1), ()), ((1/8, 5/8), (-1/2, 0, 3/4, 0)), exp_polar(-I*pi)/x**2)/(4*pi*a**(5/2)*gamma(5/4))

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDone

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(5/4)),x)
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(5/4)), x)
```

3.1210 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=82

$$-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2/5*I/a^2/(a-I*a*x)^(5/4)/(a+I*a*x)^(1/4)+6/5*(x^2+1)^(1/4)*(cos(1/2*arcta
n(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3
/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 203, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (6*(1 + x^2)^(1/4)*E
llipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{3\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{5a}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.85

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-1/5*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 107, normalized size = 1.30

method	result	size
risch	$\frac{\frac{6}{5}x^2+\frac{6}{5}ix+\frac{2}{5}}{(x+i)a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(3*I*x+3*x^2+1)/(x+I)/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/5/(a^
2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/4
)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 + 3*I*x + 1) + 5*(a^5*x^
3 + I*a^5*x^2 + a^5*x + I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)
^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 + I*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(9/4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{9/4}\,(a + a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(5/4)), x)
```

3.1211 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=115

$$-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{3a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2/9*I/a^2/(a-I*a*x)^(9/4)/(a+I*a*x)^(1/4)-2/9*I/a^3/(a-I*a*x)^(5/4)/(a+I*a
*x)^(1/4)+2/3*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))
*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {53, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{3a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4)) - ((2*I)/9)/(a^3*(a -
I*a*x)^(5/4)*(a + I*a*x)^(1/4)) + (2*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2,
 2])/(3*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{13/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} + \frac{5\int \frac{1}{(a-iax)^{9/4}(a+iax)^{5/4}}\,dx}{9a}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} - \frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\int \frac{1}{(a-iax)^{5/4}(a}}{3a^2}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} - \frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\sqrt[4]{a^2+a^2x^2}}{3a^2\sqrt[4]{a-i}}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} - \frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{\sqrt[4]{1+x^2}\int}{3a^4\sqrt[4]{a-iax}}$$

$$= -\frac{2i}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}} - \frac{2i}{9a^3(a-iax)^{5/4}\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{1+x^2}\,E}{3a^4\sqrt[4]{a-i}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.61

$$-\frac{i2^{3/4}\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{9}{4},\frac{5}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-1/9*I)*2^(3/4)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, 1/2 -
(I/2)*x])/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 113, normalized size = 0.98

method	result	size

risch	$\frac{\frac{2}{3}x^3+\frac{4}{3}ix^2-\frac{4}{9}x+\frac{4}{9}i}{(x+i)^2a^4(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}} - \frac{x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{3(a^2)^{\frac{1}{4}}a^4(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(6*I*x^2+3*x^3-2*x+2*I)/(x+I)^2/a^4/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/
4)-1/3/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^4*(-a^2*(-1+I*x)*(1+
I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(3*x^3 + 6*I*x^2 - 2*x + 2*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) +
 9*(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*x - a^6)*integral(-1/3*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(3/4)/(a^6*x^2 + a^6), x))/(a^6*x^4 + 2*I*a^6*x^3 + 2*I*a^6*
x - a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(5/4)), x)
```

3.1212 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=287

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-$$

```
[Out] 4*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(1/4)+5*I*(a-I*a*x)^(1/4)*(a+I*a*x)^(3/4)/a
+5/2*I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)-5/2*I*arct
an(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)+5/4*I*ln(1-(a-I*a*x)^
(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))*2^(1/2)-5/4*
I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1
/2))*2^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {49, 52, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{5i\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{5i\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) + ((5*I)*(a - I*a*x)^(1/4)*
(a + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I
*a*x)^(1/4)])/Sqrt[2] - ((5*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
I*a*x)^(1/4)])/Sqrt[2] + (((5*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((5*I)/2)*Log
[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a
*x)^(1/4)])/Sqrt[2]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx = \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} - 5\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{1}{2}(5a)\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right.$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 5i\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) -$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{5}{2}i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-i}}{\sqrt[4]{a+i}}\right.$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}$$

$$= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{5i\tan^{-}}{}$$

Mathematica [A]

time = 0.58, size = 117, normalized size = 0.41

$$-\frac{\sqrt[4]{a-iax}\left(\sqrt[4]{i+x}\,(-9i+x)+5i\sqrt[4]{-i+x}\,\tan^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)+5i\sqrt[4]{-i+x}\,\tanh^{-1}\left(\frac{\sqrt[4]{i+x}}{\sqrt[4]{-i+x}}\right)\right)}{\sqrt[4]{i+x}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] -(((a - I*a*x)^(1/4)*((I + x)^(1/4)*(-9*I + x) + (5*I)*(-I + x)^(1/4)*ArcTa
n[(I + x)^(1/4)/(-I + x)^(1/4)] + (5*I)*(-I + x)^(1/4)*ArcTanh[(I + x)^(1/4
)/(-I + x)^(1/4)]))/((I + x)^(1/4)*(a + I*a*x)^(1/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.57, size = 480, normalized size = 1.67

method	result
risch	$-\frac{i\left(x^2-8ix+9\right)\left(-a(ix-1)\right)^{\frac{1}{4}}}{(ix-1)\left(a(ix+1)\right)^{\frac{1}{4}}}-\frac{\left(5\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{-\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2+x^3+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(x^2+9-8*I*x)*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)-(5/2*RootOf
(_Z^2+I)*ln(-(-(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+x^3+I*RootOf(
_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x-2*I*Roo
tOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+2*I*x^2+(1-x^4-2*I*x^3-2*I*x)^(1/
2)+RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-x)/(-1+I*x)^2)+5/2*I*RootOf(_
Z^2+I)*ln(-(-I*(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+2*RootOf(_Z^2
+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+x^3+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+RootO
f(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)+I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x
)^(1/4)+2*I*x^2-(1-x^4-2*I*x^3-2*I*x)^(1/2)-x)/(-1+I*x)^2))*(-a*(-1+I*x))^(
1/4)/(-1+I*x)*(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(5/4), x)
```

Fricas [A]
time = 0.82, size = 233, normalized size = 0.81

$$-\frac{\sqrt{25i}\,(ax-i\,a)\log\left(\frac{\sqrt{25i}\,(ax-i\,a)+5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)-\sqrt{25i}\,(ax-i\,a)\log\left(-\frac{\sqrt{25i}\,(ax-i\,a)-5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)+\sqrt{-25i}\,(ax-i\,a)\log\left(\frac{\sqrt{-25i}\,(ax-i\,a)+5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)-\sqrt{-25i}\,(ax-i\,a)\log\left(-\frac{\sqrt{-25i}\,(ax-i\,a)-5\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{5\,(x-i)}\right)+2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(-i\,x-9)}{2\,(ax-i\,a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(25*I)*(a*x - I*a)*log(1/5*(sqrt(25*I)*(a*x - I*a) + 5*(I*a*x + a
)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(25*I)*(a*x - I*a)*log(-1/5*(sqr
t(25*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + sq
rt(-25*I)*(a*x - I*a)*log(1/5*(sqrt(-25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-25*I)*(a*x - I*a)*log(-1/5*(sqrt(-25
*I)*(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 2*(I*a
*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(-I*x - 9))/(a*x - I*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(5/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(-(-i)/4*16*(
(sageVARa+(-i)*sageVARa*sageVARx)^(1/4))^8/(-((sageVARa+(-i)*sageVARa*sageV
ARx)^(1/4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4),x)
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4), x)
```

3.1213 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=264

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}\right.}{\sqrt{2}}$$

```
[Out] 4*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(1/4)+1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a
+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-1/2*I*ln(1+(a-I*a*
x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)
+I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a-I*arctan(1+(
a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4))*2^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {49, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$\frac{i\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) + (I*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (I*Log[1 + Sqrt[a - I*a*x]/
Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*
a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx &= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a} \\
&= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\
&= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-}}{\sqrt[4]{a+}}\right.}{a} \\
&= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x,\right.}{a} \\
&= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\right.}{\sqrt{2}\,a} \\
&= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 109, normalized size = 0.41

$$\frac{2\left(\frac{2i\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} - \sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) + (-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] (2*(((2*I)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4) - (-1)^(1/4)*ArcTanh[((-1)^
(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)] + (-1)^(3/4)*ArcTanh[((-1)^(3/4
)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]))/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.15, size = 478, normalized size = 1.81

method	result
risch	$-\frac{4(x+i)(-a(ix-1))^{\frac{1}{4}}}{a(ix-1)(a(ix+1))^{\frac{1}{4}}}+\frac{\left(\operatorname{RootOf}\left(\_Z^2-i\right)\ln\left(-\frac{\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2-i\right)x^2+i\operatorname{RootOf}\left(\_Z^2-i\right)\left(-x^4-2ix^3-2ix\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(x+I)/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)+(RootOf(_Z^2-I)*l
n(-((1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2+I*RootOf(_Z^2-I)*(1-x^4-
2*I*x^3-2*I*x)^(3/4)+x^3+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+2*I*RootOf(_Z^2-I)
*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+2*I*x^2-(1-x^4-2*I*x^3-2*I*x)^(1/2)-RootOf(_
Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-x)/(-1+I*x)^2)+I*RootOf(_Z^2-I)*ln(-(I*(
1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf(_Z^2-I)*(1-x^4-2*I*x
^3-2*I*x)^(1/4)*x+x^3+RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-I*(1-x^4-2
*I*x^3-2*I*x)^(1/2)*x-I*RootOf(_Z^2-I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)+2*I*x^2+
(1-x^4-2*I*x^3-2*I*x)^(1/2)-x)/(-1+I*x)^2))/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)*
(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

Fricas [A]
time = 0.68, size = 296, normalized size = 1.12

$$-\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2(x-i)}\right)-(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x-ia^2)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2(x-i)}\right)+(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2(x-i)}\right)-(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x-ia^2)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2(x-i)}\right)-8(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2(a^2x-ia^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)
+ 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - (a^2*x - I*a^2)*sqrt(4
*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a
```

```
*x + a)^(1/4))/(x - I)) + (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(1/2*((a^2*x -
I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) -
(a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2
*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 8*(I*a*x + a)^(3/4)*(-I*a
*x + a)^(1/4))/(a^2*x - I*a^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4),x)
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4), x)
```

3.1214 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=31

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

```
[Out] 2*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {37}

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 1.00

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Maple [A]
time = 0.18, size = 31, normalized size = 1.00

method	result	size
risch	$\frac{2x+2i}{a(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
[Out] 2/a/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A]
time = 0.72, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a^3x-i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x - I*a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 1.16, size = 27, normalized size = 0.87

$$\frac{\left(-a\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,2\mathrm{i}}{a^2\left(a\ \left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] ((-a*(x*1i - 1))^(1/4)*2i)/(a^2*(a*(x*1i + 1))^(1/4))
```

3.1215 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}$$

```
[Out] -2/3*I/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4)+4/3*I*(a-I*a*x)^(1/4)/a^3/(a+I*a
*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{47, 37\}$

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*
x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{3a}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.67

$$\frac{2(1-2ix)(a+iax)^{3/4}}{3a^3(-i+x)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
[Out] (2*(1 - (2*I)*x)*(a + I*a*x)^(3/4))/(3*a^3*(-I + x)*(a - I*a*x)^(3/4))
```

Maple [A]
time = 0.15, size = 33, normalized size = 0.49

method	result	size
risch	$\frac{\frac{2i}{3}+\frac{4x}{3}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
[Out] 2/3/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(I+2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

Fricas [A]
time = 0.79, size = 36, normalized size = 0.54

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+i)}{3\,(a^4x^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + I)/(a^4*x^2 + a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.60, size = 40, normalized size = 0.60

$$-\frac{2\left(2\,x+1\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{3\,a^3\,\left(-1+x\,1\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -(2*(2*x + 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^3*(x*1i - 1)*(a*(x*1i + 1))^(1/4
))
```

3.1216 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}-\frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}$$

```
[Out] -2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4)-8/21*I/a^3/(a-I*a*x)^(3/4)/(a+I*
a*x)^(1/4)+16/21*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}\,(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}\,(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) - ((8*I)/21)/(a^3*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((16*I)/21)*(a - I*a*x)^(1/4))/(a^4*(a
+ I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} + \frac{4\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx}{7a}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{8\int \frac{1}{(a-iax)^{3/4}(}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}} - \frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}} + \frac{16i\sqrt[4]{a-ia}}{21a^4\sqrt[4]{a+ia}}$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 0.52

$$\frac{2(a+iax)^{3/4}\,(i+12x-8ix^2)}{21a^4(a-iax)^{3/4}\,(1+x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(I + 12*x - (8*I)*x^2))/(21*a^4*(a - I*a*x)^(3/4)*(1 +
 x^2))
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.44

method	result	size
risch	$\frac{\frac{16}{21}x^2+\frac{8}{7}ix-\frac{2}{21}}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x+i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(8*x^2+12*I*x-1)/(x+I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.83, size = 56, normalized size = 0.56

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^{2}+12i\,x-1)}{21\,(a^{5}x^{3}+i\,a^{5}x^{2}+a^{5}x+i\,a^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 12*I*x - 1)/(a^5*x^3 + I
*a^5*x^2 + a^5*x + I*a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ia\,(x-i))^{\frac{5}{4}}\,(-ia\,(x+i))^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(11/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.76, size = 46, normalized size = 0.46

$$-\frac{(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,(8\,x^{2}+x\,12\mathrm{i}-1)\;2\mathrm{i}}{21\,a^{4}\,(-1+x\,1\mathrm{i})^{2}\,(a\;(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -((-a*(x*1i - 1))^(1/4)*(x*12i + 8*x^2 - 1)*2i)/(21*a^4*(x*1i - 1)^2*(a*(x*
1i + 1))^(1/4))
```

3.1217 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=141

$$\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}+\frac{42x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}-\frac{42\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 4/5*I*(a-I*a*x)^(7/4)/a/(a+I*a*x)^(5/4)+42/5*x/(a-I*a*x)^(1/4)/(a+I*a*x)^(1
/4)-28/5*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(1/4)-42/5*(x^2+1)^(1/4)*(cos(1/2*ar
ctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/
(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]

time = 0.02, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {49, 42, 235, 233, 202}

$$-\frac{42\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}+\frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}}-\frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}}+\frac{42x}{5\sqrt[4]{a+iax}\,\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(5/4)) + (42*x)/(5*(a - I*a*x)
^(1/4)*(a + I*a*x)^(1/4)) - (((28*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(
1/4)) - (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*(a - I*a*x)^(1/4)
*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{7/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{7}{5}\int \frac{(a-iax)^{3/4}}{(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} + \frac{21}{5}\int \frac{1}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} + \frac{\left(21\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{\sqrt[4]{a^2+a^2x^2}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} + \frac{\left(21\sqrt[4]{1+x^2}\right)\int \frac{1}{\sqrt[4]{1+x^2}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} + \frac{42x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} - \frac{\left(21\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{7/4}}{5a(a+iax)^{5/4}} + \frac{42x}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{28i(a-iax)^{3/4}}{5a\sqrt[4]{a+iax}} - \frac{42\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\ldots\right)}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.50

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{11/4}\,{}_2F_1\left(\frac{9}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((I/11)*(1 + I*x)^(1/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[9/4, 11/4, 15/
4, 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.18, size = 101, normalized size = 0.72

method	result	size
risch	$-\frac{8(4x^2+ix+3)}{5(x-i)(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{21x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] -8/5*(4*x^2+3+I*x)/(x-I)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+21/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-
1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*x^2 - 30*I*x - 21) + 5*(a^2*
x^3 - 2*I*a^2*x^2 - a^2*x)*integral(42/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/
4)/(a^2*x^4 + a^2*x^2), x))/(a^2*x^3 - 2*I*a^2*x^2 - a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-ia(x+i)\right)^{\frac{7}{4}}}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(9/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-1,[1,0]%%%}+%%%{i
,[0,1]%%%}] at parameters values [44,93]ext_reduce Error: Bad Argument Type
integrate
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{7/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(9/4), x)
```

3.1218 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=115

$$\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}$$

```
[Out] 4/5*I*(a-I*a*x)^(3/4)/a/(a+I*a*x)^(5/4)-6/5*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(
1/4)-6/5*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*Elli
pticE(sin(1/2*arctan(x)),2^(1/2))/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {49, 50, 42, 203, 202}

$$-\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}-\frac{6i}{5a\sqrt[4]{a-iax}\sqrt[4]{a+iax}}+\frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(5/4)) - ((6*I)/5)/(a*(a - I*a
*x)^(1/4)*(a + I*a*x)^(1/4)) - (6*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2]
)/(5*a*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[1/(((a_) + (b_.)*(x_))^(5/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-2/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4)), x] + Dist[c, Int[1/((a + b*x)^(
```

```
5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0
] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{3/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{3}{5}\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{1}{5}(3a)\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{\left(3a\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i(a-iax)^{3/4}}{5a(a+iax)^{5/4}} - \frac{6i}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} - \frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.61

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(9/4),x]
[Out] ((I/7)*(1 + I*x)^(1/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 9/4, 11/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.21, size = 107, normalized size = 0.93

method	result	size
risch	$-\frac{2(3x^2+2ix+1)}{5(x-i)a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}+\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(3*x^2+1+2*I*x)/(x-I)/a/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)+3/5/(a^2
)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(
-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(5*I*x + 3) - 5*(a^3*x^3 - 2*I
*a^3*x^2 - a^3*x)*integral(6/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^3*x^
4 + a^3*x^2), x))/(a^3*x^3 - 2*I*a^3*x^2 - a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(9/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - a\,x\,1\mathrm{i})^{3/4}}{(a + a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(9/4),x)
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(9/4), x)
```

3.1219 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=82

$$\frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 4/5*I/a/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4)+2/5*(x^2+1)^(1/4)*(cos(1/2*arctan(x
))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^2/(a
-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {48, 42, 203, 202}

$$\frac{2\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((4*I)/5)/(a*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (2*(1 + x^2)^(1/4)*Elli
pticE[ArcTan[x]/2, 2])/(5*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{9/4}}\,dx &= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{1}{5}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx \\
&= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\sqrt[4]{1+x^2}\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}} \\
&= \frac{4i}{5a\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{2\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.85

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/3)*(1 + I*x)^(1/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4, 9/4, 7/4, 1
/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 105, normalized size = 1.28

method	result	size
risch	$\frac{\frac{2}{5}x^2-\frac{2}{5}ix+\frac{4}{5}}{(x-i)a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^2(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(x^2+2-I*x)/(x-I)/a^2/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-1/5/(a^2)^(
1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^2*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-
a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(x - 2*I) + 5*(a^4*x^2 - 2*I*a^
4*x - a^4)*integral(-1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^4*x^2 + a^
4), x))/(a^4*x^2 - 2*I*a^4*x - a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{9}{4}} \sqrt[4]{-ia(x+i)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(1/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{1/4}\,(a + a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(9/4)),x)
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(9/4)), x)
```

3.1220 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=82

$$\frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2/5*I/a^2/(a-I*a*x)^(1/4)/(a+I*a*x)^(5/4)+6/5*(x^2+1)^(1/4)*(cos(1/2*arctan
(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^3/
(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 48, 42, 203, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((2*I)/5)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(5/4)) + (6*(1 + x^2)^(1/4)*El
lipticE[ArcTan[x]/2, 2])/(5*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 48

```
Int[1/(((a_) + (b_.)*(x_))^(9/4)*((c_) + (d_.)*(x_))^(1/4)), x_Symbol] :> S
imp[-4/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4)), x] - Dist[d/(5*b), Int[1/((a
+ b*x)^(5/4)*(c + d*x)^(5/4)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c
+ a*d, 0] && NegQ[a^2*b^2]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a-iax)^{5/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{3\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{9/4}}\,dx}{a}\\
&= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{3\int \frac{1}{(a-iax)^{5/4}(a+iax)^{5/4}}\,dx}{5a}\\
&= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\
&= \frac{2i}{5a^2\sqrt[4]{a-iax}\,(a+iax)^{5/4}}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 68, normalized size = 0.83

$$
-\frac{i\sqrt[4]{1+ix}\ {}_2F_1\left(-\frac{1}{4},\frac{9}{4};\frac{3}{4};\frac{1}{2}-\frac{ix}{2}\right)}{\sqrt[4]{2}\,a^3\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(9/4)),x]
```

[Out] ((-I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-1/4, 9/4, 3/4, 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.20, size = 107, normalized size = 1.30

method	result	size
risch	$\frac{\frac{6}{5}x^2-\frac{6}{5}ix+\frac{2}{5}}{(x-i)a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{3x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{5(a^2)^{\frac{1}{4}}a^3(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)

[Out] 2/5*(-3*I*x+3*x^2+1)/(x-I)/a^3/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-3/5/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^3*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")

[Out] 1/5*(2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)*(3*x^2 - 3*I*x + 1) + 5*(a^5*x^3 - I*a^5*x^2 + a^5*x - I*a^5)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^5*x^2 + a^5), x))/(a^5*x^3 - I*a^5*x^2 + a^5*x - I*a^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(5/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{5/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(9/4)),x)
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(9/4)), x)
```

3.1221 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=88

$$\frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,\left(1+x^2\right)}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] 2/5*x/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)/(x^2+1)+6/5*(x^2+1)^(1/4)*(cos(1/
2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/
2))/a^4/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {42, 205, 203, 202}

$$\frac{6\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{2x}{5a^4\left(x^2+1\right)\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*x)/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (6*(1 + x^2)^
(1/4)*EllipticE[ArcTan[x]/2, 2])/(5*a^4*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4)
)
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx &= \frac{\sqrt[4]{a^2+a^2x^2}\int \frac{1}{(a^2+a^2x^2)^{9/4}}\,dx}{\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{\left(3\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{5/4}}\,dx}{5a^2\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{\left(3\sqrt[4]{1+x^2}\right)\int \frac{1}{(1+x^2)^{5/4}}\,dx}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= \frac{2x}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{6\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{5a^4\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 70, normalized size = 0.80

$$-\frac{i\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{5}{4},\frac{9}{4};-\frac{1}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5\sqrt[4]{2}\,a^3(a-iax)^{5/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(9/4)),x]
[Out] ((-1/5*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, 1/2 - (I/2)*x]
)/(2^(1/4)*a^3*(a - I*a*x)^(5/4)*(a + I*a*x)^(1/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{9}{4}}\,(iax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

```
[Out] int(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(9/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
[Out] 1/5*(2*(3*x^3 + 4*x)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + 5*(a^6*x^4 + 2*
a^6*x^2 + a^6)*integral(-3/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^6*x^2
+ a^6), x))/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [A]
time = 96.85, size = 95, normalized size = 1.08

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{8},\frac{13}{8},1 & \frac{1}{2},\frac{9}{4},\frac{11}{4}\\ \frac{9}{8},\frac{13}{8},\frac{7}{4},\frac{9}{4},\frac{11}{4} & 0\end{matrix}\middle|\frac{e^{-3i\pi}}{x^2}\right)e^{\frac{i\pi}{4}}}{4\pi a^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{8},\frac{9}{8},1 & \\ \frac{5}{8},\frac{9}{8} & -\frac{1}{2},0,\frac{7}{4},0\end{matrix}\middle|\frac{e^{-i\pi}}{x^2}\right)}{4\pi a^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(9/4),x)
[Out] -I*meijerg(((9/8, 13/8, 1), (1/2, 9/4, 11/4)), ((9/8, 13/8, 7/4, 9/4, 11/4)
, (0,)), exp_polar(-3*I*pi)/x**2)*exp(I*pi/4)/(4*pi*a**(9/2)*gamma(9/4)) +
I*meijerg(((-1/2, 0, 1/2, 5/8, 9/8, 1), ()), ((5/8, 9/8), (-1/2, 0, 7/4, 0)
), exp_polar(-I*pi)/x**2)/(4*pi*a**(9/2)*gamma(9/4))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{9/4}\,(a + a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(9/4)), x)
```

3.1222 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=121

$$-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{14\sqrt[4]{1+x^2}\,E\left(\frac{1}{2}\tan^{-1}(x)\middle|2\right)}{15a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}$$

```
[Out] -2/9*I/a^2/(a-I*a*x)^(9/4)/(a+I*a*x)^(5/4)+14/45*x/a^5/(a-I*a*x)^(1/4)/(a+I
*a*x)^(1/4)/(x^2+1)+14/15*(x^2+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/
2*arctan(x))*EllipticE(sin(1/2*arctan(x)),2^(1/2))/a^5/(a-I*a*x)^(1/4)/(a+I
*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 203, 202}

$$\frac{14\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{15a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{14x}{45a^5\,(x^2+1)\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/9)/(a^2*(a - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(45*a^5*(a -
I*a*x)^(1/4)*(a + I*a*x)^(1/4)*(1 + x^2)) + (14*(1 + x^2)^(1/4)*EllipticE[A
rcTan[x]/2, 2])/(15*a^5*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{7\int \frac{1}{(a-iax)^{9/4}(a+iax)^{9/4}}\,dx}{9a}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{\left(7\sqrt[4]{a^2+a^2x^2}\right)\int \frac{1}{(a^2+a^2x^2)^{9/4}}\,dx}{9a\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{(7\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{(7}{15\\ &= -\frac{2i}{9a^2(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{45a^5\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{14}{1\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.58

$$-\frac{i\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{9}{4},\frac{9}{4};-\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}\,a^3(a-iax)^{9/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(9/4)),x]

[Out] ((-1/9*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-9/4, 9/4, -5/4, 1/2 - (I/2)*x])/(2^(1/4)*a^3*(a - I*a*x)^(9/4)*(a + I*a*x)^(1/4))

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.21, size = 124, normalized size = 1.02

method	result	size
risch	$\frac{\frac{14}{15}x^4+\frac{14}{15}ix^3+\frac{56}{45}x^2+\frac{56}{45}ix+\frac{2}{9}}{(x-i)(x+i)^2a^5(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{7x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{15(a^2)^{\frac{1}{4}}a^5(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	124

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)

[Out] 2/45*(21*I*x^3+21*x^4+28*I*x+28*x^2+5)/(x-I)/(x+I)^2/a^5/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)-7/15/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],-x^2)/a^5*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")

[Out] 1/45*(2*(21*x^4 + 21*I*x^3 + 28*x^2 + 28*I*x + 5)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4) + 45*(a^7*x^5 + I*a^7*x^4 + 2*a^7*x^3 + 2*I*a^7*x^2 + a^7*x + I*a^7)*integral(-7/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(3/4)/(a^7*x^2 + a^7), x))/(a^7*x^5 + I*a^7*x^4 + 2*a^7*x^3 + 2*I*a^7*x^2 + a^7*x + I*a^7)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5458 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{13/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(9/4)), x)
```

3.1223 $\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=154

$$-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}+\frac{14x}{65a^6\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}\,(1+x^2)}+\frac{42\sqrt[4]{1+\ldots}}{65a^6\sqrt[4]{\ldots}}$$

```
[Out] -2/13*I/a^2/(a-I*a*x)^(13/4)/(a+I*a*x)^(5/4)-2/13*I/a^3/(a-I*a*x)^(9/4)/(a+
I*a*x)^(5/4)+14/65*x/a^6/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)/(x^2+1)+42/65*(x^2
+1)^(1/4)*(cos(1/2*arctan(x))^2)^(1/2)/cos(1/2*arctan(x))*EllipticE(sin(1/2
*arctan(x)),2^(1/2))/a^6/(a-I*a*x)^(1/4)/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.03, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {53, 42, 205, 203, 202}

$$\frac{42\sqrt[4]{x^2+1}\,E\left(\frac{\text{ArcTan}(x)}{2}\middle|2\right)}{65a^6\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}+\frac{14x}{65a^6(x^2+1)\sqrt[4]{a-iax}\,\sqrt[4]{a+iax}}-\frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}}-\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/13)/(a^2*(a - I*a*x)^(13/4)*(a + I*a*x)^(5/4)) - ((2*I)/13)/(a^3*(a
 - I*a*x)^(9/4)*(a + I*a*x)^(5/4)) + (14*x)/(65*a^6*(a - I*a*x)^(1/4)*(a +
I*a*x)^(1/4)*(1 + x^2)) + (42*(1 + x^2)^(1/4)*EllipticE[ArcTan[x]/2, 2])/(6
5*a^6*(a - I*a*x)^(1/4)*(a + I*a*x)^(1/4))
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{17/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} + \frac{9\int \frac{1}{(a-iax)^{13/4}(a+iax)^{9/4}}\,dx}{13a}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{7\int \frac{}{(a-}}{}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{\left(7\sqrt[4]{a}\right.}{13}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{}{65a^6\sqrt[4]{}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{}{65a^6\sqrt[4]{}}$$

$$= -\frac{2i}{13a^2(a-iax)^{13/4}(a+iax)^{5/4}} - \frac{2i}{13a^3(a-iax)^{9/4}(a+iax)^{5/4}} + \frac{}{65a^6\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.45

$$-\frac{i\sqrt[4]{1+ix}\;{}_2F_1\left(-\frac{13}{4},\frac{9}{4};-\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{13\sqrt[4]{2}\,a^3(a-iax)^{13/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(17/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-1/13*I)*(1 + I*x)^(1/4)*Hypergeometric2F1[-13/4, 9/4, -9/4, 1/2 - (I/2)*
x])/(2^(1/4)*a^3*(a - I*a*x)^(13/4)*(a + I*a*x)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.21, size = 130, normalized size = 0.84

method	result	size
risch	$\frac{\frac{42}{65}x^5+\frac{84}{65}ix^4+\frac{14}{65}x^3+\frac{112}{65}ix^2-\frac{46}{65}x+\frac{4}{13}i}{(x-i)(x+i)^3a^6(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}-\frac{21x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-x^2\right)\left(-a^2(ix-1)(ix+1)\right)^{\frac{1}{4}}}{65(a^2)^{\frac{1}{4}}a^6(-a(ix-1))^{\frac{1}{4}}(a(ix+1))^{\frac{1}{4}}}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/65*(42*I*x^4+21*x^5+56*I*x^2-23*x+7*x^3+10*I)/(x-I)/(x+I)^3/a^6/(-a*(-1+I
*x))^(1/4)/(a*(1+I*x))^(1/4)-21/65/(a^2)^(1/4)*x*hypergeom([1/4,1/2],[3/2],
-x^2)/a^6*(-a^2*(-1+I*x)*(1+I*x))^(1/4)/(-a*(-1+I*x))^(1/4)/(a*(1+I*x))^(1/
4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/65*(2*(21*x^5 + 42*I*x^4 + 7*x^3 + 56*I*x^2 - 23*x + 10*I)*(I*a*x + a)^(3
/4)*(-I*a*x + a)^(3/4) + 65*(a^8*x^6 + 2*I*a^8*x^5 + a^8*x^4 + 4*I*a^8*x^3
- a^8*x^2 + 2*I*a^8*x - a^8)*integral(-21/65*(I*a*x + a)^(3/4)*(-I*a*x + a)
^(3/4)/(a^8*x^2 + a^8), x))/(a^8*x^6 + 2*I*a^8*x^5 + a^8*x^4 + 4*I*a^8*x^3
- a^8*x^2 + 2*I*a^8*x - a^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(17/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(17/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{17/4}\,(a+a\,x\,1\mathrm{i})^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(17/4)*(a + a*x*1i)^(9/4)), x)
```

3.1224 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=297

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\log}{}$$

```
[Out] 4/5*I*(a-I*a*x)^(5/4)/a/(a+I*a*x)^(5/4)-4*I*(a-I*a*x)^(1/4)/a/(a+I*a*x)^(1/
4)-1/2*I*ln(1-(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a-I*a*x)^(1/2)/(a+I*
a*x)^(1/2))/a*2^(1/2)+1/2*I*ln(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1/4)+(a
-I*a*x)^(1/2)/(a+I*a*x)^(1/2))/a*2^(1/2)-I*arctan(1-(a-I*a*x)^(1/4)*2^(1/2)
/(a+I*a*x)^(1/4))*2^(1/2)/a+I*arctan(1+(a-I*a*x)^(1/4)*2^(1/2)/(a+I*a*x)^(1
/4))*2^(1/2)/a
```

Rubi [A]
time = 0.10, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {49, 65, 246, 217, 1179, 642, 1176, 631, 210}

$$-\frac{i\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(1
/4))/(a*(a + I*a*x)^(1/4)) - (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/
4))/(a + I*a*x)^(1/4)])/a + (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[
a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]
)/(Sqrt[2]*a)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(2i)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\text{Subst}\left(\int \right.}{} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\text{Subst}\left(\int \right.}{} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1 \right.}{} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\,\tan^{-1}\left(1 \right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 127, normalized size = 0.43

$$\frac{2\left(\frac{4(2i-3x)\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a(-i+x)^2} + 5\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\,\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 5(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)\right)}{5a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

[Out] (2*((4*(2*I - 3*x)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/(a*(-I + x)^2) + 5*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)] - 5*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]))/(5*a)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.34, size = 490, normalized size = 1.65

method	result
risch	$\frac{8(3x^2+ix+2)(-a(ix-1))^{\frac{1}{4}}}{5(x-i)a(ix-1)(a(ix+1))^{\frac{1}{4}}}-\frac{\left(\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-\left(-x^4-2ix^3-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)x^2+i\,\mathrm{RootOf}\left(\_Z^2+i\right)\left(-x^4-2ix^3\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)

[Out] 8/5*(3*x^2+2+I*x)/(x-I)/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)-(RootOf(_Z^2+I)*ln((-(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x+I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-(1-x^4-2*I*x^3-2*I*x)^(1/2)+x)/(-1+I*x)^2)+I*RootOf(_Z^2+I)*ln((-I*(1-x^4-2*I*x^3-2*I*x)^(1/4)*RootOf(_Z^2+I)*x^2+2*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)*x-x^3+RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(3/4)-I*(1-x^4-2*I*x^3-2*I*x)^(1/2)*x+I*RootOf(_Z^2+I)*(1-x^4-2*I*x^3-2*I*x)^(1/4)-2*I*x^2+(1-x^4-2*I*x^3-2*I*x)^(1/2)+x)/(-1+I*x)^2))/a*(-a*(-1+I*x))^(1/4)/(-1+I*x)*(-(-1+I*x)^3*(1+I*x))^(1/4)/(a*(1+I*x))^(1/4)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(9/4), x)

Fricas [A]
time = 0.62, size = 343, normalized size = 1.15

$$\frac{5\left(a^2x^2-2i\,a^2x-a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\,(x-i)}\right)-5\left(a^2x^2-2i\,a^2x-a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\,(x-i)}\right)+5\left(a^2x^2-2i\,a^2x-a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-i\,a^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\,(x-i)}\right)-5\left(a^2x^2-2i\,a^2x-a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-i\,a^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}}{2\,(x-i)}\right)-16\left(i\,ax+a\right)^{\frac{3}{4}}\left(-i\,ax+a\right)^{\frac{1}{4}}\left(3\,x-2i\right)}{10\left(a^2x^2-2i\,a^2x-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")

```
[Out] 1/10*(5*(a^2*x^2 - 2*I*a^2*x - a^2)*sqrt(4*I/a^2)*log(1/2*((a^2*x - I*a^2)*
sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 5*(a^2*x
^2 - 2*I*a^2*x - a^2)*sqrt(4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(4*I/a^2)
 - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) + 5*(a^2*x^2 - 2*I*a^2*
x - a^2)*sqrt(-4*I/a^2)*log(1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x
+ a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 5*(a^2*x^2 - 2*I*a^2*x - a^2)*sqr
t(-4*I/a^2)*log(-1/2*((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(x - I)) - 16*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(3*x
 - 2*I))/(a^2*x^2 - 2*I*a^2*x - a^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(9/4), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(-(-i)/4*16*(
(sageVARa+(-i)*sageVARa*sageVARx)^(1/4))^8/(((sageVARa+(-i)*sageVARa*sageVA
Rx)^(1/4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4), x)
```

3.1225 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

```
[Out] 2/5*I*(a-I*a*x)^(5/4)/a^2/(a+I*a*x)^(5/4)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{37\}$

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Mathematica [A]
time = 0.08, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.15, size = 50, normalized size = 1.52

method	result	size
risch	$\frac{2(-a(ix-1))^{\frac{1}{4}}(x^2+2ix-1)}{5a^2(ix-1)(a(ix+1))^{\frac{1}{4}}(x-i)}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/a^2*(-a*(-1+I*x))^(1/4)/(-1+I*x)/(a*(1+I*x))^(1/4)*(2*I*x+x^2-1)/(x-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

Fricas [A]
time = 0.76, size = 42, normalized size = 1.27

$$-\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(x+i)}{5\,(a^3x^2-2i\,a^3x-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -2/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(x + I)/(a^3*x^2 - 2*I*a^3*x - a^
3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(9/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

Mupad [B]
time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^2\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(9/4),x)
[Out] -(2*(x*1i - 1)*(-a*(x*1i - 1))^(1/4))/(5*a^2*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

3.1226 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=67

$$\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}+\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}$$

```
[Out] 2/5*I*(a-I*a*x)^(1/4)/a^2/(a+I*a*x)^(5/4)+4/5*I*(a-I*a*x)^(1/4)/a^3/(a+I*a*
x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{5a}$$
$$= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}} + \frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.67

$$\frac{2(3+2ix)\sqrt[4]{a-iax}}{5a^3(-i+x)\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
[Out] (2*(3 + (2*I)*x)*(a - I*a*x)^(1/4))/(5*a^3*(-I + x)*(a + I*a*x)^(1/4))
```

Maple [A]
time = 0.17, size = 44, normalized size = 0.66

method	result	size
risch	$\frac{\frac{4}{5}x^2-\frac{2}{5}ix+\frac{6}{5}}{a^2(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
[Out] 2/5/a^2/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(2*x^2+3-I*x)/(x-I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

Fricas [A]
time = 1.09, size = 44, normalized size = 0.66

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x-3i)}{5\,(a^4x^2-2i\,a^4x-a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x - 3*I)/(a^4*x^2 - 2*I*a^4*x -
 a^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(3/4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.63, size = 38, normalized size = 0.57

$$\frac{2\left(3+x\,2\mathrm{i}\right)\,{\left(-a\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^3\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(x*2i + 3)*(-a*(x*1i - 1))^(1/4))/(5*a^3*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

3.1227 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}$$

```
[Out] -2/3*I/a^2/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4)+8/15*I*(a-I*a*x)^(1/4)/a^3/(a+I*
a*x)^(5/4)+16/15*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {47, 37}

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((8*I)/15)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(5/4)) + (((16*I)/15)*(a - I*a*x)^(1/4))/(a^4*(
a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{4\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx}{3a}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,d}{15a^2}$$
$$= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}$$

Mathematica [A]
time = 0.11, size = 52, normalized size = 0.52

$$-\frac{2i(a+iax)^{3/4}\left(7-4ix+8x^2\right)}{15a^4(-i+x)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4)*(7 - (4*I)*x + 8*x^2))/(a^4*(-I + x)^2*(a - I*a*x)^(3/4))
```

Maple [A]
time = 0.15, size = 44, normalized size = 0.44

method	result	size
risch	$\frac{\frac{16}{15}x^2-\frac{8}{15}ix+\frac{14}{15}}{a^3(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^3/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(8*x^2-4*I*x+7)/(x-I)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 1.29, size = 56, normalized size = 0.56

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(8\,x^2-4i\,x+7)}{15\,(a^5x^3-i\,a^5x^2+a^5x-i\,a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2/15*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 - 4*I*x + 7)/(a^5*x^3 - I*
a^5*x^2 + a^5*x - I*a^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{9}{4}}\,(-ia\,(x+i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(7/4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**B**]
time = 0.53, size = 45, normalized size = 0.45

$$\frac{2\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}\,\left(x^2\,8\mathrm{i}+4\,x+7\mathrm{i}\right)}{15\,a^4\,\left(x^2+1\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(4*x + x^2*8i + 7i))/(15*a^4*(x^2 + 1)*(a*(x*1i +
1))^(1/4))
```

3.1228 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=133

$$-\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}+\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}$$

```
[Out] -2/7*I/a^2/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4)-4/7*I/a^3/(a-I*a*x)^(3/4)/(a+I*a
*x)^(5/4)+16/35*I*(a-I*a*x)^(1/4)/a^4/(a+I*a*x)^(5/4)+32/35*I*(a-I*a*x)^(1/
4)/a^5/(a+I*a*x)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {47, 37}

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) - ((4*I)/7)/(a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((16*I)/35)*(a - I*a*x)^(1/4))/(a^4*(a +
 I*a*x)^(5/4)) + (((32*I)/35)*(a - I*a*x)^(1/4))/(a^5*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx = -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} + \frac{6\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx}{7a}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{8\int \frac{}{(a-iax}}{}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-}}{35a^4(a+}$$
$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}} - \frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}} + \frac{16i\sqrt[4]{a-}}{35a^4(a+}$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.48

$$\frac{2(a+iax)^{3/4}\left(9-22ix+8x^2-16ix^3\right)}{35a^5(-i+x)^2(i+x)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(9 - (22*I)*x + 8*x^2 - (16*I)*x^3))/(35*a^5*(-I + x)^
2*(I + x)*(a - I*a*x)^(3/4))
```

Maple [A]
time = 0.16, size = 56, normalized size = 0.42

method	result	size
risch	$\frac{\frac{32}{35}x^3+\frac{16}{35}ix^2+\frac{44}{35}x+\frac{18}{35}i}{a^4(-a(ix-1))^{\frac{3}{4}}(a(ix+1))^{\frac{1}{4}}(x-i)(x+i)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/a^4/(-a*(-1+I*x))^(3/4)/(a*(1+I*x))^(1/4)*(16*x^3+8*I*x^2+22*x+9*I)/(x
-I)/(x+I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

Fricas [A]
time = 0.96, size = 54, normalized size = 0.41

$$\frac{2\,(16\,x^3+8i\,x^2+22\,x+9i)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{35\,(a^6x^4+2\,a^6x^2+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
[Out] 2/35*(16*x^3 + 8*I*x^2 + 22*x + 9*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(
a^6*x^4 + 2*a^6*x^2 + a^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(9/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [B]
time = 0.69, size = 56, normalized size = 0.42

$$\frac{2\,{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)}^{1/4}\,\left(x^4\,16{}\mathrm{i}+8\,x^3+x^2\,30{}\mathrm{i}+13\,x+9{}\mathrm{i}\right)}{35\,a^5\,{\left(x^2+1\right)}^2\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(9/4)),x)
[Out] (2*(-a*(x*1i - 1))^(1/4)*(13*x + x^2*30i + 8*x^3 + x^4*16i + 9i))/(35*a^5*(
x^2 + 1)^2*(a*(x*1i + 1))^(1/4))
```

3.1229 $\int (a+bx)^2(ac-bcx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}$$

```
[Out] -4*a^2*(-b*c*x+a*c)^(1+n)/b/c/(1+n)+4*a*(-b*c*x+a*c)^(2+n)/b/c^2/(2+n)-(-b*
c*x+a*c)^(3+n)/b/c^3/(3+n)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{45\}$

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] (-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))/
(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^n\,dx &= \int \left(4a^2(ac-bcx)^n-\frac{4a(ac-bcx)^{1+n}}{c}+\frac{(ac-bcx)^{2+n}}{c^2}\right)\,dx\\ &= -\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 77, normalized size = 0.93

$$\frac{(c(a-bx))^n(-a+bx)\left(a^2(14+7n+n^2)+2ab(4+5n+n^2)\,x+b^2(2+3n+n^2)\,x^2\right)}{b(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a^2*(14 + 7*n + n^2) + 2*a*b*(4 + 5*n + n^2)*x
 + b^2*(2 + 3*n + n^2)*x^2))/(b*(1 + n)*(2 + n)*(3 + n))
```

Maple [A]
time = 0.17, size = 103, normalized size = 1.24

method	result	size
gosper	$-\frac{(-bx+a)\left(b^{2} n^{2} x^{2}+2 a b\, n^{2} x+3 b^{2} n\, x^{2}+a^{2} n^{2}+10 a b n x+2 x^{2} b^{2}+7 a^{2} n+8 a b x+14 a^{2}\right)(-bcx+ac)^{n}}{b\left(n^{3}+6 n^{2}+11 n+6\right)}$	103
risch	$-\frac{\left(-b^{3} n^{2} x^{3}-a\, b^{2} n^{2} x^{2}-3 n\, b^{3} x^{3}+a^{2} b\, n^{2} x-7 a\, b^{2} n\, x^{2}-2 b^{3} x^{3}+a^{3} n^{2}+3 a^{2} b n x-6 a\, b^{2} x^{2}+7 a^{3} n-6 a^{2} b x+14 a^{3}\right)(c(-bx+a))^{n}}{(2+n)(3+n)b(1+n)}$	133
norman	$\frac{b^{2} x^{3} \mathrm{e}^{n \ln(-bcx+ac)}}{3+n}+\frac{a b(6+n) x^{2} \mathrm{e}^{n \ln(-bcx+ac)}}{n^{2}+5 n+6}-\frac{a^{2}\left(n^{2}+3 n-6\right) x\, \mathrm{e}^{n \ln(-bcx+ac)}}{n^{3}+6 n^{2}+11 n+6}-\frac{a^{3}\left(n^{2}+7 n+14\right) \mathrm{e}^{n \ln(-bcx+ac)}}{b\left(n^{3}+6 n^{2}+11 n+6\right)}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*x+a)*(b^2*n^2*x^2+2*a*b*n^2*x+3*b^2*n*x^2+a^2*n^2+10*a*b*n*x+2*b^2*x^2
+7*a^2*n+8*a*b*x+14*a^2)*(-b*c*x+a*c)^n/b/(n^3+6*n^2+11*n+6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.
time = 0.32, size = 167, normalized size = 2.01

$$\frac{2\left(b^{2} c^{n}(n+1) x^{2}-a b c^{n} n x-a^{2} c^{n}\right)(-bx+a)^{n} a}{\left(n^{2}+3 n+2\right) b}+\frac{\left(\left(n^{2}+3 n+2\right) b^{3} c^{n} x^{3}-\left(n^{2}+n\right) a b^{2} c^{n} x^{2}-2 a^{2} b c^{n} n x-2 a^{3} c^{n}\right)(-bx+a)^{n}}{\left(n^{3}+6 n^{2}+11 n+6\right) b}-\frac{(-bcx+ac)^{n+1} a^{2}}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 2*(b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n*a/((n^2 + 3*n
+ 2)*b) + ((n^2 + 3*n + 2)*b^3*c^n*x^3 - (n^2 + n)*a*b^2*c^n*x^2 - 2*a^2*b*
c^n*n*x - 2*a^3*c^n)*(-b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b) - (-b*c*x +
a*c)^(n + 1)*a^2/(b*c*(n + 1))
```

Fricas [A]
time = 0.72, size = 128, normalized size = 1.54

$$-\frac{\left(a^{3} n^{2}+7 a^{3} n-\left(b^{3} n^{2}+3 b^{3} n+2 b^{3}\right) x^{3}+14 a^{3}-\left(a b^{2} n^{2}+7 a b^{2} n+6 a b^{2}\right) x^{2}+\left(a^{2} b n^{2}+3 a^{2} b n-6 a^{2} b\right) x\right)(-bcx+ac)^{n}}{b n^{3}+6 b n^{2}+11 b n+6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^2 + 7*a^3*n - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + 14*a^3 - (a*b^2*n^2
 + 7*a*b^2*n + 6*a*b^2)*x^2 + (a^2*b*n^2 + 3*a^2*b*n - 6*a^2*b)*x)*(-b*c*x
+ a*c)^n/(b*n^3 + 6*b*n^2 + 11*b*n + 6*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(66) = 132$.
time = 0.43, size = 819, normalized size = 9.87

$$\begin{cases} a^{2}x(ac)^{n} & \text{for } b = 0 \\ -\frac{a^{2}\log\left(-\frac{a}{b}+x\right)}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}} - \frac{2a^{2}}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}} + \frac{2abx\log\left(-\frac{a}{b}+x\right)}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}} + \frac{4abx}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}} - \frac{b^{2}x^{2}\log\left(-\frac{a}{b}+x\right)}{a^{2}bc^{3}-2ab^{2}c^{3}x+b^{3}c^{3}x^{2}} & \text{for } n = -3 \\ -\frac{4a^{2}\log\left(-\frac{a}{b}+x\right)}{-abc^{2}+b^{2}c^{2}x} - \frac{5a^{2}}{-abc^{2}+b^{2}c^{2}x} + \frac{4abx\log\left(-\frac{a}{b}+x\right)}{-abc^{2}+b^{2}c^{2}x} + \frac{b^{2}x^{2}}{-abc^{2}+b^{2}c^{2}x} & \text{for } n = -2 \\ -\frac{4a^{2}\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{3ax}{c} - \frac{bx^{2}}{2c} & \text{for } n = -1 \\ -\frac{a^{3}n^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} - \frac{7a^{3}n(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} - \frac{14a^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} - \frac{a^{2}bn^{2}x(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} - \frac{3a^{2}bnx(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{6a^{2}bx(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{ab^{2}n^{2}x^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{7ab^{2}nx^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{6ab^{2}x^{2}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{b^{3}n^{2}x^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{3b^{3}nx^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} + \frac{2b^{3}x^{3}(ac-bcx)^{n}}{bn^{3}+6bn^{2}+11bn+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a**2*x*(a*c)**n, Eq(b, 0)), (-a**2*log(-a/b + x)/(a**2*b*c**3 -
2*a*b**2*c**3*x + b**3*c**3*x**2) - 2*a**2/(a**2*b*c**3 - 2*a*b**2*c**3*x +
 b**3*c**3*x**2) + 2*a*b*x*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b
**3*c**3*x**2) + 4*a*b*x/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) -
 b**2*x**2*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2),
Eq(n, -3)), (-4*a**2*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) - 5*a**2/(-a*b
*c**2 + b**2*c**2*x) + 4*a*b*x*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) + b*
*2*x**2/(-a*b*c**2 + b**2*c**2*x), Eq(n, -2)), (-4*a**2*log(-a/b + x)/(b*c)
 - 3*a*x/c - b*x**2/(2*c), Eq(n, -1)), (-a**3*n**2*(a*c - b*c*x)**n/(b*n**3
 + 6*b*n**2 + 11*b*n + 6*b) - 7*a**3*n*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2
+ 11*b*n + 6*b) - 14*a**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*
b) - a**2*b*n**2*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 3*
a**2*b*n*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a**2*b*x
*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + a*b**2*n**2*x**2*(a*
c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 7*a*b**2*n*x**2*(a*c - b
*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a*b**2*x**2*(a*c - b*c*x)**
n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + b**3*n**2*x**3*(a*c - b*c*x)**n/(b*n
**3 + 6*b*n**2 + 11*b*n + 6*b) + 3*b**3*n*x**3*(a*c - b*c*x)**n/(b*n**3 + 6
*b*n**2 + 11*b*n + 6*b) + 2*b**3*x**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 +
 11*b*n + 6*b), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(83) = 166$.
time = 0.78, size = 256, normalized size = 3.08

$$\frac{(-bcx+ac)^{n}b^{3}n^{2}x^{3} + (-bcx+ac)^{n}ab^{2}n^{2}x^{2} + 3\,(-bcx+ac)^{n}b^{3}nx^{3} - (-bcx+ac)^{n}a^{2}bn^{2}x + 7\,(-bcx+ac)^{n}ab^{2}nx^{2} + 2\,(-bcx+ac)^{n}b^{3}x^{3} - (-bcx+ac)^{n}a^{3}n^{2} - 3\,(-bcx+ac)^{n}a^{2}bnx + 6\,(-bcx+ac)^{n}ab^{2}x^{2} - 7\,(-bcx+ac)^{n}a^{3}n + 6\,(-bcx+ac)^{n}a^{2}bx - 14\,(-bcx+ac)^{n}a^{3}}{bn^{3} + 6\,bn^{2} + 11\,bn + 6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^3*n^2*x^3 + (-b*c*x + a*c)^n*a*b^2*n^2*x^2 + 3*(-b*c*x
+ a*c)^n*b^3*n*x^3 - (-b*c*x + a*c)^n*a^2*b*n^2*x + 7*(-b*c*x + a*c)^n*a*b^
2*n*x^2 + 2*(-b*c*x + a*c)^n*b^3*x^3 - (-b*c*x + a*c)^n*a^3*n^2 - 3*(-b*c*x
 + a*c)^n*a^2*b*n*x + 6*(-b*c*x + a*c)^n*a*b^2*x^2 - 7*(-b*c*x + a*c)^n*a^3
*n + 6*(-b*c*x + a*c)^n*a^2*b*x - 14*(-b*c*x + a*c)^n*a^3)/(b*n^3 + 6*b*n^2
 + 11*b*n + 6*b)
```

Mupad [B]
time = 0.49, size = 133, normalized size = 1.60

$$-(a\,c-b\,c\,x)^n\,\left(\frac{a^2\,x\,\left(n^2+3\,n-6\right)}{n^3+6\,n^2+11\,n+6}+\frac{a^3\,\left(n^2+7\,n+14\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{b^2\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}-\frac{a\,b\,x^2\,\left(n^2+7\,n+6\right)}{n^3+6\,n^2+11\,n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x)^2,x)
```

```
[Out] -(a*c - b*c*x)^n*((a^2*x*(3*n + n^2 - 6))/(11*n + 6*n^2 + n^3 + 6) + (a^3*(
7*n + n^2 + 14))/(b*(11*n + 6*n^2 + n^3 + 6)) - (b^2*x^3*(3*n + n^2 + 2))/(
11*n + 6*n^2 + n^3 + 6) - (a*b*x^2*(7*n + n^2 + 6))/(11*n + 6*n^2 + n^3 + 6
))
```

3.1230 $\int (a+bx)(ac-bcx)^n\,dx$

Optimal. Leaf size=53

$$-\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}$$

```
[Out] -2*a*(-b*c*x+a*c)^(1+n)/b/c/(1+n)+(-b*c*x+a*c)^(2+n)/b/c^2/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] (-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*(
2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^n\,dx &= \int \left(2a(ac-bcx)^n-\frac{(ac-bcx)^{1+n}}{c}\right)dx\\ &= -\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.81

$$\frac{(c(a-bx))^n(-a+bx)(a(3+n)+b(1+n)x)}{b(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a*(3 + n) + b*(1 + n)*x))/(b*(1 + n)*(2 + n))
```

Maple [A]
time = 0.12, size = 47, normalized size = 0.89

method	result	size
gosper	$-\frac{(-bcx+ac)^n(bnx+an+bx+3a)(-bx+a)}{b(n^2+3n+2)}$	47
risch	$-\frac{(-b^2n\,x^2-x^2b^2+a^2n-2abx+3a^2)(c(-bx+a))^n}{(2+n)(1+n)b}$	59
norman	$\frac{b\,x^2\mathrm{e}^{n\ln(-bcx+ac)}}{2+n}+\frac{2ax\,\mathrm{e}^{n\ln(-bcx+ac)}}{n^2+3n+2}-\frac{a^2(3+n)\mathrm{e}^{n\ln(-bcx+ac)}}{b(n^2+3n+2)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -(-b*c*x+a*c)^n*(b*n*x+a*n+b*x+3*a)*(-b*x+a)/b/(n^2+3*n+2)
```

Maxima [A]
time = 0.31, size = 81, normalized size = 1.53

$$\frac{(b^2c^n(n+1)x^2-abc^nnx-a^2c^n)(-bx+a)^n}{(n^2+3\,n+2)b}-\frac{(-bcx+ac)^{n+1}a}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] (b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n/((n^2 + 3*n + 2)
*b) - (-b*c*x + a*c)^(n + 1)*a/(b*c*(n + 1))
```

Fricas [A]
time = 0.74, size = 58, normalized size = 1.09

$$-\frac{(a^2n-2\,abx-(b^2n+b^2)x^2+3\,a^2)(-bcx+ac)^n}{bn^2+3\,bn+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n - 2*a*b*x - (b^2*n + b^2)*x^2 + 3*a^2)*(-b*c*x + a*c)^n/(b*n^2 + 3*
b*n + 2*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(41) = 82$.

time = 0.25, size = 245, normalized size = 4.62

$$\begin{cases} ax(ac)^n & \text{for } b = 0 \\ -\frac{a\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{2a}{-abc^2+b^2c^2x} + \frac{bx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} & \text{for } n = -2 \\ -\frac{2a\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{x}{c} & \text{for } n = -1 \\ -\frac{a^2n(ac-bcx)^n}{bn^2+3bn+2b} - \frac{3a^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{2abx(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2nx^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2x^2(ac-bcx)^n}{bn^2+3bn+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a*x*(a*c)**n, Eq(b, 0)), (-a*log(-a/b + x)/(-a*b*c**2 + b**2*c**
2*x) - 2*a/(-a*b*c**2 + b**2*c**2*x) + b*x*log(-a/b + x)/(-a*b*c**2 + b**2*
c**2*x), Eq(n, -2)), (-2*a*log(-a/b + x)/(b*c) - x/c, Eq(n, -1)), (-a**2*n*
(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) - 3*a**2*(a*c - b*c*x)**n/(b*n**2 +
 3*b*n + 2*b) + 2*a*b*x*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*n*x*
*2*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*x**2*(a*c - b*c*x)**n/(b*
n**2 + 3*b*n + 2*b), True))
```

Giac [A]
time = 1.13, size = 103, normalized size = 1.94

$$\frac{(-bcx+ac)^n b^2nx^2 + (-bcx+ac)^n b^2x^2 - (-bcx+ac)^n a^2n + 2\,(-bcx+ac)^n abx - 3\,(-bcx+ac)^n a^2}{bn^2 + 3\,bn + 2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^2*n*x^2 + (-b*c*x + a*c)^n*b^2*x^2 - (-b*c*x + a*c)^n*a
^2*n + 2*(-b*c*x + a*c)^n*a*b*x - 3*(-b*c*x + a*c)^n*a^2)/(b*n^2 + 3*b*n +
2*b)
```

Mupad [B]
time = 0.32, size = 66, normalized size = 1.25

$$(a\,c-b\,c\,x)^n\,\left(\frac{2\,a\,x}{n^2+3\,n+2}-\frac{a^2\,(n+3)}{b\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,(n+1)}{n^2+3\,n+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x),x)
```

```
[Out] (a*c - b*c*x)^n*((2*a*x)/(3*n + n^2 + 2) - (a^2*(n + 3))/(b*(3*n + n^2 + 2)
) + (b*x^2*(n + 1))/(3*n + n^2 + 2))
```

3.1231 $\int \frac{(ac-bcx)^n}{a+bx}\,dx$

Optimal. Leaf size=52

$$-\frac{(ac-bcx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2abc(1+n)}$$

```
[Out] -1/2*(-b*c*x+a*c)^(1+n)*hypergeom([1, 1+n],[2+n],1/2*(-b*x+a)/a)/a/b/c/(1+n
)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{70\}$

$$-\frac{(ac-bcx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a-bx}{2a}\right)}{2abc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^n/(a + b*x),x]
```

```
[Out] -1/2*((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x)/(2
*a)])/(a*b*c*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(ac-bcx)^n}{a+bx}\,dx = -\frac{(ac-bcx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2abc(1+n)}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 1.00

$$-\frac{(a-bx)(c(a-bx))^n\,{}_2F_1\left(1,1+n;2+n;\frac{a-bx}{2a}\right)}{2ab(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^n/(a + b*x),x]
```

```
[Out] -1/2*((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[1, 1 + n, 2 + n, (a - b*x
)/(2*a)])/(a*b*(1 + n))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-bcx + ac)^n}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^n/(b*x+a),x)
```

```
[Out] int((-b*c*x+a*c)^n/(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^n/(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(-a + bx))^n}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**n/(b*x+a),x)
```

```
[Out] Integral((-c*(-a + b*x))**n/(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,c-b\,c\,x)^n}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n/(a + b*x),x)
```

```
[Out] int((a*c - b*c*x)^n/(a + b*x), x)
```

3.1232 $\int \frac{(ac-bcx)^n}{(a+bx)^2} dx$

Optimal. Leaf size=52

$$-\frac{(ac-bcx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2bc(1+n)}$$

```
[Out] -1/4*(-b*c*x+a*c)^(1+n)*hypergeom([2, 1+n],[2+n],1/2*(-b*x+a)/a)/a^2/b/c/(1
+n)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{70\}$

$$-\frac{(ac-bcx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{a-bx}{2a}\right)}{4a^2bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

```
[Out] -1/4*((a*c - b*c*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x)/(2
*a)])/(a^2*b*c*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(ac-bcx)^n}{(a+bx)^2}\,dx = -\frac{(ac-bcx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2bc(1+n)}$$

Mathematica [A]
time = 0.04, size = 52, normalized size = 1.00

$$-\frac{(a-bx)(c(a-bx))^n\,{}_2F_1\left(2,1+n;2+n;\frac{a-bx}{2a}\right)}{4a^2b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^n/(a + b*x)^2,x]
```

```
[Out] -1/4*((a - b*x)*(c*(a - b*x))^n*Hypergeometric2F1[2, 1 + n, 2 + n, (a - b*x
)/(2*a)])/(a^2*b*(1 + n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(-bcx + ac)^n}{(bx + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

```
[Out] int((-b*c*x+a*c)^n/(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(-a + bx))^n}{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**n/(b*x+a)**2,x)
```

```
[Out] Integral((-c*(-a + b*x))**n/(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^n/(b*x+a)^2,x, algorithm="giac")
[Out] integrate((-b*c*x + a*c)^n/(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,c-b\,c\,x)^n}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n/(a + b*x)^2,x)
[Out] int((a*c - b*c*x)^n/(a + b*x)^2, x)
```

3.1233 $\int (a+ax)^m(c-cx)^m\,dx$

Optimal. Leaf size=41

$$x(a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

```
[Out] x*(a*x+a)^m*(-c*x+c)^m*hypergeom([1/2, -m],[3/2],x^2)/((-x^2+1)^m)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {42, 252, 251}

$$x\left(1-x^2\right)^{-m}(ax+a)^m(c-cx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^m*(c - c*x)^m,x]
```

```
[Out] (x*(a + a*x)^m*(c - c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, x^2])/(1 - x^2)^
m
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+ax)^m(c-cx)^m\,dx = \left((a+ax)^m(c-cx)^m\left(ac-acx^2\right)^{-m}\right)\int \left(ac-acx^2\right)^m\,dx$$
$$= \left((a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\right)\int \left(1-x^2\right)^m\,dx$$
$$= x(a+ax)^m(c-cx)^m\left(1-x^2\right)^{-m}\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};x^2\right)$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 1.29

$$\frac{2^m(-1+x)(1+x)^{-m}(a(1+x))^m(c-cx)^m\,{}_2F_1\left(-m,1+m;2+m;\frac{1}{2}-\frac{x}{2}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^m*(c - c*x)^m,x]
[Out] (2^m*(-1 + x)*(a*(1 + x))^m*(c - c*x)^m*Hypergeometric2F1[-m, 1 + m, 2 + m,
 1/2 - x/2])/((1 + m)*(1 + x)^m)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (ax+a)^m\,(-cx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^m*(-c*x+c)^m,x)
[Out] int((a*x+a)^m*(-c*x+c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="maxima")
[Out] integrate((a*x + a)^m*(-c*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((a*x + a)^m*(-c*x + c)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.09, size = 124, normalized size = 3.02

$$\frac{a^{m}c^{m}G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2}, \frac{1}{2}-\frac{m}{2}, 1 & \frac{1}{2}, -m, \frac{1}{2}-m \\ -m-\frac{1}{2}, -m, -\frac{m}{2}, \frac{1}{2}-m, \frac{1}{2}-\frac{m}{2} & 0 \end{matrix}\middle| \frac{e^{-2i\pi}}{x^{2}}\right)e^{-i\pi m}}{4\pi\Gamma\left(-m\right)} - \frac{a^{m}c^{m}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2}, 1 & \\ -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2} & -\frac{1}{2}, 0, -m-\frac{1}{2}, 0 \end{matrix}\middle| \frac{1}{x^{2}}\right)}{4\pi\Gamma\left(-m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**m*(-c*x+c)**m,x)
```

```
[Out] a**m*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m - 1/2, -
m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), exp_polar(-2*I*pi)/x**2)*exp(-I*pi*m)/
(4*pi*gamma(-m)) - a**m*c**m*meijerg(((-1/2, 0, 1/2, -m/2 - 1/2, -m/2, 1),
()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0)), x**(-2))/(4*pi*gamma(-m)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^m*(-c*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((a*x + a)^m*(-c*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a+a\,x\right)}^m\,{\left(c-c\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^m*(c - c*x)^m,x)
```

```
[Out] int((a + a*x)^m*(c - c*x)^m, x)
```

3.1234 $\int (a+bx)^m(ac-bcx)^m\,dx$

Optimal. Leaf size=57

$$x(a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

```
[Out] x*(b*x+a)^m*(-b*c*x+a*c)^m*hypergeom([1/2, -m],[3/2],b^2*x^2/a^2)/((1-b^2*x
^2/a^2)^m)
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {42, 252, 251}

$$x(a+bx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}(ac-bcx)^m\,{}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

```
[Out] (x*(a + b*x)^m*(a*c - b*c*x)^m*Hypergeometric2F1[1/2, -m, 3/2, (b^2*x^2)/a^
2])/(1 - (b^2*x^2)/a^2)^m
```

Rule 42

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(a
 + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^FracPart[m]), In
t[(a*c + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d,
0] &&  !IntegerQ[2*m]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int (a+bx)^m(ac-bcx)^m\,dx = \left((a+bx)^m(ac-bcx)^m\left(a^2c-b^2cx^2\right)^{-m}\right)\int \left(a^2c-b^2cx^2\right)^m\,dx$$
$$= \left((a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}\right)\int \left(1-\frac{b^2x^2}{a^2}\right)^m dx$$
$$= x(a+bx)^m(ac-bcx)^m\left(1-\frac{b^2x^2}{a^2}\right)^{-m}\, {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};\frac{b^2x^2}{a^2}\right)$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.26

$$-\frac{2^m(a-bx)(c(a-bx))^m(a+bx)^m\left(\frac{a+bx}{a}\right)^{-m}\,{}_2F_1\left(-m,1+m;2+m;\frac{a-bx}{2a}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c - b*c*x)^m,x]
```

```
[Out] -((2^m*(a - b*x)*(c*(a - b*x))^m*(a + b*x)^m*Hypergeometric2F1[-m, 1 + m, 2
 + m, (a - b*x)/(2*a)])/(b*(1 + m)*((a + b*x)/a)^m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(-bcx+ac)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

```
[Out] int((b*x+a)^m*(-b*c*x+a*c)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="maxima")
```

```
[Out] integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="fricas")
```

```
[Out] integral((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.81, size = 146, normalized size = 2.56

$$\frac{aa^{2m}c^{m}G_{6,6}^{5,3}\left(\begin{matrix} -\frac{m}{2}, \frac{1}{2}-\frac{m}{2}, 1 & \frac{1}{2}, -m, \frac{1}{2}-m \\ -m-\frac{1}{2}, -m, -\frac{m}{2}, \frac{1}{2}-m, \frac{1}{2}-\frac{m}{2} & 0 \end{matrix} \middle| \frac{a^{2}e^{-2i\pi}}{b^{2}x^{2}}\right)e^{-i\pi m}}{4\pi b\Gamma\left(-m\right)} - \frac{aa^{2m}c^{m}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2}, 1 & \\ -\frac{m}{2}-\frac{1}{2}, -\frac{m}{2} & -\frac{1}{2}, 0, -m-\frac{1}{2}, 0 \end{matrix} \middle| \frac{a^{2}}{b^{2}x^{2}}\right)}{4\pi b\Gamma\left(-m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(-b*c*x+a*c)**m,x)
```

```
[Out] a*a**(2*m)*c**m*meijerg(((-m/2, 1/2 - m/2, 1), (1/2, -m, 1/2 - m)), ((-m -
1/2, -m, -m/2, 1/2 - m, 1/2 - m/2), (0,)), a**2*exp_polar(-2*I*pi)/(b**2*x*
*2))*exp(-I*pi*m)/(4*pi*b*gamma(-m)) - a*a**(2*m)*c**m*meijerg(((-1/2, 0, 1
/2, -m/2 - 1/2, -m/2, 1), ()), ((-m/2 - 1/2, -m/2), (-1/2, 0, -m - 1/2, 0))
, a**2/(b**2*x**2))/(4*pi*b*gamma(-m))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(-b*c*x+a*c)^m,x, algorithm="giac")
```

```
[Out] integrate((-b*c*x + a*c)^m*(b*x + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a\,c-b\,c\,x\right)}^m\,{\left(a+b\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^m*(a + b*x)^m,x)
```

```
[Out] int((a*c - b*c*x)^m*(a + b*x)^m, x)
```

3.1235 $\int (3-6x)^m (2+4x)^m \, dx$

Optimal. Leaf size=20

$$6^m x \, {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; 4x^2\right)$$

```
[Out] 6^m*x*hypergeom([1/2, -m],[3/2],4*x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {41, 251}

$$6^m x \, {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; 4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (3-6x)^m (2+4x)^m \, dx &= \int \left(6-24x^2\right)^m \, dx \\ &= 6^m x \, {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; 4x^2\right) \end{aligned}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$6^m x \, {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; 4x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^m*(2 + 4*x)^m,x]
```

```
[Out] 6^m*x*Hypergeometric2F1[1/2, -m, 3/2, 4*x^2]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (3-6x)^m (2+4x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^m*(2+4*x)^m,x)
```

```
[Out] int((3-6*x)^m*(2+4*x)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="maxima")
```

```
[Out] integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="fricas")
```

```
[Out] integral((4*x + 2)^m*(-6*x + 3)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.22, size = 42, normalized size = 2.10

$$\frac{24^m \left(x+\frac{1}{2}\right) \left(x+\frac{1}{2}\right)^m \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} -m, m+1 \\ m+2 \end{matrix}\middle| \left(x+\frac{1}{2}\right) e^{2i\pi}\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**m*(4*x+2)**m,x)
```

```
[Out] 24**m*(x + 1/2)*(x + 1/2)**m*gamma(m + 1)*hyper((-m, m + 1), (m + 2,), (x +
 1/2)*exp_polar(2*I*pi))/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^m*(4*x+2)^m,x, algorithm="giac")
[Out] integrate((4*x + 2)^m*(-6*x + 3)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int (4\,x+2)^m\,(3-6\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^m*(3 - 6*x)^m,x)
[Out] int((4*x + 2)^m*(3 - 6*x)^m, x)
```

3.1236 $\int (a+bx)^4(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

```
[Out] 1/5*(-a*d+b*c)*(b*x+a)^5/b^2+1/6*d*(b*x+a)^6/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^4}{b}+\frac{d(a+bx)^5}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(38) = 76.
time = 0.01, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^4(2c+dx)+20a^3bx(3c+2dx)+15a^2b^2x^2(4c+3dx)+6ab^3x^3(5c+4dx)+b^4x^4(6c+5dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x),x]
```

[Out] (x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c + 3*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.
time = 0.12, size = 97, normalized size = 2.55

method	result
norman	$\frac{b^4 d\,x^6}{6} + \left(\frac{4}{5}a\,b^3 d + \frac{1}{5}b^4 c\right)x^5 + \left(\frac{3}{2}b^2 a^2 d + a\,b^3 c\right)x^4 + \left(\frac{4}{3}a^3 bd + 2b^2 a^2 c\right)x^3 + \left(\frac{1}{2}a^4 d + 2a^3 bc\right)x^2 + a^4$
default	$\frac{b^4 d\,x^6}{6} + \frac{(4a\,b^3 d + b^4 c)x^5}{5} + \frac{(6b^2 a^2 d + 4a\,b^3 c)x^4}{4} + \frac{(4a^3 bd + 6b^2 a^2 c)x^3}{3} + \frac{(a^4 d + 4a^3 bc)x^2}{2} + a^4 cx$
gosper	$\frac{1}{6}b^4 d\,x^6 + \frac{4}{5}x^5 a\,b^3 d + \frac{1}{5}x^5 b^4 c + \frac{3}{2}x^4 b^2 a^2 d + x^4 a\,b^3 c + \frac{4}{3}x^3 a^3 bd + 2x^3 b^2 a^2 c + \frac{1}{2}x^2 a^4 d + 2x^2 a^3 bc + a$
risch	$\frac{1}{6}b^4 d\,x^6 + \frac{4}{5}x^5 a\,b^3 d + \frac{1}{5}x^5 b^4 c + \frac{3}{2}x^4 b^2 a^2 d + x^4 a\,b^3 c + \frac{4}{3}x^3 a^3 bd + 2x^3 b^2 a^2 c + \frac{1}{2}x^2 a^4 d + 2x^2 a^3 bc + a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^4*(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/6*b^4*d*x^6+1/5*(4*a*b^3*d+b^4*c)*x^5+1/4*(6*a^2*b^2*d+4*a*b^3*c)*x^4+1/3*(4*a^3*b*d+6*a^2*b^2*c)*x^3+1/2*(a^4*d+4*a^3*b*c)*x^2+a^4*c*x

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.
time = 0.27, size = 96, normalized size = 2.53

$$\frac{1}{6}\,b^4 dx^6 + a^4 cx + \frac{1}{5}\,(b^4 c + 4\,ab^3 d)x^5 + \frac{1}{2}\,(2\,ab^3 c + 3\,a^2 b^2 d)x^4 + \frac{2}{3}\,(3\,a^2 b^2 c + 2\,a^3 bd)x^3 + \frac{1}{2}\,(4\,a^3 bc + a^4 d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="maxima")

[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d)*x^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.
time = 0.46, size = 96, normalized size = 2.53

$$\frac{1}{6}\,b^4 dx^6 + a^4 cx + \frac{1}{5}\,(b^4 c + 4\,ab^3 d)x^5 + \frac{1}{2}\,(2\,ab^3 c + 3\,a^2 b^2 d)x^4 + \frac{2}{3}\,(3\,a^2 b^2 c + 2\,a^3 bd)x^3 + \frac{1}{2}\,(4\,a^3 bc + a^4 d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="fricas")

[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d)*x^2

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.
time = 0.02, size = 100, normalized size = 2.63

$$a^4cx + \frac{b^4dx^6}{6} + x^5 \cdot \left(\frac{4ab^3d}{5} + \frac{b^4c}{5}\right) + x^4 \cdot \left(\frac{3a^2b^2d}{2} + ab^3c\right) + x^3 \cdot \left(\frac{4a^3bd}{3} + 2a^2b^2c\right) + x^2\left(\frac{a^4d}{2} + 2a^3bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c),x)
```

```
[Out] a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b*
*2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 +
 2*a**3*b*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.
time = 0.68, size = 97, normalized size = 2.55

$$\frac{1}{6}b^4dx^6 + \frac{1}{5}b^4cx^5 + \frac{4}{5}ab^3dx^5 + ab^3cx^4 + \frac{3}{2}a^2b^2dx^4 + 2a^2b^2cx^3 + \frac{4}{3}a^3bdx^3 + 2a^3bcx^2 + \frac{1}{2}a^4dx^2 + a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^2
*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^2
+ a^4*c*x
```

Mupad [B]
time = 0.19, size = 88, normalized size = 2.32

$$x^5\left(\frac{c\,b^4}{5} + \frac{4\,a\,d\,b^3}{5}\right) + x^2\left(\frac{d\,a^4}{2} + 2\,b\,c\,a^3\right) + \frac{b^4\,d\,x^6}{6} + a^4\,c\,x + \frac{2\,a^2\,b\,x^3\,(2\,a\,d + 3\,b\,c)}{3} + \frac{a\,b^2\,x^4\,(3\,a\,d + 2\,b\,c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x),x)
```

```
[Out] x^5*((b^4*c)/5 + (4*a*b^3*d)/5) + x^2*((a^4*d)/2 + 2*a^3*b*c) + (b^4*d*x^6)
/6 + a^4*c*x + (2*a^2*b*x^3*(2*a*d + 3*b*c))/3 + (a*b^2*x^4*(3*a*d + 2*b*c)
)/2
```

3.1237 $\int (a+bx)^3(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

```
[Out] 1/4*(-a*d+b*c)*(b*x+a)^4/b^2+1/5*d*(b*x+a)^5/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^3}{b}+\frac{d(a+bx)^4}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.76

$$a^3cx+\frac{1}{2}a^2(3bc+ad)x^2+ab(bc+ad)x^3+\frac{1}{4}b^2(bc+3ad)x^4+\frac{1}{5}b^3dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*a
*d)*x^4)/4 + (b^3*d*x^5)/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.
time = 0.12, size = 73, normalized size = 1.92

method	result	size
norman	$\frac{b^3 d x^5}{5}+\left(\frac{3}{4} a b^2 d+\frac{1}{4} b^3 c\right) x^4+\left(a^2 b d+a b^2 c\right) x^3+\left(\frac{1}{2} a^3 d+\frac{3}{2} a^2 b c\right) x^2+a^3 c x$	70
gosper	$\frac{1}{5} b^3 d x^5+\frac{3}{4} x^4 a b^2 d+\frac{1}{4} x^4 b^3 c+a^2 b d x^3+a b^2 c x^3+\frac{1}{2} x^2 a^3 d+\frac{3}{2} a^2 b c x^2+a^3 c x$	73
default	$\frac{b^3 d x^5}{5}+\frac{\left(3 a b^2 d+b^3 c\right) x^4}{4}+\frac{\left(3 a^2 b d+3 a b^2 c\right) x^3}{3}+\frac{\left(a^3 d+3 a^2 b c\right) x^2}{2}+a^3 c x$	73
risch	$\frac{1}{5} b^3 d x^5+\frac{3}{4} x^4 a b^2 d+\frac{1}{4} x^4 b^3 c+a^2 b d x^3+a b^2 c x^3+\frac{1}{2} x^2 a^3 d+\frac{3}{2} a^2 b c x^2+a^3 c x$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^3*d*x^5+1/4*(3*a*b^2*d+b^3*c)*x^4+1/3*(3*a^2*b*d+3*a*b^2*c)*x^3+1/2*(
a^3*d+3*a^2*b*c)*x^2+a^3*c*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.28, size = 69, normalized size = 1.82

$$\frac{1}{5}\,b^3dx^5+a^3cx+\frac{1}{4}\,\left(b^3c+3\,ab^2d\right)x^4+\left(ab^2c+a^2bd\right)x^3+\frac{1}{2}\,\left(3\,a^2bc+a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.44, size = 69, normalized size = 1.82

$$\frac{1}{5}\,b^3dx^5+a^3cx+\frac{1}{4}\,\left(b^3c+3\,ab^2d\right)x^4+\left(ab^2c+a^2bd\right)x^3+\frac{1}{2}\,\left(3\,a^2bc+a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 0.01, size = 73, normalized size = 1.92

$$a^3cx + \frac{b^3dx^5}{5} + x^4 \cdot \left(\frac{3ab^2d}{4} + \frac{b^3c}{4}\right) + x^3\left(a^2bd + ab^2c\right) + x^2\left(\frac{a^3d}{2} + \frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
+ a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.
time = 0.58, size = 72, normalized size = 1.89

$$\frac{1}{5}\,b^3dx^5 + \frac{1}{4}\,b^3cx^4 + \frac{3}{4}\,ab^2dx^4 + ab^2cx^3 + a^2bdx^3 + \frac{3}{2}\,a^2bcx^2 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^3
 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B]
time = 0.16, size = 65, normalized size = 1.71

$$x^4\left(\frac{c\,b^3}{4} + \frac{3\,a\,d\,b^2}{4}\right) + x^2\left(\frac{d\,a^3}{2} + \frac{3\,b\,c\,a^2}{2}\right) + \frac{b^3\,d\,x^5}{5} + a^3\,c\,x + a\,b\,x^3\,(a\,d + b\,c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x),x)
```

```
[Out] x^4*((b^3*c)/4 + (3*a*b^2*d)/4) + x^2*((a^3*d)/2 + (3*a^2*b*c)/2) + (b^3*d*
x^5)/5 + a^3*c*x + a*b*x^3*(a*d + b*c)
```

3.1238 $\int (a+bx)^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

```
[Out] 1/3*(-a*d+b*c)*(b*x+a)^3/b^2+1/4*d*(b*x+a)^4/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{45\}$

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x),x]
```

```
[Out] (x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12
```

Maple [A]
time = 0.12, size = 49, normalized size = 1.29

method	result	size
norman	$\frac{b^2 d x^4}{4}+\left(\frac{2}{3}abd+\frac{1}{3}b^2c\right)x^3+\left(\frac{1}{2}a^2d+abc\right)x^2+a^2cx$	48
default	$\frac{b^2 d x^4}{4}+\frac{(2abd+b^2c)x^3}{3}+\frac{(a^2d+2abc)x^2}{2}+a^2cx$	49
gosper	$\frac{1}{4}b^2d\,x^4+\frac{2}{3}x^3abd+\frac{1}{3}b^2c\,x^3+\frac{1}{2}x^2a^2d+x^2abc+a^2cx$	50
risch	$\frac{1}{4}b^2d\,x^4+\frac{2}{3}x^3abd+\frac{1}{3}b^2c\,x^3+\frac{1}{2}x^2a^2d+x^2abc+a^2cx$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2*d*x^4+1/3*(2*a*b*d+b^2*c)*x^3+1/2*(a^2*d+2*a*b*c)*x^2+a^2*c*x
```

Maxima [A]
time = 0.26, size = 48, normalized size = 1.26

$$\frac{1}{4}\,b^2dx^4+a^2cx+\frac{1}{3}\,\big(b^2c+2\,abd\big)x^3+\frac{1}{2}\,\big(2\,abc+a^2d\big)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

Fricas [A]
time = 0.49, size = 48, normalized size = 1.26

$$\frac{1}{4}\,b^2dx^4+a^2cx+\frac{1}{3}\,\big(b^2c+2\,abd\big)x^3+\frac{1}{2}\,\big(2\,abc+a^2d\big)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

Sympy [A]
time = 0.01, size = 49, normalized size = 1.29

$$a^2cx+\frac{b^2dx^4}{4}+x^3\cdot\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2d}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 + a
*b*c)
```

Giac [A]
time = 0.55, size = 49, normalized size = 1.29

$$\frac{1}{4}b^2dx^4 + \frac{1}{3}b^2cx^3 + \frac{2}{3}abdx^3 + abcx^2 + \frac{1}{2}a^2dx^2 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 a^2*c*x
```

Mupad [B]
time = 0.05, size = 47, normalized size = 1.24

$$x^2\left(\frac{d\,a^2}{2}+b\,c\,a\right)+x^3\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)+\frac{b^2\,d\,x^4}{4}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x),x)
```

```
[Out] x^2*((a^2*d)/2 + a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) + (b^2*d*x^4)/4 + a
^2*c*x
```

3.1239 $\int (a+bx)(c+dx)\,dx$

Optimal. Leaf size=28

$$acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3$$

```
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{45\}$

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)\,dx &= \int \left(ac+(bc+ad)x+bdx^2\right)\,dx\\ &= acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^2}{2}+\frac{bd\,x^3}{3}$	25
norman	$\frac{bd\,x^3}{3}+\left(\frac{ad}{2}+\frac{bc}{2}\right)x^2+acx$	26
gosper	$\frac{1}{3}bd\,x^3+\frac{1}{2}x^2ad+\frac{1}{2}bc\,x^2+acx$	27
risch	$\frac{1}{3}bd\,x^3+\frac{1}{2}x^2ad+\frac{1}{2}bc\,x^2+acx$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

Maxima [A]
time = 0.29, size = 24, normalized size = 0.86

$$\frac{1}{3}\,bdx^3+acx+\frac{1}{2}\,(bc+ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="maxima")
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

Fricas [A]
time = 0.39, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3db+\frac{1}{2}x^2cb+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="fricas")
[Out] 1/3*x^3*d*b + 1/2*x^2*c*b + 1/2*x^2*d*a + x*c*a
```

Sympy [A]
time = 0.01, size = 26, normalized size = 0.93

$$acx+\frac{bdx^3}{3}+x^2\left(\frac{ad}{2}+\frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x)
```

```
[Out] a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

Giac [A]
time = 0.93, size = 26, normalized size = 0.93

$$\frac{1}{3}\,bdx^3+\frac{1}{2}\,bcx^2+\frac{1}{2}\,adx^2+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*b*d*x^3 + 1/2*b*c*x^2 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B]
time = 0.03, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^3}{3}+\left(\frac{a\,d}{2}+\frac{b\,c}{2}\right)\,x^2+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x),x)
```

```
[Out] x^2*((a*d)/2 + (b*c)/2) + a*c*x + (b*d*x^3)/3
```

3.1240 $\int (c+dx)\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

[Out] c*x+1/2*d*x^2

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[c + d*x,x]

[Out] c*x + (d*x^2)/2

Rubi steps

$$\int (c+dx)\,dx=cx+\frac{dx^2}{2}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[c + d*x,x]

[Out] c*x + (d*x^2)/2

Maple [A]
time = 0.01, size = 11, normalized size = 0.92

method	result	size
gosper	$cx+\frac{1}{2}d\,x^2$	11
default	$cx+\frac{1}{2}d\,x^2$	11

norman	$cx + \frac{1}{2}d\,x^2$	11
risch	$cx + \frac{1}{2}d\,x^2$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c,x,method=_RETURNVERBOSE)
```

```
[Out] c*x+1/2*d*x^2
```

Maxima [A]
time = 0.31, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="maxima")
```

```
[Out] 1/2*d*x^2 + c*x
```

Fricas [A]
time = 0.42, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2d + xc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*d + x*c
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x)
```

```
[Out] c*x + d*x**2/2
```

Giac [A]
time = 1.13, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2}+c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c + d*x,x)
[Out] c*x + (d*x^2)/2
```

3.1241 $\int \frac{c+dx}{a+bx} dx$

Optimal. Leaf size=25

$$\frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}$$

```
[Out] d*x/b+(-a*d+b*c)*ln(b*x+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+bx}\,dx &= \int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx\\ &= \frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Maple [A]
time = 0.12, size = 26, normalized size = 1.04

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\ln(bx+a)}{b^2}$	26
norman	$\frac{dx}{b}-\frac{(ad-bc)\ln(bx+a)}{b^2}$	27
risch	$\frac{dx}{b}-\frac{\ln(bx+a)ad}{b^2}+\frac{c\ln(bx+a)}{b}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
[Out] d*x/b+(-a*d+b*c)*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.29, size = 25, normalized size = 1.00

$$\frac{dx}{b}+\frac{(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="maxima")
[Out] d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

Fricas [A]
time = 0.51, size = 24, normalized size = 0.96

$$\frac{bdx+(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="fricas")
[Out] (b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.80

$$\frac{dx}{b}-\frac{(ad-bc)\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x)
```

```
[Out] d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

Giac [**A**]
time = 1.33, size = 26, normalized size = 1.04

$$\frac{dx}{b}+\frac{(bc-ad)\log\left(|bx+a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] d*x/b + (b*c - a*d)*log(abs(b*x + a))/b^2
```

Mupad [**B**]
time = 0.05, size = 26, normalized size = 1.04

$$\frac{d\,x}{b}-\frac{\ln\left(a+b\,x\right)\,\left(a\,d-b\,c\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x),x)
```

```
[Out] (d*x)/b - (log(a + b*x)*(a*d - b*c))/b^2
```

3.1242 $\int \frac{c+dx}{(a+bx)^2} \, dx$

Optimal. Leaf size=32

$$-\frac{bc-ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}$$

```
[Out] (a*d-b*c)/b^2/(b*x+a)+d*ln(b*x+a)/b^2
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{d\log(a+bx)}{b^2}-\frac{bc-ad}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^2}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^2}+\frac{d}{b(a+bx)}\right)dx\\ &= -\frac{bc-ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.97

$$\frac{-bc+ad}{b^2(a+bx)}+\frac{d\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

Maple [A]
time = 0.13, size = 33, normalized size = 1.03

method	result	size
norman	$\frac{ad-bc}{b^2(bx+a)}+\frac{d\ln(bx+a)}{b^2}$	32
default	$-\frac{-ad+bc}{b^2(bx+a)}+\frac{d\ln(bx+a)}{b^2}$	33
risch	$\frac{ad}{b^2(bx+a)}-\frac{c}{b(bx+a)}+\frac{d\ln(bx+a)}{b^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -(-a*d+b*c)/b^2/(b*x+a)+d*ln(b*x+a)/b^2
```

Maxima [A]
time = 0.27, size = 35, normalized size = 1.09

$$-\frac{bc-ad}{b^3x+ab^2}+\frac{d\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="maxima")
[Out] -(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

Fricas [A]
time = 0.48, size = 39, normalized size = 1.22

$$-\frac{bc-ad-(bdx+ad)\log(bx+a)}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="fricas")
[Out] -(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.84

$$\frac{ad-bc}{ab^2+b^3x}+\frac{d\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**2,x)
```

```
[Out] (a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

Giac [A]
time = 1.03, size = 57, normalized size = 1.78

$$-\frac{d\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)}{b}-\frac{c}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="giac")
[Out] -d*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b - c/((b*x
 + a)*b)
```

Mupad [B]
time = 0.17, size = 31, normalized size = 0.97

$$\frac{a\,d-b\,c}{b^2\,\left(a+b\,x\right)}+\frac{d\,\ln\left(a+b\,x\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^2,x)
[Out] (a*d - b*c)/(b^2*(a + b*x)) + (d*log(a + b*x))/b^2
```

3.1243 $\int \frac{c+dx}{(a+bx)^3} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

```
[Out] -1/2*(d*x+c)^2/(-a*d+b*c)/(b*x+a)^2
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{37\}$

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -1/2*(c + d*x)^2/((b*c - a*d)*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx)^3} dx = -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^3,x]
```

```
[Out] -1/2*(a*d + b*(c + 2*d*x))/(b^2*(a + b*x)^2)
```

Maple [A]
time = 0.12, size = 35, normalized size = 1.25

method	result	size
gosper	$-\frac{2bdx+ad+bc}{2(bx+a)^2b^2}$	25
risch	$\frac{-\frac{dx}{b}-\frac{ad+bc}{2b^2}}{(bx+a)^2}$	29
norman	$\frac{-\frac{dx}{b}+\frac{-ad-bc}{2b^2}}{(bx+a)^2}$	31
default	$-\frac{d}{b^2(bx+a)}-\frac{-ad+bc}{2b^2(bx+a)^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -d/b^2/(b*x+a)-1/2*(-a*d+b*c)/b^2/(b*x+a)^2
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Fricas [A]
time = 0.59, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\,(b^4x^2+2\,ab^3x+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

Giac [A]
time = 0.78, size = 24, normalized size = 0.86

$$-\frac{2\,bdx+bc+ad}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.16, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,b^2}+\frac{d\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*b^2) + (d*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.1244 $\int \frac{c+dx}{(a+bx)^4} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

```
[Out] 1/3*(a*d-b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -1/3*(b*c - a*d)/(b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^4}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^4}+\frac{d}{b(a+bx)^3}\right)dx\\ &= -\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{2bc+ad+3bdx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^4,x]
```

[Out] -1/6*(2*b*c + a*d + 3*b*d*x)/(b^2*(a + b*x)^3)

Maple [A]
time = 0.13, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{3bdx+ad+2bc}{6b^2(bx+a)^3}$	26
risch	$\frac{-\frac{dx}{2b}-\frac{ad+2bc}{6b^2}}{(bx+a)^3}$	30
norman	$\frac{-\frac{dx}{2b}+\frac{-abd-2b^2c}{6b^3}}{(bx+a)^3}$	34
default	$-\frac{d}{2b^2(bx+a)^2}-\frac{-ad+bc}{3b^2(bx+a)^3}$	35

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)/(b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] -1/2*d/b^2/(b*x+a)^2-1/3*(-a*d+b*c)/b^2/(b*x+a)^3

Maxima [A]
time = 0.31, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="maxima")

[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)

Fricas [A]
time = 0.57, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\,(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="fricas")

[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)

Sympy [A]
time = 0.17, size = 53, normalized size = 1.39

$$\frac{-ad-2bc-3bdx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**4,x)
```

```
[Out] (-a*d - 2*b*c - 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

Giac [A]
time = 0.63, size = 25, normalized size = 0.66

$$-\frac{3\,bdx+2\,bc+ad}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/((b*x + a)^3*b^2)
```

Mupad [B]
time = 0.17, size = 52, normalized size = 1.37

$$-\frac{\frac{a\,d+2\,b\,c}{6\,b^2}+\frac{d\,x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^4,x)
```

```
[Out] -((a*d + 2*b*c)/(6*b^2) + (d*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x)
```

3.1245 $\int \frac{c+dx}{(a+bx)^5} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

```
[Out] 1/4*(a*d-b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -1/4*(b*c - a*d)/(b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^5}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^5}+\frac{d}{b(a+bx)^4}\right)dx\\ &= -\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{3bc+ad+4bdx}{12b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -1/12*(3*b*c + a*d + 4*b*d*x)/(b^2*(a + b*x)^4)
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{4bdx+ad+3bc}{12b^2(bx+a)^4}$	26
risch	$\frac{-\frac{dx}{3b}-\frac{ad+3bc}{12b^2}}{(bx+a)^4}$	30
default	$-\frac{-ad+bc}{4b^2(bx+a)^4}-\frac{d}{3b^2(bx+a)^3}$	35
norman	$\frac{-\frac{dx}{3b}+\frac{-a\,b^2d-3b^3c}{12b^4}}{(bx+a)^4}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-a*d+b*c)/b^2/(b*x+a)^4-1/3*d/b^2/(b*x+a)^3
```

Maxima [A]
time = 0.27, size = 61, normalized size = 1.61

$$-\frac{4\,bdx+3\,bc+ad}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Fricas [A]
time = 0.46, size = 61, normalized size = 1.61

$$-\frac{4\,bdx+3\,bc+ad}{12\,(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.
time = 0.23, size = 65, normalized size = 1.71

$$\frac{-ad-3bc-4bdx}{12a^4b^2+48a^3b^3x+72a^2b^4x^2+48ab^5x^3+12b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**5,x)
```

```
[Out] (-a*d - 3*b*c - 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2
 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

Giac [A]
time = 0.56, size = 41, normalized size = 1.08

$$-\frac{c}{4\,(bx+a)^4b}-\frac{d}{3\,(bx+a)^3b^2}+\frac{ad}{4\,(bx+a)^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*c/((b*x + a)^4*b) - 1/3*d/((b*x + a)^3*b^2) + 1/4*a*d/((b*x + a)^4*b^2
)
```

Mupad [B]
time = 0.04, size = 63, normalized size = 1.66

$$-\frac{\frac{a\,d+3\,b\,c}{12\,b^2}+\frac{d\,x}{3\,b}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^5,x)
```

```
[Out] -((a*d + 3*b*c)/(12*b^2) + (d*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)
```

3.1246 $\int (a+bx)^4(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

```
[Out] 1/5*(-a*d+b*c)^2*(b*x+a)^5/b^3+1/3*d*(-a*d+b*c)*(b*x+a)^6/b^3+1/7*d^2*(b*x+
a)^7/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^4}{b^2}+\frac{2d(bc-ad)(a+bx)^5}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(65) = 130.
time = 0.02, size = 148, normalized size = 2.28

$$a^4c^2x+a^3c(2bc+ad)x^2+\frac{1}{3}a^2\left(6b^2c^2+8abcd+a^2d^2\right)x^3+ab\left(b^2c^2+3abcd+a^2d^2\right)x^4+\frac{1}{5}b^2\left(b^2c^2+8abcd+6a^2d^2\right)x^5+\frac{1}{3}b^3d(bc+2ad)x^6+\frac{1}{7}b^4d^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2
)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*b*
c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(59) = 118$.
time = 0.13, size = 163, normalized size = 2.51

method	result
norman	$\frac{b^4 d^2 x^7}{7} + \left(\frac{2}{3} a\, b^3 d^2 + \frac{1}{3} b^4 c d\right) x^6 + \left(\frac{6}{5} b^2 a^2 d^2 + \frac{8}{5} a\, b^3 c d + \frac{1}{5} b^4 c^2\right) x^5 + \left(a^3 b\, d^2 + 3 b^2 a^2 c d + a\, b^3 c^2\right) x^4 +$
default	$\frac{b^4 d^2 x^7}{7} + \frac{(4 a\, b^3 d^2 + 2 b^4 c d) x^6}{6} + \frac{(6 b^2 a^2 d^2 + 8 a\, b^3 c d + b^4 c^2) x^5}{5} + \frac{(4 a^3 b\, d^2 + 12 b^2 a^2 c d + 4 a\, b^3 c^2) x^4}{4} + \frac{(a^4 d^2 + 8 a^3 b c d + 6 a^2 b^2 c^2) x^3}{3}$
gosper	$\frac{1}{7} b^4 d^2 x^7 + \frac{2}{3} x^6 a\, b^3 d^2 + \frac{1}{3} x^6 b^4 c d + \frac{6}{5} x^5 b^2 a^2 d^2 + \frac{8}{5} x^5 a\, b^3 c d + \frac{1}{5} b^4 c^2 x^5 + a^3 b\, d^2 x^4 + 3 a^2 b^2 c d\, x^4 + a\, b^3$
risch	$\frac{1}{7} b^4 d^2 x^7 + \frac{2}{3} x^6 a\, b^3 d^2 + \frac{1}{3} x^6 b^4 c d + \frac{6}{5} x^5 b^2 a^2 d^2 + \frac{8}{5} x^5 a\, b^3 c d + \frac{1}{5} b^4 c^2 x^5 + a^3 b\, d^2 x^4 + 3 a^2 b^2 c d\, x^4 + a\, b^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^4*d^2*x^7+1/6*(4*a*b^3*d^2+2*b^4*c*d)*x^6+1/5*(6*a^2*b^2*d^2+8*a*b^3*
c*d+b^4*c^2)*x^5+1/4*(4*a^3*b*d^2+12*a^2*b^2*c*d+4*a*b^3*c^2)*x^4+1/3*(a^4*
d^2+8*a^3*b*c*d+6*a^2*b^2*c^2)*x^3+1/2*(2*a^4*c*d+4*a^3*b*c^2)*x^2+a^4*c^2*
x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.
time = 0.27, size = 156, normalized size = 2.40

$$\frac{1}{7} b^4 d^2 x^7 + a^4 c^2 x + \frac{1}{3} \left(b^4 c d + 2 a b^3 d^2\right) x^6 + \frac{1}{5} \left(b^4 c^2 + 8 a b^3 c d + 6 a^2 b^2 d^2\right) x^5 + \left(a b^3 c^2 + 3 a^2 b^2 c d + a^3 b d^2\right) x^4 + \frac{1}{3} \left(6 a^2 b^2 c^2 + 8 a^3 b c d + a^4 d^2\right) x^3 + \left(2 a^3 b c^2 + a^4 c d\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(59) = 118$.
time = 0.46, size = 156, normalized size = 2.40

$$\frac{1}{7} b^4 d^2 x^7 + a^4 c^2 x + \frac{1}{3} \left(b^4 c d + 2 a b^3 d^2\right) x^6 + \frac{1}{5} \left(b^4 c^2 + 8 a b^3 c d + 6 a^2 b^2 d^2\right) x^5 + \left(a b^3 c^2 + 3 a^2 b^2 c d + a^3 b d^2\right) x^4 + \frac{1}{3} \left(6 a^2 b^2 c^2 + 8 a^3 b c d + a^4 d^2\right) x^3 + \left(2 a^3 b c^2 + a^4 c d\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(54) = 108$.
time = 0.02, size = 168, normalized size = 2.58

$$a^4c^2x + \frac{b^4d^2x^7}{7} + x^6 \cdot \left(\frac{2ab^3d^2}{3} + \frac{b^4cd}{3}\right) + x^5 \cdot \left(\frac{6a^2b^2d^2}{5} + \frac{8ab^3cd}{5} + \frac{b^4c^2}{5}\right) + x^4\left(a^3bd^2 + 3a^2b^2cd + ab^3c^2\right) + x^3\left(\frac{a^4d^2}{3} + \frac{8a^3bcd}{3} + 2a^2b^2c^2\right) + x^2\left(a^4cd + 2a^3bc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**2,x)
```

```
[Out] a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**5
*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2 +
3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*a**
2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(59) = 118$.
time = 0.49, size = 170, normalized size = 2.62

$$\frac{1}{7}b^4d^2x^7 + \frac{1}{3}b^4cdx^6 + \frac{2}{3}ab^3d^2x^6 + \frac{1}{5}b^4c^2x^5 + \frac{8}{5}ab^3cdx^5 + \frac{6}{5}a^2b^2d^2x^5 + ab^3c^2x^4 + 3\,a^2b^2cdx^4 + a^3bd^2x^4 + 2\,a^2b^2c^2x^3 + \frac{8}{3}a^3bcdx^3 + \frac{1}{3}a^4d^2x^3 + 2\,a^3bc^2x^2 + a^4cdx^2 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 + 8
/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^4
+ a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^3 +
 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

Mupad [B]
time = 0.07, size = 144, normalized size = 2.22

$$x^3\left(\frac{a^4\,d^2}{3} + \frac{8\,a^3\,b\,c\,d}{3} + 2\,a^2\,b^2\,c^2\right) + x^5\left(\frac{6\,a^2\,b^2\,d^2}{5} + \frac{8\,a\,b^3\,c\,d}{5} + \frac{b^4\,c^2}{5}\right) + a^4\,c^2\,x + \frac{b^4\,d^2\,x^7}{7} + a^3\,c\,x^2\,(a\,d + 2\,b\,c) + \frac{b^3\,d\,x^6\,(2\,a\,d + b\,c)}{3} + a\,b\,x^4\,\left(a^2\,d^2 + 3\,a\,b\,c\,d + b^2\,c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^2,x)
```

```
[Out] x^3*((a^4*d^2)/3 + 2*a^2*b^2*c^2 + (8*a^3*b*c*d)/3) + x^5*((b^4*c^2)/5 + (6
*a^2*b^2*d^2)/5 + (8*a*b^3*c*d)/5) + a^4*c^2*x + (b^4*d^2*x^7)/7 + a^3*c*x^
2*(a*d + 2*b*c) + (b^3*d*x^6*(2*a*d + b*c))/3 + a*b*x^4*(a^2*d^2 + b^2*c^2
+ 3*a*b*c*d)
```

3.1247 $\int (a+bx)^3(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

```
[Out] 1/4*(-a*d+b*c)^2*(b*x+a)^4/b^3+2/5*d*(-a*d+b*c)*(b*x+a)^5/b^3+1/6*d^2*(b*x+
a)^6/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^3}{b^2}+\frac{2d(bc-ad)(a+bx)^4}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 122, normalized size = 1.88

$$a^3c^2x+\frac{1}{2}a^2c(3bc+2ad)x^2+\frac{1}{3}a\left(3b^2c^2+6abcd+a^2d^2\right)x^3+\frac{1}{4}b\left(b^2c^2+6abcd+3a^2d^2\right)x^4+\frac{1}{5}b^2d(2bc+3ad)x^5+\frac{1}{6}b^3d^2x^6$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3*(c + d*x)^2,x]

[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*c + 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.13, size = 125, normalized size = 1.92

method	result
norman	$\frac{b^3d^2x^6}{6}+\left(\frac{3}{5}a\,b^2d^2+\frac{2}{5}b^3cd\right)x^5+\left(\frac{3}{4}a^2b\,d^2+\frac{3}{2}a\,b^2cd+\frac{1}{4}b^3c^2\right)x^4+\left(\frac{1}{3}a^3d^2+2a^2bcd+a\,b^2c^2\right)x^3+$
default	$\frac{b^3d^2x^6}{6}+\frac{(3a\,b^2d^2+2b^3cd)x^5}{5}+\frac{(3a^2b\,d^2+6a\,b^2cd+b^3c^2)x^4}{4}+\frac{(a^3d^2+6a^2bcd+3a\,b^2c^2)x^3}{3}+\frac{(2a^3cd+3a^2b\,c^2)x^2}{2}+a^3c^2x$
gosper	$\frac{1}{6}b^3d^2x^6+\frac{3}{5}x^5a\,b^2d^2+\frac{2}{5}x^5b^3cd+\frac{3}{4}x^4a^2b\,d^2+\frac{3}{2}x^4a\,b^2cd+\frac{1}{4}x^4b^3c^2+\frac{1}{3}x^3a^3d^2+2x^3a^2bcd+x^3a$
risch	$\frac{1}{6}b^3d^2x^6+\frac{3}{5}x^5a\,b^2d^2+\frac{2}{5}x^5b^3cd+\frac{3}{4}x^4a^2b\,d^2+\frac{3}{2}x^4a\,b^2cd+\frac{1}{4}x^4b^3c^2+\frac{1}{3}x^3a^3d^2+2x^3a^2bcd+x^3a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3*(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/6*b^3*d^2*x^6+1/5*(3*a*b^2*d^2+2*b^3*c*d)*x^5+1/4*(3*a^2*b*d^2+6*a*b^2*c*d+b^3*c^2)*x^4+1/3*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^3+1/2*(2*a^3*c*d+3*a^2*b*c^2)*x^2+a^3*c^2*x

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.28, size = 124, normalized size = 1.91

$$\frac{1}{6}b^3d^2x^6+a^3c^2x+\frac{1}{5}(2b^3cd+3ab^2d^2)x^5+\frac{1}{4}(b^3c^2+6ab^2cd+3a^2bd^2)x^4+\frac{1}{3}(3ab^2c^2+6a^2bcd+a^3d^2)x^3+\frac{1}{2}(3a^2bc^2+2a^3cd)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="maxima")

[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.50, size = 124, normalized size = 1.91

$$\frac{1}{6}b^3d^2x^6+a^3c^2x+\frac{1}{5}(2b^3cd+3ab^2d^2)x^5+\frac{1}{4}(b^3c^2+6ab^2cd+3a^2bd^2)x^4+\frac{1}{3}(3ab^2c^2+6a^2bcd+a^3d^2)x^3+\frac{1}{2}(3a^2bc^2+2a^3cd)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="fricas")

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.
time = 0.02, size = 133, normalized size = 2.05

$$a^3c^2x + \frac{b^3d^2x^6}{6} + x^5 \cdot \left(\frac{3ab^2d^2}{5} + \frac{2b^3cd}{5}\right) + x^4 \cdot \left(\frac{3a^2bd^2}{4} + \frac{3ab^2cd}{2} + \frac{b^3c^2}{4}\right) + x^3\left(\frac{a^3d^2}{3} + 2a^2bcd + ab^2c^2\right) + x^2\left(a^3cd + \frac{3a^2bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x*
*4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 + 2
*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.
time = 0.70, size = 130, normalized size = 2.00

$$\frac{1}{6}b^3d^2x^6 + \frac{2}{5}b^3cdx^5 + \frac{3}{5}ab^2d^2x^5 + \frac{1}{4}b^3c^2x^4 + \frac{3}{2}ab^2cdx^4 + \frac{3}{4}a^2bd^2x^4 + ab^2c^2x^3 + 2a^2bcdx^3 + \frac{1}{3}a^3d^2x^3 + \frac{3}{2}a^2bc^2x^2 + a^3cdx^2 + a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 + 3
/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 + 1/
3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

Mupad [B]
time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(\frac{a^3d^2}{3} + 2a^2bcd + ab^2c^2\right) + x^4\left(\frac{3a^2bd^2}{4} + \frac{3ab^2cd}{2} + \frac{b^3c^2}{4}\right) + a^3c^2x + \frac{b^3d^2x^6}{6} + \frac{a^2cx^2(2ad + 3bc)}{2} + \frac{b^2dx^5(3ad + 2bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] x^3*((a^3*d^2)/3 + a*b^2*c^2 + 2*a^2*b*c*d) + x^4*((b^3*c^2)/4 + (3*a^2*b*d
^2)/4 + (3*a*b^2*c*d)/2) + a^3*c^2*x + (b^3*d^2*x^6)/6 + (a^2*c*x^2*(2*a*d
+ 3*b*c))/2 + (b^2*d*x^5*(3*a*d + 2*b*c))/5
```

3.1248 $\int (a+bx)^2(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

```
[Out] 1/3*(-a*d+b*c)^2*(b*x+a)^3/b^3+1/2*d*(-a*d+b*c)*(b*x+a)^4/b^3+1/5*d^2*(b*x+
a)^5/b^3
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x)^4)/(2*b^3) +
 (d^2*(a + b*x)^5)/(5*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.22

$$a^2c^2x+ac(bc+ad)x^2+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3+\frac{1}{2}bd(bc+ad)x^4+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3 +
 (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

Maple [A]
time = 0.14, size = 87, normalized size = 1.34

method	result	size
norman	$\frac{b^2d^2x^5}{5}+\left(\frac{1}{2}ab\,d^2+\frac{1}{2}b^2cd\right)x^4+\left(\frac{1}{3}a^2d^2+\frac{4}{3}abcd+\frac{1}{3}b^2c^2\right)x^3+\left(a^2cd+ab\,c^2\right)x^2+a^2c^2x$	84
default	$\frac{b^2d^2x^5}{5}+\frac{(2ab\,d^2+2b^2cd)x^4}{4}+\frac{(a^2d^2+4abcd+b^2c^2)x^3}{3}+\frac{(2a^2cd+2ab\,c^2)x^2}{2}+a^2c^2x$	87
gosper	$\frac{1}{5}b^2d^2x^5+\frac{1}{2}x^4ab\,d^2+\frac{1}{2}x^4b^2cd+\frac{1}{3}x^3a^2d^2+\frac{4}{3}x^3abcd+\frac{1}{3}x^3b^2c^2+a^2cd\,x^2+ab\,c^2x^2+a^2c^2x$	90
risch	$\frac{1}{5}b^2d^2x^5+\frac{1}{2}x^4ab\,d^2+\frac{1}{2}x^4b^2cd+\frac{1}{3}x^3a^2d^2+\frac{4}{3}x^3abcd+\frac{1}{3}x^3b^2c^2+a^2cd\,x^2+ab\,c^2x^2+a^2c^2x$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*d^2*x^5+1/4*(2*a*b*d^2+2*b^2*c*d)*x^4+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^3+1/2*(2*a^2*c*d+2*a*b*c^2)*x^2+a^2*c^2*x
```

Maxima [A]
time = 0.27, size = 81, normalized size = 1.25

$$\frac{1}{5}\,b^2d^2x^5+a^2c^2x+\frac{1}{2}\,(b^2cd+abd^2)x^4+\frac{1}{3}\,(b^2c^2+4\,abcd+a^2d^2)x^3+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Fricas [A]
time = 0.39, size = 81, normalized size = 1.25

$$\frac{1}{5}\,b^2d^2x^5+a^2c^2x+\frac{1}{2}\,(b^2cd+abd^2)x^4+\frac{1}{3}\,(b^2c^2+4\,abcd+a^2d^2)x^3+\left(abc^2+a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

Sympy [A]
time = 0.02, size = 87, normalized size = 1.34

$$a^2c^2x+\frac{b^2d^2x^5}{5}+x^4\left(\frac{abd^2}{2}+\frac{b^2cd}{2}\right)+x^3\left(\frac{a^2d^2}{3}+\frac{4abcd}{3}+\frac{b^2c^2}{3}\right)+x^2\left(a^2cd+abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a**
2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

Giac [A]
time = 0.54, size = 89, normalized size = 1.37

$$\frac{1}{5}b^2d^2x^5 + \frac{1}{2}b^2cdx^4 + \frac{1}{2}abd^2x^4 + \frac{1}{3}b^2c^2x^3 + \frac{4}{3}abcdx^3 + \frac{1}{3}a^2d^2x^3 + abc^2x^2 + a^2cdx^2 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

Mupad [B]
time = 0.17, size = 74, normalized size = 1.14

$$x^3\left(\frac{a^2\,d^2}{3}+\frac{4\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{3}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^5}{5}+a\,c\,x^2\,\left(a\,d+b\,c\right)+\frac{b\,d\,x^4\,\left(a\,d+b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) + a^2*c^2*x + (b^2*d^2*x^5)
/5 + a*c*x^2*(a*d + b*c) + (b*d*x^4*(a*d + b*c))/2
```

3.1249 $\int (a+bx)(c+dx)^2\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}$$

```
[Out] -1/3*(-a*d+b*c)*(d*x+c)^3/d^2+1/4*b*(d*x+c)^4/d^2
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] -1/3*((b*c - a*d)*(c + d*x)^3)/d^2 + (b*(c + d*x)^4)/(4*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^2\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^2}{d}+\frac{b(c+dx)^3}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.24

$$\frac{1}{12}x\left(12ac^2+6c(bc+2ad)x+4d(2bc+ad)x^2+3bd^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] (x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))/
12
```

Maple [A]
time = 0.12, size = 49, normalized size = 1.29

method	result	size
norman	$\frac{b\,d^{2} x^{4}}{4}+\left(\frac{1}{3} a\,d^{2}+\frac{2}{3} b d c \right) x^{3}+\left(a c d +\frac{1}{2} b\,c^{2}\right) x^{2}+a\,c^{2} x$	48
default	$\frac{b\,d^{2} x^{4}}{4}+\frac{\left(a\,d^{2}+2 b d c \right) x^{3}}{3}+\frac{\left(2 a c d +b\,c^{2}\right) x^{2}}{2}+a\,c^{2} x$	49
gosper	$\frac{1}{4} b\,d^{2} x^{4}+\frac{1}{3} x^{3} a\,d^{2}+\frac{2}{3} x^{3} b d c +x^{2} a c d +\frac{1}{2} b\,c^{2} x^{2}+a\,c^{2} x$	50
risch	$\frac{1}{4} b\,d^{2} x^{4}+\frac{1}{3} x^{3} a\,d^{2}+\frac{2}{3} x^{3} b d c +x^{2} a c d +\frac{1}{2} b\,c^{2} x^{2}+a\,c^{2} x$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*d^2*x^4+1/3*(a*d^2+2*b*c*d)*x^3+1/2*(2*a*c*d+b*c^2)*x^2+a*c^2*x
```

Maxima [A]
time = 0.27, size = 48, normalized size = 1.26

$$\frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\,\left(2\,bcd+ad^2\right)x^3+\frac{1}{2}\,\left(bc^2+2\,acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

Fricas [A]
time = 0.41, size = 48, normalized size = 1.26

$$\frac{1}{4}\,bd^2x^4+ac^2x+\frac{1}{3}\,\left(2\,bcd+ad^2\right)x^3+\frac{1}{2}\,\left(bc^2+2\,acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

Sympy [A]
time = 0.01, size = 49, normalized size = 1.29

$$ac^2x+\frac{bd^2x^4}{4}+x^3\left(\frac{ad^2}{3}+\frac{2bcd}{3}\right)+x^2\left(acd+\frac{bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c*
*2/2)
```

Giac [A]
time = 0.62, size = 49, normalized size = 1.29

$$\frac{1}{4}\,bd^2x^4+\frac{2}{3}\,bcdx^3+\frac{1}{3}\,ad^2x^3+\frac{1}{2}\,bc^2x^2+acdx^2+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2 +
 a*c^2*x
```

Mupad [B]
time = 0.04, size = 47, normalized size = 1.24

$$x^2\,\left(\frac{b\,c^2}{2}+a\,d\,c\right)+x^3\,\left(\frac{a\,d^2}{3}+\frac{2\,b\,c\,d}{3}\right)+\frac{b\,d^2\,x^4}{4}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^2,x)
```

```
[Out] x^2*((b*c^2)/2 + a*c*d) + x^3*((a*d^2)/3 + (2*b*c*d)/3) + (b*d^2*x^4)/4 + a
*c^2*x
```

3.1250 $\int (c+dx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^3}{3d}$$

[Out] 1/3*(d*x+c)^3/d

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2,x]

[Out] (c + d*x)^3/(3*d)

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2 \, dx = \frac{(c+dx)^3}{3d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^2,x]

[Out] (c + d*x)^3/(3*d)

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(dx+c)^3}{3d}$	13
gosper	$\frac{1}{3}d^2x^3+cd\,x^2+c^2x$	21
norman	$\frac{1}{3}d^2x^3+cd\,x^2+c^2x$	21
risch	$\frac{d^2x^3}{3}+cd\,x^2+c^2x+\frac{c^3}{3d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x+c)^3/d
```

Maxima [A]
time = 0.27, size = 20, normalized size = 1.43

$$\frac{1}{3}\,d^2x^3+cdx^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Fricas [A]
time = 0.90, size = 20, normalized size = 1.43

$$\frac{1}{3}\,d^2x^3+cdx^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.
time = 0.01, size = 19, normalized size = 1.36

$$c^2x+cdx^2+\frac{d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2,x)
```

```
[Out] c**2*x + c*d*x**2 + d**2*x**3/3
```

Giac [A]
time = 0.82, size = 12, normalized size = 0.86

$$\frac{(dx+c)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(d*x + c)^3/d
```

Mupad [B]
time = 0.03, size = 20, normalized size = 1.43

$$c^2\,x+c\,d\,x^2+\frac{d^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2,x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + c*d*x^2
```

3.1251 $\int \frac{(c+dx)^2}{a+bx} dx$

Optimal. Leaf size=49

$$\frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}$$

```
[Out] d*(-a*d+b*c)*x/b^2+1/2*(d*x+c)^2/b+(-a*d+b*c)^2*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx\\ &= \frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.88

$$\frac{bdx(4bc-2ad+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

Maple [A]
time = 0.17, size = 56, normalized size = 1.14

method	result	size
default	$-\frac{d\left(-\frac{1}{2}bd\,x^{2}+adx-2bcx\right)}{b^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(bx+a\right)}{b^{3}}$	56
norman	$\frac{d^{2}x^{2}}{2b}-\frac{d\left(ad-2bc\right)x}{b^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(bx+a\right)}{b^{3}}$	59
risch	$\frac{d^{2}x^{2}}{2b}-\frac{d^{2}ax}{b^{2}}+\frac{2dcx}{b}+\frac{\ln\left(bx+a\right)a^{2}d^{2}}{b^{3}}-\frac{2\ln\left(bx+a\right)acd}{b^{2}}+\frac{\ln\left(bx+a\right)c^{2}}{b}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^2*(-1/2*b*d*x^2+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3*ln(b*x+
a)
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.24

$$\frac{bd^2x^2 + 2\,(2\,bcd - ad^2)x}{2\,b^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log{(bx + a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(b*x + a)/b^3
```

Fricas [A]
time = 0.96, size = 63, normalized size = 1.29

$$\frac{b^2d^2x^2 + 2\,(2\,b^2cd - abd^2)x + 2\,(b^2c^2 - 2\,abcd + a^2d^2)\log{(bx + a)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(b*x + a))/b^3
```

Sympy [A]
time = 0.10, size = 44, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2} + \frac{2cd}{b}\right) + \frac{d^2x^2}{2b} + \frac{(ad - bc)^2\log{(a + bx)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + d**2*x**2/(2*b) + (a*d - b*c)**2*log(a + b*x)/
b**3
```

Giac [A]
time = 0.75, size = 60, normalized size = 1.22

$$\frac{bd^2x^2 + 4\,bcdx - 2\,ad^2x}{2\,b^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log(|bx + a|)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(b*x + a))/b^3
```

Mupad [B]
time = 0.19, size = 62, normalized size = 1.27

$$\frac{\ln(a + b\,x)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{b^3} - x\,\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{d^2\,x^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x),x)
```

```
[Out] (log(a + b*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/b^3 - x*((a*d^2)/b^2 - (2*c*
d)/b) + (d^2*x^2)/(2*b)
```

3.1252 $\int \frac{(c+dx)^2}{(a+bx)^2} \, dx$

Optimal. Leaf size=51

$$\frac{d^2x}{b^2} - \frac{(bc-ad)^2}{b^3(a+bx)} + \frac{2d(bc-ad)\log(a+bx)}{b^3}$$

```
[Out] d^2*x/b^2-(-a*d+b*c)^2/b^3/(b*x+a)+2*d*(-a*d+b*c)*ln(b*x+a)/b^3
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{(bc-ad)^2}{b^3(a+bx)} + \frac{2d(bc-ad)\log(a+bx)}{b^3} + \frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^2}{(a+bx)^2} \, dx &= \int \left(\frac{d^2}{b^2} + \frac{(bc-ad)^2}{b^2(a+bx)^2} + \frac{2d(bc-ad)}{b^2(a+bx)} \right) dx \\ &= \frac{d^2x}{b^2} - \frac{(bc-ad)^2}{b^3(a+bx)} + \frac{2d(bc-ad)\log(a+bx)}{b^3} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.92

$$\frac{bd^2x - \frac{(bc-ad)^2}{a+bx} + 2d(bc-ad)\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

Maple [A]
time = 0.14, size = 63, normalized size = 1.24

method	result	size
default	$\frac{d^2 x}{b^2}-\frac{a^2 d^2-2abcd+b^2 c^2}{b^3 (bx+a)}-\frac{2d(ad-bc)\ln(bx+a)}{b^3}$	63
norman	$\frac{\frac{d^2 x^2}{b}-\frac{2a^2 d^2-2abcd+b^2 c^2}{b^3}}{bx+a}-\frac{2d(ad-bc)\ln(bx+a)}{b^3}$	68
risch	$\frac{d^2 x}{b^2}-\frac{a^2 d^2}{b^3 (bx+a)}+\frac{2acd}{b^2 (bx+a)}-\frac{c^2}{b(bx+a)}-\frac{2d^2 \ln(bx+a)a}{b^3}+\frac{2d\ln(bx+a)c}{b^2}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*x/b^2-(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)-2/b^3*d*(a*d-b*c)*ln(b*x+
a)
```

Maxima [A]
time = 0.28, size = 67, normalized size = 1.31

$$\frac{d^2x}{b^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{b^4x+ab^3}+\frac{2\,(bcd-ad^2)\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a*
d^2)*log(b*x + a)/b^3
```

Fricas [A]
time = 0.96, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2+abd^2x-b^2c^2+2\,abcd-a^2d^2+2\,(abcd-a^2d^2+(b^2cd-abd^2)x)\log\left(bx+a\right)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^2
*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A]
time = 0.17, size = 60, normalized size = 1.18

$$\frac{-a^2d^2+2abcd-b^2c^2}{ab^3+b^4x}+\frac{d^2x}{b^2}-\frac{2d(ad-bc)\log{(a+bx)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**2,x)
```

```
[Out] (-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d*
(a*d - b*c)*log(a + b*x)/b**3
```

Giac [A]
time = 0.71, size = 98, normalized size = 1.92

$$\frac{(bx+a)d^2}{b^3}-\frac{2\left(bcd-ad^2\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3}-\frac{\frac{b^3c^2}{bx+a}-\frac{2\,ab^2cd}{bx+a}+\frac{a^2bd^2}{bx+a}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2/b^3 - 2*(b*c*d - a*d^2)*log(abs(b*x + a)/((b*x + a)^2*abs(b))
)/b^3 - (b^3*c^2/(b*x + a) - 2*a*b^2*c*d/(b*x + a) + a^2*b*d^2/(b*x + a))/b
^4
```

Mupad [B]
time = 0.20, size = 71, normalized size = 1.39

$$\frac{d^2\,x}{b^2}-\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{b\,\left(x\,b^3+a\,b^2\right)}-\frac{\ln\left(a+b\,x\right)\,\left(2\,a\,d^2-2\,b\,c\,d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^2,x)
```

```
[Out] (d^2*x)/b^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(b*(a*b^2 + b^3*x)) - (log(a
+ b*x)*(2*a*d^2 - 2*b*c*d))/b^3
```

3.1253 $\int \frac{(c+dx)^2}{(a+bx)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}$$

```
[Out] -1/2*(-a*d+b*c)^2/b^3/(b*x+a)^2-2*d*(-a*d+b*c)/b^3/(b*x+a)+d^2*ln(b*x+a)/b^
3
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] -1/2*(b*c - a*d)^2/(b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) +
(d^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^3}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{d^2}{b^2(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 0.83

$$\frac{-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2}+2d^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^3,x]
[Out] (-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x])
/(2*b^3)
```

Maple [A]
time = 0.14, size = 69, normalized size = 1.17

method	result	size
risch	$\frac{\frac{2d(ad-bc)x}{b^2}+\frac{3a^2d^2-2abcd-b^2c^2}{2b^3}}{(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	67
default	$\frac{2d(ad-bc)}{b^3(bx+a)}-\frac{a^2d^2-2abcd+b^2c^2}{2b^3(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	69
norman	$\frac{\frac{3a^2d^2-2abcd-b^2c^2}{2b^3}+\frac{2\left(a\,d^2-bdc\right)x}{b^2}}{(bx+a)^2}+\frac{d^2\ln(bx+a)}{b^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 2/b^3*d*(a*d-b*c)/(b*x+a)-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^2+d^2
*ln(b*x+a)/b^3
```

Maxima [A]
time = 0.27, size = 79, normalized size = 1.34

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\,(b^2cd-abd^2)x}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}+\frac{d^2\log(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

Fricas [A]
time = 0.64, size = 99, normalized size = 1.68

$$-\frac{b^2c^2+2\,abcd-3\,a^2d^2+4\,(b^2cd-abd^2)x-2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log(bx+a)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="fricas")
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
)
```

Sympy [A]
time = 0.27, size = 80, normalized size = 1.36

$$\frac{3a^2d^2 - 2abcd - b^2c^2 + x(4abd^2 - 4b^2cd)}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{d^2\log{(a + bx)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

Giac [A]
time = 0.63, size = 68, normalized size = 1.15

$$\frac{d^2\log(|bx + a|)}{b^3} - \frac{4\,(bcd - ad^2)x + \frac{b^2c^2 + 2\,abcd - 3\,a^2d^2}{b}}{2\,(bx + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^2*log(abs(b*x + a))/b^3 - 1/2*(4*(b*c*d - a*d^2)*x + (b^2*c^2 + 2*a*b*c*d
 - 3*a^2*d^2)/b)/((b*x + a)^2*b^2)
```

Mupad [B]
time = 0.20, size = 77, normalized size = 1.31

$$\frac{d^2\,\ln\left(a + b\,x\right)}{b^3} - \frac{\frac{-3\,a^2\,d^2 + 2\,a\,b\,c\,d + b^2\,c^2}{2\,b^3} - \frac{2\,d\,x\,(a\,d - b\,c)}{b^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^3,x)
```

```
[Out] (d^2*log(a + b*x))/b^3 - ((b^2*c^2 - 3*a^2*d^2 + 2*a*b*c*d)/(2*b^3) - (2*d*
x*(a*d - b*c))/b^2)/(a^2 + b^2*x^2 + 2*a*b*x)
```

3.1254 $\int \frac{(c+dx)^2}{(a+bx)^4} \, dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

```
[Out] -1/3*(d*x+c)^3/(-a*d+b*c)/(b*x+a)^3
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(c + d*x)^3/((b*c - a*d)*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+bx)^4} \, dx = -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.89

$$-\frac{a^2d^2+abd(c+3dx)+b^2(c^2+3cdx+3d^2x^2)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(b^3*(
a + b*x)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(26) = 52$.
time = 0.14, size = 70, normalized size = 2.50

method	result	size
gosper	$-\frac{3d^2x^2b^2+3ab\,d^2x+3b^2cdx+a^2d^2+abcd+b^2c^2}{3(bx+a)^3b^3}$	60
risch	$\frac{-\frac{d^2x^2}{b}-\frac{d(ad+bc)x}{b^2}-\frac{a^2d^2+abcd+b^2c^2}{3b^3}}{(bx+a)^3}$	60
norman	$\frac{-\frac{d^2x^2}{b}+\frac{\left(-a\,d^2-bdc\right)x}{b^2}+\frac{-a^2d^2-abcd-b^2c^2}{3b^3}}{(bx+a)^3}$	66
default	$-\frac{d^2}{b^3(bx+a)}+\frac{d(ad-bc)}{b^3(bx+a)^2}-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3/(b*x+a)+1/b^3*d*(a*d-b*c)/(b*x+a)^2-1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/b^3/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.
time = 0.27, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\,(b^2cd+abd^2)x}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(26) = 52$.
time = 0.61, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\,(b^2cd+abd^2)x}{3\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(22) = 44$.
time = 0.32, size = 88, normalized size = 3.14

$$\frac{-a^2d^2 - abcd - b^2c^2 - 3b^2d^2x^2 + x(-3abd^2 - 3b^2cd)}{3a^3b^3 + 9a^2b^4x + 9ab^5x^2 + 3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**4,x)
```

```
[Out] (-a**2*d**2 - a*b*c*d - b**2*c**2 - 3*b**2*d**2*x**2 + x*(-3*a*b*d**2 - 3*b
**2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.
time = 0.61, size = 59, normalized size = 2.11

$$-\frac{3\,b^2d^2x^2 + 3\,b^2cdx + 3\,abd^2x + b^2c^2 + abcd + a^2d^2}{3\,(bx + a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*d
^2)/((b*x + a)^3*b^3)
```

Mupad [B]
time = 0.04, size = 80, normalized size = 2.86

$$-\frac{\frac{a^2\,d^2+a\,b\,c\,d+b^2\,c^2}{3\,b^3}+\frac{d^2\,x^2}{b}+\frac{d\,x\,(a\,d+b\,c)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^4,x)
```

```
[Out] -((a^2*d^2 + b^2*c^2 + a*b*c*d)/(3*b^3) + (d^2*x^2)/b + (d*x*(a*d + b*c))/b
^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.1255 $\int \frac{(c+dx)^2}{(a+bx)^5} dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}$$

```
[Out] -1/4*(-a*d+b*c)^2/b^3/(b*x+a)^4-2/3*d*(-a*d+b*c)/b^3/(b*x+a)^3-1/2*d^2/b^3/
(b*x+a)^2
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -1/4*(b*c - a*d)^2/(b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3
) - d^2/(2*b^3*(a + b*x)^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^5}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{d^2}{b^2(a+bx)^3}\right)dx\\ &= -\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 0.86

$$-\frac{a^2d^2+2abd(c+2dx)+b^2(3c^2+8cdx+6d^2x^2)}{12b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -1/12*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/(
b^3*(a + b*x)^4)
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{6d^{2}x^{2}b^{2}+4ab\,d^{2}x+8b^{2}cdx+a^{2}d^{2}+2abcd+3b^{2}c^{2}}{12b^{3}\left(bx+a\right)^{4}}$	62
risch	$\frac{-\frac{d^{2}x^{2}}{2b}-\frac{d\left(ad+2bc\right)x}{3b^{2}}-\frac{a^{2}d^{2}+2abcd+3b^{2}c^{2}}{12b^{3}}}{\left(bx+a\right)^{4}}$	63
default	$-\frac{a^{2}d^{2}-2abcd+b^{2}c^{2}}{4b^{3}\left(bx+a\right)^{4}}-\frac{d^{2}}{2b^{3}\left(bx+a\right)^{2}}+\frac{2d\left(ad-bc\right)}{3b^{3}\left(bx+a\right)^{3}}$	71
norman	$\frac{-\frac{d^{2}x^{2}}{2b}+\frac{\left(-ab\,d^{2}-2b^{2}cd\right)x}{3b^{3}}+\frac{-a^{2}b\,d^{2}-2a\,b^{2}cd-3b^{3}c^{2}}{12b^{4}}}{\left(bx+a\right)^{4}}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^4-1/2*d^2/b^3/(b*x+a)^2+2/3/b^
3*d*(a*d-b*c)/(b*x+a)^3
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.51

$$-\frac{6\,b^{2}d^{2}x^{2}+3\,b^{2}c^{2}+2\,abcd+a^{2}d^{2}+4\,(2\,b^{2}cd+abd^{2})x}{12\,(b^{7}x^{4}+4\,ab^{6}x^{3}+6\,a^{2}b^{5}x^{2}+4\,a^{3}b^{4}x+a^{4}b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Fricas [A]
time = 0.44, size = 98, normalized size = 1.51

$$-\frac{6\,b^{2}d^{2}x^{2}+3\,b^{2}c^{2}+2\,abcd+a^{2}d^{2}+4\,(2\,b^{2}cd+abd^{2})x}{12\,(b^{7}x^{4}+4\,ab^{6}x^{3}+6\,a^{2}b^{5}x^{2}+4\,a^{3}b^{4}x+a^{4}b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

Sympy [A]
time = 0.43, size = 104, normalized size = 1.60

$$\frac{-a^2d^2-2abcd-3b^2c^2-6b^2d^2x^2+x(-4abd^2-8b^2cd)}{12a^4b^3+48a^3b^4x+72a^2b^5x^2+48ab^6x^3+12b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**5,x)
```

```
[Out] (-a**2*d**2 - 2*a*b*c*d - 3*b**2*c**2 - 6*b**2*d**2*x**2 + x*(-4*a*b*d**2 -
 8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b**
6*x**3 + 12*b**7*x**4)
```

Giac [A]
time = 1.03, size = 96, normalized size = 1.48

$$-\frac{\frac{3\,c^2}{(bx+a)^4}+\frac{8\,cd}{(bx+a)^3b}-\frac{6\,acd}{(bx+a)^4b}+\frac{6\,d^2}{(bx+a)^2b^2}-\frac{8\,ad^2}{(bx+a)^3b^2}+\frac{3\,a^2d^2}{(bx+a)^4b^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*c^2/(b*x + a)^4 + 8*c*d/((b*x + a)^3*b) - 6*a*c*d/((b*x + a)^4*b)
+ 6*d^2/((b*x + a)^2*b^2) - 8*a*d^2/((b*x + a)^3*b^2) + 3*a^2*d^2/((b*x + a
)^4*b^2))/b
```

Mupad [B]
time = 0.19, size = 39, normalized size = 0.60

$$\frac{{\left(c+d\,x\right)}^3\,\left(4\,a\,d-3\,b\,c+b\,d\,x\right)}{12\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^5,x)
```

```
[Out] ((c + d*x)^3*(4*a*d - 3*b*c + b*d*x))/(12*(a*d - b*c)^2*(a + b*x)^4)
```

3.1256 $\int \frac{(c+dx)^2}{(a+bx)^6}\,dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}$$

```
[Out] -1/5*(-a*d+b*c)^2/b^3/(b*x+a)^5-1/2*d*(-a*d+b*c)/b^3/(b*x+a)^4-1/3*d^2/b^3/
(b*x+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -1/5*(b*c - a*d)^2/(b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4)
- d^2/(3*b^3*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^6}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^4}\right)dx\\ &= -\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(3c+5dx)+b^2(6c^2+15cdx+10d^2x^2)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -1/30*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))
/(b^3*(a + b*x)^5)
```

Maple [A]
time = 0.15, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{10d^2x^2b^2+5ab\,d^2x+15b^2cdx+a^2d^2+3abcd+6b^2c^2}{30b^3(bx+a)^5}$	62
risch	$\frac{-\frac{d^2x^2}{3b}-\frac{d(ad+3bc)x}{6b^2}-\frac{a^2d^2+3abcd+6b^2c^2}{30b^3}}{(bx+a)^5}$	63
default	$\frac{d(ad-bc)}{2b^3(bx+a)^4}-\frac{a^2d^2-2abcd+b^2c^2}{5b^3(bx+a)^5}-\frac{d^2}{3b^3(bx+a)^3}$	71
norman	$\frac{-\frac{d^2x^2}{3b}+\frac{\left(-a\,b^2d^2-3b^3cd\right)x}{6b^4}+\frac{-b^2a^2d^2-3a\,b^3cd-6b^4c^2}{30b^5}}{(bx+a)^5}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^3*d*(a*d-b*c)/(b*x+a)^4-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5
-1/3*d^2/b^3/(b*x+a)^3
```

Maxima [A]
time = 0.27, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\,(3\,b^2cd+abd^2)x}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Fricas [A]
time = 0.48, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2+6\,b^2c^2+3\,abcd+a^2d^2+5\,(3\,b^2cd+abd^2)x}{30\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(56) = 112$.
time = 0.52, size = 116, normalized size = 1.78

$$\frac{-a^2d^2 - 3abcd - 6b^2c^2 - 10b^2d^2x^2 + x(-5abd^2 - 15b^2cd)}{30a^5b^3 + 150a^4b^4x + 300a^3b^5x^2 + 300a^2b^6x^3 + 150ab^7x^4 + 30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**6,x)
```

```
[Out] (-a**2*d**2 - 3*a*b*c*d - 6*b**2*c**2 - 10*b**2*d**2*x**2 + x*(-5*a*b*d**2
- 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*
a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

Giac [A]
time = 1.22, size = 61, normalized size = 0.94

$$-\frac{10\,b^2d^2x^2 + 15\,b^2cdx + 5\,abd^2x + 6\,b^2c^2 + 3\,abcd + a^2d^2}{30\,(bx+a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
+ a^2*d^2)/((b*x + a)^5*b^3)
```

Mupad [B]
time = 0.20, size = 107, normalized size = 1.65

$$-\frac{\frac{a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2}{30\,b^3}+\frac{d^2\,x^2}{3\,b}+\frac{d\,x\,(a\,d+3\,b\,c)}{6\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^6,x)
```

```
[Out] -((a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d)/(30*b^3) + (d^2*x^2)/(3*b) + (d*x*(a*d
+ 3*b*c))/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b
^3*x^3 + 5*a^4*b*x)
```

3.1257 $\int \frac{(c+dx)^2}{(a+bx)^7} dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}$$

```
[Out] -1/6*(-a*d+b*c)^2/b^3/(b*x+a)^6-2/5*d*(-a*d+b*c)/b^3/(b*x+a)^5-1/4*d^2/b^3/
(b*x+a)^4
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -1/6*(b*c - a*d)^2/(b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5
) - d^2/(4*b^3*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 0.89

$$-\frac{a^2d^2+2abd(2c+3dx)+b^2(10c^2+24cdx+15d^2x^2)}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^
2))/(b^3*(a + b*x)^6)
```

Maple [A]
time = 0.16, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{15d^2x^2b^2+6ab\,d^2x+24b^2cdx+a^2d^2+4abcd+10b^2c^2}{60b^3(bx+a)^6}$	62
risch	$\frac{-\frac{d^2x^2}{4b}-\frac{d(ad+4bc)x}{10b^2}-\frac{a^2d^2+4abcd+10b^2c^2}{60b^3}}{(bx+a)^6}$	63
default	$-\frac{d^2}{4b^3(bx+a)^4}+\frac{2d(ad-bc)}{5b^3(bx+a)^5}-\frac{a^2d^2-2abcd+b^2c^2}{6b^3(bx+a)^6}$	71
norman	$\frac{-\frac{d^2x^2}{4b}+\frac{\left(-a\,b^3d^2-4b^4cd\right)x}{10b^5}+\frac{-a^2b^3d^2-4a\,b^4cd-10c^2b^5}{60b^6}}{(bx+a)^6}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2/b^3/(b*x+a)^4+2/5/b^3*d*(a*d-b*c)/(b*x+a)^5-1/6*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/b^3/(b*x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(59) = 118.
time = 0.28, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\,(4\,b^2cd+abd^2)x}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(59) = 118.
time = 0.52, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\,(4\,b^2cd+abd^2)x}{60\,(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(58) = 116$.

time = 0.65, size = 128, normalized size = 1.97

$$\frac{-a^2d^2 - 4abcd - 10b^2c^2 - 15b^2d^2x^2 + x(-6abd^2 - 24b^2cd)}{60a^6b^3 + 360a^5b^4x + 900a^4b^5x^2 + 1200a^3b^6x^3 + 900a^2b^7x^4 + 360ab^8x^5 + 60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**7,x)
```

```
[Out] (-a**2*d**2 - 4*a*b*c*d - 10*b**2*c**2 - 15*b**2*d**2*x**2 + x*(-6*a*b*d**2
 - 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 120
0*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

Giac [A]

time = 1.18, size = 61, normalized size = 0.94

$$-\frac{15\,b^2d^2x^2 + 24\,b^2cdx + 6\,abd^2x + 10\,b^2c^2 + 4\,abcd + a^2d^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)/((b*x + a)^6*b^3)
```

Mupad [B]

time = 0.09, size = 118, normalized size = 1.82

$$-\frac{\frac{a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2}{60\,b^3}+\frac{d^2\,x^2}{4\,b}+\frac{d\,x\,(a\,d+4\,b\,c)}{10\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^7,x)
```

```
[Out] -((a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d)/(60*b^3) + (d^2*x^2)/(4*b) + (d*x*(a*d
 + 4*b*c))/(10*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3
*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

3.1258 $\int (a+bx)^5(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

```
[Out] 1/6*(-a*d+b*c)^3*(b*x+a)^6/b^4+3/7*d*(-a*d+b*c)^2*(b*x+a)^7/b^4+3/8*d^2*(-a
*d+b*c)*(b*x+a)^8/b^4+1/9*d^3*(b*x+a)^9/b^4
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^5}{b^3}+\frac{3d(bc-ad)^2(a+bx)^6}{b^3}+\frac{3d^2(bc-ad)(a+bx)^7}{b^3}+\frac{d^3}{}\right.\\ &= \frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+}{9b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 235 vs. 2(92) = 184.
time = 0.05, size = 235, normalized size = 2.55

$$\frac{1}{504}x\left(126a^5\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+126a^4bx\left(10c^3+20c^2dx+15cd^2x^2+4d^3x^3\right)+84a^3b^2x^2\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(35c^3+84c^2dx+70cd^2x^2+20d^3x^3\right)+9ab^4x^4\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)+b^5x^5\left(84c^3+216c^2dx+189cd^2x^2+56d^3x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] (x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45*c
^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*x +
 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2
*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3
*x^3)))/504
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(84) = 168$.
time = 0.13, size = 281, normalized size = 3.05

method	result
norman	$\frac{b^5d^3x^9}{9} + \left(\frac{5}{8}a\,b^4d^3 + \frac{3}{8}b^5c\,d^2\right)x^8 + \left(\frac{10}{7}a^2b^3d^3 + \frac{15}{7}a\,b^4c\,d^2 + \frac{3}{7}b^5c^2d\right)x^7 + \left(\frac{5}{3}a^3b^2d^3 + 5a^2b^3c\,d^2 + \frac{5}{2}a\right.$
default	$\frac{b^5d^3x^9}{9} + \frac{(5a\,b^4d^3+3b^5c\,d^2)x^8}{8} + \frac{(10a^2b^3d^3+15a\,b^4c\,d^2+3b^5c^2d)x^7}{7} + \frac{(10a^3b^2d^3+30a^2b^3c\,d^2+15a\,b^4c^2d+b^5c^3)x^6}{6} + \frac{(5a^4b\,d^3}{}$
gosper	$\frac{1}{9}b^5d^3x^9 + \frac{5}{8}x^8a\,b^4d^3 + \frac{3}{8}x^8b^5c\,d^2 + \frac{10}{7}x^7a^2b^3d^3 + \frac{15}{7}x^7a\,b^4c\,d^2 + \frac{3}{7}x^7b^5c^2d + \frac{5}{3}x^6a^3b^2d^3 + 5x^6a^2b^3c$
risch	$\frac{1}{9}b^5d^3x^9 + \frac{5}{8}x^8a\,b^4d^3 + \frac{3}{8}x^8b^5c\,d^2 + \frac{10}{7}x^7a^2b^3d^3 + \frac{15}{7}x^7a\,b^4c\,d^2 + \frac{3}{7}x^7b^5c^2d + \frac{5}{3}x^6a^3b^2d^3 + 5x^6a^2b^3c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^5*d^3*x^9+1/8*(5*a*b^4*d^3+3*b^5*c*d^2)*x^8+1/7*(10*a^2*b^3*d^3+15*a*
b^4*c*d^2+3*b^5*c^2*d)*x^7+1/6*(10*a^3*b^2*d^3+30*a^2*b^3*c*d^2+15*a*b^4*c^
2*d+b^5*c^3)*x^6+1/5*(5*a^4*b*d^3+30*a^3*b^2*c*d^2+30*a^2*b^3*c^2*d+5*a*b^4
*c^3)*x^5+1/4*(a^5*d^3+15*a^4*b*c*d^2+30*a^3*b^2*c^2*d+10*a^2*b^3*c^3)*x^4+
1/3*(3*a^5*c*d^2+15*a^4*b*c^2*d+10*a^3*b^2*c^3)*x^3+1/2*(3*a^5*c^2*d+5*a^4*
b*c^3)*x^2+a^5*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(84) = 168$.
time = 0.32, size = 277, normalized size = 3.01

$$\frac{1}{9}b^5d^3x^9 + a^5c^3x + \frac{1}{8}\left(3\,b^5cd^2 + 5\,ab^4d^3\right)x^8 + \frac{1}{7}\left(3\,b^5c^2d + 15\,ab^4cd^2 + 10\,a^2b^3d^3\right)x^7 + \frac{1}{6}\left(b^5c^3 + 15\,ab^4c^2d + 30\,a^2b^3cd^2 + 10\,a^3b^2d^3\right)x^6 + \left(ab^4c^3 + 6\,a^2b^3c^2d + 6\,a^3b^2cd^2 + a^4bd^3\right)x^5 + \frac{1}{4}\left(10\,a^2b^3c^3 + 30\,a^3b^2c^2d + 15\,a^4bcd^2 + a^5d^3\right)x^4 + \frac{1}{3}\left(10\,a^3b^2c^3 + 15\,a^4bc^2d + 3\,a^5cd^2\right)x^3 + \frac{1}{2}\left(5\,a^4bc^3 + 3\,a^5c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
```

```
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(84) = 168$.
time = 0.43, size = 277, normalized size = 3.01

$$\frac{1}{9}\,b^5d^3x^9 + a^5c^3x + \frac{1}{8}\,(3\,b^5cd^2 + 5\,ab^4d^3)x^8 + \frac{1}{7}\,(3\,b^5c^2d + 15\,ab^4cd^2 + 10\,a^2b^3d^3)x^7 + \frac{1}{6}\,(b^5c^3 + 15\,ab^4c^2d + 30\,a^2b^3cd^2 + 10\,a^3b^2d^3)x^6 + (ab^4c^3 + 6\,a^2b^3c^2d + 6\,a^3b^2cd^2 + a^4bd^3)x^5 + \frac{1}{4}\,(10\,a^2b^3c^3 + 30\,a^3b^2c^2d + 15\,a^4bcd^2 + a^5d^3)x^4 + \frac{1}{3}\,(10\,a^3b^2c^3 + 15\,a^4bc^2d + 3\,a^5cd^2)x^3 + \frac{1}{2}\,(5\,a^4bc^3 + 3\,a^5c^2d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(82) = 164$.
time = 0.03, size = 308, normalized size = 3.35

$$a^5c^3x + \frac{b^5d^3x^9}{9} + x^8 \cdot \left(\frac{5ab^4d^3}{8} + \frac{3b^5cd^2}{8}\right) + x^7 \cdot \left(\frac{10a^2b^3d^3}{7} + \frac{15ab^4cd^2}{7} + \frac{3b^5c^2d}{7}\right) + x^6 \cdot \left(\frac{5a^3b^2d^3}{3} + 5a^2b^3cd^2 + \frac{5ab^4c^2d}{2} + \frac{b^5c^3}{6}\right) + x^5\left(a^4bd^3 + 6a^3b^2cd^2 + 6a^2b^3c^2d + ab^4c^3\right) + x^4\left(\frac{a^5d^3}{4} + \frac{15a^4bcd^2}{4} + \frac{15a^3b^2c^2d}{2} + \frac{5a^2b^3c^3}{2}\right) + x^3\left(a^5cd^2 + 5a^4bc^2d + \frac{10a^3b^2c^3}{3}\right) + x^2 \cdot \left(\frac{3a^5c^2d}{2} + \frac{5a^4bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**3,x)
```

```
[Out] a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8) +
 x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6*(
5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/6)
+ x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*c**3
) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5*a**2
*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c**3/3)
+ x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(84) = 168$.
time = 1.26, size = 303, normalized size = 3.29

$$\frac{1}{9}\,b^5d^3x^9 + \frac{3}{8}\,b^5cd^2x^8 + \frac{5}{8}\,ab^4d^3x^8 + \frac{3}{7}\,b^5c^2dx^7 + \frac{15}{7}\,ab^4cd^2x^7 + \frac{10}{7}\,a^2b^3d^3x^7 + \frac{1}{6}\,b^5c^3x^6 + \frac{5}{2}\,ab^4c^2dx^6 + 5\,a^2b^3cd^2x^6 + \frac{5}{3}\,a^3b^2d^3x^6 + ab^4c^3x^5 + 6\,a^2b^3c^2dx^5 + 6\,a^3b^2cd^2x^5 + a^4bd^3x^5 + \frac{5}{2}\,a^2b^3c^3x^4 + \frac{15}{2}\,a^3b^2c^2dx^4 + \frac{15}{4}\,a^4bcd^2x^4 + \frac{1}{4}\,a^5d^3x^4 + \frac{10}{3}\,a^3b^2c^3x^3 + 5\,a^4bc^2dx^3 + a^5cd^2x^3 + \frac{5}{2}\,a^4bc^3x^2 + \frac{3}{2}\,a^5c^2dx^2 + a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^7
 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*b^
```

```
4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5 + 6
*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*c^3*
x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4 + 10/
3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3*x^2 +
 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

Mupad [B]
time = 0.24, size = 261, normalized size = 2.84

$$x^5\,\left(a^4\,b\,d^3+6\,a^3\,b^2\,c\,d^2+6\,a^2\,b^3\,c^2\,d+a\,b^4\,c^3\right)+x^4\,\left(\frac{a^5\,d^3}{4}+\frac{15\,a^4\,b\,c\,d^2}{4}+\frac{15\,a^3\,b^2\,c^2\,d}{2}+\frac{5\,a^2\,b^3\,c^3}{2}\right)+x^6\,\left(\frac{5\,a^3\,b^2\,d^3}{3}+5\,a^2\,b^3\,c\,d^2+\frac{5\,a\,b^4\,c^2\,d}{2}+\frac{b^5\,c^3}{6}\right)+a^5\,c^3\,x+\frac{b^5\,d^3\,x^9}{9}+\frac{a^4\,c^2\,x^2\,\left(3\,a\,d+5\,b\,c\right)}{2}+\frac{b^4\,d^2\,x^8\,\left(5\,a\,d+3\,b\,c\right)}{8}+\frac{a^3\,c\,x^3\,\left(3\,a^2\,d^2+15\,a\,b\,c\,d+10\,b^2\,c^2\right)}{3}+\frac{b^3\,d\,x^7\,\left(10\,a^2\,d^2+15\,a\,b\,c\,d+3\,b^2\,c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^3,x)
```

```
[Out] x^5*(a*b^4*c^3 + a^4*b*d^3 + 6*a^2*b^3*c^2*d + 6*a^3*b^2*c*d^2) + x^4*((a^5
*d^3)/4 + (5*a^2*b^3*c^3)/2 + (15*a^3*b^2*c^2*d)/2 + (15*a^4*b*c*d^2)/4) +
x^6*((b^5*c^3)/6 + (5*a^3*b^2*d^3)/3 + 5*a^2*b^3*c*d^2 + (5*a*b^4*c^2*d)/2)
 + a^5*c^3*x + (b^5*d^3*x^9)/9 + (a^4*c^2*x^2*(3*a*d + 5*b*c))/2 + (b^4*d^2
*x^8*(5*a*d + 3*b*c))/8 + (a^3*c*x^3*(3*a^2*d^2 + 10*b^2*c^2 + 15*a*b*c*d))
/3 + (b^3*d*x^7*(10*a^2*d^2 + 3*b^2*c^2 + 15*a*b*c*d))/7
```

3.1259 $\int (a+bx)^4(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

```
[Out] 1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-a
*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^4}{b^3}+\frac{3d(bc-ad)^2(a+bx)^5}{b^3}+\frac{3d^2(bc-ad)(a+bx)^6}{b^3}+\frac{d^3\right.\\ &= \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+b}{8b^4}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.
time = 0.02, size = 217, normalized size = 2.36

$$a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2+a^2c\left(2b^2c^2+4abcd+a^2d^2\right)x^3+\frac{1}{4}a\left(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3\right)x^4+\frac{1}{5}b\left(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3\right)x^5+\frac{1}{2}b^2d\left(b^2c^2+4abcd+2a^2d^2\right)x^6+\frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)
*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5)/5
 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c + 4*a*
d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(84) = 168$.
time = 0.13, size = 229, normalized size = 2.49

method	result
norman	$\frac{b^4 d^3 x^8}{8} + \left(\frac{4}{7} a\, b^3 d^3 + \frac{3}{7} b^4 c\, d^2\right) x^7 + \left(b^2 a^2 d^3 + 2 a\, b^3 c\, d^2 + \frac{1}{2} b^4 c^2 d\right) x^6 + \left(\frac{4}{5} a^3 b\, d^3 + \frac{18}{5} b^2 a^2 c\, d^2 + \frac{12}{5} a\, b^3$
default	$\frac{b^4 d^3 x^8}{8} + \frac{(4 a\, b^3 d^3 + 3 b^4 c\, d^2) x^7}{7} + \frac{(6 b^2 a^2 d^3 + 12 a\, b^3 c\, d^2 + 3 b^4 c^2 d) x^6}{6} + \frac{(4 a^3 b\, d^3 + 18 b^2 a^2 c\, d^2 + 12 a\, b^3 c^2 d + b^4 c^3) x^5}{5} + \frac{(a^4 d^3 + 12 a^3$
gosper	$\frac{1}{8} b^4 d^3 x^8 + \frac{4}{7} x^7 a\, b^3 d^3 + \frac{3}{7} x^7 b^4 c\, d^2 + x^6 b^2 a^2 d^3 + 2 x^6 a\, b^3 c\, d^2 + \frac{1}{2} x^6 b^4 c^2 d + \frac{4}{5} x^5 a^3 b\, d^3 + \frac{18}{5} x^5 b^2 a^2 c\, d^2$
risch	$\frac{1}{8} b^4 d^3 x^8 + \frac{4}{7} x^7 a\, b^3 d^3 + \frac{3}{7} x^7 b^4 c\, d^2 + x^6 b^2 a^2 d^3 + 2 x^6 a\, b^3 c\, d^2 + \frac{1}{2} x^6 b^4 c^2 d + \frac{4}{5} x^5 a^3 b\, d^3 + \frac{18}{5} x^5 b^2 a^2 c\, d^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^4*d^3*x^8+1/7*(4*a*b^3*d^3+3*b^4*c*d^2)*x^7+1/6*(6*a^2*b^2*d^3+12*a*b
^3*c*d^2+3*b^4*c^2*d)*x^6+1/5*(4*a^3*b*d^3+18*a^2*b^2*c*d^2+12*a*b^3*c^2*d+
b^4*c^3)*x^5+1/4*(a^4*d^3+12*a^3*b*c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3)*x^4+
1/3*(3*a^4*c*d^2+12*a^3*b*c^2*d+6*a^2*b^2*c^3)*x^3+1/2*(3*a^4*c^2*d+4*a^3*b
*c^3)*x^2+a^4*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.
time = 0.28, size = 225, normalized size = 2.45

$$\frac{1}{8} b^4 d^3 x^8 + a^4 c^3 x + \frac{1}{7} \left(3 b^4 c d^2 + 4 a b^3 d^3\right) x^7 + \frac{1}{2} \left(b^4 c^2 d + 4 a b^3 c d^2 + 2 a^2 b^2 d^3\right) x^6 + \frac{1}{5} \left(b^4 c^3 + 12 a b^3 c^2 d + 18 a^2 b^2 c d^2 + 4 a^3 b d^3\right) x^5 + \frac{1}{4} \left(4 a b^3 c^3 + 18 a^2 b^2 c^2 d + 12 a^3 b c d^2 + a^4 d^3\right) x^4 + \left(2 a^2 b^2 c^3 + 4 a^3 b c^2 d + a^4 c d^2\right) x^3 + \frac{1}{2} \left(4 a^3 b c^3 + 3 a^4 c^2 d\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.
time = 0.48, size = 225, normalized size = 2.45

$$\frac{1}{8} b^4 d^3 x^8 + a^4 c^3 x + \frac{1}{7} \left(3 b^4 c d^2 + 4 a b^3 d^3\right) x^7 + \frac{1}{2} \left(b^4 c^2 d + 4 a b^3 c d^2 + 2 a^2 b^2 d^3\right) x^6 + \frac{1}{5} \left(b^4 c^3 + 12 a b^3 c^2 d + 18 a^2 b^2 c d^2 + 4 a^3 b d^3\right) x^5 + \frac{1}{4} \left(4 a b^3 c^3 + 18 a^2 b^2 c^2 d + 12 a^3 b c d^2 + a^4 d^3\right) x^4 + \left(2 a^2 b^2 c^3 + 4 a^3 b c^2 d + a^4 c d^2\right) x^3 + \frac{1}{2} \left(4 a^3 b c^3 + 3 a^4 c^2 d\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.
time = 0.03, size = 243, normalized size = 2.64

$$a^4 c^3 x + \frac{b^4 d^3 x^8}{8} + x^7 \cdot \left(\frac{4 a b^3 d^3}{7} + \frac{3 b^4 c d^2}{7}\right) + x^6 \left(a^2 b^2 d^3 + 2 a b^3 c d^2 + \frac{b^4 c^2 d}{2}\right) + x^5 \cdot \left(\frac{4 a^3 b d^3}{5} + \frac{18 a^2 b^2 c d^2}{5} + \frac{12 a b^3 c^2 d}{5} + \frac{b^4 c^3}{5}\right) + x^4 \left(\frac{a^4 d^3}{4} + 3 a^3 b c d^2 + \frac{9 a^2 b^2 c^2 d}{2} + a b^3 c^3\right) + x^3 \left(a^4 c d^2 + 4 a^3 b c^2 d + 2 a^2 b^2 c^3\right) + x^2 \cdot \left(\frac{3 a^4 c^2 d}{2} + 2 a^3 b c^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**3,x)
```

```
[Out] a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) +
 x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d
**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a
**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(
a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 +
 2*a**3*b*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.
time = 1.28, size = 245, normalized size = 2.66

$$\frac{1}{8} b^4 d^3 x^8 + \frac{3}{7} b^4 c d^2 x^7 + \frac{4}{7} a b^3 d^3 x^7 + \frac{1}{2} b^4 c^2 d x^6 + 2 a b^3 c d^2 x^6 + a^2 b^2 d^3 x^6 + \frac{1}{5} b^4 c^3 x^5 + \frac{12}{5} a b^3 c^2 d x^5 + \frac{18}{5} a^2 b^2 c d^2 x^5 + \frac{4}{5} a^3 b d^3 x^5 + a b^3 c^3 x^4 + \frac{9}{2} a^2 b^2 c^2 d x^4 + 3 a^3 b c d^2 x^4 + \frac{1}{4} a^4 d^3 x^4 + 2 a^2 b^2 c^3 x^3 + 4 a^3 b c^2 d x^3 + a^4 c d^2 x^3 + 2 a^3 b c^3 x^2 + \frac{3}{2} a^4 c^2 d x^2 + a^4 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^6
 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2*d
*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*a^2
*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^3 +
4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^2 + a
^4*c^3*x
```

Mupad [B]
time = 0.21, size = 208, normalized size = 2.26

$$x^4\left(\frac{a^4d^3}{4}+3a^3bcd^2+\frac{9a^2b^2c^2d}{2}+ab^3c^3\right)+x^5\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right)+a^4c^3x+\frac{b^4d^3x^8}{8}+\frac{a^3c^2x^2(3ad+4bc)}{2}+\frac{b^3d^2x^7(4ad+3bc)}{7}+a^2cx^3\left(a^2d^2+4abcd+2b^2c^2\right)+\frac{b^2dx^6(2a^2d^2+4abcd+b^2c^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^3,x)
```

```
[Out] x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*(
(b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5)
+ a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d^2*
x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) + (b^2
*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

3.1260 $\int (a+bx)^3(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

```
[Out] 1/4*(-a*d+b*c)^3*(b*x+a)^4/b^4+3/5*d*(-a*d+b*c)^2*(b*x+a)^5/b^4+1/2*d^2*(-a
*d+b*c)*(b*x+a)^6/b^4+1/7*d^3*(b*x+a)^7/b^4
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^4)/(4*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^5)/(5*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^6)/(2*b^4) + (d^3*(a + b*x)^7)/(7*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^3}{b^3}+\frac{3d(bc-ad)^2(a+bx)^4}{b^3}+\frac{3d^2(bc-ad)(a+bx)^5}{b^3}+\frac{d^3\right.\\ &= \frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+b}{7b^4}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 161, normalized size = 1.75

$$a^3c^3x+\frac{3}{2}a^2c^2(bc+ad)x^2+ac\left(b^2c^2+3abcd+a^2d^2\right)x^3+\frac{1}{4}\left(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^4+\frac{3}{5}bd\left(b^2c^2+3abcd+a^2d^2\right)x^5+\frac{1}{2}b^2d^2(bc+ad)x^6+\frac{1}{7}b^3d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2*
d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 + (3
*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/2 +
 (b^3*d^3*x^7)/7
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(84) = 168$.
time = 0.14, size = 177, normalized size = 1.92

method	result
norman	$\frac{b^3 d^3 x^7}{7} + \left(\frac{1}{2} a\, b^2 d^3 + \frac{1}{2} b^3 c\, d^2\right) x^6 + \left(\frac{3}{5} a^2 b\, d^3 + \frac{9}{5} a\, b^2 c\, d^2 + \frac{3}{5} b^3 c^2 d\right) x^5 + \left(\frac{1}{4} a^3 d^3 + \frac{9}{4} a^2 bc\, d^2 + \frac{9}{4} a\, b^2 c^2 d$
default	$\frac{b^3 d^3 x^7}{7} + \frac{(3a\, b^2 d^3 + 3b^3 c\, d^2) x^6}{6} + \frac{(3a^2 b\, d^3 + 9a\, b^2 c\, d^2 + 3b^3 c^2 d) x^5}{5} + \frac{(a^3 d^3 + 9a^2 bc\, d^2 + 9a\, b^2 c^2 d + b^3 c^3) x^4}{4} + \frac{(3a^3 c\, d^2 + 9a^2 b\, c^2 d +}{3}$
gosper	$\frac{1}{7} b^3 d^3 x^7 + \frac{1}{2} x^6 a\, b^2 d^3 + \frac{1}{2} x^6 b^3 c\, d^2 + \frac{3}{5} x^5 a^2 b\, d^3 + \frac{9}{5} x^5 a\, b^2 c\, d^2 + \frac{3}{5} x^5 b^3 c^2 d + \frac{1}{4} x^4 a^3 d^3 + \frac{9}{4} x^4 a^2 bc\, d^2 +$
risch	$\frac{1}{7} b^3 d^3 x^7 + \frac{1}{2} x^6 a\, b^2 d^3 + \frac{1}{2} x^6 b^3 c\, d^2 + \frac{3}{5} x^5 a^2 b\, d^3 + \frac{9}{5} x^5 a\, b^2 c\, d^2 + \frac{3}{5} x^5 b^3 c^2 d + \frac{1}{4} x^4 a^3 d^3 + \frac{9}{4} x^4 a^2 bc\, d^2 +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^3*d^3*x^7+1/6*(3*a*b^2*d^3+3*b^3*c*d^2)*x^6+1/5*(3*a^2*b*d^3+9*a*b^2*
c*d^2+3*b^3*c^2*d)*x^5+1/4*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*x^
4+1/3*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^3+1/2*(3*a^3*c^2*d+3*a^2*b*
c^3)*x^2+a^3*c^3*x
```

Maxima [A]
time = 0.28, size = 167, normalized size = 1.82

$$\frac{1}{7} b^3 d^3 x^7 + a^3 c^3 x + \frac{1}{2} (b^3 c d^2 + a b^2 d^3) x^6 + \frac{3}{5} (b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3) x^5 + \frac{1}{4} (b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3) x^4 + (a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2) x^3 + \frac{3}{2} (a^2 b c^3 + a^3 c^2 d) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Fricas [A]
time = 0.51, size = 167, normalized size = 1.82

$$\frac{1}{7} b^3 d^3 x^7 + a^3 c^3 x + \frac{1}{2} (b^3 c d^2 + a b^2 d^3) x^6 + \frac{3}{5} (b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3) x^5 + \frac{1}{4} (b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3) x^4 + (a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2) x^3 + \frac{3}{2} (a^2 b c^3 + a^3 c^2 d) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(80) = 160$.
time = 0.02, size = 190, normalized size = 2.07

$$a^3c^3x + \frac{b^3d^3x^7}{7} + x^6\left(\frac{ab^2d^3}{2} + \frac{b^3cd^2}{2}\right) + x^5\cdot\left(\frac{3a^2bd^3}{5} + \frac{9ab^2cd^2}{5} + \frac{3b^3c^2d}{5}\right) + x^4\left(\frac{a^3d^3}{4} + \frac{9a^2bcd^2}{4} + \frac{9ab^2c^2d}{4} + \frac{b^3c^3}{4}\right) + x^3\left(a^3cd^2 + 3a^2bc^2d + ab^2c^3\right) + x^2\cdot\left(\frac{3a^3c^2d}{2} + \frac{3a^2bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x**
5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d**3
/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c*d*
*2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*c**3
/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(84) = 168$.
time = 0.98, size = 188, normalized size = 2.04

$$\frac{1}{7}b^3d^3x^7 + \frac{1}{2}b^3cd^2x^6 + \frac{1}{2}ab^2d^3x^6 + \frac{3}{5}b^3c^2dx^5 + \frac{9}{5}ab^2cd^2x^5 + \frac{3}{5}a^2bd^3x^5 + \frac{1}{4}b^3c^3x^4 + \frac{9}{4}ab^2c^2dx^4 + \frac{9}{4}a^2bcd^2x^4 + \frac{1}{4}a^3d^3x^4 + ab^2c^3x^3 + 3a^2bc^2dx^3 + a^3cd^2x^3 + \frac{3}{2}a^2bc^3x^2 + \frac{3}{2}a^3c^2dx^2 + a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^5
 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*c^
2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*b*c
^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*
x
```

Mupad [B]
time = 0.06, size = 152, normalized size = 1.65

$$x^4\left(\frac{a^3\,d^3}{4} + \frac{9\,a^2\,b\,c\,d^2}{4} + \frac{9\,a\,b^2\,c^2\,d}{4} + \frac{b^3\,c^3}{4}\right) + a^3\,c^3\,x + \frac{b^3\,d^3\,x^7}{7} + a\,c\,x^3\,\left(a^2\,d^2 + 3\,a\,b\,c\,d + b^2\,c^2\right) + \frac{3\,b\,d\,x^5\,\left(a^2\,d^2 + 3\,a\,b\,c\,d + b^2\,c^2\right)}{5} + \frac{3\,a^2\,c^2\,x^2\,\left(a\,d + b\,c\right)}{2} + \frac{b^2\,d^2\,x^6\,\left(a\,d + b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] x^4*((a^3*d^3)/4 + (b^3*c^3)/4 + (9*a*b^2*c^2*d)/4 + (9*a^2*b*c*d^2)/4) + a
^3*c^3*x + (b^3*d^3*x^7)/7 + a*c*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3*b
*d*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5 + (3*a^2*c^2*x^2*(a*d + b*c))/2 +
 (b^2*d^2*x^6*(a*d + b*c))/2
```

3.1261 $\int (a+bx)^2(c+dx)^3\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

```
[Out] 1/4*(-a*d+b*c)^2*(d*x+c)^4/d^3-2/5*b*(-a*d+b*c)*(d*x+c)^5/d^3+1/6*b^2*(d*x+
c)^6/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^3\,dx&=\int\left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx\\&=\frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 122, normalized size = 1.88

$$a^2c^3x+\frac{1}{2}ac^2(2bc+3ad)x^2+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3+\frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4+\frac{1}{5}bd^2(3bc+2ad)x^5+\frac{1}{6}b^2d^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c
+ 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.14, size = 125, normalized size = 1.92

method	result
norman	$\frac{b^2d^3x^6}{6} + \left(\frac{2}{5}ab\,d^3 + \frac{3}{5}b^2c\,d^2\right)x^5 + \left(\frac{1}{4}a^2d^3 + \frac{3}{2}abc\,d^2 + \frac{3}{4}b^2c^2d\right)x^4 + \left(a^2c\,d^2 + 2ab\,c^2d + \frac{1}{3}b^2c^3\right)x^3 +$
default	$\frac{b^2d^3x^6}{6} + \frac{(2ab\,d^3+3b^2c\,d^2)x^5}{5} + \frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^4}{4} + \frac{(3a^2c\,d^2+6ab\,c^2d+b^2c^3)x^3}{3} + \frac{(3a^2c^2d+2a\,c^3b)x^2}{2} + a^2c^3x$
gosper	$\frac{1}{6}b^2d^3x^6 + \frac{2}{5}x^5ab\,d^3 + \frac{3}{5}x^5b^2c\,d^2 + \frac{1}{4}x^4a^2d^3 + \frac{3}{2}x^4abc\,d^2 + \frac{3}{4}x^4b^2c^2d + x^3a^2c\,d^2 + 2x^3ab\,c^2d + \frac{1}{3}b^2$
risch	$\frac{1}{6}b^2d^3x^6 + \frac{2}{5}x^5ab\,d^3 + \frac{3}{5}x^5b^2c\,d^2 + \frac{1}{4}x^4a^2d^3 + \frac{3}{2}x^4abc\,d^2 + \frac{3}{4}x^4b^2c^2d + x^3a^2c\,d^2 + 2x^3ab\,c^2d + \frac{1}{3}b^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*d^3*x^6+1/5*(2*a*b*d^3+3*b^2*c*d^2)*x^5+1/4*(a^2*d^3+6*a*b*c*d^2+3*
b^2*c^2*d)*x^4+1/3*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^3+1/2*(3*a^2*c^2*d+2
*a*b*c^3)*x^2+a^2*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.29, size = 124, normalized size = 1.91

$$\frac{1}{6}b^2d^3x^6 + a^2c^3x + \frac{1}{5}\left(3\,b^2cd^2 + 2\,abd^3\right)x^5 + \frac{1}{4}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^4 + \frac{1}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^3 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(59) = 118$.
time = 0.64, size = 124, normalized size = 1.91

$$\frac{1}{6}b^2d^3x^6 + a^2c^3x + \frac{1}{5}\left(3\,b^2cd^2 + 2\,abd^3\right)x^5 + \frac{1}{4}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^4 + \frac{1}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^3 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.
time = 0.02, size = 133, normalized size = 2.05

$$a^2c^3x + \frac{b^2d^3x^6}{6} + x^5 \cdot \left(\frac{2abd^3}{5} + \frac{3b^2cd^2}{5}\right) + x^4\left(\frac{a^2d^3}{4} + \frac{3abcd^2}{2} + \frac{3b^2c^2d}{4}\right) + x^3\left(a^2cd^2 + 2abc^2d + \frac{b^2c^3}{3}\right) + x^2 \cdot \left(\frac{3a^2c^2d}{2} + abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x*
*4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 + 2
*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.
time = 0.68, size = 130, normalized size = 2.00

$$\frac{1}{6}b^2d^3x^6 + \frac{3}{5}b^2cd^2x^5 + \frac{2}{5}abd^3x^5 + \frac{3}{4}b^2c^2dx^4 + \frac{3}{2}abcd^2x^4 + \frac{1}{4}a^2d^3x^4 + \frac{1}{3}b^2c^3x^3 + 2abc^2dx^3 + a^2cd^2x^3 + abc^3x^2 + \frac{3}{2}a^2c^2dx^2 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

Mupad [B]
time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(a^2\,c\,d^2 + 2\,a\,b\,c^2\,d + \frac{b^2\,c^3}{3}\right) + x^4\left(\frac{a^2\,d^3}{4} + \frac{3\,a\,b\,c\,d^2}{2} + \frac{3\,b^2\,c^2\,d}{4}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^6}{6} + \frac{a\,c^2\,x^2\,(3\,a\,d + 2\,b\,c)}{2} + \frac{b\,d^2\,x^5\,(2\,a\,d + 3\,b\,c)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b^2*c^2
*d)/4 + (3*a*b*c*d^2)/2) + a^2*c^3*x + (b^2*d^3*x^6)/6 + (a*c^2*x^2*(3*a*d
+ 2*b*c))/2 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

3.1262 $\int (a+bx)(c+dx)^3\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}$$

```
[Out] -1/4*(-a*d+b*c)*(d*x+c)^4/d^2+1/5*b*(d*x+c)^5/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*(c + d*x)^4)/d^2 + (b*(c + d*x)^5)/(5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^3}{d}+\frac{b(c+dx)^4}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 67, normalized size = 1.76

$$ac^3x+\frac{1}{2}c^2(bc+3ad)x^2+cd(bc+ad)x^3+\frac{1}{4}d^2(3bc+ad)x^4+\frac{1}{5}bd^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c + a
*d)*x^4)/4 + (b*d^3*x^5)/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.
time = 0.12, size = 73, normalized size = 1.92

method	result	size
norman	$\frac{b\,d^3x^5}{5}+\left(\frac{1}{4}a\,d^3+\frac{3}{4}bc\,d^2\right)x^4+\left(ac\,d^2+b\,c^2d\right)x^3+\left(\frac{3}{2}a\,c^2d+\frac{1}{2}b\,c^3\right)x^2+a\,c^3x$	70
gosper	$\frac{1}{5}b\,d^3x^5+\frac{1}{4}x^4a\,d^3+\frac{3}{4}x^4bc\,d^2+ac\,d^2x^3+b\,c^2d\,x^3+\frac{3}{2}x^2a\,c^2d+\frac{1}{2}b\,c^3x^2+a\,c^3x$	73
default	$\frac{b\,d^3x^5}{5}+\frac{(a\,d^3+3bc\,d^2)x^4}{4}+\frac{(3ac\,d^2+3b\,c^2d)x^3}{3}+\frac{(3a\,c^2d+b\,c^3)x^2}{2}+a\,c^3x$	73
risch	$\frac{1}{5}b\,d^3x^5+\frac{1}{4}x^4a\,d^3+\frac{3}{4}x^4bc\,d^2+ac\,d^2x^3+b\,c^2d\,x^3+\frac{3}{2}x^2a\,c^2d+\frac{1}{2}b\,c^3x^2+a\,c^3x$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b*d^3*x^5+1/4*(a*d^3+3*b*c*d^2)*x^4+1/3*(3*a*c*d^2+3*b*c^2*d)*x^3+1/2*(
3*a*c^2*d+b*c^3)*x^2+a*c^3*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.28, size = 69, normalized size = 1.82

$$\frac{1}{5}\,bd^3x^5+ac^3x+\frac{1}{4}\left(3\,bcd^2+ad^3\right)x^4+\left(bc^2d+acd^2\right)x^3+\frac{1}{2}\left(bc^3+3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.51, size = 69, normalized size = 1.82

$$\frac{1}{5}\,bd^3x^5+ac^3x+\frac{1}{4}\left(3\,bcd^2+ad^3\right)x^4+\left(bc^2d+acd^2\right)x^3+\frac{1}{2}\left(bc^3+3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.
time = 0.01, size = 73, normalized size = 1.92

$$ac^3x + \frac{bd^3x^5}{5} + x^4\left(\frac{ad^3}{4} + \frac{3bcd^2}{4}\right) + x^3\left(acd^2 + bc^2d\right) + x^2 \cdot \left(\frac{3ac^2d}{2} + \frac{bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
+ b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.
time = 0.93, size = 72, normalized size = 1.89

$$\frac{1}{5}\,bd^3x^5 + \frac{3}{4}\,bcd^2x^4 + \frac{1}{4}\,ad^3x^4 + bc^2dx^3 + acd^2x^3 + \frac{1}{2}\,bc^3x^2 + \frac{3}{2}\,ac^2dx^2 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/5*b*d^3*x^5 + 3/4*b*c*d^2*x^4 + 1/4*a*d^3*x^4 + b*c^2*d*x^3 + a*c*d^2*x^3
 + 1/2*b*c^3*x^2 + 3/2*a*c^2*d*x^2 + a*c^3*x
```

Mupad [B]
time = 0.03, size = 65, normalized size = 1.71

$$x^2\left(\frac{b\,c^3}{2} + \frac{3\,a\,d\,c^2}{2}\right) + x^4\left(\frac{a\,d^3}{4} + \frac{3\,b\,c\,d^2}{4}\right) + \frac{b\,d^3\,x^5}{5} + a\,c^3\,x + c\,d\,x^3\,(a\,d + b\,c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^3,x)
```

```
[Out] x^2*((b*c^3)/2 + (3*a*c^2*d)/2) + x^4*((a*d^3)/4 + (3*b*c*d^2)/4) + (b*d^3*
x^5)/5 + a*c^3*x + c*d*x^3*(a*d + b*c)
```

3.1263 $\int (c+dx)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^4}{4d}$$

```
[Out] 1/4*(d*x+c)^4/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3 \, dx = \frac{(c+dx)^4}{4d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3,x]
```

```
[Out] (c + d*x)^4/(4*d)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(dx+c)^4}{4d}$	13
gosper	$\frac{1}{4}d^3x^4+c\,d^2x^3+\frac{3}{2}c^2d\,x^2+c^3x$	32
norman	$\frac{1}{4}d^3x^4+c\,d^2x^3+\frac{3}{2}c^2d\,x^2+c^3x$	32
risch	$\frac{d^3x^4}{4}+c\,d^2x^3+\frac{3c^2d\,x^2}{2}+c^3x+\frac{c^4}{4d}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(d*x+c)^4/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.28, size = 31, normalized size = 2.21

$$\frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\,c^2dx^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.85, size = 31, normalized size = 2.21

$$\frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\,c^2dx^2+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.
time = 0.01, size = 32, normalized size = 2.29

$$c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3,x)
```

```
[Out] c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

Giac [A]
time = 0.83, size = 12, normalized size = 0.86

$$\frac{(dx+c)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(d*x + c)^4/d
```

Mupad [B]
time = 0.04, size = 31, normalized size = 2.21

$$c^3\,x+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3+\frac{d^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3,x)
```

```
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3
```

3.1264 $\int \frac{(c+dx)^3}{a+bx}\,dx$

Optimal. Leaf size=73

$$\frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}$$

```
[Out] d*(-a*d+b*c)^2*x/b^3+1/2*(-a*d+b*c)*(d*x+c)^2/b^2+1/3*(d*x+c)^3/b+(-a*d+b*c
)^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx\\ &= \frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.01

$$\frac{bdx(6a^2d^2-3abd(6c+dx)+b^2(18c^2+9cdx+2d^2x^2))+6(bc-ad)^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^2
)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

Maple [A]
time = 0.14, size = 109, normalized size = 1.49

method	result
norman	$\frac{d\left(a^{2} d^{2}-3abcd+3b^{2} c^{2}\right)x}{b^{3}}+\frac{d^{3} x^{3}}{3b}-\frac{d^{2}\left(ad-3bc\right)x^{2}}{2b^{2}}-\frac{\left(a^{3} d^{3}-3a^{2} bc\,d^{2}+3a\,b^{2} c^{2} d-b^{3} c^{3}\right)\ln\left(bx+a\right)}{b^{4}}$
default	$\frac{d\left(\frac{1}{3} d^{2} x^{3} b^{2}-\frac{1}{2} ab\,d^{2} x^{2}+\frac{3}{2} b^{2} cd\,x^{2}+a^{2} d^{2} x-3abcdx+3b^{2} c^{2} x\right)}{b^{3}}+\frac{\left(-a^{3} d^{3}+3a^{2} bc\,d^{2}-3a\,b^{2} c^{2} d+b^{3} c^{3}\right)\ln\left(bx+a\right)}{b^{4}}$
risch	$\frac{d^{3} x^{3}}{3b}-\frac{d^{3} a\,x^{2}}{2b^{2}}+\frac{3d^{2} c\,x^{2}}{2b}+\frac{d^{3} a^{2} x}{b^{3}}-\frac{3d^{2} acx}{b^{2}}+\frac{3d\,c^{2} x}{b}-\frac{\ln\left(bx+a\right)a^{3} d^{3}}{b^{4}}+\frac{3\ln\left(bx+a\right)a^{2} c\,d^{2}}{b^{3}}-\frac{3\ln\left(bx+a\right)a\,c^{2} d}{b^{2}}+\frac{\ln(bx+}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/3*d^2*x^3*b^2-1/2*a*b*d^2*x^2+3/2*b^2*c*d*x^2+a^2*d^2*x-3*a*b*c*d*
x+3*b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4*ln(b*x+a)
```

Maxima [A]
time = 0.28, size = 114, normalized size = 1.56

$$\frac{2\,b^{2} d^{3} x^{3}+3\,(3\,b^{2} c d^{2}-a b d^{3}) x^{2}+6\,(3\,b^{2} c^{2} d-3\,a b c d^{2}+a^{2} d^{3}) x}{6\,b^{3}}+\frac{(b^{3} c^{3}-3\,a b^{2} c^{2} d+3\,a^{2} b c d^{2}-a^{3} d^{3}) \log(b x+a)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*b
*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(b*x + a)/b^4
```

Fricas [A]
time = 1.02, size = 116, normalized size = 1.59

$$\frac{2\,b^{3} d^{3} x^{3}+3\,(3\,b^{3} c d^{2}-a b^{2} d^{3}) x^{2}+6\,(3\,b^{3} c^{2} d-3\,a b^{2} c d^{2}+a^{2} b d^{3}) x+6\,(b^{3} c^{3}-3\,a b^{2} c^{2} d+3\,a^{2} b c d^{2}-a^{3} d^{3}) \log(b x+a)}{6\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(b*x + a))/b^4
```

Sympy [A]
time = 0.16, size = 83, normalized size = 1.14

$$x^{2}\left(-\frac{ad^{3}}{2b^{2}}+\frac{3cd^{2}}{2b}\right)+x\left(\frac{a^{2}d^{3}}{b^{3}}-\frac{3acd^{2}}{b^{2}}+\frac{3c^{2}d}{b}\right)+\frac{d^{3}x^{3}}{3b}-\frac{(ad-bc)^{3}\log{(a+bx)}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a),x)
```

```
[Out] x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + d**3*x**3/(3*b) - (a*d - b*c)**3*log(a + b*x)/b**4
```

Giac [A]
time = 0.94, size = 115, normalized size = 1.58

$$\frac{2\,b^{2}d^{3}x^{3}+9\,b^{2}cd^{2}x^{2}-3\,abd^{3}x^{2}+18\,b^{2}c^{2}dx-18\,abcd^{2}x+6\,a^{2}d^{3}x}{6\,b^{3}}+\frac{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(|bx+a|\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18*
a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(b*x + a))/b^4
```

Mupad [B]
time = 0.20, size = 118, normalized size = 1.62

$$x\left(\frac{3\,c^{2}\,d}{b}+\frac{a\,\left(\frac{a\,d^{3}}{b^{2}}-\frac{3\,c\,d^{2}}{b}\right)}{b}\right)-x^{2}\left(\frac{a\,d^{3}}{2\,b^{2}}-\frac{3\,c\,d^{2}}{2\,b}\right)+\frac{d^{3}\,x^{3}}{3\,b}-\frac{\ln\left(a+b\,x\right)\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^2*((a*d^3)/(2*b^2)
- (3*c*d^2)/(2*b)) + (d^3*x^3)/(3*b) - (log(a + b*x)*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/b^4
```

3.1265 $\int \frac{(c+dx)^3}{(a+bx)^2}\,dx$

Optimal. Leaf size=75

$$\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/2*d^3*x^2/b^2-(-a*d+b*c)^3/b^4/(b*x+a)+3*d*(-a*d
+b*c)^2*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^2}\,dx &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx\\ &= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.96

$$\frac{2bd^2(3bc-2ad)x+b^2d^3x^2-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6*
d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

Maple [A]
time = 0.17, size = 109, normalized size = 1.45

method	result
default	$-\frac{d^{2}\left(-\frac{1}{2}bd\,x^{2}+2adx-3bcx\right)}{b^{3}}-\frac{-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}}{b^{4}(bx+a)}+\frac{3d\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln(bx+a)}{b^{4}}$
norman	$\frac{\frac{3a^{3}d^{3}-6a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}{b^{4}}+\frac{d^{3}x^{3}}{2b}-\frac{3d^{2}(ad-2bc)x^{2}}{2b^{2}}}{bx+a}+\frac{3d\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln(bx+a)}{b^{4}}$
risch	$\frac{d^{3}x^{2}}{2b^{2}}-\frac{2d^{3}ax}{b^{3}}+\frac{3d^{2}cx}{b^{2}}+\frac{a^{3}d^{3}}{b^{4}(bx+a)}-\frac{3a^{2}c\,d^{2}}{b^{3}(bx+a)}+\frac{3a\,c^{2}d}{b^{2}(bx+a)}-\frac{c^{3}}{b(bx+a)}+\frac{3d^{3}\ln(bx+a)a^{2}}{b^{4}}-\frac{6d^{2}\ln(bx+a)ac}{b^{3}}+3d\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^3*(-1/2*b*d*x^2+2*a*d*x-3*b*c*x)-1/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b
^2*c^2*d+b^3*c^3)/(b*x+a)+3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(b*x+a)
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.57

$$-\frac{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}{b^{5}x+ab^{4}}+\frac{bd^{3}x^{2}+2\,(3\,bcd^{2}-2\,ad^{3})x}{2\,b^{3}}+\frac{3\,(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3})\log\,(bx+a)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2*
(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(b*x + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(73) = 146$.
time = 1.01, size = 173, normalized size = 2.31

$$\frac{b^{3}d^{3}x^{3}-2\,b^{3}c^{3}+6\,ab^{2}c^{2}d-6\,a^{2}bcd^{2}+2\,a^{3}d^{3}+3\,(2\,b^{3}cd^{2}-ab^{2}d^{3})x^{2}+2\,(3\,ab^{2}cd^{2}-2\,a^{2}bd^{3})x+6\,(ab^{2}c^{2}d-2\,a^{2}bcd^{2}+a^{3}d^{3}+(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3})x)\log\,(bx+a)}{2\,(b^{5}x+ab^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(a*
```

```
b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

Sympy [A]
time = 0.27, size = 102, normalized size = 1.36

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{ab^4+b^5x}+\frac{d^3x^2}{2b^2}+\frac{3d(ad-bc)^2\log{(a+bx)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3)/(a*b**4 + b**5*x) + d**3*x**2/(2*b**2) + 3*d*(a*d - b
*c)**2*log(a + b*x)/b**4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(73) = 146.
time = 1.02, size = 167, normalized size = 2.23

$$\frac{\left(d^3+\frac{6\,(b^2cd^2-abd^3)}{(bx+a)b}\right)(bx+a)^2}{2\,b^4}-\frac{3\,(b^2c^2d-2\,abcd^2+a^2d^3)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4}-\frac{\frac{b^5c^3}{bx+a}-\frac{3\,ab^4c^2d}{bx+a}+\frac{3\,a^2b^3cd^2}{bx+a}-\frac{a^3b^2d^3}{bx+a}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x + a)*b))*(b*x + a)^2/b^4 - 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4 -
 (b^5*c^3/(b*x + a) - 3*a*b^4*c^2*d/(b*x + a) + 3*a^2*b^3*c*d^2/(b*x + a) -
 a^3*b^2*d^3/(b*x + a))/b^6
```

Mupad [B]
time = 0.21, size = 123, normalized size = 1.64

$$\frac{\ln\left(a+b\,x\right)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{b^4}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)+\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{b\,\left(x\,b^4+a\,b^3\right)}+\frac{d^3\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^2,x)
```

```
[Out] (log(a + b*x)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/b^4 - x*((2*a*d^3)/b
^3 - (3*c*d^2)/b^2) + (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
b*(a*b^3 + b^4*x)) + (d^3*x^2)/(2*b^2)
```

3.1266 $\int \frac{(c+dx)^3}{(a+bx)^3} \, dx$

Optimal. Leaf size=78

$$\frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}$$

```
[Out] d^3*x/b^3-1/2*(-a*d+b*c)^3/b^4/(b*x+a)^2-3*d*(-a*d+b*c)^2/b^4/(b*x+a)+3*d^2
*(-a*d+b*c)*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4}-\frac{3d(bc-ad)^2}{b^4(a+bx)}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}+\frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^3}\,dx&=\int \left(\frac{d^3}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)^3}+\frac{3d(bc-ad)^2}{b^3(a+bx)^2}+\frac{3d^2(bc-ad)}{b^3(a+bx)}\right)\,dx\\&=\frac{d^3x}{b^3}-\frac{(bc-ad)^3}{2b^4(a+bx)^2}-\frac{3d(bc-ad)^2}{b^4(a+bx)}+\frac{3d^2(bc-ad)\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.46

$$\frac{-5a^3d^3+a^2bd^2(9c-4dx)+ab^2d(-3c^2+12cdx+4d^2x^2)-b^3(c^3+6c^2dx-2d^3x^3)-6d^2(-bc+ad)(a+bx)^2\log(a+bx)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2*
x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

Maple [A]
time = 0.14, size = 114, normalized size = 1.46

method	result	size
default	$\frac{d^{3} x}{b^{3}}-\frac{3 d \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}{b^{4} \left(b x+a\right)}-\frac{-a^{3} d^{3}+3 a^{2} b c \,d^{2}-3 a \,b^{2} c^{2} d+b^{3} c^{3}}{2 b^{4} \left(b x+a\right)^{2}}-\frac{3 d^{2} \left(a d-b c\right) \ln \left(b x+a\right)}{b^{4}}$	114
norman	$\frac{\frac{d^{3} x^{3}}{b}-\frac{9 a^{3} d^{3}-9 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d+b^{3} c^{3}}{2 b^{4}}-\frac{\left(6 a^{2} d^{3}-6 a b c \,d^{2}+3 b^{2} c^{2} d\right) x}{b^{3}}}{\left(b x+a\right)^{2}}-\frac{3 d^{2} \left(a d-b c\right) \ln \left(b x+a\right)}{b^{4}}$	116
risch	$\frac{d^{3} x}{b^{3}}+\frac{\left(-3 a^{2} d^{3}+6 a b c \,d^{2}-3 b^{2} c^{2} d\right) x-\frac{5 a^{3} d^{3}-9 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d+b^{3} c^{3}}{2 b}}{b^{3} \left(b x+a\right)^{2}}-\frac{3 d^{3} \ln \left(b x+a\right) a}{b^{4}}+\frac{3 d^{2} \ln \left(b x+a\right) c}{b^{3}}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*x/b^3-3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)-1/2/b^4*(-a^3*d^3+3*a
^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(b*x+a)^2-3/b^4*d^2*(a*d-b*c)*ln(b*x+a)
```

Maxima [A]
time = 0.30, size = 125, normalized size = 1.60

$$\frac{d^3 x}{b^3} - \frac{b^3 c^3 + 3\,a b^2 c^2 d - 9\,a^2 b c d^2 + 5\,a^3 d^3 + 6\,(b^3 c^2 d - 2\,a b^2 c d^2 + a^2 b d^3) x}{2\,(b^6 x^2 + 2\,a b^5 x + a^2 b^4)} + \frac{3\,(b c d^2 - a d^3) \log(b x + a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(76) = 152.
time = 0.91, size = 188, normalized size = 2.41

$$\frac{2\,b^3 d^3 x^3 + 4\,a b^2 d^3 x^2 - b^3 c^3 - 3\,a b^2 c^2 d + 9\,a^2 b c d^2 - 5\,a^3 d^3 - 2\,(3\,b^3 c^2 d - 6\,a b^2 c d^2 + 2\,a^2 b d^3) x + 6\,(a^2 b c d^2 - a^3 d^3 + (b^3 c d^2 - a b^2 d^3) x^2 + 2\,(a b^2 c d^2 - a^2 b d^3) x) \log(b x + a)}{2\,(b^6 x^2 + 2\,a b^5 x + a^2 b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^2*
b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b*d^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [A]
time = 0.47, size = 128, normalized size = 1.64

$$\frac{-5a^3d^3+9a^2bcd^2-3ab^2c^2d-b^3c^3+x(-6a^2bd^3+12ab^2cd^2-6b^3c^2d)}{2a^2b^4+4ab^5x+2b^6x^2}+\frac{d^3x}{b^3}-\frac{3d^2(ad-bc)\log{(a+bx)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**3,x)
```

```
[Out] (-5*a**3*d**3 + 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - b**3*c**3 + x*(-6*a**2*
b*d**3 + 12*a*b**2*c*d**2 - 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

Giac [A]
time = 0.72, size = 112, normalized size = 1.44

$$\frac{d^3x}{b^3}+\frac{3\,(bcd^2-ad^3)\log{(|bx+a|)}}{b^4}-\frac{b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a*
b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((b*x + a)^2*b^4)
```

Mupad [B]
time = 0.82, size = 130, normalized size = 1.67

$$\frac{d^3\,x}{b^3}-\frac{\ln\left(a+b\,x\right)\,\left(3\,a\,d^3-3\,b\,c\,d^2\right)}{b^4}-\frac{\frac{5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,b}+x\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^3,x)
```

```
[Out] (d^3*x)/b^3 - (log(a + b*x)*(3*a*d^3 - 3*b*c*d^2))/b^4 - ((5*a^3*d^3 + b^3*
c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2)/(2*b) + x*(3*a^2*d^3 + 3*b^2*c^2*d - 6
*a*b*c*d^2))/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x)
```

3.1267 $\int \frac{(c+dx)^3}{(a+bx)^4} \, dx$

Optimal. Leaf size=86

$$-\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}$$

```
[Out] -1/3*(-a*d+b*c)^3/b^4/(b*x+a)^3-3/2*d*(-a*d+b*c)^2/b^4/(b*x+a)^2-3*d^2*(-a*
d+b*c)/b^4/(b*x+a)+d^3*ln(b*x+a)/b^4
```

Rubi [A]
time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] -1/3*(b*c - a*d)^3/(b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)
^2) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^4} \, dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{d^3}{b^3(a+bx)}\right) dx\\ &= -\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.93

$$\frac{-\frac{(bc-ad)\left(11a^2d^2+abd(5c+27dx)+b^2\left(2c^2+9cdx+18d^2x^2\right)\right)}{(a+bx)^3}+6d^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] (-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x +
 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)
```

Maple [A]
time = 0.14, size = 120, normalized size = 1.40

method	result	size
risch	$\frac{\frac{3d^2(ad-bc)x^2}{b^2}+\frac{3d\left(3a^2d^2-2abcd-b^2c^2\right)x}{2b^3}+\frac{11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3}{6b^4}}{(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}$	115
norman	$\frac{\frac{11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3}{6b^4}+\frac{3\left(a\,d^3-bc\,d^2\right)x^2}{b^2}+\frac{3\left(3a^2d^3-2abc\,d^2-b^2c^2d\right)x}{2b^3}}{(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}$	119
default	$\frac{3d^2(ad-bc)}{b^4(bx+a)}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{2b^4(bx+a)^2}+\frac{d^3\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{3b^4(bx+a)^3}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/b^4*d^2*(a*d-b*c)/(b*x+a)-3/2/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2
+d^3*ln(b*x+a)/b^4-1/3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(
b*x+a)^3
```

Maxima [A]
time = 0.29, size = 142, normalized size = 1.65

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\,(b^3cd^2-ab^2d^3)x^2+9\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}+\frac{d^3\log(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.
time = 0.74, size = 176, normalized size = 2.05

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\,(b^3cd^2-ab^2d^3)x^2+9\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x-6\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\log(bx+a)}{6\,(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

Sympy [A]
time = 0.65, size = 148, normalized size = 1.72

$$\frac{11a^3d^3-6a^2bcd^2-3ab^2c^2d-2b^3c^3+x^2\cdot(18ab^2d^3-18b^3cd^2)+x(27a^2bd^3-18ab^2cd^2-9b^3c^2d)}{6a^3b^4+18a^2b^5x+18ab^6x^2+6b^7x^3}+\frac{d^3\log(a+bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18*
a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*b*
*3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 d**3*log(a + b*x)/b**4
```

Giac [A]
time = 0.58, size = 118, normalized size = 1.37

$$\frac{d^3\log(|bx+a|)}{b^4}-\frac{18(b^2cd^2-abd^3)x^2+9(b^2c^2d+2abcd^2-3a^2d^3)x+\frac{2b^3c^3+3ab^2c^2d+6a^2bcd^2-11a^3d^3}{b}}{6(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2*
d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2
 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

Mupad [B]
time = 0.25, size = 138, normalized size = 1.60

$$\frac{d^3\ln(a+bx)}{b^4}-\frac{\frac{-11a^3d^3+6a^2bcd^2+3ab^2c^2d+2b^3c^3}{6b^4}+\frac{3x(-3a^2d^3+2abcd^2+b^2c^2d)}{2b^3}-\frac{3d^2x^2(ad-bc)}{b^2}}{a^3+3a^2bx+3ab^2x^2+b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^4,x)
```

```
[Out] (d^3*log(a + b*x))/b^4 - ((2*b^3*c^3 - 11*a^3*d^3 + 3*a*b^2*c^2*d + 6*a^2*b
*c*d^2)/(6*b^4) + (3*x*(b^2*c^2*d - 3*a^2*d^3 + 2*a*b*c*d^2))/(2*b^3) - (3*
d^2*x^2*(a*d - b*c))/b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

3.1268 $\int \frac{(c+dx)^3}{(a+bx)^5}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

```
[Out] -1/4*(d*x+c)^4/(-a*d+b*c)/(b*x+a)^4
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {37}

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -1/4*(c + d*x)^4/((b*c - a*d)*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^5}\,dx = -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 91 vs. 2(28) = 56.
time = 0.02, size = 91, normalized size = 3.25

$$-\frac{a^3d^3+a^2bd^2(c+4dx)+ab^2d(c^2+4cdx+6d^2x^2)+b^3(c^3+4c^2dx+6cd^2x^2+4d^3x^3)}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -1/4*(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2)
 + b^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(b^4*(a + b*x)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(26) = 52$.
time = 0.14, size = 122, normalized size = 4.36

method	result	size
risch	$\frac{-\frac{d^3x^3}{b}-\frac{3d^2(ad+bc)x^2}{2b^2}-\frac{d\left(a^2d^2+abcd+b^2c^2\right)x}{b^3}-\frac{a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3}{4b^4}}{(bx+a)^4}$	104
gosper	$-\frac{4d^3x^3b^3+6a\,b^2d^3x^2+6b^3c\,d^2x^2+4a^2b\,d^3x+4a\,b^2c\,d^2x+4b^3c^2dx+a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3}{4(bx+a)^4b^4}$	112
norman	$\frac{-\frac{d^3x^3}{b}+\frac{3\left(-a\,d^3-bc\,d^2\right)x^2}{2b^2}+\frac{\left(-a^2d^3-abc\,d^2-b^2c^2d\right)x}{b^3}+\frac{-a^3d^3-a^2bc\,d^2-a\,b^2c^2d-b^3c^3}{4b^4}}{(bx+a)^4}$	116
default	$-\frac{d^3}{b^4(bx+a)}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}+\frac{3d^2(ad-bc)}{2b^4(bx+a)^2}-\frac{d(a^2d^2-2abcd+b^2c^2)}{b^4(bx+a)^3}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3/b^4/(b*x+a)-1/4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*
x+a)^4+3/2/b^4*d^2*(a*d-b*c)/(b*x+a)^2-1/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(26) = 52$.
time = 0.34, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+ab^2cd^2+a^2bd^3)x}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(26) = 52$.
time = 0.53, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+ab^2cd^2+a^2bd^3)x}{4\,(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(22) = 44$.
time = 0.85, size = 155, normalized size = 5.54

$$\frac{-a^3d^3 - a^2bcd^2 - ab^2c^2d - b^3c^3 - 4b^3d^3x^3 + x^2(-6ab^2d^3 - 6b^3cd^2) + x(-4a^2bd^3 - 4ab^2cd^2 - 4b^3c^2d)}{4a^4b^4 + 16a^3b^5x + 24a^2b^6x^2 + 16ab^7x^3 + 4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**5,x)
```

```
[Out] (-a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d - b**3*c**3 - 4*b**3*d**3*x**3
+ x**2*(-6*a*b**2*d**3 - 6*b**3*c*d**2) + x*(-4*a**2*b*d**3 - 4*a*b**2*c*d*
*2 - 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 + 16
*a*b**7*x**3 + 4*b**8*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(26) = 52$.
time = 0.57, size = 159, normalized size = 5.68

$$-\frac{\frac{b^2c^3}{(bx+a)^4} + \frac{4\,bc^2d}{(bx+a)^3} - \frac{3\,abc^2d}{(bx+a)^4} + \frac{6\,cd^2}{(bx+a)^2} - \frac{8\,acd^2}{(bx+a)^3} + \frac{3\,a^2cd^2}{(bx+a)^4} + \frac{4\,d^3}{(bx+a)b} - \frac{6\,ad^3}{(bx+a)^2b} + \frac{4\,a^2d^3}{(bx+a)^3b} - \frac{a^3d^3}{(bx+a)^4b}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c^3/(b*x + a)^4 + 4*b*c^2*d/(b*x + a)^3 - 3*a*b*c^2*d/(b*x + a)^4
 + 6*c*d^2/(b*x + a)^2 - 8*a*c*d^2/(b*x + a)^3 + 3*a^2*c*d^2/(b*x + a)^4 +
4*d^3/((b*x + a)*b) - 6*a*d^3/((b*x + a)^2*b) + 4*a^2*d^3/((b*x + a)^3*b) -
 a^3*d^3/((b*x + a)^4*b))/b^3
```

Mupad [B]
time = 0.07, size = 135, normalized size = 4.82

$$-\frac{\frac{a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3}{4\,b^4}+\frac{d^3\,x^3}{b}+\frac{d\,x\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+b\,c\right)}{2\,b^2}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^5,x)
```

```
[Out] -((a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)/(4*b^4) + (d^3*x^3)/b + (
d*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/b^3 + (3*d^2*x^2*(a*d + b*c))/(2*b^2))/(
a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)
```

3.1269 $\int \frac{(c+dx)^3}{(a+bx)^6}\,dx$

Optimal. Leaf size=58

$$-\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5}+\frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

```
[Out] -1/5*(d*x+c)^4/(-a*d+b*c)/(b*x+a)^5+1/20*d*(d*x+c)^4/(-a*d+b*c)^2/(b*x+a)^4
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2}-\frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -1/5*(c + d*x)^4/((b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d
)^2*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^6} dx = -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} - \frac{d \int \frac{(c+dx)^3}{(a+bx)^5} dx}{5(bc-ad)}$$
$$= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5} + \frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.67

$$-\frac{a^3d^3 + a^2bd^2(2c+5dx) + ab^2d(3c^2+10cdx+10d^2x^2) + b^3(4c^3+15c^2dx+20cd^2x^2+10d^3x^3)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -1/20*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*d
^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(b^4*(a + b
*x)^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(54) = 108$.
time = 0.14, size = 121, normalized size = 2.09

method	result	size
risch	$\frac{-\frac{d^3x^3}{2b}-\frac{d^2(ad+2bc)x^2}{2b^2}-\frac{d\left(a^2d^2+2abcd+3b^2c^2\right)x}{4b^3}-\frac{a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3}{20b^4}}{(bx+a)^5}$	110
gosper	$-\frac{10d^3x^3b^3+10a\,b^2d^3x^2+20b^3c\,d^2x^2+5a^2b\,d^3x+10a\,b^2c\,d^2x+15b^3c^2dx+a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3}{20b^4(bx+a)^5}$	115
default	$-\frac{3d(a^2d^2-2abcd+b^2c^2)}{4b^4(bx+a)^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}-\frac{d^3}{2b^4(bx+a)^2}+\frac{d^2(ad-bc)}{b^4(bx+a)^3}$	121
norman	$\frac{-\frac{d^3x^3}{2b}+\frac{\left(-ab\,d^3-2b^2c\,d^2\right)x^2}{2b^3}+\frac{\left(-a^2b\,d^3-2a\,b^2c\,d^2-3b^3c^2d\right)x}{4b^4}+\frac{-a^3b\,d^3-2b^2a^2c\,d^2-3a\,b^3c^2d-4b^4c^3}{20b^5}}{(bx+a)^5}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4-1/5*(-a^3*d^3+3*a^2*b*c*d^
2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^5-1/2*d^3/b^4/(b*x+a)^2+1/b^4*d^2*(a*d
-b*c)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.
time = 0.30, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3 + 4\,b^3c^3 + 3\,ab^2c^2d + 2\,a^2bcd^2 + a^3d^3 + 10\,(2\,b^3cd^2 + ab^2d^3)x^2 + 5\,(3\,b^3c^2d + 2\,ab^2cd^2 + a^2bd^3)x}{20\,(b^9x^5 + 5\,ab^8x^4 + 10\,a^2b^7x^3 + 10\,a^3b^6x^2 + 5\,a^4b^5x + a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.
time = 0.54, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\,(2\,b^3cd^2+ab^2d^3)x^2+5\,(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3)x}{20\,(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(46) = 92$.
time = 1.13, size = 172, normalized size = 2.97

$$\frac{-a^3d^3-2a^2bcd^2-3ab^2c^2d-4b^3c^3-10b^3d^3x^3+x^2(-10ab^2d^3-20b^3cd^2)+x(-5a^2bd^3-10ab^2cd^2-15b^3c^2d)}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**6,x)
```

```
[Out] (-a**3*d**3 - 2*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 4*b**3*c**3 - 10*b**3*d**
3*x**3 + x**2*(-10*a*b**2*d**3 - 20*b**3*c*d**2) + x*(-5*a**2*b*d**3 - 10*a
*b**2*c*d**2 - 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*
b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(54) = 108$.
time = 1.11, size = 114, normalized size = 1.97

$$-\frac{10\,b^3d^3x^3+20\,b^3cd^2x^2+10\,ab^2d^3x^2+15\,b^3c^2dx+10\,ab^2cd^2x+5\,a^2bd^3x+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3}{20\,(bx+a)^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d*
x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*
c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

Mupad [B]
time = 0.08, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^4\,(5\,a\,d-4\,b\,c+b\,d\,x)}{20\,(a\,d-b\,c)^2\,(a+b\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^6,x)
[Out] ((c + d*x)^4*(5*a*d - 4*b*c + b*d*x))/(20*(a*d - b*c)^2*(a + b*x)^5)
```

3.1270 $\int \frac{(c+dx)^3}{(a+bx)^7}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}$$

```
[Out] -1/6*(-a*d+b*c)^3/b^4/(b*x+a)^6-3/5*d*(-a*d+b*c)^2/b^4/(b*x+a)^5-3/4*d^2*(-
a*d+b*c)/b^4/(b*x+a)^4-1/3*d^3/b^4/(b*x+a)^3
```

Rubi [A]

time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -1/6*(b*c - a*d)^3/(b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)
^5) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{d^3}{b^3(a+bx)^4}\right)dx\\ &= -\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+3a^2bd^2(c+2dx)+3ab^2d(2c^2+6cdx+5d^2x^2)+b^3(10c^3+36c^2dx+45cd^2x^2+20d^3x^3)}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*d
^2*x^2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(b^4*(a +
b*x)^6)
```

Maple [A]
time = 0.14, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{d^{3}x^{3}}{3b}-\frac{d^{2}(ad+3bc)x^{2}}{4b^{2}}-\frac{d\left(a^{2}d^{2}+3abcd+6b^{2}c^{2}\right)x}{10b^{3}}-\frac{a^{3}d^{3}+3a^{2}bc\,d^{2}+6a\,b^{2}c^{2}d+10b^{3}c^{3}}{60b^{4}}}{(bx+a)^{6}}$	110
gosper	$-\frac{20d^{3}x^{3}b^{3}+15a\,b^{2}d^{3}x^{2}+45b^{3}c\,d^{2}x^{2}+6a^{2}b\,d^{3}x+18a\,b^{2}c\,d^{2}x+36b^{3}c^{2}dx+a^{3}d^{3}+3a^{2}bc\,d^{2}+6a\,b^{2}c^{2}d+10b^{3}c^{3}}{60b^{4}(bx+a)^{6}}$	115
default	$\frac{3d^{2}(ad-bc)}{4b^{4}(bx+a)^{4}}-\frac{3d\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{5b^{4}(bx+a)^{5}}-\frac{-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}}{6b^{4}(bx+a)^{6}}-\frac{d^{3}}{3b^{4}(bx+a)^{3}}$	122
norman	$\frac{-\frac{d^{3}x^{3}}{3b}+\frac{\left(-a\,b^{2}d^{3}-3b^{3}c\,d^{2}\right)x^{2}}{4b^{4}}+\frac{\left(-b^{2}a^{2}d^{3}-3a\,b^{3}c\,d^{2}-6b^{4}c^{2}d\right)x}{10b^{5}}+\frac{-a^{3}b^{2}d^{3}-3a^{2}b^{3}c\,d^{2}-6a\,b^{4}c^{2}d-10b^{5}c^{3}}{60b^{6}}}{(bx+a)^{6}}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4/b^4*d^2*(a*d-b*c)/(b*x+a)^4-3/5/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+
a)^5-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6-1/3*d
^3/b^4/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.
time = 0.28, size = 171, normalized size = 1.86

$$-\frac{20\,b^{3}d^{3}x^{3}+10\,b^{3}c^{3}+6\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}+15\,(3\,b^{3}cd^{2}+ab^{2}d^{3})x^{2}+6\,(6\,b^{3}c^{2}d+3\,ab^{2}cd^{2}+a^{2}bd^{3})x}{60\,(b^{10}x^{6}+6\,ab^{9}x^{5}+15\,a^{2}b^{8}x^{4}+20\,a^{3}b^{7}x^{3}+15\,a^{4}b^{6}x^{2}+6\,a^{5}b^{5}x+a^{6}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.
time = 0.61, size = 171, normalized size = 1.86

$$-\frac{20\,b^{3}d^{3}x^{3}+10\,b^{3}c^{3}+6\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}+a^{3}d^{3}+15\,(3\,b^{3}cd^{2}+ab^{2}d^{3})x^{2}+6\,(6\,b^{3}c^{2}d+3\,ab^{2}cd^{2}+a^{2}bd^{3})x}{60\,(b^{10}x^{6}+6\,ab^{9}x^{5}+15\,a^{2}b^{8}x^{4}+20\,a^{3}b^{7}x^{3}+15\,a^{4}b^{6}x^{2}+6\,a^{5}b^{5}x+a^{6}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(83) = 166$.
time = 1.57, size = 184, normalized size = 2.00

$$\frac{-a^3d^3 - 3a^2bcd^2 - 6ab^2c^2d - 10b^3c^3 - 20b^3d^3x^3 + x^2(-15ab^2d^3 - 45b^3cd^2) + x(-6a^2bd^3 - 18ab^2cd^2 - 36b^3c^2d)}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**7,x)
```

```
[Out] (-a**3*d**3 - 3*a**2*b*c*d**2 - 6*a*b**2*c**2*d - 10*b**3*c**3 - 20*b**3*d*
*3*x**3 + x**2*(-15*a*b**2*d**3 - 45*b**3*c*d**2) + x*(-6*a**2*b*d**3 - 18*
a*b**2*c*d**2 - 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4
*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 6
0*b**10*x**6)
```

Giac [A]
time = 1.01, size = 114, normalized size = 1.24

$$-\frac{20\,b^3d^3x^3 + 45\,b^3cd^2x^2 + 15\,ab^2d^3x^2 + 36\,b^3c^2dx + 18\,ab^2cd^2x + 6\,a^2bd^3x + 10\,b^3c^3 + 6\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3}{60\,(bx+a)^6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d*
x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b
*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

Mupad [B]
time = 0.22, size = 165, normalized size = 1.79

$$-\frac{\frac{a^3\,d^3+3\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d+10\,b^3\,c^3}{60\,b^4}+\frac{d^3\,x^3}{3\,b}+\frac{d\,x\,\left(a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2\right)}{10\,b^3}+\frac{d^2\,x^2\,\left(a\,d+3\,b\,c\right)}{4\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^7,x)
```

```
[Out] -((a^3*d^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(60*b^4) + (d^3*x^
3)/(3*b) + (d*x*(a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d))/(10*b^3) + (d^2*x^2*(a*d
 + 3*b*c))/(4*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3*
b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

3.1271 $\int \frac{(c+dx)^3}{(a+bx)^8} dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}$$

```
[Out] -1/7*(-a*d+b*c)^3/b^4/(b*x+a)^7-1/2*d*(-a*d+b*c)^2/b^4/(b*x+a)^6-3/5*d^2*(-
a*d+b*c)/b^4/(b*x+a)^5-1/4*d^3/b^4/(b*x+a)^4
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -1/7*(b*c - a*d)^3/(b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6
) - (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(4c+7dx)+ab^2d(10c^2+28cdx+21d^2x^2)+b^3(20c^3+70c^2dx+84cd^2x^2+35d^3x^3)}{140b^4(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -1/140*(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 21
*d^2*x^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(b^4*(a
+ b*x)^7)
```

Maple [A]
time = 0.16, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{d^{3} x^{3}}{4 b}-\frac{3 d^{2} \left(a d+4 b c\right) x^{2}}{20 b^{2}}-\frac{d \left(a^{2} d^{2}+4 a b c d+10 b^{2} c^{2}\right) x}{20 b^{3}}-\frac{a^{3} d^{3}+4 a^{2} b c \,d^{2}+10 a \,b^{2} c^{2} d+20 b^{3} c^{3}}{140 b^{4}}}{\left(b x+a\right)^{7}}$	110
gosper	$-\frac{35 d^{3} x^{3} b^{3}+21 a \,b^{2} d^{3} x^{2}+84 b^{3} c \,d^{2} x^{2}+7 a^{2} b \,d^{3} x+28 a \,b^{2} c \,d^{2} x+70 b^{3} c^{2} d x+a^{3} d^{3}+4 a^{2} b c \,d^{2}+10 a \,b^{2} c^{2} d+20 b^{3} c^{3}}{140 b^{4} \left(b x+a\right)^{7}}$	115
default	$-\frac{d^{3}}{4 b^{4} \left(b x+a\right)^{4}}+\frac{3 d^{2} \left(a d-b c\right)}{5 b^{4} \left(b x+a\right)^{5}}-\frac{-a^{3} d^{3}+3 a^{2} b c \,d^{2}-3 a \,b^{2} c^{2} d+b^{3} c^{3}}{7 b^{4} \left(b x+a\right)^{7}}-\frac{d \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}{2 b^{4} \left(b x+a\right)^{6}}$	122
norman	$\frac{-\frac{d^{3} x^{3}}{4 b}+\frac{3 \left(-a \,b^{3} d^{3}-4 b^{4} c \,d^{2}\right) x^{2}}{20 b^{5}}+\frac{\left(-a^{2} b^{3} d^{3}-4 a \,b^{4} c \,d^{2}-10 b^{5} c^{2} d \right) x}{20 b^{6}}+\frac{-a^{3} b^{3} d^{3}-4 a^{2} b^{4} c \,d^{2}-10 a \,b^{5} c^{2} d-20 c^{3} b^{6}}{140 b^{7}}}{\left(b x+a\right)^{7}}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^3/b^4/(b*x+a)^4+3/5/b^4*d^2*(a*d-b*c)/(b*x+a)^5-1/7*(-a^3*d^3+3*a^2*
b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^7-1/2/b^4*d*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(b*x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.
time = 0.28, size = 182, normalized size = 1.98

$$-\frac{35 \, b^{3} d^{3} x^{3} + 20 \, b^{3} c^{3} + 10 \, a b^{2} c^{2} d + 4 \, a^{2} b c d^{2} + a^{3} d^{3} + 21 \, {\left(4 \, b^{3} c d^{2} + a b^{2} d^{3}\right)} x^{2} + 7 \, {\left(10 \, b^{3} c^{2} d + 4 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x}{140 \, {\left(b^{11} x^{7} + 7 \, a b^{10} x^{6} + 21 \, a^{2} b^{9} x^{5} + 35 \, a^{3} b^{8} x^{4} + 35 \, a^{4} b^{7} x^{3} + 21 \, a^{5} b^{6} x^{2} + 7 \, a^{6} b^{5} x + a^{7} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.
time = 0.49, size = 182, normalized size = 1.98

$$-\frac{35 \, b^{3} d^{3} x^{3} + 20 \, b^{3} c^{3} + 10 \, a b^{2} c^{2} d + 4 \, a^{2} b c d^{2} + a^{3} d^{3} + 21 \, {\left(4 \, b^{3} c d^{2} + a b^{2} d^{3}\right)} x^{2} + 7 \, {\left(10 \, b^{3} c^{2} d + 4 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x}{140 \, {\left(b^{11} x^{7} + 7 \, a b^{10} x^{6} + 21 \, a^{2} b^{9} x^{5} + 35 \, a^{3} b^{8} x^{4} + 35 \, a^{4} b^{7} x^{3} + 21 \, a^{5} b^{6} x^{2} + 7 \, a^{6} b^{5} x + a^{7} b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(82) = 164$.

time = 1.97, size = 196, normalized size = 2.13

$$\frac{-a^3d^3 - 4a^2bcd^2 - 10ab^2c^2d - 20b^3c^3 - 35b^3d^3x^3 + x^2(-21ab^2d^3 - 84b^3cd^2) + x(-7a^2bd^3 - 28ab^2cd^2 - 70b^3c^2d)}{140a^7b^4 + 980a^6b^5x + 2940a^5b^6x^2 + 4900a^4b^7x^3 + 4900a^3b^8x^4 + 2940a^2b^9x^5 + 980ab^{10}x^6 + 140b^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**8,x)
```

```
[Out] (-a**3*d**3 - 4*a**2*b*c*d**2 - 10*a*b**2*c**2*d - 20*b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-21*a*b**2*d**3 - 84*b**3*c*d**2) + x*(-7*a**2*b*d**3 - 28
*a*b**2*c*d**2 - 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 2940*a
**5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b**9*
x**5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

Giac [A]

time = 1.29, size = 114, normalized size = 1.24

$$-\frac{35\,b^3d^3x^3 + 84\,b^3cd^2x^2 + 21\,ab^2d^3x^2 + 70\,b^3c^2dx + 28\,ab^2cd^2x + 7\,a^2bd^3x + 20\,b^3c^3 + 10\,ab^2c^2d + 4\,a^2bcd^2 + a^3d^3}{140\,(bx+a)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*d
*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

Mupad [B]

time = 0.11, size = 176, normalized size = 1.91

$$-\frac{\frac{a^3\,d^3+4\,a^2\,b\,c\,d^2+10\,a\,b^2\,c^2\,d+20\,b^3\,c^3}{140\,b^4}+\frac{d^3\,x^3}{4\,b}+\frac{d\,x\,\left(a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2\right)}{20\,b^3}+\frac{3\,d^2\,x^2\,\left(a\,d+4\,b\,c\right)}{20\,b^2}}{a^7+7\,a^6\,b\,x+21\,a^5\,b^2\,x^2+35\,a^4\,b^3\,x^3+35\,a^3\,b^4\,x^4+21\,a^2\,b^5\,x^5+7\,a\,b^6\,x^6+b^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^8,x)
```

```
[Out] -((a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2)/(140*b^4) + (d^3*
x^3)/(4*b) + (d*x*(a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d))/(20*b^3) + (3*d^2*x^2
*(a*d + 4*b*c))/(20*b^2))/(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5*b^2*x^2 + 3
5*a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x)
```

3.1272 $\int \frac{(c+dx)^3}{(a+bx)^9}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}$$

```
[Out] -1/8*(-a*d+b*c)^3/b^4/(b*x+a)^8-3/7*d*(-a*d+b*c)^2/b^4/(b*x+a)^7-1/2*d^2*(-a*d+b*c)/b^4/(b*x+a)^6-1/5*d^3/b^4/(b*x+a)^5
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -1/8*(b*c - a*d)^3/(b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^9}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^6}\right)dx\\ &= -\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(5c+8dx)+ab^2d(15c^2+40cdx+28d^2x^2)+b^3(35c^3+120c^2dx+140cd^2x^2+56d^3x^3)}{280b^4(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -1/280*(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 28
*d^2*x^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(b^4*(
a + b*x)^8)
```

Maple [A]
time = 0.14, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{d^3x^3}{5b}-\frac{d^2(ad+5bc)x^2}{10b^2}-\frac{d\left(a^2d^2+5abcd+15b^2c^2\right)x}{35b^3}-\frac{a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3}{280b^4}}{(bx+a)^8}$	110
gosper	$-\frac{56d^3x^3b^3+28a\,b^2d^3x^2+140b^3c\,d^2x^2+8a^2b\,d^3x+40a\,b^2c\,d^2x+120b^3c^2dx+a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3}{280b^4(bx+a)^8}$	115
default	$-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{8b^4(bx+a)^8}-\frac{d^3}{5b^4(bx+a)^5}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{7b^4(bx+a)^7}+\frac{d^2(ad-bc)}{2b^4(bx+a)^6}$	122
norman	$\frac{-\frac{d^3x^3}{5b}+\frac{\left(-a\,b^4d^3-5b^5c\,d^2\right)x^2}{10b^6}+\frac{\left(-a^2b^4d^3-5a\,b^5c\,d^2-15c^2d\,b^6\right)x}{35b^7}+\frac{-a^3b^4d^3-5a^2b^5c\,d^2-15a\,b^6c^2d-35c^3b^7}{280b^8}}{(bx+a)^8}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^8-1/5*d^3/b
^4/(b*x+a)^5-3/7/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^7+1/2/b^4*d^2*(a
*d-b*c)/(b*x+a)^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.
time = 0.29, size = 193, normalized size = 2.10

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\,(5\,b^3cd^2+ab^2d^3)x^2+8\,(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3)x}{280\,(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(84) = 168$.
time = 0.50, size = 193, normalized size = 2.10

$$-\frac{56\,b^3d^3x^3+35\,b^3c^3+15\,ab^2c^2d+5\,a^2bcd^2+a^3d^3+28\,(5\,b^3cd^2+ab^2d^3)x^2+8\,(15\,b^3c^2d+5\,ab^2cd^2+a^2bd^3)x}{280\,(b^{12}x^8+8\,ab^{11}x^7+28\,a^2b^{10}x^6+56\,a^3b^9x^5+70\,a^4b^8x^4+56\,a^5b^7x^3+28\,a^6b^6x^2+8\,a^7b^5x+a^8b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(82) = 164$.
time = 3.05, size = 207, normalized size = 2.25

$$\frac{-a^3d^3 - 5a^2bcd^2 - 15ab^2c^2d - 35b^3c^3 - 56b^3d^3x^3 + x^2(-28ab^2d^3 - 140b^3cd^2) + x(-8a^2bd^3 - 40ab^2cd^2 - 120b^3c^2d)}{280a^8b^4 + 2240a^7b^5x + 7840a^6b^6x^2 + 15680a^5b^7x^3 + 19600a^4b^8x^4 + 15680a^3b^9x^5 + 7840a^2b^{10}x^6 + 2240ab^{11}x^7 + 280b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**9,x)
```

```
[Out] (-a**3*d**3 - 5*a**2*b*c*d**2 - 15*a*b**2*c**2*d - 35*b**3*c**3 - 56*b**3*d
**3*x**3 + x**2*(-28*a*b**2*d**3 - 140*b**3*c*d**2) + x*(-8*a**2*b*d**3 - 4
0*a*b**2*c*d**2 - 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x + 784
0*a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*a**3
*b**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8)
```

Giac [A]
time = 0.87, size = 114, normalized size = 1.24

$$-\frac{56\,b^3d^3x^3 + 140\,b^3cd^2x^2 + 28\,ab^2d^3x^2 + 120\,b^3c^2dx + 40\,ab^2cd^2x + 8\,a^2bd^3x + 35\,b^3c^3 + 15\,ab^2c^2d + 5\,a^2bcd^2 + a^3d^3}{280\,(bx+a)^8b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^2
*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

Mupad [B]
time = 0.23, size = 187, normalized size = 2.03

$$-\frac{\frac{a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3}{280\,b^4}+\frac{d^3\,x^3}{5\,b}+\frac{d\,x\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{35\,b^3}+\frac{d^2\,x^2\,\left(a\,d+5\,b\,c\right)}{10\,b^2}}{a^8+8\,a^7\,b\,x+28\,a^6\,b^2\,x^2+56\,a^5\,b^3\,x^3+70\,a^4\,b^4\,x^4+56\,a^3\,b^5\,x^5+28\,a^2\,b^6\,x^6+8\,a\,b^7\,x^7+b^8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^9,x)
```

```
[Out] -((a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2)/(280*b^4) + (d^3*
x^3)/(5*b) + (d*x*(a^2*d^2 + 15*b^2*c^2 + 5*a*b*c*d))/(35*b^3) + (d^2*x^2*(
a*d + 5*b*c))/(10*b^2))/(a^8 + b^8*x^8 + 8*a*b^7*x^7 + 28*a^6*b^2*x^2 + 56*
a^5*b^3*x^3 + 70*a^4*b^4*x^4 + 56*a^3*b^5*x^5 + 28*a^2*b^6*x^6 + 8*a^7*b*x)
```

3.1273 $\int (a+bx)^9(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35d^3(bc-ad)^4(a+bx)^{13}}{13b^8}+\frac{5d^4(bc$$

```
[Out] 1/10*(-a*d+b*c)^7*(b*x+a)^10/b^8+7/11*d*(-a*d+b*c)^6*(b*x+a)^11/b^8+7/4*d^2
*(-a*d+b*c)^5*(b*x+a)^12/b^8+35/13*d^3*(-a*d+b*c)^4*(b*x+a)^13/b^8+5/2*d^4*
(-a*d+b*c)^3*(b*x+a)^14/b^8+7/5*d^5*(-a*d+b*c)^2*(b*x+a)^15/b^8+7/16*d^6*(-
a*d+b*c)*(b*x+a)^16/b^8+1/17*d^7*(b*x+a)^17/b^8
```

Rubi [A]
time = 0.49, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx)^{12}(bc-ad)^5}{4b^8}+\frac{7d(a+bx)^{11}(bc-ad)^6}{11b^8}+\frac{(a+bx)^{10}(bc-ad)^7}{10b^8}+\frac{d^7(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^10)/(10*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^11)/(1
1*b^8) + (7*d^2*(b*c - a*d)^5*(a + b*x)^12)/(4*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^13)/(13*b^8) + (5*d^4*(b*c - a*d)^3*(a + b*x)^14)/(2*b^8) + (7*d
^5*(b*c - a*d)^2*(a + b*x)^15)/(5*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^16)/(
16*b^8) + (d^7*(a + b*x)^17)/(17*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^9(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^9}{b^7}+\frac{7d(bc-ad)^6(a+bx)^{10}}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{b^7}\right.\\ &= \frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35c}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 993 vs. 2(200) = 400.

time = 0.10, size = 993, normalized size = 4.96

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^9*(c + d*x)^7,x]

[Out] a^9*c^7*x + (a^8*c^6*(9*b*c + 7*a*d)*x^2)/2 + a^7*c^5*(12*b^2*c^2 + 21*a*b*c*d + 7*a^2*d^2)*x^3 + (7*a^6*c^4*(12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^5*c^3*(18*b^4*c^4 + 84*a*b^3*c^3*d + 108*a^2*b^2*c^2*d^2 + 45*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^4*c^2*(6*b^5*c^5 + 42*a*b^4*c^4*d + 84*a^2*b^3*c^3*d^2 + 60*a^3*b^2*c^2*d^3 + 15*a^4*b*c*d^4 + a^5*d^5)*x^6)/2 + a^3*c*(12*b^6*c^6 + 126*a*b^5*c^5*d + 378*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 27*a^5*b*c*d^5 + a^6*d^6)*x^7 + (a^2*(36*b^7*c^7 + 588*a*b^6*c^6*d + 2646*a^2*b^5*c^5*d^2 + 4410*a^3*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 756*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6 + a^7*d^7)*x^8)/8 + a*b*(b^7*c^7 + 28*a*b^6*c^6*d + 196*a^2*b^5*c^5*d^2 + 490*a^3*b^4*c^4*d^3 + 490*a^4*b^3*c^3*d^4 + 196*a^5*b^2*c^2*d^5 + 28*a^6*b*c*d^6 + a^7*d^7)*x^9 + (b^2*(b^7*c^7 + 63*a*b^6*c^6*d + 756*a^2*b^5*c^5*d^2 + 2940*a^3*b^4*c^4*d^3 + 4410*a^4*b^3*c^3*d^4 + 2646*a^5*b^2*c^2*d^5 + 588*a^6*b*c*d^6 + 36*a^7*d^7)*x^10)/10 + (7*b^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 180*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 12*a^6*d^6)*x^11)/11 + (7*b^4*d^2*(b^5*c^5 + 15*a*b^4*c^4*d + 60*a^2*b^3*c^3*d^2 + 84*a^3*b^2*c^2*d^3 + 42*a^4*b*c*d^4 + 6*a^5*d^5)*x^12)/4 + (7*b^5*d^3*(5*b^4*c^4 + 45*a*b^3*c^3*d + 108*a^2*b^2*c^2*d^2 + 84*a^3*b*c*d^3 + 18*a^4*d^4)*x^13)/13 + (b^6*d^4*(5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b*c*d^2 + 12*a^3*d^3)*x^14)/2 + (b^7*d^5*(7*b^2*c^2 + 21*a*b*c*d + 12*a^2*d^2)*x^15)/5 + (b^8*d^6*(7*b*c + 9*a*d)*x^16)/16 + (b^9*d^7*x^17)/17

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(184) = 368$.
time = 0.14, size = 1033, normalized size = 5.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^9*(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] 1/17*b^9*d^7*x^17+1/16*(9*a*b^8*d^7+7*b^9*c*d^6)*x^16+1/15*(36*a^2*b^7*d^7+63*a*b^8*c*d^6+21*b^9*c^2*d^5)*x^15+1/14*(84*a^3*b^6*d^7+252*a^2*b^7*c*d^6+189*a*b^8*c^2*d^5+35*b^9*c^3*d^4)*x^14+1/13*(126*a^4*b^5*d^7+588*a^3*b^6*c*d^6+756*a^2*b^7*c^2*d^5+315*a*b^8*c^3*d^4+35*b^9*c^4*d^3)*x^13+1/12*(126*a^5*b^4*d^7+882*a^4*b^5*c*d^6+1764*a^3*b^6*c^2*d^5+1260*a^2*b^7*c^3*d^4+315*a*b^8*c^4*d^3+21*b^9*c^5*d^2)*x^12+1/11*(84*a^6*b^3*d^7+882*a^5*b^4*c*d^6+2646*a^4*b^5*c^2*d^5+2940*a^3*b^6*c^3*d^4+1260*a^2*b^7*c^4*d^3+189*a*b^8*c^5*d^2+7*b^9*c^6*d)*x^11+1/10*(36*a^7*b^2*d^7+588*a^6*b^3*c*d^6+2646*a^5*b^4*c^2*d^5+4410*a^4*b^5*c^3*d^4+2940*a^3*b^6*c^4*d^3+756*a^2*b^7*c^5*d^2+63*a*b^8*c^6*d+b^9*c^7)*x^10+1/9*(9*a^8*b*d^7+252*a^7*b^2*c*d^6+1764*a^6*b^3*c^2*

```
d^5+4410*a^5*b^4*c^3*d^4+4410*a^4*b^5*c^4*d^3+1764*a^3*b^6*c^5*d^2+252*a^2*
b^7*c^6*d+9*a*b^8*c^7)*x^9+1/8*(a^9*d^7+63*a^8*b*c*d^6+756*a^7*b^2*c^2*d^5+
2940*a^6*b^3*c^3*d^4+4410*a^5*b^4*c^4*d^3+2646*a^4*b^5*c^5*d^2+588*a^3*b^6*
c^6*d+36*a^2*b^7*c^7)*x^8+1/7*(7*a^9*c*d^6+189*a^8*b*c^2*d^5+1260*a^7*b^2*c
^3*d^4+2940*a^6*b^3*c^4*d^3+2646*a^5*b^4*c^5*d^2+882*a^4*b^5*c^6*d+84*a^3*b
^6*c^7)*x^7+1/6*(21*a^9*c^2*d^5+315*a^8*b*c^3*d^4+1260*a^7*b^2*c^4*d^3+1764
*a^6*b^3*c^5*d^2+882*a^5*b^4*c^6*d+126*a^4*b^5*c^7)*x^6+1/5*(35*a^9*c^3*d^4
+315*a^8*b*c^4*d^3+756*a^7*b^2*c^5*d^2+588*a^6*b^3*c^6*d+126*a^5*b^4*c^7)*x
^5+1/4*(35*a^9*c^4*d^3+189*a^8*b*c^5*d^2+252*a^7*b^2*c^6*d+84*a^6*b^3*c^7)*
x^4+1/3*(21*a^9*c^5*d^2+63*a^8*b*c^6*d+36*a^7*b^2*c^7)*x^3+1/2*(7*a^9*c^6*d
+9*a^8*b*c^7)*x^2+a^9*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(184) = 368$.
time = 0.28, size = 1023, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(184) = 368$.

time = 0.49, size = 1023, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(184) = 368$.
time = 0.08, size = 1163, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**7,x)
```

```
[Out] a**9*c**7*x + b**9*d**7*x**17/17 + x**16*(9*a*b**8*d**7/16 + 7*b**9*c*d**6/
16) + x**15*(12*a**2*b**7*d**7/5 + 21*a*b**8*c*d**6/5 + 7*b**9*c**2*d**5/5)
 + x**14*(6*a**3*b**6*d**7 + 18*a**2*b**7*c*d**6 + 27*a*b**8*c**2*d**5/2 +
5*b**9*c**3*d**4/2) + x**13*(126*a**4*b**5*d**7/13 + 588*a**3*b**6*c*d**6/1
3 + 756*a**2*b**7*c**2*d**5/13 + 315*a*b**8*c**3*d**4/13 + 35*b**9*c**4*d**
3/13) + x**12*(21*a**5*b**4*d**7/2 + 147*a**4*b**5*c*d**6/2 + 147*a**3*b**6
```

```
*c**2*d**5 + 105*a**2*b**7*c**3*d**4 + 105*a*b**8*c**4*d**3/4 + 7*b**9*c**5
*d**2/4) + x**11*(84*a**6*b**3*d**7/11 + 882*a**5*b**4*c*d**6/11 + 2646*a**
4*b**5*c**2*d**5/11 + 2940*a**3*b**6*c**3*d**4/11 + 1260*a**2*b**7*c**4*d**
3/11 + 189*a*b**8*c**5*d**2/11 + 7*b**9*c**6*d/11) + x**10*(18*a**7*b**2*d*
*7/5 + 294*a**6*b**3*c*d**6/5 + 1323*a**5*b**4*c**2*d**5/5 + 441*a**4*b**5*
c**3*d**4 + 294*a**3*b**6*c**4*d**3 + 378*a**2*b**7*c**5*d**2/5 + 63*a*b**8
*c**6*d/10 + b**9*c**7/10) + x**9*(a**8*b*d**7 + 28*a**7*b**2*c*d**6 + 196*
a**6*b**3*c**2*d**5 + 490*a**5*b**4*c**3*d**4 + 490*a**4*b**5*c**4*d**3 + 1
96*a**3*b**6*c**5*d**2 + 28*a**2*b**7*c**6*d + a*b**8*c**7) + x**8*(a**9*d*
*7/8 + 63*a**8*b*c*d**6/8 + 189*a**7*b**2*c**2*d**5/2 + 735*a**6*b**3*c**3*
d**4/2 + 2205*a**5*b**4*c**4*d**3/4 + 1323*a**4*b**5*c**5*d**2/4 + 147*a**3
*b**6*c**6*d/2 + 9*a**2*b**7*c**7/2) + x**7*(a**9*c*d**6 + 27*a**8*b*c**2*d
**5 + 180*a**7*b**2*c**3*d**4 + 420*a**6*b**3*c**4*d**3 + 378*a**5*b**4*c**
5*d**2 + 126*a**4*b**5*c**6*d + 12*a**3*b**6*c**7) + x**6*(7*a**9*c**2*d**5
/2 + 105*a**8*b*c**3*d**4/2 + 210*a**7*b**2*c**4*d**3 + 294*a**6*b**3*c**5*
d**2 + 147*a**5*b**4*c**6*d + 21*a**4*b**5*c**7) + x**5*(7*a**9*c**3*d**4 +
 63*a**8*b*c**4*d**3 + 756*a**7*b**2*c**5*d**2/5 + 588*a**6*b**3*c**6*d/5 +
 126*a**5*b**4*c**7/5) + x**4*(35*a**9*c**4*d**3/4 + 189*a**8*b*c**5*d**2/4
 + 63*a**7*b**2*c**6*d + 21*a**6*b**3*c**7) + x**3*(7*a**9*c**5*d**2 + 21*a
**8*b*c**6*d + 12*a**7*b**2*c**7) + x**2*(7*a**9*c**6*d/2 + 9*a**8*b*c**7/2
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(184) = 368$.
time = 0.86, size = 1175, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/17*b^9*d^7*x^17 + 7/16*b^9*c*d^6*x^16 + 9/16*a*b^8*d^7*x^16 + 7/5*b^9*c^2
*d^5*x^15 + 21/5*a*b^8*c*d^6*x^15 + 12/5*a^2*b^7*d^7*x^15 + 5/2*b^9*c^3*d^4
*x^14 + 27/2*a*b^8*c^2*d^5*x^14 + 18*a^2*b^7*c*d^6*x^14 + 6*a^3*b^6*d^7*x^1
4 + 35/13*b^9*c^4*d^3*x^13 + 315/13*a*b^8*c^3*d^4*x^13 + 756/13*a^2*b^7*c^2
*d^5*x^13 + 588/13*a^3*b^6*c*d^6*x^13 + 126/13*a^4*b^5*d^7*x^13 + 7/4*b^9*c
^5*d^2*x^12 + 105/4*a*b^8*c^4*d^3*x^12 + 105*a^2*b^7*c^3*d^4*x^12 + 147*a^3
*b^6*c^2*d^5*x^12 + 147/2*a^4*b^5*c*d^6*x^12 + 21/2*a^5*b^4*d^7*x^12 + 7/11
*b^9*c^6*d*x^11 + 189/11*a*b^8*c^5*d^2*x^11 + 1260/11*a^2*b^7*c^4*d^3*x^11
+ 2940/11*a^3*b^6*c^3*d^4*x^11 + 2646/11*a^4*b^5*c^2*d^5*x^11 + 882/11*a^5*
b^4*c*d^6*x^11 + 84/11*a^6*b^3*d^7*x^11 + 1/10*b^9*c^7*x^10 + 63/10*a*b^8*c
^6*d*x^10 + 378/5*a^2*b^7*c^5*d^2*x^10 + 294*a^3*b^6*c^4*d^3*x^10 + 441*a^4
*b^5*c^3*d^4*x^10 + 1323/5*a^5*b^4*c^2*d^5*x^10 + 294/5*a^6*b^3*c*d^6*x^10
+ 18/5*a^7*b^2*d^7*x^10 + a*b^8*c^7*x^9 + 28*a^2*b^7*c^6*d*x^9 + 196*a^3*b^
6*c^5*d^2*x^9 + 490*a^4*b^5*c^4*d^3*x^9 + 490*a^5*b^4*c^3*d^4*x^9 + 196*a^6
```

```
*b^3*c^2*d^5*x^9 + 28*a^7*b^2*c*d^6*x^9 + a^8*b*d^7*x^9 + 9/2*a^2*b^7*c^7*x
^8 + 147/2*a^3*b^6*c^6*d*x^8 + 1323/4*a^4*b^5*c^5*d^2*x^8 + 2205/4*a^5*b^4*
c^4*d^3*x^8 + 735/2*a^6*b^3*c^3*d^4*x^8 + 189/2*a^7*b^2*c^2*d^5*x^8 + 63/8*
a^8*b*c*d^6*x^8 + 1/8*a^9*d^7*x^8 + 12*a^3*b^6*c^7*x^7 + 126*a^4*b^5*c^6*d*
x^7 + 378*a^5*b^4*c^5*d^2*x^7 + 420*a^6*b^3*c^4*d^3*x^7 + 180*a^7*b^2*c^3*d
^4*x^7 + 27*a^8*b*c^2*d^5*x^7 + a^9*c*d^6*x^7 + 21*a^4*b^5*c^7*x^6 + 147*a^
5*b^4*c^6*d*x^6 + 294*a^6*b^3*c^5*d^2*x^6 + 210*a^7*b^2*c^4*d^3*x^6 + 105/2
*a^8*b*c^3*d^4*x^6 + 7/2*a^9*c^2*d^5*x^6 + 126/5*a^5*b^4*c^7*x^5 + 588/5*a^
6*b^3*c^6*d*x^5 + 756/5*a^7*b^2*c^5*d^2*x^5 + 63*a^8*b*c^4*d^3*x^5 + 7*a^9*
c^3*d^4*x^5 + 21*a^6*b^3*c^7*x^4 + 63*a^7*b^2*c^6*d*x^4 + 189/4*a^8*b*c^5*d
^2*x^4 + 35/4*a^9*c^4*d^3*x^4 + 12*a^7*b^2*c^7*x^3 + 21*a^8*b*c^6*d*x^3 + 7
*a^9*c^5*d^2*x^3 + 9/2*a^8*b*c^7*x^2 + 7/2*a^9*c^6*d*x^2 + a^9*c^7*x
```

Mupad [B]
time = 0.55, size = 997, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^7,x)
```

```
[Out] x^5*((126*a^5*b^4*c^7)/5 + 7*a^9*c^3*d^4 + (588*a^6*b^3*c^6*d)/5 + 63*a^8*b
*c^4*d^3 + (756*a^7*b^2*c^5*d^2)/5) + x^13*((126*a^4*b^5*d^7)/13 + (35*b^9*
c^4*d^3)/13 + (315*a*b^8*c^3*d^4)/13 + (588*a^3*b^6*c*d^6)/13 + (756*a^2*b^
7*c^2*d^5)/13) + x^8*((a^9*d^7)/8 + (9*a^2*b^7*c^7)/2 + (147*a^3*b^6*c^6*d)
/2 + (1323*a^4*b^5*c^5*d^2)/4 + (2205*a^5*b^4*c^4*d^3)/4 + (735*a^6*b^3*c^3
*d^4)/2 + (189*a^7*b^2*c^2*d^5)/2 + (63*a^8*b*c*d^6)/8) + x^10*((b^9*c^7)/1
0 + (18*a^7*b^2*d^7)/5 + (294*a^6*b^3*c*d^6)/5 + (378*a^2*b^7*c^5*d^2)/5 +
294*a^3*b^6*c^4*d^3 + 441*a^4*b^5*c^3*d^4 + (1323*a^5*b^4*c^2*d^5)/5 + (63*
a*b^8*c^6*d)/10) + x^6*(21*a^4*b^5*c^7 + (7*a^9*c^2*d^5)/2 + 147*a^5*b^4*c^
6*d + (105*a^8*b*c^3*d^4)/2 + 294*a^6*b^3*c^5*d^2 + 210*a^7*b^2*c^4*d^3) +
x^12*((21*a^5*b^4*d^7)/2 + (7*b^9*c^5*d^2)/4 + (105*a*b^8*c^4*d^3)/4 + (147
*a^4*b^5*c*d^6)/2 + 105*a^2*b^7*c^3*d^4 + 147*a^3*b^6*c^2*d^5) + x^7*(a^9*c
*d^6 + 12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 27*a^8*b*c^2*d^5 + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4) + x^11*((7*b^9*c^6*d)/
11 + (84*a^6*b^3*d^7)/11 + (189*a*b^8*c^5*d^2)/11 + (882*a^5*b^4*c*d^6)/11
+ (1260*a^2*b^7*c^4*d^3)/11 + (2940*a^3*b^6*c^3*d^4)/11 + (2646*a^4*b^5*c^2
*d^5)/11) + x^9*(a*b^8*c^7 + a^8*b*d^7 + 28*a^2*b^7*c^6*d + 28*a^7*b^2*c*d^
6 + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d^3 + 490*a^5*b^4*c^3*d^4 + 196*a
^6*b^3*c^2*d^5) + a^9*c^7*x + (b^9*d^7*x^17)/17 + (7*a^6*c^4*x^4*(5*a^3*d^3
 + 12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c*d^2))/4 + (b^6*d^4*x^14*(12*a^3
*d^3 + 5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b*c*d^2))/2 + (a^8*c^6*x^2*(7*a*
d + 9*b*c))/2 + (b^8*d^6*x^16*(9*a*d + 7*b*c))/16 + a^7*c^5*x^3*(7*a^2*d^2
+ 12*b^2*c^2 + 21*a*b*c*d) + (b^7*d^5*x^15*(12*a^2*d^2 + 7*b^2*c^2 + 21*a*b
*c*d))/5
```

3.1274 $\int (a+bx)^8(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35d^3(bc-ad)^4(a+bx)^{12}}{12b^8}+\frac{35d^4(b$$

```
[Out] 1/9*(-a*d+b*c)^7*(b*x+a)^9/b^8+7/10*d*(-a*d+b*c)^6*(b*x+a)^10/b^8+21/11*d^2
*(-a*d+b*c)^5*(b*x+a)^11/b^8+35/12*d^3*(-a*d+b*c)^4*(b*x+a)^12/b^8+35/13*d^
4*(-a*d+b*c)^3*(b*x+a)^13/b^8+3/2*d^5*(-a*d+b*c)^2*(b*x+a)^14/b^8+7/15*d^6*
(-a*d+b*c)*(b*x+a)^15/b^8+1/16*d^7*(b*x+a)^16/b^8
```

Rubi [A]
time = 0.40, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+bx)^{11}(bc-ad)^5}{11b^8}+\frac{7d(a+bx)^{10}(bc-ad)^6}{10b^8}+\frac{(a+bx)^9(bc-ad)^7}{9b^8}+\frac{d^7(a+bx)^{16}}{16b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^9)/(9*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^10)/(10*
b^8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^11)/(11*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^12)/(12*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^13)/(13*b^8) + (3
*d^5*(b*c - a*d)^2*(a + b*x)^14)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^15)
/(15*b^8) + (d^7*(a + b*x)^16)/(16*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^8(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^8}{b^7}+\frac{7d(bc-ad)^6(a+bx)^9}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{b^7}+\right.\\ &= \frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35d}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 897 vs. 2(200) = 400.

time = 0.07, size = 897, normalized size = 4.48

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^8*(c + d*x)^7,x]

[Out] a^8*c^7*x + (a^7*c^6*(8*b*c + 7*a*d)*x^2)/2 + (7*a^6*c^5*(4*b^2*c^2 + 8*a*b *c*d + 3*a^2*d^2)*x^3)/3 + (7*a^5*c^4*(8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2* b*c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^4*c^3*(10*b^4*c^4 + 56*a*b^3*c^3*d + 84* a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^3*c^2*(8*b^5*c^ 5 + 70*a*b^4*c^4*d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c *d^4 + 3*a^5*d^5)*x^6)/6 + a^2*c*(4*b^6*c^6 + 56*a*b^5*c^5*d + 210*a^2*b^4* c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 140*a^4*b^2*c^2*d^4 + 24*a^5*b*c*d^5 + a^6* d^6)*x^7 + (a*(8*b^7*c^7 + 196*a*b^6*c^6*d + 1176*a^2*b^5*c^5*d^2 + 2450*a^ 3*b^4*c^4*d^3 + 1960*a^4*b^3*c^3*d^4 + 588*a^5*b^2*c^2*d^5 + 56*a^6*b*c*d^6 + a^7*d^7)*x^8)/8 + (b*(b^7*c^7 + 56*a*b^6*c^6*d + 588*a^2*b^5*c^5*d^2 + 1 960*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 + 1176*a^5*b^2*c^2*d^5 + 196*a^6 *b*c*d^6 + 8*a^7*d^7)*x^9)/9 + (7*b^2*d*(b^6*c^6 + 24*a*b^5*c^5*d + 140*a^2 *b^4*c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 210*a^4*b^2*c^2*d^4 + 56*a^5*b*c*d^5 + 4*a^6*d^6)*x^10)/10 + (7*b^3*d^2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3 *c^3*d^2 + 168*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^11)/11 + (7* b^4*d^3*(5*b^4*c^4 + 40*a*b^3*c^3*d + 84*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 10*a^4*d^4)*x^12)/12 + (7*b^5*d^4*(5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a^2*b*c *d^2 + 8*a^3*d^3)*x^13)/13 + (b^6*d^5*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x ^14)/2 + (b^7*d^6*(7*b*c + 8*a*d)*x^15)/15 + (b^8*d^7*x^16)/16

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(184) = 368$.
time = 0.15, size = 925, normalized size = 4.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^8*(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] 1/16*b^8*d^7*x^16+1/15*(8*a*b^7*d^7+7*b^8*c*d^6)*x^15+1/14*(28*a^2*b^6*d^7+ 56*a*b^7*c*d^6+21*b^8*c^2*d^5)*x^14+1/13*(56*a^3*b^5*d^7+196*a^2*b^6*c*d^6+ 168*a*b^7*c^2*d^5+35*b^8*c^3*d^4)*x^13+1/12*(70*a^4*b^4*d^7+392*a^3*b^5*c*d ^6+588*a^2*b^6*c^2*d^5+280*a*b^7*c^3*d^4+35*b^8*c^4*d^3)*x^12+1/11*(56*a^5* b^3*d^7+490*a^4*b^4*c*d^6+1176*a^3*b^5*c^2*d^5+980*a^2*b^6*c^3*d^4+280*a*b^ 7*c^4*d^3+21*b^8*c^5*d^2)*x^11+1/10*(28*a^6*b^2*d^7+392*a^5*b^3*c*d^6+1470* a^4*b^4*c^2*d^5+1960*a^3*b^5*c^3*d^4+980*a^2*b^6*c^4*d^3+168*a*b^7*c^5*d^2+ 7*b^8*c^6*d)*x^10+1/9*(8*a^7*b*d^7+196*a^6*b^2*c*d^6+1176*a^5*b^3*c^2*d^5+2 450*a^4*b^4*c^3*d^4+1960*a^3*b^5*c^4*d^3+588*a^2*b^6*c^5*d^2+56*a*b^7*c^6*d +b^8*c^7)*x^9+1/8*(a^8*d^7+56*a^7*b*c*d^6+588*a^6*b^2*c^2*d^5+1960*a^5*b^3* c^3*d^4+2450*a^4*b^4*c^4*d^3+1176*a^3*b^5*c^5*d^2+196*a^2*b^6*c^6*d+8*a*b^7 *c^7)*x^8+1/7*(7*a^8*c*d^6+168*a^7*b*c^2*d^5+980*a^6*b^2*c^3*d^4+1960*a^5*b

```
^3*c^4*d^3+1470*a^4*b^4*c^5*d^2+392*a^3*b^5*c^6*d+28*a^2*b^6*c^7)*x^7+1/6*(
21*a^8*c^2*d^5+280*a^7*b*c^3*d^4+980*a^6*b^2*c^4*d^3+1176*a^5*b^3*c^5*d^2+4
90*a^4*b^4*c^6*d+56*a^3*b^5*c^7)*x^6+1/5*(35*a^8*c^3*d^4+280*a^7*b*c^4*d^3+
588*a^6*b^2*c^5*d^2+392*a^5*b^3*c^6*d+70*a^4*b^4*c^7)*x^5+1/4*(35*a^8*c^4*d
^3+168*a^7*b*c^5*d^2+196*a^6*b^2*c^6*d+56*a^5*b^3*c^7)*x^4+1/3*(21*a^8*c^5*
d^2+56*a^7*b*c^6*d+28*a^6*b^2*c^7)*x^3+1/2*(7*a^8*c^6*d+8*a^7*b*c^7)*x^2+a^
8*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(184) = 368$.
time = 0.29, size = 921, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(184) = 368$.
time = 0.52, size = 921, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(184) = 368$.
time = 0.07, size = 1046, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**7,x)
```

```
[Out] a**8*c**7*x + b**8*d**7*x**16/16 + x**15*(8*a*b**7*d**7/15 + 7*b**8*c*d**6/
15) + x**14*(2*a**2*b**6*d**7 + 4*a*b**7*c*d**6 + 3*b**8*c**2*d**5/2) + x**
13*(56*a**3*b**5*d**7/13 + 196*a**2*b**6*c*d**6/13 + 168*a*b**7*c**2*d**5/1
3 + 35*b**8*c**3*d**4/13) + x**12*(35*a**4*b**4*d**7/6 + 98*a**3*b**5*c*d**
6/3 + 49*a**2*b**6*c**2*d**5 + 70*a*b**7*c**3*d**4/3 + 35*b**8*c**4*d**3/12
) + x**11*(56*a**5*b**3*d**7/11 + 490*a**4*b**4*c*d**6/11 + 1176*a**3*b**5*
c**2*d**5/11 + 980*a**2*b**6*c**3*d**4/11 + 280*a*b**7*c**4*d**3/11 + 21*b*
*8*c**5*d**2/11) + x**10*(14*a**6*b**2*d**7/5 + 196*a**5*b**3*c*d**6/5 + 14
7*a**4*b**4*c**2*d**5 + 196*a**3*b**5*c**3*d**4 + 98*a**2*b**6*c**4*d**3 +
84*a*b**7*c**5*d**2/5 + 7*b**8*c**6*d/10) + x**9*(8*a**7*b*d**7/9 + 196*a**
6*b**2*c*d**6/9 + 392*a**5*b**3*c**2*d**5/3 + 2450*a**4*b**4*c**3*d**4/9 +
1960*a**3*b**5*c**4*d**3/9 + 196*a**2*b**6*c**5*d**2/3 + 56*a*b**7*c**6*d/9
 + b**8*c**7/9) + x**8*(a**8*d**7/8 + 7*a**7*b*c*d**6 + 147*a**6*b**2*c**2*
d**5/2 + 245*a**5*b**3*c**3*d**4 + 1225*a**4*b**4*c**4*d**3/4 + 147*a**3*b*
*5*c**5*d**2 + 49*a**2*b**6*c**6*d/2 + a*b**7*c**7) + x**7*(a**8*c*d**6 + 2
```

```
4*a**7*b*c**2*d**5 + 140*a**6*b**2*c**3*d**4 + 280*a**5*b**3*c**4*d**3 + 21
0*a**4*b**4*c**5*d**2 + 56*a**3*b**5*c**6*d + 4*a**2*b**6*c**7) + x**6*(7*a
**8*c**2*d**5/2 + 140*a**7*b*c**3*d**4/3 + 490*a**6*b**2*c**4*d**3/3 + 196*
a**5*b**3*c**5*d**2 + 245*a**4*b**4*c**6*d/3 + 28*a**3*b**5*c**7/3) + x**5*
(7*a**8*c**3*d**4 + 56*a**7*b*c**4*d**3 + 588*a**6*b**2*c**5*d**2/5 + 392*a
**5*b**3*c**6*d/5 + 14*a**4*b**4*c**7) + x**4*(35*a**8*c**4*d**3/4 + 42*a**
7*b*c**5*d**2 + 49*a**6*b**2*c**6*d + 14*a**5*b**3*c**7) + x**3*(7*a**8*c**
5*d**2 + 56*a**7*b*c**6*d/3 + 28*a**6*b**2*c**7/3) + x**2*(7*a**8*c**6*d/2
+ 4*a**7*b*c**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(184) = 368$.
time = 0.63, size = 1050, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^8*d^7*x^16 + 7/15*b^8*c*d^6*x^15 + 8/15*a*b^7*d^7*x^15 + 3/2*b^8*c^2
*d^5*x^14 + 4*a*b^7*c*d^6*x^14 + 2*a^2*b^6*d^7*x^14 + 35/13*b^8*c^3*d^4*x^1
3 + 168/13*a*b^7*c^2*d^5*x^13 + 196/13*a^2*b^6*c*d^6*x^13 + 56/13*a^3*b^5*d
^7*x^13 + 35/12*b^8*c^4*d^3*x^12 + 70/3*a*b^7*c^3*d^4*x^12 + 49*a^2*b^6*c^2
*d^5*x^12 + 98/3*a^3*b^5*c*d^6*x^12 + 35/6*a^4*b^4*d^7*x^12 + 21/11*b^8*c^5
*d^2*x^11 + 280/11*a*b^7*c^4*d^3*x^11 + 980/11*a^2*b^6*c^3*d^4*x^11 + 1176/
11*a^3*b^5*c^2*d^5*x^11 + 490/11*a^4*b^4*c*d^6*x^11 + 56/11*a^5*b^3*d^7*x^1
1 + 7/10*b^8*c^6*d*x^10 + 84/5*a*b^7*c^5*d^2*x^10 + 98*a^2*b^6*c^4*d^3*x^10
 + 196*a^3*b^5*c^3*d^4*x^10 + 147*a^4*b^4*c^2*d^5*x^10 + 196/5*a^5*b^3*c*d^
6*x^10 + 14/5*a^6*b^2*d^7*x^10 + 1/9*b^8*c^7*x^9 + 56/9*a*b^7*c^6*d*x^9 + 1
96/3*a^2*b^6*c^5*d^2*x^9 + 1960/9*a^3*b^5*c^4*d^3*x^9 + 2450/9*a^4*b^4*c^3*
d^4*x^9 + 392/3*a^5*b^3*c^2*d^5*x^9 + 196/9*a^6*b^2*c*d^6*x^9 + 8/9*a^7*b*d
^7*x^9 + a*b^7*c^7*x^8 + 49/2*a^2*b^6*c^6*d*x^8 + 147*a^3*b^5*c^5*d^2*x^8 +
 1225/4*a^4*b^4*c^4*d^3*x^8 + 245*a^5*b^3*c^3*d^4*x^8 + 147/2*a^6*b^2*c^2*d
^5*x^8 + 7*a^7*b*c*d^6*x^8 + 1/8*a^8*d^7*x^8 + 4*a^2*b^6*c^7*x^7 + 56*a^3*b
^5*c^6*d*x^7 + 210*a^4*b^4*c^5*d^2*x^7 + 280*a^5*b^3*c^4*d^3*x^7 + 140*a^6*
b^2*c^3*d^4*x^7 + 24*a^7*b*c^2*d^5*x^7 + a^8*c*d^6*x^7 + 28/3*a^3*b^5*c^7*x
^6 + 245/3*a^4*b^4*c^6*d*x^6 + 196*a^5*b^3*c^5*d^2*x^6 + 490/3*a^6*b^2*c^4*
d^3*x^6 + 140/3*a^7*b*c^3*d^4*x^6 + 7/2*a^8*c^2*d^5*x^6 + 14*a^4*b^4*c^7*x^
5 + 392/5*a^5*b^3*c^6*d*x^5 + 588/5*a^6*b^2*c^5*d^2*x^5 + 56*a^7*b*c^4*d^3*
x^5 + 7*a^8*c^3*d^4*x^5 + 14*a^5*b^3*c^7*x^4 + 49*a^6*b^2*c^6*d*x^4 + 42*a^
7*b*c^5*d^2*x^4 + 35/4*a^8*c^4*d^3*x^4 + 28/3*a^6*b^2*c^7*x^3 + 56/3*a^7*b*
c^6*d*x^3 + 7*a^8*c^5*d^2*x^3 + 4*a^7*b*c^7*x^2 + 7/2*a^8*c^6*d*x^2 + a^8*c
^7*x
```

Mupad [B]

time = 0.36, size = 892, normalized size = 4.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^7,x)
```

```
[Out] x^8*((a^8*d^7)/8 + a*b^7*c^7 + (49*a^2*b^6*c^6*d)/2 + 147*a^3*b^5*c^5*d^2 +
 (1225*a^4*b^4*c^4*d^3)/4 + 245*a^5*b^3*c^3*d^4 + (147*a^6*b^2*c^2*d^5)/2 +
 7*a^7*b*c*d^6) + x^9*((b^8*c^7)/9 + (8*a^7*b*d^7)/9 + (196*a^6*b^2*c*d^6)/
9 + (196*a^2*b^6*c^5*d^2)/3 + (1960*a^3*b^5*c^4*d^3)/9 + (2450*a^4*b^4*c^3*
d^4)/9 + (392*a^5*b^3*c^2*d^5)/3 + (56*a*b^7*c^6*d)/9) + x^5*(14*a^4*b^4*c^
7 + 7*a^8*c^3*d^4 + (392*a^5*b^3*c^6*d)/5 + 56*a^7*b*c^4*d^3 + (588*a^6*b^2
*c^5*d^2)/5) + x^12*((35*a^4*b^4*d^7)/6 + (35*b^8*c^4*d^3)/12 + (70*a*b^7*c
^3*d^4)/3 + (98*a^3*b^5*c*d^6)/3 + 49*a^2*b^6*c^2*d^5) + x^6*((28*a^3*b^5*c
^7)/3 + (7*a^8*c^2*d^5)/2 + (245*a^4*b^4*c^6*d)/3 + (140*a^7*b*c^3*d^4)/3 +
 196*a^5*b^3*c^5*d^2 + (490*a^6*b^2*c^4*d^3)/3) + x^11*((56*a^5*b^3*d^7)/11
 + (21*b^8*c^5*d^2)/11 + (280*a*b^7*c^4*d^3)/11 + (490*a^4*b^4*c*d^6)/11 +
(980*a^2*b^6*c^3*d^4)/11 + (1176*a^3*b^5*c^2*d^5)/11) + x^7*(a^8*c*d^6 + 4*
a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 24*a^7*b*c^2*d^5 + 210*a^4*b^4*c^5*d^2 + 2
80*a^5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4) + x^10*((7*b^8*c^6*d)/10 + (14*a^
6*b^2*d^7)/5 + (84*a*b^7*c^5*d^2)/5 + (196*a^5*b^3*c*d^6)/5 + 98*a^2*b^6*c^
4*d^3 + 196*a^3*b^5*c^3*d^4 + 147*a^4*b^4*c^2*d^5) + a^8*c^7*x + (b^8*d^7*x
^16)/16 + (7*a^5*c^4*x^4*(5*a^3*d^3 + 8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*b
*c*d^2))/4 + (7*b^5*d^4*x^13*(8*a^3*d^3 + 5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a
^2*b*c*d^2))/13 + (a^7*c^6*x^2*(7*a*d + 8*b*c))/2 + (b^7*d^6*x^15*(8*a*d +
7*b*c))/15 + (7*a^6*c^5*x^3*(3*a^2*d^2 + 4*b^2*c^2 + 8*a*b*c*d))/3 + (b^6*d
^5*x^14*(4*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/2
```

3.1275 $\int (a+bx)^7(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d^3(bc-ad)^4(a+bx)^{11}}{11b^8}+\frac{35d^4(b\cdots}{}$$

```
[Out] 1/8*(-a*d+b*c)^7*(b*x+a)^8/b^8+7/9*d*(-a*d+b*c)^6*(b*x+a)^9/b^8+21/10*d^2*(
-a*d+b*c)^5*(b*x+a)^10/b^8+35/11*d^3*(-a*d+b*c)^4*(b*x+a)^11/b^8+35/12*d^4*
(-a*d+b*c)^3*(b*x+a)^12/b^8+21/13*d^5*(-a*d+b*c)^2*(b*x+a)^13/b^8+1/2*d^6*(
-a*d+b*c)*(b*x+a)^14/b^8+1/15*d^7*(b*x+a)^15/b^8
```

Rubi [A]
time = 0.32, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+bx)^{10}(bc-ad)^5}{10b^8}+\frac{7d(a+bx)^9(bc-ad)^6}{9b^8}+\frac{(a+bx)^8(bc-ad)^7}{8b^8}+\frac{d^7(a+bx)^{15}}{15b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^8)/(8*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^9)/(9*b^
8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^10)/(10*b^8) + (35*d^3*(b*c - a*d)^4*(
a + b*x)^11)/(11*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^12)/(12*b^8) + (21*
d^5*(b*c - a*d)^2*(a + b*x)^13)/(13*b^8) + (d^6*(b*c - a*d)*(a + b*x)^14)/(
2*b^8) + (d^7*(a + b*x)^15)/(15*b^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^7(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^7}{b^7}+\frac{7d(bc-ad)^6(a+bx)^8}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^9}{b^7}+\cdots\right. \\ &= \frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d\cdots}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 785 vs. 2(200) = 400.

time = 0.06, size = 785, normalized size = 3.92

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] a^7*c^7*x + (7*a^6*c^6*(b*c + a*d)*x^2)/2 + (7*a^5*c^5*(3*b^2*c^2 + 7*a*b*c
*d + 3*a^2*d^2)*x^3)/3 + (7*a^4*c^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*
c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^3*c^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2
*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^2*c^2*(3*b^5*c^5 +
 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^
4 + 3*a^5*d^5)*x^6)/6 + a*c*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2
 + 175*a^3*b^3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^
7 + ((b^7*c^7 + 49*a*b^6*c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3
 + 1225*a^4*b^3*c^3*d^4 + 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x
^8)/8 + (7*b*d*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 + 175*a^3*b^
3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^9)/9 + (7*b^2
*d^2*(3*b^5*c^5 + 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^
3 + 35*a^4*b*c*d^4 + 3*a^5*d^5)*x^10)/10 + (7*b^3*d^3*(5*b^4*c^4 + 35*a*b^3
*c^3*d + 63*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^11)/11 + (7*b^4
*d^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2 + 5*a^3*d^3)*x^12)/12 + (
7*b^5*d^5*(3*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x^13)/13 + (b^6*d^6*(b*c + a*
d)*x^14)/2 + (b^7*d^7*x^15)/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(184) = 368$.
time = 0.15, size = 817, normalized size = 4.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^7*d^7*x^15+1/14*(7*a*b^6*d^7+7*b^7*c*d^6)*x^14+1/13*(21*a^2*b^5*d^7+
49*a*b^6*c*d^6+21*b^7*c^2*d^5)*x^13+1/12*(35*a^3*b^4*d^7+147*a^2*b^5*c*d^6+
147*a*b^6*c^2*d^5+35*b^7*c^3*d^4)*x^12+1/11*(35*a^4*b^3*d^7+245*a^3*b^4*c*d
^6+441*a^2*b^5*c^2*d^5+245*a*b^6*c^3*d^4+35*b^7*c^4*d^3)*x^11+1/10*(21*a^5*
b^2*d^7+245*a^4*b^3*c*d^6+735*a^3*b^4*c^2*d^5+735*a^2*b^5*c^3*d^4+245*a*b^6
*c^4*d^3+21*b^7*c^5*d^2)*x^10+1/9*(7*a^6*b*d^7+147*a^5*b^2*c*d^6+735*a^4*b^
3*c^2*d^5+1225*a^3*b^4*c^3*d^4+735*a^2*b^5*c^4*d^3+147*a*b^6*c^5*d^2+7*b^7*
c^6*d)*x^9+1/8*(a^7*d^7+49*a^6*b*c*d^6+441*a^5*b^2*c^2*d^5+1225*a^4*b^3*c^3
*d^4+1225*a^3*b^4*c^4*d^3+441*a^2*b^5*c^5*d^2+49*a*b^6*c^6*d+b^7*c^7)*x^8+1
/7*(7*a^7*c*d^6+147*a^6*b*c^2*d^5+735*a^5*b^2*c^3*d^4+1225*a^4*b^3*c^4*d^3+
735*a^3*b^4*c^5*d^2+147*a^2*b^5*c^6*d+7*a*b^6*c^7)*x^7+1/6*(21*a^7*c^2*d^5+
245*a^6*b*c^3*d^4+735*a^5*b^2*c^4*d^3+735*a^4*b^3*c^5*d^2+245*a^3*b^4*c^6*d
+21*a^2*b^5*c^7)*x^6+1/5*(35*a^7*c^3*d^4+245*a^6*b*c^4*d^3+441*a^5*b^2*c^5*
d^2+245*a^4*b^3*c^6*d+35*a^3*b^4*c^7)*x^5+1/4*(35*a^7*c^4*d^3+147*a^6*b*c^5
```

```
*d^2+147*a^5*b^2*c^6*d+35*a^4*b^3*c^7)*x^4+1/3*(21*a^7*c^5*d^2+49*a^6*b*c^6
*d+21*a^5*b^2*c^7)*x^3+1/2*(7*a^7*c^6*d+7*a^6*b*c^7)*x^2+a^7*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(184) = 368$.
time = 0.30, size = 807, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(184) = 368$.
time = 0.45, size = 807, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
```

```
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(182) = 364$.

time = 0.06, size = 935, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**7,x)
```

```
[Out] a**7*c**7*x + b**7*d**7*x**15/15 + x**14*(a*b**6*d**7/2 + b**7*c*d**6/2) +
x**13*(21*a**2*b**5*d**7/13 + 49*a*b**6*c*d**6/13 + 21*b**7*c**2*d**5/13) +
 x**12*(35*a**3*b**4*d**7/12 + 49*a**2*b**5*c*d**6/4 + 49*a*b**6*c**2*d**5/
4 + 35*b**7*c**3*d**4/12) + x**11*(35*a**4*b**3*d**7/11 + 245*a**3*b**4*c*d
**6/11 + 441*a**2*b**5*c**2*d**5/11 + 245*a*b**6*c**3*d**4/11 + 35*b**7*c**
4*d**3/11) + x**10*(21*a**5*b**2*d**7/10 + 49*a**4*b**3*c*d**6/2 + 147*a**3
*b**4*c**2*d**5/2 + 147*a**2*b**5*c**3*d**4/2 + 49*a*b**6*c**4*d**3/2 + 21*
b**7*c**5*d**2/10) + x**9*(7*a**6*b*d**7/9 + 49*a**5*b**2*c*d**6/3 + 245*a*
*4*b**3*c**2*d**5/3 + 1225*a**3*b**4*c**3*d**4/9 + 245*a**2*b**5*c**4*d**3/
3 + 49*a*b**6*c**5*d**2/3 + 7*b**7*c**6*d/9) + x**8*(a**7*d**7/8 + 49*a**6*
b*c*d**6/8 + 441*a**5*b**2*c**2*d**5/8 + 1225*a**4*b**3*c**3*d**4/8 + 1225*
a**3*b**4*c**4*d**3/8 + 441*a**2*b**5*c**5*d**2/8 + 49*a*b**6*c**6*d/8 + b*
*7*c**7/8) + x**7*(a**7*c*d**6 + 21*a**6*b*c**2*d**5 + 105*a**5*b**2*c**3*d
**4 + 175*a**4*b**3*c**4*d**3 + 105*a**3*b**4*c**5*d**2 + 21*a**2*b**5*c**6
*d + a*b**6*c**7) + x**6*(7*a**7*c**2*d**5/2 + 245*a**6*b*c**3*d**4/6 + 245
*a**5*b**2*c**4*d**3/2 + 245*a**4*b**3*c**5*d**2/2 + 245*a**3*b**4*c**6*d/6
 + 7*a**2*b**5*c**7/2) + x**5*(7*a**7*c**3*d**4 + 49*a**6*b*c**4*d**3 + 441
*a**5*b**2*c**5*d**2/5 + 49*a**4*b**3*c**6*d + 7*a**3*b**4*c**7) + x**4*(35
*a**7*c**4*d**3/4 + 147*a**6*b*c**5*d**2/4 + 147*a**5*b**2*c**6*d/4 + 35*a*
*4*b**3*c**7/4) + x**3*(7*a**7*c**5*d**2 + 49*a**6*b*c**6*d/3 + 7*a**5*b**2
*c**7) + x**2*(7*a**7*c**6*d/2 + 7*a**6*b*c**7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(184) = 368$.

time = 0.64, size = 924, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*d^7*x^15 + 1/2*b^7*c*d^6*x^14 + 1/2*a*b^6*d^7*x^14 + 21/13*b^7*c^2
*d^5*x^13 + 49/13*a*b^6*c*d^6*x^13 + 21/13*a^2*b^5*d^7*x^13 + 35/12*b^7*c^3
*d^4*x^12 + 49/4*a*b^6*c^2*d^5*x^12 + 49/4*a^2*b^5*c*d^6*x^12 + 35/12*a^3*b
^4*d^7*x^12 + 35/11*b^7*c^4*d^3*x^11 + 245/11*a*b^6*c^3*d^4*x^11 + 441/11*a
^2*b^5*c^2*d^5*x^11 + 245/11*a^3*b^4*c*d^6*x^11 + 35/11*a^4*b^3*d^7*x^11 +
21/10*b^7*c^5*d^2*x^10 + 49/2*a*b^6*c^4*d^3*x^10 + 147/2*a^2*b^5*c^3*d^4*x^
10 + 147/2*a^3*b^4*c^2*d^5*x^10 + 49/2*a^4*b^3*c*d^6*x^10 + 21/10*a^5*b^2*d
^7*x^10 + 7/9*b^7*c^6*d*x^9 + 49/3*a*b^6*c^5*d^2*x^9 + 245/3*a^2*b^5*c^4*d^
3*x^9 + 1225/9*a^3*b^4*c^3*d^4*x^9 + 245/3*a^4*b^3*c^2*d^5*x^9 + 49/3*a^5*b
^2*c*d^6*x^9 + 7/9*a^6*b*d^7*x^9 + 1/8*b^7*c^7*x^8 + 49/8*a*b^6*c^6*d*x^8 +
 441/8*a^2*b^5*c^5*d^2*x^8 + 1225/8*a^3*b^4*c^4*d^3*x^8 + 1225/8*a^4*b^3*c^
3*d^4*x^8 + 441/8*a^5*b^2*c^2*d^5*x^8 + 49/8*a^6*b*c*d^6*x^8 + 1/8*a^7*d^7*
x^8 + a*b^6*c^7*x^7 + 21*a^2*b^5*c^6*d*x^7 + 105*a^3*b^4*c^5*d^2*x^7 + 175*
a^4*b^3*c^4*d^3*x^7 + 105*a^5*b^2*c^3*d^4*x^7 + 21*a^6*b*c^2*d^5*x^7 + a^7*
c*d^6*x^7 + 7/2*a^2*b^5*c^7*x^6 + 245/6*a^3*b^4*c^6*d*x^6 + 245/2*a^4*b^3*c
^5*d^2*x^6 + 245/2*a^5*b^2*c^4*d^3*x^6 + 245/6*a^6*b*c^3*d^4*x^6 + 7/2*a^7*
c^2*d^5*x^6 + 7*a^3*b^4*c^7*x^5 + 49*a^4*b^3*c^6*d*x^5 + 441/5*a^5*b^2*c^5*
d^2*x^5 + 49*a^6*b*c^4*d^3*x^5 + 7*a^7*c^3*d^4*x^5 + 35/4*a^4*b^3*c^7*x^4 +
 147/4*a^5*b^2*c^6*d*x^4 + 147/4*a^6*b*c^5*d^2*x^4 + 35/4*a^7*c^4*d^3*x^4 +
 7*a^5*b^2*c^7*x^3 + 49/3*a^6*b*c^6*d*x^3 + 7*a^7*c^5*d^2*x^3 + 7/2*a^6*b*c
^7*x^2 + 7/2*a^7*c^6*d*x^2 + a^7*c^7*x
```

Mupad [B]
time = 0.40, size = 781, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^7,x)
```

```
[Out] x^8*((a^7*d^7)/8 + (b^7*c^7)/8 + (441*a^2*b^5*c^5*d^2)/8 + (1225*a^3*b^4*c^
4*d^3)/8 + (1225*a^4*b^3*c^3*d^4)/8 + (441*a^5*b^2*c^2*d^5)/8 + (49*a*b^6*c
^6*d)/8 + (49*a^6*b*c*d^6)/8) + x^5*(7*a^3*b^4*c^7 + 7*a^7*c^3*d^4 + 49*a^4
*b^3*c^6*d + 49*a^6*b*c^4*d^3 + (441*a^5*b^2*c^5*d^2)/5) + x^11*((35*a^4*b^
3*d^7)/11 + (35*b^7*c^4*d^3)/11 + (245*a*b^6*c^3*d^4)/11 + (245*a^3*b^4*c*d
^6)/11 + (441*a^2*b^5*c^2*d^5)/11) + x^7*(a*b^6*c^7 + a^7*c*d^6 + 21*a^2*b^
5*c^6*d + 21*a^6*b*c^2*d^5 + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 10
5*a^5*b^2*c^3*d^4) + x^9*((7*a^6*b*d^7)/9 + (7*b^7*c^6*d)/9 + (49*a*b^6*c^5
*d^2)/3 + (49*a^5*b^2*c*d^6)/3 + (245*a^2*b^5*c^4*d^3)/3 + (1225*a^3*b^4*c^
3*d^4)/9 + (245*a^4*b^3*c^2*d^5)/3) + x^6*((7*a^2*b^5*c^7)/2 + (7*a^7*c^2*d
^5)/2 + (245*a^3*b^4*c^6*d)/6 + (245*a^6*b*c^3*d^4)/6 + (245*a^4*b^3*c^5*d^
2)/2 + (245*a^5*b^2*c^4*d^3)/2) + x^10*((21*a^5*b^2*d^7)/10 + (21*b^7*c^5*d
```

^2)/10 + (49*a*b^6*c^4*d^3)/2 + (49*a^4*b^3*c*d^6)/2 + (147*a^2*b^5*c^3*d^4
)/2 + (147*a^3*b^4*c^2*d^5)/2) + a^7*c^7*x + (b^7*d^7*x^15)/15 + (7*a^4*c^4
x^4(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/4 + (7*b^4*
d^4*x^12*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/12 + (7
*a^6*c^6*x^2*(a*d + b*c))/2 + (b^6*d^6*x^14*(a*d + b*c))/2 + (7*a^5*c^5*x^3
*(3*a^2*d^2 + 3*b^2*c^2 + 7*a*b*c*d))/3 + (7*b^5*d^5*x^13*(3*a^2*d^2 + 3*b^
2*c^2 + 7*a*b*c*d))/13

3.1276 $\int (a+bx)^6(c+dx)^7\,dx$

Optimal. Leaf size=173

$$\frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(bc-ad)^3(c+dx)^{11}}{11d^7}+\frac{5b^4(bc-$$

```
[Out] 1/8*(-a*d+b*c)^6*(d*x+c)^8/d^7-2/3*b*(-a*d+b*c)^5*(d*x+c)^9/d^7+3/2*b^2*(-a
*d+b*c)^4*(d*x+c)^10/d^7-20/11*b^3*(-a*d+b*c)^3*(d*x+c)^11/d^7+5/4*b^4*(-a*
d+b*c)^2*(d*x+c)^12/d^7-6/13*b^5*(-a*d+b*c)*(d*x+c)^13/d^7+1/14*b^6*(d*x+c)
^14/d^7
```

Rubi [A]
time = 0.31, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)^9(bc-ad)^5}{3d^7}+\frac{(c+dx)^8(bc-ad)^6}{8d^7}+\frac{b^6(c+dx)^{14}}{14d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^8)/(8*d^7) - (2*b*(b*c - a*d)^5*(c + d*x)^9)/(3*d^
7) + (3*b^2*(b*c - a*d)^4*(c + d*x)^10)/(2*d^7) - (20*b^3*(b*c - a*d)^3*(c
+ d*x)^11)/(11*d^7) + (5*b^4*(b*c - a*d)^2*(c + d*x)^12)/(4*d^7) - (6*b^5*(
b*c - a*d)*(c + d*x)^13)/(13*d^7) + (b^6*(c + d*x)^14)/(14*d^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^6(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^6(c+dx)^7}{d^6}-\frac{6b(bc-ad)^5(c+dx)^8}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^9}{d^6}-\right.\\ &= \frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 684 vs. 2(173) = 346.
time = 0.05, size = 684, normalized size = 3.95

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] a^6*c^7*x + (a^5*c^6*(6*b*c + 7*a*d)*x^2)/2 + a^4*c^5*(5*b^2*c^2 + 14*a*b*c
*d + 7*a^2*d^2)*x^3 + (a^3*c^4*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + a^2*c^3*(3*b^4*c^4 + 28*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 42*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (a*c^2*(2*b^5*c^5 + 35*a*b^4*c
^4*d + 140*a^2*b^3*c^3*d^2 + 175*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d
^5)*x^6)/2 + (c*(b^6*c^6 + 42*a*b^5*c^5*d + 315*a^2*b^4*c^4*d^2 + 700*a^3*b
^3*c^3*d^3 + 525*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 7*a^6*d^6)*x^7)/7 + (d
*(7*b^6*c^6 + 126*a*b^5*c^5*d + 525*a^2*b^4*c^4*d^2 + 700*a^3*b^3*c^3*d^3 +
 315*a^4*b^2*c^2*d^4 + 42*a^5*b*c*d^5 + a^6*d^6)*x^8)/8 + (b*d^2*(7*b^5*c^5
 + 70*a*b^4*c^4*d + 175*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 35*a^4*b*c*
d^4 + 2*a^5*d^5)*x^9)/3 + (b^2*d^3*(7*b^4*c^4 + 42*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 28*a^3*b*c*d^3 + 3*a^4*d^4)*x^10)/2 + (b^3*d^4*(35*b^3*c^3 + 126
*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 20*a^3*d^3)*x^11)/11 + (b^4*d^5*(7*b^2*c^2
 + 14*a*b*c*d + 5*a^2*d^2)*x^12)/4 + (b^5*d^6*(7*b*c + 6*a*d)*x^13)/13 + (b
^6*d^7*x^14)/14
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(159) = 318$.
time = 0.14, size = 709, normalized size = 4.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^6*d^7*x^14+1/13*(6*a*b^5*d^7+7*b^6*c*d^6)*x^13+1/12*(15*a^2*b^4*d^7+
42*a*b^5*c*d^6+21*b^6*c^2*d^5)*x^12+1/11*(20*a^3*b^3*d^7+105*a^2*b^4*c*d^6+
126*a*b^5*c^2*d^5+35*b^6*c^3*d^4)*x^11+1/10*(15*a^4*b^2*d^7+140*a^3*b^3*c*d
^6+315*a^2*b^4*c^2*d^5+210*a*b^5*c^3*d^4+35*b^6*c^4*d^3)*x^10+1/9*(6*a^5*b*
d^7+105*a^4*b^2*c*d^6+420*a^3*b^3*c^2*d^5+525*a^2*b^4*c^3*d^4+210*a*b^5*c^4
*d^3+21*b^6*c^5*d^2)*x^9+1/8*(a^6*d^7+42*a^5*b*c*d^6+315*a^4*b^2*c^2*d^5+70
0*a^3*b^3*c^3*d^4+525*a^2*b^4*c^4*d^3+126*a*b^5*c^5*d^2+7*b^6*c^6*d)*x^8+1/
7*(7*a^6*c*d^6+126*a^5*b*c^2*d^5+525*a^4*b^2*c^3*d^4+700*a^3*b^3*c^4*d^3+31
5*a^2*b^4*c^5*d^2+42*a*b^5*c^6*d+b^6*c^7)*x^7+1/6*(21*a^6*c^2*d^5+210*a^5*b
*c^3*d^4+525*a^4*b^2*c^4*d^3+420*a^3*b^3*c^5*d^2+105*a^2*b^4*c^6*d+6*a*b^5*
c^7)*x^6+1/5*(35*a^6*c^3*d^4+210*a^5*b*c^4*d^3+315*a^4*b^2*c^5*d^2+140*a^3*
b^3*c^6*d+15*a^2*b^4*c^7)*x^5+1/4*(35*a^6*c^4*d^3+126*a^5*b*c^5*d^2+105*a^4
*b^2*c^6*d+20*a^3*b^3*c^7)*x^4+1/3*(21*a^6*c^5*d^2+42*a^5*b*c^6*d+15*a^4*b^
2*c^7)*x^3+1/2*(7*a^6*c^6*d+6*a^5*b*c^7)*x^2+a^6*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(159) = 318$.
time = 0.27, size = 706, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(159) = 318$.
time = 0.64, size = 706, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(158) = 316$.

time = 0.06, size = 796, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**7,x)
```

```
[Out] a**6*c**7*x + b**6*d**7*x**14/14 + x**13*(6*a*b**5*d**7/13 + 7*b**6*c*d**6/
13) + x**12*(5*a**2*b**4*d**7/4 + 7*a*b**5*c*d**6/2 + 7*b**6*c**2*d**5/4) +
 x**11*(20*a**3*b**3*d**7/11 + 105*a**2*b**4*c*d**6/11 + 126*a*b**5*c**2*d*
*5/11 + 35*b**6*c**3*d**4/11) + x**10*(3*a**4*b**2*d**7/2 + 14*a**3*b**3*c*
d**6 + 63*a**2*b**4*c**2*d**5/2 + 21*a*b**5*c**3*d**4 + 7*b**6*c**4*d**3/2)
 + x**9*(2*a**5*b*d**7/3 + 35*a**4*b**2*c*d**6/3 + 140*a**3*b**3*c**2*d**5/
3 + 175*a**2*b**4*c**3*d**4/3 + 70*a*b**5*c**4*d**3/3 + 7*b**6*c**5*d**2/3)
 + x**8*(a**6*d**7/8 + 21*a**5*b*c*d**6/4 + 315*a**4*b**2*c**2*d**5/8 + 175
*a**3*b**3*c**3*d**4/2 + 525*a**2*b**4*c**4*d**3/8 + 63*a*b**5*c**5*d**2/4
+ 7*b**6*c**6*d/8) + x**7*(a**6*c*d**6 + 18*a**5*b*c**2*d**5 + 75*a**4*b**2
*c**3*d**4 + 100*a**3*b**3*c**4*d**3 + 45*a**2*b**4*c**5*d**2 + 6*a*b**5*c*
*6*d + b**6*c**7/7) + x**6*(7*a**6*c**2*d**5/2 + 35*a**5*b*c**3*d**4 + 175*
a**4*b**2*c**4*d**3/2 + 70*a**3*b**3*c**5*d**2 + 35*a**2*b**4*c**6*d/2 + a*
b**5*c**7) + x**5*(7*a**6*c**3*d**4 + 42*a**5*b*c**4*d**3 + 63*a**4*b**2*c*
*5*d**2 + 28*a**3*b**3*c**6*d + 3*a**2*b**4*c**7) + x**4*(35*a**6*c**4*d**3
/4 + 63*a**5*b*c**5*d**2/2 + 105*a**4*b**2*c**6*d/4 + 5*a**3*b**3*c**7) + x
**3*(7*a**6*c**5*d**2 + 14*a**5*b*c**6*d + 5*a**4*b**2*c**7) + x**2*(7*a**6
*c**6*d/2 + 3*a**5*b*c**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 798 vs. 2(159) = 318.
time = 0.64, size = 798, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^6*d^7*x^14 + 7/13*b^6*c*d^6*x^13 + 6/13*a*b^5*d^7*x^13 + 7/4*b^6*c^2
*d^5*x^12 + 7/2*a*b^5*c*d^6*x^12 + 5/4*a^2*b^4*d^7*x^12 + 35/11*b^6*c^3*d^4
*x^11 + 126/11*a*b^5*c^2*d^5*x^11 + 105/11*a^2*b^4*c*d^6*x^11 + 20/11*a^3*b
^3*d^7*x^11 + 7/2*b^6*c^4*d^3*x^10 + 21*a*b^5*c^3*d^4*x^10 + 63/2*a^2*b^4*c
^2*d^5*x^10 + 14*a^3*b^3*c*d^6*x^10 + 3/2*a^4*b^2*d^7*x^10 + 7/3*b^6*c^5*d^
2*x^9 + 70/3*a*b^5*c^4*d^3*x^9 + 175/3*a^2*b^4*c^3*d^4*x^9 + 140/3*a^3*b^3*
c^2*d^5*x^9 + 35/3*a^4*b^2*c*d^6*x^9 + 2/3*a^5*b*d^7*x^9 + 7/8*b^6*c^6*d*x^
8 + 63/4*a*b^5*c^5*d^2*x^8 + 525/8*a^2*b^4*c^4*d^3*x^8 + 175/2*a^3*b^3*c^3*
d^4*x^8 + 315/8*a^4*b^2*c^2*d^5*x^8 + 21/4*a^5*b*c*d^6*x^8 + 1/8*a^6*d^7*x^
8 + 1/7*b^6*c^7*x^7 + 6*a*b^5*c^6*d*x^7 + 45*a^2*b^4*c^5*d^2*x^7 + 100*a^3*
```

```
b^3*c^4*d^3*x^7 + 75*a^4*b^2*c^3*d^4*x^7 + 18*a^5*b*c^2*d^5*x^7 + a^6*c*d^6
*x^7 + a*b^5*c^7*x^6 + 35/2*a^2*b^4*c^6*d*x^6 + 70*a^3*b^3*c^5*d^2*x^6 + 17
5/2*a^4*b^2*c^4*d^3*x^6 + 35*a^5*b*c^3*d^4*x^6 + 7/2*a^6*c^2*d^5*x^6 + 3*a^
2*b^4*c^7*x^5 + 28*a^3*b^3*c^6*d*x^5 + 63*a^4*b^2*c^5*d^2*x^5 + 42*a^5*b*c^
4*d^3*x^5 + 7*a^6*c^3*d^4*x^5 + 5*a^3*b^3*c^7*x^4 + 105/4*a^4*b^2*c^6*d*x^4
 + 63/2*a^5*b*c^5*d^2*x^4 + 35/4*a^6*c^4*d^3*x^4 + 5*a^4*b^2*c^7*x^3 + 14*a
^5*b*c^6*d*x^3 + 7*a^6*c^5*d^2*x^3 + 3*a^5*b*c^7*x^2 + 7/2*a^6*c^6*d*x^2 +
a^6*c^7*x
```

Mupad [B]
time = 0.26, size = 683, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^7,x)
```

```
[Out] x^5*(3*a^2*b^4*c^7 + 7*a^6*c^3*d^4 + 28*a^3*b^3*c^6*d + 42*a^5*b*c^4*d^3 +
63*a^4*b^2*c^5*d^2) + x^10*((3*a^4*b^2*d^7)/2 + (7*b^6*c^4*d^3)/2 + 21*a*b^
5*c^3*d^4 + 14*a^3*b^3*c*d^6 + (63*a^2*b^4*c^2*d^5)/2) + x^6*(a*b^5*c^7 + (
7*a^6*c^2*d^5)/2 + (35*a^2*b^4*c^6*d)/2 + 35*a^5*b*c^3*d^4 + 70*a^3*b^3*c^5
*d^2 + (175*a^4*b^2*c^4*d^3)/2) + x^9*((2*a^5*b*d^7)/3 + (7*b^6*c^5*d^2)/3
+ (70*a*b^5*c^4*d^3)/3 + (35*a^4*b^2*c*d^6)/3 + (175*a^2*b^4*c^3*d^4)/3 + (
140*a^3*b^3*c^2*d^5)/3) + x^7*((b^6*c^7)/7 + a^6*c*d^6 + 18*a^5*b*c^2*d^5 +
 45*a^2*b^4*c^5*d^2 + 100*a^3*b^3*c^4*d^3 + 75*a^4*b^2*c^3*d^4 + 6*a*b^5*c^
6*d) + x^8*((a^6*d^7)/8 + (7*b^6*c^6*d)/8 + (63*a*b^5*c^5*d^2)/4 + (525*a^2
*b^4*c^4*d^3)/8 + (175*a^3*b^3*c^3*d^4)/2 + (315*a^4*b^2*c^2*d^5)/8 + (21*a
^5*b*c*d^6)/4) + x^4*(5*a^3*b^3*c^7 + (35*a^6*c^4*d^3)/4 + (105*a^4*b^2*c^6
*d)/4 + (63*a^5*b*c^5*d^2)/2) + x^11*((20*a^3*b^3*d^7)/11 + (35*b^6*c^3*d^4
)/11 + (126*a*b^5*c^2*d^5)/11 + (105*a^2*b^4*c*d^6)/11) + a^6*c^7*x + (b^6*
d^7*x^14)/14 + (a^5*c^6*x^2*(7*a*d + 6*b*c))/2 + (b^5*d^6*x^13*(6*a*d + 7*b
*c))/13 + a^4*c^5*x^3*(7*a^2*d^2 + 5*b^2*c^2 + 14*a*b*c*d) + (b^4*d^5*x^12*
(5*a^2*d^2 + 7*b^2*c^2 + 14*a*b*c*d))/4
```

3.1277 $\int (a+bx)^5(c+dx)^7\,dx$

Optimal. Leaf size=144

$$-\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3(bc-ad)^2(c+dx)^{11}}{11d^6}-\frac{5b^4(bc-}{}$$

```
[Out] -1/8*(-a*d+b*c)^5*(d*x+c)^8/d^6+5/9*b*(-a*d+b*c)^4*(d*x+c)^9/d^6-b^2*(-a*d+
b*c)^3*(d*x+c)^10/d^6+10/11*b^3*(-a*d+b*c)^2*(d*x+c)^11/d^6-5/12*b^4*(-a*d+
b*c)*(d*x+c)^12/d^6+1/13*b^5*(d*x+c)^13/d^6
```

Rubi [A]
time = 0.25, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-ad)^5}{8d^6}+\frac{b^5(c+dx)^{13}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] -1/8*((b*c - a*d)^5*(c + d*x)^8)/d^6 + (5*b*(b*c - a*d)^4*(c + d*x)^9)/(9*d
^6) - (b^2*(b*c - a*d)^3*(c + d*x)^10)/d^6 + (10*b^3*(b*c - a*d)^2*(c + d*x
)^11)/(11*d^6) - (5*b^4*(b*c - a*d)*(c + d*x)^12)/(12*d^6) + (b^5*(c + d*x)
^13)/(13*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^5(c+dx)^7}{d^5}+\frac{5b(bc-ad)^4(c+dx)^8}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^9}{d^5}+\right.\\ &= -\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3(b}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 574 vs. 2(144) = 288.
time = 0.04, size = 574, normalized size = 3.99

$$a^5c^7x+\frac{1}{2}a^4c^6(5bc+7ad)x^2+\frac{1}{3}a^3c^5\left(10b^2c^2+35abcd+21a^2d^2\right)x^3+\frac{5}{4}a^2c^4\left(2b^3c^3+14ab^2c^2d+21a^2bcd^2+7a^3d^3\right)x^4+ac^3\left(b^4c^4+14ab^3c^3d+42a^2b^2c^2d^2+35a^3bcd^3+7a^4d^4\right)x^5+\frac{1}{6}c^2\left(b^5c^5+35ab^4c^4d+210a^2b^3c^3d^2+350a^3b^2c^2d^3+175a^4bcd^4+21a^5d^5\right)x^6+cd\left(b^5c^5+15ab^4c^4d+50a^2b^3c^3d^2+50a^3b^2c^2d^3+15a^4bcd^4+a^5d^5\right)x^7+\frac{1}{8}d^2\left(21b^5c^5+175ab^4c^4d+350a^2b^3c^3d^2+210a^3b^2c^2d^3+35a^4bcd^4+a^5d^5\right)x^8+\frac{5}{9}bd^3\left(7b^4c^4+35ab^3c^3d+42a^2b^2c^2d^2+14a^3bcd^3+a^4d^4\right)x^9+\frac{1}{2}b^2d^4\left(7b^3c^3+21ab^2c^2d+14a^2bcd^2+2a^3d^3\right)x^{10}+\frac{1}{11}b^3d^5\left(21b^2c^2+35abcd+10a^2d^2\right)x^{11}+\frac{1}{12}b^4d^6(7bc+5ad)x^{12}+\frac{1}{13}b^5d^7x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] a^5*c^7*x + (a^4*c^6*(5*b*c + 7*a*d)*x^2)/2 + (a^3*c^5*(10*b^2*c^2 + 35*a*b
*c*d + 21*a^2*d^2)*x^3)/3 + (5*a^2*c^4*(2*b^3*c^3 + 14*a*b^2*c^2*d + 21*a^2
*b*c*d^2 + 7*a^3*d^3)*x^4)/4 + a*c^3*(b^4*c^4 + 14*a*b^3*c^3*d + 42*a^2*b^2
*c^2*d^2 + 35*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (c^2*(b^5*c^5 + 35*a*b^4*c^4*d
 + 210*a^2*b^3*c^3*d^2 + 350*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5
)*x^6)/6 + c*d*(b^5*c^5 + 15*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 + 50*a^3*b^2*
c^2*d^3 + 15*a^4*b*c*d^4 + a^5*d^5)*x^7 + (d^2*(21*b^5*c^5 + 175*a*b^4*c^4*
d + 350*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 + a^5*d^5)*x
^8)/8 + (5*b*d^3*(7*b^4*c^4 + 35*a*b^3*c^3*d + 42*a^2*b^2*c^2*d^2 + 14*a^3*
b*c*d^3 + a^4*d^4)*x^9)/9 + (b^2*d^4*(7*b^3*c^3 + 21*a*b^2*c^2*d + 14*a^2*b
*c*d^2 + 2*a^3*d^3)*x^10)/2 + (b^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^
2)*x^11)/11 + (b^4*d^6*(7*b*c + 5*a*d)*x^12)/12 + (b^5*d^7*x^13)/13
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(134) = 268$.
time = 0.14, size = 601, normalized size = 4.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^5*d^7*x^13+1/12*(5*a*b^4*d^7+7*b^5*c*d^6)*x^12+1/11*(10*a^2*b^3*d^7+
35*a*b^4*c*d^6+21*b^5*c^2*d^5)*x^11+1/10*(10*a^3*b^2*d^7+70*a^2*b^3*c*d^6+1
05*a*b^4*c^2*d^5+35*b^5*c^3*d^4)*x^10+1/9*(5*a^4*b*d^7+70*a^3*b^2*c*d^6+210
*a^2*b^3*c^2*d^5+175*a*b^4*c^3*d^4+35*b^5*c^4*d^3)*x^9+1/8*(a^5*d^7+35*a^4*
b*c*d^6+210*a^3*b^2*c^2*d^5+350*a^2*b^3*c^3*d^4+175*a*b^4*c^4*d^3+21*b^5*c^
5*d^2)*x^8+1/7*(7*a^5*c*d^6+105*a^4*b*c^2*d^5+350*a^3*b^2*c^3*d^4+350*a^2*b
^3*c^4*d^3+105*a*b^4*c^5*d^2+7*b^5*c^6*d)*x^7+1/6*(21*a^5*c^2*d^5+175*a^4*b
*c^3*d^4+350*a^3*b^2*c^4*d^3+210*a^2*b^3*c^5*d^2+35*a*b^4*c^6*d+b^5*c^7)*x^
6+1/5*(35*a^5*c^3*d^4+175*a^4*b*c^4*d^3+210*a^3*b^2*c^5*d^2+70*a^2*b^3*c^6*
d+5*a*b^4*c^7)*x^5+1/4*(35*a^5*c^4*d^3+105*a^4*b*c^5*d^2+70*a^3*b^2*c^6*d+1
0*a^2*b^3*c^7)*x^4+1/3*(21*a^5*c^5*d^2+35*a^4*b*c^6*d+10*a^3*b^2*c^7)*x^3+1
/2*(7*a^5*c^6*d+5*a^4*b*c^7)*x^2+a^5*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(134) = 268$.
time = 0.28, size = 594, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
```

```
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(134) = 268$.
time = 0.69, size = 594, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(129) = 258$.
time = 0.05, size = 673, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**7,x)
```

```
[Out] a**5*c**7*x + b**5*d**7*x**13/13 + x**12*(5*a*b**4*d**7/12 + 7*b**5*c*d**6/
12) + x**11*(10*a**2*b**3*d**7/11 + 35*a*b**4*c*d**6/11 + 21*b**5*c**2*d**5
```

```
/11) + x**10*(a**3*b**2*d**7 + 7*a**2*b**3*c*d**6 + 21*a*b**4*c**2*d**5/2 +
 7*b**5*c**3*d**4/2) + x**9*(5*a**4*b*d**7/9 + 70*a**3*b**2*c*d**6/9 + 70*a
**2*b**3*c**2*d**5/3 + 175*a*b**4*c**3*d**4/9 + 35*b**5*c**4*d**3/9) + x**8
*(a**5*d**7/8 + 35*a**4*b*c*d**6/8 + 105*a**3*b**2*c**2*d**5/4 + 175*a**2*b
**3*c**3*d**4/4 + 175*a*b**4*c**4*d**3/8 + 21*b**5*c**5*d**2/8) + x**7*(a**
5*c*d**6 + 15*a**4*b*c**2*d**5 + 50*a**3*b**2*c**3*d**4 + 50*a**2*b**3*c**4
*d**3 + 15*a*b**4*c**5*d**2 + b**5*c**6*d) + x**6*(7*a**5*c**2*d**5/2 + 175
*a**4*b*c**3*d**4/6 + 175*a**3*b**2*c**4*d**3/3 + 35*a**2*b**3*c**5*d**2 +
35*a*b**4*c**6*d/6 + b**5*c**7/6) + x**5*(7*a**5*c**3*d**4 + 35*a**4*b*c**4
*d**3 + 42*a**3*b**2*c**5*d**2 + 14*a**2*b**3*c**6*d + a*b**4*c**7) + x**4*
(35*a**5*c**4*d**3/4 + 105*a**4*b*c**5*d**2/4 + 35*a**3*b**2*c**6*d/2 + 5*a
**2*b**3*c**7/2) + x**3*(7*a**5*c**5*d**2 + 35*a**4*b*c**6*d/3 + 10*a**3*b*
*2*c**7/3) + x**2*(7*a**5*c**6*d/2 + 5*a**4*b*c**7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(134) = 268.
time = 0.55, size = 670, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^5*d^7*x^13 + 7/12*b^5*c*d^6*x^12 + 5/12*a*b^4*d^7*x^12 + 21/11*b^5*c
^2*d^5*x^11 + 35/11*a*b^4*c*d^6*x^11 + 10/11*a^2*b^3*d^7*x^11 + 7/2*b^5*c^3
*d^4*x^10 + 21/2*a*b^4*c^2*d^5*x^10 + 7*a^2*b^3*c*d^6*x^10 + a^3*b^2*d^7*x^
10 + 35/9*b^5*c^4*d^3*x^9 + 175/9*a*b^4*c^3*d^4*x^9 + 70/3*a^2*b^3*c^2*d^5*
x^9 + 70/9*a^3*b^2*c*d^6*x^9 + 5/9*a^4*b*d^7*x^9 + 21/8*b^5*c^5*d^2*x^8 + 1
75/8*a*b^4*c^4*d^3*x^8 + 175/4*a^2*b^3*c^3*d^4*x^8 + 105/4*a^3*b^2*c^2*d^5*
x^8 + 35/8*a^4*b*c*d^6*x^8 + 1/8*a^5*d^7*x^8 + b^5*c^6*d*x^7 + 15*a*b^4*c^5
*d^2*x^7 + 50*a^2*b^3*c^4*d^3*x^7 + 50*a^3*b^2*c^3*d^4*x^7 + 15*a^4*b*c^2*d
^5*x^7 + a^5*c*d^6*x^7 + 1/6*b^5*c^7*x^6 + 35/6*a*b^4*c^6*d*x^6 + 35*a^2*b^
3*c^5*d^2*x^6 + 175/3*a^3*b^2*c^4*d^3*x^6 + 175/6*a^4*b*c^3*d^4*x^6 + 7/2*a
^5*c^2*d^5*x^6 + a*b^4*c^7*x^5 + 14*a^2*b^3*c^6*d*x^5 + 42*a^3*b^2*c^5*d^2*
x^5 + 35*a^4*b*c^4*d^3*x^5 + 7*a^5*c^3*d^4*x^5 + 5/2*a^2*b^3*c^7*x^4 + 35/2
*a^3*b^2*c^6*d*x^4 + 105/4*a^4*b*c^5*d^2*x^4 + 35/4*a^5*c^4*d^3*x^4 + 10/3*
a^3*b^2*c^7*x^3 + 35/3*a^4*b*c^6*d*x^3 + 7*a^5*c^5*d^2*x^3 + 5/2*a^4*b*c^7*
x^2 + 7/2*a^5*c^6*d*x^2 + a^5*c^7*x
```

Mupad [B]
time = 0.21, size = 570, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^7,x)
```

```
[Out] x^7*(a^5*c*d^6 + b^5*c^6*d + 15*a*b^4*c^5*d^2 + 15*a^4*b*c^2*d^5 + 50*a^2*b
^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4) + x^6*((b^5*c^7)/6 + (7*a^5*c^2*d^5)/2 + (
175*a^4*b*c^3*d^4)/6 + 35*a^2*b^3*c^5*d^2 + (175*a^3*b^2*c^4*d^3)/3 + (35*a
*b^4*c^6*d)/6) + x^8*((a^5*d^7)/8 + (21*b^5*c^5*d^2)/8 + (175*a*b^4*c^4*d^3
)/8 + (175*a^2*b^3*c^3*d^4)/4 + (105*a^3*b^2*c^2*d^5)/4 + (35*a^4*b*c*d^6)/
8) + x^5*(a*b^4*c^7 + 7*a^5*c^3*d^4 + 14*a^2*b^3*c^6*d + 35*a^4*b*c^4*d^3 +
 42*a^3*b^2*c^5*d^2) + x^9*((5*a^4*b*d^7)/9 + (35*b^5*c^4*d^3)/9 + (175*a*b
^4*c^3*d^4)/9 + (70*a^3*b^2*c*d^6)/9 + (70*a^2*b^3*c^2*d^5)/3) + a^5*c^7*x
+ (b^5*d^7*x^13)/13 + (5*a^2*c^4*x^4*(7*a^3*d^3 + 2*b^3*c^3 + 14*a*b^2*c^2*
d + 21*a^2*b*c*d^2))/4 + (b^2*d^4*x^10*(2*a^3*d^3 + 7*b^3*c^3 + 21*a*b^2*c^
2*d + 14*a^2*b*c*d^2))/2 + (a^4*c^6*x^2*(7*a*d + 5*b*c))/2 + (b^4*d^6*x^12*
(5*a*d + 7*b*c))/12 + (a^3*c^5*x^3*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*b*c*d))/
3 + (b^3*d^5*x^11*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/11
```

3.1278 $\int (a+bx)^4(c+dx)^7\,dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(bc-ad)(c+dx)^{11}}{11d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

```
[Out] 1/8*(-a*d+b*c)^4*(d*x+c)^8/d^5-4/9*b*(-a*d+b*c)^3*(d*x+c)^9/d^5+3/5*b^2*(-a
*d+b*c)^2*(d*x+c)^10/d^5-4/11*b^3*(-a*d+b*c)*(d*x+c)^11/d^5+1/12*b^4*(d*x+c
)^12/d^5
```

Rubi [A]
time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^8)/(8*d^5) - (4*b*(b*c - a*d)^3*(c + d*x)^9)/(9*d^
5) + (3*b^2*(b*c - a*d)^2*(c + d*x)^10)/(5*d^5) - (4*b^3*(b*c - a*d)*(c + d
*x)^11)/(11*d^5) + (b^4*(c + d*x)^12)/(12*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^7}{d^4}-\frac{4b(bc-ad)^3(c+dx)^8}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^9}{d^4}-\right.\\ &= \frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(b\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 473 vs. 2(119) = 238.
time = 0.03, size = 473, normalized size = 3.97

$$a^4c^7x+\frac{1}{2}a^3c^6(4bc+7ad)x^2+\frac{1}{3}a^2c^5\left(6b^2c^2+28abcd+21a^2d^2\right)x^3+\frac{1}{4}ac^4\left(4b^3c^3+42ab^2c^2d+84a^2bcd^2+35a^3d^3\right)x^4+\frac{1}{5}c^3\left(b^4c^4+28ab^3c^3d+126a^2b^2c^2d^2+140a^3bcd^3+35a^4d^4\right)x^5+\frac{7}{6}c^2d\left(b^4c^4+12ab^3c^3d+30a^2b^2c^2d^2+20a^3bcd^3+3a^4d^4\right)x^6+cd^2\left(3b^4c^4+20ab^3c^3d+30a^2b^2c^2d^2+12a^3bcd^3+a^4d^4\right)x^7+\frac{1}{8}d^3\left(35b^4c^4+140ab^3c^3d+126a^2b^2c^2d^2+28a^3bcd^3+a^4d^4\right)x^8+\frac{1}{9}bd^4\left(35b^3c^3+84ab^2c^2d+42a^2bcd^2+4a^3d^3\right)x^9+\frac{1}{10}b^2d^5\left(21b^2c^2+28abcd+6a^2d^2\right)x^{10}+\frac{1}{11}b^3d^6(7bc+4ad)x^{11}+\frac{1}{12}b^4d^7x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] a^4*c^7*x + (a^3*c^6*(4*b*c + 7*a*d)*x^2)/2 + (a^2*c^5*(6*b^2*c^2 + 28*a*b*
c*d + 21*a^2*d^2)*x^3)/3 + (a*c^4*(4*b^3*c^3 + 42*a*b^2*c^2*d + 84*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + (c^3*(b^4*c^4 + 28*a*b^3*c^3*d + 126*a^2*b^2*c^2
*d^2 + 140*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + (7*c^2*d*(b^4*c^4 + 12*a*b^3*
c^3*d + 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 + 3*a^4*d^4)*x^6)/6 + c*d^2*(3*
b^4*c^4 + 20*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)*x
^7 + (d^3*(35*b^4*c^4 + 140*a*b^3*c^3*d + 126*a^2*b^2*c^2*d^2 + 28*a^3*b*c*
d^3 + a^4*d^4)*x^8)/8 + (b*d^4*(35*b^3*c^3 + 84*a*b^2*c^2*d + 42*a^2*b*c*d^
2 + 4*a^3*d^3)*x^9)/9 + (b^2*d^5*(21*b^2*c^2 + 28*a*b*c*d + 6*a^2*d^2)*x^10
)/10 + (b^3*d^6*(7*b*c + 4*a*d)*x^11)/11 + (b^4*d^7*x^12)/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(109) = 218$.
time = 0.14, size = 493, normalized size = 4.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^4*d^7*x^12+1/11*(4*a*b^3*d^7+7*b^4*c*d^6)*x^11+1/10*(6*a^2*b^2*d^7+2
8*a*b^3*c*d^6+21*b^4*c^2*d^5)*x^10+1/9*(4*a^3*b*d^7+42*a^2*b^2*c*d^6+84*a*b
^3*c^2*d^5+35*b^4*c^3*d^4)*x^9+1/8*(a^4*d^7+28*a^3*b*c*d^6+126*a^2*b^2*c^2*
d^5+140*a*b^3*c^3*d^4+35*b^4*c^4*d^3)*x^8+1/7*(7*a^4*c*d^6+84*a^3*b*c^2*d^5
+210*a^2*b^2*c^3*d^4+140*a*b^3*c^4*d^3+21*b^4*c^5*d^2)*x^7+1/6*(21*a^4*c^2*
d^5+140*a^3*b*c^3*d^4+210*a^2*b^2*c^4*d^3+84*a*b^3*c^5*d^2+7*b^4*c^6*d)*x^6
+1/5*(35*a^4*c^3*d^4+140*a^3*b*c^4*d^3+126*a^2*b^2*c^5*d^2+28*a*b^3*c^6*d+b
^4*c^7)*x^5+1/4*(35*a^4*c^4*d^3+84*a^3*b*c^5*d^2+42*a^2*b^2*c^6*d+4*a*b^3*c
^7)*x^4+1/3*(21*a^4*c^5*d^2+28*a^3*b*c^6*d+6*a^2*b^2*c^7)*x^3+1/2*(7*a^4*c^
6*d+4*a^3*b*c^7)*x^2+a^4*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(109) = 218$.
time = 0.28, size = 489, normalized size = 4.11

$$\frac{1}{12}\,b^{4}d^{7}x^{12} + a^{4}c^{7}x + \frac{1}{11}\,(7\,b^{4}cd^{6} + 4\,ab^{3}d^{7})x^{11} + \frac{1}{10}\,(21\,b^{4}c^{2}d^{5} + 28\,ab^{3}cd^{6} + 6\,a^{2}b^{2}d^{7})x^{10} + \frac{1}{9}\,(35\,b^{4}c^{3}d^{4} + 84\,ab^{3}c^{2}d^{5} + 42\,a^{2}b^{2}cd^{6} + 4\,a^{3}bd^{7})x^{9} + \frac{1}{8}\,(35\,b^{4}c^{4}d^{3} + 140\,ab^{3}c^{3}d^{4} + 126\,a^{2}b^{2}c^{2}d^{5} + 28\,a^{3}bcd^{6} + a^{4}d^{7})x^{8} + (3\,b^{4}c^{5}d^{2} + 20\,ab^{3}c^{4}d^{3} + 30\,a^{2}b^{2}c^{3}d^{4} + 12\,a^{3}bc^{2}d^{5} + a^{4}cd^{6})x^{7} + \frac{7}{6}\,(b^{4}c^{6}d + 12\,ab^{3}c^{5}d^{2} + 30\,a^{2}b^{2}c^{4}d^{3} + 20\,a^{3}bc^{3}d^{4} + 3\,a^{4}c^{2}d^{5})x^{6} + \frac{1}{5}\,(b^{4}c^{7} + 28\,ab^{3}c^{6}d + 126\,a^{2}b^{2}c^{5}d^{2} + 140\,a^{3}bc^{4}d^{3} + 35\,a^{4}c^{3}d^{4})x^{5} + \frac{1}{4}\,(4\,ab^{3}c^{7} + 42\,a^{2}b^{2}c^{6}d + 84\,a^{3}bc^{5}d^{2} + 35\,a^{4}c^{4}d^{3})x^{4} + \frac{1}{3}\,(6\,a^{2}b^{2}c^{7} + 28\,a^{3}bc^{6}d + 21\,a^{4}c^{5}d^{2})x^{3} + \frac{1}{2}\,(4\,a^{3}bc^{7} + 7\,a^{4}c^{6}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
```

```
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(109) = 218$.
time = 0.61, size = 489, normalized size = 4.11

$$\frac{1}{12} b^{4} d^{7} x^{12} + a^{4} c^{7} x + \frac{1}{11} \left(7 b^{4} c d^{6} + 4 a b^{3} d^{7}\right) x^{11} + \frac{1}{10} \left(21 b^{4} c^{2} d^{5} + 28 a b^{3} c d^{6} + 6 a^{2} b^{2} d^{7}\right) x^{10} + \frac{1}{9} \left(35 b^{4} c^{3} d^{4} + 84 a b^{3} c^{2} d^{5} + 42 a^{2} b^{2} c d^{6} + 4 a^{3} b d^{7}\right) x^{9} + \frac{1}{8} \left(35 b^{4} c^{4} d^{3} + 140 a b^{3} c^{3} d^{4} + 126 a^{2} b^{2} c^{2} d^{5} + 28 a^{3} b c d^{6} + a^{4} d^{7}\right) x^{8} + \left(3 b^{4} c^{5} d^{2} + 20 a b^{3} c^{4} d^{3} + 30 a^{2} b^{2} c^{3} d^{4} + 12 a^{3} b c^{2} d^{5} + a^{4} c d^{6}\right) x^{7} + \frac{7}{6} \left(b^{4} c^{6} d + 12 a b^{3} c^{5} d^{2} + 30 a^{2} b^{2} c^{4} d^{3} + 20 a^{3} b c^{3} d^{4} + 3 a^{4} c^{2} d^{5}\right) x^{6} + \frac{1}{5} \left(b^{4} c^{7} + 28 a b^{3} c^{6} d + 126 a^{2} b^{2} c^{5} d^{2} + 140 a^{3} b c^{4} d^{3} + 35 a^{4} c^{3} d^{4}\right) x^{5} + \frac{1}{4} \left(4 a b^{3} c^{7} + 42 a^{2} b^{2} c^{6} d + 84 a^{3} b c^{5} d^{2} + 35 a^{4} c^{4} d^{3}\right) x^{4} + \frac{1}{3} \left(6 a^{2} b^{2} c^{7} + 28 a^{3} b c^{6} d + 21 a^{4} c^{5} d^{2}\right) x^{3} + \frac{1}{2} \left(4 a^{3} b c^{7} + 7 a^{4} c^{6} d\right) x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(107) = 214$.
time = 0.04, size = 549, normalized size = 4.61

$$a^{4} c^{7} x + \frac{b^{4} d^{7} x^{12}}{12} + x^{11} \cdot \left(\frac{4 a b^{3} d^{7}}{11} + \frac{7 b^{4} c d^{6}}{11}\right) + x^{10} \cdot \left(\frac{3 a^{2} b^{2} d^{7}}{5} + \frac{14 a b^{3} c d^{6}}{5} + \frac{21 b^{4} c^{2} d^{5}}{10}\right) + x^{9} \cdot \left(\frac{4 a^{3} b d^{7}}{9} + \frac{14 a^{2} b^{2} c d^{6}}{3} + \frac{28 a b^{3} c^{2} d^{5}}{3} + \frac{35 b^{4} c^{3} d^{4}}{9}\right) + x^{8} \left(\frac{a^{4} d^{7}}{8} + \frac{7 a^{3} b c d^{6}}{2} + \frac{63 a^{2} b^{2} c^{2} d^{5}}{4} + \frac{35 a b^{3} c^{3} d^{4}}{2} + \frac{35 b^{4} c^{4} d^{3}}{8}\right) + x^{7} \left(a^{4} c d^{6} + 12 a^{3} b c^{2} d^{5} + 30 a^{2} b^{2} c^{3} d^{4} + 20 a b^{3} c^{4} d^{3} + 3 b^{4} c^{5} d^{2}\right) + x^{6} \cdot \left(\frac{7 a^{4} c^{2} d^{5}}{2} + \frac{70 a^{3} b c^{3} d^{4}}{3} + 35 a^{2} b^{2} c^{4} d^{3} + 14 a b^{3} c^{5} d^{2} + \frac{7 b^{4} c^{6} d}{6}\right) + x^{5} \cdot \left(7 a^{4} c^{3} d^{4} + 28 a^{3} b c^{4} d^{3} + \frac{126 a^{2} b^{2} c^{5} d^{2}}{5} + \frac{28 a b^{3} c^{6} d}{5} + \frac{b^{4} c^{7}}{5}\right) + x^{4} \cdot \left(\frac{35 a^{4} c^{4} d^{3}}{4} + 21 a^{3} b c^{5} d^{2} + \frac{21 a^{2} b^{2} c^{6} d}{2} + a b^{3} c^{7}\right) + x^{3} \cdot \left(7 a^{4} c^{5} d^{2} + \frac{28 a^{3} b c^{6} d}{3} + 2 a^{2} b^{2} c^{7}\right) + x^{2} \cdot \left(\frac{7 a^{4} c^{6} d}{2} + 2 a^{3} b c^{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**7,x)
```

```
[Out] a**4*c**7*x + b**4*d**7*x**12/12 + x**11*(4*a*b**3*d**7/11 + 7*b**4*c*d**6/
11) + x**10*(3*a**2*b**2*d**7/5 + 14*a*b**3*c*d**6/5 + 21*b**4*c**2*d**5/10
) + x**9*(4*a**3*b*d**7/9 + 14*a**2*b**2*c*d**6/3 + 28*a*b**3*c**2*d**5/3 +
 35*b**4*c**3*d**4/9) + x**8*(a**4*d**7/8 + 7*a**3*b*c*d**6/2 + 63*a**2*b**
2*c**2*d**5/4 + 35*a*b**3*c**3*d**4/2 + 35*b**4*c**4*d**3/8) + x**7*(a**4*c
*d**6 + 12*a**3*b*c**2*d**5 + 30*a**2*b**2*c**3*d**4 + 20*a*b**3*c**4*d**3
+ 3*b**4*c**5*d**2) + x**6*(7*a**4*c**2*d**5/2 + 70*a**3*b*c**3*d**4/3 + 35
*a**2*b**2*c**4*d**3 + 14*a*b**3*c**5*d**2 + 7*b**4*c**6*d/6) + x**5*(7*a**
4*c**3*d**4 + 28*a**3*b*c**4*d**3 + 126*a**2*b**2*c**5*d**2/5 + 28*a*b**3*c
```

```
**6*d/5 + b**4*c**7/5) + x**4*(35*a**4*c**4*d**3/4 + 21*a**3*b*c**5*d**2 +
21*a**2*b**2*c**6*d/2 + a*b**3*c**7) + x**3*(7*a**4*c**5*d**2 + 28*a**3*b*c
**6*d/3 + 2*a**2*b**2*c**7) + x**2*(7*a**4*c**6*d/2 + 2*a**3*b*c**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(109) = 218.
time = 0.54, size = 546, normalized size = 4.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^4*d^7*x^12 + 7/11*b^4*c*d^6*x^11 + 4/11*a*b^3*d^7*x^11 + 21/10*b^4*c
^2*d^5*x^10 + 14/5*a*b^3*c*d^6*x^10 + 3/5*a^2*b^2*d^7*x^10 + 35/9*b^4*c^3*d
^4*x^9 + 28/3*a*b^3*c^2*d^5*x^9 + 14/3*a^2*b^2*c*d^6*x^9 + 4/9*a^3*b*d^7*x^
9 + 35/8*b^4*c^4*d^3*x^8 + 35/2*a*b^3*c^3*d^4*x^8 + 63/4*a^2*b^2*c^2*d^5*x^
8 + 7/2*a^3*b*c*d^6*x^8 + 1/8*a^4*d^7*x^8 + 3*b^4*c^5*d^2*x^7 + 20*a*b^3*c^
4*d^3*x^7 + 30*a^2*b^2*c^3*d^4*x^7 + 12*a^3*b*c^2*d^5*x^7 + a^4*c*d^6*x^7 +
 7/6*b^4*c^6*d*x^6 + 14*a*b^3*c^5*d^2*x^6 + 35*a^2*b^2*c^4*d^3*x^6 + 70/3*a
^3*b*c^3*d^4*x^6 + 7/2*a^4*c^2*d^5*x^6 + 1/5*b^4*c^7*x^5 + 28/5*a*b^3*c^6*d
*x^5 + 126/5*a^2*b^2*c^5*d^2*x^5 + 28*a^3*b*c^4*d^3*x^5 + 7*a^4*c^3*d^4*x^5
 + a*b^3*c^7*x^4 + 21/2*a^2*b^2*c^6*d*x^4 + 21*a^3*b*c^5*d^2*x^4 + 35/4*a^4
*c^4*d^3*x^4 + 2*a^2*b^2*c^7*x^3 + 28/3*a^3*b*c^6*d*x^3 + 7*a^4*c^5*d^2*x^3
 + 2*a^3*b*c^7*x^2 + 7/2*a^4*c^6*d*x^2 + a^4*c^7*x
```

Mupad [B]
time = 0.31, size = 470, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^7,x)
```

```
[Out] x^5*((b^4*c^7)/5 + 7*a^4*c^3*d^4 + 28*a^3*b*c^4*d^3 + (126*a^2*b^2*c^5*d^2)
/5 + (28*a*b^3*c^6*d)/5) + x^8*((a^4*d^7)/8 + (35*b^4*c^4*d^3)/8 + (35*a*b^
3*c^3*d^4)/2 + (63*a^2*b^2*c^2*d^5)/4 + (7*a^3*b*c*d^6)/2) + x^4*(a*b^3*c^7
 + (35*a^4*c^4*d^3)/4 + (21*a^2*b^2*c^6*d)/2 + 21*a^3*b*c^5*d^2) + x^9*((4*
a^3*b*d^7)/9 + (35*b^4*c^3*d^4)/9 + (28*a*b^3*c^2*d^5)/3 + (14*a^2*b^2*c*d^
6)/3) + x^7*(a^4*c*d^6 + 3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 12*a^3*b*c^2*d^
5 + 30*a^2*b^2*c^3*d^4) + x^6*((7*b^4*c^6*d)/6 + (7*a^4*c^2*d^5)/2 + 14*a*b
^3*c^5*d^2 + (70*a^3*b*c^3*d^4)/3 + 35*a^2*b^2*c^4*d^3) + a^4*c^7*x + (b^4*
d^7*x^12)/12 + (a^3*c^6*x^2*(7*a*d + 4*b*c))/2 + (b^3*d^6*x^11*(4*a*d + 7*b
*c))/11 + (a^2*c^5*x^3*(21*a^2*d^2 + 6*b^2*c^2 + 28*a*b*c*d))/3 + (b^2*d^5*
x^10*(6*a^2*d^2 + 21*b^2*c^2 + 28*a*b*c*d))/10
```

3.1279 $\int (a+bx)^3(c+dx)^7\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

```
[Out] -1/8*(-a*d+b*c)^3*(d*x+c)^8/d^4+1/3*b*(-a*d+b*c)^2*(d*x+c)^9/d^4-3/10*b^2*(
-a*d+b*c)*(d*x+c)^10/d^4+1/11*b^3*(d*x+c)^11/d^4
```

Rubi [A]
time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] -1/8*((b*c - a*d)^3*(c + d*x)^8)/d^4 + (b*(b*c - a*d)^2*(c + d*x)^9)/(3*d^4
) - (3*b^2*(b*c - a*d)*(c + d*x)^10)/(10*d^4) + (b^3*(c + d*x)^11)/(11*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^7}{d^3}+\frac{3b(bc-ad)^2(c+dx)^8}{d^3}-\frac{3b^2(bc-ad)(c+dx)^9}{d^3}+\right.\\ &= -\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 360 vs. 2(92) = 184.
time = 0.03, size = 360, normalized size = 3.91

$$a^3c^7x+\frac{1}{2}a^2c^6(3bc+7ad)x^2+ac^5\left(b^2c^2+7abcd+7a^2d^2\right)x^3+\frac{1}{4}c^4\left(b^3c^3+21ab^2c^2d+63a^2bcd^2+35a^3d^3\right)x^4+\frac{7}{5}c^3d\left(b^3c^3+9ab^2c^2d+15a^2bcd^2+5a^3d^3\right)x^5+\frac{7}{2}c^2d^2\left(b^3c^3+5ab^2c^2d+5a^2bcd^2+a^3d^3\right)x^6+cd^3\left(5b^3c^3+15ab^2c^2d+9a^2bcd^2+a^3d^3\right)x^7+\frac{1}{8}d^4\left(35b^3c^3+63ab^2c^2d+21a^2bcd^2+a^3d^3\right)x^8+\frac{1}{3}bd^5\left(7b^2c^2+7abcd+a^2d^2\right)x^9+\frac{1}{10}b^2d^6(7bc+3ad)x^{10}+\frac{1}{11}b^3d^7x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] a^3*c^7*x + (a^2*c^6*(3*b*c + 7*a*d)*x^2)/2 + a*c^5*(b^2*c^2 + 7*a*b*c*d +
7*a^2*d^2)*x^3 + (c^4*(b^3*c^3 + 21*a*b^2*c^2*d + 63*a^2*b*c*d^2 + 35*a^3*d
^3)*x^4)/4 + (7*c^3*d*(b^3*c^3 + 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
)*x^5)/5 + (7*c^2*d^2*(b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*d^3)*x
^6)/2 + c*d^3*(5*b^3*c^3 + 15*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7 +
(d^4*(35*b^3*c^3 + 63*a*b^2*c^2*d + 21*a^2*b*c*d^2 + a^3*d^3)*x^8)/8 + (b*d
^5*(7*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*x^9)/3 + (b^2*d^6*(7*b*c + 3*a*d)*x^10
)/10 + (b^3*d^7*x^11)/11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(84) = 168$.
time = 0.14, size = 385, normalized size = 4.18

method	result
norman	$\frac{b^{3} d^{7} x^{11}}{11}+\left(\frac{3}{10} a\,b^{2} d^{7}+\frac{7}{10} b^{3} c\,d^{6}\right) x^{10}+\left(\frac{1}{3} a^{2} b\,d^{7}+\frac{7}{3} a\,b^{2} c\,d^{6}+\frac{7}{3} b^{3} c^{2} d^{5}\right) x^{9}+\left(\frac{1}{8} a^{3} d^{7}+\frac{21}{8} a^{2} b c\,d^{6}+\frac{63}{8}\right.$
default	$\frac{b^{3} d^{7} x^{11}}{11}+\frac{\left(3 a\,b^{2} d^{7}+7 b^{3} c\,d^{6}\right) x^{10}}{10}+\frac{\left(3 a^{2} b\,d^{7}+21 a\,b^{2} c\,d^{6}+21 b^{3} c^{2} d^{5}\right) x^{9}}{9}+\frac{\left(a^{3} d^{7}+21 a^{2} b c\,d^{6}+63 a\,b^{2} c^{2} d^{5}+35 b^{3} c^{3} d^{4}\right) x^{8}}{8}+\frac{(7 a^{3} c}{}$
gosper	$5 b^{3} c^{4} d^{3} x^{7}+7 a^{3} c^{5} d^{2} x^{3}+a\,b^{2} c^{7} x^{3}+\frac{3}{10} x^{10} a\,b^{2} d^{7}+\frac{7}{10} x^{10} b^{3} c\,d^{6}+\frac{1}{3} x^{9} a^{2} b\,d^{7}+\frac{7}{3} x^{9} b^{3} c^{2} d^{5}+\frac{35}{8} x^{8} b^{3} c^{3}$
risch	$5 b^{3} c^{4} d^{3} x^{7}+7 a^{3} c^{5} d^{2} x^{3}+a\,b^{2} c^{7} x^{3}+\frac{3}{10} x^{10} a\,b^{2} d^{7}+\frac{7}{10} x^{10} b^{3} c\,d^{6}+\frac{1}{3} x^{9} a^{2} b\,d^{7}+\frac{7}{3} x^{9} b^{3} c^{2} d^{5}+\frac{35}{8} x^{8} b^{3} c^{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^3*d^7*x^11+1/10*(3*a*b^2*d^7+7*b^3*c*d^6)*x^10+1/9*(3*a^2*b*d^7+21*a
*b^2*c*d^6+21*b^3*c^2*d^5)*x^9+1/8*(a^3*d^7+21*a^2*b*c*d^6+63*a*b^2*c^2*d^5
+35*b^3*c^3*d^4)*x^8+1/7*(7*a^3*c*d^6+63*a^2*b*c^2*d^5+105*a*b^2*c^3*d^4+35
*b^3*c^4*d^3)*x^7+1/6*(21*a^3*c^2*d^5+105*a^2*b*c^3*d^4+105*a*b^2*c^4*d^3+2
1*b^3*c^5*d^2)*x^6+1/5*(35*a^3*c^3*d^4+105*a^2*b*c^4*d^3+63*a*b^2*c^5*d^2+7
*b^3*c^6*d)*x^5+1/4*(35*a^3*c^4*d^3+63*a^2*b*c^5*d^2+21*a*b^2*c^6*d+b^3*c^7
)*x^4+1/3*(21*a^3*c^5*d^2+21*a^2*b*c^6*d+3*a*b^2*c^7)*x^3+1/2*(7*a^3*c^6*d+
3*a^2*b*c^7)*x^2+a^3*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(84) = 168$.
time = 0.29, size = 376, normalized size = 4.09

$$\frac{1}{11}\,b^3d^7x^{11}+a^3c^7x+\frac{1}{10}\,(7\,b^3cd^6+3\,ab^2d^7)x^{10}+\frac{1}{3}\,(7\,b^3c^2d^5+7\,ab^2cd^6+a^2bd^7)x^9+\frac{1}{8}\,(35\,b^3c^3d^4+63\,ab^2c^2d^5+21\,a^2bcd^6+a^3d^7)x^8+(5\,b^3c^4d^3+15\,ab^2c^3d^4+9\,a^2bc^2d^5+a^3cd^6)x^7+\frac{7}{2}\,(b^3c^5d^2+5\,ab^2c^4d^3+5\,a^2bc^3d^4+a^3c^2d^5)x^6+\frac{7}{5}\,(b^3c^6d+9\,ab^2c^5d^2+15\,a^2bc^4d^3+5\,a^3c^3d^4)x^5+\frac{1}{4}\,(b^3c^7+21\,ab^2c^6d+63\,a^2bc^5d^2+35\,a^3c^4d^3)x^4+(ab^2c^7+7\,a^2bc^6d+7\,a^3c^5d^2)x^3+\frac{1}{2}\,(3\,a^2bc^7+7\,a^3c^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(84) = 168$.

time = 0.97, size = 376, normalized size = 4.09

$$\frac{1}{11}\,b^{3}d^{7}x^{11}+a^{3}c^{7}x+\frac{1}{10}\,(7\,b^{3}cd^{6}+3\,ab^{2}d^{7})x^{10}+\frac{1}{3}\,(7\,b^{3}c^{2}d^{5}+7\,ab^{2}cd^{6}+a^{2}bd^{7})x^{9}+\frac{1}{8}\,(35\,b^{3}c^{3}d^{4}+63\,ab^{2}c^{2}d^{5}+21\,a^{2}bcd^{6}+a^{3}d^{7})x^{8}+(5\,b^{3}c^{4}d^{3}+15\,ab^{2}c^{3}d^{4}+9\,a^{2}bc^{2}d^{5}+a^{3}cd^{6})x^{7}+\frac{7}{2}\,(b^{3}c^{5}d^{2}+5\,ab^{2}c^{4}d^{3}+5\,a^{2}bc^{3}d^{4}+a^{3}c^{2}d^{5})x^{6}+\frac{7}{5}\,(b^{3}c^{6}d+9\,ab^{2}c^{5}d^{2}+15\,a^{2}bc^{4}d^{3}+5\,a^{3}c^{3}d^{4})x^{5}+\frac{1}{4}\,(b^{3}c^{7}+21\,ab^{2}c^{6}d+63\,a^{2}bc^{5}d^{2}+35\,a^{3}c^{4}d^{3})x^{4}+(ab^{2}c^{7}+7\,a^{2}bc^{6}d+7\,a^{3}c^{5}d^{2})x^{3}+\frac{1}{2}\,(3\,a^{2}bc^{7}+7\,a^{3}c^{6}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="fricas")
```


```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```


Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(80) = 160$.

time = 0.04, size = 427, normalized size = 4.64

$$a^{3}c^{7}x+\frac{b^{3}d^{7}x^{11}}{11}+x^{10}\cdot\left(\frac{3ab^{2}d^{7}}{10}+\frac{7b^{3}cd^{6}}{10}\right)+x^{9}\left(\frac{a^{2}bd^{7}}{3}+\frac{7ab^{2}cd^{6}}{3}+\frac{7b^{3}c^{2}d^{5}}{3}\right)+x^{8}\left(\frac{a^{3}d^{7}}{8}+\frac{21a^{2}bcd^{6}}{8}+\frac{63ab^{2}c^{2}d^{5}}{8}+\frac{35b^{3}c^{3}d^{4}}{8}\right)+x^{7}\left(a^{3}cd^{6}+9a^{2}bc^{2}d^{5}+15ab^{2}c^{3}d^{4}+5b^{3}c^{4}d^{3}\right)+x^{6}\cdot\left(\frac{7a^{3}c^{2}d^{5}}{2}+\frac{35a^{2}bc^{3}d^{4}}{2}+\frac{35ab^{2}c^{4}d^{3}}{2}+\frac{7b^{3}c^{5}d^{2}}{2}\right)+x^{5}\cdot\left(7a^{3}c^{3}d^{4}+21a^{2}bc^{4}d^{3}+\frac{63ab^{2}c^{5}d^{2}}{5}+\frac{7b^{3}c^{6}d}{5}\right)+x^{4}\cdot\left(\frac{35a^{3}c^{4}d^{3}}{4}+\frac{63a^{2}bc^{5}d^{2}}{4}+\frac{21ab^{2}c^{6}d}{4}+\frac{b^{3}c^{7}}{4}\right)+x^{3}\cdot\left(7a^{3}c^{5}d^{2}+7a^{2}bc^{6}d+ab^{2}c^{7}\right)+x^{2}\cdot\left(\frac{7a^{3}c^{6}d}{2}+\frac{3a^{2}bc^{7}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**7,x)
```

```
[Out] a**3*c**7*x + b**3*d**7*x**11/11 + x**10*(3*a*b**2*d**7/10 + 7*b**3*c*d**6/
10) + x**9*(a**2*b*d**7/3 + 7*a*b**2*c*d**6/3 + 7*b**3*c**2*d**5/3) + x**8*
(a**3*d**7/8 + 21*a**2*b*c*d**6/8 + 63*a*b**2*c**2*d**5/8 + 35*b**3*c**3*d*
*4/8) + x**7*(a**3*c*d**6 + 9*a**2*b*c**2*d**5 + 15*a*b**2*c**3*d**4 + 5*b*
*3*c**4*d**3) + x**6*(7*a**3*c**2*d**5/2 + 35*a**2*b*c**3*d**4/2 + 35*a*b**
2*c**4*d**3/2 + 7*b**3*c**5*d**2/2) + x**5*(7*a**3*c**3*d**4 + 21*a**2*b*c*
*4*d**3 + 63*a*b**2*c**5*d**2/5 + 7*b**3*c**6*d/5) + x**4*(35*a**3*c**4*d**
3/4 + 63*a**2*b*c**5*d**2/4 + 21*a*b**2*c**6*d/4 + b**3*c**7/4) + x**3*(7*a
**3*c**5*d**2 + 7*a**2*b*c**6*d + a*b**2*c**7) + x**2*(7*a**3*c**6*d/2 + 3*
a**2*b*c**7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(84) = 168.
time = 0.93, size = 420, normalized size = 4.57

$$\frac{1}{11}b^3d^7x^{11}+\frac{7}{10}b^3cd^6x^{10}+\frac{3}{10}ab^2d^7x^{10}+\frac{7}{3}b^3c^2d^5x^9+\frac{7}{3}ab^2cd^6x^9+\frac{1}{3}a^2bd^7x^9+\frac{35}{8}b^3c^3d^4x^8+\frac{63}{8}ab^2c^2d^5x^8+\frac{21}{8}a^2bcd^6x^8+\frac{1}{8}a^3d^7x^8+5b^3c^4d^3x^7+15ab^2c^3d^4x^7+9a^2bc^2d^5x^7+a^3cd^6x^7+\frac{7}{2}b^3c^5d^2x^6+\frac{35}{2}ab^2c^4d^3x^6+\frac{35}{2}a^2bc^3d^4x^6+\frac{7}{2}a^3c^2d^5x^6+\frac{7}{5}b^3c^6dx^5+\frac{63}{5}ab^2c^5d^2x^5+21a^2bc^4d^3x^5+7a^3c^3d^4x^5+\frac{1}{4}b^3c^7x^4+\frac{21}{4}ab^2c^6dx^4+\frac{63}{4}a^2bc^5d^2x^4+\frac{35}{4}a^3c^4d^3x^4+ab^2c^7x^3+7a^2bc^6dx^3+7a^3c^5d^2x^3+\frac{3}{2}a^2bc^7x^2+\frac{7}{2}a^3c^6dx^2+a^3c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^7*x^11 + 7/10*b^3*c*d^6*x^10 + 3/10*a*b^2*d^7*x^10 + 7/3*b^3*c^2
*d^5*x^9 + 7/3*a*b^2*c*d^6*x^9 + 1/3*a^2*b*d^7*x^9 + 35/8*b^3*c^3*d^4*x^8 +
 63/8*a*b^2*c^2*d^5*x^8 + 21/8*a^2*b*c*d^6*x^8 + 1/8*a^3*d^7*x^8 + 5*b^3*c^
4*d^3*x^7 + 15*a*b^2*c^3*d^4*x^7 + 9*a^2*b*c^2*d^5*x^7 + a^3*c*d^6*x^7 + 7/
2*b^3*c^5*d^2*x^6 + 35/2*a*b^2*c^4*d^3*x^6 + 35/2*a^2*b*c^3*d^4*x^6 + 7/2*a
^3*c^2*d^5*x^6 + 7/5*b^3*c^6*d*x^5 + 63/5*a*b^2*c^5*d^2*x^5 + 21*a^2*b*c^4*
d^3*x^5 + 7*a^3*c^3*d^4*x^5 + 1/4*b^3*c^7*x^4 + 21/4*a*b^2*c^6*d*x^4 + 63/4
*a^2*b*c^5*d^2*x^4 + 35/4*a^3*c^4*d^3*x^4 + a*b^2*c^7*x^3 + 7*a^2*b*c^6*d*x
^3 + 7*a^3*c^5*d^2*x^3 + 3/2*a^2*b*c^7*x^2 + 7/2*a^3*c^6*d*x^2 + a^3*c^7*x
```

Mupad [B]
time = 0.27, size = 356, normalized size = 3.87

$$x^7\left(a^3cd^6+9a^2bc^2d^5+15ab^2c^3d^4+5b^3c^4d^3\right)+x^5\left(7a^3c^3d^4+21a^2bc^4d^3+\frac{63ab^2c^5d^2}{5}+\frac{7b^3c^6d}{5}\right)+x^4\left(\frac{35a^3c^4d^3}{4}+\frac{63a^2bc^5d^2}{4}+\frac{21ab^2c^6d}{4}+\frac{b^3c^7}{4}\right)+x^8\left(\frac{a^3d^7}{8}+\frac{21a^2bcd^6}{8}+\frac{63ab^2c^2d^5}{8}+\frac{35b^3c^3d^4}{8}\right)+a^3c^7x+\frac{b^3d^7x^{11}}{11}+\frac{7c^2d^2x^6\left(a^3d^3+5a^2bcd^2+5ab^2c^2d+b^3c^3\right)}{2}+\frac{a^2c^6x^2\left(7ad+3bc\right)}{2}+\frac{b^2d^6x^{10}\left(3ad+7bc\right)}{10}+ac^5x^3\left(7a^2d^2+7abcd+b^2c^2\right)+\frac{bd^5x^9\left(a^2d^2+7abcd+7b^2c^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^7,x)
```

```
[Out] x^7*(a^3*c*d^6 + 5*b^3*c^4*d^3 + 15*a*b^2*c^3*d^4 + 9*a^2*b*c^2*d^5) + x^5*
((7*b^3*c^6*d)/5 + 7*a^3*c^3*d^4 + (63*a*b^2*c^5*d^2)/5 + 21*a^2*b*c^4*d^3)
 + x^4*((b^3*c^7)/4 + (35*a^3*c^4*d^3)/4 + (63*a^2*b*c^5*d^2)/4 + (21*a*b^2
*c^6*d)/4) + x^8*((a^3*d^7)/8 + (35*b^3*c^3*d^4)/8 + (63*a*b^2*c^2*d^5)/8 +
 (21*a^2*b*c*d^6)/8) + a^3*c^7*x + (b^3*d^7*x^11)/11 + (7*c^2*d^2*x^6*(a^3*
d^3 + b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2))/2 + (a^2*c^6*x^2*(7*a*d + 3
*b*c))/2 + (b^2*d^6*x^10*(3*a*d + 7*b*c))/10 + a*c^5*x^3*(7*a^2*d^2 + b^2*c
^2 + 7*a*b*c*d) + (b*d^5*x^9*(a^2*d^2 + 7*b^2*c^2 + 7*a*b*c*d))/3
```

3.1280 $\int (a+bx)^2(c+dx)^7\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

```
[Out] 1/8*(-a*d+b*c)^2*(d*x+c)^8/d^3-2/9*b*(-a*d+b*c)*(d*x+c)^9/d^3+1/10*b^2*(d*x
+c)^10/d^3
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {45}

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^8)/(8*d^3) - (2*b*(b*c - a*d)*(c + d*x)^9)/(9*d^3)
 + (b^2*(c + d*x)^10)/(10*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^7}{d^2}-\frac{2b(bc-ad)(c+dx)^8}{d^2}+\frac{b^2(c+dx)^9}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(65) = 130.
time = 0.02, size = 261, normalized size = 4.02

$$a^2c^7x+\frac{1}{2}ac^6(2bc+7ad)x^2+\frac{1}{3}c^5\left(b^2c^2+14abcd+21a^2d^2\right)x^3+\frac{7}{4}c^4d\left(b^2c^2+6abcd+5a^2d^2\right)x^4+\frac{7}{5}c^3d^2\left(3b^2c^2+10abcd+5a^2d^2\right)x^5+\frac{7}{6}c^2d^3\left(5b^2c^2+10abcd+3a^2d^2\right)x^6+cd^4\left(5b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{8}d^5\left(21b^2c^2+14abcd+a^2d^2\right)x^8+\frac{1}{9}bd^6(7bc+2ad)x^9+\frac{1}{10}b^2d^7x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] a^2*c^7*x + (a*c^6*(2*b*c + 7*a*d)*x^2)/2 + (c^5*(b^2*c^2 + 14*a*b*c*d + 21
*a^2*d^2)*x^3)/3 + (7*c^4*d*(b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*x^4)/4 + (7*c
^3*d^2*(3*b^2*c^2 + 10*a*b*c*d + 5*a^2*d^2)*x^5)/5 + (7*c^2*d^3*(5*b^2*c^2
+ 10*a*b*c*d + 3*a^2*d^2)*x^6)/6 + c*d^4*(5*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^7 + (d^5*(21*b^2*c^2 + 14*a*b*c*d + a^2*d^2)*x^8)/8 + (b*d^6*(7*b*c + 2*a
*d)*x^9)/9 + (b^2*d^7*x^10)/10
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(59) = 118$.
time = 0.13, size = 277, normalized size = 4.26

method	result
norman	$\frac{b^2 d^7 x^{10}}{10} + \left(\frac{2}{9} ab\, d^7 + \frac{7}{9} b^2 c\, d^6\right) x^9 + \left(\frac{1}{8} a^2 d^7 + \frac{7}{4} abc\, d^6 + \frac{21}{8} b^2 c^2 d^5\right) x^8 + \left(a^2 c\, d^6 + 6ab\, c^2 d^5 + 5b^2 c^3 d^4\right)$
default	$\frac{b^2 d^7 x^{10}}{10} + \frac{(2ab\, d^7 + 7b^2 c\, d^6) x^9}{9} + \frac{(a^2 d^7 + 14abc\, d^6 + 21b^2 c^2 d^5) x^8}{8} + \frac{(7a^2 c\, d^6 + 42ab\, c^2 d^5 + 35b^2 c^3 d^4) x^7}{7} + \frac{(21a^2 c^2 d^5 + 70ab\, c^3 d^4}{6}$
gosper	$\frac{1}{10} b^2 d^7 x^{10} + \frac{2}{9} x^9 ab\, d^7 + \frac{7}{9} x^9 b^2 c\, d^6 + \frac{1}{8} x^8 a^2 d^7 + \frac{7}{4} x^8 abc\, d^6 + \frac{21}{8} x^8 b^2 c^2 d^5 + a^2 c\, d^6 x^7 + 6ab\, c^2 d^5 x^7 +$
risch	$\frac{1}{10} b^2 d^7 x^{10} + \frac{2}{9} x^9 ab\, d^7 + \frac{7}{9} x^9 b^2 c\, d^6 + \frac{1}{8} x^8 a^2 d^7 + \frac{7}{4} x^8 abc\, d^6 + \frac{21}{8} x^8 b^2 c^2 d^5 + a^2 c\, d^6 x^7 + 6ab\, c^2 d^5 x^7 +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*d^7*x^10+1/9*(2*a*b*d^7+7*b^2*c*d^6)*x^9+1/8*(a^2*d^7+14*a*b*c*d^6
+21*b^2*c^2*d^5)*x^8+1/7*(7*a^2*c*d^6+42*a*b*c^2*d^5+35*b^2*c^3*d^4)*x^7+1/
6*(21*a^2*c^2*d^5+70*a*b*c^3*d^4+35*b^2*c^4*d^3)*x^6+1/5*(35*a^2*c^3*d^4+70
*a*b*c^4*d^3+21*b^2*c^5*d^2)*x^5+1/4*(35*a^2*c^4*d^3+42*a*b*c^5*d^2+7*b^2*c
^6*d)*x^4+1/3*(21*a^2*c^5*d^2+14*a*b*c^6*d+b^2*c^7)*x^3+1/2*(7*a^2*c^6*d+2*
a*b*c^7)*x^2+a^2*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(59) = 118$.
time = 0.27, size = 273, normalized size = 4.20

$$\frac{1}{10} b^2 d^7 x^{10} + a^2 c^7 x + \frac{1}{9} (7 b^2 c d^6 + 2 abd^7) x^9 + \frac{1}{8} (21 b^2 c^2 d^5 + 14 abcd^6 + a^2 d^7) x^8 + (5 b^2 c^3 d^4 + 6 abc^2 d^5 + a^2 cd^6) x^7 + \frac{7}{6} (5 b^2 c^4 d^3 + 10 abc^3 d^4 + 3 a^2 c^2 d^5) x^6 + \frac{7}{5} (3 b^2 c^5 d^2 + 10 abc^4 d^3 + 5 a^2 c^3 d^4) x^5 + \frac{7}{4} (b^2 c^6 d + 6 abc^5 d^2 + 5 a^2 c^4 d^3) x^4 + \frac{1}{3} (b^2 c^7 + 14 abc^6 d + 21 a^2 c^5 d^2) x^3 + \frac{1}{2} (2 abc^7 + 7 a^2 c^6 d) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
```

```
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(59) = 118$.
time = 0.99, size = 273, normalized size = 4.20

$$\frac{1}{10}b^2d^7x^{10} + a^2c^7x + \frac{1}{9}\left(7b^2cd^6 + 2abd^7\right)x^9 + \frac{1}{8}\left(21b^2c^2d^5 + 14abcd^6 + a^2d^7\right)x^8 + \left(5b^2c^3d^4 + 6abc^2d^5 + a^2cd^6\right)x^7 + \frac{7}{6}\left(5b^2c^4d^3 + 10abc^3d^4 + 3a^2c^2d^5\right)x^6 + \frac{7}{5}\left(3b^2c^5d^2 + 10abc^4d^3 + 5a^2c^3d^4\right)x^5 + \frac{7}{4}\left(b^2c^6d + 6abc^5d^2 + 5a^2c^4d^3\right)x^4 + \frac{1}{3}\left(b^2c^7 + 14abc^6d + 21a^2c^5d^2\right)x^3 + \frac{1}{2}\left(2abc^7 + 7a^2c^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(56) = 112$.
time = 0.03, size = 303, normalized size = 4.66

$$a^2c^7x + \frac{b^2d^7x^{10}}{10} + x^9\cdot\left(\frac{2abd^7}{9} + \frac{7b^2cd^6}{9}\right) + x^8\left(\frac{a^2d^7}{8} + \frac{7abcd^6}{4} + \frac{21b^2c^2d^5}{8}\right) + x^7\left(a^2cd^6 + 6abc^2d^5 + 5b^2c^3d^4\right) + x^6\cdot\left(\frac{7a^2c^2d^5}{2} + \frac{35abc^3d^4}{3} + \frac{35b^2c^4d^3}{6}\right) + x^5\cdot\left(7a^2c^3d^4 + 14abc^4d^3 + \frac{21b^2c^5d^2}{5}\right) + x^4\cdot\left(\frac{35a^2c^4d^3}{4} + \frac{21abc^5d^2}{2} + \frac{7b^2c^6d}{4}\right) + x^3\cdot\left(7a^2c^5d^2 + \frac{14abc^6d}{3} + \frac{b^2c^7}{3}\right) + x^2\cdot\left(\frac{7a^2c^6d}{2} + abc^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**7,x)
```

```
[Out] a**2*c**7*x + b**2*d**7*x**10/10 + x**9*(2*a*b*d**7/9 + 7*b**2*c*d**6/9) +
x**8*(a**2*d**7/8 + 7*a*b*c*d**6/4 + 21*b**2*c**2*d**5/8) + x**7*(a**2*c*d*
*6 + 6*a*b*c**2*d**5 + 5*b**2*c**3*d**4) + x**6*(7*a**2*c**2*d**5/2 + 35*a*
b*c**3*d**4/3 + 35*b**2*c**4*d**3/6) + x**5*(7*a**2*c**3*d**4 + 14*a*b*c**4
*d**3 + 21*b**2*c**5*d**2/5) + x**4*(35*a**2*c**4*d**3/4 + 21*a*b*c**5*d**2
/2 + 7*b**2*c**6*d/4) + x**3*(7*a**2*c**5*d**2 + 14*a*b*c**6*d/3 + b**2*c**
7/3) + x**2*(7*a**2*c**6*d/2 + a*b*c**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(59) = 118$.
time = 0.73, size = 294, normalized size = 4.52

$$\frac{1}{10}b^2d^7x^{10} + \frac{7}{9}b^2cd^6x^9 + \frac{2}{9}abd^7x^9 + \frac{21}{8}b^2c^2d^5x^8 + \frac{7}{4}abcd^6x^8 + \frac{1}{8}a^2d^7x^8 + 5b^2c^3d^4x^7 + 6abc^2d^5x^7 + a^2cd^6x^7 + \frac{35}{6}b^2c^4d^3x^6 + \frac{35}{3}abc^3d^4x^6 + \frac{7}{2}a^2c^2d^5x^6 + \frac{21}{5}b^2c^5d^2x^5 + 14abc^4d^3x^5 + 7a^2c^3d^4x^5 + \frac{7}{4}b^2c^6dx^4 + \frac{21}{2}abc^5d^2x^4 + \frac{35}{4}a^2c^4d^3x^4 + \frac{1}{3}b^2c^7x^3 + \frac{14}{3}abc^6dx^3 + 7a^2c^5d^2x^3 + abc^7x^2 + \frac{7}{2}a^2c^6dx^2 + a^2c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^7*x^10 + 7/9*b^2*c*d^6*x^9 + 2/9*a*b*d^7*x^9 + 21/8*b^2*c^2*d^5*
x^8 + 7/4*a*b*c*d^6*x^8 + 1/8*a^2*d^7*x^8 + 5*b^2*c^3*d^4*x^7 + 6*a*b*c^2*d
```

```
^5*x^7 + a^2*c*d^6*x^7 + 35/6*b^2*c^4*d^3*x^6 + 35/3*a*b*c^3*d^4*x^6 + 7/2*
a^2*c^2*d^5*x^6 + 21/5*b^2*c^5*d^2*x^5 + 14*a*b*c^4*d^3*x^5 + 7*a^2*c^3*d^4
*x^5 + 7/4*b^2*c^6*d*x^4 + 21/2*a*b*c^5*d^2*x^4 + 35/4*a^2*c^4*d^3*x^4 + 1/
3*b^2*c^7*x^3 + 14/3*a*b*c^6*d*x^3 + 7*a^2*c^5*d^2*x^3 + a*b*c^7*x^2 + 7/2*
a^2*c^6*d*x^2 + a^2*c^7*x
```

Mupad [B]
time = 0.11, size = 249, normalized size = 3.83

$$x^3\left(7a^2c^5d^2+\frac{14abc^6d}{3}+\frac{b^2c^7}{3}\right)+x^8\left(\frac{a^2d^7}{8}+\frac{7abcd^6}{4}+\frac{21b^2c^2d^5}{8}\right)+a^2c^7x+\frac{b^2d^7x^{10}}{10}+\frac{ac^6x^2(7ad+2bc)}{2}+\frac{bd^6x^9(2ad+7bc)}{9}+\frac{7c^4dx^4(5a^2d^2+6abcd+b^2c^2)}{4}+cd^4x^7\left(a^2d^2+6abcd+5b^2c^2\right)+\frac{7c^3d^2x^5(5a^2d^2+10abcd+3b^2c^2)}{5}+\frac{7c^2d^3x^6(3a^2d^2+10abcd+5b^2c^2)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^7,x)
```

```
[Out] x^3*((b^2*c^7)/3 + 7*a^2*c^5*d^2 + (14*a*b*c^6*d)/3) + x^8*((a^2*d^7)/8 + (
21*b^2*c^2*d^5)/8 + (7*a*b*c*d^6)/4) + a^2*c^7*x + (b^2*d^7*x^10)/10 + (a*c
^6*x^2*(7*a*d + 2*b*c))/2 + (b*d^6*x^9*(2*a*d + 7*b*c))/9 + (7*c^4*d*x^4*(5
*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/4 + c*d^4*x^7*(a^2*d^2 + 5*b^2*c^2 + 6*a*b
*c*d) + (7*c^3*d^2*x^5*(5*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/5 + (7*c^2*d^3
*x^6*(3*a^2*d^2 + 5*b^2*c^2 + 10*a*b*c*d))/6
```

3.1281 $\int (a+bx)(c+dx)^7\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}$$

```
[Out] -1/8*(-a*d+b*c)*(d*x+c)^8/d^2+1/9*b*(d*x+c)^9/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] -1/8*((b*c - a*d)*(c + d*x)^8)/d^2 + (b*(c + d*x)^9)/(9*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^7}{d}+\frac{b(c+dx)^8}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(38) = 76.
time = 0.01, size = 151, normalized size = 3.97

$$ac^7x+\frac{1}{2}c^6(bc+7ad)x^2+\frac{7}{3}c^5d(bc+3ad)x^3+\frac{7}{4}c^4d^2(3bc+5ad)x^4+7c^3d^3(bc+ad)x^5+\frac{7}{6}c^2d^4(5bc+3ad)x^6+cd^5(3bc+ad)x^7+\frac{1}{8}d^6(7bc+ad)x^8+\frac{1}{9}bd^7x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] a*c^7*x + (c^6*(b*c + 7*a*d)*x^2)/2 + (7*c^5*d*(b*c + 3*a*d)*x^3)/3 + (7*c^
4*d^2*(3*b*c + 5*a*d)*x^4)/4 + 7*c^3*d^3*(b*c + a*d)*x^5 + (7*c^2*d^4*(5*b*
c + 3*a*d)*x^6)/6 + c*d^5*(3*b*c + a*d)*x^7 + (d^6*(7*b*c + a*d)*x^8)/8 + (
b*d^7*x^9)/9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(34) = 68$.
time = 0.12, size = 169, normalized size = 4.45

method	result
norman	$\frac{b d^7 x^9}{9}+\left(\frac{1}{8}a d^7+\frac{7}{8}bc d^6\right)x^8+\left(ac d^6+3b c^2 d^5\right)x^7+\left(\frac{7}{2}a c^2 d^5+\frac{35}{6}b c^3 d^4\right)x^6+\left(7a c^3 d^4+7b c^4 d^3\right)x$
default	$\frac{b d^7 x^9}{9}+\frac{(a d^7+7bc d^6)x^8}{8}+\frac{(7ac d^6+21b c^2 d^5)x^7}{7}+\frac{(21a c^2 d^5+35b c^3 d^4)x^6}{6}+\frac{(35a c^3 d^4+35b c^4 d^3)x^5}{5}+\frac{(35a c^4 d^3+21b c^5 d^2}{4}$
gosper	$\frac{1}{9}b d^7 x^9+\frac{1}{8}x^8 a d^7+\frac{7}{8}x^8 bc d^6+ac d^6 x^7+3b c^2 d^5 x^7+\frac{7}{2}x^6 a c^2 d^5+\frac{35}{6}x^6 b c^3 d^4+7a c^3 d^4 x^5+7b c^4 d^3$
risch	$\frac{1}{9}b d^7 x^9+\frac{1}{8}x^8 a d^7+\frac{7}{8}x^8 bc d^6+ac d^6 x^7+3b c^2 d^5 x^7+\frac{7}{2}x^6 a c^2 d^5+\frac{35}{6}x^6 b c^3 d^4+7a c^3 d^4 x^5+7b c^4 d^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d^7*x^9+1/8*(a*d^7+7*b*c*d^6)*x^8+1/7*(7*a*c*d^6+21*b*c^2*d^5)*x^7+1/
6*(21*a*c^2*d^5+35*b*c^3*d^4)*x^6+1/5*(35*a*c^3*d^4+35*b*c^4*d^3)*x^5+1/4*(
35*a*c^4*d^3+21*b*c^5*d^2)*x^4+1/3*(21*a*c^5*d^2+7*b*c^6*d)*x^3+1/2*(7*a*c^
6*d+b*c^7)*x^2+a*c^7*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(34) = 68$.
time = 0.27, size = 163, normalized size = 4.29

$$\frac{1}{9}bd^7x^9+ac^7x+\frac{1}{8}(7bcd^6+ad^7)x^8+(3bc^2d^5+acd^6)x^7+\frac{7}{6}(5bc^3d^4+3ac^2d^5)x^6+7(bc^4d^3+ac^3d^4)x^5+\frac{7}{4}(3bc^5d^2+5ac^4d^3)x^4+\frac{7}{3}(bc^6d+3ac^5d^2)x^3+\frac{1}{2}(bc^7+7ac^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(34) = 68$.
time = 1.07, size = 163, normalized size = 4.29

$$\frac{1}{9}bd^7x^9+ac^7x+\frac{1}{8}(7bcd^6+ad^7)x^8+(3bc^2d^5+acd^6)x^7+\frac{7}{6}(5bc^3d^4+3ac^2d^5)x^6+7(bc^4d^3+ac^3d^4)x^5+\frac{7}{4}(3bc^5d^2+5ac^4d^3)x^4+\frac{7}{3}(bc^6d+3ac^5d^2)x^3+\frac{1}{2}(bc^7+7ac^6d)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(32) = 64$.
time = 0.02, size = 178, normalized size = 4.68

$$ac^7x + \frac{bd^7x^9}{9} + x^8\left(\frac{ad^7}{8} + \frac{7bcd^6}{8}\right) + x^7\left(acd^6 + 3bc^2d^5\right) + x^6\cdot\left(\frac{7ac^2d^5}{2} + \frac{35bc^3d^4}{6}\right) + x^5\cdot\left(7ac^3d^4 + 7bc^4d^3\right) + x^4\cdot\left(\frac{35ac^4d^3}{4} + \frac{21bc^5d^2}{4}\right) + x^3\cdot\left(7ac^5d^2 + \frac{7bc^6d}{3}\right) + x^2\cdot\left(\frac{7ac^6d}{2} + \frac{bc^7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**7,x)
```

```
[Out] a*c**7*x + b*d**7*x**9/9 + x**8*(a*d**7/8 + 7*b*c*d**6/8) + x**7*(a*c*d**6
+ 3*b*c**2*d**5) + x**6*(7*a*c**2*d**5/2 + 35*b*c**3*d**4/6) + x**5*(7*a*c*
*3*d**4 + 7*b*c**4*d**3) + x**4*(35*a*c**4*d**3/4 + 21*b*c**5*d**2/4) + x**
3*(7*a*c**5*d**2 + 7*b*c**6*d/3) + x**2*(7*a*c**6*d/2 + b*c**7/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(34) = 68$.
time = 0.62, size = 169, normalized size = 4.45

$$\frac{1}{9}\,bd^7x^9 + \frac{7}{8}\,bcd^6x^8 + \frac{1}{8}\,ad^7x^8 + 3\,bc^2d^5x^7 + acd^6x^7 + \frac{35}{6}\,bc^3d^4x^6 + \frac{7}{2}\,ac^2d^5x^6 + 7\,bc^4d^3x^5 + 7\,ac^3d^4x^5 + \frac{21}{4}\,bc^5d^2x^4 + \frac{35}{4}\,ac^4d^3x^4 + \frac{7}{3}\,bc^6dx^3 + 7\,ac^5d^2x^3 + \frac{1}{2}\,bc^7x^2 + \frac{7}{2}\,ac^6dx^2 + ac^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/9*b*d^7*x^9 + 7/8*b*c*d^6*x^8 + 1/8*a*d^7*x^8 + 3*b*c^2*d^5*x^7 + a*c*d^6
*x^7 + 35/6*b*c^3*d^4*x^6 + 7/2*a*c^2*d^5*x^6 + 7*b*c^4*d^3*x^5 + 7*a*c^3*d
^4*x^5 + 21/4*b*c^5*d^2*x^4 + 35/4*a*c^4*d^3*x^4 + 7/3*b*c^6*d*x^3 + 7*a*c^
5*d^2*x^3 + 1/2*b*c^7*x^2 + 7/2*a*c^6*d*x^2 + a*c^7*x
```

Mupad [**B**]
time = 0.08, size = 143, normalized size = 3.76

$$x^2\left(\frac{b\,c^7}{2} + \frac{7\,a\,d\,c^6}{2}\right) + x^8\left(\frac{a\,d^7}{8} + \frac{7\,b\,c\,d^6}{8}\right) + \frac{b\,d^7\,x^9}{9} + a\,c^7\,x + \frac{7\,c^5\,d\,x^3\,(3\,a\,d + b\,c)}{3} + c\,d^5\,x^7\,(a\,d + 3\,b\,c) + 7\,c^3\,d^3\,x^5\,(a\,d + b\,c) + \frac{7\,c^4\,d^2\,x^4\,(5\,a\,d + 3\,b\,c)}{4} + \frac{7\,c^2\,d^4\,x^6\,(3\,a\,d + 5\,b\,c)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^7,x)
```

```
[Out] x^2*((b*c^7)/2 + (7*a*c^6*d)/2) + x^8*((a*d^7)/8 + (7*b*c*d^6)/8) + (b*d^7*
x^9)/9 + a*c^7*x + (7*c^5*d*x^3*(3*a*d + b*c))/3 + c*d^5*x^7*(a*d + 3*b*c)
+ 7*c^3*d^3*x^5*(a*d + b*c) + (7*c^4*d^2*x^4*(5*a*d + 3*b*c))/4 + (7*c^2*d^
4*x^6*(3*a*d + 5*b*c))/6
```

3.1282 $\int (c+dx)^7 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^8}{8d}$$

```
[Out] 1/8*(d*x+c)^8/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^7 \, dx = \frac{(c+dx)^8}{8d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7,x]
```

```
[Out] (c + d*x)^8/(8*d)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{(dx+c)^8}{8d}$	13
gosper	$\frac{1}{8}d^7x^8+c\,d^6x^7+\frac{7}{2}c^2d^5x^6+7c^3d^4x^5+\frac{35}{4}c^4d^3x^4+7c^5d^2x^3+\frac{7}{2}c^6d\,x^2+c^7x$	76
norman	$\frac{1}{8}d^7x^8+c\,d^6x^7+\frac{7}{2}c^2d^5x^6+7c^3d^4x^5+\frac{35}{4}c^4d^3x^4+7c^5d^2x^3+\frac{7}{2}c^6d\,x^2+c^7x$	76
risch	$\frac{d^7x^8}{8}+c\,d^6x^7+\frac{7c^2d^5x^6}{2}+7c^3d^4x^5+\frac{35c^4d^3x^4}{4}+7c^5d^2x^3+\frac{7c^6d\,x^2}{2}+c^7x+\frac{c^8}{8d}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(d*x+c)^8/d
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$\frac{(dx+c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^8/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(12) = 24$.
time = 1.04, size = 75, normalized size = 5.36

$$\frac{1}{8}\,d^7x^8+cd^6x^7+\frac{7}{2}\,c^2d^5x^6+7\,c^3d^4x^5+\frac{35}{4}\,c^4d^3x^4+7\,c^5d^2x^3+\frac{7}{2}\,c^6dx^2+c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/8*d^7*x^8 + c*d^6*x^7 + 7/2*c^2*d^5*x^6 + 7*c^3*d^4*x^5 + 35/4*c^4*d^3*x^
4 + 7*c^5*d^2*x^3 + 7/2*c^6*d*x^2 + c^7*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.
time = 0.01, size = 83, normalized size = 5.93

$$c^7x+\frac{7c^6dx^2}{2}+7c^5d^2x^3+\frac{35c^4d^3x^4}{4}+7c^3d^4x^5+\frac{7c^2d^5x^6}{2}+cd^6x^7+\frac{d^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7,x)
```

[Out] c**7*x + 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*d**4*x**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8

Giac [A]
time = 0.87, size = 12, normalized size = 0.86

$$\frac{(dx+c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7,x, algorithm="giac")

[Out] 1/8*(d*x + c)^8/d

Mupad [B]
time = 0.06, size = 75, normalized size = 5.36

$$c^7\,x+\frac{7\,c^6\,d\,x^2}{2}+7\,c^5\,d^2\,x^3+\frac{35\,c^4\,d^3\,x^4}{4}+7\,c^3\,d^4\,x^5+\frac{7\,c^2\,d^5\,x^6}{2}+c\,d^6\,x^7+\frac{d^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^7,x)

[Out] c^7*x + (d^7*x^8)/8 + (7*c^6*d*x^2)/2 + c*d^6*x^7 + 7*c^5*d^2*x^3 + (35*c^4*d^3*x^4)/4 + 7*c^3*d^4*x^5 + (7*c^2*d^5*x^6)/2

3.1283 $\int \frac{(c+dx)^7}{a+bx} \, dx$

Optimal. Leaf size=169

$$\frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc-ad)^2(c+dx)^5}{5b^3}+\frac{(bc-$$

```
[Out] d*(-a*d+b*c)^6*x/b^7+1/2*(-a*d+b*c)^5*(d*x+c)^2/b^6+1/3*(-a*d+b*c)^4*(d*x+c
)^3/b^5+1/4*(-a*d+b*c)^3*(d*x+c)^4/b^4+1/5*(-a*d+b*c)^2*(d*x+c)^5/b^3+1/6*(
-a*d+b*c)*(d*x+c)^6/b^2+1/7*(d*x+c)^7/b+(-a*d+b*c)^7*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.05, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^7\log(a+bx)}{b^8}+\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(bc-ad)^2}{5b^3}+\frac{(c+dx)^6(bc-ad)}{6b^2}+\frac{(c+dx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^6*x)/b^7 + ((b*c - a*d)^5*(c + d*x)^2)/(2*b^6) + ((b*c - a*d
)^4*(c + d*x)^3)/(3*b^5) + ((b*c - a*d)^3*(c + d*x)^4)/(4*b^4) + ((b*c - a*
d)^2*(c + d*x)^5)/(5*b^3) + ((b*c - a*d)*(c + d*x)^6)/(6*b^2) + (c + d*x)^7
/(7*b) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^6}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)}+\frac{d(bc-ad)^5(c+dx)}{b^6}+\frac{d(bc-ad)^4(c+dx)^2}{b^5}+\frac{d(bc-a\\ &= \frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc\end{aligned}$$

Mathematica [A]
time = 0.10, size = 304, normalized size = 1.80

$$\frac{dx(420a^6d^6-210a^5bd^5(14c+dx)+70a^4b^2d^4(126c^2+21cdx+2d^2x^2)-35a^3b^3d^3(420c^3+126c^2dx+28cd^2x^2+3d^3x^3)+21a^2b^4d^2(700c^4+350c^3dx+140c^2d^2x^2+35cd^3x^3+4d^4x^4)-7ab^5d(1260c^5+1050c^4dx+700c^3d^2x^2+315c^2d^3x^3+84cd^4x^4+10d^5x^5)+b^6(2940c^6+4410c^5dx+4900c^4d^2x^2+3675c^3d^3x^3+1764c^2d^4x^4+490cd^5x^5+60d^6x^6))}{420b^7}+\frac{(bc-ad)^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*x*(420*a^6*d^6 - 210*a^5*b*d^5*(14*c + d*x) + 70*a^4*b^2*d^4*(126*c^2 +
21*c*d*x + 2*d^2*x^2) - 35*a^3*b^3*d^3*(420*c^3 + 126*c^2*d*x + 28*c*d^2*x^
2 + 3*d^3*x^3) + 21*a^2*b^4*d^2*(700*c^4 + 350*c^3*d*x + 140*c^2*d^2*x^2 +
35*c*d^3*x^3 + 4*d^4*x^4) - 7*a*b^5*d*(1260*c^5 + 1050*c^4*d*x + 700*c^3*d^
2*x^2 + 315*c^2*d^3*x^3 + 84*c*d^4*x^4 + 10*d^5*x^5) + b^6*(2940*c^6 + 4410
*c^5*d*x + 4900*c^4*d^2*x^2 + 3675*c^3*d^3*x^3 + 1764*c^2*d^4*x^4 + 490*c*d
^5*x^5 + 60*d^6*x^6)))/(420*b^7) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(157) = 314$.
time = 0.14, size = 491, normalized size = 2.91

method	result
norman	$\frac{d\left(a^{6} d^{6}-7 a^{5} b c \,d^{5}+21 a^{4} b^{2} c^{2} d^{4}-35 a^{3} b^{3} c^{3} d^{3}+35 a^{2} b^{4} c^{4} d^{2}-21 a \,b^{5} c^{5} d+7 b^{6} c^{6}\right) x}{b^{7}}+\frac{d^{7} x^{7}}{7 b}-\frac{d^{2} \left(a^{5} d^{5}-7 a^{4} b c \,d^{4}+21 a^{3} b^{2} c^{2} d^{3}-35 a^{2} b^{3} c \ldots\right)}{2 b^{6}}$
default	$d\left(-\frac{1}{6} a \,b^{5} d^{6} x^{6}+\frac{7}{6} b^{6} c \,d^{5} x^{6}+\frac{1}{5} a^{2} b^{4} d^{6} x^{5}+\frac{21}{5} b^{6} c^{2} d^{4} x^{5}-\frac{1}{4} a^{3} b^{3} d^{6} x^{4}+\frac{35}{4} b^{6} c^{3} d^{3} x^{4}+\frac{1}{3} a^{4} b^{2} d^{6} x^{3}+\frac{35}{3} b^{6} c^{4} d^{2} x^{3}-\frac{1}{2} a^{5} b \,d^{6} x^{2}+\frac{1}{7} d^{6} x^{7} b^{6}+a^{6} d \ldots\right.$
risch	$\frac{\ln\left(b x +a \right) c^{7}}{b}-\frac{d^{7} a \,x^{6}}{6 b^{2}}+\frac{7 d^{6} c \,x^{6}}{6 b}+\frac{d^{7} a^{2} x^{5}}{5 b^{3}}+\frac{21 d^{5} c^{2} x^{5}}{5 b}-\frac{d^{7} a^{3} x^{4}}{4 b^{4}}+\frac{35 d^{4} c^{3} x^{4}}{4 b}+\frac{d^{7} a^{4} x^{3}}{3 b^{5}}+\frac{35 d^{3} c^{4} x^{3}}{3 b}-\frac{d^{7} a^{5} x^{2}}{2 b^{6}}+\frac{d^{7} a^{6} \ldots}{b^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^7*(-1/6*a*b^5*d^6*x^6+7/6*b^6*c*d^5*x^6+1/5*a^2*b^4*d^6*x^5+21/5*b^6*c^
2*d^4*x^5-1/4*a^3*b^3*d^6*x^4+35/4*b^6*c^3*d^3*x^4+1/3*a^4*b^2*d^6*x^3+35/3
*b^6*c^4*d^2*x^3-1/2*a^5*b*d^6*x^2+1/7*d^6*x^7*b^6+a^6*d^6*x+7*b^6*c^6*x-21
/4*a*b^5*c^2*d^4*x^4-7/3*a^3*b^3*c*d^5*x^3+7*a^2*b^4*c^2*d^4*x^3-35/3*a*b^5
*c^3*d^3*x^3+7/2*a^4*b^2*c*d^5*x^2-21/2*a^3*b^3*c^2*d^4*x^2+35/2*a^2*b^4*c^
3*d^3*x^2-35/2*a*b^5*c^4*d^2*x^2-7*a^5*b*c*d^5*x+21*a^4*b^2*c^2*d^4*x-35*a^
3*b^3*c^3*d^3*x+35*a^2*b^4*c^4*d^2*x-21*a*b^5*c^5*d*x+21/2*b^6*c^5*d*x^2-7/
5*a*b^5*c*d^5*x^5+7/4*a^2*b^4*c*d^5*x^4)+(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2
*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c
^6*d+b^7*c^7)/b^8*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(157) = 314$.
time = 0.28, size = 460, normalized size = 2.72

$$\frac{60 b^{6} d^{7} x^{7} + 70 \left(7 b^{6} c d^{6} - a b^{5} d^{7}\right) x^{6} + 84 \left(21 b^{6} c^{2} d^{5} - 7 a b^{5} c d^{6} + a^{2} b^{4} d^{7}\right) x^{5} + 105 \left(35 b^{6} c^{3} d^{4} - 21 a b^{5} c^{2} d^{5} + 7 a^{2} b^{4} c d^{6} - a^{3} b^{3} d^{7}\right) x^{4} + 140 \left(35 b^{6} c^{4} d^{3} - 35 a b^{5} c^{3} d^{4} + 21 a^{2} b^{4} c^{2} d^{5} - 7 a^{3} b^{3} c d^{6} + a^{4} b^{2} d^{7}\right) x^{3} + 210 \left(21 b^{6} c^{5} d^{2} - 35 a b^{5} c^{4} d^{3} + 35 a^{2} b^{4} c^{3} d^{4} - 21 a^{3} b^{3} c^{2} d^{5} + 7 a^{4} b^{2} c d^{6} - a^{5} b d^{7}\right) x^{2} + 420 \left(7 b^{6} c^{6} d - 21 a b^{5} c^{5} d^{2} + 35 a^{2} b^{4} c^{4} d^{3} - 35 a^{3} b^{3} c^{3} d^{4} + 21 a^{4} b^{2} c^{2} d^{5} - 7 a^{5} b c d^{6} + a^{6} d^{7}\right) x}{420 b^{7}} + \frac{\left(b^{7} c^{7} - 7 a b^{6} c^{6} d + 21 a^{2} b^{5} c^{5} d^{2} - 35 a^{3} b^{4} c^{4} d^{3} + 35 a^{4} b^{3} c^{3} d^{4} - 21 a^{5} b^{2} c^{2} d^{5} + 7 a^{6} b c d^{6} - a^{7} d^{7}\right) \log\left(b x + a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 70*(7*b^6*c*d^6 - a*b^5*d^7)*x^6 + 84*(21*b^6*c^2*d
^5 - 7*a*b^5*c*d^6 + a^2*b^4*d^7)*x^5 + 105*(35*b^6*c^3*d^4 - 21*a*b^5*c^2*
d^5 + 7*a^2*b^4*c*d^6 - a^3*b^3*d^7)*x^4 + 140*(35*b^6*c^4*d^3 - 35*a*b^5*c
^3*d^4 + 21*a^2*b^4*c^2*d^5 - 7*a^3*b^3*c*d^6 + a^4*b^2*d^7)*x^3 + 210*(21*
b^6*c^5*d^2 - 35*a*b^5*c^4*d^3 + 35*a^2*b^4*c^3*d^4 - 21*a^3*b^3*c^2*d^5 +
7*a^4*b^2*c*d^6 - a^5*b*d^7)*x^2 + 420*(7*b^6*c^6*d - 21*a*b^5*c^5*d^2 + 35
*a^2*b^4*c^4*d^3 - 35*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6
+ a^6*d^7)*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*
b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7
*d^7)*log(b*x + a)/b^8
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(157) = 314$.
time = 1.09, size = 462, normalized size = 2.73

$$\frac{60\,b^{7}d^{7}x^{7}+70\,(7\,b^{7}cd^{6}-ab^{6}d^{7})x^{6}+84\,(21\,b^{7}c^{2}d^{5}-7\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+105\,(35\,b^{7}c^{3}d^{4}-21\,ab^{6}c^{2}d^{5}+7\,a^{2}b^{5}cd^{6}-a^{3}b^{4}d^{7})x^{4}+140\,(35\,b^{7}c^{4}d^{3}-35\,ab^{6}c^{3}d^{4}+21\,a^{2}b^{5}c^{2}d^{5}-7\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+210\,(21\,b^{7}c^{5}d^{2}-35\,ab^{6}c^{4}d^{3}+35\,a^{2}b^{5}c^{3}d^{4}-21\,a^{3}b^{4}c^{2}d^{5}+7\,a^{4}b^{3}cd^{6}-a^{5}b^{2}d^{7})x^{2}+420\,(7\,b^{7}c^{6}d-21\,ab^{6}c^{5}d^{2}+35\,a^{2}b^{5}c^{4}d^{3}-35\,a^{3}b^{4}c^{3}d^{4}+21\,a^{4}b^{3}c^{2}d^{5}-7\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x+420\,(b^{7}c^{7}-7\,ab^{6}c^{6}d+21\,a^{2}b^{5}c^{5}d^{2}-35\,a^{3}b^{4}c^{4}d^{3}+35\,a^{4}b^{3}c^{3}d^{4}-21\,a^{5}b^{2}c^{2}d^{5}+7\,a^{6}bcd^{6}-a^{7}d^{7})\log\left(bx+a\right)}{420\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/420*(60*b^7*d^7*x^7 + 70*(7*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(21*b^7*c^2*d
^5 - 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(35*b^7*c^3*d^4 - 21*a*b^6*c^2*
d^5 + 7*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(35*b^7*c^4*d^3 - 35*a*b^6*c
^3*d^4 + 21*a^2*b^5*c^2*d^5 - 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(21*
b^7*c^5*d^2 - 35*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 - 21*a^3*b^4*c^2*d^5 +
7*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 420*(7*b^7*c^6*d - 21*a*b^6*c^5*d^2 +
35*a^2*b^5*c^4*d^3 - 35*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 - 7*a^5*b^2*c*
d^6 + a^6*b*d^7)*x + 420*(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35
*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6
- a^7*d^7)*log(b*x + a))/b^8
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(144) = 288$.
time = 0.48, size = 408, normalized size = 2.41

$$x^{6}\left(-\frac{ad^{7}}{6b^{2}}+\frac{7cd^{6}}{6b}\right)+x^{5}\left(\frac{a^{2}d^{7}}{5b^{3}}-\frac{7acd^{6}}{5b^{2}}+\frac{21c^{2}d^{5}}{5b}\right)+x^{4}\left(-\frac{a^{3}d^{7}}{4b^{4}}+\frac{7a^{2}cd^{6}}{4b^{3}}-\frac{21ac^{2}d^{5}}{4b^{2}}+\frac{35c^{3}d^{4}}{4b}\right)+x^{3}\left(\frac{a^{4}d^{7}}{3b^{5}}-\frac{7a^{3}cd^{6}}{3b^{4}}+\frac{7a^{2}c^{2}d^{5}}{b^{3}}-\frac{35ac^{3}d^{4}}{3b^{2}}+\frac{35c^{4}d^{3}}{3b}\right)+x^{2}\left(-\frac{a^{5}d^{7}}{2b^{6}}+\frac{7a^{4}cd^{6}}{2b^{5}}-\frac{21a^{3}c^{2}d^{5}}{2b^{4}}+\frac{35a^{2}c^{3}d^{4}}{2b^{3}}-\frac{35ac^{4}d^{3}}{2b^{2}}+\frac{21c^{5}d^{2}}{2b}\right)+x\left(\frac{a^{6}d^{7}}{b^{7}}-\frac{7a^{5}cd^{6}}{b^{6}}+\frac{21a^{4}c^{2}d^{5}}{b^{5}}-\frac{35a^{3}c^{3}d^{4}}{b^{4}}+\frac{35a^{2}c^{4}d^{3}}{b^{3}}-\frac{21ac^{5}d^{2}}{b^{2}}+\frac{7c^{6}d}{b}\right)+\frac{d^{7}x^{7}}{7b}-\frac{(ad-bc)^{7}\log\left(a+bx\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a),x)
```

```
[Out] x**6*(-a*d**7/(6*b**2) + 7*c*d**6/(6*b)) + x**5*(a**2*d**7/(5*b**3) - 7*a*c
*d**6/(5*b**2) + 21*c**2*d**5/(5*b)) + x**4*(-a**3*d**7/(4*b**4) + 7*a**2*c
*d**6/(4*b**3) - 21*a*c**2*d**5/(4*b**2) + 35*c**3*d**4/(4*b)) + x**3*(a**4
*d**7/(3*b**5) - 7*a**3*c*d**6/(3*b**4) + 7*a**2*c**2*d**5/b**3 - 35*a*c**3
*d**4/(3*b**2) + 35*c**4*d**3/(3*b)) + x**2*(-a**5*d**7/(2*b**6) + 7*a**4*c
*d**6/(2*b**5) - 21*a**3*c**2*d**5/(2*b**4) + 35*a**2*c**3*d**4/(2*b**3) -
```

```
35*a*c**4*d**3/(2*b**2) + 21*c**5*d**2/(2*b)) + x*(a**6*d**7/b**7 - 7*a**5*
c*d**6/b**6 + 21*a**4*c**2*d**5/b**5 - 35*a**3*c**3*d**4/b**4 + 35*a**2*c**
4*d**3/b**3 - 21*a*c**5*d**2/b**2 + 7*c**6*d/b) + d**7*x**7/(7*b) - (a*d -
b*c)**7*log(a + b*x)/b**8
```

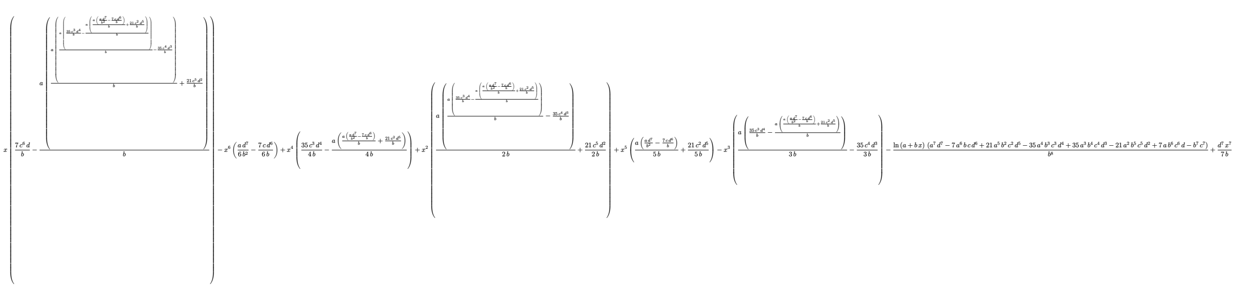
Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(157) = 314$.
time = 0.75, size = 497, normalized size = 2.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 490*b^6*c*d^6*x^6 - 70*a*b^5*d^7*x^6 + 1764*b^6*c^2
*d^5*x^5 - 588*a*b^5*c*d^6*x^5 + 84*a^2*b^4*d^7*x^5 + 3675*b^6*c^3*d^4*x^4
- 2205*a*b^5*c^2*d^5*x^4 + 735*a^2*b^4*c*d^6*x^4 - 105*a^3*b^3*d^7*x^4 + 49
00*b^6*c^4*d^3*x^3 - 4900*a*b^5*c^3*d^4*x^3 + 2940*a^2*b^4*c^2*d^5*x^3 - 98
0*a^3*b^3*c*d^6*x^3 + 140*a^4*b^2*d^7*x^3 + 4410*b^6*c^5*d^2*x^2 - 7350*a*b
^5*c^4*d^3*x^2 + 7350*a^2*b^4*c^3*d^4*x^2 - 4410*a^3*b^3*c^2*d^5*x^2 + 1470
*a^4*b^2*c*d^6*x^2 - 210*a^5*b*d^7*x^2 + 2940*b^6*c^6*d*x - 8820*a*b^5*c^5*
d^2*x + 14700*a^2*b^4*c^4*d^3*x - 14700*a^3*b^3*c^3*d^4*x + 8820*a^4*b^2*c^
2*d^5*x - 2940*a^5*b*c*d^6*x + 420*a^6*d^7*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*
d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b
^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*log(abs(b*x + a))/b^8
```

Mupad [B]
time = 0.22, size = 509, normalized size = 3.01



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x),x)
```

```
[Out] x*((7*c^6*d)/b - (a*((a*((a*((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^
6)/b))/b + (21*c^2*d^5)/b))/b))/b - (35*c^4*d^3)/b))/b + (21*c^5*d^2)/b))/b
) - x^6*((a*d^7)/(6*b^2) - (7*c*d^6)/(6*b)) + x^4*((35*c^3*d^4)/(4*b) - (a*
((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/(4*b)) + x^2*((a*((a*
((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/
b))/b - (35*c^4*d^3)/b))/(2*b) + (21*c^5*d^2)/(2*b)) + x^5*((a*((a*d^7)/b^2
```

```
 - (7*c*d^6)/b))/(5*b) + (21*c^2*d^5)/(5*b)) - x^3*((a*((35*c^3*d^4)/b - (a
*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/b))/(3*b) - (35*c^4*
d^3)/(3*b)) - (log(a + b*x)*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6))/b^8 + (d^7*x^7)/(7*b)
```

3.1284 $\int \frac{(c+dx)^7}{(a+bx)^2}\,dx$

Optimal. Leaf size=187

$$\frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\frac{21d^5(bc-ad)^2(a+bx)^4}{4b^8}$$

```
[Out] 21*d^2*(-a*d+b*c)^5*x/b^7-(-a*d+b*c)^7/b^8/(b*x+a)+35/2*d^3*(-a*d+b*c)^4*(b
*x+a)^2/b^8+35/3*d^4*(-a*d+b*c)^3*(b*x+a)^3/b^8+21/4*d^5*(-a*d+b*c)^2*(b*x+
a)^4/b^8+7/5*d^6*(-a*d+b*c)*(b*x+a)^5/b^8+1/6*d^7*(b*x+a)^6/b^8+7*d*(-a*d+b
*c)^6*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.16, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{7d(bc-ad)^6\log(a+bx)}{b^8}+\frac{d^7(a+bx)^6}{6b^8}+\frac{21d^2x(bc-ad)^5}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (21*d^2*(b*c - a*d)^5*x)/b^7 - (b*c - a*d)^7/(b^8*(a + b*x)) + (35*d^3*(b*c
 - a*d)^4*(a + b*x)^2)/(2*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^3)/(3*b^8)
 + (21*d^5*(b*c - a*d)^2*(a + b*x)^4)/(4*b^8) + (7*d^6*(b*c - a*d)*(a + b*x
)^5)/(5*b^8) + (d^7*(a + b*x)^6)/(6*b^8) + (7*d*(b*c - a*d)^6*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^2}\,dx &= \int \left(\frac{21d^2(bc-ad)^5}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^2}+\frac{7d(bc-ad)^6}{b^7(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)}{b^7}+\frac{35d^4(bc-\right.\\ &= \frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\frac{21}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 388 vs. 2(187) = 374.
time = 0.08, size = 388, normalized size = 2.07

$$\frac{60a^7d^7-60a^6bd^6(7c+6dx)+210a^5b^2d^5(6c^2+10cdx-d^2x^2)+70a^4b^3d^4(-30c^3-72c^2dx+18cd^2x^2+d^3x^3)-35a^3b^4d^3(-60c^4-180c^3dx+90c^2d^2x^2+12cd^3x^3+d^4x^4)+21a^2b^5d^2(-60c^5-200c^4dx+200c^3d^2x^2+50c^2d^3x^3+10cd^4x^4+d^5x^5)-7ab^6d(-60c^6-180c^5dx+450c^4d^2x^2+200c^3d^3x^3+75c^2d^4x^4+18cd^5x^5+2d^6x^6)+b^7(-60c^7+1260c^5d^2x^2+1050c^4d^3x^3+700c^3d^4x^4+315c^2d^5x^5+84cd^6x^6+10d^7x^7)+420d(bc-ad)^6(a+bx)\log(a+bx)}{60b^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (60*a^7*d^7 - 60*a^6*b*d^6*(7*c + 6*d*x) + 210*a^5*b^2*d^5*(6*c^2 + 10*c*d*
x - d^2*x^2) + 70*a^4*b^3*d^4*(-30*c^3 - 72*c^2*d*x + 18*c*d^2*x^2 + d^3*x^
3) - 35*a^3*b^4*d^3*(-60*c^4 - 180*c^3*d*x + 90*c^2*d^2*x^2 + 12*c*d^3*x^3
+ d^4*x^4) + 21*a^2*b^5*d^2*(-60*c^5 - 200*c^4*d*x + 200*c^3*d^2*x^2 + 50*c
^2*d^3*x^3 + 10*c*d^4*x^4 + d^5*x^5) - 7*a*b^6*d*(-60*c^6 - 180*c^5*d*x + 4
50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 75*c^2*d^4*x^4 + 18*c*d^5*x^5 + 2*d^6*x^
6) + b^7*(-60*c^7 + 1260*c^5*d^2*x^2 + 1050*c^4*d^3*x^3 + 700*c^3*d^4*x^4 +
 315*c^2*d^5*x^5 + 84*c*d^6*x^6 + 10*d^7*x^7) + 420*d*(b*c - a*d)^6*(a + b*
x)*Log[a + b*x])/(60*b^8*(a + b*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(177) = 354.
time = 0.17, size = 479, normalized size = 2.56

method	result
norman	$\frac{\frac{d^7x^7}{6b}-\frac{7d^2\left(a^5d^5-6a^4bc\,d^4+15a^3b^2c^2d^3-20a^2b^3c^3d^2+15a\,b^4c^4d-6b^5c^5\right)x^2}{2b^6}+\frac{7d^3\left(a^4d^4-6a^3bc\,d^3+15a^2b^2c^2d^2-20a\,b^3c^3d+15b^4c^4\right)x^3}{6b^5}-\frac{7d^4(}{}$
default	$-\frac{d^2\left(-\frac{1}{6}d^5x^6b^5+\frac{2}{5}a\,b^4d^5x^5-\frac{7}{5}b^5c\,d^4x^5-\frac{3}{4}a^2b^3d^5x^4+\frac{7}{2}a\,b^4c\,d^4x^4-\frac{21}{4}b^5c^2d^3x^4+\frac{4}{3}a^3b^2d^5x^3-7a^2b^3c\,d^4x^3+14a\,b^4c^2d^3x^3-\frac{35}{3}b^5c^3\right.}{}$
risch	$\frac{d^7x^6}{6b^2}-\frac{c^7}{b(bx+a)}-\frac{42d^6\ln(bx+a)a^5c}{b^7}+\frac{105d^5\ln(bx+a)a^4c^2}{b^6}-\frac{140d^4\ln(bx+a)a^3c^3}{b^5}+\frac{105d^3\ln(bx+a)a^2c^4}{b^4}-\frac{42d^2\ln(bx+}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^7*(-1/6*d^5*x^6*b^5+2/5*a*b^4*d^5*x^5-7/5*b^5*c*d^4*x^5-3/4*a^2*b^3*
d^5*x^4+7/2*a*b^4*c*d^4*x^4-21/4*b^5*c^2*d^3*x^4+4/3*a^3*b^2*d^5*x^3-7*a^2*
b^3*c*d^4*x^3+14*a*b^4*c^2*d^3*x^3-35/3*b^5*c^3*d^2*x^3-5/2*a^4*b*d^5*x^2+1
4*a^3*b^2*c*d^4*x^2-63/2*a^2*b^3*c^2*d^3*x^2+35*a*b^4*c^3*d^2*x^2-35/2*b^5*
c^4*d*x^2+6*a^5*d^5*x-35*a^4*b*c*d^4*x+84*a^3*b^2*c^2*d^3*x-105*a^2*b^3*c^3
*d^2*x+70*a*b^4*c^4*d*x-21*b^5*c^5*x)-1/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*
b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^
6*c^6*d+b^7*c^7)/(b*x+a)+7/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-
20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(177) = 354.

time = 0.35, size = 467, normalized size = 2.50

$$-\frac{b^7c^7-7\,ab^6c^6d+21\,a^2b^5c^5d^2-35\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4-21\,a^5b^2c^2d^5+7\,a^6bcd^6-a^7d^7}{b^9x+ab^8}+\frac{10\,b^5d^7x^6+12\,(7\,b^5cd^6-2\,ab^4d^7)x^5+15\,(21\,b^5c^2d^5-14\,ab^4cd^6+3\,a^2b^3d^7)x^4+20\,(35\,b^5c^3d^4-42\,ab^4c^2d^5+21\,a^2b^3cd^6-4\,a^3b^2d^7)x^3+30\,(35\,b^5c^4d^3-70\,ab^4c^3d^4+63\,a^2b^3c^2d^5-28\,a^3b^2cd^6+5\,a^4bd^7)x^2+60\,(21\,b^5c^5d^2-70\,ab^4c^4d^3+105\,a^2b^3c^3d^4-84\,a^3b^2c^2d^5+35\,a^4bcd^6-6\,a^5d^7)x}{60\,b^7}+\frac{7\,(b^6c^6d-6\,ab^5c^5d^2+15\,a^2b^4c^4d^3-20\,a^3b^3c^3d^4+15\,a^4b^2c^2d^5-6\,a^5bcd^6+a^6d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(b^9*x + a*b^
8) + 1/60*(10*b^5*d^7*x^6 + 12*(7*b^5*c*d^6 - 2*a*b^4*d^7)*x^5 + 15*(21*b^5
*c^2*d^5 - 14*a*b^4*c*d^6 + 3*a^2*b^3*d^7)*x^4 + 20*(35*b^5*c^3*d^4 - 42*a*
b^4*c^2*d^5 + 21*a^2*b^3*c*d^6 - 4*a^3*b^2*d^7)*x^3 + 30*(35*b^5*c^4*d^3 -
70*a*b^4*c^3*d^4 + 63*a^2*b^3*c^2*d^5 - 28*a^3*b^2*c*d^6 + 5*a^4*b*d^7)*x^2
 + 60*(21*b^5*c^5*d^2 - 70*a*b^4*c^4*d^3 + 105*a^2*b^3*c^3*d^4 - 84*a^3*b^2
*c^2*d^5 + 35*a^4*b*c*d^6 - 6*a^5*d^7)*x)/b^7 + 7*(b^6*c^6*d - 6*a*b^5*c^5*
d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*
b*c*d^6 + a^6*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(177) = 354$.
time = 1.25, size = 632, normalized size = 3.38

$$\frac{10\,b^7d^7x^7-60\,b^7c^7+420\,ab^6c^6d-1260\,a^2b^5c^5d^2+2100\,a^3b^4c^4d^3-2100\,a^4b^3c^3d^4+1260\,a^5b^2c^2d^5-420\,a^6bcd^6+60\,a^7d^7+14\,(6\,b^7cd^6-ab^6d^7)x^6+21\,(15\,b^7c^2d^5-6\,ab^6cd^6+a^2b^5d^7)x^5+35\,(20\,b^7c^3d^4-15\,ab^6c^2d^5+6\,a^2b^5cd^6-a^3b^4d^7)x^4+70\,(15\,b^7c^4d^3-20\,ab^6c^3d^4+15\,a^2b^5c^2d^5-6\,a^3b^4cd^6+a^4b^3d^7)x^3+210\,(6\,b^7c^5d^2-15\,ab^6c^4d^3+20\,a^2b^5c^3d^4-15\,a^3b^4c^2d^5+6\,a^4b^3cd^6-a^5b^2d^7)x^2+60\,(21\,ab^6c^5d^2-70\,a^2b^5c^4d^3+105\,a^3b^4c^3d^4-84\,a^4b^3c^2d^5+35\,a^5b^2cd^6-6\,a^6bd^7)x+420\,(ab^6c^6d-6\,a^2b^5c^5d^2+15\,a^3b^4c^4d^3-20\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5-6\,a^6bcd^6+a^7d^7+(b^7c^6d-6\,ab^6c^5d^2+15\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5-6\,a^5b^2cd^6+a^6bd^7)x)\log(bx+a)}{60\,(b^9x+ab^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*d^7*x^7 - 60*b^7*c^7 + 420*a*b^6*c^6*d - 1260*a^2*b^5*c^5*d^2
+ 2100*a^3*b^4*c^4*d^3 - 2100*a^4*b^3*c^3*d^4 + 1260*a^5*b^2*c^2*d^5 - 420*
a^6*b*c*d^6 + 60*a^7*d^7 + 14*(6*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(15*b^7*c^
2*d^5 - 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(20*b^7*c^3*d^4 - 15*a*b^6*c^
2*d^5 + 6*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 70*(15*b^7*c^4*d^3 - 20*a*b^6*
c^3*d^4 + 15*a^2*b^5*c^2*d^5 - 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(6*
b^7*c^5*d^2 - 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 15*a^3*b^4*c^2*d^5 +
6*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 60*(21*a*b^6*c^5*d^2 - 70*a^2*b^5*c^4*
d^3 + 105*a^3*b^4*c^3*d^4 - 84*a^4*b^3*c^2*d^5 + 35*a^5*b^2*c*d^6 - 6*a^6*b
*d^7)*x + 420*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d -
 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2
*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^9*x + a*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(172) = 344$.
time = 0.91, size = 428, normalized size = 2.29

$$x^5\left(-\frac{2ad^7}{5b^3}+\frac{7cd^6}{5b^2}\right)+x^4\cdot\left(\frac{3a^2d^7}{4b^4}-\frac{7acd^6}{2b^3}+\frac{21c^2d^5}{4b^2}\right)+x^3\left(-\frac{4a^3d^7}{3b^5}+\frac{7a^2cd^6}{b^4}-\frac{14ac^2d^5}{b^3}+\frac{35c^3d^4}{3b^2}\right)+x^2\cdot\left(\frac{5a^4d^7}{2b^6}-\frac{14a^3cd^6}{b^5}+\frac{63a^2c^2d^5}{2b^4}-\frac{35ac^3d^4}{b^3}+\frac{35c^4d^3}{2b^2}\right)+x\left(-\frac{6a^5d^7}{b^7}+\frac{35a^4cd^6}{b^6}-\frac{84a^3c^2d^5}{b^5}+\frac{105a^2c^3d^4}{b^4}-\frac{70ac^4d^3}{b^3}+\frac{21c^5d^2}{b^2}\right)+\frac{a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7}{ab^8+b^9x}+\frac{d^7x^6}{6b^2}+\frac{7d(ad-bc)^6\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**2,x)
```

```
[Out] x**5*(-2*a*d**7/(5*b**3) + 7*c*d**6/(5*b**2)) + x**4*(3*a**2*d**7/(4*b**4)
- 7*a*c*d**6/(2*b**3) + 21*c**2*d**5/(4*b**2)) + x**3*(-4*a**3*d**7/(3*b**5
) + 7*a**2*c*d**6/b**4 - 14*a*c**2*d**5/b**3 + 35*c**3*d**4/(3*b**2)) + x**
2*(5*a**4*d**7/(2*b**6) - 14*a**3*c*d**6/b**5 + 63*a**2*c**2*d**5/(2*b**4)
- 35*a*c**3*d**4/b**3 + 35*c**4*d**3/(2*b**2)) + x*(-6*a**5*d**7/b**7 + 35*
a**4*c*d**6/b**6 - 84*a**3*c**2*d**5/b**5 + 105*a**2*c**3*d**4/b**4 - 70*a*
c**4*d**3/b**3 + 21*c**5*d**2/b**2) + (a**7*d**7 - 7*a**6*b*c*d**6 + 21*a**
5*b**2*c**2*d**5 - 35*a**4*b**3*c**3*d**4 + 35*a**3*b**4*c**4*d**3 - 21*a**
2*b**5*c**5*d**2 + 7*a*b**6*c**6*d - b**7*c**7)/(a*b**8 + b**9*x) + d**7*x*
*6/(6*b**2) + 7*d*(a*d - b*c)**6*log(a + b*x)/b**8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(177) = 354.
time = 0.76, size = 567, normalized size = 3.03

$$\frac{\left(10\,d^7 + \frac{84\,(b^2cd^6 - abd^7)}{(bx+a)b} + \frac{315\,(b^4c^2d^5 - 2\,ab^3cd^6 + a^2b^2d^7)}{(bx+a)^2b^2} + \frac{700\,(b^6c^3d^4 - 3\,ab^5c^2d^5 + 3\,a^2b^4cd^6 - a^3b^3d^7)}{(bx+a)^3b^3} + \frac{1050\,(b^8c^4d^3 - 4\,ab^7c^3d^4 + 6\,a^2b^6c^2d^5 - 4\,a^3b^5cd^6 + a^4b^4d^7)}{(bx+a)^4b^4} + \frac{1260\,(b^{10}c^5d^2 - 5\,ab^9c^4d^3 + 10\,a^2b^8c^3d^4 - 10\,a^3b^7c^2d^5 + 5\,a^4b^6cd^6 - a^5b^5d^7)}{(bx+a)^5b^5}\right)(bx+a)^6}{60\,b^8} - \frac{7\,(b^6c^6d - 6\,ab^5c^5d^2 + 15\,a^2b^4c^4d^3 - 20\,a^3b^3c^3d^4 + 15\,a^4b^2c^2d^5 - 6\,a^5bcd^6 + a^6d^7)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^8} - \frac{\frac{b^{13}c^7}{bx+a} - \frac{7\,ab^{12}c^6d}{bx+a} + \frac{21\,a^2b^{11}c^5d^2}{bx+a} - \frac{35\,a^3b^{10}c^4d^3}{bx+a} + \frac{35\,a^4b^9c^3d^4}{bx+a} - \frac{21\,a^5b^8c^2d^5}{bx+a} + \frac{7\,a^6b^7cd^6}{bx+a} - \frac{a^7b^6d^7}{bx+a}}{b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*d^7 + 84*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 315*(b^4*c^2*d^5 -
2*a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2) + 700*(b^6*c^3*d^4 - 3*a*b^5
*c^2*d^5 + 3*a^2*b^4*c*d^6 - a^3*b^3*d^7)/((b*x + a)^3*b^3) + 1050*(b^8*c^4
*d^3 - 4*a*b^7*c^3*d^4 + 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^7)
/((b*x + a)^4*b^4) + 1260*(b^10*c^5*d^2 - 5*a*b^9*c^4*d^3 + 10*a^2*b^8*c^3*
d^4 - 10*a^3*b^7*c^2*d^5 + 5*a^4*b^6*c*d^6 - a^5*b^5*d^7)/((b*x + a)^5*b^5)
)*(b*x + a)^6/b^8 - 7*(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 2
0*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*log(abs(b
*x + a)/((b*x + a)^2*abs(b)))/b^8 - (b^13*c^7/(b*x + a) - 7*a*b^12*c^6*d/(b
*x + a) + 21*a^2*b^11*c^5*d^2/(b*x + a) - 35*a^3*b^10*c^4*d^3/(b*x + a) + 3
5*a^4*b^9*c^3*d^4/(b*x + a) - 21*a^5*b^8*c^2*d^5/(b*x + a) + 7*a^6*b^7*c*d^
6/(b*x + a) - a^7*b^6*d^7/(b*x + a))/b^14
```

Mupad [B]
time = 0.24, size = 841, normalized size = 4.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^2,x)
```

```
[Out] x^4*((a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(2*b) - (a^2*d^7)/(4*b^4) + (21*c^
2*d^5)/(4*b^2)) - x^2*((a*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3 - (
7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^7)/b
^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)/(2*b^2) + (a^2*((2*a*((2*a*d^7)
/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/(2*b^2)) - x^
5*((2*a*d^7)/(5*b^3) - (7*c*d^6)/(5*b^2)) + x*((2*a*((2*a*((35*c^3*d^4)/b^2
 - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*
d^5)/b^2))/b + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)
/b^2 + (a^2*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*
c^2*d^5)/b^2))/b^2))/b - (a^2*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3
 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^
7)/b^3 - (7*c*d^6)/b^2))/b^2))/b^2 + (21*c^5*d^2)/b^2) + x^3*((35*c^3*d^4)/
(3*b^2) - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (
21*c^2*d^5)/b^2))/(3*b) + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(3*b^2)) +
(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c
^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)/(b*(a*b^7 + b^
8*x)) + (d^7*x^6)/(6*b^2) + (log(a + b*x)*(7*a^6*d^7 + 7*b^6*c^6*d - 42*a*b
^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4 + 105*a^4*b^2*c^2*d^
5 - 42*a^5*b*c*d^6))/b^8
```

3.1285 $\int \frac{(c+dx)^7}{(a+bx)^3}\,dx$

Optimal. Leaf size=185

$$\frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-ad)^2(a+bx)^3}{b^8}+\frac{7d^6(bc-$$

```
[Out] 35*d^3*(-a*d+b*c)^4*x/b^7-1/2*(-a*d+b*c)^7/b^8/(b*x+a)^2-7*d*(-a*d+b*c)^6/b
^8/(b*x+a)+35/2*d^4*(-a*d+b*c)^3*(b*x+a)^2/b^8+7*d^5*(-a*d+b*c)^2*(b*x+a)^3
/b^8+7/4*d^6*(-a*d+b*c)*(b*x+a)^4/b^8+1/5*d^7*(b*x+a)^5/b^8+21*d^2*(-a*d+b*
c)^5*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{21d^2(bc-ad)^5\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{b^8(a+bx)}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}+\frac{d^7(a+bx)^5}{5b^8}+\frac{35d^3x(bc-ad)^4}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (35*d^3*(b*c - a*d)^4*x)/b^7 - (b*c - a*d)^7/(2*b^8*(a + b*x)^2) - (7*d*(b*
c - a*d)^6)/(b^8*(a + b*x)) + (35*d^4*(b*c - a*d)^3*(a + b*x)^2)/(2*b^8) +
(7*d^5*(b*c - a*d)^2*(a + b*x)^3)/b^8 + (7*d^6*(b*c - a*d)*(a + b*x)^4)/(4*
b^8) + (d^7*(a + b*x)^5)/(5*b^8) + (21*d^2*(b*c - a*d)^5*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^3}\,dx &= \int \left(\frac{35d^3(bc-ad)^4}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^3}+\frac{7d(bc-ad)^6}{b^7(a+bx)^2}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)}+\frac{35d^4(bc-ad)^3(a}{b^7}\right.\\ &= \frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(185) = 370.

time = 0.09, size = 389, normalized size = 2.10

$$\frac{-130a^7d^7+10a^6bd^6(77c+16dx)+10a^5b^2d^5(-189c^2-56cdx+50d^2x^2)+70a^4b^3d^4(35c^3+6c^2dx-34cd^2x^2+2d^3x^3)-35a^3b^4d^3(50c^4-20c^3dx-126c^2d^2x^2+20cd^3x^3+d^4x^4)+7a^2b^5d^2(90c^5-200c^4dx-550c^3d^2x^2+200c^2d^3x^3+25cd^4x^4+2d^5x^5)-7ab^6d(10c^6-120c^5dx-200c^4d^2x^2+200c^3d^3x^3+50c^2d^4x^4+10cd^5x^5+d^6x^6)+b^7(-10c^7-140c^6dx+700c^4d^3x^3+350c^3d^4x^4+140c^2d^5x^5+35cd^6x^6+4d^7x^7)-420d^2(-bc+ad)^5(a+bx)^2\log(a+bx)}{20b^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (-130*a^7*d^7 + 10*a^6*b*d^6*(77*c + 16*d*x) + 10*a^5*b^2*d^5*(-189*c^2 - 5
6*c*d*x + 50*d^2*x^2) + 70*a^4*b^3*d^4*(35*c^3 + 6*c^2*d*x - 34*c*d^2*x^2 +
 2*d^3*x^3) - 35*a^3*b^4*d^3*(50*c^4 - 20*c^3*d*x - 126*c^2*d^2*x^2 + 20*c*
d^3*x^3 + d^4*x^4) + 7*a^2*b^5*d^2*(90*c^5 - 200*c^4*d*x - 550*c^3*d^2*x^2
+ 200*c^2*d^3*x^3 + 25*c*d^4*x^4 + 2*d^5*x^5) - 7*a*b^6*d*(10*c^6 - 120*c^5
*d*x - 200*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 50*c^2*d^4*x^4 + 10*c*d^5*x^5 +
d^6*x^6) + b^7*(-10*c^7 - 140*c^6*d*x + 700*c^4*d^3*x^3 + 350*c^3*d^4*x^4 +
 140*c^2*d^5*x^5 + 35*c*d^6*x^6 + 4*d^7*x^7) - 420*d^2*(-(b*c) + a*d)^5*(a
+ b*x)^2*Log[a + b*x])/(20*b^8*(a + b*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(177) = 354$.
time = 0.15, size = 467, normalized size = 2.52

method	result
norman	$\frac{-\frac{63a^7d^7-315a^6bcd^6+630a^5b^2c^2d^5-630a^4b^3c^3d^4+315a^3b^4c^4d^3-63a^2b^5c^5d^2+7ab^6c^6d+b^7c^7}{2b^8}+\frac{d^7x^7}{5b}-\frac{(42a^6d^7-210a^5bcd^6+420a^4b^2c^2d^5-420}{}}{}$
default	$\frac{d^3\left(\frac{1}{5}d^4x^5b^4-\frac{3}{4}ab^3d^4x^4+\frac{7}{4}b^4cd^3x^4+2a^2b^2d^4x^3-7ab^3cd^3x^3+7b^4c^2d^2x^3-5a^3bd^4x^2+21a^2b^2cd^3x^2-\frac{63}{2}ab^3c^2d^2x^2+\frac{35}{2}b^4c^3dx^2+15c\right.}{b^7}$
risch	$\frac{d^7x^5}{5b^3}-\frac{3d^7ax^4}{4b^4}+\frac{7d^6cx^4}{4b^3}+\frac{2d^7a^2x^3}{b^5}-\frac{7d^6acx^3}{b^4}+\frac{7d^5c^2x^3}{b^3}-\frac{5d^7a^3x^2}{b^6}+\frac{21d^6a^2cx^2}{b^5}-\frac{63d^5ac^2x^2}{2b^4}+\frac{35d^4c^3x^2}{2b^3}+15$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/b^7*(1/5*d^4*x^5*b^4-3/4*a*b^3*d^4*x^4+7/4*b^4*c*d^3*x^4+2*a^2*b^2*d^4*
x^3-7*a*b^3*c*d^3*x^3+7*b^4*c^2*d^2*x^3-5*a^3*b*d^4*x^2+21*a^2*b^2*c*d^3*x^
2-63/2*a*b^3*c^2*d^2*x^2+35/2*b^4*c^3*d*x^2+15*a^4*d^4*x-70*a^3*b*c*d^3*x+1
26*a^2*b^2*c^2*d^2*x-105*a*b^3*c^3*d*x+35*b^4*c^4*x)-7/b^8*d*(a^6*d^6-6*a^5
*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c
^5*d+b^6*c^6)/(b*x+a)-1/2/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35
*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^
7)/(b*x+a)^2-21/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^
3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(177) = 354$.
time = 0.29, size = 473, normalized size = 2.56

$$-\frac{b^7c^7+7ab^6c^6d-63a^2b^5c^5d^2+175a^3b^4c^4d^3-245a^4b^3c^3d^4+189a^5b^2c^2d^5-77a^6bcd^6+13a^7d^7+14(b^7c^6d-6ab^6c^5d^2+15a^2b^5c^4d^3-20a^3b^4c^3d^4+15a^4b^3c^2d^5-6a^5b^2cd^6+a^6bd^7)x}{2(b^{10}x^2+2ab^9x+a^2b^8)}+\frac{4b^4d^7x^5+5(7b^4cd^6-3ab^3d^7)x^4+20(7b^4c^2d^5-7ab^3cd^6+2a^2b^2d^7)x^3+10(35b^4c^3d^4-63ab^3c^2d^5+42a^2b^2cd^6-10a^3bd^7)x^2+20(35b^4c^4d^3-105ab^3c^3d^4+126a^2b^2c^2d^5-70a^3bcd^6+15a^4d^7)x}{20b^7}+\frac{21(b^5c^5d^2-5ab^4c^4d^3+10a^2b^3c^3d^4-10a^3b^2c^2d^5+5a^4bcd^6-a^5d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*c^7 + 7*a*b^6*c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 -
245*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 1
4*(b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 +
15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^10*x^2 + 2*a*b^9*x
+ a^2*b^8) + 1/20*(4*b^4*d^7*x^5 + 5*(7*b^4*c*d^6 - 3*a*b^3*d^7)*x^4 + 20*(
7*b^4*c^2*d^5 - 7*a*b^3*c*d^6 + 2*a^2*b^2*d^7)*x^3 + 10*(35*b^4*c^3*d^4 - 6
3*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 - 10*a^3*b*d^7)*x^2 + 20*(35*b^4*c^4*d^3
 - 105*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 - 70*a^3*b*c*d^6 + 15*a^4*d^7)*x
)/b^7 + 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2
*c^2*d^5 + 5*a^4*b*c*d^6 - a^5*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(177) = 354$.
time = 0.98, size = 703, normalized size = 3.80

$$\frac{4\,b^{7}d^{7}x^{7} - 10\,b^{7}c^{7} - 70\,ab^{6}c^{6}d + 630\,a^{2}b^{5}c^{5}d^{2} - 1750\,a^{3}b^{4}c^{4}d^{3} + 2450\,a^{4}b^{3}c^{3}d^{4} - 1890\,a^{5}b^{2}c^{2}d^{5} + 770\,a^{6}bcd^{6} - 130\,a^{7}d^{7} + 7\,(5\,b^{7}cd^{6} - ab^{6}d^{7})x^{6} + 14\,(10\,b^{7}c^{2}d^{5} - 5\,ab^{6}cd^{6} + a^{2}b^{5}d^{7})x^{5} + 35\,(10\,b^{7}c^{3}d^{4} - 10\,ab^{6}c^{2}d^{5} + 5\,a^{2}b^{5}cd^{6} - a^{3}b^{4}d^{7})x^{4} + 140\,(5\,b^{7}c^{4}d^{3} - 10\,ab^{6}c^{3}d^{4} + 10\,a^{2}b^{5}c^{2}d^{5} - 5\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7})x^{3} + 10\,(140\,ab^{6}c^{4}d^{3} - 385\,a^{2}b^{5}c^{3}d^{4} + 441\,a^{3}b^{4}c^{2}d^{5} - 238\,a^{4}b^{3}cd^{6} + 50\,a^{5}b^{2}d^{7})x^{2} - 20\,(7\,b^{7}c^{6}d - 42\,ab^{6}c^{5}d^{2} + 70\,a^{2}b^{5}c^{4}d^{3} - 35\,a^{3}b^{4}c^{3}d^{4} - 21\,a^{4}b^{3}c^{2}d^{5} + 28\,a^{5}b^{2}cd^{6} - 8\,a^{6}bd^{7})x + 420\,(a^{2}b^{5}c^{5}d^{2} - 5\,a^{3}b^{4}c^{4}d^{3} + 10\,a^{4}b^{3}c^{3}d^{4} - 10\,a^{5}b^{2}c^{2}d^{5} + 5\,a^{6}bcd^{6} - a^{7}d^{7} + (b^{7}c^{5}d^{2} - 5\,ab^{6}c^{4}d^{3} + 10\,a^{2}b^{5}c^{3}d^{4} - 10\,a^{3}b^{4}c^{2}d^{5} + 5\,a^{4}b^{3}cd^{6} - a^{5}b^{2}d^{7})x^{2} + 2\,(ab^{6}c^{5}d^{2} - 5\,a^{2}b^{5}c^{4}d^{3} + 10\,a^{3}b^{4}c^{3}d^{4} - 10\,a^{4}b^{3}c^{2}d^{5} + 5\,a^{5}b^{2}cd^{6} - a^{6}bd^{7})x)\log(bx + a)}{20\,(b^{10}x^{2} + 2\,ab^{9}x + a^{2}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*d^7*x^7 - 10*b^7*c^7 - 70*a*b^6*c^6*d + 630*a^2*b^5*c^5*d^2 - 1
750*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 - 1890*a^5*b^2*c^2*d^5 + 770*a^6
*b*c*d^6 - 130*a^7*d^7 + 7*(5*b^7*c*d^6 - a*b^6*d^7)*x^6 + 14*(10*b^7*c^2*d
^5 - 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(10*b^7*c^3*d^4 - 10*a*b^6*c^2*d
^5 + 5*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(5*b^7*c^4*d^3 - 10*a*b^6*c^3
*d^4 + 10*a^2*b^5*c^2*d^5 - 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 10*(140*a*
b^6*c^4*d^3 - 385*a^2*b^5*c^3*d^4 + 441*a^3*b^4*c^2*d^5 - 238*a^4*b^3*c*d^6
 + 50*a^5*b^2*d^7)*x^2 - 20*(7*b^7*c^6*d - 42*a*b^6*c^5*d^2 + 70*a^2*b^5*c^
4*d^3 - 35*a^3*b^4*c^3*d^4 - 21*a^4*b^3*c^2*d^5 + 28*a^5*b^2*c*d^6 - 8*a^6*
b*d^7)*x + 420*(a^2*b^5*c^5*d^2 - 5*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 -
10*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 - a^7*d^7 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d
^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^
7)*x^2 + 2*(a*b^6*c^5*d^2 - 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 - 10*a^4
*b^3*c^2*d^5 + 5*a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^10*x^2 + 2*
a*b^9*x + a^2*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(170) = 340$.
time = 1.79, size = 447, normalized size = 2.42

$$x^{4}\left(-\frac{3ad^{7}}{4b^{4}} + \frac{7cd^{6}}{4b^{3}}\right) + x^{3}\cdot\left(\frac{2a^{2}d^{7}}{b^{5}} - \frac{7acd^{6}}{b^{4}} + \frac{7c^{2}d^{5}}{b^{3}}\right) + x^{2}\left(-\frac{5a^{3}d^{7}}{b^{6}} + \frac{21a^{2}cd^{6}}{b^{5}} - \frac{63ac^{2}d^{5}}{2b^{4}} + \frac{35c^{3}d^{4}}{2b^{3}}\right) + x\left(\frac{15a^{4}d^{7}}{b^{7}} - \frac{70a^{3}cd^{6}}{b^{6}} + \frac{126a^{2}c^{2}d^{5}}{b^{5}} - \frac{105ac^{3}d^{4}}{b^{4}} + \frac{35c^{4}d^{3}}{b^{3}}\right) + \frac{-13a^{7}d^{7} + 77a^{6}bcd^{6} - 189a^{5}b^{2}c^{2}d^{5} + 245a^{4}b^{3}c^{3}d^{4} - 175a^{3}b^{4}c^{4}d^{3} + 63a^{2}b^{5}c^{5}d^{2} - 7ab^{6}c^{6}d - b^{7}c^{7} + x(-14a^{6}bd^{7} + 84a^{5}b^{2}cd^{6} - 210a^{4}b^{3}c^{2}d^{5} + 280a^{3}b^{4}c^{3}d^{4} - 210a^{2}b^{5}c^{4}d^{3} + 84ab^{6}c^{5}d^{2} - 14b^{7}c^{6}d)}{2a^{2}b^{8} + 4ab^{9}x + 2b^{10}x^{2}} + \frac{d^{7}x^{5}}{5b^{3}} - \frac{21d^{2}(ad - bc)^{5}\log(a + bx)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**3,x)
```

```
[Out] x**4*(-3*a*d**7/(4*b**4) + 7*c*d**6/(4*b**3)) + x**3*(2*a**2*d**7/b**5 - 7*
a*c*d**6/b**4 + 7*c**2*d**5/b**3) + x**2*(-5*a**3*d**7/b**6 + 21*a**2*c*d**
6/b**5 - 63*a*c**2*d**5/(2*b**4) + 35*c**3*d**4/(2*b**3)) + x*(15*a**4*d**7
/b**7 - 70*a**3*c*d**6/b**6 + 126*a**2*c**2*d**5/b**5 - 105*a*c**3*d**4/b**
4 + 35*c**4*d**3/b**3) + (-13*a**7*d**7 + 77*a**6*b*c*d**6 - 189*a**5*b**2*
c**2*d**5 + 245*a**4*b**3*c**3*d**4 - 175*a**3*b**4*c**4*d**3 + 63*a**2*b**
5*c**5*d**2 - 7*a*b**6*c**6*d - b**7*c**7 + x*(-14*a**6*b*d**7 + 84*a**5*b*
*2*c*d**6 - 210*a**4*b**3*c**2*d**5 + 280*a**3*b**4*c**3*d**4 - 210*a**2*b*
*5*c**4*d**3 + 84*a*b**6*c**5*d**2 - 14*b**7*c**6*d))/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + d**7*x**5/(5*b**3) - 21*d**2*(a*d - b*c)**5*log(a +
b*x)/b**8
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(177) = 354.
time = 0.59, size = 477, normalized size = 2.58

$$\frac{21\,(b^5c^5d^2-5\,ab^4c^4d^3+10\,a^2b^3c^3d^4-10\,a^3b^2c^2d^5+5\,a^4bcd^6-a^5d^7)\log\left(|bx+a|\right)}{b^8}-\frac{b^7c^7+7\,ab^6c^6d-63\,a^2b^5c^5d^2+175\,a^3b^4c^4d^3-245\,a^4b^3c^3d^4+189\,a^5b^2c^2d^5-77\,a^6bcd^6+13\,a^7d^7+14\,(b^7c^6d-6\,ab^6c^5d^2+15\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5-6\,a^5b^2cd^6+a^6bd^7)x}{2\,(bx+a)^2b^8}+\frac{4\,b^{12}d^7x^5+35\,b^{12}cd^6x^4-15\,ab^{11}d^7x^4+140\,b^{12}c^2d^5x^3-140\,ab^{11}cd^6x^3+40\,a^2b^{10}d^7x^3+350\,b^{12}c^3d^4x^2-630\,ab^{11}c^2d^5x^2+420\,a^2b^{10}cd^6x^2-100\,a^3b^9d^7x^2+700\,b^{12}c^4d^3x-2100\,ab^{11}c^3d^4x+2520\,a^2b^{10}c^2d^5x-1400\,a^3b^9cd^6x+300\,a^4b^8d^7x}{20\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2*c^2*d^5
 + 5*a^4*b*c*d^6 - a^5*d^7)*log(abs(b*x + a))/b^8 - 1/2*(b^7*c^7 + 7*a*b^6*
c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 14*(b^7*c^6*d - 6*a*b^6*c
^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a
^5*b^2*c*d^6 + a^6*b*d^7)*x)/((b*x + a)^2*b^8) + 1/20*(4*b^12*d^7*x^5 + 35*
b^12*c*d^6*x^4 - 15*a*b^11*d^7*x^4 + 140*b^12*c^2*d^5*x^3 - 140*a*b^11*c*d^
6*x^3 + 40*a^2*b^10*d^7*x^3 + 350*b^12*c^3*d^4*x^2 - 630*a*b^11*c^2*d^5*x^2
 + 420*a^2*b^10*c*d^6*x^2 - 100*a^3*b^9*d^7*x^2 + 700*b^12*c^4*d^3*x - 2100
*a*b^11*c^3*d^4*x + 2520*a^2*b^10*c^2*d^5*x - 1400*a^3*b^9*c*d^6*x + 300*a^
4*b^8*d^7*x)/b^15
```

Mupad [B]
time = 0.27, size = 690, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^3,x)
```

```
[Out] x*((3*a*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)/b^5 +
(21*c^2*d^5)/b^3))/b + (a^3*d^7)/b^6 - (35*c^3*d^4)/b^3 - (3*a^2*((3*a*d^7)
/b^4 - (7*c*d^6)/b^3))/b^2))/b + (35*c^4*d^3)/b^3 + (a^3*((3*a*d^7)/b^4 - (
```

```
7*c*d^6)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a
^2*d^7)/b^5 + (21*c^2*d^5)/b^3))/b^2) - x^4*((3*a*d^7)/(4*b^4) - (7*c*d^6)/
(4*b^3)) - x^2*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)
/b^5 + (21*c^2*d^5)/b^3))/(2*b) + (a^3*d^7)/(2*b^6) - (35*c^3*d^4)/(2*b^3)
- (3*a^2*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/(2*b^2)) + x^3*((a*((3*a*d^7)/b^4
 - (7*c*d^6)/b^3))/b - (a^2*d^7)/b^5 + (7*c^2*d^5)/b^3) - ((13*a^7*d^7 + b^
7*c^7 - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 77*a^6*b*c*d^6)/(2*b) + x*(7*a^6*d^7 +
7*b^6*c^6*d - 42*a*b^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4
+ 105*a^4*b^2*c^2*d^5 - 42*a^5*b*c*d^6))/(a^2*b^7 + b^9*x^2 + 2*a*b^8*x) +
(d^7*x^5)/(5*b^3) - (log(a + b*x)*(21*a^5*d^7 - 21*b^5*c^5*d^2 + 105*a*b^4*
c^4*d^3 - 210*a^2*b^3*c^3*d^4 + 210*a^3*b^2*c^2*d^5 - 105*a^4*b*c*d^6))/b^8
```

3.1286 $\int \frac{(c+dx)^7}{(a+bx)^4} dx$

Optimal. Leaf size=187

$$\frac{35d^4(bc-ad)^3x}{b^7}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{21d^5(bc-ad)^2(a+bx)^2}{2b^8}+\frac{7d^6(bc-ad)(a+$$

```
[Out] 35*d^4*(-a*d+b*c)^3*x/b^7-1/3*(-a*d+b*c)^7/b^8/(b*x+a)^3-7/2*d*(-a*d+b*c)^6
/b^8/(b*x+a)^2-21*d^2*(-a*d+b*c)^5/b^8/(b*x+a)+21/2*d^5*(-a*d+b*c)^2*(b*x+a
)^2/b^8+7/3*d^6*(-a*d+b*c)*(b*x+a)^3/b^8+1/4*d^7*(b*x+a)^4/b^8+35*d^3*(-a*d
+b*c)^4*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.15, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}+\frac{d^7(a+bx)^4}{4b^8}+\frac{35d^4x(bc-ad)^3}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (35*d^4*(b*c - a*d)^3*x)/b^7 - (b*c - a*d)^7/(3*b^8*(a + b*x)^3) - (7*d*(b*
c - a*d)^6)/(2*b^8*(a + b*x)^2) - (21*d^2*(b*c - a*d)^5)/(b^8*(a + b*x)) +
(21*d^5*(b*c - a*d)^2*(a + b*x)^2)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^3
)/(3*b^8) + (d^7*(a + b*x)^4)/(4*b^8) + (35*d^3*(b*c - a*d)^4*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^4} dx = \int \left(\frac{35d^4(bc-ad)^3}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^4}+\frac{7d(bc-ad)^6}{b^7(a+bx)^3}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^2}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)}+\right.$$
$$=\frac{35d^4(bc-ad)^3x}{b^7}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{21d^5(bc-ad)^2(a+b}{2b^8}$$

Mathematica [A]
time = 0.07, size = 199, normalized size = 1.06

$$\frac{12bd^4(35b^3c^3-84ab^2c^2d+70a^2bcd^2-20a^3d^3)\,x+6b^2d^5(21b^2c^2-28abcd+10a^2d^2)\,x^2+4b^3d^6(7bc-4ad)x^3+3b^4d^7x^4-\frac{4(bc-ad)^7}{(a+bx)^3}-\frac{42d(bc-ad)^6}{(a+bx)^2}+\frac{252d^2(-bc+ad)^5}{a+bx}+420d^3(bc-ad)^4\log(a+bx)}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (12*b*d^4*(35*b^3*c^3 - 84*a*b^2*c^2*d + 70*a^2*b*c*d^2 - 20*a^3*d^3)*x + 6
*b^2*d^5*(21*b^2*c^2 - 28*a*b*c*d + 10*a^2*d^2)*x^2 + 4*b^3*d^6*(7*b*c - 4*
a*d)*x^3 + 3*b^4*d^7*x^4 - (4*(b*c - a*d)^7)/(a + b*x)^3 - (42*d*(b*c - a*d
)^6)/(a + b*x)^2 + (252*d^2*(-(b*c) + a*d)^5)/(a + b*x) + 420*d^3*(b*c - a*
d)^4*Log[a + b*x])/(12*b^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(177) = 354$.
time = 0.14, size = 459, normalized size = 2.45

method	result
norman	$\frac{385a^7d^7-1540a^6bc\,d^6+2310a^5b^2c^2d^5-1540a^4b^3c^3d^4+385a^3b^4c^4d^3-42a^2b^5c^5d^2-7a\,b^6c^6d-2b^7c^7}{6b^8}+\frac{d^7x^7}{4b}+\frac{3\left(35a^5d^7-140a^4bc\,d^6+210a^3b^2c^2d\right.}{}$
default	$-\frac{d^4\left(-\frac{1}{4}d^3x^4b^3+\frac{4}{3}a\,b^2d^3x^3-\frac{7}{3}b^3c\,d^2x^3-5a^2b\,d^3x^2+14a\,b^2c\,d^2x^2-\frac{21}{2}b^3c^2d\,x^2+20a^3d^3x-70a^2bc\,d^2x+84a\,b^2c^2dx-35b^3c^3x\right)}{b^7}+\frac{2}{}$
risch	$\frac{d^7x^4}{4b^4}-\frac{4d^7a\,x^3}{3b^5}+\frac{7d^6c\,x^3}{3b^4}+\frac{5d^7a^2x^2}{b^6}-\frac{14d^6ac\,x^2}{b^5}+\frac{21d^5c^2x^2}{2b^4}-\frac{20d^7a^3x}{b^7}+\frac{70d^6a^2cx}{b^6}-\frac{84d^5a\,c^2x}{b^5}+\frac{35d^4c^3x}{b^4}+($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -d^4/b^7*(-1/4*d^3*x^4*b^3+4/3*a*b^2*d^3*x^3-7/3*b^3*c*d^2*x^3-5*a^2*b*d^3*
x^2+14*a*b^2*c*d^2*x^2-21/2*b^3*c^2*d*x^2+20*a^3*d^3*x-70*a^2*b*c*d^2*x+84*
a*b^2*c^2*d*x-35*b^3*c^3*x)+21/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^
2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)-7/2/b^8*d*(a^6*d^6-
6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*
b^5*c^5*d+b^6*c^6)/(b*x+a)^2+35/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^
2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(b*x+a)-1/3/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a
^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a
*b^6*c^6*d+b^7*c^7)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(177) = 354$.
time = 0.30, size = 484, normalized size = 2.59

$$-\frac{2\,b^7c^7+7\,ab^6c^6d+42\,a^2b^5c^5d^2-385\,a^3b^4c^4d^3+910\,a^4b^3c^3d^4-987\,a^5b^2c^2d^5+518\,a^6bcd^6-107\,a^7d^7+126\,(b^7c^5d^2-5\,ab^6c^4d^3+10\,a^2b^5c^3d^4-10\,a^3b^4c^2d^5+5\,a^4b^3cd^6-a^5b^2d^7)x^2+21\,(b^7c^6d+6\,ab^6c^5d^2-45\,a^2b^5c^4d^3+100\,a^3b^4c^3d^4-105\,a^4b^3c^2d^5+54\,a^5b^2cd^6-11\,a^6bd^7)x}{6\,(b^{11}x^3+3\,ab^{10}x^2+3\,a^2b^9x+a^3b^8)}+\frac{3\,b^3d^7x^4+4\,(7\,b^3cd^6-4\,ab^2d^7)x^3+6\,(21\,b^3c^2d^5-28\,ab^2cd^6+10\,a^2bd^7)x^2+12\,(35\,b^3c^3d^4-84\,ab^2c^2d^5+70\,a^2bcd^6-20\,a^3d^7)x}{12\,b^7}+\frac{35\,(b^4c^4d^3-4\,ab^3c^3d^4+6\,a^2b^2c^2d^5-4\,a^3bcd^6+a^4d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3
+ 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 + 518*a^6*b*c*d^6 - 107*a^7*d^7
 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2
*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21*(b^7*c^6*d + 6*a*b^6*c^5*d^2
 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 - 105*a^4*b^3*c^2*d^5 + 54*a^5*
b^2*c*d^6 - 11*a^6*b*d^7)*x)/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b
^8) + 1/12*(3*b^3*d^7*x^4 + 4*(7*b^3*c*d^6 - 4*a*b^2*d^7)*x^3 + 6*(21*b^3*c
^2*d^5 - 28*a*b^2*c*d^6 + 10*a^2*b*d^7)*x^2 + 12*(35*b^3*c^3*d^4 - 84*a*b^2
*c^2*d^5 + 70*a^2*b*c*d^6 - 20*a^3*d^7)*x)/b^7 + 35*(b^4*c^4*d^3 - 4*a*b^3*
c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(177) = 354$.
time = 0.81, size = 739, normalized size = 3.95

$$\frac{3b^7d^7x^7 - 4b^7c^7 - 14ab^6c^6d - 84a^2b^5c^5d^2 + 770a^3b^4c^4d^3 - 1820a^4b^3c^3d^4 + 1974a^5b^2c^2d^5 - 1036a^6bcd^6 + 214a^7d^7 + 7(4b^7cd^6 - ab^6d^7)x^6 + 21(6b^7c^2d^5 - 4ab^6cd^6 + a^2b^5d^7)x^5 + 105(4b^7c^3d^4 - 6ab^6c^2d^5 + 4a^2b^5cd^6 - a^3b^4d^7)x^4 + 2(630ab^6c^3d^4 - 1323a^2b^5c^2d^5 + 1022a^3b^4cd^6 - 278a^4b^3d^7)x^3 - 6(42b^7c^5d^2 - 210ab^6c^4d^3 + 210a^2b^5c^3d^4 + 63a^3b^4c^2d^5 - 182a^4b^3cd^6 + 68a^5b^2d^7)x^2 - 6(7b^7c^6d + 42ab^6c^5d^2 - 315a^2b^5c^4d^3 + 630a^3b^4c^3d^4 - 567a^4b^3c^2d^5 + 238a^5b^2cd^6 - 37a^6bd^7)x + 420(a^3b^4c^4d^3 - 4a^4b^3c^3d^4 + 6a^5b^2c^2d^5 - 4a^6bcd^6 + a^7d^7 + (b^7c^4d^3 - 4ab^6c^3d^4 + 6a^2b^5c^2d^5 - 4a^3b^4cd^6 + a^4b^3d^7)x^3 + 3(ab^6c^4d^3 - 4a^2b^5c^3d^4 + 6a^3b^4c^2d^5 - 4a^4b^3cd^6 + a^5b^2d^7)x^2 + 3(a^2b^5c^4d^3 - 4a^3b^4c^3d^4 + 6a^4b^3c^2d^5 - 4a^5b^2cd^6 + a^6bd^7)x)\log(bx + a)}{12(b^{11}x^3 + 3ab^{10}x^2 + 3a^2b^9x + a^3b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*d^7*x^7 - 4*b^7*c^7 - 14*a*b^6*c^6*d - 84*a^2*b^5*c^5*d^2 + 770
*a^3*b^4*c^4*d^3 - 1820*a^4*b^3*c^3*d^4 + 1974*a^5*b^2*c^2*d^5 - 1036*a^6*b
*c*d^6 + 214*a^7*d^7 + 7*(4*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(6*b^7*c^2*d^5
- 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(4*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 +
 4*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 2*(630*a*b^6*c^3*d^4 - 1323*a^2*b^5*c
^2*d^5 + 1022*a^3*b^4*c*d^6 - 278*a^4*b^3*d^7)*x^3 - 6*(42*b^7*c^5*d^2 - 21
0*a*b^6*c^4*d^3 + 210*a^2*b^5*c^3*d^4 + 63*a^3*b^4*c^2*d^5 - 182*a^4*b^3*c*
d^6 + 68*a^5*b^2*d^7)*x^2 - 6*(7*b^7*c^6*d + 42*a*b^6*c^5*d^2 - 315*a^2*b^5
*c^4*d^3 + 630*a^3*b^4*c^3*d^4 - 567*a^4*b^3*c^2*d^5 + 238*a^5*b^2*c*d^6 -
37*a^6*b*d^7)*x + 420*(a^3*b^4*c^4*d^3 - 4*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*
d^5 - 4*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*
c^2*d^5 - 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 3*(a*b^6*c^4*d^3 - 4*a^2*b^5
*c^3*d^4 + 6*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 3*(a^2*
b^5*c^4*d^3 - 4*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 - 4*a^5*b^2*c*d^6 + a^6
*b*d^7)*x)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(172) = 344$.
time = 22.92, size = 474, normalized size = 2.53

$$x^3\left(-\frac{4ad^7}{3b^5} + \frac{7cd^6}{3b^4}\right) + x^2\cdot\left(\frac{5a^2d^7}{b^6} - \frac{14acd^6}{b^5} + \frac{21c^2d^5}{2b^4}\right) + x\left(-\frac{20a^3d^7}{b^7} + \frac{70a^2cd^6}{b^6} - \frac{84ac^2d^5}{b^5} + \frac{35c^3d^4}{b^4}\right) + \frac{107a^7d^7 - 518a^6bcd^6 + 987a^5b^2c^2d^5 - 910a^4b^3c^3d^4 + 385a^3b^4c^4d^3 - 42a^2b^5c^5d^2 - 7ab^6c^6d - 2b^7c^7 + x^2\cdot(126a^5b^2d^7 - 630a^4b^3cd^6 + 1260a^3b^4c^2d^5 - 1260a^2b^5c^3d^4 + 630ab^6c^4d^3 - 126b^7c^5d^2) + x(231a^6bd^7 - 1134a^5b^2cd^6 + 2205a^4b^3c^2d^5 - 2100a^3b^4c^3d^4 + 945a^2b^5c^4d^3 - 126ab^6c^5d^2 - 21b^7c^6d)}{6a^3b^8 + 18a^2b^9x + 18ab^{10}x^2 + 6b^{11}x^3} + \frac{d^7x^4}{4b^4} + \frac{35d^3(ad - bc)^4\log(a + bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**4,x)
```

```
[Out] x**3*(-4*a*d**7/(3*b**5) + 7*c*d**6/(3*b**4)) + x**2*(5*a**2*d**7/b**6 - 14
*a*c*d**6/b**5 + 21*c**2*d**5/(2*b**4)) + x*(-20*a**3*d**7/b**7 + 70*a**2*c
*d**6/b**6 - 84*a*c**2*d**5/b**5 + 35*c**3*d**4/b**4) + (107*a**7*d**7 - 51
8*a**6*b*c*d**6 + 987*a**5*b**2*c**2*d**5 - 910*a**4*b**3*c**3*d**4 + 385*a
**3*b**4*c**4*d**3 - 42*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 2*b**7*c**7
 + x**2*(126*a**5*b**2*d**7 - 630*a**4*b**3*c*d**6 + 1260*a**3*b**4*c**2*d*
*5 - 1260*a**2*b**5*c**3*d**4 + 630*a*b**6*c**4*d**3 - 126*b**7*c**5*d**2)
+ x*(231*a**6*b*d**7 - 1134*a**5*b**2*c*d**6 + 2205*a**4*b**3*c**2*d**5 - 2
100*a**3*b**4*c**3*d**4 + 945*a**2*b**5*c**4*d**3 - 126*a*b**6*c**5*d**2 -
21*b**7*c**6*d))/(6*a**3*b**8 + 18*a**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*
x**3) + d**7*x**4/(4*b**4) + 35*d**3*(a*d - b*c)**4*log(a + b*x)/b**8
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(177) = 354$.

time = 0.74, size = 470, normalized size = 2.51

$$\frac{35\left(b^{4}c^{4}d^{3}-4ab^{3}c^{3}d^{4}+6a^{2}b^{2}c^{2}d^{5}-4a^{3}bcd^{6}+a^{4}d^{7}\right)\log\left(|bx+a|\right)}{b^{8}}-\frac{2b^{7}c^{7}+7ab^{6}c^{6}d+42a^{2}b^{5}c^{5}d^{2}-385a^{3}b^{4}c^{4}d^{3}+910a^{4}b^{3}c^{3}d^{4}-987a^{5}b^{2}c^{2}d^{5}+518a^{6}bcd^{6}-107a^{7}d^{7}+126\left(b^{7}c^{5}d^{2}-5ab^{6}c^{4}d^{3}+10a^{2}b^{5}c^{3}d^{4}-10a^{3}b^{4}c^{2}d^{5}+5a^{4}b^{3}cd^{6}-a^{5}b^{2}d^{7}\right)x^{2}+21\left(b^{7}c^{6}d+6ab^{6}c^{5}d^{2}-45a^{2}b^{5}c^{4}d^{3}+100a^{3}b^{4}c^{3}d^{4}-105a^{4}b^{3}c^{2}d^{5}+54a^{5}b^{2}cd^{6}-11a^{6}bd^{7}\right)x}{6\left(bx+a\right)^{3}b^{8}}+\frac{3b^{12}d^{7}x^{4}+28b^{12}cd^{6}x^{3}-16ab^{11}d^{7}x^{3}+126b^{12}c^{2}d^{5}x^{2}-168ab^{11}cd^{6}x^{2}+60a^{2}b^{10}d^{7}x^{2}+420b^{12}c^{3}d^{4}x-1008ab^{11}c^{2}d^{5}x+840a^{2}b^{10}cd^{6}x-240a^{3}b^{9}d^{7}x}{12b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4
*d^7)*log(abs(b*x + a))/b^8 - 1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c
^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 +
518*a^6*b*c*d^6 - 107*a^7*d^7 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2
*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21
*(b^7*c^6*d + 6*a*b^6*c^5*d^2 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 -
105*a^4*b^3*c^2*d^5 + 54*a^5*b^2*c*d^6 - 11*a^6*b*d^7)*x)/((b*x + a)^3*b^8)
 + 1/12*(3*b^12*d^7*x^4 + 28*b^12*c*d^6*x^3 - 16*a*b^11*d^7*x^3 + 126*b^12*
c^2*d^5*x^2 - 168*a*b^11*c*d^6*x^2 + 60*a^2*b^10*d^7*x^2 + 420*b^12*c^3*d^4
*x - 1008*a*b^11*c^2*d^5*x + 840*a^2*b^10*c*d^6*x - 240*a^3*b^9*d^7*x)/b^16
```

Mupad [**B**]

time = 0.29, size = 559, normalized size = 2.99

$$x^{2}\left(\frac{2a\left(\frac{4ad^{7}}{b^{5}}-\frac{7cd^{6}}{b^{4}}\right)}{b}-\frac{3a^{2}d^{7}}{b^{6}}+\frac{21c^{2}d^{5}}{2b^{4}}\right)-x^{3}\left(\frac{4ad^{7}}{3b^{5}}-\frac{7cd^{6}}{3b^{4}}\right)-\frac{\frac{-107a^{7}d^{7}+518a^{6}bcd^{6}-987a^{5}b^{2}c^{2}d^{5}+910a^{4}b^{3}c^{3}d^{4}-385a^{3}b^{4}c^{4}d^{3}+42a^{2}b^{5}c^{5}d^{2}+7ab^{6}c^{6}d+2b^{7}c^{7}}{6b}+x\left(-\frac{77a^{6}d^{7}}{2}+189a^{5}bcd^{6}-\frac{735a^{4}b^{2}c^{2}d^{5}}{2}+350a^{3}b^{3}c^{3}d^{4}-\frac{315a^{2}b^{4}c^{4}d^{3}}{2}+21ab^{5}c^{5}d^{2}+\frac{7b^{6}c^{6}d}{2}\right)-x^{2}\left(21a^{5}bd^{7}-105a^{4}b^{2}cd^{6}+210a^{3}b^{3}c^{2}d^{5}-210a^{2}b^{4}c^{3}d^{4}+105ab^{5}c^{4}d^{3}-21b^{6}c^{5}d^{2}\right)}{a^{3}b^{7}+3a^{2}b^{8}x+3ab^{9}x^{2}+b^{10}x^{3}}-x\left(\frac{4a\left(\frac{4a\left(\frac{4ad^{7}}{b^{5}}-\frac{7cd^{6}}{b^{4}}\right)}{b}-\frac{6a^{2}d^{7}}{b^{6}}+\frac{21c^{2}d^{5}}{b^{4}}\right)}{b}+\frac{4a^{3}d^{7}}{b^{7}}-\frac{35c^{3}d^{4}}{b^{4}}-\frac{6a^{2}\left(\frac{4ad^{7}}{b^{5}}-\frac{7cd^{6}}{b^{4}}\right)}{b^{2}}\right)+\frac{\ln\left(a+bx\right)\left(35a^{4}d^{7}-140a^{3}bcd^{6}+210a^{2}b^{2}c^{2}d^{5}-140ab^{3}c^{3}d^{4}+35b^{4}c^{4}d^{3}\right)}{b^{8}}+\frac{d^{7}x^{4}}{4b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^4,x)
```

```
[Out] x^2*((2*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (3*a^2*d^7)/b^6 + (21*c^2*d^
5)/(2*b^4)) - x^3*((4*a*d^7)/(3*b^5) - (7*c*d^6)/(3*b^4)) - ((2*b^7*c^7 - 1
07*a^7*d^7 + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4
 - 987*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d + 518*a^6*b*c*d^6)/(6*b) + x*((7*b^6
```

```
*c^6*d)/2 - (77*a^6*d^7)/2 + 21*a*b^5*c^5*d^2 - (315*a^2*b^4*c^4*d^3)/2 + 3
50*a^3*b^3*c^3*d^4 - (735*a^4*b^2*c^2*d^5)/2 + 189*a^5*b*c*d^6) - x^2*(21*a
^5*b*d^7 - 21*b^6*c^5*d^2 + 105*a*b^5*c^4*d^3 - 105*a^4*b^2*c*d^6 - 210*a^2
*b^4*c^3*d^4 + 210*a^3*b^3*c^2*d^5))/(a^3*b^7 + b^10*x^3 + 3*a^2*b^8*x + 3*
a*b^9*x^2) - x*((4*a*((4*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (6*a^2*d^7)
/b^6 + (21*c^2*d^5)/b^4))/b + (4*a^3*d^7)/b^7 - (35*c^3*d^4)/b^4 - (6*a^2*(
(4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b^2) + (log(a + b*x)*(35*a^4*d^7 + 35*b^4*c
^4*d^3 - 140*a*b^3*c^3*d^4 + 210*a^2*b^2*c^2*d^5 - 140*a^3*b*c*d^6))/b^8 +
(d^7*x^4)/(4*b^4)
```

3.1287 $\int \frac{(c+dx)^7}{(a+bx)^5}\,dx$

Optimal. Leaf size=187

$$\frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+\frac{7d^6(bc-ad)(a+bx)^2}{2b^8}+$$

```
[Out] 21*d^5*(-a*d+b*c)^2*x/b^7-1/4*(-a*d+b*c)^7/b^8/(b*x+a)^4-7/3*d*(-a*d+b*c)^6
/b^8/(b*x+a)^3-21/2*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^2-35*d^3*(-a*d+b*c)^4/b^8/
(b*x+a)+7/2*d^6*(-a*d+b*c)*(b*x+a)^2/b^8+1/3*d^7*(b*x+a)^3/b^8+35*d^4*(-a*d
+b*c)^3*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.13, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}+\frac{d^7(a+bx)^3}{3b^8}+\frac{21d^5x(bc-ad)^2}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (21*d^5*(b*c - a*d)^2*x)/b^7 - (b*c - a*d)^7/(4*b^8*(a + b*x)^4) - (7*d*(b*
c - a*d)^6)/(3*b^8*(a + b*x)^3) - (21*d^2*(b*c - a*d)^5)/(2*b^8*(a + b*x)^2
) - (35*d^3*(b*c - a*d)^4)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*(a + b*x)^2
)/(2*b^8) + (d^7*(a + b*x)^3)/(3*b^8) + (35*d^4*(b*c - a*d)^3*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^5}\,dx=\int \left(\frac{21d^5(bc-ad)^2}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^5}+\frac{7d(bc-ad)^6}{b^7(a+bx)^4}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^3}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^2}+\right.$$

$$=\frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+$$

Mathematica [A]
time = 0.07, size = 173, normalized size = 0.93

$$\frac{12bd^5(21b^2c^2-35abcd+15a^2d^2)x+6b^2d^6(7bc-5ad)x^2+4b^3d^7x^3-\frac{3(bc-ad)^7}{(a+bx)^4}-\frac{28d(bc-ad)^6}{(a+bx)^3}+\frac{126d^2(-bc+ad)^5}{(a+bx)^2}-\frac{420d^3(bc-ad)^4}{a+bx}+420d^4(bc-ad)^3\log(a+bx)}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(21*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2)*x + 6*b^2*d^6*(7*b*c - 5*a
*d)*x^2 + 4*b^3*d^7*x^3 - (3*(b*c - a*d)^7)/(a + b*x)^4 - (28*d*(b*c - a*d)
^6)/(a + b*x)^3 + (126*d^2*(-(b*c) + a*d)^5)/(a + b*x)^2 - (420*d^3*(b*c -
a*d)^4)/(a + b*x) + 420*d^4*(b*c - a*d)^3*Log[a + b*x])/(12*b^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(177) = 354$.
time = 0.14, size = 453, normalized size = 2.42

method	result
norman	$-\frac{875a^7d^7-2625a^6bc\,d^6+2625a^5b^2c^2d^5-875a^4b^3c^3d^4+105a^3b^4c^4d^3+21a^2b^5c^5d^2+7a\,b^6c^6d+3b^7c^7}{12b^8}+\frac{d^7x^7}{3b}-\frac{\left(140a^4d^7-420a^3bc\,d^6+420b^2a^2c^2d^5\right.}{b^5}$
default	$\frac{d^5\left(\frac{1}{3}d^2x^3b^2-\frac{5}{2}ab\,d^2x^2+\frac{7}{2}b^2cd\,x^2+15a^2d^2x-35abcdx+21b^2c^2x\right)}{b^7}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{b^8(bx+a)}-\frac{-a^7d^7+}{}$
risch	$\frac{d^7x^3}{3b^5}-\frac{5d^7a\,x^2}{2b^6}+\frac{7d^6c\,x^2}{2b^5}+\frac{15d^7a^2x}{b^7}-\frac{35d^6acx}{b^6}+\frac{21d^5c^2x}{b^5}+\frac{\left(-35a^4b^2d^7+140a^3b^3c\,d^6-210a^2b^4c^2d^5+140a\,b^5c^3d^4-35b^6\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] d^5/b^7*(1/3*d^2*x^3*b^2-5/2*a*b*d^2*x^2+7/2*b^2*c*d*x^2+15*a^2*d^2*x-35*a*
b*c*d*x+21*b^2*c^2*x)-35/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)/(b*x+a)-1/4/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^
2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*
d+b^7*c^7)/(b*x+a)^4+21/2/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3
-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^2-35/b^8*d^4*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*x+a)-7/3/b^8*d*(a^6*d^6-6*a^5*b*c*d
^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b
^6*c^6)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(177) = 354$.
time = 0.35, size = 494, normalized size = 2.64

$$-\frac{3b^7c^7+7ab^6c^6d+21a^2b^5c^5d^2+105a^3b^4c^4d^3-875a^4b^3c^3d^4+1617a^5b^2c^2d^5-1197a^6bcd^6+319a^7d^7+420(b^7c^4d^3-4ab^6c^3d^4+6a^2b^5c^2d^5-4a^3b^4cd^6+a^4b^3d^7)x^3+126(b^7c^5d^2+5ab^6c^4d^3-30a^2b^5c^3d^4+50a^3b^4c^2d^5-35a^4b^3cd^6+9a^5b^2d^7)x^2+28(b^7c^6d+3ab^6c^5d^2+15a^2b^5c^4d^3-110a^3b^4c^3d^4+195a^4b^3c^2d^5-141a^5b^2cd^6+37a^6bd^7)x}{12(b^{12}x^4+4ab^{11}x^3+6a^2b^{10}x^2+4a^3b^9x+a^4b^8)}+\frac{2b^2d^7x^3+3(7b^2cd^6-5abd^7)x^2+6(21b^2c^2d^5-35abcd^6+15a^2d^7)x}{6b^7}+\frac{35(b^3c^3d^4-3ab^2c^2d^5+3a^2bcd^6-a^3d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^7*c^7 + 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3
 - 875*a^4*b^3*c^3*d^4 + 1617*a^5*b^2*c^2*d^5 - 1197*a^6*b*c*d^6 + 319*a^7*
d^7 + 420*(b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a^3*b^4*c*
d^6 + a^4*b^3*d^7)*x^3 + 126*(b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 - 30*a^2*b^5*c^
3*d^4 + 50*a^3*b^4*c^2*d^5 - 35*a^4*b^3*c*d^6 + 9*a^5*b^2*d^7)*x^2 + 28*(b^
7*c^6*d + 3*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 110*a^3*b^4*c^3*d^4 + 195*
a^4*b^3*c^2*d^5 - 141*a^5*b^2*c*d^6 + 37*a^6*b*d^7)*x)/(b^12*x^4 + 4*a*b^11
*x^3 + 6*a^2*b^10*x^2 + 4*a^3*b^9*x + a^4*b^8) + 1/6*(2*b^2*d^7*x^3 + 3*(7*
b^2*c*d^6 - 5*a*b*d^7)*x^2 + 6*(21*b^2*c^2*d^5 - 35*a*b*c*d^6 + 15*a^2*d^7)
*x)/b^7 + 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(
b*x + a)/b^8
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(177) = 354$.
time = 0.90, size = 754, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*d^7*x^7 - 3*b^7*c^7 - 7*a*b^6*c^6*d - 21*a^2*b^5*c^5*d^2 - 105*
a^3*b^4*c^4*d^3 + 875*a^4*b^3*c^3*d^4 - 1617*a^5*b^2*c^2*d^5 + 1197*a^6*b*c
*d^6 - 319*a^7*d^7 + 14*(3*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(3*b^7*c^2*d^5 -
 3*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 4*(252*a*b^6*c^2*d^5 - 357*a^2*b^5*c*d^
6 + 139*a^3*b^4*d^7)*x^4 - 4*(105*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4 + 252*a^2
*b^5*c^2*d^5 + 168*a^3*b^4*c*d^6 - 136*a^4*b^3*d^7)*x^3 - 6*(21*b^7*c^5*d^2
 + 105*a*b^6*c^4*d^3 - 630*a^2*b^5*c^3*d^4 + 882*a^3*b^4*c^2*d^5 - 462*a^4*
b^3*c*d^6 + 74*a^5*b^2*d^7)*x^2 - 4*(7*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 105*a
^2*b^5*c^4*d^3 - 770*a^3*b^4*c^3*d^4 + 1302*a^4*b^3*c^2*d^5 - 882*a^5*b^2*c
*d^6 + 214*a^6*b*d^7)*x + 420*(a^4*b^3*c^3*d^4 - 3*a^5*b^2*c^2*d^5 + 3*a^6*
b*c*d^6 - a^7*d^7 + (b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*
b^4*d^7)*x^4 + 4*(a*b^6*c^3*d^4 - 3*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 - a^4
*b^3*d^7)*x^3 + 6*(a^2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*d^6 -
a^5*b^2*d^7)*x^2 + 4*(a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6
 - a^6*b*d^7)*x)*log(b*x + a))/(b^12*x^4 + 4*a*b^11*x^3 + 6*a^2*b^10*x^2 +
4*a^3*b^9*x + a^4*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(177) = 354.
time = 0.61, size = 660, normalized size = 3.53

$$\frac{\left(2\,d^7+\frac{21\,(b^2cd^6-abd^7)}{(bx+a)b}+\frac{126\,(b^4c^2d^5-2\,ab^3cd^6+a^2b^2d^7)}{(bx+a)^2b^2}\right)(bx+a)^3}{6\,b^8}-\frac{35\,(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^8}-\frac{\frac{3\,b^{43}c^7}{(bx+a)^4}+\frac{28\,b^{42}c^6d}{(bx+a)^3}-\frac{21\,ab^{42}c^6d}{(bx+a)^4}+\frac{126\,b^{41}c^5d^2}{(bx+a)^2}-\frac{168\,ab^{41}c^5d^2}{(bx+a)^3}+\frac{63\,a^2b^{41}c^5d^2}{(bx+a)^4}+\frac{420\,b^{40}c^4d^3}{bx+a}-\frac{630\,ab^{40}c^4d^3}{(bx+a)^2}+\frac{420\,a^2b^{40}c^4d^3}{(bx+a)^3}-\frac{105\,a^3b^{40}c^4d^3}{(bx+a)^4}-\frac{1680\,ab^{39}c^3d^4}{bx+a}+\frac{1260\,a^2b^{39}c^3d^4}{(bx+a)^2}-\frac{560\,a^3b^{39}c^3d^4}{(bx+a)^3}+\frac{105\,a^4b^{39}c^3d^4}{(bx+a)^4}+\frac{2520\,a^2b^{38}c^2d^5}{bx+a}-\frac{1260\,a^3b^{38}c^2d^5}{(bx+a)^2}+\frac{420\,a^4b^{38}c^2d^5}{(bx+a)^3}-\frac{63\,a^5b^{38}c^2d^5}{(bx+a)^4}-\frac{1680\,a^3b^{37}cd^6}{bx+a}+\frac{630\,a^4b^{37}cd^6}{(bx+a)^2}-\frac{168\,a^5b^{37}cd^6}{(bx+a)^3}+\frac{21\,a^6b^{37}cd^6}{(bx+a)^4}+\frac{420\,a^4b^{36}d^7}{bx+a}-\frac{126\,a^5b^{36}d^7}{(bx+a)^2}+\frac{28\,a^6b^{36}d^7}{(bx+a)^3}-\frac{3\,a^7b^{36}d^7}{(bx+a)^4}}{12\,b^{44}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(2*d^7 + 21*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 126*(b^4*c^2*d^5 - 2*
a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2))*(b*x + a)^3/b^8 - 35*(b^3*c^3
*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(abs(b*x + a)/((b*x +
a)^2*abs(b)))/b^8 - 1/12*(3*b^43*c^7/(b*x + a)^4 + 28*b^42*c^6*d/(b*x + a)^
3 - 21*a*b^42*c^6*d/(b*x + a)^4 + 126*b^41*c^5*d^2/(b*x + a)^2 - 168*a*b^41
*c^5*d^2/(b*x + a)^3 + 63*a^2*b^41*c^5*d^2/(b*x + a)^4 + 420*b^40*c^4*d^3/(
b*x + a) - 630*a*b^40*c^4*d^3/(b*x + a)^2 + 420*a^2*b^40*c^4*d^3/(b*x + a)^
3 - 105*a^3*b^40*c^4*d^3/(b*x + a)^4 - 1680*a*b^39*c^3*d^4/(b*x + a) + 1260
*a^2*b^39*c^3*d^4/(b*x + a)^2 - 560*a^3*b^39*c^3*d^4/(b*x + a)^3 + 105*a^4*
b^39*c^3*d^4/(b*x + a)^4 + 2520*a^2*b^38*c^2*d^5/(b*x + a) - 1260*a^3*b^38*
c^2*d^5/(b*x + a)^2 + 420*a^4*b^38*c^2*d^5/(b*x + a)^3 - 63*a^5*b^38*c^2*d^
5/(b*x + a)^4 - 1680*a^3*b^37*c*d^6/(b*x + a) + 630*a^4*b^37*c*d^6/(b*x + a
)^2 - 168*a^5*b^37*c*d^6/(b*x + a)^3 + 21*a^6*b^37*c*d^6/(b*x + a)^4 + 420*
a^4*b^36*d^7/(b*x + a) - 126*a^5*b^36*d^7/(b*x + a)^2 + 28*a^6*b^36*d^7/(b*
x + a)^3 - 3*a^7*b^36*d^7/(b*x + a)^4)/b^44
```

Mupad [B]
time = 0.77, size = 512, normalized size = 2.74

$$x\left(\frac{5\,a\left(\frac{5\,a\,d^7}{b^6}-\frac{7\,c\,d^6}{b^5}\right)}{b}-\frac{10\,a^2\,d^7}{b^7}+\frac{21\,c^2\,d^5}{b^5}\right)-x^2\left(\frac{5\,a\,d^7}{2\,b^6}-\frac{7\,c\,d^6}{2\,b^5}\right)-\frac{\frac{319\,a^7\,d^7-1197\,a^6\,b\,c\,d^6+1617\,a^5\,b^2\,c^2\,d^5-875\,a^4\,b^3\,c^3\,d^4+105\,a^3\,b^4\,c^4\,d^3+21\,a^2\,b^5\,c^5\,d^2+7\,a\,b^6\,c^6\,d+3\,b^7\,c^7}{12\,b}+x\left(\frac{259\,a^6\,d^7}{3}-329\,a^5\,b\,c\,d^6+455\,a^4\,b^2\,c^2\,d^5-\frac{770\,a^3\,b^3\,c^3\,d^4}{3}+35\,a^2\,b^4\,c^4\,d^3+7\,a\,b^5\,c^5\,d^2+\frac{7\,b^6\,c^6\,d}{3}\right)+x^3\,(35\,a^4\,b^2\,d^7-140\,a^3\,b^3\,c\,d^6+210\,a^2\,b^4\,c^2\,d^5-140\,a\,b^5\,c^3\,d^4+35\,b^6\,c^4\,d^3)+x^2\left(\frac{189\,a^5\,b\,d^7}{2}-\frac{735\,a^4\,b^2\,c\,d^6}{2}+525\,a^3\,b^3\,c^2\,d^5-315\,a^2\,b^4\,c^3\,d^4+\frac{105\,a\,b^5\,c^4\,d^3}{2}+\frac{21\,b^6\,c^5\,d^2}{2}\right)}{a^4\,b^7+4\,a^3\,b^8\,x+6\,a^2\,b^9\,x^2+4\,a\,b^{10}\,x^3+b^{11}\,x^4}-\frac{\ln(a+b\,x)\,(35\,a^3\,d^7-105\,a^2\,b\,c\,d^6+105\,a\,b^2\,c^2\,d^5-35\,b^3\,c^3\,d^4)}{b^8}+\frac{d^7\,x^3}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^5,x)
```

```
[Out] x*((5*a*((5*a*d^7)/b^6 - (7*c*d^6)/b^5))/b - (10*a^2*d^7)/b^7 + (21*c^2*d^5
)/b^5) - x^2*((5*a*d^7)/(2*b^6) - (7*c*d^6)/(2*b^5)) - ((319*a^7*d^7 + 3*b^
7*c^7 + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 - 875*a^4*b^3*c^3*d^4 + 16
17*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 1197*a^6*b*c*d^6)/(12*b) + x*((259*a^6
*d^7)/3 + (7*b^6*c^6*d)/3 + 7*a*b^5*c^5*d^2 + 35*a^2*b^4*c^4*d^3 - (770*a^3
*b^3*c^3*d^4)/3 + 455*a^4*b^2*c^2*d^5 - 329*a^5*b*c*d^6) + x^3*(35*a^4*b^2*
d^7 + 35*b^6*c^4*d^3 - 140*a*b^5*c^3*d^4 - 140*a^3*b^3*c*d^6 + 210*a^2*b^4*
c^2*d^5) + x^2*((189*a^5*b*d^7)/2 + (21*b^6*c^5*d^2)/2 + (105*a*b^5*c^4*d^3
)/2 - (735*a^4*b^2*c*d^6)/2 - 315*a^2*b^4*c^3*d^4 + 525*a^3*b^3*c^2*d^5))/(
a^4*b^7 + b^11*x^4 + 4*a^3*b^8*x + 4*a*b^10*x^3 + 6*a^2*b^9*x^2) - (log(a +
 b*x)*(35*a^3*d^7 - 35*b^3*c^3*d^4 + 105*a*b^2*c^2*d^5 - 105*a^2*b*c*d^6))/
b^8 + (d^7*x^3)/(3*b^5)
```

3.1288 $\int \frac{(c+dx)^7}{(a+bx)^6} dx$

Optimal. Leaf size=181

$$\frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}+21c$$

```
[Out] d^6*(-6*a*d+7*b*c)*x/b^7+1/2*d^7*x^2/b^6-1/5*(-a*d+b*c)^7/b^8/(b*x+a)^5-7/4
*d*(-a*d+b*c)^6/b^8/(b*x+a)^4-7*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^3-35/2*d^3*(-a
*d+b*c)^4/b^8/(b*x+a)^2-35*d^4*(-a*d+b*c)^3/b^8/(b*x+a)+21*d^5*(-a*d+b*c)^2
*ln(b*x+a)/b^8
```

Rubi [A]

time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}+\frac{d^6x(7bc-6ad)}{b^7}+\frac{d^7x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (d^6*(7*b*c - 6*a*d)*x)/b^7 + (d^7*x^2)/(2*b^6) - (b*c - a*d)^7/(5*b^8*(a +
 b*x)^5) - (7*d*(b*c - a*d)^6)/(4*b^8*(a + b*x)^4) - (7*d^2*(b*c - a*d)^5)/
(b^8*(a + b*x)^3) - (35*d^3*(b*c - a*d)^4)/(2*b^8*(a + b*x)^2) - (35*d^4*(b
*c - a*d)^3)/(b^8*(a + b*x)) + (21*d^5*(b*c - a*d)^2*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^6}\,dx &= \int \left(\frac{d^6(7bc-6ad)}{b^7}+\frac{d^7x}{b^6}+\frac{(bc-ad)^7}{b^7(a+bx)^6}+\frac{7d(bc-ad)^6}{b^7(a+bx)^5}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^4}+\frac{35d^3(bc-}{b^7(a+bx}\right.\\ &= \frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad}{2b^8(a+bx)^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(181) = 362.

time = 0.10, size = 389, normalized size = 2.15

$$\frac{459a^7d^7+3a^6bd^6(-406c+625dx)+a^5b^2d^5(959c^2-5250cdx+2700d^2x^2)+5a^4b^3d^4(-28c^3+875c^2dx-1680cd^2x^2+260d^3x^3)-5a^3b^4d^3(7c^4+140c^3dx-1540c^2d^2x^2+1120cd^3x^3+80d^4x^4)-a^2b^5d^2(14c^5+175c^4dx+1400c^3d^2x^2-6300c^2d^3x^3+700cd^4x^4+500d^5x^5)-7ab^6d(c^6+10c^5dx+50c^4d^2x^2+200c^3d^3x^3-300c^2d^4x^4-100cd^5x^5+10d^6x^6)-b^7(4c^7+35c^6dx+140c^5d^2x^2+350c^4d^3x^3+700c^3d^4x^4-140cd^6x^6-10d^7x^7)+420d^5(bc-ad)^2(a+bx)^5\log(a+bx)}{20b^8(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (459*a^7*d^7 + 3*a^6*b*d^6*(-406*c + 625*d*x) + a^5*b^2*d^5*(959*c^2 - 5250
*c*d*x + 2700*d^2*x^2) + 5*a^4*b^3*d^4*(-28*c^3 + 875*c^2*d*x - 1680*c*d^2*
x^2 + 260*d^3*x^3) - 5*a^3*b^4*d^3*(7*c^4 + 140*c^3*d*x - 1540*c^2*d^2*x^2
+ 1120*c*d^3*x^3 + 80*d^4*x^4) - a^2*b^5*d^2*(14*c^5 + 175*c^4*d*x + 1400*c
^3*d^2*x^2 - 6300*c^2*d^3*x^3 + 700*c*d^4*x^4 + 500*d^5*x^5) - 7*a*b^6*d*(c
^6 + 10*c^5*d*x + 50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 - 300*c^2*d^4*x^4 - 100*
c*d^5*x^5 + 10*d^6*x^6) - b^7*(4*c^7 + 35*c^6*d*x + 140*c^5*d^2*x^2 + 350*c
^4*d^3*x^3 + 700*c^3*d^4*x^4 - 140*c*d^6*x^6 - 10*d^7*x^7) + 420*d^5*(b*c -
 a*d)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(173) = 346$.
time = 0.18, size = 451, normalized size = 2.49

method	result
default	$-\frac{d^6\left(-\frac{1}{2}bd\,x^2+6adx-7bcx\right)}{b^7}+\frac{35d^4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{b^8(bx+a)}-\frac{7d\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d\right.}{4b^8(bx+a)^4}$
norman	$\frac{\frac{959a^7d^7-1918a^6bc\,d^6+959a^5b^2c^2d^5-140a^4b^3c^3d^4-35a^3b^4c^4d^3-14a^2b^5c^5d^2-7a\,b^6c^6d-4b^7c^7}{20b^8}+\frac{d^7x^7}{2b}+\frac{5\left(21a^3d^7-42a^2bc\,d^6+21a\,b^2c^2d^5-7b^3c^3\right.}{b^4}}{}$
risch	$\frac{d^7x^2}{2b^6}-\frac{6d^7ax}{b^7}+\frac{7d^6cx}{b^6}+\frac{\left(35a^3b^3d^7-105a^2b^4c\,d^6+105a\,b^5c^2d^5-35b^6c^3d^4\right)x^4+\frac{35b^2d^3\left(7a^4d^4-20a^3bc\,d^3+18a^2b^2c^2d^2-4a\,b^3c^3d-b^4\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -d^6/b^7*(-1/2*b*d*x^2+6*a*d*x-7*b*c*x)+35/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)/(b*x+a)-7/4/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c
^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)
^4-1/5/b^8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35
*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/(b*x+a)^5-35/2/b
^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x
+a)^2+21/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(b*x+a)+7/b^8*d^2*(a^5*d^5-5
*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(
b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(173) = 346$.
time = 0.34, size = 504, normalized size = 2.78

$$-\frac{4\,b^7c^7+7\,ab^6c^6d+14\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+140\,a^4b^3c^3d^4-959\,a^5b^2c^2d^5+1218\,a^6bcd^6-459\,a^7d^7+700\,(b^7c^3d^4-3\,ab^6c^2d^5+3\,a^2b^5cd^6-a^3b^4d^7)x^4+350\,(b^7c^4d^3+4\,ab^6c^3d^4-18\,a^2b^5c^2d^5+20\,a^3b^4cd^6-7\,a^4b^3d^7)x^3+70\,(2\,b^7c^5d^2+5\,ab^6c^4d^3+20\,a^2b^5c^3d^4-110\,a^3b^4c^2d^5+130\,a^4b^3cd^6-47\,a^5b^2d^7)x^2+35\,(b^7c^6d+2\,ab^6c^5d^2+5\,a^2b^5c^4d^3+20\,a^3b^4c^3d^4-125\,a^4b^3c^2d^5+154\,a^5b^2cd^6-57\,a^6bd^7)x}{20\,(b^{13}x^5+5\,ab^{12}x^4+10\,a^2b^{11}x^3+10\,a^3b^{10}x^2+5\,a^4b^9x+a^5b^8)}+\frac{bd^7x^2+2\,(7\,bcd^6-6\,ad^7)x}{2\,b^7}+\frac{21\,(b^2c^2d^5-2\,abcd^6+a^2d^7)\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(4*b^7*c^7 + 7*a*b^6*c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3
+ 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^
7 + 700*(b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4
 + 350*(b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d
^6 - 7*a^4*b^3*d^7)*x^3 + 70*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*
c^3*d^4 - 110*a^3*b^4*c^2*d^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 3
5*(b^7*c^6*d + 2*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 1
25*a^4*b^3*c^2*d^5 + 154*a^5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/(b^13*x^5 + 5*a*b
^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*
(b*d^7*x^2 + 2*(7*b*c*d^6 - 6*a*d^7)*x)/b^7 + 21*(b^2*c^2*d^5 - 2*a*b*c*d^6
 + a^2*d^7)*log(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(173) = 346$.
time = 0.57, size = 732, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*d^7*x^7 - 4*b^7*c^7 - 7*a*b^6*c^6*d - 14*a^2*b^5*c^5*d^2 - 35*
a^3*b^4*c^4*d^3 - 140*a^4*b^3*c^3*d^4 + 959*a^5*b^2*c^2*d^5 - 1218*a^6*b*c*
d^6 + 459*a^7*d^7 + 70*(2*b^7*c*d^6 - a*b^6*d^7)*x^6 + 100*(7*a*b^6*c*d^6 -
 5*a^2*b^5*d^7)*x^5 - 100*(7*b^7*c^3*d^4 - 21*a*b^6*c^2*d^5 + 7*a^2*b^5*c*d
^6 + 4*a^3*b^4*d^7)*x^4 - 50*(7*b^7*c^4*d^3 + 28*a*b^6*c^3*d^4 - 126*a^2*b^
5*c^2*d^5 + 112*a^3*b^4*c*d^6 - 26*a^4*b^3*d^7)*x^3 - 10*(14*b^7*c^5*d^2 +
35*a*b^6*c^4*d^3 + 140*a^2*b^5*c^3*d^4 - 770*a^3*b^4*c^2*d^5 + 840*a^4*b^3*
c*d^6 - 270*a^5*b^2*d^7)*x^2 - 5*(7*b^7*c^6*d + 14*a*b^6*c^5*d^2 + 35*a^2*b
^5*c^4*d^3 + 140*a^3*b^4*c^3*d^4 - 875*a^4*b^3*c^2*d^5 + 1050*a^5*b^2*c*d^6
 - 375*a^6*b*d^7)*x + 420*(a^5*b^2*c^2*d^5 - 2*a^6*b*c*d^6 + a^7*d^7 + (b^7
*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 5*(a*b^6*c^2*d^5 - 2*a^2*b^5*
c*d^6 + a^3*b^4*d^7)*x^4 + 10*(a^2*b^5*c^2*d^5 - 2*a^3*b^4*c*d^6 + a^4*b^3*
d^7)*x^3 + 10*(a^3*b^4*c^2*d^5 - 2*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 5*(a^
4*b^3*c^2*d^5 - 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^13*x^5 + 5
*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(173) = 346.
time = 0.63, size = 463, normalized size = 2.56

$$\frac{21\,(b^2c^2d^5-2\,abcd^6+a^2d^7)\log\left(|bx+a|\right)}{b^8}+\frac{b^6d^7x^2+14\,b^6cd^6x-12\,ab^5d^7x}{2\,b^{12}}-\frac{4\,b^7c^7+7\,ab^6c^6d+14\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+140\,a^4b^3c^3d^4-959\,a^5b^2c^2d^5+1218\,a^6bcd^6-459\,a^7d^7+700\,(b^7c^3d^4-3\,ab^6c^2d^5+3\,a^2b^5cd^6-a^3b^4d^7)x^4+350\,(b^7c^4d^3+4\,ab^6c^3d^4-18\,a^2b^5c^2d^5+20\,a^3b^4cd^6-7\,a^4b^3d^7)x^3+70\,(2\,b^7c^5d^2+5\,ab^6c^4d^3+20\,a^2b^5c^3d^4-110\,a^3b^4c^2d^5+130\,a^4b^3cd^6-47\,a^5b^2d^7)x^2+35\,(b^7c^6d+2\,ab^6c^5d^2+5\,a^2b^5c^4d^3+20\,a^3b^4c^3d^4-125\,a^4b^3c^2d^5+154\,a^5b^2cd^6-57\,a^6bd^7)x}{20\,(bx+a)^5b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 21*(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*d
^7*x^2 + 14*b^6*c*d^6*x - 12*a*b^5*d^7*x)/b^12 - 1/20*(4*b^7*c^7 + 7*a*b^6*
c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959
*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^7 + 700*(b^7*c^3*d^4 - 3*a*
b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 350*(b^7*c^4*d^3 + 4*a*b
^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d^6 - 7*a^4*b^3*d^7)*x^3 + 7
0*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 110*a^3*b^4*c^2*d
^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 35*(b^7*c^6*d + 2*a*b^6*c^5*
d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 125*a^4*b^3*c^2*d^5 + 154*a^
5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/((b*x + a)^5*b^8)
```

Mupad [B]
time = 0.34, size = 508, normalized size = 2.81

$$\frac{\ln\left(a+b\,x\right)\,\left(21\,a^2\,d^7-42\,a\,b\,c\,d^6+21\,b^2\,c^2\,d^5\right)}{b^8}-x\,\left(\frac{6\,a\,d^7}{b^7}-\frac{7\,c\,d^6}{b^6}\right)-\frac{\frac{-459\,a^7\,d^7+1218\,a^6\,b\,c\,d^6-959\,a^5\,b^2\,c^2\,d^5+140\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+14\,a^2\,b^5\,c^5\,d^2+7\,a\,b^6\,c^6\,d+4\,b^7\,c^7}{20\,b}+x\,\left(-\frac{399\,a^6\,d^7}{4}+\frac{539\,a^5\,b\,c\,d^6}{2}-\frac{875\,a^4\,b^2\,c^2\,d^5}{4}+35\,a^3\,b^3\,c^3\,d^4+\frac{35\,a^2\,b^4\,c^4\,d^3}{4}+\frac{7\,a\,b^5\,c^5\,d^2}{2}+\frac{7\,b^6\,c^6\,d}{4}\right)+x^3\,\left(-\frac{245\,a^4\,b^2\,d^7}{2}+350\,a^3\,b^3\,c\,d^6-315\,a^2\,b^4\,c^2\,d^5+70\,a\,b^5\,c^3\,d^4+\frac{35\,b^6\,c^4\,d^3}{2}\right)+x^2\,\left(-\frac{329\,a^5\,b\,d^7}{2}+455\,a^4\,b^2\,c\,d^6-385\,a^3\,b^3\,c^2\,d^5+70\,a^2\,b^4\,c^3\,d^4+\frac{35\,a\,b^5\,c^4\,d^3}{2}+7\,b^6\,c^5\,d^2\right)-x^4\,\left(35\,a^3\,b^3\,d^7-105\,a^2\,b^4\,c\,d^6+105\,a\,b^5\,c^2\,d^5-35\,b^6\,c^3\,d^4\right)}{a^5\,b^7+5\,a^4\,b^8\,x+10\,a^3\,b^9\,x^2+10\,a^2\,b^{10}\,x^3+5\,a\,b^{11}\,x^4+b^{12}\,x^5}+\frac{d^7\,x^2}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^6,x)
```

```
[Out] (log(a + b*x)*(21*a^2*d^7 + 21*b^2*c^2*d^5 - 42*a*b*c*d^6))/b^8 - x*((6*a*d
^7)/b^7 - (7*c*d^6)/b^6) - ((4*b^7*c^7 - 459*a^7*d^7 + 14*a^2*b^5*c^5*d^2 +
 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 7*a*b^6*c
^6*d + 1218*a^6*b*c*d^6)/(20*b) + x*((7*b^6*c^6*d)/4 - (399*a^6*d^7)/4 + (7
*a*b^5*c^5*d^2)/2 + (35*a^2*b^4*c^4*d^3)/4 + 35*a^3*b^3*c^3*d^4 - (875*a^4*
b^2*c^2*d^5)/4 + (539*a^5*b*c*d^6)/2) + x^3*((35*b^6*c^4*d^3)/2 - (245*a^4*
b^2*d^7)/2 + 70*a*b^5*c^3*d^4 + 350*a^3*b^3*c*d^6 - 315*a^2*b^4*c^2*d^5) +
x^2*(7*b^6*c^5*d^2 - (329*a^5*b*d^7)/2 + (35*a*b^5*c^4*d^3)/2 + 455*a^4*b^2
*c*d^6 + 70*a^2*b^4*c^3*d^4 - 385*a^3*b^3*c^2*d^5) - x^4*(35*a^3*b^3*d^7 -
35*b^6*c^3*d^4 + 105*a*b^5*c^2*d^5 - 105*a^2*b^4*c*d^6))/(a^5*b^7 + b^12*x^
5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*a^2*b^10*x^3) + (d^7*x
^2)/(2*b^6)
```

3.1289 $\int \frac{(c+dx)^7}{(a+bx)^7}\,dx$

Optimal. Leaf size=186

$$\frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}+\frac{7d^6}{}$$

```
[Out] d^7*x/b^7-1/6*(-a*d+b*c)^7/b^8/(b*x+a)^6-7/5*d*(-a*d+b*c)^6/b^8/(b*x+a)^5-2
1/4*d^2*(-a*d+b*c)^5/b^8/(b*x+a)^4-35/3*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^3-35/2
*d^4*(-a*d+b*c)^3/b^8/(b*x+a)^2-21*d^5*(-a*d+b*c)^2/b^8/(b*x+a)+7*d^6*(-a*d
+b*c)*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.12, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}+\frac{d^7x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] (d^7*x)/b^7 - (b*c - a*d)^7/(6*b^8*(a + b*x)^6) - (7*d*(b*c - a*d)^6)/(5*b^
8*(a + b*x)^5) - (21*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^4) - (35*d^3*(b*c
- a*d)^4)/(3*b^8*(a + b*x)^3) - (35*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^2)
- (21*d^5*(b*c - a*d)^2)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*Log[a + b*x])
/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^7}\,dx &= \int \left(\frac{d^7}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^7}+\frac{7d(bc-ad)^6}{b^7(a+bx)^6}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^5}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^4}+\frac{35d^4(bc-a}{b^7(a+bx}\right.\\ &= \frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad}{2b^8(a+bx)^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 390 vs. $2(186) = 372$.

time = 0.14, size = 390, normalized size = 2.10

$$-\frac{669a^7d^7+3a^6bd^6(-343c+1198dx)+3a^5b^2d^5(70c^2-1918cdx+2575d^2x^2)+5a^4b^3d^4(14c^3+252c^2dx-2625cd^2x^2+1640d^3x^3)+5a^3b^4d^3(7c^4+84c^3dx+630c^2d^2x^2-3080cd^3x^3+810d^4x^4)+3a^2b^5d^2(7c^5+70c^4dx+350c^3d^2x^2+1400c^2d^3x^3-3150cd^4x^4+120d^5x^5)+ab^6d(14c^6+126c^5dx+525c^4d^2x^2+1400c^3d^3x^3+3150c^2d^4x^4-2520cd^5x^5-360d^6x^6)+b^7(10c^7+84c^6dx+315c^5d^2x^2+700c^4d^3x^3+1050c^3d^4x^4+1260c^2d^5x^5-60d^7x^7)+420d^6(-bc+ad)(a+bx)^6\log(a+bx)}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7*d^7 + 3*a^6*b*d^6*(-343*c + 1198*d*x) + 3*a^5*b^2*d^5*(70*c^
2 - 1918*c*d*x + 2575*d^2*x^2) + 5*a^4*b^3*d^4*(14*c^3 + 252*c^2*d*x - 2625
*c*d^2*x^2 + 1640*d^3*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 84*c^3*d*x + 630*c^2*d^
2*x^2 - 3080*c*d^3*x^3 + 810*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 70*c^4*d*x +
 350*c^3*d^2*x^2 + 1400*c^2*d^3*x^3 - 3150*c*d^4*x^4 + 120*d^5*x^5) + a*b^6
*d*(14*c^6 + 126*c^5*d*x + 525*c^4*d^2*x^2 + 1400*c^3*d^3*x^3 + 3150*c^2*d^
4*x^4 - 2520*c*d^5*x^5 - 360*d^6*x^6) + b^7*(10*c^7 + 84*c^6*d*x + 315*c^5*
d^2*x^2 + 700*c^4*d^3*x^3 + 1050*c^3*d^4*x^4 + 1260*c^2*d^5*x^5 - 60*d^7*x^
7) + 420*d^6*(-(b*c) + a*d)*(a + b*x)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(176) = 352$.

time = 0.15, size = 456, normalized size = 2.45

method	result
risch	$\frac{d^7x}{b^7} + \frac{(-21a^2b^4d^7+42ab^5cd^6-21b^6c^2d^5)x^5 - \frac{35b^3d^4\left(5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3\right)x^4}{2} - \frac{35b^2d^3\left(13a^4d^4-22a^3bcd^3+6a^2b^2c^2d^2+2ab^3 \right.}{3}}{}$
default	$\frac{d^7x}{b^7} - \frac{21d^5(a^2d^2-2abcd+b^2c^2)}{b^8(bx+a)} + \frac{21d^2(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5)}{4b^8(bx+a)^4} - \frac{7d\left(a^6d^6-6a^5bcd^5+15a^4b\right.}{}$
norman	$\frac{\frac{d^7x^7}{b} - \frac{1029a^7d^7-1029a^6bcd^6+210a^5b^2c^2d^5+70a^4b^3c^3d^4+35a^3b^4c^4d^3+21a^2b^5c^5d^2+14ab^6c^6d+10b^7c^7}{60b^8} - \frac{3\left(14a^2d^7-14abcd^6+7b^2c^2d^5\right)x^5}{b^3} - \frac{5\left(\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] d^7*x/b^7-21/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+21/4/b^8*d^2*(a^5*
d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c
^5)/(b*x+a)^4-7/5/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^
3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^5+35/2/b^8*d^4*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^2-1/6/b^8*(-a^7*d^7+7
*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^
2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/(b*x+a)^6-7/b^8*d^6*(a*d-b*c)*ln(b*x+a
)-35/3/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c
^4)/(b*x+a)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(176) = 352$.
time = 0.32, size = 516, normalized size = 2.77

$$\frac{d^7x}{b^7} - \frac{10\,b^7c^7 + 14\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 35\,a^3b^4c^4d^3 + 70\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 - 1029\,a^6bcd^6 + 669\,a^7d^7 + 1260\,(b^7c^2d^5 - 2\,ab^6cd^6 + a^2b^5d^7)x^5 + 1050\,(b^7c^3d^4 + 3\,ab^6c^2d^5 - 9\,a^2b^5cd^6 + 5\,a^3b^4d^7)x^4 + 700\,(b^7c^4d^3 + 2\,ab^6c^3d^4 + 6\,a^2b^5c^2d^5 - 22\,a^3b^4cd^6 + 13\,a^4b^3d^7)x^3 + 105\,(3\,b^7c^5d^2 + 5\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 + 30\,a^3b^4c^2d^5 - 125\,a^4b^3cd^6 + 77\,a^5b^2d^7)x^2 + 42\,(2\,b^7c^6d + 3\,ab^6c^5d^2 + 5\,a^2b^5c^4d^3 + 10\,a^3b^4c^3d^4 + 30\,a^4b^3c^2d^5 - 137\,a^5b^2cd^6 + 87\,a^6bd^7)x}{60\,(b^{14}x^6 + 6\,ab^{13}x^5 + 15\,a^2b^{12}x^4 + 20\,a^3b^{11}x^3 + 15\,a^4b^{10}x^2 + 6\,a^5b^9x + a^6b^8)} + \frac{7\,(bcd^6 - ad^7)\log(bx + a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] d^7*x/b^7 - 1/60*(10*b^7*c^7 + 14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3
*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6
+ 669*a^7*d^7 + 1260*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050
*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 70
0*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 1
3*a^4*b^3*d^7)*x^3 + 105*(3*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*
d^4 + 30*a^3*b^4*c^2*d^5 - 125*a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*
b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a
^4*b^3*c^2*d^5 - 137*a^5*b^2*c*d^6 + 87*a^6*b*d^7)*x)/(b^14*x^6 + 6*a*b^13*
x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a
^6*b^8) + 7*(b*c*d^6 - a*d^7)*log(b*x + a)/b^8
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(176) = 352$.
time = 0.49, size = 692, normalized size = 3.72

$$\frac{60\,b^7d^7x^7 + 360\,ab^6d^7x^6 - 10\,b^7c^7 - 14\,ab^6c^6d - 21\,a^2b^5c^5d^2 - 35\,a^3b^4c^4d^3 - 70\,a^4b^3c^3d^4 - 210\,a^5b^2c^2d^5 + 1029\,a^6bcd^6 - 669\,a^7d^7 - 180\,(7\,b^7c^2d^5 - 14\,ab^6cd^6 + 2\,a^2b^5d^7)x^5 - 150\,(7\,b^7c^3d^4 + 21\,ab^6c^2d^5 - 63\,a^2b^5cd^6 + 27\,a^3b^4d^7)x^4 - 100\,(7\,b^7c^4d^3 + 14\,ab^6c^3d^4 + 42\,a^2b^5c^2d^5 - 154\,a^3b^4cd^6 + 82\,a^4b^3d^7)x^3 - 15\,(21\,b^7c^5d^2 + 35\,ab^6c^4d^3 + 70\,a^2b^5c^3d^4 + 210\,a^3b^4c^2d^5 - 875\,a^4b^3cd^6 + 515\,a^5b^2d^7)x^2 - 6\,(14\,b^7c^6d + 21\,ab^6c^5d^2 + 35\,a^2b^5c^4d^3 + 70\,a^3b^4c^3d^4 + 210\,a^4b^3c^2d^5 - 959\,a^5b^2cd^6 + 599\,a^6bd^7)x + 420\,(a^6bcd^6 - a^7d^7 + (b^7cd^6 - ab^6d^7)x^6 + 6\,(ab^6cd^6 - a^2b^5d^7)x^5 + 15\,(a^2b^5cd^6 - a^3b^4d^7)x^4 + 20\,(a^3b^4cd^6 - a^4b^3d^7)x^3 + 15\,(a^4b^3cd^6 - a^5b^2d^7)x^2 + 6\,(a^5b^2cd^6 - a^6bd^7)x)\log(bx + a)}{60\,(b^{14}x^6 + 6\,ab^{13}x^5 + 15\,a^2b^{12}x^4 + 20\,a^3b^{11}x^3 + 15\,a^4b^{10}x^2 + 6\,a^5b^9x + a^6b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*d^7*x^7 + 360*a*b^6*d^7*x^6 - 10*b^7*c^7 - 14*a*b^6*c^6*d - 21
*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 - 70*a^4*b^3*c^3*d^4 - 210*a^5*b^2*c^
2*d^5 + 1029*a^6*b*c*d^6 - 669*a^7*d^7 - 180*(7*b^7*c^2*d^5 - 14*a*b^6*c*d^
6 + 2*a^2*b^5*d^7)*x^5 - 150*(7*b^7*c^3*d^4 + 21*a*b^6*c^2*d^5 - 63*a^2*b^5
*c*d^6 + 27*a^3*b^4*d^7)*x^4 - 100*(7*b^7*c^4*d^3 + 14*a*b^6*c^3*d^4 + 42*a
^2*b^5*c^2*d^5 - 154*a^3*b^4*c*d^6 + 82*a^4*b^3*d^7)*x^3 - 15*(21*b^7*c^5*d
^2 + 35*a*b^6*c^4*d^3 + 70*a^2*b^5*c^3*d^4 + 210*a^3*b^4*c^2*d^5 - 875*a^4*
b^3*c*d^6 + 515*a^5*b^2*d^7)*x^2 - 6*(14*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 35*
a^2*b^5*c^4*d^3 + 70*a^3*b^4*c^3*d^4 + 210*a^4*b^3*c^2*d^5 - 959*a^5*b^2*c*
d^6 + 599*a^6*b*d^7)*x + 420*(a^6*b*c*d^6 - a^7*d^7 + (b^7*c*d^6 - a*b^6*d^
7)*x^6 + 6*(a*b^6*c*d^6 - a^2*b^5*d^7)*x^5 + 15*(a^2*b^5*c*d^6 - a^3*b^4*d^
7)*x^4 + 20*(a^3*b^4*c*d^6 - a^4*b^3*d^7)*x^3 + 15*(a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^2 + 6*(a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^14*x^6 + 6*a*
b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*
x + a^6*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(176) = 352.
time = 0.80, size = 459, normalized size = 2.47

$$\frac{d^7x}{b^7}+\frac{7\,(bcd^6-ad^7)\log\left(|bx+a|\right)}{b^8}-\frac{10\,b^7c^7+14\,ab^6c^6d+21\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+70\,a^4b^3c^3d^4+210\,a^5b^2c^2d^5-1029\,a^6bcd^6+669\,a^7d^7+1260\,(b^7c^2d^5-2\,ab^6cd^6+a^2b^5d^7)x^5+1050\,(b^7c^3d^4+3\,ab^6c^2d^5-9\,a^2b^5cd^6+5\,a^3b^4d^7)x^4+700\,(b^7c^4d^3+2\,ab^6c^3d^4+6\,a^2b^5c^2d^5-22\,a^3b^4cd^6+13\,a^4b^3d^7)x^3+105\,(3\,b^7c^5d^2+5\,ab^6c^4d^3+10\,a^2b^5c^3d^4+30\,a^3b^4c^2d^5-125\,a^4b^3cd^6+77\,a^5b^2d^7)x^2+42\,(2\,b^7c^6d+3\,ab^6c^5d^2+5\,a^2b^5c^4d^3+10\,a^3b^4c^3d^4+30\,a^4b^3c^2d^5-137\,a^5b^2cd^6+87\,a^6bd^7)x}{60\,(bx+a)^6b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] d^7*x/b^7 + 7*(b*c*d^6 - a*d^7)*log(abs(b*x + a))/b^8 - 1/60*(10*b^7*c^7 +
14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d
^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6 + 669*a^7*d^7 + 1260*(b^7*c^2*d
^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5
 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 700*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^
4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 13*a^4*b^3*d^7)*x^3 + 105*(3*b^7
*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 - 125*
a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5
*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 - 137*a^5*b^2*c*
d^6 + 87*a^6*b*d^7)*x)/((b*x + a)^6*b^8)
```

Mupad [**B**]
time = 0.37, size = 517, normalized size = 2.78

$$\frac{d^7\,x}{b^7}-\frac{\ln\left(a+b\,x\right)\,\left(7\,a\,d^7-7\,b\,c\,d^6\right)}{b^8}-\frac{\frac{669\,a^7\,d^7-1029\,a^6\,b\,c\,d^6+210\,a^5\,b^2\,c^2\,d^5+70\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+21\,a^2\,b^5\,c^5\,d^2+14\,a\,b^6\,c^6\,d+10\,b^7\,c^7}{60\,b}+x\,\left(\frac{609\,a^6\,d^7}{10}-\frac{959\,a^5\,b\,c\,d^6}{10}+21\,a^4\,b^2\,c^2\,d^5+7\,a^3\,b^3\,c^3\,d^4+\frac{7\,a^2\,b^4\,c^4\,d^3}{2}+\frac{21\,a\,b^5\,c^5\,d^2}{10}+\frac{7\,b^6\,c^6\,d}{5}\right)+x^3\,\left(\frac{455\,a^4\,b^2\,d^7}{3}-\frac{770\,a^3\,b^3\,c\,d^6}{3}+70\,a^2\,b^4\,c^2\,d^5+\frac{70\,a\,b^5\,c^3\,d^4}{3}+\frac{35\,b^6\,c^4\,d^3}{3}\right)+x^2\,\left(\frac{539\,a^5\,b\,d^7}{4}-\frac{875\,a^4\,b^2\,c\,d^6}{4}+\frac{105\,a^3\,b^3\,c^2\,d^5}{2}+\frac{35\,a^2\,b^4\,c^3\,d^4}{2}+\frac{35\,a\,b^5\,c^4\,d^3}{4}+\frac{21\,b^6\,c^5\,d^2}{4}\right)+x^5\,\left(21\,a^2\,b^4\,d^7-42\,a\,b^5\,c\,d^6+21\,b^6\,c^2\,d^5\right)+x^4\,\left(\frac{175\,a^3\,b^3\,d^7}{2}-\frac{315\,a^2\,b^4\,c\,d^6}{2}+\frac{105\,a\,b^5\,c^2\,d^5}{2}+\frac{35\,b^6\,c^3\,d^4}{2}\right)}{a^6\,b^7+6\,a^5\,b^8\,x+15\,a^4\,b^9\,x^2+20\,a^3\,b^{10}\,x^3+15\,a^2\,b^{11}\,x^4+6\,a\,b^{12}\,x^5+b^{13}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^7,x)
```

```
[Out] (d^7*x)/b^7 - (log(a + b*x)*(7*a*d^7 - 7*b*c*d^6))/b^8 - ((669*a^7*d^7 + 10
*b^7*c^7 + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 2
10*a^5*b^2*c^2*d^5 + 14*a*b^6*c^6*d - 1029*a^6*b*c*d^6)/(60*b) + x*((609*a^
6*d^7)/10 + (7*b^6*c^6*d)/5 + (21*a*b^5*c^5*d^2)/10 + (7*a^2*b^4*c^4*d^3)/2
 + 7*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - (959*a^5*b*c*d^6)/10) + x^3*((4
55*a^4*b^2*d^7)/3 + (35*b^6*c^4*d^3)/3 + (70*a*b^5*c^3*d^4)/3 - (770*a^3*b^
3*c*d^6)/3 + 70*a^2*b^4*c^2*d^5) + x^2*((539*a^5*b*d^7)/4 + (21*b^6*c^5*d^2
```

)/4 + (35*a*b^5*c^4*d^3)/4 - (875*a^4*b^2*c*d^6)/4 + (35*a^2*b^4*c^3*d^4)/2 + (105*a^3*b^3*c^2*d^5)/2) + x^5*(21*a^2*b^4*d^7 + 21*b^6*c^2*d^5 - 42*a*b^5*c*d^6) + x^4*((175*a^3*b^3*d^7)/2 + (35*b^6*c^3*d^4)/2 + (105*a*b^5*c^2*d^5)/2 - (315*a^2*b^4*c*d^6)/2))/(a^6*b^7 + b^13*x^6 + 6*a^5*b^8*x + 6*a*b^12*x^5 + 15*a^4*b^9*x^2 + 20*a^3*b^10*x^3 + 15*a^2*b^11*x^4)

3.1290 $\int \frac{(c+dx)^7}{(a+bx)^8} \, dx$

Optimal. Leaf size=194

$$-\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{7d^6(bc-}{b^8(a+}$$

```
[Out] -1/7*(-a*d+b*c)^7/b^8/(b*x+a)^7-7/6*d*(-a*d+b*c)^6/b^8/(b*x+a)^6-21/5*d^2*(
-a*d+b*c)^5/b^8/(b*x+a)^5-35/4*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^4-35/3*d^4*(-a*
d+b*c)^3/b^8/(b*x+a)^3-21/2*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^2-7*d^6*(-a*d+b*c)
/b^8/(b*x+a)+d^7*ln(b*x+a)/b^8
```

Rubi [A]
time = 0.11, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+\frac{d^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -1/7*(b*c - a*d)^7/(b^8*(a + b*x)^7) - (7*d*(b*c - a*d)^6)/(6*b^8*(a + b*x)
^6) - (21*d^2*(b*c - a*d)^5)/(5*b^8*(a + b*x)^5) - (35*d^3*(b*c - a*d)^4)/(
4*b^8*(a + b*x)^4) - (35*d^4*(b*c - a*d)^3)/(3*b^8*(a + b*x)^3) - (21*d^5*(
b*c - a*d)^2)/(2*b^8*(a + b*x)^2) - (7*d^6*(b*c - a*d))/(b^8*(a + b*x)) + (
d^7*Log[a + b*x])/b^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^8}+\frac{7d(bc-ad)^6}{b^7(a+bx)^7}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^6}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^5}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^4}+\right.\\ &= -\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\end{aligned}$$

Mathematica [A]
time = 0.11, size = 308, normalized size = 1.59

$$-\frac{(bc-ad)\left(1089a^6d^6+3a^5bd^5(223c+2401dx)+3a^4b^2d^4\left(153c^2+1421cdx+6713d^2x^2\right)+a^3b^3d^3\left(319c^3+2793c^2dx+11319cd^2x^2+30625d^3x^3\right)+a^2b^4d^2\left(214c^4+1813c^3dx+6909c^2d^2x^2+15925cd^3x^3+26950d^4x^4\right)+ab^5d\left(130c^5+1078c^4dx+3969c^3d^2x^2+8575c^2d^3x^3+12250cd^4x^4+13230d^5x^5\right)+b^6\left(60c^6+490c^5dx+1764c^4d^2x^2+3675c^3d^3x^3+4900c^2d^4x^4+4410cd^5x^5+2940d^6x^6\right)\right)}{420b^8(a+bx)^7}+\frac{d^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -1/420*((b*c - a*d)*(1089*a^6*d^6 + 3*a^5*b*d^5*(223*c + 2401*d*x) + 3*a^4*
b^2*d^4*(153*c^2 + 1421*c*d*x + 6713*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 2793
*c^2*d*x + 11319*c*d^2*x^2 + 30625*d^3*x^3) + a^2*b^4*d^2*(214*c^4 + 1813*c
^3*d*x + 6909*c^2*d^2*x^2 + 15925*c*d^3*x^3 + 26950*d^4*x^4) + a*b^5*d*(130
*c^5 + 1078*c^4*d*x + 3969*c^3*d^2*x^2 + 8575*c^2*d^3*x^3 + 12250*c*d^4*x^4
 + 13230*d^5*x^5) + b^6*(60*c^6 + 490*c^5*d*x + 1764*c^4*d^2*x^2 + 3675*c^3
*d^3*x^3 + 4900*c^2*d^4*x^4 + 4410*c*d^5*x^5 + 2940*d^6*x^6)))/(b^8*(a + b*
x)^7) + (d^7*Log[a + b*x])/b^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(182) = 364$.
time = 0.15, size = 462, normalized size = 2.38

method	result
risch	$\frac{\frac{7d^6(ad-bc)x^6}{b^2}+\frac{21d^5\left(3a^2d^2-2abcd-b^2c^2\right)x^5}{2b^3}+\frac{35d^4\left(11a^3d^3-6a^2bc\,d^2-3a\,b^2c^2d-2b^3c^3\right)x^4}{6b^4}+\frac{35d^3\left(25a^4d^4-12a^3bc\,d^3-6a^2b^2c^2d^2-4a\,b^3c^3d-\right.}{12b^5}}{}$
norman	$\frac{\frac{1089a^7d^7-420a^6bc\,d^6-210a^5b^2c^2d^5-140a^4b^3c^3d^4-105a^3b^4c^4d^3-84a^2b^5c^5d^2-70a\,b^6c^6d-60b^7c^7}{420b^8}+\frac{7\left(a\,d^7-bc\,d^6\right)x^6}{b^2}+\frac{21\left(3a^2d^7-2abc\,d^6-b^2\right.}{2b^3}}{}$
default	$\frac{7d^6(ad-bc)}{b^8(bx+a)}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{4b^8(bx+a)^4}+\frac{21d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b\right.}{5b^8(bx+a)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 7/b^8*d^6*(a*d-b*c)/(b*x+a)-35/4/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^4+21/5/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4
+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^5-1/7
*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c
^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^7-21/2/b^8*d^5
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2-7/6/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*
a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6
)/(b*x+a)^6+d^7*ln(b*x+a)/b^8+35/3/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(182) = 364$.
time = 0.33, size = 534, normalized size = 2.75

$$-\frac{60\,b^7c^7+70\,ab^6c^6d+84\,a^2b^5c^5d^2+105\,a^3b^4c^4d^3+140\,a^4b^3c^3d^4+210\,a^5b^2c^2d^5+420\,a^6bcd^6-1089\,a^7d^7+2940\,(b^7cd^6-ab^6d^7)x^6+4410\,(b^7c^2d^5+2\,ab^6cd^6-3\,a^2b^5d^7)x^5+2450\,(2\,b^7c^3d^4+3\,ab^6c^2d^5+6\,a^2b^5cd^6-11\,a^3b^4d^7)x^4+1225\,(3\,b^7c^4d^3+4\,ab^6c^3d^4+6\,a^2b^5c^2d^5+12\,a^3b^4cd^6-25\,a^4b^3d^7)x^3+147\,(12\,b^7c^5d^2+15\,ab^6c^4d^3+20\,a^2b^5c^3d^4+30\,a^3b^4c^2d^5+60\,a^4b^3cd^6-137\,a^5b^2d^7)x^2+49\,(10\,b^7c^6d+12\,ab^6c^5d^2+15\,a^2b^5c^4d^3+20\,a^3b^4c^3d^4+30\,a^4b^3c^2d^5+60\,a^5b^2cd^6-147\,a^6bd^7)x}{420\,(b^{15}x^7+7\,ab^{14}x^6+21\,a^2b^{13}x^5+35\,a^3b^{12}x^4+35\,a^4b^{11}x^3+21\,a^5b^{10}x^2+7\,a^6b^9x+a^7b^8)}+\frac{d^7\log(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x)/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 + 35*a^3*b^12
*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8) + d^7*log
(b*x + a)/b^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(182) = 364$.
time = 0.62, size = 624, normalized size = 3.22

$$-\frac{60 b^{7} c^{7} + 70 a b^{6} c^{6} d + 84 a^{2} b^{5} c^{5} d^{2} + 105 a^{3} b^{4} c^{4} d^{3} + 140 a^{4} b^{3} c^{3} d^{4} + 210 a^{5} b^{2} c^{2} d^{5} + 420 a^{6} b c d^{6} - 1089 a^{7} d^{7} + 2940 \left(b^{7} c d^{6} - a b^{6} d^{7}\right) x^{6} + 4410 \left(b^{7} c^{2} d^{5} + 2 a b^{6} c d^{6} - 3 a^{2} b^{5} d^{7}\right) x^{5} + 2450 \left(2 b^{7} c^{3} d^{4} + 3 a b^{6} c^{2} d^{5} + 6 a^{2} b^{5} c d^{6} - 11 a^{3} b^{4} d^{7}\right) x^{4} + 1225 \left(3 b^{7} c^{4} d^{3} + 4 a b^{6} c^{3} d^{4} + 6 a^{2} b^{5} c^{2} d^{5} + 12 a^{3} b^{4} c d^{6} - 25 a^{4} b^{3} d^{7}\right) x^{3} + 147 \left(12 b^{7} c^{5} d^{2} + 15 a b^{6} c^{4} d^{3} + 20 a^{2} b^{5} c^{3} d^{4} + 30 a^{3} b^{4} c^{2} d^{5} + 60 a^{4} b^{3} c d^{6} - 137 a^{5} b^{2} d^{7}\right) x^{2} + 49 \left(10 b^{7} c^{6} d + 12 a b^{6} c^{5} d^{2} + 15 a^{2} b^{5} c^{4} d^{3} + 20 a^{3} b^{4} c^{3} d^{4} + 30 a^{4} b^{3} c^{2} d^{5} + 60 a^{5} b^{2} c d^{6} - 147 a^{6} b d^{7}\right) x - 420 \left(b^{7} d^{7} x^{7} + 7 a b^{6} d^{7} x^{6} + 21 a^{2} b^{5} d^{7} x^{5} + 35 a^{3} b^{4} d^{7} x^{4} + 35 a^{4} b^{3} d^{7} x^{3} + 21 a^{5} b^{2} d^{7} x^{2} + 7 a^{6} b d^{7} x + a^{7} d^{7}\right) \log\left(b x + a\right)}{420 \left(b^{15} x^{7} + 7 a b^{14} x^{6} + 21 a^{2} b^{13} x^{5} + 35 a^{3} b^{12} x^{4} + 35 a^{4} b^{11} x^{3} + 21 a^{5} b^{10} x^{2} + 7 a^{6} b^{9} x + a^{7} b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x - 420*(b^7*d^7*x^7 + 7*a*b^6*d^7*x^6 + 21*a^2*b^5*d^7*x^5
 + 35*a^3*b^4*d^7*x^4 + 35*a^4*b^3*d^7*x^3 + 21*a^5*b^2*d^7*x^2 + 7*a^6*b*d
^7*x + a^7*d^7)*log(b*x + a))/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 +
35*a^3*b^12*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(182) = 364$.
time = 1.02, size = 466, normalized size = 2.40

$$\frac{d^7 \log(|bx+a|)}{b^8} - \frac{2940\,(b^6cd^6 - ab^5d^7)x^6 + 4410\,(b^6c^2d^5 + 2\,ab^5cd^6 - 3\,a^2b^4d^7)x^5 + 2450\,(2\,b^6c^3d^4 + 3\,ab^5c^2d^5 + 6\,a^2b^4cd^6 - 11\,a^3b^3d^7)x^4 + 1225\,(3\,b^6c^4d^3 + 4\,ab^5c^3d^4 + 6\,a^2b^4c^2d^5 + 12\,a^3b^3cd^6 - 25\,a^4b^2d^7)x^3 + 147\,(12\,b^6c^5d^2 + 15\,ab^5c^4d^3 + 20\,a^2b^4c^3d^4 + 30\,a^3b^3c^2d^5 + 60\,a^4b^2cd^6 - 137\,a^5bd^7)x^2 + 49\,(10\,b^6c^6d + 12\,ab^5c^5d^2 + 15\,a^2b^4c^4d^3 + 20\,a^3b^3c^3d^4 + 30\,a^4b^2c^2d^5 + 60\,a^5bcd^6 - 147\,a^6d^7)x + \frac{60\,b^7c^7 + 70\,ab^6c^6d + 84\,a^2b^5c^5d^2 + 105\,a^3b^4c^4d^3 + 140\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 + 420\,a^6bcd^6 - 1089\,a^7d^7}{b}}{420\,(bx+a)^7b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] d^7*log(abs(b*x + a))/b^8 - 1/420*(2940*(b^6*c*d^6 - a*b^5*d^7)*x^6 + 4410*
(b^6*c^2*d^5 + 2*a*b^5*c*d^6 - 3*a^2*b^4*d^7)*x^5 + 2450*(2*b^6*c^3*d^4 + 3
*a*b^5*c^2*d^5 + 6*a^2*b^4*c*d^6 - 11*a^3*b^3*d^7)*x^4 + 1225*(3*b^6*c^4*d^
3 + 4*a*b^5*c^3*d^4 + 6*a^2*b^4*c^2*d^5 + 12*a^3*b^3*c*d^6 - 25*a^4*b^2*d^7
)*x^3 + 147*(12*b^6*c^5*d^2 + 15*a*b^5*c^4*d^3 + 20*a^2*b^4*c^3*d^4 + 30*a^
3*b^3*c^2*d^5 + 60*a^4*b^2*c*d^6 - 137*a^5*b*d^7)*x^2 + 49*(10*b^6*c^6*d +
12*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 + 30*a^4*b^2*c^2
*d^5 + 60*a^5*b*c*d^6 - 147*a^6*d^7)*x + (60*b^7*c^7 + 70*a*b^6*c^6*d + 84*
a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c
^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^7*d^7)/b)/((b*x + a)^7*b^7)
```

Mupad [**B**]
time = 0.35, size = 461, normalized size = 2.38

$$\frac{d^7\,\ln(a+b\,x)}{b^8} - \frac{x\,\left(-\frac{343\,a^6\,b\,d^7}{20} + 7\,a^5\,b^2\,c\,d^6 + \frac{7\,a^4\,b^3\,c^2\,d^5}{2} + \frac{7\,a^3\,b^4\,c^3\,d^4}{3} + \frac{7\,a^2\,b^5\,c^4\,d^3}{4} + \frac{7\,a\,b^6\,c^5\,d^2}{5} + \frac{7\,b^7\,c^6\,d}{6}\right) - x^6\,(7\,a\,b^6\,d^7 - 7\,b^7\,c\,d^6) + x^3\,\left(-\frac{875\,a^4\,b^3\,d^7}{12} + 35\,a^3\,b^4\,c\,d^6 + \frac{35\,a^2\,b^5\,c^2\,d^5}{2} + \frac{35\,a\,b^6\,c^3\,d^4}{3} + \frac{35\,b^7\,c^4\,d^3}{4}\right) + x^5\,\left(-\frac{63\,a^2\,b^5\,d^7}{2} + 21\,a\,b^6\,c\,d^6 + \frac{21\,b^7\,c^2\,d^5}{2}\right) + x^2\,\left(-\frac{959\,a^5\,b^2\,d^7}{20} + 21\,a^4\,b^3\,c\,d^6 + \frac{21\,a^3\,b^4\,c^2\,d^5}{2} + 7\,a^2\,b^5\,c^3\,d^4 + \frac{21\,a\,b^6\,c^4\,d^3}{4} + \frac{21\,b^7\,c^5\,d^2}{5}\right) - \frac{363\,a^7\,d^7}{140} + \frac{b^7\,c^7}{7} + x^4\,\left(-\frac{385\,a^3\,b^4\,d^7}{6} + 35\,a^2\,b^5\,c\,d^6 + \frac{35\,a\,b^6\,c^2\,d^5}{2} + \frac{35\,b^7\,c^3\,d^4}{3}\right) + \frac{a^2\,b^5\,c^5\,d^2}{5} + \frac{a^3\,b^4\,c^4\,d^3}{4} + \frac{a^4\,b^3\,c^3\,d^4}{3} + \frac{a^5\,b^2\,c^2\,d^5}{2} + \frac{a\,b^6\,c^6\,d}{6} + a^6\,b\,c\,d^6}{b^8\,(a+b\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^8,x)
```

```
[Out] (d^7*log(a + b*x))/b^8 - (x*((7*b^7*c^6*d)/6 - (343*a^6*b*d^7)/20 + (7*a*b^
6*c^5*d^2)/5 + 7*a^5*b^2*c*d^6 + (7*a^2*b^5*c^4*d^3)/4 + (7*a^3*b^4*c^3*d^4
)/3 + (7*a^4*b^3*c^2*d^5)/2) - x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*b
^7*c^4*d^3)/4 - (875*a^4*b^3*d^7)/12 + (35*a*b^6*c^3*d^4)/3 + 35*a^3*b^4*c*
d^6 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*b^7*c^2*d^5)/2 - (63*a^2*b^5*d^7)/
2 + 21*a*b^6*c*d^6) + x^2*((21*b^7*c^5*d^2)/5 - (959*a^5*b^2*d^7)/20 + (21*
a*b^6*c^4*d^3)/4 + 21*a^4*b^3*c*d^6 + 7*a^2*b^5*c^3*d^4 + (21*a^3*b^4*c^2*d
^5)/2) - (363*a^7*d^7)/140 + (b^7*c^7)/7 + x^4*((35*b^7*c^3*d^4)/3 - (385*a
^3*b^4*d^7)/6 + (35*a*b^6*c^2*d^5)/2 + 35*a^2*b^5*c*d^6) + (a^2*b^5*c^5*d^2
)/5 + (a^3*b^4*c^4*d^3)/4 + (a^4*b^3*c^3*d^4)/3 + (a^5*b^2*c^2*d^5)/2 + (a*
b^6*c^6*d)/6 + a^6*b*c*d^6)/(b^8*(a + b*x)^7)
```

3.1291 $\int \frac{(c+dx)^7}{(a+bx)^9}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

```
[Out] -1/8*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^8
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -1/8*(c + d*x)^8/((b*c - a*d)*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^9}\,dx = -\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(28) = 56.
time = 0.08, size = 353, normalized size = 12.61

$$-\frac{a^7d^7+a^6bd^6(c+8dx)+a^5b^2d^5(c^2+8cdx+28d^2x^2)+a^4b^3d^4(c^3+8c^2dx+28cd^2x^2+56d^3x^3)+a^3b^4d^3(c^4+8c^3dx+28c^2d^2x^2+56cd^3x^3+70d^4x^4)+a^2b^5d^2(c^5+8c^4dx+28c^3d^2x^2+56c^2d^3x^3+70cd^4x^4+56d^5x^5)+ab^6d(c^6+8c^5dx+28c^4d^2x^2+56c^3d^3x^3+70c^2d^4x^4+56cd^5x^5+28d^6x^6)+b^7(c^7+8c^6dx+28c^5d^2x^2+56c^4d^3x^3+70c^3d^4x^4+56c^2d^5x^5+28cd^6x^6+8d^7x^7)}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -1/8*(a^7*d^7 + a^6*b*d^6*(c + 8*d*x) + a^5*b^2*d^5*(c^2 + 8*c*d*x + 28*d^2
*x^2) + a^4*b^3*d^4*(c^3 + 8*c^2*d*x + 28*c*d^2*x^2 + 56*d^3*x^3) + a^3*b^4
*d^3*(c^4 + 8*c^3*d*x + 28*c^2*d^2*x^2 + 56*c*d^3*x^3 + 70*d^4*x^4) + a^2*b
^5*d^2*(c^5 + 8*c^4*d*x + 28*c^3*d^2*x^2 + 56*c^2*d^3*x^3 + 70*c*d^4*x^4 +
56*d^5*x^5) + a*b^6*d*(c^6 + 8*c^5*d*x + 28*c^4*d^2*x^2 + 56*c^3*d^3*x^3 +
70*c^2*d^4*x^4 + 56*c*d^5*x^5 + 28*d^6*x^6) + b^7*(c^7 + 8*c^6*d*x + 28*c^5
*d^2*x^2 + 56*c^4*d^3*x^3 + 70*c^3*d^4*x^4 + 56*c^2*d^5*x^5 + 28*c*d^6*x^6
+ 8*d^7*x^7))/(b^8*(a + b*x)^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(26) = 52$.
time = 0.14, size = 464, normalized size = 16.57

method	result
risch	$\frac{-\frac{d^{7} x^{7}}{b}-\frac{7 d^{6} (a d+b c) x^{6}}{2 b^{2}}-\frac{7 d^{5} \left(a^{2} d^{2}+a b c d+b^{2} c^{2}\right) x^{5}}{b^{3}}-\frac{35 d^{4} \left(a^{3} d^{3}+a^{2} b c \,d^{2}+a \,b^{2} c^{2} d+b^{3} c^{3}\right) x^{4}}{4 b^{4}}-\frac{7 d^{3} \left(a^{4} d^{4}+a^{3} b c \,d^{3}+a^{2} b^{2} c^{2} d^{2}+a \,b^{3} c^{3} d+b^{4} c^{4}\right) x}{b^{5}}}{}$
norman	$\frac{-\frac{d^{7} x^{7}}{b}+\frac{7 \left(-a \,d^{7}-b c \,d^{6}\right) x^{6}}{2 b^{2}}+\frac{7 \left(-a^{2} d^{7}-a b c \,d^{6}-b^{2} c^{2} d^{5}\right) x^{5}}{b^{3}}+\frac{35 \left(-a^{3} d^{7}-a^{2} b c \,d^{6}-a \,b^{2} c^{2} d^{5}-b^{3} c^{3} d^{4}\right) x^{4}}{4 b^{4}}+\frac{7 \left(-a^{4} d^{7}-a^{3} b c \,d^{6}-b^{2} a^{2} c^{2} d^{5}-a \,b^{3} c}{b^{5}}}{}$
default	$-\frac{d^{7}}{b^{8} (b x+a)}+\frac{35 d^{4} \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}\right)}{4 b^{8} (b x+a)^{4}}-\frac{-a^{7} d^{7}+7 a^{6} b c \,d^{6}-21 a^{5} b^{2} c^{2} d^{5}+35 a^{4} b^{3} c^{3} d^{4}-35 a^{3} b^{4} c^{4} d^{3}+21 a^{2} b^{5} c^{5} d^{2}-}{8 b^{8} (b x+a)^{8}}$
gosper	$-\frac{8 d^{7} x^{7} b^{7}+28 a \,b^{6} d^{7} x^{6}+28 b^{7} c \,d^{6} x^{6}+56 a^{2} b^{5} d^{7} x^{5}+56 a \,b^{6} c \,d^{6} x^{5}+56 b^{7} c^{2} d^{5} x^{5}+70 a^{3} b^{4} d^{7} x^{4}+70 a^{2} b^{5} c \,d^{6} x^{4}+70 a \,b^{6} c^{2} d^{5} x^{4}+70 b^{7} c^{3} d^{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

```
[Out] -d^7/b^8/(b*x+a)+35/4/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/(b*x+a)^4-1/8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^
4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^
8-7/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
/(b*x+a)^5-1/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3
*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^7+7/2/b^8*d^6*(a*d-b
*c)/(b*x+a)^2+7/2/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*
b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^6-7/b^8*d^5*(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(26) = 52$.
time = 0.35, size = 509, normalized size = 18.18

$$-\frac{8 b^{7} d^{7} x^{7} + b^{7} c^{7} + a b^{6} c^{6} d + a^{2} b^{5} c^{5} d^{2} + a^{3} b^{4} c^{4} d^{3} + a^{4} b^{3} c^{3} d^{4} + a^{5} b^{2} c^{2} d^{5} + a^{6} b c d^{6} + a^{7} d^{7} + 28 \,(b^{7} c d^{6} + a b^{6} d^{7}) x^{6} + 56 \,(b^{7} c^{2} d^{5} + a b^{6} c d^{6} + a^{2} b^{5} d^{7}) x^{5} + 70 \,(b^{7} c^{3} d^{4} + a b^{6} c^{2} d^{5} + a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}) x^{4} + 56 \,(b^{7} c^{4} d^{3} + a b^{6} c^{3} d^{4} + a^{2} b^{5} c^{2} d^{5} + a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}) x^{3} + 28 \,(b^{7} c^{5} d^{2} + a b^{6} c^{4} d^{3} + a^{2} b^{5} c^{3} d^{4} + a^{3} b^{4} c^{2} d^{5} + a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}) x^{2} + 8 \,(b^{7} c^{6} d + a b^{6} c^{5} d^{2} + a^{2} b^{5} c^{4} d^{3} + a^{3} b^{4} c^{3} d^{4} + a^{4} b^{3} c^{2} d^{5} + a^{5} b^{2} c d^{6} + a^{6} b d^{7}) x}{8 \,(b^{16} x^{8} + 8 \, a b^{15} x^{7} + 28 \, a^{2} b^{14} x^{6} + 56 \, a^{3} b^{13} x^{5} + 70 \, a^{4} b^{12} x^{4} + 56 \, a^{5} b^{11} x^{3} + 28 \, a^{6} b^{10} x^{2} + 8 \, a^{7} b^{9} x + a^{8} b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(26) = 52$.
time = 0.48, size = 509, normalized size = 18.18

$$-\frac{8\,b^{7}d^{7}x^{7} + b^{7}c^{7} + ab^{6}c^{6}d + a^{2}b^{5}c^{5}d^{2} + a^{3}b^{4}c^{4}d^{3} + a^{4}b^{3}c^{3}d^{4} + a^{5}b^{2}c^{2}d^{5} + a^{6}bcd^{6} + a^{7}d^{7} + 28\,(b^{7}cd^{6} + ab^{6}d^{7})x^{6} + 56\,(b^{7}c^{2}d^{5} + ab^{6}cd^{6} + a^{2}b^{5}d^{7})x^{5} + 70\,(b^{7}c^{3}d^{4} + ab^{6}c^{2}d^{5} + a^{2}b^{5}cd^{6} + a^{3}b^{4}d^{7})x^{4} + 56\,(b^{7}c^{4}d^{3} + ab^{6}c^{3}d^{4} + a^{2}b^{5}c^{2}d^{5} + a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7})x^{3} + 28\,(b^{7}c^{5}d^{2} + ab^{6}c^{4}d^{3} + a^{2}b^{5}c^{3}d^{4} + a^{3}b^{4}c^{2}d^{5} + a^{4}b^{3}cd^{6} + a^{5}b^{2}d^{7})x^{2} + 8\,(b^{7}c^{6}d + ab^{6}c^{5}d^{2} + a^{2}b^{5}c^{4}d^{3} + a^{3}b^{4}c^{3}d^{4} + a^{4}b^{3}c^{2}d^{5} + a^{5}b^{2}cd^{6} + a^{6}bd^{7})x}{8\,(b^{16}x^{8} + 8\,ab^{15}x^{7} + 28\,a^{2}b^{14}x^{6} + 56\,a^{3}b^{13}x^{5} + 70\,a^{4}b^{12}x^{4} + 56\,a^{5}b^{11}x^{3} + 28\,a^{6}b^{10}x^{2} + 8\,a^{7}b^{9}x + a^{8}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**9,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(26) = 52$.
time = 1.40, size = 489, normalized size = 17.46

$$-\frac{8\,b^{7}d^{7}x^{7} + 28\,b^{7}cd^{6}x^{6} + 28\,ab^{6}d^{7}x^{6} + 56\,b^{7}c^{2}d^{5}x^{5} + 56\,ab^{6}cd^{6}x^{5} + 56\,a^{2}b^{5}d^{7}x^{5} + 70\,b^{7}c^{3}d^{4}x^{4} + 70\,ab^{6}c^{2}d^{5}x^{4} + 70\,a^{2}b^{5}cd^{6}x^{4} + 70\,a^{3}b^{4}d^{7}x^{4} + 56\,b^{7}c^{4}d^{3}x^{3} + 56\,ab^{6}c^{3}d^{4}x^{3} + 56\,a^{2}b^{5}c^{2}d^{5}x^{3} + 56\,a^{3}b^{4}cd^{6}x^{3} + 56\,a^{4}b^{3}d^{7}x^{3} + 28\,b^{7}c^{5}d^{2}x^{2} + 28\,ab^{6}c^{4}d^{3}x^{2} + 28\,a^{2}b^{5}c^{3}d^{4}x^{2} + 28\,a^{3}b^{4}c^{2}d^{5}x^{2} + 28\,a^{4}b^{3}cd^{6}x^{2} + 28\,a^{5}b^{2}d^{7}x^{2} + 8\,b^{7}c^{6}dx + 8\,ab^{6}c^{5}d^{2}x + 8\,a^{2}b^{5}c^{4}d^{3}x + 8\,a^{3}b^{4}c^{3}d^{4}x + 8\,a^{4}b^{3}c^{2}d^{5}x + 8\,a^{5}b^{2}cd^{6}x + 8\,a^{6}bd^{7}x + b^{7}c^{7} + ab^{6}c^{6}d + a^{2}b^{5}c^{5}d^{2} + a^{3}b^{4}c^{4}d^{3} + a^{4}b^{3}c^{3}d^{4} + a^{5}b^{2}c^{2}d^{5} + a^{6}bcd^{6} + a^{7}d^{7}}{8\,(bx + a)^{8}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + 28*b^7*c*d^6*x^6 + 28*a*b^6*d^7*x^6 + 56*b^7*c^2*d^5*
x^5 + 56*a*b^6*c*d^6*x^5 + 56*a^2*b^5*d^7*x^5 + 70*b^7*c^3*d^4*x^4 + 70*a*b
^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4 + 70*a^3*b^4*d^7*x^4 + 56*b^7*c^4*d^3
*x^3 + 56*a*b^6*c^3*d^4*x^3 + 56*a^2*b^5*c^2*d^5*x^3 + 56*a^3*b^4*c*d^6*x^3
 + 56*a^4*b^3*d^7*x^3 + 28*b^7*c^5*d^2*x^2 + 28*a*b^6*c^4*d^3*x^2 + 28*a^2*
b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 28*a^4*b^3*c*d^6*x^2 + 28*a^5*b^
2*d^7*x^2 + 8*b^7*c^6*d*x + 8*a*b^6*c^5*d^2*x + 8*a^2*b^5*c^4*d^3*x + 8*a^3
*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 8*a^5*b^2*c*d^6*x + 8*a^6*b*d^7*x +
b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4
 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^8*b^8)
```

Mupad [B]
time = 0.17, size = 571, normalized size = 20.39

$$-\frac{a^7 d^7 + a^6 b c d^6 + 8 a^6 b d^7 x + a^5 b^2 c^2 d^5 + 8 a^5 b^2 c d^6 x + 28 a^5 b^2 d^7 x^2 + a^4 b^3 c^3 d^4 + 8 a^4 b^3 c^2 d^5 x + 28 a^4 b^3 c d^6 x^2 + 56 a^4 b^3 d^7 x^3 + a^3 b^4 c^4 d^3 + 8 a^3 b^4 c^3 d^4 x + 28 a^3 b^4 c^2 d^5 x^2 + 56 a^3 b^4 c d^6 x^3 + 70 a^3 b^4 d^7 x^4 + a^2 b^5 c^5 d^2 + 8 a^2 b^5 c^4 d^3 x + 28 a^2 b^5 c^3 d^4 x^2 + 56 a^2 b^5 c^2 d^5 x^3 + 70 a^2 b^5 c d^6 x^4 + 56 a^2 b^5 d^7 x^5 + a b^6 c^6 d + 8 a b^6 c^5 d^2 x + 28 a b^6 c^4 d^3 x^2 + 56 a b^6 c^3 d^4 x^3 + 70 a b^6 c^2 d^5 x^4 + 56 a b^6 c d^6 x^5 + 28 a b^6 d^7 x^6 + b^7 c^7 + 8 b^7 c^6 d x + 28 b^7 c^5 d^2 x^2 + 56 b^7 c^4 d^3 x^3 + 70 b^7 c^3 d^4 x^4 + 56 b^7 c^2 d^5 x^5 + 28 b^7 c d^6 x^6 + 8 b^7 d^7 x^7}{8 a^8 b^8 + 64 a^7 b^9 x + 224 a^6 b^{10} x^2 + 448 a^5 b^{11} x^3 + 560 a^4 b^{12} x^4 + 448 a^3 b^{13} x^5 + 224 a^2 b^{14} x^6 + 64 a b^{15} x^7 + 8 b^{16} x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^9,x)
```

```
[Out] -(a^7*d^7 + b^7*c^7 + 8*b^7*d^7*x^7 + 28*a*b^6*d^7*x^6 + 28*b^7*c*d^6*x^6 +
 a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + 28
*a^5*b^2*d^7*x^2 + 56*a^4*b^3*d^7*x^3 + 70*a^3*b^4*d^7*x^4 + 56*a^2*b^5*d^7
*x^5 + 28*b^7*c^5*d^2*x^2 + 56*b^7*c^4*d^3*x^3 + 70*b^7*c^3*d^4*x^4 + 56*b^
7*c^2*d^5*x^5 + a*b^6*c^6*d + a^6*b*c*d^6 + 8*a^6*b*d^7*x + 8*b^7*c^6*d*x +
 28*a^2*b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 56*a^2*b^5*c^2*d^5*x^3 +
 8*a*b^6*c^5*d^2*x + 8*a^5*b^2*c*d^6*x + 56*a*b^6*c*d^6*x^5 + 8*a^2*b^5*c^4
*d^3*x + 8*a^3*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 28*a*b^6*c^4*d^3*x^2 +
 28*a^4*b^3*c*d^6*x^2 + 56*a*b^6*c^3*d^4*x^3 + 56*a^3*b^4*c*d^6*x^3 + 70*a*
b^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4)/(8*a^8*b^8 + 8*b^16*x^8 + 64*a^7*b^
9*x + 64*a*b^15*x^7 + 224*a^6*b^10*x^2 + 448*a^5*b^11*x^3 + 560*a^4*b^12*x^
4 + 448*a^3*b^13*x^5 + 224*a^2*b^14*x^6)
```

3.1292 $\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=58

$$-\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9}+\frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}$$

```
[Out] -1/9*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^9+1/72*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a)^8
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2}-\frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -1/9*(c + d*x)^8/((b*c - a*d)*(a + b*x)^9) + (d*(c + d*x)^8)/(72*(b*c - a*d
)^2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx = -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{9(bc-ad)}$$
$$= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9} + \frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 367 vs. $2(58) = 116$.
time = 0.08, size = 367, normalized size = 6.33

$$-\frac{a^7d^7 + a^6bd^6(2c+9dx) + 3a^5b^2d^5(c^2+6cdx+12d^2x^2) + a^4b^3d^4(4c^3+27c^2dx+72cd^2x^2+84d^3x^3) + a^3b^4d^3(5c^4+36c^3dx+108c^2d^2x^2+168cd^3x^3+126d^4x^4) + 3a^2b^5d^2(2c^5+15c^4dx+48c^3d^2x^2+84c^2d^3x^3+84cd^4x^4+42d^5x^5) + ab^6d(7c^6+54c^5dx+180c^4d^2x^2+336c^3d^3x^3+378c^2d^4x^4+252cd^5x^5+84d^6x^6) + b^7(8c^7+63c^6dx+216c^5d^2x^2+420c^4d^3x^3+504c^3d^4x^4+378c^2d^5x^5+168cd^6x^6+36d^7x^7)}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7*d^7 + a^6*b*d^6*(2*c + 9*d*x) + 3*a^5*b^2*d^5*(c^2 + 6*c*d*x + 1
2*d^2*x^2) + a^4*b^3*d^4*(4*c^3 + 27*c^2*d*x + 72*c*d^2*x^2 + 84*d^3*x^3) +
 a^3*b^4*d^3*(5*c^4 + 36*c^3*d*x + 108*c^2*d^2*x^2 + 168*c*d^3*x^3 + 126*d^
4*x^4) + 3*a^2*b^5*d^2*(2*c^5 + 15*c^4*d*x + 48*c^3*d^2*x^2 + 84*c^2*d^3*x^
3 + 84*c*d^4*x^4 + 42*d^5*x^5) + a*b^6*d*(7*c^6 + 54*c^5*d*x + 180*c^4*d^2*
x^2 + 336*c^3*d^3*x^3 + 378*c^2*d^4*x^4 + 252*c*d^5*x^5 + 84*d^6*x^6) + b^7
*(8*c^7 + 63*c^6*d*x + 216*c^5*d^2*x^2 + 420*c^4*d^3*x^3 + 504*c^3*d^4*x^4
+ 378*c^2*d^5*x^5 + 168*c*d^6*x^6 + 36*d^7*x^7))/(b^8*(a + b*x)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(54) = 108$.
time = 0.18, size = 464, normalized size = 8.00

method	result
risch	$-\frac{d^7x^7}{2b} - \frac{7d^6(ad+2bc)x^6}{6b^2} - \frac{7d^5(a^2d^2+2abcd+3b^2c^2)x^5}{4b^3} - \frac{7d^4(a^3d^3+2a^2bcd^2+3ab^2c^2d+4b^3c^3)x^4}{4b^4} - \frac{7d^3(a^4d^4+2a^3bcd^3+3a^2b^2c^2d^2+4ab^3c^3\cdots}{6b^5}$
default	$-\frac{21d^5(a^2d^2-2abcd+b^2c^2)}{4b^8(bx+a)^4} - \frac{-a^7d^7+7a^6bcd^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7ab^6c^6d+b^7c^7}{9b^8(bx+a)^9} - \frac{7d(a^6d^6\cdots}{}$
norman	$-\frac{d^7x^7}{2b} + \frac{7(-abd^7-2b^2cd^6)x^6}{6b^3} + \frac{7(-a^2bd^7-2ab^2cd^6-3b^3c^2d^5)x^5}{4b^4} + \frac{7(-a^3bd^7-2b^2a^2cd^6-3ab^3c^2d^5-4b^4c^3d^4)x^4}{4b^5} + \frac{7(-a^4bd^7-2a^3b^2cd^6\cdots}{}$
gosper	$-\frac{36d^7x^7b^7+84ab^6d^7x^6+168b^7cd^6x^6+126a^2b^5d^7x^5+252ab^6cd^6x^5+378b^7c^2d^5x^5+126a^3b^4d^7x^4+252a^2b^5cd^6x^4+378ab^6c^2d^5x^4+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] -21/4/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4-1/9*(-a^7*d^7+7*a^6*b*c
*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^
```

```
5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^9-7/8/b^8*d*(a^6*d^6-6*a^5*b*c*d^5
+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6
*c^6)/(b*x+a)^8+7/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*
x+a)^5+3/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d
^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^7-1/2*d^7/b^8/(b*x+a)^2-35/6/b^8*d^3*(a^4
*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^6+7/3/b
^8*d^6*(a*d-b*c)/(b*x+a)^3
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(54) = 108$.
time = 0.31, size = 548, normalized size = 9.45

$$-\frac{36\,b^{7}d^{7}x^{7}+8\,b^{7}c^{7}+7\,ab^{6}c^{6}d+6\,a^{2}b^{5}c^{5}d^{2}+5\,a^{3}b^{4}c^{4}d^{3}+4\,a^{4}b^{3}c^{3}d^{4}+3\,a^{5}b^{2}c^{2}d^{5}+2\,a^{6}bcd^{6}+a^{7}d^{7}+84\,(2\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+126\,(3\,b^{7}c^{2}d^{5}+2\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+126\,(4\,b^{7}c^{3}d^{4}+3\,ab^{6}c^{2}d^{5}+2\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+84\,(5\,b^{7}c^{4}d^{3}+4\,ab^{6}c^{3}d^{4}+3\,a^{2}b^{5}c^{2}d^{5}+2\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+36\,(6\,b^{7}c^{5}d^{2}+5\,ab^{6}c^{4}d^{3}+4\,a^{2}b^{5}c^{3}d^{4}+3\,a^{3}b^{4}c^{2}d^{5}+2\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+9\,(7\,b^{7}c^{6}d+6\,ab^{6}c^{5}d^{2}+5\,a^{2}b^{5}c^{4}d^{3}+4\,a^{3}b^{4}c^{3}d^{4}+3\,a^{4}b^{3}c^{2}d^{5}+2\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{72\,(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a^{9}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(54) = 108$.
time = 0.49, size = 548, normalized size = 9.45

$$-\frac{36\,b^{7}d^{7}x^{7}+8\,b^{7}c^{7}+7\,ab^{6}c^{6}d+6\,a^{2}b^{5}c^{5}d^{2}+5\,a^{3}b^{4}c^{4}d^{3}+4\,a^{4}b^{3}c^{3}d^{4}+3\,a^{5}b^{2}c^{2}d^{5}+2\,a^{6}bcd^{6}+a^{7}d^{7}+84\,(2\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+126\,(3\,b^{7}c^{2}d^{5}+2\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+126\,(4\,b^{7}c^{3}d^{4}+3\,ab^{6}c^{2}d^{5}+2\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+84\,(5\,b^{7}c^{4}d^{3}+4\,ab^{6}c^{3}d^{4}+3\,a^{2}b^{5}c^{2}d^{5}+2\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+36\,(6\,b^{7}c^{5}d^{2}+5\,ab^{6}c^{4}d^{3}+4\,a^{2}b^{5}c^{3}d^{4}+3\,a^{3}b^{4}c^{2}d^{5}+2\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+9\,(7\,b^{7}c^{6}d+6\,ab^{6}c^{5}d^{2}+5\,a^{2}b^{5}c^{4}d^{3}+4\,a^{3}b^{4}c^{3}d^{4}+3\,a^{4}b^{3}c^{2}d^{5}+2\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{72\,(b^{17}x^{9}+9\,ab^{16}x^{8}+36\,a^{2}b^{15}x^{7}+84\,a^{3}b^{14}x^{6}+126\,a^{4}b^{13}x^{5}+126\,a^{5}b^{12}x^{4}+84\,a^{6}b^{11}x^{3}+36\,a^{7}b^{10}x^{2}+9\,a^{8}b^{9}x+a^{9}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
```

```
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**10,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(54) = 108.
time = 0.92, size = 496, normalized size = 8.55

$$-\frac{36\,b^7d^7x^7+168\,b^7cd^6x^6+84\,ab^6d^7x^6+378\,b^7c^2d^5x^5+252\,ab^6cd^6x^5+126\,a^2b^5d^7x^5+504\,b^7c^3d^4x^4+378\,ab^6c^2d^5x^4+252\,a^2b^5cd^6x^4+126\,a^3b^4d^7x^4+420\,b^7c^4d^3x^3+336\,ab^6c^3d^4x^3+252\,a^2b^5c^2d^5x^3+168\,a^3b^4cd^6x^3+84\,a^4b^3d^7x^3+216\,b^7c^5d^2x^2+180\,ab^6c^4d^3x^2+144\,a^2b^5c^3d^4x^2+108\,a^3b^4c^2d^5x^2+72\,a^4b^3cd^6x^2+36\,a^5b^2d^7x^2+63\,b^7c^6dx+54\,ab^6c^5d^2x+45\,a^2b^5c^4d^3x+36\,a^3b^4c^3d^4x+27\,a^4b^3c^2d^5x+18\,a^5b^2cd^6x+9\,a^6bd^7x+8\,b^7c^7+7\,ab^6c^6d+6\,a^2b^5c^5d^2+5\,a^3b^4c^4d^3+4\,a^4b^3c^3d^4+3\,a^5b^2c^2d^5+2\,a^6bcd^6+a^7d^7}{72\,(bx+a)^9b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 168*b^7*c*d^6*x^6 + 84*a*b^6*d^7*x^6 + 378*b^7*c^2*
d^5*x^5 + 252*a*b^6*c*d^6*x^5 + 126*a^2*b^5*d^7*x^5 + 504*b^7*c^3*d^4*x^4 +
 378*a*b^6*c^2*d^5*x^4 + 252*a^2*b^5*c*d^6*x^4 + 126*a^3*b^4*d^7*x^4 + 420*
b^7*c^4*d^3*x^3 + 336*a*b^6*c^3*d^4*x^3 + 252*a^2*b^5*c^2*d^5*x^3 + 168*a^3
*b^4*c*d^6*x^3 + 84*a^4*b^3*d^7*x^3 + 216*b^7*c^5*d^2*x^2 + 180*a*b^6*c^4*d
^3*x^2 + 144*a^2*b^5*c^3*d^4*x^2 + 108*a^3*b^4*c^2*d^5*x^2 + 72*a^4*b^3*c*d
^6*x^2 + 36*a^5*b^2*d^7*x^2 + 63*b^7*c^6*d*x + 54*a*b^6*c^5*d^2*x + 45*a^2*
b^5*c^4*d^3*x + 36*a^3*b^4*c^3*d^4*x + 27*a^4*b^3*c^2*d^5*x + 18*a^5*b^2*c*
d^6*x + 9*a^6*b*d^7*x + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7)/((b*x + a)^9*b^8)
```

Mupad [**B**]
time = 0.15, size = 39, normalized size = 0.67

$$\frac{{\left(c+d\,x\right)}^8\,\left(9\,a\,d-8\,b\,c+b\,d\,x\right)}{72\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^10,x)
```

```
[Out] ((c + d*x)^8*(9*a*d - 8*b*c + b*d*x))/(72*(a*d - b*c)^2*(a + b*x)^9)
```

3.1293 $\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx$

Optimal. Leaf size=89

$$-\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}+\frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9}-\frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}$$

```
[Out] -1/10*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^10+1/45*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a)
^9-1/360*d^2*(d*x+c)^8/(-a*d+b*c)^3/(b*x+a)^8
```

Rubi [A]
time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -1/10*(c + d*x)^8/((b*c - a*d)*(a + b*x)^10) + (d*(c + d*x)^8)/(45*(b*c - a
*d)^2*(a + b*x)^9) - (d^2*(c + d*x)^8)/(360*(b*c - a*d)^3*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx = -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{5(bc-ad)}$$
$$= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{45(bc-ad)^2}$$
$$= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} - \frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 371 vs. 2(89) = 178.

time = 0.08, size = 371, normalized size = 4.17

$$-\frac{a^7d^7 + a^6bd^6(3c+10dx) + 3a^5b^2d^5(2c^2+10cdx+15d^2x^2) + 5a^4b^3d^4(2c^3+12c^2dx+27cd^2x^2+24d^3x^3) + 5a^3b^4d^3(3c^4+20c^3dx+54c^2d^2x^2+72cd^3x^3+42d^4x^4) + 3a^2b^5d^2(7c^5+50c^4dx+150c^3d^2x^2+240c^2d^3x^3+210cd^4x^4+84d^5x^5) + ab^6d(28c^6+210c^5dx+675c^4d^2x^2+1200c^3d^3x^3+1260c^2d^4x^4+756cd^5x^5+210d^6x^6) + b^7(36c^7+280c^6dx+945c^5d^2x^2+1800c^4d^3x^3+2100c^3d^4x^4+1512c^2d^5x^5+630cd^6x^6+120d^7x^7)}{360b^8(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^11,x]
[Out] -1/360*(a^7*d^7 + a^6*b*d^6*(3*c + 10*d*x) + 3*a^5*b^2*d^5*(2*c^2 + 10*c*d*
x + 15*d^2*x^2) + 5*a^4*b^3*d^4*(2*c^3 + 12*c^2*d*x + 27*c*d^2*x^2 + 24*d^3
*x^3) + 5*a^3*b^4*d^3*(3*c^4 + 20*c^3*d*x + 54*c^2*d^2*x^2 + 72*c*d^3*x^3 +
 42*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 50*c^4*d*x + 150*c^3*d^2*x^2 + 240*c^
2*d^3*x^3 + 210*c*d^4*x^4 + 84*d^5*x^5) + a*b^6*d*(28*c^6 + 210*c^5*d*x + 6
75*c^4*d^2*x^2 + 1200*c^3*d^3*x^3 + 1260*c^2*d^4*x^4 + 756*c*d^5*x^5 + 210*
d^6*x^6) + b^7*(36*c^7 + 280*c^6*d*x + 945*c^5*d^2*x^2 + 1800*c^4*d^3*x^3 +
 2100*c^3*d^4*x^4 + 1512*c^2*d^5*x^5 + 630*c*d^6*x^6 + 120*d^7*x^7))/(b^8*(
a + b*x)^10)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(83) = 166.

time = 0.14, size = 464, normalized size = 5.21

method	result
risch	$-\frac{d^7x^7}{3b}-\frac{7d^6(ad+3bc)x^6}{12b^2}-\frac{7d^5\left(a^2d^2+3abcd+6b^2c^2\right)x^5}{10b^3}-\frac{7d^4\left(a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3\right)x^4}{12b^4}-\frac{d^3\left(a^4d^4+3a^3bc\,d^3+6a^2b^2c^2d^2+10a\,b^3c^3\right.}{3b^5}$
default	$\frac{7d^6(ad-bc)}{4b^8(bx+a)^4}-\frac{7d\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)}{9b^8(bx+a)^9}+\frac{21d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-\right.}{8b^8(bx+}$
norman	$-\frac{d^7x^7}{3b}+\frac{7\left(-a\,b^2d^7-3b^3c\,d^6\right)x^6}{12b^4}+\frac{7\left(-b^2a^2d^7-3a\,b^3c\,d^6-6b^4c^2d^5\right)x^5}{10b^5}+\frac{7\left(-a^3b^2d^7-3a^2b^3c\,d^6-6a\,b^4c^2d^5-10b^5c^3d^4\right)x^4}{12b^6}+\left(-a^4b^2d^7-3a^3b^3\right.$
gosper	$-\frac{120d^7x^7b^7+210a\,b^6d^7x^6+630b^7c\,d^6x^6+252a^2b^5d^7x^5+756a\,b^6c\,d^6x^5+1512b^7c^2d^5x^5+210a^3b^4d^7x^4+630a^2b^5c\,d^6x^4+1260a\,b^6c^2d}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^11,x,method=_RETURNVERBOSE)
```

```
[Out] 7/4/b^8*d^6*(a*d-b*c)/(b*x+a)^4-7/9/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2
*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+
a)^9+21/8/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*
d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^8-21/5/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^
2)/(b*x+a)^5-5/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3
*d+b^4*c^4)/(b*x+a)^7-1/10*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^
4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/
b^8/(b*x+a)^10+35/6/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(
b*x+a)^6-1/3*d^7/b^8/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(83) = 166$.
time = 0.30, size = 559, normalized size = 6.28

$$-\frac{120 b^7 d^7 x^7 + 36 b^7 c^7 + 28 a b^6 c^6 d + 21 a^2 b^5 c^5 d^2 + 15 a^3 b^4 c^4 d^3 + 10 a^4 b^3 c^3 d^4 + 6 a^5 b^2 c^2 d^5 + 3 a^6 b c d^6 + a^7 d^7 + 210 (3 b^7 c d^6 + a b^6 d^7) x^6 + 252 (6 b^7 c^2 d^5 + 3 a b^6 c d^6 + a^2 b^5 d^7) x^5 + 210 (10 b^7 c^3 d^4 + 6 a b^6 c^2 d^5 + 3 a^2 b^5 c d^6 + a^3 b^4 d^7) x^4 + 120 (15 b^7 c^4 d^3 + 10 a b^6 c^3 d^4 + 6 a^2 b^5 c^2 d^5 + 3 a^3 b^4 c d^6 + a^4 b^3 d^7) x^3 + 45 (21 b^7 c^5 d^2 + 15 a b^6 c^4 d^3 + 10 a^2 b^5 c^3 d^4 + 6 a^3 b^4 c^2 d^5 + 3 a^4 b^3 c d^6 + a^5 b^2 d^7) x^2 + 10 (28 b^7 c^6 d + 21 a b^6 c^5 d^2 + 15 a^2 b^5 c^4 d^3 + 10 a^3 b^4 c^3 d^4 + 6 a^4 b^3 c^2 d^5 + 3 a^5 b^2 c d^6 + a^6 b d^7) x}{360 (b^{18} x^{10} + 10 a b^{17} x^9 + 45 a^2 b^{16} x^8 + 120 a^3 b^{15} x^7 + 210 a^4 b^{14} x^6 + 252 a^5 b^{13} x^5 + 210 a^6 b^{12} x^4 + 120 a^7 b^{11} x^3 + 45 a^8 b^{10} x^2 + 10 a^9 b^9 x + a^{10} b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(83) = 166$.
time = 0.59, size = 559, normalized size = 6.28

$$-\frac{120 b^7 d^7 x^7 + 36 b^7 c^7 + 28 a b^6 c^6 d + 21 a^2 b^5 c^5 d^2 + 15 a^3 b^4 c^4 d^3 + 10 a^4 b^3 c^3 d^4 + 6 a^5 b^2 c^2 d^5 + 3 a^6 b c d^6 + a^7 d^7 + 210 (3 b^7 c d^6 + a b^6 d^7) x^6 + 252 (6 b^7 c^2 d^5 + 3 a b^6 c d^6 + a^2 b^5 d^7) x^5 + 210 (10 b^7 c^3 d^4 + 6 a b^6 c^2 d^5 + 3 a^2 b^5 c d^6 + a^3 b^4 d^7) x^4 + 120 (15 b^7 c^4 d^3 + 10 a b^6 c^3 d^4 + 6 a^2 b^5 c^2 d^5 + 3 a^3 b^4 c d^6 + a^4 b^3 d^7) x^3 + 45 (21 b^7 c^5 d^2 + 15 a b^6 c^4 d^3 + 10 a^2 b^5 c^3 d^4 + 6 a^3 b^4 c^2 d^5 + 3 a^4 b^3 c d^6 + a^5 b^2 d^7) x^2 + 10 (28 b^7 c^6 d + 21 a b^6 c^5 d^2 + 15 a^2 b^5 c^4 d^3 + 10 a^3 b^4 c^3 d^4 + 6 a^4 b^3 c^2 d^5 + 3 a^5 b^2 c d^6 + a^6 b d^7) x}{360 (b^{18} x^{10} + 10 a b^{17} x^9 + 45 a^2 b^{16} x^8 + 120 a^3 b^{15} x^7 + 210 a^4 b^{14} x^6 + 252 a^5 b^{13} x^5 + 210 a^6 b^{12} x^4 + 120 a^7 b^{11} x^3 + 45 a^8 b^{10} x^2 + 10 a^9 b^9 x + a^{10} b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**11,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(83) = 166.
time = 0.92, size = 496, normalized size = 5.57

$$-\frac{120\,b^7 d^7 x^7 + 630\,b^7 c d^6 x^6 + 210\,a b^6 d^7 x^6 + 1512\,b^7 c^2 d^5 x^5 + 756\,a b^6 c d^6 x^5 + 252\,a^2 b^5 d^7 x^5 + 2100\,b^7 c^3 d^4 x^4 + 1260\,a b^6 c^2 d^5 x^4 + 630\,a^2 b^5 c d^6 x^4 + 210\,a^3 b^4 d^7 x^4 + 1800\,b^7 c^4 d^3 x^3 + 1200\,a b^6 c^3 d^4 x^3 + 720\,a^2 b^5 c^2 d^5 x^3 + 360\,a^3 b^4 c d^6 x^3 + 120\,a^4 b^3 d^7 x^3 + 945\,b^7 c^5 d^2 x^2 + 675\,a b^6 c^4 d^3 x^2 + 450\,a^2 b^5 c^3 d^4 x^2 + 270\,a^3 b^4 c^2 d^5 x^2 + 135\,a^4 b^3 c d^6 x^2 + 45\,a^5 b^2 d^7 x^2 + 280\,b^7 c^6 d x + 210\,a b^6 c^5 d^2 x + 150\,a^2 b^5 c^4 d^3 x + 100\,a^3 b^4 c^3 d^4 x + 60\,a^4 b^3 c^2 d^5 x + 30\,a^5 b^2 c d^6 x + 10\,a^6 b d^7 x + 36\,b^7 c^7 + 28\,a b^6 c^6 d + 21\,a^2 b^5 c^5 d^2 + 15\,a^3 b^4 c^4 d^3 + 10\,a^4 b^3 c^3 d^4 + 6\,a^5 b^2 c^2 d^5 + 3\,a^6 b c d^6 + a^7 d^7}{360\,(bx+a)^{10} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 630*b^7*c*d^6*x^6 + 210*a*b^6*d^7*x^6 + 1512*b^7*
c^2*d^5*x^5 + 756*a*b^6*c*d^6*x^5 + 252*a^2*b^5*d^7*x^5 + 2100*b^7*c^3*d^4*
x^4 + 1260*a*b^6*c^2*d^5*x^4 + 630*a^2*b^5*c*d^6*x^4 + 210*a^3*b^4*d^7*x^4
+ 1800*b^7*c^4*d^3*x^3 + 1200*a*b^6*c^3*d^4*x^3 + 720*a^2*b^5*c^2*d^5*x^3 +
 360*a^3*b^4*c*d^6*x^3 + 120*a^4*b^3*d^7*x^3 + 945*b^7*c^5*d^2*x^2 + 675*a*
b^6*c^4*d^3*x^2 + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*c^2*d^5*x^2 + 135*a
^4*b^3*c*d^6*x^2 + 45*a^5*b^2*d^7*x^2 + 280*b^7*c^6*d*x + 210*a*b^6*c^5*d^2
*x + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x + 60*a^4*b^3*c^2*d^5*x +
 30*a^5*b^2*c*d^6*x + 10*a^6*b*d^7*x + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2
*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5
+ 3*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^10*b^8)
```

Mupad [**B**]
time = 0.45, size = 600, normalized size = 6.74

$$-\frac{a^7 d^7 + 3\,a^6 b c d^6 + 10\,a^6 b d^7 x + 6\,a^5 b^2 c^2 d^5 + 30\,a^5 b^2 c d^6 x + 45\,a^5 b^2 d^7 x^2 + 10\,a^4 b^3 c^3 d^4 + 60\,a^4 b^3 c^2 d^5 x + 135\,a^4 b^3 c d^6 x^2 + 120\,a^4 b^3 d^7 x^3 + 15\,a^3 b^4 c^4 d^3 + 100\,a^3 b^4 c^3 d^4 x + 270\,a^3 b^4 c^2 d^5 x^2 + 360\,a^3 b^4 c d^6 x^3 + 210\,a^3 b^4 d^7 x^4 + 21\,a^2 b^5 c^5 d^2 + 150\,a^2 b^5 c^4 d^3 x + 450\,a^2 b^5 c^3 d^4 x^2 + 720\,a^2 b^5 c^2 d^5 x^3 + 630\,a^2 b^5 c d^6 x^4 + 252\,a^2 b^5 d^7 x^5 + 28\,a b^6 c^6 d + 210\,a b^6 c^5 d^2 x + 675\,a b^6 c^4 d^3 x^2 + 1200\,a b^6 c^3 d^4 x^3 + 1260\,a b^6 c^2 d^5 x^4 + 756\,a b^6 c d^6 x^5 + 210\,a b^6 d^7 x^6 + 36\,b^7 c^7 + 280\,b^7 c^6 d x + 945\,b^7 c^5 d^2 x^2 + 1800\,b^7 c^4 d^3 x^3 + 2100\,b^7 c^3 d^4 x^4 + 1512\,b^7 c^2 d^5 x^5 + 630\,b^7 c d^6 x^6 + 120\,b^7 d^7 x^7}{360\,a^{10} b^8 + 3600\,a^9 b^9 x + 16200\,a^8 b^{10} x^2 + 43200\,a^7 b^{11} x^3 + 75600\,a^6 b^{12} x^4 + 90720\,a^5 b^{13} x^5 + 75600\,a^4 b^{14} x^6 + 43200\,a^3 b^{15} x^7 + 16200\,a^2 b^{16} x^8 + 3600\,a b^{17} x^9 + 360\,b^{18} x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^11,x)
```

```
[Out] -(a^7*d^7 + 36*b^7*c^7 + 120*b^7*d^7*x^7 + 210*a*b^6*d^7*x^6 + 630*b^7*c*d^
6*x^6 + 21*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^
5*b^2*c^2*d^5 + 45*a^5*b^2*d^7*x^2 + 120*a^4*b^3*d^7*x^3 + 210*a^3*b^4*d^7*
x^4 + 252*a^2*b^5*d^7*x^5 + 945*b^7*c^5*d^2*x^2 + 1800*b^7*c^4*d^3*x^3 + 21
00*b^7*c^3*d^4*x^4 + 1512*b^7*c^2*d^5*x^5 + 28*a*b^6*c^6*d + 3*a^6*b*c*d^6
+ 10*a^6*b*d^7*x + 280*b^7*c^6*d*x + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*
c^2*d^5*x^2 + 720*a^2*b^5*c^2*d^5*x^3 + 210*a*b^6*c^5*d^2*x + 30*a^5*b^2*c*
d^6*x + 756*a*b^6*c*d^6*x^5 + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x
 + 60*a^4*b^3*c^2*d^5*x + 675*a*b^6*c^4*d^3*x^2 + 135*a^4*b^3*c*d^6*x^2 + 1
200*a*b^6*c^3*d^4*x^3 + 360*a^3*b^4*c*d^6*x^3 + 1260*a*b^6*c^2*d^5*x^4 + 63
0*a^2*b^5*c*d^6*x^4)/(360*a^10*b^8 + 360*b^18*x^10 + 3600*a^9*b^9*x + 3600*
a*b^17*x^9 + 16200*a^8*b^10*x^2 + 43200*a^7*b^11*x^3 + 75600*a^6*b^12*x^4 +
 90720*a^5*b^13*x^5 + 75600*a^4*b^14*x^6 + 43200*a^3*b^15*x^7 + 16200*a^2*b
^16*x^8)
```

3.1294 $\int \frac{(c+dx)^7}{(a+bx)^{12}} dx$

Optimal. Leaf size=120

$$-\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}-\frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9}+\frac{d^3(c+dx)^8}{1320(bc-ad)^4(a+bx)^8}$$

```
[Out] -1/11*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^11+3/110*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a
)^10-1/165*d^2*(d*x+c)^8/(-a*d+b*c)^3/(b*x+a)^9+1/1320*d^3*(d*x+c)^8/(-a*d+
b*c)^4/(b*x+a)^8
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -1/11*(c + d*x)^8/((b*c - a*d)*(a + b*x)^11) + (3*d*(c + d*x)^8)/(110*(b*c
- a*d)^2*(a + b*x)^10) - (d^2*(c + d*x)^8)/(165*(b*c - a*d)^3*(a + b*x)^9)
+ (d^3*(c + d*x)^8)/(1320*(b*c - a*d)^4*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx = -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} - \frac{(3d)\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)}$$
$$= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} + \frac{(3d^2)\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx}{55(bc-ad)^2}$$
$$= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} - \frac{d^3\int \frac{(c+d}{(a+b}}{165(bc-}$$
$$= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} + \frac{d^3}{1320(bc}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(120) = 240.
time = 0.08, size = 369, normalized size = 3.08

$$-\frac{a^7d^7+a^6bd^6(4c+11dx)+a^5b^2d^5(10c^2+44cdx+55d^2x^2)+5a^4b^3d^4(4c^3+22c^2dx+44cd^2x^2+33d^3x^3)+5a^3b^4d^3(7c^4+44c^3dx+110c^2d^2x^2+132cd^3x^3+66d^4x^4)+a^2b^5d^2(56c^5+385c^4dx+1100c^3d^2x^2+1650c^2d^3x^3+1320cd^4x^4+462d^5x^5)+ab^6d(84c^6+616c^5dx+1925c^4d^2x^2+3300c^3d^3x^3+3300c^2d^4x^4+1848cd^5x^5+462d^6x^6)+b^7(120c^7+924c^6dx+3080c^5d^2x^2+5775c^4d^3x^3+6600c^3d^4x^4+4620c^2d^5x^5+1848cd^6x^6+330d^7x^7)}{1320b^8(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -1/1320*(a^7*d^7 + a^6*b*d^6*(4*c + 11*d*x) + a^5*b^2*d^5*(10*c^2 + 44*c*d*
x + 55*d^2*x^2) + 5*a^4*b^3*d^4*(4*c^3 + 22*c^2*d*x + 44*c*d^2*x^2 + 33*d^3
*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 44*c^3*d*x + 110*c^2*d^2*x^2 + 132*c*d^3*x^3
 + 66*d^4*x^4) + a^2*b^5*d^2*(56*c^5 + 385*c^4*d*x + 1100*c^3*d^2*x^2 + 165
0*c^2*d^3*x^3 + 1320*c*d^4*x^4 + 462*d^5*x^5) + a*b^6*d*(84*c^6 + 616*c^5*d
*x + 1925*c^4*d^2*x^2 + 3300*c^3*d^3*x^3 + 3300*c^2*d^4*x^4 + 1848*c*d^5*x^
5 + 462*d^6*x^6) + b^7*(120*c^7 + 924*c^6*d*x + 3080*c^5*d^2*x^2 + 5775*c^4
*d^3*x^3 + 6600*c^3*d^4*x^4 + 4620*c^2*d^5*x^5 + 1848*c*d^6*x^6 + 330*d^7*x
^7))/(b^8*(a + b*x)^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(112) = 224.
time = 0.14, size = 464, normalized size = 3.87

method	result
risch	$-\frac{d^7x^7}{4b}-\frac{7d^6(ad+4bc)x^6}{20b^2}-\frac{7d^5(a^2d^2+4abcd+10b^2c^2)x^5}{20b^3}-\frac{d^4(a^3d^3+4a^2bcd^2+10ab^2c^2d+20b^3c^3)x^4}{4b^4}-\frac{d^3(a^4d^4+4a^3bcd^3+10a^2b^2c^2d^2+20ab^3c^3}{8b^5}$
default	$-\frac{d^7}{4b^8(bx+a)^4}+\frac{7d^2(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5)}{3b^8(bx+a)^9}-\frac{35d^3(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4}{8b^8(bx+a)^8}$
norman	$-\frac{d^7x^7}{4b}+\frac{7(-ab^3d^7-4b^4cd^6)x^6}{20b^5}+\frac{7(-a^2b^3d^7-4ab^4cd^6-10b^5c^2d^5)x^5}{20b^6}+\frac{(-a^3b^3d^7-4a^2b^4cd^6-10ab^5c^2d^5-20b^6c^3d^4)x^4}{4b^7}+\frac{(-a^4b^3d^7-4a^3b^4c}{}$

gosper	$-\frac{330d^7x^7b^7+462a\,b^6d^7x^6+1848b^7c\,d^6x^6+462a^2b^5d^7x^5+1848a\,b^6c\,d^6x^5+4620b^7c^2d^5x^5+330a^3b^4d^7x^4+1320a^2b^5c\,d^6x^4+3300a\,b^6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^7/b^8/(b*x+a)^4+7/3/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^
3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^9-35/8/b^8*d^3*(a^4*d^4
-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^8+7/5/b^8*d
^6*(a*d-b*c)/(b*x+a)^5+5/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/(b*x+a)^7-7/10/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b
^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^10-7/2/b^8*d^5
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^6-1/11*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*
b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^
6*c^6*d+b^7*c^7)/b^8/(b*x+a)^11
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(112) = 224$.
time = 0.30, size = 570, normalized size = 4.75

$$-\frac{330\,b^7d^7x^7+120\,b^7c^7+84\,ab^6c^6d+56\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+20\,a^4b^3c^3d^4+10\,a^5b^2c^2d^5+4\,a^6bcd^6+a^7d^7+462\,(4\,b^7cd^6+ab^6d^7)x^6+462\,(10\,b^7c^2d^5+4\,ab^6cd^6+a^2b^5d^7)x^5+330\,(20\,b^7c^3d^4+10\,ab^6c^2d^5+4\,a^2b^5cd^6+a^3b^4d^7)x^4+165\,(35\,b^7c^4d^3+20\,ab^6c^3d^4+10\,a^2b^5c^2d^5+4\,a^3b^4cd^6+a^4b^3d^7)x^3+55\,(56\,b^7c^5d^2+35\,ab^6c^4d^3+20\,a^2b^5c^3d^4+10\,a^3b^4c^2d^5+4\,a^4b^3cd^6+a^5b^2d^7)x^2+11\,(84\,b^7c^6d+56\,ab^6c^5d^2+35\,a^2b^5c^4d^3+20\,a^3b^4c^3d^4+10\,a^4b^3c^2d^5+4\,a^5b^2cd^6+a^6bd^7)x}{1320\,(b^{19}x^{11}+11\,ab^{18}x^{10}+55\,a^2b^{17}x^9+165\,a^3b^{16}x^8+330\,a^4b^{15}x^7+462\,a^5b^{14}x^6+462\,a^6b^{13}x^5+330\,a^7b^{12}x^4+165\,a^8b^{11}x^3+55\,a^9b^{10}x^2+11\,a^{10}b^9x+a^{11}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(112) = 224$.
time = 0.48, size = 570, normalized size = 4.75

$$-\frac{330\,b^7d^7x^7+120\,b^7c^7+84\,ab^6c^6d+56\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+20\,a^4b^3c^3d^4+10\,a^5b^2c^2d^5+4\,a^6bcd^6+a^7d^7+462\,(4\,b^7cd^6+ab^6d^7)x^6+462\,(10\,b^7c^2d^5+4\,ab^6cd^6+a^2b^5d^7)x^5+330\,(20\,b^7c^3d^4+10\,ab^6c^2d^5+4\,a^2b^5cd^6+a^3b^4d^7)x^4+165\,(35\,b^7c^4d^3+20\,ab^6c^3d^4+10\,a^2b^5c^2d^5+4\,a^3b^4cd^6+a^4b^3d^7)x^3+55\,(56\,b^7c^5d^2+35\,ab^6c^4d^3+20\,a^2b^5c^3d^4+10\,a^3b^4c^2d^5+4\,a^4b^3cd^6+a^5b^2d^7)x^2+11\,(84\,b^7c^6d+56\,ab^6c^5d^2+35\,a^2b^5c^4d^3+20\,a^3b^4c^3d^4+10\,a^4b^3c^2d^5+4\,a^5b^2cd^6+a^6bd^7)x}{1320\,(b^{19}x^{11}+11\,ab^{18}x^{10}+55\,a^2b^{17}x^9+165\,a^3b^{16}x^8+330\,a^4b^{15}x^7+462\,a^5b^{14}x^6+462\,a^6b^{13}x^5+330\,a^7b^{12}x^4+165\,a^8b^{11}x^3+55\,a^9b^{10}x^2+11\,a^{10}b^9x+a^{11}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**12,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(112) = 224.
time = 1.12, size = 496, normalized size = 4.13

$$-\frac{330\,b^7d^7x^7+1848\,b^7cd^6x^6+462\,ab^6d^7x^6+4620\,b^7c^2d^5x^5+1848\,ab^6cd^6x^5+462\,a^2b^5d^7x^5+6600\,b^7c^3d^4x^4+3300\,ab^6c^2d^5x^4+1320\,a^2b^5cd^6x^4+330\,a^3b^4d^7x^4+5775\,b^7c^4d^3x^3+3300\,ab^6c^3d^4x^3+1650\,a^2b^5c^2d^5x^3+660\,a^3b^4cd^6x^3+165\,a^4b^3d^7x^3+3080\,b^7c^5d^2x^2+1925\,ab^6c^4d^3x^2+1100\,a^2b^5c^3d^4x^2+550\,a^3b^4c^2d^5x^2+220\,a^4b^3cd^6x^2+55\,a^5b^2d^7x^2+924\,b^7c^6dx+616\,ab^6c^5d^2x+385\,a^2b^5c^4d^3x+220\,a^3b^4c^3d^4x+110\,a^4b^3c^2d^5x+44\,a^5b^2cd^6x+11\,a^6bd^7x+120\,b^7c^7+84\,ab^6c^6d+56\,a^2b^5c^5d^2+35\,a^3b^4c^4d^3+20\,a^4b^3c^3d^4+10\,a^5b^2c^2d^5+4\,a^6bcd^6+a^7d^7}{1320\,(bx+a)^{11}b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 1848*b^7*c*d^6*x^6 + 462*a*b^6*d^7*x^6 + 4620*b^
7*c^2*d^5*x^5 + 1848*a*b^6*c*d^6*x^5 + 462*a^2*b^5*d^7*x^5 + 6600*b^7*c^3*d
^4*x^4 + 3300*a*b^6*c^2*d^5*x^4 + 1320*a^2*b^5*c*d^6*x^4 + 330*a^3*b^4*d^7*
x^4 + 5775*b^7*c^4*d^3*x^3 + 3300*a*b^6*c^3*d^4*x^3 + 1650*a^2*b^5*c^2*d^5*
x^3 + 660*a^3*b^4*c*d^6*x^3 + 165*a^4*b^3*d^7*x^3 + 3080*b^7*c^5*d^2*x^2 +
1925*a*b^6*c^4*d^3*x^2 + 1100*a^2*b^5*c^3*d^4*x^2 + 550*a^3*b^4*c^2*d^5*x^2
 + 220*a^4*b^3*c*d^6*x^2 + 55*a^5*b^2*d^7*x^2 + 924*b^7*c^6*d*x + 616*a*b^6
*c^5*d^2*x + 385*a^2*b^5*c^4*d^3*x + 220*a^3*b^4*c^3*d^4*x + 110*a^4*b^3*c^
2*d^5*x + 44*a^5*b^2*c*d^6*x + 11*a^6*b*d^7*x + 120*b^7*c^7 + 84*a*b^6*c^6*
d + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b
^2*c^2*d^5 + 4*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^11*b^8)
```

Mupad [B]
time = 0.52, size = 548, normalized size = 4.57

$$-\frac{\frac{a^7\,d^7+4\,a^6\,b\,c\,d^6+10\,a^5\,b^2\,c^2\,d^5+20\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+56\,a^2\,b^5\,c^5\,d^2+84\,a\,b^6\,c^6\,d+120\,b^7\,c^7}{1320\,b^8}+\frac{d^7\,x^7}{4\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+4\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3+20\,a^2\,b^3\,c^3\,d^2+35\,a\,b^4\,c^4\,d+56\,b^5\,c^5\right)}{24\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+4\,a^2\,b\,c\,d^2+10\,a\,b^2\,c^2\,d+20\,b^3\,c^3\right)}{4\,b^4}+\frac{7\,d^6\,x^6\,\left(a\,d+4\,b\,c\right)}{20\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+4\,a^3\,b\,c\,d^3+10\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d+35\,b^4\,c^4\right)}{8\,b^5}+\frac{d\,x\,\left(a^6\,d^6+4\,a^5\,b\,c\,d^5+10\,a^4\,b^2\,c^2\,d^4+20\,a^3\,b^3\,c^3\,d^3+35\,a^2\,b^4\,c^4\,d^2+56\,a\,b^5\,c^5\,d+84\,b^6\,c^6\right)}{120\,b^7}+\frac{7\,d^5\,x^5\,\left(a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2\right)}{20\,b^3}}{a^{11}+11\,a^{10}\,b\,x+55\,a^9\,b^2\,x^2+165\,a^8\,b^3\,x^3+330\,a^7\,b^4\,x^4+462\,a^6\,b^5\,x^5+462\,a^5\,b^6\,x^6+330\,a^4\,b^7\,x^7+165\,a^3\,b^8\,x^8+55\,a^2\,b^9\,x^9+11\,a\,b^{10}\,x^{10}+b^{11}\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^12,x)
```

```
[Out] -((a^7*d^7 + 120*b^7*c^7 + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4
*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 84*a*b^6*c^6*d + 4*a^6*b*c*d^6)/(1320*b
^8) + (d^7*x^7)/(4*b) + (d^2*x^2*(a^5*d^5 + 56*b^5*c^5 + 20*a^2*b^3*c^3*d^2
 + 10*a^3*b^2*c^2*d^3 + 35*a*b^4*c^4*d + 4*a^4*b*c*d^4))/(24*b^6) + (d^4*x^
4*(a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2))/(4*b^4) + (7*d^6
*x^6*(a*d + 4*b*c))/(20*b^2) + (d^3*x^3*(a^4*d^4 + 35*b^4*c^4 + 10*a^2*b^2*
c^2*d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3))/(8*b^5) + (d*x*(a^6*d^6 + 84*b^6
*c^6 + 35*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 10*a^4*b^2*c^2*d^4 + 56*a*
b^5*c^5*d + 4*a^5*b*c*d^5))/(120*b^7) + (7*d^5*x^5*(a^2*d^2 + 10*b^2*c^2 +
4*a*b*c*d))/(20*b^3))/(a^11 + b^11*x^11 + 11*a*b^10*x^10 + 55*a^9*b^2*x^2 +
 165*a^8*b^3*x^3 + 330*a^7*b^4*x^4 + 462*a^6*b^5*x^5 + 462*a^5*b^6*x^6 + 33
0*a^4*b^7*x^7 + 165*a^3*b^8*x^8 + 55*a^2*b^9*x^9 + 11*a^10*b*x)
```

3.1295 $\int \frac{(c+dx)^7}{(a+bx)^{13}} dx$

Optimal. Leaf size=151

$$-\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}}-\frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}}+\frac{d^3(c+dx)^8}{495(bc-ad)^4(a+bx)^9}-\frac{}{3960(}$$

```
[Out] -1/12*(d*x+c)^8/(-a*d+b*c)/(b*x+a)^12+1/33*d*(d*x+c)^8/(-a*d+b*c)^2/(b*x+a)
^11-1/110*d^2*(d*x+c)^8/(-a*d+b*c)^3/(b*x+a)^10+1/495*d^3*(d*x+c)^8/(-a*d+b
*c)^4/(b*x+a)^9-1/3960*d^4*(d*x+c)^8/(-a*d+b*c)^5/(b*x+a)^8
```

Rubi [A]
time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^8}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -1/12*(c + d*x)^8/((b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^8)/(33*(b*c - a
*d)^2*(a + b*x)^11) - (d^2*(c + d*x)^8)/(110*(b*c - a*d)^3*(a + b*x)^10) +
(d^3*(c + d*x)^8)/(495*(b*c - a*d)^4*(a + b*x)^9) - (d^4*(c + d*x)^8)/(3960
*(b*c - a*d)^5*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{13}}\,dx = -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx}{3(bc-ad)}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} + \frac{d^2\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx}{11(bc-ad)^2}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} - \frac{d^3\int \frac{(c-}{(a+}}{55(bc}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d}{495(bc}$$
$$= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d}{495(bc}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 371 vs. 2(151) = 302.

time = 0.08, size = 371, normalized size = 2.46

$$-\frac{a^7d^7+a^6bd^6(5c+12dx)+3a^5b^2d^5(5c^2+20cdx+22d^2x^2)+5a^4b^3d^4(7c^3+36c^2dx+66cd^2x^2+44d^3x^3)+5a^3b^4d^3(14c^4+84c^3dx+198c^2d^2x^2+220cd^3x^3+99d^4x^4)+3a^2b^5d^2(42c^5+280c^4dx+770c^3d^2x^2+1100c^2d^3x^3+825cd^4x^4+264d^5x^5)+ab^6d(210c^6+1512c^5dx+4620c^4d^2x^2+7700c^3d^3x^3+7425c^2d^4x^4+3960cd^5x^5+924d^6x^6)+b^7(330c^7+2520c^6dx+8316c^5d^2x^2+15400c^4d^3x^3+17325c^3d^4x^4+11880c^2d^5x^5+4620cd^6x^6+792d^7x^7)}{3960b^8(a+bx)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -1/3960*(a^7*d^7 + a^6*b*d^6*(5*c + 12*d*x) + 3*a^5*b^2*d^5*(5*c^2 + 20*c*d
*x + 22*d^2*x^2) + 5*a^4*b^3*d^4*(7*c^3 + 36*c^2*d*x + 66*c*d^2*x^2 + 44*d^
3*x^3) + 5*a^3*b^4*d^3*(14*c^4 + 84*c^3*d*x + 198*c^2*d^2*x^2 + 220*c*d^3*x
^3 + 99*d^4*x^4) + 3*a^2*b^5*d^2*(42*c^5 + 280*c^4*d*x + 770*c^3*d^2*x^2 +
1100*c^2*d^3*x^3 + 825*c*d^4*x^4 + 264*d^5*x^5) + a*b^6*d*(210*c^6 + 1512*c
^5*d*x + 4620*c^4*d^2*x^2 + 7700*c^3*d^3*x^3 + 7425*c^2*d^4*x^4 + 3960*c*d^
5*x^5 + 924*d^6*x^6) + b^7*(330*c^7 + 2520*c^6*d*x + 8316*c^5*d^2*x^2 + 154
00*c^4*d^3*x^3 + 17325*c^3*d^4*x^4 + 11880*c^2*d^5*x^5 + 4620*c*d^6*x^6 + 7
92*d^7*x^7))/(b^8*(a + b*x)^12)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(141) = 282.

time = 0.14, size = 464, normalized size = 3.07

method	result
risch	$\frac{-\frac{d^7x^7}{5b}-\frac{7d^6(ad+5bc)x^6}{30b^2}-\frac{d^5(a^2d^2+5abcd+15b^2c^2)x^5}{5b^3}-\frac{d^4(a^3d^3+5a^2bc\,d^2+15a\,b^2c^2d+35b^3c^3)x^4}{8b^4}-\frac{d^3(a^4d^4+5a^3bc\,d^3+15a^2b^2c^2d^2+35a\,b^3c}{18b^5}}{}$
default	$-\frac{-a^7d^7+7a^6bc\,d^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7a\,b^6c^6d+b^7c^7}{12b^8(bx+a)^{12}}-\frac{35d^3(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b}{9b^8(bx+a)^9}$

norman	$\frac{-\frac{d^7x^7}{5b}+\frac{7\left(-a\,b^4d^7-5b^5c\,d^6\right)x^6}{30b^6}+\frac{\left(-a^2b^4d^7-5a\,b^5c\,d^6-15b^6c^2d^5\right)x^5}{5b^7}+\frac{\left(-a^3b^4d^7-5a^2b^5c\,d^6-15a\,b^6c^2d^5-35b^7c^3d^4\right)x^4}{8b^8}+\frac{\left(-a^4b^4d^7-5a^3b^5c\,d\right.}{}}{}$
gosper	$-\frac{792d^7x^7b^7+924a\,b^6d^7x^6+4620b^7c\,d^6x^6+792a^2b^5d^7x^5+3960a\,b^6c\,d^6x^5+11880b^7c^2d^5x^5+495a^3b^4d^7x^4+2475a^2b^5c\,d^6x^4+7425a\,b^6c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*
b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^12-35/9/b
^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x
+a)^9+35/8/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^8-
1/5*d^7/b^8/(b*x+a)^5-3/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^7+21/10
/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d-b^5*c^5)/(b*x+a)^10+7/6/b^8*d^6*(a*d-b*c)/(b*x+a)^6-7/11/b^8*d*(a^
6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^
2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(141) = 282$.
time = 0.31, size = 581, normalized size = 3.85

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924\,(5\,b^7cd^6+ab^6d^7)x^6+792\,(15\,b^7c^2d^5+5\,ab^6cd^6+a^2b^5d^7)x^5+495\,(35\,b^7c^3d^4+15\,ab^6c^2d^5+5\,a^2b^5cd^6+a^3b^4d^7)x^4+220\,(70\,b^7c^4d^3+35\,ab^6c^3d^4+15\,a^2b^5c^2d^5+5\,a^3b^4cd^6+a^4b^3d^7)x^3+66\,(126\,b^7c^5d^2+70\,ab^6c^4d^3+35\,a^2b^5c^3d^4+15\,a^3b^4c^2d^5+5\,a^4b^3cd^6+a^5b^2d^7)x^2+12\,(210\,b^7c^6d+126\,ab^6c^5d^2+70\,a^2b^5c^4d^3+35\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5+5\,a^5b^2cd^6+a^6bd^7)x}{3960\,(b^{20}x^{12}+12\,ab^{19}x^{11}+66\,a^2b^{18}x^{10}+220\,a^3b^{17}x^9+495\,a^4b^{16}x^8+792\,a^5b^{15}x^7+924\,a^6b^{14}x^6+792\,a^7b^{13}x^5+495\,a^8b^{12}x^4+220\,a^9b^{11}x^3+66\,a^{10}b^{10}x^2+12\,a^{11}b^9x+a^{12}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(141) = 282$.
time = 0.45, size = 581, normalized size = 3.85

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924\,(5\,b^7cd^6+ab^6d^7)x^6+792\,(15\,b^7c^2d^5+5\,ab^6cd^6+a^2b^5d^7)x^5+495\,(35\,b^7c^3d^4+15\,ab^6c^2d^5+5\,a^2b^5cd^6+a^3b^4d^7)x^4+220\,(70\,b^7c^4d^3+35\,ab^6c^3d^4+15\,a^2b^5c^2d^5+5\,a^3b^4cd^6+a^4b^3d^7)x^3+66\,(126\,b^7c^5d^2+70\,ab^6c^4d^3+35\,a^2b^5c^3d^4+15\,a^3b^4c^2d^5+5\,a^4b^3cd^6+a^5b^2d^7)x^2+12\,(210\,b^7c^6d+126\,ab^6c^5d^2+70\,a^2b^5c^4d^3+35\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5+5\,a^5b^2cd^6+a^6bd^7)x}{3960\,(b^{20}x^{12}+12\,ab^{19}x^{11}+66\,a^2b^{18}x^{10}+220\,a^3b^{17}x^9+495\,a^4b^{16}x^8+792\,a^5b^{15}x^7+924\,a^6b^{14}x^6+792\,a^7b^{13}x^5+495\,a^8b^{12}x^4+220\,a^9b^{11}x^3+66\,a^{10}b^{10}x^2+12\,a^{11}b^9x+a^{12}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**13,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(141) = 282.
time = 1.10, size = 496, normalized size = 3.28

$$-\frac{792\,b^7d^7x^7+4620\,b^7cd^6x^6+924\,ab^6d^7x^6+11880\,b^7c^2d^5x^5+3960\,ab^6cd^6x^5+792\,a^2b^5d^7x^5+17325\,b^7c^3d^4x^4+7425\,ab^6c^2d^5x^4+2475\,a^2b^5cd^6x^4+495\,a^3b^4d^7x^4+15400\,b^7c^4d^3x^3+7700\,ab^6c^3d^4x^3+3300\,a^2b^5c^2d^5x^3+1100\,a^3b^4cd^6x^3+220\,a^4b^3d^7x^3+8316\,b^7c^5d^2x^2+4620\,ab^6c^4d^3x^2+2310\,a^2b^5c^3d^4x^2+990\,a^3b^4c^2d^5x^2+330\,a^4b^3cd^6x^2+66\,a^5b^2d^7x^2+2520\,b^7c^6dx+1512\,ab^6c^5d^2x+840\,a^2b^5c^4d^3x+420\,a^3b^4c^3d^4x+180\,a^4b^3c^2d^5x+60\,a^5b^2cd^6x+12\,a^6bd^7x+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7}{3960\,(bx+a)^{12}b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 4620*b^7*c*d^6*x^6 + 924*a*b^6*d^7*x^6 + 11880*b
^7*c^2*d^5*x^5 + 3960*a*b^6*c*d^6*x^5 + 792*a^2*b^5*d^7*x^5 + 17325*b^7*c^3
*d^4*x^4 + 7425*a*b^6*c^2*d^5*x^4 + 2475*a^2*b^5*c*d^6*x^4 + 495*a^3*b^4*d^
7*x^4 + 15400*b^7*c^4*d^3*x^3 + 7700*a*b^6*c^3*d^4*x^3 + 3300*a^2*b^5*c^2*d
^5*x^3 + 1100*a^3*b^4*c*d^6*x^3 + 220*a^4*b^3*d^7*x^3 + 8316*b^7*c^5*d^2*x^
2 + 4620*a*b^6*c^4*d^3*x^2 + 2310*a^2*b^5*c^3*d^4*x^2 + 990*a^3*b^4*c^2*d^5
*x^2 + 330*a^4*b^3*c*d^6*x^2 + 66*a^5*b^2*d^7*x^2 + 2520*b^7*c^6*d*x + 1512
*a*b^6*c^5*d^2*x + 840*a^2*b^5*c^4*d^3*x + 420*a^3*b^4*c^3*d^4*x + 180*a^4*
b^3*c^2*d^5*x + 60*a^5*b^2*c*d^6*x + 12*a^6*b*d^7*x + 330*b^7*c^7 + 210*a*b
```

```
^6*c^6*d + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 +
15*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^12*b^8)
```

Mupad [B]
time = 0.23, size = 559, normalized size = 3.70

$$-\frac{\frac{a^7 d^7+5 a^6 b c d^6+15 a^5 b^2 c^2 d^5+35 a^4 b^3 c^3 d^4+70 a^3 b^4 c^4 d^3+126 a^2 b^5 c^5 d^2+210 a b^6 c^6 d+330 b^7 c^7}{3960 b^8}+\frac{d^7 x^7}{5 b}+\frac{d^2 x^2 \left(a^5 d^5+5 a^4 b c d^4+15 a^3 b^2 c^2 d^3+35 a^2 b^3 c^3 d^2+70 a b^4 c^4 d+126 b^5 c^5\right)}{60 b^6}+\frac{d^4 x^4 \left(a^3 d^3+5 a^2 b c d^2+15 a b^2 c^2 d+35 b^3 c^3\right)}{8 b^4}+\frac{7 d^6 x^6 (a d+5 b c)}{30 b^2}+\frac{d^3 x^3 \left(a^4 d^4+5 a^3 b c d^3+15 a^2 b^2 c^2 d^2+35 a b^3 c^3 d+70 b^4 c^4\right)}{18 b^5}+\frac{d x \left(a^6 d^6+5 a^5 b c d^5+15 a^4 b^2 c^2 d^4+35 a^3 b^3 c^3 d^3+70 a^2 b^4 c^4 d^2+126 a b^5 c^5 d+210 b^6 c^6\right)}{330 b^7}+\frac{d^5 x^5 \left(a^2 d^2+5 a b c d+15 b^2 c^2\right)}{5 b^3}}{a^{12}+12 a^{11} b x+66 a^{10} b^2 x^2+220 a^9 b^3 x^3+495 a^8 b^4 x^4+792 a^7 b^5 x^5+924 a^6 b^6 x^6+792 a^5 b^7 x^7+495 a^4 b^8 x^8+220 a^3 b^9 x^9+66 a^2 b^{10} x^{10}+12 a b^{11} x^{11}+b^{12} x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^13,x)
```

```
[Out] -((a^7*d^7 + 330*b^7*c^7 + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 210*a*b^6*c^6*d + 5*a^6*b*c*d^6)/(3960
*b^8) + (d^7*x^7)/(5*b) + (d^2*x^2*(a^5*d^5 + 126*b^5*c^5 + 35*a^2*b^3*c^3*
d^2 + 15*a^3*b^2*c^2*d^3 + 70*a*b^4*c^4*d + 5*a^4*b*c*d^4))/(60*b^6) + (d^4
*x^4*(a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2))/(8*b^4) + (7*
d^6*x^6*(a*d + 5*b*c))/(30*b^2) + (d^3*x^3*(a^4*d^4 + 70*b^4*c^4 + 15*a^2*b
^2*c^2*d^2 + 35*a*b^3*c^3*d + 5*a^3*b*c*d^3))/(18*b^5) + (d*x*(a^6*d^6 + 21
0*b^6*c^6 + 70*a^2*b^4*c^4*d^2 + 35*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 +
126*a*b^5*c^5*d + 5*a^5*b*c*d^5))/(330*b^7) + (d^5*x^5*(a^2*d^2 + 15*b^2*c^
2 + 5*a*b*c*d))/(5*b^3))/(a^12 + b^12*x^12 + 12*a*b^11*x^11 + 66*a^10*b^2*x
^2 + 220*a^9*b^3*x^3 + 495*a^8*b^4*x^4 + 792*a^7*b^5*x^5 + 924*a^6*b^6*x^6
+ 792*a^5*b^7*x^7 + 495*a^4*b^8*x^8 + 220*a^3*b^9*x^9 + 66*a^2*b^10*x^10 +
12*a^11*b*x)
```

3.1296 $\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx$

Optimal. Leaf size=198

$$-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{d^6(b\cdots}{b^8(a\cdots}$$

```
[Out] -1/13*(-a*d+b*c)^7/b^8/(b*x+a)^13-7/12*d*(-a*d+b*c)^6/b^8/(b*x+a)^12-21/11*
d^2*(-a*d+b*c)^5/b^8/(b*x+a)^11-7/2*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^10-35/9*d^
4*(-a*d+b*c)^3/b^8/(b*x+a)^9-21/8*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^8-d^6*(-a*d+
b*c)/b^8/(b*x+a)^7-1/6*d^7/b^8/(b*x+a)^6
```

Rubi [A]
time = 0.11, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{d^7}{6b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -1/13*(b*c - a*d)^7/(b^8*(a + b*x)^13) - (7*d*(b*c - a*d)^6)/(12*b^8*(a + b
*x)^12) - (21*d^2*(b*c - a*d)^5)/(11*b^8*(a + b*x)^11) - (7*d^3*(b*c - a*d)
^4)/(2*b^8*(a + b*x)^10) - (35*d^4*(b*c - a*d)^3)/(9*b^8*(a + b*x)^9) - (21
*d^5*(b*c - a*d)^2)/(8*b^8*(a + b*x)^8) - (d^6*(b*c - a*d))/(b^8*(a + b*x)^
7) - d^7/(6*b^8*(a + b*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{14}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{13}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{12}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{11}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{10}}\cdots\right.\\ &= -\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}\cdots\end{aligned}$$

Mathematica [A]
time = 0.09, size = 369, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(6c+13dx)+3a^5b^2d^5(7c^2+26cdx+26d^2x^2)+a^4b^3d^4(56c^3+273c^2dx+468cd^2x^2+286d^3x^3)+a^3b^4d^3(126c^4+728c^3dx+1638c^2d^2x^2+1716cd^3x^3+715d^4x^4)+3a^2b^5d^2(84c^5+546c^4dx+1456c^3d^2x^2+2002c^2d^3x^3+1430cd^4x^4+429d^5x^5)+ab^6d(462c^6+3276c^5dx+9828c^4d^2x^2+16016c^3d^3x^3+15015c^2d^4x^4+7722cd^5x^5+1716d^6x^6)+b^7(792c^7+6006c^6dx+19656c^5d^2x^2+36036c^4d^3x^3+40040c^3d^4x^4+27027c^2d^5x^5+10296cd^6x^6+1716d^7x^7)}{10296b^8(a+bx)^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -1/10296*(a^7*d^7 + a^6*b*d^6*(6*c + 13*d*x) + 3*a^5*b^2*d^5*(7*c^2 + 26*c*
d*x + 26*d^2*x^2) + a^4*b^3*d^4*(56*c^3 + 273*c^2*d*x + 468*c*d^2*x^2 + 286
*d^3*x^3) + a^3*b^4*d^3*(126*c^4 + 728*c^3*d*x + 1638*c^2*d^2*x^2 + 1716*c*
d^3*x^3 + 715*d^4*x^4) + 3*a^2*b^5*d^2*(84*c^5 + 546*c^4*d*x + 1456*c^3*d^2
*x^2 + 2002*c^2*d^3*x^3 + 1430*c*d^4*x^4 + 429*d^5*x^5) + a*b^6*d*(462*c^6
+ 3276*c^5*d*x + 9828*c^4*d^2*x^2 + 16016*c^3*d^3*x^3 + 15015*c^2*d^4*x^4 +
 7722*c*d^5*x^5 + 1716*d^6*x^6) + b^7*(792*c^7 + 6006*c^6*d*x + 19656*c^5*d
^2*x^2 + 36036*c^4*d^3*x^3 + 40040*c^3*d^4*x^4 + 27027*c^2*d^5*x^5 + 10296*
c*d^6*x^6 + 1716*d^7*x^7))/(b^8*(a + b*x)^13)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(184) = 368.
time = 0.17, size = 463, normalized size = 2.34

method	result
risch	$-\frac{d^7x^7}{6b}-\frac{d^6(ad+6bc)x^6}{6b^2}-\frac{d^5\left(a^2d^2+6abcd+21b^2c^2\right)x^5}{8b^3}-\frac{5d^4\left(a^3d^3+6a^2bcd^2+21ab^2c^2d+56b^3c^3\right)x^4}{72b^4}-\frac{d^3\left(a^4d^4+6a^3bcd^3+21a^2b^2c^2d^2+56ab^3c^3 \right.}{36b^5}$
default	$-\frac{-a^7d^7+7a^6bcd^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7ab^6c^6d+b^7c^7}{13b^8(bx+a)^{13}}-\frac{7d\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3\right.}{12b^8(bx+}$
norman	$\frac{-\frac{d^7x^7}{6b}+\frac{\left(-ab^5d^7-6b^6cd^6\right)x^6}{6b^7}+\frac{\left(-a^2b^5d^7-6ab^6cd^6-21b^7c^2d^5\right)x^5}{8b^8}+\frac{5\left(-a^3b^5d^7-6a^2b^6cd^6-21ab^7c^2d^5-56b^8c^3d^4\right)x^4}{72b^9}+\frac{\left(-a^4b^5d^7-6a^3b^6cd\right.}{}}{}$
gosper	$-\frac{1716d^7x^7b^7+1716ab^6d^7x^6+10296b^7cd^6x^6+1287a^2b^5d^7x^5+7722ab^6cd^6x^5+27027b^7c^2d^5x^5+715a^3b^4d^7x^4+4290a^2b^5cd^6x^4+1501}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*
b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^13-7/12/b
^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^
4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^12+35/9/b^8*d^4*(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^9-21/8/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/(b*x+a)^8+1/b^8*d^6*(a*d-b*c)/(b*x+a)^7-7/2/b^8*d^3*(a^4*d^4-4*a^3*b*c*
d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^10-1/6*d^7/b^8/(b*x+a)
^6+21/11/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d
^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(184) = 368$.
time = 0.33, size = 592, normalized size = 2.99

$$-\frac{1716\,b^{7}d^{7}x^{7} + 792\,b^{7}c^{7} + 462\,ab^{6}c^{6}d + 252\,a^{2}b^{5}c^{5}d^{2} + 126\,a^{3}b^{4}c^{4}d^{3} + 56\,a^{4}b^{3}c^{3}d^{4} + 21\,a^{5}b^{2}c^{2}d^{5} + 6\,a^{6}bcd^{6} + a^{7}d^{7} + 1716\,(6\,b^{7}cd^{6} + ab^{6}d^{7})x^{6} + 1287\,(21\,b^{7}c^{2}d^{5} + 6\,ab^{6}cd^{6} + a^{2}b^{5}d^{7})x^{5} + 715\,(56\,b^{7}c^{3}d^{4} + 21\,ab^{6}c^{2}d^{5} + 6\,a^{2}b^{5}cd^{6} + a^{3}b^{4}d^{7})x^{4} + 286\,(126\,b^{7}c^{4}d^{3} + 56\,ab^{6}c^{3}d^{4} + 21\,a^{2}b^{5}c^{2}d^{5} + 6\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7})x^{3} + 78\,(252\,b^{7}c^{5}d^{2} + 126\,ab^{6}c^{4}d^{3} + 56\,a^{2}b^{5}c^{3}d^{4} + 21\,a^{3}b^{4}c^{2}d^{5} + 6\,a^{4}b^{3}cd^{6} + a^{5}b^{2}d^{7})x^{2} + 13\,(462\,b^{7}c^{6}d + 252\,ab^{6}c^{5}d^{2} + 126\,a^{2}b^{5}c^{4}d^{3} + 56\,a^{3}b^{4}c^{3}d^{4} + 21\,a^{4}b^{3}c^{2}d^{5} + 6\,a^{5}b^{2}cd^{6} + a^{6}bd^{7})x}{10296\,(b^{21}x^{13} + 13\,ab^{20}x^{12} + 78\,a^{2}b^{19}x^{11} + 286\,a^{3}b^{18}x^{10} + 715\,a^{4}b^{17}x^{9} + 1287\,a^{5}b^{16}x^{8} + 1716\,a^{6}b^{15}x^{7} + 1716\,a^{7}b^{14}x^{6} + 1287\,a^{8}b^{13}x^{5} + 715\,a^{9}b^{12}x^{4} + 286\,a^{10}b^{11}x^{3} + 78\,a^{11}b^{10}x^{2} + 13\,a^{12}b^{9}x + a^{13}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(184) = 368$.
time = 0.54, size = 592, normalized size = 2.99

$$-\frac{1716\,b^{7}d^{7}x^{7} + 792\,b^{7}c^{7} + 462\,ab^{6}c^{6}d + 252\,a^{2}b^{5}c^{5}d^{2} + 126\,a^{3}b^{4}c^{4}d^{3} + 56\,a^{4}b^{3}c^{3}d^{4} + 21\,a^{5}b^{2}c^{2}d^{5} + 6\,a^{6}bcd^{6} + a^{7}d^{7} + 1716\,(6\,b^{7}cd^{6} + ab^{6}d^{7})x^{6} + 1287\,(21\,b^{7}c^{2}d^{5} + 6\,ab^{6}cd^{6} + a^{2}b^{5}d^{7})x^{5} + 715\,(56\,b^{7}c^{3}d^{4} + 21\,ab^{6}c^{2}d^{5} + 6\,a^{2}b^{5}cd^{6} + a^{3}b^{4}d^{7})x^{4} + 286\,(126\,b^{7}c^{4}d^{3} + 56\,ab^{6}c^{3}d^{4} + 21\,a^{2}b^{5}c^{2}d^{5} + 6\,a^{3}b^{4}cd^{6} + a^{4}b^{3}d^{7})x^{3} + 78\,(252\,b^{7}c^{5}d^{2} + 126\,ab^{6}c^{4}d^{3} + 56\,a^{2}b^{5}c^{3}d^{4} + 21\,a^{3}b^{4}c^{2}d^{5} + 6\,a^{4}b^{3}cd^{6} + a^{5}b^{2}d^{7})x^{2} + 13\,(462\,b^{7}c^{6}d + 252\,ab^{6}c^{5}d^{2} + 126\,a^{2}b^{5}c^{4}d^{3} + 56\,a^{3}b^{4}c^{3}d^{4} + 21\,a^{4}b^{3}c^{2}d^{5} + 6\,a^{5}b^{2}cd^{6} + a^{6}bd^{7})x}{10296\,(b^{21}x^{13} + 13\,ab^{20}x^{12} + 78\,a^{2}b^{19}x^{11} + 286\,a^{3}b^{18}x^{10} + 715\,a^{4}b^{17}x^{9} + 1287\,a^{5}b^{16}x^{8} + 1716\,a^{6}b^{15}x^{7} + 1716\,a^{7}b^{14}x^{6} + 1287\,a^{8}b^{13}x^{5} + 715\,a^{9}b^{12}x^{4} + 286\,a^{10}b^{11}x^{3} + 78\,a^{11}b^{10}x^{2} + 13\,a^{12}b^{9}x + a^{13}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**14,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(184) = 368.
time = 0.69, size = 496, normalized size = 2.51

$$-\frac{1716\, b^7 d^7 x^7 + 10296\, b^7 c d^6 x^6 + 1716\, a b^6 d^7 x^6 + 27027\, b^7 c^2 d^5 x^5 + 7722\, a b^6 c d^6 x^5 + 1287\, a^2 b^5 d^7 x^5 + 40040\, b^7 c^3 d^4 x^4 + 15015\, a b^6 c^2 d^5 x^4 + 4290\, a^2 b^5 c d^6 x^4 + 715\, a^3 b^4 d^7 x^4 + 36036\, b^7 c^4 d^3 x^3 + 16016\, a b^6 c^3 d^4 x^3 + 6006\, a^2 b^5 c^2 d^5 x^3 + 1716\, a^3 b^4 c d^6 x^3 + 286\, a^4 b^3 d^7 x^3 + 19656\, b^7 c^5 d^2 x^2 + 9828\, a b^6 c^4 d^3 x^2 + 4368\, a^2 b^5 c^3 d^4 x^2 + 1638\, a^3 b^4 c^2 d^5 x^2 + 468\, a^4 b^3 c d^6 x^2 + 78\, a^5 b^2 d^7 x^2 + 6006\, b^7 c^6 d x + 3276\, a b^6 c^5 d^2 x + 1638\, a^2 b^5 c^4 d^3 x + 728\, a^3 b^4 c^3 d^4 x + 273\, a^4 b^3 c^2 d^5 x + 78\, a^5 b^2 c d^6 x + 13\, a^6 b d^7 x + 792\, b^7 c^7 + 462\, a b^6 c^6 d + 252\, a^2 b^5 c^5 d^2 + 126\, a^3 b^4 c^4 d^3 + 56\, a^4 b^3 c^3 d^4 + 21\, a^5 b^2 c^2 d^5 + 6\, a^6 b c d^6 + a^7 d^7}{10296\, (bx + a)^{13} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 10296*b^7*c*d^6*x^6 + 1716*a*b^6*d^7*x^6 + 270
27*b^7*c^2*d^5*x^5 + 7722*a*b^6*c*d^6*x^5 + 1287*a^2*b^5*d^7*x^5 + 40040*b^
7*c^3*d^4*x^4 + 15015*a*b^6*c^2*d^5*x^4 + 4290*a^2*b^5*c*d^6*x^4 + 715*a^3*
b^4*d^7*x^4 + 36036*b^7*c^4*d^3*x^3 + 16016*a*b^6*c^3*d^4*x^3 + 6006*a^2*b^
5*c^2*d^5*x^3 + 1716*a^3*b^4*c*d^6*x^3 + 286*a^4*b^3*d^7*x^3 + 19656*b^7*c^
5*d^2*x^2 + 9828*a*b^6*c^4*d^3*x^2 + 4368*a^2*b^5*c^3*d^4*x^2 + 1638*a^3*b^
4*c^2*d^5*x^2 + 468*a^4*b^3*c*d^6*x^2 + 78*a^5*b^2*d^7*x^2 + 6006*b^7*c^6*d
*x + 3276*a*b^6*c^5*d^2*x + 1638*a^2*b^5*c^4*d^3*x + 728*a^3*b^4*c^3*d^4*x
+ 273*a^4*b^3*c^2*d^5*x + 78*a^5*b^2*c*d^6*x + 13*a^6*b*d^7*x + 792*b^7*c^7
 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3
*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^13*b^8)
```

Mupad [B]
time = 0.40, size = 570, normalized size = 2.88

$$-\frac{\frac{a^7 d^7+6 a^6 b c d^6+21 a^5 b^2 c^2 d^5+56 a^4 b^3 c^3 d^4+126 a^3 b^4 c^4 d^3+252 a^2 b^5 c^5 d^2+462 a b^6 c^6 d+792 b^7 c^7}{10296 b^8}+\frac{d^7 x^7}{6 b}+\frac{d^2 x^2 \left(a^5 d^5+6 a^4 b c d^4+21 a^3 b^2 c^2 d^3+56 a^2 b^3 c^3 d^2+126 a b^4 c^4 d+252 b^5 c^5\right)}{132 b^6}+\frac{5 d^4 x^4 \left(a^3 d^3+6 a^2 b c d^2+21 a b^2 c^2 d+56 b^3 c^3\right)}{72 b^4}+\frac{d^6 x^6 (a d+6 b c)}{6 b^2}+\frac{d^3 x^3 \left(a^4 d^4+6 a^3 b c d^3+21 a^2 b^2 c^2 d^2+56 a b^3 c^3 d+126 b^4 c^4\right)}{36 b^5}+\frac{d x \left(a^6 d^6+6 a^5 b c d^5+21 a^4 b^2 c^2 d^4+56 a^3 b^3 c^3 d^3+126 a^2 b^4 c^4 d^2+252 a b^5 c^5 d+462 b^6 c^6\right)}{792 b^7}+\frac{d^5 x^5 \left(a^2 d^2+6 a b c d+21 b^2 c^2\right)}{8 b^3}}{a^{13}+13 a^{12} b x+78 a^{11} b^2 x^2+286 a^{10} b^3 x^3+715 a^9 b^4 x^4+1287 a^8 b^5 x^5+1716 a^7 b^6 x^6+1716 a^6 b^7 x^7+1287 a^5 b^8 x^8+715 a^4 b^9 x^9+286 a^3 b^{10} x^{10}+78 a^2 b^{11} x^{11}+13 a b^{12} x^{12}+b^{13} x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^14,x)
```

```
[Out] -((a^7*d^7 + 792*b^7*c^7 + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a
^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 462*a*b^6*c^6*d + 6*a^6*b*c*d^6)/(102
96*b^8) + (d^7*x^7)/(6*b) + (d^2*x^2*(a^5*d^5 + 252*b^5*c^5 + 56*a^2*b^3*c^
3*d^2 + 21*a^3*b^2*c^2*d^3 + 126*a*b^4*c^4*d + 6*a^4*b*c*d^4))/(132*b^6) +
(5*d^4*x^4*(a^3*d^3 + 56*b^3*c^3 + 21*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(72*b^4
) + (d^6*x^6*(a*d + 6*b*c))/(6*b^2) + (d^3*x^3*(a^4*d^4 + 126*b^4*c^4 + 21*
```

```
a^2*b^2*c^2*d^2 + 56*a*b^3*c^3*d + 6*a^3*b*c*d^3))/(36*b^5) + (d*x*(a^6*d^6
 + 462*b^6*c^6 + 126*a^2*b^4*c^4*d^2 + 56*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*
d^4 + 252*a*b^5*c^5*d + 6*a^5*b*c*d^5))/(792*b^7) + (d^5*x^5*(a^2*d^2 + 21*
b^2*c^2 + 6*a*b*c*d))/(8*b^3))/(a^13 + b^13*x^13 + 13*a*b^12*x^12 + 78*a^11
*b^2*x^2 + 286*a^10*b^3*x^3 + 715*a^9*b^4*x^4 + 1287*a^8*b^5*x^5 + 1716*a^7
*b^6*x^6 + 1716*a^6*b^7*x^7 + 1287*a^5*b^8*x^8 + 715*a^4*b^9*x^9 + 286*a^3*
b^10*x^10 + 78*a^2*b^11*x^11 + 13*a^12*b*x)
```

3.1297 $\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx$

Optimal. Leaf size=200

$$-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^6(bc-}{8b^8(a+}$$

```
[Out] -1/14*(-a*d+b*c)^7/b^8/(b*x+a)^14-7/13*d*(-a*d+b*c)^6/b^8/(b*x+a)^13-7/4*d^
2*(-a*d+b*c)^5/b^8/(b*x+a)^12-35/11*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^11-7/2*d^4
*(-a*d+b*c)^3/b^8/(b*x+a)^10-7/3*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^9-7/8*d^6*(-a
*d+b*c)/b^8/(b*x+a)^8-1/7*d^7/b^8/(b*x+a)^7
```

Rubi [A]
time = 0.10, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{d^7}{7b^8(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -1/14*(b*c - a*d)^7/(b^8*(a + b*x)^14) - (7*d*(b*c - a*d)^6)/(13*b^8*(a + b
*x)^13) - (7*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^12) - (35*d^3*(b*c - a*d)^
4)/(11*b^8*(a + b*x)^11) - (7*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^10) - (7*
d^5*(b*c - a*d)^2)/(3*b^8*(a + b*x)^9) - (7*d^6*(b*c - a*d))/(8*b^8*(a + b*
x)^8) - d^7/(7*b^8*(a + b*x)^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{15}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{14}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{13}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{12}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{11}}+\right.\\ &= -\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\end{aligned}$$

Mathematica [A]
time = 0.09, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+7a^6bd^6(c+2dx)+7a^5b^2d^5(4c^2+14cdx+13d^2x^2)+7a^4b^3d^4(12c^3+56c^2dx+91cd^2x^2+52d^3x^3)+7a^3b^4d^3(30c^4+168c^3dx+364c^2d^2x^2+364cd^3x^3+143d^4x^4)+7a^2b^5d^2(66c^5+420c^4dx+1092c^3d^2x^2+1456c^2d^3x^3+1001cd^4x^4+286d^5x^5)+7ab^6d(132c^6+924c^5dx+2730c^4d^2x^2+4368c^3d^3x^3+4004c^2d^4x^4+2002cd^5x^5+429d^6x^6)+b^7(1716c^7+12936c^6dx+42042c^5d^2x^2+76440c^4d^3x^3+84084c^3d^4x^4+56056c^2d^5x^5+21021cd^6x^6+3432d^7x^7)}{24024b^8(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -1/24024*(a^7*d^7 + 7*a^6*b*d^6*(c + 2*d*x) + 7*a^5*b^2*d^5*(4*c^2 + 14*c*d
*x + 13*d^2*x^2) + 7*a^4*b^3*d^4*(12*c^3 + 56*c^2*d*x + 91*c*d^2*x^2 + 52*d
^3*x^3) + 7*a^3*b^4*d^3*(30*c^4 + 168*c^3*d*x + 364*c^2*d^2*x^2 + 364*c*d^3
*x^3 + 143*d^4*x^4) + 7*a^2*b^5*d^2*(66*c^5 + 420*c^4*d*x + 1092*c^3*d^2*x^
2 + 1456*c^2*d^3*x^3 + 1001*c*d^4*x^4 + 286*d^5*x^5) + 7*a*b^6*d*(132*c^6 +
 924*c^5*d*x + 2730*c^4*d^2*x^2 + 4368*c^3*d^3*x^3 + 4004*c^2*d^4*x^4 + 200
2*c*d^5*x^5 + 429*d^6*x^6) + b^7*(1716*c^7 + 12936*c^6*d*x + 42042*c^5*d^2*
x^2 + 76440*c^4*d^3*x^3 + 84084*c^3*d^4*x^4 + 56056*c^2*d^5*x^5 + 21021*c*d
^6*x^6 + 3432*d^7*x^7))/(b^8*(a + b*x)^14)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(184) = 368$.
time = 0.16, size = 464, normalized size = 2.32

method	result
risch	$-\frac{d^7x^7}{7b}-\frac{d^6(ad+7bc)x^6}{8b^2}-\frac{d^5\left(a^2d^2+7abcd+28b^2c^2\right)x^5}{12b^3}-\frac{d^4\left(a^3d^3+7a^2bc\,d^2+28a\,b^2c^2d+84b^3c^3\right)x^4}{24b^4}-\frac{d^3\left(a^4d^4+7a^3bc\,d^3+28a^2b^2c^2d^2+84a\,b^3c^3\right.}{66b^5}$
default	$-\frac{7d\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)}{13b^8(bx+a)^{13}}+\frac{7d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5\right.}{4b^8(bx+a)^{12}}$
norman	$\frac{\frac{-a^7b^6d^7-7a^6b^7c\,d^6-28a^5b^8c^2d^5-84a^4b^9c^3d^4-210a^3b^{10}c^4d^3-462a^2b^{11}c^5d^2-924a\,b^{12}c^6d-1716b^{13}c^7}{24024b^{14}}+\frac{\left(-a^6b^6d^7-7a^5b^7c\,d^6-28a^4b^8c^2d^5-\right.}{}}{}$
gosper	$-\frac{3432d^7x^7b^7+3003a\,b^6d^7x^6+21021b^7c\,d^6x^6+2002a^2b^5d^7x^5+14014a\,b^6c\,d^6x^5+56056b^7c^2d^5x^5+1001a^3b^4d^7x^4+7007a^2b^5c\,d^6x^4+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^15,x,method=_RETURNVERBOSE)
```

```
[Out] -7/13/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15
*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^13+7/4/b^8*d^2*(a^5*d^5-5*a
^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*
x+a)^12-7/3/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^9+7/8/b^8*d^6*(a*d-
b*c)/(b*x+a)^8-1/14*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c
^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*
x+a)^14-1/7*d^7/b^8/(b*x+a)^7+7/2/b^8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^
2*d-b^3*c^3)/(b*x+a)^10-35/11/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*
d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(184) = 368$.
time = 0.34, size = 603, normalized size = 3.02

$$-\frac{3432\,b^{7}d^{7}x^{7}+1716\,b^{7}c^{7}+924\,ab^{6}c^{6}d+462\,a^{2}b^{5}c^{5}d^{2}+210\,a^{3}b^{4}c^{4}d^{3}+84\,a^{4}b^{3}c^{3}d^{4}+28\,a^{5}b^{2}c^{2}d^{5}+7\,a^{6}bcd^{6}+a^{7}d^{7}+3003\,(7\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+2002\,(28\,b^{7}c^{2}d^{5}+7\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+1001\,(84\,b^{7}c^{3}d^{4}+28\,ab^{6}c^{2}d^{5}+7\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+364\,(210\,b^{7}c^{4}d^{3}+84\,ab^{6}c^{3}d^{4}+28\,a^{2}b^{5}c^{2}d^{5}+7\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+91\,(462\,b^{7}c^{5}d^{2}+210\,ab^{6}c^{4}d^{3}+84\,a^{2}b^{5}c^{3}d^{4}+28\,a^{3}b^{4}c^{2}d^{5}+7\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+14\,(924\,b^{7}c^{6}d+462\,ab^{6}c^{5}d^{2}+210\,a^{2}b^{5}c^{4}d^{3}+84\,a^{3}b^{4}c^{3}d^{4}+28\,a^{4}b^{3}c^{2}d^{5}+7\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{24024\,(b^{22}x^{14}+14\,ab^{21}x^{13}+91\,a^{2}b^{20}x^{12}+364\,a^{3}b^{19}x^{11}+1001\,a^{4}b^{18}x^{10}+2002\,a^{5}b^{17}x^{9}+3003\,a^{6}b^{16}x^{8}+3432\,a^{7}b^{15}x^{7}+3003\,a^{8}b^{14}x^{6}+2002\,a^{9}b^{13}x^{5}+1001\,a^{10}b^{12}x^{4}+364\,a^{11}b^{11}x^{3}+91\,a^{12}b^{10}x^{2}+14\,a^{13}b^{9}x+a^{14}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(184) = 368$.
time = 0.49, size = 603, normalized size = 3.02

$$-\frac{3432\,b^{7}d^{7}x^{7}+1716\,b^{7}c^{7}+924\,ab^{6}c^{6}d+462\,a^{2}b^{5}c^{5}d^{2}+210\,a^{3}b^{4}c^{4}d^{3}+84\,a^{4}b^{3}c^{3}d^{4}+28\,a^{5}b^{2}c^{2}d^{5}+7\,a^{6}bcd^{6}+a^{7}d^{7}+3003\,(7\,b^{7}cd^{6}+ab^{6}d^{7})x^{6}+2002\,(28\,b^{7}c^{2}d^{5}+7\,ab^{6}cd^{6}+a^{2}b^{5}d^{7})x^{5}+1001\,(84\,b^{7}c^{3}d^{4}+28\,ab^{6}c^{2}d^{5}+7\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7})x^{4}+364\,(210\,b^{7}c^{4}d^{3}+84\,ab^{6}c^{3}d^{4}+28\,a^{2}b^{5}c^{2}d^{5}+7\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7})x^{3}+91\,(462\,b^{7}c^{5}d^{2}+210\,ab^{6}c^{4}d^{3}+84\,a^{2}b^{5}c^{3}d^{4}+28\,a^{3}b^{4}c^{2}d^{5}+7\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7})x^{2}+14\,(924\,b^{7}c^{6}d+462\,ab^{6}c^{5}d^{2}+210\,a^{2}b^{5}c^{4}d^{3}+84\,a^{3}b^{4}c^{3}d^{4}+28\,a^{4}b^{3}c^{2}d^{5}+7\,a^{5}b^{2}cd^{6}+a^{6}bd^{7})x}{24024\,(b^{22}x^{14}+14\,ab^{21}x^{13}+91\,a^{2}b^{20}x^{12}+364\,a^{3}b^{19}x^{11}+1001\,a^{4}b^{18}x^{10}+2002\,a^{5}b^{17}x^{9}+3003\,a^{6}b^{16}x^{8}+3432\,a^{7}b^{15}x^{7}+3003\,a^{8}b^{14}x^{6}+2002\,a^{9}b^{13}x^{5}+1001\,a^{10}b^{12}x^{4}+364\,a^{11}b^{11}x^{3}+91\,a^{12}b^{10}x^{2}+14\,a^{13}b^{9}x+a^{14}b^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
```

```
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**15,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(184) = 368.
time = 0.82, size = 496, normalized size = 2.48

$$-\frac{3432\,b^{7}d^{7}x^{7}+21021\,b^{7}cd^{6}x^{6}+3003\,ab^{6}d^{7}x^{6}+56056\,b^{7}c^{2}d^{5}x^{5}+14014\,ab^{6}cd^{6}x^{5}+2002\,a^{2}b^{5}d^{7}x^{5}+84084\,b^{7}c^{3}d^{4}x^{4}+28028\,ab^{6}c^{2}d^{5}x^{4}+7007\,a^{2}b^{5}cd^{6}x^{4}+1001\,a^{3}b^{4}d^{7}x^{4}+76440\,b^{7}c^{4}d^{3}x^{3}+30576\,ab^{6}c^{3}d^{4}x^{3}+10192\,a^{2}b^{5}c^{2}d^{5}x^{3}+2548\,a^{3}b^{4}cd^{6}x^{3}+364\,a^{4}b^{3}d^{7}x^{3}+42042\,b^{7}c^{5}d^{2}x^{2}+19110\,ab^{6}c^{4}d^{3}x^{2}+7644\,a^{2}b^{5}c^{3}d^{4}x^{2}+2548\,a^{3}b^{4}c^{2}d^{5}x^{2}+637\,a^{4}b^{3}cd^{6}x^{2}+91\,a^{5}b^{2}d^{7}x^{2}+12936\,b^{7}c^{6}dx+6468\,ab^{6}c^{5}d^{2}x+2940\,a^{2}b^{5}c^{4}d^{3}x+1176\,a^{3}b^{4}c^{3}d^{4}x+392\,a^{4}b^{3}c^{2}d^{5}x+98\,a^{5}b^{2}cd^{6}x+14\,a^{6}bd^{7}x+1716\,b^{7}c^{7}+924\,ab^{6}c^{6}d+462\,a^{2}b^{5}c^{5}d^{2}+210\,a^{3}b^{4}c^{4}d^{3}+84\,a^{4}b^{3}c^{3}d^{4}+28\,a^{5}b^{2}c^{2}d^{5}+7\,a^{6}bcd^{6}+a^{7}d^{7}}{24024\,(bx+a)^{14}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="giac")
[Out] -1/24024*(3432*b^7*d^7*x^7 + 21021*b^7*c*d^6*x^6 + 3003*a*b^6*d^7*x^6 + 560
56*b^7*c^2*d^5*x^5 + 14014*a*b^6*c*d^6*x^5 + 2002*a^2*b^5*d^7*x^5 + 84084*b
^7*c^3*d^4*x^4 + 28028*a*b^6*c^2*d^5*x^4 + 7007*a^2*b^5*c*d^6*x^4 + 1001*a^
3*b^4*d^7*x^4 + 76440*b^7*c^4*d^3*x^3 + 30576*a*b^6*c^3*d^4*x^3 + 10192*a^2
*b^5*c^2*d^5*x^3 + 2548*a^3*b^4*c*d^6*x^3 + 364*a^4*b^3*d^7*x^3 + 42042*b^7
*c^5*d^2*x^2 + 19110*a*b^6*c^4*d^3*x^2 + 7644*a^2*b^5*c^3*d^4*x^2 + 2548*a^
3*b^4*c^2*d^5*x^2 + 637*a^4*b^3*c*d^6*x^2 + 91*a^5*b^2*d^7*x^2 + 12936*b^7*
c^6*d*x + 6468*a*b^6*c^5*d^2*x + 2940*a^2*b^5*c^4*d^3*x + 1176*a^3*b^4*c^3*
d^4*x + 392*a^4*b^3*c^2*d^5*x + 98*a^5*b^2*c*d^6*x + 14*a^6*b*d^7*x + 1716*
b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^
14*b^8)
```

Mupad [B]
time = 1.24, size = 581, normalized size = 2.90

$$-\frac{\frac{a^7\,d^7+7\,a^6\,b\,c\,d^6+28\,a^5\,b^2\,c^2\,d^5+84\,a^4\,b^3\,c^3\,d^4+210\,a^3\,b^4\,c^4\,d^3+462\,a^2\,b^5\,c^5\,d^2+924\,a\,b^6\,c^6\,d+1716\,b^7\,c^7}{24024\,b^8}+\frac{d^7\,x^7}{7\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+7\,a^4\,b\,c\,d^4+28\,a^3\,b^2\,c^2\,d^3+84\,a^2\,b^3\,c^3\,d^2+210\,a\,b^4\,c^4\,d+462\,b^5\,c^5\right)}{264\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+7\,a^2\,b\,c\,d^2+28\,a\,b^2\,c^2\,d+84\,b^3\,c^3\right)}{24\,b^4}+\frac{d^6\,x^6\,\left(a\,d+7\,b\,c\right)}{8\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+7\,a^3\,b\,c\,d^3+28\,a^2\,b^2\,c^2\,d^2+84\,a\,b^3\,c^3\,d+210\,b^4\,c^4\right)}{66\,b^5}+\frac{d\,x\,\left(a^6\,d^6+7\,a^5\,b\,c\,d^5+28\,a^4\,b^2\,c^2\,d^4+84\,a^3\,b^3\,c^3\,d^3+210\,a^2\,b^4\,c^4\,d^2+462\,a\,b^5\,c^5\,d+924\,b^6\,c^6\right)}{1716\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+7\,a\,b\,c\,d+28\,b^2\,c^2\right)}{12\,b^3}}{a^{14}+14\,a^{13}\,b\,x+91\,a^{12}\,b^2\,x^2+364\,a^{11}\,b^3\,x^3+1001\,a^{10}\,b^4\,x^4+2002\,a^9\,b^5\,x^5+3003\,a^8\,b^6\,x^6+3432\,a^7\,b^7\,x^7+3003\,a^6\,b^8\,x^8+2002\,a^5\,b^9\,x^9+1001\,a^4\,b^{10}\,x^{10}+364\,a^3\,b^{11}\,x^{11}+91\,a^2\,b^{12}\,x^{12}+14\,a\,b^{13}\,x^{13}+b^{14}\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^15,x)
[Out] -((a^7*d^7 + 1716*b^7*c^7 + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 924*a*b^6*c^6*d + 7*a^6*b*c*d^6)/(24
024*b^8) + (d^7*x^7)/(7*b) + (d^2*x^2*(a^5*d^5 + 462*b^5*c^5 + 84*a^2*b^3*c
```

```
^3*d^2 + 28*a^3*b^2*c^2*d^3 + 210*a*b^4*c^4*d + 7*a^4*b*c*d^4))/(264*b^6) +
 (d^4*x^4*(a^3*d^3 + 84*b^3*c^3 + 28*a*b^2*c^2*d + 7*a^2*b*c*d^2))/(24*b^4)
 + (d^6*x^6*(a*d + 7*b*c))/(8*b^2) + (d^3*x^3*(a^4*d^4 + 210*b^4*c^4 + 28*a
^2*b^2*c^2*d^2 + 84*a*b^3*c^3*d + 7*a^3*b*c*d^3))/(66*b^5) + (d*x*(a^6*d^6
+ 924*b^6*c^6 + 210*a^2*b^4*c^4*d^2 + 84*a^3*b^3*c^3*d^3 + 28*a^4*b^2*c^2*d
^4 + 462*a*b^5*c^5*d + 7*a^5*b*c*d^5))/(1716*b^7) + (d^5*x^5*(a^2*d^2 + 28*
b^2*c^2 + 7*a*b*c*d))/(12*b^3))/(a^14 + b^14*x^14 + 14*a*b^13*x^13 + 91*a^1
2*b^2*x^2 + 364*a^11*b^3*x^3 + 1001*a^10*b^4*x^4 + 2002*a^9*b^5*x^5 + 3003*
a^8*b^6*x^6 + 3432*a^7*b^7*x^7 + 3003*a^6*b^8*x^8 + 2002*a^5*b^9*x^9 + 1001
*a^4*b^10*x^10 + 364*a^3*b^11*x^11 + 91*a^2*b^12*x^12 + 14*a^13*b*x)
```

3.1298 $\int \frac{(c+dx)^7}{(a+bx)^{16}} dx$

Optimal. Leaf size=200

$$-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{7d^6(\cdots}{9b^8(\cdots}$$

```
[Out] -1/15*(-a*d+b*c)^7/b^8/(b*x+a)^15-1/2*d*(-a*d+b*c)^6/b^8/(b*x+a)^14-21/13*d
^2*(-a*d+b*c)^5/b^8/(b*x+a)^13-35/12*d^3*(-a*d+b*c)^4/b^8/(b*x+a)^12-35/11*
d^4*(-a*d+b*c)^3/b^8/(b*x+a)^11-21/10*d^5*(-a*d+b*c)^2/b^8/(b*x+a)^10-7/9*d
^6*(-a*d+b*c)/b^8/(b*x+a)^9-1/8*d^7/b^8/(b*x+a)^8
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d^7}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -1/15*(b*c - a*d)^7/(b^8*(a + b*x)^15) - (d*(b*c - a*d)^6)/(2*b^8*(a + b*x)
^14) - (21*d^2*(b*c - a*d)^5)/(13*b^8*(a + b*x)^13) - (35*d^3*(b*c - a*d)^4
)/(12*b^8*(a + b*x)^12) - (35*d^4*(b*c - a*d)^3)/(11*b^8*(a + b*x)^11) - (2
1*d^5*(b*c - a*d)^2)/(10*b^8*(a + b*x)^10) - (7*d^6*(b*c - a*d))/(9*b^8*(a
+ b*x)^9) - d^7/(8*b^8*(a + b*x)^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{16}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{15}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{14}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{13}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{12}}\cdots\right.\\ &= -\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}\cdots\end{aligned}$$

Mathematica [A]
time = 0.09, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(8c+15dx)+3a^5b^2d^5(12c^2+40cdx+35d^2x^2)+5a^4b^3d^4(24c^3+108c^2dx+168cd^2x^2+91d^3x^3)+5a^3b^4d^3(66c^4+360c^3dx+756c^2d^2x^2+728cd^3x^3+273d^4x^4)+3a^2b^5d^2(264c^5+1650c^4dx+4200c^3d^2x^2+5460c^2d^3x^3+3640cd^4x^4+1001d^5x^5)+ab^6d(1716c^6+11880c^5dx+34650c^4d^2x^2+54600c^3d^3x^3+49140c^2d^4x^4+24024cd^5x^5+5005d^6x^6)+b^7(3432c^7+25740c^6dx+83160c^5d^2x^2+150150c^4d^3x^3+163800c^3d^4x^4+108108c^2d^5x^5+40040cd^6x^6+6435d^7x^7)}{51480b^8(a+bx)^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -1/51480*(a^7*d^7 + a^6*b*d^6*(8*c + 15*d*x) + 3*a^5*b^2*d^5*(12*c^2 + 40*c
*d*x + 35*d^2*x^2) + 5*a^4*b^3*d^4*(24*c^3 + 108*c^2*d*x + 168*c*d^2*x^2 +
91*d^3*x^3) + 5*a^3*b^4*d^3*(66*c^4 + 360*c^3*d*x + 756*c^2*d^2*x^2 + 728*c
*d^3*x^3 + 273*d^4*x^4) + 3*a^2*b^5*d^2*(264*c^5 + 1650*c^4*d*x + 4200*c^3*
d^2*x^2 + 5460*c^2*d^3*x^3 + 3640*c*d^4*x^4 + 1001*d^5*x^5) + a*b^6*d*(1716
*c^6 + 11880*c^5*d*x + 34650*c^4*d^2*x^2 + 54600*c^3*d^3*x^3 + 49140*c^2*d^
4*x^4 + 24024*c*d^5*x^5 + 5005*d^6*x^6) + b^7*(3432*c^7 + 25740*c^6*d*x + 8
3160*c^5*d^2*x^2 + 150150*c^4*d^3*x^3 + 163800*c^3*d^4*x^4 + 108108*c^2*d^5
*x^5 + 40040*c*d^6*x^6 + 6435*d^7*x^7))/(b^8*(a + b*x)^15)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(184) = 368$.
time = 0.14, size = 464, normalized size = 2.32

method	result
risch	$-\frac{a^7d^7+8a^6bc\,d^6+36a^5b^2c^2d^5+120a^4b^3c^3d^4+330a^3b^4c^4d^3+792a^2b^5c^5d^2+1716a\,b^6c^6d+3432b^7c^7}{51480b^8}-\frac{d\left(a^6d^6+8a^5bc\,d^5+36a^4b^2c^2d^4+120a^3b^3c^3d^3\right.}{3432b}$
default	$\frac{21d^2\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{13b^8(bx+a)^{13}}-\frac{35d^3\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{12b^8(bx+a)^{12}}+\frac{7d^6(ad-b}{9b^8(bx+a}$
norman	$\frac{-a^7b^7d^7-8a^6b^8c\,d^6-36a^5b^9c^2d^5-120a^4b^{10}c^3d^4-330a^3b^{11}c^4d^3-792a^2b^{12}c^5d^2-1716a\,b^{13}c^6d-3432b^{14}c^7}{51480b^{15}}+\frac{\left(-a^6b^7d^7-8a^5b^8c\,d^6-36a^4b^9c^2d^5-\right.}{}$
gosper	$-\frac{6435d^7x^7b^7+5005a\,b^6d^7x^6+40040b^7c\,d^6x^6+3003a^2b^5d^7x^5+24024a\,b^6c\,d^6x^5+108108b^7c^2d^5x^5+1365a^3b^4d^7x^4+10920a^2b^5c\,d^6x^4+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^16,x,method=_RETURNVERBOSE)
```

```
[Out] 21/13/b^8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+
5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^13-35/12/b^8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^
2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^12+7/9/b^8*d^6*(a*d-b*c)/(b*x+
a)^9-1/8*d^7/b^8/(b*x+a)^8-1/2/b^8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*
d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^14
-21/10/b^8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^10-1/15*(-a^7*d^7+7*a^6*
b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5
*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^15+35/11/b^8*d^4*(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^11
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(184) = 368$.

time = 0.32, size = 614, normalized size = 3.07

$$-\frac{6435\,b^7d^7x^7+3432\,b^7c^7+1716\,ab^6c^6d+792\,a^2b^5c^5d^2+330\,a^3b^4c^4d^3+120\,a^4b^3c^3d^4+36\,a^5b^2c^2d^5+8\,a^6bcd^6+a^7d^7+5005\,(8\,b^7cd^6+ab^6d^7)x^6+3003\,(36\,b^7c^2d^5+8\,ab^6cd^6+a^2b^5d^7)x^5+1365\,(120\,b^7c^3d^4+36\,ab^6c^2d^5+8\,a^2b^5cd^6+a^3b^4d^7)x^4+455\,(330\,b^7c^4d^3+120\,ab^6c^3d^4+36\,a^2b^5c^2d^5+8\,a^3b^4cd^6+a^4b^3d^7)x^3+105\,(792\,b^7c^5d^2+330\,ab^6c^4d^3+120\,a^2b^5c^3d^4+36\,a^3b^4c^2d^5+8\,a^4b^3cd^6+a^5b^2d^7)x^2+15\,(1716\,b^7c^6d+792\,ab^6c^5d^2+330\,a^2b^5c^4d^3+120\,a^3b^4c^3d^4+36\,a^4b^3c^2d^5+8\,a^5b^2cd^6+a^6bd^7)x}{51480\,(b^{23}x^{15}+15\,ab^{22}x^{14}+105\,a^2b^{21}x^{13}+455\,a^3b^{20}x^{12}+1365\,a^4b^{19}x^{11}+3003\,a^5b^{18}x^{10}+5005\,a^6b^{17}x^9+6435\,a^7b^{16}x^8+6435\,a^8b^{15}x^7+5005\,a^9b^{14}x^6+3003\,a^{10}b^{13}x^5+1365\,a^{11}b^{12}x^4+455\,a^{12}b^{11}x^3+105\,a^{13}b^{10}x^2+15\,a^{14}b^9x+a^{15}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(184) = 368$.

time = 0.45, size = 614, normalized size = 3.07

$$-\frac{6435\,b^7d^7x^7+3432\,b^7c^7+1716\,ab^6c^6d+792\,a^2b^5c^5d^2+330\,a^3b^4c^4d^3+120\,a^4b^3c^3d^4+36\,a^5b^2c^2d^5+8\,a^6bcd^6+a^7d^7+5005\,(8\,b^7cd^6+ab^6d^7)x^6+3003\,(36\,b^7c^2d^5+8\,ab^6cd^6+a^2b^5d^7)x^5+1365\,(120\,b^7c^3d^4+36\,ab^6c^2d^5+8\,a^2b^5cd^6+a^3b^4d^7)x^4+455\,(330\,b^7c^4d^3+120\,ab^6c^3d^4+36\,a^2b^5c^2d^5+8\,a^3b^4cd^6+a^4b^3d^7)x^3+105\,(792\,b^7c^5d^2+330\,ab^6c^4d^3+120\,a^2b^5c^3d^4+36\,a^3b^4c^2d^5+8\,a^4b^3cd^6+a^5b^2d^7)x^2+15\,(1716\,b^7c^6d+792\,ab^6c^5d^2+330\,a^2b^5c^4d^3+120\,a^3b^4c^3d^4+36\,a^4b^3c^2d^5+8\,a^5b^2cd^6+a^6bd^7)x}{51480\,(b^{23}x^{15}+15\,ab^{22}x^{14}+105\,a^2b^{21}x^{13}+455\,a^3b^{20}x^{12}+1365\,a^4b^{19}x^{11}+3003\,a^5b^{18}x^{10}+5005\,a^6b^{17}x^9+6435\,a^7b^{16}x^8+6435\,a^8b^{15}x^7+5005\,a^9b^{14}x^6+3003\,a^{10}b^{13}x^5+1365\,a^{11}b^{12}x^4+455\,a^{12}b^{11}x^3+105\,a^{13}b^{10}x^2+15\,a^{14}b^9x+a^{15}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
```

```
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**16,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(184) = 368$.
time = 1.06, size = 496, normalized size = 2.48

$$-\frac{6435 b^{7} d^{7} x^{7} + 40040 b^{7} c d^{6} x^{6} + 5005 a b^{6} d^{7} x^{6} + 108108 b^{7} c^{2} d^{5} x^{5} + 24024 a b^{6} c d^{6} x^{5} + 3003 a^{2} b^{5} d^{7} x^{5} + 163800 b^{7} c^{3} d^{4} x^{4} + 49140 a b^{6} c^{2} d^{5} x^{4} + 10920 a^{2} b^{5} c d^{6} x^{4} + 1365 a^{3} b^{4} d^{7} x^{4} + 150150 b^{7} c^{4} d^{3} x^{3} + 54600 a b^{6} c^{3} d^{4} x^{3} + 16380 a^{2} b^{5} c^{2} d^{5} x^{3} + 3640 a^{3} b^{4} c d^{6} x^{3} + 455 a^{4} b^{3} d^{7} x^{3} + 83160 b^{7} c^{5} d^{2} x^{2} + 34650 a b^{6} c^{4} d^{3} x^{2} + 12600 a^{2} b^{5} c^{3} d^{4} x^{2} + 3780 a^{3} b^{4} c^{2} d^{5} x^{2} + 840 a^{4} b^{3} c d^{6} x^{2} + 105 a^{5} b^{2} d^{7} x^{2} + 25740 b^{7} c^{6} d x + 11880 a b^{6} c^{5} d^{2} x + 4950 a^{2} b^{5} c^{4} d^{3} x + 1800 a^{3} b^{4} c^{3} d^{4} x + 540 a^{4} b^{3} c^{2} d^{5} x + 120 a^{5} b^{2} c d^{6} x + 15 a^{6} b d^{7} x + 3432 b^{7} c^{7} + 1716 a b^{6} c^{6} d + 792 a^{2} b^{5} c^{5} d^{2} + 330 a^{3} b^{4} c^{4} d^{3} + 120 a^{4} b^{3} c^{3} d^{4} + 36 a^{5} b^{2} c^{2} d^{5} + 8 a^{6} b c d^{6} + a^{7} d^{7}}{51480 \, {\left(b x + a\right)}^{15} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 40040*b^7*c*d^6*x^6 + 5005*a*b^6*d^7*x^6 + 108
108*b^7*c^2*d^5*x^5 + 24024*a*b^6*c*d^6*x^5 + 3003*a^2*b^5*d^7*x^5 + 163800
*b^7*c^3*d^4*x^4 + 49140*a*b^6*c^2*d^5*x^4 + 10920*a^2*b^5*c*d^6*x^4 + 1365
*a^3*b^4*d^7*x^4 + 150150*b^7*c^4*d^3*x^3 + 54600*a*b^6*c^3*d^4*x^3 + 16380
*a^2*b^5*c^2*d^5*x^3 + 3640*a^3*b^4*c*d^6*x^3 + 455*a^4*b^3*d^7*x^3 + 83160
*b^7*c^5*d^2*x^2 + 34650*a*b^6*c^4*d^3*x^2 + 12600*a^2*b^5*c^3*d^4*x^2 + 37
80*a^3*b^4*c^2*d^5*x^2 + 840*a^4*b^3*c*d^6*x^2 + 105*a^5*b^2*d^7*x^2 + 2574
0*b^7*c^6*d*x + 11880*a*b^6*c^5*d^2*x + 4950*a^2*b^5*c^4*d^3*x + 1800*a^3*b
^4*c^3*d^4*x + 540*a^4*b^3*c^2*d^5*x + 120*a^5*b^2*c*d^6*x + 15*a^6*b*d^7*x
 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*
d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7)/(
(b*x + a)^15*b^8)
```

Mupad [B]
time = 2.20, size = 592, normalized size = 2.96

$$-\frac{\frac{a^7\,d^7+8\,a^6\,b\,c\,d^6+36\,a^5\,b^2\,c^2\,d^5+120\,a^4\,b^3\,c^3\,d^4+330\,a^3\,b^4\,c^4\,d^3+792\,a^2\,b^5\,c^5\,d^2+1716\,a\,b^6\,c^6\,d+3432\,b^7\,c^7}{51480\,b^8}+\frac{d^7\,x^7}{8\,b}+\frac{7\,d^2\,x^2\,\left(a^5\,d^5+8\,a^4\,b\,c\,d^4+36\,a^3\,b^2\,c^2\,d^3+120\,a^2\,b^3\,c^3\,d^2+330\,a\,b^4\,c^4\,d+792\,b^5\,c^5\right)}{3432\,b^6}+\frac{7\,d^4\,x^4\,\left(a^3\,d^3+8\,a^2\,b\,c\,d^2+36\,a\,b^2\,c^2\,d+120\,b^3\,c^3\right)}{264\,b^4}+\frac{7\,d^6\,x^6\,\left(a\,d+8\,b\,c\right)}{72\,b^2}+\frac{7\,d^3\,x^3\,\left(a^4\,d^4+8\,a^3\,b\,c\,d^3+36\,a^2\,b^2\,c^2\,d^2+120\,a\,b^3\,c^3\,d+330\,b^4\,c^4\right)}{792\,b^5}+\frac{d\,x\,\left(a^6\,d^6+8\,a^5\,b\,c\,d^5+36\,a^4\,b^2\,c^2\,d^4+120\,a^3\,b^3\,c^3\,d^3+330\,a^2\,b^4\,c^4\,d^2+792\,a\,b^5\,c^5\,d+1716\,b^6\,c^6\right)}{3432\,b^7}+\frac{7\,d^5\,x^5\,\left(a^2\,d^2+8\,a\,b\,c\,d+36\,b^2\,c^2\right)}{120\,b^3}}{a^{15}+15\,a^{14}\,b\,x+105\,a^{13}\,b^2\,x^2+455\,a^{12}\,b^3\,x^3+1365\,a^{11}\,b^4\,x^4+3003\,a^{10}\,b^5\,x^5+5005\,a^9\,b^6\,x^6+6435\,a^8\,b^7\,x^7+6435\,a^7\,b^8\,x^8+5005\,a^6\,b^9\,x^9+3003\,a^5\,b^{10}\,x^{10}+1365\,a^4\,b^{11}\,x^{11}+455\,a^3\,b^{12}\,x^{12}+105\,a^2\,b^{13}\,x^{13}+15\,a\,b^{14}\,x^{14}+b^{15}\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^16,x)
```

```
[Out] -((a^7*d^7 + 3432*b^7*c^7 + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120
*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 1716*a*b^6*c^6*d + 8*a^6*b*c*d^6)/(
51480*b^8) + (d^7*x^7)/(8*b) + (7*d^2*x^2*(a^5*d^5 + 792*b^5*c^5 + 120*a^2*
```

b^3*c^3*d^2 + 36*a^3*b^2*c^2*d^3 + 330*a*b^4*c^4*d + 8*a^4*b*c*d^4))/(3432*
b^6) + (7*d^4*x^4*(a^3*d^3 + 120*b^3*c^3 + 36*a*b^2*c^2*d + 8*a^2*b*c*d^2))
/(264*b^4) + (7*d^6*x^6*(a*d + 8*b*c))/(72*b^2) + (7*d^3*x^3*(a^4*d^4 + 330
*b^4*c^4 + 36*a^2*b^2*c^2*d^2 + 120*a*b^3*c^3*d + 8*a^3*b*c*d^3))/(792*b^5)
 + (d*x*(a^6*d^6 + 1716*b^6*c^6 + 330*a^2*b^4*c^4*d^2 + 120*a^3*b^3*c^3*d^3
 + 36*a^4*b^2*c^2*d^4 + 792*a*b^5*c^5*d + 8*a^5*b*c*d^5))/(3432*b^7) + (7*d
^5*x^5*(a^2*d^2 + 36*b^2*c^2 + 8*a*b*c*d))/(120*b^3))/(a^15 + b^15*x^15 + 1
5*a*b^14*x^14 + 105*a^13*b^2*x^2 + 455*a^12*b^3*x^3 + 1365*a^11*b^4*x^4 + 3
003*a^10*b^5*x^5 + 5005*a^9*b^6*x^6 + 6435*a^8*b^7*x^7 + 6435*a^7*b^8*x^8 +
 5005*a^6*b^9*x^9 + 3003*a^5*b^10*x^10 + 1365*a^4*b^11*x^11 + 455*a^3*b^12*
x^12 + 105*a^2*b^13*x^13 + 15*a^14*b*x)

3.1299 $\int (a+bx)^{12}(c+dx)^{10}\,dx$

Optimal. Leaf size=275

$$\frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\frac{15d^3(bc-ad)^7(a+bx)^{16}}{2b^{11}}+\frac{210d^4(\cdots}{}$$

```
[Out] 1/13*(-a*d+b*c)^10*(b*x+a)^13/b^11+5/7*d*(-a*d+b*c)^9*(b*x+a)^14/b^11+3*d^2
*(-a*d+b*c)^8*(b*x+a)^15/b^11+15/2*d^3*(-a*d+b*c)^7*(b*x+a)^16/b^11+210/17*
d^4*(-a*d+b*c)^6*(b*x+a)^17/b^11+14*d^5*(-a*d+b*c)^5*(b*x+a)^18/b^11+210/19
*d^6*(-a*d+b*c)^4*(b*x+a)^19/b^11+6*d^7*(-a*d+b*c)^3*(b*x+a)^20/b^11+15/7*d
^8*(-a*d+b*c)^2*(b*x+a)^21/b^11+5/11*d^9*(-a*d+b*c)*(b*x+a)^22/b^11+1/23*d^
10*(b*x+a)^23/b^11
```

Rubi [A]

time = 1.01, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+bx)^{18}(bc-ad)^5}{b^{11}}+\frac{210d^4(a+bx)^{17}(bc-ad)^6}{17b^{11}}+\frac{15d^3(a+bx)^{16}(bc-ad)^7}{2b^{11}}+\frac{3d^2(a+bx)^{15}(bc-ad)^8}{b^{11}}+\frac{5d(a+bx)^{14}(bc-ad)^9}{7b^{11}}+\frac{(a+bx)^{13}(bc-ad)^{10}}{13b^{11}}+\frac{d^{10}(a+bx)^{23}}{23b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^13)/(13*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^14)/
(7*b^11) + (3*d^2*(b*c - a*d)^8*(a + b*x)^15)/b^11 + (15*d^3*(b*c - a*d)^7*
(a + b*x)^16)/(2*b^11) + (210*d^4*(b*c - a*d)^6*(a + b*x)^17)/(17*b^11) + (
14*d^5*(b*c - a*d)^5*(a + b*x)^18)/b^11 + (210*d^6*(b*c - a*d)^4*(a + b*x)^
19)/(19*b^11) + (6*d^7*(b*c - a*d)^3*(a + b*x)^20)/b^11 + (15*d^8*(b*c - a*
d)^2*(a + b*x)^21)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^22)/(11*b^11) +
(d^10*(a + b*x)^23)/(23*b^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{12}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{12}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{\cdots}}{b^{10}}\cdots\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\frac{15\cdots}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1817 vs. $2(275) = 550$.
time = 0.19, size = 1817, normalized size = 6.61

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] a^12*c^10*x + a^11*c^9*(6*b*c + 5*a*d)*x^2 + a^10*c^8*(22*b^2*c^2 + 40*a*b*
c*d + 15*a^2*d^2)*x^3 + 5*a^9*c^7*(11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 6*a^3*d^3)*x^4 + a^8*c^6*(99*b^4*c^4 + 440*a*b^3*c^3*d + 594*a^2*b^2
*c^2*d^2 + 288*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + 3*a^7*c^5*(44*b^5*c^5 + 275*
a*b^4*c^4*d + 550*a^2*b^3*c^3*d^2 + 440*a^3*b^2*c^2*d^3 + 140*a^4*b*c*d^4 +
 14*a^5*d^5)*x^6 + (3*a^6*c^4*(308*b^6*c^6 + 2640*a*b^5*c^5*d + 7425*a^2*b^
4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 4620*a^4*b^2*c^2*d^4 + 1008*a^5*b*c*d^5
+ 70*a^6*d^6)*x^7)/7 + 3*a^5*c^3*(33*b^7*c^7 + 385*a*b^6*c^6*d + 1485*a^2*b
^5*c^5*d^2 + 2475*a^3*b^4*c^4*d^3 + 1925*a^4*b^3*c^3*d^4 + 693*a^5*b^2*c^2*
d^5 + 105*a^6*b*c*d^6 + 5*a^7*d^7)*x^8 + 5*a^4*c^2*(11*b^8*c^8 + 176*a*b^7*
c^7*d + 924*a^2*b^6*c^6*d^2 + 2112*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 +
 1232*a^5*b^3*c^3*d^5 + 308*a^6*b^2*c^2*d^6 + 32*a^7*b*c*d^7 + a^8*d^8)*x^9
 + a^3*c*(22*b^9*c^9 + 495*a*b^8*c^8*d + 3564*a^2*b^7*c^7*d^2 + 11088*a^3*b
^6*c^6*d^3 + 16632*a^4*b^5*c^5*d^4 + 12474*a^5*b^4*c^4*d^5 + 4620*a^6*b^3*c
^3*d^6 + 792*a^7*b^2*c^2*d^7 + 54*a^8*b*c*d^8 + a^9*d^9)*x^10 + (a^2*(66*b^
10*c^10 + 2200*a*b^9*c^9*d + 22275*a^2*b^8*c^8*d^2 + 95040*a^3*b^7*c^7*d^3
+ 194040*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5 + 103950*a^6*b^4*c^4*d^6
+ 26400*a^7*b^3*c^3*d^7 + 2970*a^8*b^2*c^2*d^8 + 120*a^9*b*c*d^9 + a^10*d^1
0)*x^11)/11 + a*b*(b^10*c^10 + 55*a*b^9*c^9*d + 825*a^2*b^8*c^8*d^2 + 4950*
a^3*b^7*c^7*d^3 + 13860*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 + 13860*a^6
*b^4*c^4*d^6 + 4950*a^7*b^3*c^3*d^7 + 825*a^8*b^2*c^2*d^8 + 55*a^9*b*c*d^9
+ a^10*d^10)*x^12 + (b^2*(b^10*c^10 + 120*a*b^9*c^9*d + 2970*a^2*b^8*c^8*d^
2 + 26400*a^3*b^7*c^7*d^3 + 103950*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5
 + 194040*a^6*b^4*c^4*d^6 + 95040*a^7*b^3*c^3*d^7 + 22275*a^8*b^2*c^2*d^8 +
 2200*a^9*b*c*d^9 + 66*a^10*d^10)*x^13)/13 + (5*b^3*d*(b^9*c^9 + 54*a*b^8*c
^8*d + 792*a^2*b^7*c^7*d^2 + 4620*a^3*b^6*c^6*d^3 + 12474*a^4*b^5*c^5*d^4 +
 16632*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 3564*a^7*b^2*c^2*d^7 + 495
*a^8*b*c*d^8 + 22*a^9*d^9)*x^14)/7 + 3*b^4*d^2*(b^8*c^8 + 32*a*b^7*c^7*d +
308*a^2*b^6*c^6*d^2 + 1232*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 + 2112*a^
5*b^3*c^3*d^5 + 924*a^6*b^2*c^2*d^6 + 176*a^7*b*c*d^7 + 11*a^8*d^8)*x^15 +
(3*b^5*d^3*(5*b^7*c^7 + 105*a*b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1925*a^3*b^
4*c^4*d^3 + 2475*a^4*b^3*c^3*d^4 + 1485*a^5*b^2*c^2*d^5 + 385*a^6*b*c*d^6 +
 33*a^7*d^7)*x^16)/2 + (3*b^6*d^4*(70*b^6*c^6 + 1008*a*b^5*c^5*d + 4620*a^2
*b^4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 7425*a^4*b^2*c^2*d^4 + 2640*a^5*b*c*d
^5 + 308*a^6*d^6)*x^17)/17 + b^7*d^5*(14*b^5*c^5 + 140*a*b^4*c^4*d + 440*a^
2*b^3*c^3*d^2 + 550*a^3*b^2*c^2*d^3 + 275*a^4*b*c*d^4 + 44*a^5*d^5)*x^18 +
```

```
(5*b^8*d^6*(42*b^4*c^4 + 288*a*b^3*c^3*d + 594*a^2*b^2*c^2*d^2 + 440*a^3*b*
c*d^3 + 99*a^4*d^4)*x^19)/19 + b^9*d^7*(6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2
*b*c*d^2 + 11*a^3*d^3)*x^20 + (b^10*d^8*(15*b^2*c^2 + 40*a*b*c*d + 22*a^2*d
^2)*x^21)/7 + (b^11*d^9*(5*b*c + 6*a*d)*x^22)/11 + (b^12*d^10*x^23)/23
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1890 vs. 2(259) = 518.
time = 0.14, size = 1891, normalized size = 6.88

method	result	size
norman	Expression too large to display	1869
default	Expression too large to display	1891
gosper	Expression too large to display	2187
risch	Expression too large to display	2187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/23*b^12*d^10*x^23+1/22*(12*a*b^11*d^10+10*b^12*c*d^9)*x^22+1/21*(66*a^2*b
^10*d^10+120*a*b^11*c*d^9+45*b^12*c^2*d^8)*x^21+1/20*(220*a^3*b^9*d^10+660*
a^2*b^10*c*d^9+540*a*b^11*c^2*d^8+120*b^12*c^3*d^7)*x^20+1/19*(495*a^4*b^8*
d^10+2200*a^3*b^9*c*d^9+2970*a^2*b^10*c^2*d^8+1440*a*b^11*c^3*d^7+210*b^12*
c^4*d^6)*x^19+1/18*(792*a^5*b^7*d^10+4950*a^4*b^8*c*d^9+9900*a^3*b^9*c^2*d^
8+7920*a^2*b^10*c^3*d^7+2520*a*b^11*c^4*d^6+252*b^12*c^5*d^5)*x^18+1/17*(92
4*a^6*b^6*d^10+7920*a^5*b^7*c*d^9+22275*a^4*b^8*c^2*d^8+26400*a^3*b^9*c^3*d
^7+13860*a^2*b^10*c^4*d^6+3024*a*b^11*c^5*d^5+210*b^12*c^6*d^4)*x^17+1/16*(
792*a^7*b^5*d^10+9240*a^6*b^6*c*d^9+35640*a^5*b^7*c^2*d^8+59400*a^4*b^8*c^3
*d^7+46200*a^3*b^9*c^4*d^6+16632*a^2*b^10*c^5*d^5+2520*a*b^11*c^6*d^4+120*b
^12*c^7*d^3)*x^16+1/15*(495*a^8*b^4*d^10+7920*a^7*b^5*c*d^9+41580*a^6*b^6*c
^2*d^8+95040*a^5*b^7*c^3*d^7+103950*a^4*b^8*c^4*d^6+55440*a^3*b^9*c^5*d^5+1
3860*a^2*b^10*c^6*d^4+1440*a*b^11*c^7*d^3+45*b^12*c^8*d^2)*x^15+1/14*(220*a
^9*b^3*d^10+4950*a^8*b^4*c*d^9+35640*a^7*b^5*c^2*d^8+110880*a^6*b^6*c^3*d^7
+166320*a^5*b^7*c^4*d^6+124740*a^4*b^8*c^5*d^5+46200*a^3*b^9*c^6*d^4+7920*a
^2*b^10*c^7*d^3+540*a*b^11*c^8*d^2+10*b^12*c^9*d)*x^14+1/13*(66*a^10*b^2*d^
10+2200*a^9*b^3*c*d^9+22275*a^8*b^4*c^2*d^8+95040*a^7*b^5*c^3*d^7+194040*a^
6*b^6*c^4*d^6+199584*a^5*b^7*c^5*d^5+103950*a^4*b^8*c^6*d^4+26400*a^3*b^9*c
^7*d^3+2970*a^2*b^10*c^8*d^2+120*a*b^11*c^9*d+b^12*c^10)*x^13+1/12*(12*a^11
*b*d^10+660*a^10*b^2*c*d^9+9900*a^9*b^3*c^2*d^8+59400*a^8*b^4*c^3*d^7+16632
0*a^7*b^5*c^4*d^6+232848*a^6*b^6*c^5*d^5+166320*a^5*b^7*c^6*d^4+59400*a^4*b
^8*c^7*d^3+9900*a^3*b^9*c^8*d^2+660*a^2*b^10*c^9*d+12*a*b^11*c^10)*x^12+1/1
1*(a^12*d^10+120*a^11*b*c*d^9+2970*a^10*b^2*c^2*d^8+26400*a^9*b^3*c^3*d^7+1
03950*a^8*b^4*c^4*d^6+199584*a^7*b^5*c^5*d^5+194040*a^6*b^6*c^6*d^4+95040*a
^5*b^7*c^7*d^3+22275*a^4*b^8*c^8*d^2+2200*a^3*b^9*c^9*d+66*a^2*b^10*c^10)*x
^11+1/10*(10*a^12*c*d^9+540*a^11*b*c^2*d^8+7920*a^10*b^2*c^3*d^7+46200*a^9*
b^3*c^4*d^6+124740*a^8*b^4*c^5*d^5+166320*a^7*b^5*c^6*d^4+110880*a^6*b^6*c^
```

```
7*d^3+35640*a^5*b^7*c^8*d^2+4950*a^4*b^8*c^9*d+220*a^3*b^9*c^10)*x^10+1/9*(
45*a^12*c^2*d^8+1440*a^11*b*c^3*d^7+13860*a^10*b^2*c^4*d^6+55440*a^9*b^3*c^
5*d^5+103950*a^8*b^4*c^6*d^4+95040*a^7*b^5*c^7*d^3+41580*a^6*b^6*c^8*d^2+79
20*a^5*b^7*c^9*d+495*a^4*b^8*c^10)*x^9+1/8*(120*a^12*c^3*d^7+2520*a^11*b*c^
4*d^6+16632*a^10*b^2*c^5*d^5+46200*a^9*b^3*c^6*d^4+59400*a^8*b^4*c^7*d^3+35
640*a^7*b^5*c^8*d^2+9240*a^6*b^6*c^9*d+792*a^5*b^7*c^10)*x^8+1/7*(210*a^12*
c^4*d^6+3024*a^11*b*c^5*d^5+13860*a^10*b^2*c^6*d^4+26400*a^9*b^3*c^7*d^3+22
275*a^8*b^4*c^8*d^2+7920*a^7*b^5*c^9*d+924*a^6*b^6*c^10)*x^7+1/6*(252*a^12*
c^5*d^5+2520*a^11*b*c^6*d^4+7920*a^10*b^2*c^7*d^3+9900*a^9*b^3*c^8*d^2+4950
*a^8*b^4*c^9*d+792*a^7*b^5*c^10)*x^6+1/5*(210*a^12*c^6*d^4+1440*a^11*b*c^7*
d^3+2970*a^10*b^2*c^8*d^2+2200*a^9*b^3*c^9*d+495*a^8*b^4*c^10)*x^5+1/4*(120
*a^12*c^7*d^3+540*a^11*b*c^8*d^2+660*a^10*b^2*c^9*d+220*a^9*b^3*c^10)*x^4+1
/3*(45*a^12*c^8*d^2+120*a^11*b*c^9*d+66*a^10*b^2*c^10)*x^3+1/2*(10*a^12*c^9
*d+12*a^11*b*c^10)*x^2+a^12*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(259) = 518$.
time = 0.29, size = 1877, normalized size = 6.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
```

```
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(259) = 518$.
time = 0.49, size = 1877, normalized size = 6.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
```

```
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. $2(255) = 510$.
time = 0.14, size = 2088, normalized size = 7.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12*(d*x+c)**10,x)
```

```
[Out] a**12*c**10*x + b**12*d**10*x**23/23 + x**22*(6*a*b**11*d**10/11 + 5*b**12*
c*d**9/11) + x**21*(22*a**2*b**10*d**10/7 + 40*a*b**11*c*d**9/7 + 15*b**12*
c**2*d**8/7) + x**20*(11*a**3*b**9*d**10 + 33*a**2*b**10*c*d**9 + 27*a*b**1
1*c**2*d**8 + 6*b**12*c**3*d**7) + x**19*(495*a**4*b**8*d**10/19 + 2200*a**
3*b**9*c*d**9/19 + 2970*a**2*b**10*c**2*d**8/19 + 1440*a*b**11*c**3*d**7/19
 + 210*b**12*c**4*d**6/19) + x**18*(44*a**5*b**7*d**10 + 275*a**4*b**8*c*d*
*9 + 550*a**3*b**9*c**2*d**8 + 440*a**2*b**10*c**3*d**7 + 140*a*b**11*c**4*
```

```
d**6 + 14*b**12*c**5*d**5) + x**17*(924*a**6*b**6*d**10/17 + 7920*a**5*b**7
*c*d**9/17 + 22275*a**4*b**8*c**2*d**8/17 + 26400*a**3*b**9*c**3*d**7/17 +
13860*a**2*b**10*c**4*d**6/17 + 3024*a*b**11*c**5*d**5/17 + 210*b**12*c**6*
d**4/17) + x**16*(99*a**7*b**5*d**10/2 + 1155*a**6*b**6*c*d**9/2 + 4455*a**
5*b**7*c**2*d**8/2 + 7425*a**4*b**8*c**3*d**7/2 + 5775*a**3*b**9*c**4*d**6/
2 + 2079*a**2*b**10*c**5*d**5/2 + 315*a*b**11*c**6*d**4/2 + 15*b**12*c**7*d
**3/2) + x**15*(33*a**8*b**4*d**10 + 528*a**7*b**5*c*d**9 + 2772*a**6*b**6*
c**2*d**8 + 6336*a**5*b**7*c**3*d**7 + 6930*a**4*b**8*c**4*d**6 + 3696*a**3
*b**9*c**5*d**5 + 924*a**2*b**10*c**6*d**4 + 96*a*b**11*c**7*d**3 + 3*b**12
*c**8*d**2) + x**14*(110*a**9*b**3*d**10/7 + 2475*a**8*b**4*c*d**9/7 + 1782
0*a**7*b**5*c**2*d**8/7 + 7920*a**6*b**6*c**3*d**7 + 11880*a**5*b**7*c**4*d
**6 + 8910*a**4*b**8*c**5*d**5 + 3300*a**3*b**9*c**6*d**4 + 3960*a**2*b**10
*c**7*d**3/7 + 270*a*b**11*c**8*d**2/7 + 5*b**12*c**9*d/7) + x**13*(66*a**1
0*b**2*d**10/13 + 2200*a**9*b**3*c*d**9/13 + 22275*a**8*b**4*c**2*d**8/13 +
 95040*a**7*b**5*c**3*d**7/13 + 194040*a**6*b**6*c**4*d**6/13 + 199584*a**5
*b**7*c**5*d**5/13 + 103950*a**4*b**8*c**6*d**4/13 + 26400*a**3*b**9*c**7*d
**3/13 + 2970*a**2*b**10*c**8*d**2/13 + 120*a*b**11*c**9*d/13 + b**12*c**10
/13) + x**12*(a**11*b*d**10 + 55*a**10*b**2*c*d**9 + 825*a**9*b**3*c**2*d**
8 + 4950*a**8*b**4*c**3*d**7 + 13860*a**7*b**5*c**4*d**6 + 19404*a**6*b**6*
c**5*d**5 + 13860*a**5*b**7*c**6*d**4 + 4950*a**4*b**8*c**7*d**3 + 825*a**3
*b**9*c**8*d**2 + 55*a**2*b**10*c**9*d + a*b**11*c**10) + x**11*(a**12*d**1
0/11 + 120*a**11*b*c*d**9/11 + 270*a**10*b**2*c**2*d**8 + 2400*a**9*b**3*c*
*3*d**7 + 9450*a**8*b**4*c**4*d**6 + 18144*a**7*b**5*c**5*d**5 + 17640*a**6
*b**6*c**6*d**4 + 8640*a**5*b**7*c**7*d**3 + 2025*a**4*b**8*c**8*d**2 + 200
*a**3*b**9*c**9*d + 6*a**2*b**10*c**10) + x**10*(a**12*c*d**9 + 54*a**11*b*
c**2*d**8 + 792*a**10*b**2*c**3*d**7 + 4620*a**9*b**3*c**4*d**6 + 12474*a**
8*b**4*c**5*d**5 + 16632*a**7*b**5*c**6*d**4 + 11088*a**6*b**6*c**7*d**3 +
3564*a**5*b**7*c**8*d**2 + 495*a**4*b**8*c**9*d + 22*a**3*b**9*c**10) + x**
9*(5*a**12*c**2*d**8 + 160*a**11*b*c**3*d**7 + 1540*a**10*b**2*c**4*d**6 +
6160*a**9*b**3*c**5*d**5 + 11550*a**8*b**4*c**6*d**4 + 10560*a**7*b**5*c**7
*d**3 + 4620*a**6*b**6*c**8*d**2 + 880*a**5*b**7*c**9*d + 55*a**4*b**8*c**1
0) + x**8*(15*a**12*c**3*d**7 + 315*a**11*b*c**4*d**6 + 2079*a**10*b**2*c**
5*d**5 + 5775*a**9*b**3*c**6*d**4 + 7425*a**8*b**4*c**7*d**3 + 4455*a**7*b*
*5*c**8*d**2 + 1155*a**6*b**6*c**9*d + 99*a**5*b**7*c**10) + x**7*(30*a**12
*c**4*d**6 + 432*a**11*b*c**5*d**5 + 1980*a**10*b**2*c**6*d**4 + 26400*a**9
*b**3*c**7*d**3/7 + 22275*a**8*b**4*c**8*d**2/7 + 7920*a**7*b**5*c**9*d/7 +
 132*a**6*b**6*c**10) + x**6*(42*a**12*c**5*d**5 + 420*a**11*b*c**6*d**4 +
1320*a**10*b**2*c**7*d**3 + 1650*a**9*b**3*c**8*d**2 + 825*a**8*b**4*c**9*d
 + 132*a**7*b**5*c**10) + x**5*(42*a**12*c**6*d**4 + 288*a**11*b*c**7*d**3
+ 594*a**10*b**2*c**8*d**2 + 440*a**9*b**3*c**9*d + 99*a**8*b**4*c**10) + x
**4*(30*a**12*c**7*d**3 + 135*a**11*b*c**8*d**2 + 165*a**10*b**2*c**9*d + 5
5*a**9*b**3*c**10) + x**3*(15*a**12*c**8*d**2 + 40*a**11*b*c**9*d + 22*a**1
0*b**2*c**10) + x**2*(5*a**12*c**9*d + 6*a**11*b*c**10)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2186 vs.

2(259) = 518.
time = 1.16, size = 2186, normalized size = 7.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/23*b^12*d^10*x^23 + 5/11*b^12*c*d^9*x^22 + 6/11*a*b^11*d^10*x^22 + 15/7*b
^12*c^2*d^8*x^21 + 40/7*a*b^11*c*d^9*x^21 + 22/7*a^2*b^10*d^10*x^21 + 6*b^1
2*c^3*d^7*x^20 + 27*a*b^11*c^2*d^8*x^20 + 33*a^2*b^10*c*d^9*x^20 + 11*a^3*b
^9*d^10*x^20 + 210/19*b^12*c^4*d^6*x^19 + 1440/19*a*b^11*c^3*d^7*x^19 + 297
0/19*a^2*b^10*c^2*d^8*x^19 + 2200/19*a^3*b^9*c*d^9*x^19 + 495/19*a^4*b^8*d^
10*x^19 + 14*b^12*c^5*d^5*x^18 + 140*a*b^11*c^4*d^6*x^18 + 440*a^2*b^10*c^3
*d^7*x^18 + 550*a^3*b^9*c^2*d^8*x^18 + 275*a^4*b^8*c*d^9*x^18 + 44*a^5*b^7*
d^10*x^18 + 210/17*b^12*c^6*d^4*x^17 + 3024/17*a*b^11*c^5*d^5*x^17 + 13860/
17*a^2*b^10*c^4*d^6*x^17 + 26400/17*a^3*b^9*c^3*d^7*x^17 + 22275/17*a^4*b^8
*c^2*d^8*x^17 + 7920/17*a^5*b^7*c*d^9*x^17 + 924/17*a^6*b^6*d^10*x^17 + 15/
2*b^12*c^7*d^3*x^16 + 315/2*a*b^11*c^6*d^4*x^16 + 2079/2*a^2*b^10*c^5*d^5*x
^16 + 5775/2*a^3*b^9*c^4*d^6*x^16 + 7425/2*a^4*b^8*c^3*d^7*x^16 + 4455/2*a^
5*b^7*c^2*d^8*x^16 + 1155/2*a^6*b^6*c*d^9*x^16 + 99/2*a^7*b^5*d^10*x^16 + 3
*b^12*c^8*d^2*x^15 + 96*a*b^11*c^7*d^3*x^15 + 924*a^2*b^10*c^6*d^4*x^15 + 3
696*a^3*b^9*c^5*d^5*x^15 + 6930*a^4*b^8*c^4*d^6*x^15 + 6336*a^5*b^7*c^3*d^7
*x^15 + 2772*a^6*b^6*c^2*d^8*x^15 + 528*a^7*b^5*c*d^9*x^15 + 33*a^8*b^4*d^1
0*x^15 + 5/7*b^12*c^9*d*x^14 + 270/7*a*b^11*c^8*d^2*x^14 + 3960/7*a^2*b^10*
c^7*d^3*x^14 + 3300*a^3*b^9*c^6*d^4*x^14 + 8910*a^4*b^8*c^5*d^5*x^14 + 1188
0*a^5*b^7*c^4*d^6*x^14 + 7920*a^6*b^6*c^3*d^7*x^14 + 17820/7*a^7*b^5*c^2*d^
8*x^14 + 2475/7*a^8*b^4*c*d^9*x^14 + 110/7*a^9*b^3*d^10*x^14 + 1/13*b^12*c^
10*x^13 + 120/13*a*b^11*c^9*d*x^13 + 2970/13*a^2*b^10*c^8*d^2*x^13 + 26400/
13*a^3*b^9*c^7*d^3*x^13 + 103950/13*a^4*b^8*c^6*d^4*x^13 + 199584/13*a^5*b^
7*c^5*d^5*x^13 + 194040/13*a^6*b^6*c^4*d^6*x^13 + 95040/13*a^7*b^5*c^3*d^7*
x^13 + 22275/13*a^8*b^4*c^2*d^8*x^13 + 2200/13*a^9*b^3*c*d^9*x^13 + 66/13*a
^10*b^2*d^10*x^13 + a*b^11*c^10*x^12 + 55*a^2*b^10*c^9*d*x^12 + 825*a^3*b^9
*c^8*d^2*x^12 + 4950*a^4*b^8*c^7*d^3*x^12 + 13860*a^5*b^7*c^6*d^4*x^12 + 19
404*a^6*b^6*c^5*d^5*x^12 + 13860*a^7*b^5*c^4*d^6*x^12 + 4950*a^8*b^4*c^3*d^
7*x^12 + 825*a^9*b^3*c^2*d^8*x^12 + 55*a^10*b^2*c*d^9*x^12 + a^11*b*d^10*x^
12 + 6*a^2*b^10*c^10*x^11 + 200*a^3*b^9*c^9*d*x^11 + 2025*a^4*b^8*c^8*d^2*x
^11 + 8640*a^5*b^7*c^7*d^3*x^11 + 17640*a^6*b^6*c^6*d^4*x^11 + 18144*a^7*b^
5*c^5*d^5*x^11 + 9450*a^8*b^4*c^4*d^6*x^11 + 2400*a^9*b^3*c^3*d^7*x^11 + 27
0*a^10*b^2*c^2*d^8*x^11 + 120/11*a^11*b*c*d^9*x^11 + 1/11*a^12*d^10*x^11 +
22*a^3*b^9*c^10*x^10 + 495*a^4*b^8*c^9*d*x^10 + 3564*a^5*b^7*c^8*d^2*x^10 +
 11088*a^6*b^6*c^7*d^3*x^10 + 16632*a^7*b^5*c^6*d^4*x^10 + 12474*a^8*b^4*c^
5*d^5*x^10 + 4620*a^9*b^3*c^4*d^6*x^10 + 792*a^10*b^2*c^3*d^7*x^10 + 54*a^1
1*b*c^2*d^8*x^10 + a^12*c*d^9*x^10 + 55*a^4*b^8*c^10*x^9 + 880*a^5*b^7*c^9*
d*x^9 + 4620*a^6*b^6*c^8*d^2*x^9 + 10560*a^7*b^5*c^7*d^3*x^9 + 11550*a^8*b^
```

```
4*c^6*d^4*x^9 + 6160*a^9*b^3*c^5*d^5*x^9 + 1540*a^10*b^2*c^4*d^6*x^9 + 160*
a^11*b*c^3*d^7*x^9 + 5*a^12*c^2*d^8*x^9 + 99*a^5*b^7*c^10*x^8 + 1155*a^6*b^
6*c^9*d*x^8 + 4455*a^7*b^5*c^8*d^2*x^8 + 7425*a^8*b^4*c^7*d^3*x^8 + 5775*a^
9*b^3*c^6*d^4*x^8 + 2079*a^10*b^2*c^5*d^5*x^8 + 315*a^11*b*c^4*d^6*x^8 + 15
*a^12*c^3*d^7*x^8 + 132*a^6*b^6*c^10*x^7 + 7920/7*a^7*b^5*c^9*d*x^7 + 22275
/7*a^8*b^4*c^8*d^2*x^7 + 26400/7*a^9*b^3*c^7*d^3*x^7 + 1980*a^10*b^2*c^6*d^
4*x^7 + 432*a^11*b*c^5*d^5*x^7 + 30*a^12*c^4*d^6*x^7 + 132*a^7*b^5*c^10*x^6
 + 825*a^8*b^4*c^9*d*x^6 + 1650*a^9*b^3*c^8*d^2*x^6 + 1320*a^10*b^2*c^7*d^3
*x^6 + 420*a^11*b*c^6*d^4*x^6 + 42*a^12*c^5*d^5*x^6 + 99*a^8*b^4*c^10*x^5 +
 440*a^9*b^3*c^9*d*x^5 + 594*a^10*b^2*c^8*d^2*x^5 + 288*a^11*b*c^7*d^3*x^5
+ 42*a^12*c^6*d^4*x^5 + 55*a^9*b^3*c^10*x^4 + 165*a^10*b^2*c^9*d*x^4 + 135*
a^11*b*c^8*d^2*x^4 + 30*a^12*c^7*d^3*x^4 + 22*a^10*b^2*c^10*x^3 + 40*a^11*b
*c^9*d*x^3 + 15*a^12*c^8*d^2*x^3 + 6*a^11*b*c^10*x^2 + 5*a^12*c^9*d*x^2 + a
^12*c^10*x
```

Mupad [B]
time = 0.98, size = 1847, normalized size = 6.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12*(c + d*x)^10,x)
```

```
[Out] x^12*(a*b^11*c^10 + a^11*b*d^10 + 55*a^2*b^10*c^9*d + 55*a^10*b^2*c*d^9 + 8
25*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7*c^6*d^4 + 19404*a
^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3*d^7 + 825*a^9*b^3
*c^2*d^8) + x^7*(132*a^6*b^6*c^10 + 30*a^12*c^4*d^6 + (7920*a^7*b^5*c^9*d)/
7 + 432*a^11*b*c^5*d^5 + (22275*a^8*b^4*c^8*d^2)/7 + (26400*a^9*b^3*c^7*d^3
)/7 + 1980*a^10*b^2*c^6*d^4) + x^17*((924*a^6*b^6*d^10)/17 + (210*b^12*c^6*
d^4)/17 + (3024*a*b^11*c^5*d^5)/17 + (7920*a^5*b^7*c*d^9)/17 + (13860*a^2*b
^10*c^4*d^6)/17 + (26400*a^3*b^9*c^3*d^7)/17 + (22275*a^4*b^8*c^2*d^8)/17)
+ x^5*(99*a^8*b^4*c^10 + 42*a^12*c^6*d^4 + 440*a^9*b^3*c^9*d + 288*a^11*b*c
^7*d^3 + 594*a^10*b^2*c^8*d^2) + x^19*((495*a^4*b^8*d^10)/19 + (210*b^12*c^
4*d^6)/19 + (1440*a*b^11*c^3*d^7)/19 + (2200*a^3*b^9*c*d^9)/19 + (2970*a^2*
b^10*c^2*d^8)/19) + x^8*(99*a^5*b^7*c^10 + 15*a^12*c^3*d^7 + 1155*a^6*b^6*c
^9*d + 315*a^11*b*c^4*d^6 + 4455*a^7*b^5*c^8*d^2 + 7425*a^8*b^4*c^7*d^3 + 5
775*a^9*b^3*c^6*d^4 + 2079*a^10*b^2*c^5*d^5) + x^16*((99*a^7*b^5*d^10)/2 +
(15*b^12*c^7*d^3)/2 + (315*a*b^11*c^6*d^4)/2 + (1155*a^6*b^6*c*d^9)/2 + (20
79*a^2*b^10*c^5*d^5)/2 + (5775*a^3*b^9*c^4*d^6)/2 + (7425*a^4*b^8*c^3*d^7)/
2 + (4455*a^5*b^7*c^2*d^8)/2) + x^11*((a^12*d^10)/11 + 6*a^2*b^10*c^10 + 20
0*a^3*b^9*c^9*d + 2025*a^4*b^8*c^8*d^2 + 8640*a^5*b^7*c^7*d^3 + 17640*a^6*b
^6*c^6*d^4 + 18144*a^7*b^5*c^5*d^5 + 9450*a^8*b^4*c^4*d^6 + 2400*a^9*b^3*c^
3*d^7 + 270*a^10*b^2*c^2*d^8 + (120*a^11*b*c*d^9)/11) + x^13*((b^12*c^10)/1
3 + (66*a^10*b^2*d^10)/13 + (2200*a^9*b^3*c*d^9)/13 + (2970*a^2*b^10*c^8*d^
2)/13 + (26400*a^3*b^9*c^7*d^3)/13 + (103950*a^4*b^8*c^6*d^4)/13 + (199584*
```

a^5*b^7*c^5*d^5)/13 + (194040*a^6*b^6*c^4*d^6)/13 + (95040*a^7*b^5*c^3*d^7)/13 + (22275*a^8*b^4*c^2*d^8)/13 + (120*a*b^11*c^9*d)/13) + x^6*(132*a^7*b^5*c^10 + 42*a^12*c^5*d^5 + 825*a^8*b^4*c^9*d + 420*a^11*b*c^6*d^4 + 1650*a^9*b^3*c^8*d^2 + 1320*a^10*b^2*c^7*d^3) + x^18*(44*a^5*b^7*d^10 + 14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^6 + 275*a^4*b^8*c*d^9 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8) + x^9*(55*a^4*b^8*c^10 + 5*a^12*c^2*d^8 + 880*a^5*b^7*c^9*d + 160*a^11*b*c^3*d^7 + 4620*a^6*b^6*c^8*d^2 + 10560*a^7*b^5*c^7*d^3 + 11550*a^8*b^4*c^6*d^4 + 6160*a^9*b^3*c^5*d^5 + 1540*a^10*b^2*c^4*d^6) + x^15*(33*a^8*b^4*d^10 + 3*b^12*c^8*d^2 + 96*a*b^11*c^7*d^3 + 528*a^7*b^5*c*d^9 + 924*a^2*b^10*c^6*d^4 + 3696*a^3*b^9*c^5*d^5 + 6930*a^4*b^8*c^4*d^6 + 6336*a^5*b^7*c^3*d^7 + 2772*a^6*b^6*c^2*d^8) + x^10*(a^12*c*d^9 + 22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 54*a^11*b*c^2*d^8 + 3564*a^5*b^7*c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7) + x^14*((5*b^12*c^9*d)/7 + (110*a^9*b^3*d^10)/7 + (270*a*b^11*c^8*d^2)/7 + (2475*a^8*b^4*c*d^9)/7 + (3960*a^2*b^10*c^7*d^3)/7 + 3300*a^3*b^9*c^6*d^4 + 8910*a^4*b^8*c^5*d^5 + 11880*a^5*b^7*c^4*d^6 + 7920*a^6*b^6*c^3*d^7 + (17820*a^7*b^5*c^2*d^8)/7) + a^12*c^10*x + (b^12*d^10*x^23)/23 + 5*a^9*c^7*x^4*(6*a^3*d^3 + 11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c*d^2) + b^9*d^7*x^20*(11*a^3*d^3 + 6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2*b*c*d^2) + a^11*c^9*x^2*(5*a*d + 6*b*c) + (b^11*d^9*x^22*(6*a*d + 5*b*c))/11 + a^10*c^8*x^3*(15*a^2*d^2 + 22*b^2*c^2 + 40*a*b*c*d) + (b^10*d^8*x^21*(22*a^2*d^2 + 15*b^2*c^2 + 40*a*b*c*d))/7

3.1300 $\int (a+bx)^{11}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\frac{8d^3(bc-ad)^7(a+bx)^{15}}{b^{11}}+\frac{105d^4}{}$$

```
[Out] 1/12*(-a*d+b*c)^10*(b*x+a)^12/b^11+10/13*d*(-a*d+b*c)^9*(b*x+a)^13/b^11+45/
14*d^2*(-a*d+b*c)^8*(b*x+a)^14/b^11+8*d^3*(-a*d+b*c)^7*(b*x+a)^15/b^11+105/
8*d^4*(-a*d+b*c)^6*(b*x+a)^16/b^11+252/17*d^5*(-a*d+b*c)^5*(b*x+a)^17/b^11+
35/3*d^6*(-a*d+b*c)^4*(b*x+a)^18/b^11+120/19*d^7*(-a*d+b*c)^3*(b*x+a)^19/b^
11+9/4*d^8*(-a*d+b*c)^2*(b*x+a)^20/b^11+10/21*d^9*(-a*d+b*c)*(b*x+a)^21/b^1
1+1/22*d^10*(b*x+a)^22/b^11
```

Rubi [A]
time = 0.88, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(a+bx)^{17}(bc-ad)^5}{17b^{11}}+\frac{105d^4(a+bx)^{16}(bc-ad)^6}{8b^{11}}+\frac{8d^3(a+bx)^{15}(bc-ad)^7}{b^{11}}+\frac{45d^2(a+bx)^{14}(bc-ad)^8}{14b^{11}}+\frac{10d(a+bx)^{13}(bc-ad)^9}{13b^{11}}+\frac{(a+bx)^{12}(bc-ad)^{10}}{12b^{11}}+\frac{d^{10}(a+bx)^{22}}{22b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^12)/(12*b^11) + (10*d*(b*c - a*d)^9*(a + b*x)^13)
/(13*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^14)/(14*b^11) + (8*d^3*(b*c -
a*d)^7*(a + b*x)^15)/b^11 + (105*d^4*(b*c - a*d)^6*(a + b*x)^16)/(8*b^11) +
 (252*d^5*(b*c - a*d)^5*(a + b*x)^17)/(17*b^11) + (35*d^6*(b*c - a*d)^4*(a
+ b*x)^18)/(3*b^11) + (120*d^7*(b*c - a*d)^3*(a + b*x)^19)/(19*b^11) + (9*d
^8*(b*c - a*d)^2*(a + b*x)^20)/(4*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^21)
/(21*b^11) + (d^10*(a + b*x)^22)/(22*b^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{11}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{11}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{12}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)}{b^{10}}\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1702 vs. $2(279) = 558$.
time = 0.13, size = 1702, normalized size = 6.10

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] a^11*c^10*x + (a^10*c^9*(11*b*c + 10*a*d)*x^2)/2 + (5*a^9*c^8*(11*b^2*c^2 +
 22*a*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^8*c^7*(33*b^3*c^3 + 110*a*b^2*c^2*d
+ 99*a^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 3*a^7*c^6*(22*b^4*c^4 + 110*a*b^3*c
^3*d + 165*a^2*b^2*c^2*d^2 + 88*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + (a^6*c^5*(1
54*b^5*c^5 + 1100*a*b^4*c^4*d + 2475*a^2*b^3*c^3*d^2 + 2200*a^3*b^2*c^2*d^3
 + 770*a^4*b*c*d^4 + 84*a^5*d^5)*x^6)/2 + (6*a^5*c^4*(77*b^6*c^6 + 770*a*b^
5*c^5*d + 2475*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 1925*a^4*b^2*c^2*d^
4 + 462*a^5*b*c*d^5 + 35*a^6*d^6)*x^7)/7 + (15*a^4*c^3*(11*b^7*c^7 + 154*a*
b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1320*a^3*b^4*c^4*d^3 + 1155*a^4*b^3*c^3*d
^4 + 462*a^5*b^2*c^2*d^5 + 77*a^6*b*c*d^6 + 4*a^7*d^7)*x^8)/4 + (5*a^3*c^2*
(11*b^8*c^8 + 220*a*b^7*c^7*d + 1386*a^2*b^6*c^6*d^2 + 3696*a^3*b^5*c^5*d^3
 + 4620*a^4*b^4*c^4*d^4 + 2772*a^5*b^3*c^3*d^5 + 770*a^6*b^2*c^2*d^6 + 88*a
^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 + (a^2*c*(11*b^9*c^9 + 330*a*b^8*c^8*d + 297
0*a^2*b^7*c^7*d^2 + 11088*a^3*b^6*c^6*d^3 + 19404*a^4*b^5*c^5*d^4 + 16632*a
^5*b^4*c^4*d^5 + 6930*a^6*b^3*c^3*d^6 + 1320*a^7*b^2*c^2*d^7 + 99*a^8*b*c*d
^8 + 2*a^9*d^9)*x^10)/2 + (a*(11*b^10*c^10 + 550*a*b^9*c^9*d + 7425*a^2*b^8
*c^8*d^2 + 39600*a^3*b^7*c^7*d^3 + 97020*a^4*b^6*c^6*d^4 + 116424*a^5*b^5*c
^5*d^5 + 69300*a^6*b^4*c^4*d^6 + 19800*a^7*b^3*c^3*d^7 + 2475*a^8*b^2*c^2*d
^8 + 110*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (b*(b^10*c^10 + 110*a*b^9*c^9*
d + 2475*a^2*b^8*c^8*d^2 + 19800*a^3*b^7*c^7*d^3 + 69300*a^4*b^6*c^6*d^4 +
116424*a^5*b^5*c^5*d^5 + 97020*a^6*b^4*c^4*d^6 + 39600*a^7*b^3*c^3*d^7 + 74
25*a^8*b^2*c^2*d^8 + 550*a^9*b*c*d^9 + 11*a^10*d^10)*x^12)/12 + (5*b^2*d*(2
*b^9*c^9 + 99*a*b^8*c^8*d + 1320*a^2*b^7*c^7*d^2 + 6930*a^3*b^6*c^6*d^3 + 1
6632*a^4*b^5*c^5*d^4 + 19404*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 2970
*a^7*b^2*c^2*d^7 + 330*a^8*b*c*d^8 + 11*a^9*d^9)*x^13)/13 + (15*b^3*d^2*(3*
b^8*c^8 + 88*a*b^7*c^7*d + 770*a^2*b^6*c^6*d^2 + 2772*a^3*b^5*c^5*d^3 + 462
0*a^4*b^4*c^4*d^4 + 3696*a^5*b^3*c^3*d^5 + 1386*a^6*b^2*c^2*d^6 + 220*a^7*b
*c*d^7 + 11*a^8*d^8)*x^14)/14 + 2*b^4*d^3*(4*b^7*c^7 + 77*a*b^6*c^6*d + 462
*a^2*b^5*c^5*d^2 + 1155*a^3*b^4*c^4*d^3 + 1320*a^4*b^3*c^3*d^4 + 693*a^5*b^
2*c^2*d^5 + 154*a^6*b*c*d^6 + 11*a^7*d^7)*x^15 + (3*b^5*d^4*(35*b^6*c^6 + 4
62*a*b^5*c^5*d + 1925*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 2475*a^4*b^2
*c^2*d^4 + 770*a^5*b*c*d^5 + 77*a^6*d^6)*x^16)/8 + (3*b^6*d^5*(84*b^5*c^5 +
 770*a*b^4*c^4*d + 2200*a^2*b^3*c^3*d^2 + 2475*a^3*b^2*c^2*d^3 + 1100*a^4*b
*c*d^4 + 154*a^5*d^5)*x^17)/17 + (5*b^7*d^6*(14*b^4*c^4 + 88*a*b^3*c^3*d +
165*a^2*b^2*c^2*d^2 + 110*a^3*b*c*d^3 + 22*a^4*d^4)*x^18)/6 + (5*b^8*d^7*(2
4*b^3*c^3 + 99*a*b^2*c^2*d + 110*a^2*b*c*d^2 + 33*a^3*d^3)*x^19)/19 + (b^9*
```

```
d^8*(9*b^2*c^2 + 22*a*b*c*d + 11*a^2*d^2)*x^20)/4 + (b^10*d^9*(10*b*c + 11*
a*d)*x^21)/21 + (b^11*d^10*x^22)/22
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(259) = 518$.
time = 0.14, size = 1741, normalized size = 6.24

method	result	size
norman	Expression too large to display	1721
default	Expression too large to display	1741
gosper	Expression too large to display	2011
risch	Expression too large to display	2011

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*b^11*d^10*x^22+1/21*(11*a*b^10*d^10+10*b^11*c*d^9)*x^21+1/20*(55*a^2*b
^9*d^10+110*a*b^10*c*d^9+45*b^11*c^2*d^8)*x^20+1/19*(165*a^3*b^8*d^10+550*a
^2*b^9*c*d^9+495*a*b^10*c^2*d^8+120*b^11*c^3*d^7)*x^19+1/18*(330*a^4*b^7*d^
10+1650*a^3*b^8*c*d^9+2475*a^2*b^9*c^2*d^8+1320*a*b^10*c^3*d^7+210*b^11*c^4
*d^6)*x^18+1/17*(462*a^5*b^6*d^10+3300*a^4*b^7*c*d^9+7425*a^3*b^8*c^2*d^8+6
600*a^2*b^9*c^3*d^7+2310*a*b^10*c^4*d^6+252*b^11*c^5*d^5)*x^17+1/16*(462*a^
6*b^5*d^10+4620*a^5*b^6*c*d^9+14850*a^4*b^7*c^2*d^8+19800*a^3*b^8*c^3*d^7+1
1550*a^2*b^9*c^4*d^6+2772*a*b^10*c^5*d^5+210*b^11*c^6*d^4)*x^16+1/15*(330*a
^7*b^4*d^10+4620*a^6*b^5*c*d^9+20790*a^5*b^6*c^2*d^8+39600*a^4*b^7*c^3*d^7+
34650*a^3*b^8*c^4*d^6+13860*a^2*b^9*c^5*d^5+2310*a*b^10*c^6*d^4+120*b^11*c^
7*d^3)*x^15+1/14*(165*a^8*b^3*d^10+3300*a^7*b^4*c*d^9+20790*a^6*b^5*c^2*d^8
+55440*a^5*b^6*c^3*d^7+69300*a^4*b^7*c^4*d^6+41580*a^3*b^8*c^5*d^5+11550*a^
2*b^9*c^6*d^4+1320*a*b^10*c^7*d^3+45*b^11*c^8*d^2)*x^14+1/13*(55*a^9*b^2*d^
10+1650*a^8*b^3*c*d^9+14850*a^7*b^4*c^2*d^8+55440*a^6*b^5*c^3*d^7+97020*a^5
*b^6*c^4*d^6+83160*a^4*b^7*c^5*d^5+34650*a^3*b^8*c^6*d^4+6600*a^2*b^9*c^7*d
^3+495*a*b^10*c^8*d^2+10*b^11*c^9*d)*x^13+1/12*(11*a^10*b*d^10+550*a^9*b^2*
c*d^9+7425*a^8*b^3*c^2*d^8+39600*a^7*b^4*c^3*d^7+97020*a^6*b^5*c^4*d^6+1164
24*a^5*b^6*c^5*d^5+69300*a^4*b^7*c^6*d^4+19800*a^3*b^8*c^7*d^3+2475*a^2*b^9
*c^8*d^2+110*a*b^10*c^9*d+b^11*c^10)*x^12+1/11*(a^11*d^10+110*a^10*b*c*d^9+
2475*a^9*b^2*c^2*d^8+19800*a^8*b^3*c^3*d^7+69300*a^7*b^4*c^4*d^6+116424*a^6
*b^5*c^5*d^5+97020*a^5*b^6*c^6*d^4+39600*a^4*b^7*c^7*d^3+7425*a^3*b^8*c^8*d
^2+550*a^2*b^9*c^9*d+11*a*b^10*c^10)*x^11+1/10*(10*a^11*c*d^9+495*a^10*b*c^
2*d^8+6600*a^9*b^2*c^3*d^7+34650*a^8*b^3*c^4*d^6+83160*a^7*b^4*c^5*d^5+9702
0*a^6*b^5*c^6*d^4+55440*a^5*b^6*c^7*d^3+14850*a^4*b^7*c^8*d^2+1650*a^3*b^8*
c^9*d+55*a^2*b^9*c^10)*x^10+1/9*(45*a^11*c^2*d^8+1320*a^10*b*c^3*d^7+11550*
a^9*b^2*c^4*d^6+41580*a^8*b^3*c^5*d^5+69300*a^7*b^4*c^6*d^4+55440*a^6*b^5*c
^7*d^3+20790*a^5*b^6*c^8*d^2+3300*a^4*b^7*c^9*d+165*a^3*b^8*c^10)*x^9+1/8*(
120*a^11*c^3*d^7+2310*a^10*b*c^4*d^6+13860*a^9*b^2*c^5*d^5+34650*a^8*b^3*c^
6*d^4+39600*a^7*b^4*c^7*d^3+20790*a^6*b^5*c^8*d^2+4620*a^5*b^6*c^9*d+330*a^
```

```
4*b^7*c^10)*x^8+1/7*(210*a^11*c^4*d^6+2772*a^10*b*c^5*d^5+11550*a^9*b^2*c^6
*d^4+19800*a^8*b^3*c^7*d^3+14850*a^7*b^4*c^8*d^2+4620*a^6*b^5*c^9*d+462*a^5
*b^6*c^10)*x^7+1/6*(252*a^11*c^5*d^5+2310*a^10*b*c^6*d^4+6600*a^9*b^2*c^7*d
^3+7425*a^8*b^3*c^8*d^2+3300*a^7*b^4*c^9*d+462*a^6*b^5*c^10)*x^6+1/5*(210*a
^11*c^6*d^4+1320*a^10*b*c^7*d^3+2475*a^9*b^2*c^8*d^2+1650*a^8*b^3*c^9*d+330
*a^7*b^4*c^10)*x^5+1/4*(120*a^11*c^7*d^3+495*a^10*b*c^8*d^2+550*a^9*b^2*c^9
*d+165*a^8*b^3*c^10)*x^4+1/3*(45*a^11*c^8*d^2+110*a^10*b*c^9*d+55*a^9*b^2*c
^10)*x^3+1/2*(10*a^11*c^9*d+11*a^10*b*c^10)*x^2+a^11*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(259) = 518$.
time = 0.29, size = 1740, normalized size = 6.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
 462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
```

```
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(259) = 518$.
time = 0.52, size = 1740, normalized size = 6.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
 462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
```

```
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1965 vs. $2(258) = 516$.
time = 0.13, size = 1965, normalized size = 7.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11*(d*x+c)**10,x)
```

```
[Out] a**11*c**10*x + b**11*d**10*x**22/22 + x**21*(11*a*b**10*d**10/21 + 10*b**1
1*c*d**9/21) + x**20*(11*a**2*b**9*d**10/4 + 11*a*b**10*c*d**9/2 + 9*b**11*
c**2*d**8/4) + x**19*(165*a**3*b**8*d**10/19 + 550*a**2*b**9*c*d**9/19 + 49
5*a*b**10*c**2*d**8/19 + 120*b**11*c**3*d**7/19) + x**18*(55*a**4*b**7*d**1
0/3 + 275*a**3*b**8*c*d**9/3 + 275*a**2*b**9*c**2*d**8/2 + 220*a*b**10*c**3
*d**7/3 + 35*b**11*c**4*d**6/3) + x**17*(462*a**5*b**6*d**10/17 + 3300*a**4
*b**7*c*d**9/17 + 7425*a**3*b**8*c**2*d**8/17 + 6600*a**2*b**9*c**3*d**7/17
 + 2310*a*b**10*c**4*d**6/17 + 252*b**11*c**5*d**5/17) + x**16*(231*a**6*b*
*5*d**10/8 + 1155*a**5*b**6*c*d**9/4 + 7425*a**4*b**7*c**2*d**8/8 + 2475*a*
*3*b**8*c**3*d**7/2 + 5775*a**2*b**9*c**4*d**6/8 + 693*a*b**10*c**5*d**5/4
+ 105*b**11*c**6*d**4/8) + x**15*(22*a**7*b**4*d**10 + 308*a**6*b**5*c*d**9
 + 1386*a**5*b**6*c**2*d**8 + 2640*a**4*b**7*c**3*d**7 + 2310*a**3*b**8*c**
4*d**6 + 924*a**2*b**9*c**5*d**5 + 154*a*b**10*c**6*d**4 + 8*b**11*c**7*d**
3) + x**14*(165*a**8*b**3*d**10/14 + 1650*a**7*b**4*c*d**9/7 + 1485*a**6*b*
*5*c**2*d**8 + 3960*a**5*b**6*c**3*d**7 + 4950*a**4*b**7*c**4*d**6 + 2970*a
**3*b**8*c**5*d**5 + 825*a**2*b**9*c**6*d**4 + 660*a*b**10*c**7*d**3/7 + 45
*b**11*c**8*d**2/14) + x**13*(55*a**9*b**2*d**10/13 + 1650*a**8*b**3*c*d**9
/13 + 14850*a**7*b**4*c**2*d**8/13 + 55440*a**6*b**5*c**3*d**7/13 + 97020*a
**5*b**6*c**4*d**6/13 + 83160*a**4*b**7*c**5*d**5/13 + 34650*a**3*b**8*c**6
```

```
*d**4/13 + 6600*a**2*b**9*c**7*d**3/13 + 495*a*b**10*c**8*d**2/13 + 10*b**1
1*c**9*d/13) + x**12*(11*a**10*b*d**10/12 + 275*a**9*b**2*c*d**9/6 + 2475*a
**8*b**3*c**2*d**8/4 + 3300*a**7*b**4*c**3*d**7 + 8085*a**6*b**5*c**4*d**6
+ 9702*a**5*b**6*c**5*d**5 + 5775*a**4*b**7*c**6*d**4 + 1650*a**3*b**8*c**7
*d**3 + 825*a**2*b**9*c**8*d**2/4 + 55*a*b**10*c**9*d/6 + b**11*c**10/12) +
 x**11*(a**11*d**10/11 + 10*a**10*b*c*d**9 + 225*a**9*b**2*c**2*d**8 + 1800
*a**8*b**3*c**3*d**7 + 6300*a**7*b**4*c**4*d**6 + 10584*a**6*b**5*c**5*d**5
 + 8820*a**5*b**6*c**6*d**4 + 3600*a**4*b**7*c**7*d**3 + 675*a**3*b**8*c**8
*d**2 + 50*a**2*b**9*c**9*d + a*b**10*c**10) + x**10*(a**11*c*d**9 + 99*a**
10*b*c**2*d**8/2 + 660*a**9*b**2*c**3*d**7 + 3465*a**8*b**3*c**4*d**6 + 831
6*a**7*b**4*c**5*d**5 + 9702*a**6*b**5*c**6*d**4 + 5544*a**5*b**6*c**7*d**3
 + 1485*a**4*b**7*c**8*d**2 + 165*a**3*b**8*c**9*d + 11*a**2*b**9*c**10/2)
+ x**9*(5*a**11*c**2*d**8 + 440*a**10*b*c**3*d**7/3 + 3850*a**9*b**2*c**4*d
**6/3 + 4620*a**8*b**3*c**5*d**5 + 7700*a**7*b**4*c**6*d**4 + 6160*a**6*b**
5*c**7*d**3 + 2310*a**5*b**6*c**8*d**2 + 1100*a**4*b**7*c**9*d/3 + 55*a**3*
b**8*c**10/3) + x**8*(15*a**11*c**3*d**7 + 1155*a**10*b*c**4*d**6/4 + 3465*
a**9*b**2*c**5*d**5/2 + 17325*a**8*b**3*c**6*d**4/4 + 4950*a**7*b**4*c**7*d
**3 + 10395*a**6*b**5*c**8*d**2/4 + 1155*a**5*b**6*c**9*d/2 + 165*a**4*b**7
*c**10/4) + x**7*(30*a**11*c**4*d**6 + 396*a**10*b*c**5*d**5 + 1650*a**9*b*
*2*c**6*d**4 + 19800*a**8*b**3*c**7*d**3/7 + 14850*a**7*b**4*c**8*d**2/7 +
660*a**6*b**5*c**9*d + 66*a**5*b**6*c**10) + x**6*(42*a**11*c**5*d**5 + 385
*a**10*b*c**6*d**4 + 1100*a**9*b**2*c**7*d**3 + 2475*a**8*b**3*c**8*d**2/2
+ 550*a**7*b**4*c**9*d + 77*a**6*b**5*c**10) + x**5*(42*a**11*c**6*d**4 + 2
64*a**10*b*c**7*d**3 + 495*a**9*b**2*c**8*d**2 + 330*a**8*b**3*c**9*d + 66*
a**7*b**4*c**10) + x**4*(30*a**11*c**7*d**3 + 495*a**10*b*c**8*d**2/4 + 275
*a**9*b**2*c**9*d/2 + 165*a**8*b**3*c**10/4) + x**3*(15*a**11*c**8*d**2 + 1
10*a**10*b*c**9*d/3 + 55*a**9*b**2*c**10/3) + x**2*(5*a**11*c**9*d + 11*a**
10*b*c**10/2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2010 vs. $2(259) = 518$.
time = 1.33, size = 2010, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^11*d^10*x^22 + 10/21*b^11*c*d^9*x^21 + 11/21*a*b^10*d^10*x^21 + 9/4*
b^11*c^2*d^8*x^20 + 11/2*a*b^10*c*d^9*x^20 + 11/4*a^2*b^9*d^10*x^20 + 120/1
9*b^11*c^3*d^7*x^19 + 495/19*a*b^10*c^2*d^8*x^19 + 550/19*a^2*b^9*c*d^9*x^1
9 + 165/19*a^3*b^8*d^10*x^19 + 35/3*b^11*c^4*d^6*x^18 + 220/3*a*b^10*c^3*d^
7*x^18 + 275/2*a^2*b^9*c^2*d^8*x^18 + 275/3*a^3*b^8*c*d^9*x^18 + 55/3*a^4*b
^7*d^10*x^18 + 252/17*b^11*c^5*d^5*x^17 + 2310/17*a*b^10*c^4*d^6*x^17 + 660
0/17*a^2*b^9*c^3*d^7*x^17 + 7425/17*a^3*b^8*c^2*d^8*x^17 + 3300/17*a^4*b^7*
```

c*d^9*x^17 + 462/17*a^5*b^6*d^10*x^17 + 105/8*b^11*c^6*d^4*x^16 + 693/4*a*b^10*c^5*d^5*x^16 + 5775/8*a^2*b^9*c^4*d^6*x^16 + 2475/2*a^3*b^8*c^3*d^7*x^16 + 7425/8*a^4*b^7*c^2*d^8*x^16 + 1155/4*a^5*b^6*c*d^9*x^16 + 231/8*a^6*b^5*d^10*x^16 + 8*b^11*c^7*d^3*x^15 + 154*a*b^10*c^6*d^4*x^15 + 924*a^2*b^9*c^5*d^5*x^15 + 2310*a^3*b^8*c^4*d^6*x^15 + 2640*a^4*b^7*c^3*d^7*x^15 + 1386*a^5*b^6*c^2*d^8*x^15 + 308*a^6*b^5*c*d^9*x^15 + 22*a^7*b^4*d^10*x^15 + 45/14*b^11*c^8*d^2*x^14 + 660/7*a*b^10*c^7*d^3*x^14 + 825*a^2*b^9*c^6*d^4*x^14 + 2970*a^3*b^8*c^5*d^5*x^14 + 4950*a^4*b^7*c^4*d^6*x^14 + 3960*a^5*b^6*c^3*d^7*x^14 + 1485*a^6*b^5*c^2*d^8*x^14 + 1650/7*a^7*b^4*c*d^9*x^14 + 165/14*a^8*b^3*d^10*x^14 + 10/13*b^11*c^9*d*x^13 + 495/13*a*b^10*c^8*d^2*x^13 + 6600/13*a^2*b^9*c^7*d^3*x^13 + 34650/13*a^3*b^8*c^6*d^4*x^13 + 83160/13*a^4*b^7*c^5*d^5*x^13 + 97020/13*a^5*b^6*c^4*d^6*x^13 + 55440/13*a^6*b^5*c^3*d^7*x^13 + 14850/13*a^7*b^4*c^2*d^8*x^13 + 1650/13*a^8*b^3*c*d^9*x^13 + 55/13*a^9*b^2*d^10*x^13 + 1/12*b^11*c^10*x^12 + 55/6*a*b^10*c^9*d*x^12 + 825/4*a^2*b^9*c^8*d^2*x^12 + 1650*a^3*b^8*c^7*d^3*x^12 + 5775*a^4*b^7*c^6*d^4*x^12 + 9702*a^5*b^6*c^5*d^5*x^12 + 8085*a^6*b^5*c^4*d^6*x^12 + 3300*a^7*b^4*c^3*d^7*x^12 + 2475/4*a^8*b^3*c^2*d^8*x^12 + 275/6*a^9*b^2*c*d^9*x^12 + 11/12*a^10*b*d^10*x^12 + a*b^10*c^10*x^11 + 50*a^2*b^9*c^9*d*x^11 + 675*a^3*b^8*c^8*d^2*x^11 + 3600*a^4*b^7*c^7*d^3*x^11 + 8820*a^5*b^6*c^6*d^4*x^11 + 10584*a^6*b^5*c^5*d^5*x^11 + 6300*a^7*b^4*c^4*d^6*x^11 + 1800*a^8*b^3*c^3*d^7*x^11 + 225*a^9*b^2*c^2*d^8*x^11 + 10*a^10*b*c*d^9*x^11 + 1/11*a^11*d^10*x^11 + 11/2*a^2*b^9*c^10*x^10 + 165*a^3*b^8*c^9*d*x^10 + 1485*a^4*b^7*c^8*d^2*x^10 + 5544*a^5*b^6*c^7*d^3*x^10 + 9702*a^6*b^5*c^6*d^4*x^10 + 8316*a^7*b^4*c^5*d^5*x^10 + 3465*a^8*b^3*c^4*d^6*x^10 + 660*a^9*b^2*c^3*d^7*x^10 + 99/2*a^10*b*c^2*d^8*x^10 + a^11*c*d^9*x^10 + 55/3*a^3*b^8*c^10*x^9 + 1100/3*a^4*b^7*c^9*d*x^9 + 2310*a^5*b^6*c^8*d^2*x^9 + 6160*a^6*b^5*c^7*d^3*x^9 + 7700*a^7*b^4*c^6*d^4*x^9 + 4620*a^8*b^3*c^5*d^5*x^9 + 3850/3*a^9*b^2*c^4*d^6*x^9 + 440/3*a^10*b*c^3*d^7*x^9 + 5*a^11*c^2*d^8*x^9 + 165/4*a^4*b^7*c^10*x^8 + 1155/2*a^5*b^6*c^9*d*x^8 + 10395/4*a^6*b^5*c^8*d^2*x^8 + 4950*a^7*b^4*c^7*d^3*x^8 + 17325/4*a^8*b^3*c^6*d^4*x^8 + 3465/2*a^9*b^2*c^5*d^5*x^8 + 1155/4*a^10*b*c^4*d^6*x^8 + 15*a^11*c^3*d^7*x^8 + 66*a^5*b^6*c^10*x^7 + 660*a^6*b^5*c^9*d*x^7 + 14850/7*a^7*b^4*c^8*d^2*x^7 + 19800/7*a^8*b^3*c^7*d^3*x^7 + 1650*a^9*b^2*c^6*d^4*x^7 + 396*a^10*b*c^5*d^5*x^7 + 30*a^11*c^4*d^6*x^7 + 77*a^6*b^5*c^10*x^6 + 550*a^7*b^4*c^9*d*x^6 + 2475/2*a^8*b^3*c^8*d^2*x^6 + 1100*a^9*b^2*c^7*d^3*x^6 + 385*a^10*b*c^6*d^4*x^6 + 42*a^11*c^5*d^5*x^6 + 66*a^7*b^4*c^10*x^5 + 330*a^8*b^3*c^9*d*x^5 + 495*a^9*b^2*c^8*d^2*x^5 + 264*a^10*b*c^7*d^3*x^5 + 42*a^11*c^6*d^4*x^5 + 165/4*a^8*b^3*c^10*x^4 + 275/2*a^9*b^2*c^9*d*x^4 + 495/4*a^10*b*c^8*d^2*x^4 + 30*a^11*c^7*d^3*x^4 + 55/3*a^9*b^2*c^10*x^3 + 110/3*a^10*b*c^9*d*x^3 + 15*a^11*c^8*d^2*x^3 + 11/2*a^10*b*c^10*x^2 + 5*a^11*c^9*d*x^2 + a^11*c^10*x

Mupad [B]

time = 1.03, size = 1702, normalized size = 6.10

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^11*(c + d*x)^10,x)

[Out] x^7*(66*a^5*b^6*c^10 + 30*a^11*c^4*d^6 + 660*a^6*b^5*c^9*d + 396*a^10*b*c^5 *d^5 + (14850*a^7*b^4*c^8*d^2)/7 + (19800*a^8*b^3*c^7*d^3)/7 + 1650*a^9*b^2 *c^6*d^4) + x^16*((231*a^6*b^5*d^10)/8 + (105*b^11*c^6*d^4)/8 + (693*a*b^10 *c^5*d^5)/4 + (1155*a^5*b^6*c*d^9)/4 + (5775*a^2*b^9*c^4*d^6)/8 + (2475*a^3 *b^8*c^3*d^7)/2 + (7425*a^4*b^7*c^2*d^8)/8) + x^11*((a^11*d^10)/11 + a*b^10 *c^10 + 50*a^2*b^9*c^9*d + 675*a^3*b^8*c^8*d^2 + 3600*a^4*b^7*c^7*d^3 + 882 0*a^5*b^6*c^6*d^4 + 10584*a^6*b^5*c^5*d^5 + 6300*a^7*b^4*c^4*d^6 + 1800*a^8 *b^3*c^3*d^7 + 225*a^9*b^2*c^2*d^8 + 10*a^10*b*c*d^9) + x^12*((b^11*c^10)/1 2 + (11*a^10*b*d^10)/12 + (275*a^9*b^2*c*d^9)/6 + (825*a^2*b^9*c^8*d^2)/4 + 1650*a^3*b^8*c^7*d^3 + 5775*a^4*b^7*c^6*d^4 + 9702*a^5*b^6*c^5*d^5 + 8085* a^6*b^5*c^4*d^6 + 3300*a^7*b^4*c^3*d^7 + (2475*a^8*b^3*c^2*d^8)/4 + (55*a*b ^10*c^9*d)/6) + x^5*(66*a^7*b^4*c^10 + 42*a^11*c^6*d^4 + 330*a^8*b^3*c^9*d + 264*a^10*b*c^7*d^3 + 495*a^9*b^2*c^8*d^2) + x^18*((55*a^4*b^7*d^10)/3 + (35*b^11*c^4*d^6)/3 + (220*a*b^10*c^3*d^7)/3 + (275*a^3*b^8*c*d^9)/3 + (275* a^2*b^9*c^2*d^8)/2) + x^8*((165*a^4*b^7*c^10)/4 + 15*a^11*c^3*d^7 + (1155*a ^5*b^6*c^9*d)/2 + (1155*a^10*b*c^4*d^6)/4 + (10395*a^6*b^5*c^8*d^2)/4 + 495 0*a^7*b^4*c^7*d^3 + (17325*a^8*b^3*c^6*d^4)/4 + (3465*a^9*b^2*c^5*d^5)/2) + x^15*(22*a^7*b^4*d^10 + 8*b^11*c^7*d^3 + 154*a*b^10*c^6*d^4 + 308*a^6*b^5* c*d^9 + 924*a^2*b^9*c^5*d^5 + 2310*a^3*b^8*c^4*d^6 + 2640*a^4*b^7*c^3*d^7 + 1386*a^5*b^6*c^2*d^8) + x^6*(77*a^6*b^5*c^10 + 42*a^11*c^5*d^5 + 550*a^7*b ^4*c^9*d + 385*a^10*b*c^6*d^4 + (2475*a^8*b^3*c^8*d^2)/2 + 1100*a^9*b^2*c^7 *d^3) + x^17*((462*a^5*b^6*d^10)/17 + (252*b^11*c^5*d^5)/17 + (2310*a*b^10* c^4*d^6)/17 + (3300*a^4*b^7*c*d^9)/17 + (6600*a^2*b^9*c^3*d^7)/17 + (7425*a ^3*b^8*c^2*d^8)/17) + x^9*((55*a^3*b^8*c^10)/3 + 5*a^11*c^2*d^8 + (1100*a^4 *b^7*c^9*d)/3 + (440*a^10*b*c^3*d^7)/3 + 2310*a^5*b^6*c^8*d^2 + 6160*a^6*b^ 5*c^7*d^3 + 7700*a^7*b^4*c^6*d^4 + 4620*a^8*b^3*c^5*d^5 + (3850*a^9*b^2*c^4 *d^6)/3) + x^14*((165*a^8*b^3*d^10)/14 + (45*b^11*c^8*d^2)/14 + (660*a*b^10 *c^7*d^3)/7 + (1650*a^7*b^4*c*d^9)/7 + 825*a^2*b^9*c^6*d^4 + 2970*a^3*b^8*c ^5*d^5 + 4950*a^4*b^7*c^4*d^6 + 3960*a^5*b^6*c^3*d^7 + 1485*a^6*b^5*c^2*d^8) + x^10*(a^11*c*d^9 + (11*a^2*b^9*c^10)/2 + 165*a^3*b^8*c^9*d + (99*a^10*b *c^2*d^8)/2 + 1485*a^4*b^7*c^8*d^2 + 5544*a^5*b^6*c^7*d^3 + 9702*a^6*b^5*c^ 6*d^4 + 8316*a^7*b^4*c^5*d^5 + 3465*a^8*b^3*c^4*d^6 + 660*a^9*b^2*c^3*d^7) + x^13*((10*b^11*c^9*d)/13 + (55*a^9*b^2*d^10)/13 + (495*a*b^10*c^8*d^2)/13 + (1650*a^8*b^3*c*d^9)/13 + (6600*a^2*b^9*c^7*d^3)/13 + (34650*a^3*b^8*c^6 *d^4)/13 + (83160*a^4*b^7*c^5*d^5)/13 + (97020*a^5*b^6*c^4*d^6)/13 + (55440 *a^6*b^5*c^3*d^7)/13 + (14850*a^7*b^4*c^2*d^8)/13) + a^11*c^10*x + (b^11*d^ 10*x^22)/22 + (5*a^8*c^7*x^4*(24*a^3*d^3 + 33*b^3*c^3 + 110*a*b^2*c^2*d + 9 9*a^2*b*c*d^2))/4 + (5*b^8*d^7*x^19*(33*a^3*d^3 + 24*b^3*c^3 + 99*a*b^2*c^2 *d + 110*a^2*b*c*d^2))/19 + (a^10*c^9*x^2*(10*a*d + 11*b*c))/2 + (b^10*d^9* x^21*(11*a*d + 10*b*c))/21 + (5*a^9*c^8*x^3*(9*a^2*d^2 + 11*b^2*c^2 + 22*a* b*c*d))/3 + (b^9*d^8*x^20*(11*a^2*d^2 + 9*b^2*c^2 + 22*a*b*c*d))/4

3.1301 $\int (a+bx)^{10}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\frac{60d^3(bc-ad)^7(a+bx)^{14}}{7b^{11}}+\frac{14d^4}{}$$

```
[Out] 1/11*(-a*d+b*c)^10*(b*x+a)^11/b^11+5/6*d*(-a*d+b*c)^9*(b*x+a)^12/b^11+45/13
*d^2*(-a*d+b*c)^8*(b*x+a)^13/b^11+60/7*d^3*(-a*d+b*c)^7*(b*x+a)^14/b^11+14*
d^4*(-a*d+b*c)^6*(b*x+a)^15/b^11+63/4*d^5*(-a*d+b*c)^5*(b*x+a)^16/b^11+210/
17*d^6*(-a*d+b*c)^4*(b*x+a)^17/b^11+20/3*d^7*(-a*d+b*c)^3*(b*x+a)^18/b^11+4
5/19*d^8*(-a*d+b*c)^2*(b*x+a)^19/b^11+1/2*d^9*(-a*d+b*c)*(b*x+a)^20/b^11+1/
21*d^10*(b*x+a)^21/b^11
```

Rubi [A]

time = 0.77, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a+bx)^{16}(bc-ad)^5}{4b^{11}}+\frac{14d^4(a+bx)^{15}(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^{14}(bc-ad)^7}{7b^{11}}+\frac{45d^2(a+bx)^{13}(bc-ad)^8}{13b^{11}}+\frac{5d(a+bx)^{12}(bc-ad)^9}{6b^{11}}+\frac{(a+bx)^{11}(bc-ad)^{10}}{11b^{11}}+\frac{d^{10}(a+bx)^{21}}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^11)/(11*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^12)/
(6*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^13)/(13*b^11) + (60*d^3*(b*c - a
*d)^7*(a + b*x)^14)/(7*b^11) + (14*d^4*(b*c - a*d)^6*(a + b*x)^15)/b^11 + (
63*d^5*(b*c - a*d)^5*(a + b*x)^16)/(4*b^11) + (210*d^6*(b*c - a*d)^4*(a + b
*x)^17)/(17*b^11) + (20*d^7*(b*c - a*d)^3*(a + b*x)^18)/(3*b^11) + (45*d^8*
(b*c - a*d)^2*(a + b*x)^19)/(19*b^11) + (d^9*(b*c - a*d)*(a + b*x)^20)/(2*b
^11) + (d^10*(a + b*x)^21)/(21*b^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{10}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{10}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{11}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx}{b^{10}}\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1539 vs. $2(279) = 558$.
time = 0.12, size = 1539, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] a^10*c^10*x + 5*a^9*c^9*(b*c + a*d)*x^2 + (5*a^8*c^8*(9*b^2*c^2 + 20*a*b*c*
d + 9*a^2*d^2)*x^3)/3 + (15*a^7*c^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*
c*d^2 + 4*a^3*d^3)*x^4)/2 + 3*a^6*c^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^
2*b^2*c^2*d^2 + 80*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + 2*a^5*c^5*(21*b^5*c^5 +
175*a*b^4*c^4*d + 450*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d
^4 + 21*a^5*d^5)*x^6 + (30*a^4*c^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 315*a^2*b^
4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c*d^5 + 7*
a^6*d^6)*x^7)/7 + (15*a^3*c^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5
*d^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35
*a^6*b*c*d^6 + 2*a^7*d^7)*x^8)/2 + (5*a^2*c^2*(3*b^8*c^8 + 80*a*b^7*c^7*d +
 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2016*a
^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 +
 a*c*(b^9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3
 + 5292*a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540
*a^7*b^2*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^10 + ((b^10*c^10 + 100*a*b^9
*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100*a^4*b^6*c^6*d
^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^7*b^3*c^3*d^7
+ 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (5*b*d*(b^
9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3 + 5292*
a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540*a^7*b^2
*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^12)/6 + (15*b^2*d^2*(3*b^8*c^8 + 80*
a*b^7*c^7*d + 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4
*d^4 + 2016*a^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*
d^8)*x^13)/13 + (30*b^3*d^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5*d
^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35*a
^6*b*c*d^6 + 2*a^7*d^7)*x^14)/7 + 2*b^4*d^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 3
15*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c
*d^5 + 7*a^6*d^6)*x^15 + (3*b^5*d^5*(21*b^5*c^5 + 175*a*b^4*c^4*d + 450*a^2
*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5)*x^16)/4
+ (15*b^6*d^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 80*a^3*b
*c*d^3 + 14*a^4*d^4)*x^17)/17 + (5*b^7*d^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + 4*a^3*d^3)*x^18)/3 + (5*b^8*d^8*(9*b^2*c^2 + 20*a*b*c*d + 9*
a^2*d^2)*x^19)/19 + (b^9*d^9*(b*c + a*d)*x^20)/2 + (b^10*d^10*x^21)/21
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(259) = 518$.
time = 0.13, size = 1591, normalized size = 5.70

method	result	size
norman	Expression too large to display	1572
default	Expression too large to display	1591
gosper	Expression too large to display	1834
risch	Expression too large to display	1834

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b^10*d^10*x^21+1/20*(10*a*b^9*d^10+10*b^10*c*d^9)*x^20+1/19*(45*a^2*b^
8*d^10+100*a*b^9*c*d^9+45*b^10*c^2*d^8)*x^19+1/18*(120*a^3*b^7*d^10+450*a^2
*b^8*c*d^9+450*a*b^9*c^2*d^8+120*b^10*c^3*d^7)*x^18+1/17*(210*a^4*b^6*d^10+
1200*a^3*b^7*c*d^9+2025*a^2*b^8*c^2*d^8+1200*a*b^9*c^3*d^7+210*b^10*c^4*d^6
)*x^17+1/16*(252*a^5*b^5*d^10+2100*a^4*b^6*c*d^9+5400*a^3*b^7*c^2*d^8+5400*
a^2*b^8*c^3*d^7+2100*a*b^9*c^4*d^6+252*b^10*c^5*d^5)*x^16+1/15*(210*a^6*b^4
*d^10+2520*a^5*b^5*c*d^9+9450*a^4*b^6*c^2*d^8+14400*a^3*b^7*c^3*d^7+9450*a^
2*b^8*c^4*d^6+2520*a*b^9*c^5*d^5+210*b^10*c^6*d^4)*x^15+1/14*(120*a^7*b^3*d
^10+2100*a^6*b^4*c*d^9+11340*a^5*b^5*c^2*d^8+25200*a^4*b^6*c^3*d^7+25200*a^
3*b^7*c^4*d^6+11340*a^2*b^8*c^5*d^5+2100*a*b^9*c^6*d^4+120*b^10*c^7*d^3)*x^
14+1/13*(45*a^8*b^2*d^10+1200*a^7*b^3*c*d^9+9450*a^6*b^4*c^2*d^8+30240*a^5*
b^5*c^3*d^7+44100*a^4*b^6*c^4*d^6+30240*a^3*b^7*c^5*d^5+9450*a^2*b^8*c^6*d^
4+1200*a*b^9*c^7*d^3+45*b^10*c^8*d^2)*x^13+1/12*(10*a^9*b*d^10+450*a^8*b^2*
c*d^9+5400*a^7*b^3*c^2*d^8+25200*a^6*b^4*c^3*d^7+52920*a^5*b^5*c^4*d^6+5292
0*a^4*b^6*c^5*d^5+25200*a^3*b^7*c^6*d^4+5400*a^2*b^8*c^7*d^3+450*a*b^9*c^8*
d^2+10*b^10*c^9*d)*x^12+1/11*(a^10*d^10+100*a^9*b*c*d^9+2025*a^8*b^2*c^2*d^
8+14400*a^7*b^3*c^3*d^7+44100*a^6*b^4*c^4*d^6+63504*a^5*b^5*c^5*d^5+44100*a
^4*b^6*c^6*d^4+14400*a^3*b^7*c^7*d^3+2025*a^2*b^8*c^8*d^2+100*a*b^9*c^9*d+b
^10*c^10)*x^11+1/10*(10*a^10*c*d^9+450*a^9*b*c^2*d^8+5400*a^8*b^2*c^3*d^7+2
5200*a^7*b^3*c^4*d^6+52920*a^6*b^4*c^5*d^5+52920*a^5*b^5*c^6*d^4+25200*a^4*
b^6*c^7*d^3+5400*a^3*b^7*c^8*d^2+450*a^2*b^8*c^9*d+10*a*b^9*c^10)*x^10+1/9*
(45*a^10*c^2*d^8+1200*a^9*b*c^3*d^7+9450*a^8*b^2*c^4*d^6+30240*a^7*b^3*c^5*
d^5+44100*a^6*b^4*c^6*d^4+30240*a^5*b^5*c^7*d^3+9450*a^4*b^6*c^8*d^2+1200*a
^3*b^7*c^9*d+45*a^2*b^8*c^10)*x^9+1/8*(120*a^10*c^3*d^7+2100*a^9*b*c^4*d^6+
11340*a^8*b^2*c^5*d^5+25200*a^7*b^3*c^6*d^4+25200*a^6*b^4*c^7*d^3+11340*a^5
*b^5*c^8*d^2+2100*a^4*b^6*c^9*d+120*a^3*b^7*c^10)*x^8+1/7*(210*a^10*c^4*d^6
+2520*a^9*b*c^5*d^5+9450*a^8*b^2*c^6*d^4+14400*a^7*b^3*c^7*d^3+9450*a^6*b^4
*c^8*d^2+2520*a^5*b^5*c^9*d+210*a^4*b^6*c^10)*x^7+1/6*(252*a^10*c^5*d^5+210
0*a^9*b*c^6*d^4+5400*a^8*b^2*c^7*d^3+5400*a^7*b^3*c^8*d^2+2100*a^6*b^4*c^9*
d+252*a^5*b^5*c^10)*x^6+1/5*(210*a^10*c^6*d^4+1200*a^9*b*c^7*d^3+2025*a^8*b
^2*c^8*d^2+1200*a^7*b^3*c^9*d+210*a^6*b^4*c^10)*x^5+1/4*(120*a^10*c^7*d^3+4
50*a^9*b*c^8*d^2+450*a^8*b^2*c^9*d+120*a^7*b^3*c^10)*x^4+1/3*(45*a^10*c^8*d
^2+100*a^9*b*c^9*d+45*a^8*b^2*c^10)*x^3+1/2*(10*a^10*c^9*d+10*a^9*b*c^10)*x
^2+a^10*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(259) = 518$.

time = 0.28, size = 1581, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1581 vs. $2(259) = 518$.
time = 0.61, size = 1581, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1775 vs. $2(257) = 514$.
time = 0.12, size = 1775, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10*(d*x+c)**10,x)
```

```
[Out] a**10*c**10*x + b**10*d**10*x**21/21 + x**20*(a*b**9*d**10/2 + b**10*c*d**9
/2) + x**19*(45*a**2*b**8*d**10/19 + 100*a*b**9*c*d**9/19 + 45*b**10*c**2*d
**8/19) + x**18*(20*a**3*b**7*d**10/3 + 25*a**2*b**8*c*d**9 + 25*a*b**9*c**
2*d**8 + 20*b**10*c**3*d**7/3) + x**17*(210*a**4*b**6*d**10/17 + 1200*a**3*
b**7*c*d**9/17 + 2025*a**2*b**8*c**2*d**8/17 + 1200*a*b**9*c**3*d**7/17 + 2
10*b**10*c**4*d**6/17) + x**16*(63*a**5*b**5*d**10/4 + 525*a**4*b**6*c*d**9
/4 + 675*a**3*b**7*c**2*d**8/2 + 675*a**2*b**8*c**3*d**7/2 + 525*a*b**9*c**
4*d**6/4 + 63*b**10*c**5*d**5/4) + x**15*(14*a**6*b**4*d**10 + 168*a**5*b**
5*c*d**9 + 630*a**4*b**6*c**2*d**8 + 960*a**3*b**7*c**3*d**7 + 630*a**2*b**
8*c**4*d**6 + 168*a*b**9*c**5*d**5 + 14*b**10*c**6*d**4) + x**14*(60*a**7*b
**3*d**10/7 + 150*a**6*b**4*c*d**9 + 810*a**5*b**5*c**2*d**8 + 1800*a**4*b*
*6*c**3*d**7 + 1800*a**3*b**7*c**4*d**6 + 810*a**2*b**8*c**5*d**5 + 150*a*b
**9*c**6*d**4 + 60*b**10*c**7*d**3/7) + x**13*(45*a**8*b**2*d**10/13 + 1200
*a**7*b**3*c*d**9/13 + 9450*a**6*b**4*c**2*d**8/13 + 30240*a**5*b**5*c**3*d
**7/13 + 44100*a**4*b**6*c**4*d**6/13 + 30240*a**3*b**7*c**5*d**5/13 + 9450
*a**2*b**8*c**6*d**4/13 + 1200*a*b**9*c**7*d**3/13 + 45*b**10*c**8*d**2/13)
 + x**12*(5*a**9*b*d**10/6 + 75*a**8*b**2*c*d**9/2 + 450*a**7*b**3*c**2*d**
8 + 2100*a**6*b**4*c**3*d**7 + 4410*a**5*b**5*c**4*d**6 + 4410*a**4*b**6*c*
*5*d**5 + 2100*a**3*b**7*c**6*d**4 + 450*a**2*b**8*c**7*d**3 + 75*a*b**9*c*
*8*d**2/2 + 5*b**10*c**9*d/6) + x**11*(a**10*d**10/11 + 100*a**9*b*c*d**9/1
1 + 2025*a**8*b**2*c**2*d**8/11 + 14400*a**7*b**3*c**3*d**7/11 + 44100*a**6
*b**4*c**4*d**6/11 + 63504*a**5*b**5*c**5*d**5/11 + 44100*a**4*b**6*c**6*d*
*4/11 + 14400*a**3*b**7*c**7*d**3/11 + 2025*a**2*b**8*c**8*d**2/11 + 100*a*
b**9*c**9*d/11 + b**10*c**10/11) + x**10*(a**10*c*d**9 + 45*a**9*b*c**2*d**
8 + 540*a**8*b**2*c**3*d**7 + 2520*a**7*b**3*c**4*d**6 + 5292*a**6*b**4*c**
5*d**5 + 5292*a**5*b**5*c**6*d**4 + 2520*a**4*b**6*c**7*d**3 + 540*a**3*b**
7*c**8*d**2 + 45*a**2*b**8*c**9*d + a*b**9*c**10) + x**9*(5*a**10*c**2*d**8
 + 400*a**9*b*c**3*d**7/3 + 1050*a**8*b**2*c**4*d**6 + 3360*a**7*b**3*c**5*
d**5 + 4900*a**6*b**4*c**6*d**4 + 3360*a**5*b**5*c**7*d**3 + 1050*a**4*b**6
*c**8*d**2 + 400*a**3*b**7*c**9*d/3 + 5*a**2*b**8*c**10) + x**8*(15*a**10*c
**3*d**7 + 525*a**9*b*c**4*d**6/2 + 2835*a**8*b**2*c**5*d**5/2 + 3150*a**7*
b**3*c**6*d**4 + 3150*a**6*b**4*c**7*d**3 + 2835*a**5*b**5*c**8*d**2/2 + 52
5*a**4*b**6*c**9*d/2 + 15*a**3*b**7*c**10) + x**7*(30*a**10*c**4*d**6 + 360
*a**9*b*c**5*d**5 + 1350*a**8*b**2*c**6*d**4 + 14400*a**7*b**3*c**7*d**3/7
 + 1350*a**6*b**4*c**8*d**2 + 360*a**5*b**5*c**9*d + 30*a**4*b**6*c**10) + x
**6*(42*a**10*c**5*d**5 + 350*a**9*b*c**6*d**4 + 900*a**8*b**2*c**7*d**3 +
```

```
900*a**7*b**3*c**8*d**2 + 350*a**6*b**4*c**9*d + 42*a**5*b**5*c**10) + x**5
*(42*a**10*c**6*d**4 + 240*a**9*b*c**7*d**3 + 405*a**8*b**2*c**8*d**2 + 240
*a**7*b**3*c**9*d + 42*a**6*b**4*c**10) + x**4*(30*a**10*c**7*d**3 + 225*a*
*9*b*c**8*d**2/2 + 225*a**8*b**2*c**9*d/2 + 30*a**7*b**3*c**10) + x**3*(15*
a**10*c**8*d**2 + 100*a**9*b*c**9*d/3 + 15*a**8*b**2*c**10) + x**2*(5*a**10
*c**9*d + 5*a**9*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(259) = 518$.
time = 1.16, size = 1833, normalized size = 6.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*d^10*x^21 + 1/2*b^10*c*d^9*x^20 + 1/2*a*b^9*d^10*x^20 + 45/19*b^1
0*c^2*d^8*x^19 + 100/19*a*b^9*c*d^9*x^19 + 45/19*a^2*b^8*d^10*x^19 + 20/3*b
^10*c^3*d^7*x^18 + 25*a*b^9*c^2*d^8*x^18 + 25*a^2*b^8*c*d^9*x^18 + 20/3*a^3
*b^7*d^10*x^18 + 210/17*b^10*c^4*d^6*x^17 + 1200/17*a*b^9*c^3*d^7*x^17 + 20
25/17*a^2*b^8*c^2*d^8*x^17 + 1200/17*a^3*b^7*c*d^9*x^17 + 210/17*a^4*b^6*d^
10*x^17 + 63/4*b^10*c^5*d^5*x^16 + 525/4*a*b^9*c^4*d^6*x^16 + 675/2*a^2*b^8
*c^3*d^7*x^16 + 675/2*a^3*b^7*c^2*d^8*x^16 + 525/4*a^4*b^6*c*d^9*x^16 + 63/
4*a^5*b^5*d^10*x^16 + 14*b^10*c^6*d^4*x^15 + 168*a*b^9*c^5*d^5*x^15 + 630*a
^2*b^8*c^4*d^6*x^15 + 960*a^3*b^7*c^3*d^7*x^15 + 630*a^4*b^6*c^2*d^8*x^15 +
 168*a^5*b^5*c*d^9*x^15 + 14*a^6*b^4*d^10*x^15 + 60/7*b^10*c^7*d^3*x^14 + 1
50*a*b^9*c^6*d^4*x^14 + 810*a^2*b^8*c^5*d^5*x^14 + 1800*a^3*b^7*c^4*d^6*x^1
4 + 1800*a^4*b^6*c^3*d^7*x^14 + 810*a^5*b^5*c^2*d^8*x^14 + 150*a^6*b^4*c*d^
9*x^14 + 60/7*a^7*b^3*d^10*x^14 + 45/13*b^10*c^8*d^2*x^13 + 1200/13*a*b^9*c
^7*d^3*x^13 + 9450/13*a^2*b^8*c^6*d^4*x^13 + 30240/13*a^3*b^7*c^5*d^5*x^13
+ 44100/13*a^4*b^6*c^4*d^6*x^13 + 30240/13*a^5*b^5*c^3*d^7*x^13 + 9450/13*a
^6*b^4*c^2*d^8*x^13 + 1200/13*a^7*b^3*c*d^9*x^13 + 45/13*a^8*b^2*d^10*x^13
+ 5/6*b^10*c^9*d*x^12 + 75/2*a*b^9*c^8*d^2*x^12 + 450*a^2*b^8*c^7*d^3*x^12
+ 2100*a^3*b^7*c^6*d^4*x^12 + 4410*a^4*b^6*c^5*d^5*x^12 + 4410*a^5*b^5*c^4*
d^6*x^12 + 2100*a^6*b^4*c^3*d^7*x^12 + 450*a^7*b^3*c^2*d^8*x^12 + 75/2*a^8*
b^2*c*d^9*x^12 + 5/6*a^9*b*d^10*x^12 + 1/11*b^10*c^10*x^11 + 100/11*a*b^9*c
^9*d*x^11 + 2025/11*a^2*b^8*c^8*d^2*x^11 + 14400/11*a^3*b^7*c^7*d^3*x^11 +
44100/11*a^4*b^6*c^6*d^4*x^11 + 63504/11*a^5*b^5*c^5*d^5*x^11 + 44100/11*a^
6*b^4*c^4*d^6*x^11 + 14400/11*a^7*b^3*c^3*d^7*x^11 + 2025/11*a^8*b^2*c^2*d^
8*x^11 + 100/11*a^9*b*c*d^9*x^11 + 1/11*a^10*d^10*x^11 + a*b^9*c^10*x^10 +
45*a^2*b^8*c^9*d*x^10 + 540*a^3*b^7*c^8*d^2*x^10 + 2520*a^4*b^6*c^7*d^3*x^1
0 + 5292*a^5*b^5*c^6*d^4*x^10 + 5292*a^6*b^4*c^5*d^5*x^10 + 2520*a^7*b^3*c^
4*d^6*x^10 + 540*a^8*b^2*c^3*d^7*x^10 + 45*a^9*b*c^2*d^8*x^10 + a^10*c*d^9*
x^10 + 5*a^2*b^8*c^10*x^9 + 400/3*a^3*b^7*c^9*d*x^9 + 1050*a^4*b^6*c^8*d^2*
x^9 + 3360*a^5*b^5*c^7*d^3*x^9 + 4900*a^6*b^4*c^6*d^4*x^9 + 3360*a^7*b^3*c^
```

```
5*d^5*x^9 + 1050*a^8*b^2*c^4*d^6*x^9 + 400/3*a^9*b*c^3*d^7*x^9 + 5*a^10*c^2
*d^8*x^9 + 15*a^3*b^7*c^10*x^8 + 525/2*a^4*b^6*c^9*d*x^8 + 2835/2*a^5*b^5*c
^8*d^2*x^8 + 3150*a^6*b^4*c^7*d^3*x^8 + 3150*a^7*b^3*c^6*d^4*x^8 + 2835/2*a
^8*b^2*c^5*d^5*x^8 + 525/2*a^9*b*c^4*d^6*x^8 + 15*a^10*c^3*d^7*x^8 + 30*a^4
*b^6*c^10*x^7 + 360*a^5*b^5*c^9*d*x^7 + 1350*a^6*b^4*c^8*d^2*x^7 + 14400/7*
a^7*b^3*c^7*d^3*x^7 + 1350*a^8*b^2*c^6*d^4*x^7 + 360*a^9*b*c^5*d^5*x^7 + 30
*a^10*c^4*d^6*x^7 + 42*a^5*b^5*c^10*x^6 + 350*a^6*b^4*c^9*d*x^6 + 900*a^7*b
^3*c^8*d^2*x^6 + 900*a^8*b^2*c^7*d^3*x^6 + 350*a^9*b*c^6*d^4*x^6 + 42*a^10*
c^5*d^5*x^6 + 42*a^6*b^4*c^10*x^5 + 240*a^7*b^3*c^9*d*x^5 + 405*a^8*b^2*c^8
*d^2*x^5 + 240*a^9*b*c^7*d^3*x^5 + 42*a^10*c^6*d^4*x^5 + 30*a^7*b^3*c^10*x^
4 + 225/2*a^8*b^2*c^9*d*x^4 + 225/2*a^9*b*c^8*d^2*x^4 + 30*a^10*c^7*d^3*x^4
 + 15*a^8*b^2*c^10*x^3 + 100/3*a^9*b*c^9*d*x^3 + 15*a^10*c^8*d^2*x^3 + 5*a^
9*b*c^10*x^2 + 5*a^10*c^9*d*x^2 + a^10*c^10*x
```

Mupad [B]
time = 0.69, size = 1549, normalized size = 5.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10*(c + d*x)^10,x)
```

```
[Out] x^7*(30*a^4*b^6*c^10 + 30*a^10*c^4*d^6 + 360*a^5*b^5*c^9*d + 360*a^9*b*c^5*
d^5 + 1350*a^6*b^4*c^8*d^2 + (14400*a^7*b^3*c^7*d^3)/7 + 1350*a^8*b^2*c^6*d
^4) + x^15*(14*a^6*b^4*d^10 + 14*b^10*c^6*d^4 + 168*a*b^9*c^5*d^5 + 168*a^5
*b^5*c*d^9 + 630*a^2*b^8*c^4*d^6 + 960*a^3*b^7*c^3*d^7 + 630*a^4*b^6*c^2*d^
8) + x^5*(42*a^6*b^4*c^10 + 42*a^10*c^6*d^4 + 240*a^7*b^3*c^9*d + 240*a^9*b
*c^7*d^3 + 405*a^8*b^2*c^8*d^2) + x^17*((210*a^4*b^6*d^10)/17 + (210*b^10*c
^4*d^6)/17 + (1200*a*b^9*c^3*d^7)/17 + (1200*a^3*b^7*c*d^9)/17 + (2025*a^2*
b^8*c^2*d^8)/17) + x^11*((a^10*d^10)/11 + (b^10*c^10)/11 + (2025*a^2*b^8*c^
8*d^2)/11 + (14400*a^3*b^7*c^7*d^3)/11 + (44100*a^4*b^6*c^6*d^4)/11 + (6350
4*a^5*b^5*c^5*d^5)/11 + (44100*a^6*b^4*c^4*d^6)/11 + (14400*a^7*b^3*c^3*d^7
)/11 + (2025*a^8*b^2*c^2*d^8)/11 + (100*a*b^9*c^9*d)/11 + (100*a^9*b*c*d^9)
/11) + x^8*(15*a^3*b^7*c^10 + 15*a^10*c^3*d^7 + (525*a^4*b^6*c^9*d)/2 + (52
5*a^9*b*c^4*d^6)/2 + (2835*a^5*b^5*c^8*d^2)/2 + 3150*a^6*b^4*c^7*d^3 + 3150
*a^7*b^3*c^6*d^4 + (2835*a^8*b^2*c^5*d^5)/2) + x^14*((60*a^7*b^3*d^10)/7 +
(60*b^10*c^7*d^3)/7 + 150*a*b^9*c^6*d^4 + 150*a^6*b^4*c*d^9 + 810*a^2*b^8*c
^5*d^5 + 1800*a^3*b^7*c^4*d^6 + 1800*a^4*b^6*c^3*d^7 + 810*a^5*b^5*c^2*d^8)
 + x^10*(a*b^9*c^10 + a^10*c*d^9 + 45*a^2*b^8*c^9*d + 45*a^9*b*c^2*d^8 + 54
0*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*
b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540*a^8*b^2*c^3*d^7) + x^12*((5*a^9*b*
d^10)/6 + (5*b^10*c^9*d)/6 + (75*a*b^9*c^8*d^2)/2 + (75*a^8*b^2*c*d^9)/2 +
450*a^2*b^8*c^7*d^3 + 2100*a^3*b^7*c^6*d^4 + 4410*a^4*b^6*c^5*d^5 + 4410*a^
5*b^5*c^4*d^6 + 2100*a^6*b^4*c^3*d^7 + 450*a^7*b^3*c^2*d^8) + x^6*(42*a^5*b
^5*c^10 + 42*a^10*c^5*d^5 + 350*a^6*b^4*c^9*d + 350*a^9*b*c^6*d^4 + 900*a^7
```

*b^3*c^8*d^2 + 900*a^8*b^2*c^7*d^3) + x^16*((63*a^5*b^5*d^10)/4 + (63*b^10*c^5*d^5)/4 + (525*a*b^9*c^4*d^6)/4 + (525*a^4*b^6*c*d^9)/4 + (675*a^2*b^8*c^3*d^7)/2 + (675*a^3*b^7*c^2*d^8)/2) + x^9*(5*a^2*b^8*c^10 + 5*a^10*c^2*d^8 + (400*a^3*b^7*c^9*d)/3 + (400*a^9*b*c^3*d^7)/3 + 1050*a^4*b^6*c^8*d^2 + 3360*a^5*b^5*c^7*d^3 + 4900*a^6*b^4*c^6*d^4 + 3360*a^7*b^3*c^5*d^5 + 1050*a^8*b^2*c^4*d^6) + x^13*((45*a^8*b^2*d^10)/13 + (45*b^10*c^8*d^2)/13 + (1200*a*b^9*c^7*d^3)/13 + (1200*a^7*b^3*c*d^9)/13 + (9450*a^2*b^8*c^6*d^4)/13 + (30240*a^3*b^7*c^5*d^5)/13 + (44100*a^4*b^6*c^4*d^6)/13 + (30240*a^5*b^5*c^3*d^7)/13 + (9450*a^6*b^4*c^2*d^8)/13) + a^10*c^10*x + (b^10*d^10*x^21)/21 + (15*a^7*c^7*x^4*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d^2))/2 + (5*b^7*d^7*x^18*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d^2))/3 + 5*a^9*c^9*x^2*(a*d + b*c) + (b^9*d^9*x^20*(a*d + b*c))/2 + (5*a^8*c^8*x^3*(9*a^2*d^2 + 9*b^2*c^2 + 20*a*b*c*d))/3 + (5*b^8*d^8*x^19*(9*a^2*d^2 + 9*b^2*c^2 + 20*a*b*c*d))/19

3.1302 $\int (a+bx)^9(c+dx)^{10}\,dx$

Optimal. Leaf size=250

$$-\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\frac{6b^3(bc-ad)^6(c+dx)^{14}}{d^{10}}-\frac{42b^4(b$$

```
[Out] -1/11*(-a*d+b*c)^9*(d*x+c)^11/d^10+3/4*b*(-a*d+b*c)^8*(d*x+c)^12/d^10-36/13
*b^2*(-a*d+b*c)^7*(d*x+c)^13/d^10+6*b^3*(-a*d+b*c)^6*(d*x+c)^14/d^10-42/5*b
^4*(-a*d+b*c)^5*(d*x+c)^15/d^10+63/8*b^5*(-a*d+b*c)^4*(d*x+c)^16/d^10-84/17
*b^6*(-a*d+b*c)^3*(d*x+c)^17/d^10+2*b^7*(-a*d+b*c)^2*(d*x+c)^18/d^10-9/19*b
^8*(-a*d+b*c)*(d*x+c)^19/d^10+1/20*b^9*(d*x+c)^20/d^10
```

Rubi [A]
time = 0.93, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+dx)^{15}(bc-ad)^5}{5d^{10}}+\frac{6b^3(c+dx)^{14}(bc-ad)^6}{d^{10}}-\frac{36b^2(c+dx)^{13}(bc-ad)^7}{13d^{10}}+\frac{3b(c+dx)^{12}(bc-ad)^8}{4d^{10}}-\frac{(c+dx)^{11}(bc-ad)^9}{11d^{10}}+\frac{b^9(c+dx)^{20}}{20d^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^9*(c + d*x)^11)/d^10 + (3*b*(b*c - a*d)^8*(c + d*x)^12)/
(4*d^10) - (36*b^2*(b*c - a*d)^7*(c + d*x)^13)/(13*d^10) + (6*b^3*(b*c - a*
d)^6*(c + d*x)^14)/d^10 - (42*b^4*(b*c - a*d)^5*(c + d*x)^15)/(5*d^10) + (6
3*b^5*(b*c - a*d)^4*(c + d*x)^16)/(8*d^10) - (84*b^6*(b*c - a*d)^3*(c + d*x
)^17)/(17*d^10) + (2*b^7*(b*c - a*d)^2*(c + d*x)^18)/d^10 - (9*b^8*(b*c - a
*d)*(c + d*x)^19)/(19*d^10) + (b^9*(c + d*x)^20)/(20*d^10)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^9(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^9(c+dx)^{10}}{d^9}+\frac{9b(bc-ad)^8(c+dx)^{11}}{d^9}-\frac{36b^2(bc-ad)^7(c+dx)^{12}}{d^9}\right.$$

$$= -\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\frac{6}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1397 vs. $2(250) = 500$.
time = 0.12, size = 1397, normalized size = 5.59

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] a^9*c^10*x + (a^8*c^9*(9*b*c + 10*a*d)*x^2)/2 + 3*a^7*c^8*(4*b^2*c^2 + 10*a
*b*c*d + 5*a^2*d^2)*x^3 + (3*a^6*c^7*(28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^
2*b*c*d^2 + 40*a^3*d^3)*x^4)/4 + (6*a^5*c^6*(21*b^4*c^4 + 140*a*b^3*c^3*d +
 270*a^2*b^2*c^2*d^2 + 180*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + 3*a^4*c^5*(7*
b^5*c^5 + 70*a*b^4*c^4*d + 210*a^2*b^3*c^3*d^2 + 240*a^3*b^2*c^2*d^3 + 105*
a^4*b*c*d^4 + 14*a^5*d^5)*x^6 + 6*a^3*c^4*(2*b^6*c^6 + 30*a*b^5*c^5*d + 135
*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 54*a^5*b*c*d
^5 + 5*a^6*d^6)*x^7 + (3*a^2*c^3*(6*b^7*c^7 + 140*a*b^6*c^6*d + 945*a^2*b^5
*c^5*d^2 + 2520*a^3*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 1512*a^5*b^2*c^2*d
^5 + 315*a^6*b*c*d^6 + 20*a^7*d^7)*x^8)/4 + a*c^2*(b^8*c^8 + 40*a*b^7*c^7*d
 + 420*a^2*b^6*c^6*d^2 + 1680*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2352
*a^5*b^3*c^3*d^5 + 840*a^6*b^2*c^2*d^6 + 120*a^7*b*c*d^7 + 5*a^8*d^8)*x^9 +
 (c*(b^9*c^9 + 90*a*b^8*c^8*d + 1620*a^2*b^7*c^7*d^2 + 10080*a^3*b^6*c^6*d^
3 + 26460*a^4*b^5*c^5*d^4 + 31752*a^5*b^4*c^4*d^5 + 17640*a^6*b^3*c^3*d^6 +
 4320*a^7*b^2*c^2*d^7 + 405*a^8*b*c*d^8 + 10*a^9*d^9)*x^10)/10 + (d*(10*b^9
*c^9 + 405*a*b^8*c^8*d + 4320*a^2*b^7*c^7*d^2 + 17640*a^3*b^6*c^6*d^3 + 317
52*a^4*b^5*c^5*d^4 + 26460*a^5*b^4*c^4*d^5 + 10080*a^6*b^3*c^3*d^6 + 1620*a
^7*b^2*c^2*d^7 + 90*a^8*b*c*d^8 + a^9*d^9)*x^11)/11 + (3*b*d^2*(5*b^8*c^8 +
 120*a*b^7*c^7*d + 840*a^2*b^6*c^6*d^2 + 2352*a^3*b^5*c^5*d^3 + 2940*a^4*b^
4*c^4*d^4 + 1680*a^5*b^3*c^3*d^5 + 420*a^6*b^2*c^2*d^6 + 40*a^7*b*c*d^7 + a
^8*d^8)*x^12)/4 + (6*b^2*d^3*(20*b^7*c^7 + 315*a*b^6*c^6*d + 1512*a^2*b^5*c
^5*d^2 + 2940*a^3*b^4*c^4*d^3 + 2520*a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5
+ 140*a^6*b*c*d^6 + 6*a^7*d^7)*x^13)/13 + 3*b^3*d^4*(5*b^6*c^6 + 54*a*b^5*c
^5*d + 180*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 135*a^4*b^2*c^2*d^4 + 30
*a^5*b*c*d^5 + 2*a^6*d^6)*x^14 + (6*b^4*d^5*(14*b^5*c^5 + 105*a*b^4*c^4*d +
 240*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d^5)*x^
15)/5 + (3*b^5*d^6*(35*b^4*c^4 + 180*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 14
0*a^3*b*c*d^3 + 21*a^4*d^4)*x^16)/8 + (3*b^6*d^7*(40*b^3*c^3 + 135*a*b^2*c^
2*d + 120*a^2*b*c*d^2 + 28*a^3*d^3)*x^17)/17 + (b^7*d^8*(5*b^2*c^2 + 10*a*b
*c*d + 4*a^2*d^2)*x^18)/2 + (b^8*d^9*(10*b*c + 9*a*d)*x^19)/19 + (b^9*d^10*
x^20)/20
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1440 vs. $2(234) = 468$.
time = 0.14, size = 1441, normalized size = 5.76

method	result
norman	Expression too large to display
default	$\frac{b^9 d^{10} x^{20}}{20}+\frac{(9a\,b^8 d^{10}+10b^9 c\,d^9)x^{19}}{19}+\frac{(36a^2 b^7 d^{10}+90a\,b^8 c\,d^9+45b^9 c^2 d^8)x^{18}}{18}+\frac{(84a^3 b^6 d^{10}+360a^2 b^7 c\,d^9+405a\,b^8 c^2 d^8+120b^9 c^3}{17}$
gosper	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*b^9*d^10*x^20+1/19*(9*a*b^8*d^10+10*b^9*c*d^9)*x^19+1/18*(36*a^2*b^7*d
^10+90*a*b^8*c*d^9+45*b^9*c^2*d^8)*x^18+1/17*(84*a^3*b^6*d^10+360*a^2*b^7*c
*d^9+405*a*b^8*c^2*d^8+120*b^9*c^3*d^7)*x^17+1/16*(126*a^4*b^5*d^10+840*a^3
*b^6*c*d^9+1620*a^2*b^7*c^2*d^8+1080*a*b^8*c^3*d^7+210*b^9*c^4*d^6)*x^16+1/
15*(126*a^5*b^4*d^10+1260*a^4*b^5*c*d^9+3780*a^3*b^6*c^2*d^8+4320*a^2*b^7*c
^3*d^7+1890*a*b^8*c^4*d^6+252*b^9*c^5*d^5)*x^15+1/14*(84*a^6*b^3*d^10+1260*
a^5*b^4*c*d^9+5670*a^4*b^5*c^2*d^8+10080*a^3*b^6*c^3*d^7+7560*a^2*b^7*c^4*d
^6+2268*a*b^8*c^5*d^5+210*b^9*c^6*d^4)*x^14+1/13*(36*a^7*b^2*d^10+840*a^6*b
^3*c*d^9+5670*a^5*b^4*c^2*d^8+15120*a^4*b^5*c^3*d^7+17640*a^3*b^6*c^4*d^6+9
072*a^2*b^7*c^5*d^5+1890*a*b^8*c^6*d^4+120*b^9*c^7*d^3)*x^13+1/12*(9*a^8*b*
d^10+360*a^7*b^2*c*d^9+3780*a^6*b^3*c^2*d^8+15120*a^5*b^4*c^3*d^7+26460*a^4
*b^5*c^4*d^6+21168*a^3*b^6*c^5*d^5+7560*a^2*b^7*c^6*d^4+1080*a*b^8*c^7*d^3+
45*b^9*c^8*d^2)*x^12+1/11*(a^9*d^10+90*a^8*b*c*d^9+1620*a^7*b^2*c^2*d^8+100
80*a^6*b^3*c^3*d^7+26460*a^5*b^4*c^4*d^6+31752*a^4*b^5*c^5*d^5+17640*a^3*b^
6*c^6*d^4+4320*a^2*b^7*c^7*d^3+405*a*b^8*c^8*d^2+10*b^9*c^9*d)*x^11+1/10*(1
0*a^9*c*d^9+405*a^8*b*c^2*d^8+4320*a^7*b^2*c^3*d^7+17640*a^6*b^3*c^4*d^6+31
752*a^5*b^4*c^5*d^5+26460*a^4*b^5*c^6*d^4+10080*a^3*b^6*c^7*d^3+1620*a^2*b^
7*c^8*d^2+90*a*b^8*c^9*d+b^9*c^10)*x^10+1/9*(45*a^9*c^2*d^8+1080*a^8*b*c^3*
d^7+7560*a^7*b^2*c^4*d^6+21168*a^6*b^3*c^5*d^5+26460*a^5*b^4*c^6*d^4+15120*
a^4*b^5*c^7*d^3+3780*a^3*b^6*c^8*d^2+360*a^2*b^7*c^9*d+9*a*b^8*c^10)*x^9+1/
8*(120*a^9*c^3*d^7+1890*a^8*b*c^4*d^6+9072*a^7*b^2*c^5*d^5+17640*a^6*b^3*c^
6*d^4+15120*a^5*b^4*c^7*d^3+5670*a^4*b^5*c^8*d^2+840*a^3*b^6*c^9*d+36*a^2*b
^7*c^10)*x^8+1/7*(210*a^9*c^4*d^6+2268*a^8*b*c^5*d^5+7560*a^7*b^2*c^6*d^4+1
0080*a^6*b^3*c^7*d^3+5670*a^5*b^4*c^8*d^2+1260*a^4*b^5*c^9*d+84*a^3*b^6*c^1
0)*x^7+1/6*(252*a^9*c^5*d^5+1890*a^8*b*c^6*d^4+4320*a^7*b^2*c^7*d^3+3780*a^
6*b^3*c^8*d^2+1260*a^5*b^4*c^9*d+126*a^4*b^5*c^10)*x^6+1/5*(210*a^9*c^6*d^4
+1080*a^8*b*c^7*d^3+1620*a^7*b^2*c^8*d^2+840*a^6*b^3*c^9*d+126*a^5*b^4*c^10
)*x^5+1/4*(120*a^9*c^7*d^3+405*a^8*b*c^8*d^2+360*a^7*b^2*c^9*d+84*a^6*b^3*c
^10)*x^4+1/3*(45*a^9*c^8*d^2+90*a^8*b*c^9*d+36*a^7*b^2*c^10)*x^3+1/2*(10*a^
9*c^9*d+9*a^8*b*c^10)*x^2+a^9*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(234) = 468$.

time = 0.29, size = 1437, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 +
 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*
c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3
/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. $2(234) = 468$.

time = 0.55, size = 1437, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 +
 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*
c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3
/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(231) = 462$.
time = 0.11, size = 1598, normalized size = 6.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**10,x)
```

```
[Out] a**9*c**10*x + b**9*d**10*x**20/20 + x**19*(9*a*b**8*d**10/19 + 10*b**9*c*d
**9/19) + x**18*(2*a**2*b**7*d**10 + 5*a*b**8*c*d**9 + 5*b**9*c**2*d**8/2)
+ x**17*(84*a**3*b**6*d**10/17 + 360*a**2*b**7*c*d**9/17 + 405*a*b**8*c**2*
d**8/17 + 120*b**9*c**3*d**7/17) + x**16*(63*a**4*b**5*d**10/8 + 105*a**3*b
**6*c*d**9/2 + 405*a**2*b**7*c**2*d**8/4 + 135*a*b**8*c**3*d**7/2 + 105*b**
9*c**4*d**6/8) + x**15*(42*a**5*b**4*d**10/5 + 84*a**4*b**5*c*d**9 + 252*a*
*3*b**6*c**2*d**8 + 288*a**2*b**7*c**3*d**7 + 126*a*b**8*c**4*d**6 + 84*b**
9*c**5*d**5/5) + x**14*(6*a**6*b**3*d**10 + 90*a**5*b**4*c*d**9 + 405*a**4*
b**5*c**2*d**8 + 720*a**3*b**6*c**3*d**7 + 540*a**2*b**7*c**4*d**6 + 162*a*
b**8*c**5*d**5 + 15*b**9*c**6*d**4) + x**13*(36*a**7*b**2*d**10/13 + 840*a*
*6*b**3*c*d**9/13 + 5670*a**5*b**4*c**2*d**8/13 + 15120*a**4*b**5*c**3*d**7
/13 + 17640*a**3*b**6*c**4*d**6/13 + 9072*a**2*b**7*c**5*d**5/13 + 1890*a*b
**8*c**6*d**4/13 + 120*b**9*c**7*d**3/13) + x**12*(3*a**8*b*d**10/4 + 30*a*
*7*b**2*c*d**9 + 315*a**6*b**3*c**2*d**8 + 1260*a**5*b**4*c**3*d**7 + 2205*
a**4*b**5*c**4*d**6 + 1764*a**3*b**6*c**5*d**5 + 630*a**2*b**7*c**6*d**4 +
90*a*b**8*c**7*d**3 + 15*b**9*c**8*d**2/4) + x**11*(a**9*d**10/11 + 90*a**8
*b*c*d**9/11 + 1620*a**7*b**2*c**2*d**8/11 + 10080*a**6*b**3*c**3*d**7/11 +
 26460*a**5*b**4*c**4*d**6/11 + 31752*a**4*b**5*c**5*d**5/11 + 17640*a**3*b
**6*c**6*d**4/11 + 4320*a**2*b**7*c**7*d**3/11 + 405*a*b**8*c**8*d**2/11 +
10*b**9*c**9*d/11) + x**10*(a**9*c*d**9 + 81*a**8*b*c**2*d**8/2 + 432*a**7*
b**2*c**3*d**7 + 1764*a**6*b**3*c**4*d**6 + 15876*a**5*b**4*c**5*d**5/5 + 2
646*a**4*b**5*c**6*d**4 + 1008*a**3*b**6*c**7*d**3 + 162*a**2*b**7*c**8*d**
2 + 9*a*b**8*c**9*d + b**9*c**10/10) + x**9*(5*a**9*c**2*d**8 + 120*a**8*b*
c**3*d**7 + 840*a**7*b**2*c**4*d**6 + 2352*a**6*b**3*c**5*d**5 + 2940*a**5*
b**4*c**6*d**4 + 1680*a**4*b**5*c**7*d**3 + 420*a**3*b**6*c**8*d**2 + 40*a*
*2*b**7*c**9*d + a*b**8*c**10) + x**8*(15*a**9*c**3*d**7 + 945*a**8*b*c**4*
d**6/4 + 1134*a**7*b**2*c**5*d**5 + 2205*a**6*b**3*c**6*d**4 + 1890*a**5*b*
*4*c**7*d**3 + 2835*a**4*b**5*c**8*d**2/4 + 105*a**3*b**6*c**9*d + 9*a**2*b
**7*c**10/2) + x**7*(30*a**9*c**4*d**6 + 324*a**8*b*c**5*d**5 + 1080*a**7*b
**2*c**6*d**4 + 1440*a**6*b**3*c**7*d**3 + 810*a**5*b**4*c**8*d**2 + 180*a*
*4*b**5*c**9*d + 12*a**3*b**6*c**10) + x**6*(42*a**9*c**5*d**5 + 315*a**8*b
*c**6*d**4 + 720*a**7*b**2*c**7*d**3 + 630*a**6*b**3*c**8*d**2 + 210*a**5*b
**4*c**9*d + 21*a**4*b**5*c**10) + x**5*(42*a**9*c**6*d**4 + 216*a**8*b*c**
7*d**3 + 324*a**7*b**2*c**8*d**2 + 168*a**6*b**3*c**9*d + 126*a**5*b**4*c**
10/5) + x**4*(30*a**9*c**7*d**3 + 405*a**8*b*c**8*d**2/4 + 90*a**7*b**2*c**
9*d + 21*a**6*b**3*c**10) + x**3*(15*a**9*c**8*d**2 + 30*a**8*b*c**9*d + 12
*a**7*b**2*c**10) + x**2*(5*a**9*c**9*d + 9*a**8*b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(234) = 468$.
time = 0.90, size = 1656, normalized size = 6.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^9*d^10*x^20 + 10/19*b^9*c*d^9*x^19 + 9/19*a*b^8*d^10*x^19 + 5/2*b^9*
c^2*d^8*x^18 + 5*a*b^8*c*d^9*x^18 + 2*a^2*b^7*d^10*x^18 + 120/17*b^9*c^3*d^
7*x^17 + 405/17*a*b^8*c^2*d^8*x^17 + 360/17*a^2*b^7*c*d^9*x^17 + 84/17*a^3*
b^6*d^10*x^17 + 105/8*b^9*c^4*d^6*x^16 + 135/2*a*b^8*c^3*d^7*x^16 + 405/4*a
^2*b^7*c^2*d^8*x^16 + 105/2*a^3*b^6*c*d^9*x^16 + 63/8*a^4*b^5*d^10*x^16 + 8
4/5*b^9*c^5*d^5*x^15 + 126*a*b^8*c^4*d^6*x^15 + 288*a^2*b^7*c^3*d^7*x^15 +
252*a^3*b^6*c^2*d^8*x^15 + 84*a^4*b^5*c*d^9*x^15 + 42/5*a^5*b^4*d^10*x^15 +
 15*b^9*c^6*d^4*x^14 + 162*a*b^8*c^5*d^5*x^14 + 540*a^2*b^7*c^4*d^6*x^14 +
720*a^3*b^6*c^3*d^7*x^14 + 405*a^4*b^5*c^2*d^8*x^14 + 90*a^5*b^4*c*d^9*x^14
 + 6*a^6*b^3*d^10*x^14 + 120/13*b^9*c^7*d^3*x^13 + 1890/13*a*b^8*c^6*d^4*x^
13 + 9072/13*a^2*b^7*c^5*d^5*x^13 + 17640/13*a^3*b^6*c^4*d^6*x^13 + 15120/1
3*a^4*b^5*c^3*d^7*x^13 + 5670/13*a^5*b^4*c^2*d^8*x^13 + 840/13*a^6*b^3*c*d^
9*x^13 + 36/13*a^7*b^2*d^10*x^13 + 15/4*b^9*c^8*d^2*x^12 + 90*a*b^8*c^7*d^3
*x^12 + 630*a^2*b^7*c^6*d^4*x^12 + 1764*a^3*b^6*c^5*d^5*x^12 + 2205*a^4*b^5
*c^4*d^6*x^12 + 1260*a^5*b^4*c^3*d^7*x^12 + 315*a^6*b^3*c^2*d^8*x^12 + 30*a
^7*b^2*c*d^9*x^12 + 3/4*a^8*b*d^10*x^12 + 10/11*b^9*c^9*d*x^11 + 405/11*a*b
^8*c^8*d^2*x^11 + 4320/11*a^2*b^7*c^7*d^3*x^11 + 17640/11*a^3*b^6*c^6*d^4*x
^11 + 31752/11*a^4*b^5*c^5*d^5*x^11 + 26460/11*a^5*b^4*c^4*d^6*x^11 + 10080
/11*a^6*b^3*c^3*d^7*x^11 + 1620/11*a^7*b^2*c^2*d^8*x^11 + 90/11*a^8*b*c*d^9
*x^11 + 1/11*a^9*d^10*x^11 + 1/10*b^9*c^10*x^10 + 9*a*b^8*c^9*d*x^10 + 162*
a^2*b^7*c^8*d^2*x^10 + 1008*a^3*b^6*c^7*d^3*x^10 + 2646*a^4*b^5*c^6*d^4*x^1
0 + 15876/5*a^5*b^4*c^5*d^5*x^10 + 1764*a^6*b^3*c^4*d^6*x^10 + 432*a^7*b^2*
c^3*d^7*x^10 + 81/2*a^8*b*c^2*d^8*x^10 + a^9*c*d^9*x^10 + a*b^8*c^10*x^9 +
40*a^2*b^7*c^9*d*x^9 + 420*a^3*b^6*c^8*d^2*x^9 + 1680*a^4*b^5*c^7*d^3*x^9 +
 2940*a^5*b^4*c^6*d^4*x^9 + 2352*a^6*b^3*c^5*d^5*x^9 + 840*a^7*b^2*c^4*d^6*
x^9 + 120*a^8*b*c^3*d^7*x^9 + 5*a^9*c^2*d^8*x^9 + 9/2*a^2*b^7*c^10*x^8 + 10
5*a^3*b^6*c^9*d*x^8 + 2835/4*a^4*b^5*c^8*d^2*x^8 + 1890*a^5*b^4*c^7*d^3*x^8
 + 2205*a^6*b^3*c^6*d^4*x^8 + 1134*a^7*b^2*c^5*d^5*x^8 + 945/4*a^8*b*c^4*d^
6*x^8 + 15*a^9*c^3*d^7*x^8 + 12*a^3*b^6*c^10*x^7 + 180*a^4*b^5*c^9*d*x^7 +
810*a^5*b^4*c^8*d^2*x^7 + 1440*a^6*b^3*c^7*d^3*x^7 + 1080*a^7*b^2*c^6*d^4*x
^7 + 324*a^8*b*c^5*d^5*x^7 + 30*a^9*c^4*d^6*x^7 + 21*a^4*b^5*c^10*x^6 + 210
*a^5*b^4*c^9*d*x^6 + 630*a^6*b^3*c^8*d^2*x^6 + 720*a^7*b^2*c^7*d^3*x^6 + 31
5*a^8*b*c^6*d^4*x^6 + 42*a^9*c^5*d^5*x^6 + 126/5*a^5*b^4*c^10*x^5 + 168*a^6
*b^3*c^9*d*x^5 + 324*a^7*b^2*c^8*d^2*x^5 + 216*a^8*b*c^7*d^3*x^5 + 42*a^9*c
^6*d^4*x^5 + 21*a^6*b^3*c^10*x^4 + 90*a^7*b^2*c^9*d*x^4 + 405/4*a^8*b*c^8*d
^2*x^4 + 30*a^9*c^7*d^3*x^4 + 12*a^7*b^2*c^10*x^3 + 30*a^8*b*c^9*d*x^3 + 15
*a^9*c^8*d^2*x^3 + 9/2*a^8*b*c^10*x^2 + 5*a^9*c^9*d*x^2 + a^9*c^10*x
```

Mupad [B]
time = 0.79, size = 1404, normalized size = 5.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^10,x)
```

```
[Out] x^7*(12*a^3*b^6*c^10 + 30*a^9*c^4*d^6 + 180*a^4*b^5*c^9*d + 324*a^8*b*c^5*d
^5 + 810*a^5*b^4*c^8*d^2 + 1440*a^6*b^3*c^7*d^3 + 1080*a^7*b^2*c^6*d^4) + x
^14*(6*a^6*b^3*d^10 + 15*b^9*c^6*d^4 + 162*a*b^8*c^5*d^5 + 90*a^5*b^4*c*d^9
 + 540*a^2*b^7*c^4*d^6 + 720*a^3*b^6*c^3*d^7 + 405*a^4*b^5*c^2*d^8) + x^5*(
(126*a^5*b^4*c^10)/5 + 42*a^9*c^6*d^4 + 168*a^6*b^3*c^9*d + 216*a^8*b*c^7*d
^3 + 324*a^7*b^2*c^8*d^2) + x^16*((63*a^4*b^5*d^10)/8 + (105*b^9*c^4*d^6)/8
 + (135*a*b^8*c^3*d^7)/2 + (105*a^3*b^6*c*d^9)/2 + (405*a^2*b^7*c^2*d^8)/4)
 + x^8*((9*a^2*b^7*c^10)/2 + 15*a^9*c^3*d^7 + 105*a^3*b^6*c^9*d + (945*a^8*
b*c^4*d^6)/4 + (2835*a^4*b^5*c^8*d^2)/4 + 1890*a^5*b^4*c^7*d^3 + 2205*a^6*b
^3*c^6*d^4 + 1134*a^7*b^2*c^5*d^5) + x^13*((36*a^7*b^2*d^10)/13 + (120*b^9*
c^7*d^3)/13 + (1890*a*b^8*c^6*d^4)/13 + (840*a^6*b^3*c*d^9)/13 + (9072*a^2*
b^7*c^5*d^5)/13 + (17640*a^3*b^6*c^4*d^6)/13 + (15120*a^4*b^5*c^3*d^7)/13 +
 (5670*a^5*b^4*c^2*d^8)/13) + x^9*(a*b^8*c^10 + 5*a^9*c^2*d^8 + 40*a^2*b^7*
c^9*d + 120*a^8*b*c^3*d^7 + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 29
40*a^5*b^4*c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6) + x^12*((3
*a^8*b*d^10)/4 + (15*b^9*c^8*d^2)/4 + 90*a*b^8*c^7*d^3 + 30*a^7*b^2*c*d^9 +
 630*a^2*b^7*c^6*d^4 + 1764*a^3*b^6*c^5*d^5 + 2205*a^4*b^5*c^4*d^6 + 1260*a
^5*b^4*c^3*d^7 + 315*a^6*b^3*c^2*d^8) + x^6*(21*a^4*b^5*c^10 + 42*a^9*c^5*d
^5 + 210*a^5*b^4*c^9*d + 315*a^8*b*c^6*d^4 + 630*a^6*b^3*c^8*d^2 + 720*a^7*
b^2*c^7*d^3) + x^15*((42*a^5*b^4*d^10)/5 + (84*b^9*c^5*d^5)/5 + 126*a*b^8*c
^4*d^6 + 84*a^4*b^5*c*d^9 + 288*a^2*b^7*c^3*d^7 + 252*a^3*b^6*c^2*d^8) + x^
10*((b^9*c^10)/10 + a^9*c*d^9 + (81*a^8*b*c^2*d^8)/2 + 162*a^2*b^7*c^8*d^2
+ 1008*a^3*b^6*c^7*d^3 + 2646*a^4*b^5*c^6*d^4 + (15876*a^5*b^4*c^5*d^5)/5 +
 1764*a^6*b^3*c^4*d^6 + 432*a^7*b^2*c^3*d^7 + 9*a*b^8*c^9*d) + x^11*((a^9*d
^10)/11 + (10*b^9*c^9*d)/11 + (405*a*b^8*c^8*d^2)/11 + (4320*a^2*b^7*c^7*d^
3)/11 + (17640*a^3*b^6*c^6*d^4)/11 + (31752*a^4*b^5*c^5*d^5)/11 + (26460*a^
5*b^4*c^4*d^6)/11 + (10080*a^6*b^3*c^3*d^7)/11 + (1620*a^7*b^2*c^2*d^8)/11
+ (90*a^8*b*c*d^9)/11) + a^9*c^10*x + (b^9*d^10*x^20)/20 + (3*a^6*c^7*x^4*(
40*a^3*d^3 + 28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^2*b*c*d^2))/4 + (3*b^6*d^
7*x^17*(28*a^3*d^3 + 40*b^3*c^3 + 135*a*b^2*c^2*d + 120*a^2*b*c*d^2))/17 +
(a^8*c^9*x^2*(10*a*d + 9*b*c))/2 + (b^8*d^9*x^19*(9*a*d + 10*b*c))/19 + 3*a
^7*c^8*x^3*(5*a^2*d^2 + 4*b^2*c^2 + 10*a*b*c*d) + (b^7*d^8*x^18*(4*a^2*d^2
+ 5*b^2*c^2 + 10*a*b*c*d))/2
```

3.1303 $\int (a+bx)^8(c+dx)^{10}\,dx$

Optimal. Leaf size=225

$$\frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3(bc-ad)^5(c+dx)^{14}}{d^9}+\frac{14b^4(bc-$$

```
[Out] 1/11*(-a*d+b*c)^8*(d*x+c)^11/d^9-2/3*b*(-a*d+b*c)^7*(d*x+c)^12/d^9+28/13*b^
2*(-a*d+b*c)^6*(d*x+c)^13/d^9-4*b^3*(-a*d+b*c)^5*(d*x+c)^14/d^9+14/3*b^4*(-
a*d+b*c)^4*(d*x+c)^15/d^9-7/2*b^5*(-a*d+b*c)^3*(d*x+c)^16/d^9+28/17*b^6*(-a
*d+b*c)^2*(d*x+c)^17/d^9-4/9*b^7*(-a*d+b*c)*(d*x+c)^18/d^9+1/19*b^8*(d*x+c)
^19/d^9
```

Rubi [A]
time = 0.62, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+dx)^{14}(bc-ad)^5}{d^9}+\frac{28b^2(c+dx)^{13}(bc-ad)^6}{13d^9}-\frac{2b(c+dx)^{12}(bc-ad)^7}{3d^9}+\frac{(c+dx)^{11}(bc-ad)^8}{11d^9}+\frac{b^8(c+dx)^{19}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^8*(c + d*x)^11)/(11*d^9) - (2*b*(b*c - a*d)^7*(c + d*x)^12)/(3
*d^9) + (28*b^2*(b*c - a*d)^6*(c + d*x)^13)/(13*d^9) - (4*b^3*(b*c - a*d)^5
*(c + d*x)^14)/d^9 + (14*b^4*(b*c - a*d)^4*(c + d*x)^15)/(3*d^9) - (7*b^5*(
b*c - a*d)^3*(c + d*x)^16)/(2*d^9) + (28*b^6*(b*c - a*d)^2*(c + d*x)^17)/(1
7*d^9) - (4*b^7*(b*c - a*d)*(c + d*x)^18)/(9*d^9) + (b^8*(c + d*x)^19)/(19*
d^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^8(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^8(c+dx)^{10}}{d^8}-\frac{8b(bc-ad)^7(c+dx)^{11}}{d^8}+\frac{28b^2(bc-ad)^6(c+dx)^{12}}{d^8}\right.$$

$$=\frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1241 vs. $2(225) = 450$.
time = 0.11, size = 1241, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] a^8*c^10*x + a^7*c^9*(4*b*c + 5*a*d)*x^2 + (a^6*c^8*(28*b^2*c^2 + 80*a*b*c*
d + 45*a^2*d^2)*x^3)/3 + 2*a^5*c^7*(7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c
*d^2 + 15*a^3*d^3)*x^4 + 2*a^4*c^6*(7*b^4*c^4 + 56*a*b^3*c^3*d + 126*a^2*b^
2*c^2*d^2 + 96*a^3*b*c*d^3 + 21*a^4*d^4)*x^5 + (14*a^3*c^5*(2*b^5*c^5 + 25*
a*b^4*c^4*d + 90*a^2*b^3*c^3*d^2 + 120*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 + 9
*a^5*d^5)*x^6)/3 + 2*a^2*c^4*(2*b^6*c^6 + 40*a*b^5*c^5*d + 225*a^2*b^4*c^4*
d^2 + 480*a^3*b^3*c^3*d^3 + 420*a^4*b^2*c^2*d^4 + 144*a^5*b*c*d^5 + 15*a^6*
d^6)*x^7 + a*c^3*(b^7*c^7 + 35*a*b^6*c^6*d + 315*a^2*b^5*c^5*d^2 + 1050*a^3
*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*d^5 + 210*a^6*b*c*d^6
 + 15*a^7*d^7)*x^8 + (c^2*(b^8*c^8 + 80*a*b^7*c^7*d + 1260*a^2*b^6*c^6*d^2
+ 6720*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 14112*a^5*b^3*c^3*d^5 + 58
80*a^6*b^2*c^2*d^6 + 960*a^7*b*c*d^7 + 45*a^8*d^8)*x^9)/9 + c*d*(b^8*c^8 +
36*a*b^7*c^7*d + 336*a^2*b^6*c^6*d^2 + 1176*a^3*b^5*c^5*d^3 + 1764*a^4*b^4*
c^4*d^4 + 1176*a^5*b^3*c^3*d^5 + 336*a^6*b^2*c^2*d^6 + 36*a^7*b*c*d^7 + a^8
*d^8)*x^10 + (d^2*(45*b^8*c^8 + 960*a*b^7*c^7*d + 5880*a^2*b^6*c^6*d^2 + 14
112*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 6720*a^5*b^3*c^3*d^5 + 1260*a
^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + a^8*d^8)*x^11)/11 + (2*b*d^3*(15*b^7*c^7
+ 210*a*b^6*c^6*d + 882*a^2*b^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1050*a^4*b
^3*c^3*d^4 + 315*a^5*b^2*c^2*d^5 + 35*a^6*b*c*d^6 + a^7*d^7)*x^12)/3 + (14*
b^2*d^4*(15*b^6*c^6 + 144*a*b^5*c^5*d + 420*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c
^3*d^3 + 225*a^4*b^2*c^2*d^4 + 40*a^5*b*c*d^5 + 2*a^6*d^6)*x^13)/13 + 2*b^3
*d^5*(9*b^5*c^5 + 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3
 + 25*a^4*b*c*d^4 + 2*a^5*d^5)*x^14 + (2*b^4*d^6*(21*b^4*c^4 + 96*a*b^3*c^3
*d + 126*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 7*a^4*d^4)*x^15)/3 + (b^5*d^7*(
15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2 + 7*a^3*d^3)*x^16)/2 + (b^6*d^
8*(45*b^2*c^2 + 80*a*b*c*d + 28*a^2*d^2)*x^17)/17 + (b^7*d^9*(5*b*c + 4*a*d
)*x^18)/9 + (b^8*d^10*x^19)/19
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1290 vs. $2(209) = 418$.
time = 0.15, size = 1291, normalized size = 5.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*b^8*d^10*x^19+1/18*(8*a*b^7*d^10+10*b^8*c*d^9)*x^18+1/17*(28*a^2*b^6*d
^10+80*a*b^7*c*d^9+45*b^8*c^2*d^8)*x^17+1/16*(56*a^3*b^5*d^10+280*a^2*b^6*c
```

```
*d^9+360*a*b^7*c^2*d^8+120*b^8*c^3*d^7)*x^16+1/15*(70*a^4*b^4*d^10+560*a^3*
b^5*c*d^9+1260*a^2*b^6*c^2*d^8+960*a*b^7*c^3*d^7+210*b^8*c^4*d^6)*x^15+1/14
*(56*a^5*b^3*d^10+700*a^4*b^4*c*d^9+2520*a^3*b^5*c^2*d^8+3360*a^2*b^6*c^3*d
^7+1680*a*b^7*c^4*d^6+252*b^8*c^5*d^5)*x^14+1/13*(28*a^6*b^2*d^10+560*a^5*b
^3*c*d^9+3150*a^4*b^4*c^2*d^8+6720*a^3*b^5*c^3*d^7+5880*a^2*b^6*c^4*d^6+201
6*a*b^7*c^5*d^5+210*b^8*c^6*d^4)*x^13+1/12*(8*a^7*b*d^10+280*a^6*b^2*c*d^9+
2520*a^5*b^3*c^2*d^8+8400*a^4*b^4*c^3*d^7+11760*a^3*b^5*c^4*d^6+7056*a^2*b^
6*c^5*d^5+1680*a*b^7*c^6*d^4+120*b^8*c^7*d^3)*x^12+1/11*(a^8*d^10+80*a^7*b*
c*d^9+1260*a^6*b^2*c^2*d^8+6720*a^5*b^3*c^3*d^7+14700*a^4*b^4*c^4*d^6+14112
*a^3*b^5*c^5*d^5+5880*a^2*b^6*c^6*d^4+960*a*b^7*c^7*d^3+45*b^8*c^8*d^2)*x^1
1+1/10*(10*a^8*c*d^9+360*a^7*b*c^2*d^8+3360*a^6*b^2*c^3*d^7+11760*a^5*b^3*c
^4*d^6+17640*a^4*b^4*c^5*d^5+11760*a^3*b^5*c^6*d^4+3360*a^2*b^6*c^7*d^3+360
*a*b^7*c^8*d^2+10*b^8*c^9*d)*x^10+1/9*(45*a^8*c^2*d^8+960*a^7*b*c^3*d^7+588
0*a^6*b^2*c^4*d^6+14112*a^5*b^3*c^5*d^5+14700*a^4*b^4*c^6*d^4+6720*a^3*b^5*
c^7*d^3+1260*a^2*b^6*c^8*d^2+80*a*b^7*c^9*d+b^8*c^10)*x^9+1/8*(120*a^8*c^3*
d^7+1680*a^7*b*c^4*d^6+7056*a^6*b^2*c^5*d^5+11760*a^5*b^3*c^6*d^4+8400*a^4*
b^4*c^7*d^3+2520*a^3*b^5*c^8*d^2+280*a^2*b^6*c^9*d+8*a*b^7*c^10)*x^8+1/7*(2
10*a^8*c^4*d^6+2016*a^7*b*c^5*d^5+5880*a^6*b^2*c^6*d^4+6720*a^5*b^3*c^7*d^3
+3150*a^4*b^4*c^8*d^2+560*a^3*b^5*c^9*d+28*a^2*b^6*c^10)*x^7+1/6*(252*a^8*c
^5*d^5+1680*a^7*b*c^6*d^4+3360*a^6*b^2*c^7*d^3+2520*a^5*b^3*c^8*d^2+700*a^4
*b^4*c^9*d+56*a^3*b^5*c^10)*x^6+1/5*(210*a^8*c^6*d^4+960*a^7*b*c^7*d^3+1260
*a^6*b^2*c^8*d^2+560*a^5*b^3*c^9*d+70*a^4*b^4*c^10)*x^5+1/4*(120*a^8*c^7*d^
3+360*a^7*b*c^8*d^2+280*a^6*b^2*c^9*d+56*a^5*b^3*c^10)*x^4+1/3*(45*a^8*c^8*
d^2+80*a^7*b*c^9*d+28*a^6*b^2*c^10)*x^3+1/2*(10*a^8*c^9*d+8*a^7*b*c^10)*x^2
+a^8*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(209) = 418$.
time = 0.28, size = 1283, normalized size = 5.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
```

```
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1283 vs. $2(209) = 418$.
time = 0.59, size = 1283, normalized size = 5.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
```

```
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. $2(207) = 414$.
time = 0.10, size = 1428, normalized size = 6.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**10,x)
```

```
[Out] a**8*c**10*x + b**8*d**10*x**19/19 + x**18*(4*a*b**7*d**10/9 + 5*b**8*c*d**
9/9) + x**17*(28*a**2*b**6*d**10/17 + 80*a*b**7*c*d**9/17 + 45*b**8*c**2*d*
*8/17) + x**16*(7*a**3*b**5*d**10/2 + 35*a**2*b**6*c*d**9/2 + 45*a*b**7*c**
2*d**8/2 + 15*b**8*c**3*d**7/2) + x**15*(14*a**4*b**4*d**10/3 + 112*a**3*b*
*5*c*d**9/3 + 84*a**2*b**6*c**2*d**8 + 64*a*b**7*c**3*d**7 + 14*b**8*c**4*d
**6) + x**14*(4*a**5*b**3*d**10 + 50*a**4*b**4*c*d**9 + 180*a**3*b**5*c**2*
d**8 + 240*a**2*b**6*c**3*d**7 + 120*a*b**7*c**4*d**6 + 18*b**8*c**5*d**5)
+ x**13*(28*a**6*b**2*d**10/13 + 560*a**5*b**3*c*d**9/13 + 3150*a**4*b**4*c
**2*d**8/13 + 6720*a**3*b**5*c**3*d**7/13 + 5880*a**2*b**6*c**4*d**6/13 + 2
016*a*b**7*c**5*d**5/13 + 210*b**8*c**6*d**4/13) + x**12*(2*a**7*b*d**10/3
+ 70*a**6*b**2*c*d**9/3 + 210*a**5*b**3*c**2*d**8 + 700*a**4*b**4*c**3*d**7
 + 980*a**3*b**5*c**4*d**6 + 588*a**2*b**6*c**5*d**5 + 140*a*b**7*c**6*d**4
 + 10*b**8*c**7*d**3) + x**11*(a**8*d**10/11 + 80*a**7*b*c*d**9/11 + 1260*a
**6*b**2*c**2*d**8/11 + 6720*a**5*b**3*c**3*d**7/11 + 14700*a**4*b**4*c**4*
d**6/11 + 14112*a**3*b**5*c**5*d**5/11 + 5880*a**2*b**6*c**6*d**4/11 + 960*
a*b**7*c**7*d**3/11 + 45*b**8*c**8*d**2/11) + x**10*(a**8*c*d**9 + 36*a**7*
b*c**2*d**8 + 336*a**6*b**2*c**3*d**7 + 1176*a**5*b**3*c**4*d**6 + 1764*a**
4*b**4*c**5*d**5 + 1176*a**3*b**5*c**6*d**4 + 336*a**2*b**6*c**7*d**3 + 36*
a*b**7*c**8*d**2 + b**8*c**9*d) + x**9*(5*a**8*c**2*d**8 + 320*a**7*b*c**3*
d**7/3 + 1960*a**6*b**2*c**4*d**6/3 + 1568*a**5*b**3*c**5*d**5 + 4900*a**4*
b**4*c**6*d**4/3 + 2240*a**3*b**5*c**7*d**3/3 + 140*a**2*b**6*c**8*d**2 + 8
0*a*b**7*c**9*d/9 + b**8*c**10/9) + x**8*(15*a**8*c**3*d**7 + 210*a**7*b*c*
*4*d**6 + 882*a**6*b**2*c**5*d**5 + 1470*a**5*b**3*c**6*d**4 + 1050*a**4*b*
*4*c**7*d**3 + 315*a**3*b**5*c**8*d**2 + 35*a**2*b**6*c**9*d + a*b**7*c**10
```

```
) + x**7*(30*a**8*c**4*d**6 + 288*a**7*b*c**5*d**5 + 840*a**6*b**2*c**6*d**
4 + 960*a**5*b**3*c**7*d**3 + 450*a**4*b**4*c**8*d**2 + 80*a**3*b**5*c**9*d
 + 4*a**2*b**6*c**10) + x**6*(42*a**8*c**5*d**5 + 280*a**7*b*c**6*d**4 + 56
0*a**6*b**2*c**7*d**3 + 420*a**5*b**3*c**8*d**2 + 350*a**4*b**4*c**9*d/3 +
28*a**3*b**5*c**10/3) + x**5*(42*a**8*c**6*d**4 + 192*a**7*b*c**7*d**3 + 25
2*a**6*b**2*c**8*d**2 + 112*a**5*b**3*c**9*d + 14*a**4*b**4*c**10) + x**4*(
30*a**8*c**7*d**3 + 90*a**7*b*c**8*d**2 + 70*a**6*b**2*c**9*d + 14*a**5*b**
3*c**10) + x**3*(15*a**8*c**8*d**2 + 80*a**7*b*c**9*d/3 + 28*a**6*b**2*c**1
0/3) + x**2*(5*a**8*c**9*d + 4*a**7*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1478 vs. $2(209) = 418$.
time = 1.00, size = 1478, normalized size = 6.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^8*d^10*x^19 + 5/9*b^8*c*d^9*x^18 + 4/9*a*b^7*d^10*x^18 + 45/17*b^8*c
^2*d^8*x^17 + 80/17*a*b^7*c*d^9*x^17 + 28/17*a^2*b^6*d^10*x^17 + 15/2*b^8*c
^3*d^7*x^16 + 45/2*a*b^7*c^2*d^8*x^16 + 35/2*a^2*b^6*c*d^9*x^16 + 7/2*a^3*b
^5*d^10*x^16 + 14*b^8*c^4*d^6*x^15 + 64*a*b^7*c^3*d^7*x^15 + 84*a^2*b^6*c^2
*d^8*x^15 + 112/3*a^3*b^5*c*d^9*x^15 + 14/3*a^4*b^4*d^10*x^15 + 18*b^8*c^5*
d^5*x^14 + 120*a*b^7*c^4*d^6*x^14 + 240*a^2*b^6*c^3*d^7*x^14 + 180*a^3*b^5*
c^2*d^8*x^14 + 50*a^4*b^4*c*d^9*x^14 + 4*a^5*b^3*d^10*x^14 + 210/13*b^8*c^6
*d^4*x^13 + 2016/13*a*b^7*c^5*d^5*x^13 + 5880/13*a^2*b^6*c^4*d^6*x^13 + 672
0/13*a^3*b^5*c^3*d^7*x^13 + 3150/13*a^4*b^4*c^2*d^8*x^13 + 560/13*a^5*b^3*c
*d^9*x^13 + 28/13*a^6*b^2*d^10*x^13 + 10*b^8*c^7*d^3*x^12 + 140*a*b^7*c^6*d
^4*x^12 + 588*a^2*b^6*c^5*d^5*x^12 + 980*a^3*b^5*c^4*d^6*x^12 + 700*a^4*b^4
*c^3*d^7*x^12 + 210*a^5*b^3*c^2*d^8*x^12 + 70/3*a^6*b^2*c*d^9*x^12 + 2/3*a^
7*b*d^10*x^12 + 45/11*b^8*c^8*d^2*x^11 + 960/11*a*b^7*c^7*d^3*x^11 + 5880/1
1*a^2*b^6*c^6*d^4*x^11 + 14112/11*a^3*b^5*c^5*d^5*x^11 + 14700/11*a^4*b^4*c
^4*d^6*x^11 + 6720/11*a^5*b^3*c^3*d^7*x^11 + 1260/11*a^6*b^2*c^2*d^8*x^11 +
 80/11*a^7*b*c*d^9*x^11 + 1/11*a^8*d^10*x^11 + b^8*c^9*d*x^10 + 36*a*b^7*c^
8*d^2*x^10 + 336*a^2*b^6*c^7*d^3*x^10 + 1176*a^3*b^5*c^6*d^4*x^10 + 1764*a^
4*b^4*c^5*d^5*x^10 + 1176*a^5*b^3*c^4*d^6*x^10 + 336*a^6*b^2*c^3*d^7*x^10 +
 36*a^7*b*c^2*d^8*x^10 + a^8*c*d^9*x^10 + 1/9*b^8*c^10*x^9 + 80/9*a*b^7*c^9
*d*x^9 + 140*a^2*b^6*c^8*d^2*x^9 + 2240/3*a^3*b^5*c^7*d^3*x^9 + 4900/3*a^4*
b^4*c^6*d^4*x^9 + 1568*a^5*b^3*c^5*d^5*x^9 + 1960/3*a^6*b^2*c^4*d^6*x^9 + 3
20/3*a^7*b*c^3*d^7*x^9 + 5*a^8*c^2*d^8*x^9 + a*b^7*c^10*x^8 + 35*a^2*b^6*c^
9*d*x^8 + 315*a^3*b^5*c^8*d^2*x^8 + 1050*a^4*b^4*c^7*d^3*x^8 + 1470*a^5*b^3
*c^6*d^4*x^8 + 882*a^6*b^2*c^5*d^5*x^8 + 210*a^7*b*c^4*d^6*x^8 + 15*a^8*c^3
*d^7*x^8 + 4*a^2*b^6*c^10*x^7 + 80*a^3*b^5*c^9*d*x^7 + 450*a^4*b^4*c^8*d^2*
x^7 + 960*a^5*b^3*c^7*d^3*x^7 + 840*a^6*b^2*c^6*d^4*x^7 + 288*a^7*b*c^5*d^5
```

```
*x^7 + 30*a^8*c^4*d^6*x^7 + 28/3*a^3*b^5*c^10*x^6 + 350/3*a^4*b^4*c^9*d*x^6
 + 420*a^5*b^3*c^8*d^2*x^6 + 560*a^6*b^2*c^7*d^3*x^6 + 280*a^7*b*c^6*d^4*x^
6 + 42*a^8*c^5*d^5*x^6 + 14*a^4*b^4*c^10*x^5 + 112*a^5*b^3*c^9*d*x^5 + 252*
a^6*b^2*c^8*d^2*x^5 + 192*a^7*b*c^7*d^3*x^5 + 42*a^8*c^6*d^4*x^5 + 14*a^5*b
^3*c^10*x^4 + 70*a^6*b^2*c^9*d*x^4 + 90*a^7*b*c^8*d^2*x^4 + 30*a^8*c^7*d^3*
x^4 + 28/3*a^6*b^2*c^10*x^3 + 80/3*a^7*b*c^9*d*x^3 + 15*a^8*c^8*d^2*x^3 + 4
*a^7*b*c^10*x^2 + 5*a^8*c^9*d*x^2 + a^8*c^10*x
```

Mupad [B]
time = 0.71, size = 1253, normalized size = 5.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^10,x)
```

```
[Out] x^7*(4*a^2*b^6*c^10 + 30*a^8*c^4*d^6 + 80*a^3*b^5*c^9*d + 288*a^7*b*c^5*d^5
 + 450*a^4*b^4*c^8*d^2 + 960*a^5*b^3*c^7*d^3 + 840*a^6*b^2*c^6*d^4) + x^13*
((28*a^6*b^2*d^10)/13 + (210*b^8*c^6*d^4)/13 + (2016*a*b^7*c^5*d^5)/13 + (5
60*a^5*b^3*c*d^9)/13 + (5880*a^2*b^6*c^4*d^6)/13 + (6720*a^3*b^5*c^3*d^7)/1
3 + (3150*a^4*b^4*c^2*d^8)/13) + x^8*(a*b^7*c^10 + 15*a^8*c^3*d^7 + 35*a^2*
b^6*c^9*d + 210*a^7*b*c^4*d^6 + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3
+ 1470*a^5*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5) + x^12*((2*a^7*b*d^10)/3 + 10
*b^8*c^7*d^3 + 140*a*b^7*c^6*d^4 + (70*a^6*b^2*c*d^9)/3 + 588*a^2*b^6*c^5*d
^5 + 980*a^3*b^5*c^4*d^6 + 700*a^4*b^4*c^3*d^7 + 210*a^5*b^3*c^2*d^8) + x^1
0*(a^8*c*d^9 + b^8*c^9*d + 36*a*b^7*c^8*d^2 + 36*a^7*b*c^2*d^8 + 336*a^2*b^
6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*
d^6 + 336*a^6*b^2*c^3*d^7) + x^5*(14*a^4*b^4*c^10 + 42*a^8*c^6*d^4 + 112*a^
5*b^3*c^9*d + 192*a^7*b*c^7*d^3 + 252*a^6*b^2*c^8*d^2) + x^15*((14*a^4*b^4*
d^10)/3 + 14*b^8*c^4*d^6 + 64*a*b^7*c^3*d^7 + (112*a^3*b^5*c*d^9)/3 + 84*a^
2*b^6*c^2*d^8) + x^6*((28*a^3*b^5*c^10)/3 + 42*a^8*c^5*d^5 + (350*a^4*b^4*c
^9*d)/3 + 280*a^7*b*c^6*d^4 + 420*a^5*b^3*c^8*d^2 + 560*a^6*b^2*c^7*d^3) +
x^14*(4*a^5*b^3*d^10 + 18*b^8*c^5*d^5 + 120*a*b^7*c^4*d^6 + 50*a^4*b^4*c*d^
9 + 240*a^2*b^6*c^3*d^7 + 180*a^3*b^5*c^2*d^8) + x^9*((b^8*c^10)/9 + 5*a^8*
c^2*d^8 + (320*a^7*b*c^3*d^7)/3 + 140*a^2*b^6*c^8*d^2 + (2240*a^3*b^5*c^7*d
^3)/3 + (4900*a^4*b^4*c^6*d^4)/3 + 1568*a^5*b^3*c^5*d^5 + (1960*a^6*b^2*c^4
*d^6)/3 + (80*a*b^7*c^9*d)/9) + x^11*((a^8*d^10)/11 + (45*b^8*c^8*d^2)/11 +
 (960*a*b^7*c^7*d^3)/11 + (5880*a^2*b^6*c^6*d^4)/11 + (14112*a^3*b^5*c^5*d^
5)/11 + (14700*a^4*b^4*c^4*d^6)/11 + (6720*a^5*b^3*c^3*d^7)/11 + (1260*a^6*
b^2*c^2*d^8)/11 + (80*a^7*b*c*d^9)/11) + a^8*c^10*x + (b^8*d^10*x^19)/19 +
2*a^5*c^7*x^4*(15*a^3*d^3 + 7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c*d^2) +
(b^5*d^7*x^16*(7*a^3*d^3 + 15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2))/2
 + a^7*c^9*x^2*(5*a*d + 4*b*c) + (b^7*d^9*x^18*(4*a*d + 5*b*c))/9 + (a^6*c^
8*x^3*(45*a^2*d^2 + 28*b^2*c^2 + 80*a*b*c*d))/3 + (b^6*d^8*x^17*(28*a^2*d^2
 + 45*b^2*c^2 + 80*a*b*c*d))/17
```

3.1304 $\int (a+bx)^7(c+dx)^{10}\,dx$

Optimal. Leaf size=200

$$-\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+\frac{5b^3(bc-ad)^4(c+dx)^{14}}{2d^8}-\frac{7b^4(b}{}$$

```
[Out] -1/11*(-a*d+b*c)^7*(d*x+c)^11/d^8+7/12*b*(-a*d+b*c)^6*(d*x+c)^12/d^8-21/13*
b^2*(-a*d+b*c)^5*(d*x+c)^13/d^8+5/2*b^3*(-a*d+b*c)^4*(d*x+c)^14/d^8-7/3*b^4
*(-a*d+b*c)^3*(d*x+c)^15/d^8+21/16*b^5*(-a*d+b*c)^2*(d*x+c)^16/d^8-7/17*b^6
*(-a*d+b*c)*(d*x+c)^17/d^8+1/18*b^7*(d*x+c)^18/d^8
```

Rubi [A]
time = 0.54, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+dx)^{13}(bc-ad)^5}{13d^8}+\frac{7b(c+dx)^{12}(bc-ad)^6}{12d^8}-\frac{(c+dx)^{11}(bc-ad)^7}{11d^8}+\frac{b^7(c+dx)^{18}}{18d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^7*(c + d*x)^11)/d^8 + (7*b*(b*c - a*d)^6*(c + d*x)^12)/(
12*d^8) - (21*b^2*(b*c - a*d)^5*(c + d*x)^13)/(13*d^8) + (5*b^3*(b*c - a*d)
^4*(c + d*x)^14)/(2*d^8) - (7*b^4*(b*c - a*d)^3*(c + d*x)^15)/(3*d^8) + (21
*b^5*(b*c - a*d)^2*(c + d*x)^16)/(16*d^8) - (7*b^6*(b*c - a*d)*(c + d*x)^17
)/(17*d^8) + (b^7*(c + d*x)^18)/(18*d^8)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^7(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^7(c+dx)^{10}}{d^7}+\frac{7b(bc-ad)^6(c+dx)^{11}}{d^7}-\frac{21b^2(bc-ad)^5(c+dx)}{d^7}\right.$$

$$= -\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1105 vs. 2(200) = 400.

time = 0.09, size = 1105, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] a^7*c^10*x + (a^6*c^9*(7*b*c + 10*a*d)*x^2)/2 + (a^5*c^8*(21*b^2*c^2 + 70*a
*b*c*d + 45*a^2*d^2)*x^3)/3 + (5*a^4*c^7*(7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a
^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 7*a^3*c^6*(b^4*c^4 + 10*a*b^3*c^3*d + 27*
a^2*b^2*c^2*d^2 + 24*a^3*b*c*d^3 + 6*a^4*d^4)*x^5 + (7*a^2*c^5*(3*b^5*c^5 +
 50*a*b^4*c^4*d + 225*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d
^4 + 36*a^5*d^5)*x^6)/6 + a*c^4*(b^6*c^6 + 30*a*b^5*c^5*d + 225*a^2*b^4*c^4
*d^2 + 600*a^3*b^3*c^3*d^3 + 630*a^4*b^2*c^2*d^4 + 252*a^5*b*c*d^5 + 30*a^6
*d^6)*x^7 + (c^3*(b^7*c^7 + 70*a*b^6*c^6*d + 945*a^2*b^5*c^5*d^2 + 4200*a^3
*b^4*c^4*d^3 + 7350*a^4*b^3*c^3*d^4 + 5292*a^5*b^2*c^2*d^5 + 1470*a^6*b*c*d
^6 + 120*a^7*d^7)*x^8)/8 + (5*c^2*d*(2*b^7*c^7 + 63*a*b^6*c^6*d + 504*a^2*b
^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1764*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*
d^5 + 168*a^6*b*c*d^6 + 9*a^7*d^7)*x^9)/9 + (c*d^2*(9*b^7*c^7 + 168*a*b^6*c
^6*d + 882*a^2*b^5*c^5*d^2 + 1764*a^3*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 +
504*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6 + 2*a^7*d^7)*x^10)/2 + (d^3*(120*b^7*c
^7 + 1470*a*b^6*c^6*d + 5292*a^2*b^5*c^5*d^2 + 7350*a^3*b^4*c^4*d^3 + 4200*
a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5 + 70*a^6*b*c*d^6 + a^7*d^7)*x^11)/11
+ (7*b*d^4*(30*b^6*c^6 + 252*a*b^5*c^5*d + 630*a^2*b^4*c^4*d^2 + 600*a^3*b^
3*c^3*d^3 + 225*a^4*b^2*c^2*d^4 + 30*a^5*b*c*d^5 + a^6*d^6)*x^12)/12 + (7*b
^2*d^5*(36*b^5*c^5 + 210*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 + 225*a^3*b^2*c^
2*d^3 + 50*a^4*b*c*d^4 + 3*a^5*d^5)*x^13)/13 + (5*b^3*d^6*(6*b^4*c^4 + 24*a
*b^3*c^3*d + 27*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + a^4*d^4)*x^14)/2 + (b^4*
d^7*(24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2 + 7*a^3*d^3)*x^15)/3 + (b
^5*d^8*(45*b^2*c^2 + 70*a*b*c*d + 21*a^2*d^2)*x^16)/16 + (b^6*d^9*(10*b*c +
 7*a*d)*x^17)/17 + (b^7*d^10*x^18)/18
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1140 vs. $2(184) = 368$.
time = 0.13, size = 1141, normalized size = 5.70 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*b^7*d^10*x^18+1/17*(7*a*b^6*d^10+10*b^7*c*d^9)*x^17+1/16*(21*a^2*b^5*d
^10+70*a*b^6*c*d^9+45*b^7*c^2*d^8)*x^16+1/15*(35*a^3*b^4*d^10+210*a^2*b^5*c
*d^9+315*a*b^6*c^2*d^8+120*b^7*c^3*d^7)*x^15+1/14*(35*a^4*b^3*d^10+350*a^3*
b^4*c*d^9+945*a^2*b^5*c^2*d^8+840*a*b^6*c^3*d^7+210*b^7*c^4*d^6)*x^14+1/13*
(21*a^5*b^2*d^10+350*a^4*b^3*c*d^9+1575*a^3*b^4*c^2*d^8+2520*a^2*b^5*c^3*d^
7+1470*a*b^6*c^4*d^6+252*b^7*c^5*d^5)*x^13+1/12*(7*a^6*b*d^10+210*a^5*b^2*c
*d^9+1575*a^4*b^3*c^2*d^8+4200*a^3*b^4*c^3*d^7+4410*a^2*b^5*c^4*d^6+1764*a*
```

```
b^6*c^5*d^5+210*b^7*c^6*d^4)*x^12+1/11*(a^7*d^10+70*a^6*b*c*d^9+945*a^5*b^2
*c^2*d^8+4200*a^4*b^3*c^3*d^7+7350*a^3*b^4*c^4*d^6+5292*a^2*b^5*c^5*d^5+147
0*a*b^6*c^6*d^4+120*b^7*c^7*d^3)*x^11+1/10*(10*a^7*c*d^9+315*a^6*b*c^2*d^8+
2520*a^5*b^2*c^3*d^7+7350*a^4*b^3*c^4*d^6+8820*a^3*b^4*c^5*d^5+4410*a^2*b^5
*c^6*d^4+840*a*b^6*c^7*d^3+45*b^7*c^8*d^2)*x^10+1/9*(45*a^7*c^2*d^8+840*a^6
*b*c^3*d^7+4410*a^5*b^2*c^4*d^6+8820*a^4*b^3*c^5*d^5+7350*a^3*b^4*c^6*d^4+2
520*a^2*b^5*c^7*d^3+315*a*b^6*c^8*d^2+10*b^7*c^9*d)*x^9+1/8*(120*a^7*c^3*d^
7+1470*a^6*b*c^4*d^6+5292*a^5*b^2*c^5*d^5+7350*a^4*b^3*c^6*d^4+4200*a^3*b^4
*c^7*d^3+945*a^2*b^5*c^8*d^2+70*a*b^6*c^9*d+b^7*c^10)*x^8+1/7*(210*a^7*c^4*
d^6+1764*a^6*b*c^5*d^5+4410*a^5*b^2*c^6*d^4+4200*a^4*b^3*c^7*d^3+1575*a^3*b
^4*c^8*d^2+210*a^2*b^5*c^9*d+7*a*b^6*c^10)*x^7+1/6*(252*a^7*c^5*d^5+1470*a^
6*b*c^6*d^4+2520*a^5*b^2*c^7*d^3+1575*a^4*b^3*c^8*d^2+350*a^3*b^4*c^9*d+21*
a^2*b^5*c^10)*x^6+1/5*(210*a^7*c^6*d^4+840*a^6*b*c^7*d^3+945*a^5*b^2*c^8*d^
2+350*a^4*b^3*c^9*d+35*a^3*b^4*c^10)*x^5+1/4*(120*a^7*c^7*d^3+315*a^6*b*c^8
*d^2+210*a^5*b^2*c^9*d+35*a^4*b^3*c^10)*x^4+1/3*(45*a^7*c^8*d^2+70*a^6*b*c^
9*d+21*a^5*b^2*c^10)*x^3+1/2*(10*a^7*c^9*d+7*a^6*b*c^10)*x^2+a^7*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(184) = 368$.
time = 0.30, size = 1135, normalized size = 5.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
```

```
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(184) = 368$.
time = 0.50, size = 1135, normalized size = 5.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(184) = 368$.
time = 0.09, size = 1280, normalized size = 6.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**10,x)
```

```
[Out] a**7*c**10*x + b**7*d**10*x**18/18 + x**17*(7*a*b**6*d**10/17 + 10*b**7*c*d
**9/17) + x**16*(21*a**2*b**5*d**10/16 + 35*a*b**6*c*d**9/8 + 45*b**7*c**2*
d**8/16) + x**15*(7*a**3*b**4*d**10/3 + 14*a**2*b**5*c*d**9 + 21*a*b**6*c**
2*d**8 + 8*b**7*c**3*d**7) + x**14*(5*a**4*b**3*d**10/2 + 25*a**3*b**4*c*d*
*9 + 135*a**2*b**5*c**2*d**8/2 + 60*a*b**6*c**3*d**7 + 15*b**7*c**4*d**6) +
 x**13*(21*a**5*b**2*d**10/13 + 350*a**4*b**3*c*d**9/13 + 1575*a**3*b**4*c*
*2*d**8/13 + 2520*a**2*b**5*c**3*d**7/13 + 1470*a*b**6*c**4*d**6/13 + 252*b
**7*c**5*d**5/13) + x**12*(7*a**6*b*d**10/12 + 35*a**5*b**2*c*d**9/2 + 525*
a**4*b**3*c**2*d**8/4 + 350*a**3*b**4*c**3*d**7 + 735*a**2*b**5*c**4*d**6/2
 + 147*a*b**6*c**5*d**5 + 35*b**7*c**6*d**4/2) + x**11*(a**7*d**10/11 + 70*
a**6*b*c*d**9/11 + 945*a**5*b**2*c**2*d**8/11 + 4200*a**4*b**3*c**3*d**7/11
 + 7350*a**3*b**4*c**4*d**6/11 + 5292*a**2*b**5*c**5*d**5/11 + 1470*a*b**6*
c**6*d**4/11 + 120*b**7*c**7*d**3/11) + x**10*(a**7*c*d**9 + 63*a**6*b*c**2
*d**8/2 + 252*a**5*b**2*c**3*d**7 + 735*a**4*b**3*c**4*d**6 + 882*a**3*b**4
*c**5*d**5 + 441*a**2*b**5*c**6*d**4 + 84*a*b**6*c**7*d**3 + 9*b**7*c**8*d*
*2/2) + x**9*(5*a**7*c**2*d**8 + 280*a**6*b*c**3*d**7/3 + 490*a**5*b**2*c**
4*d**6 + 980*a**4*b**3*c**5*d**5 + 2450*a**3*b**4*c**6*d**4/3 + 280*a**2*b*
*5*c**7*d**3 + 35*a*b**6*c**8*d**2 + 10*b**7*c**9*d/9) + x**8*(15*a**7*c**3
*d**7 + 735*a**6*b*c**4*d**6/4 + 1323*a**5*b**2*c**5*d**5/2 + 3675*a**4*b**
3*c**6*d**4/4 + 525*a**3*b**4*c**7*d**3 + 945*a**2*b**5*c**8*d**2/8 + 35*a*
b**6*c**9*d/4 + b**7*c**10/8) + x**7*(30*a**7*c**4*d**6 + 252*a**6*b*c**5*d
**5 + 630*a**5*b**2*c**6*d**4 + 600*a**4*b**3*c**7*d**3 + 225*a**3*b**4*c**
8*d**2 + 30*a**2*b**5*c**9*d + a*b**6*c**10) + x**6*(42*a**7*c**5*d**5 + 24
5*a**6*b*c**6*d**4 + 420*a**5*b**2*c**7*d**3 + 525*a**4*b**3*c**8*d**2/2 +
175*a**3*b**4*c**9*d/3 + 7*a**2*b**5*c**10/2) + x**5*(42*a**7*c**6*d**4 + 1
68*a**6*b*c**7*d**3 + 189*a**5*b**2*c**8*d**2 + 70*a**4*b**3*c**9*d + 7*a**
3*b**4*c**10) + x**4*(30*a**7*c**7*d**3 + 315*a**6*b*c**8*d**2/4 + 105*a**5
*b**2*c**9*d/2 + 35*a**4*b**3*c**10/4) + x**3*(15*a**7*c**8*d**2 + 70*a**6*
b*c**9*d/3 + 7*a**5*b**2*c**10) + x**2*(5*a**7*c**9*d + 7*a**6*b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1302 vs. $2(184) = 368$.
time = 0.81, size = 1302, normalized size = 6.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^7*d^10*x^18 + 10/17*b^7*c*d^9*x^17 + 7/17*a*b^6*d^10*x^17 + 45/16*b^
7*c^2*d^8*x^16 + 35/8*a*b^6*c*d^9*x^16 + 21/16*a^2*b^5*d^10*x^16 + 8*b^7*c^
3*d^7*x^15 + 21*a*b^6*c^2*d^8*x^15 + 14*a^2*b^5*c*d^9*x^15 + 7/3*a^3*b^4*d^
```

```
10*x^15 + 15*b^7*c^4*d^6*x^14 + 60*a*b^6*c^3*d^7*x^14 + 135/2*a^2*b^5*c^2*d
^8*x^14 + 25*a^3*b^4*c*d^9*x^14 + 5/2*a^4*b^3*d^10*x^14 + 252/13*b^7*c^5*d^
5*x^13 + 1470/13*a*b^6*c^4*d^6*x^13 + 2520/13*a^2*b^5*c^3*d^7*x^13 + 1575/1
3*a^3*b^4*c^2*d^8*x^13 + 350/13*a^4*b^3*c*d^9*x^13 + 21/13*a^5*b^2*d^10*x^1
3 + 35/2*b^7*c^6*d^4*x^12 + 147*a*b^6*c^5*d^5*x^12 + 735/2*a^2*b^5*c^4*d^6*
x^12 + 350*a^3*b^4*c^3*d^7*x^12 + 525/4*a^4*b^3*c^2*d^8*x^12 + 35/2*a^5*b^2
*c*d^9*x^12 + 7/12*a^6*b*d^10*x^12 + 120/11*b^7*c^7*d^3*x^11 + 1470/11*a*b^
6*c^6*d^4*x^11 + 5292/11*a^2*b^5*c^5*d^5*x^11 + 7350/11*a^3*b^4*c^4*d^6*x^1
1 + 4200/11*a^4*b^3*c^3*d^7*x^11 + 945/11*a^5*b^2*c^2*d^8*x^11 + 70/11*a^6*
b*c*d^9*x^11 + 1/11*a^7*d^10*x^11 + 9/2*b^7*c^8*d^2*x^10 + 84*a*b^6*c^7*d^3
*x^10 + 441*a^2*b^5*c^6*d^4*x^10 + 882*a^3*b^4*c^5*d^5*x^10 + 735*a^4*b^3*c
^4*d^6*x^10 + 252*a^5*b^2*c^3*d^7*x^10 + 63/2*a^6*b*c^2*d^8*x^10 + a^7*c*d^
9*x^10 + 10/9*b^7*c^9*d*x^9 + 35*a*b^6*c^8*d^2*x^9 + 280*a^2*b^5*c^7*d^3*x^
9 + 2450/3*a^3*b^4*c^6*d^4*x^9 + 980*a^4*b^3*c^5*d^5*x^9 + 490*a^5*b^2*c^4*
d^6*x^9 + 280/3*a^6*b*c^3*d^7*x^9 + 5*a^7*c^2*d^8*x^9 + 1/8*b^7*c^10*x^8 +
35/4*a*b^6*c^9*d*x^8 + 945/8*a^2*b^5*c^8*d^2*x^8 + 525*a^3*b^4*c^7*d^3*x^8
+ 3675/4*a^4*b^3*c^6*d^4*x^8 + 1323/2*a^5*b^2*c^5*d^5*x^8 + 735/4*a^6*b*c^4
*d^6*x^8 + 15*a^7*c^3*d^7*x^8 + a*b^6*c^10*x^7 + 30*a^2*b^5*c^9*d*x^7 + 225
*a^3*b^4*c^8*d^2*x^7 + 600*a^4*b^3*c^7*d^3*x^7 + 630*a^5*b^2*c^6*d^4*x^7 +
252*a^6*b*c^5*d^5*x^7 + 30*a^7*c^4*d^6*x^7 + 7/2*a^2*b^5*c^10*x^6 + 175/3*a
^3*b^4*c^9*d*x^6 + 525/2*a^4*b^3*c^8*d^2*x^6 + 420*a^5*b^2*c^7*d^3*x^6 + 24
5*a^6*b*c^6*d^4*x^6 + 42*a^7*c^5*d^5*x^6 + 7*a^3*b^4*c^10*x^5 + 70*a^4*b^3*
c^9*d*x^5 + 189*a^5*b^2*c^8*d^2*x^5 + 168*a^6*b*c^7*d^3*x^5 + 42*a^7*c^6*d^
4*x^5 + 35/4*a^4*b^3*c^10*x^4 + 105/2*a^5*b^2*c^9*d*x^4 + 315/4*a^6*b*c^8*d
^2*x^4 + 30*a^7*c^7*d^3*x^4 + 7*a^5*b^2*c^10*x^3 + 70/3*a^6*b*c^9*d*x^3 + 1
5*a^7*c^8*d^2*x^3 + 7/2*a^6*b*c^10*x^2 + 5*a^7*c^9*d*x^2 + a^7*c^10*x
```

Mupad [B]
time = 0.61, size = 1106, normalized size = 5.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^10,x)
```

```
[Out] x^10*(a^7*c*d^9 + (9*b^7*c^8*d^2)/2 + 84*a*b^6*c^7*d^3 + (63*a^6*b*c^2*d^8)
/2 + 441*a^2*b^5*c^6*d^4 + 882*a^3*b^4*c^5*d^5 + 735*a^4*b^3*c^4*d^6 + 252*
a^5*b^2*c^3*d^7) + x^9*((10*b^7*c^9*d)/9 + 5*a^7*c^2*d^8 + 35*a*b^6*c^8*d^2
 + (280*a^6*b*c^3*d^7)/3 + 280*a^2*b^5*c^7*d^3 + (2450*a^3*b^4*c^6*d^4)/3 +
 980*a^4*b^3*c^5*d^5 + 490*a^5*b^2*c^4*d^6) + x^5*(7*a^3*b^4*c^10 + 42*a^7*
c^6*d^4 + 70*a^4*b^3*c^9*d + 168*a^6*b*c^7*d^3 + 189*a^5*b^2*c^8*d^2) + x^1
4*((5*a^4*b^3*d^10)/2 + 15*b^7*c^4*d^6 + 60*a*b^6*c^3*d^7 + 25*a^3*b^4*c*d^
9 + (135*a^2*b^5*c^2*d^8)/2) + x^8*((b^7*c^10)/8 + 15*a^7*c^3*d^7 + (735*a^
6*b*c^4*d^6)/4 + (945*a^2*b^5*c^8*d^2)/8 + 525*a^3*b^4*c^7*d^3 + (3675*a^4*
b^3*c^6*d^4)/4 + (1323*a^5*b^2*c^5*d^5)/2 + (35*a*b^6*c^9*d)/4) + x^11*((a^
```

7*d^10)/11 + (120*b^7*c^7*d^3)/11 + (1470*a*b^6*c^6*d^4)/11 + (5292*a^2*b^5*c^5*d^5)/11 + (7350*a^3*b^4*c^4*d^6)/11 + (4200*a^4*b^3*c^3*d^7)/11 + (945*a^5*b^2*c^2*d^8)/11 + (70*a^6*b*c*d^9)/11) + x^6*((7*a^2*b^5*c^10)/2 + 42*a^7*c^5*d^5 + (175*a^3*b^4*c^9*d)/3 + 245*a^6*b*c^6*d^4 + (525*a^4*b^3*c^8*d^2)/2 + 420*a^5*b^2*c^7*d^3) + x^13*((21*a^5*b^2*d^10)/13 + (252*b^7*c^5*d^5)/13 + (1470*a*b^6*c^4*d^6)/13 + (350*a^4*b^3*c*d^9)/13 + (2520*a^2*b^5*c^3*d^7)/13 + (1575*a^3*b^4*c^2*d^8)/13) + x^7*(a*b^6*c^10 + 30*a^7*c^4*d^6 + 30*a^2*b^5*c^9*d + 252*a^6*b*c^5*d^5 + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^3 + 630*a^5*b^2*c^6*d^4) + x^12*((7*a^6*b*d^10)/12 + (35*b^7*c^6*d^4)/2 + 147*a*b^6*c^5*d^5 + (35*a^5*b^2*c*d^9)/2 + (735*a^2*b^5*c^4*d^6)/2 + 350*a^3*b^4*c^3*d^7 + (525*a^4*b^3*c^2*d^8)/4) + a^7*c^10*x + (b^7*d^10*x^18)/18 + (5*a^4*c^7*x^4*(24*a^3*d^3 + 7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a^2*b*c*d^2))/4 + (b^4*d^7*x^15*(7*a^3*d^3 + 24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2))/3 + (a^6*c^9*x^2*(10*a*d + 7*b*c))/2 + (b^6*d^9*x^17*(7*a*d + 10*b*c))/17 + (a^5*c^8*x^3*(45*a^2*d^2 + 21*b^2*c^2 + 70*a*b*c*d))/3 + (b^5*d^8*x^16*(21*a^2*d^2 + 45*b^2*c^2 + 70*a*b*c*d))/16

3.1305 $\int (a+bx)^6(c+dx)^{10}\,dx$

Optimal. Leaf size=170

$$\frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-\frac{10b^3(bc-ad)^3(c+dx)^{14}}{7d^7}+\frac{b^4(bc-$$

```
[Out] 1/11*(-a*d+b*c)^6*(d*x+c)^11/d^7-1/2*b*(-a*d+b*c)^5*(d*x+c)^12/d^7+15/13*b^
2*(-a*d+b*c)^4*(d*x+c)^13/d^7-10/7*b^3*(-a*d+b*c)^3*(d*x+c)^14/d^7+b^4*(-a*
d+b*c)^2*(d*x+c)^15/d^7-3/8*b^5*(-a*d+b*c)*(d*x+c)^16/d^7+1/17*b^6*(d*x+c)^
17/d^7
```

Rubi [A]
time = 0.47, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^{12}(bc-ad)^5}{2d^7}+\frac{(c+dx)^{11}(bc-ad)^6}{11d^7}+\frac{b^6(c+dx)^{17}}{17d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^11)/(11*d^7) - (b*(b*c - a*d)^5*(c + d*x)^12)/(2*d
^7) + (15*b^2*(b*c - a*d)^4*(c + d*x)^13)/(13*d^7) - (10*b^3*(b*c - a*d)^3*
(c + d*x)^14)/(7*d^7) + (b^4*(b*c - a*d)^2*(c + d*x)^15)/d^7 - (3*b^5*(b*c
- a*d)*(c + d*x)^16)/(8*d^7) + (b^6*(c + d*x)^17)/(17*d^7)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^6(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^6(c+dx)^{10}}{d^6}-\frac{6b(bc-ad)^5(c+dx)^{11}}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^{12}}{d^6}\right.\\ &= \frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-\frac{10b^3}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 939 vs. 2(170) = 340.
time = 0.08, size = 939, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] a^6*c^10*x + a^5*c^9*(3*b*c + 5*a*d)*x^2 + 5*a^4*c^8*(b^2*c^2 + 4*a*b*c*d +
 3*a^2*d^2)*x^3 + (5*a^3*c^7*(2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 +
 12*a^3*d^3)*x^4)/2 + a^2*c^6*(3*b^4*c^4 + 40*a*b^3*c^3*d + 135*a^2*b^2*c^2
*d^2 + 144*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + a*c^5*(b^5*c^5 + 25*a*b^4*c^4*d
+ 150*a^2*b^3*c^3*d^2 + 300*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d^4 + 42*a^5*d^5)
*x^6 + (c^4*(b^6*c^6 + 60*a*b^5*c^5*d + 675*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*
c^3*d^3 + 3150*a^4*b^2*c^2*d^4 + 1512*a^5*b*c*d^5 + 210*a^6*d^6)*x^7)/7 + (
5*c^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 180*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d
^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 12*a^6*d^6)*x^8)/4 + 5*c^2*d^2
*(b^6*c^6 + 16*a*b^5*c^5*d + 70*a^2*b^4*c^4*d^2 + 112*a^3*b^3*c^3*d^3 + 70*
a^4*b^2*c^2*d^4 + 16*a^5*b*c*d^5 + a^6*d^6)*x^9 + c*d^3*(12*b^6*c^6 + 126*a
*b^5*c^5*d + 378*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^
4 + 27*a^5*b*c*d^5 + a^6*d^6)*x^10 + (d^4*(210*b^6*c^6 + 1512*a*b^5*c^5*d +
 3150*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*c^3*d^3 + 675*a^4*b^2*c^2*d^4 + 60*a^5
*b*c*d^5 + a^6*d^6)*x^11)/11 + (b*d^5*(42*b^5*c^5 + 210*a*b^4*c^4*d + 300*a
^2*b^3*c^3*d^2 + 150*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*x^12)/2 +
(5*b^2*d^6*(42*b^4*c^4 + 144*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 40*a^3*b*c
*d^3 + 3*a^4*d^4)*x^13)/13 + (5*b^3*d^7*(12*b^3*c^3 + 27*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 2*a^3*d^3)*x^14)/7 + b^4*d^8*(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*x^15 + (b^5*d^9*(5*b*c + 3*a*d)*x^16)/8 + (b^6*d^10*x^17)/17
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(158) = 316$.
time = 0.13, size = 991, normalized size = 5.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*b^6*d^10*x^17+1/16*(6*a*b^5*d^10+10*b^6*c*d^9)*x^16+1/15*(15*a^2*b^4*d
^10+60*a*b^5*c*d^9+45*b^6*c^2*d^8)*x^15+1/14*(20*a^3*b^3*d^10+150*a^2*b^4*c
*d^9+270*a*b^5*c^2*d^8+120*b^6*c^3*d^7)*x^14+1/13*(15*a^4*b^2*d^10+200*a^3*
b^3*c*d^9+675*a^2*b^4*c^2*d^8+720*a*b^5*c^3*d^7+210*b^6*c^4*d^6)*x^13+1/12*
(6*a^5*b*d^10+150*a^4*b^2*c*d^9+900*a^3*b^3*c^2*d^8+1800*a^2*b^4*c^3*d^7+12
60*a*b^5*c^4*d^6+252*b^6*c^5*d^5)*x^12+1/11*(a^6*d^10+60*a^5*b*c*d^9+675*a^
4*b^2*c^2*d^8+2400*a^3*b^3*c^3*d^7+3150*a^2*b^4*c^4*d^6+1512*a*b^5*c^5*d^5+
210*b^6*c^6*d^4)*x^11+1/10*(10*a^6*c*d^9+270*a^5*b*c^2*d^8+1800*a^4*b^2*c^3
*d^7+4200*a^3*b^3*c^4*d^6+3780*a^2*b^4*c^5*d^5+1260*a*b^5*c^6*d^4+120*b^6*c
^7*d^3)*x^10+1/9*(45*a^6*c^2*d^8+720*a^5*b*c^3*d^7+3150*a^4*b^2*c^4*d^6+504
0*a^3*b^3*c^5*d^5+3150*a^2*b^4*c^6*d^4+720*a*b^5*c^7*d^3+45*b^6*c^8*d^2)*x^
9+1/8*(120*a^6*c^3*d^7+1260*a^5*b*c^4*d^6+3780*a^4*b^2*c^5*d^5+4200*a^3*b^3
*c^6*d^4+1800*a^2*b^4*c^7*d^3+270*a*b^5*c^8*d^2+10*b^6*c^9*d)*x^8+1/7*(210*
a^6*c^4*d^6+1512*a^5*b*c^5*d^5+3150*a^4*b^2*c^6*d^4+2400*a^3*b^3*c^7*d^3+67
5*a^2*b^4*c^8*d^2+60*a*b^5*c^9*d+b^6*c^10)*x^7+1/6*(252*a^6*c^5*d^5+1260*a^
```

```
5*b*c^6*d^4+1800*a^4*b^2*c^7*d^3+900*a^3*b^3*c^8*d^2+150*a^2*b^4*c^9*d+6*a*
b^5*c^10)*x^6+1/5*(210*a^6*c^6*d^4+720*a^5*b*c^7*d^3+675*a^4*b^2*c^8*d^2+20
0*a^3*b^3*c^9*d+15*a^2*b^4*c^10)*x^5+1/4*(120*a^6*c^7*d^3+270*a^5*b*c^8*d^2
+150*a^4*b^2*c^9*d+20*a^3*b^3*c^10)*x^4+1/3*(45*a^6*c^8*d^2+60*a^5*b*c^9*d+
15*a^4*b^2*c^10)*x^3+1/2*(10*a^6*c^9*d+6*a^5*b*c^10)*x^2+a^6*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(158) = 316$.
time = 0.29, size = 977, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(158) = 316$.
time = 0.46, size = 977, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. $2(153) = 306$.
time = 0.08, size = 1088, normalized size = 6.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**10,x)
```

```
[Out] a**6*c**10*x + b**6*d**10*x**17/17 + x**16*(3*a*b**5*d**10/8 + 5*b**6*c*d**
9/8) + x**15*(a**2*b**4*d**10 + 4*a*b**5*c*d**9 + 3*b**6*c**2*d**8) + x**14
*(10*a**3*b**3*d**10/7 + 75*a**2*b**4*c*d**9/7 + 135*a*b**5*c**2*d**8/7 + 6
0*b**6*c**3*d**7/7) + x**13*(15*a**4*b**2*d**10/13 + 200*a**3*b**3*c*d**9/1
3 + 675*a**2*b**4*c**2*d**8/13 + 720*a*b**5*c**3*d**7/13 + 210*b**6*c**4*d*
*6/13) + x**12*(a**5*b*d**10/2 + 25*a**4*b**2*c*d**9/2 + 75*a**3*b**3*c**2*
d**8 + 150*a**2*b**4*c**3*d**7 + 105*a*b**5*c**4*d**6 + 21*b**6*c**5*d**5)
+ x**11*(a**6*d**10/11 + 60*a**5*b*c*d**9/11 + 675*a**4*b**2*c**2*d**8/11 +
 2400*a**3*b**3*c**3*d**7/11 + 3150*a**2*b**4*c**4*d**6/11 + 1512*a*b**5*c*
*5*d**5/11 + 210*b**6*c**6*d**4/11) + x**10*(a**6*c*d**9 + 27*a**5*b*c**2*d
**8 + 180*a**4*b**2*c**3*d**7 + 420*a**3*b**3*c**4*d**6 + 378*a**2*b**4*c**
5*d**5 + 126*a*b**5*c**6*d**4 + 12*b**6*c**7*d**3) + x**9*(5*a**6*c**2*d**8
 + 80*a**5*b*c**3*d**7 + 350*a**4*b**2*c**4*d**6 + 560*a**3*b**3*c**5*d**5
+ 350*a**2*b**4*c**6*d**4 + 80*a*b**5*c**7*d**3 + 5*b**6*c**8*d**2) + x**8*
```

```
(15*a**6*c**3*d**7 + 315*a**5*b*c**4*d**6/2 + 945*a**4*b**2*c**5*d**5/2 + 5
25*a**3*b**3*c**6*d**4 + 225*a**2*b**4*c**7*d**3 + 135*a*b**5*c**8*d**2/4 +
 5*b**6*c**9*d/4) + x**7*(30*a**6*c**4*d**6 + 216*a**5*b*c**5*d**5 + 450*a*
*4*b**2*c**6*d**4 + 2400*a**3*b**3*c**7*d**3/7 + 675*a**2*b**4*c**8*d**2/7
+ 60*a*b**5*c**9*d/7 + b**6*c**10/7) + x**6*(42*a**6*c**5*d**5 + 210*a**5*b
*c**6*d**4 + 300*a**4*b**2*c**7*d**3 + 150*a**3*b**3*c**8*d**2 + 25*a**2*b*
*4*c**9*d + a*b**5*c**10) + x**5*(42*a**6*c**6*d**4 + 144*a**5*b*c**7*d**3
+ 135*a**4*b**2*c**8*d**2 + 40*a**3*b**3*c**9*d + 3*a**2*b**4*c**10) + x**4
*(30*a**6*c**7*d**3 + 135*a**5*b*c**8*d**2/2 + 75*a**4*b**2*c**9*d/2 + 5*a*
*3*b**3*c**10) + x**3*(15*a**6*c**8*d**2 + 20*a**5*b*c**9*d + 5*a**4*b**2*c
**10) + x**2*(5*a**6*c**9*d + 3*a**5*b*c**10)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(158) = 316$.
time = 0.69, size = 1124, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^6*d^10*x^17 + 5/8*b^6*c*d^9*x^16 + 3/8*a*b^5*d^10*x^16 + 3*b^6*c^2*d
^8*x^15 + 4*a*b^5*c*d^9*x^15 + a^2*b^4*d^10*x^15 + 60/7*b^6*c^3*d^7*x^14 +
135/7*a*b^5*c^2*d^8*x^14 + 75/7*a^2*b^4*c*d^9*x^14 + 10/7*a^3*b^3*d^10*x^14
 + 210/13*b^6*c^4*d^6*x^13 + 720/13*a*b^5*c^3*d^7*x^13 + 675/13*a^2*b^4*c^2
*d^8*x^13 + 200/13*a^3*b^3*c*d^9*x^13 + 15/13*a^4*b^2*d^10*x^13 + 21*b^6*c^
5*d^5*x^12 + 105*a*b^5*c^4*d^6*x^12 + 150*a^2*b^4*c^3*d^7*x^12 + 75*a^3*b^3
*c^2*d^8*x^12 + 25/2*a^4*b^2*c*d^9*x^12 + 1/2*a^5*b*d^10*x^12 + 210/11*b^6*
c^6*d^4*x^11 + 1512/11*a*b^5*c^5*d^5*x^11 + 3150/11*a^2*b^4*c^4*d^6*x^11 +
2400/11*a^3*b^3*c^3*d^7*x^11 + 675/11*a^4*b^2*c^2*d^8*x^11 + 60/11*a^5*b*c*
d^9*x^11 + 1/11*a^6*d^10*x^11 + 12*b^6*c^7*d^3*x^10 + 126*a*b^5*c^6*d^4*x^1
0 + 378*a^2*b^4*c^5*d^5*x^10 + 420*a^3*b^3*c^4*d^6*x^10 + 180*a^4*b^2*c^3*d
^7*x^10 + 27*a^5*b*c^2*d^8*x^10 + a^6*c*d^9*x^10 + 5*b^6*c^8*d^2*x^9 + 80*a
*b^5*c^7*d^3*x^9 + 350*a^2*b^4*c^6*d^4*x^9 + 560*a^3*b^3*c^5*d^5*x^9 + 350*
a^4*b^2*c^4*d^6*x^9 + 80*a^5*b*c^3*d^7*x^9 + 5*a^6*c^2*d^8*x^9 + 5/4*b^6*c^
9*d*x^8 + 135/4*a*b^5*c^8*d^2*x^8 + 225*a^2*b^4*c^7*d^3*x^8 + 525*a^3*b^3*c
^6*d^4*x^8 + 945/2*a^4*b^2*c^5*d^5*x^8 + 315/2*a^5*b*c^4*d^6*x^8 + 15*a^6*c
^3*d^7*x^8 + 1/7*b^6*c^10*x^7 + 60/7*a*b^5*c^9*d*x^7 + 675/7*a^2*b^4*c^8*d^
2*x^7 + 2400/7*a^3*b^3*c^7*d^3*x^7 + 450*a^4*b^2*c^6*d^4*x^7 + 216*a^5*b*c^
5*d^5*x^7 + 30*a^6*c^4*d^6*x^7 + a*b^5*c^10*x^6 + 25*a^2*b^4*c^9*d*x^6 + 15
0*a^3*b^3*c^8*d^2*x^6 + 300*a^4*b^2*c^7*d^3*x^6 + 210*a^5*b*c^6*d^4*x^6 + 4
2*a^6*c^5*d^5*x^6 + 3*a^2*b^4*c^10*x^5 + 40*a^3*b^3*c^9*d*x^5 + 135*a^4*b^2
*c^8*d^2*x^5 + 144*a^5*b*c^7*d^3*x^5 + 42*a^6*c^6*d^4*x^5 + 5*a^3*b^3*c^10*
x^4 + 75/2*a^4*b^2*c^9*d*x^4 + 135/2*a^5*b*c^8*d^2*x^4 + 30*a^6*c^7*d^3*x^4
 + 5*a^4*b^2*c^10*x^3 + 20*a^5*b*c^9*d*x^3 + 15*a^6*c^8*d^2*x^3 + 3*a^5*b*c
^10*x^2 + 5*a^6*c^9*d*x^2 + a^6*c^10*x
```

Mupad [B]
time = 0.53, size = 953, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^10,x)
```

```
[Out] x^7*((b^6*c^10)/7 + 30*a^6*c^4*d^6 + 216*a^5*b*c^5*d^5 + (675*a^2*b^4*c^8*d
^2)/7 + (2400*a^3*b^3*c^7*d^3)/7 + 450*a^4*b^2*c^6*d^4 + (60*a*b^5*c^9*d)/7
) + x^11*((a^6*d^10)/11 + (210*b^6*c^6*d^4)/11 + (1512*a*b^5*c^5*d^5)/11 +
(3150*a^2*b^4*c^4*d^6)/11 + (2400*a^3*b^3*c^3*d^7)/11 + (675*a^4*b^2*c^2*d^
8)/11 + (60*a^5*b*c*d^9)/11) + x^9*(5*a^6*c^2*d^8 + 5*b^6*c^8*d^2 + 80*a*b^
5*c^7*d^3 + 80*a^5*b*c^3*d^7 + 350*a^2*b^4*c^6*d^4 + 560*a^3*b^3*c^5*d^5 +
350*a^4*b^2*c^4*d^6) + x^5*(3*a^2*b^4*c^10 + 42*a^6*c^6*d^4 + 40*a^3*b^3*c^
9*d + 144*a^5*b*c^7*d^3 + 135*a^4*b^2*c^8*d^2) + x^13*((15*a^4*b^2*d^10)/13
 + (210*b^6*c^4*d^6)/13 + (720*a*b^5*c^3*d^7)/13 + (200*a^3*b^3*c*d^9)/13 +
 (675*a^2*b^4*c^2*d^8)/13) + x^6*(a*b^5*c^10 + 42*a^6*c^5*d^5 + 25*a^2*b^4*
c^9*d + 210*a^5*b*c^6*d^4 + 150*a^3*b^3*c^8*d^2 + 300*a^4*b^2*c^7*d^3) + x^
12*((a^5*b*d^10)/2 + 21*b^6*c^5*d^5 + 105*a*b^5*c^4*d^6 + (25*a^4*b^2*c*d^9
)/2 + 150*a^2*b^4*c^3*d^7 + 75*a^3*b^3*c^2*d^8) + x^10*(a^6*c*d^9 + 12*b^6*
c^7*d^3 + 126*a*b^5*c^6*d^4 + 27*a^5*b*c^2*d^8 + 378*a^2*b^4*c^5*d^5 + 420*
a^3*b^3*c^4*d^6 + 180*a^4*b^2*c^3*d^7) + x^8*((5*b^6*c^9*d)/4 + 15*a^6*c^3*
d^7 + (135*a*b^5*c^8*d^2)/4 + (315*a^5*b*c^4*d^6)/2 + 225*a^2*b^4*c^7*d^3 +
 525*a^3*b^3*c^6*d^4 + (945*a^4*b^2*c^5*d^5)/2) + a^6*c^10*x + (b^6*d^10*x^
17)/17 + (5*a^3*c^7*x^4*(12*a^3*d^3 + 2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b
*c*d^2))/2 + (5*b^3*d^7*x^14*(2*a^3*d^3 + 12*b^3*c^3 + 27*a*b^2*c^2*d + 15*
a^2*b*c*d^2))/7 + a^5*c^9*x^2*(5*a*d + 3*b*c) + (b^5*d^9*x^16*(3*a*d + 5*b*
c))/8 + 5*a^4*c^8*x^3*(3*a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + b^4*d^8*x^15*(a^2
*d^2 + 3*b^2*c^2 + 4*a*b*c*d)
```

3.1306 $\int (a+bx)^5(c+dx)^{10}\,dx$

Optimal. Leaf size=146

$$-\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\frac{5b^3(bc-ad)^2(c+dx)^{14}}{7d^6}-\frac{b^4(bc-$$

```
[Out] -1/11*(-a*d+b*c)^5*(d*x+c)^11/d^6+5/12*b*(-a*d+b*c)^4*(d*x+c)^12/d^6-10/13*
b^2*(-a*d+b*c)^3*(d*x+c)^13/d^6+5/7*b^3*(-a*d+b*c)^2*(d*x+c)^14/d^6-1/3*b^4
*(-a*d+b*c)*(d*x+c)^15/d^6+1/16*b^5*(d*x+c)^16/d^6
```

Rubi [A]
time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc-ad)^5}{11d^6}+\frac{b^5(c+dx)^{16}}{16d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^5*(c + d*x)^11)/d^6 + (5*b*(b*c - a*d)^4*(c + d*x)^12)/(
12*d^6) - (10*b^2*(b*c - a*d)^3*(c + d*x)^13)/(13*d^6) + (5*b^3*(b*c - a*d)
^2*(c + d*x)^14)/(7*d^6) - (b^4*(b*c - a*d)*(c + d*x)^15)/(3*d^6) + (b^5*(c
 + d*x)^16)/(16*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^5(c+dx)^{10}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{11}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{12}}{d^5}\right.\\ &= -\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\frac{5}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 811 vs. 2(146) = 292.
time = 0.06, size = 811, normalized size = 5.55

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5*(c + d*x)^10,x]

[Out] a^5*c^10*x + (5*a^4*c^9*(b*c + 2*a*d)*x^2)/2 + (5*a^3*c^8*(2*b^2*c^2 + 10*a *b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^2*c^7*(2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^ 2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + a*c^6*(b^4*c^4 + 20*a*b^3*c^3*d + 90*a^2*b ^2*c^2*d^2 + 120*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + (c^5*(b^5*c^5 + 50*a*b^4*c ^4*d + 450*a^2*b^3*c^3*d^2 + 1200*a^3*b^2*c^2*d^3 + 1050*a^4*b*c*d^4 + 252* a^5*d^5)*x^6)/6 + (5*c^4*d*(2*b^5*c^5 + 45*a*b^4*c^4*d + 240*a^2*b^3*c^3*d^ 2 + 420*a^3*b^2*c^2*d^3 + 252*a^4*b*c*d^4 + 42*a^5*d^5)*x^7)/7 + (15*c^3*d^ 2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3*c^3*d^2 + 168*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^8)/8 + (5*c^2*d^3*(8*b^5*c^5 + 70*a*b^4*c^4* d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c*d^4 + 3*a^5*d^5) *x^9)/3 + (c*d^4*(42*b^5*c^5 + 252*a*b^4*c^4*d + 420*a^2*b^3*c^3*d^2 + 240* a^3*b^2*c^2*d^3 + 45*a^4*b*c*d^4 + 2*a^5*d^5)*x^10)/2 + (d^5*(252*b^5*c^5 + 1050*a*b^4*c^4*d + 1200*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 50*a^4*b*c *d^4 + a^5*d^5)*x^11)/11 + (5*b*d^6*(42*b^4*c^4 + 120*a*b^3*c^3*d + 90*a^2* b^2*c^2*d^2 + 20*a^3*b*c*d^3 + a^4*d^4)*x^12)/12 + (5*b^2*d^7*(24*b^3*c^3 + 45*a*b^2*c^2*d + 20*a^2*b*c*d^2 + 2*a^3*d^3)*x^13)/13 + (5*b^3*d^8*(9*b^2* c^2 + 10*a*b*c*d + 2*a^2*d^2)*x^14)/14 + (b^4*d^9*(2*b*c + a*d)*x^15)/3 + (b^5*d^10*x^16)/16

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(134) = 268$.

time = 0.14, size = 841, normalized size = 5.76 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^5*(d*x+c)^10,x,method=_RETURNVERBOSE)

[Out] 1/16*b^5*d^10*x^16+1/15*(5*a*b^4*d^10+10*b^5*c*d^9)*x^15+1/14*(10*a^2*b^3*d ^10+50*a*b^4*c*d^9+45*b^5*c^2*d^8)*x^14+1/13*(10*a^3*b^2*d^10+100*a^2*b^3*c *d^9+225*a*b^4*c^2*d^8+120*b^5*c^3*d^7)*x^13+1/12*(5*a^4*b*d^10+100*a^3*b^2 *c*d^9+450*a^2*b^3*c^2*d^8+600*a*b^4*c^3*d^7+210*b^5*c^4*d^6)*x^12+1/11*(a^ 5*d^10+50*a^4*b*c*d^9+450*a^3*b^2*c^2*d^8+1200*a^2*b^3*c^3*d^7+1050*a*b^4*c ^4*d^6+252*b^5*c^5*d^5)*x^11+1/10*(10*a^5*c*d^9+225*a^4*b*c^2*d^8+1200*a^3* b^2*c^3*d^7+2100*a^2*b^3*c^4*d^6+1260*a*b^4*c^5*d^5+210*b^5*c^6*d^4)*x^10+1 /9*(45*a^5*c^2*d^8+600*a^4*b*c^3*d^7+2100*a^3*b^2*c^4*d^6+2520*a^2*b^3*c^5* d^5+1050*a*b^4*c^6*d^4+120*b^5*c^7*d^3)*x^9+1/8*(120*a^5*c^3*d^7+1050*a^4*b *c^4*d^6+2520*a^3*b^2*c^5*d^5+2100*a^2*b^3*c^6*d^4+600*a*b^4*c^7*d^3+45*b^5 *c^8*d^2)*x^8+1/7*(210*a^5*c^4*d^6+1260*a^4*b*c^5*d^5+2100*a^3*b^2*c^6*d^4+ 1200*a^2*b^3*c^7*d^3+225*a*b^4*c^8*d^2+10*b^5*c^9*d)*x^7+1/6*(252*a^5*c^5*d ^5+1050*a^4*b*c^6*d^4+1200*a^3*b^2*c^7*d^3+450*a^2*b^3*c^8*d^2+50*a*b^4*c^9 *d+b^5*c^10)*x^6+1/5*(210*a^5*c^6*d^4+600*a^4*b*c^7*d^3+450*a^3*b^2*c^8*d^2 +100*a^2*b^3*c^9*d+5*a*b^4*c^10)*x^5+1/4*(120*a^5*c^7*d^3+225*a^4*b*c^8*d^2 +100*a^3*b^2*c^9*d+10*a^2*b^3*c^10)*x^4+1/3*(45*a^5*c^8*d^2+50*a^4*b*c^9*d+ 10*a^3*b^2*c^10)*x^3+1/2*(10*a^5*c^9*d+5*a^4*b*c^10)*x^2+a^5*c^10*x

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(134) = 268$.
time = 0.31, size = 835, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(134) = 268$.
time = 0.55, size = 835, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
```

```
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 940 vs. $2(131) = 262$.
time = 0.07, size = 940, normalized size = 6.44

$$a^5c^{10}x + \frac{b^5d^{10}x^{16}}{16} + x^{15}\left(\frac{ab^4d^{10}}{3} + \frac{2b^5cd^9}{3}\right) + x^{14}\cdot\left(\frac{5a^2b^3d^{10}}{7} + \frac{25ab^4cd^9}{7} + \frac{45b^5c^2d^8}{14}\right) + x^{13}\cdot\left(\frac{10a^3b^2d^{10}}{13} + \frac{100a^2b^3cd^9}{13} + \frac{225ab^4c^2d^8}{13} + \frac{120b^5c^3d^7}{13}\right) + x^{12}\cdot\left(\frac{5a^4bd^{10}}{12} + \frac{25a^3b^2cd^9}{3} + \frac{75a^2b^3c^2d^8}{2} + 50ab^4c^3d^7 + \frac{35b^5c^4d^6}{2}\right) + x^{11}\left(\frac{a^5d^{10}}{11} + \frac{50a^4bcd^9}{11} + \frac{450a^3b^2c^2d^8}{11} + \frac{1200a^2b^3c^3d^7}{11} + \frac{1050ab^4c^4d^6}{11} + \frac{252b^5c^5d^5}{11}\right) + x^{10}\left(a^5cd^9 + \frac{45a^4bc^2d^8}{2} + 120a^3b^2c^3d^7 + 210a^2b^3c^4d^6 + 126ab^4c^5d^5 + 21b^5c^6d^4\right) + x^9\cdot\left(5a^5c^2d^8 + \frac{200a^4bc^3d^7}{3} + \frac{700a^3b^2c^4d^6}{3} + 280a^2b^3c^5d^5 + \frac{350ab^4c^6d^4}{3} + \frac{40b^5c^7d^3}{3}\right) + x^8\cdot\left(15a^5c^3d^7 + \frac{525a^4bc^4d^6}{4} + 315a^3b^2c^5d^5 + \frac{525a^2b^3c^6d^4}{2} + 75ab^4c^7d^3 + \frac{45b^5c^8d^2}{8}\right) + x^7\cdot\left(30a^5c^4d^6 + 180a^4bc^5d^5 + 300a^3b^2c^6d^4 + \frac{1200a^2b^3c^7d^3}{7} + \frac{225ab^4c^8d^2}{7} + \frac{10b^5c^9d}{7}\right) + x^6\cdot\left(42a^5c^5d^5 + 175a^4bc^6d^4 + 200a^3b^2c^7d^3 + 75a^2b^3c^8d^2 + \frac{25ab^4c^9d}{3} + \frac{b^5c^{10}}{6}\right) + x^5\cdot\left(42a^5c^6d^4 + 120a^4bc^7d^3 + 90a^3b^2c^8d^2 + 20a^2b^3c^9d + ab^4c^{10}\right) + x^4\cdot\left(30a^5c^7d^3 + \frac{225a^4bc^8d^2}{4} + 25a^3b^2c^9d + \frac{5a^2b^3c^{10}}{2}\right) + x^3\cdot\left(15a^5c^8d^2 + \frac{50a^4bc^9d}{3} + \frac{10a^3b^2c^{10}}{3}\right) + x^2\cdot\left(5a^5c^9d + \frac{5a^4bc^{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**10,x)
```

```
[Out] a**5*c**10*x + b**5*d**10*x**16/16 + x**15*(a*b**4*d**10/3 + 2*b**5*c*d**9/
3) + x**14*(5*a**2*b**3*d**10/7 + 25*a*b**4*c*d**9/7 + 45*b**5*c**2*d**8/14
) + x**13*(10*a**3*b**2*d**10/13 + 100*a**2*b**3*c*d**9/13 + 225*a*b**4*c**
2*d**8/13 + 120*b**5*c**3*d**7/13) + x**12*(5*a**4*b*d**10/12 + 25*a**3*b**
2*c*d**9/3 + 75*a**2*b**3*c**2*d**8/2 + 50*a*b**4*c**3*d**7 + 35*b**5*c**4*
d**6/2) + x**11*(a**5*d**10/11 + 50*a**4*b*c*d**9/11 + 450*a**3*b**2*c**2*d
**8/11 + 1200*a**2*b**3*c**3*d**7/11 + 1050*a*b**4*c**4*d**6/11 + 252*b**5*
c**5*d**5/11) + x**10*(a**5*c*d**9 + 45*a**4*b*c**2*d**8/2 + 120*a**3*b**2*
c**3*d**7 + 210*a**2*b**3*c**4*d**6 + 126*a*b**4*c**5*d**5 + 21*b**5*c**6*d
**4) + x**9*(5*a**5*c**2*d**8 + 200*a**4*b*c**3*d**7/3 + 700*a**3*b**2*c**4
*d**6/3 + 280*a**2*b**3*c**5*d**5 + 350*a*b**4*c**6*d**4/3 + 40*b**5*c**7*d
**3/3) + x**8*(15*a**5*c**3*d**7 + 525*a**4*b*c**4*d**6/4 + 315*a**3*b**2*c
**5*d**5 + 525*a**2*b**3*c**6*d**4/2 + 75*a*b**4*c**7*d**3 + 45*b**5*c**8*d
**2/8) + x**7*(30*a**5*c**4*d**6 + 180*a**4*b*c**5*d**5 + 300*a**3*b**2*c**
6*d**4 + 1200*a**2*b**3*c**7*d**3/7 + 225*a*b**4*c**8*d**2/7 + 10*b**5*c**9
*d/7) + x**6*(42*a**5*c**5*d**5 + 175*a**4*b*c**6*d**4 + 200*a**3*b**2*c**7
*d**3 + 75*a**2*b**3*c**8*d**2 + 25*a*b**4*c**9*d/3 + b**5*c**10/6) + x**5*
(42*a**5*c**6*d**4 + 120*a**4*b*c**7*d**3 + 90*a**3*b**2*c**8*d**2 + 20*a**
2*b**3*c**9*d + a*b**4*c**10) + x**4*(30*a**5*c**7*d**3 + 225*a**4*b*c**8*d
**2/4 + 25*a**3*b**2*c**9*d + 5*a**2*b**3*c**10/2) + x**3*(15*a**5*c**8*d**
2 + 50*a**4*b*c**9*d/3 + 10*a**3*b**2*c**10/3) + x**2*(5*a**5*c**9*d + 5*a*
*4*b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(134) = 268$.

time = 0.65, size = 948, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="giac")

[Out] 1/16*b^5*d^10*x^16 + 2/3*b^5*c*d^9*x^15 + 1/3*a*b^4*d^10*x^15 + 45/14*b^5*c^2*d^8*x^14 + 25/7*a*b^4*c*d^9*x^14 + 5/7*a^2*b^3*d^10*x^14 + 120/13*b^5*c^3*d^7*x^13 + 225/13*a*b^4*c^2*d^8*x^13 + 100/13*a^2*b^3*c*d^9*x^13 + 10/13*a^3*b^2*d^10*x^13 + 35/2*b^5*c^4*d^6*x^12 + 50*a*b^4*c^3*d^7*x^12 + 75/2*a^2*b^3*c^2*d^8*x^12 + 25/3*a^3*b^2*c*d^9*x^12 + 5/12*a^4*b*d^10*x^12 + 252/11*b^5*c^5*d^5*x^11 + 1050/11*a*b^4*c^4*d^6*x^11 + 1200/11*a^2*b^3*c^3*d^7*x^11 + 450/11*a^3*b^2*c^2*d^8*x^11 + 50/11*a^4*b*c*d^9*x^11 + 1/11*a^5*d^10*x^11 + 21*b^5*c^6*d^4*x^10 + 126*a*b^4*c^5*d^5*x^10 + 210*a^2*b^3*c^4*d^6*x^10 + 120*a^3*b^2*c^3*d^7*x^10 + 45/2*a^4*b*c^2*d^8*x^10 + a^5*c*d^9*x^10 + 40/3*b^5*c^7*d^3*x^9 + 350/3*a*b^4*c^6*d^4*x^9 + 280*a^2*b^3*c^5*d^5*x^9 + 700/3*a^3*b^2*c^4*d^6*x^9 + 200/3*a^4*b*c^3*d^7*x^9 + 5*a^5*c^2*d^8*x^9 + 45/8*b^5*c^8*d^2*x^8 + 75*a*b^4*c^7*d^3*x^8 + 525/2*a^2*b^3*c^6*d^4*x^8 + 315*a^3*b^2*c^5*d^5*x^8 + 525/4*a^4*b*c^4*d^6*x^8 + 15*a^5*c^3*d^7*x^8 + 10/7*b^5*c^9*d*x^7 + 225/7*a*b^4*c^8*d^2*x^7 + 1200/7*a^2*b^3*c^7*d^3*x^7 + 300*a^3*b^2*c^6*d^4*x^7 + 180*a^4*b*c^5*d^5*x^7 + 30*a^5*c^4*d^6*x^7 + 1/6*b^5*c^10*x^6 + 25/3*a*b^4*c^9*d*x^6 + 75*a^2*b^3*c^8*d^2*x^6 + 200*a^3*b^2*c^7*d^3*x^6 + 175*a^4*b*c^6*d^4*x^6 + 42*a^5*c^5*d^5*x^6 + a*b^4*c^10*x^5 + 20*a^2*b^3*c^9*d*x^5 + 90*a^3*b^2*c^8*d^2*x^5 + 120*a^4*b*c^7*d^3*x^5 + 42*a^5*c^6*d^4*x^5 + 5/2*a^2*b^3*c^10*x^4 + 25*a^3*b^2*c^9*d*x^4 + 225/4*a^4*b*c^8*d^2*x^4 + 30*a^5*c^7*d^3*x^4 + 10/3*a^3*b^2*c^10*x^3 + 50/3*a^4*b*c^9*d*x^3 + 15*a^5*c^8*d^2*x^3 + 5/2*a^4*b*c^10*x^2 + 5*a^5*c^9*d*x^2 + a^5*c^10*x

Mupad [B]
time = 0.34, size = 806, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^5*(c + d*x)^10,x)

[Out] x^10*(a^5*c*d^9 + 21*b^5*c^6*d^4 + 126*a*b^4*c^5*d^5 + (45*a^4*b*c^2*d^8)/2 + 210*a^2*b^3*c^4*d^6 + 120*a^3*b^2*c^3*d^7) + x^7*((10*b^5*c^9*d)/7 + 30*a^5*c^4*d^6 + (225*a*b^4*c^8*d^2)/7 + 180*a^4*b*c^5*d^5 + (1200*a^2*b^3*c^7*d^3)/7 + 300*a^3*b^2*c^6*d^4) + x^6*((b^5*c^10)/6 + 42*a^5*c^5*d^5 + 175*a^4*b*c^6*d^4 + 75*a^2*b^3*c^8*d^2 + 200*a^3*b^2*c^7*d^3 + (25*a*b^4*c^9*d)/3) + x^11*((a^5*d^10)/11 + (252*b^5*c^5*d^5)/11 + (1050*a*b^4*c^4*d^6)/11 + (1200*a^2*b^3*c^3*d^7)/11 + (450*a^3*b^2*c^2*d^8)/11 + (50*a^4*b*c*d^9)/11

) + x^8*(15*a^5*c^3*d^7 + (45*b^5*c^8*d^2)/8 + 75*a*b^4*c^7*d^3 + (525*a^4*b*c^4*d^6)/4 + (525*a^2*b^3*c^6*d^4)/2 + 315*a^3*b^2*c^5*d^5) + x^9*(5*a^5*c^2*d^8 + (40*b^5*c^7*d^3)/3 + (350*a*b^4*c^6*d^4)/3 + (200*a^4*b*c^3*d^7)/3 + 280*a^2*b^3*c^5*d^5 + (700*a^3*b^2*c^4*d^6)/3) + x^5*(a*b^4*c^10 + 42*a^5*c^6*d^4 + 20*a^2*b^3*c^9*d + 120*a^4*b*c^7*d^3 + 90*a^3*b^2*c^8*d^2) + x^12*((5*a^4*b*d^10)/12 + (35*b^5*c^4*d^6)/2 + 50*a*b^4*c^3*d^7 + (25*a^3*b^2*c*d^9)/3 + (75*a^2*b^3*c^2*d^8)/2) + a^5*c^10*x + (b^5*d^10*x^16)/16 + (5*a^2*c^7*x^4*(24*a^3*d^3 + 2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^2*b*c*d^2))/4 + (5*b^2*d^7*x^13*(2*a^3*d^3 + 24*b^3*c^3 + 45*a*b^2*c^2*d + 20*a^2*b*c*d^2))/13 + (5*a^4*c^9*x^2*(2*a*d + b*c))/2 + (b^4*d^9*x^15*(a*d + 2*b*c))/3 + (5*a^3*c^8*x^3*(9*a^2*d^2 + 2*b^2*c^2 + 10*a*b*c*d))/3 + (5*b^3*d^8*x^14*(2*a^2*d^2 + 9*b^2*c^2 + 10*a*b*c*d))/14

3.1307 $\int (a+bx)^4(c+dx)^{10}\,dx$

Optimal. Leaf size=119

$$\frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-\frac{2b^3(bc-ad)(c+dx)^{14}}{7d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

```
[Out] 1/11*(-a*d+b*c)^4*(d*x+c)^11/d^5-1/3*b*(-a*d+b*c)^3*(d*x+c)^12/d^5+6/13*b^2
*(-a*d+b*c)^2*(d*x+c)^13/d^5-2/7*b^3*(-a*d+b*c)*(d*x+c)^14/d^5+1/15*b^4*(d*
x+c)^15/d^5
```

Rubi [A]
time = 0.31, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^11)/(11*d^5) - (b*(b*c - a*d)^3*(c + d*x)^12)/(3*d
^5) + (6*b^2*(b*c - a*d)^2*(c + d*x)^13)/(13*d^5) - (2*b^3*(b*c - a*d)*(c +
 d*x)^14)/(7*d^5) + (b^4*(c + d*x)^15)/(15*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{10}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{11}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{12}}{d^4}\right.\\ &= \frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 660 vs. 2(119) = 238.
time = 0.05, size = 660, normalized size = 5.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] a^4*c^10*x + a^3*c^9*(2*b*c + 5*a*d)*x^2 + (a^2*c^8*(6*b^2*c^2 + 40*a*b*c*d
 + 45*a^2*d^2)*x^3)/3 + a*c^7*(b^3*c^3 + 15*a*b^2*c^2*d + 45*a^2*b*c*d^2 +
30*a^3*d^3)*x^4 + (c^6*(b^4*c^4 + 40*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 48
0*a^3*b*c*d^3 + 210*a^4*d^4)*x^5)/5 + (c^5*d*(5*b^4*c^4 + 90*a*b^3*c^3*d +
360*a^2*b^2*c^2*d^2 + 420*a^3*b*c*d^3 + 126*a^4*d^4)*x^6)/3 + (3*c^4*d^2*(1
5*b^4*c^4 + 160*a*b^3*c^3*d + 420*a^2*b^2*c^2*d^2 + 336*a^3*b*c*d^3 + 70*a^
4*d^4)*x^7)/7 + 3*c^3*d^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2*b^2*c^2*d^2
+ 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^8 + (c^2*d^4*(70*b^4*c^4 + 336*a*b^3*c^3*d
+ 420*a^2*b^2*c^2*d^2 + 160*a^3*b*c*d^3 + 15*a^4*d^4)*x^9)/3 + (c*d^5*(126*
b^4*c^4 + 420*a*b^3*c^3*d + 360*a^2*b^2*c^2*d^2 + 90*a^3*b*c*d^3 + 5*a^4*d^
4)*x^10)/5 + (d^6*(210*b^4*c^4 + 480*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 + a^4*d^4)*x^11)/11 + (b*d^7*(30*b^3*c^3 + 45*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + a^3*d^3)*x^12)/3 + (b^2*d^8*(45*b^2*c^2 + 40*a*b*c*d + 6*a^2
*d^2)*x^13)/13 + (b^3*d^9*(5*b*c + 2*a*d)*x^14)/7 + (b^4*d^10*x^15)/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(109) = 218$.
time = 0.13, size = 691, normalized size = 5.81 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^4*d^10*x^15+1/14*(4*a*b^3*d^10+10*b^4*c*d^9)*x^14+1/13*(6*a^2*b^2*d^
10+40*a*b^3*c*d^9+45*b^4*c^2*d^8)*x^13+1/12*(4*a^3*b*d^10+60*a^2*b^2*c*d^9+
180*a*b^3*c^2*d^8+120*b^4*c^3*d^7)*x^12+1/11*(a^4*d^10+40*a^3*b*c*d^9+270*a
^2*b^2*c^2*d^8+480*a*b^3*c^3*d^7+210*b^4*c^4*d^6)*x^11+1/10*(10*a^4*c*d^9+1
80*a^3*b*c^2*d^8+720*a^2*b^2*c^3*d^7+840*a*b^3*c^4*d^6+252*b^4*c^5*d^5)*x^1
0+1/9*(45*a^4*c^2*d^8+480*a^3*b*c^3*d^7+1260*a^2*b^2*c^4*d^6+1008*a*b^3*c^5
*d^5+210*b^4*c^6*d^4)*x^9+1/8*(120*a^4*c^3*d^7+840*a^3*b*c^4*d^6+1512*a^2*b
^2*c^5*d^5+840*a*b^3*c^6*d^4+120*b^4*c^7*d^3)*x^8+1/7*(210*a^4*c^4*d^6+1008
*a^3*b*c^5*d^5+1260*a^2*b^2*c^6*d^4+480*a*b^3*c^7*d^3+45*b^4*c^8*d^2)*x^7+1
/6*(252*a^4*c^5*d^5+840*a^3*b*c^6*d^4+720*a^2*b^2*c^7*d^3+180*a*b^3*c^8*d^2
+10*b^4*c^9*d)*x^6+1/5*(210*a^4*c^6*d^4+480*a^3*b*c^7*d^3+270*a^2*b^2*c^8*d
^2+40*a*b^3*c^9*d+b^4*c^10)*x^5+1/4*(120*a^4*c^7*d^3+180*a^3*b*c^8*d^2+60*a
^2*b^2*c^9*d+4*a*b^3*c^10)*x^4+1/3*(45*a^4*c^8*d^2+40*a^3*b*c^9*d+6*a^2*b^2
*c^10)*x^3+1/2*(10*a^4*c^9*d+4*a^3*b*c^10)*x^2+a^4*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(109) = 218$.
time = 0.30, size = 686, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="maxima")
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(109) = 218$.
time = 0.60, size = 686, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="fricas")
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(105) = 210$.
time = 0.06, size = 748, normalized size = 6.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**10,x)
```

```
[Out] a**4*c**10*x + b**4*d**10*x**15/15 + x**14*(2*a*b**3*d**10/7 + 5*b**4*c*d**
9/7) + x**13*(6*a**2*b**2*d**10/13 + 40*a*b**3*c*d**9/13 + 45*b**4*c**2*d**
8/13) + x**12*(a**3*b*d**10/3 + 5*a**2*b**2*c*d**9 + 15*a*b**3*c**2*d**8 +
10*b**4*c**3*d**7) + x**11*(a**4*d**10/11 + 40*a**3*b*c*d**9/11 + 270*a**2*
b**2*c**2*d**8/11 + 480*a*b**3*c**3*d**7/11 + 210*b**4*c**4*d**6/11) + x**1
0*(a**4*c*d**9 + 18*a**3*b*c**2*d**8 + 72*a**2*b**2*c**3*d**7 + 84*a*b**3*c
**4*d**6 + 126*b**4*c**5*d**5/5) + x**9*(5*a**4*c**2*d**8 + 160*a**3*b*c**3
*d**7/3 + 140*a**2*b**2*c**4*d**6 + 112*a*b**3*c**5*d**5 + 70*b**4*c**6*d**
4/3) + x**8*(15*a**4*c**3*d**7 + 105*a**3*b*c**4*d**6 + 189*a**2*b**2*c**5*
d**5 + 105*a*b**3*c**6*d**4 + 15*b**4*c**7*d**3) + x**7*(30*a**4*c**4*d**6
+ 144*a**3*b*c**5*d**5 + 180*a**2*b**2*c**6*d**4 + 480*a*b**3*c**7*d**3/7 +
 45*b**4*c**8*d**2/7) + x**6*(42*a**4*c**5*d**5 + 140*a**3*b*c**6*d**4 + 12
0*a**2*b**2*c**7*d**3 + 30*a*b**3*c**8*d**2 + 5*b**4*c**9*d/3) + x**5*(42*a
**4*c**6*d**4 + 96*a**3*b*c**7*d**3 + 54*a**2*b**2*c**8*d**2 + 8*a*b**3*c**
9*d + b**4*c**10/5) + x**4*(30*a**4*c**7*d**3 + 45*a**3*b*c**8*d**2 + 15*a*
*2*b**2*c**9*d + a*b**3*c**10) + x**3*(15*a**4*c**8*d**2 + 40*a**3*b*c**9*d
/3 + 2*a**2*b**2*c**10) + x**2*(5*a**4*c**9*d + 2*a**3*b*c**10)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(109) = 218.
time = 0.55, size = 771, normalized size = 6.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^4*d^10*x^15 + 5/7*b^4*c*d^9*x^14 + 2/7*a*b^3*d^10*x^14 + 45/13*b^4*c
^2*d^8*x^13 + 40/13*a*b^3*c*d^9*x^13 + 6/13*a^2*b^2*d^10*x^13 + 10*b^4*c^3*
d^7*x^12 + 15*a*b^3*c^2*d^8*x^12 + 5*a^2*b^2*c*d^9*x^12 + 1/3*a^3*b*d^10*x^
12 + 210/11*b^4*c^4*d^6*x^11 + 480/11*a*b^3*c^3*d^7*x^11 + 270/11*a^2*b^2*c
^2*d^8*x^11 + 40/11*a^3*b*c*d^9*x^11 + 1/11*a^4*d^10*x^11 + 126/5*b^4*c^5*d
^5*x^10 + 84*a*b^3*c^4*d^6*x^10 + 72*a^2*b^2*c^3*d^7*x^10 + 18*a^3*b*c^2*d^
8*x^10 + a^4*c*d^9*x^10 + 70/3*b^4*c^6*d^4*x^9 + 112*a*b^3*c^5*d^5*x^9 + 14
0*a^2*b^2*c^4*d^6*x^9 + 160/3*a^3*b*c^3*d^7*x^9 + 5*a^4*c^2*d^8*x^9 + 15*b^
4*c^7*d^3*x^8 + 105*a*b^3*c^6*d^4*x^8 + 189*a^2*b^2*c^5*d^5*x^8 + 105*a^3*b
*c^4*d^6*x^8 + 15*a^4*c^3*d^7*x^8 + 45/7*b^4*c^8*d^2*x^7 + 480/7*a*b^3*c^7*
d^3*x^7 + 180*a^2*b^2*c^6*d^4*x^7 + 144*a^3*b*c^5*d^5*x^7 + 30*a^4*c^4*d^6*
x^7 + 5/3*b^4*c^9*d*x^6 + 30*a*b^3*c^8*d^2*x^6 + 120*a^2*b^2*c^7*d^3*x^6 +
140*a^3*b*c^6*d^4*x^6 + 42*a^4*c^5*d^5*x^6 + 1/5*b^4*c^10*x^5 + 8*a*b^3*c^9
*d*x^5 + 54*a^2*b^2*c^8*d^2*x^5 + 96*a^3*b*c^7*d^3*x^5 + 42*a^4*c^6*d^4*x^5
 + a*b^3*c^10*x^4 + 15*a^2*b^2*c^9*d*x^4 + 45*a^3*b*c^8*d^2*x^4 + 30*a^4*c^
```

```
7*d^3*x^4 + 2*a^2*b^2*c^10*x^3 + 40/3*a^3*b*c^9*d*x^3 + 15*a^4*c^8*d^2*x^3
+ 2*a^3*b*c^10*x^2 + 5*a^4*c^9*d*x^2 + a^4*c^10*x
```

Mupad [B]
time = 0.43, size = 664, normalized size = 5.58

$$x^5\left(42a^4c^6d^4+96a^3bc^7d^3+54a^2b^2c^8d^2+8ab^3c^9d+\frac{b^4c^{10}}{5}\right)+x^{11}\left(\frac{a^4d^{10}}{11}+\frac{40a^3bcd^9}{11}+\frac{270a^2b^2c^2d^8}{11}+\frac{480ab^3c^3d^7}{11}+\frac{210b^4c^4d^6}{11}\right)+x^8\left(15a^4c^3d^7+105a^3bc^4d^6+189a^2b^2c^5d^5+105ab^3c^6d^4+15b^4c^7d^3\right)+x^9\left(5a^4c^2d^8+\frac{160a^3bc^3d^7}{3}+140a^2b^2c^4d^6+112ab^3c^5d^5+\frac{70b^4c^6d^4}{3}\right)+x^7\left(30a^4c^4d^6+144a^3bc^5d^5+180a^2b^2c^6d^4+\frac{480ab^3c^7d^3}{7}+\frac{45b^4c^8d^2}{7}\right)+x^4\left(30a^4c^7d^3+45a^3bc^8d^2+15a^2b^2c^9d+ab^3c^{10}\right)+x^{12}\left(\frac{a^3bd^{10}}{3}+5a^2b^2cd^9+15ab^3c^2d^8+10b^4c^3d^7\right)+x^{10}\left(a^4cd^9+18a^3bc^2d^8+72a^2b^2c^3d^7+84ab^3c^4d^6+\frac{126b^4c^5d^5}{5}\right)+x^6\left(42a^4c^5d^5+140a^3bc^6d^4+120a^2b^2c^7d^3+30ab^3c^8d^2+\frac{5b^4c^9d}{3}\right)+a^4c^{10}x+\frac{b^4d^{10}x^{15}}{15}+a^3c^9x^2(5ad+2bc)+\frac{b^3d^9x^{14}(2ad+5bc)}{7}+\frac{a^2c^8x^3(45a^2d^2+40abcd+6b^2c^2)}{3}+\frac{b^2d^8x^{13}(6a^2d^2+40abcd+45b^2c^2)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^10,x)
```

```
[Out] x^5*((b^4*c^10)/5 + 42*a^4*c^6*d^4 + 96*a^3*b*c^7*d^3 + 54*a^2*b^2*c^8*d^2
+ 8*a*b^3*c^9*d) + x^11*((a^4*d^10)/11 + (210*b^4*c^4*d^6)/11 + (480*a*b^3*
c^3*d^7)/11 + (270*a^2*b^2*c^2*d^8)/11 + (40*a^3*b*c*d^9)/11) + x^8*(15*a^4
*c^3*d^7 + 15*b^4*c^7*d^3 + 105*a*b^3*c^6*d^4 + 105*a^3*b*c^4*d^6 + 189*a^2
*b^2*c^5*d^5) + x^9*(5*a^4*c^2*d^8 + (70*b^4*c^6*d^4)/3 + 112*a*b^3*c^5*d^5
 + (160*a^3*b*c^3*d^7)/3 + 140*a^2*b^2*c^4*d^6) + x^7*(30*a^4*c^4*d^6 + (45
*b^4*c^8*d^2)/7 + (480*a*b^3*c^7*d^3)/7 + 144*a^3*b*c^5*d^5 + 180*a^2*b^2*c
^6*d^4) + x^4*(a*b^3*c^10 + 30*a^4*c^7*d^3 + 15*a^2*b^2*c^9*d + 45*a^3*b*c^
8*d^2) + x^12*((a^3*b*d^10)/3 + 10*b^4*c^3*d^7 + 15*a*b^3*c^2*d^8 + 5*a^2*b
^2*c*d^9) + x^10*(a^4*c*d^9 + (126*b^4*c^5*d^5)/5 + 84*a*b^3*c^4*d^6 + 18*a
^3*b*c^2*d^8 + 72*a^2*b^2*c^3*d^7) + x^6*((5*b^4*c^9*d)/3 + 42*a^4*c^5*d^5
+ 30*a*b^3*c^8*d^2 + 140*a^3*b*c^6*d^4 + 120*a^2*b^2*c^7*d^3) + a^4*c^10*x
+ (b^4*d^10*x^15)/15 + a^3*c^9*x^2*(5*a*d + 2*b*c) + (b^3*d^9*x^14*(2*a*d +
 5*b*c))/7 + (a^2*c^8*x^3*(45*a^2*d^2 + 6*b^2*c^2 + 40*a*b*c*d))/3 + (b^2*d
^8*x^13*(6*a^2*d^2 + 45*b^2*c^2 + 40*a*b*c*d))/13
```

3.1308 $\int (a+bx)^3(c+dx)^{10}\,dx$

Optimal. Leaf size=92

$$-\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

```
[Out] -1/11*(-a*d+b*c)^3*(d*x+c)^11/d^4+1/4*b*(-a*d+b*c)^2*(d*x+c)^12/d^4-3/13*b^
2*(-a*d+b*c)*(d*x+c)^13/d^4+1/14*b^3*(d*x+c)^14/d^4
```

Rubi [A]
time = 0.25, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)^3*(c + d*x)^11)/d^4 + (b*(b*c - a*d)^2*(c + d*x)^12)/(4*
d^4) - (3*b^2*(b*c - a*d)*(c + d*x)^13)/(13*d^4) + (b^3*(c + d*x)^14)/(14*d
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^{10}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{11}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{12}}{d^3}\right.\\ &= -\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 511 vs. 2(92) = 184.
time = 0.04, size = 511, normalized size = 5.55

$$a^3c^{10}x+\frac{1}{2}a^2c^9(3bc+10ad)x^2+ac^8\left(b^2c^2+10abcd+15a^2d^2\right)x^3+\frac{1}{4}c^7\left(b^3c^3+30ab^2c^2d+135a^2bcd^2+120a^3d^3\right)x^4+c^6d\left(2b^3c^3+27ab^2c^2d+72a^2bcd^2+42a^3d^3\right)x^5+\frac{3}{2}c^5d^2\left(5b^3c^3+40ab^2c^2d+70a^2bcd^2+28a^3d^3\right)x^6+\frac{6}{7}c^4d^3\left(20b^3c^3+105ab^2c^2d+126a^2bcd^2+35a^3d^3\right)x^7+\frac{3}{4}c^3d^4\left(35b^3c^3+126ab^2c^2d+105a^2bcd^2+20a^3d^3\right)x^8+c^2d^5\left(28b^3c^3+70ab^2c^2d+40a^2bcd^2+5a^3d^3\right)x^9+\frac{1}{2}cd^6\left(42b^3c^3+72ab^2c^2d+27a^2bcd^2+2a^3d^3\right)x^{10}+\frac{1}{11}d^7\left(120b^3c^3+135ab^2c^2d+30a^2bcd^2+a^3d^3\right)x^{11}+\frac{1}{4}bd^8\left(15b^2c^2+10abcd+a^2d^2\right)x^{12}+\frac{1}{13}b^2d^9(10bc+3ad)x^{13}+\frac{1}{14}b^3d^{10}x^{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] a^3*c^10*x + (a^2*c^9*(3*b*c + 10*a*d)*x^2)/2 + a*c^8*(b^2*c^2 + 10*a*b*c*d
 + 15*a^2*d^2)*x^3 + (c^7*(b^3*c^3 + 30*a*b^2*c^2*d + 135*a^2*b*c*d^2 + 120
*a^3*d^3)*x^4)/4 + c^6*d*(2*b^3*c^3 + 27*a*b^2*c^2*d + 72*a^2*b*c*d^2 + 42*
a^3*d^3)*x^5 + (3*c^5*d^2*(5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2 + 28
*a^3*d^3)*x^6)/2 + (6*c^4*d^3*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d
^2 + 35*a^3*d^3)*x^7)/7 + (3*c^3*d^4*(35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^
2*b*c*d^2 + 20*a^3*d^3)*x^8)/4 + c^2*d^5*(28*b^3*c^3 + 70*a*b^2*c^2*d + 40*
a^2*b*c*d^2 + 5*a^3*d^3)*x^9 + (c*d^6*(42*b^3*c^3 + 72*a*b^2*c^2*d + 27*a^2
*b*c*d^2 + 2*a^3*d^3)*x^10)/2 + (d^7*(120*b^3*c^3 + 135*a*b^2*c^2*d + 30*a^
2*b*c*d^2 + a^3*d^3)*x^11)/11 + (b*d^8*(15*b^2*c^2 + 10*a*b*c*d + a^2*d^2)*
x^12)/4 + (b^2*d^9*(10*b*c + 3*a*d)*x^13)/13 + (b^3*d^10*x^14)/14
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(84) = 168$.
time = 0.23, size = 541, normalized size = 5.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*b^3*d^10*x^14+1/13*(3*a*b^2*d^10+10*b^3*c*d^9)*x^13+1/12*(3*a^2*b*d^10
+30*a*b^2*c*d^9+45*b^3*c^2*d^8)*x^12+1/11*(a^3*d^10+30*a^2*b*c*d^9+135*a*b^
2*c^2*d^8+120*b^3*c^3*d^7)*x^11+1/10*(10*a^3*c*d^9+135*a^2*b*c^2*d^8+360*a*
b^2*c^3*d^7+210*b^3*c^4*d^6)*x^10+1/9*(45*a^3*c^2*d^8+360*a^2*b*c^3*d^7+630
*a*b^2*c^4*d^6+252*b^3*c^5*d^5)*x^9+1/8*(120*a^3*c^3*d^7+630*a^2*b*c^4*d^6+
756*a*b^2*c^5*d^5+210*b^3*c^6*d^4)*x^8+1/7*(210*a^3*c^4*d^6+756*a^2*b*c^5*d
^5+630*a*b^2*c^6*d^4+120*b^3*c^7*d^3)*x^7+1/6*(252*a^3*c^5*d^5+630*a^2*b*c^
6*d^4+360*a*b^2*c^7*d^3+45*b^3*c^8*d^2)*x^6+1/5*(210*a^3*c^6*d^4+360*a^2*b*
c^7*d^3+135*a*b^2*c^8*d^2+10*b^3*c^9*d)*x^5+1/4*(120*a^3*c^7*d^3+135*a^2*b*
c^8*d^2+30*a*b^2*c^9*d+b^3*c^10)*x^4+1/3*(45*a^3*c^8*d^2+30*a^2*b*c^9*d+3*a
*b^2*c^10)*x^3+1/2*(10*a^3*c^9*d+3*a^2*b*c^10)*x^2+a^3*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(84) = 168$.
time = 0.29, size = 535, normalized size = 5.82

$$\frac{1}{14}\,b^{3}d^{10}x^{14}+a^{3}c^{10}x+\frac{1}{13}\,(10\,b^{3}cd^{9}+3\,ab^{2}d^{10})x^{13}+\frac{1}{4}\,(15\,b^{3}c^{2}d^{8}+10\,ab^{2}cd^{9}+a^{2}bd^{10})x^{12}+\frac{1}{11}\,(120\,b^{3}c^{3}d^{7}+135\,ab^{2}c^{2}d^{8}+30\,a^{2}bcd^{9}+a^{3}d^{10})x^{11}+\frac{1}{2}\,(42\,b^{3}c^{4}d^{6}+72\,ab^{2}c^{3}d^{7}+27\,a^{2}bc^{2}d^{8}+2\,a^{3}cd^{9})x^{10}+(28\,b^{3}c^{5}d^{5}+70\,ab^{2}c^{4}d^{6}+40\,a^{2}bc^{3}d^{7}+5\,a^{3}c^{2}d^{8})x^{9}+\frac{3}{4}\,(35\,b^{3}c^{6}d^{4}+126\,ab^{2}c^{5}d^{5}+105\,a^{2}bc^{4}d^{6}+20\,a^{3}c^{3}d^{7})x^{8}+\frac{6}{7}\,(20\,b^{3}c^{7}d^{3}+105\,ab^{2}c^{6}d^{4}+126\,a^{2}bc^{5}d^{5}+35\,a^{3}c^{4}d^{6})x^{7}+\frac{3}{2}\,(5\,b^{3}c^{8}d^{2}+40\,ab^{2}c^{7}d^{3}+70\,a^{2}bc^{6}d^{4}+28\,a^{3}c^{5}d^{5})x^{6}+(2\,b^{3}c^{9}d+27\,ab^{2}c^{8}d^{2}+72\,a^{2}bc^{7}d^{3}+42\,a^{3}c^{6}d^{4})x^{5}+\frac{1}{4}\,(b^{3}c^{10}+30\,ab^{2}c^{9}d+135\,a^{2}bc^{8}d^{2}+120\,a^{3}c^{7}d^{3})x^{4}+(ab^{2}c^{10}+10\,a^{2}bc^{9}d+15\,a^{3}c^{8}d^{2})x^{3}+\frac{1}{2}\,(3\,a^{2}bc^{10}+10\,a^{3}c^{9}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
```

```
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(84) = 168$.
time = 0.54, size = 535, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(80) = 160$.
time = 0.05, size = 586, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**10,x)
```

```
[Out] a**3*c**10*x + b**3*d**10*x**14/14 + x**13*(3*a*b**2*d**10/13 + 10*b**3*c*d
**9/13) + x**12*(a**2*b*d**10/4 + 5*a*b**2*c*d**9/2 + 15*b**3*c**2*d**8/4)
+ x**11*(a**3*d**10/11 + 30*a**2*b*c*d**9/11 + 135*a*b**2*c**2*d**8/11 + 12
0*b**3*c**3*d**7/11) + x**10*(a**3*c*d**9 + 27*a**2*b*c**2*d**8/2 + 36*a*b*
*2*c**3*d**7 + 21*b**3*c**4*d**6) + x**9*(5*a**3*c**2*d**8 + 40*a**2*b*c**3
```

```
*d**7 + 70*a*b**2*c**4*d**6 + 28*b**3*c**5*d**5) + x**8*(15*a**3*c**3*d**7
+ 315*a**2*b*c**4*d**6/4 + 189*a*b**2*c**5*d**5/2 + 105*b**3*c**6*d**4/4) +
 x**7*(30*a**3*c**4*d**6 + 108*a**2*b*c**5*d**5 + 90*a*b**2*c**6*d**4 + 120
*b**3*c**7*d**3/7) + x**6*(42*a**3*c**5*d**5 + 105*a**2*b*c**6*d**4 + 60*a*
b**2*c**7*d**3 + 15*b**3*c**8*d**2/2) + x**5*(42*a**3*c**6*d**4 + 72*a**2*b
*c**7*d**3 + 27*a*b**2*c**8*d**2 + 2*b**3*c**9*d) + x**4*(30*a**3*c**7*d**3
 + 135*a**2*b*c**8*d**2/4 + 15*a*b**2*c**9*d/2 + b**3*c**10/4) + x**3*(15*a
**3*c**8*d**2 + 10*a**2*b*c**9*d + a*b**2*c**10) + x**2*(5*a**3*c**9*d + 3*
a**2*b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(84) = 168.
time = 0.55, size = 594, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^3*d^10*x^14 + 10/13*b^3*c*d^9*x^13 + 3/13*a*b^2*d^10*x^13 + 15/4*b^3
*c^2*d^8*x^12 + 5/2*a*b^2*c*d^9*x^12 + 1/4*a^2*b*d^10*x^12 + 120/11*b^3*c^3
*d^7*x^11 + 135/11*a*b^2*c^2*d^8*x^11 + 30/11*a^2*b*c*d^9*x^11 + 1/11*a^3*d
^10*x^11 + 21*b^3*c^4*d^6*x^10 + 36*a*b^2*c^3*d^7*x^10 + 27/2*a^2*b*c^2*d^8
*x^10 + a^3*c*d^9*x^10 + 28*b^3*c^5*d^5*x^9 + 70*a*b^2*c^4*d^6*x^9 + 40*a^2
*b*c^3*d^7*x^9 + 5*a^3*c^2*d^8*x^9 + 105/4*b^3*c^6*d^4*x^8 + 189/2*a*b^2*c^
5*d^5*x^8 + 315/4*a^2*b*c^4*d^6*x^8 + 15*a^3*c^3*d^7*x^8 + 120/7*b^3*c^7*d^
3*x^7 + 90*a*b^2*c^6*d^4*x^7 + 108*a^2*b*c^5*d^5*x^7 + 30*a^3*c^4*d^6*x^7 +
 15/2*b^3*c^8*d^2*x^6 + 60*a*b^2*c^7*d^3*x^6 + 105*a^2*b*c^6*d^4*x^6 + 42*a
^3*c^5*d^5*x^6 + 2*b^3*c^9*d*x^5 + 27*a*b^2*c^8*d^2*x^5 + 72*a^2*b*c^7*d^3*
x^5 + 42*a^3*c^6*d^4*x^5 + 1/4*b^3*c^10*x^4 + 15/2*a*b^2*c^9*d*x^4 + 135/4*
a^2*b*c^8*d^2*x^4 + 30*a^3*c^7*d^3*x^4 + a*b^2*c^10*x^3 + 10*a^2*b*c^9*d*x^
3 + 15*a^3*c^8*d^2*x^3 + 3/2*a^2*b*c^10*x^2 + 5*a^3*c^9*d*x^2 + a^3*c^10*x
```

Mupad [B]
time = 0.23, size = 495, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^10,x)
```

```
[Out] x^4*((b^3*c^10)/4 + 30*a^3*c^7*d^3 + (135*a^2*b*c^8*d^2)/4 + (15*a*b^2*c^9*
d)/2) + x^11*((a^3*d^10)/11 + (120*b^3*c^3*d^7)/11 + (135*a*b^2*c^2*d^8)/11
 + (30*a^2*b*c*d^9)/11) + a^3*c^10*x + (b^3*d^10*x^14)/14 + (3*c^5*d^2*x^6*
(28*a^3*d^3 + 5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2))/2 + c^2*d^5*x^9
*(5*a^3*d^3 + 28*b^3*c^3 + 70*a*b^2*c^2*d + 40*a^2*b*c*d^2) + (6*c^4*d^3*x^
```

```
7*(35*a^3*d^3 + 20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d^2))/7 + (3*c^3
*d^4*x^8*(20*a^3*d^3 + 35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^2*b*c*d^2))/4 +
 (a^2*c^9*x^2*(10*a*d + 3*b*c))/2 + (b^2*d^9*x^13*(3*a*d + 10*b*c))/13 + a*
c^8*x^3*(15*a^2*d^2 + b^2*c^2 + 10*a*b*c*d) + (b*d^8*x^12*(a^2*d^2 + 15*b^2
*c^2 + 10*a*b*c*d))/4 + c^6*d*x^5*(42*a^3*d^3 + 2*b^3*c^3 + 27*a*b^2*c^2*d
+ 72*a^2*b*c*d^2) + (c*d^6*x^10*(2*a^3*d^3 + 42*b^3*c^3 + 72*a*b^2*c^2*d +
27*a^2*b*c*d^2))/2
```

3.1309 $\int (a+bx)^2(c+dx)^{10}\,dx$

Optimal. Leaf size=65

$$\frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

```
[Out] 1/11*(-a*d+b*c)^2*(d*x+c)^11/d^3-1/6*b*(-a*d+b*c)*(d*x+c)^12/d^3+1/13*b^2*(
d*x+c)^13/d^3
```

Rubi [A]
time = 0.18, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^11)/(11*d^3) - (b*(b*c - a*d)*(c + d*x)^12)/(6*d^3
) + (b^2*(c + d*x)^13)/(13*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{10}}{d^2}-\frac{2b(bc-ad)(c+dx)^{11}}{d^2}+\frac{b^2(c+dx)^{12}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 358 vs. 2(65) = 130.
time = 0.03, size = 358, normalized size = 5.51

$$a^2c^{10}x+ac^9(bc+5ad)x^2+\frac{1}{3}c^8\left(b^2c^2+20abcd+45a^2d^2\right)x^3+\frac{5}{2}c^7d\left(b^2c^2+9abcd+12a^2d^2\right)x^4+3c^6d^2\left(3b^2c^2+16abcd+14a^2d^2\right)x^5+2c^5d^3\left(10b^2c^2+35abcd+21a^2d^2\right)x^6+6c^4d^4\left(5b^2c^2+12abcd+5a^2d^2\right)x^7+\frac{3}{2}c^3d^5\left(21b^2c^2+35abcd+10a^2d^2\right)x^8+\frac{5}{3}c^2d^6\left(14b^2c^2+16abcd+3a^2d^2\right)x^9+cd^7\left(12b^2c^2+9abcd+a^2d^2\right)x^{10}+\frac{1}{11}d^8\left(45b^2c^2+20abcd+a^2d^2\right)x^{11}+\frac{1}{6}bd^9(5bc+ad)x^{12}+\frac{1}{13}b^2d^{10}x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] a^2*c^10*x + a*c^9*(b*c + 5*a*d)*x^2 + (c^8*(b^2*c^2 + 20*a*b*c*d + 45*a^2*
d^2)*x^3)/3 + (5*c^7*d*(b^2*c^2 + 9*a*b*c*d + 12*a^2*d^2)*x^4)/2 + 3*c^6*d^
2*(3*b^2*c^2 + 16*a*b*c*d + 14*a^2*d^2)*x^5 + 2*c^5*d^3*(10*b^2*c^2 + 35*a*
b*c*d + 21*a^2*d^2)*x^6 + 6*c^4*d^4*(5*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*x^
7 + (3*c^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^2)*x^8)/2 + (5*c^2*d^6*(
14*b^2*c^2 + 16*a*b*c*d + 3*a^2*d^2)*x^9)/3 + c*d^7*(12*b^2*c^2 + 9*a*b*c*d
 + a^2*d^2)*x^10 + (d^8*(45*b^2*c^2 + 20*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^9*(5*b*c + a*d)*x^12)/6 + (b^2*d^10*x^13)/13
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(59) = 118$.
time = 0.14, size = 391, normalized size = 6.02

method	result
norman	$\frac{b^2 d^{10} x^{13}}{13} + \left(\frac{1}{6} ab\, d^{10} + \frac{5}{6} b^2 c\, d^9\right) x^{12} + \left(\frac{1}{11} a^2 d^{10} + \frac{20}{11} abc\, d^9 + \frac{45}{11} b^2 c^2 d^8\right) x^{11} + \left(a^2 c\, d^9 + 9ab\, c^2 d^8 + 12\right.$
default	$\frac{b^2 d^{10} x^{13}}{13} + \frac{\left(2ab\, d^{10}+10b^2 c\, d^9\right) x^{12}}{12} + \frac{\left(a^2 d^{10}+20abc\, d^9+45b^2c^2d^8\right) x^{11}}{11} + \frac{\left(10a^2 c\, d^9+90ab\, c^2 d^8+120b^2c^3d^7\right) x^{10}}{10} + \frac{(45a^2c^2d^8}{}$
gosper	$\frac{1}{6} x^{12} ab\, d^{10} + \frac{5}{6} x^{12} b^2 c\, d^9 + \frac{45}{11} x^{11} b^2 c^2 d^8 + 5x^9 a^2 c^2 d^8 + \frac{70}{3} x^9 b^2 c^4 d^6 + 15x^8 a^2 c^3 d^7 + \frac{63}{2} x^8 b^2 c^5 d^5 + 30$
risch	$\frac{1}{6} x^{12} ab\, d^{10} + \frac{5}{6} x^{12} b^2 c\, d^9 + \frac{45}{11} x^{11} b^2 c^2 d^8 + 5x^9 a^2 c^2 d^8 + \frac{70}{3} x^9 b^2 c^4 d^6 + 15x^8 a^2 c^3 d^7 + \frac{63}{2} x^8 b^2 c^5 d^5 + 30$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^2*d^10*x^13+1/12*(2*a*b*d^10+10*b^2*c*d^9)*x^12+1/11*(a^2*d^10+20*a*
b*c*d^9+45*b^2*c^2*d^8)*x^11+1/10*(10*a^2*c*d^9+90*a*b*c^2*d^8+120*b^2*c^3*
d^7)*x^10+1/9*(45*a^2*c^2*d^8+240*a*b*c^3*d^7+210*b^2*c^4*d^6)*x^9+1/8*(120
*a^2*c^3*d^7+420*a*b*c^4*d^6+252*b^2*c^5*d^5)*x^8+1/7*(210*a^2*c^4*d^6+504*
a*b*c^5*d^5+210*b^2*c^6*d^4)*x^7+1/6*(252*a^2*c^5*d^5+420*a*b*c^6*d^4+120*b
^2*c^7*d^3)*x^6+1/5*(210*a^2*c^6*d^4+240*a*b*c^7*d^3+45*b^2*c^8*d^2)*x^5+1/
4*(120*a^2*c^7*d^3+90*a*b*c^8*d^2+10*b^2*c^9*d)*x^4+1/3*(45*a^2*c^8*d^2+20*
a*b*c^9*d+b^2*c^10)*x^3+1/2*(10*a^2*c^9*d+2*a*b*c^10)*x^2+a^2*c^10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(59) = 118$.
time = 0.28, size = 384, normalized size = 5.91

$$\frac{1}{13} b^2 d^{10} x^{13} + a^2 c^{10} x + \frac{1}{6} \left(5 b^2 c d^9 + ab d^{10}\right) x^{12} + \frac{1}{11} \left(45 b^2 c^2 d^8 + 20 abcd^9 + a^2 d^{10}\right) x^{11} + \left(12 b^2 c^3 d^7 + 9 abc^2 d^8 + a^2 c d^9\right) x^{10} + \frac{5}{3} \left(14 b^2 c^4 d^6 + 16 abc^3 d^7 + 3 a^2 c^2 d^8\right) x^9 + \frac{3}{2} \left(21 b^2 c^5 d^5 + 35 abc^4 d^6 + 10 a^2 c^3 d^7\right) x^8 + 6 \left(5 b^2 c^6 d^4 + 12 abc^5 d^5 + 5 a^2 c^4 d^6\right) x^7 + 2 \left(10 b^2 c^7 d^3 + 35 abc^6 d^4 + 21 a^2 c^5 d^5\right) x^6 + 3 \left(3 b^2 c^8 d^2 + 16 abc^7 d^3 + 14 a^2 c^6 d^4\right) x^5 + \frac{5}{2} \left(b^2 c^9 d + 9 abc^8 d^2 + 12 a^2 c^7 d^3\right) x^4 + \frac{1}{3} \left(b^2 c^{10} + 20 abc^9 d + 45 a^2 c^8 d^2\right) x^3 + \left(abc^{10} + 5 a^2 c^9 d\right) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
+ 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(59) = 118$.
time = 0.49, size = 384, normalized size = 5.91

$$\frac{1}{13} b^{2} d^{10} x^{13} + a^{2} c^{10} x + \frac{1}{6} \left(5 b^{2} c d^{9} + a b d^{10}\right) x^{12} + \frac{1}{11} \left(45 b^{2} c^{2} d^{8} + 20 a b c d^{9} + a^{2} d^{10}\right) x^{11} + \left(12 b^{2} c^{3} d^{7} + 9 a b c^{2} d^{8} + a^{2} c d^{9}\right) x^{10} + \frac{5}{3} \left(14 b^{2} c^{4} d^{6} + 16 a b c^{3} d^{7} + 3 a^{2} c^{2} d^{8}\right) x^{9} + \frac{3}{2} \left(21 b^{2} c^{5} d^{5} + 35 a b c^{4} d^{6} + 10 a^{2} c^{3} d^{7}\right) x^{8} + 6 \left(5 b^{2} c^{6} d^{4} + 12 a b c^{5} d^{5} + 5 a^{2} c^{4} d^{6}\right) x^{7} + 2 \left(10 b^{2} c^{7} d^{3} + 35 a b c^{6} d^{4} + 21 a^{2} c^{5} d^{5}\right) x^{6} + 3 \left(3 b^{2} c^{8} d^{2} + 16 a b c^{7} d^{3} + 14 a^{2} c^{6} d^{4}\right) x^{5} + \frac{5}{2} \left(b^{2} c^{9} d + 9 a b c^{8} d^{2} + 12 a^{2} c^{7} d^{3}\right) x^{4} + \frac{1}{3} \left(b^{2} c^{10} + 20 a b c^{9} d + 45 a^{2} c^{8} d^{2}\right) x^{3} + \left(a b c^{10} + 5 a^{2} c^{9} d\right) x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
+ 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(54) = 108$.
time = 0.04, size = 415, normalized size = 6.38

$$a^{2} c^{10} x + \frac{b^{2} d^{10} x^{13}}{13} + x^{12} \left(\frac{a b d^{10}}{6} + \frac{5 b^{2} c d^{9}}{6}\right) + x^{11} \left(\frac{a^{2} d^{10}}{11} + \frac{20 a b c d^{9}}{11} + \frac{45 b^{2} c^{2} d^{8}}{11}\right) + x^{10} \left(a^{2} c d^{9} + 9 a b c^{2} d^{8} + 12 b^{2} c^{3} d^{7}\right) + x^{9} \cdot \left(5 a^{2} c^{2} d^{8} + \frac{80 a b c^{3} d^{7}}{3} + \frac{70 b^{2} c^{4} d^{6}}{3}\right) + x^{8} \cdot \left(15 a^{2} c^{3} d^{7} + \frac{105 a b c^{4} d^{6}}{2} + \frac{63 b^{2} c^{5} d^{5}}{2}\right) + x^{7} \cdot \left(30 a^{2} c^{4} d^{6} + 72 a b c^{5} d^{5} + 30 b^{2} c^{6} d^{4}\right) + x^{6} \cdot \left(42 a^{2} c^{5} d^{5} + 70 a b c^{6} d^{4} + 20 b^{2} c^{7} d^{3}\right) + x^{5} \cdot \left(42 a^{2} c^{6} d^{4} + 48 a b c^{7} d^{3} + 9 b^{2} c^{8} d^{2}\right) + x^{4} \cdot \left(30 a^{2} c^{7} d^{3} + \frac{45 a b c^{8} d^{2}}{2} + \frac{5 b^{2} c^{9} d}{2}\right) + x^{3} \cdot \left(15 a^{2} c^{8} d^{2} + \frac{20 a b c^{9} d}{3} + \frac{b^{2} c^{10}}{3}\right) + x^{2} \cdot \left(5 a^{2} c^{9} d + a b c^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**10,x)
```

```
[Out] a**2*c**10*x + b**2*d**10*x**13/13 + x**12*(a*b*d**10/6 + 5*b**2*c*d**9/6)
+ x**11*(a**2*d**10/11 + 20*a*b*c*d**9/11 + 45*b**2*c**2*d**8/11) + x**10*(
a**2*c*d**9 + 9*a*b*c**2*d**8 + 12*b**2*c**3*d**7) + x**9*(5*a**2*c**2*d**8
 + 80*a*b*c**3*d**7/3 + 70*b**2*c**4*d**6/3) + x**8*(15*a**2*c**3*d**7 + 10
5*a*b*c**4*d**6/2 + 63*b**2*c**5*d**5/2) + x**7*(30*a**2*c**4*d**6 + 72*a*b
*c**5*d**5 + 30*b**2*c**6*d**4) + x**6*(42*a**2*c**5*d**5 + 70*a*b*c**6*d**
4 + 20*b**2*c**7*d**3) + x**5*(42*a**2*c**6*d**4 + 48*a*b*c**7*d**3 + 9*b**
2*c**8*d**2) + x**4*(30*a**2*c**7*d**3 + 45*a*b*c**8*d**2/2 + 5*b**2*c**9*d
```

```
/2) + x**3*(15*a**2*c**8*d**2 + 20*a*b*c**9*d/3 + b**2*c**10/3) + x**2*(5*a
**2*c**9*d + a*b*c**10)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(59) = 118$.
time = 0.68, size = 417, normalized size = 6.42

$$\frac{1}{13}\,b^2d^{10}x^{13}+\frac{5}{6}\,b^2cd^9x^{12}+\frac{1}{6}\,abd^{10}x^{12}+\frac{45}{11}\,b^2c^2d^8x^{11}+\frac{20}{11}\,abcd^9x^{11}+\frac{1}{11}\,a^2d^{10}x^{11}+12\,b^2c^3d^7x^{10}+9\,abc^2d^8x^{10}+a^2cd^9x^{10}+\frac{70}{3}\,b^2c^4d^6x^9+\frac{80}{3}\,abc^3d^7x^9+5\,a^2c^2d^8x^9+\frac{63}{2}\,b^2c^5d^5x^8+\frac{105}{2}\,abc^4d^6x^8+15\,a^2c^3d^7x^8+30\,b^2c^6d^4x^7+72\,abc^5d^5x^7+30\,a^2c^4d^6x^7+20\,b^2c^7d^3x^6+70\,abc^6d^4x^6+42\,a^2c^5d^5x^6+9\,b^2c^8d^2x^5+48\,abc^7d^3x^5+42\,a^2c^6d^4x^5+\frac{5}{2}\,b^2c^9dx^4+\frac{45}{2}\,abc^8d^2x^4+30\,a^2c^7d^3x^4+\frac{1}{3}\,b^2c^{10}x^3+\frac{20}{3}\,abc^9dx^3+15\,a^2c^8d^2x^3+abc^{10}x^2+5\,a^2c^9dx^2+a^2c^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^10*x^13 + 5/6*b^2*c*d^9*x^12 + 1/6*a*b*d^10*x^12 + 45/11*b^2*c^2
*d^8*x^11 + 20/11*a*b*c*d^9*x^11 + 1/11*a^2*d^10*x^11 + 12*b^2*c^3*d^7*x^10
 + 9*a*b*c^2*d^8*x^10 + a^2*c*d^9*x^10 + 70/3*b^2*c^4*d^6*x^9 + 80/3*a*b*c^
3*d^7*x^9 + 5*a^2*c^2*d^8*x^9 + 63/2*b^2*c^5*d^5*x^8 + 105/2*a*b*c^4*d^6*x^
8 + 15*a^2*c^3*d^7*x^8 + 30*b^2*c^6*d^4*x^7 + 72*a*b*c^5*d^5*x^7 + 30*a^2*c
^4*d^6*x^7 + 20*b^2*c^7*d^3*x^6 + 70*a*b*c^6*d^4*x^6 + 42*a^2*c^5*d^5*x^6 +
 9*b^2*c^8*d^2*x^5 + 48*a*b*c^7*d^3*x^5 + 42*a^2*c^6*d^4*x^5 + 5/2*b^2*c^9*
d*x^4 + 45/2*a*b*c^8*d^2*x^4 + 30*a^2*c^7*d^3*x^4 + 1/3*b^2*c^10*x^3 + 20/3
*a*b*c^9*d*x^3 + 15*a^2*c^8*d^2*x^3 + a*b*c^10*x^2 + 5*a^2*c^9*d*x^2 + a^2*
c^10*x
```

Mupad [**B**]
time = 0.32, size = 348, normalized size = 5.35

$$x^3\left(15\,a^2\,c^8\,d^2+\frac{20\,a\,b\,c^9\,d}{3}+\frac{b^2\,c^{10}}{3}\right)+x^{11}\left(\frac{a^2\,d^{10}}{11}+\frac{20\,a\,b\,c\,d^9}{11}+\frac{45\,b^2\,c^2\,d^8}{11}\right)+a^2\,c^{10}\,x+\frac{b^2\,d^{10}\,x^{13}}{13}+a\,c^9\,x^2\,(5\,a\,d+b\,c)+\frac{b\,d^9\,x^{12}\,(a\,d+5\,b\,c)}{6}+\frac{5\,c^7\,d\,x^4\,(12\,a^2\,d^2+9\,a\,b\,c\,d+b^2\,c^2)}{2}+c\,d^7\,x^{10}\,(a^2\,d^2+9\,a\,b\,c\,d+12\,b^2\,c^2)+6\,c^4\,d^4\,x^7\,(5\,a^2\,d^2+12\,a\,b\,c\,d+5\,b^2\,c^2)+3\,c^6\,d^2\,x^5\,(14\,a^2\,d^2+16\,a\,b\,c\,d+3\,b^2\,c^2)+\frac{5\,c^2\,d^6\,x^9\,(3\,a^2\,d^2+16\,a\,b\,c\,d+14\,b^2\,c^2)}{3}+2\,c^5\,d^3\,x^6\,(21\,a^2\,d^2+35\,a\,b\,c\,d+10\,b^2\,c^2)+\frac{3\,c^3\,d^5\,x^8\,(10\,a^2\,d^2+35\,a\,b\,c\,d+21\,b^2\,c^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^10,x)
```

```
[Out] x^3*((b^2*c^10)/3 + 15*a^2*c^8*d^2 + (20*a*b*c^9*d)/3) + x^11*((a^2*d^10)/1
1 + (45*b^2*c^2*d^8)/11 + (20*a*b*c*d^9)/11) + a^2*c^10*x + (b^2*d^10*x^13)
/13 + a*c^9*x^2*(5*a*d + b*c) + (b*d^9*x^12*(a*d + 5*b*c))/6 + (5*c^7*d*x^4
*(12*a^2*d^2 + b^2*c^2 + 9*a*b*c*d))/2 + c*d^7*x^10*(a^2*d^2 + 12*b^2*c^2 +
 9*a*b*c*d) + 6*c^4*d^4*x^7*(5*a^2*d^2 + 5*b^2*c^2 + 12*a*b*c*d) + 3*c^6*d^
2*x^5*(14*a^2*d^2 + 3*b^2*c^2 + 16*a*b*c*d) + (5*c^2*d^6*x^9*(3*a^2*d^2 + 1
4*b^2*c^2 + 16*a*b*c*d))/3 + 2*c^5*d^3*x^6*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*
b*c*d) + (3*c^3*d^5*x^8*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/2
```

3.1310 $\int (a+bx)(c+dx)^{10}\,dx$

Optimal. Leaf size=38

$$-\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}$$

```
[Out] -1/11*(-a*d+b*c)*(d*x+c)^11/d^2+1/12*b*(d*x+c)^12/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] -1/11*((b*c - a*d)*(c + d*x)^11)/d^2 + (b*(c + d*x)^12)/(12*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{10}}{d}+\frac{b(c+dx)^{11}}{d}\right)\,dx\\ &= -\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(38) = 76.
time = 0.02, size = 220, normalized size = 5.79

$$ac^{10}x+\frac{1}{2}c^9(bc+10ad)x^2+\frac{5}{3}c^8d(2bc+9ad)x^3+\frac{15}{4}c^7d^2(3bc+8ad)x^4+6c^6d^3(4bc+7ad)x^5+7c^5d^4(5bc+6ad)x^6+6c^4d^5(6bc+5ad)x^7+\frac{15}{4}c^3d^6(7bc+4ad)x^8+\frac{5}{3}c^2d^7(8bc+3ad)x^9+\frac{1}{2}cd^8(9bc+2ad)x^{10}+\frac{1}{11}d^9(10bc+ad)x^{11}+\frac{1}{12}bd^{10}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] a*c^10*x + (c^9*(b*c + 10*a*d)*x^2)/2 + (5*c^8*d*(2*b*c + 9*a*d)*x^3)/3 + (
15*c^7*d^2*(3*b*c + 8*a*d)*x^4)/4 + 6*c^6*d^3*(4*b*c + 7*a*d)*x^5 + 7*c^5*d
^4*(5*b*c + 6*a*d)*x^6 + 6*c^4*d^5*(6*b*c + 5*a*d)*x^7 + (15*c^3*d^6*(7*b*c
 + 4*a*d)*x^8)/4 + (5*c^2*d^7*(8*b*c + 3*a*d)*x^9)/3 + (c*d^8*(9*b*c + 2*a*
d)*x^10)/2 + (d^9*(10*b*c + a*d)*x^11)/11 + (b*d^10*x^12)/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.
time = 0.12, size = 241, normalized size = 6.34

method	result
norman	$\frac{b\,d^{10} x^{12}}{12}+\left(\frac{1}{11} a\,d^{10}+\frac{10}{11} b c\,d^{9}\right) x^{11}+\left(a c\,d^{9}+\frac{9}{2} b\,c^{2} d^{8}\right) x^{10}+\left(5 a\,c^{2} d^{8}+\frac{40}{3} b\,c^{3} d^{7}\right) x^{9}+\left(15 a\,c^{3} d^{7}+\right.$
default	$\frac{b\,d^{10} x^{12}}{12}+\frac{\left(a\,d^{10}+10 b c\,d^{9}\right) x^{11}}{11}+\frac{\left(10 a c\,d^{9}+45 b\,c^{2} d^{8}\right) x^{10}}{10}+\frac{\left(45 a\,c^{2} d^{8}+120 b\,c^{3} d^{7}\right) x^{9}}{9}+\frac{\left(120 a\,c^{3} d^{7}+210 b\,c^{4} d^{6}\right) x^{8}}{8}+\frac{(210 a\,c}{}$
gosper	$\frac{1}{12} b\,d^{10} x^{12}+\frac{1}{11} x^{11} a\,d^{10}+\frac{10}{11} x^{11} b c\,d^{9}+x^{10} a c\,d^{9}+\frac{9}{2} x^{10} b\,c^{2} d^{8}+5 x^{9} a\,c^{2} d^{8}+\frac{40}{3} x^{9} b\,c^{3} d^{7}+15 x^{8} a\,c^{3}$
risch	$\frac{1}{12} b\,d^{10} x^{12}+\frac{1}{11} x^{11} a\,d^{10}+\frac{10}{11} x^{11} b c\,d^{9}+x^{10} a c\,d^{9}+\frac{9}{2} x^{10} b\,c^{2} d^{8}+5 x^{9} a\,c^{2} d^{8}+\frac{40}{3} x^{9} b\,c^{3} d^{7}+15 x^{8} a\,c^{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b*d^10*x^12+1/11*(a*d^10+10*b*c*d^9)*x^11+1/10*(10*a*c*d^9+45*b*c^2*d^
8)*x^10+1/9*(45*a*c^2*d^8+120*b*c^3*d^7)*x^9+1/8*(120*a*c^3*d^7+210*b*c^4*d
^6)*x^8+1/7*(210*a*c^4*d^6+252*b*c^5*d^5)*x^7+1/6*(252*a*c^5*d^5+210*b*c^6*
d^4)*x^6+1/5*(210*a*c^6*d^4+120*b*c^7*d^3)*x^5+1/4*(120*a*c^7*d^3+45*b*c^8*
d^2)*x^4+1/3*(45*a*c^8*d^2+10*b*c^9*d)*x^3+1/2*(10*a*c^9*d+b*c^10)*x^2+a*c^
10*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.
time = 0.27, size = 240, normalized size = 6.32

$$\frac{1}{12}\,bd^{10}x^{12}+ac^{10}x+\frac{1}{11}\,(10\,bcd^{9}+ad^{10})x^{11}+\frac{1}{2}\,(9\,bc^{2}d^{8}+2\,acd^{9})x^{10}+\frac{5}{3}\,(8\,bc^{3}d^{7}+3\,ac^{2}d^{8})x^{9}+\frac{15}{4}\,(7\,bc^{4}d^{6}+4\,ac^{3}d^{7})x^{8}+6\,(6\,bc^{5}d^{5}+5\,ac^{4}d^{6})x^{7}+7\,(5\,bc^{6}d^{4}+6\,ac^{5}d^{5})x^{6}+6\,(4\,bc^{7}d^{3}+7\,ac^{6}d^{4})x^{5}+\frac{15}{4}\,(3\,bc^{8}d^{2}+8\,ac^{7}d^{3})x^{4}+\frac{5}{3}\,(2\,bc^{9}d+9\,ac^{8}d^{2})x^{3}+\frac{1}{2}\,(bc^{10}+10\,ac^{9}d)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(34) = 68$.
time = 0.47, size = 240, normalized size = 6.32

$$\frac{1}{12}bd^{10}x^{12}+ac^{10}x+\frac{1}{11}\left(10\,bcd^{9}+ad^{10}\right)x^{11}+\frac{1}{2}\left(9\,bc^{2}d^{8}+2\,acd^{9}\right)x^{10}+\frac{5}{3}\left(8\,bc^{3}d^{7}+3\,ac^{2}d^{8}\right)x^{9}+\frac{15}{4}\left(7\,bc^{4}d^{6}+4\,ac^{3}d^{7}\right)x^{8}+6\left(6\,bc^{5}d^{5}+5\,ac^{4}d^{6}\right)x^{7}+7\left(5\,bc^{6}d^{4}+6\,ac^{5}d^{5}\right)x^{6}+6\left(4\,bc^{7}d^{3}+7\,ac^{6}d^{4}\right)x^{5}+\frac{15}{4}\left(3\,bc^{8}d^{2}+8\,ac^{7}d^{3}\right)x^{4}+\frac{5}{3}\left(2\,bc^{9}d+9\,ac^{8}d^{2}\right)x^{3}+\frac{1}{2}\left(bc^{10}+10\,ac^{9}d\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(32) = 64$.
time = 0.03, size = 248, normalized size = 6.53

$$ac^{10}x+\frac{bd^{10}x^{12}}{12}+x^{11}\left(\frac{ad^{10}}{11}+\frac{10bcd^{9}}{11}\right)+x^{10}\left(acd^{9}+\frac{9bc^{2}d^{8}}{2}\right)+x^{9}\cdot\left(5ac^{2}d^{8}+\frac{40bc^{3}d^{7}}{3}\right)+x^{8}\cdot\left(15ac^{3}d^{7}+\frac{105bc^{4}d^{6}}{4}\right)+x^{7}\cdot\left(30ac^{4}d^{6}+36bc^{5}d^{5}\right)+x^{6}\cdot\left(42ac^{5}d^{5}+35bc^{6}d^{4}\right)+x^{5}\cdot\left(42ac^{6}d^{4}+24bc^{7}d^{3}\right)+x^{4}\cdot\left(30ac^{7}d^{3}+\frac{45bc^{8}d^{2}}{4}\right)+x^{3}\cdot\left(15ac^{8}d^{2}+\frac{10bc^{9}d}{3}\right)+x^{2}\cdot\left(5ac^{9}d+\frac{bc^{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**10,x)
```

```
[Out] a*c**10*x + b*d**10*x**12/12 + x**11*(a*d**10/11 + 10*b*c*d**9/11) + x**10*
(a*c*d**9 + 9*b*c**2*d**8/2) + x**9*(5*a*c**2*d**8 + 40*b*c**3*d**7/3) + x*
*8*(15*a*c**3*d**7 + 105*b*c**4*d**6/4) + x**7*(30*a*c**4*d**6 + 36*b*c**5*
d**5) + x**6*(42*a*c**5*d**5 + 35*b*c**6*d**4) + x**5*(42*a*c**6*d**4 + 24*
b*c**7*d**3) + x**4*(30*a*c**7*d**3 + 45*b*c**8*d**2/4) + x**3*(15*a*c**8*d
**2 + 10*b*c**9*d/3) + x**2*(5*a*c**9*d + b*c**10/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(34) = 68$.
time = 0.71, size = 241, normalized size = 6.34

$$\frac{1}{12}bd^{10}x^{12}+\frac{10}{11}bcd^{9}x^{11}+\frac{1}{11}ad^{10}x^{11}+\frac{9}{2}bc^{2}d^{8}x^{10}+acd^{9}x^{10}+\frac{40}{3}bc^{3}d^{7}x^{9}+5\,ac^{2}d^{8}x^{9}+\frac{105}{4}bc^{4}d^{6}x^{8}+15\,ac^{3}d^{7}x^{8}+36\,bc^{5}d^{5}x^{7}+30\,ac^{4}d^{6}x^{7}+35\,bc^{6}d^{4}x^{6}+42\,ac^{5}d^{5}x^{6}+24\,bc^{7}d^{3}x^{5}+42\,ac^{6}d^{4}x^{5}+\frac{45}{4}bc^{8}d^{2}x^{4}+30\,ac^{7}d^{3}x^{4}+\frac{10}{3}bc^{9}dx^{3}+15\,ac^{8}d^{2}x^{3}+\frac{1}{2}bc^{10}x^{2}+5\,ac^{9}dx^{2}+ac^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/12*b*d^10*x^12 + 10/11*b*c*d^9*x^11 + 1/11*a*d^10*x^11 + 9/2*b*c^2*d^8*x^
10 + a*c*d^9*x^10 + 40/3*b*c^3*d^7*x^9 + 5*a*c^2*d^8*x^9 + 105/4*b*c^4*d^6*
x^8 + 15*a*c^3*d^7*x^8 + 36*b*c^5*d^5*x^7 + 30*a*c^4*d^6*x^7 + 35*b*c^6*d^4
*x^6 + 42*a*c^5*d^5*x^6 + 24*b*c^7*d^3*x^5 + 42*a*c^6*d^4*x^5 + 45/4*b*c^8*
```

```
d^2*x^4 + 30*a*c^7*d^3*x^4 + 10/3*b*c^9*d*x^3 + 15*a*c^8*d^2*x^3 + 1/2*b*c^
10*x^2 + 5*a*c^9*d*x^2 + a*c^10*x
```

Mupad [B]
time = 0.13, size = 208, normalized size = 5.47

$$x^2\left(\frac{b\,c^{10}}{2}+5\,a\,d\,c^9\right)+x^{11}\left(\frac{a\,d^{10}}{11}+\frac{10\,b\,c\,d^9}{11}\right)+\frac{b\,d^{10}\,x^{12}}{12}+a\,c^{10}\,x+\frac{5\,c^8\,d\,x^3\,(9\,a\,d+2\,b\,c)}{3}+\frac{c\,d^8\,x^{10}\,(2\,a\,d+9\,b\,c)}{2}+\frac{15\,c^7\,d^2\,x^4\,(8\,a\,d+3\,b\,c)}{4}+6\,c^6\,d^3\,x^5\,(7\,a\,d+4\,b\,c)+7\,c^5\,d^4\,x^6\,(6\,a\,d+5\,b\,c)+6\,c^4\,d^5\,x^7\,(5\,a\,d+6\,b\,c)+\frac{15\,c^3\,d^6\,x^8\,(4\,a\,d+7\,b\,c)}{4}+\frac{5\,c^2\,d^7\,x^9\,(3\,a\,d+8\,b\,c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^10,x)
```

```
[Out] x^2*((b*c^10)/2 + 5*a*c^9*d) + x^11*((a*d^10)/11 + (10*b*c*d^9)/11) + (b*d^
10*x^12)/12 + a*c^10*x + (5*c^8*d*x^3*(9*a*d + 2*b*c))/3 + (c*d^8*x^10*(2*a
*d + 9*b*c))/2 + (15*c^7*d^2*x^4*(8*a*d + 3*b*c))/4 + 6*c^6*d^3*x^5*(7*a*d
+ 4*b*c) + 7*c^5*d^4*x^6*(6*a*d + 5*b*c) + 6*c^4*d^5*x^7*(5*a*d + 6*b*c) +
(15*c^3*d^6*x^8*(4*a*d + 7*b*c))/4 + (5*c^2*d^7*x^9*(3*a*d + 8*b*c))/3
```

3.1311 $\int (c+dx)^{10} \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^{11}}{11d}$$

```
[Out] 1/11*(d*x+c)^11/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{10} \, dx = \frac{(c+dx)^{11}}{11d}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Maple [A]
time = 0.11, size = 13, normalized size = 0.93

method	result
default	$\frac{(dx+c)^{11}}{11d}$
gosper	$\frac{1}{11}d^{10}x^{11}+c\,d^9x^{10}+5c^2d^8x^9+15c^3d^7x^8+30c^4d^6x^7+42c^5d^5x^6+42c^6d^4x^5+30c^7d^3x^4+15c^8d$
norman	$\frac{1}{11}d^{10}x^{11}+c\,d^9x^{10}+5c^2d^8x^9+15c^3d^7x^8+30c^4d^6x^7+42c^5d^5x^6+42c^6d^4x^5+30c^7d^3x^4+15c^8d$
risch	$\frac{d^{10}x^{11}}{11}+c\,d^9x^{10}+5c^2d^8x^9+15c^3d^7x^8+30c^4d^6x^7+42c^5d^5x^6+42c^6d^4x^5+30c^7d^3x^4+15c^8d^2x^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*(d*x+c)^11/d
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(d*x + c)^11/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(12) = 24.
time = 0.45, size = 108, normalized size = 7.71

$$\frac{1}{11}\,d^{10}x^{11}+cd^9x^{10}+5\,c^2d^8x^9+15\,c^3d^7x^8+30\,c^4d^6x^7+42\,c^5d^5x^6+42\,c^6d^4x^5+30\,c^7d^3x^4+15\,c^8d^2x^3+5\,c^9dx^2+c^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/11*d^10*x^11 + c*d^9*x^10 + 5*c^2*d^8*x^9 + 15*c^3*d^7*x^8 + 30*c^4*d^6*x
^7 + 42*c^5*d^5*x^6 + 42*c^6*d^4*x^5 + 30*c^7*d^3*x^4 + 15*c^8*d^2*x^3 + 5*
c^9*d*x^2 + c^10*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(8) = 16.
time = 0.02, size = 114, normalized size = 8.14

$$c^{10}x+5c^9dx^2+15c^8d^2x^3+30c^7d^3x^4+42c^6d^4x^5+42c^5d^5x^6+30c^4d^6x^7+15c^3d^7x^8+5c^2d^8x^9+cd^9x^{10}+\frac{d^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10,x)
```

```
[Out] c**10*x + 5*c**9*d*x**2 + 15*c**8*d**2*x**3 + 30*c**7*d**3*x**4 + 42*c**6*d
**4*x**5 + 42*c**5*d**5*x**6 + 30*c**4*d**6*x**7 + 15*c**3*d**7*x**8 + 5*c*
*2*d**8*x**9 + c*d**9*x**10 + d**10*x**11/11
```

Giac [**A**]
time = 0.68, size = 12, normalized size = 0.86

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/11*(d*x + c)^11/d
```

Mupad [**B**]
time = 0.08, size = 108, normalized size = 7.71

$$c^{10}\,x+5\,c^9\,d\,x^2+15\,c^8\,d^2\,x^3+30\,c^7\,d^3\,x^4+42\,c^6\,d^4\,x^5+42\,c^5\,d^5\,x^6+30\,c^4\,d^6\,x^7+15\,c^3\,d^7\,x^8+5\,c^2\,d^8\,x^9+c\,d^9\,x^{10}+\frac{d^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10,x)
```

```
[Out] c^10*x + (d^10*x^11)/11 + 5*c^9*d*x^2 + c*d^9*x^10 + 15*c^8*d^2*x^3 + 30*c^
7*d^3*x^4 + 42*c^6*d^4*x^5 + 42*c^5*d^5*x^6 + 30*c^4*d^6*x^7 + 15*c^3*d^7*x
^8 + 5*c^2*d^8*x^9
```

3.1312 $\int \frac{(c+dx)^{10}}{a+bx} \, dx$

Optimal. Leaf size=241

$$\frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc-ad)^5(c+dx)^5}{5b^6}+\frac{(bc-}{}$$

```
[Out] d*(-a*d+b*c)^9*x/b^10+1/2*(-a*d+b*c)^8*(d*x+c)^2/b^9+1/3*(-a*d+b*c)^7*(d*x+
c)^3/b^8+1/4*(-a*d+b*c)^6*(d*x+c)^4/b^7+1/5*(-a*d+b*c)^5*(d*x+c)^5/b^6+1/6*
(-a*d+b*c)^4*(d*x+c)^6/b^5+1/7*(-a*d+b*c)^3*(d*x+c)^7/b^4+1/8*(-a*d+b*c)^2*
(d*x+c)^8/b^3+1/9*(-a*d+b*c)*(d*x+c)^9/b^2+1/10*(d*x+c)^10/b+(-a*d+b*c)^10*
ln(b*x+a)/b^11
```

Rubi [A]

time = 0.07, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}+\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5(bc-ad)^5}{5b^6}+\frac{(c+dx)^6(bc-ad)^4}{6b^5}+\frac{(c+dx)^7(bc-ad)^3}{7b^4}+\frac{(c+dx)^8(bc-ad)^2}{8b^3}+\frac{(c+dx)^9(bc-ad)}{9b^2}+\frac{(c+dx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^9*x)/b^10 + ((b*c - a*d)^8*(c + d*x)^2)/(2*b^9) + ((b*c - a*
d)^7*(c + d*x)^3)/(3*b^8) + ((b*c - a*d)^6*(c + d*x)^4)/(4*b^7) + ((b*c - a
*d)^5*(c + d*x)^5)/(5*b^6) + ((b*c - a*d)^4*(c + d*x)^6)/(6*b^5) + ((b*c -
a*d)^3*(c + d*x)^7)/(7*b^4) + ((b*c - a*d)^2*(c + d*x)^8)/(8*b^3) + ((b*c -
 a*d)*(c + d*x)^9)/(9*b^2) + (c + d*x)^10/(10*b) + ((b*c - a*d)^10*Log[a +
b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{a+bx}\,dx=\int \left(\frac{d(bc-ad)^9}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)}+\frac{d(bc-ad)^8(c+dx)}{b^9}+\frac{d(bc-ad)^7(c+dx)^2}{b^8}+\frac{d(bc-}{}\right.$$

$$=\frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 591 vs. 2(241) = 482.
time = 0.21, size = 591, normalized size = 2.45

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^10/(a + b*x),x]

[Out] (d*x*(-2520*a^9*d^9 + 1260*a^8*b*d^8*(20*c + d*x) - 840*a^7*b^2*d^7*(135*c^2 + 15*c*d*x + d^2*x^2) + 210*a^6*b^3*d^6*(1440*c^3 + 270*c^2*d*x + 40*c*d^2*x^2 + 3*d^3*x^3) - 252*a^5*b^4*d^5*(2100*c^4 + 600*c^3*d*x + 150*c^2*d^2*x^2 + 25*c*d^3*x^3 + 2*d^4*x^4) + 210*a^4*b^5*d^4*(3024*c^5 + 1260*c^4*d*x + 480*c^3*d^2*x^2 + 135*c^2*d^3*x^3 + 24*c*d^4*x^4 + 2*d^5*x^5) - 120*a^3*b^6*d^3*(4410*c^6 + 2646*c^5*d*x + 1470*c^4*d^2*x^2 + 630*c^3*d^3*x^3 + 189*c^2*d^4*x^4 + 35*c*d^5*x^5 + 3*d^6*x^6) + 45*a^2*b^7*d^2*(6720*c^7 + 5880*c^6*d*x + 4704*c^5*d^2*x^2 + 2940*c^4*d^3*x^3 + 1344*c^3*d^4*x^4 + 420*c^2*d^5*x^5 + 80*c*d^6*x^6 + 7*d^7*x^7) - 10*a*b^8*d*(11340*c^8 + 15120*c^7*d*x + 17640*c^6*d^2*x^2 + 15876*c^5*d^3*x^3 + 10584*c^4*d^4*x^4 + 5040*c^3*d^5*x^5 + 1620*c^2*d^6*x^6 + 315*c*d^7*x^7 + 28*d^8*x^8) + b^9*(25200*c^9 + 56700*c^8*d*x + 100800*c^7*d^2*x^2 + 132300*c^6*d^3*x^3 + 127008*c^5*d^4*x^4 + 88200*c^4*d^5*x^5 + 43200*c^3*d^6*x^6 + 14175*c^2*d^7*x^7 + 2800*c*d^8*x^8 + 252*d^9*x^9)))/(2520*b^10) + ((b*c - a*d)^10*Log[a + b*x])/b^11

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2928 vs. 2(223) = 446.
time = 0.52, size = 2929, normalized size = 12.15

method	result
norman	$\frac{d^{10}x^{10}}{10b}+\frac{d^{2}\left(a^{8}d^{8}-10a^{7}bc\,d^{7}+45a^{6}b^{2}c^{2}d^{6}-120a^{5}b^{3}c^{3}d^{5}+210a^{4}b^{4}c^{4}d^{4}-252a^{3}b^{5}c^{5}d^{3}+210a^{2}b^{6}c^{6}d^{2}-120a\,b^{7}c^{7}d+45b^{8}c^{8}\right)x^{2}}{2b^{9}}-\frac{d^{3}}{}$
risch	$\frac{d^{10}x^{10}}{10b}-\frac{d^{10}x^{9}a}{9b^{2}}+\frac{10d^{9}x^{9}c}{9b}+\frac{d^{10}x^{8}a^{2}}{8b^{3}}+\frac{45d^{8}x^{8}c^{2}}{8b}-\frac{d^{10}x^{7}a^{3}}{7b^{4}}+\frac{120d^{7}x^{7}c^{3}}{7b}+\frac{d^{10}x^{6}a^{4}}{6b^{5}}+\frac{35d^{6}x^{6}c^{4}}{b}-\frac{d^{10}x^{5}a^{5}}{5b^{6}}+25$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a),x,method=_RETURNVERBOSE)

[Out] -d/b^10*(-1/10*d^9*x^10*b^9+1/9*(((a*d-2*b*c)*d^4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*d^4*b^4-b^5*d^5*(a*b^3*d^4+3*b^4*c*d^3))*x^9+1/8*(((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*d^4*b^4+((a*d-2*b*c)*d^4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a*b^3*d^4+3*b^4*c*d^3)-b^5*d^5*(a^2*b^2*d^4+a*b^3*c*d^3+4*b^4*c^2*d^2))*x^8+1/7*(((a*d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*d^4*b^4+((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*d^3

```
)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*(a*b^3*d^4+3*b^4*c*d^3)+(
(a*d-2*b*c)*d^4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a^2*b^2*d^4+a*b^3*c*d^3+
4*b^4*c^2*d^2)-b^5*d^5*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d
))*x^7+1/6*(((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^
4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4
*c^4))*d^4*b^4+((a*d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*
(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*(a*b^3*d^4+3*b^
4*c*d^3)+((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d
^3+10*b^4*c^2*d^2))*(a^2*b^2*d^4+a*b^3*c*d^3+4*b^4*c^2*d^2)+((a*d-2*b*c)*d^
4*b^4-b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^
2+2*b^4*c^3*d)-b^5*d^5*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3
*d+b^4*c^4))*x^6+1/5*((a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2
-10*a*b^3*c^3*d+5*b^4*c^4)*d^4*b^4+((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3
-10*a*b^3*c^2*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d
^2-10*a*b^3*c^3*d+5*b^4*c^4))*(a*b^3*d^4+3*b^4*c*d^3)+((a*d-2*b*c)*(a^2*b^2
*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3
*c^2*d^2+10*b^4*c^3*d))*(a^2*b^2*d^4+a*b^3*c*d^3+4*b^4*c^2*d^2)+((a*d-2*b*c
)*(-a*b^3*d^4+5*b^4*c*d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*
(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d)+((a*d-2*b*c)*d^4*b^4-
b*d*(-a*b^3*d^4+5*b^4*c*d^3))*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*
b^3*c^3*d+b^4*c^4))*x^5+1/4*((a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*
c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4)*(a*b^3*d^4+3*b^4*c*d^3)+((a*d-2*b*c)*(-a^
3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b
*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4))*(a^2*b^2*d^4+a*b^3*c*d
^3+4*b^4*c^2*d^2)+((a*d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b
*d*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*(a^3*b*d^4-a
^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d)+((a*d-2*b*c)*(-a*b^3*d^4+5*b^4*c*
d^3)-b*d*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2))*(a^4*d^4-3*a^3*b*c*d^3
+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d+b^4*c^4))*x^4+1/3*((a*d-2*b*c)*(a^4*d^4-5*
a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4)*(a^2*b^2*d^4+a*b^3
*c*d^3+4*b^4*c^2*d^2)+((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c^2
*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3*c
^3*d+5*b^4*c^4))*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b^3*c^2*d^2+2*b^4*c^3*d)+((a*
d-2*b*c)*(a^2*b^2*d^4-5*a*b^3*c*d^3+10*b^4*c^2*d^2)-b*d*(-a^3*b*d^4+5*a^2*b
^2*c*d^3-10*a*b^3*c^2*d^2+10*b^4*c^3*d))*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c
^2*d^2-2*a*b^3*c^3*d+b^4*c^4))*x^3+1/2*((a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+
10*a^2*b^2*c^2*d^2-10*a*b^3*c^3*d+5*b^4*c^4)*(a^3*b*d^4-a^2*b^2*c*d^3+2*a*b
^3*c^2*d^2+2*b^4*c^3*d)+((a*d-2*b*c)*(-a^3*b*d^4+5*a^2*b^2*c*d^3-10*a*b^3*c
^2*d^2+10*b^4*c^3*d)-b*d*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3
*c^3*d+5*b^4*c^4))*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d+b
^4*c^4))*x^2+(a*d-2*b*c)*(a^4*d^4-5*a^3*b*c*d^3+10*a^2*b^2*c^2*d^2-10*a*b^3
*c^3*d+5*b^4*c^4)*(a^4*d^4-3*a^3*b*c*d^3+4*a^2*b^2*c^2*d^2-2*a*b^3*c^3*d+b^
4*c^4)*x)+(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+
210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7
*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(223) = 446.
time = 0.30, size = 866, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 280*(10*b^9*c*d^9 - a*b^8*d^10)*x^9 + 315*(45*b
^9*c^2*d^8 - 10*a*b^8*c*d^9 + a^2*b^7*d^10)*x^8 + 360*(120*b^9*c^3*d^7 - 45
*a*b^8*c^2*d^8 + 10*a^2*b^7*c*d^9 - a^3*b^6*d^10)*x^7 + 420*(210*b^9*c^4*d^
6 - 120*a*b^8*c^3*d^7 + 45*a^2*b^7*c^2*d^8 - 10*a^3*b^6*c*d^9 + a^4*b^5*d^1
0)*x^6 + 504*(252*b^9*c^5*d^5 - 210*a*b^8*c^4*d^6 + 120*a^2*b^7*c^3*d^7 - 4
5*a^3*b^6*c^2*d^8 + 10*a^4*b^5*c*d^9 - a^5*b^4*d^10)*x^5 + 630*(210*b^9*c^6
*d^4 - 252*a*b^8*c^5*d^5 + 210*a^2*b^7*c^4*d^6 - 120*a^3*b^6*c^3*d^7 + 45*a
^4*b^5*c^2*d^8 - 10*a^5*b^4*c*d^9 + a^6*b^3*d^10)*x^4 + 840*(120*b^9*c^7*d^
3 - 210*a*b^8*c^6*d^4 + 252*a^2*b^7*c^5*d^5 - 210*a^3*b^6*c^4*d^6 + 120*a^4
*b^5*c^3*d^7 - 45*a^5*b^4*c^2*d^8 + 10*a^6*b^3*c*d^9 - a^7*b^2*d^10)*x^3 +
1260*(45*b^9*c^8*d^2 - 120*a*b^8*c^7*d^3 + 210*a^2*b^7*c^6*d^4 - 252*a^3*b^
6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 - 120*a^5*b^4*c^3*d^7 + 45*a^6*b^3*c^2*d^8
- 10*a^7*b^2*c*d^9 + a^8*b*d^10)*x^2 + 2520*(10*b^9*c^9*d - 45*a*b^8*c^8*d^
2 + 120*a^2*b^7*c^7*d^3 - 210*a^3*b^6*c^6*d^4 + 252*a^4*b^5*c^5*d^5 - 210*a
^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - 45*a^7*b^2*c^2*d^8 + 10*a^8*b*c*d^9
- a^9*d^10)*x)/b^10 + (b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 12
0*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4
*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10
*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 868 vs. 2(223) = 446.
time = 0.46, size = 868, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^10*d^10*x^10 + 280*(10*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(45
*b^10*c^2*d^8 - 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(120*b^10*c^3*d^7
- 45*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(210*b^10*c
^4*d^6 - 120*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 - 10*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 504*(252*b^10*c^5*d^5 - 210*a*b^9*c^4*d^6 + 120*a^2*b^8*c^3*d
^7 - 45*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(210*b
^10*c^6*d^4 - 252*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 - 120*a^3*b^7*c^3*d^7
```

```
 + 45*a^4*b^6*c^2*d^8 - 10*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(120*b^1
0*c^7*d^3 - 210*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 - 210*a^3*b^7*c^4*d^6 +
 120*a^4*b^6*c^3*d^7 - 45*a^5*b^5*c^2*d^8 + 10*a^6*b^4*c*d^9 - a^7*b^3*d^10
)*x^3 + 1260*(45*b^10*c^8*d^2 - 120*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 - 2
52*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 120*a^5*b^5*c^3*d^7 + 45*a^6*b^4
*c^2*d^8 - 10*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2520*(10*b^10*c^9*d - 45*
a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d^3 - 210*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5
*d^5 - 210*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 - 45*a^7*b^3*c^2*d^8 + 10*
a^8*b^2*c*d^9 - a^9*b*d^10)*x + 2520*(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b
^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^
5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9
*b*c*d^9 + a^10*d^10)*log(b*x + a))/b^11
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(206) = 412$.
time = 0.79, size = 799, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a),x)
```

```
[Out] x**9*(-a*d**10/(9*b**2) + 10*c*d**9/(9*b)) + x**8*(a**2*d**10/(8*b**3) - 5*
a*c*d**9/(4*b**2) + 45*c**2*d**8/(8*b)) + x**7*(-a**3*d**10/(7*b**4) + 10*a
**2*c*d**9/(7*b**3) - 45*a*c**2*d**8/(7*b**2) + 120*c**3*d**7/(7*b)) + x**6
*(a**4*d**10/(6*b**5) - 5*a**3*c*d**9/(3*b**4) + 15*a**2*c**2*d**8/(2*b**3)
 - 20*a*c**3*d**7/b**2 + 35*c**4*d**6/b) + x**5*(-a**5*d**10/(5*b**6) + 2*a
**4*c*d**9/b**5 - 9*a**3*c**2*d**8/b**4 + 24*a**2*c**3*d**7/b**3 - 42*a*c**
4*d**6/b**2 + 252*c**5*d**5/(5*b)) + x**4*(a**6*d**10/(4*b**7) - 5*a**5*c*d
**9/(2*b**6) + 45*a**4*c**2*d**8/(4*b**5) - 30*a**3*c**3*d**7/b**4 + 105*a*
*2*c**4*d**6/(2*b**3) - 63*a*c**5*d**5/b**2 + 105*c**6*d**4/(2*b)) + x**3*(
-a**7*d**10/(3*b**8) + 10*a**6*c*d**9/(3*b**7) - 15*a**5*c**2*d**8/b**6 + 4
0*a**4*c**3*d**7/b**5 - 70*a**3*c**4*d**6/b**4 + 84*a**2*c**5*d**5/b**3 - 7
0*a*c**6*d**4/b**2 + 40*c**7*d**3/b) + x**2*(a**8*d**10/(2*b**9) - 5*a**7*c
*d**9/b**8 + 45*a**6*c**2*d**8/(2*b**7) - 60*a**5*c**3*d**7/b**6 + 105*a**4
*c**4*d**6/b**5 - 126*a**3*c**5*d**5/b**4 + 105*a**2*c**6*d**4/b**3 - 60*a*
c**7*d**3/b**2 + 45*c**8*d**2/(2*b)) + x*(-a**9*d**10/b**10 + 10*a**8*c*d**
9/b**9 - 45*a**7*c**2*d**8/b**8 + 120*a**6*c**3*d**7/b**7 - 210*a**5*c**4*d
**6/b**6 + 252*a**4*c**5*d**5/b**5 - 210*a**3*c**6*d**4/b**4 + 120*a**2*c**
7*d**3/b**3 - 45*a*c**8*d**2/b**2 + 10*c**9*d/b) + d**10*x**10/(10*b) + (a*
d - b*c)**10*log(a + b*x)/b**11
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(223) = 446$.
time = 0.56, size = 961, normalized size = 3.99

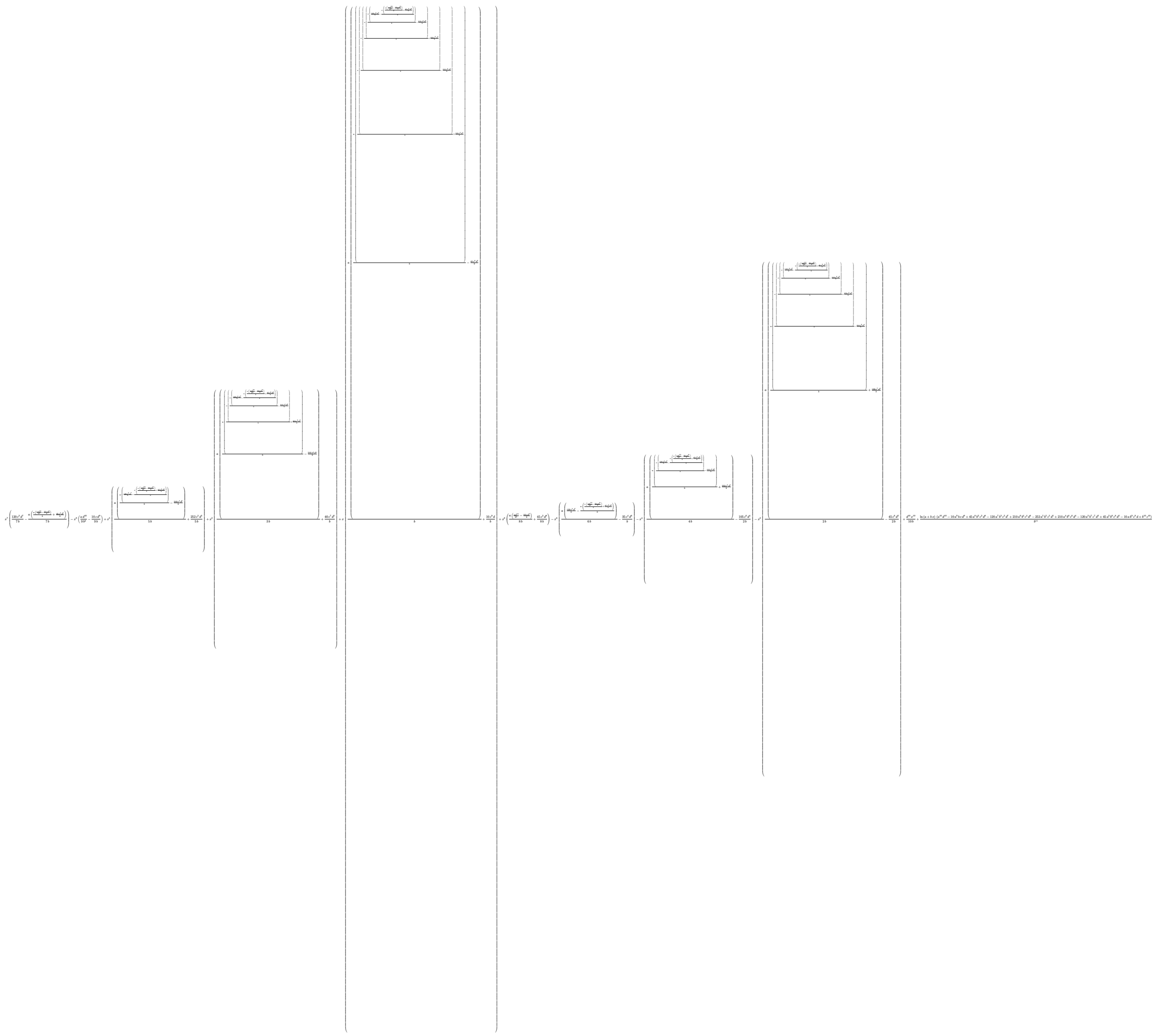
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 2800*b^9*c*d^9*x^9 - 280*a*b^8*d^10*x^9 + 14175
*b^9*c^2*d^8*x^8 - 3150*a*b^8*c*d^9*x^8 + 315*a^2*b^7*d^10*x^8 + 43200*b^9*
c^3*d^7*x^7 - 16200*a*b^8*c^2*d^8*x^7 + 3600*a^2*b^7*c*d^9*x^7 - 360*a^3*b^
6*d^10*x^7 + 88200*b^9*c^4*d^6*x^6 - 50400*a*b^8*c^3*d^7*x^6 + 18900*a^2*b^
7*c^2*d^8*x^6 - 4200*a^3*b^6*c*d^9*x^6 + 420*a^4*b^5*d^10*x^6 + 127008*b^9*
c^5*d^5*x^5 - 105840*a*b^8*c^4*d^6*x^5 + 60480*a^2*b^7*c^3*d^7*x^5 - 22680*
a^3*b^6*c^2*d^8*x^5 + 5040*a^4*b^5*c*d^9*x^5 - 504*a^5*b^4*d^10*x^5 + 13230
0*b^9*c^6*d^4*x^4 - 158760*a*b^8*c^5*d^5*x^4 + 132300*a^2*b^7*c^4*d^6*x^4 -
 75600*a^3*b^6*c^3*d^7*x^4 + 28350*a^4*b^5*c^2*d^8*x^4 - 6300*a^5*b^4*c*d^9
*x^4 + 630*a^6*b^3*d^10*x^4 + 100800*b^9*c^7*d^3*x^3 - 176400*a*b^8*c^6*d^4
*x^3 + 211680*a^2*b^7*c^5*d^5*x^3 - 176400*a^3*b^6*c^4*d^6*x^3 + 100800*a^4
*b^5*c^3*d^7*x^3 - 37800*a^5*b^4*c^2*d^8*x^3 + 8400*a^6*b^3*c*d^9*x^3 - 840
*a^7*b^2*d^10*x^3 + 56700*b^9*c^8*d^2*x^2 - 151200*a*b^8*c^7*d^3*x^2 + 2646
00*a^2*b^7*c^6*d^4*x^2 - 317520*a^3*b^6*c^5*d^5*x^2 + 264600*a^4*b^5*c^4*d^
6*x^2 - 151200*a^5*b^4*c^3*d^7*x^2 + 56700*a^6*b^3*c^2*d^8*x^2 - 12600*a^7*
b^2*c*d^9*x^2 + 1260*a^8*b*d^10*x^2 + 25200*b^9*c^9*d*x - 113400*a*b^8*c^8*
d^2*x + 302400*a^2*b^7*c^7*d^3*x - 529200*a^3*b^6*c^6*d^4*x + 635040*a^4*b^
5*c^5*d^5*x - 529200*a^5*b^4*c^4*d^6*x + 302400*a^6*b^3*c^3*d^7*x - 113400*
a^7*b^2*c^2*d^8*x + 25200*a^8*b*c*d^9*x - 2520*a^9*d^10*x)/b^10 + (b^10*c^1
0 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6
*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7
+ 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*log(abs(b*x + a))/b^11
```

Mupad [B]

time = 0.13, size = 979, normalized size = 4.06



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x),x)
```

```
[Out] x^7*((120*c^3*d^7)/(7*b) - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^
2*d^8)/b))/(7*b)) - x^9*((a*d^10)/(9*b^2) - (10*c*d^9)/(9*b)) + x^5*((a*((a
*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/
b))/b))/b - (210*c^4*d^6)/b))/(5*b) + (252*c^5*d^5)/(5*b)) + x^3*((a*((a*((
a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*
d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/
b))/(3*b) + (40*c^7*d^3)/b) + x*((a*((a*((a*((a*((a*((a*((120*c^3*d^7)/b -
(a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4
*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))
/b - (45*c^8*d^2)/b))/b + (10*c^9*d)/b) + x^8*((a*((a*d^10)/b^2 - (10*c*d^9
```

```
)/b))/(8*b) + (45*c^2*d^8)/(8*b)) - x^6*((a*((120*c^3*d^7)/b - (a*((a*((a*d
^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/(6*b) - (35*c^4*d^6)/b)
- x^4*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/
b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/(4*b)
- (105*c^6*d^4)/(2*b)) - x^2*((a*((a*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*
((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b
))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))/(2*b)
- (45*c^8*d^2)/(2*b)) + (d^10*x^10)/(10*b) + (log(a + b*x)*(a^10*d^10 + b^1
0*c^10 + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 2
52*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2
*c^2*d^8 - 10*a*b^9*c^9*d - 10*a^9*b*c*d^9))/b^11
```

3.1313 $\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx$

Optimal. Leaf size=258

$$\frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+\frac{63d^5(bc-ad)^5(a+bx)}{b^{11}}$$

```
[Out] 45*d^2*(-a*d+b*c)^8*x/b^10-(-a*d+b*c)^10/b^11/(b*x+a)+60*d^3*(-a*d+b*c)^7*(
b*x+a)^2/b^11+70*d^4*(-a*d+b*c)^6*(b*x+a)^3/b^11+63*d^5*(-a*d+b*c)^5*(b*x+a
)^4/b^11+42*d^6*(-a*d+b*c)^4*(b*x+a)^5/b^11+20*d^7*(-a*d+b*c)^3*(b*x+a)^6/b
^11+45/7*d^8*(-a*d+b*c)^2*(b*x+a)^7/b^11+5/4*d^9*(-a*d+b*c)*(b*x+a)^8/b^11+
1/9*d^10*(b*x+a)^9/b^11+10*d*(-a*d+b*c)^9*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.32, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx)^4(bc-ad)^5}{b^{11}}+\frac{70d^4(a+bx)^3(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^2(bc-ad)^7}{b^{11}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{10d(bc-ad)^9\log(a+bx)}{b^{11}}+\frac{d^{10}(a+bx)^9}{9b^{11}}+\frac{45d^2x(bc-ad)^8}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (45*d^2*(b*c - a*d)^8*x)/b^10 - (b*c - a*d)^10/(b^11*(a + b*x)) + (60*d^3*(
b*c - a*d)^7*(a + b*x)^2)/b^11 + (70*d^4*(b*c - a*d)^6*(a + b*x)^3)/b^11 +
(63*d^5*(b*c - a*d)^5*(a + b*x)^4)/b^11 + (42*d^6*(b*c - a*d)^4*(a + b*x)^5
)/b^11 + (20*d^7*(b*c - a*d)^3*(a + b*x)^6)/b^11 + (45*d^8*(b*c - a*d)^2*(a
 + b*x)^7)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^8)/(4*b^11) + (d^10*(a +
 b*x)^9)/(9*b^11) + (10*d*(b*c - a*d)^9*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx=\int \left(\frac{45d^2(bc-ad)^8}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^2}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)}+\frac{120d^3(bc-ad)^7(a+bx)}{b^{10}}+\frac{210d}{}\right.$$
$$=\frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 708 vs. 2(258) = 516.
time = 0.17, size = 708, normalized size = 2.74

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (-252*a^10*d^10 + 252*a^9*b*d^9*(10*c + 9*d*x) + 1260*a^8*b^2*d^8*(-9*c^2 -
 16*c*d*x + d^2*x^2) - 420*a^7*b^3*d^7*(-72*c^3 - 189*c^2*d*x + 27*c*d^2*x^
2 + d^3*x^3) + 210*a^6*b^4*d^6*(-252*c^4 - 864*c^3*d*x + 216*c^2*d^2*x^2 +
18*c*d^3*x^3 + d^4*x^4) - 126*a^5*b^5*d^5*(-504*c^5 - 2100*c^4*d*x + 840*c^
3*d^2*x^2 + 120*c^2*d^3*x^3 + 15*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(-12
60*c^6 - 6048*c^5*d*x + 3780*c^4*d^2*x^2 + 840*c^3*d^3*x^3 + 180*c^2*d^4*x^
4 + 27*c*d^5*x^5 + 2*d^6*x^6) - 12*a^3*b^7*d^3*(-2520*c^7 - 13230*c^6*d*x +
 13230*c^5*d^2*x^2 + 4410*c^4*d^3*x^3 + 1470*c^3*d^4*x^4 + 378*c^2*d^5*x^5
+ 63*c*d^6*x^6 + 5*d^7*x^7) + 9*a^2*b^8*d^2*(-1260*c^8 - 6720*c^7*d*x + 117
60*c^6*d^2*x^2 + 5880*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 1176*c^3*d^5*x^5 + 3
36*c^2*d^6*x^6 + 60*c*d^7*x^7 + 5*d^8*x^8) - a*b^9*d*(-2520*c^9 - 11340*c^8
*d*x + 45360*c^7*d^2*x^2 + 35280*c^6*d^3*x^3 + 26460*c^5*d^4*x^4 + 15876*c^
4*d^5*x^5 + 7056*c^3*d^6*x^6 + 2160*c^2*d^7*x^7 + 405*c*d^8*x^8 + 35*d^9*x^
9) + b^10*(-252*c^10 + 11340*c^8*d^2*x^2 + 15120*c^7*d^3*x^3 + 17640*c^6*d^
4*x^4 + 15876*c^5*d^5*x^5 + 10584*c^4*d^6*x^6 + 5040*c^3*d^7*x^7 + 1620*c^2
*d^8*x^8 + 315*c*d^9*x^9 + 28*d^10*x^10) - 2520*d*(-(b*c) + a*d)^9*(a + b*x
)*Log[a + b*x])/(252*b^11*(a + b*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(252) = 504.
time = 0.15, size = 933, normalized size = 3.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/b^10*(45*b^8*c^8*x+9*a^8*d^8*x+1/9*d^8*x^9*b^8-20/7*a*b^7*c*d^7*x^7+5*a
^2*b^6*c*d^7*x^6-15*a*b^7*c^2*d^6*x^6-8*a^3*b^5*c*d^7*x^5+27*a^2*b^6*c^2*d^
6*x^5-48*a*b^7*c^3*d^5*x^5+25/2*a^4*b^4*c*d^7*x^4-45*a^3*b^5*c^2*d^6*x^4+90
*a^2*b^6*c^3*d^5*x^4-105*a*b^7*c^4*d^4*x^4-20*a^5*b^3*c*d^7*x^3+75*a^4*b^4*
c^2*d^6*x^3-160*a^3*b^5*c^3*d^5*x^3+210*a^2*b^6*c^4*d^4*x^3-168*a*b^7*c^5*d
^3*x^3+35*a^6*b^2*c*d^7*x^2-135*a^5*b^3*c^2*d^6*x^2+300*a^4*b^4*c^3*d^5*x^2
-420*a^3*b^5*c^4*d^4*x^2+378*a^2*b^6*c^5*d^3*x^2-210*a*b^7*c^6*d^2*x^2-80*a
^7*b*c*d^7*x+315*a^6*b^2*c^2*d^6*x-720*a^5*b^3*c^3*d^5*x+1050*a^4*b^4*c^4*d
^4*x-1008*a^3*b^5*c^5*d^3*x+630*a^2*b^6*c^6*d^2*x-240*a*b^7*c^7*d*x+a^4*b^4
*d^8*x^5-1/4*a*b^7*d^8*x^8+5/4*b^8*c*d^7*x^8+3/7*a^2*b^6*d^8*x^7+45/7*b^8*c
^2*d^6*x^7-2/3*a^3*b^5*d^8*x^6+20*b^8*c^3*d^5*x^6+42*b^8*c^4*d^4*x^5-3/2*a^
5*b^3*d^8*x^4+63*b^8*c^5*d^3*x^4+7/3*a^6*b^2*d^8*x^3+70*b^8*c^6*d^2*x^3-4*a
```

```
^7*b*d^8*x^2+60*b^8*c^7*d*x^2)-(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8
-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^
6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11
/(b*x+a)-10/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3
*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*
c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(252) = 504$.
time = 0.29, size = 874, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 2
10*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^
3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)/(b^12*x + a*b^
11) + 1/252*(28*b^8*d^10*x^9 + 63*(5*b^8*c*d^9 - a*b^7*d^10)*x^8 + 36*(45*b
^8*c^2*d^8 - 20*a*b^7*c*d^9 + 3*a^2*b^6*d^10)*x^7 + 84*(60*b^8*c^3*d^7 - 45
*a*b^7*c^2*d^8 + 15*a^2*b^6*c*d^9 - 2*a^3*b^5*d^10)*x^6 + 252*(42*b^8*c^4*d
^6 - 48*a*b^7*c^3*d^7 + 27*a^2*b^6*c^2*d^8 - 8*a^3*b^5*c*d^9 + a^4*b^4*d^10
)*x^5 + 126*(126*b^8*c^5*d^5 - 210*a*b^7*c^4*d^6 + 180*a^2*b^6*c^3*d^7 - 90
*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 - 3*a^5*b^3*d^10)*x^4 + 84*(210*b^8*c^6
*d^4 - 504*a*b^7*c^5*d^5 + 630*a^2*b^6*c^4*d^6 - 480*a^3*b^5*c^3*d^7 + 225*
a^4*b^4*c^2*d^8 - 60*a^5*b^3*c*d^9 + 7*a^6*b^2*d^10)*x^3 + 252*(60*b^8*c^7*
d^3 - 210*a*b^7*c^6*d^4 + 378*a^2*b^6*c^5*d^5 - 420*a^3*b^5*c^4*d^6 + 300*a
^4*b^4*c^3*d^7 - 135*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9 - 4*a^7*b*d^10)*x^2
 + 252*(45*b^8*c^8*d^2 - 240*a*b^7*c^7*d^3 + 630*a^2*b^6*c^6*d^4 - 1008*a^3
*b^5*c^5*d^5 + 1050*a^4*b^4*c^4*d^6 - 720*a^5*b^3*c^3*d^7 + 315*a^6*b^2*c^2
*d^8 - 80*a^7*b*c*d^9 + 9*a^8*d^10)*x)/b^10 + 10*(b^9*c^9*d - 9*a*b^8*c^8*d
^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^
5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a
^9*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. $2(252) = 504$.
time = 0.48, size = 1124, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="fricas")
```

[Out] 1/252*(28*b^10*d^10*x^10 - 252*b^10*c^10 + 2520*a*b^9*c^9*d - 11340*a^2*b^8
*c^8*d^2 + 30240*a^3*b^7*c^7*d^3 - 52920*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^
5*d^5 - 52920*a^6*b^4*c^4*d^6 + 30240*a^7*b^3*c^3*d^7 - 11340*a^8*b^2*c^2*d
^8 + 2520*a^9*b*c*d^9 - 252*a^10*d^10 + 35*(9*b^10*c*d^9 - a*b^9*d^10)*x^9
+ 45*(36*b^10*c^2*d^8 - 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(84*b^10*c^3
*d^7 - 36*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(126*b^1
0*c^4*d^6 - 84*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 - 9*a^3*b^7*c*d^9 + a^4*b
^6*d^10)*x^6 + 126*(126*b^10*c^5*d^5 - 126*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 - 36*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(84*b^1
0*c^6*d^4 - 126*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 - 84*a^3*b^7*c^3*d^7 +
36*a^4*b^6*c^2*d^8 - 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 420*(36*b^10*c^7
*d^3 - 84*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 - 126*a^3*b^7*c^4*d^6 + 84*a^
4*b^6*c^3*d^7 - 36*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 +
1260*(9*b^10*c^8*d^2 - 36*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 - 126*a^3*b^7*
c^5*d^5 + 126*a^4*b^6*c^4*d^6 - 84*a^5*b^5*c^3*d^7 + 36*a^6*b^4*c^2*d^8 - 9
*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 252*(45*a*b^9*c^8*d^2 - 240*a^2*b^8*c^
7*d^3 + 630*a^3*b^7*c^6*d^4 - 1008*a^4*b^6*c^5*d^5 + 1050*a^5*b^5*c^4*d^6 -
 720*a^6*b^4*c^3*d^7 + 315*a^7*b^3*c^2*d^8 - 80*a^8*b^2*c*d^9 + 9*a^9*b*d^1
0)*x + 2520*(a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 - 84*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 - 126*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^
7 - 36*a^8*b^2*c^2*d^8 + 9*a^9*b*c*d^9 - a^10*d^10 + (b^10*c^9*d - 9*a*b^9*
c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 1
26*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*
d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^12*x + a*b^11)

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(240) = 480$.
time = 1.42, size = 816, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**10/(b*x+a)**2,x)

[Out] x**8*(-a*d**10/(4*b**3) + 5*c*d**9/(4*b**2)) + x**7*(3*a**2*d**10/(7*b**4)
- 20*a*c*d**9/(7*b**3) + 45*c**2*d**8/(7*b**2)) + x**6*(-2*a**3*d**10/(3*b*
*5) + 5*a**2*c*d**9/b**4 - 15*a*c**2*d**8/b**3 + 20*c**3*d**7/b**2) + x**5*
(a**4*d**10/b**6 - 8*a**3*c*d**9/b**5 + 27*a**2*c**2*d**8/b**4 - 48*a*c**3*
d**7/b**3 + 42*c**4*d**6/b**2) + x**4*(-3*a**5*d**10/(2*b**7) + 25*a**4*c*d
9/(2*b6) - 45*a**3*c**2*d**8/b**5 + 90*a**2*c**3*d**7/b**4 - 105*a*c**4
*d**6/b**3 + 63*c**5*d**5/b**2) + x**3*(7*a**6*d**10/(3*b**8) - 20*a**5*c*d
9/b7 + 75*a**4*c**2*d**8/b**6 - 160*a**3*c**3*d**7/b**5 + 210*a**2*c**4
*d**6/b**4 - 168*a*c**5*d**5/b**3 + 70*c**6*d**4/b**2) + x**2*(-4*a**7*d**1
0/b**9 + 35*a**6*c*d**9/b**8 - 135*a**5*c**2*d**8/b**7 + 300*a**4*c**3*d**7
/b**6 - 420*a**3*c**4*d**6/b**5 + 378*a**2*c**5*d**5/b**4 - 210*a*c**6*d**4

```
/b**3 + 60*c**7*d**3/b**2) + x*(9*a**8*d**10/b**10 - 80*a**7*c*d**9/b**9 +
315*a**6*c**2*d**8/b**8 - 720*a**5*c**3*d**7/b**7 + 1050*a**4*c**4*d**6/b**
6 - 1008*a**3*c**5*d**5/b**5 + 630*a**2*c**6*d**4/b**4 - 240*a*c**7*d**3/b*
*3 + 45*c**8*d**2/b**2) + (-a**10*d**10 + 10*a**9*b*c*d**9 - 45*a**8*b**2*c
**2*d**8 + 120*a**7*b**3*c**3*d**7 - 210*a**6*b**4*c**4*d**6 + 252*a**5*b**
5*c**5*d**5 - 210*a**4*b**6*c**6*d**4 + 120*a**3*b**7*c**7*d**3 - 45*a**2*b
**8*c**8*d**2 + 10*a*b**9*c**9*d - b**10*c**10)/(a*b**11 + b**12*x) + d**10
*x**9/(9*b**2) - 10*d*(a*d - b*c)**9*log(a + b*x)/b**11
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(252) = 504$.

time = 0.58, size = 1012, normalized size = 3.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*d^10 + 315*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 1620*(b^4*c^2*d
^8 - 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 5040*(b^6*c^3*d^7 -
3*a*b^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 10584
*(b^8*c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4
*b^4*d^10)/((b*x + a)^4*b^4) + 15876*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a
^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x
 + a)^5*b^5) + 17640*(b^12*c^6*d^4 - 6*a*b^11*c^5*d^5 + 15*a^2*b^10*c^4*d^6
 - 20*a^3*b^9*c^3*d^7 + 15*a^4*b^8*c^2*d^8 - 6*a^5*b^7*c*d^9 + a^6*b^6*d^10
)/((b*x + a)^6*b^6) + 15120*(b^14*c^7*d^3 - 7*a*b^13*c^6*d^4 + 21*a^2*b^12*
c^5*d^5 - 35*a^3*b^11*c^4*d^6 + 35*a^4*b^10*c^3*d^7 - 21*a^5*b^9*c^2*d^8 +
7*a^6*b^8*c*d^9 - a^7*b^7*d^10)/((b*x + a)^7*b^7) + 11340*(b^16*c^8*d^2 - 8
*a*b^15*c^7*d^3 + 28*a^2*b^14*c^6*d^4 - 56*a^3*b^13*c^5*d^5 + 70*a^4*b^12*c
^4*d^6 - 56*a^5*b^11*c^3*d^7 + 28*a^6*b^10*c^2*d^8 - 8*a^7*b^9*c*d^9 + a^8*
b^8*d^10)/((b*x + a)^8*b^8))*(b*x + a)^9/b^11 - 10*(b^9*c^9*d - 9*a*b^8*c^8
*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*
a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 -
 a^9*d^10)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - (b^19*c^10/(b*x +
a) - 10*a*b^18*c^9*d/(b*x + a) + 45*a^2*b^17*c^8*d^2/(b*x + a) - 120*a^3*b^
16*c^7*d^3/(b*x + a) + 210*a^4*b^15*c^6*d^4/(b*x + a) - 252*a^5*b^14*c^5*d^
5/(b*x + a) + 210*a^6*b^13*c^4*d^6/(b*x + a) - 120*a^7*b^12*c^3*d^7/(b*x +
a) + 45*a^8*b^11*c^2*d^8/(b*x + a) - 10*a^9*b^10*c*d^9/(b*x + a) + a^10*b^9
*d^10/(b*x + a))/b^20
```

Mupad [B]

time = 0.35, size = 2500, normalized size = 9.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^10/(a + b*x)^2,x)

[Out] x^7*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(7*b) - (a^2*d^10)/(7*b^4) + (45*c^2*d^8)/(7*b^2)) - x^5*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/(5*b) - (42*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/(5*b^2)) - x^8*((a*d^10)/(4*b^3) - (5*c*d^9)/(4*b^2)) + x^3*((70*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(3*b) + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/(3*b^2)) - x^2*((a*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b^2))/b - (60*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(2*b^2)) + x^6*((20*c^3*d^7)/b^2 - (a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/(3*b) + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(6*b^2)) + x*((45*c^8*d^2)/b^2 - (a^2*((210*c^6*d^4)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^

2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*
a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8
)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)
/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (
45*c^2*d^8)/b^2))/b^2))/b^2))/b^2 + (2*a*((2*a*((210*c^6*d^4)/b^2 - (2*a*((
2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2)
)/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*
d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c
*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^
3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b
^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b
^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2
*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b +
(a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2
*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)
/b^2))/b^2))/b^2))/b - (120*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/
b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (4
5*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210
*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10
)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*(
(2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b
+ (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/
b^2))/b) + x^4*((a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 -
(10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d...

3.1314 $\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx$

Optimal. Leaf size=262

$$\frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+\frac{84d^5(bc-ad)^5(a+bx)^3}{b^{11}}+\frac{105}{}$$

```
[Out] 120*d^3*(-a*d+b*c)^7*x/b^10-1/2*(-a*d+b*c)^10/b^11/(b*x+a)^2-10*d*(-a*d+b*c
)^9/b^11/(b*x+a)+105*d^4*(-a*d+b*c)^6*(b*x+a)^2/b^11+84*d^5*(-a*d+b*c)^5*(b
*x+a)^3/b^11+105/2*d^6*(-a*d+b*c)^4*(b*x+a)^4/b^11+24*d^7*(-a*d+b*c)^3*(b*x
+a)^5/b^11+15/2*d^8*(-a*d+b*c)^2*(b*x+a)^6/b^11+10/7*d^9*(-a*d+b*c)*(b*x+a)
^7/b^11+1/8*d^10*(b*x+a)^8/b^11+45*d^2*(-a*d+b*c)^8*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.31, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+bx)^3(bc-ad)^5}{b^{11}}+\frac{105d^4(a+bx)^2(bc-ad)^6}{b^{11}}+\frac{45d^2(bc-ad)^8\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}+\frac{d^{10}(a+bx)^8}{8b^{11}}+\frac{120d^3x(bc-ad)^7}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (120*d^3*(b*c - a*d)^7*x)/b^10 - (b*c - a*d)^10/(2*b^11*(a + b*x)^2) - (10*
d*(b*c - a*d)^9)/(b^11*(a + b*x)) + (105*d^4*(b*c - a*d)^6*(a + b*x)^2)/b^1
1 + (84*d^5*(b*c - a*d)^5*(a + b*x)^3)/b^11 + (105*d^6*(b*c - a*d)^4*(a + b
*x)^4)/(2*b^11) + (24*d^7*(b*c - a*d)^3*(a + b*x)^5)/b^11 + (15*d^8*(b*c -
a*d)^2*(a + b*x)^6)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^7)/(7*b^11) +
(d^10*(a + b*x)^8)/(8*b^11) + (45*d^2*(b*c - a*d)^8*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx = \int \left(\frac{120d^3(bc-ad)^7}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^3}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^2}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)}+\frac{210d^4(bc-ad}{b^{10}}\right.$$
$$= \frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+\frac{84d^5(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 708 vs. 2(262) = 524.
time = 0.16, size = 708, normalized size = 2.70

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (532*a^10*d^10 - 56*a^9*b*d^9*(85*c + 26*d*x) + 28*a^8*b^2*d^8*(675*c^2 + 3
80*c*d*x - 116*d^2*x^2) - 280*a^7*b^3*d^7*(156*c^3 + 117*c^2*d*x - 91*c*d^2
*x^2 + 3*d^3*x^3) + 210*a^6*b^4*d^6*(308*c^4 + 256*c^3*d*x - 414*c^2*d^2*x^
2 + 32*c*d^3*x^3 + d^4*x^4) - 84*a^5*b^5*d^5*(756*c^5 + 560*c^4*d*x - 2000*
c^3*d^2*x^2 + 280*c^2*d^3*x^3 + 20*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(9
80*c^6 + 336*c^5*d*x - 4760*c^4*d^2*x^2 + 1120*c^3*d^3*x^3 + 140*c^2*d^4*x^
4 + 16*c*d^5*x^5 + d^6*x^6) - 24*a^3*b^7*d^3*(700*c^7 - 490*c^6*d*x - 6174*
c^5*d^2*x^2 + 2450*c^4*d^3*x^3 + 490*c^3*d^4*x^4 + 98*c^2*d^5*x^5 + 14*c*d^
6*x^6 + d^7*x^7) + 3*a^2*b^8*d^2*(1260*c^8 - 4480*c^7*d*x - 21560*c^6*d^2*x
^2 + 15680*c^5*d^3*x^3 + 4900*c^4*d^4*x^4 + 1568*c^3*d^5*x^5 + 392*c^2*d^6*
x^6 + 64*c*d^7*x^7 + 5*d^8*x^8) - 2*a*b^9*d*(140*c^9 - 2520*c^8*d*x - 6720*
c^7*d^2*x^2 + 11760*c^6*d^3*x^3 + 5880*c^5*d^4*x^4 + 2940*c^4*d^5*x^5 + 117
6*c^3*d^6*x^6 + 336*c^2*d^7*x^7 + 60*c*d^8*x^8 + 5*d^9*x^9) + b^10*(-28*c^1
0 - 560*c^9*d*x + 6720*c^7*d^3*x^3 + 5880*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 +
2940*c^4*d^6*x^6 + 1344*c^3*d^7*x^7 + 420*c^2*d^8*x^8 + 80*c*d^9*x^9 + 7*d^
10*x^10) + 2520*d^2*(b*c - a*d)^8*(a + b*x)^2*Log[a + b*x])/(56*b^11*(a + b
*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(252) = 504.
time = 0.16, size = 914, normalized size = 3.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3/b^10*(36*a^7*d^7*x-120*b^7*c^7*x-15/4*a^4*b^3*d^7*x^4-105/2*b^7*c^4*d^
3*x^4+7*a^5*b^2*d^7*x^3-84*b^7*c^5*d^2*x^3-14*a^6*b*d^7*x^2-1/8*d^7*x^8*b^7
-105*b^7*c^6*d*x^2-675/2*a^4*b^3*c^2*d^5*x^2+600*a^3*b^4*c^3*d^4*x^2-630*a^
2*b^5*c^4*d^3*x^2+378*a*b^6*c^5*d^2*x^2-280*a^6*b*c*d^6*x+945*a^5*b^2*c^2*d
^5*x-1800*a^4*b^3*c^3*d^4*x+2100*a^3*b^4*c^4*d^3*x-1512*a^2*b^5*c^5*d^2*x+6
30*a*b^6*c^6*d*x+25*a^3*b^4*c*d^6*x^4-135/2*a^2*b^5*c^2*d^5*x^4+90*a*b^6*c^
3*d^4*x^4-50*a^4*b^3*c*d^6*x^3+150*a^3*b^4*c^2*d^5*x^3-240*a^2*b^5*c^3*d^4*
x^3+210*a*b^6*c^4*d^3*x^3+105*a^5*b^2*c*d^6*x^2+3/7*a*b^6*d^7*x^7-10/7*b^7*
c*d^6*x^7-a^2*b^5*d^7*x^6-15/2*b^7*c^2*d^5*x^6+2*a^3*b^4*d^7*x^5-24*b^7*c^3
*d^4*x^5+27*a*b^6*c^2*d^5*x^5+5*a*b^6*c*d^6*x^6-12*a^2*b^5*c*d^6*x^5)+10/b^
11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b
^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^
```

```
8*c^8*d-b^9*c^9)/(b*x+a)-1/2*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-1
20*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*
d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(
b*x+a)^2+45/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c
^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7
*d+b^8*c^8)*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(252) = 504$.
time = 0.29, size = 881, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^10*c^10 + 10*a*b^9*c^9*d - 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^
3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 15
60*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 +
 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4
 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*
b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^9*b*d^10)*x)/(b^13*x^2 + 2*a*b^12*x + a^2
*b^11) + 1/56*(7*b^7*d^10*x^8 + 8*(10*b^7*c*d^9 - 3*a*b^6*d^10)*x^7 + 28*(1
5*b^7*c^2*d^8 - 10*a*b^6*c*d^9 + 2*a^2*b^5*d^10)*x^6 + 56*(24*b^7*c^3*d^7 -
 27*a*b^6*c^2*d^8 + 12*a^2*b^5*c*d^9 - 2*a^3*b^4*d^10)*x^5 + 70*(42*b^7*c^4
*d^6 - 72*a*b^6*c^3*d^7 + 54*a^2*b^5*c^2*d^8 - 20*a^3*b^4*c*d^9 + 3*a^4*b^3
*d^10)*x^4 + 56*(84*b^7*c^5*d^5 - 210*a*b^6*c^4*d^6 + 240*a^2*b^5*c^3*d^7 -
 150*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 - 7*a^5*b^2*d^10)*x^3 + 28*(210*b^7
*c^6*d^4 - 756*a*b^6*c^5*d^5 + 1260*a^2*b^5*c^4*d^6 - 1200*a^3*b^4*c^3*d^7
+ 675*a^4*b^3*c^2*d^8 - 210*a^5*b^2*c*d^9 + 28*a^6*b*d^10)*x^2 + 56*(120*b^
7*c^7*d^3 - 630*a*b^6*c^6*d^4 + 1512*a^2*b^5*c^5*d^5 - 2100*a^3*b^4*c^4*d^6
 + 1800*a^4*b^3*c^3*d^7 - 945*a^5*b^2*c^2*d^8 + 280*a^6*b*c*d^9 - 36*a^7*d^
10)*x)/b^10 + 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a
^3*b^5*c^5*d^5 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d
^8 - 8*a^7*b*c*d^9 + a^8*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(252) = 504$.
time = 0.58, size = 1233, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*d^10*x^10 - 28*b^10*c^10 - 280*a*b^9*c^9*d + 3780*a^2*b^8*c^8*
d^2 - 16800*a^3*b^7*c^7*d^3 + 41160*a^4*b^6*c^6*d^4 - 63504*a^5*b^5*c^5*d^5
 + 64680*a^6*b^4*c^4*d^6 - 43680*a^7*b^3*c^3*d^7 + 18900*a^8*b^2*c^2*d^8 -
4760*a^9*b*c*d^9 + 532*a^10*d^10 + 10*(8*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*
(28*b^10*c^2*d^8 - 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(56*b^10*c^3*d^7
- 28*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(70*b^10*c^4*
d^6 - 56*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 - 8*a^3*b^7*c*d^9 + a^4*b^6*d^1
0)*x^6 + 84*(56*b^10*c^5*d^5 - 70*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 - 28*a
^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(28*b^10*c^6*d^4
 - 56*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 - 56*a^3*b^7*c^3*d^7 + 28*a^4*b^6*
c^2*d^8 - 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(8*b^10*c^7*d^3 - 28*a*
b^9*c^6*d^4 + 56*a^2*b^8*c^5*d^5 - 70*a^3*b^7*c^4*d^6 + 56*a^4*b^6*c^3*d^7
- 28*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 28*(480*a*b^9*
c^7*d^3 - 2310*a^2*b^8*c^6*d^4 + 5292*a^3*b^7*c^5*d^5 - 7140*a^4*b^6*c^4*d^
6 + 6000*a^5*b^5*c^3*d^7 - 3105*a^6*b^4*c^2*d^8 + 910*a^7*b^3*c*d^9 - 116*a
^8*b^2*d^10)*x^2 - 56*(10*b^10*c^9*d - 90*a*b^9*c^8*d^2 + 240*a^2*b^8*c^7*d
^3 - 210*a^3*b^7*c^6*d^4 - 252*a^4*b^6*c^5*d^5 + 840*a^5*b^5*c^4*d^6 - 960*
a^6*b^4*c^3*d^7 + 585*a^7*b^3*c^2*d^8 - 190*a^8*b^2*c*d^9 + 26*a^9*b*d^10)*
x + 2520*(a^2*b^8*c^8*d^2 - 8*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 - 56*a^5
*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 - 56*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8
 - 8*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8
*c^6*d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 2
8*a^6*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2*(a*b^9*c^8*d^2
- 8*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 - 56*a^4*b^6*c^5*d^5 + 70*a^5*b^5*
c^4*d^6 - 56*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 - 8*a^8*b^2*c*d^9 + a^9*b
*d^10)*x)*log(b*x + a))/(b^13*x^2 + 2*a*b^12*x + a^2*b^11)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(243) = 486$.
time = 6.27, size = 843, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**3,x)
```

```
[Out] x**7*(-3*a*d**10/(7*b**4) + 10*c*d**9/(7*b**3)) + x**6*(a**2*d**10/b**5 - 5
*a*c*d**9/b**4 + 15*c**2*d**8/(2*b**3)) + x**5*(-2*a**3*d**10/b**6 + 12*a**
2*c*d**9/b**5 - 27*a*c**2*d**8/b**4 + 24*c**3*d**7/b**3) + x**4*(15*a**4*d*
*10/(4*b**7) - 25*a**3*c*d**9/b**6 + 135*a**2*c**2*d**8/(2*b**5) - 90*a*c**
3*d**7/b**4 + 105*c**4*d**6/(2*b**3)) + x**3*(-7*a**5*d**10/b**8 + 50*a**4*
c*d**9/b**7 - 150*a**3*c**2*d**8/b**6 + 240*a**2*c**3*d**7/b**5 - 210*a*c**
4*d**6/b**4 + 84*c**5*d**5/b**3) + x**2*(14*a**6*d**10/b**9 - 105*a**5*c*d*
*9/b**8 + 675*a**4*c**2*d**8/(2*b**7) - 600*a**3*c**3*d**7/b**6 + 630*a**2*
c**4*d**6/b**5 - 378*a*c**5*d**5/b**4 + 105*c**6*d**4/b**3) + x*(-36*a**7*d
```

```
**10/b**10 + 280*a**6*c*d**9/b**9 - 945*a**5*c**2*d**8/b**8 + 1800*a**4*c**
3*d**7/b**7 - 2100*a**3*c**4*d**6/b**6 + 1512*a**2*c**5*d**5/b**5 - 630*a*c
**6*d**4/b**4 + 120*c**7*d**3/b**3) + (19*a**10*d**10 - 170*a**9*b*c*d**9 +
 675*a**8*b**2*c**2*d**8 - 1560*a**7*b**3*c**3*d**7 + 2310*a**6*b**4*c**4*d
**6 - 2268*a**5*b**5*c**5*d**5 + 1470*a**4*b**6*c**6*d**4 - 600*a**3*b**7*c
**7*d**3 + 135*a**2*b**8*c**8*d**2 - 10*a*b**9*c**9*d - b**10*c**10 + x*(20
*a**9*b*d**10 - 180*a**8*b**2*c*d**9 + 720*a**7*b**3*c**2*d**8 - 1680*a**6*
b**4*c**3*d**7 + 2520*a**5*b**5*c**4*d**6 - 2520*a**4*b**6*c**5*d**5 + 1680
*a**3*b**7*c**6*d**4 - 720*a**2*b**8*c**7*d**3 + 180*a*b**9*c**8*d**2 - 20*
b**10*c**9*d))/(2*a**2*b**11 + 4*a*b**12*x + 2*b**13*x**2) + d**10*x**8/(8*
b**3) + 45*d**2*(a*d - b*c)**8*log(a + b*x)/b**11
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(252) = 504.
time = 0.61, size = 924, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a^3*b^5*c^5*d^5
 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d^8 - 8*a^7*b*c
*d^9 + a^8*d^10)*log(abs(b*x + a))/b^11 - 1/2*(b^10*c^10 + 10*a*b^9*c^9*d -
 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^
5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c
^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 + 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2
+ 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b
^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^
9*b*d^10)*x)/((b*x + a)^2*b^11) + 1/56*(7*b^21*d^10*x^8 + 80*b^21*c*d^9*x^7
 - 24*a*b^20*d^10*x^7 + 420*b^21*c^2*d^8*x^6 - 280*a*b^20*c*d^9*x^6 + 56*a^
2*b^19*d^10*x^6 + 1344*b^21*c^3*d^7*x^5 - 1512*a*b^20*c^2*d^8*x^5 + 672*a^2
*b^19*c*d^9*x^5 - 112*a^3*b^18*d^10*x^5 + 2940*b^21*c^4*d^6*x^4 - 5040*a*b^
20*c^3*d^7*x^4 + 3780*a^2*b^19*c^2*d^8*x^4 - 1400*a^3*b^18*c*d^9*x^4 + 210*
a^4*b^17*d^10*x^4 + 4704*b^21*c^5*d^5*x^3 - 11760*a*b^20*c^4*d^6*x^3 + 1344
0*a^2*b^19*c^3*d^7*x^3 - 8400*a^3*b^18*c^2*d^8*x^3 + 2800*a^4*b^17*c*d^9*x^
3 - 392*a^5*b^16*d^10*x^3 + 5880*b^21*c^6*d^4*x^2 - 21168*a*b^20*c^5*d^5*x^
2 + 35280*a^2*b^19*c^4*d^6*x^2 - 33600*a^3*b^18*c^3*d^7*x^2 + 18900*a^4*b^1
7*c^2*d^8*x^2 - 5880*a^5*b^16*c*d^9*x^2 + 784*a^6*b^15*d^10*x^2 + 6720*b^21
*c^7*d^3*x - 35280*a*b^20*c^6*d^4*x + 84672*a^2*b^19*c^5*d^5*x - 117600*a^3
*b^18*c^4*d^6*x + 100800*a^4*b^17*c^3*d^7*x - 52920*a^5*b^16*c^2*d^8*x + 15
680*a^6*b^15*c*d^9*x - 2016*a^7*b^14*d^10*x)/b^24
```

Mupad [B]
time = 0.38, size = 2500, normalized size = 9.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^10/(a + b*x)^3,x)

```
[Out] x^3*((84*c^5*d^5)/b^3 - (a*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b
^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^
3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d
^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*
d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2)
)/b + (a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/
b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((
3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(3*b^3)) - x^7*((
3*a*d^10)/(7*b^4) - (10*c*d^9)/(7*b^3)) - ((b^10*c^10 - 19*a^10*d^10 - 135*
a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5
*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^
8 + 10*a*b^9*c^9*d + 170*a^9*b*c*d^9)/(2*b) - x*(10*a^9*d^10 - 10*b^9*c^9*d
 + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d^4 - 1260*a^4*
b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 360*a^7*b^2*c^2*
d^8 - 90*a^8*b*c*d^9))/(a^2*b^10 + b^12*x^2 + 2*a*b^11*x) - x^5*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/(5*b) + (a^3*d^10)/(5*b^6) - (24*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 -
 (10*c*d^9)/b^3))/(5*b^2)) + x^6*((a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/(2*
b) - (a^2*d^10)/(2*b^5) + (15*c^2*d^8)/(2*b^3)) + x^4*((3*a*((3*a*((3*a*((3
*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b
 + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)
/b^3))/b^2))/(4*b) + (105*c^4*d^6)/(2*b^3) + (a^3*((3*a*d^10)/b^4 - (10*c*d
^9)/b^3))/(4*b^3) - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*
a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(4*b^2)) - x^2*((3*a^2*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/
b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10
)/b^5 + (45*c^2*d^8)/b^3))/b^2))/(2*b^2) + (3*a*((252*c^5*d^5)/b^3 - (3*a*(
(3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
(45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^
10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b
^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))
/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3
*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b
 + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)
/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^3))/(2*b) - (105*c^6*d^4)/b^3 - (a^3*((3*
a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d
^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 -
```

(10*c*d^9)/b^3))/b^2))/(2*b^3)) + x*((3*a*((3*a^2*((3*a*((3*a*((3*a*((3*a*
d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b +
(a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^
3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b
^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
(45*c^2*d^8)/b^3))/b^2))/b^2 + (3*a*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*
((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8
)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^
4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^1
0)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2)
)/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5
+ (45*c^2*d^8)/b^3))/b^3))/b - (210*c^6*d^4)/b^3 - (a^3*((3*a*((3*a*((3*a*d
^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (
a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3
))/b^2))/b^3))/b - (a^3*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3)
)/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d
^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)
/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^1
0)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b
^3 + (120*c^7*d^3)/b^3 - (3*a^2*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*(...

3.1315 $\int \frac{(c+dx)^{10}}{(a+bx)^4} \, dx$

Optimal. Leaf size=258

$$\frac{210d^4(bc-ad)^6x}{b^{10}} - \frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3} - \frac{5d(bc-ad)^9}{b^{11}(a+bx)^2} - \frac{45d^2(bc-ad)^8}{b^{11}(a+bx)} + \frac{126d^5(bc-ad)^5(a+bx)^2}{b^{11}} + \frac{70d^6(bc-ad)^4 \ldots}{b^{1\ldots}}$$

```
[Out] 210*d^4*(-a*d+b*c)^6*x/b^10-1/3*(-a*d+b*c)^10/b^11/(b*x+a)^3-5*d*(-a*d+b*c)
^9/b^11/(b*x+a)^2-45*d^2*(-a*d+b*c)^8/b^11/(b*x+a)+126*d^5*(-a*d+b*c)^5*(b*
x+a)^2/b^11+70*d^6*(-a*d+b*c)^4*(b*x+a)^3/b^11+30*d^7*(-a*d+b*c)^3*(b*x+a)^
4/b^11+9*d^8*(-a*d+b*c)^2*(b*x+a)^5/b^11+5/3*d^9*(-a*d+b*c)*(b*x+a)^6/b^11+
1/7*d^10*(b*x+a)^7/b^11+120*d^3*(-a*d+b*c)^7*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.30, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}} + \frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}} + \frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}} + \frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}} + \frac{126d^5(a+bx)^2(bc-ad)^5}{b^{11}} + \frac{120d^3(bc-ad)^7\log(a+bx)}{b^{11}} - \frac{45d^2(bc-ad)^8}{b^{11}(a+bx)} - \frac{5d(bc-ad)^9}{b^{11}(a+bx)^2} - \frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3} + \frac{d^{10}(a+bx)^7}{7b^{11}} + \frac{210d^4x(bc-ad)^6}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (210*d^4*(b*c - a*d)^6*x)/b^10 - (b*c - a*d)^10/(3*b^11*(a + b*x)^3) - (5*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^2) - (45*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
) + (126*d^5*(b*c - a*d)^5*(a + b*x)^2)/b^11 + (70*d^6*(b*c - a*d)^4*(a + b
*x)^3)/b^11 + (30*d^7*(b*c - a*d)^3*(a + b*x)^4)/b^11 + (9*d^8*(b*c - a*d)^
2*(a + b*x)^5)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^6)/(3*b^11) + (d^10*(a +
 b*x)^7)/(7*b^11) + (120*d^3*(b*c - a*d)^7*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^4} \, dx = \int \left(\frac{210d^4(bc-ad)^6}{b^{10}} + \frac{(bc-ad)^{10}}{b^{10}(a+bx)^4} + \frac{10d(bc-ad)^9}{b^{10}(a+bx)^3} + \frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^2} + \frac{120d^3(bc-a\ldots}{b^{10}(a+bx\ldots}\right.$$
$$= \frac{210d^4(bc-ad)^6x}{b^{10}} - \frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3} - \frac{5d(bc-ad)^9}{b^{11}(a+bx)^2} - \frac{45d^2(bc-ad)^8}{b^{11}(a+bx)} + \frac{126d^5(bc-ad)^5 \ldots}{b^{11}}$$

Mathematica [A]
time = 0.12, size = 427, normalized size = 1.66

$$\frac{21bd^4(210b^6c^6-1008ab^5c^5d+2100a^2b^4c^4d^2-2400a^3b^3c^3d^3+1575a^4b^2c^2d^4-560a^5bcd^5+84a^6d^6)x+21b^2d^5(126b^5c^5-420ab^4c^4d+600a^2b^3c^3d^2-450a^3b^2c^2d^3+175a^4bcd^4-28a^5d^5)x^2+35b^3d^6(42b^4c^4-96ab^3c^3d+90a^2b^2c^2d^2-40a^3bcd^3+7a^4d^4)x^3+105b^4d^7(6b^3c^3-9ab^2c^2d+5a^2bcd^2-a^3d^3)x^4+21b^5d^8(9b^2c^2-8abcd+2a^2d^2)x^5+7b^6d^9(5bc-2ad)x^6+3b^7d^{10}x^7-\frac{7(bc-ad)^{10}}{(a+bx)^3}+\frac{105d(-bc+ad)^9}{(a+bx)^2}-\frac{945d^2(bc-ad)^8}{a+bx}+2520d^3(bc-ad)^7\log(a+bx)}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (21*b*d^4*(210*b^6*c^6 - 1008*a*b^5*c^5*d + 2100*a^2*b^4*c^4*d^2 - 2400*a^3
*b^3*c^3*d^3 + 1575*a^4*b^2*c^2*d^4 - 560*a^5*b*c*d^5 + 84*a^6*d^6)*x + 21*
b^2*d^5*(126*b^5*c^5 - 420*a*b^4*c^4*d + 600*a^2*b^3*c^3*d^2 - 450*a^3*b^2*
c^2*d^3 + 175*a^4*b*c*d^4 - 28*a^5*d^5)*x^2 + 35*b^3*d^6*(42*b^4*c^4 - 96*a
*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 + 7*a^4*d^4)*x^3 + 105*b^4
*d^7*(6*b^3*c^3 - 9*a*b^2*c^2*d + 5*a^2*b*c*d^2 - a^3*d^3)*x^4 + 21*b^5*d^8
*(9*b^2*c^2 - 8*a*b*c*d + 2*a^2*d^2)*x^5 + 7*b^6*d^9*(5*b*c - 2*a*d)*x^6 +
3*b^7*d^10*x^7 - (7*(b*c - a*d)^10)/(a + b*x)^3 + (105*d*(-(b*c) + a*d)^9)/
(a + b*x)^2 - (945*d^2*(b*c - a*d)^8)/(a + b*x) + 2520*d^3*(b*c - a*d)^7*Lo
g[a + b*x])/(21*b^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 895 vs. 2(252) = 504.
time = 0.17, size = 896, normalized size = 3.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] d^4/b^10*(-2/3*a*b^5*d^6*x^6+5/3*b^6*c*d^5*x^6+2*a^2*b^4*d^6*x^5+9*b^6*c^2*
d^4*x^5-5*a^3*b^3*d^6*x^4+30*b^6*c^3*d^3*x^4+35/3*a^4*b^2*d^6*x^3+70*b^6*c^
4*d^2*x^3-28*a^5*b*d^6*x^2+1/7*d^6*x^7*b^6+84*a^6*d^6*x+210*b^6*c^6*x-45*a*
b^5*c^2*d^4*x^4-200/3*a^3*b^3*c*d^5*x^3+150*a^2*b^4*c^2*d^4*x^3-160*a*b^5*c
^3*d^3*x^3+175*a^4*b^2*c*d^5*x^2-450*a^3*b^3*c^2*d^4*x^2+600*a^2*b^4*c^3*d^
3*x^2-420*a*b^5*c^4*d^2*x^2-560*a^5*b*c*d^5*x+1575*a^4*b^2*c^2*d^4*x-2400*a
^3*b^3*c^3*d^3*x+2100*a^2*b^4*c^4*d^2*x-1008*a*b^5*c^5*d*x+126*b^6*c^5*d*x^
2-8*a*b^5*c*d^5*x^5+25*a^2*b^4*c*d^5*x^4)-45/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^
7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d
^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)+5/b^11*d*(a^9*d^9-9*a^
8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4
*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(
b*x+a)^2-120/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*
c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)*ln(b*x
+a)-1/3*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+21
0*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d
^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(252) = 504.

time = 0.33, size = 891, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="maxima")

[Out] -1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*d^9 + 121*a^10*d^10 + 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b^9*c^8*d^2 - 108*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^5 + 1134*a^5*b^5*c^4*d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*a^8*b^2*c*d^9 + 17*a^9*b*d^10)*x)/(b^14*x^3 + 3*a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11) + 1/21*(3*b^6*d^10*x^7 + 7*(5*b^6*c*d^9 - 2*a*b^5*d^10)*x^6 + 21*(9*b^6*c^2*d^8 - 8*a*b^5*c*d^9 + 2*a^2*b^4*d^10)*x^5 + 105*(6*b^6*c^3*d^7 - 9*a*b^5*c^2*d^8 + 5*a^2*b^4*c*d^9 - a^3*b^3*d^10)*x^4 + 35*(42*b^6*c^4*d^6 - 96*a*b^5*c^3*d^7 + 90*a^2*b^4*c^2*d^8 - 40*a^3*b^3*c*d^9 + 7*a^4*b^2*d^10)*x^3 + 21*(126*b^6*c^5*d^5 - 420*a*b^5*c^4*d^6 + 600*a^2*b^4*c^3*d^7 - 450*a^3*b^3*c^2*d^8 + 175*a^4*b^2*c*d^9 - 28*a^5*b*d^10)*x^2 + 21*(210*b^6*c^6*d^4 - 1008*a*b^5*c^5*d^5 + 2100*a^2*b^4*c^4*d^6 - 2400*a^3*b^3*c^3*d^7 + 1575*a^4*b^2*c^2*d^8 - 560*a^5*b*c*d^9 + 84*a^6*d^10)*x)/b^10 + 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^6 + 35*a^4*b^3*c^3*d^7 - 21*a^5*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log(b*x + a)/b^11

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(252) = 504$.
time = 0.88, size = 1316, normalized size = 5.10

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="fricas")

[Out] 1/21*(3*b^10*d^10*x^10 - 7*b^10*c^10 - 35*a*b^9*c^9*d - 315*a^2*b^8*c^8*d^2 + 4620*a^3*b^7*c^7*d^3 - 19110*a^4*b^6*c^6*d^4 + 41454*a^5*b^5*c^5*d^5 - 54390*a^6*b^4*c^4*d^6 + 44940*a^7*b^3*c^3*d^7 - 22995*a^8*b^2*c^2*d^8 + 6685*a^9*b*c*d^9 - 847*a^10*d^10 + 5*(7*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(21*b^10*c^2*d^8 - 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 18*(35*b^10*c^3*d^7 - 21*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(35*b^10*c^4*d^6 - 35*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 - 7*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 126*(21*b^10*c^5*d^5 - 35*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 - 21*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(7*b^10*c^6*d^4 - 21*

```
a*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 - 35*a^3*b^7*c^3*d^7 + 21*a^4*b^6*c^2*d^
8 - 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 7*(1890*a*b^9*c^6*d^4 - 7938*a^2*
b^8*c^5*d^5 + 15330*a^3*b^7*c^4*d^6 - 16680*a^4*b^6*c^3*d^7 + 10575*a^5*b^5
*c^2*d^8 - 3665*a^6*b^4*c*d^9 + 539*a^7*b^3*d^10)*x^3 - 21*(45*b^10*c^8*d^2
 - 360*a*b^9*c^7*d^3 + 630*a^2*b^8*c^6*d^4 + 378*a^3*b^7*c^5*d^5 - 2730*a^4
*b^6*c^4*d^6 + 4080*a^5*b^5*c^3*d^7 - 3015*a^6*b^4*c^2*d^8 + 1145*a^7*b^3*c
*d^9 - 179*a^8*b^2*d^10)*x^2 - 21*(5*b^10*c^9*d + 45*a*b^9*c^8*d^2 - 540*a^
2*b^8*c^7*d^3 + 1890*a^3*b^7*c^6*d^4 - 3402*a^4*b^6*c^5*d^5 + 3570*a^5*b^5*
c^4*d^6 - 2220*a^6*b^4*c^3*d^7 + 765*a^7*b^3*c^2*d^8 - 115*a^8*b^2*c*d^9 +
a^9*b*d^10)*x + 2520*(a^3*b^7*c^7*d^3 - 7*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*
d^5 - 35*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 - 21*a^8*b^2*c^2*d^8 + 7*a^9*
b*c*d^9 - a^10*d^10 + (b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5
- 35*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*
c*d^9 - a^7*b^3*d^10)*x^3 + 3*(a*b^9*c^7*d^3 - 7*a^2*b^8*c^6*d^4 + 21*a^3*b
^7*c^5*d^5 - 35*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 - 21*a^6*b^4*c^2*d^8 +
 7*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 3*(a^2*b^8*c^7*d^3 - 7*a^3*b^7*c^6*d
^4 + 21*a^4*b^6*c^5*d^5 - 35*a^5*b^5*c^4*d^6 + 35*a^6*b^4*c^3*d^7 - 21*a^7*
b^3*c^2*d^8 + 7*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^14*x^3 + 3*
a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 907 vs. 2(252) = 504.
time = 0.66, size = 907, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^
6 + 35*a^4*b^3*c^3*d^7 - 21*a^5*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log
(abs(b*x + a))/b^11 - 1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a
^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*
d^9 + 121*a^10*d^10 + 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*
d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6
```

```
*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b
^9*c^8*d^2 - 108*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^
5 + 1134*a^5*b^5*c^4*d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*
a^8*b^2*c*d^9 + 17*a^9*b*d^10)*x)/((b*x + a)^3*b^11) + 1/21*(3*b^24*d^10*x^
7 + 35*b^24*c*d^9*x^6 - 14*a*b^23*d^10*x^6 + 189*b^24*c^2*d^8*x^5 - 168*a*b
^23*c*d^9*x^5 + 42*a^2*b^22*d^10*x^5 + 630*b^24*c^3*d^7*x^4 - 945*a*b^23*c^
2*d^8*x^4 + 525*a^2*b^22*c*d^9*x^4 - 105*a^3*b^21*d^10*x^4 + 1470*b^24*c^4*
d^6*x^3 - 3360*a*b^23*c^3*d^7*x^3 + 3150*a^2*b^22*c^2*d^8*x^3 - 1400*a^3*b^
21*c*d^9*x^3 + 245*a^4*b^20*d^10*x^3 + 2646*b^24*c^5*d^5*x^2 - 8820*a*b^23*
c^4*d^6*x^2 + 12600*a^2*b^22*c^3*d^7*x^2 - 9450*a^3*b^21*c^2*d^8*x^2 + 3675
*a^4*b^20*c*d^9*x^2 - 588*a^5*b^19*d^10*x^2 + 4410*b^24*c^6*d^4*x - 21168*a
*b^23*c^5*d^5*x + 44100*a^2*b^22*c^4*d^6*x - 50400*a^3*b^21*c^3*d^7*x + 330
75*a^4*b^20*c^2*d^8*x - 11760*a^5*b^19*c*d^9*x + 1764*a^6*b^18*d^10*x)/b^28
```

Mupad [B]
time = 0.39, size = 2219, normalized size = 8.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^4,x)
```

```
[Out] x^3*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b
^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*(
(4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/(3*b) - (a^4*d^10)/(3*b^8) + (70*c^
4*d^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(3*b^3) - (2*a^2*((4
*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b
^4))/b^2) - x^6*((2*a*d^10)/(3*b^5) - (5*c*d^9)/(3*b^4)) - x^4*((a*((4*a*((
4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/
b + (a^3*d^10)/b^7 - (30*c^3*d^7)/b^4 - (3*a^2*((4*a*d^10)/b^5 - (10*c*d^9)
/b^4))/(2*b^2)) + x^5*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(5*b) - (6*a
^2*d^10)/(5*b^6) + (9*c^2*d^8)/b^4) - x*((4*a*((252*c^5*d^5)/b^4 - (4*a*((4
*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (4
5*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^
10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (
4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5
 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b + (a^
4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^4 + (6*a^2*((4*a*((4*a*((4*a*d^10)/b
^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*
d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
^2))/b^2 - (4*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)
/b^6 + (45*c^2*d^8)/b^4))/b^3))/b - (210*c^6*d^4)/b^4 + (6*a^2*((4*a*((4*a*
((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8
)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 -
 (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (4*a^3*((4
```

*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b^2 - (4*a^3*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/b^3 + (a^4*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^4) + x^2*((126*c^5*d^5)/b^4 - (2*a*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b + (a^4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(2*b^4) + (3*a^2*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/b^2 - (2*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^3) - ((121*a^10*d^10 + b^10*c^10 + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 + 5*a*b^9*c^9*d - 955*a^9*b*c*d^9)/(3*b) + x*(85*a^9*d^10 + 5*b^9*c^9*d + 45*a*b^8*c^8*d^2 - 540*a^2*b^7*c^7*d^3 + 2100*a^3*b^6*c^6*d^4 - 4410*a^4*b^5*c^5*d^5 + 5670*a^5*b^4*c^4*d^6 - 4620*a^6*b^3*c^3*d^7 + 2340*a^7*b^2*c^2*d^8 - 675*a^8*b*c*d^9) + x^2*(45*a^8*b*d^10 + 45*b^9*c^8*d^2 - 360*a*b^8*c^7*d^3 - 360*a^7*b^2*c*d^9 + 1260*a^2*b^7*c^6*d^4 - 2520*a^3*b^6*c^5*d^5 + 3150*a^4*b^5*c^4*d^6 - 2520*a^5*b^4*c^3*d^7 + 1260*a^6*b^3*c^2*d^8))/(a^3*b^10 + b^13*x^3 + 3*a^2*b^11*x + 3*a*b^12*x^2) + (d^10*x^7)/(7*b^4) - (log(a + b*x)*(120*a^7*d^10 - 120*b^7*c^7*d^3 + 840*a*b^6*c^6*d^4 - 2520*a^2*b^5*c^5*d^5 + 4200*a^3*b^4*c^4*d^6 - 4200*a^4*b^3*c^3*d^7 + 2520*a^5*b^2*c^2*d^8 - 840*a^6*b*c*d^9))/b^11

3.1316 $\int \frac{(c+dx)^{10}}{(a+bx)^5}\,dx$

Optimal. Leaf size=262

$$\frac{252d^5(bc-ad)^5x}{b^{10}}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}-\frac{120d^3(bc-ad)^7}{b^{11}(a+bx)}+\frac{105d^6(bc-ad)^4(a+}{b^{11}}$$

```
[Out] 252*d^5*(-a*d+b*c)^5*x/b^10-1/4*(-a*d+b*c)^10/b^11/(b*x+a)^4-10/3*d*(-a*d+b
*c)^9/b^11/(b*x+a)^3-45/2*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^2-120*d^3*(-a*d+b*c
)^7/b^11/(b*x+a)+105*d^6*(-a*d+b*c)^4*(b*x+a)^2/b^11+40*d^7*(-a*d+b*c)^3*(b
*x+a)^3/b^11+45/4*d^8*(-a*d+b*c)^2*(b*x+a)^4/b^11+2*d^9*(-a*d+b*c)*(b*x+a)^
5/b^11+1/6*d^10*(b*x+a)^6/b^11+210*d^4*(-a*d+b*c)^6*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.30, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}}+\frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}}+\frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}}+\frac{210d^4(bc-ad)^6\log(a+bx)}{b^{11}}-\frac{120d^3(bc-ad)^7}{b^{11}(a+bx)}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}+\frac{d^{10}(a+bx)^6}{6b^{11}}+\frac{252d^5x(bc-ad)^5}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (252*d^5*(b*c - a*d)^5*x)/b^10 - (b*c - a*d)^10/(4*b^11*(a + b*x)^4) - (10*
d*(b*c - a*d)^9)/(3*b^11*(a + b*x)^3) - (45*d^2*(b*c - a*d)^8)/(2*b^11*(a +
 b*x)^2) - (120*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)) + (105*d^6*(b*c - a*d)^
4*(a + b*x)^2)/b^11 + (40*d^7*(b*c - a*d)^3*(a + b*x)^3)/b^11 + (45*d^8*(b*
c - a*d)^2*(a + b*x)^4)/(4*b^11) + (2*d^9*(b*c - a*d)*(a + b*x)^5)/b^11 + (
d^10*(a + b*x)^6)/(6*b^11) + (210*d^4*(b*c - a*d)^6*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^5}\,dx &= \int \left(\frac{252d^5(bc-ad)^5}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^5}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^4}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^3}+\frac{120d^3(bc-a}{b^{10}(a+bx)}\right.\\ &= \frac{252d^5(bc-ad)^5x}{b^{10}}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}-\frac{120d^3(bc-ad)}{b^{11}(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 359, normalized size = 1.37

$$\frac{12bd^5(252b^5c^5-1050ab^4c^4d+1800a^2b^3c^3d^2-1575a^3b^2c^2d^3+700a^4bcd^4-126a^5d^5)\,x+30b^2d^6(42b^4c^4-120ab^3c^3d+135a^2b^2c^2d^2-70a^3bcd^3+14a^4d^4)\,x^2+20b^3d^7(24b^3c^3-45ab^2c^2d+30a^2bcd^2-7a^3d^3)\,x^3+15b^4d^8(9b^2c^2-10abcd+3a^2d^2)\,x^4+12b^5d^9(2bc-ad)x^5+2b^6d^{10}x^6-\frac{3(bc-ad)^{10}}{(a+bx)^4}+\frac{40d(-bc+ad)^9}{(a+bx)^3}-\frac{270d^2(bc-ad)^8}{(a+bx)^2}+\frac{1440d^3(-bc+ad)^7}{a+bx}+2520d^4(bc-ad)^6\log(a+bx)}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(252*b^5*c^5 - 1050*a*b^4*c^4*d + 1800*a^2*b^3*c^3*d^2 - 1575*a^3
*b^2*c^2*d^3 + 700*a^4*b*c*d^4 - 126*a^5*d^5)*x + 30*b^2*d^6*(42*b^4*c^4 -
120*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 - 70*a^3*b*c*d^3 + 14*a^4*d^4)*x^2 +
20*b^3*d^7*(24*b^3*c^3 - 45*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 7*a^3*d^3)*x^3 +
 15*b^4*d^8*(9*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 12*b^5*d^9*(2*b*c -
a*d)*x^5 + 2*b^6*d^10*x^6 - (3*(b*c - a*d)^10)/(a + b*x)^4 + (40*d*(-(b*c)
+ a*d)^9)/(a + b*x)^3 - (270*d^2*(b*c - a*d)^8)/(a + b*x)^2 + (1440*d^3*(-(
b*c) + a*d)^7)/(a + b*x) + 2520*d^4*(b*c - a*d)^6*Log[a + b*x])/(12*b^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 880 vs. 2(252) = 504.
time = 0.15, size = 881, normalized size = 3.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -d^5/b^10*(-1/6*d^5*x^6*b^5+a*b^4*d^5*x^5-2*b^5*c*d^4*x^5-15/4*a^2*b^3*d^5*
x^4+25/2*a*b^4*c*d^4*x^4-45/4*b^5*c^2*d^3*x^4+35/3*a^3*b^2*d^5*x^3-50*a^2*b
^3*c*d^4*x^3+75*a*b^4*c^2*d^3*x^3-40*b^5*c^3*d^2*x^3-35*a^4*b*d^5*x^2+175*a
^3*b^2*c*d^4*x^2-675/2*a^2*b^3*c^2*d^3*x^2+300*a*b^4*c^3*d^2*x^2-105*b^5*c^
4*d*x^2+126*a^5*d^5*x-700*a^4*b*c*d^4*x+1575*a^3*b^2*c^2*d^3*x-1800*a^2*b^3
*c^3*d^2*x+1050*a*b^4*c^4*d*x-252*b^5*c^5*x)+120/b^11*d^3*(a^7*d^7-7*a^6*b*
c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c
^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)-1/4*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*
b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210
*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10
*c^10)/b^11/(b*x+a)^4-45/2/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d
^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*
d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^2+210/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15
*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^
6)*ln(b*x+a)+10/3/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b
^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^
2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(252) = 504.
time = 0.34, size = 903, normalized size = 3.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="maxima")

[Out] -1/12*(3*b^10*c^10 + 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6 + 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 8250*a^9*b*c*d^9 - 1207*a^10*d^10 + 1440*(b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 270*(b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 - 84*a^2*b^8*c^6*d^4 + 280*a^3*b^7*c^5*d^5 - 490*a^4*b^6*c^4*d^6 + 504*a^5*b^5*c^3*d^7 - 308*a^6*b^4*c^2*d^8 + 104*a^7*b^3*c*d^9 - 15*a^8*b^2*d^10)*x^2 + 20*(2*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 72*a^2*b^8*c^7*d^3 - 924*a^3*b^7*c^6*d^4 + 3276*a^4*b^6*c^5*d^5 - 5922*a^5*b^5*c^4*d^6 + 6216*a^6*b^4*c^3*d^7 - 3852*a^7*b^3*c^2*d^8 + 1314*a^8*b^2*c*d^9 - 191*a^9*b*d^10)*x)/(b^15*x^4 + 4*a*b^14*x^3 + 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11) + 1/12*(2*b^5*d^10*x^6 + 12*(2*b^5*c*d^9 - a*b^4*d^10)*x^5 + 15*(9*b^5*c^2*d^8 - 10*a*b^4*c*d^9 + 3*a^2*b^3*d^10)*x^4 + 20*(24*b^5*c^3*d^7 - 45*a*b^4*c^2*d^8 + 30*a^2*b^3*c*d^9 - 7*a^3*b^2*d^10)*x^3 + 30*(42*b^5*c^4*d^6 - 120*a*b^4*c^3*d^7 + 135*a^2*b^3*c^2*d^8 - 70*a^3*b^2*c*d^9 + 14*a^4*b*d^10)*x^2 + 12*(252*b^5*c^5*d^5 - 1050*a*b^4*c^4*d^6 + 1800*a^2*b^3*c^3*d^7 - 1575*a^3*b^2*c^2*d^8 + 700*a^4*b*c*d^9 - 126*a^5*d^10)*x)/b^10 + 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10)*log(b*x + a)/b^11

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(252) = 504$.
time = 0.93, size = 1365, normalized size = 5.21

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="fricas")

[Out] 1/12*(2*b^10*d^10*x^10 - 3*b^10*c^10 - 10*a*b^9*c^9*d - 45*a^2*b^8*c^8*d^2 - 360*a^3*b^7*c^7*d^3 + 5250*a^4*b^6*c^6*d^4 - 19404*a^5*b^5*c^5*d^5 + 35910*a^6*b^4*c^4*d^6 - 38280*a^7*b^3*c^3*d^7 + 23985*a^8*b^2*c^2*d^8 - 8250*a^9*b*c*d^9 + 1207*a^10*d^10 + 4*(6*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(15*b^10*c^2*d^8 - 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(20*b^10*c^3*d^7 - 15*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(15*b^10*c^4*d^6 - 20*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 - 6*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 504*(6*b^10*c^5*d^5 - 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 15*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + (12096*a*b^9*c^5*d^5 - 42840*a^2*b^8*c^4*d^6 + 66720*a^3*b^7*c^3*d^7 - 54765*a^4*b^6*c^2*d^8 + 23250*a^5*b^5*c*d^9 - 4043*a^6*b^4*d^10)*x^4 - 4*(360*b^10*c^7*d^3 - 2520*a*b^9*c^6*d^4 + 3024*a^2*b^8*c^5*d^5 + 5040*a^3*b^7*c^4*d^6 - 16320*a^4*b^6*c^3*d^7

```
+ 16965*a^5*b^5*c^2*d^8 - 8130*a^6*b^4*c*d^9 + 1523*a^7*b^3*d^10)*x^3 - 6*(
45*b^10*c^8*d^2 + 360*a*b^9*c^7*d^3 - 3780*a^2*b^8*c^6*d^4 + 10584*a^3*b^7*
c^5*d^5 - 13860*a^4*b^6*c^4*d^6 + 8880*a^5*b^5*c^3*d^7 - 1935*a^6*b^4*c^2*d
^8 - 570*a^7*b^3*c*d^9 + 263*a^8*b^2*d^10)*x^2 - 4*(10*b^10*c^9*d + 45*a*b^
9*c^8*d^2 + 360*a^2*b^8*c^7*d^3 - 4620*a^3*b^7*c^6*d^4 + 15624*a^4*b^6*c^5*
d^5 - 26460*a^5*b^5*c^4*d^6 + 25680*a^6*b^4*c^3*d^7 - 14535*a^7*b^3*c^2*d^8
 + 4470*a^8*b^2*c*d^9 - 577*a^9*b*d^10)*x + 2520*(a^4*b^6*c^6*d^4 - 6*a^5*b
^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 - 20*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 -
 6*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c
^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^
4*d^10)*x^4 + 4*(a*b^9*c^6*d^4 - 6*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 - 2
0*a^4*b^6*c^3*d^7 + 15*a^5*b^5*c^2*d^8 - 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^
3 + 6*(a^2*b^8*c^6*d^4 - 6*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 - 20*a^5*b^
5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 - 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 4*(a
^3*b^7*c^6*d^4 - 6*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 - 20*a^6*b^4*c^3*d^
7 + 15*a^7*b^3*c^2*d^8 - 6*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^
15*x^4 + 4*a*b^14*x^3 + 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(252) = 504$.
time = 0.53, size = 1168, normalized size = 4.46

[illegible]

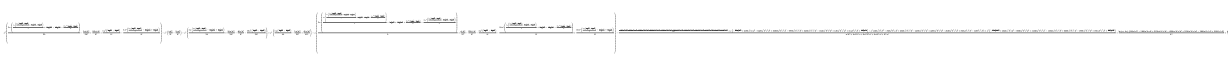
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(2*d^10 + 24*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 135*(b^4*c^2*d^8 -
 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 480*(b^6*c^3*d^7 - 3*a*b
^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 1260*(b^8*
c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4*b^4*d
^10)/((b*x + a)^4*b^4) + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*
c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x + a)^5
*b^5))*(b*x + a)^6/b^11 - 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c
^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10
)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - 1/12*(3*b^67*c^10/(b*x + a)
```

```
^4 + 40*b^66*c^9*d/(b*x + a)^3 - 30*a*b^66*c^9*d/(b*x + a)^4 + 270*b^65*c^8
*d^2/(b*x + a)^2 - 360*a*b^65*c^8*d^2/(b*x + a)^3 + 135*a^2*b^65*c^8*d^2/(b
*x + a)^4 + 1440*b^64*c^7*d^3/(b*x + a) - 2160*a*b^64*c^7*d^3/(b*x + a)^2 +
 1440*a^2*b^64*c^7*d^3/(b*x + a)^3 - 360*a^3*b^64*c^7*d^3/(b*x + a)^4 - 100
80*a*b^63*c^6*d^4/(b*x + a) + 7560*a^2*b^63*c^6*d^4/(b*x + a)^2 - 3360*a^3*
b^63*c^6*d^4/(b*x + a)^3 + 630*a^4*b^63*c^6*d^4/(b*x + a)^4 + 30240*a^2*b^6
2*c^5*d^5/(b*x + a) - 15120*a^3*b^62*c^5*d^5/(b*x + a)^2 + 5040*a^4*b^62*c^
5*d^5/(b*x + a)^3 - 756*a^5*b^62*c^5*d^5/(b*x + a)^4 - 50400*a^3*b^61*c^4*d
^6/(b*x + a) + 18900*a^4*b^61*c^4*d^6/(b*x + a)^2 - 5040*a^5*b^61*c^4*d^6/(
b*x + a)^3 + 630*a^6*b^61*c^4*d^6/(b*x + a)^4 + 50400*a^4*b^60*c^3*d^7/(b*x
 + a) - 15120*a^5*b^60*c^3*d^7/(b*x + a)^2 + 3360*a^6*b^60*c^3*d^7/(b*x + a
)^3 - 360*a^7*b^60*c^3*d^7/(b*x + a)^4 - 30240*a^5*b^59*c^2*d^8/(b*x + a) +
 7560*a^6*b^59*c^2*d^8/(b*x + a)^2 - 1440*a^7*b^59*c^2*d^8/(b*x + a)^3 + 13
5*a^8*b^59*c^2*d^8/(b*x + a)^4 + 10080*a^6*b^58*c*d^9/(b*x + a) - 2160*a^7*
b^58*c*d^9/(b*x + a)^2 + 360*a^8*b^58*c*d^9/(b*x + a)^3 - 30*a^9*b^58*c*d^9
/(b*x + a)^4 - 1440*a^7*b^57*d^10/(b*x + a) + 270*a^8*b^57*d^10/(b*x + a)^2
 - 40*a^9*b^57*d^10/(b*x + a)^3 + 3*a^10*b^57*d^10/(b*x + a)^4)/b^68
```

Mupad [B]
time = 0.38, size = 1494, normalized size = 5.70



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^5,x)
```

```
[Out] x^2*((5*a*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/
b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^
2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/(2*b) - (5*a^4*d^10)/(2*b^9) + (
105*c^4*d^6)/b^5 + (5*a^3*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (5*a^2*(
(5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8
)/b^5))/b^2) - x^5*((a*d^10)/b^6 - (2*c*d^9)/b^5) - x^3*((5*a*((5*a*((5*a*d
^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/(3*b
) + (10*a^3*d^10)/(3*b^8) - (40*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 - (1
0*c*d^9)/b^5))/(3*b^2)) + x^4*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/(4*b
) - (5*a^2*d^10)/(2*b^7) + (45*c^2*d^8)/(4*b^5)) - x*((5*a*((5*a*((5*a*((5*
a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b
^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 -
(10*c*d^9)/b^5))/b^2))/b - (5*a^4*d^10)/b^9 + (210*c^4*d^6)/b^5 + (10*a^3*(
(5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (10*a^2*((5*a*((5*a*d^10)/b^6 - (10
*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^2))/b + (a^5*d^1
0)/b^10 - (252*c^5*d^5)/b^5 - (5*a^4*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^4
 - (10*a^2*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)
/b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a
^2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/b^2 + (10*a^3*((5*a*((5*a*d^10)
```

/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^3) - ((3*b^10*c^10 - 1207*a^10*d^10 + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6 + 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 10*a*b^9*c^9*d + 8250*a^9*b*c*d^9)/(12*b) + x*((10*b^9*c^9*d)/3 - (955*a^9*d^10)/3 + 15*a*b^8*c^8*d^2 + 120*a^2*b^7*c^7*d^3 - 1540*a^3*b^6*c^6*d^4 + 5460*a^4*b^5*c^5*d^5 - 9870*a^5*b^4*c^4*d^6 + 10360*a^6*b^3*c^3*d^7 - 6420*a^7*b^2*c^2*d^8 + 2190*a^8*b*c*d^9) - x^3*(120*a^7*b^2*d^10 - 120*b^9*c^7*d^3 + 840*a*b^8*c^6*d^4 - 840*a^6*b^3*c*d^9 - 2520*a^2*b^7*c^5*d^5 + 4200*a^3*b^6*c^4*d^6 - 4200*a^4*b^5*c^3*d^7 + 2520*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)/2 - (675*a^8*b*d^10)/2 + 180*a*b^8*c^7*d^3 + 2340*a^7*b^2*c*d^9 - 1890*a^2*b^7*c^6*d^4 + 6300*a^3*b^6*c^5*d^5 - 11025*a^4*b^5*c^4*d^6 + 11340*a^5*b^4*c^3*d^7 - 6930*a^6*b^3*c^2*d^8))/(a^4*b^10 + b^14*x^4 + 4*a^3*b^11*x + 4*a*b^13*x^3 + 6*a^2*b^12*x^2) + (log(a + b*x)*(210*a^6*d^10 + 210*b^6*c^6*d^4 - 1260*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 - 4200*a^3*b^3*c^3*d^7 + 3150*a^4*b^2*c^2*d^8 - 1260*a^5*b*c*d^9))/b^11 + (d^10*x^6)/(6*b^5)

3.1317 $\int \frac{(c+dx)^{10}}{(a+bx)^6}\,dx$

Optimal. Leaf size=260

$$\frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}+\frac{60d$$

```
[Out] 210*d^6*(-a*d+b*c)^4*x/b^10-1/5*(-a*d+b*c)^10/b^11/(b*x+a)^5-5/2*d*(-a*d+b*
c)^9/b^11/(b*x+a)^4-15*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^3-60*d^3*(-a*d+b*c)^7/
b^11/(b*x+a)^2-210*d^4*(-a*d+b*c)^6/b^11/(b*x+a)+60*d^7*(-a*d+b*c)^3*(b*x+a
)^2/b^11+15*d^8*(-a*d+b*c)^2*(b*x+a)^3/b^11+5/2*d^9*(-a*d+b*c)*(b*x+a)^4/b^
11+1/5*d^10*(b*x+a)^5/b^11+252*d^5*(-a*d+b*c)^5*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.29, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{252d^5(bc-ad)^5\log(a+bx)}{b^{11}}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}+\frac{d^{10}(a+bx)^5}{5b^{11}}+\frac{210d^6x(bc-ad)^4}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (210*d^6*(b*c - a*d)^4*x)/b^10 - (b*c - a*d)^10/(5*b^11*(a + b*x)^5) - (5*d
*(b*c - a*d)^9)/(2*b^11*(a + b*x)^4) - (15*d^2*(b*c - a*d)^8)/(b^11*(a + b*
x)^3) - (60*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^2) - (210*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)) + (60*d^7*(b*c - a*d)^3*(a + b*x)^2)/b^11 + (15*d^8*(b*c
- a*d)^2*(a + b*x)^3)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^4)/(2*b^11) + (d^
10*(a + b*x)^5)/(5*b^11) + (252*d^5*(b*c - a*d)^5*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^6}\,dx &= \int \left(\frac{210d^6(bc-ad)^4}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^6}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^5}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^4}+\frac{120d^3(bc-a}{b^{10}(a+bx)}\right.\\ &= \frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 305, normalized size = 1.17

$$\frac{10bd^6\left(210b^4c^4-720ab^3c^3d+945a^2b^2c^2d^2-560a^3bcd^3+126a^4d^4\right)x+10b^2d^7\left(60b^3c^3-135ab^2c^2d+105a^2bcd^2-28a^3d^3\right)x^2+10b^3d^8\left(15b^2c^2-20abcd+7a^2d^2\right)x^3+5b^4d^9(5bc-3ad)x^4+2b^5d^{10}x^5-\frac{2(bc-ad)^{10}}{(a+bx)^5}+\frac{25d(-bc+ad)^9}{(a+bx)^4}-\frac{150d^2(bc-ad)^8}{(a+bx)^3}+\frac{600d^3(-bc+ad)^7}{(a+bx)^2}-\frac{2100d^4(bc-ad)^6}{a+bx}+2520d^5(bc-ad)^5\log(a+bx)}{10b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (10*b*d^6*(210*b^4*c^4 - 720*a*b^3*c^3*d + 945*a^2*b^2*c^2*d^2 - 560*a^3*b*
c*d^3 + 126*a^4*d^4)*x + 10*b^2*d^7*(60*b^3*c^3 - 135*a*b^2*c^2*d + 105*a^2
*b*c*d^2 - 28*a^3*d^3)*x^2 + 10*b^3*d^8*(15*b^2*c^2 - 20*a*b*c*d + 7*a^2*d^
2)*x^3 + 5*b^4*d^9*(5*b*c - 3*a*d)*x^4 + 2*b^5*d^10*x^5 - (2*(b*c - a*d)^10
)/(a + b*x)^5 + (25*d*(-(b*c) + a*d)^9)/(a + b*x)^4 - (150*d^2*(b*c - a*d)^
8)/(a + b*x)^3 + (600*d^3*(-(b*c) + a*d)^7)/(a + b*x)^2 - (2100*d^4*(b*c -
a*d)^6)/(a + b*x) + 2520*d^5*(b*c - a*d)^5*Log[a + b*x])/(10*b^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(252) = 504.
time = 0.15, size = 870, normalized size = 3.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] d^6/b^10*(1/5*d^4*x^5*b^4-3/2*a*b^3*d^4*x^4+5/2*b^4*c*d^3*x^4+7*a^2*b^2*d^4
*x^3-20*a*b^3*c*d^3*x^3+15*b^4*c^2*d^2*x^3-28*a^3*b*d^4*x^2+105*a^2*b^2*c*d
^3*x^2-135*a*b^3*c^2*d^2*x^2+60*b^4*c^3*d*x^2+126*a^4*d^4*x-560*a^3*b*c*d^3
*x+945*a^2*b^2*c^2*d^2*x-720*a*b^3*c^3*d*x+210*b^4*c^4*x)-210/b^11*d^4*(a^6
*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2
-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)+5/2/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^
2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3
*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^4-1/5*(a^10*
d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*
d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*
c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^5+60/b^11*d^3*(a^7*d^7-7*a^6
*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^
5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^2-252/b^11*d^5*(a^5*d^5-5*a^4*b*c*
d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)*ln(b*x+a)-
15/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70
*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^
8)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(252) = 504.
time = 0.37, size = 912, normalized size = 3.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/10*(2*b^10*c^10 + 5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^
3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27
540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*
d^10 + 2100*(b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b
^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600
*(b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6
 - 245*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^
3*d^10)*x^3 + 150*(b^10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 30
8*a^3*b^7*c^5*d^5 + 910*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b
^4*c^2*d^8 - 428*a^7*b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*
a*b^9*c^8*d^2 + 12*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*
d^5 + 3234*a^5*b^5*c^4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 -
1599*a^8*b^2*c*d^9 + 275*a^9*b*d^10)*x)/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b
^14*x^3 + 10*a^3*b^13*x^2 + 5*a^4*b^12*x + a^5*b^11) + 1/10*(2*b^4*d^10*x^5
 + 5*(5*b^4*c*d^9 - 3*a*b^3*d^10)*x^4 + 10*(15*b^4*c^2*d^8 - 20*a*b^3*c*d^9
 + 7*a^2*b^2*d^10)*x^3 + 10*(60*b^4*c^3*d^7 - 135*a*b^3*c^2*d^8 + 105*a^2*b
^2*c*d^9 - 28*a^3*b*d^10)*x^2 + 10*(210*b^4*c^4*d^6 - 720*a*b^3*c^3*d^7 + 9
45*a^2*b^2*c^2*d^8 - 560*a^3*b*c*d^9 + 126*a^4*d^10)*x)/b^10 + 252*(b^5*c^5
*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^8 + 5*a^4*b*
c*d^9 - a^5*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. $2(252) = 504$.
time = 1.49, size = 1395, normalized size = 5.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*d^10*x^10 - 2*b^10*c^10 - 5*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2 -
 60*a^3*b^7*c^7*d^3 - 420*a^4*b^6*c^6*d^4 + 5754*a^5*b^5*c^5*d^5 - 18270*a^
6*b^4*c^4*d^6 + 27540*a^7*b^3*c^3*d^7 - 22290*a^8*b^2*c^2*d^8 + 9395*a^9*b*
c*d^9 - 1627*a^10*d^10 + 5*(5*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*(10*b^10*c^
2*d^8 - 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(10*b^10*c^3*d^7 - 10*a*b^9*
c^2*d^8 + 5*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(5*b^10*c^4*d^6 - 10*a*
b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 - 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + (1
0500*a*b^9*c^4*d^6 - 30000*a^2*b^8*c^3*d^7 + 35250*a^3*b^7*c^2*d^8 - 19375*
a^4*b^6*c*d^9 + 4127*a^5*b^5*d^10)*x^5 - 5*(420*b^10*c^6*d^4 - 2520*a*b^9*c
^5*d^5 + 2100*a^2*b^8*c^4*d^6 + 4800*a^3*b^7*c^3*d^7 - 10050*a^4*b^6*c^2*d^
8 + 6775*a^5*b^5*c*d^9 - 1607*a^6*b^4*d^10)*x^4 - 10*(60*b^10*c^7*d^3 + 420
*a*b^9*c^6*d^4 - 3780*a^2*b^8*c^5*d^5 + 8400*a^3*b^7*c^4*d^6 - 7800*a^4*b^6
```

```
*c^3*d^7 + 2550*a^5*b^5*c^2*d^8 + 475*a^6*b^4*c*d^9 - 347*a^7*b^3*d^10)*x^3
 - 10*(15*b^10*c^8*d^2 + 60*a*b^9*c^7*d^3 + 420*a^2*b^8*c^6*d^4 - 4620*a^3*
b^7*c^5*d^5 + 12600*a^4*b^6*c^4*d^6 - 16200*a^5*b^5*c^3*d^7 + 10950*a^6*b^4
*c^2*d^8 - 3725*a^7*b^3*c*d^9 + 493*a^8*b^2*d^10)*x^2 - 5*(5*b^10*c^9*d + 1
5*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 5250*a^4*b^6*c
^5*d^5 + 15750*a^5*b^5*c^4*d^6 - 22500*a^6*b^4*c^3*d^7 + 17250*a^7*b^3*c^2*
d^8 - 6875*a^8*b^2*c*d^9 + 1123*a^9*b*d^10)*x + 2520*(a^5*b^5*c^5*d^5 - 5*a
^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 - 10*a^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 -
a^10*d^10 + (b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b
^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 5*(a*b^9*c^5*d^5 - 5*a^2
*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 - 10*a^4*b^6*c^2*d^8 + 5*a^5*b^5*c*d^9 -
a^6*b^4*d^10)*x^4 + 10*(a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^
3*d^7 - 10*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 10*(a^3*
b^7*c^5*d^5 - 5*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 - 10*a^6*b^4*c^2*d^8 +
 5*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 5*(a^4*b^6*c^5*d^5 - 5*a^5*b^5*c^4*d
^6 + 10*a^6*b^4*c^3*d^7 - 10*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 - a^9*b*d^10
)*x)*log(b*x + a))/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b^14*x^3 + 10*a^3*b^13
*x^2 + 5*a^4*b^12*x + a^5*b^11)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(252) = 504.
time = 0.60, size = 883, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 252*(b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^
8 + 5*a^4*b*c*d^9 - a^5*d^10)*log(abs(b*x + a))/b^11 - 1/10*(2*b^10*c^10 +
5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d
^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 +
 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*d^10 + 2100*(b^10*c^6
*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b
^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600*(b^10*c^7*d^3 + 7*a*
b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6 - 245*a^4*b^6*c^3*d^
```

```
7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^3*d^10)*x^3 + 150*(b^
10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 308*a^3*b^7*c^5*d^5 + 9
10*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b^4*c^2*d^8 - 428*a^7*
b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*a*b^9*c^8*d^2 + 12*a^
2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*d^5 + 3234*a^5*b^5*c^
4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 - 1599*a^8*b^2*c*d^9 +
275*a^9*b*d^10)*x)/((b*x + a)^5*b^11) + 1/10*(2*b^24*d^10*x^5 + 25*b^24*c*d
^9*x^4 - 15*a*b^23*d^10*x^4 + 150*b^24*c^2*d^8*x^3 - 200*a*b^23*c*d^9*x^3 +
 70*a^2*b^22*d^10*x^3 + 600*b^24*c^3*d^7*x^2 - 1350*a*b^23*c^2*d^8*x^2 + 10
50*a^2*b^22*c*d^9*x^2 - 280*a^3*b^21*d^10*x^2 + 2100*b^24*c^4*d^6*x - 7200*
a*b^23*c^3*d^7*x + 9450*a^2*b^22*c^2*d^8*x - 5600*a^3*b^21*c*d^9*x + 1260*a
^4*b^20*d^10*x)/b^30
```

Mupad [B]
time = 0.40, size = 1141, normalized size = 4.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^6,x)
```

```
[Out] x^3*((2*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (5*a^2*d^10)/b^8 + (15*c^2
*d^8)/b^6) - x^2*((3*a*((6*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2
*d^10)/b^8 + (45*c^2*d^8)/b^6))/b + (10*a^3*d^10)/b^9 - (60*c^3*d^7)/b^6 -
(15*a^2*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/(2*b^2)) - x^4*((3*a*d^10)/(2*b^
7) - (5*c*d^9)/(2*b^6)) - (x^4*(210*a^6*b^3*d^10 + 210*b^9*c^6*d^4 - 1260*a
*b^8*c^5*d^5 - 1260*a^5*b^4*c*d^9 + 3150*a^2*b^7*c^4*d^6 - 4200*a^3*b^6*c^3
*d^7 + 3150*a^4*b^5*c^2*d^8) + (1627*a^10*d^10 + 2*b^10*c^10 + 15*a^2*b^8*c
^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 +
18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 + 5*a
*b^9*c^9*d - 9395*a^9*b*c*d^9)/(10*b) + x*((1375*a^9*d^10)/2 + (5*b^9*c^9*d
)/2 + (15*a*b^8*c^8*d^2)/2 + 30*a^2*b^7*c^7*d^3 + 210*a^3*b^6*c^6*d^4 - 262
5*a^4*b^5*c^5*d^5 + 8085*a^5*b^4*c^4*d^6 - 11970*a^6*b^3*c^3*d^7 + 9570*a^7
*b^2*c^2*d^8 - (7995*a^8*b*c*d^9)/2) + x^3*(780*a^7*b^2*d^10 + 60*b^9*c^7*d
^3 + 420*a*b^8*c^6*d^4 - 4620*a^6*b^3*c*d^9 - 3780*a^2*b^7*c^5*d^5 + 10500*
a^3*b^6*c^4*d^6 - 14700*a^4*b^5*c^3*d^7 + 11340*a^5*b^4*c^2*d^8) + x^2*(109
5*a^8*b*d^10 + 15*b^9*c^8*d^2 + 60*a*b^8*c^7*d^3 - 6420*a^7*b^2*c*d^9 + 420
*a^2*b^7*c^6*d^4 - 4620*a^3*b^6*c^5*d^5 + 13650*a^4*b^5*c^4*d^6 - 19740*a^5
*b^4*c^3*d^7 + 15540*a^6*b^3*c^2*d^8))/(a^5*b^10 + b^15*x^5 + 5*a^4*b^11*x
+ 5*a*b^14*x^4 + 10*a^3*b^12*x^2 + 10*a^2*b^13*x^3) + x*((6*a*((6*a*((6*a*(
(6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6)
)/b + (20*a^3*d^10)/b^9 - (120*c^3*d^7)/b^6 - (15*a^2*((6*a*d^10)/b^7 - (10
*c*d^9)/b^6))/b^2))/b - (15*a^4*d^10)/b^10 + (210*c^4*d^6)/b^6 + (20*a^3*((
6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b^3 - (15*a^2*((6*a*((6*a*d^10)/b^7 - (10*
c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6))/b^2) + (d^10*x^5)/(
```

```
5*b^6) - (log(a + b*x)*(252*a^5*d^10 - 252*b^5*c^5*d^5 + 1260*a*b^4*c^4*d^6
 - 2520*a^2*b^3*c^3*d^7 + 2520*a^3*b^2*c^2*d^8 - 1260*a^4*b*c*d^9))/b^11
```

3.1318 $\int \frac{(c+dx)^{10}}{(a+bx)^7} \, dx$

Optimal. Leaf size=262

$$\frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}$$

```
[Out] 120*d^7*(-a*d+b*c)^3*x/b^10-1/6*(-a*d+b*c)^10/b^11/(b*x+a)^6-2*d*(-a*d+b*c)
^9/b^11/(b*x+a)^5-45/4*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^4-40*d^3*(-a*d+b*c)^7/
b^11/(b*x+a)^3-105*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^2-252*d^5*(-a*d+b*c)^5/b^1
1/(b*x+a)+45/2*d^8*(-a*d+b*c)^2*(b*x+a)^2/b^11+10/3*d^9*(-a*d+b*c)*(b*x+a)^
3/b^11+1/4*d^10*(b*x+a)^4/b^11+210*d^6*(-a*d+b*c)^4*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.27, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{210d^6(bc-ad)^4\log(a+bx)}{b^{11}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}+\frac{d^{10}(a+bx)^4}{4b^{11}}+\frac{120d^7x(bc-ad)^3}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (120*d^7*(b*c - a*d)^3*x)/b^10 - (b*c - a*d)^10/(6*b^11*(a + b*x)^6) - (2*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^5) - (45*d^2*(b*c - a*d)^8)/(4*b^11*(a + b*
x)^4) - (40*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^3) - (105*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)^2) - (252*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)) + (45*d^8*(b
*c - a*d)^2*(a + b*x)^2)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^3)/(3*b^1
1) + (d^10*(a + b*x)^4)/(4*b^11) + (210*d^6*(b*c - a*d)^4*Log[a + b*x])/b^1
1
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx &= \int \left(\frac{120d^7(bc-ad)^3}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^7}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^6}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^5}+\frac{120d^3(bc-a}{b^{10}(a+bx)}\right.\\ &= \frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 265, normalized size = 1.01

$$\frac{12bd^7\left(120b^3c^3-315ab^2c^2d+280a^2bcd^2-84a^3d^3\right)x+6b^2d^8\left(45b^2c^2-70abcd+28a^2d^2\right)x^2+4b^3d^9(10bc-7ad)x^3+3b^4d^{10}x^4-\frac{2(bc-ad)^{10}}{(a+bx)^6}+\frac{24d(-bc+ad)^9}{(a+bx)^5}-\frac{135d^2(bc-ad)^8}{(a+bx)^4}+\frac{480d^3(-bc+ad)^7}{(a+bx)^3}-\frac{1260d^4(bc-ad)^6}{(a+bx)^2}+\frac{3024d^5(-bc+ad)^5}{a+bx}+2520d^6(bc-ad)^4\log(a+bx)}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (12*b*d^7*(120*b^3*c^3 - 315*a*b^2*c^2*d + 280*a^2*b*c*d^2 - 84*a^3*d^3)*x
+ 6*b^2*d^8*(45*b^2*c^2 - 70*a*b*c*d + 28*a^2*d^2)*x^2 + 4*b^3*d^9*(10*b*c
- 7*a*d)*x^3 + 3*b^4*d^10*x^4 - (2*(b*c - a*d)^10)/(a + b*x)^6 + (24*d*(-(b
*c) + a*d)^9)/(a + b*x)^5 - (135*d^2*(b*c - a*d)^8)/(a + b*x)^4 + (480*d^3*
(-(b*c) + a*d)^7)/(a + b*x)^3 - (1260*d^4*(b*c - a*d)^6)/(a + b*x)^2 + (302
4*d^5*(-(b*c) + a*d)^5)/(a + b*x) + 2520*d^6*(b*c - a*d)^4*Log[a + b*x])/(1
2*b^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(252) = 504.
time = 0.15, size = 862, normalized size = 3.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -d^7/b^10*(-1/4*d^3*x^4*b^3+7/3*a*b^2*d^3*x^3-10/3*b^3*c*d^2*x^3-14*a^2*b*d
^3*x^2+35*a*b^2*c*d^2*x^2-45/2*b^3*c^2*d*x^2+84*a^3*d^3*x-280*a^2*b*c*d^2*x
+315*a*b^2*c^2*d*x-120*b^3*c^3*x)+252/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^
3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)-45/4/b^11*d
^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*
c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a
)^4+2/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+1
26*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^
2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^5-105/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a
^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
/(b*x+a)^2-1/6*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3
*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^
7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^6+210/b
^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(
b*x+a)+40/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3
*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(252) = 504.
time = 0.40, size = 925, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3
 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6174*a^6*b^4*c^4*d^6 + 16056*
a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 10036*a^9*b*c*d^9 - 2131*a^10*d^1
0 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*
c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 1260*(b^10*c^6*d^4 + 6*a*b^
9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^7*c^3*d^7 - 105*a^4*b^6*c^2*d^8
+ 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 + 240*(2*b^10*c^7*d^3 + 7*a*b^9*c
^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c^4*d^6 + 910*a^4*b^6*c^3*d^7 - 9
87*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 107*a^7*b^3*d^10)*x^3 + 45*(3*b^10
*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c^5*d^5 - 175
0*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 - 4788*a^6*b^4*c^2*d^8 + 2552*a^7*
b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 24*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c
^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9
 - 1879*a^9*b*d^10)*x)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*
b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 1/12*(3*b^3*d^10*x^
4 + 4*(10*b^3*c*d^9 - 7*a*b^2*d^10)*x^3 + 6*(45*b^3*c^2*d^8 - 70*a*b^2*c*d^
9 + 28*a^2*b*d^10)*x^2 + 12*(120*b^3*c^3*d^7 - 315*a*b^2*c^2*d^8 + 280*a^2*
b*c*d^9 - 84*a^3*d^10)*x)/b^10 + 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2
*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(252) = 504$.
time = 1.10, size = 1386, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*d^10*x^10 - 2*b^10*c^10 - 4*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
24*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 504*a^5*b^5*c^5*d^5 + 6174*a^6*b^
4*c^4*d^6 - 16056*a^7*b^3*c^3*d^7 + 18414*a^8*b^2*c^2*d^8 - 10036*a^9*b*c*d
^9 + 2131*a^10*d^10 + 10*(4*b^10*c*d^9 - a*b^9*d^10)*x^9 + 45*(6*b^10*c^2*d
^8 - 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(4*b^10*c^3*d^7 - 6*a*b^9*c^2*
d^8 + 4*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + (8640*a*b^9*c^3*d^7 - 18630*a^2
*b^8*c^2*d^8 + 14660*a^3*b^7*c*d^9 - 4043*a^4*b^6*d^10)*x^6 - 6*(504*b^10*c
^5*d^5 - 2520*a*b^9*c^4*d^6 + 1440*a^2*b^8*c^3*d^7 + 3510*a^3*b^7*c^2*d^8 -
 4580*a^4*b^6*c*d^9 + 1523*a^5*b^5*d^10)*x^5 - 15*(84*b^10*c^6*d^4 + 504*a*
b^9*c^5*d^5 - 3780*a^2*b^8*c^4*d^6 + 6480*a^3*b^7*c^3*d^7 - 4050*a^4*b^6*c^
2*d^8 + 460*a^5*b^5*c*d^9 + 263*a^6*b^4*d^10)*x^4 - 20*(24*b^10*c^7*d^3 + 8
4*a*b^9*c^6*d^4 + 504*a^2*b^8*c^5*d^5 - 4620*a^3*b^7*c^4*d^6 + 9840*a^4*b^6
*c^3*d^7 - 9090*a^5*b^5*c^2*d^8 + 3820*a^6*b^4*c*d^9 - 577*a^7*b^3*d^10)*x^
3 - 15*(9*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 504*a^3*b^
```

```
7*c^5*d^5 - 5250*a^4*b^6*c^4*d^6 + 12360*a^5*b^5*c^3*d^7 - 12870*a^6*b^4*c^
2*d^8 + 6340*a^7*b^3*c*d^9 - 1207*a^8*b^2*d^10)*x^2 - 6*(4*b^10*c^9*d + 9*a
*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^
5 - 5754*a^5*b^5*c^4*d^6 + 14376*a^6*b^4*c^3*d^7 - 15894*a^7*b^3*c^2*d^8 +
8356*a^8*b^2*c*d^9 - 1711*a^9*b*d^10)*x + 2520*(a^6*b^4*c^4*d^6 - 4*a^7*b^3
*c^3*d^7 + 6*a^8*b^2*c^2*d^8 - 4*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^4*d^6 -
4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 6*(a*b^9*c^4*d^6 - 4*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 - 4*a^4*b^6*c*d^9
 + a^5*b^5*d^10)*x^5 + 15*(a^2*b^8*c^4*d^6 - 4*a^3*b^7*c^3*d^7 + 6*a^4*b^6*
c^2*d^8 - 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 20*(a^3*b^7*c^4*d^6 - 4*a^4
*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 - 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 15
*(a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 - 4*a^7*b^3*c*d^9
 + a^8*b^2*d^10)*x^2 + 6*(a^5*b^5*c^4*d^6 - 4*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c
^2*d^8 - 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^17*x^6 + 6*a*b^1
6*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x
+ a^6*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**7,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 878 vs. 2(252) = 504.
time = 0.54, size = 878, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^
4*d^10)*log(abs(b*x + a))/b^11 - 1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*
b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5
 - 6174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 1
0036*a^9*b*c*d^9 - 2131*a^10*d^10 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 +
10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x
^5 + 1260*(b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^
7*c^3*d^7 - 105*a^4*b^6*c^2*d^8 + 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 +
 240*(2*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c
^4*d^6 + 910*a^4*b^6*c^3*d^7 - 987*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 10
```

7*a^7*b^3*d^10)*x^3 + 45*(3*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6
*d^4 + 168*a^3*b^7*c^5*d^5 - 1750*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 -
4788*a^6*b^4*c^2*d^8 + 2552*a^7*b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^
10*c^9*d + 9*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*
a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*
b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9 - 1879*a^9*b*d^10)*x)/((b*x + a)^6*b^11) +
 1/12*(3*b^21*d^10*x^4 + 40*b^21*c*d^9*x^3 - 28*a*b^20*d^10*x^3 + 270*b^21*
c^2*d^8*x^2 - 420*a*b^20*c*d^9*x^2 + 168*a^2*b^19*d^10*x^2 + 1440*b^21*c^3*
d^7*x - 3780*a*b^20*c^2*d^8*x + 3360*a^2*b^19*c*d^9*x - 1008*a^3*b^18*d^10*
x)/b^28

Mupad [B]
time = 0.42, size = 997, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^10/(a + b*x)^7,x)

[Out] x^2*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/(2*b) - (21*a^2*d^10)/(2*b^9)
+ (45*c^2*d^8)/(2*b^7)) - (x^4*(105*b^9*c^6*d^4 - 1155*a^6*b^3*d^10 + 630*a
*b^8*c^5*d^5 + 5670*a^5*b^4*c*d^9 - 4725*a^2*b^7*c^4*d^6 + 10500*a^3*b^6*c^
3*d^7 - 11025*a^4*b^5*c^2*d^8) + (2*b^10*c^10 - 2131*a^10*d^10 + 9*a^2*b^8*
c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6
174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 4*a*b
^9*c^9*d + 10036*a^9*b*c*d^9)/(12*b) + x*(2*b^9*c^9*d - (1879*a^9*d^10)/2 +
 (9*a*b^8*c^8*d^2)/2 + 12*a^2*b^7*c^7*d^3 + 42*a^3*b^6*c^6*d^4 + 252*a^4*b^
5*c^5*d^5 - 2877*a^5*b^4*c^4*d^6 + 7308*a^6*b^3*c^3*d^7 - 8262*a^7*b^2*c^2*
d^8 + 4458*a^8*b*c*d^9) + x^3*(40*b^9*c^7*d^3 - 2140*a^7*b^2*d^10 + 140*a*b
^8*c^6*d^4 + 10360*a^6*b^3*c*d^9 + 840*a^2*b^7*c^5*d^5 - 7700*a^3*b^6*c^4*d
^6 + 18200*a^4*b^5*c^3*d^7 - 19740*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)
/4 - (7995*a^8*b*d^10)/4 + 30*a*b^8*c^7*d^3 + 9570*a^7*b^2*c*d^9 + 105*a^2*
b^7*c^6*d^4 + 630*a^3*b^6*c^5*d^5 - (13125*a^4*b^5*c^4*d^6)/2 + 16170*a^5*b
^4*c^3*d^7 - 17955*a^6*b^3*c^2*d^8) - x^5*(252*a^5*b^4*d^10 - 252*b^9*c^5*d
^5 + 1260*a*b^8*c^4*d^6 - 1260*a^4*b^5*c*d^9 - 2520*a^2*b^7*c^3*d^7 + 2520*
a^3*b^6*c^2*d^8))/(a^6*b^10 + b^16*x^6 + 6*a^5*b^11*x + 6*a*b^15*x^5 + 15*a
^4*b^12*x^2 + 20*a^3*b^13*x^3 + 15*a^2*b^14*x^4) - x^3*((7*a*d^10)/(3*b^8)
- (10*c*d^9)/(3*b^7)) - x*((7*a*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b
- (21*a^2*d^10)/b^9 + (45*c^2*d^8)/b^7))/b + (35*a^3*d^10)/b^10 - (120*c^3*
d^7)/b^7 - (21*a^2*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b^2) + (log(a + b*x)*
(210*a^4*d^10 + 210*b^4*c^4*d^6 - 840*a*b^3*c^3*d^7 + 1260*a^2*b^2*c^2*d^8
- 840*a^3*b*c*d^9))/b^11 + (d^10*x^4)/(4*b^7)

3.1319 $\int \frac{(c+dx)^{10}}{(a+bx)^8} dx$

Optimal. Leaf size=258

$$\frac{45d^8(bc-ad)^2x}{b^{10}}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{126d^5(bc}{b^{11}(a+}$$

```
[Out] 45*d^8*(-a*d+b*c)^2*x/b^10-1/7*(-a*d+b*c)^10/b^11/(b*x+a)^7-5/3*d*(-a*d+b*c
)^9/b^11/(b*x+a)^6-9*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^5-30*d^3*(-a*d+b*c)^7/b^
11/(b*x+a)^4-70*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^3-126*d^5*(-a*d+b*c)^5/b^11/(
b*x+a)^2-210*d^6*(-a*d+b*c)^4/b^11/(b*x+a)+5*d^9*(-a*d+b*c)*(b*x+a)^2/b^11+
1/3*d^10*(b*x+a)^3/b^11+120*d^7*(-a*d+b*c)^3*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.25, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{120d^7(bc-ad)^3\log(a+bx)}{b^{11}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}+\frac{d^{10}(a+bx)^3}{3b^{11}}+\frac{45d^8x(bc-ad)^2}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (45*d^8*(b*c - a*d)^2*x)/b^10 - (b*c - a*d)^10/(7*b^11*(a + b*x)^7) - (5*d*
(b*c - a*d)^9)/(3*b^11*(a + b*x)^6) - (9*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
^5) - (30*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^4) - (70*d^4*(b*c - a*d)^6)/(b
^11*(a + b*x)^3) - (126*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^2) - (210*d^6*(b
*c - a*d)^4)/(b^11*(a + b*x)) + (5*d^9*(b*c - a*d)*(a + b*x)^2)/b^11 + (d^1
0*(a + b*x)^3)/(3*b^11) + (120*d^7*(b*c - a*d)^3*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^8}\,dx &= \int \left(\frac{45d^8(bc-ad)^2}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^8}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^7}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^6}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^5}\right.\\ &= \frac{45d^8(bc-ad)^2x}{b^{10}}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\end{aligned}$$

Mathematica [A]
time = 0.17, size = 239, normalized size = 0.93

$$\frac{21bd^8\left(45b^2c^2-80abcd+36a^2d^2\right)x+21b^2d^9(5bc-4ad)x^2+7b^3d^{10}x^3-\frac{3(bc-ad)^{10}}{(a+bx)^7}+\frac{35d(-bc+ad)^9}{(a+bx)^6}-\frac{189d^2(bc-ad)^8}{(a+bx)^5}+\frac{630d^3(-bc+ad)^7}{(a+bx)^4}-\frac{1470d^4(bc-ad)^6}{(a+bx)^3}+\frac{2646d^5(-bc+ad)^5}{(a+bx)^2}-\frac{4410d^6(bc-ad)^4}{a+bx}+2520d^7(bc-ad)^3\log(a+bx)}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (21*b*d^8*(45*b^2*c^2 - 80*a*b*c*d + 36*a^2*d^2)*x + 21*b^2*d^9*(5*b*c - 4*
a*d)*x^2 + 7*b^3*d^10*x^3 - (3*(b*c - a*d)^10)/(a + b*x)^7 + (35*d*(-(b*c)
+ a*d)^9)/(a + b*x)^6 - (189*d^2*(b*c - a*d)^8)/(a + b*x)^5 + (630*d^3*(-(b
*c) + a*d)^7)/(a + b*x)^4 - (1470*d^4*(b*c - a*d)^6)/(a + b*x)^3 + (2646*d^
5*(-(b*c) + a*d)^5)/(a + b*x)^2 - (4410*d^6*(b*c - a*d)^4)/(a + b*x) + 2520
*d^7*(b*c - a*d)^3*Log[a + b*x])/(21*b^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(252) = 504$.
time = 0.14, size = 856, normalized size = 3.32 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] d^8/b^10*(1/3*d^2*x^3*b^2-4*a*b*d^2*x^2+5*b^2*c*d*x^2+36*a^2*d^2*x-80*a*b*c
*d*x+45*b^2*c^2*x)-210/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*
a*b^3*c^3*d+b^4*c^4)/(b*x+a)+30/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*
c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^
6*d-b^7*c^7)/(b*x+a)^4-1/7*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120
*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^
4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*
x+a)^7-9/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*
d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+
b^8*c^8)/(b*x+a)^5+126/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-1
0*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^2+5/3/b^11*d*(a^9*d^9-9*a^
8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4
*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(
b*x+a)^6-120/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*x+
a)-70/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3
+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(252) = 504$.
time = 0.41, size = 934, normalized size = 3.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3
 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^
7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 2761*a^10*d^10
+ 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^
9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 - 30*a^2*b^8*c
^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10)*x^5 + 1470
*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3*b^7*c^3*d^7
 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x^4 + 210*(3*
b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 -
 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^9 + 319*a^7*
b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 +
 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^7 + 3654*a^6
*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(5*b^10*c^9*d
 + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*
c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^7*b^3*c^2*d^
8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/(b^18*x^7 + 7*a*b^17*x^6 + 21*
a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*x^2 + 7*a^6*
b^12*x + a^7*b^11) + 1/3*(b^2*d^10*x^3 + 3*(5*b^2*c*d^9 - 4*a*b*d^10)*x^2 +
 3*(45*b^2*c^2*d^8 - 80*a*b*c*d^9 + 36*a^2*d^10)*x)/b^10 + 120*(b^3*c^3*d^7
 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(252) = 504$.
time = 0.86, size = 1362, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*d^10*x^10 - 3*b^10*c^10 - 5*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
18*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 126*a^5*b^5*c^5*d^5 - 630*a^6*b^4
*c^4*d^6 + 6534*a^7*b^3*c^3*d^7 - 12987*a^8*b^2*c^2*d^8 + 10047*a^9*b*c*d^9
 - 2761*a^10*d^10 + 35*(3*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(3*b^10*c^2*d^
8 - 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 49*(135*a*b^9*c^2*d^8 - 195*a^2*b^8
*c*d^9 + 77*a^3*b^7*d^10)*x^7 - 49*(90*b^10*c^4*d^6 - 360*a*b^9*c^3*d^7 + 1
35*a^2*b^8*c^2*d^8 + 285*a^3*b^7*c*d^9 - 179*a^4*b^6*d^10)*x^6 - 147*(18*b^
10*c^5*d^5 + 90*a*b^9*c^4*d^6 - 540*a^2*b^8*c^3*d^7 + 675*a^3*b^7*c^2*d^8 -
 255*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 - 245*(6*b^10*c^6*d^4 + 18*a*b^9*c^5
*d^5 + 90*a^2*b^8*c^4*d^6 - 660*a^3*b^7*c^3*d^7 + 1035*a^4*b^6*c^2*d^8 - 61
5*a^5*b^5*c*d^9 + 121*a^6*b^4*d^10)*x^4 - 35*(18*b^10*c^7*d^3 + 42*a*b^9*c^
6*d^4 + 126*a^2*b^8*c^5*d^5 + 630*a^3*b^7*c^4*d^6 - 5250*a^4*b^6*c^3*d^7 +
9135*a^5*b^5*c^2*d^8 - 6195*a^6*b^4*c*d^9 + 1477*a^7*b^3*d^10)*x^3 - 21*(9*
b^10*c^8*d^2 + 18*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
```

```
+ 630*a^4*b^6*c^4*d^6 - 5754*a^5*b^5*c^3*d^7 + 10647*a^6*b^4*c^2*d^8 - 7707
*a^7*b^3*c*d^9 + 1981*a^8*b^2*d^10)*x^2 - 7*(5*b^10*c^9*d + 9*a*b^9*c^8*d^2
 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 630*a^5*
b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 11907*a^7*b^3*c^2*d^8 - 8967*a^8*b^2*c
*d^9 + 2401*a^9*b*d^10)*x + 2520*(a^7*b^3*c^3*d^7 - 3*a^8*b^2*c^2*d^8 + 3*a
^9*b*c*d^9 - a^10*d^10 + (b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9
- a^3*b^7*d^10)*x^7 + 7*(a*b^9*c^3*d^7 - 3*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^
9 - a^4*b^6*d^10)*x^6 + 21*(a^2*b^8*c^3*d^7 - 3*a^3*b^7*c^2*d^8 + 3*a^4*b^6
*c*d^9 - a^5*b^5*d^10)*x^5 + 35*(a^3*b^7*c^3*d^7 - 3*a^4*b^6*c^2*d^8 + 3*a^
5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 35*(a^4*b^6*c^3*d^7 - 3*a^5*b^5*c^2*d^8 +
 3*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 21*(a^5*b^5*c^3*d^7 - 3*a^6*b^4*c^2*
d^8 + 3*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 7*(a^6*b^4*c^3*d^7 - 3*a^7*b^3*
c^2*d^8 + 3*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^18*x^7 + 7*a*b^
17*x^6 + 21*a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*
x^2 + 7*a^6*b^12*x + a^7*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**8,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(252) = 504.
time = 0.83, size = 872, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="giac")
[Out] 120*(b^3*c^3*d^7 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(abs(b*x
+ a))/b^11 - 1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3
*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d
^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 276
1*a^10*d^10 + 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 -
30*a^2*b^8*c^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10
)*x^5 + 1470*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3
*b^7*c^3*d^7 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x
^4 + 210*(3*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b
^7*c^4*d^6 - 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^
```

```
9 + 319*a^7*b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^
7 + 3654*a^6*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(
5*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^
7*b^3*c^2*d^8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/((b*x + a)^7*b^11)
 + 1/3*(b^16*d^10*x^3 + 15*b^16*c*d^9*x^2 - 12*a*b^15*d^10*x^2 + 135*b^16*c
^2*d^8*x - 240*a*b^15*c*d^9*x + 108*a^2*b^14*d^10*x)/b^24
```

Mupad [B]
time = 0.43, size = 950, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^8,x)
```

```
[Out] x*((8*a*((8*a*d^10)/b^9 - (10*c*d^9)/b^8))/b - (28*a^2*d^10)/b^10 + (45*c^2
*d^8)/b^8) - (x^4*(2590*a^6*b^3*d^10 + 70*b^9*c^6*d^4 + 210*a*b^8*c^5*d^5 -
 9870*a^5*b^4*c*d^9 + 1050*a^2*b^7*c^4*d^6 - 7700*a^3*b^6*c^3*d^7 + 13650*a
^4*b^5*c^2*d^8) + x^6*(210*a^4*b^5*d^10 + 210*b^9*c^4*d^6 - 840*a*b^8*c^3*d
^7 - 840*a^3*b^6*c*d^9 + 1260*a^2*b^7*c^2*d^8) + (2761*a^10*d^10 + 3*b^10*c
^10 + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5
*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c
^2*d^8 + 5*a*b^9*c^9*d - 10047*a^9*b*c*d^9)/(21*b) + x*((2509*a^9*d^10)/3 +
 (5*b^9*c^9*d)/3 + 3*a*b^8*c^8*d^2 + 6*a^2*b^7*c^7*d^3 + 14*a^3*b^6*c^6*d^4
 + 42*a^4*b^5*c^5*d^5 + 210*a^5*b^4*c^4*d^6 - 2058*a^6*b^3*c^3*d^7 + 4014*a
^7*b^2*c^2*d^8 - 3069*a^8*b*c*d^9) + x^3*(3190*a^7*b^2*d^10 + 30*b^9*c^7*d^
3 + 70*a*b^8*c^6*d^4 - 11970*a^6*b^3*c*d^9 + 210*a^2*b^7*c^5*d^5 + 1050*a^3
*b^6*c^4*d^6 - 8750*a^4*b^5*c^3*d^7 + 16170*a^5*b^4*c^2*d^8) + x^2*(2229*a^
8*b*d^10 + 9*b^9*c^8*d^2 + 18*a*b^8*c^7*d^3 - 8262*a^7*b^2*c*d^9 + 42*a^2*b
^7*c^6*d^4 + 126*a^3*b^6*c^5*d^5 + 630*a^4*b^5*c^4*d^6 - 5754*a^5*b^4*c^3*d
^7 + 10962*a^6*b^3*c^2*d^8) + x^5*(1134*a^5*b^4*d^10 + 126*b^9*c^5*d^5 + 63
0*a*b^8*c^4*d^6 - 4410*a^4*b^5*c*d^9 - 3780*a^2*b^7*c^3*d^7 + 6300*a^3*b^6*
c^2*d^8))/(a^7*b^10 + b^17*x^7 + 7*a^6*b^11*x + 7*a*b^16*x^6 + 21*a^5*b^12*
x^2 + 35*a^4*b^13*x^3 + 35*a^3*b^14*x^4 + 21*a^2*b^15*x^5) - x^2*((4*a*d^10
)/b^9 - (5*c*d^9)/b^8) - (log(a + b*x)*(120*a^3*d^10 - 120*b^3*c^3*d^7 + 36
0*a*b^2*c^2*d^8 - 360*a^2*b*c*d^9))/b^11 + (d^10*x^3)/(3*b^8)
```

3.1320 $\int \frac{(c+dx)^{10}}{(a+bx)^9} \, dx$

Optimal. Leaf size=258

$$\frac{d^9(10bc-9ad)x}{b^{10}}+\frac{d^{10}x^2}{2b^9}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}-\frac{105d^4(bc-ad)}{2b^{11}(a+bx)^4}$$

```
[Out] d^9*(-9*a*d+10*b*c)*x/b^10+1/2*d^10*x^2/b^9-1/8*(-a*d+b*c)^10/b^11/(b*x+a)^
8-10/7*d*(-a*d+b*c)^9/b^11/(b*x+a)^7-15/2*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^6-2
4*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^5-105/2*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^4-84*
d^5*(-a*d+b*c)^5/b^11/(b*x+a)^3-105*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^2-120*d^7
*(-a*d+b*c)^3/b^11/(b*x+a)+45*d^8*(-a*d+b*c)^2*ln(b*x+a)/b^11
```

Rubi [A]
time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{45d^8(bc-ad)^2\log(a+bx)}{b^{11}}-\frac{120d^7(bc-ad)^3}{b^{11}(a+bx)}-\frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2}-\frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}+\frac{d^9x(10bc-9ad)}{b^{10}}+\frac{d^{10}x^2}{2b^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (d^9*(10*b*c - 9*a*d)*x)/b^10 + (d^10*x^2)/(2*b^9) - (b*c - a*d)^10/(8*b^11
*(a + b*x)^8) - (10*d*(b*c - a*d)^9)/(7*b^11*(a + b*x)^7) - (15*d^2*(b*c -
a*d)^8)/(2*b^11*(a + b*x)^6) - (24*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^5) -
(105*d^4*(b*c - a*d)^6)/(2*b^11*(a + b*x)^4) - (84*d^5*(b*c - a*d)^5)/(b^11
*(a + b*x)^3) - (105*d^6*(b*c - a*d)^4)/(b^11*(a + b*x)^2) - (120*d^7*(b*c
- a*d)^3)/(b^11*(a + b*x)) + (45*d^8*(b*c - a*d)^2*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx &= \int \left(\frac{d^9(10bc-9ad)}{b^{10}}+\frac{d^{10}x}{b^9}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^9}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^8}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^7}+\frac{120d^3(}{b^{10}(}\right.\\ &= \frac{d^9(10bc-9ad)x}{b^{10}}+\frac{d^{10}x^2}{2b^9}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{24d^3(bc}{b^{11}(a+}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 712 vs. 2(258) = 516.
time = 0.21, size = 712, normalized size = 2.76

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (3601*a^10*d^10 + 2*a^9*b*d^9*(-4609*c + 13144*d*x) + a^8*b^2*d^8*(6849*c^2
 - 68704*c*d*x + 81928*d^2*x^2) + 8*a^7*b^3*d^7*(-105*c^3 + 6534*c^2*d*x -
27538*c*d^2*x^2 + 17542*d^3*x^3) + 14*a^6*b^4*d^6*(-15*c^4 - 480*c^3*d*x +
12348*c^2*d^2*x^2 - 28112*c*d^3*x^3 + 10010*d^4*x^4) - 28*a^5*b^5*d^5*(3*c^
5 + 60*c^4*d*x + 840*c^3*d^2*x^2 - 11508*c^2*d^3*x^3 + 15050*c*d^4*x^4 - 27
44*d^5*x^5) - 14*a^4*b^6*d^4*(3*c^6 + 48*c^5*d*x + 420*c^4*d^2*x^2 + 3360*c
^3*d^3*x^3 - 26250*c^2*d^4*x^4 + 19040*c*d^5*x^5 - 1064*d^6*x^6) - 8*a^3*b^
7*d^3*(3*c^7 + 42*c^6*d*x + 294*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 7350*c^3*d
^4*x^4 - 32340*c^2*d^5*x^5 + 10780*c*d^6*x^6 + 728*d^7*x^7) - a^2*b^8*d^2*(
15*c^8 + 192*c^7*d*x + 1176*c^6*d^2*x^2 + 4704*c^5*d^3*x^3 + 14700*c^4*d^4*
x^4 + 47040*c^3*d^5*x^5 - 105840*c^2*d^6*x^6 + 4480*c*d^7*x^7 + 3248*d^8*x^
8) - 2*a*b^9*d*(5*c^9 + 60*c^8*d*x + 336*c^7*d^2*x^2 + 1176*c^6*d^3*x^3 + 2
940*c^5*d^4*x^4 + 5880*c^4*d^5*x^5 + 11760*c^3*d^6*x^6 - 10080*c^2*d^7*x^7
- 2240*c*d^8*x^8 + 140*d^9*x^9) - b^10*(7*c^10 + 80*c^9*d*x + 420*c^8*d^2*x
^2 + 1344*c^7*d^3*x^3 + 2940*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 + 5880*c^4*d^6*
x^6 + 6720*c^3*d^7*x^7 - 560*c*d^9*x^9 - 28*d^10*x^10) + 2520*d^8*(b*c - a*
d)^2*(a + b*x)^8*Log[a + b*x])/(56*b^11*(a + b*x)^8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(248) = 496.
time = 0.17, size = 854, normalized size = 3.31 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

```
[Out] -d^9/b^10*(-1/2*b*d*x^2+9*a*d*x-10*b*c*x)+120/b^11*d^7*(a^3*d^3-3*a^2*b*c*d
^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)-105/2/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*
a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6
)/(b*x+a)^4+10/7/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^
3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2
*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^7-1/8*(a^10*d^10-10*a^9*b*c*d^9
+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5
*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^
9*d+b^10*c^10)/b^11/(b*x+a)^8+24/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2
*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c
^6*d-b^7*c^7)/(b*x+a)^5-105/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^2-15/2/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28
```

```
*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+2
8*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^6+45/b^11*d^8*(a^2*d^2-2*a
*b*c*d+b^2*c^2)*ln(b*x+a)+84/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2
*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(248) = 496$.
time = 0.42, size = 945, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/56*(7*b^10*c^10 + 10*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d
^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^
7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 +
6720*(b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7
+ 5880*(b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*
d^9 - 7*a^4*b^6*d^10)*x^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2
*b^8*c^3*d^7 - 110*a^3*b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x
^5 + 2940*(b^10*c^6*d^4 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*
c^3*d^7 - 125*a^4*b^6*c^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 +
336*(4*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4
*d^6 + 140*a^4*b^6*c^3*d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459
*a^7*b^3*d^10)*x^3 + 84*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*
d^4 + 28*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*
a^6*b^4*c^2*d^8 + 2676*a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*
c^9*d + 15*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4
*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2
*d^8 + 8658*a^8*b^2*c*d^9 - 3349*a^9*b*d^10)*x)/(b^19*x^8 + 8*a*b^18*x^7 +
28*a^2*b^17*x^6 + 56*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*
a^6*b^13*x^2 + 8*a^7*b^12*x + a^8*b^11) + 1/2*(b*d^10*x^2 + 2*(10*b*c*d^9 -
 9*a*d^10)*x)/b^10 + 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(b*x + a)
/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1296 vs. $2(248) = 496$.
time = 0.70, size = 1296, normalized size = 5.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*d^10*x^10 - 7*b^10*c^10 - 10*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2
 - 24*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 84*a^5*b^5*c^5*d^5 - 210*a^6*b
^4*c^4*d^6 - 840*a^7*b^3*c^3*d^7 + 6849*a^8*b^2*c^2*d^8 - 9218*a^9*b*c*d^9
+ 3601*a^10*d^10 + 280*(2*b^10*c*d^9 - a*b^9*d^10)*x^9 + 112*(40*a*b^9*c*d^
9 - 29*a^2*b^8*d^10)*x^8 - 448*(15*b^10*c^3*d^7 - 45*a*b^9*c^2*d^8 + 10*a^2
*b^8*c*d^9 + 13*a^3*b^7*d^10)*x^7 - 392*(15*b^10*c^4*d^6 + 60*a*b^9*c^3*d^7
 - 270*a^2*b^8*c^2*d^8 + 220*a^3*b^7*c*d^9 - 38*a^4*b^6*d^10)*x^6 - 784*(6*
b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 60*a^2*b^8*c^3*d^7 - 330*a^3*b^7*c^2*d^8
+ 340*a^4*b^6*c*d^9 - 98*a^5*b^5*d^10)*x^5 - 980*(3*b^10*c^6*d^4 + 6*a*b^9*
c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 60*a^3*b^7*c^3*d^7 - 375*a^4*b^6*c^2*d^8 + 4
30*a^5*b^5*c*d^9 - 143*a^6*b^4*d^10)*x^4 - 112*(12*b^10*c^7*d^3 + 21*a*b^9*
c^6*d^4 + 42*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 -
2877*a^5*b^5*c^2*d^8 + 3514*a^6*b^4*c*d^9 - 1253*a^7*b^3*d^10)*x^3 - 28*(15
*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 84*a^3*b^7*c^5*d^5
+ 210*a^4*b^6*c^4*d^6 + 840*a^5*b^5*c^3*d^7 - 6174*a^6*b^4*c^2*d^8 + 7868*a
^7*b^3*c*d^9 - 2926*a^8*b^2*d^10)*x^2 - 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^2
 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*b
^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8588*a^8*b^2*c*d^
9 - 3286*a^9*b*d^10)*x + 2520*(a^8*b^2*c^2*d^8 - 2*a^9*b*c*d^9 + a^10*d^10
+ (b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 8*(a*b^9*c^2*d^8 - 2*
a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 28*(a^2*b^8*c^2*d^8 - 2*a^3*b^7*c*d^9 +
 a^4*b^6*d^10)*x^6 + 56*(a^3*b^7*c^2*d^8 - 2*a^4*b^6*c*d^9 + a^5*b^5*d^10)*
x^5 + 70*(a^4*b^6*c^2*d^8 - 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 56*(a^5*b
^5*c^2*d^8 - 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 28*(a^6*b^4*c^2*d^8 - 2*
a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 8*(a^7*b^3*c^2*d^8 - 2*a^8*b^2*c*d^9 +
a^9*b*d^10)*x)*log(b*x + a))/(b^19*x^8 + 8*a*b^18*x^7 + 28*a^2*b^17*x^6 + 5
6*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*a^6*b^13*x^2 + 8*a^
7*b^12*x + a^8*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**9,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(248) = 496.
time = 1.03, size = 871, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(abs(b*x + a))/b^11 + 1/2*(b^9
*d^10*x^2 + 20*b^9*c*d^9*x - 18*a*b^8*d^10*x)/b^18 - 1/56*(7*b^10*c^10 + 10
*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4
 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^
8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 + 6720*(b^10*c^3*d^7 - 3*
a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 5880*(b^10*c^4*d^6 +
4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*d^9 - 7*a^4*b^6*d^10)*x
^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 110*a^3*
b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x^5 + 2940*(b^10*c^6*d^4
 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 - 125*a^4*b^6*c
^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 + 336*(4*b^10*c^7*d^3 + 7
*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*
d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459*a^7*b^3*d^10)*x^3 + 84
*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 + 28*a^3*b^7*c^5*d^
5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*a^6*b^4*c^2*d^8 + 2676*
a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^
2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*
b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8658*a^8*b^2*c*d
^9 - 3349*a^9*b*d^10)*x)/((b*x + a)^8*b^11)
```

Mupad [B]
time = 0.26, size = 946, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^9,x)
```

```
[Out] (log(a + b*x)*(45*a^2*d^10 + 45*b^2*c^2*d^8 - 90*a*b*c*d^9))/b^11 - (x^4*((
105*b^9*c^6*d^4)/2 - (5985*a^6*b^3*d^10)/2 + 105*a*b^8*c^5*d^5 + 8085*a^5*b
^4*c*d^9 + (525*a^2*b^7*c^4*d^6)/2 + 1050*a^3*b^6*c^3*d^7 - (13125*a^4*b^5*
c^2*d^8)/2) + x^6*(105*b^9*c^4*d^6 - 735*a^4*b^5*d^10 + 420*a*b^8*c^3*d^7 +
 2100*a^3*b^6*c*d^9 - 1890*a^2*b^7*c^2*d^8) + (7*b^10*c^10 - 3601*a^10*d^10
 + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^
5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^
8 + 10*a*b^9*c^9*d + 9218*a^9*b*c*d^9)/(56*b) + x*((10*b^9*c^9*d)/7 - (3349
*a^9*d^10)/7 + (15*a*b^8*c^8*d^2)/7 + (24*a^2*b^7*c^7*d^3)/7 + 6*a^3*b^6*c^
6*d^4 + 12*a^4*b^5*c^5*d^5 + 30*a^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - (65
34*a^7*b^2*c^2*d^8)/7 + (8658*a^8*b*c*d^9)/7) + x^3*(24*b^9*c^7*d^3 - 2754*
a^7*b^2*d^10 + 42*a*b^8*c^6*d^4 + 7308*a^6*b^3*c*d^9 + 84*a^2*b^7*c^5*d^5 +
 210*a^3*b^6*c^4*d^6 + 840*a^4*b^5*c^3*d^7 - 5754*a^5*b^4*c^2*d^8) + x^2*((
15*b^9*c^8*d^2)/2 - (3069*a^8*b*d^10)/2 + 12*a*b^8*c^7*d^3 + 4014*a^7*b^2*c
*d^9 + 21*a^2*b^7*c^6*d^4 + 42*a^3*b^6*c^5*d^5 + 105*a^4*b^5*c^4*d^6 + 420*
a^5*b^4*c^3*d^7 - 3087*a^6*b^3*c^2*d^8) + x^5*(84*b^9*c^5*d^5 - 1974*a^5*b^
```

```
4*d^10 + 210*a*b^8*c^4*d^6 + 5460*a^4*b^5*c*d^9 + 840*a^2*b^7*c^3*d^7 - 462
0*a^3*b^6*c^2*d^8) - x^7*(120*a^3*b^6*d^10 - 120*b^9*c^3*d^7 + 360*a*b^8*c^
2*d^8 - 360*a^2*b^7*c*d^9))/(a^8*b^10 + b^18*x^8 + 8*a^7*b^11*x + 8*a*b^17*
x^7 + 28*a^6*b^12*x^2 + 56*a^5*b^13*x^3 + 70*a^4*b^14*x^4 + 56*a^3*b^15*x^5
 + 28*a^2*b^16*x^6) - x*((9*a*d^10)/b^10 - (10*c*d^9)/b^9) + (d^10*x^2)/(2*
b^9)
```

3.1321 $\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=257

$$\frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-7$$

```
[Out] d^10*x/b^10-1/9*(-a*d+b*c)^10/b^11/(b*x+a)^9-5/4*d*(-a*d+b*c)^9/b^11/(b*x+a
)^8-45/7*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^7-20*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^6
-42*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^5-63*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^4-70*d
^6*(-a*d+b*c)^4/b^11/(b*x+a)^3-60*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^2-45*d^8*(-
a*d+b*c)^2/b^11/(b*x+a)+10*d^9*(-a*d+b*c)*ln(b*x+a)/b^11
```

Rubi [A]

time = 0.22, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{10d^9(bc-ad)\log(a+bx)}{b^{11}}-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}+\frac{d^{10}x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] (d^10*x)/b^10 - (b*c - a*d)^10/(9*b^11*(a + b*x)^9) - (5*d*(b*c - a*d)^9)/(
4*b^11*(a + b*x)^8) - (45*d^2*(b*c - a*d)^8)/(7*b^11*(a + b*x)^7) - (20*d^3
*(b*c - a*d)^7)/(b^11*(a + b*x)^6) - (42*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)
^5) - (63*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^4) - (70*d^6*(b*c - a*d)^4)/(b
^11*(a + b*x)^3) - (60*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^2) - (45*d^8*(b*c
 - a*d)^2)/(b^11*(a + b*x)) + (10*d^9*(b*c - a*d)*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx = \int \left(\frac{d^{10}}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{10}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^9}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^8}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^7}+\frac{210d^4}{b^{10}(}\right.$$

$$=\frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-}{b^{11}(a+b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 708 vs. 2(257) = 514.
time = 0.27, size = 708, normalized size = 2.75

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10*d^10 + a^9*b*d^9*(-7129*c + 41229*d*x) + 9*a^8*b^2*d^8*(1
40*c^2 - 6849*c*d*x + 17064*d^2*x^2) + 12*a^7*b^3*d^7*(35*c^3 + 945*c^2*d*x
 - 19602*c*d^2*x^2 + 27342*d^3*x^3) + 42*a^6*b^4*d^6*(5*c^4 + 90*c^3*d*x +
1080*c^2*d^2*x^2 - 12348*c*d^3*x^3 + 10458*d^4*x^4) + 126*a^5*b^5*d^5*(c^5
+ 15*c^4*d*x + 120*c^3*d^2*x^2 + 840*c^2*d^3*x^3 - 5754*c*d^4*x^4 + 2982*d^
5*x^5) + 42*a^4*b^6*d^4*(2*c^6 + 27*c^5*d*x + 180*c^4*d^2*x^2 + 840*c^3*d^3
*x^3 + 3780*c^2*d^4*x^4 - 15750*c*d^5*x^5 + 4704*d^6*x^6) + 12*a^3*b^7*d^3*
(5*c^7 + 63*c^6*d*x + 378*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 4410*c^3*d^4*x^4
 + 13230*c^2*d^5*x^5 - 32340*c*d^6*x^6 + 4536*d^7*x^7) + 9*a^2*b^8*d^2*(5*c
^8 + 60*c^7*d*x + 336*c^6*d^2*x^2 + 1176*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 5
880*c^3*d^5*x^5 + 11760*c^2*d^6*x^6 - 15120*c*d^7*x^7 + 252*d^8*x^8) + a*b^
9*d*(35*c^9 + 405*c^8*d*x + 2160*c^7*d^2*x^2 + 7056*c^6*d^3*x^3 + 15876*c^5
*d^4*x^4 + 26460*c^4*d^5*x^5 + 35280*c^3*d^6*x^6 + 45360*c^2*d^7*x^7 - 2268
0*c*d^8*x^8 - 2268*d^9*x^9) + b^10*(28*c^10 + 315*c^9*d*x + 1620*c^8*d^2*x^
2 + 5040*c^7*d^3*x^3 + 10584*c^6*d^4*x^4 + 15876*c^5*d^5*x^5 + 17640*c^4*d^
6*x^6 + 15120*c^3*d^7*x^7 + 11340*c^2*d^8*x^8 - 252*d^10*x^10) + 2520*d^9*(
-(b*c) + a*d)*(a + b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 858 vs. 2(251) = 502.
time = 0.17, size = 859, normalized size = 3.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] d^10*x/b^10-45/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)+63/b^11*d^5*(a^
5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5
*c^5)/(b*x+a)^4-1/9*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^
3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a
^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^9-
45/7/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+
70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*
c^8)/(b*x+a)^7+5/4/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*
b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a
^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^8-42/b^11*d^4*(a^6*d^6-6*a^5*
b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^
5*d+b^6*c^6)/(b*x+a)^5+60/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
```

```
*c^3)/(b*x+a)^2+20/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^
4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/
(b*x+a)^6-10/b^11*d^9*(a*d-b*c)*ln(b*x+a)-70/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^
3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(251) = 502$.
time = 0.40, size = 957, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] d^10*x/b^10 - 1/252*(28*b^10*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 6
0*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*
c^4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4
861*a^10*d^10 + 11340*(b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1
5120*(b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^
7 + 17640*(b^10*c^4*d^6 + 2*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*
c*d^9 + 13*a^4*b^6*d^10)*x^6 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*
a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)
*x^5 + 5292*(2*b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*
b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4
 + 504*(10*b^10*c^7*d^3 + 14*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^
7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 +
 669*a^7*b^3*d^10)*x^3 + 108*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7
+ 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35
*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*
b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4609*a^9*b*d^10)*x)/(b^20*x^9 + 9*a*b^19
*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*
x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + 10*(b*
c*d^9 - a*d^10)*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(251) = 502$.
time = 0.93, size = 1216, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*d^10*x^10 + 2268*a*b^9*d^10*x^9 - 28*b^10*c^10 - 35*a*b^9*c
^9*d - 45*a^2*b^8*c^8*d^2 - 60*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 126*a
^5*b^5*c^5*d^5 - 210*a^6*b^4*c^4*d^6 - 420*a^7*b^3*c^3*d^7 - 1260*a^8*b^2*c
^2*d^8 + 7129*a^9*b*c*d^9 - 4861*a^10*d^10 - 2268*(5*b^10*c^2*d^8 - 10*a*b^
9*c*d^9 + a^2*b^8*d^10)*x^8 - 3024*(5*b^10*c^3*d^7 + 15*a*b^9*c^2*d^8 - 45*
a^2*b^8*c*d^9 + 18*a^3*b^7*d^10)*x^7 - 3528*(5*b^10*c^4*d^6 + 10*a*b^9*c^3*
d^7 + 30*a^2*b^8*c^2*d^8 - 110*a^3*b^7*c*d^9 + 56*a^4*b^6*d^10)*x^6 - 5292*
(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8
 - 125*a^4*b^6*c*d^9 + 71*a^5*b^5*d^10)*x^5 - 5292*(2*b^10*c^6*d^4 + 3*a*b^
9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 1
37*a^5*b^5*c*d^9 + 83*a^6*b^4*d^10)*x^4 - 504*(10*b^10*c^7*d^3 + 14*a*b^9*c
^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210
*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 651*a^7*b^3*d^10)*x^3 - 108*(15*b^1
0*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70
*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3
*c*d^9 + 1422*a^8*b^2*d^10)*x^2 - 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^
4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4
581*a^9*b*d^10)*x + 2520*(a^9*b*c*d^9 - a^10*d^10 + (b^10*c*d^9 - a*b^9*d^1
0)*x^9 + 9*(a*b^9*c*d^9 - a^2*b^8*d^10)*x^8 + 36*(a^2*b^8*c*d^9 - a^3*b^7*d
^10)*x^7 + 84*(a^3*b^7*c*d^9 - a^4*b^6*d^10)*x^6 + 126*(a^4*b^6*c*d^9 - a^5
*b^5*d^10)*x^5 + 126*(a^5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 84*(a^6*b^4*c*d^9
 - a^7*b^3*d^10)*x^3 + 36*(a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 9*(a^8*b^2*c
*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*
x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x
^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**10,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(251) = 502.
time = 1.16, size = 867, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] d^10*x/b^10 + 10*(b*c*d^9 - a*d^10)*log(abs(b*x + a))/b^11 - 1/252*(28*b^10
*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 84*a^4*b
^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 420*a^7*b^3*c^3*d^
7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4861*a^10*d^10 + 11340*(b^10*
c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 15120*(b^10*c^3*d^7 + 3*a*b^9
*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^7 + 17640*(b^10*c^4*d^6 + 2*
a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*c*d^9 + 13*a^4*b^6*d^10)*x^6
 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7
*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)*x^5 + 5292*(2*b^10*c^6*d^4
+ 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2
*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4 + 504*(10*b^10*c^7*d^3 + 14
*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d
^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 669*a^7*b^3*d^10)*x^3 + 108
*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178
*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d
^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^
5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c
*d^9 + 4609*a^9*b*d^10)*x)/((b*x + a)^9*b^11)
```

Mupad [B]
time = 0.50, size = 955, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^10,x)
```

```
[Out] (d^10*x)/b^10 - (log(a + b*x)*(10*a*d^10 - 10*b*c*d^9))/b^11 - (x^4*(1827*a
^6*b^3*d^10 + 42*b^9*c^6*d^4 + 63*a*b^8*c^5*d^5 - 2877*a^5*b^4*c*d^9 + 105*
a^2*b^7*c^4*d^6 + 210*a^3*b^6*c^3*d^7 + 630*a^4*b^5*c^2*d^8) + x^6*(910*a^4
*b^5*d^10 + 70*b^9*c^4*d^6 + 140*a*b^8*c^3*d^7 - 1540*a^3*b^6*c*d^9 + 420*a
^2*b^7*c^2*d^8) + (4861*a^10*d^10 + 28*b^10*c^10 + 45*a^2*b^8*c^8*d^2 + 60*
a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^
4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 35*a*b^9*c^9*d - 7129*
a^9*b*c*d^9)/(252*b) + x*((4609*a^9*d^10)/28 + (5*b^9*c^9*d)/4 + (45*a*b^8*
c^8*d^2)/28 + (15*a^2*b^7*c^7*d^3)/7 + 3*a^3*b^6*c^6*d^4 + (9*a^4*b^5*c^5*d
^5)/2 + (15*a^5*b^4*c^4*d^6)/2 + 15*a^6*b^3*c^3*d^7 + 45*a^7*b^2*c^2*d^8 -
(6849*a^8*b*c*d^9)/28) + x^8*(45*a^2*b^7*d^10 + 45*b^9*c^2*d^8 - 90*a*b^8*c
*d^9) + x^3*(1338*a^7*b^2*d^10 + 20*b^9*c^7*d^3 + 28*a*b^8*c^6*d^4 - 2058*a
^6*b^3*c*d^9 + 42*a^2*b^7*c^5*d^5 + 70*a^3*b^6*c^4*d^6 + 140*a^4*b^5*c^3*d^
7 + 420*a^5*b^4*c^2*d^8) + x^2*((4329*a^8*b*d^10)/7 + (45*b^9*c^8*d^2)/7 +
(60*a*b^8*c^7*d^3)/7 - (6534*a^7*b^2*c*d^9)/7 + 12*a^2*b^7*c^6*d^4 + 18*a^3
*b^6*c^5*d^5 + 30*a^4*b^5*c^4*d^6 + 60*a^5*b^4*c^3*d^7 + 180*a^6*b^3*c^2*d^
8) + x^5*(1617*a^5*b^4*d^10 + 63*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 - 2625*a^4
```

```
*b^5*c*d^9 + 210*a^2*b^7*c^3*d^7 + 630*a^3*b^6*c^2*d^8) + x^7*(300*a^3*b^6*
d^10 + 60*b^9*c^3*d^7 + 180*a*b^8*c^2*d^8 - 540*a^2*b^7*c*d^9))/(a^9*b^10 +
 b^19*x^9 + 9*a^8*b^11*x + 9*a*b^18*x^8 + 36*a^7*b^12*x^2 + 84*a^6*b^13*x^3
 + 126*a^5*b^14*x^4 + 126*a^4*b^15*x^5 + 84*a^3*b^16*x^6 + 36*a^2*b^17*x^7)
```

3.1322 $\int \frac{(c+dx)^{10}}{(a+bx)^{11}} \, dx$

Optimal. Leaf size=271

$$-\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-1$$

```
[Out] -1/10*(-a*d+b*c)^10/b^11/(b*x+a)^10-10/9*d*(-a*d+b*c)^9/b^11/(b*x+a)^9-45/8
*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^8-120/7*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^7-35*d
^4*(-a*d+b*c)^6/b^11/(b*x+a)^6-252/5*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^5-105/2*
d^6*(-a*d+b*c)^4/b^11/(b*x+a)^4-40*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^3-45/2*d^8
*(-a*d+b*c)^2/b^11/(b*x+a)^2-10*d^9*(-a*d+b*c)/b^11/(b*x+a)+d^10*ln(b*x+a)/
b^11
```

Rubi [A]

time = 0.19, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}+\frac{d^{10}\log(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -1/10*(b*c - a*d)^10/(b^11*(a + b*x)^10) - (10*d*(b*c - a*d)^9)/(9*b^11*(a
+ b*x)^9) - (45*d^2*(b*c - a*d)^8)/(8*b^11*(a + b*x)^8) - (120*d^3*(b*c - a
*d)^7)/(7*b^11*(a + b*x)^7) - (35*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^6) - (
252*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^5) - (105*d^6*(b*c - a*d)^4)/(2*b^
11*(a + b*x)^4) - (40*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^3) - (45*d^8*(b*c
- a*d)^2)/(2*b^11*(a + b*x)^2) - (10*d^9*(b*c - a*d))/(b^11*(a + b*x)) + (d
^10*Log[a + b*x])/b^11
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{11}} \, dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{11}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{10}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^9}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^8}+\frac{210d^4(bc-}{b^{10}(a+bx}\right.\\ &= -\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad}{b^{11}(a+bx)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 591 vs. 2(271) = 542.
time = 0.26, size = 591, normalized size = 2.18

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -1/2520*((b*c - a*d)*(7381*a^9*d^9 + a^8*b*d^8*(4861*c + 71290*d*x) + a^7*b
^2*d^7*(3601*c^2 + 46090*c*d*x + 308205*d^2*x^2) + a^6*b^3*d^6*(2761*c^3 +
33490*c^2*d*x + 194805*c*d^2*x^2 + 784080*d^3*x^3) + a^5*b^4*d^5*(2131*c^4
+ 25090*c^3*d*x + 138105*c^2*d^2*x^2 + 481680*c*d^3*x^3 + 1296540*d^4*x^4)
+ a^4*b^5*d^4*(1627*c^5 + 18790*c^4*d*x + 100305*c^3*d^2*x^2 + 330480*c^2*d
^3*x^3 + 767340*c*d^4*x^4 + 1450008*d^5*x^5) + a^3*b^6*d^3*(1207*c^6 + 1375
0*c^5*d*x + 71955*c^4*d^2*x^2 + 229680*c^3*d^3*x^3 + 502740*c^2*d^4*x^4 + 8
14968*c*d^5*x^5 + 1102500*d^6*x^6) + a^2*b^7*d^2*(847*c^7 + 9550*c^6*d*x +
49275*c^5*d^2*x^2 + 154080*c^4*d^3*x^3 + 326340*c^3*d^4*x^4 + 497448*c^2*d^
5*x^5 + 573300*c*d^6*x^6 + 554400*d^7*x^7) + a*b^8*d*(532*c^8 + 5950*c^7*d*
x + 30375*c^6*d^2*x^2 + 93600*c^5*d^3*x^3 + 194040*c^4*d^4*x^4 + 285768*c^3
*d^5*x^5 + 308700*c^2*d^6*x^6 + 252000*c*d^7*x^7 + 170100*d^8*x^8) + b^9*(2
52*c^9 + 2800*c^8*d*x + 14175*c^7*d^2*x^2 + 43200*c^6*d^3*x^3 + 88200*c^5*d
^4*x^4 + 127008*c^4*d^5*x^5 + 132300*c^3*d^6*x^6 + 100800*c^2*d^7*x^7 + 567
00*c*d^8*x^8 + 25200*d^9*x^9)))/(b^11*(a + b*x)^10) + (d^10*Log[a + b*x])/b
^11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(257) = 514.
time = 0.24, size = 865, normalized size = 3.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^11,x,method=_RETURNVERBOSE)
```

```
[Out] 10/b^11*d^9*(a*d-b*c)/(b*x+a)-105/2/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^4+10/9/b^11*d*(a^9*d^9-9*a^8*b*c*
d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c
^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)
^9+120/7/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*
d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^7-
45/8/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+
70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*
c^8)/(b*x+a)^8-1/10*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^
3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a
^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^10
+252/5/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^
2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^5-45/2/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2
```

```
)/(b*x+a)^2-35/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^
3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^6+d^10*ln(b*x+a
)/b^11+40/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(257) = 514$.
time = 0.34, size = 975, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x)/(b^21*x^10 + 10*a*b^20*x^9 + 45*a^2*b^19
*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b^16*x^5 + 210*a^6*b^1
5*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^12*x + a^10*b^11) + d
^10*log(b*x + a)/b^11
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(257) = 514$.
time = 0.98, size = 1107, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x - 2520*(b^10*d^10*x^10 + 10*a*b^9*d^10*x^
9 + 45*a^2*b^8*d^10*x^8 + 120*a^3*b^7*d^10*x^7 + 210*a^4*b^6*d^10*x^6 + 252
*a^5*b^5*d^10*x^5 + 210*a^6*b^4*d^10*x^4 + 120*a^7*b^3*d^10*x^3 + 45*a^8*b^
2*d^10*x^2 + 10*a^9*b*d^10*x + a^10*d^10)*log(b*x + a))/(b^21*x^10 + 10*a*b
^20*x^9 + 45*a^2*b^19*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b
^16*x^5 + 210*a^6*b^15*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^
12*x + a^10*b^11)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**11,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(257) = 514.
time = 0.90, size = 874, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="giac")
```

[Out] d^10*log(abs(b*x + a))/b^11 - 1/2520*(25200*(b^9*c*d^9 - a*b^8*d^10)*x^9 + 56700*(b^9*c^2*d^8 + 2*a*b^8*c*d^9 - 3*a^2*b^7*d^10)*x^8 + 50400*(2*b^9*c^3*d^7 + 3*a*b^8*c^2*d^8 + 6*a^2*b^7*c*d^9 - 11*a^3*b^6*d^10)*x^7 + 44100*(3*b^9*c^4*d^6 + 4*a*b^8*c^3*d^7 + 6*a^2*b^7*c^2*d^8 + 12*a^3*b^6*c*d^9 - 25*a^4*b^5*d^10)*x^6 + 10584*(12*b^9*c^5*d^5 + 15*a*b^8*c^4*d^6 + 20*a^2*b^7*c^3*d^7 + 30*a^3*b^6*c^2*d^8 + 60*a^4*b^5*c*d^9 - 137*a^5*b^4*d^10)*x^5 + 8820*(10*b^9*c^6*d^4 + 12*a*b^8*c^5*d^5 + 15*a^2*b^7*c^4*d^6 + 20*a^3*b^6*c^3*d^7 + 30*a^4*b^5*c^2*d^8 + 60*a^5*b^4*c*d^9 - 147*a^6*b^3*d^10)*x^4 + 720*(60*b^9*c^7*d^3 + 70*a*b^8*c^6*d^4 + 84*a^2*b^7*c^5*d^5 + 105*a^3*b^6*c^4*d^6 + 140*a^4*b^5*c^3*d^7 + 210*a^5*b^4*c^2*d^8 + 420*a^6*b^3*c*d^9 - 1089*a^7*b^2*d^10)*x^3 + 135*(105*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 140*a^2*b^7*c^6*d^4 + 168*a^3*b^6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 + 280*a^5*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 840*a^7*b^2*c*d^9 - 2283*a^8*b*d^10)*x^2 + 10*(280*b^9*c^9*d + 315*a*b^8*c^8*d^2 + 360*a^2*b^7*c^7*d^3 + 420*a^3*b^6*c^6*d^4 + 504*a^4*b^5*c^5*d^5 + 630*a^5*b^4*c^4*d^6 + 840*a^6*b^3*c^3*d^7 + 1260*a^7*b^2*c^2*d^8 + 2520*a^8*b*c*d^9 - 7129*a^9*d^10)*x + (252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^10*d^10)/b)/((b*x + a)^10*b^10)

Mupad [B]
time = 0.56, size = 866, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^10/(a + b*x)^11,x)

[Out] (d^10*log(a + b*x))/b^11 - (x^4*(35*b^10*c^6*d^4 - (1029*a^6*b^4*d^10)/2 + 42*a*b^9*c^5*d^5 + 210*a^5*b^5*c*d^9 + (105*a^2*b^8*c^4*d^6)/2 + 70*a^3*b^7*c^3*d^7 + 105*a^4*b^6*c^2*d^8) - x^9*(10*a*b^9*d^10 - 10*b^10*c*d^9) + x*((10*b^10*c^9*d)/9 - (7129*a^9*b*d^10)/252 + (5*a*b^9*c^8*d^2)/4 + 10*a^8*b^2*c*d^9 + (10*a^2*b^8*c^7*d^3)/7 + (5*a^3*b^7*c^6*d^4)/3 + 2*a^4*b^6*c^5*d^5 + (5*a^5*b^5*c^4*d^6)/2 + (10*a^6*b^4*c^3*d^7)/3 + 5*a^7*b^3*c^2*d^8) + x^6*((105*b^10*c^4*d^6)/2 - (875*a^4*b^6*d^10)/2 + 70*a*b^9*c^3*d^7 + 210*a^3*b^7*c*d^9 + 105*a^2*b^8*c^2*d^8) + x^8*((45*b^10*c^2*d^8)/2 - (135*a^2*b^8*d^10)/2 + 45*a*b^9*c*d^9) + x^3*((120*b^10*c^7*d^3)/7 - (2178*a^7*b^3*d^10)/7 + 20*a*b^9*c^6*d^4 + 120*a^6*b^4*c*d^9 + 24*a^2*b^8*c^5*d^5 + 30*a^3*b^7*c^4*d^6 + 40*a^4*b^6*c^3*d^7 + 60*a^5*b^5*c^2*d^8) + x^5*((252*b^10*c^5*d^5)/5 - (2877*a^5*b^5*d^10)/5 + 63*a*b^9*c^4*d^6 + 252*a^4*b^6*c*d^9 + 84*a^2*b^8*c^3*d^7 + 126*a^3*b^7*c^2*d^8) - (7381*a^10*d^10)/2520 + (b^10*c^10)/10 + x^7*(40*b^10*c^3*d^7 - 220*a^3*b^7*d^10 + 60*a*b^9*c^2*d^8 + 120*a^2*b^8*c*d^9) + x^2*((45*b^10*c^8*d^2)/8 - (6849*a^8*b^2*d^10)/56 + (45*a*b^9*c^7*d^3)/7 + 45*a^7*b^3*c*d^9 + (15*a^2*b^8*c^6*d^4)/2 + 9*a^3*b^7*c^5*d^5 + (45*a^4*b^6*c^4*d^6)/4 + 15*a^5*b^5*c^3*d^7 + (45*a^6*b^4*c^2*d^8)/2) +

```
(a^2*b^8*c^8*d^2)/8 + (a^3*b^7*c^7*d^3)/7 + (a^4*b^6*c^6*d^4)/6 + (a^5*b^5*
c^5*d^5)/5 + (a^6*b^4*c^4*d^6)/4 + (a^7*b^3*c^3*d^7)/3 + (a^8*b^2*c^2*d^8)/
2 + (a*b^9*c^9*d)/9 + a^9*b*c*d^9)/(b^11*(a + b*x)^10)
```

3.1323 $\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

```
[Out] -1/11*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^11
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -1/11*(c + d*x)^11/((b*c - a*d)*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx = -\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 665 vs. 2(28) = 56.
time = 0.19, size = 665, normalized size = 23.75

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -1/11*(a^10*d^10 + a^9*b*d^9*(c + 11*d*x) + a^8*b^2*d^8*(c^2 + 11*c*d*x + 5
5*d^2*x^2) + a^7*b^3*d^7*(c^3 + 11*c^2*d*x + 55*c*d^2*x^2 + 165*d^3*x^3) +
a^6*b^4*d^6*(c^4 + 11*c^3*d*x + 55*c^2*d^2*x^2 + 165*c*d^3*x^3 + 330*d^4*x^
4) + a^5*b^5*d^5*(c^5 + 11*c^4*d*x + 55*c^3*d^2*x^2 + 165*c^2*d^3*x^3 + 330
*c*d^4*x^4 + 462*d^5*x^5) + a^4*b^6*d^4*(c^6 + 11*c^5*d*x + 55*c^4*d^2*x^2
+ 165*c^3*d^3*x^3 + 330*c^2*d^4*x^4 + 462*c*d^5*x^5 + 462*d^6*x^6) + a^3*b^
7*d^3*(c^7 + 11*c^6*d*x + 55*c^5*d^2*x^2 + 165*c^4*d^3*x^3 + 330*c^3*d^4*x^
4 + 462*c^2*d^5*x^5 + 462*c*d^6*x^6 + 330*d^7*x^7) + a^2*b^8*d^2*(c^8 + 11*
c^7*d*x + 55*c^6*d^2*x^2 + 165*c^5*d^3*x^3 + 330*c^4*d^4*x^4 + 462*c^3*d^5*
x^5 + 462*c^2*d^6*x^6 + 330*c*d^7*x^7 + 165*d^8*x^8) + a*b^9*d*(c^9 + 11*c^
8*d*x + 55*c^7*d^2*x^2 + 165*c^6*d^3*x^3 + 330*c^5*d^4*x^4 + 462*c^4*d^5*x^
5 + 462*c^3*d^6*x^6 + 330*c^2*d^7*x^7 + 165*c*d^8*x^8 + 55*d^9*x^9) + b^10*
(c^10 + 11*c^9*d*x + 55*c^8*d^2*x^2 + 165*c^7*d^3*x^3 + 330*c^6*d^4*x^4 + 4
62*c^5*d^5*x^5 + 462*c^4*d^6*x^6 + 330*c^3*d^7*x^7 + 165*c^2*d^8*x^8 + 55*c
*d^9*x^9 + 11*d^10*x^10))/(b^11*(a + b*x)^11)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(26) = 52$.
time = 0.14, size = 866, normalized size = 30.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] -d^10/b^11/(b*x+a)+30/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)/(b*x+a)^4-5/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3
*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c
^7*d+b^8*c^8)/(b*x+a)^9-30/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d
^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^7+1
5/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*
a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^8+1/b^11*
d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*
c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c
^8*d-b^9*c^9)/(b*x+a)^10-42/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^5-1/11*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b
^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*
a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*
c^10)/b^11/(b*x+a)^11+5/b^11*d^9*(a*d-b*c)/(b*x+a)^2+42/b^11*d^5*(a^5*d^5-5
*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(
b*x+a)^6-15/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(26) = 52$.
time = 0.34, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(26) = 52$.
time = 1.19, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
```

```
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**12,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(26) = 52.
time = 0.77, size = 951, normalized size = 33.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + 55*b^10*c*d^9*x^9 + 55*a*b^9*d^10*x^9 + 165*b^10
*c^2*d^8*x^8 + 165*a*b^9*c*d^9*x^8 + 165*a^2*b^8*d^10*x^8 + 330*b^10*c^3*d^
7*x^7 + 330*a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7 + 330*a^3*b^7*d^10*x^
7 + 462*b^10*c^4*d^6*x^6 + 462*a*b^9*c^3*d^7*x^6 + 462*a^2*b^8*c^2*d^8*x^6
+ 462*a^3*b^7*c*d^9*x^6 + 462*a^4*b^6*d^10*x^6 + 462*b^10*c^5*d^5*x^5 + 462
*a*b^9*c^4*d^6*x^5 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 46
2*a^4*b^6*c*d^9*x^5 + 462*a^5*b^5*d^10*x^5 + 330*b^10*c^6*d^4*x^4 + 330*a*b
^9*c^5*d^5*x^4 + 330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^
4*b^6*c^2*d^8*x^4 + 330*a^5*b^5*c*d^9*x^4 + 330*a^6*b^4*d^10*x^4 + 165*b^10
*c^7*d^3*x^3 + 165*a*b^9*c^6*d^4*x^3 + 165*a^2*b^8*c^5*d^5*x^3 + 165*a^3*b^
7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 + 165*a^6
*b^4*c*d^9*x^3 + 165*a^7*b^3*d^10*x^3 + 55*b^10*c^8*d^2*x^2 + 55*a*b^9*c^7*
d^3*x^2 + 55*a^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*
d^6*x^2 + 55*a^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 55*a^7*b^3*c*d^
9*x^2 + 55*a^8*b^2*d^10*x^2 + 11*b^10*c^9*d*x + 11*a*b^9*c^8*d^2*x + 11*a^2
*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*c
^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 11*a^8*b^2*c*d^9*x
 + 11*a^9*b*d^10*x + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*b^7*c^
7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d
^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^11*b^11)
```

Mupad [B]
time = 0.46, size = 1066, normalized size = 38.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^12,x)
```

```
[Out] -(a^10*d^10 + b^10*c^10 + 11*b^10*d^10*x^10 + 55*a*b^9*d^10*x^9 + 55*b^10*c
*d^9*x^9 + a^2*b^8*c^8*d^2 + a^3*b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^
5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d^7 + a^8*b^2*c^2*d^8 + 55*a^8*b^2*d^
10*x^2 + 165*a^7*b^3*d^10*x^3 + 330*a^6*b^4*d^10*x^4 + 462*a^5*b^5*d^10*x^5
 + 462*a^4*b^6*d^10*x^6 + 330*a^3*b^7*d^10*x^7 + 165*a^2*b^8*d^10*x^8 + 55*
b^10*c^8*d^2*x^2 + 165*b^10*c^7*d^3*x^3 + 330*b^10*c^6*d^4*x^4 + 462*b^10*c
^5*d^5*x^5 + 462*b^10*c^4*d^6*x^6 + 330*b^10*c^3*d^7*x^7 + 165*b^10*c^2*d^8
*x^8 + a*b^9*c^9*d + a^9*b*c*d^9 + 11*a^9*b*d^10*x + 11*b^10*c^9*d*x + 55*a
^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*d^6*x^2 + 55*a
^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 165*a^2*b^8*c^5*d^5*x^3 + 165
*a^3*b^7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 +
330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^4*b^6*c^2*d^8*x^4
 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 462*a^2*b^8*c^2*d^8*
x^6 + 11*a*b^9*c^8*d^2*x + 11*a^8*b^2*c*d^9*x + 165*a*b^9*c*d^9*x^8 + 11*a^
2*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*
c^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 55*a*b^9*c^7*d^3*
x^2 + 55*a^7*b^3*c*d^9*x^2 + 165*a*b^9*c^6*d^4*x^3 + 165*a^6*b^4*c*d^9*x^3
+ 330*a*b^9*c^5*d^5*x^4 + 330*a^5*b^5*c*d^9*x^4 + 462*a*b^9*c^4*d^6*x^5 + 4
62*a^4*b^6*c*d^9*x^5 + 462*a*b^9*c^3*d^7*x^6 + 462*a^3*b^7*c*d^9*x^6 + 330*
a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7)/(11*a^11*b^11 + 11*b^22*x^11 + 1
21*a^10*b^12*x + 121*a*b^21*x^10 + 605*a^9*b^13*x^2 + 1815*a^8*b^14*x^3 + 3
630*a^7*b^15*x^4 + 5082*a^6*b^16*x^5 + 5082*a^5*b^17*x^6 + 3630*a^4*b^18*x^
7 + 1815*a^3*b^19*x^8 + 605*a^2*b^20*x^9)
```

3.1324 $\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx$

Optimal. Leaf size=58

$$-\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}}$$

```
[Out] -1/12*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^12+1/132*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^11
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {47, 37}

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -1/12*(c + d*x)^11/((b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^11)/(132*(b*c
- a*d)^2*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx = -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}} - \frac{d\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{12(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 684 vs. 2(58) = 116.
time = 0.18, size = 684, normalized size = 11.79

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -1/132*(a^10*d^10 + 2*a^9*b*d^9*(c + 6*d*x) + 3*a^8*b^2*d^8*(c^2 + 8*c*d*x
+ 22*d^2*x^2) + 4*a^7*b^3*d^7*(c^3 + 9*c^2*d*x + 33*c*d^2*x^2 + 55*d^3*x^3)
 + a^6*b^4*d^6*(5*c^4 + 48*c^3*d*x + 198*c^2*d^2*x^2 + 440*c*d^3*x^3 + 495*
d^4*x^4) + 6*a^5*b^5*d^5*(c^5 + 10*c^4*d*x + 44*c^3*d^2*x^2 + 110*c^2*d^3*x
^3 + 165*c*d^4*x^4 + 132*d^5*x^5) + a^4*b^6*d^4*(7*c^6 + 72*c^5*d*x + 330*c
^4*d^2*x^2 + 880*c^3*d^3*x^3 + 1485*c^2*d^4*x^4 + 1584*c*d^5*x^5 + 924*d^6*
x^6) + 4*a^3*b^7*d^3*(2*c^7 + 21*c^6*d*x + 99*c^5*d^2*x^2 + 275*c^4*d^3*x^3
 + 495*c^3*d^4*x^4 + 594*c^2*d^5*x^5 + 462*c*d^6*x^6 + 198*d^7*x^7) + 3*a^2
*b^8*d^2*(3*c^8 + 32*c^7*d*x + 154*c^6*d^2*x^2 + 440*c^5*d^3*x^3 + 825*c^4*
d^4*x^4 + 1056*c^3*d^5*x^5 + 924*c^2*d^6*x^6 + 528*c*d^7*x^7 + 165*d^8*x^8)
 + 2*a*b^9*d*(5*c^9 + 54*c^8*d*x + 264*c^7*d^2*x^2 + 770*c^6*d^3*x^3 + 1485
*c^5*d^4*x^4 + 1980*c^4*d^5*x^5 + 1848*c^3*d^6*x^6 + 1188*c^2*d^7*x^7 + 495
*c*d^8*x^8 + 110*d^9*x^9) + b^10*(11*c^10 + 120*c^9*d*x + 594*c^8*d^2*x^2 +
 1760*c^7*d^3*x^3 + 3465*c^6*d^4*x^4 + 4752*c^5*d^5*x^5 + 4620*c^4*d^6*x^6
+ 3168*c^3*d^7*x^7 + 1485*c^2*d^8*x^8 + 440*c*d^9*x^9 + 66*d^10*x^10))/(b^1
1*(a + b*x)^12)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(54) = 108.
time = 0.14, size = 867, normalized size = 14.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*
a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3
+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^12-45/4/b^11*d^8
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^4+36/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+1
```

```
0*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^7+40/3/
b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^
3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^9-105/4/b^1
1*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b
^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^8-9/2/b^11*d^2*(a^8*d^8-8*a^7*b*c
*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^
5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^10+24/b^11*d^7*(a^3
*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^5+10/11/b^11*d*(a^9*d^9-9
*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*
a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9
)/(b*x+a)^11-1/2*d^10/b^11/(b*x+a)^2-35/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a
^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^6+10/3/b^11*d^9*(a*d-b*c)/(b*
x+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(54) = 108$.

time = 0.34, size = 986, normalized size = 17.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(54) = 108$.

time = 1.49, size = 986, normalized size = 17.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**13,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(54) = 108.
time = 0.58, size = 961, normalized size = 16.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 440*b^10*c*d^9*x^9 + 220*a*b^9*d^10*x^9 + 1485*
b^10*c^2*d^8*x^8 + 990*a*b^9*c*d^9*x^8 + 495*a^2*b^8*d^10*x^8 + 3168*b^10*c
^3*d^7*x^7 + 2376*a*b^9*c^2*d^8*x^7 + 1584*a^2*b^8*c*d^9*x^7 + 792*a^3*b^7*
d^10*x^7 + 4620*b^10*c^4*d^6*x^6 + 3696*a*b^9*c^3*d^7*x^6 + 2772*a^2*b^8*c^
2*d^8*x^6 + 1848*a^3*b^7*c*d^9*x^6 + 924*a^4*b^6*d^10*x^6 + 4752*b^10*c^5*d
^5*x^5 + 3960*a*b^9*c^4*d^6*x^5 + 3168*a^2*b^8*c^3*d^7*x^5 + 2376*a^3*b^7*c
^2*d^8*x^5 + 1584*a^4*b^6*c*d^9*x^5 + 792*a^5*b^5*d^10*x^5 + 3465*b^10*c^6*
d^4*x^4 + 2970*a*b^9*c^5*d^5*x^4 + 2475*a^2*b^8*c^4*d^6*x^4 + 1980*a^3*b^7*
c^3*d^7*x^4 + 1485*a^4*b^6*c^2*d^8*x^4 + 990*a^5*b^5*c*d^9*x^4 + 495*a^6*b^
4*d^10*x^4 + 1760*b^10*c^7*d^3*x^3 + 1540*a*b^9*c^6*d^4*x^3 + 1320*a^2*b^8*
c^5*d^5*x^3 + 1100*a^3*b^7*c^4*d^6*x^3 + 880*a^4*b^6*c^3*d^7*x^3 + 660*a^5*
b^5*c^2*d^8*x^3 + 440*a^6*b^4*c*d^9*x^3 + 220*a^7*b^3*d^10*x^3 + 594*b^10*c
^8*d^2*x^2 + 528*a*b^9*c^7*d^3*x^2 + 462*a^2*b^8*c^6*d^4*x^2 + 396*a^3*b^7*
c^5*d^5*x^2 + 330*a^4*b^6*c^4*d^6*x^2 + 264*a^5*b^5*c^3*d^7*x^2 + 198*a^6*b
^4*c^2*d^8*x^2 + 132*a^7*b^3*c*d^9*x^2 + 66*a^8*b^2*d^10*x^2 + 120*b^10*c^9
*d*x + 108*a*b^9*c^8*d^2*x + 96*a^2*b^8*c^7*d^3*x + 84*a^3*b^7*c^6*d^4*x +
72*a^4*b^6*c^5*d^5*x + 60*a^5*b^5*c^4*d^6*x + 48*a^6*b^4*c^3*d^7*x + 36*a^7
*b^3*c^2*d^8*x + 24*a^8*b^2*c*d^9*x + 12*a^9*b*d^10*x + 11*b^10*c^10 + 10*a
*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*
a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8
 + 2*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^12*b^11)
```

Mupad [B]
time = 0.39, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^{11}\,(12\,a\,d-11\,b\,c+b\,d\,x)}{132\,(a\,d-b\,c)^2\,(a+b\,x)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^13,x)
```

```
[Out] ((c + d*x)^11*(12*a*d - 11*b*c + b*d*x))/(132*(a*d - b*c)^2*(a + b*x)^12)
```

3.1325 $\int \frac{(c+dx)^{10}}{(a+bx)^{14}} dx$

Optimal. Leaf size=89

$$-\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}+\frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}}-\frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}$$

```
[Out] -1/13*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^13+1/78*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x+
a)^12-1/858*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^11
```

Rubi [A]
time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -1/13*(c + d*x)^11/((b*c - a*d)*(a + b*x)^13) + (d*(c + d*x)^11)/(78*(b*c -
 a*d)^2*(a + b*x)^12) - (d^2*(c + d*x)^11)/(858*(b*c - a*d)^3*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx = -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} - \frac{(2d)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{13(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} + \frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{78(bc-ad)^2}$$
$$= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}} + \frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}} - \frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(89) = 178.
time = 0.19, size = 690, normalized size = 7.75

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -1/858*(a^10*d^10 + a^9*b*d^9*(3*c + 13*d*x) + 3*a^8*b^2*d^8*(2*c^2 + 13*c*
d*x + 26*d^2*x^2) + 2*a^7*b^3*d^7*(5*c^3 + 39*c^2*d*x + 117*c*d^2*x^2 + 143
*d^3*x^3) + a^6*b^4*d^6*(15*c^4 + 130*c^3*d*x + 468*c^2*d^2*x^2 + 858*c*d^3
*x^3 + 715*d^4*x^4) + 3*a^5*b^5*d^5*(7*c^5 + 65*c^4*d*x + 260*c^3*d^2*x^2 +
 572*c^2*d^3*x^3 + 715*c*d^4*x^4 + 429*d^5*x^5) + a^4*b^6*d^4*(28*c^6 + 273
*c^5*d*x + 1170*c^4*d^2*x^2 + 2860*c^3*d^3*x^3 + 4290*c^2*d^4*x^4 + 3861*c*
d^5*x^5 + 1716*d^6*x^6) + 2*a^3*b^7*d^3*(18*c^7 + 182*c^6*d*x + 819*c^5*d^2
*x^2 + 2145*c^4*d^3*x^3 + 3575*c^3*d^4*x^4 + 3861*c^2*d^5*x^5 + 2574*c*d^6*
x^6 + 858*d^7*x^7) + 3*a^2*b^8*d^2*(15*c^8 + 156*c^7*d*x + 728*c^6*d^2*x^2
+ 2002*c^5*d^3*x^3 + 3575*c^4*d^4*x^4 + 4290*c^3*d^5*x^5 + 3432*c^2*d^6*x^6
 + 1716*c*d^7*x^7 + 429*d^8*x^8) + a*b^9*d*(55*c^9 + 585*c^8*d*x + 2808*c^7
*d^2*x^2 + 8008*c^6*d^3*x^3 + 15015*c^5*d^4*x^4 + 19305*c^4*d^5*x^5 + 17160
*c^3*d^6*x^6 + 10296*c^2*d^7*x^7 + 3861*c*d^8*x^8 + 715*d^9*x^9) + b^10*(66
*c^10 + 715*c^9*d*x + 3510*c^8*d^2*x^2 + 10296*c^7*d^3*x^3 + 20020*c^6*d^4*
x^4 + 27027*c^5*d^5*x^5 + 25740*c^4*d^6*x^6 + 17160*c^3*d^7*x^7 + 7722*c^2*
d^8*x^8 + 2145*c*d^9*x^9 + 286*d^10*x^10))/(b^11*(a + b*x)^13)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(83) = 166.
time = 0.15, size = 867, normalized size = 9.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/13*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*
a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3
```

```
+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^13-30/b^11*d^6*(
a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^7+5/
6/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a
^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*
a*b^8*c^8*d-b^9*c^9)/(b*x+a)^12+5/2/b^11*d^9*(a*d-b*c)/(b*x+a)^4-70/3/b^11*
d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4
*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^9+63/2/b^11*d^5*(a^5*d^5-5*a^4*b*c*
d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^8+
12/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35
*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^10-9/b^1
1*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^5-45/11/b^11*d^2*(a^8*d^8-8*a^7*b
*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*
c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^11+20/b^11*d^7*(a
^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^6-1/3*d^10/b^11/(b*x+a)
^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(83) = 166$.
time = 0.37, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
 + 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
 + 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(83) = 166$.
time = 0.90, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**14,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(83) = 166$.
time = 1.20, size = 961, normalized size = 10.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="giac")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 2145*b^10*c*d^9*x^9 + 715*a*b^9*d^10*x^9 + 772
2*b^10*c^2*d^8*x^8 + 3861*a*b^9*c*d^9*x^8 + 1287*a^2*b^8*d^10*x^8 + 17160*b
^10*c^3*d^7*x^7 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7 + 1716*a
^3*b^7*d^10*x^7 + 25740*b^10*c^4*d^6*x^6 + 17160*a*b^9*c^3*d^7*x^6 + 10296*
a^2*b^8*c^2*d^8*x^6 + 5148*a^3*b^7*c*d^9*x^6 + 1716*a^4*b^6*d^10*x^6 + 2702
7*b^10*c^5*d^5*x^5 + 19305*a*b^9*c^4*d^6*x^5 + 12870*a^2*b^8*c^3*d^7*x^5 +
7722*a^3*b^7*c^2*d^8*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 1287*a^5*b^5*d^10*x^5 +
 20020*b^10*c^6*d^4*x^4 + 15015*a*b^9*c^5*d^5*x^4 + 10725*a^2*b^8*c^4*d^6*x
^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 2145*a^5*b^5*c*d
^9*x^4 + 715*a^6*b^4*d^10*x^4 + 10296*b^10*c^7*d^3*x^3 + 8008*a*b^9*c^6*d^4
*x^3 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^6*x^3 + 2860*a^4*b^6*c
^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 858*a^6*b^4*c*d^9*x^3 + 286*a^7*b^3
*d^10*x^3 + 3510*b^10*c^8*d^2*x^2 + 2808*a*b^9*c^7*d^3*x^2 + 2184*a^2*b^8*c
^6*d^4*x^2 + 1638*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*
b^5*c^3*d^7*x^2 + 468*a^6*b^4*c^2*d^8*x^2 + 234*a^7*b^3*c*d^9*x^2 + 78*a^8*
b^2*d^10*x^2 + 715*b^10*c^9*d*x + 585*a*b^9*c^8*d^2*x + 468*a^2*b^8*c^7*d^3
*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^5*c^4*d^6*x
+ 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 39*a^8*b^2*c*d^9*x + 13*a^
9*b*d^10*x + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^
7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 +
10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10)/((b*x +
 a)^13*b^11)
```

Mupad [B]
time = 0.48, size = 1098, normalized size = 12.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^14,x)
```

```
[Out] -(a^10*d^10 + 66*b^10*c^10 + 286*b^10*d^10*x^10 + 715*a*b^9*d^10*x^9 + 2145
*b^10*c*d^9*x^9 + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*
d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*
b^2*c^2*d^8 + 78*a^8*b^2*d^10*x^2 + 286*a^7*b^3*d^10*x^3 + 715*a^6*b^4*d^10
*x^4 + 1287*a^5*b^5*d^10*x^5 + 1716*a^4*b^6*d^10*x^6 + 1716*a^3*b^7*d^10*x^
7 + 1287*a^2*b^8*d^10*x^8 + 3510*b^10*c^8*d^2*x^2 + 10296*b^10*c^7*d^3*x^3
+ 20020*b^10*c^6*d^4*x^4 + 27027*b^10*c^5*d^5*x^5 + 25740*b^10*c^4*d^6*x^6
+ 17160*b^10*c^3*d^7*x^7 + 7722*b^10*c^2*d^8*x^8 + 55*a*b^9*c^9*d + 3*a^9*b
*c*d^9 + 13*a^9*b*d^10*x + 715*b^10*c^9*d*x + 2184*a^2*b^8*c^6*d^4*x^2 + 16
38*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*b^5*c^3*d^7*x^2
 + 468*a^6*b^4*c^2*d^8*x^2 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^
```

6*x^3 + 2860*a^4*b^6*c^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 10725*a^2*b^8*c^4*d^6*x^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 12870*a^2*b^8*c^3*d^7*x^5 + 7722*a^3*b^7*c^2*d^8*x^5 + 10296*a^2*b^8*c^2*d^8*x^6 + 585*a*b^9*c^8*d^2*x + 39*a^8*b^2*c*d^9*x + 3861*a*b^9*c*d^9*x^8 + 468*a^2*b^8*c^7*d^3*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^5*c^4*d^6*x + 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 2808*a*b^9*c^7*d^3*x^2 + 234*a^7*b^3*c*d^9*x^2 + 8008*a*b^9*c^6*d^4*x^3 + 858*a^6*b^4*c*d^9*x^3 + 15015*a*b^9*c^5*d^5*x^4 + 2145*a^5*b^5*c*d^9*x^4 + 19305*a*b^9*c^4*d^6*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 17160*a*b^9*c^3*d^7*x^6 + 5148*a^3*b^7*c*d^9*x^6 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7)/(858*a^13*b^11 + 858*b^24*x^13 + 11154*a^12*b^12*x + 11154*a*b^23*x^12 + 66924*a^11*b^13*x^2 + 245388*a^10*b^14*x^3 + 613470*a^9*b^15*x^4 + 1104246*a^8*b^16*x^5 + 1472328*a^7*b^17*x^6 + 1472328*a^6*b^18*x^7 + 1104246*a^5*b^19*x^8 + 613470*a^4*b^20*x^9 + 245388*a^3*b^21*x^10 + 66924*a^2*b^22*x^11)

3.1326 $\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx$

Optimal. Leaf size=120

$$-\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}-\frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}}+\frac{d^3(c+dx)^{11}}{4004(bc-ad)^4(a+bx)^{11}}$$

```
[Out] -1/14*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^14+3/182*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^13-1/364*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^12+1/4004*d^3*(d*x+c)^11/(
-a*d+b*c)^4/(b*x+a)^11
```

Rubi [A]
time = 0.02, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -1/14*(c + d*x)^11/((b*c - a*d)*(a + b*x)^14) + (3*d*(c + d*x)^11)/(182*(b*
c - a*d)^2*(a + b*x)^13) - (d^2*(c + d*x)^11)/(364*(b*c - a*d)^3*(a + b*x)^
12) + (d^3*(c + d*x)^11)/(4004*(b*c - a*d)^4*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx = -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} - \frac{(3d)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{14(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} + \frac{(3d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{91(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} - \frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}} - \frac{d^3\int \frac{(c+}{(a+}}{364(bc}$$

$$= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}} + \frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}} - \frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}} + \frac{d}{4004(bc}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. 2(120) = 240.
time = 0.19, size = 692, normalized size = 5.77

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -1/4004*(a^10*d^10 + 2*a^9*b*d^9*(2*c + 7*d*x) + a^8*b^2*d^8*(10*c^2 + 56*c
*d*x + 91*d^2*x^2) + 4*a^7*b^3*d^7*(5*c^3 + 35*c^2*d*x + 91*c*d^2*x^2 + 91*
d^3*x^3) + 7*a^6*b^4*d^6*(5*c^4 + 40*c^3*d*x + 130*c^2*d^2*x^2 + 208*c*d^3*
x^3 + 143*d^4*x^4) + 14*a^5*b^5*d^5*(4*c^5 + 35*c^4*d*x + 130*c^3*d^2*x^2 +
 260*c^2*d^3*x^3 + 286*c*d^4*x^4 + 143*d^5*x^5) + 7*a^4*b^6*d^4*(12*c^6 + 1
12*c^5*d*x + 455*c^4*d^2*x^2 + 1040*c^3*d^3*x^3 + 1430*c^2*d^4*x^4 + 1144*c
*d^5*x^5 + 429*d^6*x^6) + 4*a^3*b^7*d^3*(30*c^7 + 294*c^6*d*x + 1274*c^5*d^
2*x^2 + 3185*c^4*d^3*x^3 + 5005*c^3*d^4*x^4 + 5005*c^2*d^5*x^5 + 3003*c*d^6
*x^6 + 858*d^7*x^7) + a^2*b^8*d^2*(165*c^8 + 1680*c^7*d*x + 7644*c^6*d^2*x^
2 + 20384*c^5*d^3*x^3 + 35035*c^4*d^4*x^4 + 40040*c^3*d^5*x^5 + 30030*c^2*d
^6*x^6 + 13728*c*d^7*x^7 + 3003*d^8*x^8) + 2*a*b^9*d*(110*c^9 + 1155*c^8*d*
x + 5460*c^7*d^2*x^2 + 15288*c^6*d^3*x^3 + 28028*c^5*d^4*x^4 + 35035*c^4*d^
5*x^5 + 30030*c^3*d^6*x^6 + 17160*c^2*d^7*x^7 + 6006*c*d^8*x^8 + 1001*d^9*x
^9) + b^10*(286*c^10 + 3080*c^9*d*x + 15015*c^8*d^2*x^2 + 43680*c^7*d^3*x^3
 + 84084*c^6*d^4*x^4 + 112112*c^5*d^5*x^5 + 105105*c^4*d^6*x^6 + 68640*c^3*
d^7*x^7 + 30030*c^2*d^8*x^8 + 8008*c*d^9*x^9 + 1001*d^10*x^10))/(b^11*(a +
b*x)^14)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(112) = 224.
time = 0.15, size = 867, normalized size = 7.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^15,x,method=_RETURNVERBOSE)
```

```
[Out] 10/13/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+1
26*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^
2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^13-15/4/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28
*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+2
8*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^12-1/14*(a^10*d^10-10*a^9*
b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*
b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a
*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^14-1/4*d^10/b^11/(b*x+a)^4+28/b^11*d^5*(
a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b
^5*c^5)/(b*x+a)^9-105/4/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4
*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^8-21/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b
^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*
x+a)^10+2/b^11*d^9*(a*d-b*c)/(b*x+a)^5+120/11/b^11*d^3*(a^7*d^7-7*a^6*b*c*d
^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*
d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^11-15/2/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*
c^2)/(b*x+a)^6+120/7/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/(b*x+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(112) = 224$.
time = 0.36, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
```

```
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(112) = 224$.
time = 0.73, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**15,x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(112) = 224.
time = 1.28, size = 961, normalized size = 8.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 8008*b^10*c*d^9*x^9 + 2002*a*b^9*d^10*x^9 +
30030*b^10*c^2*d^8*x^8 + 12012*a*b^9*c*d^9*x^8 + 3003*a^2*b^8*d^10*x^8 + 68
640*b^10*c^3*d^7*x^7 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2*b^8*c*d^9*x^7 +
3432*a^3*b^7*d^10*x^7 + 105105*b^10*c^4*d^6*x^6 + 60060*a*b^9*c^3*d^7*x^6 +
 30030*a^2*b^8*c^2*d^8*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 3003*a^4*b^6*d^10*x^
6 + 112112*b^10*c^5*d^5*x^5 + 70070*a*b^9*c^4*d^6*x^5 + 40040*a^2*b^8*c^3*d
^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 2002*a^5*b^5*
d^10*x^5 + 84084*b^10*c^6*d^4*x^4 + 56056*a*b^9*c^5*d^5*x^4 + 35035*a^2*b^8
*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^2*d^8*x^4 + 4004
*a^5*b^5*c*d^9*x^4 + 1001*a^6*b^4*d^10*x^4 + 43680*b^10*c^7*d^3*x^3 + 30576
*a*b^9*c^6*d^4*x^3 + 20384*a^2*b^8*c^5*d^5*x^3 + 12740*a^3*b^7*c^4*d^6*x^3
+ 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3 + 1456*a^6*b^4*c*d^9*
x^3 + 364*a^7*b^3*d^10*x^3 + 15015*b^10*c^8*d^2*x^2 + 10920*a*b^9*c^7*d^3*x
^2 + 7644*a^2*b^8*c^6*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4
*d^6*x^2 + 1820*a^5*b^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 364*a^7*b^3
*c*d^9*x^2 + 91*a^8*b^2*d^10*x^2 + 3080*b^10*c^9*d*x + 2310*a*b^9*c^8*d^2*x
 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6*c^5*d^5*x
+ 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^2*d^8*x + 5
6*a^8*b^2*c*d^9*x + 14*a^9*b*d^10*x + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165
*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^
5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^
9*b*c*d^9 + a^10*d^10)/((b*x + a)^14*b^11)
```

Mupad [B]
time = 1.30, size = 1109, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^15,x)
```

```
[Out] -(a^10*d^10 + 286*b^10*c^10 + 1001*b^10*d^10*x^10 + 2002*a*b^9*d^10*x^9 + 8
008*b^10*c*d^9*x^9 + 165*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6
*c^6*d^4 + 56*a^5*b^5*c^5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 1
0*a^8*b^2*c^2*d^8 + 91*a^8*b^2*d^10*x^2 + 364*a^7*b^3*d^10*x^3 + 1001*a^6*b
```

^4*d^10*x^4 + 2002*a^5*b^5*d^10*x^5 + 3003*a^4*b^6*d^10*x^6 + 3432*a^3*b^7*d^10*x^7 + 3003*a^2*b^8*d^10*x^8 + 15015*b^10*c^8*d^2*x^2 + 43680*b^10*c^7*d^3*x^3 + 84084*b^10*c^6*d^4*x^4 + 112112*b^10*c^5*d^5*x^5 + 105105*b^10*c^4*d^6*x^6 + 68640*b^10*c^3*d^7*x^7 + 30030*b^10*c^2*d^8*x^8 + 220*a*b^9*c^9*d + 4*a^9*b*c*d^9 + 14*a^9*b*d^10*x + 3080*b^10*c^9*d*x + 7644*a^2*b^8*c^6*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4*d^6*x^2 + 1820*a^5*b^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 20384*a^2*b^8*c^5*d^5*x^3 + 12740*a^3*b^7*c^4*d^6*x^3 + 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3 + 35035*a^2*b^8*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^2*d^8*x^4 + 40040*a^2*b^8*c^3*d^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 30030*a^2*b^8*c^2*d^8*x^6 + 2310*a*b^9*c^8*d^2*x + 56*a^8*b^2*c*d^9*x + 12012*a*b^9*c*d^9*x^8 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6*c^5*d^5*x + 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^2*d^8*x + 10920*a*b^9*c^7*d^3*x^2 + 364*a^7*b^3*c*d^9*x^2 + 30576*a*b^9*c^6*d^4*x^3 + 1456*a^6*b^4*c*d^9*x^3 + 56056*a*b^9*c^5*d^5*x^4 + 4004*a^5*b^5*c*d^9*x^4 + 70070*a*b^9*c^4*d^6*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 60060*a*b^9*c^3*d^7*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2*b^8*c*d^9*x^7)/(4004*a^14*b^11 + 4004*b^25*x^14 + 56056*a^13*b^12*x + 56056*a*b^24*x^13 + 364364*a^12*b^13*x^2 + 1457456*a^11*b^14*x^3 + 4008004*a^10*b^15*x^4 + 8016008*a^9*b^16*x^5 + 12024012*a^8*b^17*x^6 + 13741728*a^7*b^18*x^7 + 12024012*a^6*b^19*x^8 + 8016008*a^5*b^20*x^9 + 4008004*a^4*b^21*x^10 + 1457456*a^3*b^22*x^11 + 364364*a^2*b^23*x^12)

3.1327 $\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$

Optimal. Leaf size=151

$$-\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}+\frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}}-\frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}}+\frac{d^3(c+dx)^{11}}{1365(bc-ad)^4(a+bx)^{12}}-1$$

```
[Out] -1/15*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^15+2/105*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^14-2/455*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^13+1/1365*d^3*(d*x+c)^11/(
-a*d+b*c)^4/(b*x+a)^12-1/15015*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^11
```

Rubi [A]
time = 0.03, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-\frac{(c+dx)^{11}}{15(a+bx)^{15}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -1/15*(c + d*x)^11/((b*c - a*d)*(a + b*x)^15) + (2*d*(c + d*x)^11)/(105*(b*
c - a*d)^2*(a + b*x)^14) - (2*d^2*(c + d*x)^11)/(455*(b*c - a*d)^3*(a + b*x
)^13) + (d^3*(c + d*x)^11)/(1365*(b*c - a*d)^4*(a + b*x)^12) - (d^4*(c + d*
x)^11)/(15015*(b*c - a*d)^5*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx = -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} - \frac{(4d)\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{15(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} + \frac{(2d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} - \frac{(4d^3)\int}{455(b}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d}{1365(bc}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d}{1365(bc}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(151) = 302.
time = 0.20, size = 690, normalized size = 4.57

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -1/15015*(a^10*d^10 + 5*a^9*b*d^9*(c + 3*d*x) + 15*a^8*b^2*d^8*(c^2 + 5*c*d
*x + 7*d^2*x^2) + 5*a^7*b^3*d^7*(7*c^3 + 45*c^2*d*x + 105*c*d^2*x^2 + 91*d^
3*x^3) + 35*a^6*b^4*d^6*(2*c^4 + 15*c^3*d*x + 45*c^2*d^2*x^2 + 65*c*d^3*x^3
 + 39*d^4*x^4) + 21*a^5*b^5*d^5*(6*c^5 + 50*c^4*d*x + 175*c^3*d^2*x^2 + 325
*c^2*d^3*x^3 + 325*c*d^4*x^4 + 143*d^5*x^5) + 35*a^4*b^6*d^4*(6*c^6 + 54*c^
5*d*x + 210*c^4*d^2*x^2 + 455*c^3*d^3*x^3 + 585*c^2*d^4*x^4 + 429*c*d^5*x^5
 + 143*d^6*x^6) + 5*a^3*b^7*d^3*(66*c^7 + 630*c^6*d*x + 2646*c^5*d^2*x^2 +
6370*c^4*d^3*x^3 + 9555*c^3*d^4*x^4 + 9009*c^2*d^5*x^5 + 5005*c*d^6*x^6 + 1
287*d^7*x^7) + 15*a^2*b^8*d^2*(33*c^8 + 330*c^7*d*x + 1470*c^6*d^2*x^2 + 38
22*c^5*d^3*x^3 + 6370*c^4*d^4*x^4 + 7007*c^3*d^5*x^5 + 5005*c^2*d^6*x^6 + 2
145*c*d^7*x^7 + 429*d^8*x^8) + 5*a*b^9*d*(143*c^9 + 1485*c^8*d*x + 6930*c^7
*d^2*x^2 + 19110*c^6*d^3*x^3 + 34398*c^5*d^4*x^4 + 42042*c^4*d^5*x^5 + 3503
5*c^3*d^6*x^6 + 19305*c^2*d^7*x^7 + 6435*c*d^8*x^8 + 1001*d^9*x^9) + b^10*(
1001*c^10 + 10725*c^9*d*x + 51975*c^8*d^2*x^2 + 150150*c^7*d^3*x^3 + 286650
*c^6*d^4*x^4 + 378378*c^5*d^5*x^5 + 350350*c^4*d^6*x^6 + 225225*c^3*d^7*x^7
 + 96525*c^2*d^8*x^8 + 25025*c*d^9*x^9 + 3003*d^10*x^10))/(b^11*(a + b*x)^1
5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(141) = 282.
time = 0.14, size = 867, normalized size = 5.74 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^16,x,method=_RETURNVERBOSE)
```

```
[Out] -45/13/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^
5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^
8*c^8)/(b*x+a)^13+10/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*
a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7
)/(b*x+a)^12+5/7/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^
3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2
*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^14-70/3/b^11*d^6*(a^4*d^4-4*a^3
*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^9+15/b^11*d^7*(a^
3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^8+126/5/b^11*d^5*(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)/(b*x+a)^10-1/5*d^10/b^11/(b*x+a)^5-210/11/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5
+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6
*c^6)/(b*x+a)^11-45/7/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^7+5/3/b^
11*d^9*(a*d-b*c)/(b*x+a)^6-1/15*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^
8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c
^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^1
1/(b*x+a)^15
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(141) = 282$.
time = 0.37, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
```

```
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(141) = 282$.
time = 0.79, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**16,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(141) = 282.
time = 1.44, size = 961, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 25025*b^10*c*d^9*x^9 + 5005*a*b^9*d^10*x^9
+ 96525*b^10*c^2*d^8*x^8 + 32175*a*b^9*c*d^9*x^8 + 6435*a^2*b^8*d^10*x^8 +
225225*b^10*c^3*d^7*x^7 + 96525*a*b^9*c^2*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7
 + 6435*a^3*b^7*d^10*x^7 + 350350*b^10*c^4*d^6*x^6 + 175175*a*b^9*c^3*d^7*x
^6 + 75075*a^2*b^8*c^2*d^8*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 5005*a^4*b^6*d^1
0*x^6 + 378378*b^10*c^5*d^5*x^5 + 210210*a*b^9*c^4*d^6*x^5 + 105105*a^2*b^8
*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8*x^5 + 15015*a^4*b^6*c*d^9*x^5 + 3003*a
^5*b^5*d^10*x^5 + 286650*b^10*c^6*d^4*x^4 + 171990*a*b^9*c^5*d^5*x^4 + 9555
0*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 20475*a^4*b^6*c^2*d^8*x
^4 + 6825*a^5*b^5*c*d^9*x^4 + 1365*a^6*b^4*d^10*x^4 + 150150*b^10*c^7*d^3*x
^3 + 95550*a*b^9*c^6*d^4*x^3 + 57330*a^2*b^8*c^5*d^5*x^3 + 31850*a^3*b^7*c^
4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^5*c^2*d^8*x^3 + 2275*a^6
*b^4*c*d^9*x^3 + 455*a^7*b^3*d^10*x^3 + 51975*b^10*c^8*d^2*x^2 + 34650*a*b^
9*c^7*d^3*x^2 + 22050*a^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 735
0*a^4*b^6*c^4*d^6*x^2 + 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2
 + 525*a^7*b^3*c*d^9*x^2 + 105*a^8*b^2*d^10*x^2 + 10725*b^10*c^9*d*x + 7425
*a*b^9*c^8*d^2*x + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x + 1890*a
^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x + 225*a^7
*b^3*c^2*d^8*x + 75*a^8*b^2*c*d^9*x + 15*a^9*b*d^10*x + 1001*b^10*c^10 + 71
5*a*b^9*c^9*d + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6
*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a
^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^15*b^11)
```

Mupad [B]
time = 2.28, size = 1120, normalized size = 7.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^16,x)
```

```
[Out] -(a^10*d^10 + 1001*b^10*c^10 + 3003*b^10*d^10*x^10 + 5005*a*b^9*d^10*x^9 +
25025*b^10*c*d^9*x^9 + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7
 + 15*a^8*b^2*c^2*d^8 + 105*a^8*b^2*d^10*x^2 + 455*a^7*b^3*d^10*x^3 + 1365*
a^6*b^4*d^10*x^4 + 3003*a^5*b^5*d^10*x^5 + 5005*a^4*b^6*d^10*x^6 + 6435*a^3
*b^7*d^10*x^7 + 6435*a^2*b^8*d^10*x^8 + 51975*b^10*c^8*d^2*x^2 + 150150*b^1
0*c^7*d^3*x^3 + 286650*b^10*c^6*d^4*x^4 + 378378*b^10*c^5*d^5*x^5 + 350350*
b^10*c^4*d^6*x^6 + 225225*b^10*c^3*d^7*x^7 + 96525*b^10*c^2*d^8*x^8 + 715*a
*b^9*c^9*d + 5*a^9*b*c*d^9 + 15*a^9*b*d^10*x + 10725*b^10*c^9*d*x + 22050*a
^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 7350*a^4*b^6*c^4*d^6*x^2 +
 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2 + 57330*a^2*b^8*c^5*d^
5*x^3 + 31850*a^3*b^7*c^4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^
5*c^2*d^8*x^3 + 95550*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 204
75*a^4*b^6*c^2*d^8*x^4 + 105105*a^2*b^8*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8
*x^5 + 75075*a^2*b^8*c^2*d^8*x^6 + 7425*a*b^9*c^8*d^2*x + 75*a^8*b^2*c*d^9*
x + 32175*a*b^9*c*d^9*x^8 + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x
 + 1890*a^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x
+ 225*a^7*b^3*c^2*d^8*x + 34650*a*b^9*c^7*d^3*x^2 + 525*a^7*b^3*c*d^9*x^2 +
 95550*a*b^9*c^6*d^4*x^3 + 2275*a^6*b^4*c*d^9*x^3 + 171990*a*b^9*c^5*d^5*x^
4 + 6825*a^5*b^5*c*d^9*x^4 + 210210*a*b^9*c^4*d^6*x^5 + 15015*a^4*b^6*c*d^9
*x^5 + 175175*a*b^9*c^3*d^7*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 96525*a*b^9*c^2
*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7)/(15015*a^15*b^11 + 15015*b^26*x^15 + 22
5225*a^14*b^12*x + 225225*a*b^25*x^14 + 1576575*a^13*b^13*x^2 + 6831825*a^1
2*b^14*x^3 + 20495475*a^11*b^15*x^4 + 45090045*a^10*b^16*x^5 + 75150075*a^9
*b^17*x^6 + 96621525*a^8*b^18*x^7 + 96621525*a^7*b^19*x^8 + 75150075*a^6*b^
20*x^9 + 45090045*a^5*b^21*x^10 + 20495475*a^4*b^22*x^11 + 6831825*a^3*b^23
*x^12 + 1576575*a^2*b^24*x^13)
```

3.1328 $\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx$

Optimal. Leaf size=182

$$-\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}-\frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}}+\frac{d^3(c+dx)^{11}}{728(bc-ad)^4(a+bx)^{13}}-436$$

```
[Out] -1/16*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^16+1/48*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x+
a)^15-1/168*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^14+1/728*d^3*(d*x+c)^11/(-a
*d+b*c)^4/(b*x+a)^13-1/4368*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^12+1/48048*
d^5*(d*x+c)^11/(-a*d+b*c)^6/(b*x+a)^11
```

Rubi [A]
time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{d(c+dx)^{11}}{48(a+bx)^{15}(bc-ad)^2}-\frac{(c+dx)^{11}}{16(a+bx)^{16}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -1/16*(c + d*x)^11/((b*c - a*d)*(a + b*x)^16) + (d*(c + d*x)^11)/(48*(b*c -
 a*d)^2*(a + b*x)^15) - (d^2*(c + d*x)^11)/(168*(b*c - a*d)^3*(a + b*x)^14)
 + (d^3*(c + d*x)^11)/(728*(b*c - a*d)^4*(a + b*x)^13) - (d^4*(c + d*x)^11)
/(4368*(b*c - a*d)^5*(a + b*x)^12) + (d^5*(c + d*x)^11)/(48048*(b*c - a*d)^
6*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx = -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} - \frac{(5d)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{16(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} + \frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{12(bc-ad)^2}$$
$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} - \frac{d^3\int \frac{(c+d}{(a+b}}{56(bc-}$$
$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(}{728(bc-}$$
$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(}{728(bc-}$$
$$= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(}{728(bc-}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 694 vs. 2(182) = 364.
time = 0.19, size = 694, normalized size = 3.81

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -1/48048*(a^10*d^10 + 2*a^9*b*d^9*(3*c + 8*d*x) + 3*a^8*b^2*d^8*(7*c^2 + 32
*c*d*x + 40*d^2*x^2) + 8*a^7*b^3*d^7*(7*c^3 + 42*c^2*d*x + 90*c*d^2*x^2 + 7
0*d^3*x^3) + 14*a^6*b^4*d^6*(9*c^4 + 64*c^3*d*x + 180*c^2*d^2*x^2 + 240*c*d
^3*x^3 + 130*d^4*x^4) + 84*a^5*b^5*d^5*(3*c^5 + 24*c^4*d*x + 80*c^3*d^2*x^2
 + 140*c^2*d^3*x^3 + 130*c*d^4*x^4 + 52*d^5*x^5) + 14*a^4*b^6*d^4*(33*c^6 +
 288*c^5*d*x + 1080*c^4*d^2*x^2 + 2240*c^3*d^3*x^3 + 2730*c^2*d^4*x^4 + 187
2*c*d^5*x^5 + 572*d^6*x^6) + 8*a^3*b^7*d^3*(99*c^7 + 924*c^6*d*x + 3780*c^5
*d^2*x^2 + 8820*c^4*d^3*x^3 + 12740*c^3*d^4*x^4 + 11466*c^2*d^5*x^5 + 6006*
c*d^6*x^6 + 1430*d^7*x^7) + 3*a^2*b^8*d^2*(429*c^8 + 4224*c^7*d*x + 18480*c
^6*d^2*x^2 + 47040*c^5*d^3*x^3 + 76440*c^4*d^4*x^4 + 81536*c^3*d^5*x^5 + 56
056*c^2*d^6*x^6 + 22880*c*d^7*x^7 + 4290*d^8*x^8) + 2*a*b^9*d*(1001*c^9 + 1
0296*c^8*d*x + 47520*c^7*d^2*x^2 + 129360*c^6*d^3*x^3 + 229320*c^5*d^4*x^4
+ 275184*c^4*d^5*x^5 + 224224*c^3*d^6*x^6 + 120120*c^2*d^7*x^7 + 38610*c*d^
8*x^8 + 5720*d^9*x^9) + b^10*(3003*c^10 + 32032*c^9*d*x + 154440*c^8*d^2*x^
2 + 443520*c^7*d^3*x^3 + 840840*c^6*d^4*x^4 + 1100736*c^5*d^5*x^5 + 1009008
```

```
*c^4*d^6*x^6 + 640640*c^3*d^7*x^7 + 270270*c^2*d^8*x^8 + 68640*c*d^9*x^9 +
8008*d^10*x^10))/(b^11*(a + b*x)^16)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(170) = 340$.
time = 0.21, size = 867, normalized size = 4.76 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^17,x,method=_RETURNVERBOSE)
```

```
[Out] 120/13/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^
4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^13-3
5/2/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+1
5*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^12-45/14/b^11*d^2*(a^8*d^8
-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*
a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^14+40/3/b
^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^9-45/8/b^11*d
^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^8-21/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3
+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^10+252/11/b^11*d^5*(a^5*d
^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^
5)/(b*x+a)^11-1/16*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3
*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^
3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^16-
1/6*d^10/b^11/(b*x+a)^6+2/3/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^
7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6
*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^15+10/7/b^11*d^9*(a*
d-b*c)/(b*x+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(170) = 340$.
time = 0.36, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
```

```
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(170) = 340$.
time = 0.82, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
```

```
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**17,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(170) = 340.
time = 0.96, size = 961, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 68640*b^10*c*d^9*x^9 + 11440*a*b^9*d^10*x^9
 + 270270*b^10*c^2*d^8*x^8 + 77220*a*b^9*c*d^9*x^8 + 12870*a^2*b^8*d^10*x^8
 + 640640*b^10*c^3*d^7*x^7 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9
*x^7 + 11440*a^3*b^7*d^10*x^7 + 1009008*b^10*c^4*d^6*x^6 + 448448*a*b^9*c^3
*d^7*x^6 + 168168*a^2*b^8*c^2*d^8*x^6 + 48048*a^3*b^7*c*d^9*x^6 + 8008*a^4*
b^6*d^10*x^6 + 1100736*b^10*c^5*d^5*x^5 + 550368*a*b^9*c^4*d^6*x^5 + 244608
*a^2*b^8*c^3*d^7*x^5 + 91728*a^3*b^7*c^2*d^8*x^5 + 26208*a^4*b^6*c*d^9*x^5
+ 4368*a^5*b^5*d^10*x^5 + 840840*b^10*c^6*d^4*x^4 + 458640*a*b^9*c^5*d^5*x^
4 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^3*d^7*x^4 + 38220*a^4*b^6
*c^2*d^8*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 1820*a^6*b^4*d^10*x^4 + 443520*b^1
0*c^7*d^3*x^3 + 258720*a*b^9*c^6*d^4*x^3 + 141120*a^2*b^8*c^5*d^5*x^3 + 705
60*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 + 11760*a^5*b^5*c^2*d^8*
x^3 + 3360*a^6*b^4*c*d^9*x^3 + 560*a^7*b^3*d^10*x^3 + 154440*b^10*c^8*d^2*x
^2 + 95040*a*b^9*c^7*d^3*x^2 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^
5*d^5*x^2 + 15120*a^4*b^6*c^4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6
*b^4*c^2*d^8*x^2 + 720*a^7*b^3*c*d^9*x^2 + 120*a^8*b^2*d^10*x^2 + 32032*b^1
0*c^9*d*x + 20592*a*b^9*c^8*d^2*x + 12672*a^2*b^8*c^7*d^3*x + 7392*a^3*b^7*
c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*a^6*b^4*c
^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 96*a^8*b^2*c*d^9*x + 16*a^9*b*d^10*x + 3
003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d
^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a
^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)
^16*b^11)
```

Mupad [B]
time = 0.58, size = 1131, normalized size = 6.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^17,x)
```

```
[Out] -(a^10*d^10 + 3003*b^10*c^10 + 8008*b^10*d^10*x^10 + 11440*a*b^9*d^10*x^9 +
 68640*b^10*c*d^9*x^9 + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^
4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*
d^7 + 21*a^8*b^2*c^2*d^8 + 120*a^8*b^2*d^10*x^2 + 560*a^7*b^3*d^10*x^3 + 18
20*a^6*b^4*d^10*x^4 + 4368*a^5*b^5*d^10*x^5 + 8008*a^4*b^6*d^10*x^6 + 11440
*a^3*b^7*d^10*x^7 + 12870*a^2*b^8*d^10*x^8 + 154440*b^10*c^8*d^2*x^2 + 4435
20*b^10*c^7*d^3*x^3 + 840840*b^10*c^6*d^4*x^4 + 1100736*b^10*c^5*d^5*x^5 +
1009008*b^10*c^4*d^6*x^6 + 640640*b^10*c^3*d^7*x^7 + 270270*b^10*c^2*d^8*x^
8 + 2002*a*b^9*c^9*d + 6*a^9*b*c*d^9 + 16*a^9*b*d^10*x + 32032*b^10*c^9*d*x
 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^5*d^5*x^2 + 15120*a^4*b^6*c^
4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6*b^4*c^2*d^8*x^2 + 141120*a^
2*b^8*c^5*d^5*x^3 + 70560*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 +
 11760*a^5*b^5*c^2*d^8*x^3 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^
3*d^7*x^4 + 38220*a^4*b^6*c^2*d^8*x^4 + 244608*a^2*b^8*c^3*d^7*x^5 + 91728*
a^3*b^7*c^2*d^8*x^5 + 168168*a^2*b^8*c^2*d^8*x^6 + 20592*a*b^9*c^8*d^2*x +
96*a^8*b^2*c*d^9*x + 77220*a*b^9*c*d^9*x^8 + 12672*a^2*b^8*c^7*d^3*x + 7392
*a^3*b^7*c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*
a^6*b^4*c^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 95040*a*b^9*c^7*d^3*x^2 + 720*a
^7*b^3*c*d^9*x^2 + 258720*a*b^9*c^6*d^4*x^3 + 3360*a^6*b^4*c*d^9*x^3 + 4586
40*a*b^9*c^5*d^5*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 550368*a*b^9*c^4*d^6*x^5 +
 26208*a^4*b^6*c*d^9*x^5 + 448448*a*b^9*c^3*d^7*x^6 + 48048*a^3*b^7*c*d^9*x
^6 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9*x^7)/(48048*a^16*b^11 +
 48048*b^27*x^16 + 768768*a^15*b^12*x + 768768*a*b^26*x^15 + 5765760*a^14*b
^13*x^2 + 26906880*a^13*b^14*x^3 + 87447360*a^12*b^15*x^4 + 209873664*a^11*
b^16*x^5 + 384768384*a^10*b^17*x^6 + 549669120*a^9*b^18*x^7 + 618377760*a^8
*b^19*x^8 + 549669120*a^7*b^20*x^9 + 384768384*a^6*b^21*x^10 + 209873664*a^
5*b^22*x^11 + 87447360*a^4*b^23*x^12 + 26906880*a^3*b^24*x^13 + 5765760*a^2
*b^25*x^14)
```

3.1329 $\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx$

Optimal. Leaf size=213

$$-\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}}+\frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}}-\frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}}+\frac{d^3(c+dx)^{11}}{476(bc-ad)^4(a+bx)^{14}}-61$$

```
[Out] -1/17*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^17+3/136*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^16-1/136*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^15+1/476*d^3*(d*x+c)^11/(-
a*d+b*c)^4/(b*x+a)^14-3/6188*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^13+1/12376
*d^5*(d*x+c)^11/(-a*d+b*c)^6/(b*x+a)^12-1/136136*d^6*(d*x+c)^11/(-a*d+b*c)^
7/(b*x+a)^11
```

Rubi [A]
time = 0.06, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{136(a+bx)^{15}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{136(a+bx)^{16}(bc-ad)^2}-\frac{(c+dx)^{11}}{17(a+bx)^{17}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -1/17*(c + d*x)^11/((b*c - a*d)*(a + b*x)^17) + (3*d*(c + d*x)^11)/(136*(b*
c - a*d)^2*(a + b*x)^16) - (d^2*(c + d*x)^11)/(136*(b*c - a*d)^3*(a + b*x)^
15) + (d^3*(c + d*x)^11)/(476*(b*c - a*d)^4*(a + b*x)^14) - (3*d^4*(c + d*x
)^11)/(6188*(b*c - a*d)^5*(a + b*x)^13) + (d^5*(c + d*x)^11)/(12376*(b*c -
a*d)^6*(a + b*x)^12) - (d^6*(c + d*x)^11)/(136136*(b*c - a*d)^7*(a + b*x)^1
1)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx = -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} - \frac{(6d)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{17(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} + \frac{(15d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{136(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} - \frac{d^3\int \frac{(c+}{(a+}}{34(bc}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3}{476(bc}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3}{476(bc}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3}{476(bc}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3}{476(bc}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(213) = 426.
time = 0.21, size = 690, normalized size = 3.24

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -1/136136*(a^10*d^10 + a^9*b*d^9*(7*c + 17*d*x) + a^8*b^2*d^8*(28*c^2 + 119
*c*d*x + 136*d^2*x^2) + 4*a^7*b^3*d^7*(21*c^3 + 119*c^2*d*x + 238*c*d^2*x^2
 + 170*d^3*x^3) + 14*a^6*b^4*d^6*(15*c^4 + 102*c^3*d*x + 272*c^2*d^2*x^2 +
340*c*d^3*x^3 + 170*d^4*x^4) + 14*a^5*b^5*d^5*(33*c^5 + 255*c^4*d*x + 816*c
^3*d^2*x^2 + 1360*c^2*d^3*x^3 + 1190*c*d^4*x^4 + 442*d^5*x^5) + 14*a^4*b^6*
d^4*(66*c^6 + 561*c^5*d*x + 2040*c^4*d^2*x^2 + 4080*c^3*d^3*x^3 + 4760*c^2*
d^4*x^4 + 3094*c*d^5*x^5 + 884*d^6*x^6) + 4*a^3*b^7*d^3*(429*c^7 + 3927*c^6
*d*x + 15708*c^5*d^2*x^2 + 35700*c^4*d^3*x^3 + 49980*c^3*d^4*x^4 + 43316*c^
2*d^5*x^5 + 21658*c*d^6*x^6 + 4862*d^7*x^7) + a^2*b^8*d^2*(3003*c^8 + 29172
*c^7*d*x + 125664*c^6*d^2*x^2 + 314160*c^5*d^3*x^3 + 499800*c^4*d^4*x^4 + 5
```

```
19792*c^3*d^5*x^5 + 346528*c^2*d^6*x^6 + 136136*c*d^7*x^7 + 24310*d^8*x^8)
+ a*b^9*d*(5005*c^9 + 51051*c^8*d*x + 233376*c^7*d^2*x^2 + 628320*c^6*d^3*x
^3 + 1099560*c^5*d^4*x^4 + 1299480*c^4*d^5*x^5 + 1039584*c^3*d^6*x^6 + 5445
44*c^2*d^7*x^7 + 170170*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(8008*c^10 + 8508
5*c^9*d*x + 408408*c^8*d^2*x^2 + 1166880*c^7*d^3*x^3 + 2199120*c^6*d^4*x^4
+ 2858856*c^5*d^5*x^5 + 2598960*c^4*d^6*x^6 + 1633632*c^3*d^7*x^7 + 680680*
c^2*d^8*x^8 + 170170*c*d^9*x^9 + 19448*d^10*x^10))/(b^11*(a + b*x)^17)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(199) = 398$.
time = 0.14, size = 867, normalized size = 4.07 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^18,x,method=_RETURNVERBOSE)
```

```
[Out] -210/13/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d
^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^13+21/b^11*d^5*(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)/(b*x+a)^12+60/7/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4
*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(
b*x+a)^14-5/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^9+5/4/b^11*d^9*(a*
d-b*c)/(b*x+a)^8+12/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/
(b*x+a)^10-210/11/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3
*c^3*d+b^4*c^4)/(b*x+a)^11+5/8/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2
*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*
c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^16-3/b^11*d^2*(a^
8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^
4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^15-1
/7*d^10/b^11/(b*x+a)^7-1/17*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-12
0*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d
^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b
*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(199) = 398$.
time = 0.37, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
```

```
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(199) = 398$.
time = 1.06, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
```

```
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**18,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(199) = 398.
time = 0.76, size = 961, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 170170*b^10*c*d^9*x^9 + 24310*a*b^9*d^10*
x^9 + 680680*b^10*c^2*d^8*x^8 + 170170*a*b^9*c*d^9*x^8 + 24310*a^2*b^8*d^10
*x^8 + 1633632*b^10*c^3*d^7*x^7 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8
*c*d^9*x^7 + 19448*a^3*b^7*d^10*x^7 + 2598960*b^10*c^4*d^6*x^6 + 1039584*a*
b^9*c^3*d^7*x^6 + 346528*a^2*b^8*c^2*d^8*x^6 + 86632*a^3*b^7*c*d^9*x^6 + 12
376*a^4*b^6*d^10*x^6 + 2858856*b^10*c^5*d^5*x^5 + 1299480*a*b^9*c^4*d^6*x^5
 + 519792*a^2*b^8*c^3*d^7*x^5 + 173264*a^3*b^7*c^2*d^8*x^5 + 43316*a^4*b^6*
c*d^9*x^5 + 6188*a^5*b^5*d^10*x^5 + 2199120*b^10*c^6*d^4*x^4 + 1099560*a*b^
9*c^5*d^5*x^4 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*a^3*b^7*c^3*d^7*x^4 + 6
6640*a^4*b^6*c^2*d^8*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 2380*a^6*b^4*d^10*x^4
+ 1166880*b^10*c^7*d^3*x^3 + 628320*a*b^9*c^6*d^4*x^3 + 314160*a^2*b^8*c^5*
d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3*d^7*x^3 + 19040*a^
5*b^5*c^2*d^8*x^3 + 4760*a^6*b^4*c*d^9*x^3 + 680*a^7*b^3*d^10*x^3 + 408408*
b^10*c^8*d^2*x^2 + 233376*a*b^9*c^7*d^3*x^2 + 125664*a^2*b^8*c^6*d^4*x^2 +
62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d
^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 + 952*a^7*b^3*c*d^9*x^2 + 136*a^8*b^2*d^1
0*x^2 + 85085*b^10*c^9*d*x + 51051*a*b^9*c^8*d^2*x + 29172*a^2*b^8*c^7*d^3*
```

```
x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^4*d^6
*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 119*a^8*b^2*c*d^9*x +
 17*a^9*b*d^10*x + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*a^2*b^8*c^8*d^2
 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a
^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7*a^9*b*c*d^9 +
a^10*d^10)/((b*x + a)^17*b^11)
```

Mupad [B]
time = 0.66, size = 1142, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^18,x)
```

```
[Out] -(a^10*d^10 + 8008*b^10*c^10 + 19448*b^10*d^10*x^10 + 24310*a*b^9*d^10*x^9
+ 170170*b^10*c*d^9*x^9 + 3003*a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924
*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c
^3*d^7 + 28*a^8*b^2*c^2*d^8 + 136*a^8*b^2*d^10*x^2 + 680*a^7*b^3*d^10*x^3 +
 2380*a^6*b^4*d^10*x^4 + 6188*a^5*b^5*d^10*x^5 + 12376*a^4*b^6*d^10*x^6 + 1
9448*a^3*b^7*d^10*x^7 + 24310*a^2*b^8*d^10*x^8 + 408408*b^10*c^8*d^2*x^2 +
1166880*b^10*c^7*d^3*x^3 + 2199120*b^10*c^6*d^4*x^4 + 2858856*b^10*c^5*d^5*
x^5 + 2598960*b^10*c^4*d^6*x^6 + 1633632*b^10*c^3*d^7*x^7 + 680680*b^10*c^2
*d^8*x^8 + 5005*a*b^9*c^9*d + 7*a^9*b*c*d^9 + 17*a^9*b*d^10*x + 85085*b^10*
c^9*d*x + 125664*a^2*b^8*c^6*d^4*x^2 + 62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^
4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 +
314160*a^2*b^8*c^5*d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3
*d^7*x^3 + 19040*a^5*b^5*c^2*d^8*x^3 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*
a^3*b^7*c^3*d^7*x^4 + 66640*a^4*b^6*c^2*d^8*x^4 + 519792*a^2*b^8*c^3*d^7*x^
5 + 173264*a^3*b^7*c^2*d^8*x^5 + 346528*a^2*b^8*c^2*d^8*x^6 + 51051*a*b^9*c
^8*d^2*x + 119*a^8*b^2*c*d^9*x + 170170*a*b^9*c*d^9*x^8 + 29172*a^2*b^8*c^7
*d^3*x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^
4*d^6*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 233376*a*b^9*c^7
*d^3*x^2 + 952*a^7*b^3*c*d^9*x^2 + 628320*a*b^9*c^6*d^4*x^3 + 4760*a^6*b^4*
c*d^9*x^3 + 1099560*a*b^9*c^5*d^5*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 1299480*a
*b^9*c^4*d^6*x^5 + 43316*a^4*b^6*c*d^9*x^5 + 1039584*a*b^9*c^3*d^7*x^6 + 86
632*a^3*b^7*c*d^9*x^6 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8*c*d^9*x^7
)/(136136*a^17*b^11 + 136136*b^28*x^17 + 2314312*a^16*b^12*x + 2314312*a*b^
27*x^16 + 18514496*a^15*b^13*x^2 + 92572480*a^14*b^14*x^3 + 324003680*a^13*
b^15*x^4 + 842409568*a^12*b^16*x^5 + 1684819136*a^11*b^17*x^6 + 2647572928*
a^10*b^18*x^7 + 3309466160*a^9*b^19*x^8 + 3309466160*a^8*b^20*x^9 + 2647572
928*a^7*b^21*x^10 + 1684819136*a^6*b^22*x^11 + 842409568*a^5*b^23*x^12 + 32
4003680*a^4*b^24*x^13 + 92572480*a^3*b^25*x^14 + 18514496*a^2*b^26*x^15)
```

3.1330 $\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$

Optimal. Leaf size=244

$$-\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}}+\frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}}-\frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}}+\frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx)^{15}}-$$

```
[Out] -1/18*(d*x+c)^11/(-a*d+b*c)/(b*x+a)^18+7/306*d*(d*x+c)^11/(-a*d+b*c)^2/(b*x
+a)^17-7/816*d^2*(d*x+c)^11/(-a*d+b*c)^3/(b*x+a)^16+7/2448*d^3*(d*x+c)^11/(
-a*d+b*c)^4/(b*x+a)^15-1/1224*d^4*(d*x+c)^11/(-a*d+b*c)^5/(b*x+a)^14+1/5304
*d^5*(d*x+c)^11/(-a*d+b*c)^6/(b*x+a)^13-1/31824*d^6*(d*x+c)^11/(-a*d+b*c)^7
/(b*x+a)^12+1/350064*d^7*(d*x+c)^11/(-a*d+b*c)^8/(b*x+a)^11
```

Rubi [A]
time = 0.07, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+\frac{7d^3(c+dx)^{11}}{2448(a+bx)^{15}(bc-ad)^4}-\frac{7d^2(c+dx)^{11}}{816(a+bx)^{16}(bc-ad)^3}+\frac{7d(c+dx)^{11}}{306(a+bx)^{17}(bc-ad)^2}-\frac{(c+dx)^{11}}{18(a+bx)^{18}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -1/18*(c + d*x)^11/((b*c - a*d)*(a + b*x)^18) + (7*d*(c + d*x)^11)/(306*(b*
c - a*d)^2*(a + b*x)^17) - (7*d^2*(c + d*x)^11)/(816*(b*c - a*d)^3*(a + b*x
)^16) + (7*d^3*(c + d*x)^11)/(2448*(b*c - a*d)^4*(a + b*x)^15) - (d^4*(c +
d*x)^11)/(1224*(b*c - a*d)^5*(a + b*x)^14) + (d^5*(c + d*x)^11)/(5304*(b*c
- a*d)^6*(a + b*x)^13) - (d^6*(c + d*x)^11)/(31824*(b*c - a*d)^7*(a + b*x)^
12) + (d^7*(c + d*x)^11)/(350064*(b*c - a*d)^8*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx = -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} - \frac{(7d)\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx}{18(bc-ad)}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} + \frac{(7d^2)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{51(bc-ad)^2}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} - \frac{(35d^3)}{816(}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7}{2448(bc}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7}{2448(bc}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7}{2448(bc}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7}{2448(bc}$$
$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7}{2448(bc}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 694 vs. 2(244) = 488.
time = 0.19, size = 694, normalized size = 2.84

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -1/350064*(a^10*d^10 + 2*a^9*b*d^9*(4*c + 9*d*x) + 9*a^8*b^2*d^8*(4*c^2 + 1
6*c*d*x + 17*d^2*x^2) + 24*a^7*b^3*d^7*(5*c^3 + 27*c^2*d*x + 51*c*d^2*x^2 +
 34*d^3*x^3) + 6*a^6*b^4*d^6*(55*c^4 + 360*c^3*d*x + 918*c^2*d^2*x^2 + 1088
*c*d^3*x^3 + 510*d^4*x^4) + 36*a^5*b^5*d^5*(22*c^5 + 165*c^4*d*x + 510*c^3*
d^2*x^2 + 816*c^2*d^3*x^3 + 680*c*d^4*x^4 + 238*d^5*x^5) + 6*a^4*b^6*d^4*(2
86*c^6 + 2376*c^5*d*x + 8415*c^4*d^2*x^2 + 16320*c^3*d^3*x^3 + 18360*c^2*d^
4*x^4 + 11424*c*d^5*x^5 + 3094*d^6*x^6) + 24*a^3*b^7*d^3*(143*c^7 + 1287*c^
```

```
6*d*x + 5049*c^5*d^2*x^2 + 11220*c^4*d^3*x^3 + 15300*c^3*d^4*x^4 + 12852*c^
2*d^5*x^5 + 6188*c*d^6*x^6 + 1326*d^7*x^7) + 9*a^2*b^8*d^2*(715*c^8 + 6864*
c^7*d*x + 29172*c^6*d^2*x^2 + 71808*c^5*d^3*x^3 + 112200*c^4*d^4*x^4 + 1142
40*c^3*d^5*x^5 + 74256*c^2*d^6*x^6 + 28288*c*d^7*x^7 + 4862*d^8*x^8) + 2*a*
b^9*d*(5720*c^9 + 57915*c^8*d*x + 262548*c^7*d^2*x^2 + 700128*c^6*d^3*x^3 +
 1211760*c^5*d^4*x^4 + 1413720*c^4*d^5*x^5 + 1113840*c^3*d^6*x^6 + 572832*c
^2*d^7*x^7 + 175032*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(19448*c^10 + 205920*
c^9*d*x + 984555*c^8*d^2*x^2 + 2800512*c^7*d^3*x^3 + 5250960*c^6*d^4*x^4 +
6785856*c^5*d^5*x^5 + 6126120*c^4*d^6*x^6 + 3818880*c^3*d^7*x^7 + 1575288*c
^2*d^8*x^8 + 388960*c*d^9*x^9 + 43758*d^10*x^10))/(b^11*(a + b*x)^18)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(228) = 456$.
time = 0.16, size = 867, normalized size = 3.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^19,x,method=_RETURNVERBOSE)
```

```
[Out] 252/13/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^
2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^13-35/2/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^12-15/b^11*d^4*(a^6*d^6-6*a^
5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*
c^5*d+b^6*c^6)/(b*x+a)^14+10/9/b^11*d^9*(a*d-b*c)/(b*x+a)^9-1/8*d^10/b^11/(
b*x+a)^8-9/2/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^10+120/11/b^11*d^
7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^11-45/16/b^11*d^2*(
a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*
d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^16
+8/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35
*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^15-1/18*
(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^
4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^
2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^18+10/17/b^11*d*(a^9*d
^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-
126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9
*c^9)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(228) = 456$.
time = 0.45, size = 1052, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
```

```
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(228) = 456$.
time = 0.87, size = 1052, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
```

```
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**19,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(228) = 456.
time = 0.74, size = 961, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 388960*b^10*c*d^9*x^9 + 48620*a*b^9*d^10*
x^9 + 1575288*b^10*c^2*d^8*x^8 + 350064*a*b^9*c*d^9*x^8 + 43758*a^2*b^8*d^1
0*x^8 + 3818880*b^10*c^3*d^7*x^7 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a^2*b
^8*c*d^9*x^7 + 31824*a^3*b^7*d^10*x^7 + 6126120*b^10*c^4*d^6*x^6 + 2227680*
a*b^9*c^3*d^7*x^6 + 668304*a^2*b^8*c^2*d^8*x^6 + 148512*a^3*b^7*c*d^9*x^6 +
 18564*a^4*b^6*d^10*x^6 + 6785856*b^10*c^5*d^5*x^5 + 2827440*a*b^9*c^4*d^6*
x^5 + 1028160*a^2*b^8*c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 68544*a^4*
b^6*c*d^9*x^5 + 8568*a^5*b^5*d^10*x^5 + 5250960*b^10*c^6*d^4*x^4 + 2423520*
a*b^9*c^5*d^5*x^4 + 1009800*a^2*b^8*c^4*d^6*x^4 + 367200*a^3*b^7*c^3*d^7*x^
4 + 110160*a^4*b^6*c^2*d^8*x^4 + 24480*a^5*b^5*c*d^9*x^4 + 3060*a^6*b^4*d^1
0*x^4 + 2800512*b^10*c^7*d^3*x^3 + 1400256*a*b^9*c^6*d^4*x^3 + 646272*a^2*b
^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*b^6*c^3*d^7*x^3 + 2
9376*a^5*b^5*c^2*d^8*x^3 + 6528*a^6*b^4*c*d^9*x^3 + 816*a^7*b^3*d^10*x^3 +
984555*b^10*c^8*d^2*x^2 + 525096*a*b^9*c^7*d^3*x^2 + 262548*a^2*b^8*c^6*d^4
*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 50490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b
```

```
^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 153*a^
8*b^2*d^10*x^2 + 205920*b^10*c^9*d*x + 115830*a*b^9*c^8*d^2*x + 61776*a^2*b
^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 5940*a^5
*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 144*a^8*b
^2*c*d^9*x + 18*a^9*b*d^10*x + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 6435*a
^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b^5*
c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 +
8*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^18*b^11)
```

Mupad [B]
time = 12.02, size = 1153, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^19,x)
```

```
[Out] -(a^10*d^10 + 19448*b^10*c^10 + 43758*b^10*d^10*x^10 + 48620*a*b^9*d^10*x^9
 + 388960*b^10*c*d^9*x^9 + 6435*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 17
16*a^4*b^6*c^6*d^4 + 792*a^5*b^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^
3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 + 153*a^8*b^2*d^10*x^2 + 816*a^7*b^3*d^10*x^
3 + 3060*a^6*b^4*d^10*x^4 + 8568*a^5*b^5*d^10*x^5 + 18564*a^4*b^6*d^10*x^6
+ 31824*a^3*b^7*d^10*x^7 + 43758*a^2*b^8*d^10*x^8 + 984555*b^10*c^8*d^2*x^2
 + 2800512*b^10*c^7*d^3*x^3 + 5250960*b^10*c^6*d^4*x^4 + 6785856*b^10*c^5*d
^5*x^5 + 6126120*b^10*c^4*d^6*x^6 + 3818880*b^10*c^3*d^7*x^7 + 1575288*b^10
*c^2*d^8*x^8 + 11440*a*b^9*c^9*d + 8*a^9*b*c*d^9 + 18*a^9*b*d^10*x + 205920
*b^10*c^9*d*x + 262548*a^2*b^8*c^6*d^4*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 5
0490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8
*x^2 + 646272*a^2*b^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*
b^6*c^3*d^7*x^3 + 29376*a^5*b^5*c^2*d^8*x^3 + 1009800*a^2*b^8*c^4*d^6*x^4 +
 367200*a^3*b^7*c^3*d^7*x^4 + 110160*a^4*b^6*c^2*d^8*x^4 + 1028160*a^2*b^8*
c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 668304*a^2*b^8*c^2*d^8*x^6 + 115
830*a*b^9*c^8*d^2*x + 144*a^8*b^2*c*d^9*x + 350064*a*b^9*c*d^9*x^8 + 61776*
a^2*b^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 594
0*a^5*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 5250
96*a*b^9*c^7*d^3*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 1400256*a*b^9*c^6*d^4*x^3 +
 6528*a^6*b^4*c*d^9*x^3 + 2423520*a*b^9*c^5*d^5*x^4 + 24480*a^5*b^5*c*d^9*x
^4 + 2827440*a*b^9*c^4*d^6*x^5 + 68544*a^4*b^6*c*d^9*x^5 + 2227680*a*b^9*c^
3*d^7*x^6 + 148512*a^3*b^7*c*d^9*x^6 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a
^2*b^8*c*d^9*x^7)/(350064*a^18*b^11 + 350064*b^29*x^18 + 6301152*a^17*b^12*
x + 6301152*a*b^28*x^17 + 53559792*a^16*b^13*x^2 + 285652224*a^15*b^14*x^3
+ 1071195840*a^14*b^15*x^4 + 2999348352*a^13*b^16*x^5 + 6498588096*a^12*b^1
7*x^6 + 11140436736*a^11*b^18*x^7 + 15318100512*a^10*b^19*x^8 + 17020111680
*a^9*b^20*x^9 + 15318100512*a^8*b^21*x^10 + 11140436736*a^7*b^22*x^11 + 649
8588096*a^6*b^23*x^12 + 2999348352*a^5*b^24*x^13 + 1071195840*a^4*b^25*x^14
 + 285652224*a^3*b^26*x^15 + 53559792*a^2*b^27*x^16)
```

3.1331 $\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$

Optimal. Leaf size=273

$$-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{21\ldots}{13\ldots}$$

```
[Out] -1/19*(-a*d+b*c)^10/b^11/(b*x+a)^19-5/9*d*(-a*d+b*c)^9/b^11/(b*x+a)^18-45/1
7*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^17-15/2*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^16-14
*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^15-18*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^14-210/1
3*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^13-10*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^12-45/1
1*d^8*(-a*d+b*c)^2/b^11/(b*x+a)^11-d^9*(-a*d+b*c)/b^11/(b*x+a)^10-1/9*d^10/
b^11/(b*x+a)^9
```

Rubi [A]

time = 0.21, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{d^{10}}{9b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -1/19*(b*c - a*d)^10/(b^11*(a + b*x)^19) - (5*d*(b*c - a*d)^9)/(9*b^11*(a +
 b*x)^18) - (45*d^2*(b*c - a*d)^8)/(17*b^11*(a + b*x)^17) - (15*d^3*(b*c -
a*d)^7)/(2*b^11*(a + b*x)^16) - (14*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^15)
- (18*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^14) - (210*d^6*(b*c - a*d)^4)/(13*
b^11*(a + b*x)^13) - (10*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^12) - (45*d^8*(
b*c - a*d)^2)/(11*b^11*(a + b*x)^11) - (d^9*(b*c - a*d))/(b^11*(a + b*x)^10
) - d^10/(9*b^11*(a + b*x)^9)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{20}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{19}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{18}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{17}}+\frac{210d^4(bc-\ldots}{b^{10}(a+bx\ldots}\right.\\ &= -\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-a\ldots}{b^{11}(a+bx)^{1\ldots}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. 2(273) = 546.
time = 0.19, size = 692, normalized size = 2.53

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -1/831402*(a^10*d^10 + a^9*b*d^9*(9*c + 19*d*x) + 9*a^8*b^2*d^8*(5*c^2 + 19
*c*d*x + 19*d^2*x^2) + 3*a^7*b^3*d^7*(55*c^3 + 285*c^2*d*x + 513*c*d^2*x^2
+ 323*d^3*x^3) + 3*a^6*b^4*d^6*(165*c^4 + 1045*c^3*d*x + 2565*c^2*d^2*x^2 +
 2907*c*d^3*x^3 + 1292*d^4*x^4) + 9*a^5*b^5*d^5*(143*c^5 + 1045*c^4*d*x + 3
135*c^3*d^2*x^2 + 4845*c^2*d^3*x^3 + 3876*c*d^4*x^4 + 1292*d^5*x^5) + 3*a^4
*b^6*d^4*(1001*c^6 + 8151*c^5*d*x + 28215*c^4*d^2*x^2 + 53295*c^3*d^3*x^3 +
 58140*c^2*d^4*x^4 + 34884*c*d^5*x^5 + 9044*d^6*x^6) + 3*a^3*b^7*d^3*(2145*
c^7 + 19019*c^6*d*x + 73359*c^5*d^2*x^2 + 159885*c^4*d^3*x^3 + 213180*c^3*d
^4*x^4 + 174420*c^2*d^5*x^5 + 81396*c*d^6*x^6 + 16796*d^7*x^7) + 9*a^2*b^8*
d^2*(1430*c^8 + 13585*c^7*d*x + 57057*c^6*d^2*x^2 + 138567*c^5*d^3*x^3 + 21
3180*c^4*d^4*x^4 + 213180*c^3*d^5*x^5 + 135660*c^2*d^6*x^6 + 50388*c*d^7*x^
7 + 8398*d^8*x^8) + a*b^9*d*(24310*c^9 + 244530*c^8*d*x + 1100385*c^7*d^2*x
^2 + 2909907*c^6*d^3*x^3 + 4988412*c^5*d^4*x^4 + 5755860*c^4*d^5*x^5 + 4476
780*c^3*d^6*x^6 + 2267460*c^2*d^7*x^7 + 680238*c*d^8*x^8 + 92378*d^9*x^9) +
 b^10*(43758*c^10 + 461890*c^9*d*x + 2200770*c^8*d^2*x^2 + 6235515*c^7*d^3*
x^3 + 11639628*c^6*d^4*x^4 + 14965236*c^5*d^5*x^5 + 13430340*c^4*d^6*x^6 +
8314020*c^3*d^7*x^7 + 3401190*c^2*d^8*x^8 + 831402*c*d^9*x^9 + 92378*d^10*x
^10))/(b^11*(a + b*x)^19)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(259) = 518.
time = 0.15, size = 866, normalized size = 3.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^20,x,method=_RETURNVERBOSE)
```

```
[Out] -210/13/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4
*c^4)/(b*x+a)^13+10/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/
(b*x+a)^12+18/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3
*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^14-1/9*d^10/b^11/(b*x+a)^9-1/19*(a^
10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c
^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b
^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^19+1/b^11*d^9*(a*d-b*c)/(
b*x+a)^10-45/11/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^11-14/b^11*d^4
*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^
4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^15+5/9/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+3
6*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^
```

```
4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^18+1
5/2/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+3
5*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^16-45/1
7/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*
a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8
)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(259) = 518$.
time = 0.47, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(259) = 518$.
time = 0.56, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**20,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(259) = 518.
time = 0.80, size = 961, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 831402*b^10*c*d^9*x^9 + 92378*a*b^9*d^10*
x^9 + 3401190*b^10*c^2*d^8*x^8 + 680238*a*b^9*c*d^9*x^8 + 75582*a^2*b^8*d^1
0*x^8 + 8314020*b^10*c^3*d^7*x^7 + 2267460*a*b^9*c^2*d^8*x^7 + 453492*a^2*b
^8*c*d^9*x^7 + 50388*a^3*b^7*d^10*x^7 + 13430340*b^10*c^4*d^6*x^6 + 4476780
*a*b^9*c^3*d^7*x^6 + 1220940*a^2*b^8*c^2*d^8*x^6 + 244188*a^3*b^7*c*d^9*x^6
 + 27132*a^4*b^6*d^10*x^6 + 14965236*b^10*c^5*d^5*x^5 + 5755860*a*b^9*c^4*d
^6*x^5 + 1918620*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 104652*
a^4*b^6*c*d^9*x^5 + 11628*a^5*b^5*d^10*x^5 + 11639628*b^10*c^6*d^4*x^4 + 49
88412*a*b^9*c^5*d^5*x^4 + 1918620*a^2*b^8*c^4*d^6*x^4 + 639540*a^3*b^7*c^3*
d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 34884*a^5*b^5*c*d^9*x^4 + 3876*a^6*b
^4*d^10*x^4 + 6235515*b^10*c^7*d^3*x^3 + 2909907*a*b^9*c^6*d^4*x^3 + 124710
3*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 159885*a^4*b^6*c^3*d^7
*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 969*a^7*b^3*d^1
0*x^3 + 2200770*b^10*c^8*d^2*x^2 + 1100385*a*b^9*c^7*d^3*x^2 + 513513*a^2*b
^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 2
8215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*c^2*d^8*x^2 + 1539*a^7*b^3*c*d^9*x^
2 + 171*a^8*b^2*d^10*x^2 + 461890*b^10*c^9*d*x + 244530*a*b^9*c^8*d^2*x + 1
22265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*d^5*x
 + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d^8*x
+ 171*a^8*b^2*c*d^9*x + 19*a^9*b*d^10*x + 43758*b^10*c^10 + 24310*a*b^9*c^9
*d + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 +
1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b
^2*c^2*d^8 + 9*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^19*b^11)
```

Mupad [B]
time = 25.72, size = 1164, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^20,x)
```

```
[Out] -(a^10*d^10 + 43758*b^10*c^10 + 92378*b^10*d^10*x^10 + 92378*a*b^9*d^10*x^9
 + 831402*b^10*c*d^9*x^9 + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3
003*a^4*b^6*c^6*d^4 + 1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*
b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 + 171*a^8*b^2*d^10*x^2 + 969*a^7*b^3*d^10*
x^3 + 3876*a^6*b^4*d^10*x^4 + 11628*a^5*b^5*d^10*x^5 + 27132*a^4*b^6*d^10*x
^6 + 50388*a^3*b^7*d^10*x^7 + 75582*a^2*b^8*d^10*x^8 + 2200770*b^10*c^8*d^2
*x^2 + 6235515*b^10*c^7*d^3*x^3 + 11639628*b^10*c^6*d^4*x^4 + 14965236*b^10
*c^5*d^5*x^5 + 13430340*b^10*c^4*d^6*x^6 + 8314020*b^10*c^3*d^7*x^7 + 34011
90*b^10*c^2*d^8*x^8 + 24310*a*b^9*c^9*d + 9*a^9*b*c*d^9 + 19*a^9*b*d^10*x +
 461890*b^10*c^9*d*x + 513513*a^2*b^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*
x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 28215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*
c^2*d^8*x^2 + 1247103*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 15
```

9885*a^4*b^6*c^3*d^7*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 1918620*a^2*b^8*c^4*d^6*x^4 + 639540*a^3*b^7*c^3*d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 1918620*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 1220940*a^2*b^8*c^2*d^8*x^6 + 244530*a*b^9*c^8*d^2*x + 171*a^8*b^2*c*d^9*x + 680238*a*b^9*c*d^9*x^8 + 122265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*d^5*x + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d^8*x + 1100385*a*b^9*c^7*d^3*x^2 + 1539*a^7*b^3*c*d^9*x^2 + 2909907*a*b^9*c^6*d^4*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 4988412*a*b^9*c^5*d^5*x^4 + 34884*a^5*b^5*c*d^9*x^4 + 5755860*a*b^9*c^4*d^6*x^5 + 104652*a^4*b^6*c*d^9*x^5 + 4476780*a*b^9*c^3*d^7*x^6 + 244188*a^3*b^7*c*d^9*x^6 + 2267460*a*b^9*c^2*d^8*x^7 + 453492*a^2*b^8*c*d^9*x^7)/(831402*a^19*b^11 + 831402*b^30*x^19 + 15796638*a^18*b^12*x + 15796638*a*b^29*x^18 + 142169742*a^17*b^13*x^2 + 805628538*a^16*b^14*x^3 + 3222514152*a^15*b^15*x^4 + 9667542456*a^14*b^16*x^5 + 22557599064*a^13*b^17*x^6 + 41892683976*a^12*b^18*x^7 + 62839025964*a^11*b^19*x^8 + 76803253956*a^10*b^20*x^9 + 76803253956*a^9*b^21*x^10 + 62839025964*a^8*b^22*x^11 + 41892683976*a^7*b^23*x^12 + 22557599064*a^6*b^24*x^13 + 9667542456*a^5*b^25*x^14 + 3222514152*a^4*b^26*x^15 + 805628538*a^3*b^27*x^16 + 142169742*a^2*b^28*x^17)

3.1332 $\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx$

Optimal. Leaf size=279

$$-\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-$$

```
[Out] -1/20*(-a*d+b*c)^10/b^11/(b*x+a)^20-10/19*d*(-a*d+b*c)^9/b^11/(b*x+a)^19-5/
2*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^18-120/17*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^17-
105/8*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^16-84/5*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^1
5-15*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^14-120/13*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^
13-15/4*d^8*(-a*d+b*c)^2/b^11/(b*x+a)^12-10/11*d^9*(-a*d+b*c)/b^11/(b*x+a)^
11-1/10*d^10/b^11/(b*x+a)^10
```

Rubi [A]

time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{d^{10}}{10b^{11}(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -1/20*(b*c - a*d)^10/(b^11*(a + b*x)^20) - (10*d*(b*c - a*d)^9)/(19*b^11*(a
 + b*x)^19) - (5*d^2*(b*c - a*d)^8)/(2*b^11*(a + b*x)^18) - (120*d^3*(b*c -
 a*d)^7)/(17*b^11*(a + b*x)^17) - (105*d^4*(b*c - a*d)^6)/(8*b^11*(a + b*x)
^16) - (84*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^15) - (15*d^6*(b*c - a*d)^4
)/(b^11*(a + b*x)^14) - (120*d^7*(b*c - a*d)^3)/(13*b^11*(a + b*x)^13) - (1
5*d^8*(b*c - a*d)^2)/(4*b^11*(a + b*x)^12) - (10*d^9*(b*c - a*d))/(11*b^11*
(a + b*x)^11) - d^10/(10*b^11*(a + b*x)^10)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{21}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{20}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{19}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{18}}+\frac{210d^4(bc-}{b^{10}(a+bx}\right.\\ &= -\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-}{8b^{11}(a+b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. $2(279) = 558$.
time = 0.19, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -1/1847560*(a^10*d^10 + 10*a^9*b*d^9*(c + 2*d*x) + 5*a^8*b^2*d^8*(11*c^2 +
40*c*d*x + 38*d^2*x^2) + 20*a^7*b^3*d^7*(11*c^3 + 55*c^2*d*x + 95*c*d^2*x^2
 + 57*d^3*x^3) + 5*a^6*b^4*d^6*(143*c^4 + 880*c^3*d*x + 2090*c^2*d^2*x^2 +
2280*c*d^3*x^3 + 969*d^4*x^4) + 2*a^5*b^5*d^5*(1001*c^5 + 7150*c^4*d*x + 20
900*c^3*d^2*x^2 + 31350*c^2*d^3*x^3 + 24225*c*d^4*x^4 + 7752*d^5*x^5) + 5*a
^4*b^6*d^4*(1001*c^6 + 8008*c^5*d*x + 27170*c^4*d^2*x^2 + 50160*c^3*d^3*x^3
 + 53295*c^2*d^4*x^4 + 31008*c*d^5*x^5 + 7752*d^6*x^6) + 20*a^3*b^7*d^3*(57
2*c^7 + 5005*c^6*d*x + 19019*c^5*d^2*x^2 + 40755*c^4*d^3*x^3 + 53295*c^3*d^
4*x^4 + 42636*c^2*d^5*x^5 + 19380*c*d^6*x^6 + 3876*d^7*x^7) + 5*a^2*b^8*d^2
*(4862*c^8 + 45760*c^7*d*x + 190190*c^6*d^2*x^2 + 456456*c^5*d^3*x^3 + 6928
35*c^4*d^4*x^4 + 682176*c^3*d^5*x^5 + 426360*c^2*d^6*x^6 + 155040*c*d^7*x^7
 + 25194*d^8*x^8) + 10*a*b^9*d*(4862*c^9 + 48620*c^8*d*x + 217360*c^7*d^2*x
^2 + 570570*c^6*d^3*x^3 + 969969*c^5*d^4*x^4 + 1108536*c^4*d^5*x^5 + 852720
*c^3*d^6*x^6 + 426360*c^2*d^7*x^7 + 125970*c*d^8*x^8 + 16796*d^9*x^9) + b^1
0*(92378*c^10 + 972400*c^9*d*x + 4618900*c^8*d^2*x^2 + 13041600*c^7*d^3*x^3
 + 24249225*c^6*d^4*x^4 + 31039008*c^5*d^5*x^5 + 27713400*c^4*d^6*x^6 + 170
54400*c^3*d^7*x^7 + 6928350*c^2*d^8*x^8 + 1679600*c*d^9*x^9 + 184756*d^10*x
^10))/(b^11*(a + b*x)^20)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(259) = 518$.
time = 0.16, size = 867, normalized size = 3.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^21,x,method=_RETURNVERBOSE)
```

```
[Out] 120/13/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)^13-15
/4/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^12-15/b^11*d^6*(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^14+10/19/b^11*d
*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c
^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^
8*d-b^9*c^9)/(b*x+a)^19-1/10*d^10/b^11/(b*x+a)^10+10/11/b^11*d^9*(a*d-b*c)/
(b*x+a)^11-1/20*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^
3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b
^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^20-105
/8/b^11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15
*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(b*x+a)^16+84/5/b^11*d^5*(a^5*d^5-5
```

```
*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(
b*x+a)^15-5/2/b^11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3
*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c
^7*d+b^8*c^8)/(b*x+a)^18+120/17/b^11*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*
c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^
6*d-b^7*c^7)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(259) = 518$.
time = 0.44, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="maxima")
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(259) = 518$.
time = 0.76, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="fricas")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**21,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(259) = 518.
time = 0.84, size = 961, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="giac")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 1679600*b^10*c*d^9*x^9 + 167960*a*b^9*d
^10*x^9 + 6928350*b^10*c^2*d^8*x^8 + 1259700*a*b^9*c*d^9*x^8 + 125970*a^2*b
^8*d^10*x^8 + 17054400*b^10*c^3*d^7*x^7 + 4263600*a*b^9*c^2*d^8*x^7 + 77520
0*a^2*b^8*c*d^9*x^7 + 77520*a^3*b^7*d^10*x^7 + 27713400*b^10*c^4*d^6*x^6 +
8527200*a*b^9*c^3*d^7*x^6 + 2131800*a^2*b^8*c^2*d^8*x^6 + 387600*a^3*b^7*c*
d^9*x^6 + 38760*a^4*b^6*d^10*x^6 + 31039008*b^10*c^5*d^5*x^5 + 11085360*a*b
^9*c^4*d^6*x^5 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 +
 155040*a^4*b^6*c*d^9*x^5 + 15504*a^5*b^5*d^10*x^5 + 24249225*b^10*c^6*d^4*
x^4 + 9699690*a*b^9*c^5*d^5*x^4 + 3464175*a^2*b^8*c^4*d^6*x^4 + 1065900*a^3
*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 4
845*a^6*b^4*d^10*x^4 + 13041600*b^10*c^7*d^3*x^3 + 5705700*a*b^9*c^6*d^4*x^
3 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d^6*x^3 + 250800*a^4*b
^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 1140
*a^7*b^3*d^10*x^3 + 4618900*b^10*c^8*d^2*x^2 + 2173600*a*b^9*c^7*d^3*x^2 +
950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7*c^5*d^5*x^2 + 135850*a^4*b^6*c^
4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 10450*a^6*b^4*c^2*d^8*x^2 + 1900*a^
7*b^3*c*d^9*x^2 + 190*a^8*b^2*d^10*x^2 + 972400*b^10*c^9*d*x + 486200*a*b^9
*c^8*d^2*x + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 40040*a^
4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 1100*a
^7*b^3*c^2*d^8*x + 200*a^8*b^2*c*d^9*x + 20*a^9*b*d^10*x + 92378*b^10*c^10
+ 48620*a*b^9*c^9*d + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*
a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*
c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^20*b^
11)
```

Mupad [B]
time = 0.80, size = 1175, normalized size = 4.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^21,x)
```

```
[Out] -(a^10*d^10 + 92378*b^10*c^10 + 184756*b^10*d^10*x^10 + 167960*a*b^9*d^10*x
^9 + 1679600*b^10*c*d^9*x^9 + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3
 + 5005*a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*
a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 190*a^8*b^2*d^10*x^2 + 1140*a^7*b^3*
d^10*x^3 + 4845*a^6*b^4*d^10*x^4 + 15504*a^5*b^5*d^10*x^5 + 38760*a^4*b^6*d
^10*x^6 + 77520*a^3*b^7*d^10*x^7 + 125970*a^2*b^8*d^10*x^8 + 4618900*b^10*c
^8*d^2*x^2 + 13041600*b^10*c^7*d^3*x^3 + 24249225*b^10*c^6*d^4*x^4 + 310390
08*b^10*c^5*d^5*x^5 + 27713400*b^10*c^4*d^6*x^6 + 17054400*b^10*c^3*d^7*x^7
 + 6928350*b^10*c^2*d^8*x^8 + 48620*a*b^9*c^9*d + 10*a^9*b*c*d^9 + 20*a^9*b
*d^10*x + 972400*b^10*c^9*d*x + 950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7
*c^5*d^5*x^2 + 135850*a^4*b^6*c^4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 104
```

50*a^6*b^4*c^2*d^8*x^2 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d^6*x^3 + 250800*a^4*b^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 3464175*a^2*b^8*c^4*d^6*x^4 + 1065900*a^3*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x^4 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 + 2131800*a^2*b^8*c^2*d^8*x^6 + 486200*a*b^9*c^8*d^2*x + 200*a^8*b^2*c*d^9*x + 1259700*a*b^9*c*d^9*x^8 + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 40040*a^4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 1100*a^7*b^3*c^2*d^8*x + 2173600*a*b^9*c^7*d^3*x^2 + 1900*a^7*b^3*c*d^9*x^2 + 5705700*a*b^9*c^6*d^4*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 9699690*a*b^9*c^5*d^5*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 11085360*a*b^9*c^4*d^6*x^5 + 155040*a^4*b^6*c*d^9*x^5 + 8527200*a*b^9*c^3*d^7*x^6 + 387600*a^3*b^7*c*d^9*x^6 + 4263600*a*b^9*c^2*d^8*x^7 + 775200*a^2*b^8*c*d^9*x^7)/(1847560*a^20*b^11 + 1847560*b^31*x^20 + 36951200*a^19*b^12*x + 36951200*a*b^30*x^19 + 351036400*a^18*b^13*x^2 + 2106218400*a^17*b^14*x^3 + 8951428200*a^16*b^15*x^4 + 28644570240*a^15*b^16*x^5 + 71611425600*a^14*b^17*x^6 + 143222851200*a^13*b^18*x^7 + 232737133200*a^12*b^19*x^8 + 310316177600*a^11*b^20*x^9 + 341347795360*a^10*b^21*x^10 + 310316177600*a^9*b^22*x^11 + 232737133200*a^8*b^23*x^12 + 143222851200*a^7*b^24*x^13 + 71611425600*a^6*b^25*x^14 + 28644570240*a^5*b^26*x^15 + 8951428200*a^4*b^27*x^16 + 2106218400*a^3*b^28*x^17 + 351036400*a^2*b^29*x^18)

3.1333 $\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx$

Optimal. Leaf size=279

$$-\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-1$$

```
[Out] -1/21*(-a*d+b*c)^10/b^11/(b*x+a)^21-1/2*d*(-a*d+b*c)^9/b^11/(b*x+a)^20-45/1
9*d^2*(-a*d+b*c)^8/b^11/(b*x+a)^19-20/3*d^3*(-a*d+b*c)^7/b^11/(b*x+a)^18-21
0/17*d^4*(-a*d+b*c)^6/b^11/(b*x+a)^17-63/4*d^5*(-a*d+b*c)^5/b^11/(b*x+a)^16
-14*d^6*(-a*d+b*c)^4/b^11/(b*x+a)^15-60/7*d^7*(-a*d+b*c)^3/b^11/(b*x+a)^14-
45/13*d^8*(-a*d+b*c)^2/b^11/(b*x+a)^13-5/6*d^9*(-a*d+b*c)/b^11/(b*x+a)^12-1
/11*d^10/b^11/(b*x+a)^11
```

Rubi [A]
time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d^{10}}{11b^{11}(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -1/21*(b*c - a*d)^10/(b^11*(a + b*x)^21) - (d*(b*c - a*d)^9)/(2*b^11*(a + b
*x)^20) - (45*d^2*(b*c - a*d)^8)/(19*b^11*(a + b*x)^19) - (20*d^3*(b*c - a*
d)^7)/(3*b^11*(a + b*x)^18) - (210*d^4*(b*c - a*d)^6)/(17*b^11*(a + b*x)^17
) - (63*d^5*(b*c - a*d)^5)/(4*b^11*(a + b*x)^16) - (14*d^6*(b*c - a*d)^4)/(
b^11*(a + b*x)^15) - (60*d^7*(b*c - a*d)^3)/(7*b^11*(a + b*x)^14) - (45*d^8
*(b*c - a*d)^2)/(13*b^11*(a + b*x)^13) - (5*d^9*(b*c - a*d))/(6*b^11*(a + b
*x)^12) - d^10/(11*b^11*(a + b*x)^11)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{22}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{21}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{20}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{19}}+\frac{210d^4(bc-a}{b^{10}(a+bx}\right.$$

$$= -\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-a}{17b^{11}(a+bx}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 692 vs. 2(279) = 558.
time = 0.19, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -1/3879876*(a^10*d^10 + a^9*b*d^9*(11*c + 21*d*x) + 3*a^8*b^2*d^8*(22*c^2 +
 77*c*d*x + 70*d^2*x^2) + 2*a^7*b^3*d^7*(143*c^3 + 693*c^2*d*x + 1155*c*d^2
*x^2 + 665*d^3*x^3) + 7*a^6*b^4*d^6*(143*c^4 + 858*c^3*d*x + 1980*c^2*d^2*x
^2 + 2090*c*d^3*x^3 + 855*d^4*x^4) + 21*a^5*b^5*d^5*(143*c^5 + 1001*c^4*d*x
 + 2860*c^3*d^2*x^2 + 4180*c^2*d^3*x^3 + 3135*c*d^4*x^4 + 969*d^5*x^5) + 7*
a^4*b^6*d^4*(1144*c^6 + 9009*c^5*d*x + 30030*c^4*d^2*x^2 + 54340*c^3*d^3*x^
3 + 56430*c^2*d^4*x^4 + 31977*c*d^5*x^5 + 7752*d^6*x^6) + 2*a^3*b^7*d^3*(97
24*c^7 + 84084*c^6*d*x + 315315*c^5*d^2*x^2 + 665665*c^4*d^3*x^3 + 855855*c
^3*d^4*x^4 + 671517*c^2*d^5*x^5 + 298452*c*d^6*x^6 + 58140*d^7*x^7) + 3*a^2
*b^8*d^2*(14586*c^8 + 136136*c^7*d*x + 560560*c^6*d^2*x^2 + 1331330*c^5*d^3
*x^3 + 1996995*c^4*d^4*x^4 + 1939938*c^3*d^5*x^5 + 1193808*c^2*d^6*x^6 + 42
6360*c*d^7*x^7 + 67830*d^8*x^8) + a*b^9*d*(92378*c^9 + 918918*c^8*d*x + 408
4080*c^7*d^2*x^2 + 10650640*c^6*d^3*x^3 + 17972955*c^5*d^4*x^4 + 20369349*c
^4*d^5*x^5 + 15519504*c^3*d^6*x^6 + 7674480*c^2*d^7*x^7 + 2238390*c*d^8*x^8
 + 293930*d^9*x^9) + b^10*(184756*c^10 + 1939938*c^9*d*x + 9189180*c^8*d^2*
x^2 + 25865840*c^7*d^3*x^3 + 47927880*c^6*d^4*x^4 + 61108047*c^5*d^5*x^5 +
54318264*c^4*d^6*x^6 + 33256080*c^3*d^7*x^7 + 13430340*c^2*d^8*x^8 + 323323
0*c*d^9*x^9 + 352716*d^10*x^10))/(b^11*(a + b*x)^21)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(259) = 518.
time = 0.18, size = 867, normalized size = 3.11 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^22,x,method=_RETURNVERBOSE)
```

```
[Out] -45/13/b^11*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^13+5/6/b^11*d^9*(a*d-b*
c)/(b*x+a)^12+60/7/b^11*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(
b*x+a)^14+63/4/b^11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^
3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(b*x+a)^16-45/19/b^11*d^2*(a^8*d^8-8*a^7*b
*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*
c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(b*x+a)^19-1/11*d^10/b^11
/(b*x+a)^11+1/2/b^11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3
*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*
b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(b*x+a)^20-1/21*(a^10*d^10-10*a^9*b*c*d^
9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^
5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c
```

```
^9*d+b^10*c^10)/b^11/(b*x+a)^21-14/b^11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^
2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(b*x+a)^15+20/3/b^11*d^3*(a^7*d^7-7*a^6*b*
c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c
^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(b*x+a)^18-210/17/b^11*d^4*(a^6*d^6-6*a^5*b*c
*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d
+b^6*c^6)/(b*x+a)^17
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(259) = 518$.
time = 0.43, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="maxima")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(259) = 518$.

time = 0.86, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="fricas")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**22,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 961 vs. 2(259) = 518.

time = 1.20, size = 961, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="giac")

[Out] -1/3879876*(352716*b^10*d^10*x^10 + 3233230*b^10*c*d^9*x^9 + 293930*a*b^9*d^10*x^9 + 13430340*b^10*c^2*d^8*x^8 + 2238390*a*b^9*c*d^9*x^8 + 203490*a^2*b^8*d^10*x^8 + 33256080*b^10*c^3*d^7*x^7 + 7674480*a*b^9*c^2*d^8*x^7 + 1279080*a^2*b^8*c*d^9*x^7 + 116280*a^3*b^7*d^10*x^7 + 54318264*b^10*c^4*d^6*x^6 + 15519504*a*b^9*c^3*d^7*x^6 + 3581424*a^2*b^8*c^2*d^8*x^6 + 596904*a^3*b^7*c*d^9*x^6 + 54264*a^4*b^6*d^10*x^6 + 61108047*b^10*c^5*d^5*x^5 + 20369349*a*b^9*c^4*d^6*x^5 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*x^5 + 223839*a^4*b^6*c*d^9*x^5 + 20349*a^5*b^5*d^10*x^5 + 47927880*b^10*c^6*d^4*x^4 + 17972955*a*b^9*c^5*d^5*x^4 + 5990985*a^2*b^8*c^4*d^6*x^4 + 1711710*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^2*d^8*x^4 + 65835*a^5*b^5*c*d^9*x^4 + 5985*a^6*b^4*d^10*x^4 + 25865840*b^10*c^7*d^3*x^3 + 10650640*a*b^9*c^6*d^4*x^3 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^7*c^4*d^6*x^3 + 380380*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 14630*a^6*b^4*c*d^9*x^3 + 1330*a^7*b^3*d^10*x^3 + 9189180*b^10*c^8*d^2*x^2 + 4084080*a*b^9*c^7*d^3*x^2 + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a^3*b^7*c^5*d^5*x^2 + 210210*a^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2 + 13860*a^6*b^4*c^2*d^8*x^2 + 2310*a^7*b^3*c*d^9*x^2 + 210*a^8*b^2*d^10*x^2 + 1939938*b^10*c^9*d*x + 918918*a*b^9*c^8*d^2*x + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*x + 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^7*x + 1386*a^7*b^3*c^2*d^8*x + 231*a^8*b^2*c*d^9*x + 21*a^9*b*d^10*x + 184756*b^10*c^10 + 92378*a*b^9*c^9*d + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^21*b^11)

Mupad [B]
time = 1.04, size = 1186, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^10/(a + b*x)^22,x)

[Out] -(a^10*d^10 + 184756*b^10*c^10 + 352716*b^10*d^10*x^10 + 293930*a*b^9*d^10*x^9 + 3233230*b^10*c*d^9*x^9 + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 210*a^8*b^2*d^10*x^2 + 1330*a^7*b^3*d^10*x^3 + 5985*a^6*b^4*d^10*x^4 + 20349*a^5*b^5*d^10*x^5 + 54264*a^4*b^6

*d^10*x^6 + 116280*a^3*b^7*d^10*x^7 + 203490*a^2*b^8*d^10*x^8 + 9189180*b^1
0*c^8*d^2*x^2 + 25865840*b^10*c^7*d^3*x^3 + 47927880*b^10*c^6*d^4*x^4 + 611
08047*b^10*c^5*d^5*x^5 + 54318264*b^10*c^4*d^6*x^6 + 33256080*b^10*c^3*d^7*
x^7 + 13430340*b^10*c^2*d^8*x^8 + 92378*a*b^9*c^9*d + 11*a^9*b*c*d^9 + 21*a
^9*b*d^10*x + 1939938*b^10*c^9*d*x + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a
^3*b^7*c^5*d^5*x^2 + 210210*a^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2
 + 13860*a^6*b^4*c^2*d^8*x^2 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^
7*c^4*d^6*x^3 + 380380*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 59
90985*a^2*b^8*c^4*d^6*x^4 + 1711710*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^
2*d^8*x^4 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*x^5 + 358
1424*a^2*b^8*c^2*d^8*x^6 + 918918*a*b^9*c^8*d^2*x + 231*a^8*b^2*c*d^9*x + 2
238390*a*b^9*c*d^9*x^8 + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*
x + 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^
7*x + 1386*a^7*b^3*c^2*d^8*x + 4084080*a*b^9*c^7*d^3*x^2 + 2310*a^7*b^3*c*d
^9*x^2 + 10650640*a*b^9*c^6*d^4*x^3 + 14630*a^6*b^4*c*d^9*x^3 + 17972955*a*
b^9*c^5*d^5*x^4 + 65835*a^5*b^5*c*d^9*x^4 + 20369349*a*b^9*c^4*d^6*x^5 + 22
3839*a^4*b^6*c*d^9*x^5 + 15519504*a*b^9*c^3*d^7*x^6 + 596904*a^3*b^7*c*d^9*
x^6 + 7674480*a*b^9*c^2*d^8*x^7 + 1279080*a^2*b^8*c*d^9*x^7)/(3879876*a^21*
b^11 + 3879876*b^32*x^21 + 81477396*a^20*b^12*x + 81477396*a*b^31*x^20 + 81
4773960*a^19*b^13*x^2 + 5160235080*a^18*b^14*x^3 + 23221057860*a^17*b^15*x^
4 + 78951596724*a^16*b^16*x^5 + 210537591264*a^15*b^17*x^6 + 451151981280*a
^14*b^18*x^7 + 789515967240*a^13*b^19*x^8 + 1140411952680*a^12*b^20*x^9 + 1
368494343216*a^11*b^21*x^10 + 1368494343216*a^10*b^22*x^11 + 1140411952680*
a^9*b^23*x^12 + 789515967240*a^8*b^24*x^13 + 451151981280*a^7*b^25*x^14 + 2
10537591264*a^6*b^26*x^15 + 78951596724*a^5*b^27*x^16 + 23221057860*a^4*b^2
8*x^17 + 5160235080*a^3*b^29*x^18 + 814773960*a^2*b^30*x^19)

3.1334 $\int \frac{(a+bx)^5}{c+dx}\,dx$

Optimal. Leaf size=122

$$\frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+bx)^5}{5d}-\frac{(bc-ad)^5\log(c}{d^6}$$

```
[Out] b*(-a*d+b*c)^4*x/d^5-1/2*(-a*d+b*c)^3*(b*x+a)^2/d^4+1/3*(-a*d+b*c)^2*(b*x+a
)^3/d^3-1/4*(-a*d+b*c)*(b*x+a)^4/d^2+1/5*(b*x+a)^5/d-(-a*d+b*c)^5*ln(d*x+c)
/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{(bc-ad)^5\log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{c+dx}\,dx &= \int \left(\frac{b(bc-ad)^4}{d^5}-\frac{b(bc-ad)^3(a+bx)}{d^4}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\right.\\ &= \frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+}{5}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 167, normalized size = 1.37

$$\frac{bdx(300a^4d^4+300a^3bd^3(-2c+dx)+100a^2b^2d^2(6c^2-3cdx+2d^2x^2)+25ab^3d(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3)+b^4(60c^4-30c^3dx+20c^2d^2x^2-15cd^3x^3+12d^4x^4))-60(bc-ad)^5\log(c+dx)}{60d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4
*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(114) = 228$.
time = 0.14, size = 266, normalized size = 2.18

method	result
norman	$\frac{b\left(5a^4d^4-10a^3bc\,d^3+10a^2b^2c^2d^2-5a\,b^3c^3d+b^4c^4\right)x}{d^5}+\frac{b^5x^5}{5d}+\frac{b^2\left(10a^3d^3-10a^2bc\,d^2+5a\,b^2c^2d-b^3c^3\right)x^2}{2d^4}+\frac{b^3\left(10a^2d^2-5abcd+b^2\right.}{3d^3}$
default	$\frac{b\left(\frac{1}{5}d^4x^5b^4+\frac{5}{4}a\,b^3d^4x^4-\frac{1}{4}b^4c\,d^3x^4+\frac{10}{3}a^2b^2d^4x^3-\frac{5}{3}a\,b^3c\,d^3x^3+\frac{1}{3}b^4c^2d^2x^3+5a^3b\,d^4x^2-5a^2b^2c\,d^3x^2+\frac{5}{2}a\,b^3c^2d^2x^2-\frac{1}{2}b^4c^3d\,x^2+5a^4d\right.}{d^5}$
risch	$\frac{b^5x^5}{5d}+\frac{5b^4a\,x^4}{4d}-\frac{b^5c\,x^4}{4d^2}+\frac{10b^3a^2x^3}{3d}-\frac{5b^4ac\,x^3}{3d^2}+\frac{b^5c^2x^3}{3d^3}+\frac{5b^2a^3x^2}{d}-\frac{5b^3a^2c\,x^2}{d^2}+\frac{5b^4a\,c^2x^2}{2d^3}-\frac{b^5c^3x^2}{2d^4}+\frac{5b\,a^4x}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^5*(1/5*d^4*x^5*b^4+5/4*a*b^3*d^4*x^4-1/4*b^4*c*d^3*x^4+10/3*a^2*b^2*d^4
*x^3-5/3*a*b^3*c*d^3*x^3+1/3*b^4*c^2*d^2*x^3+5*a^3*b*d^4*x^2-5*a^2*b^2*c*d^
3*x^2+5/2*a*b^3*c^2*d^2*x^2-1/2*b^4*c^3*d*x^2+5*a^4*d^4*x-10*a^3*b*c*d^3*x+
10*a^2*b^2*c^2*d^2*x-5*a*b^3*c^3*d*x+b^4*c^4*x)+(a^5*d^5-5*a^4*b*c*d^4+10*a
^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6*ln(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(114) = 228$.
time = 0.41, size = 258, normalized size = 2.11

$$\frac{12\,b^5d^4x^5-15\,(b^5cd^3-5\,ab^4d^4)x^4+20\,(b^5c^2d^2-5\,ab^4cd^3+10\,a^2b^3d^4)x^3-30\,(b^5c^3d-5\,ab^4c^2d^2+10\,a^2b^3cd^3-10\,a^3b^2d^4)x^2+60\,(b^5c^4-5\,ab^4c^3d+10\,a^2b^3c^2d^2-10\,a^3b^2cd^3+5\,a^4bd^4)x}{60\,d^5}-\frac{(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\log(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2 -
 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 + 10
*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*a^2
*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*b^4*
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
log(d*x + c)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(114) = 228$.

time = 1.25, size = 259, normalized size = 2.12

$$\frac{12\,b^5d^5x^5-15\,(b^5cd^4-5\,ab^4d^5)x^4+20\,(b^5c^2d^3-5\,ab^4cd^4+10\,a^2b^3d^5)x^3-30\,(b^5c^3d^2-5\,ab^4c^2d^3+10\,a^2b^3cd^4-10\,a^3b^2d^5)x^2+60\,(b^5c^4d-5\,ab^4c^3d^2+10\,a^2b^3c^2d^3-10\,a^3b^2cd^4+5\,a^4bd^5)x-60\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\log(dx+c)}{60\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 -
 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 +
10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2 +
10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*log(d*x + c))/d^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(104) = 208$.
time = 0.28, size = 209, normalized size = 1.71

$$\frac{b^5x^5}{5d}+x^4\cdot\left(\frac{5ab^4}{4d}-\frac{b^5c}{4d^2}\right)+x^3\cdot\left(\frac{10a^2b^3}{3d}-\frac{5ab^4c}{3d^2}+\frac{b^5c^2}{3d^3}\right)+x^2\cdot\left(\frac{5a^3b^2}{d}-\frac{5a^2b^3c}{d^2}+\frac{5ab^4c^2}{2d^3}-\frac{b^5c^3}{2d^4}\right)+x\left(\frac{5a^4b}{d}-\frac{10a^3b^2c}{d^2}+\frac{10a^2b^3c^2}{d^3}-\frac{5ab^4c^3}{d^4}+\frac{b^5c^4}{d^5}\right)+\frac{(ad-bc)^5\log(c+dx)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c),x)
```

```
[Out] b**5*x**5/(5*d) + x**4*(5*a*b**4/(4*d) - b**5*c/(4*d**2)) + x**3*(10*a**2*b
**3/(3*d) - 5*a*b**4*c/(3*d**2) + b**5*c**2/(3*d**3)) + x**2*(5*a**3*b**2/d
 - 5*a**2*b**3*c/d**2 + 5*a*b**4*c**2/(2*d**3) - b**5*c**3/(2*d**4)) + x*(5
*a**4*b/d - 10*a**3*b**2*c/d**2 + 10*a**2*b**3*c**2/d**3 - 5*a*b**4*c**3/d*
*4 + b**5*c**4/d**5) + (a*d - b*c)**5*log(c + d*x)/d**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(114) = 228$.
time = 1.35, size = 273, normalized size = 2.24

$$\frac{12\,b^5d^4x^5-15\,b^5cd^3x^4+75\,ab^4d^4x^4+20\,b^5c^2d^2x^3-100\,ab^4cd^3x^3+200\,a^2b^3d^4x^3-30\,b^5c^3dx^2+150\,ab^4c^2d^2x^2-300\,a^2b^3cd^3x^2+300\,a^3b^2d^4x^2+60\,b^5c^4x-300\,ab^4c^3dx+600\,a^2b^3c^2d^2x-600\,a^3b^2cd^3x+300\,a^4bd^4x}{60\,d^5}-\frac{(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\log(|dx+c|)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^2
*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150*a
*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*c^4
*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x + 300*
a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

Mupad [B]
time = 0.07, size = 280, normalized size = 2.30

$$x\left(\frac{5a^4b}{d}-\frac{c\left(\frac{10a^3b^2}{d}+\frac{c\left(\frac{c\left(\frac{5ab^4}{d}-\frac{b^5c}{d^2}\right)}{d}-\frac{10a^2b^3}{d}\right)}{d}\right)}{d}\right)+x^4\left(\frac{5ab^4}{4d}-\frac{b^5c}{4d^2}\right)+x^2\left(\frac{5a^3b^2}{d}+\frac{c\left(\frac{c\left(\frac{5ab^4}{d}-\frac{b^5c}{d^2}\right)}{d}-\frac{10a^2b^3}{d}\right)}{2d}\right)-x^3\left(\frac{c\left(\frac{5ab^4}{d}-\frac{b^5c}{d^2}\right)}{3d}-\frac{10a^2b^3}{3d}\right)+\frac{b^5x^5}{5d}+\frac{\ln(c+dx)\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x),x)
```

```
[Out] x*((5*a^4*b)/d - (c*((10*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d
 - (10*a^2*b^3)/d))/d))/d) + x^4*((5*a*b^4)/(4*d) - (b^5*c)/(4*d^2)) + x^2*
((5*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d - (10*a^2*b^3)/d))/(
2*d)) - x^3*((c*((5*a*b^4)/d - (b^5*c)/d^2))/(3*d) - (10*a^2*b^3)/(3*d)) +
(b^5*x^5)/(5*d) + (log(c + d*x)*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 1
0*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/d^6
```

3.1335 $\int \frac{(a+bx)^4}{c+dx}\,dx$

Optimal. Leaf size=98

$$-\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c+dx)}{d^5}$$

```
[Out] -b*(-a*d+b*c)^3*x/d^4+1/2*(-a*d+b*c)^2*(b*x+a)^2/d^3-1/3*(-a*d+b*c)*(b*x+a)
^3/d^2+1/4*(b*x+a)^4/d+(-a*d+b*c)^4*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+\\ &= -\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log\end{aligned}$$

Mathematica [A]
time = 0.03, size = 115, normalized size = 1.17

$$\frac{bdx(48a^3d^3+36a^2bd^2(-2c+dx)+8ab^2d(6c^2-3cdx+2d^2x^2)+b^3(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3))+12(bc-ad)^4\log(c+dx)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x),x]
```

```
[Out] (b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*x
 + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(
b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(92) = 184$.
time = 0.15, size = 189, normalized size = 1.93

method	result
norman	$\frac{b(4a^3d^3-6a^2bc\,d^2+4a\,b^2c^2d-b^3c^3)x}{d^4}+\frac{b^4x^4}{4d}+\frac{b^2(6a^2d^2-4abcd+b^2c^2)x^2}{2d^3}+\frac{b^3(4ad-bc)x^3}{3d^2}+\frac{(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\cdots}{d^5}$
default	$\frac{b\left(\frac{d^3x^4b^3}{4}+\frac{((2ad-bc)b^2d^2+2a\,b^2d^3)x^3}{3}+\frac{\left(2(2ad-bc)ab\,d^2+bd\left(2a^2d^2-2abcd+b^2c^2\right)\right)x^2}{2}+(2ad-bc)\left(2a^2d^2-2abcd+b^2c^2\right)x\right)}{d^4}+\frac{(a^4d^4-4a\cdots}{}$
risch	$\frac{b^4x^4}{4d}+\frac{4b^3a\,x^3}{3d}-\frac{b^4c\,x^3}{3d^2}+\frac{3b^2a^2x^2}{d}-\frac{2b^3ac\,x^2}{d^2}+\frac{b^4c^2x^2}{2d^3}+\frac{4b\,a^3x}{d}-\frac{6b^2a^2cx}{d^2}+\frac{4b^3a\,c^2x}{d^3}-\frac{b^4c^3x}{d^4}+\frac{\ln(dx+c)a^4}{d}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^4*(1/4*d^3*x^4*b^3+1/3*((2*a*d-b*c)*b^2*d^2+2*a*b^2*d^3)*x^3+1/2*(2*(2*
a*d-b*c)*a*b*d^2+b*d*(2*a^2*d^2-2*a*b*c*d+b^2*c^2))*x^2+(2*a*d-b*c)*(2*a^2*
d^2-2*a*b*c*d+b^2*c^2)*x)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)/d^5*ln(d*x+c)
```

Maxima [A]
time = 0.34, size = 177, normalized size = 1.81

$$\frac{3\,b^4d^3x^4-4\,(b^4cd^2-4\,ab^3d^3)x^3+6\,(b^4c^2d-4\,ab^3cd^2+6\,a^2b^2d^3)x^2-12\,(b^4c^3-4\,ab^3c^2d+6\,a^2b^2cd^2-4\,a^3bd^3)x}{12\,d^4}+\frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a*
b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

Fricas [A]
time = 0.76, size = 179, normalized size = 1.83

$$\frac{3\,b^4d^4x^4-4\,(b^4cd^3-4\,ab^3d^4)x^3+6\,(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2d^4)x^2-12\,(b^4c^3d-4\,ab^3c^2d^2+6\,a^2b^2cd^3-4\,a^3bd^4)x+12\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log(dx+c)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4*
a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*
b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

Sympy [A]
time = 0.20, size = 136, normalized size = 1.39

$$\frac{b^4x^4}{4d}+x^3\cdot\left(\frac{4ab^3}{3d}-\frac{b^4c}{3d^2}\right)+x^2\cdot\left(\frac{3a^2b^2}{d}-\frac{2ab^3c}{d^2}+\frac{b^4c^2}{2d^3}\right)+x\left(\frac{4a^3b}{d}-\frac{6a^2b^2c}{d^2}+\frac{4ab^3c^2}{d^3}-\frac{b^4c^3}{d^4}\right)+\frac{(ad-bc)^4\log{(c+dx)}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c),x)
```

```
[Out] b**4*x**4/(4*d) + x**3*(4*a*b**3/(3*d) - b**4*c/(3*d**2)) + x**2*(3*a**2*b*
*2/d - 2*a*b**3*c/d**2 + b**4*c**2/(2*d**3)) + x*(4*a**3*b/d - 6*a**2*b**2*
c/d**2 + 4*a*b**3*c**2/d**3 - b**4*c**3/d**4) + (a*d - b*c)**4*log(c + d*x)
/d**5
```

Giac [A]
time = 1.24, size = 184, normalized size = 1.88

$$\frac{3\,b^4d^3x^4-4\,b^4cd^2x^3+16\,ab^3d^3x^3+6\,b^4c^2dx^2-24\,ab^3cd^2x^2+36\,a^2b^2d^3x^2-12\,b^4c^3x+48\,ab^3c^2dx-72\,a^2b^2cd^2x+48\,a^3bd^3x}{12\,d^4}+\frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log(|dx+c|)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
- 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d*x
 - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

Mupad [B]
time = 0.22, size = 189, normalized size = 1.93

$$x^3\,\left(\frac{4\,a\,b^3}{3\,d}-\frac{b^4\,c}{3\,d^2}\right)+x\,\left(\frac{4\,a^3\,b}{d}+\frac{c\,\left(\frac{c\,\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{d}-\frac{6\,a^2\,b^2}{d}\right)}{d}\right)-x^2\,\left(\frac{c\,\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{2\,d}-\frac{3\,a^2\,b^2}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{b^4\,x^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x),x)
```

```
[Out] x^3*((4*a*b^3)/(3*d) - (b^4*c)/(3*d^2)) + x*((4*a^3*b)/d + (c*((c*((4*a*b^3
)/d - (b^4*c)/d^2))/d - (6*a^2*b^2)/d))/d) - x^2*((c*((4*a*b^3)/d - (b^4*c)
/d^2))/(2*d) - (3*a^2*b^2)/d) + (log(c + d*x)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d^5 + (b^4*x^4)/(4*d)
```

3.1336 $\int \frac{(a+bx)^3}{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}$$

```
[Out] b*(-a*d+b*c)^2*x/d^3-1/2*(-a*d+b*c)*(b*x+a)^2/d^2+1/3*(b*x+a)^3/d-(-a*d+b*c
)^3*ln(d*x+c)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{c+dx}\,dx&=\int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx\\&=\frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.00

$$\frac{bdx(18a^2d^2+9abd(-2c+dx)+b^2(6c^2-3cdx+2d^2x^2))-6(bc-ad)^3\log(c+dx)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^
2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

Maple [A]
time = 0.14, size = 109, normalized size = 1.47

method	result
norman	$\frac{b\left(3a^{2}d^{2}-3abcd+b^{2}c^{2}\right)x}{d^{3}}+\frac{b^{3}x^{3}}{3d}+\frac{b^{2}\left(3ad-bc\right)x^{2}}{2d^{2}}+\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\ln\left(dx+c\right)}{d^{4}}$
default	$\frac{b\left(\frac{1}{3}d^{2}x^{3}b^{2}+\frac{3}{2}ab\,d^{2}x^{2}-\frac{1}{2}b^{2}cd\,x^{2}+3a^{2}d^{2}x-3abcdx+b^{2}c^{2}x\right)}{d^{3}}+\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\ln\left(dx+c\right)}{d^{4}}$
risch	$\frac{b^{3}x^{3}}{3d}+\frac{3b^{2}a\,x^{2}}{2d}-\frac{b^{3}c\,x^{2}}{2d^{2}}+\frac{3b\,a^{2}x}{d}-\frac{3b^{2}acx}{d^{2}}+\frac{b^{3}c^{2}x}{d^{3}}+\frac{\ln\left(dx+c\right)a^{3}}{d}-\frac{3\ln\left(dx+c\right)a^{2}bc}{d^{2}}+\frac{3\ln\left(dx+c\right)a\,b^{2}c^{2}}{d^{3}}-\frac{\ln\left(dx+c\right)}{d^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(1/3*d^2*x^3*b^2+3/2*a*b*d^2*x^2-1/2*b^2*c*d*x^2+3*a^2*d^2*x-3*a*b*c*
d*x+b^2*c^2*x)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4*ln(d*x+c)
```

Maxima [A]
time = 0.31, size = 114, normalized size = 1.54

$$\frac{2\,b^{3}d^{2}x^{3}-3\,(b^{3}cd-3\,ab^{2}d^{2})x^{2}+6\,(b^{3}c^{2}-3\,ab^{2}cd+3\,a^{2}bd^{2})x}{6\,d^{3}}-\frac{(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\log\left(dx+c\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*c
*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(d*x + c)/d^4
```

Fricas [A]
time = 0.64, size = 115, normalized size = 1.55

$$\frac{2\,b^{3}d^{3}x^{3}-3\,(b^{3}cd^{2}-3\,ab^{2}d^{3})x^{2}+6\,(b^{3}c^{2}d-3\,ab^{2}cd^{2}+3\,a^{2}bd^{3})x-6\,(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3})\log\left(dx+c\right)}{6\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*b
^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(d*x + c))/d^4
```

Sympy [A]
time = 0.15, size = 83, normalized size = 1.12

$$\frac{b^3x^3}{3d} + x^2 \cdot \left(\frac{3ab^2}{2d} - \frac{b^3c}{2d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right) + \frac{(ad-bc)^3 \log{(c+dx)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c),x)
```

```
[Out] b**3*x**3/(3*d) + x**2*(3*a*b**2/(2*d) - b**3*c/(2*d**2)) + x*(3*a**2*b/d -
 3*a*b**2*c/d**2 + b**3*c**2/d**3) + (a*d - b*c)**3*log(c + d*x)/d**4
```

Giac [A]
time = 0.75, size = 116, normalized size = 1.57

$$\frac{2\,b^3d^2x^3 - 3\,b^3cdx^2 + 9\,ab^2d^2x^2 + 6\,b^3c^2x - 18\,ab^2cdx + 18\,a^2bd^2x}{6\,d^3} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log{(|dx+c|)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*b
^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(d*x + c))/d^4
```

Mupad [B]
time = 0.07, size = 118, normalized size = 1.59

$$x^2\left(\frac{3\,a\,b^2}{2\,d} - \frac{b^3\,c}{2\,d^2}\right) + x\left(\frac{3\,a^2\,b}{d} - \frac{c\left(\frac{3\,a\,b^2}{d} - \frac{b^3\,c}{d^2}\right)}{d}\right) + \frac{\ln{(c+d\,x)}\,\left(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3\right)}{d^4} + \frac{b^3\,x^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x),x)
```

```
[Out] x^2*((3*a*b^2)/(2*d) - (b^3*c)/(2*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d
- (b^3*c)/d^2))/d) + (log(c + d*x)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/d^4 + (b^3*x^3)/(3*d)
```

3.1337 $\int \frac{(a+bx)^2}{c+dx} dx$

Optimal. Leaf size=50

$$-\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}$$

```
[Out] -b*(-a*d+b*c)*x/d^2+1/2*(b*x+a)^2/d+(-a*d+b*c)^2*ln(d*x+c)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.86

$$\frac{bdx(-2bc+4ad+bdx)+2(bc-ad)^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x),x]
[Out] (b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

Maple [A]
time = 0.18, size = 56, normalized size = 1.12

method	result	size
default	$\frac{b\left(\frac{1}{2}b d\,x^{2}+2 a d x-b c x\right)}{d^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) \ln \left(d x+c\right)}{d^{3}}$	56
norman	$\frac{b\left(2 a d-b c\right) x}{d^{2}}+\frac{b^{2} x^{2}}{2 d}+\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) \ln \left(d x+c\right)}{d^{3}}$	59
risch	$\frac{b^{2} x^{2}}{2 d}+\frac{2 b a x}{d}-\frac{b^{2} c x}{d^{2}}+\frac{\ln \left(d x+c\right) a^{2}}{d}-\frac{2 \ln \left(d x+c\right) a b c}{d^{2}}+\frac{\ln \left(d x+c\right) b^{2} c^{2}}{d^{3}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
[Out] b/d^2*(1/2*b*d*x^2+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*ln(d*x+c)
```

Maxima [A]
time = 0.31, size = 60, normalized size = 1.20

$$\frac{b^{2} d x^{2}-2\,(b^{2} c-2\,a b d)x}{2\,d^{2}}+\frac{(b^{2} c^{2}-2\,a b c d+a^{2} d^{2})\log\left(d x+c\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] 1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(d*x + c)/d^3
```

Fricas [A]
time = 0.56, size = 62, normalized size = 1.24

$$\frac{b^{2} d^{2} x^{2}-2\,(b^{2} c d-2\,a b d^{2})x+2\,(b^{2} c^{2}-2\,a b c d+a^{2} d^{2})\log\left(d x+c\right)}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] 1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(d*x + c))/d^3
```

Sympy [A]
time = 0.10, size = 44, normalized size = 0.88

$$\frac{b^{2} x^{2}}{2d}+x\left(\frac{2ab}{d}-\frac{b^{2} c}{d^{2}}\right)+\frac{\left(ad-bc\right)^{2}\log\left(c+dx\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c),x)
```

```
[Out] b**2*x**2/(2*d) + x*(2*a*b/d - b**2*c/d**2) + (a*d - b*c)**2*log(c + d*x)/d
**3
```

Giac [A]
time = 0.65, size = 60, normalized size = 1.20

$$\frac{b^2dx^2 - 2\,b^2cx + 4\,abdx}{2\,d^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log{(|dx + c|)}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(d*x + c))/d^3
```

Mupad [B]
time = 0.22, size = 62, normalized size = 1.24

$$\frac{\ln{(c + d\,x)}\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{d^3} - x\,\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - x*((b^2*c)/d^2 - (2*a*
b)/d) + (b^2*x^2)/(2*d)
```

3.1338 $\int \frac{a+bx}{c+dx} dx$

Optimal. Leaf size=26

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

[Out] b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {45}

$$\frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c + d*x),x]

[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{c+dx}\,dx &= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx\\ &= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.96

$$\frac{bx}{d} + \frac{(-bc+ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(c + d*x),x]

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Maple [A]
time = 0.12, size = 26, normalized size = 1.00

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
risch	$\frac{bx}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
[Out] b*x/d+(a*d-b*c)/d^2*ln(d*x+c)
```

Maxima [A]
time = 0.38, size = 26, normalized size = 1.00

$$\frac{bx}{d}-\frac{(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Fricas [A]
time = 0.52, size = 25, normalized size = 0.96

$$\frac{bdx-(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.77

$$\frac{bx}{d}+\frac{(ad-bc)\log(c+dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x)
```

```
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Giac [A]
time = 0.61, size = 27, normalized size = 1.04

$$\frac{bx}{d} - \frac{(bc - ad)\log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

Mupad [B]
time = 0.20, size = 25, normalized size = 0.96

$$\frac{\ln(c + d\,x)\,(a\,d - b\,c)}{d^2} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

3.1339 $\int \frac{1}{c+dx} \, dx$

Optimal. Leaf size=10

$$\frac{\log(c+dx)}{d}$$

```
[Out] ln(d*x+c)/d
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{c+dx} \, dx = \frac{\log(c+dx)}{d}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

Maple [A]
time = 0.12, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(dx+c)}{d}$	11
norman	$\frac{\ln(dx+c)}{d}$	11
risch	$\frac{\ln(dx+c)}{d}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] ln(d*x+c)/d
```

Maxima [A]
time = 0.38, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(d*x + c)/d
```

Fricas [A]
time = 0.53, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="fricas")
```

```
[Out] log(d*x + c)/d
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x)
```

```
[Out] log(c + d*x)/d
```

Giac [A]
time = 0.55, size = 11, normalized size = 1.10

$$\frac{\log(|dx + c|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="giac")
[Out] log(abs(d*x + c))/d
```

Mupad [B]
time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x),x)
[Out] log(c + d*x)/d
```

3.1340 $\int \frac{1}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

```
[Out] ln(b*x+a)/(-a*d+b*c)-ln(d*x+c)/(-a*d+b*c)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {36, 31}

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)}\,dx&=\frac{b\int \frac{1}{a+bx}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx}\,dx}{bc-ad}\\&=\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.72

$$\frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

Maple [A]
time = 0.16, size = 37, normalized size = 1.03

method	result	size
default	$\frac{\ln(dx+c)}{ad-bc}-\frac{\ln(bx+a)}{ad-bc}$	37
norman	$\frac{\ln(dx+c)}{ad-bc}-\frac{\ln(bx+a)}{ad-bc}$	37
risch	$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(-dx-c)}{ad-bc}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)*ln(d*x+c)-1/(a*d-b*c)*ln(b*x+a)
```

Maxima [A]
time = 0.38, size = 36, normalized size = 1.00

$$\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

Fricas [A]
time = 0.50, size = 26, normalized size = 0.72

$$\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] (log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(26) = 52$.
time = 0.16, size = 128, normalized size = 3.56

$$\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x)
```

```
[Out] log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(
a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) - 2
*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d - b
*c)
```

Giac [A]
time = 0.55, size = 46, normalized size = 1.28

$$\frac{b\log\left(|bx+a|\right)}{b^2c-abd}-\frac{d\log\left(|dx+c|\right)}{bcd-ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/(b^2*c - a*b*d) - d*log(abs(d*x + c))/(b*c*d - a*d^2)
```

Mupad [B]
time = 0.26, size = 25, normalized size = 0.69

$$\frac{\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{a\,d-b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)),x)
```

```
[Out] log((c + d*x)/(a + b*x))/(a*d - b*c)
```

3.1341 $\int \frac{1}{(a+bx)^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -1/(-a*d+b*c)/(b*x+a)-d*ln(b*x+a)/(-a*d+b*c)^2+d*ln(d*x+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {46}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.93

$$\frac{-bc+ad-d(a+bx)\log(a+bx)+d(a+bx)\log(c+dx)}{(bc-ad)^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
 - a*d)^2*(a + b*x))
```

Maple [A]
time = 0.16, size = 57, normalized size = 1.00

method	result	size
default	$\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}$	57
risch	$\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}+\frac{d\ln(-dx-c)}{a^2d^2-2abcd+b^2c^2}$	86
norman	$-\frac{bx}{a(ad-bc)(bx+a)}+\frac{d\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A]
time = 0.32, size = 92, normalized size = 1.61

$$-\frac{d\log\left(bx+a\right)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log\left(dx+c\right)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A]
time = 0.65, size = 93, normalized size = 1.63

$$-\frac{bc-ad+(bdx+ad)\log\left(bx+a\right)-(bdx+ad)\log\left(dx+c\right)}{ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.
time = 0.40, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{\left(ad-bc\right)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{\left(ad-bc\right)^2}+\frac{1}{a^2d-abc+x\left(abd-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A]
time = 0.57, size = 78, normalized size = 1.37

$$\frac{bd\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{b}{(b^2c-abd)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

Mupad [B]
time = 0.14, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)}-\frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

3.1342 $\int \frac{1}{(a+bx)^3(c+dx)} \, dx$

Optimal. Leaf size=82

$$-\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}$$

```
[Out] -1/2/(-a*d+b*c)/(b*x+a)^2+d/(-a*d+b*c)^2/(b*x+a)+d^2*ln(b*x+a)/(-a*d+b*c)^3
-d^2*ln(d*x+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{46\}$

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] -1/2*1/((b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a
 + b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)} \, dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3}\right. \\ &= -\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(-bc+3ad+2bdx)}{(a+bx)^2}+2d^2\log(a+bx)-2d^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] (((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
 - 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

Maple [A]
time = 0.17, size = 81, normalized size = 0.99

method	result	size
default	$\frac{d^{2} \ln(dx+c)}{(ad-bc)^{3}}+\frac{1}{2(ad-bc)(bx+a)^{2}}+\frac{d}{(ad-bc)^{2}(bx+a)}-\frac{d^{2} \ln(bx+a)}{(ad-bc)^{3}}$	81
risch	$\frac{\frac{bdx}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+\frac{3ad-bc}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}}{(bx+a)^{2}}-\frac{d^{2} \ln(bx+a)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{d^{2} \ln(-dx-c)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$	172
norman	$\frac{\frac{bdx}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+\frac{3a\,b^{2}d-b^{3}c}{2b^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{(bx+a)^{2}}+\frac{d^{2} \ln(dx+c)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{d^{2} \ln(bx+a)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2
/(a*d-b*c)^3*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(80) = 160$.
time = 0.31, size = 202, normalized size = 2.46

$$\frac{d^{2} \log\left(bx+a\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}-\frac{d^{2} \log\left(dx+c\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}+\frac{2\,bdx-bc+3\,ad}{2\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{2}+2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.
time = 0.94, size = 242, normalized size = 2.95

$$-\frac{b^{2}c^{2}-4\,abcd+3\,a^{2}d^{2}-2\left(b^{2}cd-abd^{2}\right)x-2\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\log\left(bx+a\right)+2\left(b^{2}d^{2}x^{2}+2\,abd^{2}x+a^{2}d^{2}\right)\log\left(dx+c\right)}{2\left(a^{2}b^{3}c^{3}-3\,a^{3}b^{2}c^{2}d+3\,a^{4}bcd^{2}-a^{5}d^{3}+\left(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3}\right)x^{2}+2\left(ab^{4}c^{3}-3\,a^{2}b^{3}c^{2}d+3\,a^{3}b^{2}cd^{2}-a^{4}bd^{3}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2 + 2*
(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(68) = 136$.

time = 0.59, size = 381, normalized size = 4.65

$$\frac{d^2\log\left(x+\frac{-\frac{a^4d^6}{(ad-bc)^3}+\frac{4a^3bcd^5}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4}{(ad-bc)^3}+\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3-\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}-\frac{d^2\log\left(x+\frac{\frac{a^4d^6}{(ad-bc)^3}-\frac{4a^3bcd^5}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4}{(ad-bc)^3}-\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3+\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}+\frac{3ad-bc+2bdx}{2a^4d^2-4a^3bcd+2a^2b^2c^2+x^2\cdot(2a^2b^2d^2-4ab^3cd+2b^4c^2)+x\,(4a^3bd^2-8a^2b^2cd+4ab^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c),x)
```

```
[Out] d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3 +
a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*c)*
*3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*c)**
3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b*c)**
3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b
*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**2*b**2*
c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*a**3*b*d
**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.

time = 0.53, size = 165, normalized size = 2.01

$$\frac{bd^2\log(|bx+a|)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{d^3\log(|dx+c|)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4}-\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\,(b^2cd-abd^2)x}{2\,(bc-ad)^3(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d^2*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3) - d^3*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) - 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(b*x + a)^2)
```

Mupad [B]

time = 0.16, size = 182, normalized size = 2.22

$$\frac{\frac{3\,a\,d-b\,c}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{a^2+2\,a\,b\,x+b^2\,x^2}-\frac{2\,d^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{(a\,d-b\,c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)),x)
```

```
[Out] ((3*a*d - b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x)/(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a^2 + b^2*x^2 + 2*a*b*x) - (2*d^2*atanh((a^3*d^3 + b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

3.1343 $\int \frac{(a+bx)^5}{(c+dx)^2} \, dx$

Optimal. Leaf size=130

$$-\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c+dx)^4}{4d^6}+\frac{5b(bc-ad)^4 \text{l}}{d^6}$$

```
[Out] -10*b^2*(-a*d+b*c)^3*x/d^5+(-a*d+b*c)^5/d^6/(d*x+c)+5*b^3*(-a*d+b*c)^2*(d*x
+c)^2/d^6-5/3*b^4*(-a*d+b*c)*(d*x+c)^3/d^6+1/4*b^5*(d*x+c)^4/d^6+5*b*(-a*d+
b*c)^4*ln(d*x+c)/d^6
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+dx)^4}{4d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (-10*b^2*(b*c - a*d)^3*x)/d^5 + (b*c - a*d)^5/(d^6*(c + d*x)) + (5*b^3*(b*c
 - a*d)^2*(c + d*x)^2)/d^6 - (5*b^4*(b*c - a*d)*(c + d*x)^3)/(3*d^6) + (b^5
*(c + d*x)^4)/(4*d^6) + (5*b*(b*c - a*d)^4*Log[c + d*x])/d^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^2}\,dx = \int \left(-\frac{10b^2(bc-ad)^3}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^2}+\frac{5b(bc-ad)^4}{d^5(c+dx)}+\frac{10b^3(bc-ad)^2(c+dx)}{d^5}-\frac{5b^4(bc-\ldots\right.$$

$$= -\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c\ldots}{}$$

Mathematica [A]
time = 0.05, size = 228, normalized size = 1.75

$$\frac{60a^4bcd^4-12a^5d^5+120a^3b^2d^3(-c^2+cdx+d^2x^2)+60a^2b^3d^2(2c^3-4c^2dx-3cd^2x^2+d^3x^3)+20ab^4d(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4)+b^5(12c^5-48c^4dx-30c^3d^2x^2+10c^2d^3x^3-5cd^4x^4+3d^5x^5)+60b(bc-ad)^4(c+dx)\log(c+dx)}{12d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (60*a^4*b*c*d^4 - 12*a^5*d^5 + 120*a^3*b^2*d^3*(-c^2 + c*d*x + d^2*x^2) + 6
0*a^2*b^3*d^2*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + 20*a*b^4*d*(-3*
c^4 + 9*c^3*d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) + b^5*(12*c^5 - 48
*c^4*d*x - 30*c^3*d^2*x^2 + 10*c^2*d^3*x^3 - 5*c*d^4*x^4 + 3*d^5*x^5) + 60*
b*(b*c - a*d)^4*(c + d*x)*Log[c + d*x])/(12*d^6*(c + d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(126) = 252$.

time = 0.18, size = 259, normalized size = 1.99

method	result
norman	$\frac{\frac{\left(a^5 d^5-5a^4 bc\, d^4+20a^3 b^2 c^2 d^3-30a^2 b^3 c^3 d^2+20a\, b^4 c^4 d-5b^5 c^5\right)x}{d^5 c}+\frac{b^5 x^5}{4d}+\frac{5b^2\left(4a^3 d^3-6a^2 bc\, d^2+4a\, b^2 c^2 d-b^3 c^3\right)x^2}{2d^4}+\frac{5b^3\left(6a^2 d^2-4abcd+b^2 c^2\right)}{6d^3}}{dx+c}$
default	$\frac{b^2\left(\frac{1}{4}d^3 x^4 b^3+\frac{5}{3}a\, b^2 d^3 x^3-\frac{2}{3}b^3 c\, d^2 x^3+5a^2 b\, d^3 x^2-5a\, b^2 c\, d^2 x^2+\frac{3}{2}b^3 c^2 d\, x^2+10a^3 d^3 x-20a^2 bc\, d^2 x+15a\, b^2 c^2 dx-4b^3 c^3 x\right)}{d^5}-\frac{a^5 d^5-5a}{}$
risch	$\frac{b^5 x^4}{4d^2}+\frac{5b^4 a\, x^3}{3d^2}-\frac{2b^5 c\, x^3}{3d^3}+\frac{5b^3 a^2 x^2}{d^2}-\frac{5b^4 ac\, x^2}{d^3}+\frac{3b^5 c^2 x^2}{2d^4}+\frac{10b^2 a^3 x}{d^2}-\frac{20b^3 a^2 cx}{d^3}+\frac{15b^4 a\, c^2 x}{d^4}-\frac{4b^5 c^3 x}{d^5}-\frac{a^5}{d(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^5*(1/4*d^3*x^4*b^3+5/3*a*b^2*d^3*x^3-2/3*b^3*c*d^2*x^3+5*a^2*b*d^3*x^
2-5*a*b^2*c*d^2*x^2+3/2*b^3*c^2*d*x^2+10*a^3*d^3*x-20*a^2*b*c*d^2*x+15*a*b^
2*c^2*d*x-4*b^3*c^3*x)-(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3
*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)+5*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(126) = 252$.

time = 0.29, size = 264, normalized size = 2.03

$$\frac{b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5}{d^7x + cd^6} + \frac{3\,b^5d^3x^4 - 4\,(2\,b^5cd^2 - 5\,ab^4d^3)x^3 + 6\,(3\,b^5c^2d - 10\,ab^4cd^2 + 10\,a^2b^3d^3)x^2 - 12\,(4\,b^5c^3 - 15\,ab^4c^2d + 20\,a^2b^3cd^2 - 10\,a^3b^2d^3)x}{12\,d^5} + \frac{5\,(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4)\log(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*
b*c*d^4 - a^5*d^5)/(d^7*x + c*d^6) + 1/12*(3*b^5*d^3*x^4 - 4*(2*b^5*c*d^2 -
 5*a*b^4*d^3)*x^3 + 6*(3*b^5*c^2*d - 10*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^2 -
 12*(4*b^5*c^3 - 15*a*b^4*c^2*d + 20*a^2*b^3*c*d^2 - 10*a^3*b^2*d^3)*x)/d^5
 + 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4)*log(d*x + c)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(126) = 252$.
time = 1.03, size = 373, normalized size = 2.87

$$\frac{3 b^5 d^5 x^5 + 12 b^5 c^5 - 60 a b^4 c^4 d + 120 a^2 b^3 c^3 d^2 - 120 a^3 b^2 c^2 d^3 + 60 a^4 b c d^4 - 12 a^5 d^5 - 5 (b^5 c d^4 - 4 a b^4 d^5) x^4 + 10 (b^5 c^2 d^3 - 4 a b^4 c d^4 + 6 a^2 b^3 d^5) x^3 - 30 (b^5 c^3 d^2 - 4 a b^4 c^2 d^3 + 6 a^2 b^3 c d^4 - 4 a^3 b^2 d^5) x^2 - 12 (4 b^5 c^4 d - 15 a b^4 c^3 d^2 + 20 a^2 b^3 c^2 d^3 - 10 a^3 b^2 c d^4) x + 60 \left(b^5 c^5 - 4 a b^4 c^4 d + 6 a^2 b^3 c^3 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b c d^4 + (b^5 c^4 d - 4 a b^4 c^3 d^2 + 6 a^2 b^3 c^2 d^3 - 4 a^3 b^2 c d^4 + a^4 b d^5) x\right) \log(dx + c)}{12 (d^7 x + c d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*d^5*x^5 + 12*b^5*c^5 - 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 - 1
20*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 - 12*a^5*d^5 - 5*(b^5*c*d^4 - 4*a*b^4*d
^5)*x^4 + 10*(b^5*c^2*d^3 - 4*a*b^4*c*d^4 + 6*a^2*b^3*d^5)*x^3 - 30*(b^5*c^
3*d^2 - 4*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 - 4*a^3*b^2*d^5)*x^2 - 12*(4*b^5*
c^4*d - 15*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4)*x + 60*(b
^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*c*d^
4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^
4*b*d^5)*x)*log(d*x + c))/(d^7*x + c*d^6)
```

Sympy [A]
time = 0.49, size = 231, normalized size = 1.78

$$\frac{b^5 x^4}{4 d^2} + \frac{5 b (a d - b c)^4 \log(c + d x)}{d^6} + x^3 \cdot \left(\frac{5 a b^4}{3 d^2} - \frac{2 b^5 c}{3 d^3}\right) + x^2 \cdot \left(\frac{5 a^2 b^3}{d^2} - \frac{5 a b^4 c}{d^3} + \frac{3 b^5 c^2}{2 d^4}\right) + x \left(\frac{10 a^3 b^2}{d^2} - \frac{20 a^2 b^3 c}{d^3} + \frac{15 a b^4 c^2}{d^4} - \frac{4 b^5 c^3}{d^5}\right) + \frac{-a^5 d^5 + 5 a^4 b c d^4 - 10 a^3 b^2 c^2 d^3 + 10 a^2 b^3 c^3 d^2 - 5 a b^4 c^4 d + b^5 c^5}{c d^6 + d^7 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**2,x)
```

```
[Out] b**5*x**4/(4*d**2) + 5*b*(a*d - b*c)**4*log(c + d*x)/d**6 + x**3*(5*a*b**4/
(3*d**2) - 2*b**5*c/(3*d**3)) + x**2*(5*a**2*b**3/d**2 - 5*a*b**4*c/d**3 +
3*b**5*c**2/(2*d**4)) + x*(10*a**3*b**2/d**2 - 20*a**2*b**3*c/d**3 + 15*a*b
**4*c**2/d**4 - 4*b**5*c**3/d**5) + (-a**5*d**5 + 5*a**4*b*c*d**4 - 10*a**3
*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d + b**5*c**5)/(c*
d**6 + d**7*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(126) = 252$.
time = 0.54, size = 339, normalized size = 2.61

$$\frac{\left(3 b^5 - \frac{20 (b^5 c d - a b^4 d^2)}{(dx+c) d} + \frac{60 (b^5 c^2 d^2 - 2 a b^4 c d^3 + a^2 b^3 d^4)}{(dx+c)^2 d^2} - \frac{120 (b^5 c^3 d^3 - 3 a b^4 c^2 d^4 + 3 a^2 b^3 c d^5 - a^3 b^2 d^6)}{(dx+c)^3 d^3}\right) (dx+c)^4}{12 d^6} - \frac{5 (b^5 c^4 - 4 a b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c d^3 + a^4 b d^4) \log\left(\frac{|dx+c|}{(dx+c)^2 |d|}\right)}{d^6} + \frac{\frac{b^5 c^5 d^4}{dx+c} - \frac{5 a b^4 c^4 d^5}{dx+c} + \frac{10 a^2 b^3 c^3 d^6}{dx+c} - \frac{10 a^3 b^2 c^2 d^7}{dx+c} + \frac{5 a^4 b c d^8}{dx+c} - \frac{a^5 d^9}{dx+c}}{d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5 - 20*(b^5*c*d - a*b^4*d^2)/((d*x + c)*d) + 60*(b^5*c^2*d^2 - 2*
a*b^4*c*d^3 + a^2*b^3*d^4)/((d*x + c)^2*d^2) - 120*(b^5*c^3*d^3 - 3*a*b^4*c
^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6)/((d*x + c)^3*d^3))*(d*x + c)^4/d^6
```

```
- 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 + (b^5*c^5*d^4/(d*x + c) -
5*a*b^4*c^4*d^5/(d*x + c) + 10*a^2*b^3*c^3*d^6/(d*x + c) - 10*a^3*b^2*c^2*d
^7/(d*x + c) + 5*a^4*b*c*d^8/(d*x + c) - a^5*d^9/(d*x + c))/d^10
```

Mupad [B]
time = 0.25, size = 327, normalized size = 2.52

$$x^3\left(\frac{5\,a\,b^4}{3\,d^2}-\frac{2\,b^5\,c}{3\,d^3}\right)+x\left(\frac{2\,c\left(\frac{2\,c\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d}-\frac{10\,a^2\,b^3}{d^2}+\frac{b^5\,c^2}{d^4}\right)}{d}+\frac{10\,a^3\,b^2}{d^2}-\frac{c^2\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d^2}\right)-x^2\left(\frac{c\left(\frac{5\,a\,b^4}{d^2}-\frac{2\,b^5\,c}{d^3}\right)}{d}-\frac{5\,a^2\,b^3}{d^2}+\frac{b^5\,c^2}{2\,d^4}\right)+\frac{\ln\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)}{d^6}-\frac{a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5}{d\,\left(x\,d^6+c\,d^5\right)}+\frac{b^5\,x^4}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^2,x)
```

```
[Out] x^3*((5*a*b^4)/(3*d^2) - (2*b^5*c)/(3*d^3)) + x*((2*c*((2*c*((5*a*b^4)/d^2
- (2*b^5*c)/d^3))/d - (10*a^2*b^3)/d^2 + (b^5*c^2)/d^4))/d + (10*a^3*b^2)/d
^2 - (c^2*((5*a*b^4)/d^2 - (2*b^5*c)/d^3))/d^2) - x^2*((c*((5*a*b^4)/d^2 -
(2*b^5*c)/d^3))/d - (5*a^2*b^3)/d^2 + (b^5*c^2)/(2*d^4)) + (log(c + d*x)*(5
*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c
^3*d))/d^6 - (a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)/(d*(c*d^5 + d^6*x)) + (b^5*x^4)/(4*d^2)
```

3.1344 $\int \frac{(a+bx)^4}{(c+dx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}$$

```
[Out] 6*b^2*(-a*d+b*c)^2*x/d^4-(-a*d+b*c)^4/d^5/(d*x+c)-2*b^3*(-a*d+b*c)*(d*x+c)^
2/d^5+1/3*b^4*(d*x+c)^3/d^5-4*b*(-a*d+b*c)^3*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^2}\,dx &= \int \left(\frac{6b^2(bc-ad)^2}{d^4}+\frac{(-bc+ad)^4}{d^4(c+dx)^2}-\frac{4b(bc-ad)^3}{d^4(c+dx)}-\frac{4b^3(bc-ad)(c+dx)}{d^4}+\frac{b^4(c+dx)^2}{d^4}\right)\\ &= \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c}{d^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 165, normalized size = 1.59

$$\frac{12a^3bcd^3-3a^4d^4+18a^2b^2d^2(-c^2+cdx+d^2x^2)+6ab^3d(2c^3-4c^2dx-3cd^2x^2+d^3x^3)+b^4(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4)-12b(bc-ad)^3(c+dx)\log(c+dx)}{3d^5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*a
*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*d*
x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x)*L
og[c + d*x])/(3*d^5*(c + d*x))
```

Maple [A]
time = 0.14, size = 175, normalized size = 1.68

method	result
default	$\frac{b^2\left(\frac{1}{3}d^2x^3b^2+2ab\,d^2x^2-b^2cd\,x^2+6a^2d^2x-8abcdx+3b^2c^2x\right)}{d^4}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}{d^5(dx+c)}+\frac{4b\left(a^3d^3-3a^2bc\,d^2+\cdots\right.}{}$
norman	$\frac{\frac{\left(a^4d^4-4a^3bc\,d^3+12a^2b^2c^2d^2-12a\,b^3c^3d+4b^4c^4\right)x}{d^4c}+\frac{b^4x^4}{3d}+\frac{2b^2\left(3a^2d^2-3abcd+b^2c^2\right)x^2}{d^3}+\frac{2b^3(3ad-bc)x^3}{3d^2}}{dx+c}+\frac{4b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d\cdots\right.}{d^5}$
risch	$\frac{b^4x^3}{3d^2}+\frac{2b^3a\,x^2}{d^2}-\frac{b^4c\,x^2}{d^3}+\frac{6b^2a^2x}{d^2}-\frac{8b^3acx}{d^3}+\frac{3b^4c^2x}{d^4}-\frac{a^4}{d(dx+c)}+\frac{4a^3bc}{d^2(dx+c)}-\frac{6a^2b^2c^2}{d^3(dx+c)}+\frac{4a\,b^3c^3}{d^4(dx+c)}-\frac{b^4c^4}{d^5(dx+c\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^4*(1/3*d^2*x^3*b^2+2*a*b*d^2*x^2-b^2*c*d*x^2+6*a^2*d^2*x-8*a*b*c*d*x+
3*b^2*c^2*x)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)/d^5/(d*x+c)+4*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*x+
c)
```

Maxima [A]
time = 0.29, size = 183, normalized size = 1.76

$$-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{d^6x+cd^5}+\frac{b^4d^2x^3-3\,(b^4cd-2\,ab^3d^2)x^2+3\,(3\,b^4c^2-8\,ab^3cd+6\,a^2b^2d^2)x}{3\,d^4}-\frac{4\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(d
^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b^4
*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(102) = 204.
time = 1.27, size = 267, normalized size = 2.57

$$\frac{b^4d^4x^4-3\,b^4c^4+12\,ab^3c^3d-18\,a^2b^2c^2d^2+12\,a^3bcd^3-3\,a^4d^4-2\,(b^4cd^3-3\,ab^3d^4)x^3+6\,(b^4c^2d^2-3\,ab^3cd^3+3\,a^2b^2d^4)x^2+3\,(3\,b^4c^3d-8\,ab^3c^2d^2+6\,a^2b^2cd^3)x-12\,(b^4c^4-3\,ab^3c^3d+3\,a^2b^2c^2d^2-a^3bcd^3+(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x)\log\left(dx+c\right)}{3\,(d^6x+cd^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 3
*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*b*c*
d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*log(d*
x + c))/(d^6*x + c*d^5)
```

Sympy [A]
time = 0.38, size = 155, normalized size = 1.49

$$\frac{b^4x^3}{3d^2}+\frac{4b(ad-bc)^3\log{(c+dx)}}{d^5}+x^2\cdot\left(\frac{2ab^3}{d^2}-\frac{b^4c}{d^3}\right)+x\left(\frac{6a^2b^2}{d^2}-\frac{8ab^3c}{d^3}+\frac{3b^4c^2}{d^4}\right)+\frac{-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d-b^4c^4}{cd^5+d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**2,x)
```

```
[Out] b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 + x**2*(2*a*b**3/
d**2 - b**4*c/d**3) + x*(6*a**2*b**2/d**2 - 8*a*b**3*c/d**3 + 3*b**4*c**2/d
**4) + (-a**4*d**4 + 4*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 + 4*a*b**3*c**
3*d - b**4*c**4)/(c*d**5 + d**6*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(102) = 204.
time = 0.63, size = 245, normalized size = 2.36

$$\frac{\left(b^4-\frac{6\,(b^4cd-ab^3d^2)}{(dx+c)d}+\frac{18\,(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4)}{(dx+c)^2d^2}\right)(dx+c)^3}{3\,d^5}+\frac{4\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}-\frac{\frac{b^4c^4d^3}{dx+c}-\frac{4\,ab^3c^3d^4}{dx+c}+\frac{6\,a^2b^2c^2d^5}{dx+c}-\frac{4\,a^3bcd^6}{dx+c}+\frac{a^4d^7}{dx+c}}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^4 - 6*(b^4*c*d - a*b^3*d^2)/((d*x + c)*d) + 18*(b^4*c^2*d^2 - 2*a*b^
3*c*d^3 + a^2*b^2*d^4)/((d*x + c)^2*d^2))*(d*x + c)^3/d^5 + 4*(b^4*c^3 - 3*
a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c)/((d*x + c)^2*ab
s(d)))/d^5 - (b^4*c^4*d^3/(d*x + c) - 4*a*b^3*c^3*d^4/(d*x + c) + 6*a^2*b^2
*c^2*d^5/(d*x + c) - 4*a^3*b*c*d^6/(d*x + c) + a^4*d^7/(d*x + c))/d^8
```

Mupad [B]
time = 0.07, size = 203, normalized size = 1.95

$$x^2\,\left(\frac{2\,a\,b^3}{d^2}-\frac{b^4\,c}{d^3}\right)-x\,\left(\frac{2\,c\,\left(\frac{4\,a\,b^3}{d^2}-\frac{2\,b^4\,c}{d^3}\right)}{d}-\frac{6\,a^2\,b^2}{d^2}+\frac{b^4\,c^2}{d^4}\right)+\frac{b^4\,x^3}{3\,d^2}-\frac{\ln\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{d^5}-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}{d\,\left(x\,d^5+c\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^2,x)
```

```
[Out] x^2*((2*a*b^3)/d^2 - (b^4*c)/d^3) - x*((2*c*((4*a*b^3)/d^2 - (2*b^4*c)/d^3)
)/d - (6*a^2*b^2)/d^2 + (b^4*c^2)/d^4) + (b^4*x^3)/(3*d^2) - (log(c + d*x)*
(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/d^5 - (a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(d*(c*d^4
 + d^5*x))
```

3.1345 $\int \frac{(a+bx)^3}{(c+dx)^2} dx$

Optimal. Leaf size=75

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}$$

```
[Out] -b^2*(-3*a*d+2*b*c)*x/d^3+1/2*b^3*x^2/d^2+(-a*d+b*c)^3/d^4/(d*x+c)+3*b*(-a*
d+b*c)^2*ln(d*x+c)/d^4
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x}{d^2}+\frac{(-bc+ad)^3}{d^3(c+dx)^2}+\frac{3b(bc-ad)^2}{d^3(c+dx)}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 114, normalized size = 1.52

$$-\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{d^4(c+dx)}+\frac{3(b^3c^2-2ab^2cd+a^2bd^2)\log(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*Log[c + d*x])/d^4
```

Maple [A]
time = 0.14, size = 108, normalized size = 1.44

method	result
default	$\frac{b^2\left(\frac{1}{2}bd\,x^2+3adx-2bcx\right)}{d^3}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{d^4(dx+c)}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}$
norman	$\frac{-\frac{a^3d^3-3a^2bc\,d^2+6a\,b^2c^2d-3b^3c^3}{d^4}+\frac{b^3x^3}{2d}+\frac{3b^2(2ad-bc)x^2}{2d^2}}{dx+c}+\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}$
risch	$\frac{b^3x^2}{2d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}-\frac{a^3}{d(dx+c)}+\frac{3a^2bc}{d^2(dx+c)}-\frac{3a\,b^2c^2}{d^3(dx+c)}+\frac{b^3c^3}{d^4(dx+c)}+\frac{3b\ln(dx+c)a^2}{d^2}-\frac{6b^2\ln(dx+c)ac}{d^3}+\frac{3b^3\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/d^3*(1/2*b*d*x^2+3*a*d*x-2*b*c*x)-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-
b^3*c^3)/d^4/(d*x+c)+3*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*x+c)
```

Maxima [A]
time = 0.29, size = 117, normalized size = 1.56

$$\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{d^5x+cd^4}+\frac{b^3dx^2-2\,(2\,b^3c-3\,ab^2d)x}{2\,d^3}+\frac{3\,(b^3c^2-2\,ab^2cd+a^2bd^2)\log{(dx+c)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*(
b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*log(d*x + c)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(73) = 146.
time = 0.90, size = 172, normalized size = 2.29

$$\frac{b^3d^3x^3+2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2-2\,a^3d^3-3\,(b^3cd^2-2\,ab^2d^3)x^2-2\,(2\,b^3c^2d-3\,ab^2cd^2)x+6\,(b^3c^3-2\,ab^2c^2d+a^2bcd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x)\log{(dx+c)}}{2\,(d^5x+cd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(b^
3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

Sympy [A]
time = 0.26, size = 102, normalized size = 1.36

$$\frac{b^3x^2}{2d^2}+\frac{3b(ad-bc)^2\log{(c+dx)}}{d^4}+x\left(\frac{3ab^2}{d^2}-\frac{2b^3c}{d^3}\right)+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{cd^4+d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 + x*(3*a*b**2/d**
2 - 2*b**3*c/d**3) + (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3
*c**3)/(c*d**4 + d**5*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(73) = 146.
time = 0.62, size = 166, normalized size = 2.21

$$\frac{\left(b^3-\frac{6\,(b^3cd-ab^2d^2)}{(dx+c)d}\right)(dx+c)^2}{2\,d^4}-\frac{3\,(b^3c^2-2\,ab^2cd+a^2bd^2)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4}+\frac{\frac{b^3c^3d^2}{dx+c}-\frac{3\,ab^2c^2d^3}{dx+c}+\frac{3\,a^2bcd^4}{dx+c}-\frac{a^3d^5}{dx+c}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 6*(b^3*c*d - a*b^2*d^2)/((d*x + c)*d))*(d*x + c)^2/d^4 - 3*(b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 +
 (b^3*c^3*d^2/(d*x + c) - 3*a*b^2*c^2*d^3/(d*x + c) + 3*a^2*b*c*d^4/(d*x +
c) - a^3*d^5/(d*x + c))/d^6
```

Mupad [B]
time = 0.08, size = 123, normalized size = 1.64

$$x\left(\frac{3\,a\,b^2}{d^2}-\frac{2\,b^3\,c}{d^3}\right)+\frac{\ln\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}{d^4}-\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{d\,\left(x\,d^4+c\,d^3\right)}+\frac{b^3\,x^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (log(c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d))/d^4 - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
d*(c*d^3 + d^4*x)) + (b^3*x^2)/(2*d^2)
```

3.1346 $\int \frac{(a+bx)^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=51

$$\frac{b^2x}{d^2}-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}$$

```
[Out] b^2*x/d^2-(-a*d+b*c)^2/d^3/(d*x+c)-2*b*(-a*d+b*c)*ln(d*x+c)/d^3
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^2}\,dx &= \int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)dx\\ &= \frac{b^2x}{d^2}-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.92

$$\frac{b^2dx-\frac{(bc-ad)^2}{c+dx}+2b(-bc+ad)\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^2,x]
[Out] (b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

Maple [A]
time = 0.14, size = 63, normalized size = 1.24

method	result	size
default	$\frac{b^2 x}{d^2}-\frac{a^2 d^2-2abcd+b^2 c^2}{d^3 (dx+c)}+\frac{2b(ad-bc)\ln(dx+c)}{d^3}$	63
norman	$\frac{\frac{b^2 x^2}{d}-\frac{a^2 d^2-2abcd+2b^2 c^2}{d^3}}{dx+c}+\frac{2b(ad-bc)\ln(dx+c)}{d^3}$	68
risch	$\frac{b^2 x}{d^2}-\frac{a^2}{d(dx+c)}+\frac{2abc}{d^2(dx+c)}-\frac{b^2 c^2}{d^3(dx+c)}+\frac{2b\ln(dx+c)a}{d^2}-\frac{2b^2\ln(dx+c)c}{d^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] b^2*x/d^2-(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)+2*b/d^3*(a*d-b*c)*ln(d*x+
c)
```

Maxima [A]
time = 0.41, size = 67, normalized size = 1.31

$$\frac{b^2 x}{d^2}-\frac{b^2 c^2-2\,abcd+a^2 d^2}{d^4 x+cd^3}-\frac{2\,(b^2 c-abd)\log(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a*
b*d)*log(d*x + c)/d^3
```

Fricas [A]
time = 0.71, size = 92, normalized size = 1.80

$$\frac{b^2 d^2 x^2+b^2 cdx-b^2 c^2+2\,abcd-a^2 d^2-2\,(b^2 c^2-abcd+(b^2 cd-abd^2)x)\log(dx+c)}{d^4 x+cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] (b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

Sympy [A]
time = 0.17, size = 60, normalized size = 1.18

$$\frac{b^2x}{d^2} + \frac{2b(ad-bc)\log(c+dx)}{d^3} + \frac{-a^2d^2+2abcd-b^2c^2}{cd^3+d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 + (-a**2*d**2 + 2*a*b*c*d -
 b**2*c**2)/(c*d**3 + d**4*x)
```

Giac [A]
time = 0.60, size = 98, normalized size = 1.92

$$\frac{(dx+c)b^2}{d^3} + \frac{2(b^2c-abd)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3} - \frac{\frac{b^2c^2d}{dx+c} - \frac{2abcd^2}{dx+c} + \frac{a^2d^3}{dx+c}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d^3 + 2*(b^2*c - a*b*d)*log(abs(d*x + c)/((d*x + c)^2*abs(d))
)/d^3 - (b^2*c^2*d/(d*x + c) - 2*a*b*c*d^2/(d*x + c) + a^2*d^3/(d*x + c))/d
^4
```

Mupad [B]
time = 0.24, size = 71, normalized size = 1.39

$$\frac{b^2\,x}{d^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{d\,\left(x\,d^3 + c\,d^2\right)} - \frac{\ln\left(c+d\,x\right)\,\left(2\,b^2\,c - 2\,a\,b\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] (b^2*x)/d^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(d*(c*d^2 + d^3*x)) - (log(c
+ d*x)*(2*b^2*c - 2*a*b*d))/d^3
```

3.1347 $\int \frac{a+bx}{(c+dx)^2} dx$

Optimal. Leaf size=31

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

[Out] (-a*d+b*c)/d^2/(d*x+c)+b*ln(d*x+c)/d^2

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {45}

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c + d*x)^2,x]

[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^2}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^2}+\frac{b}{d(c+dx)}\right)dx\\ &= \frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(c + d*x)^2,x]

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Maple [A]
time = 0.12, size = 33, normalized size = 1.06

method	result	size
default	$-\frac{ad-bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	33
norman	$-\frac{ad-bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	33
risch	$-\frac{a}{d(dx+c)}+\frac{bc}{d^2(dx+c)}+\frac{b\ln(dx+c)}{d^2}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] -(a*d-b*c)/d^2/(d*x+c)+b*ln(d*x+c)/d^2
```

Maxima [A]
time = 0.32, size = 34, normalized size = 1.10

$$\frac{bc-ad}{d^3x+cd^2}+\frac{b\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] (b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

Fricas [A]
time = 0.83, size = 37, normalized size = 1.19

$$\frac{bc-ad+(bdx+bc)\log(dx+c)}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] (b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.87

$$\frac{b\log(c+dx)}{d^2}+\frac{-ad+bc}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**2,x)
```

```
[Out] b*log(c + d*x)/d**2 + (-a*d + b*c)/(c*d**2 + d**3*x)
```

Giac [A]
time = 0.78, size = 57, normalized size = 1.84

$$-\frac{b\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)}{d}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -b*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d))/d - a/((d*x
 + c)*d)
```

Mupad [B]
time = 0.04, size = 32, normalized size = 1.03

$$\frac{b\,\ln\left(c+d\,x\right)}{d^2}-\frac{a\,d-b\,c}{d^2\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^2,x)
```

```
[Out] (b*log(c + d*x))/d^2 - (a*d - b*c)/(d^2*(c + d*x))
```

3.1348 $\int \frac{1}{(c+dx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{d(c+dx)}$$

[Out] -1/d/(d*x+c)

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-2),x]

[Out] -(1/(d*(c + d*x)))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2} dx = -\frac{1}{d(c+dx)}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-2),x]

[Out] -(1/(d*(c + d*x)))

Maple [A]
time = 0.13, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{1}{d(dx+c)}$	13
default	$-\frac{1}{d(dx+c)}$	13
norman	$\frac{x}{c(dx+c)}$	13
risch	$-\frac{1}{d(dx+c)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/(d*x+c)
```

Maxima [A]
time = 0.31, size = 12, normalized size = 1.00

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/((d*x + c)*d)
```

Fricas [A]
time = 1.33, size = 13, normalized size = 1.08

$$-\frac{1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/(d^2*x + c*d)
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.83

$$-\frac{1}{cd+d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2,x)
```

```
[Out] -1/(c*d + d**2*x)
```

Giac [A]
time = 1.22, size = 12, normalized size = 1.00

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/((d*x + c)*d)
```

Mupad [B]
time = 0.19, size = 12, normalized size = 1.00

$$-\frac{1}{d\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^2,x)
```

```
[Out] -1/(d*(c + d*x))
```

3.1349 $\int \frac{1}{(a+bx)(c+dx)^2} dx$

Optimal. Leaf size=56

$$\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

```
[Out] 1/(-a*d+b*c)/(d*x+c)+b*ln(b*x+a)/(-a*d+b*c)^2-b*ln(d*x+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right)dx\\ &= \frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.95

$$\frac{bc-ad+b(c+dx)\log(a+bx)-b(c+dx)\log(c+dx)}{(bc-ad)^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)*(c + d*x)^2),x]

[Out] (b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c - a*d)^2*(c + d*x))

Maple [A]
time = 0.16, size = 58, normalized size = 1.04

method	result	size
default	$-\frac{1}{(ad-bc)(dx+c)}-\frac{b\ln(dx+c)}{(ad-bc)^2}+\frac{b\ln(bx+a)}{(ad-bc)^2}$	58
risch	$-\frac{1}{(ad-bc)(dx+c)}+\frac{b\ln(-bx-a)}{a^2d^2-2abcd+b^2c^2}-\frac{b\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}$	87
norman	$\frac{dx}{c(ad-bc)(dx+c)}+\frac{b\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}-\frac{b\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}$	88

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] -1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+a)

Maxima [A]
time = 0.29, size = 90, normalized size = 1.61

$$\frac{b\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}-\frac{b\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}+\frac{1}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")

[Out] b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)

Fricas [A]
time = 1.06, size = 92, normalized size = 1.64

$$\frac{bc-ad+(bdx+bc)\log(bx+a)-(bdx+bc)\log(dx+c)}{b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")

[Out] (b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46)=92$.
time = 0.36, size = 233, normalized size = 4.16

$$-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}+\frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{(ad-bc)^2}-\frac{1}{acd-bc^2+x\,(ad^2-bcd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**2,x)
```

```
[Out] -b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2
 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2
*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*
a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d -
b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c*d -
b*c**2 + x*(a*d**2 - b*c*d))
```

Giac [A]
time = 1.24, size = 77, normalized size = 1.38

$$\frac{bd\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{d}{(bcd-ad^2)(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] b*d*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3) + d/((b*c*d - a*d^2)*(d*x + c))
```

Mupad [B]
time = 0.29, size = 47, normalized size = 0.84

$$-\frac{1}{\left(a\,d-b\,c\right)\,\left(c+d\,x\right)}-\frac{b\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{{\left(a\,d-b\,c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^2),x)
```

```
[Out] - 1/((a*d - b*c)*(c + d*x)) - (b*log((c + d*x)/(a + b*x)))/(a*d - b*c)^2
```

3.1350 $\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

```
[Out] -b/(-a*d+b*c)^2/(b*x+a)-d/(-a*d+b*c)^2/(d*x+c)-2*b*d*ln(b*x+a)/(-a*d+b*c)^3
+2*b*d*ln(d*x+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] -(b/((b*c - a*d)^2*(a + b*x))) - d/((b*c - a*d)^2*(c + d*x)) - (2*b*d*Log[a
 + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\frac{\cdots}{(bc-a\cdots}\right.\\ &= -\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx\cdots}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.81

$$\frac{\frac{b(-bc+ad)}{a+bx}+\frac{d(-bc+ad)}{c+dx}-2bd\log(a+bx)+2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
 + b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Maple [A]
time = 0.16, size = 82, normalized size = 1.01

method	result	size
default	$-\frac{d}{(ad-bc)^{2}(dx+c)}-\frac{2db\ln(dx+c)}{(ad-bc)^{3}}-\frac{b}{(ad-bc)^{2}(bx+a)}+\frac{2db\ln(bx+a)}{(ad-bc)^{3}}$	82
risch	$\frac{-\frac{2bdx}{a^{2}d^{2}-2abcd+b^{2}c^{2}}-\frac{ad+bc}{a^{2}d^{2}-2abcd+b^{2}c^{2}}}{(bx+a)(dx+c)}-\frac{2bd\ln(dx+c)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}+\frac{2bd\ln(-bx-a)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$	177
norman	$\frac{\frac{-ab\,d^{2}-b^{2}cd}{db\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{2bdx}{a^{2}d^{2}-2abcd+b^{2}c^{2}}}{(bx+a)(dx+c)}+\frac{2bd\ln(bx+a)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{2bd\ln(dx+c)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$	187

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*
d/(a*d-b*c)^3*b*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(81) = 162.
time = 0.29, size = 208, normalized size = 2.57

$$-\frac{2\,bd\log\left(bx+a\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}+\frac{2\,bd\log\left(dx+c\right)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}-\frac{2\,bdx+bc+ad}{ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}+(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3})x^{2}+(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 2
*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (2*
b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(81) = 162.
time = 0.96, size = 241, normalized size = 2.98

$$-\frac{b^{2}c^{2}-a^{2}d^{2}+2\,(b^{2}cd-abd^{2})x+2\,(b^{2}d^{2}x^{2}+abcd+(b^{2}cd+abd^{2})x)\log\left(bx+a\right)-2\,(b^{2}d^{2}x^{2}+abcd+(b^{2}cd+abd^{2})x)\log\left(dx+c\right)}{ab^{3}c^{4}-3\,a^{2}b^{2}c^{3}d+3\,a^{3}bc^{2}d^{2}-a^{4}cd^{3}+(b^{4}c^{3}d-3\,ab^{3}c^{2}d^{2}+3\,a^{2}b^{2}cd^{3}-a^{3}bd^{4})x^{2}+(b^{4}c^{4}-2\,ab^{3}c^{3}d+2\,a^{3}bcd^{3}-a^{4}d^{4})x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
 a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(70) = 140$.
time = 0.60, size = 406, normalized size = 5.01

$$-\frac{2bd\log\left(x+\frac{-\frac{2a^4bd^5}{(ad-bc)^3}+\frac{8a^3b^2cd^4}{(ad-bc)^3}-\frac{12a^2b^3c^2d^3}{(ad-bc)^3}+\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2-\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3}+\frac{2bd\log\left(x+\frac{\frac{2a^4bd^5}{(ad-bc)^3}-\frac{8a^3b^2cd^4}{(ad-bc)^3}+\frac{12a^2b^3c^2d^3}{(ad-bc)^3}-\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2+\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3}+\frac{-ad-bc-2bdx}{a^3cd^2-2a^2bc^2d+ab^2c^3+x^2\left(a^2bd^3-2ab^2cd^2+b^3c^2d\right)+x\left(a^3d^3-a^2bcd^2-ab^2c^2d+b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] -2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d -
b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**2*d
**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*a**3
*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*a*b
**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)**3 +
2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x)/(a**3*c*
d**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2*c*d**2
+ b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b**3*c**3))
```

Giac [A]
time = 1.11, size = 153, normalized size = 1.89

$$\frac{2\,b^2d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3}-\frac{b^3}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)(bx+a)}+\frac{bd^2}{(bc-ad)^3\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*b^2*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*(b*x + a)) + b*d^2/((b*c - a*d)^3*(b*c/(b*x + a) - a*d/(b*x + a) + d))
```

Mupad [B]
time = 0.33, size = 74, normalized size = 0.91

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)\,(c+d\,x)}-\frac{2\,d}{(a\,d-b\,c)^2\,(c+d\,x)}-\frac{2\,b\,d\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{(a\,d-b\,c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)*(c + d*x)) - (2*d)/((a*d - b*c)^2*(c + d*x)) - (2*
b*d*log((c + d*x)/(a + b*x)))/(a*d - b*c)^3
```

3.1351 $\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}$$

```
[Out] -1/2*b/(-a*d+b*c)^2/(b*x+a)^2+2*b*d/(-a*d+b*c)^3/(b*x+a)+d^2/(-a*d+b*c)^3/(
d*x+c)+3*b*d^2*ln(b*x+a)/(-a*d+b*c)^4-3*b*d^2*ln(d*x+c)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{46\}$

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] -1/2*b/((b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^
2/((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d
^2*Log[c + d*x])/(b*c - a*d)^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^3}-\frac{2b^2d}{(bc-ad)^3(a+bx)^2}+\frac{3b^2d^2}{(bc-ad)^4(a+bx)}-\frac{}{(bc-a}\right.\\ &= -\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(}{(bc-}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.90

$$\frac{-\frac{b(bc-ad)^2}{(a+bx)^2}+\frac{4bd(bc-ad)}{a+bx}+\frac{2d^2(bc-ad)}{c+dx}+6bd^2\log(a+bx)-6bd^2\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] (-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2*
(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*(b
*c - a*d)^4)
```

Maple [A]
time = 0.16, size = 109, normalized size = 1.00

method	result
default	$-\frac{d^2}{(ad-bc)^3(dx+c)}-\frac{3d^2b\ln(dx+c)}{(ad-bc)^4}-\frac{b}{2(ad-bc)^2(bx+a)^2}+\frac{3d^2b\ln(bx+a)}{(ad-bc)^4}-\frac{2bd}{(ad-bc)^3(bx+a)}$
risch	$\frac{-\frac{3b^2d^2x^2}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}-\frac{3(3ad+bc)dbx}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{2a^2d^2+5abcd-b^2c^2}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx+a)^2(dx+c)}+\frac{3d^2b\ln(-bx-a}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-}$
norman	$\frac{-\frac{3b^2d^2x^2}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{-2a^2b^2d^3-5a\,b^3c\,d^2+b^4c^2d}{2d\,b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(-9a\,b^3d^3-3b^4c\,d^2\right)x}{2d\,b^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx+a)^2(dx+c)}+\frac{3d^2b\ln}{a^4d^4-4a^3bc\,d^3+6a^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*d-b*c)^2/(b
*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(107) = 214$.
time = 0.31, size = 386, normalized size = 3.54

$$\frac{3\,bd^2\log\left(bx+a\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{3\,bd^2\log\left(dx+c\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{6\,b^2d^2x^2-b^2c^2+5\,abcd+2\,a^2d^2+3\,(b^2cd+3\,abd^2)x}{2\,(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^3+(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4)x^2+(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b
^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5*a*
b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3
*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*
a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b
^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(107) = 214$.
time = 0.72, size = 494, normalized size = 4.53

$$-\frac{b^3c^3-6\,ab^2c^2d+3\,a^2bcd^2+2\,a^3d^3-6\,(b^3cd^2-ab^2d^3)x^2-3\,(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3)x-6\,(b^3d^3x^3+a^2bcd^2+(b^3cd^2+2\,ab^2d^3)x^2+(2\,ab^2cd^2+a^2bd^3)x)\log\left(bx+a\right)+6\,(b^3d^3x^3+a^2bcd^2+(b^3cd^2+2\,ab^2d^3)x^2+(2\,ab^2cd^2+a^2bd^3)x)\log\left(dx+c\right)}{2\,(a^2b^4c^5-4\,a^3b^3c^4d+6\,a^4b^2c^3d^2-4\,a^5bc^2d^3+a^6cd^4+(b^6c^4d-4\,ab^5c^3d^2+6\,a^2b^4c^2d^3-4\,a^3b^3cd^4+a^4b^2d^5)x^3+(b^6c^5-2\,ab^5c^4d-2\,a^2b^4c^3d^2+8\,a^3b^3c^2d^3-7\,a^4b^2cd^4+2\,a^5bd^5)x^2+(2\,ab^5c^5-7\,a^2b^4c^4d+8\,a^3b^3c^3d^2-2\,a^4b^2c^2d^3-2\,a^5bcd^4+a^6d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5 - 4*a
^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6*c^4*d
 - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^3
 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4
*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*
c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(97) = 194$.
time = 0.95, size = 634, normalized size = 5.82

$$-\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2d^5}{(ad-bc)^4}-\frac{30a^2b^4c^3d^4}{(ad-bc)^4}+\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3-\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{-2a^2d^2-5abcd+b^2c^2-6b^2d^2x^2+x(-9abd^2-3b^2cd)}{2a^5cd^3-6a^4bc^2d^2+6a^3b^2c^3d-2a^2b^3c^4+x^3\cdot(2a^3b^2d^4-6a^2b^3cd^3+6ab^4c^2d^2-2b^5c^3d)+x^2\cdot(4a^4bd^4-10a^3b^2cd^3+6a^2b^3c^2d^2+2ab^4c^3d-2b^5c^4)+x\left(2a^5d^4-2a^4bcd^3-6a^3b^2c^2d^2+10a^2b^3c^3d-4ab^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] -3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*d
 - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4
/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b**6*
c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + 3
*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a*d -
b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4/(a
*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b**6*c**
5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + (-2*
a**2*d**2 - 5*a*b*c*d + b**2*c**2 - 6*b**2*d**2*x**2 + x*(-9*a*b*d**2 - 3*b
**2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*d - 2*a**2
*b**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b**4*c**2*d*
*2 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3 + 6*a**2*b*
*3*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4 - 2*a**4*b*c
*d**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**4*c**4))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(107) = 214$.
time = 1.82, size = 216, normalized size = 1.98

$$\frac{3\,bd^3\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}+\frac{d^5}{(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)(dx+c)}+\frac{5\,b^3d^2-\frac{6\,(b^3cd^3-ab^2d^4)}{(dx+c)d}}{2\,(bc-ad)^4\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 3*b*d^3*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) + d^5/((b^3*c^3*d^3 -
3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*(d*x + c)) + 1/2*(5*b^3*d^2 - 6*
(b^3*c*d^3 - a*b^2*d^4)/((d*x + c)*d))/((b*c - a*d)^4*(b - b*c/(d*x + c) +
a*d/(d*x + c))^2)
```

Mupad [B]
time = 0.40, size = 330, normalized size = 3.03

$$\frac{6\,b\,d^2\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{\frac{2\,a^2\,d^2+5\,a\,b\,c\,d-b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,d\,x\,\left(c\,b^2+3\,a\,d\,b\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(d\,a^2+2\,b\,c\,a\right)+a^2\,c+x^2\,\left(c\,b^2+2\,a\,d\,b\right)+b^2\,d\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^2),x)
```

```
[Out] (6*b*d^2*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c*d^3)/(a*d - b
*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d
- b*c)^4))/(a*d - b*c)^4 - ((2*a^2*d^2 - b^2*c^2 + 5*a*b*c*d)/(2*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*d*x*(b^2*c + 3*a*b*d))/(2*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(x*(a^2*d + 2*a*b*c) + a^2*
c + x^2*(b^2*c + 2*a*b*d) + b^2*d*x^3)
```

3.1352 $\int \frac{(a+bx)^6}{(c+dx)^3}\,dx$

Optimal. Leaf size=158

$$-\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-ad)(c+dx)^3}{d^7}+\frac{b^6(c+d}{4d^7}$$

```
[Out] -20*b^3*(-a*d+b*c)^3*x/d^6-1/2*(-a*d+b*c)^6/d^7/(d*x+c)^2+6*b*(-a*d+b*c)^5/
d^7/(d*x+c)+15/2*b^4*(-a*d+b*c)^2*(d*x+c)^2/d^7-2*b^5*(-a*d+b*c)*(d*x+c)^3/
d^7+1/4*b^6*(d*x+c)^4/d^7+15*b^2*(-a*d+b*c)^4*ln(d*x+c)/d^7
```

Rubi [A]
time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{b^6(c+dx)^4}{4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-20*b^3*(b*c - a*d)^3*x)/d^6 - (b*c - a*d)^6/(2*d^7*(c + d*x)^2) + (6*b*(b
*c - a*d)^5)/(d^7*(c + d*x)) + (15*b^4*(b*c - a*d)^2*(c + d*x)^2)/(2*d^7) -
 (2*b^5*(b*c - a*d)*(c + d*x)^3)/d^7 + (b^6*(c + d*x)^4)/(4*d^7) + (15*b^2*
(b*c - a*d)^4*Log[c + d*x])/d^7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{(c+dx)^3}\,dx &= \int \left(-\frac{20b^3(bc-ad)^3}{d^6}+\frac{(-bc+ad)^6}{d^6(c+dx)^3}-\frac{6b(bc-ad)^5}{d^6(c+dx)^2}+\frac{15b^2(bc-ad)^4}{d^6(c+dx)}+\frac{15b^4(bc-ad)^2(}{d^6}\right.\\ &= -\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-}{}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 303, normalized size = 1.92

$$\frac{-2a^6d^6-12a^5bd^5(c+2dx)+30a^4b^2cd^4(3c+4dx)+40a^3b^3d^3(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)+30a^2b^4d^2(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4)+4ab^5d(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5)+b^6(22c^6-16c^5dx-68c^4d^2x^2-20c^3d^3x^3+5c^2d^4x^4-2cd^5x^5+d^6x^6)+60b^2(bc-ad)^4(c+dx)^2\log(c+dx)}{4d^7(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] (-2*a^6*d^6 - 12*a^5*b*d^5*(c + 2*d*x) + 30*a^4*b^2*c*d^4*(3*c + 4*d*x) + 4
0*a^3*b^3*d^3*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 30*a^2*b^4*d
^2*(7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^5*d
*(-27*c^5 + 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d
^5*x^5) + b^6*(22*c^6 - 16*c^5*d*x - 68*c^4*d^2*x^2 - 20*c^3*d^3*x^3 + 5*c^
2*d^4*x^4 - 2*c*d^5*x^5 + d^6*x^6) + 60*b^2*(b*c - a*d)^4*(c + d*x)^2*Log[c
 + d*x])/(4*d^7*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(152) = 304$.
time = 0.14, size = 351, normalized size = 2.22

method	result
norman	$\frac{-\frac{a^6d^6+6a^5bcd^5-45a^4b^2c^2d^4+180a^3b^3c^3d^3-270a^2b^4c^4d^2+180ab^5c^5d-45b^6c^6}{2d^7}+\frac{b^6x^6}{4d}-\frac{2\left(3a^5bd^5-15a^4b^2cd^4+60a^3b^3c^2d^3-90a^2b^4c^3d^2+60ab\right.}{d^6}}{(dx+c)^2}$
default	$\frac{b^3\left(\frac{1}{4}d^3x^4b^3+2ab^2d^3x^3-b^3cd^2x^3+\frac{15}{2}a^2bd^3x^2-9ab^2cd^2x^2+3b^3c^2dx^2+20a^3d^3x-45a^2bcd^2x+36ab^2c^2dx-10b^3c^3x\right)}{d^6}-\frac{6b\left(a^5d^5-5\right.}{}$
risch	$\frac{b^6x^4}{4d^3}+\frac{2b^5ax^3}{d^3}-\frac{b^6cx^3}{d^4}+\frac{15b^4a^2x^2}{2d^3}-\frac{9b^5acx^2}{d^4}+\frac{3b^6c^2x^2}{d^5}+\frac{20b^3a^3x}{d^3}-\frac{45b^4a^2cx}{d^4}+\frac{36b^5ac^2x}{d^5}-\frac{10b^6c^3x}{d^6}+\frac{(-6a^5b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d^6*(1/4*d^3*x^4*b^3+2*a*b^2*d^3*x^3-b^3*c*d^2*x^3+15/2*a^2*b*d^3*x^2-9
*a*b^2*c*d^2*x^2+3*b^3*c^2*d*x^2+20*a^3*d^3*x-45*a^2*b*c*d^2*x+36*a*b^2*c^2
*d*x-10*b^3*c^3*x)-6*b/d^7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2
*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)+15*b^2/d^7*(a^4*d^4-4*a^3*b*c*d
^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*ln(d*x+c)-1/2/d^7*(a^6*d^6-6*a^
5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*
c^5*d+b^6*c^6)/(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(152) = 304$.
time = 0.33, size = 364, normalized size = 2.30

$$\frac{11\,b^6c^6-54\,ab^5c^5d+105\,a^2b^4c^4d^2-100\,a^3b^3c^3d^3+45\,a^4b^2c^2d^4-6\,a^5bcd^5-a^6d^6+12\,(b^6c^5d-5\,ab^5c^4d^2+10\,a^2b^4c^3d^3-10\,a^3b^3c^2d^4+5\,a^4b^2cd^5-a^5bd^6)x}{2\,(d^9x^2+2\,cd^8x+c^2d^7)}+\frac{b^6d^3x^4-4\,(b^6cd^2-2\,ab^5d^3)x^3+6\,(2\,b^6c^2d-6\,ab^5cd^2+5\,a^2b^4d^3)x^2-4\,(10\,b^6c^3-36\,ab^5c^2d+45\,a^2b^4cd^2-20\,a^3b^3d^3)x}{4\,d^6}+\frac{15\,(b^6c^4-4\,ab^5c^3d+6\,a^2b^4c^2d^2-4\,a^3b^3cd^3+a^4b^2d^4)\log(dx+c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 - 100*a^3*b^3*c^3*d^
3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*d^6 + 12*(b^6*c^5*d - 5*a*b^5*
```

```
c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2*d^4 + 5*a^4*b^2*c*d^5 - a^5*b
*d^6)*x)/(d^9*x^2 + 2*c*d^8*x + c^2*d^7) + 1/4*(b^6*d^3*x^4 - 4*(b^6*c*d^2
- 2*a*b^5*d^3)*x^3 + 6*(2*b^6*c^2*d - 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^2 -
4*(10*b^6*c^3 - 36*a*b^5*c^2*d + 45*a^2*b^4*c*d^2 - 20*a^3*b^3*d^3)*x)/d^6
+ 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b
^2*d^4)*log(d*x + c)/d^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(152) = 304$.
time = 0.68, size = 548, normalized size = 3.47

$$\frac{b^6d^6x^6 + 22\,b^6c^6 - 108\,ab^5c^5d + 210\,a^2b^4c^4d^2 - 200\,a^3b^3c^3d^3 + 90\,a^4b^2c^2d^4 - 12\,a^5bcd^5 - 2\,a^6d^6 - 2\,(b^6cd^5 - 4\,ab^5d^6)x^5 + 5\,(b^6c^2d^4 - 4\,ab^5cd^5 + 6\,a^2b^4d^6)x^4 - 20\,(b^6c^3d^3 - 4\,ab^5c^2d^4 + 6\,a^2b^4cd^5 - 4\,a^3b^3d^6)x^3 - 2\,(34\,b^6c^4d^2 - 126\,ab^5c^3d^3 + 165\,a^2b^4c^2d^4 - 80\,a^3b^3cd^5)x^2 - 4\,(4\,b^6c^5d - 6\,ab^5c^4d^2 - 15\,a^2b^4c^3d^3 + 40\,a^3b^3c^2d^4 - 30\,a^4b^2cd^5 + 6\,a^5bd^6)x + 60\,(b^6c^6 - 4\,ab^5c^5d + 6\,a^2b^4c^4d^2 - 4\,a^3b^3c^3d^3 + a^4b^2c^2d^4 + (b^6c^4d^2 - 4\,ab^5c^3d^3 + 6\,a^2b^4c^2d^4 - 4\,a^3b^3cd^5 + a^4b^2d^6)x^2 + 2\,(b^6c^5d - 4\,ab^5c^4d^2 + 6\,a^2b^4c^3d^3 - 4\,a^3b^3c^2d^4 + a^4b^2cd^5)x)\log(dx + c)}{4\,(d^9x^2 + 2\,cd^8x + c^2d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*d^6*x^6 + 22*b^6*c^6 - 108*a*b^5*c^5*d + 210*a^2*b^4*c^4*d^2 - 200
*a^3*b^3*c^3*d^3 + 90*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 2*a^6*d^6 - 2*(b^6
*c*d^5 - 4*a*b^5*d^6)*x^5 + 5*(b^6*c^2*d^4 - 4*a*b^5*c*d^5 + 6*a^2*b^4*d^6)
*x^4 - 20*(b^6*c^3*d^3 - 4*a*b^5*c^2*d^4 + 6*a^2*b^4*c*d^5 - 4*a^3*b^3*d^6)
*x^3 - 2*(34*b^6*c^4*d^2 - 126*a*b^5*c^3*d^3 + 165*a^2*b^4*c^2*d^4 - 80*a^3
*b^3*c*d^5)*x^2 - 4*(4*b^6*c^5*d - 6*a*b^5*c^4*d^2 - 15*a^2*b^4*c^3*d^3 + 4
0*a^3*b^3*c^2*d^4 - 30*a^4*b^2*c*d^5 + 6*a^5*b*d^6)*x + 60*(b^6*c^6 - 4*a*b
^5*c^5*d + 6*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + (b^6*c
^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^
6)*x^2 + 2*(b^6*c^5*d - 4*a*b^5*c^4*d^2 + 6*a^2*b^4*c^3*d^3 - 4*a^3*b^3*c^2
*d^4 + a^4*b^2*c*d^5)*x)*log(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(144) = 288$.
time = 1.23, size = 340, normalized size = 2.15

$$\frac{b^6x^4}{4d^3} + \frac{15b^2(ad - bc)^4\log(c + dx)}{d^7} + x^3\cdot\left(\frac{2ab^5}{d^3} - \frac{b^6c}{d^4}\right) + x^2\cdot\left(\frac{15a^2b^4}{2d^3} - \frac{9ab^5c}{d^4} + \frac{3b^6c^2}{d^5}\right) + x\left(\frac{20a^3b^3}{d^3} - \frac{45a^2b^4c}{d^4} + \frac{36ab^5c^2}{d^5} - \frac{10b^6c^3}{d^6}\right) + \frac{-a^6d^6 - 6a^5bcd^5 + 45a^4b^2c^2d^4 - 100a^3b^3c^3d^3 + 105a^2b^4c^4d^2 - 54ab^5c^5d + 11b^6c^6 + x(-12a^5bd^6 + 60a^4b^2cd^5 - 120a^3b^3c^2d^4 + 120a^2b^4c^3d^3 - 60ab^5c^4d^2 + 12b^6c^5d)}{2c^2d^7 + 4cd^8x + 2d^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**3,x)
```

```
[Out] b**6*x**4/(4*d**3) + 15*b**2*(a*d - b*c)**4*log(c + d*x)/d**7 + x**3*(2*a*b
**5/d**3 - b**6*c/d**4) + x**2*(15*a**2*b**4/(2*d**3) - 9*a*b**5*c/d**4 + 3
*b**6*c**2/d**5) + x*(20*a**3*b**3/d**3 - 45*a**2*b**4*c/d**4 + 36*a*b**5*c
**2/d**5 - 10*b**6*c**3/d**6) + (-a**6*d**6 - 6*a**5*b*c*d**5 + 45*a**4*b**
2*c**2*d**4 - 100*a**3*b**3*c**3*d**3 + 105*a**2*b**4*c**4*d**2 - 54*a*b**5
*c**5*d + 11*b**6*c**6 + x*(-12*a**5*b*d**6 + 60*a**4*b**2*c*d**5 - 120*a**
3*b**3*c**2*d**4 + 120*a**2*b**4*c**3*d**3 - 60*a*b**5*c**4*d**2 + 12*b**6*
c**5*d))/(2*c**2*d**7 + 4*c*d**8*x + 2*d**9*x**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(152) = 304$.
time = 1.41, size = 362, normalized size = 2.29

$$\frac{15\,(b^6c^4-4\,ab^5c^3d+6\,a^2b^4c^2d^2-4\,a^3b^3cd^3+a^4b^2d^4)\log\left(|dx+c|\right)}{d^7}+\frac{11\,b^6c^6-54\,ab^5c^5d+105\,a^2b^4c^4d^2-100\,a^3b^3c^3d^3+45\,a^4b^2c^2d^4-6\,a^5bcd^5-a^6d^6+12\,(b^6c^5d-5\,ab^5c^4d^2+10\,a^2b^4c^3d^3-10\,a^3b^3c^2d^4+5\,a^4b^2cd^5-a^5bd^6)x}{2\,(dx+c)^2d^7}+\frac{b^6d^9x^4-4\,b^6cd^8x^3+8\,ab^5d^9x^3+12\,b^6c^2d^7x^2-36\,ab^5cd^8x^2+30\,a^2b^4d^9x^2-40\,b^6c^3d^6x+144\,ab^5c^2d^7x-180\,a^2b^4cd^8x+80\,a^3b^3d^9x}{4\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2
*d^4)*log(abs(d*x + c))/d^7 + 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^
4*c^4*d^2 - 100*a^3*b^3*c^3*d^3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*
d^6 + 12*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2
*d^4 + 5*a^4*b^2*c*d^5 - a^5*b*d^6)*x)/((d*x + c)^2*d^7) + 1/4*(b^6*d^9*x^4
 - 4*b^6*c*d^8*x^3 + 8*a*b^5*d^9*x^3 + 12*b^6*c^2*d^7*x^2 - 36*a*b^5*c*d^8*
x^2 + 30*a^2*b^4*d^9*x^2 - 40*b^6*c^3*d^6*x + 144*a*b^5*c^2*d^7*x - 180*a^2
*b^4*c*d^8*x + 80*a^3*b^3*d^9*x)/d^12
```

Mupad [**B**]
time = 0.27, size = 441, normalized size = 2.79

$$x^3\left(\frac{2\,a\,b^5}{d^3}-\frac{b^6\,c}{d^4}\right)-\frac{\frac{a^6\,d^6+6\,a^5\,b\,c\,d^5-45\,a^4\,b^2\,c^2\,d^4+100\,a^3\,b^3\,c^3\,d^3-105\,a^2\,b^4\,c^4\,d^2+54\,a\,b^5\,c^5\,d-11\,b^6\,c^6}{2\,d}-x\,(-6\,a^5\,b\,d^5+30\,a^4\,b^2\,c\,d^4-60\,a^3\,b^3\,c^2\,d^3+60\,a^2\,b^4\,c^3\,d^2-30\,a\,b^5\,c^4\,d+6\,b^6\,c^5)}{c^2\,d^6+2\,c\,d^7\,x+d^8\,x^2}-x^2\left(\frac{3\,c\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{2\,d}-\frac{15\,a^2\,b^4}{2\,d^3}+\frac{3\,b^6\,c^2}{2\,d^5}\right)+x\left(\frac{3\,c\left(\frac{3\,c\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d}-\frac{15\,a^2\,b^4}{d^3}+\frac{3\,b^6\,c^2}{d^5}\right)}{d}+\frac{20\,a^3\,b^3}{d^3}-\frac{b^6\,c^3}{d^6}-\frac{3\,c^2\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d^2}\right)+\frac{\ln\left(c+d\,x\right)\,(15\,a^4\,b^2\,d^4-60\,a^3\,b^3\,c\,d^3+90\,a^2\,b^4\,c^2\,d^2-60\,a\,b^5\,c^3\,d+15\,b^6\,c^4)}{d^7}+\frac{b^6\,x^4}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^3,x)
```

```
[Out] x^3*((2*a*b^5)/d^3 - (b^6*c)/d^4) - ((a^6*d^6 - 11*b^6*c^6 - 105*a^2*b^4*c^
4*d^2 + 100*a^3*b^3*c^3*d^3 - 45*a^4*b^2*c^2*d^4 + 54*a*b^5*c^5*d + 6*a^5*b
*c*d^5)/(2*d) - x*(6*b^6*c^5 - 6*a^5*b*d^5 + 30*a^4*b^2*c*d^4 + 60*a^2*b^4*
c^3*d^2 - 60*a^3*b^3*c^2*d^3 - 30*a*b^5*c^4*d))/(c^2*d^6 + d^8*x^2 + 2*c*d^
7*x) - x^2*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/(2*d) - (15*a^2*b^4)/(2*d
^3) + (3*b^6*c^2)/(2*d^5)) + x*((3*c*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))
/d - (15*a^2*b^4)/d^3 + (3*b^6*c^2)/d^5))/d + (20*a^3*b^3)/d^3 - (b^6*c^3)/
d^6 - (3*c^2*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/d^2) + (log(c + d*x)*(15*b^6*
c^4 + 15*a^4*b^2*d^4 - 60*a^3*b^3*c*d^3 + 90*a^2*b^4*c^2*d^2 - 60*a*b^5*c^3
*d))/d^7 + (b^6*x^4)/(4*d^3)
```

3.1353 $\int \frac{(a+bx)^5}{(c+dx)^3} \, dx$

Optimal. Leaf size=133

$$\frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}-\frac{10b^2(bc-ad)^3\log(c+}{d^6}$$

```
[Out] 10*b^3*(-a*d+b*c)^2*x/d^5+1/2*(-a*d+b*c)^5/d^6/(d*x+c)^2-5*b*(-a*d+b*c)^4/d
^6/(d*x+c)-5/2*b^4*(-a*d+b*c)*(d*x+c)^2/d^6+1/3*b^5*(d*x+c)^3/d^6-10*b^2*(-
a*d+b*c)^3*ln(d*x+c)/d^6
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^3}\,dx &= \int \left(\frac{10b^3(bc-ad)^2}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^3}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}-\frac{5b^4(bc-ad)(c+}{d^5}\right.\\ &= \frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 230, normalized size = 1.73

$$\frac{-3a^5d^5-15a^4bd^4(c+2dx)+30a^3b^2cd^3(3c+4dx)+30a^2b^3d^2(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)+15ab^4d(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4)+b^5(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5)-60b^2(bc-ad)^3(c+dx)^2\log(c+dx)}{6d^6(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) + 3
0*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d*(
7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c^5
+ 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5) -
60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

Maple [A]
time = 0.14, size = 254, normalized size = 1.91

method	result
default	$\frac{b^3\left(\frac{1}{3}d^2x^3b^2+\frac{5}{2}ab\,d^2x^2-\frac{3}{2}b^2cd\,x^2+10a^2d^2x-15abcdx+6b^2c^2x\right)}{d^5}-\frac{5b\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}{d^6(dx+c)}+$ $10b^2(a^3d^3-3$
norman	$\frac{-\frac{a^5d^5+5a^4bc\,d^4-30a^3b^2c^2d^3+90a^2b^3c^3d^2-90a\,b^4c^4d+30b^5c^5}{2d^6}+\frac{b^5x^5}{3d}-\frac{\left(5a^4b\,d^4-20a^3b^2c\,d^3+60a^2b^3c^2d^2-60a\,b^4c^3d+20b^5c^4\right)x}{d^5}+}{(dx+c)^2}$ $10b^3(3a^2d^2$
risch	$\frac{b^5x^3}{3d^3}+\frac{5b^4a\,x^2}{2d^3}-\frac{3b^5c\,x^2}{2d^4}+\frac{10b^3a^2x}{d^3}-\frac{15b^4acx}{d^4}+\frac{6b^5c^2x}{d^5}+\frac{\left(-5a^4b\,d^4+20a^3b^2c\,d^3-30a^2b^3c^2d^2+20a\,b^4c^3d-5b^5c^4\right)x-\frac{a^5d^5}{\cdots}}{d^5(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d^5*(1/3*d^2*x^3*b^2+5/2*a*b*d^2*x^2-3/2*b^2*c*d*x^2+10*a^2*d^2*x-15*a*
b*c*d*x+6*b^2*c^2*x)-5*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b
^3*c^3*d+b^4*c^4)/(d*x+c)+10*b^2/d^6*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)*ln(d*x+c)-1/2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*
c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(127) = 254$.
time = 0.31, size = 271, normalized size = 2.04

$$-\frac{9\,b^5c^5-35\,ab^4c^4d+50\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3+5\,a^4bcd^4+a^5d^5+10\,(b^5c^4d-4\,ab^4c^3d^2+6\,a^2b^3c^2d^3-4\,a^3b^2cd^4+a^4bd^5)x}{2\,(d^8x^2+2\,cd^7x+c^2d^6)}+\frac{2\,b^5d^2x^3-3\,(3\,b^5cd-5\,ab^4d^2)x^2+6\,(6\,b^5c^2-15\,ab^4cd+10\,a^2b^3d^2)x}{6\,d^5}-\frac{10\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)\log(dx+c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
+ 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2
*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/
6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*a*b^
4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(127) = 254$.
time = 0.59, size = 416, normalized size = 3.13

$$\frac{2 b^5 d^5 x^5 - 27 b^5 c^5 + 105 a b^4 c^4 d - 150 a^2 b^3 c^3 d^2 + 90 a^3 b^2 c^2 d^3 - 15 a^4 b c d^4 - 3 a^5 d^5 - 5 (b^5 c d^4 - 3 a b^4 d^5) x^4 + 20 (b^5 c^2 d^3 - 3 a b^4 c d^4 + 3 a^2 b^3 d^5) x^3 + 3 (21 b^5 c^3 d^2 - 55 a b^4 c^2 d^3 + 40 a^2 b^3 c d^4) x^2 + 6 (b^5 c^4 d + 5 a b^4 c^3 d^2 - 20 a^2 b^3 c^2 d^3 + 20 a^3 b^2 c d^4 - 5 a^4 b d^5) x - 60 (b^5 c^5 - 3 a b^4 c^4 d + 3 a^2 b^3 c^3 d^2 - a^3 b^2 c^2 d^3 + (b^5 c^3 d^2 - 3 a b^4 c^2 d^3 + 3 a^2 b^3 c d^4 - a^3 b^2 d^5) x^2 + 2 (b^5 c^4 d - 3 a b^4 c^3 d^2 + 3 a^2 b^3 c^2 d^3 - a^3 b^2 c d^4) x) \log (dx + c)}{6 (d^8 x^2 + 2 c d^7 x + c^2 d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 + 9
0*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d^5
)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5*c^
3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b^4*c
^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(b^5*c
^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d^2 - 3
*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d - 3*a*b^
4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^8*x^2 +
2*c*d^7*x + c^2*d^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(121) = 242$.
time = 0.94, size = 258, normalized size = 1.94

$$\frac{b^5 x^3}{3 d^3} + \frac{10 b^2 (ad - bc)^3 \log (c + dx)}{d^6} + x^2 \cdot \left(\frac{5 a b^4}{2 d^3} - \frac{3 b^5 c}{2 d^4}\right) + x \left(\frac{10 a^2 b^3}{d^3} - \frac{15 a b^4 c}{d^4} + \frac{6 b^5 c^2}{d^5}\right) + \frac{-a^5 d^5 - 5 a^4 b c d^4 + 30 a^3 b^2 c^2 d^3 - 50 a^2 b^3 c^3 d^2 + 35 a b^4 c^4 d - 9 b^5 c^5 + x (-10 a^4 b d^5 + 40 a^3 b^2 c d^4 - 60 a^2 b^3 c^2 d^3 + 40 a b^4 c^3 d^2 - 10 b^5 c^4 d)}{2 c^2 d^6 + 4 c d^7 x + 2 d^8 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**3,x)
```

```
[Out] b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 + x**2*(5*a*b
**4/(2*d**3) - 3*b**5*c/(2*d**4)) + x*(10*a**2*b**3/d**3 - 15*a*b**4*c/d**4
 + 6*b**5*c**2/d**5) + (-a**5*d**5 - 5*a**4*b*c*d**4 + 30*a**3*b**2*c**2*d*
*3 - 50*a**2*b**3*c**3*d**2 + 35*a*b**4*c**4*d - 9*b**5*c**5 + x*(-10*a**4*
b*d**5 + 40*a**3*b**2*c*d**4 - 60*a**2*b**3*c**2*d**3 + 40*a*b**4*c**3*d**2
 - 10*b**5*c**4*d))/(2*c**2*d**6 + 4*c*d**7*x + 2*d**8*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(127) = 254$.
time = 0.93, size = 264, normalized size = 1.98

$$-\frac{10 (b^5 c^3 - 3 a b^4 c^2 d + 3 a^2 b^3 c d^2 - a^3 b^2 d^3) \log (|dx + c|)}{d^6} - \frac{9 b^5 c^5 - 35 a b^4 c^4 d + 50 a^2 b^3 c^3 d^2 - 30 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 + a^5 d^5 + 10 (b^5 c^4 d - 4 a b^4 c^3 d^2 + 6 a^2 b^3 c^2 d^3 - 4 a^3 b^2 c d^4 + a^4 b d^5) x}{2 (dx + c)^2 d^6} + \frac{2 b^5 d^6 x^3 - 9 b^5 c d^5 x^2 + 15 a b^4 d^6 x^2 + 36 b^5 c^2 d^4 x - 90 a b^4 c d^5 x + 60 a^2 b^3 d^6 x}{6 d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^
```

```
2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a
^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2
*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x - 90*a
*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

Mupad [B]
time = 0.10, size = 291, normalized size = 2.19

$$x^2\left(\frac{5\,a\,b^4}{2\,d^3}-\frac{3\,b^5\,c}{2\,d^4}\right)-\frac{\frac{a^5\,d^5+5\,a^4\,b\,c\,d^4-30\,a^3\,b^2\,c^2\,d^3+50\,a^2\,b^3\,c^3\,d^2-35\,a\,b^4\,c^4\,d+9\,b^5\,c^5}{2\,d}+x\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)}{c^2\,d^5+2\,c\,d^6\,x+d^7\,x^2}-x\,\left(\frac{3\,c\,\left(\frac{5\,a\,b^4}{d^3}-\frac{3\,b^5\,c}{d^4}\right)}{d}-\frac{10\,a^2\,b^3}{d^3}+\frac{3\,b^5\,c^2}{d^5}\right)-\frac{\ln\left(c+d\,x\right)\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)}{d^6}+\frac{b^5\,x^3}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^3,x)
```

```
[Out] x^2*((5*a*b^4)/(2*d^3) - (3*b^5*c)/(2*d^4)) - ((a^5*d^5 + 9*b^5*c^5 + 50*a^
2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 35*a*b^4*c^4*d + 5*a^4*b*c*d^4)/(2*d)
+ x*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a
*b^4*c^3*d))/(c^2*d^5 + d^7*x^2 + 2*c*d^6*x) - x*((3*c*((5*a*b^4)/d^3 - (3*
b^5*c)/d^4))/d - (10*a^2*b^3)/d^3 + (3*b^5*c^2)/d^5) - (log(c + d*x)*(10*b^
5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d))/d^6 + (b^5*x^3
)/(3*d^3)
```

3.1354 $\int \frac{(a+bx)^4}{(c+dx)^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}$$

```
[Out] -b^3*(-4*a*d+3*b*c)*x/d^4+1/2*b^4*x^2/d^3-1/2*(-a*d+b*c)^4/d^5/(d*x+c)^2+4*
b*(-a*d+b*c)^3/d^5/(d*x+c)+6*b^2*(-a*d+b*c)^2*ln(d*x+c)/d^5
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^3}\,dx &= \int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{b^4x}{d^3}+\frac{(-bc+ad)^4}{d^4(c+dx)^3}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}\right)dx\\ &= -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 167, normalized size = 1.62

$$\frac{-a^4d^4-4a^3bd^3(c+2dx)+6a^2b^2cd^2(3c+4dx)+4ab^3d(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)+b^4(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4)+12b^2(bc-ad)^2(c+dx)^2\log(c+dx)}{2d^5(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*a
*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^3*
d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c + d
*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

Maple [A]
time = 0.14, size = 172, normalized size = 1.67

method	result
default	$\frac{b^3\left(\frac{1}{2}bd\,x^2+4adx-3bcx\right)}{d^4}-\frac{4b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^5(dx+c)}+\frac{6b^2\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^5}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d}{2d^5(dx+c)}$
norman	$\frac{-\frac{a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+36a\,b^3c^3d-18b^4c^4}{2d^5}+\frac{b^4x^4}{2d}-\frac{2\left(2a^3b\,d^3-6b^2a^2c\,d^2+12a\,b^3c^2d-6b^4c^3\right)x}{d^4}+\frac{2b^3(2ad-bc)x^3}{d^2}}{(dx+c)^2}+\frac{6b^2\left(a^2d^2-2abcd+\right.}{d^5}$
risch	$\frac{b^4x^2}{2d^3}+\frac{4a\,b^3x}{d^3}-\frac{3b^4cx}{d^4}+\frac{\left(-4a^3b\,d^3+12b^2a^2c\,d^2-12a\,b^3c^2d+4b^4c^3\right)x-\frac{a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+20a\,b^3c^3d-7b^4c^4}{2d}}{d^4(dx+c)^2}+\frac{6b^2\ln(dx}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3/d^4*(1/2*b*d*x^2+4*a*d*x-3*b*c*x)-4*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^
2*c^2*d-b^3*c^3)/(d*x+c)+6*b^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*x+c)-1/
2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^5/(d*x+
c)^2
```

Maxima [A]
time = 0.29, size = 191, normalized size = 1.85

$$\frac{7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4+8\,(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x}{2\,(d^7x^2+2\,cd^6x+c^2d^5)}+\frac{b^4dx^2-2\,(3\,b^4c-4\,ab^3d)x}{2\,d^4}+\frac{6\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\log{(dx+c)}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*
d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d^7
*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x)/d
^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(99) = 198$.
time = 0.82, size = 291, normalized size = 2.83

$$\frac{b^4d^4x^4+7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4-4\,(b^4cd^3-2\,ab^3d^4)x^3-(11\,b^4c^2d^2-16\,ab^3cd^3)x^2+2\,(b^4c^3d-8\,ab^3c^2d^2+12\,a^2b^2cd^3-4\,a^3bd^4)x+12\,(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2+(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4)x^2+2\,(b^4c^3d-2\,ab^3c^2d^2+a^2b^2cd^3)x)\log{(dx+c)}}{2\,(d^7x^2+2\,cd^6x+c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 16*
a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4*a^
3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d^2 -
 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a^2*b^
2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A]
time = 0.67, size = 185, normalized size = 1.80

$$\frac{b^4x^2}{2d^3}+\frac{6b^2(ad-bc)^2\log{(c+dx)}}{d^5}+x\left(\frac{4ab^3}{d^3}-\frac{3b^4c}{d^4}\right)+\frac{-a^4d^4-4a^3bcd^3+18a^2b^2c^2d^2-20ab^3c^3d+7b^4c^4+x(-8a^3bd^4+24a^2b^2cd^3-24ab^3c^2d^2+8b^4c^3d)}{2c^2d^5+4cd^6x+2d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**3,x)
```

```
[Out] b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 + x*(4*a*b**3/
d**3 - 3*b**4*c/d**4) + (-a**4*d**4 - 4*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d
**2 - 20*a*b**3*c**3*d + 7*b**4*c**4 + x*(-8*a**3*b*d**4 + 24*a**2*b**2*c*d
**3 - 24*a*b**3*c**2*d**2 + 8*b**4*c**3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d
**7*x**2)
```

Giac [A]
time = 0.61, size = 183, normalized size = 1.78

$$\frac{6\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\log\left(|dx+c|\right)}{d^5}+\frac{b^4d^3x^2-6\,b^4cd^2x+8\,ab^3d^3x}{2\,d^6}+\frac{7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4+8\,(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)x}{2\,(dx+c)^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^
3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*
d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)
```

Mupad [B]
time = 0.10, size = 196, normalized size = 1.90

$$x\,\left(\frac{4\,a\,b^3}{d^3}-\frac{3\,b^4\,c}{d^4}\right)-\frac{\frac{a^4\,d^4+4\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d-7\,b^4\,c^4}{2\,d}-x\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{c^2\,d^4+2\,c\,d^5\,x+d^6\,x^2}+\frac{b^4\,x^2}{2\,d^3}+\frac{\ln\left(c+d\,x\right)\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^3,x)
```

```
[Out] x*((4*a*b^3)/d^3 - (3*b^4*c)/d^4) - ((a^4*d^4 - 7*b^4*c^4 - 18*a^2*b^2*c^2*
d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3)/(2*d) - x*(4*b^4*c^3 - 4*a^3*b*d^3 +
12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(c^2*d^4 + d^6*x^2 + 2*c*d^5*x) + (b^4*
x^2)/(2*d^3) + (log(c + d*x)*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d))/d^
5
```

3.1355 $\int \frac{(a+bx)^3}{(c+dx)^3}\,dx$

Optimal. Leaf size=78

$$\frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}$$

```
[Out] b^3*x/d^3+1/2*(-a*d+b*c)^3/d^4/(d*x+c)^2-3*b*(-a*d+b*c)^2/d^4/(d*x+c)-3*b^2
*(-a*d+b*c)*ln(d*x+c)/d^4
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = {45}

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^3}\,dx &= \int \left(\frac{b^3}{d^3}+\frac{(-bc+ad)^3}{d^3(c+dx)^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}-\frac{3b^2(bc-ad)}{d^3(c+dx)}\right)dx\\ &= \frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 114, normalized size = 1.46

$$\frac{-a^3d^3-3a^2bd^2(c+2dx)+3ab^2cd(3c+4dx)+b^3(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3)-6b^2(bc-ad)(c+dx)^2\log(c+dx)}{2d^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-5
*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)^2
*Log[c + d*x])/(2*d^4*(c + d*x)^2)
```

Maple [A]
time = 0.15, size = 114, normalized size = 1.46

method	result	size
default	$\frac{b^{3} x}{d^{3}}-\frac{3 b \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}{d^{4} \left(d x+c\right)}+\frac{3 b^{2} \left(a d-b c\right) \ln \left(d x+c\right)}{d^{4}}-\frac{a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d-b^{3} c^{3}}{2 d^{4} \left(d x+c\right)^{2}}$	114
norman	$\frac{\frac{b^{3} x^{3}}{d}-\frac{a^{3} d^{3}+3 a^{2} b c \,d^{2}-9 a \,b^{2} c^{2} d+9 b^{3} c^{3}}{2 d^{4}}-\frac{\left(3 a^{2} b \,d^{2}-6 a \,b^{2} c d+6 b^{3} c^{2}\right) x}{d^{3}}}{\left(d x+c\right)^{2}}+\frac{3 b^{2} \left(a d-b c\right) \ln \left(d x+c\right)}{d^{4}}$	116
risch	$\frac{b^{3} x}{d^{3}}+\frac{\left(-3 a^{2} b \,d^{2}+6 a \,b^{2} c d-3 b^{3} c^{2}\right) x-\frac{a^{3} d^{3}+3 a^{2} b c \,d^{2}-9 a \,b^{2} c^{2} d+5 b^{3} c^{3}}{2 d}}{d^{3} \left(d x+c\right)^{2}}+\frac{3 b^{2} \ln \left(d x+c\right) a}{d^{3}}-\frac{3 b^{3} \ln \left(d x+c\right) c}{d^{4}}$	121

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3-3*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)+3*b^2/d^4*(a*d-b*c)*l
n(d*x+c)-1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^2
```

Maxima [A]
time = 0.30, size = 125, normalized size = 1.60

$$\frac{b^{3} x}{d^{3}} - \frac{5 \, b^{3} c^{3} - 9 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} + a^{3} d^{3} + 6 \, {\left(b^{3} c^{2} d - 2 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x}{2 \, {\left(d^{6} x^{2} + 2 \, c d^{5} x + c^{2} d^{4}\right)}} - \frac{3 \, {\left(b^{3} c - a b^{2} d\right)} \log\left(d x + c\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) -
3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(76) = 152.
time = 0.65, size = 188, normalized size = 2.41

$$\frac{2 \, b^{3} d^{3} x^{3} + 4 \, b^{3} c d^{2} x^{2} - 5 \, b^{3} c^{3} + 9 \, a b^{2} c^{2} d - 3 \, a^{2} b c d^{2} - a^{3} d^{3} - 2 \, {\left(2 \, b^{3} c^{2} d - 6 \, a b^{2} c d^{2} + 3 \, a^{2} b d^{3}\right)} x - 6 \, {\left(b^{3} c^{3} - a b^{2} c^{2} d + {\left(b^{3} c d^{2} - a b^{2} d^{3}\right)} x^{2} + 2 \, {\left(b^{3} c^{2} d - a b^{2} c d^{2}\right)} x\right)} \log\left(d x + c\right)}{2 \, {\left(d^{6} x^{2} + 2 \, c d^{5} x + c^{2} d^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

[Out] 1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c*d^2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)

Sympy [A]
time = 0.46, size = 128, normalized size = 1.64

$$\frac{b^3x}{d^3}+\frac{3b^2(ad-bc)\log{(c+dx)}}{d^4}+\frac{-a^3d^3-3a^2bcd^2+9ab^2c^2d-5b^3c^3+x(-6a^2bd^3+12ab^2cd^2-6b^3c^2d)}{2c^2d^4+4cd^5x+2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**3/(d*x+c)**3,x)

[Out] b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 + (-a**3*d**3 - 3*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 5*b**3*c**3 + x*(-6*a**2*b*d**3 + 12*a*b**2*c*d**2 - 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)

Giac [A]
time = 0.69, size = 112, normalized size = 1.44

$$\frac{b^3x}{d^3}-\frac{3\,(b^3c-ab^2d)\log{(|dx+c|)}}{d^4}-\frac{5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3+6\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x}{2\,(dx+c)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="giac")

[Out] b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/((d*x + c)^2*d^4)

Mupad [B]
time = 0.11, size = 130, normalized size = 1.67

$$\frac{b^3\,x}{d^3}-\frac{\ln{(c+d\,x)}\,\left(3\,b^3\,c-3\,a\,b^2\,d\right)}{d^4}-\frac{\frac{a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3}{2\,d}+x\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}{c^2\,d^3+2\,c\,d^4\,x+d^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^3/(c + d*x)^3,x)

[Out] (b^3*x)/d^3 - (log(c + d*x)*(3*b^3*c - 3*a*b^2*d))/d^4 - ((a^3*d^3 + 5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(2*d) + x*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d))/(c^2*d^3 + d^5*x^2 + 2*c*d^4*x)

3.1356 $\int \frac{(a+bx)^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}$$

```
[Out] -1/2*(-a*d+b*c)^2/d^3/(d*x+c)^2+2*b*(-a*d+b*c)/d^3/(d*x+c)+b^2*ln(d*x+c)/d^
3
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{2b(bc-ad)}{d^3(c+dx)}-\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] -1/2*(b*c - a*d)^2/(d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) +
(b^2*Log[c + d*x])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^3}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3}-\frac{2b(bc-ad)}{d^2(c+dx)^2}+\frac{b^2}{d^2(c+dx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2d^3(c+dx)^2}+\frac{2b(bc-ad)}{d^3(c+dx)}+\frac{b^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 48, normalized size = 0.81

$$\frac{\frac{(bc-ad)(3bc+ad+4bdx)}{(c+dx)^2}+2b^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(2
*d^3)
```

Maple [A]
time = 0.13, size = 69, normalized size = 1.17

method	result	size
risch	$\frac{-\frac{2b(ad-bc)x}{d^2}-\frac{a^2d^2+2abcd-3b^2c^2}{2d^3}}{(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}$	66
norman	$\frac{-\frac{a^2d^2+2abcd-3b^2c^2}{2d^3}-\frac{2\left(abd-b^2c\right)x}{d^2}}{(dx+c)^2}+\frac{b^2\ln(dx+c)}{d^3}$	68
default	$-\frac{2b(ad-bc)}{d^3(dx+c)}+\frac{b^2\ln(dx+c)}{d^3}-\frac{a^2d^2-2abcd+b^2c^2}{2d^3(dx+c)^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/d^3*(a*d-b*c)/(d*x+c)+b^2*ln(d*x+c)/d^3-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/d^3/(d*x+c)^2
```

Maxima [A]
time = 0.29, size = 80, normalized size = 1.36

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}+\frac{b^2\log{(dx+c)}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

Fricas [A]
time = 0.79, size = 100, normalized size = 1.69

$$\frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log{(dx+c)}}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [A]
time = 0.24, size = 80, normalized size = 1.36

$$\frac{b^2\log{(c+dx)}}{d^3}+\frac{-a^2d^2-2abcd+3b^2c^2+x(-4abd^2+4b^2cd)}{2c^2d^3+4cd^4x+2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c + d*x)/d**3 + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x*(-4*a*b*
d**2 + 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

Giac [A]
time = 0.70, size = 69, normalized size = 1.17

$$\frac{b^2\log{(|dx+c|)}}{d^3}+\frac{4\,(b^2c-abd)x+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{2\,(dx+c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

Mupad [B]
time = 0.23, size = 77, normalized size = 1.31

$$\frac{b^2\,\ln{(c+d\,x)}}{d^3}-\frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{2\,d^3}+\frac{2\,b\,x\,(a\,d-b\,c)}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3 - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(2*d^3) + (2*b*
x*(a*d - b*c))/d^2)/(c^2 + d^2*x^2 + 2*c*d*x)
```

3.1357 $\int \frac{a+bx}{(c+dx)^3} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

[Out] 1/2*(b*x+a)^2/(-a*d+b*c)/(d*x+c)^2

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{37\}$

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)/(c + d*x)^3,x]

[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{(c+dx)^3} dx = \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)/(c + d*x)^3,x]

[Out] -1/2*(a*d + b*(c + 2*d*x))/(d^2*(c + d*x)^2)

Maple [A]
time = 0.12, size = 35, normalized size = 1.25

method	result	size
gosper	$-\frac{2bdx+ad+bc}{2d^2(dx+c)^2}$	25
norman	$\frac{-\frac{bx}{d}-\frac{ad+bc}{2d^2}}{(dx+c)^2}$	29
risch	$\frac{-\frac{bx}{d}-\frac{ad+bc}{2d^2}}{(dx+c)^2}$	29
default	$-\frac{b}{d^2(dx+c)}-\frac{ad-bc}{2d^2(dx+c)^2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/d^2/(d*x+c)-1/2*(a*d-b*c)/d^2/(d*x+c)^2
```

Maxima [A]
time = 0.29, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Fricas [A]
time = 0.69, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\,(d^4x^2+2\,cd^3x+c^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2c^2d^2+4cd^3x+2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

Giac [A]
time = 1.89, size = 24, normalized size = 0.86

$$-\frac{2\,bdx+bc+ad}{2\,(dx+c)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

Mupad [B]
time = 0.03, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,d^2}+\frac{b\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*d^2) + (b*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x)
```

3.1358 $\int \frac{1}{(c+dx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2d(c+dx)^2}$$

```
[Out] -1/2/d/(d*x+c)^2
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3),x]
```

```
[Out] -1/2*1/(d*(c + d*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3}\,dx = -\frac{1}{2d(c+dx)^2}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3),x]
```

```
[Out] -1/2*1/(d*(c + d*x)^2)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{2d(dx+c)^2}$	13
default	$-\frac{1}{2d(dx+c)^2}$	13
norman	$-\frac{1}{2d(dx+c)^2}$	13
risch	$-\frac{1}{2d(dx+c)^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/(d*x+c)^2
```

Maxima [A]
time = 0.32, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((d*x + c)^2*d)
```

Fricas [A]
time = 0.58, size = 24, normalized size = 1.71

$$-\frac{1}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3,x)
```

```
[Out] -1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

Giac [A]
time = 1.64, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(dx+c\right)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2/((d*x + c)^2*d)
```

Mupad [B]
time = 0.02, size = 26, normalized size = 1.86

$$-\frac{1}{2\,c^2\,d+4\,c\,d^2\,x+2\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^3,x)
```

```
[Out] -1/(2*c^2*d + 2*d^3*x^2 + 4*c*d^2*x)
```

3.1359 $\int \frac{1}{(a+bx)(c+dx)^3}\,dx$

Optimal. Leaf size=82

$$\frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}$$

```
[Out] 1/2/(-a*d+b*c)/(d*x+c)^2+b/(-a*d+b*c)^2/(d*x+c)+b^2*ln(b*x+a)/(-a*d+b*c)^3-
b^2*ln(d*x+c)/(-a*d+b*c)^3
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a +
b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b}{(bc-ad)}\right.\\ &= \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(3bc-ad+2bdx)}{(c+dx)^2}+2b^2\log(a+bx)-2b^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)*(c + d*x)^3),x]

[Out] (((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] - 2*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)

Maple [A]
time = 0.17, size = 81, normalized size = 0.99

method	result	size
default	$-\frac{1}{2(ad-bc)(dx+c)^{2}}+\frac{b^{2}\ln(dx+c)}{(ad-bc)^{3}}+\frac{b}{(ad-bc)^{2}(dx+c)}-\frac{b^{2}\ln(bx+a)}{(ad-bc)^{3}}$	81
risch	$\frac{\frac{bdx}{a^{2}d^{2}-2abcd+b^{2}c^{2}}-\frac{ad-3bc}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{(dx+c)^{2}}+\frac{b^{2}\ln(-dx-c)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$	171
norman	$\frac{\frac{bdx}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+\frac{-a\,d^{3}+3bc\,d^{2}}{2d^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{(dx+c)^{2}}+\frac{b^{2}\ln(dx+c)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$	177

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(80) = 160.
time = 0.30, size = 202, normalized size = 2.46

$$\frac{b^{2}\log(bx+a)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}-\frac{b^{2}\log(dx+c)}{b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}}+\frac{2\,bdx+3\,bc-ad}{2\,(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4})x^{2}+2\,(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")

[Out] b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(80) = 160.
time = 0.49, size = 242, normalized size = 2.95

$$\frac{3\,b^{2}c^{2}-4\,abcd+a^{2}d^{2}+2\,(b^{2}cd-abd^{2})x+2\,(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2})\log(bx+a)-2\,(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2})\log(dx+c)}{2\,(b^{3}c^{5}-3\,ab^{2}c^{4}d+3\,a^{2}bc^{3}d^{2}-a^{3}c^{2}d^{3}+(b^{3}c^{3}d^{2}-3\,ab^{2}c^{2}d^{3}+3\,a^{2}bcd^{4}-a^{3}d^{5})x^{2}+2\,(b^{3}c^{4}d-3\,ab^{2}c^{3}d^{2}+3\,a^{2}bc^{2}d^{3}-a^{3}cd^{4})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^
2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 + 2*(
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(68) = 136$.
time = 0.57, size = 381, normalized size = 4.65

$$\frac{b^2 \log\left(x + \frac{-\frac{a^4b^2d^4}{(ad-bc)^3} + \frac{4a^3b^3cd^3}{(ad-bc)^3} - \frac{6a^2b^4c^2d^2}{(ad-bc)^3} + \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d - \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3} - \frac{b^2 \log\left(x + \frac{\frac{a^4b^2d^4}{(ad-bc)^3} - \frac{4a^3b^3cd^3}{(ad-bc)^3} + \frac{6a^2b^4c^2d^2}{(ad-bc)^3} - \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d + \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{(ad-bc)^3} + \frac{-ad+3bc+2bdx}{2a^2c^2d^2 - 4abc^3d + 2b^2c^4 + x^2 \cdot (2a^2d^4 - 4abcd^3 + 2b^2c^2d^2) + x\,(4a^2cd^3 - 8abc^2d^2 + 4b^2c^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**3,x)
```

```
[Out] b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b*
c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c)*
*3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*c)*
*3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b
*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b
*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d + 2*b**2
*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4*a**2*c*
d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(80) = 160$.
time = 1.48, size = 165, normalized size = 2.01

$$\frac{b^3 \log(|bx+a|)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^2d\log(|dx+c|)}{b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4} + \frac{3\,b^2c^2 - 4\,abcd + a^2d^2 + 2\,(b^2cd - abd^2)x}{2\,(bc-ad)^3(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(d*x + c)^2)
```

Mupad [B]
time = 0.30, size = 183, normalized size = 2.23

$$-\frac{\frac{a\,d-3\,b\,c}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)} - \frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{c^2+2\,c\,d\,x+d^2\,x^2} - \frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{(a\,d-b\,c)^3} + \frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{(a\,d-b\,c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^3),x)
```

```
[Out] - ((a*d - 3*b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(c^2 + d^2*x^2 + 2*c*d*x) - (2*b^2*atanh((a^3*d^3 + b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

3.1360 $\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx$

Optimal. Leaf size=110

$$-\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}$$

```
[Out] -b^2/(-a*d+b*c)^3/(b*x+a)-1/2*d/(-a*d+b*c)^2/(d*x+c)^2-2*b*d/(-a*d+b*c)^3/(
d*x+c)-3*b^2*d*ln(b*x+a)/(-a*d+b*c)^4+3*b^2*d*ln(d*x+c)/(-a*d+b*c)^4
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{46\}$

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^3}\,dx = \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{}{(bc-a}\right.$$
$$=-\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(}{(bc-}$$

Mathematica [A]
time = 0.07, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bc-ad)}{a+bx}+\frac{d(bc-ad)^2}{(c+dx)^2}+\frac{4bd(bc-ad)}{c+dx}+6b^2d\log(a+bx)-6b^2d\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/2*((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b*
d*(b*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(b*
c - a*d)^4
```

Maple [A]
time = 0.17, size = 108, normalized size = 0.98

method	result
default	$-\frac{d}{2(ad-bc)^2(dx+c)^2}+\frac{3d\,b^2\ln(dx+c)}{(ad-bc)^4}+\frac{2db}{(ad-bc)^3(dx+c)}+\frac{b^2}{(ad-bc)^3(bx+a)}-\frac{3d\,b^2\ln(bx+a)}{(ad-bc)^4}$
risch	$\frac{\frac{3b^2d^2x^2}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{3(ad+3bc)bdx}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}-\frac{a^2d^2-5abcd-2b^2c^2}{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx+a)(dx+c)^2}+\frac{3b^2d\ln(-dx-c)}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4}$
norman	$\frac{\frac{3b^2d^2x^2}{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}+\frac{-a^2b\,d^4+5a\,b^2c\,d^3+2b^3c^2d^2}{2b\,d^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(3a\,b^2d^4+9b^3c\,d^3\right)x}{2b\,d^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(bx+a)(dx+c)^2}-\frac{3b^2d\ln(b}{a^4d^4-4a^3bc\,d^3+6a^2b^2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*
b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(108) = 216.
time = 0.31, size = 386, normalized size = 3.51

$$-\frac{3\,b^2d\log\left(bx+a\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}+\frac{3\,b^2d\log\left(dx+c\right)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}-\frac{6\,b^2d^2x^2+2\,b^2c^2+5\,abcd-a^2d^2+3\,(3\,b^2cd+abd^2)x}{2\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^3+(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^2+(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2 + 5
*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(108) = 216.
time = 0.46, size = 495, normalized size = 4.50

$$-\frac{2\,b^3c^3+3\,ab^2c^2d-6\,a^2bcd^2+a^3d^3+6\,(b^3cd^2-ab^2d^3)x^2+3\,(3\,b^3c^2d-2\,ab^2cd^2-a^2bd^3)x+6\,(b^3d^3x^3+ab^2c^2d+(2\,b^3cd^2+ab^2d^3)x^2+(b^3c^2d+2\,ab^2cd^2)x)\log\left(bx+a\right)-6\,(b^3d^3x^3+ab^2c^2d+(2\,b^3cd^2+ab^2d^3)x^2+(b^3c^2d+2\,ab^2cd^2)x)\log\left(dx+c\right)}{2\,(ab^4c^6-4\,a^2b^3c^5d+6\,a^3b^2c^4d^2-4\,a^4bc^3d^3+a^5c^2d^4+(b^5c^4d^2-4\,ab^4c^3d^3+6\,a^2b^3c^2d^4-4\,a^3b^2cd^5+a^4bd^6)x^3+(2\,b^5c^5d-7\,ab^4c^4d^2+8\,a^2b^3c^3d^3-2\,a^3b^2c^2d^4-2\,a^4bcd^5+a^5d^6)x^2+(b^5c^6-2\,ab^4c^5d-2\,a^2b^3c^4d^2+8\,a^3b^2c^3d^3-7\,a^4bc^2d^4+2\,a^5cd^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 - 4*a^2
*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d
^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^3
 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 -
 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^
2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(97) = 194$.
time = 0.93, size = 632, normalized size = 5.75

$$\frac{3b^2d\log\left(x+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}-\frac{3b^2d\log\left(x+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\frac{30a^2b^5c^3d^3}{(ad-bc)^4}+\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2-\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}+\frac{-a^2d^2+5abcd+2b^2c^2+6b^2d^2x^2+x(3abd^2+9b^2cd)}{2a^4c^2d^3-6a^3bc^3d^2+6a^2b^2c^4d-2ab^3c^5+x^3\cdot(2a^3bd^5-6a^2b^2cd^4+6ab^3c^2d^3-2b^4c^3d^2)+x^2\cdot(2a^4d^5-2a^3bcd^4-6a^2b^2c^2d^3+10ab^3c^3d^2-4b^4c^4d)+x\left(4a^4cd^4-10a^3bc^2d^3+6a^2b^2c^3d^2+2ab^3c^4d-2b^4c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] 3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(a
*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d*
*3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 + 3*
b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 - 3*
b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d
- b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 - 3*b**
7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 + (-a**
2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**2 + 9*b**
2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4*d - 2*a*b
**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c**2*d**3 -
2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**2*b**2*c**2*
d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4 - 10*a**3*b*
c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*c**5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(108) = 216$.
time = 2.29, size = 217, normalized size = 1.97

$$\frac{3\,b^3d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}-\frac{b^5}{(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)(bx+a)}+\frac{5\,b^2d^3+\frac{6\,(b^4cd^2-ab^3d^3)}{(bx+a)b}}{2\,(bc-ad)^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 3*b^3*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^5*c^4 - 4*a*b^4*c^3*
d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - b^5/((b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x + a)) + 1/2*(5*b^2*d^3 + 6*
(b^4*c*d^2 - a*b^3*d^3)/((b*x + a)*b))/((b*c - a*d)^4*(b*c/(b*x + a) - a*d/
(b*x + a) + d)^2)
```

Mupad [B]
time = 0.40, size = 329, normalized size = 2.99

$$\frac{\frac{-a^2\,d^2+5\,a\,b\,c\,d+2\,b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b\,x\,\left(a\,d^2+3\,b\,c\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(b\,c^2+2\,a\,d\,c\right)+a\,c^2+x^2\,\left(a\,d^2+2\,b\,c\,d\right)+b\,d^2\,x^3}-\frac{6\,b^2\,d\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] ((2*b^2*c^2 - a^2*d^2 + 5*a*b*c*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (3*b*x*(a*d^2 + 3*b*c*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(x*(b*c^2 + 2*a*c*d) + a*c^2 + x^2*(a*d^2 + 2*b*c*d) +
 b*d^2*x^3) - (6*b^2*d*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c
*d^3)/(a*d - b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))/(a*d - b*c)^4))/(a*d - b*c)^4
```

3.1361 $\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+dx)}+\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}$$

```
[Out] -1/2*b^2/(-a*d+b*c)^3/(b*x+a)^2+3*b^2*d/(-a*d+b*c)^4/(b*x+a)+1/2*d^2/(-a*d+
b*c)^3/(d*x+c)^2+3*b*d^2/(-a*d+b*c)^4/(d*x+c)+6*b^2*d^2*ln(b*x+a)/(-a*d+b*c
)^5-6*b^2*d^2*ln(d*x+c)/(-a*d+b*c)^5
```

Rubi [A]
time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {46}

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/2*b^2/((b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d)/((b*c - a*d)^4*(a + b*x))
+ d^2/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2)/((b*c - a*d)^4*(c + d*x)) +
 (6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b*c - a
*d)^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^3}-\frac{3b^3d}{(bc-ad)^4(a+bx)^2}+\frac{6b^3d^2}{(bc-ad)^5(a+bx)}-\frac{}{(bc-a}\right.\\ &= -\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{}{(bc-a}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 128, normalized size = 0.90

$$\frac{-\frac{b^2(bc-ad)^2}{(a+bx)^2}+\frac{6b^2d(bc-ad)}{a+bx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}+\frac{6bd^2(bc-ad)}{c+dx}+12b^2d^2\log(a+bx)-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] (-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d^
2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d^2
*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

Maple [**A**]
time = 0.18, size = 140, normalized size = 0.98

method	result
default	$-\frac{d^2}{2(ad-bc)^3(dx+c)^2}+\frac{6d^2b^2\ln(dx+c)}{(ad-bc)^5}+\frac{3d^2b}{(ad-bc)^4(dx+c)}+\frac{b^2}{2(ad-bc)^3(bx+a)^2}-\frac{6d^2b^2\ln(bx+a)}{(ad-bc)^5}+\frac{3b^2d}{(ad-bc)^4(bx+a)}$
risch	$\frac{\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}+\frac{9b^2d^2(ad+bc)x^2}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}+\frac{2\left(a^2d^2+7abcd+b^2c^2\right)bdx}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4}-\frac{\ldots}{2\left(a^4d^{\ldots}\right.}}{(bx+a)^2(dx+c)^2}$
norman	$\frac{\frac{\left(9a\,b^4d^5+9b^5c\,d^4\right)x^2}{d^2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{-a^3b^2d^5+7a^2b^3c\,d^4+7a\,b^4c^2d^3-b^5c^3d^2}{2d^2b^2\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{6b^3d^3x^3}{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d\ldots}}{(bx+a)^2(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3*d^2/(a*d-b
*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*b^2*ln(b*x+
a)+3*b^2/(a*d-b*c)^4*d/(b*x+a)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(139) = 278$.
time = 0.34, size = 594, normalized size = 4.15

$$\frac{6\,b^2d^2\log\left(bx+a\right)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}-\frac{6\,b^2d^2\log\left(dx+c\right)}{b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5}+\frac{12\,b^3d^3x^3-b^3c^3+7\,ab^2c^2d+7\,a^2bcd^2-a^3d^3+18\,(b^3cd^2+ab^2d^3)x^2+4\,(b^3c^2d+7\,ab^2cd^2+a^2bd^3)x}{2\,(a^2b^4c^6-4\,a^3b^3c^5d+6\,a^4b^2c^4d^2-4\,a^5bc^3d^3+a^6c^2d^4+(b^6c^4d^2-4\,ab^5c^3d^3+6\,a^2b^4c^2d^4-4\,a^3b^3cd^5+a^4b^2d^6)x^4+2\,(b^6c^5d-3\,ab^5c^4d^2+2\,a^2b^4c^3d^3+2\,a^3b^3c^2d^4-3\,a^4b^2cd^5+a^5bd^6)x^3+(b^6c^6-9\,a^2b^4c^4d^2+16\,a^3b^3c^3d^3-9\,a^4b^2c^2d^4+a^6d^6)x^2+2\,(ab^5c^6-3\,a^2b^4c^5d+2\,a^3b^3c^4d^2+2\,a^4b^2c^3d^3-3\,a^5bc^2d^4+a^6cd^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
- a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
- a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 +
 a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4
 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*
a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b
^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d
```

```
^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^
3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(139) = 278$.
time = 0.47, size = 760, normalized size = 5.31

$$-\frac{b^4c^4 - 8\,ab^3c^3d + 8\,a^3bcd^3 - a^4d^4 - 12\,(b^4cd^3 - ab^3d^4)x^3 - 18\,(b^4c^2d^2 - a^2b^2d^4)x^2 - 4\,(b^4c^3d + 6\,ab^3c^2d^2 - 6\,a^2b^2cd^3 - a^3bd^4)x - 12\,(b^4d^4x^4 + a^2b^2c^2d^2 + 2\,(b^4cd^3 + ab^3d^4)x^3 + (b^4c^2d^2 + 4\,ab^3cd^3 + a^2b^2d^4)x^2 + 2\,(ab^3c^2d^2 + a^2b^2cd^3)x)\log(bx + a) + 12\,(b^4d^4x^4 + a^2b^2c^2d^2 + 2\,(b^4cd^3 + ab^3d^4)x^3 + (b^4c^2d^2 + 4\,ab^3cd^3 + a^2b^2d^4)x^2 + 2\,(ab^3c^2d^2 + a^2b^2cd^3)x)\log(dx + c)}{2\,(a^2b^5c^7 - 5\,a^3b^4c^6d + 10\,a^4b^3c^5d^2 - 10\,a^5b^2c^4d^3 + 5\,a^6bc^3d^4 - a^7c^2d^5 + (b^7c^5d^2 - 5\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 - 10\,a^3b^4c^2d^5 + 5\,a^4b^3cd^6 - a^5b^2d^7)x^4 + 2\,(b^7c^6d - 4\,ab^6c^5d^2 + 5\,a^2b^5c^4d^3 - 5\,a^4b^3c^2d^5 + 4\,a^5b^2cd^6 - a^6bd^7)x^3 + (b^7c^7 - ab^6c^6d - 9\,a^2b^5c^5d^2 + 25\,a^3b^4c^4d^3 - 25\,a^4b^3c^3d^4 + 9\,a^5b^2c^2d^5 + a^6bcd^6 - a^7d^7)x^2 + 2\,(ab^6c^7 - 4\,a^2b^5c^6d + 5\,a^3b^4c^5d^2 - 5\,a^5b^2c^3d^4 + 4\,a^6bc^2d^5 - a^7cd^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 - a
*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b^3
*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d
^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2
*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^4*d^4
*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a
*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*
x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^
4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10
*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^4
+ 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 +
4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d
^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*
d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5
*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(128) = 256$.
time = 1.40, size = 881, normalized size = 6.16

$$\frac{6b^2d^2\log\left(x + \frac{-\frac{6a^6b^2d^8}{(ad-bc)^5} + \frac{36a^5b^3cd^7}{(ad-bc)^5} - \frac{90a^4b^4c^2d^6}{(ad-bc)^5} + \frac{120a^3b^5c^3d^5}{(ad-bc)^5} - \frac{90a^2b^6c^4d^4}{(ad-bc)^5} + \frac{36ab^7c^5d^3}{(ad-bc)^5} + 6ab^2d^3 - \frac{6b^8c^6d^2}{(ad-bc)^5} + 6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5} - \frac{6b^2d^2\log\left(x + \frac{\frac{6a^6b^2d^8}{(ad-bc)^5} - \frac{36a^5b^3cd^7}{(ad-bc)^5} + \frac{90a^4b^4c^2d^6}{(ad-bc)^5} - \frac{120a^3b^5c^3d^5}{(ad-bc)^5} + \frac{90a^2b^6c^4d^4}{(ad-bc)^5} - \frac{36ab^7c^5d^3}{(ad-bc)^5} + 6ab^2d^3 + \frac{6b^8c^6d^2}{(ad-bc)^5} + 6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5} + \frac{-a^3d^3 + 7a^2bcd^2 + 7ab^2c^2d - b^3c^3 + 12b^3d^3x^3 + x^2\cdot(18ab^2d^3 + 18b^3cd^2) + x(4a^2bd^3 + 28ab^2cd^2 + 4b^3c^2d)}{2a^6c^2d^4 - 8a^5bc^3d^3 + 12a^4b^2c^4d^2 - 8a^3b^3c^5d + 2a^2b^4c^6 + x^4\cdot(2a^4b^2d^6 - 8a^3b^3cd^5 + 12a^2b^4c^2d^4 - 8ab^5c^3d^3 + 2b^6c^4d^2) + x^3\cdot(4a^5bd^6 - 12a^4b^2cd^5 + 8a^3b^3c^2d^4 + 8a^2b^4c^3d^3 - 12ab^5c^4d^2 + 4b^6c^5d) + x^2\cdot(2a^6d^6 - 18a^4b^2c^2d^4 + 32a^3b^3c^3d^3 - 18a^2b^4c^4d^2 + 2b^6c^6) + x(4a^6cd^5 - 12a^5bc^2d^4 + 8a^4b^2c^3d^3 + 8a^3b^3c^4d^2 - 12a^2b^4c^5d + 4ab^5c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] 6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**7
/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c**
3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**7*c
**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)**5 +
 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (6*a**
6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*a**4*b
**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*
a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6
*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(12*b**3*d*
```

```
*3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**
3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d**2) + x*(4*
a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d**4 - 8*a**5
*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2*a**2*b**4*c*
*6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b**4*c**2*d**4 -
 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**6 - 12*a**4*b**
2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 - 12*a*b**5*c**4*d
**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c**2*d**4 + 32*a**3
*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) + x*(4*a**6*c*d**5
- 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b**3*c**4*d**2 - 12*
a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(139) = 278.

time = 1.62, size = 345, normalized size = 2.41

$$\frac{6\,b^3d^2\log\left(|bx+a|\right)}{b^6c^5-5\,ab^5c^4d+10\,a^2b^4c^3d^2-10\,a^3b^3c^2d^3+5\,a^4b^2cd^4-a^5bd^5}-\frac{6\,b^2d^3\log\left(|dx+c|\right)}{b^5c^5d-5\,ab^4c^4d^2+10\,a^2b^3c^3d^3-10\,a^3b^2c^2d^4+5\,a^4bcd^5-a^5d^6}+\frac{12\,b^3d^3x^3+18\,b^3cd^2x^2+18\,ab^2d^3x^2+4\,b^3c^2dx+28\,ab^2cd^2x+4\,a^2bd^3x-b^3c^3+7\,ab^2c^2d+7\,a^2bcd^2-a^3d^3}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(bdx^2+bcx+adx+ac)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*b^3*d^2*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 -
 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - 6*b^2*d^3*log(abs(d*x
+ c))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^
4 + 5*a^4*b*c*d^5 - a^5*d^6) + 1/2*(12*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*
a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 28*a*b^2*c*d^2*x + 4*a^2*b*d^3*x - b^3*c^3
+ 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*d*x^2 + b*c*x + a*d*x + a*c)^2)
```

Mupad [B]

time = 0.53, size = 542, normalized size = 3.79

$$\frac{\frac{6\,b^3\,d^3\,x^3}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}-\frac{a^3\,d^3-7\,a^2\,b\,c\,d^2-7\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}+\frac{9\,b\,d\,x^2\,(c\,b^2\,d+a\,b\,d^2)}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}+\frac{2\,b\,d\,x\,(a^2\,d^2+7\,a\,b\,c\,d+b^2\,c^2)}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}}{x\,(2\,d\,a^2\,c+2\,b\,a\,c^2)+x^2\,(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)+x^3\,(2\,c\,b^2\,d+2\,a\,b\,d^2)+a^2\,c^2+b^2\,d^2\,x^4}-\frac{12\,b^2\,d^2\,\mathrm{atanh}\left(\frac{a^5\,d^5-3\,a^4\,b\,c\,d^4+2\,a^3\,b^2\,c^2\,d^3+2\,a^2\,b^3\,c^3\,d^2-3\,a\,b^4\,c^4\,d+b^5\,c^5}{{(a\,d-b\,c)}^5}+\frac{2\,b\,d\,x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{{(a\,d-b\,c)}^5}\right)}{{(a\,d-b\,c)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^3),x)
```

```
[Out] ((6*b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3) - (a^3*d^3 + b^3*c^3 - 7*a*b^2*c^2*d - 7*a^2*b*c*d^2)/(2*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (9*b
*d*x^2*(a*b*d^2 + b^2*c*d))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 + 7*a*b*c*d))/(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(x*(2*a*
b*c^2 + 2*a^2*c*d) + x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^3*(2*a*b*d^2 +
 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^4) - (12*b^2*d^2*atanh((a^5*d^5 + b^5*c^5
 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3*a*b^4*c^4*d - 3*a^4*b*c*d^4)/(
a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d - b*c)^5
```

3.1362 $\int \frac{(a+bx)^9}{(c+dx)^8}\,dx$

Optimal. Leaf size=232

$$-\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-$$

```
[Out] -b^8*(-9*a*d+8*b*c)*x/d^9+1/2*b^9*x^2/d^8+1/7*(-a*d+b*c)^9/d^10/(d*x+c)^7-3
/2*b*(-a*d+b*c)^8/d^10/(d*x+c)^6+36/5*b^2*(-a*d+b*c)^7/d^10/(d*x+c)^5-21*b^
3*(-a*d+b*c)^6/d^10/(d*x+c)^4+42*b^4*(-a*d+b*c)^5/d^10/(d*x+c)^3-63*b^5*(-a
*d+b*c)^4/d^10/(d*x+c)^2+84*b^6*(-a*d+b*c)^3/d^10/(d*x+c)+36*b^7*(-a*d+b*c)
^2*ln(d*x+c)/d^10
```

Rubi [A]
time = 0.25, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{36b^7(bc-ad)^2\log(c+dx)}{d^{10}}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}+\frac{b^9x^2}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -((b^8*(8*b*c - 9*a*d)*x)/d^9) + (b^9*x^2)/(2*d^8) + (b*c - a*d)^9/(7*d^10*
(c + d*x)^7) - (3*b*(b*c - a*d)^8)/(2*d^10*(c + d*x)^6) + (36*b^2*(b*c - a*
d)^7)/(5*d^10*(c + d*x)^5) - (21*b^3*(b*c - a*d)^6)/(d^10*(c + d*x)^4) + (4
2*b^4*(b*c - a*d)^5)/(d^10*(c + d*x)^3) - (63*b^5*(b*c - a*d)^4)/(d^10*(c +
 d*x)^2) + (84*b^6*(b*c - a*d)^3)/(d^10*(c + d*x)) + (36*b^7*(b*c - a*d)^2*
Log[c + d*x])/d^10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^9}{(c+dx)^8}\,dx = \int \left(-\frac{b^8(8bc-9ad)}{d^9}+\frac{b^9x}{d^8}+\frac{(-bc+ad)^9}{d^9(c+dx)^8}+\frac{9b(bc-ad)^8}{d^9(c+dx)^7}-\frac{36b^2(bc-ad)^7}{d^9(c+dx)^6}+\frac{84b^3(bc-}{d^9(c+}\right.$$

$$=-\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-}{d^{10}(c+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 584 vs. 2(232) = 464.
time = 0.18, size = 584, normalized size = 2.52

$$-\frac{10a^9d^9+15a^8bd^8(c+7dx)+24a^7b^2d^7(c^2+7cdx+21d^2x^2)+42a^6b^3d^6(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+84a^5b^4d^5(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+210a^4b^5d^4(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+840a^3b^6d^3(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)-6a^2b^7cd^2(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)+6ab^8d(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8)-b^9(3349c^9+20923c^8dx+53949c^7d^2x^2+72275c^6d^3x^3+50225c^5d^4x^4+12495c^4d^5x^5-4655c^3d^6x^6-3185c^2d^7x^7-315cd^8x^8+35d^9x^9)-2520b^7(bc-ad)^2(c+dx)^7\log(c+dx)}{70d^{10}(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -1/70*(10*a^9*d^9 + 15*a^8*b*d^8*(c + 7*d*x) + 24*a^7*b^2*d^7*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 42*a^6*b^3*d^6*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x
^3) + 84*a^5*b^4*d^5*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*
d^4*x^4) + 210*a^4*b^5*d^4*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x
^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 840*a^3*b^6*d^3*(c^6 + 7*c^5*d*x + 21*c^4
*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 6*
a^2*b^7*c*d^2*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*
x^3 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + 6*a*b^8*d*(1443
*c^8 + 9261*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4
*x^4 + 11025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) -
 b^9*(3349*c^9 + 20923*c^8*d*x + 53949*c^7*d^2*x^2 + 72275*c^6*d^3*x^3 + 50
225*c^5*d^4*x^4 + 12495*c^4*d^5*x^5 - 4655*c^3*d^6*x^6 - 3185*c^2*d^7*x^7 -
 315*c*d^8*x^8 + 35*d^9*x^9) - 2520*b^7*(b*c - a*d)^2*(c + d*x)^7*Log[c + d
*x])/(d^10*(c + d*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(224) = 448.
time = 0.14, size = 705, normalized size = 3.04

method	result
default	$\frac{b^8\left(\frac{1}{2}bd\,x^2+9adx-8bcx\right)}{d^9}-\frac{42b^4\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{d^{10}(dx+c)^3}-\frac{3b\left(a^8d^8-8a^7bc\,d^7+28a^6b^2c^2d^6-56\right.}{}$
norman	$-\frac{10a^9d^9+15a^8bc\,d^8+24a^7b^2c^2d^7+42a^6b^3c^3d^6+84a^5b^4c^4d^5+210a^4b^5c^5d^4+840a^3b^6d^3c^6-6534a^2b^7d^2c^7+13068a\,b^8c^8d-6534b^9c^9}{70d^{10}}+\frac{b^9x^9}{2d}-\frac{7\left(12c\right.}{}$
risch	$\frac{b^9x^2}{2d^8}+\frac{9b^8ax}{d^8}-\frac{8b^9cx}{d^9}+\frac{\left(-84a^3b^6d^8+252a^2b^7c\,d^7-252a\,b^8c^2d^6+84b^9c^3d^5\right)x^6-63b^5d^4\left(a^4d^4+4a^3bc\,d^3-18a^2b^2c^2d^2+20a\,b^3c^3d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] b^8/d^9*(1/2*b*d*x^2+9*a*d*x-8*b*c*x)-42*b^4/d^10*(a^5*d^5-5*a^4*b*c*d^4+10
*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)^3-3/2*b/
d^10*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^
4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/(d*x
+c)^6-36/5*b^2/d^10*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^
```

```
3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(d*x+c)^
5-21*b^3/d^10*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+
15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/(d*x+c)^4-84*b^6/d^10*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)+36*b^7/d^10*(a^2*d^2-2*a*b*c*d+b
^2*c^2)*ln(d*x+c)-63*b^5/d^10*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*
b^3*c^3*d+b^4*c^4)/(d*x+c)^2-1/7/d^10*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2
*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*
c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(224) = 448$.
time = 0.42, size = 786, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/70*(3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*
c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 2
4*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b
^8*c^2*d^7 + 3*a^2*b^7*c*d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*
a*b^8*c^3*d^6 + 18*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1
470*(47*b^9*c^5*d^4 - 130*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*
c^2*d^7 - 5*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154
*a*b^8*c^5*d^4 + 125*a^2*b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d
^7 - 2*a^5*b^4*c*d^8 - a^6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8
*c^6*d^3 + 959*a^2*b^7*c^5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 -
 14*a^5*b^4*c^2*d^7 - 7*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c
^8*d - 2676*a*b^8*c^7*d^2 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70
*a^4*b^5*c^4*d^5 - 28*a^5*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^
8 - 5*a^8*b*d^9)*x)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^1
4*x^4 + 35*c^4*d^13*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10) + 1/2*
(b^9*d*x^2 - 2*(8*b^9*c - 9*a*b^8*d)*x)/d^9 + 36*(b^9*c^2 - 2*a*b^8*c*d + a
^2*b^7*d^2)*log(d*x + c)/d^10
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(224) = 448$.
time = 0.54, size = 1093, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/70*(35*b^9*d^9*x^9 + 3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d
^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^
6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 - 315*(b^9
*c*d^8 - 2*a*b^8*d^9)*x^8 - 245*(13*b^9*c^2*d^7 - 18*a*b^8*c*d^8)*x^7 - 245
*(19*b^9*c^3*d^6 + 18*a*b^8*c^2*d^7 - 72*a^2*b^7*c*d^8 + 24*a^3*b^6*d^9)*x^
6 + 735*(17*b^9*c^4*d^5 - 90*a*b^8*c^3*d^6 + 108*a^2*b^7*c^2*d^7 - 24*a^3*b
^6*c*d^8 - 6*a^4*b^5*d^9)*x^5 + 245*(205*b^9*c^5*d^4 - 690*a*b^8*c^4*d^5 +
660*a^2*b^7*c^3*d^6 - 120*a^3*b^6*c^2*d^7 - 30*a^4*b^5*c*d^8 - 12*a^5*b^4*d
^9)*x^4 + 245*(295*b^9*c^6*d^3 - 870*a*b^8*c^5*d^4 + 750*a^2*b^7*c^4*d^5 -
120*a^3*b^6*c^3*d^6 - 30*a^4*b^5*c^2*d^7 - 12*a^5*b^4*c*d^8 - 6*a^6*b^3*d^9
)*x^3 + 21*(2569*b^9*c^7*d^2 - 7098*a*b^8*c^6*d^3 + 5754*a^2*b^7*c^5*d^4 -
840*a^3*b^6*c^4*d^5 - 210*a^4*b^5*c^3*d^6 - 84*a^5*b^4*c^2*d^7 - 42*a^6*b^3
*c*d^8 - 24*a^7*b^2*d^9)*x^2 + 7*(2989*b^9*c^8*d - 7938*a*b^8*c^7*d^2 + 617
4*a^2*b^7*c^6*d^3 - 840*a^3*b^6*c^5*d^4 - 210*a^4*b^5*c^4*d^5 - 84*a^5*b^4*
c^3*d^6 - 42*a^6*b^3*c^2*d^7 - 24*a^7*b^2*c*d^8 - 15*a^8*b*d^9)*x + 2520*(b
^9*c^9 - 2*a*b^8*c^8*d + a^2*b^7*c^7*d^2 + (b^9*c^2*d^7 - 2*a*b^8*c*d^8 + a
^2*b^7*d^9)*x^7 + 7*(b^9*c^3*d^6 - 2*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 2
1*(b^9*c^4*d^5 - 2*a*b^8*c^3*d^6 + a^2*b^7*c^2*d^7)*x^5 + 35*(b^9*c^5*d^4 -
 2*a*b^8*c^4*d^5 + a^2*b^7*c^3*d^6)*x^4 + 35*(b^9*c^6*d^3 - 2*a*b^8*c^5*d^4
 + a^2*b^7*c^4*d^5)*x^3 + 21*(b^9*c^7*d^2 - 2*a*b^8*c^6*d^3 + a^2*b^7*c^5*d
^4)*x^2 + 7*(b^9*c^8*d - 2*a*b^8*c^7*d^2 + a^2*b^7*c^6*d^3)*x)*log(d*x + c)
)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^14*x^4 + 35*c^4*d^1
3*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(224) = 448.
time = 1.13, size = 723, normalized size = 3.12

$$\frac{36\,(b^9c^2 - 2\,ab^8cd + a^2b^7d^2)\log(|dx + c|)}{d^{10}} + \frac{b^9d^8x^2 - 16\,b^9cd^7x + 18\,ab^8d^8x}{2\,d^{16}} + \frac{3349\,b^9c^9 - 8658\,ab^8c^8d + 6534\,a^2b^7c^7d^2 - 840\,a^3b^6c^6d^3 - 210\,a^4b^5c^5d^4 - \cdots}{\cdots}$$
[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*(b^9*c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*log(abs(d*x + c))/d^10 + 1/2*(b^9*
d^8*x^2 - 16*b^9*c*d^7*x + 18*a*b^8*d^8*x)/d^16 + 1/70*(3349*b^9*c^9 - 8658
*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5
```

*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^
8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b^8*c^2*d^7 + 3*a^2*b^7*c*
d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*a*b^8*c^3*d^6 + 18*a^2*b^
7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1470*(47*b^9*c^5*d^4 - 130
*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*c^2*d^7 - 5*a^4*b^5*c*d^8
 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154*a*b^8*c^5*d^4 + 125*a^2*
b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d^7 - 2*a^5*b^4*c*d^8 - a^
6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8*c^6*d^3 + 959*a^2*b^7*c^
5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 - 14*a^5*b^4*c^2*d^7 - 7*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c^8*d - 2676*a*b^8*c^7*d^2
 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70*a^4*b^5*c^4*d^5 - 28*a^5
*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^8 - 5*a^8*b*d^9)*x)/((d*x
 + c)^7*d^10)

Mupad [B]
time = 0.26, size = 784, normalized size = 3.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^9/(c + d*x)^8,x)

[Out] x*((9*a*b^8)/d^8 - (8*b^9*c)/d^9) - ((10*a^9*d^9 - 3349*b^9*c^9 - 6534*a^2*
b^7*c^7*d^2 + 840*a^3*b^6*c^6*d^3 + 210*a^4*b^5*c^5*d^4 + 84*a^5*b^4*c^4*d^
5 + 42*a^6*b^3*c^3*d^6 + 24*a^7*b^2*c^2*d^7 + 8658*a*b^8*c^8*d + 15*a^8*b*c
*d^8)/(70*d) + x*((3*a^8*b*d^8)/2 - (3069*b^9*c^8)/10 + (12*a^7*b^2*c*d^7)/
5 - (3087*a^2*b^7*c^6*d^2)/5 + 84*a^3*b^6*c^5*d^3 + 21*a^4*b^5*c^4*d^4 + (4
2*a^5*b^4*c^3*d^5)/5 + (21*a^6*b^3*c^2*d^6)/5 + (4014*a*b^8*c^7*d)/5) + x^3
*(21*a^6*b^3*d^8 - 1197*b^9*c^6*d^2 + 3234*a*b^8*c^5*d^3 + 42*a^5*b^4*c*d^7
 - 2625*a^2*b^7*c^4*d^4 + 420*a^3*b^6*c^3*d^5 + 105*a^4*b^5*c^2*d^6) + x^2*
((36*a^7*b^2*d^8)/5 - (4131*b^9*c^7*d)/5 + (10962*a*b^8*c^6*d^2)/5 + (63*a^
6*b^3*c*d^7)/5 - (8631*a^2*b^7*c^5*d^3)/5 + 252*a^3*b^6*c^4*d^4 + 63*a^4*b^
5*c^3*d^5 + (126*a^5*b^4*c^2*d^6)/5) + x^5*(63*a^4*b^5*d^8 - 441*b^9*c^4*d^
4 + 1260*a*b^8*c^3*d^5 + 252*a^3*b^6*c*d^7 - 1134*a^2*b^7*c^2*d^6) + x^4*(4
2*a^5*b^4*d^8 - 987*b^9*c^5*d^3 + 2730*a*b^8*c^4*d^4 + 105*a^4*b^5*c*d^7 -
2310*a^2*b^7*c^3*d^5 + 420*a^3*b^6*c^2*d^6) + x^6*(84*a^3*b^6*d^8 - 84*b^9*
c^3*d^5 + 252*a*b^8*c^2*d^6 - 252*a^2*b^7*c*d^7))/(c^7*d^9 + d^16*x^7 + 7*c
^6*d^10*x + 7*c*d^15*x^6 + 21*c^5*d^11*x^2 + 35*c^4*d^12*x^3 + 35*c^3*d^13*
x^4 + 21*c^2*d^14*x^5) + (b^9*x^2)/(2*d^8) + (log(c + d*x)*(36*b^9*c^2 + 36
*a^2*b^7*d^2 - 72*a*b^8*c*d))/d^10

3.1363 $\int \frac{(a+bx)^8}{(c+dx)^8} dx$

Optimal. Leaf size=209

$$\frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{28b^6(\ldots}{d^9(\ldots}$$

```
[Out] b^8*x/d^8-1/7*(-a*d+b*c)^8/d^9/(d*x+c)^7+4/3*b*(-a*d+b*c)^7/d^9/(d*x+c)^6-2
8/5*b^2*(-a*d+b*c)^6/d^9/(d*x+c)^5+14*b^3*(-a*d+b*c)^5/d^9/(d*x+c)^4-70/3*b
^4*(-a*d+b*c)^4/d^9/(d*x+c)^3+28*b^5*(-a*d+b*c)^3/d^9/(d*x+c)^2-28*b^6*(-a*
d+b*c)^2/d^9/(d*x+c)-8*b^7*(-a*d+b*c)*ln(d*x+c)/d^9
```

Rubi [A]

time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{b^8x}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] (b^8*x)/d^8 - (b*c - a*d)^8/(7*d^9*(c + d*x)^7) + (4*b*(b*c - a*d)^7)/(3*d^
9*(c + d*x)^6) - (28*b^2*(b*c - a*d)^6)/(5*d^9*(c + d*x)^5) + (14*b^3*(b*c
- a*d)^5)/(d^9*(c + d*x)^4) - (70*b^4*(b*c - a*d)^4)/(3*d^9*(c + d*x)^3) +
(28*b^5*(b*c - a*d)^3)/(d^9*(c + d*x)^2) - (28*b^6*(b*c - a*d)^2)/(d^9*(c +
 d*x)) - (8*b^7*(b*c - a*d)*Log[c + d*x])/d^9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^8}{(c+dx)^8}\,dx = \int \left(\frac{b^8}{d^8}+\frac{(-bc+ad)^8}{d^8(c+dx)^8}-\frac{8b(bc-ad)^7}{d^8(c+dx)^7}+\frac{28b^2(bc-ad)^6}{d^8(c+dx)^6}-\frac{56b^3(bc-ad)^5}{d^8(c+dx)^5}+\frac{70b^4(bc-ad\ldots}{d^8(c+dx)^4\ldots}\right.$$

$$=\frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}\ldots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 474 vs. $2(209) = 418$.

time = 0.13, size = 474, normalized size = 2.27

$$-\frac{15a^8d^8+20a^7bd^7(c+7dx)+28a^6b^2d^6(c^2+7cdx+21d^2x^2)+42a^5b^3d^5(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+70a^4b^4d^4(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+140a^3b^5d^3(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+420a^2b^6d^2(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)-2ab^7cd(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)+b^8(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8)+840b^7(bc-ad)(c+dx)^7\log(c+dx)}{105d^9(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^8*d^8 + 20*a^7*b*d^7*(c + 7*d*x) + 28*a^6*b^2*d^6*(c^2 + 7*c*d
*x + 21*d^2*x^2) + 42*a^5*b^3*d^5*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*
x^3) + 70*a^4*b^4*d^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35
*d^4*x^4) + 140*a^3*b^5*d^3*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*
x^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 420*a^2*b^6*d^2*(c^6 + 7*c^5*d*x + 21*c^
4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 2
*a*b^7*c*d*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3
 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + b^8*(1443*c^8 + 92
61*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4*x^4 + 11
025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) + 840*b^7*
(b*c - a*d)*(c + d*x)^7*Log[c + d*x])/(d^9*(c + d*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(201) = 402$.

time = 0.14, size = 576, normalized size = 2.76

method	result
risch	$\frac{b^8x}{d^8}+\frac{(-28a^2b^6d^7+56ab^7cd^6-28b^8c^2d^5)x^6-28b^5d^4(a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3)x^5-\frac{70b^4d^3(a^4d^4+2a^3bcd^3+6a^2b^2c^2d^2-22a}{3}}{}$
default	$\frac{b^8x}{d^8}-\frac{4b(a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7)}{3d^9(dx+c)^6}-\frac{28b^6(a^2d^2-2abcd+b^2c^2)}{d^9(dx+c)}-$
norman	$\frac{\frac{b^8x^8}{d}-\frac{15a^8d^8+20a^7bcd^7+28a^6b^2c^2d^6+42a^5b^3c^3d^5+70a^4b^4c^4d^4+140a^3b^5c^5d^3+420a^2b^6c^6d^2-2178ab^7c^7d+2178b^8c^8}{105d^9}-\frac{7(4a^2b^6d^2-8ab^7cd+}{d^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] b^8*x/d^8-4/3*b/d^9*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^
3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/(d*x+c)^
6-28*b^6/d^9*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-28/5*b^2/d^9*(a^6*d^6-6*a^
5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*
c^5*d+b^6*c^6)/(d*x+c)^5+8*b^7/d^9*(a*d-b*c)*ln(d*x+c)-70/3*b^4/d^9*(a^4*d^
4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^3-14*b^3/d
^9*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4
*d-b^5*c^5)/(d*x+c)^4-28*b^5/d^9*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
```

```
^3)/(d*x+c)^2-1/7/d^9*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*
c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^
7*d+b^8*c^8)/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(201) = 402$.
time = 0.35, size = 649, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^8*x/d^8 - 1/105*(1443*b^8*c^8 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 +
140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 + 20*a^7*b*c*d^7 + 15*a^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 +
 a^2*b^6*d^8)*x^6 + 2940*(5*b^8*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7
 + a^3*b^5*d^8)*x^5 + 2450*(13*b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c
^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b
^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*
a^5*b^3*d^8)*x^3 + 294*(87*b^8*c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4
*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2
*d^8)*x^2 + 14*(669*b^8*c^7*d - 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 +
70*a^3*b^5*c^4*d^4 + 35*a^4*b^4*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c
*d^7 + 10*a^7*b*d^8)*x)/(d^16*x^7 + 7*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3
*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*d^11*x^2 + 7*c^6*d^10*x + c^7*d^9) - 8
*(b^8*c - a*b^7*d)*log(d*x + c)/d^9
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(201) = 402$.
time = 0.48, size = 852, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d^8*x^8 + 735*b^8*c*d^7*x^7 - 1443*b^8*c^8 + 2178*a*b^7*c^7*
d - 420*a^2*b^6*c^6*d^2 - 140*a^3*b^5*c^5*d^3 - 70*a^4*b^4*c^4*d^4 - 42*a^5
*b^3*c^3*d^5 - 28*a^6*b^2*c^2*d^6 - 20*a^7*b*c*d^7 - 15*a^8*d^8 - 735*(b^8*
c^2*d^6 - 8*a*b^7*c*d^7 + 4*a^2*b^6*d^8)*x^6 - 735*(15*b^8*c^3*d^5 - 36*a*b
^7*c^2*d^6 + 12*a^2*b^6*c*d^7 + 4*a^3*b^5*d^8)*x^5 - 1225*(23*b^8*c^4*d^4 -
 44*a*b^7*c^3*d^5 + 12*a^2*b^6*c^2*d^6 + 4*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x
^4 - 245*(145*b^8*c^5*d^3 - 250*a*b^7*c^4*d^4 + 60*a^2*b^6*c^3*d^5 + 20*a^3
*b^5*c^2*d^6 + 10*a^4*b^4*c*d^7 + 6*a^5*b^3*d^8)*x^3 - 147*(169*b^8*c^6*d^2
 - 274*a*b^7*c^5*d^3 + 60*a^2*b^6*c^4*d^4 + 20*a^3*b^5*c^3*d^5 + 10*a^4*b^4
```

```
*c^2*d^6 + 6*a^5*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 - 7*(1323*b^8*c^7*d - 2058*
a*b^7*c^6*d^2 + 420*a^2*b^6*c^5*d^3 + 140*a^3*b^5*c^4*d^4 + 70*a^4*b^4*c^3*
d^5 + 42*a^5*b^3*c^2*d^6 + 28*a^6*b^2*c*d^7 + 20*a^7*b*d^8)*x - 840*(b^8*c^
8 - a*b^7*c^7*d + (b^8*c*d^7 - a*b^7*d^8)*x^7 + 7*(b^8*c^2*d^6 - a*b^7*c*d^
7)*x^6 + 21*(b^8*c^3*d^5 - a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 - a*b^7*c^3
*d^5)*x^4 + 35*(b^8*c^5*d^3 - a*b^7*c^4*d^4)*x^3 + 21*(b^8*c^6*d^2 - a*b^7*
c^5*d^3)*x^2 + 7*(b^8*c^7*d - a*b^7*c^6*d^2)*x)*log(d*x + c))/(d^16*x^7 + 7
*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*
d^11*x^2 + 7*c^6*d^10*x + c^7*d^9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(201) = 402.
time = 1.69, size = 581, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*x/d^8 - 8*(b^8*c - a*b^7*d)*log(abs(d*x + c))/d^9 - 1/105*(1443*b^8*c^8
 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 + 20*a^7*b*c*d^7 + 15*a
^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 + a^2*b^6*d^8)*x^6 + 2940*(5*b^8
*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 2450*(13*
b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*
b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5
 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 294*(87*b^8*
c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a
^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2*d^8)*x^2 + 14*(669*b^8*c^7*d -
 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 + 70*a^3*b^5*c^4*d^4 + 35*a^4*b^4
*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x)/((d*x +
 c)^7*d^9)
```

Mupad [**B**]
time = 0.43, size = 649, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^8/(c + d*x)^8,x)

[Out] (b^8*x)/d^8 - (log(c + d*x)*(8*b^8*c - 8*a*b^7*d))/d^9 - (x^4*((70*a^4*b^4*d^7)/3 + (910*b^8*c^4*d^3)/3 - (1540*a*b^7*c^3*d^4)/3 + (140*a^3*b^5*c*d^6)/3 + 140*a^2*b^6*c^2*d^5) + x^6*(28*a^2*b^6*d^7 + 28*b^8*c^2*d^5 - 56*a*b^7*c*d^6) + (15*a^8*d^8 + 1443*b^8*c^8 + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 2178*a*b^7*c^7*d + 20*a^7*b*c*d^7)/(105*d) + x*((446*b^8*c^7)/5 + (4*a^7*b*d^7)/3 + (28*a^6*b^2*c*d^6)/15 + 28*a^2*b^6*c^5*d^2 + (28*a^3*b^5*c^4*d^3)/3 + (14*a^4*b^4*c^3*d^4)/3 + (14*a^5*b^3*c^2*d^5)/5 - (686*a*b^7*c^6*d)/5) + x^3*(14*a^5*b^3*d^7 + (1078*b^8*c^5*d^2)/3 - (1750*a*b^7*c^4*d^3)/3 + (70*a^4*b^4*c*d^6)/3 + 140*a^2*b^6*c^3*d^4 + (140*a^3*b^5*c^2*d^5)/3) + x^2*((1218*b^8*c^6*d)/5 + (28*a^6*b^2*d^7)/5 - (1918*a*b^7*c^5*d^2)/5 + (42*a^5*b^3*c*d^6)/5 + 84*a^2*b^6*c^4*d^3 + 28*a^3*b^5*c^3*d^4 + 14*a^4*b^4*c^2*d^5) + x^5*(28*a^3*b^5*d^7 + 140*b^8*c^3*d^4 - 252*a*b^7*c^2*d^5 + 84*a^2*b^6*c*d^6))/(c^7*d^8 + d^15*x^7 + 7*c^6*d^9*x + 7*c*d^14*x^6 + 21*c^5*d^10*x^2 + 35*c^4*d^11*x^3 + 35*c^3*d^12*x^4 + 21*c^2*d^13*x^5)

3.1364 $\int \frac{(a+bx)^7}{(c+dx)^8}\,dx$

Optimal. Leaf size=194

$$\frac{(bc-ad)^7}{7d^8(c+dx)^7}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{7b^6(bc-a}{d^8(c+dx}$$

```
[Out] 1/7*(-a*d+b*c)^7/d^8/(d*x+c)^7-7/6*b*(-a*d+b*c)^6/d^8/(d*x+c)^6+21/5*b^2*(-
a*d+b*c)^5/d^8/(d*x+c)^5-35/4*b^3*(-a*d+b*c)^4/d^8/(d*x+c)^4+35/3*b^4*(-a*d
+b*c)^3/d^8/(d*x+c)^3-21/2*b^5*(-a*d+b*c)^2/d^8/(d*x+c)^2+7*b^6*(-a*d+b*c)/
d^8/(d*x+c)+b^7*ln(d*x+c)/d^8
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{7b^6(bc-ad)}{d^8(c+dx)}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{(bc-ad)^7}{7d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^7/(7*d^8*(c + d*x)^7) - (7*b*(b*c - a*d)^6)/(6*d^8*(c + d*x)^6)
 + (21*b^2*(b*c - a*d)^5)/(5*d^8*(c + d*x)^5) - (35*b^3*(b*c - a*d)^4)/(4*d
^8*(c + d*x)^4) + (35*b^4*(b*c - a*d)^3)/(3*d^8*(c + d*x)^3) - (21*b^5*(b*c
 - a*d)^2)/(2*d^8*(c + d*x)^2) + (7*b^6*(b*c - a*d))/(d^8*(c + d*x)) + (b^7
*Log[c + d*x])/d^8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^7}{d^7(c+dx)^8}+\frac{7b(bc-ad)^6}{d^7(c+dx)^7}-\frac{21b^2(bc-ad)^5}{d^7(c+dx)^6}+\frac{35b^3(bc-ad)^4}{d^7(c+dx)^5}-\frac{35b^4(bc-ad)^3}{d^7(c+dx)^4}+\right.\\ &= \frac{(bc-ad)^7}{7d^8(c+dx)^7}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{21}{2}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 308, normalized size = 1.59

$$\frac{(bc-ad)\left(60a^6d^6+10a^5bd^5(13c+49dx)+2a^4b^2d^4(107c^2+539cdx+882d^2x^2)+a^3b^3d^3(319c^3+1813c^2dx+3969cd^2x^2+3675d^3x^3)+a^2b^4d^2(459c^4+2793c^3dx+6909c^2d^2x^2+8575cd^3x^3+4900d^4x^4)+ab^5d(669c^5+4263c^4dx+11319c^3d^2x^2+15925c^2d^3x^3+12250cd^4x^4+4410d^5x^5)+b^6(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)\right)}{420d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] ((b*c - a*d)*(60*a^6*d^6 + 10*a^5*b*d^5*(13*c + 49*d*x) + 2*a^4*b^2*d^4*(10
7*c^2 + 539*c*d*x + 882*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 1813*c^2*d*x + 39
69*c*d^2*x^2 + 3675*d^3*x^3) + a^2*b^4*d^2*(459*c^4 + 2793*c^3*d*x + 6909*c
^2*d^2*x^2 + 8575*c*d^3*x^3 + 4900*d^4*x^4) + a*b^5*d*(669*c^5 + 4263*c^4*d
*x + 11319*c^3*d^2*x^2 + 15925*c^2*d^3*x^3 + 12250*c*d^4*x^4 + 4410*d^5*x^5
) + b^6*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 +
26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6)))/(420*d^8*(c + d*x)^7)
 + (b^7*Log[c + d*x])/d^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(182) = 364.
time = 0.14, size = 462, normalized size = 2.38

method	result
risch	$-\frac{7b^6(ad-bc)x^6}{d^2}-\frac{21b^5\left(a^2d^2+2abcd-3b^2c^2\right)x^5}{2d^3}-\frac{35b^4\left(2a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d-11b^3c^3\right)x^4}{6d^4}-\frac{35b^3\left(3a^4d^4+4a^3bc\,d^3+6a^2b^2c^2d^2+12a\,b^3c^3d-2\right.}{12d^5}$
norman	$-\frac{60a^7d^7+70a^6bc\,d^6+84a^5b^2c^2d^5+105a^4b^3c^3d^4+140a^3b^4c^4d^3+210a^2b^5c^5d^2+420a\,b^6c^6d-1089b^7c^7}{420d^8}-\frac{7\left(a\,b^6d-b^7c\right)x^6}{d^2}-\frac{21\left(a^2b^5d^2+2a\,b^6cd-3b^7\right.}{2d^3}$
default	$-\frac{7b\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)}{6d^8(dx+c)^6}-\frac{7b^6(ad-bc)}{d^8(dx+c)}-\frac{35b^4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3\right.}{3d^8(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -7/6*b*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*
b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/d^8/(d*x+c)^6-7*b^6/d^8*(a*d-b*c)/(d*x+c
)-35/3*b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^8/(d*x+c)^3+b^7*
ln(d*x+c)/d^8-35/4*b^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3
*d+b^4*c^4)/d^8/(d*x+c)^4-21/5*b^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^
3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^8/(d*x+c)^5-21/2*b^5*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/d^8/(d*x+c)^2-1/7*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2
*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d
-b^7*c^7)/d^8/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(182) = 364.
time = 0.33, size = 535, normalized size = 2.76

$$\frac{1089\,b^7c^7-420\,ab^6c^6d-210\,a^2b^5c^5d^2-140\,a^3b^4c^4d^3-105\,a^4b^3c^3d^4-84\,a^5b^2c^2d^5-70\,a^6bcd^6-60\,a^7d^7+2940\,(b^7cd^6-ab^6d^7)x^6+4410\,(3\,b^7c^2d^5-2\,ab^6cd^6-a^2b^5d^7)x^5+2450\,(11\,b^7c^3d^4-6\,ab^6c^2d^5-3\,a^2b^5cd^6-2\,a^3b^4d^7)x^4+1225\,(25\,b^7c^4d^3-12\,ab^6c^3d^4-6\,a^2b^5c^2d^5-4\,a^3b^4cd^6-3\,a^4b^3d^7)x^3+147\,(137\,b^7c^5d^2-60\,ab^6c^4d^3-30\,a^2b^5c^3d^4-20\,a^3b^4c^2d^5-15\,a^4b^3cd^6-12\,a^5b^2d^7)x^2+49\,(147\,b^7c^6d-60\,ab^6c^5d^2-30\,a^2b^5c^4d^3-20\,a^3b^4c^3d^4-15\,a^4b^3c^2d^5-12\,a^5b^2cd^6-10\,a^6bd^7)x}{420\,(d^{15}x^7+7\,cd^{14}x^6+21\,c^2d^{13}x^5+35\,c^3d^{12}x^4+35\,c^4d^{11}x^3+21\,c^5d^{10}x^2+7\,c^6d^9x+c^7d^8)}+\frac{b^7\log(dx+c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x)/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 35*c^3*d^12*
x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8) + b^7*log(
d*x + c)/d^8
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(182) = 364$.
time = 0.75, size = 625, normalized size = 3.22

$$\frac{1089\,b^7c^7 - 420\,ab^6c^6d - 210\,a^2b^5c^5d^2 - 140\,a^3b^4c^4d^3 - 105\,a^4b^3c^3d^4 - 84\,a^5b^2c^2d^5 - 70\,a^6bcd^6 - 60\,a^7d^7 + 2940\,(b^7cd^6 - ab^6d^7)x^6 + 4410\,(3\,b^7c^2d^5 - 2\,ab^6cd^6 - a^2b^5d^7)x^5 + 2450\,(11\,b^7c^3d^4 - 6\,ab^6c^2d^5 - 3\,a^2b^5cd^6 - 2\,a^3b^4d^7)x^4 + 1225\,(25\,b^7c^4d^3 - 12\,ab^6c^3d^4 - 6\,a^2b^5c^2d^5 - 4\,a^3b^4cd^6 - 3\,a^4b^3d^7)x^3 + 147\,(137\,b^7c^5d^2 - 60\,ab^6c^4d^3 - 30\,a^2b^5c^3d^4 - 20\,a^3b^4c^2d^5 - 15\,a^4b^3cd^6 - 12\,a^5b^2d^7)x^2 + 49\,(147\,b^7c^6d - 60\,ab^6c^5d^2 - 30\,a^2b^5c^4d^3 - 20\,a^3b^4c^3d^4 - 15\,a^4b^3c^2d^5 - 12\,a^5b^2cd^6 - 10\,a^6bd^7)x + 420\,(b^7d^7x^7 + 7\,b^7cd^6x^6 + 21\,b^7c^2d^5x^5 + 35\,b^7c^3d^4x^4 + 35\,b^7c^4d^3x^3 + 21\,b^7c^5d^2x^2 + 7\,b^7c^6dx + b^7c^7)\log\left(dx + c\right)}{420\,(d^{15}x^7 + 7\,cd^{14}x^6 + 21\,c^2d^{13}x^5 + 35\,c^3d^{12}x^4 + 35\,c^4d^{11}x^3 + 21\,c^5d^{10}x^2 + 7\,c^6d^9x + c^7d^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 21*b^7*c^2*d^5*x^5
+ 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^2*x^2 + 7*b^7*c^6*
d*x + b^7*c^7)*log(d*x + c))/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 3
5*c^3*d^12*x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(182) = 364.
time = 1.66, size = 467, normalized size = 2.41

$$\frac{b^7\log\left(|dx+c|\right)}{d^8}+\frac{2940\,(b^7cd^5-ab^6d^6)x^6+4410\,(3\,b^7c^2d^4-2\,ab^6cd^5-a^2b^5d^6)x^5+2450\,(11\,b^7c^3d^3-6\,ab^6c^2d^4-3\,a^2b^5cd^5-2\,a^3b^4d^6)x^4+1225\,(25\,b^7c^4d^2-12\,ab^6c^3d^3-6\,a^2b^5c^2d^4-4\,a^3b^4cd^5-3\,a^4b^3d^6)x^3+147\,(137\,b^7c^5d-60\,ab^6c^4d^2-30\,a^2b^5c^3d^3-20\,a^3b^4c^2d^4-15\,a^4b^3cd^5-12\,a^5b^2d^6)x^2+49\,(147\,b^7c^6-60\,ab^6c^5d-30\,a^2b^5c^4d^2-20\,a^3b^4c^3d^3-15\,a^4b^3c^2d^4-12\,a^5b^2cd^5-10\,a^6bd^6)x+\frac{1089\,b^7c^7-420\,ab^6c^6d-210\,a^2b^5c^5d^2-140\,a^3b^4c^4d^3-105\,a^4b^3c^3d^4-84\,a^5b^2c^2d^5-70\,a^6bcd^6-60\,a^7d^7}{d}}{420\,(dx+c)^7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(d*x + c))/d^8 + 1/420*(2940*(b^7*c*d^5 - a*b^6*d^6)*x^6 + 4410*
(3*b^7*c^2*d^4 - 2*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 2450*(11*b^7*c^3*d^3 -
6*a*b^6*c^2*d^4 - 3*a^2*b^5*c*d^5 - 2*a^3*b^4*d^6)*x^4 + 1225*(25*b^7*c^4*d
^2 - 12*a*b^6*c^3*d^3 - 6*a^2*b^5*c^2*d^4 - 4*a^3*b^4*c*d^5 - 3*a^4*b^3*d^6
)*x^3 + 147*(137*b^7*c^5*d - 60*a*b^6*c^4*d^2 - 30*a^2*b^5*c^3*d^3 - 20*a^3
*b^4*c^2*d^4 - 15*a^4*b^3*c*d^5 - 12*a^5*b^2*d^6)*x^2 + 49*(147*b^7*c^6 - 6
0*a*b^6*c^5*d - 30*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 - 15*a^4*b^3*c^2*d^
4 - 12*a^5*b^2*c*d^5 - 10*a^6*b*d^6)*x + (1089*b^7*c^7 - 420*a*b^6*c^6*d -
210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^
2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7*d^7)/d)/((d*x + c)^7*d^7)
```

Mupad [**B**]
time = 0.38, size = 460, normalized size = 2.37

$$\frac{b^7\,\ln\left(c+d\,x\right)}{d^8}-\frac{x\,\left(\frac{7\,a^6\,b\,d^7}{6}+\frac{7\,a^5\,b^2\,c\,d^6}{5}+\frac{7\,a^4\,b^3\,c^2\,d^5}{4}+\frac{7\,a^3\,b^4\,c^3\,d^4}{3}+\frac{7\,a^2\,b^5\,c^4\,d^3}{2}+7\,a\,b^6\,c^5\,d^2-\frac{343\,b^7\,c^6\,d}{20}\right)+x^6\,\left(7\,a\,b^6\,d^7-7\,b^7\,c\,d^6\right)+x^3\,\left(\frac{35\,a^4\,b^3\,d^7}{4}+\frac{35\,a^3\,b^4\,c\,d^6}{3}+\frac{35\,a^2\,b^5\,c^2\,d^5}{2}+35\,a\,b^6\,c^3\,d^4-\frac{875\,b^7\,c^4\,d^3}{12}\right)+x^5\,\left(\frac{21\,a^2\,b^5\,d^7}{2}+21\,a\,b^6\,c\,d^6-\frac{63\,b^7\,c^2\,d^5}{2}\right)+x^2\,\left(\frac{21\,a^5\,b^2\,d^7}{5}+\frac{21\,a^4\,b^3\,c\,d^6}{4}+7\,a^3\,b^4\,c^2\,d^5+\frac{21\,a^2\,b^5\,c^3\,d^4}{2}+21\,a\,b^6\,c^4\,d^3-\frac{959\,b^7\,c^5\,d^2}{20}\right)+\frac{a^7\,d^7}{7}-\frac{363\,b^7\,c^7}{140}+x^4\,\left(\frac{35\,a^3\,b^4\,d^7}{3}+\frac{35\,a^2\,b^5\,c\,d^6}{2}+35\,a\,b^6\,c^2\,d^5-\frac{385\,b^7\,c^3\,d^4}{6}\right)+\frac{a^2\,b^5\,c^5\,d^2}{2}+\frac{a^3\,b^4\,c^4\,d^3}{3}+\frac{a^4\,b^3\,c^3\,d^4}{4}+\frac{a^5\,b^2\,c^2\,d^5}{5}+a\,b^6\,c^6\,d+\frac{a^6\,b\,c\,d^6}{6}}{d^8\,{\left(c+d\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/(c + d*x)^8,x)
```

```
[Out] (b^7*log(c + d*x))/d^8 - (x*((7*a^6*b*d^7)/6 - (343*b^7*c^6*d)/20 + 7*a*b^6
*c^5*d^2 + (7*a^5*b^2*c*d^6)/5 + (7*a^2*b^5*c^4*d^3)/2 + (7*a^3*b^4*c^3*d^4
)/3 + (7*a^4*b^3*c^2*d^5)/4) + x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*a
^4*b^3*d^7)/4 - (875*b^7*c^4*d^3)/12 + 35*a*b^6*c^3*d^4 + (35*a^3*b^4*c*d^6
)/3 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*a^2*b^5*d^7)/2 - (63*b^7*c^2*d^5)/
2 + 21*a*b^6*c*d^6) + x^2*((21*a^5*b^2*d^7)/5 - (959*b^7*c^5*d^2)/20 + 21*a
*b^6*c^4*d^3 + (21*a^4*b^3*c*d^6)/4 + (21*a^2*b^5*c^3*d^4)/2 + 7*a^3*b^4*c^
2*d^5) + (a^7*d^7)/7 - (363*b^7*c^7)/140 + x^4*((35*a^3*b^4*d^7)/3 - (385*b
^7*c^3*d^4)/6 + 35*a*b^6*c^2*d^5 + (35*a^2*b^5*c*d^6)/2) + (a^2*b^5*c^5*d^2
)/2 + (a^3*b^4*c^4*d^3)/3 + (a^4*b^3*c^3*d^4)/4 + (a^5*b^2*c^2*d^5)/5 + a*b
^6*c^6*d + (a^6*b*c*d^6)/6)/(d^8*(c + d*x)^7)
```

3.1365 $\int \frac{(a+bx)^6}{(c+dx)^8} \, dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

```
[Out] 1/7*(b*x+a)^7/(-a*d+b*c)/(d*x+c)^7
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{37\}$

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^7/(7*(b*c - a*d)*(c + d*x)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^8} \, dx = \frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 271 vs. 2(28) = 56.
time = 0.06, size = 271, normalized size = 9.68

$$-\frac{a^6d^6+a^5bd^5(c+7dx)+a^4b^2d^4(c^2+7cdx+21d^2x^2)+a^3b^3d^3(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+a^2b^4d^2(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+ab^5d(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+b^6(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)}{7d^7(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] -1/7*(a^6*d^6 + a^5*b*d^5*(c + 7*d*x) + a^4*b^2*d^4*(c^2 + 7*c*d*x + 21*d^2
*x^2) + a^3*b^3*d^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + a^2*b^4
*d^2*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4) + a*b^5
*d*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 21*d
^5*x^5) + b^6*(c^6 + 7*c^5*d*x + 21*c^4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d
^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6))/(d^7*(c + d*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(26) = 52$.
time = 0.14, size = 357, normalized size = 12.75

method	result
risch	$\frac{-\frac{b^6x^6}{d}-\frac{3b^5(ad+bc)x^5}{d^2}-\frac{5b^4\left(a^2d^2+abcd+b^2c^2\right)x^4}{d^3}-\frac{5b^3\left(a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{d^4}-\frac{3b^2\left(a^4d^4+a^3bc\,d^3+a^2b^2c^2d^2+a\,b^3c^3d+b^4c^4\right)x^2}{d^5}-\cdots}{(dx+c)^7}$
norman	$\frac{-\frac{b^6x^6}{d}-\frac{3\left(a\,b^5d+b^6c\right)x^5}{d^2}-\frac{5\left(a^2b^4d^2+a\,b^5cd+c^2b^6\right)x^4}{d^3}-\frac{5\left(a^3b^3d^3+a^2c\,b^4d^2+a\,c^2b^5d+c^3b^6\right)x^3}{d^4}-\frac{3\left(a^4b^2d^4+a^3b^3c\,d^3+a^2b^4c^2d^2+a\,b^5c^3d+b^6c^4\cdots\right.}{d^5}\cdots}{(dx+c)^7}$
default	$-\frac{b\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}{d^7(dx+c)^6}-\frac{b^6}{d^7(dx+c)}-\frac{5b^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^7(dx+c)^4}-\frac{3b^2\left(a^4d^4\cdots\right.}{\cdots}$
gosper	$-\frac{7x^6b^6d^6+21a\,b^5d^6x^5+21b^6c\,d^5x^5+35a^2b^4d^6x^4+35a\,b^5c\,d^5x^4+35b^6c^2d^4x^4+35a^3b^3d^6x^3+35a^2b^4c\,d^5x^3+35a\,b^5c^2d^4x^3+35b^6c^3d^3x^3\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -b*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4
*d-b^5*c^5)/d^7/(d*x+c)^6-b^6/d^7/(d*x+c)-5*b^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*
b^2*c^2*d-b^3*c^3)/d^7/(d*x+c)^4-3*b^2/d^7*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^5-5*b^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/d^7/(d*x+c)^3-3*b^5*(a*d-b*c)/d^7/(d*x+c)^2-1/7*(a^6*d^6-6*a^5*b*c*d^5+15*
a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6
)/d^7/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(26) = 52$.
time = 0.30, size = 398, normalized size = 14.21

$$-\frac{7\,b^6d^6x^6+b^6c^6+ab^5c^5d+a^2b^4c^4d^2+a^3b^3c^3d^3+a^4b^2c^2d^4+a^5bcd^5+a^6d^6+21\,(b^6cd^5+ab^5d^6)x^5+35\,(b^6c^2d^4+ab^5cd^5+a^2b^4d^6)x^4+35\,(b^6c^3d^3+ab^5c^2d^4+a^2b^4cd^5+a^3b^3d^6)x^3+21\,(b^6c^4d^2+ab^5c^3d^3+a^2b^4c^2d^4+a^3b^3cd^5+a^4b^2d^6)x^2+7\,(b^6c^5d+ab^5c^4d^2+a^2b^4c^3d^3+a^3b^3c^2d^4+a^4b^2cd^5+a^5bd^6)x}{7\,(d^{14}x^7+7\,cd^{13}x^6+21\,c^2d^{12}x^5+35\,c^3d^{11}x^4+35\,c^4d^{10}x^3+21\,c^5d^9x^2+7\,c^6d^8x+c^7d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
```

```
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(26) = 52.
time = 0.92, size = 398, normalized size = 14.21

$$-\frac{7\,b^6d^6x^6+b^6c^6+ab^5c^5d+a^2b^4c^4d^2+a^3b^3c^3d^3+a^4b^2c^2d^4+a^5bcd^5+a^6d^6+21\,(b^6cd^5+ab^5d^6)x^5+35\,(b^6c^2d^4+ab^5cd^5+a^2b^4d^6)x^4+35\,(b^6c^3d^3+ab^5c^2d^4+a^2b^4cd^5+a^3b^3d^6)x^3+21\,(b^6c^4d^2+ab^5c^3d^3+a^2b^4c^2d^4+a^3b^3cd^5+a^4b^2d^6)x^2+7\,(b^6c^5d+ab^5c^4d^2+a^2b^4c^3d^3+a^3b^3c^2d^4+a^4b^2cd^5+a^5bd^6)x}{7\,(d^{14}x^7+7\,cd^{13}x^6+21\,c^2d^{12}x^5+35\,c^3d^{11}x^4+35\,c^4d^{10}x^3+21\,c^5d^9x^2+7\,c^6d^8x+c^7d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(26) = 52.
time = 3.22, size = 369, normalized size = 13.18

$$-\frac{7\,b^6d^6x^6+21\,b^6cd^5x^5+21\,ab^5d^6x^5+35\,b^6c^2d^4x^4+35\,ab^5cd^5x^4+35\,a^2b^4d^6x^4+35\,b^6c^3d^3x^3+35\,ab^5c^2d^4x^3+35\,a^2b^4cd^5x^3+35\,a^3b^3d^6x^3+21\,b^6c^4d^2x^2+21\,ab^5c^3d^3x^2+21\,a^2b^4c^2d^4x^2+21\,a^3b^3cd^5x^2+21\,a^4b^2d^6x^2+7\,b^6c^5dx+7\,ab^5c^4d^2x+7\,a^2b^4c^3d^3x+7\,a^3b^3c^2d^4x+7\,a^4b^2cd^5x+7\,a^5bd^6x+b^6c^6+ab^5c^5d+a^2b^4c^4d^2+a^3b^3c^3d^3+a^4b^2c^2d^4+a^5bcd^5+a^6d^6}{7\,(dx+c)^7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + 21*b^6*c*d^5*x^5 + 21*a*b^5*d^6*x^5 + 35*b^6*c^2*d^4*
x^4 + 35*a*b^5*c*d^5*x^4 + 35*a^2*b^4*d^6*x^4 + 35*b^6*c^3*d^3*x^3 + 35*a*b
^5*c^2*d^4*x^3 + 35*a^2*b^4*c*d^5*x^3 + 35*a^3*b^3*d^6*x^3 + 21*b^6*c^4*d^2
*x^2 + 21*a*b^5*c^3*d^3*x^2 + 21*a^2*b^4*c^2*d^4*x^2 + 21*a^3*b^3*c*d^5*x^2
```

```
 + 21*a^4*b^2*d^6*x^2 + 7*b^6*c^5*d*x + 7*a*b^5*c^4*d^2*x + 7*a^2*b^4*c^3*d
^3*x + 7*a^3*b^3*c^2*d^4*x + 7*a^4*b^2*c*d^5*x + 7*a^5*b*d^6*x + b^6*c^6 +
a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c
*d^5 + a^6*d^6)/((d*x + c)^7*d^7)
```

Mupad [B]
time = 0.15, size = 378, normalized size = 13.50

$$-\frac{\frac{a^6\,d^6+a^5\,b\,c\,d^5+a^4\,b^2\,c^2\,d^4+a^3\,b^3\,c^3\,d^3+a^2\,b^4\,c^4\,d^2+a\,b^5\,c^5\,d+b^6\,c^6}{7\,d^7}+\frac{b^6\,x^6}{d}+\frac{5\,b^3\,x^3\,\left(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}{d^4}+\frac{b\,x\,\left(a^5\,d^5+a^4\,b\,c\,d^4+a^3\,b^2\,c^2\,d^3+a^2\,b^3\,c^3\,d^2+a\,b^4\,c^4\,d+b^5\,c^5\right)}{d^6}+\frac{3\,b^5\,x^5\,\left(a\,d+b\,c\right)}{d^2}+\frac{3\,b^2\,x^2\,\left(a^4\,d^4+a^3\,b\,c\,d^3+a^2\,b^2\,c^2\,d^2+a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{5\,b^4\,x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^8,x)
```

```
[Out] -((a^6*d^6 + b^6*c^6 + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4
+ a*b^5*c^5*d + a^5*b*c*d^5)/(7*d^7) + (b^6*x^6)/d + (5*b^3*x^3*(a^3*d^3 +
b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2))/d^4 + (b*x*(a^5*d^5 + b^5*c^5 + a^2*b
^3*c^3*d^2 + a^3*b^2*c^2*d^3 + a*b^4*c^4*d + a^4*b*c*d^4))/d^6 + (3*b^5*x^5
*(a*d + b*c))/d^2 + (3*b^2*x^2*(a^4*d^4 + b^4*c^4 + a^2*b^2*c^2*d^2 + a*b^3
*c^3*d + a^3*b*c*d^3))/d^5 + (5*b^4*x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/d^3)
/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^
4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1366 $\int \frac{(a+bx)^5}{(c+dx)^8} dx$

Optimal. Leaf size=58

$$\frac{(a+bx)^6}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}$$

```
[Out] 1/7*(b*x+a)^6/(-a*d+b*c)/(d*x+c)^7+1/42*b*(b*x+a)^6/(-a*d+b*c)^2/(d*x+c)^6
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^6/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^6)/(42*(b*c - a*d)^2
*(c + d*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^8}\,dx = \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7} + \frac{b\int \frac{(a+bx)^5}{(c+dx)^7}\,dx}{7(bc-ad)}$$
$$= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(58) = 116.
time = 0.04, size = 205, normalized size = 3.53

$$-\frac{6a^5d^5+5a^4bd^4(c+7dx)+4a^3b^2d^3(c^2+7cdx+21d^2x^2)+3a^2b^3d^2(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+2ab^4d(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+b^5(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)}{42d^6(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a^5*d^5 + 5*a^4*b*d^4*(c + 7*d*x) + 4*a^3*b^2*d^3*(c^2 + 7*c*d*x +
 21*d^2*x^2) + 3*a^2*b^3*d^2*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3)
+ 2*a*b^4*d*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4)
+ b^5*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 2
1*d^5*x^5))/(d^6*(c + d*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(54) = 108.
time = 0.16, size = 265, normalized size = 4.57

method	result
risch	$\frac{-\frac{b^5x^5}{2d}-\frac{5b^4(2ad+bc)x^4}{6d^2}-\frac{5b^3(3a^2d^2+2abcd+b^2c^2)x^3}{6d^3}-\frac{b^2(4a^3d^3+3a^2bc\,d^2+2a\,b^2c^2d+b^3c^3)x^2}{2d^4}-\frac{b(5a^4d^4+4a^3bc\,d^3+3a^2b^2c^2d^2+2a\,b^3c^3d+b^4c}{6d^5}}{(dx+c)^7}$
default	$-\frac{5b^4(ad-bc)}{3d^6(dx+c)^3}-\frac{5b(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}{6d^6(dx+c)^6}-\frac{2b^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{d^6(dx+c)^5}-\frac{5b^3(a^2d^2-2abcd+b^2}{2d^6(dx+c)^4}$
norman	$\frac{-\frac{b^5x^5}{2d}-\frac{5(2a\,b^4d^2+b^5cd)x^4}{6d^3}-\frac{5(3a^2b^3d^3+2a\,b^4c\,d^2+b^5c^2d)x^3}{6d^4}-\frac{(4a^3b^2d^4+3a^2b^3c\,d^3+2a\,b^4c^2d^2+b^5c^3d)x^2}{2d^5}-\frac{(5a^4b\,d^5+4a^3b^2c\,d^4+3a^2b^3c^2d^3}{6d^6}}{(dx+c)^7}$
gosper	$-\frac{21b^5x^5d^5+70a\,b^4d^5x^4+35b^5c\,d^4x^4+105a^2b^3d^5x^3+70a\,b^4c\,d^4x^3+35b^5c^2d^3x^3+84a^3b^2d^5x^2+63a^2b^3c\,d^4x^2+42a\,b^4c^2d^3x^2+21b^5c^3d^2}{42d^6(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -5/3*b^4*(a*d-b*c)/d^6/(d*x+c)^3-5/6*b/d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^6-2*b^2/d^6*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^5-5/2*b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^6/(d
```

```
*x+c)^4-1/2*b^5/d^6/(d*x+c)^2-1/7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3
-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^6/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(54) = 108$.
time = 0.30, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,(b^5cd^4+2\,ab^4d^5)x^4+35\,(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5)x^3+21\,(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5)x^2+7\,(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5)x}{42\,(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(54) = 108$.
time = 0.95, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,(b^5cd^4+2\,ab^4d^5)x^4+35\,(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5)x^3+21\,(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5)x^2+7\,(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5)x}{42\,(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(54) = 108.
time = 1.85, size = 271, normalized size = 4.67

$$-\frac{21\,b^5d^5x^5+35\,b^5cd^4x^4+70\,ab^4d^5x^4+35\,b^5c^2d^3x^3+70\,ab^4cd^4x^3+105\,a^2b^3d^5x^3+21\,b^5c^3d^2x^2+42\,ab^4c^2d^3x^2+63\,a^2b^3cd^4x^2+84\,a^3b^2d^5x^2+7\,b^5c^4dx+14\,ab^4c^3d^2x+21\,a^2b^3c^2d^3x+28\,a^3b^2cd^4x+35\,a^4bd^5x+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5}{42\,(dx+c)^7d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + 35*b^5*c*d^4*x^4 + 70*a*b^4*d^5*x^4 + 35*b^5*c^2*d^
3*x^3 + 70*a*b^4*c*d^4*x^3 + 105*a^2*b^3*d^5*x^3 + 21*b^5*c^3*d^2*x^2 + 42*
a*b^4*c^2*d^3*x^2 + 63*a^2*b^3*c*d^4*x^2 + 84*a^3*b^2*d^5*x^2 + 7*b^5*c^4*d
*x + 14*a*b^4*c^3*d^2*x + 21*a^2*b^3*c^2*d^3*x + 28*a^3*b^2*c*d^4*x + 35*a^
4*b*d^5*x + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3*b^2*c^2*d^3
 + 5*a^4*b*c*d^4 + 6*a^5*d^5)/((d*x + c)^7*d^6)
```

Mupad [B]
time = 0.28, size = 39, normalized size = 0.67

$$\frac{(a+b\,x)^6\,(7\,b\,c-6\,a\,d+b\,d\,x)}{42\,(a\,d-b\,c)^2\,(c+d\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^8,x)
```

```
[Out] ((a + b*x)^6*(7*b*c - 6*a*d + b*d*x))/(42*(a*d - b*c)^2*(c + d*x)^7)
```

3.1367 $\int \frac{(a+bx)^4}{(c+dx)^8} dx$

Optimal. Leaf size=89

$$\frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}$$

```
[Out] 1/7*(b*x+a)^5/(-a*d+b*c)/(d*x+c)^7+1/21*b*(b*x+a)^5/(-a*d+b*c)^2/(d*x+c)^6+
1/105*b^2*(b*x+a)^5/(-a*d+b*c)^3/(d*x+c)^5
```

Rubi [A]

time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {47, 37}

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^5/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^5)/(21*(b*c - a*d)^2
*(c + d*x)^6) + (b^2*(a + b*x)^5)/(105*(b*c - a*d)^3*(c + d*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^4}{(c+dx)^8}\,dx = \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7} + \frac{(2b)\int \frac{(a+bx)^4}{(c+dx)^7}\,dx}{7(bc-ad)}$$
$$= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6} + \frac{b^2\int \frac{(a+bx)^4}{(c+dx)^6}\,dx}{21(bc-ad)^2}$$
$$= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7} + \frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6} + \frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}$$

Mathematica [A]
time = 0.03, size = 144, normalized size = 1.62

$$-\frac{15a^4d^4+10a^3bd^3(c+7dx)+6a^2b^2d^2(c^2+7cdx+21d^2x^2)+3ab^3d(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+b^4(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)}{105d^5(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 6*a^2*b^2*d^2*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 3*a*b^3*d*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) +
 b^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4))/(d^5*(
c + d*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(83) = 166.
time = 0.14, size = 186, normalized size = 2.09

method	result
risch	$\frac{-\frac{b^4x^4}{3d}-\frac{b^3(3ad+bc)x^3}{3d^2}-\frac{b^2\left(6a^2d^2+3abcd+b^2c^2\right)x^2}{5d^3}-\frac{b\left(10a^3d^3+6a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)x}{15d^4}-\frac{15a^4d^4+10a^3bc\,d^3+6a^2b^2c^2d^2+3a\,b^3c^3d+b^4c^4}{105d^5}}{(dx+c)^7}$
gosper	$-\frac{35d^4x^4b^4+105a\,b^3d^4x^3+35b^4c\,d^3x^3+126a^2b^2d^4x^2+63a\,b^3c\,d^3x^2+21b^4c^2d^2x^2+70a^3b\,d^4x+42a^2b^2c\,d^3x+21a\,b^3c^2d^2x+7b^4c^3dx+15}{105d^5(dx+c)^7}$
default	$-\frac{b^4}{3d^5(dx+c)^3}-\frac{2b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{3d^5(dx+c)^6}-\frac{b^3(ad-bc)}{d^5(dx+c)^4}-\frac{6b^2\left(a^2d^2-2abcd+b^2c^2\right)}{5d^5(dx+c)^5}-\frac{a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4}{7d^5(dx+c)^7}$
norman	$\frac{-\frac{b^4x^4}{3d}-\frac{\left(3a\,b^3d^3+b^4c\,d^2\right)x^3}{3d^4}-\frac{\left(6b^2a^2d^4+3a\,b^3c\,d^3+b^4c^2d^2\right)x^2}{5d^5}-\frac{\left(10a^3b\,d^5+6a^2b^2c\,d^4+3a\,b^3c^2d^3+b^4c^3d^2\right)x}{15d^6}-\frac{15a^4d^6+10a^3bc\,d^5+6a^2b^2c^2d^4+}{105d^7}}{(dx+c)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^4/d^5/(d*x+c)^3-2/3*b/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/(d*x+c)^6-b^3*(a*d-b*c)/d^5/(d*x+c)^4-6/5*b^2/d^5*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/(d*x+c)^5-1/7*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d
+b^4*c^4)/d^5/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.
time = 0.31, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\,(b^4cd^3+3\,ab^3d^4)x^3+21\,(b^4c^2d^2+3\,ab^3cd^3+6\,a^2b^2d^4)x^2+7\,(b^4c^3d+3\,ab^3c^2d^2+6\,a^2b^2cd^3+10\,a^3bd^4)x}{105\,(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+7\,c^6d^6x+c^7d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.
time = 0.78, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4+35\,(b^4cd^3+3\,ab^3d^4)x^3+21\,(b^4c^2d^2+3\,ab^3cd^3+6\,a^2b^2d^4)x^2+7\,(b^4c^3d+3\,ab^3c^2d^2+6\,a^2b^2cd^3+10\,a^3bd^4)x}{105\,(d^{12}x^7+7\,cd^{11}x^6+21\,c^2d^{10}x^5+35\,c^3d^9x^4+35\,c^4d^8x^3+21\,c^5d^7x^2+7\,c^6d^6x+c^7d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(73) = 146$.
time = 111.24, size = 267, normalized size = 3.00

$$\frac{-15a^4d^4-10a^3bcd^3-6a^2b^2c^2d^2-3ab^3c^3d-b^4c^4-35b^4d^4x^4+x^3(-105ab^3d^4-35b^4cd^3)+x^2(-126a^2b^2d^4-63ab^3cd^3-21b^4c^2d^2)+x(-70a^3bd^4-42a^2b^2cd^3-21ab^3c^2d^2-7b^4c^3d)}{105c^7d^5+735c^6d^6x+2205c^5d^7x^2+3675c^4d^8x^3+3675c^3d^9x^4+2205c^2d^{10}x^5+735cd^{11}x^6+105d^{12}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**8,x)
```

```
[Out] (-15*a**4*d**4 - 10*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 - 3*a*b**3*c**3*d
 - b**4*c**4 - 35*b**4*d**4*x**4 + x**3*(-105*a*b**3*d**4 - 35*b**4*c*d**3)
 + x**2*(-126*a**2*b**2*d**4 - 63*a*b**3*c*d**3 - 21*b**4*c**2*d**2) + x*(-
70*a**3*b*d**4 - 42*a**2*b**2*c*d**3 - 21*a*b**3*c**2*d**2 - 7*b**4*c**3*d)
```

```
)/(105*c**7*d**5 + 735*c**6*d**6*x + 2205*c**5*d**7*x**2 + 3675*c**4*d**8*x
**3 + 3675*c**3*d**9*x**4 + 2205*c**2*d**10*x**5 + 735*c*d**11*x**6 + 105*d
**12*x**7)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(83) = 166.
time = 1.33, size = 184, normalized size = 2.07

$$-\frac{35\,b^4d^4x^4+35\,b^4cd^3x^3+105\,ab^3d^4x^3+21\,b^4c^2d^2x^2+63\,ab^3cd^3x^2+126\,a^2b^2d^4x^2+7\,b^4c^3dx+21\,ab^3c^2d^2x+42\,a^2b^2cd^3x+70\,a^3bd^4x+b^4c^4+3\,ab^3c^3d+6\,a^2b^2c^2d^2+10\,a^3bcd^3+15\,a^4d^4}{105\,(dx+c)^7d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + 35*b^4*c*d^3*x^3 + 105*a*b^3*d^4*x^3 + 21*b^4*c^2*
d^2*x^2 + 63*a*b^3*c*d^3*x^2 + 126*a^2*b^2*d^4*x^2 + 7*b^4*c^3*d*x + 21*a*b
^3*c^2*d^2*x + 42*a^2*b^2*c*d^3*x + 70*a^3*b*d^4*x + b^4*c^4 + 3*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + 15*a^4*d^4)/((d*x + c)^7*d^5)
```

Mupad [B]
time = 0.11, size = 237, normalized size = 2.66

$$-\frac{\frac{15\,a^4\,d^4+10\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+3\,a\,b^3\,c^3\,d+b^4\,c^4}{105\,d^5}+\frac{b^4\,x^4}{3\,d}+\frac{b^3\,x^3\,\left(3\,a\,d+b\,c\right)}{3\,d^2}+\frac{b\,x\,\left(10\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{15\,d^4}+\frac{b^2\,x^2\,\left(6\,a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5\,d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^8,x)
```

```
[Out] -((15*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 3*a*b^3*c^3*d + 10*a^3*b*c*d^
3)/(105*d^5) + (b^4*x^4)/(3*d) + (b^3*x^3*(3*a*d + b*c))/(3*d^2) + (b*x*(10
*a^3*d^3 + b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(15*d^4) + (b^2*x^2*(6
*a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/(5*d^3))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21
*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x
)
```

3.1368 $\int \frac{(a+bx)^3}{(c+dx)^8} \, dx$

Optimal. Leaf size=92

$$\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b^3}{4d^4(c+dx)^4}$$

```
[Out] 1/7*(-a*d+b*c)^3/d^4/(d*x+c)^7-1/2*b*(-a*d+b*c)^2/d^4/(d*x+c)^6+3/5*b^2*(-a
*d+b*c)/d^4/(d*x+c)^5-1/4*b^3/d^4/(d*x+c)^4
```

Rubi [A]
time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b^3}{4d^4(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^3/(7*d^4*(c + d*x)^7) - (b*(b*c - a*d)^2)/(2*d^4*(c + d*x)^6) +
 (3*b^2*(b*c - a*d))/(5*d^4*(c + d*x)^5) - b^3/(4*d^4*(c + d*x)^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^8}+\frac{3b(bc-ad)^2}{d^3(c+dx)^7}-\frac{3b^2(bc-ad)}{d^3(c+dx)^6}+\frac{b^3}{d^3(c+dx)^5}\right)dx\\ &= \frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b^3}{4d^4(c+dx)^4}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 94, normalized size = 1.02

$$-\frac{20a^3d^3+10a^2bd^2(c+7dx)+4ab^2d(c^2+7cdx+21d^2x^2)+b^3(c^3+7c^2dx+21cd^2x^2+35d^3x^3)}{140d^4(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] -1/140*(20*a^3*d^3 + 10*a^2*b*d^2*(c + 7*d*x) + 4*a*b^2*d*(c^2 + 7*c*d*x +
21*d^2*x^2) + b^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3))/(d^4*(c +
d*x)^7)
```

Maple [A]
time = 0.14, size = 122, normalized size = 1.33

method	result	size
risch	$\frac{-\frac{b^{3} x^{3}}{4 d}-\frac{3 b^{2} (4 a d+b c) x^{2}}{20 d^{2}}-\frac{b \left(10 a^{2} d^{2}+4 a b c d+b^{2} c^{2}\right) x}{20 d^{3}}-\frac{20 a^{3} d^{3}+10 a^{2} b c\, d^{2}+4 a\, b^{2} c^{2} d+b^{3} c^{3}}{140 d^{4}}}{(d x+c)^{7}}$	110
gosper	$-\frac{35 b^{3} x^{3} d^{3}+84 a\, b^{2} d^{3} x^{2}+21 b^{3} c\, d^{2} x^{2}+70 a^{2} b\, d^{3} x+28 a\, b^{2} c\, d^{2} x+7 b^{3} c^{2} d x+20 a^{3} d^{3}+10 a^{2} b c\, d^{2}+4 a\, b^{2} c^{2} d+b^{3} c^{3}}{140 d^{4} (d x+c)^{7}}$	115
default	$-\frac{b \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}{2 d^{4} (d x+c)^{6}}-\frac{3 b^{2} (a d-b c)}{5 d^{4} (d x+c)^{5}}-\frac{b^{3}}{4 d^{4} (d x+c)^{4}}-\frac{a^{3} d^{3}-3 a^{2} b c\, d^{2}+3 a\, b^{2} c^{2} d-b^{3} c^{3}}{7 d^{4} (d x+c)^{7}}$	122
norman	$\frac{-\frac{b^{3} x^{3}}{4 d}-\frac{3 \left(4 a\, b^{2} d^{4}+b^{3} c\, d^{3}\right) x^{2}}{20 d^{5}}-\frac{\left(10 a^{2} b\, d^{5}+4 a\, b^{2} c\, d^{4}+b^{3} c^{2} d^{3}\right) x}{20 d^{6}}-\frac{20 a^{3} d^{6}+10 a^{2} b c\, d^{5}+4 a\, b^{2} c^{2} d^{4}+b^{3} c^{3} d^{3}}{140 d^{7}}}{(d x+c)^{7}}$	129

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/d^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^6-3/5*b^2/d^4*(a*d-b*c)/(d*x
+c)^5-1/4*b^3/d^4/(d*x+c)^4-1/7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)/d^4/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(84) = 168.
time = 0.28, size = 182, normalized size = 1.98

$$-\frac{35\, b^3 d^3 x^3 + b^3 c^3 + 4\, a b^2 c^2 d + 10\, a^2 b c d^2 + 20\, a^3 d^3 + 21\, (b^3 c d^2 + 4\, a b^2 d^3) x^2 + 7\, (b^3 c^2 d + 4\, a b^2 c d^2 + 10\, a^2 b d^3) x}{140\, (d^{11} x^7 + 7\, c d^{10} x^6 + 21\, c^2 d^9 x^5 + 35\, c^3 d^8 x^4 + 35\, c^4 d^7 x^3 + 21\, c^5 d^6 x^2 + 7\, c^6 d^5 x + c^7 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(84) = 168.
time = 0.80, size = 182, normalized size = 1.98

$$-\frac{35\, b^3 d^3 x^3 + b^3 c^3 + 4\, a b^2 c^2 d + 10\, a^2 b c d^2 + 20\, a^3 d^3 + 21\, (b^3 c d^2 + 4\, a b^2 d^3) x^2 + 7\, (b^3 c^2 d + 4\, a b^2 c d^2 + 10\, a^2 b d^3) x}{140\, (d^{11} x^7 + 7\, c d^{10} x^6 + 21\, c^2 d^9 x^5 + 35\, c^3 d^8 x^4 + 35\, c^4 d^7 x^3 + 21\, c^5 d^6 x^2 + 7\, c^6 d^5 x + c^7 d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(80) = 160$.

time = 1.94, size = 196, normalized size = 2.13

$$\frac{-20a^3d^3 - 10a^2bcd^2 - 4ab^2c^2d - b^3c^3 - 35b^3d^3x^3 + x^2(-84ab^2d^3 - 21b^3cd^2) + x(-70a^2bd^3 - 28ab^2cd^2 - 7b^3c^2d)}{140c^7d^4 + 980c^6d^5x + 2940c^5d^6x^2 + 4900c^4d^7x^3 + 4900c^3d^8x^4 + 2940c^2d^9x^5 + 980cd^{10}x^6 + 140d^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] (-20*a**3*d**3 - 10*a**2*b*c*d**2 - 4*a*b**2*c**2*d - b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-84*a*b**2*d**3 - 21*b**3*c*d**2) + x*(-70*a**2*b*d**3 - 2
8*a*b**2*c*d**2 - 7*b**3*c**2*d))/(140*c**7*d**4 + 980*c**6*d**5*x + 2940*c
**5*d**6*x**2 + 4900*c**4*d**7*x**3 + 4900*c**3*d**8*x**4 + 2940*c**2*d**9*
x**5 + 980*c*d**10*x**6 + 140*d**11*x**7)
```

Giac [A]

time = 0.85, size = 114, normalized size = 1.24

$$-\frac{35\,b^3d^3x^3 + 21\,b^3cd^2x^2 + 84\,ab^2d^3x^2 + 7\,b^3c^2dx + 28\,ab^2cd^2x + 70\,a^2bd^3x + b^3c^3 + 4\,ab^2c^2d + 10\,a^2bcd^2 + 20\,a^3d^3}{140\,(dx+c)^7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 21*b^3*c*d^2*x^2 + 84*a*b^2*d^3*x^2 + 7*b^3*c^2*d*
x + 28*a*b^2*c*d^2*x + 70*a^2*b*d^3*x + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*
c*d^2 + 20*a^3*d^3)/((d*x + c)^7*d^4)
```

Mupad [B]

time = 0.10, size = 176, normalized size = 1.91

$$-\frac{\frac{20\,a^3\,d^3+10\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d+b^3\,c^3}{140\,d^4}+\frac{b^3\,x^3}{4\,d}+\frac{b\,x\,\left(10\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{20\,d^3}+\frac{3\,b^2\,x^2\,\left(4\,a\,d+b\,c\right)}{20\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^8,x)
```

```
[Out] -((20*a^3*d^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2)/(140*d^4) + (b^3*
x^3)/(4*d) + (b*x*(10*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/(20*d^3) + (3*b^2*x^2
*(4*a*d + b*c))/(20*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 3
5*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1369 $\int \frac{(a+bx)^2}{(c+dx)^8} dx$

Optimal. Leaf size=65

$$-\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5}$$

```
[Out] -1/7*(-a*d+b*c)^2/d^3/(d*x+c)^7+1/3*b*(-a*d+b*c)/d^3/(d*x+c)^6-1/5*b^2/d^3/
(d*x+c)^5
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -1/7*(b*c - a*d)^2/(d^3*(c + d*x)^7) + (b*(b*c - a*d))/(3*d^3*(c + d*x)^6)
- b^2/(5*d^3*(c + d*x)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^8}-\frac{2b(bc-ad)}{d^2(c+dx)^7}+\frac{b^2}{d^2(c+dx)^6}\right)dx\\ &= -\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.85

$$-\frac{15a^2d^2+5abd(c+7dx)+b^2(c^2+7cdx+21d^2x^2)}{105d^3(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(c^2 + 7*c*d*x + 21*d^2*x^2)
)/(d^3*(c + d*x)^7)
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.09

method	result	size
gosper	$-\frac{21b^2x^2d^2+35ab\,d^2x+7b^2cdx+15a^2d^2+5abcd+b^2c^2}{105d^3(dx+c)^7}$	62
risch	$\frac{-\frac{b^2x^2}{5d}-\frac{b(5ad+bc)x}{15d^2}-\frac{15a^2d^2+5abcd+b^2c^2}{105d^3}}{(dx+c)^7}$	63
default	$-\frac{b(ad-bc)}{3d^3(dx+c)^6}-\frac{b^2}{5d^3(dx+c)^5}-\frac{a^2d^2-2abcd+b^2c^2}{7d^3(dx+c)^7}$	71
norman	$\frac{-\frac{b^2x^2}{5d}-\frac{\left(5ab\,d^5+b^2c\,d^4\right)x}{15d^6}-\frac{15a^2d^6+5abc\,d^5+b^2c^2d^4}{105d^7}}{(dx+c)^7}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b/d^3*(a*d-b*c)/(d*x+c)^6-1/5*b^2/d^3/(d*x+c)^5-1/7*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/d^3/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(59) = 118.
time = 0.30, size = 131, normalized size = 2.02

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\,(b^2cd+5\,abd^2)x}{105\,(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(59) = 118.
time = 0.66, size = 131, normalized size = 2.02

$$-\frac{21\,b^2d^2x^2+b^2c^2+5\,abcd+15\,a^2d^2+7\,(b^2cd+5\,abd^2)x}{105\,(d^{10}x^7+7\,cd^9x^6+21\,c^2d^8x^5+35\,c^3d^7x^4+35\,c^4d^6x^3+21\,c^5d^5x^2+7\,c^6d^4x+c^7d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(54) = 108$.
time = 0.81, size = 139, normalized size = 2.14

$$\frac{-15a^2d^2 - 5abcd - b^2c^2 - 21b^2d^2x^2 + x(-35abd^2 - 7b^2cd)}{105c^7d^3 + 735c^6d^4x + 2205c^5d^5x^2 + 3675c^4d^6x^3 + 3675c^3d^7x^4 + 2205c^2d^8x^5 + 735cd^9x^6 + 105d^{10}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] (-15*a**2*d**2 - 5*a*b*c*d - b**2*c**2 - 21*b**2*d**2*x**2 + x*(-35*a*b*d**
2 - 7*b**2*c*d))/(105*c**7*d**3 + 735*c**6*d**4*x + 2205*c**5*d**5*x**2 + 3
675*c**4*d**6*x**3 + 3675*c**3*d**7*x**4 + 2205*c**2*d**8*x**5 + 735*c*d**9
*x**6 + 105*d**10*x**7)
```

Giac [A]
time = 1.68, size = 61, normalized size = 0.94

$$-\frac{21\,b^2d^2x^2 + 7\,b^2cdx + 35\,abd^2x + b^2c^2 + 5\,abcd + 15\,a^2d^2}{105\,(dx+c)^7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 7*b^2*c*d*x + 35*a*b*d^2*x + b^2*c^2 + 5*a*b*c*d +
 15*a^2*d^2)/((d*x + c)^7*d^3)
```

Mupad [B]
time = 0.09, size = 129, normalized size = 1.98

$$-\frac{\frac{15\,a^2\,d^2+5\,a\,b\,c\,d+b^2\,c^2}{105\,d^3}+\frac{b^2\,x^2}{5\,d}+\frac{b\,x\,(5\,a\,d+b\,c)}{15\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^8,x)
```

```
[Out] -((15*a^2*d^2 + b^2*c^2 + 5*a*b*c*d)/(105*d^3) + (b^2*x^2)/(5*d) + (b*x*(5*
a*d + b*c))/(15*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^
4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1370 $\int \frac{a+bx}{(c+dx)^8} \, dx$

Optimal. Leaf size=38

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

```
[Out] 1/7*(-a*d+b*c)/d^2/(d*x+c)^7-1/6*b/d^2/(d*x+c)^6
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)/(7*d^2*(c + d*x)^7) - b/(6*d^2*(c + d*x)^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{(c+dx)^8} \, dx &= \int \left(\frac{-bc+ad}{d(c+dx)^8}+\frac{b}{d(c+dx)^7}\right) dx \\ &= \frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{6ad+b(c+7dx)}{42d^2(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a*d + b*(c + 7*d*x))/(d^2*(c + d*x)^7)
```

Maple [A]
time = 0.12, size = 35, normalized size = 0.92

method	result	size
gosper	$-\frac{7bdx+6ad+bc}{42d^2(dx+c)^7}$	26
risch	$\frac{-\frac{bx}{6d}-\frac{6ad+bc}{42d^2}}{(dx+c)^7}$	30
default	$-\frac{b}{6d^2(dx+c)^6}-\frac{ad-bc}{7d^2(dx+c)^7}$	35
norman	$\frac{-\frac{bx}{6d}-\frac{6a\,d^6+bc\,d^5}{42d^7}}{(dx+c)^7}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*b/d^2/(d*x+c)^6-1/7*(a*d-b*c)/d^2/(d*x+c)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(34) = 68.
time = 0.29, size = 94, normalized size = 2.47

$$-\frac{7\,bdx+bc+6\,ad}{42\,(d^9x^7+7\,cd^8x^6+21\,c^2d^7x^5+35\,c^3d^6x^4+35\,c^4d^5x^3+21\,c^5d^4x^2+7\,c^6d^3x+c^7d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(34) = 68.
time = 0.53, size = 94, normalized size = 2.47

$$-\frac{7\,bdx+bc+6\,ad}{42\,(d^9x^7+7\,cd^8x^6+21\,c^2d^7x^5+35\,c^3d^6x^4+35\,c^4d^5x^3+21\,c^5d^4x^2+7\,c^6d^3x+c^7d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.
time = 0.51, size = 100, normalized size = 2.63

$$\frac{-6ad - bc - 7bdx}{42c^7d^2 + 294c^6d^3x + 882c^5d^4x^2 + 1470c^4d^5x^3 + 1470c^3d^6x^4 + 882c^2d^7x^5 + 294cd^8x^6 + 42d^9x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**8,x)
```

```
[Out] (-6*a*d - b*c - 7*b*d*x)/(42*c**7*d**2 + 294*c**6*d**3*x + 882*c**5*d**4*x*
*2 + 1470*c**4*d**5*x**3 + 1470*c**3*d**6*x**4 + 882*c**2*d**7*x**5 + 294*c
*d**8*x**6 + 42*d**9*x**7)
```

Giac [A]
time = 1.71, size = 25, normalized size = 0.66

$$-\frac{7\,bdx + bc + 6\,ad}{42\,(dx + c)^7 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/((d*x + c)^7*d^2)
```

Mupad [B]
time = 0.23, size = 96, normalized size = 2.53

$$-\frac{\frac{6\,a\,d+b\,c}{42\,d^2} + \frac{b\,x}{6\,d}}{c^7 + 7\,c^6\,d\,x + 21\,c^5\,d^2\,x^2 + 35\,c^4\,d^3\,x^3 + 35\,c^3\,d^4\,x^4 + 21\,c^2\,d^5\,x^5 + 7\,c\,d^6\,x^6 + d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^8,x)
```

```
[Out] -((6*a*d + b*c)/(42*d^2) + (b*x)/(6*d))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c
^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

3.1371 $\int \frac{1}{(c+dx)^8} dx$

Optimal. Leaf size=14

$$-\frac{1}{7d(c+dx)^7}$$

```
[Out] -1/7/d/(d*x+c)^7
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-8),x]
```

```
[Out] -1/7*1/(d*(c + d*x)^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^8} dx = -\frac{1}{7d(c+dx)^7}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-8),x]
```

```
[Out] -1/7*1/(d*(c + d*x)^7)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{1}{7d(dx+c)^7}$	13
default	$-\frac{1}{7d(dx+c)^7}$	13
norman	$-\frac{1}{7d(dx+c)^7}$	13
risch	$-\frac{1}{7d(dx+c)^7}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/d/(d*x+c)^7
```

Maxima [A]
time = 0.31, size = 12, normalized size = 0.86

$$-\frac{1}{7\,(dx+c)^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(12) = 24.
time = 0.45, size = 79, normalized size = 5.64

$$-\frac{1}{7\,(d^8x^7+7\,cd^7x^6+21\,c^2d^6x^5+35\,c^3d^5x^4+35\,c^4d^4x^3+21\,c^5d^3x^2+7\,c^6d^2x+c^7d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(d^8*x^7 + 7*c*d^7*x^6 + 21*c^2*d^6*x^5 + 35*c^3*d^5*x^4 + 35*c^4*d^4*
x^3 + 21*c^5*d^3*x^2 + 7*c^6*d^2*x + c^7*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(12) = 24.
time = 0.22, size = 85, normalized size = 6.07

$$-\frac{1}{7c^7d+49c^6d^2x+147c^5d^3x^2+245c^4d^4x^3+245c^3d^5x^4+147c^2d^6x^5+49cd^7x^6+7d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**8,x)
```

```
[Out] -1/(7*c**7*d + 49*c**6*d**2*x + 147*c**5*d**3*x**2 + 245*c**4*d**4*x**3 + 2
45*c**3*d**5*x**4 + 147*c**2*d**6*x**5 + 49*c*d**7*x**6 + 7*d**8*x**7)
```

Giac [A]
time = 1.00, size = 12, normalized size = 0.86

$$-\frac{1}{7\,(dx+c)^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

Mupad [B]
time = 0.22, size = 81, normalized size = 5.79

$$-\frac{1}{7\,c^7\,d+49\,c^6\,d^2\,x+147\,c^5\,d^3\,x^2+245\,c^4\,d^4\,x^3+245\,c^3\,d^5\,x^4+147\,c^2\,d^6\,x^5+49\,c\,d^7\,x^6+7\,d^8\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^8,x)
```

```
[Out] -1/(7*c^7*d + 7*d^8*x^7 + 49*c^6*d^2*x + 49*c*d^7*x^6 + 147*c^5*d^3*x^2 + 2
45*c^4*d^4*x^3 + 245*c^3*d^5*x^4 + 147*c^2*d^6*x^5)
```

3.1372 $\int \frac{1}{(a+bx)(c+dx)^8} \, dx$

Optimal. Leaf size=202

$$\frac{1}{7(bc-ad)(c+dx)^7}+\frac{b}{6(bc-ad)^2(c+dx)^6}+\frac{b^2}{5(bc-ad)^3(c+dx)^5}+\frac{b^3}{4(bc-ad)^4(c+dx)^4}+\frac{b^4}{3(bc-ad)^5(c+$$

```
[Out] 1/7/(-a*d+b*c)/(d*x+c)^7+1/6*b/(-a*d+b*c)^2/(d*x+c)^6+1/5*b^2/(-a*d+b*c)^3/
(d*x+c)^5+1/4*b^3/(-a*d+b*c)^4/(d*x+c)^4+1/3*b^4/(-a*d+b*c)^5/(d*x+c)^3+1/2
*b^5/(-a*d+b*c)^6/(d*x+c)^2+b^6/(-a*d+b*c)^7/(d*x+c)+b^7*ln(b*x+a)/(-a*d+b*
c)^8-b^7*ln(d*x+c)/(-a*d+b*c)^8
```

Rubi [A]
time = 0.12, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{46\}$

$$\frac{b^7\log(a+bx)}{(bc-ad)^8}-\frac{b^7\log(c+dx)}{(bc-ad)^8}+\frac{b^6}{(c+dx)(bc-ad)^7}+\frac{b^5}{2(c+dx)^2(bc-ad)^6}+\frac{b^4}{3(c+dx)^3(bc-ad)^5}+\frac{b^3}{4(c+dx)^4(bc-ad)^4}+\frac{b^2}{5(c+dx)^5(bc-ad)^3}+\frac{b}{6(c+dx)^6(bc-ad)^2}+\frac{1}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] 1/(7*(b*c - a*d)*(c + d*x)^7) + b/(6*(b*c - a*d)^2*(c + d*x)^6) + b^2/(5*(b
*c - a*d)^3*(c + d*x)^5) + b^3/(4*(b*c - a*d)^4*(c + d*x)^4) + b^4/(3*(b*c
- a*d)^5*(c + d*x)^3) + b^5/(2*(b*c - a*d)^6*(c + d*x)^2) + b^6/((b*c - a*d
)^7*(c + d*x)) + (b^7*Log[a + b*x])/(b*c - a*d)^8 - (b^7*Log[c + d*x])/(b*c
 - a*d)^8
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^8} \, dx &= \int \left(\frac{b^8}{(bc-ad)^8(a+bx)}-\frac{d}{(bc-ad)(c+dx)^8}-\frac{bd}{(bc-ad)^2(c+dx)^7}-\frac{b}{(bc-ad)}\right. \\ &= \frac{1}{7(bc-ad)(c+dx)^7}+\frac{b}{6(bc-ad)^2(c+dx)^6}+\frac{b^2}{5(bc-ad)^3(c+dx)^5}+\frac{}{4(bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 196, normalized size = 0.97

$$\frac{60(bc-ad)^7+70b(bc-ad)^6(c+dx)+84b^2(bc-ad)^5(c+dx)^2+105b^3(bc-ad)^4(c+dx)^3+140b^4(bc-ad)^3(c+dx)^4+210b^5(bc-ad)^2(c+dx)^5+420b^6(bc-ad)(c+dx)^6+420b^7(c+dx)^7\log(a+bx)-420b^7(c+dx)^7\log(c+dx)}{420(bc-ad)^8(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] (60*(b*c - a*d)^7 + 70*b*(b*c - a*d)^6*(c + d*x) + 84*b^2*(b*c - a*d)^5*(c
+ d*x)^2 + 105*b^3*(b*c - a*d)^4*(c + d*x)^3 + 140*b^4*(b*c - a*d)^3*(c + d
*x)^4 + 210*b^5*(b*c - a*d)^2*(c + d*x)^5 + 420*b^6*(b*c - a*d)*(c + d*x)^6
 + 420*b^7*(c + d*x)^7*Log[a + b*x] - 420*b^7*(c + d*x)^7*Log[c + d*x])/(42
0*(b*c - a*d)^8*(c + d*x)^7)
```

Maple [A]
time = 0.20, size = 192, normalized size = 0.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/(a*d-b*c)/(d*x+c)^7-1/5*b^2/(a*d-b*c)^3/(d*x+c)^5-1/3*b^4/(a*d-b*c)^5/
(d*x+c)^3-b^6/(a*d-b*c)^7/(d*x+c)+1/4*b^3/(a*d-b*c)^4/(d*x+c)^4+1/2*b^5/(a*
d-b*c)^6/(d*x+c)^2-b^7/(a*d-b*c)^8*ln(d*x+c)+1/6*b/(a*d-b*c)^2/(d*x+c)^6+b^
7/(a*d-b*c)^8*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. $2(190) = 380$.
time = 0.44, size = 1418, normalized size = 7.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^7*log(b*x + a)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8) - b^7*log(d*x + c)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8) + 1/420*(420*b^6*d^6*x^6
+ 1089*b^6*c^6 - 1851*a*b^5*c^5*d + 2559*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3
*d^3 + 1334*a^4*b^2*c^2*d^4 - 430*a^5*b*c*d^5 + 60*a^6*d^6 + 210*(13*b^6*c*
d^5 - a*b^5*d^6)*x^5 + 70*(107*b^6*c^2*d^4 - 19*a*b^5*c*d^5 + 2*a^2*b^4*d^6
)*x^4 + 35*(319*b^6*c^3*d^3 - 101*a*b^5*c^2*d^4 + 25*a^2*b^4*c*d^5 - 3*a^3*
b^3*d^6)*x^3 + 21*(459*b^6*c^4*d^2 - 241*a*b^5*c^3*d^3 + 109*a^2*b^4*c^2*d^
4 - 31*a^3*b^3*c*d^5 + 4*a^4*b^2*d^6)*x^2 + 7*(669*b^6*c^5*d - 591*a*b^5*c^
4*d^2 + 459*a^2*b^4*c^3*d^3 - 241*a^3*b^3*c^2*d^4 + 74*a^4*b^2*c*d^5 - 10*a
^5*b*d^6)*x)/(b^7*c^14 - 7*a*b^6*c^13*d + 21*a^2*b^5*c^12*d^2 - 35*a^3*b^4*
c^11*d^3 + 35*a^4*b^3*c^10*d^4 - 21*a^5*b^2*c^9*d^5 + 7*a^6*b*c^8*d^6 - a^7
*c^7*d^7 + (b^7*c^7*d^7 - 7*a*b^6*c^6*d^8 + 21*a^2*b^5*c^5*d^9 - 35*a^3*b^4
*c^4*d^10 + 35*a^4*b^3*c^3*d^11 - 21*a^5*b^2*c^2*d^12 + 7*a^6*b*c*d^13 - a^
7*d^14)*x^7 + 7*(b^7*c^8*d^6 - 7*a*b^6*c^7*d^7 + 21*a^2*b^5*c^6*d^8 - 35*a^
```

```
3*b^4*c^5*d^9 + 35*a^4*b^3*c^4*d^10 - 21*a^5*b^2*c^3*d^11 + 7*a^6*b*c^2*d^1
2 - a^7*c*d^13)*x^6 + 21*(b^7*c^9*d^5 - 7*a*b^6*c^8*d^6 + 21*a^2*b^5*c^7*d^
7 - 35*a^3*b^4*c^6*d^8 + 35*a^4*b^3*c^5*d^9 - 21*a^5*b^2*c^4*d^10 + 7*a^6*b
*c^3*d^11 - a^7*c^2*d^12)*x^5 + 35*(b^7*c^10*d^4 - 7*a*b^6*c^9*d^5 + 21*a^2
*b^5*c^8*d^6 - 35*a^3*b^4*c^7*d^7 + 35*a^4*b^3*c^6*d^8 - 21*a^5*b^2*c^5*d^9
 + 7*a^6*b*c^4*d^10 - a^7*c^3*d^11)*x^4 + 35*(b^7*c^11*d^3 - 7*a*b^6*c^10*d
^4 + 21*a^2*b^5*c^9*d^5 - 35*a^3*b^4*c^8*d^6 + 35*a^4*b^3*c^7*d^7 - 21*a^5*
b^2*c^6*d^8 + 7*a^6*b*c^5*d^9 - a^7*c^4*d^10)*x^3 + 21*(b^7*c^12*d^2 - 7*a*
b^6*c^11*d^3 + 21*a^2*b^5*c^10*d^4 - 35*a^3*b^4*c^9*d^5 + 35*a^4*b^3*c^8*d^
6 - 21*a^5*b^2*c^7*d^7 + 7*a^6*b*c^6*d^8 - a^7*c^5*d^9)*x^2 + 7*(b^7*c^13*d
 - 7*a*b^6*c^12*d^2 + 21*a^2*b^5*c^11*d^3 - 35*a^3*b^4*c^10*d^4 + 35*a^4*b^
3*c^9*d^5 - 21*a^5*b^2*c^8*d^6 + 7*a^6*b*c^7*d^7 - a^7*c^6*d^8)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1589 vs. $2(190) = 380$.
time = 0.60, size = 1589, normalized size = 7.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^
4*c^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 -
 60*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*
b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21
*a^2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d
^4 + 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*
b^7*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5
 + 35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^
5*d^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 -
84*a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 2
1*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^
2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(b*x + a) - 420*(b^7*d^7*x^7 + 7*b^7*c*
d^6*x^6 + 21*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21
*b^7*c^5*d^2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(d*x + c))/(b^8*c^15 - 8*a*b
^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4
 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8
 + (b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^1
0 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7
*b*c*d^14 + a^8*d^15)*x^7 + 7*(b^8*c^9*d^6 - 8*a*b^7*c^8*d^7 + 28*a^2*b^6*c
^7*d^8 - 56*a^3*b^5*c^6*d^9 + 70*a^4*b^4*c^5*d^10 - 56*a^5*b^3*c^4*d^11 + 2
8*a^6*b^2*c^3*d^12 - 8*a^7*b*c^2*d^13 + a^8*c*d^14)*x^6 + 21*(b^8*c^10*d^5
- 8*a*b^7*c^9*d^6 + 28*a^2*b^6*c^8*d^7 - 56*a^3*b^5*c^7*d^8 + 70*a^4*b^4*c^
6*d^9 - 56*a^5*b^3*c^5*d^10 + 28*a^6*b^2*c^4*d^11 - 8*a^7*b*c^3*d^12 + a^8*
```

```
c^2*d^13)*x^5 + 35*(b^8*c^11*d^4 - 8*a*b^7*c^10*d^5 + 28*a^2*b^6*c^9*d^6 -
56*a^3*b^5*c^8*d^7 + 70*a^4*b^4*c^7*d^8 - 56*a^5*b^3*c^6*d^9 + 28*a^6*b^2*c
^5*d^10 - 8*a^7*b*c^4*d^11 + a^8*c^3*d^12)*x^4 + 35*(b^8*c^12*d^3 - 8*a*b^7
*c^11*d^4 + 28*a^2*b^6*c^10*d^5 - 56*a^3*b^5*c^9*d^6 + 70*a^4*b^4*c^8*d^7 -
 56*a^5*b^3*c^7*d^8 + 28*a^6*b^2*c^6*d^9 - 8*a^7*b*c^5*d^10 + a^8*c^4*d^11)
*x^3 + 21*(b^8*c^13*d^2 - 8*a*b^7*c^12*d^3 + 28*a^2*b^6*c^11*d^4 - 56*a^3*b
^5*c^10*d^5 + 70*a^4*b^4*c^9*d^6 - 56*a^5*b^3*c^8*d^7 + 28*a^6*b^2*c^7*d^8
- 8*a^7*b*c^6*d^9 + a^8*c^5*d^10)*x^2 + 7*(b^8*c^14*d - 8*a*b^7*c^13*d^2 +
28*a^2*b^6*c^12*d^3 - 56*a^3*b^5*c^11*d^4 + 70*a^4*b^4*c^10*d^5 - 56*a^5*b^
3*c^9*d^6 + 28*a^6*b^2*c^8*d^7 - 8*a^7*b*c^7*d^8 + a^8*c^6*d^9)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1776 vs. $2(170) = 340$.
time = 8.34, size = 1776, normalized size = 8.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**8,x)
```

```
[Out] -b**7*log(x + (-a**9*b**7*d**9/(a*d - b*c)**8 + 9*a**8*b**8*c*d**8/(a*d - b
*c)**8 - 36*a**7*b**9*c**2*d**7/(a*d - b*c)**8 + 84*a**6*b**10*c**3*d**6/(a
*d - b*c)**8 - 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 + 126*a**4*b**12*c**
5*d**4/(a*d - b*c)**8 - 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 + 36*a**2*b*
*14*c**7*d**2/(a*d - b*c)**8 - 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d +
 b**16*c**9/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + b**7*log(
x + (a**9*b**7*d**9/(a*d - b*c)**8 - 9*a**8*b**8*c*d**8/(a*d - b*c)**8 + 36
*a**7*b**9*c**2*d**7/(a*d - b*c)**8 - 84*a**6*b**10*c**3*d**6/(a*d - b*c)**
8 + 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 - 126*a**4*b**12*c**5*d**4/(a*d
 - b*c)**8 + 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 - 36*a**2*b**14*c**7*d*
*2/(a*d - b*c)**8 + 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d - b**16*c**9
/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + (-60*a**6*d**6 + 430
*a**5*b*c*d**5 - 1334*a**4*b**2*c**2*d**4 + 2341*a**3*b**3*c**3*d**3 - 2559
*a**2*b**4*c**4*d**2 + 1851*a*b**5*c**5*d - 1089*b**6*c**6 - 420*b**6*d**6*
x**6 + x**5*(210*a*b**5*d**6 - 2730*b**6*c*d**5) + x**4*(-140*a**2*b**4*d**
6 + 1330*a*b**5*c*d**5 - 7490*b**6*c**2*d**4) + x**3*(105*a**3*b**3*d**6 -
875*a**2*b**4*c*d**5 + 3535*a*b**5*c**2*d**4 - 11165*b**6*c**3*d**3) + x**2
*(-84*a**4*b**2*d**6 + 651*a**3*b**3*c*d**5 - 2289*a**2*b**4*c**2*d**4 + 50
61*a*b**5*c**3*d**3 - 9639*b**6*c**4*d**2) + x*(70*a**5*b*d**6 - 518*a**4*b
**2*c*d**5 + 1687*a**3*b**3*c**2*d**4 - 3213*a**2*b**4*c**3*d**3 + 4137*a*b
**5*c**4*d**2 - 4683*b**6*c**5*d))/(420*a**7*c**7*d**7 - 2940*a**6*b*c**8*d
**6 + 8820*a**5*b**2*c**9*d**5 - 14700*a**4*b**3*c**10*d**4 + 14700*a**3*b*
*4*c**11*d**3 - 8820*a**2*b**5*c**12*d**2 + 2940*a*b**6*c**13*d - 420*b**7*
c**14 + x**7*(420*a**7*d**14 - 2940*a**6*b*c*d**13 + 8820*a**5*b**2*c**2*d*
*12 - 14700*a**4*b**3*c**3*d**11 + 14700*a**3*b**4*c**4*d**10 - 8820*a**2*b
```

```
**5*c**5*d**9 + 2940*a*b**6*c**6*d**8 - 420*b**7*c**7*d**7) + x**6*(2940*a*
*7*c*d**13 - 20580*a**6*b*c**2*d**12 + 61740*a**5*b**2*c**3*d**11 - 102900*
a**4*b**3*c**4*d**10 + 102900*a**3*b**4*c**5*d**9 - 61740*a**2*b**5*c**6*d*
*8 + 20580*a*b**6*c**7*d**7 - 2940*b**7*c**8*d**6) + x**5*(8820*a**7*c**2*d
**12 - 61740*a**6*b*c**3*d**11 + 185220*a**5*b**2*c**4*d**10 - 308700*a**4*
b**3*c**5*d**9 + 308700*a**3*b**4*c**6*d**8 - 185220*a**2*b**5*c**7*d**7 +
61740*a*b**6*c**8*d**6 - 8820*b**7*c**9*d**5) + x**4*(14700*a**7*c**3*d**11
 - 102900*a**6*b*c**4*d**10 + 308700*a**5*b**2*c**5*d**9 - 514500*a**4*b**3
*c**6*d**8 + 514500*a**3*b**4*c**7*d**7 - 308700*a**2*b**5*c**8*d**6 + 1029
00*a*b**6*c**9*d**5 - 14700*b**7*c**10*d**4) + x**3*(14700*a**7*c**4*d**10
- 102900*a**6*b*c**5*d**9 + 308700*a**5*b**2*c**6*d**8 - 514500*a**4*b**3*c
**7*d**7 + 514500*a**3*b**4*c**8*d**6 - 308700*a**2*b**5*c**9*d**5 + 102900
*a*b**6*c**10*d**4 - 14700*b**7*c**11*d**3) + x**2*(8820*a**7*c**5*d**9 - 6
1740*a**6*b*c**6*d**8 + 185220*a**5*b**2*c**7*d**7 - 308700*a**4*b**3*c**8*
d**6 + 308700*a**3*b**4*c**9*d**5 - 185220*a**2*b**5*c**10*d**4 + 61740*a*b
**6*c**11*d**3 - 8820*b**7*c**12*d**2) + x*(2940*a**7*c**6*d**8 - 20580*a**
6*b*c**7*d**7 + 61740*a**5*b**2*c**8*d**6 - 102900*a**4*b**3*c**9*d**5 + 10
2900*a**3*b**4*c**10*d**4 - 61740*a**2*b**5*c**11*d**3 + 20580*a*b**6*c**12
*d**2 - 2940*b**7*c**13*d))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(190) = 380$.
time = 1.83, size = 703, normalized size = 3.48

$$\frac{b^8\log\left(|bx+a|\right)}{b^9c^8-8ab^8c^7d+28a^2b^7c^6d^2-56a^3b^6c^5d^3+70a^4b^5c^4d^4-56a^5b^4c^3d^5+28a^6b^3c^2d^6-8a^7b^2cd^7+a^8bd^8}-\frac{b^7d\log\left(|dx+c|\right)}{b^8c^8d-8ab^7c^7d^2+28a^2b^6c^6d^3-56a^3b^5c^5d^4+70a^4b^4c^4d^5-56a^5b^3c^3d^6+28a^6b^2c^2d^7-8a^7bcd^8+a^8d^9}+\frac{1089b^7c^7-2940ab^6c^6d+4410a^2b^5c^5d^2-4900a^3b^4c^4d^3+3675a^4b^3c^3d^4-1764a^5b^2c^2d^5+490a^6bcd^6-60a^7d^7+420(b^7cd^6-ab^6d^7)x^6+210(13b^7c^2d^5-14ab^6cd^6+a^2b^5d^7)x^5+70(107b^7c^3d^4-126ab^6c^2d^5+21a^2b^5cd^6-2a^3b^4d^7)x^4+35(319b^7c^4d^3-420ab^6c^3d^4+126a^2b^5c^2d^5-28a^3b^4cd^6+3a^4b^3d^7)x^3+21(459b^7c^5d^2-700ab^6c^4d^3+350a^2b^5c^3d^4-140a^3b^4c^2d^5+35a^4b^3cd^6-4a^5b^2d^7)x^2+7(669b^7c^6d-1260ab^6c^5d^2+1050a^2b^5c^4d^3-700a^3b^4c^3d^4+315a^4b^3c^2d^5-84a^5b^2cd^6+10a^6bd^7)x}{420(bc-ad)^8(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*log(abs(b*x + a))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^
3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^
6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8) - b^7*d*log(abs(d*x + c))/(b^8*c^8*d - 8*a
*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5
 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8 + a^8*d^9) + 1/4
20*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^4*c
^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 - 60
*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*b^6
*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21*a^
2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4
+ 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*b^7
*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5 +
35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^5*d
^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 - 84*
a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x)/((b*c - a*d)^8*(d*x + c)^7)
```

Mupad [B]

time = 0.87, size = 1299, normalized size = 6.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^8),x)

[Out] (2*b^7*atanh((a^8*d^8 - b^8*c^8 - 14*a^2*b^6*c^6*d^2 + 14*a^3*b^5*c^5*d^3 -
 14*a^5*b^3*c^3*d^5 + 14*a^6*b^2*c^2*d^6 + 6*a*b^7*c^7*d - 6*a^7*b*c*d^7)/(
a*d - b*c)^8 + (2*b*d*x*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^
4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6
*b*c*d^6))/(a*d - b*c)^8))/(a*d - b*c)^8 - ((60*a^6*d^6 + 1089*b^6*c^6 + 25
59*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3*d^3 + 1334*a^4*b^2*c^2*d^4 - 1851*a*b
^5*c^5*d - 430*a^5*b*c*d^5)/(420*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 +
35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*
d - 7*a^6*b*c*d^6)) - (b^3*x^3*(3*a^3*d^6 - 319*b^3*c^3*d^3 + 101*a*b^2*c^2
*d^4 - 25*a^2*b*c*d^5))/(12*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6)) + (b^6*d^6*x^6)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35
*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d
- 7*a^6*b*c*d^6) - (b^5*x^5*(a*d^6 - 13*b*c*d^5))/(2*(a^7*d^7 - b^7*c^7 - 2
1*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^
2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^2*x^2*(4*a^4*d^6 + 459*b^4*c^4
*d^2 - 241*a*b^3*c^3*d^3 + 109*a^2*b^2*c^2*d^4 - 31*a^3*b*c*d^5))/(20*(a^7*
d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^
4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^4*x^4*(2*a^2*
d^6 + 107*b^2*c^2*d^4 - 19*a*b*c*d^5))/(6*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c
^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a
*b^6*c^6*d - 7*a^6*b*c*d^6)) - (b*x*(10*a^5*d^6 - 669*b^5*c^5*d + 591*a*b^4
*c^4*d^2 - 459*a^2*b^3*c^3*d^3 + 241*a^3*b^2*c^2*d^4 - 74*a^4*b*c*d^5))/(60
*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*
c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)))/(c^7 + d^7*
x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c
^2*d^5*x^5 + 7*c^6*d*x)
```

3.1373 $\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$

Optimal. Leaf size=231

$$-\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{3b^2d}{5(bc-ad)^4(c+dx)^5}-\frac{b^3d}{(bc-ad)^5(c+$$

```
[Out] -b^7/(-a*d+b*c)^8/(b*x+a)-1/7*d/(-a*d+b*c)^2/(d*x+c)^7-1/3*b*d/(-a*d+b*c)^3
/(d*x+c)^6-3/5*b^2*d/(-a*d+b*c)^4/(d*x+c)^5-b^3*d/(-a*d+b*c)^5/(d*x+c)^4-5/
3*b^4*d/(-a*d+b*c)^6/(d*x+c)^3-3*b^5*d/(-a*d+b*c)^7/(d*x+c)^2-7*b^6*d/(-a*d
+b*c)^8/(d*x+c)-8*b^7*d*ln(b*x+a)/(-a*d+b*c)^9+8*b^7*d*ln(d*x+c)/(-a*d+b*c)
^9
```

Rubi [A]

time = 0.19, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{46\}$

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{8b^7d\log(a+bx)}{(bc-ad)^9}+\frac{8b^7d\log(c+dx)}{(bc-ad)^9}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc-ad)^6}-\frac{b^3d}{(c+dx)^4(bc-ad)^5}-\frac{3b^2d}{5(c+dx)^5(bc-ad)^4}-\frac{bd}{3(c+dx)^6(bc-ad)^3}-\frac{d}{7(c+dx)^7(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -(b^7/((b*c - a*d)^8*(a + b*x))) - d/(7*(b*c - a*d)^2*(c + d*x)^7) - (b*d)/
(3*(b*c - a*d)^3*(c + d*x)^6) - (3*b^2*d)/(5*(b*c - a*d)^4*(c + d*x)^5) - (
b^3*d)/((b*c - a*d)^5*(c + d*x)^4) - (5*b^4*d)/(3*(b*c - a*d)^6*(c + d*x)^3
) - (3*b^5*d)/((b*c - a*d)^7*(c + d*x)^2) - (7*b^6*d)/((b*c - a*d)^8*(c + d
*x)) - (8*b^7*d*Log[a + b*x])/(b*c - a*d)^9 + (8*b^7*d*Log[c + d*x])/(b*c -
 a*d)^9
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx &= \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^2}-\frac{8b^8d}{(bc-ad)^9(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^8}+\frac{}{(bc-a}\right.\\ &= -\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{}{5(bc-}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 213, normalized size = 0.92

$$-\frac{\frac{105b^7(bc-ad)}{a+bx}-\frac{15d(-bc+ad)^7}{(c+dx)^7}+\frac{35bd(bc-ad)^6}{(c+dx)^6}+\frac{63b^2d(bc-ad)^5}{(c+dx)^5}+\frac{105b^3d(bc-ad)^4}{(c+dx)^4}+\frac{175b^4d(bc-ad)^3}{(c+dx)^3}+\frac{315b^5d(bc-ad)^2}{(c+dx)^2}+\frac{735b^6d(bc-ad)}{c+dx}+840b^7d\log(a+bx)-840b^7d\log(c+dx)}{105(bc-ad)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -1/105*((105*b^7*(b*c - a*d))/(a + b*x) - (15*d*(-(b*c) + a*d)^7)/(c + d*x)
^7 + (35*b*d*(b*c - a*d)^6)/(c + d*x)^6 + (63*b^2*d*(b*c - a*d)^5)/(c + d*x
)^5 + (105*b^3*d*(b*c - a*d)^4)/(c + d*x)^4 + (175*b^4*d*(b*c - a*d)^3)/(c
+ d*x)^3 + (315*b^5*d*(b*c - a*d)^2)/(c + d*x)^2 + (735*b^6*d*(b*c - a*d))/
(c + d*x) + 840*b^7*d*Log[a + b*x] - 840*b^7*d*Log[c + d*x])/(b*c - a*d)^9
```

Maple [A]
time = 0.20, size = 223, normalized size = 0.97 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*d/(a*d-b*c)^2/(d*x+c)^7-8*d/(a*d-b*c)^9*b^7*ln(d*x+c)-7*d/(a*d-b*c)^8*
b^6/(d*x+c)+3*d/(a*d-b*c)^7*b^5/(d*x+c)^2-5/3*d/(a*d-b*c)^6*b^4/(d*x+c)^3+d
/(a*d-b*c)^5*b^3/(d*x+c)^4-3/5*d/(a*d-b*c)^4*b^2/(d*x+c)^5+1/3*d/(a*d-b*c)^
3*b/(d*x+c)^6-b^7/(a*d-b*c)^8/(b*x+a)+8*d/(a*d-b*c)^9*b^7*ln(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1881 vs. 2(223) = 446.
time = 0.56, size = 1881, normalized size = 8.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -8*b^7*d*log(b*x + a)/(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9) + 8*b^7*d*log(d*x + c)/
(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^
4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d
^7 + 9*a^8*b*c*d^8 - a^9*d^9) - 1/105*(840*b^7*d^7*x^7 + 105*b^7*c^7 + 1443
*a*b^6*c^6*d - 1497*a^2*b^5*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c
^3*d^4 + 463*a^5*b^2*c^2*d^5 - 125*a^6*b*c*d^6 + 15*a^7*d^7 + 420*(13*b^7*c
*d^6 + a*b^6*d^7)*x^6 + 140*(107*b^7*c^2*d^5 + 20*a*b^6*c*d^6 - a^2*b^5*d^7
)*x^5 + 70*(319*b^7*c^3*d^4 + 113*a*b^6*c^2*d^5 - 13*a^2*b^5*c*d^6 + a^3*b^
4*d^7)*x^4 + 14*(1377*b^7*c^4*d^3 + 872*a*b^6*c^3*d^4 - 178*a^2*b^5*c^2*d^5
 + 32*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 14*(669*b^7*c^5*d^2 + 786*a*b^6*
```

```
c^4*d^3 - 264*a^2*b^5*c^3*d^4 + 86*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 2*a
^5*b^2*d^7)*x^2 + 2*(1089*b^7*c^6*d + 2832*a*b^6*c^5*d^2 - 1578*a^2*b^5*c^4
*d^3 + 872*a^3*b^4*c^3*d^4 - 353*a^4*b^3*c^2*d^5 + 88*a^5*b^2*c*d^6 - 10*a^
6*b*d^7)*x)/(a*b^8*c^15 - 8*a^2*b^7*c^14*d + 28*a^3*b^6*c^13*d^2 - 56*a^4*b
^5*c^12*d^3 + 70*a^5*b^4*c^11*d^4 - 56*a^6*b^3*c^10*d^5 + 28*a^7*b^2*c^9*d^
6 - 8*a^8*b*c^8*d^7 + a^9*c^7*d^8 + (b^9*c^8*d^7 - 8*a*b^8*c^7*d^8 + 28*a^2
*b^7*c^6*d^9 - 56*a^3*b^6*c^5*d^10 + 70*a^4*b^5*c^4*d^11 - 56*a^5*b^4*c^3*d
^12 + 28*a^6*b^3*c^2*d^13 - 8*a^7*b^2*c*d^14 + a^8*b*d^15)*x^8 + (7*b^9*c^9
*d^6 - 55*a*b^8*c^8*d^7 + 188*a^2*b^7*c^7*d^8 - 364*a^3*b^6*c^6*d^9 + 434*a
^4*b^5*c^5*d^10 - 322*a^5*b^4*c^4*d^11 + 140*a^6*b^3*c^3*d^12 - 28*a^7*b^2*
c^2*d^13 - a^8*b*c*d^14 + a^9*d^15)*x^7 + 7*(3*b^9*c^10*d^5 - 23*a*b^8*c^9*
d^6 + 76*a^2*b^7*c^8*d^7 - 140*a^3*b^6*c^7*d^8 + 154*a^4*b^5*c^6*d^9 - 98*a
^5*b^4*c^5*d^10 + 28*a^6*b^3*c^4*d^11 + 4*a^7*b^2*c^3*d^12 - 5*a^8*b*c^2*d^
13 + a^9*c*d^14)*x^6 + 7*(5*b^9*c^11*d^4 - 37*a*b^8*c^10*d^5 + 116*a^2*b^7*
c^9*d^6 - 196*a^3*b^6*c^8*d^7 + 182*a^4*b^5*c^7*d^8 - 70*a^5*b^4*c^6*d^9 -
28*a^6*b^3*c^5*d^10 + 44*a^7*b^2*c^4*d^11 - 19*a^8*b*c^3*d^12 + 3*a^9*c^2*d
^13)*x^5 + 35*(b^9*c^12*d^3 - 7*a*b^8*c^11*d^4 + 20*a^2*b^7*c^10*d^5 - 28*a
^3*b^6*c^9*d^6 + 14*a^4*b^5*c^8*d^7 + 14*a^5*b^4*c^7*d^8 - 28*a^6*b^3*c^6*d
^9 + 20*a^7*b^2*c^5*d^10 - 7*a^8*b*c^4*d^11 + a^9*c^3*d^12)*x^4 + 7*(3*b^9*
c^13*d^2 - 19*a*b^8*c^12*d^3 + 44*a^2*b^7*c^11*d^4 - 28*a^3*b^6*c^10*d^5 -
70*a^4*b^5*c^9*d^6 + 182*a^5*b^4*c^8*d^7 - 196*a^6*b^3*c^7*d^8 + 116*a^7*b^
2*c^6*d^9 - 37*a^8*b*c^5*d^10 + 5*a^9*c^4*d^11)*x^3 + 7*(b^9*c^14*d - 5*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 + 28*a^3*b^6*c^11*d^4 - 98*a^4*b^5*c^10*d^
5 + 154*a^5*b^4*c^9*d^6 - 140*a^6*b^3*c^8*d^7 + 76*a^7*b^2*c^7*d^8 - 23*a^8
*b*c^6*d^9 + 3*a^9*c^5*d^10)*x^2 + (b^9*c^15 - a*b^8*c^14*d - 28*a^2*b^7*c^
13*d^2 + 140*a^3*b^6*c^12*d^3 - 322*a^4*b^5*c^11*d^4 + 434*a^5*b^4*c^10*d^5
 - 364*a^6*b^3*c^9*d^6 + 188*a^7*b^2*c^8*d^7 - 55*a^8*b*c^7*d^8 + 7*a^9*c^6
*d^9)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. $2(223) = 446$.
time = 0.60, size = 2264, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(105*b^8*c^8 + 1338*a*b^7*c^7*d - 2940*a^2*b^6*c^6*d^2 + 2940*a^3*b^
5*c^5*d^3 - 2450*a^4*b^4*c^4*d^4 + 1470*a^5*b^3*c^3*d^5 - 588*a^6*b^2*c^2*d
^6 + 140*a^7*b*c*d^7 - 15*a^8*d^8 + 840*(b^8*c*d^7 - a*b^7*d^8)*x^7 + 420*(
13*b^8*c^2*d^6 - 12*a*b^7*c*d^7 - a^2*b^6*d^8)*x^6 + 140*(107*b^8*c^3*d^5 -
 87*a*b^7*c^2*d^6 - 21*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 70*(319*b^8*c^4*d
^4 - 206*a*b^7*c^3*d^5 - 126*a^2*b^6*c^2*d^6 + 14*a^3*b^5*c*d^7 - a^4*b^4*d
^8)*x^4 + 14*(1377*b^8*c^5*d^3 - 505*a*b^7*c^4*d^4 - 1050*a^2*b^6*c^3*d^5 +
```

```
 210*a^3*b^5*c^2*d^6 - 35*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 14*(669*b^8*
c^6*d^2 + 117*a*b^7*c^5*d^3 - 1050*a^2*b^6*c^4*d^4 + 350*a^3*b^5*c^3*d^5 -
105*a^4*b^4*c^2*d^6 + 21*a^5*b^3*c*d^7 - 2*a^6*b^2*d^8)*x^2 + 2*(1089*b^8*c
^7*d + 1743*a*b^7*c^6*d^2 - 4410*a^2*b^6*c^5*d^3 + 2450*a^3*b^5*c^4*d^4 - 1
225*a^4*b^4*c^3*d^5 + 441*a^5*b^3*c^2*d^6 - 98*a^6*b^2*c*d^7 + 10*a^7*b*d^8
)*x + 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d^8)*x^7 + 7*(3
*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*c^2*d^6)*x^5 +
 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*a*b^7*c^4*d^4)
*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7*a*b^7*c^6*d^2
)*x)*log(b*x + a) - 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d
^8)*x^7 + 7*(3*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*
c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*
a*b^7*c^4*d^4)*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7
*a*b^7*c^6*d^2)*x)*log(d*x + c))/(a*b^9*c^16 - 9*a^2*b^8*c^15*d + 36*a^3*b^
7*c^14*d^2 - 84*a^4*b^6*c^13*d^3 + 126*a^5*b^5*c^12*d^4 - 126*a^6*b^4*c^11*
d^5 + 84*a^7*b^3*c^10*d^6 - 36*a^8*b^2*c^9*d^7 + 9*a^9*b*c^8*d^8 - a^10*c^7
*d^9 + (b^10*c^9*d^7 - 9*a*b^9*c^8*d^8 + 36*a^2*b^8*c^7*d^9 - 84*a^3*b^7*c^
6*d^10 + 126*a^4*b^6*c^5*d^11 - 126*a^5*b^5*c^4*d^12 + 84*a^6*b^4*c^3*d^13
- 36*a^7*b^3*c^2*d^14 + 9*a^8*b^2*c*d^15 - a^9*b*d^16)*x^8 + (7*b^10*c^10*d
^6 - 62*a*b^9*c^9*d^7 + 243*a^2*b^8*c^8*d^8 - 552*a^3*b^7*c^7*d^9 + 798*a^4
*b^6*c^6*d^10 - 756*a^5*b^5*c^5*d^11 + 462*a^6*b^4*c^4*d^12 - 168*a^7*b^3*c
^3*d^13 + 27*a^8*b^2*c^2*d^14 + 2*a^9*b*c*d^15 - a^10*d^16)*x^7 + 7*(3*b^10
*c^11*d^5 - 26*a*b^9*c^10*d^6 + 99*a^2*b^8*c^9*d^7 - 216*a^3*b^7*c^8*d^8 +
294*a^4*b^6*c^7*d^9 - 252*a^5*b^5*c^6*d^10 + 126*a^6*b^4*c^5*d^11 - 24*a^7*
b^3*c^4*d^12 - 9*a^8*b^2*c^3*d^13 + 6*a^9*b*c^2*d^14 - a^10*c*d^15)*x^6 + 7
*(5*b^10*c^12*d^4 - 42*a*b^9*c^11*d^5 + 153*a^2*b^8*c^10*d^6 - 312*a^3*b^7*
c^9*d^7 + 378*a^4*b^6*c^8*d^8 - 252*a^5*b^5*c^7*d^9 + 42*a^6*b^4*c^6*d^10 +
 72*a^7*b^3*c^5*d^11 - 63*a^8*b^2*c^4*d^12 + 22*a^9*b*c^3*d^13 - 3*a^10*c^2
*d^14)*x^5 + 35*(b^10*c^13*d^3 - 8*a*b^9*c^12*d^4 + 27*a^2*b^8*c^11*d^5 - 4
8*a^3*b^7*c^10*d^6 + 42*a^4*b^6*c^9*d^7 - 42*a^6*b^4*c^7*d^9 + 48*a^7*b^3*c
^6*d^10 - 27*a^8*b^2*c^5*d^11 + 8*a^9*b*c^4*d^12 - a^10*c^3*d^13)*x^4 + 7*(
3*b^10*c^14*d^2 - 22*a*b^9*c^13*d^3 + 63*a^2*b^8*c^12*d^4 - 72*a^3*b^7*c^11
*d^5 - 42*a^4*b^6*c^10*d^6 + 252*a^5*b^5*c^9*d^7 - 378*a^6*b^4*c^8*d^8 + 31
2*a^7*b^3*c^7*d^9 - 153*a^8*b^2*c^6*d^10 + 42*a^9*b*c^5*d^11 - 5*a^10*c^4*d
^12)*x^3 + 7*(b^10*c^15*d - 6*a*b^9*c^14*d^2 + 9*a^2*b^8*c^13*d^3 + 24*a^3*
b^7*c^12*d^4 - 126*a^4*b^6*c^11*d^5 + 252*a^5*b^5*c^10*d^6 - 294*a^6*b^4*c^
9*d^7 + 216*a^7*b^3*c^8*d^8 - 99*a^8*b^2*c^7*d^9 + 26*a^9*b*c^6*d^10 - 3*a^
10*c^5*d^11)*x^2 + (b^10*c^16 - 2*a*b^9*c^15*d - 27*a^2*b^8*c^14*d^2 + 168*
a^3*b^7*c^13*d^3 - 462*a^4*b^6*c^12*d^4 + 756*a^5*b^5*c^11*d^5 - 798*a^6*b^
4*c^10*d^6 + 552*a^7*b^3*c^9*d^7 - 243*a^8*b^2*c^8*d^8 + 62*a^9*b*c^7*d^9 -
 7*a^10*c^6*d^10)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2336 vs. $2(209) = 418$.

time = 26.93, size = 2336, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] -8*b**7*d*log(x + (-8*a**10*b**7*d**11/(a*d - b*c)**9 + 80*a**9*b**8*c*d**1
0/(a*d - b*c)**9 - 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 + 960*a**7*b**10*
c**3*d**8/(a*d - b*c)**9 - 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 + 2016*
a**5*b**12*c**5*d**6/(a*d - b*c)**9 - 1680*a**4*b**13*c**6*d**5/(a*d - b*c)
**9 + 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 - 360*a**2*b**15*c**8*d**3/(a
*d - b*c)**9 + 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 - 8*b**1
7*c**10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + 8*b
**7*d*log(x + (8*a**10*b**7*d**11/(a*d - b*c)**9 - 80*a**9*b**8*c*d**10/(a*
d - b*c)**9 + 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 - 960*a**7*b**10*c**3*
d**8/(a*d - b*c)**9 + 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 - 2016*a**5*
b**12*c**5*d**6/(a*d - b*c)**9 + 1680*a**4*b**13*c**6*d**5/(a*d - b*c)**9 -
 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 + 360*a**2*b**15*c**8*d**3/(a*d -
b*c)**9 - 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 + 8*b**17*c**
10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + (-15*a**
7*d**7 + 125*a**6*b*c*d**6 - 463*a**5*b**2*c**2*d**5 + 1007*a**4*b**3*c**3*
d**4 - 1443*a**3*b**4*c**4*d**3 + 1497*a**2*b**5*c**5*d**2 - 1443*a*b**6*c*
*6*d - 105*b**7*c**7 - 840*b**7*d**7*x**7 + x**6*(-420*a*b**6*d**7 - 5460*b
**7*c*d**6) + x**5*(140*a**2*b**5*d**7 - 2800*a*b**6*c*d**6 - 14980*b**7*c*
*2*d**5) + x**4*(-70*a**3*b**4*d**7 + 910*a**2*b**5*c*d**6 - 7910*a*b**6*c*
*2*d**5 - 22330*b**7*c**3*d**4) + x**3*(42*a**4*b**3*d**7 - 448*a**3*b**4*c
*d**6 + 2492*a**2*b**5*c**2*d**5 - 12208*a*b**6*c**3*d**4 - 19278*b**7*c**4
*d**3) + x**2*(-28*a**5*b**2*d**7 + 266*a**4*b**3*c*d**6 - 1204*a**3*b**4*c
**2*d**5 + 3696*a**2*b**5*c**3*d**4 - 11004*a*b**6*c**4*d**3 - 9366*b**7*c*
*5*d**2) + x*(20*a**6*b*d**7 - 176*a**5*b**2*c*d**6 + 706*a**4*b**3*c**2*d*
*5 - 1744*a**3*b**4*c**3*d**4 + 3156*a**2*b**5*c**4*d**3 - 5664*a*b**6*c**5
*d**2 - 2178*b**7*c**6*d))/(105*a**9*c**7*d**8 - 840*a**8*b*c**8*d**7 + 294
0*a**7*b**2*c**9*d**6 - 5880*a**6*b**3*c**10*d**5 + 7350*a**5*b**4*c**11*d*
*4 - 5880*a**4*b**5*c**12*d**3 + 2940*a**3*b**6*c**13*d**2 - 840*a**2*b**7*
c**14*d + 105*a*b**8*c**15 + x**8*(105*a**8*b*d**15 - 840*a**7*b**2*c*d**14
 + 2940*a**6*b**3*c**2*d**13 - 5880*a**5*b**4*c**3*d**12 + 7350*a**4*b**5*c
**4*d**11 - 5880*a**3*b**6*c**5*d**10 + 2940*a**2*b**7*c**6*d**9 - 840*a*b*
*8*c**7*d**8 + 105*b**9*c**8*d**7) + x**7*(105*a**9*d**15 - 105*a**8*b*c*d*
*14 - 2940*a**7*b**2*c**2*d**13 + 14700*a**6*b**3*c**3*d**12 - 33810*a**5*b
**4*c**4*d**11 + 45570*a**4*b**5*c**5*d**10 - 38220*a**3*b**6*c**6*d**9 + 1
9740*a**2*b**7*c**7*d**8 - 5775*a*b**8*c**8*d**7 + 735*b**9*c**9*d**6) + x*
*6*(735*a**9*c*d**14 - 3675*a**8*b*c**2*d**13 + 2940*a**7*b**2*c**3*d**12 +
 20580*a**6*b**3*c**4*d**11 - 72030*a**5*b**4*c**5*d**10 + 113190*a**4*b**5
*c**6*d**9 - 102900*a**3*b**6*c**7*d**8 + 55860*a**2*b**7*c**8*d**7 - 16905
```

```
*a*b**8*c**9*d**6 + 2205*b**9*c**10*d**5) + x**5*(2205*a**9*c**2*d**13 - 13
965*a**8*b*c**3*d**12 + 32340*a**7*b**2*c**4*d**11 - 20580*a**6*b**3*c**5*d
**10 - 51450*a**5*b**4*c**6*d**9 + 133770*a**4*b**5*c**7*d**8 - 144060*a**3
*b**6*c**8*d**7 + 85260*a**2*b**7*c**9*d**6 - 27195*a*b**8*c**10*d**5 + 367
5*b**9*c**11*d**4) + x**4*(3675*a**9*c**3*d**12 - 25725*a**8*b*c**4*d**11 +
 73500*a**7*b**2*c**5*d**10 - 102900*a**6*b**3*c**6*d**9 + 51450*a**5*b**4*
c**7*d**8 + 51450*a**4*b**5*c**8*d**7 - 102900*a**3*b**6*c**9*d**6 + 73500*
a**2*b**7*c**10*d**5 - 25725*a*b**8*c**11*d**4 + 3675*b**9*c**12*d**3) + x*
*3*(3675*a**9*c**4*d**11 - 27195*a**8*b*c**5*d**10 + 85260*a**7*b**2*c**6*d
**9 - 144060*a**6*b**3*c**7*d**8 + 133770*a**5*b**4*c**8*d**7 - 51450*a**4*
b**5*c**9*d**6 - 20580*a**3*b**6*c**10*d**5 + 32340*a**2*b**7*c**11*d**4 -
13965*a*b**8*c**12*d**3 + 2205*b**9*c**13*d**2) + x**2*(2205*a**9*c**5*d**1
0 - 16905*a**8*b*c**6*d**9 + 55860*a**7*b**2*c**7*d**8 - 102900*a**6*b**3*c
**8*d**7 + 113190*a**5*b**4*c**9*d**6 - 72030*a**4*b**5*c**10*d**5 + 20580*
a**3*b**6*c**11*d**4 + 2940*a**2*b**7*c**12*d**3 - 3675*a*b**8*c**13*d**2 +
 735*b**9*c**14*d) + x*(735*a**9*c**6*d**9 - 5775*a**8*b*c**7*d**8 + 19740*
a**7*b**2*c**8*d**7 - 38220*a**6*b**3*c**9*d**6 + 45570*a**5*b**4*c**10*d**
5 - 33810*a**4*b**5*c**11*d**4 + 14700*a**3*b**6*c**12*d**3 - 2940*a**2*b**
7*c**13*d**2 - 105*a*b**8*c**14*d + 105*b**9*c**15))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(223) = 446.
time = 1.19, size = 714, normalized size = 3.09

$$-\frac{b^{15}}{(b^{16}c^8-8ab^{15}c^7d+28a^2b^{14}c^6d^2-56a^3b^{13}c^5d^3+70a^4b^{12}c^4d^4-56a^5b^{11}c^3d^5+28a^6b^{10}c^2d^6-8a^7b^9cd^7+a^8b^8d^8)(bx+a)}+\frac{8b^8d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^{10}c^9-9ab^9c^8d+36a^2b^8c^7d^2-84a^3b^7c^6d^3+126a^4b^6c^5d^4-126a^5b^5c^4d^5+84a^6b^4c^3d^6-36a^7b^3c^2d^7+9a^8b^2cd^8-a^9bd^9}+\frac{1443b^7d^8+\frac{9366(b^9cd^7-ab^8d^8)}{(bx+a)b}+\frac{25578(b^{11}c^2d^6-2ab^{10}cd^7+a^2b^9d^8)}{(bx+a)^2b^2}+\frac{37730(b^{13}c^3d^5-3ab^{12}c^2d^6+3a^2b^{11}cd^7-a^3b^{10}d^8)}{(bx+a)^3b^3}+\frac{31850(b^{15}c^4d^4-4ab^{14}c^3d^5+6a^2b^{13}c^2d^6-4a^3b^{12}cd^7+a^4b^{11}d^8)}{(bx+a)^4b^4}+\frac{14700(b^{17}c^5d^3-5ab^{16}c^4d^4+10a^2b^{15}c^3d^5-10a^3b^{14}c^2d^6+5a^4b^{13}cd^7-a^5b^{12}d^8)}{(bx+a)^5b^5}+\frac{2940(b^{19}c^6d^2-6ab^{18}c^5d^3+15a^2b^{17}c^4d^4-20a^3b^{16}c^3d^5+15a^4b^{15}c^2d^6-6a^5b^{14}cd^7+a^6b^{13}d^8)}{(bx+a)^6b^6}}{105(bc-ad)^9\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -b^15/((b^16*c^8 - 8*a*b^15*c^7*d + 28*a^2*b^14*c^6*d^2 - 56*a^3*b^13*c^5*d
^3 + 70*a^4*b^12*c^4*d^4 - 56*a^5*b^11*c^3*d^5 + 28*a^6*b^10*c^2*d^6 - 8*a^
7*b^9*c*d^7 + a^8*b^8*d^8)*(b*x + a)) + 8*b^8*d*log(abs(b*c/(b*x + a) - a*d
/(b*x + a) + d))/(b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^
7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6
- 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9) + 1/105*(1443*b^7*d^8 +
 9366*(b^9*c*d^7 - a*b^8*d^8)/((b*x + a)*b) + 25578*(b^11*c^2*d^6 - 2*a*b^1
0*c*d^7 + a^2*b^9*d^8)/((b*x + a)^2*b^2) + 37730*(b^13*c^3*d^5 - 3*a*b^12*c
^2*d^6 + 3*a^2*b^11*c*d^7 - a^3*b^10*d^8)/((b*x + a)^3*b^3) + 31850*(b^15*c
^4*d^4 - 4*a*b^14*c^3*d^5 + 6*a^2*b^13*c^2*d^6 - 4*a^3*b^12*c*d^7 + a^4*b^1
1*d^8)/((b*x + a)^4*b^4) + 14700*(b^17*c^5*d^3 - 5*a*b^16*c^4*d^4 + 10*a^2*
b^15*c^3*d^5 - 10*a^3*b^14*c^2*d^6 + 5*a^4*b^13*c*d^7 - a^5*b^12*d^8)/((b*x
 + a)^5*b^5) + 2940*(b^19*c^6*d^2 - 6*a*b^18*c^5*d^3 + 15*a^2*b^17*c^4*d^4
- 20*a^3*b^16*c^3*d^5 + 15*a^4*b^15*c^2*d^6 - 6*a^5*b^14*c*d^7 + a^6*b^13*d
^8)/((b*x + a)^6*b^6))/((b*c - a*d)^9*(b*c/(b*x + a) - a*d/(b*x + a) + d)^7
)
```

Mupad [B]
time = 1.39, size = 1738, normalized size = 7.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^8),x)
```

```
[Out] (16*b^7*d*atanh((a^9*d^9 + b^9*c^9 + 20*a^2*b^7*c^7*d^2 - 28*a^3*b^6*c^6*d^
3 + 14*a^4*b^5*c^5*d^4 + 14*a^5*b^4*c^4*d^5 - 28*a^6*b^3*c^3*d^6 + 20*a^7*b
^2*c^2*d^7 - 7*a*b^8*c^8*d - 7*a^8*b*c*d^8)/(a*d - b*c)^9 + (2*b*d*x*(a^8*d
^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)
)/(a*d - b*c)^9))/(a*d - b*c)^9 - ((15*a^7*d^7 + 105*b^7*c^7 - 1497*a^2*b^5
*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c^3*d^4 + 463*a^5*b^2*c^2*d^
5 + 1443*a*b^6*c^6*d - 125*a^6*b*c*d^6)/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*b^
6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (4*b^5*x^5*(107*b^2*
c^2*d^5 - a^2*d^7 + 20*a*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^2*x^2*(2*a^5*d^7 + 669
*b^5*c^5*d^2 + 786*a*b^4*c^4*d^3 - 264*a^2*b^3*c^3*d^4 + 86*a^3*b^2*c^2*d^5
 - 19*a^4*b*c*d^6))/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^4*x^4*(a^3*d^7 + 319*b^3*c^3*d^4 + 1
13*a*b^2*c^2*d^5 - 13*a^2*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b*x*(1089*b^6*c^6*d - 1
0*a^6*d^7 + 2832*a*b^5*c^5*d^2 - 1578*a^2*b^4*c^4*d^3 + 872*a^3*b^3*c^3*d^4
 - 353*a^4*b^2*c^2*d^5 + 88*a^5*b*c*d^6))/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*
b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (8*b^7*d^7*x^7)/(a
^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*
d^7) + (4*b^6*x^6*(a*d^7 + 13*b*c*d^6))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^
6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (2*b^3*x^3*(1377*b^4*c^4*d
^3 - 3*a^4*d^7 + 872*a*b^3*c^3*d^4 - 178*a^2*b^2*c^2*d^5 + 32*a^3*b*c*d^6))
/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a
^7*b*c*d^7)))/(x^7*(a*d^7 + 7*b*c*d^6) + x^3*(35*a*c^4*d^3 + 21*b*c^5*d^2)
+ x^5*(21*a*c^2*d^5 + 35*b*c^3*d^4) + x^4*(35*a*c^3*d^4 + 35*b*c^4*d^3) + a
*c^7 + x*(b*c^7 + 7*a*c^6*d) + x^2*(21*a*c^5*d^2 + 7*b*c^6*d) + x^6*(21*b*c
^2*d^5 + 7*a*c*d^6) + b*d^7*x^8)
```

3.1374 $\int \frac{1}{(a+bx)^3(c+dx)^8} \, dx$

Optimal. Leaf size=276

$$-\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+\frac{bd^2}{2(bc-ad)^4(c+dx)^6}+\frac{6b^2d^2}{5(bc-ad)^5(c+d}$$

```
[Out] -1/2*b^7/(-a*d+b*c)^8/(b*x+a)^2+8*b^7*d/(-a*d+b*c)^9/(b*x+a)+1/7*d^2/(-a*d+
b*c)^3/(d*x+c)^7+1/2*b*d^2/(-a*d+b*c)^4/(d*x+c)^6+6/5*b^2*d^2/(-a*d+b*c)^5/
(d*x+c)^5+5/2*b^3*d^2/(-a*d+b*c)^6/(d*x+c)^4+5*b^4*d^2/(-a*d+b*c)^7/(d*x+c)
^3+21/2*b^5*d^2/(-a*d+b*c)^8/(d*x+c)^2+28*b^6*d^2/(-a*d+b*c)^9/(d*x+c)+36*b
^7*d^2*ln(b*x+a)/(-a*d+b*c)^10-36*b^7*d^2*ln(d*x+c)/(-a*d+b*c)^10
```

Rubi [A]

time = 0.25, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {46}

$$\frac{36b^7d^2\log(a+bx)}{(bc-ad)^{10}}-\frac{36b^7d^2\log(c+dx)}{(bc-ad)^{10}}+\frac{8b^7d}{(a+bx)(bc-ad)^9}-\frac{b^7}{2(a+bx)^2(bc-ad)^8}+\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21b^5d^2}{2(c+dx)^2(bc-ad)^8}+\frac{5b^4d^2}{(c+dx)^3(bc-ad)^7}+\frac{5b^3d^2}{2(c+dx)^4(bc-ad)^6}+\frac{6b^2d^2}{5(c+dx)^5(bc-ad)^5}+\frac{bd^2}{2(c+dx)^6(bc-ad)^4}+\frac{d^2}{7(c+dx)^7(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] -1/2*b^7/((b*c - a*d)^8*(a + b*x)^2) + (8*b^7*d)/((b*c - a*d)^9*(a + b*x))
+ d^2/(7*(b*c - a*d)^3*(c + d*x)^7) + (b*d^2)/(2*(b*c - a*d)^4*(c + d*x)^6)
 + (6*b^2*d^2)/(5*(b*c - a*d)^5*(c + d*x)^5) + (5*b^3*d^2)/(2*(b*c - a*d)^6
*(c + d*x)^4) + (5*b^4*d^2)/((b*c - a*d)^7*(c + d*x)^3) + (21*b^5*d^2)/(2*(
b*c - a*d)^8*(c + d*x)^2) + (28*b^6*d^2)/((b*c - a*d)^9*(c + d*x)) + (36*b^
7*d^2*Log[a + b*x])/(b*c - a*d)^10 - (36*b^7*d^2*Log[c + d*x])/(b*c - a*d)^
10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^8}\,dx &= \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^3}-\frac{8b^8d}{(bc-ad)^9(a+bx)^2}+\frac{36b^8d^2}{(bc-ad)^{10}(a+bx)}-\frac{}{(bc-ad}\right.\\ &= -\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+\frac{}{2(bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 254, normalized size = 0.92

$$\frac{-\frac{35b^7(bc-ad)^2}{(a+bx)^2}+\frac{560b^7d(bc-ad)}{a+bx}+\frac{10d^2(bc-ad)^7}{(c+dx)^7}+\frac{35bd^2(bc-ad)^6}{(c+dx)^6}+\frac{84b^2d^2(bc-ad)^5}{(c+dx)^5}+\frac{175b^3d^2(bc-ad)^4}{(c+dx)^4}+\frac{350b^4d^2(bc-ad)^3}{(c+dx)^3}+\frac{735b^5d^2(bc-ad)^2}{(c+dx)^2}+\frac{1960b^6d^2(bc-ad)}{c+dx}+2520b^7d^2\log(a+bx)-2520b^7d^2\log(c+dx)}{70(bc-ad)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] ((-35*b^7*(b*c - a*d)^2)/(a + b*x)^2 + (560*b^7*d*(b*c - a*d))/(a + b*x) +
(10*d^2*(b*c - a*d)^7)/(c + d*x)^7 + (35*b*d^2*(b*c - a*d)^6)/(c + d*x)^6 +
 (84*b^2*d^2*(b*c - a*d)^5)/(c + d*x)^5 + (175*b^3*d^2*(b*c - a*d)^4)/(c +
d*x)^4 + (350*b^4*d^2*(b*c - a*d)^3)/(c + d*x)^3 + (735*b^5*d^2*(b*c - a*d)
^2)/(c + d*x)^2 + (1960*b^6*d^2*(b*c - a*d))/(c + d*x) + 2520*b^7*d^2*Log[a
 + b*x] - 2520*b^7*d^2*Log[c + d*x])/(70*(b*c - a*d)^10)
```

Maple [A]
time = 0.24, size = 265, normalized size = 0.96

method	result
default	$-\frac{d^2}{7(ad-bc)^3(dx+c)^7}-\frac{36d^2b^7\ln(dx+c)}{(ad-bc)^{10}}-\frac{28d^2b^6}{(ad-bc)^9(dx+c)}+\frac{21d^2b^5}{2(ad-bc)^8(dx+c)^2}-\frac{5d^2b^4}{(ad-bc)^7(dx+c)^3}+\frac{5d^2b^3}{2(ad-bc)^6(dx+c)}$
risch	Expression too large to display
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*d^2/(a*d-b*c)^3/(d*x+c)^7-36*d^2/(a*d-b*c)^10*b^7*ln(d*x+c)-28*d^2/(a*
d-b*c)^9*b^6/(d*x+c)+21/2*d^2/(a*d-b*c)^8*b^5/(d*x+c)^2-5*d^2/(a*d-b*c)^7*b
^4/(d*x+c)^3+5/2*d^2/(a*d-b*c)^6*b^3/(d*x+c)^4-6/5*d^2/(a*d-b*c)^5*b^2/(d*x
+c)^5+1/2*d^2/(a*d-b*c)^4*b/(d*x+c)^6-1/2*b^7/(a*d-b*c)^8/(b*x+a)^2+36*d^2/
(a*d-b*c)^10*b^7*ln(b*x+a)-8*b^7/(a*d-b*c)^9*d/(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2399 vs. 2(264) = 528.
time = 0.67, size = 2399, normalized size = 8.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 36*b^7*d^2*log(b*x + a)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b
^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^
```

10*d^10) - 36*b^7*d^2*log(d*x + c)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10) + 1/70*(2520*b^8*d^8*x^8 - 35*b^8*c^8 + 595*a*b^7*c^7*d + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b^4*c^4*d^4 - 1061*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 - 95*a^7*b*c*d^7 + 10*a^8*d^8 + 1260*(13*b^8*c*d^7 + 3*a*b^7*d^8)*x^7 + 420*(107*b^8*c^2*d^6 + 59*a*b^7*c*d^7 + 2*a^2*b^6*d^8)*x^6 + 210*(319*b^8*c^3*d^5 + 327*a*b^7*c^2*d^6 + 27*a^2*b^6*c*d^7 - a^3*b^5*d^8)*x^5 + 42*(1377*b^8*c^4*d^4 + 2467*a*b^7*c^3*d^5 + 387*a^2*b^6*c^2*d^6 - 33*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x^4 + 42*(669*b^8*c^5*d^3 + 2163*a*b^7*c^4*d^4 + 608*a^2*b^6*c^3*d^5 - 92*a^3*b^5*c^2*d^6 + 13*a^4*b^4*c*d^7 - a^5*b^3*d^8)*x^3 + 6*(1089*b^8*c^6*d^2 + 7515*a*b^7*c^5*d^3 + 3924*a^2*b^6*c^4*d^4 - 976*a^3*b^5*c^3*d^5 + 249*a^4*b^4*c^2*d^6 - 45*a^5*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 + 3*(105*b^8*c^7*d + 3621*a*b^7*c^6*d^2 + 4167*a^2*b^6*c^5*d^3 - 1713*a^3*b^5*c^4*d^4 + 737*a^4*b^4*c^3*d^5 - 243*a^5*b^3*c^2*d^6 + 51*a^6*b^2*c*d^7 - 5*a^7*b*d^8)*x)/(a^2*b^9*c^16 - 9*a^3*b^8*c^15*d + 36*a^4*b^7*c^14*d^2 - 84*a^5*b^6*c^13*d^3 + 126*a^6*b^5*c^12*d^4 - 126*a^7*b^4*c^11*d^5 + 84*a^8*b^3*c^10*d^6 - 36*a^9*b^2*c^9*d^7 + 9*a^10*b*c^8*d^8 - a^11*c^7*d^9 + (b^11*c^9*d^7 - 9*a*b^10*c^8*d^8 + 36*a^2*b^9*c^7*d^9 - 84*a^3*b^8*c^6*d^10 + 126*a^4*b^7*c^5*d^11 - 126*a^5*b^6*c^4*d^12 + 84*a^6*b^5*c^3*d^13 - 36*a^7*b^4*c^2*d^14 + 9*a^8*b^3*c*d^15 - a^9*b^2*d^16)*x^9 + (7*b^11*c^10*d^6 - 61*a*b^10*c^9*d^7 + 234*a^2*b^9*c^8*d^8 - 516*a^3*b^8*c^7*d^9 + 714*a^4*b^7*c^6*d^10 - 630*a^5*b^6*c^5*d^11 + 336*a^6*b^5*c^4*d^12 - 84*a^7*b^4*c^3*d^13 - 9*a^8*b^3*c^2*d^14 + 11*a^9*b^2*c*d^15 - 2*a^10*b*d^16)*x^8 + (21*b^11*c^11*d^5 - 175*a*b^10*c^10*d^6 + 631*a^2*b^9*c^9*d^7 - 1269*a^3*b^8*c^8*d^8 + 1506*a^4*b^7*c^7*d^9 - 966*a^5*b^6*c^6*d^10 + 126*a^6*b^5*c^5*d^11 + 294*a^7*b^4*c^4*d^12 - 231*a^8*b^3*c^3*d^13 + 69*a^9*b^2*c^2*d^14 - 5*a^10*b*c*d^15 - a^11*d^16)*x^7 + 7*(5*b^11*c^12*d^4 - 39*a*b^10*c^11*d^5 + 127*a^2*b^9*c^10*d^6 - 213*a^3*b^8*c^9*d^7 + 162*a^4*b^7*c^8*d^8 + 42*a^5*b^6*c^7*d^9 - 210*a^6*b^5*c^6*d^10 + 198*a^7*b^4*c^5*d^11 - 87*a^8*b^3*c^4*d^12 + 13*a^9*b^2*c^3*d^13 + 3*a^10*b*c^2*d^14 - a^11*c*d^15)*x^6 + 7*(5*b^11*c^13*d^3 - 35*a*b^10*c^12*d^4 + 93*a^2*b^9*c^11*d^5 - 87*a^3*b^8*c^10*d^6 - 102*a^4*b^7*c^9*d^7 + 378*a^5*b^6*c^8*d^8 - 462*a^6*b^5*c^7*d^9 + 282*a^7*b^4*c^6*d^10 - 63*a^8*b^3*c^5*d^11 - 23*a^9*b^2*c^4*d^12 + 17*a^10*b*c^3*d^13 - 3*a^11*c^2*d^14)*x^5 + 7*(3*b^11*c^14*d^2 - 17*a*b^10*c^13*d^3 + 23*a^2*b^9*c^12*d^4 + 63*a^3*b^8*c^11*d^5 - 282*a^4*b^7*c^10*d^6 + 462*a^5*b^6*c^9*d^7 - 378*a^6*b^5*c^8*d^8 + 102*a^7*b^4*c^7*d^9 + 87*a^8*b^3*c^6*d^10 - 93*a^9*b^2*c^5*d^11 + 35*a^10*b*c^4*d^12 - 5*a^11*c^3*d^13)*x^4 + 7*(b^11*c^15*d - 3*a*b^10*c^14*d^2 - 13*a^2*b^9*c^13*d^3 + 87*a^3*b^8*c^12*d^4 - 198*a^4*b^7*c^11*d^5 + 210*a^5*b^6*c^10*d^6 - 42*a^6*b^5*c^9*d^7 - 162*a^7*b^4*c^8*d^8 + 213*a^8*b^3*c^7*d^9 - 127*a^9*b^2*c^6*d^10 + 39*a^10*b*c^5*d^11 - 5*a^11*c^4*d^12)*x^3 + (b^11*c^16 + 5*a*b^10*c^15*d - 69*a^2*b^9*c^14*d^2 + 231*a^3*b^8*c^13*d^3 - 294*a^4*b^7*c^12*d^4 - 126*a^5*b^6*c^11*d^5 + 966*a^6*b^5*c^10*d^6 - 1506*a^7*b^4*c^9*d^7 + 1269*a^8*b^3*c^8*d^8 - 631*a^9*b^2*c^7*d^9 + 175*a^10*b*c^6*d^10 - 21*a^11*c^5*d^11)*

```
x^2 + (2*a*b^10*c^16 - 11*a^2*b^9*c^15*d + 9*a^3*b^8*c^14*d^2 + 84*a^4*b^7*
c^13*d^3 - 336*a^5*b^6*c^12*d^4 + 630*a^6*b^5*c^11*d^5 - 714*a^7*b^4*c^10*d
^6 + 516*a^8*b^3*c^9*d^7 - 234*a^9*b^2*c^8*d^8 + 61*a^10*b*c^7*d^9 - 7*a^11
*c^6*d^10)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3016 vs. $2(264) = 528$.
time = 0.68, size = 3016, normalized size = 10.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/70*(35*b^9*c^9 - 630*a*b^8*c^8*d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c
^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 2940*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6
 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a
*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7
- 420*(107*b^9*c^3*d^6 - 48*a*b^8*c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^
9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*
a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 - 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^
5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b
^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3 + 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d
^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3
*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 + 6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4
 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 - 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 +
 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^
5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x - 2
520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*
b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^
8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^
2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^
6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c
^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*
a^2*b^7*c^6*d^3)*x)*log(b*x + a) + 2520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7
*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*
d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b
^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 +
 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d
^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*
c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(d*x + c))/(a^2*
b^10*c^17 - 10*a^3*b^9*c^16*d + 45*a^4*b^8*c^15*d^2 - 120*a^5*b^7*c^14*d^3
+ 210*a^6*b^6*c^13*d^4 - 252*a^7*b^5*c^12*d^5 + 210*a^8*b^4*c^11*d^6 - 120*
a^9*b^3*c^10*d^7 + 45*a^10*b^2*c^9*d^8 - 10*a^11*b*c^8*d^9 + a^12*c^7*d^10
```

```
+ (b^12*c^10*d^7 - 10*a*b^11*c^9*d^8 + 45*a^2*b^10*c^8*d^9 - 120*a^3*b^9*c^
7*d^10 + 210*a^4*b^8*c^6*d^11 - 252*a^5*b^7*c^5*d^12 + 210*a^6*b^6*c^4*d^13
 - 120*a^7*b^5*c^3*d^14 + 45*a^8*b^4*c^2*d^15 - 10*a^9*b^3*c*d^16 + a^10*b^
2*d^17)*x^9 + (7*b^12*c^11*d^6 - 68*a*b^11*c^10*d^7 + 295*a^2*b^10*c^9*d^8
- 750*a^3*b^9*c^8*d^9 + 1230*a^4*b^8*c^7*d^10 - 1344*a^5*b^7*c^6*d^11 + 966
*a^6*b^6*c^5*d^12 - 420*a^7*b^5*c^4*d^13 + 75*a^8*b^4*c^3*d^14 + 20*a^9*b^3
*c^2*d^15 - 13*a^10*b^2*c*d^16 + 2*a^11*b*d^17)*x^8 + (21*b^12*c^12*d^5 - 1
96*a*b^11*c^11*d^6 + 806*a^2*b^10*c^10*d^7 - 1900*a^3*b^9*c^9*d^8 + 2775*a^
4*b^8*c^8*d^9 - 2472*a^5*b^7*c^7*d^10 + 1092*a^6*b^6*c^6*d^11 + 168*a^7*b^5
*c^5*d^12 - 525*a^8*b^4*c^4*d^13 + 300*a^9*b^3*c^3*d^14 - 74*a^10*b^2*c^2*d
^15 + 4*a^11*b*c*d^16 + a^12*d^17)*x^7 + 7*(5*b^12*c^13*d^4 - 44*a*b^11*c^1
2*d^5 + 166*a^2*b^10*c^11*d^6 - 340*a^3*b^9*c^10*d^7 + 375*a^4*b^8*c^9*d^8
- 120*a^5*b^7*c^8*d^9 - 252*a^6*b^6*c^7*d^10 + 408*a^7*b^5*c^6*d^11 - 285*a
^8*b^4*c^5*d^12 + 100*a^9*b^3*c^4*d^13 - 10*a^10*b^2*c^3*d^14 - 4*a^11*b*c^
2*d^15 + a^12*c*d^16)*x^6 + 7*(5*b^12*c^14*d^3 - 40*a*b^11*c^13*d^4 + 128*a
^2*b^10*c^12*d^5 - 180*a^3*b^9*c^11*d^6 - 15*a^4*b^8*c^10*d^7 + 480*a^5*b^7
*c^9*d^8 - 840*a^6*b^6*c^8*d^9 + 744*a^7*b^5*c^7*d^10 - 345*a^8*b^4*c^6*d^1
1 + 40*a^9*b^3*c^5*d^12 + 40*a^10*b^2*c^4*d^13 - 20*a^11*b*c^3*d^14 + 3*a^1
2*c^2*d^15)*x^5 + 7*(3*b^12*c^15*d^2 - 20*a*b^11*c^14*d^3 + 40*a^2*b^10*c^1
3*d^4 + 40*a^3*b^9*c^12*d^5 - 345*a^4*b^8*c^11*d^6 + 744*a^5*b^7*c^10*d^7 -
 840*a^6*b^6*c^9*d^8 + 480*a^7*b^5*c^8*d^9 - 15*a^8*b^4*c^7*d^10 - 180*a^9*
b^3*c^6*d^11 + 128*a^10*b^2*c^5*d^12 - 40*a^11*b*c^4*d^13 + 5*a^12*c^3*d^14
)*x^4 + 7*(b^12*c^16*d - 4*a*b^11*c^15*d^2 - 10*a^2*b^10*c^14*d^3 + 100*a^3
*b^9*c^13*d^4 - 285*a^4*b^8*c^12*d^5 + 408*a^5*b^7*c^11*d^6 - 252*a^6*b^6*c
^10*d^7 - 120*a^7*b^5*c^9*d^8 + 375*a^8*b^4*c^8*d^9 - 340*a^9*b^3*c^7*d^10
+ 166*a^10*b^2*c^6*d^11 - 44*a^11*b*c^5*d^12 + 5*a^12*c^4*d^13)*x^3 + (b^12
*c^17 + 4*a*b^11*c^16*d - 74*a^2*b^10*c^15*d^2 + 300*a^3*b^9*c^14*d^3 - 525
*a^4*b^8*c^13*d^4 + 168*a^5*b^7*c^12*d^5 + 1092*a^6*b^6*c^11*d^6 - 2472*a^7
*b^5*c^10*d^7 + 2775*a^8*b^4*c^9*d^8 - 1900*a^9*b^3*c^8*d^9 + 806*a^10*b^2*
c^7*d^10 - 196*a^11*b*c^6*d^11 + 21*a^12*c^5*d^12)*x^2 + (2*a*b^11*c^17 - 1
3*a^2*b^10*c^16*d + 20*a^3*b^9*c^15*d^2 + 75*a^4*b^8*c^14*d^3 - 420*a^5*b^7
*c^13*d^4 + 966*a^6*b^6*c^12*d^5 - 1344*a^7*b^5*c^11*d^6 + 1230*a^8*b^4*c^1
0*d^7 - 750*a^9*b^3*c^9*d^8 + 295*a^10*b^2*c^8*d^9 - 68*a^11*b*c^7*d^10 + 7
*a^12*c^6*d^11)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. 2(264) = 528.

time = 1.29, size = 1029, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*b^8*d^2*log(abs(b*x + a))/(b^11*c^10 - 10*a*b^10*c^9*d + 45*a^2*b^9*c^8*
d^2 - 120*a^3*b^8*c^7*d^3 + 210*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 + 210
*a^6*b^5*c^4*d^6 - 120*a^7*b^4*c^3*d^7 + 45*a^8*b^3*c^2*d^8 - 10*a^9*b^2*c*
d^9 + a^10*b*d^10) - 36*b^7*d^3*log(abs(d*x + c))/(b^10*c^10*d - 10*a*b^9*c
^9*d^2 + 45*a^2*b^8*c^8*d^3 - 120*a^3*b^7*c^7*d^4 + 210*a^4*b^6*c^6*d^5 - 2
52*a^5*b^5*c^5*d^6 + 210*a^6*b^4*c^4*d^7 - 120*a^7*b^3*c^3*d^8 + 45*a^8*b^2
*c^2*d^9 - 10*a^9*b*c*d^10 + a^10*d^11) - 1/70*(35*b^9*c^9 - 630*a*b^8*c^8*
d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 29
40*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b
*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d
^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7 - 420*(107*b^9*c^3*d^6 - 48*a*b^8*
c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*
a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 -
 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3
*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3
+ 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4
*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 +
6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4
*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2
- 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 + 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^
6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^
7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x)/((b*c - a*d)^10*(b*x + a)^2*(d*x + c
)^7)
```

Mupad [B]
time = 1.91, size = 2224, normalized size = 8.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^8),x)
```

```
[Out] (72*b^7*d^2*atanh((a^10*d^10 - b^10*c^10 - 27*a^2*b^8*c^8*d^2 + 48*a^3*b^7*
c^7*d^3 - 42*a^4*b^6*c^6*d^4 + 42*a^6*b^4*c^4*d^6 - 48*a^7*b^3*c^3*d^7 + 27
*a^8*b^2*c^2*d^8 + 8*a*b^9*c^9*d - 8*a^9*b*c*d^9)/(a*d - b*c)^10 + (2*b*d*x
*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5
*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
9*a*b^8*c^8*d - 9*a^8*b*c*d^8))/(a*d - b*c)^10))/(a*d - b*c)^10 - ((10*a^8*
```

d^8 - 35*b^8*c^8 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b^4*c^4*d^4 - 1061*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 + 595*a*b^7*c^7*d - 95*a^7*b*c*d^7)/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b^2*x^2*(4*a^6*d^8 + 1089*b^6*c^6*d^2 + 7515*a*b^5*c^5*d^3 + 3924*a^2*b^4*c^4*d^4 - 976*a^3*b^3*c^3*d^5 + 249*a^4*b^2*c^2*d^6 - 45*a^5*b*c*d^7))/(35*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b^4*x^4*(2*a^4*d^8 + 1377*b^4*c^4*d^4 + 2467*a*b^3*c^3*d^5 + 387*a^2*b^2*c^2*d^6 - 33*a^3*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b*x*(105*b^7*c^7*d - 5*a^7*d^8 + 3621*a*b^6*c^6*d^2 + 4167*a^2*b^5*c^5*d^3 - 1713*a^3*b^4*c^4*d^4 + 737*a^4*b^3*c^3*d^5 - 243*a^5*b^2*c^2*d^6 + 51*a^6*b*c*d^7))/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (6*b^6*x^6*(2*a^2*d^8 + 107*b^2*c^2*d^6 + 59*a*b*c*d^7))/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*b^3*x^3*(669*b^5*c^5*d^3 - a^5*d^8 + 2163*a*b^4*c^4*d^4 + 608*a^2*b^3*c^3*d^5 - 92*a^3*b^2*c^2*d^6 + 13*a^4*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b^5*x^5*(319*b^3*c^3*d^5 - a^3*d^8 + 327*a*b^2*c^2*d^6 + 27*a^2*b*c*d^7))/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (36*b^8*d^8*x^8)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (18*b^6*d*x^7*(13*b^2*c*d^6 + 3*a*b*d^7))/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8))/(x^3*(7*b^2*c^6*d + 35*a^2*c^4*d^3 + 42*a*b*c^5*d^2) + x^6*(7*a^2*c*d^6 + 35*b^2*c^3*d^4 + 42*a*b*c^2*d^5) + x*(7*a^2*c^6*d + 2*a*b*c^7) + x^2*(b^2*c^7 + 21*a^2*c^5*d^2 + 14*a*b*c^6*d) + x^7*(a^2*d^7 + 21*b^2*c^2*d^5 + 14*a*b*c*d^6) + x^4*(35*a^2*c^3*d^4 + 21*b^2*c^5*d^2 + 70*a*b*c^4*d^3) + x^5*(21*a^2*c^2*d^5 + 35*b^2*c^4*d^3 + 70*a*b*c^3*d^4) + x^8*(7*b^2*c*d^6 + 2*a*b*d^7) + a^2*c^7 + b^2*d^7*x^9)

3.1375 $\int (a+bx)^5 \sqrt{c+dx}\,dx$

Optimal. Leaf size=156

$$-\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{9/2}}{9d^6}-$$

```
[Out] -2/3*(-a*d+b*c)^5*(d*x+c)^(3/2)/d^6+2*b*(-a*d+b*c)^4*(d*x+c)^(5/2)/d^6-20/7
*b^2*(-a*d+b*c)^3*(d*x+c)^(7/2)/d^6+20/9*b^3*(-a*d+b*c)^2*(d*x+c)^(9/2)/d^6
-10/11*b^4*(-a*d+b*c)*(d*x+c)^(11/2)/d^6+2/13*b^5*(d*x+c)^(13/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+dx)^{3/2}(bc-ad)^5}{3d^6}+\frac{2b^5(c+dx)^{13/2}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b*c - a*d)^4*(c + d*x)^(
5/2))/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(b*c -
 a*d)^2*(c + d*x)^(9/2))/(9*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(11/2))/(1
1*d^6) + (2*b^5*(c + d*x)^(13/2))/(13*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^5\sqrt{c+dx}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{3/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^5}{d^5}\right.\\ &= -\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 217, normalized size = 1.39

$$\frac{2(c+dx)^{3/2}\left(3003a^5d^5+3003a^4bd^4(-2c+3dx)+858a^3b^2d^3(8c^2-12cdx+15d^2x^2)+286a^2b^3d^2(-16c^3+24c^2dx-30cd^2x^2+35d^3x^3)+13ab^4d(128c^4-192c^3dx+240c^2d^2x^2-280cd^3x^3+315d^4x^4)+b^5(-256c^5+384c^4dx-480c^3d^2x^2+560c^2d^3x^3-630cd^4x^4+693d^5x^5)\right)}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(3003*a^5*d^5 + 3003*a^4*b*d^4*(-2*c + 3*d*x) + 858*a^3*
b^2*d^3*(8*c^2 - 12*c*d*x + 15*d^2*x^2) + 286*a^2*b^3*d^2*(-16*c^3 + 24*c^2
*d*x - 30*c*d^2*x^2 + 35*d^3*x^3) + 13*a*b^4*d*(128*c^4 - 192*c^3*d*x + 240
*c^2*d^2*x^2 - 280*c*d^3*x^3 + 315*d^4*x^4) + b^5*(-256*c^5 + 384*c^4*d*x -
 480*c^3*d^2*x^2 + 560*c^2*d^3*x^3 - 630*c*d^4*x^4 + 693*d^5*x^5)))/(9009*d
^6)
```

Maple [A]
time = 0.14, size = 121, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{13}{2}}}{13}+\frac{10(ad-bc)b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{7}{2}}}{7}+2(ad-bc)^4b(dx+c)^{\frac{5}{2}}+\frac{2(ad-bc)^5(dx+c}{3}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{13}{2}}}{13}+\frac{10(ad-bc)b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{7}{2}}}{7}+2(ad-bc)^4b(dx+c)^{\frac{5}{2}}+\frac{2(ad-bc)^5(dx+c}{3}}{d^6}$
gosper	$2(dx+c)^{\frac{3}{2}}(693b^5x^5d^5+4095a\,b^4d^5x^4-630b^5c\,d^4x^4+10010a^2b^3d^5x^3-3640a\,b^4c\,d^4x^3+560b^5c^2d^3x^3+12870a^3b^2d^5x^2-858$
trager	$2(693b^5d^6x^6+4095a\,b^4d^6x^5+63b^5c\,d^5x^5+10010a^2b^3d^6x^4+455a\,b^4c\,d^5x^4-70b^5c^2d^4x^4+12870a^3b^2d^6x^3+1430a^2b^3c\,d^5x^3$
risch	$2(693b^5d^6x^6+4095a\,b^4d^6x^5+63b^5c\,d^5x^5+10010a^2b^3d^6x^4+455a\,b^4c\,d^5x^4-70b^5c^2d^4x^4+12870a^3b^2d^6x^3+1430a^2b^3c\,d^5x^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/13*b^5*(d*x+c)^(13/2)+5/11*(a*d-b*c)*b^4*(d*x+c)^(11/2)+10/9*(a*d-
b*c)^2*b^3*(d*x+c)^(9/2)+10/7*(a*d-b*c)^3*b^2*(d*x+c)^(7/2)+(a*d-b*c)^4*b*(
d*x+c)^(5/2)+1/3*(a*d-b*c)^5*(d*x+c)^(3/2))
```

Maxima [A]
time = 0.29, size = 259, normalized size = 1.66

$$\frac{2\left(693(dx+c)^{\frac{13}{2}}b^5-4095(b^5c-ab^4d)(dx+c)^{\frac{11}{2}}+10010(b^5c^2-2ab^4cd+a^2b^3d^2)(dx+c)^{\frac{9}{2}}-12870(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{7}{2}}+9009(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4)(dx+c)^{\frac{5}{2}}-3003(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5)(dx+c)^{\frac{3}{2}}\right)}{9009\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*(d*x + c)^(13/2)*b^5 - 4095*(b^5*c - a*b^4*d)*(d*x + c)^(11/2)
+ 10010*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(9/2) - 12870*(b^5*
c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(7/2) + 9009
*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
)*(d*x + c)^(5/2) - 3003*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(3/2))/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(134) = 268$.

time = 0.44, size = 338, normalized size = 2.17

$$\frac{2\left(693\,b^{5}d^{6}x^{6} - 256\,b^{5}c^{6} + 1664\,ab^{4}c^{5}d - 4576\,a^{2}b^{3}c^{4}d^{2} + 6864\,a^{3}b^{2}c^{3}d^{3} - 6006\,a^{4}bc^{2}d^{4} + 3003\,a^{5}cd^{5} + 63\,(b^{5}cd^{5} + 65\,ab^{4}d^{6})x^{5} - 35\,(2\,b^{5}c^{2}d^{4} - 13\,ab^{4}cd^{5} - 286\,a^{2}b^{3}d^{6})x^{4} + 10\,(8\,b^{5}c^{3}d^{3} - 52\,ab^{4}c^{2}d^{4} + 143\,a^{2}b^{3}cd^{5} + 1287\,a^{3}b^{2}d^{6})x^{3} - 3\,(32\,b^{5}c^{4}d^{2} - 208\,ab^{4}c^{3}d^{3} + 572\,a^{2}b^{3}c^{2}d^{4} - 858\,a^{3}b^{2}cd^{5} - 3003\,a^{4}bd^{6})x^{2} + (128\,b^{5}c^{5}d - 832\,ab^{4}c^{4}d^{2} + 2288\,a^{2}b^{3}c^{3}d^{3} - 3432\,a^{3}b^{2}c^{2}d^{4} + 3003\,a^{4}bcd^{5} + 3003\,a^{5}d^{6})x\right)\sqrt{dx + c}}{9009\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9009*(693*b^5*d^6*x^6 - 256*b^5*c^6 + 1664*a*b^4*c^5*d - 4576*a^2*b^3*c^4
*d^2 + 6864*a^3*b^2*c^3*d^3 - 6006*a^4*b*c^2*d^4 + 3003*a^5*c*d^5 + 63*(b^5
*c*d^5 + 65*a*b^4*d^6)*x^5 - 35*(2*b^5*c^2*d^4 - 13*a*b^4*c*d^5 - 286*a^2*b
^3*d^6)*x^4 + 10*(8*b^5*c^3*d^3 - 52*a*b^4*c^2*d^4 + 143*a^2*b^3*c*d^5 + 12
87*a^3*b^2*d^6)*x^3 - 3*(32*b^5*c^4*d^2 - 208*a*b^4*c^3*d^3 + 572*a^2*b^3*c
^2*d^4 - 858*a^3*b^2*c*d^5 - 3003*a^4*b*d^6)*x^2 + (128*b^5*c^5*d - 832*a*b
^4*c^4*d^2 + 2288*a^2*b^3*c^3*d^3 - 3432*a^3*b^2*c^2*d^4 + 3003*a^4*b*c*d^5
 + 3003*a^5*d^6)*x)*sqrt(d*x + c)/d^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(144) = 288$.

time = 2.33, size = 314, normalized size = 2.01

$$\frac{2\left(\frac{b^{5}(c+dx)^{\frac{13}{2}}}{13d^{5}} + \frac{(c+dx)^{\frac{11}{2}}\cdot(5ab^{4}d-5b^{5}c)}{11d^{5}} + \frac{(c+dx)^{\frac{9}{2}}\cdot(10a^{2}b^{3}d^{2}-20ab^{4}cd+10b^{5}c^{2})}{9d^{5}} + \frac{(c+dx)^{\frac{7}{2}}\cdot(10a^{3}b^{2}d^{3}-30a^{2}b^{3}cd^{2}+30ab^{4}c^{2}d-10b^{5}c^{3})}{7d^{5}} + \frac{(c+dx)^{\frac{5}{2}}\cdot(5a^{4}bd^{4}-20a^{3}b^{2}cd^{3}+30a^{2}b^{3}c^{2}d^{2}-20ab^{4}c^{3}d+5b^{5}c^{4})}{5d^{5}} + \frac{(c+dx)^{\frac{3}{2}}\left(a^{5}d^{5}-5a^{4}bcd^{4}+10a^{3}b^{2}c^{2}d^{3}-10a^{2}b^{3}c^{3}d^{2}+5ab^{4}c^{4}d-b^{5}c^{5}\right)}{3d^{5}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**5*(c + d*x)**(13/2)/(13*d**5) + (c + d*x)**(11/2)*(5*a*b**4*d - 5*b**
5*c)/(11*d**5) + (c + d*x)**(9/2)*(10*a**2*b**3*d**2 - 20*a*b**4*c*d + 10*b
**5*c**2)/(9*d**5) + (c + d*x)**(7/2)*(10*a**3*b**2*d**3 - 30*a**2*b**3*c*d
**2 + 30*a*b**4*c**2*d - 10*b**5*c**3)/(7*d**5) + (c + d*x)**(5/2)*(5*a**4*
b*d**4 - 20*a**3*b**2*c*d**3 + 30*a**2*b**3*c**2*d**2 - 20*a*b**4*c**3*d +
5*b**5*c**4)/(5*d**5) + (c + d*x)**(3/2)*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*
a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)
/(3*d**5))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(134) = 268$.

time = 1.84, size = 641, normalized size = 4.11

$$\frac{2\left(9009\sqrt{dx+c}\,a^{5}c + 3003\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)a^{5} + \frac{15015\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^{4}bc}{d} + \frac{6006\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^{2}\right)a^{3}b^{2}c}{d^{2}} + \frac{3003\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^{2}\right)a^{4}b}{d} + \frac{2574\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^{2}-35\sqrt{dx+c}\,c^{3}\right)a^{2}b^{3}c}{d^{3}} + \frac{2574\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^{2}-35\sqrt{dx+c}\,c^{3}\right)a^{3}b^{2}}{d^{2}} + \frac{143\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^{2}-420(dx+c)^{\frac{3}{2}}c^{3}+315\sqrt{dx+c}\,c^{4}\right)ab^{4}c}{d^{4}} + \frac{286\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^{2}-420(dx+c)^{\frac{3}{2}}c^{3}+315\sqrt{dx+c}\,c^{4}\right)a^{2}b^{3}}{d^{3}} + \frac{13\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^{2}-1386(dx+c)^{\frac{5}{2}}c^{3}+1155(dx+c)^{\frac{3}{2}}c^{4}-693\sqrt{dx+c}\,c^{5}\right)b^{5}c}{d^{5}} + \frac{65\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^{2}-1386(dx+c)^{\frac{5}{2}}c^{3}+1155(dx+c)^{\frac{3}{2}}c^{4}-693\sqrt{dx+c}\,c^{5}\right)ab^{4}}{d^{4}} + \frac{3\left(231(dx+c)^{\frac{13}{2}}-1638(dx+c)^{\frac{11}{2}}c+5005(dx+c)^{\frac{9}{2}}c^{2}-8580(dx+c)^{\frac{7}{2}}c^{3}+9009(dx+c)^{\frac{5}{2}}c^{4}-6006(dx+c)^{\frac{3}{2}}c^{5}+3003\sqrt{dx+c}\,c^{6}\right)b^{5}}{d^{5}}\right)}{9009\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9009*(9009*sqrt(d*x + c)*a^5*c + 3003*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^5 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c/d + 6006*(3*(d
*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*b^2*c/d^2
+ 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
4*b/d + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)
*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(
d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2/d
^2 + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*
c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c/d^4 + 286*(3
5*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(
d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^3/d^3 + 13*(63*(d*x + c)^
(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(
5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c/d^5 + 65
*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1
386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)
*a*b^4/d^4 + 3*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5/d^5)/d
```

Mupad [B]
time = 0.08, size = 137, normalized size = 0.88

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{5/2}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(13/2))/(13*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(11/
2))/(11*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(3/2))/(3*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(9/2))/(9*
d^6) + (2*b*(a*d - b*c)^4*(c + d*x)^(5/2))/d^6
```

3.1376 $\int (a+bx)^4 \sqrt{c+dx}\, dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-\frac{8b^3(bc-ad)(c+dx)^{9/2}}{9d^5}+\frac{2b^4}{}$$

```
[Out] 2/3*(-a*d+b*c)^4*(d*x+c)^(3/2)/d^5-8/5*b*(-a*d+b*c)^3*(d*x+c)^(5/2)/d^5+12/
7*b^2*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^5-8/9*b^3*(-a*d+b*c)*(d*x+c)^(9/2)/d^5+2
/11*b^4*(d*x+c)^(11/2)/d^5
```

Rubi [A]
time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(3/2))/(3*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(5
/2))/(5*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^5) - (8*b^3*(b*c
 - a*d)*(c + d*x)^(9/2))/(9*d^5) + (2*b^4*(c + d*x)^(11/2))/(11*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^4\sqrt{c+dx}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{3/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{5/}}{d^4}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-\end{aligned}$$

Mathematica [A]
time = 0.08, size = 154, normalized size = 1.19

$$\frac{2(c+dx)^{3/2}\left(1155a^4d^4+924a^3bd^3(-2c+3dx)+198a^2b^2d^2(8c^2-12cdx+15d^2x^2)+44ab^3d(-16c^3+24c^2dx-30cd^2x^2+35d^3x^3)+b^4(128c^4-192c^3dx+240c^2d^2x^2-280cd^3x^3+315d^4x^4)\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*a^4*d^4 + 924*a^3*b*d^3*(-2*c + 3*d*x) + 198*a^2*b
^2*d^2*(8*c^2 - 12*c*d*x + 15*d^2*x^2) + 44*a*b^3*d*(-16*c^3 + 24*c^2*d*x -
 30*c*d^2*x^2 + 35*d^3*x^3) + b^4*(128*c^4 - 192*c^3*d*x + 240*c^2*d^2*x^2
- 280*c*d^3*x^3 + 315*d^4*x^4)))/(3465*d^5)
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{8(ad-bc)^3b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^4(dx+c)^{\frac{3}{2}}}{3}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{8(ad-bc)^3b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^4(dx+c)^{\frac{3}{2}}}{3}}{d^5}$
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(315d^4x^4b^4+1540a\,b^3d^4x^3-280b^4c\,d^3x^3+2970a^2b^2d^4x^2-1320a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+2772a^3b\,d^4x-2376a^2}{3465d^5}$
trager	$2(315b^4d^5x^5+1540a\,b^3d^5x^4+35b^4c\,d^4x^4+2970a^2b^2d^5x^3+220a\,b^3c\,d^4x^3-40b^4c^2d^3x^3+2772a^3b\,d^5x^2+594a^2b^2c\,d^4x^2-26$
risch	$2(315b^4d^5x^5+1540a\,b^3d^5x^4+35b^4c\,d^4x^4+2970a^2b^2d^5x^3+220a\,b^3c\,d^4x^3-40b^4c^2d^3x^3+2772a^3b\,d^5x^2+594a^2b^2c\,d^4x^2-26$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/11*b^4*(d*x+c)^(11/2)+4/9*(a*d-b*c)*b^3*(d*x+c)^(9/2)+6/7*(a*d-b*c
)^2*b^2*(d*x+c)^(7/2)+4/5*(a*d-b*c)^3*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)^4*(d*x+
c)^(3/2))
```

Maxima [A]
time = 0.29, size = 181, normalized size = 1.40

$$\frac{2\left(315\,(dx+c)^{\frac{11}{2}}b^4-1540\,(b^4c-ab^3d)(dx+c)^{\frac{9}{2}}+2970\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(dx+c)^{\frac{7}{2}}-2772\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)(dx+c)^{\frac{5}{2}}+1155\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{3}{2}}\right)}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(d*x + c)^(11/2)*b^4 - 1540*(b^4*c - a*b^3*d)*(d*x + c)^(9/2) +
 2970*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(7/2) - 2772*(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(5/2) + 1155*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x +
c)^(3/2))/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(109) = 218$.

time = 0.43, size = 245, normalized size = 1.90

$$\frac{2\left(315\,b^{4}d^{5}x^{5}+128\,b^{4}c^{5}-704\,ab^{3}c^{4}d+1584\,a^{2}b^{2}c^{3}d^{2}-1848\,a^{3}bc^{2}d^{3}+1155\,a^{4}cd^{4}+35\left(b^{4}cd^{4}+44\,ab^{3}d^{5}\right)x^{4}-10\left(4\,b^{4}c^{2}d^{3}-22\,ab^{3}cd^{4}-297\,a^{2}b^{2}d^{5}\right)x^{3}+6\left(8\,b^{4}c^{3}d^{2}-44\,ab^{3}c^{2}d^{3}+99\,a^{2}b^{2}cd^{4}+462\,a^{3}bd^{5}\right)x^{2}-\left(64\,b^{4}c^{4}d-352\,ab^{3}c^{3}d^{2}+792\,a^{2}b^{2}c^{2}d^{3}-924\,a^{3}bcd^{4}-1155\,a^{4}d^{5}\right)x\right)\sqrt{dx+c}}{3465\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^4*d^5*x^5 + 128*b^4*c^5 - 704*a*b^3*c^4*d + 1584*a^2*b^2*c^3*
d^2 - 1848*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 + 35*(b^4*c*d^4 + 44*a*b^3*d^5)*x
^4 - 10*(4*b^4*c^2*d^3 - 22*a*b^3*c*d^4 - 297*a^2*b^2*d^5)*x^3 + 6*(8*b^4*c
^3*d^2 - 44*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 + 462*a^3*b*d^5)*x^2 - (64*b^4
*c^4*d - 352*a*b^3*c^3*d^2 + 792*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 - 1155*a
^4*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A]
time = 1.87, size = 223, normalized size = 1.73

$$\frac{2\left(\frac{b^{4}(c+dx)^{\frac{11}{2}}}{11d^{4}}+\frac{(c+dx)^{\frac{9}{2}}\cdot(4ab^{3}d-4b^{4}c)}{9d^{4}}+\frac{(c+dx)^{\frac{7}{2}}\cdot(6a^{2}b^{2}d^{2}-12ab^{3}cd+6b^{4}c^{2})}{7d^{4}}+\frac{(c+dx)^{\frac{5}{2}}\cdot(4a^{3}bd^{3}-12a^{2}b^{2}cd^{2}+12ab^{3}c^{2}d-4b^{4}c^{3})}{5d^{4}}+\frac{(c+dx)^{\frac{3}{2}}\,(a^{4}d^{4}-4a^{3}bcd^{3}+6a^{2}b^{2}c^{2}d^{2}-4ab^{3}c^{3}d+b^{4}c^{4})}{3d^{4}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**4*(c + d*x)**(11/2)/(11*d**4) + (c + d*x)**(9/2)*(4*a*b**3*d - 4*b**4
*c)/(9*d**4) + (c + d*x)**(7/2)*(6*a**2*b**2*d**2 - 12*a*b**3*c*d + 6*b**4*
c**2)/(7*d**4) + (c + d*x)**(5/2)*(4*a**3*b*d**3 - 12*a**2*b**2*c*d**2 + 12
*a*b**3*c**2*d - 4*b**4*c**3)/(5*d**4) + (c + d*x)**(3/2)*(a**4*d**4 - 4*a*
*3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*d**4)
)/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(109) = 218.
time = 1.21, size = 470, normalized size = 3.64

$$\frac{2\left(3465\,\sqrt{dx+c}\,a^{4}c+1155\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^{4}+\frac{4620\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^{3}bc}{d}+\frac{1386\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^{2}\right)a^{2}b^{2}c}{d^{2}}+\frac{924\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^{2}\right)a^{3}b}{d}+\frac{396\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\sqrt{dx+c}\,c^{3}\right)ab^{3}c}{d^{3}}+\frac{594\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\sqrt{dx+c}\,c^{3}\right)a^{2}b^{2}}{d^{2}}+\frac{11\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^{2}-420\,(dx+c)^{\frac{3}{2}}c^{3}+315\sqrt{dx+c}\,c^{4}\right)b^{4}c}{d^{4}}+\frac{44\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^{2}-420\,(dx+c)^{\frac{3}{2}}c^{3}+315\sqrt{dx+c}\,c^{4}\right)ab^{3}}{d^{3}}+\frac{5\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^{2}-1386\,(dx+c)^{\frac{5}{2}}c^{3}+1155\,(dx+c)^{\frac{3}{2}}c^{4}-693\sqrt{dx+c}\,c^{5}\right)b^{4}}{d^{4}}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^4*c + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^4 + 4620*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c/d + 1386*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b^2*c/d^2 +
 924*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*
b/d + 396*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c/d^3 + 594*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2/d^2 + 1
```

```
1*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 4
20*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c/d^4 + 44*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3/d^3 + 5*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4/d^4)/d
```

Mupad [B]
time = 0.22, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(11/2))/(11*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(9/2))
/(9*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^5) + (12*b^2*(a*d - b*c)^
2*(c + d*x)^(7/2))/(7*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^5)
```

3.1377 $\int (a+bx)^3\sqrt{c+dx}\,dx$

Optimal. Leaf size=100

$$-\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

```
[Out] -2/3*(-a*d+b*c)^3*(d*x+c)^(3/2)/d^4+6/5*b*(-a*d+b*c)^2*(d*x+c)^(5/2)/d^4-6/
7*b^2*(-a*d+b*c)*(d*x+c)^(7/2)/d^4+2/9*b^3*(d*x+c)^(9/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
5/2))/(5*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^4) + (2*b^3*(c + d
*x)^(9/2))/(9*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^3\sqrt{c+dx}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{3/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{5/2}}{d^3}\right.\\ &= -\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+\end{aligned}$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 1.02

$$\frac{2(c+dx)^{3/2}\left(105a^3d^3+63a^2bd^2(-2c+3dx)+9ab^2d(8c^2-12cdx+15d^2x^2)+b^3(-16c^3+24c^2dx-30cd^2x^2+35d^3x^3)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(105*a^3*d^3 + 63*a^2*b*d^2*(-2*c + 3*d*x) + 9*a*b^2*d*(
8*c^2 - 12*c*d*x + 15*d^2*x^2) + b^3*(-16*c^3 + 24*c^2*d*x - 30*c*d^2*x^2 +
 35*d^3*x^3)))/(315*d^4)
```

Maple [A]
time = 0.15, size = 78, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6(ad-bc)b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)^2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^3(dx+c)^{\frac{3}{2}}}{3}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{9}{2}}}{9}+\frac{6(ad-bc)b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)^2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^3(dx+c)^{\frac{3}{2}}}{3}}{d^4}$
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(35b^3x^3d^3+135a\,b^2d^3x^2-30b^3c\,d^2x^2+189a^2b\,d^3x-108a\,b^2c\,d^2x+24b^3c^2dx+105a^3d^3-126a^2bc\,d^2+72a\,b^2c^2d-}{315d^4}$
trager	$\frac{2(35b^3d^4x^4+135a\,b^2d^4x^3+5b^3c\,d^3x^3+189a^2b\,d^4x^2+27a\,b^2c\,d^3x^2-6b^3c^2d^2x^2+105a^3d^4x+63a^2bc\,d^3x-36a\,b^2c^2d^2x+8b^3}{315d^4}$
risch	$\frac{2(35b^3d^4x^4+135a\,b^2d^4x^3+5b^3c\,d^3x^3+189a^2b\,d^4x^2+27a\,b^2c\,d^3x^2-6b^3c^2d^2x^2+105a^3d^4x+63a^2bc\,d^3x-36a\,b^2c^2d^2x+8b^3}{315d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/9*b^3*(d*x+c)^(9/2)+3/7*(a*d-b*c)*b^2*(d*x+c)^(7/2)+3/5*(a*d-b*c)^
2*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)^3*(d*x+c)^(3/2))
```

Maxima [A]
time = 0.30, size = 118, normalized size = 1.18

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^3-135\,(b^3c-ab^2d)(dx+c)^{\frac{7}{2}}+189\,(b^3c^2-2\,ab^2cd+a^2bd^2)(dx+c)^{\frac{5}{2}}-105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx+c)^{\frac{3}{2}}\right)}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^3 - 135*(b^3*c - a*b^2*d)*(d*x + c)^(7/2) + 189
*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(5/2) - 105*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))/d^4
```

Fricas [A]
time = 0.39, size = 164, normalized size = 1.64

$$\frac{2\,(35\,b^3d^4x^4-16\,b^3c^4+72\,ab^2c^3d-126\,a^2bc^2d^2+105\,a^3cd^3+5\,(b^3cd^3+27\,ab^2d^4)x^3-3\,(2\,b^3c^2d^2-9\,ab^2cd^3-63\,a^2bd^4)x^2+(8\,b^3c^3d-36\,ab^2c^2d^2+63\,a^2bcd^3+105\,a^3d^4)x)\sqrt{dx+c}}{315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*d^4*x^4 - 16*b^3*c^4 + 72*a*b^2*c^3*d - 126*a^2*b*c^2*d^2 + 1
05*a^3*c*d^3 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*x^3 - 3*(2*b^3*c^2*d^2 - 9*a*b^
2*c*d^3 - 63*a^2*b*d^4)*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*
d^3 + 105*a^3*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A]
time = 1.47, size = 146, normalized size = 1.46

$$\frac{2\left(\frac{b^3(c+dx)^{\frac{9}{2}}}{9d^3}+\frac{(c+dx)^{\frac{7}{2}}\cdot(3ab^2d-3b^3c)}{7d^3}+\frac{(c+dx)^{\frac{5}{2}}\cdot(3a^2bd^2-6ab^2cd+3b^3c^2)}{5d^3}+\frac{(c+dx)^{\frac{3}{2}}\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3d^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**3*(c + d*x)**(9/2)/(9*d**3) + (c + d*x)**(7/2)*(3*a*b**2*d - 3*b**3*c
)/(7*d**3) + (c + d*x)**(5/2)*(3*a**2*b*d**2 - 6*a*b**2*c*d + 3*b**3*c**2)/
(5*d**3) + (c + d*x)**(3/2)*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3)/(3*d**3))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(84) = 168.
time = 1.18, size = 322, normalized size = 3.22

$$\frac{2\left(315\sqrt{dx+c}\,a^3c+105\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^3+\frac{315\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^2bc}{d}+\frac{63\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ab^2c}{d^2}+\frac{63\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^2b}{d}+\frac{9\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)b^3c}{d^3}+\frac{27\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)ab^2}{d^2}+\frac{\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)b^3}{d^3}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^3*c + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c/d^2 + 63*(3*(
d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b/d + 9*(
5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt
(d*x + c)*c^3)*b^3*c/d^3 + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2/d^2 + (35*(d*x + c)^(9/
2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*
c^3 + 315*sqrt(d*x + c)*c^4)*b^3/d^3)/d
```

Mupad [B]
time = 0.07, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(9/2))/(9*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(7/2))/(
7*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(a*d - b*c)^2*(c
+ d*x)^(5/2))/(5*d^4)
```

3.1378 $\int (a+bx)^2\sqrt{c+dx}\,dx$

Optimal. Leaf size=71

$$\frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

```
[Out] 2/3*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^3-4/5*b*(-a*d+b*c)*(d*x+c)^(5/2)/d^3+2/7*b
^2*(d*x+c)^(7/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(5/2
))/(5*d^3) + (2*b^2*(c + d*x)^(7/2))/(7*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(-2c+3dx)+b^2(8c^2-12cdx+15d^2x^2)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x) + b^2*(8*c^2 - 12*
c*d*x + 15*d^2*x^2)))/(105*d^3)
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{4(ad-bc)b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^2(dx+c)^{\frac{3}{2}}}{3}}{d^3}$	56
default	$\frac{\frac{2b^2(dx+c)^{\frac{7}{2}}}{7}+\frac{4(ad-bc)b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)^2(dx+c)^{\frac{3}{2}}}{3}}{d^3}$	56
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(15b^2x^2d^2+42ab\,d^2x-12b^2cdx+35a^2d^2-28abcd+8b^2c^2\right)}{105d^3}$	63
trager	$\frac{2\left(15b^2d^3x^3+42ab\,d^3x^2+3b^2c\,d^2x^2+35a^2d^3x+14abc\,d^2x-4b^2c^2dx+35a^2c\,d^2-28ab\,c^2d+8b^2c^3\right)\sqrt{dx+c}}{105d^3}$	100
risch	$\frac{2\left(15b^2d^3x^3+42ab\,d^3x^2+3b^2c\,d^2x^2+35a^2d^3x+14abc\,d^2x-4b^2c^2dx+35a^2c\,d^2-28ab\,c^2d+8b^2c^3\right)\sqrt{dx+c}}{105d^3}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/7*b^2*(d*x+c)^(7/2)+2/5*(a*d-b*c)*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)^2*
(d*x+c)^(3/2))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.96

$$\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^2-42\,(b^2c-abd)(dx+c)^{\frac{5}{2}}+35\,(b^2c^2-2\,abcd+a^2d^2)(dx+c)^{\frac{3}{2}}\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(d*x + c)^(7/2)*b^2 - 42*(b^2*c - a*b*d)*(d*x + c)^(5/2) + 35*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))/d^3
```

Fricas [A]
time = 0.64, size = 99, normalized size = 1.39

$$\frac{2\,(15\,b^2d^3x^3+8\,b^2c^3-28\,abc^2d+35\,a^2cd^2+3\,(b^2cd^2+14\,abd^3)x^2-(4\,b^2c^2d-14\,abcd^2-35\,a^2d^3)x)\sqrt{dx+c}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d^3*x^3 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^2 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x)*sqrt(d
*x + c)/d^3
```

Sympy [A]
time = 1.12, size = 85, normalized size = 1.20

$$\frac{2\left(\frac{b^2(c+dx)^{\frac{7}{2}}}{7d^2}+\frac{(c+dx)^{\frac{5}{2}}\cdot(2abd-2b^2c)}{5d^2}+\frac{(c+dx)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}{3d^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**2*(c + d*x)**(7/2)/(7*d**2) + (c + d*x)**(5/2)*(2*a*b*d - 2*b**2*c)/(
5*d**2) + (c + d*x)**(3/2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*d**2))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(59) = 118.
time = 1.30, size = 200, normalized size = 2.82

$$\frac{2\left(105\sqrt{dx+c}\,a^2c+35\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^2+\frac{70\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)abc}{d}+\frac{7\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b^2c}{d^2}+\frac{14\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ab}{d}+\frac{3\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)b^2}{d^2}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a^2*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c/d^2 + 14*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b/d + 3*(5*(d*x +
c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)
*c^3)*b^2/d^2)/d
```

Mupad [B]
time = 0.24, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}\,\left(15\,b^2\,{\left(c+d\,x\right)}^2+35\,a^2\,d^2+35\,b^2\,c^2-42\,b^2\,c\,\left(c+d\,x\right)+42\,a\,b\,d\,\left(c+d\,x\right)-70\,a\,b\,c\,d\right)}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(15*b^2*(c + d*x)^2 + 35*a^2*d^2 + 35*b^2*c^2 - 42*b^2*c
*(c + d*x) + 42*a*b*d*(c + d*x) - 70*a*b*c*d))/(105*d^3)
```

3.1379 $\int (a+bx)\sqrt{c+dx}\,dx$

Optimal. Leaf size=42

$$-\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}$$

```
[Out] -2/3*(-a*d+b*c)*(d*x+c)^(3/2)/d^2+2/5*b*(d*x+c)^(5/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(3/2))/(3*d^2) + (2*b*(c + d*x)^(5/2))/(5*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{3/2}(-2bc+5ad+3bdx)}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x],x]
[Out] (2*(c + d*x)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x))/(15*d^2)
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(3bdx+5ad-2bc)}{15d^{2}}$	27
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)(dx+c)^{\frac{3}{2}}}{3}}{d^{2}}$	34
default	$\frac{\frac{2b(dx+c)^{\frac{5}{2}}}{5}+\frac{2(ad-bc)(dx+c)^{\frac{3}{2}}}{3}}{d^{2}}$	34
trager	$\frac{2(3b\,d^{2}x^{2}+5a\,d^{2}x+bcdx+5acd-2b\,c^{2})\sqrt{dx+c}}{15d^{2}}$	46
risch	$\frac{2(3b\,d^{2}x^{2}+5a\,d^{2}x+bcdx+5acd-2b\,c^{2})\sqrt{dx+c}}{15d^{2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/5*b*(d*x+c)^(5/2)+1/3*(a*d-b*c)*(d*x+c)^(3/2))
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.79

$$\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b-5\,(bc-ad)(dx+c)^{\frac{3}{2}}\right)}{15\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*(d*x + c)^(5/2)*b - 5*(b*c - a*d)*(d*x + c)^(3/2))/d^2
```

Fricas [A]
time = 0.64, size = 46, normalized size = 1.10

$$\frac{2\,(3\,bd^{2}x^{2}-2\,bc^{2}+5\,acd+(bcd+5\,ad^{2})x)\sqrt{dx+c}}{15\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b*d^2*x^2 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x)*sqrt(d*x + c)/
d^2
```

Sympy [A]
time = 0.88, size = 36, normalized size = 0.86

$$\frac{2\left(\frac{b(c+dx)^{\frac{5}{2}}}{5d}+\frac{(c+dx)^{\frac{3}{2}}(ad-bc)}{3d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b*(c + d*x)**(5/2)/(5*d) + (c + d*x)**(3/2)*(a*d - b*c)/(3*d))/d
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(34) = 68.
time = 1.13, size = 100, normalized size = 2.38

$$\frac{2\left(15\sqrt{dx+c}\,ac+5\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a+\frac{5\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)bc}{d}+\frac{\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b}{d}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a*c + 5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a + 5*
((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c/d + (3*(d*x + c)^(5/2) - 10*(d*x
+ c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b/d)/d
```

Mupad [B]
time = 0.04, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}\,\left(5\,a\,d-5\,b\,c+3\,b\,\left(c+d\,x\right)\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(5*a*d - 5*b*c + 3*b*(c + d*x)))/(15*d^2)
```

3.1380 $\int \sqrt{c+dx}\, dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{3/2}}{3d}$$

```
[Out] 2/3*(d*x+c)^(3/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+dx}\, dx = \frac{2(c+dx)^{3/2}}{3d}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
derivativedivides	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
default	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
trager	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13
risch	$\frac{2(dx+c)^{\frac{3}{2}}}{3d}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(d*x+c)^(3/2)/d
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(d*x + c)^(3/2)/d
```

Fricas [A]
time = 0.69, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(d*x + c)^(3/2)/d
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(c+dx)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*d)
```

Giac [A]
time = 1.54, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,(c+d\,x)^{3/2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

3.1381 $\int \frac{\sqrt{c+dx}}{a+bx}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(3/2)
+2*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 65, 214}

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{a+bx}\,dx &= \frac{2\sqrt{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b}\\ &= \frac{2\sqrt{c+dx}}{b}+\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{bd}\\ &= \frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x),x]
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[-(b*c) + a*d]])/b^(3/2)
```

Maple [A]
time = 0.16, size = 61, normalized size = 0.98

method	result	size
derivativedivides	$\frac{2\sqrt{dx+c}}{b}+\frac{2(-ad+bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{b\sqrt{(ad-bc)\,b}}$	61
default	$\frac{2\sqrt{dx+c}}{b}+\frac{2(-ad+bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{b\sqrt{(ad-bc)\,b}}$	61

risch	$\frac{2\sqrt{dx+c}}{b}-\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)ad}{b\sqrt{(ad-bc)\,b}}+\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)c}{\sqrt{(ad-bc)\,b}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)^(1/2)/b+2*(-a*d+b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)
/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.94, size = 143, normalized size = 2.31

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,\sqrt{dx+c}}{b},\,-\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx+c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*
c - a*d)/b))/(b*x + a)) + 2*sqrt(d*x + c))/b, -2*(sqrt(-(b*c - a*d)/b)*arct
an(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x + c))/b]
```

Sympy [A]

time = 1.88, size = 61, normalized size = 0.98

$$\frac{2\left(\frac{d\sqrt{c+dx}}{b}-\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x)/b - d*(a*d - b*c)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b)
)/(b**2*sqrt((a*d - b*c)/b)))/d
```

Giac [A]
time = 1.31, size = 62, normalized size = 1.00

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}+\frac{2\,\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*c - a*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + 2*sqrt(d*x + c)/b
```

Mupad [B]
time = 0.07, size = 50, normalized size = 0.81

$$\frac{2\sqrt{c+d\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(1/2))/b - (2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)
)*(a*d - b*c)^(1/2))/b^(3/2)
```

3.1382 $\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\sqrt{c+dx}}{b(a+bx)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

```
[Out] -d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
-(d*x+c)^(1/2)/b/(b*x+a)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {43, 65, 214}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx = -\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b}$$
$$= -\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b}$$
$$= -\frac{\sqrt{c+dx}}{b(a+bx)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A]
time = 0.19, size = 69, normalized size = 0.99

$$-\frac{\sqrt{c+dx}}{b(a+bx)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b
*c) + a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

Maple [A]
time = 0.16, size = 73, normalized size = 1.04

method	result	size
derivativedivides	$2d\left(-\frac{\sqrt{dx+c}}{2b((dx+c)b+ad-bc)}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2b\sqrt{(ad-bc)\,b}}\right)$	73
default	$2d\left(-\frac{\sqrt{dx+c}}{2b((dx+c)b+ad-bc)}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2b\sqrt{(ad-bc)\,b}}\right)$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/2/b*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+1/2/b/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.73, size = 232, normalized size = 3.31

$$\left[\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(b^2c-abd)\sqrt{dx+c}}{2\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x)},\ \frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(b^2c-abd)\sqrt{dx+c}}{ab^3c-a^2b^2d+(b^4c-ab^3d)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b
^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(b^2*c - a*b*d)*sqrt(d*x + c))/
(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x), (sqrt(-b^2*c + a*b*d)*(b*d*x +
 a*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (b^2*c - a
*b*d)*sqrt(d*x + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(58) = 116$.
time = 21.74, size = 573, normalized size = 8.19

$$-\frac{2ad^2\sqrt{c+dx}}{2a^2bd^2-2ab^2cd+2ab^2d^2x-2b^3cdx}+\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{2cd\sqrt{c+dx}}{2a^2d^2-2abcd+2abd^2x-2b^2cdx}+\frac{2d\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad}{b}-c}}\right)}{b^2\sqrt{\frac{ad}{b}-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2
*b**3*c*d*x) + a*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b
*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(
-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*
```

```
b) - c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)*
*3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d -
b*c)**3)) + sqrt(c + d*x))/2 + c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**
2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**
2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*c*d*sqrt(c + d*x)
/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2*d*atan(sqrt(c
+ d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c))
```

Giac [A]
time = 1.09, size = 72, normalized size = 1.03

$$\frac{d \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\, b} - \frac{\sqrt{dx+c}\, d}{((dx+c)b-bc+ad)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - s
qrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*b)
```

Mupad [B]
time = 0.24, size = 61, normalized size = 0.87

$$\frac{d \operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}} - \frac{d\,\sqrt{c+d\,x}}{d\,x\,b^2+a\,d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^2,x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(3/2)*(a*d - b*c)^
(1/2)) - (d*(c + d*x)^(1/2))/(a*b*d + b^2*d*x)
```

3.1383 $\int \frac{\sqrt{c+dx}}{(a+bx)^3} dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(3/2)-1/2*(d*x+c)^(1/2)/b/(b*x+a)^2-1/4*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b*x+a
)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)}-\frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/2*Sqrt[c + d*x]/(b*(a + b*x)^2) - (d*Sqrt[c + d*x])/(4*b*(b*c - a*d)*(a
+ b*x)) + (d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(3/2)
*(b*c - a*d)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}+\frac{d\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4b}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}-\frac{d\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2b(a+bx)^2}-\frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 99, normalized size = 0.90

$$\frac{\frac{\sqrt{b}\,\sqrt{c+dx}\,(2bc-ad+bdx)}{(-bc+ad)(a+bx)^2}+\frac{d^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x]*(2*b*c - a*d + b*d*x))/((-(b*c) + a*d)*(a + b*x)^2)
 + (d^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^
(3/2))/(4*b^(3/2))
```

Maple [A]
time = 0.14, size = 106, normalized size = 0.96

method	result	size
derivativedivides	$2d^2\left(\frac{\frac{(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\sqrt{dx+c}}{8b}}{((dx+c)b+ad-bc)^2}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8(ad-bc)b\sqrt{(ad-bc)b}}\right)$	106
default	$2d^2\left(\frac{\frac{(dx+c)^{\frac{3}{2}}}{8ad-8bc}-\frac{\sqrt{dx+c}}{8b}}{((dx+c)b+ad-bc)^2}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8(ad-bc)b\sqrt{(ad-bc)b}}\right)$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*((1/8/(a*d-b*c)*(d*x+c)^(3/2)-1/8*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c
)^2+1/8/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b
)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(90) = 180.
time = 0.51, size = 456, normalized size = 4.15

$$\left[-\frac{(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(2\,b^3c^2-3\,ab^2cd+a^2bd^2+(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{8\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^2+2\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)x)},-\frac{(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(2\,b^3c^2-3\,ab^2cd+a^2bd^2+(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{4\,(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2+(b^6c^2-2\,ab^5cd+a^2b^4d^2)x^2+2\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x
 + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(2*b^3
*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
```

```
*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x), -1/4*((b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*
d)*sqrt(d*x + c)/(b*d*x + b*c)) + (2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b
^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^
4*c*d + a^3*b^3*d^2)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. $2(88) = 176$.
time = 93.91, size = 1658, normalized size = 15.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**3,x)
```

```
[Out] -10*a**2*d**4*sqrt(c + d*x)/(8*a**4*b*d**4 - 16*a**3*b**2*c*d**3 + 16*a**3*
b**2*d**4*x - 48*a**2*b**3*c*d**3*x + 8*a**2*b**3*d**2*(c + d*x)**2 + 16*a*
b**4*c**3*d + 48*a*b**4*c**2*d**2*x - 16*a*b**4*c*d*(c + d*x)**2 - 8*b**5*c
**4 - 16*b**5*c**3*d*x + 8*b**5*c**2*(c + d*x)**2) + 20*a*c*d**3*sqrt(c + d
*x)/(8*a**4*d**4 - 16*a**3*b*c*d**3 + 16*a**3*b*d**4*x - 48*a**2*b**2*c*d**
3*x + 8*a**2*b**2*d**2*(c + d*x)**2 + 16*a*b**3*c**3*d + 48*a*b**3*c**2*d**
2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*
c**2*(c + d*x)**2) - 6*a*d**3*(c + d*x)**(3/2)/(8*a**4*d**4 - 16*a**3*b*c*d
**3 + 16*a**3*b*d**4*x - 48*a**2*b**2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)
**2 + 16*a*b**3*c**3*d + 48*a*b**3*c**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2
 - 8*b**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*c**2*(c + d*x)**2) + 3*a*d**3*sq
rt(-1/(b*(a*d - b*c)**5))*log(-a**3*d**3*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a*
*2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**5)) - 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d -
 b*c)**5)) + b**3*c**3*sqrt(-1/(b*(a*d - b*c)**5)) + sqrt(c + d*x))/(8*b) -
 3*a*d**3*sqrt(-1/(b*(a*d - b*c)**5))*log(a**3*d**3*sqrt(-1/(b*(a*d - b*c)*
*5)) - 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a*b**2*c**2*d*sqrt(-
1/(b*(a*d - b*c)**5)) - b**3*c**3*sqrt(-1/(b*(a*d - b*c)**5)) + sqrt(c + d*
x))/(8*b) - 10*b*c**2*d**2*sqrt(c + d*x)/(8*a**4*d**4 - 16*a**3*b*c*d**3 +
16*a**3*b*d**4*x - 48*a**2*b**2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)**2 +
16*a*b**3*c**3*d + 48*a*b**3*c**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b
**4*c**4 - 16*b**4*c**3*d*x + 8*b**4*c**2*(c + d*x)**2) + 6*b*c*d**2*(c + d
*x)**(3/2)/(8*a**4*d**4 - 16*a**3*b*c*d**3 + 16*a**3*b*d**4*x - 48*a**2*b**
2*c*d**3*x + 8*a**2*b**2*d**2*(c + d*x)**2 + 16*a*b**3*c**3*d + 48*a*b**3*c
**2*d**2*x - 16*a*b**3*c*d*(c + d*x)**2 - 8*b**4*c**4 - 16*b**4*c**3*d*x +
8*b**4*c**2*(c + d*x)**2) - 3*c*d**2*sqrt(-1/(b*(a*d - b*c)**5))*log(-a**3*
d**3*sqrt(-1/(b*(a*d - b*c)**5)) + 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d - b*c)**
5)) - 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d - b*c)**5)) + b**3*c**3*sqrt(-1/(b*(a
*d - b*c)**5)) + sqrt(c + d*x))/8 + 3*c*d**2*sqrt(-1/(b*(a*d - b*c)**5))*lo
g(a**3*d**3*sqrt(-1/(b*(a*d - b*c)**5)) - 3*a**2*b*c*d**2*sqrt(-1/(b*(a*d -
```

```
b*c)**5)) + 3*a*b**2*c**2*d*sqrt(-1/(b*(a*d - b*c)**5)) - b**3*c**3*sqrt(-
1/(b*(a*d - b*c)**5)) + sqrt(c + d*x))/8 + 2*d**2*sqrt(c + d*x)/(2*a**2*b*d
**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) - d**2*sqrt(-1/(b*(a*d
 - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) + d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d -
 b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(
a*d - b*c)**3)) + sqrt(c + d*x))/(2*b)
```

Giac [A]
time = 1.15, size = 126, normalized size = 1.15

$$-\frac{d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^2c-abd)\sqrt{-b^2c+abd}}-\frac{(dx+c)^{\frac{3}{2}}bd^2+\sqrt{dx+c}\,bcd^2-\sqrt{dx+c}\,ad^3}{4\,(b^2c-abd)\big((dx+c)b-bc+ad\big)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt
(-b^2*c + a*b*d)) - 1/4*((d*x + c)^(3/2)*b*d^2 + sqrt(d*x + c)*b*c*d^2 - sq
rt(d*x + c)*a*d^3)/((b^2*c - a*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [B]
time = 0.30, size = 135, normalized size = 1.23

$$\frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{d^2\,\sqrt{c+d\,x}}{4\,b}-\frac{d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,\left(a\,d-b\,c\right)}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^3,x)
```

```
[Out] (d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(3/2)*(a*d - b
*c)^(3/2)) - ((d^2*(c + d*x)^(1/2))/(4*b) - (d^2*(c + d*x)^(3/2))/(4*(a*d -
 b*c)))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)
```

3.1384 $\int \frac{\sqrt{c+dx}}{(a+bx)^4} dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}$$

```
[Out] -1/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(5/2)-1/3*(d*x+c)^(1/2)/b/(b*x+a)^3-1/12*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b*x
+a)^2+1/8*d^2*(d*x+c)^(1/2)/b/(-a*d+b*c)^2/(b*x+a)
```

Rubi [A]

time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] -1/3*Sqrt[c + d*x]/(b*(a + b*x)^3) - (d*Sqrt[c + d*x])/(12*b*(b*c - a*d)*(a
 + b*x)^2) + (d^2*Sqrt[c + d*x])/(8*b*(b*c - a*d)^2*(a + b*x)) - (d^3*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx &= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}+\frac{d\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6b} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}-\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b(bc-ad)} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}+\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b(bc-ad)^2} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}+\frac{d^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b(bc-ad)^2} \\
&= -\frac{\sqrt{c+dx}}{3b(a+bx)^3}-\frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2}+\frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)}-\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.64, size = 131, normalized size = 0.90

$$\frac{\sqrt{c+dx}\,(-3a^2d^2+2abd(7c+4dx)+b^2(-8c^2-2cdx+3d^2x^2))}{24b(bc-ad)^2(a+bx)^3}+\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8b^{3/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (Sqrt[c + d*x]*(-3*a^2*d^2 + 2*a*b*d*(7*c + 4*d*x) + b^2*(-8*c^2 - 2*c*d*x
+ 3*d^2*x^2)))/(24*b*(b*c - a*d)^2*(a + b*x)^3) + (d^3*ArcTan[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*b^(3/2)*(-(b*c) + a*d)^(5/2))
```

Maple [A]
time = 0.15, size = 152, normalized size = 1.04

method	result	size
derivativedivides	$2d^3\left(\frac{\frac{b(dx+c)^{\frac{5}{2}}}{16a^2d^2-32abcd+16b^2c^2}+\frac{(dx+c)^{\frac{3}{2}}}{6ad-6bc}-\frac{\sqrt{dx+c}}{16b}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16b(a^2d^2-2abcd+b^2c^2)\sqrt{(ad-bc)b}}\right)$	152
default	$2d^3\left(\frac{\frac{b(dx+c)^{\frac{5}{2}}}{16a^2d^2-32abcd+16b^2c^2}+\frac{(dx+c)^{\frac{3}{2}}}{6ad-6bc}-\frac{\sqrt{dx+c}}{16b}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16b(a^2d^2-2abcd+b^2c^2)\sqrt{(ad-bc)b}}\right)$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*((1/16*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+1/6/(a*d-b*c)*(d*x
+c)^(3/2)-1/16*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c)^3+1/16/b/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(122) = 244$.
time = 0.63, size = 785, normalized size = 5.38

$$\left[\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(8\,b^4c^3-22\,ab^3c^2d+17\,a^2b^2cd^2-3\,a^3bd^3-3\,(b^4cd^2-ab^3d^3)x^2+2\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^3+3\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^2+3\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x)},\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(8\,b^4c^3-22\,ab^3c^2d+17\,a^2b^2cd^2-3\,a^3bd^3-3\,(b^4cd^2-ab^3d^3)x^2+2\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^3+3\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^2+3\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
```

```
/(b*x + a)) - 2*(8*b^4*c^3 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^
3 - 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^
2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^
3 + 3*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3
*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x), 1/24*(
3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a
*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3
 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^3 - 3*(b^4*c*d^2 - a*b^3*d
^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^3 + 3*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3*(a^2*b^6*c^3 - 3*a^3*b^5*
c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.47, size = 207, normalized size = 1.42

$$\frac{d^3 \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{5}{2}}b^2d^3-8\,(dx+c)^{\frac{3}{2}}b^2cd^3-3\,\sqrt{dx+c}\,b^2c^2d^3+8\,(dx+c)^{\frac{3}{2}}abd^4+6\,\sqrt{dx+c}\,abcd^4-3\,\sqrt{dx+c}\,a^2d^5}{24\,(b^3c^2-2\,ab^2cd+a^2bd^2)((dx+c)b-bc+ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 - 8*
(d*x + c)^(3/2)*b^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 + 8*(d*x + c)^(3/2)
*a*b*d^4 + 6*sqrt(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*((d*x + c)*b - b*c + a*d)^3)
```

Mupad [B]
time = 0.37, size = 207, normalized size = 1.42

$$\frac{\frac{d^3\,{\left(c+d\,x\right)}^{3/2}}{3\,\left(a\,d-b\,c\right)}-\frac{d^3\,\sqrt{c+d\,x}}{8\,b}+\frac{b\,d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,{\left(a\,d-b\,c\right)}^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}+\frac{d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^4,x)
```

```
[Out] ((d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)) - (d^3*(c + d*x)^(1/2))/(8*b) + (b*d
^3*(c + d*x)^(5/2))/(8*(a*d - b*c)^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (d^3*atan((b^(1/2)*(c + d*
x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(3/2)*(a*d - b*c)^(5/2))
```

3.1385 $\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}}$$

```
[Out] 5/64*d^4*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(7/2)-1/4*(d*x+c)^(1/2)/b/(b*x+a)^4-1/24*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b*x
+a)^3+5/96*d^2*(d*x+c)^(1/2)/b/(-a*d+b*c)^2/(b*x+a)^2-5/64*d^3*(d*x+c)^(1/2
)/b/(-a*d+b*c)^3/(b*x+a)
```

Rubi [A]
time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}}-\frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3}+\frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2}-\frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)}-\frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -1/4*Sqrt[c + d*x]/(b*(a + b*x)^4) - (d*Sqrt[c + d*x])/(24*b*(b*c - a*d)*(a
 + b*x)^3) + (5*d^2*Sqrt[c + d*x])/(96*b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^
3*Sqrt[c + d*x])/(64*b*(b*c - a*d)^3*(a + b*x)) + (5*d^4*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(64*b^(3/2)*(b*c - a*d)^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx = -\frac{\sqrt{c+dx}}{4b(a+bx)^4}+\frac{d\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8b}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}-\frac{(5d^2)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}+\frac{(5d^3)\int \frac{1}{(a+bx)^2\sqrt{c+}}}{64b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+b}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+b}$$

$$= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+b}$$

Mathematica [A]
time = 1.00, size = 170, normalized size = 0.93

$$\frac{\sqrt{c+dx}\,(-15a^3d^3+a^2bd^2(118c+73dx)+ab^2d(-136c^2-36cdx+55d^2x^2)+b^3(48c^3+8c^2dx-10cd^2x^2+15d^3x^3))}{192b(-bc+ad)^3(a+bx)^4}+\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{64b^{3/2}(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] (Sqrt[c + d*x]*(-15*a^3*d^3 + a^2*b*d^2*(118*c + 73*d*x) + a*b^2*d*(-136*c^
2 - 36*c*d*x + 55*d^2*x^2) + b^3*(48*c^3 + 8*c^2*d*x - 10*c*d^2*x^2 + 15*d^
```

```
3*x^3)))/(192*b*(-(b*c) + a*d)^3*(a + b*x)^4) + (5*d^4*ArcTan[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[-(b*c) + a*d]])/(64*b^(3/2)*(-(b*c) + a*d)^(7/2))
```

Maple [A]
time = 0.14, size = 217, normalized size = 1.19

method	result
derivativedivides	$2d^4\left(\frac{\frac{5b^2(dx+c)^{\frac{7}{2}}}{128\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{55b(dx+c)^{\frac{5}{2}}}{384\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{73(dx+c)^{\frac{3}{2}}}{384(ad-bc)}-\frac{5\sqrt{dx+c}}{128b}}{((dx+c)b+ad-bc)^4}+\frac{5\arctan}{128b(a^3d^3-3a^2bc\,d^2}\right.$
default	$2d^4\left(\frac{\frac{5b^2(dx+c)^{\frac{7}{2}}}{128\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{55b(dx+c)^{\frac{5}{2}}}{384\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{73(dx+c)^{\frac{3}{2}}}{384(ad-bc)}-\frac{5\sqrt{dx+c}}{128b}}{((dx+c)b+ad-bc)^4}+\frac{5\arctan}{128b(a^3d^3-3a^2bc\,d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^4*((5/128*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d*x+c)^(7/
2)+55/384*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+73/384/(a*d-b*c)*(d*x
+c)^(3/2)-5/128*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c)^4+5/128/b/(a^3*d^3-3*a
^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2
)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(154) = 308$.
time = 0.50, size = 1176, normalized size = 6.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 184*a*b^4*c^3*d + 254*
a^2*b^3*c^2*d^2 - 133*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*
d^4)*x^3 - 5*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5
*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3
 + a^8*b^2*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*
c*d^3 + a^4*b^6*d^4)*x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d
^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d
+ 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 -
 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x), -
1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)
/(b*d*x + b*c)) + (48*b^5*c^4 - 184*a*b^4*c^3*d + 254*a^2*b^3*c^2*d^2 - 133
*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*x^3 - 5*(2*b^5*c
^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5*c^3*d - 44*a*b^4*c^2
*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4 + (b^1
0*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3
+ a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 -
 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*
a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(154) = 308.
time = 2.45, size = 311, normalized size = 1.71

$$-\frac{5\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^3d^4-55\,(dx+c)^{\frac{5}{2}}b^3cd^4+73\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+15\,\sqrt{dx+c}\,b^3c^3d^4+55\,(dx+c)^{\frac{5}{2}}ab^2d^5-146\,(dx+c)^{\frac{3}{2}}ab^2cd^5-45\,\sqrt{dx+c}\,ab^2c^2d^5+73\,(dx+c)^{\frac{3}{2}}a^2bd^6+45\,\sqrt{dx+c}\,a^2bcd^6-15\,\sqrt{dx+c}\,a^3d^7}{192\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="giac")
```

[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x + c)^(7/2)*b^3*d^4 - 55*(d*x + c)^(5/2)*b^3*c*d^4 + 73*(d*x + c)^(3/2)*b^3*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^4 + 55*(d*x + c)^(5/2)*a*b^2*d^5 - 146*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqrt(d*x + c)*a*b^2*c^2*d^5 + 73*(d*x + c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^7)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*((d*x + c)*b - b*c + a*d)^4)

Mupad [B]
time = 0.22, size = 297, normalized size = 1.63

$$\frac{\frac{73\,d^4\,{\left(c+d\,x\right)}^{3/2}}{192\,\left(a\,d-b\,c\right)}-\frac{5\,d^4\,\sqrt{c+d\,x}}{64\,b}+\frac{5\,b^2\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^3}+\frac{55\,b\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(1/2)/(a + b*x)^5,x)

[Out] ((73*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (5*d^4*(c + d*x)^(1/2))/(64*b) + (5*b^2*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^3) + (55*b*d^4*(c + d*x)^(5/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + (5*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(3/2)*(a*d - b*c)^(7/2))

3.1386 $\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx$

Optimal. Leaf size=218

$$-\frac{\sqrt{c+dx}}{5b(a+bx)^5}-\frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4}+\frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3}-\frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2}+\frac{7d^4\sqrt{c+}}{128b(bc-ad)^4}$$

```
[Out] -7/128*d^5*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*
c)^(9/2)-1/5*(d*x+c)^(1/2)/b/(b*x+a)^5-1/40*d*(d*x+c)^(1/2)/b/(-a*d+b*c)/(b
*x+a)^4+7/240*d^2*(d*x+c)^(1/2)/b/(-a*d+b*c)^2/(b*x+a)^3-7/192*d^3*(d*x+c)^
(1/2)/b/(-a*d+b*c)^3/(b*x+a)^2+7/128*d^4*(d*x+c)^(1/2)/b/(-a*d+b*c)^4/(b*x+
a)
```

Rubi [A]
time = 0.10, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{7d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c+dx}}{40b(a+bx)^4(bc-ad)}-\frac{\sqrt{c+dx}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] -1/5*Sqrt[c + d*x]/(b*(a + b*x)^5) - (d*Sqrt[c + d*x])/(40*b*(b*c - a*d)*(a
 + b*x)^4) + (7*d^2*Sqrt[c + d*x])/(240*b*(b*c - a*d)^2*(a + b*x)^3) - (7*d
^3*Sqrt[c + d*x])/(192*b*(b*c - a*d)^3*(a + b*x)^2) + (7*d^4*Sqrt[c + d*x])
/(128*b*(b*c - a*d)^4*(a + b*x)) - (7*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[b*c - a*d]])/(128*b^(3/2)*(b*c - a*d)^(9/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx = -\frac{\sqrt{c+dx}}{5b(a+bx)^5} + \frac{d\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx}{10b}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} - \frac{(7d^2)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} + \frac{(7d^3)\int \frac{1}{(a+bx)^3\sqrt{c+\cdots}}}{96b(bc-ad)^2}\cdots$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}$$

$$= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+\cdots}$$

Mathematica [A]
time = 1.41, size = 224, normalized size = 1.03

$$\frac{\sqrt{c+dx}\left(-105a^4d^4+10a^3bd^3(121c+79dx)+2a^2b^2d^2(-1052c^2-289cdx+448d^2x^2)+2ab^3d(744c^3+128c^2dx-161cd^2x^2+245d^3x^3)+b^4(-384c^4-48c^3dx+56c^2d^2x^2-70cd^3x^3+105d^4x^4)\right)}{1920b(bc-ad)^4(a+bx)^5}+\frac{7d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{128b^{3/2}(-bc+ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (Sqrt[c + d*x]*(-105*a^4*d^4 + 10*a^3*b*d^3*(121*c + 79*d*x) + 2*a^2*b^2*d^
2*(-1052*c^2 - 289*c*d*x + 448*d^2*x^2) + 2*a*b^3*d*(744*c^3 + 128*c^2*d*x
- 161*c*d^2*x^2 + 245*d^3*x^3) + b^4*(-384*c^4 - 48*c^3*d*x + 56*c^2*d^2*x^
2 - 70*c*d^3*x^3 + 105*d^4*x^4)))/(1920*b*(b*c - a*d)^4*(a + b*x)^5) + (7*d
^5*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(128*b^(3/2)*(-(b*c)
 + a*d)^(9/2))
```

Maple [A]
time = 0.16, size = 293, normalized size = 1.34

method	result
derivativedivides	$2d^5\left(\frac{\frac{7b^3(dx+c)^{\frac{9}{2}}}{256\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{49b^2(dx+c)^{\frac{7}{2}}}{384\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7b(dx+c)^{\frac{5}{2}}}{30\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{79(}{384}}{((dx+c)b+ad-bc)^5}\right.$
default	$2d^5\left(\frac{\frac{7b^3(dx+c)^{\frac{9}{2}}}{256\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{49b^2(dx+c)^{\frac{7}{2}}}{384\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7b(dx+c)^{\frac{5}{2}}}{30\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{79(}{384}}{((dx+c)b+ad-bc)^5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^5*((7/256*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^
4*c^4)*(d*x+c)^(9/2)+49/384*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)*(d*x+c)^(7/2)+7/30*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+79/384/(a
*d-b*c)*(d*x+c)^(3/2)-7/256*(d*x+c)^(1/2)/b)/((d*x+c)*b+a*d-b*c)^5+7/256/b/
(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/((a*d-b*c)*
b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

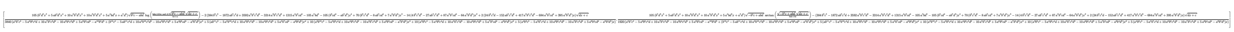
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(186) = 372$.
time = 0.45, size = 1673, normalized size = 7.67



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(384*b^6*c^5
- 1872*a*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4
*b^2*c*d^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*
d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2
*d^3 + 87*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5
*c^3*d^2 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sq
rt(d*x + c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b
^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d +
10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x
^5 + 5*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2
*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^
4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d
^5)*x^3 + 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b
^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^
7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b
^3*d^5)*x), 1/1920*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3
 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)*arcta
n(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (384*b^6*c^5 - 1872*a
*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4*b^2*c*d
^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 8
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5*c^3*d^2
 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sqrt(d*x +
 c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d
^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b
^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x^5 + 5*(
a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5
*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10
*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*x^3
+ 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d
^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d
+ 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*
x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(186) = 372.
time = 2.11, size = 432, normalized size = 1.98

$$\frac{7 d^5 \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^2c+abd}}\right)}{128\,(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{9}{2}}b^4d^5-490\,(dx+c)^{\frac{7}{2}}b^4cd^5+896\,(dx+c)^{\frac{5}{2}}b^4c^2d^5-790\,(dx+c)^{\frac{3}{2}}b^4c^3d^5-105\sqrt{dx+c}\,b^4c^4d^5+490\,(dx+c)^{\frac{7}{2}}ab^3d^6-1792\,(dx+c)^{\frac{5}{2}}ab^3cd^6+2370\,(dx+c)^{\frac{3}{2}}ab^3c^2d^6+420\sqrt{dx+c}\,ab^3c^3d^6+896\,(dx+c)^{\frac{5}{2}}a^2b^2d^7-2370\,(dx+c)^{\frac{3}{2}}a^2b^2cd^7-630\sqrt{dx+c}\,a^2b^2c^2d^7+790\,(dx+c)^{\frac{3}{2}}a^3bd^8+420\sqrt{dx+c}\,a^3bcd^8-105\sqrt{dx+c}\,a^4d^9}{1920\,(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4)((dx+c)b-bc+ad)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 7/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*c + a*b*
d)) + 1/1920*(105*(d*x + c)^(9/2)*b^4*d^5 - 490*(d*x + c)^(7/2)*b^4*c*d^5 +
 896*(d*x + c)^(5/2)*b^4*c^2*d^5 - 790*(d*x + c)^(3/2)*b^4*c^3*d^5 - 105*sq
rt(d*x + c)*b^4*c^4*d^5 + 490*(d*x + c)^(7/2)*a*b^3*d^6 - 1792*(d*x + c)^(5
/2)*a*b^3*c*d^6 + 2370*(d*x + c)^(3/2)*a*b^3*c^2*d^6 + 420*sqrt(d*x + c)*a*
b^3*c^3*d^6 + 896*(d*x + c)^(5/2)*a^2*b^2*d^7 - 2370*(d*x + c)^(3/2)*a^2*b^
2*c*d^7 - 630*sqrt(d*x + c)*a^2*b^2*c^2*d^7 + 790*(d*x + c)^(3/2)*a^3*b*d^8
 + 420*sqrt(d*x + c)*a^3*b*c*d^8 - 105*sqrt(d*x + c)*a^4*d^9)/((b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*((d*x + c)*
b - b*c + a*d)^5)
```

Mupad [**B**]
time = 0.49, size = 401, normalized size = 1.84

$$\frac{\frac{79\,d^5\,{(c+d\,x)}^{3/2}}{192\,(a\,d-b\,c)}-\frac{7\,d^5\,\sqrt{c+d\,x}}{128\,b}+\frac{49\,b^2\,d^5\,{(c+d\,x)}^{7/2}}{192\,{(a\,d-b\,c)}^3}+\frac{7\,b^3\,d^5\,{(c+d\,x)}^{9/2}}{128\,{(a\,d-b\,c)}^4}+\frac{7\,b\,d^5\,{(c+d\,x)}^{5/2}}{15\,{(a\,d-b\,c)}^2}}{b^5\,{(c+d\,x)}^5-{(c+d\,x)}^2\,(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3)-(5\,b^5\,c-5\,a\,b^4\,d)\,{(c+d\,x)}^4+a^5\,d^5-b^5\,c^5+{(c+d\,x)}^3\,(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2)+(c+d\,x)\,(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{7\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{3/2}\,{(a\,d-b\,c)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^6,x)
```

```
[Out] ((79*d^5*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (7*d^5*(c + d*x)^(1/2))/(128*
b) + (49*b^2*d^5*(c + d*x)^(7/2))/(192*(a*d - b*c)^3) + (7*b^3*d^5*(c + d*x
)^(9/2))/(128*(a*d - b*c)^4) + (7*b*d^5*(c + d*x)^(5/2))/(15*(a*d - b*c)^2)
)/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*
c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5
*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)
*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^
4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*
b*c*d^4) + (7*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b
^(3/2)*(a*d - b*c)^(9/2))
```

3.1387 $\int (a+bx)^5(c+dx)^{3/2}\,dx$

Optimal. Leaf size=158

$$-\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/2}}{9d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{11/2}}{11d^6}$$

```
[Out] -2/5*(-a*d+b*c)^5*(d*x+c)^(5/2)/d^6+10/7*b*(-a*d+b*c)^4*(d*x+c)^(7/2)/d^6-2
0/9*b^2*(-a*d+b*c)^3*(d*x+c)^(9/2)/d^6+20/11*b^3*(-a*d+b*c)^2*(d*x+c)^(11/2
)/d^6-10/13*b^4*(-a*d+b*c)*(d*x+c)^(13/2)/d^6+2/15*b^5*(d*x+c)^(15/2)/d^6
```

Rubi [A]

time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c+dx)^{5/2}(bc-ad)^5}{5d^6}+\frac{2b^5(c+dx)^{15/2}}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(5/2))/(5*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(7/2))/(7*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(
b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(13
/2))/(13*d^6) + (2*b^5*(c + d*x)^(15/2))/(15*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{3/2}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{3/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{5/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx}{d^5}\right.$$
$$=-\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/}}{9d^6}$$

Mathematica [A]

time = 0.13, size = 217, normalized size = 1.37

$$\frac{2(c+dx)^{5/2}\left(9009a^5d^5+6435a^4bd^4(-2c+5dx)+1430a^3b^2d^3(8c^2-20cdx+35d^2x^2)+390a^2b^3d^2(-16c^3+40c^2dx-70cd^2x^2+105d^3x^3)+15ab^4d(128c^4-320c^3dx+560c^2d^2x^2-840cd^3x^3+1155d^4x^4)+b^5(-256c^5+640c^4dx-1120c^3d^2x^2+1680c^2d^3x^3-2310cd^4x^4+3003d^5x^5)\right)}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(9009*a^5*d^5 + 6435*a^4*b*d^4*(-2*c + 5*d*x) + 1430*a^3
*b^2*d^3*(8*c^2 - 20*c*d*x + 35*d^2*x^2) + 390*a^2*b^3*d^2*(-16*c^3 + 40*c^
2*d*x - 70*c*d^2*x^2 + 105*d^3*x^3) + 15*a*b^4*d*(128*c^4 - 320*c^3*d*x + 5
60*c^2*d^2*x^2 - 840*c*d^3*x^3 + 1155*d^4*x^4) + b^5*(-256*c^5 + 640*c^4*d*
x - 1120*c^3*d^2*x^2 + 1680*c^2*d^3*x^3 - 2310*c*d^4*x^4 + 3003*d^5*x^5)))/
(45045*d^6)
```

Maple [A]
time = 0.16, size = 122, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{15}{2}}}{15}+\frac{10(ad-bc)b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{10(ad-bc)^4b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^5(d}{5}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{15}{2}}}{15}+\frac{10(ad-bc)b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{10(ad-bc)^4b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^5(d}{5}}{d^6}$
gosper	$2(dx+c)^{\frac{5}{2}}(3003b^5x^5d^5+17325a\,b^4d^5x^4-2310b^5c\,d^4x^4+40950a^2b^3d^5x^3-12600a\,b^4c\,d^4x^3+1680b^5c^2d^3x^3+50050a^3b^2d$
trager	$2(3003b^5d^7x^7+17325a\,b^4d^7x^6+3696b^5c\,d^6x^6+40950a^2b^3d^7x^5+22050a\,b^4c\,d^6x^5+63b^5c^2d^5x^5+50050a^3b^2d^7x^4+54600$
risch	$2(3003b^5d^7x^7+17325a\,b^4d^7x^6+3696b^5c\,d^6x^6+40950a^2b^3d^7x^5+22050a\,b^4c\,d^6x^5+63b^5c^2d^5x^5+50050a^3b^2d^7x^4+54600$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/15*b^5*(d*x+c)^(15/2)+5/13*(a*d-b*c)*b^4*(d*x+c)^(13/2)+10/11*(a*d
-b*c)^2*b^3*(d*x+c)^(11/2)+10/9*(a*d-b*c)^3*b^2*(d*x+c)^(9/2)+5/7*(a*d-b*c)
^4*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^5*(d*x+c)^(5/2))
```

Maxima [A]
time = 0.30, size = 259, normalized size = 1.64

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^5-17325\,(b^5c-ab^4d)(dx+c)^{\frac{13}{2}}+40950\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)(dx+c)^{\frac{11}{2}}-50050\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{9}{2}}+32175\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)(dx+c)^{\frac{7}{2}}-9009\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)(dx+c)^{\frac{5}{2}}\right)}{45045\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^5 - 17325*(b^5*c - a*b^4*d)*(d*x + c)^(13/
2) + 40950*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(11/2) - 50050*(
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(9/2) +
32175*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*(d*x + c)^(7/2) - 9009*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2
 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(5/2))/d^6
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(134) = 268$.
time = 0.49, size = 418, normalized size = 2.65

$$\frac{2\,\left(3003\,b^{5}d^{7}x^{7}-256\,b^{5}c^{7}+1920\,ab^{4}c^{6}d-6240\,a^{2}b^{3}c^{5}d^{2}+11440\,a^{3}b^{2}c^{4}d^{3}-12870\,a^{4}bc^{3}d^{4}+9009\,a^{5}c^{2}d^{5}+231\,\left(16\,b^{5}cd^{6}+75\,ab^{4}d^{7}\right)x^{6}+63\,\left(b^{5}c^{2}d^{5}+350\,ab^{4}cd^{6}+650\,a^{2}b^{3}d^{7}\right)x^{5}-35\,\left(2\,b^{5}c^{3}d^{4}-15\,ab^{4}c^{2}d^{5}-1560\,a^{2}b^{3}cd^{6}-1430\,a^{3}b^{2}d^{7}\right)x^{4}+5\,\left(16\,b^{5}c^{4}d^{3}-120\,ab^{4}c^{3}d^{4}+390\,a^{2}b^{3}c^{2}d^{5}+14300\,a^{3}b^{2}cd^{6}+6435\,a^{4}bd^{7}\right)x^{3}-3\,\left(32\,b^{5}c^{5}d^{2}-240\,ab^{4}c^{4}d^{3}+780\,a^{2}b^{3}c^{3}d^{4}-1430\,a^{3}b^{2}c^{2}d^{5}-17160\,a^{4}bcd^{6}-3003\,a^{5}d^{7}\right)x^{2}+\left(128\,b^{5}c^{6}d-960\,ab^{4}c^{5}d^{2}+3120\,a^{2}b^{3}c^{4}d^{3}-5720\,a^{3}b^{2}c^{3}d^{4}+6435\,a^{4}bc^{2}d^{5}+18018\,a^{5}cd^{6}\right)x\right)\sqrt{dx+c}}{45045\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^5*d^7*x^7 - 256*b^5*c^7 + 1920*a*b^4*c^6*d - 6240*a^2*b^3*c
^5*d^2 + 11440*a^3*b^2*c^4*d^3 - 12870*a^4*b*c^3*d^4 + 9009*a^5*c^2*d^5 + 2
31*(16*b^5*c*d^6 + 75*a*b^4*d^7)*x^6 + 63*(b^5*c^2*d^5 + 350*a*b^4*c*d^6 +
650*a^2*b^3*d^7)*x^5 - 35*(2*b^5*c^3*d^4 - 15*a*b^4*c^2*d^5 - 1560*a^2*b^3*
c*d^6 - 1430*a^3*b^2*d^7)*x^4 + 5*(16*b^5*c^4*d^3 - 120*a*b^4*c^3*d^4 + 390
*a^2*b^3*c^2*d^5 + 14300*a^3*b^2*c*d^6 + 6435*a^4*b*d^7)*x^3 - 3*(32*b^5*c^
5*d^2 - 240*a*b^4*c^4*d^3 + 780*a^2*b^3*c^3*d^4 - 1430*a^3*b^2*c^2*d^5 - 17
160*a^4*b*c*d^6 - 3003*a^5*d^7)*x^2 + (128*b^5*c^6*d - 960*a*b^4*c^5*d^2 +
3120*a^2*b^3*c^4*d^3 - 5720*a^3*b^2*c^3*d^4 + 6435*a^4*b*c^2*d^5 + 18018*a^
5*c*d^6)*x)*sqrt(d*x + c)/d^6
```

Sympy [**A**]
time = 13.83, size = 763, normalized size = 4.83

$$a^{5}c\left(\begin{cases}\sqrt{c}\,x & \text{for}\: d = 0\\\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^{5}\left(-\frac{c\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{\left(c+dx\right)^{\frac{5}{2}}}{5}\right)}{d}+\frac{10a^{4}bc\left(-\frac{c\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{\left(c+dx\right)^{\frac{5}{2}}}{5}\right)}{d^{2}}+\frac{10a^{4}b\left(\frac{c^{2}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{\left(c+dx\right)^{\frac{7}{2}}}{7}\right)}{d^{2}}+\frac{20a^{3}b^{2}c\left(\frac{c^{2}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{\left(c+dx\right)^{\frac{7}{2}}}{7}\right)}{d^{3}}+\frac{20a^{3}b^{2}\left(-\frac{c^{3}\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{3c^{2}\left(c+dx\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+dx\right)^{\frac{7}{2}}}{7}+\frac{\left(c+dx\right)^{\frac{9}{2}}}{9}\right)}{d^{3}}+\frac{20a^{2}b^{3}c\left(-\frac{c^{3}\left(c+dx\right)^{\frac{3}{2}}}{3}+\frac{3c^{2}\left(c+dx\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+dx\right)^{\frac{7}{2}}}{7}+\frac{\left(c+dx\right)^{\frac{9}{2}}}{9}\right)}{d^{4}}+\frac{20a^{2}b^{3}\left(\frac{c^{4}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{4c^{3}\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{6c^{2}\left(c+dx\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+dx\right)^{\frac{9}{2}}}{9}+\frac{\left(c+dx\right)^{\frac{11}{2}}}{11}\right)}{d^{4}}+\frac{10ab^{4}c\left(\frac{c^{4}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{4c^{3}\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{6c^{2}\left(c+dx\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+dx\right)^{\frac{9}{2}}}{9}+\frac{\left(c+dx\right)^{\frac{11}{2}}}{11}\right)}{d^{5}}+\frac{10ab^{4}\left(-\frac{c^{5}\left(c+dx\right)^{\frac{3}{2}}}{3}+c^{4}\left(c+dx\right)^{\frac{5}{2}}-\frac{10c^{3}\left(c+dx\right)^{\frac{7}{2}}}{7}+\frac{10c^{2}\left(c+dx\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+dx\right)^{\frac{11}{2}}}{11}+\frac{\left(c+dx\right)^{\frac{13}{2}}}{13}\right)}{d^{5}}+\frac{2b^{5}c\left(-\frac{c^{5}\left(c+dx\right)^{\frac{3}{2}}}{3}+c^{4}\left(c+dx\right)^{\frac{5}{2}}-\frac{10c^{3}\left(c+dx\right)^{\frac{7}{2}}}{7}+\frac{10c^{2}\left(c+dx\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+dx\right)^{\frac{11}{2}}}{11}+\frac{\left(c+dx\right)^{\frac{13}{2}}}{13}\right)}{d^{6}}+\frac{2b^{5}\left(\frac{c^{6}\left(c+dx\right)^{\frac{3}{2}}}{3}-\frac{6c^{5}\left(c+dx\right)^{\frac{5}{2}}}{5}+\frac{15c^{4}\left(c+dx\right)^{\frac{7}{2}}}{7}-\frac{20c^{3}\left(c+dx\right)^{\frac{9}{2}}}{9}+\frac{15c^{2}\left(c+dx\right)^{\frac{11}{2}}}{11}-\frac{6c\left(c+dx\right)^{\frac{13}{2}}}{13}+\frac{\left(c+dx\right)^{\frac{15}{2}}}{15}\right)}{d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(3/2),x)
```

```
[Out] a**5*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**5*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 10*a**4*b*c*(-c*(c
 + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 10*a**4*b*(c**2*(c + d*x)**(3
/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 20*a**3*b**2*c*
(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**
3 + 20*a**3*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*
c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**2*b**3*c*(-c**3*(c
+ d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c +
 d*x)**(9/2)/9)/d**4 + 20*a**2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c +
d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d
*x)**(11/2)/11)/d**4 + 10*a*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d
*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*
x)**(11/2)/11)/d**5 + 10*a*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**5*c*(-c**5*(c + d*x)**
(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c +
 d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**6 + 2*
```

```
b**5*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*
x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6
*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(134) = 268$.
time = 1.46, size = 1084, normalized size = 6.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^5*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^5*c + 75075*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^5 +
 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
3*b^2*c^2/d^2 + 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^4*b*c/d + 12870*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c^2/d^3 + 25740*(5*(
d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*a^3*b^2*c/d^2 + 6435*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c
+ 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4*b/d + 715*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c^2/d^4 + 2860*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a^2*b^3*c/d^3 + 1430*(35*(d*x + c)^(9/2) - 180*(d
*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*s
qrt(d*x + c)*c^4)*a^3*b^2/d^2 + 65*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/
2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^
(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c^2/d^5 + 650*(63*(d*x + c)^(11/2) -
 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3
 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c/d^4 + 650*(63*
(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(
d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*
b^3/d^3 + 30*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x +
c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(
d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c/d^5 + 75*(231*(d*x + c)^
(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c
)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sq
rt(d*x + c)*c^6)*a*b^4/d^4 + 7*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2
)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x +
 c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 643
5*sqrt(d*x + c)*c^7)*b^5/d^5)/d
```

Mupad [B]

time = 0.24, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,(c+d\,x)^{15/2}}{15\,d^6}-\frac{(10\,b^5\,c-10\,a\,b^4\,d)\,(c+d\,x)^{13/2}}{13\,d^6}+\frac{2\,(a\,d-b\,c)^5\,(c+d\,x)^{5/2}}{5\,d^6}+\frac{20\,b^2\,(a\,d-b\,c)^3\,(c+d\,x)^{9/2}}{9\,d^6}+\frac{20\,b^3\,(a\,d-b\,c)^2\,(c+d\,x)^{11/2}}{11\,d^6}+\frac{10\,b\,(a\,d-b\,c)^4\,(c+d\,x)^{7/2}}{7\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(15/2))/(15*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(13/
2))/(13*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(5/2))/(5*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(11/2))/(1
1*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^6)
```

3.1388 $\int (a+bx)^4(c+dx)^{3/2}\,dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}-\frac{8b^3(bc-ad)(c+dx)^{11/2}}{11d^5}+\frac{2b^4(\ldots}{}$$

```
[Out] 2/5*(-a*d+b*c)^4*(d*x+c)^(5/2)/d^5-8/7*b*(-a*d+b*c)^3*(d*x+c)^(7/2)/d^5+4/3
*b^2*(-a*d+b*c)^2*(d*x+c)^(9/2)/d^5-8/11*b^3*(-a*d+b*c)*(d*x+c)^(11/2)/d^5+
2/13*b^4*(d*x+c)^(13/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(5/2))/(5*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(7
/2))/(7*d^5) + (4*b^2*(b*c - a*d)^2*(c + d*x)^(9/2))/(3*d^5) - (8*b^3*(b*c
- a*d)*(c + d*x)^(11/2))/(11*d^5) + (2*b^4*(c + d*x)^(13/2))/(13*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{3/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{5/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx\ldots}{d^4}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}\ldots\end{aligned}$$

Mathematica [A]
time = 0.09, size = 154, normalized size = 1.19

$$\frac{2(c+dx)^{5/2}\left(3003a^4d^4+1716a^3bd^3(-2c+5dx)+286a^2b^2d^2(8c^2-20cdx+35d^2x^2)+52ab^3d(-16c^3+40c^2dx-70cd^2x^2+105d^3x^3)+b^4(128c^4-320c^3dx+560c^2d^2x^2-840cd^3x^3+1155d^4x^4)\right)}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*a^4*d^4 + 1716*a^3*b*d^3*(-2*c + 5*d*x) + 286*a^2*
b^2*d^2*(8*c^2 - 20*c*d*x + 35*d^2*x^2) + 52*a*b^3*d*(-16*c^3 + 40*c^2*d*x
- 70*c*d^2*x^2 + 105*d^3*x^3) + b^4*(128*c^4 - 320*c^3*d*x + 560*c^2*d^2*x^
2 - 840*c*d^3*x^3 + 1155*d^4*x^4)))/(15015*d^5)
```

Maple [A]
time = 0.16, size = 100, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{8(ad-bc)b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)^2b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{8(ad-bc)^3b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^4(dx+c)^{\frac{5}{2}}}{5}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{13}{2}}}{13}+\frac{8(ad-bc)b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)^2b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{8(ad-bc)^3b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^4(dx+c)^{\frac{5}{2}}}{5}}{d^5}$
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(1155d^4x^4b^4+5460ab^3d^4x^3-840b^4cd^3x^3+10010a^2b^2d^4x^2-3640ab^3cd^3x^2+560b^4c^2d^2x^2+8580a^3bd^4x-5720}{15015d^5}$
trager	$2(1155b^4d^6x^6+5460ab^3d^6x^5+1470b^4cd^5x^5+10010a^2b^2d^6x^4+7280ab^3cd^5x^4+35b^4c^2d^4x^4+8580a^3bd^6x^3+14300a^2b^2ca$
risch	$2(1155b^4d^6x^6+5460ab^3d^6x^5+1470b^4cd^5x^5+10010a^2b^2d^6x^4+7280ab^3cd^5x^4+35b^4c^2d^4x^4+8580a^3bd^6x^3+14300a^2b^2ca$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/13*b^4*(d*x+c)^(13/2)+4/11*(a*d-b*c)*b^3*(d*x+c)^(11/2)+2/3*(a*d-b
*c)^2*b^2*(d*x+c)^(9/2)+4/7*(a*d-b*c)^3*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^4*(d*
x+c)^(5/2))
```

Maxima [A]
time = 0.30, size = 181, normalized size = 1.40

$$\frac{2\left(1155\,(dx+c)^{\frac{13}{2}}b^4-5460\,(b^4c-ab^3d)(dx+c)^{\frac{11}{2}}+10010\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(dx+c)^{\frac{9}{2}}-8580\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)(dx+c)^{\frac{7}{2}}+3003\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{5}{2}}\right)}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(d*x + c)^(13/2)*b^4 - 5460*(b^4*c - a*b^3*d)*(d*x + c)^(11/2
) + 10010*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(9/2) - 8580*(b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(7/2) + 3003*
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*
x + c)^(5/2))/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(109) = 218$.

time = 0.44, size = 311, normalized size = 2.41

$$\frac{2\left(1155 b^4 d^6 x^6+128 b^4 c^6-832 a b^3 c^5 d+2288 a^2 b^2 c^4 d^2-3432 a^3 b c^3 d^3+3003 a^4 c^2 d^4+210\left(7 b^4 c d^5+26 a b^3 d^6\right) x^5+35\left(b^4 c^2 d^4+208 a b^3 c d^5+286 a^2 b^2 d^6\right) x^4-20\left(2 b^4 c^3 d^3-13 a b^3 c^2 d^4-715 a^2 b^2 c d^5-429 a^3 b d^6\right) x^3+3\left(16 b^4 c^4 d^2-104 a b^3 c^3 d^3+286 a^2 b^2 c^2 d^4+4576 a^3 b c d^5+1001 a^4 d^6\right) x^2-2\left(32 b^4 c^5 d-208 a b^3 c^4 d^2+572 a^2 b^2 c^3 d^3-858 a^3 b c^2 d^4-3003 a^4 c d^5\right) x\right) \sqrt{d x+c}}{15015 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^4*d^6*x^6 + 128*b^4*c^6 - 832*a*b^3*c^5*d + 2288*a^2*b^2*c^
4*d^2 - 3432*a^3*b*c^3*d^3 + 3003*a^4*c^2*d^4 + 210*(7*b^4*c*d^5 + 26*a*b^3
*d^6)*x^5 + 35*(b^4*c^2*d^4 + 208*a*b^3*c*d^5 + 286*a^2*b^2*d^6)*x^4 - 20*(
2*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4 - 715*a^2*b^2*c*d^5 - 429*a^3*b*d^6)*x^3 +
 3*(16*b^4*c^4*d^2 - 104*a*b^3*c^3*d^3 + 286*a^2*b^2*c^2*d^4 + 4576*a^3*b*c
*d^5 + 1001*a^4*d^6)*x^2 - 2*(32*b^4*c^5*d - 208*a*b^3*c^4*d^2 + 572*a^2*b^
2*c^3*d^3 - 858*a^3*b*c^2*d^4 - 3003*a^4*c*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A]
time = 10.52, size = 559, normalized size = 4.33

$$a^4 c\left(\begin{cases}\sqrt{c}\, x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^4\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{8a^3bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{8a^3b\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{12a^2b^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}+\frac{12a^2b^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{8ab^3c\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}+\frac{8ab^3\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^4}+\frac{2b^4c\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^5}+\frac{2b^4\left(-\frac{c^5(c+dx)^{\frac{3}{2}}}{3}+c^4(c+dx)^{\frac{5}{2}}-\frac{10c^3(c+dx)^{\frac{7}{2}}}{7}+\frac{10c^2(c+dx)^{\frac{9}{2}}}{9}-\frac{5c(c+dx)^{\frac{11}{2}}}{11}+\frac{(c+dx)^{\frac{13}{2}}}{13}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(3/2),x)
```

```
[Out] a**4*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**4*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 8*a**3*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 8*a**3*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 12*a**2*b**2*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3
+ 12*a**2*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*
(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 8*a*b**3*c*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 8*a*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2
)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2
)/11)/d**4 + 2*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
+ 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11
)/d**5 + 2*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3
*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11
 + (c + d*x)**(13/2)/13)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(109) = 218.
time = 1.09, size = 807, normalized size = 6.26

$$2\left(45045\sqrt{dx+c}\,a^4c^2+30030\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^4c+\ldots\right.$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="giac")

[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*c + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^2/d + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4 + 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b^2*c^2/d^2 + 24024*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*b*c/d + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c^2/d^3 + 15444*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2*c/d^2 + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^2/d^4 + 1144*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3*c/d^3 + 858*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^2/d^2 + 130*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4*c/d^4 + 260*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^3/d^3 + 15*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^4/d^4)/d

Mupad [B]
time = 0.24, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^4*(c + d*x)^(3/2),x)

[Out] (2*b^4*(c + d*x)^(13/2))/(13*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(11/2))/(11*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(5/2))/(5*d^5) + (4*b^2*(a*d - b*c)^2*(c + d*x)^(9/2))/(3*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^5)

3.1389 $\int (a+bx)^3(c+dx)^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

```
[Out] -2/5*(-a*d+b*c)^3*(d*x+c)^(5/2)/d^4+6/7*b*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^4-2/
3*b^2*(-a*d+b*c)*(d*x+c)^(9/2)/d^4+2/11*b^3*(d*x+c)^(11/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
7/2))/(7*d^4) - (2*b^2*(b*c - a*d)*(c + d*x)^(9/2))/(3*d^4) + (2*b^3*(c + d
*x)^(11/2))/(11*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{3/2}\,dx = \int \left(\frac{(-bc+ad)^3(c+dx)^{3/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{5/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)}{d^3}\right.$$
$$= -\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}$$

Mathematica [A]
time = 0.07, size = 102, normalized size = 1.02

$$\frac{2(c+dx)^{5/2}\left(231a^3d^3+99a^2bd^2(-2c+5dx)+11ab^2d(8c^2-20cdx+35d^2x^2)+b^3(-16c^3+40c^2dx-70cd^2x^2+105d^3x^3)\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(231*a^3*d^3 + 99*a^2*b*d^2*(-2*c + 5*d*x) + 11*a*b^2*d*
(8*c^2 - 20*c*d*x + 35*d^2*x^2) + b^3*(-16*c^3 + 40*c^2*d*x - 70*c*d^2*x^2
+ 105*d^3*x^3)))/(1155*d^4)
```

Maple [A]
time = 0.17, size = 78, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{6(ad-bc)^2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^3(dx+c)^{\frac{5}{2}}}{5}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)b^2(dx+c)^{\frac{9}{2}}}{3}+\frac{6(ad-bc)^2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^3(dx+c)^{\frac{5}{2}}}{5}}{d^4}$
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(105b^3x^3d^3+385a\,b^2d^3x^2-70b^3c\,d^2x^2+495a^2b\,d^3x-220a\,b^2c\,d^2x+40b^3c^2dx+231a^3d^3-198a^2bc\,d^2+88a\,b^2c^2d}{1155d^4}$
trager	$\frac{2(105b^3d^5x^5+385a\,b^2d^5x^4+140b^3c\,d^4x^4+495a^2b\,d^5x^3+550a\,b^2c\,d^4x^3+5b^3c^2d^3x^3+231a^3d^5x^2+792a^2bc\,d^4x^2+33a\,b^2c^2d}{1}$
risch	$\frac{2(105b^3d^5x^5+385a\,b^2d^5x^4+140b^3c\,d^4x^4+495a^2b\,d^5x^3+550a\,b^2c\,d^4x^3+5b^3c^2d^3x^3+231a^3d^5x^2+792a^2bc\,d^4x^2+33a\,b^2c^2d}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/11*b^3*(d*x+c)^(11/2)+1/3*(a*d-b*c)*b^2*(d*x+c)^(9/2)+3/7*(a*d-b*c
)^2*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^3*(d*x+c)^(5/2))
```

Maple [A]

Maxima [A]
time = 0.33, size = 118, normalized size = 1.18

$$\frac{2\left(105\,(dx+c)^{\frac{11}{2}}b^3-385\,(b^3c-ab^2d)(dx+c)^{\frac{9}{2}}+495\,(b^3c^2-2\,ab^2cd+a^2bd^2)(dx+c)^{\frac{7}{2}}-231\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx+c)^{\frac{5}{2}}\right)}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*(d*x + c)^(11/2)*b^3 - 385*(b^3*c - a*b^2*d)*(d*x + c)^(9/2) +
495*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(7/2) - 231*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(5/2))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(84) = 168.
time = 0.48, size = 216, normalized size = 2.16

$$\frac{2\,(105\,b^3d^5x^5-16\,b^3c^5+88\,ab^2c^4d-198\,a^2bc^3d^2+231\,a^3c^2d^3+35\,(4\,b^3cd^4+11\,ab^2d^5)x^4+5\,(b^3c^2d^3+110\,ab^2cd^4+99\,a^2bd^5)x^3-3\,(2\,b^3c^3d^2-11\,ab^2c^2d^3-264\,a^2bcd^4-77\,a^3d^5)x^2+(8\,b^3c^4d-44\,ab^2c^3d^2+99\,a^2bc^2d^3+462\,a^3cd^4)x)\sqrt{dx+c}}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*d^5*x^5 - 16*b^3*c^5 + 88*a*b^2*c^4*d - 198*a^2*b*c^3*d^2 +
 231*a^3*c^2*d^3 + 35*(4*b^3*c*d^4 + 11*a*b^2*d^5)*x^4 + 5*(b^3*c^2*d^3 + 1
10*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - 3*(2*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3 -
264*a^2*b*c*d^4 - 77*a^3*d^5)*x^2 + (8*b^3*c^4*d - 44*a*b^2*c^3*d^2 + 99*a^
2*b*c^2*d^3 + 462*a^3*c*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [**A**]
time = 7.57, size = 386, normalized size = 3.86

$$a^3c\left(\begin{cases}\sqrt{c}\,x & \text{for } d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^3\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{6a^2bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{6a^2b\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{6ab^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}+\frac{6ab^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{2b^3c\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}+\frac{2b^3\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(3/2),x)
```

```
[Out] a**3*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**3*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 6*a**2*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 6*a**2*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 6*a*b**2*c*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 6*
a*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x
)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**3*c*(-c**3*(c + d*x)**(3/2)/3
+ 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/
d**4 + 2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2
*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**4
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(84) = 168.
time = 1.46, size = 566, normalized size = 5.66

$$\frac{2\left(3465\sqrt{dx+c}\,a^3c^2+2310\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^3c+\frac{3465\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^2bc^2}{d}+231\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^3+\frac{693\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ab^2c^2}{d^2}+\frac{1386\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^2bc}{d}+\frac{99\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)b^3c^2}{d^3}+\frac{594\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)ab^2c}{d^2}+\frac{297\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a^2b}{d}+\frac{22\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)b^3c}{d^3}+\frac{33\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)ab^2}{d^2}+\frac{5\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^2-1386(dx+c)^{\frac{5}{2}}c^3+1155(dx+c)^{\frac{3}{2}}c^4-693\sqrt{dx+c}\,c^5\right)b^3}{d^3}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^3*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^3*c + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^2/d + 231*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3 + 693*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c^2
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b*c/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^3*c^2/d^3 + 594*(5*(d*x + c)^(7/2) - 21
*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c
```

```
/d^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c
^2 - 35*sqrt(d*x + c)*c^3)*a^2*b/d + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)
^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
 + c)*c^4)*b^3*c/d^3 + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378
*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b
^2/d^2 + 5*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/
2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x
 + c)*c^5)*b^3/d^3)/d
```

Mupad [B]
time = 0.25, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(11/2))/(11*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(9/2))
/(9*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(a*d - b*c)^2*(
c + d*x)^(7/2))/(7*d^4)
```

3.1390 $\int (a+bx)^2(c+dx)^{3/2}\,dx$

Optimal. Leaf size=71

$$\frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

```
[Out] 2/5*(-a*d+b*c)^2*(d*x+c)^(5/2)/d^3-4/7*b*(-a*d+b*c)*(d*x+c)^(7/2)/d^3+2/9*b
^2*(d*x+c)^(9/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = $\{45\}$

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(7/2
))/(7*d^3) + (2*b^2*(c + d*x)^(9/2))/(9*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(-2c+5dx)+b^2(8c^2-20cdx+35d^2x^2)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x) + b^2*(8*c^2 - 20*
c*d*x + 35*d^2*x^2)))/(315*d^3)
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{4(ad-bc)b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^2(dx+c)^{\frac{5}{2}}}{5}}{d^3}$
default	$\frac{\frac{2b^2(dx+c)^{\frac{9}{2}}}{9}+\frac{4(ad-bc)b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)^2(dx+c)^{\frac{5}{2}}}{5}}{d^3}$
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(35b^2x^2d^2+90ab\,d^2x-20b^2cdx+63a^2d^2-36abcd+8b^2c^2)}{315d^3}$
trager	$\frac{2(35b^2d^4x^4+90ab\,d^4x^3+50b^2c\,d^3x^3+63a^2d^4x^2+144abc\,d^3x^2+3b^2c^2d^2x^2+126a^2c\,d^3x+18ab\,c^2d^2x-4b^2c^3dx+63a^2c^2d^2-}{315d^3}$
risch	$\frac{2(35b^2d^4x^4+90ab\,d^4x^3+50b^2c\,d^3x^3+63a^2d^4x^2+144abc\,d^3x^2+3b^2c^2d^2x^2+126a^2c\,d^3x+18ab\,c^2d^2x-4b^2c^3dx+63a^2c^2d^2-}{315d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/9*b^2*(d*x+c)^(9/2)+2/7*(a*d-b*c)*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)^2*
(d*x+c)^(5/2))
```

Maxima [A]
time = 0.30, size = 68, normalized size = 0.96

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^2-90\,(b^2c-abd)(dx+c)^{\frac{7}{2}}+63\,(b^2c^2-2\,abcd+a^2d^2)(dx+c)^{\frac{5}{2}}\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^2 - 90*(b^2*c - a*b*d)*(d*x + c)^(7/2) + 63*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(5/2))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(59) = 118$.
time = 0.47, size = 137, normalized size = 1.93

$$\frac{2\,(35\,b^2d^4x^4+8\,b^2c^4-36\,abc^3d+63\,a^2c^2d^2+10\,(5\,b^2cd^3+9\,abd^4)x^3+3\,(b^2c^2d^2+48\,abcd^3+21\,a^2d^4)x^2-2\,(2\,b^2c^3d-9\,abc^2d^2-63\,a^2cd^3)x)\sqrt{dx+c}}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*d^4*x^4 + 8*b^2*c^4 - 36*a*b*c^3*d + 63*a^2*c^2*d^2 + 10*(5*b
^2*c*d^3 + 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^2
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x)*sqrt(d*x + c)/d^3
```

Sympy [**A**]
time = 6.44, size = 240, normalized size = 3.38

$$a^2c\left(\begin{cases}\sqrt{c}\,x & \text{for } d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^2\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{4abc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{2b^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}+\frac{2b^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 4*a*b*c*(-c*(c + d
*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 -
2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*c*(c**2*(c + d*x
)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 2*b**2*(-c
**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7
 + (c + d*x)**(9/2)/9)/d**3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(59) = 118$.
time = 0.84, size = 360, normalized size = 5.07

$$\frac{2\left(315\sqrt{dx+c}\,a^2c^2+210\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^2c+\frac{210\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)abc^2}{d}+21\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^2+\frac{21\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b^2c^2}{d^2}+\frac{84\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)abc}{d}+\frac{18\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)b^2c}{d^2}+\frac{18\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)ab}{d}+\frac{\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)b^2}{d^2}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^2*c^2 + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c
)*a^2*c + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^2/d + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2 + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^2/d^2 + 84*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d +
 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35
*sqrt(d*x + c)*c^3)*b^2*c/d^2 + 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*
c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b/d + (35*(d*x + c)^(9
/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)
*c^3 + 315*sqrt(d*x + c)*c^4)*b^2/d^2)/d
```

Mupad [**B**]
time = 0.06, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}\,\left(35\,b^2\,{\left(c+d\,x\right)}^2+63\,a^2\,d^2+63\,b^2\,c^2-90\,b^2\,c\,\left(c+d\,x\right)+90\,a\,b\,d\,\left(c+d\,x\right)-126\,a\,b\,c\,d\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(35*b^2*(c + d*x)^2 + 63*a^2*d^2 + 63*b^2*c^2 - 90*b^2*c
*(c + d*x) + 90*a*b*d*(c + d*x) - 126*a*b*c*d))/(315*d^3)
```

3.1391 $\int (a+bx)(c+dx)^{3/2}\,dx$

Optimal. Leaf size=42

$$-\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}$$

```
[Out] -2/5*(-a*d+b*c)*(d*x+c)^(5/2)/d^2+2/7*b*(d*x+c)^(7/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = $\{45\}$

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(3/2),x]
[Out] (-2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^2) + (2*b*(c + d*x)^(7/2))/(7*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{5/2}(-2bc+7ad+5bdx)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(3/2),x]
[Out] (2*(c + d*x)^(5/2)*(-2*b*c + 7*a*d + 5*b*d*x))/(35*d^2)
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(5bdx+7ad-2bc)}{35d^2}$	27
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)(dx+c)^{\frac{5}{2}}}{5}}{d^2}$	34
default	$\frac{\frac{2b(dx+c)^{\frac{7}{2}}}{7}+\frac{2(ad-bc)(dx+c)^{\frac{5}{2}}}{5}}{d^2}$	34
trager	$\frac{2(5b\,d^3x^3+7a\,d^3x^2+8bc\,d^2x^2+14ac\,d^2x+b\,c^2dx+7a\,c^2d-2b\,c^3)\sqrt{dx+c}}{35d^2}$	70
risch	$\frac{2(5b\,d^3x^3+7a\,d^3x^2+8bc\,d^2x^2+14ac\,d^2x+b\,c^2dx+7a\,c^2d-2b\,c^3)\sqrt{dx+c}}{35d^2}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/7*b*(d*x+c)^(7/2)+1/5*(a*d-b*c)*(d*x+c)^(5/2))
```

Maxima [A]
time = 0.30, size = 33, normalized size = 0.79

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}b-7\,(bc-ad)(dx+c)^{\frac{5}{2}}\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 2/35*(5*(d*x + c)^(7/2)*b - 7*(b*c - a*d)*(d*x + c)^(5/2))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.40, size = 69, normalized size = 1.64

$$\frac{2\,(5\,bd^3x^3-2\,bc^3+7\,ac^2d+(8\,bcd^2+7\,ad^3)x^2+(bc^2d+14\,acd^2)x)\sqrt{dx+c}}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 2/35*(5*b*d^3*x^3 - 2*b*c^3 + 7*a*c^2*d + (8*b*c*d^2 + 7*a*d^3)*x^2 + (b*c^
2*d + 14*a*c*d^2)*x)*sqrt(d*x + c)/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(41) = 82$.
time = 0.14, size = 146, normalized size = 3.48

$$\begin{cases} \frac{2ac^2\sqrt{c+dx}}{5d} + \frac{4acx\sqrt{c+dx}}{5} + \frac{2adx^2\sqrt{c+dx}}{5} - \frac{4bc^3\sqrt{c+dx}}{35d^2} + \frac{2bc^2x\sqrt{c+dx}}{35d} + \frac{16bcx^2\sqrt{c+dx}}{35} + \frac{2bdx^3\sqrt{c+dx}}{7} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(ax + \frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((2*a*c**2*sqrt(c + d*x)/(5*d) + 4*a*c*x*sqrt(c + d*x)/5 + 2*a*d*x
**2*sqrt(c + d*x)/5 - 4*b*c**3*sqrt(c + d*x)/(35*d**2) + 2*b*c**2*x*sqrt(c
+ d*x)/(35*d) + 16*b*c*x**2*sqrt(c + d*x)/35 + 2*b*d*x**3*sqrt(c + d*x)/7,
Ne(d, 0)), (c**(3/2)*(a*x + b*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(34) = 68$.
time = 0.77, size = 192, normalized size = 4.57

$$\frac{2\left(105\sqrt{dx+c}\,ac^2 + 70\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)ac + \frac{35\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)bc^2}{d} + 7\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)a + \frac{14\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)bc}{d} + \frac{3\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)b}{d}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a*c^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^2/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a + 14*(3*(d*x + c)^(5/2)
- 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c/d + 3*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b/
d)/d
```

Mupad [B]
time = 0.21, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}\,\left(7\,a\,d-7\,b\,c+5\,b\,\left(c+d\,x\right)\right)}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(7*a*d - 7*b*c + 5*b*(c + d*x)))/(35*d^2)
```

3.1392 $\int (c+dx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{5/2}}{5d}$$

```
[Out] 2/5*(d*x+c)^(5/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{3/2}\,dx = \frac{2(c+dx)^{5/2}}{5d}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{5}{2}}}{5d}$	13
derivativedivides	$\frac{2(dx+c)^{\frac{5}{2}}}{5d}$	13
default	$\frac{2(dx+c)^{\frac{5}{2}}}{5d}$	13
trager	$\frac{2(d^2x^2+2cdx+c^2)\sqrt{dx+c}}{5d}$	29
risch	$\frac{2(d^2x^2+2cdx+c^2)\sqrt{dx+c}}{5d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(d*x+c)^(5/2)/d
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(d*x + c)^(5/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 0.40, size = 28, normalized size = 1.75

$$\frac{2\,(d^2x^2+2\,cdx+c^2)\sqrt{dx+c}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(d*x + c)/d
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(c+dx)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*d)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.
time = 1.10, size = 58, normalized size = 3.62

$$\frac{2\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 30\sqrt{dx+c}\,c^2 + 10\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 30*sqrt(d*x + c)*c^2 + 10*
((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c)/d
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

3.1393 $\int \frac{(c+dx)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=86

$$\frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] 2/3*(d*x+c)^(3/2)/b-2*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/b^(5/2)+2*(-a*d+b*c)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 65, 214}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)}{b^2}+\frac{2(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x])/b^2 + (2*(c + d*x)^(3/2))/(3*b) - (2*(b*c - a
*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{a+bx}\,dx = \frac{2(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b}$$
$$= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^2}$$
$$= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} + \frac{(2(bc-ad)^2)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b^2d}$$
$$= \frac{2(bc-ad)\sqrt{c+dx}}{b^2} + \frac{2(c+dx)^{3/2}}{3b} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Mathematica [A]
time = 0.16, size = 77, normalized size = 0.90

$$\frac{2\sqrt{c+dx}\,(4bc-3ad+bdx)}{3b^2} + \frac{2(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(4*b*c - 3*a*d + b*d*x))/(3*b^2) + (2*(-(b*c) + a*d)^(3/2)
*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

Maple [A]
time = 0.17, size = 99, normalized size = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{b(dx+c)^{\frac{3}{2}}}{3}+ad\sqrt{dx+c}-bc\sqrt{dx+c}\right)}{b^2}+\frac{2(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{b^2\sqrt{(ad-bc)b}}$
default	$-\frac{2\left(-\frac{b(dx+c)^{\frac{3}{2}}}{3}+ad\sqrt{dx+c}-bc\sqrt{dx+c}\right)}{b^2}+\frac{2(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{b^2\sqrt{(ad-bc)b}}$
risch	$-\frac{2(-bdx+3ad-4bc)\sqrt{dx+c}}{3b^2}+\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)a^2d^2}{b^2\sqrt{(ad-bc)b}}-\frac{4\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)acd}{b\sqrt{(ad-bc)b}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b^2*(-1/3*b*(d*x+c)^(3/2)+a*d*(d*x+c)^(1/2)-b*c*(d*x+c)^(1/2))+2*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b
*c)*b)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]

time = 0.46, size = 188, normalized size = 2.19

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\,(bdx+4\,bc-3\,ad)\sqrt{dx+c}}{3\,b^2},\ -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(bdx+4\,bc-3\,ad)\sqrt{dx+c}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(
d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(b*d*x + 4*b*c - 3*a*d)*sqrt
(d*x + c))/b^2, -2/3*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x + 4*b*c - 3*a*d)*sqrt(d*x
+ c))/b^2]
```

Sympy [A]

time = 7.12, size = 82, normalized size = 0.95

$$\frac{2(c+dx)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx}\,(-2ad+2bc)}{b^2}+\frac{2(ad-bc)^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*b) + sqrt(c + d*x)*(-2*a*d + 2*b*c)/b**2 + 2*(a*d - b
*c)**2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 1.20, size = 105, normalized size = 1.22

$$\frac{2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{2}}+\frac{2\left((dx+c)^{\frac{3}{2}}b^{2}+3\sqrt{dx+c}\,b^{2}c-3\sqrt{dx+c}\,abd\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/3*((d*x + c)^(3/2)*b^2 + 3*sqrt(d*x + c)
*b^2*c - 3*sqrt(d*x + c)*a*b*d)/b^3
```

Mupad [B]
time = 0.07, size = 93, normalized size = 1.08

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,b}-\frac{2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{b^2}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b) - (2*(a*d - b*c)*(c + d*x)^(1/2))/b^2 + (2*atan((
b^(1/2)*(a*d - b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
*(a*d - b*c)^(3/2))/b^(5/2)
```

3.1394 $\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$\frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

```
[Out] -(d*x+c)^(3/2)/b/(b*x+a)-3*d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)
)*(-a*d+b*c)^(1/2)/b^(5/2)+3*d*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 52, 65, 214}

$$-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/b^2 - (c + d*x)^(3/2)/(b*(a + b*x)) - (3*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b}\\ &= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^2}\\ &= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b^2}\\ &= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 83, normalized size = 0.98

$$\frac{\sqrt{c+dx}\,(-bc+3ad+2bdx)}{b^2(a+bx)}-\frac{3d\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(b^2*(a + b*x)) - (3*d*Sqrt[-(b*
c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

Maple [A]
time = 0.22, size = 100, normalized size = 1.18

method	result

derivativedivides	$2d\left(\frac{\sqrt{dx+c}}{b^2}-\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{3(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}}{b^2}\right)$
default	$2d\left(\frac{\sqrt{dx+c}}{b^2}-\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{3(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}}{b^2}\right)$
risch	$\frac{2d\sqrt{dx+c}}{b^2}+\frac{d^2\sqrt{dx+c}\,a}{b^2(bdx+ad)}-\frac{d\sqrt{dx+c}\,c}{b(bdx+ad)}-\frac{3d^2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)a}{b^2\sqrt{(ad-bc)b}}+\frac{3d\arctan\left(\frac{b\sqrt{}}{\sqrt{(}}\right.}{b\sqrt{(ad}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)

[Out] 2*d*(1/b^2*(d*x+c)^(1/2)-1/b^2*((-1/2*a*d+1/2*b*c)*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+3/2*(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more detail

Fricas [A]
time = 0.76, size = 210, normalized size = 2.47

$$\left[\frac{3\,(bdx+ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{2\,(b^3x+ab^2)},-\frac{3\,(bdx+ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d*x + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(2*b*d*x - b*c + 3*a*d)*sqr
t(d*x + c))/(b^3*x + a*b^2), -(3*(b*d*x + a*d)*sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b*d*x - b*c + 3*a*d
)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(73) = 146$.
time = 70.02, size = 923, normalized size = 10.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**2,x)
```

```
[Out] 2*a**2*d**3*sqrt(c + d*x)/(2*a**2*b**2*d**2 - 2*a*b**3*c*d + 2*a*b**3*d**2*
x - 2*b**4*c*d*x) - a**2*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sq
rt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c*
*2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + a**2*d**3*sqrt(-
1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d
*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt
(c + d*x))/(2*b**2) - 4*a*c*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*
d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) + a*c*d**2*sqrt(-1/(b*(a*d - b*c)**3))*
log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*
c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/b - a*c*d*
*2*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) -
2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3
)) + sqrt(c + d*x))/b - 4*a*d**2*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**3*
sqrt(a*d/b - c)) - c**2*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c**2*d*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 +
 2*c**2*d*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*
d*x) + 4*c*d*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + 2
*d*sqrt(c + d*x)/b**2
```

Giac [A]
time = 2.40, size = 113, normalized size = 1.33

$$\frac{2\sqrt{dx+c}\,d}{b^2}+\frac{3\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx+c}\,bcd-\sqrt{dx+c}\,ad^2}{((dx+c)b-bc+ad)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d/b^2 + 3*(b*c*d - a*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - (sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*a*d^2)/(((d*x + c)*b - b*c + a*d)*b^2)
```

Mupad [B]
time = 0.11, size = 109, normalized size = 1.28

$$\frac{(a\,d^2-b\,c\,d)\,\sqrt{c+d\,x}}{b^3\,(c+d\,x)-b^3\,c+a\,b^2\,d}+\frac{2\,d\,\sqrt{c+d\,x}}{b^2}-\frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^2,x)
```

```
[Out] ((a*d^2 - b*c*d)*(c + d*x)^(1/2))/(b^3*(c + d*x) - b^3*c + a*b^2*d) + (2*d*
(c + d*x)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(a*d - b*c)^(1/2)*(c + d*x)^(1/
2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/b^(5/2)
```

3.1395 $\int \frac{(c+dx)^{3/2}}{(a+bx)^3} \, dx$

Optimal. Leaf size=100

$$-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

```
[Out] -1/2*(d*x+c)^(3/2)/b/(b*x+a)^2-3/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-3/4*d*(d*x+c)^(1/2)/b^2/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {43, 65, 214}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(4*b^2*(a + b*x)) - (c + d*x)^(3/2)/(2*b*(a + b*x)^2)
- (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx &= -\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^2}\\
&= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b^2}\\
&= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A]
time = 0.36, size = 90, normalized size = 0.90

$$
-\frac{\sqrt{c+dx}\,(2bc+3ad+5bdx)}{4b^2(a+bx)^2}+\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{4b^{5/2}\sqrt{-bc+ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*(2*b*c + 3*a*d + 5*b*d*x))/(b^2*(a + b*x)^2) + (3*d^2*A
rcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) +
 a*d])
```

Maple [A]
time = 0.16, size = 97, normalized size = 0.97

method	result	size
derivativedivides	$2d^2\left(\frac{-\frac{5(dx+c)^{\frac{3}{2}}}{8b}-\frac{3(ad-bc)\sqrt{dx+c}}{8b^2}}{((dx+c)b+ad-bc)^2}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8b^2\sqrt{(ad-bc)\,b}}\right)$	97
default	$2d^2\left(\frac{-\frac{5(dx+c)^{\frac{3}{2}}}{8b}-\frac{3(ad-bc)\sqrt{dx+c}}{8b^2}}{((dx+c)b+ad-bc)^2}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8b^2\sqrt{(ad-bc)\,b}}\right)$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*((-5/8*(d*x+c)^(3/2)/b-3/8*(a*d-b*c)/b^2*(d*x+c)^(1/2))/((d*x+c)*b+a*
d-b*c)^2+3/8/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(
1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(80) = 160$.
time = 0.61, size = 383, normalized size = 3.83

$$\left[\frac{3\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2+5\,(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{8\,(a^2b^4c-a^3b^3d+(b^6c-ab^5d)x^2+2\,(ab^5c-a^2b^4d)x)},\frac{3\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(2\,b^3c^2+ab^2cd-3\,a^2bd^2+5\,(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{4\,(a^2b^4c-a^3b^3d+(b^6c-ab^5d)x^2+2\,(ab^5c-a^2b^4d)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))
/(a^2*b^4*c - a^3*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x
), 1/4*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (2*b^3*c^2 + a*b^2*c*d
 - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c - a^3
*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.94, size = 108, normalized size = 1.08

$$\frac{3\, d^2 \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}\, b^2} - \frac{5\,(dx+c)^{\frac{3}{2}} bd^2 - 3\,\sqrt{dx+c}\, bcd^2 + 3\,\sqrt{dx+c}\, ad^3}{4\,\big((dx+c)b - bc + ad\big)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^2) - 1/4*(5*(d*x + c)^(3/2)*b*d^2 - 3*sqrt(d*x + c)*b*c*d^2 + 3*sqrt(d*x
+ c)*a*d^3)/(((d*x + c)*b - b*c + a*d)^2*b^2)
```

Mupad [B]
time = 0.28, size = 135, normalized size = 1.35

$$\frac{3\, d^2 \operatorname{atan}\left(\frac{\sqrt{b}\, \sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\, b^{5/2}\, \sqrt{a\,d-b\,c}} - \frac{\frac{5\,d^2\,(c+d\,x)^{3/2}}{4\,b} + \frac{3\,d^2\,(a\,d-b\,c)\,\sqrt{c+d\,x}}{4\,b^2}}{b^2\,(c+d\,x)^2 - (2\,b^2\,c - 2\,a\,b\,d)\,(c+d\,x) + a^2\,d^2 + b^2\,c^2 - 2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(5/2)*(a*d -
 b*c)^(1/2)) - ((5*d^2*(c + d*x)^(3/2))/(4*b) + (3*d^2*(a*d - b*c)*(c + d*x
)^(1/2))/(4*b^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)
```

3.1396 $\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx$

Optimal. Leaf size=136

$$-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/3*(d*x+c)^(3/2)/b/(b*x+a)^3+1/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(3/2)-1/4*d*(d*x+c)^(1/2)/b^2/(b*x+a)^2-1/8*
d^2*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)}-\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -1/4*(d*Sqrt[c + d*x])/(b^2*(a + b*x)^2) - (d^2*Sqrt[c + d*x])/(8*b^2*(b*c
- a*d)*(a + b*x)) - (c + d*x)^(3/2)/(3*b*(a + b*x)^3) + (d^3*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{2b}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b^2}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^2(bc-ad)}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c}\right.}{8b^2(bc-ad)}\\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 129, normalized size = 0.95

$$\frac{\sqrt{c+dx}\,(-3a^2d^2-2abd(c+4dx)+b^2(8c^2+14cdx+3d^2x^2))}{24b^2(-bc+ad)(a+bx)^3}+\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8b^{5/2}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] (Sqrt[c + d*x]*(-3*a^2*d^2 - 2*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 14*c*d*x +
3*d^2*x^2)))/(24*b^2*(-(b*c) + a*d)*(a + b*x)^3) + (d^3*ArcTan[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*b^(5/2)*(-(b*c) + a*d)^(3/2))
```

Maple [A]
time = 0.16, size = 126, normalized size = 0.93

method	result	size
derivativedivides	$2d^3\left(\frac{\frac{(dx+c)^{\frac{5}{2}}}{16ad-16bc}-\frac{(dx+c)^{\frac{3}{2}}}{6b}-\frac{(ad-bc)\sqrt{dx+c}}{16b^2}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16(ad-bc)b^2\sqrt{(ad-bc)\,b}}\right)$	126
default	$2d^3\left(\frac{\frac{(dx+c)^{\frac{5}{2}}}{16ad-16bc}-\frac{(dx+c)^{\frac{3}{2}}}{6b}-\frac{(ad-bc)\sqrt{dx+c}}{16b^2}}{((dx+c)b+ad-bc)^3}+\frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16(ad-bc)b^2\sqrt{(ad-bc)\,b}}\right)$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*((1/16/(a*d-b*c)*(d*x+c)^(5/2)-1/6*(d*x+c)^(3/2)/b-1/16*(a*d-b*c)/b^2
*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^3+1/16/(a*d-b*c)/b^2/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(112) = 224$.
time = 0.49, size = 666, normalized size = 4.90

$$\left[-\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(8\,b^4c^3-10\,ab^3c^2d-a^2b^2cd^2+3\,a^3bd^3+3\,(b^4cd^2-ab^3d^3)x^2+2\,(7\,b^4c^2d-11\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c^2-2\,a^4b^4cd+a^5b^3d^2+(b^8c^2-2\,ab^7cd+a^2b^6d^2)x^3+3\,(ab^7c^2-2\,a^2b^6cd+a^3b^5d^2)x^2+3\,(a^2b^6c^2-2\,a^3b^5cd+a^4b^4d^2)x)},\ -\frac{3\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(8\,b^4c^3-10\,ab^3c^2d-a^2b^2cd^2+3\,a^3bd^3+3\,(b^4cd^2-ab^3d^3)x^2+2\,(7\,b^4c^2d-11\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c^2-2\,a^4b^4cd+a^5b^3d^2+(b^8c^2-2\,ab^7cd+a^2b^6d^2)x^3+3\,(ab^7c^2-2\,a^2b^6cd+a^3b^5d^2)x^2+3\,(a^2b^6c^2-2\,a^3b^5cd+a^4b^4d^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*(8*b^4*c^3 - 10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3
+ 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b
```

```
^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*
b^5*d^2)*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*x), -1/24*(3*(
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*
d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (8*b^4*c^3 -
10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3 + 3*(b^4*c*d^2 - a*b^3*d^3)*x^
2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3
*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d
^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*x^2 + 3*(a^2*b^6*c^2
- 2*a^3*b^5*c*d + a^4*b^4*d^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.67, size = 185, normalized size = 1.36

$$-\frac{d^{3} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2}c+abd}}\right)}{8\,(b^{3}c-ab^{2}d)\sqrt{-b^{2}c+abd}}-\frac{3\,(dx+c)^{\frac{5}{2}}b^{2}d^{3}+8\,(dx+c)^{\frac{3}{2}}b^{2}cd^{3}-3\,\sqrt{dx+c}\,b^{2}c^{2}d^{3}-8\,(dx+c)^{\frac{3}{2}}abd^{4}+6\,\sqrt{dx+c}\,abcd^{4}-3\,\sqrt{dx+c}\,a^{2}d^{5}}{24\,(b^{3}c-ab^{2}d)((dx+c)b-bc+ad)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c - a*b^2*d)*sq
rt(-b^2*c + a*b*d)) - 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 + 8*(d*x + c)^(3/2)*b
^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 - 8*(d*x + c)^(3/2)*a*b*d^4 + 6*sqrt
(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c - a*b^2*d)*((d*x + c
)*b - b*c + a*d)^3)
```

Mupad [**B**]
time = 0.34, size = 209, normalized size = 1.54

$$\frac{d^{3}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{d^{3}\,{\left(c+d\,x\right)}^{3/2}}{3\,b}-\frac{d^{3}\,{\left(c+d\,x\right)}^{5/2}}{8\,\left(a\,d-b\,c\right)}+\frac{d^{3}\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{8\,b^{2}}}{\left(c+d\,x\right)\,\left(3\,a^{2}\,b\,d^{2}-6\,a\,b^{2}\,c\,d+3\,b^{3}\,c^{2}\right)+b^{3}\,{\left(c+d\,x\right)}^{3}-\left(3\,b^{3}\,c-3\,a\,b^{2}\,d\right)\,{\left(c+d\,x\right)}^{2}+a^{3}\,d^{3}-b^{3}\,c^{3}+3\,a\,b^{2}\,c^{2}\,d-3\,a^{2}\,b\,c\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^4,x)
```

```
[Out] (d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(5/2)*(a*d - b
*c)^(3/2)) - ((d^3*(c + d*x)^(3/2))/(3*b) - (d^3*(c + d*x)^(5/2))/(8*(a*d -
 b*c)) + (d^3*(a*d - b*c)*(c + d*x)^(1/2))/(8*b^2))/((c + d*x)*(3*b^3*c^2 +
 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c +
d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
```

3.1397 $\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx$

Optimal. Leaf size=172

$$-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2}+\frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)/b/(b*x+a)^4-3/64*d^4*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d
+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(5/2)-1/8*d*(d*x+c)^(1/2)/b^2/(b*x+a)^3-1/3
2*d^2*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)^2+3/64*d^3*(d*x+c)^(1/2)/b^2/(-a
*d+b*c)^2/(b*x+a)
```

Rubi [A]

time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}+\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -1/8*(d*Sqrt[c + d*x])/(b^2*(a + b*x)^3) - (d^2*Sqrt[c + d*x])/(32*b^2*(b*c
 - a*d)*(a + b*x)^2) + (3*d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x
)) - (c + d*x)^(3/2)/(4*b*(a + b*x)^4) - (3*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[b*c - a*d]])/(64*b^(5/2)*(b*c - a*d)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx &= -\frac{(c+dx)^{3/2}}{4b(a+bx)^4}+\frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{8b}\\ &= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}+\frac{d^2\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{16b^2}\\ &= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}-\frac{(3d^3)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^2(bc-ad)}\\ &= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2}+\frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}+\frac{(3}{}\\ &= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2}+\frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}+\frac{(3}{}\\ &= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2}+\frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}-\frac{3}{}\end{aligned}$$

Mathematica [A]
time = 0.95, size = 171, normalized size = 0.99

$$-\frac{\sqrt{c+dx}\,\left(3a^3d^3+a^2bd^2(2c+11dx)-ab^2d(24c^2+44cdx+11d^2x^2)+b^3(16c^3+24c^2dx+2cd^2x^2-3d^3x^3)\right)}{64b^2(bc-ad)^2(a+bx)^4}+\frac{3d^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{64b^{5/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^5,x]

[Out] -1/64*(Sqrt[c + d*x]*(3*a^3*d^3 + a^2*b*d^2*(2*c + 11*d*x) - a*b^2*d*(24*c^
2 + 44*c*d*x + 11*d^2*x^2) + b^3*(16*c^3 + 24*c^2*d*x + 2*c*d^2*x^2 - 3*d^3
```

```
*x^3)))/(b^2*(b*c - a*d)^2*(a + b*x)^4) + (3*d^4*ArcTan[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[-(b*c) + a*d]])/(64*b^(5/2)*(-(b*c) + a*d)^(5/2))
```

Maple [A]
time = 0.16, size = 172, normalized size = 1.00

method	result
derivativedivides	$2d^4\left(\frac{\frac{3b(dx+c)^{\frac{7}{2}}}{128\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{11(dx+c)^{\frac{5}{2}}}{128(ad-bc)}-\frac{11(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{128b^2}}{((dx+c)b+ad-bc)^4}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{128(a^2d^2-2abcd+b^2c^2)b^2\sqrt{(ad-}}\right.$
default	$2d^4\left(\frac{\frac{3b(dx+c)^{\frac{7}{2}}}{128\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{11(dx+c)^{\frac{5}{2}}}{128(ad-bc)}-\frac{11(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{128b^2}}{((dx+c)b+ad-bc)^4}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{128(a^2d^2-2abcd+b^2c^2)b^2\sqrt{(ad-}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^4*((3/128/(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(d*x+c)^(7/2)+11/128/(a*d-b*c)*
(d*x+c)^(5/2)-11/128*(d*x+c)^(3/2)/b-3/128*(a*d-b*c)/b^2*(d*x+c)^(1/2))/((d
*x+c)*b+a*d-b*c)^4+3/128/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(144) = 288.
time = 0.54, size = 1043, normalized size = 6.06

$$\left[\frac{3\,(b^4d^4x^4+4\,ab^3d^4x^3+6\,a^2b^2d^4x^2+4\,a^3bd^4x+a^4d^4)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(16\,b^5c^4-40\,ab^4c^3d+26\,a^2b^3c^2d^2+a^3b^2cd^3-3\,a^4bd^4-3\,(b^5cd^3-ab^4d^4)x^3+(2\,b^5c^2d^2-13\,ab^4cd^3+11\,a^2b^3d^4)x^2+(24\,b^5c^3d-68\,ab^4c^2d^2+55\,a^2b^3cd^3-11\,a^3b^2d^4)x)\sqrt{dx+c}}{128\,(a^4b^6c^3-3\,a^5b^5c^2d+3\,a^6b^4cd^2-a^7b^3d^3+(b^{10}c^3-3\,ab^9c^2d+3\,a^2b^8cd^2-a^3b^7d^3)x^4+4\,(ab^9c^3-3\,a^2b^8c^2d+3\,a^3b^7cd^2-a^4b^6d^3)x^3+6\,(a^2b^8c^3-3\,a^3b^7c^2d+3\,a^4b^6cd^2-a^5b^5d^3)x^2+4\,(a^3b^7c^3-3\,a^4b^6c^2d+3\,a^5b^5cd^2-a^6b^4d^3)x)},\ \frac{3\,(b^4d^4x^4+4\,ab^3d^4x^3+6\,a^2b^2d^4x^2+4\,a^3bd^4x+a^4d^4)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(16\,b^5c^4-40\,ab^4c^3d+26\,a^2b^3c^2d^2+a^3b^2cd^3-3\,a^4bd^4-3\,(b^5cd^3-ab^4d^4)x^3+(2\,b^5c^2d^2-13\,ab^4cd^3+11\,a^2b^3d^4)x^2+(24\,b^5c^3d-68\,ab^4c^2d^2+55\,a^2b^3cd^3-11\,a^3b^2d^4)x)\sqrt{dx+c}}{64\,(a^4b^6c^3-3\,a^5b^5c^2d+3\,a^6b^4cd^2-a^7b^3d^3+(b^{10}c^3-3\,ab^9c^2d+3\,a^2b^8cd^2-a^3b^7d^3)x^4+4\,(ab^9c^3-3\,a^2b^8c^2d+3\,a^3b^7cd^2-a^4b^6d^3)x^3+6\,(a^2b^8c^3-3\,a^3b^7c^2d+3\,a^4b^6cd^2-a^5b^5d^3)x^2+4\,(a^3b^7c^3-3\,a^4b^6c^2d+3\,a^5b^5cd^2-a^6b^4d^3)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/128*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c -
a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(16*b^5*c^4 - 40*a*b^4*c^3*d + 26*a^2*
b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 - a*b^4*d^4)*x^3 +
 (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (24*b^5*c^3*d - 68
*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*
b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 + (b^10*c^3 - 3*a
*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*
c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2
*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d
+ 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x), 1/64*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3
 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (16*b^5*c^4 - 40*a*b^4
*c^3*d + 26*a^2*b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 -
a*b^4*d^4)*x^3 + (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (2
4*b^5*c^3*d - 68*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt
(d*x + c))/(a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 +
 (b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*
c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3
 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 -
3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.01, size = 285, normalized size = 1.66

$$\frac{3\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{7}{2}}b^3d^4-11\,(dx+c)^{\frac{5}{2}}b^3cd^4-11\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+3\,\sqrt{dx+c}\,b^3c^3d^4+11\,(dx+c)^{\frac{5}{2}}ab^2d^5+22\,(dx+c)^{\frac{3}{2}}ab^2cd^5-9\,\sqrt{dx+c}\,ab^2c^2d^5-11\,(dx+c)^{\frac{3}{2}}a^2bd^6+9\,\sqrt{dx+c}\,a^2bcd^6-3\,\sqrt{dx+c}\,a^3d^7}{64\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 3/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/64*(3*(d*x + c)^(7/2)*b^3*d^4 -
 11*(d*x + c)^(5/2)*b^3*c*d^4 - 11*(d*x + c)^(3/2)*b^3*c^2*d^4 + 3*sqrt(d*x
 + c)*b^3*c^3*d^4 + 11*(d*x + c)^(5/2)*a*b^2*d^5 + 22*(d*x + c)^(3/2)*a*b^2
*c*d^5 - 9*sqrt(d*x + c)*a*b^2*c^2*d^5 - 11*(d*x + c)^(3/2)*a^2*b*d^6 + 9*s
```

```
qrt(d*x + c)*a^2*b*c*d^6 - 3*sqrt(d*x + c)*a^3*d^7)/((b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*((d*x + c)*b - b*c + a*d)^4)
```

Mupad [B]
time = 0.37, size = 296, normalized size = 1.72

$$\frac{3\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{11\,d^4\,{\left(c+d\,x\right)}^{3/2}}{64\,b}-\frac{11\,d^4\,{\left(c+d\,x\right)}^{5/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^4\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{64\,b^2}-\frac{3\,b\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^5,x)
```

```
[Out] (3*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(5/2)*(a*d
- b*c)^(5/2)) - ((11*d^4*(c + d*x)^(3/2))/(64*b) - (11*d^4*(c + d*x)^(5/2))
/(64*(a*d - b*c)) + (3*d^4*(a*d - b*c)*(c + d*x)^(1/2))/(64*b^2) - (3*b*d^4
*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3
*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 1
2*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2
 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
```

3.1398 $\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx$

Optimal. Leaf size=208

$$-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2}-\frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+bx)}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}$$

```
[Out] -1/5*(d*x+c)^(3/2)/b/(b*x+a)^5+3/128*d^5*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*
d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(7/2)-3/40*d*(d*x+c)^(1/2)/b^2/(b*x+a)^4-1
/80*d^2*(d*x+c)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)^3+1/64*d^3*(d*x+c)^(1/2)/b^2/(
-a*d+b*c)^2/(b*x+a)^2-3/128*d^4*(d*x+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x+a)
```

Rubi [A]
time = 0.07, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}}-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)}-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(40*b^2*(a + b*x)^4) - (d^2*Sqrt[c + d*x])/(80*b^2*(b*
c - a*d)*(a + b*x)^3) + (d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x)
^2) - (3*d^4*Sqrt[c + d*x])/(128*b^2*(b*c - a*d)^3*(a + b*x)) - (c + d*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(128*b^(5/2)*(b*c - a*d)^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{(3d^2)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} - \frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \cdots$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+\cdots}}{128b^2(bc-ad)^3\cdots}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+\cdots}}{128b^2(bc-ad)^3\cdots}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+\cdots}}{128b^2(bc-ad)^3\cdots}$$

Mathematica [A]
time = 1.72, size = 223, normalized size = 1.07

$$\frac{\frac{\sqrt{b}\,\sqrt{c+dx}\,\left(-15a^4d^4-10a^3bd^3(c+7dx)+2a^2b^2d^2\left(124c^2+233cdx+64d^2x^2\right)-2ab^3d\left(168c^3+256c^2dx+23cd^2x^2-35d^3x^3\right)+b^4\left(128c^4+176c^3dx+8c^2d^2x^2-10cd^3x^3+15d^4x^4\right)\right)}{(-bc+ad)^3(a+bx)^5}+\frac{15d^5\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c + d*x]*(-15*a^4*d^4 - 10*a^3*b*d^3*(c + 7*d*x) + 2*a^2*b^2
*d^2*(124*c^2 + 233*c*d*x + 64*d^2*x^2) - 2*a*b^3*d*(168*c^3 + 256*c^2*d*x
+ 23*c*d^2*x^2 - 35*d^3*x^3) + b^4*(128*c^4 + 176*c^3*d*x + 8*c^2*d^2*x^2 -
 10*c*d^3*x^3 + 15*d^4*x^4)))/((-(b*c) + a*d)^3*(a + b*x)^5) + (15*d^5*ArcT
an[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2))/(640*
b^(5/2))
```

Maple [A]
time = 0.16, size = 237, normalized size = 1.14

method	result
derivativedivides	$2d^5\left(\frac{\frac{3b^2(dx+c)^{\frac{9}{2}}}{256(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{7b(dx+c)^{\frac{7}{2}}}{128(a^2d^2-2abcd+b^2c^2)}+\frac{(dx+c)^{\frac{5}{2}}}{10ad-10bc}-\frac{7(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{256b^2}}{((dx+c)b+ad-bc)^5}+\right.$
default	$2d^5\left(\frac{\frac{3b^2(dx+c)^{\frac{9}{2}}}{256(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{7b(dx+c)^{\frac{7}{2}}}{128(a^2d^2-2abcd+b^2c^2)}+\frac{(dx+c)^{\frac{5}{2}}}{10ad-10bc}-\frac{7(dx+c)^{\frac{3}{2}}}{128b}-\frac{3(ad-bc)\sqrt{dx+c}}{256b^2}}{((dx+c)b+ad-bc)^5}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^5*((3/256/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^2*(d*x+c)^(9/
2)+7/128/(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(d*x+c)^(7/2)+1/10/(a*d-b*c)*(d*x+c)
^(5/2)-7/128*(d*x+c)^(3/2)/b-3/256*(a*d-b*c)/b^2*(d*x+c)^(1/2))/((d*x+c)*b+
a*d-b*c)^5+3/256/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^2/((a*d-b*
c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(176) = 352$.

time = 0.79, size = 1492, normalized size = 7.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(128*b^6*c^5
- 464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c
*d^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87
*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2
+ 489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c
))/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a
^9*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*
d^3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d
^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*
d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^
4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^
2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3
+ a^8*b^4*d^4)*x), -1/640*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d
^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)
*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (128*b^6*c^5 -
464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c*d
^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*a*
b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87*a
^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2 +
489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c))
/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9
*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^
3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2
 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d
+ 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^4
- 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^2
+ 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 +
a^8*b^4*d^4)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(176) = 352.
time = 1.73, size = 410, normalized size = 1.97

$$-\frac{3\,d^{5}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{128\,(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3})\sqrt{-b^{2}c+abd}}-\frac{15\,(dx+c)^{\frac{9}{2}}b^{4}d^{5}-70\,(dx+c)^{\frac{7}{2}}b^{4}cd^{5}+128\,(dx+c)^{\frac{5}{2}}b^{4}c^{2}d^{5}+70\,(dx+c)^{\frac{3}{2}}b^{4}c^{3}d^{5}-15\,\sqrt{dx+c}\,b^{4}c^{4}d^{5}+70\,(dx+c)^{\frac{7}{2}}ab^{3}d^{6}-256\,(dx+c)^{\frac{5}{2}}ab^{3}cd^{6}-210\,(dx+c)^{\frac{3}{2}}ab^{3}c^{2}d^{6}+60\,\sqrt{dx+c}\,ab^{3}c^{3}d^{6}+128\,(dx+c)^{\frac{5}{2}}a^{2}b^{2}d^{7}+210\,(dx+c)^{\frac{3}{2}}a^{2}b^{2}cd^{7}-90\,\sqrt{dx+c}\,a^{2}b^{2}c^{2}d^{7}-70\,(dx+c)^{\frac{3}{2}}a^{3}bd^{8}+60\,\sqrt{dx+c}\,a^{3}bcd^{8}-15\,\sqrt{dx+c}\,a^{4}d^{9}}{640\,(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3})((dx+c)b-bc+ad)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^3 - 3*a*b^4
*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*c + a*b*d)) - 1/640*(15*(
d*x + c)^(9/2)*b^4*d^5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 + 128*(d*x + c)^(5/2)
*b^4*c^2*d^5 + 70*(d*x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^
5 + 70*(d*x + c)^(7/2)*a*b^3*d^6 - 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d
*x + c)^(3/2)*a*b^3*c^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 + 128*(d*x + c
)^(5/2)*a^2*b^2*d^7 + 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*
a^2*b^2*c^2*d^7 - 70*(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d
^8 - 15*sqrt(d*x + c)*a^4*d^9)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
- a^3*b^2*d^3)*((d*x + c)*b - b*c + a*d)^5)
```

Mupad [B]
time = 0.47, size = 398, normalized size = 1.91

$$\frac{\frac{d^{5}\,(c+d\,x)^{5/2}}{5\,(a\,d-b\,c)}-\frac{7\,d^{5}\,(c+d\,x)^{3/2}}{64\,b}+\frac{3\,b^{2}\,d^{5}\,(c+d\,x)^{9/2}}{128\,(a\,d-b\,c)^{3}}-\frac{3\,d^{5}\,(a\,d-b\,c)\,\sqrt{c+d\,x}}{128\,b^{2}}+\frac{7\,b\,d^{5}\,(c+d\,x)^{7/2}}{64\,(a\,d-b\,c)^{2}}}{b^{5}\,(c+d\,x)^{5}-(c+d\,x)^{2}\,(-10\,a^{3}\,b^{2}\,d^{3}+30\,a^{2}\,b^{3}\,c\,d^{2}-30\,a\,b^{4}\,c^{2}\,d+10\,b^{5}\,c^{3})-(5\,b^{5}\,c-5\,a\,b^{4}\,d)\,(c+d\,x)^{4}+a^{5}\,d^{5}-b^{5}\,c^{5}+(c+d\,x)^{3}\,(10\,a^{2}\,b^{3}\,d^{2}-20\,a\,b^{4}\,c\,d+10\,b^{5}\,c^{2})+(c+d\,x)\,(5\,a^{4}\,b\,d^{4}-20\,a^{3}\,b^{2}\,c\,d^{3}+30\,a^{2}\,b^{3}\,c^{2}\,d^{2}-20\,a\,b^{4}\,c^{3}\,d+5\,b^{5}\,c^{4})-10\,a^{2}\,b^{3}\,c^{3}\,d^{2}+10\,a^{3}\,b^{2}\,c^{2}\,d^{3}+5\,a\,b^{4}\,c^{4}\,d-5\,a^{4}\,b\,c\,d^{4}}+\frac{3\,d^{5}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{5/2}\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^6,x)
```

```
[Out] ((d^5*(c + d*x)^(5/2))/(5*(a*d - b*c)) - (7*d^5*(c + d*x)^(3/2))/(64*b) + (
3*b^2*d^5*(c + d*x)^(9/2))/(128*(a*d - b*c)^3) - (3*d^5*(a*d - b*c)*(c + d*
x)^(1/2))/(128*b^2) + (7*b*d^5*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^5*(c
 + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 3
0*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c
 + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c
^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d)
- 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
+ (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(5/2)*(a
*d - b*c)^(7/2))
```

3.1399 $\int (a+bx)^5(c+dx)^{5/2}\,dx$

Optimal. Leaf size=158

$$-\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11/2}}{11d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{13/2}}{13d^6}$$

```
[Out] -2/7*(-a*d+b*c)^5*(d*x+c)^(7/2)/d^6+10/9*b*(-a*d+b*c)^4*(d*x+c)^(9/2)/d^6-2
0/11*b^2*(-a*d+b*c)^3*(d*x+c)^(11/2)/d^6+20/13*b^3*(-a*d+b*c)^2*(d*x+c)^(13
/2)/d^6-2/3*b^4*(-a*d+b*c)*(d*x+c)^(15/2)/d^6+2/17*b^5*(d*x+c)^(17/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c+dx)^{7/2}(bc-ad)^5}{7d^6}+\frac{2b^5(c+dx)^{17/2}}{17d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(7/2))/(7*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(9/2))/(9*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3
*(b*c - a*d)^2*(c + d*x)^(13/2))/(13*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(1
5/2))/(3*d^6) + (2*b^5*(c + d*x)^(17/2))/(17*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^5(c+dx)^{5/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{7/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx}{d^5}\right.\\ &= -\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11}}{11d^6}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 217, normalized size = 1.37

$$\frac{2(c+dx)^{7/2}\left(21879a^5d^5+12155a^4bd^4(-2c+7dx)+2210a^3b^2d^3(8c^2-28cdx+63d^2x^2)+510a^2b^3d^2(-16c^3+56c^2dx-126cd^2x^2+231d^3x^3)+17ab^4d(128c^4-448c^3dx+1008c^2d^2x^2-1848cd^3x^3+3003d^4x^4)+b^5(-256c^5+896c^4dx-2016c^3d^2x^2+3696c^2d^3x^3-6006cd^4x^4+9009d^5x^5)\right)}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(21879*a^5*d^5 + 12155*a^4*b*d^4*(-2*c + 7*d*x) + 2210*a
^3*b^2*d^3*(8*c^2 - 28*c*d*x + 63*d^2*x^2) + 510*a^2*b^3*d^2*(-16*c^3 + 56*
c^2*d*x - 126*c*d^2*x^2 + 231*d^3*x^3) + 17*a*b^4*d*(128*c^4 - 448*c^3*d*x
+ 1008*c^2*d^2*x^2 - 1848*c*d^3*x^3 + 3003*d^4*x^4) + b^5*(-256*c^5 + 896*c
^4*d*x - 2016*c^3*d^2*x^2 + 3696*c^2*d^3*x^3 - 6006*c*d^4*x^4 + 9009*d^5*x^
5)))/(153153*d^6)
```

Maple [A]
time = 0.15, size = 122, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{17}{2}}}{17}+\frac{2(ad-bc)b^4(dx+c)^{\frac{15}{2}}}{3}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)^4b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^5(d\ldots}{7}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{17}{2}}}{17}+\frac{2(ad-bc)b^4(dx+c)^{\frac{15}{2}}}{3}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{20(ad-bc)^3b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)^4b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^5(d\ldots}{7}}{d^6}$
gosper	$2(dx+c)^{\frac{7}{2}}(9009b^5x^5d^5+51051a\,b^4d^5x^4-6006b^5c\,d^4x^4+117810a^2b^3d^5x^3-31416a\,b^4c\,d^4x^3+3696b^5c^2d^3x^3+139230a^3b^2\ldots$
trager	$2(9009b^5d^8x^8+51051a\,b^4d^8x^7+21021b^5c\,d^7x^7+117810a^2b^3d^8x^6+121737a\,b^4c\,d^7x^6+12705b^5c^2d^6x^6+139230a^3b^2d^8x^5\ldots$
risch	$2(9009b^5d^8x^8+51051a\,b^4d^8x^7+21021b^5c\,d^7x^7+117810a^2b^3d^8x^6+121737a\,b^4c\,d^7x^6+12705b^5c^2d^6x^6+139230a^3b^2d^8x^5\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/17*b^5*(d*x+c)^(17/2)+1/3*(a*d-b*c)*b^4*(d*x+c)^(15/2)+10/13*(a*d-
b*c)^2*b^3*(d*x+c)^(13/2)+10/11*(a*d-b*c)^3*b^2*(d*x+c)^(11/2)+5/9*(a*d-b*c
)^4*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^5*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.29, size = 259, normalized size = 1.64

$$\frac{2\left(9009\,(dx+c)^{\frac{17}{2}}b^5-51051\,(b^5c-ab^4d)(dx+c)^{\frac{15}{2}}+117810\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)(dx+c)^{\frac{13}{2}}-139230\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{11}{2}}+85085\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)(dx+c)^{\frac{9}{2}}-21879\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)(dx+c)^{\frac{7}{2}}\right)}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/153153*(9009*(d*x + c)^(17/2)*b^5 - 51051*(b^5*c - a*b^4*d)*(d*x + c)^(15
/2) + 117810*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(13/2) - 13923
0*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(11/2
) + 85085*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*(d*x + c)^(9/2) - 21879*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(7/2))/d^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(134) = 268$.

time = 0.74, size = 497, normalized size = 3.15

$$\frac{2\left(9009\,b^5d^8x^8-256\,b^5c^8+2176\,ab^4c^7d-8160\,a^2b^3c^6d^2+17680\,a^3b^2c^5d^3-24310\,a^4bc^4d^4+21879\,a^5c^3d^5+3003\left(7\,b^5cd^7+17\,ab^4d^8\right)x^7+231\left(55\,b^5c^2d^6+527\,ab^4cd^7+510\,a^2b^3d^8\right)x^6+63\left(b^5c^3d^5+1207\,ab^4c^2d^6+4590\,a^2b^3cd^7+2210\,a^3b^2d^8\right)x^5-35\left(2\,b^5c^4d^4-17\,ab^4c^3d^5-5406\,a^2b^3c^2d^6-10166\,a^3b^2cd^7-2431\,a^4bd^8\right)x^4+\left(80\,b^5c^5d^3-680\,ab^4c^4d^4+2550\,a^2b^3c^3d^5+249730\,a^3b^2c^2d^6+230945\,a^4bcd^7+21879\,a^5d^8\right)x^3-3\left(32\,b^5c^6d^2-272\,ab^4c^5d^3+1020\,a^2b^3c^4d^4-2210\,a^3b^2c^3d^5-60775\,a^4bc^2d^6-21879\,a^5cd^7\right)x^2+\left(128\,b^5c^7d-1088\,ab^4c^6d^2+4080\,a^2b^3c^5d^3-8840\,a^3b^2c^4d^4+12155\,a^4bc^3d^5+65637\,a^5c^2d^6\right)x\right)\sqrt{dx+c}}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/153153*(9009*b^5*d^8*x^8 - 256*b^5*c^8 + 2176*a*b^4*c^7*d - 8160*a^2*b^3*
c^6*d^2 + 17680*a^3*b^2*c^5*d^3 - 24310*a^4*b*c^4*d^4 + 21879*a^5*c^3*d^5 +
 3003*(7*b^5*c*d^7 + 17*a*b^4*d^8)*x^7 + 231*(55*b^5*c^2*d^6 + 527*a*b^4*c*
d^7 + 510*a^2*b^3*d^8)*x^6 + 63*(b^5*c^3*d^5 + 1207*a*b^4*c^2*d^6 + 4590*a^
2*b^3*c*d^7 + 2210*a^3*b^2*d^8)*x^5 - 35*(2*b^5*c^4*d^4 - 17*a*b^4*c^3*d^5
- 5406*a^2*b^3*c^2*d^6 - 10166*a^3*b^2*c*d^7 - 2431*a^4*b*d^8)*x^4 + (80*b^
5*c^5*d^3 - 680*a*b^4*c^4*d^4 + 2550*a^2*b^3*c^3*d^5 + 249730*a^3*b^2*c^2*d
^6 + 230945*a^4*b*c*d^7 + 21879*a^5*d^8)*x^3 - 3*(32*b^5*c^6*d^2 - 272*a*b^
4*c^5*d^3 + 1020*a^2*b^3*c^4*d^4 - 2210*a^3*b^2*c^3*d^5 - 60775*a^4*b*c^2*d
^6 - 21879*a^5*c*d^7)*x^2 + (128*b^5*c^7*d - 1088*a*b^4*c^6*d^2 + 4080*a^2*
b^3*c^5*d^3 - 8840*a^3*b^2*c^4*d^4 + 12155*a^4*b*c^3*d^5 + 65637*a^5*c^2*d^
6)*x)*sqrt(d*x + c)/d^6
```

Sympy [A]
time = 22.83, size = 1292, normalized size = 8.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(5/2),x)
```

```
[Out] a**5*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**5*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**5*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 10*a*
*4*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 20*a**4*b*c*(
c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
 + 10*a**4*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c
 + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 20*a**3*b**2*c**2*(c**2*(c +
d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 40*a**3
*b**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*
x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**3*b**2*(c**4*(c + d*x)**(3/2
)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)
**(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 20*a**2*b**3*c**2*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 40*a**2*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)
**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)*
```

```
*(11/2)/11)/d**4 + 20*a**2*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**4 + 10*a*b**4*c**2*(c**4*(c + d
*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*
(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**5 + 20*a*b**4*c*(-c**5*(c + d
*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2
*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5
 + 10*a*b**4*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4
*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2
)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**5 + 2*b**5*c**2*
(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2
)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(1
3/2)/13)/d**6 + 4*b**5*c*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)
/5 + 15*c**4*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c +
 d*x)**(11/2)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6 +
2*b**5*(-c**7*(c + d*x)**(3/2)/3 + 7*c**6*(c + d*x)**(5/2)/5 - 3*c**5*(c +
d*x)**(7/2) + 35*c**4*(c + d*x)**(9/2)/9 - 35*c**3*(c + d*x)**(11/2)/11 + 2
1*c**2*(c + d*x)**(13/2)/13 - 7*c*(c + d*x)**(15/2)/15 + (c + d*x)**(17/2)/
17)/d**6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1599 vs. $2(134) = 268$.
time = 1.81, size = 1599, normalized size = 10.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/765765*(765765*sqrt(d*x + c)*a^5*c^3 + 765765*((d*x + c)^(3/2) - 3*sqrt(d
*x + c)*c)*a^5*c^2 + 1276275*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^
3/d + 153153*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^5*c + 510510*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x
+ c)*c^2)*a^3*b^2*c^3/d^2 + 765765*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*
c + 15*sqrt(d*x + c)*c^2)*a^4*b*c^2/d + 21879*(5*(d*x + c)^(7/2) - 21*(d*x
+ c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^5 + 218790*
(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqr
t(d*x + c)*c^3)*a^2*b^3*c^3/d^3 + 656370*(5*(d*x + c)^(7/2) - 21*(d*x + c)^
(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2*c^2/d^2 +
328185*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*a^4*b*c/d + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c
)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*
x + c)*c^4)*a*b^4*c^3/d^4 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)
*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*
c^4)*a^2*b^3*c^2/d^3 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c +
```

```
378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*
a^3*b^2*c/d^2 + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*
x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^4*b/d
 + 1105*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*
c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x +
c)*c^5)*b^5*c^3/d^5 + 16575*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c^2/d^4 + 33150*(63*(d*x + c)^(11/2) - 38
5*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 +
1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*b^3*c/d^3 + 11050*(63
*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*
(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^3
*b^2/d^2 + 765*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c^2/d^5 + 3825*(231*(d*x
 + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d
*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3
003*sqrt(d*x + c)*c^6)*a*b^4*c/d^4 + 2550*(231*(d*x + c)^(13/2) - 1638*(d*x
 + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009
*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a
^2*b^3/d^3 + 357*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12285*(d
*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2)*c^4
- 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d*x + c
)*c^7)*b^5*c/d^5 + 595*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12
285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2
)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d
*x + c)*c^7)*a*b^4/d^4 + 7*(6435*(d*x + c)^(17/2) - 58344*(d*x + c)^(15/2)*
c + 235620*(d*x + c)^(13/2)*c^2 - 556920*(d*x + c)^(11/2)*c^3 + 850850*(d*x
 + c)^(9/2)*c^4 - 875160*(d*x + c)^(7/2)*c^5 + 612612*(d*x + c)^(5/2)*c^6 -
 291720*(d*x + c)^(3/2)*c^7 + 109395*sqrt(d*x + c)*c^8)*b^5/d^5)/d
```

Mupad [B]
time = 0.27, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{17/2}}{17\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{15/2}}{15\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(17/2))/(17*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(15/
2))/(15*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(7/2))/(7*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(13/2))/
(13*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(9/2))/(9*d^6)
```

3.1400 $\int (a+bx)^4(c+dx)^{5/2}\,dx$

Optimal. Leaf size=129

$$\frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}-\frac{8b^3(bc-ad)(c+dx)^{13/2}}{13d^5}+\frac{2b^4(c+dx)^{15/2}}{15d^5}$$

```
[Out] 2/7*(-a*d+b*c)^4*(d*x+c)^(7/2)/d^5-8/9*b*(-a*d+b*c)^3*(d*x+c)^(9/2)/d^5+12/
11*b^2*(-a*d+b*c)^2*(d*x+c)^(11/2)/d^5-8/13*b^3*(-a*d+b*c)*(d*x+c)^(13/2)/d
^5+2/15*b^4*(d*x+c)^(15/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(7/2))/(7*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(9
/2))/(9*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^5) - (8*b^3*(b
*c - a*d)*(c + d*x)^(13/2))/(13*d^5) + (2*b^4*(c + d*x)^(15/2))/(15*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{5/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{7/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)}{d^4}\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 154, normalized size = 1.19

$$\frac{2(c+dx)^{7/2}\left(6435a^4d^4+2860a^3bd^3(-2c+7dx)+390a^2b^2d^2(8c^2-28cdx+63d^2x^2)+60ab^3d(-16c^3+56c^2dx-126cd^2x^2+231d^3x^3)+b^4(128c^4-448c^3dx+1008c^2d^2x^2-1848cd^3x^3+3003d^4x^4)\right)}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*a^4*d^4 + 2860*a^3*b*d^3*(-2*c + 7*d*x) + 390*a^2*
b^2*d^2*(8*c^2 - 28*c*d*x + 63*d^2*x^2) + 60*a*b^3*d*(-16*c^3 + 56*c^2*d*x
- 126*c*d^2*x^2 + 231*d^3*x^3) + b^4*(128*c^4 - 448*c^3*d*x + 1008*c^2*d^2*
x^2 - 1848*c*d^3*x^3 + 3003*d^4*x^4)))/(45045*d^5)
```

Maple [A]
time = 0.15, size = 100, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{15}{2}}}{15}+\frac{8(ad-bc)b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)^3b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^4(dx+c)^{\frac{7}{2}}}{7}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{15}{2}}}{15}+\frac{8(ad-bc)b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{8(ad-bc)^3b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^4(dx+c)^{\frac{7}{2}}}{7}}{d^5}$
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(3003d^4x^4b^4+13860a\,b^3d^4x^3-1848b^4c\,d^3x^3+24570a^2b^2d^4x^2-7560a\,b^3c\,d^3x^2+1008b^4c^2d^2x^2+20020a^3b\,d^4x-}{45045d^5}$
trager	$2(3003b^4d^7x^7+13860a\,b^3d^7x^6+7161b^4c\,d^6x^6+24570b^2a^2d^7x^5+34020a\,b^3c\,d^6x^5+4473b^4c^2d^5x^5+20020a^3b\,d^7x^4+62790b$
risch	$2(3003b^4d^7x^7+13860a\,b^3d^7x^6+7161b^4c\,d^6x^6+24570b^2a^2d^7x^5+34020a\,b^3c\,d^6x^5+4473b^4c^2d^5x^5+20020a^3b\,d^7x^4+62790b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/15*b^4*(d*x+c)^(15/2)+4/13*(a*d-b*c)*b^3*(d*x+c)^(13/2)+6/11*(a*d-
b*c)^2*b^2*(d*x+c)^(11/2)+4/9*(a*d-b*c)^3*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^4*(
d*x+c)^(7/2))
```

Maxima [A]
time = 0.32, size = 181, normalized size = 1.40

$$\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}b^4-13860\,(b^4c-ab^3d)(dx+c)^{\frac{13}{2}}+24570\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(dx+c)^{\frac{11}{2}}-20020\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)(dx+c)^{\frac{9}{2}}+6435\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{7}{2}}\right)}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^4 - 13860*(b^4*c - a*b^3*d)*(d*x + c)^(13/
2) + 24570*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(11/2) - 20020*(
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(9/2) + 64
35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
(d*x + c)^(7/2))/d^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(109) = 218$.

time = 0.89, size = 377, normalized size = 2.92

$$\frac{2\,{\left(3003\,b^{4}d^{7}x^{7}+128\,b^{4}c^{7}-960\,ab^{3}c^{6}d+3120\,a^{2}b^{2}c^{5}d^{2}-5720\,a^{3}bc^{4}d^{3}+6435\,a^{4}c^{3}d^{4}+231\,{\left(31\,b^{4}cd^{6}+60\,ab^{3}d^{7}\right)}x^{6}+63\,{\left(71\,b^{4}c^{2}d^{5}+540\,ab^{3}cd^{6}+390\,a^{2}b^{2}d^{7}\right)}x^{5}+35\,{\left(b^{4}c^{3}d^{4}+636\,ab^{3}c^{2}d^{5}+1794\,a^{2}b^{2}cd^{6}+572\,a^{3}bd^{7}\right)}x^{4}-5\,{\left(8\,b^{4}c^{4}d^{3}-60\,ab^{3}c^{3}d^{4}-8814\,a^{2}b^{2}c^{2}d^{5}-10868\,a^{3}bcd^{6}-1287\,a^{4}d^{7}\right)}x^{3}+3\,{\left(16\,b^{4}c^{5}d^{2}-120\,ab^{3}c^{4}d^{3}+390\,a^{2}b^{2}c^{3}d^{4}+14300\,a^{3}bc^{2}d^{5}+6435\,a^{4}cd^{6}\right)}x^{2}-{\left(64\,b^{4}c^{6}d-480\,ab^{3}c^{5}d^{2}+1560\,a^{2}b^{2}c^{4}d^{3}-2860\,a^{3}bc^{3}d^{4}-19305\,a^{4}c^{2}d^{5}\right)}x\right)}\sqrt{dx+c}}{45045\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^4*d^7*x^7 + 128*b^4*c^7 - 960*a*b^3*c^6*d + 3120*a^2*b^2*c^
5*d^2 - 5720*a^3*b*c^4*d^3 + 6435*a^4*c^3*d^4 + 231*(31*b^4*c*d^6 + 60*a*b^
3*d^7)*x^6 + 63*(71*b^4*c^2*d^5 + 540*a*b^3*c*d^6 + 390*a^2*b^2*d^7)*x^5 +
35*(b^4*c^3*d^4 + 636*a*b^3*c^2*d^5 + 1794*a^2*b^2*c*d^6 + 572*a^3*b*d^7)*x
^4 - 5*(8*b^4*c^4*d^3 - 60*a*b^3*c^3*d^4 - 8814*a^2*b^2*c^2*d^5 - 10868*a^3
*b*c*d^6 - 1287*a^4*d^7)*x^3 + 3*(16*b^4*c^5*d^2 - 120*a*b^3*c^4*d^3 + 390*
a^2*b^2*c^3*d^4 + 14300*a^3*b*c^2*d^5 + 6435*a^4*c*d^6)*x^2 - (64*b^4*c^6*d
 - 480*a*b^3*c^5*d^2 + 1560*a^2*b^2*c^4*d^3 - 2860*a^3*b*c^3*d^4 - 19305*a^
4*c^2*d^5)*x)*sqrt(d*x + c)/d^5
```

Sympy [A]
time = 16.76, size = 960, normalized size = 7.44

$$a^{4} c^{2} \left(\begin{cases} \sqrt{c} x & \text{for}\: d = 0 \\ \frac{2 \left(c + d x\right)^{\frac{3}{2}}}{3 d} & \text{otherwise} \end{cases}\right) + \frac{4 a^{4} c \left(- \frac{c \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{\left(c + d x\right)^{\frac{5}{2}}}{5}\right)}{d} + \frac{2 a^{4} \left(\frac{c^{2} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{2 c \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{\left(c + d x\right)^{\frac{7}{2}}}{7}\right)}{d} + \frac{8 a^{3} b c^{2} \left(- \frac{c \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{\left(c + d x\right)^{\frac{5}{2}}}{5}\right)}{d^{2}} + \frac{16 a^{3} b c \left(\frac{c^{2} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{2 c \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{\left(c + d x\right)^{\frac{7}{2}}}{7}\right)}{d^{2}} + \frac{8 a^{3} b \left(- \frac{c^{3} \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{3 c^{2} \left(c + d x\right)^{\frac{5}{2}}}{5} - \frac{3 c \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{\left(c + d x\right)^{\frac{9}{2}}}{9}\right)}{d^{2}} + \frac{12 a^{2} b^{2} c^{2} \left(\frac{c^{2} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{2 c \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{\left(c + d x\right)^{\frac{7}{2}}}{7}\right)}{d^{3}} + \frac{24 a^{2} b^{2} c \left(- \frac{c^{3} \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{3 c^{2} \left(c + d x\right)^{\frac{5}{2}}}{5} - \frac{3 c \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{\left(c + d x\right)^{\frac{9}{2}}}{9}\right)}{d^{3}} + \frac{12 a^{2} b^{2} \left(\frac{c^{4} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{4 c^{3} \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{6 c^{2} \left(c + d x\right)^{\frac{7}{2}}}{7} - \frac{4 c \left(c + d x\right)^{\frac{9}{2}}}{9} + \frac{\left(c + d x\right)^{\frac{11}{2}}}{11}\right)}{d^{3}} + \frac{8 a b^{3} c^{2} \left(- \frac{c^{3} \left(c + d x\right)^{\frac{3}{2}}}{3} + \frac{3 c^{2} \left(c + d x\right)^{\frac{5}{2}}}{5} - \frac{3 c \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{\left(c + d x\right)^{\frac{9}{2}}}{9}\right)}{d^{4}} + \frac{16 a b^{3} c \left(\frac{c^{4} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{4 c^{3} \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{6 c^{2} \left(c + d x\right)^{\frac{7}{2}}}{7} - \frac{4 c \left(c + d x\right)^{\frac{9}{2}}}{9} + \frac{\left(c + d x\right)^{\frac{11}{2}}}{11}\right)}{d^{4}} + \frac{8 a b^{3} \left(- \frac{c^{5} \left(c + d x\right)^{\frac{3}{2}}}{3} + c^{4} \left(c + d x\right)^{\frac{5}{2}} - \frac{10 c^{3} \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{10 c^{2} \left(c + d x\right)^{\frac{9}{2}}}{9} - \frac{5 c \left(c + d x\right)^{\frac{11}{2}}}{11} + \frac{\left(c + d x\right)^{\frac{13}{2}}}{13}\right)}{d^{4}} + \frac{2 b^{4} c^{2} \left(\frac{c^{4} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{4 c^{3} \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{6 c^{2} \left(c + d x\right)^{\frac{7}{2}}}{7} - \frac{4 c \left(c + d x\right)^{\frac{9}{2}}}{9} + \frac{\left(c + d x\right)^{\frac{11}{2}}}{11}\right)}{d^{5}} + \frac{4 b^{4} c \left(- \frac{c^{5} \left(c + d x\right)^{\frac{3}{2}}}{3} + c^{4} \left(c + d x\right)^{\frac{5}{2}} - \frac{10 c^{3} \left(c + d x\right)^{\frac{7}{2}}}{7} + \frac{10 c^{2} \left(c + d x\right)^{\frac{9}{2}}}{9} - \frac{5 c \left(c + d x\right)^{\frac{11}{2}}}{11} + \frac{\left(c + d x\right)^{\frac{13}{2}}}{13}\right)}{d^{5}} + \frac{2 b^{4} \left(\frac{c^{6} \left(c + d x\right)^{\frac{3}{2}}}{3} - \frac{6 c^{5} \left(c + d x\right)^{\frac{5}{2}}}{5} + \frac{15 c^{4} \left(c + d x\right)^{\frac{7}{2}}}{7} - \frac{20 c^{3} \left(c + d x\right)^{\frac{9}{2}}}{9} + \frac{15 c^{2} \left(c + d x\right)^{\frac{11}{2}}}{11} - \frac{6 c \left(c + d x\right)^{\frac{13}{2}}}{13} + \frac{\left(c + d x\right)^{\frac{15}{2}}}{15}\right)}{d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**4*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**4*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 8*a**
3*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 16*a**3*b*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
+ 8*a**3*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c +
 d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 12*a**2*b**2*c**2*(c**2*(c + d*
x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 24*a**2*b
**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)
**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 12*a**2*b**2*(c**4*(c + d*x)**(3/2)/
3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**
(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 8*a*b**3*c**2*(-c**3*(c + d*x)**(3/2
)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)
/9)/d**4 + 16*a*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/1
1)/d**4 + 8*a*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c
**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)
/11 + (c + d*x)**(13/2)/13)/d**4 + 2*b**4*c**2*(c**4*(c + d*x)**(3/2)/3 - 4
*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)
/9 + (c + d*x)**(11/2)/11)/d**5 + 4*b**4*c*(-c**5*(c + d*x)**(3/2)/3 + c**4
```

```
*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9
 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**4*(c**6*(c
+ d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*x)**(7/2)/7 -
20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6*c*(c + d*x)**
(13/2)/13 + (c + d*x)**(15/2)/15)/d**5
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(109) = 218$.
time = 1.79, size = 1204, normalized size = 9.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^3 + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c^2 + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^3/d +
 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4
*c + 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2
)*a^2*b^2*c^3/d^2 + 36036*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sq
rt(d*x + c)*c^2)*a^3*b*c^2/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)
*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4 + 5148*(5*(d*x + c)
^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*a*b^3*c^3/d^3 + 23166*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d
*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2*c^2/d^2 + 15444*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a^3*b*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d
*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^
3/d^4 + 1716*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5
/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3*c^2/d^3 +
2574*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2
- 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^2*c/d^2 + 572*(35*
(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*
x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^3*b/d + 195*(63*(d*x + c)^(11/2
) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*
c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4*c^2/d^4 + 780*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a
*b^3*c/d^3 + 390*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x +
c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sq
rt(d*x + c)*c^5)*a^2*b^2/d^2 + 45*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(1
1/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x +
c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^4*c/d^4
 + 60*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2
)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c
```

```
)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a*b^3/d^3 + 7*(429*(d*x + c)^(15/2) -
 3465*(d*x + c)^(13/2)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/
2)*c^3 + 32175*(d*x + c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x
 + c)^(3/2)*c^6 - 6435*sqrt(d*x + c)*c^7)*b^4/d^4)/d
```

Mupad [B]
time = 0.23, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{15/2}}{15\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(15/2))/(15*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(13/2)
)/(13*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^5) + (12*b^2*(a*d - b*c
)^2*(c + d*x)^(11/2))/(11*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(9/2))/(9*d^5
)
```

3.1401 $\int (a+bx)^3(c+dx)^{5/2}\,dx$

Optimal. Leaf size=100

$$-\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

```
[Out] -2/7*(-a*d+b*c)^3*(d*x+c)^(7/2)/d^4+2/3*b*(-a*d+b*c)^2*(d*x+c)^(9/2)/d^4-6/
11*b^2*(-a*d+b*c)*(d*x+c)^(11/2)/d^4+2/13*b^3*(d*x+c)^(13/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(b*c - a*d)^2*(c + d*x)^(
9/2))/(3*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(11/2))/(11*d^4) + (2*b^3*(c +
 d*x)^(13/2))/(13*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^{5/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{7/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{9/}}{d^3}\right.\\ &= -\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 102, normalized size = 1.02

$$\frac{2(c+dx)^{7/2}\left(429a^3d^3+143a^2bd^2(-2c+7dx)+13ab^2d(8c^2-28cdx+63d^2x^2)+b^3(-16c^3+56c^2dx-126cd^2x^2+231d^3x^3)\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(429*a^3*d^3 + 143*a^2*b*d^2*(-2*c + 7*d*x) + 13*a*b^2*d
*(8*c^2 - 28*c*d*x + 63*d^2*x^2) + b^3*(-16*c^3 + 56*c^2*d*x - 126*c*d^2*x^
2 + 231*d^3*x^3)))/(3003*d^4)
```

Maple [A]
time = 0.14, size = 78, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{6(ad-bc)b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)^2b(dx+c)^{\frac{9}{2}}}{3}+\frac{2(ad-bc)^3(dx+c)^{\frac{7}{2}}}{7}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{13}{2}}}{13}+\frac{6(ad-bc)b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{2(ad-bc)^2b(dx+c)^{\frac{9}{2}}}{3}+\frac{2(ad-bc)^3(dx+c)^{\frac{7}{2}}}{7}}{d^4}$
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(231b^3x^3d^3+819a\,b^2d^3x^2-126b^3c\,d^2x^2+1001a^2b\,d^3x-364a\,b^2c\,d^2x+56b^3c^2dx+429a^3d^3-286a^2bc\,d^2+104a}{3003d^4}$
trager	$2(231b^3d^6x^6+819a\,b^2d^6x^5+567b^3c\,d^5x^5+1001a^2b\,d^6x^4+2093a\,b^2c\,d^5x^4+371b^3c^2d^4x^4+429a^3d^6x^3+2717a^2bc\,d^5x^3+1$
risch	$2(231b^3d^6x^6+819a\,b^2d^6x^5+567b^3c\,d^5x^5+1001a^2b\,d^6x^4+2093a\,b^2c\,d^5x^4+371b^3c^2d^4x^4+429a^3d^6x^3+2717a^2bc\,d^5x^3+1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/13*b^3*(d*x+c)^(13/2)+3/11*(a*d-b*c)*b^2*(d*x+c)^(11/2)+1/3*(a*d-b
*c)^2*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^3*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.18

$$\frac{2\left(231\,(dx+c)^{\frac{13}{2}}b^3-819\,(b^3c-ab^2d)(dx+c)^{\frac{11}{2}}+1001\,(b^3c^2-2\,ab^2cd+a^2bd^2)(dx+c)^{\frac{9}{2}}-429\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx+c)^{\frac{7}{2}}\right)}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*(d*x + c)^(13/2)*b^3 - 819*(b^3*c - a*b^2*d)*(d*x + c)^(11/2) +
 1001*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(9/2) - 429*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(7/2))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(84) = 168$.
time = 0.61, size = 268, normalized size = 2.68

$$\frac{2\,(231\,b^3d^6x^6-16\,b^3c^6+104\,ab^2c^5d-286\,a^2bc^4d^2+429\,a^3c^3d^3+63\,(9\,b^3cd^5+13\,ab^2d^6)x^5+7\,(53\,b^3c^2d^4+299\,ab^2cd^5+143\,a^2bd^6)x^4+(5\,b^3c^3d^3+1469\,ab^2c^2d^4+2717\,a^2bcd^5+429\,a^3d^6)x^3-3\,(2\,b^3c^4d^2-13\,ab^2c^3d^3-715\,a^2bc^2d^4-429\,a^3cd^5)x^2+(8\,b^3c^5d-52\,ab^2c^4d^2+143\,a^2bc^3d^3+1287\,a^3c^2d^4)x)\sqrt{dx+c}}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*d^6*x^6 - 16*b^3*c^6 + 104*a*b^2*c^5*d - 286*a^2*b*c^4*d^2
+ 429*a^3*c^3*d^3 + 63*(9*b^3*c*d^5 + 13*a*b^2*d^6)*x^5 + 7*(53*b^3*c^2*d^4
 + 299*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 + (5*b^3*c^3*d^3 + 1469*a*b^2*c^2*d
^4 + 2717*a^2*b*c*d^5 + 429*a^3*d^6)*x^3 - 3*(2*b^3*c^4*d^2 - 13*a*b^2*c^3*
d^3 - 715*a^2*b*c^2*d^4 - 429*a^3*c*d^5)*x^2 + (8*b^3*c^5*d - 52*a*b^2*c^4*
d^2 + 143*a^2*b*c^3*d^3 + 1287*a^3*c^2*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(92) = 184$.
time = 0.41, size = 549, normalized size = 5.49

$$\begin{cases} \frac{2a^3c^3\sqrt{c+dx}}{7d} + \frac{6a^3c^2x\sqrt{c+dx}}{7} + \frac{6a^3cdx^2\sqrt{c+dx}}{7} + \frac{2a^3d^2x^3\sqrt{c+dx}}{7} - \frac{4a^2bc^4\sqrt{c+dx}}{21d^2} + \frac{2a^2bc^3x\sqrt{c+dx}}{21d} + \frac{10a^2bc^2x^2\sqrt{c+dx}}{7} + \frac{38a^2bcdx^3\sqrt{c+dx}}{21} + \frac{2a^2bd^2x^4\sqrt{c+dx}}{3} + \frac{16ab^2c^5\sqrt{c+dx}}{231d^3} - \frac{8ab^2c^4x\sqrt{c+dx}}{231d^2} + \frac{2ab^2c^3x^2\sqrt{c+dx}}{77d} + \frac{226ab^2c^2x^3\sqrt{c+dx}}{231} + \frac{46ab^2cdx^4\sqrt{c+dx}}{33} + \frac{6ab^2d^2x^5\sqrt{c+dx}}{11} - \frac{32b^3c^6\sqrt{c+dx}}{3003d^4} + \frac{16b^3c^5x\sqrt{c+dx}}{3003d^3} - \frac{4b^3c^4x^2\sqrt{c+dx}}{1001d^2} + \frac{10b^3c^3x^3\sqrt{c+dx}}{3003d} + \frac{106b^3c^2x^4\sqrt{c+dx}}{429} + \frac{54b^3cdx^5\sqrt{c+dx}}{143} + \frac{2b^3d^2x^6\sqrt{c+dx}}{13} & \text{for}\ d \neq 0 \\ c^{\frac{5}{2}}\left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**3*c**3*sqrt(c + d*x)/(7*d) + 6*a**3*c**2*x*sqrt(c + d*x)/7
+ 6*a**3*c*d*x**2*sqrt(c + d*x)/7 + 2*a**3*d**2*x**3*sqrt(c + d*x)/7 - 4*a*
*2*b*c**4*sqrt(c + d*x)/(21*d**2) + 2*a**2*b*c**3*x*sqrt(c + d*x)/(21*d) +
10*a**2*b*c**2*x**2*sqrt(c + d*x)/7 + 38*a**2*b*c*d*x**3*sqrt(c + d*x)/21 +
 2*a**2*b*d**2*x**4*sqrt(c + d*x)/3 + 16*a*b**2*c**5*sqrt(c + d*x)/(231*d**
3) - 8*a*b**2*c**4*x*sqrt(c + d*x)/(231*d**2) + 2*a*b**2*c**3*x**2*sqrt(c +
 d*x)/(77*d) + 226*a*b**2*c**2*x**3*sqrt(c + d*x)/231 + 46*a*b**2*c*d*x**4*
sqrt(c + d*x)/33 + 6*a*b**2*d**2*x**5*sqrt(c + d*x)/11 - 32*b**3*c**6*sqrt(
c + d*x)/(3003*d**4) + 16*b**3*c**5*x*sqrt(c + d*x)/(3003*d**3) - 4*b**3*c*
*4*x**2*sqrt(c + d*x)/(1001*d**2) + 10*b**3*c**3*x**3*sqrt(c + d*x)/(3003*d
) + 106*b**3*c**2*x**4*sqrt(c + d*x)/429 + 54*b**3*c*d*x**5*sqrt(c + d*x)/1
43 + 2*b**3*d**2*x**6*sqrt(c + d*x)/13, Ne(d, 0)), (c**(5/2)*(a**3*x + 3*a*
*2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(84) = 168$.
time = 1.60, size = 857, normalized size = 8.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(d*x + c)*a^3*c^3 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^3*c^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^3/d +
 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3
```

```
*c + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b^2*c^3/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^2*b*c^2/d + 429*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3 + 429*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^
3*c^3/d^3 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(
3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c^2/d^2 + 3861*(5*(d*x + c)^(7/2) -
21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b
*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2
)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^3*c^2/d^3 + 429*
(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420
*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^2*c/d^2 + 143*(35*(d*x +
c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^
(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b/d + 65*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^3*c/d^3 + 65*(63*(d*x + c
)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)
^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^2/d^2 +
5*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^
2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3
/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^3/d^3)/d
```

Mupad [B]
time = 0.08, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{13/2}}{13\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(13/2))/(13*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(11/2)
)/(11*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(a*d - b*c)^2
*(c + d*x)^(9/2))/(3*d^4)
```

3.1402 $\int (a+bx)^2(c+dx)^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

```
[Out] 2/7*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^3-4/9*b*(-a*d+b*c)*(d*x+c)^(9/2)/d^3+2/11*
b^2*(d*x+c)^(11/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(9/2
))/(9*d^3) + (2*b^2*(c + d*x)^(11/2))/(11*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(-2c+7dx)+b^2(8c^2-28cdx+63d^2x^2)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x) + b^2*(8*c^2 - 28*
c*d*x + 63*d^2*x^2)))/(693*d^3)
```

Maple [A]
time = 0.15, size = 56, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^2(dx+c)^{\frac{7}{2}}}{7}}{d^3}$
default	$\frac{\frac{2b^2(dx+c)^{\frac{11}{2}}}{11}+\frac{4(ad-bc)b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)^2(dx+c)^{\frac{7}{2}}}{7}}{d^3}$
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(63b^2x^2d^2+154ab\,d^2x-28b^2cdx+99a^2d^2-44abcd+8b^2c^2)}{693d^3}$
trager	$\frac{2(63b^2d^5x^5+154ab\,d^5x^4+161b^2c\,d^4x^4+99a^2d^5x^3+418abc\,d^4x^3+113b^2c^2d^3x^3+297a^2c\,d^4x^2+330ab\,c^2d^3x^2+3b^2c^3d^2x^2}{693d^3}$
risch	$\frac{2(63b^2d^5x^5+154ab\,d^5x^4+161b^2c\,d^4x^4+99a^2d^5x^3+418abc\,d^4x^3+113b^2c^2d^3x^3+297a^2c\,d^4x^2+330ab\,c^2d^3x^2+3b^2c^3d^2x^2}{693d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/11*b^2*(d*x+c)^(11/2)+2/9*(a*d-b*c)*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)^
2*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.30, size = 68, normalized size = 0.96

$$\frac{2\left(63\,(dx+c)^{\frac{11}{2}}b^2-154\,(b^2c-abd)(dx+c)^{\frac{9}{2}}+99\,(b^2c^2-2\,abcd+a^2d^2)(dx+c)^{\frac{7}{2}}\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(d*x + c)^(11/2)*b^2 - 154*(b^2*c - a*b*d)*(d*x + c)^(9/2) + 99*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(7/2))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(59) = 118.
time = 0.62, size = 174, normalized size = 2.45

$$\frac{2\,(63\,b^2d^5x^5+8\,b^2c^5-44\,abc^4d+99\,a^2c^3d^2+7\,(23\,b^2cd^4+22\,abd^5)x^4+(113\,b^2c^2d^3+418\,abcd^4+99\,a^2d^5)x^3+3\,(b^2c^3d^2+110\,abc^2d^3+99\,a^2cd^4)x^2-(4\,b^2c^4d-22\,abc^3d^2-297\,a^2c^2d^3)x)\sqrt{dx+c}}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*d^5*x^5 + 8*b^2*c^5 - 44*a*b*c^4*d + 99*a^2*c^3*d^2 + 7*(23*b
^2*c*d^4 + 22*a*b*d^5)*x^4 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5)
*x^3 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^2 - (4*b^2*c^4*d
- 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x)*sqrt(d*x + c)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(65) = 130$.
time = 0.34, size = 355, normalized size = 5.00

$$\begin{cases} \frac{2a^2c^3\sqrt{c+dx}}{7d} + \frac{6a^2c^2x\sqrt{c+dx}}{7} + \frac{6a^2cdx^2\sqrt{c+dx}}{7} + \frac{2a^2d^2x^3\sqrt{c+dx}}{7} - \frac{8abc^4\sqrt{c+dx}}{63d^2} + \frac{4abc^3x\sqrt{c+dx}}{63d} + \frac{20abc^2x^2\sqrt{c+dx}}{21} + \frac{76abcdx^3\sqrt{c+dx}}{63} + \frac{4abd^2x^4\sqrt{c+dx}}{9} + \frac{16b^2c^5\sqrt{c+dx}}{693d^3} - \frac{8b^2c^4x\sqrt{c+dx}}{693d^2} + \frac{2b^2c^3x^2\sqrt{c+dx}}{231d} + \frac{226b^2c^2x^3\sqrt{c+dx}}{693} + \frac{46b^2cdx^4\sqrt{c+dx}}{99} + \frac{2b^2d^2x^5\sqrt{c+dx}}{11} & \text{for}\ d \neq 0 \\ c^{\frac{5}{2}}\left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**2*c**3*sqrt(c + d*x)/(7*d) + 6*a**2*c**2*x*sqrt(c + d*x)/7
+ 6*a**2*c*d*x**2*sqrt(c + d*x)/7 + 2*a**2*d**2*x**3*sqrt(c + d*x)/7 - 8*a*
b*c**4*sqrt(c + d*x)/(63*d**2) + 4*a*b*c**3*x*sqrt(c + d*x)/(63*d) + 20*a*b
*c**2*x**2*sqrt(c + d*x)/21 + 76*a*b*c*d*x**3*sqrt(c + d*x)/63 + 4*a*b*d**2
*x**4*sqrt(c + d*x)/9 + 16*b**2*c**5*sqrt(c + d*x)/(693*d**3) - 8*b**2*c**4
*x*sqrt(c + d*x)/(693*d**2) + 2*b**2*c**3*x**2*sqrt(c + d*x)/(231*d) + 226*
b**2*c**2*x**3*sqrt(c + d*x)/693 + 46*b**2*c*d*x**4*sqrt(c + d*x)/99 + 2*b*
*2*d**2*x**5*sqrt(c + d*x)/11, Ne(d, 0)), (c**(5/2)*(a**2*x + a*b*x**2 + b*
*2*x**3/3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(59) = 118$.
time = 0.81, size = 558, normalized size = 7.86

$$\frac{2\left(3465\sqrt{dx+c}\,a^2c^3 + 3465\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)a^2c^2 + \frac{2310\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)abc^3}{d} + 693\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)a^2c + \frac{231\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)b^2c^3}{d^2} + \frac{1386\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)abc^2}{d} + 99\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)a^2 + \frac{297\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)b^2c^2}{d^2} + \frac{594\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)abc}{d} + \frac{33\left(35(dx+c)^{\frac{9}{2}} - 180(dx+c)^{\frac{7}{2}}c + 378(dx+c)^{\frac{5}{2}}c^2 - 420(dx+c)^{\frac{3}{2}}c^3 + 315\sqrt{dx+c}\,c^4\right)b^2c}{d^2} + \frac{22\left(35(dx+c)^{\frac{9}{2}} - 180(dx+c)^{\frac{7}{2}}c + 378(dx+c)^{\frac{5}{2}}c^2 - 420(dx+c)^{\frac{3}{2}}c^3 + 315\sqrt{dx+c}\,c^4\right)ab}{d} + \frac{5\left(63(dx+c)^{\frac{11}{2}} - 385(dx+c)^{\frac{9}{2}}c + 990(dx+c)^{\frac{7}{2}}c^2 - 1386(dx+c)^{\frac{5}{2}}c^3 + 1155(dx+c)^{\frac{3}{2}}c^4 - 693\sqrt{dx+c}\,c^5\right)b^2}{d^2}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^2*c^3 + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^2*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^3/d + 693*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*c + 23
1*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^3
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a*b*c^2/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^2*c^2/d^2 + 5
94*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*
sqrt(d*x + c)*c^3)*a*b*c/d + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c
 + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^
```

```
4)*b^2*c/d^2 + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x +
c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d + 5*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b
^2/d^2)/d
```

Mupad [B]
time = 0.07, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(63\,b^2\,{\left(c+d\,x\right)}^2+99\,a^2\,d^2+99\,b^2\,c^2-154\,b^2\,c\,\left(c+d\,x\right)+154\,a\,b\,d\,\left(c+d\,x\right)-198\,a\,b\,c\,d\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(63*b^2*(c + d*x)^2 + 99*a^2*d^2 + 99*b^2*c^2 - 154*b^2*
c*(c + d*x) + 154*a*b*d*(c + d*x) - 198*a*b*c*d))/(693*d^3)
```

3.1403 $\int (a+bx)(c+dx)^{5/2}\,dx$

Optimal. Leaf size=42

$$-\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}$$

```
[Out] -2/7*(-a*d+b*c)*(d*x+c)^(7/2)/d^2+2/9*b*(d*x+c)^(9/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^2) + (2*b*(c + d*x)^(9/2))/(9*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{5/2}}{d}+\frac{b(c+dx)^{7/2}}{d}\right)dx\\ &= -\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{7/2}(-2bc+9ad+7bdx)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2)*(-2*b*c + 9*a*d + 7*b*d*x))/(63*d^2)
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(7bdx+9ad-2bc)}{63d^2}$	27
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)(dx+c)^{\frac{7}{2}}}{7}}{d^2}$	34
default	$\frac{\frac{2b(dx+c)^{\frac{9}{2}}}{9}+\frac{2(ad-bc)(dx+c)^{\frac{7}{2}}}{7}}{d^2}$	34
trager	$\frac{2(7b\,d^4x^4+9a\,d^4x^3+19bc\,d^3x^3+27ac\,d^3x^2+15b\,c^2d^2x^2+27a\,c^2d^2x+b\,c^3dx+9a\,c^3d-2b\,c^4)\sqrt{dx+c}}{63d^2}$	94
risch	$\frac{2(7b\,d^4x^4+9a\,d^4x^3+19bc\,d^3x^3+27ac\,d^3x^2+15b\,c^2d^2x^2+27a\,c^2d^2x+b\,c^3dx+9a\,c^3d-2b\,c^4)\sqrt{dx+c}}{63d^2}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/9*b*(d*x+c)^(9/2)+1/7*(a*d-b*c)*(d*x+c)^(7/2))
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.79

$$\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b-9\,(bc-ad)(dx+c)^{\frac{7}{2}}\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="maxima")
[Out] 2/63*(7*(d*x + c)^(9/2)*b - 9*(b*c - a*d)*(d*x + c)^(7/2))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(34) = 68$.
time = 0.54, size = 93, normalized size = 2.21

$$\frac{2\,(7\,bd^4x^4-2\,bc^4+9\,ac^3d+(19\,bcd^3+9\,ad^4)x^3+3\,(5\,bc^2d^2+9\,acd^3)x^2+(bc^3d+27\,ac^2d^2)x)\sqrt{dx+c}}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/63*(7*b*d^4*x^4 - 2*b*c^4 + 9*a*c^3*d + (19*b*c*d^3 + 9*a*d^4)*x^3 + 3*(5
*b*c^2*d^2 + 9*a*c*d^3)*x^2 + (b*c^3*d + 27*a*c^2*d^2)*x)*sqrt(d*x + c)/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(41) = 82$.
time = 0.28, size = 194, normalized size = 4.62

$$\begin{cases} \frac{2ac^3\sqrt{c+dx}}{7d} + \frac{6ac^2x\sqrt{c+dx}}{7} + \frac{6acdx^2\sqrt{c+dx}}{7} + \frac{2ad^2x^3\sqrt{c+dx}}{7} - \frac{4bc^4\sqrt{c+dx}}{63d^2} + \frac{2bc^3x\sqrt{c+dx}}{63d} + \frac{10bc^2x^2\sqrt{c+dx}}{21} + \frac{38bcdx^3\sqrt{c+dx}}{63} + \frac{2bd^2x^4\sqrt{c+dx}}{9} & \text{for } d \neq 0 \\ c^{\frac{5}{2}}\left(ax + \frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a*c**3*sqrt(c + d*x)/(7*d) + 6*a*c**2*x*sqrt(c + d*x)/7 + 6*a*
c*d*x**2*sqrt(c + d*x)/7 + 2*a*d**2*x**3*sqrt(c + d*x)/7 - 4*b*c**4*sqrt(c
+ d*x)/(63*d**2) + 2*b*c**3*x*sqrt(c + d*x)/(63*d) + 10*b*c**2*x**2*sqrt(c
+ d*x)/21 + 38*b*c*d*x**3*sqrt(c + d*x)/63 + 2*b*d**2*x**4*sqrt(c + d*x)/9,
 Ne(d, 0)), (c**(5/2)*(a*x + b*x**2/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(34) = 68$.
time = 1.93, size = 306, normalized size = 7.29

$$\frac{2\left(315\sqrt{dx+c}\,ac^3 + 315\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)ac^2 + \frac{105\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)bc^3}{d} + 63\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)ac + \frac{63\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)bc^2}{d} + 9\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)a + \frac{27\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)bc}{d} + \frac{\left(35(dx+c)^{\frac{9}{2}} - 180(dx+c)^{\frac{7}{2}}c + 378(dx+c)^{\frac{5}{2}}c^2 - 420(dx+c)^{\frac{3}{2}}c^3 + 315\sqrt{dx+c}\,c^4\right)b}{d}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a*c^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a*c^2 + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^3/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*c + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c^2/d + 9*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2
)*c^2 - 35*sqrt(d*x + c)*c^3)*b*c/d + (35*(d*x + c)^(9/2) - 180*(d*x + c)^(
7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x +
 c)*c^4)*b/d)/d
```

Mupad [B]
time = 0.05, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(9\,a\,d-9\,b\,c+7\,b\,\left(c+d\,x\right)\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(9*a*d - 9*b*c + 7*b*(c + d*x)))/(63*d^2)
```

3.1404 $\int (c+dx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{7/2}}{7d}$$

```
[Out] 2/7*(d*x+c)^(7/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{5/2}\,dx = \frac{2(c+dx)^{7/2}}{7d}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.81

method	result	size
gosper	$\frac{2(dx+c)^{\frac{7}{2}}}{7d}$	13
derivativedivides	$\frac{2(dx+c)^{\frac{7}{2}}}{7d}$	13
default	$\frac{2(dx+c)^{\frac{7}{2}}}{7d}$	13
trager	$\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)\sqrt{dx+c}}{7d}$	40
risch	$\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)\sqrt{dx+c}}{7d}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(d*x+c)^(7/2)/d
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="maxima")
[Out] 2/7*(d*x + c)^(7/2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.
time = 0.44, size = 39, normalized size = 2.44

$$\frac{2\,(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3)\sqrt{dx+c}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="fricas")
[Out] 2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(d*x + c)/d
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.75

$$\frac{2(c+dx)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2),x)
```

```
[Out] 2*(c + d*x)**(7/2)/(7*d)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(12) = 24$.
time = 1.59, size = 95, normalized size = 5.94

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}+35\,\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)c^{2}+7\,\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 + 3
5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c^2 + 7*(3*(d*x + c)^(5/2) - 10*(d*
x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*c)/d
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

3.1405 $\int \frac{(c+dx)^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=112

$$\frac{2(bc-ad)^2\sqrt{c+dx}}{b^3}+\frac{2(bc-ad)(c+dx)^{3/2}}{3b^2}+\frac{2(c+dx)^{5/2}}{5b}-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

```
[Out] 2/3*(-a*d+b*c)*(d*x+c)^(3/2)/b^2+2/5*(d*x+c)^(5/2)/b-2*(-a*d+b*c)^(5/2)*arc
tanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+2*(-a*d+b*c)^2*(d*x+c)
^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {52, 65, 214}

$$-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}+\frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/b^3 + (2*(b*c - a*d)*(c + d*x)^(3/2))/(3*b^
2) + (2*(c + d*x)^(5/2))/(5*b) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{a+bx}\,dx = \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{b}$$
$$= \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b^2}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx)\sqrt{c}}}{b^3}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(2(bc-ad)^3)\,\text{Subst}\Big(\int}{}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} - \frac{2(bc-ad)^{5/2}\tanh^{-1}\Big(}{b^{7/2}}$$

Mathematica [A]
time = 0.13, size = 108, normalized size = 0.96

$$\frac{2\sqrt{c+dx}\,(15a^2d^2-5abd(7c+dx)+b^2(23c^2+11cdx+3d^2x^2))}{15b^3} - \frac{2(-bc+ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x),x]
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 - 5*a*b*d*(7*c + d*x) + b^2*(23*c^2 + 11*c*d*x
 + 3*d^2*x^2)))/(15*b^3) - (2*(-(b*c) + a*d)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[-(b*c) + a*d]])/b^(7/2)
```

Maple [A]
time = 0.17, size = 161, normalized size = 1.44

method	result
derivativedivides	$\frac{\frac{2(dx+c)^{\frac{5}{2}}b^2}{5}-\frac{2abd(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c(dx+c)^{\frac{3}{2}}}{3}+2a^2d^2\sqrt{dx+c}-4abcd\sqrt{dx+c}+2b^2c^2\sqrt{dx+c}}{b^3}+\frac{2(-a^3d^3+3}{}$

default	$\frac{\frac{2(dx+c)^{\frac{5}{2}}b^2}{5}-\frac{2abd(dx+c)^{\frac{3}{2}}}{3}+\frac{2b^2c(dx+c)^{\frac{3}{2}}}{3}+2a^2d^2\sqrt{dx+c}-4abcd\sqrt{dx+c}+2b^2c^2\sqrt{dx+c}}{b^3}+\frac{2\left(-a^3d^3+3a^2\right.}{}$
risch	$\frac{2(3b^2x^2d^2-5ab\,d^2x+11b^2cdx+15a^2d^2-35abcd+23b^2c^2)\sqrt{dx+c}}{15b^3}-\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)a^3d^3}{b^3\sqrt{(ad-bc)\,b}}+\frac{6\,\mathrm{arc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(1/5*(d*x+c)^(5/2)*b^2-1/3*a*b*d*(d*x+c)^(3/2)+1/3*b^2*c*(d*x+c)^(3/2
)+a^2*d^2*(d*x+c)^(1/2)-2*a*b*c*d*(d*x+c)^(1/2)+b^2*c^2*(d*x+c)^(1/2))+2*(-
a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/((a*d-b*c)*b)^(1/2)*arctan
(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.46, size = 290, normalized size = 2.59

$$\left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(3\,b^2d^2x^2+23\,b^2c^2-35\,abcd+15\,a^2d^2+(11\,b^2cd-5\,abd^2)x)\sqrt{dx+c}}{15\,b^3},\ -\frac{2\left(15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^2d^2x^2+23\,b^2c^2-35\,abcd+15\,a^2d^2+(11\,b^2cd-5\,abd^2)x)\sqrt{dx+c}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x +
2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(3*b^2*
d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x
)*sqrt(d*x + c))/b^3, -2/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^
2*d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)
*x)*sqrt(d*x + c))/b^3]
```

Sympy [A]
time = 13.00, size = 121, normalized size = 1.08

$$\frac{2(c+dx)^{\frac{5}{2}}}{5b}+\frac{(c+dx)^{\frac{3}{2}}(-2ad+2bc)}{3b^2}+\frac{\sqrt{c+dx}\,(2a^2d^2-4abcd+2b^2c^2)}{b^3}-\frac{2(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*b) + (c + d*x)**(3/2)*(-2*a*d + 2*b*c)/(3*b**2) + sqr
t(c + d*x)*(2*a**2*d**2 - 4*a*b*c*d + 2*b**2*c**2)/b**3 - 2*(a*d - b*c)**3*
atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

Giac [A]
time = 1.69, size = 171, normalized size = 1.53

$$\frac{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b^4+5\,(dx+c)^{\frac{3}{2}}b^4c+15\,\sqrt{dx+c}\,b^4c^2-5\,(dx+c)^{\frac{3}{2}}ab^3d-30\,\sqrt{dx+c}\,ab^3cd+15\,\sqrt{dx+c}\,a^2b^2d^2\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/15*(3*(d*x + c)^(5/2
)*b^4 + 5*(d*x + c)^(3/2)*b^4*c + 15*sqrt(d*x + c)*b^4*c^2 - 5*(d*x + c)^(3
/2)*a*b^3*d - 30*sqrt(d*x + c)*a*b^3*c*d + 15*sqrt(d*x + c)*a^2*b^2*d^2)/b^
5
```

Mupad [B]
time = 0.08, size = 130, normalized size = 1.16

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{2\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}+\frac{2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{b^3}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}\,\sqrt{c+d\,x}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) - (2*(a*d - b*c)*(c + d*x)^(3/2))/(3*b^2) + (2*(a
*d - b*c)^2*(c + d*x)^(1/2))/b^3 - (2*atan((b^(1/2)*(a*d - b*c)^(5/2)*(c +
d*x)^(1/2))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c
)^(5/2))/b^(7/2)
```

3.1406 $\int \frac{(c+dx)^{5/2}}{(a+bx)^2} dx$

Optimal. Leaf size=110

$$\frac{5d(bc-ad)\sqrt{c+dx}}{b^3}+\frac{5d(c+dx)^{3/2}}{3b^2}-\frac{(c+dx)^{5/2}}{b(a+bx)}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

```
[Out] 5/3*d*(d*x+c)^(3/2)/b^2-(d*x+c)^(5/2)/b/(b*x+a)-5*d*(-a*d+b*c)^(3/2)*arctan
h(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+5*d*(-a*d+b*c)*(d*x+c)^(1
/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 52, 65, 214}

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{5d\sqrt{c+dx}\,(bc-ad)}{b^3}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x])/b^3 + (5*d*(c + d*x)^(3/2))/(3*b^2) - (c +
d*x)^(5/2)/(b*(a + b*x)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx = -\frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d(bc-ad))\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b^2}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d(bc-ad)^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}}{2b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{b^3}$$

$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{bc-}}\right.}{b^{7/2}}$$

Mathematica [A]
time = 0.32, size = 116, normalized size = 1.05

$$-\frac{\sqrt{c+dx}\,(15a^2d^2+10abd(-2c+dx)+b^2(3c^2-14cdx-2d^2x^2))}{3b^3(a+bx)} + \frac{5d(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] -1/3*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(3*c^2 - 14*c
*d*x - 2*d^2*x^2)))/(b^3*(a + b*x)) + (5*d*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(7/2)
```

Maple [A]
time = 0.19, size = 152, normalized size = 1.38

method	result
derivativedivides	$2d\left(-\frac{-\frac{b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{b^3}+\frac{\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{5\left(a^2d^2-2abcd+b^2c^2\right)}{2\sqrt{(}}}{b^3}\right.$
default	$2d\left(-\frac{-\frac{b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{b^3}+\frac{\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)\sqrt{dx+c}}{(dx+c)b+ad-bc}+\frac{5\left(a^2d^2-2abcd+b^2c^2\right)}{2\sqrt{(}}}{b^3}\right.$
risch	$-\frac{2d(-bdx+6ad-7bc)\sqrt{dx+c}}{3b^3}-\frac{d^3\sqrt{dx+c}\,a^2}{b^3(bdx+ad)}+\frac{2d^2\sqrt{dx+c}\,ac}{b^2(bdx+ad)}-\frac{d\sqrt{dx+c}\,c^2}{b(bdx+ad)}+\frac{5d^3\arctan\left(\right.}{b^3\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/b^3*(-1/3*b*(d*x+c)^(3/2)+2*a*d*(d*x+c)^(1/2)-2*b*c*(d*x+c)^(1/2))+
1/b^3*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)
+5/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)
/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.42, size = 330, normalized size = 3.00

$$\left[-\frac{15\left(abcd-a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad+2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\left(2b^{2}d^{2}x^{2}-3b^{2}c^{2}+20abcd-15a^{2}d^{2}+2\left(7b^{2}cd-5abd^{2}\right)x\right)\sqrt{dx+c}}{6\left(b^{4}x+ab^{3}\right)},\ -\frac{15\left(abcd-a^{2}d^{2}+\left(b^{2}cd-abd^{2}\right)x\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2b^{2}d^{2}x^{2}-3b^{2}c^{2}+20abcd-15a^{2}d^{2}+2\left(7b^{2}cd-5abd^{2}\right)x\right)\sqrt{dx+c}}{3\left(b^{4}x+ab^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x)*sqrt((b*c - a*d)/b)*l
og((b*d*x + 2*b*c - a*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a))
 - 2*(2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3), -1/3*(15*(a*b*c*d - a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*
a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. $2(97) = 194$.
time = 124.93, size = 1312, normalized size = 11.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**2,x)
```

```
[Out] -2*a**3*d**4*sqrt(c + d*x)/(2*a**2*b**3*d**2 - 2*a*b**4*c*d + 2*a*b**4*d**2
*x - 2*b**5*c*d*x) + a**3*d**4*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*s
qrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c
**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**3) - a**3*d**4*sqrt(
-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*
d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqr
t(c + d*x))/(2*b**3) + 6*a**2*c*d**3*sqrt(c + d*x)/(2*a**2*b**2*d**2 - 2*a*
b**3*c*d + 2*a*b**3*d**2*x - 2*b**4*c*d*x) - 3*a**2*c*d**3*sqrt(-1/(b*(a*d
- b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/
(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x)
)/(2*b**2) + 3*a**2*c*d**3*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b**2) + 6*a**2*d**3*atan(sqr
t(c + d*x)/sqrt(a*d/b - c))/(b**4*sqrt(a*d/b - c)) - 6*a*c**2*d**2*sqrt(c +
 d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2*b**3*c*d*x) + 3*a
*c**2*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c
)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d
- b*c)**3)) + sqrt(c + d*x))/(2*b) - 3*a*c**2*d**2*sqrt(-1/(b*(a*d - b*c)**
3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d -
```

```
b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) -
12*a*c*d**2*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**3*sqrt(a*d/b - c)) - 4
*a*d**2*sqrt(c + d*x)/b**3 - c**3*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d
**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b
**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c**3*d*sqrt(-1/(b
*(a*d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqr
t(-1/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c +
d*x))/2 + 2*c**3*d*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x -
2*b**2*c*d*x) + 6*c**2*d*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*
d/b - c)) + 4*c*d*sqrt(c + d*x)/b**2 + 2*d*(c + d*x)**(3/2)/(3*b**2)
```

Giac [A]
time = 1.95, size = 181, normalized size = 1.65

$$\frac{5\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{3}}-\frac{\sqrt{dx+c}\,b^{2}c^{2}d-2\,\sqrt{dx+c}\,abcd^{2}+\sqrt{dx+c}\,a^{2}d^{3}}{((dx+c)b-bc+ad)b^{3}}+\frac{2\left((dx+c)^{\frac{3}{2}}b^{4}d+6\,\sqrt{dx+c}\,b^{4}cd-6\,\sqrt{dx+c}\,ab^{3}d^{2}\right)}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - (sqrt(d*x + c)*b^2*c^2*d - 2*sqrt(d*x
+ c)*a*b*c*d^2 + sqrt(d*x + c)*a^2*d^3)/(((d*x + c)*b - b*c + a*d)*b^3) + 2
/3*((d*x + c)^(3/2)*b^4*d + 6*sqrt(d*x + c)*b^4*c*d - 6*sqrt(d*x + c)*a*b^3
*d^2)/b^6
```

Mupad [B]
time = 0.12, size = 161, normalized size = 1.46

$$\frac{2\,d\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}-\frac{\sqrt{c+d\,x}\,\left(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{b^4\,\left(c+d\,x\right)-b^4\,c+a\,b^3\,d}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{7/2}}+\frac{2\,d\,\left(2\,b^2\,c-2\,a\,b\,d\right)\,\sqrt{c+d\,x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*d*(c + d*x)^(3/2))/(3*b^2) - ((c + d*x)^(1/2)*(a^2*d^3 + b^2*c^2*d - 2*a
*b*c*d^2))/(b^4*(c + d*x) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/2)*d*(a*d -
b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))*(a*d - b*c
)^(3/2))/b^(7/2) + (2*d*(2*b^2*c - 2*a*b*d)*(c + d*x)^(1/2))/b^4
```

3.1407 $\int \frac{(c+dx)^{5/2}}{(a+bx)^3} dx$

Optimal. Leaf size=119

$$\frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} - \frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}$$

```
[Out] -5/4*d*(d*x+c)^(3/2)/b^2/(b*x+a)-1/2*(d*x+c)^(5/2)/b/(b*x+a)^2-15/4*d^2*arc
tanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+15/4*
d^2*(d*x+c)^(1/2)/b^3
```

Rubi [A]

time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 52, 65, 214}

$$-\frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{15d^2\sqrt{c+dx}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x])/(4*b^3) - (5*d*(c + d*x)^(3/2))/(4*b^2*(a + b*x)) -
(c + d*x)^(5/2)/(2*b*(a + b*x)^2) - (15*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(7/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^3}\,dx &= -\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d^2)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{8b^2}\\
&= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d^2(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^3}\\
&= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{(15d(bc-ad))\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\right.}{4b^3}\\
&= \frac{15d^2\sqrt{c+dx}}{4b^3}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}-\frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 119, normalized size = 1.00

$$\frac{\sqrt{c+dx}\,(15a^2d^2-5abd(c-5dx)+b^2(-2c^2-9cdx+8d^2x^2))}{4b^3(a+bx)^2}-\frac{15d^2\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (Sqrt[c + d*x]*(15*a^2*d^2 - 5*a*b*d*(c - 5*d*x) + b^2*(-2*c^2 - 9*c*d*x +
8*d^2*x^2)))/(4*b^3*(a + b*x)^2) - (15*d^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(7/2))
```

Maple [A]
time = 0.20, size = 138, normalized size = 1.16

method	result
derivativedivides	$2d^2\left(\frac{\sqrt{dx+c}}{b^3}-\frac{\frac{\left(-\frac{9}{8}abd+\frac{9}{8}b^2c\right)(dx+c)^{\frac{3}{2}}+\left(-\frac{7}{8}a^2d^2+\frac{7}{4}abcd-\frac{7}{8}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)b}}}{b^3}\right)$
default	$2d^2\left(\frac{\sqrt{dx+c}}{b^3}-\frac{\frac{\left(-\frac{9}{8}abd+\frac{9}{8}b^2c\right)(dx+c)^{\frac{3}{2}}+\left(-\frac{7}{8}a^2d^2+\frac{7}{4}abcd-\frac{7}{8}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15(ad-bc)\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)b}}}{b^3}\right)$
risch	$\frac{2d^2\sqrt{dx+c}}{b^3}+\frac{9d^3(dx+c)^{\frac{3}{2}}a}{4b^2(bdx+ad)^2}-\frac{9d^2(dx+c)^{\frac{3}{2}}c}{4b(bdx+ad)^2}+\frac{7d^4\sqrt{dx+c}\,a^2}{4b^3(bdx+ad)^2}-\frac{7d^3\sqrt{dx+c}\,ac}{2b^2(bdx+ad)^2}+\frac{7d^2\sqrt{dx+}}{4b(bdx+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(1/b^3*(d*x+c)^(1/2)-1/b^3*(((-9/8*a*b*d+9/8*b^2*c)*(d*x+c)^(3/2)+(-7
/8*a^2*d^2+7/4*a*b*c*d-7/8*b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^2+15
/8*(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.45, size = 344, normalized size = 2.89

$$\left[\frac{15\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-(9\,b^2cd-25\,abd^2)x)\sqrt{dx+c}}{8\,(b^5x^2+2\,ab^4x+a^2b^3)},\ -\frac{15\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(8\,b^2d^2x^2-2\,b^2c^2-5\,abcd+15\,a^2d^2-(9\,b^2cd-25\,abd^2)x)\sqrt{dx+c}}{4\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d
*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(8
*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2
)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(
-(b*c - a*d)/b)/(b*c - a*d)) - (8*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*
a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.71, size = 171, normalized size = 1.44

$$\frac{2\,\sqrt{dx+c}\,d^2}{b^3}+\frac{15\,(bcd^2-ad^3)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}\,b^3}-\frac{9\,(dx+c)^{\frac{3}{2}}b^2cd^2-7\,\sqrt{dx+c}\,b^2c^2d^2-9\,(dx+c)^{\frac{3}{2}}abd^3+14\,\sqrt{dx+c}\,abcd^3-7\,\sqrt{dx+c}\,a^2d^4}{4\,((dx+c)b-bc+ad)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d^2/b^3 + 15/4*(b*c*d^2 - a*d^3)*arctan(sqrt(d*x + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/4*(9*(d*x + c)^(3/2)*b^2*
c*d^2 - 7*sqrt(d*x + c)*b^2*c^2*d^2 - 9*(d*x + c)^(3/2)*a*b*d^3 + 14*sqrt(d
*x + c)*a*b*c*d^3 - 7*sqrt(d*x + c)*a^2*d^4)/(((d*x + c)*b - b*c + a*d)^2*b
^3)
```

Mupad [B]
time = 0.16, size = 199, normalized size = 1.67

$$\frac{2\,d^2\,\sqrt{c+d\,x}}{b^3}-\frac{\left(\frac{9\,b^2\,c\,d^2}{4}-\frac{9\,a\,b\,d^3}{4}\right)\,{\left(c+d\,x\right)}^{3/2}-\sqrt{c+d\,x}\,\left(\frac{7\,a^2\,d^4}{4}-\frac{7\,a\,b\,c\,d^3}{2}+\frac{7\,b^2\,c^2\,d^2}{4}\right)}{b^5\,{\left(c+d\,x\right)}^2-\left(2\,b^5\,c-2\,a\,b^4\,d\right)\,\left(c+d\,x\right)+b^5\,c^2+a^2\,b^3\,d^2-2\,a\,b^4\,c\,d}-\frac{15\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,d^2\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^3-b\,c\,d^2}\right)\,\sqrt{a\,d-b\,c}}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(5/2)/(a + b*x)^3,x)

[Out] (2*d^2*(c + d*x)^(1/2))/b^3 - (((9*b^2*c*d^2)/4 - (9*a*b*d^3)/4)*(c + d*x)^(3/2) - (c + d*x)^(1/2)*((7*a^2*d^4)/4 + (7*b^2*c^2*d^2)/4 - (7*a*b*c*d^3)/2))/(b^5*(c + d*x)^2 - (2*b^5*c - 2*a*b^4*d)*(c + d*x) + b^5*c^2 + a^2*b^3*d^2 - 2*a*b^4*c*d) - (15*d^2*atan((b^(1/2)*d^2*(a*d - b*c)^(1/2)*(c + d*x)^(1/2))/(a*d^3 - b*c*d^2))*(a*d - b*c)^(1/2))/(4*b^(7/2))

3.1408 $\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx$

Optimal. Leaf size=126

$$-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}$$

```
[Out] -5/12*d*(d*x+c)^(3/2)/b^2/(b*x+a)^2-1/3*(d*x+c)^(5/2)/b/(b*x+a)^3-5/8*d^3*a
rctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(1/2)-5/8
*d^2*(d*x+c)^(1/2)/b^3/(b*x+a)
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {43, 65, 214}

$$-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(8*b^3*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (c + d*x)^(5/2)/(3*b*(a + b*x)^3) - (5*d^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx}{6b}\\
&= -\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{8b^2}\\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d^3)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^3}\\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+d}\right.}{8b^3}\\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.57, size = 119, normalized size = 0.94

$$-\frac{\sqrt{c+dx}\,(15a^2d^2+10abd(c+4dx)+b^2(8c^2+26cdx+33d^2x^2))}{24b^3(a+bx)^3}+\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8b^{7/2}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 26*c
*d*x + 33*d^2*x^2)))/(b^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*
x])/Sqrt[-(b*c) + a*d]])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

Maple [A]
time = 0.16, size = 130, normalized size = 1.03

method	result	s

derivativedivides	$2d^3\left(\frac{-\frac{11(dx+c)^{\frac{5}{2}}}{16b}-\frac{5(ad-bc)(dx+c)^{\frac{3}{2}}}{6b^2}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{16b^3}}{\left((dx+c)b+ad-bc\right)^3}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16b^3\sqrt{(ad-bc)\,b}}\right)$	13
default	$2d^3\left(\frac{-\frac{11(dx+c)^{\frac{5}{2}}}{16b}-\frac{5(ad-bc)(dx+c)^{\frac{3}{2}}}{6b^2}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{16b^3}}{\left((dx+c)b+ad-bc\right)^3}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{16b^3\sqrt{(ad-bc)\,b}}\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*((-11/16*(d*x+c)^(5/2)/b-5/6*(a*d-b*c)/b^2*(d*x+c)^(3/2)-5/16/b^3*(a^
2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^3+5/16/b^3/((a*
d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(102) = 204$.
time = 0.46, size = 563, normalized size = 4.47

$$\left[\frac{15\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(8\,b^4c^3+2\,ab^3c^2d+5\,a^2b^2cd^2-15\,a^3bd^3+33\,(b^4cd^2-ab^3d^3)x^2+2\,(13\,b^4c^2d+7\,ab^3cd^2-20\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c-a^4b^4d+(b^8c-ab^7d)x^3+3\,(ab^7c-a^2b^6d)x^2+3\,(a^2b^6c-a^3b^5d)x)},\ \frac{15\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(8\,b^4c^3+2\,ab^3c^2d+5\,a^2b^2cd^2-15\,a^3bd^3+33\,(b^4cd^2-ab^3d^3)x^2+2\,(13\,b^4c^2d+7\,ab^3cd^2-20\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c-a^4b^4d+(b^8c-ab^7d)x^3+3\,(ab^7c-a^2b^6d)x^2+3\,(a^2b^6c-a^3b^5d)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) - 2*(8*b^4*c^3 + 2*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^
3 + 33*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^
```

```
3 + 3*(a*b^7*c - a^2*b^6*d)*x^2 + 3*(a^2*b^6*c - a^3*b^5*d)*x), 1/24*(15*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*d
)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3 + 2
*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^3 + 33*(b^4*c*d^2 - a*b^3*d^3)*
x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^3 + 3*(a*b^7*c - a^2*b^6*d)*x^2
 + 3*(a^2*b^6*c - a^3*b^5*d)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.56, size = 161, normalized size = 1.28

$$\frac{5\,d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,\sqrt{-b^2c+abd}\,b^3}-\frac{33\,(dx+c)^{\frac{5}{2}}b^2d^3-40\,(dx+c)^{\frac{3}{2}}b^2cd^3+15\,\sqrt{dx+c}\,b^2c^2d^3+40\,(dx+c)^{\frac{3}{2}}abd^4-30\,\sqrt{dx+c}\,abcd^4+15\,\sqrt{dx+c}\,a^2d^5}{24\,((dx+c)b-bc+ad)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^3) - 1/24*(33*(d*x + c)^(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 15
*sqrt(d*x + c)*b^2*c^2*d^3 + 40*(d*x + c)^(3/2)*a*b*d^4 - 30*sqrt(d*x + c)*
a*b*c*d^4 + 15*sqrt(d*x + c)*a^2*d^5)/(((d*x + c)*b - b*c + a*d)^3*b^3)
```

Mupad [**B**]
time = 0.36, size = 222, normalized size = 1.76

$$\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{7/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{11\,d^3\,{\left(c+d\,x\right)}^{5/2}}{8\,b}+\frac{5\,d^3\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,b^3}+\frac{5\,d^3\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^4,x)
```

```
[Out] (5*d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(7/2)*(a*d -
 b*c)^(1/2)) - ((11*d^3*(c + d*x)^(5/2))/(8*b) + (5*d^3*(c + d*x)^(1/2)*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d))/(8*b^3) + (5*d^3*(a*d - b*c)*(c + d*x)^(3/2))
/(3*b^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x
)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)
```

3.1409 $\int \frac{(c+dx)^{5/2}}{(a+bx)^5} \, dx$

Optimal. Leaf size=162

$$-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}$$

```
[Out] -5/24*d*(d*x+c)^(3/2)/b^2/(b*x+a)^3-1/4*(d*x+c)^(5/2)/b/(b*x+a)^4+5/64*d^4*
arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(3/2)-5/
32*d^2*(d*x+c)^(1/2)/b^3/(b*x+a)^2-5/64*d^3*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b
*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {43, 44, 65, 214}

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)}-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(32*b^3*(a + b*x)^2) - (5*d^3*Sqrt[c + d*x])/(64*b^3
*(b*c - a*d)*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(24*b^2*(a + b*x)^3) - (c +
 d*x)^(5/2)/(4*b*(a + b*x)^4) + (5*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[b*c - a*d]])/(64*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{(5d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{16b^2}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{(5d^3)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^3}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} - \frac{(5d^4)\int \frac{}{(a+}}{128b^3}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} - \frac{(5d^3)\,\mathrm{Subs}}{}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{5d^4\tanh^{-1}}{64b^{7/}}$$

Mathematica [A]
time = 0.97, size = 172, normalized size = 1.06

$$\frac{\sqrt{c+dx}\,(15a^3d^3+5a^2bd^2(2c+11dx)+ab^2d(8c^2+36cdx+73d^2x^2)-b^3(48c^3+136c^2dx+118cd^2x^2+15d^3x^3))}{192b^3(bc-ad)(a+bx)^4}+\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{64b^{7/2}(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (Sqrt[c + d*x]*(15*a^3*d^3 + 5*a^2*b*d^2*(2*c + 11*d*x) + a*b^2*d*(8*c^2 +
36*c*d*x + 73*d^2*x^2) - b^3*(48*c^3 + 136*c^2*d*x + 118*c*d^2*x^2 + 15*d^3
```

```
*x^3)))/(192*b^3*(b*c - a*d)*(a + b*x)^4) + (5*d^4*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[-(b*c) + a*d]])/(64*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

Maple [A]
time = 0.16, size = 159, normalized size = 0.98

method	result
derivativedivides	$2d^4\left(\frac{\frac{5(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{73(dx+c)^{\frac{5}{2}}}{384b}-\frac{55(ad-bc)(dx+c)^{\frac{3}{2}}}{384b^2}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{128b^3}}{\left((dx+c)b+ad-bc\right)^4}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{128(ad-bc)b^3\sqrt{(ad-bc)b}}\right)$
default	$2d^4\left(\frac{\frac{5(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{73(dx+c)^{\frac{5}{2}}}{384b}-\frac{55(ad-bc)(dx+c)^{\frac{3}{2}}}{384b^2}-\frac{5\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{dx+c}}{128b^3}}{\left((dx+c)b+ad-bc\right)^4}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{128(ad-bc)b^3\sqrt{(ad-bc)b}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^4*((5/128/(a*d-b*c)*(d*x+c)^(7/2)-73/384*(d*x+c)^(5/2)/b-55/384*(a*d-b*
c)/b^2*(d*x+c)^(3/2)-5/128/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(1/2))/(
(d*x+c)*b+a*d-b*c)^4+5/128/(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+
c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(134) = 268.
time = 0.42, size = 894, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a^2
*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*
x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5*c
^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x +
 c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d +
 a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(a^
2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6*c
*d + a^5*b^5*d^2)*x), -1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
 + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a
^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4
)*x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5
*c^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d
 + a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(
a^2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6
*c*d + a^5*b^5*d^2)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.53, size = 259, normalized size = 1.60

$$-\frac{5\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\,(b^4c-ab^3d)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^3d^4+73\,(dx+c)^{\frac{5}{2}}b^3cd^4-55\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+15\,\sqrt{dx+c}\,b^3c^3d^4-73\,(dx+c)^{\frac{5}{2}}ab^2d^5+110\,(dx+c)^{\frac{3}{2}}ab^2cd^5-45\,\sqrt{dx+c}\,ab^2c^2d^5-55\,(dx+c)^{\frac{3}{2}}a^2bd^6+45\,\sqrt{dx+c}\,a^2bcd^6-15\,\sqrt{dx+c}\,a^3d^7}{192\,(b^4c-ab^3d)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c - a*b^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x + c)^(7/2)*b^3*d^4 + 73*(d*x + c)^(5/
2)*b^3*c*d^4 - 55*(d*x + c)^(3/2)*b^3*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^
4 - 73*(d*x + c)^(5/2)*a*b^2*d^5 + 110*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqr
t(d*x + c)*a*b^2*c^2*d^5 - 55*(d*x + c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*
a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^7)/((b^4*c - a*b^3*d)*((d*x + c)*b - b
*c + a*d)^4)
```

Mupad [B]
time = 0.41, size = 309, normalized size = 1.91

$$\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{73\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,b}-\frac{5\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{5\,d^4\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{64\,b^3}+\frac{55\,d^4\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{192\,b^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^5,x)
```

```
[Out] (5*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(7/2)*(a*d
- b*c)^(3/2)) - ((73*d^4*(c + d*x)^(5/2))/(192*b) - (5*d^4*(c + d*x)^(7/2))
/(64*(a*d - b*c)) + (5*d^4*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
/(64*b^3) + (55*d^4*(a*d - b*c)*(c + d*x)^(3/2))/(192*b^2))/(b^4*(c + d*x)^
4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3
+ 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b
^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)
```

3.1410 $\int \frac{(c+dx)^{5/2}}{(a+bx)^6} \, dx$

Optimal. Leaf size=198

$$-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}-\frac{3d^5\tanh^{-}}{128b}$$

```
[Out] -1/8*d*(d*x+c)^(3/2)/b^2/(b*x+a)^4-1/5*(d*x+c)^(5/2)/b/(b*x+a)^5-3/128*d^5*
arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(5/2)-1/
16*d^2*(d*x+c)^(1/2)/b^3/(b*x+a)^3-1/64*d^3*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b
*x+a)^2+3/128*d^4*(d*x+c)^(1/2)/b^3/(-a*d+b*c)^2/(b*x+a)
```

Rubi [A]

time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = $\{43, 44, 65, 214\}$

$$-\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}+\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -1/16*(d^2*Sqrt[c + d*x])/(b^3*(a + b*x)^3) - (d^3*Sqrt[c + d*x])/(64*b^3*(
b*c - a*d)*(a + b*x)^2) + (3*d^4*Sqrt[c + d*x])/(128*b^3*(b*c - a*d)^2*(a +
 b*x)) - (d*(c + d*x)^(3/2))/(8*b^2*(a + b*x)^4) - (c + d*x)^(5/2)/(5*b*(a
+ b*x)^5) - (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(128*b
^(7/2)*(b*c - a*d)^(5/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^6}\,dx &= -\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx}{2b}\\
&= -\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+\frac{(3d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{16b^2}\\
&= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}+\frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^3}\\
&= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}-\frac{(3d^4)\int \frac{}{(a+bx}}{128b^3(}\\
&= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\\
&= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\\
&= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2}+\frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-
\end{aligned}$$

Mathematica [A]
time = 1.32, size = 222, normalized size = 1.12

$$-\frac{\sqrt{c+dx}\,(15a^4d^4+10a^3bd^3(c+7dx)+2a^2b^2d^2(4c^2+23cdx+64d^2x^2)-2ab^3d(88c^3+256c^2dx+233cd^2x^2+35d^3x^3)+b^4(128c^4+336c^3dx+248c^2d^2x^2+10cd^3x^3-15d^4x^4))}{640b^3(bc-ad)^2(a+bx)^5}+\frac{3d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{128b^{7/2}(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -1/640*(Sqrt[c + d*x]*(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 2*a^2*b^2*d^
2*(4*c^2 + 23*c*d*x + 64*d^2*x^2) - 2*a*b^3*d*(88*c^3 + 256*c^2*d*x + 233*c
*d^2*x^2 + 35*d^3*x^3) + b^4*(128*c^4 + 336*c^3*d*x + 248*c^2*d^2*x^2 + 10*
c*d^3*x^3 - 15*d^4*x^4)))/(b^3*(b*c - a*d)^2*(a + b*x)^5) + (3*d^5*ArcTan[(
Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(128*b^(7/2)*(-(b*c) + a*d)^(5/
2))
```

Maple [A]
time = 0.16, size = 205, normalized size = 1.04

method	result
derivativedivides	$2d^5\left(\frac{\frac{3b(dx+c)^{\frac{9}{2}}}{256(a^2d^2-2abcd+b^2c^2)}+\frac{7(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{(dx+c)^{\frac{5}{2}}}{10b}-\frac{7(ad-bc)(dx+c)^{\frac{3}{2}}}{128b^2}-\frac{3(a^2d^2-2abcd+b^2c^2)\sqrt{dx+c}}{256b^3}}{((dx+c)b+ad-bc)^5}+\frac{}{256b^3(}\right.$
default	$2d^5\left(\frac{\frac{3b(dx+c)^{\frac{9}{2}}}{256(a^2d^2-2abcd+b^2c^2)}+\frac{7(dx+c)^{\frac{7}{2}}}{128(ad-bc)}-\frac{(dx+c)^{\frac{5}{2}}}{10b}-\frac{7(ad-bc)(dx+c)^{\frac{3}{2}}}{128b^2}-\frac{3(a^2d^2-2abcd+b^2c^2)\sqrt{dx+c}}{256b^3}}{((dx+c)b+ad-bc)^5}+\frac{}{256b^3(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^5*((3/256*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(9/2)+7/128/(a*d-b*c)*(
d*x+c)^(7/2)-1/10*(d*x+c)^(5/2)/b-7/128*(a*d-b*c)/b^2*(d*x+c)^(3/2)-3/256/b
^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^5+3/256/b
^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/(
(a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(166) = 332$.

time = 0.45, size = 1337, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^
2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c
 - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(128*b^6*c^5 -
 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^
4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 10*(b^6*c^2*d^3 - 8*a*b
^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3*d^2 - 357*a*b^5*c^2*d^3 + 29
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168*b^6*c^4*d - 424*a*b^5*c^3*d^
2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 35*a^4*b^2*d^5)*x)*sqrt(d*x +
c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5*c*d^2 - a^8*b^4*d^3 + (b^12*
c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*b^9*d^3)*x^5 + 5*(a*b^11*c^3
- 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8*d^3)*x^4 + 10*(a^2*b^10*c^3
- 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*d^3)*x^3 + 10*(a^3*b^9*c^3 -
3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^3)*x^2 + 5*(a^4*b^8*c^3 - 3*a
^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*x), 1/640*(15*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
 a^5*d^5)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b
*d*x + b*c)) - (128*b^6*c^5 - 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*
x^4 + 10*(b^6*c^2*d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3
*d^2 - 357*a*b^5*c^2*d^3 + 297*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168
*b^6*c^4*d - 424*a*b^5*c^3*d^2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 3
5*a^4*b^2*d^5)*x)*sqrt(d*x + c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5
*c*d^2 - a^8*b^4*d^3 + (b^12*c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*
b^9*d^3)*x^5 + 5*(a*b^11*c^3 - 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8
*d^3)*x^4 + 10*(a^2*b^10*c^3 - 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*
d^3)*x^3 + 10*(a^3*b^9*c^3 - 3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^
3)*x^2 + 5*(a^4*b^8*c^3 - 3*a^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*
x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(166) = 332.

time = 1.51, size = 380, normalized size = 1.92

$$\frac{3\,d^5\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{128\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)\sqrt{-b^2c+abd}}+\frac{15\,(dx+c)^{\frac{9}{2}}b^4d^5-70\,(dx+c)^{\frac{7}{2}}b^4cd^5-128\,(dx+c)^{\frac{5}{2}}b^4c^2d^5+70\,(dx+c)^{\frac{3}{2}}b^4c^3d^5-15\,\sqrt{dx+c}\,b^4c^4d^5+70\,(dx+c)^{\frac{7}{2}}ab^3d^6+256\,(dx+c)^{\frac{5}{2}}ab^3cd^6-210\,(dx+c)^{\frac{3}{2}}ab^3c^2d^6+60\,\sqrt{dx+c}\,ab^3c^3d^6-128\,(dx+c)^{\frac{5}{2}}a^2b^2d^7+210\,(dx+c)^{\frac{3}{2}}a^2b^2cd^7-90\,\sqrt{dx+c}\,a^2b^2c^2d^7-70\,(dx+c)^{\frac{3}{2}}a^3bd^8+60\,\sqrt{dx+c}\,a^3bcd^8-15\,\sqrt{dx+c}\,a^4d^9}{640\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)((dx+c)b-bc+ad)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^2 - 2*a*b^4*
c*d + a^2*b^3*d^2)*sqrt(-b^2*c + a*b*d)) + 1/640*(15*(d*x + c)^(9/2)*b^4*d^
5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 - 128*(d*x + c)^(5/2)*b^4*c^2*d^5 + 70*(d*
x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^5 + 70*(d*x + c)^(7/2
)*a*b^3*d^6 + 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d*x + c)^(3/2)*a*b^3*c
^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 - 128*(d*x + c)^(5/2)*a^2*b^2*d^7 +
 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*a^2*b^2*c^2*d^7 - 70*
(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d^8 - 15*sqrt(d*x + c)
*a^4*d^9)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*((d*x + c)*b - b*c + a*d)^
5)
```

Mupad [B]

time = 0.50, size = 411, normalized size = 2.08

$$\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{d^5\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{7\,d^5\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^5\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{128\,b^3}+\frac{7\,d^5\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{64\,b^2}-\frac{3\,b\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^6,x)
```

```
[Out] (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(7/2)*(a*d
 - b*c)^(5/2)) - ((d^5*(c + d*x)^(5/2))/(5*b) - (7*d^5*(c + d*x)^(7/2))/(64
*(a*d - b*c)) + (3*d^5*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12
8*b^3) + (7*d^5*(a*d - b*c)*(c + d*x)^(3/2))/(64*b^2) - (3*b*d^5*(c + d*x)^
(9/2))/(128*(a*d - b*c)^2))/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10
*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(
c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 -
 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30
*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^
3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
```

3.1411 $\int \frac{\sqrt{-1+x}}{(1+x)^2} \, dx$

Optimal. Leaf size=35

$$-\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-(-1+x)^(1/2)/(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {43, 65, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx &= -\frac{\sqrt{-1+x}}{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx\\
&= -\frac{\sqrt{-1+x}}{1+x}+\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= -\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 1.00

$$
-\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^2,x]
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Maple [A]
time = 0.17, size = 30, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\sqrt{-1+x}}{1+x}$	30
default	$\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\sqrt{-1+x}}{1+x}$	30
risch	$\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\sqrt{-1+x}}{1+x}$	30
trager	$-\frac{\sqrt{-1+x}}{1+x}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+2\right)x+4\sqrt{-1+x}+3\operatorname{RootOf}\left(\_Z^2+2\right)}{1+x}\right)}{4}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-(-1+x)^(1/2)/(1+x)
```

Maxima [A]
time = 0.50, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="maxima")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

Fricas [A]
time = 0.45, size = 33, normalized size = 0.94

$$\frac{\sqrt{2}\,(x+1)\arctan\left(\frac{1}{2}\sqrt{2}\,\sqrt{x-1}\right)-2\sqrt{x-1}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - 2*sqrt(x - 1))/(x +
1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.88, size = 105, normalized size = 3.00

$$\begin{cases} \frac{\sqrt{2}\,i\,\operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2}+\frac{i}{\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}}-\frac{2i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} & \text{for } \frac{1}{|x+1|}>\frac{1}{2} \\ -\frac{\sqrt{1-\frac{2}{x+1}}}{\sqrt{x+1}}-\frac{\sqrt{2}\,\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**2,x)
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/2 + I/(sqrt(-1 + 2/(x + 1))
*sqrt(x + 1)) - 2*I/(sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)), 1/Abs(x + 1) > 1
/2), (-sqrt(1 - 2/(x + 1))/sqrt(x + 1) - sqrt(2)*asin(sqrt(2)/sqrt(x + 1))/
2, True))
```

Giac [A]
time = 1.64, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

Mupad [B]
time = 0.06, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)}{2}-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^2,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/2 - (x - 1)^(1/2)/(x + 1)
```

3.1412 $\int \frac{\sqrt{-1+x}}{(1+x)^3} dx$

Optimal. Leaf size=56

$$-\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/16*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)-1/2*(-1+x)^(1/2)/(1+x)^2+1/8*
(-1+x)^(1/2)/(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {43, 44, 65, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{\sqrt{x-1}}{8(x+1)}-\frac{\sqrt{x-1}}{2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] -1/2*Sqrt[-1 + x]/(1 + x)^2 + Sqrt[-1 + x]/(8*(1 + x)) + ArcTan[Sqrt[-1 + x
]/Sqrt[2]]/(8*Sqrt[2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{(1+x)^3}\,dx &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,(1+x)^2}\,dx\\ &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{16}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx\\ &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\ &= -\frac{\sqrt{-1+x}}{2(1+x)^2}+\frac{\sqrt{-1+x}}{8(1+x)}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 0.77

$$\frac{(-3+x)\sqrt{-1+x}}{8(1+x)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^3,x]
[Out] ((-3 + x)*Sqrt[-1 + x])/(8*(1 + x)^2) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/(8*Sqr
t[2])
```

Maple [A]
time = 0.20, size = 40, normalized size = 0.71

method	result	siz

risch	$\frac{x^2-4x+3}{8(1+x)^2\sqrt{-1+x}}+\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
derivativedivides	$\frac{\frac{(-1+x)^{\frac{3}{2}}}{8}-\frac{\sqrt{-1+x}}{4}}{(1+x)^2}+\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{16}$	40
default	$\frac{\frac{(-1+x)^{\frac{3}{2}}}{8}-\frac{\sqrt{-1+x}}{4}}{(1+x)^2}+\frac{\arctan\left(\frac{\sqrt{-1+x}\,\sqrt{2}}{2}\right)\sqrt{2}}{16}$	40
trager	$\frac{(-3+x)\sqrt{-1+x}}{8(1+x)^2}+\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+2\right)x+4\sqrt{-1+x}+3\operatorname{RootOf}\left(\_Z^2+2\right)}{1+x}\right)}{32}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/16*(-1+x)^(3/2)-1/8*(-1+x)^(1/2))/(1+x)^2+1/16*arctan(1/2*(-1+x)^(1/2)
*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.53, size = 43, normalized size = 0.77

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\sqrt{x-1}}{8\left((x-1)^2+4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/((x - 1)^2 + 4*x)
```

Fricas [A]
time = 0.42, size = 46, normalized size = 0.82

$$\frac{\sqrt{2}\left(x^2+2x+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}\,(x-3)}{16\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 + 2*x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x -
1)*(x - 3))/(x^2 + 2*x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.63, size = 168, normalized size = 3.00

$$\begin{cases} \frac{\sqrt{2}\, i \operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} - \frac{i}{8\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}} + \frac{3i}{4\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} - \frac{i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{5}{2}}} & \text{for } \frac{1}{|x+1|} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16} + \frac{1}{8\sqrt{1-\frac{2}{x+1}}\,\sqrt{x+1}} - \frac{3}{4\sqrt{1-\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} + \frac{1}{\sqrt{1-\frac{2}{x+1}}\,(x+1)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**3,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/16 - I/(8*sqrt(-1 + 2/(x +
1))*sqrt(x + 1)) + 3*I/(4*sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)) - I/(sqrt(-1
 + 2/(x + 1))*(x + 1)**(5/2)), 1/Abs(x + 1) > 1/2), (-sqrt(2)*asin(sqrt(2)/
sqrt(x + 1))/16 + 1/(8*sqrt(1 - 2/(x + 1))*sqrt(x + 1)) - 3/(4*sqrt(1 - 2/(
x + 1))*(x + 1)**(3/2)) + 1/(sqrt(1 - 2/(x + 1))*(x + 1)**(5/2)), True))
```

Giac [A]
time = 1.23, size = 37, normalized size = 0.66

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right) + \frac{(x-1)^{\frac{3}{2}} - 2\sqrt{x-1}}{8(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/(x + 1)^2
```

Mupad [B]
time = 0.04, size = 45, normalized size = 0.80

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{16} - \frac{\frac{\sqrt{x-1}}{4} - \frac{(x-1)^{3/2}}{8}}{4x + (x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^3,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/16 - ((x - 1)^(1/2)/4 - (x - 1)^(
3/2)/8)/(4*x + (x - 1)^2)
```

3.1413 $\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=154

$$-\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{7/2}}{7d^6}-10$$

```
[Out] 10/3*b*(-a*d+b*c)^4*(d*x+c)^(3/2)/d^6-4*b^2*(-a*d+b*c)^3*(d*x+c)^(5/2)/d^6+
20/7*b^3*(-a*d+b*c)^2*(d*x+c)^(7/2)/d^6-10/9*b^4*(-a*d+b*c)*(d*x+c)^(9/2)/d
^6+2/11*b^5*(d*x+c)^(11/2)/d^6-2*(-a*d+b*c)^5*(d*x+c)^(1/2)/d^6
```

Rubi [A]
time = 0.03, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+dx}\,(bc-ad)^5}{d^6}+\frac{2b^5(c+dx)^{11/2}}{11d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*Sqrt[c + d*x])/d^6 + (10*b*(b*c - a*d)^4*(c + d*x)^(3/2))
/(3*d^6) - (4*b^2*(b*c - a*d)^3*(c + d*x)^(5/2))/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(7/2))/(7*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(9/2))/(9*d^6) +
 (2*b^5*(c + d*x)^(11/2))/(11*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5\sqrt{c+dx}}+\frac{5b(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{3/2}}{d^5}+\frac{10b^3(bc-ad)^2(}{d^5}\right.\\ &= -\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(bc}{}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 216, normalized size = 1.40

$$\frac{2\sqrt{c+dx}\,(693a^5d^5+1155a^4bd^4(-2c+dx)+462a^3b^2d^3(8c^2-4cdx+3d^2x^2)+198a^2b^3d^2(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3)+11ab^4d(128c^4-64c^3dx+48c^2d^2x^2-40cd^3x^3+35d^4x^4)+b^5(-256c^5+128c^4dx-96c^3d^2x^2+80c^2d^3x^3-70cd^4x^4+63d^5x^5))}{693d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(693*a^5*d^5 + 1155*a^4*b*d^4*(-2*c + d*x) + 462*a^3*b^2*d
^3*(8*c^2 - 4*c*d*x + 3*d^2*x^2) + 198*a^2*b^3*d^2*(-16*c^3 + 8*c^2*d*x - 6
*c*d^2*x^2 + 5*d^3*x^3) + 11*a*b^4*d*(128*c^4 - 64*c^3*d*x + 48*c^2*d^2*x^2
 - 40*c*d^3*x^3 + 35*d^4*x^4) + b^5*(-256*c^5 + 128*c^4*d*x - 96*c^3*d^2*x^
2 + 80*c^2*d^3*x^3 - 70*c*d^4*x^4 + 63*d^5*x^5)))/(693*d^6)
```

Maple [A]
time = 0.15, size = 121, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{7}{2}}}{7}+4(ad-bc)^3b^2(dx+c)^{\frac{5}{2}}+\frac{10(ad-bc)^4b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^5\sqrt{\,}}{d^6}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{11}{2}}}{11}+\frac{10(ad-bc)b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{20(ad-bc)^2b^3(dx+c)^{\frac{7}{2}}}{7}+4(ad-bc)^3b^2(dx+c)^{\frac{5}{2}}+\frac{10(ad-bc)^4b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^5\sqrt{\,}}{d^6}$
gosper	$2\sqrt{dx+c}\,(63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a$
trager	$2\sqrt{dx+c}\,(63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a$
risch	$2\sqrt{dx+c}\,(63b^5x^5d^5+385a\,b^4d^5x^4-70b^5c\,d^4x^4+990a^2b^3d^5x^3-440a\,b^4c\,d^4x^3+80b^5c^2d^3x^3+1386a^3b^2d^5x^2-1188a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/11*b^5*(d*x+c)^(11/2)+5/9*(a*d-b*c)*b^4*(d*x+c)^(9/2)+10/7*(a*d-b*
c)^2*b^3*(d*x+c)^(7/2)+2*(a*d-b*c)^3*b^2*(d*x+c)^(5/2)+5/3*(a*d-b*c)^4*b*(d
*x+c)^(3/2)+(a*d-b*c)^5*(d*x+c)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.
time = 0.29, size = 283, normalized size = 1.84

$$\frac{2\left(693\sqrt{dx+c}\,a^5+\frac{1155\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^4b}{d}+\frac{462\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^3b^2}{d^2}+\frac{198\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a^2b^3}{d^3}+\frac{11\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)ab^4}{d^4}+\frac{\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^2-1386(dx+c)^{\frac{5}{2}}c^3+1155(dx+c)^{\frac{3}{2}}c^4-693\sqrt{dx+c}\,c^5\right)b^5}{d^5}\right)}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
```

```
+ 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

Fricas [A]
time = 0.67, size = 261, normalized size = 1.69

$$\frac{2\left(63\,b^{5}d^{5}x^{5} - 256\,b^{5}c^{5} + 1408\,ab^{4}c^{4}d - 3168\,a^{2}b^{3}c^{3}d^{2} + 3696\,a^{3}b^{2}c^{2}d^{3} - 2310\,a^{4}bcd^{4} + 693\,a^{5}d^{5} - 35\left(2\,b^{5}cd^{4} - 11\,ab^{4}d^{5}\right)x^{4} + 10\left(8\,b^{5}c^{2}d^{3} - 44\,ab^{4}cd^{4} + 99\,a^{2}b^{3}d^{5}\right)x^{3} - 6\left(16\,b^{5}c^{3}d^{2} - 88\,ab^{4}c^{2}d^{3} + 198\,a^{2}b^{3}cd^{4} - 231\,a^{3}b^{2}d^{5}\right)x^{2} + \left(128\,b^{5}c^{4}d - 704\,ab^{4}c^{3}d^{2} + 1584\,a^{2}b^{3}c^{2}d^{3} - 1848\,a^{3}b^{2}cd^{4} + 1155\,a^{4}bd^{5}\right)x\right)\sqrt{dx+c}}{693\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*d^5*x^5 - 256*b^5*c^5 + 1408*a*b^4*c^4*d - 3168*a^2*b^3*c^3*d
^2 + 3696*a^3*b^2*c^2*d^3 - 2310*a^4*b*c*d^4 + 693*a^5*d^5 - 35*(2*b^5*c*d^
4 - 11*a*b^4*d^5)*x^4 + 10*(8*b^5*c^2*d^3 - 44*a*b^4*c*d^4 + 99*a^2*b^3*d^5
)*x^3 - 6*(16*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 + 198*a^2*b^3*c*d^4 - 231*a^3*
b^2*d^5)*x^2 + (128*b^5*c^4*d - 704*a*b^4*c^3*d^2 + 1584*a^2*b^3*c^2*d^3 -
1848*a^3*b^2*c*d^4 + 1155*a^4*b*d^5)*x)*sqrt(d*x + c)/d^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(143) = 286$.
time = 38.65, size = 728, normalized size = 4.73

$$\begin{cases} \frac{-\frac{2a^{5}c}{\sqrt{c+dx}} - 2a^{5}\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{10a^{4}bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{10a^{4}b\left(\frac{c^{2}}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{20a^{3}b^{2}c\left(\frac{c^{2}}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^{2}} - \frac{20a^{3}b^{2}\left(-\frac{c^{3}}{\sqrt{c+dx}} - 3c^{2}\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^{2}} - \frac{20a^{2}b^{3}c\left(-\frac{c^{3}}{\sqrt{c+dx}} - 3c^{2}\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^{3}} - \frac{20a^{2}b^{3}\left(\frac{c^{4}}{\sqrt{c+dx}} + 4c^{3}\sqrt{c+dx} - 2c^{2}(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^{3}} - \frac{10ab^{4}c\left(\frac{c^{4}}{\sqrt{c+dx}} + 4c^{3}\sqrt{c+dx} - 2c^{2}(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^{4}} - \frac{10ab^{4}\left(-\frac{c^{5}}{\sqrt{c+dx}} - 5c^{4}\sqrt{c+dx} + \frac{10c^{3}(c+dx)^{\frac{3}{2}}}{3} - 2c^{2}(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^{4}} - \frac{2b^{5}c\left(-\frac{c^{5}}{\sqrt{c+dx}} - 5c^{4}\sqrt{c+dx} + \frac{10c^{3}(c+dx)^{\frac{3}{2}}}{3} - 2c^{2}(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^{5}} - \frac{2b^{5}\left(\frac{c^{6}}{\sqrt{c+dx}} + 6c^{5}\sqrt{c+dx} - 5c^{4}(c+dx)^{\frac{3}{2}} + 4c^{3}(c+dx)^{\frac{5}{2}} - \frac{15c^{2}(c+dx)^{\frac{7}{2}}}{7} + \frac{2c(c+dx)^{\frac{9}{2}}}{3} - \frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^{5}}}{d} & \text{for}\: d \neq 0 \\ \frac{\begin{cases} a^{5}x & \text{for}\: b = 0 \\ \frac{(a+bx)^{6}}{6b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**5*c/sqrt(c + d*x) - 2*a**5*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 10*a**4*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 10*a**4*b*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 20*a**3*b**2*c*(
c**2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 20*a**3
*b**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c
 + d*x)**(5/2)/5)/d**2 - 20*a**2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(
c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 20*a**2*b**3*(c*
*4/sqrt(c + d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c
+ d*x)**(5/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 10*a*b**4*c*(c**4/sqrt(c + d*x
) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5
 - (c + d*x)**(7/2)/7)/d**4 - 10*a*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(
c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c +
d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**4 - 2*b**5*c*(-c**5/sqrt(c + d*x) -
5*c**4*sqrt(c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2)
 + 5*c*(c + d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**5 - 2*b**5*(c**6/sqrt(c
+ d*x) + 6*c**5*sqrt(c + d*x) - 5*c**4*(c + d*x)**(3/2) + 4*c**3*(c + d*x)*
*(5/2) - 15*c**2*(c + d*x)**(7/2)/7 + 2*c*(c + d*x)**(9/2)/3 - (c + d*x)**(
```

```
11/2)/11)/d**5)/d, Ne(d, 0)), (Piecewise((a**5*x, Eq(b, 0)), ((a + b*x)**6/
(6*b), True))/sqrt(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(134) = 268.
time = 1.64, size = 283, normalized size = 1.84

$$\frac{2\left(693\sqrt{dx+c}\,a^{5}+\frac{1155\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^{4}b}{d}+\frac{462\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^{2}\right)a^{3}b^{2}}{d^{2}}+\frac{198\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^{2}-35\sqrt{dx+c}\,c^{3}\right)a^{2}b^{3}}{d^{3}}+\frac{11\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^{2}-420(dx+c)^{\frac{3}{2}}c^{3}+315\sqrt{dx+c}\,c^{4}\right)ab^{4}}{d^{4}}+\frac{\left(63(dx+c)^{\frac{11}{2}}-385(dx+c)^{\frac{9}{2}}c+990(dx+c)^{\frac{7}{2}}c^{2}-1386(dx+c)^{\frac{5}{2}}c^{3}+1155(dx+c)^{\frac{3}{2}}c^{4}-693\sqrt{dx+c}\,c^{5}\right)b^{5}}{d^{5}}\right)}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

Mupad [B]
time = 0.07, size = 137, normalized size = 0.89

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,\sqrt{c+d\,x}}{d^6}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(11/2))/(11*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(9/2
))/(9*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(1/2))/d^6 + (4*b^2*(a*d - b*c)^3*(
c + d*x)^(5/2))/d^6 + (20*b^3*(a*d - b*c)^2*(c + d*x)^(7/2))/(7*d^6) + (10*
b*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^6)
```

3.1414 $\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=127

$$\frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-ad)(c+dx)^{7/2}}{7d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

```
[Out] -8/3*b*(-a*d+b*c)^3*(d*x+c)^(3/2)/d^5+12/5*b^2*(-a*d+b*c)^2*(d*x+c)^(5/2)/d
^5-8/7*b^3*(-a*d+b*c)*(d*x+c)^(7/2)/d^5+2/9*b^4*(d*x+c)^(9/2)/d^5+2*(-a*d+b
*c)^4*(d*x+c)^(1/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}\,(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*Sqrt[c + d*x])/d^5 - (8*b*(b*c - a*d)^3*(c + d*x)^(3/2))/(
3*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^5) - (8*b^3*(b*c - a*d
)*(c + d*x)^(7/2))/(7*d^5) + (2*b^4*(c + d*x)^(9/2))/(9*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4\sqrt{c+dx}}-\frac{4b(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{5/2}}{d^4}+\frac{b^4(c+dx)^{7/2}}{d^4}\right)dx\\ &= \frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-ad)(c+dx)^{7/2}}{7d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 153, normalized size = 1.20

$$\frac{2\sqrt{c+dx}\,(315a^4d^4+420a^3bd^3(-2c+dx)+126a^2b^2d^2(8c^2-4cdx+3d^2x^2)+36ab^3d(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3)+b^4(128c^4-64c^3dx+48c^2d^2x^2-40cd^3x^3+35d^4x^4))}{315d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*a^4*d^4 + 420*a^3*b*d^3*(-2*c + d*x) + 126*a^2*b^2*d^
2*(8*c^2 - 4*c*d*x + 3*d^2*x^2) + 36*a*b^3*d*(-16*c^3 + 8*c^2*d*x - 6*c*d^2
*x^2 + 5*d^3*x^3) + b^4*(128*c^4 - 64*c^3*d*x + 48*c^2*d^2*x^2 - 40*c*d^3*x
^3 + 35*d^4*x^4)))/(315*d^5)
```

Maple [A]
time = 0.14, size = 99, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{8(ad-bc)b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{5}{2}}}{5}+\frac{8(ad-bc)^3b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^4\sqrt{dx+c}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{9}{2}}}{9}+\frac{8(ad-bc)b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{12(ad-bc)^2b^2(dx+c)^{\frac{5}{2}}}{5}+\frac{8(ad-bc)^3b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^4\sqrt{dx+c}}{d^5}$
gosper	$\frac{2\sqrt{dx+c}\,(35d^4x^4b^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c}{315d^5}$
trager	$\frac{2\sqrt{dx+c}\,(35d^4x^4b^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c}{315d^5}$
risch	$\frac{2\sqrt{dx+c}\,(35d^4x^4b^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c}{315d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/9*b^4*(d*x+c)^(9/2)+4/7*(a*d-b*c)*b^3*(d*x+c)^(7/2)+6/5*(a*d-b*c)^
2*b^2*(d*x+c)^(5/2)+4/3*(a*d-b*c)^3*b*(d*x+c)^(3/2)+(a*d-b*c)^4*(d*x+c)^(1/
2))
```

Maxima [A]
time = 0.28, size = 204, normalized size = 1.61

$$\frac{2\left(315\sqrt{dx+c}\,a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^3b}{d}+\frac{126\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)a^2b^2}{d^2}+\frac{36\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)ab^3}{d^3}+\frac{\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)b^4}{d^4}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

Fricas [**A**]
time = 1.03, size = 182, normalized size = 1.43

$$\frac{2\,(35\,b^4d^4x^4+128\,b^4c^4-576\,ab^3c^3d+1008\,a^2b^2c^2d^2-840\,a^3bcd^3+315\,a^4d^4-20\,(2\,b^4cd^3-9\,ab^3d^4)x^3+6\,(8\,b^4c^2d^2-36\,ab^3cd^3+63\,a^2b^2d^4)x^2-4\,(16\,b^4c^3d-72\,ab^3c^2d^2+126\,a^2b^2cd^3-105\,a^3bd^4)x)\sqrt{dx+c}}{315\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*d^4*x^4 + 128*b^4*c^4 - 576*a*b^3*c^3*d + 1008*a^2*b^2*c^2*d^
2 - 840*a^3*b*c*d^3 + 315*a^4*d^4 - 20*(2*b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 6*
(8*b^4*c^2*d^2 - 36*a*b^3*c*d^3 + 63*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 7
2*a*b^3*c^2*d^2 + 126*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(d*x + c)/d^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(117) = 234$.
time = 27.18, size = 532, normalized size = 4.19

$$\begin{cases} \frac{-\frac{2a^4c}{\sqrt{c+dx}}-2a^4\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{8a^3bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{8a^3b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}-\frac{12a^2b^2c\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2}-\frac{12a^2b^2\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{8ab^3c\left(-\frac{c^3}{\sqrt{c+dx}}-3c^2\sqrt{c+dx}+c(c+dx)^{\frac{3}{2}}-\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3}-\frac{8ab^3\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{2b^4c\left(\frac{c^4}{\sqrt{c+dx}}+4c^3\sqrt{c+dx}-2c^2(c+dx)^{\frac{3}{2}}+\frac{4c(c+dx)^{\frac{5}{2}}}{5}-\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4}-\frac{2b^4\left(-\frac{c^5}{\sqrt{c+dx}}-5c^4\sqrt{c+dx}+\frac{10c^3(c+dx)^{\frac{3}{2}}}{3}-2c^2(c+dx)^{\frac{5}{2}}+\frac{5c(c+dx)^{\frac{7}{2}}}{7}-\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}}{d} & \text{for}\ d \neq 0 \\ \frac{\begin{cases} a^4x & \text{for}\ b = 0 \\ \frac{(a+bx)^5}{5b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**4*c/sqrt(c + d*x) - 2*a**4*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 8*a**3*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 8*a**3*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 12*a**2*b**2*c*(c*
*2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 12*a**2*b
**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c +
 d*x)**(5/2)/5)/d**2 - 8*a*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*
x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 8*a*b**3*(c**4/sqrt(c
+ d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5
/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 2*b**4*c*(c**4/sqrt(c + d*x) + 4*c**3*sq
rt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c + d*x)*
*(7/2)/7)/d**4 - 2*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(c + d*x) + 10*c*
*3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c + d*x)**(7/2)/7 -
(c + d*x)**(9/2)/9)/d**4)/d, Ne(d, 0)), (Piecewise((a**4*x, Eq(b, 0)), ((a
+ b*x)**5/(5*b), True))/sqrt(c), True))
```

Giac [**A**]
time = 1.22, size = 204, normalized size = 1.61

$$\frac{2\left(315\,\sqrt{dx+c}\,a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3b}{d}+\frac{126\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b^2}{d^2}+\frac{36\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^3}{d^3}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^4}{d^4}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

Mupad [B]
time = 0.24, size = 112, normalized size = 0.88

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(9/2))/(9*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(7/2))/(
7*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(1/2))/d^5 + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(5/2))/(5*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^5)
```

3.1415 $\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=96

$$-\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

```
[Out] 2*b*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^4-6/5*b^2*(-a*d+b*c)*(d*x+c)^(5/2)/d^4+2/7
*b^3*(d*x+c)^(7/2)/d^4-2*(-a*d+b*c)^3*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}\,(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*Sqrt[c + d*x])/d^4 + (2*b*(b*c - a*d)^2*(c + d*x)^(3/2))/
d^4 - (6*b^2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^4) + (2*b^3*(c + d*x)^(7/2))
/(7*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3\sqrt{c+dx}}+\frac{3b(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{b^3(c+dx)^{5/2}}{d^3}\right) dx\\ &= -\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 101, normalized size = 1.05

$$\frac{2\sqrt{c+dx}\,(35a^3d^3+35a^2bd^2(-2c+dx)+7ab^2d(8c^2-4cdx+3d^2x^2)+b^3(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3))}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^3*d^3 + 35*a^2*b*d^2*(-2*c + d*x) + 7*a*b^2*d*(8*c^2
 - 4*c*d*x + 3*d^2*x^2) + b^3*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^2 + 5*d^3*x^
3)))/(35*d^4)
```

Maple [A]
time = 0.15, size = 76, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)b^2(dx+c)^{\frac{5}{2}}}{5}+2(ad-bc)^2b(dx+c)^{\frac{3}{2}}+2(ad-bc)^3\sqrt{dx+c}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{7}{2}}}{7}+\frac{6(ad-bc)b^2(dx+c)^{\frac{5}{2}}}{5}+2(ad-bc)^2b(dx+c)^{\frac{3}{2}}+2(ad-bc)^3\sqrt{dx+c}}{d^4}$
gosper	$\frac{2\sqrt{dx+c}\,(5b^3x^3d^3+21d^3a\,x^2b^2-6b^3c\,d^2x^2+35a^2b\,d^3x-28a\,b^2c\,d^2x+8b^3c^2dx+35a^3d^3-70a^2bc\,d^2+56a\,b^2c^2d-16}{35d^4}$
trager	$\frac{2\sqrt{dx+c}\,(5b^3x^3d^3+21d^3a\,x^2b^2-6b^3c\,d^2x^2+35a^2b\,d^3x-28a\,b^2c\,d^2x+8b^3c^2dx+35a^3d^3-70a^2bc\,d^2+56a\,b^2c^2d-16}{35d^4}$
risch	$\frac{2\sqrt{dx+c}\,(5b^3x^3d^3+21d^3a\,x^2b^2-6b^3c\,d^2x^2+35a^2b\,d^3x-28a\,b^2c\,d^2x+8b^3c^2dx+35a^3d^3-70a^2bc\,d^2+56a\,b^2c^2d-16}{35d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/7*b^3*(d*x+c)^(7/2)+3/5*(a*d-b*c)*b^2*(d*x+c)^(5/2)+(a*d-b*c)^2*b*
(d*x+c)^(3/2)+(a*d-b*c)^3*(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 137, normalized size = 1.43

$$\frac{2\left(35\sqrt{dx+c}\,a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a^2b}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ab^2}{d^2}+\frac{\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

Fricas [A]
time = 1.25, size = 115, normalized size = 1.20

$$\frac{2\,(5\,b^3d^3x^3-16\,b^3c^3+56\,ab^2c^2d-70\,a^2bcd^2+35\,a^3d^3-3\,(2\,b^3cd^2-7\,ab^2d^3)x^2+(8\,b^3c^2d-28\,ab^2cd^2+35\,a^2bd^3)x)\sqrt{dx+c}}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*d^3*x^3 - 16*b^3*c^3 + 56*a*b^2*c^2*d - 70*a^2*b*c*d^2 + 35*a^3
*d^3 - 3*(2*b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 28*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(d*x + c)/d^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(88) = 176$.

time = 17.60, size = 366, normalized size = 3.81

$$\begin{cases} \frac{-\frac{2a^3c}{\sqrt{c+dx}} - 2a^3\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{6a^2bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{6a^2b\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{6ab^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{6ab^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{2b^3c\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3} - \frac{2b^3\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for } d \neq 0 \\ \frac{\begin{cases} a^3x & \text{for } b = 0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**3*c/sqrt(c + d*x) - 2*a**3*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 6*a**2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 6*a**2*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 6*a*b**2*c*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 6*a*b**2*(-c*
*3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(
5/2)/5)/d**2 - 2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c
+ d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 2*b**3*(c**4/sqrt(c + d*x) + 4*c
**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c +
 d*x)**(7/2)/7)/d**3)/d, Ne(d, 0)), (Piecewise((a**3*x, Eq(b, 0)), ((a + b*
x)**4/(4*b), True))/sqrt(c), True))
```

Giac [A]

time = 2.26, size = 137, normalized size = 1.43

$$\frac{2\left(35\sqrt{dx+c}\,a^3 + \frac{35\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)a^2b}{d} + \frac{7\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)ab^2}{d^2} + \frac{\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b
/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*
b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d
```

Mupad [B]

time = 0.26, size = 87, normalized size = 0.91

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(7/2))/(7*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(5/2))/(
5*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^4 + (2*b*(a*d - b*c)^2*(c + d*
x)^(3/2))/d^4
```

3.1416 $\int \frac{(a+bx)^2}{\sqrt{c+dx}} dx$

Optimal. Leaf size=69

$$\frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

```
[Out] -4/3*b*(-a*d+b*c)*(d*x+c)^(3/2)/d^3+2/5*b^2*(d*x+c)^(5/2)/d^3+2*(-a*d+b*c)^
2*(d*x+c)^(1/2)/d^3
```

Rubi [A]

time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/d^3 - (4*b*(b*c - a*d)*(c + d*x)^(3/2))/(3*
d^3) + (2*b^2*(c + d*x)^(5/2))/(5*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}\end{aligned}$$

Mathematica [A]

time = 0.04, size = 60, normalized size = 0.87

$$\frac{2\sqrt{c+dx}\,(15a^2d^2+10abd(-2c+dx)+b^2(8c^2-4cdx+3d^2x^2))}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(8*c^2 - 4*c*d*x
 + 3*d^2*x^2)))/(15*d^3)
```

Maple [A]
time = 0.14, size = 55, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{2(dx+c)^{\frac{5}{2}} b^2}{5}+\frac{4(ad-bc)b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^2\sqrt{dx+c}}{d^3}$	55
default	$\frac{\frac{2(dx+c)^{\frac{5}{2}} b^2}{5}+\frac{4(ad-bc)b(dx+c)^{\frac{3}{2}}}{3}+2(ad-bc)^2\sqrt{dx+c}}{d^3}$	55
gosper	$\frac{2\sqrt{dx+c}\,(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$	63
trager	$\frac{2\sqrt{dx+c}\,(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$	63
risch	$\frac{2\sqrt{dx+c}\,(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2)}{15d^3}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/5*(d*x+c)^(5/2)*b^2+2/3*(a*d-b*c)*b*(d*x+c)^(3/2)+(a*d-b*c)^2*(d*x
+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.19

$$\frac{2\left(15\sqrt{dx+c}\,a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)ab}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

Fricas [A]
time = 0.95, size = 64, normalized size = 0.93

$$\frac{2\,(3\,b^2d^2x^2+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\,(2\,b^2cd-5\,abd^2)x)\sqrt{dx+c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c)/d^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(63) = 126$.

time = 10.29, size = 231, normalized size = 3.35

$$\begin{cases} \frac{-\frac{2a^2c}{\sqrt{c+dx}} - 2a^2\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{4abc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{4ab\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{2b^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{2b^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for } d \neq 0 \\ \frac{\begin{cases} a^2x & \text{for } b = 0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**2*c/sqrt(c + d*x) - 2*a**2*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 4*a*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 4*a*b*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 2*b**2*c*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 2*b**2*(-c**3/sqrt(c
+ d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d*
*2)/d, Ne(d, 0)), (Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), True)
)/sqrt(c), True))
```

Giac [A]

time = 1.60, size = 82, normalized size = 1.19

$$\frac{2\left(15\sqrt{dx+c}\,a^2 + \frac{10\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)ab}{d} + \frac{\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d
 + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^
2)/d
```

Mupad [B]

time = 0.07, size = 68, normalized size = 0.99

$$\frac{2\sqrt{c+d\,x}\,\left(3\,b^2\,(c+d\,x)^2 + 15\,a^2\,d^2 + 15\,b^2\,c^2 - 10\,b^2\,c\,(c+d\,x) + 10\,a\,b\,d\,(c+d\,x) - 30\,a\,b\,c\,d\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2)*(3*b^2*(c + d*x)^2 + 15*a^2*d^2 + 15*b^2*c^2 - 10*b^2*c*
(c + d*x) + 10*a*b*d*(c + d*x) - 30*a*b*c*d))/(15*d^3)
```

3.1417 $\int \frac{a+bx}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=40

$$-\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}$$

```
[Out] 2/3*b*(d*x+c)^(3/2)/d^2-2*(-a*d+b*c)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c + d*x],x]
[Out] (-2*(b*c - a*d)*Sqrt[c + d*x])/d^2 + (2*b*(c + d*x)^(3/2))/(3*d^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{c+dx}}\,dx &= \int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx\\ &= -\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.72

$$\frac{2\sqrt{c+dx}\,(-2bc+3ad+bdx)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/(3*d^2)
```

Maple [A]
time = 0.13, size = 38, normalized size = 0.95

method	result	size
gosper	$\frac{2\sqrt{dx+c}\,(bdx+3ad-2bc)}{3d^{2}}$	26
trager	$\frac{2\sqrt{dx+c}\,(bdx+3ad-2bc)}{3d^{2}}$	26
risch	$\frac{2\sqrt{dx+c}\,(bdx+3ad-2bc)}{3d^{2}}$	26
derivativedivides	$\frac{\frac{2b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{d^{2}}$	38
default	$\frac{\frac{2b(dx+c)^{\frac{3}{2}}}{3}+2ad\sqrt{dx+c}-2bc\sqrt{dx+c}}{d^{2}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(1/3*b*(d*x+c)^(3/2)+a*d*(d*x+c)^(1/2)-b*c*(d*x+c)^(1/2))
```

Maxima [A]
time = 0.29, size = 39, normalized size = 0.98

$$\frac{2\left(3\sqrt{dx+c}\,a+\frac{\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

Fricas [A]
time = 0.68, size = 25, normalized size = 0.62

$$\frac{2\,(bdx-2\,bc+3\,ad)\sqrt{dx+c}}{3\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x - 2*b*c + 3*a*d)*sqrt(d*x + c)/d^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(37) = 74$.
time = 2.27, size = 121, normalized size = 3.02

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{c+dx}} - 2a\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{2bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{2b\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{bx^2}{2}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c/sqrt(c + d*x) - 2*a*(-c/sqrt(c + d*x) - sqrt(c + d*x)) -
 2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 2*b*(c**2/sqrt(c + d*x) + 2*c
*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d)/d, Ne(d, 0)), ((a*x + b*x**2/2)/sqr
t(c), True))
```

Giac [A]
time = 1.43, size = 39, normalized size = 0.98

$$\frac{2\left(3\sqrt{dx+c}\,a + \frac{\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.70

$$\frac{2\sqrt{c+d\,x}\,\left(3\,a\,d-3\,b\,c+b\,\left(c+d\,x\right)\right)}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2)*(3*a*d - 3*b*c + b*(c + d*x)))/(3*d^2)
```

3.1418 $\int \frac{1}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{c+dx}}{d}$$

```
[Out] 2*(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x])/d
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}}{d}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x])/d
```

Maple [A]
time = 0.13, size = 13, normalized size = 0.93

method	result	size
gosper	$\frac{2\sqrt{dx+c}}{d}$	13
derivativedivides	$\frac{2\sqrt{dx+c}}{d}$	13
default	$\frac{2\sqrt{dx+c}}{d}$	13
trager	$\frac{2\sqrt{dx+c}}{d}$	13
risch	$\frac{2\sqrt{dx+c}}{d}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)^(1/2)/d
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Fricas [A]
time = 0.93, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(1/2),x)
```

```
[Out] 2*sqrt(c + d*x)/d
```

Giac [A]
time = 1.63, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)/d
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{c+d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2))/d
```

3.1419 $\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c)
 + a*d])
```

Maple [A]
time = 0.16, size = 37, normalized size = 0.79

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{\sqrt{(ad-bc)\,b}}$	37
default	$\frac{2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{\sqrt{(ad-bc)\,b}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.74, size = 119, normalized size = 2.53

$$\left[\frac{\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)}{\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)}{b^2c-abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a))
/sqrt(b^2*c - a*b*d), 2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sq
rt(d*x + c)/(b*d*x + b*c))/(b^2*c - a*b*d)]
```

Sympy [A]
time = 2.30, size = 44, normalized size = 0.94

$$-\frac{2\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{b}{ad-bc}}\,\sqrt{c+dx}}\right)}{\sqrt{\frac{b}{ad-bc}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] -2*atan(1/(sqrt(b/(a*d - b*c))*sqrt(c + d*x)))/(sqrt(b/(a*d - b*c))*(a*d -
b*c))
```

Giac [A]
time = 1.03, size = 38, normalized size = 0.81

$$\frac{2\,\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B]
time = 0.27, size = 38, normalized size = 0.81

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\sqrt{c+d\,x}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/2)),x)
[Out] (2*atan((b*(c + d*x)^(1/2))/(a*b*d - b^2*c)^(1/2)))/(a*b*d - b^2*c)^(1/2)
```

3.1420 $\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}$$

[Out] `d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^(1/2)-(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)`

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + b*x)^2*Sqrt[c + d*x]),x]`

[Out] `-(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))`

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{bc-ad}$$
$$= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.99

$$\frac{\sqrt{c+dx}}{(-bc+ad)(a+bx)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] Sqrt[c + d*x]/((-(b*c) + a*d)*(a + b*x)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

Maple [A]
time = 0.16, size = 87, normalized size = 1.14

method	result	size
derivativedivides	$2d\left(\frac{\sqrt{dx+c}}{2(ad-bc)((dx+c)b+ad-bc)} + \frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2(ad-bc)\sqrt{(ad-bc)\,b}}\right)$	87
default	$2d\left(\frac{\sqrt{dx+c}}{2(ad-bc)((dx+c)b+ad-bc)} + \frac{\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2(ad-bc)\sqrt{(ad-bc)\,b}}\right)$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(1/2*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*
c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(64) = 128$.
time = 0.98, size = 280, normalized size = 3.68

$$\left[-\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(b^2c-abd)\sqrt{dx+c}}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x)},\ -\frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(b^2c-abd)\sqrt{dx+c}}{ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(
b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(b^2*c - a*b*d)*sqrt(d*x + c))
/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*x), -(sqrt(-b^2*c + a*b*d)*(b*d*x + a*d)*arctan(sqrt(-b^2*c + a*b*d)*s
qrt(d*x + c)/(b*d*x + b*c)) + (b^2*c - a*b*d)*sqrt(d*x + c))/(a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*sqrt(c + d*x)), x)
```

Giac [A]
time = 1.46, size = 87, normalized size = 1.14

$$-\frac{d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx+c}\,d}{((dx+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
- a*d)) - sqrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*(b*c - a*d))
```

Mupad [B]
time = 0.09, size = 74, normalized size = 0.97

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\,\left(a\,d-b\,c\right)^{3/2}}+\frac{d\,\sqrt{c+d\,x}}{\left(a\,d-b\,c\right)\,\left(a\,d-b\,c+b\,\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^
(3/2)) + (d*(c + d*x)^(1/2))/((a*d - b*c)*(a*d - b*c + b*(c + d*x)))
```

3.1421 $\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}$$

```
[Out] -3/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(5/2)/b
^(1/2)-1/2*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^2+3/4*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)
```

Rubi [A]

time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] -1/2*Sqrt[c + d*x]/((b*c - a*d)*(a + b*x)^2) + (3*d*Sqrt[c + d*x])/(4*(b*c
- a*d)^2*(a + b*x)) - (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(4*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx &= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}-\frac{(3d)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4(bc-ad)}\\ &= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}+\frac{(3d^2)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8(bc-ad)^2}\\ &= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}+\frac{(3d)\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{\right.}{4(bc-ad)^2}\\ &= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2}+\frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 96, normalized size = 0.84

$$\frac{1}{4}\left(\frac{\sqrt{c+dx}\,(-2bc+5ad+3bdx)}{(bc-ad)^2(a+bx)^2}+\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] ((Sqrt[c + d*x]*(-2*b*c + 5*a*d + 3*b*d*x))/((b*c - a*d)^2*(a + b*x)^2) + (
3*d^2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c)
+ a*d)^(5/2)))/4
```

Maple [A]
time = 0.16, size = 138, normalized size = 1.21

method	result	size

derivativedivides	$2d^2\left(\frac{\sqrt{dx+c}}{4(ad-bc)((dx+c)b+ad-bc)^2}+\frac{\frac{3\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8(ad-bc)\sqrt{(ad-bc)\,b}}}{ad-bc}\right)$	138
default	$2d^2\left(\frac{\sqrt{dx+c}}{4(ad-bc)((dx+c)b+ad-bc)^2}+\frac{\frac{3\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{8(ad-bc)\sqrt{(ad-bc)\,b}}}{ad-bc}\right)$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(1/4*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^2+3/4/(a*d-b*c)*(1/2
*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(94) = 188$.
time = 0.99, size = 549, normalized size = 4.82

$$\left[\frac{3\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\,(2\,b^3c^2-7\,ab^2cd+5\,a^2bd^2-3\,(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{8\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^2+2\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x)},\ \frac{3\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-(2\,b^3c^2-7\,ab^2cd+5\,a^2bd^2-3\,(b^3cd-ab^2d^2)x)\sqrt{dx+c}}{4\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^2+2\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c
))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3 + (b^6*c^3
- 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^2 + 2*(a*b^5*c^3 - 3*a^2
*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x), 1/4*(3*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x
 + c)/(b*d*x + b*c)) - (2*b^3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d
- a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c
*d^2 - a^5*b*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*x^2 + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3 \sqrt{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**3*sqrt(c + d*x)), x)
```

Giac [A]
time = 1.49, size = 148, normalized size = 1.30

$$\frac{3\, d^2 \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{3}{2}}bd^2-5\,\sqrt{dx+c}\,bcd^2+5\,\sqrt{dx+c}\,ad^3}{4\,(b^2c^2-2\,abcd+a^2d^2)((dx+c)b-bc+ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/4*(3*(d*x + c)^(3/2)*b*d^2 - 5*sqrt(d*
x + c)*b*c*d^2 + 5*sqrt(d*x + c)*a*d^3)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((
d*x + c)*b - b*c + a*d)^2)
```

Mupad [B]
time = 0.33, size = 142, normalized size = 1.25

$$\frac{\frac{5\,d^2\,\sqrt{c+d\,x}}{4\,(a\,d-b\,c)}+\frac{3\,b\,d^2\,(c+d\,x)^{3/2}}{4\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^2-(2\,b^2\,c-2\,a\,b\,d)\,(c+d\,x)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}+\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(1/2)),x)
```

```
[Out] ((5*d^2*(c + d*x)^(1/2))/(4*(a*d - b*c)) + (3*b*d^2*(c + d*x)^(3/2))/(4*(a*
d - b*c)^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) + (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/
2)))/(4*b^(1/2)*(a*d - b*c)^(5/2))
```

3.1422 $\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx$

Optimal. Leaf size=147

$$-\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3}+\frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2}-\frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)}+\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}$$

```
[Out] 5/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(7/2)/b^
(1/2)-1/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^3+5/12*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)^2-5/8*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)
```

Rubi [A]
time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {44, 65, 214}

$$\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] -1/3*Sqrt[c + d*x]/((b*c - a*d)*(a + b*x)^3) + (5*d*Sqrt[c + d*x])/(12*(b*c
 - a*d)^2*(a + b*x)^2) - (5*d^2*Sqrt[c + d*x])/(8*(b*c - a*d)^3*(a + b*x))
+ (5*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c
- a*d)^(7/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} - \frac{(5d)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} + \frac{(5d^2)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{(5d^3)\int}{}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{(5d^2)\,\mathrm{S}}{}$$
$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} + \frac{5d^3\tan}{8}$$

Mathematica [A]
time = 0.36, size = 128, normalized size = 0.87

$$\frac{\sqrt{c+dx}\,(33a^2d^2+2abd(-13c+20dx)+b^2(8c^2-10cdx+15d^2x^2))}{24(-bc+ad)^3(a+bx)^3} + \frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8\sqrt{b}\,(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(33*a^2*d^2 + 2*a*b*d*(-13*c + 20*d*x) + b^2*(8*c^2 - 10*c*d
*x + 15*d^2*x^2)))/(24*(-(b*c) + a*d)^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*Sqrt[b]*(-(b*c) + a*d)^(7/2))
```

Maple [A]
time = 0.16, size = 187, normalized size = 1.27

method	result

derivativedivides	$2d^3\left(\frac{\sqrt{dx+c}}{6(ad-bc)((dx+c)b+ad-bc)^3}+\frac{\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2}+\frac{5\left(\frac{3\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)}+\frac{3\arctan\left(\frac{b\sqrt{dx}}{\sqrt{(ad-}}\right.}{8(ad-bc)\sqrt{(ad-}}\right.}{6(ad-bc)}}{ad-bc}\right.$
default	$2d^3\left(\frac{\sqrt{dx+c}}{6(ad-bc)((dx+c)b+ad-bc)^3}+\frac{\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2}+\frac{5\left(\frac{3\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)}+\frac{3\arctan\left(\frac{b\sqrt{dx}}{\sqrt{(ad-}}\right.}{8(ad-bc)\sqrt{(ad-}}\right.}{6(ad-bc)}}{ad-bc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(1/6*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^3+5/6/(a*d-b*c)*(1/4
*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^2+3/4/(a*d-b*c)*(1/2*(d*x+c)^(
1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan
(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(123) = 246$.

time = 1.05, size = 884, normalized size = 6.01

$$\left[-\frac{15\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\,(8\,b^4c^3-34\,ab^3c^2d+59\,a^2b^2cd^2-33\,a^3bd^3+15\,(b^4cd^2-ab^3d^3)x^2-10\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{48\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^3+3\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^2+3\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x)},\ -\frac{15\,(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+(8\,b^4c^3-34\,ab^3c^2d+59\,a^2b^2cd^2-33\,a^3bd^3+15\,(b^4cd^2-ab^3d^3)x^2-10\,(b^4c^2d-5\,ab^3cd^2+4\,a^2b^2d^3)x)\sqrt{dx+c}}{24\,(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4+(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4)x^3+3\,(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4)x^2+3\,(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b
^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c
))/(b*x + a)) + 2*(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b
*d^3 + 15*(b^4*c*d^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*
c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3
*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^
4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)
, -1/24*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) +
(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b*d^3 + 15*(b^4*c*d
^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqr
t(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*
c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^
5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2
*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*
d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.79, size = 231, normalized size = 1.57

$$-\frac{5\,d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{5}{2}}b^2d^3-40\,(dx+c)^{\frac{3}{2}}b^2cd^3+33\,\sqrt{dx+c}\,b^2c^2d^3+40\,(dx+c)^{\frac{3}{2}}abd^4-66\,\sqrt{dx+c}\,abcd^4+33\,\sqrt{dx+c}\,a^2d^5}{24\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx+c)b-bc+ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/24*(15*(d*x + c)^
```

```
(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 33*sqrt(d*x + c)*b^2*c^2*d^3
 + 40*(d*x + c)^(3/2)*a*b*d^4 - 66*sqrt(d*x + c)*a*b*c*d^4 + 33*sqrt(d*x +
c)*a^2*d^5)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)
*b - b*c + a*d)^3)
```

Mupad [B]
time = 0.39, size = 218, normalized size = 1.48

$$\frac{\frac{11\,d^3\,\sqrt{c+d\,x}}{8\,(a\,d-b\,c)}+\frac{5\,b^2\,d^3\,(c+d\,x)^{5/2}}{8\,(a\,d-b\,c)^3}+\frac{5\,b\,d^3\,(c+d\,x)^{3/2}}{3\,(a\,d-b\,c)^2}}{(c+d\,x)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,(c+d\,x)^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,(c+d\,x)^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}+\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(1/2)),x)
```

```
[Out] ((11*d^3*(c + d*x)^(1/2))/(8*(a*d - b*c)) + (5*b^2*d^3*(c + d*x)^(5/2))/(8*
(a*d - b*c)^3) + (5*b*d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)^2))/((c + d*x)*(3
*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^
2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (5*
d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(7/2))
```

3.1423 $\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx$

Optimal. Leaf size=180

$$-\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4}+\frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3}-\frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2}+\frac{35d^3\sqrt{c+dx}}{64(bc-ad)^4(a+bx)}-\frac{35d^4\tanh^{-1}\left(\ldots\right.}{64\sqrt{b}\,(\ldots}$$

```
[Out] -35/64*d^4*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(9/2)
/b^(1/2)-1/4*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^4+7/24*d*(d*x+c)^(1/2)/(-a*d+
b*c)^2/(b*x+a)^3-35/96*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^2+35/64*d^3*(
d*x+c)^(1/2)/(-a*d+b*c)^4/(b*x+a)
```

Rubi [A]
time = 0.05, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {44, 65, 214}

$$-\frac{35d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}\,(bc-ad)^{9/2}}+\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] -1/4*Sqrt[c + d*x]/((b*c - a*d)*(a + b*x)^4) + (7*d*Sqrt[c + d*x])/(24*(b*c
 - a*d)^2*(a + b*x)^3) - (35*d^2*Sqrt[c + d*x])/(96*(b*c - a*d)^3*(a + b*x)
^2) + (35*d^3*Sqrt[c + d*x])/(64*(b*c - a*d)^4*(a + b*x)) - (35*d^4*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(64*Sqrt[b]*(b*c - a*d)^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} - \frac{(7d)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} + \frac{(35d^2)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} - \frac{(35\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{\cdots}{64(\cdots}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{\cdots}{64(\cdots}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{\cdots}{64(\cdots}$$

Mathematica [A]
time = 0.57, size = 166, normalized size = 0.92

$$\frac{1}{192}\left(\frac{\sqrt{c+dx}\,(279a^3d^3+a^2bd^2(-326c+511dx)+ab^2d(200c^2-252cdx+385d^2x^2)+b^3(-48c^3+56c^2dx-70cd^2x^2+105d^3x^3))}{(bc-ad)^4(a+bx)^4}+\frac{105d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\,(-bc+ad)^{9/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] ((Sqrt[c + d*x]*(279*a^3*d^3 + a^2*b*d^2*(-326*c + 511*d*x) + a*b^2*d*(200*
c^2 - 252*c*d*x + 385*d^2*x^2) + b^3*(-48*c^3 + 56*c^2*d*x - 70*c*d^2*x^2 +
 105*d^3*x^3)))/((b*c - a*d)^4*(a + b*x)^4) + (105*d^4*ArcTan[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(9/2)))/192
```

Maple [A]
time = 0.16, size = 236, normalized size = 1.31

method	result
derivativedivides	$2d^4\left(\frac{\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)^4}+\frac{\frac{7\sqrt{dx+c}}{48(ad-bc)((dx+c)b+ad-bc)^3}+\frac{7\left(\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2}+\frac{5\left(\frac{3\sqrt{dx+}}{8(ad-bc)((dx+c)b}\right.}{}\right.}{8(ad-bc}}{ad-bc}\right.$
default	$2d^4\left(\frac{\sqrt{dx+c}}{8(ad-bc)((dx+c)b+ad-bc)^4}+\frac{\frac{7\sqrt{dx+c}}{48(ad-bc)((dx+c)b+ad-bc)^3}+\frac{7\left(\frac{5\sqrt{dx+c}}{24(ad-bc)((dx+c)b+ad-bc)^2}+\frac{5\left(\frac{3\sqrt{dx+}}{8(ad-bc)((dx+c)b}\right.}{}\right.}{8(ad-bc}}{ad-bc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^5/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^4*(1/8*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^4+7/8/(a*d-b*c)*(1/6
*(d*x+c)^(1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^3+5/6/(a*d-b*c)*(1/4*(d*x+c)^(
1/2)/(a*d-b*c)/((d*x+c)*b+a*d-b*c)^2+3/4/(a*d-b*c)*(1/2*(d*x+c)^(1/2)/(a*d-
```

```
b*c)/((d*x+c)*b+a*d-b*c)+1/2/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(152) = 304$.
time = 0.77, size = 1325, normalized size = 7.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(48*b^5*c^4 - 248*a*b^4*c^3*d + 526*
a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*(b^5*c*d^3 - a*b^
4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 - 7*(
8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqr
t(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^
3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^
2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^4 + 4
*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5
*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a
^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^2 +
4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3
+ 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x), 1/192*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*
x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arc
tan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (48*b^5*c^4 - 248*a
*b^4*c^3*d + 526*a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*
(b^5*c*d^3 - a*b^4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b
^3*d^4)*x^2 - 7*(8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^
3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^
3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*
```

```
b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5
*b^5*d^5)*x^4 + 4*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^
4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3
*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^
7*b^3*d^5)*x^2 + 4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10
*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(152) = 304.
time = 2.25, size = 331, normalized size = 1.84

$$\frac{35\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{7}{2}}b^3d^4-385\,(dx+c)^{\frac{5}{2}}b^3cd^4+511\,(dx+c)^{\frac{3}{2}}b^3c^2d^4-279\,\sqrt{dx+c}\,b^3c^3d^4+385\,(dx+c)^{\frac{5}{2}}ab^2d^5-1022\,(dx+c)^{\frac{3}{2}}ab^2cd^5+837\,\sqrt{dx+c}\,ab^2c^2d^5+511\,(dx+c)^{\frac{3}{2}}a^2bd^6-837\,\sqrt{dx+c}\,a^2bcd^6+279\,\sqrt{dx+c}\,a^3d^7}{192\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 35/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d))
+ 1/192*(105*(d*x + c)^(7/2)*b^3*d^4 - 385*(d*x + c)^(5/2)*b^3*c*d^4 + 511*
(d*x + c)^(3/2)*b^3*c^2*d^4 - 279*sqrt(d*x + c)*b^3*c^3*d^4 + 385*(d*x + c)
^(5/2)*a*b^2*d^5 - 1022*(d*x + c)^(3/2)*a*b^2*c*d^5 + 837*sqrt(d*x + c)*a*b
^2*c^2*d^5 + 511*(d*x + c)^(3/2)*a^2*b*d^6 - 837*sqrt(d*x + c)*a^2*b*c*d^6
+ 279*sqrt(d*x + c)*a^3*d^7)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b - b*c + a*d)^4)
```

Mupad [**B**]
time = 0.46, size = 307, normalized size = 1.71

$$\frac{\frac{93\,d^4\,\sqrt{c+d\,x}}{64\,(a\,d-b\,c)}+\frac{385\,b^2\,d^4\,{(c+d\,x)}^{5/2}}{192\,{(a\,d-b\,c)}^3}+\frac{35\,b^3\,d^4\,{(c+d\,x)}^{7/2}}{64\,{(a\,d-b\,c)}^4}+\frac{511\,b\,d^4\,{(c+d\,x)}^{3/2}}{192\,{(a\,d-b\,c)}^2}}{b^4\,{(c+d\,x)}^4-(4\,b^4\,c-4\,a\,b^3\,d)\,{(c+d\,x)}^3-(c+d\,x)\,(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3)+a^4\,d^4+b^4\,c^4+{(c+d\,x)}^2\,(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{35\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,\sqrt{b}\,{(a\,d-b\,c)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^5*(c + d*x)^(1/2)),x)
```

```
[Out] ((93*d^4*(c + d*x)^(1/2))/(64*(a*d - b*c)) + (385*b^2*d^4*(c + d*x)^(5/2))/
(192*(a*d - b*c)^3) + (35*b^3*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^4) + (51
1*b*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c -
```

```
 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c
*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2
*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3) + (35*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(1/2)
*(a*d - b*c)^(9/2))
```

3.1424 $\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+dx)^{5/2}}{d^6}-\frac{10b^4(bc-ad)(}{7d^6}$$

[Out] -20/3*b^2*(-a*d+b*c)^3*(d*x+c)^(3/2)/d^6+4*b^3*(-a*d+b*c)^2*(d*x+c)^(5/2)/d^6-10/7*b^4*(-a*d+b*c)*(d*x+c)^(7/2)/d^6+2/9*b^5*(d*x+c)^(9/2)/d^6+2*(-a*d+b*c)^5/d^6/(d*x+c)^(1/2)+10*b*(-a*d+b*c)^4*(d*x+c)^(1/2)/d^6

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}\,(bc-ad)^4}{d^6}+\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{2b^5(c+dx)^{9/2}}{9d^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(c + d*x)^(3/2),x]

[Out] (2*(b*c - a*d)^5)/(d^6*Sqrt[c + d*x]) + (10*b*(b*c - a*d)^4*Sqrt[c + d*x])/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(b*c - a*d)^2*(c + d*x)^(5/2))/d^6 - (10*b^4*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^6) + (2*b^5*(c + d*x)^(9/2))/(9*d^6)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx = \int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{3/2}}+\frac{5b(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{10b^2(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^3}{d^5}\right.$$

$$=\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+}{d^6}$$

Mathematica [A]
time = 0.11, size = 214, normalized size = 1.41

$$\frac{2(-63a^5d^5+315a^4bd^4(2c+dx)+210a^3b^2d^3(-8c^2-4cdx+d^2x^2)+126a^2b^3d^2(16c^3+8c^2dx-2cd^2x^2+d^3x^3)+9ab^4d(-128c^4-64c^3dx+16c^2d^2x^2-8cd^3x^3+5d^4x^4)+b^5(256c^5+128c^4dx-32c^3d^2x^2+16c^2d^3x^3-10cd^4x^4+7d^5x^5))}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-63*a^5*d^5 + 315*a^4*b*d^4*(2*c + d*x) + 210*a^3*b^2*d^3*(-8*c^2 - 4*c
*d*x + d^2*x^2) + 126*a^2*b^3*d^2*(16*c^3 + 8*c^2*d*x - 2*c*d^2*x^2 + d^3*x
^3) + 9*a*b^4*d*(-128*c^4 - 64*c^3*d*x + 16*c^2*d^2*x^2 - 8*c*d^3*x^3 + 5*d
^4*x^4) + b^5*(256*c^5 + 128*c^4*d*x - 32*c^3*d^2*x^2 + 16*c^2*d^3*x^3 - 10
*c*d^4*x^4 + 7*d^5*x^5)))/(63*d^6*Sqrt[c + d*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(134) = 268$.
time = 0.15, size = 324, normalized size = 2.13

method	result
risch	$\frac{2b(7d^4x^4b^4+45a\,b^3d^4x^3-17b^4c\,d^3x^3+126a^2b^2d^4x^2-117a\,b^3c\,d^3x^2+33b^4c^2d^2x^2+210a^3b\,d^4x-378a^2b^2c\,d^3x+261a\,b^3c}{63d^6}$
gosper	$-\frac{2(-7b^5x^5d^5-45a\,b^4d^5x^4+10b^5c\,d^4x^4-126a^2b^3d^5x^3+72a\,b^4c\,d^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+252a^2b^3c\,d^4x^2-144}{}$
trager	$-\frac{2(-7b^5x^5d^5-45a\,b^4d^5x^4+10b^5c\,d^4x^4-126a^2b^3d^5x^3+72a\,b^4c\,d^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+252a^2b^3c\,d^4x^2-144}{}$
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{9}{2}}}{9}+\frac{10a\,b^4d(dx+c)^{\frac{7}{2}}}{7}-\frac{10b^5c(dx+c)^{\frac{7}{2}}}{7}+4a^2b^3d^2(dx+c)^{\frac{5}{2}}-8a\,b^4cd(dx+c)^{\frac{5}{2}}+4b^5c^2(dx+c)^{\frac{5}{2}}+\frac{20a^3b^2d^3(dx+c)^{\frac{3}{2}}}{3}-}{}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{9}{2}}}{9}+\frac{10a\,b^4d(dx+c)^{\frac{7}{2}}}{7}-\frac{10b^5c(dx+c)^{\frac{7}{2}}}{7}+4a^2b^3d^2(dx+c)^{\frac{5}{2}}-8a\,b^4cd(dx+c)^{\frac{5}{2}}+4b^5c^2(dx+c)^{\frac{5}{2}}+\frac{20a^3b^2d^3(dx+c)^{\frac{3}{2}}}{3}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/9*b^5*(d*x+c)^(9/2)+5/7*a*b^4*d*(d*x+c)^(7/2)-5/7*b^5*c*(d*x+c)^(7
/2)+2*a^2*b^3*d^2*(d*x+c)^(5/2)-4*a*b^4*c*d*(d*x+c)^(5/2)+2*b^5*c^2*(d*x+c)
^(5/2)+10/3*a^3*b^2*d^3*(d*x+c)^(3/2)-10*a^2*b^3*c*d^2*(d*x+c)^(3/2)+10*a*b
^4*c^2*d*(d*x+c)^(3/2)-10/3*b^5*c^3*(d*x+c)^(3/2)+5*a^4*b*d^4*(d*x+c)^(1/2)
-20*a^3*b^2*c*d^3*(d*x+c)^(1/2)+30*a^2*b^3*c^2*d^2*(d*x+c)^(1/2)-20*a*b^4*c
^3*d*(d*x+c)^(1/2)+5*b^5*c^4*(d*x+c)^(1/2)-(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^
2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 267, normalized size = 1.76

$$\frac{2\left(\frac{7(dx+c)^{\frac{9}{2}}b^5-45(b^5c-ab^4d)(dx+c)^{\frac{7}{2}}+126(b^5c^2-2ab^4cd+a^2b^3d^2)(dx+c)^{\frac{5}{2}}-210(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3)(dx+c)^{\frac{3}{2}}+315(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4)\sqrt{dx+c}}{d^5}+\frac{63(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5)}{\sqrt{dx+c}\,d^5}\right)}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*((7*(d*x + c)^(9/2)*b^5 - 45*(b^5*c - a*b^4*d)*(d*x + c)^(7/2) + 126*(
b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(5/2) - 210*(b^5*c^3 - 3*a*b
^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(3/2) + 315*(b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(d*x +
 c))/d^5 + 63*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(134) = 268.
time = 0.61, size = 271, normalized size = 1.78

$$\frac{2\,(7\,b^5d^5x^5 + 256\,b^5c^5 - 1152\,ab^4c^4d + 2016\,a^2b^3c^3d^2 - 1680\,a^3b^2c^2d^3 + 630\,a^4bcd^4 - 63\,a^5d^5 - 5\,(2\,b^5cd^4 - 9\,ab^4d^5)x^4 + 2\,(8\,b^5c^2d^3 - 36\,ab^4cd^4 + 63\,a^2b^3d^5)x^3 - 2\,(16\,b^5c^3d^2 - 72\,ab^4c^2d^3 + 126\,a^2b^3cd^4 - 105\,a^3b^2d^5)x^2 + (128\,b^5c^4d - 576\,ab^4c^3d^2 + 1008\,a^2b^3c^2d^3 - 840\,a^3b^2cd^4 + 315\,a^4bd^5)x)\sqrt{dx+c}}{63\,(d^7x + cd^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^5*d^5*x^5 + 256*b^5*c^5 - 1152*a*b^4*c^4*d + 2016*a^2*b^3*c^3*d^2
 - 1680*a^3*b^2*c^2*d^3 + 630*a^4*b*c*d^4 - 63*a^5*d^5 - 5*(2*b^5*c*d^4 - 9
*a*b^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 36*a*b^4*c*d^4 + 63*a^2*b^3*d^5)*x^3 -
 2*(16*b^5*c^3*d^2 - 72*a*b^4*c^2*d^3 + 126*a^2*b^3*c*d^4 - 105*a^3*b^2*d^5
)*x^2 + (128*b^5*c^4*d - 576*a*b^4*c^3*d^2 + 1008*a^2*b^3*c^2*d^3 - 840*a^3
*b^2*c*d^4 + 315*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^7*x + c*d^6)
```

Sympy [A]
time = 18.34, size = 243, normalized size = 1.60

$$\frac{2b^5(c+dx)^{\frac{9}{2}}}{9d^6} + \frac{(c+dx)^{\frac{7}{2}}\cdot(10ab^4d - 10b^5c)}{7d^6} + \frac{(c+dx)^{\frac{5}{2}}\cdot(20a^2b^3d^2 - 40ab^4cd + 20b^5c^2)}{5d^6} + \frac{(c+dx)^{\frac{3}{2}}\cdot(20a^3b^2d^3 - 60a^2b^3cd^2 + 60ab^4c^2d - 20b^5c^3)}{3d^6} + \frac{\sqrt{c+dx}\,(10a^4bd^4 - 40a^3b^2cd^3 + 60a^2b^3c^2d^2 - 40ab^4c^3d + 10b^5c^4)}{d^6} - \frac{2(ad-bc)^5}{d^6\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(9/2)/(9*d**6) + (c + d*x)**(7/2)*(10*a*b**4*d - 10*b**5*
c)/(7*d**6) + (c + d*x)**(5/2)*(20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5
*c**2)/(5*d**6) + (c + d*x)**(3/2)*(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2
 + 60*a*b**4*c**2*d - 20*b**5*c**3)/(3*d**6) + sqrt(c + d*x)*(10*a**4*b*d**
4 - 40*a**3*b**2*c*d**3 + 60*a**2*b**3*c**2*d**2 - 40*a*b**4*c**3*d + 10*b*
*5*c**4)/d**6 - 2*(a*d - b*c)**5/(d**6*sqrt(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(134) = 268.
time = 1.12, size = 350, normalized size = 2.30

$$\frac{2\,(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)}{\sqrt{dx+c}\,d^6} + \frac{2\left(7\,(dx+c)^{\frac{9}{2}}b^5d^{48} - 45\,(dx+c)^{\frac{7}{2}}b^5cd^{48} + 126\,(dx+c)^{\frac{5}{2}}b^5c^2d^{48} - 210\,(dx+c)^{\frac{3}{2}}b^5c^3d^{48} + 315\,\sqrt{dx+c}\,b^5c^4d^{48} + 45\,(dx+c)^{\frac{7}{2}}ab^4d^{49} - 252\,(dx+c)^{\frac{5}{2}}ab^4cd^{49} + 630\,(dx+c)^{\frac{3}{2}}ab^4c^2d^{49} - 1260\,\sqrt{dx+c}\,ab^4c^3d^{49} + 126\,(dx+c)^{\frac{5}{2}}a^2b^3d^{50} - 630\,(dx+c)^{\frac{3}{2}}a^2b^3cd^{50} + 1890\,\sqrt{dx+c}\,a^2b^3c^2d^{50} + 210\,(dx+c)^{\frac{3}{2}}a^3b^2d^{51} - 1260\,\sqrt{dx+c}\,a^3b^2cd^{51} + 315\,\sqrt{dx+c}\,a^4bd^{52}\right)}{63\,d^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^
4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^6) + 2/63*(7*(d*x + c)^(9/2)*b^5*d^48
 - 45*(d*x + c)^(7/2)*b^5*c*d^48 + 126*(d*x + c)^(5/2)*b^5*c^2*d^48 - 210*(
d*x + c)^(3/2)*b^5*c^3*d^48 + 315*sqrt(d*x + c)*b^5*c^4*d^48 + 45*(d*x + c)
^(7/2)*a*b^4*d^49 - 252*(d*x + c)^(5/2)*a*b^4*c*d^49 + 630*(d*x + c)^(3/2)*
a*b^4*c^2*d^49 - 1260*sqrt(d*x + c)*a*b^4*c^3*d^49 + 126*(d*x + c)^(5/2)*a^
2*b^3*d^50 - 630*(d*x + c)^(3/2)*a^2*b^3*c*d^50 + 1890*sqrt(d*x + c)*a^2*b^
3*c^2*d^50 + 210*(d*x + c)^(3/2)*a^3*b^2*d^51 - 1260*sqrt(d*x + c)*a^3*b^2*
c*d^51 + 315*sqrt(d*x + c)*a^4*b*d^52)/d^54
```

Mupad [B]
time = 0.08, size = 192, normalized size = 1.26

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{2\,a^5\,d^5-10\,a^4\,b\,c\,d^4+20\,a^3\,b^2\,c^2\,d^3-20\,a^2\,b^3\,c^3\,d^2+10\,a\,b^4\,c^4\,d-2\,b^5\,c^5}{d^6\,\sqrt{c+d\,x}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{4\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(9/2))/(9*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(7/2))
/(7*d^6) - (2*a^5*d^5 - 2*b^5*c^5 - 20*a^2*b^3*c^3*d^2 + 20*a^3*b^2*c^2*d^3
 + 10*a*b^4*c^4*d - 10*a^4*b*c*d^4)/(d^6*(c + d*x)^(1/2)) + (20*b^2*(a*d -
b*c)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(a*d - b*c)^2*(c + d*x)^(5/2))/d^6
 + (10*b*(a*d - b*c)^4*(c + d*x)^(1/2))/d^6
```

3.1425 $\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{5/2}}{5d^5}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

```
[Out] 4*b^2*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^5-8/5*b^3*(-a*d+b*c)*(d*x+c)^(5/2)/d^5+2
/7*b^4*(d*x+c)^(7/2)/d^5-2*(-a*d+b*c)^4/d^5/(d*x+c)^(1/2)-8*b*(-a*d+b*c)^3*
(d*x+c)^(1/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}\,(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(d^5*Sqrt[c + d*x]) - (8*b*(b*c - a*d)^3*Sqrt[c + d*x])/
d^5 + (4*b^2*(b*c - a*d)^2*(c + d*x)^(3/2))/d^5 - (8*b^3*(b*c - a*d)*(c + d
*x)^(5/2))/(5*d^5) + (2*b^4*(c + d*x)^(7/2))/(7*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{3/2}}-\frac{4b(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b^2(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{3/2}}{d^4}\right.\\ &= -\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+a}{5d^5}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 151, normalized size = 1.23

$$\frac{2(-35a^4d^4+140a^3bd^3(2c+dx)+70a^2b^2d^2(-8c^2-4cdx+d^2x^2)+28ab^3d(16c^3+8c^2dx-2cd^2x^2+d^3x^3)+b^4(-128c^4-64c^3dx+16c^2d^2x^2-8cd^3x^3+5d^4x^4))}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*a^4*d^4 + 140*a^3*b*d^3*(2*c + d*x) + 70*a^2*b^2*d^2*(-8*c^2 - 4*c*
d*x + d^2*x^2) + 28*a*b^3*d*(16*c^3 + 8*c^2*d*x - 2*c*d^2*x^2 + d^3*x^3) +
b^4*(-128*c^4 - 64*c^3*d*x + 16*c^2*d^2*x^2 - 8*c*d^3*x^3 + 5*d^4*x^4)))/(3
5*d^5*Sqrt[c + d*x])
```

Maple [A]
time = 0.16, size = 219, normalized size = 1.78

method	result
risch	$\frac{2b\left(5b^3x^3d^3+28d^3a\,x^2b^2-13b^3c\,d^2x^2+70a^2b\,d^3x-84a\,b^2c\,d^2x+29b^3c^2dx+140a^3d^3-350a^2bc\,d^2+308a\,b^2c^2d-93b^3c^3\right)\sqrt{}}{35d^5}$
gosper	$-\frac{2\left(-5d^4x^4b^4-28a\,b^3d^4x^3+8b^4c\,d^3x^3-70a^2b^2d^4x^2+56a\,b^3c\,d^3x^2-16b^4c^2d^2x^2-140a^3b\,d^4x+280a^2b^2c\,d^3x-224a\,b^3c^2\right.}{35\sqrt{dx+c}\,d^5}$
trager	$-\frac{2\left(-5d^4x^4b^4-28a\,b^3d^4x^3+8b^4c\,d^3x^3-70a^2b^2d^4x^2+56a\,b^3c\,d^3x^2-16b^4c^2d^2x^2-140a^3b\,d^4x+280a^2b^2c\,d^3x-224a\,b^3c^2\right.}{35\sqrt{dx+c}\,d^5}$
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{7}{2}}}{7}+\frac{8a\,b^3d(dx+c)^{\frac{5}{2}}}{5}-\frac{8b^4c(dx+c)^{\frac{5}{2}}}{5}+4a^2b^2d^2(dx+c)^{\frac{3}{2}}-8a\,b^3cd(dx+c)^{\frac{3}{2}}+4b^4c^2(dx+c)^{\frac{3}{2}}+8a^3b\,d^3\sqrt{dx+c}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{7}{2}}}{7}+\frac{8a\,b^3d(dx+c)^{\frac{5}{2}}}{5}-\frac{8b^4c(dx+c)^{\frac{5}{2}}}{5}+4a^2b^2d^2(dx+c)^{\frac{3}{2}}-8a\,b^3cd(dx+c)^{\frac{3}{2}}+4b^4c^2(dx+c)^{\frac{3}{2}}+8a^3b\,d^3\sqrt{dx+c}}{d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/7*b^4*(d*x+c)^(7/2)+4/5*a*b^3*d*(d*x+c)^(5/2)-4/5*b^4*c*(d*x+c)^(5
/2)+2*a^2*b^2*d^2*(d*x+c)^(3/2)-4*a*b^3*c*d*(d*x+c)^(3/2)+2*b^4*c^2*(d*x+c)
^(3/2)+4*a^3*b*d^3*(d*x+c)^(1/2)-12*a^2*b^2*c*d^2*(d*x+c)^(1/2)+12*a*b^3*c^
2*d*(d*x+c)^(1/2)-4*b^4*c^3*(d*x+c)^(1/2)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 189, normalized size = 1.54

$$\frac{2\left(\frac{5(dx+c)^{\frac{7}{2}}b^4-28(b^4c-ab^3d)(dx+c)^{\frac{5}{2}}+70(b^4c^2-2ab^3cd+a^2b^2d^2)(dx+c)^{\frac{3}{2}}-140(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)\sqrt{dx+c}}{d^4}-\frac{35(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)}{\sqrt{dx+c}\,d^4}\right)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(d*x + c)^(7/2)*b^4 - 28*(b^4*c - a*b^3*d)*(d*x + c)^(5/2) + 70*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(3/2) - 140*(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(d*x + c))/d^4 - 35*(b^4*c^4 - 4
```

```
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(sqrt(d*x + c)*
d^4))/d
```

Fricas [A]
time = 0.82, size = 192, normalized size = 1.56

$$\frac{2\left(5\,b^{4}d^{4}x^{4}-128\,b^{4}c^{4}+448\,ab^{3}c^{3}d-560\,a^{2}b^{2}c^{2}d^{2}+280\,a^{3}bcd^{3}-35\,a^{4}d^{4}-4\left(2\,b^{4}cd^{3}-7\,ab^{3}d^{4}\right)x^{3}+2\left(8\,b^{4}c^{2}d^{2}-28\,ab^{3}cd^{3}+35\,a^{2}b^{2}d^{4}\right)x^{2}-4\left(16\,b^{4}c^{3}d-56\,ab^{3}c^{2}d^{2}+70\,a^{2}b^{2}cd^{3}-35\,a^{3}bd^{4}\right)x\right)\sqrt{dx+c}}{35\left(d^{6}x+cd^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*d^4*x^4 - 128*b^4*c^4 + 448*a*b^3*c^3*d - 560*a^2*b^2*c^2*d^2 +
 280*a^3*b*c*d^3 - 35*a^4*d^4 - 4*(2*b^4*c*d^3 - 7*a*b^3*d^4)*x^3 + 2*(8*b^
4*c^2*d^2 - 28*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 56*a*b
^3*c^2*d^2 + 70*a^2*b^2*c*d^3 - 35*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^6*x + c*d
^5)
```

Sympy [A]
time = 13.06, size = 168, normalized size = 1.37

$$\frac{2b^{4}(c+dx)^{\frac{7}{2}}}{7d^{5}}+\frac{(c+dx)^{\frac{5}{2}}\cdot(8ab^{3}d-8b^{4}c)}{5d^{5}}+\frac{(c+dx)^{\frac{3}{2}}\cdot(12a^{2}b^{2}d^{2}-24ab^{3}cd+12b^{4}c^{2})}{3d^{5}}+\frac{\sqrt{c+dx}\,(8a^{3}bd^{3}-24a^{2}b^{2}cd^{2}+24ab^{3}c^{2}d-8b^{4}c^{3})}{d^{5}}-\frac{2(ad-bc)^{4}}{d^{5}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(7/2)/(7*d**5) + (c + d*x)**(5/2)*(8*a*b**3*d - 8*b**4*c)
/(5*d**5) + (c + d*x)**(3/2)*(12*a**2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c
**2)/(3*d**5) + sqrt(c + d*x)*(8*a**3*b*d**3 - 24*a**2*b**2*c*d**2 + 24*a*b
**3*c**2*d - 8*b**4*c**3)/d**5 - 2*(a*d - b*c)**4/(d**5*sqrt(c + d*x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(109) = 218.
time = 1.47, size = 240, normalized size = 1.95

$$-\frac{2\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}\right)}{\sqrt{dx+c}\,d^{5}}+\frac{2\left(5\,(dx+c)^{\frac{7}{2}}b^{4}d^{30}-28\,(dx+c)^{\frac{5}{2}}b^{4}cd^{30}+70\,(dx+c)^{\frac{3}{2}}b^{4}c^{2}d^{30}-140\,\sqrt{dx+c}\,b^{4}c^{3}d^{30}+28\,(dx+c)^{\frac{5}{2}}ab^{3}d^{31}-140\,(dx+c)^{\frac{3}{2}}ab^{3}cd^{31}+420\,\sqrt{dx+c}\,ab^{3}c^{2}d^{31}+70\,(dx+c)^{\frac{3}{2}}a^{2}b^{2}d^{32}-420\,\sqrt{dx+c}\,a^{2}b^{2}cd^{32}+140\,\sqrt{dx+c}\,a^{3}bd^{33}\right)}{35\,d^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
(sqrt(d*x + c)*d^5) + 2/35*(5*(d*x + c)^(7/2)*b^4*d^30 - 28*(d*x + c)^(5/2)
*b^4*c*d^30 + 70*(d*x + c)^(3/2)*b^4*c^2*d^30 - 140*sqrt(d*x + c)*b^4*c^3*d
^30 + 28*(d*x + c)^(5/2)*a*b^3*d^31 - 140*(d*x + c)^(3/2)*a*b^3*c*d^31 + 42
0*sqrt(d*x + c)*a*b^3*c^2*d^31 + 70*(d*x + c)^(3/2)*a^2*b^2*d^32 - 420*sqrt
(d*x + c)*a^2*b^2*c*d^32 + 140*sqrt(d*x + c)*a^3*b*d^33)/d^35
```

Mupad [B]
time = 0.06, size = 153, normalized size = 1.24

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{2\,a^4\,d^4-8\,a^3\,b\,c\,d^3+12\,a^2\,b^2\,c^2\,d^2-8\,a\,b^3\,c^3\,d+2\,b^4\,c^4}{d^5\,\sqrt{c+d\,x}}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(7/2))/(7*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(5/2))/(
5*d^5) - (2*a^4*d^4 + 2*b^4*c^4 + 12*a^2*b^2*c^2*d^2 - 8*a*b^3*c^3*d - 8*a^
3*b*c*d^3)/(d^5*(c + d*x)^(1/2)) + (4*b^2*(a*d - b*c)^2*(c + d*x)^(3/2))/d^
5 + (8*b*(a*d - b*c)^3*(c + d*x)^(1/2))/d^5
```

3.1426 $\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

```
[Out] -2*b^2*(-a*d+b*c)*(d*x+c)^(3/2)/d^4+2/5*b^3*(d*x+c)^(5/2)/d^4+2*(-a*d+b*c)^
3/d^4/(d*x+c)^(1/2)+6*b*(-a*d+b*c)^2*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}\,(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(d^4*Sqrt[c + d*x]) + (6*b*(b*c - a*d)^2*Sqrt[c + d*x])/d
^4 - (2*b^2*(b*c - a*d)*(c + d*x)^(3/2))/d^4 + (2*b^3*(c + d*x)^(5/2))/(5*d
^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{3/2}}+\frac{3b(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{3b^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{b^3(c+dx)^{3/2}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 1.05

$$\frac{2(-5a^3d^3+15a^2bd^2(2c+dx)+5ab^2d(-8c^2-4cdx+d^2x^2)+b^3(16c^3+8c^2dx-2cd^2x^2+d^3x^3))}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/(c + d*x)^(3/2),x]

[Out] (2*(-5*a^3*d^3 + 15*a^2*b*d^2*(2*c + d*x) + 5*a*b^2*d*(-8*c^2 - 4*c*d*x + d^2*x^2) + b^3*(16*c^3 + 8*c^2*d*x - 2*c*d^2*x^2 + d^3*x^3)))/(5*d^4*Sqrt[c + d*x])

Maple [A]
time = 0.19, size = 136, normalized size = 1.45

method	result
risch	$\frac{2b\left(b^2x^2d^2+5ab\,d^2x-3b^2cdx+15a^2d^2-25abcd+11b^2c^2\right)\sqrt{dx+c}}{5d^4}-\frac{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{d^4\sqrt{dx+c}}$
gosper	$-\frac{2\left(-b^3x^3d^3-5d^3a\,x^2b^2+2b^3c\,d^2x^2-15a^2b\,d^3x+20a\,b^2c\,d^2x-8b^3c^2dx+5a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d-16b^3c^3\right)}{5\sqrt{dx+c}\,d^4}$
trager	$-\frac{2\left(-b^3x^3d^3-5d^3a\,x^2b^2+2b^3c\,d^2x^2-15a^2b\,d^3x+20a\,b^2c\,d^2x-8b^3c^2dx+5a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d-16b^3c^3\right)}{5\sqrt{dx+c}\,d^4}$
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{5}{2}}}{5}+2a\,b^2d(dx+c)^{\frac{3}{2}}-2b^3c(dx+c)^{\frac{3}{2}}+6a^2b\,d^2\sqrt{dx+c}-12a\,b^2cd\sqrt{dx+c}+6b^3c^2\sqrt{dx+c}-\frac{2(a^3d^3}{}}{d^4}$
default	$\frac{\frac{2b^3(dx+c)^{\frac{5}{2}}}{5}+2a\,b^2d(dx+c)^{\frac{3}{2}}-2b^3c(dx+c)^{\frac{3}{2}}+6a^2b\,d^2\sqrt{dx+c}-12a\,b^2cd\sqrt{dx+c}+6b^3c^2\sqrt{dx+c}-\frac{2(a^3d^3}{}}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d^4*(1/5*b^3*(d*x+c)^(5/2)+a*b^2*d*(d*x+c)^(3/2)-b^3*c*(d*x+c)^(3/2)+3*a^2*b*d^2*(d*x+c)^(1/2)-6*a*b^2*c*d*(d*x+c)^(1/2)+3*b^3*c^2*(d*x+c)^(1/2)-(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^(1/2))

Maxima [A]
time = 0.28, size = 125, normalized size = 1.33

$$\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^3-5\,(b^3c-ab^2d)\,(dx+c)^{\frac{3}{2}}+15\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{dx+c}}{d^3}+\frac{5\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}{\sqrt{dx+c}\,d^3}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 2/5*(((d*x + c)^(5/2)*b^3 - 5*(b^3*c - a*b^2*d)*(d*x + c)^(3/2) + 15*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(d*x + c))/d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^3))/d

Fricas [A]
time = 1.21, size = 124, normalized size = 1.32

$$\frac{2\,{\left(b^{3} d^{3} x^{3} + 16\,b^{3} c^{3} - 40\,a b^{2} c^{2} d + 30\,a^{2} b c d^{2} - 5\,a^{3} d^{3} - {\left(2\,b^{3} c d^{2} - 5\,a b^{2} d^{3}\right)} x^{2} + {\left(8\,b^{3} c^{2} d - 20\,a b^{2} c d^{2} + 15\,a^{2} b d^{3}\right)} x\right)} \sqrt{d x + c}}{5\,{\left(d^{5} x + c d^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*d^3*x^3 + 16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3
 - (2*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 20*a*b^2*c*d^2 + 15*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^5*x + c*d^4)
```

Sympy [A]
time = 8.90, size = 109, normalized size = 1.16

$$\frac{2b^{3}(c+dx)^{\frac{5}{2}}}{5d^{4}} + \frac{(c+dx)^{\frac{3}{2}}\cdot(6ab^{2}d-6b^{3}c)}{3d^{4}} + \frac{\sqrt{c+dx}\,(6a^{2}bd^{2}-12ab^{2}cd+6b^{3}c^{2})}{d^{4}} - \frac{2(ad-bc)^{3}}{d^{4}\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**3*(c + d*x)**(5/2)/(5*d**4) + (c + d*x)**(3/2)*(6*a*b**2*d - 6*b**3*c)
/(3*d**4) + sqrt(c + d*x)*(6*a**2*b*d**2 - 12*a*b**2*c*d + 6*b**3*c**2)/d**
4 - 2*(a*d - b*c)**3/(d**4*sqrt(c + d*x))
```

Giac [A]
time = 1.15, size = 152, normalized size = 1.62

$$\frac{2\,{\left(b^{3} c^{3} - 3\,a b^{2} c^{2} d + 3\,a^{2} b c d^{2} - a^{3} d^{3}\right)}}{\sqrt{d x + c} d^{4}} + \frac{2\,{\left({\left(d x + c\right)}^{\frac{5}{2}} b^{3} d^{16} - 5\,{\left(d x + c\right)}^{\frac{3}{2}} b^{3} c d^{16} + 15\,\sqrt{d x + c} b^{3} c^{2} d^{16} + 5\,{\left(d x + c\right)}^{\frac{3}{2}} a b^{2} d^{17} - 30\,\sqrt{d x + c} a b^{2} c d^{17} + 15\,\sqrt{d x + c} a^{2} b d^{18}\right)}}{5\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^4) +
 2/5*((d*x + c)^(5/2)*b^3*d^16 - 5*(d*x + c)^(3/2)*b^3*c*d^16 + 15*sqrt(d*x
 + c)*b^3*c^2*d^16 + 5*(d*x + c)^(3/2)*a*b^2*d^17 - 30*sqrt(d*x + c)*a*b^2*
c*d^17 + 15*sqrt(d*x + c)*a^2*b*d^18)/d^20
```

Mupad [B]
time = 0.08, size = 114, normalized size = 1.21

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}-\frac{2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-2\,b^3\,c^3}{d^4\,\sqrt{c+d\,x}}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(5/2))/(5*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(3/2))/(
3*d^4) - (2*a^3*d^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(d^4*(c +
d*x)^(1/2)) + (6*b*(a*d - b*c)^2*(c + d*x)^(1/2))/d^4
```

3.1427 $\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

```
[Out] 2/3*b^2*(d*x+c)^(3/2)/d^3-2*(-a*d+b*c)^2/d^3/(d*x+c)^(1/2)-4*b*(-a*d+b*c)*(
d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {45}

$$-\frac{4b\sqrt{c+dx}\,(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x]) - (4*b*(b*c - a*d)*Sqrt[c + d*x])/d^
3 + (2*b^2*(c + d*x)^(3/2))/(3*d^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx\\ &= -\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.88

$$\frac{2(-3a^2d^2+6abd(2c+dx)+b^2(-8c^2-4cdx+d^2x^2))}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2*d^2 + 6*a*b*d*(2*c + d*x) + b^2*(-8*c^2 - 4*c*d*x + d^2*x^2)))/(
3*d^3*Sqrt[c + d*x])
```

Maple [A]
time = 0.17, size = 74, normalized size = 1.10

method	result	size
risch	$\frac{2b(bdx+6ad-5bc)\sqrt{dx+c}}{3d^3}-\frac{2(a^2d^2-2abcd+b^2c^2)}{d^3\sqrt{dx+c}}$	61
gosper	$-\frac{2(-b^2x^2d^2-6ab\,d^2x+4b^2cdx+3a^2d^2-12abcd+8b^2c^2)}{3\sqrt{dx+c}\,d^3}$	63
trager	$-\frac{2(-b^2x^2d^2-6ab\,d^2x+4b^2cdx+3a^2d^2-12abcd+8b^2c^2)}{3\sqrt{dx+c}\,d^3}$	63
derivativedivides	$\frac{\frac{2b^2(dx+c)^{\frac{3}{2}}}{3}+4adb\sqrt{dx+c}-4b^2c\sqrt{dx+c}-\frac{2(a^2d^2-2abcd+b^2c^2)}{\sqrt{dx+c}}}{d^3}$	74
default	$\frac{\frac{2b^2(dx+c)^{\frac{3}{2}}}{3}+4adb\sqrt{dx+c}-4b^2c\sqrt{dx+c}-\frac{2(a^2d^2-2abcd+b^2c^2)}{\sqrt{dx+c}}}{d^3}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/3*b^2*(d*x+c)^(3/2)+2*a*d*b*(d*x+c)^(1/2)-2*b^2*c*(d*x+c)^(1/2)-(a
^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 75, normalized size = 1.12

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2-6\,(b^2c-abd)\sqrt{dx+c}}{d^2}-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)}{\sqrt{dx+c}\,d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^2 - 6*(b^2*c - a*b*d)*sqrt(d*x + c))/d^2 - 3*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^2))/d
```

Fricas [A]
time = 1.18, size = 73, normalized size = 1.09

$$\frac{2\,(b^2d^2x^2-8\,b^2c^2+12\,abcd-3\,a^2d^2-2\,(2\,b^2cd-3\,abd^2)x)\sqrt{dx+c}}{3\,(d^4x+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*d^2*x^2 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x)*sqrt(d*x + c)/(d^4*x + c*d^3)
```

Sympy [A]
time = 5.63, size = 65, normalized size = 0.97

$$\frac{2b^2(c+dx)^{\frac{3}{2}}}{3d^3}+\frac{\sqrt{c+dx}\,(4abd-4b^2c)}{d^3}-\frac{2(ad-bc)^2}{d^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**2*(c + d*x)**(3/2)/(3*d**3) + sqrt(c + d*x)*(4*a*b*d - 4*b**2*c)/d**3
- 2*(a*d - b*c)**2/(d**3*sqrt(c + d*x))
```

Giac [A]
time = 1.50, size = 84, normalized size = 1.25

$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)}{\sqrt{dx+c}\,d^3}+\frac{2\left((dx+c)^{\frac{3}{2}}b^2d^6-6\,\sqrt{dx+c}\,b^2cd^6+6\,\sqrt{dx+c}\,abd^7\right)}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^3) + 2/3*((d*x + c)^(3/
2)*b^2*d^6 - 6*sqrt(d*x + c)*b^2*c*d^6 + 6*sqrt(d*x + c)*a*b*d^7)/d^9
```

Mupad [B]
time = 0.26, size = 67, normalized size = 1.00

$$\frac{\frac{2\,b^2\,(c+d\,x)^2}{3}-2\,a^2\,d^2-2\,b^2\,c^2-4\,b^2\,c\,(c+d\,x)+4\,a\,b\,d\,(c+d\,x)+4\,a\,b\,c\,d}{d^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(3/2),x)
```

```
[Out] ((2*b^2*(c + d*x)^2)/3 - 2*a^2*d^2 - 2*b^2*c^2 - 4*b^2*c*(c + d*x) + 4*a*b*
d*(c + d*x) + 4*a*b*c*d)/(d^3*(c + d*x)^(1/2))
```

3.1428 $\int \frac{a+bx}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

```
[Out] 2*(-a*d+b*c)/d^2/(d*x+c)^(1/2)+2*b*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d))/(d^2*Sqrt[c + d*x]) + (2*b*Sqrt[c + d*x])/d^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right)dx\\ &= \frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.71

$$\frac{2(2bc-ad+bdx)}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(3/2),x]
[Out] (2*(2*b*c - a*d + b*d*x))/(d^2*Sqrt[c + d*x])
```

Maple [A]
time = 0.16, size = 33, normalized size = 0.87

method	result	size
gosper	$-\frac{2(-bdx+ad-2bc)}{\sqrt{dx+c}\,d^2}$	26
trager	$-\frac{2(-bdx+ad-2bc)}{\sqrt{dx+c}\,d^2}$	26
derivativedivides	$\frac{2b\sqrt{dx+c}-\frac{2(ad-bc)}{\sqrt{dx+c}}}{d^2}$	33
default	$\frac{2b\sqrt{dx+c}-\frac{2(ad-bc)}{\sqrt{dx+c}}}{d^2}$	33
risch	$\frac{2b\sqrt{dx+c}}{d^2}-\frac{2(ad-bc)}{d^2\sqrt{dx+c}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(b*(d*x+c)^(1/2)-(a*d-b*c)/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.29, size = 37, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{dx+c}\,b}{d}+\frac{bc-ad}{\sqrt{dx+c}\,d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 2*(sqrt(d*x + c)*b/d + (b*c - a*d)/(sqrt(d*x + c)*d))/d
```

Fricas [A]
time = 0.81, size = 35, normalized size = 0.92

$$\frac{2\,(bdx+2\,bc-ad)\sqrt{dx+c}}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] 2*(b*d*x + 2*b*c - a*d)*sqrt(d*x + c)/(d^3*x + c*d^2)
```

Sympy [A]
time = 0.27, size = 60, normalized size = 1.58

$$\begin{cases} -\frac{2a}{d\sqrt{c+dx}} + \frac{4bc}{d^2\sqrt{c+dx}} + \frac{2bx}{d\sqrt{c+dx}} & \text{for } d \neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a/(d*sqrt(c + d*x)) + 4*b*c/(d**2*sqrt(c + d*x)) + 2*b*x/(d*s
qrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(3/2), True))
```

Giac [A]
time = 1.08, size = 34, normalized size = 0.89

$$\frac{2\sqrt{dx+c}\,b}{d^2} + \frac{2\,(bc-ad)}{\sqrt{dx+c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b/d^2 + 2*(b*c - a*d)/(sqrt(d*x + c)*d^2)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.66

$$\frac{4\,b\,c-2\,a\,d+2\,b\,d\,x}{d^2\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] (4*b*c - 2*a*d + 2*b*d*x)/(d^2*(c + d*x)^(1/2))
```

3.1429 $\int \frac{1}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{d\sqrt{c+dx}}$$

[Out] -2/d/(d*x+c)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{32\}$

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(-3/2),x]

[Out] -2/(d*Sqrt[c + d*x])

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{3/2}}\,dx = -\frac{2}{d\sqrt{c+dx}}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(-3/2),x]

[Out] -2/(d*Sqrt[c + d*x])

Maple [A]
time = 0.14, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{2}{d\sqrt{dx+c}}$	13
derivativedivides	$-\frac{2}{d\sqrt{dx+c}}$	13
default	$-\frac{2}{d\sqrt{dx+c}}$	13
trager	$-\frac{2}{d\sqrt{dx+c}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d/(d*x+c)^(1/2)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

Fricas [A]
time = 1.09, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{dx+c}}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*x + c)/(d^2*x + c*d)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.86

$$-\frac{2}{d\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(3/2),x)
```

```
[Out] -2/(d*sqrt(c + d*x))
```

Giac [**A**]
time = 1.26, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(d*x + c)*d)
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{d\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(3/2),x)
```

```
[Out] -2/(d*(c + d*x)^(1/2))
```

3.1430 $\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

```
[Out] -2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(3/2)
+2/(-a*d+b*c)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {53, 65, 214}

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx &= \frac{2}{(bc-ad)\sqrt{c+dx}}+\frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{bc-ad}\\
&= \frac{2}{(bc-ad)\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d(bc-ad)}\\
&= \frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 1.00

$$\frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)),x]
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2)
```

Maple [A]
time = 0.19, size = 68, normalized size = 0.99

method	result	size
derivativedivides	$-\frac{2}{(ad-bc)\sqrt{dx+c}}-\frac{2b\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)\sqrt{(ad-bc)\,b}}$	68
default	$-\frac{2}{(ad-bc)\sqrt{dx+c}}-\frac{2b\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)\sqrt{(ad-bc)\,b}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a*d-b*c)/(d*x+c)^(1/2)-2*b/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+
c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.01, size = 214, normalized size = 3.10

$$\left[-\frac{(dx+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,\sqrt{dx+c}}{bc^2-acd+(bcd-ad^2)x},\ -\frac{2\left((dx+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\sqrt{dx+c}\right)}{bc^2-acd+(bcd-ad^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((d*x + c)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*s
qrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*sqrt(d*x + c))/(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x), -2*((d*x + c)*sqrt(-b/(b*c - a*d))*arctan(-(b*c -
 a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - sqrt(d*x + c))/(b
*c^2 - a*c*d + (b*c*d - a*d^2)*x)]
```

Sympy [A]
time = 4.51, size = 60, normalized size = 0.87

$$-\frac{2}{\sqrt{c+dx}\,(ad-bc)}-\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] -2/(sqrt(c + d*x)*(a*d - b*c)) - 2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/
(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

Giac [**A**]
time = 1.66, size = 69, normalized size = 1.00

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{2}{(bc-ad)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 2/((b*c - a*d)*sqrt(d*x + c))
```

Mupad [**B**]
time = 0.27, size = 57, normalized size = 0.83

$$-\frac{2}{(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{2\,\sqrt{b}\,\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{{(a\,d-b\,c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(3/2)),x)
```

```
[Out] - 2/((a*d - b*c)*(c + d*x)^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*(c + d*x)^(1/2
))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

3.1431 $\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3d}{(bc-ad)^2\sqrt{c+dx}}-\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] 3*d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2
)-3*d/(-a*d+b*c)^2/(d*x+c)^(1/2)-1/(-a*d+b*c)/(b*x+a)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {44, 53, 65, 214}

$$-\frac{3d}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d)/((b*c - a*d)^2*Sqrt[c + d*x]) - 1/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c -
 a*d)^(5/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx &= -\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)} \\
&= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3bd)\int \frac{1}{(a+bx)\sqrt{c+dx}}}{2(bc-ad)^2} \\
&= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{(bc-a} \\
&= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} + \frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{bc}}\right.}{(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 90, normalized size = 0.91

$$-\frac{2ad+b(c+3dx)}{(bc-ad)^2(a+bx)\sqrt{c+dx}} - \frac{3\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] -((2*a*d + b*(c + 3*d*x))/((b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x])) - (3*Sqr
t[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(
5/2)
```

Maple [A]
time = 0.19, size = 100, normalized size = 1.01

method	result	size
derivativedivides	$2d\left(-\frac{b\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)^2}-\frac{1}{(ad-bc)^2\sqrt{dx+c}}\right)$	100
default	$2d\left(-\frac{b\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{3\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{2\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)^2}-\frac{1}{(ad-bc)^2\sqrt{dx+c}}\right)$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/(a*d-b*c)^2*b*(1/2*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+3/2/((a*d-b*c)
*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))-1/(a*d-b*c)^2/(d*x+c
)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(85) = 170$.
time = 0.74, size = 423, normalized size = 4.27

$$\left[\frac{3\left(bd^2x^2+acd+(bcd+ad^2)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x\right)},\ \frac{3\left(bd^2x^2+acd+(bcd+ad^2)x\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/2*(3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*(3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x), (3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2 (c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**2/(d*x+c)**(3/2),x)

[Out] Integral(1/((a + b*x)**2*(c + d*x)**(3/2)), x)

Giac [A]
time = 1.20, size = 143, normalized size = 1.44

$$-\frac{3\,bd\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}-\frac{3\,(dx+c)bd-2\,bcd+2\,ad^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}\,bc+\sqrt{dx+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")

[Out] -3*b*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) - (3*(d*x + c)*b*d - 2*b*c*d + 2*a*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + sqrt(d*x + c)*a*d))

Mupad [B]
time = 0.19, size = 123, normalized size = 1.24

$$-\frac{\frac{2\,d}{a\,d-b\,c}+\frac{3\,b\,d\,(c+d\,x)}{(a\,d-b\,c)^2}}{b\,(c+d\,x)^{3/2}+(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)}{(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d)/(a*d - b*c) + (3*b*d*(c + d*x))/(a*d - b*c)^2)/(b*(c + d*x)^(3/2)
+ (a*d - b*c)*(c + d*x)^(1/2)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

3.1432 $\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}}-\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}}+\frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}$$

```
[Out] -15/4*d^2*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c
)^(7/2)+15/4*d^2/(-a*d+b*c)^3/(d*x+c)^(1/2)-1/2/(-a*d+b*c)/(b*x+a)^2/(d*x+c
)^(1/2)+5/4*d/(-a*d+b*c)^2/(b*x+a)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$\frac{15d^2}{4\sqrt{c+dx}\,(bc-ad)^3}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (15*d^2)/(4*(b*c - a*d)^3*Sqrt[c + d*x]) - 1/(2*(b*c - a*d)*(a + b*x)^2*Sqr
t[c + d*x]) + (5*d)/(4*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) - (15*Sqrt[b]
*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(7/2)
)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{(15d^2)\int \frac{1}{(a+\cdots}}{8(bc\cdots}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{\cdots}}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{\cdots}}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{\cdots}}$$

Mathematica [A]
time = 0.55, size = 126, normalized size = 0.90

$$\frac{1}{4}\left(\frac{8a^2d^2+abd(9c+25dx)+b^2(-2c^2+5cdx+15d^2x^2)}{(bc-ad)^3(a+bx)^2\sqrt{c+dx}} - \frac{15\sqrt{b}\,d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] ((8*a^2*d^2 + a*b*d*(9*c + 25*d*x) + b^2*(-2*c^2 + 5*c*d*x + 15*d^2*x^2))/(
(b*c - a*d)^3*(a + b*x)^2*Sqrt[c + d*x]) - (15*Sqrt[b]*d^2*ArcTan[(Sqrt[b]*
Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2))/4
```

Maple [A]
time = 0.20, size = 122, normalized size = 0.87

method	result
derivativedivides	$2d^2\left(-\frac{1}{(ad-bc)^3\sqrt{dx+c}}-\frac{b\left(\frac{\frac{7b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{9ad}{8}-\frac{9bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\right)$
default	$2d^2\left(-\frac{1}{(ad-bc)^3\sqrt{dx+c}}-\frac{b\left(\frac{\frac{7b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{9ad}{8}-\frac{9bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{15\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-1/(a*d-b*c)^3/(d*x+c)^(1/2)-b/(a*d-b*c)^3*((7/8*b*(d*x+c)^(3/2)+(9/
8*a*d-9/8*b*c)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^2+15/8/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(116) = 232$.
time = 0.83, size = 782, normalized size = 5.59

$$\left[-\frac{15\,(b^2d^3x^3+a^2cd^2+(b^2cd^2+2\,abd^3)x^2+(2\,abcd^2+a^2d^3)x)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,(15\,b^2d^2x^2-2\,b^2c^2+9\,abcd+8\,a^2d^2+5\,(b^2cd+5\,abd^2)x)\sqrt{dx+c}}{8\,(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^3+(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4)x^2+(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4)x)}, -\frac{15\,(b^2d^3x^3+a^2cd^2+(b^2cd^2+2\,abd^3)x^2+(2\,abcd^2+a^2d^3)x)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(15\,b^2d^2x^2-2\,b^2c^2+9\,abcd+8\,a^2d^2+5\,(b^2cd+5\,abd^2)x)\sqrt{dx+c}}{4\,(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4)x^3+(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4)x^2+(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c
*d^2 + a^2*d^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c -
a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*(15*b^2*d^2*x^2 - 2*
b^2*c^2 + 9*a*b*c*d + 8*a^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))
/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c
^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^
4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x), -1/4*(
15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c*d^2 +
a^2*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/
(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 - 2*b^2*c^2 + 9*a*b*c*d + 8*a
^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))/(a^2*b^3*c^4 - 3*a^3*b^2
*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2
*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2
+ 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a
^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(116) = 232$.
time = 1.57, size = 234, normalized size = 1.67

$$\frac{15\,bd^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}+\frac{2\,d^2}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{dx+c}}+\frac{7\,(dx+c)^{\frac{3}{2}}b^2d^2-9\,\sqrt{dx+c}\,b^2cd^2+9\,\sqrt{dx+c}\,abd^3}{4\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx+c)b-bc+ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*d^2/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(d*x + c)) + 1/4*(7*(d*x + c)
^(3/2)*b^2*d^2 - 9*sqrt(d*x + c)*b^2*c*d^2 + 9*sqrt(d*x + c)*a*b*d^3)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c + a*d)^2
)
```

Mupad [B]
time = 0.44, size = 205, normalized size = 1.46

$$-\frac{\frac{2\,d^2}{a\,d-b\,c}+\frac{15\,b^2\,d^2\,{\left(c+d\,x\right)}^2}{4\,{\left(a\,d-b\,c\right)}^3}+\frac{25\,b\,d^2\,\left(c+d\,x\right)}{4\,{\left(a\,d-b\,c\right)}^2}}{b^2\,{\left(c+d\,x\right)}^{5/2}-\left(2\,b^2\,c-2\,a\,b\,d\right)\,{\left(c+d\,x\right)}^{3/2}+\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{15\,\sqrt{b}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^{7/2}}\right)}{4\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^2)/(a*d - b*c) + (15*b^2*d^2*(c + d*x)^2)/(4*(a*d - b*c)^3) + (25*b
*d^2*(c + d*x))/(4*(a*d - b*c)^2))/(b^2*(c + d*x)^(5/2) - (2*b^2*c - 2*a*b*
d)*(c + d*x)^(3/2) + (c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (15
*b^(1/2)*d^2*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(4*(a*d - b*c)^(7/2))
```

3.1433 $\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}}-\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}}+\frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}}-\frac{35d^2}{24(bc-ad)^3(a+b}$$

```
[Out] 35/8*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)
^(9/2)-35/8*d^3/(-a*d+b*c)^4/(d*x+c)^(1/2)-1/3/(-a*d+b*c)/(b*x+a)^3/(d*x+c)
^(1/2)+7/12*d/(-a*d+b*c)^2/(b*x+a)^2/(d*x+c)^(1/2)-35/24*d^2/(-a*d+b*c)^3/(
b*x+a)/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$-\frac{35d^3}{8\sqrt{c+dx}\,(bc-ad)^4}+\frac{35\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}\,(bc-ad)^3}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{3(a+bx)^3\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*Sqrt[c + d*x]) - 1/(3*(b*c - a*d)*(a + b*x)^3*Sq
rt[c + d*x]) + (7*d)/(12*(b*c - a*d)^2*(a + b*x)^2*Sqrt[c + d*x]) - (35*d^2
)/(24*(b*c - a*d)^3*(a + b*x)*Sqrt[c + d*x]) + (35*Sqrt[b]*d^3*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*(b*c - a*d)^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx}{6(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} + \frac{(35d^2)\int}{24}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{}{24(bc-}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+b}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+b}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+b}$$

Mathematica [A]
time = 0.71, size = 170, normalized size = 0.98

$$\frac{-48a^3d^3-3a^2bd^2(29c+77dx)-2ab^2d(-19c^2+49cdx+140d^2x^2)-b^3(8c^3-14c^2dx+35cd^2x^2+105d^3x^3)}{24(bc-ad)^4(a+bx)^3\sqrt{c+dx}} - \frac{35\sqrt{b}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{8(-bc+ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

[Out] (-48*a^3*d^3 - 3*a^2*b*d^2*(29*c + 77*d*x) - 2*a*b^2*d*(-19*c^2 + 49*c*d*x + 140*d^2*x^2) - b^3*(8*c^3 - 14*c^2*d*x + 35*c*d^2*x^2 + 105*d^3*x^3))/(24*(b*c - a*d)^4*(a + b*x)^3*Sqrt[c + d*x]) - (35*Sqrt[b]*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*(-(b*c) + a*d)^(9/2))

Maple [A]
time = 0.19, size = 156, normalized size = 0.90

method	result
derivativedivides	$2d^3\left(-\frac{b\left(\frac{\frac{19(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{17(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{29}{16}a^2d^2-\frac{29}{8}abcd+\frac{29}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{35\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^4}\right.$
default	$2d^3\left(-\frac{b\left(\frac{\frac{19(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{17(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{29}{16}a^2d^2-\frac{29}{8}abcd+\frac{29}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{35\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^4/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*d^3*(-1/(a*d-b*c)^4*b*((19/16*(d*x+c)^(5/2)*b^2+17/6*(a*d-b*c)*b*(d*x+c)^(3/2)+(29/16*a^2*d^2-29/8*a*b*c*d+29/16*b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^3+35/16/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))-1/(a*d-b*c)^4/(d*x+c)^(1/2))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(145) = 290$.
time = 0.60, size = 1204, normalized size = 6.96

$$\left[\frac{105\,(b^3d^4x^4+a^3cd^3+(b^3cd^3+3\,ab^2d^4)x^3+3\,(ab^2cd^3+a^2bd^4)x^2+(3\,a^2bcd^3+a^3d^4)x)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,(105\,b^3d^3x^3+8\,b^3c^3-38\,ab^2c^2d+87\,a^2bcd^2+48\,a^3d^3+35\,(b^3cd^2+8\,ab^2d^3)x^2-7\,(2\,b^3c^2d-14\,ab^2cd^2-33\,a^2bd^3)x)\sqrt{dx+c}}{48\,(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5)x^4+(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5)x^3+3\,(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5)x^2+(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5)x)},\ \frac{105\,(b^3d^4x^4+a^3cd^3+(b^3cd^3+3\,ab^2d^4)x^3+3\,(ab^2cd^3+a^2bd^4)x^2+(3\,a^2bcd^3+a^3d^4)x)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(105\,b^3d^3x^3+8\,b^3c^3-38\,ab^2c^2d+87\,a^2bcd^2+48\,a^3d^3+35\,(b^3cd^2+8\,ab^2d^3)x^2-7\,(2\,b^3c^2d-14\,ab^2cd^2-33\,a^2bd^3)x)\sqrt{dx+c}}{24\,(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5)x^4+(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5)x^3+3\,(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5)x^2+(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*x^3 + 3*(a*
b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b/(b*c - a*d
))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d
)))/(b*x + a)) - 2*(105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b
*c*d^2 + 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 1
4*a*b^2*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x), 1/24*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*
x^3 + 3*(a*b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(-
b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*
x + b*c)) - (105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b*c*d^2
+ 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 14*a*b^2
*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6
*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*
b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a
^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^
4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*
b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)
*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(145) = 290.
time = 1.30, size = 326, normalized size = 1.88

$$-\frac{35\,bd^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-b^2c+abd}}-\frac{2\,d^3}{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{dx+c}}-\frac{57\,(dx+c)^{\frac{5}{2}}b^3d^3-136\,(dx+c)^{\frac{3}{2}}b^3cd^3+87\,\sqrt{dx+c}\,b^3c^2d^3+136\,(dx+c)^{\frac{3}{2}}ab^2d^4-174\,\sqrt{dx+c}\,ab^2cd^4+87\,\sqrt{dx+c}\,a^2bd^5}{24\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)((dx+c)b-bc+ad)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)
) - 2*d^3/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(d*x + c)) - 1/24*(57*(d*x + c)^(5/2)*b^3*d^3 - 136*(d*x + c)^(
3/2)*b^3*c*d^3 + 87*sqrt(d*x + c)*b^3*c^2*d^3 + 136*(d*x + c)^(3/2)*a*b^2*d
^4 - 174*sqrt(d*x + c)*a*b^2*c*d^4 + 87*sqrt(d*x + c)*a^2*b*d^5)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b
 - b*c + a*d)^3)
```

Mupad [B]
time = 0.54, size = 294, normalized size = 1.70

$$-\frac{\frac{2\,d^3}{a\,d-b\,c}+\frac{35\,b^2\,d^3\,(c+d\,x)^2}{3\,(a\,d-b\,c)^3}+\frac{35\,b^3\,d^3\,(c+d\,x)^3}{8\,(a\,d-b\,c)^4}+\frac{77\,b\,d^3\,(c+d\,x)}{8\,(a\,d-b\,c)^2}}{\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{(c+d\,x)}^{7/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{(c+d\,x)}^{5/2}+{(c+d\,x)}^{3/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}-\frac{35\,\sqrt{b}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{{(a\,d-b\,c)}^{9/2}}\right)}{8\,{(a\,d-b\,c)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^3)/(a*d - b*c) + (35*b^2*d^3*(c + d*x)^2)/(3*(a*d - b*c)^3) + (35*b
^3*d^3*(c + d*x)^3)/(8*(a*d - b*c)^4) + (77*b*d^3*(c + d*x))/(8*(a*d - b*c)
^2))/((c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) +
 b^3*(c + d*x)^(7/2) - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^(5/2) + (c + d*x)^(3
/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d)) - (35*b^(1/2)*d^3*atan((b^(1/2
)*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
4*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(8*(a*d - b*c)^(9/2))
```

3.1434 $\int \frac{(a+bx)^5}{(c+dx)^{5/2}} \, dx$

Optimal. Leaf size=152

$$\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/2}}{3d^6}-\frac{2b^4(bc-ad)(c+dx)}{d^6}$$

```
[Out] 2/3*(-a*d+b*c)^5/d^6/(d*x+c)^(3/2)+20/3*b^3*(-a*d+b*c)^2*(d*x+c)^(3/2)/d^6-
2*b^4*(-a*d+b*c)*(d*x+c)^(5/2)/d^6+2/7*b^5*(d*x+c)^(7/2)/d^6-10*b*(-a*d+b*c
)^4/d^6/(d*x+c)^(1/2)-20*b^2*(-a*d+b*c)^3*(d*x+c)^(1/2)/d^6
```

Rubi [A]
time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}\,(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b^5(c+dx)^{7/2}}{7d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(3*d^6*(c + d*x)^(3/2)) - (10*b*(b*c - a*d)^4)/(d^6*Sqrt[
c + d*x]) - (20*b^2*(b*c - a*d)^3*Sqrt[c + d*x])/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(3/2))/(3*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(5/2))/d^6 + (2*b
^5*(c + d*x)^(7/2))/(7*d^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{5/2}}+\frac{5b(bc-ad)^4}{d^5(c+dx)^{3/2}}-\frac{10b^2(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{10b^3(bc-ad)^2\sqrt{c+dx}}{d^5}-5b^4\right.\\ &= \frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/}}{3d^6}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 217, normalized size = 1.43

$$-\frac{2(7a^5d^5+35a^4bd^4(2c+3dx)-70a^3b^2d^3(8c^2+12cdx+3d^2x^2)+70a^2b^3d^2(16c^3+24c^2dx+6cd^2x^2-d^3x^3)-7ab^4d(128c^4+192c^3dx+48c^2d^2x^2-8cd^3x^3+3d^4x^4)+b^5(256c^5+384c^4dx+96c^3d^2x^2-16c^2d^3x^3+6cd^4x^4-3d^5x^5))}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(7*a^5*d^5 + 35*a^4*b*d^4*(2*c + 3*d*x) - 70*a^3*b^2*d^3*(8*c^2 + 12*c*
d*x + 3*d^2*x^2) + 70*a^2*b^3*d^2*(16*c^3 + 24*c^2*d*x + 6*c*d^2*x^2 - d^3*
x^3) - 7*a*b^4*d*(128*c^4 + 192*c^3*d*x + 48*c^2*d^2*x^2 - 8*c*d^3*x^3 + 3*
d^4*x^4) + b^5*(256*c^5 + 384*c^4*d*x + 96*c^3*d^2*x^2 - 16*c^2*d^3*x^3 + 6
*c*d^4*x^4 - 3*d^5*x^5)))/(21*d^6*(c + d*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(134) = 268$.
time = 0.17, size = 294, normalized size = 1.93

method	result
risch	$\frac{2b^2(3b^3x^3d^3+21d^3a\,x^2b^2-12b^3c\,d^2x^2+70a^2b\,d^3x-98a\,b^2c\,d^2x+37b^3c^2dx+210a^3d^3-560a^2bc\,d^2+511a\,b^2c^2d-158b^3c^3)}{21d^6}$
gosper	$-\frac{2(-3b^5x^5d^5-21a\,b^4d^5x^4+6b^5c\,d^4x^4-70a^2b^3d^5x^3+56a\,b^4c\,d^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+420a^2b^3c\,d^4x^2-336a\,b^4}{}$
trager	$-\frac{2(-3b^5x^5d^5-21a\,b^4d^5x^4+6b^5c\,d^4x^4-70a^2b^3d^5x^3+56a\,b^4c\,d^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+420a^2b^3c\,d^4x^2-336a\,b^4}{}$
derivativedivides	$\frac{\frac{2b^5(dx+c)^{\frac{7}{2}}}{7}+2a\,b^4d(dx+c)^{\frac{5}{2}}-2b^5c(dx+c)^{\frac{5}{2}}+\frac{20a^2b^3d^2(dx+c)^{\frac{3}{2}}}{3}-\frac{40a\,b^4cd(dx+c)^{\frac{3}{2}}}{3}+\frac{20b^5c^2(dx+c)^{\frac{3}{2}}}{3}+20a^3b^2d^3\sqrt{dx+c}}{}$
default	$\frac{\frac{2b^5(dx+c)^{\frac{7}{2}}}{7}+2a\,b^4d(dx+c)^{\frac{5}{2}}-2b^5c(dx+c)^{\frac{5}{2}}+\frac{20a^2b^3d^2(dx+c)^{\frac{3}{2}}}{3}-\frac{40a\,b^4cd(dx+c)^{\frac{3}{2}}}{3}+\frac{20b^5c^2(dx+c)^{\frac{3}{2}}}{3}+20a^3b^2d^3\sqrt{dx+c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^6*(1/7*b^5*(d*x+c)^(7/2)+a*b^4*d*(d*x+c)^(5/2)-b^5*c*(d*x+c)^(5/2)+10/3
*a^2*b^3*d^2*(d*x+c)^(3/2)-20/3*a*b^4*c*d*(d*x+c)^(3/2)+10/3*b^5*c^2*(d*x+c
)^(3/2)+10*a^3*b^2*d^3*(d*x+c)^(1/2)-30*a^2*b^3*c*d^2*(d*x+c)^(1/2)+30*a*b^
4*c^2*d*(d*x+c)^(1/2)-10*b^5*c^3*(d*x+c)^(1/2)-5*b*(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(d*x+c)^(1/2)-1/3*(a^5*d^5-5*a^4*b*
c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(d*x+c)^
(3/2))
```

Maxima [A]
time = 0.29, size = 265, normalized size = 1.74

$$\frac{2\left(\frac{3(dx+c)^{\frac{7}{2}}b^5-21(b^5c-ab^4d)(dx+c)^{\frac{5}{2}}+70(b^5c^2-2ab^4cd+a^2b^3d^2)(dx+c)^{\frac{3}{2}}-210(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3)\sqrt{dx+c}}{d^5}+\frac{7(b^5c^5-5ab^4c^4d+10a^2b^3c^3d^2-10a^3b^2c^2d^3+5a^4bcd^4-a^5d^5-15(b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4)(dx+c))}{(dx+c)^{\frac{3}{2}}d^5}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*((3*(d*x + c)^(7/2)*b^5 - 21*(b^5*c - a*b^4*d)*(d*x + c)^(5/2) + 70*(b
^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(3/2) - 210*(b^5*c^3 - 3*a*b^
4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(d*x + c))/d^5 + 7*(b^5*c^5 -
5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a
^5*d^5 - 15*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4)*(d*x + c))/((d*x + c)^(3/2)*d^5))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.
time = 0.47, size = 283, normalized size = 1.86

$$\frac{2\,{\left(3\,b^{5}d^{5}x^{5} - 256\,b^{5}c^{5} + 896\,ab^{4}c^{4}d - 1120\,a^{2}b^{3}c^{3}d^{2} + 560\,a^{3}b^{2}c^{2}d^{3} - 70\,a^{4}bcd^{4} - 7\,a^{5}d^{5} - 3\,{\left(2\,b^{5}cd^{4} - 7\,ab^{4}d^{5}\right)}x^{4} + 2\,{\left(8\,b^{5}c^{2}d^{3} - 28\,ab^{4}cd^{4} + 35\,a^{2}b^{3}d^{5}\right)}x^{3} - 6\,{\left(16\,b^{5}c^{3}d^{2} - 56\,ab^{4}c^{2}d^{3} + 70\,a^{2}b^{3}cd^{4} - 35\,a^{3}b^{2}d^{5}\right)}x^{2} - 3\,{\left(128\,b^{5}c^{4}d - 448\,ab^{4}c^{3}d^{2} + 560\,a^{2}b^{3}c^{2}d^{3} - 280\,a^{3}b^{2}cd^{4} + 35\,a^{4}bd^{5}\right)}x\right)}\sqrt{dx + c}}{21\,{\left(d^{8}x^{2} + 2\,cd^{7}x + c^{2}d^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^5*d^5*x^5 - 256*b^5*c^5 + 896*a*b^4*c^4*d - 1120*a^2*b^3*c^3*d^2
+ 560*a^3*b^2*c^2*d^3 - 70*a^4*b*c*d^4 - 7*a^5*d^5 - 3*(2*b^5*c*d^4 - 7*a*b
^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 28*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 6*(
16*b^5*c^3*d^2 - 56*a*b^4*c^2*d^3 + 70*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
- 3*(128*b^5*c^4*d - 448*a*b^4*c^3*d^2 + 560*a^2*b^3*c^2*d^3 - 280*a^3*b^2*
c*d^4 + 35*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6)
```

Sympy [**A**]
time = 24.13, size = 196, normalized size = 1.29

$$\frac{2b^{5}(c + dx)^{\frac{7}{2}}}{7d^{6}} - \frac{10b(ad - bc)^{4}}{d^{6}\sqrt{c + dx}} + \frac{(c + dx)^{\frac{5}{2}} \cdot (10ab^{4}d - 10b^{5}c)}{5d^{6}} + \frac{(c + dx)^{\frac{3}{2}} \cdot (20a^{2}b^{3}d^{2} - 40ab^{4}cd + 20b^{5}c^{2})}{3d^{6}} + \frac{\sqrt{c + dx}\,(20a^{3}b^{2}d^{3} - 60a^{2}b^{3}cd^{2} + 60ab^{4}c^{2}d - 20b^{5}c^{3})}{d^{6}} - \frac{2(ad - bc)^{5}}{3d^{6}\,(c + dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(7/2)/(7*d**6) - 10*b*(a*d - b*c)**4/(d**6*sqrt(c + d*x))
 + (c + d*x)**(5/2)*(10*a*b**4*d - 10*b**5*c)/(5*d**6) + (c + d*x)**(3/2)*(
20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5*c**2)/(3*d**6) + sqrt(c + d*x)*
(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2 + 60*a*b**4*c**2*d - 20*b**5*c**3)
/d**6 - 2*(a*d - b*c)**5/(3*d**6*(c + d*x)**(3/2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(134) = 268$.
time = 0.88, size = 335, normalized size = 2.20

$$-\frac{2\,{\left(15\,{\left(dx + c\right)}b^{5}c^{4} - b^{5}c^{5} - 60\,{\left(dx + c\right)}ab^{4}c^{3}d + 5\,ab^{4}c^{4}d + 90\,{\left(dx + c\right)}a^{2}b^{3}c^{2}d^{2} - 10\,a^{2}b^{3}c^{3}d^{2} - 60\,{\left(dx + c\right)}a^{3}b^{2}cd^{3} + 10\,a^{3}b^{2}c^{2}d^{3} + 15\,{\left(dx + c\right)}a^{4}bd^{4} - 5\,a^{4}bcd^{4} + a^{5}d^{5}\right)}}{3\,{\left(dx + c\right)}^{\frac{3}{2}}d^{6}} + \frac{2\,{\left(3\,{\left(dx + c\right)}^{\frac{7}{2}}b^{5}d^{36} - 21\,{\left(dx + c\right)}^{\frac{5}{2}}b^{5}cd^{36} + 70\,{\left(dx + c\right)}^{\frac{3}{2}}b^{5}c^{2}d^{36} - 210\,\sqrt{dx + c}\,b^{5}c^{3}d^{36} + 21\,{\left(dx + c\right)}^{\frac{5}{2}}ab^{4}d^{37} - 140\,{\left(dx + c\right)}^{\frac{3}{2}}ab^{4}cd^{37} + 630\,\sqrt{dx + c}\,ab^{4}c^{2}d^{37} + 70\,{\left(dx + c\right)}^{\frac{3}{2}}a^{2}b^{3}d^{38} - 630\,\sqrt{dx + c}\,a^{2}b^{3}cd^{38} + 210\,\sqrt{dx + c}\,a^{3}b^{2}d^{39}\right)}}{21\,d^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="giac")
```

[Out] -2/3*(15*(d*x + c)*b^5*c^4 - b^5*c^5 - 60*(d*x + c)*a*b^4*c^3*d + 5*a*b^4*c^4*d + 90*(d*x + c)*a^2*b^3*c^2*d^2 - 10*a^2*b^3*c^3*d^2 - 60*(d*x + c)*a^3*b^2*c*d^3 + 10*a^3*b^2*c^2*d^3 + 15*(d*x + c)*a^4*b*d^4 - 5*a^4*b*c*d^4 + a^5*d^5)/((d*x + c)^(3/2)*d^6) + 2/21*(3*(d*x + c)^(7/2)*b^5*d^36 - 21*(d*x + c)^(5/2)*b^5*c*d^36 + 70*(d*x + c)^(3/2)*b^5*c^2*d^36 - 210*sqrt(d*x + c)*b^5*c^3*d^36 + 21*(d*x + c)^(5/2)*a*b^4*d^37 - 140*(d*x + c)^(3/2)*a*b^4*c*d^37 + 630*sqrt(d*x + c)*a*b^4*c^2*d^37 + 70*(d*x + c)^(3/2)*a^2*b^3*d^38 - 630*sqrt(d*x + c)*a^2*b^3*c*d^38 + 210*sqrt(d*x + c)*a^3*b^2*d^39)/d^42

Mupad [B]
time = 0.08, size = 229, normalized size = 1.51

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^6}-\frac{\frac{2\,a^5\,d^5}{3}-\frac{2\,b^5\,c^5}{3}+\left(c+d\,x\right)\,\left(10\,a^4\,b\,d^4-40\,a^3\,b^2\,c\,d^3+60\,a^2\,b^3\,c^2\,d^2-40\,a\,b^4\,c^3\,d+10\,b^5\,c^4\right)-\frac{20\,a^2\,b^3\,c^3\,d^2}{3}+\frac{20\,a^3\,b^2\,c^2\,d^3}{3}+\frac{10\,a\,b^4\,c^4\,d}{3}-\frac{10\,a^4\,b\,c\,d^4}{3}}{d^6\,{\left(c+d\,x\right)}^{3/2}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^5/(c + d*x)^(5/2),x)

[Out] (2*b^5*(c + d*x)^(7/2))/(7*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(5/2))/(5*d^6) - ((2*a^5*d^5)/3 - (2*b^5*c^5)/3 + (c + d*x)*(10*b^5*c^4 + 10*a^4*b*d^4 - 40*a^3*b^2*c*d^3 + 60*a^2*b^3*c^2*d^2 - 40*a*b^4*c^3*d) - (20*a^2*b^3*c^3*d^2)/3 + (20*a^3*b^2*c^2*d^3)/3 + (10*a*b^4*c^4*d)/3 - (10*a^4*b*c*d^4)/3)/(d^6*(c + d*x)^(3/2)) + (20*b^2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^6 + (20*b^3*(a*d - b*c)^2*(c + d*x)^(3/2))/(3*d^6)

3.1435 $\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

```
[Out] -2/3*(-a*d+b*c)^4/d^5/(d*x+c)^(3/2)-8/3*b^3*(-a*d+b*c)*(d*x+c)^(3/2)/d^5+2/
5*b^4*(d*x+c)^(5/2)/d^5+8*b*(-a*d+b*c)^3/d^5/(d*x+c)^(1/2)+12*b^2*(-a*d+b*c
)^2*(d*x+c)^(1/2)/d^5
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}\,(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(3*d^5*(c + d*x)^(3/2)) + (8*b*(b*c - a*d)^3)/(d^5*Sqrt[
c + d*x]) + (12*b^2*(b*c - a*d)^2*Sqrt[c + d*x])/d^5 - (8*b^3*(b*c - a*d)*(
c + d*x)^(3/2))/(3*d^5) + (2*b^4*(c + d*x)^(5/2))/(5*d^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{5/2}}-\frac{4b(bc-ad)^3}{d^4(c+dx)^{3/2}}+\frac{6b^2(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{4b^3(bc-ad)\sqrt{c+dx}}{d^4}+\frac{b^4(c+\right.\\ &= -\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 153, normalized size = 1.22

$$\frac{2(-5a^4d^4-20a^3bd^3(2c+3dx)+30a^2b^2d^2(8c^2+12cdx+3d^2x^2)+20ab^3d(-16c^3-24c^2dx-6cd^2x^2+d^3x^3)+b^4(128c^4+192c^3dx+48c^2d^2x^2-8cd^3x^3+3d^4x^4))}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*a^4*d^4 - 20*a^3*b*d^3*(2*c + 3*d*x) + 30*a^2*b^2*d^2*(8*c^2 + 12*c*
d*x + 3*d^2*x^2) + 20*a*b^3*d*(-16*c^3 - 24*c^2*d*x - 6*c*d^2*x^2 + d^3*x^3
) + b^4*(128*c^4 + 192*c^3*d*x + 48*c^2*d^2*x^2 - 8*c*d^3*x^3 + 3*d^4*x^4))
)/(15*d^5*(c + d*x)^(3/2))
```

Maple [A]
time = 0.16, size = 198, normalized size = 1.58

method	result
risch	$\frac{2b^2(3b^2x^2d^2+20ab\,d^2x-14b^2cdx+90a^2d^2-160abcd+73b^2c^2)\sqrt{dx+c}}{15d^5}-\frac{2(12bdx+ad+11bc)(a^3d^3-3a^2bc\,d^2+3a\,b^2}{3d^5(dx+c)^{\frac{3}{2}}}$
gosper	$-\frac{2(-3d^4x^4b^4-20a\,b^3d^4x^3+8b^4c\,d^3x^3-90a^2b^2d^4x^2+120a\,b^3c\,d^3x^2-48b^4c^2d^2x^2+60a^3b\,d^4x-360a^2b^2c\,d^3x+480a\,b^3c^2d}{15(dx+c)^{\frac{3}{2}}d^5}$
trager	$-\frac{2(-3d^4x^4b^4-20a\,b^3d^4x^3+8b^4c\,d^3x^3-90a^2b^2d^4x^2+120a\,b^3c\,d^3x^2-48b^4c^2d^2x^2+60a^3b\,d^4x-360a^2b^2c\,d^3x+480a\,b^3c^2d}{15(dx+c)^{\frac{3}{2}}d^5}$
derivativedivides	$\frac{\frac{2b^4(dx+c)^{\frac{5}{2}}}{5}+\frac{8a\,b^3d(dx+c)^{\frac{3}{2}}}{3}-\frac{8b^4c(dx+c)^{\frac{3}{2}}}{3}+12a^2b^2d^2\sqrt{dx+c}-24a\,b^3cd\sqrt{dx+c}+12b^4c^2\sqrt{dx+c}-\frac{2(a^4d^4-}{}}{d^5}$
default	$\frac{\frac{2b^4(dx+c)^{\frac{5}{2}}}{5}+\frac{8a\,b^3d(dx+c)^{\frac{3}{2}}}{3}-\frac{8b^4c(dx+c)^{\frac{3}{2}}}{3}+12a^2b^2d^2\sqrt{dx+c}-24a\,b^3cd\sqrt{dx+c}+12b^4c^2\sqrt{dx+c}-\frac{2(a^4d^4-}{}}{d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^5*(1/5*b^4*(d*x+c)^(5/2)+4/3*a*b^3*d*(d*x+c)^(3/2)-4/3*b^4*c*(d*x+c)^(3
/2)+6*a^2*b^2*d^2*(d*x+c)^(1/2)-12*a*b^3*c*d*(d*x+c)^(1/2)+6*b^4*c^2*(d*x+c
)^(1/2)-1/3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
/(d*x+c)^(3/2)-4*b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^(1
/2))
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.50

$$\frac{2\left(\frac{3(dx+c)^{\frac{5}{2}}b^4-20(b^4c-ab^3d)(dx+c)^{\frac{3}{2}}+90(b^4c^2-2ab^3cd+a^2b^2d^2)\sqrt{dx+c}}{d^4}-\frac{5(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4-12(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)(dx+c))}{(dx+c)^{\frac{3}{2}}d^4}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(d*x + c)^(5/2)*b^4 - 20*(b^4*c - a*b^3*d)*(d*x + c)^(3/2) + 90*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(d*x + c))/d^4 - 5*(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^3 - 3*a*
```

```
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c))/((d*x + c)^(3/2)*d^4))/
d
```

Fricas [A]
time = 0.48, size = 203, normalized size = 1.62

$$\frac{2\,(3\,b^4d^4x^4+128\,b^4c^4-320\,ab^3c^3d+240\,a^2b^2c^2d^2-40\,a^3bcd^3-5\,a^4d^4-4\,(2\,b^4cd^3-5\,ab^3d^4)x^3+6\,(8\,b^4c^2d^2-20\,ab^3cd^3+15\,a^2b^2d^4)x^2+12\,(16\,b^4c^3d-40\,ab^3c^2d^2+30\,a^2b^2cd^3-5\,a^3bd^4)x)\sqrt{dx+c}}{15\,(d^7x^2+2\,cd^6x+c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*d^4*x^4 + 128*b^4*c^4 - 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 -
 40*a^3*b*c*d^3 - 5*a^4*d^4 - 4*(2*b^4*c*d^3 - 5*a*b^3*d^4)*x^3 + 6*(8*b^4*
c^2*d^2 - 20*a*b^3*c*d^3 + 15*a^2*b^2*d^4)*x^2 + 12*(16*b^4*c^3*d - 40*a*b^
3*c^2*d^2 + 30*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^7*x^2 + 2*c
*d^6*x + c^2*d^5)
```

Sympy [A]
time = 18.51, size = 136, normalized size = 1.09

$$\frac{2b^4(c+dx)^{\frac{5}{2}}}{5d^5}-\frac{8b(ad-bc)^3}{d^5\sqrt{c+dx}}+\frac{(c+dx)^{\frac{3}{2}}\cdot(8ab^3d-8b^4c)}{3d^5}+\frac{\sqrt{c+dx}\,(12a^2b^2d^2-24ab^3cd+12b^4c^2)}{d^5}-\frac{2(ad-bc)^4}{3d^5\,(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(5/2)/(5*d**5) - 8*b*(a*d - b*c)**3/(d**5*sqrt(c + d*x))
+ (c + d*x)**(3/2)*(8*a*b**3*d - 8*b**4*c)/(3*d**5) + sqrt(c + d*x)*(12*a**
2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c**2)/d**5 - 2*(a*d - b*c)**4/(3*d**5
*(c + d*x)**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(109) = 218.
time = 0.98, size = 229, normalized size = 1.83

$$\frac{2\,(12\,(dx+c)b^4c^3-b^4c^4-36\,(dx+c)ab^3c^2d+4\,ab^3c^3d+36\,(dx+c)a^2b^2cd^2-6\,a^2b^2c^2d^2-12\,(dx+c)a^3bd^3+4\,a^3bcd^3-a^4d^4)}{3\,(dx+c)^{\frac{3}{2}}d^5}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b^4d^{20}-20\,(dx+c)^{\frac{3}{2}}b^4cd^{20}+90\,\sqrt{dx+c}\,b^4c^2d^{20}+20\,(dx+c)^{\frac{3}{2}}ab^3d^{21}-180\,\sqrt{dx+c}\,ab^3cd^{21}+90\,\sqrt{dx+c}\,a^2b^2d^{22}\right)}{15\,d^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(d*x + c)*b^4*c^3 - b^4*c^4 - 36*(d*x + c)*a*b^3*c^2*d + 4*a*b^3*c^
3*d + 36*(d*x + c)*a^2*b^2*c*d^2 - 6*a^2*b^2*c^2*d^2 - 12*(d*x + c)*a^3*b*d
^3 + 4*a^3*b*c*d^3 - a^4*d^4)/((d*x + c)^(3/2)*d^5) + 2/15*(3*(d*x + c)^(5/
2)*b^4*d^20 - 20*(d*x + c)^(3/2)*b^4*c*d^20 + 90*sqrt(d*x + c)*b^4*c^2*d^20
 + 20*(d*x + c)^(3/2)*a*b^3*d^21 - 180*sqrt(d*x + c)*a*b^3*c*d^21 + 90*sqrt
(d*x + c)*a^2*b^2*d^22)/d^25
```

Mupad [B]
time = 0.30, size = 175, normalized size = 1.40

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{\left(c+d\,x\right)\,\left(-8\,a^3\,b\,d^3+24\,a^2\,b^2\,c\,d^2-24\,a\,b^3\,c^2\,d+8\,b^4\,c^3\right)-\frac{2\,a^4\,d^4}{3}-\frac{2\,b^4\,c^4}{3}-4\,a^2\,b^2\,c^2\,d^2+\frac{8\,a\,b^3\,c^3\,d}{3}+\frac{8\,a^3\,b\,c\,d^3}{3}}{d^5\,{\left(c+d\,x\right)}^{3/2}}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(5/2))/(5*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(3/2))/(
3*d^5) + ((c + d*x)*(8*b^4*c^3 - 8*a^3*b*d^3 + 24*a^2*b^2*c*d^2 - 24*a*b^3*
c^2*d) - (2*a^4*d^4)/3 - (2*b^4*c^4)/3 - 4*a^2*b^2*c^2*d^2 + (8*a*b^3*c^3*d
)/3 + (8*a^3*b*c*d^3)/3)/(d^5*(c + d*x)^(3/2)) + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(1/2))/d^5
```

3.1436 $\int \frac{(a+bx)^3}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=96

$$\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

```
[Out] 2/3*(-a*d+b*c)^3/d^4/(d*x+c)^(3/2)+2/3*b^3*(d*x+c)^(3/2)/d^4-6*b*(-a*d+b*c)
^2/d^4/(d*x+c)^(1/2)-6*b^2*(-a*d+b*c)*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {45}

$$-\frac{6b^2\sqrt{c+dx}\,(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(3*d^4*(c + d*x)^(3/2)) - (6*b*(b*c - a*d)^2)/(d^4*Sqrt[c
 + d*x]) - (6*b^2*(b*c - a*d)*Sqrt[c + d*x])/d^4 + (2*b^3*(c + d*x)^(3/2))/
(3*d^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{5/2}}+\frac{3b(bc-ad)^2}{d^3(c+dx)^{3/2}}-\frac{3b^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{b^3\sqrt{c+dx}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 101, normalized size = 1.05

$$-\frac{2(a^3d^3+3a^2bd^2(2c+3dx)-3ab^2d(8c^2+12cdx+3d^2x^2)+b^3(16c^3+24c^2dx+6cd^2x^2-d^3x^3))}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^3/(c + d*x)^(5/2),x]

[Out] (-2*(a^3*d^3 + 3*a^2*b*d^2*(2*c + 3*d*x) - 3*a*b^2*d*(8*c^2 + 12*c*d*x + 3*d^2*x^2) + b^3*(16*c^3 + 24*c^2*d*x + 6*c*d^2*x^2 - d^3*x^3)))/(3*d^4*(c + d*x)^(3/2))

Maple [A]
time = 0.17, size = 122, normalized size = 1.27

method	result	size
risch	$\frac{2b^2(bdx+9ad-8bc)\sqrt{dx+c}}{3d^4}-\frac{2(9bdx+ad+8bc)(a^2d^2-2abcd+b^2c^2)}{3d^4(dx+c)^{\frac{3}{2}}}$	76
gosper	$-\frac{2(-b^3x^3d^3-9d^3a\,x^2b^2+6b^3c\,d^2x^2+9a^2b\,d^3x-36a\,b^2c\,d^2x+24b^3c^2dx+a^3d^3+6a^2bc\,d^2-24a\,b^2c^2d+16b^3c^3)}{3(dx+c)^{\frac{3}{2}}d^4}$	115
trager	$-\frac{2(-b^3x^3d^3-9d^3a\,x^2b^2+6b^3c\,d^2x^2+9a^2b\,d^3x-36a\,b^2c\,d^2x+24b^3c^2dx+a^3d^3+6a^2bc\,d^2-24a\,b^2c^2d+16b^3c^3)}{3(dx+c)^{\frac{3}{2}}d^4}$	115
derivativedivides	$\frac{\frac{2b^3(dx+c)^{\frac{3}{2}}}{3}+6ad\,b^2\sqrt{dx+c}-6b^3c\sqrt{dx+c}-\frac{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{3(dx+c)^{\frac{3}{2}}}-\frac{6b(a^2d^2-2abcd+b^2c^2)}{\sqrt{dx+c}}}{d^4}$	122
default	$\frac{\frac{2b^3(dx+c)^{\frac{3}{2}}}{3}+6ad\,b^2\sqrt{dx+c}-6b^3c\sqrt{dx+c}-\frac{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}{3(dx+c)^{\frac{3}{2}}}-\frac{6b(a^2d^2-2abcd+b^2c^2)}{\sqrt{dx+c}}}{d^4}$	122

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/d^4*(1/3*b^3*(d*x+c)^(3/2)+3*a*d*b^2*(d*x+c)^(1/2)-3*b^3*c*(d*x+c)^(1/2)-1/3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)^(3/2)-3*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^(1/2))

Maxima [A]
time = 0.28, size = 122, normalized size = 1.27

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^3-9\,(b^3c-ab^2d)\sqrt{dx+c}}{d^3}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3-9\,(b^3c^2-2\,ab^2cd+a^2bd^2)(dx+c)}{(dx+c)^{\frac{3}{2}}d^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 2/3*(((d*x + c)^(3/2)*b^3 - 9*(b^3*c - a*b^2*d)*sqrt(d*x + c))/d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c))/((d*x + c)^(3/2)*d^3))/d

Fricas [A]
time = 0.42, size = 136, normalized size = 1.42

$$\frac{2\,(b^3d^3x^3 - 16\,b^3c^3 + 24\,ab^2c^2d - 6\,a^2bcd^2 - a^3d^3 - 3\,(2\,b^3cd^2 - 3\,ab^2d^3)x^2 - 3\,(8\,b^3c^2d - 12\,ab^2cd^2 + 3\,a^2bd^3)x)\sqrt{dx + c}}{3\,(d^6x^2 + 2\,cd^5x + c^2d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*d^3*x^3 - 16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a^3*d^3 -
3*(2*b^3*c*d^2 - 3*a*b^2*d^3)*x^2 - 3*(8*b^3*c^2*d - 12*a*b^2*c*d^2 + 3*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(88) = 176$.
time = 0.45, size = 461, normalized size = 4.80

$$\begin{cases} -\frac{2a^3d^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{12a^2bcd^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{18a^2bd^3x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{48ab^2c^2d}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{72ab^2cd^2x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{18ab^2d^3x^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{32b^3c^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{48b^3c^2dx}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{12b^3cd^2x^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{2b^3d^3x^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} & \text{for}\: d \neq 0 \\ \frac{a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**3*d**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) -
 12*a**2*b*c*d**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 18*a*
*2*b*d**3*x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 48*a*b**2*c
**2*d/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 72*a*b**2*c*d**2*
x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 18*a*b**2*d**3*x**2/(
3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 32*b**3*c**3/(3*c*d**4*s
qrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 48*b**3*c**2*d*x/(3*c*d**4*sqrt(c
+ d*x) + 3*d**5*x*sqrt(c + d*x)) - 12*b**3*c*d**2*x**2/(3*c*d**4*sqrt(c + d
*x) + 3*d**5*x*sqrt(c + d*x)) + 2*b**3*d**3*x**3/(3*c*d**4*sqrt(c + d*x) +
3*d**5*x*sqrt(c + d*x)), Ne(d, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**
3 + b**3*x**4/4)/c**(5/2), True))
```

Giac [A]
time = 1.58, size = 141, normalized size = 1.47

$$-\frac{2\,(9\,(dx + c)b^3c^2 - b^3c^3 - 18\,(dx + c)ab^2cd + 3\,ab^2c^2d + 9\,(dx + c)a^2bd^2 - 3\,a^2bcd^2 + a^3d^3)}{3\,(dx + c)^{\frac{3}{2}}d^4} + \frac{2\,\left((dx + c)^{\frac{3}{2}}b^3d^8 - 9\,\sqrt{dx + c}\,b^3cd^8 + 9\,\sqrt{dx + c}\,ab^2d^9\right)}{3\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(d*x + c)*b^3*c^2 - b^3*c^3 - 18*(d*x + c)*a*b^2*c*d + 3*a*b^2*c^2*
d + 9*(d*x + c)*a^2*b*d^2 - 3*a^2*b*c*d^2 + a^3*d^3)/((d*x + c)^(3/2)*d^4)
+ 2/3*((d*x + c)^(3/2)*b^3*d^8 - 9*sqrt(d*x + c)*b^3*c*d^8 + 9*sqrt(d*x + c
)*a*b^2*d^9)/d^12
```

Mupad [B]
time = 0.09, size = 128, normalized size = 1.33

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^3-2\,a^3\,d^3+2\,b^3\,c^3-18\,b^3\,c\,{\left(c+d\,x\right)}^2-18\,b^3\,c^2\,\left(c+d\,x\right)+18\,a\,b^2\,d\,{\left(c+d\,x\right)}^2-18\,a^2\,b\,d^2\,\left(c+d\,x\right)-6\,a\,b^2\,c^2\,d+6\,a^2\,b\,c\,d^2+36\,a\,b^2\,c\,d\,\left(c+d\,x\right)}{3\,d^4\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^3 - 2*a^3*d^3 + 2*b^3*c^3 - 18*b^3*c*(c + d*x)^2 - 18*b^3*
c^2*(c + d*x) + 18*a*b^2*d*(c + d*x)^2 - 18*a^2*b*d^2*(c + d*x) - 6*a*b^2*c
^2*d + 6*a^2*b*c*d^2 + 36*a*b^2*c*d*(c + d*x))/(3*d^4*(c + d*x)^(3/2))
```

3.1437 $\int \frac{(a+bx)^2}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=67

$$-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

```
[Out] -2/3*(-a*d+b*c)^2/d^3/(d*x+c)^(3/2)+4*b*(-a*d+b*c)/d^3/(d*x+c)^(1/2)+2*b^2*
(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{45\}$

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}-\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{2b^2\sqrt{c+dx}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(3*d^3*(c + d*x)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x]) + (2*b^2*Sqrt[c + d*x])/d^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}}-\frac{2b(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{b^2}{d^2\sqrt{c+dx}}\right)dx\\ &= -\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}}+\frac{4b(bc-ad)}{d^3\sqrt{c+dx}}+\frac{2b^2\sqrt{c+dx}}{d^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.93

$$\frac{-2a^2d^2-4abd(2c+3dx)+2b^2(8c^2+12cdx+3d^2x^2)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*a^2*d^2 - 4*a*b*d*(2*c + 3*d*x) + 2*b^2*(8*c^2 + 12*c*d*x + 3*d^2*x^2))
/(3*d^3*(c + d*x)^(3/2))
```

Maple [A]
time = 0.16, size = 66, normalized size = 0.99

method	result	size
risch	$\frac{2b^2\sqrt{dx+c}}{d^3}-\frac{2(6bdx+ad+5bc)(ad-bc)}{3d^3(dx+c)^{\frac{3}{2}}}$	50
gosper	$-\frac{2\left(-3b^2x^2d^2+6ab\,d^2x-12b^2cdx+a^2d^2+4abcd-8b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}d^3}$	62
trager	$-\frac{2\left(-3b^2x^2d^2+6ab\,d^2x-12b^2cdx+a^2d^2+4abcd-8b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}d^3}$	62
derivativedivides	$\frac{2b^2\sqrt{dx+c}-\frac{4b(ad-bc)}{\sqrt{dx+c}}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}}}{d^3}$	66
default	$\frac{2b^2\sqrt{dx+c}-\frac{4b(ad-bc)}{\sqrt{dx+c}}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)}{3(dx+c)^{\frac{3}{2}}}}{d^3}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(b^2*(d*x+c)^(1/2)-2*b*(a*d-b*c)/(d*x+c)^(1/2)-1/3*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/(d*x+c)^(3/2))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 1.07

$$\frac{2\left(\frac{3\sqrt{dx+c}\,b^2}{d^2}-\frac{b^2c^2-2\,abcd+a^2d^2-6\,(b^2c-abd)(dx+c)}{(dx+c)^{\frac{3}{2}}d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*b^2/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 6*(b^2*c -
a*b*d)*(d*x + c))/((d*x + c)^(3/2)*d^2))/d
```

Fricas [A]
time = 0.44, size = 85, normalized size = 1.27

$$\frac{2\,(3\,b^2d^2x^2+8\,b^2c^2-4\,abcd-a^2d^2+6\,(2\,b^2cd-abd^2)x)\sqrt{dx+c}}{3\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x)*sqrt(d*x + c)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.
time = 0.40, size = 265, normalized size = 3.96

$$\begin{cases} -\frac{2a^2d^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} - \frac{8abcd}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} - \frac{12abd^2x}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} + \frac{16b^2c^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} + \frac{24b^2cdx}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} + \frac{6b^2d^2x^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} & \text{for } d \neq 0 \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*d**2/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) -
 8*a*b*c*d/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) - 12*a*b*d**2*
x/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 16*b**2*c**2/(3*c*d**
3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 24*b**2*c*d*x/(3*c*d**3*sqrt(c
+ d*x) + 3*d**4*x*sqrt(c + d*x)) + 6*b**2*d**2*x**2/(3*c*d**3*sqrt(c + d*x)
 + 3*d**4*x*sqrt(c + d*x)), Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/c
**(5/2), True))
```

Giac [A]
time = 2.05, size = 72, normalized size = 1.07

$$\frac{2\sqrt{dx+c}\,b^2}{d^3} + \frac{2\left(6\,(dx+c)b^2c - b^2c^2 - 6\,(dx+c)abd + 2\,abcd - a^2d^2\right)}{3\,(dx+c)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b^2/d^3 + 2/3*(6*(d*x + c)*b^2*c - b^2*c^2 - 6*(d*x + c)*a*
b*d + 2*a*b*c*d - a^2*d^2)/((d*x + c)^(3/2)*d^3)
```

Mupad [B]
time = 0.07, size = 68, normalized size = 1.01

$$\frac{6\,b^2\,{\left(c+d\,x\right)}^2-2\,a^2\,d^2-2\,b^2\,c^2+12\,b^2\,c\,\left(c+d\,x\right)-12\,a\,b\,d\,\left(c+d\,x\right)+4\,a\,b\,c\,d}{3\,d^3\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(5/2),x)
```

```
[Out] (6*b^2*(c + d*x)^2 - 2*a^2*d^2 - 2*b^2*c^2 + 12*b^2*c*(c + d*x) - 12*a*b*d*
(c + d*x) + 4*a*b*c*d)/(3*d^3*(c + d*x)^(3/2))
```

3.1438 $\int \frac{a+bx}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

```
[Out] 2/3*(-a*d+b*c)/d^2/(d*x+c)^(3/2)-2*b/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {45}

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d))/(3*d^2*(c + d*x)^(3/2)) - (2*b)/(d^2*Sqrt[c + d*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{5/2}}+\frac{b}{d(c+dx)^{3/2}}\right)dx\\ &= \frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.72

$$-\frac{2(2bc+ad+3bdx)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(2*b*c + a*d + 3*b*d*x))/(3*d^2*(c + d*x)^(3/2))
```

Maple [A]
time = 0.14, size = 34, normalized size = 0.85

method	result	size
gosper	$-\frac{2(3bdx+ad+2bc)}{3(dx+c)^{\frac{3}{2}}d^2}$	26
trager	$-\frac{2(3bdx+ad+2bc)}{3(dx+c)^{\frac{3}{2}}d^2}$	26
derivativedivides	$\frac{-\frac{2(ad-bc)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b}{\sqrt{dx+c}}}{d^2}$	34
default	$\frac{-\frac{2(ad-bc)}{3(dx+c)^{\frac{3}{2}}}-\frac{2b}{\sqrt{dx+c}}}{d^2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/d^2*(-1/3*(a*d-b*c)/(d*x+c)^(3/2)-b/(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.70

$$-\frac{2\left(3\left(dx+c\right)b-bc+ad\right)}{3\left(dx+c\right)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

Fricas [A]
time = 0.45, size = 46, normalized size = 1.15

$$-\frac{2\left(3\,bdx+2\,bc+ad\right)\sqrt{dx+c}}{3\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*d*x + 2*b*c + a*d)*sqrt(d*x + c)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(39) = 78.
time = 0.39, size = 124, normalized size = 3.10

$$\begin{cases} -\frac{2ad}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}}-\frac{4bc}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}}-\frac{6bdx}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a*d/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 4*b*c
/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 6*b*d*x/(3*c*d**2*sqrt
(c + d*x) + 3*d**3*x*sqrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(5/2),
 True))
```

Giac [A]
time = 2.40, size = 28, normalized size = 0.70

$$-\frac{2\left(3\left(dx+c\right)b-bc+ad\right)}{3\left(dx+c\right)^{\frac{3}{2}}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

Mupad [B]
time = 0.25, size = 29, normalized size = 0.72

$$-\frac{2\,a\,d-2\,b\,c+6\,b\,\left(c+d\,x\right)}{3\,d^2\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(5/2),x)
```

```
[Out] -(2*a*d - 2*b*c + 6*b*(c + d*x))/(3*d^2*(c + d*x)^(3/2))
```

3.1439 $\int \frac{1}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{3d(c+dx)^{3/2}}$$

```
[Out] -2/3/d/(d*x+c)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{5/2}}\,dx = -\frac{2}{3d(c+dx)^{3/2}}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.81

method	result	size
gosper	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13
derivativedivides	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13
default	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13
trager	$-\frac{2}{3d(dx+c)^{\frac{3}{2}}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d/(d*x+c)^(3/2)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.42, size = 31, normalized size = 1.94

$$-\frac{2\,\sqrt{dx+c}}{3\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d*x + c)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.88

$$-\frac{2}{3d\,(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(5/2),x)
```

```
[Out] -2/(3*d*(c + d*x)**(3/2))
```

Giac [A]
time = 1.67, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((d*x + c)^(3/2)*d)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.75

$$-\frac{2}{3\,d\,(c+d\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(5/2),x)
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

3.1440 $\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] 2/3/(-a*d+b*c)/(d*x+c)^(3/2)-2*b^(3/2)*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+
b*c)^(1/2))/(-a*d+b*c)^(5/2)+2*b/(-a*d+b*c)^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {53, 65, 214}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}\,(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b)/((b*c - a*d)^2*Sqrt[c + d*x]) - (
2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx &= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{b\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{bc-ad}\\
&= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{(bc-ad)^2}\\
&= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\right.}{d(bc-ad)^2}\\
&= \frac{2}{3(bc-ad)(c+dx)^{3/2}}+\frac{2b}{(bc-ad)^2\sqrt{c+dx}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 85, normalized size = 0.91

$$\frac{2(4bc-ad+3bdx)}{3(bc-ad)^2(c+dx)^{3/2}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)),x]
[Out] (2*(4*b*c - a*d + 3*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) + (2*b^(3/2)*
ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2)
```

Maple [A]
time = 0.17, size = 90, normalized size = 0.97

method	result	size
derivativedivides	$-\frac{2}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^2\sqrt{dx+c}}+\frac{2b^2\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)^2\sqrt{(ad-bc)\,b}}$	90

default	$-\frac{2}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^{2}\sqrt{dx+c}}+\frac{2b^{2}\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)\,b}}\right)}{(ad-bc)^{2}\sqrt{(ad-bc)\,b}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)+2/(a*d-b*c)^2*b/(d*x+c)^(1/2)+2*b^2/(a*d-b*c)^
2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(77) = 154$.
time = 0.43, size = 398, normalized size = 4.28

$$\left[\frac{3\,(bd^{2}x^{2}+2\,bcdx+bc^{2})\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\,(3\,bdx+4\,bc-ad)\sqrt{dx+c}}{3\,(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4})x^{2}+2\,(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3})x)},\,-\frac{2\left(3\,(bd^{2}x^{2}+2\,bcdx+bc^{2})\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+4\,bc-ad)\sqrt{dx+c}\right)}{3\,(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}+(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4})x^{2}+2\,(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*
b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2
*(3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^
2 + a^2*c*d^3)*x), -2/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/(b*c - a
*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c))
- (3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d
^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x)]
```

Sympy [A]
time = 5.97, size = 83, normalized size = 0.89

$$\frac{2b}{\sqrt{c+dx}\,(ad-bc)^2}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{2}{3\,(c+dx)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] 2*b/(sqrt(c + d*x)*(a*d - b*c)**2) + 2*b*atan(sqrt(c + d*x)/sqrt((a*d - b*c
)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 2/(3*(c + d*x)**(3/2)*(a*d - b
*c))
```

Giac [A]
time = 2.67, size = 113, normalized size = 1.22

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{2\,(3\,(dx+c)b+bc-ad)}{3\,(b^2c^2-2\,abcd+a^2d^2)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 2/3*(3*(d*x + c)*b + b*c - a*d)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))
```

Mupad [B]
time = 0.33, size = 100, normalized size = 1.08

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{2}{3\,\left(a\,d-b\,c\right)}-\frac{2\,b\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(5/2)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(
a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (2/(3*(a*d - b*c)) - (2*b*(c + d*x))
/(a*d - b*c)^2)/(c + d*x)^(3/2)
```

3.1441 $\int \frac{1}{(a+bx)^2(c+dx)^{5/2}} dx$

Optimal. Leaf size=124

$$-\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}}-\frac{5bd}{(bc-ad)^3\sqrt{c+dx}}+\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}$$

```
[Out] -5/3*d/(-a*d+b*c)^2/(d*x+c)^(3/2)-1/(-a*d+b*c)/(b*x+a)/(d*x+c)^(3/2)+5*b^(3
/2)*d*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(7/2)-5*b*
d/(-a*d+b*c)^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}\,(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-5*d)/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - 1/((b*c - a*d)*(a + b*x)*(c + d*
x)^(3/2)) - (5*b*d)/((b*c - a*d)^3*Sqrt[c + d*x]) + (5*b^(3/2)*d*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx &= -\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx}{2(bc-ad)} \\
&= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5bd)\int \frac{1}{(a+bx)(c+dx}}{2(bc-ad)^2} \\
&= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} \\
&= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} \\
&= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 125, normalized size = 1.01

$$\frac{2a^2d^2-2abd(7c+5dx)-b^2(3c^2+20cdx+15d^2x^2)}{3(bc-ad)^3(a+bx)(c+dx)^{3/2}}+\frac{5b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 2*a*b*d*(7*c + 5*d*x) - b^2*(3*c^2 + 20*c*d*x + 15*d^2*x^2))/(
3*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) + (5*b^(3/2)*d*ArcTan[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2)
```

Maple [A]
time = 0.17, size = 121, normalized size = 0.98

method	result
derivativedivides	$2d\left(-\frac{1}{3(ad-bc)^2(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^3\sqrt{dx+c}}+\frac{b^2\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\right)$
default	$2d\left(-\frac{1}{3(ad-bc)^2(dx+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^3\sqrt{dx+c}}+\frac{b^2\left(\frac{\sqrt{dx+c}}{2(dx+c)b+2ad-2bc}+\frac{5\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{2\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-1/3/(a*d-b*c)^2/(d*x+c)^(3/2)+2/(a*d-b*c)^3*b/(d*x+c)^(1/2)+1/(a*d-b*
c)^3*b^2*(1/2*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+5/2/((a*d-b*c)*b)^(1/2)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(106) = 212.
time = 0.41, size = 782, normalized size = 6.31

$$\left[-\frac{15\,(b^2d^3x^3+abc^2d+(2\,b^2cd^2+abd^3)x^2+(b^2c^2d+2\,abcd^2)x)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\,(15\,b^2d^2x^2+3\,b^2c^2+14\,abcd-2\,a^2d^2+10\,(2\,b^2cd+abd^2)x)\sqrt{dx+c}}{6\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^3+(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^2+(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4)x)}, \frac{15\,(b^2d^3x^3+abc^2d+(2\,b^2cd^2+abd^3)x^2+(b^2c^2d+2\,abcd^2)x)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(15\,b^2d^2x^2+3\,b^2c^2+14\,abcd-2\,a^2d^2+10\,(2\,b^2cd+abd^2)x)\sqrt{dx+c}}{3\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^3+(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^2+(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2
*d + 2*a*b*c*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c
- a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(15*b^2*d^2*x^2 +
3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x +
 c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d -
 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^
5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x), 1/
3*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2*d +
 2*a*b*c*d^2)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqr
t(-b/(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 + 3*b^2*c^2 + 14*a*b*c*d
 - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x + c))/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(5/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(106) = 212.
time = 1.34, size = 216, normalized size = 1.74

$$-\frac{5\,b^2d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}-\frac{\sqrt{dx+c}\,b^2d}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx+c)b-bc+ad)}-\frac{2\,(6\,(dx+c)bd+bcd-ad^2)}{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b^2*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x + c)*b^2*d
/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c +
a*d)) - 2/3*(6*(d*x + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))
```

Mupad [B]
time = 0.38, size = 161, normalized size = 1.30

$$\frac{\frac{10\,b\,d\,(c+d\,x)}{3\,(a\,d-b\,c)^2}-\frac{2\,d}{3\,(a\,d-b\,c)}+\frac{5\,b^2\,d\,(c+d\,x)^2}{(a\,d-b\,c)^3}}{b\,(c+d\,x)^{5/2}+(a\,d-b\,c)\,(c+d\,x)^{3/2}}+\frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^{7/2}}\right)}{(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(5/2)),x)
```

```
[Out] ((10*b*d*(c + d*x))/(3*(a*d - b*c)^2) - (2*d)/(3*(a*d - b*c)) + (5*b^2*d*(c
 + d*x)^2)/(a*d - b*c)^3)/(b*(c + d*x)^(5/2) + (a*d - b*c)*(c + d*x)^(3/2))
 + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(a*d - b*c)^(7/2)
```

3.1442 $\int \frac{1}{(a+bx)^3(c+dx)^{5/2}} \, dx$

Optimal. Leaf size=167

$$\frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}} + \frac{35bd^2}{4(bc-ad)^4\sqrt{c}}$$

```
[Out] 35/12*d^2/(-a*d+b*c)^3/(d*x+c)^(3/2)-1/2/(-a*d+b*c)/(b*x+a)^2/(d*x+c)^(3/2)
+7/4*d/(-a*d+b*c)^2/(b*x+a)/(d*x+c)^(3/2)-35/4*b^(3/2)*d^2*arctanh(b^(1/2)*
(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(9/2)+35/4*b*d^2/(-a*d+b*c)^4/(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}} + \frac{35bd^2}{4\sqrt{c+dx}\,(bc-ad)^4} + \frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3} + \frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2} - \frac{1}{2(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (35*d^2)/(12*(b*c - a*d)^3*(c + d*x)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x)^2*
(c + d*x)^(3/2)) + (7*d)/(4*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(3/2)) + (35*
b*d^2)/(4*(b*c - a*d)^4*Sqrt[c + d*x]) - (35*b^(3/2)*d^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(9/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}} + \frac{(35d^2)\int}{8(}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+b}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+b}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+b}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+b}$$

Mathematica [A]
time = 0.70, size = 168, normalized size = 1.01

$$\frac{-8a^3d^3+8a^2bd^2(10c+7dx)+ab^2d(39c^2+238cdx+175d^2x^2)+b^3(-6c^3+21c^2dx+140cd^2x^2+105d^3x^3)}{12(bc-ad)^4(a+bx)^2(c+dx)^{3/2}} + \frac{35b^{3/2}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{4(-bc+ad)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (-8*a^3*d^3 + 8*a^2*b*d^2*(10*c + 7*d*x) + a*b^2*d*(39*c^2 + 238*c*d*x + 17
5*d^2*x^2) + b^3*(-6*c^3 + 21*c^2*d*x + 140*c*d^2*x^2 + 105*d^3*x^3))/(12*(
b*c - a*d)^4*(a + b*x)^2*(c + d*x)^(3/2)) + (35*b^(3/2)*d^2*ArcTan[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*(-(b*c) + a*d)^(9/2))
```

Maple [A]
time = 0.17, size = 143, normalized size = 0.86

method	result
derivativedivides	$2d^2\left(-\frac{1}{3(ad-bc)^3(dx+c)^{\frac{3}{2}}}+\frac{3b}{(ad-bc)^4\sqrt{dx+c}}+\frac{b^2\left(\frac{\frac{11b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{13ad}{8}-\frac{13bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{35\arctan\left(\right.}{8\sqrt{(}}\right.}{(ad-bc)^4}\right.$
default	$2d^2\left(-\frac{1}{3(ad-bc)^3(dx+c)^{\frac{3}{2}}}+\frac{3b}{(ad-bc)^4\sqrt{dx+c}}+\frac{b^2\left(\frac{\frac{11b(dx+c)^{\frac{3}{2}}}{8}+\left(\frac{13ad}{8}-\frac{13bc}{8}\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^2}+\frac{35\arctan\left(\right.}{8\sqrt{(}}\right.}{(ad-bc)^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-1/3/(a*d-b*c)^3/(d*x+c)^(3/2)+3/(a*d-b*c)^4*b/(d*x+c)^(1/2)+b^2/(a*
d-b*c)^4*((11/8*b*(d*x+c)^(3/2)+(13/8*a*d-13/8*b*c)*(d*x+c)^(1/2))/((d*x+c)
*b+a*d-b*c)^2+35/8/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(139) = 278$.
time = 0.41, size = 1226, normalized size = 7.34

$$\left[\frac{105\,(b^3d^4x^4 + a^2bc^2d^2 + 2\,(b^3cd^3 + ab^2d^4)x^3 + (b^3c^2d^2 + 4\,ab^2cd^3 + a^2bd^4)x^2 + 2\,(ab^2c^2d^2 + a^2bcd^3)x)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx + 2\,bc - ad - 2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right) + 2\,(105\,b^3d^3x^3 - 6\,b^3c^3 + 39\,ab^2c^2d + 80\,a^2bcd^2 - 8\,a^3d^3 + 35\,(4\,b^3cd^2 + 5\,ab^2d^3)x^2 + 7\,(3\,b^3c^2d + 34\,ab^2cd^2 + 8\,a^2bd^3)x)\sqrt{dx+c}}{24\,(a^2b^4c^6 - 4\,a^3b^3c^5d + 6\,a^4b^2c^4d^2 - 4\,a^5bc^3d^3 + a^6c^2d^4 + (b^6c^4d^2 - 4\,ab^5c^3d^3 + 6\,a^2b^4c^2d^4 - 4\,a^3b^3cd^5 + a^4b^2d^6)x^4 + 2\,(b^6c^5d - 3\,ab^5c^4d^2 + 2\,a^2b^4c^3d^3 + 2\,a^3b^3c^2d^4 - 3\,a^4b^2cd^5 + a^5bd^6)x^3 + (b^6c^6 - 9\,a^2b^4c^4d^2 + 16\,a^3b^3c^3d^3 - 9\,a^4b^2c^2d^4 + a^6d^6)x^2 + 2\,(ab^5c^6 - 3\,a^2b^4c^5d + 2\,a^3b^3c^4d^2 + 2\,a^4b^2c^3d^3 - 3\,a^5bc^2d^4 + a^6cd^5)x)}, -\frac{105\,(b^3d^4x^4 + a^2bc^2d^2 + 2\,(b^3cd^3 + ab^2d^4)x^3 + (b^3c^2d^2 + 4\,ab^2cd^3 + a^2bd^4)x^2 + 2\,(ab^2c^2d^2 + a^2bcd^3)x)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right) - (105\,b^3d^3x^3 - 6\,b^3c^3 + 39\,ab^2c^2d + 80\,a^2bcd^2 - 8\,a^3d^3 + 35\,(4\,b^3cd^2 + 5\,ab^2d^3)x^2 + 7\,(3\,b^3c^2d + 34\,ab^2cd^2 + 8\,a^2bd^3)x)\sqrt{dx+c}}{12\,(a^2b^4c^6 - 4\,a^3b^3c^5d + 6\,a^4b^2c^4d^2 - 4\,a^5bc^3d^3 + a^6c^2d^4 + (b^6c^4d^2 - 4\,ab^5c^3d^3 + 6\,a^2b^4c^2d^4 - 4\,a^3b^3cd^5 + a^4b^2d^6)x^4 + 2\,(b^6c^5d - 3\,ab^5c^4d^2 + 2\,a^2b^4c^3d^3 + 2\,a^3b^3c^2d^4 - 3\,a^4b^2cd^5 + a^5bd^6)x^3 + (b^6c^6 - 9\,a^2b^4c^4d^2 + 16\,a^3b^3c^3d^3 - 9\,a^4b^2c^2d^4 + a^6d^6)x^2 + 2\,(ab^5c^6 - 3\,a^2b^4c^5d + 2\,a^3b^3c^4d^2 + 2\,a^4b^2c^3d^3 - 3\,a^5bc^2d^4 + a^6cd^5)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d^3 + a*b^2*d^4)*x^3 + (
b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*b^2*c^2*d^2 + a^2*b*c*d
^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*
x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(105*b^3*d^3*x^3 - 6*b^3*c^3 + 3
9*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2*d^3)
*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^
2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2
*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 +
2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4
*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5
*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^
2*d^4 + a^6*c*d^5)*x), -1/12*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d
^3 + a*b^2*d^4)*x^3 + (b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*
b^2*c^2*d^2 + a^2*b*c*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (105*b^3*d^3*x^3 - 6*b^3*c^
3 + 39*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2
*d^3)*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c)
)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^
6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*
c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d
^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^
2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(
a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5
*b*c^2*d^4 + a^6*c*d^5)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(139) = 278.
time = 1.51, size = 298, normalized size = 1.78

$$\frac{35\,b^2d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-b^2c+abd}}+\frac{2\,(9\,(dx+c)bd^2+bcd^2-ad^3)}{3\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)(dx+c)^{\frac{3}{2}}}+\frac{11\,(dx+c)^{\frac{3}{2}}b^3d^2-13\,\sqrt{dx+c}\,b^3cd^2+13\,\sqrt{dx+c}\,ab^2d^3}{4\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)((dx+c)b-bc+ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d
)) + 2/3*(9*(d*x + c)*b*d^2 + b*c*d^2 - a*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x + c)^(3/2)) + 1/4*(11*(d*
x + c)^(3/2)*b^3*d^2 - 13*sqrt(d*x + c)*b^3*c*d^2 + 13*sqrt(d*x + c)*a*b^2*
d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [B]
time = 0.28, size = 243, normalized size = 1.46

$$\frac{\frac{175\,b^2\,d^2\,(c+d\,x)^2}{12\,(a\,d-b\,c)^3}-\frac{2\,d^2}{3\,(a\,d-b\,c)}+\frac{35\,b^3\,d^2\,(c+d\,x)^3}{4\,(a\,d-b\,c)^4}+\frac{14\,b\,d^2\,(c+d\,x)}{3\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^{7/2}-(2\,b^2\,c-2\,a\,b\,d)\,(c+d\,x)^{5/2}+(c+d\,x)^{3/2}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{35\,b^{3/2}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{(a\,d-b\,c)^{9/2}}\right)}{4\,(a\,d-b\,c)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(5/2)),x)
```

```
[Out] ((175*b^2*d^2*(c + d*x)^2)/(12*(a*d - b*c)^3) - (2*d^2)/(3*(a*d - b*c)) + (
35*b^3*d^2*(c + d*x)^3)/(4*(a*d - b*c)^4) + (14*b*d^2*(c + d*x))/(3*(a*d -
b*c)^2))/(b^2*(c + d*x)^(7/2) - (2*b^2*c - 2*a*b*d)*(c + d*x)^(5/2) + (c +
d*x)^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (35*b^(3/2)*d^2*atan((b^(1/2)
*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(4*(a*d - b*c)^(9/2))
```

3.1443 $\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}}-\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}}+\frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}}-\frac{21}{8(bc-ad)^3(a+}$$

```
[Out] -35/8*d^3/(-a*d+b*c)^4/(d*x+c)^(3/2)-1/3/(-a*d+b*c)/(b*x+a)^3/(d*x+c)^(3/2)
+3/4*d/(-a*d+b*c)^2/(b*x+a)^2/(d*x+c)^(3/2)-21/8*d^2/(-a*d+b*c)^3/(b*x+a)/(
d*x+c)^(3/2)+105/8*b^(3/2)*d^3*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/
2))/(-a*d+b*c)^(11/2)-105/8*b*d^3/(-a*d+b*c)^5/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {44, 53, 65, 214}

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}\,(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{3d}{4(a+bx)^2(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{3(a+bx)^3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*(c + d*x)^(3/2)) - 1/(3*(b*c - a*d)*(a + b*x)^3*
(c + d*x)^(3/2)) + (3*d)/(4*(b*c - a*d)^2*(a + b*x)^2*(c + d*x)^(3/2)) - (2
1*d^2)/(8*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - (105*b*d^3)/(8*(b*c -
a*d)^5*Sqrt[c + d*x]) + (105*b^(3/2)*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/(8*(b*c - a*d)^(11/2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} - \frac{(3d)\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} + \frac{(21d^2)}{}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} - \frac{}{8(bc-}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3}{4(bc-ad)^2(a+}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3}{4(bc-ad)^2(a+}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3}{4(bc-ad)^2(a+}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3}{4(bc-ad)^2(a+}$$

Mathematica [A]
time = 0.99, size = 220, normalized size = 1.10

$$\frac{1}{24}\left(\frac{-16a^4d^4+16a^3bd^3(13c+9dx)+3a^2b^2d^2(55c^2+318cdx+231d^2x^2)+2ab^3d(-25c^3+90c^2dx+567cd^2x^2+420d^3x^3)+b^4(8c^4-18c^3dx+63c^2d^2x^2+420cd^3x^3+315d^4x^4)}{(-bc+ad)^5(a+bx)^3(c+dx)^{3/2}}+\frac{315b^{3/2}d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{11/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] ((-16*a^4*d^4 + 16*a^3*b*d^3*(13*c + 9*d*x) + 3*a^2*b^2*d^2*(55*c^2 + 318*c
*d*x + 231*d^2*x^2) + 2*a*b^3*d*(-25*c^3 + 90*c^2*d*x + 567*c*d^2*x^2 + 420
*d^3*x^3) + b^4*(8*c^4 - 18*c^3*d*x + 63*c^2*d^2*x^2 + 420*c*d^3*x^3 + 315*
d^4*x^4))/((-(b*c) + a*d)^5*(a + b*x)^3*(c + d*x)^(3/2)) + (315*b^(3/2)*d^3
*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(11/2))
/24
```

Maple [A]
time = 0.17, size = 177, normalized size = 0.88

method	result
derivativedivides	$2d^3\left(\frac{b^2\left(\frac{\frac{41(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{35(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{55}{16}a^2d^2-\frac{55}{8}abcd+\frac{55}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{105\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^5}\right.$
default	$2d^3\left(\frac{b^2\left(\frac{\frac{41(dx+c)^{\frac{5}{2}}b^2}{16}+\frac{35(ad-bc)b(dx+c)^{\frac{3}{2}}}{6}+\left(\frac{55}{16}a^2d^2-\frac{55}{8}abcd+\frac{55}{16}b^2c^2\right)\sqrt{dx+c}}{((dx+c)b+ad-bc)^3}+\frac{105\arctan\left(\frac{b\sqrt{dx+c}}{\sqrt{(ad-bc)b}}\right)}{16\sqrt{(ad-bc)b}}\right)}{(ad-bc)^5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^3*(1/(a*d-b*c)^5*b^2*((41/16*(d*x+c)^(5/2)*b^2+35/6*(a*d-b*c)*b*(d*x+c)
^(3/2)+(55/16*a^2*d^2-55/8*a*b*c*d+55/16*b^2*c^2)*(d*x+c)^(1/2))/((d*x+c)*b
+a*d-b*c)^3+105/16/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2)))-1/3/(a*d-b*c)^4/(d*x+c)^(3/2)+4/(a*d-b*c)^5*b/(d*x+c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(168) = 336$.
time = 0.47, size = 1840, normalized size = 9.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 + (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4
+ (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*
a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqr
t(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqr
t(b/(b*c - a*d)))/(b*x + a)) + 2*(315*b^4*d^4*x^4 + 8*b^4*c^4 - 50*a*b^3*c^
3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 420*(b^4*c*d^3 +
 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2
- 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*x)*sqr
t(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^
2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3
+ 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*
x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d
^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7
+ a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^
4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9
*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4
+ 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3
*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 +
7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x), 1/24*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 +
 (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4 + (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2
*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^
2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (315*b^4*d^4*x^4 + 8*b^4*c^
4 - 50*a*b^3*c^3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 4
20*(b^4*c*d^3 + 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^
2*b^2*d^4)*x^2 - 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^
3*b*d^4)*x)*sqrt(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*
d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5
*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6
- a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 1
0*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*
```

```
x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25
*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (
3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a
^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b
^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6
*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 432 vs. 2(168) = 336.
time = 1.40, size = 432, normalized size = 2.16

$$-\frac{105\,b^2d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\sqrt{-b^2c+abd}}-\frac{315\,(dx+c)^4b^4d^3-840\,(dx+c)^3b^4cd^3+693\,(dx+c)^2b^4c^2d^3-144\,(dx+c)b^4c^3d^3-16\,b^4c^4d^3+840\,(dx+c)^3ab^3d^4-1386\,(dx+c)^2ab^3cd^4+432\,(dx+c)ab^3c^2d^4+64\,ab^3c^3d^4+693\,(dx+c)^2a^2b^2d^5-432\,(dx+c)a^2b^2cd^5-96\,a^2b^2c^2d^5+144\,(dx+c)a^3bd^6+64\,a^3bcd^6-16\,a^4d^7}{24\,(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}\,bc+\sqrt{dx+c}\,ad\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -105/8*b^2*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^5 - 5*a
*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*
d^5)*sqrt(-b^2*c + a*b*d)) - 1/24*(315*(d*x + c)^4*b^4*d^3 - 840*(d*x + c)^
3*b^4*c*d^3 + 693*(d*x + c)^2*b^4*c^2*d^3 - 144*(d*x + c)*b^4*c^3*d^3 - 16*
b^4*c^4*d^3 + 840*(d*x + c)^3*a*b^3*d^4 - 1386*(d*x + c)^2*a*b^3*c*d^4 + 43
2*(d*x + c)*a*b^3*c^2*d^4 + 64*a*b^3*c^3*d^4 + 693*(d*x + c)^2*a^2*b^2*d^5
- 432*(d*x + c)*a^2*b^2*c*d^5 - 96*a^2*b^2*c^2*d^5 + 144*(d*x + c)*a^3*b*d^
6 + 64*a^3*b*c*d^6 - 16*a^4*d^7)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*((d*x + c)^(3/2)*b - s
qrt(d*x + c)*b*c + sqrt(d*x + c)*a*d)^3)
```

Mupad [**B**]
time = 0.64, size = 334, normalized size = 1.67

$$\frac{\frac{231\,b^2\,d^3\,{\left(c+d\,x\right)}^2}{8\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^3}{3\,\left(a\,d-b\,c\right)}+\frac{35\,b^3\,d^3\,{\left(c+d\,x\right)}^3}{{\left(a\,d-b\,c\right)}^4}+\frac{105\,b^4\,d^3\,{\left(c+d\,x\right)}^4}{8\,{\left(a\,d-b\,c\right)}^5}+\frac{6\,b\,d^3\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{\left(c+d\,x\right)}^{9/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}+{\left(c+d\,x\right)}^{5/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}+\frac{105\,b^{3/2}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{{\left(a\,d-b\,c\right)}^{11/2}}\right)}{8\,{\left(a\,d-b\,c\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(5/2)),x)
```

[Out] ((231*b^2*d^3*(c + d*x)^2)/(8*(a*d - b*c)^3) - (2*d^3)/(3*(a*d - b*c)) + (35*b^3*d^3*(c + d*x)^3)/(a*d - b*c)^4 + (105*b^4*d^3*(c + d*x)^4)/(8*(a*d - b*c)^5) + (6*b*d^3*(c + d*x))/(a*d - b*c)^2)/((c + d*x)^(3/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + b^3*(c + d*x)^(9/2) - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^(7/2) + (c + d*x)^(5/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d)) + (105*b^(3/2)*d^3*atan((b^(1/2)*(c + d*x)^(1/2)*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(a*d - b*c)^(11/2)))/(8*(a*d - b*c)^(11/2))

3.1444 $\int (a+bx)^5(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

[Out] 2/15*(b*c*x+a*c)^(15/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]

[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^{3/2}\,dx &= \frac{\int (ac+bcx)^{13/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{15/2}}{15bc^6}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6(c(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6*(c*(a + b*x))^(3/2))/(15*b)
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{15}{2}}}{15b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{15}{2}}}{15b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}(bcx+ac)^{\frac{3}{2}}}{15b}$	23
trager	$\frac{2c\left(b^{7}x^{7}+7a\,b^{6}x^{6}+21a^{2}b^{5}x^{5}+35a^{3}b^{4}x^{4}+35a^{4}b^{3}x^{3}+21a^{5}b^{2}x^{2}+7a^{6}bx+a^{7}\right)\sqrt{bcx+ac}}{15b}$	88
risch	$\frac{2c^{2}\left(b^{7}x^{7}+7a\,b^{6}x^{6}+21a^{2}b^{5}x^{5}+35a^{3}b^{4}x^{4}+35a^{4}b^{3}x^{3}+21a^{5}b^{2}x^{2}+7a^{6}bx+a^{7}\right)(bx+a)}{15b\sqrt{c\,(bx+a)}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(b*c*x+a*c)^(15/2)/b/c^6
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{15}{2}}}{15\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*c*x + a*c)^(15/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(18) = 36$.
time = 0.38, size = 95, normalized size = 4.32

$$\frac{2\,(b^{7}cx^{7}+7\,ab^{6}cx^{6}+21\,a^{2}b^{5}cx^{5}+35\,a^{3}b^{4}cx^{4}+35\,a^{4}b^{3}cx^{3}+21\,a^{5}b^{2}cx^{2}+7\,a^{6}bcx+a^{7}c)\sqrt{bcx+ac}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

[Out] 2/15*(b^7*c*x^7 + 7*a*b^6*c*x^6 + 21*a^2*b^5*c*x^5 + 35*a^3*b^4*c*x^4 + 35*a^4*b^3*c*x^3 + 21*a^5*b^2*c*x^2 + 7*a^6*b*c*x + a^7*c)*sqrt(b*c*x + a*c)/b

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.
time = 0.58, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{13}{2}}c^{\frac{3}{2}}\left(\frac{a}{b}+x\right)^{\frac{15}{2}}}{15} & \text{for } \left|\frac{a}{b}+x\right| < 1 \\ b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{17}{2} \\ \frac{15}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right) + b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{17}{2}, 1 & \\ & \frac{15}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**5*(b*c*x+a*c)**(3/2),x)

[Out] Piecewise((2*b**(13/2)*c**(3/2)*(a/b + x)**(15/2)/15, Abs(a/b + x) < 1), (b**(13/2)*c**(3/2)*meijerg(((1,), (17/2,)), ((15/2,), (0,)), a/b + x) + b**(13/2)*c**(3/2)*meijerg(((17/2, 1), ()), ((), (15/2, 0)), a/b + x), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(18) = 36$.
time = 1.15, size = 637, normalized size = 28.95

$$\frac{2\left(6435\sqrt{bcx+ac}\,a^7c - 15015\left(3\sqrt{bcx+ac}\,ac - (bcx+ac)^{\frac{3}{2}}\right)a^6 + \frac{9009\left(15\sqrt{bcx+ac}\,a^2c^2 - 10(bcx+ac)^{\frac{3}{2}}ac + 3(bcx+ac)^{\frac{5}{2}}\right)a^5}{c} - \frac{6435\left(35\sqrt{bcx+ac}\,a^3c^3 - 35(bcx+ac)^{\frac{3}{2}}a^2c^2 + 21(bcx+ac)^{\frac{5}{2}}ac - 5(bcx+ac)^{\frac{7}{2}}\right)a^4}{c^2} + \frac{715\left(315\sqrt{bcx+ac}\,a^4c^4 - 420(bcx+ac)^{\frac{3}{2}}a^3c^3 + 378(bcx+ac)^{\frac{5}{2}}a^2c^2 - 180(bcx+ac)^{\frac{7}{2}}ac + 35(bcx+ac)^{\frac{9}{2}}\right)a^3}{c^3} - \frac{195\left(693\sqrt{bcx+ac}\,a^5c^5 - 1155(bcx+ac)^{\frac{3}{2}}a^4c^4 + 1386(bcx+ac)^{\frac{5}{2}}a^3c^3 - 990(bcx+ac)^{\frac{7}{2}}a^2c^2 + 385(bcx+ac)^{\frac{9}{2}}ac - 63(bcx+ac)^{\frac{11}{2}}\right)a^2}{c^4} + \frac{15\left(3003\sqrt{bcx+ac}\,a^6c^6 - 6006(bcx+ac)^{\frac{3}{2}}a^5c^5 + 9009(bcx+ac)^{\frac{5}{2}}a^4c^4 - 8580(bcx+ac)^{\frac{7}{2}}a^3c^3 + 5005(bcx+ac)^{\frac{9}{2}}a^2c^2 - 1638(bcx+ac)^{\frac{11}{2}}ac + 231(bcx+ac)^{\frac{13}{2}}\right)a}{c^5} - \frac{6435\sqrt{bcx+ac}\,a^7c^7 - 15015(bcx+ac)^{\frac{3}{2}}a^6c^6 + 27027(bcx+ac)^{\frac{5}{2}}a^5c^5 - 32175(bcx+ac)^{\frac{7}{2}}a^4c^4 + 25025(bcx+ac)^{\frac{9}{2}}a^3c^3 - 12285(bcx+ac)^{\frac{11}{2}}a^2c^2 + 3465(bcx+ac)^{\frac{13}{2}}ac - 429(bcx+ac)^{\frac{15}{2}}}{c^6}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="giac")

[Out] 2/6435*(6435*sqrt(b*c*x + a*c)*a^7*c - 15015*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2))*a^6 + 9009*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^5/c - 6435*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^4/c^2 + 715*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^3/c^3 - 195*(693*sqrt(b*c*x + a*c)*a^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^(11/2))*a^2/c^4 + 15*(3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a/c^5 - (6435*sqrt(b*c*x + a*c)*a^7*c^7 - 15015*(b*c*x + a*c)^(3/2)*a^6*c^6 + 27027*(b*c*x + a*c)^(5/2)*a^5*c^5 - 32175*(b*c*x + a*c)^(7/2)*a^4*c^4 + 25025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 12285*(b*c*x + a*c)^(11/2)*a^2*c^2 + 3465*(b*c*x + a*c)^(13/2)*a*c - 429*(b*c*x + a*c)^(15/2))/c^6)/b

Mupad [B]
time = 0.05, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{15/2}}{15\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(3/2)*(a + b*x)^5,x)
[Out] (2*(c*(a + b*x))^(15/2))/(15*b*c^6)
```

3.1445 $\int (a+bx)^5 \sqrt{ac+bcx}\, dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

[Out] 2/13*(b*c*x+a*c)^(13/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5*Sqrt[a*c + b*c*x],x]

[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a+bx)^5 \sqrt{ac+bcx}\, dx &= \frac{\int (ac+bcx)^{11/2}\, dx}{c^5} \\ &= \frac{2(ac+bcx)^{13/2}}{13bc^6} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6 \sqrt{c(a+bx)}}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6*Sqrt[c*(a + b*x)])/(13*b)
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{13}{2}}}{13b\,c^6}$	19
default	$\frac{2(bcx+ac)^{\frac{13}{2}}}{13b\,c^6}$	19
gosper	$\frac{2(bx+a)^6\sqrt{bcx+ac}}{13b}$	23
trager	$\frac{2\left(x^6b^6+6a\,x^5b^5+15a^2x^4b^4+20a^3b^3x^3+15a^4x^2b^2+6a^5xb+a^6\right)\sqrt{bcx+ac}}{13b}$	76
risch	$\frac{2c\left(x^6b^6+6a\,x^5b^5+15a^2x^4b^4+20a^3b^3x^3+15a^4x^2b^2+6a^5xb+a^6\right)(bx+a)}{13b\sqrt{c\,(bx+a)}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/13*(b*c*x+a*c)^(13/2)/b/c^6
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{13}{2}}}{13\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*c*x + a*c)^(13/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(18) = 36$.
time = 0.41, size = 75, normalized size = 3.41

$$\frac{2\,(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6)\sqrt{bcx+ac}}{13\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*
x^2 + 6*a^5*b*x + a^6)*sqrt(b*c*x + a*c)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.
time = 0.48, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{11}{2}}\sqrt{c}\left(\frac{a}{b}+x\right)^{\frac{13}{2}}}{13} & \text{for } \left|\frac{a}{b}+x\right| < 1 \\ b^{\frac{11}{2}}\sqrt{c}\, G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{15}{2} \\ \frac{13}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right) + b^{\frac{11}{2}}\sqrt{c}\, G_{2,2}^{0,2}\left(\begin{matrix} \frac{15}{2}, 1 & \\ & \frac{13}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(11/2)*sqrt(c)*(a/b + x)**(13/2)/13, Abs(a/b + x) < 1), (b*
*(11/2)*sqrt(c)*meijerg(((1,), (15/2,)), ((13/2,), (0,)), a/b + x) + b**(11
/2)*sqrt(c)*meijerg(((15/2, 1), ()), ((), (13/2, 0)), a/b + x), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(18) = 36$.
time = 1.18, size = 495, normalized size = 22.50

$$\frac{2\left(3003\sqrt{bcx+ac}\,a^6 - \frac{6006\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^5}{c} + \frac{3003\left(15\sqrt{bcx+ac}\,a^2c^2-10(bcx+ac)^{\frac{3}{2}}ac+3(bcx+ac)^{\frac{5}{2}}\right)a^4}{c^2} - \frac{1716\left(35\sqrt{bcx+ac}\,a^3c^3-35(bcx+ac)^{\frac{3}{2}}a^2c^2+21(bcx+ac)^{\frac{5}{2}}ac-5(bcx+ac)^{\frac{7}{2}}\right)a^3}{c^3} + \frac{143\left(315\sqrt{bcx+ac}\,a^4c^4-420(bcx+ac)^{\frac{3}{2}}a^3c^3+378(bcx+ac)^{\frac{5}{2}}a^2c^2-180(bcx+ac)^{\frac{7}{2}}ac+35(bcx+ac)^{\frac{9}{2}}\right)a^2}{c^4} - \frac{26\left(693\sqrt{bcx+ac}\,a^5c^5-1155(bcx+ac)^{\frac{3}{2}}a^4c^4+1386(bcx+ac)^{\frac{5}{2}}a^3c^3-990(bcx+ac)^{\frac{7}{2}}a^2c^2+385(bcx+ac)^{\frac{9}{2}}ac-63(bcx+ac)^{\frac{11}{2}}\right)a}{c^5} + \frac{3003\sqrt{bcx+ac}\,a^6c^6-6006(bcx+ac)^{\frac{3}{2}}a^5c^5+9009(bcx+ac)^{\frac{5}{2}}a^4c^4-8580(bcx+ac)^{\frac{7}{2}}a^3c^3+5005(bcx+ac)^{\frac{9}{2}}a^2c^2-1638(bcx+ac)^{\frac{11}{2}}ac+231(bcx+ac)^{\frac{13}{2}}}{c^6}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3003*(3003*sqrt(b*c*x + a*c)*a^6 - 6006*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x
 + a*c)^(3/2))*a^5/c + 3003*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c
)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^4/c^2 - 1716*(35*sqrt(b*c*x + a*c)*a
^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b
*c*x + a*c)^(7/2))*a^3/c^3 + 143*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*
x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c
)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^2/c^4 - 26*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a/c^5 + (3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*x + a
*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a*c)^(
7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(11/2)
*a*c + 231*(b*c*x + a*c)^(13/2))/c^6)/b
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\left(c\left(a+b\,x\right)\right)^{13/2}}{13\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(1/2)*(a + b*x)^5,x)
[Out] (2*(c*(a + b*x))^(13/2))/(13*b*c^6)
```

3.1446 $\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

```
[Out] 2/11*(b*c*x+a*c)^(11/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx &= \frac{\int (ac+bcx)^{9/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{11/2}}{11bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{11b\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

```
[Out] (2*(a + b*x)^6)/(11*b*Sqrt[c*(a + b*x)])
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{11}{2}}}{11b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{11}{2}}}{11b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{11b\sqrt{bcx+ac}}$	23
trager	$\frac{2\left(b^{5}x^{5}+5a\,b^{4}x^{4}+10a^{2}b^{3}x^{3}+10a^{3}b^{2}x^{2}+5a^{4}bx+a^{5}\right)\sqrt{bcx+ac}}{11cb}$	68
risch	$\frac{2\left(b^{5}x^{5}+5a\,b^{4}x^{4}+10a^{2}b^{3}x^{3}+10a^{3}b^{2}x^{2}+5a^{4}bx+a^{5}\right)(bx+a)}{11b\sqrt{c\,(bx+a)}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11*(b*c*x+a*c)^(11/2)/b/c^6
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(18) = 36.
time = 0.29, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^{5}-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^{4}}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^{2}c^{2}-10(bcx+ac)^{\frac{3}{2}}ac+3(bcx+ac)^{\frac{5}{2}}\right)a^{3}}{c^{2}}-\frac{198\left(35\sqrt{bcx+ac}\,a^{3}c^{3}-35(bcx+ac)^{\frac{3}{2}}a^{2}c^{2}+21(bcx+ac)^{\frac{5}{2}}ac-5(bcx+ac)^{\frac{7}{2}}\right)a^{2}}{c^{3}}+\frac{11\left(315\sqrt{bcx+ac}\,a^{4}c^{4}-420(bcx+ac)^{\frac{3}{2}}a^{3}c^{3}+378(bcx+ac)^{\frac{5}{2}}a^{2}c^{2}-180(bcx+ac)^{\frac{7}{2}}ac+35(bcx+ac)^{\frac{9}{2}}\right)a}{c^{4}}-\frac{693\sqrt{bcx+ac}\,a^{5}c^{5}-1155(bcx+ac)^{\frac{3}{2}}a^{4}c^{4}+1386(bcx+ac)^{\frac{5}{2}}a^{3}c^{3}-990(bcx+ac)^{\frac{7}{2}}a^{2}c^{2}+385(bcx+ac)^{\frac{9}{2}}ac-63(bcx+ac)^{\frac{11}{2}}}{c^{5}}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(18) = 36$.
time = 0.74, size = 67, normalized size = 3.05

$$\frac{2\,(b^5x^5 + 5\,ab^4x^4 + 10\,a^2b^3x^3 + 10\,a^3b^2x^2 + 5\,a^4bx + a^5)\sqrt{bcx + ac}}{11\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*c*x + a*c)/(b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(19) = 38$.
time = 0.48, size = 88, normalized size = 4.00

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \wedge \left|\frac{a}{b}+x\right| < 1 \\ \frac{2b^{\frac{9}{2}}\left(\frac{a}{b}+x\right)^{\frac{11}{2}}}{11\sqrt{c}} & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \vee \left|\frac{a}{b}+x\right| < 1 \\ \frac{b^{\frac{9}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{13}{2} \\ \frac{11}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x \right)}{\sqrt{c}} + \frac{b^{\frac{9}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{13}{2}, 1 & \\ & \frac{11}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x \right)}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((0, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (2*b**(9/2)*(a/b
+ x)**(11/2)/(11*sqrt(c)), (Abs(a/b + x) < 1) | (1/Abs(a/b + x) < 1)), (b**
(9/2)*meijerg(((1,), (13/2,)), ((11/2,), (0,)), a/b + x)/sqrt(c) + b**(9/2)
*meijerg(((13/2, 1), ()), ((), (11/2, 0)), a/b + x)/sqrt(c), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(18) = 36$.
time = 0.67, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^5 - \frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^4}{c} + \frac{462\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^3}{c^2} - \frac{198\left(35\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^2}{c^3} + \frac{11\left(315\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a}{c^4} - \frac{693\sqrt{bcx+ac}\,a^5c^5-1155\,(bcx+ac)^{\frac{3}{2}}a^4c^4+1386\,(bcx+ac)^{\frac{5}{2}}a^3c^3-990\,(bcx+ac)^{\frac{7}{2}}a^2c^2+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}}{c^5}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
 a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(
3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c
```

```
^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x
 + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a
*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/
2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1
155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b
*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^
(11/2))/c^5)/(b*c)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{11/2}}{11\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(1/2),x)
[Out] (2*(c*(a + b*x))^(11/2))/(11*b*c^6)
```

3.1447 $\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

```
[Out] 2/9*(b*c*x+a*c)^(9/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx &= \frac{\int (ac+bcx)^{7/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{9/2}}{9bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{9b(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6)/(9*b*(c*(a + b*x))^(3/2))
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{9}{2}}}{9b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{9}{2}}}{9b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{9b(bcx+ac)^{\frac{3}{2}}}$	23
trager	$\frac{2\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4}\right)\sqrt{bcx+ac}}{9c^{2}b}$	57
risch	$\frac{2\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4a^{3}bx+a^{4}\right)(bx+a)}{9cb\sqrt{c\left(bx+a\right)}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*(b*c*x+a*c)^(9/2)/b/c^6
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{9}{2}}}{9\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*c*x + a*c)^(9/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.
time = 0.92, size = 56, normalized size = 2.55

$$\frac{2\,(b^{4}x^{4}+4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+4\,a^{3}bx+a^{4})\sqrt{bcx+ac}}{9\,bc^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(b*c*x +
a*c)/(b*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(19) = 38$.
time = 0.49, size = 88, normalized size = 4.00

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \wedge \left|\frac{a}{b}+x\right| < 1 \\ \frac{2b^{\frac{7}{2}}\left(\frac{a}{b}+x\right)^{\frac{9}{2}}}{9c^{\frac{3}{2}}} & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \vee \left|\frac{a}{b}+x\right| < 1 \\ \frac{b^{\frac{7}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{11}{2} \\ \frac{9}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x \right)}{c^{\frac{3}{2}}} + \frac{b^{\frac{7}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{11}{2}, 1 & \\ & \frac{9}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x \right)}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((0, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (2*b**(7/2)*(a/b
+ x)**(9/2)/(9*c**(3/2)), (Abs(a/b + x) < 1) | (1/Abs(a/b + x) < 1)), (b**(
7/2)*meijerg(((1,), (11/2,)), ((9/2,), (0,)), a/b + x)/c**(3/2) + b**(7/2)*
meijerg(((11/2, 1), ()), ((), (9/2, 0)), a/b + x)/c**(3/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(18) = 36$.
time = 0.57, size = 266, normalized size = 12.09

$$\frac{2\left(315\sqrt{bcx+ac}\,a^4 - \frac{420\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^3}{c} + \frac{126\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^2}{c^2} - \frac{36\left(35\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a}{c^3} + \frac{315\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}}{c^4}\right)}{315\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(b*c*x + a*c)*a^4 - 420*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x +
a*c)^(3/2))*a^3/c + 126*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3
/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^2/c^2 - 36*(35*sqrt(b*c*x + a*c)*a^3*c^3
 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x +
 a*c)^(7/2))*a/c^3 + (315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/
2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c
+ 35*(b*c*x + a*c)^(9/2))/c^4)/(b*c^2)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{9/2}}{9\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(3/2),x)
[Out] (2*(c*(a + b*x))^(9/2))/(9*b*c^6)
```

3.1448 $\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

[Out] 2/7*(b*c*x+a*c)^(7/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]

[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx &= \frac{\int (ac+bcx)^{5/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{7/2}}{7bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{7b(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^6)/(7*b*(c*(a + b*x))^(5/2))
```

Maple [A]
time = 0.15, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{7}{2}}}{7b\,c^6}$	19
default	$\frac{2(bcx+ac)^{\frac{7}{2}}}{7b\,c^6}$	19
gosper	$\frac{2(bx+a)^6}{7b(bcx+ac)^{\frac{5}{2}}}$	23
trager	$\frac{2(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)\sqrt{bcx+ac}}{7c^3b}$	46
risch	$\frac{2(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)(bx+a)}{7c^2b\sqrt{c\,(bx+a)}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(b*c*x+a*c)^(7/2)/b/c^6
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{7}{2}}}{7\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*c*x + a*c)^(7/2)/(b*c^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.
time = 1.29, size = 45, normalized size = 2.05

$$\frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bcx+ac}}{7\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*c*x + a*c)/(b*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(19) = 38$.
time = 0.63, size = 88, normalized size = 4.00

$$\begin{cases} 0 & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \wedge \left|\frac{a}{b}+x\right| < 1 \\ \frac{2b^{\frac{5}{2}}\left(\frac{a}{b}+x\right)^{\frac{7}{2}}}{7c^{\frac{5}{2}}} & \text{for}\ \frac{1}{\left|\frac{a}{b}+x\right|} < 1 \vee \left|\frac{a}{b}+x\right| < 1 \\ \frac{b^{\frac{5}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{9}{2} \\ \frac{7}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{c^{\frac{5}{2}}} + \frac{b^{\frac{5}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{9}{2}, 1 & \\ & \frac{7}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(5/2),x)
```

```
[Out] Piecewise((0, (Abs(a/b + x) < 1) & (1/Abs(a/b + x) < 1)), (2*b**(5/2)*(a/b
+ x)**(7/2)/(7*c**(5/2)), (Abs(a/b + x) < 1) | (1/Abs(a/b + x) < 1)), (b**(
5/2)*meijerg(((1,), (9/2,)), ((7/2,), (0,)), a/b + x)/c**(5/2) + b**(5/2)*m
eijerg(((9/2, 1), ()), ((), (7/2, 0)), a/b + x)/c**(5/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(18) = 36$.
time = 0.84, size = 178, normalized size = 8.09

$$\frac{2\left(35\sqrt{bcx+ac}\,a^3 - \frac{35\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^2}{c} + \frac{7\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a}{c^2} - \frac{35\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}}{c^3}\right)}{35\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(b*c*x + a*c)*a^3 - 35*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a^2/c + 7*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a
*c + 3*(b*c*x + a*c)^(5/2))*a/c^2 - (35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c
*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2
))/c^3)/(b*c^3)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{7/2}}{7\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(5/2),x)
```

```
[Out] (2*(c*(a + b*x))^(7/2))/(7*b*c^6)
```

3.1449 $\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}} dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

[Out] 2/5*(b*c*x+a*c)^(5/2)/b/c^6

Rubi [A]

time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]

[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx &= \frac{\int (ac+bcx)^{3/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{5/2}}{5bc^6}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{5b(c(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
[Out] (2*(a + b*x)^6)/(5*b*(c*(a + b*x))^(7/2))
```

Maple [A]
time = 0.18, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{5}{2}}}{5b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{5}{2}}}{5b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{5b(bcx+ac)^{\frac{7}{2}}}$	23
trager	$\frac{2(x^{2}b^{2}+2abx+a^{2})\sqrt{bcx+ac}}{5c^{4}b}$	35
risch	$\frac{2(x^{2}b^{2}+2abx+a^{2})(bx+a)}{5c^{3}b\sqrt{c\left(bx+a\right)}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(b*c*x+a*c)^(5/2)/b/c^6
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{5}{2}}}{5\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="maxima")
[Out] 2/5*(b*c*x + a*c)^(5/2)/(b*c^6)
```

Fricas [A]
time = 1.07, size = 34, normalized size = 1.55

$$\frac{2\,(b^{2}x^{2}+2\,abx+a^{2})\sqrt{bcx+ac}}{5\,bc^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="fricas")
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*c*x + a*c)/(b*c^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(19) = 38$.
time = 0.68, size = 80, normalized size = 3.64

$$\begin{cases} \frac{2a^2\sqrt{ac+bcx}}{5bc^4} + \frac{4ax\sqrt{ac+bcx}}{5c^4} + \frac{2bx^2\sqrt{ac+bcx}}{5c^4} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(7/2),x)
```

```
[Out] Piecewise((2*a**2*sqrt(a*c + b*c*x)/(5*b*c**4) + 4*a*x*sqrt(a*c + b*c*x)/(5
*c**4) + 2*b*x**2*sqrt(a*c + b*c*x)/(5*c**4), Ne(b, 0)), (a**5*x/(a*c)**(7/
2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(18) = 36$.
time = 0.92, size = 106, normalized size = 4.82

$$\frac{2\left(15\sqrt{bcx+ac}\,a^2 - \frac{10\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a}{c} + \frac{15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c +
 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c^4)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{5/2}}{5\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(7/2),x)
```

```
[Out] (2*(c*(a + b*x))^(5/2))/(5*b*c^6)
```

3.1450 $\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}} dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

[Out] 2/3*(b*c*x+a*c)^(3/2)/b/c^6

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]

[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}}\,dx &= \frac{\int \sqrt{ac+bcx}\,dx}{c^5}\\
&= \frac{2(ac+bcx)^{3/2}}{3bc^6}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.18

$$\frac{2(a+bx)\sqrt{c(a+bx)}}{3bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)])/(3*b*c^5)
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(bcx+ac)^{\frac{3}{2}}}{3b\,c^{6}}$	19
default	$\frac{2(bcx+ac)^{\frac{3}{2}}}{3b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{3b(bcx+ac)^{\frac{9}{2}}}$	23
trager	$\frac{2(bx+a)\sqrt{bcx+ac}}{3c^{5}b}$	24
risch	$\frac{2(bx+a)^{2}}{3c^{4}b\sqrt{c\,(bx+a)}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*c*x+a*c)^(3/2)/b/c^6
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{3}{2}}}{3\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*c*x + a*c)^(3/2)/(b*c^6)
```

Fricas [A]
time = 0.90, size = 23, normalized size = 1.05

$$\frac{2\,\sqrt{bcx+ac}\,(bx+a)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*c*x + a*c)*(b*x + a)/(b*c^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(19) = 38$.
time = 1.03, size = 53, normalized size = 2.41

$$\begin{cases} \frac{2a\sqrt{ac+bcx}}{3bc^5} + \frac{2x\sqrt{ac+bcx}}{3c^5} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(9/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*c + b*c*x)/(3*b*c**5) + 2*x*sqrt(a*c + b*c*x)/(3*c**5
), Ne(b, 0)), (a**5*x/(a*c)**(9/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(18) = 36$.
time = 0.99, size = 54, normalized size = 2.45

$$\frac{2\left(3\sqrt{bcx+ac}\,a - \frac{3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}}{c}\right)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2)
)/c)/(b*c^5)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\left(c\left(a+b\,x\right)\right)^{3/2}}{3\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(9/2),x)
```

```
[Out] (2*(c*(a + b*x))^(3/2))/(3*b*c^6)
```

3.1451 $\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} \, dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

```
[Out] 2*(b*c*x+a*c)^(1/2)/b/c^6
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} \, dx &= \frac{\int \frac{1}{\sqrt{ac+bcx}} \, dx}{c^5} \\ &= \frac{2\sqrt{ac+bcx}}{bc^6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.20

$$\frac{2(a+bx)}{bc^5\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
[Out] (2*(a + b*x))/(b*c^5*Sqrt[c*(a + b*x)])
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.95

method	result	size
derivativedivides	$\frac{2\sqrt{bcx+ac}}{b\,c^{6}}$	19
default	$\frac{2\sqrt{bcx+ac}}{b\,c^{6}}$	19
trager	$\frac{2\sqrt{bcx+ac}}{b\,c^{6}}$	19
gosper	$\frac{2(bx+a)^{6}}{b(bcx+ac)^{\frac{11}{2}}}$	23
risch	$\frac{2bx+2a}{c^{5}b\sqrt{c\,(bx+a)}}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(11/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*c*x+a*c)^(1/2)/b/c^6
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="maxima")
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

Fricas [A]
time = 0.82, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="fricas")
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

Sympy [**A**]
time = 1.51, size = 29, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{ac+bcx}}{bc^6} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{11}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(11/2),x)
[Out] Piecewise((2*sqrt(a*c + b*c*x)/(b*c**6), Ne(b, 0)), (a**5*x/(a*c)**(11/2),
True))
```

Giac [**A**]
time = 1.18, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="giac")
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

Mupad [**B**]
time = 0.03, size = 17, normalized size = 0.85

$$\frac{2\sqrt{c\,(a+b\,x)}}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(11/2),x)
[Out] (2*(c*(a + b*x))^(1/2))/(b*c^6)
```

3.1452 $\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}} dx$

Optimal. Leaf size=20

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

```
[Out] -2/b/c^6/(b*c*x+a*c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] -2/(b*c^6*Sqrt[a*c + b*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx &= \frac{\int \frac{1}{(ac+bcx)^{3/2}}\,dx}{c^5}\\ &= -\frac{2}{bc^6\sqrt{ac+bcx}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.20

$$-\frac{2(a+bx)}{bc^5(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x))/(b*c^5*(c*(a + b*x))^(3/2))
```

Maple [A]
time = 0.17, size = 19, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2}{b\,c^{6}\sqrt{bcx+ac}}$	19
default	$-\frac{2}{b\,c^{6}\sqrt{bcx+ac}}$	19
gosper	$-\frac{2(bx+a)^{6}}{b(bcx+ac)^{\frac{13}{2}}}$	23
trager	$-\frac{2\sqrt{bcx+ac}}{c^{7}b(bx+a)}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/c^6/(b*c*x+a*c)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx+ac}\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

Fricas [A]
time = 0.86, size = 29, normalized size = 1.45

$$-\frac{2\sqrt{bcx+ac}}{b^{2}c^{7}x+abc^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*c*x + a*c)/(b^2*c^7*x + a*b*c^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 2.31, size = 41, normalized size = 2.05

$$\begin{cases} -\frac{2\sqrt{ac+bcx}}{abc^7+b^2c^7x} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{13}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(13/2),x)
```

```
[Out] Piecewise((-2*sqrt(a*c + b*c*x)/(a*b*c**7 + b**2*c**7*x), Ne(b, 0)), (a**5*
x/(a*c)**(13/2), True))
```

Giac [A]
time = 1.10, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx+ac}\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.85

$$-\frac{2}{b\,c^6\,\sqrt{c\,(a+b\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(13/2),x)
```

```
[Out] -2/(b*c^6*(c*(a + b*x))^(1/2))
```

3.1453 $\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx$

Optimal. Leaf size=14

$$-\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)$$

```
[Out] -arctanh(1/2*(2+x)^(1/2))
```

Rubi [A]

time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 213}

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx &= 2\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= -\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.16, size = 22, normalized size = 1.57

method	result	size
trager	$-\frac{\ln\left(-\frac{6+x+4\sqrt{2+x}}{-2+x}\right)}{2}$	21
derivativedivides	$\frac{\ln\left(\sqrt{2+x}-2\right)}{2}-\frac{\ln\left(\sqrt{2+x}+2\right)}{2}$	22
default	$\frac{\ln\left(\sqrt{2+x}-2\right)}{2}-\frac{\ln\left(\sqrt{2+x}+2\right)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln((2+x)^(1/2)-2)-1/2*ln((2+x)^(1/2)+2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.28, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.81, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.29, size = 26, normalized size = 1.86

$$\begin{cases} -\operatorname{acoth}\left(\frac{\sqrt{x+2}}{2}\right) & \text{for } |x+2| > 4 \\ -\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)**(1/2),x)
```

```
[Out] Piecewise((-acoth(sqrt(x + 2)/2), Abs(x + 2) > 4), (-atanh(sqrt(x + 2)/2),
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.92, size = 22, normalized size = 1.57

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(abs(sqrt(x + 2) - 2))
```

Mupad [B]
time = 0.05, size = 10, normalized size = 0.71

$$-\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(x + 2)^(1/2)),x)
```

```
[Out] -atanh((x + 2)^(1/2)/2)
```

3.1454 $\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}$$

```
[Out] 2/21*arctan(1/7*21^(1/2)*(1+5*x)^(1/2))*21^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 209}

$$\frac{2\text{ArcTan}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx = \frac{2}{5}\text{Subst}\left(\int \frac{1}{\frac{7}{5}+\frac{3x^2}{5}}\,dx, x, \sqrt{1+5x}\right)$$
$$= \frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Maple [A]
time = 0.16, size = 19, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{21}\,\sqrt{5x+1}}{7}\right)\sqrt{21}}{21}$	19
default	$\frac{2\arctan\left(\frac{\sqrt{21}\,\sqrt{5x+1}}{7}\right)\sqrt{21}}{21}$	19
trager	$\frac{\text{RootOf}\left(\_Z^2+21\right)\ln\left(-\frac{15\,\text{RootOf}\left(\_Z^2+21\right)x-4\,\text{RootOf}\left(\_Z^2+21\right)-42\sqrt{5x+1}}{2+3x}\right)}{21}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x)/(5*x+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21*arctan(1/7*21^(1/2)*(5*x+1)^(1/2))*21^(1/2)
```

Maxima [A]
time = 0.53, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5\,x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="maxima")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Fricas [A]
time = 0.73, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="fricas")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.51, size = 61, normalized size = 2.44

$$\begin{cases} \frac{2\sqrt{21}\, i \operatorname{acosh}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{for}\ \frac{1}{\left|x+\frac{2}{3}\right|} > \frac{15}{7} \\ -\frac{2\sqrt{21}\operatorname{asin}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)**(1/2),x)
[Out] Piecewise((2*sqrt(21)*I*acosh(sqrt(105)/(15*sqrt(x + 2/3)))/21, 1/Abs(x + 2
/3) > 15/7), (-2*sqrt(21)*asin(sqrt(105)/(15*sqrt(x + 2/3)))/21, True))
```

Giac [A]
time = 0.94, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="giac")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

Mupad [B]
time = 0.06, size = 15, normalized size = 0.60

$$\frac{2\sqrt{21}\,\mathrm{atan}\left(\frac{\sqrt{105\,x+21}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x + 2)*(5*x + 1)^(1/2)),x)
[Out] (2*21^(1/2)*atan((105*x + 21)^(1/2)/7))/21
```

3.1455 $\int \frac{\sqrt[3]{1-x}}{1+x}\,dx$

Optimal. Leaf size=84

$$3\sqrt[3]{1-x}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}$$

```
[Out] 3*(1-x)^(1/3)+3/2*ln(2^(1/3)-(1-x)^(1/3))*2^(1/3)-1/2*ln(1+x)*2^(1/3)-2^(1/
3)*arctan(1/3*(1+2^(2/3)*(1-x)^(1/3))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {52, 59, 631, 210, 31}

$$-\sqrt[3]{2}\sqrt{3}\,\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)+3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + (3*Log[2^(1/3) - (1 - x)^(1/3)])/2^(2/3) - Log[1 + x]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
```

```
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{1-x}}{1+x}\,dx &= 3\sqrt[3]{1-x}+2\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx\\ &= 3\sqrt[3]{1-x}-\frac{\log(1+x)}{2^{2/3}}-\frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{3\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,d\right.}{\sqrt[3]{2}}\\ &= 3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}+\left(3\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.\\ &= 3\sqrt[3]{1-x}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(1+x)}{2^{2/3}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 112, normalized size = 1.33

$$3\sqrt[3]{1-x}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right)+\sqrt[3]{2}\log\left(-2+2^{2/3}\sqrt[3]{1-x}\right)-\frac{\log\left(2+2^{2/3}\sqrt[3]{1-x}+\sqrt[3]{2}(1-x)^{2/3}\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + 2^(1/3)*Log[-2 + 2^(2/3)*(1 - x)^(1/3)] - Log[2 + 2^(2/3)*(1 - x)^(1/3
) + 2^(1/3)*(1 - x)^(2/3)]/2^(2/3)
```

Maple [A]
time = 3.09, size = 84, normalized size = 1.00

method	result
derivativedivides	$3(1-x)^{\frac{1}{3}}+2^{\frac{1}{3}}\ln\left((1-x)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\frac{2^{\frac{1}{3}}\ln\left((1-x)^{\frac{2}{3}}+2^{\frac{1}{3}}(1-x)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{2}-2^{\frac{1}{3}}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(1-x)\right.}{3}\right.$
default	$3(1-x)^{\frac{1}{3}}+2^{\frac{1}{3}}\ln\left((1-x)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\frac{2^{\frac{1}{3}}\ln\left((1-x)^{\frac{2}{3}}+2^{\frac{1}{3}}(1-x)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{2}-2^{\frac{1}{3}}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}(1-x)\right.}{3}\right.$
trager	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/3)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] 3*(1-x)^(1/3)+2^(1/3)*ln((1-x)^(1/3)-2^(1/3))-1/2*2^(1/3)*ln((1-x)^(2/3)+2^
(1/3)*(1-x)^(1/3)+2^(2/3))-2^(1/3)*arctan(1/3*(1+2^(2/3)*(1-x)^(1/3))*3^(1/
2))*3^(1/2)
```

Maxima [A]
time = 0.53, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\,(-x+1)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3\,(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

Fricas [A]
time = 0.84, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3\,(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(-x + 1)^(1/3) + 1/3*sqrt(3)) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.23, size = 170, normalized size = 2.02

$$\frac{4\sqrt[3]{-1}\sqrt[3]{x-1}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{-\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{-2}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{i\pi}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{5i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)/(1+x),x)
```

```
[Out] 4*(-1)**(1/3)*(x - 1)**(1/3)*gamma(4/3)/gamma(7/3) + 4*(-2)**(1/3)*exp(-I*p
i/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi/3)/2 + 1)*gamma(4/3)/(3*ga
mma(7/3)) - 4*(-2)**(1/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi)/2 +
1)*gamma(4/3)/(3*gamma(7/3)) + 4*(-2)**(1/3)*exp(I*pi/3)*log(-2**(2/3)*(x -
 1)**(1/3)*exp_polar(5*I*pi/3)/2 + 1)*gamma(4/3)/(3*gamma(7/3))
```

Giac [A]
time = 0.98, size = 87, normalized size = 1.04

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x+1\right)^{\frac{1}{3}}+\left(-x+1\right)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+\left(-x+1\right)^{\frac{1}{3}}\right|\right)+3\left(-x+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(abs(-2^(1/3) + (-x + 1)^(1/3))) + 3*(-x + 1)^(1/3)
```

Mupad [B]
time = 0.07, size = 104, normalized size = 1.24

$$2^{1/3}\ln\left(18\left(1-x\right)^{1/3}-18\,2^{1/3}\right)+3\left(1-x\right)^{1/3}+\frac{2^{1/3}\ln\left(18\left(1-x\right)^{1/3}-9\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{2^{1/3}\ln\left(18\left(1-x\right)^{1/3}+9\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)/(x + 1),x)
```

```
[Out] 2^(1/3)*log(18*(1 - x)^(1/3) - 18*2^(1/3)) + 3*(1 - x)^(1/3) + (2^(1/3)*log
(18*(1 - x)^(1/3) - 9*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/2 - (2^(1
/3)*log(18*(1 - x)^(1/3) + 9*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/2
```

3.1456 $\int \sqrt[3]{3-2x}\,(7+x)\,dx$

Optimal. Leaf size=27

$$-\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}$$

```
[Out] -51/16*(3-2*x)^(4/3)+3/28*(3-2*x)^(7/3)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(1/3)*(7 + x),x]
[Out] (-51*(3 - 2*x)^(4/3))/16 + (3*(3 - 2*x)^(7/3))/28
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{3-2x}\,(7+x)\,dx &= \int \left(\frac{17}{2}\sqrt[3]{3-2x}-\frac{1}{2}(3-2x)^{4/3}\right)dx\\ &= -\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.67

$$-\frac{3}{112}(3-2x)^{4/3}(107+8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(1/3)*(7 + x),x]
[Out] (-3*(3 - 2*x)^(4/3)*(107 + 8*x))/112
```

Maple [A]
time = 0.15, size = 20, normalized size = 0.74

method	result	size
gosper	$-\frac{3(8x+107)(3-2x)^{\frac{4}{3}}}{112}$	15
trager	$\left(\frac{3}{7}x^{2}+\frac{285}{56}x-\frac{963}{112}\right)(3-2x)^{\frac{1}{3}}$	19
derivativedivides	$-\frac{51(3-2x)^{\frac{4}{3}}}{16}+\frac{3(3-2x)^{\frac{7}{3}}}{28}$	20
default	$-\frac{51(3-2x)^{\frac{4}{3}}}{16}+\frac{3(3-2x)^{\frac{7}{3}}}{28}$	20
risch	$-\frac{3(16x^{2}+190x-321)(2x-3)}{112(3-2x)^{\frac{2}{3}}}$	25
meijerg	$7\,3^{\frac{1}{3}}x\,\text{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],\frac{2x}{3}\right)+\frac{3^{\frac{1}{3}}x^{2}\,\text{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],\frac{2x}{3}\right)}{2}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(1/3)*(7+x),x,method=_RETURNVERBOSE)
[Out] -51/16*(3-2*x)^(4/3)+3/28*(3-2*x)^(7/3)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.70

$$\frac{3}{28}\,(-2\,x+3)^{\frac{7}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="maxima")
[Out] 3/28*(-2*x + 3)^(7/3) - 51/16*(-2*x + 3)^(4/3)
```

Fricas [A]
time = 0.78, size = 19, normalized size = 0.70

$$\frac{3}{112}\,\left(16\,x^{2}+190\,x-321\right)(-2\,x+3)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="fricas")
[Out] 3/112*(16*x^2 + 190*x - 321)*(-2*x + 3)^(1/3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.52, size = 112, normalized size = 4.15

$$\begin{cases} \frac{3(x+7)^{2}\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{7}-\frac{51(x+7)\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{56}-\frac{2601\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{112} & \text{for}\ |x+7|>\frac{17}{2} \\ \frac{3\sqrt[3]{3-2x}\,(x+7)^{2}}{7}-\frac{51\sqrt[3]{3-2x}\,(x+7)}{56}-\frac{2601\sqrt[3]{3-2x}}{112} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(1/3)*(7+x),x)
```

```
[Out] Piecewise((3*(x + 7)**2*(2*x - 3)**(1/3)*exp(I*pi/3)/7 - 51*(x + 7)*(2*x -
3)**(1/3)*exp(I*pi/3)/56 - 2601*(2*x - 3)**(1/3)*exp(I*pi/3)/112, Abs(x + 7
) > 17/2), (3*(3 - 2*x)**(1/3)*(x + 7)**2/7 - 51*(3 - 2*x)**(1/3)*(x + 7)/5
6 - 2601*(3 - 2*x)**(1/3)/112, True))
```

Giac [A]
time = 1.23, size = 26, normalized size = 0.96

$$\frac{3}{28}\left(2x-3\right)^{2}\left(-2x+3\right)^{\frac{1}{3}}-\frac{51}{16}\left(-2x+3\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="giac")
```

```
[Out] 3/28*(2*x - 3)^2*(-2*x + 3)^(1/3) - 51/16*(-2*x + 3)^(4/3)
```

Mupad [B]
time = 0.26, size = 14, normalized size = 0.52

$$-\frac{3\left(3-2x\right)^{4/3}\left(8x+107\right)}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 2*x)^(1/3)*(x + 7),x)
```

```
[Out] -(3*(3 - 2*x)^(4/3)*(8*x + 107))/112
```

3.1457 $\int \sqrt[3]{1-x}\,(1+x)^2\,dx$

Optimal. Leaf size=38

$$-3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}$$

```
[Out] -3*(1-x)^(4/3)+12/7*(1-x)^(7/3)-3/10*(1-x)^(10/3)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{45\}$

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)*(1 + x)^2,x]
```

```
[Out] -3*(1 - x)^(4/3) + (12*(1 - x)^(7/3))/7 - (3*(1 - x)^(10/3))/10
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{1-x}\,(1+x)^2\,dx &= \int \left(4\sqrt[3]{1-x}-4(1-x)^{4/3}+(1-x)^{7/3}\right)\,dx\\ &= -3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.61

$$-\frac{3}{70}(1-x)^{4/3}\left(37+26x+7x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)*(1 + x)^2,x]
```

[Out] (-3*(1 - x)^(4/3)*(37 + 26*x + 7*x^2))/70

Maple [A]
time = 0.17, size = 29, normalized size = 0.76

method	result
gosper	$-\frac{3(7x^2+26x+37)(1-x)^{\frac{4}{3}}}{70}$
trager	$\left(\frac{3}{10}x^3+\frac{57}{70}x^2+\frac{33}{70}x-\frac{111}{70}\right)(1-x)^{\frac{1}{3}}$
risch	$-\frac{3(7x^3+19x^2+11x-37)(-1+x)}{70(1-x)^{\frac{2}{3}}}$
derivativedivides	$-3(1-x)^{\frac{4}{3}}+\frac{12(1-x)^{\frac{7}{3}}}{7}-\frac{3(1-x)^{\frac{10}{3}}}{10}$
default	$-3(1-x)^{\frac{4}{3}}+\frac{12(1-x)^{\frac{7}{3}}}{7}-\frac{3(1-x)^{\frac{10}{3}}}{10}$
meijerg	$x\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},1\right],[2],x\right)+x^2\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},2\right],[3],x\right)+\frac{x^3\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},3\right],[4],x\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)^(1/3)*(1+x)^2,x,method=_RETURNVERBOSE)

[Out] -3*(1-x)^(4/3)+12/7*(1-x)^(7/3)-3/10*(1-x)^(10/3)

Maxima [A]
time = 0.27, size = 28, normalized size = 0.74

$$-\frac{3}{10}(-x+1)^{\frac{10}{3}}+\frac{12}{7}(-x+1)^{\frac{7}{3}}-3(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="maxima")

[Out] -3/10*(-x + 1)^(10/3) + 12/7*(-x + 1)^(7/3) - 3*(-x + 1)^(4/3)

Fricas [A]
time = 0.71, size = 24, normalized size = 0.63

$$\frac{3}{70}\left(7\,x^3+19\,x^2+11\,x-37\right)(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="fricas")

[Out] 3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(-x + 1)^(1/3)

Sympy [C] Result contains complex when optimal does not.
time = 0.69, size = 144, normalized size = 3.79

$$\begin{cases} -\frac{3\sqrt[3]{x-1}\,(x+1)^3e^{-\frac{2i\pi}{3}}}{10}+\frac{3\sqrt[3]{x-1}\,(x+1)^2e^{-\frac{2i\pi}{3}}}{35}+\frac{9\sqrt[3]{x-1}\,(x+1)e^{-\frac{2i\pi}{3}}}{35}+\frac{54\sqrt[3]{x-1}\,e^{-\frac{2i\pi}{3}}}{35} & \text{for } |x+1|>2 \\ \frac{3\sqrt[3]{1-x}\,(x+1)^3}{10}-\frac{3\sqrt[3]{1-x}\,(x+1)^2}{35}-\frac{9\sqrt[3]{1-x}\,(x+1)}{35}-\frac{54\sqrt[3]{1-x}}{35} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)*(1+x)**2,x)
```

```
[Out] Piecewise((-3*(x - 1)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(x - 1)**(1/3
)*(x + 1)**2*exp(-2*I*pi/3)/35 + 9*(x - 1)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35
 + 54*(x - 1)**(1/3)*exp(-2*I*pi/3)/35, Abs(x + 1) > 2), (3*(1 - x)**(1/3)*
(x + 1)**3/10 - 3*(1 - x)**(1/3)*(x + 1)**2/35 - 9*(1 - x)**(1/3)*(x + 1)/3
5 - 54*(1 - x)**(1/3)/35, True))
```

Giac [A]
time = 1.84, size = 38, normalized size = 1.00

$$\frac{3}{10}\,(x-1)^{3}(-x+1)^{\frac{1}{3}}+\frac{12}{7}\,(x-1)^{2}(-x+1)^{\frac{1}{3}}-3\,(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 3/10*(x - 1)^3*(-x + 1)^(1/3) + 12/7*(x - 1)^2*(-x + 1)^(1/3) - 3*(-x + 1)^
(4/3)
```

Mupad [B]
time = 0.05, size = 21, normalized size = 0.55

$$-\frac{3\,(1-x)^{4/3}\,\left(40\,x+7\,(x-1)^2+30\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)*(x + 1)^2,x)
```

```
[Out] -(3*(1 - x)^(4/3)*(40*x + 7*(x - 1)^2 + 30))/70
```

3.1458 $\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/2*ln(b*x+a)/b^(2/3)/(-a*d+b*c)^(1/3)+3/2*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))/b^(2/3)/(-a*d+b*c)^(1/3)+arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/
(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/(-a*d+b*c)^(1/3)
```

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {57, 631, 210, 31}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(b^(2/3)*(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3))
 + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a
*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx = -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2b} - \text{—}$$

$$= -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}} - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{b^{2/3}\sqrt[3]{b}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}} - \frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{\cdot}\right.}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A]
time = 0.20, size = 154, normalized size = 1.11

$$\frac{-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{-bc+ad}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{-bc+ad}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)+\log\left((-bc+ad)^{2/3}-\sqrt[3]{b}\sqrt[3]{-bc+ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)}{2b^{2/3}\sqrt[3]{-bc+ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/S
qrt[3]] - 2*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)] + Log[(-(b*
c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
 + d*x)^(2/3)])/(2*b^(2/3)*(-(b*c) + a*d)^(1/3))
```

Maple [A]
time = 0.17, size = 161, normalized size = 1.16

method	result
derivativedivides	$-\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)}\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}$
default	$-\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)}\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/((a*d-b*c)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/2/b/((a*d-
b*c)/b)^(1/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)
/b)^(2/3))+3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b
)^(1/3)*(d*x+c)^(1/3)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(108) = 216.
time = 0.84, size = 570, normalized size = 4.10

$$\left[\frac{\sqrt{3}\,(b^2c-abd)\sqrt{-\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2b^2dx+3b^2c-abd-\sqrt{3}\left((b^3c-ab^2d)^{\frac{1}{3}}(bc-ad)+(b^2c-abd)(dx+c)^{\frac{1}{3}}-2(b^3c-ab^2d)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\right)\sqrt{-\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}-3(b^3c-ab^2d)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)-(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^2+(b^3c-ab^2d)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{2}{3}}\right)+2(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-(b^3c-ab^2d)^{\frac{1}{3}}\right)}{2(b^3c-ab^2d)}, \frac{2\sqrt{3}\,(b^2c-abd)\sqrt{\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{3}\left(2(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{1}{3}}\right)\sqrt{\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}}{3b}\right)-(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^2+(b^3c-ab^2d)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{2}{3}}\right)+2(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-(b^3c-ab^2d)^{\frac{1}{3}}\right)}{2(b^3c-ab^2d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))*lo
g((2*b^2*d*x + 3*b^2*c - a*b*d - sqrt(3)*((b^3*c - a*b^2*d)^(1/3)*(b*c - a*
d) + (b^2*c - a*b*d)*(d*x + c)^(1/3) - 2*(b^3*c - a*b^2*d)^(2/3)*(d*x + c)^
(2/3))*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d)) - 3*(b^3*c - a*b^2*d)^(2/
3)*(d*x + c)^(1/3))/(b*x + a)) - (b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(2/3
)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)*b + (b^3*c - a*b^2*d)^(2/3)
) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/3)*b - (b^3*c - a*b^2*d)^(1/
3)))/(b^3*c - a*b^2*d), 1/2*(2*sqrt(3)*(b^2*c - a*b*d)*sqrt((b^3*c - a*b^2*
d)^(1/3)/(b*c - a*d))*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3)*b + (b^3*c - a*
b^2*d)^(1/3))*sqrt((b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))/b) - (b^3*c - a*b^2
*d)^(2/3)*log((d*x + c)^(2/3)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)
*b + (b^3*c - a*b^2*d)^(2/3)) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/
3)*b - (b^3*c - a*b^2*d)^(1/3)))/(b^3*c - a*b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)\sqrt[3]{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(1/3)), x)
```

Giac [A]
time = 1.52, size = 196, normalized size = 1.41

$$\frac{3\left(b^3c-ab^2d\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^3c-\sqrt{3}\,ab^2d}-\frac{\log\left((dx+c)^{\frac{2}{3}}+(dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^3c-ab^2d\right)^{\frac{1}{3}}}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\log\left(\left|(dx+c)^{\frac{1}{3}}-\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] 3*(b^3*c - a*b^2*d)^(2/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a
*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b^2*d) - 1/
2*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d
)/b)^(2/3))/(b^3*c - a*b^2*d)^(1/3) + ((b*c - a*d)/b)^(2/3)*log(abs((d*x +
c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

Mupad [B]
time = 0.21, size = 204, normalized size = 1.47

$$\frac{\ln\left(9\,b\,{\left(c+d\,x\right)}^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{4/3}\,{\left(b\,c-a\,d\right)}^{2/3}}\right)}{b^{2/3}\,{\left(b\,c-a\,d\right)}^{1/3}}+\frac{\ln\left(9\,b\,{\left(c+d\,x\right)}^{1/3}-\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{4\,b^{4/3}\,{\left(b\,c-a\,d\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{2/3}\,{\left(b\,c-a\,d\right)}^{1/3}}-\frac{\ln\left(9\,b\,{\left(c+d\,x\right)}^{1/3}-\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{4\,b^{4/3}\,{\left(b\,c-a\,d\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{2/3}\,{\left(b\,c-a\,d\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/3)),x)
```

```
[Out] log(9*b*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(4/3)*(b*c - a*d)^(2/3))
)/(b^(2/3)*(b*c - a*d)^(1/3)) + (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i - 1
)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i - 1))
/(2*b^(2/3)*(b*c - a*d)^(1/3)) - (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i +
1)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i + 1)
)/(2*b^(2/3)*(b*c - a*d)^(1/3))
```

3.1459 $\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}$$

```
[Out] -1/2*ln(b*x+a)/b^(1/3)/(-a*d+b*c)^(2/3)+3/2*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))/b^(1/3)/(-a*d+b*c)^(2/3)-arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/
(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)/(-a*d+b*c)^(2/3)
```

Rubi [A]
time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {59, 631, 210, 31}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx &= -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\text{Subst}\left(\int \right.}{} \\ &= -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{\sqrt[3]{b}\,(b} \\ &= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{\right.}{2\sqrt[3]{b}\,(bc-ad}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 154, normalized size = 1.10

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{-bc+ad}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{-bc+ad}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)+\log\left((-bc+ad)^{2/3}-\sqrt[3]{b}\sqrt[3]{-bc+ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)}{2\sqrt[3]{b}\,(-bc+ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3
))/Sqrt[3]] - 2*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)] + Log[(
-(b*c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3
)*(c + d*x)^(2/3)])/(b^(1/3)*(-(b*c) + a*d)^(2/3))
```

Maple [A]
time = 0.17, size = 160, normalized size = 1.14

method	result
derivativedivides	$\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}$
default	$\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{b\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b/((a*d-b*c)/b)^(2/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))-1/2/b/((a*d-b
*c)/b)^(2/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/
b)^(2/3))+1/b/((a*d-b*c)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/
b)^(1/3)*(d*x+c)^(1/3)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(109) = 218.
time = 0.80, size = 900, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1
/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + sq
rt(3)*(2*(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)
^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + (b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3)
 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2), -1/2*(2*sqrt(3)*
(b^2*c - a*b*d)*sqrt((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*arctan(1/
3*sqrt(3)*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) + 2*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)^(1/3)/b)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(
2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(2/3)), x)
```

Giac [A]
time = 0.76, size = 207, normalized size = 1.48

$$-\frac{3\,(b^3c-ab^2d)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c-\sqrt{3}\,abd}-\frac{(b^3c-ab^2d)^{\frac{1}{3}}\log\left((dx+c)^{\frac{2}{3}}+(dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\,(b^2c-abd)}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\log\left(\left|(dx+c)^{\frac{1}{3}}-\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] -3*(b^3*c - a*b^2*d)^(1/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c -
a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b*d) - 1/2
*(b^3*c - a*b^2*d)^(1/3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)
```

```
/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/(b^2*c - a*b*d) + ((b*c - a*d)/b)^(1/3)*
log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

Mupad [B]
time = 0.37, size = 206, normalized size = 1.47

$$\frac{\ln\left(9\,b^2\,{\left(c+d\,x\right)}^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(9\,b^2\,{\left(c+d\,x\right)}^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{\ln\left(9\,b^2\,{\left(c+d\,x\right)}^{1/3}+\frac{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(2/3)),x)
```

```
[Out] log(9*b^2*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(1/3)*(a*d - b*c)^(2/3
)))/(b^(1/3)*(a*d - b*c)^(2/3)) + (log(9*b^2*(c + d*x)^(1/3) - ((3^(1/2)*1i
 - 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i - 1
))/(2*b^(1/3)*(a*d - b*c)^(2/3)) - (log(9*b^2*(c + d*x)^(1/3) + ((3^(1/2)*1
i + 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i +
1))/(2*b^(1/3)*(a*d - b*c)^(2/3))
```

3.1460 $\int (a+bx)^{7/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=230

$$-\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4}+\frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3}-\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2}+\frac{(bc-$$

```
[Out] 7/128*(-a*d+b*c)^5*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(
3/2)/d^(9/2)+7/192*(-a*d+b*c)^3*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d^3-7/240*(-a
*d+b*c)^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b/d^2+1/40*(-a*d+b*c)*(b*x+a)^(7/2)*(
d*x+c)^(1/2)/b/d+1/5*(b*x+a)^(9/2)*(d*x+c)^(1/2)/b-7/128*(-a*d+b*c)^4*(b*x+
a)^(1/2)*(d*x+c)^(1/2)/b/d^4
```

Rubi [A]
time = 0.11, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{240bd^2}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40bd}+\frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] (-7*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b*d^4) + (7*(b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(192*b*d^3) - (7*(b*c - a*d)^2*(a + b*x)^
(5/2)*Sqrt[c + d*x])/(240*b*d^2) + ((b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*
x])/(40*b*d) + ((a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b) + (7*(b*c - a*d)^5*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(3/2)*d^(9/2)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{7/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} + \frac{(bc-ad)\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx}{10b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} - \frac{(7(bc-ad)^2)\int \frac{(a+bx}{\sqrt{c+}}}{80bd}$$

$$= -\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)}{}$$

$$= \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-a}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-}{}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-}{}$$

Mathematica [A]

time = 0.45, size = 166, normalized size = 0.72

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(105d^4(a+bx)^4+790bd^3(a+bx)^3(c+dx)-896b^2d^2(a+bx)^2(c+dx)^2+490b^3d(a+bx)(c+dx)^3-105b^4(c+dx)^4)}{1920bd^4}+\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{128b^{3/2}d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(105*d^4*(a + b*x)^4 + 790*b*d^3*(a + b*x)^3*(
c + d*x) - 896*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 490*b^3*d*(a + b*x)*(c + d
*x)^3 - 105*b^4*(c + d*x)^4))/(1920*b*d^4) + (7*(b*c - a*d)^5*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(3/2)*d^(9/2))
```

Maple [A]
time = 0.16, size = 239, normalized size = 1.04

method	result

default	$\frac{(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}}{5d} - \frac{7(-ad+bc)\left(\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}{4d} - \frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d} - \frac{(-ad+bc)\left(\frac{\sqrt{}}{}\right.}{}\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(b*x+a)^(7/2)*(d*x+c)^(3/2)-7/10*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(5/2)*(d
*x+c)^(3/2)-5/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)-1/2*(-a*d+b
*c)/d*(1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d
*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)
^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/
2))/(b*d)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 0.70, size = 702, normalized size = 3.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*
x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 49
0*a*b^4*c^3*d^2 - 896*a^2*b^3*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 +
 48*(b^5*c*d^4 + 31*a*b^4*d^5)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 26
3*a^2*b^3*d^5)*x^2 + 2*(35*b^5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*
d^4 + 605*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^5), -1/3840*(
105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*
d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x)) - 2*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 490*a*b^4*c^3*d^2 - 896*a^2*b^3
*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 + 48*(b^5*c*d^4 + 31*a*b^4*d^5
)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 263*a^2*b^3*d^5)*x^2 + 2*(35*b^
5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*d^4 + 605*a^3*b^2*d^5)*x)*sqr
t(b*x + a)*sqrt(d*x + c))/(b^2*d^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. 2(186) = 372.
time = 0.85, size = 1107, normalized size = 4.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="giac")

[Out] 1/1920*(480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*abs(b) - 1920*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^4*abs(b)/b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*b*abs(b) + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*b^2*abs(b) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*abs(b)/b^2)/b

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{7/2}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(7/2)*(c + d*x)^(1/2),x)

[Out] int((a + b*x)^(7/2)*(c + d*x)^(1/2), x)

3.1461 $\int (a+bx)^{5/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=192

$$\frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3}-\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2}+\frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd}+\frac{(a+bx)^{7/}}{4}$$

```
[Out] -5/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(
3/2)/d^(7/2)-5/96*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d^2+1/24*(-a*d
+b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b/d+1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b+5/6
4*(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^3
```

Rubi [A]
time = 0.07, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24bd}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b*d^3) - (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*b*d^2) + ((b*c - a*d)*(a + b*x)^(5/2)*S
qrt[c + d*x])/(24*b*d) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - (5*(b*c -
a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(3/2
)*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} - \frac{(5(bc-ad)^2)\int \frac{(a+bx}{\sqrt{c+}}$$

$$= -\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)}{}$$

Mathematica [A]
time = 0.36, size = 144, normalized size = 0.75

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(15d^3(a+bx)^3+73bd^2(a+bx)^2(c+dx)-55b^2d(a+bx)(c+dx)^2+15b^3(c+dx)^3)}{192bd^3} - \frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^3*(a + b*x)^3 + 73*b*d^2*(a + b*x)^2*(c
+ d*x) - 55*b^2*d*(a + b*x)*(c + d*x)^2 + 15*b^3*(c + d*x)^3))/(192*b*d^3)
- (5*(b*c - a*d)^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])]
)/(64*b^(3/2)*d^(7/2))
```

Maple [A]
time = 0.18, size = 206, normalized size = 1.07

method	result
default	$\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}{4d}-\frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-b}{}\right.}{}\right.}{}\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)-5/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(3/2)*(d*
x+c)^(3/2)-1/2*(-a*d+b*c)/d*(1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*
c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)
/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^
2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more detail

Fricas [A]
time = 0.57, size = 540, normalized size = 2.81

$$\left[\frac{15\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(48\,b^4d^4x^3+15\,b^4c^3d-55\,ab^3c^2d^2+73\,a^2b^2cd^3+15\,a^3bd^4+8\,(b^4cd^3+17\,ab^3d^4)x^2-2\,(5\,b^4c^2d^2-18\,ab^3cd^3-59\,a^2b^2d^4)x)\sqrt{bx+a}\sqrt{dx+c}}{768\,b^2d^4},\ \frac{15\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(48\,b^4d^4x^3+15\,b^4c^3d-55\,ab^3c^2d^2+73\,a^2b^2cd^3+15\,a^3bd^4+8\,(b^4cd^3+17\,ab^3d^4)x^2-2\,(5\,b^4c^2d^2-18\,ab^3cd^3-59\,a^2b^2d^4)x)\sqrt{bx+a}\sqrt{dx+c}}{384\,b^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2
 - 18*a*b^3*c*d^3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^
4), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2 - 18*a*b^3*c*d^
3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(154) = 308.
time = 0.97, size = 726, normalized size = 3.78

$$\frac{24\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{b^6cd^3-13\,ab^5d^4}{b^7d^4}\right)-\frac{3\,(b^7c^2d^2+2\,ab^6cd^3-11\,a^2b^5d^4)}{b^7d^4}\right)-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)a|b|-\frac{192\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)a^3|b|}{b^2}+\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)}{b^3}+\frac{b^{13}cd^5-25\,ab^{12}d^6}{b^{14}d^6}\right)-\frac{5\,b^{13}c^2d^4+14\,ab^{12}cd^5-163\,a^2b^{11}d^6}{b^{14}d^6}\right)+\frac{3\,(5\,b^{14}c^3d^3+9\,ab^{13}c^2d^4+15\,a^2b^{12}cd^5-93\,a^3b^{11}d^6)}{b^{14}d^6}\right)\sqrt{bx+a}+\frac{3\,(5\,b^4c^4+4\,ab^3c^3d+6\,a^2b^2c^2d^2+20\,a^3bcd^3-35\,a^4d^4)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,b^2d^3}\right)b|b|+\frac{144\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)a^2|b|}{b^2}}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
```

```
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*abs(b) - 192*((b^2*c - a*b*d)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt
(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*abs(b)/b^2 +
 (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a
)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b
^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^
2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*
(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^
4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*b^2*d^3))*b*abs(b) + 144*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(
2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
 - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)))/(sqrt(b*d)*d))*a^2*abs(b)/b^2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{5/2}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(1/2), x)
```

3.1462 $\int (a+bx)^{3/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=154

$$-\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2}+\frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}+\frac{(bc-ad)^3\tanh^{-1}\left(\ldots\right.}{8b^{3/2}d\ldots}$$

```
[Out] 1/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/
2)/d^(5/2)+1/12*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d+1/3*(b*x+a)^(5/2
)*(d*x+c)^(1/2)/b-1/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^2
```

Rubi [A]
time = 0.05, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -1/8*((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(b*d^2) + ((b*c - a*d)*(a
+ b*x)^(3/2)*Sqrt[c + d*x])/(12*b*d) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b
) + ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(8*b^(3/2)*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} - \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{8bd}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}}{3b}$$

Mathematica [**A**]
time = 0.27, size = 128, normalized size = 0.83

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(3a^2d^2+2abd(4c+7dx)+b^2(-3c^2+2cdx+8d^2x^2))}{24bd^2} + \frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{8b^{3/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(3*a^2*d^2 + 2*a*b*d*(4*c + 7*d*x) + b^2*(-3*c
^2 + 2*c*d*x + 8*d^2*x^2)))/(24*b*d^2) + ((b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*b^(3/2)*d^(5/2))
```

Maple [A]
time = 0.15, size = 173, normalized size = 1.12

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)} \ldots}{2b\sqrt{\ldots}}\right.}{4d}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)-1/2*(-a*d+b*c)/d*(1/2/d*(b*x+a)^(1/2)*(d*
x+c)^(3/2)-1/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*
((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d
*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.93, size = 410, normalized size = 2.66

$$\left[-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2-3\,b^3c^2d+8\,ab^2cd^2+3\,a^2bd^3+2\,(b^3cd^2+7\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,b^2d^3},-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(8\,b^3d^3x^2-3\,b^3c^2d+8\,ab^2cd^2+3\,a^2bd^3+2\,(b^3cd^2+7\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
```

```
^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b*d^3 + 2*(b^3*c*d^2 + 7*a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3), -1/48*(3*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a
*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b
*d^3 + 2*(b^3*c*d^2 + 7*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3
)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(122) = 244.
time = 1.50, size = 438, normalized size = 2.84

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{b^6cd^3-13\,ab^5d^4}{b^7d^4}\right)-\frac{3\,(b^7c^2d^2+2\,ab^6cd^3-11\,a^2b^5d^4)}{b^7d^4}\right)-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)|b|-\frac{24\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a^2|b|}{b^2}+\frac{12\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^3c^2+2\,ab^2cd-3\,a^2bd^2)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)a|b|}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*abs(b) - 24*((b^2*c - a*b*d)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*abs(b)/b^2 + 12*(sq
rt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqr
t(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a*abs(b)/b^
2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/2), x)
```

3.1463 $\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$\frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

```
[Out] -1/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3
/2)/d^(3/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b+1/4*(-a*d+b*c)*(b*x+a)^(1/2)*
(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2 \int \frac{1}{\sqrt{a+bx}\,\sqrt{c \ldots}}}{8bd}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{\ldots}{\sqrt{c \ldots}} \ldots\right.}{\ldots}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}} \ldots\right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\ldots}{\sqrt{b}\ldots}\right.}{4b^{3/2}d^{3/2}}$$

Mathematica [A]
time = 0.08, size = 95, normalized size = 0.82

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(ad+b(c+2dx))}{4bd} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(a*d + b*(c + 2*d*x)))/(4*b*d) - ((b*c - a*d)^
2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^(3
/2))
```

Maple [A]
time = 0.16, size = 140, normalized size = 1.21

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}-\frac{(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x}\right.}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)-1/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(
1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*
ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*
d)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.70, size = 300, normalized size = 2.59

$$\left[\frac{(b^2c^2-2abcd+a^2d^2)\sqrt{bd}\,\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8(b^2cd+abd^2)x\right)+4(2b^2d^2x+b^2cd+abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16b^2d^2},\ \frac{(b^2c^2-2abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2(2b^2d^2x+b^2cd+abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(90) = 180.
time = 0.88, size = 232, normalized size = 2.00

$$-\frac{\frac{4\left(\frac{\left(b^{2}c-abd\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^{2}c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a|b|}{b^{2}}-\frac{\left(\sqrt{b^{2}c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^{2}}{d^{2}}\right)\sqrt{bx+a}+\frac{\left(b^{3}c^{2}+2\,ab^{2}cd-3\,a^{2}bd^{2}\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)|b|}{b^{2}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

Mupad [B]
time = 0.14, size = 88, normalized size = 0.76

$$\left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

```
[Out] (x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*
c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c
)^2)/(8*b^(3/2)*d^(3/2))
```

3.1464 $\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}$$

```
[Out] (-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)/d^(
1/2)+(b*x+a)^(1/2)*(d*x+c)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx &= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^2}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 74, normalized size = 1.03

$$\frac{b\sqrt{a+bx}\,\sqrt{c+dx}+\sqrt{\frac{b}{d}}\,(-bc+ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x] + Sqrt[b/d]*(-(b*c) + a*d)*Log[Sqrt[a + b*x]
 - Sqrt[b/d]*Sqrt[c + d*x]])/b^2
```

Maple [A]
time = 0.16, size = 107, normalized size = 1.49

method	result	s
default	$\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}+(ad+bc)\,x+ac}\right)}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.67, size = 236, normalized size = 3.28

$$\left[\frac{4\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{2}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^{2}cd+abd^{2})x\right)}{4\,b^{2}d},\ \frac{2\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^{2}d^{2}x^{2}+abcd+(b^{2}cd+abd^{2})x)}\right)}{2\,b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b^2*d), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/sqrt(a + b*x), x)
```

Giac [A]
time = 0.99, size = 93, normalized size = 1.29

$$-\frac{\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
a))*abs(b)/b^3
```

Mupad [B]
time = 4.01, size = 260, normalized size = 3.61

$$\frac{\frac{(2ad+2bc)\left(\sqrt{a+bx}-\sqrt{a}\right)}{d^2\left(\sqrt{c+dx}-\sqrt{c}\right)}+\frac{(2ad+2bc)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{bd\left(\sqrt{c+dx}-\sqrt{c}\right)^3}-\frac{8\sqrt{a}\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2b\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{c+dx}-\sqrt{c}\right)}\right)(ad-bc)}{b^{3/2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2)))/(d^2*((c + d*x)^(1/2) - c^(1
/2))) + ((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*d*((c + d*x)^(1/
2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c
 + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d*x)^(1/2
) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c + d*x
)^(1/2) - c^(1/2))^2)) - (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^
(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(3/2)*d^(1/2))
```

3.1465 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}$$

```
[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*d^(1/2)/b^(3/2)-2*(d
*x+c)^(1/2)/b/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 223, 212}

$$\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b}\\ &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^2}\\ &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}\\ &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 1.00

$$-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(3/2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(50) = 100$.
time = 0.70, size = 241, normalized size = 3.65

$$\left[\frac{(bx+a)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2x+ab)},\,-\frac{(bx+a)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)+2\,\sqrt{bx+a}\,\sqrt{dx+c}}{b^2x+ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x + a)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*x + a*b), -((b
*x + a)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(b^2*x + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(50) = 100.

time = 0.81, size = 131, normalized size = 1.98

$$-\frac{\left(\frac{\sqrt{bd}\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b}+\frac{4\left(\sqrt{bd}\,bc-\sqrt{bd}\,ad\right)}{b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(b*d)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^2)/b + 4*(sqrt(b*d)*b*c - sqrt(b*d)*a*d)/(b^2*c - a*b*d - (sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2))*abs(b)/b^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c+d\,x}}{{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/2)/(a + b*x)^(3/2), x)
```

3.1466 $\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

```
[Out] -2/3*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.
time = 0.16, size = 88, normalized size = 2.75

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}}{3(bx+a)^{\frac{3}{2}}(ad-bc)}$	27
default	$-\frac{\sqrt{dx+c}}{b(bx+a)^{\frac{3}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{2b}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(d*x+c)^(1/2)/(b*x+a)^(3/2)+1/2*(a*d-b*c)/b*(-2/3*(d*x+c)^(1/2)/(-a*d+
b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.74, size = 65, normalized size = 2.03

$$-\frac{2\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{3\left(a^2bc-a^3d+(b^3c-ab^2d)x^2+2\,(ab^2c-a^2bd)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)*(d*x + c)^(3/2)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2
 + 2*(a*b^2*c - a^2*b*d)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(26) = 52.
time = 1.09, size = 152, normalized size = 4.75

$$-\frac{4\left(\sqrt{bd}\,b^4c^2d-2\,\sqrt{bd}\,ab^3cd^2+\sqrt{bd}\,a^2b^2d^3+3\,\sqrt{bd}\,\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4d\right)|b|}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(sqrt(b*d)*b^4*c^2*d - 2*sqrt(b*d)*a*b^3*c*d^2 + sqrt(b*d)*a^2*b^2*d^3
 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*d)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^2)
```

Mupad [**B**]
time = 0.72, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{\left(3\,a\,d-3\,b\,c\right)\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/((3*a*d - 3*b*c)*(a + b*x)^(3/2))
```

3.1467 $\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}}+\frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}$$

```
[Out] -2/5*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(5/2)+4/15*d*(d*x+c)^(3/2)/(-a*d+b*c)
^2/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2)
)/(15*(b*c - a*d)^2*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{5(bc-ad)}$$
$$= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} + \frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{3/2}(-3bc+5ad+2bdx)}{15(bc-ad)^2(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3*b*c + 5*a*d + 2*b*d*x))/(15*(b*c - a*d)^2*(a + b*x)^
(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(54) = 108$.
time = 0.16, size = 128, normalized size = 1.94

method	result	size
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(2bdx+5ad-3bc)}{15(bx+a)^{\frac{5}{2}}(a^2d^2-2abcd+b^2c^2)}$	54
default	$-\frac{\sqrt{dx+c}}{2b(bx+a)^{\frac{5}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{4b}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*x+c)^(1/2)/(b*x+a)^(5/2)+1/4*(a*d-b*c)/b*(-2/5*(d*x+c)^(1/2)/(-a*
d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a
)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(54) = 108$.
time = 0.90, size = 175, normalized size = 2.65

$$\frac{2\,{\left(2\,b d^{2} x^{2} - 3\,b c^{2} + 5\,a c d - {\left(b c d - 5\,a d^{2}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{15\,{\left(a^{3} b^{2} c^{2} - 2\,a^{4} b c d + a^{5} d^{2} + {\left(b^{5} c^{2} - 2\,a b^{4} c d + a^{2} b^{3} d^{2}\right)} x^{3} + 3\,{\left(a b^{4} c^{2} - 2\,a^{2} b^{3} c d + a^{3} b^{2} d^{2}\right)} x^{2} + 3\,{\left(a^{2} b^{3} c^{2} - 2\,a^{3} b^{2} c d + a^{4} b d^{2}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b*d^2*x^2 - 3*b*c^2 + 5*a*c*d - (b*c*d - 5*a*d^2)*x)*sqrt(b*x + a)*
sqrt(d*x + c)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d x}}{\left(a + b x\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(7/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(54) = 108$.
time = 0.79, size = 447, normalized size = 6.77

$$\frac{8\,{\left(\sqrt{b d} b^{7} c^{3} d^{2} - 3\,\sqrt{b d} a b^{6} c^{2} d^{3} + 3\,\sqrt{b d} a^{2} b^{5} c d^{4} - \sqrt{b d} a^{3} b^{4} d^{5} - 5\,\sqrt{b d} {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2} b^{5} c^{2} d^{2} + 10\,\sqrt{b d} {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2} a b^{4} c d^{3} - 5\,\sqrt{b d} {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2} a^{2} b^{3} d^{4} - 5\,\sqrt{b d} {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{4} b^{3} c d^{2} + 5\,\sqrt{b d} {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{4} a b^{2} d^{3} - 15\,\sqrt{b d} {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{6} b d^{2}\right)} {\left| b \right|}}{15\,{\left(b^{2} c - a b d - {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2}\right)}^{5} b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(sqrt(b*d)*b^7*c^3*d^2 - 3*sqrt(b*d)*a*b^6*c^2*d^3 + 3*sqrt(b*d)*a^2*b
^5*c*d^4 - sqrt(b*d)*a^3*b^4*d^5 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c^2*d^2 + 10*sqrt(b*d)*(sqrt(b*d)
```

```
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^4*c*d^3 - 5*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
2*b^3*d^4 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*b^3*c*d^2 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a*b^2*d^3 - 15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b*d^2)*abs(b)/((b^2*c - a*b*d
 - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^2
)
```

Mupad [B]
time = 0.82, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(10\,a\,d^2-2\,b\,c\,d\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{6\,b\,c^2-10\,a\,c\,d}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,d^2\,x^2}{15\,b\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(7/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(10*a*d^2 - 2*b*c*d))/(15*b^2*(a*d - b*c)^2) - (6*b*c^
2 - 10*a*c*d)/(15*b^2*(a*d - b*c)^2) + (4*d^2*x^2)/(15*b*(a*d - b*c)^2)))/(
x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*x)^(1/2))/b
)
```

3.1468 $\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}$$

```
[Out] -2/7*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(7/2)+8/35*d*(d*x+c)^(3/2)/(-a*d+b*c)
^2/(b*x+a)^(5/2)-16/105*d^2*(d*x+c)^(3/2)/(-a*d+b*c)^3/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (8*d*(c + d*x)^(3/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(4d)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}$$
$$= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{(8d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx}{35(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.09, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(-3c+2dx)+b^2\left(15c^2-12cdx+8d^2x^2\right)\right)}{105(bc-ad)^3(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-3*c + 2*d*x) + b^2*(15*c^2 - 1
2*c*d*x + 8*d^2*x^2)))/(105*(b*c - a*d)^3*(a + b*x)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(83) = 166.
time = 0.16, size = 168, normalized size = 1.66

method	result
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(8b^2x^2d^2+28ab\,d^2x-12b^2cdx+35a^2d^2-42abcd+15b^2c^2\right)}{105(bx+a)^{\frac{7}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$
default	$-\frac{\sqrt{dx+c}}{3b(bx+a)^{\frac{7}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{7(-ad+bc)}\right)}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(d*x+c)^(1/2)/(b*x+a)^(7/2)+1/6*(a*d-b*c)/b*(-2/7*(d*x+c)^(1/2)/(-a*
d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a
```

```
)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d
*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(83) = 166$.
time = 1.51, size = 337, normalized size = 3.34

$$-\frac{2\,(8\,b^2d^3x^3+15\,b^2c^3-42\,abc^2d+35\,a^2cd^2-4\,(b^2cd^2-7\,abd^3)x^2+(3\,b^2c^2d-14\,abcd^2+35\,a^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{105\,(a^4b^3c^3-3\,a^5b^2c^2d+3\,a^6bcd^2-a^7d^3+(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)x^4+4\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)x^3+6\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)x^2+4\,(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^2*d^3*x^3 + 15*b^2*c^3 - 42*a*b*c^2*d + 35*a^2*c*d^2 - 4*(b^2*c
*d^2 - 7*a*b*d^3)*x^2 + (3*b^2*c^2*d - 14*a*b*c*d^2 + 35*a^2*d^3)*x)*sqrt(b
*x + a)*sqrt(d*x + c)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^
3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(83) = 166$.
time = 2.01, size = 689, normalized size = 6.82

$$-\frac{32\left(\sqrt{bd}\,b^{10}c^4d^3-4\,\sqrt{bd}\,ab^9c^3d^4+6\,\sqrt{bd}\,a^2b^8c^2d^5-4\,\sqrt{bd}\,a^3b^7cd^6+\sqrt{bd}\,a^4b^6d^7-7\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^8c^3d^3+21\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^7c^2d^4-21\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^6cd^5+7\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3b^5d^6+21\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^6c^2d^3-42\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^5cd^4+21\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^4d^5+35\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6b^4cd^3-35\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6ab^3d^4+70\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8b^2d^3\right)|b|}{105\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="giac")

[Out] -32/105*(sqrt(b*d)*b^10*c^4*d^3 - 4*sqrt(b*d)*a*b^9*c^3*d^4 + 6*sqrt(b*d)*a^2*b^8*c^2*d^5 - 4*sqrt(b*d)*a^3*b^7*c*d^6 + sqrt(b*d)*a^4*b^6*d^7 - 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8*c^3*d^3 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^7*c^2*d^4 - 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*d^5 + 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^5*d^6 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6*c^2*d^3 - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^5*c*d^4 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^5 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^4*c*d^3 - 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^3*d^4 + 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^2*d^3)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^2)

Mupad [B]
time = 0.97, size = 203, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{70\,a^2\,c\,d^2-84\,a\,b\,c^2\,d+30\,b^2\,c^3}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x\,\left(70\,a^2\,d^3-28\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^3\,x^3}{105\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^3}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(1/2)/(a + b*x)^(9/2),x)

[Out] ((c + d*x)^(1/2)*((30*b^2*c^3 + 70*a^2*c*d^2 - 84*a*b*c^2*d)/(105*b^3*(a*d - b*c)^3) + (x*(70*a^2*d^3 + 6*b^2*c^2*d - 28*a*b*c*d^2))/(105*b^3*(a*d - b*c)^3) + (16*d^3*x^3)/(105*b*(a*d - b*c)^3) + (8*d^2*x^2*(7*a*d - b*c))/(105*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)

3.1469 $\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{32d^3(c+dx)^{3/2}}{315(bc-ad)^4(a+bx)^{3/2}}$$

```
[Out] -2/9*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(9/2)+4/21*d*(d*x+c)^(3/2)/(-a*d+b*c)
^2/(b*x+a)^(7/2)-16/105*d^2*(d*x+c)^(3/2)/(-a*d+b*c)^3/(b*x+a)^(5/2)+32/315
*d^3*(d*x+c)^(3/2)/(-a*d+b*c)^4/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(3/2)
)/(21*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(5/2)) + (32*d^3*(c + d*x)^(3/2))/(315*(b*c - a*d)^4*(a +
 b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} + \frac{(8d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{21(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} - \frac{(16c}{}$$

$$= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}} + \frac{}{315(}$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 0.68

$$-\frac{2(c+dx)^{3/2}\left(-105d^3(a+bx)^3+189bd^2(a+bx)^2(c+dx)-135b^2d(a+bx)(c+dx)^2+35b^3(c+dx)^3\right)}{315(bc-ad)^4(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(-105*d^3*(a + b*x)^3 + 189*b*d^2*(a + b*x)^2*(c + d*x)
 - 135*b^2*d*(a + b*x)*(c + d*x)^2 + 35*b^3*(c + d*x)^3))/(315*(b*c - a*d)^
4*(a + b*x)^(9/2))
```

Maple [A]
time = 0.16, size = 208, normalized size = 1.53

method	result
gosper	$\frac{2(dx+c)^{\frac{3}{2}}\left(16b^3x^3d^3+72d^3a\,x^2b^2-24b^3c\,d^2x^2+126a^2b\,d^3x-108a\,b^2c\,d^2x+30b^3c^2dx+105a^3d^3-189a^2bc\,d^2+135a\,b^2c^2d-35b^3c^3\right)}{315(bx+a)^{\frac{9}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

default	$-\frac{\sqrt{dx+c}}{4b\left(bx+a\right)^{\frac{9}{2}}}+\frac{\left(ad-bc\right)\left(-\frac{2\sqrt{dx+c}}{9\left(-ad+bc\right)\left(bx+a\right)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7\left(-ad+bc\right)\left(bx+a\right)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5\left(-ad+bc\right)\left(bx+a\right)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3\left(-ad+bc\right)\left(bx+a\right)^{\frac{3}{2}}}+\frac{\cdots}{3(-\cdots}\right)}{5(-ad+b\cdots}\right)}{7\left(-ad+bc\right)\cdots}\right)}{9\left(-ad+bc\right)\cdots}\right)}{8b\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(d*x+c)^(1/2)/(b*x+a)^(9/2)+1/8*(a*d-b*c)/b*(-2/9*(d*x+c)^(1/2)/(-a*
d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a
)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d
/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2
)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(112) = 224$.
time = 4.56, size = 532, normalized size = 3.91

$$\frac{2\,{\left(16\,b^{3} d^{4} x^{4} - 35\,b^{3} c^{4} + 135\,a b^{2} c^{3} d - 189\,a^{2} b c^{2} d^{2} + 105\,a^{3} c d^{3} - 8\,{\left(b^{3} c d^{3} - 9\,a b^{2} d^{4}\right)} x^{3} + 6\,{\left(b^{3} c^{2} d^{2} - 6\,a b^{2} c d^{3} + 21\,a^{2} b d^{4}\right)} x^{2} - {\left(5\,b^{3} c^{3} d - 27\,a b^{2} c^{2} d^{2} + 63\,a^{2} b c d^{3} - 105\,a^{3} d^{4}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{315\,{\left(a^{5} b^{4} c^{4} - 4\,a^{6} b^{3} c^{3} d + 6\,a^{7} b^{2} c^{2} d^{2} - 4\,a^{8} b c d^{3} + a^{9} d^{4} + {\left(b^{9} c^{4} - 4\,a b^{8} c^{3} d + 6\,a^{2} b^{7} c^{2} d^{2} - 4\,a^{3} b^{6} c d^{3} + a^{4} b^{5} d^{4}\right)} x^{5} + 5\,{\left(a b^{8} c^{4} - 4\,a^{2} b^{7} c^{3} d + 6\,a^{3} b^{6} c^{2} d^{2} - 4\,a^{4} b^{5} c d^{3} + a^{5} b^{4} d^{4}\right)} x^{4} + 10\,{\left(a^{2} b^{7} c^{4} - 4\,a^{3} b^{6} c^{3} d + 6\,a^{4} b^{5} c^{2} d^{2} - 4\,a^{5} b^{4} c d^{3} + a^{6} b^{3} d^{4}\right)} x^{3} + 10\,{\left(a^{3} b^{6} c^{4} - 4\,a^{4} b^{5} c^{3} d + 6\,a^{5} b^{4} c^{2} d^{2} - 4\,a^{6} b^{3} c d^{3} + a^{7} b^{2} d^{4}\right)} x^{2} + 5\,{\left(a^{4} b^{5} c^{4} - 4\,a^{5} b^{4} c^{3} d + 6\,a^{6} b^{3} c^{2} d^{2} - 4\,a^{7} b^{2} c d^{3} + a^{8} b d^{4}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^3*d^4*x^4 - 35*b^3*c^4 + 135*a*b^2*c^3*d - 189*a^2*b*c^2*d^2 +
105*a^3*c*d^3 - 8*(b^3*c*d^3 - 9*a*b^2*d^4)*x^3 + 6*(b^3*c^2*d^2 - 6*a*b^2*
c*d^3 + 21*a^2*b*d^4)*x^2 - (5*b^3*c^3*d - 27*a*b^2*c^2*d^2 + 63*a^2*b*c*d^
3 - 105*a^3*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 - 4*a*b^8*c^3*
d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5*(a*b^8*c^4 -
 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^4 +
 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a
^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(112) = 224.
time = 1.93, size = 989, normalized size = 7.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 64/315*(sqrt(b*d)*b^13*c^5*d^4 - 5*sqrt(b*d)*a*b^12*c^4*d^5 + 10*sqrt(b*d)*
a^2*b^11*c^3*d^6 - 10*sqrt(b*d)*a^3*b^10*c^2*d^7 + 5*sqrt(b*d)*a^4*b^9*c*d^
8 - sqrt(b*d)*a^5*b^8*d^9 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*b^11*c^4*d^4 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^10*c^3*d^5 - 54*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^9*c^2*d^6 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^3*b^8*c*d^7 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^7*d^8 + 36*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^9*c^3*d^4 - 108*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^8*c^2*d^5 + 108*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^4*a^2*b^7*c*d^6 - 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
```

```
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^6*d^7 - 84*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^7*c^2*d^4 + 168
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a*b^6*c*d^5 - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^6*a^2*b^5*d^6 - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^5*c*d^4 + 189*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^4*d^5 - 315*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b
^3*d^4)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)^9*b^2)
```

Mupad [B]
time = 1.18, size = 292, normalized size = 2.15

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^4\,x^4}{315\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{-210\,a^3\,c\,d^3+378\,a^2\,b\,c^2\,d^2-270\,a\,b^2\,c^3\,d+70\,b^3\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(210\,a^3\,d^4-126\,a^2\,b\,c\,d^3+54\,a\,b^2\,c^2\,d^2-10\,b^3\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^4}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^4*x^4)/(315*b*(a*d - b*c)^4) - (70*b^3*c^4 - 210*a^
3*c*d^3 + 378*a^2*b*c^2*d^2 - 270*a*b^2*c^3*d)/(315*b^4*(a*d - b*c)^4) + (x
*(210*a^3*d^4 - 10*b^3*c^3*d + 54*a*b^2*c^2*d^2 - 126*a^2*b*c*d^3))/(315*b^
4*(a*d - b*c)^4) + (16*d^3*x^3*(9*a*d - b*c))/(315*b^2*(a*d - b*c)^4) + (4*
d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^3*(a*d - b*c)^4)))/(x^4*
(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b
^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

3.1470 $\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{128d^3(c+dx)^{3/2}}{1155(bc-ad)^4(a+bx)^{5/2}}$$

```
[Out] -2/11*(d*x+c)^(3/2)/(-a*d+b*c)/(b*x+a)^(11/2)+16/99*d*(d*x+c)^(3/2)/(-a*d+b
*c)^2/(b*x+a)^(9/2)-32/231*d^2*(d*x+c)^(3/2)/(-a*d+b*c)^3/(b*x+a)^(7/2)+128
/1155*d^3*(d*x+c)^(3/2)/(-a*d+b*c)^4/(b*x+a)^(5/2)-256/3465*d^4*(d*x+c)^(3/
2)/(-a*d+b*c)^5/(b*x+a)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (16*d*(c + d*x)^(3
/2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (32*d^2*(c + d*x)^(3/2))/(231*(b*
c - a*d)^3*(a + b*x)^(7/2)) + (128*d^3*(c + d*x)^(3/2))/(1155*(b*c - a*d)^4
*(a + b*x)^(5/2)) - (256*d^4*(c + d*x)^(3/2))/(3465*(b*c - a*d)^5*(a + b*x)
^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(8d)\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{(16d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{(64\ldots}{}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{}{115\ldots}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{}{115\ldots}$$

Mathematica [A]
time = 0.15, size = 117, normalized size = 0.68

$$-\frac{2(c+dx)^{3/2}\left(1155d^4 - \frac{2772bd^3(c+dx)}{a+bx} + \frac{2970b^2d^2(c+dx)^2}{(a+bx)^2} - \frac{1540b^3d(c+dx)^3}{(a+bx)^3} + \frac{315b^4(c+dx)^4}{(a+bx)^4}\right)}{3465(bc-ad)^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*d^4 - (2772*b*d^3*(c + d*x))/(a + b*x) + (2970*b^
2*d^2*(c + d*x)^2)/(a + b*x)^2 - (1540*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (31
5*b^4*(c + d*x)^4)/(a + b*x)^4))/(3465*(b*c - a*d)^5*(a + b*x)^(3/2))
```

Maple [A]
time = 0.16, size = 248, normalized size = 1.45

method	result

default	$-\frac{\sqrt{dx+c}}{5b(bx+a)^{\frac{11}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}-\frac{10d\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}\right.}{}\right)}{9}\right)}{11(-ad+bc)}\right)}{10b}$
gosper	$\frac{2(dx+c)^{\frac{3}{2}}(128d^4x^4b^4+704a\,b^3d^4x^3-192b^4c\,d^3x^3+1584a^2b^2d^4x^2-1056a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+1848a^3b\,d^4x-2376a^2b^2c\,d^3x+132}{3465(bx+a)^{\frac{11}{2}}(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*(d*x+c)^(1/2)/(b*x+a)^(11/2)+1/10*(a*d-b*c)/b*(-2/11/(-a*d+b*c)/(b*x
+a)^(11/2)*(d*x+c)^(1/2)-10/11*d/(-a*d+b*c)*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/
(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)
-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+
b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d
+b*c)^2/(b*x+a)^(1/2))))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. $2(141) = 282$.
time = 7.14, size = 781, normalized size = 4.57

$$-\frac{2\,{\left(128\,b^{4}d^{5}x^{5}+315\,b^{4}c^{5}-1540\,ab^{3}c^{4}d+2970\,a^{2}b^{2}c^{3}d^{2}-2772\,a^{3}bc^{2}d^{3}+1155\,a^{4}cd^{4}-64\,{\left(b^{4}cd^{4}-11\,ab^{3}d^{5}\right)}x^{4}+16\,{\left(3\,b^{4}c^{2}d^{3}-22\,ab^{3}cd^{4}+99\,a^{2}b^{2}d^{5}\right)}x^{3}-8\,{\left(5\,b^{4}c^{3}d^{2}-33\,ab^{3}c^{2}d^{3}+99\,a^{2}b^{2}cd^{4}-231\,a^{3}bd^{5}\right)}x^{2}+{\left(35\,b^{4}c^{4}d-220\,ab^{3}c^{3}d^{2}+594\,a^{2}b^{2}c^{2}d^{3}-924\,a^{3}bcd^{4}+1155\,a^{4}d^{5}\right)}x\right)}\sqrt{bx+a}\,\sqrt{dx+c}}{3465\,{\left(a^{6}b^{5}c^{5}-5\,a^{7}b^{4}c^{4}d+10\,a^{8}b^{3}c^{3}d^{2}-10\,a^{9}b^{2}c^{2}d^{3}+5\,a^{10}bcd^{4}-a^{11}d^{5}+{\left(b^{11}c^{5}-5\,ab^{10}c^{4}d+10\,a^{2}b^{9}c^{3}d^{2}-10\,a^{3}b^{8}c^{2}d^{3}+5\,a^{4}b^{7}cd^{4}-a^{5}b^{6}d^{5}\right)}x^{6}+6\,{\left(ab^{10}c^{5}-5\,a^{2}b^{9}c^{4}d+10\,a^{3}b^{8}c^{3}d^{2}-10\,a^{4}b^{7}c^{2}d^{3}+5\,a^{5}b^{6}cd^{4}-a^{6}b^{5}d^{5}\right)}x^{5}+15\,{\left(a^{2}b^{9}c^{5}-5\,a^{3}b^{8}c^{4}d+10\,a^{4}b^{7}c^{3}d^{2}-10\,a^{5}b^{6}c^{2}d^{3}+5\,a^{6}b^{5}cd^{4}-a^{7}b^{4}d^{5}\right)}x^{4}+20\,{\left(a^{3}b^{8}c^{5}-5\,a^{4}b^{7}c^{4}d+10\,a^{5}b^{6}c^{3}d^{2}-10\,a^{6}b^{5}c^{2}d^{3}+5\,a^{7}b^{4}cd^{4}-a^{8}b^{3}d^{5}\right)}x^{3}+15\,{\left(a^{4}b^{7}c^{5}-5\,a^{5}b^{6}c^{4}d+10\,a^{6}b^{5}c^{3}d^{2}-10\,a^{7}b^{4}c^{2}d^{3}+5\,a^{8}b^{3}cd^{4}-a^{9}b^{2}d^{5}\right)}x^{2}+6\,{\left(a^{5}b^{6}c^{5}-5\,a^{6}b^{5}c^{4}d+10\,a^{7}b^{4}c^{3}d^{2}-10\,a^{8}b^{3}c^{2}d^{3}+5\,a^{9}b^{2}cd^{4}-a^{10}bd^{5}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/3465*(128*b^4*d^5*x^5 + 315*b^4*c^5 - 1540*a*b^3*c^4*d + 2970*a^2*b^2*c^
3*d^2 - 2772*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 - 64*(b^4*c*d^4 - 11*a*b^3*d^5)
*x^4 + 16*(3*b^4*c^2*d^3 - 22*a*b^3*c*d^4 + 99*a^2*b^2*d^5)*x^3 - 8*(5*b^4*
c^3*d^2 - 33*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 - 231*a^3*b*d^5)*x^2 + (35*b^
4*c^4*d - 220*a*b^3*c^3*d^2 + 594*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 + 1155*
a^4*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^5*c^5 - 5*a^7*b^4*c^4*d + 10
*a^8*b^3*c^3*d^2 - 10*a^9*b^2*c^2*d^3 + 5*a^10*b*c*d^4 - a^11*d^5 + (b^11*c
^5 - 5*a*b^10*c^4*d + 10*a^2*b^9*c^3*d^2 - 10*a^3*b^8*c^2*d^3 + 5*a^4*b^7*c
*d^4 - a^5*b^6*d^5)*x^6 + 6*(a*b^10*c^5 - 5*a^2*b^9*c^4*d + 10*a^3*b^8*c^3*
d^2 - 10*a^4*b^7*c^2*d^3 + 5*a^5*b^6*c*d^4 - a^6*b^5*d^5)*x^5 + 15*(a^2*b^9
*c^5 - 5*a^3*b^8*c^4*d + 10*a^4*b^7*c^3*d^2 - 10*a^5*b^6*c^2*d^3 + 5*a^6*b^
5*c*d^4 - a^7*b^4*d^5)*x^4 + 20*(a^3*b^8*c^5 - 5*a^4*b^7*c^4*d + 10*a^5*b^6
*c^3*d^2 - 10*a^6*b^5*c^2*d^3 + 5*a^7*b^4*c*d^4 - a^8*b^3*d^5)*x^3 + 15*(a^
4*b^7*c^5 - 5*a^5*b^6*c^4*d + 10*a^6*b^5*c^3*d^2 - 10*a^7*b^4*c^2*d^3 + 5*a
^8*b^3*c*d^4 - a^9*b^2*d^5)*x^2 + 6*(a^5*b^6*c^5 - 5*a^6*b^5*c^4*d + 10*a^7
*b^4*c^3*d^2 - 10*a^8*b^3*c^2*d^3 + 5*a^9*b^2*c*d^4 - a^10*b*d^5)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(13/2),x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(141) = 282$.
time = 1.54, size = 1345, normalized size = 7.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -512/3465*(sqrt(b*d)*b^16*c^6*d^5 - 6*sqrt(b*d)*a*b^15*c^5*d^6 + 15*sqrt(b*
d)*a^2*b^14*c^4*d^7 - 20*sqrt(b*d)*a^3*b^13*c^3*d^8 + 15*sqrt(b*d)*a^4*b^12
*c^2*d^9 - 6*sqrt(b*d)*a^5*b^11*c*d^10 + sqrt(b*d)*a^6*b^10*d^11 - 11*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14
*c^5*d^5 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^13*c^4*d^6 - 110*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^3*d^7 + 110*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^2*d
^8 - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^2*a^4*b^10*c*d^9 + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*d^10 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^4*d^5 - 220*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^11
*c^3*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a^2*b^10*c^2*d^7 - 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c*d^8 + 55*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^8*d^9 - 1
65*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^10*c^3*d^5 + 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^9*c^2*d^6 - 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^8*c*d^7 + 165*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*d
^8 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^8*b^8*c^2*d^5 - 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^8*a*b^7*c*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*d^7 + 924*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c*d
^5 - 924*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^10*a*b^5*d^6 + 1386*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^12*b^4*d^5)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^11*b^2)
```

Mupad [B]
time = 1.43, size = 397, normalized size = 2.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{2310\,a^4\,c\,d^4-5544\,a^3\,b\,c^2\,d^3+5940\,a^2\,b^2\,c^3\,d^2-3080\,a\,b^3\,c^4\,d+630\,b^4\,c^5}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(2310\,a^4\,d^5-1848\,a^3\,b\,c\,d^4+1188\,a^2\,b^2\,c^2\,d^3-440\,a\,b^3\,c^3\,d^2+70\,b^4\,c^4\,d\right)}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,d^5\,x^5}{3465\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{16\,d^2\,x^2\,\left(231\,a^3\,d^3-99\,a^2\,b\,c\,d^2+33\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}{3465\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{3465\,b^2\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3465\,b^3\,{\left(a\,d-b\,c\right)}^5}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((630*b^4*c^5 + 2310*a^4*c*d^4 - 5544*a^3*b*c^2*d^3 + 5940
*a^2*b^2*c^3*d^2 - 3080*a*b^3*c^4*d)/(3465*b^5*(a*d - b*c)^5) + (x*(2310*a^
4*d^5 + 70*b^4*c^4*d - 440*a*b^3*c^3*d^2 + 1188*a^2*b^2*c^2*d^3 - 1848*a^3*
b*c*d^4))/(3465*b^5*(a*d - b*c)^5) + (256*d^5*x^5)/(3465*b*(a*d - b*c)^5) +
 (16*d^2*x^2*(231*a^3*d^3 - 5*b^3*c^3 + 33*a*b^2*c^2*d - 99*a^2*b*c*d^2))/(
3465*b^4*(a*d - b*c)^5) + (128*d^4*x^4*(11*a*d - b*c))/(3465*b^2*(a*d - b*c
)^5) + (32*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2 - 22*a*b*c*d))/(3465*b^3*(a*d -
b*c)^5)))/(x^5*(a + b*x)^(1/2) + (a^5*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a
 + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^
(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

3.1471 $\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=227

$$\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3}-\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d}+\frac{3(bc-a}{}$$

```
[Out] 1/5*(b*x+a)^(7/2)*(d*x+c)^(3/2)/b-3/128*(-a*d+b*c)^5*arctanh(d^(1/2)*(b*x+a
)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(7/2)-1/64*(-a*d+b*c)^3*(b*x+a)^(3
/2)*(d*x+c)^(1/2)/b^2/d^2+1/80*(-a*d+b*c)^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^2
/d+3/40*(-a*d+b*c)*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b^2+3/128*(-a*d+b*c)^4*(b*x+
a)^(1/2)*(d*x+c)^(1/2)/b^2/d^3
```

Rubi [A]
time = 0.08, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{80b^2d}+\frac{3(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40b^2}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] (3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^2*d^3) - ((b*c - a*d)^
3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^2*(a + b*x)^(5
/2)*Sqrt[c + d*x])/(80*b^2*d) + (3*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x
])/(40*b^2) + ((a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b) - (3*(b*c - a*d)^5*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2
))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad))\int (a+bx)^{5/2}\sqrt{c+dx}\,dx}{10b}$$

$$= \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad)^2)\int \frac{(a}{\sqrt{c}}}{80b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/}}{}$$

$$= -\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-a}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2}{}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2}{}$$

Mathematica [A]

time = 0.52, size = 166, normalized size = 0.73

$$-\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(15d^4(a+bx)^4-70bd^3(a+bx)^3(c+dx)-128b^2d^2(a+bx)^2(c+dx)^2+70b^3d(a+bx)(c+dx)^3-15b^4(c+dx)^4)}{640b^2d^3}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] -1/640*(Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^4*(a + b*x)^4 - 70*b*d^3*(a + b*x
)^3*(c + d*x) - 128*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 70*b^3*d*(a + b*x)*(c
 + d*x)^3 - 15*b^4*(c + d*x)^4))/(b^2*d^3) - (3*(b*c - a*d)^5*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2))
```

Maple [A]
time = 0.16, size = 239, normalized size = 1.05

method	result

default	$\frac{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{2}}}{5d} - \frac{(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{4d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad-bc)\left(\right.}{}\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(b*x+a)^(5/2)*(d*x+c)^(5/2)-1/2*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(3/2)*(d*
x+c)^(5/2)-3/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)-1/6*(-a*d+b*
c)/d*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x
+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.24, size = 702, normalized size = 3.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(
11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2
*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^4), 1/1280*(15*(b^5*c^
5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(1
28*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70
*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b
^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a
*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x
 + c))/(b^3*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{5}{2}} (c + dx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. 2(183) = 366.
time = 2.55, size = 1740, normalized size = 7.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*(240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c*abs(b
)/b^2 + 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^
4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9
*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x
 + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 -
 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c*abs(b) + 30*(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b
^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6
)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 -
93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b
*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d*ab
s(b) + 240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*d*abs(b)/b + (sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*
c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 51
3*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^
2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4
 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^
19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b
^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3
*d^4))*b*d*abs(b) + 1440*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
+ (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d
^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*d))*a^2*c*abs(b)/b^2 + 480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*d*abs(b)/b^3)/b
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(3/2),x)
[Out] int((a + b*x)^(5/2)*(c + d*x)^(3/2), x)
```

3.1472 $\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=189

$$-\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2}+\frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d}+\frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}+\frac{(a+bx)^5}{}$$

```
[Out] 1/4*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b+3/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)
^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(5/2)+1/32*(-a*d+b*c)^2*(b*x+a)^(3/
2)*(d*x+c)^(1/2)/b^2/d+1/8*(-a*d+b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^2-3/64*
(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d^2
```

Rubi [A]

time = 0.06, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-3*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^2*d) + ((b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(5
/2)*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(3(bc-ad))\int (a+bx)^{3/2}\sqrt{c+dx}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(bc-ad)^2\int \frac{(a+bx}{\sqrt{c+}}}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5}}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-a}{}$$

Mathematica [A]
time = 0.40, size = 144, normalized size = 0.76

$$-\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(3d^3(a+bx)^3-11bd^2(a+bx)^2(c+dx)-11b^2d(a+bx)(c+dx)^2+3b^3(c+dx)^3)}{64b^2d^2}+\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64b^{5/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] -1/64*(Sqrt[a + b*x]*Sqrt[c + d*x]*(3*d^3*(a + b*x)^3 - 11*b*d^2*(a + b*x)^
2*(c + d*x) - 11*b^2*d*(a + b*x)*(c + d*x)^2 + 3*b^3*(c + d*x)^3))/(b^2*d^2
) + (3*(b*c - a*d)^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x]
)])/(64*b^(5/2)*d^(5/2))
```

Maple [A]
time = 0.16, size = 206, normalized size = 1.09

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{4d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b} - \cdots\right)}{\cdots}\right)}{\cdots}\right)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)-3/8*(-a*d+b*c)/d*(1/3/d*(b*x+a)^(1/2)*(d*
x+c)^(5/2)-1/6*(-a*d+b*c)/d*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c
)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/
(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2
+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [A]
time = 1.02, size = 534, normalized size = 2.83

$$\left[\frac{3\,{\left(b^{4} c^{4} - 4\,a b^{3} c^{3} d + 6\,a^{2} b^{2} c^{2} d^{2} - 4\,a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{b d} \log\left(8\,b^{2} d^{2} x^{2} + b^{2} c^{2} + 6\,a b c d + a^{2} d^{2} + 4\,{\left(2\,b d x + b c + a d\right)} \sqrt{b d} \sqrt{b x + a} \sqrt{d x + c} + 8\,{\left(b^{2} c d + a b d^{2}\right)} x\right) + 4\,{\left(16\,b^{4} d^{4} x^{3} - 3\,b^{4} c^{3} d + 11\,a b^{3} c^{2} d^{2} + 11\,a^{2} b^{2} c d^{3} - 3\,a^{3} b d^{4} + 24\,{\left(b^{4} c d^{3} + a b^{3} d^{4}\right)} x^{2} + 2\,{\left(b^{4} c^{2} d^{2} + 22\,a b^{3} c d^{3} + a^{2} b^{2} d^{4}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{256\,b^{3} d^{3}}, -\frac{3\,{\left(b^{4} c^{4} - 4\,a b^{3} c^{3} d + 6\,a^{2} b^{2} c^{2} d^{2} - 4\,a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{-b d} \arctan\left(\frac{{\left(2\,b d x + b c + a d\right)} \sqrt{-b d} \sqrt{b x + a} \sqrt{d x + c}}{2\,{\left(b^{2} d^{2} x^{2} + a b c d + {\left(b^{2} c d + a b d^{2}\right)} x\right)}}\right) - 2\,{\left(16\,b^{4} d^{4} x^{3} - 3\,b^{4} c^{3} d + 11\,a b^{3} c^{2} d^{2} + 11\,a^{2} b^{2} c d^{3} - 3\,a^{3} b d^{4} + 24\,{\left(b^{4} c d^{3} + a b^{3} d^{4}\right)} x^{2} + 2\,{\left(b^{4} c^{2} d^{2} + 22\,a b^{3} c d^{3} + a^{2} b^{2} d^{4}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{128\,b^{3} d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(16*b^4*d^4*x^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c
*d^3 - 3*a^3*b*d^4 + 24*(b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a
*b^3*c*d^3 + a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3), -1/128
*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(
d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(16*b^4*d^4*x
^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 3*a^3*b*d^4 + 24*(
b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a*b^3*c*d^3 + a^2*b^2*d^4)
*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b x\right)^{\frac{3}{2}} \left(c + d x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(151) = 302$.
time = 1.49, size = 1071, normalized size = 5.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*(8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4
*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2
```

```
*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*abs(b) - 192*((b^2*c - a*b*d)*log
(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(
b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c*abs(b)/b^2
+ (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x +
a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*
b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c
^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3
*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d
^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*b^2*d^3))*d*abs(b) + 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d*abs
(b)/b + 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a
*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-s
qrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d
))*a*c*abs(b)/b^2 + 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
(b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*d))*a^2*d*abs(b)/b^3)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(3/2),x)
[Out] int((a + b*x)^(3/2)*(c + d*x)^(3/2), x)
```

3.1473 $\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx$

Optimal. Leaf size=151

$$\frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d}+\frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}-\frac{(bc-ad)^3\tanh^{-1}\Big(}{8b^{5/2}d}$$

```
[Out] 1/3*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b-1/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^
(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(3/2)+1/4*(-a*d+b*c)*(b*x+a)^(3/2)*(
d*x+c)^(1/2)/b^2+1/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^2*d) + ((b*c - a*d)*(a + b
*x)^(3/2)*Sqrt[c + d*x])/(4*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b)
- ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(5/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2 \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{8b^2}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c}{3b}$$

Mathematica [A]
time = 0.25, size = 127, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(-3a^2d^2+2abd(4c+dx)+b^2(3c^2+14cdx+8d^2x^2))}{24b^2d} - \frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(-3*a^2*d^2 + 2*a*b*d*(4*c + d*x) + b^2*(3*c^2
 + 14*c*d*x + 8*d^2*x^2)))/(24*b^2*d) - ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(5/2)*d^(3/2))
```

Maple [A]
time = 0.16, size = 173, normalized size = 1.15

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d}-\frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+ \cdots}}{2 \cdots}\right)}{4b \cdots}\right)}{6d \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)-1/6*(-a*d+b*c)/d*(1/2*(d*x+c)^(3/2)*(b*x+
a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*(
(b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*
x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.93, size = 410, normalized size = 2.72

$$\left[-\frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2+3\,b^3c^2d+8\,ab^2cd^2-3\,a^2bd^3+2\,(7\,b^3cd^2+ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,b^3d^2},\ \frac{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(8\,b^3d^3x^2+3\,b^3c^2d+8\,ab^2cd^2-3\,a^2bd^3+2\,(7\,b^3cd^2+ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,b^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
```

```
^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*d^3 + 2*(7*b^3*c*d^2 + a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2), 1/48*(3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*
d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*
d^3 + 2*(7*b^3*c*d^2 + a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(119) = 238.
time = 1.19, size = 576, normalized size = 3.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*a*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d*abs(b)/b - 6
*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)
*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*s
qrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*abs(b
)/b^2 - 6*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*
d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d)
)*a*d*abs(b)/b^3)/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(3/2), x)
```

3.1474 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=113

$$\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

```
[Out] 3/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/
2)/d^(1/2)+1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b+3/4*(-a*d+b*c)*(b*x+a)^(1/2)*(
d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^2) + (Sqrt[a + b*x]*(c + d
*x)^(3/2))/(2*b) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt[d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+\cdots}}}{8b^2}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{c-\cdots}}\right.}{4b\cdots}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\cdots\right.}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a\cdots}}{\sqrt{b}\,\sqrt{c+\cdots}}\right.}{4b^{5/2}\sqrt{d}}$$

Mathematica [A]
time = 0.17, size = 94, normalized size = 0.83

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(5bc-3ad+2bdx)}{4b^2} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(5*b*c - 3*a*d + 2*b*d*x))/(4*b^2) + (3*(b*c -
 a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2
)*Sqrt[d])
```

Maple [A]
time = 0.16, size = 140, normalized size = 1.24

method	result
default	$\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}}\right.}{2b\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1
/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*l
n((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d
)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.94, size = 306, normalized size = 2.71

$$\left[\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x+5\,b^2cd-3\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,b^3d},\,-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(2\,b^2d^2x+5\,b^2cd-3\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + 5*b^2*c*d - 3*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x + 5*b^2*c*d - 3*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(87) = 174.
time = 1.04, size = 233, normalized size = 2.06

$$-\frac{\frac{4\left(\frac{\left(b^{2}c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^{2}c+(bx+a)bd-abd}\sqrt{bx+a}\right)c|b|}{b^{2}}-\frac{\left(\sqrt{b^{2}c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^{2}}{d^{2}}\right)\sqrt{bx+a}+\frac{\left(b^{3}c^{2}+2\,ab^{2}cd-3\,a^{2}bd^{2}\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)d|b|}{b^{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*d*abs(b)/b^3)/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c+d\,x)}^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(1/2), x)
```

3.1475 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}$$

```
[Out] 3*(-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*d^(1/2)/b
^(5/2)-2*(d*x+c)^(3/2)/b/(b*x+a)^(1/2)+3*d*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}+\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (3*d*Sqrt[a + b*x]*Sqrt[c + d*x])/b^2 - (2*(c + d*x)^(3/2))/(b*Sqrt[a + b*x
]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x
}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.43, size = 89, normalized size = 0.91

$$\frac{\frac{b\sqrt{c+dx}\,(-2bc+3ad+bdx)}{\sqrt{a+bx}}+3\sqrt{\frac{b}{d}}\,d(-bc+ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] ((b*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/Sqrt[a + b*x] + 3*Sqrt[b/d]*d*(
-(b*c) + a*d)*Log[Sqrt[a + b*x] - Sqrt[b/d]*Sqrt[c + d*x]])/b^3
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**A**]
time = 1.29, size = 311, normalized size = 3.17

$$\left[-\frac{3\,(abc-a^2d+(b^2c-abd)x)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,(bdx-2\,bc+3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\,(b^3x+ab^2)},\ -\frac{3\,(abc-a^2d+(b^2c-abd)x)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)-2\,(bdx-2\,bc+3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^3x+ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b*c - a^2*d + (b^2*c - a*b*d)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x - 2*b*c + 3*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2), -1/2*(3*(a*b*c - a^2*d +
 (b^2*c - a*b*d)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(b
*d*x - 2*b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(78) = 156.
time = 1.36, size = 204, normalized size = 2.08

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\,d|b|}{b^4}-\frac{3\left(\sqrt{bd}\,bc|b|-\sqrt{bd}\,ad|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2\,b^4}-\frac{4\left(\sqrt{bd}\,b^2c^2|b|-2\,\sqrt{bd}\,abcd|b|+\sqrt{bd}\,a^2d^2|b|\right)}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d*abs(b)/b^4 - 3/2*(sqrt(
b*d)*b*c*abs(b) - sqrt(b*d)*a*d*abs(b))*log((sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^4 - 4*(sqrt(b*d)*b^2*c^2*abs(b) - 2*s
qrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b))/((b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(3/2), x)
```

3.1476 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}$$

```
[Out] -2/3*(d*x+c)^(3/2)/b/(b*x+a)^(3/2)+2*d^(3/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/
b^(1/2)/(d*x+c)^(1/2))/b^(5/2)-2*d*(d*x+c)^(1/2)/b^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {49, 65, 223, 212}

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*d*Sqrt[c + d*x])/(b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(3/2))/(3*b*(a + b*
x)^(3/2)) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/b^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b^2}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 81, normalized size = 0.88

$$-\frac{2\sqrt{c+dx}\,(3ad+b(c+4dx))}{3b^2(a+bx)^{3/2}}+\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(3*a*d + b*(c + 4*d*x)))/(3*b^2*(a + b*x)^(3/2)) + (2*d^(
3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(5/2)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(70) = 140$.
time = 1.67, size = 325, normalized size = 3.53

$$\left[\frac{3\left(b^{2}dx^{2}+2abdx+a^{2}d\right)\sqrt{\frac{d}{b}}\log\left(8b^{2}d^{2}x^{2}+b^{2}c^{2}+6abcd+a^{2}d^{2}+4\left(2b^{2}dx+b^{2}c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^{2}cd+abd^{2}\right)x\right)-4\left(4bdx+bc+3ad\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(b^{4}x^{2}+2ab^{3}x+a^{2}b^{2}\right)},\ -\frac{3\left(b^{2}dx^{2}+2abdx+a^{2}d\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^{2}x^{2}+acd+\left(bcd+ad^{2}\right)x\right)}\right)+2\left(4bdx+bc+3ad\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(b^{4}x^{2}+2ab^{3}x+a^{2}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + b*c + 3*a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2), -1/3*(3*(b^2*
d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x))
 + 2*(4*b*d*x + b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^
3*x + a^2*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(70) = 140.
time = 1.37, size = 455, normalized size = 4.95

$$-\frac{\sqrt{bd}\,d|b|\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b^4}-\frac{8\left(2\,\sqrt{bd}\,b^5c^3d|b|-6\,\sqrt{bd}\,ab^4c^2d^2|b|+6\,\sqrt{bd}\,a^2b^3cd^3|b|-2\,\sqrt{bd}\,a^3b^2d^4|b|-3\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^3c^2d|b|+6\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^2cd^2|b|-3\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2bd^3|b|+3\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4bcd|b|-3\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ad^2|b|\right)}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2)/b^4 - 8/3*(2*sqrt(b*d)*b^5*c^3*d*abs(b) - 6*sqrt(b*d)*a*b^4
*c^2*d^2*abs(b) + 6*sqrt(b*d)*a^2*b^3*c*d^3*abs(b) - 2*sqrt(b*d)*a^3*b^2*d^
4*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^3*c^2*d*abs(b) + 6*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^2*c*d^2*abs(b) - 3*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b*d^3*abs(b
) + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*b*c*d*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^4*a*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{3/2}}{{\left(a+b\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(5/2), x)
```

3.1477 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

```
[Out] -2/5*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(26) = 52$.
time = 0.18, size = 161, normalized size = 5.03

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}}{5(bx+a)^{\frac{5}{2}}(ad-bc)}$
default	$-\frac{(dx+c)^{\frac{3}{2}}}{b(bx+a)^{\frac{5}{2}}}+\frac{3(ad-bc)\left(-\frac{\sqrt{dx+c}}{2b(bx+a)^{\frac{5}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{4b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(d*x+c)^(3/2)/(b*x+a)^(5/2)+3/2*(a*d-b*c)/b*(-1/2/b*(d*x+c)^(1/2)/(b*x
+a)^(5/2)+1/4*(a*d-b*c)/b*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*
d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/
2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(26) = 52$.
time = 2.15, size = 104, normalized size = 3.25

$$-\frac{2\left(d^{2}x^{2}+2\,cdx+c^{2}\right)\sqrt{bx+a}\,\sqrt{dx+c}}{5\left(a^{3}bc-a^{4}d+(b^{4}c-ab^{3}d)x^{3}+3\,(ab^{3}c-a^{2}b^{2}d)x^{2}+3\,(a^{2}b^{2}c-a^{3}bd)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b*c - a^4*d
 + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c - a^3
*b*d)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(26) = 52.
time = 1.07, size = 374, normalized size = 11.69

$$-\frac{4\left(\sqrt{bd}\,b^{8}c^{4}d^{2}|b| - 4\sqrt{bd}\,ab^{7}c^{3}d^{3}|b| + 6\sqrt{bd}\,a^{2}b^{6}c^{2}d^{4}|b| - 4\sqrt{bd}\,a^{3}b^{5}cd^{5}|b| + \sqrt{bd}\,a^{4}b^{4}d^{6}|b| + 10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{4}c^{2}d^{2}|b| - 20\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}ab^{3}cd^{3}|b| + 10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}a^{2}b^{2}d^{4}|b| + 5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{8}d^{2}|b|\right)}{5\left(b^{2}c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{5}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -4/5*(sqrt(b*d)*b^8*c^4*d^2*abs(b) - 4*sqrt(b*d)*a*b^7*c^3*d^3*abs(b) + 6*s
qrt(b*d)*a^2*b^6*c^2*d^4*abs(b) - 4*sqrt(b*d)*a^3*b^5*c*d^5*abs(b) + sqrt(b
*d)*a^4*b^4*d^6*abs(b) + 10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^4*b^4*c^2*d^2*abs(b) - 20*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d^3*abs(b) +
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^2*b^2*d^4*abs(b) + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^3)
```

Mupad [**B**]
time = 0.80, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(a+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(7/2),x)
```

```
[Out] (2*(c + d*x)^(5/2))/((5*a*d - 5*b*c)*(a + b*x)^(5/2))
```

3.1478 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}$$

```
[Out] -2/7*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(7/2)+4/35*d*(d*x+c)^(5/2)/(-a*d+b*c)
^2/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (4*d*(c + d*x)^(5/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(2d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}} + \frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}$$

Mathematica [A]
time = 0.10, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{5/2}(-5bc+7ad+2bdx)}{35(bc-ad)^2(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
[Out] (2*(c + d*x)^(5/2)*(-5*b*c + 7*a*d + 2*b*d*x))/(35*(b*c - a*d)^2*(a + b*x)^
(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(54) = 108.
time = 0.16, size = 201, normalized size = 3.05

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}(2bdx+7ad-5bc)}{35(bx+a)^{\frac{7}{2}}(a^2d^2-2abcd+b^2c^2)}$
default	$-\frac{(dx+c)^{\frac{3}{2}}}{2b(bx+a)^{\frac{7}{2}}} + \frac{3(ad-bc)\left(-\frac{\sqrt{dx+c}}{3b(bx+a)^{\frac{7}{2}}} + \frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}} - \frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}} - \frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}} + \frac{}{3(-}\right.}{5(-ad+bc}\right.}{7(-ad+bc)}\right.}{6b}\right.}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*x+c)^(3/2)/(b*x+a)^(7/2)+3/4*(a*d-b*c)/b*(-1/3/b*(d*x+c)^(1/2)/(b
*x+a)^(7/2)+1/6*(a*d-b*c)/b*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/
7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c
)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*
c)^2/(b*x+a)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 2.07, size = 235, normalized size = 3.56

$$\frac{2\,(2\,bd^3x^3 - 5\,bc^3 + 7\,ac^2d - (bcd^2 - 7\,ad^3)x^2 - 2\,(4\,bc^2d - 7\,acd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{35\,(a^4b^2c^2 - 2\,a^5bcd + a^6d^2 + (b^6c^2 - 2\,ab^5cd + a^2b^4d^2)x^4 + 4\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)x^3 + 6\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)x^2 + 4\,(a^3b^3c^2 - 2\,a^4b^2cd + a^5bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b*d^3*x^3 - 5*b*c^3 + 7*a*c^2*d - (b*c*d^2 - 7*a*d^3)*x^2 - 2*(4*b*
c^2*d - 7*a*c*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^2*c^2 - 2*a^5*b*c*
d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*c^2 - 2*
a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(54) = 108$.

time = 0.87, size = 1024, normalized size = 15.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 8/35*(sqrt(b*d)*b^10*c^5*d^3*abs(b) - 5*sqrt(b*d)*a*b^9*c^4*d^4*abs(b) + 10
*sqrt(b*d)*a^2*b^8*c^3*d^5*abs(b) - 10*sqrt(b*d)*a^3*b^7*c^2*d^6*abs(b) + 5
*sqrt(b*d)*a^4*b^6*c*d^7*abs(b) - sqrt(b*d)*a^5*b^5*d^8*abs(b) - 7*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8*c^4
*d^3*abs(b) + 28*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a*b^7*c^3*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c^2*d^5*abs(b) + 28*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^3*b^5*c*d^6*abs(b) - 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*a^4*b^4*d^7*abs(b) - 14*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6*c^3*d^3*abs(b) + 42*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^5*c^2*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^4*c*d^5*abs(b) + 14*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^3*d^6*abs(b) -
70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^4*c^2*d^3*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*a*b^3*c*d^4*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^2*d^5*abs(b) -
 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*b^2*c*d^3*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*a*b*d^4*abs(b) - 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*d^3*abs(b))/((b^2*c - a*b
*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b
^2)
```

Mupad [B]

time = 0.93, size = 178, normalized size = 2.70

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^3\,x^3}{35\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{10\,b\,c^3-14\,a\,c^2\,d}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{x^2\,\left(14\,a\,d^3-2\,b\,c\,d^2\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,c\,d\,x\,\left(7\,a\,d-4\,b\,c\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^3*x^3)/(35*b^2*(a*d - b*c)^2) - (10*b*c^3 - 14*a*c^2
*d)/(35*b^3*(a*d - b*c)^2) + (x^2*(14*a*d^3 - 2*b*c*d^2))/(35*b^3*(a*d - b*
c)^2) + (4*c*d*x*(7*a*d - 4*b*c))/(35*b^3*(a*d - b*c)^2)))/(x^3*(a + b*x)^(
1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(
a + b*x)^(1/2))/b^2)
```

3.1479 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}$$

```
[Out] -2/9*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(9/2)+8/63*d*(d*x+c)^(5/2)/(-a*d+b*c)
^2/(b*x+a)^(7/2)-16/315*d^2*(d*x+c)^(5/2)/(-a*d+b*c)^3/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (8*d*(c + d*x)^(5/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(5/2))/(315*(b*c -
 a*d)^3*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(4d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{(8d^2)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{63(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{9/2}\left(35b^2+\frac{63d^2(a+bx)^2}{(c+dx)^2}-\frac{90bd(a+bx)}{c+dx}\right)}{315(bc-ad)^3(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(35*b^2 + (63*d^2*(a + b*x)^2)/(c + d*x)^2 - (90*b*d*(a
 + b*x))/(c + d*x)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(83) = 166$.
time = 0.17, size = 241, normalized size = 2.39

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}\left(8b^2x^2d^2+36ab\,d^2x-20b^2cdx+63a^2d^2-90abcd+35b^2c^2\right)}{315(bx+a)^{\frac{9}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$

default	$-\frac{(dx+c)^{\frac{3}{2}}}{3b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{\sqrt{dx+c}}{4b(bx+a)^{\frac{9}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\cdots\right)}{\cdots}\right)}{\cdots}\right)}{9(-ad+b\cdots}\right)}{8b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(d*x+c)^(3/2)/(b*x+a)^(9/2)+1/2*(a*d-b*c)/b*(-1/4/b*(d*x+c)^(1/2)/(b
*x+a)^(9/2)+1/8*(a*d-b*c)/b*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/
9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c
)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+
c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^
(1/2))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(83) = 166$.
time = 4.07, size = 426, normalized size = 4.22

$$-\frac{2\,{\left(8\,b^{2}d^{4}x^{4} + 35\,b^{2}c^{4} - 90\,abc^{3}d + 63\,a^{2}c^{2}d^{2} - 4\,{\left(b^{2}cd^{3} - 9\,abd^{4}\right)}x^{3} + 3\,{\left(b^{2}c^{2}d^{2} - 6\,abcd^{3} + 21\,a^{2}d^{4}\right)}x^{2} + 2\,{\left(25\,b^{2}c^{3}d - 72\,abc^{2}d^{2} + 63\,a^{2}cd^{3}\right)}x\right)}\sqrt{bx + a}\,\sqrt{dx + c}}{315\,{\left(a^{5}b^{3}c^{3} - 3\,a^{6}b^{2}c^{2}d + 3\,a^{7}bcd^{2} - a^{8}d^{3} + {\left(b^{8}c^{3} - 3\,ab^{7}c^{2}d + 3\,a^{2}b^{6}cd^{2} - a^{3}b^{5}d^{3}\right)}x^{5} + 5\,{\left(ab^{7}c^{3} - 3\,a^{2}b^{6}c^{2}d + 3\,a^{3}b^{5}cd^{2} - a^{4}b^{4}d^{3}\right)}x^{4} + 10\,{\left(a^{2}b^{6}c^{3} - 3\,a^{3}b^{5}c^{2}d + 3\,a^{4}b^{4}cd^{2} - a^{5}b^{3}d^{3}\right)}x^{3} + 10\,{\left(a^{3}b^{5}c^{3} - 3\,a^{4}b^{4}c^{2}d + 3\,a^{5}b^{3}cd^{2} - a^{6}b^{2}d^{3}\right)}x^{2} + 5\,{\left(a^{4}b^{4}c^{3} - 3\,a^{5}b^{3}c^{2}d + 3\,a^{6}b^{2}cd^{2} - a^{7}bd^{3}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="fricas")

[Out] -2/315*(8*b^2*d^4*x^4 + 35*b^2*c^4 - 90*a*b*c^3*d + 63*a^2*c^2*d^2 - 4*(b^2
*c*d^3 - 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 - 6*a*b*c*d^3 + 21*a^2*d^4)*x^2 +
2*(25*b^2*c^3*d - 72*a*b*c^2*d^2 + 63*a^2*c*d^3)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c^3 - 3*a^3*b^5
*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3 - 3*a^4*b^4*c
^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*
d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2)/(b*x+a)**(11/2),x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(83) = 166$.
time = 0.81, size = 1394, normalized size = 13.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="giac")

```
[Out] -32/315*(sqrt(b*d)*b^12*c^6*d^4*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^5*abs(b)
+ 15*sqrt(b*d)*a^2*b^10*c^4*d^6*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^7*abs(b
) + 15*sqrt(b*d)*a^4*b^8*c^2*d^8*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^9*abs(b)
+ sqrt(b*d)*a^6*b^6*d^10*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^10*c^5*d^4*abs(b) + 45*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^9*c^4*d^
5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^2*b^8*c^3*d^6*abs(b) + 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^7*c^2*d^7*abs(b) - 45*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
4*b^6*c*d^8*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*a^5*b^5*d^9*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^4*abs(b) - 144*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^7*c^3*d^5*abs(b) + 216*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*a^2*b^6*c^2*d^6*abs(b) - 144*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^7*abs
(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^4*d^8*abs(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^6*c^3*d^4*abs(b) - 378*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^5*c^2*d^5
*abs(b) + 378*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^6*a^2*b^4*c*d^6*abs(b) - 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^3*d^7*abs(b) + 441*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^4*c^
2*d^4*abs(b) - 882*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^8*a*b^3*c*d^5*abs(b) + 441*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2*d^6*abs(b) + 315*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^
2*c*d^4*abs(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*a*b*d^5*abs(b) + 210*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*d^4*abs(b))/((b^2*c - a*b*d -
 (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^9*b)
```

Mupad [B]
time = 1.11, size = 268, normalized size = 2.65

$$\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^2\,c^2\,d^2-180\,a\,b\,c^3\,d+70\,b^2\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,\left(126\,a^2\,d^4-36\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^4\,x^4}{315\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,c\,d\,x\,\left(63\,a^2\,d^2-72\,a\,b\,c\,d+25\,b^2\,c^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((70*b^2*c^4 + 126*a^2*c^2*d^2 - 180*a*b*c^3*d)/(315*b^4*(
a*d - b*c)^3) + (x^2*(126*a^2*d^4 + 6*b^2*c^2*d^2 - 36*a*b*c*d^3))/(315*b^4
```

```
*(a*d - b*c)^3) + (16*d^4*x^4)/(315*b^2*(a*d - b*c)^3) + (8*d^3*x^3*(9*a*d
- b*c))/(315*b^3*(a*d - b*c)^3) + (4*c*d*x*(63*a^2*d^2 + 25*b^2*c^2 - 72*a*
b*c*d))/(315*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/
2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (
4*a^3*x*(a + b*x)^(1/2))/b^3)
```

3.1480 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{32d^3(c+dx)^{5/2}}{1155(bc-ad)^4(a+bx)^{5/2}}$$

```
[Out] -2/11*(d*x+c)^(5/2)/(-a*d+b*c)/(b*x+a)^(11/2)+4/33*d*(d*x+c)^(5/2)/(-a*d+b*
c)^2/(b*x+a)^(9/2)-16/231*d^2*(d*x+c)^(5/2)/(-a*d+b*c)^3/(b*x+a)^(7/2)+32/1
155*d^3*(d*x+c)^(5/2)/(-a*d+b*c)^4/(b*x+a)^(5/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (4*d*(c + d*x)^(5/
2))/(33*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(5/2))/(231*(b*c
 - a*d)^3*(a + b*x)^(7/2)) + (32*d^3*(c + d*x)^(5/2))/(1155*(b*c - a*d)^4*(
a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(6d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} + \frac{(8d^2)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} - \frac{(1}{}$$
$$= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}} + \frac{}{11}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.70

$$-\frac{2(c+dx)^{11/2}\left(105b^3-\frac{231d^3(a+bx)^3}{(c+dx)^3}+\frac{495bd^2(a+bx)^2}{(c+dx)^2}-\frac{385b^2d(a+bx)}{c+dx}\right)}{1155(bc-ad)^4(a+bx)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(11/2)*(105*b^3 - (231*d^3*(a + b*x)^3)/(c + d*x)^3 + (495*b*
d^2*(a + b*x)^2)/(c + d*x)^2 - (385*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
 - a*d)^4*(a + b*x)^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(112) = 224$.
time = 0.17, size = 281, normalized size = 2.07

method	result
gosper	$\frac{2(dx+c)^{\frac{5}{2}}\left(16b^3x^3d^3+88d^3a\,x^2b^2-40b^3c\,d^2x^2+198a^2b\,d^3x-220a\,b^2c\,d^2x+70b^3c^2dx+231a^3d^3-495a^2bc\,d^2+385a\,b^2c^2d-105b^3c^3\right)}{1155(bx+a)^{\frac{11}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

$$8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\right.$$

$$10d\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}-\right.$$

$$3(ad-bc)\left(-\frac{\sqrt{dx+c}}{5b(bx+a)^{\frac{11}{2}}}+\frac{\cdots}{1\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(d*x+c)^(3/2)/(b*x+a)^(11/2)+3/8*(a*d-b*c)/b*(-1/5/b*(d*x+c)^(1/2)/(
b*x+a)^(11/2)+1/10*(a*d-b*c)/b*(-2/11/(-a*d+b*c)/(b*x+a)^(11/2)*(d*x+c)^(1/
2)-10/11*d/(-a*d+b*c)*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-
a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/
5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/
2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))
)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(112) = 224$.
time = 8.29, size = 649, normalized size = 4.77

$$\frac{2\,{\left(16\,b^{3}d^{5}x^{5} - 105\,b^{3}c^{5} + 385\,ab^{2}c^{4}d - 495\,a^{2}bc^{3}d^{2} + 231\,a^{3}c^{2}d^{3} - 8\,{\left(b^{3}cd^{4} - 11\,ab^{2}d^{5}\right)}x^{4} + 2\,{\left(3\,b^{3}c^{2}d^{3} - 22\,ab^{2}cd^{4} + 99\,a^{2}bd^{5}\right)}x^{3} - {\left(5\,b^{3}c^{3}d^{2} - 33\,ab^{2}c^{2}d^{3} + 99\,a^{2}bcd^{4} - 231\,a^{3}d^{5}\right)}x^{2} - 2\,{\left(70\,b^{3}c^{4}d - 275\,ab^{2}c^{3}d^{2} + 396\,a^{2}bc^{2}d^{3} - 231\,a^{3}cd^{4}\right)}x\right)}\sqrt{bx + a}\,\sqrt{dx + c}}{1155\,{\left(a^{6}b^{4}c^{4} - 4\,a^{7}b^{3}c^{3}d + 6\,a^{8}b^{2}c^{2}d^{2} - 4\,a^{9}bcd^{3} + a^{10}d^{4} + {\left(b^{10}c^{4} - 4\,ab^{9}c^{3}d + 6\,a^{2}b^{8}c^{2}d^{2} - 4\,a^{3}b^{7}cd^{3} + a^{4}b^{6}d^{4}\right)}x^{6} + 6\,{\left(ab^{9}c^{4} - 4\,a^{2}b^{8}c^{3}d + 6\,a^{3}b^{7}c^{2}d^{2} - 4\,a^{4}b^{6}cd^{3} + a^{5}b^{5}d^{4}\right)}x^{5} + 15\,{\left(a^{2}b^{8}c^{4} - 4\,a^{3}b^{7}c^{3}d + 6\,a^{4}b^{6}c^{2}d^{2} - 4\,a^{5}b^{5}cd^{3} + a^{6}b^{4}d^{4}\right)}x^{4} + 20\,{\left(a^{3}b^{7}c^{4} - 4\,a^{4}b^{6}c^{3}d + 6\,a^{5}b^{5}c^{2}d^{2} - 4\,a^{6}b^{4}cd^{3} + a^{7}b^{3}d^{4}\right)}x^{3} + 15\,{\left(a^{4}b^{6}c^{4} - 4\,a^{5}b^{5}c^{3}d + 6\,a^{6}b^{4}c^{2}d^{2} - 4\,a^{7}b^{3}cd^{3} + a^{8}b^{2}d^{4}\right)}x^{2} + 6\,{\left(a^{5}b^{5}c^{4} - 4\,a^{6}b^{4}c^{3}d + 6\,a^{7}b^{3}c^{2}d^{2} - 4\,a^{8}b^{2}cd^{3} + a^{9}bd^{4}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^3*d^5*x^5 - 105*b^3*c^5 + 385*a*b^2*c^4*d - 495*a^2*b*c^3*d^2
+ 231*a^3*c^2*d^3 - 8*(b^3*c*d^4 - 11*a*b^2*d^5)*x^4 + 2*(3*b^3*c^2*d^3 - 2
2*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - (5*b^3*c^3*d^2 - 33*a*b^2*c^2*d^3 + 99*
a^2*b*c*d^4 - 231*a^3*d^5)*x^2 - 2*(70*b^3*c^4*d - 275*a*b^2*c^3*d^2 + 396*
a^2*b*c^2*d^3 - 231*a^3*c*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^4*c^4
- 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 + a^10*d^4 + (b^10*c^
4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^6
+ 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^
5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*
a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^
```

```
5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*(a^4*b^6*c^4 - 4*a^
5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*x^2 + 6*(a
^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*
d^4)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1823 vs. $2(112) = 224$.
time = 1.80, size = 1823, normalized size = 13.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] 64/1155*(sqrt(b*d)*b^14*c^7*d^5*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^6*abs(b)
+ 21*sqrt(b*d)*a^2*b^12*c^5*d^7*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^8*abs(
b) + 35*sqrt(b*d)*a^4*b^10*c^3*d^9*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^10*a
bs(b) + 7*sqrt(b*d)*a^6*b^8*c*d^11*abs(b) - sqrt(b*d)*a^7*b^7*d^12*abs(b) -
 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^12*c^6*d^5*abs(b) + 66*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^6*abs(b) - 165*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^
7*abs(b) + 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2*a^3*b^9*c^3*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^9*abs(b) + 66*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a^5*b^7*c*d^10*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2*a^6*b^6*d^11*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^5*abs(b) -
 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^9*c^4*d^6*abs(b) + 550*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^7*abs(b) - 550*sqrt(b*d)*(s
qrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2
*d^8*abs(b) + 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^4*a^4*b^6*c*d^9*abs(b) - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^10*abs(b) - 165*sqr
```

```
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^4*d^5*abs(b) + 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^7*c^3*d^6*abs(b) - 990*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^7*abs(b)
+ 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^6*a^3*b^5*c*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^9*abs(b) - 825*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^5*ab
s(b) + 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^8*a*b^5*c^2*d^6*abs(b) - 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^7*abs(b) + 825*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b
^3*d^8*abs(b) - 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*b^4*c^2*d^5*abs(b) + 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^6*abs(b) - 254
1*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^10*a^2*b^2*d^7*abs(b) - 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^12*b^2*c*d^5*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^6*abs(b) -
1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^14*d^5*abs(b))/(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2)^11
```

Mupad [B]
time = 1.33, size = 376, normalized size = 2.76

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(462\,a^3\,d^5-198\,a^2\,b\,c\,d^4+66\,a\,b^2\,c^2\,d^3-10\,b^3\,c^3\,d^2\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}-\frac{-462\,a^3\,c^2\,d^3+990\,a^2\,b\,c^3\,d^2-770\,a\,b^2\,c^4\,d+210\,b^3\,c^5}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(924\,a^3\,c\,d^4-1584\,a^2\,b\,c^2\,d^3+1100\,a\,b^2\,c^3\,d^2-280\,b^3\,c^4\,d\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^5\,x^5}{1155\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{1155\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{1155\,b^4\,{\left(a\,d-b\,c\right)}^4}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(462*a^3*d^5 - 10*b^3*c^3*d^2 + 66*a*b^2*c^2*d^3 - 1
98*a^2*b*c*d^4))/(1155*b^5*(a*d - b*c)^4) - (210*b^3*c^5 - 462*a^3*c^2*d^3
+ 990*a^2*b*c^3*d^2 - 770*a*b^2*c^4*d)/(1155*b^5*(a*d - b*c)^4) + (x*(924*a
^3*c*d^4 - 280*b^3*c^4*d + 1100*a*b^2*c^3*d^2 - 1584*a^2*b*c^2*d^3))/(1155*
b^5*(a*d - b*c)^4) + (32*d^5*x^5)/(1155*b^2*(a*d - b*c)^4) + (16*d^4*x^4*(1
1*a*d - b*c))/(1155*b^3*(a*d - b*c)^4) + (4*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2
 - 22*a*b*c*d))/(1155*b^4*(a*d - b*c)^4)))/(x^5*(a + b*x)^(1/2) + (a^5*(a +
 b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)
^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

3.1481 $\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3}-\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2}+\frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d}+\frac{(bc-ad}{}$$

```
[Out] 1/12*(-a*d+b*c)*(b*x+a)^(7/2)*(d*x+c)^(3/2)/b^2+1/6*(b*x+a)^(7/2)*(d*x+c)^(
5/2)/b-5/512*(-a*d+b*c)^6*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/
2))/b^(7/2)/d^(7/2)-5/768*(-a*d+b*c)^4*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d^2+
1/192*(-a*d+b*c)^3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3/d+1/32*(-a*d+b*c)^2*(b*x
+a)^(7/2)*(d*x+c)^(1/2)/b^3+5/512*(-a*d+b*c)^5*(b*x+a)^(1/2)*(d*x+c)^(1/2)/
b^3/d^3
```

Rubi [A]
time = 0.11, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}{768b^3d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}{192b^3d}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x])/(512*b^3*d^3) - (5*(b*c - a*d
)^4*(a + b*x)^(3/2)*Sqrt[c + d*x])/(768*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)
^(5/2)*Sqrt[c + d*x])/(192*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(32*b^3) + ((b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(12*b^2) +
((a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b) - (5*(b*c - a*d)^6*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(512*b^(7/2)*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(5(bc-ad))\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(bc-ad)^2\int (a+$$

$$= \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)}{}$$

$$= \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad}{}$$

$$= -\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad}{}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\,\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad}{}$$

Mathematica [A]
time = 0.67, size = 191, normalized size = 0.73

$$\frac{\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx}\,\sqrt{c+dx}\,(15d^5(a+bx)^5-85bd^4(a+bx)^4(c+dx)+198b^2d^3(a+bx)^3(c+dx)^2+198b^3d^2(a+bx)^2(c+dx)^3-85b^4d(a+bx)(c+dx)^4+15b^5(c+dx)^5)-15(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{1536b^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^5*(a + b*x)^5 - 85*b*d^4
*(a + b*x)^4*(c + d*x) + 198*b^2*d^3*(a + b*x)^3*(c + d*x)^2 + 198*b^3*d^2*
(a + b*x)^2*(c + d*x)^3 - 85*b^4*d*(a + b*x)*(c + d*x)^4 + 15*b^5*(c + d*x)
^5) - 15*(b*c - a*d)^6*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*
x])])/(1536*b^(7/2)*d^(7/2))
```

Maple [A]
time = 0.16, size = 272, normalized size = 1.04

method	result

$$5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{2}}}{5d}-\frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d}-\frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5(ad-bc\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(b*x+a)^(5/2)*(d*x+c)^(7/2)-5/12*(-a*d+b*c)/d*(1/5/d*(b*x+a)^(3/2)*(d
*x+c)^(7/2)-3/10*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)-1/8*(-a*d+
b*c)/d*(1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*c)/b*(1/2*(d*x+c)^(3/2
)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b
*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b
*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(212) = 424$.
time = 1.07, size = 882, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6144*(15*(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sq
rt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(256*b^6*d^6*x^5 +
 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^3*d^3 + 198*a^3*b^3*c^2*d^
4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*d^5 + a*b^5*d^6)*x^4 + 16*
(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^6)*x^3 + 8*(b^6*c^3*d^3 +
159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^2 - 2*(5*b^6*c^4*d^2
 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^3*b^3*c*d^5 + 5*a^4*b^2*d^
6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4), 1/3072*(15*(b^6*c^6 - 6*a*b^5
*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
```

```
^5*b*c*d^5 + a^6*d^6)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d
)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)) + 2*(256*b^6*d^6*x^5 + 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^
3*d^3 + 198*a^3*b^3*c^2*d^4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*
d^5 + a*b^5*d^6)*x^4 + 16*(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^
6)*x^3 + 8*(b^6*c^3*d^3 + 159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d
^6)*x^2 - 2*(5*b^6*c^4*d^2 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^
3*b^3*c*d^5 + 5*a^4*b^2*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}} (c+dx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3120 vs. $2(212) = 424$.
time = 1.81, size = 3120, normalized size = 11.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7680*(960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c^2*abs(b) - 7680*((b^2*c - a*
b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c^2*a
bs(b)/b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x +
a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^
2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt
(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d
^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*
b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c^2*abs(b) + 240*(sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 -
 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b
^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c
```

*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)
*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))
*a*c*d*abs(b) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*
(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7
*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*
c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*c*d*abs(b)/b + 8*(s
qrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a
)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b
^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c
^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(
7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^
7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c
^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*b^3*d^4))*b*c*d*abs(b) + 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
(2(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^
8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^2
3*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a
^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5
 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*
sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*
b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*a*d^2*abs(b)
+ 320*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^3*d^2*abs(b)/b^2 + 120*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*
c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 16
3*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2
*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^
2*d^3))*a^2*d^2*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(2*(b
x + a)(8*(b*x + a)*(10*(b*x + a)/b^5 + (b^30*c*d^9 - 61*a*b^29*d^10)/(b^3
4*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*c*d^9 - 417*a^2*b^29*d^10)/(b^34*d
^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2*d^8 + 183*a^2*b^30*c*d^9 - 3481*a^
3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5*(21*b^33*c^4*d^6 + 56*a*b^32*c^3*d^
7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^30*c*d^9 - 2279*a^4*b^29*d^10)/(b^34*d
^10))*(b*x + a) + 15*(21*b^34*c^5*d^5 + 35*a*b^33*c^4*d^6 + 50*a^2*b^32*c^3
*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4*b^30*c*d^9 - 793*a^5*b^29*d^10)/(b^34*
d^10))*sqrt(b*x + a) + 15*(21*b^6*c^6 + 14*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2

```
 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 - 231*a^6*d^6)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b^4*d^5))*b*d^2*abs(b) + 5760*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x ...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(5/2), x)
```

3.1482 $\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=224

$$-\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2}+\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d}+\frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3}+\frac{(bc-ad}{}$$

[Out] 1/8*(-a*d+b*c)*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b^2+1/5*(b*x+a)^(5/2)*(d*x+c)^(5/2)/b+3/128*(-a*d+b*c)^5*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(5/2)+1/64*(-a*d+b*c)^3*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d+1/16*(-a*d+b*c)^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3-3/128*(-a*d+b*c)^4*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d^2

Rubi [A]
time = 0.08, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}{128b^3d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{16b^3}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]

[Out] (-3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(5/2)*Sqrt[c + d*x])/(16*b^3) + ((b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b) + (3*(b*c - a*d)^5*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(7/2)*d^(5/2))

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(3(bc-ad)^2)\int (c$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)}{}$$

$$= \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-a}{}$$

Mathematica [A]
time = 0.53, size = 169, normalized size = 0.75

$$\frac{\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx}\,\sqrt{c+dx}\,(15d^4(a+bx)^4-70bd^3(a+bx)^3(c+dx)+128b^2d^2(a+bx)^2(c+dx)^2+70b^3d(a+bx)(c+dx)^3-15b^4(c+dx)^4)+15(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{640b^{7/2}d^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]

[Out] (Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^4*(a + b*x)^4 - 70*b*d^3*(a + b*x)^3*(c + d*x) + 128*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 70*b^3*d*(a + b*x)*(c + d*x)^3 - 15*b^4*(c + d*x)^4) + 15*(b*c - a*d)^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(640*b^(7/2)*d^(5/2))

Maple [A]
time = 0.17, size = 239, normalized size = 1.07

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{2}}}{5d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b} - \frac{5(ad-bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(3/2)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] 1/5/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)-3/10*(-a*d+b*c)/d*(1/4/d*(b*x+a)^(1/2)*(d
*x+c)^(7/2)-1/8*(-a*d+b*c)/d*(1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*
c)/b*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x
+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2)
)/(b*d)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.02, size = 702, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 70*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(
21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2
*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^3), -1/1280*(15*(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^
4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*
x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(
128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 7
0*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(
31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233
*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*
x + c))/(b^4*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1962 vs. $2(180) = 360$.
time = 1.75, size = 1962, normalized size = 8.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1920*(80*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c^2*abs(b) - 1920*((b^2*c - a*b*d
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c^2*abs(
b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*
(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d
^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 +
9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*
x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3
- 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)))/(sqrt(b*d)*b^2*d^3))*c*d*abs(b) + 320*(sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a
*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^
7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*
d^2))*a*c*d*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)
*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) -
 (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b
^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^
23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c
^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) -
15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35
```

```
*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*d^2*abs(b) + 80*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*
c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*
b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b*d^2))*a^2*d^2*abs(b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^
6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^1
4*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3
*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d^2*abs(b)
/b + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a*c^2*abs(b)/b^2 + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*a^2*c*d*abs(b)/b^3)/b
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/2),x)
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/2), x)
```

3.1483 $\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx$

Optimal. Leaf size=186

$$\frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d}+\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3}+\frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2}+\frac{(a+bx)}{}$$

[Out] 5/24*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^2+1/4*(b*x+a)^(3/2)*(d*x+c)^(5/2)/b-5/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(3/2)+5/32*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3+5/64*(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d

Rubi [A]

time = 0.07, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}{24b^2}+\frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^3*d) + (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^3) + (5*(b*c - a*d)*(a + b*x)^(3/2)*(
c + d*x)^(3/2))/(24*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b) - (5*(b*
c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^
(7/2)*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad))\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad)^2)\int \sqrt{}}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad}{}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad}{}$$

Mathematica [A]
time = 0.35, size = 144, normalized size = 0.77

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(15d^3(a+bx)^3-55bd^2(a+bx)^2(c+dx)+73b^2d(a+bx)(c+dx)^2+15b^3(c+dx)^3)}{192b^3d} - \frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(15*d^3*(a + b*x)^3 - 55*b*d^2*(a + b*x)^2*(c + d*x) + 73*b^2*d*(a + b*x)*(c + d*x)^2 + 15*b^3*(c + d*x)^3))/(192*b^3*d) - (5*(b*c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(7/2)*d^(3/2))

Maple [A]
time = 0.16, size = 206, normalized size = 1.11

method	result
default	$\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d} - \frac{(-ad+bc)\left(\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b} - \frac{5(ad-bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b} - \frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b} \cdots\right.}{\cdots}\right.}{\cdots}\right.}{8d \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/2)*(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)-1/8*(-a*d+b*c)/d*(1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*c)/b*(1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more detail

Fricas [A]
time = 1.16, size = 540, normalized size = 2.90

$$\left[\frac{15\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(48\,b^4d^4x^3+15\,b^4c^3d+73\,ab^3c^2d^2-55\,a^2b^2cd^3+15\,a^3bd^4+8\,(17\,b^4cd^3+ab^3d^4)x^2+2\,(59\,b^4c^2d^2+18\,ab^3cd^3-5\,a^2b^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{768\,b^4d^2},\ \frac{15\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(48\,b^4d^4x^3+15\,b^4c^3d+73\,ab^3c^2d^2-55\,a^2b^2cd^3+15\,a^3bd^4+8\,(17\,b^4cd^3+ab^3d^4)x^2+2\,(59\,b^4c^2d^2+18\,ab^3cd^3-5\,a^2b^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{384\,b^4d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^
2 + 18*a*b^3*c*d^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^
2), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^2 + 18*a*b^3*c*d
^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. 2(148) = 296.
time = 1.22, size = 1083, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/192*(192*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*
sqrt(b*x + a))*a*c^2*abs(b)/b^2 - 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
```

```
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*d*abs
(b)/b - 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d^2*abs(b)/b^2 - (sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*
d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*
a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b
^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt
(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*
d^3))*d^2*abs(b)/b - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*c^2*abs(b)/b^2 - 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2
*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
- 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d))*a*c*d*abs(b)/b^3)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/2),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/2), x)
```

3.1484 $\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=148

$$\frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3}+\frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}+\frac{5(bc-ad)^3\tanh^{-1}}{8b^{7/2}}$$

```
[Out] 5/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/
2)/d^(1/2)+5/12*(-a*d+b*c)*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b^2+1/3*(d*x+c)^(5/2
)*(b*x+a)^(1/2)/b+5/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3
```

Rubi [A]

time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^3) + (5*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2))/(12*b^2) + (Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b)
 + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*b^(7/2)*Sqrt[d])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad))\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad)^2)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

Mathematica [A]
time = 0.24, size = 124, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(15a^2d^2-10abd(4c+dx)+b^2(33c^2+26cdx+8d^2x^2))}{24b^3} + \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(4*c + d*x) + b^2*(33*c
^2 + 26*c*d*x + 8*d^2*x^2)))/(24*b^3) + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*S
qrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(7/2)*Sqrt[d])
```

Maple [A]
time = 0.16, size = 173, normalized size = 1.17

method	result
default	$\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}-\frac{5(ad-bc)\left(\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}-\frac{3(ad-bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}-\frac{(ad-bc)\sqrt{(bx+a)(dx+}}{2b}\right.}{4b}\right.}{6b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/6*(a*d-b*c)/b*(1/2*(d*x+c)^(3/2)*(b*x+a
)^(1/2)/b-3/4*(a*d-b*c)/b*((b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((
b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x
)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.94, size = 412, normalized size = 2.78

$$\left[-\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2+33\,b^3c^2d-40\,ab^2cd^2+15\,a^2bd^3+2\,(13\,b^3cd^2-5\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,b^4d},\ -\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(8\,b^3d^3x^2+33\,b^3c^2d-40\,ab^2cd^2+15\,a^2bd^3+2\,(13\,b^3cd^2-5\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 + 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 -
5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d), -1/48*(15*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b
*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (
b^2*c*d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 +
 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 - 5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x +
c))/(b^4*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(116) = 232.
time = 1.38, size = 446, normalized size = 3.01

$$-\frac{\frac{24\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)c^2|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)}{b^2}+\frac{b^6cd^3-13ab^5d^4}{b^7d^4}\right)-\frac{3(b^7c^2d^2+2ab^6cd^3-11a^2b^5d^4)}{b^7d^4}\right)-\frac{3(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)d^2|b|}{b^2}-\frac{12\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^3c^2+2ab^2cd-3a^2bd^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)cd|b|}{b^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*c^2*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d^2*abs(b)/b^2
 - 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)
/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b
*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*
d*abs(b)/b^3)/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(1/2), x)
```

3.1485 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a}\cdots}{\sqrt{b}\,\sqrt{c}\cdots}\right.}{4b^{7/2}}$$

```
[Out] 15/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*d^(1
/2)/b^(7/2)-2*(d*x+c)^(5/2)/b/(b*x+a)^(1/2)+5/2*d*(d*x+c)^(3/2)*(b*x+a)^(1/
2)/b^2+15/4*d*(-a*d+b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}+\frac{15d\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (15*d*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^3) + (5*d*Sqrt[a + b*x]
*(c + d*x)^(3/2))/(2*b^2) - (2*(c + d*x)^(5/2))/(b*Sqrt[a + b*x]) + (15*Sqr
t[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(4*b^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(15d(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b^2}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(15d(bc-}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(15d(bc-}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(15d(bc-}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{15\sqrt{d}\,(bc}{}$$

Mathematica [A]
time = 0.26, size = 124, normalized size = 0.90

$$\frac{\sqrt{c+dx}\,(-15a^2d^2-5abd(-5c+dx)+b^2(-8c^2+9cdx+2d^2x^2))}{4b^3\sqrt{a+bx}}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x]*(-15*a^2*d^2 - 5*a*b*d*(-5*c + d*x) + b^2*(-8*c^2 + 9*c*d*x
+ 2*d^2*x^2)))/(4*b^3*Sqrt[a + b*x]) + (15*Sqrt[d]*(b*c - a*d)^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(7/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.01, size = 439, normalized size = 3.18

$$\left[\frac{15\,(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x)\sqrt{\frac{d}{b}}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x^2-8\,b^2c^2+25\,abcd-15\,a^2d^2+(9\,b^2cd-5\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,(b^4x+ab^3)},\ -\frac{15\,(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x)\sqrt{-\frac{d}{b}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)-2\,(2\,b^2d^2x^2-8\,b^2c^2+25\,abcd-15\,a^2d^2+(9\,b^2cd-5\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,(b^4x+ab^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2
+ (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3),
-1/8*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(2*b^2*d^2*x
^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2 + (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(108) = 216.
time = 1.49, size = 287, normalized size = 2.08

$$\frac{1}{4}\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)d^2|b|}{b^5}+\frac{9\,(b^{10}cd^3|b|-ab^9d^4|b|)}{b^{14}d^2}\right)-\frac{15\left(\sqrt{bd}\,b^2c^2|b|-2\sqrt{bd}\,abcd|b|+\sqrt{bd}\,a^2d^2|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{8\,b^5}-\frac{4\left(\sqrt{bd}\,b^3c^3|b|-3\sqrt{bd}\,ab^2c^2d|b|+3\sqrt{bd}\,a^2bcd^2|b|-\sqrt{bd}\,a^3d^3|b|\right)}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*d^2*abs(
b)/b^5 + 9*(b^10*c*d^3*abs(b) - a*b^9*d^4*abs(b))/(b^14*d^2)) - 15/8*(sqrt(
b*d)*b^2*c^2*abs(b) - 2*sqrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b)
)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^
5 - 4*(sqrt(b*d)*b^3*c^3*abs(b) - 3*sqrt(b*d)*a*b^2*c^2*d*abs(b) + 3*sqrt(b
*d)*a^2*b*c*d^2*abs(b) - sqrt(b*d)*a^3*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(3/2), x)
```

3.1486 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}$$

```
[Out] -2/3*(d*x+c)^(5/2)/b/(b*x+a)^(3/2)+5*d^(3/2)*(-a*d+b*c)*arctanh(d^(1/2)*(b*
x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)-10/3*d*(d*x+c)^(3/2)/b^2/(b*x+a)^
(1/2)+5*d^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}+\frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (5*d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/b^3 - (10*d*(c + d*x)^(3/2))/(3*b^2*Sqr
t[a + b*x]) - (2*(c + d*x)^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*d^(3/2)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}$$

$$= -\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d^2)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b^2}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d^2(bc-ad))\int \frac{}{\sqrt{a+bx}}}{2b^3}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d^2(bc-ad))\,\mathrm{Subst}\left(\int -\right.}{}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d^2(bc-ad))\,\mathrm{Subst}\left(\int \frac{}{1-}\right.}{b^4}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{b^{7/2}}$$

Mathematica [A]

time = 0.73, size = 121, normalized size = 0.95

$$\frac{\frac{\sqrt{c+dx}\,(15a^2d^2-10abd(c-2dx)+b^2(-2c^2-14cdx+3d^2x^2))}{(a+bx)^{3/2}}+\frac{15d(-bc+ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{\sqrt{\frac{b}{d}}}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] ((Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(-2*c^2 - 14*c*d*x
 + 3*d^2*x^2)))/(a + b*x)^(3/2) + (15*d*(-(b*c) + a*d)*Log[Sqrt[a + b*x] -
Sqrt[b/d]*Sqrt[c + d*x]])/Sqrt[b/d])/(3*b^3)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(100) = 200.
time = 1.42, size = 475, normalized size = 3.71

$$\left[-\frac{15\,(a^2bcd-a^3d^2+(b^3cd-ab^2d^2)x^2+2\,(ab^2cd-a^2bd^2)x)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,(3\,b^2d^2x^2-2\,b^2c^2-10\,abcd+15\,a^2d^2-2\,(7\,b^2cd-10\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{12\,(b^5x^2+2\,ab^4x+a^2b^3)},\ -\frac{15\,(a^2bcd-a^3d^2+(b^3cd-ab^2d^2)x^2+2\,(ab^2cd-a^2bd^2)x)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)-2\,(3\,b^2d^2x^2-2\,b^2c^2-10\,abcd+15\,a^2d^2-2\,(7\,b^2cd-10\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{6\,(b^5x^2+2\,ab^4x+a^2b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^2)*x^2 + 2*(a*b^2*c*d
- a^2*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2
*d^2 - 2*(7*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/6*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^
2)*x^2 + 2*(a*b^2*c*d - a^2*b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c
+ a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d +
 a*d^2)*x)) - 2*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 2*(7
*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{2}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(100) = 200.
time = 1.36, size = 650, normalized size = 5.08

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\,d^2|b|}{b^5}-\frac{5\left(\sqrt{bd}\,bcd|b|-\sqrt{bd}\,ad^2|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2\,b^5}-\frac{4\left(7\,\sqrt{bd}\,b^6c^4d|b|-28\,\sqrt{bd}\,ab^5c^3d^2|b|+42\,\sqrt{bd}\,a^2b^4c^2d^3|b|-28\,\sqrt{bd}\,a^3b^3cd^4|b|+7\,\sqrt{bd}\,a^4b^2d^5|b|-12\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^4c^3d|b|+36\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^3c^2d^2|b|-36\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^2cd^3|b|+12\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3bd^4|b|+9\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^2c^2d|b|-18\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4abcd^2|b|+9\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2d^3|b|\right)}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d^2*abs(b)/b^5 - 5/2*(sqr
t(b*d)*b*c*d*abs(b) - sqrt(b*d)*a*d^2*abs(b))*log((sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^5 - 4/3*(7*sqrt(b*d)*b^6*c^4*d*
abs(b) - 28*sqrt(b*d)*a*b^5*c^3*d^2*abs(b) + 42*sqrt(b*d)*a^2*b^4*c^2*d^3*a
bs(b) - 28*sqrt(b*d)*a^3*b^3*c*d^4*abs(b) + 7*sqrt(b*d)*a^4*b^2*d^5*abs(b)
- 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^4*c^3*d*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a*b^3*c^2*d^2*abs(b) - 36*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*c*d^3*abs(b
) + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
```

```
b*d))^2*a^3*b*d^4*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*b^2*c^2*d*abs(b) - 18*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b*c*d^2*abs(b) + 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a^2*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(5/2), x)
```

3.1487 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}} dx$

Optimal. Leaf size=120

$$-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}$$

```
[Out] -2/3*d*(d*x+c)^(3/2)/b^2/(b*x+a)^(3/2)-2/5*(d*x+c)^(5/2)/b/(b*x+a)^(5/2)+2*
d^(5/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)-2*d^2*
(d*x+c)^(1/2)/b^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 223, 212}

$$\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*d^2*Sqrt[c + d*x])/(b^3*Sqrt[a + b*x]) - (2*d*(c + d*x)^(3/2))/(3*b^2*(
a + b*x)^(3/2)) - (2*(c + d*x)^(5/2))/(5*b*(a + b*x)^(5/2)) + (2*d^(5/2)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx}{b}\\
&= -\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^2\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b^2}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^3\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b^3}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{(2d^3)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx\right.}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{(2d^3)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right.}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 111, normalized size = 0.92

$$-\frac{2\sqrt{c+dx}\,(15a^2d^2+5abd(c+7dx)+b^2(3c^2+11cdx+23d^2x^2))}{15b^3(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(3*c^2 + 11*c*d*x
 + 23*d^2*x^2)))/(15*b^3*(a + b*x)^(5/2)) + (2*d^(5/2)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(7/2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(92) = 184$.
time = 1.93, size = 463, normalized size = 3.86

$$\left[\frac{15\,(b^3d^2x^3+3\,ab^2d^2x^2+3\,a^2bd^2x+a^3d^2)\sqrt{\frac{d}{b}}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,b^2dx+b^2c+abd)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\,(b^2cd+abd^2)x\right)-4\,(23\,b^2d^2x^2+3\,b^2c^2+5\,abcd+15\,a^2d^2+(11\,b^2cd+35\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{30\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)},\ -\frac{15\,(b^3d^2x^3+3\,ab^2d^2x^2+3\,a^2bd^2x+a^3d^2)\sqrt{-\frac{d}{b}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\,(bd^2x^2+acd+(bcd+ad^2)x)}\right)+2\,(23\,b^2d^2x^2+3\,b^2c^2+5\,abcd+15\,a^2d^2+(11\,b^2cd+35\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{15\,(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x + a^3*d^2)*sqrt(d/
b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c
 + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x)
- 4*(23*b^2*d^2*x^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^
4*x + a^3*b^3), -1/15*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x +
a^3*d^2)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
 + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*(23*b^2*d^2*x
```

```
^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35*a*b*d^2)*x)*sqrt
(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. 2(92) = 184.
time = 1.95, size = 1025, normalized size = 8.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d^2*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^2)/b^5 - 4/15*(23*sqrt(b*d)*b^9*c^5*d^2*abs(b) - 115*sqrt(b*
d)*a*b^8*c^4*d^3*abs(b) + 230*sqrt(b*d)*a^2*b^7*c^3*d^4*abs(b) - 230*sqrt(b
*d)*a^3*b^6*c^2*d^5*abs(b) + 115*sqrt(b*d)*a^4*b^5*c*d^6*abs(b) - 23*sqrt(b
*d)*a^5*b^4*d^7*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*b^7*c^4*d^2*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^6*c^3*d^3*abs(b)
 - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^2*a^2*b^5*c^2*d^4*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^4*c*d^5*abs(b) - 70*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^3*d^
6*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^5*c^3*d^2*abs(b) - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^4*c^2*d^3*abs(b) + 420*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b
^3*c*d^4*abs(b) - 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^4*a^3*b^2*d^5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^3*c^2*d^2*abs(b) + 180*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a
*b^2*c*d^3*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a^2*b*d^4*abs(b) + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b*c*d^2*abs(b) - 45*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*d^3*a
```

```
bs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2)^5*b^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(7/2), x)
```

3.1488 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

```
[Out] -2/7*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(26) = 52$.

time = 0.16, size = 234, normalized size = 7.31

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}}{7(bx+a)^{\frac{7}{2}}(ad-bc)}$
default	$-\frac{(dx+c)^{\frac{5}{2}}}{b(bx+a)^{\frac{7}{2}}}+\frac{5(ad-bc)\left(-\frac{(dx+c)^{\frac{3}{2}}}{2b(bx+a)^{\frac{7}{2}}}+\frac{3(ad-bc)\left(-\frac{\sqrt{dx+c}}{3b(bx+a)^{\frac{7}{2}}}+\frac{(ad-bc)\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{6b}\right.}{4b}\right.}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)/(b*x+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(d*x+c)^(5/2)/(b*x+a)^(7/2)+5/2*(a*d-b*c)/b*(-1/2/b*(d*x+c)^(3/2)/(b*x+a)^(7/2)+3/4*(a*d-b*c)/b*(-1/3/b*(d*x+c)^(1/2)/(b*x+a)^(7/2)+1/6*(a*d-b*c)

```
/b*(-2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x
+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a
*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(26) = 52.
time = 1.47, size = 138, normalized size = 4.31

$$-\frac{2\,(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3)\sqrt{bx+a}\,\sqrt{dx+c}}{7\,(a^4bc-a^5d+(b^5c-ab^4d)x^4+4\,(ab^4c-a^2b^3d)x^3+6\,(a^2b^3c-a^3b^2d)x^2+4\,(a^3b^2c-a^4bd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(b*x + a)*sqrt(d*x + c)/
(a^4*b*c - a^5*d + (b^5*c - a*b^4*d)*x^4 + 4*(a*b^4*c - a^2*b^3*d)*x^3 + 6*
(a^2*b^3*c - a^3*b^2*d)*x^2 + 4*(a^3*b^2*c - a^4*b*d)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(26) = 52.
time = 1.18, size = 706, normalized size = 22.06

$$-\frac{4\left(\sqrt{bd}\,b^{12}c^6d^3|b|-6\sqrt{bd}\,ab^{11}c^5d^4|b|+15\sqrt{bd}\,a^2b^{10}c^4d^5|b|-20\sqrt{bd}\,a^3b^9c^3d^6|b|+15\sqrt{bd}\,a^4b^8c^2d^7|b|-6\sqrt{bd}\,a^5b^7cd^8|b|+\sqrt{bd}\,a^6b^6d^9|b|+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^8c^4d^3|b|-84\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^7c^3d^4|b|+126\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^6c^2d^5|b|-84\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^3b^5cd^6|b|+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^4b^4d^7|b|+35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8b^4c^2d^3|b|-70\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8ab^3cd^4|b|+35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8a^2b^2d^5|b|+7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^{12}d^3|b|\right)}{7\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^7b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="giac")

[Out] -4/7*(sqrt(b*d)*b^12*c^6*d^3*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^4*abs(b) + 15*sqrt(b*d)*a^2*b^10*c^4*d^5*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^6*abs(b) + 15*sqrt(b*d)*a^4*b^8*c^2*d^7*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^8*abs(b) + sqrt(b*d)*a^6*b^6*d^9*abs(b) + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^3*abs(b) - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^7*c^3*d^4*abs(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^6*c^2*d^5*abs(b) - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^6*abs(b) + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^4*d^7*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^4*c^2*d^3*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^3*c*d^4*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2*d^5*abs(b) + 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^4)

Mupad [B]
time = 0.97, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{\left(7\,a\,d-7\,b\,c\right)\,{\left(a+b\,x\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(5/2)/(a + b*x)^(9/2),x)

[Out] (2*(c + d*x)^(7/2))/((7*a*d - 7*b*c)*(a + b*x)^(7/2))

3.1489 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}$$

```
[Out] -2/9*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(9/2)+4/63*d*(d*x+c)^(7/2)/(-a*d+b*c)
^2/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(7/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx = -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(2d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}} + \frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}$$

Mathematica [A]
time = 0.11, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{7/2}(-7bc+9ad+2bdx)}{63(bc-ad)^2(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-7*b*c + 9*a*d + 2*b*d*x))/(63*(b*c - a*d)^2*(a + b*x)^
(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(54) = 108.
time = 0.16, size = 274, normalized size = 4.15

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}(2bdx+9ad-7bc)}{63(bx+a)^{\frac{9}{2}}(a^2d^2-2abcd+b^2c^2)}$

$$8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\right.\quad 6d\left(-\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\right.$$

$$(ad-bc)\left(-\frac{\sqrt{dx+c}}{4b(bx+a)^{\frac{9}{2}}}+\right.$$

$$5(ad-bc)\left(-\frac{(dx+c)^{\frac{3}{2}}}{3b(bx+a)^{\frac{9}{2}}}+\frac{\cdots}{2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d*x+c)^(5/2)/(b*x+a)^(9/2)+5/4*(a*d-b*c)/b*(-1/3/b*(d*x+c)^(3/2)/(b
*x+a)^(9/2)+1/2*(a*d-b*c)/b*(-1/4/b*(d*x+c)^(1/2)/(b*x+a)^(9/2)+1/8*(a*d-b*
c)/b*(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d
*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(
-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*
x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(54) = 108$.
time = 5.92, size = 295, normalized size = 4.47

$$\frac{2\,{\left(2\,b d^{4} x^{4} - 7\,b c^{4} + 9\,a c^{3} d - {\left(b c d^{3} - 9\,a d^{4}\right)} x^{3} - 3\,{\left(5\,b c^{2} d^{2} - 9\,a c d^{3}\right)} x^{2} - {\left(19\,b c^{3} d - 27\,a c^{2} d^{2}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{63\,{\left(a^{5} b^{2} c^{2} - 2\,a^{6} b c d + a^{7} d^{2} + {\left(b^{7} c^{2} - 2\,a b^{6} c d + a^{2} b^{5} d^{2}\right)} x^{5} + 5\,{\left(a b^{6} c^{2} - 2\,a^{2} b^{5} c d + a^{3} b^{4} d^{2}\right)} x^{4} + 10\,{\left(a^{2} b^{5} c^{2} - 2\,a^{3} b^{4} c d + a^{4} b^{3} d^{2}\right)} x^{3} + 10\,{\left(a^{3} b^{4} c^{2} - 2\,a^{4} b^{3} c d + a^{5} b^{2} d^{2}\right)} x^{2} + 5\,{\left(a^{4} b^{3} c^{2} - 2\,a^{5} b^{2} c d + a^{6} b d^{2}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b*d^4*x^4 - 7*b*c^4 + 9*a*c^3*d - (b*c*d^3 - 9*a*d^4)*x^3 - 3*(5*b*
c^2*d^2 - 9*a*c*d^3)*x^2 - (19*b*c^3*d - 27*a*c^2*d^2)*x)*sqrt(b*x + a)*sqr
t(d*x + c)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (b^7*c^2 - 2*a*b^6*c*d +
a^2*b^5*d^2)*x^5 + 5*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*x^4 + 10*(a^
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*x^3 + 10*(a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*x^2 + 5*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1826 vs. $2(54) = 108$.
time = 1.72, size = 1826, normalized size = 27.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 8/63*(sqrt(b*d)*b^14*c^7*d^4*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^5*abs(b) + 2
1*sqrt(b*d)*a^2*b^12*c^5*d^6*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^7*abs(b)
+ 35*sqrt(b*d)*a^4*b^10*c^3*d^8*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^9*abs(b
) + 7*sqrt(b*d)*a^6*b^8*c*d^10*abs(b) - sqrt(b*d)*a^7*b^7*d^11*abs(b) - 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
b^12*c^6*d^4*abs(b) + 54*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^5*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^6*abs
(b) + 180*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^9*c^3*d^7*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^8*abs(b) + 54*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b
^7*c*d^9*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^2*a^6*b^6*d^10*abs(b) - 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^4*abs(b) + 135*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^9*c^4*d^5*abs(b) - 270*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^6*abs(b) + 270*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2*d^7*ab
s(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^4*b^6*c*d^8*abs(b) + 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^9*abs(b) - 189*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^8*c^4*d^
4*abs(b) + 756*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^6*a*b^7*c^3*d^5*abs(b) - 1134*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^6*abs(b) + 756*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
a^3*b^5*c*d^7*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^8*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^4*abs(b) +
567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
```

```
))^8*a*b^5*c^2*d^5*abs(b) - 567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^6*abs(b) + 189*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^3*d^7*ab
s(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^10*b^4*c^2*d^4*abs(b) + 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^5*abs(b) - 315*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^2*
d^6*abs(b) - 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^12*b^2*c*d^4*abs(b) + 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^5*abs(b) - 63*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*d^4*abs(b
))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^2)^9*b^3)
```

Mupad [B]
time = 1.14, size = 229, normalized size = 3.47

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^4\,x^4}{63\,b^3\,{\left(a\,d-b\,c\right)}^2}-\frac{14\,b\,c^4-18\,a\,c^3\,d}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{x^3\,\left(18\,a\,d^4-2\,b\,c\,d^3\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c^2\,d\,x\,\left(27\,a\,d-19\,b\,c\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c\,d^2\,x^2\,\left(9\,a\,d-5\,b\,c\right)}{21\,b^4\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^4*x^4)/(63*b^3*(a*d - b*c)^2) - (14*b*c^4 - 18*a*c^3
*d)/(63*b^4*(a*d - b*c)^2) + (x^3*(18*a*d^4 - 2*b*c*d^3))/(63*b^4*(a*d - b*
c)^2) + (2*c^2*d*x*(27*a*d - 19*b*c))/(63*b^4*(a*d - b*c)^2) + (2*c*d^2*x^2
*(9*a*d - 5*b*c))/(21*b^4*(a*d - b*c)^2)))/(x^4*(a + b*x)^(1/2) + (a^4*(a +
 b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/
2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

3.1490 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}$$

```
[Out] -2/11*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(11/2)+8/99*d*(d*x+c)^(7/2)/(-a*d+b*
c)^2/(b*x+a)^(9/2)-16/693*d^2*(d*x+c)^(7/2)/(-a*d+b*c)^3/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (8*d*(c + d*x)^(7/
2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(7/2))/(693*(b*c
 - a*d)^3*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} - \frac{(4d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$
$$= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}} + \frac{(8d^2)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{99(bc-ad)^2}$$
$$= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}} + \frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}} - \frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{7/2}\left(99d^2 - \frac{154bd(c+dx)}{a+bx} + \frac{63b^2(c+dx)^2}{(a+bx)^2}\right)}{693(bc-ad)^3(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(99*d^2 - (154*b*d*(c + d*x))/(a + b*x) + (63*b^2*(c +
d*x)^2)/(a + b*x)^2))/(693*(b*c - a*d)^3*(a + b*x)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(83) = 166.
time = 0.16, size = 314, normalized size = 3.11

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}\left(8b^2x^2d^2+44ab\,d^2x-28b^2cdx+99a^2d^2-154abcd+63b^2c^2\right)}{693(bx+a)^{\frac{11}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$

$$10d\left(-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}}-\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}-\right.$$

$$3(ad-bc)\left(-\frac{\sqrt{dx+c}}{5b(bx+a)^{\frac{11}{2}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(d*x+c)^(5/2)/(b*x+a)^(11/2)+5/6*(a*d-b*c)/b*(-1/4/b*(d*x+c)^(3/2)/(
b*x+a)^(11/2)+3/8*(a*d-b*c)/b*(-1/5/b*(d*x+c)^(1/2)/(b*x+a)^(11/2)+1/10*(a*
d-b*c)/b*(-2/11/(-a*d+b*c)/(b*x+a)^(11/2)*(d*x+c)^(1/2)-10/11*d/(-a*d+b*c)*
(-2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)
^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+
b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^
(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(83) = 166$.
time = 10.56, size = 513, normalized size = 5.08

$$-\frac{2\,{\left(8\,b^{2}d^{5}x^{5} + 63\,b^{2}c^{5} - 154\,abc^{4}d + 99\,a^{2}c^{3}d^{2} - 4\,{\left(b^{2}cd^{4} - 11\,abd^{5}\right)}x^{4} + {\left(3\,b^{2}c^{2}d^{3} - 22\,abcd^{4} + 99\,a^{2}d^{5}\right)}x^{3} + {\left(113\,b^{2}c^{3}d^{2} - 330\,abc^{2}d^{3} + 297\,a^{2}cd^{4}\right)}x^{2} + {\left(161\,b^{2}c^{4}d - 418\,abc^{3}d^{2} + 297\,a^{2}c^{2}d^{3}\right)}x\right)}\sqrt{bx + a}\,\sqrt{dx + c}}{693\,{\left(a^{6}b^{3}c^{3} - 3\,a^{7}b^{2}c^{2}d + 3\,a^{8}bcd^{2} - a^{9}d^{3} + {\left(b^{9}c^{3} - 3\,ab^{8}c^{2}d + 3\,a^{2}b^{7}cd^{2} - a^{3}b^{6}d^{3}\right)}x^{6} + 6\,{\left(ab^{8}c^{3} - 3\,a^{2}b^{7}c^{2}d + 3\,a^{3}b^{6}cd^{2} - a^{4}b^{5}d^{3}\right)}x^{5} + 15\,{\left(a^{2}b^{7}c^{3} - 3\,a^{3}b^{6}c^{2}d + 3\,a^{4}b^{5}cd^{2} - a^{5}b^{4}d^{3}\right)}x^{4} + 20\,{\left(a^{3}b^{6}c^{3} - 3\,a^{4}b^{5}c^{2}d + 3\,a^{5}b^{4}cd^{2} - a^{6}b^{3}d^{3}\right)}x^{3} + 15\,{\left(a^{4}b^{5}c^{3} - 3\,a^{5}b^{4}c^{2}d + 3\,a^{6}b^{3}cd^{2} - a^{7}b^{2}d^{3}\right)}x^{2} + 6\,{\left(a^{5}b^{4}c^{3} - 3\,a^{6}b^{3}c^{2}d + 3\,a^{7}b^{2}cd^{2} - a^{8}bd^{3}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^2*d^5*x^5 + 63*b^2*c^5 - 154*a*b*c^4*d + 99*a^2*c^3*d^2 - 4*(b^
2*c*d^4 - 11*a*b*d^5)*x^4 + (3*b^2*c^2*d^3 - 22*a*b*c*d^4 + 99*a^2*d^5)*x^3
 + (113*b^2*c^3*d^2 - 330*a*b*c^2*d^3 + 297*a^2*c*d^4)*x^2 + (161*b^2*c^4*d
 - 418*a*b*c^3*d^2 + 297*a^2*c^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b
^3*c^3 - 3*a^7*b^2*c^2*d + 3*a^8*b*c*d^2 - a^9*d^3 + (b^9*c^3 - 3*a*b^8*c^2
*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*x^6 + 6*(a*b^8*c^3 - 3*a^2*b^7*c^2*d +
3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*x^5 + 15*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*x^4 + 20*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2 - a^6*b^3*d^3)*x^3 + 15*(a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*
b^3*c*d^2 - a^7*b^2*d^3)*x^2 + 6*(a^5*b^4*c^3 - 3*a^6*b^3*c^2*d + 3*a^7*b^2
*c*d^2 - a^8*b*d^3)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(5/2)/(b*x+a)**(13/2),x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2316 vs. 2(83) = 166.
time = 1.91, size = 2316, normalized size = 22.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="giac")

[Out] -32/693*(sqrt(b*d)*b^16*c^8*d^5*abs(b) - 8*sqrt(b*d)*a*b^15*c^7*d^6*abs(b) + 28*sqrt(b*d)*a^2*b^14*c^6*d^7*abs(b) - 56*sqrt(b*d)*a^3*b^13*c^5*d^8*abs(b) + 70*sqrt(b*d)*a^4*b^12*c^4*d^9*abs(b) - 56*sqrt(b*d)*a^5*b^11*c^3*d^10*abs(b) + 28*sqrt(b*d)*a^6*b^10*c^2*d^11*abs(b) - 8*sqrt(b*d)*a^7*b^9*c*d^12*abs(b) + sqrt(b*d)*a^8*b^8*d^13*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14*c^7*d^5*abs(b) + 77*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^13*c^6*d^6*abs(b) - 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^5*d^7*abs(b) + 385*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^4*d^8*abs(b) - 385*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^10*c^3*d^9*abs(b) + 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*c^2*d^10*abs(b) - 77*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^6*b^8*c*d^11*abs(b) + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^7*d^12*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^6*d^5*abs(b) - 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^11*c^5*d^6*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^10*c^4*d^7*abs(b) - 1100*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c^3*d^8*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^8*c^2*d^9*abs(b) - 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^7*c*d^10*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^6*d^11*abs(b) + 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2

```
*c + (b*x + a)*b*d - a*b*d))^6*b^10*c^5*d^5*abs(b) - 1485*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^9*c^4*d^6*ab
s(b) + 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^6*a^2*b^8*c^3*d^7*abs(b) - 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*c^2*d^8*abs(b) + 1485*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
a^4*b^6*c*d^9*abs(b) - 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^5*b^5*d^10*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^8*c^4*d^5*abs(b)
- 5940*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^8*a*b^7*c^3*d^6*abs(b) + 8910*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*c^2*d^7*abs(b) - 5940*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^5
*c*d^8*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^8*a^4*b^4*d^9*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c^3*d^5*abs(b) - 6237
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
10*a*b^5*c^2*d^6*abs(b) + 6237*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^4*c*d^7*abs(b) - 2079*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^3*d^8*
abs(b) + 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^12*b^4*c^2*d^5*abs(b) - 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^3*c*d^6*abs(b) + 2541*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2
*b^2*d^7*abs(b) + 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^14*b^2*c*d^5*abs(b) - 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b*d^6*abs(b) + 462*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*d
^5*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^2)^11*b^2)
```

Mupad [B]
time = 1.35, size = 333, normalized size = 3.30

$$\frac{\sqrt{c+d\,x}\,\left(\frac{198\,a^2\,c^3\,d^2-308\,a\,b\,c^4\,d+126\,b^2\,c^5}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{x^3\,\left(198\,a^2\,d^5-44\,a\,b\,c\,d^4+6\,b^2\,c^2\,d^3\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^5\,x^5}{693\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{693\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c\,d^2\,x^2\,\left(297\,a^2\,d^2-330\,a\,b\,c\,d+113\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c^2\,d\,x\,\left(297\,a^2\,d^2-418\,a\,b\,c\,d+161\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((126*b^2*c^5 + 198*a^2*c^3*d^2 - 308*a*b*c^4*d)/(693*b^5*
(a*d - b*c)^3) + (x^3*(198*a^2*d^5 + 6*b^2*c^2*d^3 - 44*a*b*c*d^4))/(693*b^
5*(a*d - b*c)^3) + (16*d^5*x^5)/(693*b^3*(a*d - b*c)^3) + (8*d^4*x^4*(11*a*
d - b*c))/(693*b^4*(a*d - b*c)^3) + (2*c*d^2*x^2*(297*a^2*d^2 + 113*b^2*c^2
 - 330*a*b*c*d))/(693*b^5*(a*d - b*c)^3) + (2*c^2*d*x*(297*a^2*d^2 + 161*b^
2*c^2 - 418*a*b*c*d))/(693*b^5*(a*d - b*c)^3)))/(x^5*(a + b*x)^(1/2) + (a^5
```

```
*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a +
 b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/
b^4)
```

3.1491 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}}+\frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}}-\frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}}+\frac{32d^3(c+dx)^{7/2}}{3003(bc-ad)^4(a+bx)^{7/2}}$$

```
[Out] -2/13*(d*x+c)^(7/2)/(-a*d+b*c)/(b*x+a)^(13/2)+12/143*d*(d*x+c)^(7/2)/(-a*d+
b*c)^2/(b*x+a)^(11/2)-16/429*d^2*(d*x+c)^(7/2)/(-a*d+b*c)^3/(b*x+a)^(9/2)+3
2/3003*d^3*(d*x+c)^(7/2)/(-a*d+b*c)^4/(b*x+a)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(13*(b*c - a*d)*(a + b*x)^(13/2)) + (12*d*(c + d*x)^(7
/2))/(143*(b*c - a*d)^2*(a + b*x)^(11/2)) - (16*d^2*(c + d*x)^(7/2))/(429*(
b*c - a*d)^3*(a + b*x)^(9/2)) + (32*d^3*(c + d*x)^(7/2))/(3003*(b*c - a*d)^
4*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx = -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} - \frac{(6d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx}{13(bc-ad)}$$

$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} + \frac{(24d^2)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{143(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} - \cdots$$

$$= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}} + \frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}} - \frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}} + \cdots$$

Mathematica [A]
time = 0.16, size = 95, normalized size = 0.70

$$-\frac{2(c+dx)^{7/2}\left(-429d^3+\frac{1001bd^2(c+dx)}{a+bx}-\frac{819b^2d(c+dx)^2}{(a+bx)^2}+\frac{231b^3(c+dx)^3}{(a+bx)^3}\right)}{3003(bc-ad)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(-429*d^3 + (1001*b*d^2*(c + d*x))/(a + b*x) - (819*b^2
*d*(c + d*x)^2)/(a + b*x)^2 + (231*b^3*(c + d*x)^3)/(a + b*x)^3))/(3003*(b*
c - a*d)^4*(a + b*x)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(112) = 224.
time = 0.17, size = 354, normalized size = 2.60

method	result
gosper	$\frac{2(dx+c)^{\frac{7}{2}}\left(16b^3x^3d^3+104d^3a\,x^2b^2-56b^3c\,d^2x^2+286a^2b\,d^3x-364a\,b^2c\,d^2x+126b^3c^2dx+429a^3d^3-1001a^2bc\,d^2+819a\,b^2c^2d-231b^3c^3\right)}{3003(bx+a)^{\frac{13}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

$$12d\left(-\frac{2\sqrt{dx+c}}{11(-ad+bc)(bx+a)^{\frac{11}{2}}}\right.$$

$$(ad-bc)\left(-\frac{2\sqrt{dx+c}}{13(-ad+bc)(bx+a)^{\frac{13}{2}}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/b*(d*x+c)^(5/2)/(b*x+a)^(13/2)+5/8*(a*d-b*c)/b*(-1/5/b*(d*x+c)^(3/2)/(
b*x+a)^(13/2)+3/10*(a*d-b*c)/b*(-1/6/b*(d*x+c)^(1/2)/(b*x+a)^(13/2)+1/12*(a
*d-b*c)/b*(-2/13/(-a*d+b*c)/(b*x+a)^(13/2)*(d*x+c)^(1/2)-12/13*d/(-a*d+b*c)
*(-2/11/(-a*d+b*c)/(b*x+a)^(11/2)*(d*x+c)^(1/2)-10/11*d/(-a*d+b*c)*(-2/9*(d
*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^(1/2)/(
-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*
x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/
3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))))))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(112) = 224$.
time = 16.07, size = 765, normalized size = 5.62

$$\frac{2\,(16\,b^3d^6x^6 - 231\,b^3c^6 + 819\,ab^2c^5d - 1001\,a^2bc^4d^2 + 429\,a^3c^3d^3 - 8\,(b^3cd^5 - 13\,ab^2d^6)x^5 + 2\,(3\,b^3c^2d^4 - 26\,ab^2cd^5 + 143\,a^2bd^6)x^4 - (5\,b^3c^3d^3 - 39\,ab^2c^2d^4 + 143\,a^2bcd^5 - 429\,a^3d^6)x^3 - (371\,b^3c^4d^2 - 1469\,ab^2c^3d^3 + 2145\,a^2bc^2d^4 - 1287\,a^3cd^5)x^2 - (567\,b^3c^5d - 2093\,ab^2c^4d^2 + 2717\,a^2bc^3d^3 - 1287\,a^3c^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{3003\,(a^7b^4c^4 - 4\,a^8b^3c^3d + 6\,a^9b^2c^2d^2 - 4\,a^{10}bcd^3 + a^{11}d^4 + (b^{11}c^4 - 4\,ab^{10}c^3d + 6\,a^2b^9c^2d^2 - 4\,a^3b^8cd^3 + a^4b^7d^4)x^7 + 7\,(ab^{10}c^4 - 4\,a^2b^9c^3d + 6\,a^3b^8c^2d^2 - 4\,a^4b^7cd^3 + a^5b^6d^4)x^6 + 21\,(a^2b^9c^4 - 4\,a^3b^8c^3d + 6\,a^4b^7c^2d^2 - 4\,a^5b^6cd^3 + a^6b^5d^4)x^5 + 35\,(a^3b^8c^4 - 4\,a^4b^7c^3d + 6\,a^5b^6c^2d^2 - 4\,a^6b^5cd^3 + a^7b^4d^4)x^4 + 35\,(a^4b^7c^4 - 4\,a^5b^6c^3d + 6\,a^6b^5c^2d^2 - 4\,a^7b^4cd^3 + a^8b^3d^4)x^3 + 21\,(a^5b^6c^4 - 4\,a^6b^5c^3d + 6\,a^7b^4c^2d^2 - 4\,a^8b^3cd^3 + a^9b^2d^4)x^2 + 7\,(a^6b^5c^4 - 4\,a^7b^4c^3d + 6\,a^8b^3c^2d^2 - 4\,a^9b^2cd^3 + a^{10}bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^3*d^6*x^6 - 231*b^3*c^6 + 819*a*b^2*c^5*d - 1001*a^2*b*c^4*d^2
 + 429*a^3*c^3*d^3 - 8*(b^3*c*d^5 - 13*a*b^2*d^6)*x^5 + 2*(3*b^3*c^2*d^4 -
26*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 - (5*b^3*c^3*d^3 - 39*a*b^2*c^2*d^4 + 1
43*a^2*b*c*d^5 - 429*a^3*d^6)*x^3 - (371*b^3*c^4*d^2 - 1469*a*b^2*c^3*d^3 +
 2145*a^2*b*c^2*d^4 - 1287*a^3*c*d^5)*x^2 - (567*b^3*c^5*d - 2093*a*b^2*c^4
*d^2 + 2717*a^2*b*c^3*d^3 - 1287*a^3*c^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
)/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^1
1*d^4 + (b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 +
a^4*b^7*d^4)*x^7 + 7*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*
```

```
a^4*b^7*c*d^3 + a^5*b^6*d^4)*x^6 + 21*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^
4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*x^5 + 35*(a^3*b^8*c^4 - 4*a^
4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*x^4 + 35*(
a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b
^3*d^4)*x^3 + 21*(a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8
*b^3*c*d^3 + a^9*b^2*d^4)*x^2 + 7*(a^6*b^5*c^4 - 4*a^7*b^4*c^3*d + 6*a^8*b^
3*c^2*d^2 - 4*a^9*b^2*c*d^3 + a^10*b*d^4)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(15/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2868 vs. $2(112) = 224$.
time = 2.17, size = 2868, normalized size = 21.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="giac")
```

```
[Out] 64/3003*(sqrt(b*d)*b^18*c^9*d^6*abs(b) - 9*sqrt(b*d)*a*b^17*c^8*d^7*abs(b)
+ 36*sqrt(b*d)*a^2*b^16*c^7*d^8*abs(b) - 84*sqrt(b*d)*a^3*b^15*c^6*d^9*abs(
b) + 126*sqrt(b*d)*a^4*b^14*c^5*d^10*abs(b) - 126*sqrt(b*d)*a^5*b^13*c^4*d^
11*abs(b) + 84*sqrt(b*d)*a^6*b^12*c^3*d^12*abs(b) - 36*sqrt(b*d)*a^7*b^11*c
^2*d^13*abs(b) + 9*sqrt(b*d)*a^8*b^10*c*d^14*abs(b) - sqrt(b*d)*a^9*b^9*d^1
5*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*b^16*c^8*d^6*abs(b) + 104*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^7*d^7*abs(b) - 364*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^14*c^6*d^8*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^13*c^5*d^9*abs(b) - 910*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^4*d^10
*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^5*b^11*c^3*d^11*abs(b) - 364*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^6*b^10*c^2*d^12*abs(b) + 1
04*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^7*b^9*c*d^13*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2*a^8*b^8*d^14*abs(b) + 78*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^7*d^6*abs(
```

b) - 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^13*c^6*d^7*abs(b) + 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^5*d^8*abs(b) - 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^4*d^9*abs(b) + 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^10*c^3*d^10*abs(b) - 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c^2*d^11*abs(b) + 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*c*d^12*abs(b) - 78*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^7*b^7*d^13*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^12*c^6*d^6*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^11*c^5*d^7*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^4*d^8*abs(b) + 5720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^9*c^3*d^9*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c^2*d^10*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*c*d^11*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^6*b^6*d^12*abs(b) - 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10*c^5*d^6*abs(b) + 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^9*c^4*d^7*abs(b) - 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^3*d^8*abs(b) + 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c^2*d^9*abs(b) - 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^4*b^6*c*d^10*abs(b) + 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^5*b^5*d^11*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^4*d^6*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^3*d^7*abs(b) - 61776*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^6*c^2*d^8*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*c*d^9*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^4*b^4*d^10*abs(b) - 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^3*d^6*abs(b) + 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^5*c^2*d^7*abs(b) - 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2*b^4*c*d^8*abs(b) + 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^3*b^3*d^9*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c^2*d^6*abs(b) + 36036*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*c*d^7*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a^2*b^2*d^8*abs(b) - 9009*s

```
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16
*b^2*c*d^6*abs(b) + 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^16*a*b*d^7*abs(b) - 300...
```

Mupad [B]
time = 1.62, size = 459, normalized size = 3.38

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(2574\,a^3\,c\,d^5-4290\,a^2\,b\,c^2\,d^4+2938\,a\,b^2\,c^3\,d^3-742\,b^3\,c^4\,d^2\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}-\frac{-858\,a^3\,c^3\,d^3+2002\,a^2\,b\,c^4\,d^2-1638\,a\,b^2\,c^5\,d+462\,b^3\,c^6}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{x^3\,\left(858\,a^3\,d^6-286\,a^2\,b\,c\,d^5+78\,a\,b^2\,c^2\,d^4-10\,b^3\,c^3\,d^3\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^6\,x^6}{3003\,b^3\,{\left(a\,d-b\,c\right)}^4}-\frac{x\,\left(-2574\,a^3\,c^2\,d^4+5434\,a^2\,b\,c^3\,d^3-4186\,a\,b^2\,c^4\,d^2+1134\,b^3\,c^5\,d\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^5\,x^5\,\left(13\,a\,d-b\,c\right)}{3003\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^4\,x^4\,\left(143\,a^2\,d^2-26\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3003\,b^5\,{\left(a\,d-b\,c\right)}^4}\right)}{x^6\,\sqrt{a+b\,x}+\frac{a^6\,\sqrt{a+b\,x}}{b^6}+\frac{15\,a^2\,x^4\,\sqrt{a+b\,x}}{b^2}+\frac{20\,a^3\,x^3\,\sqrt{a+b\,x}}{b^3}+\frac{15\,a^4\,x^2\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a\,x^5\,\sqrt{a+b\,x}}{b}+\frac{6\,a^5\,x\,\sqrt{a+b\,x}}{b^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(15/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(2574*a^3*c*d^5 - 742*b^3*c^4*d^2 + 2938*a*b^2*c^3*d
^3 - 4290*a^2*b*c^2*d^4))/(3003*b^6*(a*d - b*c)^4) - (462*b^3*c^6 - 858*a^3
*c^3*d^3 + 2002*a^2*b*c^4*d^2 - 1638*a*b^2*c^5*d)/(3003*b^6*(a*d - b*c)^4)
+ (x^3*(858*a^3*d^6 - 10*b^3*c^3*d^3 + 78*a*b^2*c^2*d^4 - 286*a^2*b*c*d^5))
/(3003*b^6*(a*d - b*c)^4) + (32*d^6*x^6)/(3003*b^3*(a*d - b*c)^4) - (x*(113
4*b^3*c^5*d - 2574*a^3*c^2*d^4 - 4186*a*b^2*c^4*d^2 + 5434*a^2*b*c^3*d^3))/
(3003*b^6*(a*d - b*c)^4) + (16*d^5*x^5*(13*a*d - b*c))/(3003*b^4*(a*d - b*c
)^4) + (4*d^4*x^4*(143*a^2*d^2 + 3*b^2*c^2 - 26*a*b*c*d))/(3003*b^5*(a*d -
b*c)^4)))/(x^6*(a + b*x)^(1/2) + (a^6*(a + b*x)^(1/2))/b^6 + (15*a^2*x^4*(a
 + b*x)^(1/2))/b^2 + (20*a^3*x^3*(a + b*x)^(1/2))/b^3 + (15*a^4*x^2*(a + b*
x)^(1/2))/b^4 + (6*a*x^5*(a + b*x)^(1/2))/b + (6*a^5*x*(a + b*x)^(1/2))/b^5
)
```

3.1492 $\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=183

$$-\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4}+\frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3}-\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2}+(a+$$

```
[Out] 35/64*(-a*d+b*c)^4*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(
9/2)/b^(1/2)+35/96*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^3-7/24*(-a*d+
b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d^2+1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d-35/6
4*(-a*d+b*c)^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A]

time = 0.07, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64\sqrt{b}\,d^{9/2}}-\frac{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24d^2}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (-35*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*d^4) + (35*(b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*d^3) - (7*(b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(24*d^2) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (35*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*Sqrt
[b]*d^(9/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} + \frac{(35(bc-ad)^2)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}}{48d^2}$$

$$= \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+}}{4d}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a}{}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a}{}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a}{}$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a}{}$$

Mathematica [A]
time = 0.21, size = 164, normalized size = 0.90

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(279a^3d^3+a^2bd^2(-511c+326dx)+ab^2d(385c^2-252cdx+200d^2x^2)+b^3(-105c^3+70c^2dx-56cd^2x^2+48d^3x^3))}{192d^4} + \frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{64\sqrt{b}\,d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(279*a^3*d^3 + a^2*b*d^2*(-511*c + 326*d*x) +
a*b^2*d*(385*c^2 - 252*c*d*x + 200*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*x -
56*c*d^2*x^2 + 48*d^3*x^3)))/(192*d^4) + (35*(b*c - a*d)^4*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*Sqrt[b]*d^(9/2))
```

Maple [A]
time = 0.16, size = 206, normalized size = 1.13

method	result
default	$\frac{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}{4d} - \frac{7(-ad+bc)\left(\frac{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}{3d} - \frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d} - \frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\sqrt{dx+}}{d}\right.}{}\right.}{}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d-7/8*(-a*d+b*c)/d*(1/3*(b*x+a)^(5/2)*(d*x+
c)^(1/2)/d-5/6*(-a*d+b*c)/d*(1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d-3/4*(-a*d+b*
c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2
)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x
^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.07, size = 542, normalized size = 2.96

$$\left[\frac{105\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(48\,b^4d^4x^3-105\,b^4c^3d+385\,ab^3c^2d^2-511\,a^2b^2cd^3+279\,a^3bd^4-8\,(7\,b^4cd^3-25\,ab^3d^4)x^2+2\,(35\,b^4c^2d^2-126\,ab^3cd^3+163\,a^2b^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{768\,bd^5}, -\frac{105\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(48\,b^4d^4x^3-105\,b^4c^3d+385\,ab^3c^2d^2-511\,a^2b^2cd^3+279\,a^3bd^4-8\,(7\,b^4cd^3-25\,ab^3d^4)x^2+2\,(35\,b^4c^2d^2-126\,ab^3cd^3+163\,a^2b^2d^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{384\,bd^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2
*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*
b*d^2)*x) + 4*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2
*b^2*c*d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4
*c^2*d^2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^5), -1/384*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)
*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)) - 2*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2*b^2*c*
d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4*c^2*d^
2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d
^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.22, size = 268, normalized size = 1.46

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)}{bd}-\frac{7\,(bcd^5-ad^6)}{bd^7}\right)+\frac{35\,(b^2c^2d^4-2\,abcd^5+a^2d^6)}{bd^7}\right)-\frac{105\,(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)}{bd^7}\right)\sqrt{bx+a}-\frac{105\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^4}\right)b}{192\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)/(b*d) - 7*(b*c*d^5 - a*d^6)/(b*d^7)) + 35*(b^2*c^2*d^4 - 2*a*b*c*d^5
 + a^2*d^6)/(b*d^7)) - 105*(b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 -
 a^3*d^6)/(b*d^7))*sqrt(b*x + a) - 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4))*b/abs(b)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(1/2), x)
```

3.1493 $\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=148

$$\frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3}-\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\frac{5(bc-ad)^3\tanh^{-1}}{8\sqrt{b}}$$

```
[Out] -5/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(7
/2)/b^(1/2)-5/12*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^2+1/3*(b*x+a)^(5/
2)*(d*x+c)^(1/2)/d+5/8*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^3
```

Rubi [A]

time = 0.05, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^3) - (5*(b*c - a*d)*(a +
 b*x)^(3/2)*Sqrt[c + d*x])/(12*d^2) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d)
 - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*Sqrt[b]*d^(7/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} + \frac{(5(bc-ad)^2)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Mathematica [A]
time = 0.20, size = 125, normalized size = 0.84

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(33a^2d^2+2abd(-20c+13dx)+b^2(15c^2-10cdx+8d^2x^2))}{24d^3} - \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{8\sqrt{b}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x) + b^2*(
15*c^2 - 10*c*d*x + 8*d^2*x^2)))/(24*d^3) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*Sqrt[b]*d^(7/2))
```

Maple [A]
time = 0.16, size = 173, normalized size = 1.17

method	result
default	$\frac{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}{3d}-\frac{5(-ad+bc)\left(\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d}-\frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}-\frac{(-ad+bc)\sqrt{(bx+a)(dx}}{}\right.}{}\right.}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d-5/6*(-a*d+b*c)/d*(1/2*(b*x+a)^(3/2)*(d*x+
c)^(1/2)/d-3/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d
*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*
d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.24, size = 412, normalized size = 2.78

$$\left[-\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)-4\,(8\,b^3d^3x^2+15\,b^3c^2d-40\,ab^2cd^2+33\,a^2bd^3-2\,(5\,b^3cd^2-13\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,bd^4},\ \frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\,(8\,b^3d^3x^2+15\,b^3c^2d-40\,ab^2cd^2+33\,a^2bd^3-2\,(5\,b^3cd^2-13\,ab^2d^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 1
3*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^4), 1/48*(15*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*
c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 +
33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.75, size = 198, normalized size = 1.34

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\,\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{bd}-\frac{5\,(bcd^3-ad^4)}{bd^5}\right)+\frac{15\,(b^2c^2d^2-2\,abcd^3+a^2d^4)}{bd^5}\right)+\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^3}\right)b}{24\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d^3))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/2), x)
```

3.1494 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=113

$$-\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}$$

```
[Out] 3/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(5/
2)/b^(1/2)+1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d-3/4*(-a*d+b*c)*(b*x+a)^(1/2)*(
d*x+c)^(1/2)/d^2
```

Rubi [A]

time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 223, 212}

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^2) + ((a + b*x)^(3/2)*Sqr
t[c + d*x])/(2*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[
b]*Sqrt[c + d*x])])/(4*Sqrt[b]*d^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{(3(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c-}}}{8d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{(3(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{c-}}\right.}{4b}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{(3(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-}}{\sqrt{b}\,\sqrt{c-}}\right.}{4\sqrt{b}\,d^{5/2}}$$

Mathematica [A]
time = 0.18, size = 94, normalized size = 0.83

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(-3bc+5ad+2bdx)}{4d^2} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4\sqrt{b}\,d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*(-3*b*c + 5*a*d + 2*b*d*x))/(4*d^2) + (3*(b*c
- a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*Sqrt[
b]*d^(5/2))
```

Maple [A]
time = 0.15, size = 140, normalized size = 1.24

method	result
default	$\frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{2d}-\frac{3(-ad+bc)\left(\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}-\frac{(-ad+bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\ldots}\right.}{2d\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{b\ldots}}\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d-3/4*(-a*d+b*c)/d*((b*x+a)^(1/2)*(d*x+c)^(
1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)
*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b
*d)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.82, size = 306, normalized size = 2.71

$$\left[\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x-3\,b^2cd+5\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,bd^3},-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)-2\,(2\,b^2d^2x-3\,b^2cd+5\,abd^2)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x - 3*b^2*c*d + 5*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/sqrt(c + d*x), x)
```

Giac [A]
time = 1.33, size = 139, normalized size = 1.23

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)}{bd}-\frac{3\,(bcd-ad^2)}{bd^3}\right)-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2}\right)b}{4\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b*d) -
 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^
2))*b/abs(b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/2), x)
```

3.1495 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

```
[Out] -(-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(3/2)/b^
(1/2)+(b*x+a)^(1/2)*(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 223, 212}

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx &= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{bd}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{bd}\\
&= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 74, normalized size = 1.01

$$\frac{d\sqrt{a+bx}\,\sqrt{c+dx}+\frac{(bc-ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{\sqrt{\frac{b}{d}}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (d*Sqrt[a + b*x]*Sqrt[c + d*x] + ((b*c - a*d)*Log[Sqrt[a + b*x] - Sqrt[b/d]
*Sqrt[c + d*x]])/Sqrt[b/d])/d^2
```

Maple [A]
time = 0.16, size = 107, normalized size = 1.47

method	result
default	$\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}-\frac{(-ad+bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^{2}+(ad+bc)\,x+ac}\right)}{2d\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x
+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*
d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.01, size = 235, normalized size = 3.22

$$\left[\frac{4\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^{2}d^{2}x^{2}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^{2}cd+abd^{2})x\right)}{4\,bd^{2}},\ \frac{2\sqrt{bx+a}\,\sqrt{dx+c}\,bd+(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^{2}d^{2}x^{2}+abcd+(b^{2}cd+abd^{2})x)}\right)}{2\,bd^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b*d^2), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/sqrt(c + d*x), x)
```

Giac [A]
time = 1.32, size = 97, normalized size = 1.33

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}}{bd}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*((b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
 a)/(b*d))/abs(b)
```

Mupad [B]
time = 3.80, size = 261, normalized size = 3.58

$$\frac{\frac{(2ad+2bc)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^3}+\frac{(2cb^2+2adb)\left(\sqrt{a+bx}-\sqrt{a}\right)}{d^3\left(\sqrt{c+dx}-\sqrt{c}\right)}-\frac{8\sqrt{a}\,b\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2b\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{d\left(\sqrt{c+dx}-\sqrt{c}\right)^2}}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{c+dx}-\sqrt{c}\right)}\right)(ad-bc)}{\sqrt{b}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(d^2*((c + d*x)^(1/2) - c^
(1/2))^3) + ((2*b^2*c + 2*a*b*d)*((a + b*x)^(1/2) - a^(1/2)))/(d^3*((c + d*
x)^(1/2) - c^(1/2))) - (8*a^(1/2)*b*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/
(d^2*((c + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d
*x)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*(
(c + d*x)^(1/2) - c^(1/2))^2)) + (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/
2)))/(b^(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(1/2)*d^(3/2))
```

3.1496 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {65, 223, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [**A**]
time = 0.06, size = 42, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[
d])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.
time = 0.16, size = 76, normalized size = 1.81

method	result	size
default	$\frac{\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{bd\,x^2+(ad+bc)\,x+ac}\right)}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d
*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(30) = 60$.
time = 0.76, size = 178, normalized size = 4.24

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)}{2\,bd}, -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d
*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^
2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b
*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x))/(b*d
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)), x)
```

Giac [A]
time = 1.37, size = 50, normalized size = 1.19

$$-\frac{2\,b\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*abs(b))
```

Mupad [B]
time = 0.29, size = 45, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] -(4*atan((b*((c + d*x)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x)^(1/2) - a
^(1/2)))))/(-b*d)^(1/2)
```

3.1497 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])

Maple [A]
time = 0.16, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{2\sqrt{dx+c}}{\sqrt{bx+a}\,(ad-bc)}$	27
default	$-\frac{2\sqrt{dx+c}}{(-ad+bc)\sqrt{bx+a}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.77, size = 42, normalized size = 1.40

$$-\frac{2\sqrt{bx+a}\,\sqrt{dx+c}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)*sqrt(d*x + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2),x)

[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.
time = 1.28, size = 66, normalized size = 2.20

$$-\frac{4\sqrt{bd}\,b}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")

[Out] -4*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*abs(b))

Mupad [B]
time = 0.73, size = 26, normalized size = 0.87

$$\frac{2\sqrt{c+d\,x}}{(a\,d-b\,c)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/2)),x)

[Out] (2*(c + d*x)^(1/2))/((a*d - b*c)*(a + b*x)^(1/2))

3.1498 $\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^
2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (4*d*Sqrt[c + d*x])/(3
*(b*c - a*d)^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{3(bc-ad)}$$
$$= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{2\sqrt{c+dx}\,(-bc+3ad+2bdx)}{3(bc-ad)^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(a + b*x)^(3/
2))
```

Maple [A]
time = 0.16, size = 55, normalized size = 0.83

method	result	size
gosper	$\frac{2\sqrt{dx+c}\,(2bdx+3ad-bc)}{3(bx+a)^{\frac{3}{2}}(a^2d^2-2abcd+b^2c^2)}$	54
default	$-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^
2/(b*x+a)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(54) = 108.
time = 1.11, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx-bc+3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x - b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^2*c^2 - 2*a^
3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^
2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(54) = 108.
time = 1.26, size = 121, normalized size = 1.83

$$\frac{8\left(b^2c-abd-3\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)\sqrt{bd}\,b^2d}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(b^2*c - a*b*d - 3*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*abs(b))
```

Mupad [B]
time = 0.89, size = 71, normalized size = 1.08

$$\frac{\left(\frac{4\,d\,x}{3\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d-2\,b\,c}{3\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{a\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (((4*d*x)/(3*(a*d - b*c)^2) + (6*a*d - 2*b*c)/(3*b*(a*d - b*c)^2))*(c + d*x
)^(1/2))/(x*(a + b*x)^(1/2) + (a*(a + b*x)^(1/2))/b)
```

3.1499 $\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}+\frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}}-\frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}$$

```
[Out] -2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)+8/15*d*(d*x+c)^(1/2)/(-a*d+b*c)
^2/(b*x+a)^(3/2)-16/15*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}\,(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (8*d*Sqrt[c + d*x])/(1
5*(b*c - a*d)^2*(a + b*x)^(3/2)) - (16*d^2*Sqrt[c + d*x])/(15*(b*c - a*d)^3
*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} + \frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}} + \frac{(8d^2)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}}{15(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}} + \frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}} - \frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.74

$$-\frac{2\sqrt{c+dx}\,(15a^2d^2-10abd(c-2dx)+b^2(3c^2-4cdx+8d^2x^2))}{15(bc-ad)^3(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(3*c^2 - 4*c*d*x
 + 8*d^2*x^2)))/(15*(b*c - a*d)^3*(a + b*x)^(5/2))
```

Maple [A]
time = 0.16, size = 95, normalized size = 0.94

method	result	size
default	$-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}\right)}{5(-ad+bc)}$	95
gosper	$\frac{2\sqrt{dx+c}\,(8b^2x^2d^2+20ab\,d^2x-4b^2cdx+15a^2d^2-10abcd+3b^2c^2)}{15(bx+a)^{\frac{5}{2}}(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^
(1/2)/(-a*d+b*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/
2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(83) = 166$.
time = 0.86, size = 251, normalized size = 2.49

$$-\frac{2\,{\left(8\,b^{2} d^{2} x^{2} + 3\,b^{2} c^{2} - 10\,a b c d + 15\,a^{2} d^{2} - 4\,{\left(b^{2} c d - 5\,a b d^{2}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{15\,{\left(a^{3} b^{3} c^{3} - 3\,a^{4} b^{2} c^{2} d + 3\,a^{5} b c d^{2} - a^{6} d^{3} + {\left(b^{6} c^{3} - 3\,a b^{5} c^{2} d + 3\,a^{2} b^{4} c d^{2} - a^{3} b^{3} d^{3}\right)} x^{3} + 3\,{\left(a b^{5} c^{3} - 3\,a^{2} b^{4} c^{2} d + 3\,a^{3} b^{3} c d^{2} - a^{4} b^{2} d^{3}\right)} x^{2} + 3\,{\left(a^{2} b^{4} c^{3} - 3\,a^{3} b^{3} c^{2} d + 3\,a^{4} b^{2} c d^{2} - a^{5} b d^{3}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*d^2*x^2 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3
*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^
3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b x\right)^{\frac{7}{2}} \sqrt{c + d x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(83) = 166$.
time = 1.05, size = 227, normalized size = 2.25

$$-\frac{32\,{\left(b^{4} c^{2} - 2\,a b^{3} c d + a^{2} b^{2} d^{2} - 5\,{\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2} b^{2} c + 5\,{\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2} a b d + 10\,{\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{4}\right)} \sqrt{b d} b^{3} d^{2}}{15\,{\left(b^{2} c - a b d - {\left(\sqrt{b d} \sqrt{b x + a} - \sqrt{b^{2} c + {\left(b x + a\right)} b d - a b d}\right)}^{2}\right)}^{5} {\left| b \right|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -32/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*d)*sqrt(b*x + a) -
```

```
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((b^2*c - a*b*
d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*ab
s(b))
```

Mupad [B]
time = 1.01, size = 133, normalized size = 1.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d^2\,x^2}{15\,{\left(a\,d-b\,c\right)}^3}+\frac{30\,a^2\,d^2-20\,a\,b\,c\,d+6\,b^2\,c^2}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d\,x\,\left(5\,a\,d-b\,c\right)}{15\,b\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
[Out] ((c + d*x)^(1/2)*((16*d^2*x^2)/(15*(a*d - b*c)^3) + (30*a^2*d^2 + 6*b^2*c^2
 - 20*a*b*c*d)/(15*b^2*(a*d - b*c)^3) + (8*d*x*(5*a*d - b*c))/(15*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*
x)^(1/2))/b)
```

3.1500 $\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}}+\frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}+\frac{32d^3\sqrt{c+dx}}{35(bc-ad)^4\sqrt{a+bx}}$$

```
[Out] -2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)+12/35*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)^(5/2)-16/35*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(3/2)+32/35*
d^3*(d*x+c)^(1/2)/(-a*d+b*c)^4/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}\,(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (12*d*Sqrt[c + d*x])/(
35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*Sqrt[c + d*x])/(35*(b*c - a*d)^
3*(a + b*x)^(3/2)) + (32*d^3*Sqrt[c + d*x])/(35*(b*c - a*d)^4*Sqrt[a + b*x]
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx}{7(bc-ad)}$$
$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} + \frac{(24d^2)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+d}}}{35(bc-ad)^2}$$
$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}$$
$$= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}} + \frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}} - \frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.68

$$-\frac{2\sqrt{c+dx}\,(-35d^3(a+bx)^3+35bd^2(a+bx)^2(c+dx)-21b^2d(a+bx)(c+dx)^2+5b^3(c+dx)^3)}{35(bc-ad)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(-35*d^3*(a + b*x)^3 + 35*b*d^2*(a + b*x)^2*(c + d*x) - 2
1*b^2*d*(a + b*x)*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35*(b*c - a*d)^4*(a +
b*x)^(7/2))
```

Maple [A]
time = 0.20, size = 135, normalized size = 0.99

method	result	size
default	$-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}}-\frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}}-\frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}}+\frac{4d\sqrt{dx+c}}{3(-ad+bc)^{2}\sqrt{bx+a}}\right)}{5(-ad+bc)}\right)}{7(-ad+bc)}$	135
gosper	$\frac{2\sqrt{dx+c}\,(16b^3x^3d^3+56d^3a\,x^2b^2-8b^3c\,d^2x^2+70a^2b\,d^3x-28a\,b^2c\,d^2x+6b^3c^2dx+35a^3d^3-35a^2bc\,d^2+21a\,b^2c^2d-5b^3c^3)}{35(bx+a)^{\frac{7}{2}}(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^
(1/2)/(-a*d+b*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b
*c)/(b*x+a)^(3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(112) = 224.
time = 1.56, size = 419, normalized size = 3.08

$$\frac{2\,{\left(16\,b^{3}d^{3}x^{3}-5\,b^{3}c^{3}+21\,ab^{2}c^{2}d-35\,a^{2}bcd^{2}+35\,a^{3}d^{3}-8\,{\left(b^{3}cd^{2}-7\,ab^{2}d^{3}\right)}x^{2}+2\,{\left(3\,b^{3}c^{2}d-14\,ab^{2}cd^{2}+35\,a^{2}bd^{3}\right)}x\right)}\sqrt{bx+a}\,\sqrt{dx+c}}{35\,{\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+{\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)}x^{4}+4\,{\left(ab^{7}c^{4}-4\,a^{2}b^{6}c^{3}d+6\,a^{3}b^{5}c^{2}d^{2}-4\,a^{4}b^{4}cd^{3}+a^{5}b^{3}d^{4}\right)}x^{3}+6\,{\left(a^{2}b^{6}c^{4}-4\,a^{3}b^{5}c^{3}d+6\,a^{4}b^{4}c^{2}d^{2}-4\,a^{5}b^{3}cd^{3}+a^{6}b^{2}d^{4}\right)}x^{2}+4\,{\left(a^{3}b^{5}c^{4}-4\,a^{4}b^{4}c^{3}d+6\,a^{5}b^{3}c^{2}d^{2}-4\,a^{6}b^{2}cd^{3}+a^{7}bd^{4}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*d^3*x^3 - 5*b^3*c^3 + 21*a*b^2*c^2*d - 35*a^2*b*c*d^2 + 35*a^3
*d^3 - 8*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 2*(3*b^3*c^2*d - 14*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d
 + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d +
 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*
a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*
(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*
b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6
*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(112) = 224.
time = 1.22, size = 386, normalized size = 2.84

$$\frac{64\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}-7\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{4}c^{2}+14\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}ab^{3}cd-7\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}a^{2}b^{2}d^{2}+21\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{2}c-21\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}abd-35\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{6}\right)\sqrt{bd}\,b^{4}d^{3}}{35\left(b^{2}c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{7}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3 - 7*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2 + 14*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^3*c*d - 7
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*
d^2 + 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
b^2*c - 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
4*a*b*d - 35*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6)*sqrt(b*d)*b^4*d^3/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2)^7*abs(b))
```

Mupad [B]
time = 1.19, size = 209, normalized size = 1.54

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^3\,x^3}{35\,{\left(a\,d-b\,c\right)}^4}+\frac{70\,a^3\,d^3-70\,a^2\,b\,c\,d^2+42\,a\,b^2\,c^2\,d-10\,b^3\,c^3}{35\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d\,x\,\left(35\,a^2\,d^2-14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{35\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{35\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^3*x^3)/(35*(a*d - b*c)^4) + (70*a^3*d^3 - 10*b^3*c^
3 + 42*a*b^2*c^2*d - 70*a^2*b*c*d^2)/(35*b^3*(a*d - b*c)^4) + (4*d*x*(35*a^
2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(35*b^2*(a*d - b*c)^4) + (16*d^2*x^2*(7*a*
d - b*c))/(35*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2
))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

3.1501 $\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=171

$$-\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}+\frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{128d^3\sqrt{c+dx}}{315(bc-ad)^4(a+bx)^{3/2}}-\frac{}{315}$$

```
[Out] -2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)+16/63*d*(d*x+c)^(1/2)/(-a*d+b*c
)^2/(b*x+a)^(7/2)-32/105*d^2*(d*x+c)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(5/2)+128/3
15*d^3*(d*x+c)^(1/2)/(-a*d+b*c)^4/(b*x+a)^(3/2)-256/315*d^4*(d*x+c)^(1/2)/(
-a*d+b*c)^5/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}\,(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (16*d*Sqrt[c + d*x])/(
63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (32*d^2*Sqrt[c + d*x])/(105*(b*c - a*d)
^3*(a + b*x)^(5/2)) + (128*d^3*Sqrt[c + d*x])/(315*(b*c - a*d)^4*(a + b*x)^
(3/2)) - (256*d^4*Sqrt[c + d*x])/(315*(b*c - a*d)^5*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} + \frac{(16d^2)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+\cdots}}}{21(bc-ad)^2}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5\cdots}}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5\cdots}}$$

$$= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}} + \frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}} - \frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5\cdots}}$$

Mathematica [A]

time = 0.14, size = 115, normalized size = 0.67

$$-\frac{2\sqrt{c+dx}\,(315d^4(a+bx)^4 - 420bd^3(a+bx)^3(c+dx) + 378b^2d^2(a+bx)^2(c+dx)^2 - 180b^3d(a+bx)(c+dx)^3 + 35b^4(c+dx)^4)}{315(bc-ad)^5(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(315*d^4*(a + b*x)^4 - 420*b*d^3*(a + b*x)^3*(c + d*x) +
378*b^2*d^2*(a + b*x)^2*(c + d*x)^2 - 180*b^3*d*(a + b*x)*(c + d*x)^3 + 35*
b^4*(c + d*x)^4))/(315*(b*c - a*d)^5*(a + b*x)^(9/2))
```

Maple [A]

time = 0.16, size = 175, normalized size = 1.02

method	result
default	$-\frac{2\sqrt{dx+c}}{9(-ad+bc)(bx+a)^{\frac{9}{2}}} - \frac{8d\left(-\frac{2\sqrt{dx+c}}{7(-ad+bc)(bx+a)^{\frac{7}{2}}} - \frac{6d\left(-\frac{2\sqrt{dx+c}}{5(-ad+bc)(bx+a)^{\frac{5}{2}}} - \frac{4d\left(-\frac{2\sqrt{dx+c}}{3(-ad+bc)(bx+a)^{\frac{3}{2}}} + \frac{4d\sqrt{dx+c}}{3(-ad+bc)^2\sqrt{bx+a}}\right)}{5(-ad+bc)}\cdots\right)}{7(-ad+bc)}\cdots\right)}{9(-ad+bc)}\cdots$
gosper	$\frac{2\sqrt{dx+c}\,(128d^4x^4b^4+576a\,b^3d^4x^3-64b^4c\,d^3x^3+1008a^2b^2d^4x^2-288a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+840a^3b\,d^4x-504a^2b^2c\,d^3x+216a\cdots}{315(bx+a)^{\frac{9}{2}}(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(9/2)-8/9*d/(-a*d+b*c)*(-2/7*(d*x+c)^
(1/2)/(-a*d+b*c)/(b*x+a)^(7/2)-6/7*d/(-a*d+b*c)*(-2/5*(d*x+c)^(1/2)/(-a*d+b
*c)/(b*x+a)^(5/2)-4/5*d/(-a*d+b*c)*(-2/3*(d*x+c)^(1/2)/(-a*d+b*c)/(b*x+a)^(
3/2)+4/3*d*(d*x+c)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(141) = 282.
time = 3.68, size = 638, normalized size = 3.73

$$-\frac{2\,{\left(128\,b^{4} d^{4} x^{4} + 35\,b^{4} c^{4} - 180\,a b^{3} c^{3} d + 378\,a^{2} b^{2} c^{2} d^{2} - 420\,a^{3} b c d^{3} + 315\,a^{4} d^{4} - 64\,{\left(b^{4} c d^{3} - 9\,a b^{3} d^{4}\right)} x^{3} + 48\,{\left(b^{4} c^{2} d^{2} - 6\,a b^{3} c d^{3} + 21\,a^{2} b^{2} d^{4}\right)} x^{2} - 8\,{\left(5\,b^{4} c^{3} d - 27\,a b^{3} c^{2} d^{2} + 63\,a^{2} b^{2} c d^{3} - 105\,a^{3} b d^{4}\right)} x\right)} \sqrt{b x + a} \sqrt{d x + c}}{315\,{\left(a^{5} b^{5} c^{5} - 5\,a^{6} b^{4} c^{4} d + 10\,a^{7} b^{3} c^{3} d^{2} - 10\,a^{8} b^{2} c^{2} d^{3} + 5\,a^{9} b c d^{4} - a^{10} d^{5} + {\left(b^{10} c^{5} - 5\,a b^{9} c^{4} d + 10\,a^{2} b^{8} c^{3} d^{2} - 10\,a^{3} b^{7} c^{2} d^{3} + 5\,a^{4} b^{6} c d^{4} - a^{5} b^{5} d^{5}\right)} x^{5} + 5\,{\left(a b^{9} c^{5} - 5\,a^{2} b^{8} c^{4} d + 10\,a^{3} b^{7} c^{3} d^{2} - 10\,a^{4} b^{6} c^{2} d^{3} + 5\,a^{5} b^{5} c d^{4} - a^{6} b^{4} d^{5}\right)} x^{4} + 10\,{\left(a^{2} b^{8} c^{5} - 5\,a^{3} b^{7} c^{4} d + 10\,a^{4} b^{6} c^{3} d^{2} - 10\,a^{5} b^{5} c^{2} d^{3} + 5\,a^{6} b^{4} c d^{4} - a^{7} b^{3} d^{5}\right)} x^{3} + 10\,{\left(a^{3} b^{7} c^{5} - 5\,a^{4} b^{6} c^{4} d + 10\,a^{5} b^{5} c^{3} d^{2} - 10\,a^{6} b^{4} c^{2} d^{3} + 5\,a^{7} b^{3} c d^{4} - a^{8} b^{2} d^{5}\right)} x^{2} + 5\,{\left(a^{4} b^{6} c^{5} - 5\,a^{5} b^{5} c^{4} d + 10\,a^{6} b^{4} c^{3} d^{2} - 10\,a^{7} b^{3} c^{2} d^{3} + 5\,a^{8} b^{2} c d^{4} - a^{9} b d^{5}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*d^4*x^4 + 35*b^4*c^4 - 180*a*b^3*c^3*d + 378*a^2*b^2*c^2*d^
2 - 420*a^3*b*c*d^3 + 315*a^4*d^4 - 64*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 48*(
b^4*c^2*d^2 - 6*a*b^3*c*d^3 + 21*a^2*b^2*d^4)*x^2 - 8*(5*b^4*c^3*d - 27*a*b
^3*c^2*d^2 + 63*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x +
c)/(a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3
 + 5*a^9*b*c*d^4 - a^10*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^
2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^5 + 5*(a*b^9*c^5
- 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d
^4 - a^6*b^4*d^5)*x^4 + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*
d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^3 + 10*(a^3*b^7
*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^
3*c*d^4 - a^8*b^2*d^5)*x^2 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*
c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(141) = 282.
time = 2.20, size = 596, normalized size = 3.49

$$-\frac{512\left(b^8c^4-4ab^7c^3d+6a^2b^6c^2d^2-4a^3b^5cd^3+a^4b^4d^4-9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^6c^3+27\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^5c^2d-27\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^4cd^2+9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3b^3d^3+36\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^4c^2-72\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^3cd+36\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^2d^2-84\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6b^2c+84\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6abd+126\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8\right)\sqrt{bd}\,b^5d^4}{315\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^9|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -512/315*(b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a
^4*b^4*d^4 - 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^6*c^3 + 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^5*c^2*d - 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^3*d^3 + 36*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^4*c^2 - 72*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d + 36*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 84*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^2*c + 84*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b*d + 126*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8)*sqrt(b*d)*b^5*d^
4/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2)^9*abs(b))
```

Mupad [**B**]
time = 1.37, size = 303, normalized size = 1.77

$$\frac{\sqrt{c+d\,x}\,\left(\frac{256\,d^4\,x^4}{315\,{\left(a\,d-b\,c\right)}^5}+\frac{630\,a^4\,d^4-840\,a^3\,b\,c\,d^3+756\,a^2\,b^2\,c^2\,d^2-360\,a\,b^3\,c^3\,d+70\,b^4\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(1680\,a^3\,b\,d^4-1008\,a^2\,b^2\,c\,d^3+432\,a\,b^3\,c^2\,d^2-80\,b^4\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((256*d^4*x^4)/(315*(a*d - b*c)^5) + (630*a^4*d^4 + 70*b^4
*c^4 + 756*a^2*b^2*c^2*d^2 - 360*a*b^3*c^3*d - 840*a^3*b*c*d^3)/(315*b^4*(a
*d - b*c)^5) + (x*(1680*a^3*b*d^4 - 80*b^4*c^3*d + 432*a*b^3*c^2*d^2 - 1008
```

```
*a^2*b^2*c*d^3))/(315*b^4*(a*d - b*c)^5) + (128*d^3*x^3*(9*a*d - b*c))/(315
*b*(a*d - b*c)^5) + (32*d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^
2*(a*d - b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^
2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*
x)^(1/2))/b^3)
```

3.1502 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=174

$$-\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}}+\frac{35b(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8d^4}-\frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}+\frac{7b(a+bx)^{5/2}\sqrt{c+d}}{3d^2}$$

```
[Out] -35/8*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(
1/2)/d^(9/2)-2*(b*x+a)^(7/2)/d/(d*x+c)^(1/2)-35/12*b*(-a*d+b*c)*(b*x+a)^(3/
2)*(d*x+c)^(1/2)/d^3+7/3*b*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d^2+35/8*b*(-a*d+b*c
)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{35\sqrt{b}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8d^{9/2}}+\frac{35b\sqrt{a+bx}\sqrt{c+dx}(bc-ad)^2}{8d^4}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{12d^3}+\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}-\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(d*Sqrt[c + d*x]) + (35*b*(b*c - a*d)^2*Sqrt[a + b*x]*
Sqrt[c + d*x])/(8*d^4) - (35*b*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(
12*d^3) + (7*b*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d^2) - (35*Sqrt[b]*(b*c -
a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2)
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} - \frac{(35b(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} + \frac{(35b(bc}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3}$$

Mathematica [A]
time = 0.31, size = 166, normalized size = 0.95

$$\frac{\sqrt{a+bx}\,(-48a^3d^3+3a^2bd^2(77c+29dx)+2ab^2d(-140c^2-49cdx+19d^2x^2)+b^3(105c^3+35c^2dx-14cd^2x^2+8d^3x^3))}{24d^4\sqrt{c+dx}} - \frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-48*a^3*d^3 + 3*a^2*b*d^2*(77*c + 29*d*x) + 2*a*b^2*d*(-140
*c^2 - 49*c*d*x + 19*d^2*x^2) + b^3*(105*c^3 + 35*c^2*d*x - 14*c*d^2*x^2 +
8*d^3*x^3)))/(24*d^4*Sqrt[c + d*x]) - (35*Sqrt[b]*(b*c - a*d)^3*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(138) = 276$.
time = 1.07, size = 603, normalized size = 3.47

$$\left[-\frac{105\,(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3 + (b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bd^2x + bcd + ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}} + 8\,(b^2cd + abd^2)x\right) - 4\,(8\,b^3d^3x^3 + 105\,b^3c^3 - 280\,ab^2c^2d + 231\,a^2bcd^2 - 48\,a^3d^3 - 2\,(7\,b^3cd^2 - 19\,ab^2d^3)x^2 + (35\,b^3c^2d - 98\,ab^2cd^2 + 87\,a^2bd^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{96\,(d^5x + cd^4)},\ \frac{105\,(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3 + (b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx + bc + ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2 + abc + (b^2c + abd)x)}\right) + 2\,(8\,b^3d^3x^3 + 105\,b^3c^3 - 280\,ab^2c^2d + 231\,a^2bcd^2 - 48\,a^3d^3 - 2\,(7\,b^3cd^2 - 19\,ab^2d^3)x^2 + (35\,b^3c^2d - 98\,ab^2cd^2 + 87\,a^2bd^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{48\,(d^5x + cd^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(105*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(
b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d^3*
x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48*a^3*d^3 - 2*(7*b
^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*d^2 + 87*a^2*b*d^
3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4), 1/48*(105*(b^3*c^4 - 3*
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqr
t(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)
) + 2*(8*b^3*d^3*x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48
*a^3*d^3 - 2*(7*b^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*
d^2 + 87*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^{\frac{7}{2}}}{(c + dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(138) = 276.
time = 1.26, size = 279, normalized size = 1.60

$$\frac{\left((bx+a)\left(2\,(bx+a)\left(\frac{4\,(bx+a)b^2}{d|b|}-\frac{7\,(b^3cd^5-ab^2d^6)}{d^7|b|}\right)+\frac{35\,(b^4c^2d^4-2\,ab^3cd^5+a^2b^2d^6)}{d^7|b|}\right)+\frac{105\,(b^5c^3d^3-3\,ab^4c^2d^4+3\,a^2b^3cd^5-a^3b^2d^6)}{d^7|b|}\right)\sqrt{bx+a}}{24\,\sqrt{b^2c+(bx+a)bd-abd}}+\frac{35\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{8\,\sqrt{bd}\,d^4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((b*x + a)*(2*(b*x + a)*(4*(b*x + a)*b^2/(d*abs(b)) - 7*(b^3*c*d^5 - a
*b^2*d^6)/(d^7*abs(b))) + 35*(b^4*c^2*d^4 - 2*a*b^3*c*d^5 + a^2*b^2*d^6)/(d
^7*abs(b))) + 105*(b^5*c^3*d^3 - 3*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^
2*d^6)/(d^7*abs(b)))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 35
/8*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(-sqrt(
b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4*a
bs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(3/2), x)
```

3.1503 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=138

$$-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}-\frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}+\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}$$

```
[Out] 15/4*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(1
/2)/d^(7/2)-2*(b*x+a)^(5/2)/d/(d*x+c)^(1/2)+5/2*b*(b*x+a)^(3/2)*(d*x+c)^(1/
2)/d^2-15/4*b*(-a*d+b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{15b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(d*Sqrt[c + d*x]) - (15*b*(b*c - a*d)*Sqrt[a + b*x]*Sq
rt[c + d*x])/(4*d^3) + (5*b*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d^2) + (15*Sq
rt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(4*d^(7/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}+\frac{(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{(15b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}-\frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}+\frac{(15b(bc\cdots}{\cdots}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}-\frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}+\frac{(15(bc-\cdots}{\cdots}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}-\frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}+\frac{(15(bc-\cdots}{\cdots}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}-\frac{15b(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}+\frac{15\sqrt{b}\,(b\cdots}{\cdots}$$

Mathematica [A]
time = 0.25, size = 124, normalized size = 0.90

$$\frac{\sqrt{a+bx}\left(-8a^2d^2+abd(25c+9dx)+b^2(-15c^2-5cdx+2d^2x^2)\right)}{4d^3\sqrt{c+dx}}+\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-8*a^2*d^2 + a*b*d*(25*c + 9*d*x) + b^2*(-15*c^2 - 5*c*d*x
+ 2*d^2*x^2)))/(4*d^3*Sqrt[c + d*x]) + (15*Sqrt[b]*(b*c - a*d)^2*ArcTanh[(S
qrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(7/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.67, size = 441, normalized size = 3.20

$$\left[\frac{15\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)+4\,(2\,b^2d^2x^2-15\,b^2c^2+25\,abcd-8\,a^2d^2-(5\,b^2cd-9\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,(d^4x+cd^3)},\ -\frac{15\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)-2\,(2\,b^2d^2x^2-15\,b^2c^2+25\,abcd-8\,a^2d^2-(5\,b^2cd-9\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,(d^4x+cd^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2
- (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3),
-1/8*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(2*b^2*d^2*x
^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2 - (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(3/2), x)
```

Giac [A]
time = 1.64, size = 201, normalized size = 1.46

$$\frac{\sqrt{bx+a}\left((bx+a)\left(\frac{2\,(bx+a)b^2}{d|b|}-\frac{5\,(b^3cd^3-ab^2d^4)}{d^5|b|}\right)-\frac{15\,(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4)}{d^5|b|}\right)}{4\,\sqrt{b^2c+(bx+a)bd-abd}}-\frac{15\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{4\,\sqrt{bd}\,d^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*((b*x + a)*(2*(b*x + a)*b^2/(d*abs(b)) - 5*(b^3*c*d^3 - a
*b^2*d^4)/(d^5*abs(b))) - 15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(d
^5*abs(b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) - 15/4*(b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(3/2), x)
```

3.1504 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}$$

```
[Out] -3*(-a*d+b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(1/2)/
d^(5/2)-2*(b*x+a)^(3/2)/d/(d*x+c)^(1/2)+3*b*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^2
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x]) + (3*b*Sqrt[a + b*x]*Sqrt[c + d*x])/
d^2 - (3*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[
c + d*x])])/d^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{(3b)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d^2}$$

$$= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x\right.}{d^2}$$

$$= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}$$

$$= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}$$

Mathematica [A]

time = 0.39, size = 87, normalized size = 0.89

$$\frac{\frac{\sqrt{a+bx}\,(3bc-2ad+bdx)}{\sqrt{c+dx}}+3\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] ((Sqrt[a + b*x]*(3*b*c - 2*a*d + b*d*x))/Sqrt[c + d*x] + 3*Sqrt[b/d]*(b*c -
 a*d)*Log[Sqrt[a + b*x] - Sqrt[b/d]*Sqrt[c + d*x]])/d^2
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.31, size = 311, normalized size = 3.17

$$\left[-\frac{3\,(bc^2-acd+(bcd-ad^2)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\,(d^3x+cd^2)}, \frac{3\,(bc^2-acd+(bcd-ad^2)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(d^3x+cd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x + 3*b*c - 2*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2), 1/2*(3*(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a
)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(b*
d*x + 3*b*c - 2*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/2), x)
```

Giac [A]
time = 1.02, size = 137, normalized size = 1.40

$$\frac{\sqrt{bx+a}\left(\frac{(bx+a)b^2}{d|b|}+\frac{3\,(b^3cd-ab^2d^2)}{d^3|b|}\right)}{\sqrt{b^2c+(bx+a)bd-abd}}+\frac{3\,(b^3c-ab^2d)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x + a)*((b*x + a)*b^2/(d*abs(b)) + 3*(b^3*c*d - a*b^2*d^2)/(d^3*abs(
b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 3*(b^3*c - a*b^2*d)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2
*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(3/2), x)
```

3.1505 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}$$

```
[Out] 2*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))*b^(1/2)/d^(3/2)-2*(b
*x+a)^(1/2)/d/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 223, 212}

$$\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{b\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d}\\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d}\\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 66, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(50) = 100.
time = 0.94, size = 241, normalized size = 3.65

$$\left[\frac{(dx+c)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(d^2x+cd)}, -\frac{(dx+c)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,\sqrt{bx+a}\,\sqrt{dx+c}}{d^2x+cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((d*x + c)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(d^2*x + c*d), -((d
*x + c)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(d^2*x + c*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [A]
time = 1.04, size = 96, normalized size = 1.45

$$-\frac{2\,b^2\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}\,+\sqrt{b^2c+(bx+a)bd-abd}\,\right|\right)}{\sqrt{bd}\,d|b|}-\frac{2\,\sqrt{bx+a}\,b^2}{\sqrt{b^2c+(bx+a)bd-abd}\,d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d*abs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*d*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(3/2), x)
```

3.1506 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

```
[Out] 2*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Maple [A]
time = 0.16, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{2\sqrt{bx+a}}{\sqrt{dx+c}\,(ad-bc)}$	27
default	$-\frac{2\sqrt{bx+a}}{\sqrt{dx+c}\,(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 1.26, size = 42, normalized size = 1.40

$$\frac{2\sqrt{bx+a}\,\sqrt{dx+c}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(d*x + c)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/2)), x)
```

Giac [A]
time = 1.32, size = 47, normalized size = 1.57

$$\frac{2\sqrt{bx+a}\,b^2}{\sqrt{b^2c+(bx+a)bd-abd}\,(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(b*c*abs(b) - a*d*
abs(b)))
```

Mupad [B]
time = 0.74, size = 26, normalized size = 0.87

$$-\frac{2\sqrt{a+b\,x}}{(a\,d-b\,c)\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(2*(a + b*x)^(1/2))/((a*d - b*c)*(c + d*x)^(1/2))
```

3.1507 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}-\frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}$$

```
[Out] -2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)-4*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/(d
*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x]) - (4*d*Sqrt[a + b*x])/((b*c -
a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{bc-ad}$$
$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}} - \frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}$$

Mathematica [A]
time = 0.09, size = 42, normalized size = 0.68

$$-\frac{2(ad+b(c+2dx))}{(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(a*d + b*(c + 2*d*x)))/((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])
```

Maple [A]
time = 0.16, size = 65, normalized size = 1.05

method	result	size
gosper	$-\frac{2(2bdx+ad+bc)}{\sqrt{bx+a}\,\sqrt{dx+c}\,(a^2d^2-2abcd+b^2c^2)}$	52
default	$-\frac{2}{(-ad+bc)\sqrt{bx+a}\,\sqrt{dx+c}}+\frac{4d\sqrt{bx+a}}{(-ad+bc)\sqrt{dx+c}\,(ad-bc)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x
+c)^(1/2)/(a*d-b*c)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(54) = 108.
time = 1.19, size = 125, normalized size = 2.02

$$-\frac{2\,(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(54) = 108.
time = 1.85, size = 142, normalized size = 2.29

$$-\frac{2\,\sqrt{bx+a}\,b^2d}{(b^2c^2|b|-2\,abcd|b|+a^2d^2|b|)\sqrt{b^2c+(bx+a)bd-abd}}-\frac{4\,\sqrt{bd}\,b^2}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b)
)*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - 4*sqrt(b*d)*b^2/((b^2*c - a*b*d -
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*(b*c*abs
(b) - a*d*abs(b)))
```

Mupad [B]
time = 0.86, size = 71, normalized size = 1.15

$$-\frac{\left(\frac{4\,b\,x}{{\left(a\,d-b\,c\right)}^2}+\frac{2\,a\,d+2\,b\,c}{d\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{c\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(((4*b*x)/(a*d - b*c)^2 + (2*a*d + 2*b*c)/(d*(a*d - b*c)^2))*(c + d*x)^(1/
2))/(x*(a + b*x)^(1/2) + (c*(a + b*x)^(1/2))/d)
```

3.1508 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}+\frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}+\frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/2)+8/3*d/(-a*d+b*c)^2/(b*x+a)^(1/2
)/(d*x+c)^(1/2)+16/3*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (8*d)/(3*(b*c - a*d)^2*S
qrt[a + b*x]*Sqrt[c + d*x]) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx}{3(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} + \frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{(8d^2)\int}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}} + \frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}} + \frac{16d}{3(bc-}$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.70

$$-\frac{2(c+dx)^{3/2}\left(b^2-\frac{3d^2(a+bx)^2}{(c+dx)^2}-\frac{6bd(a+bx)}{c+dx}\right)}{3(bc-ad)^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(b^2 - (3*d^2*(a + b*x)^2)/(c + d*x)^2 - (6*b*d*(a + b*
x))/(c + d*x)))/(3*(b*c - a*d)^3*(a + b*x)^(3/2))
```

Maple [A]
time = 0.16, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{2\left(8b^2x^2d^2+12ab\,d^2x+4b^2cdx+3a^2d^2+6abcd-b^2c^2\right)}{3(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105
default	$-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}-\frac{4d\left(-\frac{2}{(-ad+bc)\sqrt{bx+a}\,\sqrt{dx+c}}+\frac{4d\sqrt{bx+a}}{(-ad+bc)\sqrt{dx+c}\,(ad-bc)}\right)}{3(-ad+bc)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)
/(b*x+a)^(1/2)/(d*x+c)^(1/2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*
d-b*c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(83) = 166$.
time = 0.95, size = 273, normalized size = 2.70

$$\frac{2\,(8\,b^2d^2x^2 - b^2c^2 + 6\,abcd + 3\,a^2d^2 + 4\,(b^2cd + 3\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(a^2b^3c^4 - 3\,a^3b^2c^3d + 3\,a^4bc^2d^2 - a^5cd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^3 + (b^5c^4 - ab^4c^3d - 3\,a^2b^3c^2d^2 + 5\,a^3b^2cd^3 - 2\,a^4bd^4)x^2 + (2\,ab^4c^4 - 5\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 + a^4bcd^3 - a^5d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*d^2*x^2 - b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2 + 4*(b^2*c*d + 3*a*b*d
^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b
*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3
*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 +
 a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(83) = 166$.
time = 1.97, size = 368, normalized size = 3.64

$$\frac{2\,\sqrt{bx+a}\,b^2d^2}{(b^3c^3|b| - 3\,ab^2c^2d|b| + 3\,a^2bcd^2|b| - a^3d^3|b|)\sqrt{b^2c+(bx+a)bd-abd}} + \frac{4\left(5\,\sqrt{bd}\,b^6c^2d - 10\,\sqrt{bd}\,ab^5cd^2 + 5\,\sqrt{bd}\,a^2b^4d^3 - 12\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2b^4cd + 12\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^3d^2 + 3\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^4b^2d\right)}{3\,(b^2c^2|b| - 2\,abcd|b| + a^2d^2|b|)\left(b^2c - abd - \left(\sqrt{bd}\,\sqrt{bx+a} - \sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^2/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c
*d^2*abs(b) - a^3*d^3*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/3*(5
```

```
*sqrt(b*d)*b^6*c^2*d - 10*sqrt(b*d)*a*b^5*c*d^2 + 5*sqrt(b*d)*a^2*b^4*d^3 -
 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^4*c*d + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^2*a*b^3*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^2*d)/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b
) + a^2*d^2*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)^3)
```

Mupad [B]
time = 1.06, size = 141, normalized size = 1.40

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{8\,x\,\left(3\,a\,d+b\,c\right)}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,b\,d\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{6\,a^2\,d^2+12\,a\,b\,c\,d-2\,b^2\,c^2}{3\,b\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a\,c\,\sqrt{a+b\,x}}{b\,d}+\frac{x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((8*x*(3*a*d + b*c))/(3*(a*d - b*c)^3) + (16*b*d*x^2)/(3*
(a*d - b*c)^3) + (6*a^2*d^2 - 2*b^2*c^2 + 12*a*b*c*d)/(3*b*d*(a*d - b*c)^3)
))/(x^2*(a + b*x)^(1/2) + (a*c*(a + b*x)^(1/2))/(b*d) + (x*(a*d + b*c)*(a +
 b*x)^(1/2))/(b*d))
```

3.1509 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}+\frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}-\frac{16d^2}{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}-\frac{32}{5(bc+}$$

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(1/2)+4/5*d/(-a*d+b*c)^2/(b*x+a)^(3/2
)/(d*x+c)^(1/2)-16/5*d^2/(-a*d+b*c)^3/(b*x+a)^(1/2)/(d*x+c)^(1/2)-32/5*d^3*
(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}\,(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*Sqrt[c + d*x]) + (4*d)/(5*(b*c - a*d)^2*(
a + b*x)^(3/2)*Sqrt[c + d*x]) - (16*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqr
t[c + d*x]) - (32*d^3*Sqrt[a + b*x])/(5*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx}{5(bc-ad)}$$
$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} + \frac{(8d^2)}{}$$
$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{}{5(bc-}$$
$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}} + \frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{}{5(bc-}$$

Mathematica [A]
time = 0.12, size = 93, normalized size = 0.68

$$-\frac{2(c+dx)^{5/2}\left(b^3+\frac{5d^3(a+bx)^3}{(c+dx)^3}+\frac{15bd^2(a+bx)^2}{(c+dx)^2}-\frac{5b^2d(a+bx)}{c+dx}\right)}{5(bc-ad)^4(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
[Out] (-2*(c + d*x)^(5/2)*(b^3 + (5*d^3*(a + b*x)^3)/(c + d*x)^3 + (15*b*d^2*(a +
 b*x)^2)/(c + d*x)^2 - (5*b^2*d*(a + b*x))/(c + d*x)))/(5*(b*c - a*d)^4*(a
+ b*x)^(5/2))
```

Maple [A]
time = 0.16, size = 145, normalized size = 1.07

method	result
default	$-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}-\frac{6d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}-\frac{4d\left(-\frac{2}{(-ad+bc)\sqrt{bx+a}\,\sqrt{dx+c}}+\frac{4d\sqrt{}}{(-ad+bc)\sqrt{}}\right)}{3(-ad+bc)}\right)}{5(-ad+bc)}$
gosper	$-\frac{2\left(16b^3x^3d^3+40d^3a\,x^2b^2+8b^3c\,d^2x^2+30a^2b\,d^3x+20a\,b^2c\,d^2x-2b^3c^2dx+5a^3d^3+15a^2bc\,d^2-5a\,b^2c^2d+b^3c^3\right)}{5(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(1/2)-6/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*
c)/(b*x+a)^(3/2)/(d*x+c)^(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(b*x+a)^(1/2
)/(d*x+c)^(1/2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: ValueError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(112) = 224.
time = 1.59, size = 455, normalized size = 3.35

$$-\frac{2\,{\left(16\,b^{3}d^{3}x^{3}+b^{3}c^{3}-5\,ab^{2}c^{2}d+15\,a^{2}bcd^{2}+5\,a^{3}d^{3}+8\,{\left(b^{3}cd^{2}+5\,ab^{2}d^{3}\right)}x^{2}-2\,{\left(b^{3}c^{2}d-10\,ab^{2}cd^{2}-15\,a^{2}bd^{3}\right)}x\right)}\sqrt{bx+a}\,\sqrt{dx+c}}{5\,{\left(a^{3}b^{4}c^{5}-4\,a^{4}b^{3}c^{4}d+6\,a^{5}b^{2}c^{3}d^{2}-4\,a^{6}bc^{2}d^{3}+a^{7}cd^{4}+{\left(b^{7}c^{4}d-4\,ab^{6}c^{3}d^{2}+6\,a^{2}b^{5}c^{2}d^{3}-4\,a^{3}b^{4}cd^{4}+a^{4}b^{3}d^{5}\right)}x^{4}+{\left(b^{7}c^{5}-ab^{6}c^{4}d-6\,a^{2}b^{5}c^{3}d^{2}+14\,a^{3}b^{4}c^{2}d^{3}-11\,a^{4}b^{3}cd^{4}+3\,a^{5}b^{2}d^{5}\right)}x^{3}+3\,{\left(ab^{6}c^{5}-3\,a^{2}b^{5}c^{4}d+2\,a^{3}b^{4}c^{3}d^{2}+2\,a^{4}b^{3}c^{2}d^{3}-3\,a^{5}b^{2}cd^{4}+a^{6}bd^{5}\right)}x^{2}+{\left(3\,a^{2}b^{5}c^{5}-11\,a^{3}b^{4}c^{4}d+14\,a^{4}b^{3}c^{3}d^{2}-6\,a^{5}b^{2}c^{2}d^{3}-a^{6}bcd^{4}+a^{7}d^{5}\right)}x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
 + 8*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 10*a*b^2*c*d^2 - 15*a^2
*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a
^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d^2
 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*b^
6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5
*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*
b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{7}{2}}\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(112) = 224.
time = 2.80, size = 830, normalized size = 6.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^3/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^
2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)) - 4/5*(11*sqrt(b*d)*b^10*c^4*d^2 - 44*sqrt(b*d)*a*b^9*c
^3*d^3 + 66*sqrt(b*d)*a^2*b^8*c^2*d^4 - 44*sqrt(b*d)*a^3*b^7*c*d^5 + 11*sqr
t(b*d)*a^4*b^6*d^6 - 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*b^8*c^3*d^2 + 150*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^7*c^2*d^3 - 150*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*
d^4 + 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a^3*b^5*d^5 + 80*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*b^6*c^2*d^2 - 160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^5*c*d^3 + 80*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^
4 - 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^6*b^4*c*d^2 + 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^3*d^3 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^2*d^2)/((b^3*c^3*abs(b) - 3*a*b^2*
c^2*d*abs(b) + 3*a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

Mupad [B]
time = 1.31, size = 227, normalized size = 1.67

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d\,x^2\,\left(5\,a\,d+b\,c\right)}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,a^3\,d^3+6\,a^2\,b\,c\,d^2-2\,a\,b^2\,c^2\,d+\frac{2\,b^3\,c^3}{5}}{b^2\,d\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b\,d^2\,x^3}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(15\,a^2\,d^2+10\,a\,b\,c\,d-b^2\,c^2\right)}{5\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^2\,c\,\sqrt{a+b\,x}}{b^2\,d}+\frac{x^2\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a\,x\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((16*d*x^2*(5*a*d + b*c))/(5*(a*d - b*c)^4) + (2*a^3*d^3
 + (2*b^3*c^3)/5 - 2*a*b^2*c^2*d + 6*a^2*b*c*d^2)/(b^2*d*(a*d - b*c)^4) + (3
2*b*d^2*x^3)/(5*(a*d - b*c)^4) + (4*x*(15*a^2*d^2 - b^2*c^2 + 10*a*b*c*d))/
(5*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^2*c*(a + b*x)^(1/2))/(b^2*d
) + (x^2*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a*x*(a*d + 2*b*c)*(a + b*x
)^(1/2))/(b^2*d))
```

3.1510 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}+\frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}-\frac{32d^2}{35(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}+\frac{3}{}$$

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(1/2)+16/35*d/(-a*d+b*c)^2/(b*x+a)^(5
/2)/(d*x+c)^(1/2)-32/35*d^2/(-a*d+b*c)^3/(b*x+a)^(3/2)/(d*x+c)^(1/2)+128/35
*d^3/(-a*d+b*c)^4/(b*x+a)^(1/2)/(d*x+c)^(1/2)+256/35*d^4*(b*x+a)^(1/2)/(-a*
d+b*c)^5/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}\,(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x]) + (16*d)/(35*(b*c - a*d)^2
*(a + b*x)^(5/2)*Sqrt[c + d*x]) - (32*d^2)/(35*(b*c - a*d)^3*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + (128*d^3)/(35*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (256*d^4*Sqrt[a + b*x])/(35*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} + \frac{(48d$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{}{35(b}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{}{35(b}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}} + \frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}} - \frac{}{35(b}$$

Mathematica [A]
time = 0.15, size = 117, normalized size = 0.68

$$-\frac{2(c+dx)^{7/2}\left(5b^4-\frac{35d^4(a+bx)^4}{(c+dx)^4}-\frac{140bd^3(a+bx)^3}{(c+dx)^3}+\frac{70b^2d^2(a+bx)^2}{(c+dx)^2}-\frac{28b^3d(a+bx)}{c+dx}\right)}{35(bc-ad)^5(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(5*b^4 - (35*d^4*(a + b*x)^4)/(c + d*x)^4 - (140*b*d^3*
(a + b*x)^3)/(c + d*x)^3 + (70*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 - (28*b^3*d
*(a + b*x))/(c + d*x)))/(35*(b*c - a*d)^5*(a + b*x)^(7/2))
```

Maple [A]
time = 0.16, size = 185, normalized size = 1.08

method	result
default	$-\frac{2}{7(-ad+bc)(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}-\frac{8d\left(-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}-\frac{6d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}-\frac{4d\left(-\frac{}{(-ad+bc)}\right.}{}\right)}{5}\right)}{7(-ad+bc)}$

gosper	$-\frac{2(128d^4x^4b^4+448a\,b^3d^4x^3+64b^4c\,d^3x^3+560a^2b^2d^4x^2+224a\,b^3c\,d^3x^2-16b^4c^2d^2x^2+280a^3b\,d^4x+280a^2b^2c\,d^3x-56a\,b^3c^2d^2x+8b^4}{35(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\,(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(1/2)-8/7*d/(-a*d+b*c)*(-2/5/(-a*d+b*
c)/(b*x+a)^(5/2)/(d*x+c)^(1/2)-6/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*c)/(b*x+a)^(3
/2)/(d*x+c)^(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/
2)+4*d/(-a*d+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(141) = 282$.
time = 2.73, size = 689, normalized size = 4.03

$$\frac{2\,(128\,b^4d^4x^4-5\,b^4c^4+28\,ab^3c^3d-70\,a^2b^2c^2d^2+140\,a^3bcd^3+35\,a^4d^4+64\,(b^4cd^3+7\,ab^3d^4)x^3-16\,(b^4c^2d^2-14\,ab^3cd^3-35\,a^2b^2d^4)x^2+8\,(b^4c^3d-7\,ab^3c^2d^2+35\,a^2b^2cd^3+35\,a^3bd^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{35\,(a^4b^5c^6-5\,a^5b^4c^5d+10\,a^6b^3c^4d^2-10\,a^7b^2c^3d^3+5\,a^8bc^2d^4-a^9cd^5+(b^9c^5d-5\,ab^8c^4d^2+10\,a^2b^7c^3d^3-10\,a^3b^6c^2d^4+5\,a^4b^5cd^5-a^5b^4d^6)x^5+(b^9c^6-ab^8c^5d-10\,a^2b^7c^4d^2+30\,a^3b^6c^3d^3-35\,a^4b^5c^2d^4+19\,a^5b^4cd^5-4\,a^6b^3d^6)x^4+2\,(2\,ab^8c^6-7\,a^2b^7c^5d+5\,a^3b^6c^4d^2+10\,a^4b^5c^3d^3-20\,a^5b^4c^2d^4+13\,a^6b^3cd^5-3\,a^7b^2d^6)x^3+2\,(3\,a^2b^7c^6-13\,a^3b^6c^5d+20\,a^4b^5c^4d^2-10\,a^5b^4c^3d^3-5\,a^6b^3c^2d^4+7\,a^7b^2cd^5-2\,a^8bd^6)x^2+(4\,a^3b^6c^6-19\,a^4b^5c^5d+35\,a^5b^4c^4d^2-30\,a^6b^3c^3d^3+10\,a^7b^2c^2d^4+a^8bcd^5-a^9d^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(128*b^4*d^4*x^4 - 5*b^4*c^4 + 28*a*b^3*c^3*d - 70*a^2*b^2*c^2*d^2 + 1
40*a^3*b*c*d^3 + 35*a^4*d^4 + 64*(b^4*c*d^3 + 7*a*b^3*d^4)*x^3 - 16*(b^4*c^
2*d^2 - 14*a*b^3*c*d^3 - 35*a^2*b^2*d^4)*x^2 + 8*(b^4*c^3*d - 7*a*b^3*c^2*d
^2 + 35*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b
^5*c^6 - 5*a^5*b^4*c^5*d + 10*a^6*b^3*c^4*d^2 - 10*a^7*b^2*c^3*d^3 + 5*a^8*
b*c^2*d^4 - a^9*c*d^5 + (b^9*c^5*d - 5*a*b^8*c^4*d^2 + 10*a^2*b^7*c^3*d^3 -
 10*a^3*b^6*c^2*d^4 + 5*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^5 + (b^9*c^6 - a*b^8
*c^5*d - 10*a^2*b^7*c^4*d^2 + 30*a^3*b^6*c^3*d^3 - 35*a^4*b^5*c^2*d^4 + 19*
a^5*b^4*c*d^5 - 4*a^6*b^3*d^6)*x^4 + 2*(2*a*b^8*c^6 - 7*a^2*b^7*c^5*d + 5*a
^3*b^6*c^4*d^2 + 10*a^4*b^5*c^3*d^3 - 20*a^5*b^4*c^2*d^4 + 13*a^6*b^3*c*d^5
 - 3*a^7*b^2*d^6)*x^3 + 2*(3*a^2*b^7*c^6 - 13*a^3*b^6*c^5*d + 20*a^4*b^5*c^
4*d^2 - 10*a^5*b^4*c^3*d^3 - 5*a^6*b^3*c^2*d^4 + 7*a^7*b^2*c*d^5 - 2*a^8*b*
```

d^6)*x^2 + (4*a^3*b^6*c^6 - 19*a^4*b^5*c^5*d + 35*a^5*b^4*c^4*d^2 - 30*a^6*b^3*c^3*d^3 + 10*a^7*b^2*c^2*d^4 + a^8*b*c*d^5 - a^9*d^6)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(3/2),x)

[Out] Integral(1/((a + b*x)**(9/2)*(c + d*x)**(3/2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(141) = 282$.
time = 2.39, size = 1518, normalized size = 8.88

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="giac")

[Out] 2*sqrt(b*x + a)*b^2*d^4/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) - 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/35*(93*sqrt(b*d)*b^14*c^6*d^3 - 558*sqrt(b*d)*a*b^13*c^5*d^4 + 1395*sqrt(b*d)*a^2*b^12*c^4*d^5 - 1860*sqrt(b*d)*a^3*b^11*c^3*d^6 + 1395*sqrt(b*d)*a^4*b^10*c^2*d^7 - 558*sqrt(b*d)*a^5*b^9*c*d^8 + 93*sqrt(b*d)*a^6*b^8*d^9 - 616*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^12*c^5*d^3 + 3080*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^11*c^4*d^4 - 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^3*d^5 + 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^9*c^2*d^6 - 3080*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c*d^7 + 616*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^7*d^8 + 1673*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^4*d^3 - 6692*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^9*c^3*d^4 + 10038*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^2*d^5 - 6692*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c*d^6 + 1673*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^6*d^7 - 2240*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^8*c^3*d^3 + 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^7*c^2*d^4 - 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -

```
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c*d^5 + 2240*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^5*d^6 +
 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^8*b^6*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^8*a*b^5*c*d^4 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*d^5 - 280*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^4*c*d^
3 + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^10*a*b^3*d^4 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^12*b^2*d^3)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b)
 + 6*a^2*b^2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c
 - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2)^7)
```

Mupad [B]

time = 1.50, size = 337, normalized size = 1.97

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{256\,b\,d^3\,x^4}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^2\,x^3\,\left(7\,a\,d+b\,c\right)}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{70\,a^4\,d^4+280\,a^3\,b\,c\,d^3-140\,a^2\,b^2\,c^2\,d^2+56\,a\,b^3\,c^3\,d-10\,b^4\,c^4}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(560\,a^3\,b\,d^4+560\,a^2\,b^2\,c\,d^3-112\,a\,b^3\,c^2\,d^2+16\,b^4\,c^3\,d\right)}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d\,x^2\,\left(35\,a^2\,d^2+14\,a\,b\,c\,d-b^2\,c^2\right)}{35\,b\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^3\,c\,\sqrt{a+b\,x}}{b^3\,d}+\frac{x^3\,\left(3\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{3\,a\,x^2\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^2\,x\,\left(a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((256*b*d^3*x^4)/(35*(a*d - b*c)^5) + (128*d^2*x^3*(7*a*d
 + b*c))/(35*(a*d - b*c)^5) + (70*a^4*d^4 - 10*b^4*c^4 - 140*a^2*b^2*c^2*d^
2 + 56*a*b^3*c^3*d + 280*a^3*b*c*d^3)/(35*b^3*d*(a*d - b*c)^5) + (x*(560*a^
3*b*d^4 + 16*b^4*c^3*d - 112*a*b^3*c^2*d^2 + 560*a^2*b^2*c*d^3))/(35*b^3*d*
(a*d - b*c)^5) + (32*d*x^2*(35*a^2*d^2 - b^2*c^2 + 14*a*b*c*d))/(35*b*(a*d
- b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^3*c*(a + b*x)^(1/2))/(b^3*d) + (x^3*(
3*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (3*a*x^2*(a*d + b*c)*(a + b*x)^(1/2))
/(b^2*d) + (a^2*x*(a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

3.1511 $\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}}+\frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}-\frac{32d^2}{63(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}+\frac{}{63\ldots}$$

```
[Out] -2/9/(-a*d+b*c)/(b*x+a)^(9/2)/(d*x+c)^(1/2)+20/63*d/(-a*d+b*c)^2/(b*x+a)^(7
/2)/(d*x+c)^(1/2)-32/63*d^2/(-a*d+b*c)^3/(b*x+a)^(5/2)/(d*x+c)^(1/2)+64/63*
d^3/(-a*d+b*c)^4/(b*x+a)^(3/2)/(d*x+c)^(1/2)-256/63*d^4/(-a*d+b*c)^5/(b*x+a
)^(1/2)/(d*x+c)^(1/2)-512/63*d^5*(b*x+a)^(1/2)/(-a*d+b*c)^6/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{20d}{63(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{9(a+bx)^{9/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) - (256*d^4)/(63*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x]) - (512*d^5*Sqr
t[a + b*x])/(63*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx = -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} + \frac{(8\cdots}{\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{\cdots}{63\cdots}$$

Mathematica [A]
time = 0.14, size = 139, normalized size = 0.67

$$-\frac{2(c+dx)^{9/2}\left(7b^5+\frac{63d^5(a+bx)^5}{(c+dx)^5}+\frac{315bd^4(a+bx)^4}{(c+dx)^4}-\frac{210b^2d^3(a+bx)^3}{(c+dx)^3}+\frac{126b^3d^2(a+bx)^2}{(c+dx)^2}-\frac{45b^4d(a+bx)}{c+dx}\right)}{63(bc-ad)^6(a+bx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(7*b^5 + (63*d^5*(a + b*x)^5)/(c + d*x)^5 + (315*b*d^4*
(a + b*x)^4)/(c + d*x)^4 - (210*b^2*d^3*(a + b*x)^3)/(c + d*x)^3 + (126*b^3
*d^2*(a + b*x)^2)/(c + d*x)^2 - (45*b^4*d*(a + b*x))/(c + d*x)))/(63*(b*c -
 a*d)^6*(a + b*x)^(9/2))
```

Maple [A]
time = 0.16, size = 225, normalized size = 1.09

method	result

default	$-\frac{2}{9(-ad+bc)(bx+a)^{\frac{9}{2}}\sqrt{dx+c}}-\frac{10d\left(-\frac{2}{7(-ad+bc)(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}-\frac{8d\left(-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}-\frac{6d\left(-\frac{\cdots}{3(-ad+b\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$
gosper	$-\frac{2(256b^5d^5x^5+1152a\,b^4d^5x^4+128b^5c\,d^4x^4+2016a^2b^3d^5x^3+576a\,b^4c\,d^4x^3-32b^5c^2d^3x^3+1680a^3b^2d^5x^2+1008a^2b^3c\,d^4x^2-144a\,b^4c^2\cdots}{63(bx+a)^{\frac{9}{2}}\sqrt{dx+c}\,(a^6d^6-6a^5\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9/(-a*d+b*c)/(b*x+a)^(9/2)/(d*x+c)^(1/2)-10/9*d/(-a*d+b*c)*(-2/7/(-a*d+b
*c)/(b*x+a)^(7/2)/(d*x+c)^(1/2)-8/7*d/(-a*d+b*c)*(-2/5/(-a*d+b*c)/(b*x+a)^(
5/2)/(d*x+c)^(1/2)-6/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^
(1/2)-4/3*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+4*d/(-a*d
+b*c)*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 955 vs. 2(170) = 340.
time = 5.33, size = 955, normalized size = 4.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(256*b^5*d^5*x^5 + 7*b^5*c^5 - 45*a*b^4*c^4*d + 126*a^2*b^3*c^3*d^2 -
 210*a^3*b^2*c^2*d^3 + 315*a^4*b*c*d^4 + 63*a^5*d^5 + 128*(b^5*c*d^4 + 9*a*
b^4*d^5)*x^4 - 32*(b^5*c^2*d^3 - 18*a*b^4*c*d^4 - 63*a^2*b^3*d^5)*x^3 + 16*
(b^5*c^3*d^2 - 9*a*b^4*c^2*d^3 + 63*a^2*b^3*c*d^4 + 105*a^3*b^2*d^5)*x^2 -
2*(5*b^5*c^4*d - 36*a*b^4*c^3*d^2 + 126*a^2*b^3*c^2*d^3 - 420*a^3*b^2*c*d^4
 - 315*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^6*c^7 - 6*a^6*b^5*c
^6*d + 15*a^7*b^4*c^5*d^2 - 20*a^8*b^3*c^4*d^3 + 15*a^9*b^2*c^3*d^4 - 6*a^1
0*b*c^2*d^5 + a^11*c*d^6 + (b^11*c^6*d - 6*a*b^10*c^5*d^2 + 15*a^2*b^9*c^4*
d^3 - 20*a^3*b^8*c^3*d^4 + 15*a^4*b^7*c^2*d^5 - 6*a^5*b^6*c*d^6 + a^6*b^5*d
^7)*x^6 + (b^11*c^7 - a*b^10*c^6*d - 15*a^2*b^9*c^5*d^2 + 55*a^3*b^8*c^4*d^
3 - 85*a^4*b^7*c^3*d^4 + 69*a^5*b^6*c^2*d^5 - 29*a^6*b^5*c*d^6 + 5*a^7*b^4*
d^7)*x^5 + 5*(a*b^10*c^7 - 4*a^2*b^9*c^6*d + 3*a^3*b^8*c^5*d^2 + 10*a^4*b^7
*c^4*d^3 - 25*a^5*b^6*c^3*d^4 + 24*a^6*b^5*c^2*d^5 - 11*a^7*b^4*c*d^6 + 2*a
^8*b^3*d^7)*x^4 + 10*(a^2*b^9*c^7 - 5*a^3*b^8*c^6*d + 9*a^4*b^7*c^5*d^2 - 5
*a^5*b^6*c^4*d^3 - 5*a^6*b^5*c^3*d^4 + 9*a^7*b^4*c^2*d^5 - 5*a^8*b^3*c*d^6
+ a^9*b^2*d^7)*x^3 + 5*(2*a^3*b^8*c^7 - 11*a^4*b^7*c^6*d + 24*a^5*b^6*c^5*d
^2 - 25*a^6*b^5*c^4*d^3 + 10*a^7*b^4*c^3*d^4 + 3*a^8*b^3*c^2*d^5 - 4*a^9*b^
2*c*d^6 + a^10*b*d^7)*x^2 + (5*a^4*b^7*c^7 - 29*a^5*b^6*c^6*d + 69*a^6*b^5*
c^5*d^2 - 85*a^7*b^4*c^4*d^3 + 55*a^8*b^3*c^3*d^4 - 15*a^9*b^2*c^2*d^5 - a^
10*b*c*d^6 + a^11*d^7)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(11/2)*(c + d*x)**(3/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2438 vs. $2(170) = 340$.
time = 3.08, size = 2438, normalized size = 11.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^5/((b^6*c^6*abs(b) - 6*a*b^5*c^5*d*abs(b) + 15*a^2*b
^4*c^4*d^2*abs(b) - 20*a^3*b^3*c^3*d^3*abs(b) + 15*a^4*b^2*c^2*d^4*abs(b) -
```

6*a^5*b*c*d^5*abs(b) + a^6*d^6*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
) - 4/63*(193*sqrt(b*d)*b^18*c^8*d^4 - 1544*sqrt(b*d)*a*b^17*c^7*d^5 + 5404
*sqrt(b*d)*a^2*b^16*c^6*d^6 - 10808*sqrt(b*d)*a^3*b^15*c^5*d^7 + 13510*sqrt
(b*d)*a^4*b^14*c^4*d^8 - 10808*sqrt(b*d)*a^5*b^13*c^3*d^9 + 5404*sqrt(b*d)*
a^6*b^12*c^2*d^10 - 1544*sqrt(b*d)*a^7*b^11*c*d^11 + 193*sqrt(b*d)*a^8*b^10
*d^12 - 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^16*c^7*d^4 + 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^6*d^5 - 35154*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^14*c^5*d^
6 + 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a^3*b^13*c^4*d^7 - 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^3*d^8 + 35154*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^11*c^2
*d^9 - 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*a^6*b^10*c*d^10 + 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^9*d^11 + 6318*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^6*d^4 - 3
7908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^13*c^5*d^5 + 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^4*d^6 - 126360*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^3*d^7 +
94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^4*a^4*b^10*c^2*d^8 - 37908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c*d^9 + 6318*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*d^10 - 1331
4*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^6*b^12*c^5*d^4 + 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^6*a*b^11*c^4*d^5 - 133140*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^3*d^6 + 133140
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a^3*b^9*c^2*d^7 - 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^6*a^4*b^8*c*d^8 + 13314*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*d^9 + 16128*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10
*c^4*d^4 - 64512*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*a*b^9*c^3*d^5 + 96768*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^2*d^6 - 64512*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c
*d^7 + 16128*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^8*a^4*b^6*d^8 - 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^3*d^4 + 24570*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^2*d^5 - 24
570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^10*a^2*b^6*c*d^6 + 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^10*a^3*b^5*d^7 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b

```
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^2*d^4 - 5796*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^
5*c*d^5 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^12*a^2*b^4*d^6 - 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c*d^4 + 630*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*d^5 + 63*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2
*d^4)/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) -
 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c
- a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
)^9)
```

Mupad [B]
time = 1.96, size = 454, normalized size = 2.20

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^5\,d^5+630\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+252\,a^2\,b^3\,c^3\,d^2-90\,a\,b^4\,c^4\,d+14\,b^5\,c^5}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b\,d^4\,x^5}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,d^3\,x^4\,\left(9\,a\,d+b\,c\right)}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(1260\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4-504\,a^2\,b^3\,c^2\,d^3+144\,a\,b^4\,c^3\,d^2-20\,b^5\,c^4\,d\right)}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d^2\,x^3\,\left(63\,a^2\,d^2+18\,a\,b\,c\,d-b^2\,c^2\right)}{63\,b\,{\left(a\,d-b\,c\right)}^6}+\frac{32\,d\,x^2\,\left(105\,a^3\,d^3+63\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{63\,b^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^4\,c\,\sqrt{a+b\,x}}{b^4\,d}+\frac{x^4\,\left(4\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{2\,a\,x^3\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^3\,x\,\left(a\,d+4\,b\,c\right)\,\sqrt{a+b\,x}}{b^4\,d}+\frac{2\,a^2\,x^2\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((126*a^5*d^5 + 14*b^5*c^5 + 252*a^2*b^3*c^3*d^2 - 420*a^
3*b^2*c^2*d^3 - 90*a*b^4*c^4*d + 630*a^4*b*c*d^4)/(63*b^4*d*(a*d - b*c)^6)
+ (512*b*d^4*x^5)/(63*(a*d - b*c)^6) + (256*d^3*x^4*(9*a*d + b*c))/(63*(a*d
 - b*c)^6) + (x*(1260*a^4*b*d^5 - 20*b^5*c^4*d + 144*a*b^4*c^3*d^2 + 1680*a
^3*b^2*c*d^4 - 504*a^2*b^3*c^2*d^3))/(63*b^4*d*(a*d - b*c)^6) + (64*d^2*x^3
*(63*a^2*d^2 - b^2*c^2 + 18*a*b*c*d))/(63*b*(a*d - b*c)^6) + (32*d*x^2*(105
*a^3*d^3 + b^3*c^3 - 9*a*b^2*c^2*d + 63*a^2*b*c*d^2))/(63*b^2*(a*d - b*c)^6
)))/(x^5*(a + b*x)^(1/2) + (a^4*c*(a + b*x)^(1/2))/(b^4*d) + (x^4*(4*a*d +
b*c)*(a + b*x)^(1/2))/(b*d) + (2*a*x^3*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b^
2*d) + (a^3*x*(a*d + 4*b*c)*(a + b*x)^(1/2))/(b^4*d) + (2*a^2*x^2*(2*a*d +
3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

3.1512 $\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}}-\frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}}+\frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5}-\frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4}+7b$$

```
[Out] -2/3*(b*x+a)^(9/2)/d/(d*x+c)^(3/2)-105/8*b^(3/2)*(-a*d+b*c)^3*arctanh(d^(1/
2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(11/2)-6*b*(b*x+a)^(7/2)/d^2/(d*x
+c)^(1/2)-35/4*b^2*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^4+7*b^2*(b*x+a)
^(5/2)*(d*x+c)^(1/2)/d^3+105/8*b^2*(-a*d+b*c)^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)
/d^5
```

Rubi [A]
time = 0.08, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{11/2}}+\frac{105b^2\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^5}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}-\frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(9/2))/(3*d*(c + d*x)^(3/2)) - (6*b*(a + b*x)^(7/2))/(d^2*Sqr
t[c + d*x]) + (105*b^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^5) -
 (35*b^2*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(4*d^4) + (7*b^2*(a + b
*x)^(5/2)*Sqrt[c + d*x])/d^3 - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(11/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} + \frac{(3b)\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{(21b^2)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3} - \frac{(35b^2(bc-ad))\int \frac{(a+bx)^{3/}}{\sqrt{c+d}}}{2d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} + \frac{7b^2(a+bx)^{5/2}\sqrt{}}{d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(}{}$$

Mathematica [A]
time = 0.39, size = 165, normalized size = 0.81

$$-\frac{(a+bx)^{9/2}\left(16d^4+\frac{144bd^3(c+dx)}{a+bx}-\frac{693b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{840b^3d(c+dx)^3}{(a+bx)^3}-\frac{315b^4(c+dx)^4}{(a+bx)^4}\right)}{24d^5(c+dx)^{3/2}}-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{8d^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/24*((a + b*x)^(9/2)*(16*d^4 + (144*b*d^3*(c + d*x))/(a + b*x) - (693*b^2
*d^2*(c + d*x)^2)/(a + b*x)^2 + (840*b^3*d*(c + d*x)^3)/(a + b*x)^3 - (315*
b^4*(c + d*x)^4)/(a + b*x)^4))/(d^5*(c + d*x)^(3/2)) - (105*b^(3/2)*(b*c -
a*d)^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*d^(11/2
))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{9}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(164) = 328.
time = 1.46, size = 879, normalized size = 4.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 +
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*
c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 - a^3*b*c*d^4)*x)*sqrt(b/d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d
^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5), 1/48*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a
^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2
*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^
```

```
3 - a^3*b*c*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(164) = 328.
time = 0.96, size = 500, normalized size = 2.45

$$\frac{\left(\left(\left(2\,(bx+a)\left(\frac{4\,(b^6cd^8-ab^5d^9)(bx+a)}{b^2cd^9|b|-abd^{10}|b|}-\frac{9\,(b^7c^2d^7-2\,ab^6cd^8+a^2b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)+\frac{63\,(b^8c^3d^6-3\,ab^7c^2d^7+3\,a^2b^6cd^8-a^3b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{420\,(b^9c^4d^5-4\,ab^8c^3d^6+6\,a^2b^7c^2d^7-4\,a^3b^6cd^8+a^4b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{315\,(b^{10}c^5d^4-5\,ab^9c^4d^5+10\,a^2b^8c^3d^6-10\,a^3b^7c^2d^7+5\,a^4b^6cd^8-a^5b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)\sqrt{bx+a}}{24\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}+\frac{105\,(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{8\,\sqrt{bd}\,d^5|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(b*x + a)*(4*(b^6*c*d^8 - a*b^5*d^9)*(b*x + a)/(b^2*c*d^9*abs(b)
- a*b*d^10*abs(b)) - 9*(b^7*c^2*d^7 - 2*a*b^6*c*d^8 + a^2*b^5*d^9)/(b^2*c*d
^9*abs(b) - a*b*d^10*abs(b))) + 63*(b^8*c^3*d^6 - 3*a*b^7*c^2*d^7 + 3*a^2*b
^6*c*d^8 - a^3*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 4
20*(b^9*c^4*d^5 - 4*a*b^8*c^3*d^6 + 6*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 + a
^4*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 315*(b^10*c^5
*d^4 - 5*a*b^9*c^4*d^5 + 10*a^2*b^8*c^3*d^6 - 10*a^3*b^7*c^2*d^7 + 5*a^4*b^
6*c*d^8 - a^5*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*sqrt(b*x + a)/
(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 105/8*(b^6*c^3 - 3*a*b^5*c^2*d + 3*
a^2*b^4*c*d^2 - a^3*b^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^5*abs(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{9/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(9/2)/(c + d*x)^(5/2), x)
```

3.1513 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}+\frac{35b^{3/2}(bc-$$

```
[Out] -2/3*(b*x+a)^(7/2)/d/(d*x+c)^(3/2)+35/4*b^(3/2)*(-a*d+b*c)^2*arctanh(d^(1/2
)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(9/2)-14/3*b*(b*x+a)^(5/2)/d^2/(d*
x+c)^(1/2)+35/6*b^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d^3-35/4*b^2*(-a*d+b*c)*(b*
x+a)^(1/2)*(d*x+c)^(1/2)/d^4
```

Rubi [A]
time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{35b^2\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/2)) - (14*b*(a + b*x)^(5/2))/(3*d^2*
Sqrt[c + d*x]) - (35*b^2*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^4) +
 (35*b^2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(6*d^3) + (35*b^(3/2)*(b*c - a*d)^2
*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(9/2))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{(35b^2)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} + \frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3} - \frac{(35b^2(bc-ad))\int \frac{\sqrt{a}}{\sqrt{c}}}{4d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/}}{6d^3}$$

$$= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}} - \frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4} + \frac{35b^2(a+bx)^{3/}}{6d^3}$$

Mathematica [A]
time = 0.27, size = 166, normalized size = 0.98

$$-\frac{\sqrt{a+bx}\,(8a^3d^3+8a^2bd^2(7c+10dx)-ab^2d(175c^2+238cdx+39d^2x^2)+b^3(105c^3+140c^2dx+21cd^2x^2-6d^3x^3))}{12d^4(c+dx)^{3/2}} + \frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[a + b*x]*(8*a^3*d^3 + 8*a^2*b*d^2*(7*c + 10*d*x) - a*b^2*d*(175
*c^2 + 238*c*d*x + 39*d^2*x^2) + b^3*(105*c^3 + 140*c^2*d*x + 21*c*d^2*x^2
- 6*d^3*x^3)))/(d^4*(c + d*x)^(3/2)) + (35*b^(3/2)*(b*c - a*d)^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*d^(9/2))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)

[Out] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(132) = 264$.
time = 1.54, size = 657, normalized size = 3.86

$$\left[\frac{105\,(b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 + (b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4)x^2 + 2\,(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2 + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\,bd^2x + bcd + ad^2)\sqrt{bx + a}\,\sqrt{dx + c}\,\sqrt{\frac{b}{d}} + 8\,(b^2cd + abd^2)x\right) + 4\,(6\,b^3d^3x^3 - 105\,b^3c^3 + 175\,ab^2c^2d - 56\,a^2bcd^2 - 8\,a^3d^3 - 3\,(7\,b^3cd^2 - 13\,ab^2d^3)x^2 - 2\,(70\,b^3c^2d - 119\,ab^2cd^2 + 40\,a^2bd^3)x)\sqrt{bx + a}\,\sqrt{dx + c}}{48\,(d^6x^2 + 2\,cd^5x + c^2d^4)}, -\frac{105\,(b^3c^4 - 2\,ab^2c^3d + a^2bc^2d^2 + (b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4)x^2 + 2\,(b^3c^3d - 2\,ab^2c^2d^2 + a^2bcd^3)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx + bc + ad)\sqrt{bx + a}\,\sqrt{dx + c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2 + abc + (b^2c + abd)x)}\right) - 2\,(6\,b^3d^3x^3 - 105\,b^3c^3 + 175\,ab^2c^2d - 56\,a^2bcd^2 - 8\,a^3d^3 - 3\,(7\,b^3cd^2 - 13\,ab^2d^3)x^2 - 2\,(70\,b^3c^2d - 119\,ab^2cd^2 + 40\,a^2bd^3)x)\sqrt{bx + a}\,\sqrt{dx + c}}{24\,(d^6x^2 + 2\,cd^5x + c^2d^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/48*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3*c^2*d^2 - 2*a*b^
2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3)*x)
*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x
 + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(6*b^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2
 - 8*a^3*d^3 - 3*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a
*b^2*c*d^2 + 40*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d
^5*x + c^2*d^4), -1/24*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3
*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2
+ a^2*b*c*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(6*b
^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2 - 8*a^3*d^3 - 3
*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a*b^2*c*d^2 + 40*
a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^{\frac{7}{2}}}{(c + dx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(132) = 264$.

time = 0.80, size = 380, normalized size = 2.24

$$\frac{\left(\left(3\,(bx+a)\left(\frac{2\,(b^6cd^6-ab^5d^7)(bx+a)}{b^2cd^7|b|-abd^8|b|}-\frac{7\,(b^7c^2d^5-2\,ab^6cd^6+a^2b^5d^7)}{b^2cd^7|b|-abd^8|b|}\right)-\frac{140\,(b^8c^3d^4-3\,ab^7c^2d^5+3\,a^2b^6cd^6-a^3b^5d^7)}{b^2cd^7|b|-abd^8|b|}\right)(bx+a)-\frac{105\,(b^9c^4d^3-4\,ab^8c^3d^4+6\,a^2b^7c^2d^5-4\,a^3b^6cd^6+a^4b^5d^7)}{b^2cd^7|b|-abd^8|b|}\right)\sqrt{bx+a}}{12\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}-\frac{35\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{4\,\sqrt{bd}\,d^4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*(b*x + a)*(2*(b^6*c*d^6 - a*b^5*d^7)*(b*x + a)/(b^2*c*d^7*abs(b) -
 a*b*d^8*abs(b)) - 7*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)/(b^2*c*d^7
*abs(b) - a*b*d^8*abs(b))) - 140*(b^8*c^3*d^4 - 3*a*b^7*c^2*d^5 + 3*a^2*b^6
*c*d^6 - a^3*b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*(b*x + a) - 105*
(b^9*c^4*d^3 - 4*a*b^8*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 4*a^3*b^6*c*d^6 + a^4*
b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x +
 a)*b*d - a*b*d)^(3/2) - 35/4*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*log(abs
(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d
)*d^4*abs(b))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(5/2), x)
```

3.1514 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}$$

```
[Out] -2/3*(b*x+a)^(5/2)/d/(d*x+c)^(3/2)-5*b^(3/2)*(-a*d+b*c)*arctanh(d^(1/2)*(b*
x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(7/2)-10/3*b*(b*x+a)^(3/2)/d^2/(d*x+c)^
(1/2)+5*b^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 223, 212}

$$-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*(a + b*x)^(3/2))/(3*d^2*
Sqrt[c + d*x]) + (5*b^2*Sqrt[a + b*x]*Sqrt[c + d*x])/d^3 - (5*b^(3/2)*(b*c
- a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(7/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} + \frac{(5b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx}{3d}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{(5b^2)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{(5b^2(bc-ad))\int \frac{1}{\sqrt{a+bx}\ldots}}{2d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{(5b(bc-ad))\mathrm{Subst}\left(\int \ldots\right)}{\ldots}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{(5b(bc-ad))\mathrm{Subst}\left(\int \frac{\ldots}{1-\ldots}\ldots\right)}{d^3}$$

$$= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}} - \frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}} + \frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3} - \frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\ldots\right)}{d^{7/2}}$$

Mathematica [A]

time = 0.78, size = 123, normalized size = 0.96

$$\frac{\frac{\sqrt{a+bx}\,(-2a^2d^2-2abd(5c+7dx)+b^2(15c^2+20cdx+3d^2x^2))}{(c+dx)^{3/2}}+15b\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] ((Sqrt[a + b*x]*(-2*a^2*d^2 - 2*a*b*d*(5*c + 7*d*x) + b^2*(15*c^2 + 20*c*d*
x + 3*d^2*x^2)))/(c + d*x)^(3/2) + 15*b*Sqrt[b/d]*(b*c - a*d)*Log[Sqrt[a +
b*x] - Sqrt[b/d]*Sqrt[c + d*x]])/(3*d^3)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(100) = 200.
time = 0.84, size = 475, normalized size = 3.71

$$\left[-\frac{15\,(b^2c^3-abc^2d+(b^2cd^2-abd^3)x^2+2\,(b^2c^2d-abcd^2)x)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,(3\,b^2d^2x^2+15\,b^2c^2-10\,abcd-2\,a^2d^2+2\,(10\,b^2cd-7\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{12\,(d^5x^2+2\,cd^4x+c^2d^3)},\ \frac{15\,(b^2c^3-abc^2d+(b^2cd^2-abd^3)x^2+2\,(b^2c^2d-abcd^2)x)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,(3\,b^2d^2x^2+15\,b^2c^2-10\,abcd-2\,a^2d^2+2\,(10\,b^2cd-7\,abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{6\,(d^5x^2+2\,cd^4x+c^2d^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3)*x^2 + 2*(b^2*c^2*d
- a*b*c*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2
*d^2 + 2*(10*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2
+ 2*c*d^4*x + c^2*d^3), 1/6*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3
)*x^2 + 2*(b^2*c^2*d - a*b*c*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c +
 a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c +
a*b*d)*x)) + 2*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2*d^2 + 2*(10
*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(100) = 200.
time = 1.31, size = 276, normalized size = 2.16

$$\frac{\left((bx+a)\left(\frac{3\,(b^6cd^4-ab^5d^5)(bx+a)}{b^2cd^5|b|-abd^6|b|}+\frac{20\,(b^7c^2d^3-2\,ab^6cd^4+a^2b^5d^5)}{b^2cd^5|b|-abd^6|b|}\right)+\frac{15\,(b^8c^3d^2-3\,ab^7c^2d^3+3\,a^2b^6cd^4-a^3b^5d^5)}{b^2cd^5|b|-abd^6|b|}\right)\sqrt{bx+a}}{3\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}+\frac{5\,(b^4c-ab^3d)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)*(3*(b^6*c*d^4 - a*b^5*d^5)*(b*x + a)/(b^2*c*d^5*abs(b) - a*b
*d^6*abs(b)) + 20*(b^7*c^2*d^3 - 2*a*b^6*c*d^4 + a^2*b^5*d^5)/(b^2*c*d^5*ab
s(b) - a*b*d^6*abs(b))) + 15*(b^8*c^3*d^2 - 3*a*b^7*c^2*d^3 + 3*a^2*b^6*c*d
^4 - a^3*b^5*d^5)/(b^2*c*d^5*abs(b) - a*b*d^6*abs(b)))*sqrt(b*x + a)/(b^2*c
 + (b*x + a)*b*d - a*b*d)^(3/2) + 5*(b^4*c - a*b^3*d)*log(abs(-sqrt(b*d)*sq
rt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/2), x)
```

3.1515 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}$$

```
[Out] -2/3*(b*x+a)^(3/2)/d/(d*x+c)^(3/2)+2*b^(3/2)*arctanh(d^(1/2)*(b*x+a)^(1/2)/
b^(1/2)/(d*x+c)^(1/2))/d^(5/2)-2*b*(b*x+a)^(1/2)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {49, 65, 223, 212}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*Sqrt[a + b*x])/(d^2*Sqrt[
c + d*x]) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/d^(5/2)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}+\frac{b\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{b^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{(2b)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 81, normalized size = 0.88

$$-\frac{2\sqrt{a+bx}\,(3bc+ad+4bdx)}{3d^2(c+dx)^{3/2}}+\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(3*b*c + a*d + 4*b*d*x))/(3*d^2*(c + d*x)^(3/2)) + (2*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/d^(5/2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(70) = 140$.
time = 1.23, size = 325, normalized size = 3.53

$$\left[\frac{3\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bd^2x+bcd+ad^2)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\,(b^2cd+abd^2)x\right)-4\,(4\,bdx+3\,bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{6\,(d^4x^2+2\,cd^3x+c^2d^2)},\ -\frac{3\,(bd^2x^2+2\,bcdx+bc^2)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\,(b^2dx^2+abc+(b^2c+abd)x)}\right)+2\,(4\,bdx+3\,bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(d^4x^2+2\,cd^3x+c^2d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + 3*b*c + a*d)*
sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2), -1/3*(3*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x))
 + 2*(4*b*d*x + 3*b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^
3*x + c^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(70) = 140.
time = 1.46, size = 181, normalized size = 1.97

$$-\frac{2\,b^3\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}\,+\sqrt{b^2c+(bx+a)bd-abd}\,\right|\right)}{\sqrt{bd}\,d^2|b|}-\frac{2\,\sqrt{bx+a}\,\left(\frac{4\,(b^5cd^2-ab^4d^3)(bx+a)}{bcd^3|b|-ad^4|b|}+\frac{3\,(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3)}{bcd^3|b|-ad^4|b|}\right)}{3\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2*b^3*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d^2*abs(b)) - 2/3*sqrt(b*x + a)*(4*(b^5*c*d^2 - a*b^4*d^3)*
(b*x + a)/(b*c*d^3*abs(b) - a*d^4*abs(b)) + 3*(b^6*c^2*d - 2*a*b^5*c*d^2 +
a^2*b^4*d^3)/(b*c*d^3*abs(b) - a*d^4*abs(b)))/(b^2*c + (b*x + a)*b*d - a*b*
d)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/2), x)
```

3.1516 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

```
[Out] 2/3*(b*x+a)^(3/2)/(-a*d+b*c)/(d*x+c)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx = \frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(26) = 52$.
time = 0.16, size = 88, normalized size = 2.75

method	result	size
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}}{3(dx+c)^{\frac{3}{2}}(ad-bc)}$	27
default	$-\frac{\sqrt{bx+a}}{d(dx+c)^{\frac{3}{2}}}+\frac{(-ad+bc)\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^{2}\sqrt{dx+c}}\right)}{2d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(b*x+a)^(1/2)/(d*x+c)^(3/2)+1/2*(-a*d+b*c)/d*(-2/3/(a*d-b*c)/(d*x+c)^(
3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 1.34, size = 65, normalized size = 2.03

$$\frac{2\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{3\,(bc^3-ac^2d+(bcd^2-ad^3)x^2+2\,(bc^2d-acd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)*sqrt(d*x + c)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2
+ 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [A]
time = 1.46, size = 51, normalized size = 1.59

$$\frac{2\,(bx+a)^{\frac{3}{2}}b^4d}{3\,(bcd|b|-ad^2|b|)(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)*b^4*d/((b*c*d*abs(b) - a*d^2*abs(b))*(b^2*c + (b*x + a)
*b*d - a*b*d)^(3/2))
```

Mupad [B]
time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{2\,a\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{2\,b\,x\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}\right)\,\sqrt{c+d\,x}}{x^2-\frac{3\,b\,c^3-3\,a\,c^2\,d}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{6\,c\,d\,x\,\left(a\,d-b\,c\right)}{3\,a\,d^3-3\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/2),x)
```

```
[Out] -(((2*a*(a + b*x)^(1/2))/(3*a*d^3 - 3*b*c*d^2) + (2*b*x*(a + b*x)^(1/2))/(3
*a*d^3 - 3*b*c*d^2))*(c + d*x)^(1/2))/(x^2 - (3*b*c^3 - 3*a*c^2*d)/(3*a*d^3
 - 3*b*c*d^2) + (6*c*d*x*(a*d - b*c))/(3*a*d^3 - 3*b*c*d^2))
```

3.1517 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}}+\frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}$$

```
[Out] 2/3*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(3/2)+4/3*b*(b*x+a)^(1/2)/(-a*d+b*c)^2
/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = $\{47, 37\}$

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b*x])/(3*
(b*c - a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx = \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{3(bc-ad)}$$
$$= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}} + \frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}$$

Mathematica [A]
time = 0.08, size = 46, normalized size = 0.70

$$\frac{2\sqrt{a+bx}\,(3bc-ad+2bdx)}{3(bc-ad)^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
[Out] (2*Sqrt[a + b*x]*(3*b*c - a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2))
```

Maple [A]
time = 0.16, size = 55, normalized size = 0.83

method	result	size
gosper	$-\frac{2\sqrt{bx+a}\,(-2bdx+ad-3bc)}{3(dx+c)^{\frac{3}{2}}(a^2d^2-2abcd+b^2c^2)}$	53
default	$-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^2\sqrt{dx+c}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/
(d*x+c)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see 'assume?' for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 1.35, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx+3\,bc-ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x + 3*b*c - a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*c^4 - 2*a*b*c^
3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*
d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.
time = 1.25, size = 126, normalized size = 1.91

$$\frac{2\left(\frac{2\,(bx+a)b^4d^2}{b^2c^2d|b|-2\,abcd^2|b|+a^2d^3|b|}+\frac{3\,(b^5cd-ab^4d^2)}{b^2c^2d|b|-2\,abcd^2|b|+a^2d^3|b|}\right)\sqrt{bx+a}}{3\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(b*x + a)*b^4*d^2/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a^2*d^3*a
bs(b)) + 3*(b^5*c*d - a*b^4*d^2)/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a
^2*d^3*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

Mupad [B]
time = 0.90, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,(6\,c\,b^2+2\,a\,d\,b)}{3\,d^2\,(a\,d-b\,c)^2}-\frac{2\,a^2\,d-6\,a\,b\,c}{3\,d^2\,(a\,d-b\,c)^2}+\frac{4\,b^2\,x^2}{3\,d\,(a\,d-b\,c)^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(6*b^2*c + 2*a*b*d))/(3*d^2*(a*d - b*c)^2) - (2*a^2*d
- 6*a*b*c)/(3*d^2*(a*d - b*c)^2) + (4*b^2*x^2)/(3*d*(a*d - b*c)^2)))/(x^2*(
a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(1/2))/d)
```

3.1518 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}-\frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{16bd\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(3/2)/(b*x+a)^(1/2)-8/3*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/
(d*x+c)^(3/2)-16/3*b*d*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) - (8*d*Sqrt[a + b*x])/(3*(b*
c - a*d)^2*(c + d*x)^(3/2)) - (16*b*d*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{(4d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx}{bc-ad}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{(8bd)\int \frac{1}{\sqrt{a+bx}\ldots}}{3(bc-a\ldots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}} - \frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{16bd\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+\ldots}}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.74

$$-\frac{2(a+bx)^{3/2}\left(-d^2+\frac{6bd(c+dx)}{a+bx}+\frac{3b^2(c+dx)^2}{(a+bx)^2}\right)}{3(bc-ad)^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-d^2 + (6*b*d*(c + d*x))/(a + b*x) + (3*b^2*(c + d*x)^
2)/(a + b*x)^2))/(3*(b*c - a*d)^3*(c + d*x)^(3/2))
```

Maple [A]
time = 0.16, size = 95, normalized size = 0.97

method	result	size
default	$-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4d\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^2\sqrt{dx+c}}\right)}{-ad+bc}$	95
gosper	$-\frac{2\left(-8b^2x^2d^2-4ab\,d^2x-12b^2cdx+a^2d^2-6abcd-3b^2c^2\right)}{3\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(3/2)/(b*x+a)^(1/2)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d
*x+c)^(3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(82) = 164$.
time = 1.19, size = 273, normalized size = 2.79

$$-\frac{2\,(8\,b^2d^2x^2+3\,b^2c^2+6\,abcd-a^2d^2+4\,(3\,b^2cd+abd^2)x)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^3+(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^2+(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*d^2*x^2 + 3*b^2*c^2 + 6*a*b*c*d - a^2*d^2 + 4*(3*b^2*c*d + a*b*
d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(82) = 164$.
time = 1.12, size = 373, normalized size = 3.81

$$-\frac{4\,\sqrt{bd}\,b^3}{(b^2c^2|b|-2\,abcd|b|+a^2d^2|b|)\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}-\frac{2\,\sqrt{bx+a}\left(\frac{5\,(b^6c^2d^3|b|-2\,ab^5cd^4|b|+a^2b^4d^5|b|)(bx+a)}{b^7c^5d-5\,ab^6c^4d^2+10\,a^2b^5c^3d^3-10\,a^3b^4c^2d^4+5\,a^4b^3cd^5-a^5b^2d^6}+\frac{6\,(b^7c^3d^2|b|-3\,ab^6c^2d^3|b|+3\,a^2b^5cd^4|b|-a^3b^4d^5|b|)}{b^7c^5d-5\,ab^6c^4d^2+10\,a^2b^5c^3d^3-10\,a^3b^4c^2d^4+5\,a^4b^3cd^5-a^5b^2d^6}\right)}{3\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(b*d)*b^3/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b))*(b^2
*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
```

```
)^2)) - 2/3*sqrt(b*x + a)*(5*(b^6*c^2*d^3*abs(b) - 2*a*b^5*c*d^4*abs(b) + a
^2*b^4*d^5*abs(b))*(b*x + a)/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*
d^3 - 10*a^3*b^4*c^2*d^4 + 5*a^4*b^3*c*d^5 - a^5*b^2*d^6) + 6*(b^7*c^3*d^2*
abs(b) - 3*a*b^6*c^2*d^3*abs(b) + 3*a^2*b^5*c*d^4*abs(b) - a^3*b^4*d^5*abs(
b))/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*d^3 - 10*a^3*b^4*c^2*d^4
+ 5*a^4*b^3*c*d^5 - a^5*b^2*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

Mupad [B]
time = 1.03, size = 132, normalized size = 1.35

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b^2\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{-2\,a^2\,d^2+12\,a\,b\,c\,d+6\,b^2\,c^2}{3\,d^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,b\,x\,\left(a\,d+3\,b\,c\right)}{3\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b^2*x^2)/(3*(a*d - b*c)^3) + (6*b^2*c^2 - 2*a^2*d^2 +
 12*a*b*c*d)/(3*d^2*(a*d - b*c)^3) + (8*b*x*(a*d + 3*b*c))/(3*d*(a*d - b*c)
^3)))/(x^2*(a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(
1/2))/d)
```

3.1519 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}+\frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}}+\frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3(c+dx)^{3/2}}+\frac{32bd^2\sqrt{a+bx}}{3(bc-ad)^4\sqrt{c+dx}}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(3/2)+4*d/(-a*d+b*c)^2/(d*x+c)^(3/2)/
(b*x+a)^(1/2)+16/3*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(3/2)+32/3*b*d^2*
(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/2)) + (4*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/2)) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*(c
+ d*x)^(3/2)) + (32*b*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx}{bc-ad}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{(8d^2)}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{1}{3(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}} + \frac{1}{3(bc-}$$

Mathematica [A]
time = 0.14, size = 92, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(d^3-\frac{9bd^2(c+dx)}{a+bx}-\frac{9b^2d(c+dx)^2}{(a+bx)^2}+\frac{b^3(c+dx)^3}{(a+bx)^3}\right)}{3(bc-ad)^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(d^3 - (9*b*d^2*(c + d*x))/(a + b*x) - (9*b^2*d*(c + d*
x)^2)/(a + b*x)^2 + (b^3*(c + d*x)^3)/(a + b*x)^3))/(3*(b*c - a*d)^4*(c + d
*x)^(3/2))
```

Maple [A]
time = 0.16, size = 135, normalized size = 1.00

method	result	siz
default	$-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4d\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\frac{4b\sqrt{bx+a}}{3(ad-bc)^2\sqrt{dx+c}}\right)}{-ad+bc}\right)}{-ad+bc}$	135
gosper	$-\frac{2(-16b^3x^3d^3-24d^3a\,x^2b^2-24b^3c\,d^2x^2-6a^2b\,d^3x-36a\,b^2c\,d^2x-6b^3c^2dx+a^3d^3-9a^2bc\,d^2-9a\,b^2c^2d+b^3c^3)}{3(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}$	169

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(3/2)-2*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(
d*x+c)^(3/2)/(b*x+a)^(1/2)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*
x+a)^(1/2)+4/3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(113) = 226.
time = 1.66, size = 447, normalized size = 3.31

$$\frac{2\,(16\,b^3d^3x^3-b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2-a^3d^3+24\,(b^3cd^2+ab^2d^3)x^2+6\,(b^3c^2d+6\,ab^2cd^2+a^2bd^3)x)\sqrt{bx+a}\,\sqrt{dx+c}}{3\,(a^2b^4c^6-4\,a^3b^3c^5d+6\,a^4b^2c^4d^2-4\,a^5bc^3d^3+a^6c^2d^4+(b^6c^4d^2-4\,ab^5c^3d^3+6\,a^2b^4c^2d^4-4\,a^3b^3cd^5+a^4b^2d^6)x^4+2\,(b^6c^5d-3\,ab^5c^4d^2+2\,a^2b^4c^3d^3+2\,a^3b^3c^2d^4-3\,a^4b^2cd^5+a^5bd^6)x^3+(b^6c^6-9\,a^2b^4c^4d^2+16\,a^3b^3c^3d^3-9\,a^4b^2c^2d^4+a^6d^6)x^2+2\,(ab^5c^6-3\,a^2b^4c^5d+2\,a^3b^3c^4d^2+2\,a^4b^2c^3d^3-3\,a^5bc^2d^4+a^6cd^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*d^3*x^3 - b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - a^3*d^3 + 2
4*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(b^3*c^2*d + 6*a*b^2*c*d^2 + a^2*b*d^3)*x
)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^
4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*
a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b
^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*
b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(113) = 226.
time = 1.88, size = 670, normalized size = 4.96

$$\frac{2\,\sqrt{bx+a}\left(\frac{8\,(b^7c^3d^4|b|-3\,ab^6c^2d^5|b|+3\,a^2b^5cd^6|b|-a^3b^4d^7|b|)(bx+a)}{b^9c^7d-7\,ab^8c^6d^2+21\,a^2b^7c^5d^3-35\,a^3b^6c^4d^4+35\,a^4b^5c^3d^5-21\,a^5b^4c^2d^6+7\,a^6b^3cd^7-a^7b^2d^8}+\frac{9\,(b^8c^4d^3|b|-4\,ab^7c^3d^4|b|+6\,a^2b^6c^2d^5|b|-4\,a^3b^5cd^6|b|+a^4b^4d^7|b|)}{b^9c^7d-7\,ab^8c^6d^2+21\,a^2b^7c^5d^3-35\,a^3b^6c^4d^4+35\,a^4b^5c^3d^5-21\,a^5b^4c^2d^6+7\,a^6b^3cd^7-a^7b^2d^8}\right)}{3\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}+\frac{8\left(4\,\sqrt{bd}\,b^7c^2d-8\,\sqrt{bd}\,ab^6cd^2+4\,\sqrt{bd}\,a^2b^5d^3-9\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^5cd+9\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^4d^2+3\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^3d\right)}{3\,(b^3c^3|b|-3\,ab^2c^2d|b|+3\,a^2bcd^2|b|-a^3d^3|b|)\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b^7*c^3*d^4*abs(b) - 3*a*b^6*c^2*d^5*abs(b) + 3*a^2*b
^5*c*d^6*abs(b) - a^3*b^4*d^7*abs(b))*(b*x + a)/(b^9*c^7*d - 7*a*b^8*c^6*d^
2 + 21*a^2*b^7*c^5*d^3 - 35*a^3*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b
^4*c^2*d^6 + 7*a^6*b^3*c*d^7 - a^7*b^2*d^8) + 9*(b^8*c^4*d^3*abs(b) - 4*a*b
^7*c^3*d^4*abs(b) + 6*a^2*b^6*c^2*d^5*abs(b) - 4*a^3*b^5*c*d^6*abs(b) + a^4
*b^4*d^7*abs(b))/(b^9*c^7*d - 7*a*b^8*c^6*d^2 + 21*a^2*b^7*c^5*d^3 - 35*a^3
*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b^4*c^2*d^6 + 7*a^6*b^3*c*d^7 -
a^7*b^2*d^8))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 8/3*(4*sqrt(b*d)*b^7*
c^2*d - 8*sqrt(b*d)*a*b^6*c*d^2 + 4*sqrt(b*d)*a^2*b^5*d^3 - 9*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c*d + 9*
sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
*a*b^4*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^3*d)/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c*
d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3)
```

Mupad [B]
time = 1.29, size = 224, normalized size = 1.66

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b\,x^2\,\left(a\,d+b\,c\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{2\,a^3\,d^3-18\,a^2\,b\,c\,d^2-18\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{3\,b\,d^2\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b^2\,d\,x^3}{3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{d\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a\,c^2\,\sqrt{a+b\,x}}{b\,d^2}+\frac{x^2\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{c\,x\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b*x^2*(a*d + b*c))/(a*d - b*c)^4 - (2*a^3*d^3 + 2*b^3
*c^3 - 18*a*b^2*c^2*d - 18*a^2*b*c*d^2)/(3*b*d^2*(a*d - b*c)^4) + (32*b^2*d
*x^3)/(3*(a*d - b*c)^4) + (4*x*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(d*(a*d - b
*c)^4)))/(x^3*(a + b*x)^(1/2) + (a*c^2*(a + b*x)^(1/2))/(b*d^2) + (x^2*(a*d
 + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (c*x*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d
^2))
```

3.1520 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}+\frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}-\frac{32d^2}{5(bc-ad)^3\sqrt{a+bx}\,(c+dx)^{3/2}}-$$

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(3/2)+16/15*d/(-a*d+b*c)^2/(b*x+a)^(3
/2)/(d*x+c)^(3/2)-32/5*d^2/(-a*d+b*c)^3/(d*x+c)^(3/2)/(b*x+a)^(1/2)-128/15*
d^3*(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(3/2)-256/15*b*d^3*(b*x+a)^(1/2)/(-a
*d+b*c)^5/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}\,(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2)) + (16*d)/(15*(b*c - a*d)
^2*(a + b*x)^(3/2)*(c + d*x)^(3/2)) - (32*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*
x]*(c + d*x)^(3/2)) - (128*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^4*(c + d*x)^(
3/2)) - (256*b*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} + \frac{(1}{}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{}{5(}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{}{5(}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}} - \frac{}{5(}$$

Mathematica [A]

time = 0.16, size = 117, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(-5d^4+\frac{60bd^3(c+dx)}{a+bx}+\frac{90b^2d^2(c+dx)^2}{(a+bx)^2}-\frac{20b^3d(c+dx)^3}{(a+bx)^3}+\frac{3b^4(c+dx)^4}{(a+bx)^4}\right)}{15(bc-ad)^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-5*d^4 + (60*b*d^3*(c + d*x))/(a + b*x) + (90*b^2*d^2*
(c + d*x)^2)/(a + b*x)^2 - (20*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (3*b^4*(c +
 d*x)^4)/(a + b*x)^4))/(15*(b*c - a*d)^5*(c + d*x)^(3/2))
```

Maple [A]

time = 0.16, size = 175, normalized size = 1.02

method	result
default	$-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}-\frac{8d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}-\frac{4d\left(-\frac{2\sqrt{bx+a}}{3(ad-bc)(dx+c)^{\frac{3}{2}}}+\right.}{-ad}\right.}{-ad+bc}\right.}{5(-ad+bc)}$

gosper	$-\frac{2(-128d^4x^4b^4-320a\,b^3d^4x^3-192b^4c\,d^3x^3-240a^2b^2d^4x^2-480a\,b^3c\,d^3x^2-48b^4c^2d^2x^2-40a^3b\,d^4x-360a^2b^2c\,d^3x-120a\,b^3c^2d^2x+}{15(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/(-a*d+b*c)/(b*x+a)^(5/2)/(d*x+c)^(3/2)-8/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*
c)/(b*x+a)^(3/2)/(d*x+c)^(3/2)-2*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(d*x+c)^(3/2)/
(b*x+a)^(1/2)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*x+a)^(1/2)+4/
3*b/(a*d-b*c)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(142) = 284.
time = 3.27, size = 715, normalized size = 4.16

$$-\frac{2\,(128\,b^4d^4x^4+3\,b^4c^4-20\,ab^3c^3d+90\,a^2b^2c^2d^2+60\,a^3bcd^3-5\,a^4d^4+64\,(3\,b^4cd^3+5\,ab^3d^4)x^3+48\,(b^4c^2d^2+10\,ab^3cd^3+5\,a^2b^2d^4)x^2-8\,(b^4c^3d-15\,ab^3c^2d^2-45\,a^2b^2cd^3-5\,a^3bd^4)x)\sqrt{bx+a}\,\sqrt{dx+c}}{15\,(a^3b^5c^7-5\,a^4b^4c^6d+10\,a^5b^3c^5d^2-10\,a^6b^2c^4d^3+5\,a^7bc^3d^4-a^8c^2d^5+(b^8c^5d^2-5\,ab^7c^4d^3+10\,a^2b^6c^3d^4-10\,a^3b^5c^2d^5+5\,a^4b^4cd^6-a^5b^3d^7)x^5+(2\,b^8c^6d-7\,ab^7c^5d^2+5\,a^2b^6c^4d^3+10\,a^3b^5c^3d^4-20\,a^4b^4c^2d^5+13\,a^5b^3cd^6-3\,a^6b^2d^7)x^4+(b^8c^7+ab^7c^6d-17\,a^2b^6c^5d^2+35\,a^3b^5c^4d^3-25\,a^4b^4c^3d^4-a^5b^3c^2d^5+9\,a^6b^2cd^6-3\,a^7bd^7)x^3+(3\,ab^7c^7-9\,a^2b^6c^6d+a^3b^5c^5d^2+25\,a^4b^4c^4d^3-35\,a^5b^3c^3d^4+17\,a^6b^2c^2d^5-a^7bcd^6-a^8d^7)x^2+(3\,a^2b^6c^7-13\,a^3b^5c^6d+20\,a^4b^4c^5d^2-10\,a^5b^3c^4d^3-5\,a^6b^2c^3d^4+7\,a^7bc^2d^5-2\,a^8cd^6)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*d^4*x^4 + 3*b^4*c^4 - 20*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 +
60*a^3*b*c*d^3 - 5*a^4*d^4 + 64*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 48*(b^4*c
^2*d^2 + 10*a*b^3*c*d^3 + 5*a^2*b^2*d^4)*x^2 - 8*(b^4*c^3*d - 15*a*b^3*c^2*
d^2 - 45*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b
^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*
b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d
^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d
 - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^
2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17
*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^
5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a
^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d
```

```
^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^
4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 -
2*a^8*c*d^6)*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(5/2)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. 2(142) = 284.
time = 2.91, size = 1203, normalized size = 6.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + a)*(11*(b^8*c^4*d^5*abs(b) - 4*a*b^7*c^3*d^6*abs(b) + 6*a^2
*b^6*c^2*d^7*abs(b) - 4*a^3*b^5*c*d^8*abs(b) + a^4*b^4*d^9*abs(b))*(b*x + a
)/(b^11*c^9*d - 9*a*b^10*c^8*d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4
+ 126*a^4*b^7*c^5*d^5 - 126*a^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b
^4*c^2*d^8 + 9*a^8*b^3*c*d^9 - a^9*b^2*d^10) + 12*(b^9*c^5*d^4*abs(b) - 5*a
*b^8*c^4*d^5*abs(b) + 10*a^2*b^7*c^3*d^6*abs(b) - 10*a^3*b^6*c^2*d^7*abs(b)
 + 5*a^4*b^5*c*d^8*abs(b) - a^5*b^4*d^9*abs(b))/(b^11*c^9*d - 9*a*b^10*c^8*
d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4 + 126*a^4*b^7*c^5*d^5 - 126*a
^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b^4*c^2*d^8 + 9*a^8*b^3*c*d^9
- a^9*b^2*d^10))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 4/15*(73*sqrt(b*d)
*b^11*c^4*d^2 - 292*sqrt(b*d)*a*b^10*c^3*d^3 + 438*sqrt(b*d)*a^2*b^9*c^2*d^
4 - 292*sqrt(b*d)*a^3*b^8*c*d^5 + 73*sqrt(b*d)*a^4*b^7*d^6 - 320*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^9*c^3*d
^2 + 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^8*c^2*d^3 - 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^7*c*d^4 + 320*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^6*d^5 + 490*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^7*
c^2*d^2 - 980*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^6*c*d^3 + 490*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^5*d^4 - 240*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^5*c*d^2 + 240*sqrt(b
```

```
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^4
*d^3 + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^8*b^3*d^2)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^2*c^2
*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

Mupad [B]

time = 1.53, size = 346, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,x^2\,\left(5\,a^2\,d^2+10\,a\,b\,c\,d+b^2\,c^2\right)}{5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,b^2\,d^2\,x^4}{15\,{\left(a\,d-b\,c\right)}^5}+\frac{-10\,a^4\,d^4+120\,a^3\,b\,c\,d^3+180\,a^2\,b^2\,c^2\,d^2-40\,a\,b^3\,c^3\,d+6\,b^4\,c^4}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(80\,a^3\,b\,d^4+720\,a^2\,b^2\,c\,d^3+240\,a\,b^3\,c^2\,d^2-16\,b^4\,c^3\,d\right)}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,b\,d\,x^3\,\left(5\,a\,d+3\,b\,c\right)}{15\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{2\,x^3\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^2\,c^2\,\sqrt{a+b\,x}}{b^2\,d^2}+\frac{2\,a\,c\,x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(5*a^2*d^2 + b^2*c^2 + 10*a*b*c*d))/(5*(a*d - b*c
)^5) + (256*b^2*d^2*x^4)/(15*(a*d - b*c)^5) + (6*b^4*c^4 - 10*a^4*d^4 + 180
*a^2*b^2*c^2*d^2 - 40*a*b^3*c^3*d + 120*a^3*b*c*d^3)/(15*b^2*d^2*(a*d - b*c
)^5) + (x*(80*a^3*b*d^4 - 16*b^4*c^3*d + 240*a*b^3*c^2*d^2 + 720*a^2*b^2*c*
d^3))/(15*b^2*d^2*(a*d - b*c)^5) + (128*b*d*x^3*(5*a*d + 3*b*c))/(15*(a*d -
 b*c)^5)))/(x^4*(a + b*x)^(1/2) + (x^2*(a + b*x)^(1/2)*(a^2*d^2 + b^2*c^2 +
 4*a*b*c*d))/(b^2*d^2) + (2*x^3*(a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a^2*c
^2*(a + b*x)^(1/2))/(b^2*d^2) + (2*a*c*x*(a*d + b*c)*(a + b*x)^(1/2))/(b^2*
d^2))
```

3.1521 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}+\frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}}-\frac{32d^2}{21(bc-ad)^3(a+bx)^{3/2}(c+dx)^{3/2}}+$$

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(3/2)+4/7*d/(-a*d+b*c)^2/(b*x+a)^(5/2
)/(d*x+c)^(3/2)-32/21*d^2/(-a*d+b*c)^3/(b*x+a)^(3/2)/(d*x+c)^(3/2)+64/7*d^3
/(-a*d+b*c)^4/(d*x+c)^(3/2)/(b*x+a)^(1/2)+256/21*d^4*(b*x+a)^(1/2)/(-a*d+b*
c)^5/(d*x+c)^(3/2)+512/21*b*d^4*(b*x+a)^(1/2)/(-a*d+b*c)^6/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}\,(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{7(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2)) + (4*d)/(7*(b*c - a*d)^2
*(a + b*x)^(5/2)*(c + d*x)^(3/2)) - (32*d^2)/(21*(b*c - a*d)^3*(a + b*x)^(3
/2)*(c + d*x)^(3/2)) + (64*d^3)/(7*(b*c - a*d)^4*Sqrt[a + b*x]*(c + d*x)^(3
/2)) + (256*d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^5*(c + d*x)^(3/2)) + (512*b*
d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} - \frac{(10d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} + \cdots$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \cdots$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \cdots$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \cdots$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \cdots$$

Mathematica [A]
time = 0.15, size = 139, normalized size = 0.67

$$-\frac{2(a+bx)^{3/2}\left(7d^5 - \frac{105bd^4(c+dx)}{a+bx} - \frac{210b^2d^3(c+dx)^2}{(a+bx)^2} + \frac{70b^3d^2(c+dx)^3}{(a+bx)^3} - \frac{21b^4d(c+dx)^4}{(a+bx)^4} + \frac{3b^5(c+dx)^5}{(a+bx)^5}\right)}{21(bc-ad)^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(7*d^5 - (105*b*d^4*(c + d*x))/(a + b*x) - (210*b^2*d^3
*(c + d*x)^2)/(a + b*x)^2 + (70*b^3*d^2*(c + d*x)^3)/(a + b*x)^3 - (21*b^4*
d*(c + d*x)^4)/(a + b*x)^4 + (3*b^5*(c + d*x)^5)/(a + b*x)^5))/(21*(b*c - a
*d)^6*(c + d*x)^(3/2))
```

Maple [A]
time = 0.18, size = 215, normalized size = 1.04

method	result

default	$-\frac{2}{7(-ad+bc)(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}}-\frac{10d\left(-\frac{2}{5(-ad+bc)(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{3}{2}}}-\frac{8d\left(-\frac{2}{3(-ad+bc)(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}-\frac{2d\left(-\frac{2}{(-ad+bc)(dx+c)^{\frac{3}{2}}\sqrt{b}}\right.}{}\right.}{5(-}\right.}{7(-ad+bc)}$
gosper	$-\frac{2(-256b^5d^5x^5-896a\,b^4d^5x^4-384b^5c\,d^4x^4-1120a^2b^3d^5x^3-1344a\,b^4c\,d^4x^3-96b^5c^2d^3x^3-560a^3b^2d^5x^2-1680a^2b^3c\,d^4x^2-336a\,b^4c^2}{21(bx+a)^{\frac{7}{2}}(dx+c)^{\frac{3}{2}}(a^6d^6-6a^5bc\,d^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/(-a*d+b*c)/(b*x+a)^(7/2)/(d*x+c)^(3/2)-10/7*d/(-a*d+b*c)*(-2/5/(-a*d+b
*c)/(b*x+a)^(5/2)/(d*x+c)^(3/2)-8/5*d/(-a*d+b*c)*(-2/3/(-a*d+b*c)/(b*x+a)^(
3/2)/(d*x+c)^(3/2)-2*d/(-a*d+b*c)*(-2/(-a*d+b*c)/(d*x+c)^(3/2)/(b*x+a)^(1/2
)-4*d/(-a*d+b*c)*(-2/3/(a*d-b*c)/(d*x+c)^(3/2)*(b*x+a)^(1/2)+4/3*b/(a*d-b*c
)^2*(b*x+a)^(1/2)/(d*x+c)^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. $2(171) = 342$.
time = 6.93, size = 999, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 2/21*(256*b^5*d^5*x^5 - 3*b^5*c^5 + 21*a*b^4*c^4*d - 70*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 105*a^4*b*c*d^4 - 7*a^5*d^5 + 128*(3*b^5*c*d^4 + 7*a*b^4*d^5)*x^4 + 32*(3*b^5*c^2*d^3 + 42*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 16*(b^5*c^3*d^2 - 21*a*b^4*c^2*d^3 - 105*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2 + 2*(3*b^5*c^4*d - 28*a*b^4*c^3*d^2 + 210*a^2*b^3*c^2*d^3 + 420*a^3*b^2*c*d^4 + 35*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^6*c^8 - 6*a^5*b^5*c^7*d + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4 - 6*a^9*b*c^3*d^5 + a^10*c^2*d^6 + (b^10*c^6*d^2 - 6*a*b^9*c^5*d^3 + 15*a^2*b^8*c^4*d^4 - 20*a^3*b^7*c^3*d^5 + 15*a^4*b^6*c^2*d^6 - 6*a^5*b^5*c*d^7 + a^6*b^4*d^8)*x^6 + 2*(b^10*c^7*d - 4*a*b^9*c^6*d^2 + 3*a^2*b^8*c^5*d^3 + 10*a^3*b^7*c^4*d^4 - 25*a^4*b^6*c^3*d^5 + 24*a^5*b^5*c^2*d^6 - 11*a^6*b^4*c*d^7 + 2*a^7*b^3*d^8)*x^5 + (b^10*c^8 + 2*a*b^9*c^7*d - 27*a^2*b^8*c^6*d^2 + 64*a^3*b^7*c^5*d^3 - 55*a^4*b^6*c^4*d^4 - 6*a^5*b^5*c^3*d^5 + 43*a^6*b^4*c^2*d^6 - 28*a^7*b^3*c*d^7 + 6*a^8*b^2*d^8)*x^4 + 4*(a*b^9*c^8 - 3*a^2*b^8*c^7*d - 2*a^3*b^7*c^6*d^2 + 19*a^4*b^6*c^5*d^3 - 30*a^5*b^5*c^4*d^4 + 19*a^6*b^4*c^3*d^5 - 2*a^7*b^3*c^2*d^6 - 3*a^8*b^2*c*d^7 + a^9*b*d^8)*x^3 + (6*a^2*b^8*c^8 - 28*a^3*b^7*c^7*d + 43*a^4*b^6*c^6*d^2 - 6*a^5*b^5*c^5*d^3 - 55*a^6*b^4*c^4*d^4 + 64*a^7*b^3*c^3*d^5 - 27*a^8*b^2*c^2*d^6 + 2*a^9*b*c*d^7 + a^10*d^8)*x^2 + 2*(2*a^3*b^7*c^8 - 11*a^4*b^6*c^7*d + 24*a^5*b^5*c^6*d^2 - 25*a^6*b^4*c^5*d^3 + 10*a^7*b^3*c^4*d^4 + 3*a^8*b^2*c^3*d^5 - 4*a^9*b*c^2*d^6 + a^10*c*d^7)*x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(5/2),x)

[Out] Integral(1/((a + b*x)**(9/2)*(c + d*x)**(5/2)), x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. 2(171) = 342.
time = 2.88, size = 1964, normalized size = 9.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")

```
[Out] 2/3*sqrt(b*x + a)*(14*(b^9*c^5*d^6*abs(b) - 5*a*b^8*c^4*d^7*abs(b) + 10*a^2
*b^7*c^3*d^8*abs(b) - 10*a^3*b^6*c^2*d^9*abs(b) + 5*a^4*b^5*c*d^10*abs(b) -
 a^5*b^4*d^11*abs(b))*(b*x + a)/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*
b^11*c^9*d^3 - 165*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6
*d^6 + 462*a^6*b^7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55
*a^9*b^4*c^2*d^10 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12) + 15*(b^10*c^6*d^5*
abs(b) - 6*a*b^9*c^5*d^6*abs(b) + 15*a^2*b^8*c^4*d^7*abs(b) - 20*a^3*b^7*c^
3*d^8*abs(b) + 15*a^4*b^6*c^2*d^9*abs(b) - 6*a^5*b^5*c*d^10*abs(b) + a^6*b^
4*d^11*abs(b))/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*b^11*c^9*d^3 - 16
5*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6*d^6 + 462*a^6*b^
7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55*a^9*b^4*c^2*d^10
 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/
2) + 8/21*(79*sqrt(b*d)*b^15*c^6*d^3 - 474*sqrt(b*d)*a*b^14*c^5*d^4 + 1185*
sqrt(b*d)*a^2*b^13*c^4*d^5 - 1580*sqrt(b*d)*a^3*b^12*c^3*d^6 + 1185*sqrt(b*
d)*a^4*b^11*c^2*d^7 - 474*sqrt(b*d)*a^5*b^10*c*d^8 + 79*sqrt(b*d)*a^6*b^9*d
^9 - 511*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*b^13*c^5*d^3 + 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^12*c^4*d^4 - 5110*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^11*c^3*d^5 + 51
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^3*b^10*c^2*d^6 - 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a^4*b^9*c*d^7 + 511*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^8*d^8 + 1344*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^11*
c^4*d^3 - 5376*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a*b^10*c^3*d^4 + 8064*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^9*c^2*d^5 - 5376*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^8*c*d^
6 + 1344*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^7*d^7 - 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*b^9*c^3*d^3 + 5250*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^8*c^2*d^4 - 5250*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2
*b^7*c*d^5 + 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^6*a^3*b^6*d^6 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^7*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^6*c*d^4 + 10
15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^8*a^2*b^5*d^5 - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^10*b^5*c*d^3 + 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^4*d^4 + 42*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^3*d^3)/((b^5*c
^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) - 10*a^3*b^2*c
^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c - a*b*d - (sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7)
```

Mupad [B]

time = 1.91, size = 478, normalized size = 2.31

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,x^2\,\left(35\,a^3\,d^3+105\,a^2\,b\,c\,d^2+21\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{21\,b\,{\left(a\,d-b\,c\right)}^6}-\frac{14\,a^5\,d^5-210\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+140\,a^2\,b^3\,c^3\,d^2-42\,a\,b^4\,c^4\,d+6\,b^5\,c^5}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d\,x^3\,\left(35\,a^2\,d^2+42\,a\,b\,c\,d+3\,b^2\,c^2\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b^2\,d^3\,x^5}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,b\,d^2\,x^4\,\left(7\,a\,d+3\,b\,c\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(140\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4+840\,a^2\,b^3\,c^2\,d^3-112\,a\,b^4\,c^3\,d^2+12\,b^5\,c^4\,d\right)}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{x^3\,\sqrt{a+b\,x}\,\left(3\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{x^4\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^3\,c^2\,\sqrt{a+b\,x}}{b^3\,d^2}+\frac{a\,x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+6\,a\,b\,c\,d+3\,b^2\,c^2\right)}{b^3\,d^2}+\frac{a^2\,c\,x\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(35*a^3*d^3 - b^3*c^3 + 21*a*b^2*c^2*d + 105*a^2*
b*c*d^2))/(21*b*(a*d - b*c)^6) - (14*a^5*d^5 + 6*b^5*c^5 + 140*a^2*b^3*c^3*
d^2 - 420*a^3*b^2*c^2*d^3 - 42*a*b^4*c^4*d - 210*a^4*b*c*d^4)/(21*b^3*d^2*(
a*d - b*c)^6) + (64*d*x^3*(35*a^2*d^2 + 3*b^2*c^2 + 42*a*b*c*d))/(21*(a*d -
 b*c)^6) + (512*b^2*d^3*x^5)/(21*(a*d - b*c)^6) + (256*b*d^2*x^4*(7*a*d + 3
*b*c))/(21*(a*d - b*c)^6) + (x*(140*a^4*b*d^5 + 12*b^5*c^4*d - 112*a*b^4*c^
3*d^2 + 1680*a^3*b^2*c*d^4 + 840*a^2*b^3*c^2*d^3))/(21*b^3*d^2*(a*d - b*c)^
6)))/(x^5*(a + b*x)^(1/2) + (x^3*(a + b*x)^(1/2)*(3*a^2*d^2 + b^2*c^2 + 6*a
*b*c*d))/(b^2*d^2) + (x^4*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (a^3*c^2
*(a + b*x)^(1/2))/(b^3*d^2) + (a*x^2*(a + b*x)^(1/2)*(a^2*d^2 + 3*b^2*c^2 +
 6*a*b*c*d))/(b^3*d^2) + (a^2*c*x*(2*a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d^2
))
```

3.1522 $\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x+a)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 1.37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{4+a+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*x]/Sqrt[4 + a + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(15) = 30$.
time = 0.17, size = 86, normalized size = 4.53

method	result	size
default	$\frac{\sqrt{(bx+a)(bx+a+4)}\,\ln\left(\frac{\frac{ab}{2}+\frac{b(a+4)}{2}+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+(ab+b(a+4))x+a(a+4)}\right)}{\sqrt{bx+a}\,\sqrt{bx+a+4}\,\sqrt{b^2}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+a)*(b*x+a+4))^(1/2)/(b*x+a)^(1/2)/(b*x+a+4)^(1/2)*ln((1/2*a*b+1/2*b*(
a+4)+b^2*x)/(b^2)^(1/2)+(x^2*b^2+(a*b+b*(a+4))*x+a*(a+4))^(1/2))/(b^2)^(1/2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.
time = 0.29, size = 48, normalized size = 2.53

$$\frac{\log\left(2\,b^2x+2\,ab+2\,\sqrt{b^2x^2+a^2+2\,(ab+2\,b)x+4\,a}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + a^2 + 2*(a*b + 2*b)*x + 4*a)*b + 4*b
)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 0.75, size = 31, normalized size = 1.63

$$-\frac{\log\left(-bx+\sqrt{bx+a+4}\,\sqrt{bx+a}\,-a-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + a + 4)*sqrt(b*x + a) - a - 2)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{a+bx+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*x+a+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(a + b*x + 4)), x)
```

Giac [A]
time = 2.40, size = 24, normalized size = 1.26

$$-\frac{2\log\left(\sqrt{bx+a+4}-\sqrt{bx+a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + a + 4) - sqrt(b*x + a))/b
```

Mupad [B]
time = 0.31, size = 50, normalized size = 2.63

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a+4}-\sqrt{a+b\,x+4}\right)}{\sqrt{-b^2}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(a + b*x + 4)^(1/2)),x)
```

```
[Out] (4*atan((b*((a + 4)^(1/2) - (a + b*x + 4)^(1/2)))/((-b^2)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(-b^2)^(1/2)
```

3.1523 $\int \frac{1}{\sqrt{2+bx}\sqrt{6+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}$$

[Out] 2*arcsinh(1/2*(b*x+2)^(1/2))/b

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]

[Out] (2*ArcSinh[Sqrt[2 + b*x]/2])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\sqrt{6+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{6+bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[6 + b*x]/Sqrt[2 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(15) = 30.
time = 0.17, size = 66, normalized size = 3.47

method	result	size
default	$\frac{\sqrt{(bx+2)(bx+6)}\,\ln\left(\frac{b^2x+4b}{\sqrt{b^2}}+\sqrt{x^2b^2+8bx+12}\right)}{\sqrt{bx+2}\,\sqrt{bx+6}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+2)*(b*x+6))^(1/2)/(b*x+2)^(1/2)/(b*x+6)^(1/2)*ln((b^2*x+4*b)/(b^2)^(1
/2)+(b^2*x^2+8*b*x+12)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.29, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+8\,bx+12}\,b+8\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 8*b*x + 12)*b + 8*b)/b
```

Fricas [A]
time = 0.79, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+6}\,\sqrt{bx+2}-4\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 6)*sqrt(b*x + 2) - 4)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+2}\,\sqrt{bx+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+6)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 6)), x)
```

Giac [A]
time = 1.23, size = 23, normalized size = 1.21

$$-\frac{2\,\log\left(\sqrt{bx+6}-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 6) - sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.34, size = 47, normalized size = 2.47

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{6}-\sqrt{b\,x+6}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 6)^(1/2)),x)
```

```
[Out] -(4*atan((b*(6^(1/2) - (b*x + 6)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

3.1524 $\int \frac{1}{\sqrt{1+bx}\sqrt{5+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}$$

[Out] 2*arcsinh(1/2*(b*x+1)^(1/2))/b

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]

[Out] (2*ArcSinh[Sqrt[1 + b*x]/2])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{5+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{5+bx}}{\sqrt{1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[5 + b*x]/Sqrt[1 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(15) = 30$.
time = 0.16, size = 66, normalized size = 3.47

method	result	size
default	$\frac{\sqrt{(bx+1)(bx+5)}\,\ln\left(\frac{b^2x+3b}{\sqrt{b^2}}+\sqrt{x^2b^2+6bx+5}\right)}{\sqrt{bx+1}\,\sqrt{bx+5}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+1)*(b*x+5))^(1/2)/(b*x+1)^(1/2)/(b*x+5)^(1/2)*ln((b^2*x+3*b)/(b^2)^(1
/2)+(b^2*x^2+6*b*x+5)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.30, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+6\,bx+5}\,b+6\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 6*b*x + 5)*b + 6*b)/b
```

Fricas [A]
time = 0.85, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+5}\,\sqrt{bx+1}-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 5)*sqrt(b*x + 1) - 3)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+1}\,\sqrt{bx+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+5)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 5)), x)
```

Giac [A]
time = 0.92, size = 23, normalized size = 1.21

$$-\frac{2\,\log\left(\sqrt{bx+5}-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 5) - sqrt(b*x + 1))/b
```

Mupad [B]
time = 0.33, size = 43, normalized size = 2.26

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{5}-\sqrt{b\,x+5}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 5)^(1/2)),x)
```

```
[Out] (4*atan((b*(5^(1/2) - (b*x + 5)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

3.1525 $\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=17

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.
time = 0.04, size = 42, normalized size = 2.47

$$-\frac{2\sqrt{x}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{4+bx}\right)}{\sqrt{b}\,\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
[Out] (-2*Sqrt[x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[4 + b*x]])/(Sqrt[b]*Sqrt[b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(13) = 26.
time = 0.14, size = 60, normalized size = 3.53

method	result	size
meijerg	$\frac{2\sqrt{x}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}\,\sqrt{bx}}$	23
default	$\frac{\sqrt{bx\,(bx+4)}\,\ln\left(\frac{b^{2}x+2b}{\sqrt{b^{2}}}+\sqrt{x^{2}b^{2}+4bx}\right)}{\sqrt{bx}\,\sqrt{bx+4}\,\sqrt{b^{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+4)^(1/2),x,method=_RETURNVERBOSE)
[Out] (b*x*(b*x+4))^(1/2)/(b*x)^(1/2)/(b*x+4)^(1/2)*ln((b^2*x+2*b)/(b^2)^(1/2)+(b
^2*x^2+4*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.
time = 0.29, size = 32, normalized size = 1.88

$$\frac{\log\left(2\,b^{2}x+2\,\sqrt{b^{2}x^{2}+4\,bx}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 4*b*x)*b + 4*b)/b
```

Fricas [A]
time = 0.80, size = 25, normalized size = 1.47

$$-\frac{\log\left(-bx+\sqrt{bx+4}\,\sqrt{bx}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x) - 2)/b
```

Sympy [A]
time = 0.57, size = 15, normalized size = 0.88

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/2)/b
```

Giac [A]
time = 2.60, size = 21, normalized size = 1.24

$$-\frac{2\log\left(\sqrt{bx+4}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x))/b
```

Mupad [B]
time = 0.31, size = 33, normalized size = 1.94

$$-\frac{4\operatorname{atan}\left(\frac{b\left(\sqrt{b\,x+4}-2\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x + 4)^(1/2) - 2))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)^(1/2
)
```

3.1526 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{3+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x-1)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\,\sqrt{3+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+bx}}{\sqrt{-1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[-1 + b*x]])/b
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.
time = 0.16, size = 64, normalized size = 3.37

method	result	size
default	$\frac{\sqrt{(bx-1)(bx+3)}\,\ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{x^2b^2+2bx-3}\right)}{\sqrt{bx-1}\,\sqrt{bx+3}\,\sqrt{b^2}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-1)*(b*x+3))^(1/2)/(b*x-1)^(1/2)/(b*x+3)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2
)+(b^2*x^2+2*b*x-3)^(1/2))/(b^2)^(1/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.
time = 0.28, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+2\,bx-3}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x - 3)*b + 2*b)/b
```

Fricas [**A**]
time = 0.74, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+3}\,\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 3)*sqrt(b*x - 1) - 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-1}\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+3)**(1/2),x)
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 3)), x)
```

Giac [A]
time = 1.25, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+3}-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x - 1))/b
```

Mupad [B]
time = 0.32, size = 44, normalized size = 2.32

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{3}-\sqrt{b\,x+3}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 3)^(1/2)),x)
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((3^(1/2) - (b*x + 3)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

3.1527 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] arccosh(1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{54\}$

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.04, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(9) = 18$.
time = 0.16, size = 57, normalized size = 5.18

method	result	size
default	$\frac{\sqrt{(bx-2)(bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.27, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.84, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 13.71, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{4e^{2i\pi}}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{4}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.
time = 1.52, size = 23, normalized size = 2.09

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

Mupad [B]
time = 0.30, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{b\,x-2}+\sqrt{2}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1528 $\int \frac{1}{\sqrt{-3+bx}\,\sqrt{1+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x-3)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/2])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\,\sqrt{1+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{1+bx}}{\sqrt{-3+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[1 + b*x]/Sqrt[-3 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(15) = 30.
time = 0.16, size = 66, normalized size = 3.47

method	result	size
default	$\frac{\sqrt{(bx-3)(bx+1)}\,\ln\left(\frac{b^2x-b}{\sqrt{b^2}}+\sqrt{x^2b^2-2bx-3}\right)}{\sqrt{bx-3}\,\sqrt{bx+1}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-3)*(b*x+1))^(1/2)/(b*x-3)^(1/2)/(b*x+1)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2
)+(b^2*x^2-2*b*x-3)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.
time = 0.29, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-2\,bx-3}\,b-2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x - 3)*b - 2*b)/b
```

Fricas [A]
time = 0.63, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+1}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 1)*sqrt(b*x - 3) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 1)), x)
```

Giac [A]
time = 1.19, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+1}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 1) - sqrt(b*x - 3))/b
```

Mupad [B]
time = 0.29, size = 46, normalized size = 2.42

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x - 3)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1
/2))))/(-b^2)^(1/2)
```

3.1529 $\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b}$$

```
[Out] 2*arcsinh((b*x+2)^(1/2))/b
```

Rubi [A]

time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[2 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(13) = 26$.
time = 0.16, size = 66, normalized size = 4.40

method	result	size
default	$\frac{\sqrt{(bx+2)(bx+3)}\,\ln\left(\frac{\frac{5}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+5bx+6}\right)}{\sqrt{bx+2}\,\sqrt{bx+3}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+2)*(b*x+3))^(1/2)/(b*x+2)^(1/2)/(b*x+3)^(1/2)*ln((5/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+5*b*x+6)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.
time = 0.29, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+5\,bx+6}\,b+5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 5*b*x + 6)*b + 5*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 1.11, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+3}\,\sqrt{bx+2}\,-5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 3)*sqrt(b*x + 2) - 5)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+2}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 3)), x)
```

Giac [A]
time = 1.18, size = 23, normalized size = 1.53

$$-\frac{2\log\left(\sqrt{bx+3}-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.29, size = 47, normalized size = 3.13

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{b\,x+3}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (b*x + 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

3.1530 $\int \frac{1}{2+bx} \, dx$

Optimal. Leaf size=10

$$\frac{\log(2 + bx)}{b}$$

```
[Out] ln(b*x+2)/b
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {31}

$$\frac{\log(bx + 2)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(-1),x]
```

```
[Out] Log[2 + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2 + bx} \, dx = \frac{\log(2 + bx)}{b}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(2 + bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(-1),x]
```

```
[Out] Log[2 + b*x]/b
```

Maple [A]
time = 0.14, size = 11, normalized size = 1.10

method	result	size
default	$\frac{\ln(bx+2)}{b}$	11
norman	$\frac{\ln(bx+2)}{b}$	11
risch	$\frac{\ln(bx+2)}{b}$	11
meijerg	$\frac{\ln\left(\frac{bx}{2}+1\right)}{b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+2)/b
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="maxima")
```

```
[Out] log(b*x + 2)/b
```

Fricas [A]
time = 0.90, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="fricas")
```

```
[Out] log(b*x + 2)/b
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.70

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x)
```

```
[Out] log(b*x + 2)/b
```

Giac [A]
time = 1.15, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx + 2|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + 2))/b
```

Mupad [B]
time = 0.26, size = 10, normalized size = 1.00

$$\frac{\ln\left(b\,x + 2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + 2),x)
```

```
[Out] log(b*x + 2)/b
```

3.1531 $\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}$$

```
[Out] 2*arcsinh((b*x+1)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[1 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(13) = 26$.
time = 0.16, size = 66, normalized size = 4.40

method	result	size
default	$\frac{\sqrt{(bx+1)(bx+2)}\,\ln\left(\frac{\frac{3}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+3bx+2}\right)}{\sqrt{bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+1)*(b*x+2))^(1/2)/(b*x+1)^(1/2)/(b*x+2)^(1/2)*ln((3/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+3*b*x+2)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.
time = 0.31, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 + 3\,bx + 2}\,b + 3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 3*b*x + 2)*b + 3*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.89, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx + 2\,\sqrt{bx+2}\,\sqrt{bx+1} - 3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x + 1) - 3)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 1.53, size = 23, normalized size = 1.53

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x + 1))/b
```

Mupad [B]
time = 0.29, size = 43, normalized size = 2.87

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{b\,x+1}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

3.1532 $\int \frac{1}{\sqrt{bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

```
[Out] 2*arcsinh(1/2*(b*x)^(1/2)*2^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/Sqrt[2]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\,\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.
time = 0.01, size = 42, normalized size = 2.21

$$-\frac{2\sqrt{x}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Sqrt[x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(Sqrt[b]*Sqrt[b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(16) = 32.
time = 0.14, size = 58, normalized size = 3.05

method	result	size
meijerg	$\frac{2\sqrt{x}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{\sqrt{b}\sqrt{bx}}$	26
default	$\frac{\sqrt{bx(bx+2)}\,\ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{x^2b^2+2bx}\right)}{\sqrt{bx}\sqrt{bx+2}\sqrt{b^2}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x*(b*x+2))^(1/2)/(b*x)^(1/2)/(b*x+2)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2)+(b^2
*x^2+2*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.29, size = 32, normalized size = 1.68

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+2\,bx}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x)*b + 2*b)/b
```

Fricas [A]
time = 0.63, size = 25, normalized size = 1.32

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x) - 1)/b
```

Sympy [A]
time = 0.59, size = 20, normalized size = 1.05

$$\frac{2\,\text{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

Giac [A]
time = 1.18, size = 21, normalized size = 1.11

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x))/b
```

Mupad [B]
time = 0.28, size = 37, normalized size = 1.95

$$\frac{4\,\text{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)
^(1/2)
```

3.1533 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}$$

```
[Out] 2*arcsinh(1/3*(b*x-1)^(1/2)*3^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/Sqrt[3]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-1+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-1 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(18) = 36$.
time = 0.15, size = 65, normalized size = 3.10

method	result	size
default	$\frac{\sqrt{(bx-1)(bx+2)}\,\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+bx-2}\right)}{\sqrt{bx-1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-1)*(b*x+2))^(1/2)/(b*x-1)^(1/2)/(b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+b*x-2)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.43

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-2}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 2)*b + b)/b
```

Fricas [A]
time = 0.63, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx-1}\,-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 1) - 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 0.88, size = 23, normalized size = 1.10

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 1))/b
```

Mupad [B]
time = 0.29, size = 44, normalized size = 2.10

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

3.1534 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] arccosh(1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{54\}$

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(9) = 18$.
time = 0.15, size = 57, normalized size = 5.18

method	result	size
default	$\frac{\sqrt{(bx-2)(bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.29, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.70, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 13.74, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.
time = 1.12, size = 23, normalized size = 2.09

$$-\frac{2 \log \left(\sqrt{bx+2} - \sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

Mupad [B]
time = 0.00, size = 50, normalized size = 4.55

$$-\frac{4 \operatorname{atan}\left(\frac{b\left(-\sqrt{b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1535 $\int \frac{1}{\sqrt{-3+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}$$

[Out] 2*arcsinh(1/5*(b*x-3)^(1/2)*5^(1/2))/b

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]

[Out] (2*ArcSinh[Sqrt[-3 + b*x]/Sqrt[5]])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{-3+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-3 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(18) = 36$.
time = 0.16, size = 66, normalized size = 3.14

method	result	size
default	$\frac{\sqrt{(bx-3)(bx+2)}\,\ln\left(\frac{-\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-bx-6}\right)}{\sqrt{bx-3}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-3)*(b*x+2))^(1/2)/(b*x-3)^(1/2)/(b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(b^2)
^(1/2)+(b^2*x^2-b*x-6)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 1.57

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-bx-6}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 6)*b - b)/b
```

Fricas [A]
time = 0.83, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 3) + 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 1.46, size = 23, normalized size = 1.10

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 3))/b
```

Mupad [B]
time = 0.28, size = 50, normalized size = 2.38

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x - 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1536 $\int \frac{1}{\sqrt{3-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

```
[Out] arcsin(2/5*b*x-1/5)/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 633, 222}

$$-\frac{\text{ArcSin}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(1 - 2*b*x)/5]/b)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{6+bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25b^2}}}\,dx, x, b-2b^2x\right)}{5b^2}$$
$$= -\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Mathematica [A]
time = 0.05, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[3 - b*x]/Sqrt[2 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(11) = 22$.
time = 0.16, size = 65, normalized size = 4.06

method	result	size
default	$\frac{\sqrt{(-bx+3)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x-\frac{1}{2b}\right)}{\sqrt{-x^2b^2+bx+6}}\right)}{\sqrt{-bx+3}\,\sqrt{bx+2}\,\sqrt{b^2}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+3)*(b*x+2))^(1/2)/(-b*x+3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x+6)^(1/2))
```

Maxima [A]
time = 0.49, size = 21, normalized size = 1.31

$$-\frac{\arcsin\left(-\frac{2\,b^2x-b}{5\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/5*(2*b^2*x - b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.
time = 1.00, size = 44, normalized size = 2.75

$$-\frac{\arctan\left(\frac{(2\,bx-1)\sqrt{bx+2}\,\sqrt{-bx+3}}{2\,(b^2x^2-bx-6)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x - 1)*sqrt(b*x + 2)*sqrt(-b*x + 3)/(b^2*x^2 - b*x - 6))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 3)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 1.26, size = 18, normalized size = 1.12

$$\frac{2\,\arcsin\left(\frac{1}{5}\,\sqrt{5}\,\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/5*sqrt(5)*sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.08, size = 44, normalized size = 2.75

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

3.1537 $\int \frac{1}{\sqrt{2-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

```
[Out] arcsin(1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {41, 222}

$$\frac{\text{ArcSin}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\,\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{4-b^2x^2}}\,dx\\ &= \frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(11) = 22.
time = 0.04, size = 39, normalized size = 3.55

$$-\frac{\log\left(-\sqrt{-b^2}\,x+\sqrt{4-b^2x^2}\right)}{\sqrt{-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
[Out] -(Log[-(Sqrt[-b^2]*x) + Sqrt[4 - b^2*x^2]]/Sqrt[-b^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(9) = 18$.
time = 0.16, size = 56, normalized size = 5.09

method	result	size
default	$\frac{\sqrt{(-bx+2)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-x^2b^2+4}}\right)}{\sqrt{-bx+2}\,\sqrt{bx+2}\,\sqrt{b^2}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-b*x+2)*(b*x+2))^(1/2)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*x/(-b^2*x^2+4)^(1/2))
```

Maxima [A]
time = 0.50, size = 9, normalized size = 0.82

$$\frac{\arcsin\left(\frac{1}{2}\,bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] arcsin(1/2*b*x)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(9) = 18$.
time = 0.86, size = 31, normalized size = 2.82

$$-\frac{2\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-bx+2}-2}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(b*x + 2)*sqrt(-b*x + 2) - 2)/(b*x))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 13.88, size = 76, normalized size = 6.91

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\,\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\,\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
4/(b**2*x**2))/(4*pi**(3/2)*b) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [A]
time = 1.17, size = 15, normalized size = 1.36

$$\frac{2 \arcsin\left(\frac{1}{2} \sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.08, size = 44, normalized size = 4.00

$$-\frac{4 \operatorname{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

3.1538 $\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}$$

```
[Out] arcsin(2/3*b*x+1/3)/b
```

Rubi [A]

time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 633, 222}

$$-\frac{\text{ArcSin}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(-1 - 2*b*x)/3]/b)
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{2-bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9b^2}}}\,dx, x, -b-2b^2x\right)}{3b^2}$$
$$= -\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}$$

Mathematica [**A**]
time = 0.05, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[1 - b*x]/Sqrt[2 + b*x]])/b
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(11) = 22$.
time = 0.17, size = 66, normalized size = 4.12

method	result	size
default	$\frac{\sqrt{(-bx+1)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{1}{2b}\right)}{\sqrt{-x^2b^2-bx+2}}\right)}{\sqrt{-bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((-b*x+1)*(b*x+2))^(1/2)/(-b*x+1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+1/2/b)/(-b^2*x^2-b*x+2)^(1/2))
```

Maxima [**A**]
time = 0.49, size = 19, normalized size = 1.19

$$-\frac{\arcsin\left(-\frac{2\,b^2x+b}{3\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*(2*b^2*x + b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(11) = 22$.
time = 0.65, size = 43, normalized size = 2.69

$$-\frac{\arctan\left(\frac{(2\,bx+1)\sqrt{bx+2}\,\sqrt{-bx+1}}{2\,(b^2x^2+bx-2)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(-b*x + 1)/(b^2*x^2 + b*x - 2))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 1.09, size = 18, normalized size = 1.12

$$\frac{2\,\arcsin\left(\frac{1}{3}\,\sqrt{3}\,\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/3*sqrt(3)*sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.32, size = 40, normalized size = 2.50

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{1-b\,x}-1\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(b^2)^(1/2)
)))/(b^2)^(1/2)
```

3.1539 $\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=10

$$\frac{\sin^{-1}(1+bx)}{b}$$

```
[Out] arcsin(b*x+1)/b
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {55, 633, 222}

$$\frac{\text{ArcSin}(bx+1)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[1 + b*x]/b
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}$$
$$= \frac{\sin^{-1}(1+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(10) = 20.
time = 0.01, size = 57, normalized size = 5.70

$$-\frac{2\sqrt{x}\sqrt{2+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{2+bx}\right)}{\sqrt{b}\sqrt{-bx(2+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[2 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(Sqrt[b]
*Sqrt[-(b*x*(2 + b*x))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(10) = 20.
time = 0.15, size = 58, normalized size = 5.80

method	result	size
meijerg	$\frac{2\sqrt{x}\,\text{arcsinh}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{2}}{2}\right)}{\sqrt{b}\sqrt{-bx}}$	27
default	$\frac{\sqrt{-bx\,(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{1}{b}\right)}{\sqrt{-x^2b^2-2bx}}\right)}{\sqrt{-bx}\sqrt{bx+2}\sqrt{b^2}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x*(b*x+2))^(1/2)/(-b*x)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

Maxima [A]
time = 0.50, size = 18, normalized size = 1.80

$$-\frac{\arcsin\left(-\frac{b^2x+b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(b^2*x + b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.76, size = 26, normalized size = 2.60

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(b*x + 2)*sqrt(-b*x)/(b*x))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 24, normalized size = 2.40

$$-\frac{2i\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*I*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

Giac [A]
time = 1.15, size = 17, normalized size = 1.70

$$-\frac{2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{-bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arcsin(1/2*sqrt(2)*sqrt(-b*x))/b
```

Mupad [B]
time = 0.29, size = 34, normalized size = 3.40

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{-b\,x}\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(b*x + 2)^(1/2)),x)
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((-b*x)^(1/2)*(b^2)^(1/2))))/(b^2)
^(1/2)
```

3.1540 $\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(3+2bx)}{b}$$

```
[Out] arcsin(2*b*x+3)/b
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {55, 633, 222}

$$\frac{\text{ArcSin}(2bx+3)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[3 + 2*b*x]/b
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-bx}\sqrt{2+bx}}\,dx = \int \frac{1}{\sqrt{-2-3bx-b^2x^2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, -3b-2b^2x\right)}{b^2}$$
$$= \frac{\sin^{-1}(3+2bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(11) = 22.
time = 0.02, size = 59, normalized size = 5.36

$$\frac{2\sqrt{1+bx}\sqrt{2+bx}\tanh^{-1}\left(\frac{\sqrt{2+bx}}{\sqrt{1+bx}}\right)}{b\sqrt{-((1+bx)(2+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*Sqrt[2 + b*x]*ArcTanh[Sqrt[2 + b*x]/Sqrt[1 + b*x]])/(b*Sqr
t[-((1 + b*x)*(2 + b*x))])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(11) = 22.
time = 0.17, size = 66, normalized size = 6.00

method	result	size
default	$\frac{\sqrt{(-bx-1)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{3}{2b}\right)}{\sqrt{-x^2b^2-3bx-2}}\right)}{\sqrt{-bx-1}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-1)*(b*x+2))^(1/2)/(-b*x-1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+3/2/b)/(-b^2*x^2-3*b*x-2)^(1/2))
```

Maxima [A]
time = 0.49, size = 21, normalized size = 1.91

$$-\frac{\arcsin\left(-\frac{2\,b^2x+3\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 3*b)/b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(11) = 22$.
time = 0.96, size = 44, normalized size = 4.00

$$-\frac{\arctan\left(\frac{(2\,bx+3)\sqrt{bx+2}\,\sqrt{-bx-1}}{2\,(b^2x^2+3\,bx+2)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 3)*sqrt(b*x + 2)*sqrt(-b*x - 1)/(b^2*x^2 + 3*b*x + 2))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(b*x + 2)), x)
```

Giac [A]
time = 1.44, size = 13, normalized size = 1.18

$$\frac{2\,\arcsin\left(\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.30, size = 41, normalized size = 3.73

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)^(1/
2))))/(b^2)^(1/2)
```

3.1541 $\int \frac{1}{\sqrt{-2-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{2+bx}\,\log(2+bx)}{b\sqrt{-2-bx}}$$

```
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {23, 31}

$$\frac{\sqrt{bx+2}\,\log(bx+2)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*Log[2 + b*x])/(b*Sqrt[-2 - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-bx}\sqrt{2+bx}}\,dx &= \frac{\sqrt{2+bx}\int \frac{1}{2+bx}\,dx}{\sqrt{-2-bx}}\\ &= \frac{\sqrt{2+bx}\,\log(2+bx)}{b\sqrt{-2-bx}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.97

$$\frac{(2+bx)\log(2+bx)}{b\sqrt{-(2+bx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ((2 + b*x)*Log[2 + b*x])/(b*Sqrt[-(2 + b*x)^2])
```

Maple [A]
time = 0.24, size = 26, normalized size = 0.90

method	result	size
default	$\frac{\ln(bx+2)\sqrt{bx+2}}{b\sqrt{-bx-2}}$	26
meijerg	$\frac{\sqrt{\mathrm{signum}\left(\frac{bx}{2}+1\right)}\,\ln\left(\frac{bx}{2}+1\right)}{\sqrt{-\mathrm{signum}\left(\frac{bx}{2}+1\right)}\,b}$	32
risch	$-\frac{i\sqrt{\frac{-bx-2}{bx+2}}\,\sqrt{bx+2}\,\ln(bx+2)}{\sqrt{-bx-2}\,b}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.55

$$\sqrt{-\frac{1}{b^2}}\log\left(x+\frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-1/b^2)*log(x + 2/b)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 0
```

Sympy [C] Result contains complex when optimal does not.
time = 0.92, size = 87, normalized size = 3.00

$$\begin{cases} \frac{i\log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} - \frac{i\log\left(x+\frac{2}{b}\right)}{b} & \text{for } \frac{1}{\left|x+\frac{2}{b}\right|} < 1 \wedge \left|x+\frac{2}{b}\right| < 1 \\ -\frac{i\log\left(x+\frac{2}{b}\right)}{b} & \text{for } \left|x+\frac{2}{b}\right| < 1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} & \text{for } \frac{1}{\left|x+\frac{2}{b}\right|} < 1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix} \middle| x+\frac{2}{b}\right)}{b} - \frac{iG_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix} \middle| x+\frac{2}{b}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*log(1/(x + 2/b))/b - I*log(x + 2/b)/b, (Abs(x + 2/b) < 1) & (1
/Abs(x + 2/b) < 1)), (-I*log(x + 2/b)/b, Abs(x + 2/b) < 1), (I*log(1/(x + 2
/b))/b, 1/Abs(x + 2/b) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/b
)/b - I*meijerg(((1, 1), ()), ((), (0, 0)), x + 2/b)/b, True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.97, size = 16, normalized size = 0.55

$$-\frac{i\log\left(|bx+2|\right)\operatorname{sgn}(b)\operatorname{sgn}(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -I*log(abs(b*x + 2))*sgn(b)*sgn(x)/b
```

Mupad [B]
time = 0.07, size = 47, normalized size = 1.62

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

3.1542 $\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

[Out] -2*arctan((-b*x-3)^(1/2)/(b*x+2)^(1/2))/b

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {65, 223, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]

[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-bx}\sqrt{2+bx}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

Mathematica [A]
time = 0.04, size = 26, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.
time = 0.16, size = 66, normalized size = 2.54

method	result	size
default	$\frac{\sqrt{(-bx-3)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{5}{2b}\right)}{\sqrt{-x^2b^2-5bx-6}}\right)}{\sqrt{-bx-3}\,\sqrt{bx+2}\,\sqrt{b^2}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-3)*(b*x+2))^(1/2)/(-b*x-3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+5/2/b)/(-b^2*x^2-5*b*x-6)^(1/2))
```

Maxima [A]
time = 0.50, size = 21, normalized size = 0.81

$$-\frac{\arcsin\left(-\frac{2\,b^2x+5\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 5*b)/b)/b
```

Fricas [A]
time = 0.99, size = 44, normalized size = 1.69

$$-\frac{\arctan\left(\frac{(2\,bx+5)\sqrt{bx+2}\,\sqrt{-bx-3}}{2\,(b^2x^2+5\,bx+6)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 5)*sqrt(b*x + 2)*sqrt(-b*x - 3)/(b^2*x^2 + 5*b*x + 6))
/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(b*x + 2)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 2.16, size = 23, normalized size = 0.88

$$\frac{2i\,\log\left(\sqrt{bx+3}-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*I*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

Mupad [B]
time = 0.30, size = 47, normalized size = 1.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

3.1543 $\int \frac{1}{\sqrt{2-bx}\,\sqrt{3-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

```
[Out] -2*arcsinh((-b*x+2)^(1/2))/b
```

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\,\sqrt{3-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. $2(16) = 32$.
time = 0.04, size = 59, normalized size = 3.69

$$\frac{\log\left(-1+\frac{\sqrt{3-bx}}{\sqrt{2-bx}}\right)}{b}-\frac{\log\left(b+\frac{b\sqrt{3-bx}}{\sqrt{2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[3 - b*x]/Sqrt[2 - b*x]]/b - Log[b + (b*Sqrt[3 - b*x])/Sqrt[2
 - b*x]]/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.
time = 0.16, size = 70, normalized size = 4.38

method	result	size
default	$\frac{\sqrt{(-bx+2)(-bx+3)}\,\ln\left(\frac{-\frac{5}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-5bx+6}\right)}{\sqrt{-bx+2}\,\sqrt{-bx+3}\,\sqrt{b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+2)*(-b*x+3))^(1/2)/(-b*x+2)^(1/2)/(-b*x+3)^(1/2)*ln((-5/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-5*b*x+6)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 0.33, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-5\,bx+6}\,b-5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 5*b*x + 6)*b - 5*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 1.01, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+3}\,\sqrt{-bx+2}+5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 3)*sqrt(-b*x + 2) + 5)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+2}\,\sqrt{-bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(-b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 2)*sqrt(-b*x + 3)), x)
```

Giac [A]
time = 0.90, size = 25, normalized size = 1.56

$$\frac{2\,\log\left(\sqrt{-bx+3}-\sqrt{-bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 3) - sqrt(-b*x + 2))/b
```

Mupad [B]
time = 0.31, size = 49, normalized size = 3.06

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)
^(1/2))))/(-b^2)^(1/2)
```

3.1544 $\int \frac{1}{2-bx} \, dx$

Optimal. Leaf size=12

$$-\frac{\log(2-bx)}{b}$$

[Out] -ln(-b*x+2)/b

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {31}

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(2 - b*x)^(-1),x]

[Out] -(Log[2 - b*x]/b)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2-bx} \, dx = -\frac{\log(2-bx)}{b}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - b*x)^(-1),x]

[Out] -(Log[2 - b*x]/b)

Maple [A]
time = 0.15, size = 13, normalized size = 1.08

method	result	size
norman	$-\frac{\ln(bx-2)}{b}$	12
risch	$-\frac{\ln(bx-2)}{b}$	12
default	$-\frac{\ln(-bx+2)}{b}$	13
meijerg	$-\frac{\ln\left(-\frac{bx}{2}+1\right)}{b}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2),x,method=_RETURNVERBOSE)
[Out] -ln(-b*x+2)/b
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="maxima")
[Out] -log(b*x - 2)/b
```

Fricas [A]
time = 1.39, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="fricas")
[Out] -log(b*x - 2)/b
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.67

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x)
[Out] -log(b*x - 2)/b
```

Giac [A]
time = 0.65, size = 12, normalized size = 1.00

$$-\frac{\log\left(|bx-2|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="giac")
```

```
[Out] -log(abs(b*x - 2))/b
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.92

$$-\frac{\ln\left(b\,x-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x - 2),x)
```

```
[Out] -log(b*x - 2)/b
```

3.1545 $\int \frac{1}{\sqrt{1-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

```
[Out] -2*arcsinh((-b*x+1)^(1/2))/b
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 59 vs. $2(16) = 32$.
time = 0.04, size = 59, normalized size = 3.69

$$\frac{\log\left(-1+\frac{\sqrt{2-bx}}{\sqrt{1-bx}}\right)}{b}-\frac{\log\left(b+\frac{b\sqrt{2-bx}}{\sqrt{1-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[2 - b*x]/Sqrt[1 - b*x]]/b - Log[b + (b*Sqrt[2 - b*x])/Sqrt[1
- b*x]]/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.
time = 0.16, size = 70, normalized size = 4.38

method	result	size
default	$\frac{\sqrt{(-bx+1)(-bx+2)}\,\ln\left(\frac{-\frac{3}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-3bx+2}\right)}{\sqrt{-bx+1}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+1)*(-b*x+2))^(1/2)/(-b*x+1)^(1/2)/(-b*x+2)^(1/2)*ln((-3/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-3*b*x+2)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.
time = 0.28, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-3\,bx+2}\,b-3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 3*b*x + 2)*b - 3*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 1.49, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx+1}+3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x + 1) + 3)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(-b*x + 2)), x)
```

Giac [A]
time = 1.07, size = 25, normalized size = 1.56

$$\frac{2\log\left(\sqrt{-bx+2}-\sqrt{-bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x + 1))/b
```

Mupad [B]
time = 0.31, size = 45, normalized size = 2.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{1-b\,x}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

3.1546 $\int \frac{1}{\sqrt{-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

```
[Out] -2*arcsinh(1/2*(-b*x)^(1/2)*2^(1/2))/b
```

Rubi [A]

time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-(b*x)]/Sqrt[2]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-bx}\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.
time = 0.01, size = 48, normalized size = 2.40

$$-\frac{2\sqrt{x}\,\log\left(-\sqrt{-b}\,\sqrt{x}+\sqrt{2-bx}\right)}{\sqrt{-b}\,\sqrt{-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*Sqrt[x]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(Sqrt[-b]*Sqrt[-(b*x)
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.
time = 0.14, size = 64, normalized size = 3.20

method	result	size
meijerg	$\frac{2\sqrt{x}\,\arcsin\left(\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{2}}{2}\right)}{\sqrt{b}\,\sqrt{-bx}}$	27
default	$\frac{\sqrt{-bx\,(-bx+2)}\,\ln\left(\frac{b^2x-b}{\sqrt{b^2}}+\sqrt{x^2b^2-2bx}\right)}{\sqrt{-bx}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x*(-b*x+2))^(1/2)/(-b*x)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2)+
(b^2*x^2-2*b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.30, size = 32, normalized size = 1.60

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-2\,bx}\,b-2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x)*b - 2*b)/b
```

Fricas [A]
time = 0.97, size = 27, normalized size = 1.35

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-bx}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x) + 1)/b
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 51, normalized size = 2.55

$$\begin{cases} -\frac{2\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b} & \text{for } |bx| > 2 \\ -\frac{2i\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, Abs(b*x) > 2), (-2*I*asin
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, True))
```

Giac [A]
time = 1.52, size = 23, normalized size = 1.15

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x))/b
```

Mupad [B]
time = 0.28, size = 39, normalized size = 1.95

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\sqrt{-b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((-b*x)^(1/2)*(-b^2)^(1/2))))/(-b^
2)^(1/2)
```

3.1547 $\int \frac{1}{\sqrt{-1-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}$$

```
[Out] -2*arcsinh(1/3*(-b*x-1)^(1/2)*3^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-1 - b*x]/Sqrt[3]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-bx}\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-1-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-1 - b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(19) = 38$.
time = 0.16, size = 70, normalized size = 3.18

method	result	size
default	$\frac{\sqrt{(-bx-1)(-bx+2)}\,\ln\left(\frac{-\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-bx-2}\right)}{\sqrt{-bx-1}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-1)*(-b*x+2))^(1/2)/(-b*x-1)^(1/2)/(-b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-b*x-2)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 1.50

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-bx-2}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 2)*b - b)/b
```

Fricas [A]
time = 1.18, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-1}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 1) + 1)/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(-b*x + 2)), x)
```

Giac [**A**]
time = 1.55, size = 25, normalized size = 1.14

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 1))/b
```

Mupad [**B**]
time = 0.28, size = 46, normalized size = 2.09

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)^(
1/2))))/(-b^2)^(1/2)
```

3.1548 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

```
[Out] -arccosh(-1/2*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{54\}$

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] -(ArcCosh[-1/2*(b*x)]/b)
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx = -\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.
time = 0.04, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-2 - b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(10) = 20$.
time = 0.16, size = 61, normalized size = 5.08

method	result	size
default	$\frac{\sqrt{(-bx-2)(-bx+2)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-4}\right)}{\sqrt{-bx-2}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-2)*(-b*x+2))^(1/2)/(-b*x-2)^(1/2)/(-b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2
)+(b^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.
time = 0.27, size = 26, normalized size = 2.17

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(10) = 20$.
time = 1.31, size = 28, normalized size = 2.33

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x - 2))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 14.38, size = 78, normalized size = 6.50

$$-\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\,\frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\,\frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] -meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4/
(b**2*x**2))/(4*pi**(3/2)*b) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.
time = 1.41, size = 25, normalized size = 2.08

$$\frac{2 \log \left(\sqrt{-bx+2} - \sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 2))/b
```

Mupad [B]
time = 0.29, size = 52, normalized size = 4.33

$$\frac{4 \operatorname{atan}\left(\frac{b \left(-\sqrt{-b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1549 $\int \frac{1}{\sqrt{-3-bx}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}$$

```
[Out] -2*arcsinh(1/5*(-b*x-3)^(1/2)*5^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {65, 221}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-3 - b*x]/Sqrt[5]])/b
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3-bx}\sqrt{2-bx}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-3-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-3 - b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.
time = 0.17, size = 69, normalized size = 3.14

method	result	size
default	$\frac{\sqrt{(-bx-3)(-bx+2)}\,\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2+bx-6}\right)}{\sqrt{-bx-3}\,\sqrt{-bx+2}\,\sqrt{b^2}}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x-3)*(-b*x+2))^(1/2)/(-b*x-3)^(1/2)/(-b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b
^2)^(1/2)+(b^2*x^2+b*x-6)^(1/2))/(b^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 30, normalized size = 1.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-6}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 6)*b + b)/b
```

Fricas [A]
time = 1.37, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-3}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 3) - 1)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-3}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(-b*x + 2)), x)
```

Giac [A]
time = 1.47, size = 25, normalized size = 1.14

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 3))/b
```

Mupad [B]
time = 0.29, size = 52, normalized size = 2.36

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 3)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

3.1550 $\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

```
[Out] arccosh(1/4*b*x)/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {54}

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/4]/b
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.04, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{4+bx}}{\sqrt{-4+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[4 + b*x]/Sqrt[-4 + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(9) = 18$.
time = 0.17, size = 57, normalized size = 5.18

method	result	size
default	$\frac{\sqrt{(bx-4)(bx+4)}\,\ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{x^2b^2-16}\right)}{\sqrt{bx-4}\,\sqrt{bx+4}\,\sqrt{b^2}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x-4)*(b*x+4))^(1/2)/(b*x-4)^(1/2)/(b*x+4)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-16)^(1/2))/(b^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.28, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-16}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 16)*b)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(9) = 18$.
time = 0.89, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+4}\,\sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x - 4))/b
```

Sympy [C] Result contains complex when optimal does not.
time = 13.44, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\,\frac{16e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\,\frac{16}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 16*
exp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 16/(b**2*x**2))/(4*pi**
(3/2)*b)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.
time = 0.97, size = 23, normalized size = 2.09

$$-\frac{2 \log\left(\sqrt{bx+4}-\sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x - 4))/b
```

Mupad [B]
time = 0.32, size = 40, normalized size = 3.64

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x-4}-2{}\mathrm{i}\right)}{\left(\sqrt{b\,x+4}-2\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 4)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x - 4)^(1/2) - 2i))/(((b*x + 4)^(1/2) - 2)*(-b^2)^(1/2))))/
(-b^2)^(1/2)
```

3.1551 $\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] 2*arcsinh(d^(1/2)*(-b*(1-c)/d+b*x)^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {65, 221}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[d]*Sqrt[-((b*(1 - c))/d) + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d
])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{-b+bc}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{-b+bc}{d}+bx}\right)}{b}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 1.19

$$\frac{2\sqrt{-1+c+dx}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{-1+c+dx}}\right)}{d\sqrt{\frac{b(-1+c+dx)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-1 + c + d*x]*ArcTanh[Sqrt[c + d*x]/Sqrt[-1 + c + d*x]])/(d*Sqrt[(b
*(-1 + c + d*x))/d])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(33) = 66$.
time = 0.19, size = 100, normalized size = 2.33

method	result	size
default	$\frac{\sqrt{\left(bx+\frac{b(c-1)}{d}\right)(dx+c)}\,\ln\left(\frac{\frac{b(c-1)}{2}+\frac{bc}{2}+bdx}{\sqrt{bd}}+\sqrt{bd\,x^2+(b\,(c-1)+bc)\,x+\frac{b(c-1)c}{d}}\right)}{\sqrt{bx+\frac{b(c-1)}{d}}\,\sqrt{dx+c}\,\sqrt{bd}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x+b*(c-1)/d)*(d*x+c))^(1/2)/(b*x+b*(c-1)/d)^(1/2)/(d*x+c)^(1/2)*ln((1/2
*b*(c-1)+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(b*(c-1)+b*c)*x+b*(c-1)/d*c)^(
1/2))/(b*d)^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(30) = 60$.
time = 1.18, size = 175, normalized size = 4.07

$$\left[\frac{\sqrt{bd}\,\log\left(8\,bd^{2}x^{2}+8\,bc^{2}+8\,(2\,bc-b)dx+4\,\sqrt{bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}}{2\,(bd^{2}x^{2}+bc^{2}+(2\,bc-b)dx-bc)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b*d^2*x^2 + 8*b*c^2 + 8*(2*b*c - b)*d*x + 4*sqrt(b*d)*
(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((b*d*x + b*c - b)/d) - 8*b*c + b)/(b*d
), -sqrt(-b*d)*arctan(1/2*sqrt(-b*d)*(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((
b*d*x + b*c - b)/d)/(b*d^2*x^2 + b*c^2 + (2*b*c - b)*d*x - b*c))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\frac{c}{d}+x-\frac{1}{d}\right)}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(c/d + x - 1/d))*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(30) = 60$.
time = 0.99, size = 66, normalized size = 1.53

$$-\frac{2\,|b|\log\left(-\sqrt{bd^{2}x+bcd-bd}\,\sqrt{bd}+\sqrt{b^{2}d^{2}+(bd^{2}x+bcd-bd)bd}\right)}{\sqrt{bd}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*abs(b)*log(-sqrt(b*d^2*x + b*c*d - b*d)*sqrt(b*d) + sqrt(b^2*d^2 + (b*d^
2*x + b*c*d - b*d)*b*d))/(sqrt(b*d)*b)
```

Mupad [B]
time = 0.50, size = 66, normalized size = 1.53

$$\frac{4\,\mathrm{atan}\left(-\frac{d\,\left(\sqrt{b\,x-\frac{b-b\,c}{d}}-\sqrt{-\frac{b-b\,c}{d}}\right)}{\sqrt{-b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - (b - b*c)/d)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (4*atan(-(d*((b*x - (b - b*c)/d)^(1/2) - (-(b - b*c)/d)^(1/2)))/((-b*d)^(1/
2)*((c + d*x)^(1/2) - c^(1/2)))))/(-b*d)^(1/2)
```

3.1552 $\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{2}\,\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)$$

[Out] arcsinh(1/3*(-3+2*x)^(1/2)*3^(1/2))*2^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {56, 221}

$$\sqrt{2}\,\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]

[Out] Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{2}\,\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.36

$$-\sqrt{2}\,\log\left(-\sqrt{2}\sqrt{x}+\sqrt{-3+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] -(Sqrt[2]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[-3 + 2*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.
time = 0.16, size = 48, normalized size = 2.18

method	result	size
meijerg	$\frac{\sqrt{2}\,\sqrt{-\mathrm{signum}\left(x-\frac{3}{2}\right)}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{3}\,\sqrt{2}}{3}\right)}{\sqrt{\mathrm{signum}\left(x-\frac{3}{2}\right)}}$	31
default	$\frac{\sqrt{x\,(2x-3)}\,\ln\left(\frac{\left(-\frac{3}{2}+2x\right)\sqrt{2}}{2}+\sqrt{2x^{2}-3x}\right)\sqrt{2}}{2\sqrt{x}\,\sqrt{2x-3}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(2*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x*(2*x-3))^(1/2)/x^(1/2)/(2*x-3)^(1/2)*ln(1/2*(-3/2+2*x)*2^(1/2)+(2*x^
2-3*x)^(1/2))*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.49, size = 41, normalized size = 1.86

$$-\frac{1}{2}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2\,x-3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2\,x-3}}{\sqrt{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x - 3)/sqrt(x))/(sqrt(2) + sqrt(2*x - 3
)/sqrt(x)))
```

Fricas [A]
time = 1.35, size = 26, normalized size = 1.18

$$\frac{1}{2}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\,\sqrt{2\,x-3}\,\sqrt{x}-4\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(2*x - 3)*sqrt(x) - 4*x + 3)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.47, size = 42, normalized size = 1.91

$$\begin{cases} \sqrt{2}\,\operatorname{acosh}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{for}\ |x| > \frac{3}{2} \\ -\sqrt{2}\,i\operatorname{asin}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-3+2*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x)/3), Abs(x) > 3/2), (-sqrt(2)*I*asi
n(sqrt(6)*sqrt(x)/3), True))
```

Giac [A]
time = 1.58, size = 23, normalized size = 1.05

$$-\sqrt{2}\,\log\left(\sqrt{2}\,\sqrt{x} - \sqrt{2\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(sqrt(2)*sqrt(x) - sqrt(2*x - 3))
```

Mupad [B]
time = 0.44, size = 30, normalized size = 1.36

$$-2\,\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\left(-\sqrt{2\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\,\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2*x - 3)^(1/2)),x)
```

```
[Out] -2*2^(1/2)*atanh((2^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*x^(1/2)))
```

3.1553 $\int \frac{1}{\sqrt{-3+2x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)$$

```
[Out] 1/3*arcsinh(1/13*39^(1/2)*(-3+2*x)^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {56, 221}

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+2x}\,\sqrt{2+3x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13+3x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 26, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-9+6x}{4+6x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
[Out] Sqrt[2/3]*ArcTanh[1/Sqrt[(-9 + 6*x)/(4 + 6*x)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.
time = 0.17, size = 57, normalized size = 2.19

method	result	size
default	$\frac{\sqrt{(2x-3)(2+3x)}\,\ln\left(\frac{\left(-\frac{5}{2}+6x\right)\sqrt{6}}{6}+\sqrt{6x^2-5x-6}\right)\sqrt{6}}{6\sqrt{2x-3}\,\sqrt{2+3x}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-3)^(1/2)/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*((2*x-3)*(2+3*x))^(1/2)/(2*x-3)^(1/2)/(2+3*x)^(1/2)*ln(1/6*(-5/2+6*x)*6
^(1/2)+(6*x^2-5*x-6)^(1/2))*6^(1/2)
```

Maxima [A]
time = 0.50, size = 28, normalized size = 1.08

$$\frac{1}{6}\sqrt{6}\log\left(2\sqrt{6}\sqrt{6x^2-5x-6}+12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
[Out] 1/6*sqrt(6)*log(2*sqrt(6)*sqrt(6*x^2 - 5*x - 6) + 12*x - 5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.
time = 1.24, size = 46, normalized size = 1.77

$$\frac{1}{12}\sqrt{3}\sqrt{2}\log\left(4\sqrt{3}\sqrt{2}(12x-5)\sqrt{3x+2}\sqrt{2x-3}+288x^2-240x-119\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*log(4*sqrt(3)*sqrt(2)*(12*x - 5)*sqrt(3*x + 2)*sqrt(2*
x - 3) + 288*x^2 - 240*x - 119)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.50, size = 56, normalized size = 2.15

$$\begin{cases} \frac{\sqrt{6}\operatorname{acosh}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{for } \left|x+\frac{2}{3}\right| > \frac{13}{6} \\ -\frac{\sqrt{6}\, i \operatorname{asin}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(6)*acosh(sqrt(78)*sqrt(x + 2/3)/13)/3, Abs(x + 2/3) > 13/6)
, (-sqrt(6)*I*asin(sqrt(78)*sqrt(x + 2/3)/13)/3, True))
```

Giac [A]
time = 1.28, size = 30, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\sqrt{2}\log\left(\left|-\sqrt{2}\sqrt{3x+2}+\sqrt{6x-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*sqrt(2)*log(abs(-sqrt(2)*sqrt(3*x + 2) + sqrt(6*x - 9)))
```

Mupad [B]
time = 0.12, size = 43, normalized size = 1.65

$$\frac{2\sqrt{6}\operatorname{atanh}\left(\frac{\sqrt{6}\left(-\sqrt{2x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\left(\sqrt{2}-\sqrt{3x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 3)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] (2*6^(1/2)*atanh((6^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*(2^(1/2) - (3*
x + 2)^(1/2)))))/3
```

3.1554 $\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] 2*arcsin(d^(1/2)*(b*(1-c)/d+b*x)^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {65, 222}

$$\frac{2\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[d]*Sqrt[(b*(1 - c))/d + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\,\sqrt{c-dx}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{b-bc}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{b-bc}{d}+bx}\right)}{b}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Mathematica [**A**]
time = 0.07, size = 58, normalized size = 1.38

$$-\frac{2\sqrt{1-c+dx}\,\tan^{-1}\left(\frac{\sqrt{c-dx}}{\sqrt{1-c+dx}}\right)}{d\sqrt{\frac{b(1-c+dx)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (-2*Sqrt[1 - c + d*x]*ArcTan[Sqrt[c - d*x]/Sqrt[1 - c + d*x]])/(d*Sqrt[(b*(
1 - c + d*x))/d])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(32) = 64$.
time = 0.18, size = 109, normalized size = 2.60

method	result	size
default	$\frac{\sqrt{\left(bx-\frac{b(c-1)}{d}\right)(-dx+c)}\,\arctan\left(\frac{\sqrt{bd}\,\left(x-\frac{b(c-1)+bc}{2bd}\right)}{\sqrt{-bd\,x^2+(b\,(c-1)+bc)\,x-\frac{b(c-1)c}{d}}}\right)}{\sqrt{bx-\frac{b(c-1)}{d}}\,\sqrt{-dx+c}\,\sqrt{bd}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] ((b*x-b*(c-1)/d)*(-d*x+c))^(1/2)/(b*x-b*(c-1)/d)^(1/2)/(-d*x+c)^(1/2)/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(b*(c-1)+b*c)/b/d)/(-b*d*x^2+(b*(c-1)+b*c)*x-b*(c-1)/d*c)^(1/2))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more details)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.
time = 1.36, size = 176, normalized size = 4.19

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,bd^{2}x^{2}+8\,bc^{2}-8\,(2\,bc-b)dx-4\,\sqrt{-bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{bd}\,\arctan\left(\frac{\sqrt{bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}}{2\,(bd^{2}x^{2}+bc^{2}-(2\,bc-b)dx-bc)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-b*d)*log(8*b*d^2*x^2 + 8*b*c^2 - 8*(2*b*c - b)*d*x - 4*sqrt(-b*d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqrt((b*d*x - b*c + b)/d) - 8*b*c + b)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(b*d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqrt((b*d*x - b*c + b)/d)/(b*d^2*x^2 + b*c^2 - (2*b*c - b)*d*x - b*c))/(b*d)]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\left(-\frac{c}{d}+x+\frac{1}{d}\right)}\,\sqrt{c-dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b*c+b)/d+b*x)**(1/2)/(-d*x+c)**(1/2),x)

[Out] Integral(1/(sqrt(b*(-c/d + x + 1/d))*sqrt(c - d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(31) = 62$.
time = 1.75, size = 69, normalized size = 1.64

$$-\frac{2\,|b|\log\left(-\sqrt{bd^2x-bcd+bd}\,\sqrt{-bd}+\sqrt{b^2d^2-(bd^2x-bcd+bd)bd}\right)}{\sqrt{-bd}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*abs(b)*log(-sqrt(b*d^2*x - b*c*d + b*d)*sqrt(-b*d) + sqrt(b^2*d^2 - (b*d
^2*x - b*c*d + b*d)*b*d))/(sqrt(-b*d)*b)
```

Mupad [B]
time = 0.51, size = 63, normalized size = 1.50

$$-\frac{4\,\mathrm{atan}\left(-\frac{d\left(\sqrt{\frac{b-b\,c}{d}+b\,x}-\sqrt{\frac{b-b\,c}{d}}\right)}{\sqrt{b\,d}\,\left(\sqrt{c-d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((b - b*c)/d + b*x)^(1/2)*(c - d*x)^(1/2)),x)
```

```
[Out] -(4*atan(-(d*(((b - b*c)/d + b*x)^(1/2) - ((b - b*c)/d)^(1/2)))/((b*d)^(1/2
)*((c - d*x)^(1/2) - c^(1/2)))))/(b*d)^(1/2)
```

3.1555 $\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] arcsin(-1+1/2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {55, 633, 222}

$$-\text{ArcSin}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

```
[Out] -ArcSin[1 - x/2]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx = \int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)\right)$$
$$= -\sin^{-1}\left(1-\frac{x}{2}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(10) = 20.
time = 0.03, size = 38, normalized size = 3.80

$$\frac{2\sqrt{-4+x}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-4+x}}\right)}{\sqrt{-((-4+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]
[Out] (2*Sqrt[-4 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-4 + x]])/Sqrt[-((-4 + x)*x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.
time = 0.15, size = 27, normalized size = 2.70

method	result	size
meijerg	$2\arcsin\left(\frac{\sqrt{x}}{2}\right)$	9
default	$\frac{\sqrt{(4-x)\,x}\ \arcsin\left(-1+\frac{x}{2}\right)}{\sqrt{4-x}\,\sqrt{x}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] ((4-x)*x)^(1/2)/(4-x)^(1/2)/x^(1/2)*arcsin(-1+1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.51, size = 14, normalized size = 1.40

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 1.16, size = 14, normalized size = 1.40

$$-2 \arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 24, normalized size = 2.40

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{x}}{2}\right) & \text{for } |x| > 4 \\ 2 \operatorname{asin}\left(\frac{\sqrt{x}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)**(1/2)/x**(1/2),x)
[Out] Piecewise((-2*I*acosh(sqrt(x)/2), Abs(x) > 4), (2*asin(sqrt(x)/2), True))
```

Giac [A]
time = 1.39, size = 8, normalized size = 0.80

$$2 \arcsin\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(x))
```

Mupad [B]
time = 0.29, size = 16, normalized size = 1.60

$$-4 \operatorname{atan}\left(\frac{\sqrt{4-x}-2}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(4 - x)^(1/2)),x)
[Out] -4*atan(((4 - x)^(1/2) - 2)/x^(1/2))
```

3.1556 $\int \frac{1}{\sqrt{3-2x}\,\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{2}\,\sin^{-1}\left(\sqrt{\frac{2}{3}}\,\sqrt{x}\right)$$

```
[Out] arcsin(1/3*6^(1/2)*x^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {56, 222}

$$\sqrt{2}\,\text{ArcSin}\left(\sqrt{\frac{2}{3}}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

```
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\,\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{2}\,\sin^{-1}\left(\sqrt{\frac{2}{3}}\,\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 1.90

$$-2\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{3}-\sqrt{3-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
```

```
[Out] -2*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[x])/(Sqrt[3] - Sqrt[3 - 2*x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.
time = 0.15, size = 31, normalized size = 1.55

method	result	size
meijerg	$\sqrt{2}\ \arcsin\left(\frac{\sqrt{x}\ \sqrt{3}\ \sqrt{2}}{3}\right)$	17
default	$\frac{\sqrt{(3-2x)\,x}\ \sqrt{2}\ \arcsin\left(\frac{4x}{3}-1\right)}{2\sqrt{3-2x}\ \sqrt{x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((3-2*x)*x)^(1/2)/(3-2*x)^(1/2)/x^(1/2)*2^(1/2)*arcsin(4/3*x-1)
```

Maxima [A]
time = 0.49, size = 21, normalized size = 1.05

$$-\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{-2\,x+3}}{2\ \sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Fricas [A]
time = 1.59, size = 21, normalized size = 1.05

$$-\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\ \sqrt{-2\,x+3}}{2\ \sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.45, size = 42, normalized size = 2.10

$$\begin{cases} -\sqrt{2}\, i \operatorname{acosh}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{for } |x| > \frac{3}{2} \\ \sqrt{2} \operatorname{asin}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x)/3), Abs(x) > 3/2), (sqrt(2)*asi
n(sqrt(6)*sqrt(x)/3), True))
```

Giac [A]
time = 1.00, size = 13, normalized size = 0.65

$$\sqrt{2} \arcsin\left(\frac{1}{3}\sqrt{6}\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arcsin(1/3*sqrt(6)*sqrt(x))
```

Mupad [B]
time = 0.30, size = 27, normalized size = 1.35

$$2\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt{3}-\sqrt{3-2x}\right)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(3 - 2*x)^(1/2)),x)
```

```
[Out] 2*2^(1/2)*atan((2^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*x^(1/2)))
```

3.1557 $\int \frac{1}{\sqrt{3-2x}\,\sqrt{3+5x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{3+5x}\right)$$

```
[Out] 1/5*arcsin(1/21*42^(1/2)*(3+5*x)^(1/2))*10^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {56, 222}

$$\sqrt{\frac{2}{5}}\,\text{ArcSin}\left(\sqrt{\frac{2}{21}}\sqrt{5x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] Sqrt[2/5]*ArcSin[Sqrt[2/21]*Sqrt[3 + 5*x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\,\sqrt{3+5x}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{21-2x^2}}\,dx, x, \sqrt{3+5x}\right)}{\sqrt{5}}\\ &= \sqrt{\frac{2}{5}}\sin^{-1}\left(\sqrt{\frac{2}{21}}\sqrt{3+5x}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 1.19

$$-\sqrt{\frac{2}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{15}{2}-5x}}{\sqrt{3+5x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -(Sqrt[2/5]*ArcTan[Sqrt[15/2 - 5*x]/Sqrt[3 + 5*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.17, size = 39, normalized size = 1.50

method	result	size
default	$\frac{\sqrt{(3-2x)(3+5x)}\,\sqrt{10}\,\arcsin\left(\frac{20x}{21}-\frac{3}{7}\right)}{10\sqrt{3-2x}\,\sqrt{3+5x}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*((3-2*x)*(3+5*x))^(1/2)/(3-2*x)^(1/2)/(3+5*x)^(1/2)*10^(1/2)*arcsin(20
/21*x-3/7)
```

Maxima [A]
time = 0.50, size = 11, normalized size = 0.42

$$-\frac{1}{10}\sqrt{10}\arcsin\left(-\frac{20}{21}x+\frac{3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(10)*arcsin(-20/21*x + 3/7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.
time = 1.23, size = 44, normalized size = 1.69

$$-\frac{1}{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{5x+3}\sqrt{-2x+3}-3\sqrt{5}\sqrt{2}}{10x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2)*arctan(1/10*(sqrt(5)*sqrt(2)*sqrt(5*x + 3)*sqrt(-2*x +
 3) - 3*sqrt(5)*sqrt(2))/x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 56, normalized size = 2.15

$$\begin{cases} -\frac{\sqrt{10}\, i \operatorname{acosh}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{for } \left|x+\frac{3}{5}\right| > \frac{21}{10} \\ \frac{\sqrt{10}\operatorname{asin}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/(3+5*x)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(10)*I*acosh(sqrt(210)*sqrt(x + 3/5)/21)/5, Abs(x + 3/5) >
21/10), (sqrt(10)*asin(sqrt(210)*sqrt(x + 3/5)/21)/5, True))
```

Giac [A]
time = 0.85, size = 21, normalized size = 0.81

$$\frac{1}{5}\sqrt{5}\sqrt{2}\arcsin\left(\frac{1}{21}\sqrt{42}\sqrt{5x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arcsin(1/21*sqrt(42)*sqrt(5*x + 3))
```

Mupad [B]
time = 0.08, size = 40, normalized size = 1.54

$$-\frac{2\sqrt{10}\operatorname{atan}\left(\frac{\sqrt{10}\left(\sqrt{3}-\sqrt{3-2x}\right)}{2\left(\sqrt{3}-\sqrt{5x+3}\right)}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(1/2)*(5*x + 3)^(1/2)),x)
```

```
[Out] -(2*10^(1/2)*atan((10^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*(3^(1/2) - (5*x
 + 3)^(1/2)))))/5
```

3.1558 $\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a-bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[Out] -2*arctan(d^(1/2)*(-b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {65, 223, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{a-bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx}\right)}{b}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx}}{\sqrt{c+dx}}\right)}{b}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [A]
time = 0.07, size = 43, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a-bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a - b*x])])/(Sqrt[b]*Sqrt[d
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(31) = 62$.
time = 0.17, size = 84, normalized size = 1.95

method	result	size
default	$\frac{\sqrt{(-bx+a)(dx+c)}\,\arctan\left(\frac{\sqrt{bd}\,\left(x-\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\,x^2+(ad-bc)\,x+ac}}\right)}{\sqrt{-bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b*x+a)*(d*x+c))^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2)/(b*d)^(1/2)*arctan((b
*d)^(1/2)*(x-1/2*(a*d-b*c)/b/d)/(-b*d*x^2+(a*d-b*c)*x+a*c)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(31) = 62.
time = 1.10, size = 185, normalized size = 4.30

$$\left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,(2\,bdx+bc-ad)\sqrt{-bd}\,\sqrt{-bx+a}\,\sqrt{dx+c}+8\,(b^2cd-abd^2)x\right)}{2\,bd},-\frac{\sqrt{bd}\arctan\left(\frac{(2\,bdx+bc-ad)\sqrt{bd}\,\sqrt{-bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2-abcd+(b^2cd-abd^2)x)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*(2*b
*d*x + b*c - a*d)*sqrt(-b*d)*sqrt(-b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d - a*
b*d^2)*x))/(b*d), -sqrt(b*d)*arctan(1/2*(2*b*d*x + b*c - a*d)*sqrt(b*d)*sqrt
(-b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x))/(
b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-bx}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x)*sqrt(c + d*x)), x)
```

Giac [A]
time = 2.09, size = 54, normalized size = 1.26

$$\frac{2\,b\log\left(\left|-\sqrt{-bd}\,\sqrt{-bx+a}+\sqrt{b^2c+(bx-a)bd+abd}\right|\right)}{\sqrt{-bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*b*log(abs(-sqrt(-b*d)*sqrt(-b*x + a) + sqrt(b^2*c + (b*x - a)*b*d + a*b*d
)))/(sqrt(-b*d)*abs(b))
```

Mupad [B]
time = 0.34, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{a-b\,x}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((d*((a - b*x)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x)^(1/2) - c^
(1/2)))))/(b*d)^(1/2)
```

3.1559 $\int (a+bx)^{3/2}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=457

$$-\frac{108(bc-ad)^2\sqrt{a+bx}\,\sqrt[3]{c+dx}}{935bd^2}+\frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}-\frac{108\ 3^{3/4}\sqrt{2}}{}$$

[Out] 12/187*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/3)/b/d+6/17*(b*x+a)^(5/2)*(d*x+c)^(1/3)/b-108/935*(-a*d+b*c)^2*(d*x+c)^(1/3)*(b*x+a)^(1/2)/b/d^2-108/935*3^(3/4)*(-a*d+b*c)^3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(4/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.60, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$-\frac{108\ 3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^3\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{935b^{4/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{108\sqrt{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)^2}{935bd^2}+\frac{12(a+bx)^{3/2}\sqrt[3]{c+dx}\,(bc-ad)}{187bd}+\frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]

[Out] (-108*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/3))/(935*b*d^2) + (12*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/3))/(187*b*d) + (6*(a + b*x)^(5/2)*(c + d*x)^(1/3))/(17*b) - (108*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^3*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(935*b^(4/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[3]{c+dx}\,dx = \frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b} + \frac{(2(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}dx}{17b}$$

$$= \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + \frac{6(a+bx)^{5/2}\sqrt[3]{c+dx}}{17b} - \frac{(18(bc-ad)^2)\int}{187bd}$$

$$= -\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2} + \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + \frac{6(a+}{}$$

$$= -\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2} + \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + \frac{6(a+}{}$$

$$= -\frac{108(bc-ad)^2\sqrt{a+bx}\sqrt[3]{c+dx}}{935bd^2} + \frac{12(bc-ad)(a+bx)^{3/2}\sqrt[3]{c+dx}}{187bd} + \frac{6(a+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.16

$$\frac{2(a+bx)^{5/2}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/3), x)
```

3.1560 $\int \sqrt{a+bx}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=419

$$\frac{12(bc-ad)\sqrt{a+bx}\sqrt[3]{c+dx}}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}+\frac{12\ 3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c}\right.}{}$$

```
[Out] 6/11*(b*x+a)^(3/2)*(d*x+c)^(1/3)/b+12/55*(-a*d+b*c)*(d*x+c)^(1/3)*(b*x+a)^(
1/2)/b/d+12/55*3^(3/4)*(-a*d+b*c)^2*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
)*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2
/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1
/2))/b^(4/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

Rubi [A]

time = 0.27, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$\frac{12\ 3^{3/4}\sqrt{2-\sqrt{3}}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{55b^{4/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{12\sqrt{a+bx}\sqrt[3]{c+dx}(bc-ad)}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/3),x]
```

```
[Out] (12*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*b*d) + (6*(a + b*x)^(3/2
)*(c + d*x)^(1/3))/(11*b) + (12*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(4/3)*d^2*Sq
rt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt[3]{c+dx}\,dx = \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}+\frac{(2(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}dx}{11b}$$

$$=\frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}-\frac{(6(bc-ad)^2)\int \frac{}{\sqrt{a+}}}{55bd}$$

$$=\frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}-\frac{(18(bc-ad)^2)\,\text{Subst}\Big(}{}$$

$$=\frac{12(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55bd}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11b}+\frac{12\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.17

$$\frac{2(a+bx)^{3/2}\sqrt[3]{c+dx}\ {}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/3),x)
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(1/3), x)
```

3.1561 $\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=381

$$\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-\cdots}\right.}}}{5b^{4/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

```
[Out] 6/5*(d*x+c)^(1/3)*(b*x+a)^(1/2)/b-4/5*3^(3/4)*(-a*d+b*c)*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(
1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*
x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))/b^(4/3)/d/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b}-\frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{5b^{4/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/Sqrt[a + b*x],x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*b) - (4*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(4
/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b} + \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{5b}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b} + \frac{(6(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{5bd}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5b} - \frac{4\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-}{}}}{5b^{4/3}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$\frac{2\sqrt{a+bx}\sqrt[3]{c+dx}\ {}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{1/3}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(1/2), x)
```

3.1562 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=366

$$-\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}}-\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\cdots}}}{\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-a\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-a\cdots}\right.}}}$$

```
[Out] -2*(d*x+c)^(1/3)/b/(b*x+a)^(1/2)-4/3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3
)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(
2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3
)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(
1/2))*3^(3/4)/b^(4/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^
2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 65, 225}

$$-\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/(b*Sqrt[a + b*x]) - (4*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sqrt[a + b*
x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))
/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{3b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{b\sqrt{a+bx}} - \frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc\cdots}\right.}}}{\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$-\frac{2\sqrt[3]{c+dx}\ {}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\ \sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

<u>**Sympy**</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(3/2),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(3/2), x)
```

<u>**Giac**</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(3/2), x)
```

<u>**Mupad**</u> **[F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(3/2), x)
```

3.1563 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=417

$$-\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}-\frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}+\frac{4\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}\cdots}{9\sqrt[4]{3}\,b^{4/3}(bc-ad)\sqrt{a+\cdots}}$$

[Out] -2/3*(d*x+c)^(1/3)/b/(b*x+a)^(3/2)-4/9*d*(d*x+c)^(1/3)/b/(-a*d+b*c)/(b*x+a)^(1/2)+4/27*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(4/3)/(-a*d+b*c)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.26, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 53, 65, 225}

$$\frac{4\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\,(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{4d\sqrt[3]{c+dx}}{9b\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]

[Out] (-2*(c + d*x)^(1/3))/(3*b*(a + b*x)^(3/2)) - (4*d*(c + d*x)^(1/3))/(9*b*(b*c - a*d)*Sqrt[a + b*x]) + (4*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(4/3)*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} + \frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{9b}$$
$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} - \frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(2d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{27b(bc-ad)}$$
$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} - \frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(2d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right.}{9b(bc-ad)}$$
$$= -\frac{2\sqrt[3]{c+dx}}{3b(a+bx)^{3/2}} - \frac{4d\sqrt[3]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} + \frac{4\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.18

$$-\frac{2\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(1/3)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{(a + bx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+dx)^{1/3}}{(a+bx)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(5/2), x)
```

3.1564 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=457

$$-\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}-\frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}}+\frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}}-\frac{28\sqrt{2-\sqrt{3}}\,d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[Out] -2/5*(d*x+c)^(1/3)/b/(b*x+a)^(5/2)-4/45*d*(d*x+c)^(1/3)/b/(-a*d+b*c)/(b*x+a
)^(3/2)+28/135*d^2*(d*x+c)^(1/3)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)-28/405*d^2*((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-
a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(
1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(
1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(
1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(4/3)/(-a*d+b*c)^2/(b*x
+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(
1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 53, 65, 225}

$$-\frac{28\sqrt{2-\sqrt{3}}\,d^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{135\sqrt[4]{3}\,b^{4/3}\sqrt{a+bx}\,(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{28d^2\sqrt[3]{c+dx}}{135b\sqrt{a+bx}\,(bc-ad)^2}-\frac{4d\sqrt[3]{c+dx}}{45b(a+bx)^{3/2}(bc-ad)}-\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/2)) - (4*d*(c + d*x)^(1/3))/(45*b*(b
*c - a*d)*(a + b*x)^(3/2)) + (28*d^2*(c + d*x)^(1/3))/(135*b*(b*c - a*d)^2*
Sqrt[a + b*x]) - (28*Sqrt[2 - Sqrt[3]]*d^2*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)
^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))], -7 + 4*Sqrt[3]])/(135*3^(1/4)*b^(4/3)*(b*c - a*d)^2*Sqrt[a + b*x
]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} + \frac{(2d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx}{15b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} - \frac{(14d^2)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{135b(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} + \frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}} + \frac{(14d^3)\int \frac{}{\sqrt{a-}}}{405b(b}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} + \frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}} + \frac{(14d^2)\,\text{Subst}\Big(}{}$$

$$= -\frac{2\sqrt[3]{c+dx}}{5b(a+bx)^{5/2}} - \frac{4d\sqrt[3]{c+dx}}{45b(bc-ad)(a+bx)^{3/2}} + \frac{28d^2\sqrt[3]{c+dx}}{135b(bc-ad)^2\sqrt{a+bx}} - \frac{28\sqrt{2-\sqrt{3}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.16

$$-\frac{2\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{3};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/3)*Hypergeometric2F1[-5/2, -1/3, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{(a + bx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(7/2), x)
```

3.1565 $\int \frac{(a+bx)^{3/2}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=839

$$-\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2}+\frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d}-\frac{162(bc-ad)^2\sqrt{a+bx}}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right.}$$

```
[Out] 6/13*(b*x+a)^(3/2)*(d*x+c)^(2/3)/d-54/91*(-a*d+b*c)*(d*x+c)^(2/3)*(b*x+a)^(
1/2)/d^2-162/91*(-a*d+b*c)^2*(b*x+a)^(1/2)/b^(2/3)/d^2/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))-54/91*3^(3/4)*(-a*d+b*c)^(7/3)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2
*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(
1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(
1/2)))^2)^(1/2)/b^(2/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1
/2)))^2)^(1/2)+81/91*3^(1/4)*(-a*d+b*c)^(7/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a
*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))/b^(2/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 310, 225, 1893}

$$\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)(bc-ad)^{7/3}}{91b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{54\sqrt{2}\,3^{3/4}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)(bc-ad)^{7/3}}{91b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{162\sqrt{a+bx}\,(bc-ad)^2}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{54\sqrt{a+bx}\,(c+dx)^{2/3}(bc-ad)}{91d^2}+\frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]
```

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(2/3))/(91*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(2/3))/(13*d) - (162*(b*c - a*d)^2*Sqrt[a + b*x])/(91*b^(2/3)*
d^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) + (81*3^(1
```

```
/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))], -7 + 4*Sqrt[3]])/(91*b^(2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^
(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (54*Sqrt[2]*3^(3/4)*(b*c - a*d
)^(7/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(91*b^(
2/3)*d^3*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[3]{c+dx}}\,dx = \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx}{13d}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} + \frac{(27(bc-ad)^2)\int \frac{}{\sqrt{a+b}}}{91d^2}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} + \frac{(81(bc-ad)^2)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{(81(bc-ad)^2)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,(c+dx)^{2/3}}{91d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{2/3}}{13d} - \frac{162(bc-a}{91b^{2/3}d^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/3),x]
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/3), x)
```

3.1566 $\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=804

$$\frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d}+\frac{18(bc-ad)\sqrt{a+bx}}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(bc-ad)^{4/3}\left(\sqrt[3]{b}\right.}{}$$

```
[Out] 6/7*(d*x+c)^(2/3)*(b*x+a)^(1/2)/d+18/7*(-a*d+b*c)*(b*x+a)^(1/2)/b^(2/3)/d/(
-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))+6/7*3^(3/4)*(-a*d+b*c)
^(4/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)
^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/
3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c
)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+
b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^(2/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/
3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b
*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)-9/7*3^(1/4)*(-a*d+b*c)^(4/3)*((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/
3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I
*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)
*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 804, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 310, 225, 1893}

$$-\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)(bc-ad)^{4/3}}{7b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{6\sqrt{2}\,3^{3/4}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)(bc-ad)^{4/3}}{7b^{2/3}d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{18\sqrt{a+bx}\,(bc-ad)}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(2/3))/(7*d) + (18*(b*c - a*d)*Sqrt[a + b*x])/(7
*b^(2/3)*d*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (
9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*
```

```
(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d
*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c -
a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (6*Sqrt[2]*3^(3/4)*(b*c -
 a*d)^(4/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(7*
b^(2/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[3]{c+dx}}\,dx}{7d}$$

$$= \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{7d^2}$$

$$= \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} + \frac{(9(bc-ad))\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right.}{7\sqrt[3]{b}\,d^2}$$

$$= \frac{6\sqrt{a+bx}\,(c+dx)^{2/3}}{7d} + \frac{18(bc-ad)\sqrt{a+bx}}{7b^{2/3}d\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{9\sqrt[4]{3}\,\sqrt{2+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/3), x)
```

3.1567 $\int \frac{1}{\sqrt{a+bx}\,\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=762

$$-\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\ \cdots}{\cdots}$$

```
[Out] -6*(b*x+a)^(1/2)/b^(2/3)/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2
)))-2*3^(3/4)*(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*Ell
ipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x
+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)
^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1
/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)/b^(2/3)/d/(b*x+a)^
(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)+3*3^(1/4)*(-a*d+b*c)^
(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^
(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3
)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(
d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3
)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/d/(b*x+a)^(1/2)/(
-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c
)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 762, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {65, 310, 225, 1893}

$$-\frac{2\sqrt{2}\,3^{3/4}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/3)),x]
```

```
[Out] (-6*Sqrt[a + b*x])/(b^(2/3)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))) + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^(1/3)*((b*c - a*d)
^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(
```

```
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sq
rt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1
- Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*3
^(3/4)*(b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*
Sqrt[3]])/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx = \frac{3\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{\sqrt[3]{b}\,d} + \frac{\left(3\sqrt{2\left(2+\sqrt{3}\right)}\ \sqrt[3]{\cdots}\right.}{\cdots}$$

$$= -\frac{6\sqrt{a+bx}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\ \sqrt[3]{bc-ad}\ \left(\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\ (dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx}\,\sqrt[3]{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,\left(c+d\,x\right)^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/3)), x)
```

3.1568 $\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=796

$$-\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}}-\frac{2d\sqrt{a+bx}}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}\right.}{}$$

[Out] -2*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(1/2)-2*d*(b*x+a)^(1/2)/b^(2/3)/(-a*d+b*c)/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))-2/3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)+3^(1/4)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.50, antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 310, 225, 1893}

$$-\frac{2\sqrt{a+bx}\,d}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt{2}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]

[Out] (-2*(c + d*x)^(2/3))/((b*c - a*d)*Sqrt[a + b*x]) - (2*d*Sqrt[a + b*x])/(b^(2/3)*(b*c - a*d)*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^

```
(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c +
 d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))],
-7 + 4*Sqrt[3]])/(b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*
d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) - (2*Sqrt[2]*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2
/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
```

```
/; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx = -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,x}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{\sqrt[3]{b}\,(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt{a+bx}}{b^{2/3}(bc-ad)\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 71, normalized size = 0.09

$$-\frac{2\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/3)), x)
```

3.1569 $\int \frac{1}{(a+bx)^{5/2}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=842

$$-\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}}+\frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}}+\frac{10d^2\sqrt{a+bx}}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\cdots$$

[Out] -2/3*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(3/2)+10/9*d*(d*x+c)^(2/3)/(-a*d+b*c)^2/(b*x+a)^(1/2)+10/9*d^2*(b*x+a)^(1/2)/b^(2/3)/(-a*d+b*c)^2/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))+10/27*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)-5/9*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticE((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/b^(2/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.57, antiderivative size = 842, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 310, 225, 1893}

$$\frac{10\sqrt{a+bx}\,d^2}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)d}{3\,3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{10\sqrt{2}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)d}{9\sqrt[4]{3}\,b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{10(c+dx)^{2/3}d}{9(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]

[Out] (-2*(c + d*x)^(2/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (10*d*(c + d*x)^(2/3))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (10*d^2*Sqrt[a + b*x])/(9*b^(2/3)*(b*c - a*d)^2*((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))) - (5

```
*Sqrt[2 + Sqrt[3]]*d*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b
*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c +
d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*
EllipticE[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt
[3]])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d
)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c
- a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)]) + (10*Sqrt[2]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(2/3)*(b*c - a*d
)^(5/3)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
```

```
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[3]{c+dx}}\,dx = -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)^{3/2}\sqrt[3]{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[3]{c+dx}}\,d}{27(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(5d)\mathrm{Subst}\left(\int \frac{x}{\sqrt{a-\frac{bc}{d}+\frac{b}{}}}\right.}{9(bc-ad}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(5d)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc}}{\sqrt{a-\frac{bc}{d}}}\right.}{9\sqrt[3]{b}\,(b}$$

$$= -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{10d^2\sqrt{}}{9b^{2/3}(bc-ad)^2\left(\left(1-\sqrt{3}\right)\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.09

$$-\frac{2\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x]
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(2/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/3)), x)
```

3.1570 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=416

$$-\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d}-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{}$$

```
[Out] 6/11*(b*x+a)^(3/2)*(d*x+c)^(1/3)/d-54/55*(-a*d+b*c)*(d*x+c)^(1/3)*(b*x+a)^(
1/2)/d^2-54/55*3^(3/4)*(-a*d+b*c)^2*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
)*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2
/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)
*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1
/2))/b^(1/3)/d^3/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

Rubi [A]
time = 0.28, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$-\frac{54\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)^2\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{55\sqrt[3]{b}\,d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{54\sqrt{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{55d^2}+\frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]
```

```
[Out] (-54*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/3))/(55*d^2) + (6*(a + b*x)^(3/
2)*(c + d*x)^(1/3))/(11*d) - (54*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^2*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(55*b^(1/3)*d^3*S
qrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}}\,dx = \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx}{11d}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} + \frac{(27(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}}}{55d^2}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} + \frac{(81(bc-ad)^2)\,\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{54(bc-ad)\sqrt{a+bx}\,\sqrt[3]{c+dx}}{55d^2} + \frac{6(a+bx)^{3/2}\sqrt[3]{c+dx}}{11d} - \frac{54\ 3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(2/3),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(2/3), x)
```

3.1571 $\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=381

$$\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d}+\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc\cdots}\right.}}}{5\sqrt[3]{b}\,d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{bc\cdots}}{\left(\left(1-\cdots\right.\right.}}}$$

```
[Out] 6/5*(d*x+c)^(1/3)*(b*x+a)^(1/2)/d+6/5*3^(3/4)*(-a*d+b*c)*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(
1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*
x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/3)/d^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/
3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 225}

$$\frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{5\sqrt[3]{b}\,d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] (6*Sqrt[a + b*x]*(c + d*x)^(1/3))/(5*d) + (6*3^(3/4)*Sqrt[2 - Sqrt[3]]*(b*c
 - a*d)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/
3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/(
(1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[
3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^(1
/3)*d^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3
)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}}\,dx = \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{5d}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{5d^2}$$

$$= \frac{6\sqrt{a+bx}\,\sqrt[3]{c+dx}}{5d} + \frac{6\,3^{3/4}\sqrt{2-\sqrt{3}}\,(bc-ad)\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-}{}}}{5\sqrt[3]{b}\,d^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.19

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(2/3),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(2/3), x)
```

3.1572 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=345

$$-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}{\sqrt[3]{b}\,d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{}\right.}}}$$

```
[Out] -2*3^(3/4)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF((-b^(1/3)*(d*
x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1
/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))/b^(1/3)/d/(b*x+a)^(1
/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(
d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 225}

$$-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\,d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*3^(3/4)*Sqrt[2 - Sqrt[3]]*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2
/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7
 + 4*Sqrt[3]])/(b^(1/3)*d*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^
(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx = \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{2\;3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-\cdots}\right.}}}{\sqrt[3]{b}\,d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\cdots}{\left(\left(1\cdots\right.\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.21

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(2/3)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[1/2, 2
/3, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{2}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\left(c+dx\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(2/3),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(2/3)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(2/3)), x)
```

3.1573 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=383

$$-\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}}}{\sqrt[4]{3}\sqrt[3]{b}(bc-ad)\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{bc-}}{\left(\left(1-\right.\right.}}}$$

```
[Out] -2*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(1/2)+2/3*((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))*EllipticF((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a
*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))
/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2
)-1/2*2^(1/2))*3^(3/4)/b^(1/3)/(-a*d+b*c)/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}(bc-ad)\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[3]{c+dx}}{\sqrt{a+bx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[2 - Sqrt[3]]*((b
*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3
)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3
])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*(
b*c - a*d)*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(
1/3)*(c + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx = -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}\,dx}{3(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^3}{d}}}\,dx, x, \sqrt[3]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[3]{c+dx}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-a\cdots}{\cdots}}}{\sqrt[4]{3}\,\sqrt[3]{b}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.19

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(2/3)), x)
```

3.1574 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=421

$$-\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}}-\frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^2 \cdots}{((\cdots}}}{9\sqrt[4]{3}\sqrt[3]{b}\,(bc \cdots}$$

```
[Out] -2/3*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(3/2)+14/9*d*(d*x+c)^(1/3)/(-a*d+b*c)
^2/(b*x+a)^(1/2)-14/27*d*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*EllipticF
((-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1+3^(1/2)))/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)
*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/(-b^(1/3)*(d*x+c)^(1
/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4
)/b^(1/3)/(-a*d+b*c)^2/(b*x+a)^(1/2)/(-(-a*d+b*c)^(1/3)*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3))/(-b^(1/3)*(d*x+c)^(1/3)+(-a*d+b*c)^(1/3)*(1-3^(1/2)))
^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 225}

$$-\frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}{\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\middle|-7+4\sqrt{3}\right)}{9\sqrt[4]{3}\sqrt[3]{b}\sqrt{a+bx}\,(bc-ad)^2\sqrt{-\frac{\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{14d\sqrt[3]{c+dx}}{9\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt[3]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*(c + d*x)^(1/3))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (14*d*(c + d*x)^(1/3
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) - (14*Sqrt[2 - Sqrt[3]]*d*((b*c - a*d)^(
1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((1 - Sqrt[3])*(b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*(b*
c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))/((1 - Sqrt[3])*(b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))], -7 + 4*Sqrt[3]])/(9*3^(1/4)*b^(1/3)*(b*c - a*d
)^2*Sqrt[a + b*x]*Sqrt[-(((b*c - a*d)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((1 - Sqrt[3])*(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{2/3}}\,dx = -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{2/3}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(7d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{2/3}}}{27(bc-ad)^2}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(7d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}}}\right.}{9(bc-}$$

$$= -\frac{2\sqrt[3]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d\sqrt[3]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{14\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{bc-a}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.17

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(-\frac{3}{2},\frac{2}{3};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(2/3)), x)
```

3.1575 $\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=219

$$\frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}}+\frac{(bc-$$

```
[Out] 1/6*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(1/3)/b/d+1/2*(b*x+a)^(5/3)*(d*x+c)^(1
/3)/b+1/18*(-a*d+b*c)^2*ln(d*x+c)/b^(4/3)/d^(5/3)+1/6*(-a*d+b*c)^2*ln(-1+d^
(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(4/3)/d^(5/3)+1/9*(-a*d+b*c)^2
*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3)*3^(1/2)
)/b^(4/3)/d^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {52, 61}

$$\frac{(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*b*d) + ((a + b*x)^(5/3)*(c
 + d*x)^(1/3))/(2*b) + ((b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*
x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) +
 ((b*c - a*d)^2*Log[c + d*x])/(18*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[-1
+ (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(4/3)*d^(5/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6b}$$

$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a+bx} \ldots}}{9bd}$$

$$= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}} \ldots\right.}{3\sqrt{3} \ldots}$$

Mathematica [A]
time = 0.60, size = 220, normalized size = 1.00

$$\frac{(bc-ad)^2\left(\frac{3\sqrt[3]{b}\,d^{2/3}(a+bx)^{2/3}\sqrt[3]{c+dx}\,(2ad+b(c+3dx))}{(bc-ad)^2} - 2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{d} - \frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right) - \log\left(d^{2/3} + \frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}} + \frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)\right)}{18b^{4/3}d^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
[Out] ((b*c - a*d)^2*((3*b^(1/3)*d^(2/3)*(a + b*x)^(2/3)*(c + d*x)^(1/3)*(2*a*d +
 b*(c + 3*d*x)))/(b*c - a*d)^2 - 2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)
^(1/3))/(d^(1/3)*(a + b*x)^(1/3)))/Sqrt[3]] + 2*Log[d^(1/3) - (b^(1/3)*(c +
 d*x)^(1/3))/(a + b*x)^(1/3)] - Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1
/3))/(a + b*x)^(1/3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x)^(2/3)]))/(18*b^(
4/3)*d^(5/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

Fricas [**A**]
time = 1.00, size = 717, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] [1/18*(3*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(b*d^2)^(1
/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1/3)*(b*x + a)^(2/3)*(
d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (b*
d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(1/3)*(b*d*x + a*d))*s
qrt(-(b*d^2)^(1/3)/b)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*lo
g(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)
) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(
1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x + b^2*c*d^2 + 2*a*b*d^3)*(b
*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3), -1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt(
(b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*
d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x +
a))/(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3
) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) - 3*(3*b^2*d^3*x + b^2*c*d^2 +
2*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)*(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(2/3)*(c + d*x)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{2/3}\,(c+d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(2/3)*(c + d*x)^(1/3), x)
```

3.1576 $\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=172

$$\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(-\ldots\right.}{2b\ldots}$$

```
[Out] (b*x+a)^(2/3)*(d*x+c)^(1/3)/b-1/6*(-a*d+b*c)*ln(d*x+c)/b^(4/3)/d^(2/3)-1/2*
(-a*d+b*c)*ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(4/3)/d^(2/
3)-1/3*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x
+c)^(1/3)*3^(1/2))/b^(4/3)/d^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$-\frac{(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/b - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^
(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(4/3)
*d^(2/3)) - ((b*c - a*d)*Log[c + d*x])/(6*b^(4/3)*d^(2/3)) - ((b*c - a*d)*L
og[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3)*d^
(2/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3b} \\
&= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}} - \frac{(bc-ad)\log(c+}{6b^{4/3}d^{2/3}}
\end{aligned}
$$

Mathematica [A]
time = 9.45, size = 278, normalized size = 1.62

$$
\frac{(a+bx)^{2/3}\left(6\sqrt[3]{b}\,(d(a+bx))^{2/3}\sqrt[3]{c+dx}+2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}{2\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)+(-2bc+2ad)\log\left(\sqrt[3]{d(a+bx)}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)+bc\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\sqrt[3]{d(a+bx)}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)-ad\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\sqrt[3]{d(a+bx)}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)\right)}{6b^{4/3}(d(a+bx))^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(6*b^(1/3)*(d*(a + b*x))^(2/3)*(c + d*x)^(1/3) + 2*Sqrt[3]
*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(2*(d*(a + b*x))^(1/3
) + b^(1/3)*(c + d*x)^(1/3))] + (-2*b*c + 2*a*d)*Log[(d*(a + b*x))^(1/3) -
b^(1/3)*(c + d*x)^(1/3)] + b*c*Log[(d*(a + b*x))^(2/3) + b^(1/3)*(d*(a + b*
x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)] - a*d*Log[(d*(a + b*x)
)^(2/3) + b^(1/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(
2/3)]))/(6*b^(4/3)*(d*(a + b*x))^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(132) = 264$.
time = 0.77, size = 596, normalized size = 3.47

$$\left[\frac{6\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 3\sqrt{\frac{1}{3}}\,(b^2cd-abd^2)\sqrt{-\frac{(bd^2)^{\frac{1}{3}}}{b}}\log\left(-3\,bd^2x - 2\,bcd - ad^2 + 3\,(bd^2)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d + 3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd - (bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - (bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{(bd^2)^{\frac{1}{3}}}{b}}\right) - 2\,(bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + (bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{6\,b^2d^2}, \frac{6\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 + 6\sqrt{\frac{1}{3}}\,(b^2cd-abd^2)\sqrt{\frac{(bd^2)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{(bd^2)^{\frac{1}{3}}}{b}}}{bd^2x+ad^2}\right) - 2\,(bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + (bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{6\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt(-(b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1
/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d - (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(
1/3)*(b*d*x + a*d))*sqrt(-(b*d^2)^(1/3)/b)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*l
og(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a
)) + (b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (
b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))
/(b*x + a)))/(b^2*d^2), 1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 + 6*sq
rt(1/3)*(b^2*c*d - a*b*d^2)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^
2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqr
t((b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + (
b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x
+ a)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{\sqrt[3]{a + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(1/3),x)
```

```
[Out] int((c + d*x)^(1/3)/(a + b*x)^(1/3), x)
```

3.1577 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{b^{4/3}}-\frac{\sqrt[3]{d}\,\log(c+dx)}{2b^{4/3}}-\frac{3\sqrt[3]{d}\,\log\left(-1+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{2b^{4/3}}$$

```
[Out] -3*(d*x+c)^(1/3)/b/(b*x+a)^(1/3)-1/2*d^(1/3)*ln(d*x+c)/b^(4/3)-3/2*d^(1/3)*
ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(4/3)-d^(1/3)*arctan(1
/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3)*3^(1/2))*3^(1/2)
/b^(4/3)
```

Rubi [A]

time = 0.02, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 61}

$$-\frac{\sqrt{3}\,\sqrt[3]{d}\,\text{ArcTan}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{d}\,\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\,\log(c+dx)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) - (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/b^(4/3)
 - (d^(1/3)*Log[c + d*x])/(2*b^(4/3)) - (3*d^(1/3)*Log[-1 + (d^(1/3)*(a + b
*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{b}$$
$$= -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}} - \frac{\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{b^{4/3}} - \frac{\sqrt[3]{d}\,\log(c+dx)}{2b^{4/3}} - \frac{3\sqrt[3]{d}}{\ldots}$$

Mathematica [A]
time = 0.18, size = 191, normalized size = 1.28

$$\frac{-\frac{6\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}} + 2\sqrt{3}\,\sqrt[3]{d}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}}{\sqrt{3}}\right) - 2\sqrt[3]{d}\,\log\left(\sqrt[3]{d} - \frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right) + \sqrt[3]{d}\,\log\left(d^{2/3} + \frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}} + \frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
[Out] ((-6*b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + 2*Sqrt[3]*d^(1/3)*ArcTan[(1
 + (2*b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3)))/Sqrt[3]] - 2*d^(1
/3)*Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)] + d^(1/3)*Log[
d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c +
 d*x)^(2/3))/(a + b*x)^(2/3)])/(2*b^(4/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
[Out] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.
time = 0.88, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}b\left(-\frac{d}{b}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right)+(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{2}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bx+a}\right)-2(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)+6(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{2(b^2x+ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(b*x + a)*(-d/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)*b*(-d/b)^(2/3) + sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) +
(b*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(2/3) - (b*x + a)^(2/3)*(d*x +
 c)^(1/3)*(-d/b)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(b*x + a)) - 2*(b
*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(1/3) + (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a)) + 6*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*x + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(4/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(c+d\,x)^{1/3}}{(a+b\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(4/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(4/3), x)
```

3.1578 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

```
[Out] -3/4*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx = -\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{3(dx+c)^{\frac{4}{3}}}{4(bx+a)^{\frac{4}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/3),x,method=_RETURNVERBOSE)
[Out] 3/4/(b*x+a)^(4/3)*(d*x+c)^(4/3)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.62, size = 65, normalized size = 2.03

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}{4\,(a^2bc-a^3d+(b^3c-ab^2d)x^2+2\,(ab^2c-a^2bd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
[Out] -3/4*(b*x + a)^(2/3)*(d*x + c)^(4/3)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x
^2 + 2*(a*b^2*c - a^2*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

Mupad [B]
time = 0.71, size = 92, normalized size = 2.88

$$-\frac{\left(\frac{3\,c}{4\,b^2\,c-4\,a\,b\,d}+\frac{3\,d\,x}{4\,b^2\,c-4\,a\,b\,d}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}-\frac{\left(4\,a^2\,d-4\,a\,b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}{4\,b^2\,c-4\,a\,b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(7/3),x)
```

```
[Out] -(((3*c)/(4*b^2*c - 4*a*b*d) + (3*d*x)/(4*b^2*c - 4*a*b*d))*(c + d*x)^(1/3)
)/(x*(a + b*x)^(1/3) - ((4*a^2*d - 4*a*b*c)*(a + b*x)^(1/3))/(4*b^2*c - 4*a
*b*d))
```

3.1579 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}$$

```
[Out] -3/7*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(7/3)+9/28*d*(d*x+c)^(4/3)/(-a*d+b*c)
^2/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(4/3)
)/(28*(b*c - a*d)^2*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx = -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{7(bc-ad)}$$
$$= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{4/3}(-4bc+7ad+3bdx)}{28(bc-ad)^2(a+bx)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
[Out] (3*(c + d*x)^(4/3)*(-4*b*c + 7*a*d + 3*b*d*x))/(28*(b*c - a*d)^2*(a + b*x)^
(7/3))
```

Maple [A]
time = 0.19, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{3(dx+c)^{\frac{4}{3}}(3bdx+7ad-4bc)}{28(bx+a)^{\frac{7}{3}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(10/3),x,method=_RETURNVERBOSE)
[Out] 3/28*(d*x+c)^(4/3)*(3*b*d*x+7*a*d-4*b*c)/(b*x+a)^(7/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(54) = 108.
time = 0.88, size = 175, normalized size = 2.65

$$\frac{3\left(3\,bd^{2}x^{2}-4\,bc^{2}+7\,acd-(bcd-7\,ad^{2})x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{28\left(a^{3}b^{2}c^{2}-2\,a^{4}bcd+a^{5}d^{2}+(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2})x^{3}+3\,(ab^{4}c^{2}-2\,a^{2}b^{3}cd+a^{3}b^{2}d^{2})x^{2}+3\,(a^{2}b^{3}c^{2}-2\,a^{3}b^{2}cd+a^{4}bd^{2})x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 3/28*(3*b*d^2*x^2 - 4*b*c^2 + 7*a*c*d - (b*c*d - 7*a*d^2)*x)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4
*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 +
 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(10/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(10/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

Mupad [B]
time = 1.03, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/3}\left(\frac{x\,(21\,a\,d^{2}-3\,b\,c\,d)}{28\,b^{2}\,(a\,d-b\,c)^{2}}-\frac{12\,b\,c^{2}-21\,a\,c\,d}{28\,b^{2}\,(a\,d-b\,c)^{2}}+\frac{9\,d^{2}\,x^{2}}{28\,b\,(a\,d-b\,c)^{2}}\right)}{x^{2}\,(a+b\,x)^{1/3}+\frac{a^{2}\,(a+b\,x)^{1/3}}{b^{2}}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(10/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((x*(21*a*d^2 - 3*b*c*d))/(28*b^2*(a*d - b*c)^2) - (12*b*c
^2 - 21*a*c*d)/(28*b^2*(a*d - b*c)^2) + (9*d^2*x^2)/(28*b*(a*d - b*c)^2)))/
(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*x)^(1/3))/
b)
```

3.1580 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}+\frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}}-\frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}$$

```
[Out] -3/10*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(10/3)+9/35*d*(d*x+c)^(4/3)/(-a*d+b*
c)^2/(b*x+a)^(7/3)-27/140*d^2*(d*x+c)^(4/3)/(-a*d+b*c)^3/(b*x+a)^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (9*d*(c + d*x)^(4/
3))/(35*(b*c - a*d)^2*(a + b*x)^(7/3)) - (27*d^2*(c + d*x)^(4/3))/(140*(b*c
 - a*d)^3*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx = -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{5(bc-ad)}$$
$$= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} + \frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}} + \frac{(9d^2)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{35(bc-ad)^2}$$
$$= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}} + \frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}} - \frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{4/3}\left(35d^2-\frac{40bd(c+dx)}{a+bx}+\frac{14b^2(c+dx)^2}{(a+bx)^2}\right)}{140(bc-ad)^3(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(35*d^2 - (40*b*d*(c + d*x))/(a + b*x) + (14*b^2*(c + d
*x)^2)/(a + b*x)^2))/(140*(b*c - a*d)^3*(a + b*x)^(4/3))
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{3(dx+c)^{\frac{4}{3}}\left(9b^2x^2d^2+30ab\,d^2x-12b^2cdx+35a^2d^2-40abcd+14b^2c^2\right)}{140(bx+a)^{\frac{10}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(13/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/140*(d*x+c)^(4/3)*(9*b^2*d^2*x^2+30*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-40*
a*b*c*d+14*b^2*c^2)/(b*x+a)^(10/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(83) = 166$.
time = 0.93, size = 337, normalized size = 3.34

$$-\frac{3\,(9\,b^2d^3x^3+14\,b^2c^3-40\,abc^2d+35\,a^2cd^2-3\,(b^2cd^2-10\,abd^3)x^2+(2\,b^2c^2d-10\,abcd^2+35\,a^2d^3)x)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\,(a^4b^3c^3-3\,a^5b^2c^2d+3\,a^6bcd^2-a^7d^3+(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)x^4+4\,(ab^6c^3-3\,a^2b^5c^2d+3\,a^3b^4cd^2-a^4b^3d^3)x^3+6\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3)x^2+4\,(a^3b^4c^3-3\,a^4b^3c^2d+3\,a^5b^2cd^2-a^6bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

```
[Out] -3/140*(9*b^2*d^3*x^3 + 14*b^2*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2 - 3*(b^2*c
*d^2 - 10*a*b*d^3)*x^2 + (2*b^2*c^2*d - 10*a*b*c*d^2 + 35*a^2*d^3)*x)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 -
 a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 +
4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^
2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b
^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(13/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(13/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

Mupad [B]
time = 1.02, size = 203, normalized size = 2.01

$$\frac{(c+d\,x)^{1/3}\left(\frac{105\,a^2\,c\,d^2-120\,a\,b\,c^2\,d+42\,b^2\,c^3}{140\,b^3\,(a\,d-b\,c)^3}+\frac{x\,(105\,a^2\,d^3-30\,a\,b\,c\,d^2+6\,b^2\,c^2\,d)}{140\,b^3\,(a\,d-b\,c)^3}+\frac{27\,d^3\,x^3}{140\,b\,(a\,d-b\,c)^3}+\frac{9\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(13/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((42*b^2*c^3 + 105*a^2*c*d^2 - 120*a*b*c^2*d)/(140*b^3*(a*
d - b*c)^3) + (x*(105*a^2*d^3 + 6*b^2*c^2*d - 30*a*b*c*d^2))/(140*b^3*(a*d
- b*c)^3) + (27*d^3*x^3)/(140*b*(a*d - b*c)^3) + (9*d^2*x^2*(10*a*d - b*c))
/(140*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/3) + (a^3*(a + b*x)^(1/3))/b^3
 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/b^2)
```

3.1581 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}}+\frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}}-\frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}}+\frac{243d^3(c+dx)^{4/3}}{1820(bc-ad)^4(a+bx)^{4/3}}$$

```
[Out] -3/13*(d*x+c)^(4/3)/(-a*d+b*c)/(b*x+a)^(13/3)+27/130*d*(d*x+c)^(4/3)/(-a*d+
b*c)^2/(b*x+a)^(10/3)-81/455*d^2*(d*x+c)^(4/3)/(-a*d+b*c)^3/(b*x+a)^(7/3)+2
43/1820*d^3*(d*x+c)^(4/3)/(-a*d+b*c)^4/(b*x+a)^(4/3)
```

Rubi [A]

time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(13*(b*c - a*d)*(a + b*x)^(13/3)) + (27*d*(c + d*x)^(4
/3))/(130*(b*c - a*d)^2*(a + b*x)^(10/3)) - (81*d^2*(c + d*x)^(4/3))/(455*(
b*c - a*d)^3*(a + b*x)^(7/3)) + (243*d^3*(c + d*x)^(4/3))/(1820*(b*c - a*d)
^4*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx = -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} - \frac{(9d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx}{13(bc-ad)}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} + \frac{(27d^2)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{65(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} -$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}} + \frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}} - \frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}} +$$

Mathematica [A]
time = 0.14, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{4/3}\left(-455d^3+\frac{780bd^2(c+dx)}{a+bx}-\frac{546b^2d(c+dx)^2}{(a+bx)^2}+\frac{140b^3(c+dx)^3}{(a+bx)^3}\right)}{1820(bc-ad)^4(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(-455*d^3 + (780*b*d^2*(c + d*x))/(a + b*x) - (546*b^2*
d*(c + d*x)^2)/(a + b*x)^2 + (140*b^3*(c + d*x)^3)/(a + b*x)^3))/(1820*(b*c
 - a*d)^4*(a + b*x)^(4/3))
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$\frac{3(dx+c)^{\frac{4}{3}}(81b^3x^3d^3+351d^3ax^2b^2-108b^3cd^2x^2+585a^2bd^3x-468ab^2cd^2x+126b^3c^2dx+455a^3d^3-780a^2bcd^2+546ab^2c^2d-140b^3c}{1820(bx+a)^{\frac{13}{3}}(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(16/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/1820*(d*x+c)^(4/3)*(81*b^3*d^3*x^3+351*a*b^2*d^3*x^2-108*b^3*c*d^2*x^2+58
5*a^2*b*d^3*x-468*a*b^2*c*d^2*x+126*b^3*c^2*d*x+455*a^3*d^3-780*a^2*b*c*d^2
+546*a*b^2*c^2*d-140*b^3*c^3)/(b*x+a)^(13/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(112) = 224$.
time = 1.05, size = 533, normalized size = 3.92

$$\frac{3\left(81\,b^{3}d^{4}x^{4}-140\,b^{3}c^{4}+546\,ab^{2}c^{3}d-780\,a^{2}bc^{2}d^{2}+455\,a^{3}cd^{3}-27\left(b^{3}cd^{3}-13\,ab^{2}d^{4}\right)x^{3}+9\left(2\,b^{3}c^{2}d^{2}-13\,ab^{2}cd^{3}+65\,a^{2}bd^{4}\right)x^{2}-\left(14\,b^{3}c^{3}d-78\,ab^{2}c^{2}d^{2}+195\,a^{2}bcd^{3}-455\,a^{3}d^{4}\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{1820\left(a^{5}b^{4}c^{4}-4\,a^{6}b^{3}c^{3}d+6\,a^{7}b^{2}c^{2}d^{2}-4\,a^{8}bcd^{3}+a^{9}d^{4}+\left(b^{9}c^{4}-4\,ab^{8}c^{3}d+6\,a^{2}b^{7}c^{2}d^{2}-4\,a^{3}b^{6}cd^{3}+a^{4}b^{5}d^{4}\right)x^{5}+5\left(ab^{8}c^{4}-4\,a^{2}b^{7}c^{3}d+6\,a^{3}b^{6}c^{2}d^{2}-4\,a^{4}b^{5}cd^{3}+a^{5}b^{4}d^{4}\right)x^{4}+10\left(a^{2}b^{7}c^{4}-4\,a^{3}b^{6}c^{3}d+6\,a^{4}b^{5}c^{2}d^{2}-4\,a^{5}b^{4}cd^{3}+a^{6}b^{3}d^{4}\right)x^{3}+10\left(a^{3}b^{6}c^{4}-4\,a^{4}b^{5}c^{3}d+6\,a^{5}b^{4}c^{2}d^{2}-4\,a^{6}b^{3}cd^{3}+a^{7}b^{2}d^{4}\right)x^{2}+5\left(a^{4}b^{5}c^{4}-4\,a^{5}b^{4}c^{3}d+6\,a^{6}b^{3}c^{2}d^{2}-4\,a^{7}b^{2}cd^{3}+a^{8}bd^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(81*b^3*d^4*x^4 - 140*b^3*c^4 + 546*a*b^2*c^3*d - 780*a^2*b*c^2*d^2
+ 455*a^3*c*d^3 - 27*(b^3*c*d^3 - 13*a*b^2*d^4)*x^3 + 9*(2*b^3*c^2*d^2 - 13
*a*b^2*c*d^3 + 65*a^2*b*d^4)*x^2 - (14*b^3*c^3*d - 78*a*b^2*c^2*d^2 + 195*a
^2*b*c*d^3 - 455*a^3*d^4)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^5*b^4*c^4 -
 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 -
 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5
*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b
^4*d^4)*x^4 + 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(16/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4496 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

Mupad [B]
time = 1.15, size = 293, normalized size = 2.15

$$\frac{(c+d\,x)^{1/3}\left(\frac{243\,d^4\,x^4}{1820\,b\,(a\,d-b\,c)^4}-\frac{-1365\,a^3\,c\,d^3+2340\,a^2\,b\,c^2\,d^2-1638\,a\,b^2\,c^3\,d+420\,b^3\,c^4}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{x\,(1365\,a^3\,d^4-585\,a^2\,b\,c\,d^3+234\,a\,b^2\,c^2\,d^2-42\,b^3\,c^3\,d)}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{81\,d^3\,x^3\,(13\,a\,d-b\,c)}{1820\,b^2\,(a\,d-b\,c)^4}+\frac{27\,d^2\,x^2\,(65\,a^2\,d^2-13\,a\,b\,c\,d+2\,b^2\,c^2)}{1820\,b^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/3}+\frac{a^4\,(a+b\,x)^{1/3}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/3}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/3}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/3}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(16/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^4*x^4)/(1820*b*(a*d - b*c)^4) - (420*b^3*c^4 - 136
5*a^3*c*d^3 + 2340*a^2*b*c^2*d^2 - 1638*a*b^2*c^3*d)/(1820*b^4*(a*d - b*c)^
4) + (x*(1365*a^3*d^4 - 42*b^3*c^3*d + 234*a*b^2*c^2*d^2 - 585*a^2*b*c*d^3)
)/(1820*b^4*(a*d - b*c)^4) + (81*d^3*x^3*(13*a*d - b*c))/(1820*b^2*(a*d - b
*c)^4) + (27*d^2*x^2*(65*a^2*d^2 + 2*b^2*c^2 - 13*a*b*c*d))/(1820*b^3*(a*d
- b*c)^4)))/(x^4*(a + b*x)^(1/3) + (a^4*(a + b*x)^(1/3))/b^4 + (6*a^2*x^2*(
a + b*x)^(1/3))/b^2 + (4*a*x^3*(a + b*x)^(1/3))/b + (4*a^3*x*(a + b*x)^(1/3
))/b^3)
```

3.1582 $\int (a+bx)^{4/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=655

$$-\frac{3(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20bd^2}+\frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(b}{}$$

```
[Out] -3/20*(-a*d+b*c)^2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/b/d^2+3/40*(-a*d+b*c)*(b*x+a
)^(4/3)*(d*x+c)^(1/3)/b/d+3/8*(b*x+a)^(7/3)*(d*x+c)^(1/3)/b+1/20*3^(3/4)*(-
a*d+b*c)^3*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3
)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b
*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d
^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*(
(b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-
a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(4/3)/d^(7/3)/(b*x+a)^(2/3)/
(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 1.02, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{10\,2^{2/3}b^{4/3}d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)^2}{20bd^2}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}\,(bc-ad)}{40bd}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]
```

```
[Out] (-3*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*b*d) + (3*(a + b*x)^(7/3)*(c + d*x
)^(1/3))/(8*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c + d
*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
```

```
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(4/3)*d^(7/3)*(
a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int (a+bx)^{4/3}\sqrt[3]{c+dx}\,dx = \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} + \frac{(bc-ad)\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx}{8b}$$

$$= \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8b} - \frac{(bc-ad)^2\int \frac{\sqrt[3]{a+b}}{(c+dx)^{2/}}}{10bd}$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + 3(a+bx)$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + 3(a+bx)$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + 3(a+bx)$$

$$= -\frac{3(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}{20bd^2} + \frac{3(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40bd} + 3(a+bx)$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.11

$$\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 7/3, 10/3, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{4}{3}}\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)*(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)*(d*x + c)^(1/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{4/3}\,(c+d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(4/3)*(c + d*x)^(1/3), x)
```

3.1583 $\int \sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=617

$$\frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(b\cdots}}{\cdots}$$

```
[Out] 3/10*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(1/3)/b/d+3/5*(b*x+a)^(4/3)*(d*x+c)^(
1/3)/b-1/10*3^(3/4)*(-a*d+b*c)^2*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+
2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(
1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(
4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(
1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*
x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(4/3)/
d^(4/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)
^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c
)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5\,2^{2/3}b^{4/3}d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{10bd}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(10*b*d) + (3*(a + b*x)^(4/
3)*(c + d*x)^(1/3))/(5*b) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^2*((a +
b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
```

```
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)
*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,dx = \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} + \frac{(bc-ad)\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}dx}{5b}$$
$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{(bc-ad)^2\int \frac{1}{(a+bx)^{2/3}(c}}{10bd}$$
$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{((bc-ad)^2((a+bx)(}{10bd(a}$$
$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{\left(3(bc-ad)^2((a+bx)\right.}{}$$
$$= \frac{3(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{10bd} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5b} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.12

$$\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(4*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)*(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)*(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(1/3)*(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)*(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/3)*(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{1/3}\,(c+d\,x)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)*(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/3)*(c + d*x)^(1/3), x)
```

3.1584 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=576

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2 \cdots\right.}{2^{2/3}b^{4/} \cdots}$$

```
[Out] 3/2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/b+1/2*3^(3/4)*(-a*d+b*c)*((b*x+a)*(d*x+c))^
(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x
+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)*2^(1/3)/b^(4/3)/d^(1/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*
c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 576, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2^{2/3}b^{4/3}\sqrt[3]{d}\,(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,\sqrt{(ad+b(c+2dx))^2}}+\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*b) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c
 - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
```

```
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(4/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2b}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{\left((bc-ad)((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{\left(3(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\Big(}{2b(a+bx)^{2/3}(c+dx}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2b} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bd}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.12

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{(a + bx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(2/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x)^{1/3}}{(a + b\,x)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(2/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(2/3), x)
```

3.1585 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/3}} \, dx$

Optimal. Leaf size=568

$$-\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\, d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\right.}{2^{2/3}b^{4/3}(a+}$$

```
[Out] -3/2*(d*x+c)^(1/3)/b/(b*x+a)^(2/3)+1/2*3^(3/4)*d^(2/3)*((b*x+a)*(d*x+c))^(2
/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*Elli
pticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-
3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2
^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a
)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^
(1/2)*2^(1/3)/b^(4/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b
*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 64, 637, 224}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\, d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\, b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\, F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(2*b*(a + b*x)^(2/3)) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(
2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
```

```
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(2/3
)*b^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
- a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{5/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{d\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2b}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{\left(d((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{\left(3d((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd}}\right.}{2b(a+bx)^{2/3}(c+dx)^{2/3}(bc+}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2b(a+bx)^{2/3}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.13

$$-\frac{3\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{3};\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b(a+bx)^{2/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(5/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, (d*(a + b*x))/(-(b*c
) + a*d)])/(2*b*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(5/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(5/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(5/3),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(5/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(5/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(5/3), x)
```

3.1586 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{8/3}}\,dx$

Optimal. Leaf size=617

$$-\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}}-\frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\Big(\Big(}{}$$

```
[Out] -3/5*(d*x+c)^(1/3)/b/(b*x+a)^(5/3)-3/10*d*(d*x+c)^(1/3)/b/(-a*d+b*c)/(b*x+a
)^(2/3)-1/10*3^(3/4)*d^(5/3)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(
1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)
-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)
*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(4/3)/(-a*
d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^
(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 64, 637, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{5\,2^{2/3}b^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3d\sqrt[3]{c+dx}}{10b(a+bx)^{2/3}(bc-ad)}-\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(5*b*(a + b*x)^(5/3)) - (3*d*(c + d*x)^(1/3))/(10*b*(b
*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(5/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
```

```
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*2^(2/3)*b^(4/3)*(b*c
 - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
- 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{8/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} + \frac{d\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{5b}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{d^2\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{10b(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{\left(d^2((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bc}}{10b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{\left(3d^2((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2b}\right.}{10b(bc}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5b(a+bx)^{5/3}} - \frac{3d\sqrt[3]{c+dx}}{10b(bc-ad)(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.12

$$-\frac{3\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{5}{3},-\frac{1}{3};-\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(8/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(8/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(8/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c + dx}}{(a + bx)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(8/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(8/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(8/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{8/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(8/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(8/3), x)
```

3.1587 $\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=216

$$-\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}-$$

```
[Out] -2/3*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(2/3)/d^2+1/2*(b*x+a)^(4/3)*(d*x+c)^(
2/3)/d-1/9*(-a*d+b*c)^2*ln(b*x+a)/b^(2/3)/d^(7/3)-1/3*(-a*d+b*c)^2*ln(-1+b^
(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/b^(2/3)/d^(7/3)-2/9*(-a*d+b*c)^2
*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2)
)/b^(2/3)/d^(7/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$-\frac{2(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-2*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(2/3))/(3*d^2) + ((a + b*x)^(4/3)
*(c + d*x)^(2/3))/(2*d) - (2*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c
 + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/
3)) - ((b*c - a*d)^2*Log[a + b*x])/(9*b^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*b^(2/3)*d^(7
/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
```

```
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx = \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{(2(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d}$$

$$= -\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} + \frac{(2(bc-ad)^2)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{\cdots}}}{9d^2}$$

$$= -\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2} + \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\cdots\right.}{3\sqrt{3}\,b^{2/\cdots}}$$

Mathematica [A]
time = 0.58, size = 231, normalized size = 1.07

$$\frac{3b^{2/3}\sqrt[3]{d}\sqrt[3]{a+bx}\,(c+dx)^{2/3}(-4bc+7ad+3bdx)+4\sqrt{3}\,(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}}{\sqrt{3}}\right)-4(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)+2(bc-ad)^2\log\left(b^{2/3}+\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{18b^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*b^(2/3)*d^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(2/3)*(-4*b*c + 7*a*d + 3*b*d*
x) + 4*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/
3)*(c + d*x)^(1/3)))/Sqrt[3]] - 4*(b*c - a*d)^2*Log[b^(1/3) - (d^(1/3)*(a +
 b*x)^(1/3))/(c + d*x)^(1/3)] + 2*(b*c - a*d)^2*Log[b^(2/3) + (d^(2/3)*(a +
 b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3)])/(18*b^(2/3)*d^(7/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(166) = 332$.
time = 1.76, size = 740, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b^2*d)^(1
/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)^(1/3)*(b*d*x + b*c))
*sqrt((-b^2*d)^(1/3)/d)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)
*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d
 - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*b^3*d^2*x - 4*b^3*c*d + 7*a*
b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*d^3), 1/18*(12*sqrt(1/3)*(b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*
x + b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + 2*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))
/(d*x + c)) - 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*
b^3*d^2*x - 4*b^3*c*d + 7*a*b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*
d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(4/3)/(c + d*x)^(1/3), x)
```

3.1588 $\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(-\right.}{2b}$$

```
[Out] (b*x+a)^(1/3)*(d*x+c)^(2/3)/d+1/6*(-a*d+b*c)*ln(b*x+a)/b^(2/3)/d^(4/3)+1/2*
(-a*d+b*c)*ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/b^(2/3)/d^(4/
3)+1/3*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x
+a)^(1/3)*3^(1/2))/b^(2/3)/d^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$\frac{(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(1/3)*(c + d*x)^(2/3))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^
(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*b^(2/3)
*d^(4/3)) + ((b*c - a*d)*Log[a + b*x])/(6*b^(2/3)*d^(4/3)) + ((b*c - a*d)*L
og[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3)*d^
(4/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}} + \frac{(bc-ad)\log(a +}{6b^{2/3}d^{4/3}}$$

Mathematica [A]
time = 10.02, size = 278, normalized size = 1.63

$$\frac{(a+bx)^{4/3}\left(6b^{2/3}\sqrt[3]{d(a+bx)}\,(c+dx)^{2/3}+2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{d(a+bx)}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)+2(bc-ad)\log\left(\sqrt[3]{d(a+bx)}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)-bc\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)+ad\log\left((d(a+bx))^{2/3}+\sqrt[3]{b}\,\sqrt[3]{d(a+bx)}\,\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}\right)\right)}{6b^{2/3}(d(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(4/3)*(6*b^(2/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(2/3) + 2*Sqrt[3]
*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(2*(d*(a + b*x))^(1/3
) + b^(1/3)*(c + d*x)^(1/3))] + 2*(b*c - a*d)*Log[(d*(a + b*x))^(1/3) - b^(
1/3)*(c + d*x)^(1/3)] - b*c*Log[(d*(a + b*x))^(2/3) + b^(1/3)*(d*(a + b*x))
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)] + a*d*Log[(d*(a + b*x))^(
2/3) + b^(1/3)*(d*(a + b*x))^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
)]))/(6*b^(2/3)*(d*(a + b*x))^(4/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(132) = 264$.
time = 1.35, size = 618, normalized size = 3.61

$$\left[\frac{6\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b^2d-3\,\sqrt{\frac{1}{3}}\,(b^2cd-abd^2)\sqrt{\frac{(-b^2d)^{\frac{1}{3}}}{d}}\log\left(3\,b^2dx+b^2c+2\,abd+3\,(-b^2d)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b+3\,\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}+(-b^2d)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{\frac{(-b^2d)^{\frac{1}{3}}}{d}}\right)-(-b^2d)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-(-b^2d)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)+2\,(-b^2d)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-(-b^2d)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{6\,b^2d^2},\ \frac{6\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b^2d-6\,\sqrt{\frac{1}{3}}\,(b^2cd-abd^2)\sqrt{-\frac{(-b^2d)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-(-b^2d)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{-\frac{(-b^2d)^{\frac{1}{3}}}{d}}}{b^2dx+b^2c}\right)-(-b^2d)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-(-b^2d)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)+2\,(-b^2d)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-(-b^2d)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{6\,b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1
/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x
 + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)
^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) - (-b^2*d)^(2/3)*(b*c - a*d)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)^(2/3
)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*
x + c))/(d*x + c)))/(b^2*d^2), 1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d
 - 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) - (-b^2*d)^(2/3)*(b*c -
a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1
/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)
^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3
)*(d*x + c))/(d*x + c)))/(b^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/3)/(c + d*x)^(1/3), x)
```

3.1589 $\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

```
[Out] -1/2*ln(b*x+a)/b^(2/3)/d^(1/3)-3/2*ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x
+a)^(1/3))/b^(2/3)/d^(1/3)-arctan(1/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(
1/3)/(b*x+a)^(1/3)*3^(1/2))*3^(1/2)/b^(2/3)/d^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {61}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*
(a + b*x)^(1/3))])/(b^(2/3)*d^(1/3))) - Log[a + b*x]/(2*b^(2/3)*d^(1/3)) -
(3*Log[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3
)*d^(1/3))
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{}}{\sqrt[3]{d}\sqrt[3]{}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

Mathematica [A]
time = 0.15, size = 155, normalized size = 1.23

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)+\log\left(b^{2/3}+\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3)
))/Sqrt[3]] - 2*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)] +
Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(2*b^(2/3)*d^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(90) = 180.
time = 1.28, size = 519, normalized size = 4.12

$$\left[\frac{\sqrt{3}\,bd\sqrt{\frac{(-b^2d)^{\frac{1}{3}}}{d}}\log\left(3b^2dx+b^2c+2abd+3(-b^2d)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}b+\sqrt{3}\left(2(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}+(-b^2d)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{\frac{(-b^2d)^{\frac{1}{3}}}{d}}\right)+(-b^2d)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-(-b^2d)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)-2(-b^2d)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-(-b^2d)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{2b^2d}, \frac{2\sqrt{3}\,bd\sqrt{-\frac{(-b^2d)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{3}\left(2(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-(-b^2d)^{\frac{1}{3}}(bdx+bc)\right)\sqrt{-\frac{(-b^2d)^{\frac{1}{3}}}{d}}}{3(b^2dx+b^2c)}\right)+(-b^2d)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd+(-b^2d)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}-(-b^2d)^{\frac{1}{3}}(bdx+bc)}{dx+c}\right)-2(-b^2d)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-(-b^2d)^{\frac{2}{3}}(dx+c)}{dx+c}\right)}{2b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d +
3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + sqrt(3)*(2*(b*x + a)^(
2/3)*(d*x + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) +
 (-b^2*d)^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) + (-b^2*d)^(2/3)*log
(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x
 + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*l
og(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x +
c)))/(b^2*d), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b^2*d)^(1/3)/d)*arctan(1/3*sqrt(3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + (-b^2*d)^(2/3)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*log((
(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)))
/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(1/3),x)
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{2/3}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)), x)
```

3.1590 $\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

```
[Out] -3/2*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))

Maple [A]
time = 0.18, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{3(dx+c)^{\frac{2}{3}}}{2(bx+a)^{\frac{2}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)

[Out] 3/2/(b*x+a)^(2/3)*(d*x+c)^(2/3)/(a*d-b*c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)

Fricas [A]
time = 0.91, size = 42, normalized size = 1.31

$$-\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\,(abc-a^2d+(b^2c-abd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")

[Out] -3/2*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{5}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(1/3),x)

[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(1/3)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/3}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)), x)
```

3.1591 $\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=66

$$-\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}}+\frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}$$

```
[Out] -3/5*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(5/3)+9/10*d*(d*x+c)^(2/3)/(-a*d+b*c)
^2/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (9*d*(c + d*x)^(2/3)
)/(10*(b*c - a*d)^2*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{5(bc-ad)}$$
$$= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}} + \frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}$$

Mathematica [**A**]
time = 0.13, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{2/3}(-2bc+5ad+3bdx)}{10(bc-ad)^2(a+bx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
[Out] (3*(c + d*x)^(2/3)*(-2*b*c + 5*a*d + 3*b*d*x))/(10*(b*c - a*d)^2*(a + b*x)^
(5/3))
```

Maple [**A**]
time = 0.16, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{3(dx+c)^{\frac{2}{3}}(3bdx+5ad-2bc)}{10(bx+a)^{\frac{5}{3}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/10*(d*x+c)^(2/3)*(3*b*d*x+5*a*d-2*b*c)/(b*x+a)^(5/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 1.22, size = 118, normalized size = 1.79

$$\frac{3\left(3\,bdx-2\,bc+5\,ad\right)\left(bx+a\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}}{10\left(a^{2}b^{2}c^{2}-2\,a^{3}bcd+a^{4}d^{2}+\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)x^{2}+2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(3*b*d*x - 2*b*c + 5*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{8}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{8/3}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)), x)
```

3.1592 $\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}+\frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}}-\frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}$$

```
[Out] -3/8*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(8/3)+9/20*d*(d*x+c)^(2/3)/(-a*d+b*c)
^2/(b*x+a)^(5/3)-27/40*d^2*(d*x+c)^(2/3)/(-a*d+b*c)^3/(b*x+a)^(2/3)
```

Rubi [A]

time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (9*d*(c + d*x)^(2/3
))/(20*(b*c - a*d)^2*(a + b*x)^(5/3)) - (27*d^2*(c + d*x)^(2/3))/(40*(b*c -
a*d)^3*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{4(bc-ad)}$$
$$= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}} + \frac{(9d^2)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}}{20(bc-ad)^2}$$
$$= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}} - \frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}$$

Mathematica [A]
time = 0.17, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{8/3}\left(5b^2+\frac{20d^2(a+bx)^2}{(c+dx)^2}-\frac{16bd(a+bx)}{c+dx}\right)}{40(bc-ad)^3(a+bx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^2 + (20*d^2*(a + b*x)^2)/(c + d*x)^2 - (16*b*d*(a
+ b*x))/(c + d*x)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

Maple [A]
time = 0.18, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{3(dx+c)^{\frac{2}{3}}\left(9b^2x^2d^2+24ab\,d^2x-6b^2cdx+20a^2d^2-16abcd+5b^2c^2\right)}{40(bx+a)^{\frac{8}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/40*(d*x+c)^(2/3)*(9*b^2*d^2*x^2+24*a*b*d^2*x-6*b^2*c*d*x+20*a^2*d^2-16*a*
b*c*d+5*b^2*c^2)/(b*x+a)^(8/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(83) = 166.
time = 1.43, size = 251, normalized size = 2.49

$$-\frac{3\,(9\,b^2d^2x^2+5\,b^2c^2-16\,abcd+20\,a^2d^2-6\,(b^2cd-4\,abd^2)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/40*(9*b^2*d^2*x^2 + 5*b^2*c^2 - 16*a*b*c*d + 20*a^2*d^2 - 6*(b^2*c*d - 4
*a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a
^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^
2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/3}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)), x)
```

3.1593 $\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=136

$$-\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}}+\frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}}-\frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}}+\frac{243d^3(c+dx)^{2/3}}{440(bc-ad)^4(a+bx)^{2/3}}$$

```
[Out] -3/11*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(11/3)+27/88*d*(d*x+c)^(2/3)/(-a*d+b
*c)^2/(b*x+a)^(8/3)-81/220*d^2*(d*x+c)^(2/3)/(-a*d+b*c)^3/(b*x+a)^(5/3)+243
/440*d^3*(d*x+c)^(2/3)/(-a*d+b*c)^4/(b*x+a)^(2/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(11*(b*c - a*d)*(a + b*x)^(11/3)) + (27*d*(c + d*x)^(2
/3))/(88*(b*c - a*d)^2*(a + b*x)^(8/3)) - (81*d^2*(c + d*x)^(2/3))/(220*(b*
c - a*d)^3*(a + b*x)^(5/3)) + (243*d^3*(c + d*x)^(2/3))/(440*(b*c - a*d)^4*
(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c-}}}{44(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx}$$

$$= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}} + \frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}} - \frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx}$$

Mathematica [A]
time = 0.17, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{11/3}\left(40b^3-\frac{220d^3(a+bx)^3}{(c+dx)^3}+\frac{264bd^2(a+bx)^2}{(c+dx)^2}-\frac{165b^2d(a+bx)}{c+dx}\right)}{440(bc-ad)^4(a+bx)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(11/3)*(40*b^3 - (220*d^3*(a + b*x)^3)/(c + d*x)^3 + (264*b*d
^2*(a + b*x)^2)/(c + d*x)^2 - (165*b^2*d*(a + b*x))/(c + d*x)))/(440*(b*c -
 a*d)^4*(a + b*x)^(11/3))
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$\frac{3(dx+c)^{\frac{2}{3}}\left(81b^3x^3d^3+297d^3a\,x^2b^2-54b^3c\,d^2x^2+396a^2b\,d^3x-198a\,b^2c\,d^2x+45b^3c^2dx+220a^3d^3-264a^2bc\,d^2+165a\,b^2c^2d-40b^3c^3\right)}{440(bx+a)^{\frac{11}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/440*(d*x+c)^(2/3)*(81*b^3*d^3*x^3+297*a*b^2*d^3*x^2-54*b^3*c*d^2*x^2+396*
a^2*b*d^3*x-198*a*b^2*c*d^2*x+45*b^3*c^2*d*x+220*a^3*d^3-264*a^2*b*c*d^2+16
5*a*b^2*c^2*d-40*b^3*c^3)/(b*x+a)^(11/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(112) = 224$.
time = 1.71, size = 420, normalized size = 3.09

$$\frac{3\left(81\,b^{3}d^{3}x^{3}-40\,b^{3}c^{3}+165\,ab^{2}c^{2}d-264\,a^{2}bcd^{2}+220\,a^{3}d^{3}-27\left(2\,b^{3}cd^{2}-11\,ab^{2}d^{3}\right)x^{2}+9\left(5\,b^{3}c^{2}d-22\,ab^{2}cd^{2}+44\,a^{2}bd^{3}\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{440\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)x^{4}+4\left(ab^{7}c^{4}-4\,a^{2}b^{6}c^{3}d+6\,a^{3}b^{5}c^{2}d^{2}-4\,a^{4}b^{4}cd^{3}+a^{5}b^{3}d^{4}\right)x^{3}+6\left(a^{2}b^{6}c^{4}-4\,a^{3}b^{5}c^{3}d+6\,a^{4}b^{4}c^{2}d^{2}-4\,a^{5}b^{3}cd^{3}+a^{6}b^{2}d^{4}\right)x^{2}+4\left(a^{3}b^{5}c^{4}-4\,a^{4}b^{4}c^{3}d+6\,a^{5}b^{3}c^{2}d^{2}-4\,a^{6}b^{2}cd^{3}+a^{7}bd^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(81*b^3*d^3*x^3 - 40*b^3*c^3 + 165*a*b^2*c^2*d - 264*a^2*b*c*d^2 + 22
0*a^3*d^3 - 27*(2*b^3*c*d^2 - 11*a*b^2*d^3)*x^2 + 9*(5*b^3*c^2*d - 22*a*b^2
*c*d^2 + 44*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(14/3)/(d*x+c)**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{14/3}\,{\left(c+d\,x\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)), x)
```

3.1594 $\int \frac{(a+bx)^{8/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1365

$$\frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3}-\frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2}+\frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d}-\frac{}{7b^{2/3}d^{11/3}\sqrt[3]{}}$$

```
[Out] 3/7*(-a*d+b*c)^2*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d^3-12/35*(-a*d+b*c)*(b*x+a)^(
5/3)*(d*x+c)^(2/3)/d^2+3/10*(b*x+a)^(8/3)*(d*x+c)^(2/3)/d-3/7*2^(2/3)*(-a*d
+b*c)^3*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+
c))^2)^(1/2)/b^(2/3)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)
/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/
2)))-2/7*2^(1/6)*3^(3/4)*(-a*d+b*c)^(11/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(
1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(
2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^(11/3)/(b*x+a)^(1/3)/(
d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3
)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)+3/14*3^(1/4)*(-a*d+b*c)^(11/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(
2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^
(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b
*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+
2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(
2/3)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+
c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a
*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.92, antiderivative size = 1365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{11/3}}{7\sqrt[3]{2}b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}} - \frac{2\sqrt[6]{2}3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{11/3}}{7b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}} - \frac{3\,2^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)^3}{7b^{2/3}d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)} + \frac{3(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^2}{7d^3} - \frac{12(a+bx)^{5/3}(c+dx)^{2/3}(bc-ad)}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(7*d^3) - (12*(b*c - a*d)
*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(35*d^2) + (3*(a + b*x)^(8/3)*(c + d*x)^(
2/3))/(10*d) - (3*2^(2/3)*(b*c - a*d)^3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*b^(2/3)*d^(11/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (3*3^(
1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(11/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(
b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((
a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(1/3)*b^(2/3)*d^(11/3)*(a + b*x)^(1/3)*(
c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2
]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*2^(1/6)*3^(3/4)*(b*c - a*d)^(11/3)*((
a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4
/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
 + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b
*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*b^(2/3)*d^(
11/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*
d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))
^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{8/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} - \frac{(4(bc-ad))\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx}{5d}$$

$$= -\frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(c+dx)^{2/3}}{10d} + \frac{(4(bc-ad)^2)\int \frac{(a+bx}{\sqrt[3]{c+}}}{7d^2}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(}{10a}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(}{10a}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(}{10a}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(}{10a}$$

$$= \frac{3(bc-ad)^2(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} - \frac{12(bc-ad)(a+bx)^{5/3}(c+dx)^{2/3}}{35d^2} + \frac{3(a+bx)^{8/3}(}{10a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{11/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{d(a+bx)}{-bc+ad}\right)}{11b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(8/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(11/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3
, 11/3, 14/3, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{8}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(8/3)/(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(8/3)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{8/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(8/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(8/3)/(c + d*x)^(1/3), x)
```

3.1595 $\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1330

$$-\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2}+\frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d}+\frac{15(bc-ad)^2\sqrt[3]{(a+bx)(c+}}{14\sqrt[3]{2}\,b^{2/3}d^{8/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2}$$

```
[Out] -15/28*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d^2+3/7*(b*x+a)^(5/3)*(d*x+c)
^(2/3)/d+15/28*(-a*d+b*c)^2*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(
1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x
+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+5/14*3^(3/4)*(-a*d+b*c)^(8/3)*((b*x+a)*(d*x
+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2
/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c
)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c
)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*
2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/b^(2/
3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))
^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x
+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+
b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-15/56*3^(1/4)*(-a*d+b*c)^(8/3)*((b*x+a)*(d
*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^
(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b
*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1
/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3
)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3)
)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1
/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x
+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.39, antiderivative size = 1330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {52, 64, 637, 309, 224, 1891}

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{28\sqrt[3]{2}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{5\,3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{7\,2^{5/6}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{15\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)^2}{14\sqrt[3]{2}b^{2/3}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{15(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)}{28d^2}+\frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-15*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(28*d^2) + (3*(a + b*x)^(
5/3)*(c + d*x)^(2/3))/(7*d) + (15*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b
^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(8/3)*((a + b*x)*(c + d
*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b
^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^
(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3
)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
)/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*(b*c - a*d)^(
8/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5
/6)*b^(2/3)*d^(8/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{(5(bc-ad)^2)\int \frac{}{\sqrt[3]{a+}}}{14d^2}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(5(bc-ad)^2\sqrt[3]{(a+b}\right.}{}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(15(bc-ad)^2\sqrt[3]{(a+}\right.}{}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{\left(15(bc-ad)^2\sqrt[3]{(a+}\right.}{}$$

$$= -\frac{15(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{28d^2} + \frac{3(a+bx)^{5/3}(c+dx)^{2/3}}{7d} + \frac{15(}{14\sqrt[3]{2}\,b^{2/3}d^{8/3}\sqrt[3]{a+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{8/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{-bc+ad}\right)}{8b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(1/3),x]
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(5/3)/(c + d*x)^(1/3), x)
```

3.1596 $\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1293

$$\frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d}-\frac{3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c-}{2\sqrt[3]{2}\,b^{2/3}d^{5/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{}\right.}$$

```
[Out] 3/4*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d-3/4*(-a*d+b*c)*((b*x+a)*(d*x+c))^(1/3)*((
2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(5/
3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))-1/2*3^(3/4)*(-a*d+b*c)
^(5/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^
2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+
a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)*2^(1/6)/b^(2/3)/d^(5/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c
)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b
^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)+3/8*3^(1/4)*(-a*d+b*c)
^(5/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^
2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/
3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x
+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+
b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/d^(5/3)/(b*x+a)^(1/3)/(d*x
+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*(
(-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A]
time = 1.49, antiderivative size = 1293, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {52, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{4\sqrt[3]{2}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{2^{5/6}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)}{2\sqrt[3]{2}b^{2/3}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(2/3))/(4*d) - (3*(b*c - a*d)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2
*2^(1/3)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d
*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*d)^(5/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*d
^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a
*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*(b*c -
a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((
b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
+ d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(
2^(5/6)*b^(2/3)*d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx = \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx}{2d}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left((bc-ad)\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}}{2d\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left(3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{2d\sqrt[3]{a+bx}\,\sqrt[3]{c+}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{\left(3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{2\,2^{2/3}\sqrt[3]{b}\,d^{4/3}\sqrt[3]{a+bx}}$$

$$= \frac{3(a+bx)^{2/3}(c+dx)^{2/3}}{4d} - \frac{3(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad}}{2\sqrt[3]{2}\,b^{2/3}d^{5/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{5/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(2/3)/(c + d*x)^(1/3), x)
```

3.1597 $\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1257

$$\frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}\,b^{2/3}d^{2/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}$$

```
[Out] 3/2*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^
2)^(1/2)*2^(2/3)/b^(2/3)/d^(2/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b
*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^
(1/2)))+2^(1/6)*3^(3/4)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c
)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(
2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/
3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/
3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^(2/3)/(b*x+a)^(1/3)/(d*x
+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*(
(-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1
/2)-3/4*3^(1/4)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+
2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(
1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(
4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(
1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*
x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/
d^(2/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)
^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c
)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 1257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$,

Rules used = {64, 637, 309, 224, 1891}

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*
(c + 2*d*x))^2])/(2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(
b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*(b*c - a*
d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*
c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 +
Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*
2^(1/3)*b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*
x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) +
 (2^(1/6)*3^(3/4)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c +
 a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c -
a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*
x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))], -7 - 4*Sqrt[3]])/(b^(2/3)*d^(2/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b
*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(
c + 2*d*x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx = \frac{\sqrt[3]{(a+bx)(c+dx)}\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}$$

$$= \frac{\left(3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-4abcd+(bc+ad)^2}}\right.}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)}$$

$$= \frac{\left(3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}}{\sqrt{-4abcd+(bc+ad)^2}}\right.}{2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2b}$$

$$= \frac{3\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+}}{\sqrt[3]{2}\,b^{2/3}d^{2/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{2/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 2/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(1/3),x)
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/3}\,{\left(c+d\,x\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(1/3)), x)
```

3.1598 $\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1297

$$-\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{3\sqrt[3]{d}\ \sqrt[3]{(a+bx)(c+dx)}\ \sqrt{(bc+ad+2bdx)^2}\ \sqrt{(ad+b(c+2d}}{\sqrt[3]{2}\ b^{2/3}(bc-ad)\sqrt[3]{a+bx}\ \sqrt[3]{c+dx}\ (bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt{}\right.}$$

```
[Out] -3*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(1/3)+3/2*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/
(-a*d+b*c)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d
^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+2^(1/6)*3^(3/4
)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+
c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*
c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b
*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^
(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))
^2)^(1/2)/b^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d
+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+
a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-3/4*3^(1/4)*d^(1/3
)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a
)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^
(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-
a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(
d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^
(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/
(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 1.72, antiderivative size = 1297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[3]{2}b^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{\sqrt[6]{2}3^{3/4}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}}+\frac{3\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}b^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/((b*c - a*d)*(a + b*x)^(1/3)) + (3*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2
])/(2^(1/3)*b^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d
+ 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
 b*x)*(c + d*x))^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*((a + b*x)*
(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*b^(2/3)*(b*c
 - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((
b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2^(1/6)*
3^(3/4)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sq
rt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x
)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(
2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3
]])/(b^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3
)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{d\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}dx}{bc-ad}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{\left(d\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{\left(3d\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}(}{(bc-ad)\sqrt[3]{a+b}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{\left(3d^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Sub}}{2^{2/3}\sqrt[3]{b}(bc-ad)\sqrt[3]{}}$$

$$= -\frac{3(c+dx)^{2/3}}{(bc-ad)\sqrt[3]{a+bx}} + \frac{3\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc}}{\sqrt[3]{2}b^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/3)*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{4/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(4/3)*(c + d*x)^(1/3)), x)
```

3.1599 $\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1335

$$-\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}}+\frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}}-\frac{3d^{4/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad}}{2\sqrt[3]{2}\,b^{2/3}(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1\right.\right.}$$

```
[Out] -3/4*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(4/3)+3/2*d*(d*x+c)^(2/3)/(-a*d+b*c)^
2/(b*x+a)^(1/3)-3/4*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(
1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^2/(b*x+a)^(1/3)
/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))-1/2*3^(3/4)*d^(4/3)*((b*x+a)*(d*x+c))^
(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/
3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/
3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+
a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/b^(2/3)/(-
a*d+b*c)^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x
+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-
a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)+3/8*3^(1/4)*d^(4/3)*((b*x+a)*(d*x+c))^
(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2
*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x
+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d
*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)
+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.41, antiderivative size = 1335, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{4\sqrt[3]{2}b^{2/3}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{2^{5/6}b^{2/3}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{4/3}}{2\sqrt[3]{2}b^{2/3}(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{3(c+dx)^{2/3}d}{2(bc-ad)^2\sqrt[3]{a+bx}}-\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (3*d*(c + d*x)^(2/3)
)/(2*(b*c - a*d)^2*(a + b*x)^(1/3)) - (3*d^(4/3)*((a + b*x)*(c + d*x))^(1/3
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*b
^(2/3)*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*b^(2/3)*(b*c - a*d)^(4/
3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^
(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (3^(3/4)*d^(4/3)*((a
 + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/
3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)
+ 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*b^(2/3
)*(b*c - a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*S
qrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

<u>Rule 53</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} - \frac{d\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{d^2\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(d^2\sqrt[3]{(a+bx)(c+dx)}\right)\int}{2(bc-ad)^2\sqrt[3]{}}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(3d^2\sqrt[3]{(a+bx)(c+dx)}\right.}{}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{\left(3d^{5/3}\sqrt[3]{(a+bx)(c+dx)}\right.}{}$$

$$= -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2\sqrt[3]{a+bx}} - \frac{3d}{2\sqrt[3]{2}\,b^{2/3}(bc-ad)^2\sqrt[3]{a+b}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{4}{3},\frac{1}{3};-\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b(a+bx)^{4/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(4*b*(a + b*x)^(4/3)*(c + d*x)^(1/3))
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}\,(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(1/3)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{3}}\,\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{7/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(7/3)*(c + d*x)^(1/3)), x)
```

3.1600 $\int \frac{1}{(a+bx)^{10/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=1372

$$-\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}}+\frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}}-\frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}+\frac{15d^{7/3}\sqrt[3]{}}{14\sqrt[3]{2}\,b^{2/3}(bc-ad)^3\sqrt[3]{a+bx}\,\sqrt[3]{c}}$$

```
[Out] -3/7*(d*x+c)^(2/3)/(-a*d+b*c)/(b*x+a)^(7/3)+15/28*d*(d*x+c)^(2/3)/(-a*d+b*c
)^2/(b*x+a)^(4/3)-15/14*d^2*(d*x+c)^(2/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)+15/28*
d^(7/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+
c))^2)^(1/2)*2^(2/3)/b^(2/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*
d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2
/3)*(1+3^(1/2)))+5/14*3^(3/4)*d^(7/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(
2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^
(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*
d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*
((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(
-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/b^(2/3)/(-a*d+b*c)^(7/3)/(b*x
+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*
d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3
^(1/2)))^2)^(1/2)-15/56*3^(1/4)*d^(7/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)
^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d
+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3
)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/
3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/b
^(2/3)/(-a*d+b*c)^(7/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d
+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*
d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.75, antiderivative size = 1372, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$-\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{7/3}}{28\sqrt[3]{2}b^{2/3}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{5\,3^{3/4}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{7/3}}{7\,2^{5/6}b^{2/3}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{15\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{7/3}}{14\sqrt[3]{2}b^{2/3}(bc-ad)^3\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{15(c+dx)^{2/3}d^2}{14(bc-ad)^3\sqrt[3]{a+bx}}+\frac{15(c+dx)^{2/3}d}{28(bc-ad)^2(a+bx)^{4/3}}-\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (15*d*(c + d*x)^(2/3
))/(28*(b*c - a*d)^2*(a + b*x)^(4/3)) - (15*d^2*(c + d*x)^(2/3))/(14*(b*c -
 a*d)^3*(a + b*x)^(1/3)) + (15*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(14*2^(1/3)*b^(2/3)*(b
*c - a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(7/3)*((a + b*x)*(c + d*x))^(1/3)*Sq
rt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3
)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)
*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))
/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3))], -7 - 4*Sqrt[3]])/(28*2^(1/3)*b^(2/3)*(b*c - a*d)^(7/3)*(a +
 b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*(
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/(
(1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d
*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (5*3^(3/4)*d^(7/3)*((a + b*
x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) -
2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2
^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*2^(5/6)*b^(2/3)*(
b*c - a*d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt
[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(5d)\int \frac{1}{(a+bx)^{7/3}\sqrt[3]{c+dx}}\,dx}{7(bc-ad)}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} + \frac{(5d^2)\int \frac{1}{(a+bx)^{4/3}\sqrt[3]{c+d}}}{14(bc-ad)^2}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

$$= -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.05

$$-\frac{3\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};-\frac{4}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[-7/3, 1/3, -4/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(7*b*(a + b*x)^(7/3)*(c + d*x)^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{10}{3}} (dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
[Out] int(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(1/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^4*d*x^5 + a^4*c + (b^4*c + 4*a*
b^3*d)*x^4 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 2*a^3*b*d)*
x^2 + (4*a^3*b*c + a^4*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{10}{3}} \sqrt[3]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(1/3),x)
[Out] Integral(1/((a + b*x)**(10/3)*(c + d*x)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(1/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{10/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(10/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(10/3)*(c + d*x)^(1/3)), x)
```

3.1601 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}} dx$

Optimal. Leaf size=216

$$-\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}\sqrt[3]{b}d^{8/3}}-5($$

```
[Out] -5/6*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(1/3)/d^2+1/2*(b*x+a)^(5/3)*(d*x+c)^(
1/3)/d-5/18*(-a*d+b*c)^2*ln(d*x+c)/b^(1/3)/d^(8/3)-5/6*(-a*d+b*c)^2*ln(-1+d
^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(1/3)/d^(8/3)-5/9*(-a*d+b*c)^
2*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3)*3^(1/2
))/b^(1/3)/d^(8/3)*3^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$-\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}d^{8/3}}-\frac{5(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)}{6d^2}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*d^2) + ((a + b*x)^(5/3)
*(c + d*x)^(1/3))/(2*d) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a
 + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^(8/
3)) - (5*(b*c - a*d)^2*Log[c + d*x])/(18*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^
2*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(1/3)
*d^(8/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
```

```
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} + \frac{(5(bc-ad)^2)\int \frac{1}{\sqrt[3]{a+bx}\,(c}}{9d^2}$$

$$= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2} + \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\right.}{3\sqrt{3}\,\sqrt[3]{b}\,d}$$

Mathematica [A]
time = 0.41, size = 231, normalized size = 1.07

$$\frac{3\sqrt[3]{b}\,d^{2/3}(a+bx)^{2/3}\sqrt[3]{c+dx}\,(-5bc+8ad+3bdx)+10\sqrt{3}\,(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}}{\sqrt{3}}\right)-10(bc-ad)^2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)+5(bc-ad)^2\log\left(d^{2/3}+\frac{\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)}{18\sqrt[3]{b}\,d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*b^(1/3)*d^(2/3)*(a + b*x)^(2/3)*(c + d*x)^(1/3)*(-5*b*c + 8*a*d + 3*b*d*
x) + 10*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(d^(1
/3)*(a + b*x)^(1/3)))/Sqrt[3]] - 10*(b*c - a*d)^2*Log[d^(1/3) - (b^(1/3)*(c
 + d*x)^(1/3))/(a + b*x)^(1/3)] + 5*(b*c - a*d)^2*Log[d^(2/3) + (b^(1/3)*d^
(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x
)^(2/3)])/(18*b^(1/3)*d^(8/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(166) = 332.
time = 1.00, size = 741, normalized size = 3.43

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(b^3c^2d-2ab^2cd^2+a^2bd^3\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\log\left(-3bd^2x-2bcd-ad^2-3\left(-bd^2\right)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\right)-10\left(b^2c^2-2abcd+a^2d^2\right)\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+5\left(b^2c^2-2abcd+a^2d^2\right)\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)+3\left(3b^2d^3x-5b^2cd^2+8abd^3\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{18\,bd^4},\ \frac{30\sqrt{\frac{1}{3}}\left(b^3c^2d-2ab^2cd^2+a^2bd^3\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}}{bd^2x+ad^2}\right)-10\left(b^2c^2-2abcd+a^2d^2\right)\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-\left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+5\left(b^2c^2-2abcd+a^2d^2\right)\left(-bd^2\right)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-\left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)+3\left(3b^2d^3x-5b^2cd^2+8abd^3\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{18\,bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b*d^2)^(
1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)
*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2)^(1/3)*(b*d*x + a*d
))*sqrt((-b*d^2)^(1/3)/b)) - 10*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2
/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b
*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
- (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x - 5*b^2*c*d^2 +
 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*d^4), 1/18*(30*sqrt(1/3)*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d
*x + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 10*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b
*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(
3*b^2*d^3*x - 5*b^2*c*d^2 + 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*
d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(5/3)/(c + d*x)^(2/3), x)
```

3.1602 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=169

$$\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}+\frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\right.}{}$$

```
[Out] (b*x+a)^(2/3)*(d*x+c)^(1/3)/d+1/3*(-a*d+b*c)*ln(d*x+c)/b^(1/3)/d^(5/3)+(-a*
d+b*c)*ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^(1/3))/b^(1/3)/d^(5/3)+2
/3*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x+c)^
(1/3)*3^(1/2))/b^(1/3)/d^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {52, 61}

$$\frac{2(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}\,d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/d + (2*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*
d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(1/
3)*d^(5/3)) + ((b*c - a*d)*Log[c + d*x])/(3*b^(1/3)*d^(5/3)) + ((b*c - a*d)
*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(b^(1/3)*d^
(5/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx = \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3d}$$

$$= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d} + \frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}} + \frac{(bc-ad)\log(\ldots}{3\sqrt[3]{b}\,d^{5/\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.04, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(131) = 262$.
time = 1.23, size = 619, normalized size = 3.66

$$\left[\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 3\sqrt{\frac{1}{3}}\,(b^2cd - abd^2)\sqrt{\frac{(-bd^2)^{\frac{1}{3}}}{b}}\log\left(-3\,bd^2x - 2\,bcd - ad^2 - 3\,(-bd^2)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d - 3\sqrt{\frac{1}{3}}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd - (-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} + (-bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{(-bd^2)^{\frac{1}{3}}}{b}}\right) + 2\,(-bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd - (-bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) - (-bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd + (-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - (-bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{3\,bd^3}, \frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd^2 - 6\sqrt{\frac{1}{3}}\,(b^2cd - abd^2)\sqrt{-\frac{(-bd^2)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - (-bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{(-bd^2)^{\frac{1}{3}}}{b}}}{bd^2x+ad^2}\right) + 2\,(-bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd - (-bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right) - (-bd^2)^{\frac{2}{3}}(bc-ad)\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd + (-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - (-bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{3\,bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(
1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*
x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2
)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) + 2*(-b*d^2)^(2/3)*(b*c - a*
d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*
x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b
*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))/(b*x + a)))/(b*d^3), 1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2
- 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x +
 a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) + 2*(-b*d^2)^(2/3)*(b*c -
 a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/
(b*x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3
)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*
d*x + a*d))/(b*x + a)))/(b*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(2/3),x)
```

```
[Out] int((a + b*x)^(2/3)/(c + d*x)^(2/3), x)
```

3.1603 $\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=126

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2\sqrt[3]{b}\,d^{2/3}}$$

```
[Out] -1/2*ln(d*x+c)/b^(1/3)/d^(2/3)-3/2*ln(-1+d^(1/3)*(b*x+a)^(1/3)/b^(1/3)/(d*x
+c)^(1/3))/b^(1/3)/d^(2/3)-arctan(1/3*3^(1/2)+2/3*d^(1/3)*(b*x+a)^(1/3)/b^(
1/3)/(d*x+c)^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)/d^(2/3)
```

Rubi [A]
time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {61}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*
(c + d*x)^(1/3))])/(b^(1/3)*d^(2/3))) - Log[c + d*x]/(2*b^(1/3)*d^(2/3)) -
(3*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(1/3
)*d^(2/3))
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}\,d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}\,d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{b}\sqrt[3]{}}\right.}{2\sqrt[3]{b}\,d^{2/3}}$$

Mathematica [A]
time = 0.15, size = 155, normalized size = 1.23

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)+\log\left(d^{2/3}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}\right)}{2\sqrt[3]{b}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3)
))/Sqrt[3]] - 2*Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)] +
Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*
(c + d*x)^(2/3))/(a + b*x)^(2/3)])/(2*b^(1/3)*d^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(90) = 180.
time = 1.21, size = 521, normalized size = 4.13

$$\left[\frac{\sqrt{3}\,bd\sqrt{\frac{(-bd^2)^{\frac{1}{3}}}{b}}\log\left(-3\,bd^2x-2\,bcd-ad^2-3\,(-bd^2)^{\frac{1}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}d-\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd-(-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}+(-bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{(-bd^2)^{\frac{1}{3}}}{b}}\right)-2\,(-bd^2)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(-bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+(-bd^2)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-(-bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\,bd^2},\ \frac{2\,\sqrt{3}\,bd\sqrt{-\frac{(-bd^2)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{3}\left(2\,(-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-(-bd^2)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{(-bd^2)^{\frac{1}{3}}}{b}}}{3\,(bd^2x+ad^2)}\right)-2\,(-bd^2)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}bd-(-bd^2)^{\frac{2}{3}}(bx+a)}{bx+a}\right)+(-bd^2)^{\frac{2}{3}}\log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}bd+(-bd^2)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}-(-bd^2)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 -
 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - sqrt(3)*(2*(b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (-b*d^2)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) - 2*(-b*d^2)^(2/3)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x +
 a)) + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(
2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x +
 a)))/(b*d^2), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b*d^2)^(1/3)/b)*arctan(1/3*sqrt(3
)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(-b*d^2)^(2/3)*log(
((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x + a))
 + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)
*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a))
)/(b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/3}\,{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)), x)
```

3.1604 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=30

$$-\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

```
[Out] -3*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{3(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/(b*x+a)^(1/3)*(d*x+c)^(1/3)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

Fricas [A]
time = 1.23, size = 42, normalized size = 1.40

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

Mupad [B]
time = 0.83, size = 26, normalized size = 0.87

$$\frac{3\,{\left(c+d\,x\right)}^{1/3}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x)
[Out] (3*(c + d*x)^(1/3))/((a*d - b*c)*(a + b*x)^(1/3))
```

3.1605 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}}+\frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}$$

```
[Out] -3/4*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(4/3)+9/4*d*(d*x+c)^(1/3)/(-a*d+b*c)^
2/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {47, 37}

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}\,(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (9*d*(c + d*x)^(1/3)
)/(4*(b*c - a*d)^2*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}} - \frac{(3d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{4(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}} + \frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}$$

Mathematica [A]
time = 0.10, size = 46, normalized size = 0.70

$$\frac{3\sqrt[3]{c+dx}\,(-bc+4ad+3bdx)}{4(bc-ad)^2(a+bx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(-(b*c) + 4*a*d + 3*b*d*x))/(4*(b*c - a*d)^2*(a + b*x)^(
4/3))
```

Maple [A]
time = 0.24, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{3(dx+c)^{\frac{1}{3}}(3bdx+4ad-bc)}{4(bx+a)^{\frac{4}{3}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(d*x+c)^(1/3)*(3*b*d*x+4*a*d-b*c)/(b*x+a)^(4/3)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.82, size = 118, normalized size = 1.79

$$\frac{3\left(3\,bdx - bc + 4\,ad\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{4\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(3*b*d*x - b*c + 4*a*d)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

Mupad [B]
time = 0.98, size = 71, normalized size = 1.08

$$\frac{\left(\frac{9\,d\,x}{4\,{\left(a\,d-b\,c\right)}^2}+\frac{12\,a\,d-3\,b\,c}{4\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}+\frac{a\,{\left(a+b\,x\right)}^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x)
```

```
[Out] (((9*d*x)/(4*(a*d - b*c)^2) + (12*a*d - 3*b*c)/(4*b*(a*d - b*c)^2))*(c + d*
x)^(1/3))/(x*(a + b*x)^(1/3) + (a*(a + b*x)^(1/3))/b)
```

3.1606 $\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}}-\frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

```
[Out] -3/7*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(7/3)+9/14*d*(d*x+c)^(1/3)/(-a*d+b*c)
^2/(b*x+a)^(4/3)-27/14*d^2*(d*x+c)^(1/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}\,(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(1/3)
)/(14*(b*c - a*d)^2*(a + b*x)^(4/3)) - (27*d^2*(c + d*x)^(1/3))/(14*(b*c -
a*d)^3*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{7(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}} + \frac{(9d^2)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}}{14(bc-ad)^2}$$
$$= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}} + \frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}} - \frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}$$

Mathematica [**A**]
time = 0.12, size = 75, normalized size = 0.74

$$-\frac{3\sqrt[3]{c+dx}\,(14a^2d^2-7abd(c-3dx)+b^2(2c^2-3cdx+9d^2x^2))}{14(bc-ad)^3(a+bx)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(14*a^2*d^2 - 7*a*b*d*(c - 3*d*x) + b^2*(2*c^2 - 3*c*d*
x + 9*d^2*x^2)))/(14*(b*c - a*d)^3*(a + b*x)^(7/3))
```

Maple [**A**]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{3(dx+c)^{\frac{1}{3}}\left(9b^2x^2d^2+21ab\,d^2x-3b^2cdx+14a^2d^2-7abcd+2b^2c^2\right)}{14(bx+a)^{\frac{7}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/14*(d*x+c)^(1/3)*(9*b^2*d^2*x^2+21*a*b*d^2*x-3*b^2*c*d*x+14*a^2*d^2-7*a*b
*c*d+2*b^2*c^2)/(b*x+a)^(7/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(83) = 166$.
time = 1.06, size = 251, normalized size = 2.49

$$-\frac{3\,(9\,b^2d^2x^2+2\,b^2c^2-7\,abcd+14\,a^2d^2-3\,(b^2cd-7\,abd^2)x)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{14\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/14*(9*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 14*a^2*d^2 - 3*(b^2*c*d - 7*
a*b*d^2)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
+ 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^
3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2
*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{10}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(10/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

Mupad [B]
time = 1.51, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{27\,d^2\,x^2}{14\,(a\,d-b\,c)^3}+\frac{42\,a^2\,d^2-21\,a\,b\,c\,d+6\,b^2\,c^2}{14\,b^2\,(a\,d-b\,c)^3}+\frac{9\,d\,x\,(7\,a\,d-b\,c)}{14\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((27*d^2*x^2)/(14*(a*d - b*c)^3) + (42*a^2*d^2 + 6*b^2*c^2
 - 21*a*b*c*d)/(14*b^2*(a*d - b*c)^3) + (9*d*x*(7*a*d - b*c))/(14*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*
x)^(1/3))/b)
```

3.1607 $\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}}+\frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}}-\frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}}+\frac{243d^3\sqrt[3]{c+dx}}{140(bc-ad)^4\sqrt[3]{a+bx}}$$

```
[Out] -3/10*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(10/3)+27/70*d*(d*x+c)^(1/3)/(-a*d+b
*c)^2/(b*x+a)^(7/3)-81/140*d^2*(d*x+c)^(1/3)/(-a*d+b*c)^3/(b*x+a)^(4/3)+243
/140*d^3*(d*x+c)^(1/3)/(-a*d+b*c)^4/(b*x+a)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}\,(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (27*d*(c + d*x)^(1
/3))/(70*(b*c - a*d)^2*(a + b*x)^(7/3)) - (81*d^2*(c + d*x)^(1/3))/(140*(b*
c - a*d)^3*(a + b*x)^(4/3)) + (243*d^3*(c + d*x)^(1/3))/(140*(b*c - a*d)^4*
(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} - \frac{(9d)\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx}{10(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} + \frac{(27d^2)\int \frac{1}{(a+bx)^{7/3}(c+}}{35(bc-ad)}$$
$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+b}$$
$$= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}} + \frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}} - \frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+b}$$

Mathematica [A]
time = 0.15, size = 95, normalized size = 0.70

$$-\frac{3\sqrt[3]{c+dx}\left(-140d^3+\frac{105bd^2(c+dx)}{a+bx}-\frac{60b^2d(c+dx)^2}{(a+bx)^2}+\frac{14b^3(c+dx)^3}{(a+bx)^3}\right)}{140(bc-ad)^4\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(-140*d^3 + (105*b*d^2*(c + d*x))/(a + b*x) - (60*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (14*b^3*(c + d*x)^3)/(a + b*x)^3))/(140*(b*c -
a*d)^4*(a + b*x)^(1/3))
```

Maple [A]
time = 0.19, size = 171, normalized size = 1.26

method	result
gosper	$\frac{3(dx+c)^{\frac{1}{3}}\left(81b^3x^3d^3+270d^3a\,x^2b^2-27b^3c\,d^2x^2+315a^2b\,d^3x-90a\,b^2c\,d^2x+18b^3c^2dx+140a^3d^3-105a^2bc\,d^2+60a\,b^2c^2d-14b^3c^3\right)}{140(bx+a)^{\frac{10}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/140*(d*x+c)^(1/3)*(81*b^3*d^3*x^3+270*a*b^2*d^3*x^2-27*b^3*c*d^2*x^2+315*
a^2*b*d^3*x-90*a*b^2*c*d^2*x+18*b^3*c^2*d*x+140*a^3*d^3-105*a^2*b*c*d^2+60*
a*b^2*c^2*d-14*b^3*c^3)/(b*x+a)^(10/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(112) = 224$.
time = 0.81, size = 419, normalized size = 3.08

$$\frac{3\,(81\,b^3d^3x^3 - 14\,b^3c^3 + 60\,ab^2c^2d - 105\,a^2bcd^2 + 140\,a^3d^3 - 27\,(b^3cd^2 - 10\,ab^2d^3)x^2 + 9\,(2\,b^3c^2d - 10\,ab^2cd^2 + 35\,a^2bd^3)x)(bx + a)^{\frac{2}{3}}(dx + c)^{\frac{1}{3}}}{140\,(a^4b^4c^4 - 4\,a^5b^3c^3d + 6\,a^6b^2c^2d^2 - 4\,a^7bcd^3 + a^8d^4 + (b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4b^4d^4)x^4 + 4\,(ab^7c^4 - 4\,a^2b^6c^3d + 6\,a^3b^5c^2d^2 - 4\,a^4b^4cd^3 + a^5b^3d^4)x^3 + 6\,(a^2b^6c^4 - 4\,a^3b^5c^3d + 6\,a^4b^4c^2d^2 - 4\,a^5b^3cd^3 + a^6b^2d^4)x^2 + 4\,(a^3b^5c^4 - 4\,a^4b^4c^3d + 6\,a^5b^3c^2d^2 - 4\,a^6b^2cd^3 + a^7bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(81*b^3*d^3*x^3 - 14*b^3*c^3 + 60*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 140
*a^3*d^3 - 27*(b^3*c*d^2 - 10*a*b^2*d^3)*x^2 + 9*(2*b^3*c^2*d - 10*a*b^2*c*
d^2 + 35*a^2*b*d^3)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^4*c^4 - 4*a^5
*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b
^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^
7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4
)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*
d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d
^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{13}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(13/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

Mupad [B]
time = 1.27, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/3}\left(\frac{243\,d^3\,x^3}{140\,(a\,d-b\,c)^4}+\frac{420\,a^3\,d^3-315\,a^2\,b\,c\,d^2+180\,a\,b^2\,c^2\,d-42\,b^3\,c^3}{140\,b^3\,(a\,d-b\,c)^4}+\frac{27\,d\,x\,(35\,a^2\,d^2-10\,a\,b\,c\,d+2\,b^2\,c^2)}{140\,b^2\,(a\,d-b\,c)^4}+\frac{81\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^3*x^3)/(140*(a*d - b*c)^4) + (420*a^3*d^3 - 42*b^3
*c^3 + 180*a*b^2*c^2*d - 315*a^2*b*c*d^2)/(140*b^3*(a*d - b*c)^4) + (27*d*x
*(35*a^2*d^2 + 2*b^2*c^2 - 10*a*b*c*d))/(140*b^2*(a*d - b*c)^4) + (81*d^2*x
^2*(10*a*d - b*c))/(140*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/3) + (a^3*(a +
 b*x)^(1/3))/b^3 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/
b^2)
```

3.1608 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=649

$$\frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3}-\frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d}-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}}{}$$

```
[Out] 21/20*(-a*d+b*c)^2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/d^3-21/40*(-a*d+b*c)*(b*x+a)
^(4/3)*(d*x+c)^(1/3)/d^2+3/8*(b*x+a)^(7/3)*(d*x+c)^(1/3)/d-7/20*3^(3/4)*(-a
*d+b*c)^3*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+
c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*
c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^
(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((
b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a
*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/d^(10/3)/(b*x+a)^(2/3)/
(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 649, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^3((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{10\,2^{2/3}\sqrt[3]{b}\,d^{10/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{21\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc-ad)^2}{20d^3}-\frac{21(a+bx)^{4/3}\sqrt[3]{c+dx}(bc-ad)}{40d^2}+\frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]
```

```
[Out] (21*(b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(20*d^3) - (21*(b*c - a*
d)*(a + b*x)^(4/3)*(c + d*x)^(1/3))/(40*d^2) + (3*(a + b*x)^(7/3)*(c + d*x)
^(1/3))/(8*d) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)^3*((a + b*x)*(c +
d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1
/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*
b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b
```

```
^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3
) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*d^(10/3)
*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}}\,dx = \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}dx}{8d}$$

$$= -\frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+dx}}{8d} + \frac{(7(bc-ad)^2)\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}}{10d^2}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

$$= \frac{21(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{20d^3} - \frac{21(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}{40d^2} + \frac{3(a+bx)^{7/3}\sqrt[3]{c+}}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.11

$$\frac{3(a+bx)^{10/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{10}{3};\frac{13}{3};\frac{d(a+bx)}{-bc+ad}\right)}{10b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(10/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3
, 10/3, 13/3, (d*(a + b*x))/(-(b*c) + a*d)])/(10*b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/3)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(7/3)/(c + d*x)^(2/3), x)
```

3.1609 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=614

$$-\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d}+\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}}{\ldots}$$

```
[Out] -6/5*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(1/3)/d^2+3/5*(b*x+a)^(4/3)*(d*x+c)^(
1/3)/d+2/5*2^(1/3)*3^(3/4)*(-a*d+b*c)^2*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)
^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d
+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3
)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/
3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/d
^(7/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^
(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)^2((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\,d^{7/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{6\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc-ad)}{5d^2}+\frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]
```

```
[Out] (-6*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(5*d^2) + (3*(a + b*x)^(4/
3)*(c + d*x)^(1/3))/(5*d) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d
)^2*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a
*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))
^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
```

```
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b^(1/
3)*d^(7/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{2/3}}\,dx = \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} - \frac{(4(bc-ad))\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx}{5d}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{(2(bc-ad)^2)\int \frac{1}{(a+bx)^{2/3}(c+d}}{5d^2}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{(2(bc-ad)^2((a+bx)(c+}{5d^2(a+bx}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{\left(6(bc-ad)^2((a+bx)(c+\right.}{}$$

$$= -\frac{6(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{5d^2} + \frac{3(a+bx)^{4/3}\sqrt[3]{c+dx}}{5d} + \frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.12

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,x\right)}^{4/3}}{{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(4/3)/(c + d*x)^(2/3), x)
```

3.1610 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=577

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\,\Big((bc-ad)^{2/3}+2\cdots}{2^{2/3}\sqrt[3]{b}\cdots}$$

[Out] 3/2*(b*x+a)^(1/3)*(d*x+c)^(1/3)/d-1/2*3^(3/4)*(-a*d+b*c)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/d^(4/3)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.38, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 224}

$$\frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{2^{2/3}\sqrt[3]{b}\,d^{4/3}(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]

[Out] (3*(a + b*x)^(1/3)*(c + d*x)^(1/3))/(2*d) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*(b*c - a*d)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3

```
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(
2/3)*b^(1/3)*d^(4/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2d}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{\left((bc-ad)((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2d(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{\left(3(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\Big(}{2d(a+bx)^{2/3}(c+dx}$$

$$= \frac{3\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}{2d} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(bc-ad)((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bd}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.13

$$\frac{3(a+bx)^{4/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(1/3)/(c + d*x)^(2/3), x)
```

3.1611 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=542

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+d}\right.}{\sqrt[3]{b}\,\sqrt[3]{d}\,(a+bx)^{2/3}(c+dx)^{2/3}}$$

```
[Out] 2^(1/3)*3^(3/4)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d
^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*
x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+
a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1
/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2
/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/d^(1/3)/(b*x+a)^(2/3)/(d*
x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*
((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {64, 637, 224}

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\,\sqrt[3]{d}\,(a+bx)^{2/3}(c+dx)^{2/3}(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c +
a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a
*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x
)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
```

```
3))], -7 - 4*Sqrt[3]])/(b^(1/3)*d^(1/3)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*
c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx = \frac{((a+bx)(c+dx))^{2/3}\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= \frac{\left(3((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+ad}}\right.}{(a+bx)^{2/3}(c+dx)^{2/3}(bc+ad+2bdx}$$

$$= \frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.13

$$\frac{3\sqrt[3]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[1/3,
 2/3, 4/3, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{2}{3}}\left(c+dx\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{2/3}\,{\left(c+d\,x\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(2/3)), x)
```

3.1612 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=586

$$-\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}}-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\right.}{2^{2/3}\sqrt[3]{b}\,(}$$

```
[Out] -3/2*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(2/3)-1/2*3^(3/4)*d^(2/3)*((b*x+a)*(d
*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1
/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^
(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b
*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1
/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3
)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3)
)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1
/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/(-a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*
d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3
)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)
*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 586, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 224}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}\,b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{c+dx}}{2(a+bx)^{2/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(2*(b*c - a*d)*(a + b*x)^(2/3)) - (3^(3/4)*Sqrt[2 + Sq
rt[3]]*d^(2/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
```

```
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(2^(2/3)*b^(1/3)*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d
+ 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x
))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{d\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}\,dx}{2(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{\left(d((a+bx)(c+dx))^{2/3}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{2/3}}\,dx}{2(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{\left(3d((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad+2bdx)^2}\right)\text{S}}{2(bc-ad)(a+}$$

$$= -\frac{3\sqrt[3]{c+dx}}{2(bc-ad)(a+bx)^{2/3}} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,d^{2/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.12

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},\frac{2}{3};\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b(a+bx)^{2/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(2*b*(a + b*x)^(2/3)*(c + d*x)^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}} (c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/3}\,(c+d\,x)^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(2/3)), x)
```

3.1613 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=621

$$-\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}}+\frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}}+\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(bc+ad}}{}$$

```
[Out] -3/5*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(5/3)+6/5*d*(d*x+c)^(1/3)/(-a*d+b*c)^
2/(b*x+a)^(2/3)+2/5*2^(1/3)*3^(3/4)*d^(5/3)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+
b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(
2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((
-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^
(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d
^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/
3)/(-a*d+b*c)^2/(b*x+a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*
x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(
-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 224}

$$\frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^2(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{6d\sqrt[3]{c+dx}}{5(a+bx)^{2/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{5(a+bx)^{5/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (6*d*(c + d*x)^(1/3)
)/(5*(b*c - a*d)^2*(a + b*x)^(2/3)) + (2*2^(1/3)*3^(3/4)*Sqrt[2 + Sqrt[3]]*
d^(5/3)*((a + b*x)*(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a
*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c
 - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d
*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
```

```
1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(5*b
^(1/3)*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*
Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*
d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}\,dx}{5(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{(2d^2)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{2/3}}}{5(bc-ad)^2}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{\left(2d^2((a+bx)(c+dx))\right.}{5(bc-ad)^2(a}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{\left(6d^2((a+bx)(c+dx))\right.}{}$$

$$= -\frac{3\sqrt[3]{c+dx}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d\sqrt[3]{c+dx}}{5(bc-ad)^2(a+bx)^{2/3}} + \frac{2\sqrt[3]{2}\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{5/}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.12

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{2}{3};-\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}\,(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(2/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{8/3}\,{\left(c+d\,x\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(2/3)), x)
```

3.1614 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=656

$$-\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}}+\frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}}-\frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/3}}-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{8/3}((a+bx)(}{}$$

```
[Out] -3/8*(d*x+c)^(1/3)/(-a*d+b*c)/(b*x+a)^(8/3)+21/40*d*(d*x+c)^(1/3)/(-a*d+b*c
)^2/(b*x+a)^(5/3)-21/20*d^2*(d*x+c)^(1/3)/(-a*d+b*c)^3/(b*x+a)^(2/3)-7/20*3
^(3/4)*d^(8/3)*((b*x+a)*(d*x+c))^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x
+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a
*d+b*c)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/
3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/
3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/3)/b^(1/3)/(-a*d+b*c)^3/(b*x+
a)^(2/3)/(d*x+c)^(2/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d
+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 656, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {53, 64, 637, 224}

$$-\frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,d^{8/3}((a+bx)(c+dx))^{2/3}\sqrt{(ad+bc+2bdx)^2}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)\sqrt{\frac{2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{4/3}}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{10\,2^{2/3}\sqrt[3]{b}(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)^3(ad+bc+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+(bc-ad)^{2/3}\right)}{\left(2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}+\left(1+\sqrt{3}\right)(bc-ad)^{2/3}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{21d^2\sqrt[3]{c+dx}}{20(a+bx)^{2/3}(bc-ad)^3}+\frac{21d\sqrt[3]{c+dx}}{40(a+bx)^{5/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{8(a+bx)^{8/3}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (21*d*(c + d*x)^(1/3
))/(40*(b*c - a*d)^2*(a + b*x)^(5/3)) - (21*d^2*(c + d*x)^(1/3))/(20*(b*c -
 a*d)^3*(a + b*x)^(2/3)) - (7*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^(8/3)*((a + b*x)*
(c + d*x))^(2/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(
2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1
```

```
/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x
)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1
/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(10*2^(2/3)*b^(1/3)*(b*
c - a*d)^3*(a + b*x)^(2/3)*(c + d*x)^(2/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c
 - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} - \frac{(7d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{2/3}}\,dx}{8(bc-ad)}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} + \frac{(7d^2)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{2/3}}}{10(bc-ad)^2}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

$$= -\frac{3\sqrt[3]{c+dx}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d\sqrt[3]{c+dx}}{40(bc-ad)^2(a+bx)^{5/3}} - \frac{21d^2\sqrt[3]{c+dx}}{20(bc-ad)^3(a+bx)^{2/}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.11

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(-\frac{8}{3},\frac{2}{3};-\frac{5}{3};\frac{d(a+bx)}{-bc+ad}\right)}{8b(a+bx)^{8/3}(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[-8/3, 2/3, -5/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(8*b*(a + b*x)^(8/3)*(c + d*x)^(2/3))
```

Maple [**F**]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(2/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/3)*(d*x + c)^(1/3)/(b^4*d*x^5 + a^4*c + (b^4*c + 4*a*
b^3*d)*x^4 + 2*(2*a*b^3*c + 3*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 2*a^3*b*d)*
x^2 + (4*a^3*b*c + a^4*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(2/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(2/3)), x)
```

3.1615 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=241

$$-\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}}-\frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3}+\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{3\sqrt{3}\,d^{10}}$$

```
[Out] -3*(b*x+a)^(7/3)/d/(d*x+c)^(1/3)-14/3*b*(-a*d+b*c)*(b*x+a)^(1/3)*(d*x+c)^(2
/3)/d^3+7/2*b*(b*x+a)^(4/3)*(d*x+c)^(2/3)/d^2-7/9*b^(1/3)*(-a*d+b*c)^2*ln(b
*x+a)/d^(10/3)-7/3*b^(1/3)*(-a*d+b*c)^2*ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)
/(b*x+a)^(1/3))/d^(10/3)-14/9*b^(1/3)*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*b
^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2))/d^(10/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {49, 52, 61}

$$-\frac{14\sqrt[3]{b}\,(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log(a+bx)}{9d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3d^{10/3}}-\frac{14b\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^3}+\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2}-\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(7/3))/(d*(c + d*x)^(1/3)) - (14*b*(b*c - a*d)*(a + b*x)^(1/3
)*(c + d*x)^(2/3))/(3*d^3) + (7*b*(a + b*x)^(4/3)*(c + d*x)^(2/3))/(2*d^2)
- (14*b^(1/3)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/
(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*d^(10/3)) - (7*b^(1/3)*(b*c
- a*d)^2*Log[a + b*x])/(9*d^(10/3)) - (7*b^(1/3)*(b*c - a*d)^2*Log[-1 + (b^
(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*d^(10/3))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{(14b(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} + \frac{(14b(bc$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14\sqrt[3]{b}\,(}{}$$

Mathematica [A]
time = 0.58, size = 267, normalized size = 1.11

$$\frac{-\frac{3\sqrt[3]{d}\sqrt[3]{a+bx}\left(18a^2d^2-abd(49c+13dx)+b^2\left(28c^2+7cdx-3d^2x^2\right)\right)}{\sqrt[3]{c+dx}}+28\sqrt{3}\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}}{\sqrt{3}}\right)-28\sqrt[3]{b}\,(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)+14\sqrt[3]{b}\,(bc-ad)^2\log\left(b^{2/3}+\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{18d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] ((-3*d^(1/3)*(a + b*x)^(1/3)*(18*a^2*d^2 - a*b*d*(49*c + 13*d*x) + b^2*(28*
c^2 + 7*c*d*x - 3*d^2*x^2)))/(c + d*x)^(1/3) + 28*Sqrt[3]*b^(1/3)*(b*c - a*
d)^2*ArcTan[(1 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3)))/Sqr
t[3]] - 28*b^(1/3)*(b*c - a*d)^2*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c
 + d*x)^(1/3)] + 14*b^(1/3)*(b*c - a*d)^2*Log[b^(2/3) + (d^(2/3)*(a + b*x)^
```

```
(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)]
)/(18*d^(10/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
[Out] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(187) = 374$.
time = 0.72, size = 423, normalized size = 1.76

$$-\frac{28\,\sqrt{3}\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right)+14\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-28\,(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)-3\,(3\,b^2d^2x^2-28\,b^2c^2+49\,abcd-18\,a^2d^2-(7\,b^2cd-13\,abd^2)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{18\,(d^4x+cd^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] -1/18*(28*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) + 14*(b^2
*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b
/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b
/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 28*(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*
log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) -
 3*(3*b^2*d^2*x^2 - 28*b^2*c^2 + 49*a*b*c*d - 18*a^2*d^2 - (7*b^2*c*d - 13*
a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^4*x + c*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(7/3)/(c + d*x)^(4/3), x)
```

3.1616 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=195

$$-\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}+\frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2}+\frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{2\sqrt[3]{b}\,(bc-ad}{3d^7}$$

```
[Out] -3*(b*x+a)^(4/3)/d/(d*x+c)^(1/3)+4*b*(b*x+a)^(1/3)*(d*x+c)^(2/3)/d^2+2/3*b^
(1/3)*(-a*d+b*c)*ln(b*x+a)/d^(7/3)+2*b^(1/3)*(-a*d+b*c)*ln(-1+b^(1/3)*(d*x+
c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/d^(7/3)+4/3*b^(1/3)*(-a*d+b*c)*arctan(1/3*3
^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2))/d^(7/3)*3^(
1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 52, 61}

$$\frac{4\sqrt[3]{b}\,(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}+\frac{2\sqrt[3]{b}\,(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}\,(bc-ad)\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2}-\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(4/3))/(d*(c + d*x)^(1/3)) + (4*b*(a + b*x)^(1/3)*(c + d*x)^(
2/3))/d^2 + (4*b^(1/3)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^
(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*d^(7/3)) + (2*b^(1/3)*(
b*c - a*d)*Log[a + b*x])/(3*d^(7/3)) + (2*b^(1/3)*(b*c - a*d)*Log[-1 + (b^(
1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/d^(7/3)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{(4b)\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2} - \frac{(4b(bc-ad))\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2} + \frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+}}\right.}{\sqrt{3}\,d^{7/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.05, size = 73, normalized size = 0.37

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

Fricas [A]
time = 0.73, size = 306, normalized size = 1.57

$$\frac{4\sqrt{3}\,(bc^2 - acd + (bcd - ad^2)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right) + 2\,(bc^2 - acd + (bcd - ad^2)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right) - 4\,(bc^2 - acd + (bcd - ad^2)x)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right) + 3\,(bdx + 4\,bc - 3\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{3\,(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)*arctan(1/3*
(2*sqrt(3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x
+ b*c))/(b*d*x + b*c)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)
*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b/d)^(1/3)
 + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 4*(b*c^2 - a*c*d + (b*c*d
- a*d^2)*x)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x
 + c)^(2/3))/(d*x + c)) + 3*(b*d*x + 4*b*c - 3*a*d)*(b*x + a)^(1/3)*(d*x +
c)^(2/3))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^{\frac{4}{3}}}{(c + dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(4/3)/(c + d*x)^(4/3), x)
```

3.1617 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{d^{4/3}}-\frac{\sqrt[3]{b}\,\log(a+bx)}{2d^{4/3}}-\frac{3\sqrt[3]{b}\,\log\left(-1+\frac{\sqrt[3]{b}\,\sqrt[3]{c+d}}{\sqrt[3]{d}\,\sqrt[3]{a+b}}\right.}{2d^{4/3}}$$

```
[Out] -3*(b*x+a)^(1/3)/d/(d*x+c)^(1/3)-1/2*b^(1/3)*ln(b*x+a)/d^(4/3)-3/2*b^(1/3)*
ln(-1+b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3))/d^(4/3)-b^(1/3)*arctan(1
/3*3^(1/2)+2/3*b^(1/3)*(d*x+c)^(1/3)/d^(1/3)/(b*x+a)^(1/3)*3^(1/2))*3^(1/2)
/d^(4/3)
```

Rubi [A]
time = 0.02, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {49, 61}

$$-\frac{\sqrt{3}\,\sqrt[3]{b}\,\text{ArcTan}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{b}\,\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\,\log(a+bx)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) - (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))]])/d^(4/3)
 - (b^(1/3)*Log[a + b*x])/(2*d^(4/3)) - (3*b^(1/3)*Log[-1 + (b^(1/3)*(c + d
*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*d^(4/3))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx = -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{d}$$
$$= -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}} - \frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{d^{4/3}} - \frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}} - \frac{3\sqrt[3]{b}\ \mathrm{l}}{}$$

Mathematica [A]
time = 0.21, size = 191, normalized size = 1.28

$$\frac{-\frac{6\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} + 2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}}{\sqrt{3}}\right) - 2\sqrt[3]{b}\log\left(\sqrt[3]{b} - \frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right) + \sqrt[3]{b}\log\left(b^{2/3} + \frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}} + \frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] ((-6*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + 2*Sqrt[3]*b^(1/3)*ArcTan[(1
 + (2*d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3)))/Sqrt[3]] - 2*b^(1
/3)*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)] + b^(1/3)*Log[
b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a +
 b*x)^(1/3))/(c + d*x)^(1/3)])/(2*d^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(109) = 218$.
time = 0.95, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}\,(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}\,(bdx+bc)}{3\,(bdx+bc)}\right)+(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-2\,(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)+6\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\,(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(d*x + c)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3
)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) +
(d*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x +
 c)^(2/3)*(-b/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 2*(d
*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c)) + 6*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(4/3),x)
[Out] int((a + b*x)^(1/3)/(c + d*x)^(4/3), x)
```

3.1618 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}} dx$

Optimal. Leaf size=30

$$\frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

[Out] 3*(b*x+a)^(1/3)/(-a*d+b*c)/(d*x+c)^(1/3)

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]

[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]

```
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Maple [A]
time = 0.20, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{3(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
[Out] -3*(b*x+a)^(1/3)/(d*x+c)^(1/3)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

Fricas [A]
time = 1.13, size = 42, normalized size = 1.40

$$\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] 3*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(4/3),x)
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a+b\,x)^{2/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)), x)
```

3.1619 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}$$

```
[Out] -3/2/(-a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)-9/2*d*(b*x+a)^(1/3)/(-a*d+b*c)^
2/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(2*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3)) - (9*d*(a + b*x)^(1/3))/
(2*(b*c - a*d)^2*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx = -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx}{2(bc-ad)}$$
$$= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}} - \frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.68

$$-\frac{3(2ad+b(c+3dx))}{2(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(2*a*d + b*(c + 3*d*x)))/(2*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(1/
3))
```

Maple [A]
time = 0.16, size = 53, normalized size = 0.80

method	result	size
gosper	$-\frac{3(3bdx+2ad+bc)}{2(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*(3*b*d*x+2*a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.

time = 0.83, size = 126, normalized size = 1.91

$$-\frac{3\,(3\,bdx+bc+2\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(3*b*d*x + b*c + 2*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(4/3)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)), x)
```

3.1620 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}+\frac{27d^2\sqrt[3]{a+bx}}{5(bc-ad)^3\sqrt[3]{c+dx}}$$

```
[Out] -3/5/(-a*d+b*c)/(b*x+a)^(5/3)/(d*x+c)^(1/3)+9/5*d/(-a*d+b*c)^2/(b*x+a)^(2/3
)/(d*x+c)^(1/3)+27/5*d^2*(b*x+a)^(1/3)/(-a*d+b*c)^3/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(5*(b*c - a*d)*(a + b*x)^(5/3)*(c + d*x)^(1/3)) + (9*d)/(5*(b*c - a*d)^2
*(a + b*x)^(2/3)*(c + d*x)^(1/3)) + (27*d^2*(a + b*x)^(1/3))/(5*(b*c - a*d)
^3*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx = -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx}{5(bc-ad)}$$
$$= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}} + \frac{(9d^2)}{}$$
$$= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}} + \frac{27}{5(bc-}$$

Mathematica [A]
time = 0.13, size = 71, normalized size = 0.70

$$-\frac{3(c+dx)^{5/3}\left(b^2-\frac{5d^2(a+bx)^2}{(c+dx)^2}-\frac{5bd(a+bx)}{c+dx}\right)}{5(bc-ad)^3(a+bx)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(5/3)*(b^2 - (5*d^2*(a + b*x)^2)/(c + d*x)^2 - (5*b*d*(a + b*
x))/(c + d*x)))/(5*(b*c - a*d)^3*(a + b*x)^(5/3))
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{3\left(9b^2x^2d^2+15ab\,d^2x+3b^2cdx+5a^2d^2+5abcd-b^2c^2\right)}{5(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/5*(9*b^2*d^2*x^2+15*a*b*d^2*x+3*b^2*c*d*x+5*a^2*d^2+5*a*b*c*d-b^2*c^2)/(
b*x+a)^(5/3)/(d*x+c)^(1/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(83) = 166$.
time = 0.99, size = 273, normalized size = 2.70

$$\frac{3\,(9\,b^2d^2x^2 - b^2c^2 + 5\,abcd + 5\,a^2d^2 + 3\,(b^2cd + 5\,abd^2)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{5\,(a^2b^3c^4 - 3\,a^3b^2c^3d + 3\,a^4bc^2d^2 - a^5cd^3 + (b^5c^3d - 3\,ab^4c^2d^2 + 3\,a^2b^3cd^3 - a^3b^2d^4)x^3 + (b^5c^4 - ab^4c^3d - 3\,a^2b^3c^2d^2 + 5\,a^3b^2cd^3 - 2\,a^4bd^4)x^2 + (2\,ab^4c^4 - 5\,a^2b^3c^3d + 3\,a^3b^2c^2d^2 + a^4bcd^3 - a^5d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(9*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 5*a^2*d^2 + 3*(b^2*c*d + 5*a*b*d
^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a
^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 -
 a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*
c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{8/3}\,{\left(c+d\,x\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)), x)
```

3.1621 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}}+\frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}}-\frac{81d^2}{40(bc-ad)^3(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{}{40(}$$

```
[Out] -3/8/(-a*d+b*c)/(b*x+a)^(8/3)/(d*x+c)^(1/3)+27/40*d/(-a*d+b*c)^2/(b*x+a)^(5
/3)/(d*x+c)^(1/3)-81/40*d^2/(-a*d+b*c)^3/(b*x+a)^(2/3)/(d*x+c)^(1/3)-243/40
*d^3*(b*x+a)^(1/3)/(-a*d+b*c)^4/(d*x+c)^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}\,(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(8*(b*c - a*d)*(a + b*x)^(8/3)*(c + d*x)^(1/3)) + (27*d)/(40*(b*c - a*d)
^2*(a + b*x)^(5/3)*(c + d*x)^(1/3)) - (81*d^2)/(40*(b*c - a*d)^3*(a + b*x)^
(2/3)*(c + d*x)^(1/3)) - (243*d^3*(a + b*x)^(1/3))/(40*(b*c - a*d)^4*(c + d
*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx = -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} - \frac{(9d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx}{8(bc-ad)}$$
$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} + \frac{(2\cdots}{\cdots}$$
$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{\cdots}{40\cdots}$$
$$= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}} + \frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}} - \frac{\cdots}{40\cdots}$$

Mathematica [A]
time = 0.14, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{8/3}\left(5b^3+\frac{40d^3(a+bx)^3}{(c+dx)^3}+\frac{60bd^2(a+bx)^2}{(c+dx)^2}-\frac{24b^2d(a+bx)}{c+dx}\right)}{40(bc-ad)^4(a+bx)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^3 + (40*d^3*(a + b*x)^3)/(c + d*x)^3 + (60*b*d^2*(
a + b*x)^2)/(c + d*x)^2 - (24*b^2*d*(a + b*x))/(c + d*x)))/(40*(b*c - a*d)^
4*(a + b*x)^(8/3))
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.26

method	result	size
gosper	$-\frac{3\left(81b^3x^3d^3+216d^3a\,x^2b^2+27b^3c\,d^2x^2+180a^2b\,d^3x+72a\,b^2c\,d^2x-9b^3c^2dx+40a^3d^3+60a^2bc\,d^2-24a\,b^2c^2d+5b^3c^3\right)}{40(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/40*(81*b^3*d^3*x^3+216*a*b^2*d^3*x^2+27*b^3*c*d^2*x^2+180*a^2*b*d^3*x+72
*a*b^2*c*d^2*x-9*b^3*c^2*d*x+40*a^3*d^3+60*a^2*b*c*d^2-24*a*b^2*c^2*d+5*b^3
*c^3)/(b*x+a)^(8/3)/(d*x+c)^(1/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(112) = 224$.
time = 1.45, size = 456, normalized size = 3.35

$$-\frac{3\,(81\,b^3d^3x^3+5\,b^3c^3-24\,ab^2c^2d+60\,a^2bcd^2+40\,a^3d^3+27\,(b^3cd^2+8\,ab^2d^3)x^2-9\,(b^3c^2d-8\,ab^2cd^2-20\,a^2bd^3)x)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\,(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5)x^4+(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5)x^3+3\,(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5)x^2+(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/40*(81*b^3*d^3*x^3 + 5*b^3*c^3 - 24*a*b^2*c^2*d + 60*a^2*b*c*d^2 + 40*a^
3*d^3 + 27*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 9*(b^3*c^2*d - 8*a*b^2*c*d^2 - 2
0*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)), x)
```

3.1622 $\int \frac{(a+bx)^{8/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1355

$$-\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}}-\frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3}+\frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2}+\frac{30\ 2^{2/3}\sqrt[3]{b}\,(bc-a}{7d^{11/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc}$$

```
[Out] -3*(b*x+a)^(8/3)/d/(d*x+c)^(1/3)-30/7*b*(-a*d+b*c)*(b*x+a)^(2/3)*(d*x+c)^(2
/3)/d^3+24/7*b*(b*x+a)^(5/3)*(d*x+c)^(2/3)/d^2+30/7*2^(2/3)*b^(1/3)*(-a*d+b
*c)^2*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c)
)^2)^(1/2)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*
b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+20/7*
2^(1/6)*3^(3/4)*b^(1/3)*(-a*d+b*c)^(8/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c
)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(
2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/
3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/
3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/d^(11/3)/(b*x+a)^(1/3)/(d*x+c)^(1/
3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)
*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-15/
7*2^(2/3)*3^(1/4)*b^(1/3)*(-a*d+b*c)^(8/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-
a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(
1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/d^(11/
3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2
)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.73, antiderivative size = 1355, normalized size of antiderivative = 1.00, number

of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 64, 637, 309, 224, 1891}

$$-\frac{15\ 2^{2/3}\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{20\sqrt[6]{2}\,3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{8/3}}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{30\ 2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)^2}{7d^{11/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{30b(a+bx)^{2/3}(c+dx)^{2/3}(bc-ad)}{7d^3}+\frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2}-\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(8/3))/(d*(c + d*x)^(1/3)) - (30*b*(b*c - a*d)*(a + b*x)^(2/3
)*(c + d*x)^(2/3))/(7*d^3) + (24*b*(a + b*x)^(5/3)*(c + d*x)^(2/3))/(7*d^2)
 + (30*2^(2/3)*b^(1/3)*(b*c - a*d)^2*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c
+ a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(7*d^(11/3)*(a + b*x)^(1
/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3)
+ 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) - (15*2^(2/3)*3^(1/
4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(8/3)*((a + b*x)*(c + d*x))^(1/3)*
Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1
/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/
3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a + b*x)^(1/3)*(c + d*x)^(1
/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d
)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d
 + b*(c + 2*d*x))^2]) + (20*2^(1/6)*3^(3/4)*b^(1/3)*(b*c - a*d)^(8/3)*((a +
 b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2
^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
 - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
- a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
 + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3
)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(7*d^(11/3)*(a +
b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((
1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

<u>Rule 49</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{8/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} + \frac{(8b)\int \frac{(a+bx)^{5/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} - \frac{(40b(bc-ad))\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{7d^2}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{(20}{}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{(2}{}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{(6}{}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{(3}{}$$

$$= -\frac{3(a+bx)^{8/3}}{d\sqrt[3]{c+dx}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \frac{}{7d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.08, size = 73, normalized size = 0.05

$$\frac{3(a+bx)^{11/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{4}{3},\frac{11}{3};\frac{14}{3};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(8/3)/(c + d*x)^(4/3),x]
[Out] (3*(a + b*x)^(11/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3
, 11/3, 14/3, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{8}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
[Out] int((b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(2/3)*(d*x + c)^(2/3)/(d^2*x^2
 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{8}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(8/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(8/3)/(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(8/3)/(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{8/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(8/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(8/3)/(c + d*x)^(4/3), x)
```

3.1623 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1317

$$-\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{15\sqrt[3]{b}\,(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2\cdots}}{2\sqrt[3]{2}\,d^{8/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-\cdots\right.}$$

```
[Out] -3*(b*x+a)^(5/3)/d/(d*x+c)^(1/3)+15/4*b*(b*x+a)^(2/3)*(d*x+c)^(2/3)/d^2-15/
4*b^(1/3)*(-a*d+b*c)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((
a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*
d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2
/3)*(1+3^(1/2)))-5/2*3^(3/4)*b^(1/3)*(-a*d+b*c)^(5/3)*((b*x+a)*(d*x+c))^(1/
3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*Ellip
ticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3
^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-
2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*
b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/d^(8/3)/(b*x+
a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d
+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^
(1/2)))^2)^(1/2)+15/8*3^(1/4)*b^(1/3)*(-a*d+b*c)^(5/3)*((b*x+a)*(d*x+c))^(1
/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*Elli
pticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-
3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2
^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a
)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^
(1/2)*2^(2/3)/d^(8/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b
*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 1.41, antiderivative size = 1317, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$,

Rules used = {49, 52, 64, 637, 309, 224, 1891}

$$\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{4\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{5\,3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)(bc-ad)^{5/3}}{2^{5/6}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{15\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}(bc-ad)}{2\sqrt[3]{2}d^{8/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2}-\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(5/3))/(d*(c + d*x)^(1/3)) + (15*b*(a + b*x)^(2/3)*(c + d*x)^
(2/3))/(4*d^2) - (15*b^(1/3)*(b*c - a*d)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*d^(8/3)*(
a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])*(b*c -
a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))) + (15*3^
(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(5/3)*((a + b*x)*(c + d*x))^(1/
3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d
^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^
(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/
3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*d^(8/3)*(a + b*x)^(1/3)*(c
 + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*
(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]
*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*(b*c - a*d)^(5/3)*((a
+ b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) +
2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3
) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) +
 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((
a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/
3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*d^(8/3)
*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2
/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
```

```
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{(5b)\int \frac{(a+bx)^{2/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{(5b(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}\,dx}{2d^2}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{\left(5b(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{}{\sqrt[3]{a}}}{2d^2\sqrt[3]{a+bx}\,\sqrt[3]{c+}}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{\left(15b(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc-}\right.}{}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{\left(15b^{2/3}(bc-ad)\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(b}\right.}{}$$

$$= -\frac{3(a+bx)^{5/3}}{d\sqrt[3]{c+dx}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \frac{15\sqrt[3]{b}\,(bc-ad)\sqrt[3]{(a+bx)(}}{2\sqrt[3]{2}\,d^{8/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{8/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(\frac{4}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{-bc+ad}\right)}{8b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/3)*(d*x + c)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(4/3),x)
[Out] int((a + b*x)^(5/3)/(c + d*x)^(4/3), x)
```

3.1624 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1279

$$-\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}}+\frac{3\,2^{2/3}\sqrt[3]{b}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{d^{5/3}\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c}\right.}$$

```
[Out] -3*(b*x+a)^(2/3)/d/(d*x+c)^(1/3)+3*2^(2/3)*b^(1/3)*((b*x+a)*(d*x+c))^(1/3)*
((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(5/3)/(b*x+a)^(1/
3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c
))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+2*2^(1/6)*3^(3/4)*b^(1/3)*(-a*d+b*c)
^(2/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((
b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))
^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x
+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^
2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+
a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)/d^(5/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x
+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*(
(b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-
a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*b^(1/3)*(-a*d+b*c)^(2/3)*(
(b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(
-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)
*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d
+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x
+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/d^(5/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d
*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)
+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.06, antiderivative size = 1279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {49, 64, 637, 309, 224, 1891}

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{2}d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt[6]{2}3^{3/4}\sqrt[3]{b}(bc-ad)^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(2/3))/(d*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*((a + b*x)*(c
 + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
/(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[
3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)
)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c +
 d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(
1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)
*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*
b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/
3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
 + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*d^(5/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*(b*c -
 a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c
 - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[(
(b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c
 + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1
 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x
))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^
(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3)
 + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/
(d^(5/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{(2b)\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(2b\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{d\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(6b\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd+}}\right.}{d\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{\left(3\sqrt[3]{2}\,b^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{(1-\sqrt{}}{\sqrt{-4}}\right.}{d^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+}$$

$$= -\frac{3(a+bx)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(}}{d^{5/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{4}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(2/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(4/3),x)
```

```
[Out] int((a + b*x)^(2/3)/(c + d*x)^(4/3), x)
```

3.1625 $\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1298

$$\frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}}-\frac{3\sqrt[3]{b}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx}{\sqrt[3]{2}\,d^{2/3}(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}}$$

```
[Out] 3*(b*x+a)^(2/3)/(-a*d+b*c)/(d*x+c)^(1/3)-3/2*b^(1/3)*((b*x+a)*(d*x+c))^(1/3
)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)*2^(2/3)/d^(2/3)/(
-a*d+b*c)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))-2^(1/6)*3^(3/4)
*b^(1/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*
((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c
))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d
*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c
)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*
x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(
2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^
2)^(1/2)/d^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+
b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a
)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)+3/4*3^(1/4)*b^(1/3)
*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3))*EllipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)
+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/
2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a
*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d
*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(
2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)/d^(2/3)/(-a*d+b*c)^(1/3)/(b*x+a)^(1/3)/(
d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3
)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A]

time = 1.09, antiderivative size = 1298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[3]{2}d^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{\sqrt[6]{2}3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)}{d^{2/3}\sqrt[3]{bc-ad}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt[3]{2}d^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3))/((b*c - a*d)*(c + d*x)^(1/3)) - (3*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]
)/(2^(1/3)*d^(2/3)*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
 2*b*d*x)*((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a +
b*x)*(c + d*x))^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*((a + b*x)*(
c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*
b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2
/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/
3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^
(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)
*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2*2^(1/3)*d^(2/3)*(b*c
- a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b
*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c
+ d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*
((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2^(1/6)*3
^(3/4)*b^(1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((
b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqr
t[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)
*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/
((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2
/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]
])/(d^(2/3)*(b*c - a*d)^(1/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d +
2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/
3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))
^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{4/3}}\,dx = \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{b\int \frac{1}{\sqrt[3]{a+bx}\sqrt[3]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(b\sqrt[3]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[3]{ac+(bc+ad)x+bdx^2}}\,dx}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(3b\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int\right.}{(bc-ad)\sqrt[3]{a+bx}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{\left(3b^{2/3}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{2^{2/3}\sqrt[3]{d}\,(bc-ad)\sqrt[3]{a}}$$

$$= \frac{3(a+bx)^{2/3}}{(bc-ad)\sqrt[3]{c+dx}} - \frac{3\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc}}{\sqrt[3]{2}\,d^{2/3}(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.06

$$\frac{3(a+bx)^{2/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{5}{3};\frac{d(a+bx)}{-bc+ad}\right)}{2b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[2/3,
 4/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx}\,(c + dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(4/3)), x)
```

3.1626 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1327

$$-\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}-\frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}}+\frac{3\,2^{2/3}\sqrt[3]{b}\,\sqrt[3]{d}\,\sqrt[3]{(a+bx)(c+dx)}\,\sqrt{(bc}}{(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}\,(bc+ad+2bdx)\left(\left(1+\right.\right.}$$

```
[Out] -3/(-a*d+b*c)/(b*x+a)^(1/3)/(d*x+c)^(1/3)-6*d*(b*x+a)^(2/3)/(-a*d+b*c)^2/(d
*x+c)^(1/3)+3*2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d
+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^2/(b*x+a)^(1/3)/(d*x+
c)^(1/3)/(2*b*d*x+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)
+(-a*d+b*c)^(2/3)*(1+3^(1/2)))+2*2^(1/6)*3^(3/4)*b^(1/3)*d^(1/3)*((b*x+a)*(
d*x+c))^(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(
1/3))*EllipticF((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+
b*c)^(2/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+
b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)
+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3
)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)/(-a*d+b*c)
^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^
(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*
(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)
^(2/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3^(1/4)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^
(1/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*El
lipticE((2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(
1-3^(1/2)))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2
*2^(1/2))*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x
+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2
/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2
)^(1/2)*2^(2/3)/(-a*d+b*c)^(4/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b
*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)
*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)
*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.38, antiderivative size = 1327, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$-\frac{6(a+bx)^{2/3}d}{(bc-ad)^2\sqrt[3]{c+dx}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{2}(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt[6]{2}3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)\sqrt[3]{d}}{(bc-ad)^{4/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}+\frac{3\,2^{2/3}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}\sqrt[3]{d}}{(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}-\frac{3}{(bc-ad)\sqrt[3]{a+bx}\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/((b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(1/3)) - (6*d*(a + b*x)^(2/3))/((
b*c - a*d)^2*(c + d*x)^(1/3)) + (3*2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((
b*c - a*d)^2*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sq
rt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1
/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^
(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^
(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3
)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)
*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1
- Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b
*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(1/3)*(b*c - a*d)^(4/3)*(a + b*
x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*
c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1
+ Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x)
)^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*2^(1/6)*3^(3/4)*b^(1/3)*d^(
1/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)
^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c -
a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x)
)^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt
[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3
))^2]*EllipticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d
^(1/3)*((a + b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2
/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/((b*c -
 a*d)^(4/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*
c - a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c +
 d*x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*(
(a + b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx = -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{4/3}}\,dx}{bc-ad}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{(2bd)\int \frac{1}{\sqrt[3]{a+bx}\,\sqrt[3]{}}}{(bc-ad)^{}}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(2bd\sqrt[3]{(a+bx)(c+}\right.}{(bc-}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(6bd\sqrt[3]{(a+bx)(c+}\right.}{}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{\left(3\sqrt[3]{2}\,b^{2/3}d^{2/3}\sqrt[3]{(a}\right.}{}$$

$$= -\frac{3}{(bc-ad)\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2\sqrt[3]{c+dx}} + \frac{3\ 2^{2}}{(bc-ad)^2\sqrt[3]{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.05

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{2}{3};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[3]{a+bx}\,(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x]
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/3)*(c + d*x)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}\,(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

```
[Out] int(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{4/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(4/3)*(c + d*x)^(4/3)), x)
```

3.1627 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=1370

$$-\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}+\frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}}+\frac{15d^2(a+bx)^{2/3}}{2(bc-ad)^3\sqrt[3]{c+dx}}-\frac{}{2\sqrt[3]{2}\,(bc-ad)^3\sqrt[3]{}}$$

```
[Out] -3/4/(-a*d+b*c)/(b*x+a)^(4/3)/(d*x+c)^(1/3)+15/4*d/(-a*d+b*c)^2/(b*x+a)^(1/
3)/(d*x+c)^(1/3)+15/2*d^2*(b*x+a)^(2/3)/(-a*d+b*c)^3/(d*x+c)^(1/3)-15/4*b^(
1/3)*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2
*d*x+c))^2)^(1/2)*2^(2/3)/(-a*d+b*c)^3/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x
+a*d+b*c)/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)
*(1+3^(1/2)))-5/2*3^(3/4)*b^(1/3)*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b*
c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticF((2^(2/3
)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2^
(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(((-a*d+b*c)^(4/3)-2^(2/3)*b^(1
/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(1/3)+2*2^(1/3)*b^(2/3)*d^(2
/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/
3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/6)/(-a*d+b*c)^(7/3)/(b*x+a)^
(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)/((-a*d+b*
c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))
/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/
2)))^2)^(1/2)+15/8*3^(1/4)*b^(1/3)*d^(4/3)*((b*x+a)*(d*x+c))^(1/3)*((-a*d+b
*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3))*EllipticE((2^(2/
3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1-3^(1/2)))/(2
^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*((2*b*d*x+a*d+b*c)^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(((-
a*d+b*c)^(4/3)-2^(2/3)*b^(1/3)*d^(1/3)*(-a*d+b*c)^(2/3)*((b*x+a)*(d*x+c))^(
1/3)+2*2^(1/3)*b^(2/3)*d^(2/3)*((b*x+a)*(d*x+c))^(2/3))/(2^(2/3)*b^(1/3)*d^
(1/3)*((b*x+a)*(d*x+c))^(1/3)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3
)/(-a*d+b*c)^(7/3)/(b*x+a)^(1/3)/(d*x+c)^(1/3)/(2*b*d*x+a*d+b*c)/((a*d+b*(2
*d*x+c))^2)^(1/2)/((-a*d+b*c)^(2/3)*((-a*d+b*c)^(2/3)+2^(2/3)*b^(1/3)*d^(1/
3)*((b*x+a)*(d*x+c))^(1/3))/(2^(2/3)*b^(1/3)*d^(1/3)*((b*x+a)*(d*x+c))^(1/3
)+(-a*d+b*c)^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.70, antiderivative size = 1370, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$,

Rules used = {53, 64, 637, 309, 224, 1891}

$$\frac{15(a+bx)^{2/3}d^2}{2(bc-ad)^3\sqrt[3]{c+dx}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}E\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{4\sqrt[3]{2}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{5\,3^{3/4}\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)\sqrt{\frac{(bc-ad)^{4/3}-2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}(bc-ad)^{2/3}+2\sqrt[3]{2}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3}}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}{\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}}\right)|-7-4\sqrt{3}\right)d^{4/3}}{2^{5/6}(bc-ad)^{7/3}\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\sqrt{\frac{(bc-ad)^{2/3}\left((bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)^2}}\sqrt{(ad+b(c+2dx))^2}}-\frac{15\sqrt[3]{b}\sqrt[3]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}d^{4/3}}{2\sqrt[3]{2}(bc-ad)^3\sqrt[3]{a+bx}\sqrt[3]{c+dx}(bc+ad+2bdx)\left(\left(1+\sqrt{3}\right)(bc-ad)^{2/3}+2^{2/3}\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{(a+bx)(c+dx)}\right)}+\frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\sqrt[3]{c+dx}}-\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(4*(b*c - a*d)*(a + b*x)^(4/3)*(c + d*x)^(1/3)) + (15*d)/(4*(b*c - a*d)^
2*(a + b*x)^(1/3)*(c + d*x)^(1/3)) + (15*d^2*(a + b*x)^(2/3))/(2*(b*c - a*d
)^3*(c + d*x)^(1/3)) - (15*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(2*2^(1/3)*(b*c -
a*d)^3*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*((1 + Sqrt[3])
*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))
+ (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*d^(4/3)*((a + b*x)*(c + d*x))^(1/3)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)
*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3) - 2^(2/3)*b^(1/3)*d^(
1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2*2^(1/3)*b^(2/3)*d^(2
/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)
*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/
3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(
c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(4*2^(1/3)*(b*c - a*d)^(7/3)*(a + b*x)^
(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c - a*d)^(2/3)*((b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3)))/((1 + S
qrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(
1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (5*3^(3/4)*b^(1/3)*d^(4/3)*((a +
b*x)*(c + d*x))^(1/3)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*((b*c - a*d)^(2/3) + 2^
(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))*Sqrt[((b*c - a*d)^(4/3)
- 2^(2/3)*b^(1/3)*d^(1/3)*(b*c - a*d)^(2/3)*((a + b*x)*(c + d*x))^(1/3) + 2
*2^(1/3)*b^(2/3)*d^(2/3)*((a + b*x)*(c + d*x))^(2/3))/((1 + Sqrt[3])*(b*c -
 a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)
*d^(1/3)*((a + b*x)*(c + d*x))^(1/3))], -7 - 4*Sqrt[3]])/(2^(5/6)*(b*c - a*
d)^(7/3)*(a + b*x)^(1/3)*(c + d*x)^(1/3)*(b*c + a*d + 2*b*d*x)*Sqrt[((b*c -
 a*d)^(2/3)*((b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a + b*x)*(c + d*
x))^(1/3)))/((1 + Sqrt[3])*(b*c - a*d)^(2/3) + 2^(2/3)*b^(1/3)*d^(1/3)*((a
+ b*x)*(c + d*x))^(1/3))^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/3}(c+dx)^{4/3}}\,dx = -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{4/3}}\,dx}{4(bc-ad)}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{(5d^2)}{}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d}{2(bc-}$$

$$= -\frac{3}{4(bc-ad)(a+bx)^{4/3}\sqrt[3]{c+dx}} + \frac{15d}{4(bc-ad)^2\sqrt[3]{a+bx}\,\sqrt[3]{c+dx}} + \frac{15d}{2(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.05

$$-\frac{3\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(-\frac{4}{3},\frac{4}{3};-\frac{1}{3};\frac{d(a+bx)}{-bc+ad}\right)}{4b(a+bx)^{4/3}(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[-4/3, 4/3, -1/3, (d
*(a + b*x))/(-(b*c) + a*d)])/(4*b*(a + b*x)^(4/3)*(c + d*x)^(4/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}} (dx+c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)
[Out] int(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(4/3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] integral((b*x + a)^(2/3)*(d*x + c)^(2/3)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*
d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{3}} (c+dx)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(4/3),x)
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(4/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(4/3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{7/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(7/3)*(c + d*x)^(4/3)), x)
```

3.1628 $\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=77

$$\sqrt[3]{-1+x}\,(1+x)^{2/3}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{\sqrt{3}}+\frac{1}{3}\log(-1+x)+\log\left(-1+\frac{\sqrt[3]{1+x}}{\sqrt[3]{-1+x}}\right)$$

```
[Out] (-1+x)^(1/3)*(1+x)^(2/3)+1/3*ln(-1+x)+ln(-1+(1+x)^(1/3)/(-1+x)^(1/3))+2/3*a
rctan(1/3*3^(1/2)+2/3*(1+x)^(1/3)/(-1+x)^(1/3)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {52, 61}

$$\frac{2\text{ArcTan}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\,\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqr
t[3]*(-1 + x)^(1/3))])/Sqrt[3] + Log[-1 + x]/3 + Log[-1 + (1 + x)^(1/3)/(-1
 + x)^(1/3)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx = \sqrt[3]{-1+x}\,(1+x)^{2/3} - \frac{2}{3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{1+x}}\,dx$$
$$= \sqrt[3]{-1+x}\,(1+x)^{2/3} + \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{1+x}}{\sqrt{3}\,\sqrt[3]{-1+x}}\right)}{\sqrt{3}} + \frac{1}{3}\log(-1+x) + \log\left(-1+\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(77) = 154.
time = 0.22, size = 158, normalized size = 2.05

$$\frac{\sqrt[3]{\frac{-1+x}{1+x}}\left(3\sqrt[3]{-1+x}+3\sqrt[3]{-1+x}\,x+2\sqrt{3}\,\sqrt[3]{1+x}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{1+x}}{2\sqrt[3]{-1+x}+\sqrt[3]{1+x}}\right)+2\sqrt[3]{1+x}\,\log\left(\sqrt[3]{-1+x}-\sqrt[3]{1+x}\right)-\sqrt[3]{1+x}\,\log\left((-1+x)^{2/3}+\sqrt[3]{-1+x}\,\sqrt[3]{1+x}+(1+x)^{2/3}\right)\right)}{3\sqrt[3]{-1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (((-1 + x)/(1 + x))^(1/3)*(3*(-1 + x)^(1/3) + 3*(-1 + x)^(1/3)*x + 2*Sqrt[3
]*(1 + x)^(1/3)*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2*(-1 + x)^(1/3) + (1 + x)^
(1/3))] + 2*(1 + x)^(1/3)*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)] - (1 + x)^(1/
3)*Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]))/(3*
(-1 + x)^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 577, normalized size = 7.49

method	result
risch	$(-1+x)^{\frac{1}{3}}(1+x)^{\frac{2}{3}} + \frac{\left(2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x^2-2\,\mathrm{RootOf}\left(\_Z^2+\_Z+1\right)^2 x+3\,\mathrm{RootOf}(\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/3)/(1+x)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x)^(1/3)*(1+x)^(2/3)+(2/3*RootOf(_Z^2+_Z+1)*ln(-(2*RootOf(_Z^2+_Z+1)^2*
x^2-2*RootOf(_Z^2+_Z+1)^2*x+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+3*RootO
f(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x+5*RootOf(_Z^2+_Z+1)*x^2-3*RootOf(_Z^2+_Z
+1)*(x^3-x^2-x+1)^(1/3)-4*RootOf(_Z^2+_Z+1)*x+2*x^2-RootOf(_Z^2+_Z+1)-2)/(-
```

```
1+x))-2/3*ln(-(2*RootOf(_Z^2+_Z+1)^2*x^2-2*RootOf(_Z^2+_Z+1)^2*x-3*RootOf(_
Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x-Roo
tOf(_Z^2+_Z+1)*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)-3*(x^3-x^2-x+1)^
(2/3)-3*(x^3-x^2-x+1)^(1/3)*x-x^2+RootOf(_Z^2+_Z+1)+3*(x^3-x^2-x+1)^(1/3)+2
*x-1)/(-1+x))*RootOf(_Z^2+_Z+1)-2/3*ln(-(2*RootOf(_Z^2+_Z+1)^2*x^2-2*RootOf
(_Z^2+_Z+1)^2*x-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)
*(x^3-x^2-x+1)^(1/3)*x-RootOf(_Z^2+_Z+1)*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x
+1)^(1/3)-3*(x^3-x^2-x+1)^(2/3)-3*(x^3-x^2-x+1)^(1/3)*x-x^2+RootOf(_Z^2+_Z+
1)+3*(x^3-x^2-x+1)^(1/3)+2*x-1)/(-1+x)))/(-1+x)^(2/3)*((-1+x)^2*(1+x))^(1/3
)/(1+x)^(1/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="maxima")
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

Fricas [A]
time = 1.11, size = 107, normalized size = 1.39

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1)+2\sqrt{3}(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}}{3(x+1)}\right)+(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-\frac{1}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}+(x+1)^{\frac{1}{3}}(x-1)^{\frac{2}{3}}+x+1}{x+1}\right)+\frac{2}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-x-1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x + 1)^(2/3)*(x - 1)^
(1/3))/(x + 1)) + (x + 1)^(2/3)*(x - 1)^(1/3) - 1/3*log(((x + 1)^(2/3)*(x -
 1)^(1/3) + (x + 1)^(1/3)*(x - 1)^(2/3) + x + 1)/(x + 1)) + 2/3*log(((x + 1
)^(2/3)*(x - 1)^(1/3) - x - 1)/(x + 1))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.59, size = 39, normalized size = 0.51

$$\frac{2^{\frac{2}{3}}(x-1)^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{(x-1)e^{i\pi}}{2}\right)}{2\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/3)/(1+x)**(1/3),x)
```

```
[Out] 2**(2/3)*(x - 1)**(4/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), (x - 1)*exp_po
lar(I*pi)/2)/(2*gamma(7/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="giac")
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)^{1/3}}{(x+1)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/3)/(x + 1)^(1/3),x)
[Out] int((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

3.1629 $\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx$

Optimal. Leaf size=185

$$-\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b}+\frac{16(bc-ad)^{13/4}\sqrt{}}{}$$

```
[Out] 4/77*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b/d+4/11*(b*x+a)^(5/2)*(d*x+c)^
(1/4)/b-8/77*(-a*d+b*c)^2*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b/d^2+16/77*(-a*d+b*c
)^(13/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-
a*d+b*c))^(1/2)/b^(5/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{16(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{77b^{5/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{77bd^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}{77bd}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]
```

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(77*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(1/4))/(11*b) + (16*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(77*b^(5
/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx = \frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b}+\frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{11b}$$

$$= \frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}}{11b}-\frac{(6(bc-ad)^2)\int \frac{\sqrt{a}}{(c+}}{77bd}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx}{}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{77bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77bd}+\frac{4(a+bx}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 73, normalized size = 0.39

$$\frac{2(a+bx)^{5/2}\sqrt[4]{c+dx}\ {}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} \sqrt[4]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/4), x)
```

3.1630 $\int \sqrt{a+bx}\, \sqrt[4]{c+dx}\, dx$

Optimal. Leaf size=147

$$\frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b}-\frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{21b^{5/4}d^2\sqrt{a+bx}}$$

[Out] 4/7*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b+4/21*(-a*d+b*c)*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b/d-8/21*(-a*d+b*c)^(9/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/d^2/(b*x+a)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$-\frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{21b^{5/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{21bd}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/4),x]

[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b*d) + (4*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(7*b) - (8*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(21*b^(5/4)*d^2*Sqrt[a + b*x])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt[4]{c+dx}\,dx = \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{7b}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{(2(bc-ad)^2)\int \frac{}{\sqrt{a+}}}{21bd}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{(8(bc-ad)^2)\,\mathrm{Subst}\Big(}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{\Big(8(bc-ad)^2\sqrt{\frac{d(a+}{-bc-}}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21bd} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7b} - \frac{8(bc-ad)^{9/4}\sqrt{-\frac{d(a}{bc}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.50

$$\frac{2(a+bx)^{3/2}\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt[4]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/4),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,\,{\left(c+d\,x\right)}^{1/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(1/4), x)
```

3.1631 $\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=111

$$\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b}+\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}$$

```
[Out] 4/3*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b+4/3*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/d/(b*
x+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 230, 227}

$$\frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/Sqrt[a + b*x],x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b) + (4*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(5/4)*d*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3b}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{(4(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3bd}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{\left(4(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{}\right.}{3bd\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b} + \frac{4(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}d\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\,\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/4)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(1/4)/(a + b*x)^(1/2), x)
```

3.1632 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(1/4)/b/(b*x+a)^(1/2)+2*(-a*d+b*c)^(1/4)*EllipticF(b^(1/4)*(d*x+
c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/(b*x+a)^
(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {49, 65, 230, 227}

$$\frac{2\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}-\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(b*Sqrt[a + b*x]) + (2*(b*c - a*d)^(1/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(5/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{2b}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{b\sqrt{a+bx}}\\
&= -\frac{2\sqrt[4]{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{5/4}\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 71, normalized size = 0.68

$$
-\frac{2\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}
$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(3/2),x]

[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/4))

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx + c)^{\frac{1}{4}}}{(bx + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(1/4)/(b*x+a)^(3/2),x)

[Out] int((d*x+c)^(1/4)/(b*x+a)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*x^2 + 2*a*b*x + a^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c + dx}}{(a + bx)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(1/4)/(b*x+a)**(3/2),x)

[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(3/2), x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(3/2), x)
```

3.1633 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}-\frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

[Out] -2/3*(d*x+c)^(1/4)/b/(b*x+a)^(3/2)-1/3*d*(d*x+c)^(1/4)/b/(-a*d+b*c)/(b*x+a)^(1/2)-1/3*d*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(5/4)/(-a*d+b*c)^(3/4)/(b*x+a)^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{5/4}\sqrt{a+bx}\,(bc-ad)^{3/4}}-\frac{d\sqrt[4]{c+dx}}{3b\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(1/4))/(3*b*(b*c
 - a*d)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(5/4)*(b*c - a*
d)^(3/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{6b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d^2\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{12b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)c}}}\right.}{3b(bc-ad)\sqrt{a+bx}}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3b(a+bx)^{3/2}} - \frac{d\sqrt[4]{c+dx}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-}{3b^{5/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.50

$$-\frac{2\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c + dx}}{(a + bx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(5/2), x)
```

3.1634 $\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}-\frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}}+\frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}}+\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right.}{6b^{5/4}(bc-ad)^{7/4}\sqrt{a+bx}}$$

```
[Out] -2/5*(d*x+c)^(1/4)/b/(b*x+a)^(5/2)-1/15*d*(d*x+c)^(1/4)/b/(-a*d+b*c)/(b*x+a
)^(3/2)+1/6*d^2*(d*x+c)^(1/4)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)+1/6*d^2*Elliptic
F(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b
^(5/4)/(-a*d+b*c)^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{5/4}\sqrt{a+bx}\,(bc-ad)^{7/4}}+\frac{d^2\sqrt[4]{c+dx}}{6b\sqrt{a+bx}\,(bc-ad)^2}-\frac{d\sqrt[4]{c+dx}}{15b(a+bx)^{3/2}(bc-ad)}-\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(1/4))/(15*b*(b*c
 - a*d)*(a + b*x)^(3/2)) + (d^2*(c + d*x)^(1/4))/(6*b*(b*c - a*d)^2*Sqrt[a
+ b*x]) + (d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)
*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(5/4)*(b*c - a*d)^(7/4)*Sqr
t[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx = -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx}{10b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} - \frac{d^2\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{12b(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^3\int \frac{1}{\sqrt{a+bx}\,(c\cdots}}{24b(bc-a\cdots}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^2\text{Subst}\left(\int \frac{}{\sqrt{a\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\cdots\right.}{\cdots}$$

$$= -\frac{2\sqrt[4]{c+dx}}{5b(a+bx)^{5/2}} - \frac{d\sqrt[4]{c+dx}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2\sqrt[4]{c+dx}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\cdots}{6b^{5/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.39

$$-\frac{2\sqrt[4]{c+dx}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{4};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/4))
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(1/4)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/4)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{c + dx}}{(a + bx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(1/4)/(a + b*x)**(7/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/4)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,4,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/4}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/4)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(1/4)/(a + b*x)^(7/2), x)
```

3.1635 $\int (a+bx)^{3/2}(c+dx)^{3/4}\,dx$

Optimal. Leaf size=270

$$-\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2}+\frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}+\frac{16(bc-ad)^{1}}{}$$

```
[Out] 4/39*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/4)/b/d+4/13*(b*x+a)^(5/2)*(d*x+c)^
(3/4)/b-8/65*(-a*d+b*c)^2*(d*x+c)^(3/4)*(b*x+a)^(1/2)/b/d^2+16/65*(-a*d+b*c
)^(15/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-
a*d+b*c))^(1/2)/b^(7/4)/d^3/(b*x+a)^(1/2)-16/65*(-a*d+b*c)^(15/4)*EllipticF
(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^
(7/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{65b^{7/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)^2}{65bd^2}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}{39bd}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]
```

```
[Out] (-8*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(65*b*d^2) + (4*(b*c - a*d
)*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(39*b*d) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*b) + (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(65*b^(7
/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(65*b^(7/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/4}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} + \frac{(3(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{13b}$$

$$= \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} - \frac{(2(bc-ad)^2)\int}{13bc}$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + 4(a+$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + 4(a+$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + 4(a+$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + 4(a+$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + 4(a+$$

$$= -\frac{8(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + 4(a+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.27

$$\frac{2(a+bx)^{5/2}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/4),x]

[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(3/4))

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(3/2)*(d*x+c)^(3/4),x)

[Out] int((b*x+a)^(3/2)*(d*x+c)^(3/4),x)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="maxima")

[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(3/4), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="fricas")

[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(3/4), x)
```

3.1636 $\int \sqrt{a+bx}\,(c+dx)^{3/4}\,dx$

Optimal. Leaf size=232

$$\frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b}-\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c}}{\sqrt[4]{bc-}}\right.\right.}{15b^{7/4}d^2\sqrt{a+bx}}$$

```
[Out] 4/9*(b*x+a)^(3/2)*(d*x+c)^(3/4)/b+4/15*(-a*d+b*c)*(d*x+c)^(3/4)*(b*x+a)^(1/
2)/b/d-8/15*(-a*d+b*c)^(11/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1
/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d^2/(b*x+a)^(1/2)+8/15*(-a*d+b
*c)^(11/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/
(-a*d+b*c))^(1/2)/b^(7/4)/d^2/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{7/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{15bd}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/4),x]
```

```
[Out] (4*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*b*d) + (4*(a + b*x)^(3/2)
*(c + d*x)^(3/4))/(9*b) - (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(15*b^(7/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x)
)/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4
)], -1])/(15*b^(7/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{3/4}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3b}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{(2(bc-ad)^2)\int}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{(8(bc-ad)^2)\,\mathrm{Sub}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{\left(8(bc-ad)^{5/2}\right)\,\mathrm{S}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{\left(8(bc-ad)^{5/2}\sqrt{}\right.}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} + \frac{8(bc-ad)^{11/4}\sqrt{}}{}$$

$$= \frac{4(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15bd} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9b} - \frac{8(bc-ad)^{11/4}\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.31

$$\frac{2(a+bx)^{3/2}(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,(c+dx)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/4),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(3/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(3/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,\left(c+d\,x\right)^{3/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(3/4), x)
```

3.1637 $\int \frac{(c+dx)^{3/4}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=196

$$\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b}+\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}-\frac{12(bc-ad)^{7/4}\sqrt{-\ldots}}{}$$

```
[Out] 4/5*(d*x+c)^(3/4)*(b*x+a)^(1/2)/b+12/5*(-a*d+b*c)^(7/4)*EllipticE(b^(1/4)*(
d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d/(b
*x+a)^(1/2)-12/5*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c
)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/d/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/Sqrt[a + b*x],x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*b) + (12*(b*c - a*d)^(7/4)*Sqrt[-((d*(
a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a
*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x]) - (12*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(7/4)*d*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} + \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5b}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} + \frac{(12(bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5bd}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} - \frac{\left(12(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5b^{3/2}d}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} - \frac{\left(12(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx\right.}{5b^{3/2}d\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} - \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5b} + \frac{12(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{7/4}d\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.36

$$\frac{2\sqrt{a+bx}\,(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(1/2),x)
[Out] int((d*x+c)^(3/4)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(3/4)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((d*x + c)^(3/4)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(3/4)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/4)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/4}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(3/4)/(a + b*x)^(1/2), x)
```

3.1638 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx}{bc-ad}}}{b^{7/}}$$

```
[Out] -2*(d*x+c)^(3/4)/b/(b*x+a)^(1/2)+6*(-a*d+b*c)^(3/4)*EllipticE(b^(1/4)*(d*x+
c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(b*x+a)^
(1/2)-6*(-a*d+b*c)^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I
)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {49, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}+\frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}}-\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(b*Sqrt[a + b*x]) + (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x]) - (6*(b*c - a*d)^(3/4)*Sqrt[-((d*(a
+ b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d
)^(1/4)], -1])/(b^(7/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{(3d)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{2b}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{6\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} - \frac{\left(6\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b^{3/2}} + \frac{\left(6\sqrt{b\right.}}{}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} - \frac{\left(6\sqrt{bc-ad}\,\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+\right.}{b^{3/2}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} - \frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}} + \frac{\left(6\sqrt{\right.}}{}$$

$$= -\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{6(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}} - \frac{6(bc}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.39

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(3/2),x]
```

[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/4))

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/4)/(b*x+a)^(3/2),x)

[Out] int((d*x+c)^(3/4)/(b*x+a)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="maxima")

[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*x^2 + 2*a*b*x + a^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/4)/(b*x+a)**(3/2),x)

[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{3/4}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(3/4)/(a + b*x)^(3/2), x)
```

3.1639 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}-\frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}}{b^{7/4}}$$

```
[Out] -2/3*(d*x+c)^(3/4)/b/(b*x+a)^(3/2)-d*(d*x+c)^(3/4)/b/(-a*d+b*c)/(b*x+a)^(1/
2)+d*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+
b*c))^(1/2)/b^(7/4)/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)-d*EllipticF(b^(1/4)*(d*x
+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(-a*d+b
*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\,\sqrt[4]{bc-ad}}+\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{7/4}\sqrt{a+bx}\,\sqrt[4]{bc-ad}}-\frac{d(c+dx)^{3/4}}{b\sqrt{a+bx}\,(bc-ad)}-\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(3*b*(a + b*x)^(3/2)) - (d*(c + d*x)^(3/4))/(b*(b*c -
a*d)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSi
n[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(7/4)*(b*c - a*d)^(
1/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx}{2b}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d^2\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{4b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{b^{3/2}\sqrt{bc-ad}}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} - \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\right.}{b^{3/2}\sqrt{bc-ad}\,\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle| -\right.}{b^{7/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle| -\right.}{b^{7/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.33

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(5/2),x]

[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/4))

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/4)/(b*x+a)^(5/2),x)

[Out] int((d*x+c)^(3/4)/(b*x+a)^(5/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="maxima")

[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(5/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{3/4}}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(3/4)/(a + b*x)^(5/2), x)
```

3.1640 $\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}}-\frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}}+\frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}}-\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+}}{\sqrt[4]{bc-a}}\right.\right.}{10b^{7/4}(bc-ad)^{5/4}\sqrt{a+bx}}$$

```
[Out] -2/5*(d*x+c)^(3/4)/b/(b*x+a)^(5/2)-1/5*d*(d*x+c)^(3/4)/b/(-a*d+b*c)/(b*x+a)
^(3/2)+3/10*d^2*(d*x+c)^(3/4)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)-3/10*d^2*Ellipti
cE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/
b^(7/4)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)+3/10*d^2*EllipticF(b^(1/4)*(d*x+c)^(
1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(7/4)/(-a*d+b*c)^(
5/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{10b^{7/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{3d^2(c+dx)^{3/4}}{10b\sqrt{a+bx}\,(bc-ad)^2}-\frac{d(c+dx)^{3/4}}{5b(a+bx)^{3/2}(bc-ad)}-\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(5*b*(a + b*x)^(5/2)) - (d*(c + d*x)^(3/4))/(5*b*(b*c
- a*d)*(a + b*x)^(3/2)) + (3*d^2*(c + d*x)^(3/4))/(10*b*(b*c - a*d)^2*Sqrt[
a + b*x]) - (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(
1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*d)^(5/4
)*Sqrt[a + b*x]) + (3*d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(10*b^(7/4)*(b*c - a*
d)^(5/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} + \frac{(3d)\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx}{10b}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} - \frac{(3d^2)\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx}{20b(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{(3d^3)\int \frac{1}{\sqrt{a+bx} \cdots}}{40b(bc- \cdots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{(3d^2)\,\mathrm{Subst}\left(\int - \cdots\right)}{\cdots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{(3d^2)\,\mathrm{Subst}\left(\int - \cdots\right)}{1\cdots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{\left(3d^2\sqrt{\frac{d(a+bx)}{-bc+ad \cdots}} \cdots\right.}{\cdots}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} + \frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}} \cdots}{10b^{7/\cdots}}$$

$$= -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}} \cdots}{10b^{7/\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.27

$$-\frac{2(c+dx)^{3/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{4};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/4))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(3/4)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{4}}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/4)/(b*x+a)**(7/2),x)
[Out] Integral((c + d*x)**(3/4)/(a + b*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/4)/(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(3/4)/(b*x + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{3/4}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/4)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(3/4)/(a + b*x)^(7/2), x)
```

3.1641 $\int (a+bx)^{3/2}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=220

$$-\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2}+\frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2}+\frac{4(a+$$

```
[Out] 4/231*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b^2/d+4/33*(-a*d+b*c)*(b*x+a
)^(5/2)*(d*x+c)^(1/4)/b^2+4/15*(b*x+a)^(5/2)*(d*x+c)^(5/4)/b-8/231*(-a*d+b*
c)^3*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2/d^2+16/231*(-a*d+b*c)^(17/4)*EllipticF
(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^
(9/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{16(bc-ad)^{17/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{231b^{9/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^3}{231b^2d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)^2}{231b^2d}+\frac{4(a+bx)^{5/2}\sqrt[4]{c+dx}\,(bc-ad)}{33b^2}+\frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]
```

```
[Out] (-8*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d^2) + (4*(b*c -
a*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(231*b^2*d) + (4*(b*c - a*d)*(a + b
*x)^(5/2)*(c + d*x)^(1/4))/(33*b^2) + (4*(a + b*x)^(5/2)*(c + d*x)^(5/4))/(
15*b) + (16*(b*c - a*d)^(17/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF
[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b^(9/4)*d^3
*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/4}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b}+\frac{(bc-ad)\int (a+bx)^{3/2}\sqrt[4]{c+dx}\,dx}{3b}$$

$$= \frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2}+\frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b}+\frac{(bc-ad)^2\int \frac{(a+bx}{(c+dx}}{33b^2}$$

$$= \frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-ad)(a+bx)^{5/2}\sqrt[4]{c+dx}}{33b^2}+\frac{4(a+bx}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2}+\frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2}+\frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2}+\frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-}{}$$

$$= -\frac{8(bc-ad)^3\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d^2}+\frac{4(bc-ad)^2(a+bx)^{3/2}\sqrt[4]{c+dx}}{231b^2d}+\frac{4(bc-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/2)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*sqrt(b*x + a)*(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/4), x)
```

3.1642 $\int \sqrt{a+bx}\,(c+dx)^{5/4}\,dx$

Optimal. Leaf size=182

$$\frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d}+\frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}-\frac{40(bc-ad)^{13}}{}$$

```
[Out] 20/77*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/4)/b^2+4/11*(b*x+a)^(3/2)*(d*x+c)
^(5/4)/b+20/231*(-a*d+b*c)^2*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2/d-40/231*(-a*d
+b*c)^(13/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a
)/(-a*d+b*c))^(1/2)/b^(9/4)/d^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$-\frac{40(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{231b^{9/4}d^2\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{231b^2d}+\frac{20(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}{77b^2}+\frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/4),x]
```

```
[Out] (20*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/4))/(231*b^2*d) + (20*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4))/(77*b^2) + (4*(a + b*x)^(3/2)*(c + d*x
)^(5/4))/(11*b) - (40*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(231*b
^(9/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/4}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b} + \frac{(5(bc-ad))\int \sqrt{a+bx}\,\sqrt[4]{c+dx}\,dx}{11b}$$

$$= \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+bx)^{3/2}(c+dx)^{5/4}}{11b} + \frac{(5(bc-ad)^2)\int \cdots}{77b^2}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+bx\cdots}{\cdots}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+bx\cdots}{\cdots}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+bx\cdots}{\cdots}$$

$$= \frac{20(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{231b^2d} + \frac{20(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{77b^2} + \frac{4(a+bx\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.40

$$\frac{2(a+bx)^{3/2}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Sympy [A]
time = 8.33, size = 218, normalized size = 1.20

$$-\frac{2ad(a+bx)^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\,\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{3b^2}+\frac{2c(a+bx)^{\frac{3}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\,\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{3b}+\frac{2d(a+bx)^{\frac{5}{2}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ade^{i\pi}}{b\,\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}+\frac{dxe^{i\pi}}{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}\right)\sqrt[4]{\text{polar\_lift}\left(-\frac{ad}{b}+c\right)}}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/4),x)
```

```
[Out] -2*a*d*(a + b*x)**(3/2)*hyper((-1/4, 3/2), (5/2,), a*d*exp_polar(I*pi)/(b*p
olar_lift(-a*d/b + c)) + d*x*exp_polar(I*pi)/polar_lift(-a*d/b + c))*polar_
```

```
lift(-a*d/b + c)**(1/4)/(3*b**2) + 2*c*(a + b*x)**(3/2)*hyper((-1/4, 3/2),
(5/2,), a*d*exp_polar(I*pi)/(b*polar_lift(-a*d/b + c)) + d*x*exp_polar(I*pi
)/polar_lift(-a*d/b + c))*polar_lift(-a*d/b + c)**(1/4)/(3*b) + 2*d*(a + b*
x)**(5/2)*hyper((-1/4, 5/2), (7/2,), a*d*exp_polar(I*pi)/(b*polar_lift(-a*d
/b + c)) + d*x*exp_polar(I*pi)/polar_lift(-a*d/b + c))*polar_lift(-a*d/b +
c)**(1/4)/(5*b**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/4), x)
```

3.1643 $\int \frac{(c+dx)^{5/4}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=144

$$\frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2}+\frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b}+\frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{21b^{9/4}d\sqrt{a+bx}}$$

```
[Out] 20/21*(-a*d+b*c)*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2+4/7*(d*x+c)^(5/4)*(b*x+a)^
(1/2)/b+20/21*(-a*d+b*c)^(9/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(
1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/d/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{21b^{9/4}d\sqrt{a+bx}}+\frac{20\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{21b^2}+\frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/Sqrt[a + b*x],x]
```

```
[Out] (20*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(21*b^2) + (4*Sqrt[a + b*x]*
(c + d*x)^(5/4))/(7*b) + (20*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(21*b^(9/4)*d*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt{a+bx}}\,dx = \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx}{7b}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,(c+d}}{21b^2}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{(20(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{21}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{\left(20(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)}{}$$

$$= \frac{20(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{21b^2} + \frac{4\sqrt{a+bx}\,(c+dx)^{5/4}}{7b} + \frac{20(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{21b^{9/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 71, normalized size = 0.49

$$\frac{2\sqrt{a+bx}\,(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{\sqrt{bx+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt{a+bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(1/2), x)
```

3.1644 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=132

$$\frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}}+\frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(5/4)/b/(b*x+a)^(1/2)+10/3*d*(d*x+c)^(1/4)*(b*x+a)^(1/2)/b^2+10/
3*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 230, 227}

$$\frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}+\frac{10d\sqrt{a+bx}\sqrt[4]{c+dx}}{3b^2}-\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]
```

```
[Out] (10*d*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*b^2) - (2*(c + d*x)^(5/4))/(b*Sqrt[
a + b*x]) + (10*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellipt
icF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/4)*Sq
rt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt{a+bx}}\,dx}{2b}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{6b^2}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{(10(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x\right.}{3b^2}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{\left(10(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{3b^2\sqrt{a+bx}}$$

$$= \frac{10d\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{2(c+dx)^{5/4}}{b\sqrt{a+bx}} + \frac{10(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{b}}\right.\right.}{3b^{9/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.54

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(3/2), x)
```

3.1645 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}+\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(5/4)/b/(b*x+a)^(3/2)-5/3*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(1/2)+5/
3*d*(-a*d+b*c)^(1/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-
d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 230, 227}

$$\frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3b^{9/4}\sqrt{a+bx}}-\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-5*d*(c + d*x)^(1/4))/(3*b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(3*b*(a
+ b*x)^(3/2)) + (5*d*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(9/
4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/2}}\,dx}{6b}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{12b^2}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{(5d)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3b^2}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{\left(5d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,\right.}{3b^2\sqrt{a+bx}}$$

$$= -\frac{5d\sqrt[4]{c+dx}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{5d\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{3b^{9/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.54

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(5/2), x)
```

3.1646 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{9/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] -1/3*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(3/2)-2/5*(d*x+c)^(5/4)/b/(b*x+a)^(5/2)-1/
6*d^2*(d*x+c)^(1/4)/b^2/(-a*d+b*c)/(b*x+a)^(1/2)-1/6*d^2*EllipticF(b^(1/4)*
(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(-a
*d+b*c)^(3/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$-\frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{6b^{9/4}\sqrt{a+bx}\,(bc-ad)^{3/4}}-\frac{d^2\sqrt[4]{c+dx}}{6b^2\sqrt{a+bx}\,(bc-ad)}-\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]
```

```
[Out] -1/3*(d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(3/2)) - (d^2*(c + d*x)^(1/4))/(6*b
^2*(b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(5*b*(a + b*x)^(5/2)) -
 (d^2*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x
)^(1/4))/(b*c - a*d)^(1/4)], -1])/(6*b^(9/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x
])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/2}}\,dx}{2b}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} + \frac{d^2\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{12b^2}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^3\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{24b^2(bc-ad)}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx}{d}}}\right.}{6b^2(bc-}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{\left(d^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}}{6b^2(}$$

$$= -\frac{d\sqrt[4]{c+dx}}{3b^2(a+bx)^{3/2}} - \frac{d^2\sqrt[4]{c+dx}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\right.}{6b^{9/4}(bc-ad)^{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.42

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{4};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(7/2),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(7/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(7/2), x)
```

3.1647 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}} dx$

Optimal. Leaf size=213

$$-\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}}-\frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}}+\frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}+\frac{5d^3\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F}{84b^{9/4}(b}$$

```
[Out] -1/7*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(5/2)-1/42*d^2*(d*x+c)^(1/4)/b^2/(-a*d+b*c
)/(b*x+a)^(3/2)-2/7*(d*x+c)^(5/4)/b/(b*x+a)^(7/2)+5/84*d^3*(d*x+c)^(1/4)/b^
2/(-a*d+b*c)^2/(b*x+a)^(1/2)+5/84*d^3*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d
+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(9/4)/(-a*d+b*c)^(7/4)/(b*x+
a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 65, 230, 227}

$$\frac{5d^3\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{84b^{9/4}\sqrt{a+bx}\,(bc-ad)^{7/4}}+\frac{5d^3\sqrt[4]{c+dx}}{84b^2\sqrt{a+bx}\,(bc-ad)^2}-\frac{d^2\sqrt[4]{c+dx}}{42b^2(a+bx)^{3/2}(bc-ad)}-\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}}-\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]
```

```
[Out] -1/7*(d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(5/2)) - (d^2*(c + d*x)^(1/4))/(42*
b^2*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d^3*(c + d*x)^(1/4))/(84*b^2*(b*c - a
*d)^2*Sqrt[a + b*x]) - (2*(c + d*x)^(5/4))/(7*b*(a + b*x)^(7/2)) + (5*d^3*S
qrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4)
)/(b*c - a*d)^(1/4)], -1])/(84*b^(9/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/2}}\,dx}{14b}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} + \frac{d^2\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx}{28b^2}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} - \frac{2(c+dx)^{5/4}}{7b(a+bx)^{7/2}} - \frac{(5d^3)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}}{168b^2(bc-ad)}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/}}{7b(a+bx)^{7/}}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/}}{7b(a+bx)^{7/}}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/}}{7b(a+bx)^{7/}}$$

$$= -\frac{d\sqrt[4]{c+dx}}{7b^2(a+bx)^{5/2}} - \frac{d^2\sqrt[4]{c+dx}}{42b^2(bc-ad)(a+bx)^{3/2}} + \frac{5d^3\sqrt[4]{c+dx}}{84b^2(bc-ad)^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/}}{7b(a+bx)^{7/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 73, normalized size = 0.34

$$-\frac{2(c+dx)^{5/4}\,{}_2F_1\left(-\frac{7}{2},-\frac{5}{4};-\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/4)*Hypergeometric2F1[-7/2, -5/4, -5/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(7*b*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/4)/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*
x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/2),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(9/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(9/2),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(9/2), x)
```

3.1648 $\int \frac{(a+bx)^{5/2}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=264

$$\frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3}-\frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d}-\frac{32(bc-ad)}{}$$

```
[Out] -40/117*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d^2+4/13*(b*x+a)^(5/2)*(d*x+
c)^(3/4)/d+16/39*(-a*d+b*c)^2*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^3-32/39*(-a*d+b
*c)^(15/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/
(-a*d+b*c))^(1/2)/b^(3/4)/d^4/(b*x+a)^(1/2)+32/39*(-a*d+b*c)^(15/4)*Ellipti
cF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/
b^(3/4)/d^4/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}-\frac{32(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{39b^{3/4}d^4\sqrt{a+bx}}+\frac{16\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)^2}{39d^3}-\frac{40(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}{117d^2}+\frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]
```

```
[Out] (16*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(39*d^3) - (40*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(117*d^2) + (4*(a + b*x)^(5/2)*(c + d*x)^
(3/4))/(13*d) - (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(39*b^(3
/4)*d^4*Sqrt[a + b*x]) + (32*(b*c - a*d)^(15/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(39*b^(3/4)*d^4*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt[4]{c+dx}}\,dx = \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} - \frac{(10(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{13d}$$

$$= -\frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13d} + \frac{(20(bc-ad)^2)\int \frac{\sqrt{a+}}{\sqrt[4]{c+}}}{39d^2}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c}{13d}$$

$$= \frac{16(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}{39d^3} - \frac{40(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{117d^2} + \frac{4(a+bx)^{5/2}(c}{13d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 73, normalized size = 0.28

$$\frac{2(a+bx)^{7/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/4),x]
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/4), x)
```

3.1649 $\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=229

$$-\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d}+\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{\ }}{\sqrt[4]{b}}\right.\right.}{15b^{3/4}d^3\sqrt{a+bx}}$$

```
[Out] 4/9*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d-8/15*(-a*d+b*c)*(d*x+c)^(3/4)*(b*x+a)^(1/
2)/d^2+16/15*(-a*d+b*c)^(11/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(
1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d^3/(b*x+a)^(1/2)-16/15*(-a*d
+b*c)^(11/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a
)/(-a*d+b*c))^(1/2)/b^(3/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}+\frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15b^{3/4}d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{15d^2}+\frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]
```

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*d^2) + (4*(a + b*x)^(3/2
)*(c + d*x)^(3/4))/(9*d) + (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c
 - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1
])/(15*b^(3/4)*d^3*Sqrt[a + b*x]) - (16*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b
*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(
1/4)], -1])/(15*b^(3/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx = \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{(2(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3d}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{(4(bc-ad)^2)\int \frac{\ldots}{\sqrt{a+bx}\ldots}}{15d^2}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{(16(bc-ad)^2)\,\mathrm{Subst}\left(\int \ldots\right)}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{(16(bc-ad)^{5/2})\,\mathrm{Subst}\left(\ldots\right)}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{\left(16(bc-ad)^{5/2}\sqrt{\frac{d(a+\ldots}{-bc\ldots}}\ldots\right)}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} - \frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a\ldots}{bc\ldots}}\ldots}{1\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{3/4}}{9d} + \frac{16(bc-ad)^{11/4}\sqrt{-\frac{d(a\ldots}{bc\ldots}}\ldots}{1\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.32

$$\frac{2(a+bx)^{5/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/4),x]
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/4),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/4), x)
```

3.1650 $\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=196

$$\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d}-\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}+\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a}{bc}}}{}$$

```
[Out] 4/5*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d-8/5*(-a*d+b*c)^(7/4)*EllipticE(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d^2/(
b*x+a)^(1/2)+8/5*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c
)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d^2/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}-\frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}}+\frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/4),x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d) - (8*(b*c - a*d)^(7/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x]) + (8*(b*c - a*d)^(7/4)*Sqrt[-
((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*
c - a*d)^(1/4)], -1])/(5*b^(3/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{5d}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} - \frac{(8(bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^2}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} + \frac{\left(8(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5\sqrt{b}\,d^2} - $$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} + \frac{\left(8(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x,\right.}{5\sqrt{b}\,d^2\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} + \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}} - $$

$$= \frac{4\sqrt{a+bx}\,(c+dx)^{3/4}}{5d} - \frac{8(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5b^{3/4}d^2\sqrt{a+bx}} + $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.37

$$\frac{2(a+bx)^{3/2}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/4), x)
```

3.1651 $\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}-\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\ldots}{\ldots}\right.\right.}{b^{3/4}d\sqrt{a+bx}}$$

```
[Out] 4*(-a*d+b*c)^(3/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/d/(b*x+a)^(1/2)-4*(-a*d+b*c)^(3/4)*Ellipt
icF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)
/b^(3/4)/d/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {65, 313, 230, 227, 1214, 1213, 435}

$$\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}-\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x])
- (4*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[
(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*d*Sqrt[a + b*x]
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx = \frac{4\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d}$$

$$= -\frac{\left(4\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\,d} + \frac{\left(4\sqrt{bc-ad}\right.}{}$$

$$= -\frac{\left(4\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\,d\sqrt{a+bx}}$$

$$= -\frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}} + \frac{\left(4\sqrt{bc-ad}\right.}{}$$

$$= \frac{4(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a+bx}} - \frac{4(bc-ad)^{3/4}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.43

$$\frac{2\sqrt{a+bx}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/4),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,\left(c+d\,x\right)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/4)), x)
```

3.1652 $\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=191

$$-\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}}+\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(1/2)+2*EllipticE(b^(1/4)*(d*x+c)^(1/4)
/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/(-a*d+b*c)^(1/4)
/(b*x+a)^(1/2)-2*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b
*x+a)/(-a*d+b*c))^(1/2)/b^(3/4)/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,\sqrt[4]{bc-ad}}+\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,\sqrt[4]{bc-ad}}-\frac{2(c+dx)^{3/4}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/((b*c - a*d)*Sqrt[a + b*x]) + (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(3/4)*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}\,dx = -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}\,dx}{2(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{\sqrt{b}\sqrt{bc-ad}} + \text{—} \quad 2\text{Sub}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{\right.}{\sqrt{b}\sqrt{bc-ad}\sqrt{a+bx}}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}} + \text{—}$$

$$= -\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt[4]{bc-ad}\sqrt{a+bx}} - \text{—} \quad 2$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.37

$$-\frac{2\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x]
```

[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/4))

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)

[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/4)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}} \sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/4),x)

[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/4)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/4)), x)
```

3.1653 $\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=224

$$-\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}}+\frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}}+\frac{d\sqrt{-\frac{d}{}}}{}$$

```
[Out] -2/3*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(3/2)+d*(d*x+c)^(3/4)/(-a*d+b*c)^2/(b
*x+a)^(1/2)-d*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+
a)/(-a*d+b*c))^(1/2)/b^(3/4)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)+d*EllipticF(b^(
1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(3/4
)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{d(c+dx)^{3/4}}{\sqrt{a+bx}\,(bc-ad)^2}-\frac{2(c+dx)^{3/4}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (d*(c + d*x)^(3/4))/
((b*c - a*d)^2*Sqrt[a + b*x]) - (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x]) + (d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(3/4)*(b*
c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[4]{c+dx}}\,dx = -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} - \frac{d\int \frac{1}{(a+bx)^{3/2}\sqrt[4]{c+dx}}dx}{2(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt[4]{c+dx}}dx}{4(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\right.}{\sqrt{b}\,(bc-ad}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{\left(d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\right.}{\sqrt{b}\,(bc}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} + \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\right.\right.}{b^{3/4}(bc-ad)^{5/4}}$$

$$= -\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} - \frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\right.\right.}{b^{3/4}(bc-ad)^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.33

$$-\frac{2\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{1}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}} \sqrt[4]{c+dx}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/4)), x)
```

3.1654 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}} dx$

Optimal. Leaf size=144

$$-\frac{8(bc-ad)\sqrt{a+bx}\sqrt[4]{c+dx}}{7d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d}+\frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{7\sqrt[4]{b}\,d^3\sqrt{a+bx}}$$

```
[Out] 4/7*(b*x+a)^(3/2)*(d*x+c)^(1/4)/d-8/7*(-a*d+b*c)*(d*x+c)^(1/4)*(b*x+a)^(1/2
)/d^2+16/7*(-a*d+b*c)^(9/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4
),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/d^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 65, 230, 227}

$$\frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\right|-1\right)}{7\sqrt[4]{b}\,d^3\sqrt{a+bx}}-\frac{8\sqrt{a+bx}\sqrt[4]{c+dx}(bc-ad)}{7d^2}+\frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]
```

```
[Out] (-8*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4))/(7*d^2) + (4*(a + b*x)^(3/2)
*(c + d*x)^(1/4))/(7*d) + (16*(b*c - a*d)^(9/4)*Sqrt[-((d*(a + b*x))/(b*c -
 a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])
/(7*b^(1/4)*d^3*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx = \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} - \frac{(6(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{7d}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{(4(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,(c\ldots}}{7d^2}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{(16(bc-ad)^2)\,\mathrm{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{\left(16(bc-ad)^2\sqrt{\frac{d(a+bx)}{-bc+ad}}\ldots\right.}{\ldots}$$

$$= -\frac{8(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}{7d^2} + \frac{4(a+bx)^{3/2}\sqrt[4]{c+dx}}{7d} + \frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\ldots}{7\sqrt[4]{b}\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.51

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(3/4), x)
```

3.1655 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=111

$$\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d}-\frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,d^2\sqrt{a+bx}}$$

```
[Out] 4/3*(d*x+c)^(1/4)*(b*x+a)^(1/2)/d-8/3*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/d^2/(
b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 65, 230, 227}

$$\frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d}-\frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,d^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/4),x]
```

```
[Out] (4*Sqrt[a + b*x]*(c + d*x)^(1/4))/(3*d) - (8*(b*c - a*d)^(5/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*b^(1/4)*d^2*Sqrt[a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx = \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{(2(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3d}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{(8(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{3d^2}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{\left(8(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x,\right.}{3d^2\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d} - \frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,d^2\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/4),x]
```

[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(3/4))

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(3/4), x)
```

3.1656 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=83

$$\frac{4\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,d\sqrt{a+bx}}$$

```
[Out] 4*(-a*d+b*c)^(1/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/d/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {65, 230, 227}

$$\frac{4\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,d\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(b*c - a*d)^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b
^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(b^(1/4)*d*Sqrt[a + b*x])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx = \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d}$$

$$= \frac{\left(4\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{a+bx}}$$

$$= \frac{4\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,d\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.86

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[1/2, 3
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(3/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/4),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/4)), x)
```

3.1657 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=111

$$-\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}}-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] -2*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(1/2)-2*EllipticF(b^(1/4)*(d*x+c)^(1/4)
/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/(-a*d+b*c)^(3/4)
/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$-\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,\sqrt{a+bx}\,(bc-ad)^{3/4}}-\frac{2\sqrt[4]{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/((b*c - a*d)*Sqrt[a + b*x]) - (2*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx = -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{\left(2\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x,\right.}{(bc-ad)\sqrt{a+bx}}$$

$$= -\frac{2\sqrt[4]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.64

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x]
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(3/4))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+a\right)^{\frac{3}{2}} \left(dx+c\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{3}{2}} \left(c+dx\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/4)), x)
```

3.1658 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}+\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{a+bx}}$$

```
[Out] -2/3*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(3/2)+5/3*d*(d*x+c)^(1/4)/(-a*d+b*c)^
2/(b*x+a)^(1/2)+5/3*d*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(
-d*(b*x+a)/(-a*d+b*c))^(1/2)/b^(1/4)/(-a*d+b*c)^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$\frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt[4]{b}\,\sqrt{a+bx}\,(bc-ad)^{7/4}}+\frac{5d\sqrt[4]{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt[4]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (5*d*(c + d*x)^(1/4)
)/(3*(b*c - a*d)^2*Sqrt[a + b*x]) + (5*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*b^(
1/4)*(b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/4}}\,dx = -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{(5d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\, d}{12(bc-ad)^2}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{(5d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+}}\right.}{3(bc-}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{\left(5d\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}}{3(b}$$

$$= -\frac{2\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{5d\sqrt[4]{c+dx}}{3(bc-ad)^2\sqrt{a+bx}} + \frac{5d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\right.}{3\sqrt[4]{b}\,(bc-ad)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.49

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}} (c+dx)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/4)), x)
```

3.1659 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=254

$$-\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}}-\frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3}+\frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2}+\frac{32\sqrt[4]{b}\,(bc-ad)^{11/4}\sqrt{-\frac{d(a}{bc}}}{3}$$

```
[Out] -4*(b*x+a)^(5/2)/d/(d*x+c)^(1/4)+40/9*b*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d^2-16/
3*b*(-a*d+b*c)*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^3+32/3*b^(1/4)*(-a*d+b*c)^(11/
4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*
c))^(1/2)/d^4/(b*x+a)^(1/2)-32/3*b^(1/4)*(-a*d+b*c)^(11/4)*EllipticF(b^(1/4
)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^4/(b*x+
a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{32\sqrt[4]{b}\,(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^4\sqrt{a+bx}}+\frac{32\sqrt[4]{b}\,(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^4\sqrt{a+bx}}-\frac{16b\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{3d^3}+\frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2}-\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/2))/(d*(c + d*x)^(1/4)) - (16*b*(b*c - a*d)*Sqrt[a + b*x]*
(c + d*x)^(3/4))/(3*d^3) + (40*b*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^2) +
 (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Elliptic
E[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*Sqrt[a +
 b*x]) - (32*b^(1/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*
EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*d^4*
Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} + \frac{(10b)\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{(20b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{(8b(}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{(32b}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{\left(32\right.}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{\left(32\right.}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} - \frac{32\sqrt[4]{}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{d\sqrt[4]{c+dx}} - \frac{16b(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \frac{32\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 10.05, size = 73, normalized size = 0.29

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(3/4)/(d^2*x^2 +
 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/4), x)
```

3.1660 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=220

$$-\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}}+\frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2}-\frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}$$

[Out] -4*(b*x+a)^(3/2)/d/(d*x+c)^(1/4)+24/5*b*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^2-48/5*b^(1/4)*(-a*d+b*c)^(7/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)+48/5*b^(1/4)*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}+\frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2}-\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(d*(c + d*x)^(1/4)) + (24*b*Sqrt[a + b*x]*(c + d*x)^(3
/4))/(5*d^2) - (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*
d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5
*d^3*Sqrt[a + b*x]) + (48*b^(1/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b
*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)],
-1])/(5*d^3*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{(6b)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} - \frac{(12b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} - \frac{(48b(bc-ad))\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx \ldots\right.}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} + \frac{\left(48\sqrt{b}\,(bc-ad)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\ldots}}\ldots\right.}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} + \frac{\left(48\sqrt{b}\,(bc-ad)^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\ldots\right.}{5d^3\sqrt{a+\ldots}}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} + \frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\ldots\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{d\sqrt[4]{c+dx}} + \frac{24b\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^2} - \frac{48\sqrt[4]{b}\,(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\ldots\right.}{5d^3\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/4), x)
```

3.1661 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}} dx$

Optimal. Leaf size=190

$$-\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}}+\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}-\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,\cdots}{\cdots}$$

```
[Out] -4*(b*x+a)^(1/2)/d/(d*x+c)^(1/4)+8*b^(1/4)*(-a*d+b*c)^(3/4)*EllipticE(b^(1/
4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^2/(b*x
+a)^(1/2)-8*b^(1/4)*(-a*d+b*c)^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+
b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^2/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {49, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}+\frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(d*(c + d*x)^(1/4)) + (8*b^(1/4)*(b*c - a*d)^(3/4)*Sqrt[
-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b
*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x]) - (8*b^(1/4)*(b*c - a*d)^(3/4)*S
qrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4)
)/(b*c - a*d)^(1/4)], -1])/(d^2*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx = -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{(8b)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{\left(8\sqrt{b}\,\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d^2} + \frac{(8\sqrt{}\ldots}{\ldots}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{\left(8\sqrt{b}\,\sqrt{bc-ad}\,\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x, \sqrt[4]{c\ldots}\right.}{d^2\sqrt{a+bx}}$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} - \frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}} + \ldots$$

$$= -\frac{4\sqrt{a+bx}}{d\sqrt[4]{c+dx}} + \frac{8\sqrt[4]{b}\,(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d^2\sqrt{a+bx}} - \frac{8\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.38

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/4),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(5/4), x)
```

3.1662 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=197

$$\frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}-\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\sqrt{a+bx}}+\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\right.\right.}{d\sqrt[4]{bc-ad}\sqrt{a}}$$

```
[Out] 4*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(1/4)-4*b^(1/4)*EllipticE(b^(1/4)*(d*x+c
)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^(1/4
)/(b*x+a)^(1/2)+4*b^(1/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),
I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{4\sqrt{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/4)) - (4*b^(1/4)*Sqrt[-((d*(a +
 b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)
^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x]) + (4*b^(1/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(d*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx = \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{b\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{(4b)\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(4\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{d\sqrt{bc-ad}} -$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(4\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx, x\right.}{d\sqrt{bc-ad}\,\sqrt{a+bx}}$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} + \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\,\sqrt{a+bx}} -$$

$$= \frac{4\sqrt{a+bx}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{4\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{d\sqrt[4]{bc-ad}\,\sqrt{a+bx}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.36

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[1/2, 5
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/4),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/4)), x)
```

3.1663 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=222

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}}-\frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}}+\frac{6\sqrt[4]{b}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{(bc-ad)^{5/4}\sqrt{a+bx}}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(1/4)/(b*x+a)^(1/2)-6*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/(d
*x+c)^(1/4)+6*b^(1/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(
-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)-6*b^(1/4)*Ellip
ticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2
)/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{6\sqrt[4]{b}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{6\sqrt[4]{b}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{6d\sqrt{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/4)) - (6*d*Sqrt[a + b*x])/((b*c
- a*d)^2*(c + d*x)^(1/4)) + (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x]) - (6*b^(1/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*E
llipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c -
a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{(3d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(3bd)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{}}}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(6b)\text{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{\left(6\sqrt{b}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{\left(6\sqrt{b}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} - \frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{(bc}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[4]{c+dx}} - \frac{6d\sqrt{a+bx}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{6\sqrt[4]{b}\sqrt{-\frac{d(a+bx)}{bc-ad}}}{(bc}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.32

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/4)), x)
```

3.1664 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=261

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}+\frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}+\frac{7d^2\sqrt{a+bx}}{(bc-ad)^3\sqrt[4]{c+dx}}-\frac{7\sqrt[4]{b}\,d\sqrt{-\frac{d(a+bx)}{bc-ad}}\cdots}{(b\cdots}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/4)+7/3*d/(-a*d+b*c)^2/(d*x+c)^(1/4
)/(b*x+a)^(1/2)+7*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/4)-7*b^(1/4)*d*
EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))
^(1/2)/(-a*d+b*c)^(9/4)/(b*x+a)^(1/2)+7*b^(1/4)*d*EllipticF(b^(1/4)*(d*x+c)
^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(9/4)/(
b*x+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{7\sqrt[4]{b}\,d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{9/4}}-\frac{7\sqrt[4]{b}\,d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{9/4}}+\frac{7d^2\sqrt{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{7d}{3\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/4)) + (7*d)/(3*(b*c - a*d)^2
*Sqrt[a + b*x]*(c + d*x)^(1/4)) + (7*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c +
 d*x)^(1/4)) - (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Ar
cSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4)*
Sqrt[a + b*x]) + (7*b^(1/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(9/4
)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{(7d^2)\int}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}} + \frac{7d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}} + \frac{7d^2}{(bc-a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 73, normalized size = 0.28

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}\,(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/4)), x)
```

3.1665 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=207

$$-\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}}+\frac{160b(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{33d^4}-\frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3}+\frac{56b(a+bx)^{5/2}\sqrt[4]{c}}{33d^2}$$

```
[Out] -4/3*(b*x+a)^(7/2)/d/(d*x+c)^(3/4)-80/33*b*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)
^(1/4)/d^3+56/33*b*(b*x+a)^(5/2)*(d*x+c)^(1/4)/d^2+160/33*b*(-a*d+b*c)^2*(d
*x+c)^(1/4)*(b*x+a)^(1/2)/d^4-320/33*b^(3/4)*(-a*d+b*c)^(13/4)*EllipticF(b^
(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^5/(
b*x+a)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 230, 227}

$$-\frac{320b^{3/4}(bc-ad)^{13/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{33d^5\sqrt{a+bx}}+\frac{160b\sqrt{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{33d^4}-\frac{80b(a+bx)^{3/2}\sqrt[4]{c+dx}\,(bc-ad)}{33d^3}+\frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2}-\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]
```

```
[Out] (-4*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/4)) + (160*b*(b*c - a*d)^2*Sqrt[a +
b*x]*(c + d*x)^(1/4))/(33*d^4) - (80*b*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x
)^(1/4))/(33*d^3) + (56*b*(a + b*x)^(5/2)*(c + d*x)^(1/4))/(33*d^2) - (320*
b^(3/4)*(b*c - a*d)^(13/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(33*d^5*Sqrt[a + b*x
])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{7/4}}\,dx = -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{(14b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/4}}\,dx}{3d}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2} - \frac{(140b(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}}\,dx}{33d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+dx}}{33d^3} + \frac{56b(a+bx)^{5/2}\sqrt[4]{c+dx}}{33d^2} + \frac{(40b$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{3d(c+dx)^{3/4}} + \frac{160b(bc-ad)^2\sqrt{a+bx}\,\sqrt[4]{c+dx}}{33d^4} - \frac{80b(bc-ad)(a+bx)^{3/2}\sqrt[4]{c+d}}{33d^3}$$

<u>**Mathematica**</u> **[C]** Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.35

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}{}_2F_1\left(\frac{7}{4},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(7/4),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[7/4,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(7/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
[Out] int((b*x+a)^(7/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="fricas")
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*(d*x + c)^
(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{7}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(7/4),x)
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(7/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(7/4),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(7/4), x)
```

3.1666 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=137

$$-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}+\frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2}-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^3\sqrt{a+bx}}$$

```
[Out] -4/3*(b*x+a)^(3/2)/d/(d*x+c)^(3/4)+8/3*b*(d*x+c)^(1/4)*(b*x+a)^(1/2)/d^2-16
/3*b^(3/4)*(-a*d+b*c)^(5/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4
),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 65, 230, 227}

$$-\frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^3\sqrt{a+bx}}+\frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2}-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/4)) + (8*b*Sqrt[a + b*x]*(c + d*x)^(
1/4))/(3*d^2) - (16*b^(3/4)*(b*c - a*d)^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
3*d^3*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}}\,dx = -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{(2b)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{(4b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{(16b(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,\right.}{3d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{\left(16b(bc-ad)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{3d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}\,\sqrt[4]{c+dx}}{3d^2} - \frac{16b^{3/4}(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\right.\right.}{3d^3\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.53

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[7/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(7/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(7/4),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(7/4), x)
```

3.1667 $\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}} dx$

Optimal. Leaf size=111

$$-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}+\frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}$$

```
[Out] -4/3*(b*x+a)^(1/2)/d/(d*x+c)^(3/4)+8/3*b^(3/4)*(-a*d+b*c)^(1/4)*EllipticF(b
^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^2/
(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 65, 230, 227}

$$\frac{8b^{3/4}\sqrt[4]{bc-ad}\sqrt{-\frac{d(a+bx)}{bc-ad}}F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(7/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(3*d*(c + d*x)^(3/4)) + (8*b^(3/4)*(b*c - a*d)^(1/4)*Sqr
t[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/
(b*c - a*d)^(1/4)], -1])/(3*d^2*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}}\,dx &= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}+\frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3d}\\
&= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}+\frac{(8b)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d^2}\\
&= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}+\frac{\left(8b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d^2\sqrt{a+bx}}\\
&= -\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}}+\frac{8b^{3/4}\sqrt[4]{bc-ad}\,\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d^2\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.66

$$
\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}{}_2F_1\left(\frac{3}{2},\frac{7}{4};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(7/4),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[3/2,
 7/4, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(7/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(7/4),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(7/4),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(7/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(7/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(7/4), x)
```

3.1668 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=118

$$\frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d(bc-ad)^{3/4}\sqrt{a+bx}}$$

```
[Out] 4/3*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(3/4)+4/3*b^(3/4)*EllipticF(b^(1/4)*(d
*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^
(3/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d\sqrt{a+bx}\,(bc-ad)^{3/4}}+\frac{4\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(7/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/4)) + (4*b^(3/4)*Sqrt[-((d*(a
 + b*x))/(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*
d)^(1/4)], -1])/(3*d*(b*c - a*d)^(3/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/4}}\,dx &= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{b\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/4}}\,dx}{3(bc-ad)}\\
&= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx,x,\sqrt[4]{c+dx}\right)}{3d(bc-ad)}\\
&= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{\left(4b\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{bx^4}{\left(a-\frac{bc}{d}\right)d}}}\,dx,x,\right.}{3d(bc-ad)\sqrt{a+bx}}\\
&= \frac{4\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/4}}+\frac{4b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3d(bc-ad)^{3/4}\sqrt{a+bx}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.60

$$
\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(7/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[1/2, 7
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(7/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(7/4),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(7/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/4),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/4)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/4)), x)
```

3.1669 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=146

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}-\frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}}-\frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|\right.}{3(bc-ad)^{7/4}\sqrt{a+bx}}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(3/4)/(b*x+a)^(1/2)-10/3*d*(b*x+a)^(1/2)/(-a*d+b*c)^2
/(d*x+c)^(3/4)-10/3*b^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4
),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(7/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {53, 65, 230, 227}

$$-\frac{10b^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{3\sqrt{a+bx}\,(bc-ad)^{7/4}}-\frac{10d\sqrt{a+bx}}{3(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4)) - (10*d*Sqrt[a + b*x])/(3*(b
*c - a*d)^2*(c + d*x)^(3/4)) - (10*b^(3/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d)
)]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(3*(
b*c - a*d)^(7/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{(5d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{(5bd)\int \frac{1}{\sqrt{a+bx}\cdots}}{6(bc-a\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{(10b)\text{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{\left(10b\sqrt{\frac{d(a+bx\cdots}{-bc+a\cdots}}\right.}{\cdots}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}} - \frac{10d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/4}} - \frac{10b^{3/4}\sqrt{-\frac{d(a+\cdots}{bc-\cdots}}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.49

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(-\frac{1}{2},\frac{7}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[-1/2, 7/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(7/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d
+ a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d
)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(7/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{7/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/4)), x)
```

3.1670 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/4}}\,dx$

Optimal. Leaf size=178

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}+\frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}}+\frac{5d^2\sqrt{a+bx}}{(bc-ad)^3(c+dx)^{3/4}}+\frac{5b^{3/4}d\sqrt{-\frac{d(a}{bc}}}{}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(3/4)+3*d/(-a*d+b*c)^2/(d*x+c)^(3/4)/
(b*x+a)^(1/2)+5*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(3/4)+5*b^(3/4)*d*El
lipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(
1/2)/(-a*d+b*c)^(11/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 65, 230, 227}

$$\frac{5b^{3/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{\sqrt{a+bx}\,(bc-ad)^{11/4}}+\frac{5d^2\sqrt{a+bx}}{(c+dx)^{3/4}(bc-ad)^3}+\frac{3d}{\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/4)) + (3*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/4)) + (5*d^2*Sqrt[a + b*x])/((b*c - a*d)^3*(c + d
*x)^(3/4)) + (5*b^(3/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/((b*c - a*d)^(11/4)*S
qrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} - \frac{(3d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{(15d^2}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}} + \frac{3d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/4}} + \frac{5}{(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 73, normalized size = 0.41

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/4}\,{}_2F_1\left(-\frac{3}{2},\frac{7}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/4)*Hypergeometric2F1[-3/2, 7/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(7/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}\,(dx+c)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(7/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(7/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/4)), x)
```

3.1671 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=286

$$-\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}}-\frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}}-\frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4}+\frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3}+\frac{448b^5}{}$$

```
[Out] -4/5*(b*x+a)^(7/2)/d/(d*x+c)^(5/4)-56/5*b*(b*x+a)^(5/2)/d^2/(d*x+c)^(1/4)+1
12/9*b^2*(b*x+a)^(3/2)*(d*x+c)^(3/4)/d^3-224/15*b^2*(-a*d+b*c)*(d*x+c)^(3/4
)*(b*x+a)^(1/2)/d^4+448/15*b^(5/4)*(-a*d+b*c)^(11/4)*EllipticE(b^(1/4)*(d*x
+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^5/(b*x+a)^(1/
2)-448/15*b^(5/4)*(-a*d+b*c)^(11/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b
*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^5/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{448b^{5/4}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15d^5\sqrt{a+bx}}+\frac{448b^{5/4}(bc-ad)^{11/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{15d^5\sqrt{a+bx}}-\frac{224b^2\sqrt{a+bx}\,(c+dx)^{3/4}(bc-ad)}{15d^4}+\frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3}-\frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}}-\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(7/2))/(5*d*(c + d*x)^(5/4)) - (56*b*(a + b*x)^(5/2))/(5*d^2*
(c + d*x)^(1/4)) - (224*b^2*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/4))/(15*
d^4) + (112*b^2*(a + b*x)^(3/2)*(c + d*x)^(3/4))/(9*d^3) + (448*b^(5/4)*(b*
c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^(1/4
)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]) - (448*b
^(5/4)*(b*c - a*d)^(11/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(15*d^5*Sqrt[a + b*x]
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} + \frac{(14b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} + \frac{(28b^2)\int \frac{(a+bx)^{3/2}}{\sqrt[4]{c+dx}}\,dx}{d^2}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} + \frac{112b^2(a+bx)^{3/2}(c+dx)^{3/4}}{9d^3} - \frac{(56b^2(bc-ad))\int \frac{\sqrt{a}}{\sqrt[4]{c}}}{3d^3}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

$$= -\frac{4(a+bx)^{7/2}}{5d(c+dx)^{5/4}} - \frac{56b(a+bx)^{5/2}}{5d^2\sqrt[4]{c+dx}} - \frac{224b^2(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/4}}{15d^4} + \frac{112b^2(a+bx}{9}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.08, size = 73, normalized size = 0.26

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(9/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*(d*x + c)^
(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(9/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/2)/(d*x + c)^(9/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{7/2}}{{\left(c+d\,x\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(9/4),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(9/4), x)
```

3.1672 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=248

$$-\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}}-\frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}}+\frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3}-\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\sqrt[4]{\cdots}\right.\right.}{5d^4\sqrt{a+bx}}$$

```
[Out] -4/5*(b*x+a)^(5/2)/d/(d*x+c)^(5/4)-8*b*(b*x+a)^(3/2)/d^2/(d*x+c)^(1/4)+48/5
*b^2*(d*x+c)^(3/4)*(b*x+a)^(1/2)/d^3-96/5*b^(5/4)*(-a*d+b*c)^(7/4)*Elliptic
E(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d
^4/(b*x+a)^(1/2)+96/5*b^(5/4)*(-a*d+b*c)^(7/4)*EllipticF(b^(1/4)*(d*x+c)^(1
/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^4/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 52, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^4\sqrt{a+bx}}-\frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^4\sqrt{a+bx}}+\frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3}-\frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}}-\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(5/2))/(5*d*(c + d*x)^(5/4)) - (8*b*(a + b*x)^(3/2))/(d^2*(c
+ d*x)^(1/4)) + (48*b^2*Sqrt[a + b*x]*(c + d*x)^(3/4))/(5*d^3) - (96*b^(5/4
)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]) + (96
*b^(5/4)*(b*c - a*d)^(7/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^4*Sqrt[a + b*x]
)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} + \frac{(2b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{(12b^2)\int \frac{\sqrt{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d^2}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} - \frac{(24b^2(bc-ad))\int \frac{}{\sqrt{a+}}}{5d^3}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} - \frac{(96b^2(bc-ad))\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} + \frac{\left(96b^{3/2}(bc-ad)^{3/2}\right)\mathrm{Su}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} + \frac{\Big(96b^{3/2}(bc-ad)^{3/2}\sqrt{\frac{c}{}}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} + \frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{c}{}}}{}$$

$$= -\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2\sqrt[4]{c+dx}} + \frac{48b^2\sqrt{a+bx}\,(c+dx)^{3/4}}{5d^3} - \frac{96b^{5/4}(bc-ad)^{7/4}\sqrt{-\frac{c}{}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

4 in optimal.
time = 10.06, size = 73, normalized size = 0.29

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(\frac{9}{4},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(9/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(3/4)/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(9/4),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(9/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(9/4),x)
[Out] int((a + b*x)^(5/2)/(c + d*x)^(9/4), x)
```

3.1673 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=222

$$-\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}}-\frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}}+\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{48b^{5/4}(}{}$$

```
[Out] -4/5*(b*x+a)^(3/2)/d/(d*x+c)^(5/4)-24/5*b*(b*x+a)^(1/2)/d^2/(d*x+c)^(1/4)+4
8/5*b^(5/4)*(-a*d+b*c)^(3/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/
4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)-48/5*b^(5/4)*(-a*d+b*
c)^(3/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-
a*d+b*c))^(1/2)/d^3/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {49, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}+\frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^3\sqrt{a+bx}}-\frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}}-\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]
```

```
[Out] (-4*(a + b*x)^(3/2))/(5*d*(c + d*x)^(5/4)) - (24*b*Sqrt[a + b*x])/(5*d^2*(c
 + d*x)^(1/4)) + (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/(b*c -
a*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/
(5*d^3*Sqrt[a + b*x]) - (48*b^(5/4)*(b*c - a*d)^(3/4)*Sqrt[-((d*(a + b*x))/
(b*c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)]
, -1])/(5*d^3*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}}\,dx = -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} + \frac{(6b)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{(12b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5d^2}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{(48b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+dx}\right)}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{\left(48b^{3/2}\sqrt{bc-ad}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x,\right.}{5d^3}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{\left(48b^{3/2}\sqrt{bc-ad}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{}{(a}}}\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} - \frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+}}{\sqrt[4]{bc-}}\right.\right.}{5d^3\sqrt{a+bx}}$$

$$= -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2\sqrt[4]{c+dx}} + \frac{48b^{5/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+}}{\sqrt[4]{bc-}}\right.\right.}{5d^3\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.33

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(9/4),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[9/4,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(9/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(9/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(9/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(9/4),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(9/4), x)
```

3.1674 $\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}} \, dx$

Optimal. Leaf size=232

$$-\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}}+\frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}}-\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt[4]{bc-ad}\sqrt{a+bx}}+\frac{8b^{5/4}\sqrt{-\frac{d}{}}}{}$$

```
[Out] -4/5*(b*x+a)^(1/2)/d/(d*x+c)^(5/4)+8/5*b*(b*x+a)^(1/2)/d/(-a*d+b*c)/(d*x+c)
^(1/4)-8/5*b^(5/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*
(b*x+a)/(-a*d+b*c))^(1/2)/d^2/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)+8/5*b^(5/4)*El
lipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(
1/2)/d^2/(-a*d+b*c)^(1/4)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\sqrt[4]{bc-ad}}-\frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d^2\sqrt{a+bx}\sqrt[4]{bc-ad}}+\frac{8b\sqrt{a+bx}}{5d\sqrt[4]{c+dx}(bc-ad)}-\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(9/4),x]
```

```
[Out] (-4*Sqrt[a + b*x])/(5*d*(c + d*x)^(5/4)) + (8*b*Sqrt[a + b*x])/(5*d*(b*c -
a*d)*(c + d*x)^(1/4)) - (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*Ellip
ticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2*(b*c
- a*d)^(1/4)*Sqrt[a + b*x]) + (8*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]
*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*d^2
*(b*c - a*d)^(1/4)*Sqrt[a + b*x])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
```

```
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}}\,dx = -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx}{5d}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{(2b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5d(bc-ad)}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{(8b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c+
}{5d^2(bc-ad)}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{(8b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^4}{d}}}\,dx, x, \sqrt[4]{c
}{5d^2\sqrt{bc-ad}}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{\left(8b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{b
}{(a-
}}}{5d^2\sqrt{bc-ad}\,\sqrt{a+b
}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} + \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{5d^2\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

$$= -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)\sqrt[4]{c+dx}} - \frac{8b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right.\right.}{5d^2\sqrt[4]{bc-ad}\,\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.06, size = 73, normalized size = 0.31

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}{}_2F_1\left(\frac{3}{2},\frac{9}{4};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(9/4),x]
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[3/2,
 9/4, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(9/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(9/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(9/4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(9/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(9/4),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(9/4), x)
```

3.1675 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=236

$$\frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}}+\frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}}-\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d(bc-ad)^{5/4}\sqrt{a+bx}}+\frac{12b^{5/}}{}$$

```
[Out] 4/5*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(5/4)+12/5*b*(b*x+a)^(1/2)/(-a*d+b*c)^
2/(d*x+c)^(1/4)-12/5*b^(5/4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/
4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/d/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)+12/5*b
^(5/4)*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*
d+b*c))^(1/2)/d/(-a*d+b*c)^(5/4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}\,(bc-ad)^{5/4}}-\frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5d\sqrt{a+bx}\,(bc-ad)^{5/4}}+\frac{12b\sqrt{a+bx}}{5\sqrt[4]{c+dx}\,(bc-ad)^2}+\frac{4\sqrt{a+bx}}{5(c+dx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(9/4)),x]
```

```
[Out] (4*Sqrt[a + b*x])/(5*(b*c - a*d)*(c + d*x)^(5/4)) + (12*b*Sqrt[a + b*x])/(5
*(b*c - a*d)^2*(c + d*x)^(1/4)) - (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a
*d))]*EllipticE[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(
5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x]) + (12*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*
c - a*d))]*EllipticF[ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -
1])/(5*d*(b*c - a*d)^(5/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{9/4}}\,dx = \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{(3b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/4}}\,dx}{5(bc-ad)}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{(3b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt[4]{c+dx}}\,dx}{5(bc-ad)^2}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{(12b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a-\frac{bc}{d}+\cdots}}\right.}{5d(bc-ad\cdots}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(12b^{3/2})\,\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\cdots}}\right.}{5d(bc-ad\cdots}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(12b^{3/2}\sqrt{\frac{d(a+bx)}{-bc+ad}}\right)\text{Subst}\cdots}{5d(bc-\cdots}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} + \frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\sin^{-\cdots}\right.}{5d(bc-ad)^{5/4}\cdots}$$

$$= \frac{4\sqrt{a+bx}}{5(bc-ad)(c+dx)^{5/4}} + \frac{12b\sqrt{a+bx}}{5(bc-ad)^2\sqrt[4]{c+dx}} - \frac{12b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\sin^{-\cdots}\right.}{5d(bc-ad)^{5/4}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.30

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(9/4)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[1/2, 9
/4, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(9/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(9/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 + a*
d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\left(c+dx\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(9/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(9/4),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(9/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(9/4)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(9/4)), x)
```

3.1676 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=262

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}}-\frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}}-\frac{42bd\sqrt{a+bx}}{5(bc-ad)^3\sqrt[4]{c+dx}}+\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\Big(}{5(bc-ad}$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(5/4)/(b*x+a)^(1/2)-14/5*d*(b*x+a)^(1/2)/(-a*d+b*c)^2
/(d*x+c)^(5/4)-42/5*b*d*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/4)+42/5*b^(5/
4)*EllipticE(b^(1/4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*
c))^(1/2)/(-a*d+b*c)^(9/4)/(b*x+a)^(1/2)-42/5*b^(5/4)*EllipticF(b^(1/4)*(d*
x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(9/
4)/(b*x+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$-\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{9/4}}+\frac{42b^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{9/4}}-\frac{42bd\sqrt{a+bx}}{5\sqrt[4]{c+dx}\,(bc-ad)^3}-\frac{14d\sqrt{a+bx}}{5(c+dx)^{5/4}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/4)) - (14*d*Sqrt[a + b*x])/(5*(b
*c - a*d)^2*(c + d*x)^(5/4)) - (42*b*d*Sqrt[a + b*x])/(5*(b*c - a*d)^3*(c +
 d*x)^(1/4)) + (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[Arc
Sin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9/4)
*Sqrt[a + b*x]) - (42*b^(5/4)*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[
ArcSin[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(9
/4)*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{(7d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{9/4}}\,dx}{2(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{(21bd)\int \frac{1}{\sqrt{a+}}}{10(bc-}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/4}} - \frac{14d\sqrt{a+bx}}{5(bc-ad)^2(c+dx)^{5/4}} - \frac{42bd\sqrt{a+b}}{5(bc-ad)^3\sqrt[4]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 71, normalized size = 0.27

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4} {}_2F_1\left(-\frac{1}{2},\frac{9}{4};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[-1/2, 9/4, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(9/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(9/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^2*d^3*x^5 + a^2*c^3 + (3*b^2*c*d^
2 + 2*a*b*d^3)*x^4 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 +
 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(9/4),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(9/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(9/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(9/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(9/4)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(9/4)), x)
```

3.1677 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{9/4}}\,dx$

Optimal. Leaf size=303

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}}+\frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}}+\frac{77d^2\sqrt{a+bx}}{15(bc-ad)^3(c+dx)^{5/4}}+\frac{77bd^2\sqrt{a}}{5(bc-ad)^4}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(5/4)+11/3*d/(-a*d+b*c)^2/(d*x+c)^(5/
4)/(b*x+a)^(1/2)+77/15*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(5/4)+77/5*b*
d^2*(b*x+a)^(1/2)/(-a*d+b*c)^4/(d*x+c)^(1/4)-77/5*b^(5/4)*d*EllipticE(b^(1/
4)*(d*x+c)^(1/4)/(-a*d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*
c)^(13/4)/(b*x+a)^(1/2)+77/5*b^(5/4)*d*EllipticF(b^(1/4)*(d*x+c)^(1/4)/(-a*
d+b*c)^(1/4),I)*(-d*(b*x+a)/(-a*d+b*c))^(1/2)/(-a*d+b*c)^(13/4)/(b*x+a)^(1/
2)
```

Rubi [A]
time = 0.21, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {53, 65, 313, 230, 227, 1214, 1213, 435}

$$\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{13/4}}-\frac{77b^{5/4}d\sqrt{-\frac{d(a+bx)}{bc-ad}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{bc-ad}}\right)\middle|-1\right)}{5\sqrt{a+bx}\,(bc-ad)^{13/4}}+\frac{77bd^2\sqrt{a+bx}}{5\sqrt[4]{c+dx}\,(bc-ad)^4}+\frac{77d^2\sqrt{a+bx}}{15(c+dx)^{5/4}(bc-ad)^3}+\frac{11d}{3\sqrt{a+bx}\,(c+dx)^{5/4}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(5/4)) + (11*d)/(3*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(5/4)) + (77*d^2*Sqrt[a + b*x])/(15*(b*c - a*d)^3
*(c + d*x)^(5/4)) + (77*b*d^2*Sqrt[a + b*x])/(5*(b*c - a*d)^4*(c + d*x)^(1/
4)) - (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticE[ArcSin[(b^
(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)*Sqrt[
a + b*x]) + (77*b^(5/4)*d*Sqrt[-((d*(a + b*x))/(b*c - a*d))]*EllipticF[ArcS
in[(b^(1/4)*(c + d*x)^(1/4))/(b*c - a*d)^(1/4)], -1])/(5*(b*c - a*d)^(13/4)
*Sqrt[a + b*x])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{9/4}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} - \frac{(11d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{9/4}}\,dx}{6(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{(77d}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/4}} + \frac{11d}{3(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/4}} + \frac{}{15(b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.24

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{9/4} {}_2F_1\left(-\frac{3}{2},\frac{9}{4};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(9/4)*Hypergeometric2F1[-3/2, 9/4, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(9/4))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(9/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(3/4)/(b^3*d^3*x^6 + a^3*c^3 + 3*(b^3*c*d^
2 + a*b^2*d^3)*x^5 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c
^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^3 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*x^2 + 3*(a^2*b*c^3 + a^3*c^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(9/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(9/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(9/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(9/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(9/4)), x)
```

3.1678 $\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=205

$$\frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d}+\frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}+\frac{5(bc-ad)^3\tan}{64b}$$

```
[Out] 5/96*(-a*d+b*c)^2*(b*x+a)^(3/4)*(d*x+c)^(1/4)/b^2/d+5/24*(-a*d+b*c)*(b*x+a)
^(7/4)*(d*x+c)^(1/4)/b^2+1/3*(b*x+a)^(7/4)*(d*x+c)^(5/4)/b+5/64*(-a*d+b*c)^
3*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)/d^(7/4)-5/64*
(-a*d+b*c)^3*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)/d
^(7/4)
```

Rubi [A]
time = 0.10, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$\frac{5(bc-ad)^3\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (5*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(96*b^2*d) + (5*(b*c - a*
d)*(a + b*x)^(7/4)*(c + d*x)^(1/4))/(24*b^2) + ((a + b*x)^(7/4)*(c + d*x)^(
5/4))/(3*b) + (5*(b*c - a*d)^3*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c
 + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c - a*d)^3*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx = \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int (a+bx)^{3/4}\sqrt[4]{c+dx}\,dx}{12b}$$

$$= \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad)^2)\int}{96b^2}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx}{}$$

Mathematica [A]
time = 0.55, size = 174, normalized size = 0.85

$$\frac{(bc-ad)^3\left(\frac{2\sqrt[4]{b}\,d^{3/4}(a+bx)^{3/4}\sqrt[4]{c+dx}\,(-15a^2d^2+6abd(7c+2dx)+b^2(5c^2+52cdx+32d^2x^2))}{(bc-ad)^3} - 15\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right) - 15\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)\right)}{192b^{9/4}d^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^3*((2*b^(1/4)*d^(3/4)*(a + b*x)^(3/4)*(c + d*x)^(1/4)*(-15*a^2
*d^2 + 6*a*b*d*(7*c + 2*d*x) + b^2*(5*c^2 + 52*c*d*x + 32*d^2*x^2)))/(b*c -
 a*d)^3 - 15*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))] -
15*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]))/(192*b^(9
/4)*d^(7/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}\,(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2151 vs. $2(159) = 318$.
time = 0.87, size = 2151, normalized size = 10.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/384*(60*b^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*arc
tan(((b^10*c^3*d^5 - 3*a*b^9*c^2*d^6 + 3*a^2*b^8*c*d^7 - a^3*b^7*d^8)*(b*x
+ a)^(3/4)*(d*x + c)^(1/4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^
3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))
^(3/4) + (b^8*d^5*x + a*b^7*d^5)*sqrt(((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^
4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d
^6)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^4*x + a*b^4*d^4)*sqrt((b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^
11*b*c*d^11 + a^12*d^12)/(b^9*d^7)))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^9*d^7))^(3/4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^1
0*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^
5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a
^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^
```

```
13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b
^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^
10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 15*b^2*d*((b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d^2*x + a*b^2*d^2
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4))/(b*x + a)) - 15*b
^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4
) - (b^3*d^2*x + a*b^2*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7)
)^(1/4))/(b*x + a)) - 4*(32*b^2*d^2*x^2 + 5*b^2*c^2 + 42*a*b*c*d - 15*a^2*d
^2 + 4*(13*b^2*c*d + 3*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{4}} (c+dx)^{\frac{5}{4}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)*(c + d*x)**(5/4), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, need to choose a branch for the root of a polyno
```

```
mial with parameters. This might be wrong.The choice was done assuming [sag
eVARa,sag
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/4}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(3/4)*(c + d*x)^(5/4), x)
```

3.1679 $\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal. Leaf size=167

$$\frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2}{}$$

```
[Out] 5/8*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^(1/4)/b^2+1/2*(b*x+a)^(3/4)*(d*x+c)^(5
/4)/b-5/16*(-a*d+b*c)^2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))
/b^(9/4)/d^(3/4)+5/16*(-a*d+b*c)^2*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d
*x+c)^(1/4))/b^(9/4)/d^(3/4)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$-\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*b^2) + ((a + b*x)^(3/4)*
(c + d*x)^(5/4))/(2*b) - (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/
(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh
[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx = \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\int \frac{1}{\sqrt[4]{a+bx}\,(c+d}}{32b^2}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\left(c-\frac{ad}{b}\right.}\right.}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\right.}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{b}-}\right.}{16b^2\sqrt{}}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a-}}{\sqrt[4]{b}\,\sqrt[4]{c+}}\right.}{16b^{9/4}d^{3/4}}$$

Mathematica [A]
time = 0.44, size = 143, normalized size = 0.86

$$\frac{(bc-ad)^2\left(\frac{2\sqrt[4]{b}(a+bx)^{3/4}\sqrt[4]{c+dx}(9bc-5ad+4bdx)}{(bc-ad)^2}+\frac{5\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{d^{3/4}}+\frac{5\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{d^{3/4}}\right)}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] ((b*c - a*d)^2*((2*b^(1/4)*(a + b*x)^(3/4)*(c + d*x)^(1/4)*(9*b*c - 5*a*d +
 4*b*d*x))/(b*c - a*d)^2 + (5*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a
+ b*x)^(1/4))])/d^(3/4) + (5*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a
+ b*x)^(1/4))])/d^(3/4)))/(16*b^(9/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. 2(127) = 254.
time = 1.40, size = 1468, normalized size = 8.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*arctan(-((b^9*c^2*d^2 - 2*a*b^8*c*d^
3 + a^2*b^7*d^4)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4) -
 (b^8*d^2*x + a*b^7*d^2)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^2*x + a*b^
4*d^2)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^9*d^3)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4))/(a*b^8*
c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^
4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d
^8 + (b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 7
0*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d
^7 + a^8*b*d^8)*x)) - 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d*x + a*b^2*d)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^9*d^3))^(1/4))/(b*x + a)) + 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b^3*d*x +
a*b^2*d)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4))/(b*x + a)) - 4*(4*b*d*x + 9*b*c - 5*a*d)
*(b*x + a)^(3/4)*(d*x + c)^(1/4))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt[4]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(1/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(1/4), x)
```

3.1680 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}-\frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}$$

```
[Out] 5*d*(b*x+a)^(3/4)*(d*x+c)^(1/4)/b^2-4*(d*x+c)^(5/4)/b/(b*x+a)^(1/4)-5/2*d^(
1/4)*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)
+5/2*d^(1/4)*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4)
)/b^(9/4)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 65, 338, 304, 211, 214}

$$-\frac{5\sqrt[4]{d}\,(bc-ad)\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (5*d*(a + b*x)^(3/4)*(c + d*x)^(1/4))/b^2 - (4*(c + d*x)^(5/4))/(b*(a + b*x
)^(1/4)) - (5*d^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)
*(c + d*x)^(1/4))])/(2*b^(9/4)) + (5*d^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(9/4))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx = -\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{b}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4b^2}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a}\right.}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{\left(5\sqrt{d}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \right.}{2b^2}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} - \frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}} + \frac{5\sqrt[4]{d}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.05, size = 71, normalized size = 0.47

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(116) = 232$.
time = 1.50, size = 857, normalized size = 5.64

$$\frac{20\,(b^3x+ab^2)\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{(b^8c-ab^7d)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{3}{4}}+(b^8x+ab^7)\sqrt{\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bx+a}\,\sqrt{dx+c}+(b^5x+ab^4)\sqrt{\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}}}{bx+a}}\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{3}{4}}}{ab^4c^4d-4\,a^2b^3c^3d^2+6\,a^3b^2c^2d^3-4\,a^4bcd^4+a^5d^5+(b^5c^4d-4\,ab^4c^3d^2+6\,a^2b^3c^2d^3-4\,a^3b^2cd^4+a^4bd^5)x}\right)+5\,(b^3x+ab^2)\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}+(b^3x+ab^2)\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-5\,(b^3x+ab^2)\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}-(b^3x+ab^2)\left(\frac{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}{b^9}\right)^{\frac{1}{4}}\right)}{bx+a}\right)+4\,(bdx-4\,bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{4\,(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")
[Out] 1/4*(20*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*arctan(((b^8*c - a*b^7*d)*(b*x + a)^(3
/4)*(d*x + c)^(1/4)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a
^3*b*c*d^4 + a^4*d^5)/b^9)^(3/4) + (b^8*x + a*b^7)*sqrt(((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*x + a*b^4)*sqrt((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9))/(b*
x + a))*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
 a^4*d^5)/b^9)^(3/4))/(a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3
- 4*a^4*b*c*d^4 + a^5*d^5 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^
3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-
5*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*x + a*b^2)*((b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/
4))/(b*x + a)) - 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^
2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
a)^(3/4)*(d*x + c)^(1/4) - (b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4))/(b*x + a)) + 4*(b*
d*x - 4*b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^3*x + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(5/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(5/4), x)
```

3.1681 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx$

Optimal. Leaf size=134

$$-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}$$

```
[Out] -4*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(1/4)-4/5*(d*x+c)^(5/4)/b/(b*x+a)^(5/4)-2*d^
(5/4)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)+2*d^(5/4)
*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(9/4)
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 65, 338, 304, 211, 214}

$$-\frac{2d^{5/4}\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(1/4)) - (4*(c + d*x)^(5/4))/(5*b*(a
+ b*x)^(5/4)) - (2*d^(5/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d
*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/b^(9/4)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx = -\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/4}}\,dx}{b}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{b^2}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(4d^2)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b^3}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(4d^2)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} + \frac{(2d^{3/2})\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^2} - \frac{(2d$$

$$= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} - \frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{9/4}} + \frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{}}{\sqrt[4]{b}\,\sqrt[4]{}}\right.}{b^{9/4}}$$

Mathematica [A]
time = 0.30, size = 122, normalized size = 0.91

$$\frac{2\left(-\frac{2\sqrt[4]{b}\,\sqrt[4]{c+dx}\,(bc+5ad+6bdx)}{(a+bx)^{5/4}} + 5d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right) + 5d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)\right)}{5b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (2*((-2*b^(1/4)*(c + d*x)^(1/4)*(b*c + 5*a*d + 6*b*d*x))/(a + b*x)^(5/4) +
5*d^(5/4)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))] + 5*d
^(5/4)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]))/(5*b^
(9/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(100) = 200$.
time = 1.20, size = 368, normalized size = 2.75

$$-\frac{20\left(b^{4}x^{2}+2ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}b^{7}d\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}-\left(b^{8}x+ab^{7}\right)\sqrt{\frac{\sqrt{bx+a}\,\sqrt{dx+c}\,d^{2}+\left(b^{5}x+ab^{4}\right)\sqrt{\frac{d^{5}}{b^{9}}}}{bx+a}}\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}}{bd^{5}x+ad^{5}}\right)-5\left(b^{4}x^{2}+2ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}d+\left(b^{3}x+ab^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+5\left(b^{4}x^{2}+2ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}d-\left(b^{3}x+ab^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+4\left(6bdx+bc+5ad\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(b^{4}x^{2}+2ab^{3}x+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="fricas")
[Out] -1/5*(20*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*arctan(-((b*x + a)
^(3/4)*(d*x + c)^(1/4)*b^7*d*(d^5/b^9)^(3/4) - (b^8*x + a*b^7)*sqrt((sqrt(b
*x + a)*sqrt(d*x + c)*d^2 + (b^5*x + a*b^4)*sqrt(d^5/b^9))/(b*x + a))*(d^5/
b^9)^(3/4))/(b*d^5*x + a*d^5)) - 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9
)^(1/4)*log(((b*x + a)^(3/4)*(d*x + c)^(1/4)*d + (b^3*x + a*b^2)*(d^5/b^9)^
(1/4))/(b*x + a)) + 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*log((
(b*x + a)^(3/4)*(d*x + c)^(1/4)*d - (b^3*x + a*b^2)*(d^5/b^9)^(1/4))/(b*x +
 a)) + 4*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(9/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(9/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(9/4), x)
```

3.1682 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

```
[Out] -4/9*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx = -\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

Mathematica [A]
time = 0.06, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Maple [A]
time = 0.17, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{4(dx+c)^{\frac{9}{4}}}{9(bx+a)^{\frac{9}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(13/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/9/(b*x+a)^(9/4)*(d*x+c)^(9/4)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(26) = 52.
time = 0.90, size = 104, normalized size = 3.25

$$-\frac{4\left(d^2x^2+2\,cdx+c^2\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{9\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="fricas")
```

```
[Out] -4/9*(d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b*c - a
^4*d + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c -
 a^3*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(13/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(13/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

Mupad [B]
time = 0.81, size = 99, normalized size = 3.09

$$\frac{4\,c^2\,{\left(c+d\,x\right)}^{1/4}+4\,d^2\,x^2\,{\left(c+d\,x\right)}^{1/4}+8\,c\,d\,x\,{\left(c+d\,x\right)}^{1/4}}{{\left(a+b\,x\right)}^{1/4}\,\left(9\,d\,a^3+18\,d\,a^2\,b\,x-9\,c\,a^2\,b+9\,d\,a\,b^2\,x^2-18\,c\,a\,b^2\,x-9\,c\,b^3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(13/4),x)
```

```
[Out] (4*c^2*(c + d*x)^(1/4) + 4*d^2*x^2*(c + d*x)^(1/4) + 8*c*d*x*(c + d*x)^(1/4
))/((a + b*x)^(1/4)*(9*a^3*d - 9*b^3*c*x^2 - 9*a^2*b*c - 18*a*b^2*c*x + 18*
a^2*b*d*x + 9*a*b^2*d*x^2))
```

3.1683 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}} dx$

Optimal. Leaf size=66

$$-\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}$$

```
[Out] -4/13*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(13/4)+16/117*d*(d*x+c)^(9/4)/(-a*d+
b*c)^2/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(9
/4))/(117*(b*c - a*d)^2*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx = -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{13(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{9/4}(-9bc+13ad+4bdx)}{117(bc-ad)^2(a+bx)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(-9*b*c + 13*a*d + 4*b*d*x))/(117*(b*c - a*d)^2*(a + b*x
)^(13/4))
```

Maple [A]
time = 0.22, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{4(dx+c)^{\frac{9}{4}}(4bdx+13ad-9bc)}{117(bx+a)^{\frac{13}{4}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(17/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/117*(d*x+c)^(9/4)*(4*b*d*x+13*a*d-9*b*c)/(b*x+a)^(13/4)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 0.88, size = 235, normalized size = 3.56

$$\frac{4\,(4\,bd^3x^3 - 9\,bc^3 + 13\,ac^2d - (bcd^2 - 13\,ad^3)x^2 - 2\,(7\,bc^2d - 13\,acd^2)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{117\,(a^4b^2c^2 - 2\,a^5bcd + a^6d^2 + (b^6c^2 - 2\,ab^5cd + a^2b^4d^2)x^4 + 4\,(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2)x^3 + 6\,(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2)x^2 + 4\,(a^3b^3c^2 - 2\,a^4b^2cd + a^5bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="fricas")
```

```
[Out] 4/117*(4*b*d^3*x^3 - 9*b*c^3 + 13*a*c^2*d - (b*c*d^2 - 13*a*d^3)*x^2 - 2*(7
*b*c^2*d - 13*a*c*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^2*c^2 - 2*
a^5*b*c*d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(17/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4961 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

Mupad [**B**]
time = 0.95, size = 178, normalized size = 2.70

$$\frac{(c+d\,x)^{1/4}\left(\frac{16\,d^3\,x^3}{117\,b^2\,(a\,d-b\,c)^2}-\frac{36\,b\,c^3-52\,a\,c^2\,d}{117\,b^3\,(a\,d-b\,c)^2}+\frac{x^2\,(52\,a\,d^3-4\,b\,c\,d^2)}{117\,b^3\,(a\,d-b\,c)^2}+\frac{8\,c\,d\,x\,(13\,a\,d-7\,b\,c)}{117\,b^3\,(a\,d-b\,c)^2}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(17/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((16*d^3*x^3)/(117*b^2*(a*d - b*c)^2) - (36*b*c^3 - 52*a*c
^2*d)/(117*b^3*(a*d - b*c)^2) + (x^2*(52*a*d^3 - 4*b*c*d^2))/(117*b^3*(a*d
- b*c)^2) + (8*c*d*x*(13*a*d - 7*b*c))/(117*b^3*(a*d - b*c)^2)))/(x^3*(a +
b*x)^(1/4) + (a^3*(a + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a
^2*x*(a + b*x)^(1/4))/b^2)
```

3.1684 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}} dx$

Optimal. Leaf size=101

$$-\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}+\frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}}-\frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}$$

```
[Out] -4/17*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(17/4)+32/221*d*(d*x+c)^(9/4)/(-a*d+
b*c)^2/(b*x+a)^(13/4)-128/1989*d^2*(d*x+c)^(9/4)/(-a*d+b*c)^3/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(17*(b*c - a*d)*(a + b*x)^(17/4)) + (32*d*(c + d*x)^(9
/4))/(221*(b*c - a*d)^2*(a + b*x)^(13/4)) - (128*d^2*(c + d*x)^(9/4))/(1989
*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx = -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} - \frac{(8d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{17(bc-ad)}$$
$$= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} + \frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}} + \frac{(32d^2)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{221(bc-ad)^2}$$
$$= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}} + \frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}} - \frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}$$

Mathematica [A]
time = 0.19, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{9/4}\left(221d^2-\frac{306bd(c+dx)}{a+bx}+\frac{117b^2(c+dx)^2}{(a+bx)^2}\right)}{1989(bc-ad)^3(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*d^2 - (306*b*d*(c + d*x))/(a + b*x) + (117*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(1989*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Maple [A]
time = 0.22, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{4(dx+c)^{\frac{9}{4}}\left(32b^2x^2d^2+136ab\,d^2x-72b^2cdx+221a^2d^2-306abcd+117b^2c^2\right)}{1989(bx+a)^{\frac{17}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(21/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/1989*(d*x+c)^(9/4)*(32*b^2*d^2*x^2+136*a*b*d^2*x-72*b^2*c*d*x+221*a^2*d^2
-306*a*b*c*d+117*b^2*c^2)/(b*x+a)^(17/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(83) = 166$.
time = 0.86, size = 426, normalized size = 4.22

$$-\frac{4\,(32\,b^2d^4x^4+117\,b^2c^4-306\,abc^3d+221\,a^2c^2d^2-8\,(b^2cd^3-17\,abd^4)x^3+(5\,b^2c^2d^2-34\,abcd^3+221\,a^2d^4)x^2+2\,(81\,b^2c^3d-238\,abc^2d^2+221\,a^2cd^3)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{1989\,(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3)x^5+5\,(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3)x^4+10\,(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3)x^3+10\,(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3)x^2+5\,(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="fricas")
```

```
[Out] -4/1989*(32*b^2*d^4*x^4 + 117*b^2*c^4 - 306*a*b*c^3*d + 221*a^2*c^2*d^2 - 8
*(b^2*c*d^3 - 17*a*b*d^4)*x^3 + (5*b^2*c^2*d^2 - 34*a*b*c*d^3 + 221*a^2*d^4
)*x^2 + 2*(81*b^2*c^3*d - 238*a*b*c^2*d^2 + 221*a^2*c*d^3)*x)*(b*x + a)^(3/
4)*(d*x + c)^(1/4)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3
 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7
*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c
^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(21/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 9881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

Mupad [B]
time = 1.13, size = 268, normalized size = 2.65

$$\frac{(c+d\,x)^{1/4}\left(\frac{884\,a^2\,c^2\,d^2-1224\,a\,b\,c^3\,d+468\,b^2\,c^4}{1989\,b^4\,(a\,d-b\,c)^3}+\frac{x^2\,(884\,a^2\,d^4-136\,a\,b\,c\,d^3+20\,b^2\,c^2\,d^2)}{1989\,b^4\,(a\,d-b\,c)^3}+\frac{128\,d^4\,x^4}{1989\,b^2\,(a\,d-b\,c)^3}+\frac{32\,d^3\,x^3\,(17\,a\,d-b\,c)}{1989\,b^3\,(a\,d-b\,c)^3}+\frac{8\,c\,d\,x\,(221\,a^2\,d^2-238\,a\,b\,c\,d+81\,b^2\,c^2)}{1989\,b^4\,(a\,d-b\,c)^3}\right)}{x^4\,(a+b\,x)^{1/4}+\frac{a^4\,(a+b\,x)^{1/4}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/4}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/4}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/4}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(21/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((468*b^2*c^4 + 884*a^2*c^2*d^2 - 1224*a*b*c^3*d)/(1989*b^
4*(a*d - b*c)^3) + (x^2*(884*a^2*d^4 + 20*b^2*c^2*d^2 - 136*a*b*c*d^3))/(19
89*b^4*(a*d - b*c)^3) + (128*d^4*x^4)/(1989*b^2*(a*d - b*c)^3) + (32*d^3*x^
3*(17*a*d - b*c))/(1989*b^3*(a*d - b*c)^3) + (8*c*d*x*(221*a^2*d^2 + 81*b^2
*c^2 - 238*a*b*c*d))/(1989*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/4) + (a^4
*(a + b*x)^(1/4))/b^4 + (6*a^2*x^2*(a + b*x)^(1/4))/b^2 + (4*a*x^3*(a + b*x
)^(1/4))/b + (4*a^3*x*(a + b*x)^(1/4))/b^3)
```

3.1685 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}}+\frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}}-\frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}}+\frac{512d^3(c+dx)^{9/4}}{13923(bc-ad)^4(a+bx)^{9/4}}$$

```
[Out] -4/21*(d*x+c)^(9/4)/(-a*d+b*c)/(b*x+a)^(21/4)+16/119*d*(d*x+c)^(9/4)/(-a*d+
b*c)^2/(b*x+a)^(17/4)-128/1547*d^2*(d*x+c)^(9/4)/(-a*d+b*c)^3/(b*x+a)^(13/4
)+512/13923*d^3*(d*x+c)^(9/4)/(-a*d+b*c)^4/(b*x+a)^(9/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(21*(b*c - a*d)*(a + b*x)^(21/4)) + (16*d*(c + d*x)^(9
/4))/(119*(b*c - a*d)^2*(a + b*x)^(17/4)) - (128*d^2*(c + d*x)^(9/4))/(1547
*(b*c - a*d)^3*(a + b*x)^(13/4)) + (512*d^3*(c + d*x)^(9/4))/(13923*(b*c -
a*d)^4*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx = -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} - \frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} + \frac{(32d^2)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{119(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}}$$

$$= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}} + \frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}} - \frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}}$$

Mathematica [A]
time = 0.19, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{9/4}\left(-1547d^3+\frac{3213bd^2(c+dx)}{a+bx}-\frac{2457b^2d(c+dx)^2}{(a+bx)^2}+\frac{663b^3(c+dx)^3}{(a+bx)^3}\right)}{13923(bc-ad)^4(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
[Out] (-4*(c + d*x)^(9/4)*(-1547*d^3 + (3213*b*d^2*(c + d*x))/(a + b*x) - (2457*b
^2*d*(c + d*x)^2)/(a + b*x)^2 + (663*b^3*(c + d*x)^3)/(a + b*x)^3))/(13923*
(b*c - a*d)^4*(a + b*x)^(9/4))
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.26

method	result
gosper	$\frac{4(dx+c)^{\frac{9}{4}}\left(128b^3x^3d^3+672d^3a\,x^2b^2-288b^3c\,d^2x^2+1428a^2b\,d^3x-1512a\,b^2c\,d^2x+468b^3c^2dx+1547a^3d^3-3213a^2bc\,d^2+2457a\,b^2c^2d-\right.}{13923(bx+a)^{\frac{21}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(25/4),x,method=_RETURNVERBOSE)
[Out] 4/13923*(d*x+c)^(9/4)*(128*b^3*d^3*x^3+672*a*b^2*d^3*x^2-288*b^3*c*d^2*x^2+
1428*a^2*b*d^3*x-1512*a*b^2*c*d^2*x+468*b^3*c^2*d*x+1547*a^3*d^3-3213*a^2*b
*c*d^2+2457*a*b^2*c^2*d-663*b^3*c^3)/(b*x+a)^(21/4)/(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(112) = 224$.
time = 1.06, size = 649, normalized size = 4.77

$$\frac{4\,{\left(128\,b^{3} d^{5} x^{5} - 663\,b^{3} c^{5} + 2457\,a b^{2} c^{4} d - 3213\,a^{2} b c^{3} d^{2} + 1547\,a^{3} c^{2} d^{3} - 32\,{\left(b^{3} c d^{4} - 21\,a b^{2} d^{5}\right)} x^{4} + 4\,{\left(5\,b^{3} c^{2} d^{3} - 42\,a b^{2} c d^{4} + 357\,a^{2} b d^{5}\right)} x^{3} - {\left(15\,b^{3} c^{3} d^{2} - 105\,a b^{2} c^{2} d^{3} + 357\,a^{2} b c d^{4} - 1547\,a^{3} d^{5}\right)} x^{2} - 2\,{\left(429\,b^{3} c^{4} d - 1701\,a b^{2} c^{3} d^{2} + 2499\,a^{2} b c^{2} d^{3} - 1547\,a^{3} c d^{4}\right)} x\right)} {\left(b x + a\right)}^{\frac{3}{4}} {\left(d x + c\right)}^{\frac{1}{4}}}{13923\,{\left(a^{6} b^{4} c^{4} - 4\,a^{7} b^{3} c^{3} d + 6\,a^{8} b^{2} c^{2} d^{2} - 4\,a^{9} b c d^{3} + a^{10} d^{4} + {\left(b^{10} c^{4} - 4\,a b^{9} c^{3} d + 6\,a^{2} b^{8} c^{2} d^{2} - 4\,a^{3} b^{7} c d^{3} + a^{4} b^{6} d^{4}\right)} x^{6} + 6\,{\left(a b^{9} c^{4} - 4\,a^{2} b^{8} c^{3} d + 6\,a^{3} b^{7} c^{2} d^{2} - 4\,a^{4} b^{6} c d^{3} + a^{5} b^{5} d^{4}\right)} x^{5} + 15\,{\left(a^{2} b^{8} c^{4} - 4\,a^{3} b^{7} c^{3} d + 6\,a^{4} b^{6} c^{2} d^{2} - 4\,a^{5} b^{5} c d^{3} + a^{6} b^{4} d^{4}\right)} x^{4} + 20\,{\left(a^{3} b^{7} c^{4} - 4\,a^{4} b^{6} c^{3} d + 6\,a^{5} b^{5} c^{2} d^{2} - 4\,a^{6} b^{4} c d^{3} + a^{7} b^{3} d^{4}\right)} x^{3} + 15\,{\left(a^{4} b^{6} c^{4} - 4\,a^{5} b^{5} c^{3} d + 6\,a^{6} b^{4} c^{2} d^{2} - 4\,a^{7} b^{3} c d^{3} + a^{8} b^{2} d^{4}\right)} x^{2} + 6\,{\left(a^{5} b^{5} c^{4} - 4\,a^{6} b^{4} c^{3} d + 6\,a^{7} b^{3} c^{2} d^{2} - 4\,a^{8} b^{2} c d^{3} + a^{9} b d^{4}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="fricas")
```

```
[Out] 4/13923*(128*b^3*d^5*x^5 - 663*b^3*c^5 + 2457*a*b^2*c^4*d - 3213*a^2*b*c^3*
d^2 + 1547*a^3*c^2*d^3 - 32*(b^3*c*d^4 - 21*a*b^2*d^5)*x^4 + 4*(5*b^3*c^2*d
^3 - 42*a*b^2*c*d^4 + 357*a^2*b*d^5)*x^3 - (15*b^3*c^3*d^2 - 105*a*b^2*c^2*
d^3 + 357*a^2*b*c*d^4 - 1547*a^3*d^5)*x^2 - 2*(429*b^3*c^4*d - 1701*a*b^2*c
^3*d^2 + 2499*a^2*b*c^2*d^3 - 1547*a^3*c*d^4)*x)*(b*x + a)^(3/4)*(d*x + c)^
(1/4)/(a^6*b^4*c^4 - 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 +
a^10*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*x^6 + 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4
*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a
^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a
^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*
(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*
b^2*d^4)*x^2 + 6*(a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8
*b^2*c*d^3 + a^9*b*d^4)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(25/4),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

Mupad [B]
time = 1.36, size = 376, normalized size = 2.76

$$\frac{(c+d\,x)^{1/4}\left(\frac{x^2\,\left(6188\,a^3\,d^5-1428\,a^2\,b\,c\,d^4+420\,a\,b^2\,c^2\,d^3-60\,b^3\,c^3\,d^2\right)}{13923\,b^5\,(a\,d-b\,c)^4}-\frac{-6188\,a^3\,c^2\,d^3+12852\,a^2\,b\,c^3\,d^2-9828\,a\,b^2\,c^4\,d+2652\,b^3\,c^5}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{x\,\left(12376\,a^3\,c\,d^4-19992\,a^2\,b\,c^2\,d^3+13608\,a\,b^2\,c^3\,d^2-3432\,b^3\,c^4\,d\right)}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{512\,d^5\,x^5}{13923\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^4\,x^4\,(21\,a\,d-b\,c)}{13923\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d^3\,x^3\,\left(357\,a^2\,d^2-42\,a\,b\,c\,d+5\,b^2\,c^2\right)}{13923\,b^4\,(a\,d-b\,c)^4}\right)}{x^5\,(a+b\,x)^{1/4}+\frac{a^5\,(a+b\,x)^{1/4}}{b^5}+\frac{10\,a^2\,x^3\,(a+b\,x)^{1/4}}{b^2}+\frac{10\,a^3\,x^2\,(a+b\,x)^{1/4}}{b^3}+\frac{5\,a\,x^4\,(a+b\,x)^{1/4}}{b}+\frac{5\,a^4\,x\,(a+b\,x)^{1/4}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(25/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((x^2*(6188*a^3*d^5 - 60*b^3*c^3*d^2 + 420*a*b^2*c^2*d^3 -
 1428*a^2*b*c*d^4))/(13923*b^5*(a*d - b*c)^4) - (2652*b^3*c^5 - 6188*a^3*c^
2*d^3 + 12852*a^2*b*c^3*d^2 - 9828*a*b^2*c^4*d)/(13923*b^5*(a*d - b*c)^4) +
 (x*(12376*a^3*c*d^4 - 3432*b^3*c^4*d + 13608*a*b^2*c^3*d^2 - 19992*a^2*b*c
^2*d^3))/(13923*b^5*(a*d - b*c)^4) + (512*d^5*x^5)/(13923*b^2*(a*d - b*c)^4
) + (128*d^4*x^4*(21*a*d - b*c))/(13923*b^3*(a*d - b*c)^4) + (16*d^3*x^3*(3
57*a^2*d^2 + 5*b^2*c^2 - 42*a*b*c*d))/(13923*b^4*(a*d - b*c)^4)))/(x^5*(a +
 b*x)^(1/4) + (a^5*(a + b*x)^(1/4))/b^5 + (10*a^2*x^3*(a + b*x)^(1/4))/b^2
+ (10*a^3*x^2*(a + b*x)^(1/4))/b^3 + (5*a*x^4*(a + b*x)^(1/4))/b + (5*a^4*x
*(a + b*x)^(1/4))/b^4)
```

3.1686 $\int (a+bx)^{5/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=408

$$-\frac{5(bc-ad)^3\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{84b^2d^2}+\frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d}+\frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2}+\frac{2(a+bx)}{}$$

```
[Out] -5/84*(-a*d+b*c)^3*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2/d^2+1/42*(-a*d+b*c)^2*(b
*x+a)^(5/4)*(d*x+c)^(1/4)/b^2/d+1/7*(-a*d+b*c)*(b*x+a)^(9/4)*(d*x+c)^(1/4)/
b^2+2/7*(b*x+a)^(9/4)*(d*x+c)^(5/4)/b+5/336*(-a*d+b*c)^(9/2)*((b*x+a)*(d*x+
c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a
*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/
4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+
a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1
/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b
*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a
*d+b*c))^2)^(1/2)/b^(9/4)/d^(9/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+
b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {52, 64, 637, 226}

$$\frac{5(bc-ad)^{9/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{168\sqrt{2}\,b^{9/4}d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{5\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^3}{84b^2d^2}+\frac{(a+bx)^{5/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{42b^2d}+\frac{(a+bx)^{9/4}\sqrt[4]{c+dx}\,(bc-ad)}{7b^2}+\frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]
```

```
[Out] (-5*(b*c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(84*b^2*d^2) + ((b*c - a
*d)^2*(a + b*x)^(5/4)*(c + d*x)^(1/4))/(42*b^2*d) + ((b*c - a*d)*(a + b*x)^
(9/4)*(c + d*x)^(1/4))/(7*b^2) + (2*(a + b*x)^(9/4)*(c + d*x)^(5/4))/(7*b)
+ (5*(b*c - a*d)^(9/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*
x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[
(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*
x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(168*Sqrt[2]*b^(9/4)*
d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b
*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int (a+bx)^{5/4}(c+dx)^{5/4}\,dx = \frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b} + \frac{(5(bc-ad))\int (a+bx)^{5/4}\sqrt[4]{c+dx}\,dx}{14b}$$
$$= \frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2} + \frac{2(a+bx)^{9/4}(c+dx)^{5/4}}{7b} + \frac{(bc-ad)^2\int \frac{(a+bx)^{5}}{(c+dx)^{3}}}{28b^2}$$
$$= \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-ad)(a+bx)^{9/4}\sqrt[4]{c+dx}}{7b^2} + \frac{2(a+bx)^{9/}}{7}$$
$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-ad}{}$$
$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-ad}{}$$
$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-ad}{}$$
$$= -\frac{5(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}}{84b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/4}\sqrt[4]{c+dx}}{42b^2d} + \frac{(bc-ad}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 73, normalized size = 0.18

$$\frac{4(a+bx)^{9/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{-bc+ad}\right)}{9b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, (d*(a
 + b*x))/(-(b*c) + a*d)])/(9*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(5/4)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/4)*(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="fricas")
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(1/4)*(d*x + c)^(1/4), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{\frac{5}{4}} (c + dx)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)*(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(5/4)*(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)*(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/4)*(d*x + c)^(5/4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/4}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)*(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(5/4)*(c + d*x)^(5/4), x)
```

3.1687 $\int \sqrt[4]{a+bx}\,(c+dx)^{5/4}\,dx$

Optimal. Leaf size=370

$$\frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d}+\frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2}+\frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b}-\frac{(bc-ad)^{7/2}((a+\cdots}{\cdots}$$

```
[Out] 1/6*(-a*d+b*c)^2*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2/d+1/3*(-a*d+b*c)*(b*x+a)^(
5/4)*(d*x+c)^(1/4)/b^2+2/5*(b*x+a)^(5/4)*(d*x+c)^(5/4)/b-1/24*(-a*d+b*c)^(7
/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))
^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b
*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))
*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c
)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*
(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/d^(5/4)/(b*x+a)^(3/4)/(d*x+c)^(
3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$-\frac{(bc-ad)^{7/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{12\sqrt{2}\,b^{9/4}d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^2}{6b^2d}+\frac{(a+bx)^{5/4}\sqrt[4]{c+dx}\,(bc-ad)}{3b^2}+\frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(6*b^2*d) + ((b*c - a*d)*(a
 + b*x)^(5/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(5/4)*(c + d*x)^(5/4)
)/(5*b) - ((b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(12*Sqrt[2]*b^(
9/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*
d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \sqrt[4]{a+bx}\,(c+dx)^{5/4}\,dx = \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} + \frac{(bc-ad)\int \sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c+dx)^{5/4}}{5b} + \frac{(bc-ad)^2 \int \frac{\sqrt[4]{a+b}}{(c+dx)^3}}{12b^2}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c}{5b}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c}{5b}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c}{5b}$$

$$= \frac{(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{6b^2d} + \frac{(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}{3b^2} + \frac{2(a+bx)^{5/4}(c}{5b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.20

$$\frac{4(a+bx)^{5/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 5/4, 9/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{4}} (dx+c)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/4)*(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{a+bx}\,(c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)*(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)*(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)*(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/4)*(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,x\right)}^{1/4}\,{\left(c+d\,x\right)}^{5/4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)*(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/4)*(c + d*x)^(5/4), x)
```

3.1688 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}}\,dx$

Optimal. Leaf size=332

$$\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2}+\frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b}+\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bd}}{}$$

```
[Out] 5/3*(-a*d+b*c)*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2+2/3*(b*x+a)^(1/4)*(d*x+c)^(5
/4)/b+5/12*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d
^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*Ell
ipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b
*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d
+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b
^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/d^(1/4)
/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2
)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{6\sqrt{2}\,b^{9/4}\sqrt[4]{d}\,(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{5\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{3b^2}+\frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) + (2*(a + b*x)^(1/4
)*(c + d*x)^(5/4))/(3*b) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4)
*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqr
t[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan
[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/
2])/(6*Sqrt[2]*b^(9/4)*d^(1/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}}\,dx = \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/4}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad)^2)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/}}}{12b^2}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad)^2((a+bx)(c+dx))}{12b^2(a+bx)^{3/}}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{\Big(5(bc-ad)^2((a+bx)(c+dx)}{}$$

$$= \frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} + \frac{2\sqrt[4]{a+bx}\,(c+dx)^{5/4}}{3b} + \frac{5(bc-ad)^{5/2}((a+bx)(c+dx)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 71, normalized size = 0.21

$$\frac{4\sqrt[4]{a+bx}\,(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(1/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 1/4, 5/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(3/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(3/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(3/4), x)
```

3.1689 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/4}}\,dx$

Optimal. Leaf size=325

$$\frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}+\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\,\Big(1+\frac{2\ldots}{}}{3\ldots}$$

[Out] 10/3*d*(b*x+a)^(1/4)*(d*x+c)^(1/4)/b^2-4/3*(d*x+c)^(5/4)/b/(b*x+a)^(3/4)+5/6*d^(3/4)*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 52, 64, 637, 226}

$$\frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)\middle|\frac{1}{2}\right)}{3\sqrt{2}\,b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2}-\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]

[Out] (10*d*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*b^2) - (4*(c + d*x)^(5/4))/(3*b*(a + b*x)^(3/4)) + (5*d^(3/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(3*Sqrt[2]*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/4}}\,dx = -\frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{3/4}}\,dx}{3b}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{(5d(bc-ad))\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{6b^2}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{(5d(bc-ad)((a+bx)(c+dx))^{3/4})\int \frac{}{(ac+(bc+}}{6b^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{\left(10d(bc-ad)((a+bx)(c+dx))^{3/4}\sqrt{(bc+}\right.}{3}$$

$$= \frac{10d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3b^2} - \frac{4(c+dx)^{5/4}}{3b(a+bx)^{3/4}} + \frac{5d^{3/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.22

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{3}{4};\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(7/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -3/4, 1/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(3*b*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(7/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(7/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(7/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(7/4),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(7/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(7/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(7/4), x)
```

3.1690 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{11/4}}\,dx$

Optimal. Leaf size=325

$$-\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{5\sqrt{2}\,d^{7/4}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\cdots\right.}{21b^{9}\cdots}$$

```
[Out] -20/21*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(3/4)-4/7*(d*x+c)^(5/4)/b/(b*x+a)^(7/4)+
5/21*d^(7/4)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)
*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^
(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*ar
ctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2
*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(
1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c
)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/
4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2
)
```

Rubi [A]
time = 0.21, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 64, 637, 226}

$$\frac{5\sqrt{2}\,d^{7/4}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{21b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]
```

```
[Out] (-20*d*(c + d*x)^(1/4))/(21*b^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(7*b
*(a + b*x)^(7/4)) + (5*Sqrt[2]*d^(7/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x)
)^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2
*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*
d]], 1/2])/(21*b^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{11/4}}\,dx = -\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{7/4}}\,dx}{7b}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{(5d^2)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{21b^2}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{\left(5d^2((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{21b^2(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{\left(20d^2((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\cdots}{21b^2(a+bx\cdots}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{21b^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{7b(a+bx)^{7/4}}+\frac{5\sqrt{2}\,d^{7/4}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+a\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.22

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{7}{4},-\frac{5}{4};-\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(11/4),x]
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-7/4, -5/4, -3/4, (d*(a + b*x))/(-(b*
c) + a*d)])/(7*b*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
[Out] int((d*x+c)^(5/4)/(b*x+a)^(11/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(11/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(11/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(11/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(11/4),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(11/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{11/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(11/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(11/4), x)
```

3.1691 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{15/4}}\,dx$

Optimal. Leaf size=363

$$-\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}}-\frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}}-\frac{10\sqrt{2}\,d^{11/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+a}}{}$$

```
[Out] -20/77*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(7/4)-20/231*d^2*(d*x+c)^(1/4)/b^2/(-a*d
+b*c)/(b*x+a)^(3/4)-4/11*(d*x+c)^(5/4)/b/(b*x+a)^(11/4)-10/231*d^(11/4)*((b
*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*
2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(
d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(
1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2
)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*
c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)
*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(9/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2
*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 53, 64, 637, 226}

$$-\frac{10\sqrt{2}\,d^{11/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{20d^2\sqrt[4]{c+dx}}{231b^2(a+bx)^{3/4}(bc-ad)}-\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}}-\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]
```

```
[Out] (-20*d*(c + d*x)^(1/4))/(77*b^2*(a + b*x)^(7/4)) - (20*d^2*(c + d*x)^(1/4))
/(231*b^2*(b*c - a*d)*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(11*b*(a + b*x
)^(11/4)) - (10*Sqrt[2]*d^(11/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*
d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a
*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sq
rt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/
4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4
)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqr
t[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{15/4}}\,dx = -\frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{11/4}}\,dx}{11b}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} + \frac{(5d^2)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{77b^2}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{(10d^3)\int \frac{1}{(a+bx)^{3/4}(}}{231b^2(bc-}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{(10d^3((a+bx)(c}{231b^2(bc-}$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{\left(40d^3((a+bx)(c\right.}{}$$

$$10\sqrt{2}\,d^{11/4}((a+$$

$$= -\frac{20d\sqrt[4]{c+dx}}{77b^2(a+bx)^{7/4}} - \frac{20d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{3/4}} - \frac{4(c+dx)^{5/4}}{11b(a+bx)^{11/4}} - \frac{}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.20

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{11}{4},-\frac{5}{4};-\frac{7}{4};\frac{d(a+bx)}{-bc+ad}\right)}{11b(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(15/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-11/4, -5/4, -7/4, (d*(a + b*x))/(-(b
*c) + a*d)])/(11*b*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(15/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(15/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2
*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(15/4),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3277 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(15/4),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(15/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(c+d\,x)}^{5/4}}{{(a+b\,x)}^{15/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(15/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(15/4), x)
```

3.1692 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{19/4}}\,dx$

Optimal. Leaf size=401

$$-\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}+\frac{4\sqrt{2}\,d^{15/4}((\cdots}{\cdots}$$

```
[Out] -4/33*d*(d*x+c)^(1/4)/b^2/(b*x+a)^(11/4)-4/231*d^2*(d*x+c)^(1/4)/b^2/(-a*d+
b*c)/(b*x+a)^(7/4)+8/231*d^3*(d*x+c)^(1/4)/b^2/(-a*d+b*c)^2/(b*x+a)^(3/4)-4
/15*(d*x+c)^(5/4)/b/(b*x+a)^(15/4)+4/231*d^(15/4)*((b*x+a)*(d*x+c))^(3/4)*(
cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/
2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/
(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))
^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)
*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2
*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+
b*c))^2)^(1/2)/b^(9/4)/(-a*d+b*c)^(3/2)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*
x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {49, 53, 64, 637, 226}

$$\frac{4\sqrt{2}\,d^{15/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{231b^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{8d^3\sqrt[4]{c+dx}}{231b^2(a+bx)^{3/4}(bc-ad)^2}-\frac{4d^2\sqrt[4]{c+dx}}{231b^2(a+bx)^{7/4}(bc-ad)}-\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}}-\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(33*b^2*(a + b*x)^(11/4)) - (4*d^2*(c + d*x)^(1/4))/
(231*b^2*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d^3*(c + d*x)^(1/4))/(231*b^2*(b
*c - a*d)^2*(a + b*x)^(3/4)) - (4*(c + d*x)^(5/4))/(15*b*(a + b*x)^(15/4))
+ (4*Sqrt[2]*d^(15/4)*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x
)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(
a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x
)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*
((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(231*b^(9/4)*(b*c - a*
d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d +
b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{19/4}}\,dx = -\frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}} + \frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{15/4}}\,dx}{3b}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}} + \frac{d^2\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx}{33b^2}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} - \frac{4(c+dx)^{5/4}}{15b(a+bx)^{15/4}} - \frac{(2d^3)\int \frac{1}{(a+bx)^{7/4}(c}}{77b^2(bc-}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c}{15b(a}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c}{15b(a}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c}{15b(a}$$

$$= -\frac{4d\sqrt[4]{c+dx}}{33b^2(a+bx)^{11/4}} - \frac{4d^2\sqrt[4]{c+dx}}{231b^2(bc-ad)(a+bx)^{7/4}} + \frac{8d^3\sqrt[4]{c+dx}}{231b^2(bc-ad)^2(a+bx)^{3/4}} - \frac{4(c}{15b(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.18

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{15}{4},-\frac{5}{4};-\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{15b(a+bx)^{15/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(19/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-15/4, -5/4, -11/4, (d*(a + b*x))/(-(
b*c) + a*d)])/(15*b*(a + b*x)^(15/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(19/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(19/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(5/4)/(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^
3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(19/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7141 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(19/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(19/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{19/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(19/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(19/4), x)
```

3.1693 $\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$-\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2}{}$$

```
[Out] -5/8*(-a*d+b*c)*(b*x+a)^(1/4)*(d*x+c)^(3/4)/d^2+1/2*(b*x+a)^(5/4)*(d*x+c)^(
3/4)/d+5/16*(-a*d+b*c)^2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4)
)/b^(3/4)/d^(9/4)+5/16*(-a*d+b*c)^2*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(
d*x+c)^(1/4))/b^(3/4)/d^(9/4)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 246, 218, 214, 211}

$$\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}\,(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(3/4))/(8*d^2) + ((a + b*x)^(5/4)
*(c + d*x)^(3/4))/(2*d) + (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))
/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTan
h[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx = \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{8d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c}}}{32d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{\sqrt[4]{c}}\right.}{}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{1-d}\right.}{8bd^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{(5(bc-ad)^2)\,\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{16\sqrt{}}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a}}{\sqrt[4]{b}\,\sqrt[4]{c}}\right.}{16b^{3/4}d^{9/4}}$$

Mathematica [A]
time = 0.40, size = 143, normalized size = 0.86

$$\frac{2\sqrt[4]{d}\,\sqrt[4]{a+bx}\,(c+dx)^{3/4}(-5bc+9ad+4bdx) + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}} + \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{b^{3/4}}}{16d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
[Out] (2*d^(1/4)*(a + b*x)^(1/4)*(c + d*x)^(3/4)*(-5*b*c + 9*a*d + 4*b*d*x) + (5*
(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
b^(3/4) + (5*(b*c - a*d)^2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c +
d*x)^(1/4))])/b^(3/4))/(16*d^(9/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. $2(127) = 254$.
time = 0.78, size = 1468, normalized size = 8.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] -1/32*(20*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*arctan(-((b^4*c^2*d^7 - 2*a*b^3*c*d^
8 + a^2*b^2*d^9)*(b*x + a)^(1/4)*(d*x + c)^(3/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4) -
 (b^2*d^8*x + b^2*c*d^7)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^5*x + b^2*
c*d^4)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^3*d^9)))/(d*x + c))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4))/(b^8*c^
9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^
5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d
^8 + (b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4
 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c
*d^8 + a^8*d^9)*x)) - 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^3*x + b*c*d^2)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
```

```
^8)/(b^3*d^9))^(1/4))/(d*x + c)) + 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) - (b*d^3*x +
b*c*d^2)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4))/(d*x + c)) - 4*(4*b*d*x - 5*b*c + 9*a*d)
*(b*x + a)^(1/4)*(d*x + c)^(3/4))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(5/4)/(c + d*x)^(1/4), x)
```

3.1694 $\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}$$

```
[Out] (b*x+a)^(1/4)*(d*x+c)^(3/4)/d-1/2*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b
^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(5/4)-1/2*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a
)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(5/4)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 246, 218, 214, 211}

$$-\frac{(bc-ad)\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((a + b*x)^(1/4)*(c + d*x)^(3/4))/d - ((b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x
)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/4)) - ((b*c - a*d)*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/
4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\mathrm{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2\sqrt{b}\,d} - \frac{(bc-ad)\mathrm{Subs}}{}$$

$$= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+}}{\sqrt[4]{b}\,\sqrt[4]{c+}}\right.}{2b^{3/4}d^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.03, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{5/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(95) = 190.
time = 0.93, size = 814, normalized size = 6.41

$$-\frac{4\,d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{(b^3cd^4-ab^2d^5)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{3}{4}}+(b^2d^5x+b^2cd^4)\sqrt{\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bx+a}\,\sqrt{dx+c}+(b^2d^3x+b^2cd^2)\sqrt{\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}}}{dx+c}}\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{3}{4}}}{b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4+(b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5)x}\right)+d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}\log\left(-\frac{(bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}+(bd^2x+bcd)\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}}{dx+c}\right)-d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}\log\left(-\frac{(bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}-(bd^2x+bcd)\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{b^3d^5}\right)^{\frac{1}{4}}}{dx+c}\right)-4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b^3*d^5))^(1/4)*arctan(((b^3*c*d^4 - a*b^2*d^5)*(b*x + a)^(1/4)*(d
*x + c)^(3/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(b^3*d^5))^(3/4) + (b^2*d^5*x + b^2*c*d^4)*sqrt(((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^3*x + b^2*c*d^2)*sqr
t((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
b^3*d^5)))/(d*x + c))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(3/4))/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*
c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4 + (b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*x)) + d*((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c
- a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/
(d*x + c)) - d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(
3/4) - (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/(d*x + c)) - 4*(b*x + a)^(1/4)*(d*
x + c)^(3/4))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(1/4),x)
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{1/4}}{{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(1/4)/(c + d*x)^(1/4), x)
```

3.1695 $\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

```
[Out] 2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(1/4)+2*arc
tanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(3/4)/d^(1/4)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {65, 246, 218, 214, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx &= \frac{4\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{b} \\
&= \frac{4\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}} \\
&= \frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.86

$$\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
[Out] (2*(ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))] + ArcTanh[(
d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))]))/(b^(3/4)*d^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}} (dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(61) = 122$.
time = 0.63, size = 234, normalized size = 2.75

$$-4 \left(\frac{1}{b^3 d}\right)^{\frac{1}{4}} \arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}} b^2 d \left(\frac{1}{b^3 d}\right)^{\frac{3}{4}} - (b^2 d^2 x + b^2 c d)\sqrt{\frac{(b^2 dx + b^2 c)\sqrt{\frac{1}{b^3 d}} + \sqrt{bx+a}\,\sqrt{dx+c}}{dx+c}}\left(\frac{1}{b^3 d}\right)^{\frac{3}{4}}}{dx+c}\right) + \left(\frac{1}{b^3 d}\right)^{\frac{1}{4}} \log\left(\frac{(bdx+bc)\left(\frac{1}{b^3 d}\right)^{\frac{1}{4}} + (bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right) - \left(\frac{1}{b^3 d}\right)^{\frac{1}{4}} \log\left(-\frac{(bdx+bc)\left(\frac{1}{b^3 d}\right)^{\frac{1}{4}} - (bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b^3*d))^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*b^2*d*(1/(b^3
*d))^(3/4) - (b^2*d^2*x + b^2*c*d)*sqrt(((b^2*d*x + b^2*c)*sqrt(1/(b^3*d))
+ sqrt(b*x + a)*sqrt(d*x + c))/(d*x + c))*(1/(b^3*d))^(3/4))/(d*x + c)) + (
1/(b^3*d))^(1/4)*log(((b*d*x + b*c)*(1/(b^3*d))^(1/4) + (b*x + a)^(1/4)*(d*
x + c)^(3/4))/(d*x + c)) - (1/(b^3*d))^(1/4)*log(-((b*d*x + b*c)*(1/(b^3*d)
)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}} \sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/4}\,{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)), x)
```

3.1696 $\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

```
[Out] -4/3*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Maple [A]
time = 0.19, size = 27, normalized size = 0.84

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}}{3(bx+a)^{\frac{3}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/3/(b*x+a)^(3/4)*(d*x+c)^(3/4)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

Fricas [A]
time = 0.74, size = 42, normalized size = 1.31

$$-\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\,(abc-a^2d+(b^2c-abd)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] -4/3*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{7}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)), x)
```

3.1697 $\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=66

$$-\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}}+\frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}$$

```
[Out] -4/7*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(7/4)+16/21*d*(d*x+c)^(3/4)/(-a*d+b*c
)^2/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (16*d*(c + d*x)^(3/4
))/(21*(b*c - a*d)^2*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{7(bc-ad)}$$
$$= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}} + \frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}$$

Mathematica [A]
time = 0.12, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{3/4}(-3bc+7ad+4bdx)}{21(bc-ad)^2(a+bx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
[Out] (4*(c + d*x)^(3/4)*(-3*b*c + 7*a*d + 4*b*d*x))/(21*(b*c - a*d)^2*(a + b*x)^
(7/4))
```

Maple [A]
time = 0.22, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}(4bdx+7ad-3bc)}{21(bx+a)^{\frac{7}{4}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/21*(d*x+c)^(3/4)*(4*b*d*x+7*a*d-3*b*c)/(b*x+a)^(7/4)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.79, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx - 3\,bc + 7\,ad)(bx + a)^{\frac{1}{4}}(dx + c)^{\frac{3}{4}}}{21\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + (b^4c^2 - 2\,ab^3cd + a^2b^2d^2)x^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(4*b*d*x - 3*b*c + 7*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{11}{4}}\sqrt[4]{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/4}\,{\left(c+d\,x\right)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)), x)
```

3.1698 $\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}+\frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}}-\frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+bx)^{3/4}}$$

```
[Out] -4/11*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(11/4)+32/77*d*(d*x+c)^(3/4)/(-a*d+b
*c)^2/(b*x+a)^(7/4)-128/231*d^2*(d*x+c)^(3/4)/(-a*d+b*c)^3/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(11*(b*c - a*d)*(a + b*x)^(11/4)) + (32*d*(c + d*x)^(3
/4))/(77*(b*c - a*d)^2*(a + b*x)^(7/4)) - (128*d^2*(c + d*x)^(3/4))/(231*(b
*c - a*d)^3*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} - \frac{(8d)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{11(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} + \frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+}}}{77(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}} + \frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}} - \frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+bx)}$$

Mathematica [A]
time = 0.15, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{11/4}\left(21b^2+\frac{77d^2(a+bx)^2}{(c+dx)^2}-\frac{66bd(a+bx)}{c+dx}\right)}{231(bc-ad)^3(a+bx)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(21*b^2 + (77*d^2*(a + b*x)^2)/(c + d*x)^2 - (66*b*d*(
a + b*x))/(c + d*x)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

Maple [A]
time = 0.21, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{4(dx+c)^{\frac{3}{4}}\left(32b^2x^2d^2+88ab\,d^2x-24b^2cdx+77a^2d^2-66abcd+21b^2c^2\right)}{231(bx+a)^{\frac{11}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/231*(d*x+c)^(3/4)*(32*b^2*d^2*x^2+88*a*b*d^2*x-24*b^2*c*d*x+77*a^2*d^2-66
*a*b*c*d+21*b^2*c^2)/(b*x+a)^(11/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(83) = 166$.
time = 1.69, size = 252, normalized size = 2.50

$$-\frac{4\,(32\,b^2d^2x^2+21\,b^2c^2-66\,abcd+77\,a^2d^2-8\,(3\,b^2cd-11\,abd^2)x)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{231\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/231*(32*b^2*d^2*x^2 + 21*b^2*c^2 - 66*a*b*c*d + 77*a^2*d^2 - 8*(3*b^2*c*
d - 11*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^3*c^3 - 3*a^4*b^2
*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d
^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5
*b*d^3)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(1/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(a+b\,x)}^{15/4}\,{(c+d\,x)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)), x)
```

3.1699 $\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=136

$$-\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}}+\frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}}-\frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}}+\frac{512d^3(c+dx)^{3/4}}{1155(bc-ad)^4(a+bx)^{3/4}}$$

```
[Out] -4/15*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(15/4)+16/55*d*(d*x+c)^(3/4)/(-a*d+b
*c)^2/(b*x+a)^(11/4)-128/385*d^2*(d*x+c)^(3/4)/(-a*d+b*c)^3/(b*x+a)^(7/4)+5
12/1155*d^3*(d*x+c)^(3/4)/(-a*d+b*c)^4/(b*x+a)^(3/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(15*(b*c - a*d)*(a + b*x)^(15/4)) + (16*d*(c + d*x)^(3
/4))/(55*(b*c - a*d)^2*(a + b*x)^(11/4)) - (128*d^2*(c + d*x)^(3/4))/(385*(
b*c - a*d)^3*(a + b*x)^(7/4)) + (512*d^3*(c + d*x)^(3/4))/(1155*(b*c - a*d)
^4*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$
$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c}}}{55(bc-ad)}$$
$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/}}{385(bc-ad)^3(a+b}$$
$$= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}} + \frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}} - \frac{128d^2(c+dx)^{3/}}{385(bc-ad)^3(a+b}$$

Mathematica [A]
time = 0.19, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{15/4}\left(77b^3 - \frac{385d^3(a+bx)^3}{(c+dx)^3} + \frac{495bd^2(a+bx)^2}{(c+dx)^2} - \frac{315b^2d(a+bx)}{c+dx}\right)}{1155(bc-ad)^4(a+bx)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
[Out] (-4*(c + d*x)^(15/4)*(77*b^3 - (385*d^3*(a + b*x)^3)/(c + d*x)^3 + (495*b*d
^2*(a + b*x)^2)/(c + d*x)^2 - (315*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
- a*d)^4*(a + b*x)^(15/4))
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$\frac{4(dx+c)^{\frac{3}{4}}\left(128b^3x^3d^3+480d^3a\,x^2b^2-96b^3c\,d^2x^2+660a^2b\,d^3x-360a\,b^2c\,d^2x+84b^3c^2dx+385a^3d^3-495a^2bc\,d^2+315a\,b^2c^2d-77b^3c^3\right)}{1155(bx+a)^{\frac{15}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/1155*(d*x+c)^(3/4)*(128*b^3*d^3*x^3+480*a*b^2*d^3*x^2-96*b^3*c*d^2*x^2+66
0*a^2*b*d^3*x-360*a*b^2*c*d^2*x+84*b^3*c^2*d*x+385*a^3*d^3-495*a^2*b*c*d^2+
315*a*b^2*c^2*d-77*b^3*c^3)/(b*x+a)^(15/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(112) = 224$.
time = 3.05, size = 419, normalized size = 3.08

$$\frac{4\,(128\,b^3d^3x^3 - 77\,b^3c^3 + 315\,ab^2c^2d - 495\,a^2bcd^2 + 385\,a^3d^3 - 96\,(b^3cd^2 - 5\,ab^2d^3)x^2 + 12\,(7\,b^3c^2d - 30\,ab^2cd^2 + 55\,a^2bd^3)x)(bx + a)^{\frac{1}{4}}(dx + c)^{\frac{3}{4}}}{1155\,(a^4b^4c^4 - 4\,a^5b^3c^3d + 6\,a^6b^2c^2d^2 - 4\,a^7bcd^3 + a^8d^4 + (b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4b^4d^4)x^4 + 4\,(ab^7c^4 - 4\,a^2b^6c^3d + 6\,a^3b^5c^2d^2 - 4\,a^4b^4cd^3 + a^5b^3d^4)x^3 + 6\,(a^2b^6c^4 - 4\,a^3b^5c^3d + 6\,a^4b^4c^2d^2 - 4\,a^5b^3cd^3 + a^6b^2d^4)x^2 + 4\,(a^3b^5c^4 - 4\,a^4b^4c^3d + 6\,a^5b^3c^2d^2 - 4\,a^6b^2cd^3 + a^7bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(128*b^3*d^3*x^3 - 77*b^3*c^3 + 315*a*b^2*c^2*d - 495*a^2*b*c*d^2 +
385*a^3*d^3 - 96*(b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 12*(7*b^3*c^2*d - 30*a*b^2
*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(19/4)/(d*x+c)**(1/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7772 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{19/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)), x)
```

3.1700 $\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=751

$$-\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2}+\frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d}+\frac{7(bc-ad)\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad}}{10\sqrt{b}\,d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)}$$

```
[Out] -7/15*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^(3/4)/d^2+2/5*(b*x+a)^(7/4)*(d*x+c)^
(3/4)/d+7/10*(-a*d+b*c)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)
*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(5/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a
*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))-7/
20*(-a*d+b*c)^(7/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*(
(b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(s
in(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2
))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*(
(2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d
^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(11/4)/(b*x+a
)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
+7/40*(-a*d+b*c)^(7/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4
)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(
b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*Elliptic
F(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(
1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c)
)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2
)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(11/4)/(b*
x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1
/2)
```

Rubi [A]

time = 0.54, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 64, 637, 311, 226, 1210}

$$\frac{7(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{20\sqrt{2}\,b^{3/4}d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{7(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{10\sqrt{2}\,b^{3/4}d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{7(bc-ad)\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{10\sqrt{b}\,d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{7(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)}{15d^2}+\frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(15*d^2) + (2*(a + b*x)^(7
/4)*(c + d*x)^(3/4))/(5*d) + (7*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[
```

```
(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*Sqrt[b]*d^(5/2)
*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (7*(b*c - a*d)^(7/2)*((a + b
*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c
- a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)
]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/
Sqrt[b*c - a*d]], 1/2])/(10*Sqrt[2]*b^(3/4)*d^(11/4)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (7*(b*c -
a*d)^(7/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (
2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c
+ 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*
(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*b^(3/4)*d^(11/4)*(a +
 b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x)
)^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx = \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{10d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{(7(bc-ad)^2)\int \frac{}{\sqrt[4]{a+bx}}}{20d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\sqrt[4]{(a+bx)}\right.}{20}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^2\sqrt[4]{(a+bx)}\right.}{}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{\left(7(bc-ad)^3\sqrt[4]{(a+bx)}\right.}{}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^2} + \frac{2(a+bx)^{7/4}(c+dx)^{3/4}}{5d} + \frac{7(bc-ad)\sqrt{(a+b}}{10\sqrt{b}\,d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{11/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{-bc+ad}\right)}{11b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(1/4),x]
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
[Out] int((b*x+a)^(7/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(7/4)/(c + d*x)^(1/4), x)
```

3.1701 $\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=705

$$\frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d}-\frac{\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}-$$

```
[Out] 2/3*(b*x+a)^(3/4)*(d*x+c)^(3/4)/d-((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c
)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(3/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/
(2*b*d*x+a*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*
d+b*c))+1/2*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*El
lipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+
b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*
d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*
b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(7/4
)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^
2)^(1/2)-1/4*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)
*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*E
llipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d
+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a
*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2
*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(7/
4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))
^2)^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 705, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 64, 637, 311, 226, 1210}

$$-\frac{(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\,b^{3/4}d^{7/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,b^{3/4}d^{7/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]
```

```
[Out] (2*(a + b*x)^(3/4)*(c + d*x)^(3/4))/(3*d) - (Sqrt[(a + b*x)*(c + d*x)]*Sqrt
[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(Sqrt[b]*d^(3/2)*(
```

```
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d
]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + ((b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4
)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - ((b*c - a*d)^(5/2)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(2*Sqrt[2]*b^(3/4)*d^(7/4)*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
- 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx = \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{2d}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left((bc-ad)\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}}{2d\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left(2(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{d\sqrt[4]{a+bx}\sqrt[4]{c+}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\left((bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{}}$$

$$= \frac{2(a+bx)^{3/4}(c+dx)^{3/4}}{3d} - \frac{\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b}}{\sqrt{b}\,d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}}{}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{7/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{\sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(1/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(1/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(1/4),x)
[Out] int((a + b*x)^(3/4)/(c + d*x)^(1/4), x)
```

3.1702 $\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=688

$$\frac{2\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,\sqrt{d}\,(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)} - \frac{\sqrt{2}\,(bc-ad)^{3/2}\cdots}{\cdots}$$

```
[Out] 2*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)
^(1/2)/(-a*d+b*c)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/b^(1/2)/d^(
1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))+1/2*(-a*d+b*c
)^(3/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x
+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)
*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(
b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1
/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d
+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x
+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(3/4)/(b*x+a)^(1/4)/(d*x+
c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)-(-a*d+b*c)^(
3/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c)
)^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2)
)*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*
x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*
((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/d^(3/4)/(b*x+a)^(1/4)/
(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 688, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {64, 637, 311, 226, 1210}

$$\frac{(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,b^{3/4}d^{3/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}} - \frac{\sqrt{2}\,(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{b^{3/4}d^{3/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}} + \frac{2\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\sqrt{d}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c
 + 2*d*x))^2])/(Sqrt[b]*Sqrt[d]*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)
*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
```

```
b*c - a*d))) - (Sqrt[2]*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[
(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*S
qrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt
[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b
^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(
a*d + b*(c + 2*d*x))^2]) + ((b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*S
qrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[
b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(
Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2]
)/(Sqrt[2]*b^(3/4)*d^(3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b
*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx = \frac{\sqrt[4]{(a+bx)(c+dx)}\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

$$= \frac{\left(4\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{-4abcd+(bc+ad)^2+}}\right.}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)}$$

$$= \frac{\left(2(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd+(bc}}\right.}{\sqrt{b}\,\sqrt{d}\,\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2}$$

$$= \frac{2\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,\sqrt{d}\,(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(}}{bc-ad}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 73, normalized size = 0.11

$$\frac{4(a+bx)^{3/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x]
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}\,(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(1/4)), x)
```

3.1703 $\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=718

$$-\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}}+\frac{4\sqrt{d}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\,(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right.}$$

```
[Out] -4*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(1/4)+4*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)
*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^2/(b*x+
a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x
+a)*(d*x+c))^(1/2)/(-a*d+b*c))-2*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arc
tan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c
)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^
(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)
*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((
a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2
)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c
)/((a*d+b*(2*d*x+c))^2)^(1/2)+d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan
(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(
1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/
2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^
(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d
+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(
-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(
(a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$\frac{\sqrt{2}\sqrt[4]{d}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{2\sqrt{2}\sqrt[4]{d}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4(c+dx)^{3/4}}{\sqrt[4]{a+bx}(bc-ad)}+\frac{4\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{b}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^2(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/((b*c - a*d)*(a + b*x)^(1/4)) + (4*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

```
/(Sqrt[b]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*
x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (2*Sq
rt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d)
)*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[
(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*
d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(3/4)*(a +
b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))
^2]) + (Sqrt[2]*d^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b
*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(
b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2
]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(
3/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
 + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
- 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
<= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{(2d)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(2d\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}}{(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(8d\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}}{(bc-ad)\sqrt[4]{a+b}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{\left(4\sqrt{d}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\right)\text{Sub}}{\sqrt{b}\sqrt[4]{a+bx}}$$

$$= -\frac{4(c+dx)^{3/4}}{(bc-ad)\sqrt[4]{a+bx}} + \frac{4\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx}}{\sqrt{b}(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.10

$$-\frac{4\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{a+bx}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/4)*(c + d*x)^(1/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}} (dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] int(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}} \sqrt[4]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(1/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(1/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(5/4)*(c + d*x)^(1/4)), x)
```

3.1704 $\int \frac{1}{(a+bx)^{9/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=760

$$-\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}}+\frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}}-\frac{8d^{3/2}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(}}{5\sqrt{b}(bc-ad)^3\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2}{}\right.}$$

```
[Out] -4/5*(d*x+c)^(3/4)/(-a*d+b*c)/(b*x+a)^(5/4)+8/5*d*(d*x+c)^(3/4)/(-a*d+b*c)^
2/(b*x+a)^(1/4)-8/5*d^(3/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(
1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(
2*b*d*x+a*d+b*c)/b^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d
+b*c))+4/5*d^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(si
n(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)
)),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+
b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^
(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)^
(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2
)^(1/2)-2/5*d^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*(
(b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(
1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(s
in(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2
))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d
+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b
^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(3/4)/(b*x+a)
^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^
2)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$-\frac{2\sqrt{2}d^{5/4}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{4\sqrt{2}d^{5/4}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}\sqrt{bc-ad}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{8d^{3/2}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{5\sqrt{b}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^3(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{8d(c+dx)^{3/4}}{5\sqrt[4]{a+bx}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (8*d*(c + d*x)^(3/4)
)/(5*(b*c - a*d)^2*(a + b*x)^(1/4)) - (8*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*
```

```
Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*Sqrt[b]*(b*
c - a*d)^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (4*Sqrt[2]*d^(5/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a + b*x)^(1/4)*
(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (2*S
qrt[2]*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b
*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b
*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*b^(3/4)*Sqrt[b*c - a*d]*(a
 + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} - \frac{(2d)\int \frac{1}{(a+bx)^{5/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{(4d^2)\int \frac{1}{\sqrt[4]{a+bx}\sqrt[4]{c+dx}}\,d}{5(bc-ad)^2}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(4d^2\sqrt[4]{(a+bx)(c+dx)}\right)\int}{5(bc-ad)^2\sqrt[4]{}}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(16d^2\sqrt[4]{(a+bx)(c+dx)}\sqrt{}\right.}{}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{\left(8d^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{}\right.}{}$$

$$= -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2\sqrt[4]{a+bx}} - \frac{8d^{3/2}\sqrt{(a+bx)(c}}{5\sqrt{b}\,(bc-ad)^3\sqrt[4]{a+bx}\,\sqrt[4]{c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.04, size = 73, normalized size = 0.10

$$-\frac{4\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};-\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/4}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x]
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/4)*(c + d*x)^(1/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
[Out] int(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(1/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(1/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{9/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(9/4)*(c + d*x)^(1/4)), x)
```

3.1705 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}} \, dx$

Optimal. Leaf size=167

$$-\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad}{}$$

```
[Out] -7/8*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^(1/4)/d^2+1/2*(b*x+a)^(7/4)*(d*x+c)^(
1/4)/d-21/16*(-a*d+b*c)^2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4
))/b^(1/4)/d^(11/4)+21/16*(-a*d+b*c)^2*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4
)/(d*x+c)^(1/4))/b^(1/4)/d^(11/4)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$-\frac{21(bc-ad)^2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8d^2}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*d^2) + ((a + b*x)^(7/4)
*(c + d*x)^(1/4))/(2*d) - (21*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4)
)/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4)) + (21*(b*c - a*d)^2*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(1
1/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\int \frac{1}{\sqrt[4]{a+bx}\,(}}{32d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\,\mathrm{Subst}\Big(\int \frac{}{(c}}{8}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\,\mathrm{Subst}\Big(\int \frac{}{1-}}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{(21(bc-ad)^2)\,\mathrm{Subst}\Big(\int \frac{}{\sqrt{}}}{16d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{}}{\sqrt[4]{b}\,\sqrt[4]{}}\right.}{16\sqrt[4]{b}\,d^{11/4}}$$

Mathematica [A]
time = 0.28, size = 143, normalized size = 0.86

$$\frac{2d^{3/4}(a+bx)^{3/4}\sqrt[4]{c+dx}\,(-7bc+11ad+4bdx) + \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}} + \frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}}}{16d^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (2*d^(3/4)*(a + b*x)^(3/4)*(c + d*x)^(1/4)*(-7*b*c + 11*a*d + 4*b*d*x) + (2
1*(b*c - a*d)^2*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]
)/b^(1/4) + (21*(b*c - a*d)^2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a
 + b*x)^(1/4))])/b^(1/4))/(16*d^(11/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. $2(127) = 254$.
time = 1.01, size = 1457, normalized size = 8.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/32*(84*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*arctan(-((b^3*c^2*d^8 - 2*a*b^2*c*d^9
 + a^2*b*d^10)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4) - (b
^2*d^8*x + a*b*d^8)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^6*x + a*d^6)*sqrt
((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(b*d^11)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4))/(a*b^8*c^8 - 8*a^2*b
^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 5
6*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8 + (b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8)*x)) - 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b*d^3*x + a*d^3)*((b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^
```

```
(1/4))/(b*x + a)) + 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^3*x + a*d^3)*((b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(b*d^11))^(1/4))/(b*x + a)) - 4*(4*b*d*x - 7*b*c + 11*a*d)*(b*x + a)^(3/4)
*(d*x + c)^(1/4))/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(7/4)/(c + d*x)^(3/4), x)
```

3.1706 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=127

$$\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}+\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}$$

```
[Out] (b*x+a)^(3/4)*(d*x+c)^(1/4)/d+3/2*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b
^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(7/4)-3/2*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a
)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(7/4)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {52, 65, 338, 304, 211, 214}

$$\frac{3(bc-ad)\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((a + b*x)^(3/4)*(c + d*x)^(1/4))/d + (3*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b
*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)
*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d
^(7/4))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4d}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^{3/2}} + \frac{(3(bc-}{}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} + \frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}} - \frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{b}}\right.}{2\sqrt[4]{b}\,d^{7/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.03, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. 2(95) = 190.
time = 0.95, size = 808, normalized size = 6.36

$$-\frac{12\,d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{1}{4}}\arctan\left(\frac{(b^2cd^5-abd^6)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{3}{4}}+(b^2d^5x+abd^5)\sqrt{\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{bx+a}\,\sqrt{dx+c}+(bd^4x+ad^4)\sqrt{\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}}}{bx+a}}\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{3}{4}}}{ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4+(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)x}\right)+3\,d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}+(bd^2x+ad^2)\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-3\,d\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left((bc-ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}-(bd^2x+ad^2)\left(\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{bd^7}\right)^{\frac{1}{4}}\right)}{bx+a}\right)-4\,(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/4*(12*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(1/4)*arctan(((b^2*c*d^5 - a*b*d^6)*(b*x + a)^(3/4)*(d*x
+ c)^(1/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
```

```
a^4*d^4)/(b*d^7))^(3/4) + (b^2*d^5*x + a*b*d^5)*sqrt(((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^4*x + a*d^4)*sqrt((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7)))/(b*x
 + a))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)/(b*d^7))^(3/4))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a
^4*b*c*d^3 + a^5*d^4 + (b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4)*x)) + 3*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a
)^(3/4)*(d*x + c)^(1/4) + (b*d^2*x + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 3*d*
((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b
*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^2*x
 + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 4*(b*x + a)^(3/4)*(d*x + c)^(1/4))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(3/4)/(c + d*x)^(3/4), x)
```

3.1707 $\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

```
[Out] -2*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(3/4)+2*ar
ctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/b^(1/4)/d^(3/4)
```

Rubi [A]
time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {65, 338, 304, 211, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}-\frac{2\text{ArcTan}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(1/4)*d
^(3/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
(b^(1/4)*d^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx = \frac{4\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$

$$= \frac{4\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Mathematica [**A**]
time = 0.11, size = 73, normalized size = 0.86

$$\frac{2\left(\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (2*(ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))] + ArcTanh[(
b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))]))/(b^(1/4)*d^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx+a\right)^{\frac{1}{4}} \left(dx+c\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(61) = 122.
time = 1.21, size = 234, normalized size = 2.75

$$-4 \left(\frac{1}{bd^3}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(bx+a\right)^{\frac{3}{4}} \left(dx+c\right)^{\frac{1}{4}} bd^2 \left(\frac{1}{bd^3}\right)^{\frac{3}{4}} - \left(b^2d^2x+abd^2\right)\sqrt{\frac{\left(bd^2x+ad^2\right)\sqrt{\frac{1}{bd^3}} + \sqrt{bx+a}\,\sqrt{dx+c}}{bx+a}}\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}}{bx+a}\right) + \left(\frac{1}{bd^3}\right)^{\frac{1}{4}} \log\left(\frac{\left(bdx+ad\right)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}} + \left(bx+a\right)^{\frac{3}{4}}\left(dx+c\right)^{\frac{1}{4}}}{bx+a}\right) - \left(\frac{1}{bd^3}\right)^{\frac{1}{4}} \log\left(-\frac{\left(bdx+ad\right)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}} - \left(bx+a\right)^{\frac{3}{4}}\left(dx+c\right)^{\frac{1}{4}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b*d^3))^(1/4)*arctan(-((b*x + a)^(3/4)*(d*x + c)^(1/4)*b*d^2*(1/(b*d
^3))^(3/4) - (b^2*d^2*x + a*b*d^2)*sqrt(((b*d^2*x + a*d^2)*sqrt(1/(b*d^3))
+ sqrt(b*x + a)*sqrt(d*x + c))/(b*x + a))*(1/(b*d^3))^(3/4))/(b*x + a)) + (
1/(b*d^3))^(1/4)*log(((b*d*x + a*d)*(1/(b*d^3))^(1/4) + (b*x + a)^(3/4)*(d*
x + c)^(1/4))/(b*x + a)) - (1/(b*d^3))^(1/4)*log(-((b*d*x + a*d)*(1/(b*d^3)
)^(1/4) - (b*x + a)^(3/4)*(d*x + c)^(1/4))/(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,\left(c+dx\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/4}\,{\left(c+d\,x\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x)
```

```
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)), x)
```

3.1708 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

```
[Out] -4*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
[Out] 4/(b*x+a)^(1/4)*(d*x+c)^(1/4)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

Fricas [A]
time = 1.16, size = 42, normalized size = 1.40

$$-\frac{4\,(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] -4*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{5}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

Mupad [B]
time = 0.71, size = 26, normalized size = 0.87

$$\frac{4\,{\left(c+d\,x\right)}^{1/4}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x)
[Out] (4*(c + d*x)^(1/4))/((a*d - b*c)*(a + b*x)^(1/4))
```

3.1709 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}}+\frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}$$

```
[Out] -4/5*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(5/4)+16/5*d*(d*x+c)^(1/4)/(-a*d+b*c)
^2/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}\,(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (16*d*(c + d*x)^(1/4
))/(5*(b*c - a*d)^2*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}} - \frac{(4d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{5(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}} + \frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}$$

Mathematica [A]
time = 0.10, size = 46, normalized size = 0.70

$$\frac{4\sqrt[4]{c+dx}\,(-bc+5ad+4bdx)}{5(bc-ad)^2(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(-(b*c) + 5*a*d + 4*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(
5/4))
```

Maple [A]
time = 0.19, size = 54, normalized size = 0.82

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}(4bdx+5ad-bc)}{5(bx+a)^{\frac{5}{4}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/5*(d*x+c)^(1/4)*(4*b*d*x+5*a*d-b*c)/(b*x+a)^(5/4)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 1.28, size = 118, normalized size = 1.79

$$\frac{4 (4 bdx - bc + 5 ad)(bx + a)^{\frac{3}{4}}(dx + c)^{\frac{1}{4}}}{5 (a^2b^2c^2 - 2 a^3bcd + a^4d^2 + (b^4c^2 - 2 ab^3cd + a^2b^2d^2)x^2 + 2 (ab^3c^2 - 2 a^2b^2cd + a^3bd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(4*b*d*x - b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{9}{4}} (c + dx)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

Mupad [B]
time = 0.87, size = 71, normalized size = 1.08

$$\frac{\left(\frac{16\,d\,x}{5\,{\left(a\,d-b\,c\right)}^2}+\frac{20\,a\,d-4\,b\,c}{5\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/4}}{x\,{\left(a+b\,x\right)}^{1/4}+\frac{a\,{\left(a+b\,x\right)}^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x)
```

```
[Out] (((16*d*x)/(5*(a*d - b*c)^2) + (20*a*d - 4*b*c)/(5*b*(a*d - b*c)^2))*(c + d
*x)^(1/4))/(x*(a + b*x)^(1/4) + (a*(a + b*x)^(1/4))/b)
```

3.1710 $\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}} \, dx$

Optimal. Leaf size=101

$$-\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}+\frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}}-\frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}$$

```
[Out] -4/9*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(9/4)+32/45*d*(d*x+c)^(1/4)/(-a*d+b*c
)^2/(b*x+a)^(5/4)-128/45*d^2*(d*x+c)^(1/4)/(-a*d+b*c)^3/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}\,(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(9*(b*c - a*d)*(a + b*x)^(9/4)) + (32*d*(c + d*x)^(1/4
))/(45*(b*c - a*d)^2*(a + b*x)^(5/4)) - (128*d^2*(c + d*x)^(1/4))/(45*(b*c
- a*d)^3*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} - \frac{(8d)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{9(bc-ad)}$$
$$= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} + \frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{5/4}(c+dx}}{45(bc-ad)^2}$$
$$= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}} + \frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}} - \frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}$$

Mathematica [A]
time = 0.11, size = 75, normalized size = 0.74

$$-\frac{4\sqrt[4]{c+dx}\,(45a^2d^2-18abd(c-4dx)+b^2(5c^2-8cdx+32d^2x^2))}{45(bc-ad)^3(a+bx)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(45*a^2*d^2 - 18*a*b*d*(c - 4*d*x) + b^2*(5*c^2 - 8*c*d
*x + 32*d^2*x^2)))/(45*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$\frac{4(dx+c)^{\frac{1}{4}}\left(32b^2x^2d^2+72ab\,d^2x-8b^2cdx+45a^2d^2-18abcd+5b^2c^2\right)}{45(bx+a)^{\frac{9}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/45*(d*x+c)^(1/4)*(32*b^2*d^2*x^2+72*a*b*d^2*x-8*b^2*c*d*x+45*a^2*d^2-18*a
*b*c*d+5*b^2*c^2)/(b*x+a)^(9/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(83) = 166$.
time = 0.71, size = 251, normalized size = 2.49

$$-\frac{4\,(32\,b^2d^2x^2+5\,b^2c^2-18\,abcd+45\,a^2d^2-8\,(b^2cd-9\,abd^2)x)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{45\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^3+3\,(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3)x^2+3\,(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/45*(32*b^2*d^2*x^2 + 5*b^2*c^2 - 18*a*b*c*d + 45*a^2*d^2 - 8*(b^2*c*d -
9*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*
d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b
^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^
3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{13}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(13/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

Mupad [B]
time = 1.02, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/4}\left(\frac{128\,d^2\,x^2}{45\,(a\,d-b\,c)^3}+\frac{180\,a^2\,d^2-72\,a\,b\,c\,d+20\,b^2\,c^2}{45\,b^2\,(a\,d-b\,c)^3}+\frac{32\,d\,x\,(9\,a\,d-b\,c)}{45\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/4}+\frac{a^2\,(a+b\,x)^{1/4}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((128*d^2*x^2)/(45*(a*d - b*c)^3) + (180*a^2*d^2 + 20*b^2*
c^2 - 72*a*b*c*d)/(45*b^2*(a*d - b*c)^3) + (32*d*x*(9*a*d - b*c))/(45*b*(a*
d - b*c)^3)))/(x^2*(a + b*x)^(1/4) + (a^2*(a + b*x)^(1/4))/b^2 + (2*a*x*(a
+ b*x)^(1/4))/b)
```

3.1711 $\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}}-\frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}}+\frac{512d^3\sqrt[4]{c+dx}}{195(bc-ad)^4\sqrt[4]{a+bx}}$$

```
[Out] -4/13*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(13/4)+16/39*d*(d*x+c)^(1/4)/(-a*d+b
*c)^2/(b*x+a)^(9/4)-128/195*d^2*(d*x+c)^(1/4)/(-a*d+b*c)^3/(b*x+a)^(5/4)+51
2/195*d^3*(d*x+c)^(1/4)/(-a*d+b*c)^4/(b*x+a)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}\,(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(1
/4))/(39*(b*c - a*d)^2*(a + b*x)^(9/4)) - (128*d^2*(c + d*x)^(1/4))/(195*(b
*c - a*d)^3*(a + b*x)^(5/4)) + (512*d^3*(c + d*x)^(1/4))/(195*(b*c - a*d)^4
*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} - \frac{(12d)\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx}{13(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} + \frac{(32d^2)\int \frac{1}{(a+bx)^{9/4}(c+}}{39(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+}$$

$$= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}} + \frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}} - \frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+}$$

Mathematica [**A**]
time = 0.15, size = 93, normalized size = 0.68

$$-\frac{4\sqrt[4]{c+dx}\left(-195d^3(a+bx)^3+117bd^2(a+bx)^2(c+dx)-65b^2d(a+bx)(c+dx)^2+15b^3(c+dx)^3\right)}{195(bc-ad)^4(a+bx)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(-195*d^3*(a + b*x)^3 + 117*b*d^2*(a + b*x)^2*(c + d*x)
 - 65*b^2*d*(a + b*x)*(c + d*x)^2 + 15*b^3*(c + d*x)^3))/(195*(b*c - a*d)^4
*(a + b*x)^(13/4))
```

Maple [**A**]
time = 0.18, size = 171, normalized size = 1.26

method	result
gosper	$\frac{4(dx+c)^{\frac{1}{4}}\left(128b^3x^3d^3+416d^3a\,x^2b^2-32b^3c\,d^2x^2+468a^2b\,d^3x-104a\,b^2c\,d^2x+20b^3c^2dx+195a^3d^3-117a^2bc\,d^2+65a\,b^2c^2d-15b^3c^3\right)}{195(bx+a)^{\frac{13}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/195*(d*x+c)^(1/4)*(128*b^3*d^3*x^3+416*a*b^2*d^3*x^2-32*b^3*c*d^2*x^2+468
*a^2*b*d^3*x-104*a*b^2*c*d^2*x+20*b^3*c^2*d*x+195*a^3*d^3-117*a^2*b*c*d^2+6
5*a*b^2*c^2*d-15*b^3*c^3)/(b*x+a)^(13/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(112) = 224$.
time = 0.79, size = 419, normalized size = 3.08

$$\frac{4\,(128\,b^3d^3x^3 - 15\,b^3c^3 + 65\,ab^2c^2d - 117\,a^2bcd^2 + 195\,a^3d^3 - 32\,(b^3cd^2 - 13\,ab^2d^3)x^2 + 4\,(5\,b^3c^2d - 26\,ab^2cd^2 + 117\,a^2bd^3)x)(bx + a)^{\frac{3}{4}}(dx + c)^{\frac{1}{4}}}{195\,(a^4b^4c^4 - 4\,a^5b^3c^3d + 6\,a^6b^2c^2d^2 - 4\,a^7bcd^3 + a^8d^4 + (b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4b^4d^4)x^4 + 4\,(ab^7c^4 - 4\,a^2b^6c^3d + 6\,a^3b^5c^2d^2 - 4\,a^4b^4cd^3 + a^5b^3d^4)x^3 + 6\,(a^2b^6c^4 - 4\,a^3b^5c^3d + 6\,a^4b^4c^2d^2 - 4\,a^5b^3cd^3 + a^6b^2d^4)x^2 + 4\,(a^3b^5c^4 - 4\,a^4b^4c^3d + 6\,a^5b^3c^2d^2 - 4\,a^6b^2cd^3 + a^7bd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/195*(128*b^3*d^3*x^3 - 15*b^3*c^3 + 65*a*b^2*c^2*d - 117*a^2*b*c*d^2 + 19
5*a^3*d^3 - 32*(b^3*c*d^2 - 13*a*b^2*d^3)*x^2 + 4*(5*b^3*c^2*d - 26*a*b^2*c
*d^2 + 117*a^2*b*d^3)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^4*c^4 - 4*a
^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a
*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*
b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d
^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*
c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2
*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(17/4)/(d*x+c)**(3/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

Mupad [B]
time = 1.26, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/4}\left(\frac{512\,d^3\,x^3}{195\,(a\,d-b\,c)^4}+\frac{780\,a^3\,d^3-468\,a^2\,b\,c\,d^2+260\,a\,b^2\,c^2\,d-60\,b^3\,c^3}{195\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d\,x\,(117\,a^2\,d^2-26\,a\,b\,c\,d+5\,b^2\,c^2)}{195\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^2\,x^2\,(13\,a\,d-b\,c)}{195\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((512*d^3*x^3)/(195*(a*d - b*c)^4) + (780*a^3*d^3 - 60*b^3
*c^3 + 260*a*b^2*c^2*d - 468*a^2*b*c*d^2)/(195*b^3*(a*d - b*c)^4) + (16*d*x
*(117*a^2*d^2 + 5*b^2*c^2 - 26*a*b*c*d))/(195*b^2*(a*d - b*c)^4) + (128*d^2
*x^2*(13*a*d - b*c))/(195*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/4) + (a^3*(a
 + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a^2*x*(a + b*x)^(1/4)
)/b^2)
```

3.1712 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=332

$$-\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2}+\frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d}+\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bd}}{}$$

[Out] -5/3*(-a*d+b*c)*(b*x+a)^(1/4)*(d*x+c)^(1/4)/d^2+2/3*(b*x+a)^(5/4)*(d*x+c)^(
1/4)/d+5/12*(-a*d+b*c)^(5/2)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*El
lipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+
b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*
d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*
b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)/d^(9/4
)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^
2)^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{6\sqrt{2}\sqrt[4]{b}\,d^{9/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{5\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}{3d^2}+\frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]

[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4))/(3*d^2) + (2*(a + b*x)^(5/
4)*(c + d*x)^(1/4))/(3*d) + (5*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(3/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(6*Sqrt[2]*b^(1/4)*d^(9/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d
+ 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}}\,dx = \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{(5(bc-ad)^2)\int \frac{1}{(a+bx)^{3/4}(c+d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{(5(bc-ad)^2((a+bx)(c+}{12d^2(a+b}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{\Big(5(bc-ad)^2((a+bx)(c+}{}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{3d^2} + \frac{2(a+bx)^{5/4}\sqrt[4]{c+dx}}{3d} + \frac{5(bc-ad)^{5/2}((a+bx)(c+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.22

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(5/4)/(c + d*x)^(3/4), x)
```

3.1713 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=295

$$\frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d}-\frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+d}}{bc-ad}\right.}{\sqrt{2}\,\sqrt[4]{b}\,d^{5/4}(a+bx)^{3/4}(c}$$

```
[Out] 2*(b*x+a)^(1/4)*(d*x+c)^(1/4)/d-1/2*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(3/4
)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^
(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/
2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+
c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*
x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+
c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))
^2)^(1/2)/b^(1/4)/d^(5/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)*2^(
1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {52, 64, 637, 226}

$$\frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d}-\frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt[4]{b}\,d^{5/4}(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]
```

```
[Out] (2*(a + b*x)^(1/4)*(c + d*x)^(1/4))/d - ((b*c - a*d)^(3/2)*((a + b*x)*(c +
d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*
(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*Ellipti
cF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c
- a*d]], 1/2])/(Sqrt[2]*b^(1/4)*d^(5/4)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{3/4}}\,dx = \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{2d}$$

$$= \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{\left((bc-ad)((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{2d(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{\left(2(bc-ad)((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\Big(}{d(a+bx)^{3/4}(c+dx)}$$

$$= \frac{2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}{d} - \frac{(bc-ad)^{3/2}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\Big(1+\frac{2\sqrt{}}{}}{\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 73, normalized size = 0.25

$$\frac{4(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(3/4),x]
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/4))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
[Out] int((b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(3/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(3/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(3/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(3/4), x)
```

3.1714 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2}\,\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)\sqrt{\frac{\quad}{(bc-ad)^2\,(}}}{\sqrt[4]{b}\,\sqrt[4]{d}\,(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bc}$$

```
[Out] ((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/
4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a
)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(
1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*
c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1
/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)/d^(1/4)/(b
*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {64, 637, 226}

$$\frac{\sqrt{2}\,\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\,\sqrt[4]{d}\,(a+bx)^{3/4}(c+dx)^{3/4}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*
d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqr
t[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/
4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(b^(1/4)*d^(1/4)*(a
 + b*x)^(3/4)*(c + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*
x))^2])
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx = \frac{((a+bx)(c+dx))^{3/4}\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= \frac{\left(4((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-4abcd+(bc+}\right.}{(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bd}$$

$$= \frac{\sqrt{2}\sqrt{bc-ad}\,((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(}}{b}\right.}{\sqrt[4]{b}\sqrt[4]{d}\,(a}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 71, normalized size = 0.26

$$\frac{4\sqrt[4]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[1/4,
 3/4, 5/4, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(3/4)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/4}\,(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(3/4)), x)
```

3.1715 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=306

$$-\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}}-\frac{2\sqrt{2}\,d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+}}{bc-ad}\right.}{3\sqrt[4]{b}\sqrt{bc-ad}\,(a+bx)^3}$$

```
[Out] -4/3*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(3/4)-2/3*d^(3/4)*((b*x+a)*(d*x+c))^(
3/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*
c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^
(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d
*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d
^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*
d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/
(-a*d+b*c))^2)^(1/2)/b^(1/4)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/
(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 226}

$$-\frac{2\sqrt{2}\,d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,(a+bx)^{3/4}(c+dx)^{3/4}\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4\sqrt[4]{c+dx}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(3*(b*c - a*d)*(a + b*x)^(3/4)) - (2*Sqrt[2]*d^(3/4)*(
(a + b*x)*(c + d*x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/
((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*
d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(
1/4))/Sqrt[b*c - a*d]], 1/2])/(3*b^(1/4)*Sqrt[b*c - a*d]*(a + b*x)^(3/4)*(c
 + d*x)^(3/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{(2d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}\,dx}{3(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{\left(2d((a+bx)(c+dx))^{3/4}\right)\int \frac{1}{(ac+(bc+ad)x+bdx^2)^{3/4}}\,dx}{3(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}$$

$$= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{\left(8d((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\right) \mathrm{S}}{3(bc-ad)(a+}$$

$$= -\frac{4\sqrt[4]{c+dx}}{3(bc-ad)(a+bx)^{3/4}} - \frac{2\sqrt{2}\,d^{3/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.24

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/4}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/4)*(c + d*x)^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}} (dx+c)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}} (c+dx)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/4}\,{\left(c+d\,x\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(3/4)), x)
```

3.1716 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=339

$$-\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}}+\frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}}+\frac{4\sqrt{2}\,d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}\left(1+\right.}{7}$$

```
[Out] -4/7*(d*x+c)^(1/4)/(-a*d+b*c)/(b*x+a)^(7/4)+8/7*d*(d*x+c)^(1/4)/(-a*d+b*c)^
2/(b*x+a)^(3/4)+4/7*d^(7/4)*((b*x+a)*(d*x+c))^(3/4)*(cos(2*arctan(b^(1/4)*d
^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*Ell
ipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b
*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/
2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^
2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/b^(1/4)
/(-a*d+b*c)^(3/2)/(b*x+a)^(3/4)/(d*x+c)^(3/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*
d*x+c))^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {53, 64, 637, 226}

$$\frac{4\sqrt{2}\,d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)^{3/2}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{8d\sqrt[4]{c+dx}}{7(a+bx)^{3/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (8*d*(c + d*x)^(1/4)
)/(7*(b*c - a*d)^2*(a + b*x)^(3/4)) + (4*Sqrt[2]*d^(7/4)*((a + b*x)*(c + d*
x))^(3/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b
*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1
 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF
[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c -
a*d]], 1/2])/(7*b^(1/4)*(b*c - a*d)^(3/2)*(a + b*x)^(3/4)*(c + d*x)^(3/4)*(
b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} - \frac{(6d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{3/4}}\,dx}{7(bc-ad)}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{(4d^2)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{3/4}}}{7(bc-ad)^2}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{(4d^2((a+bx)(c+dx))}{7(bc-ad)^2(a}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{\left(16d^2((a+bx)(c+dx\right.}{}$$

$$= -\frac{4\sqrt[4]{c+dx}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d\sqrt[4]{c+dx}}{7(bc-ad)^2(a+bx)^{3/4}} + \frac{4\sqrt{2}\,d^{7/4}((a+bx)(c+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.22

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(-\frac{7}{4},\frac{3}{4};-\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(a+bx)^{7/4}(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(7*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
```

```
[Out] int(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(3/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/4)*(d*x + c)^(1/4)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*
b^2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{4}}(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/4}\,{\left(c+d\,x\right)}^{3/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(3/4)), x)
```

3.1717 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\cdots\right.}{2d^{9/4}}$$

```
[Out] -4*(b*x+a)^(5/4)/d/(d*x+c)^(1/4)+5*b*(b*x+a)^(1/4)*(d*x+c)^(3/4)/d^2-5/2*b^
(1/4)*(-a*d+b*c)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4))/d^(9/4
)-5/2*b^(1/4)*(-a*d+b*c)*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)/(d*x+c)^(1/4
))/d^(9/4)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 65, 246, 218, 214, 211}

$$-\frac{5\sqrt[4]{b}\,(bc-ad)\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/4))/(d*(c + d*x)^(1/4)) + (5*b*(a + b*x)^(1/4)*(c + d*x)^(
3/4))/d^2 - (5*b^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/(2*d^(9/4)) - (5*b^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*d^(9/4))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{(5b)\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5b(bc-ad))\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x\right.}{d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{\left(5\sqrt{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x,\right.}{2d^2}$$

$$= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}} + \frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2} - \frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}} - \frac{5\sqrt[4]{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.06, size = 73, normalized size = 0.48

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{-bc+ad}\right)}{9b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(116) = 232$.
time = 1.00, size = 857, normalized size = 5.64

$$-\frac{20\,(d^{3}x+cd^{2})\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{(bcd^{7}-ad^{8})(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{3}{4}}+(d^{8}x+cd^{7})\sqrt{\frac{(b^{2}c^{2}-2\,abcd+a^{2}d^{2})\sqrt{bx+a}\,\sqrt{dx+c}+(d^{5}x+cd^{4})\sqrt{\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}}}{dx+c}}\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{3}{4}}}{b^{5}c^{5}-4\,ab^{4}c^{4}d+6\,a^{2}b^{3}c^{3}d^{2}-4\,a^{3}b^{2}c^{2}d^{3}+a^{4}bcd^{4}+(b^{5}c^{4}d-4\,ab^{4}c^{3}d^{2}+6\,a^{2}b^{3}c^{2}d^{3}-4\,a^{3}b^{2}cd^{4}+a^{4}bd^{5})x}\right)+5\,(d^{3}x+cd^{2})\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}+(d^{3}x+cd^{2})\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\right)}{dx+c}\right)-5\,(d^{3}x+cd^{2})\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}-(d^{3}x+cd^{2})\left(\frac{b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\right)}{dx+c}\right)-4\,(bdx+5\,bc-4\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{4\,(d^{3}x+cd^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/4*(20*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*arctan(((b*c*d^7 - a*d^8)*(b*x + a)^(
1/4)*(d*x + c)^(3/4)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)/d^9)^(3/4) + (d^8*x + c*d^7)*sqrt(((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (d^5*x + c*d^4)*sqrt((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9))/(d
*x + c))*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)/d^9)^(3/4))/(b^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3
*b^2*c^2*d^3 + a^4*b*c*d^4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d
^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(
-5*((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (d^3*x + c*d^2)*((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1
/4))/(d*x + c)) - 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
 a)^(1/4)*(d*x + c)^(3/4) - (d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4))/(d*x + c)) - 4*(b
*d*x + 5*b*c - 4*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(5/4)/(c + d*x)^(5/4), x)
```

3.1718 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=108

$$-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}$$

```
[Out] -4*(b*x+a)^(1/4)/d/(d*x+c)^(1/4)+2*b^(1/4)*arctan(d^(1/4)*(b*x+a)^(1/4)/b^(
1/4)/(d*x+c)^(1/4))/d^(5/4)+2*b^(1/4)*arctanh(d^(1/4)*(b*x+a)^(1/4)/b^(1/4)
/(d*x+c)^(1/4))/d^(5/4)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 65, 246, 218, 214, 211}

$$\frac{2\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx &= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{b\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx,x,\sqrt[4]{a+bx}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}+\frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{2\sqrt[4]{b}\,\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\,\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{d^{5/4}}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 105, normalized size = 0.97

$$\frac{2\left(-\frac{2\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}+\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)+\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)\right)}{d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (2*((-2*d^(1/4)*(a + b*x)^(1/4))/(c + d*x)^(1/4) + b^(1/4)*ArcTan[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))] + b^(1/4)*ArcTanh[(d^(1/4)*(a +
 b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))]))/d^(5/4)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(80) = 160$.
time = 0.66, size = 273, normalized size = 2.53

$$-\frac{4\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}d^{4}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}-(d^{5}x+cd^{4})\sqrt{\frac{(d^{3}x+cd^{2})\sqrt{\frac{b}{d^{5}}}+\sqrt{bx+a}\sqrt{dx+c}}{dx+c}}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}}{bdx+bc}\right)-\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(\frac{(d^{2}x+cd)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(-\frac{(d^{2}x+cd)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -(4*(d^2*x + c*d)*(b/d^5)^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*d^
4*(b/d^5)^(3/4) - (d^5*x + c*d^4)*sqrt(((d^3*x + c*d^2)*sqrt(b/d^5) + sqrt(
b*x + a)*sqrt(d*x + c))/(d*x + c))*(b/d^5)^(3/4))/(b*d*x + b*c)) - (d^2*x +
 c*d)*(b/d^5)^(1/4)*log(((d^2*x + c*d)*(b/d^5)^(1/4) + (b*x + a)^(1/4)*(d*x
 + c)^(3/4))/(d*x + c)) + (d^2*x + c*d)*(b/d^5)^(1/4)*log(-((d^2*x + c*d)*(
b/d^5)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c)) + 4*(b*x + a)^(1
/4)*(d*x + c)^(3/4))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(5/4), x)
```

3.1719 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=30

$$\frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

[Out] 4*(b*x+a)^(1/4)/(-a*d+b*c)/(d*x+c)^(1/4)

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]

[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx = \frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Maple [A]
time = 0.21, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{4(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
[Out] -4*(b*x+a)^(1/4)/(d*x+c)^(1/4)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

Fricas [A]
time = 0.73, size = 42, normalized size = 1.40

$$\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] 4*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a+b\,x)^{3/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)), x)
```

3.1720 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}$$

```
[Out] -4/3/(-a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)-16/3*d*(b*x+a)^(1/4)/(-a*d+b*c)
^2/(d*x+c)^(1/4)
```

Rubi [A]

time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(3*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4)) - (16*d*(a + b*x)^(1/4))
/(3*(b*c - a*d)^2*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx = -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx}{3(bc-ad)}$$
$$= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}} - \frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.68

$$-\frac{4(3ad+b(c+4dx))}{3(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
[Out] (-4*(3*a*d + b*(c + 4*d*x)))/(3*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/
4))
```

Maple [A]
time = 0.18, size = 53, normalized size = 0.80

method	result	size
gosper	$-\frac{4(4bdx+3ad+bc)}{3(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
[Out] -4/3*(4*b*d*x+3*a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.
time = 0.58, size = 126, normalized size = 1.91

$$-\frac{4\,(4\,bdx + bc + 3\,ad)(bx + a)^{\frac{1}{4}}(dx + c)^{\frac{3}{4}}}{3\,(ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/3*(4*b*d*x + b*c + 3*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{7}{4}}\,(c + dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)), x)
```

3.1721 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}+\frac{128d^2\sqrt[4]{a+bx}}{21(bc-ad)^3\sqrt[4]{c+dx}}$$

```
[Out] -4/7/(-a*d+b*c)/(b*x+a)^(7/4)/(d*x+c)^(1/4)+32/21*d/(-a*d+b*c)^2/(b*x+a)^(3
/4)/(d*x+c)^(1/4)+128/21*d^2*(b*x+a)^(1/4)/(-a*d+b*c)^3/(d*x+c)^(1/4)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(7*(b*c - a*d)*(a + b*x)^(7/4)*(c + d*x)^(1/4)) + (32*d)/(21*(b*c - a*d)
^2*(a + b*x)^(3/4)*(c + d*x)^(1/4)) + (128*d^2*(a + b*x)^(1/4))/(21*(b*c -
a*d)^3*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx = -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} - \frac{(8d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx}{7(bc-ad)}$$
$$= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}} + \frac{(32c}{}$$
$$= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}} + \frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}} + \frac{1}{21(b}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{7/4}\left(3b^2-\frac{21d^2(a+bx)^2}{(c+dx)^2}-\frac{14bd(a+bx)}{c+dx}\right)}{21(bc-ad)^3(a+bx)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(7/4)*(3*b^2 - (21*d^2*(a + b*x)^2)/(c + d*x)^2 - (14*b*d*(a
+ b*x))/(c + d*x)))/(21*(b*c - a*d)^3*(a + b*x)^(7/4))
```

Maple [A]
time = 0.19, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{4\left(32b^2x^2d^2+56ab\,d^2x+8b^2cdx+21a^2d^2+14abcd-3b^2c^2\right)}{21(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4/21*(32*b^2*d^2*x^2+56*a*b*d^2*x+8*b^2*c*d*x+21*a^2*d^2+14*a*b*c*d-3*b^2*
c^2)/(b*x+a)^(7/4)/(d*x+c)^(1/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(83) = 166$.
time = 0.73, size = 273, normalized size = 2.70

$$\frac{4\,{\left(32\,b^{2} d^{2} x^{2} - 3\,b^{2} c^{2} + 14\,a b c d + 21\,a^{2} d^{2} + 8\,{\left(b^{2} c d + 7\,a b d^{2}\right)} x\right)} {\left(b x + a\right)}^{\frac{1}{4}} {\left(d x + c\right)}^{\frac{3}{4}}}{21\,{\left(a^{2} b^{3} c^{4} - 3\,a^{3} b^{2} c^{3} d + 3\,a^{4} b c^{2} d^{2} - a^{5} c d^{3} + {\left(b^{5} c^{3} d - 3\,a b^{4} c^{2} d^{2} + 3\,a^{2} b^{3} c d^{3} - a^{3} b^{2} d^{4}\right)} x^{3} + {\left(b^{5} c^{4} - a b^{4} c^{3} d - 3\,a^{2} b^{3} c^{2} d^{2} + 5\,a^{3} b^{2} c d^{3} - 2\,a^{4} b d^{4}\right)} x^{2} + {\left(2\,a b^{4} c^{4} - 5\,a^{2} b^{3} c^{3} d + 3\,a^{3} b^{2} c^{2} d^{2} + a^{4} b c d^{3} - a^{5} d^{4}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4/21*(32*b^2*d^2*x^2 - 3*b^2*c^2 + 14*a*b*c*d + 21*a^2*d^2 + 8*(b^2*c*d + 7
*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c
*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^
3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b x\right)^{\frac{11}{4}} \left(c + d x\right)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{11/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)), x)
```

3.1722 $\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}}+\frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}}-\frac{128d^2}{77(bc-ad)^3(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{7}{}$$

```
[Out] -4/11/(-a*d+b*c)/(b*x+a)^(11/4)/(d*x+c)^(1/4)+48/77*d/(-a*d+b*c)^2/(b*x+a)^
(7/4)/(d*x+c)^(1/4)-128/77*d^2/(-a*d+b*c)^3/(b*x+a)^(3/4)/(d*x+c)^(1/4)-512
/77*d^3*(b*x+a)^(1/4)/(-a*d+b*c)^4/(d*x+c)^(1/4)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}\,(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(11*(b*c - a*d)*(a + b*x)^(11/4)*(c + d*x)^(1/4)) + (48*d)/(77*(b*c - a*
d)^2*(a + b*x)^(7/4)*(c + d*x)^(1/4)) - (128*d^2)/(77*(b*c - a*d)^3*(a + b*
x)^(3/4)*(c + d*x)^(1/4)) - (512*d^3*(a + b*x)^(1/4))/(77*(b*c - a*d)^4*(c
+ d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx = -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} - \frac{(12d)\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx}{11(bc-ad)}$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} + \cdots$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \cdots$$
$$= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}} + \frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}} - \cdots$$

Mathematica [A]
time = 0.14, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{11/4}\left(7b^3+\frac{77d^3(a+bx)^3}{(c+dx)^3}+\frac{77bd^2(a+bx)^2}{(c+dx)^2}-\frac{33b^2d(a+bx)}{c+dx}\right)}{77(bc-ad)^4(a+bx)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(7*b^3 + (77*d^3*(a + b*x)^3)/(c + d*x)^3 + (77*b*d^2*
(a + b*x)^2)/(c + d*x)^2 - (33*b^2*d*(a + b*x))/(c + d*x)))/(77*(b*c - a*d)
^4*(a + b*x)^(11/4))
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result	size
gosper	$-\frac{4\left(128b^3x^3d^3+352d^3a\,x^2b^2+32b^3c\,d^2x^2+308a^2b\,d^3x+88a\,b^2c\,d^2x-12b^3c^2dx+77a^3d^3+77a^2bc\,d^2-33a\,b^2c^2d+7b^3c^3\right)}{77(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{1}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$	171

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x,method=_RETURNVERBOSE)
```

```
[Out] -4/77*(128*b^3*d^3*x^3+352*a*b^2*d^3*x^2+32*b^3*c*d^2*x^2+308*a^2*b*d^3*x+8
8*a*b^2*c*d^2*x-12*b^3*c^2*d*x+77*a^3*d^3+77*a^2*b*c*d^2-33*a*b^2*c^2*d+7*b
^3*c^3)/(b*x+a)^(11/4)/(d*x+c)^(1/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(112) = 224$.
time = 0.83, size = 457, normalized size = 3.36

$$-\frac{4\,(128\,b^3d^3x^3+7\,b^3c^3-33\,ab^2c^2d+77\,a^2bcd^2+77\,a^3d^3+32\,(b^3cd^2+11\,ab^2d^3)x^2-4\,(3\,b^3c^2d-22\,ab^2cd^2-77\,a^2bd^3)x)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{77\,(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5)x^4+(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5)x^3+3\,(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5)x^2+(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/77*(128*b^3*d^3*x^3 + 7*b^3*c^3 - 33*a*b^2*c^2*d + 77*a^2*b*c*d^2 + 77*a
^3*d^3 + 32*(b^3*c*d^2 + 11*a*b^2*d^3)*x^2 - 4*(3*b^3*c^2*d - 22*a*b^2*c*d^
2 - 77*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^4*c^5 - 4*a^4*b
^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4
*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (
b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^
3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c
^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^
4 + a^7*d^5)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(5/4),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5458 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{15/4}\,(c+d\,x)^{5/4}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)), x)
```

3.1723 $\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=776

$$-\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}}-\frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3}+\frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2}+\frac{77\sqrt{b}\,(bc-ad)\sqrt{(a+bx)}}{10d^{7/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(b}$$

```
[Out] -4*(b*x+a)^(11/4)/d/(d*x+c)^(1/4)-77/15*b*(-a*d+b*c)*(b*x+a)^(3/4)*(d*x+c)^
(3/4)/d^3+22/5*b*(b*x+a)^(7/4)*(d*x+c)^(3/4)/d^2+77/10*(-a*d+b*c)*b^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1
/2)/d^(7/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1
/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))-77/20*b^(1/4)*(-a*d+b*c)^(7/2)*((b*
x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2
^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d
*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1
/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^
(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/
2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))
^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(15/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a
*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)+77/40*b^(1/4)*(-a*d+b*c)^(7/2)*
((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/
4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a
)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*
d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+
2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)
^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x
+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(15/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d
*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 64, 637, 311, 226, 1210}

$$\frac{77\sqrt[4]{b}\,(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{20\sqrt{2}\,d^{15/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{77\sqrt[4]{b}\,(bc-ad)^{7/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{10\sqrt{2}\,d^{15/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{77\sqrt{b}\,(bc-ad)\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{10d^{7/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{77b(a+bx)^{3/4}(c+dx)^{3/4}(bc-ad)}{15d^3}+\frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2}-\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(11/4))/(d*(c + d*x)^(1/4)) - (77*b*(b*c - a*d)*(a + b*x)^(3/
4)*(c + d*x)^(3/4))/(15*d^3) + (22*b*(a + b*x)^(7/4)*(c + d*x)^(3/4))/(5*d^
```

```
2) + (77*Sqrt[b]*(b*c - a*d)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*
b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(10*d^(7/2)*(a + b*x)^(1/4)*(c + d
*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))) - (77*b^(1/4)*(b*c - a*d)^(7/2)*((a + b*x)*(c + d*x))
^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)
*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 +
(2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*
ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d
]], 1/2])/(10*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d +
 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (77*b^(1/4)*(b*c - a*d)^(7/2)*((
a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqr
t[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/(
(b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d
))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1
/4))/Sqrt[b*c - a*d]], 1/2])/(20*Sqrt[2]*d^(15/4)*(a + b*x)^(1/4)*(c + d*x)
^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} + \frac{(11b)\int \frac{(a+bx)^{7/4}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} - \frac{(77b(bc-ad))\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{10d^2}$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \cdots$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \cdots$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \cdots$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \cdots$$

$$= -\frac{4(a+bx)^{11/4}}{d\sqrt[4]{c+dx}} - \frac{77b(bc-ad)(a+bx)^{3/4}(c+dx)^{3/4}}{15d^3} + \frac{22b(a+bx)^{7/4}(c+dx)^{3/4}}{5d^2} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.09

$$\frac{4(a+bx)^{15/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{15}{4};\frac{19}{4};\frac{d(a+bx)}{-bc+ad}\right)}{15b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(11/4)/(c + d*x)^(5/4),x]
[Out] (4*(a + b*x)^(15/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4
, 15/4, 19/4, (d*(a + b*x))/(-(b*c) + a*d)])/(15*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{11}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(11/4)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(3/4)*(d*x + c)^(3/4)/(d^2*x^2
 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{11}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(11/4)/(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(11/4)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(11/4)/(d*x + c)^(5/4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(11/4)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(11/4)/(c + d*x)^(5/4), x)
```

3.1724 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=730

$$-\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}}+\frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2}-\frac{7\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+}}{d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx}}{bc-ad}\right.}$$

```
[Out] -4*(b*x+a)^(7/4)/d/(d*x+c)^(1/4)+14/3*b*(b*x+a)^(3/4)*(d*x+c)^(3/4)/d^2-7*b
^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c
))^2)^(1/2)/d^(5/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1
/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))+7/2*b^(1/4)*(-a*d+b*c)^(5/2
)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(
1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x
+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4
)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(
1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^
2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d
*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(11/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b
*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)-7/4*b^(1/4)*(-a*d+b*c)^(5
/2)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))
^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b
*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))
*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c
)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*
(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/d^(11/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2
*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {49, 52, 64, 637, 311, 226, 1210}

$$-\frac{7\sqrt[4]{b}\,(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\,d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{7\sqrt[4]{b}\,(bc-ad)^{5/2}\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,d^{11/4}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{7\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2}-\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(7/4))/(d*(c + d*x)^(1/4)) + (14*b*(a + b*x)^(3/4)*(c + d*x)^
(3/4))/(3*d^2) - (7*Sqrt[b]*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b
```

```
*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(5/2)*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x
)])/(b*c - a*d))) + (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*(c + d*x))^(1/4
)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c +
 d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sq
rt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTa
n[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1
/2])/(Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (7*b^(1/4)*(b*c - a*d)^(5/2)*((a + b*x)*
(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt
[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*
d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*El
lipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt
[b*c - a*d]], 1/2])/(2*Sqrt[2]*d^(11/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*
c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{3/4}}{\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{(7b(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx}{2d^2}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(7b(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{}{\sqrt[4]{a}}}{2d^2\sqrt[4]{a+bx}\,\sqrt[4]{c+}}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(14b(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc-}\right.}{}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{\left(7\sqrt{b}\,(bc-ad)^2\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(}\right.}{}$$

$$= -\frac{4(a+bx)^{7/4}}{d\sqrt[4]{c+dx}} + \frac{14b(a+bx)^{3/4}(c+dx)^{3/4}}{3d^2} - \frac{7\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad}}{d^{5/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(7/4)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(7/4)/(c + d*x)^(5/4), x)
```

3.1725 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=712

$$3\sqrt{2}$$

$$-\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}}+\frac{6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{3/2}(bc-ad)\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)}-$$

```
[Out] -4*(b*x+a)^(3/4)/d/(d*x+c)^(1/4)+6*b^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*
x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/d^(3/2)/(-a*d+b*c)/(b*x+a)^
(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c)
)^(1/2)/(-a*d+b*c))+3/2*b^(1/4)*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(1/4)*(c
os(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(
-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^
(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*(1+2*b^(1/2)*d^(1/2)*((b*x+a)
*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^
2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^
(1/2)/d^(7/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)*2^(1/2)/((a*d+b
*(2*d*x+c))^2)^(1/2)-3*b^(1/4)*(-a*d+b*c)^(3/2)*((b*x+a)*(d*x+c))^(1/4)*(co
s(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-
a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(
1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d
*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*
c))^2)^(1/2)/d^(7/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*
(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 64, 637, 311, 226, 1210}

$$\frac{3\sqrt[4]{b}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt{2}d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{3\sqrt{2}\sqrt[4]{b}(bc-ad)^{3/2}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{d^{7/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{d^{3/2}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(3/4))/(d*(c + d*x)^(1/4)) + (6*Sqrt[b]*Sqrt[(a + b*x)*(c + d
*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(d^(3/2)*
```

```
(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*S
qrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (3*Sqrt[2]*b^(1/4
)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^
2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*
d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*
(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((
a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(7/4)*(a + b*x)^(1/4)
*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (3*
b^(1/4)*(b*c - a*d)^(3/2)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b
*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sq
rt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a +
 b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1
/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[2]*d^(7/4)*(
a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d
*x))^2])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{5/4}}\,dx = -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{(3b)\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx}{d}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(3b\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,dx}{d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(12b\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{-4abcd+}}\right.}{d\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{\left(6\sqrt{b}\,(bc-ad)\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{d^{3/2}\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc}$$

$$= -\frac{4(a+bx)^{3/4}}{d\sqrt[4]{c+dx}} + \frac{6\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2}}{d^{3/2}(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+}}{bc-}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4} {}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(5/4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(5/4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(5/4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(3/4)/(c + d*x)^(5/4), x)
```

3.1726 $\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=719

$$\frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}}-\frac{4\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt{d}\,(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}\right.}$$

```
[Out] 4*(b*x+a)^(3/4)/(-a*d+b*c)/(d*x+c)^(1/4)-4*b^(1/2)*((b*x+a)*(d*x+c))^(1/2)*
((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^2/(b*x+a
)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/d^(1/2)/(1+2*b^(1/2)*d^(1/2)*((b*x+
a)*(d*x+c))^(1/2)/(-a*d+b*c))+2*b^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arct
an(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)
^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(
1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*
d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a
*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)
/(-a*d+b*c))^2)^(1/2)/d^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)
/((a*d+b*(2*d*x+c))^2)^(1/2)-b^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(
b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1
/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2
)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(-a*d+b*c)^(1/2)*(1+2*b^(1/2)*d^(
1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+
b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-
a*d+b*c))^2)^(1/2)/d^(3/4)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((
a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}+\frac{2\sqrt{2}\sqrt[4]{b}\sqrt{bc-ad}\sqrt[4]{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{d^{3/4}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(ad+bc+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{\sqrt{d}\sqrt[4]{a+bx}\sqrt[4]{c+dx}(bc-ad)^2(ad+bc+2bdx)\left(\frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}+\frac{4(a+bx)^{3/4}}{\sqrt[4]{c+dx}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4))/((b*c - a*d)*(c + d*x)^(1/4)) - (4*Sqrt[b]*Sqrt[(a + b*
x)*(c + d*x)]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/
```

```
(Sqrt[d]*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x
)*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (2*Sqr
t[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d +
2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(
a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d
^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3/4)*(a + b
*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^
2]) - (Sqrt[2]*b^(1/4)*Sqrt[b*c - a*d]*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*
c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b
*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt
[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]
*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(d^(3
/4)*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c
+ 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{5/4}}\,dx = \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{(2b)\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(2b\sqrt[4]{(a+bx)(c+dx)}\right)\int \frac{1}{\sqrt[4]{ac+(bc+ad)x+bdx^2}}\,d}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(8b\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}\left(\int\right.}{(bc-ad)\sqrt[4]{a+bx}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{\left(4\sqrt{b}\,\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}\right)\mathrm{Subst}}{\sqrt{d}\,\sqrt[4]{a+bx}\,\sqrt[4]{}}$$

$$= \frac{4(a+bx)^{3/4}}{(bc-ad)\sqrt[4]{c+dx}} - \frac{4\sqrt{b}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)^2}}{\sqrt{d}\,(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.10

$$\frac{4(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[3/4,
 5/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}\,(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(5/4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(5/4)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(5/4)), x)
```

3.1727 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=750

$$-\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}}-\frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}}+\frac{8\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(bc+ad+2bdx)}}{(bc-ad)^3\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc+ad+2bdx)\left(1+\frac{2}{}\right.}$$

```
[Out] -4/(-a*d+b*c)/(b*x+a)^(1/4)/(d*x+c)^(1/4)-8*d*(b*x+a)^(3/4)/(-a*d+b*c)^2/(d
*x+c)^(1/4)+8*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)
^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^3/(b*x+a)^(1/4)/(d*x+c)^(1/4)
/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))
-4*b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((
b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(si
n(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)
)),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+
b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^
(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(b*x+a)^(1/4)/(d
*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)+
2*b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b
*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin
(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))
),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b
*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(
1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(b*x+a)^(1/4)/(d*
x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(-a*d+b*c)^(1/2)/((a*d+b*(2*d*x+c))^2)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 750, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

$$\frac{2\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{4\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{d}\,\sqrt[4]{(a+bx)(c+dx)}}{\sqrt{bc-ad}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,\sqrt{bc-ad}\,(ad+bc+2bdx)\sqrt{(ad+b(c+2dx))^2}}-\frac{8d(a+bx)^{3/4}}{\sqrt[4]{c+dx}\,(bc-ad)^2}+\frac{8\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}\,\sqrt{(ad+bc+2bdx)^2}\,\sqrt{(ad+b(c+2dx))^2}}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)^3(ad+bc+2bdx)\left(\frac{2\sqrt{b}\,\sqrt{d}\,\sqrt{(a+bx)(c+dx)}}{bc-ad}+1\right)}-\frac{4}{\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/((b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(1/4)) - (8*d*(a + b*x)^(3/4))/((
b*c - a*d)^2*(c + d*x)^(1/4)) + (8*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)
]*Sqrt[(b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/((b*c - a*d)
```

```
^3*(a + b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sq
rt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) - (4*Sqrt[2]*b^(1/4)*d^(1/4)
*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*
Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^
2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c -
a*d))^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))
^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4)*(c + d*x)^
(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) + (2*Sqrt[2]*b^(
1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(Sqrt[b*c - a*d]*(a + b*x)^(1/4
)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
```

```
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx = -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{(2d)\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{5/4}}\,dx}{bc-ad}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{(4bd)\int \frac{1}{\sqrt[4]{a+bx}\,\sqrt[4]{c}}}{(bc-ad)^2}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(4bd\sqrt[4]{(a+bx)(c+}\right.}{(bc-}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(16bd\sqrt[4]{(a+bx)(c+}\right.}{}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{\left(8\sqrt{b}\,\sqrt{d}\,\sqrt[4]{(a+bx}\right.}{}$$

$$= -\frac{4}{(bc-ad)\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2\sqrt[4]{c+dx}} + \frac{8\sqrt{b}\,\sqrt{d}\,\sqrt{(a-}}{(bc-ad)^3\sqrt[4]{a+bx}\,\sqrt[4]{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{3}{4};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[4]{a+bx}\,(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/4)*(c + d*x)^(5/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}} (dx+c)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}} (c+dx)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(5/4)*(c + d*x)^(5/4)), x)
```

3.1728 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=795

$$-\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}}+\frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\sqrt[4]{c+dx}}+\frac{48d^2(a+bx)^{3/4}}{5(bc-ad)^3\sqrt[4]{c+dx}}-\frac{48\sqrt{b}\,d^{3/2}}{5(bc-ad)^4\sqrt[4]{a+}}$$

[Out] -4/5/(-a*d+b*c)/(b*x+a)^(5/4)/(d*x+c)^(1/4)+24/5*d/(-a*d+b*c)^2/(b*x+a)^(1/4)/(d*x+c)^(1/4)+48/5*d^2*(b*x+a)^(3/4)/(-a*d+b*c)^3/(d*x+c)^(1/4)-48/5*d^(3/2)*b^(1/2)*((b*x+a)*(d*x+c))^(1/2)*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2)^(1/2)/(-a*d+b*c)^4/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))+24/5*b^(1/4)*d^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(-a*d+b*c)^(3/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)-12/5*b^(1/4)*d^(5/4)*((b*x+a)*(d*x+c))^(1/4)*(cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*d^(1/4)*((b*x+a)*(d*x+c))^(1/4)*2^(1/2)/(-a*d+b*c)^(1/2))),1/2*2^(1/2))*2^(1/2)*(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))*((2*b*d*x+a*d+b*c)^2)^(1/2)*((a*d+b*(2*d*x+c))^2/(-a*d+b*c)^2/(1+2*b^(1/2)*d^(1/2)*((b*x+a)*(d*x+c))^(1/2)/(-a*d+b*c))^2)^(1/2)/(-a*d+b*c)^(3/2)/(b*x+a)^(1/4)/(d*x+c)^(1/4)/(2*b*d*x+a*d+b*c)/((a*d+b*(2*d*x+c))^2)^(1/2)

Rubi [A]

time = 0.62, antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {53, 64, 637, 311, 226, 1210}

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]

[Out] -4/(5*(b*c - a*d)*(a + b*x)^(5/4)*(c + d*x)^(1/4)) + (24*d)/(5*(b*c - a*d)^2*(a + b*x)^(1/4)*(c + d*x)^(1/4)) + (48*d^2*(a + b*x)^(3/4))/(5*(b*c - a*d

```
)^3*(c + d*x)^(1/4)) - (48*Sqrt[b]*d^(3/2)*Sqrt[(a + b*x)*(c + d*x)]*Sqrt[(
b*c + a*d + 2*b*d*x)^2]*Sqrt[(a*d + b*(c + 2*d*x))^2])/(5*(b*c - a*d)^4*(a
+ b*x)^(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*(1 + (2*Sqrt[b]*Sqrt[d]*
Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))) + (24*Sqrt[2]*b^(1/4)*d^(5/4)*((a
+ b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 + (2*Sqrt[b]*Sqrt[
d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(c + 2*d*x))^2/((b
*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))
^2)]*EllipticE[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x)*(c + d*x))^(1/4
))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^(1/4)*(c + d*x)^(
1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2]) - (12*Sqrt[2]*b^(
1/4)*d^(5/4)*((a + b*x)*(c + d*x))^(1/4)*Sqrt[(b*c + a*d + 2*b*d*x)^2]*(1 +
 (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])/(b*c - a*d))*Sqrt[(a*d + b*(
c + 2*d*x))^2/((b*c - a*d)^2*(1 + (2*Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*
x)])/(b*c - a*d))^2)]*EllipticF[2*ArcTan[(Sqrt[2]*b^(1/4)*d^(1/4)*((a + b*x
)*(c + d*x))^(1/4))/Sqrt[b*c - a*d]], 1/2])/(5*(b*c - a*d)^(3/2)*(a + b*x)^
(1/4)*(c + d*x)^(1/4)*(b*c + a*d + 2*b*d*x)*Sqrt[(a*d + b*(c + 2*d*x))^2])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 64

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/((a + b*x)*(c + d*x))^m), Int[(a*c + (b*c + a*d)*x
+ b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[
-1, m, 0] && LeQ[3, Denominator[m], 4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 637

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{d = Denomi
nator[p]}, Dist[d*(Sqrt[(b + 2*c*x)^2]/(b + 2*c*x)), Subst[Int[x^(d*(p + 1)
 - 1)/Sqrt[b^2 - 4*a*c + 4*c*x^d], x], x, (a + b*x + c*x^2)^(1/d)], x] /; 3
 <= d <= 4] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && RationalQ[p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/4}(c+dx)^{5/4}}\,dx = -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} - \frac{(6d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{5/4}}\,dx}{5(bc-ad)}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{(12d^2)}{}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d}{5(bc-}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d}{5(bc-}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d}{5(bc-}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d}{5(bc-}$$

$$= -\frac{4}{5(bc-ad)(a+bx)^{5/4}\sqrt[4]{c+dx}} + \frac{24d}{5(bc-ad)^2\sqrt[4]{a+bx}\,\sqrt[4]{c+dx}} + \frac{48d}{5(bc-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.03, size = 73, normalized size = 0.09

$$-\frac{4\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(-\frac{5}{4},\frac{5}{4};-\frac{1}{4};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/4}(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, (d
*(a + b*x))/(-(b*c) + a*d)])/(5*b*(a + b*x)^(5/4)*(c + d*x)^(5/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

```
[Out] int(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(5/4)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/4)*(d*x + c)^(3/4)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*
d + 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2
*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{9}{4}}\left(c+dx\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(5/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(5/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{9/4}\,{\left(c+d\,x\right)}^{5/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(9/4)*(c + d*x)^(5/4)), x)
```

3.1729 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx$

Optimal. Leaf size=279

$$\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\sqrt{\ }\right.}{\sqrt{2}\sqrt[4]{a}\,b^{3/}}$$

```
[Out] -1/2*ln(-a^(1/4)*b^(1/4)*(-a*x+1)^(1/4)*2^(1/2)/(b*x+1)^(1/4)+a^(1/2)+b^(1/
2)*(-a*x+1)^(1/2)/(b*x+1)^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)+1/2*ln(a^(1/4)*b^(
1/4)*(-a*x+1)^(1/4)*2^(1/2)/(b*x+1)^(1/4)+a^(1/2)+b^(1/2)*(-a*x+1)^(1/2)/(b
*x+1)^(1/2))/a^(1/4)/b^(3/4)*2^(1/2)+arctan(1-b^(1/4)*(-a*x+1)^(1/4)*2^(1/2
)/a^(1/4)/(b*x+1)^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)-arctan(1+b^(1/4)*(-a*x+1)^
(1/4)*2^(1/2)/a^(1/4)/(b*x+1)^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)
```

Rubi [A]
time = 0.20, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}+1\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*(1 - a*x)^(1/4))/(a^(1/4)*(1 + b*x)^(1
/4))])/(a^(1/4)*b^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*(1 - a*x)^(
1/4))/(a^(1/4)*(1 + b*x)^(1/4))])/(a^(1/4)*b^(3/4)) - Log[Sqrt[a] + (Sqrt[b
]*Sqrt[1 - a*x])/Sqrt[1 + b*x] - (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/
(1 + b*x)^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqrt[a] + (Sqrt[b]*Sqrt[1
- a*x])/Sqrt[1 + b*x] + (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/(1 + b*x)
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx = -\frac{4\text{Subst}\left(\int \frac{x^2}{\left(1+\frac{b}{a}-\frac{bx^4}{a}\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a}$$

$$= \frac{2\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}} - \frac{2\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}\right.}{b}$$

$$= -\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1}}{\sqrt{1+}}\right.}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} -$$

Mathematica [A]
time = 0.21, size = 156, normalized size = 0.56

$$\frac{\sqrt{2}\left(\tan^{-1}\left(\frac{-\sqrt{b}\sqrt{1-ax}+\sqrt{a}\sqrt{1+bx}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}\sqrt[4]{1+bx}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}\sqrt[4]{1+bx}}{\sqrt{b}\sqrt{1-ax}+\sqrt{a}\sqrt{1+bx}}\right)\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*(ArcTan[(-(Sqrt[b]*Sqrt[1 - a*x]) + Sqrt[a]*Sqrt[1 + b*x])/(Sqrt[2
]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4)*(1 + b*x)^(1/4))] + ArcTanh[(Sqrt[2]*a^(1
/4)*b^(1/4)*(1 - a*x)^(1/4)*(1 + b*x)^(1/4))/(Sqrt[b]*Sqrt[1 - a*x] + Sqrt[
a]*Sqrt[1 + b*x])]))/(a^(1/4)*b^(3/4))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}\,(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

Fricas [A]
time = 0.79, size = 247, normalized size = 0.89

$$-4\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}-(a^2b^2x-ab^2)\sqrt{\frac{(ab^2x-b^2)\sqrt{-\frac{1}{ab^3}}-\sqrt{-ax+1}\sqrt{bx+1}}{ax-1}}\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}}{ax-1}\right)-\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)+\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}-(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(-1/(a*b^3))^(1/4)*arctan(-((-a*x + 1)^(3/4)*(b*x + 1)^(1/4)*a*b^2*(-1/(
a*b^3))^(3/4) - (a^2*b^2*x - a*b^2)*sqrt(((a*b^2*x - b^2)*sqrt(-1/(a*b^3))
- sqrt(-a*x + 1)*sqrt(b*x + 1))/(a*x - 1))*(-1/(a*b^3))^(3/4))/(a*x - 1)) -
 (-1/(a*b^3))^(1/4)*log(((a*b*x - b)*(-1/(a*b^3))^(1/4) + (-a*x + 1)^(3/4)*
(b*x + 1)^(1/4))/(a*x - 1)) + (-1/(a*b^3))^(1/4)*log(-((a*b*x - b)*(-1/(a*b
^3))^(1/4) - (-a*x + 1)^(3/4)*(b*x + 1)^(1/4))/(a*x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(b*x+1)**(3/4),x)
```

```
[Out] Integral(1/((-a*x + 1)**(1/4)*(b*x + 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-a\,x)^{1/4}\,(b\,x+1)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)),x)
```

```
[Out] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)), x)
```

3.1730 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$

Optimal. Leaf size=193

$$\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}-\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a}$$

```
[Out] -1/2*ln(1-(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a*x+1)^(1/2)
)/a*2^(1/2)+1/2*ln(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4)+(-a*x+1)^(1/2)/(a
*x+1)^(1/2))/a*2^(1/2)-arctan(-1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1
/2)/a-arctan(1+(-a*x+1)^(1/4)*2^(1/2)/(a*x+1)^(1/4))*2^(1/2)/a
```

Rubi [A]
time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {65, 338, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx = -\frac{4\text{Subst}\left(\int \frac{x^2}{(2-x^4)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= \frac{2\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt[4]{1-}}{\sqrt[4]{1+}}\right.}{a}$$

$$= -\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}\,a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\,\sqrt[4]{1-a}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}\,a}$$

$$= \frac{\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\log\left(\right.}{}$$

<u>**Mathematica**</u> [A]
time = 0.11, size = 96, normalized size = 0.50

$$\frac{\sqrt{2}\left(\tan^{-1}\left(\frac{-\sqrt{1-ax}+\sqrt{1+ax}}{\sqrt{2}\,\sqrt[4]{1-a^2x^2}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{1-a^2x^2}}{\sqrt{1-ax}+\sqrt{1+ax}}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*(ArcTan[(-Sqrt[1 - a*x] + Sqrt[1 + a*x])/(Sqrt[2]*(1 - a^2*x^2)^(1
/4))] + ArcTanh[(Sqrt[2]*(1 - a^2*x^2)^(1/4))/(Sqrt[1 - a*x] + Sqrt[1 + a*x
])]))/a
```

<u>**Maple**</u> [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(154) = 308$.
time = 1.01, size = 448, normalized size = 2.32

$$2\sqrt{2}{\frac{1}{a^{4}}}^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^{3}{\frac{1}{a^{4}}}^{\frac{3}{4}}-\sqrt{2}(a^{4}x-a^{3})\sqrt{\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a{\frac{1}{a^{4}}}^{\frac{1}{4}}+(a^{3}x-a^{2})\sqrt{\frac{1}{a^{4}}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}{\frac{1}{a^{4}}}^{\frac{3}{4}}+ax-1}{ax-1}\right)+2\sqrt{2}{\frac{1}{a^{4}}}^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^{3}{\frac{1}{a^{4}}}^{\frac{3}{4}}-\sqrt{2}(a^{4}x-a^{3})\sqrt{-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a{\frac{1}{a^{4}}}^{\frac{1}{4}}-(a^{3}x-a^{2})\sqrt{\frac{1}{a^{4}}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}{\frac{1}{a^{4}}}^{\frac{3}{4}}-ax+1}{ax-1}\right)-\frac{1}{2}\sqrt{2}{\frac{1}{a^{4}}}^{\frac{1}{4}}\log\left(\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a{\frac{1}{a^{4}}}^{\frac{1}{4}}+(a^{3}x-a^{2})\sqrt{\frac{1}{a^{4}}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)+\frac{1}{2}\sqrt{2}{\frac{1}{a^{4}}}^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a{\frac{1}{a^{4}}}^{\frac{1}{4}}-(a^{3}x-a^{2})\sqrt{\frac{1}{a^{4}}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a^3*(a^(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt((sqrt(2)*(a*x + 1)^(1/4)*(-
a*x + 1)^(3/4)*a*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1
)*sqrt(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) + a*x - 1)/(a*x - 1)) + 2*sqrt(
2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a^3*(a^
(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x +
1)^(3/4)*a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt
(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) - a*x + 1)/(a*x - 1)) - 1/2*sqrt(2)*(
a^(-4))^(1/4)*log((sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a*(a^(-4))^(1/4
) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x - 1)) +
 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt(-a*x + 1
))/(a*x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(a*x+1)**(3/4),x)
```

```
[Out] Integral(1/((-a*x + 1)**(1/4)*(a*x + 1)**(3/4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="giac")
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1-a\,x)^{1/4}\,(a\,x+1)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)),x)
[Out] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)), x)
```

3.1731 $\int \frac{(a+bx)^{3/2}}{\sqrt[5]{c+dx}} \, dx$

Optimal. Leaf size=74

$$\frac{2(a+bx)^{5/2} \sqrt[5]{\frac{b(c+dx)}{bc-ad}} \, {}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

```
[Out] 2/5*(b*x+a)^(5/2)*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([1/5, 5/2],[7/2],-
d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/5)
```

Rubi [A]

time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{2(a+bx)^{5/2} \sqrt[5]{\frac{b(c+dx)}{bc-ad}} \, {}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{(a+bx)^{3/2}}{\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$

$$= \frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{2(a+bx)^{5/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/5),x]
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/5))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/5),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/5), x)
```

3.1732 $\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

```
[Out] 2/3*(b*x+a)^(3/2)*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([1/5, 3/2],[5/2],-
d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/5)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/5),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 3/2, 5/2, -((d*(a + b*x))/(b*c - a*d))])/(3*b*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{\sqrt{a+bx}}{\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= \frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{2(a+bx)^{3/2}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{5},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/5),x]
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/5))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx}}{\sqrt[5]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/5),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/5), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/5),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/5), x)
```

3.1733 $\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

```
[Out] 2*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([1/5, 1/2],[3/2],-d*(b*x+a)/(-a*d+
b*c))*(b*x+a)^(1/2)/b/(d*x+c)^(1/5)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/5)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt{a+bx}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$

$$= \frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.04, size = 71, normalized size = 0.99

$$\frac{2\sqrt{a+bx}\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{5},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/5)),x]
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[1/5, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/5))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/5),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/5)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/5),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/5)),x)
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/5)), x)
```

3.1734 $\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

```
[Out] -2*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([-1/2, 1/5],[1/2],-d*(b*x+a)/(-a*
d+b*c))/b/(d*x+c)^(1/5)/(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, -((
d*(a + b*x))/(b*c - a*d))])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{3/2}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= -\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.04, size = 71, normalized size = 0.99

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{5};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x]
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-1/2, 1/5, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/5))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/5),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/5)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/5),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/5)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/5)), x)
```

3.1735 $\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{c+dx}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

```
[Out] -2/3*(b*(d*x+c)/(-a*d+b*c))^(1/5)*hypergeom([-3/2, 1/5],[-1/2],-d*(b*x+a)/(
-a*d+b*c))/b/(b*x+a)^(3/2)/(d*x+c)^(1/5)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, -(
(d*(a + b*x))/(b*c - a*d))])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{c+dx}}\,dx = \frac{\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{5/2}\sqrt[5]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[5]{c+dx}}$$
$$= -\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};-\frac{d(a+bx)}{bc-ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$-\frac{2\sqrt[5]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{3}{2},\frac{1}{5};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[5]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x]
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/5)*Hypergeometric2F1[-3/2, 1/5, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/5))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/5)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(4/5)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[5]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/5),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/5)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/5),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/5)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{1/5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/5)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/5)), x)
```

3.1736 $\int (a+bx)^{5/2}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=487

$$\frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+$$

```
[Out] -9/352*(-a*d+b*c)^2*(b*x+a)^(3/2)*(d*x+c)^(1/6)/b/d^2+3/176*(-a*d+b*c)*(b*x
+a)^(5/2)*(d*x+c)^(1/6)/b/d+3/11*(b*x+a)^(7/2)*(d*x+c)^(1/6)/b+81/1408*(-a*
d+b*c)^3*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b/d^3-81/2816*3^(3/4)*(-a*d+b*c)^(11/3
)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)
))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(
2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b
*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d^4/(b*x+a)^(1/2)/(
-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$-\frac{81\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{11/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816bd^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{81\sqrt{a+bx}\sqrt[6]{c+dx}\,(bc-ad)^3}{1408bd^3}-\frac{9(a+bx)^{3/2}\sqrt[6]{c+dx}\,(bc-ad)^2}{352bd^2}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}\,(bc-ad)}{176bd}+\frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]
```

```
[Out] (81*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(1408*b*d^3) - (9*(b*c - a
*d)^2*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(352*b*d^2) + (3*(b*c - a*d)*(a + b*
x)^(5/2)*(c + d*x)^(1/6))/(176*b*d) + (3*(a + b*x)^(7/2)*(c + d*x)^(1/6))/(
11*b) - (81*3^(3/4)*(b*c - a*d)^(11/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
 b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/
3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
```

```
- Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2816*b*d^4*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}+\frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}dx}{22b}$$

$$= \frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+bx)^{7/2}\sqrt[6]{c+dx}}{11b}-\frac{(15(bc-ad)^2)\int \frac{(a\cdots}{(c\cdots}}{352bd}$$

$$= -\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-ad)(a+bx)^{5/2}\sqrt[6]{c+dx}}{176bd}+\frac{3(a+b\cdots}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-a\cdots}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-a\cdots}{}$$

$$= \frac{81(bc-ad)^3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{1408bd^3}-\frac{9(bc-ad)^2(a+bx)^{3/2}\sqrt[6]{c+dx}}{352bd^2}+\frac{3(bc-a\cdots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.07, size = 73, normalized size = 0.15

$$\frac{2(a+bx)^{7/2}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/2, 9/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/2)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/2)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)*(d*x + c)^(1/6), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(5/2)*(c + d*x)^(1/6), x)
```

3.1737 $\int (a+bx)^{3/2}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=449

$$-\frac{27(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{320bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}+\frac{27\ 3^{3/4}(bc-ad)}{\quad}$$

[Out] 3/80*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/6)/b/d+3/8*(b*x+a)^(5/2)*(d*x+c)^(1/6)/b-27/320*(-a*d+b*c)^2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b/d^2+27/640*3^(3/4)*(-a*d+b*c)^(8/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.24, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{27\ 3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640bd^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{27\sqrt{a+bx}\sqrt[6]{c+dx}(bc-ad)^2}{320bd^2}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}(bc-ad)}{80bd}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]

[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(320*b*d^2) + (3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(80*b*d) + (3*(a + b*x)^(5/2)*(c + d*x)^(1/6))/(8*b) + (27*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(640*b*d^3*Sqrt[a + b*x]*S

```
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}dx}{16b}$$

$$= \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8b} - \frac{(9(bc-ad)^2)\int \frac{\sqrt{a\ldots}}{(c+d\ldots}}{160bd}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{320bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+bx\ldots}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{320bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+bx\ldots}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}{320bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{80bd} + \frac{3(a+bx\ldots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.16

$$\frac{2(a+bx)^{5/2}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(1/6),x]
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(3/2)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/6), x)
```

3.1738 $\int \sqrt{a+bx}\,\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=411

$$\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20bd}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b}-\frac{3\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+\right.}{}$$

[Out] 3/5*(b*x+a)^(3/2)*(d*x+c)^(1/6)/b+3/20*(-a*d+b*c)*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b/d-3/40*3^(3/4)*(-a*d+b*c)^(5/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.20, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$-\frac{3\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40bd^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{20bd}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*x]*(c + d*x)^(1/6),x]

[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*b*d) + (3*(a + b*x)^(3/2)*(c + d*x)^(1/6))/(5*b) - (3*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*b*d^2*Sqrt[a +

```
b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt[6]{c+dx}\,dx = \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}dx}{10b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{(3(bc-ad)^2)\int \frac{}{\sqrt{a+}}}{40bd}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{(9(bc-ad)^2)\,\text{Subst}\Bigg(}{}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20bd} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5b} - \frac{3\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{3/2}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/6),x)
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(1/6), x)
```

3.1739 $\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=375

$$\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b}+\frac{3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-c}}{\left(\sqrt[3]{bc-ad}-\left(1+\right.\right.}}}{4bd\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+}}{\left(\sqrt[3]{bc-\right.}}}}$$

[Out] 3/2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/b+1/4*3^(3/4)*(-a*d+b*c)^(2/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.17, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4bd\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(1/6)/Sqrt[a + b*x],x]

[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*b) + (3^(3/4)*(b*c - a*d)^(2/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4*b

```
*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx = \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{4b}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{2bd}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2b} + \frac{3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-a}{}}}{4bd\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 71, normalized size = 0.19

$$\frac{2\sqrt{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (d*(a +
b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(1/6)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(1/2), x)
```

3.1740 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}+\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+\cdots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}{\sqrt[4]{3}\,b\sqrt[3]{bc-ad}\,\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{b\cdots\right.}{\left(\sqrt[3]{bc-ad}-\left(1\cdots\right.}}}$$

```
[Out] -2*(d*x+c)^(1/6)/b/(b*x+a)^(1/2)+1/3*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1
/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/b/(-a*d+b*c)^(1/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1
/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {49, 65, 231}

$$\frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,b\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(b*Sqrt[a + b*x]) + ((c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b*(b*c - a*d)^(1/3)*Sq
```

```
rt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
 + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}} + \frac{d\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{3b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}} + \frac{2\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{b\sqrt{a+bx}} + \frac{\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}}{\left(\sqrt[3]{bc-ad}-\left(1+\right.\right.}}}{\sqrt[4]{3}\,b\sqrt[3]{bc-ad}\sqrt{a+bx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 71, normalized size = 0.19

$$-\frac{2\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(3/2),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(3/2),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(3/2), x)
```

3.1741 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=409

$$-\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}}-\frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}}-\frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{b}\cdots}{\left(\sqrt[3]{bc-ad}-\cdots\right.}}\cdots}{9\sqrt[4]{3}\,b(bc-ad)^{4/3}\sqrt{a\cdots}}$$

```
[Out] -2/3*(d*x+c)^(1/6)/b/(b*x+a)^(3/2)-2/9*d*(d*x+c)^(1/6)/b/(-a*d+b*c)/(b*x+a)
^(1/2)-2/27*d*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipti
cF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((
(-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/
3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b
/(-a*d+b*c)^(4/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {49, 53, 65, 231}

$$-\frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\,b\sqrt{a+bx}\,(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2d\sqrt[6]{c+dx}}{9b\sqrt{a+bx}\,(bc-ad)}-\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(3*b*(a + b*x)^(3/2)) - (2*d*(c + d*x)^(1/6))/(9*b*(b*
c - a*d)*Sqrt[a + b*x]) - (2*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
```

```
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b*(b*c - a*d)^(4/3)*Sqrt[a + b*
x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(
1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx}{9b}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(2d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(4d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+\cdot\right.}{9b(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3b(a+bx)^{3/2}} - \frac{2d\sqrt[6]{c+dx}}{9b(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.18

$$-\frac{2\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c + dx}}{(a + bx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(5/2), x)
```

3.1742 $\int (a+bx)^{3/2}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=896

$$-\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2}+\frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b}-\frac{81\Big(1+}{448b^{5/3}d^2\Big(}$$

```
[Out] 3/28*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(5/6)/b/d+3/10*(b*x+a)^(5/2)*(d*x+c)^
(5/6)/b-27/224*(-a*d+b*c)^2*(d*x+c)^(5/6)*(b*x+a)^(1/2)/b/d^2-81/448*(-a*d+
b*c)^3*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)/d^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-81/448*3^(1/4)*(-a*d+b*c)^(10/3)*(d*x+
c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^
(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d^3/(b*x+a)^(1/2)/(-
b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/896*3^(3/4)*(-a*d+b*c)
^(10/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+
c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/
2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+
c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(
5/3)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 896, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$-\frac{81\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{448b^{5/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{896b^{5/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{81\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}(bc-ad)^3}{448b^{5/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{27\sqrt{a+bx}(c+dx)^{5/6}(bc-ad)^2}{224bd^2}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}(bc-ad)}{28bd}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]
```

```
[Out] (-27*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*b*d^2) + (3*(b*c - a
*d)*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*b*d) + (3*(a + b*x)^(5/2)*(c + d*x
)^(5/6))/(10*b) - (81*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(
1/6))/(448*b^(5/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(5/3)*d^3*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (27*
3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(5/3)*d^3*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
```

```
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/6}\,dx = \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10b} - \frac{(9(bc-ad)^2)\,\text{J}}{56}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(}{}$$

$$= -\frac{27(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224bd^2} + \frac{3(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28bd} + \frac{3(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{5/2}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/2, 7/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{2}} (dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(3/2)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)*(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/6), x)
```

3.1743 $\int \sqrt{a+bx}\,(c+dx)^{5/6}\,dx$

Optimal. Leaf size=858

$$\frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b}+\frac{45\left(1+\sqrt{3}\right)(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{112b^{5/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right.}$$

```
[Out] 3/7*(b*x+a)^(3/2)*(d*x+c)^(5/6)/b+15/56*(-a*d+b*c)*(d*x+c)^(5/6)*(b*x+a)^(1
/2)/b/d+45/112*(-a*d+b*c)^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)
/d/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+45/112*3^(1/4)*(-a*
d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Elliptic
E((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((
-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3
))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d^
2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+15/224*
3^(3/4)*(-a*d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1
/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2
)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1
/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(5/3)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.55, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$\frac{45\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{112b^{5/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{15\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{224b^{5/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{45\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{112b^{5/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{15\sqrt{a+bx}\,(c+dx)^{5/6}(bc-ad)}{56bd}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/6),x]
```

```
[Out] (15*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*b*d) + (3*(a + b*x)^(3/2
)*(c + d*x)^(5/6))/(7*b) + (45*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(c
 + d*x)^(1/6))/(112*b^(5/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))) + (45*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d
)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c -
 a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1
/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(5/3)*d^2*Sqrt[a +
 b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
+ (15*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(5/3)*d^2*Sqrt[a + b
*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^
(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/6}\,dx = \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{(5(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14b}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} - \frac{(15(bc-ad)^2)\int}{}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} - \frac{(45(bc-ad)^2)\,S}{}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{(45(bc-ad)^2)\,S}{}$$

$$= \frac{15(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56bd} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7b} + \frac{45\left(1+\sqrt{3}\right.}{112b^{5/3}d\left(\sqrt[3]{bc-}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 3/2, 5/2, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx+a}\,(dx+c)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/2)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,(c+dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/6),x)
```

```
[Out] Integral(sqrt(a + b*x)*(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)*(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,x}\,\,{\left(c+d\,x\right)}^{5/6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/6), x)
```

3.1744 $\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=817

$$\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b}-\frac{15\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{15\sqrt[4]{3}\,(bc-ad)^{4/3}\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-\right.}{}$$

```
[Out] 3/4*(d*x+c)^(5/6)*(b*x+a)^(1/2)/b-15/8*(-a*d+b*c)*(d*x+c)^(1/6)*(1+3^(1/2))
*(b*x+a)^(1/2)/b^(5/3)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))
-15/8*3^(1/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b
*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))
^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^
(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2
)^(1/2)/b^(5/3)/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^
2)^(1/2)-5/16*3^(3/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1
/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)
*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 817, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$-\frac{15\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{8b^{5/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{16b^{5/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{15\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}(bc-ad)}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/Sqrt[a + b*x],x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*b) - (15*(1 + Sqrt[3])*(b*c - a*d)*Sqr
t[a + b*x]*(c + d*x)^(1/6))/(8*b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))) - (15*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(5/3)*d*Sqr
t[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c
+ d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^
2)]) - (5*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(5/3)*d*Sqrt[a +
b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)
^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}}\,dx = \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} + \frac{(5(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{8b}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} + \frac{(15(bc-ad))\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4bd}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} - \frac{(15(bc-ad))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+d
}{8b^{5/3}d}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4b} - \frac{15\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{15\sqrt[4]{3}\,(bc-a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\,(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(1/2),x)
[Out] Integral((c + d*x)**(5/6)/sqrt(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/6)/sqrt(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(1/2),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(1/2), x)
```

3.1745 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=798

$$-\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}}-\frac{5\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{5\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{}$$

```
[Out] -2*(d*x+c)^(5/6)/b/(b*x+a)^(1/2)-5*d*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2
)/b^(5/3)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-5*3^(1/4)*(-
a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipt
icE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2
/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/
(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/6*(-a*d
+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+
b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*
(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF
((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3
^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(
d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 798, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 65, 314, 231, 1895}

$$-\frac{5\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{5\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{5/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{5/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(b*Sqrt[a + b*x]) - (5*(1 + Sqrt[3])*d*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(b^(5/3)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (5*3^(1/4)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(5/3)*Sqrt[a + b*x]*Sqrt[-((
b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(1 - Sqrt
[3])*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/
3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1
/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(
c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^
(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{3b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} + \frac{10\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} - \frac{5\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{5/3}} - \frac{\left(5\left(1-\sqrt{3}\right.\right.\cdots}{\cdots}$$

$$= -\frac{2(c+dx)^{5/6}}{b\sqrt{a+bx}} - \frac{5\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{5/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{5\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.09

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^{\frac{5}{6}}}{(a + bx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x)^{5/6}}{(a + b\,x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(5/6)/(a + b*x)^(3/2), x)
```

3.1746 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=854

$$-\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}}-\frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}}-\frac{10\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{10d\sqrt[6]{c+\cdots}}{\cdots}$$

```
[Out] -2/3*(d*x+c)^(5/6)/b/(b*x+a)^(3/2)-10/9*d*(d*x+c)^(5/6)/b/(-a*d+b*c)/(b*x+a
)^(1/2)-10/9*d^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)/(-a*d+b*c)
/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-10/9*d*(d*x+c)^(1/6)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^
2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a
*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(-a*d+b*c)^(2/3)/(b*x
+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/27*d*(d*x+c
)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*
c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(
-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 854, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 53, 65, 314, 231, 1895}

$$-\frac{10\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^2}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{10\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{3\,3^{3/4}b^{5/3}(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{5\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{9\sqrt[4]{3}\,b^{5/3}(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{10(c+dx)^{5/6}d}{9b(bc-ad)\sqrt{a+bx}}-\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(3*b*(a + b*x)^(3/2)) - (10*d*(c + d*x)^(5/6))/(9*b*(b
*c - a*d)*Sqrt[a + b*x]) - (10*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*b^(5/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))) - (10*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x
)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3
) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
+ d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*
Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (5*(
1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
)*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(
2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c +
d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (
2 + Sqrt[3])/4])/(9*3^(1/4)*b^(5/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{9b}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} + \frac{(10d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} + \frac{(20d)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+\right.}{9b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \frac{(10d)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx\right.}{9b^{5/3}(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \frac{10\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{9b^{5/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.09

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-3/2, -5/6, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
 a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/2),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(5/2), x)
```

3.1747 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=896

$$-\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}}-\frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}}+\frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}}+\frac{8\left(1+\sqrt{3}\right)d^3\sqrt{a+bx}\,\sqrt[6]{c+}}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right.\right.}$$

```
[Out] -2/5*(d*x+c)^(5/6)/b/(b*x+a)^(5/2)-2/9*d*(d*x+c)^(5/6)/b/(-a*d+b*c)/(b*x+a)
^(3/2)+8/27*d^2*(d*x+c)^(5/6)/b/(-a*d+b*c)^2/(b*x+a)^(1/2)+8/27*d^3*(d*x+c)
^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(5/3)/(-a*d+b*c)^2/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+8/27*d^2*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^
2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c
)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(1/4)/b^(5/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*
(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/81*d^2*(d*x+c)^(1/6)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-
3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/(
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))
^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(
-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(-a*d+b*c)^(5/3)/(b
*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 896, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 53, 65, 314, 231, 1895}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^3}{27b^{5/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d^2}{9\,3^{3/4}b^{5/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{4\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d^2}{27\sqrt[4]{3}\,b^{5/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{8(c+dx)^{5/6}d^2}{27b(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/6}d}{9b(bc-ad)(a+bx)^{3/2}}-\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(5*b*(a + b*x)^(5/2)) - (2*d*(c + d*x)^(5/6))/(9*b*(b*
c - a*d)*(a + b*x)^(3/2)) + (8*d^2*(c + d*x)^(5/6))/(27*b*(b*c - a*d)^2*Sqr
t[a + b*x]) + (8*(1 + Sqrt[3])*d^3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(27*b^(5/
3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)) + (8*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*S
qrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3
)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x
)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 +
 Sqrt[3])/4])/(9*3^(3/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (4*(1 - Sqrt[3
])*d^2*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[(
(b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
+ d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2
]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/
3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt
[3])/4])/(27*3^(1/4)*b^(5/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3
)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} + \frac{d\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx}{3b}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} - \frac{(4d^2)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{27b(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} - \frac{(8d^3)\int \frac{}{\sqrt{a+bx}}}{81b(bc-}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} - \frac{(16d^2)\,\text{Subst}\Bigg(\int}{}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \frac{(8d^2)\,\text{Subst}\Bigg(\int}{}$$

$$= -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \frac{8\Big(1}{27b^{5/3}(bc-ad)^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.08

$$-\frac{2(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{6};-\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/6)*Hypergeometric2F1[-5/2, -5/6, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(7/2),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(5/6)/(a + b*x)^(7/2), x)
```

3.1748 $\int \frac{(a+bx)^{5/2}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=890

$$\frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3}-\frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d}+\frac{243\Big(1+\cdots}{448b^{2/3}d^3\Big(\sqrt[3]{\cdots}}$$

```
[Out] -9/28*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(5/6)/d^2+3/10*(b*x+a)^(5/2)*(d*x+c)
^(5/6)/d+81/224*(-a*d+b*c)^2*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d^3+243/448*(-a*d+
b*c)^3*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)/d^3/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+243/448*3^(1/4)*(-a*d+b*c)^(10/3)*(d*x
+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b
^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^4/(b*x+a)^(1/2)/(
-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+81/896*3^(3/4)*(-a*d+b*c
)^(10/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-
3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1
-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1
/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x
+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^
(2/3)/d^4/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 890, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$\frac{243\sqrt[4]{3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{448b^{2/3}d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{81\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{10/3}}{896b^{2/3}d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{243\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}(bc-ad)^3}{448b^{2/3}d^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{81\sqrt{a+bx}(c+dx)^{5/6}(bc-ad)^2}{224d^3}-\frac{9(a+bx)^{3/2}(c+dx)^{5/6}(bc-ad)}{28d^2}+\frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]
```

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(5/6))/(224*d^3) - (9*(b*c - a*d)
*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(28*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(
5/6))/(10*d) + (243*(1 + Sqrt[3])*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(1/
6))/(448*b^(2/3)*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))) + (243*3^(1/4)*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
 - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(
1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (
1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(448*b^(2/3)*d^4*Sqrt[a + b*x]*S
qrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (81*3
^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(10/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(896*b^(2/3)*d^4*Sqrt[a + b*x]*Sq
rt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
)/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
```

```
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt[6]{c+dx}}\,dx = \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} - \frac{(3(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{4d}$$

$$= -\frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c+dx)^{5/6}}{10d} + \frac{(27(bc-ad)^2)\int \frac{\sqrt{a+}}{\sqrt[6]{c+}}}{56d^2}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{5/6}}{224d^3} - \frac{9(bc-ad)(a+bx)^{3/2}(c+dx)^{5/6}}{28d^2} + \frac{3(a+bx)^{5/2}(c}{10d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{7/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{\sqrt[6]{c+dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/6), x)
```

3.1749 $\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=855

$$-\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d}-\frac{81\left(1+\sqrt{3}\right)(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right.}$$

```
[Out] 3/7*(b*x+a)^(3/2)*(d*x+c)^(5/6)/d-27/56*(-a*d+b*c)*(d*x+c)^(5/6)*(b*x+a)^(1
/2)/d^2-81/112*(-a*d+b*c)^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)
/d^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-81/112*3^(1/4)*(-
a*d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipt
icE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2
/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/
d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/22
4*3^(3/4)*(-a*d+b*c)^(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/(
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^
(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)/b^(2/3)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 855, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$-\frac{81\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{112b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{7/3}}{224b^{2/3}d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{81\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{27\sqrt{a+bx}\,(c+dx)^{5/6}(bc-ad)}{56d^2}+\frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]
```

```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(5/6))/(56*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(5/6))/(7*d) - (81*(1 + Sqrt[3])*(b*c - a*d)^2*Sqrt[a + b*x]*(
c + d*x)^(1/6))/(112*b^(2/3)*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)
*(c + d*x)^(1/3))) - (81*3^(1/4)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*b^(2/3)*d^3*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
]) - (27*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*
d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
- a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
- (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(
1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqr
t[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*b^(2/3)*d^3*Sqrt[a
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx = \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14d}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} + \frac{(27(bc-ad)^2)\int \frac{}{\sqrt{a+}}}{112d^2}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} + \frac{(81(bc-ad)^2)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{(81(bc-ad)^2)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \frac{81\left(1+\sqrt{3}\right)(bc-}{112b^{2/3}d^2\left(\sqrt[3]{bc-ad}\right.-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/6), x)
```

3.1750 $\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=820

$$\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d}+\frac{9\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{9\sqrt[4]{3}\,(bc-ad)^{4/3}\sqrt[6]{c+dx}\,\left(\sqrt[3]{bc-\right.}{}$$

```
[Out] 3/4*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d+9/8*(-a*d+b*c)*(d*x+c)^(1/6)*(1+3^(1/2))*
(b*x+a)^(1/2)/b^(2/3)/d/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)+9/8*3^(1/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b
*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+
3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))
^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^
(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2
)^(1/2)/b^(2/3)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)+3/16*3^(3/4)*(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^
2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^
(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c
)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(
1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 820, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {52, 65, 314, 231, 1895}

$$\frac{9\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{8b^{2/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)(bc-ad)^{4/3}}{16b^{2/3}d^2\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{9\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(1/6),x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(5/6))/(4*d) + (9*(1 + Sqrt[3])*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(1/6))/(8*b^(2/3)*d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))) + (9*3^(1/4)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/
3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d
)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c
- a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
(1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*b^(2/3)*d^2*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)]) + (3*3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c -
 a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b
*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/
3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d
)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*b^(2/3)*d^2*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
])
```

<u>Rule 52</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 231</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx = \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{8d}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{4d^2}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} + \frac{(9(bc-ad))\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{8b^{2/3}d^2}$$

$$= \frac{3\sqrt{a+bx}\,(c+dx)^{5/6}}{4d} + \frac{9\left(1+\sqrt{3}\right)(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8b^{2/3}d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} + \frac{9\sqrt[4]{3}\,(bc-ad}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(1/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(1/6), x)
```

3.1751 $\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=780

$$-\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{3\sqrt[4]{3}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\quad}}{}$$

```
[Out] -3*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))-3*3^(1/4)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1
/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*
c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^
(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(3/4)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6
)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+
c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/
3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/d/(b*x+a)^(1/2)
/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {65, 314, 231, 1895}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt[6]{c+dx}\sqrt[3]{bc-ad}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}d\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(1/6)),x]
```

```
[Out] (-3*(1 + Sqrt[3])*Sqrt[a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (3*3^(1/4)*(b*c - a*d)^(1/3)*
(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c -
a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^
(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*Ellip
ticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4]
)/(b^(2/3)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2)]) - (3^(3/4)*(1 - Sqrt[3])*(b*c - a*d)^(1/3)*(c + d*x)^(
1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3)
+ b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*
c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos
[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*b^(2/3
)*d*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1
/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx = \frac{6\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{b^{2/3}d} - \frac{\left(3\left(1-\sqrt{3}\right)(bc-\right.}{}$$

$$= -\frac{3\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{2/3}\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{3\sqrt[4]{3}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+dx}\left(\sqrt[3]{\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\,\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(1/6)),x]
```

[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6, 1
/2, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/6))

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)

[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/6)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/6),x)

[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(1/6)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(1/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,\left(c+d\,x\right)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/6)), x)
```

3.1752 $\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=813

$$-\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}-\frac{2\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{2\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\right.}{}$$

```
[Out] -2*(d*x+c)^(5/6)/(-a*d+b*c)/(b*x+a)^(1/2)-2*d*(d*x+c)^(1/6)*(1+3^(1/2))*(b*
x+a)^(1/2)/b^(2/3)/(-a*d+b*c)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))-2*3^(1/4)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*((
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ell
ipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)
^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)-1/3*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellipt
icF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3
)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/b^(2/3)/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 813, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$-\frac{2\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{2\sqrt[4]{3}\,\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(1 + Sqrt[3])*d*Sqrt[
a + b*x]*(c + d*x)^(1/6))/(b^(2/3)*(b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))) - (2*3^(1/4)*(c + d*x)^(1/6)*((b*c - a*d)^
(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a
*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (
1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3
) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(b^(2/3)*(b*c - a*d)^(2/3)*
Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*
(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3
))^2)]) - ((1 - Sqrt[3])*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(
1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*
(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*b^(2/3)*(b*c - a*d)^(2/3)*Sqrt[a + b*x]
*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/
3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx = -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}+\frac{(2d)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{3(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}+\frac{4\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx,x,\sqrt[6]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}-\frac{2\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx,x,\sqrt[6]{c+dx}\right)}{b^{2/3}(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{(bc-ad)\sqrt{a+bx}}-\frac{2\left(1+\sqrt{3}\right)d\sqrt{a+bx}\,\sqrt[6]{c+dx}}{b^{2/3}(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$-\frac{2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(1/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/6),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(1/6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(1/6)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/6)), x)
```

3.1753 $\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=858

$$-\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}}+\frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}}+\frac{8\left(1+\sqrt{3}\right)d^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+$$

```
[Out] -2/3*(d*x+c)^(5/6)/(-a*d+b*c)/(b*x+a)^(3/2)+8/9*d*(d*x+c)^(5/6)/(-a*d+b*c)^
2/(b*x+a)^(1/2)+8/9*d^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/b^(2/3)/(-a
*d+b*c)^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+8/9*d*(d*x+c
)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)
*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^
(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(
1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(-a*d+b*c)^(5
/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/27*
d*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)
))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((
(-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/
3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b
^(2/3)/(-a*d+b*c)^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{a+bx}\,\sqrt[6]{c+dx}\,d^2}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{8\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{3\,3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{4\left(1-\sqrt{3}\right)\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{9\sqrt[4]{3}\,b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}+\frac{8(c+dx)^{5/6}d}{9(bc-ad)^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*(c + d*x)^(5/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (8*d*(c + d*x)^(5/6)
)/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (8*(1 + Sqrt[3])*d^2*Sqrt[a + b*x]*(c +
 d*x)^(1/6))/(9*b^(2/3)*(b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))) + (8*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)
*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c +
d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^
(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3
])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c +
 d*x)^(1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[
a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c +
d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)
]) + (4*(1 - Sqrt[3])*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*b^(2/3)*(b*c - a*d)^(5/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}\sqrt[6]{c+dx}}\,dx = -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(4d)\int \frac{1}{(a+bx)^{3/2}\sqrt[6]{c+dx}}\,dx}{9(bc-ad)}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(8d^2)\int \frac{1}{\sqrt{a+bx}\sqrt[6]{c+dx}}\, d}{27(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} - \frac{(16d)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+}}\right.}{9(bc-a}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(8d)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-}{\sqrt{a-\frac{bc}{d}}}\right.}{9b^{2/3}(b}$$

$$= -\frac{2(c+dx)^{5/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{8d(c+dx)^{5/6}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{8\left(1+\sqrt{3}\right)d^2\sqrt{}}{9b^{2/3}(bc-ad)^2\left(\sqrt[3]{bc-ad}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.05, size = 73, normalized size = 0.09

$$-\frac{2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{3}{2},\frac{1}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(1/6))
```

Maple [F]

time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(1/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(1/6)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/2}\,(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/6)), x)
```

3.1754 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=440

$$\frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3}-\frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d}-\frac{81\ 3^{3/4}(bc-ad)}{\ }$$

```
[Out] -9/16*(-a*d+b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/6)/d^2+3/8*(b*x+a)^(5/2)*(d*x+c)^
(1/6)/d+81/64*(-a*d+b*c)^2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/d^3-81/128*3^(3/4)*(
-a*d+b*c)^(8/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1
/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Ellip
ticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(
2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^4/(b*
x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))
/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$-\frac{81\ 3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{8/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{128d^4\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{81\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}{64d^3}-\frac{9(a+bx)^{3/2}\sqrt[6]{c+dx}\,(bc-ad)}{16d^2}+\frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]
```

```
[Out] (81*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))/(64*d^3) - (9*(b*c - a*d)*
(a + b*x)^(3/2)*(c + d*x)^(1/6))/(16*d^2) + (3*(a + b*x)^(5/2)*(c + d*x)^(1
/6))/(8*d) - (81*3^(3/4)*(b*c - a*d)^(8/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
```

```
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(128*d^4*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}}\,dx = \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} - \frac{(15(bc-ad))\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx}{16d}$$

$$= -\frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+dx}}{8d} + \frac{(27(bc-ad)^2)\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}}{32d^2}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+}}{8d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+}}{8d}$$

$$= \frac{81(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}}{64d^3} - \frac{9(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}{16d^2} + \frac{3(a+bx)^{5/2}\sqrt[6]{c+}}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 73, normalized size = 0.17

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/6), x)
```

3.1755 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}} \, dx$

Optimal. Leaf size=405

$$-\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d}+\frac{27\,3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c\right.}}{}$$

```
[Out] 3/5*(b*x+a)^(3/2)*(d*x+c)^(1/6)/d-27/20*(-a*d+b*c)*(d*x+c)^(1/6)*(b*x+a)^(1
/2)/d^2+27/40*3^(3/4)*(-a*d+b*c)^(5/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^
(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{27\,3^{3/4}\sqrt[6]{c+dx}\,(bc-ad)^{5/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40d^3\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{27\sqrt{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}{20d^2}+\frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]
```

```
[Out] (-27*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6))/(20*d^2) + (3*(a + b*x)^(3/
2)*(c + d*x)^(1/6))/(5*d) + (27*3^(3/4)*(b*c - a*d)^(5/3)*(c + d*x)^(1/6)*(
(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1
/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*
d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c
 - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) -
 (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(40*d^3*Sqrt[a
```

```
+ b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*
x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}}\,dx = \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} - \frac{(9(bc-ad))\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx}{10d}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{(27(bc-ad)^2)\int \frac{1}{\sqrt{a+bx}\,(\cdots}}{40d^2}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{(81(bc-ad)^2)\,\text{Subst}\left(\int \frac{\cdots}{\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{27(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{20d^2} + \frac{3(a+bx)^{3/2}\sqrt[6]{c+dx}}{5d} + \frac{27\ 3^{3/4}(bc-ad)^{5/3}\sqrt[6]{c+dx}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.18

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/6), x)
```

3.1756 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=372

$$\frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d}-\frac{3\,3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-}}{\left(\sqrt[3]{bc-ad}-\left(1+\right.\right.}}}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+}}{\left(\sqrt[3]{bc-}\right.}}}$$

```
[Out] 3/2*(d*x+c)^(1/6)*(b*x+a)^(1/2)/d-3/4*3^(3/4)*(-a*d+b*c)^(2/3)*(d*x+c)^(1/6
)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+
c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*
(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(
-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)
^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {52, 65, 231}

$$\frac{3\sqrt{a+bx}\sqrt[6]{c+dx}}{2d}-\frac{3\,3^{3/4}\sqrt[6]{c+dx}(bc-ad)^{2/3}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/6),x]
```

```
[Out] (3*Sqrt[a + b*x]*(c + d*x)^(1/6))/(2*d) - (3*3^(3/4)*(b*c - a*d)^(2/3)*(c +
 d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)
^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3
))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF
[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c -
 a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(4
*d^2*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(
```

```
1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)
^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/6}}\,dx = \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{4d}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d} - \frac{(9(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{2d^2}$$

$$= \frac{3\sqrt{a+bx}\,\sqrt[6]{c+dx}}{2d} - \frac{3\,3^{3/4}(bc-ad)^{2/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-}{}}}{4d^2\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 73, normalized size = 0.20

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/6),x]
```

```
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/6),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(5/6), x)
```

3.1757 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=343

$$\frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{c+dx}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{c}\cdots\right.}{d\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{\cdots}\right.}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c\cdots}\right.}}}$$

```
[Out] 3^(3/4)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+
c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3
^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-
((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+
b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d/(-a*d+b*c)^(1
/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {65, 231}

$$\frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d\sqrt{a+bx}\sqrt[3]{bc-ad}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/6)),x]
```

```
[Out] (3^(3/4)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt
[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(
c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sq
rt[3])/4])/(d*(b*c - a*d)^(1/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/
3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx = \frac{6\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d}$$

$$= \frac{3^{3/4}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[3]{\cdots}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\cdots\right.\right.}}}{d\sqrt[3]{bc-ad}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\cdots}{\left(\sqrt[3]{\cdots}\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.21

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/2, 5
/6, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/6),x)
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(5/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/6)), x)
```

3.1758 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=372

$$-\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}}-\frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[3]{c+dx}+b\cdots}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+\cdots}\right.}}\cdots}{\sqrt[4]{3}\,(bc-ad)^{4/3}\sqrt{a+bx}\,\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+\cdots}}{\left(\sqrt[3]{bc-a\cdots}\right.}}}$$

```
[Out] -2*(d*x+c)^(1/6)/(-a*d+b*c)/(b*x+a)^(1/2)-2/3*(d*x+c)^(1/6)*((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/
2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(
d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(-a*d+b*c)^(4/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*
x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/
3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 231}

$$-\frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt{a+bx}\,(bc-ad)^{4/3}\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}}-\frac{2\sqrt[6]{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/((b*c - a*d)*Sqrt[a + b*x]) - (2*(c + d*x)^(1/6)*((b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*
(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a
*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1
+ Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d
)^(4/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) -
```

```
b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx = -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}\,dx}{3(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{bc-ad}$$

$$= -\frac{2\sqrt[6]{c+dx}}{(bc-ad)\sqrt{a+bx}} - \frac{2\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2}\ldots}{\left(\sqrt[3]{\ldots}\right.}}}{\sqrt[4]{3}\,(bc-\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 71, normalized size = 0.19

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*b*
d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(5/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{5/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/6)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/6)), x)
```

3.1759 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=410

$$-\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}}+\frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\cdots}{\left(\sqrt[3]{bc-\cdots}\right.\cdots}}\cdots}{9\sqrt[4]{3}(bc-ad)\cdots}$$

```
[Out] -2/3*(d*x+c)^(1/6)/(-a*d+b*c)/(b*x+a)^(3/2)+16/9*d*(d*x+c)^(1/6)/(-a*d+b*c)
^2/(b*x+a)^(1/2)+16/27*d*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1
/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2
)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*
(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2
)*3^(3/4)/(-a*d+b*c)^(7/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {53, 65, 231}

$$\frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\sqrt[4]{3}\sqrt{a+bx}(bc-ad)^{7/3}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{16d\sqrt[6]{c+dx}}{9\sqrt{a+bx}(bc-ad)^2}-\frac{2\sqrt[6]{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*(c + d*x)^(1/6))/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (16*d*(c + d*x)^(1/6
))/(9*(b*c - a*d)^2*Sqrt[a + b*x]) + (16*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(7/3)*S
```

```
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/6}}\,dx = -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} - \frac{(8d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/6}}\,dx}{9(bc-ad)}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(16d^2)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/6}}}{27(bc-ad)^2}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{(32d)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{bc}{d}+}}\right.}{9(bc-}$$

$$= -\frac{2\sqrt[6]{c+dx}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d\sqrt[6]{c+dx}}{9(bc-ad)^2\sqrt{a+bx}} + \frac{16d\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}\right.-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.18

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(-\frac{3}{2},\frac{5}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(1/6)/(b^3*d*x^4 + a^3*c + (b^3*c + 3*a*b^
2*d)*x^3 + 3*(a*b^2*c + a^2*b*d)*x^2 + (3*a^2*b*c + a^3*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(5/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/6)),x)
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/6)), x)
```

3.1760 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=880

$$-\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}}-\frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3}+\frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2}-\frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-\cdots}{112d^3\left(\sqrt[3]{bc-ad}-\left(1+\cdots\right.\right.}$$

```
[Out] -6*(b*x+a)^(5/2)/d/(d*x+c)^(1/6)+45/7*b*(b*x+a)^(3/2)*(d*x+c)^(5/6)/d^2-405
/56*b*(-a*d+b*c)*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d^3-1215/112*b^(1/3)*(-a*d+b*c
)^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/d^3/((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3)*(1+3^(1/2)))-1215/112*3^(1/4)*b^(1/3)*(-a*d+b*c)^(7/3)*(d*x+c)
^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/
3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1
/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)
-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^4/(b*x+a)^(1/2)/(-b^(1/3)*(d
*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-405/224*3^(3/4)*b^(1/3)*(-a*d+b*c)^
(7/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2)
)*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)
^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^4/(
b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 880, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 52, 65, 314, 231, 1895}

$$-\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}}+\frac{45b(c+dx)^{5/6}(a+bx)^{3/2}}{7d^2}-\frac{405b(bc-ad)(c+dx)^{5/6}\sqrt{a+bx}}{56d^3}-\frac{1215\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)^2\sqrt[6]{c+dx}\,\sqrt{a+bx}}{112d^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}-\frac{1215\sqrt[4]{3}\,\sqrt[3]{b}\,(bc-ad)^{7/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112d^4\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,\sqrt{a+bx}}-\frac{405\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)^{7/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224d^4\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(5/2))/(d*(c + d*x)^(1/6)) - (405*b*(b*c - a*d)*Sqrt[a + b*x]
*(c + d*x)^(5/6))/(56*d^3) + (45*b*(a + b*x)^(3/2)*(c + d*x)^(5/6))/(7*d^2)
 - (1215*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)^2*Sqrt[a + b*x]*(c + d*x)^(1/6))
/(112*d^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))) - (1
215*3^(1/4)*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(112*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (405*3^(3/4)*(1
- Sqrt[3])*b^(1/3)*(b*c - a*d)^(7/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b
^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)
*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3
)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(224*d^4*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} + \frac{(15b)\int \frac{(a+bx)^{3/2}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{(135b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{14d^2}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} + \frac{(405}{}$$

$$(121$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} + \frac{}{}$$

$$\Big(121$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{}{}$$

$$= -\frac{6(a+bx)^{5/2}}{d\sqrt[6]{c+dx}} - \frac{405b(bc-ad)\sqrt{a+bx}\,(c+dx)^{5/6}}{56d^3} + \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{1215}{112}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.08

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(7/6),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(7/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
[Out] int((b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*(d*x + c)^(5/6)/(d^2*x^2 +
 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/2)/(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(7/6),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(7/6), x)
```

3.1761 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=844

$$-\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}}+\frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2}+\frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{81\sqrt[4]{3}\,\sqrt[3]{b}\,(bc-}{}$$

```
[Out] -6*(b*x+a)^(3/2)/d/(d*x+c)^(1/6)+27/4*b*(d*x+c)^(5/6)*(b*x+a)^(1/2)/d^2+81/
8*b^(1/3)*(-a*d+b*c)*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/d^2/((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+81/8*3^(1/4)*b^(1/3)*(-a*d+b*c)^
(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(
1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*
c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^3/(b*x+a)^(1/2)
/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+27/16*3^(3/4)*b^(1/3)*
(-a*d+b*c)^(4/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1
/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*Elli
pticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)
^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2
/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^
(1/2)/d^3/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(
d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.55, antiderivative size = 844, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {49, 52, 65, 314, 231, 1895}

$$-\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}}+\frac{27b(c+dx)^{5/6}\sqrt{a+bx}}{4d^2}+\frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)\sqrt[6]{c+dx}\,\sqrt{a+bx}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}+\frac{81\sqrt[4]{3}\,\sqrt[3]{b}\,(bc-ad)^{4/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8d^3\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,\sqrt{a+bx}}+\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)^{4/3}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16d^3\sqrt{-\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)^2}}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(3/2))/(d*(c + d*x)^(1/6)) + (27*b*Sqrt[a + b*x]*(c + d*x)^(5
/6))/(4*d^2) + (81*(1 + Sqrt[3])*b^(1/3)*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x
)^(1/6))/(8*d^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))
) + (81*3^(1/4)*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3
) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^
(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) -
(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b
^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(8*d^3*Sqrt[a + b*x]*Sqrt[-((b^
(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
- a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (27*3^(3/4)*(1
 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(4/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) -
b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3
)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 -
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/
3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(16*d^3*Sqrt[a + b*x]*Sqrt[-((b^(1/
3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a
*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{(9b)\int \frac{\sqrt{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} - \frac{(27b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{8d^2}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} - \frac{(81b(bc-ad))\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx\right.}{4d^3}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} + \frac{\left(81\sqrt[3]{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-a}{\sqrt{a-\frac{bc}{d}}}\right.}{8d^3}$$

$$= -\frac{6(a+bx)^{3/2}}{d\sqrt[6]{c+dx}} + \frac{27b\sqrt{a+bx}\,(c+dx)^{5/6}}{4d^2} + \frac{81\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+d}}{8d^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(7/6),x]
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(3/2)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/2)/(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(3/2)/(c + d*x)^(7/6), x)
```

3.1762 $\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=806

$$-\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}}-\frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{9\sqrt[4]{3}\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\right.}{}$$

```
[Out] -6*(b*x+a)^(1/2)/d/(d*x+c)^(1/6)-9*b^(1/3)*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a
)^(1/2)/d/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-9*3^(1/4)*b^
(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^
(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d
*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/2*3
^(3/4)*b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+
b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^
(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1
+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2))
)^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*
x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(
1+3^(1/2)))^2)^(1/2)/d^2/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 806, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {49, 65, 314, 231, 1895}

$$-\frac{9\sqrt[4]{3}\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{bc-ad}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2d^2\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}}-\frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(7/6),x]
```

```
[Out] (-6*Sqrt[a + b*x])/(d*(c + d*x)^(1/6)) - (9*(1 + Sqrt[3])*b^(1/3)*Sqrt[a +
b*x]*(c + d*x)^(1/6))/(d*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))) - (9*3^(1/4)*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a
*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c
 - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3)
 - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^
(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d^2*Sqrt[a + b*x]*Sqrt
[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/
((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (3*3^(3/
4)*(1 - Sqrt[3])*b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)
^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 +
 Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) -
 (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(2*d^2*Sqrt[a + b*x]*Sqrt[-((b
^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c
 - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} + \frac{(3b)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} + \frac{(18b)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d^2}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\left(9\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d^2} - \frac{\left(9\left(1\right.\right.}{}$$

$$= -\frac{6\sqrt{a+bx}}{d\sqrt[6]{c+dx}} - \frac{9\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{d\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} - \frac{9\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt[3]{bc-ad}\,\sqrt[6]{c+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 73, normalized size = 0.09

$$\frac{2(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(7/6),x]
[Out] (2*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 3/2, 5/2, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
[Out] int((b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(7/6),x)
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate(sqrt(b*x + a)/(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(1/2)/(c + d*x)^(7/6), x)
```

3.1763 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=817

$$\frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}+\frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{6\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\cdots}{\cdots}$$

```
[Out] 6*(b*x+a)^(1/2)/(-a*d+b*c)/(d*x+c)^(1/6)+6*b^(1/3)*(d*x+c)^(1/6)*(1+3^(1/2)
)*(b*x+a)^(1/2)/(-a*d+b*c)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))+6*3^(1/4)*b^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3
)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*
x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2))
)*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*
d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^
(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d
*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-
b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))
^2)^(1/2)+3^(3/4)*b^(1/3)*(d*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(
1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*
(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4
*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(
1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1
/2)))^2)^(1/2)/d/(-a*d+b*c)^(2/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 817, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$\frac{6\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d(bc-ad)^{2/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}+\frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(7/6)),x]
```

```
[Out] (6*Sqrt[a + b*x])/((b*c - a*d)*(c + d*x)^(1/6)) + (6*(1 + Sqrt[3])*b^(1/3)*
Sqrt[a + b*x]*(c + d*x)^(1/6))/((b*c - a*d)*((b*c - a*d)^(1/3) - (1 + Sqrt[
3])*b^(1/3)*(c + d*x)^(1/3))) + (6*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c -
a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*
c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3
) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)
^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + S
qrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*S
qrt[a + b*x]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(
c + d*x)^(1/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3)
)^2)]) + (3^(3/4)*(1 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3)
- b^(1/3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1
/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sq
rt[3])*b^(1/3)*(c + d*x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1
 - Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(
1/3)*(c + d*x)^(1/3))], (2 + Sqrt[3])/4])/(d*(b*c - a*d)^(2/3)*Sqrt[a + b*x
]*Sqrt[-((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3)))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
```

]

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/6}}\,dx = \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} - \frac{(2b)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c+dx}}\,dx}{bc-ad}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} - \frac{(12b)\text{Subst}\left(\int \frac{x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+dx}\right)}{d(bc-ad)}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} + \frac{\left(6\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)(bc-ad)^{2/3}-2b^{2/3}x^4}{\sqrt{a-\frac{bc}{d}+\frac{bx^6}{d}}}\,dx, x, \sqrt[6]{c+\right.}{d(bc-ad)}$$

$$6\sqrt[4]{}$$

$$= \frac{6\sqrt{a+bx}}{(bc-ad)\sqrt[6]{c+dx}} + \frac{6\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt{a+bx}\,\sqrt[6]{c+dx}}{(bc-ad)\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)} + -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.09

$$\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(7/6)),x]
```

```
[Out] (2*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[1/2, 7
/6, 3/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^
2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(7/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*(d*x + c)^(7/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/2)*(c + d*x)^(7/6)), x)
```

3.1764 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=844

$$-\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}-\frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}}-\frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}\,d\sqrt{a+bx}\sqrt[6]{c+dx}}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-$$

```
[Out] -2/(-a*d+b*c)/(d*x+c)^(1/6)/(b*x+a)^(1/2)-8*d*(b*x+a)^(1/2)/(-a*d+b*c)^2/(d
*x+c)^(1/6)-8*b^(1/3)*d*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/(-a*d+b*c)^
2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))-8*3^(1/4)*b^(1/3)*(d
*x+c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(
1+3^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*(
(-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)
+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^
(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)/(-a*d+b*c)^(5/3)/(b*x+a)^
(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)-4/3*b^(1/3)*(d*x+
c)^(1/6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3
)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3
^(1/2)))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)
^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b
*c)^(2/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-
a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(-a*d+b*
c)^(5/3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 844, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$-\frac{8\sqrt{a+bx}\,d}{(bc-ad)^2\sqrt[6]{c+dx}}-\frac{8\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}\,d}{(bc-ad)^2\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}-\frac{8\sqrt[4]{3}\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{4\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}-\frac{2}{(bc-ad)\sqrt{a+bx}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(1/6)) - (8*d*Sqrt[a + b*x])/((b*c
- a*d)^2*(c + d*x)^(1/6)) - (8*(1 + Sqrt[3])*b^(1/3)*d*Sqrt[a + b*x]*(c + d
*x)^(1/6))/((b*c - a*d)^2*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))) - (8*3^(1/4)*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/
3)*(c + d*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c
+ d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*
b^(1/3)*(c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt
[3])*b^(1/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))], (2 + Sqrt[3])/4])/((b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-(
(b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b
*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) - (4*(1 - Sqr
t[3])*b^(1/3)*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3))
*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2
/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c + d
*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))], (2
 + Sqrt[3])/4])/(3^(1/4)*(b*c - a*d)^(5/3)*Sqrt[a + b*x]*Sqrt[-((b^(1/3)*(c
 + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c - a*d)^(
1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx = -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{(4d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{7/6}}\,dx}{3(bc-ad)}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} + \frac{(8bd)\int \frac{1}{\sqrt{a+bx}\,\sqrt[6]{c}}}{3(bc-ad)^2}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} + \frac{(16b)\mathrm{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{(}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} - \frac{\left(8\sqrt[3]{b}\right)\mathrm{Subst}\left(\int \frac{(-}{}\right.}{}$$

$$= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt[6]{c+dx}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2\sqrt[6]{c+dx}} - \frac{8\left(1+\sqrt{3}\right)}{(bc-ad)^2\left(\sqrt[3]{bc-a}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 71, normalized size = 0.08

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(-\frac{1}{2},\frac{7}{6};\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt{a+bx}\,(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*Sqrt[a + b*x]*(c + d*x)^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)

[Out] int(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="maxima")

[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/6)), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="fricas")

[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c*d)*x), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(7/6),x)

[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(7/6)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(7/6),x, algorithm="giac")

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(7/6)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{3/2}\,(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/6)),x)
[Out] int(1/((a + b*x)^(3/2)*(c + d*x)^(7/6)), x)
```

3.1765 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=893

$$-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}+\frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}+\frac{80d^2\sqrt{a+bx}}{9(bc-ad)^3\sqrt[6]{c+dx}}+\frac{80\left(1+\sqrt{\right.}}{9(bc-ad)^3\left(\sqrt[3]{b\right.}}$$

```
[Out] -2/3/(-a*d+b*c)/(b*x+a)^(3/2)/(d*x+c)^(1/6)+20/9*d/(-a*d+b*c)^2/(d*x+c)^(1/
6)/(b*x+a)^(1/2)+80/9*d^2*(b*x+a)^(1/2)/(-a*d+b*c)^3/(d*x+c)^(1/6)+80/9*b^(
1/3)*d^2*(d*x+c)^(1/6)*(1+3^(1/2))*(b*x+a)^(1/2)/(-a*d+b*c)^3/((-a*d+b*c)^(
1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))+80/9*b^(1/3)*d*(d*x+c)^(1/6)*((-a*d
+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/
2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticE((1-((-a*d+b*c)^(1/3)-b^(1/3)*(d
*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/
2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(((-a*d+b*c)^(2/3)+b^(1/3)*(-a*d+b*c
)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/(-a*d+b*c)^(8/3)/(b*x+a)^(1/2)/(-b^
(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/((-a*d+b*c)^(1
/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)+40/27*b^(1/3)*d*(d*x+c)^(1/
6)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))*(((-a*d+b*c)^(1/3)-b^(1/3)*(d*x
+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)
))^2)^(1/2)/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))*((-a*d+b*c
)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))*EllipticF((1-((-a*d+b*c)^(1/3)-b
^(1/3)*(d*x+c)^(1/3)*(1-3^(1/2)))^2/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(((-a*d+b*c)^(2
/3)+b^(1/3)*(-a*d+b*c)^(1/3)*(d*x+c)^(1/3)+b^(2/3)*(d*x+c)^(2/3))/((-a*d+b*
c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(-a*d+b*c)^(8/
3)/(b*x+a)^(1/2)/(-b^(1/3)*(d*x+c)^(1/3)*((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^
(1/3))/((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 893, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {53, 65, 314, 231, 1895}

$$\frac{80\sqrt{a+bx}\,d^2}{9(bc-ad)^3\sqrt[6]{c+dx}}+\frac{80\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{a+bx}\sqrt[6]{c+dx}\,d^2}{9(bc-ad)^3\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)}+\frac{80\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{3\,3^{3/4}(bc-ad)^{8/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{40\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[6]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)\sqrt{\frac{(bc-ad)^{2/3}+\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+b^{2/3}(c+dx)^{2/3}}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{bc-ad}-\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)d}{9\sqrt[4]{3}(bc-ad)^{8/3}\sqrt{a+bx}\sqrt{-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\left(\sqrt[3]{bc-ad}-\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt[3]{c+dx}\right)^2}}}+\frac{20d}{9(bc-ad)^2\sqrt{a+bx}\sqrt[6]{c+dx}}-\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(1/6)) + (20*d)/(9*(b*c - a*d)^
2*Sqrt[a + b*x]*(c + d*x)^(1/6)) + (80*d^2*Sqrt[a + b*x])/(9*(b*c - a*d)^3*
(c + d*x)^(1/6)) + (80*(1 + Sqrt[3])*b^(1/3)*d^2*Sqrt[a + b*x]*(c + d*x)^(1
/6))/(9*(b*c - a*d)^3*((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^
(1/3))) + (80*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d
*x)^(1/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1
/3) + b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(
c + d*x)^(1/3))^2]*EllipticE[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1
/3)*(c + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(
1/3))], (2 + Sqrt[3])/4])/(3*3^(3/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-
((b^(1/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((
b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)]) + (40*(1 - S
qrt[3])*b^(1/3)*d*(c + d*x)^(1/6)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1
/3))*Sqrt[((b*c - a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) +
b^(2/3)*(c + d*x)^(2/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*
x)^(1/3))^2]*EllipticF[ArcCos[((b*c - a*d)^(1/3) - (1 - Sqrt[3])*b^(1/3)*(c
 + d*x)^(1/3))/((b*c - a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))]
, (2 + Sqrt[3])/4])/(9*3^(1/4)*(b*c - a*d)^(8/3)*Sqrt[a + b*x]*Sqrt[-((b^(1
/3)*(c + d*x)^(1/3)*((b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)))/((b*c -
a*d)^(1/3) - (1 + Sqrt[3])*b^(1/3)*(c + d*x)^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
```

```
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{7/6}}\,dx = -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{7/6}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{(40d^2)\ldots}{}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80d^2\ldots}{9(bc-\ldots}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80d^2\ldots}{9(bc-\ldots}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80d^2\ldots}{9(bc-\ldots}$$

$$= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt[6]{c+dx}} + \frac{20d}{9(bc-ad)^2\sqrt{a+bx}\,\sqrt[6]{c+dx}} + \frac{80d^2\ldots}{9(bc-\ldots}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 73, normalized size = 0.08

$$-\frac{2\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6} {}_2F_1\left(-\frac{3}{2},\frac{7}{6};-\frac{1}{2};\frac{d(a+bx)}{-bc+ad}\right)}{3b(a+bx)^{3/2}(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x]
```

```
[Out] (-2*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, (d
*(a + b*x))/(-(b*c) + a*d)])/(3*b*(a + b*x)^(3/2)*(c + d*x)^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{2}} (dx+c)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*(d*x + c)^(5/6)/(b^3*d^2*x^5 + a^3*c^2 + (2*b^3*c*d
+ 3*a*b^2*d^2)*x^4 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^3 + (3*a*b^2*c
^2 + 6*a^2*b*c*d + a^3*d^2)*x^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}} (c+dx)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(7/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/2)*(d*x + c)^(7/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/2}\,(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/2)*(c + d*x)^(7/6)), x)
```

3.1766 $\int \sqrt[6]{a+bx}\,(c+dx)^{13/6}\,dx$

Optimal. Leaf size=84

$$\frac{6(bc-ad)^2(a+bx)^{7/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/7*(-a*d+b*c)^2*(b*x+a)^(7/6)*(d*x+c)^(1/6)*hypergeom([-13/6, 7/6],[13/6],
-d*(b*x+a)/(-a*d+b*c))/b^3/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,(bc-ad)^2\ {}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 7
/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^3*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{13/6}\,dx = \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int \sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)^2(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.07, size = 73, normalized size = 0.87

$$\frac{6(a+bx)^{7/6}(c+dx)^{13/6}\;{}_2F_1\left(-\frac{13}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 7/6, 13/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
[Out] int((b*x+a)^(1/6)*(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(13/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(13/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5984 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(13/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{13/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(13/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(13/6), x)
```

3.1767 $\int \sqrt[6]{a+bx}\,(c+dx)^{7/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/7*(-a*d+b*c)*(b*x+a)^(7/6)*(d*x+c)^(1/6)*hypergeom([-7/6, 7/6],[13/6],-d*
(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\,(bc-ad)\;{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 7/6,
 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b^2*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{7/6}\,dx = \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int \sqrt[6]{a+bx}\,\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}\,dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{7/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(7/6),x]
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
[Out] int((b*x+a)^(1/6)*(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,(c+dx)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{7/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(7/6), x)
```

3.1768 $\int \sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/7*(b*x+a)^(7/6)*(d*x+c)^(1/6)*hypergeom([-1/6, 7/6],[13/6],-d*(b*x+a)/(-a
*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, -((d*
(a + b*x))/(b*c - a*d))])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,dx=\frac{\sqrt[6]{c+dx}\int \sqrt[6]{a+bx}\,\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$=\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{7/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(1/6),x]
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(1/6)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(1/6), x)
```

3.1769 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

```
[Out] 6/7*(b*x+a)^(7/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([5/6, 7/6],[13/6],
-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*b*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{5/6}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}} = \frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7b(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(5/6),x]
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(a+b\,x)}^{1/6}}{{(c+d\,x)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(5/6),x)
```

```
[Out] int((a + b*x)^(1/6)/(c + d*x)^(5/6), x)
```

3.1770 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6/7*(b*x+a)^(7/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([7/6, 11/6],[13/6]
,-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/6,
 11/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{11/6}}\,dx = \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}$$
$$= \frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}\,{}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(11/6),x]
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[7/6
, 11/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(11/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
[Out] int((b*x+a)^(1/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(11/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(11/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(11/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(11/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(11/6), x)
```

3.1771 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6/7*b*(b*x+a)^(7/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([7/6, 17/6],[13/
6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, -((d*(a + b*x))/(b*c - a*d))])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{\sqrt[6]{a+bx}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$
$$= \frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};-\frac{d(a+bx)}{bc-ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 81, normalized size = 0.99

$$\frac{6b(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{7}{6},\frac{17}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7(bc-ad)^2(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(17/6),x]
[Out] (6*b*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[7/
6, 17/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*(b*c - a*d)^2*(c + d*x)^(5
/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
[Out] int((b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(17/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(17/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(17/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{17/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(17/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(17/6), x)
```

3.1772 $\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx$

Optimal. Leaf size=427

$$\frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\cdots$$

```
[Out] 5/12*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(5/6)/b/d+1/2*(b*x+a)^(7/6)*(d*x+c)^(
5/6)/b-5/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6
))/b^(11/6)/d^(7/6)+5/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*
x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(7/6)-5/
144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(
1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(7/6)-5/72*(-a*d+b*c)^2*arctan
(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(1
1/6)/d^(7/6)*3^(1/2)-5/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+
a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(11/6)/d^(7/6)*3^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}}+\frac{5\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12bd}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b*d) + ((a + b*x)^(7/6)
*(c + d*x)^(5/6))/(2*b) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(
7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(11/6)*d^(7/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(11/6)*d^(7/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12b}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\int \frac{\quad}{(a+}}{72}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\,\mathrm{Subs}}{}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\,\mathrm{Subs}}{}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{(5(bc-ad)^2)\,\mathrm{Subs}}{}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}}{36b^{11}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}}{36b^{11}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{5(bc-ad)^2\tan^{-1}}{24\sqrt{3}}$$

Mathematica [A]
time = 0.87, size = 267, normalized size = 0.63

$$\frac{(bc-ad)^2\left(\frac{6b^{5/6}\sqrt[6]{d}\,\sqrt[6]{a+bx}\,(c+dx)^{5/6}(5bc+ad+6bdx)}{(bc-ad)^2}+5\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)-5\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)-10\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)-5\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\right)}{72b^{11/6}d^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(5/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(5/6)*(5*b*c +
 a*d + 6*b*d*x))/(b*c - a*d)^2 + 5*Sqrt[3]*ArcTan[(1 - (2*d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] - 5*Sqrt[3]*ArcTan[(1 + (2*d^(1
/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] - 10*ArcTanh[(d^(1
/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))] - 5*ArcTanh[(b^(1/6)*d^(1/6
)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c +
d*x)^(1/3))]))/(72*b^(11/6)*d^(7/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(5/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. 2(321) = 642.
time = 1.14, size = 5633, normalized size = 13.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="fricas")
```

[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d^7 + a^2*b^9*d^8)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d^7 + a^2*b^9*d^8)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -

```
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^11*d^7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(-((
b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
 c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^11*d^7))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 -
220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*
b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d
^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d
 - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - ...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(5/6), x)
```

3.1773 $\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=378

$$\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+b}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+}}\right.}{2\sqrt{3}\,b^{5/6}d^{7/6}}$$

```
[Out] (b*x+a)^(1/6)*(d*x+c)^(5/6)/d-1/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(7/6)+1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(
b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(
5/6)/d^(7/6)-1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)
+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(7/6)-1/6*(-a*d+b*c
)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/
2))/b^(5/6)/d^(7/6)*3^(1/2)-1/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(
b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(5/6)/d^(7/6)*3^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$\frac{(bc-ad)\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}-\frac{(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}+\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/
6)*d^(7/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/6)*d^(7/6)) - ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(5/6)
*d^(7/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(
7/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(7/6)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{5/6}d} - \frac{(bc}{}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{}}{\sqrt[3]{b}-\sqrt[6]{b}}\right.}{12b^{}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{}}{\sqrt[3]{c}}\right.}{12b^{}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}}{\sqrt[6]{b}}}{\sqrt{}}\right.}{2\sqrt{3}\,b^{5/6}d^{7/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.03, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{7/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{-bc+ad}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3025 vs. 2(280) = 560.
time = 1.09, size = 3025, normalized size = 8.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(b*x + a)^(1/6)*(d*x + c)^(
```

5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + 2*sqrt(3)*(b^4*d^7*x + b^4*c*d^6)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + 2*sqrt(3)*(b^4*d^7*x + b^4*c*d^6)*sqrt(-((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) - sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)) - d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7

```
))^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)) + 2*d*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6) + (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^5*d^7))^(1/6))/(d*x + c)) - 2*d*((b^6*c^6 - 6*a*b^5*c^5*d +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^
5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(
5/6) - (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7)
)^(1/6))/(d*x + c)) - 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{1/6}}{{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(1/6), x)
```

3.1774 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+2\cdots$$

```
[Out] -6*(b*x+a)^(1/6)/d/(d*x+c)^(1/6)+2*b^(1/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^
(1/6)/(d*x+c)^(1/6))/d^(7/6)-1/2*b^(1/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(
d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(7/6)+1/2*b^(1/
6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(
1/6)/(d*x+c)^(1/6))/d^(7/6)+b^(1/6)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)
^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(7/6)+b^(1/6)*arctan(1/3*3^
(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(7
/6)
```

Rubi [A]
time = 0.33, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{\sqrt{3}\,\sqrt[6]{b}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\,\sqrt[6]{b}\,\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{7/6}}-\frac{\sqrt[6]{b}\,\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\,\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{2\sqrt[6]{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{7/6}}-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) - (b^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*d^(7/6)) + (b^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*d^(7/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

```
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}+\frac{b\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}+\frac{6\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx,x,\sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}+\frac{6\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}+\frac{\left(2\sqrt[6]{b}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}+\frac{\left(2\sqrt[6]{b}\right)\text{Su}}{}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}+\frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}-\frac{\sqrt[6]{b}\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx\right.}{2d^{7/6}}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}+\frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}-\frac{\sqrt[6]{b}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\ldots}{}\right.}{2d^{7/6}}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}}+\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}}$$

Mathematica [A]

time = 0.30, size = 239, normalized size = 0.72

$$\frac{-\frac{6\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}-\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+\sqrt{3}\,\sqrt[6]{b}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+2\sqrt[6]{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)+\sqrt[6]{b}\,\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{d^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] ((-6*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6) - Sqrt[3]*b^(1/6)*ArcTan[(1 -
 (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] + Sqrt[3]*
b^(1/6)*ArcTan[(1 + (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/
Sqrt[3]] + 2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(
1/6))] + b^(1/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/
(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/d^(7/6)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(239) = 478.
time = 1.49, size = 663, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sqrt(3)*(d^7*x + c*d^6)*sqrt(
((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3*x + c*d^2)*(b/d^7)^
(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(b/d^7)^(5/6) + sqrt(3)
*(b*d*x + b*c))/(b*d*x + b*c)) + 4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sq
rt(3)*(d^7*x + c*d^6)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6
) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x +
 c))*(b/d^7)^(5/6) - sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) - (d^2*x + c*d)*
(b/d^7)^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3
*x + c*d^2)*(b/d^7)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) + (
d^2*x + c*d)*(b/d^7)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7
)^(1/6) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/
(d*x + c)) - 2*(d^2*x + c*d)*(b/d^7)^(1/6)*log(((d^2*x + c*d)*(b/d^7)^(1/6)
 + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) + 2*(d^2*x + c*d)*(b/d^7)^(1
/6)*log(-((d^2*x + c*d)*(b/d^7)^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d
*x + c)) + 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a + bx}}{(c + dx)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,x)^{1/6}}{(c + d\,x)^{7/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(7/6), x)
```

3.1775 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

```
[Out] 6/7*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx = \frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
```

[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))

Maple [A]
time = 0.17, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}}{7(dx+c)^{\frac{7}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/6)/(d*x+c)^(13/6),x,method=_RETURNVERBOSE)

[Out] -6/7*(b*x+a)^(7/6)/(d*x+c)^(7/6)/(a*d-b*c)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")

[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.66, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}{7\,(bc^3-ac^2d+(bcd^2-ad^3)x^2+2\,(bc^2d-acd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")

[Out] 6/7*(b*x + a)^(7/6)*(d*x + c)^(5/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2 + 2*(b*c^2*d - a*c*d^2)*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(1/6)/(d*x+c)**(13/6),x)

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(13/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)
```

Mupad [B]
time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,{\left(a+b\,x\right)}^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{6\,b\,x\,{\left(a+b\,x\right)}^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}\right)\,{\left(c+d\,x\right)}^{5/6}}{x^2-\frac{7\,b\,c^3-7\,a\,c^2\,d}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{14\,c\,d\,x\,\left(a\,d-b\,c\right)}{7\,a\,d^3-7\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(13/6),x)
[Out] -(((6*a*(a + b*x)^(1/6))/(7*a*d^3 - 7*b*c*d^2) + (6*b*x*(a + b*x)^(1/6))/(7
*a*d^3 - 7*b*c*d^2))*(c + d*x)^(5/6))/(x^2 - (7*b*c^3 - 7*a*c^2*d)/(7*a*d^3
 - 7*b*c*d^2) + (14*c*d*x*(a*d - b*c))/(7*a*d^3 - 7*b*c*d^2))
```

3.1776 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}}+\frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}$$

```
[Out] 6/13*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(13/6)+36/91*b*(b*x+a)^(7/6)/(-a*d+b*
c)^2/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (36*b*(a + b*x)^(7/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}} + \frac{(6b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{13(bc-ad)}$$
$$= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}} + \frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}$$

Mathematica [A]
time = 0.13, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}(13bc-7ad+6bdx)}{91(bc-ad)^2(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
[Out] (6*(a + b*x)^(7/6)*(13*b*c - 7*a*d + 6*b*d*x))/(91*(b*c - a*d)^2*(c + d*x)^
(13/6))
```

Maple [A]
time = 0.20, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}(-6bdx+7ad-13bc)}{91(dx+c)^{\frac{13}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(19/6),x,method=_RETURNVERBOSE)
[Out] -6/91*(b*x+a)^(7/6)*(-6*b*d*x+7*a*d-13*b*c)/(d*x+c)^(13/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(54) = 108$.
time = 0.67, size = 175, normalized size = 2.65

$$\frac{6\,(6\,b^2dx^2+13\,abc-7\,a^2d+(13\,b^2c-abd)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^2d^3-2\,abcd^4+a^2d^5)x^3+3\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2+3\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(6*b^2*d*x^2 + 13*a*b*c - 7*a^2*d + (13*b^2*c - a*b*d)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 - 2
*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2
 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

Mupad [B]
time = 0.75, size = 137, normalized size = 2.08

$$\frac{{\left(c+d\,x\right)}^{5/6}\,\left(\frac{36\,b^2\,x^2\,{\left(a+b\,x\right)}^{1/6}}{91\,d^2\,{\left(a\,d-b\,c\right)}^2}-\frac{\left(42\,a^2\,d-78\,a\,b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}{91\,d^3\,{\left(a\,d-b\,c\right)}^2}+\frac{x\,\left(78\,b^2\,c-6\,a\,b\,d\right)\,{\left(a+b\,x\right)}^{1/6}}{91\,d^3\,{\left(a\,d-b\,c\right)}^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(19/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((36*b^2*x^2*(a + b*x)^(1/6))/(91*d^2*(a*d - b*c)^2) - ((4
2*a^2*d - 78*a*b*c)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2) + (x*(78*b^2*c
- 6*a*b*d)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 + (3*c*
x^2)/d + (3*c^2*x)/d^2)
```

3.1777 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}$$

```
[Out] 6/19*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(19/6)+72/247*b*(b*x+a)^(7/6)/(-a*d+b
*c)^2/(d*x+c)^(13/6)+432/1729*b^2*(b*x+a)^(7/6)/(-a*d+b*c)^3/(d*x+c)^(7/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (72*b*(a + b*x)^(7/
6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (432*b^2*(a + b*x)^(7/6))/(1729*
(b*c - a*d)^3*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx = \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(12b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$
$$= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{(72b^2)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{247(bc-ad)^2}$$
$$= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}$$

Mathematica [A]
time = 0.15, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{7/6}\left(247b^2+\frac{91d^2(a+bx)^2}{(c+dx)^2}-\frac{266bd(a+bx)}{c+dx}\right)}{1729(bc-ad)^3(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(247*b^2 + (91*d^2*(a + b*x)^2)/(c + d*x)^2 - (266*b*d*(
a + b*x))/(c + d*x)))/(1729*(b*c - a*d)^3*(c + d*x)^(7/6))
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}\left(72b^2x^2d^2-84ab\,d^2x+228b^2cdx+91a^2d^2-266abcd+247b^2c^2\right)}{1729(dx+c)^{\frac{19}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(25/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/1729*(b*x+a)^(7/6)*(72*b^2*d^2*x^2-84*a*b*d^2*x+228*b^2*c*d*x+91*a^2*d^2
-266*a*b*c*d+247*b^2*c^2)/(d*x+c)^(19/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(83) = 166$.
time = 1.18, size = 338, normalized size = 3.35

$$\frac{6\,(72\,b^3d^2x^3 + 247\,ab^2c^2 - 266\,a^2bcd + 91\,a^3d^2 + 12\,(19\,b^3cd - ab^2d^2)x^2 + (247\,b^3c^2 - 38\,ab^2cd + 7\,a^2bd^2)x)(bx + a)^{\frac{1}{6}}(dx + c)^{\frac{5}{6}}}{1729\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3 + (b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3d^7)x^4 + 4\,(b^3c^4d^3 - 3\,ab^2c^3d^4 + 3\,a^2bc^2d^5 - a^3cd^6)x^3 + 6\,(b^3c^5d^2 - 3\,ab^2c^4d^3 + 3\,a^2bc^3d^4 - a^3c^2d^5)x^2 + 4\,(b^3c^6d - 3\,ab^2c^5d^2 + 3\,a^2bc^4d^3 - a^3c^3d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(72*b^3*d^2*x^3 + 247*a*b^2*c^2 - 266*a^2*b*c*d + 91*a^3*d^2 + 12*(1
9*b^3*c*d - a*b^2*d^2)*x^2 + (247*b^3*c^2 - 38*a*b^2*c*d + 7*a^2*b*d^2)*x)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
- a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*x
^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x^3 +
6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2 + 4*(
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

Mupad [**B**]
time = 0.95, size = 213, normalized size = 2.11

$$-\frac{{\left(c+d\,x\right)}^{5/6}\,\left(\frac{{\left(a+b\,x\right)}^{1/6}\,\left(546\,a^3\,d^2-1596\,a^2\,b\,c\,d+1482\,a\,b^2\,c^2\right)}{1729\,d^4\,{\left(a\,d-b\,c\right)}^3}+\frac{432\,b^3\,x^3\,{\left(a+b\,x\right)}^{1/6}}{1729\,d^2\,{\left(a\,d-b\,c\right)}^3}+\frac{x\,{\left(a+b\,x\right)}^{1/6}\,\left(42\,a^2\,b\,d^2-228\,a\,b^2\,c\,d+1482\,b^3\,c^2\right)}{1729\,d^4\,{\left(a\,d-b\,c\right)}^3}-\frac{72\,b^2\,x^2\,\left(a\,d-19\,b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}{1729\,d^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(546*a^3*d^2 + 1482*a*b^2*c^2 - 1596*a^
2*b*c*d))/(1729*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(1/6))/(1729*d^
2*(a*d - b*c)^3) + (x*(a + b*x)^(1/6)*(1482*b^3*c^2 + 42*a^2*b*d^2 - 228*a*
b^2*c*d))/(1729*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(a*d - 19*b*c)*(a + b*x)^(
1/6))/(1729*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d
^3 + (6*c^2*x^2)/d^2)
```

3.1778 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}}+\frac{7776b^3(a+bx)^{7/6}}{43225(bc-ad)^4(c+dx)}$$

```
[Out] 6/25*(b*x+a)^(7/6)/(-a*d+b*c)/(d*x+c)^(25/6)+108/475*b*(b*x+a)^(7/6)/(-a*d+
b*c)^2/(d*x+c)^(19/6)+1296/6175*b^2*(b*x+a)^(7/6)/(-a*d+b*c)^3/(d*x+c)^(13/
6)+7776/43225*b^3*(b*x+a)^(7/6)/(-a*d+b*c)^4/(d*x+c)^(7/6)
```

Rubi [A]

time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (108*b*(a + b*x)^(7
/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (1296*b^2*(a + b*x)^(7/6))/(617
5*(b*c - a*d)^3*(c + d*x)^(13/6)) + (7776*b^3*(a + b*x)^(7/6))/(43225*(b*c
- a*d)^4*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(18b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{(216b^2)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} + \frac{(1}{}$$

$$= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}} + \frac{}{4}$$

Mathematica [A]
time = 0.17, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}\left(6175b^3-\frac{1729d^3(a+bx)^3}{(c+dx)^3}+\frac{6825bd^2(a+bx)^2}{(c+dx)^2}-\frac{9975b^2d(a+bx)}{c+dx}\right)}{43225(bc-ad)^4(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(6175*b^3 - (1729*d^3*(a + b*x)^3)/(c + d*x)^3 + (6825*b
*d^2*(a + b*x)^2)/(c + d*x)^2 - (9975*b^2*d*(a + b*x))/(c + d*x)))/(43225*(
b*c - a*d)^4*(c + d*x)^(7/6))
```

Maple [A]
time = 0.21, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{7}{6}}\left(-1296b^3x^3d^3+1512d^3a\,x^2b^2-5400b^3c\,d^2x^2-1638a^2b\,d^3x+6300a\,b^2c\,d^2x-8550b^3c^2dx+1729a^3d^3-6825a^2bc\,d^2+9975a\right.}{43225(dx+c)^{\frac{25}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(31/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/43225*(b*x+a)^(7/6)*(-1296*b^3*d^3*x^3+1512*a*b^2*d^3*x^2-5400*b^3*c*d^2
*x^2-1638*a^2*b*d^3*x+6300*a*b^2*c*d^2*x-8550*b^3*c^2*d*x+1729*a^3*d^3-6825
*a^2*b*c*d^2+9975*a*b^2*c^2*d-6175*b^3*c^3)/(d*x+c)^(25/6)/(a^4*d^4-4*a^3*b
*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(112) = 224$.
time = 1.02, size = 533, normalized size = 3.92

$$\frac{6\,(1296\,b^4d^3x^4+6175\,ab^3c^3-9975\,a^2b^2c^2d+6825\,a^3bcd^2-1729\,a^4d^3+216\,(25\,b^4cd^2-ab^3d^3)x^3+18\,(475\,b^4c^2d-50\,ab^3cd^2+7\,a^2b^2d^3)x^2+(6175\,b^4c^3-1425\,ab^3c^2d+525\,a^2b^2cd^2-91\,a^3bd^3)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{43225\,(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9)x^5+5\,(b^4c^5d^4-4\,ab^3c^4d^5+6\,a^2b^2c^3d^6-4\,a^3bc^2d^7+a^4cd^8)x^4+10\,(b^4c^6d^3-4\,ab^3c^5d^4+6\,a^2b^2c^4d^5-4\,a^3bc^3d^6+a^4c^2d^7)x^3+10\,(b^4c^7d^2-4\,ab^3c^6d^3+6\,a^2b^2c^5d^4-4\,a^3bc^4d^5+a^4c^3d^6)x^2+5\,(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/43225*(1296*b^4*d^3*x^4 + 6175*a*b^3*c^3 - 9975*a^2*b^2*c^2*d + 6825*a^3*
b*c*d^2 - 1729*a^4*d^3 + 216*(25*b^4*c*d^2 - a*b^3*d^3)*x^3 + 18*(475*b^4*c
^2*d - 50*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + (6175*b^4*c^3 - 1425*a*b^3*c^2
*d + 525*a^2*b^2*c*d^2 - 91*a^3*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4
*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*
c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^
4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 + 5*(b^4*c^8*d
 - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

Mupad [B]
time = 1.15, size = 302, normalized size = 2.22

$$\frac{(c+d\,x)^{5/6}\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{1/6}}{43225\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\,(10374\,a^4\,d^3-40950\,a^3\,b\,c\,d^2+59850\,a^2\,b^2\,c^2\,d-37050\,a\,b^3\,c^3)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\,(-546\,a^3\,b\,d^3+3150\,a^2\,b^2\,c\,d^2-8550\,a\,b^3\,c^2\,d+37050\,b^4\,c^3)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{1/6}\,(7\,a^2\,d^2-50\,a\,b\,c\,d+475\,b^2\,c^2)}{43225\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{43225\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(31/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^4*x^4*(a + b*x)^(1/6))/(43225*d^2*(a*d - b*c)^4)
- ((a + b*x)^(1/6)*(10374*a^4*d^3 - 37050*a*b^3*c^3 + 59850*a^2*b^2*c^2*d -
 40950*a^3*b*c*d^2))/(43225*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(37050*
b^4*c^3 - 546*a^3*b*d^3 + 3150*a^2*b^2*c*d^2 - 8550*a*b^3*c^2*d))/(43225*d^
5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(1/6)*(7*a^2*d^2 + 475*b^2*c^2 -
50*a*b*c*d))/(43225*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(a*d - 25*b*c)*(a +
b*x)^(1/6))/(43225*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*x^4)/d + (5*c
^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

3.1779 $\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=427

$$\frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}+\frac{5(b}{}$$

```
[Out] 1/12*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b/d+1/2*(b*x+a)^(11/6)*(d*x+c)^
(1/6)/b-5/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/
6))/b^(7/6)/d^(11/6)+5/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d
*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(11/6)-5
/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^
(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(11/6)+5/72*(-a*d+b*c)^2*arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(
7/6)/d^(11/6)*3^(1/2)+5/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x
+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(7/6)/d^(11/6)*3^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}+\frac{5(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}+\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b*d) + ((a + b*x)^(11/6)*
(c + d*x)^(1/6))/(2*b) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
+ b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11
/6)) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqr
t[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a
*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(
7/6)*d^(11/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(
c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b
^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(7/6)*d^(11/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx = \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{(5(bc-ad)^2)\int \frac{}{\sqrt[6]{a-}}}{72bd}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{(5(bc-ad)^2)\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{(5(bc-ad)^2)\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{(5(bc-ad)^2)\,\mathrm{Subst}\Big(}{}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\Big(}{36b^{7/6}d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\Big(}{36b^{7/6}d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\Bigg(}{24\sqrt{3}\,b^{7}}$$

Mathematica [A]
time = 0.89, size = 267, normalized size = 0.63

$$\frac{(bc-ad)^2\left(\frac{6\sqrt[6]{b}\,d^{5/6}(a+bx)^{5/6}\sqrt[6]{c+dx}\,(5ad+b(c+6dx))}{(bc-ad)^2}+5\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right)-5\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right)-10\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)-5\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)\right)}{72b^{7/6}d^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(1/6)*d^(5/6)*(a + b*x)^(5/6)*(c + d*x)^(1/6)*(5*a*d +
 b*(c + 6*d*x)))/(b*c - a*d)^2 + 5*Sqrt[3]*ArcTan[(1 - (2*b^(1/6)*(c + d*x)
^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] - 5*Sqrt[3]*ArcTan[(1 + (2*b^(1
/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] - 10*ArcTanh[(b^(1
/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))] - 5*ArcTanh[(b^(1/6)*d^(1/6
)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c +
d*x)^(1/3))]))/(72*b^(7/6)*d^(11/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. 2(321) = 642.
time = 1.10, size = 5633, normalized size = 13.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(((b^3*c^2*d^2 - 2*a*b^2*
c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x
 + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x
 + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) + sqrt(3)*(a
*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a
^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c
^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66
*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b
^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^
8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*
b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*
a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c
^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 +
 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^
12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d
^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*
a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*
c*d^11 + a^12*b*d^12)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^7*d^11))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^1
0 + a^2*b^6*d^11)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
```

```
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^7*d^11))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(-((
b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
 c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^7*d^11))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^
2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a
^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c
^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 -
 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(5/6)*(c + d*x)^(1/6), x)
```

3.1780 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=378

$$\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c}}\right.}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}$$

```
[Out] (b*x+a)^(5/6)*(d*x+c)^(1/6)/d-5/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(11/6)+5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*
(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^
(1/6)/d^(11/6)-5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/
3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(11/6)+5/6*(-a*d+
b*c)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^
(1/2))/b^(1/6)/d^(11/6)*3^(1/2)+5/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/
6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(1/6)/d^(11/6)*3^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$-\frac{5(bc-ad)\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/d - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
1/6)*d^(11/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(1/6)*d^(11/6)) - (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(1/6)*d^(11/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(1/6)*d^(11/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6d}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3d^{5/3}} - \frac{(5(bc-a}{}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{}{\sqrt[3]{b}}\right.}{}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{}}{}\right.}{1}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2}{}}{}\right.}{2\sqrt{3}\,\sqrt[6]{b}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.05, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2997 vs. 2(280) = 560.
time = 1.12, size = 2997, normalized size = 7.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -1/12*(20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
```

5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b
^2*d^9*x + a*b*d^9)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*
c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 -
6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b
^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6
*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a
*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a
^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^
2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
 a^6*b*d^6)*x)) + 20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*
d^11))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2
sqrt(3)(b^2*d^9*x + a*b*d^9)*sqrt(-((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*
x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) - (b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^4*x + a
d^4)((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) - sqrt(3)*(a*b^6*c
^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2
*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*
c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b
*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^
3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c
^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5
*b^2*c*d^5 + a^6*b*d^6)*x)) + 5*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b*d^11))^(1/6)*log(25*((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a)) - 5*d*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-25*((b*c*d^2 - a*d^3)
*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4

```
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^
(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11)
)^(1/3))/(b*x + a)) + 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11
))^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b*d^2*x + a
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a))
- 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-5*((b*
c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b*d^2*x + a*d^2)*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a)) - 12*(b*x + a)^(5/6)
*(d*x + c)^(1/6))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx)^{\frac{5}{6}}}{(c + dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(5/6), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{5/6}}{{\left(c+d\,x\right)}^{5/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(5/6), x)
```

3.1781 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=334

$$-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}+$$

```
[Out] -6/5*(b*x+a)^(5/6)/d/(d*x+c)^(5/6)+2*b^(5/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/d^(11/6)-1/2*b^(5/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3
)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(11/6)+1/2*b
^(5/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+
a)^(1/6)/(d*x+c)^(1/6))/d^(11/6)-b^(5/6)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b
*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(11/6)-b^(5/6)*arctan(
1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2
)/d^(11/6)
```

Rubi [A]
time = 0.36, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{\sqrt{3}\,b^{5/6}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{\sqrt{3}\,b^{5/6}\text{ArcTan}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{11/6}}-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11
/6) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt
[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(d^(1/6)*(a +
b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) - (b^(5/6)*Log[b^(1/3) + (
d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6)
)/(c + d*x)^(1/6)])/(2*d^(11/6)) + (b^(5/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(2*d^(11/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx = -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{b\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{5/3}} + \frac{\left(2b^{5/6}\right)\text{S}}{}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\right.}{2d^{11/6}}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\sqrt[6]{}\right.}{2d^{11/6}}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} - \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+}}{\sqrt[6]{b}\,\sqrt[6]{c+}}}{\sqrt{3}}\right.}{d^{11/6}}$$

Mathematica [A]
time = 0.39, size = 244, normalized size = 0.73

$$\frac{-\frac{6d^{5/6}(a+bx)^{5/6}}{(c+dx)^{5/6}} - 5\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 5\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 10b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right) + 5b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{5d^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] ((-6*d^(5/6)*(a + b*x)^(5/6))/(c + d*x)^(5/6) - 5*Sqrt[3]*b^(5/6)*ArcTan[(1
 - (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] + 5*Sqrt
[3]*b^(5/6)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6
)))/Sqrt[3]] + 10*b^(5/6)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b
*x)^(1/6))] + 5*b^(5/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^
(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(5*d^(11/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(241) = 482.
time = 1.31, size = 755, normalized size = 2.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -1/10*(20*sqrt(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x +
a*d^9)*sqrt(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) + (b*x
+ a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x +
 a))*(b^5/d^11)^(5/6) + sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) + 20*sqrt
```

```
(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x + a*d^9)*sqrt(-(
(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d
*x + c)^(1/3)*b^2 - (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a))*(b^5/d^1
1)^(5/6) - sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) - 5*(d^2*x + c*d)*(b^5
/d^11)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6)
+ (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))
/(b*x + a)) + 5*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x
 + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 -
(b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a)) - 10*(d^2*x + c*d)*(b^5/d^11
)^(1/6)*log(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b + (b*d^2*x + a*d^2)*(b^5/d^1
1)^(1/6))/(b*x + a)) + 10*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(((b*x + a)^(5/
6)*(d*x + c)^(1/6)*b - (b*d^2*x + a*d^2)*(b^5/d^11)^(1/6))/(b*x + a)) + 12*
(b*x + a)^(5/6)*(d*x + c)^(1/6))/(d^2*x + c*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(11/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(11/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(11/6), x)
```

3.1782 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

```
[Out] 6/11*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(11/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx = \frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Maple [A]
time = 0.18, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}}{11(dx+c)^{\frac{11}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(17/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/11*(b*x+a)^(11/6)/(d*x+c)^(11/6)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 1.02, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{11}{6}}(dx+c)^{\frac{1}{6}}}{11\,(bc^3-ac^2d+(bcd^2-ad^3)x^2+2\,(bc^2d-acd^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/11*(b*x + a)^(11/6)*(d*x + c)^(1/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*
x^2 + 2*(b*c^2*d - a*c*d^2)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(17/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

Mupad [B]
time = 0.59, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,{\left(a+b\,x\right)}^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{6\,b\,x\,{\left(a+b\,x\right)}^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}\right)\,{\left(c+d\,x\right)}^{1/6}}{x^2-\frac{11\,b\,c^3-11\,a\,c^2\,d}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{22\,c\,d\,x\,\left(a\,d-b\,c\right)}{11\,a\,d^3-11\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(17/6),x)
```

```
[Out] -(((6*a*(a + b*x)^(5/6))/(11*a*d^3 - 11*b*c*d^2) + (6*b*x*(a + b*x)^(5/6))/
(11*a*d^3 - 11*b*c*d^2))*(c + d*x)^(1/6))/(x^2 - (11*b*c^3 - 11*a*c^2*d)/(1
1*a*d^3 - 11*b*c*d^2) + (22*c*d*x*(a*d - b*c))/(11*a*d^3 - 11*b*c*d^2))
```

3.1783 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} \, dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}$$

```
[Out] 6/17*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(17/6)+36/187*b*(b*x+a)^(11/6)/(-a*d
+b*c)^2/(d*x+c)^(11/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (36*b*(a + b*x)^(1
1/6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx = \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{(6b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{17(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{11/6}(17bc-11ad+6bdx)}{187(bc-ad)^2(c+dx)^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
[Out] (6*(a + b*x)^(11/6)*(17*b*c - 11*a*d + 6*b*d*x))/(187*(b*c - a*d)^2*(c + d*
x)^(17/6))
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}(-6bdx+11ad-17bc)}{187(dx+c)^{\frac{17}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(23/6),x,method=_RETURNVERBOSE)
[Out] -6/187*(b*x+a)^(11/6)*(-6*b*d*x+11*a*d-17*b*c)/(d*x+c)^(17/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(54) = 108$.
time = 1.08, size = 175, normalized size = 2.65

$$\frac{6\,(6\,b^2dx^2+17\,abc-11\,a^2d+(17\,b^2c-5\,abd)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{187\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^2d^3-2\,abcd^4+a^2d^5)x^3+3\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2+3\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="fricas")
[Out] 6/187*(6*b^2*d*x^2 + 17*a*b*c - 11*a^2*d + (17*b^2*c - 5*a*b*d)*x)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3
 - 2*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)
*x^2 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(23/6),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

Mupad [B]
time = 0.74, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{1/6}\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{5/6}}{187\,d^2\,(a\,d-b\,c)^2}-\frac{(66\,a^2\,d-102\,a\,b\,c)\,(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}+\frac{x\,(102\,b^2\,c-30\,a\,b\,d)\,(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(23/6),x)
[Out] ((c + d*x)^(1/6)*((36*b^2*x^2*(a + b*x)^(5/6))/(187*d^2*(a*d - b*c)^2) - ((
66*a^2*d - 102*a*b*c)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2) + (x*(102*b^
2*c - 30*a*b*d)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 +
 (3*c*x^2)/d + (3*c^2*x)/d^2)
```

3.1784 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}$$

```
[Out] 6/23*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(23/6)+72/391*b*(b*x+a)^(11/6)/(-a*d
+b*c)^2/(d*x+c)^(17/6)+432/4301*b^2*(b*x+a)^(11/6)/(-a*d+b*c)^3/(d*x+c)^(11
/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (72*b*(a + b*x)^(1
1/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (432*b^2*(a + b*x)^(11/6))/(43
01*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(12b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{(72b^2)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}$$
$$= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}$$

Mathematica [A]
time = 0.17, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{23/6}\left(187d^2 - \frac{506bd(c+dx)}{a+bx} + \frac{391b^2(c+dx)^2}{(a+bx)^2}\right)}{4301(bc-ad)^3(c+dx)^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(23/6)*(187*d^2 - (506*b*d*(c + d*x))/(a + b*x) + (391*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}\left(72b^2x^2d^2-132ab\,d^2x+276b^2cdx+187a^2d^2-506abcd+391b^2c^2\right)}{4301(dx+c)^{\frac{23}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(29/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/4301*(b*x+a)^(11/6)*(72*b^2*d^2*x^2-132*a*b*d^2*x+276*b^2*c*d*x+187*a^2*
d^2-506*a*b*c*d+391*b^2*c^2)/(d*x+c)^(23/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(83) = 166$.
time = 1.29, size = 338, normalized size = 3.35

$$\frac{6\,(72\,b^3d^2x^3 + 391\,ab^2c^2 - 506\,a^2bcd + 187\,a^3d^2 + 12\,(23\,b^3cd - 5\,ab^2d^2)x^2 + (391\,b^3c^2 - 230\,ab^2cd + 55\,a^2bd^2)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{4301\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3 + (b^3c^3d^4 - 3\,ab^2c^2d^5 + 3\,a^2bcd^6 - a^3d^7)x^4 + 4\,(b^3c^4d^3 - 3\,ab^2c^3d^4 + 3\,a^2bc^2d^5 - a^3cd^6)x^3 + 6\,(b^3c^5d^2 - 3\,ab^2c^4d^3 + 3\,a^2bc^3d^4 - a^3c^2d^5)x^2 + 4\,(b^3c^6d - 3\,ab^2c^5d^2 + 3\,a^2bc^4d^3 - a^3c^3d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/4301*(72*b^3*d^2*x^3 + 391*a*b^2*c^2 - 506*a^2*b*c*d + 187*a^3*d^2 + 12*(
23*b^3*c*d - 5*a*b^2*d^2)*x^2 + (391*b^3*c^2 - 230*a*b^2*c*d + 55*a^2*b*d^2
)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5
*d^2 - a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d
^7)*x^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x
^3 + 6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2
+ 4*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

Mupad [B]
time = 0.94, size = 214, normalized size = 2.12

$$-\frac{(c+d\,x)^{1/6}\left(\frac{(a+b\,x)^{5/6}\,(1122\,a^3\,d^2-3036\,a^2\,b\,c\,d+2346\,a\,b^2\,c^2)}{4301\,d^4\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3\,(a+b\,x)^{5/6}}{4301\,d^2\,(a\,d-b\,c)^3}+\frac{x\,(a+b\,x)^{5/6}\,(330\,a^2\,b\,d^2-1380\,a\,b^2\,c\,d+2346\,b^3\,c^2)}{4301\,d^4\,(a\,d-b\,c)^3}-\frac{72\,b^2\,x^2\,(5\,a\,d-23\,b\,c)\,(a+b\,x)^{5/6}}{4301\,d^3\,(a\,d-b\,c)^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(29/6),x)
```

```
[Out] -((c + d*x)^(1/6)*(((a + b*x)^(5/6)*(1122*a^3*d^2 + 2346*a*b^2*c^2 - 3036*a
^2*b*c*d))/(4301*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(5/6))/(4301*d
^2*(a*d - b*c)^3) + (x*(a + b*x)^(5/6)*(2346*b^3*c^2 + 330*a^2*b*d^2 - 1380
*a*b^2*c*d))/(4301*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(5*a*d - 23*b*c)*(a + b
*x)^(5/6))/(4301*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3
*x)/d^3 + (6*c^2*x^2)/d^2)
```

3.1785 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}}+\frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}}+\frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}}+\frac{7776b^3(a+bx)^{11/6}}{124729(bc-ad)^4(c+d$$

```
[Out] 6/29*(b*x+a)^(11/6)/(-a*d+b*c)/(d*x+c)^(29/6)+108/667*b*(b*x+a)^(11/6)/(-a*
d+b*c)^2/(d*x+c)^(23/6)+1296/11339*b^2*(b*x+a)^(11/6)/(-a*d+b*c)^3/(d*x+c)^
(17/6)+7776/124729*b^3*(b*x+a)^(11/6)/(-a*d+b*c)^4/(d*x+c)^(11/6)
```

Rubi [A]

time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(29*(b*c - a*d)*(c + d*x)^(29/6)) + (108*b*(a + b*x)^(
11/6))/(667*(b*c - a*d)^2*(c + d*x)^(23/6)) + (1296*b^2*(a + b*x)^(11/6))/(
11339*(b*c - a*d)^3*(c + d*x)^(17/6)) + (7776*b^3*(a + b*x)^(11/6))/(124729
*(b*c - a*d)^4*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx = \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{(18b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx}{29(bc-ad)}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{(216b^2)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{667(bc-ad)^2}$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} + \cdot$$
$$= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}} + \frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}} + \frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}} + \cdot$$

Mathematica [A]
time = 0.18, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{29/6}\left(-4301d^3+\frac{16269bd^2(c+dx)}{a+bx}-\frac{22011b^2d(c+dx)^2}{(a+bx)^2}+\frac{11339b^3(c+dx)^3}{(a+bx)^3}\right)}{124729(bc-ad)^4(c+dx)^{29/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
[Out] (6*(a + b*x)^(29/6)*(-4301*d^3 + (16269*b*d^2*(c + d*x))/(a + b*x) - (22011
*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (11339*b^3*(c + d*x)^3)/(a + b*x)^3))/(12
4729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

Maple [A]
time = 0.20, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{11}{6}}\left(-1296b^3x^3d^3+2376d^3a\,x^2b^2-6264b^3c\,d^2x^2-3366a^2b\,d^3x+11484a\,b^2c\,d^2x-12006b^3c^2dx+4301a^3d^3-16269a^2bc\,d^2+22\right.}{124729(dx+c)^{\frac{29}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(35/6),x,method=_RETURNVERBOSE)
[Out] -6/124729*(b*x+a)^(11/6)*(-1296*b^3*d^3*x^3+2376*a*b^2*d^3*x^2-6264*b^3*c*d
^2*x^2-3366*a^2*b*d^3*x+11484*a*b^2*c*d^2*x-12006*b^3*c^2*d*x+4301*a^3*d^3-
16269*a^2*b*c*d^2+22011*a*b^2*c^2*d-11339*b^3*c^3)/(d*x+c)^(29/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(112) = 224$.
time = 1.32, size = 533, normalized size = 3.92

$$\frac{6\left(1296\,b^{4}d^{3}x^{4}+11339\,ab^{3}c^{3}-22011\,a^{2}b^{2}c^{2}d+16269\,a^{3}bcd^{2}-4301\,a^{4}d^{3}+216\,(29\,b^{4}cd^{2}-5\,ab^{3}d^{3})x^{3}+18\,(667\,b^{4}c^{2}d-290\,ab^{3}cd^{2}+55\,a^{2}b^{2}d^{3})x^{2}+(11339\,b^{4}c^{3}-10005\,ab^{3}c^{2}d+4785\,a^{2}b^{2}cd^{2}-935\,a^{3}bd^{3})x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{124729\left(b^{4}c^{9}-4\,ab^{3}c^{8}d+6\,a^{2}b^{2}c^{7}d^{2}-4\,a^{3}bc^{6}d^{3}+a^{4}c^{5}d^{4}+(b^{4}c^{4}d^{5}-4\,ab^{3}c^{3}d^{6}+6\,a^{2}b^{2}c^{2}d^{7}-4\,a^{3}bcd^{8}+a^{4}d^{9})x^{5}+5\,(b^{4}c^{5}d^{4}-4\,ab^{3}c^{4}d^{5}+6\,a^{2}b^{2}c^{3}d^{6}-4\,a^{3}bc^{2}d^{7}+a^{4}cd^{8})x^{4}+10\,(b^{4}c^{6}d^{3}-4\,ab^{3}c^{5}d^{4}+6\,a^{2}b^{2}c^{4}d^{5}-4\,a^{3}bc^{3}d^{6}+a^{4}c^{2}d^{7})x^{3}+10\,(b^{4}c^{7}d^{2}-4\,ab^{3}c^{6}d^{3}+6\,a^{2}b^{2}c^{5}d^{4}-4\,a^{3}bc^{4}d^{5}+a^{4}c^{3}d^{6})x^{2}+5\,(b^{4}c^{8}d-4\,ab^{3}c^{7}d^{2}+6\,a^{2}b^{2}c^{6}d^{3}-4\,a^{3}bc^{5}d^{4}+a^{4}c^{4}d^{5})x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="fricas")
```

```
[Out] 6/124729*(1296*b^4*d^3*x^4 + 11339*a*b^3*c^3 - 22011*a^2*b^2*c^2*d + 16269*
a^3*b*c*d^2 - 4301*a^4*d^3 + 216*(29*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 18*(667
*b^4*c^2*d - 290*a*b^3*c*d^2 + 55*a^2*b^2*d^3)*x^2 + (11339*b^4*c^3 - 10005
*a*b^3*c^2*d + 4785*a^2*b^2*c*d^2 - 935*a^3*b*d^3)*x)*(b*x + a)^(5/6)*(d*x
+ c)^(1/6)/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^
6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 +
6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 -
4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 +
5*(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*
c^4*d^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(35/6),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

Mupad [B]

time = 1.16, size = 303, normalized size = 2.23

$$\frac{(c+d\,x)^{1/6}\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{5/6}}{124729\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{5/6}\,\left(25806\,a^4\,d^3-97614\,a^3\,b\,c\,d^2+132066\,a^2\,b^2\,c^2\,d-68034\,a\,b^3\,c^3\right)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{5/6}\,\left(-5610\,a^3\,b\,d^3+28710\,a^2\,b^2\,c\,d^2-60030\,a\,b^3\,c^2\,d+68034\,b^4\,c^3\right)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{5/6}\,\left(55\,a^2\,d^2-290\,a\,b\,c\,d+667\,b^2\,c^2\right)}{124729\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(5\,a\,d-29\,b\,c)\,(a+b\,x)^{5/6}}{124729\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(35/6),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4*(a + b*x)^(5/6))/(124729*d^2*(a*d - b*c)^4)
 - ((a + b*x)^(5/6)*(25806*a^4*d^3 - 68034*a*b^3*c^3 + 132066*a^2*b^2*c^2*d
 - 97614*a^3*b*c*d^2))/(124729*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(5/6)*(680
34*b^4*c^3 - 5610*a^3*b*d^3 + 28710*a^2*b^2*c*d^2 - 60030*a*b^3*c^2*d))/(12
4729*d^5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(5/6)*(55*a^2*d^2 + 667*b^
2*c^2 - 290*a*b*c*d))/(124729*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(5*a*d - 2
9*b*c)*(a + b*x)^(5/6))/(124729*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*
x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

3.1786 $\int (a+bx)^{5/6}(c+dx)^{11/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/11*(-a*d+b*c)*(b*x+a)^(11/6)*(d*x+c)^(5/6)*hypergeom([-11/6, 11/6],[17/6]
,-d*(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 11
/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b^2*((b*(c + d*x))/(b*c - a*d)
)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{5/6}(c+dx)^{11/6}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int (a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= \frac{6(bc-ad)(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.07, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{11/6}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 11/6, 17/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)*(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(11/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(11/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(11/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5984 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(11/6),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,x)^{5/6}\,(c + d\,x)^{11/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(5/6)*(c + d*x)^(11/6), x)
```

3.1787 $\int (a+bx)^{5/6}(c+dx)^{5/6}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/11*(b*x+a)^(11/6)*(d*x+c)^(5/6)*hypergeom([-5/6, 11/6],[17/6],-d*(b*x+a)/
(-a*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{72, 71\}$

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, -((
d*(a + b*x))/(b*c - a*d))])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{5/6}(c+dx)^{5/6}\,dx = \frac{(c+dx)^{5/6}\int (a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= \frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{11/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(5/6),x]
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 11/6, 17/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{5}{6}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(5/6)*(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{6}}(c+dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(5/6),x)
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(5/6), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(5/6),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{5/6}\,(c+d\,x)^{5/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(5/6)*(c + d*x)^(5/6), x)
```

3.1788 $\int \frac{(a+bx)^{5/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

```
[Out] 6/11*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([1/6, 11/6],[17/
6],-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*b*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{(a+bx)^{5/6}}{\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11b\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(1/6),x]
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(1/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(1/6), x)
```

3.1789 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)\sqrt[6]{c+dx}}$$

```
[Out] 6/11*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([7/6, 11/6],[17/
6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)*(c + d*x)^(1/6
))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{7/6}}\,dx = \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\ {}_2F_1\left(\frac{7}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(7/6),x]
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(7/6), x)
```

3.1790 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^2\sqrt[6]{c+dx}}$$

```
[Out] 6/11*b*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([11/6, 13/6],[
17/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1
1/6, 13/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^2*(c + d*x)
^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{13/6}}\,dx = \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}$$
$$= \frac{6b(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^2\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.07, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\ {}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[11
/6, 13/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(13/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(13/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(13/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(13/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(13/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(13/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(13/6), x)
```

3.1791 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=84

$$\frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^3\sqrt[6]{c+dx}}$$

```
[Out] 6/11*b^2*(b*x+a)^(11/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([11/6, 19/6]
,[17/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11\sqrt[6]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*b^2*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1
[11/6, 19/6, 17/6, -((d*(a + b*x))/(b*c - a*d))])/(11*(b*c - a*d)^3*(c + d*
x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{19/6}}\,dx = \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{(a+bx)^{5/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}$$

$$= \frac{6b^2(a+bx)^{11/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};-\frac{d(a+bx)}{bc-ad}\right)}{11(bc-ad)^3\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 81, normalized size = 0.96

$$\frac{6b(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{11}{6},\frac{19}{6};\frac{17}{6};\frac{d(a+bx)}{-bc+ad}\right)}{11(bc-ad)^2(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(19/6),x]
[Out] (6*b*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[1
1/6, 19/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*(b*c - a*d)^2*(c + d*x)
^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(19/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2
*x^2 + 4*c^3*d*x + c^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(19/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(19/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(19/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(19/6), x)
```

3.1792 $\int (a+bx)^{7/6}(c+dx)^{13/6}\,dx$

Optimal. Leaf size=84

$$\frac{6(bc-ad)^2(a+bx)^{13/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/13*(-a*d+b*c)^2*(b*x+a)^(13/6)*(d*x+c)^(1/6)*hypergeom([-13/6, 13/6],[19/
6],-d*(b*x+a)/(-a*d+b*c))/b^3/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,(bc-ad)^2\ {}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6,
13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^3*((b*(c + d*x))/(b*c - a*
d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{7/6}(c+dx)^{13/6}\,dx = \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int (a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)^2(a+bx)^{13/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [**A**]
time = 10.08, size = 73, normalized size = 0.87

$$\frac{6(a+bx)^{13/6}(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 13/6, 19/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{13}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
[Out] int((b*x+a)^(7/6)*(d*x+c)^(13/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(13/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^3 + a*c^2 + (2*b*c*d + a*d^2)*x^2 + (b*c^2 + 2*a*c*d)*x)*
(b*x + a)^(1/6)*(d*x + c)^(1/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(13/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 9880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(13/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{7/6}\,(c+d\,x)^{13/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)*(c + d*x)^(13/6),x)
```

```
[Out] int((a + b*x)^(7/6)*(c + d*x)^(13/6), x)
```

3.1793 $\int (a+bx)^{7/6}(c+dx)^{7/6}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/13*(-a*d+b*c)*(b*x+a)^(13/6)*(d*x+c)^(1/6)*hypergeom([-7/6, 13/6],[19/6],
-d*(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\;(bc-ad)\;{}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 13/
6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b^2*((b*(c + d*x))/(b*c - a*d))
^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{7/6}(c+dx)^{7/6}\,dx = \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int (a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)(a+bx)^{13/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{13/6}(c+dx)^{7/6}\ {}_2F_1\left(-\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 13/6, 19/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{7}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)*(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="fricas")
[Out] integral((b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(1/6), x
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(7/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3276 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{7/6}\,(c+d\,x)^{7/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)*(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(7/6)*(c + d*x)^(7/6), x)
```

3.1794 $\int (a+bx)^{7/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6/13*(b*x+a)^(13/6)*(d*x+c)^(1/6)*hypergeom([-1/6, 13/6],[19/6],-d*(b*x+a)/
(-a*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, -((
d*(a + b*x))/(b*c - a*d))])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{7/6}\sqrt[6]{c+dx}\,dx = \frac{\sqrt[6]{c+dx}\int (a+bx)^{7/6}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{13/6}\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)*(c + d*x)^(1/6),x]
[Out] (6*(a + b*x)^(13/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 13/6, 19/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(13*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(7/6)*(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{7}{6}}\sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(7/6)*(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)*(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)*(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{7/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(7/6)*(c + d*x)^(1/6), x)
```

3.1795 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

```
[Out] 6/13*(b*x+a)^(13/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([5/6, 13/6],[19/
6],-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*b*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{5/6}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}$$
$$= \frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13b(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(5/6),x]
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(5/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")

[Out] integral((b*x + a)^(7/6)/(d*x + c)^(5/6), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(7/6)/(d*x+c)**(5/6),x)

[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(5/6), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="giac")

[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(5/6), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(7/6)/(c + d*x)^(5/6),x)

[Out] int((a + b*x)^(7/6)/(c + d*x)^(5/6), x)

3.1796 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6/13*(b*x+a)^(13/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([11/6, 13/6],[19
/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[11/
6, 13/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)*(c + d*x)^(5/
6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{11/6}}\,dx = \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}$$
$$= \frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.05, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}{}_2F_1\left(\frac{11}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(11/6),x]
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[11
/6, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(11/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(11/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(11/6),x)
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(11/6), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(11/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{7/6}}{{\left(c+d\,x\right)}^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(11/6), x)
```

3.1797 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6/13*b*(b*x+a)^(13/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([13/6, 17/6],[
19/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]
```

```
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, -((d*(a + b*x))/(b*c - a*d))])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{(a+bx)^{7/6}}{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$
$$= \frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};-\frac{d(a+bx)}{bc-ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 81, normalized size = 0.99

$$\frac{6b(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{13}{6},\frac{17}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13(bc-ad)^2(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(17/6),x]
[Out] (6*b*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1
3/6, 17/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*(b*c - a*d)^2*(c + d*x)
^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(17/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(7/6)*(d*x + c)^(1/6)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(17/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4496 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(17/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{17/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(17/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(17/6), x)
```

3.1798 $\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=424

$$-\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}-$$

```
[Out] -7/12*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(5/6)/d^2+1/2*(b*x+a)^(7/6)*(d*x+c)^
(5/6)/d+7/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/
6))/b^(5/6)/d^(13/6)-7/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d
*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(13/6)+7
/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^
(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(13/6)+7/72*(-a*d+b*c)^2*arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(
5/6)/d^(13/6)*3^(1/2)+7/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x
+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(5/6)/d^(13/6)*3^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{7(bc-ad)^2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}}-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}}-\frac{7\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12d^2}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*d^2) + ((a + b*x)^(7/6
)*(c + d*x)^(5/6))/(2*d) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(
13/6)) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(S
qrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c -
 a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b
^(5/6)*d^(13/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(1
44*b^(5/6)*d^(13/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{(7(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12d}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{}}}{72d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\,\mathrm{Subst}\left(\int -\right.}{}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{1-}\right.}{12bd}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{(7(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{}{\sqrt[3]{}}\right.}{}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}}{\sqrt[6]{b}}\right.}{36b^{5/6}d^{13/6}}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}}{\sqrt[6]{b}}\right.}{36b^{5/6}d^{13/6}}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{}}{\sqrt[6]{}}}{}\right.}{24\sqrt{3}\,b^{5/6}d^{13}}$$

Mathematica [A]
time = 0.77, size = 288, normalized size = 0.68

$$\frac{6b^{5/6}\sqrt[6]{d}\sqrt[6]{a+bx}(c+dx)^{5/6}(-7bc+13ad+6bdx)-7\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+7\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+14(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{72b^{5/6}d^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*b^(5/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(5/6)*(-7*b*c + 13*a*d + 6*b*d
*x) - 7*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1
/6)*(c + d*x)^(1/6)))/Sqrt[3]] + 7*Sqrt[3]*(b*c - a*d)^2*ArcTan[(1 + (2*d^(
1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] + 14*(b*c - a*d)^
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))] + 7*(b*c - a
*d)^2*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a
 + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(72*b^(5/6)*d^(13/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. $2(318) = 636$.
time = 1.37, size = 5633, normalized size = 13.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")

[Out] -1/144*(28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) + 28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*

```
c*d^12 + a^2*b^4*d^13)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*s
qrt(-((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)
*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^5*d^13))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^5*d^13))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11
*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 9
24*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^
3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12
*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(1/6), x)
```

3.1799 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$-\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}+\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-7\sqrt[6]{b}\,(bc-ad)\,t$$

```
[Out] -6*(b*x+a)^(7/6)/d/(d*x+c)^(1/6)+7*b*(b*x+a)^(1/6)*(d*x+c)^(5/6)/d^2-7/3*b^
(1/6)*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/d^(13
/6)+7/12*b^(1/6)*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-
b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(13/6)-7/12*b^(1/6)*(-a*d+b*
c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(
1/6)/(d*x+c)^(1/6))/d^(13/6)-7/6*b^(1/6)*(-a*d+b*c)*arctan(-1/3*3^(1/2)+2/3
*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/d^(13/6)*3^(1/2)-7/6*
b^(1/6)*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*
x+c)^(1/6)*3^(1/2))/d^(13/6)*3^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {49, 52, 65, 246, 216, 648, 632, 210, 642, 214}

$$\frac{7\sqrt[6]{b}\,(bc-ad)\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}}+\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}}+\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}-\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(d*(c + d*x)^(1/6)) + (7*b*(a + b*x)^(1/6)*(c + d*x)^(
5/6))/d^2 + (7*b^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6
)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/
6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6)*(b*c - a*d)*ArcTanh
[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*d^(13/6)) + (7*b^
(1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
 (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6)) - (7*b^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) +
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{(7b)\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7b(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx,\right.}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right.}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{\left(7\sqrt[6]{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+}\right.}{3d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}} +$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}} +$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} + \frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.05, size = 73, normalized size = 0.18

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{-bc+ad}\right)}{13b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(7/6))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3084 vs. $2(301) = 602$.
time = 0.95, size = 3084, normalized size = 7.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/12*(28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^
4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d
^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*d^11 - a*d^12)*(b*x + a)^(1/6)*(
d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + 2
*sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c
)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*(
(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4
*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c))*((b^7*c
^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c
^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + sqrt(3)*(b^7*c^7 - 6*a*
```

b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5 *c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6* b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d ^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6* a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^ 5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2 *d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c* d^11 - a*d^12)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c* d^5 + a^6*b*d^6)/d^13)^(5/6) + 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2 *b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d *x + c)^(2/3) - (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4* d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c))*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) - sqrt(3)*(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^ 4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6 *d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3 *c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a ^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^ 5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^ 3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4* c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log (49*((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6* c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^ 5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 1 5*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d ^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c)) - 7*(d^3*x + c*d^2)*((b^7*c^6 - 6*a *b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log(-49*((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^ 4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/ (d*x + c)) + 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4* d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^

```
3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c
)) - 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^
(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (d^3*x + c*d^2)
*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a
^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c)) - 12*
(b*d*x + 7*b*c - 6*a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(7/6), x)
```

3.1800 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=358

$$-\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}}{\sqrt{3}\,\sqrt[6]{\cdot}}\right.}{d^{13/6}}$$

```
[Out] -6/7*(b*x+a)^(7/6)/d/(d*x+c)^(7/6)-6*b*(b*x+a)^(1/6)/d^2/(d*x+c)^(1/6)+2*b^
(7/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/d^(13/6)-1/2*b^(
7/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)
^(1/6)/(d*x+c)^(1/6))/d^(13/6)+1/2*b^(7/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)
/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/d^(13/6)+b^(7/6
)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/
2))*3^(1/2)/d^(13/6)+b^(7/6)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b
^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/d^(13/6)
```

Rubi [A]
time = 0.34, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {49, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{\sqrt{3}\,b^{7/6}\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{13/6}}-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(7*d*(c + d*x)^(7/6)) - (6*b*(a + b*x)^(1/6))/(d^2*(c
+ d*x)^(1/6)) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1
/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcTan
[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]
)/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x
)^(1/6))])/d^(13/6) - (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c +
 d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/
6)) + (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx = -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}+\frac{b\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{b^2\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{(6b)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx,x,\sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{(6b)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{\left(2b^{7/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx,x,\frac{\sqrt[6]{a+b}}{\sqrt[6]{c+}}\right.}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{b^{7/6}\text{Subst}\left(\int \frac{-\sqrt[6]{}}{\sqrt[3]{b}-}\right.}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{b^{7/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{}}{}\right.}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-}}{}$$

Mathematica [A]

time = 0.50, size = 257, normalized size = 0.72

$$\frac{-\frac{6\sqrt[6]{d}\sqrt[6]{a+bx}(7bc+ad+8bdx)}{(c+dx)^{7/6}}-7\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+7\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+14b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+7b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{7d^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] ((-6*d^(1/6)*(a + b*x)^(1/6)*(7*b*c + a*d + 8*b*d*x))/(c + d*x)^(7/6) - 7*S
qrt[3]*b^(7/6)*ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(
1/6)))/Sqrt[3]] + 7*Sqrt[3]*b^(7/6)*ArcTan[(1 + (2*d^(1/6)*(a + b*x)^(1/6))
/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]] + 14*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x
)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))] + 7*b^(7/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a
 + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)
^(1/3))])/(7*d^(13/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(259) = 518$.
time = 0.99, size = 855, normalized size = 2.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] -1/14*(28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*arctan(-
1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/6) - 2*
sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/
d^13)^(1/6) + (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^
13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) + sqrt(3)*(b^7*d*x + b^7*c))/(b^7*d*
x + b^7*c)) + 28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*a
rctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/
6) - 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d
^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 - (d^5*x + c*d^4)
*(b^7/d^13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) - sqrt(3)*(b^7*d*x + b^7*c))
/(b^7*d*x + b^7*c)) - 7*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*lo
g(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) + (b*x + a)^(1/
3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) + 7*(
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*
x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 -
 (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) - 14*(d^4*x^2 + 2*c*d^3*x + c
^2*d^2)*(b^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b + (d^3*x +
c*d^2)*(b^7/d^13)^(1/6))/(d*x + c)) + 14*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b
^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b - (d^3*x + c*d^2)*(b^
7/d^13)^(1/6))/(d*x + c)) + 12*(8*b*d*x + 7*b*c + a*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(13/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(13/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(13/6),x)
[Out] int((a + b*x)^(7/6)/(c + d*x)^(13/6), x)
```

3.1801 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

```
[Out] 6/13*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(13/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Maple [A]
time = 0.17, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}}{13(dx+c)^{\frac{13}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(19/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/13*(b*x+a)^(13/6)/(d*x+c)^(13/6)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(26) = 52$.
time = 0.76, size = 104, normalized size = 3.25

$$\frac{6\,(b^2x^2+2\,abx+a^2)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{13\,(bc^4-ac^3d+(bcd^3-ad^4)x^3+3\,(bc^2d^2-acd^3)x^2+3\,(bc^3d-ac^2d^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/13*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^4 - a*c
^3*d + (b*c*d^3 - a*d^4)*x^3 + 3*(b*c^2*d^2 - a*c*d^3)*x^2 + 3*(b*c^3*d - a
*c^2*d^2)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 5985 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

Mupad [B]
time = 0.76, size = 199, normalized size = 6.22

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{6\,a^2\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{6\,b^2\,x^2\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{12\,a\,b\,x\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}\right)}{x^3-\frac{13\,b\,c^4-13\,a\,c^3\,d}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c\,d^2\,x^2\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c^2\,d\,x\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(19/6),x)
[Out] -((c + d*x)^(5/6)*((6*a^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (6*b^2
*x^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (12*a*b*x*(a + b*x)^(1/6))/
(13*a*d^4 - 13*b*c*d^3)))/(x^3 - (13*b*c^4 - 13*a*c^3*d)/(13*a*d^4 - 13*b*c
*d^3) + (39*c*d^2*x^2*(a*d - b*c))/(13*a*d^4 - 13*b*c*d^3) + (39*c^2*d*x*(a
*d - b*c))/(13*a*d^4 - 13*b*c*d^3))
```

3.1802 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}} dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}$$

```
[Out] 6/19*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(19/6)+36/247*b*(b*x+a)^(13/6)/(-a*d
+b*c)^2/(d*x+c)^(13/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (36*b*(a + b*x)^(1
3/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx = \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(6b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}} + \frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}$$

Mathematica [A]
time = 0.15, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}(19bc-13ad+6bdx)}{247(bc-ad)^2(c+dx)^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
[Out] (6*(a + b*x)^(13/6)*(19*b*c - 13*a*d + 6*b*d*x))/(247*(b*c - a*d)^2*(c + d*
x)^(19/6))
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}(-6bdx+13ad-19bc)}{247(dx+c)^{\frac{19}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(25/6),x,method=_RETURNVERBOSE)
[Out] -6/247*(b*x+a)^(13/6)*(-6*b*d*x+13*a*d-19*b*c)/(d*x+c)^(19/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(54) = 108$.
time = 0.64, size = 235, normalized size = 3.56

$$\frac{6\,(6\,b^{3}dx^{3}+19\,a^{2}bc-13\,a^{3}d+(19\,b^{3}c-ab^{2}d)x^{2}+2\,(19\,ab^{2}c-10\,a^{2}bd)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{247\,(b^{2}c^{6}-2\,abc^{5}d+a^{2}c^{4}d^{2}+(b^{2}c^{2}d^{4}-2\,abcd^{5}+a^{2}d^{6})x^{4}+4\,(b^{2}c^{3}d^{3}-2\,abc^{2}d^{4}+a^{2}cd^{5})x^{3}+6\,(b^{2}c^{4}d^{2}-2\,abc^{3}d^{3}+a^{2}c^{2}d^{4})x^{2}+4\,(b^{2}c^{5}d-2\,abc^{4}d^{2}+a^{2}c^{3}d^{3})x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/247*(6*b^3*d*x^3 + 19*a^2*b*c - 13*a^3*d + (19*b^3*c - a*b^2*d)*x^2 + 2*(
19*a*b^2*c - 10*a^2*b*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^6 - 2*a*
b*c^5*d + a^2*c^4*d^2 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^4 + 4*(b^2*
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x^3 + 6*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 +
 a^2*c^2*d^4)*x^2 + 4*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

Mupad [B]
time = 0.91, size = 189, normalized size = 2.86

$$-\frac{(c+d\,x)^{5/6}\left(\frac{(78\,a^3\,d-114\,a^2\,b\,c)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}-\frac{36\,b^3\,x^3\,(a+b\,x)^{1/6}}{247\,d^3\,(a\,d-b\,c)^2}-\frac{x^2\,(114\,b^3\,c-6\,a\,b^2\,d)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}+\frac{12\,a\,b\,x\,(10\,a\,d-19\,b\,c)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((78*a^3*d - 114*a^2*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d
 - b*c)^2) - (36*b^3*x^3*(a + b*x)^(1/6))/(247*d^3*(a*d - b*c)^2) - (x^2*(1
14*b^3*c - 6*a*b^2*d)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2) + (12*a*b*x*
(10*a*d - 19*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2)))/(x^4 + c^4/d^4
 + (4*c*x^3)/d + (4*c^3*x)/d^3 + (6*c^2*x^2)/d^2)
```

3.1803 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}} dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}$$

```
[Out] 6/25*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(25/6)+72/475*b*(b*x+a)^(13/6)/(-a*d
+b*c)^2/(d*x+c)^(19/6)+432/6175*b^2*(b*x+a)^(13/6)/(-a*d+b*c)^3/(d*x+c)^(13
/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (72*b*(a + b*x)^(1
3/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (432*b^2*(a + b*x)^(13/6))/(61
75*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx = \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{(12b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{(72b^2)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}$$
$$= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}} + \frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}} + \frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}$$

Mathematica [A]
time = 0.19, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{13/6}\left(475b^2+\frac{247d^2(a+bx)^2}{(c+dx)^2}-\frac{650bd(a+bx)}{c+dx}\right)}{6175(bc-ad)^3(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(475*b^2 + (247*d^2*(a + b*x)^2)/(c + d*x)^2 - (650*b*d
*(a + b*x))/(c + d*x)))/(6175*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}\left(72b^2x^2d^2-156ab\,d^2x+300b^2cdx+247a^2d^2-650abcd+475b^2c^2\right)}{6175(dx+c)^{\frac{25}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(31/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/6175*(b*x+a)^(13/6)*(72*b^2*d^2*x^2-156*a*b*d^2*x+300*b^2*c*d*x+247*a^2*
d^2-650*a*b*c*d+475*b^2*c^2)/(d*x+c)^(25/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(83) = 166$.
time = 1.65, size = 427, normalized size = 4.23

$$\frac{6\,(72\,b^4d^2x^4 + 475\,a^2b^2c^2 - 650\,a^3bcd + 247\,a^4d^2 + 12\,(25\,b^4cd - ab^3d^2)x^3 + (475\,b^4c^2 - 50\,ab^3cd + 7\,a^2b^2d^2)x^2 + 2\,(475\,ab^3c^2 - 500\,a^2b^2cd + 169\,a^3bd^2)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{6175\,(b^3c^8 - 3\,ab^2c^7d + 3\,a^2bc^6d^2 - a^3c^5d^3 + (b^3c^3d^5 - 3\,ab^2c^2d^6 + 3\,a^2bcd^7 - a^3d^8)x^5 + 5\,(b^3c^4d^4 - 3\,ab^2c^3d^5 + 3\,a^2bc^2d^6 - a^3cd^7)x^4 + 10\,(b^3c^5d^3 - 3\,ab^2c^4d^4 + 3\,a^2bc^3d^5 - a^3c^2d^6)x^3 + 10\,(b^3c^6d^2 - 3\,ab^2c^5d^3 + 3\,a^2bc^4d^4 - a^3c^3d^5)x^2 + 5\,(b^3c^7d - 3\,ab^2c^6d^2 + 3\,a^2bc^5d^3 - a^3c^4d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="fricas")

[Out] 6/6175*(72*b^4*d^2*x^4 + 475*a^2*b^2*c^2 - 650*a^3*b*c*d + 247*a^4*d^2 + 12*(25*b^4*c*d - a*b^3*d^2)*x^3 + (475*b^4*c^2 - 50*a*b^3*c*d + 7*a^2*b^2*d^2)*x^2 + 2*(475*a*b^3*c^2 - 500*a^2*b^2*c*d + 169*a^3*b*d^2)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^3 + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*x^5 + 5*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 10*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^3 + 10*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2 + 5*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(7/6)/(d*x+c)**(31/6),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="giac")

[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)

Mupad [B]
time = 1.14, size = 278, normalized size = 2.75

$$-\frac{{\left(c+d\,x\right)}^{5/6}\,\left(\frac{{\left(a+b\,x\right)}^{1/6}\,\left(1482\,a^4\,d^2-3900\,a^3\,b\,c\,d+2850\,a^2\,b^2\,c^2\right)}{6175\,d^5\,{\left(a\,d-b\,c\right)}^3}+\frac{432\,b^4\,x^4\,{\left(a+b\,x\right)}^{1/6}}{6175\,d^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,{\left(a+b\,x\right)}^{1/6}\,\left(42\,a^2\,b^2\,d^2-300\,a\,b^3\,c\,d+2850\,b^4\,c^2\right)}{6175\,d^5\,{\left(a\,d-b\,c\right)}^3}-\frac{72\,b^3\,x^3\,\left(a\,d-25\,b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}{6175\,d^4\,{\left(a\,d-b\,c\right)}^3}+\frac{12\,a\,b\,x\,{\left(a+b\,x\right)}^{1/6}\,\left(169\,a^2\,d^2-500\,a\,b\,c\,d+475\,b^2\,c^2\right)}{6175\,d^5\,{\left(a\,d-b\,c\right)}^3}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(31/6),x)

[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(1482*a^4*d^2 + 2850*a^2*b^2*c^2 - 3900
*a^3*b*c*d))/(6175*d^5*(a*d - b*c)^3) + (432*b^4*x^4*(a + b*x)^(1/6))/(6175
*d^3*(a*d - b*c)^3) + (x^2*(a + b*x)^(1/6)*(2850*b^4*c^2 + 42*a^2*b^2*d^2 -
 300*a*b^3*c*d))/(6175*d^5*(a*d - b*c)^3) - (72*b^3*x^3*(a*d - 25*b*c)*(a +
 b*x)^(1/6))/(6175*d^4*(a*d - b*c)^3) + (12*a*b*x*(a + b*x)^(1/6)*(169*a^2*
d^2 + 475*b^2*c^2 - 500*a*b*c*d))/(6175*d^5*(a*d - b*c)^3)))/(x^5 + c^5/d^5
 + (5*c*x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

3.1804 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}} dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}}+\frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}}+\frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}}+\frac{7776b^3(a+bx)^{13/6}}{191425(bc-ad)^4(c+d$$

```
[Out] 6/31*(b*x+a)^(13/6)/(-a*d+b*c)/(d*x+c)^(31/6)+108/775*b*(b*x+a)^(13/6)/(-a*
d+b*c)^2/(d*x+c)^(25/6)+1296/14725*b^2*(b*x+a)^(13/6)/(-a*d+b*c)^3/(d*x+c)^
(19/6)+7776/191425*b^3*(b*x+a)^(13/6)/(-a*d+b*c)^4/(d*x+c)^(13/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(31*(b*c - a*d)*(c + d*x)^(31/6)) + (108*b*(a + b*x)^(
13/6))/(775*(b*c - a*d)^2*(c + d*x)^(25/6)) + (1296*b^2*(a + b*x)^(13/6))/(
14725*(b*c - a*d)^3*(c + d*x)^(19/6)) + (7776*b^3*(a + b*x)^(13/6))/(191425
*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx = \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{(18b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx}{31(bc-ad)}$$
$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{(216b^2)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{775(bc-ad)^2}$$
$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} +$$
$$= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}} + \frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}} + \frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}} +$$

Mathematica [A]
time = 0.24, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}\left(14725b^3 - \frac{6175d^3(a+bx)^3}{(c+dx)^3} + \frac{22971bd^2(a+bx)^2}{(c+dx)^2} - \frac{30225b^2d(a+bx)}{c+dx}\right)}{191425(bc-ad)^4(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(14725*b^3 - (6175*d^3*(a + b*x)^3)/(c + d*x)^3 + (2297
1*b*d^2*(a + b*x)^2)/(c + d*x)^2 - (30225*b^2*d*(a + b*x))/(c + d*x)))/(191
425*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Maple [A]
time = 0.18, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{13}{6}}\left(-1296b^3x^3d^3+2808d^3a\,x^2b^2-6696b^3c\,d^2x^2-4446a^2b\,d^3x+14508a\,b^2c\,d^2x-13950b^3c^2dx+6175a^3d^3-22971a^2bc\,d^2+30\right.}{191425(dx+c)^{\frac{31}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(37/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/191425*(b*x+a)^(13/6)*(-1296*b^3*d^3*x^3+2808*a*b^2*d^3*x^2-6696*b^3*c*d
^2*x^2-4446*a^2*b*d^3*x+14508*a*b^2*c*d^2*x-13950*b^3*c^2*d*x+6175*a^3*d^3-
22971*a^2*b*c*d^2+30225*a*b^2*c^2*d-14725*b^3*c^3)/(d*x+c)^(31/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(112) = 224$.
time = 0.89, size = 649, normalized size = 4.77

$$\frac{6\,(1296\,b^5d^3x^5 + 14725\,a^2b^3c^3 - 30225\,a^3b^2c^2d + 22971\,a^4bcd^2 - 6175\,a^5d^3 + 216\,(31\,b^5cd^2 - ab^4d^3)x^4 + 18\,(775\,b^5c^2d - 62\,ab^4cd^2 + 7\,a^2b^3d^3)x^3 + (14725\,b^5c^3 - 2325\,ab^4c^2d + 651\,a^2b^3cd^2 - 91\,a^3b^2d^3)x^2 + 2\,(14725\,ab^4c^3 - 23250\,a^2b^3c^2d + 15717\,a^3b^2cd^2 - 3952\,a^4bd^3)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{191425\,(b^4c^{10} - 4\,ab^3c^9d + 6\,a^2b^2c^8d^2 - 4\,a^3bc^7d^3 + a^4c^6d^4 + (b^4c^4d^6 - 4\,ab^3c^3d^7 + 6\,a^2b^2c^2d^8 - 4\,a^3bcd^9 + a^4d^{10})x^6 + 6\,(b^4c^5d^5 - 4\,ab^3c^4d^6 + 6\,a^2b^2c^3d^7 - 4\,a^3bc^2d^8 + a^4cd^9)x^5 + 15\,(b^4c^6d^4 - 4\,ab^3c^5d^5 + 6\,a^2b^2c^4d^6 - 4\,a^3bc^3d^7 + a^4c^2d^8)x^4 + 20\,(b^4c^7d^3 - 4\,ab^3c^6d^4 + 6\,a^2b^2c^5d^5 - 4\,a^3bc^4d^6 + a^4c^3d^7)x^3 + 15\,(b^4c^8d^2 - 4\,ab^3c^7d^3 + 6\,a^2b^2c^6d^4 - 4\,a^3bc^5d^5 + a^4c^4d^6)x^2 + 6\,(b^4c^9d - 4\,ab^3c^8d^2 + 6\,a^2b^2c^7d^3 - 4\,a^3bc^6d^4 + a^4c^5d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="fricas")
```

```
[Out] 6/191425*(1296*b^5*d^3*x^5 + 14725*a^2*b^3*c^3 - 30225*a^3*b^2*c^2*d + 2297
1*a^4*b*c*d^2 - 6175*a^5*d^3 + 216*(31*b^5*c*d^2 - a*b^4*d^3)*x^4 + 18*(775
*b^5*c^2*d - 62*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*x^3 + (14725*b^5*c^3 - 2325*a*
b^4*c^2*d + 651*a^2*b^3*c*d^2 - 91*a^3*b^2*d^3)*x^2 + 2*(14725*a*b^4*c^3 -
23250*a^2*b^3*c^2*d + 15717*a^3*b^2*c*d^2 - 3952*a^4*b*d^3)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b
*c^7*d^3 + a^4*c^6*d^4 + (b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8
 - 4*a^3*b*c*d^9 + a^4*d^10)*x^6 + 6*(b^4*c^5*d^5 - 4*a*b^3*c^4*d^6 + 6*a^2
*b^2*c^3*d^7 - 4*a^3*b*c^2*d^8 + a^4*c*d^9)*x^5 + 15*(b^4*c^6*d^4 - 4*a*b^3
*c^5*d^5 + 6*a^2*b^2*c^4*d^6 - 4*a^3*b*c^3*d^7 + a^4*c^2*d^8)*x^4 + 20*(b^4
*c^7*d^3 - 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^5*d^5 - 4*a^3*b*c^4*d^6 + a^4*c^3*
d^7)*x^3 + 15*(b^4*c^8*d^2 - 4*a*b^3*c^7*d^3 + 6*a^2*b^2*c^6*d^4 - 4*a^3*b*
c^5*d^5 + a^4*c^4*d^6)*x^2 + 6*(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7
*d^3 - 4*a^3*b*c^6*d^4 + a^4*c^5*d^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(37/6),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

Mupad [B]
time = 1.43, size = 385, normalized size = 2.83

$$\frac{(c+d\,x)^{5/6}\,\left(\frac{7776\,b^5\,x^5\,(a+b\,x)^{1/6}}{191425\,d^3\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\,(37050\,a^5\,d^3-137826\,a^4\,b\,c\,d^2+181350\,a^3\,b^2\,c^2\,d-88350\,a^2\,b^3\,c^3)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x^2\,(a+b\,x)^{1/6}\,(-546\,a^3\,b^2\,d^3+3906\,a^2\,b^3\,c\,d^2-13950\,a\,b^4\,c^2\,d+88350\,b^5\,c^3)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\,(-47424\,a^4\,b\,d^3+188604\,a^3\,b^2\,c\,d^2-279000\,a^2\,b^3\,c^2\,d+176700\,a\,b^4\,c^3)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{108\,b^3\,x^3\,(a+b\,x)^{1/6}\,(7\,a^2\,d^2-62\,a\,b\,c\,d+775\,b^2\,c^2)}{191425\,d^5\,(a\,d-b\,c)^4}-\frac{1296\,b^4\,x^4\,(a\,d-31\,b\,c)\,(a+b\,x)^{1/6}}{191425\,d^4\,(a\,d-b\,c)^4}\right)}{x^6+\frac{c^6}{d^6}+\frac{6\,c\,x^5}{d}+\frac{6\,c^5\,x}{d^5}+\frac{15\,c^2\,x^4}{d^2}+\frac{20\,c^3\,x^3}{d^3}+\frac{15\,c^4\,x^2}{d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(37/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^5*x^5*(a + b*x)^(1/6))/(191425*d^3*(a*d - b*c)^4)
 - ((a + b*x)^(1/6)*(37050*a^5*d^3 - 88350*a^2*b^3*c^3 + 181350*a^3*b^2*c^2
*d - 137826*a^4*b*c*d^2))/(191425*d^6*(a*d - b*c)^4) + (x^2*(a + b*x)^(1/6)
*(88350*b^5*c^3 - 546*a^3*b^2*d^3 + 3906*a^2*b^3*c*d^2 - 13950*a*b^4*c^2*d)
)/(191425*d^6*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(176700*a*b^4*c^3 - 47424
*a^4*b*d^3 - 279000*a^2*b^3*c^2*d + 188604*a^3*b^2*c*d^2))/(191425*d^6*(a*d
 - b*c)^4) + (108*b^3*x^3*(a + b*x)^(1/6)*(7*a^2*d^2 + 775*b^2*c^2 - 62*a*b
*c*d))/(191425*d^5*(a*d - b*c)^4) - (1296*b^4*x^4*(a*d - 31*b*c)*(a + b*x)^
(1/6))/(191425*d^4*(a*d - b*c)^4)))/(x^6 + c^6/d^6 + (6*c*x^5)/d + (6*c^5*x
)/d^5 + (15*c^2*x^4)/d^2 + (20*c^3*x^3)/d^3 + (15*c^4*x^2)/d^4)
```

3.1805 $\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}} dx$

Optimal. Leaf size=424

$$\frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2}+\frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{13/6}d^{5/6}}-$$

```
[Out] 7/12*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b^2+1/2*(b*x+a)^(5/6)*(d*x+c)^(
7/6)/b+7/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6
))/b^(13/6)/d^(5/6)-7/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*
x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(13/6)/d^(5/6)+7/
144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(
1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(13/6)/d^(5/6)-7/72*(-a*d+b*c)^2*arctan
(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(1
3/6)/d^(5/6)*3^(1/2)-7/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+
a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(13/6)/d^(5/6)*3^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{7(bc-ad)^2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{13/6}d^{5/6}}-\frac{7(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{13/6}d^{5/6}}-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}}+\frac{7(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12b^2}+\frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (7*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^2) + ((a + b*x)^(5/6)
*(c + d*x)^(7/6))/(2*b) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(
5/6)) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(13/6)*d^(5/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(13/6)*d^(5/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad)^2)\int \frac{1}{\sqrt[6]{a+bx}\,(c\cdots}}{72b^2}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{\cdots}{\left(c-\cdots\right.}\cdots\right.}{12\cdots}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{x^4}{1-\frac{dx\cdots}{b}}\cdots\right.}{12b^3}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt[3]{b}\cdots}\cdots\right.}{3\cdots}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{\cdots}}{\sqrt[6]{b}\,\sqrt[6]{\cdots}}\right.}{36b^{13/6}d^{5/6}}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{\cdots}}{\sqrt[6]{b}\,\sqrt[6]{\cdots}}\right.}{36b^{13/6}d^{5/6}}$$

$$= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}}{\cdots}\right.}{24\sqrt{3}\,b^{13/6}d^{5/6}\cdots}$$

Mathematica [A]
time = 0.80, size = 278, normalized size = 0.66

$$\frac{(bc-ad)^2\left(\frac{6\sqrt[6]{b}(a+bx)^{5/6}\sqrt[6]{c+dx}(13bc-7ad+6bdx)}{(bc-ad)^2}-\frac{7\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)}{d^{5/6}}+\frac{7\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)}{d^{5/6}}+\frac{14\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{d^{5/6}}+\frac{7\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{d^{5/6}}\right)}{72b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(1/6)*(a + b*x)^(5/6)*(c + d*x)^(1/6)*(13*b*c - 7*a*d
+ 6*b*d*x))/(b*c - a*d)^2 - (7*Sqrt[3]*ArcTan[(1 - (2*b^(1/6)*(c + d*x)^(1/
6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]])/d^(5/6) + (7*Sqrt[3]*ArcTan[(1 + (
2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]])/d^(5/6) + (
14*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/d^(5/6) +
(7*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a +
b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/d^(5/6)))/(72*b^(13/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5633 vs. 2(318) = 636.
time = 1.32, size = 5633, normalized size = 13.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="fricas")

[Out] -1/144*(28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*sqrt(((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) + sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*

```
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12
*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*s
qrt(-((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(
1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)
*(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^13*d^5))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
+ 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^1
0*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13
*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(1/6),x)
```

```
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(1/6), x)
```

3.1806 $\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=378

$$\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+b}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+}}\right.}{2\sqrt{3}\,b^{7/6}d^{5/6}}$$

```
[Out] (b*x+a)^(5/6)*(d*x+c)^(1/6)/b+1/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(5/6)-1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(
b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(
7/6)/d^(5/6)+1/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)
+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)/d^(5/6)-1/6*(-a*d+b*c
)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/
2))/b^(7/6)/d^(5/6)*3^(1/2)-1/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(
b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(7/6)/d^(5/6)*3^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{(bc-ad)\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}-\frac{(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/b + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/
6)*d^(5/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/6)*d^(5/6)) + ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(7/6)
*d^(5/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(
5/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(5/6)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{2/3}} + \frac{(bc}{}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{}}{\sqrt[3]{b}-\sqrt[6]{b}}\right.}{12b^{}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}} - \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{}}{\sqrt[3]{c}}\right.}{12b^{}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}}{\sqrt[6]{b}}}{\sqrt{}}\right.}{2\sqrt{3}\,b^{7/6}d^{5/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.03, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{5/6}\sqrt[6]{c+dx}\ {}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3025 vs. 2(280) = 560.
time = 1.32, size = 3025, normalized size = 8.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1
```

/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) - b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5)

```
)^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) + 2*b*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6) + (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^7*d^5))^(1/6))/(b*x + a)) - 2*b*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1
/6) - (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))
^(1/6))/(b*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{1/6}}{{\left(a+b\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(1/6), x)
```

3.1807 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

```
[Out] 2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(5/6)-1/2*
ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6
)/(d*x+c)^(1/6))/b^(1/6)/d^(5/6)+1/2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+
c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(1/6)/d^(5/6)-arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(
1/2)/b^(1/6)/d^(5/6)-arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(
d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(1/6)/d^(5/6)
```

Rubi [A]
time = 0.35, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c
 + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) + (
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^
(5/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)
*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6)) + Log[b^(1/3
) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^
(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx = \frac{6\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{2/3}} + \frac{2\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx\right.}{\sqrt[6]{b}\,d^{2/3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} + \frac{2\tanh^{-}}{}$$

Mathematica [A]
time = 0.19, size = 194, normalized size = 0.63

$$\frac{\sqrt{3}\left(-\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)+\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*(-ArcTan[(1 - (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)
))/Sqrt[3]] + ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1
/6)))/Sqrt[3]]) + 2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1
/6))] + ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*
(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(b^(1/6)*d^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(222) = 444.
time = 1.34, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}bd^4\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}-2\sqrt{3}(b^2d^4x+abd^4)\sqrt{\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}+(bd^2x+ad^2)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}+\sqrt{3}(bx+a)}{3(bx+a)}\right)-2\sqrt{3}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}bd^4\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}-2\sqrt{3}(b^2d^4x+abd^4)\sqrt{-\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}-(bd^2x+ad^2)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{1}{bd^5}\right)^{\frac{5}{6}}-\sqrt{3}(bx+a)}{3(bx+a)}\right)+\frac{1}{2}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}+(bd^2x+ad^2)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)-\frac{1}{2}\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}d\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}-(bd^2x+ad^2)\left(\frac{1}{bd^5}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)+\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+ad)\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}+(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)-\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+ad)\left(\frac{1}{bd^5}\right)^{\frac{1}{6}}-(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x +
c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqrt(3)*(b^2*d^4*x + a*b*d^4)*sqrt(((b
*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/6) + (b*d^2*x + a*d^2)*(1/(b
*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6
) + sqrt(3)*(b*x + a))/(b*x + a)) - 2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqr
t(3)*(b^2*d^4*x + a*b*d^4)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d
^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6) - sqrt(3)*(b*x + a))/(b*x + a)) + 1/2
*(1/(b*d^5))^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/
6) + (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))
/(b*x + a)) - 1/2*(1/(b*d^5))^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)
*d*(1/(b*d^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3
)*(d*x + c)^(1/3))/(b*x + a)) + (1/(b*d^5))^(1/6)*log(((b*d*x + a*d)*(1/(b*
d^5))^(1/6) + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) - (1/(b*d^5))^(1/
```

```
6)*log(-((b*d*x + a*d)*(1/(b*d^5))^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))
/(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(5/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)), x)
```

3.1808 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6/5*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx = \frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Maple [A]
time = 0.19, size = 27, normalized size = 0.84

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}}{5(dx+c)^{\frac{5}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x,method=_RETURNVERBOSE)
[Out] -6/5*(b*x+a)^(5/6)/(d*x+c)^(5/6)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

Fricas [A]
time = 1.22, size = 42, normalized size = 1.31

$$\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\,(bc^2-acd+(bcd-ad^2)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] 6/5*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(11/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(11/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

Mupad [B]
time = 0.76, size = 27, normalized size = 0.84

$$-\frac{6\,{\left(a+b\,x\right)}^{5/6}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(c+d\,x\right)}^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x)
[Out] -(6*(a + b*x)^(5/6))/((5*a*d - 5*b*c)*(c + d*x)^(5/6))
```

3.1809 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}}+\frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] 6/11*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(11/6)+36/55*b*(b*x+a)^(5/6)/(-a*d+b*
c)^2/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(11*(b*c - a*d)*(c + d*x)^(11/6)) + (36*b*(a + b*x)^(5/
6))/(55*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx = \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}} + \frac{(6b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{11(bc-ad)}$$
$$= \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}} + \frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{5/6}(11bc-5ad+6bdx)}{55(bc-ad)^2(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
[Out] (6*(a + b*x)^(5/6)*(11*b*c - 5*a*d + 6*b*d*x))/(55*(b*c - a*d)^2*(c + d*x)^
(11/6))
```

Maple [A]
time = 0.20, size = 54, normalized size = 0.82

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}(-6bdx+5ad-11bc)}{55(dx+c)^{\frac{11}{6}}(a^2d^2-2abcd+b^2c^2)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x,method=_RETURNVERBOSE)
[Out] -6/55*(b*x+a)^(5/6)*(-6*b*d*x+5*a*d-11*b*c)/(d*x+c)^(11/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.

time = 0.95, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx+11\,bc-5\,ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/55*(6*b*d*x + 11*b*c - 5*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^2*c^4 -
2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(
b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(17/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4497 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

Mupad [B]

time = 0.86, size = 127, normalized size = 1.92

$$\frac{{\left(c+d\,x\right)}^{1/6}\,\left(\frac{x\,\left(66\,c\,b^2+6\,a\,d\,b\right)}{55\,d^2\,{\left(a\,d-b\,c\right)}^2}-\frac{30\,a^2\,d-66\,a\,b\,c}{55\,d^2\,{\left(a\,d-b\,c\right)}^2}+\frac{36\,b^2\,x^2}{55\,d\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,{\left(a+b\,x\right)}^{1/6}+\frac{c^2\,{\left(a+b\,x\right)}^{1/6}}{d^2}+\frac{2\,c\,x\,{\left(a+b\,x\right)}^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((x*(66*b^2*c + 6*a*b*d))/(55*d^2*(a*d - b*c)^2) - (30*a^2
*d - 66*a*b*c)/(55*d^2*(a*d - b*c)^2) + (36*b^2*x^2)/(55*d*(a*d - b*c)^2)))
/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(a + b*x)^(1/6))
/d)
```

3.1810 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}}+\frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)^{5/6}}$$

```
[Out] 6/17*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(17/6)+72/187*b*(b*x+a)^(5/6)/(-a*d+b
*c)^2/(d*x+c)^(11/6)+432/935*b^2*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (72*b*(a + b*x)^(5/
6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6)) + (432*b^2*(a + b*x)^(5/6))/(935*(
b*c - a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx = \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{(12b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{17(bc-ad)}$$

$$= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}} + \frac{(72b^2)\int \frac{1}{\sqrt[6]{a+bx}\,(c+d}}{187(bc-ad)^2}$$

$$= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}} + \frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}} + \frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{17/6}\left(55d^2 - \frac{170bd(c+dx)}{a+bx} + \frac{187b^2(c+dx)^2}{(a+bx)^2}\right)}{935(bc-ad)^3(c+dx)^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(17/6)*(55*d^2 - (170*b*d*(c + d*x))/(a + b*x) + (187*b^2*(c +
 d*x)^2)/(a + b*x)^2))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}\left(72b^2x^2d^2-60ab\,d^2x+204b^2cdx+55a^2d^2-170abcd+187b^2c^2\right)}{935(dx+c)^{\frac{17}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/935*(b*x+a)^(5/6)*(72*b^2*d^2*x^2-60*a*b*d^2*x+204*b^2*c*d*x+55*a^2*d^2-
170*a*b*c*d+187*b^2*c^2)/(d*x+c)^(17/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(83) = 166$.
time = 1.27, size = 252, normalized size = 2.50

$$\frac{6\,(72\,b^2d^2x^2 + 187\,b^2c^2 - 170\,abcd + 55\,a^2d^2 + 12\,(17\,b^2cd - 5\,abd^2)x)(bx + a)^{\frac{5}{6}}(dx + c)^{\frac{1}{6}}}{935\,(b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3 + (b^3c^3d^3 - 3\,ab^2c^2d^4 + 3\,a^2bcd^5 - a^3d^6)x^3 + 3\,(b^3c^4d^2 - 3\,ab^2c^3d^3 + 3\,a^2bc^2d^4 - a^3cd^5)x^2 + 3\,(b^3c^5d - 3\,ab^2c^4d^2 + 3\,a^2bc^3d^3 - a^3c^2d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/935*(72*b^2*d^2*x^2 + 187*b^2*c^2 - 170*a*b*c*d + 55*a^2*d^2 + 12*(17*b^2
*c*d - 5*a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^6 - 3*a*b^2*c^5
*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2
*b*c*d^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^
4 - a^3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3
*c^2*d^4)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

Mupad [B]
time = 1.03, size = 203, normalized size = 2.01

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{330\,a^3\,d^2-1020\,a^2\,b\,c\,d+1122\,a\,b^2\,c^2}{935\,d^3\,(a\,d-b\,c)^3}+\frac{x\,(-30\,a^2\,b\,d^2+204\,a\,b^2\,c\,d+1122\,b^3\,c^2)}{935\,d^3\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3}{935\,d\,(a\,d-b\,c)^3}+\frac{72\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((330*a^3*d^2 + 1122*a*b^2*c^2 - 1020*a^2*b*c*d)/(935*d^3
*(a*d - b*c)^3) + (x*(1122*b^3*c^2 - 30*a^2*b*d^2 + 204*a*b^2*c*d))/(935*d^
3*(a*d - b*c)^3) + (432*b^3*x^3)/(935*d*(a*d - b*c)^3) + (72*b^2*x^2*(a*d +
 17*b*c))/(935*d^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/6) + (c^3*(a + b*x)^(
1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*x)^(1/6))/d^2)
```

3.1811 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}}+\frac{7776b^3(a+bx)^{5/6}}{21505(bc-ad)^4(c+dx)$$

```
[Out] 6/23*(b*x+a)^(5/6)/(-a*d+b*c)/(d*x+c)^(23/6)+108/391*b*(b*x+a)^(5/6)/(-a*d+
b*c)^2/(d*x+c)^(17/6)+1296/4301*b^2*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(11/
6)+7776/21505*b^3*(b*x+a)^(5/6)/(-a*d+b*c)^4/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (108*b*(a + b*x)^(5
/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (1296*b^2*(a + b*x)^(5/6))/(430
1*(b*c - a*d)^3*(c + d*x)^(11/6)) + (7776*b^3*(a + b*x)^(5/6))/(21505*(b*c
- a*d)^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{29/6}}\,dx = \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{(18b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{23(bc-ad)}$$

$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{(216b^2)\int \frac{1}{\sqrt[6]{a+bx}\,(c+}}{391(bc-ad)^2}$$

$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx}$$

$$= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}} + \frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}} + \frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx}$$

Mathematica [A]
time = 0.14, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{23/6}\left(-935d^3+\frac{3795bd^2(c+dx)}{a+bx}-\frac{5865b^2d(c+dx)^2}{(a+bx)^2}+\frac{4301b^3(c+dx)^3}{(a+bx)^3}\right)}{21505(bc-ad)^4(c+dx)^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(23/6)*(-935*d^3 + (3795*b*d^2*(c + d*x))/(a + b*x) - (5865*b^
2*d*(c + d*x)^2)/(a + b*x)^2 + (4301*b^3*(c + d*x)^3)/(a + b*x)^3))/(21505*
(b*c - a*d)^4*(c + d*x)^(23/6))
```

Maple [A]
time = 0.23, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{5}{6}}\left(-1296b^3x^3d^3+1080d^3a\,x^2b^2-4968b^3c\,d^2x^2-990a^2b\,d^3x+4140a\,b^2c\,d^2x-7038b^3c^2dx+935a^3d^3-3795a^2bc\,d^2+5865a\,b^2\right.}{21505(dx+c)^{\frac{23}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/21505*(b*x+a)^(5/6)*(-1296*b^3*d^3*x^3+1080*a*b^2*d^3*x^2-4968*b^3*c*d^2
*x^2-990*a^2*b*d^3*x+4140*a*b^2*c*d^2*x-7038*b^3*c^2*d*x+935*a^3*d^3-3795*a
^2*b*c*d^2+5865*a*b^2*c^2*d-4301*b^3*c^3)/(d*x+c)^(23/6)/(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(112) = 224$.
time = 1.02, size = 420, normalized size = 3.09

$$\frac{6\,(1296\,b^3d^3x^3+4301\,b^3c^3-5865\,ab^2c^2d+3795\,a^2bcd^2-935\,a^3d^3+216\,(23\,b^3cd^2-5\,ab^2d^3)x^2+18\,(391\,b^3c^2d-230\,ab^2cd^2+55\,a^2bd^3)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{21505\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8)x^4+4\,(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7)x^3+6\,(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6)x^2+4\,(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/21505*(1296*b^3*d^3*x^3 + 4301*b^3*c^3 - 5865*a*b^2*c^2*d + 3795*a^2*b*c*
d^2 - 935*a^3*d^3 + 216*(23*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 18*(391*b^3*c^2*
d - 230*a*b^2*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^4
*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 +
(b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^
8)*x^4 + 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2
*d^6 + a^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^
4 - 4*a^3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

Mupad [**B**]
time = 1.20, size = 292, normalized size = 2.15

$$\frac{{\left(c+d\,x\right)}^{1/6}\,\left(\frac{7776\,b^4\,x^4}{21505\,d\,{\left(a\,d-b\,c\right)}^4}-\frac{5610\,a^4\,d^3-22770\,a^3\,b\,c\,d^2+35190\,a^2\,b^2\,c^2\,d-25806\,a\,b^3\,c^3}{21505\,d^4\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(330\,a^3\,b\,d^3-2070\,a^2\,b^2\,c\,d^2+7038\,a\,b^3\,c^2\,d+25806\,b^4\,c^3\right)}{21505\,d^4\,{\left(a\,d-b\,c\right)}^4}+\frac{1296\,b^3\,x^3\,\left(a\,d+23\,b\,c\right)}{21505\,d^2\,{\left(a\,d-b\,c\right)}^4}+\frac{108\,b^2\,x^2\,\left(-5\,a^2\,d^2+46\,a\,b\,c\,d+391\,b^2\,c^2\right)}{21505\,d^3\,{\left(a\,d-b\,c\right)}^4}\right)}{x^4\,{\left(a+b\,x\right)}^{1/6}+\frac{c^4\,{\left(a+b\,x\right)}^{1/6}}{d^4}+\frac{6\,c^2\,x^2\,{\left(a+b\,x\right)}^{1/6}}{d^2}+\frac{4\,c\,x^3\,{\left(a+b\,x\right)}^{1/6}}{d}+\frac{4\,c^3\,x\,{\left(a+b\,x\right)}^{1/6}}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4)/(21505*d*(a*d - b*c)^4) - (5610*a^4*d^3 -
25806*a*b^3*c^3 + 35190*a^2*b^2*c^2*d - 22770*a^3*b*c*d^2)/(21505*d^4*(a*d
- b*c)^4) + (x*(25806*b^4*c^3 + 330*a^3*b*d^3 - 2070*a^2*b^2*c*d^2 + 7038*a
*b^3*c^2*d))/(21505*d^4*(a*d - b*c)^4) + (1296*b^3*x^3*(a*d + 23*b*c))/(215
05*d^2*(a*d - b*c)^4) + (108*b^2*x^2*(391*b^2*c^2 - 5*a^2*d^2 + 46*a*b*c*d)
)/(21505*d^3*(a*d - b*c)^4)))/(x^4*(a + b*x)^(1/6) + (c^4*(a + b*x)^(1/6))/
d^4 + (6*c^2*x^2*(a + b*x)^(1/6))/d^2 + (4*c*x^3*(a + b*x)^(1/6))/d + (4*c^
3*x*(a + b*x)^(1/6))/d^3)
```

3.1812 $\int \frac{(c+dx)^{11/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/5*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(5/6)*hypergeom([-11/6, 5/6],[11/6],-d
*(b*x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, 5/6
, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b^2*((b*(c + d*x))/(b*c - a*d))^(
5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{\sqrt[6]{a+bx}}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}{\sqrt[6]{a+bx}}\,dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= \frac{6(bc-ad)(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{5/6}(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(1/6),x]
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 5/6, 11/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
[Out] int((d*x+c)^(11/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(11/6)/(b*x + a)^(1/6), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(1/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(1/6),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(11/6)/(a + b*x)^(1/6), x)
```

3.1813 $\int \frac{(c+dx)^{5/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] 6/5*(b*x+a)^(5/6)*(d*x+c)^(5/6)*hypergeom([-5/6, 5/6],[11/6],-d*(b*x+a)/(-a
*d+b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{72, 71\}$

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, -((d*
(a + b*x))/(b*c - a*d))])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{\sqrt[6]{a+bx}}\,dx = \frac{(c+dx)^{5/6}\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}{\sqrt[6]{a+bx}}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

$$= \frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{5/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(1/6),x]
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(1/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(1/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(1/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(1/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(1/6), x)
```

3.1814 $\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=74

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

```
[Out] 6/5*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([1/6, 5/6],[11/6],
-d*(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(1/6)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*b*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{\sqrt[6]{a+bx}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5b\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.02, size = 73, normalized size = 0.99

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x]
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 5/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(1/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(1/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(1/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(1/6)), x)
```

3.1815 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=81

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)\sqrt[6]{c+dx}}$$

```
[Out] 6/5*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([5/6, 7/6],[11/6],
-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{7/6}}\,dx = \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}$$
$$= \frac{6(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 73, normalized size = 0.90

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\ {}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[5/6,
 7/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(7/6))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(7/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(7/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(7/6)), x)
```

3.1816 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^2\sqrt[6]{c+dx}}$$

```
[Out] 6/5*b*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([5/6, 13/6],[11/
6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[5/
6, 13/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^2*(c + d*x)^(1
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{13/6}}\,dx = \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}$$
$$= \frac{6b(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^2\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 10.05, size = 73, normalized size = 0.89

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[5/6
, 13/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(13/6))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(13/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 +
a*d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(13/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(13/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(13/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(13/6)),x)
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(13/6)), x)
```

3.1817 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=84

$$\frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^3\sqrt[6]{c+dx}}$$

```
[Out] 6/5*b^2*(b*x+a)^(5/6)*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([5/6, 19/6],[1
1/6],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5\sqrt[6]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*b^2*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[
5/6, 19/6, 11/6, -((d*(a + b*x))/(b*c - a*d))])/(5*(b*c - a*d)^3*(c + d*x)^
(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{19/6}}\,dx = \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt[6]{a+bx}\,\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}$$
$$= \frac{6b^2(a+bx)^{5/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};-\frac{d(a+bx)}{bc-ad}\right)}{5(bc-ad)^3\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 81, normalized size = 0.96

$$\frac{6b(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{5}{6},\frac{19}{6};\frac{11}{6};\frac{d(a+bx)}{-bc+ad}\right)}{5(bc-ad)^2(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*b*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[5/
6, 19/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*(b*c - a*d)^2*(c + d*x)^(7
/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(19/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b*d^4*x^5 + a*c^4 + (4*b*c*d^3 +
a*d^4)*x^4 + 2*(3*b*c^2*d^2 + 2*a*c*d^3)*x^3 + 2*(2*b*c^3*d + 3*a*c^2*d^2)*
x^2 + (b*c^4 + 4*a*c^3*d)*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(19/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/6}\,{\left(c+d\,x\right)}^{19/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(19/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(19/6)), x)
```

3.1818 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=82

$$\frac{6(bc-ad)^2\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6*(-a*d+b*c)^2*(b*x+a)^(1/6)*(d*x+c)^(1/6)*hypergeom([-13/6, 1/6],[7/6],-d*
(b*x+a)/(-a*d+b*c))/b^3/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2\;{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-13/6, 1
/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^3*((b*(c + d*x))/(b*c - a*d))^(1
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{5/6}}\,dx = \frac{\left((bc-ad)^2\sqrt[6]{c+dx}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}{(a+bx)^{5/6}}\,dx}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$
$$= \frac{6(bc-ad)^2\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.06, size = 71, normalized size = 0.87

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, 1/6, 7/6, (d*(
a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(13/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(5/6), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="fricas")
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(5/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(5/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{13/6}}{{\left(a+b\,x\right)}^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(13/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(13/6)/(a + b*x)^(5/6), x)
```

3.1819 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=80

$$\frac{6(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(1/6)*hypergeom([-7/6, 1/6],[7/6],-d*(b*
x+a)/(-a*d+b*c))/b^2/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)\;{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-7/6, 1/6,
 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)
)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{5/6}}\,dx = \frac{\left((bc-ad)\sqrt[6]{c+dx}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}{(a+bx)^{5/6}}\,dx}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

$$= \frac{6(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.04, size = 71, normalized size = 0.89

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(5/6),x]
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
[Out] int((d*x+c)^(7/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(5/6), x)
```

3.1820 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

```
[Out] 6*(b*x+a)^(1/6)*(d*x+c)^(1/6)*hypergeom([-1/6, 1/6],[7/6],-d*(b*x+a)/(-a*d+
b*c))/b/(b*(d*x+c)/(-a*d+b*c))^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, -((d*(
a + b*x))/(b*c - a*d))])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{c+dx}\int \frac{\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}{(a+bx)^{5/6}}\,dx}{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}} = \frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Mathematica [A]
time = 10.02, size = 71, normalized size = 0.99

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(5/6),x]
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="fricas")
[Out] integral((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(5/6), x)
```

3.1821 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

```
[Out] 6*(b*x+a)^(1/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([1/6, 5/6],[7/6],-d*
(b*x+a)/(-a*d+b*c))/b/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/(b*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{5/6}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}\,dx}{(c+dx)^{5/6}}$$
$$= \frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b(c+dx)^{5/6}}$$

Mathematica [**A**]
time = 10.03, size = 71, normalized size = 0.99

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{5}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x]
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 5/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(5/6))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(5/6)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d*x^2 + a*c + (b*c + a*d)*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(5/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(5/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{5/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(5/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(5/6)), x)
```

3.1822 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=79

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(c+dx)^{5/6}}$$

```
[Out] 6*(b*x+a)^(1/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([1/6, 11/6],[7/6],-d
*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {72, 71}

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6,
 11/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(c + d*x)^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{11/6}}\,dx = \frac{\left(b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}\,dx}{(bc-ad)(c+dx)^{5/6}}$$
$$= \frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.06, size = 71, normalized size = 0.90

$$\frac{6\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6} {}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x]
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[1/6
, 11/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(11/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(11/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d^2*x^3 + a*c^2 + (2*b*c*d + a*
d^2)*x^2 + (b*c^2 + 2*a*c*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(11/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(11/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{11/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(11/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(11/6)), x)
```

3.1823 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=80

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

[Out] 6*b*(b*x+a)^(1/6)*(b*(d*x+c)/(-a*d+b*c))^(5/6)*hypergeom([1/6, 17/6],[7/6], -d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^2/(d*x+c)^(5/6)

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]

[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/6, 17/6, 7/6, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(c + d*x)^(5/6))

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{17/6}}\,dx = \frac{\left(b^2\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\right)\int \frac{1}{(a+bx)^{5/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{17/6}}\,dx}{(bc-ad)^2(c+dx)^{5/6}}$$
$$= \frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A]
time = 10.04, size = 79, normalized size = 0.99

$$\frac{6b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)^2(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x]
[Out] (6*b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[1/
6, 17/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(c + d*x)^(5/6)
)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(17/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(1/6)*(d*x + c)^(1/6)/(b*d^3*x^4 + a*c^3 + (3*b*c*d^2 +
a*d^3)*x^3 + 3*(b*c^2*d + a*c*d^2)*x^2 + (b*c^3 + 3*a*c^2*d)*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(17/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6547 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(17/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{17/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(17/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(17/6)), x)
```

3.1824 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=424

$$\frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b}-\frac{55(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+$$

```
[Out] 11/12*(-a*d+b*c)*(b*x+a)^(1/6)*(d*x+c)^(5/6)/b^2+1/2*(b*x+a)^(1/6)*(d*x+c)^
(11/6)/b+55/36*(-a*d+b*c)^2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(
1/6))/b^(17/6)/d^(1/6)-55/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)
/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(17/6)/d^(1/6
)+55/144*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6
)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(17/6)/d^(1/6)+55/72*(-a*d+b*c)^2*
arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2)
)/b^(17/6)/d^(1/6)*3^(1/2)+55/72*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6
)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(17/6)/d^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{55(bc-ad)^2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}}-\frac{55(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}}+\frac{11\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12b^2}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (11*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b^2) + ((a + b*x)^(1/6
)*(c + d*x)^(11/6))/(2*b) - (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*
d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))
/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b
*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(
36*b^(17/6)*d^(1/6)) - (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(
1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])
/(144*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(144*b^(17/6)*d^(1/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(11(bc-ad))\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx}{12b}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{\cdots}}}{72b^2}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\,\mathrm{Subst}\left(\int \frac{\cdots}{1\cdots}\right.}{12b^3\cdots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(55(bc-ad)^2)\,\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}\right.}{36b^{17/6}\sqrt[6]{d}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}\right.}{36b^{17/6}\sqrt[6]{d}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} - \frac{55(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{\cdots}}{\sqrt[6]{\cdots}}}{\cdots}\right.}{24\sqrt{3}\,b^{17/6}\sqrt[6]{\cdots}}$$

Mathematica [A]
time = 0.55, size = 278, normalized size = 0.66

$$\frac{(bc-ad)^2\left(\frac{6b^{5/6}\sqrt[6]{a+bx}\,(c+dx)^{5/6}(17bc-11ad+6bdx)}{(bc-ad)^2}-\frac{55\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{d}}+\frac{55\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{d}}+\frac{110\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{d}}+\frac{55\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[6]{d}}\right)}{72b^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(5/6)*(a + b*x)^(1/6)*(c + d*x)^(5/6)*(17*b*c - 11*a*d
 + 6*b*d*x))/(b*c - a*d)^2 - (55*Sqrt[3]*ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(
1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]])/d^(1/6) + (55*Sqrt[3]*ArcTan[(1
+ (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)))/Sqrt[3]])/d^(1/6)
+ (110*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(1/6
) + (55*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*
(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/d^(1/6)))/(72*b^(17/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(5/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5591 vs. 2(318) = 636.
time = 1.14, size = 5591, normalized size = 13.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="fricas")

[Out] -1/144*(220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) + 220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*

```
c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d)
)^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d +
 a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12)/(b^17*d))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)
*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b
^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*
d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495
*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c
^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^1
0*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(5/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(11/6)/(a + b*x)^(5/6), x)
```

3.1825 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}} \, dx$

Optimal. Leaf size=378

$$\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c}}\right.}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}$$

```
[Out] (b*x+a)^(1/6)*(d*x+c)^(5/6)/b+5/3*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/
b^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(1/6)-5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*
(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^
(11/6)/d^(1/6)+5/12*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/
3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(11/6)/d^(1/6)+5/6*(-a*d+
b*c)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^
(1/2))/b^(11/6)/d^(1/6)*3^(1/2)+5/6*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/
6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(11/6)/d^(1/6)*3^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {52, 65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{5(bc-ad)\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/b - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
11/6)*d^(1/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(11/6)*d^(1/6)) + (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(11/6)*d^(1/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(11/6)*d^(1/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(11/6)*d^(1/6))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6b}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{11/6}} +$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} + \frac{(5(bc-ad))\text{Subst}\Big(\int \frac{}{\sqrt[3]{b}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} - \frac{5(bc-ad)\log\Big(\sqrt[3]{b}+\sqrt[3]{}}{1}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}} + \frac{5(bc-ad)\tan^{-1}\Big(\frac{1+2}{}}{2\sqrt{3}\,b^{11/6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 10.02, size = 71, normalized size = 0.19

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2997 vs. 2(280) = 560.
time = 0.88, size = 2997, normalized size = 7.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/12*(20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/
```

6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15
*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*sqrt(3)*(b^
9*d^2*x + b^9*c*d)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + sqrt(3)*(b^6*c^7 - 6*a*b^5*c^
6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*
b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^
6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^
2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15
*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6
 + a^6*d^7)*x)) + 20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^1
1*d))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d
x + c)^(5/6)((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*
sqrt(3)*(b^9*d^2*x + b^9*c*d)*sqrt(-((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) - (b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^4*d*x + b^
4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) - sqrt(3)*(b^6*c^7
- 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*
d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b
^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*
d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3
 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*
c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*
a^5*b*c*d^6 + a^6*d^7)*x)) + 5*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^11*d))^(1/6)*log(25*((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*
b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c)) - 5*b*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-25*((b^3*c - a*b^2*d)*

```
(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b
^11*d))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(
2/3) - (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))
^(1/3))/(d*x + c)) + 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d)
)^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (b^2*d*x + b^
2*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) -
 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-5*((b*c
 - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (b^2*d*x + b^2*c)*((b^6*c^6 - 6*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) + 12*(b*x + a)^(1/6)*
(d*x + c)^(5/6))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(5/6), x)
```

3.1826 $\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=309

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

```
[Out] 2*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(1/6)-1/2*
ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6
)/(d*x+c)^(1/6))/b^(5/6)/d^(1/6)+1/2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+
c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(5/6)/d^(1/6)+arcta
n(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(
1/2)/b^(5/6)/d^(1/6)+arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(
d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(5/6)/d^(1/6)
```

Rubi [A]
time = 0.33, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {65, 246, 216, 648, 632, 210, 642, 214}

$$-\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\,\mathrm{ArcTan}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}}-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1
/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6)) + Log[b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*
x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx = \frac{6\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{5/6}} + \frac{2\text{Subst}\left(\int \frac{\sqrt[6]{b}+\frac{\sqrt[6]{d}}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}}\right.}{b}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\sqrt[3]{d}x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right.}{2b^{2/3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{5/6}\sqrt[6]{d}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{2\tanh}{}$$

Mathematica [A]
time = 0.18, size = 194, normalized size = 0.63

$$\frac{\sqrt{3}\left(-\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)\right)+2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)+\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (Sqrt[3]*(-ArcTan[(1 - (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6)
))/Sqrt[3]] + ArcTan[(1 + (2*d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1
/6)))/Sqrt[3]]) + 2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1
/6))] + ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*
(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/(b^(5/6)*d^(1/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(220) = 440$.
time = 1.12, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4 d\left(\frac{1}{b^5 d}\right)^{\frac{5}{6}} - 2\sqrt{3}(b^4d^2x+b^4cd)\sqrt{\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}+(b^2dx+b^2c)\left(\frac{1}{b^5 d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5 d}\right)^{\frac{5}{6}}+\sqrt{3}(dx+c)}{3(dx+c)}\right) - 2\sqrt{3}\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4 d\left(\frac{1}{b^5 d}\right)^{\frac{5}{6}} - 2\sqrt{3}(b^4d^2x+b^4cd)\sqrt{-\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}-(b^2dx+b^2c)\left(\frac{1}{b^5 d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5 d}\right)^{\frac{5}{6}}-\sqrt{3}(dx+c)}{3(dx+c)}\right) + \frac{1}{2}\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}+(b^2dx+b^2c)\left(\frac{1}{b^5 d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right) - \frac{1}{2}\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}-(b^2dx+b^2c)\left(\frac{1}{b^5 d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right) + \left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}+(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right) - \left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^5 d}\right)^{\frac{1}{6}}-(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x +
c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqrt(3)*(b^4*d^2*x + b^4*c*d)*sqrt(((b
*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/6) + (b^2*d*x + b^2*c)*(1/(b
^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6
) + sqrt(3)*(d*x + c))/(d*x + c)) - 2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqr
t(3)*(b^4*d^2*x + b^4*c*d)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5
*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6) - sqrt(3)*(d*x + c))/(d*x + c)) + 1/2
*(1/(b^5*d))^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/
6) + (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))
/(d*x + c)) - 1/2*(1/(b^5*d))^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)
*b*(1/(b^5*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3
)*(d*x + c)^(2/3))/(d*x + c)) + (1/(b^5*d))^(1/6)*log(((b*d*x + b*c)*(1/(b^
5*d))^(1/6) + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) - (1/(b^5*d))^(1/
```

```
6)*log(-((b*d*x + b*c)*(1/(b^5*d))^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))
/(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(1/6),x)
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(1/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)), x)
```

3.1827 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=30

$$\frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

```
[Out] 6*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
```

[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))

Maple [A]
time = 0.17, size = 27, normalized size = 0.90

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x,method=_RETURNVERBOSE)
[Out] -6*(b*x+a)^(1/6)/(d*x+c)^(1/6)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

Fricas [A]
time = 0.59, size = 42, normalized size = 1.40

$$\frac{6\,(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{bc^2-acd+(bcd-ad^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] 6*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(7/6),x)
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(7/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)), x)
```

3.1828 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=66

$$\frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}}+\frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}$$

```
[Out] 6/7*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(7/6)+36/7*b*(b*x+a)^(1/6)/(-a*d+b*c)^
2/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}\,(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(7*(b*c - a*d)*(c + d*x)^(7/6)) + (36*b*(a + b*x)^(1/6)
)/(7*(b*c - a*d)^2*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}} + \frac{(6b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx}{7(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}} + \frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}$$

Mathematica [**A**]
time = 0.09, size = 46, normalized size = 0.70

$$\frac{6\sqrt[6]{a+bx}\,(7bc-ad+6bdx)}{7(bc-ad)^2(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*b*c - a*d + 6*b*d*x))/(7*(b*c - a*d)^2*(c + d*x)^(7/6
))
```

Maple [**A**]
time = 0.17, size = 53, normalized size = 0.80

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}(-6bdx+ad-7bc)}{7(dx+c)^{\frac{7}{6}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/7*(b*x+a)^(1/6)*(-6*b*d*x+a*d-7*b*c)/(d*x+c)^(7/6)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(54) = 108$.
time = 0.89, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx + 7\,bc - ad)(bx + a)^{\frac{1}{6}}(dx + c)^{\frac{5}{6}}}{7\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2 + (b^2c^2d^2 - 2\,abcd^3 + a^2d^4)x^2 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] 6/7*(6*b*d*x + 7*b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*
c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx)^{\frac{5}{6}}\,(c + dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(13/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(13/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(a+b\,x)}^{5/6}\,{(c+d\,x)}^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)), x)
```

3.1829 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=101

$$\frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}}+\frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}$$

```
[Out] 6/13*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(13/6)+72/91*b*(b*x+a)^(1/6)/(-a*d+b*
c)^2/(d*x+c)^(7/6)+432/91*b^2*(b*x+a)^(1/6)/(-a*d+b*c)^3/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 37}

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}\,(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (72*b*(a + b*x)^(1/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6)) + (432*b^2*(a + b*x)^(1/6))/(91*(b*c
 - a*d)^3*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{(12b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx}{13(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}} + \frac{(72b^2)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7}}}{91(bc-ad)^2}$$
$$= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}} + \frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}} + \frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 0.11, size = 77, normalized size = 0.76

$$\frac{6\sqrt[6]{a+bx}\,(7a^2d^2-2abd(13c+6dx)+b^2(91c^2+156cdx+72d^2x^2))}{91(bc-ad)^3(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*a^2*d^2 - 2*a*b*d*(13*c + 6*d*x) + b^2*(91*c^2 + 156*
c*d*x + 72*d^2*x^2)))/(91*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Maple [A]
time = 0.20, size = 105, normalized size = 1.04

method	result	size
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}\left(72b^2x^2d^2-12ab\,d^2x+156b^2cdx+7a^2d^2-26abcd+91b^2c^2\right)}{91(dx+c)^{\frac{13}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/91*(b*x+a)^(1/6)*(72*b^2*d^2*x^2-12*a*b*d^2*x+156*b^2*c*d*x+7*a^2*d^2-26
*a*b*c*d+91*b^2*c^2)/(d*x+c)^(13/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(83) = 166$.
time = 0.94, size = 252, normalized size = 2.50

$$\frac{6\,(72\,b^2d^2x^2+91\,b^2c^2-26\,abcd+7\,a^2d^2+12\,(13\,b^2cd-abd^2)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)x^3+3\,(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5)x^2+3\,(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(72*b^2*d^2*x^2 + 91*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2 + 12*(13*b^2*c*d
 - a*b*d^2)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^6 - 3*a*b^2*c^5*d + 3
*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d
^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^
3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d
^4)*x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(19/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 9141 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{19/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)), x)
```

3.1830 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=136

$$\frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}}+\frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}}+\frac{7776b^3\sqrt[6]{a+bx}}{1729(bc-ad)^4\sqrt[6]{c+dx}}$$

```
[Out] 6/19*(b*x+a)^(1/6)/(-a*d+b*c)/(d*x+c)^(19/6)+108/247*b*(b*x+a)^(1/6)/(-a*d+
b*c)^2/(d*x+c)^(13/6)+1296/1729*b^2*(b*x+a)^(1/6)/(-a*d+b*c)^3/(d*x+c)^(7/6
)+7776/1729*b^3*(b*x+a)^(1/6)/(-a*d+b*c)^4/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.02, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}\,(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (108*b*(a + b*x)^(1
/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (1296*b^2*(a + b*x)^(1/6))/(172
9*(b*c - a*d)^3*(c + d*x)^(7/6)) + (7776*b^3*(a + b*x)^(1/6))/(1729*(b*c -
a*d)^4*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{(18b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx}{19(bc-ad)}$$
$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{(216b^2)\int \frac{1}{(a+bx)^{5/6}(c}{247(bc-ac}$$
$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+b}}{1729(bc-ad)^3(c+}$$
$$= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}} + \frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}} + \frac{1296b^2\sqrt[6]{a+b}}{1729(bc-ad)^3(c+}$$

Mathematica [A]
time = 0.14, size = 93, normalized size = 0.68

$$\frac{6\sqrt[6]{a+bx}\,(-91d^3(a+bx)^3+399bd^2(a+bx)^2(c+dx)-741b^2d(a+bx)(c+dx)^2+1729b^3(c+dx)^3)}{1729(bc-ad)^4(c+dx)^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(-91*d^3*(a + b*x)^3 + 399*b*d^2*(a + b*x)^2*(c + d*x) -
 741*b^2*d*(a + b*x)*(c + d*x)^2 + 1729*b^3*(c + d*x)^3))/(1729*(b*c - a*d)
^4*(c + d*x)^(19/6))
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.26

method	result
gosper	$-\frac{6(bx+a)^{\frac{1}{6}}(-1296b^3x^3d^3+216d^3ax^2b^2-4104b^3cd^2x^2-126a^2bd^3x+684ab^2cd^2x-4446b^3c^2dx+91a^3d^3-399a^2bcd^2+741ab^2c^2d-}{1729(dx+c)^{\frac{19}{6}}(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/1729*(b*x+a)^(1/6)*(-1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2-4104*b^3*c*d^2*x
^2-126*a^2*b*d^3*x+684*a*b^2*c*d^2*x-4446*b^3*c^2*d*x+91*a^3*d^3-399*a^2*b*
c*d^2+741*a*b^2*c^2*d-1729*b^3*c^3)/(d*x+c)^(19/6)/(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(112) = 224$.
time = 1.18, size = 420, normalized size = 3.09

$$\frac{6\,(1296\,b^3d^3x^3 + 1729\,b^3c^3 - 741\,ab^2c^2d + 399\,a^2bcd^2 - 91\,a^3d^3 + 216\,(19\,b^3cd^2 - ab^2d^3)x^2 + 18\,(247\,b^3c^2d - 38\,ab^2cd^2 + 7\,a^2bd^3)x)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\,(b^4c^8 - 4\,ab^3c^7d + 6\,a^2b^2c^6d^2 - 4\,a^3bc^5d^3 + a^4c^4d^4 + (b^4c^4d^4 - 4\,ab^3c^3d^5 + 6\,a^2b^2c^2d^6 - 4\,a^3bcd^7 + a^4d^8)x^4 + 4\,(b^4c^5d^3 - 4\,ab^3c^4d^4 + 6\,a^2b^2c^3d^5 - 4\,a^3bc^2d^6 + a^4cd^7)x^3 + 6\,(b^4c^6d^2 - 4\,ab^3c^5d^3 + 6\,a^2b^2c^4d^4 - 4\,a^3bc^3d^5 + a^4c^2d^6)x^2 + 4\,(b^4c^7d - 4\,ab^3c^6d^2 + 6\,a^2b^2c^5d^3 - 4\,a^3bc^4d^4 + a^4c^3d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(1296*b^3*d^3*x^3 + 1729*b^3*c^3 - 741*a*b^2*c^2*d + 399*a^2*b*c*d^2
 - 91*a^3*d^3 + 216*(19*b^3*c*d^2 - a*b^2*d^3)*x^2 + 18*(247*b^3*c^2*d - 38
*a*b^2*c*d^2 + 7*a^2*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^4*c^8 - 4
*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (b^4*c^4
*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*x^4 +
 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2*d^6 + a
^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^4 - 4*a^
3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2
*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{5/6}\,{\left(c+d\,x\right)}^{25/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)), x)
```

3.1831 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=449

$$\frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3}+\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}-\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right.}{24\sqrt{3}\,b^{19}}$$

```
[Out] 91/12*d*(-a*d+b*c)*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b^3+13/2*d*(b*x+a)^(5/6)*(d*
x+c)^(7/6)/b^2-6*(d*x+c)^(13/6)/b/(b*x+a)^(1/6)+91/36*d^(1/6)*(-a*d+b*c)^2*
arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(19/6)-91/144*d^(1/6
)*(-a*d+b*c)^2*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b^(1/6)*d^(1/
6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(19/6)+91/144*d^(1/6)*(-a*d+b*c)^2*ln(b^(
1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x
+c)^(1/6))/b^(19/6)-91/72*d^(1/6)*(-a*d+b*c)^2*arctan(-1/3*3^(1/2)+2/3*d^(1
/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(19/6)*3^(1/2)-91/72*d^(
1/6)*(-a*d+b*c)^2*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x
+c)^(1/6)*3^(1/2))/b^(19/6)*3^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {49, 52, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{91\sqrt[6]{d}\,(bc-ad)^2\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{19/6}}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{19/6}}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{19/6}}+\frac{91d(a+bx)^{5/6}\sqrt[6]{c+dx}\,(bc-ad)}{12b^3}+\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}-\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (91*d*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^3) + (13*d*(a + b*
x)^(5/6)*(c + d*x)^(7/6))/(2*b^2) - (6*(c + d*x)^(13/6))/(b*(a + b*x)^(1/6)
) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6)
)/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) - (91*d^(1/6)*(
b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcT
anh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(19/6)) - (
91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6))
+ (91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*
x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6
))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
```

```
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(13d)\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b^2}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d \cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d \cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d \cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\left(91 \cdots\right.}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{\cdots}}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{\cdots}}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{\cdots}}{\cdots}$$

Mathematica [A]
time = 0.92, size = 308, normalized size = 0.69

$$\frac{(bc-ad)^2\left(\frac{6\sqrt[6]{b}\sqrt[6]{c+dx}\left(-91a^2d^2-13abd(-13c+dx)+b^2\left(-72c^2+25cdx+6d^2x^2\right)\right)}{(bc-ad)^2\sqrt[6]{a+bx}} - 91\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 91\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right) + 182\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right) + 91\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)\right)}{72b^{19/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] ((b*c - a*d)^2*((6*b^(1/6)*(c + d*x)^(1/6)*(-91*a^2*d^2 - 13*a*b*d*(-13*c +
 d*x) + b^2*(-72*c^2 + 25*c*d*x + 6*d^2*x^2)))/((b*c - a*d)^2*(a + b*x)^(1/
6)) - 91*Sqrt[3]*d^(1/6)*ArcTan[(1 - (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(
a + b*x)^(1/6)))/Sqrt[3]] + 91*Sqrt[3]*d^(1/6)*ArcTan[(1 + (2*b^(1/6)*(c +
d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] + 182*d^(1/6)*ArcTanh[(b^(1
/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))] + 91*d^(1/6)*ArcTanh[(b^(1/
6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/(d^(1/3)*(a + b*x)^(1/3) + b^(1
/3)*(c + d*x)^(1/3))]))/(72*b^(19/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5690 vs. $2(339) = 678$.
time = 1.35, size = 5690, normalized size = 12.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/144*(364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66
*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^
7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^
13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*
d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
```

^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) + sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) + 364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^

```
2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) - (b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x
+ c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^
10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d
^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*
a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^1
9)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*
d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924
*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*
c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6)
 - sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 2
20*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^
6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^
10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 1
2*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*...
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(7/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5457 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^{13/6}}{{\left(a+b\,x\right)}^{7/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(13/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(13/6)/(a + b*x)^(7/6), x)
```

3.1832 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)}{}$$

```
[Out] 7*d*(b*x+a)^(5/6)*(d*x+c)^(1/6)/b^2-6*(d*x+c)^(7/6)/b/(b*x+a)^(1/6)+7/3*d^(
1/6)*(-a*d+b*c)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6))/b^(13/
6)-7/12*d^(1/6)*(-a*d+b*c)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)-b
^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(13/6)+7/12*d^(1/6)*(-a*d+b*c
)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(1
/6)/(d*x+c)^(1/6))/b^(13/6)-7/6*d^(1/6)*(-a*d+b*c)*arctan(-1/3*3^(1/2)+2/3*
d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))/b^(13/6)*3^(1/2)-7/6*d
^(1/6)*(-a*d+b*c)*arctan(1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x
+c)^(1/6)*3^(1/2))/b^(13/6)*3^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {49, 52, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{7\sqrt[6]{d}\,(bc-ad)\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)\text{ArcTan}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}}+\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (7*d*(a + b*x)^(5/6)*(c + d*x)^(1/6))/b^2 - (6*(c + d*x)^(7/6))/(b*(a + b*x
)^(1/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(
1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(13/6)) - (7*d^(1/6)
*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(13/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTanh[
(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(13/6)) - (7*d^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(13/6)) + (7*d^(1
/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (
b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(13/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
```

```
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
```

```
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d)\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b^2}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a}\right.}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{\left(7\sqrt[3]{d}\,(bc-ad)\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\right.}{3b^{13/6}}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{(7}{}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{7\sqrt[6]{}}{}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 10.06, size = 71, normalized size = 0.18

$$-\frac{6(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3084 vs. $2(301) = 602$.
time = 1.47, size = 3084, normalized size = 7.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] 1/12*(28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4
*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^
7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*c - a*b^11*d)*(b*x + a)^(5/6)*(d
*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + 2*
sqrt(3)*(b^12*x + a*b^11)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)
^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((
b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*
a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a))*((b^6*c^
6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^
2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + sqrt(3)*(a*b^6*c^6*d - 6
```

*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*c - a*b^11*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + 2*sqrt(3)*(b^12*x + a*b^11)*sqrt(-((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a))*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) - sqrt(3)*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(49*((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a)) - 7*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(-49*((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a)) + 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3

```
*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)
) - 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(
1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^3*x + a*b^2)*
((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 1
5*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)) + 12*(
b*d*x - 6*b*c + 7*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^3*x + a*b^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(7/6),x)
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(7/6), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(7/6), x)
```

3.1833 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}} dx$

Optimal. Leaf size=332

$$-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}+2$$

```
[Out] -6*(d*x+c)^(1/6)/b/(b*x+a)^(1/6)+2*d^(1/6)*arctanh(d^(1/6)*(b*x+a)^(1/6)/b^
(1/6)/(d*x+c)^(1/6))/b^(7/6)-1/2*d^(1/6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(
d*x+c)^(1/3)-b^(1/6)*d^(1/6)*(b*x+a)^(1/6)/(d*x+c)^(1/6))/b^(7/6)+1/2*d^(1/
6)*ln(b^(1/3)+d^(1/3)*(b*x+a)^(1/3)/(d*x+c)^(1/3)+b^(1/6)*d^(1/6)*(b*x+a)^(
1/6)/(d*x+c)^(1/6))/b^(7/6)-d^(1/6)*arctan(-1/3*3^(1/2)+2/3*d^(1/6)*(b*x+a)
^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(7/6)-d^(1/6)*arctan(1/3*3^
(1/2)+2/3*d^(1/6)*(b*x+a)^(1/6)/b^(1/6)/(d*x+c)^(1/6)*3^(1/2))*3^(1/2)/b^(7
/6)
```

Rubi [A]
time = 0.37, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {49, 65, 338, 302, 648, 632, 210, 642, 214}

$$\frac{\sqrt{3}\sqrt[6]{d}\,\text{ArcTan}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt{3}\sqrt[6]{d}\,\text{ArcTan}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{7/6}}-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6)
 - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) + (2*d^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) - (d^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*b^(7/6)) + (d^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*b^(7/6))
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
```

& IntLinearQ[a, b, c, d, m, n, x]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 338

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx &= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{d\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{b}\\
&= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{(6d)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx,x,\sqrt[6]{a+bx}\right)}{b^2}\\
&= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{(6d)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}\\
&= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{7/6}}+\frac{\left(2\sqrt[3]{d}\right)\text{Su}\cdots}{\cdots}\\
&= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt[6]{d}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx\cdots\right)}{2b^{7/6}}\cdots\\
&= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt[6]{d}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\cdots}{\cdots}\right)}{2b^{7/6}}\cdots\\
&= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}}-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}}\cdots
\end{aligned}
$$

Mathematica [A]

time = 0.28, size = 239, normalized size = 0.72

$$\frac{-\frac{6\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}-\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)+\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}}{\sqrt{3}}\right)+2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)+\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}\sqrt[6]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{b^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] ((-6*b^(1/6)*(c + d*x)^(1/6))/(a + b*x)^(1/6) - Sqrt[3]*d^(1/6)*ArcTan[(1 -
 (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/Sqrt[3]] + Sqrt[3]*
d^(1/6)*ArcTan[(1 + (2*b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6)))/
Sqrt[3]] + 2*d^(1/6)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(
1/6))] + d^(1/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6)*(c + d*x)^(1/6))/
(d^(1/3)*(a + b*x)^(1/3) + b^(1/3)*(c + d*x)^(1/3))])/b^(7/6)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(241) = 482.
time = 1.15, size = 663, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sqrt(3)*(b^7*x + a*b^6)*sqrt(
((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3*x + a*b^2)*(d/b^7)^
(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(d/b^7)^(5/6) + sqrt(3)
*(b*d*x + a*d))/(b*d*x + a*d)) + 4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sq
rt(3)*(b^7*x + a*b^6)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6
) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x +
 a))*(d/b^7)^(5/6) - sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) - (b^2*x + a*b)*
(d/b^7)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3
*x + a*b^2)*(d/b^7)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a)) + (
b^2*x + a*b)*(d/b^7)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7
)^(1/6) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/
(b*x + a)) - 2*(b^2*x + a*b)*(d/b^7)^(1/6)*log(((b^2*x + a*b)*(d/b^7)^(1/6)
 + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) + 2*(b^2*x + a*b)*(d/b^7)^(1
/6)*log(-((b^2*x + a*b)*(d/b^7)^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b
*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^2*x + a*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c + dx}}{(a + bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(7/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(c+d\,x)}^{1/6}}{{(a+b\,x)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(7/6), x)
```

3.1834 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=30

$$-\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

```
[Out] -6*(d*x+c)^(1/6)/(-a*d+b*c)/(b*x+a)^(1/6)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 1.00

$$-\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

Maple [A]
time = 0.17, size = 27, normalized size = 0.90

method	result	size
gosper	$\frac{6(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}(ad-bc)}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x,method=_RETURNVERBOSE)
[Out] 6/(b*x+a)^(1/6)*(d*x+c)^(1/6)/(a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

Fricas [A]
time = 0.92, size = 42, normalized size = 1.40

$$-\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] -6*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(5/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

Mupad [B]
time = 0.68, size = 26, normalized size = 0.87

$$\frac{6\,{\left(c+d\,x\right)}^{1/6}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x)
[Out] (6*(c + d*x)^(1/6))/((a*d - b*c)*(a + b*x)^(1/6))
```

3.1835 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=64

$$-\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}-\frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}$$

```
[Out] -6/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)-36/5*d*(b*x+a)^(5/6)/(-a*d+b*c)^2
/(d*x+c)^(5/6)
```

Rubi [A]

time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6)) - (36*d*(a + b*x)^(5/6))/(
5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}} - \frac{(6d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{bc-ad}$$
$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}} - \frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.70

$$-\frac{6(5bc+ad+6bdx)}{5(bc-ad)^2\sqrt[6]{a+bx}\,(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (-6*(5*b*c + a*d + 6*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(5/
6))
```

Maple [A]
time = 0.18, size = 53, normalized size = 0.83

method	result	size
gosper	$-\frac{6(6bdx+ad+5bc)}{5(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}(a^2d^2-2abcd+b^2c^2)}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/5*(6*b*d*x+a*d+5*b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(54) = 108$.
time = 0.65, size = 126, normalized size = 1.97

$$-\frac{6\,(6\,bdx+5\,bc+ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -6/5*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(11/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

Mupad [B]
time = 0.83, size = 72, normalized size = 1.12

$$-\frac{\left(\frac{36\,b\,x}{5\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d+30\,b\,c}{5\,d\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/6}}{x\,{\left(a+b\,x\right)}^{1/6}+\frac{c\,{\left(a+b\,x\right)}^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x)
```

```
[Out] -(((36*b*x)/(5*(a*d - b*c)^2) + (6*a*d + 30*b*c)/(5*d*(a*d - b*c)^2))*(c +
d*x)^(1/6))/(x*(a + b*x)^(1/6) + (c*(a + b*x)^(1/6))/d)
```

3.1836 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=98

$$-\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}}-\frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}}-\frac{432bd(a+bx)^{5/6}}{55(bc-ad)^3(c+dx)^{5/6}}$$

```
[Out] -6/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(11/6)-72/11*d*(b*x+a)^(5/6)/(-a*d+b*c)
^2/(d*x+c)^(11/6)-432/55*b*d*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(11/6)) - (72*d*(a + b*x)^(5/6))/
(11*(b*c - a*d)^2*(c + d*x)^(11/6)) - (432*b*d*(a + b*x)^(5/6))/(55*(b*c -
a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{(12d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{bc-ad}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}} - \frac{(72bd)\int \frac{1}{\sqrt[6]{a}}}{11(b}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}} - \frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}} - \frac{432bd(a-}{55(bc-ad)^3}$$

Mathematica [A]
time = 0.12, size = 73, normalized size = 0.74

$$-\frac{6(a+bx)^{11/6}\left(-5d^2+\frac{22bd(c+dx)}{a+bx}+\frac{55b^2(c+dx)^2}{(a+bx)^2}\right)}{55(bc-ad)^3(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(a + b*x)^(11/6)*(-5*d^2 + (22*b*d*(c + d*x))/(a + b*x) + (55*b^2*(c +
d*x)^2)/(a + b*x)^2))/(55*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Maple [A]
time = 0.17, size = 105, normalized size = 1.07

method	result	size
gosper	$-\frac{6\left(-72b^2x^2d^2-12ab\,d^2x-132b^2cdx+5a^2d^2-22abcd-55b^2c^2\right)}{55(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/55*(-72*b^2*d^2*x^2-12*a*b*d^2*x-132*b^2*c*d*x+5*a^2*d^2-22*a*b*c*d-55*b
^2*c^2)/(b*x+a)^(1/6)/(d*x+c)^(11/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(82) = 164$.
time = 1.56, size = 273, normalized size = 2.79

$$-\frac{6\,(72\,b^2d^2x^2+55\,b^2c^2+22\,abcd-5\,a^2d^2+12\,(11\,b^2cd+abd^2)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^3+(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5)x^2+(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] -6/55*(72*b^2*d^2*x^2 + 55*b^2*c^2 + 22*a*b*c*d - 5*a^2*d^2 + 12*(11*b^2*c*
d + a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(17/6),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7772 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

Mupad [**B**]
time = 0.96, size = 132, normalized size = 1.35

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{432\,b^2\,x^2}{55\,(a\,d-b\,c)^3}+\frac{-30\,a^2\,d^2+132\,a\,b\,c\,d+330\,b^2\,c^2}{55\,d^2\,(a\,d-b\,c)^3}+\frac{72\,b\,x\,(a\,d+11\,b\,c)}{55\,d\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/6}+\frac{c^2\,(a+b\,x)^{1/6}}{d^2}+\frac{2\,c\,x\,(a+b\,x)^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((432*b^2*x^2)/(55*(a*d - b*c)^3) + (330*b^2*c^2 - 30*a^2*
d^2 + 132*a*b*c*d)/(55*d^2*(a*d - b*c)^3) + (72*b*x*(a*d + 11*b*c))/(55*d*(
a*d - b*c)^3)))/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(
a + b*x)^(1/6))/d)
```

3.1837 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=134

$$-\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}}-\frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}}-\frac{1296bd(a+bx)^{5/6}}{187(bc-ad)^3(c+dx)^{11/6}}-\frac{7776b^2d(a+bx)^{5/6}}{935(bc-ad)^4(c+dx)^{5/6}}$$

```
[Out] -6/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(17/6)-108/17*d*(b*x+a)^(5/6)/(-a*d+b*c
)^2/(d*x+c)^(17/6)-1296/187*b*d*(b*x+a)^(5/6)/(-a*d+b*c)^3/(d*x+c)^(11/6)-7
776/935*b^2*d*(b*x+a)^(5/6)/(-a*d+b*c)^4/(d*x+c)^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(17/6)) - (108*d*(a + b*x)^(5/6))
/(17*(b*c - a*d)^2*(c + d*x)^(17/6)) - (1296*b*d*(a + b*x)^(5/6))/(187*(b*c
 - a*d)^3*(c + d*x)^(11/6)) - (7776*b^2*d*(a + b*x)^(5/6))/(935*(b*c - a*d)
^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx = -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{(18d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{bc-ad}$$
$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{(216bd)\int \frac{}{\sqrt[6]{}}}{17(}$$
$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd(a}{187(bc-ad)}$$
$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}} - \frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}} - \frac{1296bd(a}{187(bc-ad)}$$

Mathematica [A]
time = 0.13, size = 95, normalized size = 0.71

$$-\frac{6(a+bx)^{17/6}\left(55d^3-\frac{255bd^2(c+dx)}{a+bx}+\frac{561b^2d(c+dx)^2}{(a+bx)^2}+\frac{935b^3(c+dx)^3}{(a+bx)^3}\right)}{935(bc-ad)^4(c+dx)^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(a + b*x)^(17/6)*(55*d^3 - (255*b*d^2*(c + d*x))/(a + b*x) + (561*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (935*b^3*(c + d*x)^3)/(a + b*x)^3))/(935*(b*c -
 a*d)^4*(c + d*x)^(17/6))
```

Maple [A]
time = 0.17, size = 171, normalized size = 1.28

method	result	s
gosper	$-\frac{6\left(1296b^3x^3d^3+216d^3a\,x^2b^2+3672b^3c\,d^2x^2-90a^2b\,d^3x+612a\,b^2c\,d^2x+3366b^3c^2dx+55a^3d^3-255a^2bc\,d^2+561a\,b^2c^2d+935b^3c^3\right)}{935(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$	

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x,method=_RETURNVERBOSE)
```

```
[Out] -6/935*(1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2+3672*b^3*c*d^2*x^2-90*a^2*b*d^3*
x+612*a*b^2*c*d^2*x+3366*b^3*c^2*d*x+55*a^3*d^3-255*a^2*b*c*d^2+561*a*b^2*c
^2*d+935*b^3*c^3)/(b*x+a)^(1/6)/(d*x+c)^(17/6)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(112) = 224$.
time = 1.39, size = 457, normalized size = 3.41

$$-\frac{6\,(1296\,b^3d^3x^3 + 935\,b^3c^3 + 561\,ab^2c^2d - 255\,a^2bcd^2 + 55\,a^3d^3 + 216\,(17\,b^3cd^2 + ab^2d^3)x^2 + 18\,(187\,b^3c^2d + 34\,ab^2cd^2 - 5\,a^2bd^3)x)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\,(ab^4c^7 - 4\,a^2b^3c^6d + 6\,a^3b^2c^5d^2 - 4\,a^4bc^4d^3 + a^5c^3d^4 + (b^5c^4d^3 - 4\,ab^4c^3d^4 + 6\,a^2b^3c^2d^5 - 4\,a^3b^2cd^6 + a^4bd^7)x^4 + (3\,b^5c^5d^2 - 11\,ab^4c^4d^3 + 14\,a^2b^3c^3d^4 - 6\,a^3b^2c^2d^5 - a^4bcd^6 + a^5d^7)x^3 + 3\,(b^5c^6d - 3\,ab^4c^5d^2 + 2\,a^2b^3c^4d^3 + 2\,a^3b^2c^3d^4 - 3\,a^4bc^2d^5 + a^5cd^6)x^2 + (b^5c^7 - ab^4c^6d - 6\,a^2b^3c^5d^2 + 14\,a^3b^2c^4d^3 - 11\,a^4bc^3d^4 + 3\,a^5c^2d^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] -6/935*(1296*b^3*d^3*x^3 + 935*b^3*c^3 + 561*a*b^2*c^2*d - 255*a^2*b*c*d^2
+ 55*a^3*d^3 + 216*(17*b^3*c*d^2 + a*b^2*d^3)*x^2 + 18*(187*b^3*c^2*d + 34*
a*b^2*c*d^2 - 5*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^4*c^7 -
4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*
c^4*d^3 - 4*a*b^4*c^3*d^4 + 6*a^2*b^3*c^2*d^5 - 4*a^3*b^2*c*d^6 + a^4*b*d^7
)*x^4 + (3*b^5*c^5*d^2 - 11*a*b^4*c^4*d^3 + 14*a^2*b^3*c^3*d^4 - 6*a^3*b^2*
c^2*d^5 - a^4*b*c*d^6 + a^5*d^7)*x^3 + 3*(b^5*c^6*d - 3*a*b^4*c^5*d^2 + 2*a
^2*b^3*c^4*d^3 + 2*a^3*b^2*c^3*d^4 - 3*a^4*b*c^2*d^5 + a^5*c*d^6)*x^2 + (b^
5*c^7 - a*b^4*c^6*d - 6*a^2*b^3*c^5*d^2 + 14*a^3*b^2*c^4*d^3 - 11*a^4*b*c^3
*d^4 + 3*a^5*c^2*d^5)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

Mupad [B]
time = 1.15, size = 209, normalized size = 1.56

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{7776\,b^3\,x^3}{935\,(a\,d-b\,c)^4}+\frac{330\,a^3\,d^3-1530\,a^2\,b\,c\,d^2+3366\,a\,b^2\,c^2\,d+5610\,b^3\,c^3}{935\,d^3\,(a\,d-b\,c)^4}+\frac{108\,b\,x\,(-5\,a^2\,d^2+34\,a\,b\,c\,d+187\,b^2\,c^2)}{935\,d^2\,(a\,d-b\,c)^4}+\frac{1296\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((7776*b^3*x^3)/(935*(a*d - b*c)^4) + (330*a^3*d^3 + 5610
*b^3*c^3 + 3366*a*b^2*c^2*d - 1530*a^2*b*c*d^2)/(935*d^3*(a*d - b*c)^4) + (
108*b*x*(187*b^2*c^2 - 5*a^2*d^2 + 34*a*b*c*d))/(935*d^2*(a*d - b*c)^4) + (
1296*b^2*x^2*(a*d + 17*b*c))/(935*d*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/6) +
 (c^3*(a + b*x)^(1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*
x)^(1/6))/d^2)
```

3.1838 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{7/6}} dx$

Optimal. Leaf size=80

$$-\frac{6(bc-ad)(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] -6*(-a*d+b*c)*(d*x+c)^(5/6)*hypergeom([-11/6, -1/6],[5/6],-d*(b*x+a)/(-a*d+
b*c))/b^2/(b*x+a)^(1/6)/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6(c+dx)^{5/6}(bc-ad)\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(b*c - a*d)*(c + d*x)^(5/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, -((d*(a
 + b*x))/(b*c - a*d))])/(b^2*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5
/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{7/6}}\,dx = \frac{\left((bc-ad)(c+dx)^{5/6}\right)\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{11/6}}{(a+bx)^{7/6}}\,dx}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$
$$= -\frac{6(bc-ad)(c+dx)^{5/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b^2\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

<u>**Mathematica**</u> [**A**]
time = 10.06, size = 71, normalized size = 0.89

$$-\frac{6(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(7/6),x]
[Out] (-6*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, -1/6, 5/6, (d*(a + b*x))/(-(b
*c) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

<u>**Maple**</u> [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(11/6)/(b*x+a)^(7/6),x)
```

<u>**Maxima**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(11/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(7/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3655 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(11/6)/(a + b*x)^(7/6), x)
```

3.1839 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=72

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

```
[Out] -6*(d*x+c)^(5/6)*hypergeom([-5/6, -1/6],[5/6],-d*(b*x+a)/(-a*d+b*c))/b/(b*x
+a)^(1/6)/(b*(d*x+c)/(-a*d+b*c))^(5/6)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, -((d*(a + b*x))/(b*c
 - a*d))])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/6}}\,dx = \frac{(c+dx)^{5/6}\int \frac{\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{5/6}}{(a+bx)^{7/6}}\,dx}{\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

$$= -\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Mathematica [A]
time = 10.02, size = 71, normalized size = 0.99

$$-\frac{6(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(7/6),x]
[Out] (-6*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(7/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(7/6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(7/6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/6}}{{\left(a+b\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(7/6), x)
```

3.1840 $\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=72

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 1/6],[5/6],-d*(b*x+a)/(-a*
d+b*c))/b/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\int \frac{1}{(a+bx)^{7/6}\sqrt[6]{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}\,dx}{\sqrt[6]{c+dx}}$$
$$= -\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.03, size = 71, normalized size = 0.99

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(1/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d*x^3 + a^2*c + (b^2*c + 2*a*
b*d)*x^2 + (2*a*b*c + a^2*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(1/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(1/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/6}\,{\left(c+d\,x\right)}^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(1/6)),x)
```

```
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(1/6)), x)
```

3.1841 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=79

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 7/6],[5/6],-d*(b*x+a)/(-a*
d+b*c))/(-a*d+b*c)/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, -((
d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{7/6}}\,dx = \frac{\left(b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{7/6}}\,dx}{(bc-ad)\sqrt[6]{c+dx}}$$
$$= -\frac{6\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.04, size = 71, normalized size = 0.90

$$-\frac{6\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\ {}_2F_1\left(-\frac{1}{6},\frac{7}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\,(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x]
```

```
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/6, 7/6, 5/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(7/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}\,(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(7/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^2*x^4 + a^2*c^2 + 2*(b^2*c*
d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + 2*(a*b*c^2 + a^2*c
*d)*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(7/6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(7/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{7/6}\,(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(7/6)),x)
```

```
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(7/6)), x)
```

3.1842 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=80

$$-\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*b*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 13/6],[5/6],-d*(b*x+a)/(
-a*d+b*c))/(-a*d+b*c)^2/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]

time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 13/6, 5/6,
-((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(1/
6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{13/6}}\,dx = \frac{\left(b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{13/6}}\,dx}{(bc-ad)^2\sqrt[6]{c+dx}}$$
$$= -\frac{6b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\ \sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.05, size = 71, normalized size = 0.89

$$-\frac{6\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}\ {}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{b\sqrt[6]{a+bx}\ (c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x]
[Out] (-6*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[-1/6, 13/6, 5/6, (
d*(a + b*x))/(-(b*c) + a*d)])/(b*(a + b*x)^(1/6)*(c + d*x)^(13/6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}\,(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(13/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^3*x^5 + a^2*c^3 + (3*b^2*c*
d^2 + 2*a*b*d^3)*x^4 + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3
 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^2 + (2*a*b*c^3 + 3*a^2*c^2*d)*x), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(13/6),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(13/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{7/6}\,{\left(c+d\,x\right)}^{13/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(13/6)),x)
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(13/6)), x)
```

3.1843 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{19/6}} dx$

Optimal. Leaf size=82

$$-\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}}$$

```
[Out] -6*b^2*(b*(d*x+c)/(-a*d+b*c))^(1/6)*hypergeom([-1/6, 19/6],[5/6],-d*(b*x+a)
/(-a*d+b*c))/(-a*d+b*c)^3/(b*x+a)^(1/6)/(d*x+c)^(1/6)
```

Rubi [A]
time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{\sqrt[6]{a+bx}\,\sqrt[6]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (-6*b^2*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[-1/6, 19/6, 5/6
, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(
1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{19/6}}\,dx = \frac{\left(b^3\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{(a+bx)^{7/6}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{19/6}}\,dx}{(bc-ad)^3\sqrt[6]{c+dx}}$$
$$= -\frac{6b^2\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\ {}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3\sqrt[6]{a+bx}\ \sqrt[6]{c+dx}}$$

Mathematica [A]
time = 10.04, size = 79, normalized size = 0.96

$$-\frac{6b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\ {}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{d(a+bx)}{-bc+ad}\right)}{(bc-ad)^2\sqrt[6]{a+bx}\ (c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (-6*b*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[-1/6, 19/6, 5/6,
(d*(a + b*x))/(-(b*c) + a*d)])/((b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(7/
6))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}\,(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] int(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(19/6)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(5/6)*(d*x + c)^(5/6)/(b^2*d^4*x^6 + a^2*c^4 + 2*(2*b^2*
c*d^3 + a*b*d^4)*x^5 + (6*b^2*c^2*d^2 + 8*a*b*c*d^3 + a^2*d^4)*x^4 + 4*(b^2
*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^3)*x^3 + (b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^
2*d^2)*x^2 + 2*(a*b*c^4 + 2*a^2*c^3*d)*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(19/6)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{7/6}\,(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(19/6)),x)
```

```
[Out] int(1/((a + b*x)^(7/6)*(c + d*x)^(19/6)), x)
```

3.1844 $\int (a+bx)^m(a+b(2+m)x)\,dx$

Optimal. Leaf size=11

$$x(a+bx)^{1+m}$$

```
[Out] x*(b*x+a)^(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {34}

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int (a+bx)^m(a+b(2+m)x)\,dx = x(a+bx)^{1+m}$$

Mathematica [A]
time = 0.05, size = 11, normalized size = 1.00

$$x(a+bx)^{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

Maple [A]
time = 0.15, size = 12, normalized size = 1.09

method	result	size
gosper	$x(bx+a)^{1+m}$	12
risch	$(bx+a)^m x(bx+a)$	15
norman	$ax\,e^{m\ln(bx+a)}+x^2b\,e^{m\ln(bx+a)}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a+b*(2+m)*x),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*x+a)^(1+m)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(11) = 22$.
time = 0.30, size = 106, normalized size = 9.64

$$\frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m m}{(m^2+3m+2)b}+\frac{2\left(b^2(m+1)x^2+abmx-a^2\right)(bx+a)^m}{(m^2+3m+2)b}+\frac{(bx+a)^{m+1}a}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*m/((m^2 + 3*m + 2)*b) + 2*(b^
2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m/((m^2 + 3*m + 2)*b) + (b*x + a)^
(m + 1)*a/(b*(m + 1))
```

Fricas [A]
time = 0.57, size = 17, normalized size = 1.55

$$\left(bx^2+ax\right)(bx+a)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^m
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.
time = 0.09, size = 20, normalized size = 1.82

$$ax(a+bx)^m+bx^2(a+bx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a+b*(2+m)*x),x)
```

```
[Out] a*x*(a + b*x)**m + b*x**2*(a + b*x)**m
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 3.71, size = 23, normalized size = 2.09

$$(bx + a)^m bx^2 + (bx + a)^m ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="giac")
```

```
[Out] (b*x + a)^m*b*x^2 + (b*x + a)^m*a*x
```

Mupad [B]
time = 0.46, size = 11, normalized size = 1.00

$$x (a + b x)^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x*(m + 2))*(a + b*x)^m,x)
```

```
[Out] x*(a + b*x)^(m + 1)
```

3.1845 $\int (a+bx)^m(c+dx)^n\,dx$

Optimal. Leaf size=61

$$-\frac{(a+bx)^{1+m}(c+dx)^{1+n}\,{}_2F_1\left(1,2+m+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+n],b*(d*x+c)/(-a*d+b*c
))/(-a*d+b*c)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {72, 71}

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a
+ b*x))/(b*c - a*d))])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(1+m,-n;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+m)}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 1.20

$$\frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(1+m,-n;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n,x]
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n,x)
[Out] int((b*x+a)^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*(d*x + c)^n, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (a+b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^m*(c + d*x)^n, x)
```

3.1846 $\int (a+bx)^m (c+dx)^3 \, dx$

Optimal. Leaf size=110

$$\frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}+\frac{d^3(a+bx)^{4+m}}{b^4(4+m)}$$

```
[Out] (-a*d+b*c)^3*(b*x+a)^(1+m)/b^4/(1+m)+3*d*(-a*d+b*c)^2*(b*x+a)^(2+m)/b^4/(2+
m)+3*d^2*(-a*d+b*c)*(b*x+a)^(3+m)/b^4/(3+m)+d^3*(b*x+a)^(4+m)/b^4/(4+m)
```

Rubi [A]
time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^(1 + m))/(b^4*(1 + m)) + (3*d*(b*c - a*d)^2*(a + b
*x)^(2 + m))/(b^4*(2 + m)) + (3*d^2*(b*c - a*d)*(a + b*x)^(3 + m))/(b^4*(3
+ m)) + (d^3*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^m(c+dx)^3\,dx = \int \left(\frac{(bc-ad)^3(a+bx)^m}{b^3}+\frac{3d(bc-ad)^2(a+bx)^{1+m}}{b^3}+\frac{3d^2(bc-ad)(a+bx)^{2+}}{b^3}\right.$$
$$= \frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.85

$$\frac{(a+bx)^{1+m}\left(\frac{(bc-ad)^3}{1+m}+\frac{3d(bc-ad)^2(a+bx)}{2+m}+\frac{3d^2(bc-ad)(a+bx)^2}{3+m}+\frac{d^3(a+bx)^3}{4+m}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^3/(1 + m) + (3*d*(b*c - a*d)^2*(a + b*x))/(
2 + m) + (3*d^2*(b*c - a*d)*(a + b*x)^2)/(3 + m) + (d^3*(a + b*x)^3)/(4 + m
)))/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(110) = 220$.
time = 0.18, size = 389, normalized size = 3.54

method	result
gosper	$-\frac{(bx+a)^{1+m}(-b^3d^3m^3x^3-3b^3c\,d^2m^3x^2-6b^3d^3m^2x^3+3a\,b^2d^3m^2x^2-3b^3c^2d\,m^3x-21b^3c\,d^2m^2x^2-11b^3d^3m\,x^3+6a\,b^2c\,d^2m^2x+9a\,b}{}$
norman	$\frac{d^3x^4e^{m\ln(bx+a)}}{4+m}+\frac{(3a\,b^2c^2d\,m^3+b^3c^3m^3-6a^2bc\,d^2m^2+21a\,b^2c^2d\,m^2+9b^3c^3m^2+6a^3d^3m-24a^2bc\,d^2m+36a\,b^2c^2dm+26b^3c^3m+24}{b^3(m^4+10m^3+35m^2+50m+24)}$
risch	$-\frac{(-b^4d^3m^3x^4-a\,b^3d^3m^3x^3-3b^4c\,d^2m^3x^3-6b^4d^3m^2x^4-3a\,b^3c\,d^2m^3x^2-3a\,b^3d^3m^2x^3-3b^4c^2d\,m^3x^2-21b^4c\,d^2m^2x^3-11b^4d^3m\,x^4+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(-b^3*d^3*m^3*x^3-3*b^3*c*d^2*m^3*x^2-6*b^3*d^3*m^2*x^3+3*a*
b^2*d^3*m^2*x^2-3*b^3*c^2*d*m^3*x-21*b^3*c*d^2*m^2*x^2-11*b^3*d^3*m*x^3+6*a
*b^2*c*d^2*m^2*x+9*a*b^2*d^3*m*x^2-b^3*c^3*m^3-24*b^3*c^2*d*m^2*x-42*b^3*c*
d^2*m*x^2-6*b^3*d^3*x^3-6*a^2*b*d^3*m*x+3*a*b^2*c^2*d*m^2+30*a*b^2*c*d^2*m*
x+6*a*b^2*d^3*x^2-9*b^3*c^3*m^2-57*b^3*c^2*d*m*x-24*b^3*c*d^2*x^2-6*a^2*b*c
*d^2*m-6*a^2*b*d^3*x+21*a*b^2*c^2*d*m+24*a*b^2*c*d^2*x-26*b^3*c^3*m-36*b^3*
c^2*d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/b^4/(m^4+10*m^3
+35*m^2+50*m+24)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(110) = 220$.
time = 0.32, size = 246, normalized size = 2.24

$$\frac{3\,(b^2(m+1)x^2+abmx-a^2)(bx+a)^m c^2 d}{(m^2+3\,m+2)b^2}+\frac{(bx+a)^{m+1}c^3}{b(m+1)}+\frac{3\,((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m cd^2}{(m^3+6\,m^2+11\,m+6)b^3}+\frac{((m^3+6\,m^2+11\,m+6)b^4x^4+(m^3+3\,m^2+2\,m)ab^3x^3-3\,(m^2+m)a^2b^2x^2+6\,a^3bmx-6\,a^4)(bx+a)^m d^3}{(m^4+10\,m^3+35\,m^2+50\,m+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c^2*d/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c^3/(b*(m + 1)) + 3*((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*d^2/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m^2 + 2*m)*a*b^3*x
^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x + a)^m*d^3/((m^4 +
 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(110) = 220$.
time = 0.66, size = 497, normalized size = 4.52

$$\frac{\left(ab^3c^3m^3 + 24\,ab^3c^3 - 36\,a^2b^2c^2d + 24\,a^3bcd^2 - 6\,a^4d^3 + \left(b^4d^3m^3 + 6\,b^4d^3m^2 + 11\,b^4d^3m + 6\,b^4d^3\right)x^4 + \left(24\,b^4cd^2 + \left(3\,b^4cd^2 + ab^3d^3\right)m^3 + 3\left(7\,b^4cd^2 + ab^3d^3\right)m^2 + 2\left(21\,b^4cd^2 + ab^3d^3\right)m\right)x^3 + 3\left(3\,ab^3c^3 - a^2b^2c^2d\right)m^2 + 3\left(12\,b^4c^2d + \left(b^4c^2d + ab^3cd^2\right)m^3 + \left(8\,b^4c^2d + 5\,ab^3cd^2 - a^2b^2d^3\right)m^2 + \left(19\,b^4c^2d + 4\,ab^3cd^2 - a^2b^2d^3\right)m\right)x^2 + \left(26\,ab^3c^3 - 21\,a^2b^2c^2d + 6\,a^3bcd^2\right)m + \left(24\,b^4c^3 + \left(b^4c^3 + 3\,ab^3c^2d\right)m^3 + 3\left(3\,b^4c^3 + 7\,ab^3c^2d - 2\,a^2b^2cd^2\right)m^2 + 2\left(13\,b^4c^3 + 18\,ab^3c^2d - 12\,a^2b^2cd^2 + 3\,a^3bd^3\right)m\right)x\right)(bx+a)^m}{b^4m^4 + 10\,b^4m^3 + 35\,b^4m^2 + 50\,b^4m + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^3*c^3*m^3 + 24*a*b^3*c^3 - 36*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 6*a^4*d
^3 + (b^4*d^3*m^3 + 6*b^4*d^3*m^2 + 11*b^4*d^3*m + 6*b^4*d^3)*x^4 + (24*b^4
*c*d^2 + (3*b^4*c*d^2 + a*b^3*d^3)*m^3 + 3*(7*b^4*c*d^2 + a*b^3*d^3)*m^2 +
2*(21*b^4*c*d^2 + a*b^3*d^3)*m)*x^3 + 3*(3*a*b^3*c^3 - a^2*b^2*c^2*d)*m^2 +
 3*(12*b^4*c^2*d + (b^4*c^2*d + a*b^3*c*d^2)*m^3 + (8*b^4*c^2*d + 5*a*b^3*c
*d^2 - a^2*b^2*d^3)*m^2 + (19*b^4*c^2*d + 4*a*b^3*c*d^2 - a^2*b^2*d^3)*m)*x
^2 + (26*a*b^3*c^3 - 21*a^2*b^2*c^2*d + 6*a^3*b*c*d^2)*m + (24*b^4*c^3 + (b
^4*c^3 + 3*a*b^3*c^2*d)*m^3 + 3*(3*b^4*c^3 + 7*a*b^3*c^2*d - 2*a^2*b^2*c*d^
2)*m^2 + 2*(13*b^4*c^3 + 18*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*m
)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4058 vs. $2(95) = 190$.
time = 1.14, size = 4058, normalized size = 36.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**3,x)
```

```
[Out] Piecewise((a**m*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(
b, 0)), (6*a**3*d**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 11*a**3*d**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 6*a**2*b*c*d**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d**3*x*log(a/b + x)/(6*a**3*b**4 +
18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d**3*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*a*b**2*c**2*d/(6*a
**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 18*a*b**2*c*d**
2*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b*
*2*d**3*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3) + 18*a*b**2*d**3*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) - 2*b**3*c**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 9*b**3*c**2*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) - 18*b**3*c*d**2*x**2/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d**3*x**3*log(a/b + x)/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(m, -4)), (-6*a*
```

```
*3*d**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d**3
/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 6*a**2*b*c*d**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 9*a**2*b*c*d**2/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b
**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*
x**2) - 3*a*b**2*c**2*d/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d**3*x**2*lo
g(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c**3/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 6*b**3*c**2*d*x/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 6*b**3*c*d**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 2*b**3*d**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2),
Eq(m, -3)), (6*a**3*d**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d**3/(
2*a*b**4 + 2*b**5*x) - 12*a**2*b*c*d**2*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
- 12*a**2*b*c*d**2/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d**3*x*log(a/b + x)/(2*
a*b**4 + 2*b**5*x) + 6*a*b**2*c**2*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6
*a*b**2*c**2*d/(2*a*b**4 + 2*b**5*x) - 12*a*b**2*c*d**2*x*log(a/b + x)/(2*a
*b**4 + 2*b**5*x) - 3*a*b**2*d**3*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c**3/
(2*a*b**4 + 2*b**5*x) + 6*b**3*c**2*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 6*b**3*c*d**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d**3*x**3/(2*a*b**4 + 2*b
**5*x), Eq(m, -2)), (-a**3*d**3*log(a/b + x)/b**4 + 3*a**2*c*d**2*log(a/b +
 x)/b**3 + a**2*d**3*x/b**3 - 3*a*c**2*d*log(a/b + x)/b**2 - 3*a*c*d**2*x/b
**2 - a*d**3*x**2/(2*b**2) + c**3*log(a/b + x)/b + 3*c**2*d*x/b + 3*c*d**2*
x**2/(2*b) + d**3*x**3/(3*b), Eq(m, -1)), (-6*a**4*d**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*a**3*b*c*d**2
*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 24*a**3*b*c*d**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m*
*2 + 50*b**4*m + 24*b**4) + 6*a**3*b*d**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*c**2*d*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 21*a**2*b**2*c**2*d*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) - 36*a**2*b**2*c**2*d*(a + b*x)**m/(b**4*m**4 +
10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 6*a**2*b**2*c*d**2*m**
2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) - 24*a**2*b**2*c*d**2*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35
*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d**3*m**2*x**2*(a + b*x)**m
/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b
**2*d**3*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + a*b**3*c**3*m**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c**3*m**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*a*b**3*c
**3*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 24*a*b**3*c**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c**2*d*m**3*x*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*a*b**3*c**2*d*m*
```

```
*2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 36*a*b**3*c**2*d*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c*d**2*m**3*x**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 15*a*b**3*c
*d**2*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*d**2*m*x**2*(a ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(110) = 220.
time = 3.07, size = 833, normalized size = 7.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d^3*m^3*x^4 + 3*(b*x + a)^m*b^4*c*d^2*m^3*x^3 + (b*x + a)^
m*a*b^3*d^3*m^3*x^3 + 6*(b*x + a)^m*b^4*d^3*m^2*x^4 + 3*(b*x + a)^m*b^4*c^2
*d*m^3*x^2 + 3*(b*x + a)^m*a*b^3*c*d^2*m^3*x^2 + 21*(b*x + a)^m*b^4*c*d^2*m
^2*x^3 + 3*(b*x + a)^m*a*b^3*d^3*m^2*x^3 + 11*(b*x + a)^m*b^4*d^3*m*x^4 + (
b*x + a)^m*b^4*c^3*m^3*x + 3*(b*x + a)^m*a*b^3*c^2*d*m^3*x + 24*(b*x + a)^m
*b^4*c^2*d*m^2*x^2 + 15*(b*x + a)^m*a*b^3*c*d^2*m^2*x^2 - 3*(b*x + a)^m*a^2
*b^2*d^3*m^2*x^2 + 42*(b*x + a)^m*b^4*c*d^2*m*x^3 + 2*(b*x + a)^m*a*b^3*d^3
*m*x^3 + 6*(b*x + a)^m*b^4*d^3*x^4 + (b*x + a)^m*a*b^3*c^3*m^3 + 9*(b*x + a
)^m*b^4*c^3*m^2*x + 21*(b*x + a)^m*a*b^3*c^2*d*m^2*x - 6*(b*x + a)^m*a^2*b^
2*c*d^2*m^2*x + 57*(b*x + a)^m*b^4*c^2*d*m*x^2 + 12*(b*x + a)^m*a*b^3*c*d^2
*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d^3*m*x^2 + 24*(b*x + a)^m*b^4*c*d^2*x^3 + 9
*(b*x + a)^m*a*b^3*c^3*m^2 - 3*(b*x + a)^m*a^2*b^2*c^2*d*m^2 + 26*(b*x + a)
^m*b^4*c^3*m*x + 36*(b*x + a)^m*a*b^3*c^2*d*m*x - 24*(b*x + a)^m*a^2*b^2*c*
d^2*m*x + 6*(b*x + a)^m*a^3*b*d^3*m*x + 36*(b*x + a)^m*b^4*c^2*d*x^2 + 26*(
b*x + a)^m*a*b^3*c^3*m - 21*(b*x + a)^m*a^2*b^2*c^2*d*m + 6*(b*x + a)^m*a^3
*b*c*d^2*m + 24*(b*x + a)^m*b^4*c^3*x + 24*(b*x + a)^m*a*b^3*c^3 - 36*(b*x
+ a)^m*a^2*b^2*c^2*d + 24*(b*x + a)^m*a^3*b*c*d^2 - 6*(b*x + a)^m*a^4*d^3)/
(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

Mupad [B]
time = 0.94, size = 478, normalized size = 4.35

$$\frac{d^3\,x^4\,{\left(a+b\,x\right)}^m\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{a\,{\left(a+b\,x\right)}^m\,\left(-6\,a^3\,d^3+6\,a^2\,b\,c\,d^2\,m+24\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d\,m^2-21\,a\,b^2\,c^2\,d\,m-36\,a\,b^2\,c^2\,d+b^3\,c^3\,m^3+9\,b^3\,c^3\,m^2+26\,b^3\,c^3\,m+24\,b^3\,c^3\right)}{b^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}+\frac{x\,{\left(a+b\,x\right)}^m\,\left(6\,a^3\,b\,d^3\,m-6\,a^2\,b^2\,c\,d^2\,m^2-24\,a^2\,b^2\,c\,d^2\,m+3\,a\,b^3\,c^2\,d\,m^3+21\,a\,b^3\,c^2\,d\,m^2+36\,a\,b^3\,c^2\,d\,m+b^4\,c^3\,m^3+9\,b^4\,c^3\,m^2+26\,b^4\,c^3\,m+24\,b^4\,c^3\right)}{b^4\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}+\frac{3\,d\,x^2\,\left(m+1\right)\,{\left(a+b\,x\right)}^m\,\left(-a^2\,d^2\,m+a\,b\,c\,d\,m^2+4\,a\,b\,c\,d\,m+b^2\,c^2\,m^2+7\,b^2\,c^2\,m+12\,b^2\,c^2\right)}{b^2\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}+\frac{d^2\,x^3\,{\left(a+b\,x\right)}^m\,\left(12\,b\,c+a\,d\,m+3\,b\,c\,m\right)\,\left(m^2+3\,m+2\right)}{b\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^3,x)
```

```
[Out] (d^3*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (a*(a + b*x)^m*(24*b^3*c^3 - 6*a^3*d^3 + 26*b^3*c^3*m + 9*b^3*c^3
*m^2 + b^3*c^3*m^3 - 36*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 21*a*b^2*c^2*d*m + 6
```

```
*a^2*b*c*d^2*m - 3*a*b^2*c^2*d*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4)) + (x*(a + b*x)^m*(24*b^4*c^3 + 26*b^4*c^3*m + 9*b^4*c^3*m^2 + b^4*c^3*m
^3 + 6*a^3*b*d^3*m + 36*a*b^3*c^2*d*m - 24*a^2*b^2*c*d^2*m + 21*a*b^3*c^2*d
*m^2 + 3*a*b^3*c^2*d*m^3 - 6*a^2*b^2*c*d^2*m^2))/(b^4*(50*m + 35*m^2 + 10*m
^3 + m^4 + 24)) + (3*d*x^2*(m + 1)*(a + b*x)^m*(12*b^2*c^2 - a^2*d^2*m + 7*
b^2*c^2*m + b^2*c^2*m^2 + 4*a*b*c*d*m + a*b*c*d*m^2))/(b^2*(50*m + 35*m^2 +
 10*m^3 + m^4 + 24)) + (d^2*x^3*(a + b*x)^m*(12*b*c + a*d*m + 3*b*c*m)*(3*m
 + m^2 + 2))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

3.1847 $\int (a+bx)^m (c+dx)^2 \, dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}$$

```
[Out] (-a*d+b*c)^2*(b*x+a)^(1+m)/b^3/(1+m)+2*d*(-a*d+b*c)*(b*x+a)^(2+m)/b^3/(2+m)
+d^2*(b*x+a)^(3+m)/b^3/(3+m)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1 + m))/(b^3*(1 + m)) + (2*d*(b*c - a*d)*(a + b*x
)^(2 + m))/(b^3*(2 + m)) + (d^2*(a + b*x)^(3 + m))/(b^3*(3 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^m}{b^2}+\frac{2d(bc-ad)(a+bx)^{1+m}}{b^2}+\frac{d^2(a+bx)^{2+m}}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.86

$$\frac{(a+bx)^{1+m}\left(\frac{(bc-ad)^2}{1+m}+\frac{2d(bc-ad)(a+bx)}{2+m}+\frac{d^2(a+bx)^2}{3+m}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^2/(1 + m) + (2*d*(b*c - a*d)*(a + b*x))/(2
+ m) + (d^2*(a + b*x)^2)/(3 + m)))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.
time = 0.19, size = 159, normalized size = 2.04

method	result
gosper	$\frac{(bx+a)^{1+m}(b^2d^2m^2x^2+2b^2cd\,m^2x+3b^2d^2m\,x^2-2ab\,d^2mx+b^2c^2m^2+8b^2cdmx+2b^2x^2d^2-2abcdm-2ab\,d^2x+5b^2c^2m+6b^2cdx+2a^2d^2}{b^3(m^3+6m^2+11m+6)}$
norman	$\frac{d^2x^3e^{m\ln(bx+a)}}{3+m}+\frac{a(b^2c^2m^2-2abcdm+5b^2c^2m+2a^2d^2-6abcd+6b^2c^2)e^{m\ln(bx+a)}}{b^3(m^3+6m^2+11m+6)}+\frac{(adm+2bcm+6bc)d\,x^2e^{m\ln(bx+a)}}{b(m^2+5m+6)}-\frac{(-2a}{}$
risch	$\frac{(b^3d^2m^2x^3+a\,b^2d^2m^2x^2+2b^3cd\,m^2x^2+3b^3d^2m\,x^3+2a\,b^2cd\,m^2x+a\,b^2d^2m\,x^2+b^3c^2m^2x+8b^3cdm\,x^2+2d^2x^3b^3-2a^2b\,d^2mx+a\,b^2c^2m}{(2+m)(3+m)(1+m)b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+m)*(b^2*d^2*m^2*x^2+2*b^2*c*d*m^2*x+3*b^2*d^2*m*x^2-2*a*b*d^2*m*
x+b^2*c^2*m^2+8*b^2*c*d*m*x+2*b^2*d^2*x^2-2*a*b*c*d*m-2*a*b*d^2*x+5*b^2*c^2
*m+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/b^3/(m^3+6*m^2+11*m+6)
```

Maxima [A]
time = 0.31, size = 138, normalized size = 1.77

$$\frac{2\,(b^2(m+1)x^2+abmx-a^2)(bx+a)^m cd}{(m^2+3\,m+2)b^2}+\frac{(bx+a)^{m+1}c^2}{b(m+1)}+\frac{((m^2+3\,m+2)b^3x^3+(m^2+m)ab^2x^2-2\,a^2bmx+2\,a^3)(bx+a)^m d^2}{(m^3+6\,m^2+11\,m+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*d/((m^2 + 3*m + 2)*b^2) +
 (b*x + a)^(m + 1)*c^2/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a
*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d^2/((m^3 + 6*m^2 + 11*m + 6)*b
^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(78) = 156$.
time = 0.63, size = 235, normalized size = 3.01

$$\frac{(ab^2c^2m^2+6\,ab^2c^2-6\,a^2bcd+2\,a^3d^2+(b^3d^2m^2+3\,b^3d^2m+2\,b^3d^2)x^3+(6\,b^3cd+(2\,b^3cd+ab^2d^2)m^2+(8\,b^3cd+ab^2d^2)m)x^2+(5\,ab^2c^2-2\,a^2bcd)m+(6\,b^3c^2+(b^3c^2+2\,ab^2cd)m^2+(5\,b^3c^2+6\,ab^2cd-2\,a^2bd^2)m)x)(bx+a)^m}{b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2*m^2 + 6*a*b^2*c^2 - 6*a^2*b*c*d + 2*a^3*d^2 + (b^3*d^2*m^2 + 3*b
^3*d^2*m + 2*b^3*d^2)*x^3 + (6*b^3*c*d + (2*b^3*c*d + a*b^2*d^2)*m^2 + (8*b
^3*c*d + a*b^2*d^2)*m)*x^2 + (5*a*b^2*c^2 - 2*a^2*b*c*d)*m + (6*b^3*c^2 + (
b^3*c^2 + 2*a*b^2*c*d)*m^2 + (5*b^3*c^2 + 6*a*b^2*c*d - 2*a^2*b*d^2)*m)*x)*
(b*x + a)^m/(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(66) = 132$.

time = 0.58, size = 1506, normalized size = 19.31

$$\begin{cases} a^{m}\left(c^{2}x + cdx^{2} + \frac{d^{2}x^{3}}{3}\right) & \text{for}\ b = 0 \\ \frac{2a^{2}d^{2}\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} + \frac{3a^{2}d^{2}}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} - \frac{2abcd}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} + \frac{4abd^{2}x\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} + \frac{4abd^{2}x}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} - \frac{b^{2}c^{2}}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} - \frac{4b^{2}cdx}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} + \frac{2b^{2}d^{2}x^{2}\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} & \text{for}\ m = -3 \\ -\frac{2a^{2}d^{2}\log\left(\frac{a}{b}+x\right)}{ab^{3}+b^{4}x} - \frac{2a^{2}d^{2}}{ab^{3}+b^{4}x} + \frac{2abcd\log\left(\frac{a}{b}+x\right)}{ab^{3}+b^{4}x} + \frac{2abcd}{ab^{3}+b^{4}x} - \frac{2abd^{2}x\log\left(\frac{a}{b}+x\right)}{ab^{3}+b^{4}x} - \frac{b^{2}c^{2}}{ab^{3}+b^{4}x} + \frac{2b^{2}cdx\log\left(\frac{a}{b}+x\right)}{ab^{3}+b^{4}x} + \frac{b^{2}d^{2}x^{2}}{ab^{3}+b^{4}x} & \text{for}\ m = -2 \\ \frac{a^{2}d^{2}\log\left(\frac{a}{b}+x\right)}{b^{3}} - \frac{2acd\log\left(\frac{a}{b}+x\right)}{b^{2}} - \frac{ad^{2}x}{b^{2}} + \frac{c^{2}\log\left(\frac{a}{b}+x\right)}{b} + \frac{2cdx}{b} + \frac{d^{2}x^{2}}{2b} & \text{for}\ m = -1 \\ \frac{2a^{3}d^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} - \frac{2a^{2}bcdm(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} - \frac{6a^{2}bcd(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} - \frac{2a^{2}bd^{2}mx(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{ab^{2}c^{2}m^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{5ab^{2}c^{2}m(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{6ab^{2}c^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{2ab^{2}cdm^{2}x(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{6ab^{2}cdmx(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{ab^{2}d^{2}m^{2}x^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{ab^{2}d^{2}mx^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{b^{3}c^{2}m^{2}x(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{5b^{3}c^{2}mx(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{6b^{3}c^{2}x(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{2b^{3}cdm^{2}x^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{8b^{3}cdmx^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{6b^{3}cdx^{2}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{b^{3}d^{2}m^{2}x^{3}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{3b^{3}d^{2}mx^{3}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} + \frac{2b^{3}d^{2}x^{3}(a+bx)^{m}}{b^{3}m^{3}+6b^{3}m^{2}+11b^{3}m+6b^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**2,x)
```

```
[Out] Piecewise((a**m*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(b, 0)), (2*a**2*d**2*
log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d**2/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*a*b*c*d/(2*a**2*b**3 + 4*a*b**4*x + 2
*b**5*x**2) + 4*a*b*d**2*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 4*a*b*d**2*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c**2/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 4*b**2*c*d*x/(2*a**2*b**3 + 4*a*b
**4*x + 2*b**5*x**2) + 2*b**2*d**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**
4*x + 2*b**5*x**2), Eq(m, -3)), (-2*a**2*d**2*log(a/b + x)/(a*b**3 + b**4*x
) - 2*a**2*d**2/(a*b**3 + b**4*x) + 2*a*b*c*d*log(a/b + x)/(a*b**3 + b**4*x
) + 2*a*b*c*d/(a*b**3 + b**4*x) - 2*a*b*d**2*x*log(a/b + x)/(a*b**3 + b**4*
x) - b**2*c**2/(a*b**3 + b**4*x) + 2*b**2*c*d*x*log(a/b + x)/(a*b**3 + b**4
*x) + b**2*d**2*x**2/(a*b**3 + b**4*x), Eq(m, -2)), (a**2*d**2*log(a/b + x)
/b**3 - 2*a*c*d*log(a/b + x)/b**2 - a*d**2*x/b**2 + c**2*log(a/b + x)/b + 2
*c*d*x/b + d**2*x**2/(2*b), Eq(m, -1)), (2*a**3*d**2*(a + b*x)**m/(b**3*m**
3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*c*d*m*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 6*a**2*b*c*d*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*d**2*m*x*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*c**2*m**2*(a + b*x)**
m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 5*a*b**2*c**2*m*(a + b*x
)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c**2*(a + b*
x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*a*b**2*c*d*m**2*x*
(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c*d*
m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*d*
*2*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) +
a*b**2*d**2*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b*
*3) + b**3*c**2*m**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m +
6*b**3) + 5*b**3*c**2*m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m
 + 6*b**3) + 6*b**3*c**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*
m + 6*b**3) + 2*b**3*c*d*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 +
11*b**3*m + 6*b**3) + 8*b**3*c*d*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m*
```

```
*2 + 11*b**3*m + 6*b**3) + 6*b**3*c*d*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3
*m**2 + 11*b**3*m + 6*b**3) + b**3*d**2*m**2*x**3*(a + b*x)**m/(b**3*m**3 +
 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 3*b**3*d**2*m*x**3*(a + b*x)**m/(b**3*
m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*b**3*d**2*x**3*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(78) = 156.
time = 3.58, size = 385, normalized size = 4.94

$$\frac{(bx+a)^m b^3 d^2 m^2 x^3 + 2\,(bx+a)^m b^3 c d m^2 x^2 + (bx+a)^m a b^2 d^2 m^2 x^2 + 3\,(bx+a)^m b^3 d^2 m x^3 + (bx+a)^m b^3 c^2 m^2 x + 2\,(bx+a)^m a b^2 c d m^2 x + 8\,(bx+a)^m b^3 c d m x^2 + (bx+a)^m a b^2 d^2 m x^2 + 2\,(bx+a)^m b^3 d^2 x^3 + (bx+a)^m a b^2 c^2 m^2 + 5\,(bx+a)^m b^3 c^2 m x + 6\,(bx+a)^m a b^2 c d m x - 2\,(bx+a)^m a^2 b d^2 m x + 6\,(bx+a)^m b^3 c d x^2 + 5\,(bx+a)^m a b^2 c^2 m - 2\,(bx+a)^m a^2 b c d m + 6\,(bx+a)^m b^3 c^2 x + 6\,(bx+a)^m a b^2 c^2 - 6\,(bx+a)^m a^2 b c d + 2\,(bx+a)^m a^3 d^2}{b^3 m^3 + 6\,b^3 m^2 + 11\,b^3 m + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^3*d^2*m^2*x^3 + 2*(b*x + a)^m*b^3*c*d*m^2*x^2 + (b*x + a)^m*
a*b^2*d^2*m^2*x^2 + 3*(b*x + a)^m*b^3*d^2*m*x^3 + (b*x + a)^m*b^3*c^2*m^2*x
 + 2*(b*x + a)^m*a*b^2*c*d*m^2*x + 8*(b*x + a)^m*b^3*c*d*m*x^2 + (b*x + a)^
m*a*b^2*d^2*m*x^2 + 2*(b*x + a)^m*b^3*d^2*x^3 + (b*x + a)^m*a*b^2*c^2*m^2 +
 5*(b*x + a)^m*b^3*c^2*m*x + 6*(b*x + a)^m*a*b^2*c*d*m*x - 2*(b*x + a)^m*a^
2*b*d^2*m*x + 6*(b*x + a)^m*b^3*c*d*x^2 + 5*(b*x + a)^m*a*b^2*c^2*m - 2*(b*
x + a)^m*a^2*b*c*d*m + 6*(b*x + a)^m*b^3*c^2*x + 6*(b*x + a)^m*a*b^2*c^2 -
6*(b*x + a)^m*a^2*b*c*d + 2*(b*x + a)^m*a^3*d^2)/(b^3*m^3 + 6*b^3*m^2 + 11*
b^3*m + 6*b^3)
```

Mupad [B]
time = 0.66, size = 226, normalized size = 2.90

$$(a+b\,x)^m\,\left(\frac{a\,\left(2\,a^2\,d^2-2\,a\,b\,c\,d\,m-6\,a\,b\,c\,d+b^2\,c^2\,m^2+5\,b^2\,c^2\,m+6\,b^2\,c^2\right)}{b^3\,\left(m^3+6\,m^2+11\,m+6\right)}+\frac{d^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{x\,\left(-2\,a^2\,b\,d^2\,m+2\,a\,b^2\,c\,d\,m^2+6\,a\,b^2\,c\,d\,m+b^3\,c^2\,m^2+5\,b^3\,c^2\,m+6\,b^3\,c^2\right)}{b^3\,\left(m^3+6\,m^2+11\,m+6\right)}+\frac{d\,x^2\,\left(m+1\right)\,\left(6\,b\,c+a\,d\,m+2\,b\,c\,m\right)}{b\,\left(m^3+6\,m^2+11\,m+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^2,x)
```

```
[Out] (a + b*x)^m*((a*(2*a^2*d^2 + 6*b^2*c^2 + 5*b^2*c^2*m + b^2*c^2*m^2 - 6*a*b*
c*d - 2*a*b*c*d*m))/(b^3*(11*m + 6*m^2 + m^3 + 6)) + (d^2*x^3*(3*m + m^2 +
2))/(11*m + 6*m^2 + m^3 + 6) + (x*(6*b^3*c^2 + 5*b^3*c^2*m + b^3*c^2*m^2 -
2*a^2*b*d^2*m + 2*a*b^2*c*d*m^2 + 6*a*b^2*c*d*m))/(b^3*(11*m + 6*m^2 + m^3
+ 6)) + (d*x^2*(m + 1)*(6*b*c + a*d*m + 2*b*c*m))/(b*(11*m + 6*m^2 + m^3 +
6)))
```

3.1848 $\int (a+bx)^m(c+dx)\,dx$

Optimal. Leaf size=46

$$\frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}$$

```
[Out] (-a*d+b*c)*(b*x+a)^(1+m)/b^2/(1+m)+d*(b*x+a)^(2+m)/b^2/(2+m)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{45\}$

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(1 + m))/(b^2*(1 + m)) + (d*(a + b*x)^(2 + m))/(b^2*
(2 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^m}{b}+\frac{d(a+bx)^{1+m}}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 0.89

$$\frac{(a+bx)^{1+m}(-ad+bc(2+m)+bd(1+m)x)}{b^2(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(a*d) + b*c*(2 + m) + b*d*(1 + m)*x))/(b^2*(1 + m)*(2 + m))
```

Maple [A]
time = 0.15, size = 49, normalized size = 1.07

method	result	size
gosper	$-\frac{(bx+a)^{1+m}(-bdmx-bcm-bdx+ad-2bc)}{b^2(m^2+3m+2)}$	49
risch	$-\frac{(-d\,x^2b^2m-abdmx-b^2cmx-d\,x^2b^2-abcm-2b^2cx+a^2d-2abc)(bx+a)^m}{b^2(2+m)(1+m)}$	81
norman	$\frac{d\,x^2\mathrm{e}^{m\ln(bx+a)}}{2+m}+\frac{(adm+bcm+2bc)x\,\mathrm{e}^{m\ln(bx+a)}}{b(m^2+3m+2)}-\frac{a(-bcm+ad-2bc)\mathrm{e}^{m\ln(bx+a)}}{b^2(m^2+3m+2)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+m)*(-b*d*m*x-b*c*m-b*d*x+a*d-2*b*c)/b^2/(m^2+3*m+2)
```

Maxima [A]
time = 0.26, size = 63, normalized size = 1.37

$$\frac{(b^2(m+1)x^2+abmx-a^2)(bx+a)^m d}{(m^2+3\,m+2)b^2}+\frac{(bx+a)^{m+1}c}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d/((m^2 + 3*m + 2)*b^2) + (b*x + a)^(m + 1)*c/(b*(m + 1))
```

Fricas [A]
time = 1.00, size = 83, normalized size = 1.80

$$\frac{(abcm+2\,abc-a^2d+(b^2dm+b^2d)x^2+(2\,b^2c+(b^2c+abd)m)x)(bx+a)^m}{b^2m^2+3\,b^2m+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*b*c*m + 2*a*b*c - a^2*d + (b^2*d*m + b^2*d)*x^2 + (2*b^2*c + (b^2*c + a*b*d)*m)*x)*(b*x + a)^m/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(37) = 74$.

time = 0.29, size = 377, normalized size = 8.20

$$\begin{cases} a^m\left(cx+\frac{dx^2}{2}\right) & \text{for } b=0 \\ \frac{ad\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{ad}{ab^2+b^3x}-\frac{bc}{ab^2+b^3x}+\frac{bdx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } m=-2 \\ -\frac{ad\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx}{b} & \text{for } m=-1 \\ -\frac{a^2d(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{abcm(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{2abc(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{abdmx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2cmx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{2b^2cx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2dmx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2dx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c),x)
```

```
[Out] Piecewise((a**m*(c*x + d*x**2/2), Eq(b, 0)), (a*d*log(a/b + x)/(a*b**2 + b*
*3*x) + a*d/(a*b**2 + b**3*x) - b*c/(a*b**2 + b**3*x) + b*d*x*log(a/b + x)/
(a*b**2 + b**3*x), Eq(m, -2)), (-a*d*log(a/b + x)/b**2 + c*log(a/b + x)/b +
 d*x/b, Eq(m, -1)), (-a**2*d*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 a*b*c*m*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + 2*a*b*c*(a + b*x)**
m/(b**2*m**2 + 3*b**2*m + 2*b**2) + a*b*d*m*x*(a + b*x)**m/(b**2*m**2 + 3*b
**2*m + 2*b**2) + b**2*c*m*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 2*b**2*c*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*m*x**2*(a
 + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*x**2*(a + b*x)**m/(b**2
*m**2 + 3*b**2*m + 2*b**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(46) = 92.
time = 2.26, size = 132, normalized size = 2.87

$$\frac{(bx+a)^m b^2dmx^2+(bx+a)^m b^2cmx+(bx+a)^m abdmx+(bx+a)^m b^2dx^2+(bx+a)^m abcm+2\,(bx+a)^m b^2cx+2\,(bx+a)^m abc-(bx+a)^m a^2d}{b^2m^2+3\,b^2m+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^2*d*m*x^2 + (b*x + a)^m*b^2*c*m*x + (b*x + a)^m*a*b*d*m*x +
(b*x + a)^m*b^2*d*x^2 + (b*x + a)^m*a*b*c*m + 2*(b*x + a)^m*b^2*c*x + 2*(b*
x + a)^m*a*b*c - (b*x + a)^m*a^2*d)/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

Mupad [B]
time = 0.48, size = 88, normalized size = 1.91

$$(a+b\,x)^m\,\left(\frac{a\,(2\,b\,c-a\,d+b\,c\,m)}{b^2\,\left(m^2+3\,m+2\right)}+\frac{x\,(2\,b^2\,c+b^2\,c\,m+a\,b\,d\,m)}{b^2\,\left(m^2+3\,m+2\right)}+\frac{d\,x^2\,(m+1)}{m^2+3\,m+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x),x)
```

```
[Out] (a + b*x)^m*((a*(2*b*c - a*d + b*c*m))/(b^2*(3*m + m^2 + 2)) + (x*(2*b^2*c
+ b^2*c*m + a*b*d*m))/(b^2*(3*m + m^2 + 2)) + (d*x^2*(m + 1))/(3*m + m^2 +
2))
```

3.1849 $\int \frac{(a+bx)^m}{c+dx}\,dx$

Optimal. Leaf size=51

$$\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

```
[Out] (b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(1
+m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {70}

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/((b*c - a*d)*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{c+dx}\,dx = \frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 1.00

$$-\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d)])/((-(b*c) + a*d)*(1 + m)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c),x)
```

```
[Out] int((b*x+a)^m/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c),x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="giac")
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x),x)
[Out] int((a + b*x)^m/(c + d*x), x)
```

3.1850 $\int \frac{(a+bx)^m}{(c+dx)^2} \, dx$

Optimal. Leaf size=52

$$\frac{b(a+bx)^{1+m} {}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

```
[Out] b*(b*x+a)^(1+m)*hypergeom([2, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^
2/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$\frac{b(a+bx)^{m+1} {}_2F_1\left(2,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x)^2,x]
```

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)^2} \, dx = \frac{b(a+bx)^{1+m} {}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 1.00

$$\frac{b(a+bx)^{1+m} {}_2F_1\left(2,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x)^2,x]
```

```
[Out] (b*(a + b*x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(a + b*x))/(b*
c - a*d))])/((b*c - a*d)^2*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)^2,x)
```

```
[Out] int((b*x+a)^m/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*x + a)^m/(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x)^2,x)
[Out] int((a + b*x)^m/(c + d*x)^2, x)
```

3.1851 $\int \frac{(a+bx)^m}{(c+dx)^3}\,dx$

Optimal. Leaf size=54

$$\frac{b^2(a+bx)^{1+m}\,{}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

```
[Out] b^2*(b*x+a)^(1+m)*hypergeom([3, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c
)^3/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$\frac{b^2(a+bx)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/(c + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+bx)^m}{(c+dx)^3}\,dx = \frac{b^2(a+bx)^{1+m}\,{}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 1.00

$$\frac{b^2(a+bx)^{1+m}\,{}_2F_1\left(3,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)^3(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/(c + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)^3*(1 + m))
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)^3,x)
```

```
[Out] int((b*x+a)^m/(d*x+c)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)^3,x, algorithm="giac")
[Out] integrate((b*x + a)^m/(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x)^3,x)
[Out] int((a + b*x)^m/(c + d*x)^3, x)
```

3.1852 $\int (a+bx)^3(c+dx)^n\,dx$

Optimal. Leaf size=111

$$-\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}+\frac{b^3(c+dx)^{4+n}}{d^4(4+n)}$$

```
[Out] -(-a*d+b*c)^3*(d*x+c)^(1+n)/d^4/(1+n)+3*b*(-a*d+b*c)^2*(d*x+c)^(2+n)/d^4/(2
+n)-3*b^2*(-a*d+b*c)*(d*x+c)^(3+n)/d^4/(3+n)+b^3*(d*x+c)^(4+n)/d^4/(4+n)
```

Rubi [A]

time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {45}

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)^3*(c + d*x)^(1 + n))/(d^4*(1 + n))) + (3*b*(b*c - a*d)^2*(c
+ d*x)^(2 + n))/(d^4*(2 + n)) - (3*b^2*(b*c - a*d)*(c + d*x)^(3 + n))/(d^4*
(3 + n)) + (b^3*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^n}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{1+n}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{2+}}{d^3}\right.\\ &= -\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}\end{aligned}$$

Mathematica [A]

time = 0.10, size = 95, normalized size = 0.86

$$\frac{(c+dx)^{1+n}\left(-\frac{(bc-ad)^3}{1+n}+\frac{3b(bc-ad)^2(c+dx)}{2+n}-\frac{3b^2(bc-ad)(c+dx)^2}{3+n}+\frac{b^3(c+dx)^3}{4+n}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-((b*c - a*d)^3/(1 + n)) + (3*b*(b*c - a*d)^2*(c + d*x)
)/(2 + n) - (3*b^2*(b*c - a*d)*(c + d*x)^2)/(3 + n) + (b^3*(c + d*x)^3)/(4
+ n)))/d^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(111) = 222$.
time = 0.18, size = 386, normalized size = 3.48

method	result
gosper	$(dx+c)^{1+n}(b^3d^3n^3x^3+3ab^2d^3n^3x^2+6b^3d^3n^2x^3+3a^2bd^3n^3x+21ab^2d^3n^2x^2-3b^3cd^2n^2x^2+11b^3d^3nx^3+a^3d^3n^3+24a^2bd^3n^2x-6ab^2$
norman	$\frac{b^3x^4e^{n\ln(dx+c)}}{4+n}+\frac{c(a^3d^3n^3+9a^3d^3n^2-3a^2bcd^2n^2+26a^3d^3n-21a^2bcd^2n+6ab^2c^2dn+24a^3d^3-36a^2bcd^2+24ab^2c^2d-6b^3c^3)e^{n\ln(d}}{d^4(n^4+10n^3+35n^2+50n+24)}$
risch	$(b^3d^4n^3x^4+3ab^2d^4n^3x^3+b^3cd^3n^3x^3+6b^3d^4n^2x^4+3a^2bd^4n^3x^2+3ab^2cd^3n^3x^2+21ab^2d^4n^2x^3+3b^3cd^3n^2x^3+11b^3d^4nx^4+a^3d^4n^3x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)*(b^3*d^3*n^3*x^3+3*a*b^2*d^3*n^3*x^2+6*b^3*d^3*n^2*x^3+3*a^2*
b*d^3*n^3*x+21*a*b^2*d^3*n^2*x^2-3*b^3*c*d^2*n^2*x^2+11*b^3*d^3*n*x^3+a^3*d
^3*n^3+24*a^2*b*d^3*n^2*x-6*a*b^2*c*d^2*n^2*x+42*a*b^2*d^3*n*x^2-9*b^3*c*d^
2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-3*a^2*b*c*d^2*n^2+57*a^2*b*d^3*n*x-30*a
*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2+6*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*
n-21*a^2*b*c*d^2*n+36*a^2*b*d^3*x+6*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x+6*b^3*c^
2*d*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/d^4/(n^4+10*n^3+3
5*n^2+50*n+24)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(111) = 222$.
time = 0.31, size = 246, normalized size = 2.22

$$\frac{3\left(d^2(n+1)x^2+cdnx-c^2\right)(dx+c)^n a^2 b}{(n^2+3n+2)d^2}+\frac{(dx+c)^{n+1}a^3}{d(n+1)}+\frac{3\left((n^2+3n+2)d^3x^3+(n^2+n)cd^2x^2-2c^2dnx+2c^3\right)(dx+c)^n ab^2}{(n^3+6n^2+11n+6)d^3}+\frac{\left((n^3+6n^2+11n+6)d^4x^4+(n^3+3n^2+2n)cd^3x^3-3(n^2+n)c^2d^2x^2+6c^3dnx-6c^4\right)(dx+c)^n b^3}{(n^4+10n^3+35n^2+50n+24)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 3*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a^2*b/((n^2 + 3*n + 2)*d^2)
 + (d*x + c)^(n + 1)*a^3/(d*(n + 1)) + 3*((n^2 + 3*n + 2)*d^3*x^3 + (n^2 +
n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*a*b^2/((n^3 + 6*n^2 + 11*n
+ 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x
^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*b^3/((n^4 +
 10*n^3 + 35*n^2 + 50*n + 24)*d^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(111) = 222$.
time = 0.99, size = 496, normalized size = 4.47

$$\frac{(a^3cd^3n^3 - 6\,b^3c^4 + 24\,ab^2c^3d - 36\,a^2bc^2d^2 + 24\,a^3cd^3 + (b^3d^4n^3 + 6\,b^3d^4n^2 + 11\,b^3d^4n + 6\,b^3d^4)x^4 + (24\,ab^2d^4 + (b^3cd^3 + 3\,ab^2d^4)n^3 + 3\,(b^3cd^3 + 7\,ab^2d^4)n^2 + 2\,(b^3cd^3 + 21\,ab^2d^4)n)x^3 - 3\,(a^2bc^2d^2 - 3\,a^3cd^3)n^2 + 3\,(12\,a^2bd^4 + (ab^2cd^3 + a^2bd^4)n^3 - (b^3c^2d^2 - 5\,ab^2cd^3 - 8\,a^2bd^4)n^2 - (b^3c^2d^2 - 4\,ab^2cd^3 - 19\,a^2bd^4)n)x^2 + (6\,ab^2c^3d - 21\,a^2bc^2d^2 + 26\,a^3cd^3)n + (24\,a^3d^4 + (3\,a^2bcd^3 + a^3d^4)n^3 - 3\,(2\,ab^2c^2d^2 - 7\,a^2bcd^3 - 3\,a^3d^4)n^2 + 2\,(3\,b^3c^3d - 12\,ab^2c^2d^2 + 18\,a^2bcd^3 + 13\,a^3d^4)n)x)(dx + c)^n}{d^4n^4 + 10\,d^4n^3 + 35\,d^4n^2 + 50\,d^4n + 24\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^3*c*d^3*n^3 - 6*b^3*c^4 + 24*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 24*a^3*c*d
^3 + (b^3*d^4*n^3 + 6*b^3*d^4*n^2 + 11*b^3*d^4*n + 6*b^3*d^4)*x^4 + (24*a*b
^2*d^4 + (b^3*c*d^3 + 3*a*b^2*d^4)*n^3 + 3*(b^3*c*d^3 + 7*a*b^2*d^4)*n^2 +
2*(b^3*c*d^3 + 21*a*b^2*d^4)*n)*x^3 - 3*(a^2*b*c^2*d^2 - 3*a^3*c*d^3)*n^2 +
 3*(12*a^2*b*d^4 + (a*b^2*c*d^3 + a^2*b*d^4)*n^3 - (b^3*c^2*d^2 - 5*a*b^2*c
*d^3 - 8*a^2*b*d^4)*n^2 - (b^3*c^2*d^2 - 4*a*b^2*c*d^3 - 19*a^2*b*d^4)*n)*x
^2 + (6*a*b^2*c^3*d - 21*a^2*b*c^2*d^2 + 26*a^3*c*d^3)*n + (24*a^3*d^4 + (3
*a^2*b*c*d^3 + a^3*d^4)*n^3 - 3*(2*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 - 3*a^3*d^
4)*n^2 + 2*(3*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 + 13*a^3*d^4)*n
)*x)*(d*x + c)^n/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4058 vs. $2(95) = 190$.
time = 1.06, size = 4058, normalized size = 36.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), Eq(
d, 0)), (-2*a**3*d**3/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d*
*7*x**3) - 3*a**2*b*c*d**2/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 +
 6*d**7*x**3) - 9*a**2*b*d**3*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x
**2 + 6*d**7*x**3) - 6*a*b**2*c**2*d/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d
**6*x**2 + 6*d**7*x**3) - 18*a*b**2*c*d**2*x/(6*c**3*d**4 + 18*c**2*d**5*x
+ 18*c*d**6*x**2 + 6*d**7*x**3) - 18*a*b**2*d**3*x**2/(6*c**3*d**4 + 18*c**
2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*c**3*log(c/d + x)/(6*c**3
*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 11*b**3*c**3/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c**2*d*x
*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3)
 + 27*b**3*c**2*d*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7
*x**3) + 18*b**3*c*d**2*x**2*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 1
8*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c*d**2*x**2/(6*c**3*d**4 + 18*c**2*d
**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*d**3*x**3*log(c/d + x)/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3), Eq(n, -4)), (-a**
```

3*d**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 3*a**2*b*c*d**2/(2*c**2*d
4 + 4*c*d5*x + 2*d**6*x**2) - 6*a**2*b*d**3*x/(2*c**2*d**4 + 4*c*d**5*x
 + 2*d**6*x**2) + 6*a*b**2*c**2*d*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x +
2*d**6*x**2) + 9*a*b**2*c**2*d/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 6*a*b**2*d**3*
x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*b**3*c**3*lo
g(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 9*b**3*c**3/(2*c**2*d
4 + 4*c*d5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x*log(c/d + x)/(2*c**2*d**
4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) - 6*b**3*c*d**2*x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) + 2*b**3*d**3*x**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2),
 Eq(n, -3)), (-2*a**3*d**3/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2*log(c/d
+ x)/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2/(2*c*d**4 + 2*d**5*x) + 6*a**2
*b*d**3*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d*log(c/d + x
)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d/(2*c*d**4 + 2*d**5*x) - 12*a*b**
2*c*d**2*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*a*b**2*d**3*x**2/(2*c*d**
4 + 2*d**5*x) + 6*b**3*c**3*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**
3/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**2*d*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x
) - 3*b**3*c*d**2*x**2/(2*c*d**4 + 2*d**5*x) + b**3*d**3*x**3/(2*c*d**4 + 2
*d**5*x), Eq(n, -2)), (a**3*log(c/d + x)/d - 3*a**2*b*c*log(c/d + x)/d**2 +
 3*a**2*b*x/d + 3*a*b**2*c**2*log(c/d + x)/d**3 - 3*a*b**2*c*x/d**2 + 3*a*b
2*x2/(2*d) - b**3*c**3*log(c/d + x)/d**4 + b**3*c**2*x/d**3 - b**3*c*x*
*2/(2*d**2) + b**3*x**3/(3*d), Eq(n, -1)), (a**3*c*d**3*n**3*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9*a**3*c*d
3*n2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n
+ 24*d**4) + 26*a**3*c*d**3*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d
4*n2 + 50*d**4*n + 24*d**4) + 24*a**3*c*d**3*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + a**3*d**4*n**3*x*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9
*a**3*d**4*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 5
0*d**4*n + 24*d**4) + 26*a**3*d**4*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n*
*3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*d**4*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*a**2*b*c**2
*d**2*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*
n + 24*d**4) - 21*a**2*b*c**2*d**2*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 36*a**2*b*c**2*d**2*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**2*b*c
*d**3*n**3*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**
4*n + 24*d**4) + 21*a**2*b*c*d**3*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*
n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 36*a**2*b*c*d**3*n*x*(c + d*x)
n/(d4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**
2*b*d**4*n**3*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
50*d**4*n + 24*d**4) + 24*a**2*b*d**4*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 1
0*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 57*a**2*b*d**4*n*x**2*(

```
c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4)
 + 36*a**2*b*d**4*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**
2 + 50*d**4*n + 24*d**4) + 6*a*b**2*c**3*d*n*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a*b**2*c**3*d*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 6*a*b
**2*c**2*d**2*n**2*x*(c + d*x)**n/(d**4*n**4 + ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(111) = 222$.
time = 2.16, size = 833, normalized size = 7.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^3*d^4*n^3*x^4 + (d*x + c)^n*b^3*c*d^3*n^3*x^3 + 3*(d*x + c)^
n*a*b^2*d^4*n^3*x^3 + 6*(d*x + c)^n*b^3*d^4*n^2*x^4 + 3*(d*x + c)^n*a*b^2*c
*d^3*n^3*x^2 + 3*(d*x + c)^n*a^2*b*d^4*n^3*x^2 + 3*(d*x + c)^n*b^3*c*d^3*n^
2*x^3 + 21*(d*x + c)^n*a*b^2*d^4*n^2*x^3 + 11*(d*x + c)^n*b^3*d^4*n*x^4 + 3
*(d*x + c)^n*a^2*b*c*d^3*n^3*x + (d*x + c)^n*a^3*d^4*n^3*x - 3*(d*x + c)^n*
b^3*c^2*d^2*n^2*x^2 + 15*(d*x + c)^n*a*b^2*c*d^3*n^2*x^2 + 24*(d*x + c)^n*a
^2*b*d^4*n^2*x^2 + 2*(d*x + c)^n*b^3*c*d^3*n*x^3 + 42*(d*x + c)^n*a*b^2*d^4
*n*x^3 + 6*(d*x + c)^n*b^3*d^4*x^4 + (d*x + c)^n*a^3*c*d^3*n^3 - 6*(d*x + c
)^n*a*b^2*c^2*d^2*n^2*x + 21*(d*x + c)^n*a^2*b*c*d^3*n^2*x + 9*(d*x + c)^n*
a^3*d^4*n^2*x - 3*(d*x + c)^n*b^3*c^2*d^2*n*x^2 + 12*(d*x + c)^n*a*b^2*c*d^
3*n*x^2 + 57*(d*x + c)^n*a^2*b*d^4*n*x^2 + 24*(d*x + c)^n*a*b^2*d^4*x^3 - 3
*(d*x + c)^n*a^2*b*c^2*d^2*n^2 + 9*(d*x + c)^n*a^3*c*d^3*n^2 + 6*(d*x + c)^
n*b^3*c^3*d*n*x - 24*(d*x + c)^n*a*b^2*c^2*d^2*n*x + 36*(d*x + c)^n*a^2*b*c
*d^3*n*x + 26*(d*x + c)^n*a^3*d^4*n*x + 36*(d*x + c)^n*a^2*b*d^4*x^2 + 6*(d
*x + c)^n*a*b^2*c^3*d*n - 21*(d*x + c)^n*a^2*b*c^2*d^2*n + 26*(d*x + c)^n*a
^3*c*d^3*n + 24*(d*x + c)^n*a^3*d^4*x - 6*(d*x + c)^n*b^3*c^4 + 24*(d*x + c
)^n*a*b^2*c^3*d - 36*(d*x + c)^n*a^2*b*c^2*d^2 + 24*(d*x + c)^n*a^3*c*d^3)/
(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

Mupad [B]
time = 0.91, size = 478, normalized size = 4.31

$$\frac{x\,{\left(c+d\,x\right)}^n\,\left(a^3\,d^4\,n^3+9\,a^3\,d^4\,n^2+26\,a^3\,d^4\,n+24\,a^3\,d^4+3\,a^2\,b\,c\,d^3\,n^3+21\,a^2\,b\,c\,d^3\,n^2+36\,a^2\,b\,c\,d^3\,n-6\,a\,b^2\,c^2\,d^2\,n^2-24\,a\,b^2\,c^2\,d^2\,n+6\,b^3\,c^3\,d\,n\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^3\,x^4\,{\left(c+d\,x\right)}^n\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{c\,{\left(c+d\,x\right)}^n\,\left(a^3\,d^3\,n^3+9\,a^3\,d^3\,n^2+26\,a^3\,d^3\,n+24\,a^3\,d^3-3\,a^2\,b\,c\,d^2\,n^2-21\,a^2\,b\,c\,d^2\,n-36\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d\,n+24\,a\,b^2\,c^2\,d-6\,b^3\,c^3\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,b\,x^2\,\left(n+1\right)\,{\left(c+d\,x\right)}^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2+a\,b\,c\,d\,n^2+4\,a\,b\,c\,d\,n-b^2\,c^2\,n\right)}{d^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^2\,x^3\,{\left(c+d\,x\right)}^n\,\left(12\,a\,d+3\,a\,d\,n+b\,c\,n\right)\,\left(n^2+3\,n+2\right)}{d\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^n,x)
```

```
[Out] (x*(c + d*x)^n*(24*a^3*d^4 + 26*a^3*d^4*n + 9*a^3*d^4*n^2 + a^3*d^4*n^3 + 6
*b^3*c^3*d*n + 36*a^2*b*c*d^3*n - 24*a*b^2*c^2*d^2*n + 21*a^2*b*c*d^3*n^2 +
 3*a^2*b*c*d^3*n^3 - 6*a*b^2*c^2*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n^3 + n
```

```
^4 + 24)) + (b^3*x^4*(c + d*x)^n*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) + (c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n
 + 9*a^3*d^3*n^2 + a^3*d^3*n^3 + 24*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 6*a*b^2*
c^2*d*n - 21*a^2*b*c*d^2*n - 3*a^2*b*c*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n
^3 + n^4 + 24)) + (3*b*x^2*(n + 1)*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n -
b^2*c^2*n + a^2*d^2*n^2 + 4*a*b*c*d*n + a*b*c*d*n^2))/(d^2*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24)) + (b^2*x^3*(c + d*x)^n*(12*a*d + 3*a*d*n + b*c*n)*(3*n
 + n^2 + 2))/(d*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

3.1853 $\int (a+bx)^2(c+dx)^n\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}$$

```
[Out] (-a*d+b*c)^2*(d*x+c)^(1+n)/d^3/(1+n)-2*b*(-a*d+b*c)*(d*x+c)^(2+n)/d^3/(2+n)
+b^2*(d*x+c)^(3+n)/d^3/(3+n)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{45\}$

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^(1 + n))/(d^3*(1 + n)) - (2*b*(b*c - a*d)*(c + d*x
)^(2 + n))/(d^3*(2 + n)) + (b^2*(c + d*x)^(3 + n))/(d^3*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^n}{d^2}-\frac{2b(bc-ad)(c+dx)^{1+n}}{d^2}+\frac{b^2(c+dx)^{2+n}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 0.86

$$\frac{(c+dx)^{1+n}\left(\frac{(bc-ad)^2}{1+n}-\frac{2b(bc-ad)(c+dx)}{2+n}+\frac{b^2(c+dx)^2}{3+n}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*((b*c - a*d)^2/(1 + n) - (2*b*(b*c - a*d)*(c + d*x))/(2
+ n) + (b^2*(c + d*x)^2)/(3 + n)))/d^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(78) = 156$.
time = 0.19, size = 159, normalized size = 2.04

method	result
gosper	$\frac{(dx+c)^{1+n}(b^2d^2n^2x^2+2ab\,d^2n^2x+3b^2d^2n\,x^2+a^2d^2n^2+8ab\,d^2nx-2b^2cdnx+2b^2x^2d^2+5a^2d^2n-2abcdn+6ab\,d^2x-2b^2cdx+6a^2d^2-6a}{d^3(n^3+6n^2+11n+6)}$
norman	$\frac{b^2x^3e^{n\ln(dx+c)}}{3+n}+\frac{c(a^2d^2n^2+5a^2d^2n-2abcdn+6a^2d^2-6abcd+2b^2c^2)e^{n\ln(dx+c)}}{d^3(n^3+6n^2+11n+6)}+\frac{(a^2d^2n^2+2abcd\,n^2+5a^2d^2n+6abcdn-2b^2c^2n+}{d^2(n^3+6n^2+11n+6)}$
risch	$\frac{(b^2d^3n^2x^3+2ab\,d^3n^2x^2+b^2c\,d^2n^2x^2+3b^2d^3n\,x^3+a^2d^3n^2x+2abc\,d^2n^2x+8ab\,d^3n\,x^2+b^2c\,d^2n\,x^2+2b^2x^3d^3+a^2c\,d^2n^2+5a^2d^3nx+6abc}{(2+n)(3+n)(1+n)d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)*(b^2*d^2*n^2*x^2+2*a*b*d^2*n^2*x+3*b^2*d^2*n*x^2+a^2*d^2*n^2+
8*a*b*d^2*n*x-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-2*a*b*c*d*n+6*a*b*d^2
*x-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/d^3/(n^3+6*n^2+11*n+6)
```

Maxima [A]
time = 0.31, size = 138, normalized size = 1.77

$$\frac{2\,(d^2(n+1)x^2+cdnx-c^2)(dx+c)^n ab}{(n^2+3\,n+2)d^2}+\frac{(dx+c)^{n+1}a^2}{d(n+1)}+\frac{((n^2+3\,n+2)d^3x^3+(n^2+n)cd^2x^2-2\,c^2dnx+2\,c^3)(dx+c)^n b^2}{(n^3+6\,n^2+11\,n+6)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 2*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a*b/((n^2 + 3*n + 2)*d^2) +
 (d*x + c)^(n + 1)*a^2/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c
*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b^2/((n^3 + 6*n^2 + 11*n + 6)*d
^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(78) = 156$.
time = 0.72, size = 237, normalized size = 3.04

$$\frac{(a^2cd^2n^2+2\,b^2c^3-6\,abc^2d+6\,a^2cd^2+(b^2d^3n^2+3\,b^2d^3n+2\,b^2d^3)x^3+(6\,abd^3+(b^2cd^2+2\,abd^3)n^2+(b^2cd^2+8\,abd^3)n)x^2-(2\,abc^2d-5\,a^2cd^2)n+(6\,a^2d^3+(2\,abcd^2+a^2d^3)n^2-(2\,b^2c^2d-6\,abcd^2-5\,a^2d^3)n)x)(dx+c)^n}{d^3n^3+6\,d^3n^2+11\,d^3n+6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^2*c*d^2*n^2 + 2*b^2*c^3 - 6*a*b*c^2*d + 6*a^2*c*d^2 + (b^2*d^3*n^2 + 3*b
^2*d^3*n + 2*b^2*d^3)*x^3 + (6*a*b*d^3 + (b^2*c*d^2 + 2*a*b*d^3)*n^2 + (b^2
*c*d^2 + 8*a*b*d^3)*n)*x^2 - (2*a*b*c^2*d - 5*a^2*c*d^2)*n + (6*a^2*d^3 + (
2*a*b*c*d^2 + a^2*d^3)*n^2 - (2*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*n)*x)*
(d*x + c)^n/(d^3*n^3 + 6*d^3*n^2 + 11*d^3*n + 6*d^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(66) = 132$.
time = 0.55, size = 1506, normalized size = 19.31

$$\begin{cases} c^{n}\left(a^{2}x+abx^{2}+\frac{b^{2}x^{3}}{3}\right) & \text{for } d=0 \\ -\frac{a^{2}d^{2}}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}-\frac{2abcd}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}-\frac{4abd^{2}x}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}+\frac{2b^{2}c^{2}\log\left(\frac{c}{d}+x\right)}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}+\frac{3b^{2}c^{2}}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}+\frac{4b^{2}cdx\log\left(\frac{c}{d}+x\right)}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}+\frac{4b^{2}cdx}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}}+\frac{2b^{2}d^{2}x^{2}\log\left(\frac{c}{d}+x\right)}{2c^{2}d^{3}+4cd^{4}x+2d^{5}x^{2}} & \text{for } n=-3 \\ -\frac{a^{2}d^{2}}{cd^{3}+d^{4}x}+\frac{2abcd\log\left(\frac{c}{d}+x\right)}{cd^{3}+d^{4}x}+\frac{2abcd}{cd^{3}+d^{4}x}+\frac{2abd^{2}x\log\left(\frac{c}{d}+x\right)}{cd^{3}+d^{4}x}-\frac{2b^{2}c^{2}\log\left(\frac{c}{d}+x\right)}{cd^{3}+d^{4}x}-\frac{2b^{2}c^{2}}{cd^{3}+d^{4}x}-\frac{2b^{2}cdx\log\left(\frac{c}{d}+x\right)}{cd^{3}+d^{4}x}+\frac{b^{2}d^{2}x^{2}}{cd^{3}+d^{4}x} & \text{for } n=-2 \\ \frac{a^{2}\log\left(\frac{c}{d}+x\right)}{d}-\frac{2abc\log\left(\frac{c}{d}+x\right)}{d^{2}}+\frac{2abx}{d}+\frac{b^{2}c^{2}\log\left(\frac{c}{d}+x\right)}{d^{3}}-\frac{b^{2}cx}{d^{2}}+\frac{b^{2}x^{2}}{2d} & \text{for } n=-1 \\ \frac{a^{2}cd^{2}n^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{5a^{2}cd^{2}n(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{6a^{2}cd^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{a^{2}d^{3}n^{2}x(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{5a^{2}d^{3}nx(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{6a^{2}d^{3}x(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}-\frac{2abc^{2}dn(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}-\frac{6abc^{2}d(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{2abcd^{2}n^{2}x(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{6abcd^{2}nx(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{2abd^{3}n^{2}x^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{8abd^{3}nx^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{6abd^{3}x^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{2b^{2}c^{3}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}-\frac{2b^{2}c^{2}dnx(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{b^{2}cd^{2}n^{2}x^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{b^{2}cd^{2}nx^{2}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{b^{2}d^{3}n^{2}x^{3}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{3b^{2}d^{3}nx^{3}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}}+\frac{2b^{2}d^{3}x^{3}(c+dx)^{n}}{d^{3}n^{3}+6d^{3}n^{2}+11d^{3}n+6d^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(d, 0)), (-a**2*d**2/(
2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) - 2*a*b*c*d/(2*c**2*d**3 + 4*c*d**4
*x + 2*d**5*x**2) - 4*a*b*d**2*x/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) +
 2*b**2*c**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 3*b**2
*c**2/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x*log(c/d + x)/
(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x/(2*c**2*d**3 + 4*c*
d**4*x + 2*d**5*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d*
*4*x + 2*d**5*x**2), Eq(n, -3)), (-a**2*d**2/(c*d**3 + d**4*x) + 2*a*b*c*d*
log(c/d + x)/(c*d**3 + d**4*x) + 2*a*b*c*d/(c*d**3 + d**4*x) + 2*a*b*d**2*x
*log(c/d + x)/(c*d**3 + d**4*x) - 2*b**2*c**2*log(c/d + x)/(c*d**3 + d**4*x
) - 2*b**2*c**2/(c*d**3 + d**4*x) - 2*b**2*c*d*x*log(c/d + x)/(c*d**3 + d**
4*x) + b**2*d**2*x**2/(c*d**3 + d**4*x), Eq(n, -2)), (a**2*log(c/d + x)/d -
 2*a*b*c*log(c/d + x)/d**2 + 2*a*b*x/d + b**2*c**2*log(c/d + x)/d**3 - b**2
*c*x/d**2 + b**2*x**2/(2*d), Eq(n, -1)), (a**2*c*d**2*n**2*(c + d*x)**n/(d*
*3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*c*d**2*n*(c + d*x)**n/
(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*c*d**2*(c + d*x)**n
/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + a**2*d**3*n**2*x*(c + d*x
)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*d**3*n*x*(c +
d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*d**3*x*(c +
 d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 2*a*b*c**2*d*n*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 6*a*b*c**2*d*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*b*c*d**2*n*
*2*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a*b*c*
d**2*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*
b*d**3*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3
) + 8*a*b*d**3*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6
*d**3) + 6*a*b*d**3*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n
+ 6*d**3) + 2*b**2*c**3*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n +
```

```
6*d**3) - 2*b**2*c**2*d*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**
3*n + 6*d**3) + b**2*c*d**2*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2
 + 11*d**3*n + 6*d**3) + b**2*c*d**2*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**
3*n**2 + 11*d**3*n + 6*d**3) + b**2*d**3*n**2*x**3*(c + d*x)**n/(d**3*n**3
+ 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 3*b**2*d**3*n*x**3*(c + d*x)**n/(d**3
*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*b**2*d**3*x**3*(c + d*x)**n/(
d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(78) = 156.
time = 1.61, size = 385, normalized size = 4.94

$$\frac{(dx+c)^n b^2 d^3 n^2 x^3 + (dx+c)^n b^2 c d^2 n^2 x^2 + 2\,(dx+c)^n a b d^3 n^2 x^2 + 3\,(dx+c)^n b^2 d^3 n x^3 + 2\,(dx+c)^n a b c d^2 n^2 x + (dx+c)^n a^2 d^3 n^2 x + (dx+c)^n b^2 c d^2 n x^2 + 8\,(dx+c)^n a b d^3 n x^2 + 2\,(dx+c)^n b^2 d^3 x^3 + (dx+c)^n a^2 c d^2 n^2 - 2\,(dx+c)^n b^2 c^2 d n x + 6\,(dx+c)^n a b c d^2 n x + 5\,(dx+c)^n a^2 d^3 n x + 6\,(dx+c)^n a b d^3 x^2 - 2\,(dx+c)^n a b c^2 d n + 5\,(dx+c)^n a^2 c d^2 n + 6\,(dx+c)^n a^2 d^3 x + 2\,(dx+c)^n b^2 c^3 - 6\,(dx+c)^n a b c^2 d + 6\,(dx+c)^n a^2 c d^2}{d^3 n^3 + 6\,d^3 n^2 + 11\,d^3 n + 6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^2*d^3*n^2*x^3 + (d*x + c)^n*b^2*c*d^2*n^2*x^2 + 2*(d*x + c)^
n*a*b*d^3*n^2*x^2 + 3*(d*x + c)^n*b^2*d^3*n*x^3 + 2*(d*x + c)^n*a*b*c*d^2*n
^2*x + (d*x + c)^n*a^2*d^3*n^2*x + (d*x + c)^n*b^2*c*d^2*n*x^2 + 8*(d*x + c
)^n*a*b*d^3*n*x^2 + 2*(d*x + c)^n*b^2*d^3*x^3 + (d*x + c)^n*a^2*c*d^2*n^2 -
 2*(d*x + c)^n*b^2*c^2*d*n*x + 6*(d*x + c)^n*a*b*c*d^2*n*x + 5*(d*x + c)^n*
a^2*d^3*n*x + 6*(d*x + c)^n*a*b*d^3*x^2 - 2*(d*x + c)^n*a*b*c^2*d*n + 5*(d*
x + c)^n*a^2*c*d^2*n + 6*(d*x + c)^n*a^2*d^3*x + 2*(d*x + c)^n*b^2*c^3 - 6*
(d*x + c)^n*a*b*c^2*d + 6*(d*x + c)^n*a^2*c*d^2)/(d^3*n^3 + 6*d^3*n^2 + 11*
d^3*n + 6*d^3)
```

Mupad [B]
time = 0.62, size = 226, normalized size = 2.90

$$(c+d\,x)^n\,\left(\frac{c\,\left(a^2\,d^2\,n^2+5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+2\,b^2\,c^2\right)}{d^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b^2\,x^3\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{x\,\left(a^2\,d^3\,n^2+5\,a^2\,d^3\,n+6\,a^2\,d^3+2\,a\,b\,c\,d^2\,n^2+6\,a\,b\,c\,d^2\,n-2\,b^2\,c^2\,d\,n\right)}{d^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,x^2\,\left(n+1\right)\,\left(6\,a\,d+2\,a\,d\,n+b\,c\,n\right)}{d\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(6*a^2*d^2 + 2*b^2*c^2 + 5*a^2*d^2*n + a^2*d^2*n^2 - 6*a*b*
c*d - 2*a*b*c*d*n))/(d^3*(11*n + 6*n^2 + n^3 + 6)) + (b^2*x^3*(3*n + n^2 +
2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*a^2*d^3 + 5*a^2*d^3*n + a^2*d^3*n^2 -
2*b^2*c^2*d*n + 2*a*b*c*d^2*n^2 + 6*a*b*c*d^2*n))/(d^3*(11*n + 6*n^2 + n^3
+ 6)) + (b*x^2*(n + 1)*(6*a*d + 2*a*d*n + b*c*n))/(d*(11*n + 6*n^2 + n^3 +
6)))
```

3.1854 $\int (a+bx)(c+dx)^n \, dx$

Optimal. Leaf size=47

$$-\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}$$

```
[Out] -(-a*d+b*c)*(d*x+c)^(1+n)/d^2/(1+n)+b*(d*x+c)^(2+n)/d^2/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = $\{45\}$

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)*(c + d*x)^(1 + n))/(d^2*(1 + n))) + (b*(c + d*x)^(2 + n))/(d
^2*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^n \, dx &= \int \left(\frac{(-bc+ad)(c+dx)^n}{d}+\frac{b(c+dx)^{1+n}}{d}\right) dx\\ &= -\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 0.87

$$\frac{(c+dx)^{1+n}(-bc+ad(2+n)+bd(1+n)x)}{d^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(b*c) + a*d*(2 + n) + b*d*(1 + n)*x))/(d^2*(1 + n)*(2
+ n))
```

Maple [A]
time = 0.16, size = 46, normalized size = 0.98

method	result	size
gosper	$\frac{(dx+c)^{1+n}(bdnx+adn+bdx+2ad-bc)}{d^2(n^2+3n+2)}$	46
risch	$\frac{(x^2b\,d^2n+a\,d^2nx+bcdnx+x^2b\,d^2+acdn+2a\,d^2x+2acd-b\,c^2)(dx+c)^n}{d^2(2+n)(1+n)}$	76
norman	$\frac{b\,x^2e^{n\ln(dx+c)}}{2+n}+\frac{c(adn+2ad-bc)e^{n\ln(dx+c)}}{d^2(n^2+3n+2)}+\frac{(adn+bcn+2ad)x\,e^{n\ln(dx+c)}}{d(n^2+3n+2)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)*(b*d*n*x+a*d*n+b*d*x+2*a*d-b*c)/d^2/(n^2+3*n+2)
```

Maxima [A]
time = 0.29, size = 63, normalized size = 1.34

$$\frac{(d^2(n+1)x^2+cdnx-c^2)(dx+c)^nb}{(n^2+3\,n+2)d^2}+\frac{(dx+c)^{n+1}a}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] (d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*b/((n^2 + 3*n + 2)*d^2) + (d*
x + c)^(n + 1)*a/(d*(n + 1))
```

Fricas [A]
time = 0.71, size = 83, normalized size = 1.77

$$\frac{(acdn-bc^2+2\,acd+(bd^2n+bd^2)x^2+(2\,ad^2+(bcd+ad^2)n)x)(dx+c)^n}{d^2n^2+3\,d^2n+2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a*c*d*n - b*c^2 + 2*a*c*d + (b*d^2*n + b*d^2)*x^2 + (2*a*d^2 + (b*c*d + a*
d^2)*n)*x)*(d*x + c)^n/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(37) = 74$.

time = 0.28, size = 377, normalized size = 8.02

$$\begin{cases} c^n\left(ax + \frac{bx^2}{2}\right) & \text{for } d = 0 \\ -\frac{ad}{cd^2+d^3x} + \frac{bc\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} + \frac{bc}{cd^2+d^3x} + \frac{bdx\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} & \text{for } n = -2 \\ \frac{a\log\left(\frac{c}{d}+x\right)}{d} - \frac{bc\log\left(\frac{c}{d}+x\right)}{d^2} + \frac{bx}{d} & \text{for } n = -1 \\ \frac{acdn(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{2acd(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{ad^2nx(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{2ad^2x(c+dx)^n}{d^2n^2+3d^2n+2d^2} - \frac{bc^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{bcdnx(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{bd^2nx^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} + \frac{bd^2x^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a*x + b*x**2/2), Eq(d, 0)), (-a*d/(c*d**2 + d**3*x) + b*c*
log(c/d + x)/(c*d**2 + d**3*x) + b*c/(c*d**2 + d**3*x) + b*d*x*log(c/d + x)
/(c*d**2 + d**3*x), Eq(n, -2)), (a*log(c/d + x)/d - b*c*log(c/d + x)/d**2 +
 b*x/d, Eq(n, -1)), (a*c*d*n*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 2*a*c*d*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + a*d**2*n*x*(c + d*x
)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + 2*a*d**2*x*(c + d*x)**n/(d**2*n**2 +
 3*d**2*n + 2*d**2) - b*c**2*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 b*c*d*n*x*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*n*x**2*(c
+ d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*x**2*(c + d*x)**n/(d**2*
n**2 + 3*d**2*n + 2*d**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(47) = 94.
time = 6.13, size = 132, normalized size = 2.81

$$\frac{(dx+c)^n bd^2nx^2 + (dx+c)^n bcdnx + (dx+c)^n ad^2nx + (dx+c)^n bd^2x^2 + (dx+c)^n acdn + 2\,(dx+c)^n ad^2x - (dx+c)^n bc^2 + 2\,(dx+c)^n acd}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b*d^2*n*x^2 + (d*x + c)^n*b*c*d*n*x + (d*x + c)^n*a*d^2*n*x +
(d*x + c)^n*b*d^2*x^2 + (d*x + c)^n*a*c*d*n + 2*(d*x + c)^n*a*d^2*x - (d*x
+ c)^n*b*c^2 + 2*(d*x + c)^n*a*c*d)/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

Mupad [B]
time = 0.49, size = 88, normalized size = 1.87

$$(c+d\,x)^n\left(\frac{c\,(2\,a\,d-b\,c+a\,d\,n)}{d^2\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,(n+1)}{n^2+3\,n+2}+\frac{x\,(2\,a\,d^2+a\,d^2\,n+b\,c\,d\,n)}{d^2\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(2*a*d - b*c + a*d*n))/(d^2*(3*n + n^2 + 2)) + (b*x^2*(n +
1))/(3*n + n^2 + 2) + (x*(2*a*d^2 + a*d^2*n + b*c*d*n))/(d^2*(3*n + n^2 + 2
)))
```

3.1855 $\int (c+dx)^n \, dx$

Optimal. Leaf size=18

$$\frac{(c+dx)^{1+n}}{d(1+n)}$$

[Out] (d*x+c)^(1+n)/d/(1+n)

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{32\}$

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^n,x]

[Out] (c + d*x)^(1 + n)/(d*(1 + n))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^n \, dx = \frac{(c+dx)^{1+n}}{d(1+n)}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 0.94

$$\frac{(c+dx)^{1+n}}{d+dn}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)^n,x]

[Out] (c + d*x)^(1 + n)/(d + d*n)

Maple [A]
time = 0.15, size = 19, normalized size = 1.06

method	result	size
gosper	$\frac{(dx+c)^{1+n}}{d(1+n)}$	19
default	$\frac{(dx+c)^{1+n}}{d(1+n)}$	19
risch	$\frac{(dx+c)(dx+c)^{n}}{d(1+n)}$	22
norman	$\frac{x\,e^{n\ln(dx+c)}}{1+n}+\frac{c\,e^{n\ln(dx+c)}}{d(1+n)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (d*x+c)^(1+n)/d/(1+n)
```

Maxima [A]
time = 0.30, size = 18, normalized size = 1.00

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

Fricas [A]
time = 1.15, size = 20, normalized size = 1.11

$$\frac{(dx+c)(dx+c)^{n}}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (d*x + c)*(d*x + c)^n/(d*n + d)
```

Sympy [A]
time = 0.01, size = 20, normalized size = 1.11

$$\frac{\begin{cases}\frac{(c+dx)^{n+1}}{n+1} & \text{for}\ n\neq -1\\ \log(c+dx) & \text{otherwise}\end{cases}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n,x)
```

```
[Out] Piecewise(((c + d*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(c + d*x), True))/d
```

Giac [A]
time = 4.33, size = 18, normalized size = 1.00

$$\frac{(dx + c)^{n+1}}{d(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="giac")
```

```
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

Mupad [B]
time = 0.38, size = 18, normalized size = 1.00

$$\frac{(c + d\,x)^{n+1}}{d\,(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n,x)
```

```
[Out] (c + d*x)^(n + 1)/(d*(n + 1))
```

3.1856 $\int \frac{(c+dx)^n}{a+bx}\,dx$

Optimal. Leaf size=51

$$-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

```
[Out] -(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c)/(1
+n)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x),x]
```

```
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{a+bx}\,dx = -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.00

$$-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x),x]
[Out] -(((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)*(1 + n)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^n}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a),x)
[Out] int((d*x+c)^n/(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^n/(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^n/(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^n}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a),x)
[Out] Integral((c + d*x)**n/(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^n/(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c+d\,x)^n}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x),x)
[Out] int((c + d*x)^n/(a + b*x), x)
```

3.1857 $\int \frac{(c+dx)^n}{(a+bx)^2} dx$

Optimal. Leaf size=51

$$\frac{d(c+dx)^{1+n} \, _2F_1\left(2,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^2(1+n)}$$

```
[Out] d*(d*x+c)^(1+n)*hypergeom([2, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c)^2
/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{70\}$

$$\frac{d(c+dx)^{n+1} \, _2F_1\left(2,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^2,x]
```

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (b*(c + d*x))/(b*c
- a*d)])/((b*c - a*d)^2*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{(a+bx)^2} \, dx = \frac{d(c+dx)^{1+n} \, _2F_1\left(2,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^2(1+n)}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 1.02

$$\frac{d(c+dx)^{1+n} \, _2F_1\left(2,1+n;2+n;-\frac{b(c+dx)}{-bc+ad}\right)}{(-bc+ad)^2(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^2,x]
```

```
[Out] (d*(c + d*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, -((b*(c + d*x))/(-(
b*c) + a*d))])/((-(b*c) + a*d)^2*(1 + n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^n}{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^n/(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^n}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**n/(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^n/(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c+d\,x\right)}^n}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^2,x)
[Out] int((c + d*x)^n/(a + b*x)^2, x)
```

3.1858 $\int \frac{(c+dx)^n}{(a+bx)^3} dx$

Optimal. Leaf size=54

$$-\frac{d^2(c+dx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^3(1+n)}$$

```
[Out] -d^2*(d*x+c)^(1+n)*hypergeom([3, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/(-a*d+b*c
)^3/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {70}

$$-\frac{d^2(c+dx)^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{(n+1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^3,x]
```

```
[Out] -((d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, (b*(c + d*x))/(
b*c - a*d)])/((b*c - a*d)^3*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(c+dx)^n}{(a+bx)^3}\,dx = -\frac{d^2(c+dx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)^3(1+n)}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 1.00

$$\frac{d^2(c+dx)^{1+n}\,{}_2F_1\left(3,1+n;2+n;-\frac{b(c+dx)}{-bc+ad}\right)}{(-bc+ad)^3(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^3,x]
```

```
[Out] (d^2*(c + d*x)^(1 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, -((b*(c + d*x))/(
-(b*c) + a*d))])/((-(b*c) + a*d)^3*(1 + n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^n}{(bx+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^n/(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^n}{(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**n/(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c+d\,x\right)}^n}{{\left(a+b\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^3,x)
```

```
[Out] int((c + d*x)^n/(a + b*x)^3, x)
```

3.1859 $\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=143

$$-\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}+\frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)}-\frac{2d^2(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^3(1-n)(2-n)(3-n)}$$

```
[Out] -(b*x+a)^(-3+n)*(d*x+c)^(1-n)/(-a*d+b*c)/(3-n)+2*d*(b*x+a)^(-2+n)*(d*x+c)^(
1-n)/(-a*d+b*c)^2/(2-n)/(3-n)-2*d^2*(b*x+a)^(-1+n)*(d*x+c)^(1-n)/(-a*d+b*c)
^3/(1-n)/(2-n)/(3-n)
```

Rubi [A]
time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(3 - n))) + (2*d*(a +
 b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(2 - n)*(3 - n)) - (2*d^2*
(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^3*(1 - n)*(2 - n)*(3 - n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} - \frac{(2d)\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx}{(bc-ad)(3-n)}$$
$$= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} + \frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)} + \frac{(2d^2)\int (a+bx}{(bc-ad)^2}$$
$$= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)} + \frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)} - \frac{2d^2(a+bx)^{-}}{(bc-ad)^3(1-}$$

Mathematica [A]
time = 0.08, size = 112, normalized size = 0.78

$$\frac{(a+bx)^{-3+n}(c+dx)^{1-n}\left(a^2d^2(6-5n+n^2)-2abd(-3+n)(c(-1+n)+dx)+b^2(c^2(2-3n+n^2)+2cd(-1+n)x+2d^2x^2)\right)}{(bc-ad)^3(-3+n)(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-3 + n)*(c + d*x)^(1 - n)*(a^2*d^2*(6 - 5*n + n^2) - 2*a*b*d*(-
3 + n)*(c*(-1 + n) + d*x) + b^2*(c^2*(2 - 3*n + n^2) + 2*c*d*(-1 + n)*x + 2
*d^2*x^2)))/((b*c - a*d)^3*(-3 + n)*(-2 + n)*(-1 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(143) = 286.
time = 0.22, size = 322, normalized size = 2.25

method	result
gosper	$-\frac{(bx+a)^{-3+n}(dx+c)\left(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2-5a^2d^2n+8abcdn+6ab\,d^2x-3b^2c^2n-2b^2c\right.}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3-6a^3d^3n^2+18a^2bc\,d^2n^2-18a\,b^2c^2d\,n^2+6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4+n)/((d*x+c)^n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(-3+n)*(d*x+c)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*c^2*n^
2+2*b^2*c*d*n*x+2*b^2*d^2*x^2-5*a^2*d^2*n+8*a*b*c*d*n+6*a*b*d^2*x-3*b^2*c^2
*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d^2*n^
3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3-6*a^3*d^3*n^2+18*a^2*b*c*d^2*n^2-18*a*b^2*c
^2*d*n^2+6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-11*b^
3*c^3*n-6*a^3*d^3+18*a^2*b*c*d^2-18*a*b^2*c^2*d+6*b^3*c^3)/((d*x+c)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(130) = 260$.
time = 0.87, size = 512, normalized size = 3.58

$$-\frac{\left(2\,b^3d^3x^4 + 2\,ab^2c^3 - 6\,a^2bc^2d + 6\,a^3cd^2 + 2\,\left(4\,ab^2d^3 + (b^3cd^2 - ab^2d^3)n\right)x^3 + (ab^2c^3 - 2\,a^2bc^2d + a^3cd^2)n^2 + \left(12\,a^2bd^3 + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)n^2 - (b^3c^2d - 8\,ab^2cd^2 + 7\,a^2bd^3)n\right)x^2 - (3\,ab^2c^3 - 8\,a^2bc^2d + 5\,a^3cd^2)n + \left(2\,b^3c^3 - 6\,ab^2c^2d + 6\,a^2bcd^2 + 6\,a^3d^3 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)n^2 - (3\,b^3c^3 - 7\,ab^2c^2d - a^2bcd^2 + 5\,a^3d^3)n\right)x\right)(bx+a)^{n-4}}{\left(6\,b^3c^3 - 18\,ab^2c^2d + 18\,a^2bcd^2 - 6\,a^3d^3 - (b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)n^3 + 6\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)n^2 - 11\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)n\right)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 - (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 - (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 - (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(n - 4)/((6*b^3*c^3 - 18*
a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*n^2 - 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n)*(d*x +
 c)^n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

Mupad [B]
time = 1.10, size = 528, normalized size = 3.69

$$-\frac{x\,{\left(a+b\,x\right)}^{n-4}\,\left(a^3\,d^3\,n^2-5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2+a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2+7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2-3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{a\,c\,{\left(a+b\,x\right)}^{n-4}\,\left(a^2\,d^2\,n^2-5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2-3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^3\,d^3\,x^4\,{\left(a+b\,x\right)}^{n-4}}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{b\,d\,x^2\,{\left(a+b\,x\right)}^{n-4}\,\left(a^2\,d^2\,n^2-7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2-b^2\,c^2\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^2\,d^2\,x^3\,{\left(a+b\,x\right)}^{n-4}\,\left(4\,a\,d-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^n\,\left(n^3-6\,n^2+11\,n-6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 4)/(c + d*x)^n,x)
```

```
[Out] - (x*(a + b*x)^(n - 4)*(6*a^3*d^3 + 2*b^3*c^3 - 5*a^3*d^3*n - 3*b^3*c^3*n +
 a^3*d^3*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 7*a*b^2*c^2*d*
n + a^2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c +
 d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (a*c*(a + b*x)^(n - 4)*(6*a^2*d^2 + 2*b
^2*c^2 - 5*a^2*d^2*n - 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d
+ 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 +
n^3 - 6)) - (2*b^3*d^3*x^4*(a + b*x)^(n - 4))/((a*d - b*c)^3*(c + d*x)^n*(1
1*n - 6*n^2 + n^3 - 6)) - (b*d*x^2*(a + b*x)^(n - 4)*(12*a^2*d^2 - 7*a^2*d^
2*n - b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 + 8*a*b*c*d*n - 2*a*b*c*d*n^2))
/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^(n - 4)*(4*a*d - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6
*n^2 + n^3 - 6))
```

3.1860 $\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=86

$$-\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}+\frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}$$

```
[Out] -(b*x+a)^(-2+n)*(d*x+c)^(1-n)/(-a*d+b*c)/(2-n)+d*(b*x+a)^(-1+n)*(d*x+c)^(1-
n)/(-a*d+b*c)^2/(1-n)/(2-n)
```

Rubi [A]
time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(2 - n))) + (d*(a + b
*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)} - \frac{d\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx}{(bc-ad)(2-n)}$$
$$= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)} + \frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.69

$$\frac{(a+bx)^{-2+n}(c+dx)^{1-n}(-ad(-2+n)+bc(-1+n)+bdx)}{(bc-ad)^2(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 + n)*(c + d*x)^(1 - n)*(-(a*d*(-2 + n)) + b*c*(-1 + n) + b*d
*x))/((b*c - a*d)^2*(-2 + n)*(-1 + n))
```

Maple [A]
time = 0.19, size = 127, normalized size = 1.48

method	result	size
gosper	$-\frac{(bx+a)^{-2+n}(dx+c)(adn-bcn-bdx-2ad+bc)(dx+c)^{-n}}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2-3a^2d^2n+6abcdn-3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3+n)/((d*x+c)^n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(-2+n)*(d*x+c)*(a*d*n-b*c*n-b*d*x-2*a*d+b*c)/(a^2*d^2*n^2-2*a*b*c*
d*n^2+b^2*c^2*n^2-3*a^2*d^2*n+6*a*b*c*d*n-3*b^2*c^2*n+2*a^2*d^2-4*a*b*c*d+2
*b^2*c^2)/((d*x+c)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.
time = 0.96, size = 206, normalized size = 2.40

$$\frac{(b^2d^2x^3 - abc^2 + 2\,a^2cd + (3\,abd^2 + (b^2cd - abd^2)n)x^2 + (abc^2 - a^2cd)n - (b^2c^2 - 2\,abcd - 2\,a^2d^2 - (b^2c^2 - a^2d^2)n)x)(bx+a)^{n-3}}{(2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + (b^2c^2 - 2\,abcd + a^2d^2)n^2 - 3\,(b^2c^2 - 2\,abcd + a^2d^2)n)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 - (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(n - 3)/((2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*n^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n)*(d*x
+ c)^n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

Mupad [B]
time = 0.77, size = 220, normalized size = 2.56

$$(a+b\,x)^{n-3}\,\left(\frac{x\,(2\,a^2\,d^2-b^2\,c^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d)}{{(a\,d-b\,c)}^2\,{(c+d\,x)}^n\,(n^2-3\,n+2)}+\frac{b^2\,d^2\,x^3}{{(a\,d-b\,c)}^2\,{(c+d\,x)}^n\,(n^2-3\,n+2)}+\frac{a\,c\,(2\,a\,d-b\,c-a\,d\,n+b\,c\,n)}{{(a\,d-b\,c)}^2\,{(c+d\,x)}^n\,(n^2-3\,n+2)}+\frac{b\,d\,x^2\,(3\,a\,d-a\,d\,n+b\,c\,n)}{{(a\,d-b\,c)}^2\,{(c+d\,x)}^n\,(n^2-3\,n+2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 3)/(c + d*x)^n,x)
```

```
[Out] (a + b*x)^(n - 3)*((x*(2*a^2*d^2 - b^2*c^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*
c*d))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b^2*d^2*x^3)/((a*d - b
*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (a*c*(2*a*d - b*c - a*d*n + b*c*n))/((
a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b*d*x^2*(3*a*d - a*d*n + b*c*n
))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)))
```

3.1861 $\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

```
[Out] -(b*x+a)^(-1+n)*(d*x+c)^(1-n)/(-a*d+b*c)/(1-n)
```

Rubi [A]
time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 0.92

$$\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(-1 + n))
```

Maple [A]
time = 0.19, size = 45, normalized size = 1.15

method	result	size
gosper	$-\frac{(bx+a)^{-1+n}(dx+c)(dx+c)^{-n}}{adn-bcn-ad+bc}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2+n)/((d*x+c)^n),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(-1+n)*(d*x+c)/(a*d*n-b*c*n-a*d+b*c)/((d*x+c)^n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

Fricas [A]
time = 1.30, size = 60, normalized size = 1.54

$$-\frac{(bdx^2+ac+(bc+ad)x)(bx+a)^{n-2}}{(bc-ad-(bc-ad)n)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="fricas")
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(n - 2)/((b*c - a*d - (b*c - a*d
)*n)*(d*x + c)^n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)/((d*x+c)**n),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

Mupad [B]
time = 0.56, size = 102, normalized size = 2.62

$$-(a+b\,x)^{n-2}\left(\frac{a\,c}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}+\frac{x\,(a\,d+b\,c)}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}+\frac{b\,d\,x^2}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 2)/(c + d*x)^n,x)
```

```
[Out] -(a + b*x)^(n - 2)*((a*c)/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (x*(a*d + b*c
))/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (b*d*x^2)/((a*d - b*c)*(n - 1)*(c +
d*x)^n))
```

3.1862 $\int (a+bx)^{-1+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=66

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;1+n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

```
[Out] (b*x+a)^n*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, n],[1+n],-d*(b*x+a)/(-a*d+
b*c))/b/n/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{(a+bx)^n(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n;n+1;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, -
((d*(a + b*x))/(b*c - a*d))])/(b*n*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-1+n}(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{-1+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,n;1+n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.98

$$\frac{(a+bx)^{n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,n;1+n;\frac{d(a+bx)}{-bc+ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 + n)/(c + d*x)^n,x]
[Out] ((a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, n, 1 + n, (
d*(a + b*x))/(-(b*c) + a*d)])/(b*n*(c + d*x)^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{-1+n}(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1+n)/((d*x+c)^n),x)
[Out] int((b*x+a)^(-1+n)/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 1)/(d*x + c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{n-1}}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 1)/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^(n - 1)/(c + d*x)^n, x)
```

3.1863 $\int (a+bx)^n(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

```
[Out] (b*x+a)^(1+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 1+n],[2+n],-d*(b*x+a)/
(-a*d+b*c))/b/(1+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^n\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c + d*x)^n,x]
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + n)*(c + d*x)^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/((d*x+c)^n),x)
[Out] int((b*x+a)^n/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^n}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^n/(c + d*x)^n, x)
```

3.1864 $\int (a+bx)^{1+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{2+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,2+n;3+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(2+n)}$$

```
[Out] (b*x+a)^(2+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 2+n],[3+n],-d*(b*x+a)/
(-a*d+b*c))/b/(2+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+2}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+2;n+3;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(2 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 2 + n
, 3 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(2 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{1+n}(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{1+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{2+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}{}_2F_1\left(n,2+n;3+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(2+n)}$$

Mathematica [A]
time = 0.07, size = 89, normalized size = 1.24

$$\frac{(bc-ad)(a+bx)^{n}\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{1-n}{}_2F_1\left(-1-n,1-n;2-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1 + n)/(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^n*(c + d*x)^(1 - n)*Hypergeometric2F1[-1 - n, 1 - n,
 2 - n, (b*(c + d*x))/(b*c - a*d)])/(d^2*(-1 + n)*((d*(a + b*x))/(-(b*c) +
a*d))^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(bx+a\right)^{1+n}\left(dx+c\right)^{-n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1+n)/((d*x+c)^n),x)
```

```
[Out] int((b*x+a)^(1+n)/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n + 1)/(d*x + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{n+1}}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n + 1)/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^(n + 1)/(c + d*x)^n, x)
```

3.1865 $\int (a+bx)^{2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{3+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,3+n;4+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(3+n)}$$

```
[Out] (b*x+a)^(3+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 3+n],[4+n],-d*(b*x+a)/
(-a*d+b*c))/b/(3+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+3}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,n+3;n+4;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(3 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 3 + n
, 4 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(3 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{2+n}(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{2+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{3+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(n,3+n;4+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(3+n)}$$

Mathematica [A]
time = 0.06, size = 92, normalized size = 1.28

$$-\frac{(bc-ad)^2(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{1-n}\,{}_2F_1\left(-2-n,1-n;2-n;\frac{b(c+dx)}{bc-ad}\right)}{d^3(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2 + n)/(c + d*x)^n,x]
[Out] -(((b*c - a*d)^2*(a + b*x)^n*(c + d*x)^(1 - n)*Hypergeometric2F1[-2 - n, 1
- n, 2 - n, (b*(c + d*x))/(b*c - a*d)])/(d^3*(-1 + n)*((d*(a + b*x))/(-(b*c
) + a*d))^n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{2+n}(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2+n)/((d*x+c)^n),x)
[Out] int((b*x+a)^(2+n)/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n + 2)/(d*x + c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{n+2}}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n + 2)/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^(n + 2)/(c + d*x)^n, x)
```

3.1866 $\int (a+bx)^{-n}(c+dx)^n\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^n(c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

```
[Out] (-d*(b*x+a)/(-a*d+b*c))^n*(d*x+c)^(1+n)*hypergeom([n, 1+n],[2+n],b*(d*x+c)/
(-a*d+b*c))/d/(1+n)/((b*x+a)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {72, 71}

$$\frac{(a+bx)^{-n}(c+dx)^{n+1}\left(-\frac{d(a+bx)}{bc-ad}\right)^n\,{}_2F_1\left(n,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n/(a + b*x)^n,x]
```

```
[Out] ((-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1
+ n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-n}(c+dx)^n\,dx = \left((a+bx)^{-n}\left(\frac{d(a+bx)}{-bc+ad}\right)^n\right)\int (c+dx)^n\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^n(c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{-n}\left(\frac{d(a+bx)}{-bc+ad}\right)^n(c+dx)^{1+n}\,{}_2F_1\left(n,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n/(a + b*x)^n,x]
[Out] (((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^(1 + n)*Hypergeometric2F1[n, 1
+ n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(d*(1 + n)*(a + b*x)^n)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (dx+c)^n\,(bx+a)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n/((b*x+a)^n),x)
[Out] int((d*x+c)^n/((b*x+a)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="maxima")
[Out] integrate((d*x + c)^n/(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n/(b*x + a)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n/((b*x+a)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n/(b*x + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^n}{(a+b\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^n,x)
```

```
[Out] int((c + d*x)^n/(a + b*x)^n, x)
```

3.1867 $\int (a+bx)^{-1-n}(c+dx)^n\,dx$

Optimal. Leaf size=75

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

```
[Out] -(d*x+c)^n*hypergeom([-n, -n],[1-n],-d*(b*x+a)/(-a*d+b*c))/b/n/((b*x+a)^n)/
((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - n)*(c + d*x)^n,x]
```

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, -((d*(a + b*x))/(b*c - a*d)
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-1-n}(c+dx)^n\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{-1-n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx$$
$$= -\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{d(a+bx)}{bc-ad}\right)}{bn}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 0.99

$$-\frac{(a+bx)^{-n}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{d(a+bx)}{-bc+ad}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - n)*(c + d*x)^n,x]
```

```
[Out] -(((c + d*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (d*(a + b*x))/(-(b*c) + a*d
)])/(b*n*(a + b*x)^n*((b*(c + d*x))/(b*c - a*d))^n))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (bx+a)^{-1-n}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

```
[Out] int((b*x+a)^(-1-n)*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 1)*(d*x + c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^n}{(a+b\,x)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 1),x)
```

```
[Out] int((c + d*x)^n/(a + b*x)^(n + 1), x)
```

3.1868 $\int (a+bx)^{-2-n}(c+dx)^n\,dx$

Optimal. Leaf size=37

$$-\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

```
[Out] -(b*x+a)^(-1-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(1 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.03

$$\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(-1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(-1 - n))
```

Maple [A]
time = 0.19, size = 41, normalized size = 1.11

method	result	size
gosper	$\frac{(bx+a)^{-1-n}(dx+c)^{1+n}}{adn-bcn+ad-bc}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-1-n)*(d*x+c)^(1+n)/(a*d*n-b*c*n+a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

Fricas [A]
time = 0.88, size = 59, normalized size = 1.59

$$-\frac{(bdx^2+ac+(bc+ad)x)(bx+a)^{-n-2}(dx+c)^n}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(-n - 2)*(d*x + c)^n/(b*c - a*d
+ (b*c - a*d)*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2-n)*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

Mupad [B]
time = 0.53, size = 97, normalized size = 2.62

$$\frac{\frac{a\,c\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{x\,\left(a\,d+b\,c\right)\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{b\,d\,x^2\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}}{{\left(a+b\,x\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 2),x)
```

```
[Out] ((a*c*(c + d*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(c + d*x)^n)/((a*
d - b*c)*(n + 1)) + (b*d*x^2*(c + d*x)^n)/((a*d - b*c)*(n + 1)))/(a + b*x)^
(n + 2)
```

3.1869 $\int (a+bx)^{-3-n}(c+dx)^n\,dx$

Optimal. Leaf size=80

$$-\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}+\frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}$$

```
[Out] -(b*x+a)^(-2-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(2+n)+d*(b*x+a)^(-1-n)*(d*x+c)^(1+
n)/(-a*d+b*c)^2/(1+n)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(2 + n))) + (d*(a + b
*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-3-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)} - \frac{d\int (a+bx)^{-2-n}(c+dx)^n\,dx}{(bc-ad)(2+n)}$$
$$= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)} + \frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}$$

Mathematica [**A**]
time = 0.05, size = 60, normalized size = 0.75

$$\frac{(a+bx)^{-2-n}(c+dx)^{1+n}(ad(2+n)-b(c+cn-dx))}{(bc-ad)^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 - n)*(c + d*x)^(1 + n)*(a*d*(2 + n) - b*(c + c*n - d*x)))/((
b*c - a*d)^2*(1 + n)*(2 + n))
```

Maple [**A**]
time = 0.19, size = 123, normalized size = 1.54

method	result	size
gosper	$\frac{(bx+a)^{-2-n}(dx+c)^{1+n}(adn-bcn+bdx+2ad-bc)}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-2-n)*(d*x+c)^(1+n)*(a*d*n-b*c*n+b*d*x+2*a*d-b*c)/(a^2*d^2*n^2-2*a
*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b*
c*d+2*b^2*c^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(80) = 160$.
time = 1.01, size = 207, normalized size = 2.59

$$\frac{(b^2d^2x^3 - abc^2 + 2a^2cd + (3abd^2 - (b^2cd - abd^2)n)x^2 - (abc^2 - a^2cd)n - (b^2c^2 - 2abcd - 2a^2d^2 + (b^2c^2 - a^2d^2)n)x)(bx + a)^{-n-3}(dx + c)^n}{2b^2c^2 - 4abcd + 2a^2d^2 + (b^2c^2 - 2abcd + a^2d^2)n^2 + 3(b^2c^2 - 2abcd + a^2d^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 - (b^2*c*d - a*b*d^2)*n)*x^
2 - (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(-n - 3)*(d*x + c)^n/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

Mupad [**B**]
time = 0.74, size = 214, normalized size = 2.68

$$\frac{\frac{x\,(c+d\,x)^n\,(2\,a^2\,d^2-b^2\,c^2+a^2\,d^2\,n-b^2\,c^2\,n+2\,a\,b\,c\,d)}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}+\frac{a\,c\,(c+d\,x)^n\,(2\,a\,d-b\,c+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}+\frac{b^2\,d^2\,x^3\,(c+d\,x)^n}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,(3\,a\,d+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,(n^2+3\,n+2)}}{(a+b\,x)^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 3),x)
```

```
[Out] ((x*(c + d*x)^n*(2*a^2*d^2 - b^2*c^2 + a^2*d^2*n - b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) + (a*c*(c + d*x)^n*(2*a*d - b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(c + d*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(c + d*x)^n*(3*a*d + a*d*n - b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(a + b*x)^(n + 3)
```

3.1870 $\int (a+bx)^{-4-n}(c+dx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}+\frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)}-\frac{2d^2(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

```
[Out] -(b*x+a)^(-3-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(3+n)+2*d*(b*x+a)^(-2-n)*(d*x+c)^(
1+n)/(-a*d+b*c)^2/(2+n)/(3+n)-2*d^2*(b*x+a)^(-1-n)*(d*x+c)^(1+n)/(-a*d+b*c)
^3/(1+n)/(2+n)/(3+n)
```

Rubi [A]
time = 0.03, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(3 + n))) + (2*d*(a +
 b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(2 + n)*(3 + n)) - (2*d^2*
(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-4-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} - \frac{(2d)\int (a+bx)^{-3-n}(c+dx)^n\,dx}{(bc-ad)(3+n)}$$
$$= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} + \frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)} + \frac{(2d^2)\int (a+bx)^{\cdots}}{(bc-ad)^2(2\cdots}$$
$$= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)} + \frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)} - \frac{2d^2(a+bx)^{-1\cdots}}{(bc-ad)^3(1+\cdots}$$

Mathematica [A]
time = 0.09, size = 113, normalized size = 0.86

$$-\frac{(a+bx)^{-3-n}(c+dx)^{1+n}\left(a^2d^2(6+5n+n^2)-2abd(3+n)(c+cn-dx)+b^2(c^2(2+3n+n^2)-2cd(1+n)x+2d^2x^2)\right)}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n)*(a^2*d^2*(6 + 5*n + n^2) - 2*a*b*d*
(3 + n)*(c + c*n - d*x) + b^2*(c^2*(2 + 3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(131) = 262.
time = 0.19, size = 318, normalized size = 2.43

method	result
gosper	$\frac{(bx+a)^{-3-n}(dx+c)^{1+n}(a^2d^2n^2-2abcd\,n^2+2ab\,d^2nx+b^2c^2n^2-2b^2cdnx+2b^2x^2d^2+5a^2d^2n-8abcdn+6ab\,d^2x+3b^2c^2n-\cdots}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-3-n)*(d*x+c)^(1+n)*(a^2*d^2*n^2-2*a*b*c*d*n^2+2*a*b*d^2*n*x+b^2*c
^2*n^2-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-8*a*b*c*d*n+6*a*b*d^2*x+3*b^
2*c^2*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d
^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a*
b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-
11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(131) = 262$.
time = 0.76, size = 509, normalized size = 3.89

$$-\frac{\left(2 b^{3} d^{3} x^{4}+2 a b^{2} c^{3}-6 a^{2} b c^{2} d+6 a^{3} c d^{2}+2 \left(4 a b^{2} d^{3}-\left(b^{3} c d^{2}-a b^{2} d^{3}\right) n\right) x^{3}+\left(a b^{2} c^{3}-2 a^{2} b c^{2} d+a^{3} c d^{2}\right) n^{2}+\left(12 a^{2} b d^{3}+\left(b^{3} c^{2} d-2 a b^{2} c d^{2}+a^{2} b d^{3}\right) n^{2}+\left(b^{3} c^{2} d-8 a b^{2} c d^{2}+7 a^{2} b d^{3}\right) n\right) x^{2}+\left(3 a b^{2} c^{3}-8 a^{2} b c^{2} d+5 a^{3} c d^{2}\right) n+\left(2 b^{3} c^{3}-6 a b^{2} c^{2} d+6 a^{2} b c d^{2}+6 a^{3} d^{3}+\left(b^{3} c^{3}-a b^{2} c^{2} d-a^{2} b c d^{2}+a^{3} d^{3}\right) n^{2}+\left(3 b^{3} c^{3}-7 a b^{2} c^{2} d-a^{2} b c d^{2}+5 a^{3} d^{3}\right) n\right) x\right) (b x+a)^{-n-4} (d x+c)^{n}}{6 b^{3} c^{3}-18 a b^{2} c^{2} d+18 a^{2} b c d^{2}-6 a^{3} d^{3}+\left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) n^{3}+6 \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) n^{2}+11 \left(b^{3} c^{3}-3 a b^{2} c^{2} d+3 a^{2} b c d^{2}-a^{3} d^{3}\right) n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 - (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 + (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(-n - 4)*(d*x + c)^n/(6*b
^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

Mupad [B]
time = 0.99, size = 525, normalized size = 4.01

$$\frac{x\,(c+d\,x)^n\,(a^3\,d^3\,n^2+5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+3\,b^3\,c^3\,n+2\,b^3\,c^3)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}+\frac{a\,c\,(c+d\,x)^n\,(a^2\,d^2\,n^2+5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+3\,b^2\,c^2\,n+2\,b^2\,c^2)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}+\frac{2\,b^3\,d^3\,x^4\,(c+d\,x)^n}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+b^2\,c^2\,n)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}+\frac{2\,b^2\,d^2\,x^3\,(c+d\,x)^n\,(4\,a\,d+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 4),x)
```

```
[Out] (x*(c + d*x)^n*(6*a^3*d^3 + 2*b^3*c^3 + 5*a^3*d^3*n + 3*b^3*c^3*n + a^3*d^3
*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 7*a*b^2*c^2*d*n - a^2*
b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(a + b*x)^(n
 + 4)*(11*n + 6*n^2 + n^3 + 6)) + (a*c*(c + d*x)^n*(6*a^2*d^2 + 2*b^2*c^2 +
 5*a^2*d^2*n + 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*b*
c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^
3 + 6)) + (2*b^3*d^3*x^4*(c + d*x)^n)/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*
n + 6*n^2 + n^3 + 6)) + (b*d*x^2*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n + b^
2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d -
 b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (2*b^2*d^2*x^3*(c + d
*x)^n*(4*a*d + a*d*n - b*c*n))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n
^2 + n^3 + 6))
```

3.1871 $\int (a+bx)^{-5-n}(c+dx)^n\,dx$

Optimal. Leaf size=186

$$-\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)}+\frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)}-\frac{6d^2(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^3(2+n)(3+n)(4+n)}+\frac{6d^3(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^4(1+n)(2+n)(3+n)(4+n)}$$

```
[Out] -(b*x+a)^(-4-n)*(d*x+c)^(1+n)/(-a*d+b*c)/(4+n)+3*d*(b*x+a)^(-3-n)*(d*x+c)^(
1+n)/(-a*d+b*c)^2/(3+n)/(4+n)-6*d^2*(b*x+a)^(-2-n)*(d*x+c)^(1+n)/(-a*d+b*c)
^3/(2+n)/(3+n)/(4+n)+6*d^3*(b*x+a)^(-1-n)*(d*x+c)^(1+n)/(-a*d+b*c)^4/(1+n)/
(2+n)/(3+n)/(4+n)
```

Rubi [A]
time = 0.06, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-4 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(4 + n))) + (3*d*(a +
 b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(3 + n)*(4 + n)) - (6*d^2*
(a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n
)) + (6*d^3*(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-5-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} - \frac{(3d)\int (a+bx)^{-4-n}(c+dx)^n\,dx}{(bc-ad)(4+n)}$$
$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} + \frac{(6d^2)\int (a+bx)}{(bc-ad)^2(3}$$
$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2}}{(bc-ad)^3(2+}$$
$$= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)} + \frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)} - \frac{6d^2(a+bx)^{-2}}{(bc-ad)^3(2+}$$

Mathematica [**A**]
time = 0.11, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{-4-n}(c+dx)^{1+n}\left(a^3d^3(24+26n+9n^2+n^3)-3a^2bd^2(12+7n+n^2)(c+cn-dx)+3ab^2d(4+n)(c^2(2+3n+n^2)-2cd(1+n)x+2d^2x^2)-b^3(c^3(6+11n+6n^2+n^3)-3c^2d(2+3n+n^2)x+6cd^2(1+n)x^2-6d^3x^3)\right)}{(bc-ad)^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-4 - n)*(c + d*x)^(1 + n)*(a^3*d^3*(24 + 26*n + 9*n^2 + n^3) -
3*a^2*b*d^2*(12 + 7*n + n^2)*(c + c*n - d*x) + 3*a*b^2*d*(4 + n)*(c^2*(2 +
3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^2*x^2) - b^3*(c^3*(6 + 11*n + 6*n^2 + n^
3) - 3*c^2*d*(2 + 3*n + n^2)*x + 6*c*d^2*(1 + n)*x^2 - 6*d^3*x^3)))/((b*c -
 a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(186) = 372$.
time = 0.21, size = 661, normalized size = 3.55

method	result
gosper	$\frac{(bx+a)^{-4-n}(dx+c)^{1+n}(a^3d^3n^3-3a^2bc\,d^2n^3+3a^2b\,d^3n^2x+3a\,b^2c^2d\,n^3-6a\,b^2c\,d^2n^2x+6a\,b^2d^3n\,x^2-b^3c^3n^3+3b^3c^2d\,n^2x-6b^3c\,d^2n}{a^4d^4n^4-4a^3bc\,d^3n^4+6a^2b^2c^2d^2n^4-4a\,b^3c^3d\,n^4+b^4c^4n^4+10a^4d^4n^3-40a^3bc\,d^3n^3+60a^2b^2c^2d^2n^3-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-5-n)*(d*x+c)^n,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(-4-n)*(d*x+c)^(1+n)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3+3*a^2*b*d^3*n^2
*x+3*a*b^2*c^2*d*n^3-6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3+3*b^
3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-24*a^2*b*c*d^2*
n^2+21*a^2*b*d^3*n*x+21*a*b^2*c^2*d*n^2-30*a*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2
-6*b^3*c^3*n^2+9*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*n-57*a^2*b*c*d^2*
n+36*a^2*b*d^3*x+42*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x-11*b^3*c^3*n+6*b^3*c^2*d
*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/(a^4*d^4*n^4-4*a^3*b
```

```
*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*n
^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4*
n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c^
3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*n
-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d
^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(186) = 372$.
time = 0.93, size = 959, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 - 6*a*b^3*c^4 + 24*a^2*b^2*c^3*d - 36*a^3*b*c^2*d^2 + 24*a^4
*c*d^3 + 6*(5*a*b^3*d^4 - (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 - (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*a^2*b^2*d^4 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (b^4*c^2*d^2 - 10*a*b^3*c*d^3
+ 9*a^2*b^2*d^4)*n)*x^3 - 3*(2*a*b^3*c^4 - 7*a^2*b^2*c^3*d + 8*a^3*b*c^2*d^
2 - 3*a^4*c*d^3)*n^2 + (60*a^3*b*d^4 - (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 - 3*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 9*a^2*b^2*c*
d^3 - 4*a^3*b*d^4)*n^2 - (2*b^4*c^3*d - 15*a*b^3*c^2*d^2 + 60*a^2*b^2*c*d^3
 - 47*a^3*b*d^4)*n)*x^2 - (11*a*b^3*c^4 - 42*a^2*b^2*c^3*d + 57*a^3*b*c^2*d
^2 - 26*a^4*c*d^3)*n - (6*b^4*c^4 - 24*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 2
4*a^3*b*c*d^3 - 24*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(2*b^4*c^4 - 6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 - 3*a^4*d^4)*n^2 + (11*b^4*c^4 - 40*a*b^3*c^3*d + 45*a^2*b^2*c^2*d^2 + 10
*a^3*b*c*d^3 - 26*a^4*d^4)*n)*x)*(b*x + a)^(-n - 5)*(d*x + c)^n/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-5-n)*(d*x+c)**n,x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="giac")
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

Mupad [**B**]
time = 1.64, size = 944, normalized size = 5.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 5),x)
[Out] (a*c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n - 11*b^3*c^3*n + 9*
a^3*d^3*n^2 - 6*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 24*a*b^2*c^2*d -
36*a^2*b*c*d^2 + 42*a*b^2*c^2*d*n - 57*a^2*b*c*d^2*n + 21*a*b^2*c^2*d*n^2 -
 24*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (x*(c + d*x)^n*(
6*b^4*c^4 - 24*a^4*d^4 - 26*a^4*d^4*n + 11*b^4*c^4*n - 9*a^4*d^4*n^2 + 6*b^
4*c^4*n^2 - a^4*d^4*n^3 + b^4*c^4*n^3 + 36*a^2*b^2*c^2*d^2 - 24*a*b^3*c^3*d
 - 24*a^3*b*c*d^3 - 40*a*b^3*c^3*d*n + 10*a^3*b*c*d^3*n + 9*a^2*b^2*c^2*d^2
*n^2 - 18*a*b^3*c^3*d*n^2 + 12*a^3*b*c*d^3*n^2 - 2*a*b^3*c^3*d*n^3 + 2*a^3*
b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n
 + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^4*d^4*x^5*(c + d*x)^n)/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
c + d*x)^n*(20*a^2*d^2 + 9*a^2*d^2*n + b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(c + d*x)^n*(5*a*d + a*d*n -
 b*c*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(c + d*x)^n*(60*a^3*d^3 + 47*a^3*d^3*n - 2*b^3*c^3*n + 12*a^
3*d^3*n^2 - 3*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 15*a*b^2*c^2*d*n -
60*a^2*b*c*d^2*n + 18*a*b^2*c^2*d*n^2 - 27*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*
n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

3.1872 $\int (a+bx)^n (c+dx)^{-n}\,dx$

Optimal. Leaf size=72

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

```
[Out] (b*x+a)^(1+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([n, 1+n],[2+n],-d*(b*x+a)/
(-a*d+b*c))/b/(1+n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {72, 71}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,n+1;n+2;-\frac{d(a+bx)}{bc-ad}\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 + n)*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-n}\,dx = \left((c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n\right)\int (a+bx)^n\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{-n}dx$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;-\frac{d(a+bx)}{bc-ad}\right)}{b(1+n)}$$

Mathematica [A]
time = 0.01, size = 71, normalized size = 0.99

$$\frac{(a+bx)^{1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n,1+n;2+n;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n/(c + d*x)^n,x]
[Out] ((a + b*x)^(1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeometric2F1[n, 1 + n
, 2 + n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + n)*(c + d*x)^n)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n/((d*x+c)^n),x)
[Out] int((b*x+a)^n/((d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="maxima")
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n/(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n/(d*x + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^n}{(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^n,x)
```

```
[Out] int((a + b*x)^n/(c + d*x)^n, x)
```

3.1873 $\int (a+bx)^n (c+dx)^{-1-n} \, dx$

Optimal. Leaf size=75

$$-\frac{(a+bx)^n \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} (c+dx)^{-n} \, {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

```
[Out] -(b*x+a)^n*hypergeom([-n, -n],[1-n],b*(d*x+c)/(-a*d+b*c))/d/n/((-d*(b*x+a)/
(-a*d+b*c))^n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {72, 71}

$$-\frac{(a+bx)^n (c+dx)^{-n} \left(-\frac{d(a+bx)}{bc-ad}\right)^{-n} \, {}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-1 - n),x]
```

```
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-1-n}\,dx = \left((a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}\right)\int (c+dx)^{-1-n}\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^n dx$$
$$= -\frac{(a+bx)^n\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 0.99

$$-\frac{(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-1 - n),x]
```

```
[Out] -(((a + b*x)^n*Hypergeometric2F1[-n, -n, 1 - n, (b*(c + d*x))/(b*c - a*d)])
/(d*n*((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-1-n),x)
```

```
[Out] int((b*x+a)^n*(d*x+c)^(-1-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^n}{(c+d\,x)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 1),x)
```

```
[Out] int((a + b*x)^n/(c + d*x)^(n + 1), x)
```

3.1874 $\int (a+bx)^n (c+dx)^{-2-n} \, dx$

Optimal. Leaf size=36

$$\frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-1-n)/(-a*d+b*c)/(1+n)
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{37\}$

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-2-n} \, dx = \frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 1.00

$$\frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Maple [A]
time = 0.19, size = 42, normalized size = 1.17

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-1-n}}{adn-bcn+ad-bc}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-2-n),x,method=_RETURNVERBOSE)
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(a*d*n-b*c*n+a*d-b*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

Fricas [A]
time = 0.84, size = 58, normalized size = 1.61

$$\frac{(bdx^2 + ac + (bc + ad)x)(bx + a)^n(dx + c)^{-n-2}}{bc - ad + (bc - ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="fricas")
[Out] (b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^n*(d*x + c)^(-n - 2)/(b*c - a*d +
 (b*c - a*d)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-2-n),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

Mupad [B]
time = 0.56, size = 98, normalized size = 2.72

$$-\frac{\frac{a\,c\,{\left(a+b\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{x\,\left(a\,d+b\,c\right)\,{\left(a+b\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{b\,d\,x^2\,{\left(a+b\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}}{{\left(c+d\,x\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 2),x)
```

```
[Out] -((a*c*(a + b*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(a + b*x)^n)/((a
*d - b*c)*(n + 1)) + (b*d*x^2*(a + b*x)^n)/((a*d - b*c)*(n + 1)))/(c + d*x)
^(n + 2)
```

3.1875 $\int (a+bx)^n (c+dx)^{-3-n}\,dx$

Optimal. Leaf size=79

$$\frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-2-n)/(-a*d+b*c)/(2+n)+b*(b*x+a)^(1+n)*(d*x+c)^(-1-n
)/(-a*d+b*c)^2/(1+n)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)*(2 + n)) + (b*(a + b*x)
^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-3-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)} + \frac{b\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)(2+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)} + \frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 0.75

$$\frac{(a+bx)^{1+n}(c+dx)^{-2-n}(-ad(1+n)+bc(2+n)+bdx)}{(bc-ad)^2(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n)*(-(a*d*(1 + n)) + b*c*(2 + n) + b*d*x
))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Maple [A]
time = 0.23, size = 124, normalized size = 1.57

method	result	size
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-2-n}(adn-bcn-bdx+ad-2bc)}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-3-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-2-n)*(a*d*n-b*c*n-b*d*x+a*d-2*b*c)/(a^2*d^2*n^2-2*
a*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b
*c*d+2*b^2*c^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(79) = 158$.
time = 0.84, size = 205, normalized size = 2.59

$$\frac{(b^2d^2x^3 + 2\,abc^2 - a^2cd + (3\,b^2cd + (b^2cd - abd^2)n)x^2 + (abc^2 - a^2cd)n + (2\,b^2c^2 + 2\,abcd - a^2d^2 + (b^2c^2 - a^2d^2)n)x)(bx + a)^n(dx + c)^{-n-3}}{2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + (b^2c^2 - 2\,abcd + a^2d^2)n^2 + 3\,(b^2c^2 - 2\,abcd + a^2d^2)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 + 2*a*b*c^2 - a^2*c*d + (3*b^2*c*d + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n + (2*b^2*c^2 + 2*a*b*c*d - a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 3)/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-3-n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

Mupad [**B**]
time = 0.74, size = 214, normalized size = 2.71

$$\frac{\frac{x\,{\left(a+b\,x\right)}^n\,\left(2\,b^2\,c^2-a^2\,d^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}-\frac{a\,c\,{\left(a+b\,x\right)}^n\,\left(a\,d-2\,b\,c+a\,d\,n-b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}+\frac{b^2\,d^2\,x^3\,{\left(a+b\,x\right)}^n}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}+\frac{b\,d\,x^2\,{\left(a+b\,x\right)}^n\,\left(3\,b\,c-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^2\,\left(n^2+3\,n+2\right)}}{{\left(c+d\,x\right)}^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 3),x)
```

```
[Out] ((x*(a + b*x)^n*(2*b^2*c^2 - a^2*d^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) - (a*c*(a + b*x)^n*(a*d - 2*b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(a + b*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(a + b*x)^n*(3*b*c - a*d*n + b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(c + d*x)^(n + 3)
```

3.1876 $\int (a+bx)^n (c+dx)^{-4-n}\,dx$

Optimal. Leaf size=130

$$\frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)}+\frac{2b^2(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-3-n)/(-a*d+b*c)/(3+n)+2*b*(b*x+a)^(1+n)*(d*x+c)^(-2
-n)/(-a*d+b*c)^2/(2+n)/(3+n)+2*b^2*(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(-a*d+b*c)^
3/(1+n)/(2+n)/(3+n)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)*(3 + n)) + (2*b*(a + b*
x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^2*(2 + n)*(3 + n)) + (2*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-4-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{(2b)\int (a+bx)^n(c+dx)^{-3-n}\,dx}{(bc-ad)(3+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)} + \frac{(2b^2)\int (a+bx)^n(c}{(bc-ad)^2(2+}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)} + \frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)} + \frac{2b^2(a+bx)^{1+n}(}{(bc-ad)^3(1+n)(}$$

Mathematica [A]
time = 0.11, size = 112, normalized size = 0.86

$$\frac{(a+bx)^{1+n}(c+dx)^{-3-n}\left(a^2d^2(2+3n+n^2)-2abd(1+n)(c(3+n)+dx)+b^2(c^2(6+5n+n^2)+2cd(3+n)x+2d^2x^2)\right)}{(bc-ad)^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n)*(a^2*d^2*(2 + 3*n + n^2) - 2*a*b*d*(1
 + n)*(c*(3 + n) + d*x) + b^2*(c^2*(6 + 5*n + n^2) + 2*c*d*(3 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(130) = 260$.
time = 0.24, size = 319, normalized size = 2.45

method	result
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-3-n}(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2+3a^2d^2n-8abcdn-2ab\,d^2x+5b^2c^2}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-4-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-3-n)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*
c^2*n^2+2*b^2*c*d*n*x+2*b^2*d^2*x^2+3*a^2*d^2*n-8*a*b*c*d*n-2*a*b*d^2*x+5*b
^2*c^2*n+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*
d^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a
*b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n
-11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(130) = 260$.
time = 0.63, size = 507, normalized size = 3.90

$$\frac{(2\,b^3d^3x^4 + 6\,ab^2c^3 - 6\,a^2bc^2d + 2\,a^3cd^2 + 2\,(4\,b^3cd^2 + (b^3cd^2 - ab^2d^3)n)x^3 + (ab^2c^3 - 2\,a^2bc^2d + a^3cd^2)n^2 + (12\,b^3c^2d + (b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)n^2 + (7\,b^3c^2d - 8\,ab^2cd^2 + a^2bd^3)n)x^2 + (5\,ab^2c^3 - 8\,a^2bc^2d + 3\,a^3cd^2)n + (6\,b^3c^3 + 6\,ab^2c^2d - 6\,a^2bcd^2 + 2\,a^3d^3 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)n^2 + (5\,b^3c^3 - ab^2c^2d - 7\,a^2bcd^2 + 3\,a^3d^3)n)x)(bx + a)^n(dx + c)^{-n-4}}{6\,b^3c^3 - 18\,ab^2c^2d + 18\,a^2bcd^2 - 6\,a^3d^3 + (b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)n^3 + 6\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)n^2 + 11\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="fricas")
```

```
[Out] (2*b^3*d^3*x^4 + 6*a*b^2*c^3 - 6*a^2*b*c^2*d + 2*a^3*c*d^2 + 2*(4*b^3*c*d^2
 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)
*n^2 + (12*b^3*c^2*d + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (7*b^3
*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*n)*x^2 + (5*a*b^2*c^3 - 8*a^2*b*c^2*d +
 3*a^3*c*d^2)*n + (6*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (5*b^3*c^3 - a*b^2*c^
2*d - 7*a^2*b*c*d^2 + 3*a^3*d^3)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 4)/(6*b^
3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-4-n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

Mupad [B]
time = 1.02, size = 528, normalized size = 4.06

$$-\frac{x\,(a+b\,x)^n\,(a^3\,d^3\,n^2+3\,a^3\,d^3\,n+2\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-7\,a^2\,b\,c\,d^2\,n-6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-a\,b^2\,c^2\,d\,n+6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+5\,b^3\,c^3\,n+6\,b^3\,c^3)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}-\frac{a\,c\,(a+b\,x)^n\,(a^2\,d^2\,n^2+3\,a^2\,d^2\,n+2\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+5\,b^2\,c^2\,n+6\,b^2\,c^2)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}-\frac{2\,b^3\,d^3\,x^4\,(a+b\,x)^n}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}-\frac{b\,d\,x^2\,(a+b\,x)^n\,(a^2\,d^2\,n^2+a^2\,d^2\,n-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+7\,b^2\,c^2\,n+12\,b^2\,c^2)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}-\frac{2\,b^2\,d^2\,x^3\,(a+b\,x)^n\,(4\,b\,c-a\,d\,n+b\,c\,n)}{(a\,d-b\,c)^3\,(c+d\,x)^{n+4}\,(n^3+6\,n^2+11\,n+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 4),x)
```

```
[Out] - (x*(a + b*x)^n*(2*a^3*d^3 + 6*b^3*c^3 + 3*a^3*d^3*n + 5*b^3*c^3*n + a^3*d
^3*n^2 + b^3*c^3*n^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a*b^2*c^2*d*n - 7*a^
2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c + d*x)^
(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (a*c*(a + b*x)^n*(2*a^2*d^2 + 6*b^2*c^2
 + 3*a^2*d^2*n + 5*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*
b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 +
n^3 + 6)) - (2*b^3*d^3*x^4*(a + b*x)^n)/((a*d - b*c)^3*(c + d*x)^(n + 4)*(1
1*n + 6*n^2 + n^3 + 6)) - (b*d*x^2*(a + b*x)^n*(12*b^2*c^2 + a^2*d^2*n + 7*
b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d
 - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^n*(4*b*c - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6
*n^2 + n^3 + 6))
```

3.1877 $\int (a+bx)^n(c+dx)^{-5-n}\,dx$

Optimal. Leaf size=185

$$\frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)}+\frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)}+\frac{6b^2(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^3(2+n)(3+n)(4+n)}+\frac{6b^3(a+bx)^{1}}{(bc-ad)^4(1+n)(}$$

```
[Out] (b*x+a)^(1+n)*(d*x+c)^(-4-n)/(-a*d+b*c)/(4+n)+3*b*(b*x+a)^(1+n)*(d*x+c)^(-3
-n)/(-a*d+b*c)^2/(3+n)/(4+n)+6*b^2*(b*x+a)^(1+n)*(d*x+c)^(-2-n)/(-a*d+b*c)^
3/(2+n)/(3+n)/(4+n)+6*b^3*(b*x+a)^(1+n)*(d*x+c)^(-1-n)/(-a*d+b*c)^4/(1+n)/(
2+n)/(3+n)/(4+n)
```

Rubi [A]
time = 0.04, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {47, 37}

$$\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n))/((b*c - a*d)*(4 + n)) + (3*b*(a + b*
x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)^2*(3 + n)*(4 + n)) + (6*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n))
+ (6*b^3*(a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^4*(1 + n)*(2 +
n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n(c+dx)^{-5-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{(3b)\int (a+bx)^n(c+dx)^{-4-n}\,dx}{(bc-ad)(4+n)}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{(6b^2)\int (a+bx)^n(c}{(bc-ad)^2(3+}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(}{(bc-ad)^3(2+n)(}$$
$$= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)} + \frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)} + \frac{6b^2(a+bx)^{1+n}(}{(bc-ad)^3(2+n)(}$$

Mathematica [A]
time = 0.12, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{1+n}(c+dx)^{-4-n}\left(-a^3d^3(6+11n+6n^2+n^3)+3a^2bd^2(2+3n+n^2)\left(c(4+n)+dx\right)-3ab^2d(1+n)\left(c^2(12+7n+n^2)+2cd(4+n)x+2d^2x^2\right)+b^3\left(c^3(24+26n+9n^2+n^3)+3c^2d(12+7n+n^2)\,x+6cd^2(4+n)x^2+6d^3x^3\right)\right)}{(bc-ad)^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n)*(-(a^3*d^3*(6 + 11*n + 6*n^2 + n^3))
+ 3*a^2*b*d^2*(2 + 3*n + n^2)*(c*(4 + n) + d*x) - 3*a*b^2*d*(1 + n)*(c^2*(1
2 + 7*n + n^2) + 2*c*d*(4 + n)*x + 2*d^2*x^2) + b^3*(c^3*(24 + 26*n + 9*n^2
 + n^3) + 3*c^2*d*(12 + 7*n + n^2)*x + 6*c*d^2*(4 + n)*x^2 + 6*d^3*x^3)))/(
(b*c - a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(185) = 370$.
time = 0.23, size = 662, normalized size = 3.58

method	result
gosper	$-\frac{(bx+a)^{1+n}(dx+c)^{-4-n}(a^3d^3n^3-3a^2bc\,d^2n^3-3a^2b\,d^3n^2x+3a\,b^2c^2d\,n^3+6a\,b^2c\,d^2n^2x+6a\,b^2d^3n\,x^2-b^3c^3n^3-3b^3c^2d\,n^2x-6b^3c\,d}{a^4d^4n^4-4a^3bc\,d^3n^4+6a^2b^2c^2d^2n^4-4a\,b^3c^3d\,n^4+b^4c^4n^4+10a^4d^4n^3-40a^3bc\,d^3n^3+60a^2b^2c^2d^2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-5-n),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(d*x+c)^(-4-n)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3-3*a^2*b*d^3*n^
2*x+3*a*b^2*c^2*d*n^3+6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3-3*b
^3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2-6*b^3*d^3*x^3+6*a^3*d^3*n^2-21*a^2*b*c*d^2
*n^2-9*a^2*b*d^3*n*x+24*a*b^2*c^2*d*n^2+30*a*b^2*c*d^2*n*x+6*a*b^2*d^3*x^2-
9*b^3*c^3*n^2-21*b^3*c^2*d*n*x-24*b^3*c*d^2*x^2+11*a^3*d^3*n-42*a^2*b*c*d^2
*n-6*a^2*b*d^3*x+57*a*b^2*c^2*d*n+24*a*b^2*c*d^2*x-26*b^3*c^3*n-36*b^3*c^2*
d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/(a^4*d^4*n^4-4*a^3*
```

```
b*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*
n^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4
*n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c
^3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*
n-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*
d^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="maxima")
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(185) = 370.
time = 0.90, size = 954, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="fricas")
[Out] (6*b^4*d^4*x^5 + 24*a*b^3*c^4 - 36*a^2*b^2*c^3*d + 24*a^3*b*c^2*d^2 - 6*a^4
*c*d^3 + 6*(5*b^4*c*d^3 + (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 + (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*b^4*c^2*d^2 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (9*b^4*c^2*d^2 - 10*a*b^3*c*d^
3 + a^2*b^2*d^4)*n)*x^3 + 3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^
2 - 2*a^4*c*d^3)*n^2 + (60*b^4*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 + 3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*
c*d^3 - a^3*b*d^4)*n^2 + (47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2*b^2*c*d^
3 - 2*a^3*b*d^4)*n)*x^2 + (26*a*b^3*c^4 - 57*a^2*b^2*c^3*d + 42*a^3*b*c^2*d
^2 - 11*a^4*c*d^3)*n + (24*b^4*c^4 + 24*a*b^3*c^3*d - 36*a^2*b^2*c^2*d^2 +
24*a^3*b*c*d^3 - 6*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c*d^
3 - 2*a^4*d^4)*n^2 + (26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 - 11*a^4*d^4)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 5)/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-5-n),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="giac")
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

Mupad [**B**]
time = 1.61, size = 945, normalized size = 5.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 5),x)
[Out] (6*b^4*d^4*x^5*(a + b*x)^n)/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2
 + 10*n^3 + n^4 + 24)) - (a*c*(a + b*x)^n*(6*a^3*d^3 - 24*b^3*c^3 + 11*a^3*
d^3*n - 26*b^3*c^3*n + 6*a^3*d^3*n^2 - 9*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^
3*n^3 + 36*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 57*a*b^2*c^2*d*n - 42*a^2*b*c*d^2
*n + 24*a*b^2*c^2*d*n^2 - 21*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*
c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24)) - (x*(a + b*x)^n*(6*a^4*d^4 - 24*b^4*c^4 + 11*a^4*d^4*n - 26*b^4*c^4
*n + 6*a^4*d^4*n^2 - 9*b^4*c^4*n^2 + a^4*d^4*n^3 - b^4*c^4*n^3 + 36*a^2*b^2
*c^2*d^2 - 24*a*b^3*c^3*d - 24*a^3*b*c*d^3 + 10*a*b^3*c^3*d*n - 40*a^3*b*c*
d^3*n + 9*a^2*b^2*c^2*d^2*n^2 + 12*a*b^3*c^3*d*n^2 - 18*a^3*b*c*d^3*n^2 + 2
*a*b^3*c^3*d*n^3 - 2*a^3*b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^
4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
a + b*x)^n*(20*b^2*c^2 + a^2*d^2*n + 9*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(a + b*x)^n*(5*b*c - a*d*n +
 b*c*n))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(a + b*x)^n*(60*b^3*c^3 - 2*a^3*d^3*n + 47*b^3*c^3*n - 3*a^3
*d^3*n^2 + 12*b^3*c^3*n^2 - a^3*d^3*n^3 + b^3*c^3*n^3 - 60*a*b^2*c^2*d*n +
15*a^2*b*c*d^2*n - 27*a*b^2*c^2*d*n^2 + 18*a^2*b*c*d^2*n^2 - 3*a*b^2*c^2*d*
n^3 + 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

3.1878 $\int (a+bx)^{-2+n}(c+dx)^{1-n}\,dx$

Optimal. Leaf size=83

$$-\frac{(bc-ad)(a+bx)^{-1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(-1+n,-1+n;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}$$

```
[Out] -(-a*d+b*c)*(b*x+a)^(-1+n)*(b*(d*x+c)/(-a*d+b*c))^n*hypergeom([-1+n, -1+n],
[n],-d*(b*x+a)/(-a*d+b*c))/b^2/(1-n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {72, 71}

$$-\frac{(bc-ad)(a+bx)^{n-1}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^n {}_2F_1\left(n-1,n-1;n;-\frac{d(a+bx)}{bc-ad}\right)}{b^2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)*(c + d*x)^(1 - n),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x)^(-1 + n)*((b*(c + d*x))/(b*c - a*d))^n*Hypergeomet
ric2F1[-1 + n, -1 + n, n, -((d*(a + b*x))/(b*c - a*d))])/(b^2*(1 - n)*(c +
d*x)^n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{1-n}\,dx = \frac{\left((bc-ad)(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\right)\int (a+bx)^{-2+n}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^{1-n}\,dx}{b}$$
$$= -\frac{(bc-ad)(a+bx)^{-1+n}(c+dx)^{-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{n}\,{}_2F_1\left(-1+n,-1+n;n;-\frac{d(}{b}\right.}{b^2(1-n)}$$

Mathematica [A]
time = 0.08, size = 75, normalized size = 0.90

$$\frac{(a+bx)^{-1+n}(c+dx)^{1-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-1+n}\,{}_2F_1\left(-1+n,-1+n;n;\frac{d(a+bx)}{-bc+ad}\right)}{b(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)*(c + d*x)^(1 - n),x]
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n)*((b*(c + d*x))/(b*c - a*d))^(-1 + n)*
Hypergeometric2F1[-1 + n, -1 + n, n, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(-1
+ n))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{-2+n}(dx+c)^{1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
[Out] int((b*x+a)^(-2+n)*(d*x+c)^(1-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)*(d*x+c)**(1-n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)*(d*x+c)^(1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 2)*(d*x + c)^(-n + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{n-2}\,(c+d\,x)^{1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 2)*(c + d*x)^(1 - n),x)
```

```
[Out] int((a + b*x)^(n - 2)*(c + d*x)^(1 - n), x)
```

3.1879 $\int (a+bx)^{1+n}(c+dx)^{-1-n}\,dx$

Optimal. Leaf size=84

$$\frac{(bc-ad)(a+bx)^n\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

```
[Out] (-a*d+b*c)*(b*x+a)^n*hypergeom([-n, -1-n],[1-n],b*(d*x+c)/(-a*d+b*c))/d^2/n
/((-d*(b*x+a)/(-a*d+b*c))^n)/((d*x+c)^n)
```

Rubi [A]
time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {72, 71}

$$\frac{(bc-ad)(a+bx)^n(c+dx)^{-n}\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(-n-1,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1 + n)*(c + d*x)^(-1 - n),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*(-((d*(a + b*x))/(b*c - a*d)))^n*(c + d*x)^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rubi steps

$$\int (a+bx)^{1+n}(c+dx)^{-1-n}\,dx = \frac{\left((-bc+ad)(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}\right)\int (c+dx)^{-1-n}\left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^{1+n}dx}{d}$$
$$= \frac{(bc-ad)(a+bx)^n\left(-\frac{d(a+bx)}{bc-ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Mathematica [A]
time = 0.07, size = 83, normalized size = 0.99

$$\frac{(bc-ad)(a+bx)^n\left(\frac{d(a+bx)}{-bc+ad}\right)^{-n}(c+dx)^{-n}\,{}_2F_1\left(-1-n,-n;1-n;\frac{b(c+dx)}{bc-ad}\right)}{d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1 + n)*(c + d*x)^(-1 - n),x]
[Out] ((b*c - a*d)*(a + b*x)^n*Hypergeometric2F1[-1 - n, -n, 1 - n, (b*(c + d*x))
/(b*c - a*d)])/(d^2*n*((d*(a + b*x))/(-(b*c) + a*d))^n*(c + d*x)^n)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{1+n}(dx+c)^{-1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
[Out] int((b*x+a)^(1+n)*(d*x+c)^(-1-n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="maxima")
[Out] integrate((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1+n)*(d*x+c)**(-1-n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1+n)*(d*x+c)^(-1-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n + 1)*(d*x + c)^(-n - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{n+1}}{(c+d\,x)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n + 1)/(c + d*x)^(n + 1),x)
```

```
[Out] int((a + b*x)^(n + 1)/(c + d*x)^(n + 1), x)
```

3.1880 $\int (a+bx)^m (c+dx)^{1+2n-2(1+n)}\,dx$

Optimal. Leaf size=51

$$\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}$$

```
[Out] (b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)/(1
+m)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7, 70}

$$\frac{(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]
```

```
[Out] ((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/((b*c - a*d)*(1 + m))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m (c+dx)^{1+2n-2(1+n)}\,dx &= \int \frac{(a+bx)^m}{c+dx}\,dx\\ &= \frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.00

$$-\frac{(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{(-bc+ad)(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 + 2*n - 2*(1 + n)),x]
[Out] -(((a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(-(b*
c) + a*d)])/((-(b*c) + a*d)*(1 + m)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c),x)
[Out] int((b*x+a)^m/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="maxima")
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="fricas")
[Out] integral((b*x + a)^m/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c),x)
```

```
[Out] Integral((a + b*x)**m/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^m}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(c + d*x),x)
```

```
[Out] int((a + b*x)^m/(c + d*x), x)
```

3.1881 $\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx$

Optimal. Leaf size=57

$$-\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

```
[Out] -1/(-a*d+b*c)/(b*x+a)-d*ln(b*x+a)/(-a*d+b*c)^2+d*ln(d*x+c)/(-a*d+b*c)^2
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7, 46}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2}\,dx &= \int \frac{1}{(a+bx)^2(c+dx)}\,dx\\ &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.93

$$\frac{-bc+ad-d(a+bx)\log(a+bx)+d(a+bx)\log(c+dx)}{(bc-ad)^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
 - a*d)^2*(a + b*x))
```

Maple [A]
time = 0.19, size = 57, normalized size = 1.00

method	result	size
default	$\frac{d\ln(dx+c)}{(ad-bc)^2}+\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{(ad-bc)^2}$	57
risch	$\frac{1}{(ad-bc)(bx+a)}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}+\frac{d\ln(-dx-c)}{a^2d^2-2abcd+b^2c^2}$	86
norman	$-\frac{bx}{a(ad-bc)(bx+a)}+\frac{d\ln(dx+c)}{a^2d^2-2abcd+b^2c^2}-\frac{d\ln(bx+a)}{a^2d^2-2abcd+b^2c^2}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

Maxima [A]
time = 0.26, size = 92, normalized size = 1.61

$$-\frac{d\log(bx+a)}{b^2c^2-2\,abcd+a^2d^2}+\frac{d\log(dx+c)}{b^2c^2-2\,abcd+a^2d^2}-\frac{1}{abc-a^2d+(b^2c-abd)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

Fricas [A]
time = 0.92, size = 93, normalized size = 1.63

$$-\frac{bc-ad+(bdx+ad)\log(bx+a)-(bdx+ad)\log(dx+c)}{ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(46) = 92$.
time = 0.37, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}+\frac{1}{a^2d-abc+x\,(abd-b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

Giac [A]
time = 1.99, size = 78, normalized size = 1.37

$$\frac{bd\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{b}{(b^2c-abd)(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

Mupad [B]
time = 0.44, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d-b\,c)\,(a+b\,x)}-\frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d-b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

3.1882 $\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx$

Optimal. Leaf size=95

$$-\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)}+\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-1-m}}{a^2bc^2(1+m)(2+m)}$$

```
[Out] -(b*x+a)^(1+m)*(a*c*(1+m)+b*c*(2+m)*x)^(-2-m)/a/b/c/(2+m)+(b*x+a)^(1+m)*(a*
c*(1+m)+b*c*(2+m)*x)^(-1-m)/a^2/b/c^2/(m^2+3*m+2)
```

Rubi [A]
time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {47, 37}

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-2 - m))/(a*b*c*(2 + m)
)) + ((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-1 - m))/(a^2*b*c^2*
(1 + m)*(2 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx = -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)} - \frac{\int (a+bx}{}$$
$$= -\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)} + \frac{(a+bx)}{}$$

Mathematica [A]
time = 0.19, size = 54, normalized size = 0.57

$$\frac{x(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-m}}{a^2c^3(1+m)(a(1+m)+b(2+m)x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] (x*(a + b*x)^(1 + m))/(a^2*c^3*(1 + m)*(a*(1 + m) + b*(2 + m)*x)^2*(a*c*(1
+ m) + b*c*(2 + m)*x)^m)
```

Maple [A]
time = 0.28, size = 57, normalized size = 0.60

method	result	size
gosper	$\frac{(bx+a)^{1+m}(bxm+am+2bx+a)x(bcxm+acm+2bcx+ac)^{-3-m}}{a^2(1+m)}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+m)*(b*m*x+a*m+2*b*x+a)/a^2/(1+m)*x*(b*c*m*x+a*c*m+2*b*c*x+a*c)^(
-3-m)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="maxima")
```

```
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

Fricas [A]
time = 0.93, size = 85, normalized size = 0.89

$$\frac{((b^2m+2\,b^2)x^3+(2\,abm+3\,ab)x^2+(a^2m+a^2)x)(acm+ac+(bcm+2\,bc)x)^{-m-3}(bx+a)^m}{a^2m+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="fricas")

[Out] ((b^2*m + 2*b^2)*x^3 + (2*a*b*m + 3*a*b)*x^2 + (a^2*m + a^2)*x)*(a*c*m + a*c + (b*c*m + 2*b*c)*x)^(-m - 3)*(b*x + a)^m/(a^2*m + a^2)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**m*(a*c*(1+m)+b*c*(2+m)*x)**(-3-m),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="giac")

[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)

Mupad [B]
time = 1.04, size = 81, normalized size = 0.85

$$\frac{x\,{\left(a+b\,x\right)}^m+\frac{b\,x^2\,\left(2\,m+3\right)\,{\left(a+b\,x\right)}^m}{a\,\left(m+1\right)}+\frac{b^2\,x^3\,\left(m+2\right)\,{\left(a+b\,x\right)}^m}{a^2\,\left(m+1\right)}}{{\left(a\,c\,\left(m+1\right)+b\,c\,x\,\left(m+2\right)\right)}^{m+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^m/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3),x)

[Out] (x*(a + b*x)^m + (b*x^2*(2*m + 3)*(a + b*x)^m)/(a*(m + 1)) + (b^2*x^3*(m + 2)*(a + b*x)^m)/(a^2*(m + 1)))/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3)

3.1883 $\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$

Optimal. Leaf size=97

$$-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

```
[Out] -(d*x+c)^(a*d/(-a*d+b*c))/b/c/((b*x+a)^(b*c/(-a*d+b*c)))+(d*x+c)^(a*d/(-a*d
+b*c))/a/b/c/((b*x+a)^(a*d/(-a*d+b*c)))
```

Rubi [A]
time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {47, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a*d)),x
]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx = -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} - \frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,d}{bc}$$

$$= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} + \frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

Mathematica [A]
time = 0.27, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a
*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

Maple [A]
time = 0.20, size = 66, normalized size = 0.68

method	result	size
gosper	$\frac{(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}x}{ac}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x,method=_RETUR
NVERBOSE)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

Fricas [A]
time = 1.30, size = 84, normalized size = 0.87

$$\frac{bdx^3 + acx + (bc + ad)x^2}{(bx + a)^{\frac{2bc - ad}{bc - ad}}(dx + c)^{\frac{bc - 2ad}{bc - ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^{-\frac{bc}{-ad + bc} - 1}(c + dx)^{\frac{ad}{-ad + bc} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-b*c/(-a*d+b*c))*(d*x+c)**(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] Integral((a + b*x)**(-b*c/(-a*d + b*c) - 1)*(c + d*x)**(a*d/(-a*d + b*c) -
1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="giac")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

Mupad [B]
time = 2.14, size = 119, normalized size = 1.23

$$\frac{x\,{\left(a+b\,x\right)}^{\frac{b\,c}{a\,d-b\,c}-1}+\frac{x^2\,\left(a\,d+b\,c\right)\,{\left(a+b\,x\right)}^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}+\frac{b\,d\,x^3\,{\left(a+b\,x\right)}^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}}{{\left(c+d\,x\right)}^{\frac{a\,d}{a\,d-b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^((b*c)/(a*d - b*c) - 1)/(c + d*x)^((a*d)/(a*d - b*c) + 1),x)
```

```
[Out] (x*(a + b*x)^((b*c)/(a*d - b*c) - 1) + (x^2*(a*d + b*c)*(a + b*x)^((b*c)/(a
*d - b*c) - 1))/(a*c) + (b*d*x^3*(a + b*x)^((b*c)/(a*d - b*c) - 1))/(a*c))/
(c + d*x)^((a*d)/(a*d - b*c) + 1)
```

3.1884 $\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$

Optimal. Leaf size=97

$$-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

```
[Out] -(d*x+c)^(a*d/(-a*d+b*c))/b/c/((b*x+a)^(b*c/(-a*d+b*c)))+(d*x+c)^(a*d/(-a*d
+b*c))/a/b/c/((b*x+a)^(a*d/(-a*d+b*c)))
```

Rubi [A]
time = 0.01, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {47, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(-(b*c)
 + a*d)),x]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx = -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} - \frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx}{bc}$$
$$= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc} + \frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}$$

Mathematica [A]
time = 0.13, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(
-(b*c) + a*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

Maple [A]
time = 0.21, size = 66, normalized size = 0.68

method	result	size
gosper	$\frac{(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}x}{ac}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c)),x,met
hod=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

Fricas [**A**]
time = 1.06, size = 84, normalized size = 0.87

$$\frac{bdx^3+acx+(bc+ad)x^2}{(bx+a)^{\frac{2bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)**((-2*a*d+b*c)/(a*d-b*c
)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

Mupad [**B**]
time = 0.85, size = 142, normalized size = 1.46

$$\frac{\frac{x}{(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{x^2\,(a\,d+b\,c)}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{b\,d\,x^3}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}}{(c+d\,x)^{\frac{2\,a\,d-b\,c}{a\,d-b\,c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^((a*d - 2*b*c)/(a*d - b*c))*(c + d*x)^((2*a*d - b*c)/(a*d
- b*c))),x)
```

```
[Out] (x/(a + b*x)^((a*d - 2*b*c)/(a*d - b*c)) + (x^2*(a*d + b*c))/(a*c*(a + b*x)
^((a*d - 2*b*c)/(a*d - b*c))) + (b*d*x^3)/(a*c*(a + b*x)^((a*d - 2*b*c)/(a*
d - b*c))))/(c + d*x)^((2*a*d - b*c)/(a*d - b*c))
```

3.1885 $\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=30

$$2^{1+n}\sqrt{1+x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

```
[Out] 2^(1+n)*hypergeom([1/2, -n],[3/2],1/2+1/2*x)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{71\}$

$$2^{n+1}\sqrt{x+1}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^n/Sqrt[1 + x],x]
```

```
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx = 2^{1+n}\sqrt{1+x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

Mathematica [A]
time = 0.08, size = 30, normalized size = 1.00

$$2^{1+n}\sqrt{1+x}\ {}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1+x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^n/Sqrt[1 + x],x]
[Out] 2^(1 + n)*Sqrt[1 + x]*Hypergeometric2F1[1/2, -n, 3/2, (1 + x)/2]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(1-x)^n}{\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^n/(1+x)^(1/2),x)
[Out] int((1-x)^n/(1+x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="maxima")
[Out] integrate((-x + 1)^n/sqrt(x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="fricas")
[Out] integral((-x + 1)^n/sqrt(x + 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.17, size = 29, normalized size = 0.97

$$2\cdot 2^n\sqrt{x+1}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, -n\\ \frac{3}{2}\end{matrix}\right|\frac{(x+1)\,e^{2i\pi}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**n/(1+x)**(1/2),x)
[Out] 2*2**n*sqrt(x + 1)*hyper((1/2, -n), (3/2,), (x + 1)*exp_polar(2*I*pi)/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n/(1+x)^(1/2),x, algorithm="giac")
[Out] integrate((-x + 1)^n/sqrt(x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(1-x)^n}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^n/(x + 1)^(1/2),x)
[Out] int((1 - x)^n/(x + 1)^(1/2), x)
```

3.1886 $\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=35

$$-2^{1+n}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

```
[Out] -2^(1+n)*hypergeom([1/2, -n],[3/2],1/2-1/2*x)*(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{71\}$

$$-2^{n+1}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^n/Sqrt[1 - x],x]
```

```
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx=-2^{1+n}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 1.00

$$-2^{1+n}\sqrt{1-x}\;{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^n/Sqrt[1 - x],x]
[Out] -(2^(1 + n)*Sqrt[1 - x]*Hypergeometric2F1[1/2, -n, 3/2, (1 - x)/2])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(1+x)^n}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^n/(1-x)^(1/2),x)
[Out] int((1+x)^n/(1-x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)^n/sqrt(-x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="fricas")
[Out] integral(-(x + 1)^n*sqrt(-x + 1)/(x - 1), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.10, size = 31, normalized size = 0.89

$$-2\cdot 2^n i\sqrt{x-1}\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, -n\\ \frac{3}{2}\end{matrix}\middle|\,\frac{(x-1)\,e^{i\pi}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**n/(1-x)**(1/2),x)
[Out] -2*2**n*I*sqrt(x - 1)*hyper((1/2, -n), (3/2,), (x - 1)*exp_polar(I*pi)/2)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^n/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)^n/sqrt(-x + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(x+1)^n}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^n/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^n/(1 - x)^(1/2), x)
```

3.1887 $\int (1-x)^n (1+x)^{7/3}\,dx$

Optimal. Leaf size=33

$$\frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

```
[Out] 3/5*2^(-1+n)*(1+x)^(10/3)*hypergeom([10/3, -n],[13/3],1/2+1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{71\}$

$$\frac{3}{5}2^{n-1}(x+1)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^n*(1 + x)^(7/3),x]
```

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int (1-x)^n(1+x)^{7/3}\,dx = \frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

Mathematica [A]
time = 0.09, size = 33, normalized size = 1.00

$$\frac{3}{5}2^{-1+n}(1+x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1+x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^n*(1 + x)^(7/3),x]
```

```
[Out] (3*2^(-1 + n)*(1 + x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 + x)/2])/
5
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (1-x)^{n} (1+x)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^n*(1+x)^(7/3),x)
```

```
[Out] int((1-x)^n*(1+x)^(7/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(7/3)*(-x + 1)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((x^2 + 2*x + 1)*(x + 1)^(1/3)*(-x + 1)^n, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 52.97, size = 37, normalized size = 1.12

$$\frac{2^{n} (x+1)^{\frac{10}{3}}\, \Gamma\left(\frac{10}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{10}{3}, -n \\ \frac{13}{3} \end{matrix}\middle| \frac{(x+1)e^{2i\pi}}{2}\right)}{\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**n*(1+x)**(7/3),x)
```

```
[Out] 2**n*(x + 1)**(10/3)*gamma(10/3)*hyper((10/3, -n), (13/3,), (x + 1)*exp_pol
ar(2*I*pi)/2)/gamma(13/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^n*(1+x)^(7/3),x, algorithm="giac")
[Out] integrate((x + 1)^(7/3)*(-x + 1)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (1-x)^n\,(x+1)^{7/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^n*(x + 1)^(7/3),x)
[Out] int((1 - x)^n*(x + 1)^(7/3), x)
```

3.1888 $\int (1-x)^{7/3}(1+x)^n\,dx$

Optimal. Leaf size=37

$$-\frac{3}{5}2^{-1+n}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

```
[Out] -3/5*2^(-1+n)*(1-x)^(10/3)*hypergeom([10/3, -n],[13/3],1/2-1/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{71\}$

$$-\frac{3}{5}2^{n-1}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/3)*(1 + x)^n,x]
```

```
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int (1-x)^{7/3}(1+x)^n\,dx=-\frac{3}{5}2^{-1+n}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 1.00

$$-\frac{3}{5}2^{-1+n}(1-x)^{10/3}\,{}_2F_1\left(\frac{10}{3},-n;\frac{13}{3};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/3)*(1 + x)^n,x]
[Out] (-3*2^(-1 + n)*(1 - x)^(10/3)*Hypergeometric2F1[10/3, -n, 13/3, (1 - x)/2])
/5
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (1-x)^{\frac{7}{3}} (1+x)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(7/3)*(1+x)^n,x)
[Out] int((1-x)^(7/3)*(1+x)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="maxima")
[Out] integrate((x + 1)^n*(-x + 1)^(7/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="fricas")
[Out] integral((x^2 - 2*x + 1)*(x + 1)^n*(-x + 1)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 53.26, size = 42, normalized size = 1.14

$$\frac{\sqrt[3]{-1} \cdot 2^n (x-1)^{\frac{10}{3}} \Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} \frac{10}{3}, -n \\ \frac{13}{3} \end{matrix} \middle| \frac{(x-1)e^{i\pi}}{2}\right)}{\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/3)*(1+x)**n,x)
```

```
[Out] (-1)**(1/3)*2**n*(x - 1)**(10/3)*gamma(10/3)*hyper((10/3, -n), (13/3,), (x
- 1)*exp_polar(I*pi)/2)/gamma(13/3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/3)*(1+x)^n,x, algorithm="giac")
```

```
[Out] integrate((x + 1)^n*(-x + 1)^(7/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int (1-x)^{7/3}\,(x+1)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/3)*(x + 1)^n,x)
```

```
[Out] int((1 - x)^(7/3)*(x + 1)^n, x)
```

3.1889 $\int (1+2x)^{-m}(2+3x)^m\,dx$

Optimal. Leaf size=47

$$\frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

```
[Out] 2^(-1-m)*(1+2*x)^(1-m)*hypergeom([-m, 1-m],[2-m],-3-6*x)/(1-m)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{71\}$

$$\frac{2^{-m-1}(2x+1)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(2x+1))}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^m/(1 + 2*x)^m,x]
```

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rubi steps

$$\int (1+2x)^{-m}(2+3x)^m\,dx = \frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.00

$$\frac{2^{-1-m}(1+2x)^{1-m}\,{}_2F_1(1-m,-m;2-m;-3(1+2x))}{1-m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^m/(1 + 2*x)^m,x]
```

```
[Out] (2^(-1 - m)*(1 + 2*x)^(1 - m)*Hypergeometric2F1[1 - m, -m, 2 - m, -3*(1 + 2
*x)])/(1 - m)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (2+3x)^m (1+2x)^{-m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^m/((1+2*x)^m),x)
[Out] int((2+3*x)^m/((1+2*x)^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="maxima")
[Out] integrate((3*x + 2)^m/(2*x + 1)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="fricas")
[Out] integral((3*x + 2)^m/(2*x + 1)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.38, size = 42, normalized size = 0.89

$$\frac{3^{2m} \left(x+\frac{2}{3}\right) \left(x+\frac{2}{3}\right)^m e^{-i\pi m} \Gamma(m+1)\, {}_2F_1\left(\begin{matrix} m, m+1 \\ m+2 \end{matrix} \middle| 6x+4\right)}{\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**m/((1+2*x)**m),x)
[Out] 3**(2*m)*(x + 2/3)*(x + 2/3)**m*exp(-I*pi*m)*gamma(m + 1)*hyper((m, m + 1),
 (m + 2,), 6*x + 4)/gamma(m + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^m/((1+2*x)^m),x, algorithm="giac")
[Out] integrate((3*x + 2)^m/(2*x + 1)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(3\,x+2)^m}{(2\,x+1)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^m/(2*x + 1)^m,x)
[Out] int((3*x + 2)^m/(2*x + 1)^m, x)
```

3.1890 $\int \left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n \, dx$

Optimal. Leaf size=45

$$\frac{(c+dx)^{1+n} \, {}_2F_1\left(-m,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}$$

```
[Out] (d*x+c)^(1+n)*hypergeom([-m, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/d/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {192, 71}

$$\frac{(c+dx)^{n+1} \, {}_2F_1\left(-m,n+1;n+2;\frac{b(c+dx)}{bc-ad}\right)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*Hypergeometric2F1[-m, 1 + n, 2 + n, (b*(c + d*x))/(b*c -
 a*d)])/(d*(1 + n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 192

```
Int[(u_)^(m_.)*(v_)^(n_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^n, x] /; FreeQ[{m, n}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u
, v}, x]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n \, dx &= \int (c+dx)^n \left(-\frac{ad}{bc-ad}-\frac{bdx}{bc-ad}\right)^m \, dx\\ &= \frac{(c+dx)^{1+n} \, {}_2F_1\left(-m,1+n;2+n;\frac{b(c+dx)}{bc-ad}\right)}{d(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 1.96

$$\frac{(a+bx)\left(\frac{d(a+bx)}{-bc+ad}\right)^m (c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} {}_2F_1\left(1+m,-n;2+m;\frac{d(a+bx)}{-bc+ad}\right)}{b(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n,x]
[Out] ((a + b*x)*((d*(a + b*x))/(-(b*c) + a*d))^m*(c + d*x)^n*Hypergeometric2F1[1
 + m, -n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(1 + m)*((b*(c + d*x))/(
b*c - a*d))^n)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\frac{d(bx+a)}{ad-bc}\right)^m (dx+c)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
[Out] int((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^n*(-(b*x + a)*d/(b*c - a*d))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="fricas")
[Out] integral((d*x + c)^n*(-(b*d*x + a*d)/(b*c - a*d))^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))**m*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*(b*x+a)/(a*d-b*c))^m*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*(-(b*x + a)*d/(b*c - a*d))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c+d\,x)^n\,\left(\frac{d\,(a+b\,x)}{a\,d-b\,c}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n*((d*(a + b*x))/(a*d - b*c))^m,x)
```

```
[Out] int((c + d*x)^n*((d*(a + b*x))/(a*d - b*c))^m, x)
```

3.1891 $\int (a+bx+cx^2+dx^3) \, dx$

Optimal. Leaf size=28

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/4*d*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x + c*x^2 + d*x^3,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Rubi steps

$$\int (a+bx+cx^2+dx^3) \, dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x + c*x^2 + d*x^3,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Maple [A]
time = 0.01, size = 23, normalized size = 0.82

method	result	size
gosper	$ax+\frac{1}{2}x^2b+\frac{1}{3}c\,x^3+\frac{1}{4}d\,x^4$	23
default	$ax+\frac{1}{2}x^2b+\frac{1}{3}c\,x^3+\frac{1}{4}d\,x^4$	23

norman	$ax+\frac{1}{2}x^2b+\frac{1}{3}c\,x^3+\frac{1}{4}d\,x^4$	23
risch	$ax+\frac{1}{2}x^2b+\frac{1}{3}c\,x^3+\frac{1}{4}d\,x^4$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3+c*x^2+b*x+a,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/2*x^2*b+1/3*c*x^3+1/4*d*x^4
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^4+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="maxima")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A]
time = 1.13, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^4+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="fricas")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.79

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3+c*x**2+b*x+a,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + d*x**4/4
```

Giac [A]
time = 1.91, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^4+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="giac")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.79

$$\frac{d\,x^4}{4}+\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x + c*x^2 + d*x^3,x)
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

3.1892 $\int(-x^3+x^4)\,dx$

Optimal. Leaf size=15

$$-\frac{x^4}{4}+\frac{x^5}{5}$$

```
[Out] -1/4*x^4+1/5*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[-x^3 + x^4,x]
```

```
[Out] -1/4*x^4 + x^5/5
```

Rubi steps

$$\int(-x^3+x^4)\,dx=-\frac{x^4}{4}+\frac{x^5}{5}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{x^4}{4}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^3 + x^4,x]
```

```
[Out] -1/4*x^4 + x^5/5
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.80

method	result	size
gosper	$\frac{x^4(-5+4x)}{20}$	11
default	$-\frac{1}{4}x^4+\frac{1}{5}x^5$	12

norman	$-\frac{1}{4}x^4+\frac{1}{5}x^5$	12
risch	$-\frac{1}{4}x^4+\frac{1}{5}x^5$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4-x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4+1/5*x^5
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5-\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

Fricas [A]
time = 1.07, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5-\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.53

$$\frac{x^5}{5}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4-x**3,x)
```

```
[Out] x**5/5 - x**4/4
```

Giac [A]
time = 1.53, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5-\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 1/4*x^4
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^4\,\left(4\,x-5\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4 - x^3,x)
```

```
[Out] (x^4*(4*x - 5))/20
```

3.1893 $\int (-1+x^5)\,dx$

Optimal. Leaf size=11

$$-x+\frac{x^6}{6}$$

```
[Out] -x+1/6*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

```
[In] Int[-1 + x^5,x]
[Out] -x + x^6/6
```

Rubi steps

$$\int (-1+x^5)\,dx = -x+\frac{x^6}{6}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-x+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + x^5,x]
[Out] -x + x^6/6
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
gosper	$\frac{x(x^5-6)}{6}$	9
default	$-x+\frac{1}{6}x^6$	10

norman	$-x+\frac{1}{6}x^6$	10
risch	$-x+\frac{1}{6}x^6$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5-1,x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/6*x^6
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - x
```

Fricas [A]
time = 0.60, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - x
```

Sympy [A]
time = 0.00, size = 5, normalized size = 0.45

$$\frac{x^6}{6}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5-1,x)
```

```
[Out] x**6/6 - x
```

Giac [A]
time = 1.51, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="giac")
```

```
[Out] 1/6*x^6 - x
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.73

$$\frac{x\,\left(x^5-6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5 - 1,x)
```

```
[Out] (x*(x^5 - 6))/6
```

3.1894 $\int (7+4x)\,dx$

Optimal. Leaf size=9

$$7x+2x^2$$

[Out] 2*x^2+7*x

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$2x^2+7x$$

Antiderivative was successfully verified.

[In] Int[7 + 4*x,x]

[Out] 7*x + 2*x^2

Rubi steps

$$\int (7+4x)\,dx = 7x+2x^2$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$7x+2x^2$$

Antiderivative was successfully verified.

[In] Integrate[7 + 4*x,x]

[Out] 7*x + 2*x^2

Maple [A]
time = 0.01, size = 10, normalized size = 1.11

method	result	size
gosper	$2x^2+7x$	10
default	$2x^2+7x$	10
norman	$2x^2+7x$	10
risch	$2x^2+7x$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7+4*x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^2+7*x
```

Maxima [A]
time = 0.27, size = 9, normalized size = 1.00

$$2\,x^2+7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="maxima")
```

```
[Out] 2*x^2 + 7*x
```

Fricas [A]
time = 0.68, size = 9, normalized size = 1.00

$$2\,x^2+7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="fricas")
```

```
[Out] 2*x^2 + 7*x
```

Sympy [A]
time = 0.00, size = 7, normalized size = 0.78

$$2x^2+7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x)
```

```
[Out] 2*x**2 + 7*x
```

Giac [A]
time = 1.07, size = 9, normalized size = 1.00

$$2\,x^2+7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="giac")
```

```
[Out] 2*x^2 + 7*x
```

Mupad [B]
time = 0.03, size = 7, normalized size = 0.78

$$x\,(2\,x+7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 7,x)
[Out] x*(2*x + 7)
```

3.1895 $\int (4x + \pi x^3)\, dx$

Optimal. Leaf size=14

$$2x^2 + \frac{\pi x^4}{4}$$

[Out] 2*x^2+1/4*Pi*x^4

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{\pi x^4}{4} + 2x^2$$

Antiderivative was successfully verified.

[In] Int[4*x + Pi*x^3,x]

[Out] 2*x^2 + (Pi*x^4)/4

Rubi steps

$$\int (4x + \pi x^3)\, dx = 2x^2 + \frac{\pi x^4}{4}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$2x^2 + \frac{\pi x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[4*x + Pi*x^3,x]

[Out] 2*x^2 + (Pi*x^4)/4

Maple [A]
time = 0.04, size = 15, normalized size = 1.07

method	result	size
gosper	$\frac{x^2(\pi x^2+8)}{4}$	13
norman	$2x^2 + \frac{1}{4}\pi x^4$	13

risch	$2x^2 + \frac{1}{4}\pi x^4$	13
default	$\frac{(\pi x^2+4)^2}{4\pi}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi*x^3+4*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(Pi*x^2+4)^2/Pi
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.86

$$\frac{1}{4}\pi x^4 + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

Fricas [A]
time = 0.56, size = 12, normalized size = 0.86

$$\frac{1}{4}\pi x^4 + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="fricas")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.71

$$\frac{\pi x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x**3+4*x,x)
```

```
[Out] pi*x**4/4 + 2*x**2
```

Giac [A]
time = 1.55, size = 12, normalized size = 0.86

$$\frac{1}{4}\pi x^4 + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="giac")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.86

$$\frac{\Pi x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + Pi*x^3,x)
```

```
[Out] (Pi*x^4)/4 + 2*x^2
```

3.1896 $\int (2x + 5x^2) \, dx$

Optimal. Leaf size=11

$$x^2 + \frac{5x^3}{3}$$

[Out] x^2+5/3*x^3

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{5x^3}{3} + x^2$$

Antiderivative was successfully verified.

[In] Int[2*x + 5*x^2,x]

[Out] x^2 + (5*x^3)/3

Rubi steps

$$\int (2x + 5x^2) \, dx = x^2 + \frac{5x^3}{3}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x^2 + \frac{5x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[2*x + 5*x^2,x]

[Out] x^2 + (5*x^3)/3

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
default	$x^2 + \frac{5}{3}x^3$	10
norman	$x^2 + \frac{5}{3}x^3$	10

risch	$x^2 + \frac{5}{3}x^3$	10
gosper	$\frac{x^2(3+5x)}{3}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x^2+2*x,x,method=_RETURNVERBOSE)
```

```
[Out] x^2+5/3*x^3
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + x^2
```

Fricas [A]
time = 0.57, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="fricas")
```

```
[Out] 5/3*x^3 + x^2
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$\frac{5x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x**2+2*x,x)
```

```
[Out] 5*x**3/3 + x**2
```

Giac [A]
time = 2.30, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + x^2
```

Mupad [B]
time = 0.02, size = 10, normalized size = 0.91

$$\frac{x^2\,\left(5\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 5*x^2,x)
```

```
[Out] (x^2*(5*x + 3))/3
```

3.1897 $\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$

Optimal. Leaf size=15

$$\frac{x^3}{6}+\frac{x^4}{12}$$

```
[Out] 1/6*x^3+1/12*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

Rubi steps

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx=\frac{x^3}{6}+\frac{x^4}{12}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^3}{6}+\frac{x^4}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.80

method	result	size

gosper	$\frac{x^3(2+x)}{12}$	9
default	$\frac{1}{6}x^3 + \frac{1}{12}x^4$	12
norman	$\frac{1}{6}x^3 + \frac{1}{12}x^4$	12
risch	$\frac{1}{6}x^3 + \frac{1}{12}x^4$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^2+1/3*x^3,x,method=_RETURNVERBOSE)
[Out] 1/6*x^3+1/12*x^4
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^4 + \frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="maxima")
[Out] 1/12*x^4 + 1/6*x^3
```

Fricas [A]
time = 0.84, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^4 + \frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="fricas")
[Out] 1/12*x^4 + 1/6*x^3
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.53

$$\frac{x^4}{12} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**2+1/3*x**3,x)
[Out] x**4/12 + x**3/6
```

Giac [A]
time = 1.01, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^4+\frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="giac")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.53

$$\frac{x^3\,\left(x+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/2 + x^3/3,x)
```

```
[Out] (x^3*(x + 2))/12
```

3.1898 $\int (3-5x+2x^2)\,dx$

Optimal. Leaf size=18

$$3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

[Out] 3*x-5/2*x^2+2/3*x^3

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

[In] Int[3 - 5*x + 2*x^2,x]

[Out] 3*x - (5*x^2)/2 + (2*x^3)/3

Rubi steps

$$\int (3-5x+2x^2)\,dx = 3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[3 - 5*x + 2*x^2,x]

[Out] 3*x - (5*x^2)/2 + (2*x^3)/3

Maple [A]
time = 0.01, size = 15, normalized size = 0.83

method	result	size
gosper	$\frac{x(4x^2-15x+18)}{6}$	14
default	$3x-\frac{5}{2}x^2+\frac{2}{3}x^3$	15

norman	$3x - \frac{5}{2}x^2 + \frac{2}{3}x^3$	15
risch	$3x - \frac{5}{2}x^2 + \frac{2}{3}x^3$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2-5*x+3,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x-5/2*x^2+2/3*x^3
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.78

$$\frac{2}{3}\,x^3 - \frac{5}{2}\,x^2 + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Fricas [A]
time = 0.67, size = 14, normalized size = 0.78

$$\frac{2}{3}\,x^3 - \frac{5}{2}\,x^2 + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.83

$$\frac{2x^3}{3} - \frac{5x^2}{2} + 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x**2-5*x+3,x)
```

```
[Out] 2*x**3/3 - 5*x**2/2 + 3*x
```

Giac [A]
time = 0.86, size = 14, normalized size = 0.78

$$\frac{2}{3}\,x^3 - \frac{5}{2}\,x^2 + 3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="giac")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

Mupad [B]
time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(4\,x^2-15\,x+18\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2 - 5*x + 3,x)
```

```
[Out] (x*(4*x^2 - 15*x + 18))/6
```

3.1899 $\int (-2x + x^2 + x^3) \, dx$

Optimal. Leaf size=20

$$-x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int (-2x + x^2 + x^3) \, dx = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$-x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]
```

```
[Out] -x^2 + x^3/3 + x^4/4
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
gosper	$\frac{x^2(3x^2+4x-12)}{12}$	16
default	$-x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$	17

norman	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17
risch	$-x^2+\frac{1}{3}x^3+\frac{1}{4}x^4$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x,method=_RETURNVERBOSE)
```

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.80

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Fricas [A]
time = 0.69, size = 16, normalized size = 0.80

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.60

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

Giac [A]
time = 1.67, size = 16, normalized size = 0.80

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.75

$$\frac{x^2\,\left(3\,x^2+4\,x-12\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2 - 2*x + x^3,x)
```

```
[Out] (x^2*(4*x + 3*x^2 - 12))/12
```

3.1900 $\int (1 - x^2 - 3x^5) \, dx$

Optimal. Leaf size=16

$$x - \frac{x^3}{3} - \frac{x^6}{2}$$

[Out] x-1/3*x^3-1/2*x^6

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Antiderivative was successfully verified.

[In] Int[1 - x^2 - 3*x^5,x]

[Out] x - x^3/3 - x^6/2

Rubi steps

$$\int (1 - x^2 - 3x^5) \, dx = x - \frac{x^3}{3} - \frac{x^6}{2}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$x - \frac{x^3}{3} - \frac{x^6}{2}$$

Antiderivative was successfully verified.

[In] Integrate[1 - x^2 - 3*x^5,x]

[Out] x - x^3/3 - x^6/2

Maple [A]
time = 0.01, size = 13, normalized size = 0.81

method	result	size
default	$x - \frac{1}{3}x^3 - \frac{1}{2}x^6$	13
norman	$x - \frac{1}{3}x^3 - \frac{1}{2}x^6$	13

risch	$x-\frac{1}{3}x^3-\frac{1}{2}x^6$	13
gosper	$-\frac{x(3x^5+2x^2-6)}{6}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3*x^5-x^2+1,x,method=_RETURNVERBOSE)
[Out] x-1/3*x^3-1/2*x^6
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6-\frac{1}{3}x^3+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="maxima")
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Fricas [A]
time = 0.63, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6-\frac{1}{3}x^3+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="fricas")
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.62

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x**5-x**2+1,x)
[Out] -x**6/2 - x**3/3 + x
```

Giac [A]
time = 2.44, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6-\frac{1}{3}x^3+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="giac")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.75

$$-\frac{x^6}{2}-\frac{x^3}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - 3*x^5 - x^2,x)
```

```
[Out] x - x^3/3 - x^6/2
```

3.1901 $\int (5+2x+3x^2+4x^3) \, dx$

Optimal. Leaf size=13

$$5x+x^2+x^3+x^4$$

```
[Out] x^4+x^3+x^2+5*x
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x^4+x^3+x^2+5x$$

Antiderivative was successfully verified.

```
[In] Int[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] 5*x + x^2 + x^3 + x^4
```

Rubi steps

$$\int (5+2x+3x^2+4x^3) \, dx = 5x+x^2+x^3+x^4$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$5x+x^2+x^3+x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] 5*x + x^2 + x^3 + x^4
```

Maple [A]
time = 0.01, size = 14, normalized size = 1.08

method	result	size
gosper	$x^4+x^3+x^2+5x$	14
default	$x^4+x^3+x^2+5x$	14
norman	$x^4+x^3+x^2+5x$	14
risch	$x^4+x^3+x^2+5x$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^3+3*x^2+2*x+5,x,method=_RETURNVERBOSE)
```

```
[Out] x^4+x^3+x^2+5*x
```

Maxima [A]
time = 0.27, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="maxima")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Fricas [A]
time = 0.90, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="fricas")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.92

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**3+3*x**2+2*x+5,x)
```

```
[Out] x**4 + x**3 + x**2 + 5*x
```

Giac [A]
time = 1.75, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="giac")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

Mupad [B]
time = 0.03, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 3*x^2 + 4*x^3 + 5,x)
[Out] 5*x + x^2 + x^3 + x^4
```

3.1902 $\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$

Optimal. Leaf size=22

$$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

```
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+b\log(x)-\frac{c}{x}-\frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[a + d/x^3 + c/x^2 + b/x,x]
[Out] -1/2*d/x^2 - c/x + a*x + b*Log[x]
```

Rubi steps

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx=-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 1.00

$$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + d/x^3 + c/x^2 + b/x,x]
[Out] -1/2*d/x^2 - c/x + a*x + b*Log[x]
```

Maple [A]
time = 0.01, size = 21, normalized size = 0.95

method	result	size
default	$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\ln(x)$	21
risch	$-\frac{d}{2x^2}-\frac{c}{x}+ax+b\ln(x)$	21

norman	$\frac{a\,x^3-\frac{1}{2}d-cx}{x^2}+b\ln\left(x\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+d/x^3+c/x^2+b/x,x,method=_RETURNVERBOSE)
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.91

$$ax+b\log\left(x\right)-\frac{c}{x}-\frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="maxima")
[Out] a*x + b*log(x) - c/x - 1/2*d/x^2
```

Fricas [A]
time = 0.96, size = 27, normalized size = 1.23

$$\frac{2\,ax^3+2\,bx^2\log\left(x\right)-2\,cx-d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="fricas")
[Out] 1/2*(2*a*x^3 + 2*b*x^2*log(x) - 2*c*x - d)/x^2
```

Sympy [A]
time = 0.07, size = 20, normalized size = 0.91

$$ax+b\log\left(x\right)+\frac{-2cx-d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x**3+c/x**2+b/x,x)
[Out] a*x + b*log(x) + (-2*c*x - d)/(2*x**2)
```

Giac [A]
time = 1.25, size = 21, normalized size = 0.95

$$ax+b\log\left(|x|\right)-\frac{c}{x}-\frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="giac")
[Out] a*x + b*log(abs(x)) - c/x - 1/2*d/x^2
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.91

$$a\,x-\frac{\frac{d}{2}+c\,x}{x^2}+b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x + c/x^2 + d/x^3,x)
[Out] a*x - (d/2 + c*x)/x^2 + b*log(x)
```

3.1903 $\int \left(\frac{1}{x^5}+x+x^5\right) dx$

Optimal. Leaf size=22

$$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

```
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{x^6}{6}-\frac{1}{4x^4}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5) + x + x^5,x]
```

```
[Out] -1/4*1/x^4 + x^2/2 + x^6/6
```

Rubi steps

$$\int \left(\frac{1}{x^5}+x+x^5\right) dx=-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5) + x + x^5,x]
```

```
[Out] -1/4*1/x^4 + x^2/2 + x^6/6
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
default	$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$	17
norman	$\frac{\frac{1}{6}x^{10}+\frac{1}{2}x^6-\frac{1}{4}}{x^4}$	17

risch	$-\frac{1}{4x^4}+\frac{x^2}{2}+\frac{x^6}{6}$	17
gosper	$\frac{2x^{10}+6x^6-3}{12x^4}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5+x+x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.73

$$\frac{1}{6}x^6+\frac{1}{2}x^2-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

Fricas [A]
time = 0.80, size = 17, normalized size = 0.77

$$\frac{2x^{10}+6x^6-3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^10 + 6*x^6 - 3)/x^4
```

Sympy [A]
time = 0.02, size = 15, normalized size = 0.68

$$\frac{x^6}{6}+\frac{x^2}{2}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5+x+x**5,x)
```

```
[Out] x**6/6 + x**2/2 - 1/(4*x**4)
```

Giac [A]
time = 1.20, size = 16, normalized size = 0.73

$$\frac{1}{6}x^6+\frac{1}{2}x^2-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="giac")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.77

$$\frac{2x^{10}+6x^6-3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x^5 + x^5,x)
```

```
[Out] (6*x^6 + 2*x^10 - 3)/(12*x^4)
```

3.1904 $\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

```
[Out] -1/2/x^2-1/x+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] -1/2*1/x^2 - x^(-1) + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)\,dx=-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3) + x^(-2) + x^(-1),x]
```

```
[Out] -1/2*1/x^2 - x^(-1) + Log[x]
```

Maple [A]
time = 0.01, size = 14, normalized size = 0.93

method	result	size
norman	$\frac{-\frac{1}{2}-x}{x^2}+\ln(x)$	13
default	$-\frac{1}{2x^2}-\frac{1}{x}+\ln(x)$	14

risch	$-\frac{1}{2x^2}-\frac{1}{x}+\ln(x)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3+1/x^2+1/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/x+ln(x)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{x}-\frac{1}{2\,x^2}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="maxima")
```

```
[Out] -1/x - 1/2/x^2 + log(x)
```

Fricas [A]
time = 0.86, size = 17, normalized size = 1.13

$$\frac{2\,x^2\log\left(x\right)-2\,x-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x) - 2*x - 1)/x^2
```

Sympy [A]
time = 0.02, size = 14, normalized size = 0.93

$$\log\left(x\right)+\frac{-2x-1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3+1/x**2+1/x,x)
```

```
[Out] log(x) + (-2*x - 1)/(2*x**2)
```

Giac [A]
time = 1.15, size = 14, normalized size = 0.93

$$-\frac{1}{x}-\frac{1}{2\,x^2}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="giac")
```

```
[Out] -1/x - 1/2/x^2 + log(abs(x))
```

Mupad [**B**]
time = 0.03, size = 11, normalized size = 0.73

$$\ln(x) - \frac{x + \frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x + 1/x^2 + 1/x^3,x)
```

```
[Out] log(x) - (x + 1/2)/x^2
```

3.1905 $\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$

Optimal. Leaf size=10

$$\frac{2}{x}+3\log(x)$$

```
[Out] 2/x+3*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x^2 + 3/x,x]
```

```
[Out] 2/x + 3*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx=\frac{2}{x}+3\log(x)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x^2 + 3/x,x]
```

```
[Out] 2/x + 3*Log[x]
```

Maple [A]
time = 0.01, size = 11, normalized size = 1.10

method	result	size
default	$\frac{2}{x}+3\ln\left(x\right)$	11
norman	$\frac{2}{x}+3\ln\left(x\right)$	11

risch	$\frac{2}{x}+3\ln(x)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x^2+3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/x+3*ln(x)
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="maxima")
```

```
[Out] 2/x + 3*log(x)
```

Fricas [A]
time = 0.80, size = 11, normalized size = 1.10

$$\frac{3x\log(x)+2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="fricas")
```

```
[Out] (3*x*log(x) + 2)/x
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.70

$$3\log(x)+\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x**2+3/x,x)
```

```
[Out] 3*log(x) + 2/x
```

Giac [A]
time = 1.31, size = 11, normalized size = 1.10

$$\frac{2}{x}+3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="giac")
```

```
[Out] 2/x + 3*log(abs(x))
```

Mupad [B]

time = 0.03, size = 10, normalized size = 1.00

$$3\,\ln\left(x\right)+\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/x - 2/x^2,x)
```

```
[Out] 3*log(x) + 2/x
```

3.1906 $\int \left(-\frac{1}{7x^6}+x^6\right) dx$

Optimal. Leaf size=15

$$\frac{1}{35x^5}+\frac{x^7}{7}$$

[Out] 1/35/x^5+1/7*x^7

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

[In] Int[-1/7*1/x^6 + x^6,x]

[Out] 1/(35*x^5) + x^7/7

Rubi steps

$$\int \left(-\frac{1}{7x^6}+x^6\right) dx=\frac{1}{35x^5}+\frac{x^7}{7}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{1}{35x^5}+\frac{x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[-1/7*1/x^6 + x^6,x]

[Out] 1/(35*x^5) + x^7/7

Maple [A]
time = 0.02, size = 12, normalized size = 0.80

method	result	size
default	$\frac{1}{35x^5}+\frac{x^7}{7}$	12
norman	$\frac{\frac{1}{35}+\frac{x^{12}}{7}}{x^5}$	12

risch	$\frac{1}{35x^5} + \frac{x^7}{7}$	12
gosper	$\frac{5x^{12}+1}{35x^5}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/7/x^6+x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/35/x^5+1/7*x^7
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^7 + \frac{1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="maxima")
```

```
[Out] 1/7*x^7 + 1/35/x^5
```

Fricas [A]
time = 0.61, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*x^12 + 1)/x^5
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^7}{7} + \frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x**6+x**6,x)
```

```
[Out] x**7/7 + 1/(35*x**5)
```

Giac [A]
time = 1.55, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^7 + \frac{1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="giac")
```

```
[Out] 1/7*x^7 + 1/35/x^5
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6 - 1/(7*x^6),x)
```

```
[Out] (5*x^12 + 1)/(35*x^5)
```

3.1907 $\int \left(1+\frac{1}{x}+x\right) \, dx$

Optimal. Leaf size=11

$$x+\frac{x^2}{2}+\log(x)$$

```
[Out] x+1/2*x^2+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^(-1) + x,x]
[Out] x + x^2/2 + Log[x]
```

Rubi steps

$$\int \left(1+\frac{1}{x}+x\right) \, dx = x+\frac{x^2}{2}+\log(x)$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x+\frac{x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^(-1) + x,x]
[Out] x + x^2/2 + Log[x]
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
default	$x+\frac{x^2}{2}+\ln(x)$	10
norman	$x+\frac{x^2}{2}+\ln(x)$	10

risch	$x+\frac{x^2}{2}+\ln(x)$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+1/x+x,x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*x^2+ln(x)
```

Maxima [A]
time = 0.27, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Fricas [A]
time = 0.54, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + log(x)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.73

$$\frac{x^2}{2}+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x)
```

```
[Out] x**2/2 + x + log(x)
```

Giac [A]
time = 1.40, size = 10, normalized size = 0.91

$$\frac{1}{2}x^2+x+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="giac")
[Out] 1/2*x^2 + x + log(abs(x))
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.82

$$x + \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x + 1,x)
[Out] x + log(x) + x^2/2
```

3.1908 $\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx$

Optimal. Leaf size=13

$$\frac{3}{2x^2}-\frac{4}{x}$$

```
[Out] 3/2/x^2-4/x
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-3/x^3 + 4/x^2,x]
```

```
[Out] 3/(2*x^2) - 4/x
```

Rubi steps

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx=\frac{3}{2x^2}-\frac{4}{x}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/x^3 + 4/x^2,x]
```

```
[Out] 3/(2*x^2) - 4/x
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
norman	$\frac{-4x+\frac{3}{2}}{x^2}$	10
gosper	$-\frac{8x-3}{2x^2}$	11

default	$\frac{3}{2x^2} - \frac{4}{x}$	12
risch	$\frac{3}{2x^2} - \frac{4}{x}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/x^3+4/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/x^2-4/x
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.85

$$-\frac{4}{x} + \frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="maxima")
```

```
[Out] -4/x + 3/2/x^2
```

Fricas [A]
time = 0.58, size = 10, normalized size = 0.77

$$-\frac{8\,x - 3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*x - 3)/x^2
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.62

$$\frac{3 - 8x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x**3+4/x**2,x)
```

```
[Out] (3 - 8*x)/(2*x**2)
```

Giac [A]
time = 1.57, size = 11, normalized size = 0.85

$$-\frac{4}{x} + \frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="giac")
```

```
[Out] -4/x + 3/2/x^2
```

Mupad [B]
time = 0.03, size = 10, normalized size = 0.77

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4/x^2 - 3/x^3,x)
```

```
[Out] -(8*x - 3)/(2*x^2)
```

3.1909 $\int \left(\frac{1}{x}+2x+x^2\right)\,dx$

Optimal. Leaf size=13

$$x^2+\frac{x^3}{3}+\log(x)$$

```
[Out] x^2+1/3*x^3+ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^3}{3}+x^2+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x}+2x+x^2\right)\,dx=x^2+\frac{x^3}{3}+\log(x)$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$x^2+\frac{x^3}{3}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.92

method	result	size
default	$x^2+\frac{x^3}{3}+\ln(x)$	12
norman	$x^2+\frac{x^3}{3}+\ln(x)$	12

risch	$x^2 + \frac{x^3}{3} + \ln(x)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x+2*x+x^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2+1/3*x^3+ln(x)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.85

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

Fricas [A]
time = 0.74, size = 11, normalized size = 0.85

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x**2,x)
```

```
[Out] x**3/3 + x**2 + log(x)
```

Giac [A]
time = 1.84, size = 12, normalized size = 0.92

$$\frac{1}{3}x^3 + x^2 + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="giac")
[Out] 1/3*x^3 + x^2 + log(abs(x))
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.85

$$\ln(x) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 1/x + x^2,x)
[Out] log(x) + x^2 + x^3/3
```

3.1910 $\int \left(x^{5/6}-x^3\right) \, dx$

Optimal. Leaf size=17

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

[Out] 6/11*x^(11/6)-1/4*x^4

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[x^(5/6) - x^3,x]

[Out] (6*x^(11/6))/11 - x^4/4

Rubi steps

$$\int \left(x^{5/6}-x^3\right) \, dx=\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/6) - x^3,x]

[Out] (6*x^(11/6))/11 - x^4/4

Maple [A]
time = 0.04, size = 12, normalized size = 0.71

method	result	size

derivativedivides	$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$	12
default	$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$	12
risch	$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$	12
trager	$-\frac{(x^3+x^2+x+1)(-1+x)}{4}+\frac{6x^{\frac{11}{6}}}{11}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6)-x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 6/11*x^(11/6)-1/4*x^4
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.65

$$-\frac{1}{4}x^4+\frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Fricas [A]
time = 0.68, size = 11, normalized size = 0.65

$$-\frac{1}{4}x^4+\frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.71

$$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/6)-x**3,x)
```

```
[Out] 6*x**(11/6)/11 - x**4/4
```

Giac [A]
time = 1.89, size = 11, normalized size = 0.65

$$-\frac{1}{4}\,x^{4}+\frac{6}{11}\,x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="giac")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.65

$$\frac{6\,x^{11/6}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6) - x^3,x)
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

3.1911 $\int \left(33+\sqrt[33]{x}\right) dx$

Optimal. Leaf size=13

$$33x+\frac{33x^{34/33}}{34}$$

[Out] 33*x+33/34*x^(34/33)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

[In] Int[33 + x^(1/33),x]

[Out] 33*x + (33*x^(34/33))/34

Rubi steps

$$\int \left(33+\sqrt[33]{x}\right) dx = 33x+\frac{33x^{34/33}}{34}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$33x+\frac{33x^{34/33}}{34}$$

Antiderivative was successfully verified.

[In] Integrate[33 + x^(1/33),x]

[Out] 33*x + (33*x^(34/33))/34

Maple [A]
time = 0.17, size = 10, normalized size = 0.77

method	result	size

derivativedivides	$33x + \frac{33x^{\frac{34}{33}}}{34}$	10
default	$33x + \frac{33x^{\frac{34}{33}}}{34}$	10
risch	$33x + \frac{33x^{\frac{34}{33}}}{34}$	10
trager	$-33 + 33x + \frac{33x^{\frac{34}{33}}}{34}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(33+x^(1/33),x,method=_RETURNVERBOSE)
[Out] 33*x+33/34*x^(34/33)
```

Maxima [A]
time = 0.26, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="maxima")
[Out] 33/34*x^(34/33) + 33*x
```

Fricas [A]
time = 0.91, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="fricas")
[Out] 33/34*x^(34/33) + 33*x
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34} + 33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x**(1/33),x)
[Out] 33*x**(34/33)/34 + 33*x
```

Giac [A]
time = 1.35, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="giac")
```

```
[Out] 33/34*x^(34/33) + 33*x
```

Mupad [B]
time = 0.02, size = 8, normalized size = 0.62

$$\frac{33\,x\,\left(x^{1/33}+34\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/33) + 33,x)
```

```
[Out] (33*x*(x^(1/33) + 34))/34
```

3.1912 $\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right) dx$

Optimal. Leaf size=15

$$\sqrt{x}+\frac{4x^{3/2}}{3}$$

```
[Out] 4/3*x^(3/2)+x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{4x^{3/2}}{3}+\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] Sqrt[x] + (4*x^(3/2))/3
```

Rubi steps

$$\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right) dx=\sqrt{x}+\frac{4x^{3/2}}{3}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.93

$$\frac{1}{3}\sqrt{x}\,(3+4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] (Sqrt[x]*(3 + 4*x))/3
```

Maple [A]
time = 0.02, size = 10, normalized size = 0.67

method	result	size

derivativedivides	$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$	10
default	$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$	10
risch	$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$	10
gosper	$\frac{\sqrt{x}\,(3+4x)}{3}$	11
trager	$\frac{\left(2+\frac{8x}{3}\right)\sqrt{x}}{2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/x^(1/2)+2*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*x^(3/2)+x^(1/2)
```

Maxima [A]
time = 0.27, size = 9, normalized size = 0.60

$$\frac{4}{3}\,x^{\frac{3}{2}}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*x^(3/2) + sqrt(x)
```

Fricas [A]
time = 0.80, size = 10, normalized size = 0.67

$$\frac{1}{3}\,(4\,x+3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*x + 3)*sqrt(x)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.80

$$\frac{4x^{\frac{3}{2}}}{3}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x**(1/2)+2*x**(1/2),x)
```

```
[Out] 4*x**(3/2)/3 + sqrt(x)
```

Giac [**A**]
time = 1.36, size = 9, normalized size = 0.60

$$\frac{4}{3}x^{\frac{3}{2}}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*x^(3/2) + sqrt(x)
```

Mupad [**B**]
time = 0.02, size = 10, normalized size = 0.67

$$\frac{\sqrt{x}\,\left(4\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)) + 2*x^(1/2),x)
```

```
[Out] (x^(1/2)*(4*x + 3))/3
```

3.1913 $\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx$

Optimal. Leaf size=15

$$\frac{1}{x}+4x^{3/2}+10\log(x)$$

```
[Out] 1/x+4*x^(3/2)+10*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-x^(-2) + 10/x + 6*Sqrt[x],x]
```

```
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

Rubi steps

$$\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx = \frac{1}{x}+4x^{3/2}+10\log(x)$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 1.00

$$\frac{1}{x}+4x^{3/2}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^(-2) + 10/x + 6*Sqrt[x],x]
```

```
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

Maple [A]
time = 0.05, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{1}{x}+4x^{\frac{3}{2}}+10\ln\left(x\right)$	14
default	$\frac{1}{x}+4x^{\frac{3}{2}}+10\ln\left(x\right)$	14

risch	$\frac{1}{x}+4x^{\frac{3}{2}}+10\ln\left(x\right)$	14
trager	$-\frac{-1+x}{x}+4x^{\frac{3}{2}}-10\ln\left(\frac{1}{x}\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/x^2+10/x+6*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x+4*x^(3/2)+10*ln(x)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="maxima")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(x)
```

Fricas [A]
time = 0.57, size = 18, normalized size = 1.20

$$\frac{4\,x^{\frac{5}{2}}+20\,x\log\left(\sqrt{x}\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="fricas")
```

```
[Out] (4*x^(5/2) + 20*x*log(sqrt(x)) + 1)/x
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}}+10\log\left(x\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x**2+10/x+6*x**(1/2),x)
```

```
[Out] 4*x**(3/2) + 10*log(x) + 1/x
```

Giac [A]
time = 1.96, size = 14, normalized size = 0.93

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="giac")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(abs(x))
```

Mupad [B]
time = 0.29, size = 15, normalized size = 1.00

$$20\,\ln\left(\sqrt{x}\right)+\frac{1}{x}+4\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/x - 1/x^2 + 6*x^(1/2),x)
```

```
[Out] 20*log(x^(1/2)) + 1/x + 4*x^(3/2)
```

3.1914 $\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx$

Optimal. Leaf size=17

$$-\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

[Out] 2/5*x^(5/2)-2/x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[x^(-3/2) + x^(3/2),x]

[Out] -2/Sqrt[x] + (2*x^(5/2))/5

Rubi steps

$$\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx=-\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.82

$$\frac{2(-5+x^3)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3/2) + x^(3/2),x]

[Out] (2*(-5 + x^3))/(5*Sqrt[x])

Maple [A]
time = 0.03, size = 12, normalized size = 0.71

method	result	size

gosper	$\frac{\frac{2x^3}{5}-2}{\sqrt{x}}$	11
trager	$\frac{\frac{2x^3}{5}-2}{\sqrt{x}}$	11
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$	12
default	$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$	12
risch	$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)+x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^(5/2)-2/x^(1/2)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

Fricas [A]
time = 0.95, size = 10, normalized size = 0.59

$$\frac{2\,(x^3-5)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(x^3 - 5)/sqrt(x)
```

Sympy [A]
time = 0.01, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5}-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)+x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5 - 2/sqrt(x)
```

Giac [A]
time = 1.66, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.71

$$\frac{2\,x^3 - 10}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2) + x^(3/2),x)
```

```
[Out] (2*x^3 - 10)/(5*x^(1/2))
```

3.1915 $\int \left(-5x^{3/2}+7x^{5/2}\right) dx$

Optimal. Leaf size=15

$$-2x^{5/2}+2x^{7/2}$$

[Out] -2*x^(5/2)+2*x^(7/2)

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$2x^{7/2}-2x^{5/2}$$

Antiderivative was successfully verified.

[In] Int[-5*x^(3/2) + 7*x^(5/2),x]

[Out] -2*x^(5/2) + 2*x^(7/2)

Rubi steps

$$\int \left(-5x^{3/2}+7x^{5/2}\right) dx = -2x^{5/2}+2x^{7/2}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 0.67

$$2(-1+x)x^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[-5*x^(3/2) + 7*x^(5/2),x]

[Out] 2*(-1 + x)*x^(5/2)

Maple [A]
time = 0.04, size = 12, normalized size = 0.80

method	result	size
gosper	$2x^{\frac{5}{2}}(-1+x)$	9
trager	$2x^{\frac{5}{2}}(-1+x)$	9
derivativedivides	$-2x^{\frac{5}{2}}+2x^{\frac{7}{2}}$	12
default	$-2x^{\frac{5}{2}}+2x^{\frac{7}{2}}$	12

risch	$-2x^{\frac{5}{2}}+2x^{\frac{7}{2}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-5*x^(3/2)+7*x^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x^(5/2)+2*x^(7/2)
```

Maxima [A]
time = 0.26, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}}-2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

Fricas [A]
time = 0.60, size = 14, normalized size = 0.93

$$2\left(x^{3}-x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2*(x^3 - x^2)*sqrt(x)
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.80

$$2x^{\frac{7}{2}}-2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x**(3/2)+7*x**(5/2),x)
```

```
[Out] 2*x**(7/2) - 2*x**(5/2)
```

Giac [A]
time = 1.83, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}}-2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="giac")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

Mupad [B]
time = 0.03, size = 8, normalized size = 0.53

$$2\,x^{5/2}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7*x^(5/2) - 5*x^(3/2),x)
[Out] 2*x^(5/2)*(x - 1)
```

3.1916 $\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx$

Optimal. Leaf size=24

$$4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

```
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{2x^{3/2}}{3}-\frac{x^2}{4}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[2/Sqrt[x] + Sqrt[x] - x/2,x]
```

```
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Rubi steps

$$\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx = 4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/Sqrt[x] + Sqrt[x] - x/2,x]
```

```
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.71

method	result	size

derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}+4\sqrt{x}$	17
default	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}+4\sqrt{x}$	17
risch	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{4}+4\sqrt{x}$	17
trager	$-\frac{(1+x)(-1+x)}{4}+\frac{\left(8+\frac{4x}{3}\right)\sqrt{x}}{2}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*x+2/x^(1/2)+x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.67

$$-\frac{1}{4}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}+4\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

Fricas [A]
time = 0.75, size = 14, normalized size = 0.58

$$-\frac{1}{4}\,x^2+\frac{2}{3}\,(x+6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="fricas")
[Out] -1/4*x^2 + 2/3*(x + 6)*sqrt(x)
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.79

$$\frac{2x^{\frac{3}{2}}}{3}+4\sqrt{x}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x**(1/2)+x**(1/2),x)
[Out] 2*x**(3/2)/3 + 4*sqrt(x) - x**2/4
```

Giac [A]
time = 1.63, size = 16, normalized size = 0.67

$$-\frac{1}{4}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}+4\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.62

$$\frac{\sqrt{x}\,\left(8\,x-3\,x^{3/2}+48\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/x^(1/2) - x/2 + x^(1/2),x)
```

```
[Out] (x^(1/2)*(8*x - 3*x^(3/2) + 48))/12
```

3.1917 $\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) \, dx$

Optimal. Leaf size=23

$$\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

```
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x + Sqrt[x]/5 + x^(3/2),x]
```

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) \, dx=\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x + Sqrt[x]/5 + x^(3/2),x]
```

```
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Maple [A]
time = 0.04, size = 16, normalized size = 0.70

method	result	size

derivativedivides	$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\ln\left(x\right)$	16
default	$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\ln\left(x\right)$	16
trager	$\frac{2x^{\frac{3}{2}}(1+3x)}{15}-2\ln\left(x\right)$	16
risch	$\frac{2x^{\frac{3}{2}}}{15}+\frac{2x^{\frac{5}{2}}}{5}-2\ln\left(x\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x+x^(3/2)+1/5*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{15}\,x^{\frac{3}{2}}-2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(x)
```

Fricas [A]
time = 1.01, size = 19, normalized size = 0.83

$$\frac{2}{15}\left(3\,x^{2}+x\right)\sqrt{x}\;-4\,\log\left(\sqrt{x}\,\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x)*sqrt(x) - 4*log(sqrt(x))
```

Sympy [A]
time = 0.01, size = 20, normalized size = 0.87

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{15}-2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x**(3/2)+1/5*x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/15 - 2*log(x)
```

Giac [A]
time = 1.49, size = 16, normalized size = 0.70

$$\frac{2}{5}\,x^{\frac{5}{2}} + \frac{2}{15}\,x^{\frac{3}{2}} - 2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(abs(x))
```

Mupad [B]
time = 0.28, size = 17, normalized size = 0.74

$$\frac{2\,x^{3/2}}{15} - 4\,\ln\left(\sqrt{x}\right) + \frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/5 - 2/x + x^(3/2),x)
[Out] (2*x^(3/2))/15 - 4*log(x^(1/2)) + (2*x^(5/2))/5
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```